



Progressive cracking of ceramic-matrix composites  
by Magadi Ravinder Joshi

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in  
Mechanical Engineering  
Montana State University  
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Abstract:

Ceramic-matrix composites have the potential of being used in many applications particularly as high temperature structural materials for engines. Although ceramics have high strength at temperatures exceeding the melting points of the superalloys, as well as oxidation, wear and corrosion resistance, they have the drawback of low flaw tolerance. The addition of reinforcing fibers greatly improves the toughness of ceramics, but local cracking can still develop in the brittle matrix and fiber/matrix interface. Recent studies have shown that cracking of simple crossply laminates initially occurs in the transverse plies at strains considerably less than the matrix cracking strain of the longitudinal plies; longitudinal ply cracking follows. Most of the present theoretical models for cracking in crossplied composites are for polymer matrices, in which the longitudinal plies remain intact and have a constraining effect on the cracking process in the transverse plies. In ceramic-matrix composites, development of a network of transverse ply cracks is followed by longitudinal matrix cracking normal to the fibers. This cracking of the longitudinal ply causes a decrease in the ply stiffness and thus the composite stiffness. Two theoretical models are developed in the present study. These are built on the lines of the existing theoretical models for polymer matrices. The classical theory of Aveston, Cooper and Kelly is used to account for longitudinal ply matrix cracking, which is coupled with the theories of Laws and Dvorak and Nairn for transverse ply cracking. Good agreement is shown between the analytical results and the existing experimental data using the Laws and Dvorak analysis. The theory should provide a guide to the engineering use of these materials, as well as a basis for improving the materials through better understanding the parameters controlling the damage process.

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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## ABSTRACT

Ceramic-matrix composites have the potential of being used in many applications particularly as high temperature structural materials for engines. Although ceramics have high strength at temperatures exceeding the melting points of the superalloys, as well as oxidation, wear and corrosion resistance, they have the drawback of low flaw tolerance. The addition of reinforcing fibers greatly improves the toughness of ceramics, but local cracking can still develop in the brittle matrix and fiber/matrix interface. Recent studies have shown that cracking of simple crossply laminates initially occurs in the transverse plies at strains considerably less than the matrix cracking strain of the longitudinal plies; longitudinal ply cracking follows. Most of the present theoretical models for cracking in crossplied composites are for polymer matrices, in which the longitudinal plies remain intact and have a constraining effect on the cracking process in the transverse plies. In ceramic-matrix composites, development of a network of transverse ply cracks is followed by longitudinal matrix cracking normal to the fibers. This cracking of the longitudinal ply causes a decrease in the ply stiffness and thus the composite stiffness. Two theoretical models are developed in the present study. These are built on the lines of the existing theoretical models for polymer matrices. The classical theory of Aveston, Cooper and Kelly is used to account for longitudinal ply matrix cracking, which is coupled with the theories of Laws and Dvorak and Nairn for transverse ply cracking. Good agreement is shown between the analytical results and the existing experimental data using the Laws and Dvorak analysis. The theory should provide a guide to the engineering use of these materials, as well as a basis for improving the materials through better understanding the parameters controlling the damage process.

## CHAPTER 1

## INTRODUCTION

Ceramic-matrix composites (CMC's) are under development for structural and nonstructural applications such as heat engines, turbine blades, wear parts, and heat exchangers, at use temperatures far beyond the capabilities of metals. If CMC's maintain toughness -- resistance to rapid crack propagation -- and are able to resist attack by aggressive environments, then the low densities and low coefficients of thermal expansion of these materials make them attractive for many heat engines and hot aerospace applications. These materials have the potential of improving efficiency since they may allow operation of engineering components at higher temperatures, with lower friction, weight and inertia, as well as high thrust-to-weight ratios.

Monolithic ceramics, though they have high temperature capability and strength, are extremely brittle. A flaw as small as 10 micrometers can reduce strength to a few percent of the theoretical strength (1). This problem of brittleness can be overcome by using either (2): (i) whisker-and-particulate reinforced ceramic-matrix composites or (ii) continuous fiber-reinforced ceramics. Whisker-and-particulate composites can fail catastrophically with failure generally being matrix-controlled, while fiber-reinforced ceramics can be made to fail noncatastrophically if the fiber-matrix bonding is optimized, so that the failure is mostly controlled

by the fibers, much like current fiberglass materials.

The present work is focussed on understanding the cracking behavior of continuous fiber-reinforced ceramics. Extensive experimental work has been done previously (3), both at ambient and high temperatures, to understand the behavior of these materials. The work in Ref.3 is quite important to the present study as explained further in the ensuing chapters. A theoretical model is developed to predict the cracking behavior of CMC's. The different CMC matrix materials that were taken into consideration were: (i) lithium aluminosilicate (LAS) (ii) calcium aluminosilicate (CAS) and (iii) Corning code 1723 glass (an aluminosilicate glass). The fiber reinforcement was Nicalon (crystalline, primarily silicon carbide fibers, marketed by Nippon Carbon Company). Nicalon/1723 and Nicalon/CAS have moderate bond strength and high matrix thermal expansion coefficients, while Nicalon/LAS is less strongly bonded and has a low matrix thermal expansion coefficient. This variation in the bond strength and expansion coefficient makes it possible to compare the effects of bond strength and residual stresses on the failure behavior.

Most of the existing theoretical models of cracking are for crossplied polymer matrix composites, in which longitudinal ( $0^{\circ}$ ) ply matrix cracking is not observed, due to the high failure strain of the matrix relative to the fibers. But in CMC's, longitudinal ply matrix cracking accompanies the

cracking in the transverse ( $90^{\circ}$ ) plies. The goal of the present work is to improve the modelling of these composites by taking into account the  $0^{\circ}$  ply cracking, and its interaction with transverse ply cracking through its effect on stiffness. The approach is to use two current models for transverse cracking, and to combine these with an earlier model for  $0^{\circ}$  ply cracking. A computer routine then simulates the influence of one array of cracks on the other as the stress on the composite is increased. The predicted cracking patterns and stiffness changes are then compared with existing experimental data for three materials.

## CHAPTER 2

## BACKGROUND

Reinforced Ceramics

Ceramics have received widespread attention because of their inertness and stability at high temperatures combined with their low densities, low thermal expansion and relatively low thermal conductivity (2). Theoretical strengths of ceramics over a wide range of temperature compare well with the low temperature strengths of the strongest materials in common use (1). In practice, most ceramics have flaws and low toughness which lead to low tensile strength and poor resistance to thermal and mechanical shock.

Among the ways of increasing the fracture surface energy - the energy required to create a new surface in a material - and thus decreasing the brittleness of ceramics is reinforcement with particles or, especially, fibers. The toughening mechanisms in these composites are believed to be crack deflection - as the crack approaches the fiber, it cannot pass through it, as a result of which it is deflected parallel to the reinforcement - or debonding at the fiber-matrix interface. Although whisker and particulate reinforcements provide isotropic mechanical properties, improvements in fracture toughness in these composites is not as great as in continuous fiber reinforced composites. Fiber reinforced composites are therefore of considerable

importance. The fibers can increase the macroscopic flaw tolerance in the fiber direction, as well as strength, stiffness and thermal shock resistance (4). Fiber reinforced composites can provide high dimensional stability over a broad temperature range, resistance to attack by atomic oxygen, and damage tolerance (2).

### Behavior of Ceramic Composites

The ceramic matrices are very brittle, having failure strains far less than the fiber failure strains. Hence, initial damage in a CMC is controlled by the matrix. Depending on the orientation of the fibers in the matrix, these composites can be classified into (i) unidirectional, in which all the fibers are oriented in the same direction and (ii) multidirectional, in which the fibers are oriented in different directions in the different layers. The behavior of these two types of composites is explained in this section.

#### Unidirectional Composites

In Unidirectional Composites, all of the fibers are aligned in the same direction. A representative unidirectional lamina is shown in Fig.1. In this kind of composite, usually fibers having high tensile strength are incorporated into the matrix. As a result of this, a unidirectional lamina has excellent properties in the direction of the reinforcement, since most of the applied load is carried by the fibers. However, the laminate has poor properties in the other

directions. The modes of longitudinal tensile failure that can be observed in CMC's are shown in Fig.2. If a tensile load is

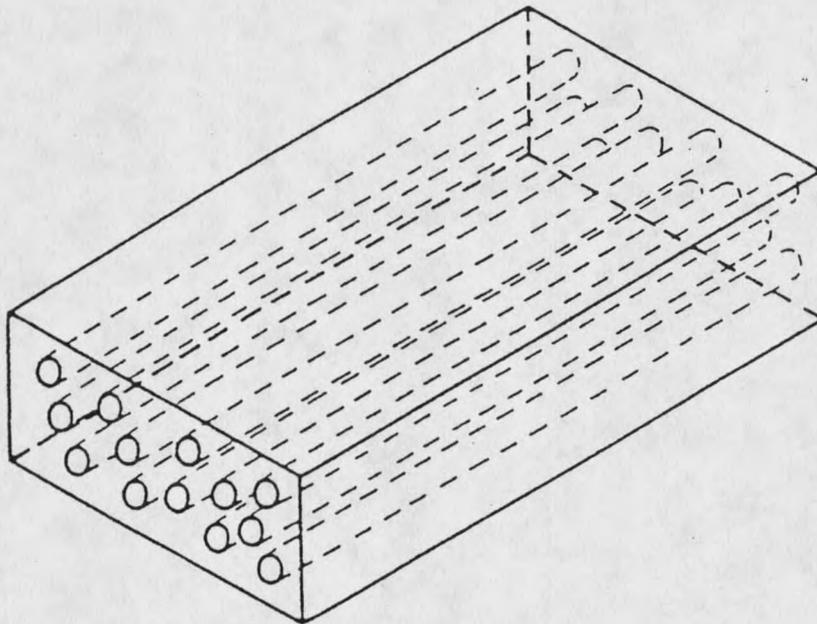


Fig.1 A Unidirectional Lamina, the Tubular Structures  
Shown are the Continuous Fibers Reinforced in the  
Matrix Material (14)

applied in the direction of the reinforcing fibers, the brittle matrix cracks, and if the fiber-matrix interface strength is too high the crack may propagate right through the fibers, thus resulting in the failure of the material. In a weakly-bonded material, this may not be the case, and if the proportion of the fibers is sufficient to carry the additional load shed onto them, multiple cracking of the matrix will

result. This is explained in more detail later in this chapter. For a matrix crack to occur, two conditions must be satisfied (5):

- (i) The matrix stress has to be equal to its breaking stress and

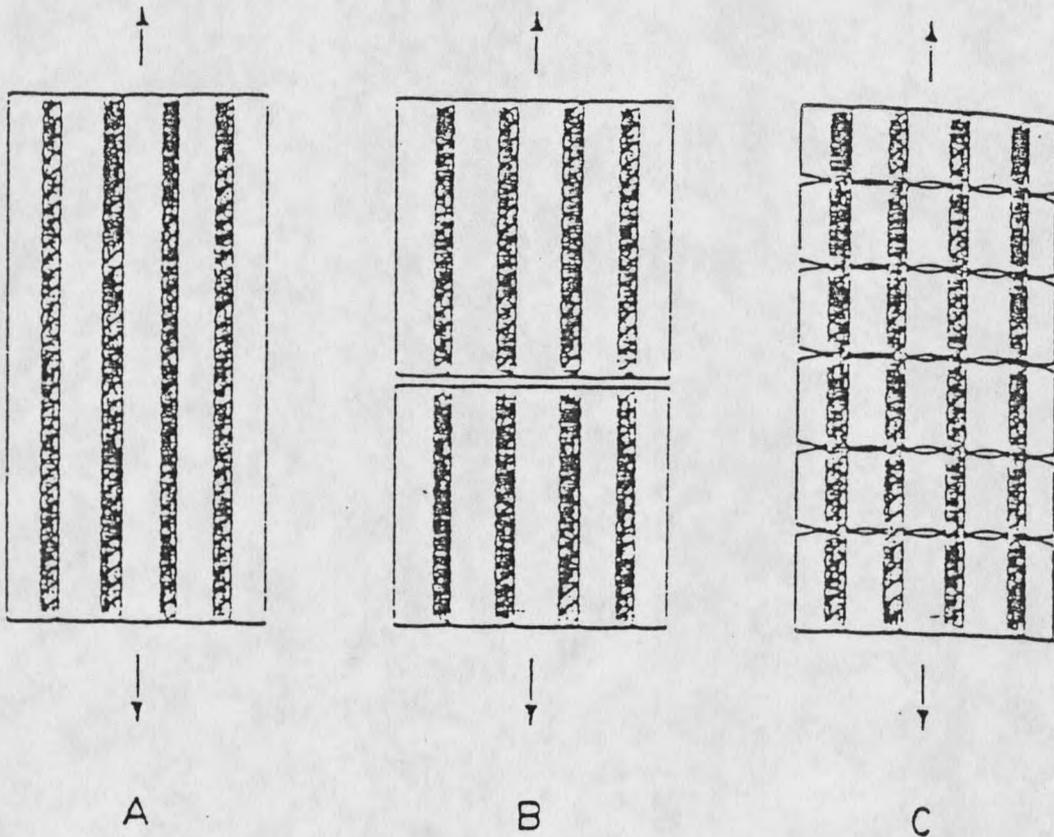


Fig.2 Types of Failure in a Typical Unidirectional CMC in the Longitudinal Direction (adapted from 14)

- (A) Before Failure  
 (B) A Single Matrix Crack Propagating Through the Fibers  
 (C) Multiple Matrix Cracks Bridged by the Unbroken Fibers

- (ii) There must be sufficient stored elastic strain energy to propagate the crack.

If a crack occurs in the matrix, some load is shed onto the bridging fibers. If the volume fraction of the reinforcing fibers is not sufficient to sustain the additional load thrown onto them, failure of the composite occurs at the failure strain of the matrix (4). However, if the fiber volume fraction is high, the fibers can bear the additional load, and multiple cracking occurs in the matrix. One effect of the reinforcing fibers may be to increase the failing strain of a brittle matrix by modifying the rates of release and absorption of energy during the propagation of a matrix crack (4), but this has not been well demonstrated experimentally. The critical fiber volume fraction at which a transition from single matrix fracture to stable multiple fracture occurs is given by (6)

$$V_f = \sigma_{mu} / (\sigma_{mu} + \sigma_{fu} - \sigma_f') \text{ ----- (2.1)}$$

where

- $\sigma_{mu}$  = breaking stress of the matrix
- $\sigma_{fu}$  = ultimate stress of the fibers
- $\sigma_f'$  = stress on the fibers when the matrix breaks

This is explained in more detail in the modelling section of this chapter. The interface (or interphase) between the fibers and the matrix is subjected to very high shear stresses, whose values depend on the difference in elastic modulus between the fibers and matrix and on the fiber volume fraction (6). At the site of the matrix crack, geometrical

stress concentrations arise, which increase the shear stresses. If the energy required to fail the interfacial bond can be supplied by the loading system or by the relaxation of stresses in the composite material, local debonding of the interface might result. Mechanical frictional forces at the interfaces help in shear stress transfer between the fibers and the matrix near the crack face, after the fibers debond. At the crack face, the reinforcing fibers carry the whole load applied to the composite, while the matrix carries zero load. The additional load thrown onto the fibers is transferred back to the matrix over a certain distance along the fiber via shear stress transfer. There are two mechanisms by which this stress transfer occurs: (i) frictional interaction, in which the interfacial stress is assumed to be constant and (ii) bonded condition, in which the fibers remain either fully or partially bonded to the matrix. These two cases have been studied by Aveston, Cooper and Kelly (ACK) (5) and Aveston and Kelly (7) respectively. The behavior of the unidirectional composites under these two kinds of interfacial stress transfer is explained briefly below.

Frictional Interaction. As explained above, the stress transfer between the fibers and the matrix is assumed to be through frictional forces in this kind of interface (see Fig.3). The fibers carry the whole load applied to the composite at the crack site, while the matrix carries no load. Over a certain distance from the crack site the additional

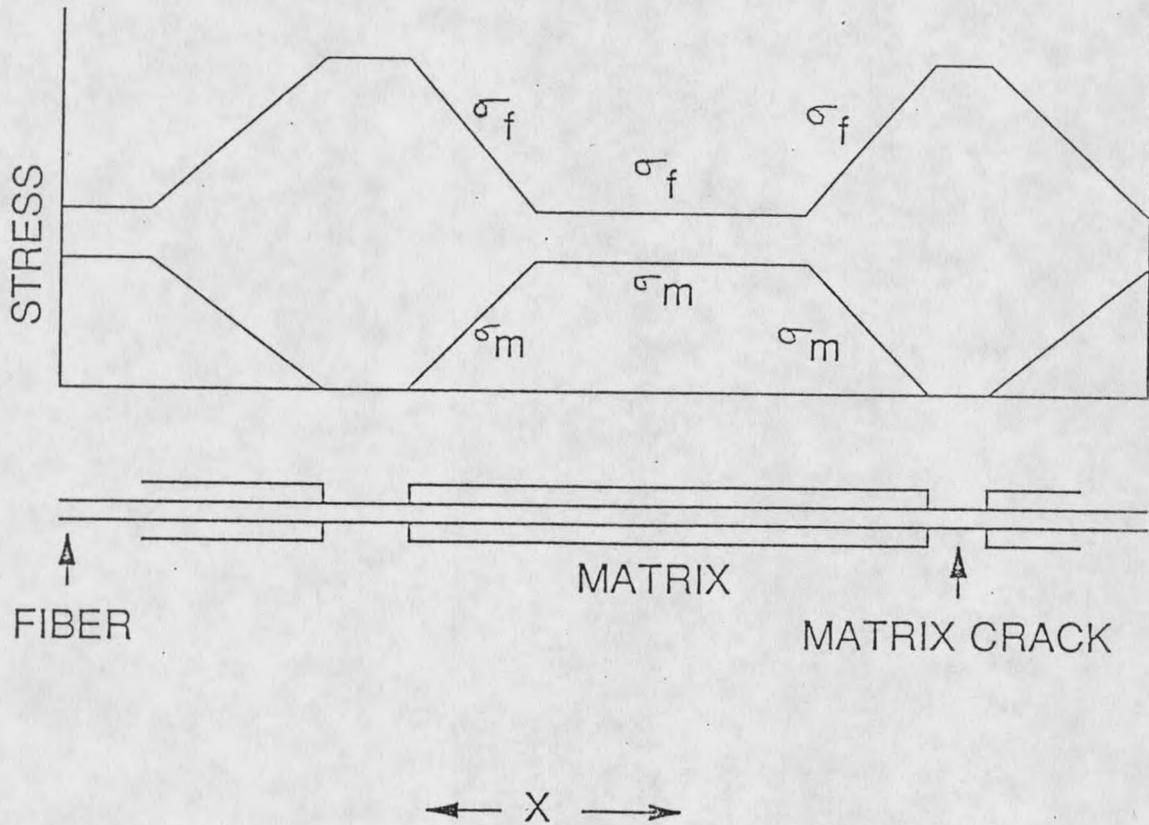


Fig.3 Load Transfer Between the Fibers and the Matrix  
(adapted from 6)

load is transferred to the matrix through frictional forces until the matrix assumes its original load at some distance away from the crack. The various energy changes occurring during matrix cracking under a fixed load are given as (5):

(i) Work done by the applied stress, given as

$$\Delta W = E_c \epsilon_{mu}^2 x / \alpha \text{ ----- (2.2)}$$

(the various terms are explained in the ensuing discussion)

(ii) Work absorbed in debonding the fiber-matrix interface,  $\Gamma_{db}$

(iii) The displacement of the fibers differs from that of the matrix. This implies that work done against the frictional force,  $\tau$ , is equal to this force multiplied by the difference in displacement ( $U_s$ )

(iv) Reduction in strain energy in the matrix ( $\Delta U_m$ )

(v) Increase in elastic strain energy of the fibers ( $\Delta U_f$ )

Thus, a crack will form only if the following condition is satisfied

$$2 \Gamma_m V_m + \Gamma_{db} + U_s + \Delta U_f \leq \Delta W + \Delta U_m \quad \text{----- (2.3)}$$

where  $\Gamma_m$  is the fracture surface work in forming a crack.

As the load is increased, the crack density - number of cracks per unit length - increases. The limiting crack spacing, after which no more matrix cracks occur, is given by

(5)

$$x' = \left( \frac{V_m}{V_f} \right) \frac{\sigma_{mu} r}{2\tau} \quad \text{----- (2.4)}$$

where

$V_m$  = volume fraction of matrix

$V_f$  = volume fraction of fibers

$\sigma_{mu}$  = ultimate strength of matrix

$r$  = radius of fiber

$\tau$  = shear stress at fiber-matrix interface

Fully Bonded Condition. In the frictional interaction

case, the fibers and the matrix are assumed to move relative to each other, and the shear stress at the fiber-matrix interface is assumed to be constant along the fiber. In many composite systems, the fibers are well-bonded to the matrix so that the system is essentially elastic, and the interfacial shear stress is not constant, but is given by (5)

$$\tau = \frac{\tau}{2} \Delta \sigma_0 \sqrt{\phi} \exp(-\sqrt{\phi} y) \text{ ----- (2.5)}$$

where

$$\phi = \text{sqrt}(2G_m E_c / E_f E_m V_m) \ 1 / (r \ \text{sqrt}(\ln(R/r)))$$

$G_m$  = shear modulus of the matrix

$2R$  = distance between the fiber centers

$y$  = distance from the matrix crack face

The stress distribution at a distance  $y$  from the crack face is given by

$$\Delta \sigma = \Delta \sigma_0 \exp(-\sqrt{\phi} y) \text{ ----- (2.6)}$$

where  $\Delta \sigma_0$  is the stress on the fibers at the crack face.

If there is debonding near the crack site, stress transfer takes place by frictional effects over the debonded length of the fiber and by elastic effects over the rest of the length.

### Multidirectional Composites

Unidirectional composites have excellent properties in the axial direction, i.e., in the direction of the reinforcing fibers. But they have poor off-axis properties. Some of the factors causing these poor properties are (4):

- (1) strain concentrations from the fibers
- (2) residual stresses
- (3) weak interfaces
- (4) porosity and other flaws

Because of these poor off-axis properties, ceramic composites are likely to find applicability mostly with multidirectional reinforcement. In this kind of composite, each ply may have different orientation of fibers. A representative multidirectional composite is shown in Fig.4. The orientation of the fibers in the various layers controls the elastic characteristics of the laminate. When a tensile stress component is perpendicular to the direction of the fiber alignment, the lamina shows poor strength and the matrix cracks at very low transverse strains, with crack propagation parallel to the fibers. The simplest form of multidirectional composites is cross-plyed composites, in which the plies have fibers oriented at an angle of  $0^{\circ}$  and  $90^{\circ}$  alternately. This kind of composite is shown in Fig.5. In the case of crossplyed laminates, the strength of an individual layer in the transverse direction is much lower than in the longitudinal direction. Hence, cracking occurs in the transverse plies (plies perpendicular to the applied tensile load) at much lower strains than in the longitudinal plies. The residual stresses complicate this by changing the stress state, and possibly causing debonding or cracking. Thus, there is often damage at very low strains, and the stress-strain curve is



























































































































































