



Multiple embodiment instructional sequence using the computer as the interfacing agent in the instruction of volume of rectangular solids
by Ruth Mary Regling Johnson

A thesis submitted in partial fulfillment of the requirement for the degree of Doctor of Education
Montana State University
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Abstract:

This study measured the effect of two multiple embodiment instructional sequences on the topic of volume of rectangular solids upon student achievement. Instructional sequences investigated were (1) the sequence with computer, (2) the same sequence without computer ; and a textbook-based sequence. Independent variables were sequence, ability level, and gender. Dependent variables were scores on post-instruction and post-retention criterion-referenced tests. Test items were divided into these categories: knowledge and comprehension, application and analysis, and total test.

A pretest determined all sequence groups were equal before instruction.

The four-week study was conducted in 21 southwestern Montana classrooms. Classes were randomly assigned to the sequences. Classroom teachers conducted all instruction and testing activities. Post-instruction test was administered after one week of instruction; post-retention test after three weeks of retention activities, one activity each week.

Factorial analysis of scores established the following conclusion. Both multiple embodiment sequences were superior to the textbook sequence except for application and analysis post-instruction scores. The highest achievement was among high ability groups; the lowest achievement was among low ability. Males outperformed females on knowledge and comprehension questions; females outperformed males on the computer sequence; and on both tests, males and females demonstrated equal achievement on application and analysis and total test scores. Two trends were noted: (1) students in the embodiment sequence without computer produced higher scores post-instruction, but students in the computer embodiment sequence scored higher post-retention, and (2) low ability students in the computer sequence scored lower than their counterparts in the other sequences.

The following recommendations were made concerning instruction of volume of rectangular solids. Carefully sequenced multiple embodiment instruction should be used. Use of the computer should be considered in embodiment instruction. Assumption that male achievement is superior to female achievement in this area should not be made. Instruction on this topic should be reinforced over time and should include multiple embodiments.

MULTIPLE EMBODIMENT INSTRUCTIONAL SEQUENCE
USING THE COMPUTER AS THE INTERFACING AGENT
IN THE INSTRUCTION OF VOLUME OF RECTANGULAR SOLIDS

by

Ruth Mary Regling Johnson

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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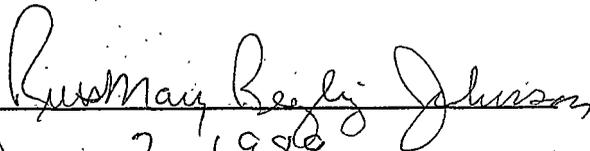
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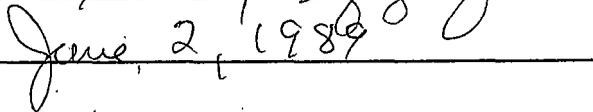
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ABSTRACT

This study measured the effect of two multiple embodiment instructional sequences on the topic of volume of rectangular solids upon student achievement. Instructional sequences investigated were (1) the sequence with computer, (2) the same sequence without computer; and a textbook-based sequence. Independent variables were sequence, ability level, and gender. Dependent variables were scores on post-instruction and post-retention criterion-referenced tests. Test items were divided into these categories: knowledge and comprehension, application and analysis, and total test. A pretest determined all sequence groups were equal before instruction.

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Factorial analysis of scores established the following conclusion. Both multiple embodiment sequences were superior to the textbook sequence except for application and analysis post-instruction scores. The highest achievement was among high ability groups; the lowest achievement was among low ability. Males outperformed females on knowledge and comprehension questions; females outperformed males on the computer sequence; and on both tests, males and females demonstrated equal achievement on application and analysis and total test scores. Two trends were noted: (1) students in the embodiment sequence without computer produced higher scores post-instruction, but students in the computer embodiment sequence scored higher post-retention, and (2) low ability students in the computer sequence scored lower than their counterparts in the other sequences.

The following recommendations were made concerning instruction of volume of rectangular solids. Carefully sequenced multiple embodiment instruction should be used. Use of the computer should be considered in embodiment instruction. Assumption that male achievement is superior to female achievement in this area should not be made. Instruction on this topic should be reinforced over time and should include multiple embodiments.

CHAPTER I

INTRODUCTION

Introduction

Mathematics educators, psychologists, and learning theorists have long searched for the processes by which students learn mathematics in order to develop instructional methods that will effectively facilitate such learning. Yet, concern over declining test scores of mathematical skill and concept development has been documented in the past two decades (Flake, 1982; National Assessment of Educational Progress Report (NAEP), 1979a; NAEP, 1979b; NAEP, 1983; National Commission on Excellence in Education, 1983; National Research Council, 1989; Willoughby, 1987). Even though some gain in mathematics achievement was shown in the 1986 NAEP test, it was primarily reflected in lower-order skills (Dossey et al., 1988). Concurrent with the fall of math scores has been a call by several national educational organizations to improve application and analysis skills, to link these to problem solving skills, and to explore ways to incorporate computers into instruction as a means

of enhancing mathematics learning (Committee on Research in Mathematics, Science and Technology, Commission on Behavioral and Social Sciences and Education, National Research Council, 1985; Dolan, 1987; Dossey et al., 1988; Heddens, 1984; National Council of Teachers of Mathematics (NCTM), 1985; National Research Council, 1989; National Science Board Commission on Pre-College Education in Mathematics, Science and Technology, 1983; Suydam, 1984b). Consequently, it is no surprise that a current topic of research is studying how to use the computer as a supplement to mathematics instruction to increase student achievement.

Until the 1960s much of mathematics instruction emphasized teacher show student tell environments, memorization, and drill and practice to promote, reinforce, and retain mathematics knowledge (Byers & Erlwanger, 1985; Heddens, 1984; Shuell, 1986). Traditional elementary school mathematics instruction has consisted of lecture, chalkboard demonstration, and assignment of numerous homework problems from textbooks. Traditional elementary school mathematics learning has consisted of listening, copying, memorizing algorithms, and time-consuming hand calculations to solve the homework assignments. Frequently, this approach has emphasized lower-level computation and comprehension

skills at the expense of the development of higher level application and analytical skills (DeVault, 1981; Dossey, 1989; Dugdale & Kibbey, 1983, 1984; NAEP, 1979a; NAEP, 1979b; NAEP, 1983; NCTM, 1985). This behavioral approach reflected the research of Skinner and Pavlov and most educators viewed mathematics learning and mathematics memory as a "quantitative" measure of learning (Byers & Erlwanger, 1985; Davis, 1983; Gagne, 1985; Shuell, 1986).

However, in the late 1960s and early 1970s a renewed emphasis on "understanding" the concept versus "rote learning" surfaced. This is evidenced by the influence of "meaningful verbal learning" (Ausubel, 1963), developmental stages of learning (Piaget, 1960), "discovery learning" (Bruner, 1975), Rathmell's symbolic-oral-written concept of learning (Payne & Rathmell, 1980), and others. Brownell's (1956) "meaningful arithmetic" was an early harbinger. These process-oriented learning theories produced instructional strategies involving advanced organizers, concrete manipulatives, multiple embodiment instruction, and a shift from teacher-dominated to child-centered, activity-oriented classrooms and laboratories. The "qualitative" instructional approach with regard to meaning and organization of mathematics learning surfaced as the major emphasis in mathematics education during

this time (Wheeler, 1971). It was hoped this approach would develop higher cognitive levels of mathematics learning as well as higher levels of student achievement. However, mathematics instruction in 1983 still consisted primarily of teacher-dominated lecture and demonstration and reliance on textbooks and worksheets (Dossey et al., 1988). Most teachers use a narrow repertoire of instructional practices, though research shows manipulatives used in mathematics instruction increases achievement (McCrary, 1988).

The idea of introducing concepts via concrete materials in elementary mathematics instruction is not new and has been prevalent in the literature since the early 1900s. In 1920, C. L. Hull, for example, noted "a combination of abstract presentations and concrete examples yield a distinctly greater functional efficiency than either method alone" (p. 28). Again, in 1927, J. P. Haynes stated:

It is often necessary for pupils to work with concrete materials; for it is only through actual contact with things, events, qualities, and relations that words, numbers, signs, and symbols acquire meaning. Concrete material is a means of thinking, computing, and verifying results. It provides for learning through several senses (p. 107).

Bruner's (1971) theory of intellectual development is one theory advocating the use of concrete manipulatives for initial concept building. He believed

knowledge can be represented by a set of actions (enactive), a set of images (iconic), and a set of symbols representing the knowledge concepts and including rules for manipulating the mathematics concept (symbolic). Further, learners proceed through the learning experience in three stages: concrete, semi-concrete and abstract. Piaget (1960, 1967, 1973), Freudenthal (1981), and Van Heile (Hoffer, 1983; Van Heile, 1986) are among the math psychologists and educators who agree with Bruner that students need experiences at the concrete or manipulative level, before they reach the representative or pictorial level. Only then can the abstract level of understanding be reached (Groen & Kieran, 1983; Heddens, 1984).

The Multiple Embodiment Principle proposed by Z. P. Dienes in the 1960s is built upon Bruner's and Piaget's theories. The primary focus of Dienes' theory is that learners acquire mathematical structures through manipulation of physical embodiments, concrete devices that embody the mathematical structure to be learned. The amount of abstraction acquired by the learner is directly proportional to the number of varied experiences embodying the concept (Dienes, 1967).

Retention of knowledge is as important to mathematics instruction as the acquisition of knowledge.

Byers and Erlwanger (1985) contend that "memory plays an essential role in the understanding of mathematics (p. 259)". Gagne and White (1978) acknowledged the importance of memory when they incorporated memory structure into their learning paradigm. Memory involving mathematics learning may be one key to understanding why some students "learn" mathematics by memorizing, others by understanding, and still others learn not at all.

Like mathematics learning, mathematics memory viewed as a quantitative measure (i.e., recall of math facts) was prevalent until the 1960s (Byers & Erlanger, 1985; Norman, 1982). The Ebbinghaus retention curve of verbal memory reflects the rote notion of memory popular in the first part of the 20th century (Underwood, 1982). During this period of behavioral emphasis, drill and practice was a major instructional method. Recently, cognitive psychology has placed a new emphasis on the role of meaning and organization in learning and retention of learning. Byers and Erlanger (1985, p. 261) summarize the current view of memory in mathematics learning:

The crucial question is not whether memory plays a role in understanding mathematics but 'what' it is that is remembered and 'how' it is remembered by those who understand it - as well as by those who do not.

National testing results also indicate the concepts and computation involving the subject of volume are a

matter of concern (Gardner, Rudman, Karlson, & Merwin, 1983a, 1983b; Michigan Department of Education (MEAP), 1983; NAEP, 1983). One reason for this concern may be the minimal attention given to three-dimensional space and geometry in mathematics curriculum (Ben-Haim, Lappan, & Houang, 1985). Shumway (1980) notes the nominal amount of attention given to space and geometry in mathematics research (Shumway, 1980). Tyson-Bernstein (1988) noted the negative effect on the quality of education when curriculum and textbook standards are set by political rather than knowledgeable synthesis.

In addition to the political influence on textbook content, the overuse of textual representation may not be adequate for all learners. Many students learn better with other forms of communication such as graphics, motion, tactile motions, and computer representations (Taylor & Cunniff, 1988). Unfortunately, instructional content is based on textbook content (Dossey, 1988; Nicely, 1985; Taylor & Cunniff, 1988). Two-thirds of all mathematics classes are taught using only one textbook (Committee on Indicators of Pre-College Science and Mathematics, Commission of Behavioral and Social Studies and Education, & National Research Council, 1985). Therefore, supplement of mathematics textbooks to achieve higher level mathematics skills is mandatory (Nicely,

1985). Fortunately, the NCTM is attempting to address this problem by defining appropriate standards for future needs; it is the only educational group to do so (Tyson-Bernstein, 1988).

Another reason for poor achievement on the topic of volume may pertain to poor spatial ability, the ability to visualize and manipulate three-dimensional space (Bishop, 1983; Ben-Haim et al., 1985). One study by Ben Haim et al. (1985) analyzed errors made by students who took the 1983 Michigan Educational Assessment Program test. The test items reviewed measured ability to calculate the volume of rectangular solids. Their findings suggest visualization of three-dimensional objects and mathematical problems dealing with visualization is an area in which students lack skills. The authors suggest initial instruction should include concrete activities. The traditional textbook approach to volume of rectangular solids that uses a two-dimensional semi-concrete format followed by drill and practice type activities is not sufficient (Ben-Haim et al., 1983; Hart, 1981; Nicely, 1985).

Additionally, the traditional mathematics textbook approach follows an instructional progression of length, area, and volume. Some research seems to demonstrate this may not be the best sequence for all students (Hart,

1984). Other research indicates a visual approach to volume is more beneficial to students who have low visualization skills (Gabel & Enochs, 1987). When students do not understand the concept, volume will be interpreted only as a memorized formula (Dugdale & Kibbey, 1983; Gabel & Enochs, 1987; NAEP, 1983).

Just as mathematics learning theories and practices have shifted from quantitative to qualitative emphasis, so has computer-assisted instruction (Chambers & Sprecher, 1983). Until the late 1960s CAI consisted primarily of drill and practice formats. This form of CAI improved memorization and recall of facts, which are lower levels of learning (Bialo & Erickson, 1985; Kulik, Kulik, & Cohen, 1980). Recently, however, development of a new type of software, the simulation, or microworld, offers promise in enhancing concept development in math education (Alesandrini, 1985; Henderson et al., 1983; Chambers & Sprecher, 1983; NCTM, 1985). The ability of the computer to quickly create visual images that represent an abstract mathematical concept holds great promise in helping students acquire those abstract concepts and promoting learning at higher cognitive levels (Bork, 1980; Franz, 1986; Marks, 1985; Reed, 1985; Smith, 1982).

The lack of accessibility of computers for classroom use, lack of funds for software and hardware, and the poor quality of computer software has, in the past, stifled attempts to respond to these national concerns over declining mathematics scores and recommendations for incorporating computer aided instruction into mathematics classrooms (DeVault, 1981; DeVault & Chapin, 1980). Today, however, problems with accessibility, funding, and the quality of software programs have diminished and, in some cases, been completely resolved. A 1984-85 comprehensive national survey revealed that 94% of all public school districts in the United States use computers in instruction; 82% of elementary schools have microcomputers in their classrooms (Hood, 1985). In Montana, 94% of the state's classrooms have access to or contain permanently placed computers (Office of Public Instruction (OPI), 1985b). In Montana's elementary school classrooms the percentage is 84 (OPI, 1987). Public domain software, site licenses, and labs in which all students in a school have access to computers have helped defray costs for both software and hardware purchases. A few studies indicate mathematics instruction incorporating CAI is less expensive than without CAI (Adam, 1989).

A consensus of Montana educational experts concluded top priorities for computer literacy for Montana public schools are: (a) students should be able to use the computer as a tool for inquiry, problem solving, and recreation; and (b) students should participate in a variety of experiences (hands on) in several subject areas (Bruwelheide, 1982). A review of the literature demonstrates that the use of computers as a supplement to instruction can also enhance learning in mathematics (Burns & Bozeman, 1981; Henderson et al., 1983; Kelman et al., 1983).

The impetus for this study came from an internship completed in the Spring of 1987 involving a field test of a multiple embodiment instructional sequence on the subject of volume of rectangular solids. A computer simulation was included as one embodiment. Forty-five students from one town and two rural Montana schools (see Appendix C) participated in the study which consisted of a four-day instructional unit and three weeks of retention activities. Results of the study indicated that use of computer-assisted instruction (CAI) may have been a factor in the increase in student achievement. The positive reaction of both teachers and students indicated to this researcher a need to replicate the pilot study in a more controlled environment. Because

several of the instructional days required more than a 50-minute period, this researcher also decided to increase the length of the instruction time to five 50-minute periods, reconfiguring the research study to a five-day instructional time period.

Statement of the Problem

Concern over the declining test scores in mathematical skill and concept development, including the concept of volume of rectangular solids, has been documented (Gardner et al., 1983; MEAP, 1983; NAEP, 1983). In addition to improving mathematics achievement, educators are concerned about developing mathematics ability at higher cognitive levels. Many national educational organizations urge inclusion of computer-assisted instruction as a means of improving the learning of mathematics.

The problem in this study was to determine the effect of three methods of instruction on student achievement. The three methods included: (a) a multiple-embodiment teaching sequence including the computer as one embodiment, (b) the same sequence with embodiments but without the computer embodiment, and (c) a teaching sequence based on traditional textbook procedures. The dependent variables were mean scores on

criterion-referenced tests. Independent variables were method of instruction, gender, and ability grouping. Instruction consisted of the teaching of volume of rectangular solids. The population were fifth grade students enrolled in school districts in southwestern Montana.

Need for the Study

National testing services have documented the need for an increase in mathematics achievement, including the subject of volume (Gardner et al., 1983; MEAP, 1983; NAEP, 1979a; NAEP, 1979b; NAEP, 1983). Educators are also concerned that much of mathematics instruction does not promote higher cognitive levels of learning (Bialo & Erickson, 1985; MCTM, 1985; Wheeler, 1971). The importance of retention, as well as initial acquisition of mathematics knowledge, is another matter for concern (Byers & Erlwanger, 1985).

The National Science Board of Pre-College Education in Mathematics, Science and Technology (1983), the National Council of Teachers of Mathematics (1985), the National Commission on Excellence in Education (1983), and the National Council of Supervisors of Mathematics (1988) were among the groups who have urged new and continued research to develop improved curricula and

