



Quality box charts : a modern process monitoring tool
by Eric Damon Mott

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
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Abstract:

Modern industry monitors its processes to guarantee quality of products and services. Control Charts are one of the most effective methods for process monitoring. Yet traditional control charts can be difficult to implement and interpret. Quality Box Charts, a generalized class of process monitors was developed to address these concerns. .

The present research builds upon previous Quality Box Chart results. Using statistical resampling techniques, assumptions that hampered previous research were no longer necessary. Control limit calculations were possible for a wide variety of applications.

Quality Box Charts offer increased flexibility along with applications that were not previously practical. In addition, Quality Box Charts are usually more sensitive in detecting shifts in the underlying process than traditional \bar{x}/R charts. In-control run properties are also greater for the Quality Box Chart. Furthermore, Quality Box Charts are significantly easier to interpret than \bar{x}/R charts for all levels of users.

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

Modern industry monitors its processes to guarantee quality of products and services. Control Charts are one of the most effective methods for process monitoring. Yet traditional control charts can be difficult to implement and interpret. Quality Box Charts, a generalized class of process monitors was developed to address these concerns.

The present research builds upon previous Quality Box Chart results. Using statistical resampling techniques, assumptions that hampered previous research were no longer necessary. Control limit calculations were possible for a wide variety of applications.

Quality Box Charts offer increased flexibility along with applications that were not previously practical. In addition, Quality Box Charts are usually more sensitive in detecting shifts in the underlying process than traditional \bar{x}/R charts. In-control run properties are also greater for the Quality Box Chart. Furthermore, Quality Box Charts are significantly easier to interpret than \bar{x}/R charts for all levels of users.

CHAPTER 1

BUSINESS PROCESSES AND PROCESS MONITORING

Today's business environment is becoming less and less forgiving. Companies often competed on cost alone, but this is rarely possible now. World markets are integrating at higher and higher levels all the time. With the evolution of global markets, the world has witnessed the evolution of competitive position. Markets are now being fought with the weapons of cost, quality, and timing. If a company does not possess all three of these weapons in its arsenal, it may not survive. A central philosophy of this global, business evolution is quality. Quality profoundly affects cost and timing. It was believed for most of the industrial revolution that cost and quality were mutually exclusive concepts. Yet, low cost and high quality can and often do exist together. Nearly every industry has shown that proper attention to quality lowers cost and aids in the design and introduction of new products [Deming, 3]. One of the breakthrough ideas of quality and the modern industrial age is the Process Mentality which establishes every business function as a process.

A process is defined as a set of interrelated inputs that produce an output. A process is simply a set of actions and materials that join together to produce an outcome. Therefore one sees that every action taken in a business setting is a step in some process. No longer is it sufficient to consider singular actions being disjointed or unimportant because all functions are processes in a larger system. Processes follow one another to a final destination.

When an organization makes the fundamental step of recognizing its activities as a series of processes, several questions naturally begin to occur.

- What is the first step of a process?
- What is the last step of a process?
- Is the process operating in the optimum way and at the proper level?
- How can the process be monitored to provide the information necessary to answer questions about its performance?

The natural outcome of process definition is process improvement. Process improvement leads to higher quality which, when done properly, will reduce costs and improve the timeliness of operations.

In 1950, Dr. W. Edwards Deming presented the Process Mentality to the Japanese industrial leaders [Deming, 3]. He showed the leaders that raw material suppliers were just an earlier activity in the same process and the customer was the final step in the process. Every action from the earliest raw material source to the final customer is simply a series of processes that meld together. If any one step or process is neglected, all others suffer.

Deming reported, "Management learned about their responsibilities for improvement at every stage. Engineers learned theirs, and learned simple but powerful statistical methods by which to detect the existence of special (assignable) causes of variation, and that continual improvement of processes is essential . . . Once management in Japan adopted the chain reaction, everyone then from 1950 onward had one common aim, namely, quality." [Deming

Process Monitoring

Accepting the Process Mentality creates a paradigm shift. The model of disjoint activities is replaced by related processes. It is a natural reaction to define the process and to want to monitor it—the action of defining a process leads to improvement [Goetsch 8]. To understand how a process operates, it must be monitored. As it is monitored through time, it may be possible to discover trends or actions that greatly influence the performance of the process:

Motorola's Six Sigma Quality approach has been one of the most successful arguments for process monitoring. It claims that everything known about a process should be expressed as numbers. If it is not able to be expressed as numbers, then really not much is known about the process [Hahn, 9].

Fundamentally, there are two types of process monitoring. The first method monitors all inputs to the process. Inputs may be raw materials, people, actions, machinery, or similar categories. Deming believed that the important inputs were: people, materials, methods, equipment, and the environment [Roehm, 15]. Usually monitoring inputs is very complicated. Often, there are no metrics to measure the necessary qualities. Furthermore, there is a great deal of variation in each input and it is difficult to know what is appropriate and what is not. In addition, the process may be altered by the monitoring process itself as in the Hawthorne effect. In general, it is difficult and expensive to monitor the inputs to a process.

The second method of process monitoring monitors the outputs. In general, the process outputs are easier to measure. In fact, measuring can often be automated or process operators can perform the monitoring themselves. Relatively little cost is usually associated with monitoring the output. Most process monitoring involves go/no-go actions, visual go/no-go, or caliper measurements which are usually not time consuming. If output is monitored, then only limited measuring tasks are involved which should be simpler than monitoring multiple inputs to a system.

Furthermore, output monitoring is efficient for uncommon occurrences. For example, incoming raw materials are frequently sampled to guard against receiving a poor batch. Receiving inspection can often be replaced by proper process output monitoring. Output monitoring is sensitive to raw material failure. Even if value has been added to the product before it is discovered to be defective, the value of the lost product will probably be less than the cost of the continuous receiving inspection.

In addition, process monitoring is useful for detecting when and where problems occur in the process. For example, if a piece of equipment begins to malfunction or if an employee is doing something to affect the output quality, process monitoring of the output will indicate a problem.

The Politics of Variation

When variation is against business goals of an organization, the politics of variation become a factor. If quality is indeed a business priority, then the language of variation must become a common language of the organization.

Every process is variable. Process variation is universally categorized as one of two types. The first type is often called special or removable variation. Special cause variation is the variation in the process that is attributable to things not working or being done properly. Therefore, improper machine settings, power spikes, operator error, poor raw materials, machine malfunction, all are considered special causes of variations. Special causes of variation are considered acts of nature or uninformed actions. Special causes are often singular or cyclical occurrences and are not always present. Special causes generally cause large spikes in the quality of the process output.

The second type of variation is common cause variation. Common cause variation is a result of uncontrollable, natural deviations in the environment, machinery or personnel. Common cause variation is always present in a system and is not a singular or spike occurrence.

A key tenant of Dr. Deming's work is special causes do not occur regularly and when they do occur, they can usually be eliminated by supervisory level personnel [Moen, 12]. Yet, if the variation is caused by a common cause source of variation, the fault is inherent in the system itself. Only management has the ability to change the system. Therefore, it is directly the responsibility of management to improve the system and systematically eliminate the variation of a process—everyone “trying harder” will not affect the variation of the system. In his career, Deming estimated that 94% of all variation he saw was attributable to common cause variation, that is only 6% of the variation is attributable to special causes [Deming, 3]. There is a subtle undercurrent here that the workers who are usually blamed for poor quality really have no control over it and managers who have the control do not believe it is their

responsibility. Neither workers or management can distinguish between the types of variation if the process is not being monitored properly.

Attributes of a Process Monitor

The development of a process mentality, output monitoring, and the politics of variation leads one to question how a process monitor should function. The ideal process monitor indicates a special cause of variation as soon as the special cause influences the process. For example, if a batch of defective raw material was introduced into a manufacturing process, the ideal process monitor would detect the first unit affected by the defective raw material. In addition, the ideal process monitor would never make a mistake, that is, every time it indicated a problem there would indeed be a problem.

Yet, since a process contains variation, a monitor can be fooled. If the process produces output such that common cause variation looks like special cause variation, the monitor will be fooled. The fooling of a process monitor in this way is known as a *false alarm*. A false alarm occurs when the process monitor signals the influence of a special cause of variation when one is not present. In scientific terminology this is a false positive. The ideal process monitor would never suffer from a false alarm; it would have the power of perfect discernment.

A second problem faced in practice is where special cause variation affects the process and the monitor fails to detect the problem. This can be called a *late alarm* or in scientific terminology, false negative. An ideal process monitor would not have late alarms.

A third issue is that an ideal process monitor would be easy enough to use that all of the necessary personnel could use it with little or no training. In today's world, there are many capable process monitors available, yet many of these are so complicated that only professionals can gain relevant information from them which is not usually the optimum situation. Therefore, every process monitor must be evaluated on at least three characteristics: False alarm rate, sensitivity or late alarm rate, and ease of use.

Therefore, the goal of this research was to develop an accurate process monitoring tool which is more accurately stated:

Research Objective

To develop a generalized class of distribution free, variable control charts that are easy to construct and simple to interpret.

The present research develops a new generalized family of Quality Box Charts (QBC) from the singular case of White and Schroeder [White, 22]. Quality Box Charts are process monitoring tools that are easy to use and simple to interpret. QBC's also have better false alarm discriminate power than standard process monitors. In addition, QBC's are generally more sensitive with fewer late alarms than more traditional process monitors. Most important, QBC's are a general family that can be adapted to specialized needs of users. The present research greatly extends the ideas previously presented and uses Monte Carlo simulation to overcome problems encountered by previous researchers of Quality Box Charts.

Quality Box Charts are able to be used by all members of an organization in monitoring important aspects of the business process in an effort to gain an advantage with quality, productivity, cost, and timeliness. Quality Box Charts can be an important tool in today's business environment.

CHAPTER 2

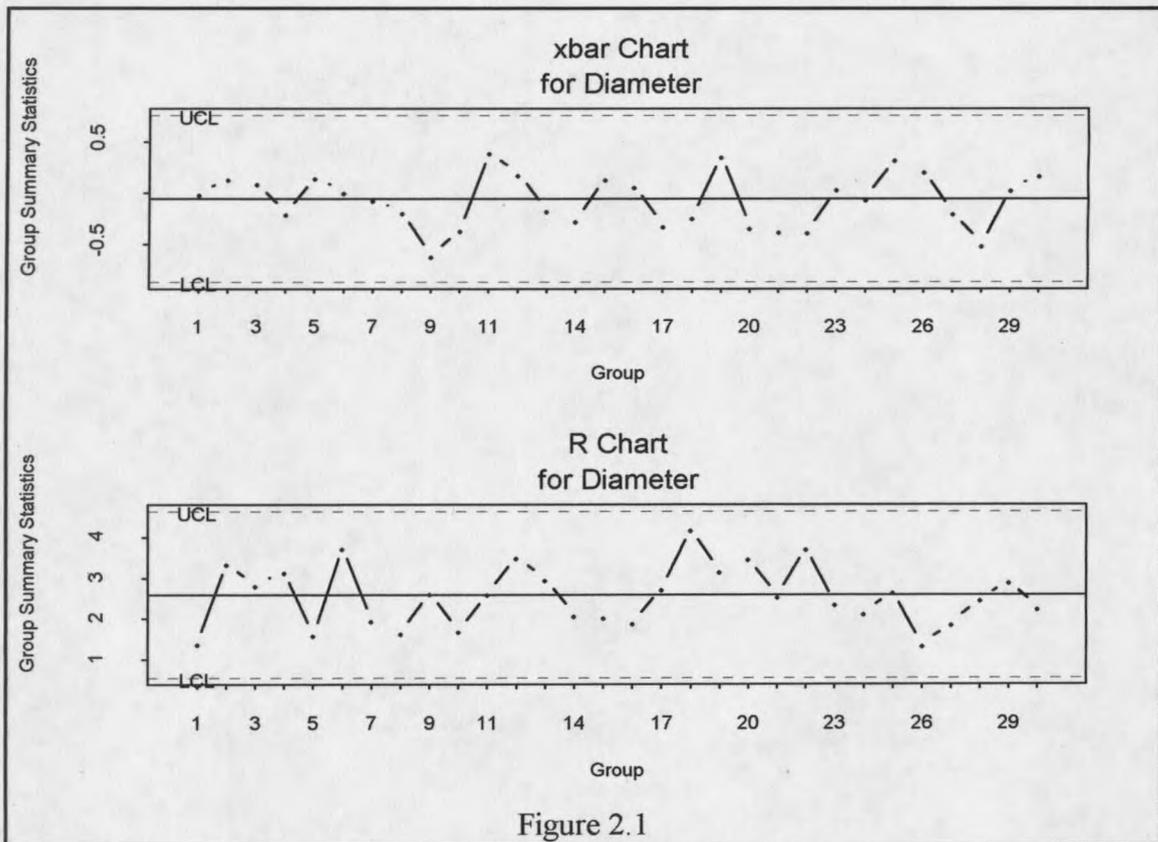
QUALITY BOX CHARTS

Quality Box Charts are process monitors which are simple to implement and use. Quality Box Charts (QBC) are output monitors based upon the most common type of process monitors known as control charts. QBC's are developed by considering the general form of all control charts followed by an introduction to the boxplot. The generalized QBC family is also developed in this chapter.

Control Charts

In the 1920's Walter Shewhart of Bell Laboratories developed a method of process monitoring known as control charting [Shewhart, 17]. Control charting allows operators to distinguish common cause variation from special cause variation by using statistics and probability models. Since it helps to identify when special causes of variation are present, control charting generally provides superior process monitoring compared to other methods. Traditional methods such as operator experience or intuition usually increase the process variation.

Many types of control charts exist, yet all of the charts have basic properties relating back to the first charts developed by Shewhart. Control charts are constructed by measuring a quantity of interest and plotting the value on a time plot. The natural variation the process



is expected to exhibit is indicated by upper and lower lines called control limits and the quantity of interest is expected to lie between these control limit values. If the measured quantity goes outside of the control limit values, a special cause of variation may be present.

The original control chart developed by Walter Shewhart is called an \bar{x}/R (\bar{x} -bar/range) chart. It consists of two plots used simultaneously—one for the sample mean and another for the sample range. The example shown in Figure 2.1 of an \bar{x}/R chart for needle diameters from a medical device manufacturer. Each point of the top plot is the average needle diameter from a random sample of needles while the range of each sample is plotted in the bottom chart. The random variation due to common cause variation is shown by the up and down bounce of the points. The control limits are also present to show the presence of special causes of variation.

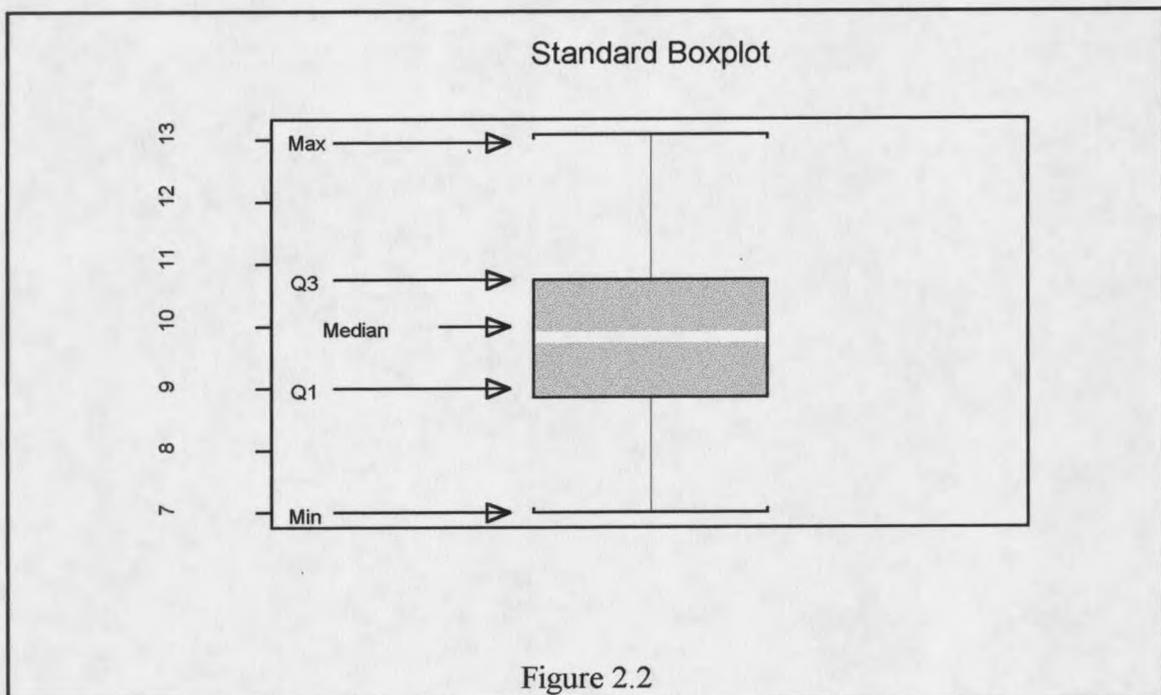
To construct an \bar{x}/R chart, it is necessary to calculate the mean and range of each sample and plot each one. More important, the control limits require knowing the statistical distribution of the sample average. The control limits are key to understanding the difference between common and special cause variation—their placement is important. Control limits cannot be estimated. How would one know where to set them? Experience may help but is not sufficient. Probability theory is used to set control limits.

QBC's are Shewhart control charts where a boxplot is used to characterize the sample output rather than the mean or range. QBC's require only one graph rather than two as with the \bar{x}/R chart because the sample center, range, and other information are conveyed simply in the boxplots. In addition, QBC's do not need to make assumptions about the process distribution or the distributions of the statistics. Because of this, QBC's use different methods to calculate the control limits (Chapter 3). Quality Box Charts are based on the box plot developed by [Tukey, 20].

The Boxplot

Boxplots are a standard method of examining data. Since their introduction [Tukey, 19], they have been incorporated into nearly all statistical software packages and most textbooks. Several modifications have been suggested, but few offer the ease of use given by the original design [McGill, 11; Benjamini, 1]. Boxplots are most informative when placed side-by-side in a single graph. Side-by-side orientation illustrates both within sample information as well as between sample information.

A boxplot is a graphical representation of a data set's minimum, maximum, and middle values along with its first and third quartiles. The quartiles are found by simply finding the middle value of both the upper and lower halves of the data set. The five numbers plotted by the boxplot are collectively called the statistical five number summary of a data set.



One advantage of using boxplots is their ease of interpretation. Because quartiles calculated by finding the middle of each sample half, they represent the quarter points of the ordered data ($1/2 * 1/2 = 1/4$). In addition, the median is the middle value of the data. The median can also be called Q_2 since it is the second quarter. The boxplot has the property that one-half of the data is represented by the box and 25% of the data is on each side of the box. A boxplot divides every sample into quarters. The distance that each quarter segment fills

indicates its density, that is, each segment always contains 25% of the data but may be spread out or tightly packed.

When used side-by-side, boxplots provide a large amount of information. Other than providing the five number summary values: minimum, maximum, median, and quartiles, boxplots present the range, the inter-quartile range and the mean, if desired. Boxplots also track skewness, kurtosis, trend, and within and between sample variation. Simple modifications to the boxplot allow confidence intervals and the tracking of any percentile of interest.

Even though boxplots experienced such widespread use, it took a long time for researchers to apply them to control charts. In standard control charting, all of the information in a sample is collapsed down to a single number such as the mean. Boxplots allow users to retain the original structure of the data as well as having statistically viable summaries.

Quality Box Charts

Quality Box Charts were first examined by White and Schroeder [White, 22]. White and Schroeder introduced the topic of QBC's by using the five number summary boxplot with normally distributed data. Their QBC was simply a Shewhart median chart using a boxplot median instead of a singular point. Other than using resistant measures, White and Schroeder did not show the QBC was robust to distributional assumptions. The first serious consideration of the QBC's was given by Iglewicz and Hoaglin [10]. They developed the traditional statistical theory justifying the box chart as a viable method of control charting; even though their presentation dealt with "process evaluation." Nagasawa and Ootaki [15]

consider box plot charts with independent upper and lower control limits based upon a normally distributed process. The present research builds on these researchers' work, yet it varies in its scope, monitoring approach, implications, and calculation methods. All previous research was based on the explicit assumption of a normally distributed process which is often infeasible [Shainin, 18].

Quality Box Chart Interpretation

The interpretation of QBC's is very simple. If the quantity of interest (e.g., the median) crosses the control limit, the process may be out of control, that is, special causes of variation may be present. A process is said to be in statistical control when only common cause sources of variation are present. More formally,

Quality Box Chart Run Rule: Let θ_x be the statistic of interest for which the Lower Control Limit (LCL) or Upper Control Limit (UCL) was calculated. The process is not in statistical control whenever

$$\theta_x > UCL \quad \text{or}$$

$$\theta_x < LCL$$

otherwise, the process is in statistical control.

Interpretation of the QBC is easy with this single rule. As long as the statistic(s) that is(are) being monitored stay(s) within the control limit, the process is in statistical control.

Quality Box Chart Construction

Two issues are involved in the construction of QBC's. The first issue is the construction of the boxplot itself. The boxplot can be constructed using either available

software, or it can be quickly created and sketched by hand. The sample values must only be placed in order and the appropriate five numbers (minimum, maximum, median, quartile₁, and quartile₃) chosen from the list. The five numbers are simply graphed as horizontal lines with the vertical lines added to the middle three vertical lines to construct a box. It is customary to connect the upper and lower points by a horizontal line.

Example: Consider the following process sample: 67, 56, 42, 69, 56, 66, 61, 64, 64

Step 1: Order the sample 42, 56, 56, 61, 64, 64, 66, 67, 69

Step 2: Find min and max values 42, 69

Step 3: Find the median 64

Step 4: Find the quartiles 56, 66

Now the boxplot can be drawn from these values

The second issue involved in QBC construction is the calculation of the probability control limits. The calculation method is discussed in Chapter 3. Once the control limits are calculated, they are added to the side-by-side boxplot graph to make a Shewhart style Quality Box Chart.

The Generalized Family of Quality Box Charts

The present form of the Quality Box Chart is a definite extension of the charts discussed by previous researchers. Two distinct differences exist between the previous methods and the present QBC method. First, the present method does not make distributional assumptions concerning the underlying process (discussed in Chapter 3). Secondly, the present method develops a generalized family of control charts, which are not limited by only considering the

