



Determining the change in snow microstructure during large deformation processes by the method of quantitative stereology
by Michael Quast Edens

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering Mechanics
Montana State University
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Abstract:

Modern constitutive theories for snow are now using microstructural parameters in their formulation. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed for the evaluation of empirical parameters.

The microstructural variables selected to characterize the behavior of snow must be able to represent dominant mechanisms such as pore collapse, bond fracture, and neck and bond growth due to pressure sintering as well as effects of pore pressure, a mechanism to account for a reduction in grain mobility, coupling of deviatoric and volumetric responses, work hardening, and local inertial effects.

In this thesis a set microstructural variables that meet these criteria and corresponding mathematical relations from quantitative stereology are reviewed along with relations and techniques required for numerical evaluation. An experimental investigation is carried out to understand the effect changes in these variables have on the behavior of snow subjected to large deformations. Measurements at several stages of deformation are used to understand microstructure changes, dominant mechanisms, and effects on bulk behavior.

Microstructure measurements of six snow samples subjected to confined compression tests are presented for precompressed and compressed states corresponding to final loads of 11.2 KN, 22.4 KN, or 44.8 KN. Order of magnitude changes in microstructural parameters are compared with corresponding magnitude changes in stress level for densities ranging from 0.5 g/cc to 0.64 g/cc. Generally, microstructural variables underwent order of 2 changes compared with 16 for the stress level.

This experimental investigation presents changes in microstructural variables that result from large deformations and high deformation rate. These results go a long-way toward filling in a gap in the presently available data. Microstructural variables which, to the present, have not been adequately measured and mathematically described are evaluated.

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PROCESSES BY THE METHOD OF QUANTITATIVE STEREOLOGY**

**by
Michael Quast Edens**

**A thesis submitted in partial fulfillment
of the requirements for the degree**

**of
Master of Science
in
Engineering Mechanics**

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Bozeman, Montana**

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Approval

of a thesis submitted by

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency and is ready for submission to the College of Graduate Studies.

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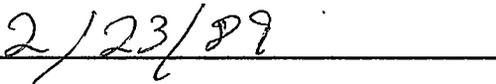


TABLE OF CONTENTS

	Page
APPROVAL	ii
STATEMENT OF PERMISSION TO USE	iii
TABLE OF CONTENTS	iv
LIST OF TABLES	vi
LIST OF FIGURES	vii
ABSTRACT	ix
INTRODUCTION	1
Purpose	2
STEREOLOGY AND MICROSTRUCTURAL PARAMETERS	4
Requirements for Microstructure Variables	4
Two Dimensional Parameters and Measurements	5
Number of Grains Per Unit Length of Test Line; N_{gL}	7
Number of Bonds Per Unit Area; N_{bA}	7
The Number of Grains Per Unit Area; N_{gA}	8
Air, Bond and Ice Area Fractions; A_{aA} , A_{bA} , A_{gA}	8
Mean 2-D Bond Diameter and Harmonic Mean of 2-D Bond Diameters; D_2 , E	9
Bond Frequency; f_0, f_1, f_2, f_3	9
Density Ratio; α	9
3-D Parameters	10
Mean Bond Radius; R_3	10
Mean Neck Length; h	11
Mean Pore Length; λ	11
Mean Number of Bonds Per Unit Volume; N_{bV}	11
Mean Intercept Length; L_3	12
Mean Grain Surface Area Per Volume; S_V	12
3-D Microstructure Requiring a Grain Shape Factor	12
EXPERIMENT AND RESULTS	17
Experimental Procedure	17
Experimental Results	20

TABLE OF CONTENTS-Continued

	Page
Discussion and Conclusions	37
RECOMMENDATIONS AND SUMMARY	39
Recommendations	39
Summary	39
REFERENCES CITED	41
APPENDICES	43
Appendix A-Surface Section Preparation	44
Appendix B-Surface Section Analysis	48
Preliminaries	49
Analysis Procedure	50
Appendix C- State Variable Calculation	70

LIST OF TABLES

Table	Page
1. Digitized Image Resolution and Analysis Area Sizes	22
2. Measured and Section Density	24
3. Mean Intercept Length, Mean Pore Length and Mean Neck Length	26
4. Values Obtained Directly From Measured 2-D Bonds in the Plane	27
5. Number of Features Per Length, Per Area and Per Volume	30
6. Measured Bond Frequency Distributions and Relative Errors	34
7. N_3 , N_{gV} , V and S_V for the Precompressed Samples	34
8. N_3 , N_{gV} , V and S_V for the Compressed Samples	35

LIST OF FIGURES

Figure	Page
1. An idealized picture of bonded ice grains	7
2. An example of basic counting techniques	8
3. Schematic of Compression Ring with Upper and Lower Platens	19
4. Final Screen Image of Analyzed Surface Section 2k502p	21
5. L/L_0 VS. $\Delta\rho/\rho_0$	24
6. Number VS. 2-D Bond Diameter for Sample 10k01	27
7. Number VS. 2-D Bond Diameter for Sample 5k001	28
8. Number VS. 2-D Bond Diameter for Sample 5k002	28
9. Number VS. 2-D Bond Diameter for Sample 2k501	29
10. Number VS. 2-D Bond Diameter for Sample 2k502	29
11. R/R_0 VS. $\Delta\rho/\rho_0$	30
12. $N_b V^* L^2$ VS. λ	31
13. Three Dimensional Coordination Number VS. Density	36
14. Ratio of Surface Area Per Unit Volume VS. $\Delta\rho/\rho_0$	36
15. Axial and Radial Stress VS. Lagrangian Strain	37
16. Cursor Movement During a Typical Grain Bond Coloring Procedure	52
17. Surface Section Analysis Program	54
18. State Variable Output after First Iteration	74
19. State Variable Output after Second Iteration	75
20. State Variable Output after Third Iteration	76

Figure	Page
21. State Variable Output after Fourth Iteration	77
22. State Variable Program Listing	78

ABSTRACT

Modern constitutive theories for snow are now using microstructural parameters in their formulation. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed for the evaluation of empirical parameters.

The microstructural variables selected to characterize the behavior of snow must be able to represent dominant mechanisms such as pore collapse, bond fracture, and neck and bond growth due to pressure sintering as well as effects of pore pressure, a mechanism to account for a reduction in grain mobility, coupling of deviatoric and volumetric responses, work hardening, and local inertial effects.

In this thesis a set microstructural variables that meet these criteria and corresponding mathematical relations from quantitative stereology are reviewed along with relations and techniques required for numerical evaluation. An experimental investigation is carried out to understand the effect changes in these variables have on the behavior of snow subjected to large deformations. Measurements at several stages of deformation are used to understand microstructure changes, dominant mechanisms, and effects on bulk behavior.

Microstructure measurements of six snow samples subjected to confined compression tests are presented for precompressed and compressed states corresponding to final loads of 11.2 KN, 22.4 KN, or 44.8 KN. Order of magnitude changes in microstructural parameters are compared with corresponding magnitude changes in stress level for densities ranging from 0.5 g/cc to 0.64 g/cc. Generally, microstructural variables underwent order of 2 changes compared with 16 for the stress level.

This experimental investigation presents changes in microstructural variables that result from large deformations and high deformation rate. These results go a long-way toward filling in a gap in the presently available data. Microstructural variables which, to the present, have not been adequately measured and mathematically described are evaluated.

INTRODUCTION

A material that will affect nearly everyone living in polar climates at some time is snow. Its mechanical properties can exhibit a strong influence on everything from vehicle travel (Brown, 1979) and winter recreation to avalanche prediction and control. One of the features most commonly observed by those living in regions where snow ground cover occurs is its compressibility. Snow can undergo large volumetric deformations due to moderate bulk stresses. These deformations are basically irreversible. The ability to undergo large deformation is due in large part to its porosity. Brown (1979) demonstrated the ability of a constitutive law based on a pore collapse model to accurately represent the observed behavior of snow subjected to large strains and strain rates.

In its natural state snow exhibits a wide range of grain shapes and densities. Able and Gow (1975 and 1976) have demonstrated the affects initial snow density has on load bearing ability. From their experiments, it was clearly demonstrated that for undisturbed snow with densities from 0.1 to 0.27 g/cc and subjected to high rate deformation, lower initial densities resulted in lower final densities for a given final stress than that with a higher initial density. They attributed this to the very irregular shaped grains found in very low density snow and the more uniform rounded nature of the higher density material. It was found that the initial density was a significant factor for final stresses < 1 Mpa. Beyond this, the effects of higher initial density became less obvious. Their studies also showed the rate dependent nature of snow's load bearing ability.

Snow is a thermodynamically active material. This has been clearly observed in collected samples left in storage longer than two months. Volume reductions on the order of 0.5 have been observed. This reduction is a result of the snow trying to reduce its surface area to volume ratio. This same mechanism is important in neck growth due to pressure sintering. A volumetric constitutive law based on a neck growth model was developed by Brown (1980). This model included structural parameters that account for changes in neck length, grain radius, and coordination number. The resulting equation accurately described the behavior of snow subjected to a given deformation rate for a range of densities.

A constitutive law based upon nonlinear thermodynamics that accounts for the mechanical properties of snow subjected to finite strain under high rate uniaxial deformation has been developed by Hansen (1985). The microstructural properties of snow in this theory are included in the form of internal state variables. This theory was shown to predict the behavior of mechanical properties such as rate dependence, stress relaxation, and strain recovery.

Modern constitutive theories are now using microstructural parameters. In order to improve the theories, more advanced methods of describing the microstructural behavior are needed, thus this thesis.

Purpose

In all of the theories previously mentioned, accuracy is dependent on understanding the behavior of the microstructural variable included. This is particularly important since simplifying assumptions must be made in order that the resulting theory is manageable. To confirm understanding of microstructural behavior and validity of assumptions of such, it is necessary to obtain experimental data pertinent to the density range, deformation range, and deformation rate being modeled. This data is also needed

for the evaluation of empirical parameters. At the present time, little or no data on microstructural behavior exists for snow densified by large deformation to densities in the 0.5 to 0.65 g/cc range. The work of Able and Gow mentioned earlier provides a strong basis for understanding bulk behavior of snow subjected to high deformation rates and large strains, but little or no data was obtained relating the microstructure to the deformation. That obtained was for samples that had been unloaded for several hours. The resulting data may not accurately reflect the actual microstructure existing at the end of the deformation process since crystallization had time to occur in significant amounts. Gubler (1978a, 1978b) and Kry (1975a, 1975b) carried out extensive microstructural evaluation in their work, but it was generally confined to final densities below 0.5 g/cc and very low deformation rates.

It is the major purpose of this experimental investigation to evaluate the changes in microstructural variables that result from large deformations and high deformation rate. The results obtained will go a long way toward filling in a gap in the presently available data. In addition, it is intended to measure microstructural variables which to the present have not been adequately measured and mathematically described. An empirical equation relating the number of bonds per volume to the mean pore length will be given and its physical significance discussed. A new definition of bandwidth in Gubler's (1978a) work will also be given.

STEREOLOGY AND MICROSTRUCTURAL PARAMETERS

Stereology is a statistically based method by which quantities measured directly from a two dimensional plane, intersecting a three dimensional body, are used to obtain average values for the body's three dimensional structure. The relationships between two and three dimensions and the resulting numerical values can be used as a means for understanding the average microstructural behavior of the body. The task then is to select those parameters most important in characterizing the average behavior and making the pertinent section measurements. In this chapter, possible microstructural variables discussed by Hansen (1985) are reviewed along with the necessary two dimensional measurements. A new definition of the bandwidth defined by Gubler (1978a) for his method of calculating mean three dimensional coordination number is also presented.

Requirements for Microstructure Variables

The variables selected to represent the microstructure of snow under large deformations and high rate deformation must be able to characterize the average rearrangement of the microstructure. These variables must be able to represent dominant mechanisms such as pore collapse, bond fracture, and neck and bond growth due to pressure sintering. Hansen (1985) lists 5 other phenomena that these parameters must account for : (1) effects of pore pressure, (2) a locking mechanism to account for a reduction in grain mobility, (3) coupling of deviatoric and volumetric responses, (4) work hardening and (5) local inertial effects.

Two Dimensional Parameters and Measurements

Measurements from a section plane are generally based upon three counting methods: point, lineal and areal. The main assumptions necessary for validity of measurements are homogeneity and isotropy. Homogeneity requires that the material exhibits the same properties throughout the body. This may be difficult to meet in small regions, but if large enough areas are measured this requirement can be satisfied. Isotropy requires that the orientation of material structures are not predominate in any direction. For oriented structures, a series of randomly placed and oriented test lines or grids may be used.

Hansen (1985) considered the microstructure of snow by representing it as a three phase mixture. The phases were air, neck, and ice grain. The subscripts used to represent each phase and a brief description of each follow.

- a: air phase The air phase represents the pore space of the material matrix.
- b: neck phase The neck phase represents the material joining grains together.
- g: ice grain phase The grain phase represents the ice grains.

The required measurements of each phase in the section plane are represented by the following variables:

- N_{gL} : number of grains intercepted per unit length of test line
- N_{bA} : number of bonds per unit area
- N_{gA} : number of grains per unit area
- A_{aA} : area fraction of air phase.
- A_{bA} : area fraction of neck phase.
- A_{gA} : area fraction of ice grain phase.

The remaining surface section measurements required are

- α : ratio of ice density to snow density, ρ_i / ρ_s

- D₂: mean bond diameter
- E: harmonic mean of bond diameter
- f₀: fraction of grains with 0 bonds appearing in the section plane
- f₁: fraction of grains with 1 bond appearing in the section plane
- f₂: fraction of grains with 2 bonds appearing in the section plane
- f₃: fraction of grains with 3 bonds appearing in the section plane.

Before proceeding further, two definitions are needed. The first is that of a grain bond. The definition used is that given by Kry (1975a). A grain bond intercepted by the section plane will appear as a line joining two opposite edges of the ice in the plane. To identify this line as a bond, three criteria must be met. First, there must be a minimum constriction of the ice in the section plane. Secondly, the constriction must be present on both edges of the ice. Finally, the constrictions must point approximately towards each other. See Figure 1. In practice, applying this definition will have some subjectivity involved. As long as consistency in applying the definition is observed, comparison of measurements should not be seriously affected.

The second definition is that of the neck region shown in Figure 1. This definition comes from Hansen (1985). The neck is considered to be that portion of the ice joining two ice grains and lying between the points where the surface goes from convex to concave with respect to the outward normal. The application of this definition also contains an element of subjectivity. These regions tend to contain less than 2% of the total section area. Errors in location of the change in curvature or identification will have only small affect on other measurements. The region enclosed by the large rectangle in Figure 2 clearly shows all phases of interest for an image enhanced surface section. Now the section variables and their measurement can be discussed. In discussing each variable, it is indicated whether the measurement is visual or automatic using computer analysis software.

Number of Grains Per Unit
Length of Test Line: N_{GL}

N_{GL} is found by taking a number test lines spaced evenly across the section, counting the total number of grains intercepted by these lines and dividing this by the total length of the test lines. In the surface analysis program, a test line is displaced parallel to itself across the entire area being analyzed in steps of 0.2 scale markings. At each step, the number of intercepted grains are then counted. Grains intercepted by the left region boundary are not counted; those on the right are counted as 1 each. Refer to Figure 1 for a graphic representation of the counting procedure.

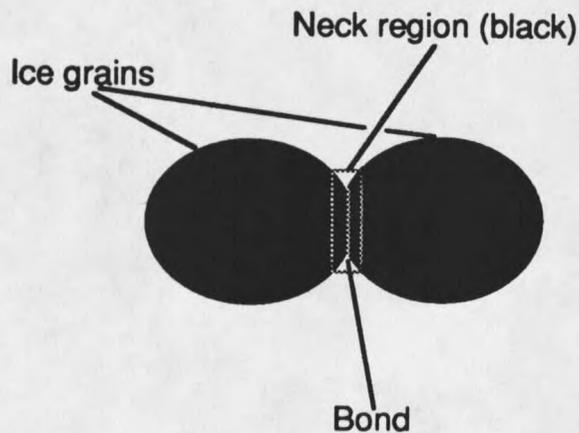


Figure 1. An idealized picture of bonded ice grains. This shows the line representing the bond in two dimensions and the neck region, the black portion within the rectangle.

Number of Bonds Per Unit Area: N_{bA}

N_{bA} is simply the number of bonds in the test area divided by the test area. Each bond is automatically counted as the bond diameter is measured. Bonds that are intercepted by the region boundary are counted as 1/2 and are not measured.

The Number of Grains
Per Unit Area: N_{gA}

N_{gA} is similar to N_{bA} . In this case the number of grains must be counted manually. The grains are actually counted in five separate categories, grains having 0, 1, 2, 3, or more than 3 bonds appearing in the surface section. Each category of grain intercepted by the region boundary is counted as 1/2. Summing all the categories gives the number of grains. Figure 1. shows an example of this for the top boundary.

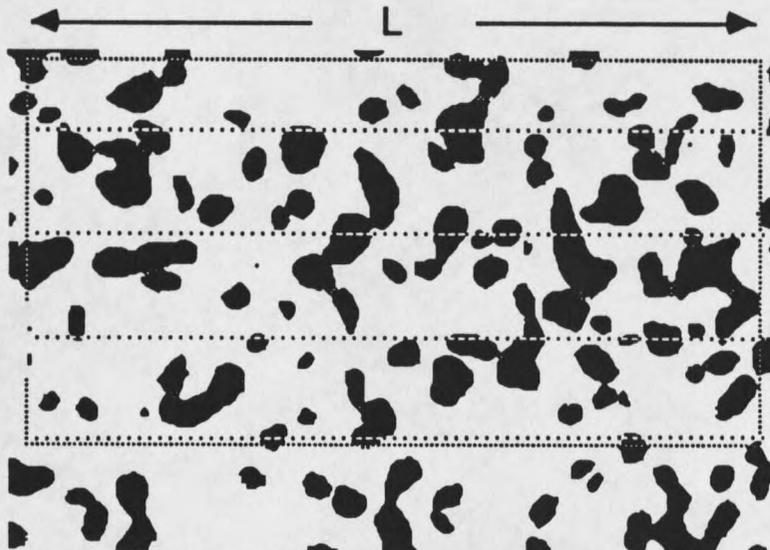


Figure 2. An example of basic counting techniques. 21 grains are intercepted by a total line length of $4 \cdot L$. The resulting value of N_{gL} is $21/(4 \cdot L)$. Looking at the top boundary line, the count for 0 bonds is 1.5, for 1 bond $1/2 + 1/2 = 1$, 2 bonds $1/2$, 3 and >3 bonds both are 0. The total number of grains along the top boundary is then 3.

Air. Bond and Ice Area Fractions:

$$A_{aA} \cdot A_{bA} \cdot A_{gA}$$

Area fraction measurements are simply the total area of the phase of interest divided by the total analysis area. The neck regions are delineated when the bond is measured and appear gray on the section. These measurements are an automatic feature of the analysis program.

Mean 2-D Bond Diameter and Harmonic
Mean of 2-D Bond Diameters: D_2 , E

The 2-D bond diameter D_2 is the average of all bonds within the analysis region. Those intercepted by the boundary are excluded from this measurement.

The Harmonic mean E , given by Fullman (1953) is

$$E = \frac{1}{N} \sum_{i=1}^N \frac{1}{D_{2i}} \quad (1)$$

where N is the number of 2-D bonds in the section plane and D_{2i} is length of the i^{th} bond as it is measured in the surface section. Though E has no readily apparent physical meaning it is important as a variable in equations for polydispersed spheres and disks. In the work presented, bonds in 3-D are approximated by disks, and it is in this context that E is used.

Bond Frequency: f_0, f_1, f_2, f_3

$f_0, f_1, f_2,$ and f_3 represent the relative frequency of grains with 0, 1, 2, and 3 bonds respectively. This measurement requires visually counting the total number of bonds in each category. This is the same counting measurement used for N_{gA} . The importance of these measurements will become more evident when the method for calculating the 3-D coordination number is discussed later in this chapter.

Density Ratio: α

The last parameter discussed in this section is the density ratio α . It is the ratio of ice density ρ_i to the snow density ρ . For ice at -10°C ρ_i has a value of 0.917 g/cc. The stereologic relationship for α is given by the following (Underwood, 1970):

$$\alpha = \frac{1}{A_{gA} + A_{bA}} \quad (2)$$

As well as being a variable in several of the 3-D parameters, α serves as a check on the accuracy of the image enhancement process, in particular, any interpretation required of section features. It is often necessary to determine whether pits that occur in the plane are grains that have been dislodged, grains or pore space which have been poorly polished, or air bubbles sliced by the section.

3-D Parameters

Two types of parameters will be discussed here, those that come directly from the 2-D values and those that normally require some assumption of a particle shape factor. Those which need only 2-D values are based purely on statistical assumptions and are therefore valid for particles of any shape. Before proceeding, the definition of a bond in three dimensions is needed. As given by Kry (1975a), a grain bond is defined to be the plane surface of minimum area located in the neck region of two joined grains.

Mean Bond Radius: R_3

The neck regions within the three dimensional snow structure represent locations of high stress concentration when the bulk material is subjected to external loading. These regions may have large constrictions relative to the grain structure. Since the grain bond is the location of the largest constriction, it will then have the highest stress concentration. Due to this, R_3 along with the mean three dimensional coordination number N_3 are important in determining the load bearing ability of the snow. R_3 is also important in characterizing bond growth due to pressure sintering. It was used by Brown (1979) as a parameter in a volumetric constitutive law based on a neck growth model. Kry (1975a) suggested that the bonds in three dimensions may be represented by circular disks. Based upon this assumption, the necessary relationship for the bond radius as derived by Fullman (1953) is

$$R_3 = \frac{\Pi}{4E} \quad (3)$$

Mean Neck Length: h

Hansen (1985) gives the following relation for h:

$$h = \frac{A_b A}{N_b \Pi r^2} \quad (4)$$

h was used by Brown in his neck growth constitutive law. An increase in h occurs during pressure sintering and from mass transport as snow attempts to reduce its surface area to volume ratio.

Mean Pore Length: λ

The mean pore length is important in characterizing grain mobility within the material matrix. This grain mobility is critical in controlling the amount of deformation the material can undergo for a given load condition. Pressure sintering becomes more dominant as λ decreases under external loading. Another important result is an increase in the number of grain contacts. The equation for λ , derived by Fullman (1953) is valid regardless of grain shape, size, or distribution, and is given as

$$\lambda = \frac{\alpha - 1}{\alpha N_{gL}} \quad (5)$$

Mean Number of Bonds
Per Unit Volume: $N_b V$

When a load is placed upon snow, not only bond area but also the number of bonds per grain affect the stress concentration at a bond. An increase in the number of bonds prior to an increase in load is effectively like increasing the area of a fewer number of bonds. $N_b V$'s importance is as a variable in the mean three dimensional coordination number N_3 . Based on the assumption of bonds being circular disks, Fullman (1953) has

derived the following relation necessary for determining the mean number of bonds per volume:

$$N_{bV} = \frac{8EN_{bA}}{\Pi^2} \quad (6)$$

Mean Intercept Length: L_3

The equation for the mean intercept length derived by Underwood (1970) is valid without assumptions of grain shape, size, or distribution. It is given by

$$L_3 = \frac{A_{gA} + A_{bA}}{N_{gL}} \quad (7)$$

L_3 provides a measure of the grain length. It is useful along with the mean pore length in characterizing grain mobility.

Mean Grain Surface Area Per Volume: S_V

The mean surface area per unit volume is related to the free energy in the sample. As grain bonds break there will be an increase in surface area and, as a result, an increase in free energy. This will affect the neck and bond growth occurring during pressure sintering. The equation derived by Hansen (1985) is

$$S_V = 4N_{gL} - 2\Pi R^2 N_3 N_{gV} \quad (8)$$

This represents the mean surface area per volume of detached grains, minus, the mean surface area of the grain bonds.

3-D Microstructure Requiring a Grain Shape Factor

The parameters that remain to be calculated are N_{gV} , the mean number of grains per unit volume; N_3 , the mean number of bonds per grain; and V , the mean grain volume.

These parameters require some knowledge of grain shape. Attempting to characterize alpine snow by a grain shape factor may not be a valid generalization due to the wide range of shapes present within a given sample. Gubler (1978a) and Hansen (1985) provided a method for determining these parameters without any assumption of grain shape, other than for an initial guess.

Gubler's method for evaluating N_3 was based upon several parameters. The first was the probability p defined to be the probability that if a grain with coordination number $N_3=1$ is cut by the section plane, its bond lies in the section plane. Next, the distribution of coordination number about the mean coordination number N_3 was used. This distribution could be changed by an adjustable free parameter i . i has no apparent physical meaning.

The procedure was then to adjust N_3 and i until the theoretical values of the two dimensional distribution given by

$$f_2(l) = N_2 \sum_{n=l}^{12} \binom{n}{l} p(1-p)^{n-l} f_3(n) \quad (9)$$

converged to the measured values. N_2 is a normalizing constant introduced by Hansen (1985) and is given by

$$N_2 \sum_{l=0}^6 f_2(l) = 1. \quad (10)$$

$\binom{n}{l}$, the number of combinations is given by

$$\binom{n}{l} = \frac{n!}{l!(n-l)!} \quad (11)$$

$f_3(n)$, the three dimensional probability distribution function of a grain having a coordination number n , derived by Gubler (1978a), is given by

$$f_3(n) = N_3 n^{3i-1} \exp(-\beta n^{2i}) \quad (12)$$

N_3 is a normalizing constant given by

$$N_3 = 4i \frac{\beta^2}{\sqrt{\pi}} \quad (13)$$

β is given by

$$\beta = \left[\frac{2}{\sqrt{\pi}} \frac{1}{N_3} \Gamma \left(\frac{3}{2} + \frac{1}{2i} \right) \right]^{2i} \quad (14)$$

where Γ is the gamma function.

The definition of p is given by

$$p = \frac{s_\beta}{s_k} \quad (15)$$

where

$$s_\beta = \frac{\pi N_g L b}{N_{gA}} \quad (16)$$

and

$$s_k = \frac{4 N_g L}{N_{gV}} \quad (17)$$

S_β is the area of a band of width b whose center line is the section plane. S_k

is the mean grain surface area. b is given by

$$b = 2 \left[R_3^2 - \left(\frac{D_2}{2} \right)^2 \right]^{\frac{1}{2}} \quad (18)$$

Hansen adjusts p and N_3 by varying N_{gV} and using the equation

$$N_3 = \frac{2 N_b V}{N_{gV}} \quad (19)$$

which is neglected by Gubler. It is by using equation 19 and directly varying N_{gV} and i

to force equation 9 to converge to the measured values, rather than N_3 and i , that Hansen is able to determine N_{gv} without resorting to the assumption of a shape factor. One problem with Gubler's approach is that he has fixed N_{gv} by assuming a shape factor C and using

$$N_{gv} = C \frac{N_{gA}^2}{N_{gL}} \quad (20)$$

This fixes the value of N_3 since N_{bV} is known, yet he still allows N_3 to vary which may lead to uniqueness problems.

A slight digression will now be taken in order to introduce a new definition of the bandwidth b . In equation 18, $b/2$ is the height of a right triangle necessary to yield a line of length $D_2/2$ lying in the intersecting plane, when R_3 , the hypotenuse is projected onto the plane. The small bonds intersected by the section plane provided the most weight in determining the magnitude of R_3 , with the largest necks having much less influence. This occurs, because R_3 is based upon the sum of the reciprocal of the two dimensional bonds (equation 1 and 3). With this in mind it is then reasonable to assume that when R_3 is projected onto the section plane, one should find those values which most affect its magnitude. The definition as given does not allow for this. Only those values lying between $D_2/2$ and R_3 will be found within the resulting bandwidth. If this band is narrow, it may happen that all but a small number of measured necks will lie within the band. By replacing the term in parenthesis in equation 18 with the minimum value found in the section plane, b will then contain most of the necks found in the section plane, particularly those that most heavily influence its magnitude. The new definition for bandwidth becomes

$$b = 2 \left[R_3^2 - \left(\frac{D_{2min}}{2} \right)^2 \right]^{\frac{1}{2}} \quad (21)$$

Once N_{gV} is determined using Hansen's approach along with the new definition of bandwidth, the mean grain volume can be determined. The governing equation given by Hansen (1985) is

$$V = \frac{A_g A + A_b A}{N_{gV}} \quad (22)$$

Hansen included the neck area fraction since the neck becomes an integral part of the grain when the bond fractures. This term is important in measuring inertial effects, particularly at very high rate deformation.

The equations in this chapter have been incorporated in to two computer programs. A new program written in Forth contains code that will allow evaluation of all surface section variables except those based upon the work of Gubler. A Fortran program written by Hansen and since modified to run on a Macintosh computer and incorporating the new bandwidth, contains code for the remainder of the variables. These will be discussed further in the next chapter along with results.

EXPERIMENT AND RESULTS

In this chapter the experimental procedure, results, and conclusions will be presented. The data provided in this chapter will fill a gap in that presently available by providing extensive measurements of snow microstructure over a wide density range, resulting from a high deformation rate and large strains.

Experimental Procedure

Alpine snow was collected and stored in a cold room at -10° C. Confined compression tests were carried out within a period of two to six months after the samples were first placed in the cold room. After two months, the original sample volume had decreased by nearly 50%, and volume changes occurring after this were generally very small. The snow samples consisted of snow from two years, 1987 and 1988. The large grained samples 10k01, 5k001 and 2k501 were from February 1987, 5k002 and 2k502 were from February 1988.

The first part of the experiment involved subjecting samples to a constant deformation rate, in this case 1" per minute, until a predetermined maximum load value was reached. The possible maximums were 44.8 KN, 22.4 KN, or 11.2 KN. Once this load was attained, the test was stopped. Surface sections were then immediately prepared from the uncompressed and compressed samples.

The compressive tests were carried out using an MTS 880 materials test system capable of loadings as high as 222.5 KN. The crosshead and samples were enclosed in an environmental chamber maintained at -10° C. Samples were prepared in the cold room for testing by cutting them in square segments with edge dimensions slightly larger

than the compression ring diameter. The compression ring was then pushed onto the cubed sample and the protruding snow trimmed flush with the top of the ring. The bottom edge of the compression ring has a beveled edge which facilitates cutting through the sample without damage. The sample and compression ring were then weighed together. The weight of the ring was subtracted, and the initial density calculated using the ring volume.

The compression ring consists of four separate rings placed on top of one another and held together by three binding posts which were loosened during testing. Four strain gauges were mounted to the rings, two (dummy) on the top and two (active) on the bottom ring. These were used to measure hoop strain. A schematic of the test setup is shown in Figure 3. Three voltages from the MTS system were fed into a GW Instruments MacAdios Data Acquisition System connected to a Macintosh Plus computer. These voltages corresponded to the axial displacement, axial load, and the hoop strain. Conversion of voltages to stress and strain data and data manipulation were carried out on the Macintosh.

Surface sections were prepared from portions of each sample in the uncompressed and the compressed states. Surface preparation was based on the procedure presented by Perla (1982). The sections were prepared in 3.8X3.8X1.7 mm³ trays. A solution of dimethylphthalate and methyl blue was allowed to cool to between -2° and -4° C, then slowly poured into the samples. After the solution had completely frozen, pedestals were attached, the sample was removed from the tray, then microtomed.

The top millimeter of the sample was removed before polishing began. The exposed surface was then allowed to sublimate for 3 minutes before it was polished with Lamp Black. Once the polishing procedure was complete, the section was photographed using a 35 mm camera with 55 mm lens and a flash ring light source. Instructions and recommendations for surface section preparation are presented in appendix A.

