



An investigation of load entry into the edge beam of a concrete hyperbolic paraboloid
by Allen G Thurman

A THESIS Submitted to the graduate faculty in partial fulfillment of the requirements for the degree of
Master of Science in Civil Engineering
Montana State University
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Abstract:

This thesis is a report of load tests made on a hyperbolic paraboloid model. The model, made of concrete, was 10 ft 0 in. x 10 ft 0 in. x 0 ft 2 in. thick with an edge beam rise of 4 ft 0 in. The encompassing edge beams, 3 in. x 4 in. deep, extended into rigid buttresses, 1 ft 10 in. from the low corners of the shell. Uniform, unsymmetrical and concentrated incremental loads were applied to the shell with sand bags. Throughout the model, strains were measured by SR=4 strain gages and a Baldwin-Lima-Hamilton strain indicator at each increment of load. Hook-up and test procedures are thoroughly described. Tests were made to determine the modulus of elasticity of the model. After E was determined, mean strains were calculated and converted to stress by an IBM 650 computer. Stresses in the edge beams and shell have been plotted in several manners for each loading condition.

Theoretical action for uniform load was derived by employing several simplifications, particularly in regard to boundary conditions. This analysis declares that the applied load is distributed equally to tension and compression parabolas where it is transferred into edge beams without causing bending stresses; further, that the load in the edge beams is axial and continues into the buttresses with uniform compressive stress. Conflicting with this theory, the model studied; had large bending stresses in the edge beam, in the shell, and between the shell and the edge beam. Although compression and tension parabolas were created, their stresses were not equal. Primary reasons for the discrepancies between theory and tests appeared to be the existence of a strong rigid arch between buttresses, distortions of the shell and edge beams, and eccentric entry of diaphragm shear into the edge beams.

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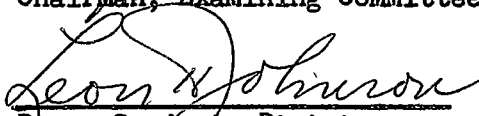
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Montana State College

Approved:


Head, Major Department


Chairman, Examining Committee


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Bozeman, Montana
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ALLEN G. THURMAN

ABSTRACT

This thesis is a report of load tests made on a hyperbolic paraboloid model. The model, made of concrete, was 10 ft 0 in. x 10 ft 0 in. x 0 ft 2 in. thick with an edge beam rise of 4 ft 0 in. The encompassing edge beams, 3 in. x 4 in. deep, extended into rigid buttresses, 1 ft 10 in. from the low corners of the shell. Uniform, unsymmetrical and concentrated incremental loads were applied to the shell with sand bags. Throughout the model, strains were measured by SR-4 strain gages and a Baldwin-Lima-Hamilton strain indicator at each increment of load. Hook-up and test procedures are thoroughly described. Tests were made to determine the modulus of elasticity of the model. After E was determined, mean strains were calculated and converted to stress by an IBM 650 computer. Stresses in the edge beams and shell have been plotted in several manners for each loading condition.

Theoretical action for uniform load was derived by employing several simplifications, particularly in regard to boundary conditions. This analysis declares that the applied load is distributed equally to tension and compression parabolas where it is transferred into edge beams without causing bending stresses; further, that the load in the edge beams is axial and continues into the buttresses with uniform compressive stress. Conflicting with this theory, the model studied had large bending stresses in the edge beam, in the shell, and between the shell and the edge beam. Although compression and tension parabolas were created, their stresses were not equal. Primary reasons for the discrepancies between theory and tests appeared to be the existence of a strong rigid arch between buttresses, distortions of the shell and edge beams, and eccentric entry of diaphragm shear into the edge beams.

INTRODUCTION

Problem Defined

This thesis is an attempt to evaluate the entry of load into the edge beam of a hyperbolic paraboloid shell by measuring accumulative shears and bending moments in the edge beams and the diaphragm.

An endless hyperbolic paraboloid structure of infinitesimal thickness would have pure compressive stress in the convex paraboloid and pure tensile stress in the concave paraboloid,¹ but a real structure must be given a location of final support. This usually is done by encompassing the shell surface with edge beams, an enlargement at the periphery of the structure, and extending these beams into two buttresses located at the two lowest points of the curved surface.² The forces caused by the dead and live loads are transferred from the diaphragm into the beams and then into these buttresses.

A real structure must have a specific thickness, the amount depending upon the material used and its strength. For ease of placing concrete and for fire protection of the reinforcing steel, three inches is often considered a minimum thickness for concrete shell construction. Such a thickness is capable of a sizable bending moment and one will be created

¹See General Analysis, p. 18.

²See Fig. 4, p. 19.

at any shell edge.¹ An edge beam, which is quite stiff compared to a free shell edge, will greatly increase this bending moment. Therefore, only interior portions of large hyperbolic paraboloids approach the condition of pure compressive and tensile stress, that is, freedom from bending stresses.

Concrete Hyperbolic Paraboloid Use

The hyperbolic paraboloid shape is beautiful in itself, uttering simplicity and rejecting the ornamentation found on many buildings. Its natural strength, created by the double curvature, allows it to economically span large areas entirely uninterrupted by interior supports -- an awesome spectacle to the engineer as well as the layman. With support required at only two points, the walls become mere barriers of the elements, allowing windows of any size to be placed at any point including the corners under the high peaks.

A building which fully utilizes these advantages is a Roman Catholic Church recently completed in Spokane, Washington.² The simplicity of its smooth, curving lines, its humbling spaciousness, its strong, powerfully uplifting altar give it a functional and esthetic value seldom achieved in a building. To see it after looking at other buildings is like seeing a figure by Michelangelo after walking through a park viewing

¹H. H. Bleich and M. G. Salvadori, "Bending Moments on Shell Boundaries," Journal of Structural Division, Proceedings of the American Society of Civil Engineers, LXXXVIII, No. ST8, (October 1959), pp. 91-100.

²St. Charles Catholic Church, Spokane, Washington; Funk, Murray & Johnson, Architect; T. Y. Lin, Consulting Engineer; Johnson, Busboom & Rauh, General Contractor.

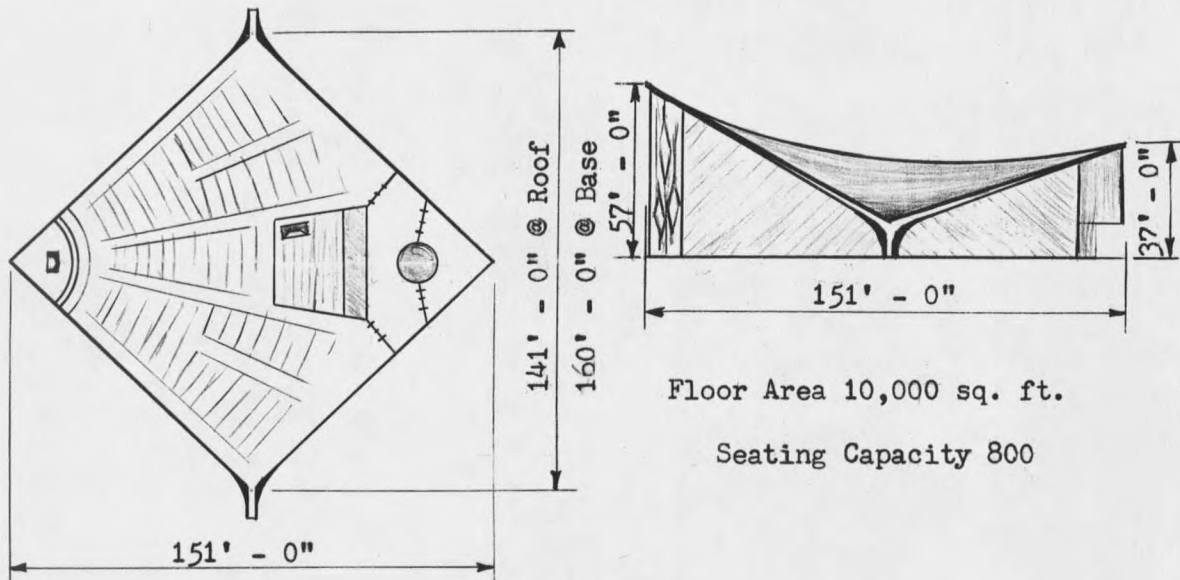
the cold, dead statues of forgotten generals. Similar opinions are held by many non-Catholics whose judgments have a minimum of biased distortion.

The building's structure is a hyperbolic paraboloid, diamond shaped in plan (Fig. 1, p. 11). The buttresses, at the two corners closest together, carry the gentle curve of the shell into the ground with undoubted strength. By rotating the entire shape about the buttresses, one corner has been raised, greatly emphasizing its majestic cantilever. Edge beams exist, but their usual heavy appearance has been entirely eliminated by putting a reversed curve on the outside face leaving a distinct edge only two or three inches thick, and by graduating the total thickness from a minimum at the peak to a maximum at the buttresses. Smooth, white, architectural concrete complements the building shape.

The entrance curves inward under the lower cantilevered peak. The vertical freedom beneath this peak is emphasized by the simplicity of the entry hall and ante-rooms which fill this portion of the plan. Thus, the lower peak is well utilized without detracting from the grandeur of the main peak.

Lines which are formed by the sanctuary floor sloping downward toward a curved altar rail and the parabolic roof thrusting to a peak high above, create a natural altar having an atmosphere of aspiration and reverent mystery. Intensifying this impression is a wide, modern, stained glass corner window extending from the floor to the very roof peak and providing a major portion of the light for the building.

The organ and choir are located on a balcony centered under the roof area and open to all sides including the entry hall. Thus, with



Schematic Plan and Elevation - No Scale

ST. CHARLES CATHOLIC CHURCH

FIGURE 1

minor exceptions in the rear of the church, the entire area enclosed by the exterior walls is open to the roof, transmitting sound and light with unusual freedom.

This use of the hyperbolic paraboloid, employing the shape as a complete structure in itself, is perhaps the classic use of this shape as it combines its natural strength with its inherent beauty. Two or more hyperbolic paraboloids often are connected to form one large roof area. While these arrangements maintain their strength allowing large areas to be economically covered with few or no interior obstructions, more than four hyperbolic paraboloids joined together lose much intrinsic beauty as the identity of the shape is lost. The hyperbolic paraboloid has also

been used for foundation work where the shape, which is buried in the earth, becomes economically valuable only for its natural strength.¹

Model Studies of Hyperbolic Paraboloids

So that the functional beauty of the hyperbolic paraboloid may be properly utilized, it is important to attain a complete understanding of the structural properties of this shape. Being very complicated, exact mathematical analyses must be substantiated by thorough test model studies. A review of literature on this subject at the Montana State College Library revealed a conspicuous lack of previous formal testing of concrete hyperbolic paraboloids. American Doctoral dissertation abstracts from 1952 through 1960 show that there has been no dissertation devoted to this subject,² although one discusses stress theory and a few mention hyperbolic paraboloids along with discussion of other shells. The past two and one-half years of government translations of foreign material³ include no treatment of hyperbolic paraboloids. Dr. T. Y. Lin, while speaking before the

¹Arthur R. Anderson, "Precast Prestressed Stadium Floats on Hyperbolic Paraboloids," Engineering News-Record, CLXIV, No. 7 (February 18, 1960), 62.

²Dissertation Abstracts, (Ann Arbor, Michigan: University Microfilms, Inc., Jan. 1956-Oct. 1960), Vol. 16-22. Doctoral Dissertations Accepted by American Universities, The Association of Research Libraries, New York: H. W. Wilson Co., 1952-1955), No. 20-22. The idea of using hyperbolic paraboloids for structures dates back to at least 1930, but almost all use of the shape has been since World War II, originating primarily in Latin America. Therefore, these volumes appear to include all the work done in the U.S.A.

³Technical Translations, U.S. Dept. of Commerce, Office of Technical Services, (Washington: Government Printing Office, Jan. 2, 1959-March 28, 1961), Vol. 1, No. 1-Vol. 5, No. 6.

Architect-Engineers' Convention in Great Falls, Montana, January 1958, mentioned that tests utilizing prestressing of concrete hyperbolic paraboloids were being conducted at the University of California. No report of this work was found. Although the amount of library material is limited, this survey confirms the lack of research on concrete hyperbolic paraboloids.

The Portland Cement Association has completed some experiments and has made public some important aspects of these tests.¹ Some discussions of theory have recently appeared in the Proceedings of the American Society of Civil Engineers and other well circulated engineering publications. A British journal, Concrete and Construction Engineering,² notes several uses of hyperbolic paraboloids and also some tests of scale models of structures which were built following these tests. Unfortunately almost no design data is given nor are sources of further information listed.

Test reports of wooden hyperbolic paraboloids are of some value, however such structures do not have the inherent stiffness of component parts, such as slab to edge beams, found in concrete structures. In addition, there is an enormous difference in the ability of the two materials to yield in bending without fracture, thus redistributing the loads. Therefore, extreme caution must be used when considering concrete design based on results of tests of wooden structures.

¹Elementary Analysis of Hyperbolic Paraboloid Shells, Reinforced Concrete Bulletin 35, (Chicago, Illinois: Portland Cement Assoc., 1960).

²Concrete and Construction Engineering, London: Concrete Publications Ltd., Monthly.

Hyperbolic paraboloid model study at Montana State College was initiated during 1959-60. Dennis Nottingham, the Ideal Cement Company Fellow for that school year, constructed the model and started testing it. His test report¹ gave indications of definite disagreement with some theoretical ideas.

The tests described in this report are in many ways a duplication as well as an extension of the work done by Nottingham. The duplication was warranted because:

1. More strain gages were required to better describe the action of the model under load.
2. Total strains were small, indicating greater accuracy could be obtained safely by applying larger loads.
3. Some strains were erratic, indicating a more accurate strain measurement was desirable.
4. Because stress (pounds per square inch) is a unit more easily compared with theoretical analyses than strain (inches per inch), determination of the modulus of elasticity of the model and conversion from strain to stress made the tests more valuable.

Thus, by extending and increasing the precision of Nottingham's model studies, the results became more convincing by their greater reliability and utility.

¹Dennis Nottingham, "Experimental Testing of a Saddle Type Hyperbolic Paraboloid Using Three Different Load Conditions" (Unpublished M.S. thesis, Dept. of Civil Engineering, Montana State College, 1960).

REVIEW OF THEORY

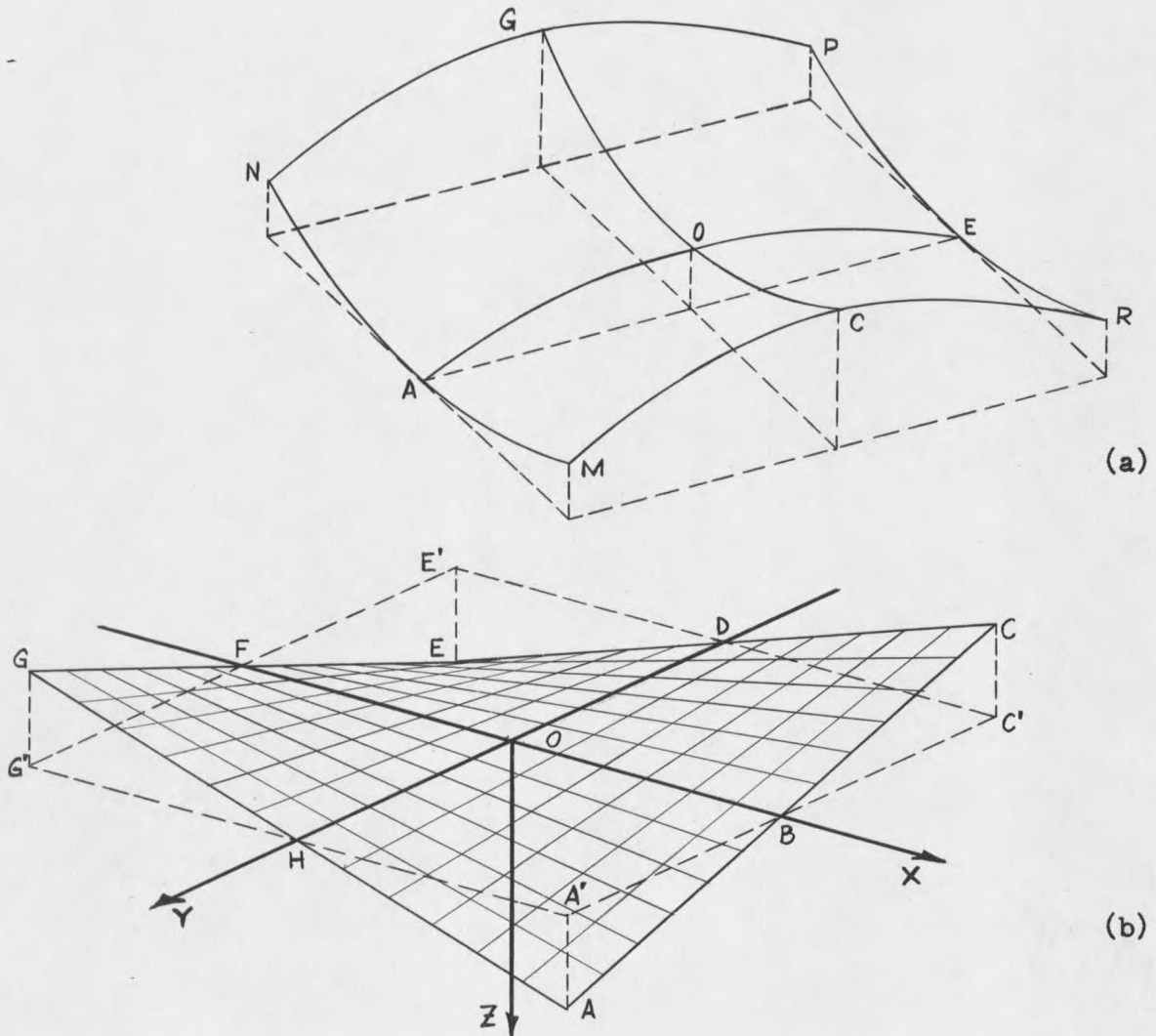
A complete structural analysis, discussing the effects of various boundary and loading conditions is not within the scope of this report. Only a limited, elementary discussion using a uniformly applied load will be presented. However, for those interested in more thorough treatment of theoretical analyses, a list of suggested references is included at the end of this section.

General Analysis

This material is from the Portland Cement Association bulletin, Elementary Analysis of Hyperbolic Paraboloid Shells.¹ As presented here and in the bulletin, the analysis employs several simplifications that are not entirely correct. Some of these points will be discussed briefly following this analysis.

The hyperbolic paraboloid is a doubly curved surface which may be generated in either of two ways. One method which may be described by using Fig. 2a, p. 16, is: A saddle-shaped surface called a hyperbolic paraboloid will be generated if a convex parabola AOE is draped over a fixed concave parabola GOC and moved keeping the parabolas at right angles and keeping the vertex O of parabola AOE on the parabola GOC. The other method may best be described by using Fig. 2b, p. 16: A hyperbolic paraboloid is formed by

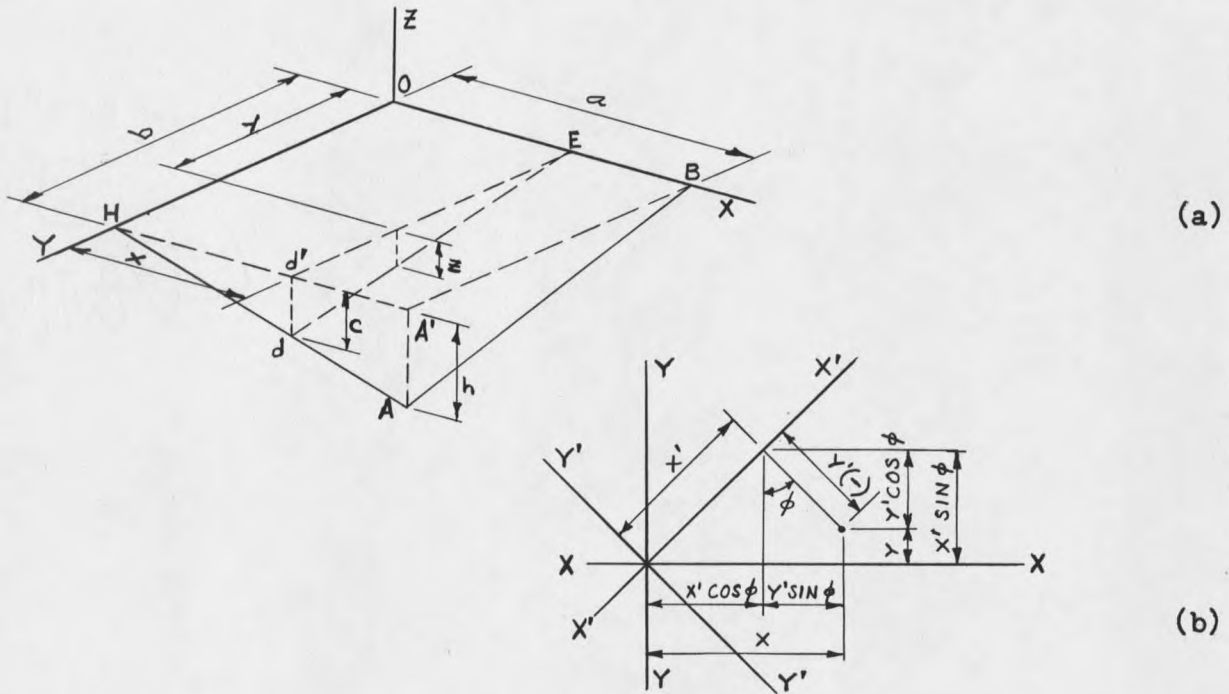
¹Elementary Analysis of Hyperbolic Paraboloid Shells, Reinforced Concrete Bulletin 35, (Chicago, Illinois: Portland Cement Association, 1960), pp. 1-8.



SURFACE DEFINITIONS

FIGURE 2

moving a line GHA , which is parallel to the XZ plane, over a line ABC , which passes through the X axis and is parallel to the YZ plane, but non-parallel to the Y axis. Similarly, the same surface is generated by moving line ABC over line AHG and the X axis. Points O , A , G , E and C are common to both portions of Figure 2.



GEOMETRY

FIGURE 3

Using the expanded quadrant ABOH of Figure 2 in Figure 3a, the surface may be defined in terms of x , y and z . By similar triangles:

$$\begin{aligned} \frac{c}{h} &= \frac{x}{a} & c &= \frac{xh}{a} \\ \frac{z}{c} &= \frac{y}{b} & z &= \frac{yc}{b} = \frac{y}{b} \frac{xh}{a} = xy \frac{h}{ab} \end{aligned}$$

$$\left(\text{using } k = \frac{h}{ab}\right) \quad z = kxy \quad (1)$$

To simplify the analysis, the axes are rotated 45 degrees in Figure 3b. This gives

$$x = x' \cos \phi - y' \sin \phi = 0.707 (x' - y')$$

$$y = y' \cos \phi + x' \sin \phi = 0.707 (x' + y')$$

Substituting into equation (1),

$$z = 0.5k (x' + y') (x' - y') = 0.5k [(x')^2 - (y')^2] \quad (2)$$

When x' is constant,

$$z - 0.5k (x')^2 = z - k_1 = z' = -0.5k (y')^2 \quad (3)$$

When y' is constant,

$$z + 0.5k (y')^2 = z + k_2 = z'' = 0.5k (x')^2 \quad (4)$$

Equations (3) and (4) are general expressions for parabolas lying in or parallel to the $Y'Z$ plane and $X'Z$ plane respectively. The vertex of these parabolas will lie in the $X'Z$ plane and $Y'Z$ plane respectively. Thus, the parabola in the $Y'Z$ plane and the one in the $X'Z$ plane have the origin of the axes as a common vertex. Hereafter these two parabolas will be referred to as the main parabolas. The negative sign of equation (3) indicates concave parabolas, whereas the positive sign of equation (4) indicates convex parabolas.

If z is held constant in equation (2),

$$1 = k_3 [(x')^2 - (y')^2] \quad (5)$$

Thus, any horizontal plane cutting the warped surface forms two hyperbolas. For this reason the surface has been named hyperbolic paraboloid.

The bending moment equals zero throughout a two-hinged parabolic arch supporting only a uniform load. The moment in any two-hinged arch is equal to the simple beam bending moment minus the moment due to the horizontal reaction H . If it is assumed that the convex and concave parabolic arches forming the surface of a hyperbolic paraboloid each carry $1/2$ an

applied total uniform load w , the thin parabolic arch, parallel to the $Y'Z$ plane, shown in Figure 4, can be analyzed:

$$\frac{w}{2} \times \frac{L^2}{8} + Hh_{xy} = 0$$

or $H = - \frac{w}{4} \frac{L^2}{4h_{xy}}$ (6)

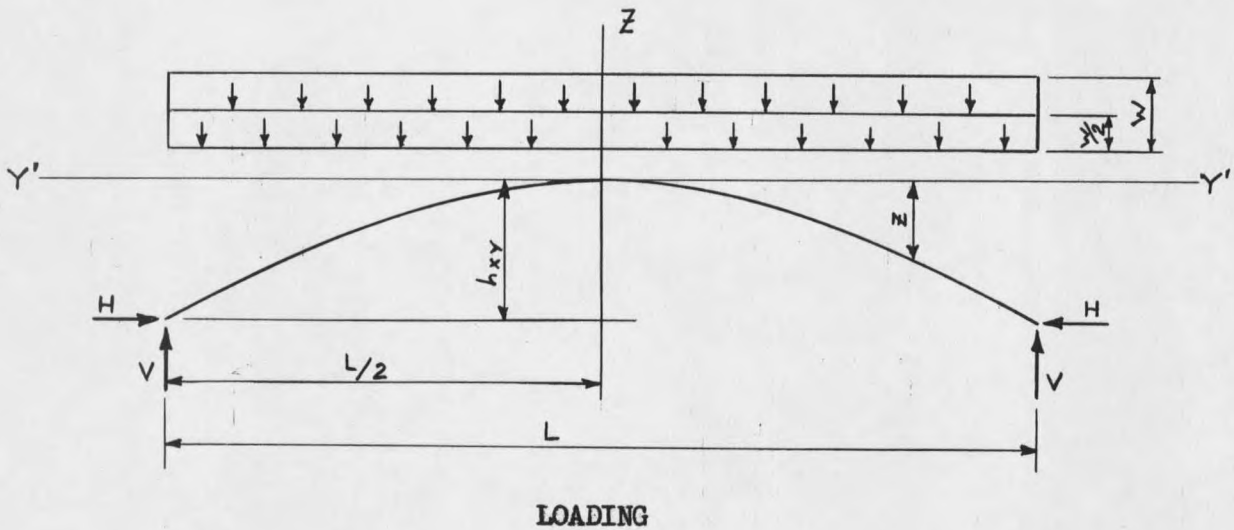


FIGURE 4

Using $h_{xy} = z'$ and $\frac{L}{2} = y'$ equation (3) can be written:

$$h_{xy} = -0.5k\left(\frac{L^2}{4}\right)$$

or $\frac{L^2}{4h_{xy}} = - \frac{1}{0.5k}$

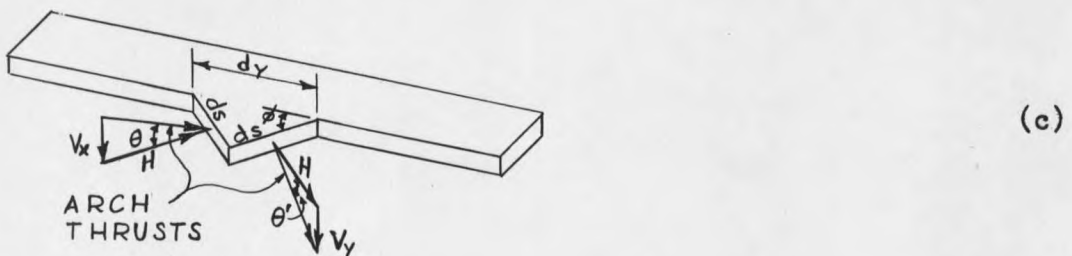
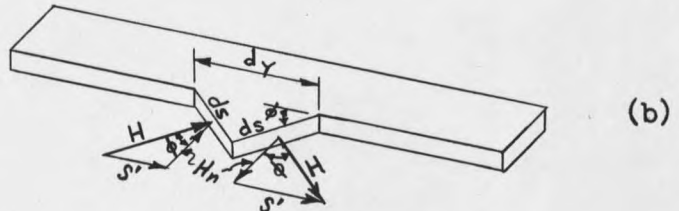
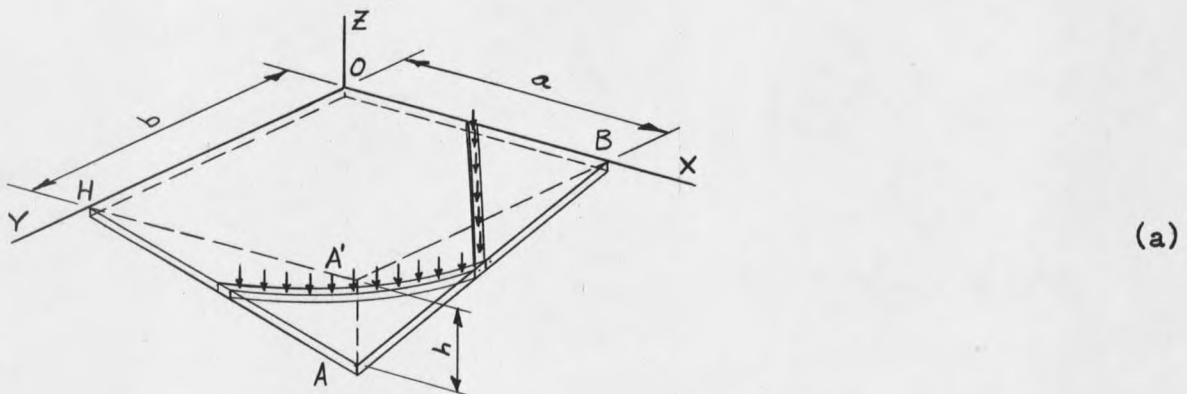
Substituting this in equation (6) gives

$$H = - \frac{w}{4} \left[- \frac{1}{0.5k} \right] = \frac{w}{2k} = \frac{wab}{2h} \quad (7)$$

Equation (7) gives the compressive thrust in the convex parabola of the shell. Similar analysis shows that the tensile action in the concave parabola has the same magnitude. Since a , b and h are constants found by

the total size of the shell, H is proven to be equal for all component arches of the hyperbolic paraboloid.

Typical parabolic arches of width ds are shown in Figure 5a. The two perpendicular arches intersect side AB at 45 degree angles. Referring



FORCES ON EDGE BEAMS

FIGURE 5

to Figure 5b, because H has the same magnitude for both parabolas, the two H_n forces will cancel each other, resulting in no force normal to the edge. If H is in the form of force per unit of area, the longitudinal components of each parabola per unit of thickness are:

$$S' = H \sin \phi \, ds$$

Because the forces from the two parabolas are additive

$$Sdy = 2S' = 2H \sin \phi \, ds$$

$$\text{or } S = 2H \sin \phi \frac{ds}{dy} = 2H \sin \phi \cos \phi$$

From equation (7), $H = \frac{wab}{2h}$ and using $\phi = 45$ degrees the intensity of force per unit horizontal length of edge beam becomes:

$$S = 2 \left[H \right] \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = H \quad (8a)$$

$$\text{or } S = \frac{wab}{2h} \quad (8b)$$

Referring to Figure 5c, where θ and θ' are the angles in a vertical plane between the arch thrust lines and a horizontal plane:

$$V = V_x + V_y = H \tan \theta + H \tan \theta'$$

$$\text{or } V = H \frac{dz}{dy'} + H \frac{dz}{dx'} \quad (9)$$

From equation (2)

$$\frac{dz}{dy'} = (-0.5k)(2y') = -ky' \quad (10a)$$

$$\text{and } \frac{dz}{dx'} = (0.5k)(2x') = +kx' \quad (10b)$$

Along the sloping edge AB, x' and y' are not equal. The equation of line AB from the general equation $y = mx + b$ (remembering the $X'Y'$ axes are rotated 45 degrees from the XY axes) is

$$y' = x' - a\sqrt{2} \quad (11)$$

Substituting this value in equations (10)

$$\frac{dz}{dy'} = -k (x' - a\sqrt{2}) \quad (12a)$$

$$\text{and } \frac{dz}{dy'} = kx' \quad (12b)$$

Applying equation (9), the net vertical component of arch thrust at the edge is

$$V = H [-k(x' - a\sqrt{2}) + H(kx')] = Hka\sqrt{2} \quad (13)$$

but $k = \frac{h}{ab}$ so equation (13) can be written

$$V = H \frac{h}{ab} a\sqrt{2} = \frac{Hh\sqrt{2}}{b} \quad (14)$$

Remembering the force V is the net force of the two unit lengths ds , it is readily seen that V' , the intensity of force per unit horizontal length of edge beam, is:

$$V' = \frac{V}{\sqrt{2}}$$

Therefore, from equation (14) and $H = \frac{wab}{2h}$

$$V' = \frac{Hh\sqrt{2}}{b} \left(\frac{1}{\sqrt{2}}\right) = \frac{Hh}{b} \quad (15a)$$

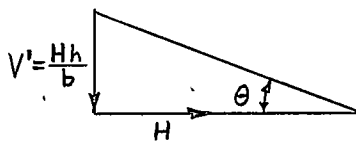
$$\text{or } V' = \frac{wb}{2} \quad (15b)$$

In a similar manner, the vertical force exerted by the shell along the edge HA is

$$V' = \frac{Hh}{a} \quad (16a)$$

$$\text{or } V' = \frac{wb}{2} \quad (16b)$$

Figure 6 graphically pictures equations (8a) and (15a) which represent horizontal and vertical forces per unit horizontal length of edge beam.



EDGE BEAM FORCE COMPONENTS

FIGURE 6

From Figure 6

$$\tan \theta = \frac{Hh}{b} \times \frac{1}{H} = \frac{h}{b} \quad (17)$$

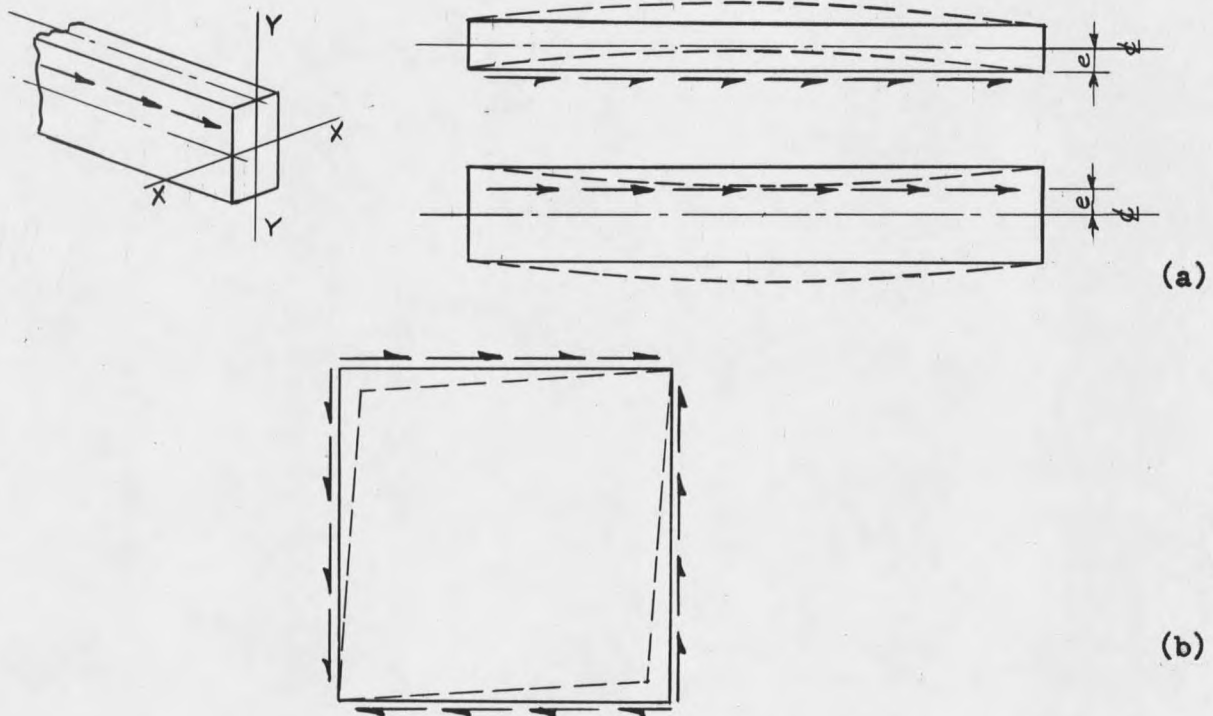
This is the slope of the edge beam and thus the forces combine to produce a shearing force parallel to the edge beam on the inside face of the edge beam at the center of the shell.

Discussion of Analysis

The elementary analysis just concluded contained several oversimplifications which will be discussed briefly with no attempt to derive more inclusive or exact equations.

The first major simplification was that the increment width parabolas were two-hinged arches. Edge beams, no matter how small, provide some degree of resistance to shell rotation. Several things which influence the degree of this resistance include: (1) the relative size of the edge beams compared to the shell, (2) the length of the beams, and (3) the extent to which the edge beams are fixed against rotation at their ends.

A second simplification was that deflections distorting the theoretical geometry did not exist. The falsity of this assumption will be shown with the use of Fig. 7, p. 24. Part (a) shows the effect of shear being applied to a rectangular edge beam. The shell, being attached to the face of the beam, must apply a shearing force eccentric in the x direction a distance equal to 1/2 the width of the beam. Eccentricity can be eliminated in the vertical direction, but this often is not done due to the desirability of having the shell flush to one surface of the beam. For example, if a shell 1/2 the beam depth in thickness is flush with the top of



SHAPE DISTORTIONS

FIGURE 7

the beam, there will be a shearing force eccentric in the y direction a distance of $1/4$ the depth of the beam. Part (b) shows how the accumulation of edge shear warps the shell surface by shortening the edges.

The assumption that the load is perfectly uniform leads to some error in almost every case. The dead load, being a function of real surface area rather than a horizontal projection, increases as the surface steepens. Even if the dead load is small compared to the total load, its effect may be important, for if it warped the surface even slightly, subsequent application of uniform loads would not lead to action free of bending moments. Of course, application of any other type of load would rapid-

ly lead to large deviations from the theory presented. The effects of forces normal to the edge beams become very important in these cases.

Indication of the importance of considerations beyond those presented in the General Analysis is found in more technical presentations. For instance, "...the dampening of edge disturbances is slower in hyperbolic paraboloidal shells than in cylindrical shells and elliptic paraboloidal shells, which in general indicates that edge disturbances produce relatively more significant bending effects in the hyperbolic paraboloidal shells."¹ Citing one example, "...the bending stresses σ_B which occur in the edge zone are of the same order of magnitude as the membrane shear stresses."²

It is important to note that while the more theoretical analysis requires mathematical abilities beyond most engineers, simplifications in the forms of graphs and tables are beginning to appear. Apeland's article contains several of these tables. Further, he says, "A general program for such table computation by means of an automatic digital computer is in process. An assembly of such tables for a suitable choice of parameter values will be the subject of a separate publication."³

¹Kristoffer Apeland, "Stress Analysis of Transitional Shells," Journal of Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, LXXXVII, No. EM1, (February 1961), p. 137.

²Bleich and Salvadori, op. cit., p. 100.

³Apeland, op. cit., p. 127.

For those wishing to delve further in the theory of hyperbolic paraboloid analysis, a list of suggested references follows:

- Apeland, Kristoffer. "Stress Analysis of Transitional Shells," Journal of Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, LXXXVII, No. EM1, February 1961.
- Bleich, H. H. and Salvadori, M. G. "Bending Moments on Shell Boundaries," Journal of Structural Division, Proceedings of the American Society of Civil Engineers, LXXXVIII, No. ST8, October 1959.
- Sobotka, Zdeněk. Discussion of "Bending Moments on Shell Boundaries," by Bleich and Salvadori. Journal of Structural Div., Proceedings of A.S.C.E., LXXXVI, No. ST5, May 1960.
- Candela, Felix. "Structural Applications of Hyperbolic Paraboloidal Shells," Journal of the American Concrete Institute, XXVI, No. 51-20, January 1955.
- Levy, M. P. "Thin Shells; Some Basic References for Architects and Engineers," Architectural Record, 127:216-21; 246-50, April 1960.
- Candela, Felix. "General Formula for Membrane Stresses in Hyperbolic Paraboloidal Shells," American Concrete Institute, 32:353-71 (Bibliog. 370-71), October 1960.
- Harrenstein, H. P. "Hyperbolic Paraboloidal Umbrella Shells Under Vertical Loads," Amer. Concr. Inst. J., 32:385-401, October 1960.

TEST SET-UP

Previous Work on Test Model

The south end of Ryon Laboratory was chosen for the location of the concrete hyperbolic paraboloid model. Placing the model inside limited its size but freed it from weather conditions allowing greater accuracy and freedom of time for testing. Nottingham constructed the model, located some gages and conducted preliminary tests during the school year 1959-60.

The model was 10 ft 0 in. square in plan with a 4 ft 0 in. rise. The diaphragm slab was 2 in. thick and was flush with the top surface of edge beams 3 in. wide by 4 in. deep. Buttresses were made by dropping the edge beams vertically at a point 1 ft 10 in. from the corner. The footings of the buttresses were 3 ft 6 in. x 5 ft 0 in. x 0 ft 5-1/2 in. thick.¹

Wood forming was used following standard construction procedures. Six by 6, 10/10 gage, welded wire mesh was placed in the slab. A number 3 reinforcing rod was centered in each edge beam 2 in. from the bottom. The footings were reinforced with number 5 rods 6 in. on center each way. A 7/8 in. tie rod with a turnbuckle in the center was extended between the

¹Nottingham, op. cit., pp. 16-18: Also see Fig. 8, p. 30, Fig. 9, p. 31, and Fig. 10, p. 32.

buttresses. This later proved unnecessary because the footing to floor slab friction prevented spreading of the buttresses.¹

After the mesh and other reinforcing steel were in position, ready mix concrete having a slump varying from 1 in. to 2 in. was poured, vibrated, troweled, and broomed. A wet canvas cover kept the structure damp until the fifth day when the canvas and forms were removed.² Test cylinders were made of the same concrete. Some of these were cured in the moist room and some beside the model.

Fifty-eight type A-1 and 5 type AR-1 Baldwin SR-4 strain gages were applied to the concrete after preparation of the surface at each gage location. The methods used were similar to those described later. Type A-1 and AR-1 compensating gages were attached to two of the test cylinders. Wiring from all the gages to switching-balancing units and from the units to a Baldwin SR-4 strain indicator completed the set-up.³

After the model was constructed and strain gages were attached, Nottingham conducted tests similar to those described in this report. His tests revealed where additional strain gages could be placed and how testing methods could be modified to increase the reliability and value of test results.

¹ Nottingham, op. cit., p. 18.

² Ibid., pp. 19-20.

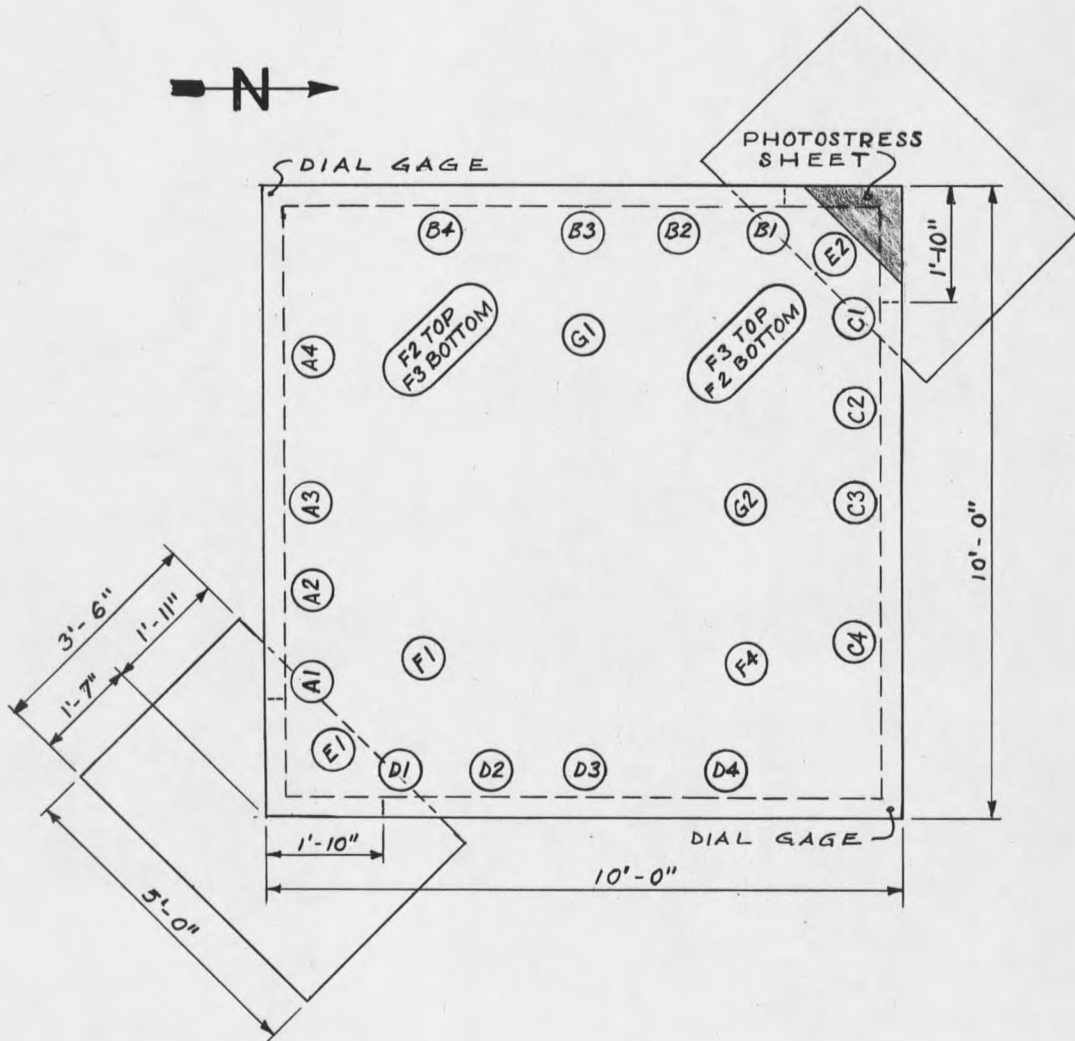
³ Ibid., pp. 20-26.

Preliminary Investigation

After Nottingham completed his experiments, the instruments were disconnected and stored. To become familiar with the previous test procedures, the author rewired the testing apparatus and ran a uniform load test following the methods used by Nottingham. The results of this test, combined with Nottingham's report, were useful in determining the way to conduct succeeding tests. Figures 8, 9 and 10, on the following three pages, present the gage notation that is used throughout this report and the location of each gage. Not all of the subscripted locations have gages. Table I, p. 33, is a complete listing of the gages. Gage notation of previous work is noted in parentheses for the purpose of cross reference.¹ Also, this clearly distinguishes the newly applied gages from the original gages.

Professor Robert C. Smith observed a reflective photostress sheet during the preliminary test. This sheet had been placed on the northwest corner of the structure as shown by the shaded portion of Fig. 8, p. 30. Low stress, due to the size ratios of the structure and the location of the photostress sheet, invalidated this method of stress measurement. Therefore further testing by photoelastic methods was abandoned.

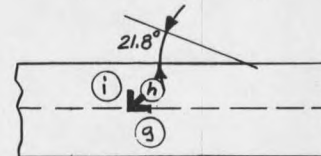
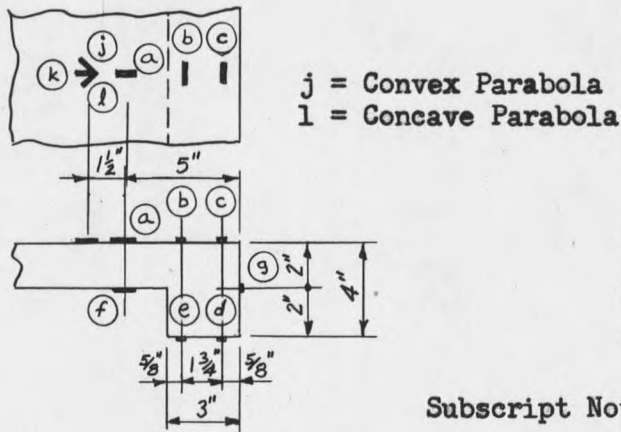
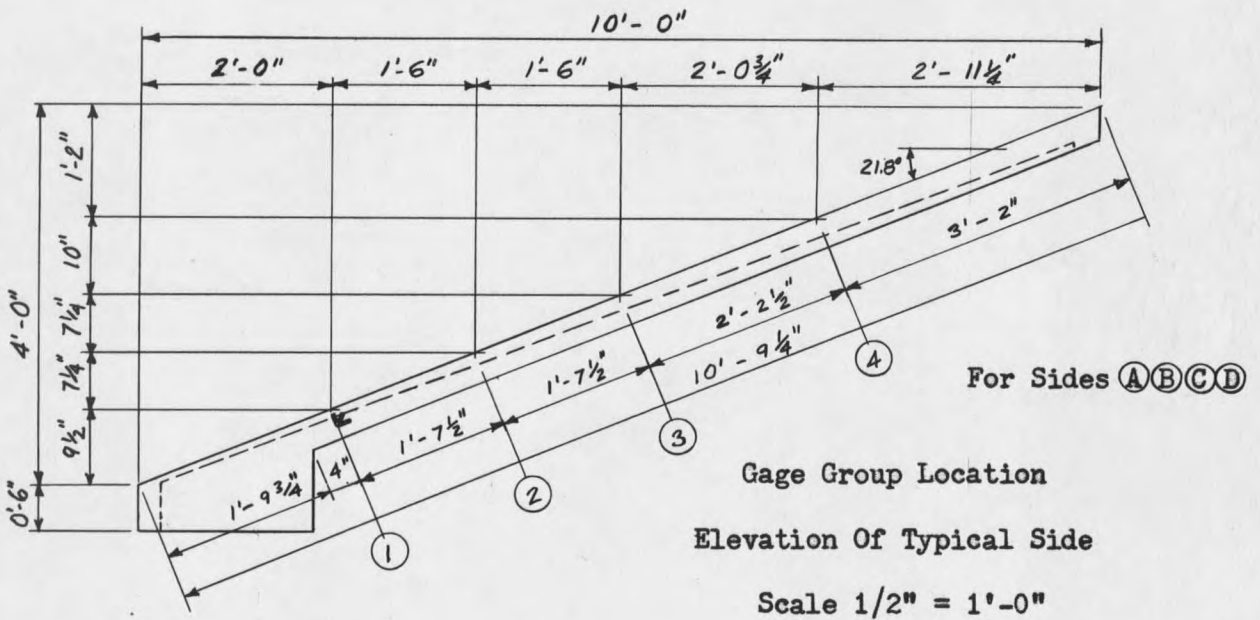
¹Nottingham, op. cit., pp. 22, 25, 26.



GAGE NUMBERING PLAN

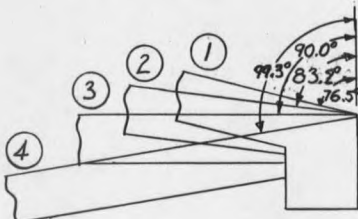
With Basic Plan Dimensions - Scale 1" = 3'-0"

FIGURE 8



Subscript Notation For Sides (A)(B)(C)(D)

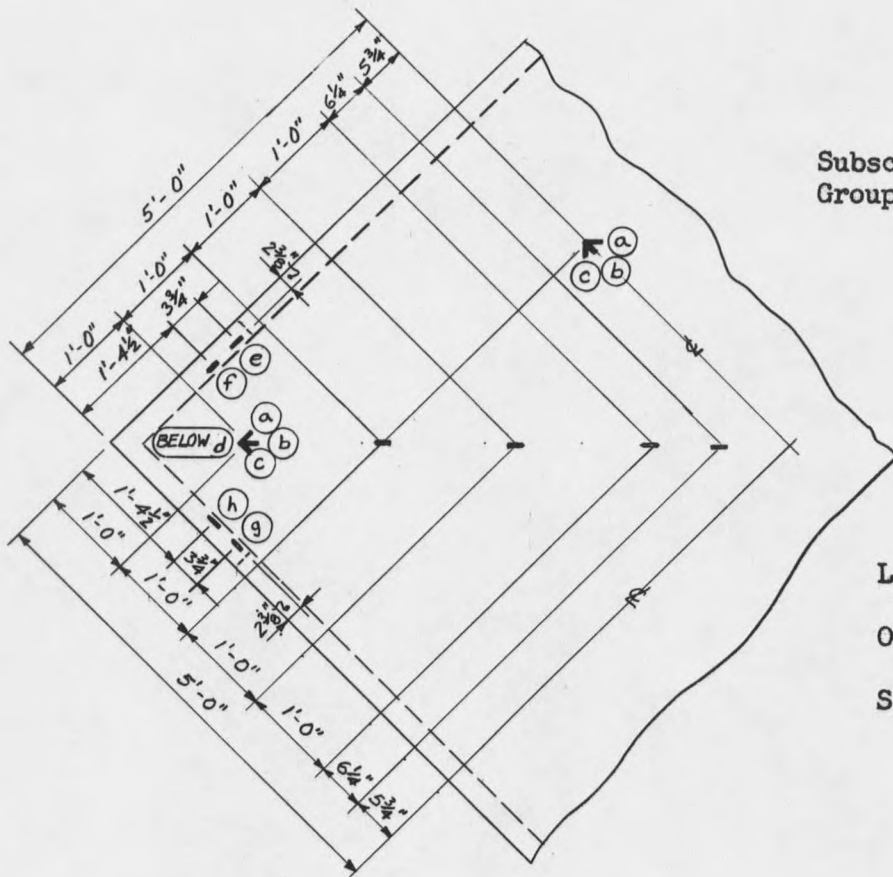
Scale 1-1/2" = 1'-0"



Vertical Plane Angles
At Edge Beam Gage Groups

LOCATION AND NOTATION OF EDGE BEAM GAGES

FIGURE 9



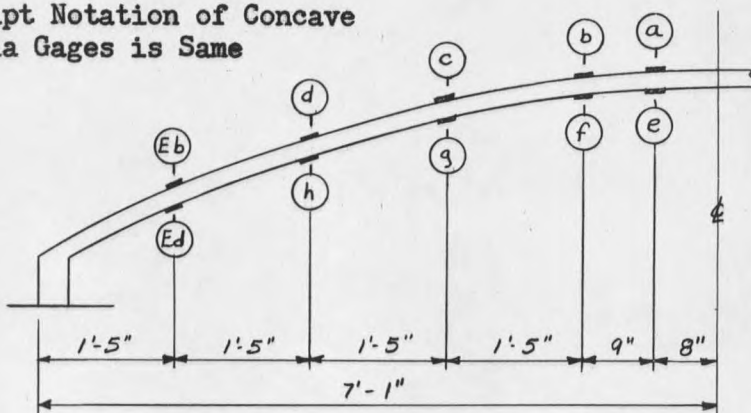
Subscript Notation of Groups $\textcircled{E}$ & $\textcircled{G}$ Given

Location Of Gages

Of Groups (E)(F)(G)

Scale 1/2"= 1'-0"

Subscript Notation of Concave Parabola Gages is Same



Subscript Notation
for Groups (F1) (F2)
(F3) (F4) Start At The
Center With (a) to (d)
On Top and (e) to (h)
On The Bottom

Subscript Notation On

Diagonal Center Line

Scale 1/2" = 1'-0"

LOCATION AND NOTATION OF SURFACE GAGES

FIGURE 10

TABLE I GAGES USED

A1a (22)	B2k	C4a	F1a
A1b (21)	B2l	C4b	F1b (56)
A1c (23)		C4e	F1c
A1d (24)	B3a (33)	C4f	F1d (55)
A1e	B3b (32)	C4g	F1e
A1f	B3c (31)		F1f (58)
A1g (74)	B3d (34)	D1a	F1g
A1h (75)	B3e (35)	D1b (18)	F1h (57)
A1i (76)	B3f (36)	D1c (17)	
	B3g (510)	D1d (19)	F2a
A2a	B3j (85)	D1e (110)	F2b (62)
A2b	B3k (84)	D1f	F2c
A2e	B3l (86)	D1g (71)	F2d (61)
A2f		D1h (72)	F2e
	B4a	D1i (73)	F2f (67)
A3a (27)	B4b		F2g
A3b (26)	B4e	D2a	F2h (68)
A3c (25)	B4f	D2b	
A3d (28)	B4g	D2e	F3a
A3e (29)	B4j	D2f	F3b (66)
A3f (210)	B4k		F3c
	B4l	D3a (13)	F3d (65)
A4a		D3b (12)	F3e
A4b	C1a	D3c (11)	F3f (64)
A4e	C1b (42)	D3d (14)	F3g
A4f	C1c (41)	D3e (15)	F3h (63)
	C1d (43)	D3f (16)	
B1a	C1e (44)		F4a
B1b (38)	C1f	D4a	F4b (52)
B1c (37)	C1g (81)	D4b	F4c
B1d (39)	C1h (82)	D4e	F4d (51)
B1e (310)	C1i (83)	D4f	F4e
B1f			F4f (54)
B1g (77)	C2a	E1a	F4g
B1h (78)	C2b	E1b	F4h (53)
B1i (79)	C2e	E1c	
B1j	C2f	E1d	G1a
B1k	C2g	E1e	G1b
B1l		E1f	G1c
	C3a (47)	E1g	
B2a	C3b (46)	E1h	G2a
B2b	C3c (45)		G2b
B2e	C3d (48)	E2a	G2c
B2f	C3e (49)	E2b	
B2g	C3f (410)	E2c	
B2j	C3g (59)	E2d	

Materials Used

The concrete mixture used in the hyperbolic paraboloid has been well proven in commercial construction throughout the Bozeman area; however, the exact strength of the pour used in the model is questionable. Although slump is only a measurement of consistency, the fact that the slump was low (see page 28), together with the assumption that the cement content was similar to typical mixtures produced by the ready mix supplier, suggests that the mix used in the model had a good water-cement ratio. Further, since the five day moist curing probably bettered typical field curing methods, it has been assumed that the present concrete strength is high.

Nottingham's report mentions the results of several concrete cylinder tests:

<u>Age</u>	<u>Strength</u>	<u>Curing Method</u>
5 days	1,950 psi	Unknown ¹
5 days	3,910 psi	Unknown ¹
30 days	4,660 psi	Near Model ²
30 days	9,560 psi	Moist Room ²
30 days	9,050 psi	Moist Room ²

Two, 1 year old cylinders tested by the author had strengths of 4,400 psi and 6,340 psi. Because these cylinders had compensating gages and Nottingham's name on them, and had been stored beside the model, it is very probably that they are of the same pour used in the construction

¹Nottingham, op. cit., p. 20.

²Ibid., p. 33.

of the model. No curing method was recorded, but it has been presumed that they had been located by the model at least since Nottingham initiated his tests.

Selecting the probable strength of the model was a problem. The most rational pattern of readings was derived by: (1) eliminating the readings of the two cylinders known to be moist room cured because the thin model probably dried too fast to attain such strength, and (2) eliminating the lower 5 day and 1 year readings because these readings are so much lower than the others, indicating the possibility of cylinder flaws. The remaining readings are: 3,910 psi at 5 days, 4,660 psi at 30 days, and 6,340 psi at 1 year. Therefore, the estimated strength of the model at the time of these tests was 6,000 psi.

The standard Baldwin-Lima-Hamilton strain indicator which was used allowed strains to be read in micro-inches per inch. Simplified, the principle of this instrument is that of a Wheatstone bridge, using the strained gage as the unknown and the compensating gage as the known resistance. Ten, 10-channel switching-balancing units allowed a reading capacity of one hundred gages. These switching-balancing units were built at Montana State College by Dean Herman. Problems encountered with the instrumentation are discussed in Accuracy Evaluation.

Other materials and equipment used included:

1. SR-4 Strain Gages
 - a. 12 Type AR-1 gage factor 2.06
 - b. 114 Type A-1 gage factor 2.02
 - c. 10 Type A-1 gage factor 2.07

2. Wire

- a. 20 gage, single strand, solid core, plastic coated bell wire (4,000 ft)
- b. 22 gage, double strand, solid core, plastic coated bell wire (about 400 ft)

3. Load

- a. 150 forty pound sand bags
- b. 10 twenty pound sand bags

4. Portable concrete grinder borrowed from Pioneer Ready Mix Concrete Company, Bozeman, Montana.

5. Miscellaneous

- a. Instrument table
- b. Loading platform for concentrated load
- c. Gypsum
- d. SR-4 cement and SR-4 pre-coat solvent
- e. Sulphur capping compound
- f. 2 dial gages
- g. Riehle 300,000 lb compression machine

Preparation and Instrument Hook-up

When formed by standard methods, concrete surfaces are never as smooth as required for attachment of strain gages. The test structure was formed not only by standard methods, but was broomed producing an extremely rough surface. A portable concrete grinder was used to smooth the general areas where gages were to be placed. Small, deep irregularities were filled with a gypsum mixture and then resanded.

After marking its exact location, each strain gage was attached by: (1) cleaning the surface with SR-4 pre-coat solvent, (2) trimming the gage, (3) coating the gage and concrete surface with SR-4 cement, (4) placing the gage on the structure, and (5) applying pressure to the gage until the cement was set. Gages placed on top of the structure were easily attached. Pressure was applied by placing a piece of 1/2 in. thick synthetic sponge

over the gage and then compressing the sponge with a weight. Pressure was applied to the gages on the underside of the structure by two thicknesses of sponge covered by a $3/4$ in. piece of wood to give a greater angle to three pieces of $3/4$ in. drafting tape which compressed the sponge with a steady, uniform pressure. The number of type A-1 gages was increased from 58 to 124. The number of type AR-1 gages was increased from 5 to 12.

While preparing for the preliminary test it became apparent that with a greater number of gages, a more efficient system of wire identification would be imperative. With the number of gages more than doubled, the capacity of the switching-balancing units was exceeded. Therefore many wires had to be detached and others attached during each test. This necessitated easy wire identification to eliminate errors during this changing of wires.

To facilitate wire identification, the original wiring was disentangled and organized into a logical pattern. Subsequent wiring was integrated into this same pattern: the two wires from each gage were twisted together; all the pairs of twisted wires from each group of gages ($A_1, A_2, A_3, \dots$ etc.) were tied together at one foot intervals; all groups of wires from each area (side A, side B, top, $\dots$ etc.) were tied together and brought to the instruments as a major group. As a result of this organization, no identification problems were encountered during either the final hook-up or the changing of wires during each test.

During the preliminary test, one gage short-circuited when the gage leads came in contact with each other. The probability of this re-occurring was reduced by gluing the leads to the concrete. Because of the

tendency of structures to conduct electricity this could not be done under normal conditions, but in this case the concrete was so dry that no current was transmitted through it. Further protection was achieved by gluing blocks of wood to the concrete at close intervals and tying the groups of wires to them. Each gage on the top surface was protected by a covering made from a piece of masonite attached to wooden blocks which were glued to the concrete.

The most critical problem pinpointed by the preliminary test was that of faulty connections at the switching-balancing units. The wire used for gage hook-up was solid core, 20 gage bell wire. This wire rolled as the connection knobs were screwed down on the wire. This rolling of the wire created enough torque to cause the knobs to release. It was impossible to manually turn the knobs tight enough to prevent this release and subsequent poor circuit. This problem was solved by hammering flat the ends of all wires involved.

A test for air bubbles under the gages was made by applying light thumb pressures to the gages and observing the movement of the strain needle on the Baldwin strain indicator. Irregular "jumping" of the needle would indicate that a bubble did exist. All gages proved to be properly attached. During this test, gage C2f indicated a short circuit which was not corrected.

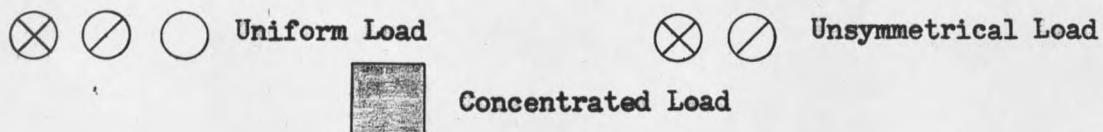
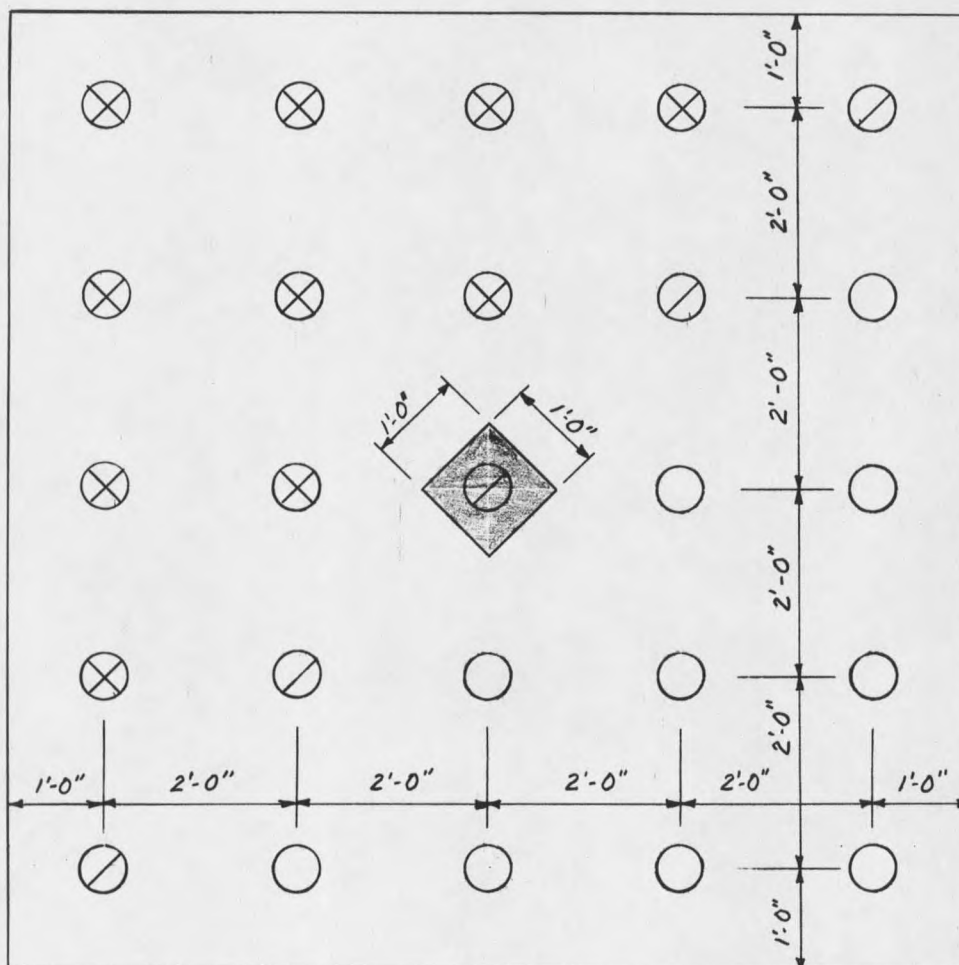
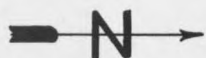
TEST PROCEDURE

Uniform Load

A uniform load of 60 psf was applied to the hyperbolic paraboloid in increments of 10 psf. This load was applied on top of the structure with 40 lb sand bags located 2 ft on center each way as shown in Fig. 11, p. 40. Twenty five bags thus constituted the increment load. One hundred fifty bags weighing a total of three tons were required at full load.

The 10 switching-balancing units had a combined capacity of 100 gages. Type AR-1 gages are rosettes or essentially three gages in one. This meant that the 124 type A-1 and 12 AR-1 gages required 160 hook-ups. Therefore two wirings were required for each complete test. The first run, called Test A, consisted of 76 A-1 and 24 AR-1 gages. Forty-eight A-1 and 12 AR-1 gages were then changed and a second run, called Test B, was made. The gage notations and switching-balancing units for Tests A and B are noted in Table II, p. 41. All gages attached during Test B were read, creating some duplication.

During Test A, strain gage readings were recorded for loads of 0, 10, 20, 30, 40, 50, 60, 60, 50, 40, 30, 20, 10, and 0 psf. During Test B, strain gage readings were recorded for loads of 0, 10, 20, 30, 40, 50, 60, 50, 40, 30, 20, 10, and 0 psf. In addition to the time required to transpose the wires between the test runs, the two tests required a total of about ten hours for completion.



LOADING POINTS

FIGURE 11

TABLE II
GAGE HOOK-UP

Unit & Switch	Test A	Test B	Unit & Switch	Test A	Test B	Unit & Switch	Test A	Test B
1-1	B2a	F1d	4-5	A3a	F4e	7-8	C1d	
1-2	B2b	F1c	4-6	A3b	F4f	7-9	C1a	
1-3	B2e	F1b	4-7	A3c	F4g	7-10	C1b	
1-4	B2f	F1a	4-8	A3d	F4h			
1-5	B1a	F4d	4-9	A3e	F2f	8-1	C1c	
1-6	B1b	F4c	4-10	A3f	F2e	8-2	C2a	
1-7	B1c	F4b				8-3	C2b	
1-8	B1d	F4a	5-1	A2a	F1f	8-4	C2e	
1-9	B1e	F3d	5-2	A2b	F1e	8-5	C2f	(Short)
1-10	B1f	F3c	5-3	A2e	E1d	8-6	C2g	
			5-4	A2f	E1e	8-7	E1a	B1j
2-1	D4a	F3b	5-5	A1a	E1f	8-8	E1b	B1k
2-2	D4b	F3a	5-6	A1b	E1g	8-9	E1c	B1l
2-3	D4e	F2d	5-7	A1c	E1h	8-10	E2a	B2j
2-4	D4f	F2c	5-8	A1d	F1g			
2-5	D1a	F2a	5-9	A1e	F1h	9-1	E2b	B2k
2-6	D1b	F2b	5-10	A1f	---	9-2	E2c	B2l
2-7	D1c	B2g				9-3	G1a	B3j
2-8	D1d	B4e	6-1	C4a		9-4	G1b	B3k
2-9	D1e	B4f	6-2	C4b		9-5	G1c	B3l
2-10	D1f	B3g	6-3	C4e		9-6	G2a	B4j
			6-4	C4f		9-7	G2b	B4k
3-1	D2a	B3a	6-5	C4g		9-8	G2c	B4l
3-2	D2b	B3b	6-6	B4a		9-9	A1g	
3-3	D2e	B3c	6-7	B4b		9-10	A1h	
3-4	D2f	B3d	6-8	B4g	Used 8-5 Some Tests			
3-5	D3a	B3f	6-9	C3g		10-1	A1i	
3-6	D3b	B3e	6-10	C3f		10-2	B1g	
3-7	D3c	E2d				10-3	B1h	
3-8	D3d	F2h	7-1	C3e		10-4	B1i	
3-9	D3e	F2g	7-2	C3d		10-5	C1g	
3-10	D3f	---	7-3	C3c		10-6	C1h	
			7-4	C3b		10-7	C1i	
4-1	A4a	F3e	7-5	C3a		10-8	D1g	
4-2	A4b	F3f	7-6	C1f		10-9	D1h	
4-3	A4e	F3g	7-7	C1e		10-10	D1i	
4-4	A4f	F3h						

Besides making strain gage readings, two dial gages were read at the same load increments noted above. These gages were located under the cantilevered peaks and indicate the total deflection of the structure.

Unsymmetrical Load

An unsymmetrical load of 60 psf was applied to one-half of the hyperbolic paraboloid in increments of 10 psf. The direction of unsymmetry was that causing torsion about the buttresses as shown in Fig. 11, p. 40. Twenty pound bags on the line of symmetry and 40 lb bags at the other loading points constituted one load increment. One and one-half tons were required for full test load.

Gage hook-up remained as described for the Uniform Load and noted in Table II, p. 41. Readings for all gages attached to the switching-balancing units during Tests A and B were again recorded.

During both Test A and Test B, strain gage readings were recorded for loads of 0, 10, 20, 30, 40, 50, 60, 60, 50, 40, 30, 20, 10, and 0 psf. Each test required about five hours to complete in addition to the time required to transpose the wires between the test runs.

Dial readings of the deflections of the cantilevered peaks were read as described for the Uniform Load.

Concentrated Load

A concentrated load of 3,600 lbs was applied to the hyperbolic paraboloid in increments of 600 lbs. By the use of a loading frame the load was applied over an area 1 ft square in the center of the saddle

as shown in Fig. 11, p. 40. Fifteen 40 lb bags constituted one increment. Ninety bags were required for a complete run.

Gage hook-up remained as described for the Uniform Load and noted in Table II, p. 41. Readings for all gages attached to the switching-balancing units during Tests A and B were again recorded.

During both Test A and Test B, strain gage readings were recorded for loads of 0, 600, 1200, 1800, 2400, 3000, 3600, 3600, 3000, 2400, 1800, 1200, 600, and 0 lbs. Each test required about five hours to complete in addition to the time required to transpose the wires between the test runs.

Dial readings of the deflections of the cantilevered peaks were read as described for the Uniform Load.

No major problems occurred during any of the tests. Several minor problems and irregularities that occurred are discussed in Accuracy Evaluation.

DERIVATION OF MODULUS OF ELASTICITY

Introduction

Because the engineer usually works in terms of stress in pounds per square inch, interpretation of test data of experimental stress analysis is simplified if the data is presented in that form. A Baldwin strain indicator, of the type used in these tests, is calibrated in terms of strain in micro-inches per inch to render the instrument more versatile. To convert data to the more desirable form it is necessary to determine the stress-strain relationship, i.e., the modulus of elasticity, E , in pounds per square inch.

Concrete has no absolute value of E . The ACI Code¹ specifies the following empirical formula:

$$E = 1,000 f'_c \quad (1)$$

where f'_c is the compressive strength of the concrete at 28 days. This simple approximation yields excessively high values of E for concrete strengths greater than 4,000 psi. In addition, "...it is possible to produce concretes having the same modulus but having strengths differing by more than

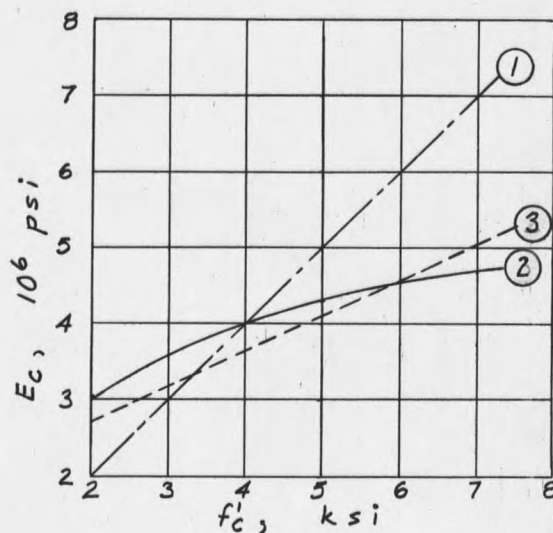
¹Building Code Requirements for Reinforced Concrete, (ACI 318-56; Detroit, Michigan: American Concrete Institute, 1956), p. 935.

100 per cent."¹ Other empirical formulas that have been employed are:²

$$E = \frac{6 \times 10^6}{1 + (2,000/f'_c)} \quad (2)$$

$$E = 1,800,000 + 460f'_c \quad (3)$$

The results of these three formulas are graphically described in the following figure.



EMPIRICAL FORMULAS FOR E

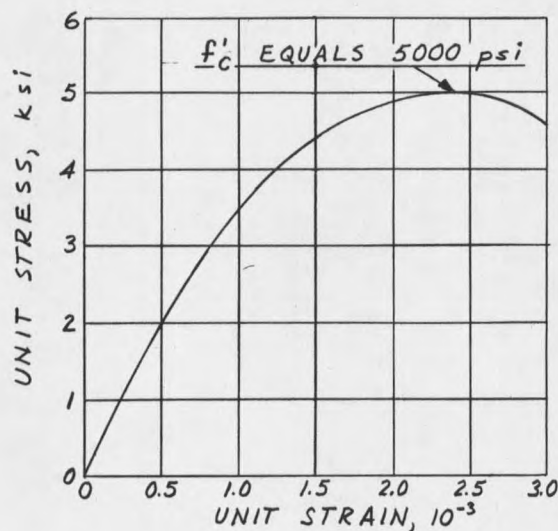
FIGURE 12

One of three tests is usually employed to determine experimentally the value of E. The tests are: (1) measurement of vibration frequencies of concrete subjected to sonic forces, (2) measurement of standard beam deflections under applied loads, and (3) measurement of standard compression cylinder deflections under applied loads. The first method determines an

¹G. E. Troxell and H. E. Davis, Composition and Properties of Concrete, (New York: McGraw-Hill Book Co., 1956), p. 261.

²T. Y. Lin, Prestressed Concrete Structures, (New York: John Wiley & Sons, Inc., 1955), p. 32.

initial modulus of elasticity, whereas the second and third methods are affected by time-creep actions. Normally the E value is determined 28 days after the concrete is made. At this time the sample is nearly saturated from being stored in a standard moist room during the curing period. A typical stress-strain curve is shown in the following graph.



TYPICAL CONCRETE STRESS-STRAIN CURVE

FIGURE 13

Modulus of elasticity is the ratio of change of load (stress) to change of length (strain) and is commonly expressed as a single-valued, linear relationship. The curvature of the real stress-strain relationship in concrete has led to the three different expressions shown by the graph. The common self-defining terms are: (E_i) initial tangent modulus, (E_t) tangent modulus, and (E_s) secant modulus. The stress at which the tangent

modulus is taken or through which the secant modulus is drawn can be arbitrarily chosen.

By employing special testing methods, Hrennikoff¹ has attempted to separate the effects of creep and elastic bending. The primary feature of his methods is to very rapidly apply several increment loads to a cylinder and then maintain the load during periods of creep. Each load increment is held until the creep rate is reduced to at least 1/10 the initial rate. Readings are taken during both the loading and the unloading of the specimens. In this manner Hrennikoff tested both moist and dry cylinders of the same material. Two essential differences he observed were: (1) creep of dry cylinders was very small and (2) comparatively flat portions existed at both the beginning and ending of the dry cylinder test, whereas the center portion was essentially like that of the moist condition. The diagonal parts of the center portions of these tests were essentially parallel and represent the modulus of elasticity. Point (2) is explained, "The reason for this anomaly is the presence in the cylinder of fine transverse cracks which form in the course of drying and are virtually unavoidable. Closure of those cracks at the commencement of loading and their opening and (upon) unloading is responsible for the additional strain deformation present."² His specimens were made of a cement paste, but it is believed that the inclusion of aggregate would not significantly affect the resulting informa-

¹A. Hrennikoff, "Incremental Compression Test for Cement Research", Proceedings of the American Society of Civil Engineering, Journal of Engineering Mechanics Division, LXXXIV, No. EM2, April 1958, pp. 1604-1 to 1604-13.

²Ibid., p. 1604-12.

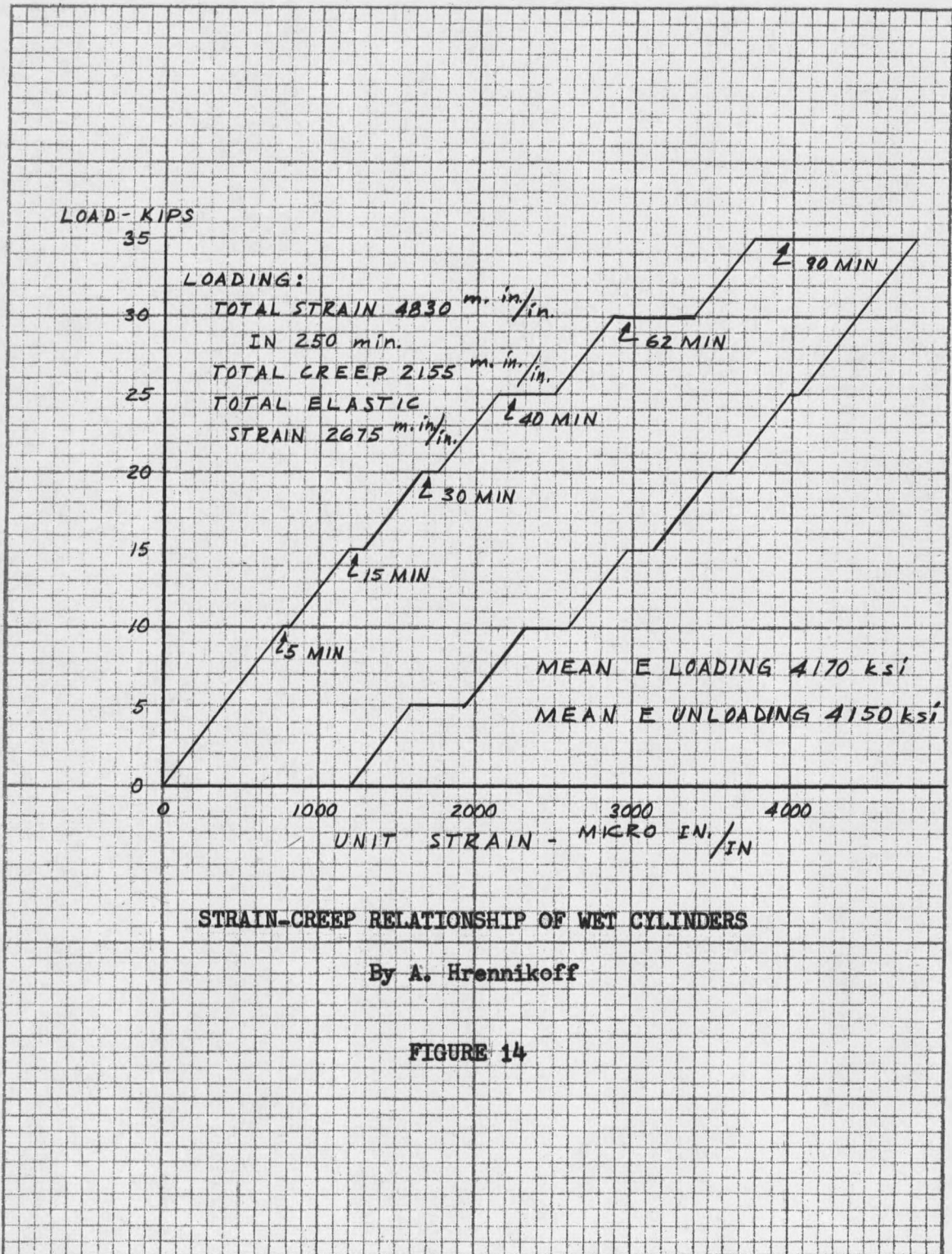
tion. Graphically presenting Hrennikoff's results are Fig. 14, p. 49 and Fig. 15, p. 50.

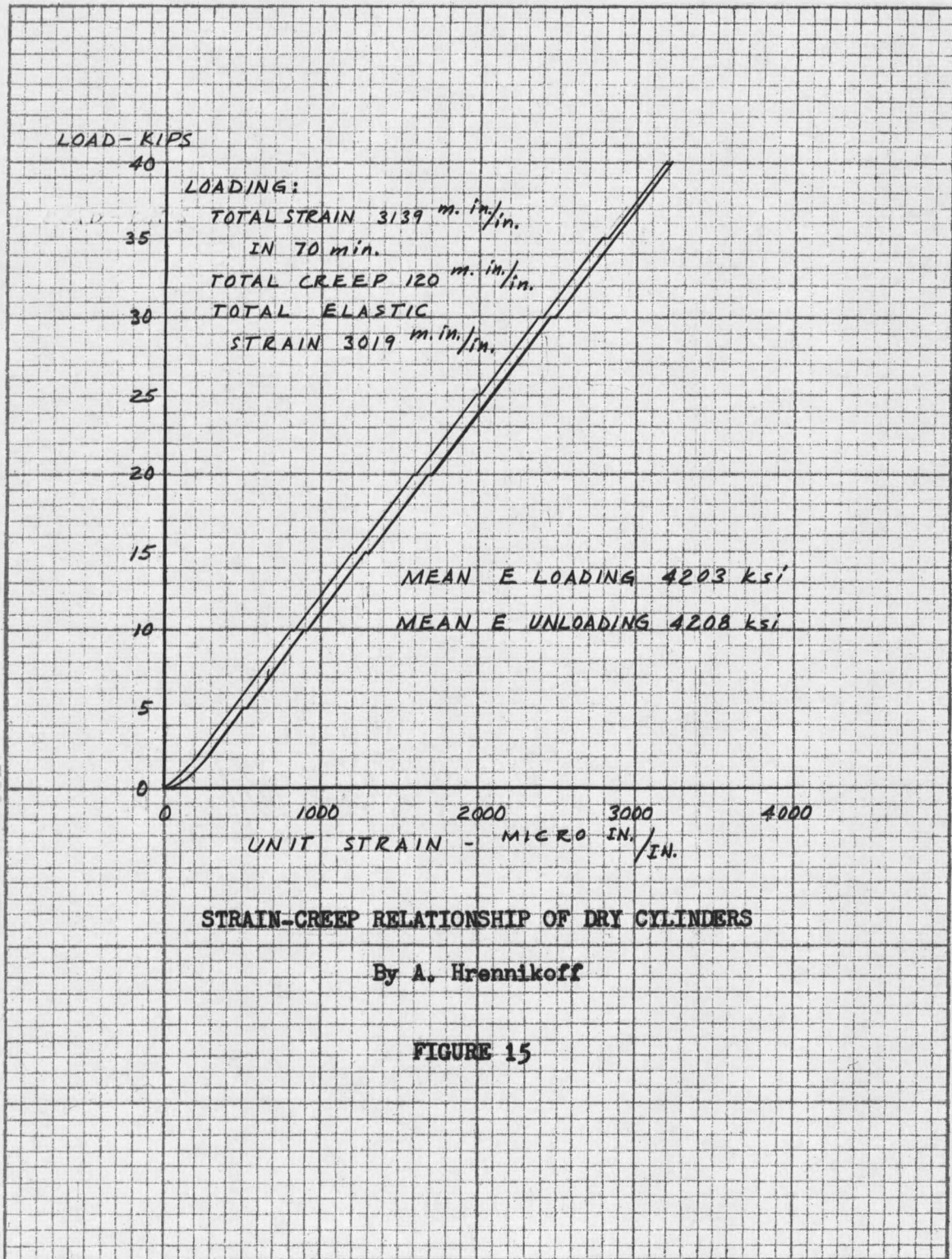
Preparation and Testing

Tests were made to determine the probable modulus of elasticity of the hyperbolic paraboloid model, using three 6 in. x 12 in. concrete cylinders which were taken by Nottingham when the model was made. Information known about these cylinders has been discussed in Materials Used, page 34. Although it is known that both moist room and site condition curing methods were used on these cylinders, no records or markings remain to indicate which method was used on each cylinder. However, because the model was moist cured for five days and all cylinders have been stored beside the model since an initial 30 day curing period, the model and cylinders probably have a similar strength and definitely have the same degree of dryness.

After applying a standard sulphur cap to the cylinders, two A-1, SR-4 strain gages wired in series, were placed diametrically opposite each other at the center of each cylinder. A compensating gage was formed by wiring, in series, two A-1 gages already attached to the model. These gages, combined to form one compensating gage and one measuring gage per cylinder, were wired directly to the Baldwin strain indicator.

Loads were applied to the first cylinder in increments of 20,000 pounds by a Riehle 300,000 lb compression machine. The machine was stopped between load increments only long enough to obtain a strain reading. As the strength of the concrete was expected to be greater than 6,000 psi, an





applied load of 120,000 lbs (4,240 psi) was expected to be well within the capacity of the cylinder, but failure occurred during the third loading. Readings were taken only during the loading cycle of the tests.

The other two cylinders were loaded in increments of 12,500 lbs to a total load of 100,000 lbs (3,640 psi). Again, the machine was stopped between load increments only long enough to obtain a strain reading and readings were taken during the loading cycle only. Four test cycles were run on each cylinder. The third cylinder was loaded a fifth time. This run was continued in increments of 12,500 lbs until failure occurred at 179,500 lbs (6,350 psi).

Discussion of Results

The stress-strain relationships of test cylinders 2 and 3 were almost exactly alike, while 1 exhibited much more strain per unit stress. Because test cylinder 3 (and apparently 2) had an ultimate compressive strength much closer to the expected value than did cylinder 1, the values of cylinders 2 and 3 were used for determining the estimated value of E, the modulus of elasticity.

Test cylinder 1 did exhibit characteristics of interest. During the initial loading, the stress-strain relationship had a convex curvature essentially like the Typical Concrete Stress-Strain Curve shown in Fig. 13, p. 46. The second loading had an initial relatively flat portion similar to Hrennikoff's findings for dry cylinders, Fig. 15, p. 50. Apparently, initial failure took place when the last increment load of the first test

run was applied, leaving small transverse failure cracks that had to be re-compressed before the more linear stress-strain relationship reoccurred.

All loadings of test cylinders 2 and 3 had initial relatively flat portions in the stress-strain curves. The most reasonable explanation for the differences between the initial portions of the curve from cylinder 1 and the averaged curve from cylinders 2 and 3 seems to be that cylinder 1 had been moist room cured, prohibiting the growth of drying cracks until the cylinder had gained a major portion of its strength, and that cylinders 2 and 3 had been site cured, allowing them to dry more rapidly, thus increasing the growth of drying cracks. As all cylinders had been stored by the model for nearly a year and the model is in a laboratory which is dry and quite warm, it follows that all cylinders were very dry when tested. Assuming that dry cylinders exhibit properties similar to those shown by Hrennikoff, creep played a very minor role in the testing of these cylinders.

The stress-strain curve shown on the following page was derived by averaging all test runs of cylinders 2 and 3. Deviations from this curve by the individual test runs were small. Using the various methods of calculating the modulus of elasticity, the following were obtained:

Initial Modulus = 3,400,000 psi

Secant Modulus at 400 psi = 3,440,000 psi

Secant Modulus at 2,000 psi = 3,720,000 psi

Tangent Modulus at 2,000 psi = 3,920,000 psi

Hrennikoff's data indicates that for high values of stress, the tangent modulus is the most realistic value for dry cylinders. Assuming that f'_c

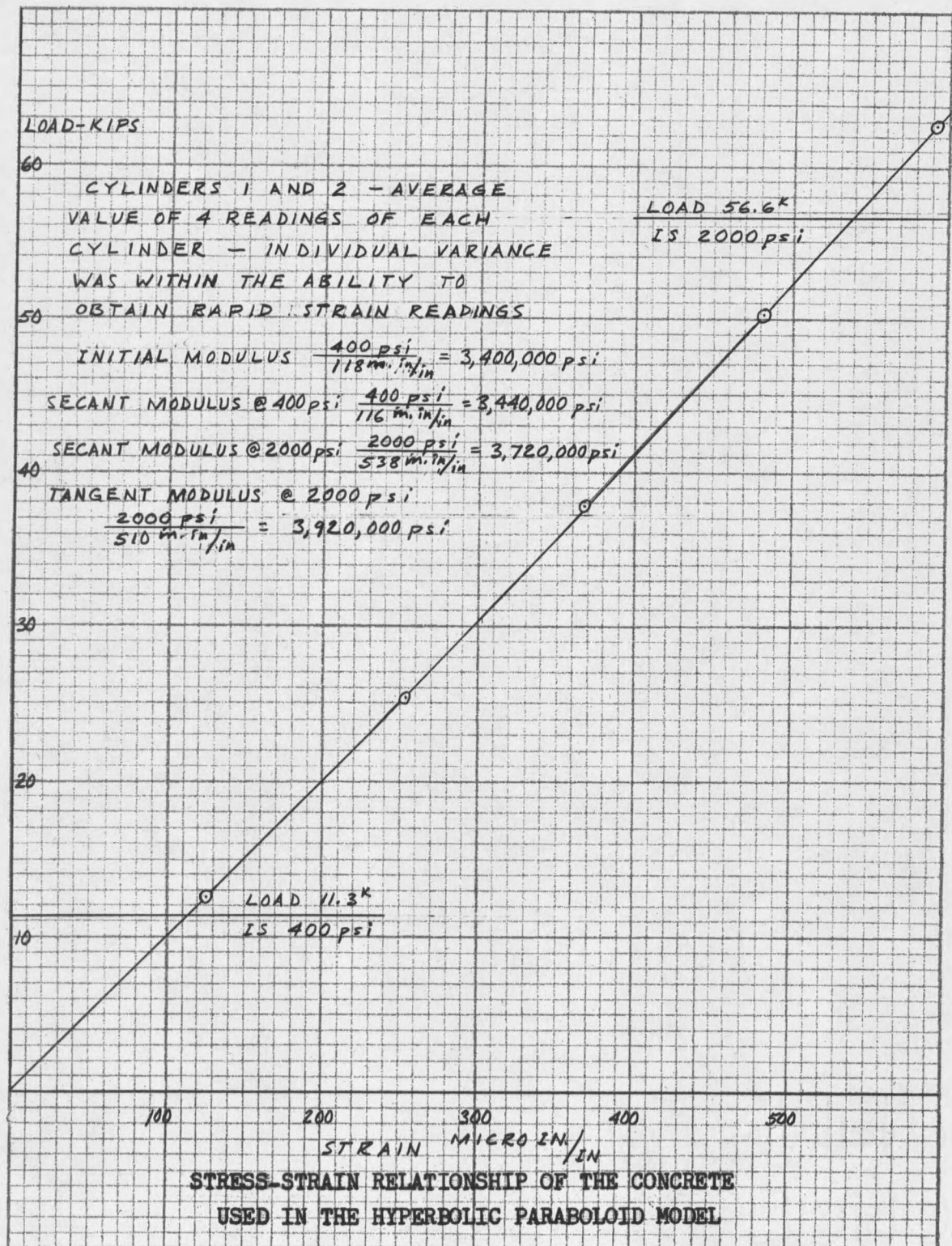


FIGURE 16

is 6,350 psi as found by breaking cylinder 3 and referring to Fig. 16, p.53 and Fig. 12, p. 45, it is found that cylinders 2 and 3 had an E value of 3.92×10^6 , about 15 per cent lower than that given by empirical formulas 2 and 3. This variance is justified by Davis' statement concerning the possible variance in the relationship of the modulus of elasticity to strength of concrete.

Because the model was certainly as dry as the test cylinders, it can be assumed that its stress-strain relationship was essentially like that of the cylinders. The loads applied to the hyperbolic paraboloid model created only small strains; therefore, it is reasonable to use the E value which best describes the stress-strain relationship at small values of strain rather than the large values of strain used in the comparison of this test with the empirical formulas. The chosen value was the secant modulus at 400 psi which was 3.44×10^6 .

It should be noted that even if the E value determined above is in error, it should provide a good figure for transposing the measured strains of the test model to stresses, because the same instrumentation and measurement methods were used to find both E and the strains of the model. Because of small stresses and the reduced creep in dry concrete, creep strains were probably non-existent or very small even though each model test required about five hours.

It was assumed in the design analysis that the concave and convex parabolas equally distributed the applied load. Lin says, "Not too much work has been done for the modulus of elasticity of concrete in tension, but it is generally assumed that, before cracking, the average modulus over

a length of several inches is the same as in compression, although the local modulus in tension is known to vary greatly."¹ Hyperbolic paraboloid design would be greatly affected if the modulus in tension did vary from the modulus in compression, but apparently no educated estimate can be made of this possible effect.

¹Lin, op.cit., p.33.

TEST DATA

Accuracy Evaluation

To increase the utility of the test results, these tests duplicate some of Nottingham's work as noted in INTRODUCTION, page 14. Primarily, this increased utility was accomplished by increasing the accuracy of the data and expressing the data in stress rather than strain. The increased accuracy was accomplished by:

1. More than doubling the number of strain gages to measure more exactly the action of the model under load.
2. Increasing the total strain by extending the applied loads from 50 psf to 60 psf for the uniform and unsymmetrical tests and from 2,000 lbs to 3,600 lbs for the concentrated test.
3. Increasing the probability of having correct test readings by recording strains while loading and unloading and, in some cases, duplicating the loading and unloading.
4. Determining the mean unit strain of the strain readings by the method of least squares, thus decreasing the variance of the mean.
5. Averaging the mean unit strains at the locations receiving the same action by the symmetry of the model and load, thus further decreasing the probable error.
6. Graphically plotting each line of action, e.g., gages 1a, 2a, 3a and 4a of sides A, B, C and D for the uniform load test, to visi-

bly describe the response of the entire model under load rather than merely the response at singular points.

Thus, by repetition, duplication, and mathematical and graphical interpretation, the strain measurements have been evaluated as carefully as possible. Therefore the accuracy and value of the test results are directly related to the accuracy of the individual strain measurement. In other words, the reliability of the test data is proportional to the precision of the instrumentation, the conditions surrounding the test model and the accuracy of the construction and the homogeneity of the test model. Each of these points — instrument accuracy, test environs and model consistency — warrant further discussion.

The Baldwin-Lima-Hamilton strain indicator used was an obvious source of instrument error. The Civil Engineering Department spent considerable money trying to repair this device last year and has recently purchased a new instrument. Thus, others recognized the inadequacies of this instrument. Three problems were particularly evident:

1. The strain indicator ON-OFF switch apparently did not properly close a circuit, for when the switch was OFF the needle deflected about 15 degrees.
2. The ON-OFF switch had some play in the ON location. When the switch was turned far to the right a different reading would be obtained than when the switch was turned just far enough to be called ON rather than OFF.

3. At times, the needle danced wildly while the instrument was turned ON. This action could occur at any time during a test run and usually had a duration of a few seconds.

The first malfunction probably had little or no effect on the test results. If the current leakage remained constant, it would have a constant effect and would not invalidate the data. Even if the current did fluctuate, the strain indicator, if working correctly, would measure the correct difference between the active and compensating gages.

The second malfunction appeared to have a more definite effect. The switch was turned OFF between readings while more load was being applied to the model. The probability of the switch being turned to the same spot each time was small. Because different values were obtainable by the play in the ON switch, different increment readings resulted. This effect showed up very strongly when plotting typical diagrams of load versus strain. Increment readings which were higher or lower than average for one gage tended to remain high or low for all gage readings of that load increment.

The third malfunction was the probable source of another error. In the results of several tests, there exists a series of one to couple dozen readings which are entirely out of bounds. Both before and after one of these series, the readings fit a rather good pattern. Even a strain gage within the questioned series usually produced logical readings at the preceding and following load increments.

Other instrument errors existed in the switching-balancing units. The most critical disturbance was caused by an apparent change in resistance

due to the heating of the resistance coils. This caused the strain indicator needle to continually float from right to left. This floating was much more evident immediately after turning to a new switch. As there was no apparent remedy for this problem other than new switches, all readings were taken after the needle had slowed, having floated an equal length of time.

Since measurement of exact time elements was impossible without employing a second recorder, care was taken to read the measurements at a constant pace. This method was successful to this extent: Time readings, to the nearest minute, were taken at the beginning and ending of each set of readings, i.e., the readings of the 100 switches for each load increment. During three test runs, all but one or two readings were accomplished within one minute of the same length of time. During the other three test runs, all but one reading were accomplished within two minutes of the same length of time. The most typical times required for reading the 100 switches were 14 and 15 minutes. Reading times for each of the 100 individual switches undoubtedly had a much larger variance.

Another problem encountered in the switching-balancing units was that some individual switches were extremely sensitive to the slightest movement. Several would readily move from their point of balance with an accidental jar of the table or instruments, while a few moved for no apparent reason. This happened consistently at unit 6, switch 9 and unit 10, switches 1 through 10. Generally, data obtained from these switches was disregarded.

Problems which occurred while tightening the connections of the switching-balancing units are described in TEST SET-UP. Although the method finally used appeared to eliminate all obvious problems, there still remains the possibility that these gage connections may have permitted some variance.

During the uniform load test, the dial gages placed under the high corners of the model were expected to deflect equally and in even amounts when the load increments were applied, but instead, they deflected unequally and were quite inconsistent. The two gages were removed from their supports at the model and given exact deflections by placing them in the compression machine and moving the machine head several small amounts. During this test the two gages recorded exactly the same deflections. It was later discovered that both gages had sufficient internal points of friction to create a tendency to resist deflection. Apparently the gage-supporting apparatus had enough give or slack to cause a gage to occasionally "hang up" during part of the deflection of the model.

In TEST SET-UP it was mentioned that the model was constructed inside the engineering laboratory. Without a doubt, this was the best location available, but this laboratory is not equipped for precise structural model study. The lack of temperature control definitely affected the test results, and the slab support may have affected the results.

The arrangement of compensating and active strain gages was self-adjusting for equal temperature differentials occurring during a test. However, unequal temperature changes caused by slight air movements caused erratic readings because the gage resistance was extremely sensitive to tem-

perature variations. During the uniform load test, the air currents were strong enough to be felt by the author who was wearing a short sleeve shirt. Erratic readings occurred simultaneously with each observed movement of the air. Many active gages on the model received considerable temperature protection from the coverings used to shield them from the applied loads. It was found that a light wrapping of gauze around the compensating gages gave similar temperature protection during later tests.

The floor of the engineering laboratory is believed to be a 4 in. slab, reinforced with light wire mesh, on 4 or 6 in. of gravel on grade. A heavier, structurally reinforced floor may be more desirable for structural model study as it would reduce any tendency toward movement of the model support. Whether or not the slab under the hyperbolic paraboloid test model did move is not known, and even if it did, the strains in the model may not have been affected. However, as a person walked near the model, the dial gages fluctuated. This may have been only a vibration action which would not occur during a static test. But since three tons of sand bags were moved from a position near the structure onto the surface of the structure during a test, it is entirely possible that settlements occurred, causing strains independent of the loads applied.

The hyperbolic paraboloid model appeared clean and neat, but careful inspection revealed flat spots in the warped surface, indicating some variation in thickness. Of course it is impossible to create a homogeneous concrete structure due to the composition of cement paste and aggregate. A Schmidt Rebound Hammer applied to various parts of the structure had a large range of dynamic response due to either or both the varying thickness and

the unhomogeneous material. Thus, although the accuracy of the construction of the model and the homogeneity of the structural material was definitely within the limits of actual field conditions, known irregularities existed. By placing the gages at several points of symmetry and by averaging the strain readings obtained, it was hoped that the effect of model inaccuracies was diminished.

In conclusion, it can be said that the test data had several specifically inaccurate aspects and several questionable aspects. Errors in human judgment during testing and data extension also could have caused some inaccuracies. Counteracting these inaccuracies are the large number of gages used, the extensive readings taken and the particular method of data extension and interpretation. It will be shown later, by the consistency of the test results, that these counteractions have made the results valuable if not exact.

Summary of Calculation Methods

The basic intent of this section of the report is to describe the methods used to increase the reliability of the test results. Although the use of the method of least squares to fit a line to the data and the averaging of results at the points of symmetry have been the most important additives to accuracy, all the methods will be reviewed in chronological order. Examples of specific calculations are in the APPENDIX.

During each test, loads were applied in six increments, providing seven points of load including zero load. Strain recordings were made during both the loading and unloading of the structure, furnishing two read-

ings at each increment. In addition, gages attached to units 6, 7, 10, unit 8, switches 1 through 6, and unit 9, switches 9 and 10, were read for both tests A and B as previously described. This supplied four readings at each increment for nearly one-fourth of the gages.

Each of the 100 readings for tests A and B of the uniform load and unsymmetrical load tests were individually plotted. For the concentrated load test, gages common to tests A and B were plotted together while all others were plotted individually. During the process of plotting the data, individual readings markedly different from the general trend were discarded. When a series of readings appeared to be invalid, as described in Accuracy Evaluation, it was usually discarded, but in a few instances the series was given an equal arbitrary change to put the majority of readings in general agreement with their related readings. An example of this modification is presented in the APPENDIX.

When an erratic reading was discarded, a straight line was drawn from the preceeding to the following reading to obtain an estimated value for the increment in question. An example is given in the sample calculation presented in the APPENDIX. An average of the loading and unloading values of strain was determined. When applicable, the averaged values of tests A and B were again plotted and a final average was calculated. The plotting of A and B averages showed a marked increase in consistency, supporting the value of this method of determining the increment readings.

To determine a mean unit strain it was necessary to fit a line to these averaged values. True deflection may not be exactly linear, but due

to the low stress levels and other reasons discussed in DERIVATION OF MODULUS OF ELASTICITY, a straight line of the form $Y = A + BX$ was used.

The method of least squares was chosen to derive the values of A and B for this equation, using X as the independent variable or loading increment and Y as the dependent variable or average increment strain reading. The constant A is the Y intercept. It was calculated but was used only to plot the trend line over the plotted readings. The constant B is the slope of the trend line. The slope, that is, the rate of change of strain, is the value sought.

The gage factor for most of the strain gages was 2.02, but the AR-1 gages had a factor of 2.06 and some of the A-1 gages had a factor of 2.07. Because the strain indicator was adjusted for a 2.02 gage factor during all tests, a modification of B was made to obtain a true value of the rate of change of strain.

The two normal equations¹ for the method of least squares for N separate Y readings are:

$$\sum Y = NA + B\sum X \quad (1)$$

$$\sum XY = A\sum X + B\sum X^2 \quad (2)$$

There was an odd number (N=7) of items equidistant along the X axis. By taking the origin at the mean (4th) X value, the normal equations become:

$$\sum Y = NA \quad * \quad (3)$$

$$\sum XY = B\sum X^2 \quad * \quad (4)$$

¹F. E. Croxton and D. J. Cowden, Practical Business Statistics (New York: Prentice-Hall, Inc., 1952), pp. 249-254.

*In the modified equations, A is now the intercept of the new Y axis and X is measured from the new Y axis.

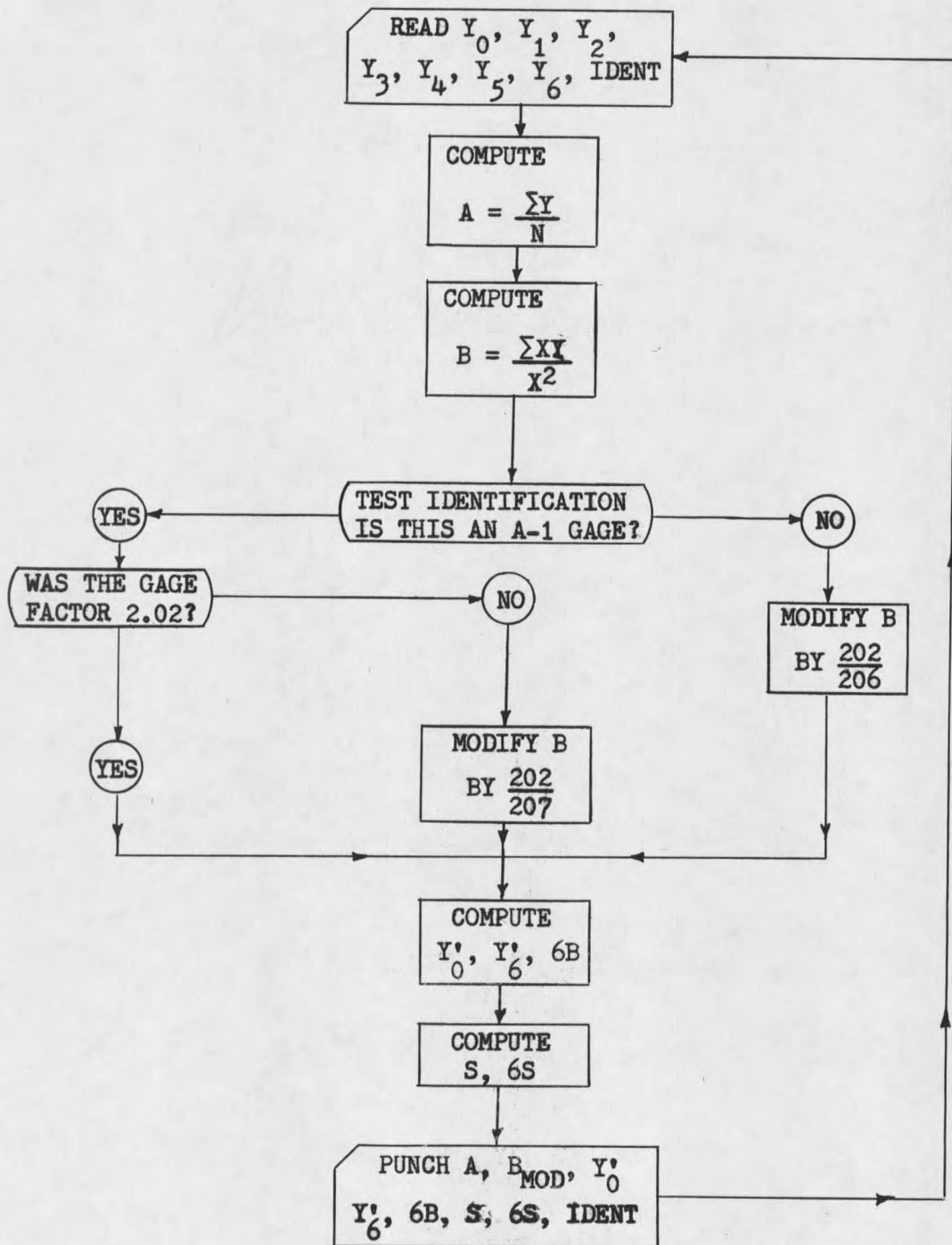
The laborious task of extending the nearly 500 sets of data was delegated to an IBM 650 digital computer. The seven average increment readings and an identification word describing the test, switches, gage and gage factor were punched on IBM data cards.

The total machine time required was minimized by writing the IBM program in SOAP II form.¹ The resulting optimization allowed all computations to be done within the time required for the machine to punch output cards, i.e., the machine produced the answers as fast as it could punch cards — 100 cards per minute.

In addition to determining B, the computer was programmed to modify B for the different gage factors, compute the strain at maximum load, convert strain to stress and compute the stress at maximum load. A flow diagram of the computer process is shown in Fig. 17, p. 66. The terms used are:

1. $Y_0, Y_1, \dots, Y_6$ are the seven averaged increment readings.
2. IDENT is identification of the gage factor, test, switch hook-up and gage.
3. A is the Y intercept or value at the 4th load increment.
4. B is the rate of change of strain.
5. Y_0' and Y_6' are the beginning and ending values of the trend line.
6. $6B$ is the strain at maximum load.
7. S is the rate of change of stress.
8. $6S$ is the stress at maximum load.

¹See APPENDIX, p. 110.



FLOW CHART OF COMPUTER PROGRAM

FIGURE 17

TEST RESULTS

Graphical Presentation

The next several pages are a graphical presentation of the test data. The points of the stress curves were determined as previously described in Summary of Calculation Methods. All data presented is in the form of stress (psi) with total load applied. Unless specifically marked, all curves pass directly through the calculated points. TEST SET-UP, pages 28-33, should be referred to for a complete definition of notation and location of the various gages, although the graphs are essentially self-defined. For convenience the graphs are listed below.

UNIFORM LOAD

Figure Page

Diaphragm to Edge Beam Action	18	69
Vertical Action of Edge Beams	19	70
Horizontal Action of Edge Beams	20	71
Action of Main Convex Parabola	21	72
Action of Main Concave Parabola	22	73
Diaphragm Rosettes.	23	74
Edge Beam Rosettes at the Buttresses	24	74

UNSYMMETRICAL LOAD

Diaphragm to Edge Beam Action	25	75
Vertical Action of Loaded Edge Beams	26	76
Vertical Action of Unloaded Edge Beams	27	77
Horizontal Action of Edge Beams.	28	78
Action of Main Convex Parabola	29	79
Action of Main Concave Parabola.	30	80
Diaphragm Rosettes.	31	81
Edge Beam Rosettes at the Buttresses	32	81

CONCENTRATED LOAD

	Figure	Page
Diaphragm to Edge Beam Action	33	82
Vertical Action of Edge Beams.	34	83
Horizontal Action of Edge Beams.	35	84
Action of Main Convex Parabola	36	85
Action of Main Concave Parabola.	37	86
Diaphragm Rosettes.	38	87
Edge Beam Rosettes at the Buttresses	39	87

To indicate the variance of the values used to calculate these final points, one additional graph is presented in the APPENDIX, Fig. 41, p. 109. Here the values of each of the four sides are given to describe the Vertical Action of Edge Beams under Uniform Load.

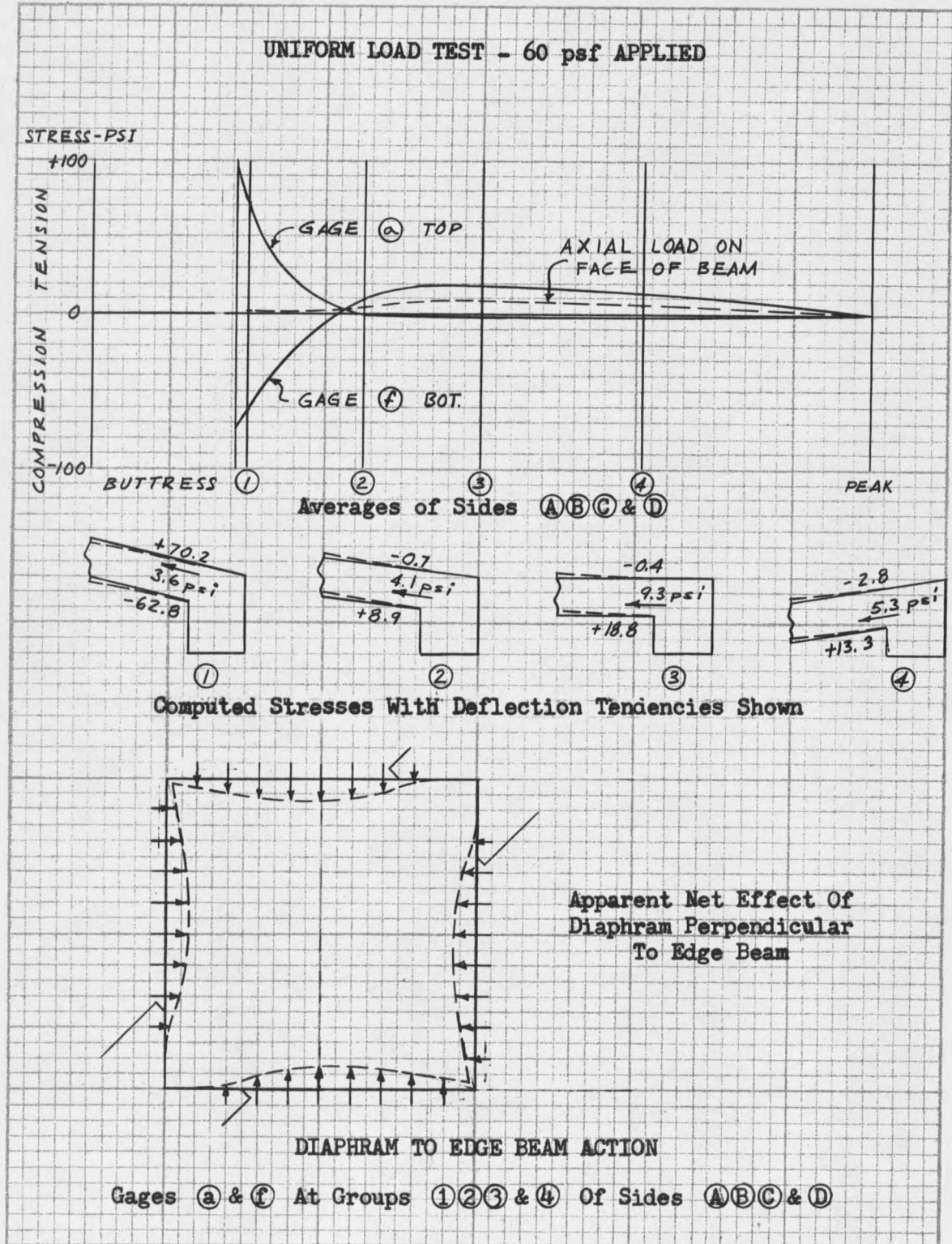


FIGURE 18

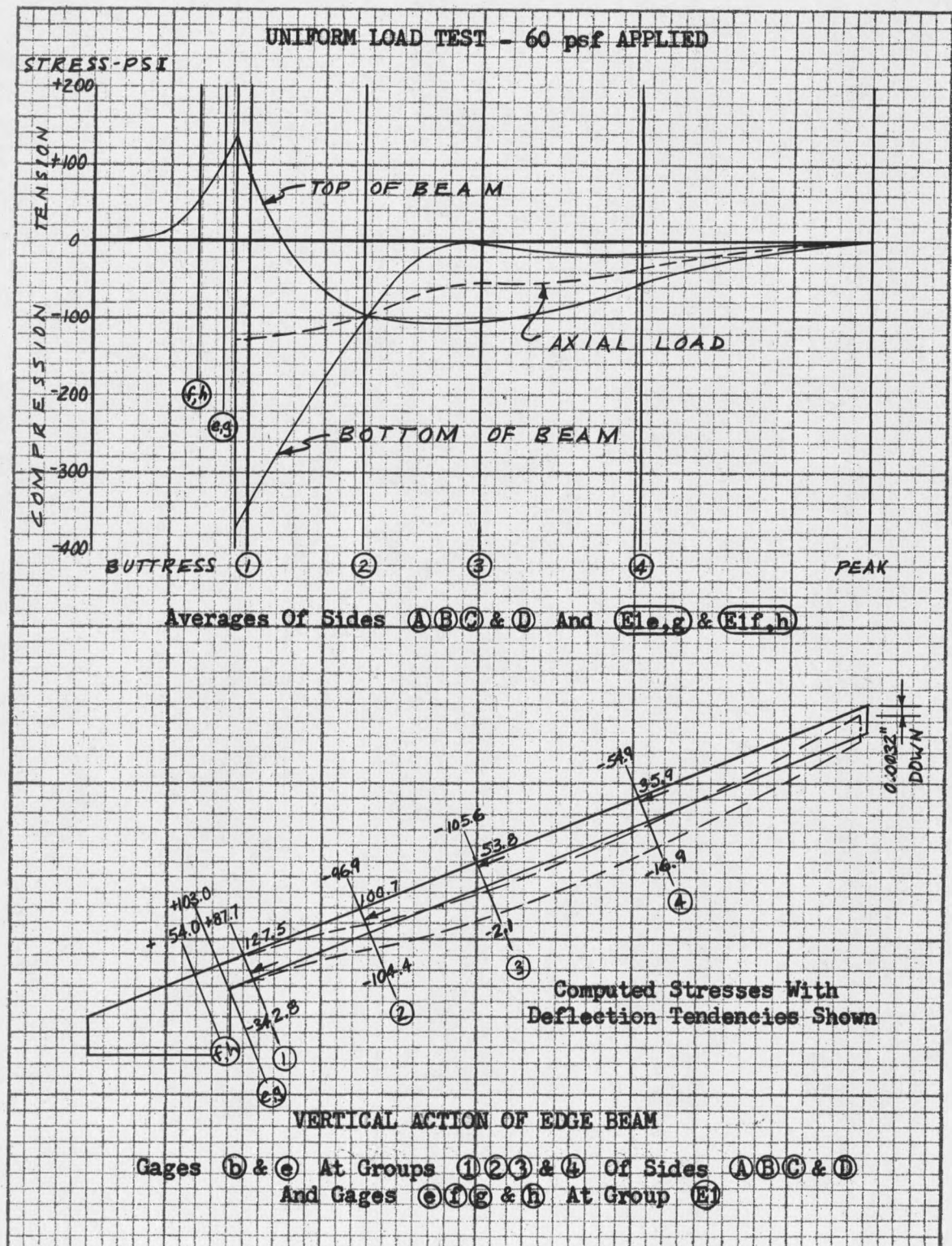


FIGURE 19

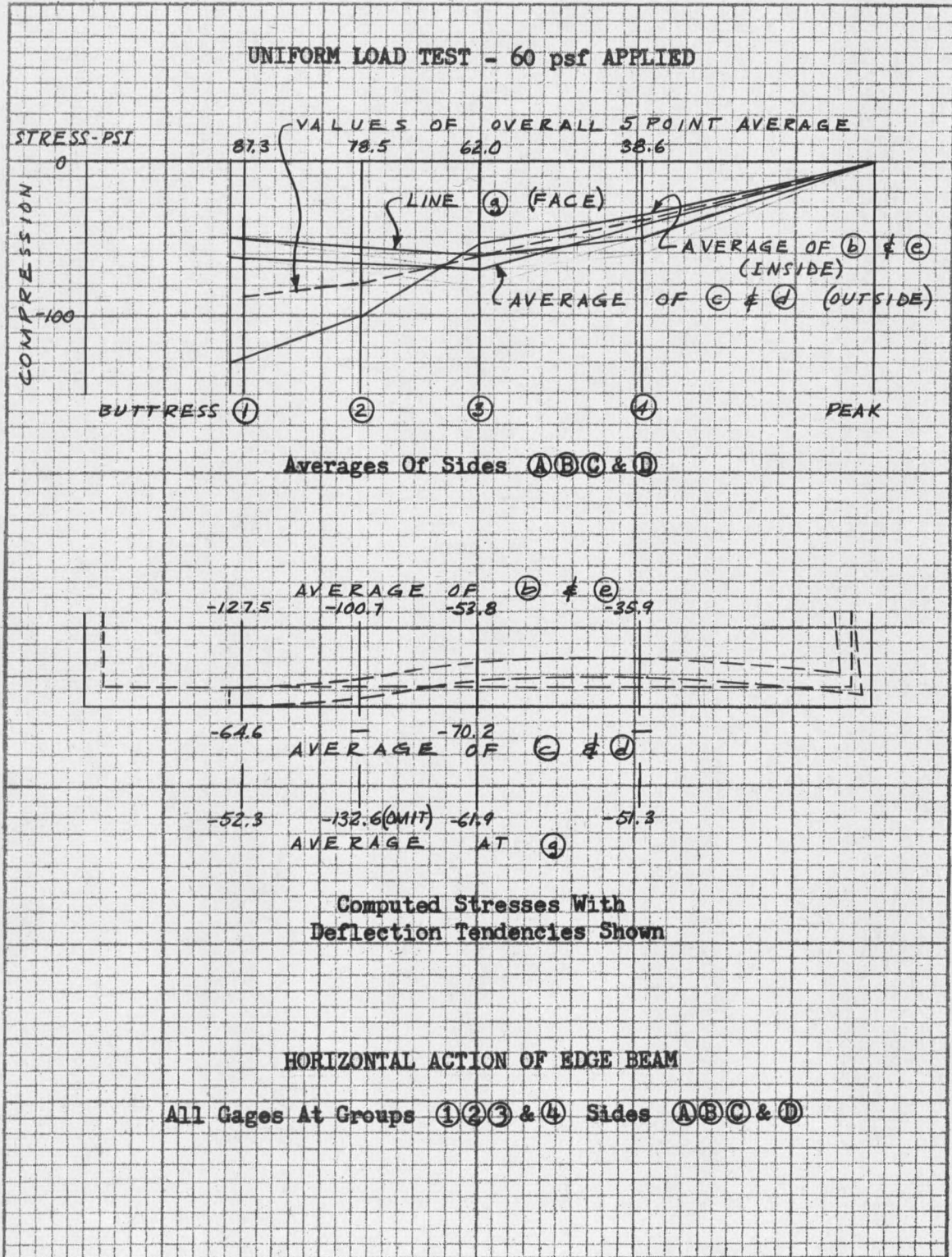
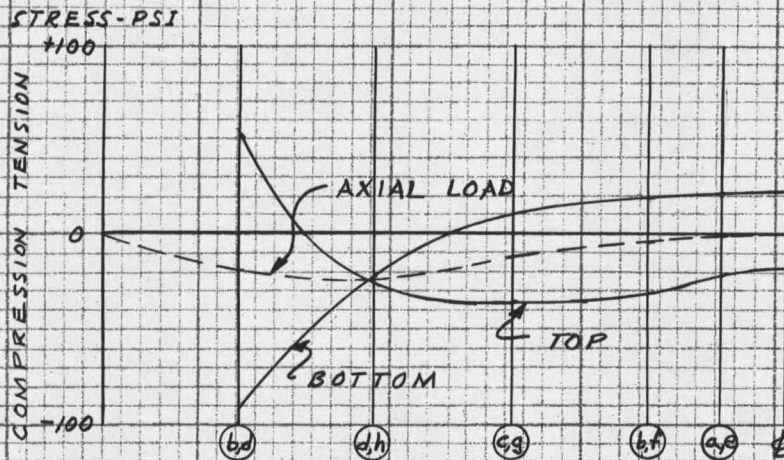
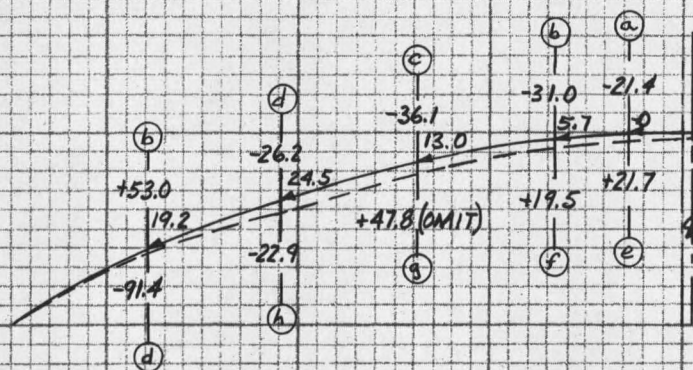


FIGURE 20

UNIFORM LOAD TEST - 60 psf APPLIED



Averages Of Right And Left Sides



Computed Stresses With Deflection Tendencies Shown

All Gages In Groups (F1) & (F3) Top - F2 Bottom

And (b) & (d) At (E1) & (E2)

ACTION OF MAIN CONVEX PARABOLA

FIGURE 21

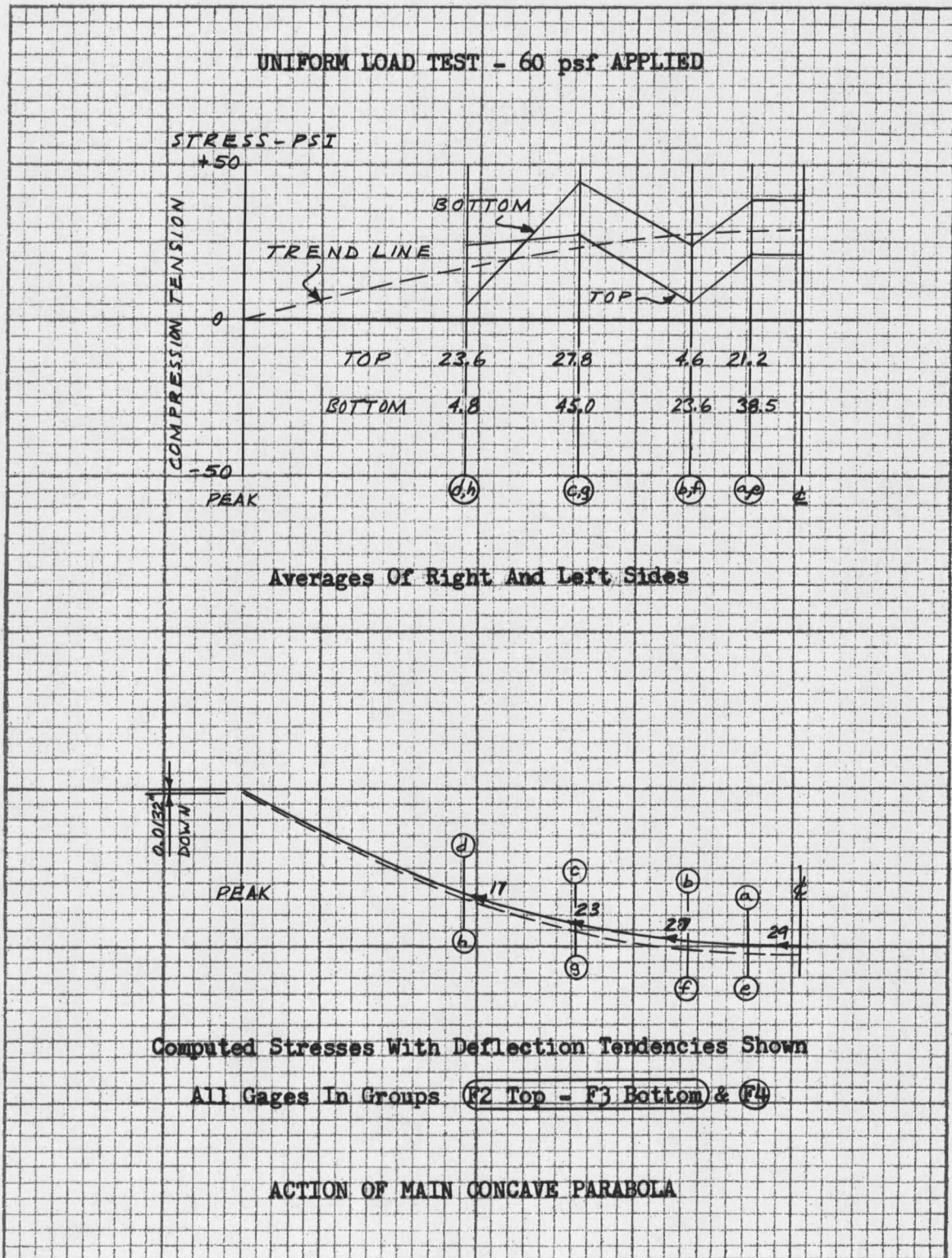


FIGURE 22

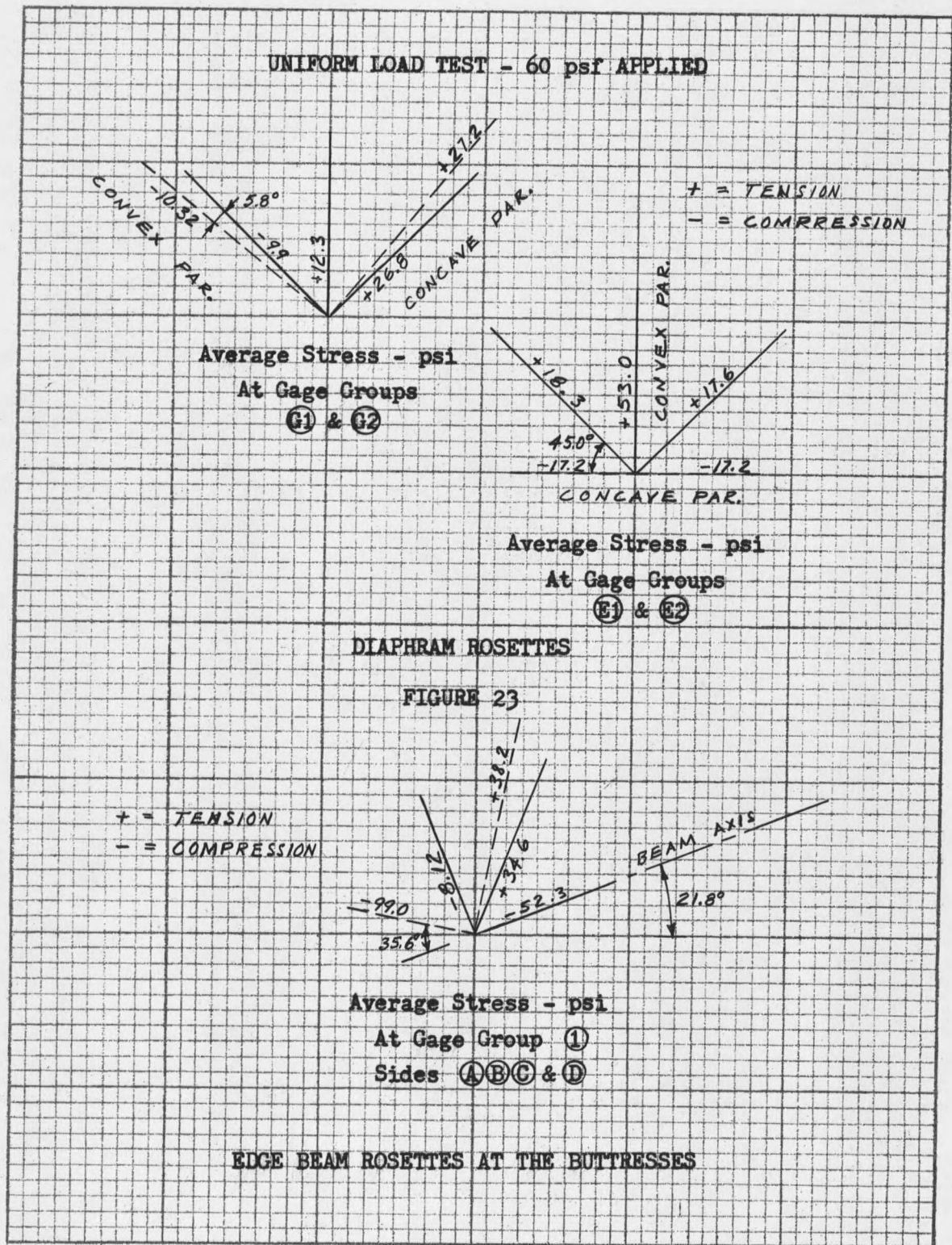


FIGURE 24

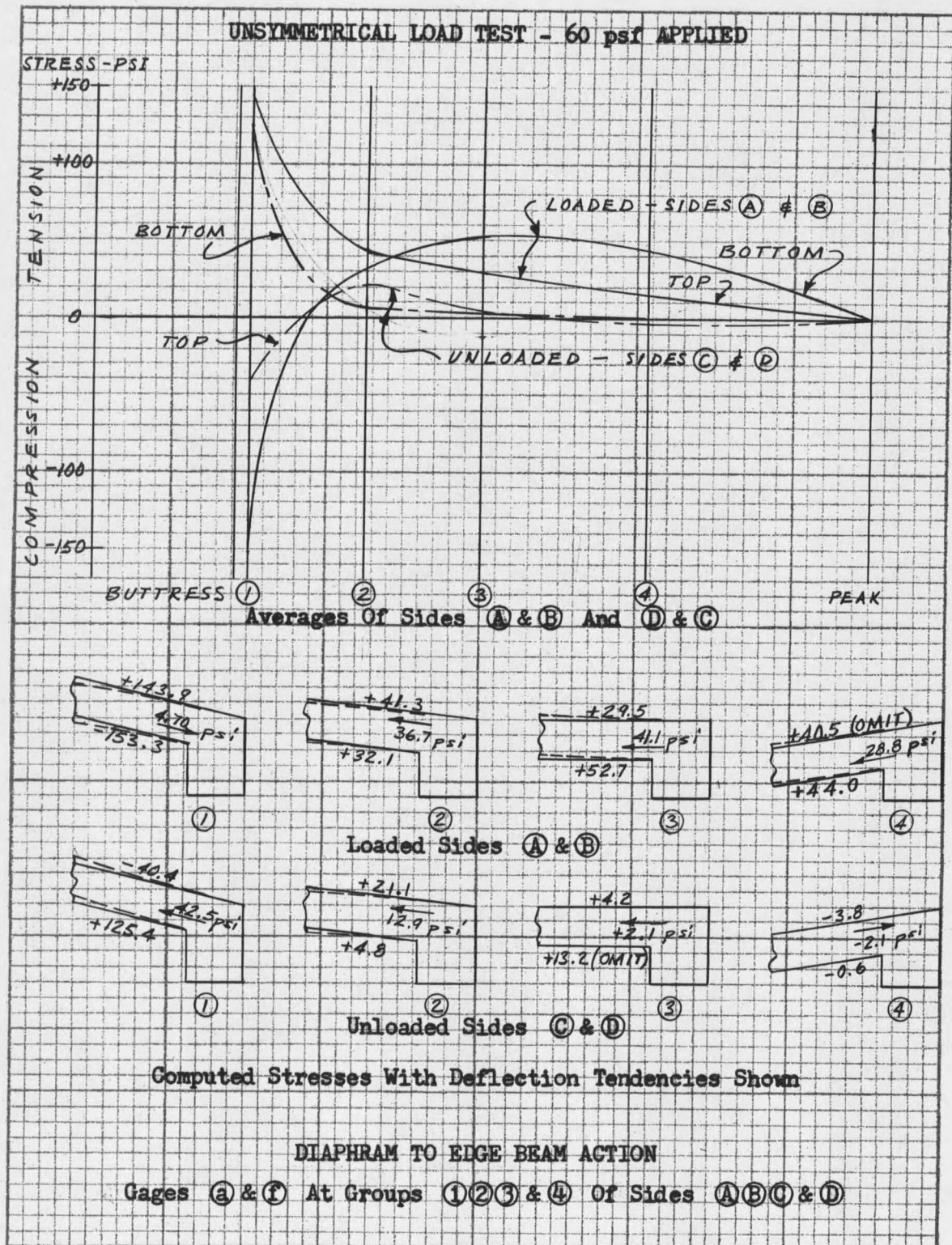


FIGURE 25

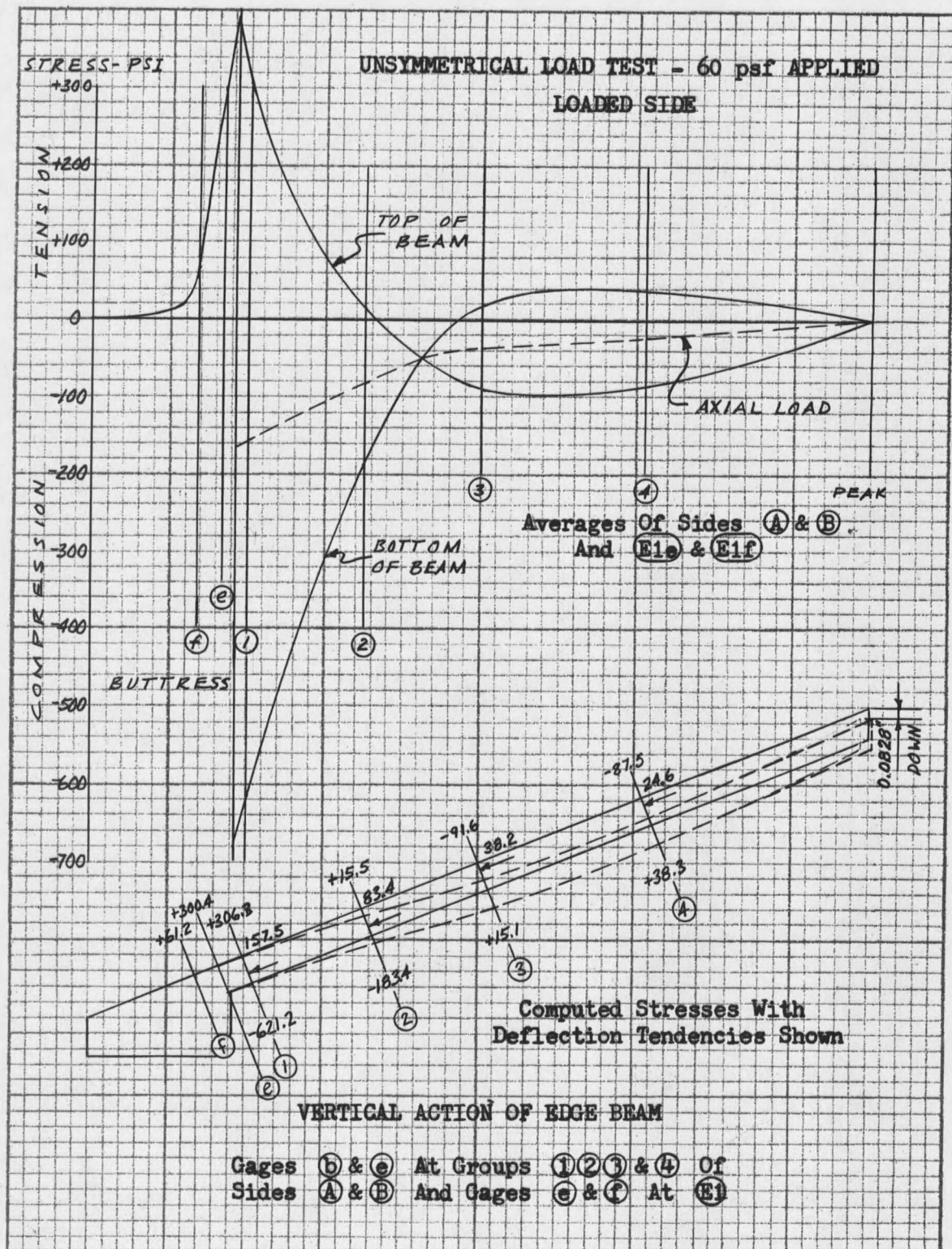


FIGURE 26

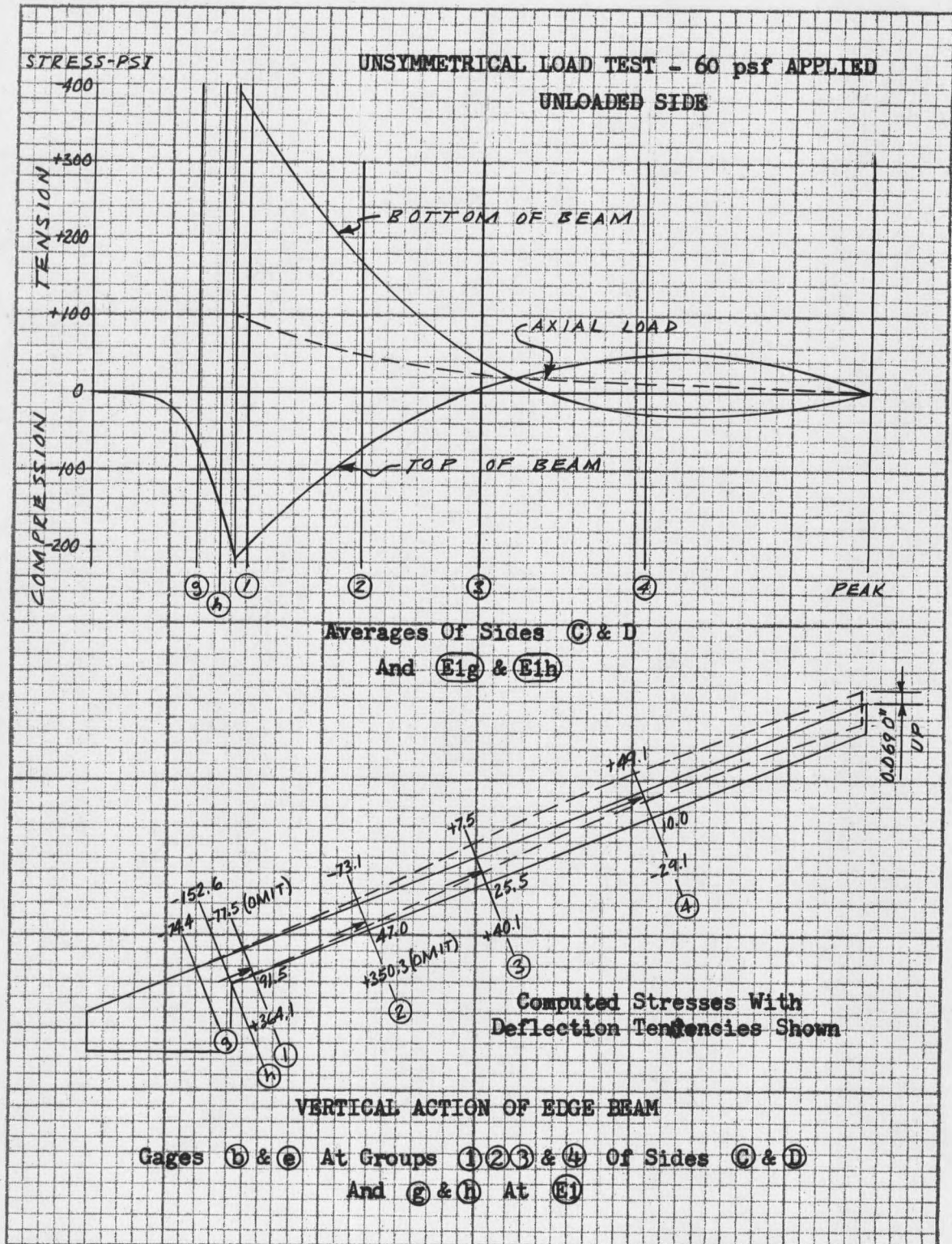


FIGURE 27

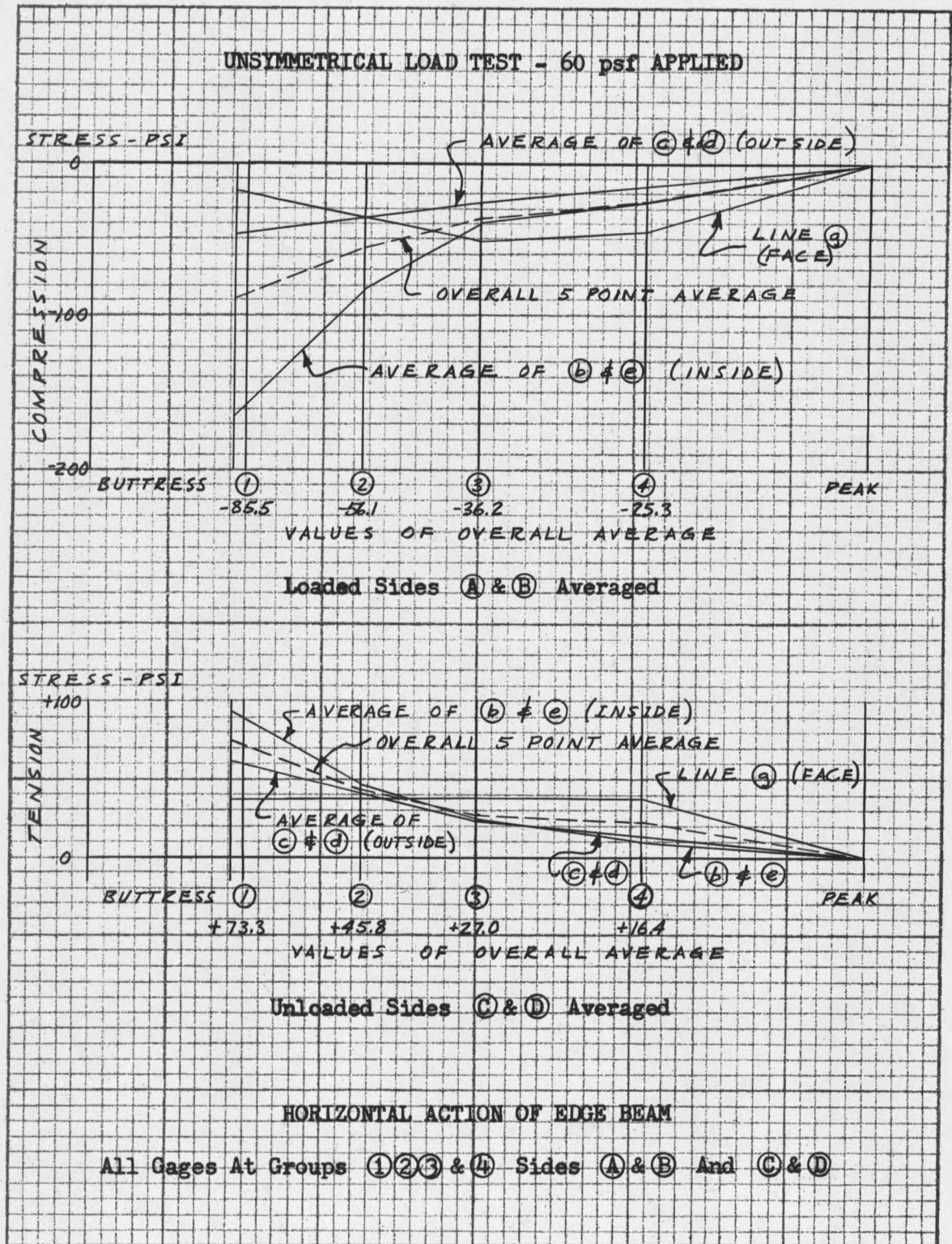


FIGURE 28

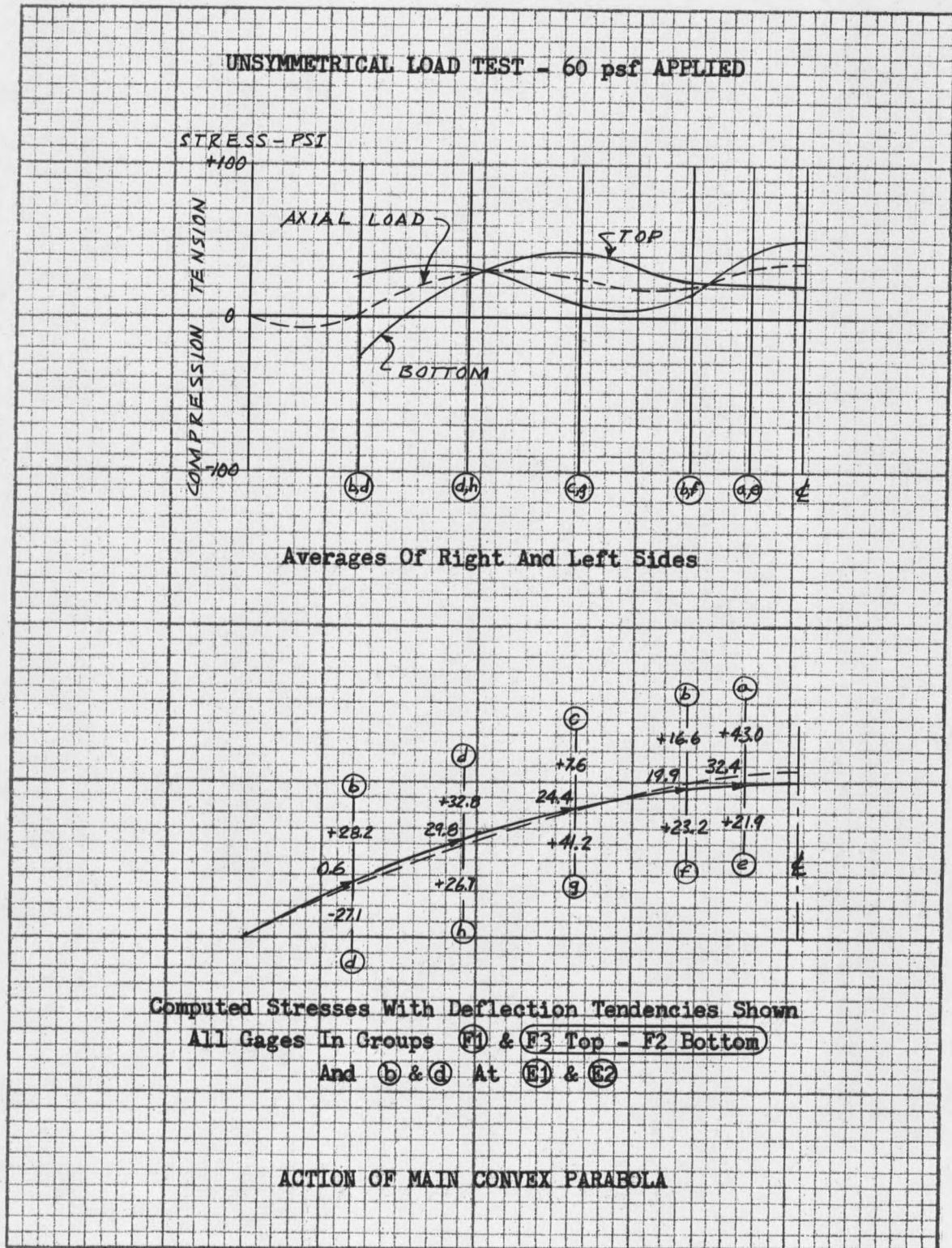
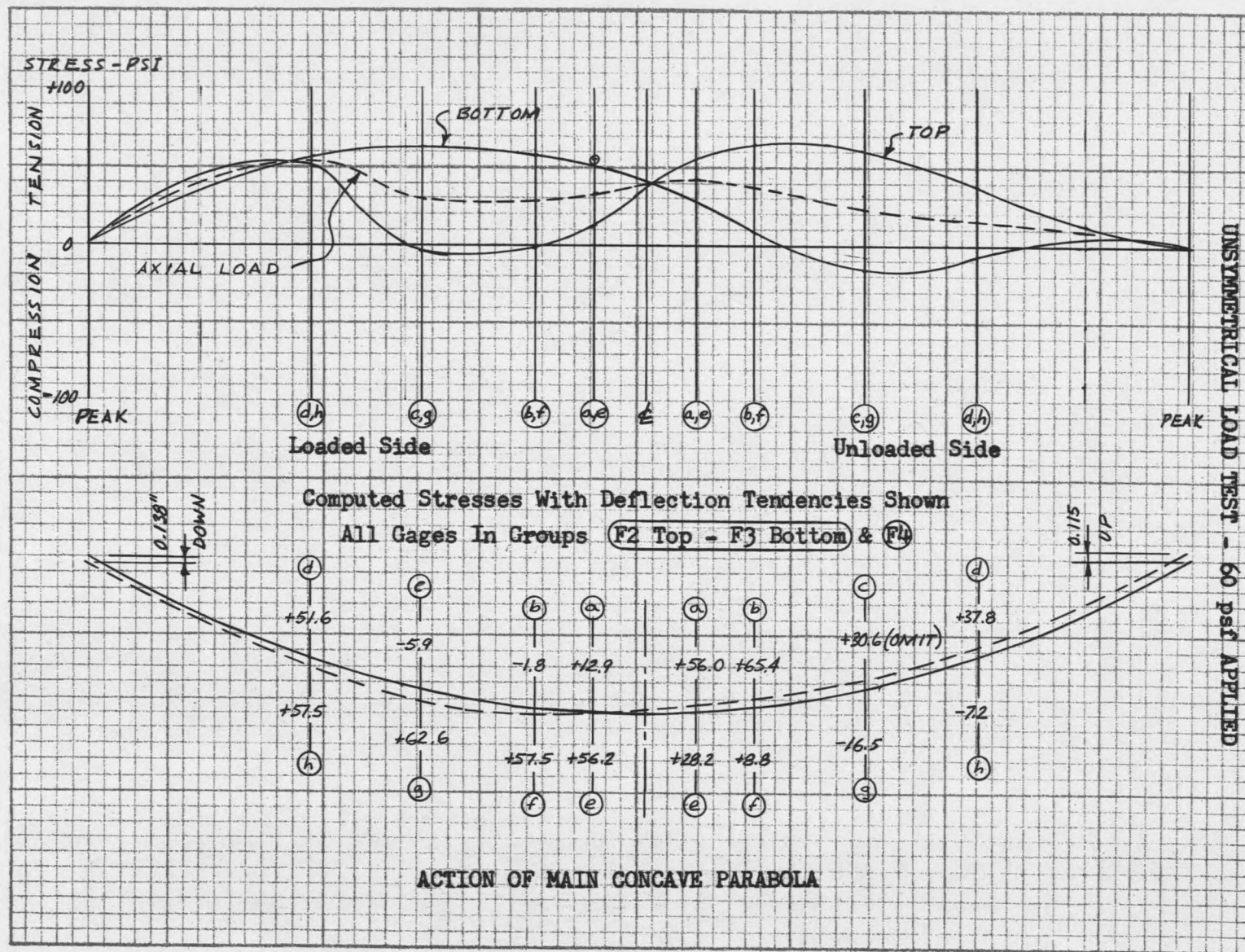
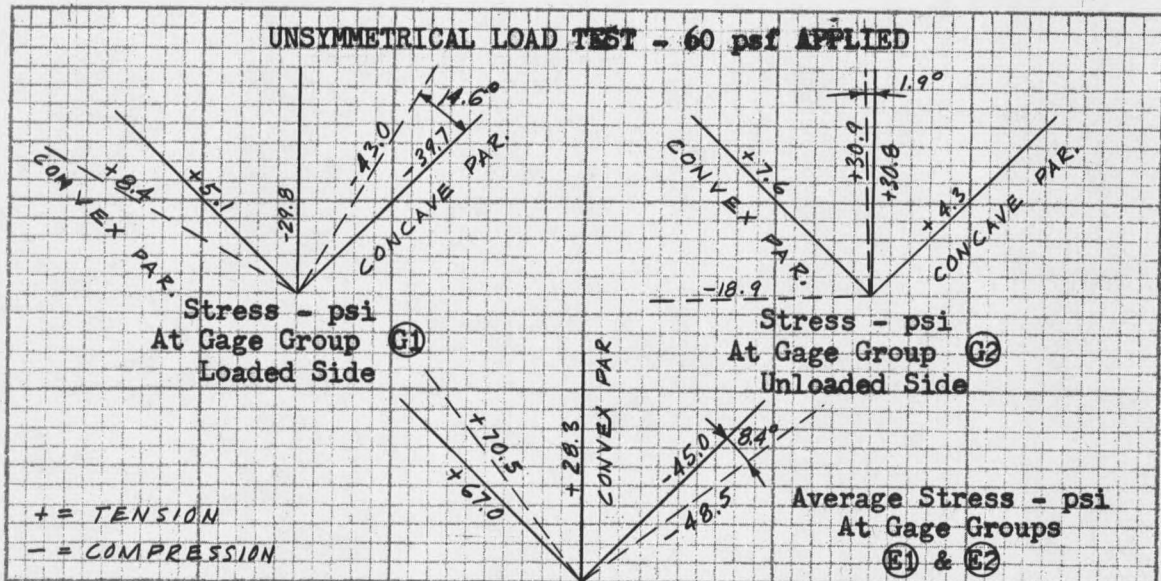


FIGURE 29

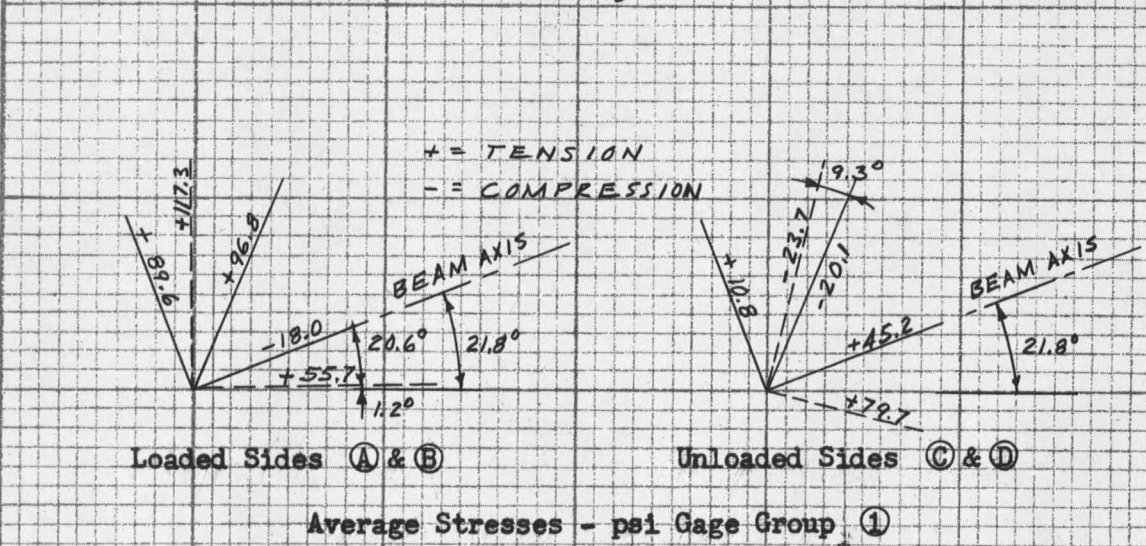
FIGURE 30





DIAPHRAM ROSETTES

FIGURE 31



EDGE BEAM ROSETTES AT THE BUTTRESSES

FIGURE 32

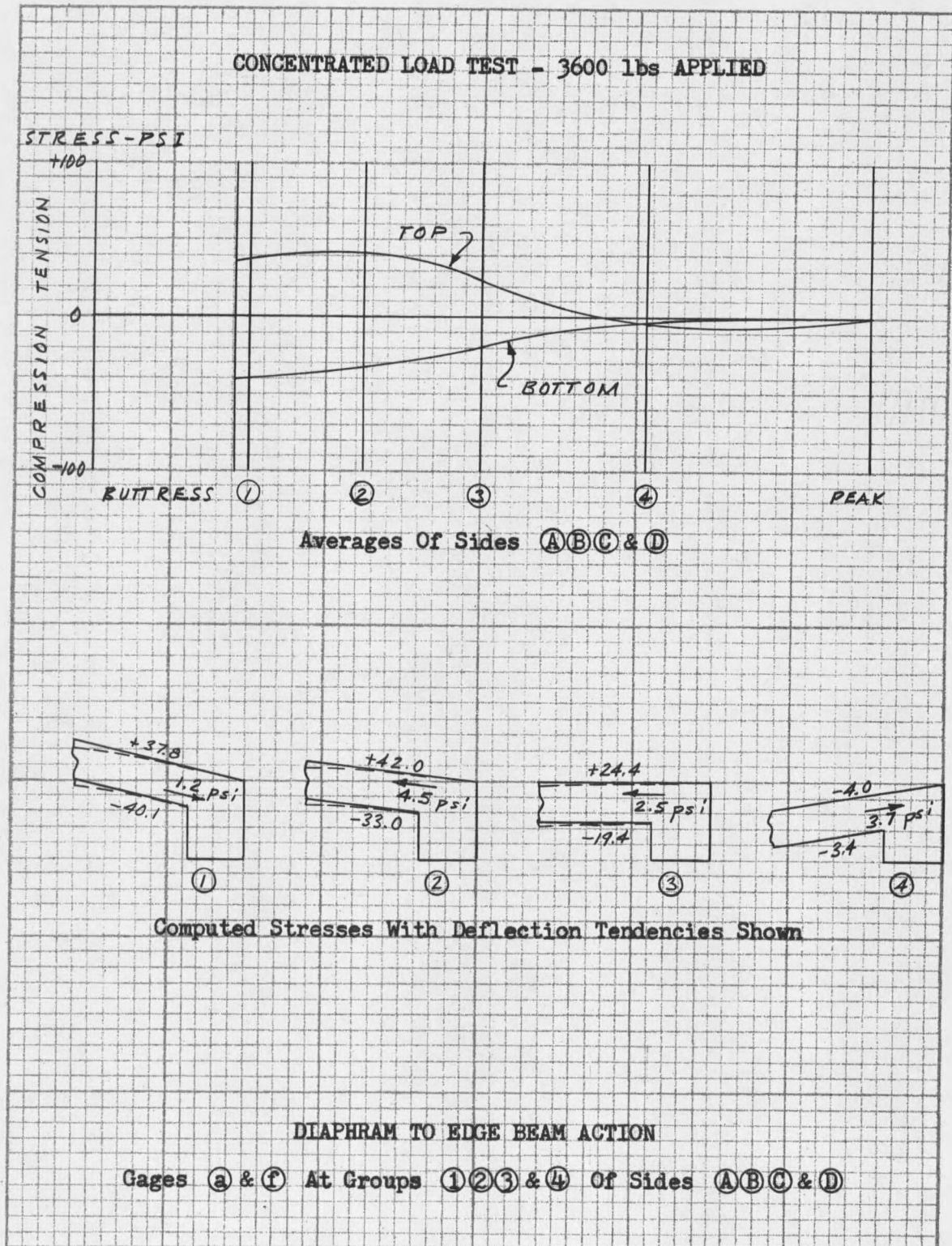


FIGURE 33

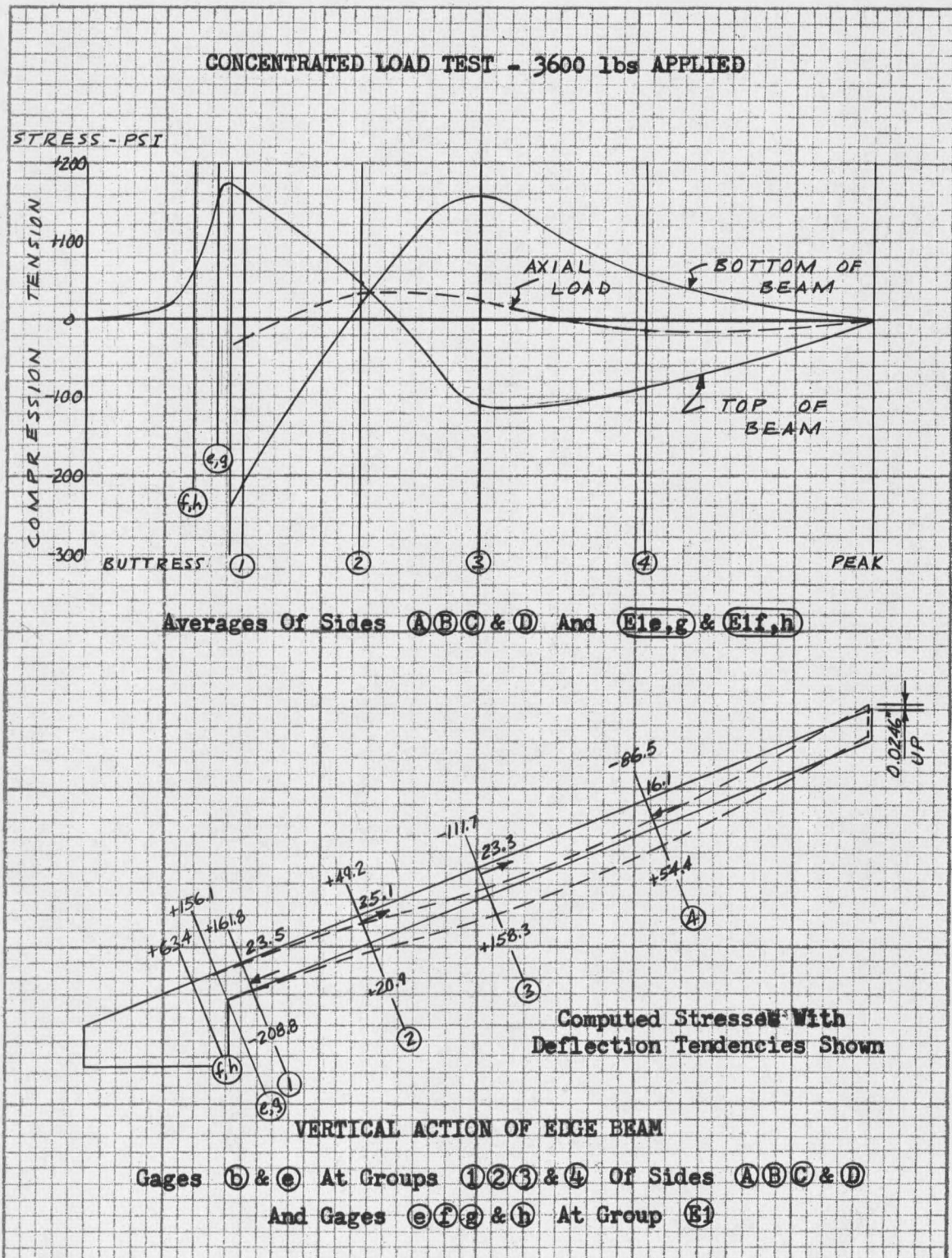


FIGURE 34

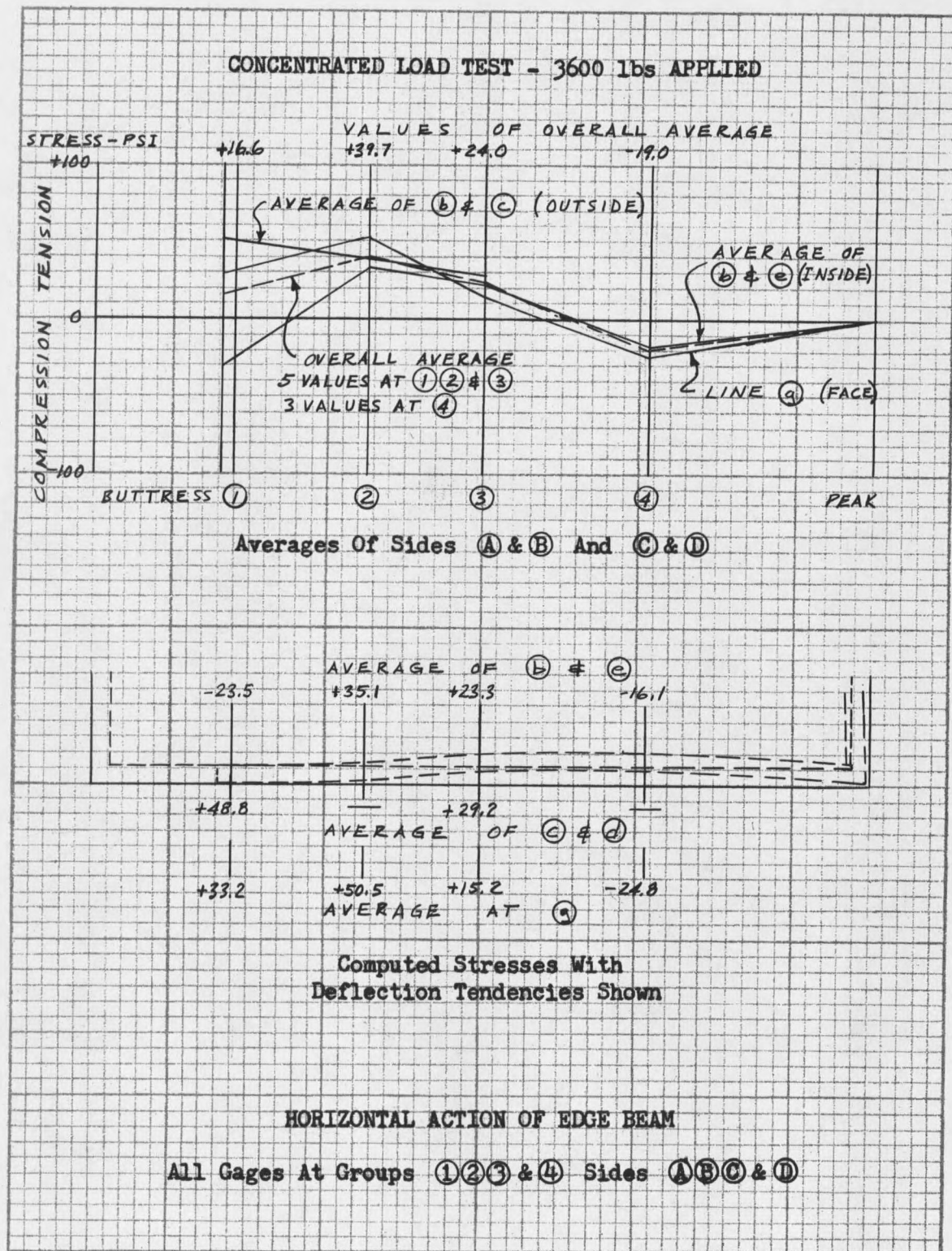


FIGURE 35

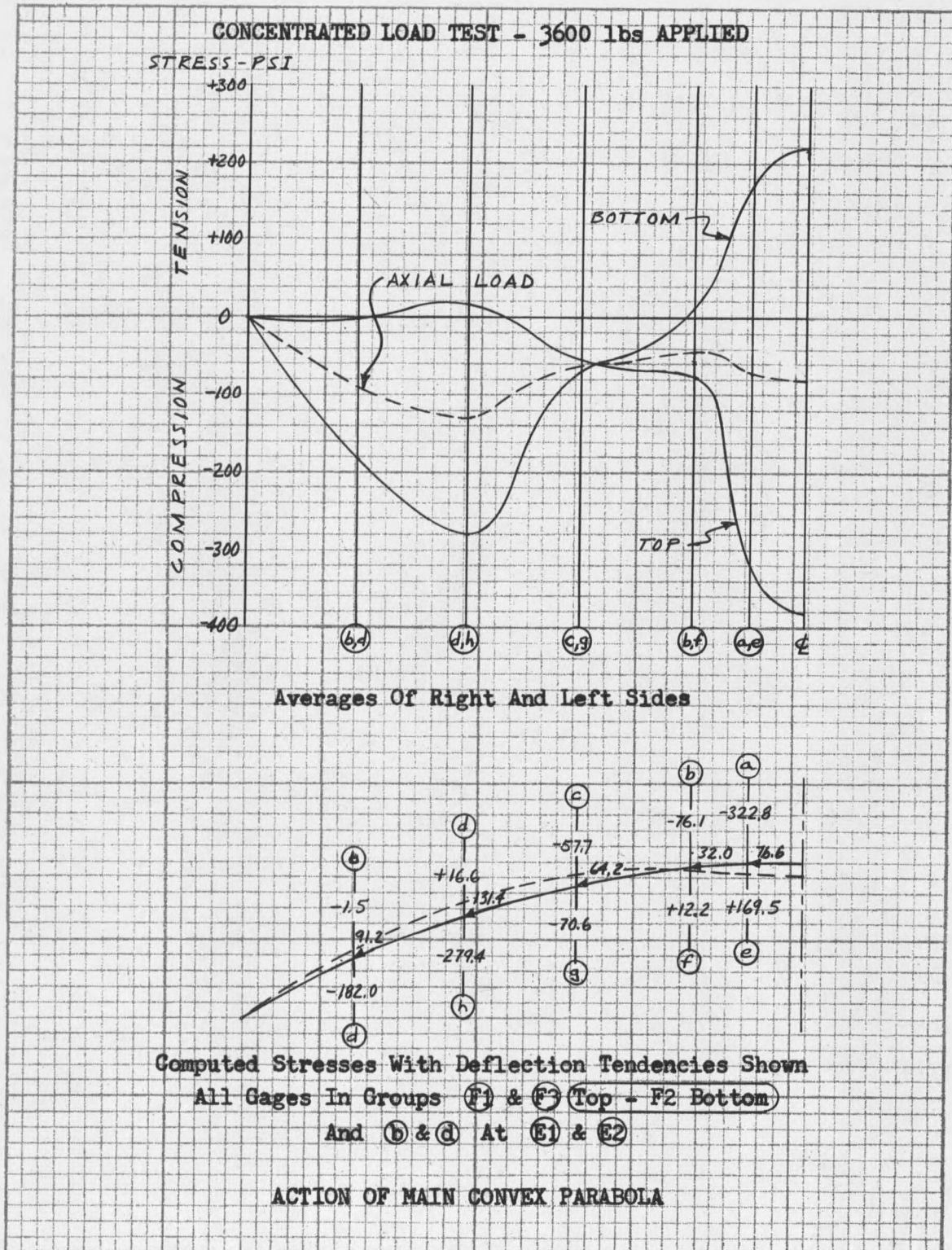


FIGURE 36

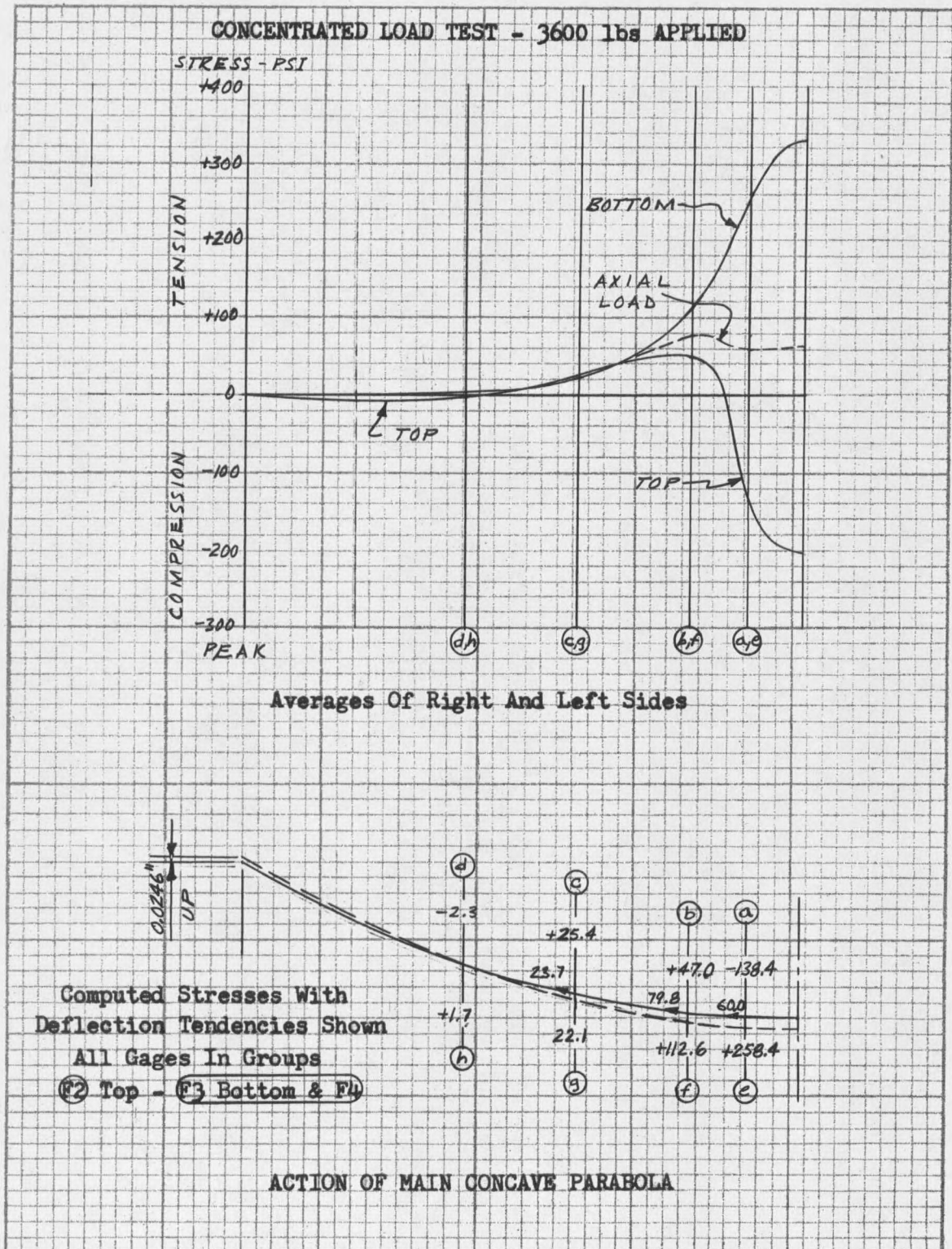


FIGURE 37

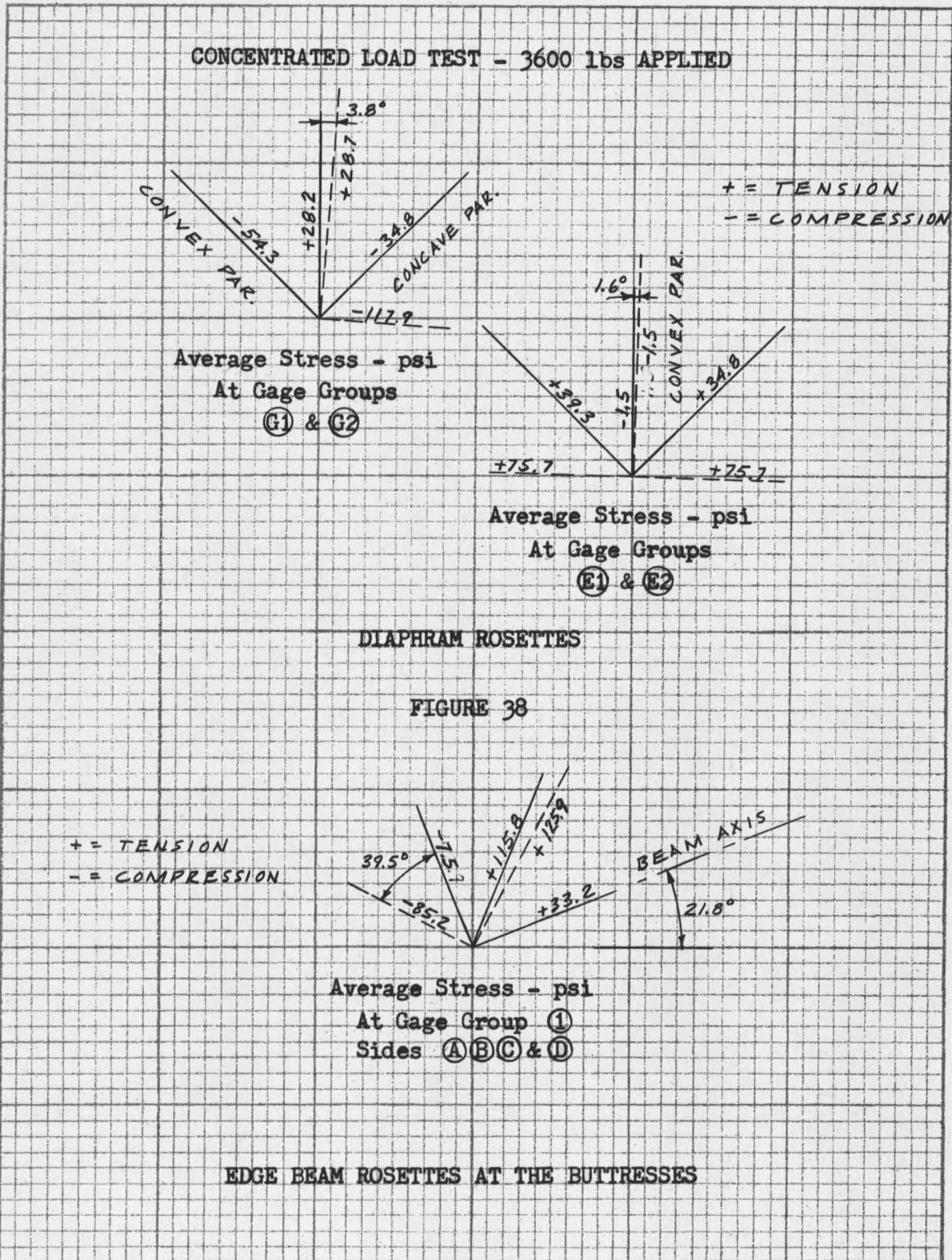


FIGURE 39

Discussion of Results

To facilitate a cross reference with the Graphical Presentation, this discussion follows the order used there. However, comparative statements are included without regard to this sequence. Because a uniform load is most common, the actions of the model under unsymmetrical and concentrated loads are compared with its actions under uniform load.

Uniform Load

Diaphragm to Edge Beam Action, Fig. 18, p. 69: (This concerns action perpendicular to the edge beams — not parallel to the planes of the parabolas.) As indicated by the graph, some moment existed over nearly the entire edge length. The magnitude of the moment was very small except in the vicinity of the stiff buttresses. Here the measured bending stress reached +70 psi and -63 psi at a point where the stress was increasing rapidly, indicating that much higher stresses existed than were measured. This bending moment corresponds to Vertical Action of Edge Beams, i.e., where the edge beam deflected upward, tension existed in the bottom of the diaphragm, while at the buttresses where the edge beam deflected downward the tension was in the top of the diaphragm. Surprisingly, a net tension action of the diaphragm on the edge beam existed over its entire length, reaching a maximum value of +9.3 psi at the center of each edge. The existence of this force is substantiated by Horizontal Action of Edge Beams where it is shown that the edge beams tend to bend inward.

Vertical Action of Edge Beams, Fig. 19, p. 70: It is indicated in the general theory that a lineally accumulative axial load existed at the

edge beam in the form of shear from the diaphragm. The tests confirmed this with the exception of a deviation from the expected straight line accumulation at the peak and at the center where the rate of accumulation decreased. Because this shear accumulated in the upper half of the edge beam, an eccentric axial load existed that bent the beam upward. Approaching the buttresses, this effect was counteracted by an apparent cantilevering effect of the beam. Here the bending stresses reached their maximum value of -380 psi and +150 psi. To account for this reversed bending action, it must be assumed that the diaphragm carried a vertical shear as well as an axial load into the edge beam. Further, this shearing force was a major contributor to the action of the beam. After the faces of the buttresses were reached the stress dropped rapidly. At a point slightly over 4 in. from the face, the stress on top reduced to only $1/3$ the maximum value.

Horizontal Action of Edge Beams, Fig. 20, p. 71: Contrary to available theory, forces existed that tended to bend the edge beams in near the center of their length. This was first apparent in Diaphragm to Edge Beam Action with the existence of net tensile forces acting on the beams. The horizontal action again indicated this movement. Near the center of the beams, the gages on the outer part of the beam measured greater compressive stresses than did those on the inner part, -70 psi compared to -54 psi. A cantilevering effect at the buttresses reversed this to -65 psi on the outside compared with -128 psi on the inside. Stresses on the far outside face (gages subscripted g) very nearly comply to this idea, but be-

cause these gages were placed on only two sides, not as many strains were averaged to form the final computation, making these calculations less refined than the others.

Action of Main Convex Parabola, Fig. 21, p. 72: If the parabolic arch had been free to rotate at the ends, it is entirely possible that the stress action would have been primarily axial with little or no bending — agreeing with the concept presented in the theoretical analysis. But with the buttresses providing full fixity, bending stresses were created reaching a maximum of -91 psi and +53 psi. Although the graph indicates that the stress was increasing at this point, it is believed that the gages were located near the point of maximum bending. Axial load accumulated nearly uniformly to a point approximately $\frac{3}{5}$ the distance from the center. As this point is nearly 90 degrees from the face of the buttress, it is logical that loads would start dispersing, seeking the easiest path into the support.

Action of Main Concave Parabola, Fig. 22, p. 73: In contrast to the convex parabola the tests indicated little bending in the concave parabola. There appeared to be some inconsistency in the test results, but since the graph presents an average of two sides and those two sides had very nearly the same results, it is possible that some unusual action occurred. Assuming the trend line is accurate, the accumulation of axial load has two interesting features. First, it reached a maximum value at the center of the saddle, whereas the axial load in the convex parabola was zero at this point. Secondly, it approached zero at the peak. The stress levels of the perpendicular parabolas were expected to be equal

and of opposite sign — each increasing toward the edge beams, but the increase in one accompanied by a decrease in the other still produces a rational pattern. Also, the zero stress at the peak conforms to the accumulative axial load in the edge beams as pictured in Vertical Action of Edge Beams. Since neither curve is supported by test results near the peak, either or both graphs may be in error.

Diaphragm Rosettes, Fig. 23, p. 74: At gages G1 and G2 the direction of principal stress was only 5.8 degrees from the expected direction, i.e., 5.8 degrees from the planes of the concave and convex parabolas. This small value could be the result of errors. The values of the stress were +27 psi in the concave parabola and -10 psi in the convex one. These values disagree with the expected equality of stress but agree with the plottings of the main parabolas. The gages E1 and E2 had precisely the expected direction of principal stress. As indicated in Action of Convex Parabola, the stress in the top surface (+53 psi) was caused by bending action. The minor stress was -17.2 psi.

Edge Beam Rosettes at the Buttresses, Fig. 24, p. 74: It was mentioned in Vertical Action of Edge Beams that vertical shear was a major factor in producing the actions observed there. The direction of the principal stresses at the edge beam rosette substantiate this, as the major stress is 35.6 degrees from the axis of the beam.

Unsymmetrical Load

Diaphragm to Edge Beam Action, Fig. 25, p. 75: (This concerns action perpendicular to the edge beams — not parallel to the planes of

the parabolas.) The strain measurements of this test indicated (just as in the uniform load test) that a net tension force existed between the diaphragm and the edge beam and that bending stresses existed between the diaphragm and edge beam, but were small except near the buttresses. The magnitude of the stresses was increased considerably on the loaded side, e.g., net tension at mid-beam was +41.1 psi and maximum bending stress at the buttress was -153 psi compared with +9.3 psi and +70 psi, respectively, during uniform load. On the unloaded side stresses were almost non-existent in the upper half. At the buttresses the bending stress reached a maximum of +125 psi. A maximum net tension of +42.5 psi was reached near the buttress, whereas the loaded side and the sides under uniform load had their maximum values at the center.

Vertical Action of Loaded Edge Beams, Fig. 26, p. 76: The graph of this action is very similar to the graph of the same action under uniform load although the magnitudes vary. The maximum bending stresses at the buttresses were +380 psi and -700 psi as compared with +150 psi and -380 psi during uniform load. Bending stresses in the upper portions of the beam due to eccentric axial action were also larger. The reason for this is not apparent because the axial load in this portion was lower instead of higher. The axial load accumulated lineally in the upper half of the beam's length, reaching -38 psi at the center. From the center the accumulation was at an increasing rate, reaching -158 psi, 4 in. from the buttresses compared with -128 psi, 4 in. from the buttresses during uniform load.

Vertical Action of Unloaded Edge Beams, Fig. 27, p. 77: As expected the direction of bending of the unloaded edge beams was opposite that of the

loaded edge beams, due to the torsion of the load about the buttresses. Also, the magnitude of the bending stresses was lower for the unloaded side, reaching +400 psi and -195 psi as compared with +380 psi and -700 psi for the loaded side. The ability of the edge beams to act as free cantilevers is shown by this reduction in stress from one side to the other. The accumulative tension axial stresses reached +92 psi four inches from the buttresses. It is interesting to note that the net compressive stress at the buttress under unsymmetrical load (+66 psi) is almost exactly half the stress under uniform load (+128 psi). As half the load was applied, half the stress was expected.

Horizontal Action of Edge Beams, Fig. 28, p. 78: (A) Loaded Sides; Disregarding line c-d, the edge beam again appears to have deflected inward. This is shown by greater compressive stresses on the outside of the beam than on the inside for the upper portions of the beam, followed by greater compressive stress on the inside of the beam as the buttress is approached. Line c-d is drawn through calculated values only at points 1 and 3. The value at point 1 is in agreement with the other data while the value at point 3 contradicts it. If this one discrepancy can be overlooked due to the extent of the other data, it seems safe to say that inward deflection occurred. (B) Unloaded Sides; Because there is no general trend of inside stresses being greater or lesser than outside stresses, it must be assumed that the edge beams on the unloaded sides remained essentially straight. This corresponds with the results of the Diaphragm to Edge Beam Action where the net stress between the diaphragm and the edge beam was very small over most of the length of the beam.

Action of Main Convex Parabola, Fig. 29, p. 79: The most important feature of this action is that tension can exist in the convex parabolas. Apparently across this line of the structure the diaphragm acts as a tie-down, helping to resist the overturning action of the applied load. It is probable that this tie-down effect reaches its maximum value in the vicinity of the face of the buttresses, i.e., a convex parabolic line about 14 in. off center towards the unloaded side would have maximum tensile stress.

Action of Main Concave Parabola, Fig. 30, p. 80: The warping of the diaphragm in this plane was exactly as expected, i.e., the loaded side bent into a tighter curve, causing increased tensile stress on the bottom, while the unloaded side was pulled straighter, causing increased tensile stress to occur on the top. As was noted under Uniform Load, it is only assumed that the stress diminished to zero at the boundaries. Because this idea fits the known points of stress so well, the idea seems well founded. At equal distances from the center of the model, a change in axial stress in the concave parabola is accompanied by a compensating change in axial stress in the convex parabola. As the force in the convex parabola appears to be a reactive action, this is a logical result. During the uniform load test, both lines of load support were additive, creating an effect opposite to that of the unsymmetrical load test.

Diaphragm Rosettes, Fig. 31, p. 81: A marked difference resulted between the G gages of the loaded side and the G gages of the unloaded side. On the loaded side the direction of principal stress was similar to that caused by a uniform load (8.4 degrees from parabolic line as compared with 5.8 degrees under uniform load), but the values are of opposite sign.

Both the main convex and concave parabolas had a tension axial stress and apparently this was counteracted at the medium length parabolas by compressive axial stress. On the unloaded side, the direction of principal stress is nearly 45 degrees to the parabolic lines — an action similar to pure torsion and therefore very logical. The action at E1 and E2 is characteristic of both torsion and direct load.

Edge Beam Rosettes at the Buttresses, Fig. 32, p. 81: Similarities exist between the unsymmetrical load and uniform load at these gages, particularly with respect to the direction of principal stresses. The reversal of the major and minor stresses corresponds to the reversal of bending action in the edge beams.

Concentrated Load

Diaphragm to Edge Beam Action, Fig. 33, p. 82: (This concerns action perpendicular to the edge beams — not parallel to the planes of the parabolas.) Compared with the action under uniform load, relatively high bending stresses existed over the lower portion of the beams, but almost no bending existed over the upper portions. A similar effect appears in Action of Main Concave Parabola. Most of the load resistance appears to be in the lower portions of the model. Because the buttresses are fixed while the peaks are free to move, this is not illogical. Although the net forces between the diaphragm and the beams are small and may be a result of error, the major force does agree with the results obtained under uniform load, i.e., there is a tensile force tending to deflect the edge beam inward.

Vertical Action of Edge Beams, Fig. 34, p. 83: Under uniform load the magnitude of stress at the center of the edge beam was less than $1/5$ the stress at the buttress, but under concentrated load it was over $3/5$ the stress at the buttress. Because the concentration of load was in the center of the saddle, the concentration of bending stress at the center of the beam was a logical result. An unexpected phenomenon was the double reversal of accumulative axial load. The consistency of the several averaged results confirms that this is not an error, but there is no apparent explanation for such a reversal.

Horizontal Action of Edge Beams, Fig. 35, p. 84: Deflection tendencies are again revealed by the difference of stress from the inside to the outside of the edge beam. A distinct pattern is expressed using the values of line b-e and line g. Values of line c-d were obtained at only points 1 and 3. At point 1 the indication of bending is confirmed although line g was expected to be on the outside of line c-d. At point 3 a discrepancy occurs. Because the points defining lines b-e and c-d are the average of four sides, they are probably more accurate than line g which is defined by the average of two sides. Because of the reversal of stress, line c-d was not extended past point 3 and the overall average at point 4 was calculated by using the three points, b, c, and g. This reversal in stress in the horizontal direction was partially indicated by the net diaphragm-beam forces shown in Diaphragm to Edge Beam Action.

Action of Main Convex Parabola, Fig. 36, p. 85: Very high bending stresses occurred directly under the point of concentration of the load. The curves rise rapidly near the last point of strain measurement — 8 in.

from the center line. Because the load was applied over the last 6 in., it was believed that the stress level would flatten soon after passing the 8 in. point. If this was true, the maximum stress values were about +220 psi and -380 psi. The concentrated load caused the diaphragm to bend downward at the center forcing an upward bend near the quarter points. There a second major bending plane was created.

Action of Main Concave Parabola, Fig. 37, p. 86: Bending action at the center of this parabola was similar to Action of Main Convex Parabola, causing stresses near +330 psi and -210 psi. The applied load again caused the diaphragm to bend down at the center, but because the peaks were not restrained, the diaphragm was free to rotate and a second major bending point was not created. Net axial load was significant for only about 30 in. from the center of the saddle. Apparently the center convex portion of the structure supplied the major support of the load due to fixity at the buttresses and freedom at the peaks. During the uniform load test, the applied loads created a restraint at the peaks, keeping them from rotating and thus forcing an axial stress to be present over the entire length of the parabola.

Diaphragm Rosettes, Fig. 38, p. 87: If a flat circular plate is subjected to a concentrated load, principal stresses extend radially from the point of load and perpendicularly to the radial lines. The same action apparently occurred to the hyperbolic paraboloid model under a concentrated load, at least for a considerable distance from the point of application, because the directions of the principal stresses at G1 and G2 are only 3.8 degrees from a radial line through the load. This is 41.2 degrees from

the theoretical direction under uniform load. Gages E1 and E2 agreed very closely with the expected results, as they were only 1.6 degrees from the planes of the parabolas.

Edge Beam Rosettes at the Buttresses, Fig. 39, p. 87: The direction of principal stress is 27.3 degrees from vertical and 39.5 degrees from the axis of the edge beam. Vertical shear must be a major factor in determining this angle, since pure compressive stress would have caused it to be parallel to the axis of the beam. This action is very similar to the action under uniform load.

CONCLUSIONS

In General Analysis, page 21, the horizontal force per foot in the edge beam of a hyperbolic paraboloid was found to be:

$$S = wab/2h \quad (8b)$$

If the action of the test model conformed to this theory, the change in edge beam compression per horizontal foot with a 60 psf uniform load applied would have been:

$$\begin{aligned} F &= (wab/2h) (\sqrt{a^2 + h^2}/a) \\ F &= [(60)(5)(5)/2(2)] [\sqrt{(5)^2 + (2)^2}/5] \\ F &= 403 \text{ plf} \end{aligned}$$

Four inches from the face of the buttress, the total compressive force would have been:

$$F_T = (403)(7.83) = 3,160 \text{ lbs}$$

Using 3 in. x 4 in. = 12 sq in. as the edge beam area, the unit stress would have been:

$$\sigma = 3,160/12 = 263 \text{ psi}$$

The value obtained from the test data for the edge beam stress, 4 in. from the buttress, was 128 psi — almost exactly 1/2 the theoretical value. Two factors must account for this discrepancy. The first is that some undeterminable amount of the edge of the shell was acting as part of the edge beam, e.g., if 6 sq in. of shell were effective, the theoretical unit stress would be reduced to 175 psi. The second is that the boundary

conditions redistributed the load paths, causing the axial stress in the edge beam to be reduced.

Each graph presented in the test data had several features contradicting the simplified General Analysis. Many of these contradictions appear to have been caused by one or more of the points mentioned in Discussion of Analysis. Three major points, with comments, are noted for the uniform load test:

1. Normal forces and bending moments existed between the shell and the edge beam: movement of the edge beam and/or warping of the shell caused the increment arches to have different horizontal reactions.
2. Edge beams deflected both horizontally and vertically: shear, eccentric vertically and horizontally to the beam, applied bending moments; warping of the shell caused the increment arches to have unequal horizontal reactions creating vertical shears and forces normal to the edge beam.
3. The shell had bending moments and non-theoretical axial forces: variable degrees of fixity at the edge beams and movement of the edge beams destroyed the two-hinged arch theory; warping of the shell in one spot affected other spots.

The same comments can be applied to the data for the unsymmetrical and concentrated load tests. During these tests the variation from General Analysis was even greater. The reason was: two-hinged parabolic arches do not have zero bending moments when loads other than uniform are applied.

I believe two conditions caused most of the redistribution of load in the test model. First, a large center portion of the convex surface acted like a strong fixed arch, both because the buttresses were quite wide and fully fixed, and because the shell was relatively thick compared to its span, rendering it capable of resisting a rather large moment. Second, bending moment and vertical shear were created in the edge beams when they deflected under the loads caused by the eccentrically applied shears, and by the warped shell which provided forces normal to the edge beams and axial loads non-parallel to the axes of the beams.

The first point played an extremely important part in the concentrated load test because the load was all applied to this effective fixed end arch. By its action as a tension tie-down, and undoubtedly a compressive arch, this center portion also played an important part in the unsymmetrical load test. In the uniform load test, the moment created in this portion was sufficient to eliminate axial thrust at the center -- a very good demonstration of its effectiveness.

The value of the second point was best shown during the unsymmetrical load test. At the face of the buttresses on the loaded side, the applied loads caused a bending moment of 46,000 in. lbs. The two 3 in. x 4 in. edge beams on that side resisted 5,300 in. lbs in bending alone. Although this moment was the largest recorded for the various tests, all had sizeable bending moment values. In all three tests the strain gage rosettes near the buttresses clearly indicated the presence of vertical shear.

The smoothness of most stress curves and the general consistency within individual tests and between tests indicate that the many problems

which complicated the gathering of valid test data were solved, providing valuable, accurate information regarding the entry of loads into the edge beams of thick concrete hyperbolic paraboloids.

If the model had a more common shell thickness to span ratio, it is probable that the results observed under uniform load would have had only small deviations from the General Analysis. The theory presented there employed membrane analysis -- an invalid method when boundary conditions play a major role in the load distribution as they do when spans are small and thicknesses large. Although unsymmetrical and concentrated loads are encountered in practice, they are usually quite small in structures utilizing shell construction. Thus, because of the size ratios of the model and the loadings used, the data is primarily of academic value. However, in addition to providing interesting comparisons between actions under a concentrated or unsymmetrical load and a uniform load, the tests are a forewarning of problems which would be encountered if thick small hyperbolic paraboloids were used.

RECOMMENDATIONS FOR FURTHER STUDY

The existing test model could be used for further studies by jacking one buttress vertically, spreading the buttresses axially and eccentrically, and by applying a vertical bending moment to the buttresses by sandbagging the footing. Should destruction tests be made, it is suggested that they be conducted using buttress movements or an unsymmetrical concentrated load. The strength of the model makes destructive uniform loads impractical.

After more tests have been run on the model, it would be interesting to have all results analyzed by more complete theoretical methods. By the time this could be accomplished, it is possible that these complicated analyses will have been simplified by graphs and charts or that a computer program could be run in the campus computer laboratory.

If other test models are built, I recommend that every effort be made to see that the size ratios conform to those of general construction. And if such models are made, I further recommend that more complete information be recorded and kept, e.g., concrete test cylinders. This should be done even at the sacrifice of testing time.

APPENDIX

Sample Calculation Method

The data and computations presented here are for Gage G2a, attached to switching-balancing unit 8, switch 2. The tests were tests A and B of the Unsymmetrical Load Test.

Strain Readings, Test A:

Load, psf	Strain, micro in./in.
0	1011
10	1012
20	1008
30	1011
40	1009
50	1009
60	1011
60	1010
50	1010
40	1010
30	1008
20	1004
10	1002
0	1000

Strain Readings, Test B:

Load, psf	Strain, micro in./in.
0	982
10	979
20	981
30	982
40	981
50	982
60	987
60	983
50	986
40	979
30	979
20	968
10	975
0	974

Data of Test A was plotted as originally read. Data of Test B was plotted with two modifications: (1) the reading 968 at 20 psf unloading was discarded, (2) the reading 982 at 0 psf loading was arbitrarily lowered 4 micro in./in. to 978. Both modifications were made for all Test B readings from unit 1, switch 1 through unit 8, switch 6 as explained in Summary of Calculation Methods. This data, with the noted modifications, is presented graphically in Figure 40 on the next page.

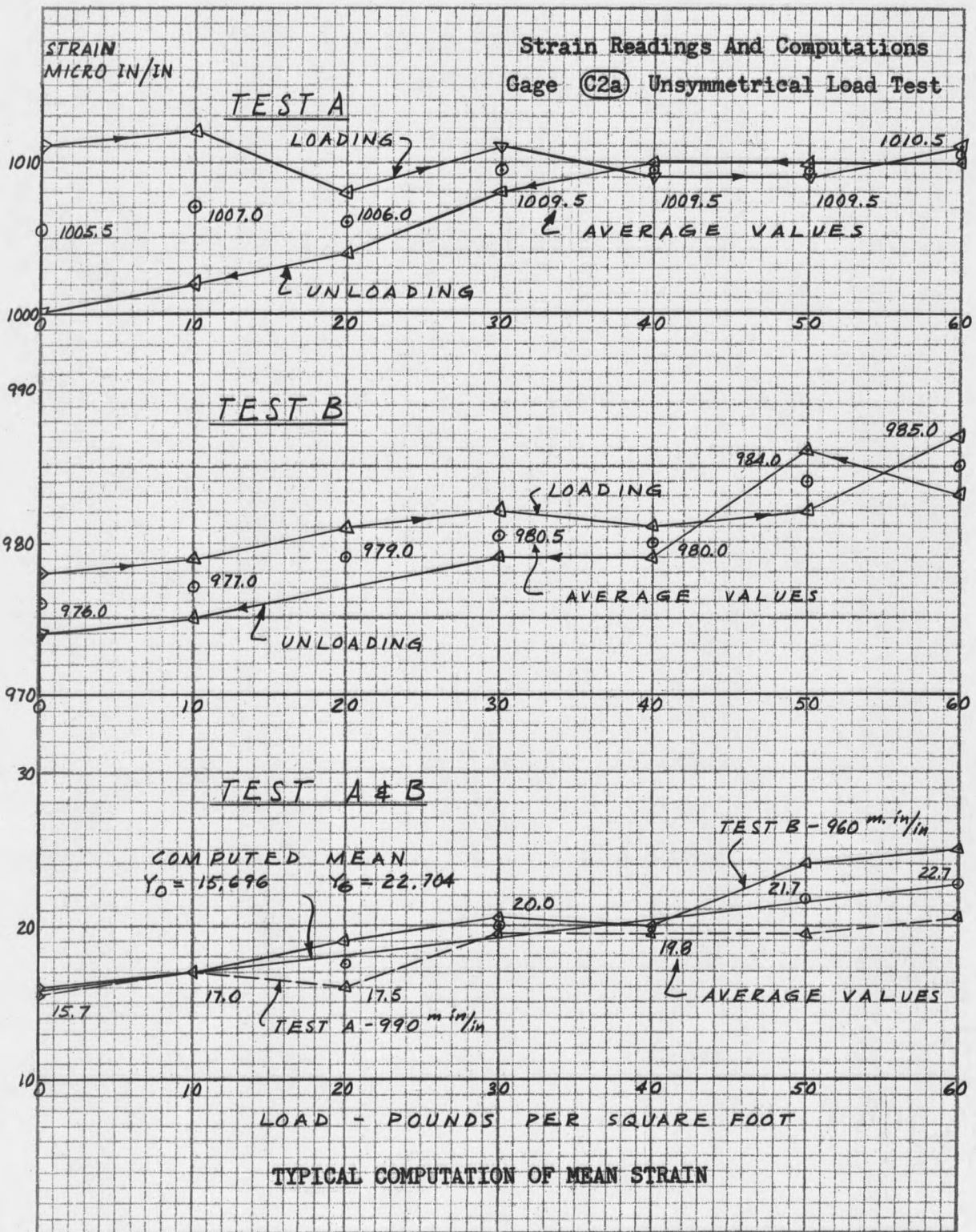


FIGURE 40

When the gage strain was read during both Tests A and B, to simplify the computation the initial average strains were modified by a constant factor before the final average was taken. The computer was given these modified strain averages. When the gage strain was read in only Test A or B, the computer was given the real averages.

Data given the IBM 650 Computer¹:

$Y_0 = 15.7$ micro in./in.	$Y_4 = 19.8$ micro in./in.
$Y_1 = 17.0$ "	$Y_5 = 21.7$ "
$Y_2 = 17.5$ "	$Y_6 = 22.7$ "
$Y_3 = 20.0$ "	IDENT = 0238203201

The identification is a code describing gage factor, test, wiring and gage. Numbering the digits 1 through 10 from right to left, the code reads:

1. Digits 1 and 2; Gage subscript

1 = a	7 = g
2 = b	8 = h
3 = c	9 = i
4 = d	10 = j
5 = e	11 = k
6 = f	12 = l

2. Digit 3; Gage number

1 = 1	3 = 3
2 = 2	4 = 4

3. Digit 4; Gage area location

1 = A	5 = E
2 = B	6 = F
3 = C	7 = G
4 = D	

4. Digit 5; Not used

¹See Summary of Calculation Methods for terminology and other explanations.

5. Digit 6; Switch number of switching-balancing unit

1 = 1	6 = 6
2 = 2	7 = 7
3 = 3	8 = 8
4 = 4	9 = 9
5 = 5	10 = 0

6. Digit 7; Number of switching-balancing unit

1 = 1	6 = 6
2 = 2	7 = 7
3 = 3	8 = 8
4 = 4	9 = 9
5 = 5	10 = 0

7. Digit 8; Subtest

1 = A
2 = B
3 = A & B

8. Digit 9; Test

1 = Uniform Load Test
2 = Unsymmetrical Load Test
3 = Concentrated Load Test

9. Digit 10; Gage Factor

0 = 2.02
1 = 2.06
3 = 2.07

Using this notation, the identification number 0238203201 means:

1. Gage factor is 2.02	(Digit 10 = 0)
2. Test A & B of Unsymmetrical Load Test	(Digit 9 = 2)
	Digit 8 = 3)
3. Wiring Hook-Up is unit 8, switch 2	(Digit 7 = 8)
	Digit 6 = 2)
4. Gage is C2a	(Digit 4 = 3)
	Digit 3 = 2
	Digits 1
	and 2 = 1)

Computed by the IBM 650 Computer:

$$\begin{aligned}
 Y &= 134.400 \text{ micro in./in.} \\
 A &= \sum Y/N = 134.4/7 = 19.200 \text{ micro in./in.} \\
 XY &= 32.701 \text{ micro in./in.} \\
 B &= \sum XY/X^2 = 32.701/28 = 1.168 \text{ micro in./in./unit of x} \\
 Y_0 &= A - 3B = 19.200 - 3.504 = 15.696 \text{ micro in./in.} \\
 Y_0^2 &= A + 3B = 19.200 + 3.504 = 22.704 \text{ micro in./in.}
 \end{aligned}$$

$$\begin{aligned}6B &= 6 \times 1.168 = 7.008 \text{ micro in./in./6 units of } x \\S &= EB = (3.44 \times 10^6)(1.168 \times 10^6) = 4.017 \text{ psi/unit of } x \\6S &= 6 \times 4.007 = 24.102 \text{ psi/6 units of } x \\1 \text{ unit } x &= 10 \text{ psf}\end{aligned}$$

Data returned by the IBM 650 Computer:

$$\begin{array}{ll}A = 19.2000 & 6B = 7.0072 \\B = 1.1679 & S = 4.0174 \\Y_0 = 15.6964 & 6S = 24.1046 \\Y_6 = 22.7036 & IDENT = 0238203201\end{array}$$

Gage D2a is symmetrical to Gage C2a. The total stress, 6S, for D2a was 18.060 psi / 60 psf load. Therefore the average stress at the symmetrical points was 21.08 psi / 60 psf load. The values for average total stress at symmetrical points were used in the diagrams in this report.

Figure 41 on the following page presents the computed stress during uniform load at gages b and e for each of the four sides as well as the average stress of these symmetrical points. The variance of the values of stress is typical and is presented to indicate the importance of making this final average.

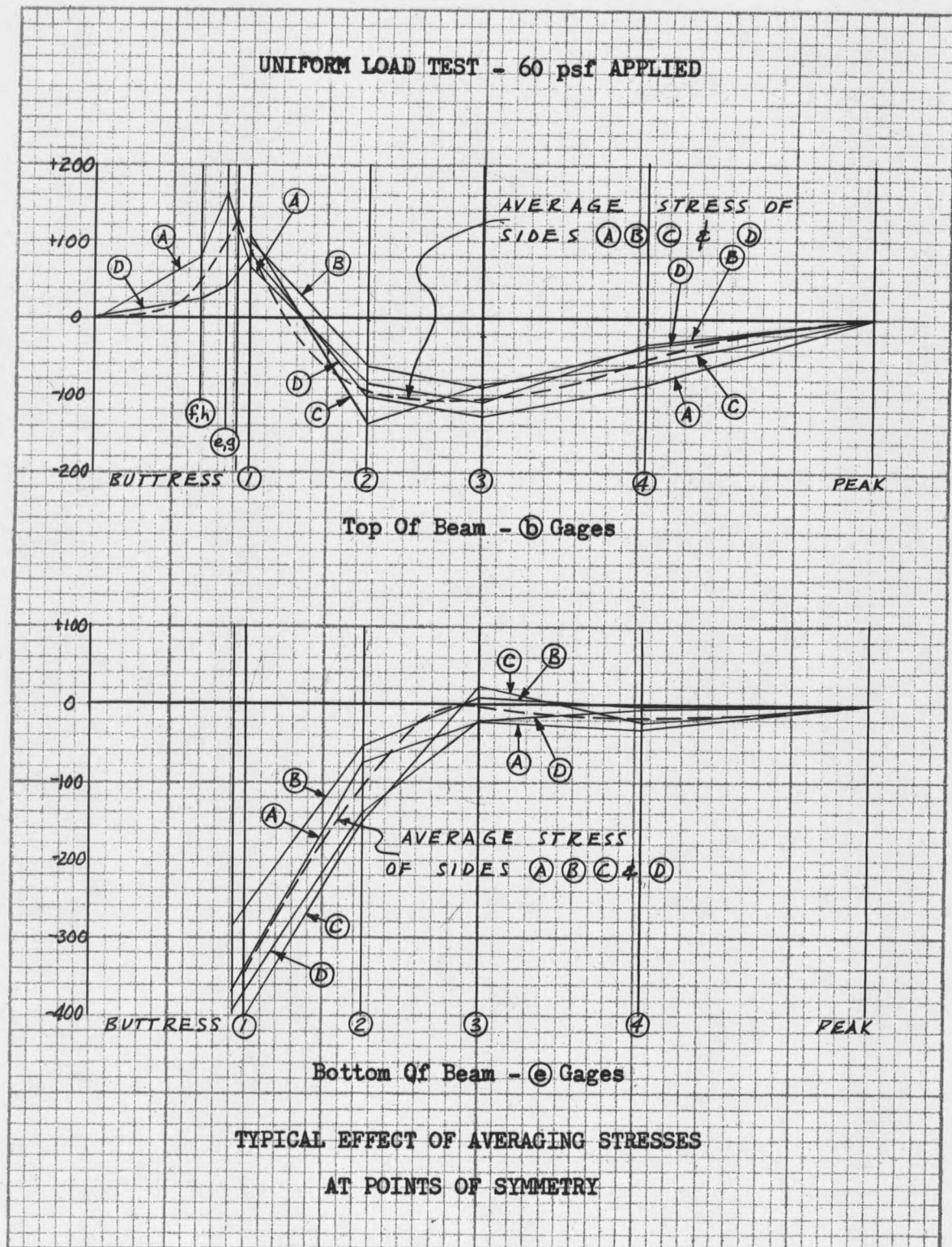


FIGURE 41

IBM 650 PROGRAM

SOAP II PROGRAM

BASIC LANGUAGE

LOC. OF INST.	INSTRUCTION			REMARKS	INST NO.	LOC. OF INST.	INSTRUCTION		
	OP CODE	DATA LOC.	INST. LOC.				OP LOC.	DATA LOC.	INST. LOC.
	REG	R1951	1960	READ ZONE	1	0000	00	0000	0000
	REG	P1927	1934	PUNCH ZONE	2	0000	00	0000	0000
	SYN	START	0000		3	0000	00	0000	0000
SEVN	70	0000	0000	N TERMS	4	0050	70	0000	0000
SUMX	28	0000	0000	SUMMATION X SQUARED	5	0100	28	0000	0000
SIX	00	0000	0006	6 LOAD INCREMENTS	6	0150	00	0000	0006
ONE	00	0000	0001	G. F. TEST	7	0200	00	0000	0001
AR1	98	0580	0000	G. F. 2.06 CORRECTION	8	0250	98	0580	0000
A1	97	5850	0000	G. F. 2.07 CORRECTION	9	0300	97	5850	0000
E	00	0000	0344	MODULUS OF ELASTICITY	10	0350	00	0000	0344
START	RDI	R0001		READ DATA CARD	11	0000	70	1951	0006
	RAU	R0001			12	0006	60	1951	0005
	AUP	R0002			13	0005	10	1952	0007
	AUP	R0003			14	0007	10	1953	0057
	AUP	R0004		SUMMATION OF	15	0057	10	1954	0009
	AUP	R0005		Y TERMS - FORM	16	0009	10	1955	0059
	AUP	R0006		5-5	17	0059	10	1956	0011
	AUP	R0007			18	0011	10	1957	0061
	DVR	SEVN		$\Sigma Y/N$ - FORM 4-6	19	0061	64	0050	0111
	SRD	0001		FORM 5-5	20	0111	31	0001	0017
	STL	P0001		A IN WORD 1	21	0017	20	1927	0030
	RSU	R0001			22	0030	61	1951	0055
	SUP	8001			23	0055	11	8001	0013
	SUP	8001		$X_0 Y_0$	24	0013	11	8001	0021
	SUP	R0002			25	0021	11	1952	0107
	SUP	8001		$X_1 Y_1$	26	0107	11	8001	0015
	SUP	R0003		$X_2 Y_2$	27	0015	11	1953	0157
	AUP	R0005		$X_4 Y_4$	28	0157	10	1955	0109
	AUP	R0006			29	0109	10	1956	0161
	AUP	8001		$X_5 Y_5$	30	0161	10	8001	0019
	AUP	R0007			31	0019	10	1957	0211
	AUP	8001			32	0211	10	8001	0069
	AUP	8001		$X_6 Y_6$	33	0069	10	8001	0027
	DVR	SUMX		$\Sigma XY / X^2$ - FORM 3-7	34	0027	64	0100	0261
	SRD	0002		FORM 5-5	35	0261	31	0002	0119
	STL	P0002		B IN WORD 2	36	0119	20	1928	0031
	RAL	R0008		IDENTIFICATION	37	0031	65	1958	0063
	STD	P0008		IDENT. IN WORD 8	38	0063	24	1934	0037
	SLT	0001		} GAGE FACTOR TEST	39	0037	35	0001	0043
	NZU		OKAY	} ZERO IN UPPER IS 2.02	40	0043	44	0047	0048

SOAP II PROGRAM

BASIC LANGUAGE

LOC. OF INST.	INSTRUCTION			REMARKS	INST NO.	LOC. OF INST.	INSTRUCTION		
	OP CODE	DATA LOC.	INST. LOC.				OP	DATA LOC.	INST. LOC.
ZERO	SUP	ONE		} GAGE FACTOR TEST	41	0047	11	0200	0105
	NZU		ZERO		42	0105	44	0159	0010
	RAU	P0002		} MODIFY B FOR	43	0159	60	1928	0033
	MPY	A1			44	0033	19	0300	0207
	STU	P0002	OKAY	MODIFIED B IN WORD 2	45	0207	21	1928	0048
	RAU	P0002		} MODIFY B FOR	46	0010	60	1928	0083
	MPY	AR1			47	0083	19	0086	0257
	STU	P0002	OKAY	MODIFIED B IN WORD 2	48	0257	21	1928	0048
	RAL	P0002		} Y ₆	49	0048	65	1928	0133
	ALO	8001			50	0133	15	8001	0041
OKAY	ALO	8001		} Y ₆	51	0041	15	8001	0049
	ALO	P0001			52	0049	15	1927	0081
	STL	P0004		Y ₆ IN WORD 4	53	0081	20	1930	0183
	RAU	P0002		} 6B	54	0183	60	1928	0233
	MPY	SIX			55	0233	19	0150	0071
	STL	P0005		6B IN WORD 5	56	0071	20	1931	0034
	RSL	8001		} Y ₀	57	0034	66	8001	0091
	ALO	P0004			58	0091	15	1930	0035
	STL	P0003		Y ₀ IN WORD 3	59	0035	20	1929	0032
	RAU	P0002		} S IN FORM 3-7	60	0032	60	1928	0283
	MPY	E			61	0283	19	0350	0121
	SRD	0002		FORM 5-5	62	0121	31	0002	0029
	STL	P0006		S IN WORD 6	63	0029	20	1932	0085
	RAU	8001		} 6S	64	0085	60	8001	0093
	MPY	SIX			65	0093	19	0150	0171
	STL	P0007		6S IN WORD 7	66	0171	20	1933	0136
	WRI	P0001	START	PUNCH AND LOOP	67	0136	71	1927	0000

LITERATURE CITED AND CONSULTED

- Anderson, Arthur R. "Precast Prestressed Stadium Floats on Hyperbolic Paraboloids," Engineering News-Record, Vol. CLXIV, n. 7, February 18, 1960.
- Apeland, Kristoffer. "Stress Analysis of Transitional Shell," Journal of Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, LXXXVII, No. EM1, February 1961.
- Bleich, H. H. and Salvadori, M. G. "Bending Moments on Shell Boundaries," Journal of Structural Division, Proceedings of the American Society of Civil Engineers, LXXXVIII, No. ST8, October 1959.
- Building Code Requirements for Reinforced Concrete. ACI 318-56, Detroit, Michigan: American Concrete Institute, 1956.
- Concrete and Construction Engineering. London: Concrete Publications Ltd.
- Croxton, F. E. and Cowden, D. J. Practical Business Statistics. New York: Prentice-Hall, Inc., 1952.
- Dissertation Abstracts. Ann Arbor, Michigan: University Microfilms, Inc., Vol. 16-22, January 1956 - October 1960.
- Doctoral Dissertations Accepted by American Universities. The Association of Research Libraries, New York: H. W. Wilson Co., No. 20-22, 1952-1955.
- Elementary Analysis of Hyperbolic Paraboloid Shells. Reinforced Concrete Bulletin 35, Chicago, Illinois: Portland Cement Association, 1960.
- Hrennikoff, A. "Incremental Compression Test for Cement Research," Proceedings of the American Society of Civil Engineering, Journal of Engineering Mechanics Division, LXXXIV, No. EM2, April 1958.
- Lin, T. Y. Prestressed Concrete Structures, New York: John Wiley & Sons, Inc., 1955.
- Nottingham, Dennis. "Experimental Testing of a Saddle Type Hyperbolic Paraboloid Using Three Different Load Conditions". Unpublished Master's thesis, Department of Civil Engineering, Montana State College, 1960.

LITERATURE CITED AND CONSULTED (cont'd)

Technical Translations. U.S. Department of Commerce, Office of Technical Services, Vol 1, No. 1, -Vol. 5, No. 6, January 2, 1959-March 28, 1961.

Troxell, G. E. and Davis, H. E. Composition and Properties of Concrete. New York: McGraw-Hill Book Co., 1956.



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An investigation of load entry
into the edge beam of a concrete

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