



The dynamics of Montana beef cattle inventories
by Randal Ray Rucker

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Applied Economics

Montana State University

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Abstract:

Models for estimating beef cattle herd sizes in Montana are estimated using a nonlinear least squares algorithm. The use of this algorithm allows the estimation of "nonstochastic difference equations" in which the stochastic component is purged from the lagged values of the dependent variable. Models are developed to explain the levels of total herd sizes as well as the levels of breeding herds. Explanatory variables in these models include the levels of hay production, beef to corn price ratio and calf (or feeder steer) prices in preceding periods. Interpretations of the estimated distributed lag patterns and of the nonstochastic forms of the difference equations are offered. The models estimated in this thesis appear to offer an improvement over previous attempts to explain and model beef cattle herd sizes.

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Date January 2, 1981

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by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

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in

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ABSTRACT

Models for estimating beef cattle herd sizes in Montana are estimated using a nonlinear least squares algorithm. The use of this algorithm allows the estimation of "nonstochastic difference equations" in which the stochastic component is purged from the lagged values of the dependent variable. Models are developed to explain the levels of total herd sizes as well as the levels of breeding herds. Explanatory variables in these models include the levels of hay production, beef to corn price ratio and calf (or feeder steer) prices in preceding periods. Interpretations of the estimated distributed lag patterns and of the nonstochastic forms of the difference equations are offered. The models estimated in this thesis appear to offer an improvement over previous attempts to explain and model beef cattle herd sizes.

Chapter 1

INTRODUCTION

Introduction

The purpose of this thesis is to develop a model to explain and predict the level of cattle numbers in Montana. A detailed discussion of the cattle production industry will be used to help construct a theoretical model. The model will describe the various factors that determine or influence changes in the level of cattle numbers in different time periods. The model will then be estimated empirically and the results will be analyzed in detail.

One of the primary focuses of this study will be on the dynamics of the cattle raising industry. As Nerlove suggested more than two decades ago, agricultural supply functions might be estimated most effectively with distributed lag models. Even a very cursory survey of the literature since that time indicates that his suggestion was a most useful one. Certain assumptions about the form of the lag distributions in these systems lead directly to more estimable dynamic systems, like the ones to be developed in this thesis.

It will be assumed that most of the readers have some familiarity with the field of agricultural economics and have some rudimentary notions of what cattle production entails. However, to help clarify the discussions that follow for those not particularly familiar with cattle raising, a list of terms related to the beef industry will be pre-

sented and defined. Some of the more frequently used terms in discussions of cattle raising and beef production include,¹

Cattle numbers - The total inventory of cattle on farms, either in a particular state or in a country (e.g. the U.S. or Canada) as of January 1 of a given year. In this paper, cattle inventories in Montana will be the center of focus.

Breeding herd - The total inventory of cows and heifers one to two years old) as of January 1 of a given year. The difference between breeding herds and total cattle numbers is that steers, heifers (less than one year old) and bulls are included in the latter but not in the former.

Cow herd - The number of cows that have calved.

Calf crop - The number of calves born in a given year.

Calf - Generally refers to cattle less than 7 months old.

Yearling - Generally refers to cattle about a year old. Often the distinction between calves and yearlings is made on the basis of weights, with the division between the two classes being made at 500 lbs.

Heifers - Female cattle that have not yet calved. These are either kept in the herd for purposes of reproduction or sold

¹The definitions of several of these terms were taken directly from Rosine (p. 20).

for slaughter or feeding.

Steers - Castrated cattle. These are generally young animals sold either to feedlots for fattening or directly to slaughter-houses.

Feeder Cattle - Cattle supplied to feedlot operators by ranchers for fattening. Ranchers generally make the decision to sell to feedlots when the calves are between eight and ten months old.

Fat cattle - Cattle that have been fattened for slaughter on grain or high protein supplements.

Grass-fed (or non-fed) cattle - Cattle that have been on range and pasture before being sold for slaughter. These cattle do not pass through feedlots.

Cull cows or bulls - Old cows and bulls that are no longer considered useful for reproductive purposes and are therefore marketed for slaughter.

Cow-calf operator - Cattle producer or rancher who generally markets his cattle as calves (except for those retained in the breeding herd). Most of the cattle operations in Montana are of this type.

Cow-yearling operator - Cattle producer who markets his cattle as yearlings.

Feed-lot operator - Cattle feeder who purchases cattle from ranchers and fattens them for slaughter. Most feeder cattle in Montana are sold to feed-lot operators in the Midwest market (Nebraska, Iowa, and Kansas).

Beef - The final product of the cattle industry, i.e., all meat produced from cattle.

Statement of the Problem and Structure of the Industry

The estimation of beef cattle inventories is, by itself, a stimulating problem, particularly from an academic point of view. Cattle producers' decisions regarding desired herd sizes emanate from their expectations of future prices of cattle and calves, but these expectations are not observable. To the economist interested in developing an empirical model for cattle numbers, specification of a proxy for these expectations is a problem of considerable importance. Another problem stems from the fact that planned changes in herd sizes do not correspond perfectly with actual changes. If it is assumed that herd size in a given year is determined in part by herd sizes in previous years, then it may be that ranchers base their desired herd size for this year on what would have been the average herd size last year if it were not for certain random disturbances, rather than on actual herd size. This problem and the methods to account for it in an empirical model have

received little attention in the literature.² An attempt to deal with this problem will be made in this thesis.

From a nonacademic point of view, estimation of beef cattle inventories is of interest for at least two reasons. First, cattle prices as well as cattle numbers seem to follow a cyclical pattern with some modicum of regularity. For many purposes, cyclical or seasonal fluctuations around the trend in the level of a variable can be ignored. For example, if the problem being addressed is to predict long term growth rates in the level of national income, then the problem of predicting cyclical fluctuations around the trend is not of interest. But for other problems, the prediction of these cyclical movements is of vital importance. For individual cattle operators, the trend level in cattle prices is of little relevance to them if they do not have the financial resources to survive the trough of the current cycle. Prediction of turning points in the levels of variables over time is one of the most difficult tasks faced by econometricians and is thus of

² This problem appears to have been recognized first by Griliches (1960, p. 291). Recently, work related to this problem has been done by Burt (1978, 1980) and LaFrance. Methods of handling this problem have been discussed primarily in the context of estimating crop responses. In the present problem, livestock responses are being estimated. The interpretation of the nature and causes of the differences between the expected level (or systematic component) and the actual level may prove to be somewhat different for a livestock response model than for an acreage or yield response model. This problem will be discussed in detail in later chapters.

interest from an academic standpoint. But from a practical or policy-oriented point of view, the problem is also of great importance. If the turning points (and possibly the amplitudes) of these cycles can be predicted with some accuracy, then it may be possible for ranchers to alter their actions in ways that will enable them to survive the low points in the cycles or for policy makers to intervene with actions that ease the burden on cattle producers during hard times.

The development of a model to estimate Montana cattle numbers would seem at first glance to be an endeavor of limited value to anyone other than Montana policy makers. But it is hoped that after the Montana model is developed it will be useful, without major alterations, for estimating and predicting cattle numbers in other states and possibly on the national level. It might be expected that the structure of the cattle industry in Montana would be less complex than the structure of the industry on a national level because of more homogeneous conditions and a relatively small dairy industry. An additional simplification for a state model is that cattle prices can be treated as exogenous with little risk of joint dependence in the relationship between inventories and prices. Often, valuable insights to a problem can be gained by solving more simplified versions of the problem first. A logical approach to the problem of estimating a national model for beef cattle inventories might thus be to estimate a model on a state level and then use the knowledge obtained to help in constructing the more complicated

national model.

Beef cattle inventories are important determinants of the number of cattle that enter feedlots and are slaughtered. The number of cattle slaughtered is in turn the primary determinant of the supply of beef on retail meat counters. The ultimate usefulness of beef cattle inventory models on the state and national levels therefore lies in the role they play in obtaining estimates of retail beef supply. This is the second nonacademic reason for developing a model of Montana cattle herd sizes.

It has long been recognized that hog prices and numbers follow a cyclical path over time. There have been numerous attempts to develop models that effectively account for this type of variation and accurately predict upcoming phases of these cycles. As was mentioned above, cattle prices and numbers also appear to follow a cyclical pattern. Yet there has been considerably less attention directed towards the development of models to explain and predict cattle numbers.³ It is not clear why these cattle cycles have been neglected relative to the hog cycles. One possible explanation is that because the cattle cycles have longer periods, more data would be required to develop a model that

³The studies that have been done on this topic include Maki (1962 and 1963), Tryfos, Martin and Haack, and Arzac and Wilkinson. In most of these studies, the equations for estimating cattle numbers are parts of larger systems for estimating livestock or meat supplies. In these articles, the authors spend little time interpreting the results of their cattle number equations. These articles and the results they contain are analysed in detail in the next chapter of this thesis.

effectively deals with these cyclical movements.⁴ The longer time series increases the hazards of structural change during the sample period.

On a national level, the United States cattle industry is currently going through the final phases of the seventh cycle of this century.⁵ These cycles have typically lasted about ten to twelve years. The average cycle has had an expansionary phase of about six or seven years and a contractionary phase of three to four years. However, these cycles have not by any means been uniform in these traits. The second cycle of the century, for example, lasted from 1912 to 1928 and had expansionary and contractionary phases of six and ten years respectively. Most of the other cycles have peaked near the midpoint of a decade and hit low points late in the same decade. For example, in the past 50 years, cycles have peaked in 1934, 1945, 1956, and 1965, and have bottomed out in 1928, 1938, 1949, 1958, and 1967.

Figure A-1 in Appendix A reveals that cattle numbers on the state

⁴That is, since hog cycles appear to be about four years long, twenty-five to thirty years of data will span about seven complete cycles. If there is any regularity at all to these cycles, an econometric model should be able to isolate it quite effectively with data on that many cycles. But, since cattle cycles seem to last about ten years, twenty-five years of data only spans two or three cycles. It is much more difficult for a model to pick out regularities with data on so few cycles.

⁵See Rosine (p. 13) for a plot of the path of total cattle numbers on a national level during this century.

level in Montana have followed the peaks and troughs of the national cattle cycle quite closely since World War II. This would be expected if cattle numbers are determined largely by cattle prices, since there is no reason the prices received by Montana ranchers should not follow the same path over time as the prices received by ranchers in other parts of the country.

An important question is "Why do cattle numbers follow this cyclical path?" Is this cyclical movement a result of factors external to the cattle production industry or is it due to forces that are internal to the process? It appears that these cycles can be attributed largely to biological constraints and lags in economic adjustments inherent in the cattle raising process. Before describing the characteristics of cattle cycles in detail, it will be useful to sidetrack somewhat and outline the process by which beef on the hoof becomes beef on the retail meat counter.⁶

For descriptive purposes, it is convenient to view the cattle industry as consisting of three sequential markets - the markets for feeder cattle, fat cattle, and beef. Each of these markets, like any other market, has a supply and a demand side. In the feeder cattle market, the suppliers are the cow-calf-yearling operators and the

⁶A significant portion of the following discussion was adapted from Kohls and Uhl and Rosine.

demanders are feedlot operators. The feedlot operators are the suppliers of fat cattle to the packer, wholesale and retail demanders. The latter then act as suppliers in the beef market while consumers play the role of demanders. Not all beef goes through each of these steps, of course. Some cattle are slaughtered immediately after purchase from the cow-calf operator without being fattened in a feedlot. The different functions in these three markets are not always performed by separate enterprises. For example, some operations in the Midwest raise feedgrains as well as cattle. In fact, before the 1950's, most cattle production took place on these diversified crop-livestock farms. The last three decades, however, have seen the development of large specialized cattle feedlots in the West and Southwest regions.⁷

The focus in this thesis is on the suppliers in the feeder-cattle market, i.e. the cattle ranchers. It is important to point out that the organization and structure of these producing units is highly diversified. Some ranchers raise sheep and/or hogs as well as cattle. Some raise grain crops in addition to their livestock. Others view their livestock production as a secondary source of income and use it to occupy otherwise unused resources like labor and buildings. The size

⁷ For further discussion of some of the different aspects of the livestock marketing industry, the different roles played by large highly specialized feedlot operations in different sections of the country, and the movement towards decentralization of the livestock and meat packing industry see Kohls and Uhl (Chapter 23).

of these operations also varies widely, from small to very large and highly specialized. This diversity is pointed out to emphasize the problems that may be encountered when trying to develop a single equation to model such a heterogeneous group of operations. Again the merits of constructing a model for a single state initially should be clear. Cattle production operations in a given state would be expected to be fairly homogeneous since they face similar climatic and topographic as well as economic (distance to markets, etc.) conditions. Thus, it should be relatively easier to construct a model on a state level. Again, it is hoped that insights can be gained from this exercise that will be valuable in modeling the cattle industry on a national level.

The cattle production portion of the cattle industry consists primarily of cow-calf and cow-yearling operations. In Montana, the majority of cattle ranches are of the former type. Calves are generally born late in the winter and early in the spring. Ranchers will typically keep these calves until they are about seven to ten months old. At that time, a number of options are available. Steers can be sold to feedlots for backgrounding and then fattening or directly to slaughter. Ranchers may also choose to carry the steers through the upcoming winter. This option might be chosen if ranchers felt that the price of steers would be higher in the spring and/or if winter pasture and feed conditions looked promising. A rancher would probably not choose to

carry steers through the winter if the summer hay yields were low and if forage conditions for the winter look poor. Male calves are not generally kept as part of the breeding herd since most herds have only one bull for every thirty to forty-five cows. These bulls are generally purchased rather than raised. In Montana the majority of steer calves appear to be sold to feedlots.⁸

Ranchers face basically the same set of alternatives for their heifers with the additional option of keeping a certain portion in their herds for breeding purposes. Heifers might be added to the breeding herd either as replacements for old cows that are culled from the herd or as net additions to the herd. If the heifers are "plowed back" into the herd, it usually takes about two years for them to have the first calf. The gestation period for cows is about nine or ten months. The useful reproductive life of a cow is generally about six to eight years. After that the cows are "culled" from the herd and sold for slaughter. The age at which these cows are culled will depend in part

⁸ Whether ranchers sell their cattle to feedlots or for direct slaughter depends on which buyer offers them a higher price. Changes in the price of feeder cattle relative to non-fed slaughter cattle will depend in part on changes in the demand by consumers for high quality as compared to low quality cuts of meat. However, the critical factor in the determination of these relative cattle prices is probably the composition of beef made available by suppliers. This composition will be affected by where the economy is located in the cattle cycle at the time in question and by the number of older animals (which rarely go to feedlots) being marketed.

on whether the rancher is trying to expand or contract his herd. If cattle prices are rising and the rancher is expanding his herd, he will keep these older cows in the herd longer. If prices are falling and he is reducing his herd size, he will be more likely to cull the older cows earlier.

Steers and heifers sold to feedlots are fattened on high concentrate rations (e.g, soybeans and/or cottonseed oil meals) and grains for approximately 130 to 160 days, after which they are referred to as fat, fed, or slaughter cattle. The length of time the feedlot operator keeps the cattle in the feedlot will depend partly on the price of feed relative to the price of fat cattle. From the feedlots, fed cattle are sold to packing plants where they are slaughtered and fabricated, then sold to wholesalers or directly to retail stores (possibly for further processing), and then purchased by consumers.

At this point it is helpful to describe a "typical" cattle cycle. An ordinary cycle consists of three distinct phases:⁹ (1) the rapid growth stage, (2) the deceleration stage, and (3) the turnaround stage.

The rapid growth stage is characterized by (1) favorable beef prices, (2) rapid increases in cattle numbers, (3) a low ratio of slaughter to inventory, and (4) higher than average financial returns

⁹These classifications, as well as much of the information content in the discussion and description of cattle cycles was obtained from Hasbargen and Egertson.

to beef producers. During this stage favorable beef prices induce ranchers to increase their breeding herds to try to capitalize on those prices. This increase is accomplished by keeping older cows in the herd longer and by retaining more than the normal number of heifers. Calf slaughter drops because more heifers are kept in the breeding herds and because feedlot operators (who are also benefiting from high beef prices) bid more of the lower quality heifers away from slaughter accounts.¹⁰ The holding back of heifers and reduction in calf slaughter results in reduced production of beef which causes prices to rise still more. The normal ratio of cattle and calf slaughter to January 1 inventories for the United States is about 36 percent. During the rapid growth stage this ratio falls to about 30 percent. Notice that according to economic theory, the normal response of a supplier to an increased price is to increase output. However, the biological characteristics of cattle production dictate that an increase in output can only be accomplished by first reducing current beef supplies by selling fewer calves to feedlots. Thus, the short run effect of rising prices

¹⁰ Actually, this representation of the relationship between the prices of feeder cattle and the profitability of feedlot operations is probably an oversimplification of reality. Rising prices of feeder cattle will mean increased costs for feedlot operators. Unless fat cattle prices rise by a similar amount, feeding margins will be pinched. Similarly, in the deceleration stage, when feeder cattle prices start falling, margins will improve if fat cattle prices fall at a slower rate.

on beef production would be expected to be negative. Prices and herd sizes will continue to rise until breeding herds become large enough that (although heifers are still being held back) the number of cattle being slaughtered begins to increase. When this happens, the production of beef will increase and prices will begin to fall. Falling prices are then passed from the retailers on down the market channel to the cattle producers. These falling prices and the resulting cutbacks in the rate at which herd sizes are being increased mark the end of the rapid growth stage and the start of the deceleration stage.

The characteristics of the deceleration stage include (1) unfavorable cattle prices, (2) the growth rate in herd sizes declines sharply and then inventories actually drop, (3) the ratio of slaughter to inventory begins to increase, and (4) returns to beef producers are below average. The falling prices that marked the end of the rapid growth stage affect cattle feeders. They receive lower fed cattle prices and must then lower prices for feeder cattle to protect feeding margins. Ranchers then begin to slow down the rate of growth of their herds. They do this by plowing back fewer heifers into the breeding herd and culling more of their older cows. Since cattle producers are trying to cut back on production to reduce their losses, this period is marked by an increase in slaughter relative to inventories. Examples of this rise in recent deceleration stages includes a jump from a 30 percent slaughter rate in 1973 to a 36 percent rate in 1975 and from 32 percent

in 1952 to 42 percent in 1955-56. Between 1973 and 1975 the culling rate rose from 12 to 20 percent which implies an accompanying increase in cow slaughter. Probably the only type of cattle not showing an increase in slaughter during this stage is choice finished slaughter cattle. This is because reduced demand and prices paid by feedlots allow slaughter accounts to outbid feeder accounts.

At the beginning of this stage, when prices initially start to fall, ranchers do not immediately decrease their herd sizes. This is because prices have been rising for several years and ranchers expect them to continue rising. They initially view the falling prices as temporary and are thus hesitant to decrease their herd sizes. If ranchers did reduce their herds and prices started rising again, they would not be able to capitalize on those high prices.¹¹ But the increased slaughter rates at the beginning of this stage lead to increased beef supplies which lead in turn to further price reductions.

¹¹One interesting characteristic of the cattle production process is that increases in herd sizes are constrained by the rate at which cattle can reproduce but that decreases in herd sizes can occur as rapidly as producers are able to liquidate their herds. The tendency for producers' price expectations to adjust slowly (which causes them to hesitate in liquidating their herds) may act to make the cattle cycle somewhat more symmetrical than it would be if its shape was dictated solely by the biological constraints of the industry. It was noted by Lorie that during the beginnings of this stage, prices are falling, but they are still above their average or "normal" level. As long as this is the case, one might expect producers to continue increasing their herds, depending of course, on how much their price expectations are affected by anticipated cyclical behavior of prices,

After a while, cattle producers realize that the falling prices are not an aberration and actually reduce their herd sizes. This, of course, leads to further decreases in prices. This continues until herd sizes are small enough that the supply of beef begins to fall and prices start to recover. When this happens, the cattle cycle enters the turnaround stage.

The basic characteristic of the turnaround stage is "normality"; prices recover from the low levels of the deceleration stage, herd sizes stabilize, the slaughter to inventory ratio is near normal, and returns to beef producers are about average. This stage begins when prices start to recover (even though producers are still marketing more than a normal proportion of heifers and may be culling at fairly high rates) due to herd sizes being small enough that beef production is reduced. Prices and herd sizes level off for awhile, but if population and per capita incomes continue to grow, the demand for beef will increase. Prices will begin to rise again and feedlot operators will respond by increasing their production levels and bidding up the price of feeder cattle. These rising prices induce cattle producers to start increasing their herds again. The cycle has entered the rapid growth stage once more.

From the preceding discussion, it can be seen that the cattle cycle is caused basically by the physical and biological constraints of the production process and by the manner in which producers form their

price expectations. That is, cattle ranchers decide to increase (decrease) their herd sizes because current and past price increases (decreases) lead them to expect future increases (decreases). One influence that is required for the continuation of the cattle cycle but is not inherent in the industry is the increase in population and incomes that acts to start prices rising and moves the cycle from the turnaround to the rapid growth stage. (Even in a rather stagnated economy, it seems that the phasing of the business cycle would sooner or later disturb the cattle cycle from the "normality" of the turnaround stage.) Other "outside" factors also affect the shape and length of the cycles even though they are not necessary for the continuation of the cyclical path. These factors may explain why the stages of different cycles vary so much in magnitude and duration. Examples of these influences include the end of the Korean War in 1952 which resulted in a slowdown in demand for beef and a break in beef prices, and the economy-wide boom of the late 1960's that kept consumer demand high and resulted in a shortening of the deceleration stage that started around 1965. Random variations in weather and feedgrain crop yields also affect feeder cattle demand and hence, cattle prices.

Another factor that may be contributing to the cyclical nature of this industry is related to forage conditions. When prices are rising, producers are likely to overgraze their range lands. When prices begin to fall, the depleted condition of this pasture acreage will

contribute to incentives to reduce herd sizes. By the time prices bottom out, pasture conditions should have recovered. Thus, readily available forage will contribute to the incentives provided by rising prices to increase herd sizes.

The econometric model that will be used in this thesis will be a distributed lag model. These models are employed when there is a time lag between the point at which a change in an independent variable occurs and the point at which all of the effects of this change on the dependent variable have been felt. This type of a model seems appropriate in the estimation of cattle numbers for at least three reasons. First, if an event occurs that induces ranchers to decide to change their herd sizes, it takes a significant length of time for these desired changes to be implemented. Cattle do not reproduce instantaneously and it is not possible, in the aggregate, for all ranchers to buy cattle to enlarge their breeding herds at the same time. Second, ranchers face rigidities in addition to those associated with the biological constraints of cattle reproduction. For example, if cattle producers decide they want to enlarge their herds, they will want to be reasonably certain of having adequate feed available. This may require developing additional hay acreage or purchasing (or leasing) more grazing land, which again takes time. Third, ranchers' actions with regard to expanding or contracting their herd sizes will depend largely on their expectations about future price levels. Although little is

known about the precise manner in which these price expectations are formed, it is generally agreed that they are based on some type of weighted average of past prices. Thus the complete effects of a price change in a given time period on a decision maker's future price expectations will not be felt until several periods have elapsed. The uses and precise forms of the distributed lag models to be employed in this paper will be discussed in detail in ensuing chapters.

The next chapter of this thesis contains a review of the literature related to the estimation of cattle (and other livestock) cycles and the use of distributed lag models. In the third chapter, a number of the theoretical aspects of the problem of estimating cattle numbers are considered. The estimated models are presented and discussed in the fourth chapter. The final chapter summarizes the accomplishments of this project and suggests some possibilities for extending our work. The raw data used to estimate the models and graphs of this data are presented in Appendix A. Appendix B contains plots of the lag distributions implied by the estimated models.

Chapter 2

REVIEW OF THE LITERATURE

This chapter contains summaries and discussions of many of the important past works related to the subject of cattle cycles. The literature that will be reviewed can be divided into two sections. The first contains articles that dealt directly with discussions and modeling of cattle numbers and the cyclical nature of the industry. These articles include studies that attempted to develop empirical models and make predictions as well as works that discussed the theoretical nature of the industry without attempting to do any quantitative work. Prices and numbers of other types of livestock besides cattle have been observed to follow cyclical paths over time. As was mentioned in the previous chapter, much attention has been directed toward the study of hog cycles. Since the nature of the cycles for cattle and hogs is essentially the same (the basic difference is the length of the cycles) and since a significant portion of the early work on livestock cycles was done on hog cycles, a few articles dealing only with hogs will also be reviewed. The analysis in these studies is easily modified to apply to cattle numbers.

The second section of literature to be reviewed deals with studies on the subject of distributed lags. The topics of these

studies range from theoretical discussions of when the use of a distributed lag model is appropriate to quite technical works discussing how such models might be estimated under conditions of widely varying complexity. Within this section of the literature, a topic of particular relevance to later work in this thesis pertains to the use of "stochastic" vs. "non-stochastic" difference equations.

Literature on Cattle and Other Livestock Cycles

The first section of this literature review will be further divided into two sub-sections. The first sub-section will review articles that deal specifically with cattle numbers and cattle cycles. The articles in this sub-section will be reviewed in chronological order and will consist of, (1) studies of a purely qualitative nature which are of value because of the ideas presented or because they contain exceptional descriptions of some aspect of the cattle industry or its cyclical nature, and (2) works of a quantitative nature that attempted empirical analysis and prediction of the cattle cycles. The second sub-section will contain descriptions of articles that have focused on other types of animal cycles. Once again, the reason for looking at these studies is that the techniques they used for estimating cycles may prove to be useful in studying the cattle cycle.

One of the first studies that attempted to develop a concrete

analytical framework for analyzing livestock and crop cycles was conducted by Mordecai Ezekiel (1938). The framework he used to look at these cycles is referred to as the "cobweb theorem". The basic idea of this theorem is that cycles in prices and/or production can be attributed largely to lags between the time a production decision is made and the time the output from that decision is actually produced.¹ Ezekiel began his article by giving credit to the originators of the "theorem" and then discussed the conditions that must be satisfied for the theorem to be appropriate for analyzing cyclical fluctuations in the "real world". These conditions essentially require that supply in a given period is inelastic and is determined by price in the preceding period and that demand in a given period is determined by the price in that period. After this, Ezekiel gave verbal and graphical descriptions of the three types of fluctuations (continuous, convergent, and divergent) that might occur. The type of fluctuation that actually occurs depends on the relative slopes of the supply and demand curves.

He then pointed out that his theory explained short cycles, or cycles in which high and low levels of production alternate year

¹For a more detailed description of the cobweb theorem, see Mansfield (pp. 240-242).

after year, but did not fully explain the long cycles observed in many commodities.² To remedy this, Ezekiel extended the analysis of the theorem by changing the assumptions about the length of time it takes production to respond to price changes. In the original formulation of the theorem, it was assumed that this lag was one year. This resulted in a two year cycle. By assuming that production in a given year depends solely on prices two years earlier, a four year cycle results. Using this technique, cycles of any length can be obtained using the framework of the cobweb theorem, simply by adjusting the assumption about the time lag between price changes and production response. He then made a visual comparison between these theoretically derived cycles and the actual price and production cycles of hogs and cattle. He pointed out the evident similarities, noted that the actual cycles differ from the theoretical cycles in that they are more irregular, and left the reader with the impression that his modified cobweb theorem could be used to explain the actual cycles.

Ezekiel then stressed that "the cobweb theory can apply exactly only to commodities which fulfill three conditions: (1) where production is completely determined by the producers' response to price

²A long cycle would be one in which production levels are higher than normal for several consecutive periods and then below normal for several consecutive periods.

under conditions of pure competition (where the producer makes plans for future production on the assumption present prices will continue and that his own production plans will not affect the market); (2) where the time needed for production requires at least one full period before production can be changed, once the plans are made; and (3) where the price is set by the supply available" (p. 272). Finally Ezekiel discussed some of the shortcomings of the theory.

These included,

- there is some elasticity of response in the supply of most commodities, especially on the contractionary side.³
- for most commodities, production in a given period depends on price levels in more than one preceding period.
- intended and actual output may differ due to weather factors. These influences, if they occur frequently, may tend to distort the shape of the actual cycle so that it does not appear to follow a pattern predicted by the "cobweb theory".
- frequently, production levels do not bounce from very high to very low levels in succeeding years in response to fluctuations in prices as the theory predicts. For example, producers may expand herd sizes slowly in response to rising prices, but contract rapidly in response to falling prices.

³ For example, once a production decision has been made it may not be possible to significantly increase output but it may be feasible to decrease output by plowing a crop under rather than harvesting it or by slaughtering cattle or hogs earlier than would be the case if prices were higher.

- there is no commodity for which supply alone determines the price. Other factors, like prices and availability of substitutes and income levels, also affect prices. Because these other factors are usually not constant, it is difficult to isolate any underlying cyclical movements.

It seems apparent that one of the most obvious shortcomings of this theory is related to the second fault Ezekiel discussed. Producers make their production decisions on the basis of the prices they expect to receive at harvest time. The assumption that is made in the cobweb theory about the manner in which these expectations are formulated is remarkably naive.⁴ For this theory to be useful as a tool for analyzing real-world cycles, it must be adapted to incorporate a more complex model of the formation of price expectations.

In a 1947 publication, James Lorie presented an extensive discussion of the nature and mechanics of the cattle cycle. Actually this study had a considerably broader scope than simply looking at the cattle cycle. Lorie looked first at the fluctuations in feed consumption and then focused on fluctuations in the number of animal units on farms (he developed an index to measure these units in his opening chapter). He found that there were four distinct cycles in these numbers between 1891 and 1945. He then attempted

⁴The assumption, once again, is that the price in a single period in the past (usually the immediately preceding period) will continue into the current period.

to determine which of the component cycles (cattle, hogs, or sheep) seemed to be accounting for the largest proportion of the deviations in the total number of animal units. He found that movements in cattle numbers accounted for the major cyclical fluctuations in numbers of all animal units on farms. Because of this finding, he devoted considerable effort to trying to determine the factors that cause the cattle cycle.

He opened his discussion of the cattle cycle by reviewing and critiquing previous theoretical explanations of the cycle. Two prominent theories for explaining the cycle at that time were the theories of exogenous and endogenous causation. As the names imply the first theory attributed cycles in cattle numbers to factors that were not an inherent part of the cattle cycle, while the second credited the cycle to the nature of the cattle production process itself. The leading proponent of the theory of exogenous causation was John A. Hopkins Jr.. In his A Statistical Study of the Prices and Production of Beef Cattle, Hopkins states that

" The cattle cycles of the past 60 years are apparently due to forces outside of the cattle industry, but these forces or conditions which have caused the major crises in the cattle industry do not seem to be related to any regularly recurrent phenomena. The prices of cattle are affected, of course, by the activity or depression of business, and are therefore influenced by the general business cycles. This will probably account for a minor series of cycles but not for the major cycles which have been 12 to 15 years in length. No phenomena of a regularly recurrent nature

and with a periodicity of 12 to 15 years have come to light to explain the major cycles. The cattle price cycles are irregular in length and in amplitude, and for each there seems to be a different reason."(p. 351)

He then attempted to determine the different causes of the individual cycles of the six decades preceding his study. Lorie levels some extremely harsh criticism on Hopkins' explanation of the factors that caused the cattle cycles. The first cycle Hopkins attempted to explain peaked in 1870. Lorie questioned Hopkins' factual data concerning the timing of the booms and depressions of the 1870's, then stated that "Hopkins explains subsequent cycles with an equal imprecision and lack of cogency"(p. 50). After criticizing Hopkins' analysis of the cattle cycle of the 1880's, Lorie dismissed his discussions of subsequent cycles with the comments,

"At no time is statistical evidence presented and seldom is the influence of factors, other than the somewhat arbitrarily chosen causal agent, considered. In the absence of any reliable measure of the demand for cattle, or the effect of population changes, or of changes in business activity, one must view with extreme caution any conclusion which posits a causal relationship between these various factors and changes in cattle prices or numbers. Hopkins seems to have proceeded on the basis of 'reasonable' assumption rather than upon a rigorous examination of the available evidence. It seems probable that a more consistent and convincing explanation of the fairly regular fourteen - to sixteen-year cycles in cattle numbers can be found." (p. 51)

Lorie then looked at the work of Professor F. L. Thomsen, a proponent of the endogenous causation explanation of the cattle cycle.

Thomsen made the following remark with regard to works such as Hopkins',

"It appears to many . . . that these episodic influences have served merely to affect the length or emphasize the degree of the cyclical movements of cattle production and prices and that there is a residual movement of cattle prices which is attributable to the truly cyclical nature of cattle production." (p. 360)

Lorie criticized Thomsen for not being more precise in his delineation of the causes of the cyclical nature of the production process and for not discussing the effects of exogenous factors such as the weather on these cycles.

After looking at these two theories of the cattle cycle, Lorie then turned his attention to "more sophisticated" attempts at modelling the cattle cycle through the use of the cobweb theorem. In particular, he focused on Ezekiel's 1938 article. He took particular exception to Ezekiel's implication that the cobweb theorem provided a framework for analyzing the cattle cycle. Lorie, upon substituting "more precise phrasing for Ezekiel's ambiguities," found that "the actual cycle is far longer than the theorem would have led one to expect" (p. 52). In support of his argument, Lorie cited a study of the hog cycle by Coase and Fowler that concluded that the cobweb theorem simply did not explain the observed facts.

Lorie then developed his own theory to explain the cattle

cycle and the interrelationships between value, marketings, and number of cattle on farms. One important aspect of Lorie's theory was his emphasis that the separate cycles in these three components of the cattle industry were really part of the same cycle. Lorie's theory of the cattle cycle began with the assumption that all exogenous factors were constant, so that the system was initially in equilibrium with no cyclical motion. Then a change occurs; perhaps the weather is exceptionally good or there is a change in tastes (which leads to a greater demand for beef). As a result of this, ranchers decide to increase their herd sizes. This leads to a decline in marketings and increased cattle prices. At this point, Lorie pointed out that theoretically, farmers could react to these higher prices either by increasing sales to take advantage of high prices immediately, or by decreasing sales to build their herds so they can benefit to a greater extent from continued high prices in the future. Lorie felt that the available evidence indicated that ranchers most often took the latter course. The result is further declines in marketings, accumulation of cattle on ranches, and increases in values. In other words, the initial forces set off other forces that move the system away from equilibrium. This trend continues for three or four years until "the increased production on farms resulting from the greater breeding capacity of the herds can be expected to reverse the downward trend

in marketing" (Lorie, p. 54). This increase in marketings will (other factors constant) lead to a fall in beef prices. Lorie maintained that ranchers would continue to increase their herd sizes (although at a slower rate), despite falling prices, as long as prices remained above their "normal" level. Because of increasing herd sizes, marketings would continue to increase and prices would continue to fall. When prices fall below their equilibrium level, ranchers would begin to cut back on their herd sizes. According to Lorie's theory, this decline in cattle numbers would begin about seven years after the initial disturbance that moved the system away from its equilibrium position.

As the liquidation of herds continues, marketings continue to rise and prices continue to fall. These trends continue until the herds are small enough that "the productive capacity of breeding herds will have declined so much that the lessening in the flow to market of young animals will more than offset the continued liquidation of breeding stock" (Lorie, p. 57). At this point, marketings begin to fall, prices begin to rise and the rate of liquidation is lessened (although liquidation does continue as long as prices are below their equilibrium level). When prices eventually rise above their equilibrium values, ranchers begin to expand their inventories

once again and the cycle repeats itself.⁵ Lorie's cycle took about fourteen years to complete a full rotation. The length of this cycle is determined by the assumption that cattle are marketed at an average age of about two and one-half years. The development of large scale feed-lots, technological changes in feeding, and possibly shifts in consumer demand have caused a reduction in average marketing age. This may explain part of the discrepancy between Lorie's cycle length and observed cycle lengths in recent decades of ten to twelve years.

Lorie ran some fairly crude tests and found that his theory was confirmed by the observation that accumulation (liquidation) of cattle seems to occur "with great regularity" when cattle values are above (below) normal. He then tried to determine the effects of exogenous factors on the cycle and concluded that weather fluctuations seemed to have a considerable effect on cattle numbers while the business cycle seemed to have had a relatively minor

⁵ It is interesting to note that Lorie's breakdown of the cattle cycle is essentially the same as that of Hasbargen and Egertson (described in the preceding chapter). The basic difference between their analyses is that in Lorie's story the cycle, once kicked off by an initial exogenous shock, seems to continue indefinitely while in Hasbargen and Egertson's model, each period of the cycle is begun with exogenous boosts, e.g. increasing population and/or incomes.

effect on the cattle cycle.

In 1955, Harold Breimyer published an article in which he suggested a number of different ways of looking at data on cattle prices, inventories, and slaughter. His hope was that by breaking the data down in a variety of ways, different aspects of the industry and cycle could be concentrated on and improved forecasts of different variables could be obtained. Breimyer opened his article by describing the price-decline phase of the cycle that started in 1952 and then commented on the inability of economists of the 1920's to empirically demonstrate a connection between cattle prices in one time period and cattle numbers in later periods. He felt that the cattle cycle, "more than any recurring fluctuation within agriculture resembles cycles of industrial origin. Basic to all such cycles is the management of capital goods of high investment cost and long productive life. In cattle those 'goods' are breeding stock" (p.2). Next, Breimyer commented on the conflict between those who felt the cattle cycle was self-generated and those who felt it was due to outside forces. He concluded this discussion by saying that

"In making their responses to *all* factors, producers are affected by the complicating features of large financial investment, long life cycle, and scarcity of alternatives. As a result, responses by cattlemen are not quick, simple and direct but take on the slow evolutions known as the cattle cycle. It is the special features of the cattle industry, converting all responses into cyclical responses, that largely account for the historical cattle cycle. The

cyclical effect comes about regardless of whether the forces responded to arise from within or outside the cattle industry." (p. 3)

Breimyer did not specify what "special features" of the industry he was referring to. His statement implied that he did not consider the discussion of endogenous or exogenous causation of the cattle cycle to be very relevant since the cycle would occur in either case. In the next section of this paper, Breimyer presented some of the devices he thought would be useful as tools for predicting upcoming trends in the cycle. These devices included free-hand graphic extensions of the trend and tabular arrangements of the data that he referred to, as "balance sheets" and "progressive balance sheets" of cattle numbers. The balance sheets basically broke total cattle numbers down into different categories according to how they were disposed of (death loss, slaughter, exports, etc). He felt these could be used as aids for prediction. The progressive balance sheets traced different categories of cattle (milk heifer calves, beef heifer calves, and cows and heifers) through time to see how they were disposed of. Breimyer felt there was enough regularity in these patterns of disposition that "forecasts based on this progressive balance sheet would have much validity" (p. 4). He did not make any actual predictions using these balance sheets, probably because the data base necessary to construct them was too weak. In the final section of his paper, Breimyer looked at the

changes in prices, price ratios, and the composition of inventories and slaughter during previous cattle cycles. He felt that the uniformity between the cycles, though far from complete, was close enough to be informative. Breimyer did not attempt to make any actual forecasts or develop any regression models, but he hoped that his exposition would "make possible more productive analyses of trends in cattle and improve the accuracy of forecasts" (p. 10).

In 1962, Wilbur Maki reported the results of a study in which he developed an econometric model for the beef and pork industries. He estimated nine multiple-variable equations and used the results of these to "generate the time paths, from 1949 to 1964, of selected output and price variables within the livestock-meat economy" (1962, p. 731). One novel aspect of Maki's work was that in his study, livestock numbers were disaggregated to a greater extent than in any previous study. He estimated separate inventory equations for calves, heifers, cows and steers. In these equations,

"the number of various classes of cattle on farms was related to the number of other specified cattle on farms a year earlier, the number of cattle slaughtered under federal inspection during the preceding year, and the average Kansas City feeder calf price during the last two years." (Maki, 1962, p. 731)

For example, the change in beef calves in year t was regressed on the change in beef cows lagged one year and the change in feeder calf prices lagged two years. Maki then used the results of these

equations to obtain an equivalent equation in which changes in beef cow inventories were a function of changes in beef cow numbers lagged three years, federally inspected slaughter lagged one year, changes in hog prices lagged one year, and changes in feeder calf prices lagged three and four years (since he obtained the estimates of the coefficients in this equation indirectly, he did not report any standard errors or other summary statistics). He then gave a very sketchy interpretation of the meanings of the resulting coefficients. Next, Maki estimated equations for slaughter prediction and feeder calf prices and described the causal chain of events that linked feeder calf prices, cattle inventories, cattle slaughter, and beef prices. In this description he emphasized the importance of the role played by changes in beef cow inventories in accounting for both the period and the amplitude of the beef cycle.

In a 1963 article, Maki extended the work in his 1962 article. At the beginning of the 1963 article, before he started actually discussing his results, Maki made two important observations on points related to the cattle cycle. First, he pointed out that for certain purposes, the prediction of seasonal and cyclical fluctuations is not important, but for other forecasting purposes, "the seasonal and cyclical fluctuations are quite important since they determine the economic life of businesses that lack the resources to cope

successfully with market instabilities and uncertainty" (1963, p. 612). Second, he made the observation that when ranchers respond to price information, we do not have any form of data that indicates their true production response. The proxy that is generally used as a measure of this response is changes in the number of cattle and calves on ranches on January 1. It is important to keep in mind that differences between intended and actual changes may exist for a number of reasons, including unexpected death loss, unusual weather conditions, and unusual breeding success (or lack of it).

To describe the producing sectors of the beef and pork industry, Maki developed a series of nine equation systems (a total of 37 individual equations). The system of interest for the purposes of this thesis is the series that obtained predictions for the number of cattle on hand on January 1. He estimated individual equations for dairy cows, beef cows, calves and heifers, steers, and cattle and calves on feed. He estimated most of these as relatively simple first order difference equations with only one or two right hand side variables in addition to the lagged value of the dependent variable. For example, the number of beef cows in a given period was a function of the number of beef cows in the preceding period and the change in the number of beef heifers lagged one period. Maki then combined the results of all these equations to obtain a series of two and four year forecasts and very briefly gave a

verbal description of his results. He also stated that a series of simulation runs were made for the beef and pork sectors under varying levels of aggregate demand. These resulted in "well defined cyclical patterns" with a cattle cycle whose period was seven years.

In 1966, an article by Schlomo Reutlinger was published in which he constructed and tested a model to estimate the short-run supply elasticity of beef. Prior to this time, researchers had been puzzled when their empirical results had indicated zero or negative elasticities of slaughter response with respect to prices.⁶

Reutlinger pointed out that one property of the cow and heifer components of beef supply was that a demand existed for them as investment goods as well as consumption goods. "Hence, while in response to a rising beef-feed price ratio, more beef is produced, also more of it is demanded for inventory buildup. The correlation between the beef-feed price ratio and market supplies of beef depends on the balance between the increased supply of beef and demand for inventory build up" (p. 909). Reutlinger's approach to the problems was to develop and then empirically test separate models for the individual

⁶This thesis is concerned with estimating a model for breeding herds. However, a negative relationship would be expected to exist between slaughter and herd sizes. Thus, Reutlinger's discussion of slaughter response also provided useful insights into the responses of ranchers with respect to expanding or contracting their herd sizes.

components (steers, cows, and heifers) of beef supply response. His models predict positive elasticity of response to price changes for steer slaughter and negative elasticity for cow slaughter. His model for heifer supply did not yield a determinate sign for the elasticity of heifer response. The coefficients in his models had the expected signs when he tested them empirically (the coefficient on price in the heifer equation was positive). When he combined the component equations to get an estimate of the aggregate elasticity of response, he (like previous researchers) got a very small, negative value.

An attempt was made to estimate the simultaneous relationships among demand, supply, price and export variables in the Canadian beef cattle sector by Kulshreshtha and Wilson in a 1972 article. One of the equations estimated in their system was a beef cattle inventory equation. The inventory variable was then used as an explanatory variable in estimating the supply of beef in two other equations in the model. In their inventory equation, levels of cattle inventories at the start of a given year were explained by the number of live feeder cattle exported from Canada (in the given year, not the preceding year, according to the authors' definition of their variables), the farm price of beef cattle in the given year, the expected price of beef cattle, the number of heifers, steers, and calves in previous years, the feed grain supply lagged one period, and total cattle slaughter lagged one period. The variable for farm price of beef cattle is actually a ratio of beef

cattle prices to feed grain prices, while "expected price" is measured by the "average ratio of the beef prices deflated by feed grain price index for the years $t-3$ to $t-5$ " (Kulshreshtha and Wilson, p. 86).

Their choice of the periods $t-3$ to $t-5$ was based on the length of the gestation period of cattle. This inventory model does not appear to have been very carefully formulated (or possibly the authors just presented their description of it poorly). In light of this, it does not seem too surprising that their empirical results were not very good. The only estimated coefficients that were statistically significant were those on heifers, steers and calves in previous years and the lagged feed grain supply variables. Despite the fact that the coefficients on the expected beef price and current farm price of beef variables were not significantly different from zero, the authors calculated and reported the elasticities of response of beef inventories to changes in these variables. No qualifications were made about the validity of these estimates as the authors noted their similarity to estimates in a previous study.⁷ To conclude their article, the authors used their estimation results to obtain a projection of the beef cattle sector for 1975.

⁷ Kulshreshtha and Wilson claim that their estimates of elasticities of inventory response "approximate those obtained by Lamgemeier and Thompson for the U.S. beef economy" (p. 88). It is not clear what estimates by Lamgemeier and Thompson (1967) they are referring to, since Lamgemeier and Thompson made no attempts to estimate any inventory models.

In 1974 Lovell Jarvis reported the results of a study of the Argentine cattle sector. He was concerned with slaughter response and did not develop or estimate equations specific to cattle inventories, but his analysis contained many important insights into the production sector of the cattle industry. He described cattle as "capital goods which are held by producers as long as their capital value in production exceeds their slaughter value. In essence, producers become portfolio managers seeking the optimal combination of different categories of animals to complement their non-cattle assets, given existing conditions and future expectations" (p. 489).

In his introductory discussion of the industry he pointed out the following important characteristics of the cattle production process,

"The cattle sector presents an interesting feature insofar as the slaughter of animals responds negatively to a price increase in the short run. This behavior contrasts with the supply response of most other agricultural products, such as field crops, whose desired output (in the Nerlovian sense) and actual output are both expected to rise immediately in response to a price increase. Although actual output for these crops is expected to adjust only gradually to the long-run desired output, there is never reason to expect output to fall when its price increases. But, because cattle production can be increased only by increasing the size of the breeding herd and/or withholding animals for further fattening, producers must bid animals away from consumers to increase the capital stock which is the source of higher future beef production. And the slow rate of

biological reproduction causes the negative supply response to persist for some time." (p. 491)

Jarvis set up a theoretical model to determine optimal slaughter time that was similar to the Fisherian model for the determination of the optimal harvest time for trees. He developed different models for different classes of cattle (e.g. steers, cows, heifers, etc.), derived the expected direction of response in the decision variables to changes in different parameters, and also discussed possible rankings of the elasticity estimates for the different cattle classes (e.g. he hypothesized that the price elasticity of female slaughter would be greater than that of males). He also discussed such topics as the probable differences in response to temporary and permanent price changes, the differences between slaughter and production response, and some possible estimation problems that might occur because of the time lapse between data observations being different from that for which the animals moved between categories.⁸ Since it was "not certain how producers might form their (price) expectations, or whether the same variables would have the same effect in different equations," (p. 509)

⁸For example, a price increase may lead producers to hold onto certain categories of cattle just enough longer for them to move into a different class when they are slaughtered than previously. This could make it appear (incorrectly) as though the price-slaughter response of the older class was positive.

Jarvis discussed and tested several different specifications of the price coefficients. Among other results, he found that the rate of change of price was consistently a significant determinant of slaughter. When he estimated his model with data on the Argentine cattle industry, his equations explained high proportions of the variations in slaughter of the different classes and most of the estimated coefficients had the expected algebraic signs and were significant. For those coefficients that did not have the expected signs, Jarvis went through perceptive discussions of why the discrepancies may have occurred.

In another 1974 article, Peter Tryfos attempted to conduct a comprehensive study of supply and inventories in the Canadian livestock sector. Tryfos briefly developed theoretical models for livestock supply and inventories and then estimated equations for each of these for four different categories of livestock: cattle, calves, sheep and lambs, and pigs. In developing his model for stock formations, Tryfos assumed that 'desired' livestock inventories depended on the expected live animal price and expected feed costs. In developing a model for the formation of expectations, he considered various models of 'adaptive expectations', but found that a simple model in which current price and feed costs were used as proxies for expected price and feed costs generally performed better than the more complex models. Thus, the stock formation models he estimated were first

order difference equations in which inventory at the end of a given period was a function of average prices and feed costs during that period and inventory at the end of the preceding period. Upon estimating these models, he found that, as hypothesized, all price coefficients were positive and all feed coefficients were negative. Also, the coefficients were clearly significant in all the equations estimated, except for his beef cattle equation. He suggested that this lack of significance may have been due to "the aggregation of the components of beef supply and inventories" (p. 110). Obviously, Tryfos' attempt to estimate an equation for cattle numbers was not very successful.

In a 1974 publication of the Federal Reserve Bank of Minneapolis, John Rosine discussed various aspects of the cattle industry including past cycles, the structure of the industry, and some possible developments that might have been expected at that time. He began his article with a brief overview of previous cycles of this century, (including a description of the typical cycle), and then focused his attention on the state of the industry in 1974. He predicted that the cattle cycle was entering a contractionary phase (which turned out to be a correct prediction). His reasons for making that prediction included high feed, labor, and energy costs, and lagging consumer demand. To determine whether the industry was headed for a contraction and if so, how prolonged and serious it

would be, Rosine compared conditions in 1974 with conditions in the early 1950's (the last time an extended break in prices occurred). He decided that, although in many ways conditions in 1974 looked like those in 1952, there were also noticeable differences between the two periods. To illustrate some of the factors that complicated the 1974 cattle cycle, Rosine made some simplifying assumptions about the structure and workings of the cattle industry. In this model (which was used in the discussion in the first chapter of this thesis), he viewed the cattle industry as consisting of three sequential markets - the markets for feeder cattle, fat cattle, and beef. Using this framework, he discussed the possible effects on different parts of the industry of such factors as shifting consumer demand, increases in marketing costs, rising feed costs and guaranteed loan programs. Rosine's framework for looking at the industry is quite useful as an analytical device for deriving testable hypotheses. However, he did not make any attempt to derive empirical estimates of the magnitudes of the effects he discussed. His "forecast" of the probable course of the industry consisted of a verbal description of some of the possible effects of the factors that complicated the analysis of the industry in 1974.

In a 1976 publication by the Ag Extension Service at the University of Minnesota, Paul Hasbargen and Kenneth Egertson discussed

various general characteristics of past cattle cycles and made some predictions about the immediate future courses of different prices and quantities in the cattle industry. They began by giving a detailed description of the various phases of the cattle cycle (their breakdown of the phases of the cycle was used in the discussion of cattle cycles in the previous chapter of this paper). They then analyzed the deceleration stage of the cycle that was occurring at that time. Their analysis included looking at the causes of the downturn and comparing it with the similar downturn of the mid 1950's. They described the similarities and differences between the two cycles and then made some fairly detailed predictions about corn prices, cattle and calf slaughter, per capita beef consumption, feeder cattle prices, and choice steer prices. Actually, they made dual predictions. One set of predictions would apply if weather conditions were good, and the other set would be relevant if the weather was bad (they were concerned about the possibility of an impending drought). All of the predictions made were of a very short term nature (three to six months) and no attempts were made to develop or test any empirical models.

In a 1977 article, Larry Martin and Richard Haack constructed a model of North American beef supply response in which they tried to capture "the nature of supply response in a manner that fully

explains the sequential linkages among decisions in beef production" (p. 29). Because of its importance as a component of beef supply response, the authors estimated a beef breeding inventory equation in an attempt to model the decisions of cow-calf operators concerning the sizes of their breeding herds. They viewed cows as "capital goods which are transformed into an intermediate product - stocker calves - which are sold to feed lots for further transformation"

(p. 33). They hypothesized that the decisions of ranchers pertaining to changes in their herd sizes were based primarily on their expectations about the future prices of stocker calves. Thus, the model they set up attempted to explain breeding herd inventories at the end of the year as a function of expected stocker calf prices.

They assumed that price expectations were formed according to a geometric lag specification. After transformation, the equation they estimated had breeding inventories in a given year as a function of stocker prices and breeding herds in the preceeding year. To form their price variable, they took an average of stocker prices in the four quarters immediately preceding the time at which the decision on the number of cows to breed is made. This average is weighted by the proportion of annual sales of stocker calves through public stockyards in each quarter. The authors presented an interesting justification for not including variables like slaughter

cattle prices and feed costs in their equation. Their argument was that the price feedlot operators pay for stocker calves is determined by their (the feedlot operators') expectations of fattened steer and feed prices. "In other words, expectations of steer and feed prices as formulated by feedlot operators are embedded in the stocker prices" (p. 34). Separate equations were estimated for the U.S. and Eastern and Western Canada. The results indicated that all coefficients had the expected signs and were statistically significant and that their models explained high proportions of the variation in the dependent variables. The authors reported their estimates of the short and long-run elasticities of breeding herd responses to changes in stocker prices. They also estimated equations for stocker prices as a function of steer and feed prices. From these, they derived estimates of the elasticity of breeding herds with respect to steer prices which they felt may be of more importance than the other elasticity estimates (they did not explain why these may be more important). They also suggested that interesting extensions of their study might include examining the response to steer prices in different phases of the cycles and replacing the geometric lag specification for the formation of price expectations

with more general polynomial specifications.⁹

A quarterly econometric model of the United States feed and livestock markets was presented by Enrique Arzac and Maurice Wilkinson in a 1979 article. The primary purpose of the study was to provide quarterly forecasts for different variables associated with the livestock and feed sectors. One of the forty-two equations in their system explained breeding herd inventories. In their formulation, inventories in year t were a function of prices of feeder calves in periods $t-1$ and $t-2$, prices of non-fed beef in period $t-1$, and inventories in $t-1$. The estimated price coefficients had the expected signs, but only the coefficient on feeder calf prices in period $t-2$ was significantly different from zero. The estimated coefficient on lagged inventory was positive and highly significant. When the inventory model was tested to see how well it predicted, it was found that a simple fourth-order autoregressive model yielded better results than the econometric model developed by Arzac and Wilkinson.

This concludes the review and discussion of articles dealing directly with the cattle cycle. Next, in chronological order, a number of articles dealing with different types of animal cycles are

⁹The results of this study by Martin and Haack were incorporated into a larger project by the Policy and Economics Branch of Agriculture Canada whose purpose was to conduct an empirical study of the major livestock and grain commodities and purchased farm inputs (See Hassan and Huff, pp. 29-60).

reviewed. The purpose of this section will not be to present a complete review of the "animal cycle literature," but rather to simply give an idea of some of the different approaches that have been used to study other cycles. It may be that some of these techniques might prove useful for studying the cattle cycle.

The animal cycle that has probably been discussed, estimated, and analyzed more than any other is the hog cycle. The reasons for all this attention may be that, (1) pork is a fairly important source of protein, so fluctuations in its price and production affect a significant segment of the economy, (2) the fluctuations of the hog cycle were distinct in their uniformity, particularly in the two decades following World War II, and (3) the period of the hog cycle was short enough (about 4 years) that a relatively short series of data would hopefully be sufficient for analyzing its cyclical characteristics. The cobweb theorem was probably the first formal model used to analyze animal (particularly hog) cycles. The basic problem with using this theorem to analyze the hog cycle was that the length of the cycle that emerged most naturally from the cobweb model was two years, while data indicated that the hog cycle actually had a period of four years.

As early as 1937, the cobweb theorem was used by Coase and Fowler as a possible theoretical explanation of the hog cycle in Great Britain. In 1938, Ezekiel presented one of the first formal

discussions of the cobweb theorem and its potential usefulness. As was noted earlier in this chapter, he also showed how the basic formulation of the theorem had to be changed in order to handle cycles whose periods were longer than two years. This was followed by a discussion of the shortcomings of the theorem as a tool for analyzing and estimating "real-world" cycles.

Among the articles on the topic of the hog cycle that have been published in the last two decades, one of the first was by Arthur Harlow in 1960. He attempted to explain the four year hog cycle as a result of factors inherent in the industry. He began by pointing out the interrelationship among hog prices, pig crop, and slaughter, and then told a story describing the mechanism of the hog cycle that was remarkably similar to Lorie's 1947 description of the cattle cycle mechanism. In Harlow's story, price in one period determines the size of the pig crop in the next period, which determines slaughter in the following period, which determines price in that period, which determines the pig crop in the following period, etc. He then tried to determine how well the cobweb theorem works in explaining the hog cycle. He described the conditions necessary for the theorem to be applicable, pointed out that one of the stumbling blocks for previous researchers had been that the two year cycle implied by the theorem did not correspond to the four year cycle actually observed, and then showed how the theorem could be adopted to yield a four

year cycle. His "adaptation" was to assume that the time lag between price changes and marketing response was two years instead of the one year lag typically assumed in the formulation of the theorem. As justification for this assumption, he referred to evidence indicating "there has been an approximate two year lag between price and its effects on marketings in recent years" (p. 849). He did not bother to cite his evidence.. The problems with this formulation are obvious. The major one is it required that marketing response in a given period depended solely on price levels two periods earlier. The simplicity of this model for the formulation of price expectations was criticized earlier in connection with Ezekiel's article. Harlow's modification of the cobweb theorem to handle four year cycles was the same as that suggested by Ezekiel over twenty years earlier. The only contribution of this article seemed to be to point out that if the proper assumptions were made (absurd though they may be), the cobweb theorem could be used as a model for explaining cycles of any length.

In 1964, Arnold Larson presented a theory of the hog cycle as "true harmonic motion" that arose from "a widely occurring phenomenon called feedback" (p. 376). The basic requirement for feedback is "an unvarying response to a signal, which acts through a fixed delay to alter the signal in a predetermined manner. The chief objection to use of the theory in economics has been its insistence on such a

rigid behavior pattern" (Larson, p. 377). In the context of the hog cycle, these assumptions would require that a given price change would, after an unvarying time period, lead to a change in marketings that would lead to another change in price. This theory requires that a given price change always leads to the same marketing response and that the subsequent price change always be the same. After looking at the characteristics of hog cycles over the period from 1949-1962, Larson decided that most of the assumptions of his theory were probably reasonably realistic. He then gave two possible explanations of the mechanism of the hog cycle that led to theoretical cycles that resembled the observed cycles. Near the end of the paper, after describing the similarities and differences between his model and the cobweb model, Larson warned that, if his model is correct, "economists may need to turn to measurement of mass, force, and other truly dynamic concepts rather than trace the reflection of static parameters whose relevance (or even existence) is dubious" (p. 386). Larson's paper is primarily theoretical in nature and he made no attempt to develop any numerical predictions. In his concluding section he suggested that the harmonic motion model may be appropriate for analyzing the cattle cycle. Apparently, no successful attempts have been made to follow up this suggestion.

In 1970, Rausser and Cargill applied spectral analysis to time series data on the broiler chick industry. Previous authors had

found evidence of an approximate 30-month cycle in broiler prices and placements. The objective of the authors was to try to determine whether such a cycle did exist and whether or not they could detect any noticeable lag-lead relationships between changes in prices and changes in flock supplies. Their results indicated that there was no 30 month cycle present and that there was very little association between price and output series. The technique of spectral analysis is relatively new in its application to economic time series data. Among other things, it can be used in the specification and estimation of distributed-lag models. It appears that this technique may be useful in analyzing certain aspects of cattle cycles. No articles were located that made any attempts in this direction.

In a brief article published in 1973 Mark Jelavich used data from the hog industry to demonstrate the use of distributed lag models to test for evidence of harmonic motion. Upon estimating a second order difference equation, he found that the null hypothesis of stable harmonic oscillation of the hog cycle could not be rejected. Jelavich pointed out that his results implied a four year hog cycle, which in turn led to certain implications about the manner in which producers formed their price expectations. Jelavich's article was notable insofar as it was one of very few attempts to empirically test the harmonic hypothesis. It appears that the technique he suggested might be an easy way to check for the existence of harmonic

motion in the cattle cycle (although it is not clear whether the simple specification he used to get the coefficient estimates he needed to test his hypothesis would yield unbiased estimates).

This concludes the first section of this chapter. The next section will consist of reviews and discussions of the articles that were important in the development of the theory and estimation techniques for distributed lag models.

Literature on Distributed Lags

When dealing with time series data the situation often arises in which a change in an independent variable in a given period results in changes in the dependent variable in several subsequent time periods. That is, the effects of the change in the independent variable on the dependent variable are distributed over several time periods. A model that attempts to account for this phenomenon is referred to as a distributed lag model. The most simplistic formulation of such a model has the dependent variable in a given time period as a function of values of the independent variable in previous time periods. The primary problem in developing and estimating such a model is that it is usually not known a priori how far back in time the effects of the independent variable on the dependent variable are felt. The decision to include only a given number of lagged values in the equation will usually be arbitrary and without any

theoretical basis. If it is decided the lagged effects will continue indefinitely the model will not be estimatable unless one has an infinite data sample. Thus, the problems most articles on distributed lag models are concerned with is how to formulate the model in such a way that coefficient estimates are obtainable (and have desirable properties) without placing "unduly" arbitrary (or unrealistic) restrictions on the form of the lag distribution.

The first person to explicitly recognize and discuss the distributed lag phenomenon appears to have been Irving Fisher. In 1925 he wrote an article whose purpose was to determine the relationship between fluctuations in the value of the dollar and in the level of business activity. Fisher had the novel idea of including not one, but several, lagged values of rates of price change as independent variables in his equations. In previous studies, researchers had included a single lagged value of the independent variable. The length of the lag was determined by which lagged value had the highest correlation with the dependent variable (e.g. Fisher found that the price variable lagged seven months yielded a higher correlation value than a lag of six or eight or any other). Fisher pointed out that the degree of correlation could be greatly increased "simply by substituting for this fixed lag of seven months a distributed lag spread over one, two, three, etc. months according to the

principles of probability" (p. 183) and then argued that

"the reason for distributing the lag is that the full effect of each P' item is extremely unlikely to be felt at only one instant, such as seven months later, and not felt at any other time either earlier or later than this seven months. Can we imagine that the sharp rise of prices in April, 1917, did not begin to affect trade until November, 1917, and that then the whole effect suddenly exploded, as it were? It is far more probable that the influence began at once, showing itself in the very next month, May, and that it then gradually increased to a maximum a few months later and thereafter tapered off indefinitely according to the probability distribution." (p. 184)

Fisher estimated his model (he did not say how many lagged values of the price variable were included in his final formulation) and then attempted to determine the nature of the probability distribution of the lag. He found that the distribution that most closely fit his estimated results was a transformed normal distribution in which the time axis was scaled as a geometric (rather than an arithmetic) progression. Fisher's analogy of a "gunner shooting bullets along a time road" provided a useful source for obtaining an intuitive feel for the concepts of his distributed lag model.

Another early discussion of the problem of distributed lags was presented by Franz Alt in 1942. The problems on which he focused were the determination of the length of the lag and of the function by which the lagged values were weighted. His method for determining the length of the lag was quite straightforward. Using time series data he simply kept adding lagged values of the independent variable

to his equation until the estimated coefficients no longer made sense. If the addition of another lagged value caused the previously estimated coefficients values to become erratic, Alt took that as an indication that too many independent variables were being introduced, and that the process should be stopped (he did not seem to be concerned with whether or not the added lagged values were statistically significant). Once the length of the lag was determined in this way, Alt described a method by which an approximation of the weight function could be obtained.

In a book published in 1954, L.M. Koyck suggested the use of a particular lag scheme for studying investment behavior and other similar problems in econometrics. Prior to this, the lag scheme most frequently used was a general distributed lag of the form

$$(1) \quad y_t = \sum_{i=0}^{\infty} a_i x_{t-i} + u_t$$

There are at least two problems with this formulation of the lag. First, the lagged values of the x 's must be truncated at some point so that enough degrees of freedom remain to obtain estimates of the a_i 's. This truncation will usually be arbitrary. Second, it is likely that the x 's will be highly correlated with each other, implying that the values of the coefficient estimates will be small relative to their estimated standard errors.

To avoid these problems, Koyck proposed a more restrictive

formulation. The simplest form of his scheme was

$$(2) \quad y_t = a \sum_{i=0}^{\infty} \lambda^i x_{t-i} + u_t$$

In this model, the coefficients on the x 's are constrained to decline geometrically.¹⁰ This formulation can be easily transformed into the equivalent relationship

$$(3) \quad y_t = ax_t + \lambda y_{t-1} + u_t - \lambda u_{t-1}$$

which involves only three observed variables and estimation of two parameters. The problem with estimating this equation using ordinary least squares is that y_{t-1} is contemporaneously correlated with u_{t-1} in the composite error term. This implies that the estimates of a and λ will be biased and inconsistent. Koyck developed a method for obtaining consistent estimates of a and λ but his method depended on an assumed value for ξ in

$$(4) \quad u_t = \xi u_{t-1} + w_t$$

That is, he assumed that the error term in the original formulation (2) may be autocorrelated but he did not suggest a method for obtaining estimates of the value of ξ from the sample data.

In 1956, in a book edited by Milton Friedman, a study of hyperinflation by Philip Cagan was published along with a number of

¹⁰In an article published in 1937, Fisher had suggested a scheme similar to Koyck's in which the coefficients declined arithmetically.

other studies on the quantity theory of money. The important result of this article from the standpoint of this thesis was that Cagan developed what has been referred to as the "adaptive expectations" model for the formulation of price expectations. One of the most appealing attributes of Cagan's formulation was that it supplied economists with a theoretical justification for the use of distributed lag models.

In this study, Cagan first pointed out that one conspicuous characteristic of the hyperinflations he surveyed was that real cash balances tended to decline markedly. He discussed the determination of real balances and decided that the expected returns from holding wealth in different forms was the only factor that varied enough during hyperinflations to explain the observed changes in real balances. Cagan looked at the opportunity costs of holding cash balances, decided that the real rates of return on bonds, equities, and non-perishable consumers' goods did not vary dramatically during these times and that therefore

"the only cost of holding cash balances that seems to fluctuate widely enough to account for the drastic changes in real cash balances during hyperinflation is the rate of depreciation in the value of money or, equivalently, the rate of change in prices. This observation suggests the hypothesis that changes in real cash balances in hyperinflation result from variations in the expected rate of change in prices." (pp. 31-33)

In other words, people altered the composition of their asset portfolios away from cash and into other types of assets because of the expected rapid devaluation in the real value of money.

Cagan felt that to test his hypothesis he had to relate desired real cash balances and the expected rate of change in prices to observable phenomena. To handle the former, he made the assumption that desired real cash balances were equal to actual balances at all times, i.e. any differences that existed between these two quantities would be erased immediately by changes in prices. With respect to the latter, he postulated that "the expected rate of change in prices seems to depend in some way on what the actual rates of change were in the past" (p. 37). He therefore assumed that "the expected rate of change in prices is revised per period of time in proportion to the difference between the actual rate of change in price and the rate of change that was expected" (p. 37). This assumption, which Cagan formulated in a continuous-time framework, is usually expressed in a discrete-time framework as

$$(5) \quad (p_t^e - p_{t-1}^e) = \beta(p_{t-1} - p_{t-1}^e) \quad 0 \leq \beta \leq 1$$

where p_t^e is the expected price level, p_t is the actual price level, and β is the "coefficient of expectation." β values close to one imply rapid adjustment of expectations while β 's close to zero imply

that expectations adjust slowly.¹¹ After Cagan tested his hypotheses (and found that they received strong statistical support from the data), he discussed his assumption about the relationship between desired and real cash balances. He explained that if it was assumed that desired balances adjusted to real balances with a lag (and if the form of this adjustment was analogous to that of the adjustment in price expectations)¹² and that there was no lag in expectations then the equation to be estimated would be identical to the equation derived in footnote 11 above. This implied that it was not possible to determine from his empirical results whether the lag between the

¹¹In a discrete time framework, Cagan's hypothesis about the relationship between desired real cash balances and expected price levels might be expressed as

$$(5a) \quad \left(\frac{m}{p}\right)_t^d = a + b p_t^e + u_t$$

where $\left(\frac{m}{p}\right)_t^d$ is desired real cash balances in period t and u_t is an error term that follows the classical assumptions. Under the assumptions that $\left(\frac{m}{p}\right)_t^d = \left(\frac{m}{p}\right)_t$ (where $\left(\frac{m}{p}\right)_t$ is actual real cash balances) and that price expectations adjust as in (5), (5a) can be rewritten in terms of observable variables as

$$(5b) \quad \left(\frac{m}{p}\right)_t = a' + b\beta p_t + (1-\beta)\left(\frac{m}{p}\right)_{t-1} + v_t$$

$$\text{where} \quad v_t = u_t - (1-\beta)u_{t-1}$$

¹²At this point, Cagan was discussing what is now referred to as a "partial adjustment" model.

adjustment in actual cash balances behind actual price changes was due to the lag in adjustment of price expectations or to the lag in adjusting desired to actual cash balances. He also displayed the model that results if both of these lags are assumed to exist. In this model it is also impossible to distinguish empirically between the lags. Because of these factors, he referred to the estimated coefficient on lagged real balances that he obtained empirically as an "amalgam" of the two lags and stated that "to some extent the estimates of the coefficient of expectation also reflect a lag in the balances and are to that extent too low as a measure of the lag in expectations alone" (p. 76).

In a frequently cited 1958 article, L. R. Klein addressed the problem of finding consistent estimates for the parameters in the distributed lag model proposed by Koyck (see equation (3) above). Koyck had developed consistent estimates under the assumption that the autocorrelation parameter of the error term in the original equation (2) was known. Klein showed that Koyck's estimates can be interpreted as either generalized least squares or maximum likelihood estimates and developed an alternative computational method for deriving Koyck's estimates. Klein then proposed an iterative technique for obtaining parameter estimates when the autocorrelation parameter

of the original error term is not known.¹³

A series of articles written by Marc Nerlove between 1956 and 1958 dealt with the problems of estimating agricultural supply functions and long and short-run demand and supply elasticities. These works provided a solid foundation for the theory of distributed lag models. In the first of these articles, Nerlove attempted to explain why previous studies had found such low response elasticities of acreage with respect to prices and to devise an estimation technique that yielded estimates that were "more in line with what we know from studies made on production functions and on farmers' reactions to the allotment and price support programs" (1956, p. 496). Nerlove argued that the models from which these previous estimates had been derived had assumed that farmers based their acreage decisions solely on the price in the previous year. Nerlove felt that "farmers react, not to last year's price, but rather to the price they expect, and this expected price depends only to a limited extent on what last year's price was" (1956, p. 498). To express this in a form that was estimable, Nerlove required a formulation that allowed a declining weighting system on the coefficients of past prices. For these purposes, Nerlove adapted Cagan's model to the form presented

¹³ In a comment published about three years after this article, Malinvaud showed that Klein's technique for finding estimates of the parameters under these latter conditions did not yield consistent estimates.

in equation (5). That equation is equivalent to the following

$$(6) \quad P_t^e = \beta P_{t-1} + (1-\beta) \beta P_{t-2} + (1-\beta)^2 \beta P_{t-3} + \dots$$

where $0 < \beta \leq 1$

which implies that expected price in a given year is a weighted moving average of past prices where the weights are a function only of the coefficient of expectation (β). The weights on past prices decline geometrically in this formulation. Nerlove transformed his hypothetical model in which acreage in a given period depended on expected prices into an estimable model in which acreage depended on actual price and acreage both lagged one period. In a footnote, Nerlove pointed out that although the formulation he estimated was identical to that suggested by Koyck's work, there was an important difference conceptually in the way the models are derived since Koyck simply postulated the existence of a distributed lag whereas Nerlove's method explained the existence of the lag. Nerlove, like Cagan, also pointed out the problems in interpreting the results of the model that arise because of the inability to distinguish between the model he proposed and a model in which it is postulated that expected price was equal to last year's price but that desired acreage was not the same as observed acreage. After making some comments on the possible properties of the coefficient estimates in his model, Nerlove presented his results. These results were very promising in that (1) his method explained a

considerably higher proportion of the variation in the dependent variable than the method used in previous studies and (2) the estimated long-run response elasticities were larger than previous estimates and therefore more in line with the evidence from other sources.

In the first of two 1958 articles, Nerlove presented a theoretical discussion of the problems of estimating short- and long-run elasticities using static models. He showed that under conditions in which "it takes time for producers and consumers to adjust to changed conditions and wherever the period which is required for full adjustment exceeds the interval of observation, then statistical relationships among observations on the relevant variables, each of which is taken at the same time, tell us little about the long-run elasticity or any of the short-run elasticities" (May, 1958, p. 306). To circumvent this problem, Nerlove advocated the use of distributed lag models. As a method of attacking the problem of estimating these models, he supported the development of "an explicit dynamic model of producer or consumer behavior which implies a distributed lag only incidentally." This approach would lead to "a direct interpretation of the distribution of lag in terms of producer or consumer behavior, and therefore in terms of the difference between short- and long-run elasticities of supply and demand" (May, 1958, p. 308).

The model Nerlove proposed for obtaining estimates of long-run supply elasticities was (under the assumption that farmers had static

expectations and based their production plans on prices in the preceding year)

$$(7) \quad \bar{x}_t = dp_{t-1} + e_t$$

where $\bar{x}_t$ is planned long-run equilibrium output. The postulated dynamic adjustment process of actual to planned output was

$$(8) \quad x_t - x_{t-1} = \alpha(\bar{x}_t - x_{t-1})$$

where x_t is current planned output and α is the coefficient of adjustment. By solving (8) for $\bar{x}_t$ and substituting into (7), Nerlove obtained

$$(9) \quad x_t = \alpha dp_{t-1} + (1-\alpha) x_{t-1} + e_t$$

Long-run supply elasticities can be easily estimated from the coefficient estimates of (9). Nerlove then commented about the assumption in this model of static price expectations. He cryptically asserted that non-static expectations were actually introduced "through the back door" in the above model and (in contrast to his position in his 1956 article) that there seemed to be little to be gained by the explicit introduction of non-static expectations into the model.

In the sequel to the first 1958 paper, Nerlove and Addison had an article published in November of that year in which they presented the results of empirical tests of Nerlove's earlier hypotheses. They used data on consumption of different commodities in the United Kingdom to estimate a series of demand relations and data on vegetables produced for fresh markets in the United States to estimate supply functions. The dynamic models they used were quite simple, but resulted in

significant improvements over similar static models. The authors felt that the most important finding in their study was that the serial correlation among the error terms in their dynamic models was almost nonexistent.¹⁴ This was in marked contrast with the results of their static models. Their explanation for this was that if the dynamic formulation was the true form of the model, but a static model was actually estimated, then one would expect to see positive serial correlation in the error terms (if the dependent variable is positively serially correlated). They then comment on the virtues of treating such problems (how to handle serial correlation) as economic problems rather than as purley statistical problems.

In 1960, Robert Solow proposed the use of a more general form of lag distribution than the geometrically declining form used by Koyck. The simplest form of Koyck's model for distributed lags was shown in equation (2) above. Solow criticized the conceptual basis of this form because it imposed the constraint that the largest effect of a change in x on the level of y would be felt immediately. Solow suggested that

¹⁴Part of the reason for this result may be that Nerlove and Addison used the Durbin-Watson statistic to test for serial correlation in both the static and dynamic models. The use of this statistic is not appropriate when lagged values of the dependent variable are present as right hand side variables. Under these conditions, the Durbin-Watson statistic is biased towards non-autocorrelation.

when observations on the data being used are taken more frequently than annually then "most of the reasons which make a distributed lag scheme seem like a good idea in the first place also suggest that the effect of x on y may very likely build up to a peak before tailing off" (p. 394). Koyck's method of handling this problem was to use the more general formulation

$$(10) \quad y_t = \sum_{i=0}^m \beta_i x_{t-i} + \alpha \sum_{i=m+1}^{\infty} \lambda^{i-(m+1)} x_{t-i} + u_t$$

which can be transformed into the more easily estimated form¹⁵

$$(11) \quad y_t = \beta_0 x_t + \sum_{i=1}^m (\beta_i - \lambda \beta_{i-1}) x_{t-i} + (\alpha - \lambda \beta_m) x_{t-(m+1)} \\ + \lambda y_{t-1} + (u_t - \lambda u_{t-1})$$

The problem with this formulation of the model is that if m is larger than two or three, then the same problems in estimation occur as with equation (1) above, i.e., a proliferation of parameters and multicollinear variables. In particular, if the x_t 's are autocorrelated, then the coefficient estimates will not be very precise. As a remedy to these problems, Solow suggested the use of the Pascal distribution as a

¹⁵To obtain (11) from (10), lag equation (10) by one period and multiply both sides by λ . When this transformed equation is subtracted from (10), the result is equation (11). The advantage of (11) lies in the reduced number of parameters to be estimated.

constraint on the shape taken by the lag distribution. The advantage of the Pascal distribution (which includes Koyck's geometric distribution as a special case) is that it is considerably more flexible in the shape it assumes and allows the peak in the response function to occur several periods after the initial change in the independent variable. If this constraint is imposed on the form of the lag distribution, then it turns out that only three parameters have to be estimated. Thus, this technique "provides a way of getting a wider range of response profiles without having to make very highly intercorrelated time series bear the burden of estimating a large number of independent coefficients" (Solow, p. 397). One problem with using this constraint is that simple estimation techniques are not appropriate. The reasons for this are, (1) two of the three parameters enter the formulation of the equation to be estimated in a highly non-linear fashion, and (2) the residuals are autocorrelated. Solow discussed these problems and some of their possible solutions extensively in the second half of his paper.

In a 1961 article, Zvi Griliches focused on the problem of serial correlation in the error terms in distributed lag models that have lagged dependent variables as explanatory variables. Griliches derived expressions showing the direction of bias in the coefficient estimates of a distributed lag model when ordinary least squares methods are used in the presence of serially correlated error terms. In addition, he

found expressions for calculating lower bounds on the magnitude of the bias for different values of the serial correlation parameter. He also discussed circumstances under which estimation of a distributed lag model resulted in a reasonable and significant coefficient on the lagged dependent variable and a reduction in the serial correlation of the errors even though the distributed lag model was not the "true" model generating the data. Griliches closed his article with the following comment on the problem of serial correlation and the various methods of dealing with it,

"I believe that as long as we have serial correlation in the disturbances there is still something systematic in the world which our model has not incorporated. It seems to me that it is much more desirable to find the economic reasons behind this correlation and incorporate them into the model than to pursue very complicated estimation techniques designed to deal with this problem. The research strategy should be directed toward eliminating serial correlation by including its causes explicitly within our models, rather than to devising new methods of living with it. On the other hand one should be careful in interpreting the results of such models. Just because the serial correlation has been "taken care of" by the particular technique does not prove that it is generated by the assumed mechanism. Serial correlation in the disturbances could be due to many different reasons, and its identification with one mechanism rather than another may require more careful investigation and much more detailed data than are usually available." (pp. 71-72)

In another 1961 article, Wayne Fuller and James Martin were concerned with essentially the same problem as Griliches. In this paper, they suggested an iterative estimation technique for finding maximum likelihood estimates of the coefficients in first order difference

equation models with a first order autoregressive error term. They also derived confidence intervals for the estimates of long-run elasticities obtained from these models. After this, they estimated demand equations using United Kingdom consumption, price and income data and compared the results using ordinary least squares methods with the results using their suggested technique. They then discussed the biases in the estimates from the ordinary least squares model and remarked on the invalidation of the Durbin-Watson statistic when lagged endogenous variables are present in the equation.

In 1963, Nissan Liviatan suggested a solution to the problem of developing a consistent estimation technique for equations with a lagged dependent variable and autocorrelated error terms. He mentioned the shortcomings of Koyck's solution to this problem and then criticized Klein's iterative procedure as being somewhat messy computationally. He also cited Malinvaud's contention that Klein's procedure was not consistent. Liviatan's alternative estimation technique was to use an instrumental variable (or two stage least squares) estimator. The main advantages of his method were that it resulted in consistent estimates and was easy to apply. The disadvantage was that his method did not yield efficient estimates. He also pointed out that his method did not require any assumptions of homoscedasticity, normality, or the form of the correlation in the error terms. Liviatan initially proposed the use of a simple instrumental variable (the lagged value of an

independent variable) and then discussed the possibility of using more complex variables in an effort to improve the efficiency of his estimator. He also developed some tests for determining whether the losses (in terms of inconsistency) from using OLS were great enough to warrant the use of his instrumental variables technique and the accompanying losses from inefficiency.

Two years after Liviatan's work, E.J. Hannan derived the asymptotic distribution of Liviatan's estimators under general conditions. He also used spectral methods to develop efficient estimators for distributed lag models with serially correlated error terms.

In 1966, Dale Jorgenson wrote an article whose purpose was to find a class of lag distributions that (1) could be used to estimate an arbitrary distributed lag function (of the form shown in equation (1) above) to any desired degree of accuracy and (2) would have at most the number of parameters that a finite distributed lag function approximation of a comparable degree of accuracy would have.¹⁶ Jorgenson felt that the problem with using finite distributed lag functions as approximations of (infinite) distributed lag functions was that "the number of

¹⁶ Jorgenson defined a finite distributed lag function as one in which all of the coefficients after a certain point were zero. That is, it would have the form,

$$y_t = \alpha_0 x_t + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \dots + \alpha_m x_{t-m} + u_t$$

parameters required for a satisfactory approximation of this type may be quite large" (p. 136). He mentioned that specific solutions to the problem of reducing the number of parameters to be estimated had been proposed by Fisher (1937), Koyck (1954) and Solow (1960), but that each of the classes of estimators that they suggested had the undesirable property of not being capable of approximating an arbitrary distributed lag function. Jorgenson suggested the use of rational distributed lag functions to solve the problem. A distributed lag function qualifies as a member of this class "if and only if it may be written with a finite number of lags in both dependent and independent variables" (p. 138). He then demonstrated that finite distributed lag functions, geometric distributed lag functions, and Solow's Pascal distributed lag functions were all special cases of the class of rational distributed lag functions. Jorgenson also derived estimators (with desirable large sample properties) for the parameters in his lag function under two different specifications of the stochastic nature of the system.

In another 1966 article Yair Mundlak examined the link between the adjustment process suggested by Nerlove and comparative static economic theory. He found that the Nerlove adjustment process and other more complex processes assumed "intrinsic relationships between economically endogenous variables. Such relationships cannot be justified by economic theory, and their existence may imply very strong restrictions on comparative statics theory" (p. 51). To demonstrate,

Mundlak devised examples of perfectly feasible response paths which violated the constraints on the coefficient of adjustment in Nerlove's partial adjustment model. He also discussed similar problems with Koyck's and Jorgenson's investment models and went through an example in which the LeChatelier principle was violated.

In a frequently cited 1967 survey article, Zvi Griliches presented an extensive discussion of the topic of distributed lags. The purpose of his paper was to

"draw attention to the practical difficulties of distinguishing between different lag schemes and lag models from the data. We may be asking too much of our data. We want them to test our theories, provide us with estimates of important parameters, and disclose to us the exact time form of the interrelationships between the various variables. Progress in this area is likely to slow until we have a much better theoretical base for imposing a time-lag structure on the data." (pp. 17-18)

The contents of this paper included,

- a description of lag generating functions and lag operators.
- a review and discussion of efforts in the direction of developing generalized forms for the lag distribution (e.g. Koyck, Solow, Jorgenson).
- a discussion of the problems inherent in trying to determine the form of the lag distribution from the data generally available. Griliches demonstrated how relatively small variations in parameter estimates result in radically different shapes for the lag distribution.
- discussions of the nature and direction of biases in coefficient estimates due to serially correlated error terms and several possible methods (and their respective shortcomings) of handling the problem.

-comments on the "theoretical underpinnings" for distributed lag models. His opinion was that, in general, they were pretty weak.

In a 1968 article Roger Waud looked at the bias that resulted from misspecification of the "partial adjustment" and/or "adaptive expectations" models. Cagan's "adaptive expectations" model and Nerlove's "partial adjustment" model are not distinguishable from each other when the models are estimated empirically, i.e. both of them can be reduced to first order difference equations and it is not possible (from the estimation results) to determine whether one is obtaining an estimate of the "coefficient of expectation" or of the "coefficient of adjustment." If the two models are combined, the resulting estimable form of the model is a second order difference equation (in this equation it is still not possible to distinguish between the coefficients of expectation and adaptation since they enter the model symmetrically). Waud pointed out that the Cagan and Nerlove models were really special cases of the more general "combination" model. He described the rather unrealistic assumptions that had to be made, either explicitly or implicitly, in order for one of the "simple" models to be the correct model to estimate.¹⁷ If the "combination model" is the truth and one

¹⁷ For example, the "adaptive expectations" formulation is the true specification of the model if the coefficient of adjustment is unity. Similarly, the "partial adjustment" formulation is correct if expectations are formed statically, i.e. if the coefficient of expectations is equal to one.

of the simple models is estimated then the estimated coefficients will be biased (this "omitted variable bias" results because the second-order lagged value of the dependent variable has been excluded as an explanatory variable). In this paper, Waud looked at the extent of the resulting bias in the estimated coefficients, in their standard errors, and in the estimated average lags under a variety of assumptions about parameter values when one of the simple models was inappropriately specified as the "correct" model.

Phoebus Dhrymes developed a procedure for estimating distributed lag models with autocorrelated error terms in a 1969 article. By using this procedure, which he felt was simpler than other available methods, he was able to obtain maximum likelihood estimators that "globally maximize the likelihood function for every sample size" (p. 48). The procedure was developed under the assumptions of a geometrically declining lag distribution on one of the independent variables, and a first-order Markov process on the normally distributed error term. This method, which involved a two-stage iterative process yielded consistent and efficient estimates.

In a comment on Dhrymes' paper, Y.P. Gupta suggested the use of a "somewhat simpler two-step estimator" (p. 112) for the model considered by Dhrymes. This technique involved the application of the instrumental variables method followed by ordinary least squares. Gupta did not derive or mention the properties of his estimator.

In 1971 G.S. Maddala and A.S. Rao used an extension of the estimation technique developed by Klein (1958) to obtain maximum likelihood estimates for Solow's Pascal distribution and Jorgenson's rational distributed lags. They developed these estimators first under the assumption of independent normally distributed error terms and then under the assumption that the error term followed a first-order autoregressive scheme. In their technique, it was necessary to deal with the presample values of their independent variables in one way or another. They found that their results were improved greatly when they estimated these initial values as parameters rather than simply ignoring them (setting the presample values equal to zero). They also demonstrated how the likelihood ratio test can be used to test for equality of the roots of the polynomial in the denominator of Jorgenson's rational lags. This test is of interest since Jorgenson's model reduces to Solow's Pascal distribution when these roots are equal.

In a 1972 survey paper, Marc Nerlove discussed developments in the relationship between economic theory and measurement over the three and a half decades preceding his paper. In particular, he described the progress (or lack thereof) in the development of a dynamic theory of economic behavior and how this theory was being used in concert with empirical work. He was quite concerned with the haphazard manner in which distributed lag structures were frequently tacked onto models that were otherwise static in nature. Nerlove, of course, felt that

the models should have been based completely on dynamic theory.

Nerlove focused largely on the studies that had been done on investment. In particular, he discussed the work done by Jorgenson in that area. Nerlove repeatedly labelled the dynamic portions of Jorgenson's models as being ad hoc. In fact, after reading Nerlove's critique of Jorgenson's dynamic formulations, it is not at all clear what type of a dynamic theory would not be considered ad hoc by Nerlove. Most of Nerlove's efforts in this paper are directed toward trying to develop a theory to deal with the dynamic characteristics of investment. Nerlove also spends time in this article expounding on his own philosophy about the proper relationship between empirical work and theory. One important facet of his view has to do with how the shape of the lag distribution should be determined. Often in empirical studies, researchers have let the data determine the shape of the lag. Nerlove's view was that time series data is usually not adequately detailed to determine the shape of the lag with any exactitude, thus theories of dynamic behavior needed to be developed which would specify the shape of the lag in advance. He was not certain whether this could be accomplished, and in fact his efforts in that direction in this paper do not appear to meet with much success (judging by comments on those efforts made in later studies).

In another survey paper, published two years after Nerlove's, Christopher Sims discussed various developments in the study of

distributed lags since the time of Griliches' 1967 survey article. Sims, like Nerlove, was concerned about developments in the relationship between dynamic theory and empirical estimation. He felt that Muth's (1961) article on rational expectations was an important contribution to dynamic theory but that "the difficult program of bringing the rational expectations hypothesis usefully into the optimizing theories used to generate applied distributed lag models is still only beginning" (p. 292). Among the many items discussed by Sims in this paper were

- comments on the validity of interpreting the sum of the coefficients in an estimated lag distribution as the long-run effect.

- descriptions and comments on estimation techniques for difference equations under various assumptions about the nature of the error term.

- a review of works on the use of spectral methods of estimation.

- a discussion of the effects of placing restrictions on infinite lag distributions in order to make them estimable and some critical comments on the implications of the assumption that the lag distribution is smooth.

- descriptions of recent advances in methods of testing for specification error.

In 1977, Askari and Cummings published a survey of studies that used Nerlove's partial adjustments and/or adaptive expectations model to estimate agricultural supply response. They discussed the general modifications that have been made in Nerlove's basic model, as well as some of the problems encountered in finding the empirical analogues of

the theoretical variables in Nerlove's models (e.g. "Which prices should be used and how should they be deflated?", "What should be done if the variables to which decision makers respond change over time?" and "What is the proper measure of desired output response?"). The authors then presented the long- and short-run elasticity estimates from an enormous number of studies on agricultural supply response (their bibliography contains 190 references!).

In a 1978 paper Oscar Burt discussed some of the problems in estimating agricultural supply response and suggested a new approach to the problem. To begin with, he discussed some justifications for using difference equations. These included Nerlove's "theoretical" justifications, i.e. the partial adjustments and adaptive expectations models. Burt also pointed out that a complete history on the lagged values of the exogenous variables in the model probably contains the same information as a few lagged values of the dependent variable. Thus, a more pragmatic justification for the use of difference equations is that since time series data is not generally available on the entire history of the exogenous variables, a relatively low order difference equation will probably accomplish the same purpose. Burt also felt that a relatively low order difference equation that included a couple of lagged price variables should do a good job of capturing the essence of the technological, economic, biological and behavioral factors and constraints that act to determine supply response (but which are

probably impossible to measure more directly). Burt's new approach to the supply-response estimation problem was to use what he referred to as "nonstochastic difference equations." A typical first order stochastic difference equation is

$$(12) \quad y_t = \alpha + \beta x_{t-1} + \lambda y_{t-1} + u_t$$

This expression has two stochastic components, the random error term u_t , and random portion of the lagged dependent variable, y_{t-1} . Burt defined a non-stochastic difference equation as

$$(13) \quad y_t = \alpha + \beta x_{t-1} + \lambda E(y_{t-1}) + u_t$$

where $E(y_{t-1})$ can be thought of as the lagged value of the dependent variable minus its non-systematic components, i.e. $E(y_t) = y_t - u_t$.

He discussed the uses and interpretation of these difference equations and also developed a non-linear least squares algorithm for estimating the parameters under general assumptions about the nature of the error term. Burt also presented summary results for applications of his method to U.S. wheat and soybean acreage, U.S. wheat yield and Montana beef breeding herd inventories. The results using the non-stochastic difference equations appeared to be markedly better than the results using the stochastic specifications.

In a 1980 paper on the same topic, Burt extended the analysis and discussion in his 1978 paper. In this paper, he discussed further the distinctions between stochastic and non-stochastic difference equations and then showed, for some simple cases, that the two models can be

made equivalent through reparameterization, i.e. he showed that a stochastic difference equation with a classic error term is equivalent to a nonstochastic difference equation whose error term follows a first order autoregressive process. The bulk of this paper was directed toward explaining and describing the particular estimation procedure and algorithm developed by Burt. This algorithm (which was used for the empirical work in this thesis) is designed to estimate rational distributed lag functions with very general specifications of the error term.

This concludes the second portion of the literature review. The preceding discussion of distributed lags is not considered to be a complete review of that literature. To get some idea of the futility of undertaking the task of completely reviewing the distributed lag literature, one has only to look at the bibliographies in such publications as Griliches (1967), Nerlove (1972), Sims (1974), and Askari and Cummings (1977). Many important works have been left out of this paper for one reason or another. For example, no discussion of Friedman's (1957) work on the consumption function and the permanent income hypothesis has been presented, even though that work along with Cagan's are the seminal articles in the development of the adaptive expectations literature. Various studies that dealt with different types of constraints on the shape of lag distributions have not been discussed. Examples of these include DeLeeuw's "inverted V"

distribution and Almon's work with polynomial constraints on the lag distribution. A great deal of work on estimation techniques for distributed lag models has not been discussed. One of the most important works in this area was probably Dhrymes' 1971 book. The list of important works that have been deleted could probably go on indefinitely.

The articles that have been discussed describe some of the more important developments in the distributed lag literature as it applies to the work done in this thesis, and provides a basis to interpret and evaluate the empirical results.

Chapter 3

DEVELOPMENT OF THE MODEL

In the first section of the preceding chapter a skeletal description of several earlier attempts at estimating models of beef cattle inventories was given. In those models, a price variable (or variables) was generally present (although not necessarily statistically significant) and several of the models were specified with lagged dependent variables acting as explanatory variables. The second section described some of the developments on the topic of distributed lags that seem pertinent to the model to be constructed in this thesis. The most common theoretical justifications for the use of distributed lag models are the "partial adjustment" and "adaptive expectations" hypotheses. The partial adjustment model is used in situations where the adjustment in the level of some decision variable does not take place instantaneously. This slow adjustment process may be due to such factors as technological and/or biological constraints, habit persistence, or institutional rigidities. The adaptive expectations model is appropriate in situations where the level of a decision variable is a function of the expected level of an independent variable. If the expected future value of the independent variable is some type of a weighted average of past and

present values of that variable, then a distributed lag model is implied. When developing a model to explain cattle herd sizes, the use of a distributed lag model can be justified using either or both of these hypotheses. For example, the partial adjustment model makes sense in a situation in which herd size is the decision variable since biological factors as well as physical factors (like capacity or feed availability constraints) act to make adjustments occur slowly. Similarly, an adaptive expectations model is appropriate since decisions with regard to herd sizes will be made on the basis of ranchers expectations about the future level of cattle prices and the availability (and/or price) of feed.

Models in which the distributed lag on the independent variable (or variables) is infinite can generally be converted to low order difference equations (see Cagan and Nerlove (1956)). In fact, Jorgenson showed that any arbitrary infinite distributed lag function can be approximated to the desired degree of accuracy by a rational distributed lag function. These rational lag functions can frequently be expressed as low order difference equations with relatively low order lags on the independent variables in the model. The model to be estimated in this thesis will be a member of the class of rational distributed lag functions.

There are two views in the economics profession as to how the

shape of lag distributions should be determined for purposes of empirical work. The first group bases their argument on demonstrations like those presented by Griliches (1967, pp. 29-31) in which he showed that relatively small changes in parameter estimates lead to a wide variety of implied shapes for the lag distributions. This group, whose leaders include Marc Nerlove, argues that time series data is virtually always inexact enough that rough approximations of the shape of the lag distribution will be the most that the data can be expected to yield. They maintain, therefore, that the shape of the lag distribution must be derived from a theoretical basis. A well developed dynamic theory of economics is required before theory can dictate these shapes to empirical workers. Unfortunately, no such theory exists and although efforts to develop one continue, it does not appear that any major breakthroughs have occurred up to this point in time (see Nerlove (1972) and Sims for a discussion of the developments in this area). The flaw in the view held by this group is that even if an adequate dynamic theory existed, it would still have to be tested using inexact data. Parameter estimates from this data will have wide confidence bands implying, (1) that it will be difficult to reject a given hypothesis about the shape of the lag distribution and (2) that it may be nearly impossible to distinguish among alternative theoretical explanations of the possible

shapes of these distributions.

The second group holds the view that the data should be allowed to determine the shape of the lag distribution. Their argument is that since no adequate theory exists to explain how expectations are formed or how dynamic adjustments take place, there is no way the forms of the lag distributions can be obtained from this source. The only available method to determine these shapes is thus to let the data do it. This will be the approach taken in this thesis for approximating the form of cattle ranchers' response functions. However, it will be kept in mind that the estimated lag distributions are only inexact approximations. Tests to give indications of the limits of the inexactness will be run when possible.

The method of estimation suggested by Burt (1978, 1980), using "nonstochastic difference equations" will be tested and used in this thesis. This method basically involves removing the stochastic component from the lagged values of the dependent variable and using the resulting nonstochastic variable as a regressor. One advantage of this technique is that the stochastic and nonstochastic components of the equation are clearly delineated. In a stochastic difference equation, the lagged values of the dependent variables contain a random, as well as a systematic element, implying that the delineation is not well defined. In a nonstochastic difference equation, the

lagged dependent variables only contain a systematic component. Thus all random elements on the right hand side of the equation are contained in the error term. This is a helpful feature in certain instances when interpreting the model under consideration. Another positive attribute of the nonstochastic formulation is that coefficient estimates will be consistent regardless of whether the second moment specification on the error term is correct. This nonstochastic difference equation formulation can be viewed as an alternative to the method developed by Maddala and Rao for computing maximum likelihood estimates.

In his study attempting to estimate aggregate agricultural supply, LaFrance offered a feasible theoretical interpretation of the meaning of these nonstochastic lagged values of the dependent variable. Using a comment by Griliches (1960, p. 291) as a basis for his argument, LaFrance suggested that the correct behavioral response variable for farmers in a distributed lag model was expected normal (or planned) output rather than actual output. The proxy used for planned output is approximated in the aggregate for the industry by the nonstochastic lagged dependent variable. LaFrance's argument which was couched primarily in terms of crop production, was that actual output differed from planned output because of the effects of random factors like weather variation. Planned output in

turn differed from desired long-run equilibrium output because of adjustment costs and asset fixities. LaFrance demonstrated that the systematic, measurable component of planned output is equivalent to the "expected" level of production, given average weather conditions. In other words, a nonstochastic difference equation might be interpreted as implying that output in the current period is affected by the "planned" or "expected" levels of output in previous periods rather than by the actual levels. This interpretation seems quite reasonable for crop output where the difference between planned and actual output due to random weather effects may be proportionately quite large. Also, for crops, unusually good weather conditions in a given year will lead to unusually high output in that year but will probably not have much effect on the level of output in following years. Thus, it makes sense to remove this random component of output from the lagged value of the dependent variable. That is, the use of a nonstochastic difference equation seems to be appropriate when estimating models where crop yield has a major influence on the dependent variable.

The above interpretations for crop production would appear not to apply as well to a livestock model. It may not make sense to remove the random component from the lagged value of the dependent variable for the same reasons as in the crop models since the random

component of output in a given year may affect the level of output in subsequent years to a considerable degree (e.g. an exceptionally large calf crop in a given year may imply exceptionally large herd sizes in following years).¹ This characteristic of the livestock breeding industry implies that the use of nonstochastic difference equations may not be theoretically justifiable when estimating beef inventory models. However, there are other factors that enter into the determination of the "appropriate" forms of the difference equations. It might be instructive at this point to discuss in more detail the elements that will generally determine whether stochastic or nonstochastic difference equations yield better results.

One crucial consideration here is the composition of the error term. The degree of correspondence between the error term in the aggregate model and the random variations that cause differences

¹The causes of the differences that do exist between planned and actual herd sizes (these differences should generally be much smaller proportionately for cattle than for most field crops) are not as obvious for cattle as they were for crops. For example, weather variations would appear to have a proportionately smaller direct effect on the size of beef breeding herds than on crop production. Some factors that may result in divergences between planned and actual herd sizes include unanticipated disease and death loss, variations in the availability of feed because of random weather factors, and variations in calf crops are a result of the random nature of the cattle breeding process.

between planned and actual output play an important role in determining the correct specification of the form of the difference equation. To see this, consider the following model:

$$(1) \quad y_t = \alpha + \beta X_t + \lambda y_{t-1} + u_t$$

Assume that this difference equation has been derived from a partial adjustment model. The question to be addressed, once again, is whether the stochastic or the nonstochastic formulation of this model is appropriate. The answer appears to depend largely on the composition of the error term. This can be demonstrated by partitioning the error term into two components,

$$(2) \quad u_t = w_t + \epsilon_t$$

where w_t is the portion of the error term due to fluctuations in factors beyond the control of ranchers (e.g. weather variations) that cause actual output to differ from planned output. ϵ_t is other types of errors in our model (e.g. maybe some minor variables that have systematic effects on the level of y_t have not been included). Now, compare the two extreme situations to clarify the effect of the composition of the error term.

First, look at the situation in which $\epsilon_t = 0$ (assume also that the average effect of the w_t 's is zero). In this case, the model is correctly specified to the extent that all relevant explanatory

variables have been included. The only source of differences between planned and actual levels of y_t are random factors (like weather) beyond the control of ranchers. Under these circumstances, the variable that best reflects the constraints faced by ranchers (or alternatively, the planned level of y_{t-1}) is $\bar{y}_{t-1} = \alpha + \beta X_t + \lambda \bar{y}_{t-2}$. The best example of this type of a situation might be a crop yield model. For these farmers, the variable that has the greatest effect on y_t will be the average (or planned) level of y_{t-1} since the actual level of y_{t-1} is influenced by random weather fluctuations (w_{t-1}) which, by themselves, will not have much effect on y_t . A nonstochastic specification of the difference equation would seem correct in this scenario.

At the other extreme, suppose that $w_t = 0$. That is, all of the differences between the actual values of y_t and the values predicted by the model are due to variables that ranchers take into account when making production decisions but which have not been included in the model. In this case, the error component of y_{t-1} will have a systematic effect on the level of y_t . That is, $\bar{y}_{t-1}$ would not do a

good job of reflecting constraints or rigidities,² since its value depends only on previous values of X_t , when in reality, there are systematic effects from other variables that also affect previous levels of y_t . Thus, the stochastic formulation of the difference equation would seem appropriate under these circumstances.³

²The use of a difference equation can be justified directly if the lagged values of the dependent variable are thought of as proxy variables for the rigidities faced by ranchers. At any given point in time, each rancher will be constrained by the levels of certain "state variables." These variables include such factors as, condition of the range, age distribution of the breeding herd, and availability of specialized labor and machinery. If a one dimensional measure of the information contained in these "n" state variables is desired (either because of prohibitive costs of measuring the state variables, or because of limited degrees of freedom) then the level of herd sizes in the preceding period is probably the leading candidate. This discussion justifies the use of a difference equation, but does not reveal whether the stochastic or nonstochastic specification is appropriate. This will depend on how well the X_t 's in the model reflect the state variables actually taken into account by producers. If the X_t 's do a poor job of reflecting the levels of these variables, then the stochastic specification should be used.

³The interpretation of the lagged value of the dependent variable (in either its stochastic or nonstochastic form) as a measure of production rigidities avoids any discussion of producers' subjectively formed expectations. This is probably desirable, since this notion is quite different from the "expected value of a random variable," which is what is obtained when we find the nonstochastic value of the lagged dependent variable in our models. It might also be noted that a set of consecutive lagged values of the dependent variable (or its expectation) would perform essentially the same role as a single value, but jointly, could potentially capture more information about the underlying rigidities and constraints on production.

The true composition of the error term will be somewhere between these two extremes. If the error term consists mainly of the first type of error, then a nonstochastic difference equation will probably yield the best empirical results. If the second type of error is predominant, then a stochastic difference equation should yield better results. Normally, the true composition of the error term will not be known. In such cases, it might be possible to make inferences about this composition by comparing the empirical results from the estimation of the two specifications of the difference equation.

In the preceding discussion of the elements that determine the appropriate specification of the difference equation, it was assumed that a partial adjustments model led directly to the presence of the lagged dependent variable. Difference equations can also result "indirectly" from adaptive expectations models.⁴ To see which specification is appropriate when the difference equation is implied by an adaptive expectations model, consider the model,

$$(3) \quad y_t = \alpha + \beta X_t^* + u_t$$

⁴That is, given certain assumptions about the weights given to preceding values in the formation of expectations of future values of a variable, a difference equation can be estimated as a way of reducing the number of parameters in the model.

where u_t has the classic properties and X_t^* , the expected level of X_t , is formulated in the following fashion

$$(4) \quad X_t^* - X_{t-1}^* = \gamma(X_t - X_{t-1}^*)$$

The difference equation that results from solving (4) for X_t^* , substituting the result into equation (3) and then performing a Koyck transformation on the resulting geometric lag model is

$$(5) \quad y_t = \alpha + \beta\gamma X_t + (1-\gamma)y_{t-1} + u_t - (1-\gamma)u_{t-1}$$

Noting that $y_{t-1} - u_{t-1} = E(y_{t-1})$, (5) can be rewritten as the non-stochastic difference equation

$$(6) \quad y_t = \alpha + \beta\gamma X_t + (1-\gamma)E(y_{t-1}) + u_t$$

The advantage of equation (6) over equation (5) is that the stochastic and systematic components of the model are clearly delineated in (6) and that the error term in (6) has the same properties as in the original equation (3). The latter implies that the structure of the error term in (6) will be simpler than in (5). This is a straightforward justification for the use of a nonstochastic difference equation when the model to be estimated is derived from an adaptive expectations model.

Suppose now, that a difference equation results from a combination partial adjustment - adaptive expectation model. In this situation it

is unlikely that, a priori, one will be able to specify the form of the difference equation. Comparing results from the empirical estimation of the nonstochastic and stochastic formulations of the models will yield certain insights into the structure of the system. For example, for the cattle numbers models estimated in this thesis, the nonstochastic formulation of the difference equation was clearly superior to similar stochastic specifications. This might indicate that the factors implying a nonstochastic difference equation (random variations in variables beyond the control of producers and the influence of the adaptive expectation portion of the system) "outweighed" the elements that lead to a stochastic difference equation (left-out state variables).

Suppose the model to be estimated is of the form

$$(7) \quad y_t = \alpha + \beta X_t + \lambda \bar{y}_{t-1} + u_t$$

where y_t is herd size in period t , X_t is the level of an independent variable (or vector of variables) that is hypothesized to affect herd size, $\bar{y}_{t-1}$ is the nonstochastic component of the lagged dependent variable, and u_t is the error term. The best estimate of the average value of the lagged dependent variable is

$$(8) \quad \bar{y}_{t-1} = \alpha + \beta X_{t-1} + \lambda \bar{y}_{t-2}$$

If (8) is substituted into (7) and then (7) is solved recur-

sively, the result is

$$(9) \quad y_t = (\alpha + \lambda\alpha + \lambda^2\alpha + \dots + \lambda^{t-1}\alpha) + (\beta x_t + \lambda\beta x_{t-1} + \lambda^2\beta x_{t-2} + \dots + \lambda^{t-1}\beta x_0) + \lambda^t \bar{y}_0 + u_t$$

This equation is nonlinear in the parameters and thus cannot be estimated using linear least squares techniques. If the error term does not have the classical properties (e.g. if it is autoregressive) then the model is nonlinear for that reason as well. To handle this, the computer program used for estimating the model obtains least squares estimates using a modified Marquardt nonlinear least squares algorithm.⁵ The program is capable of estimating stochastic as well as nonstochastic difference equations with an autoregressive/moving average error term.

After a difference equation has been estimated, one of the phenomena that can be discussed is the implied shape of the response function of the dependent variable to changes in the exogenous variables. Different values of the estimated coefficients on the lagged dependent variables imply different adjustment paths. Some set of values of these coefficients implies cyclical adjustment patterns, in particular, the case of complex roots in a second order difference

⁵For a description of some common methods of nonlinear estimation, see Draper and Smith (Chapter 10).

equation. (The methods for solving difference equations are described in Baumol and Chiang.) One simple, and plausible, explanation of the meaning of these cyclical types of response paths is that decision makers have a tendency to overadjust in their responses to exogenous shocks in prices or other variables.

The most common theoretical justifications for using difference equations are the partial adjustment, adaptive expectations, and combination partial adjustment-adaptive expectations models. It can be demonstrated that all of these models implicitly constrain the values of the coefficients of the lagged dependent variables in such a way as to exclude the possibility of a cyclical adjustment pattern. Thus, there is no theoretical model in the literature on distributed lags that would be consistent with a cyclical adjustment pattern. In addition, the interpretation of a lag distribution as a probability density function is no longer valid when the response function is cyclical, implying that the concepts of "the mean lag" and "the variance of the lag" lose whatever meaning they had in the case of asymptotic response functions. As a result, there has been very little discussion of cyclical lag patterns.

Yet observable quantities do follow cyclical paths outside of theoretical models and there are at least two reasons for suspecting that the difference equations to be estimated in this paper may have

complex roots. First, cattle prices have tended to be cyclical in the past. Rational producers probably expect these cyclical fluctuations to continue in the future. Thus, even if prices stopped their cyclic fluctuations, ranchers would continue to expect such fluctuations for a certain period of time (until their expectations, adapted to the new path of prices) and would adjust their herd sizes accordingly. In such a case, one might expect the response function for cattle numbers to follow a damped oscillatory path, with the oscillations dying out as ranchers realize that prices are no longer cyclic.

The second reason for expecting complex roots derives from the possibility of specification error in the model. Practically speaking, it is unlikely that any empirical model will be correctly specified. In the model to be estimated in this paper, one likely source of specification error is in the functional form. The estimation procedure to be used is constrained to linear difference equations. It is probably naive to presume that the true relationship is linear. Thus, if the true model is represented by a nonlinear difference equation, it seems likely that in order to achieve the best empirical fit of the nonlinear system, the estimated model may take on a form that implies a cyclical response function, particularly in view of the cyclical pattern of prices within the sample. Because of these

factors the coefficients on the difference equations estimated in this thesis will not be constrained to values that imply only real roots.

The Inventory Model

The basic form of the model for estimating cattle numbers will be

$$y_t^D = f(P_t^e, F_t, W_t) + \epsilon_t$$

where y_t^D is the desired herd size in period t , P_t^e is expected future prices of cattle, F_t is feed price (or availability) in period t , W_t is weather conditions in period t , and ϵ_t is an error term.

It is hypothesized that ranchers base their decisions about herd size on their expectations of the future price of cattle. Using the approach suggested by Jarvis (1974), in which cattle are viewed as capital goods, it can be seen that price changes will have two opposing effects on ranchers' decisions. An increase in price will probably cause ranchers to expect higher prices in the future. This should lead them to want to increase the size of their breeding herds so they can better take advantage of these future prices. Jarvis referred to this as the investment demand for cattle. On the other hand, the price increase will put pressure on cattle producers to sell cattle immediately to take advantage of the current high price. This effect is analogous to Jarvis' consumption demand for cattle. A priori, there

is no theoretical justification for expecting one of these effects to be greater, although the results of previous empirical studies generally seem to indicate that the former effect outweighs the latter (which makes sense if the rapid growth stage is going to occur).

The second variable that is hypothesized to have an effect on cattle inventories is the price or availability of feed. Ranchers will look at factors like the prices of high protein supplements, the availability of winter forage and the price of hay when deciding how many cattle to keep through the winter. It is not possible to tell in advance which of these variables will have significant effects empirically, but it would be expected that an increase in the price of protein supplements (like soybean oil meal) will cause ranchers to decrease the size of their herds (*ceteris paribus*, of course) because of the implied higher costs of production. Poor pasture conditions, i.e. a shortage of winter forage, will be expected to cause ranchers to carry fewer cattle through the winter than otherwise. A poor hay crop during the summer will imply that ranchers will have trouble finding this type of feed for their cattle during the winter. This would be expected to cause them to reduce their herd sizes. There should be a strong negative correlation between the availability and price of hay, so that it is difficult to predict which of these variables will do the best job empirically. It might be argued though,

that if most ranchers grow their own hay (rather than buying it) then a measure of availability might be more effective than a measure of price. Another reason for suspecting that the availability variable may be more effective is that aggregate price statistics on hay have tended to be rather poor because of the local nature of hay markets.

The third variable is weather conditions. Poor conditions would be expected to cause ranchers to reduce their herd sizes. There is probably a variety of forms that "poor weather conditions" can take. For example, an unusually dry summer would probably imply a dearth of winter forage and hay availability. Or unusually cold and snowy conditions during the months of October, November and December might lead ranchers to expect the same for the rest of the winter and cause them to cut back on their herd sizes. Bad weather in the spring during calving season would also have an adverse effect.

In the next chapter, some of the problems encountered in trying to obtain empirical counterparts of the theoretical variables described above are discussed. The results and implications of the final versions of the models estimated are then analyzed in detail.

Chapter 4

RESULTS

In the preceding chapter, the theoretical considerations in the development of the model for this thesis were discussed. In this chapter, the estimated results of the theoretical model are described.

Three final models were developed in this study. The first model attempts to explain the level of breeding herds in Montana, while the other two attempt to explain total cattle numbers in Montana (the definition and discussion of these categories is contained in Appendix A). All three of these models will be discussed, since the potential uses of a total cattle numbers model may differ from the uses for a breeding herd model.¹

The breeding herd model and both of the total cattle numbers models will be described, analyzed, and discussed in detail. Both total

¹For example, predictions of total cattle numbers may be used to forecast slaughter, which in turn might lead to estimates of wholesale or retail beef supplies. On the other hand, one potential use for the breeding herd model might be as a tool to better understand how cattle producers form their expectations of the future, and the criteria they use in their production decisions. It may also turn out that breeding herd models are the best indicator of future total cattle numbers, since changes in breeding herds should be a strong indication of what ranchers plan to do with their overall herd sizes. Obviously, breeding herd size is the crucial population variable for forecasting next year's calf crop.

cattle numbers models will be discussed since an evaluation of the results of the tests on these two models indicated that neither was clearly superior.

This chapter will be divided into three sections. The first section will contain a description of the specification and results of several of the preliminary models tested in the search for the final models. The second section contains detailed analyses and discussions of the final models. The third section describes several attempts to improve the final models, as well as the results of these attempts.

Preliminary Models

A major problem in the estimation of the models was to find the combination of measurable proxies for the theoretical variables described at the end of the preceding chapter that resulted in the "best empirical model." There is no formal definition outlining the characteristics of the "best empirical model," so frequently the choice among model specifications was made on the basis of subjective factors. At times, these choices were made using nothing more concrete than the intuition that was developed from working with different versions of the model. In most cases, the early versions of the models were so different from the final versions (discussed later in this chapter) that a comparison of the actual empirical estimates would be of little value. For this reason, most of the following discussion is basically descriptive in nature and very few actual estimation results are

presented.

The first theoretical variable in the model was the price of cattle. There are many potential empirical analogs to this variable. As was discussed in the preceding chapter, a change in the price of cattle should have two effects: (1) it should cause a change in ranchers' expectations of future price levels and (2) it should imply a change in the pressure to sell immediately. Much of the effort to find proxies for the price variable was devoted to trying to isolate and distinguish between these effects. The remaining work with this variable (as well as with the other variables in the model) was directed toward finding the order of the lag on the proxy variables that resulted in the best empirical fit.²

One price variable that was tested in a number of early models was the price received for utility cows. It was hypothesized that changes in this price might induce ranchers to sell their older cows at different ages, thereby resulting in changes in the size of their herds. However, empirical results indicated that its coefficient was not generally statistically significant, i.e. it added little to the

²The inclusion of lagged values was justified since we postulated that expected future prices were determined by current and past price levels. The search for the lag that yielded the best empirical fit was warranted by our decision to let the data determine the shape of the response function. A rational lag structure is determined jointly by the order of the difference equation and the order of the lag on the independent variable.

explanatory power of the model. Another price variable that was included in a number of early models was the November price of feeder steers at Kansas City. It was thought that this variable might reflect the immediate pressure on ranchers to sell their cattle before the winter began. Although the estimated coefficient on this variable was significantly different from zero in many of the early models, it did not seem to contribute much to the more refined models and is therefore not included in any of the final models. Two of the following three price variables appear in each of the final models: (1) a twelve month average of Kansas City feeder steer prices, (2) the October through December average of prices received for calves by Montana ranchers, and (3) the ratio of the average wholesale price of choice steers at Omaha to the wholesale price of corn at Omaha. These variables and the roles they play in the final models are discussed in the next section of this chapter and in the final chapter.

The second variable in the theoretical model is the price, or availability, of feed. It was hypothesized that the price of protein supplements might affect ranchers' decisions to increase or decrease herd sizes. To test for this, a protein price variable (the average annual price of cottonseed meal at Memphis) was included in preliminary specifications of the model. The coefficients on this variable were frequently significant, but the values of the coefficient were extremely sensitive to the specification of the model. In several cases,

the addition or subtraction of one variable from the model caused the coefficient on the protein price variable to change from a value significantly greater than zero to a value significantly less than zero. Because of this instability, the protein variable was eventually dropped from the model. Another protein price variable (price of soybean oil meal at Chicago) was also tested in a couple of specifications of the model. This variable showed little potential and was quickly discarded. A possible explanation for these variables not playing a strong role is that Montana cattle ranchers rely on other sources (forage and hay) for most of their feed so the price of protein has little effect on their decisions regarding herd sizes. This result may not generalize to areas where ranchers rely less on their own sources of feed.

Another hypothesized feed variable was the availability of forage for winter feeding. Unfortunately, no direct measure of this variable is available. Data are not kept on the number of acres left as pasture and rangeland (although it may be possible to estimate this acreage as a residual) or on the quantity of grass that grows each year. It might be posited, though, that the availability of forage would be positively correlated with the condition of the range. To test this hypothesis, a range index variable was included in the early models. Although different modifications of this variable were run in different models (e.g. the average of the April through December levels of the index and

the November level of the index lagged either one or two years), it never entered with strong significance. This may have been because this variable contained the same type of information as the hay variables in the model or because the information it did contain was simply not detailed enough to pick up the desired effects.

The third hypothesized feed variable was the availability of hay. The total tonnage of hay (which includes both irrigated and non-irrigated hay) was used as a measure of this variable. This was an important variable throughout the preliminary specifications and is included in the final version of the models. Since it was not clear whether ranchers base their decisions on the availability of hay or on its price, two different series of hay prices were used in place of the production variable in early runs. With these price series, the models either had trouble converging, or the price variables were not significant, or the introduction of these variables resulted in considerable alteration in the signs and significance of the other variables in the model. As a result, it was decided to use the production series in later versions of the model.

The final variable in the theoretical version of the model was weather conditions. It would be expected that the range index might act as a reasonable proxy for this variable. The results of attempts to incorporate this variable into the model were described above. It is also possible that the hay production variable acts as a surrogate

for weather conditions. Attempts to test for this possibility are described in detail in the next section of this chapter. The results of these tests indicate that the primary role of the production variable was not as a proxy for weather conditions. Thus, there is no weather variable, per se, in the final specification of the model.

Stochastic and Nonstochastic Model Tests

Tests were run on the revised versions of the models to compare the results of the stochastic and nonstochastic formulations. These comparisons are displayed in Tables 1 and 2. Looking first at the comparisons of the two formulations under the assumption that the error term has the classic properties, it is obvious that the nonstochastic specification is superior for all three of the final models. In every case, the adjusted multiple correlation coefficient ($\bar{R}^2$) and the standard error of the estimate are considerably lower for the nonstochastic difference equations. The average t-ratios in the tables also indicate that the explanatory power of the independent variables is greater in the nonstochastic models.³

From these observations it can be seen that the nonstochastic difference equation specification of the model has at least two advantages

³The calculated average t-ratios do not have a clear meaning and thus no great emphasis should be placed on them. They might be interpreted as giving a rough indication of the nominal significance of the explanatory variables, and nothing more.

TABLE 1

Comparison of Summary Statistics from Stochastic and Nonstochastic
Specification of the Breeding Herd Model*

Model Description	$\bar{R}^2$	Standard Error of the Estimate	Average t-ratio on Independent Variables**
Classical Error Term			
Stochastic Dif. Eq.	.9832	46,700	2.90
Nonstochastic Dif. Eq.	.9946	26,700	5.73
Second Order Serial Correlation			
Stochastic Dif. Eq.	.9904	32,600	3.49
Nonstochastic Dif. Eq.	.9946	24,400	9.46

*This model is described in detail in the ensuing section of this chapter. The models in this table were estimated using actual breeding herd numbers as pre-sample values.

**Independent variables include hay production, Montana calf prices, and the ratio of fattened steer prices to corn prices.

TABLE 2

Comparison of Summary Statistics from Stochastic and Nonstochastic Specifications of the Total Cattle Numbers Models*

Model Description	$\bar{R}^2$	Standard Error of the Estimate	Average t-ratio on Independent Variables**
<u>Model TA</u>			
Classical Error Term			
Stochastic Dif. Eq.	.9628	89,500	2.12
Nonstochastic Dif. Eq.	.9851	57,800	5.84
Second Order Serial Correlation			
Stochastic Dif. Eq.	.9768	64,500	3.41
Nonstochastic Dif. Eq.	.9902	42,000	10.61
<u>Model TM</u>			
Classical Error Term			
Stochastic Dif. Eq.	.9674	83,700	2.72
Nonstochastic Dif. Eq.	.9849	58,400	4.97
Second Order Serial Correlation			
Stochastic Dif. Eq.	.9791	61,300	3.16
Nonstochastic Dif. Eq.	.9898	43,000	8.46

*These models are described in detail in the ensuing section of this chapter. The models in this table were estimated using actual total cattle numbers as pre-sample values.

**Independent variables in model TA are hay production, price of feeder steers at Kansas City and the ratio of the prices of fattened steers to corn. Independent variables in Model TM are hay production, Montana calf prices, and the ratio of the prices of fattened steers to corn.

over the stochastic specification. First, considerably less of the variation in the dependent variable remains to be explained as systematic behavior in the error term. This is advantageous since (1) the error term consists of variations in the dependent variable, the causes of which cannot be attributed to any factors predicted by theory, and (2) fewer parameters are required in the model with a simpler structure on the error term. Second, if the specification of the model with a classical error term is not the "true" form of the model generating the data, then the estimated coefficients in the stochastic difference equation will not be consistent. On the other hand, the estimated coefficients in the nonstochastic difference equation model with the classical error term are consistent regardless of whether the specified structure of the error term is the "truth." Thus, the nonstochastic difference equation is more robust with respect to specification errors on the structure of the error term.

It is also apparent from Tables 1 and 2 that for the models with second order autoregressive specifications of the error term, the nonstochastic versions are, in all three cases, superior to the stochastic versions. It can be shown that there exists a reparameterization of any nonstochastic difference equation (regardless of the specification of the error term) that results in an equivalent stochastic difference equation (see Burt 1980, pp. 3-5). If it is assumed that the nonstochastic difference equation models with second order

autoregressive error terms are the true specifications (which is what the comparison of the results of the numerous models estimated would imply), then there exists a stochastic difference equation specification that is equivalent and would yield comparable results. In this case, the error term in the stochastic difference equation would be a combination autoregressive, moving average process. The advantages of estimating the nonstochastic difference equation are that (1) the structure of the error term is less complex, making the interpretation of this component of the model easier, and (2) it is less of a problem numerically to estimate models with autoregressive error terms than it is to estimate models with moving average errors.

In the estimation procedure used for the models in this thesis (and for any distributed lag model) it was necessary to specify pre-sample values for the variables in the model. For example, if the first observation on the dependent variable included in the data set is for year t , then values for the dependent variable must be specified for years $t-1$ and $t-2$ when estimating a second order difference equation. For the nonstochastic difference equations in this thesis, two possible estimates of these presample values were considered. The first possibility was to use the actual values of the dependent variables for these two years. The second possibility was to estimate these presample values as parameters, i.e. to find the values of the dependent variable in those two years that would minimize the residual

sum of squares. A comparison of the summary statistics from these alternative specifications is presented in Table 3. For the breeding herd model, the specification in which the presample values are estimated as parameters is only marginally better than the alternative. For the total cattle numbers models, this specification is clearly superior. It is worthwhile to speculate as to why these results were obtained.

If the structure of the system was the same during the presample period as during the sample period and if no unusual events occurred during the presample years, then one would expect the actual presample values of the dependent variable to be quite close to the values of that variable that minimize the residual sum of squares. In this case, it would not make much difference which method was used to obtain the presample values. It might be hypothesized that this was the situation for the breeding herd model. This can be tested by observing how close the estimated presample parameter values were to the actual levels of herd sizes. The results indicated that the actual values fell near the centers of the confidence intervals for the parameter estimates of the presample values⁴, i.e., the hypothesis that no unusual events occurred

⁴To construct confidence intervals of the initial value parameters, it was necessary to have estimates of the standard deviations of $y_0 - E(y_0) = \epsilon_0$ and $y_{-1} - E(y_{-1}) = \epsilon_{-1}$. Under the assumption that the properties of the error terms in the presample period are the same as during the sample period, a reasonable approximation of the standard

TABLE 3

Comparisons of Summary Statistics from Models Estimating
Pre-sample Values as Parameters with Models using
Actual Herd Sizes as Pre-sample Values

Model Description*	$\bar{R}^2$	Standard Error of the Estimate
<u>Breeding Herd Model</u>		
Actual Herd Sizes Used as Pre-sample Values	.9946	24,400
Pre-sample Values Estimated as Parameters	.9955	24,100
<u>Model TA</u>		
Actual Herd Sizes Used as Pre-sample Values	.9902	42,000
Pre-sample Values Estimated as Parameters	.9955	31,200
<u>Model TM</u>		
Actual Herd Sizes Used as Pre-Sample Values	.9898	43,000
Pre-Sample Values Estimated as Parameters	.9961	29,100

*All models in this table use nonstochastic difference equations and were run with second-order serial correlation specifications on the error term.

and that the structure of the system did not change during the presample years would be accepted. On the other hand, if the structure of the system was different for the presample period or if unusual events occurred during these two years, then it would be expected that the difference between the estimated and actual presample values will be considerable. This would appear to be the situation for the total cattle numbers models. The results of the models showed that this was indeed the case. The first actual presample value was outside of the confidence interval for that parameter estimate. From this, the hypothesis that the structure of the system changed or that the actual values of the dependent variables were affected by unusual events during the presample years would be accepted.⁵

⁴(cont.) deviations of ϵ_0 and ϵ_{-1} might be the standard error of the estimate of the estimated model. Using these, approximate 95 percent two-tailed confidence intervals were constructed to test the hypothesis discussed in the body of the paper. It might be noted that the standard error of the estimate only includes the variance due to the movements in the dependent variable. Strictly speaking, the desired measure of variation would also include variation due to estimating the parameters. Inclusion of this variation would increase the width of the confidence interval and make it more difficult to reject the hypothesis that there is no specification error.

⁵The preceding is related to a Monte Carlo study by Schmidt (1975). Except for the presample parameter estimates, the estimates of the parameters in the model are consistent regardless of how the initial conditions are obtained. Schmidt found that it made little difference whether the actual values were used or whether the initial values were estimated as parameters. Our results indicate that under certain conditions the choice between using actual values or estimating the initial values as parameters may make a considerable difference in the

Another way of looking at the comparisons in Table 3 is that they provide us with a test for specification error. If the results from the two specifications of the model are quite different it can be taken as an indication that the structure of the system changed between the presample years and the years included in the sample. This implies that it might not be appropriate to start the sample in a year that was preceded by years from a system with a different structure since the use of initial values from systems with a different structure may prove detrimental to the results of the model (especially in small samples).

The next section of this chapter contains detailed descriptions and analyses of the final versions of the estimated models.

⁵ (cont.) fit of the model; apparently the total cattle numbers equations suffer some specification error.

Analysis of Final Models

As mentioned above, three final models were developed, one breeding herd model and two total cattle numbers models. In this section, the breeding herd model will be presented and analyzed first. Two different breeding herd models were initially developed. Early tests did not show either of them to be better than the other, so both were tested in more detail. A close analysis of these more complete results indicated that one of the models (the model that used Montana calf prices rather than Kansas City feeder steer prices) did appear to be superior. Each of these models will be presented individually, justification will be given for choosing one of the models over the other, and then the chosen model will be analyzed in detail. Following this, each of the total cattle numbers models will be presented and analyzed in detail. These models will then be compared and contrasted. Both of the total cattle numbers models will be analyzed in detail since neither of these models was clearly superior.

The definitions of the variables used in this section are:⁶

AF\$ - average Kansas City feeder steer price (dollars per hundred weight). This is an annual average of all weights and grades shipped.

⁶For further discussion of these variables and the construction of the data series see Appendix A.

B/C - beef to corn price ratio. The beef price used was the annual average wholesale price of choice steers (dollars per hundred weight) in Omaha. A weighted average annual price (dollars per bushel) of #2 yellow corn at Omaha was used as the 'corn price.' This latter annual average was calculated for a twelve month period starting in October and ending in September.

HP - hay production. This variable was defined as the total production of all hay (includes irrigated and nonirrigated hay) in tons in a given year.

MC\$ - price received by Montana ranchers for their calves (dollars per hundred weight). A weighted average of October through December prices was used.

w_t - total head of cattle in Montana on January 1 of year t .

y_t - number of cows and heifers included in the breeding herd category on January 1 of year t .

$E(y_{t-j})$ or $E(w_{t-j})$ - the 'expected value' of the dependent variable lagged j periods. This variable can be viewed as the actual lagged value of the dependent variable with the non-systematic, or random, components netted out.

u_t - the error term. In the three final models the estimated error term followed a second order autoregressive pattern.

That is, in these models, $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + v_t$,

where v_t has the properties of the classic error term.

The annual feeder price and Montana calf price variables have been adjusted for inflation in all of the final models (note that it is not necessary to deflate the B/C variable, since the deflator enters both the numerator and denominator). The price index used was 1967 base 'prices paid by farmers for commodities, services, interest, taxes and wage rates.'

The following abbreviations will be used to identify the models to be discussed.

BM - the breeding herd model that uses Montana calf prices as the price variable.

BA - the breeding herd model that uses average annual Kansas City feeder steer prices as the price variable.

TM - the total cattle numbers model that uses Montana calf prices.

TA - the total cattle numbers model that uses annual Kansas City feeder prices.

Breeding Herd Models

The sample period used for both of the breeding herd models was 1949-1979. There were two reasons for beginning the sample period around 1949. First, several of the data series did not exist prior to this time. Second, it was felt that the general structure of the industry may have been quite different before and during World War II than it has been since that time. There was no particular reason for

beginning the sample in 1949 rather than in 1948 or 1950. Later results will indicate that the model was not particularly sensitive to the year in which the sample was started.

Table 4 presents the breeding herd models.⁷ The basic difference between these models, with respect to their specifications, is that in model BA lagged average annual Kansas City feeder steer prices were used as proxies for price expectations, while in model BM weighted averages of October through December Montana calf prices were used. It should be noted that the lag structures on the price variables in these models is different. Model BA has a second order lag on the beef to corn price ratio and a third order lag on the annual feeder price variable, while BM has a first order lag on the beef to corn ratio and a second order lag on Montana calf prices. Model BM displays second order serial correlation, compared to third order serial correlation in

⁷The summary statistics calculated by the program used to do the empirical estimation for this thesis (R^2 , $\bar{R}^2$, S.E.E., etc.) do take account of the variation in the dependent variable that is attributable to the systematic portion of the error term. In a nonlinear model, there are problems interpreting the meaning of the conventional definition of R^2 . To make this measure more intuitively meaningful, R^2 was defined and computed as the square of the correlation coefficient between the actual and predicted values of the dependent variable. For a discussion of this topic, see Dhrymes (1971, pp. 338-343).

TABLE 4
SUMMARY OF BREEDING HERD MODELS

MODEL

BM $y_t = -118,470 + .16888HP_{t-1} - .10935HP_{t-2} + 3580.8(B/C)_{t-1} + 3213.7MC\$_{t-1}$
 (6.025) (10.87) (4.564) (7.841) (6.996)

$- 2146.6MC\$_{t-2} + 1.7102E(y_{t-1}) - .83200E(y_{t-2}) - .69622u_{t-1} - .39377u_{t-2}$
 (8.449) (50.69) (43.45) (4.217) (2.385)

$R^2 = .9972$ $\bar{R}^2 = .9955$ S.E.E. = 24,139 S.E.P. = 32,186

 d.f. = 19 Sample Period = 1949 - 1979 S/M = .00153

BA $y_t = -178,140 + .15567HP_{t-1} - .078689HP_{t-2} + 8952.2(B/C)_{t-1} - 3807.6(B/C)_{t-2}$
 (10.29) (14.65) (4.998) (10.08) (4.667)

$+ 2456.4AF\$_{t-1} - 3757.2AF\$_{t-2} + 2620.1AF\$_{t-3} + 1.7256E(y_{t-1}) - .87090E(y_{t-2})$
 (2.270) (2.301) (3.082) (83.25) (66.01)

$- 1.3284u_{t-1} - 1.1740u_{t-2} - .52421u_{t-3}$
 (8.544) (6.099) (3.372)

$R^2 = .9986$ $\bar{R}^2 = .9973$ S.E.E. = 18,197 S.E.P. = 25,052

 d.f. = 15 Sample Period = 1949 - 1979 S/M = .00115

Numbers in parentheses are estimated t-values. S.E.E. is the standard error of the estimate; S.E.P. is the approximate standard error of prediction; d.f. is the number of degrees of freedom; and S/M is the ratio of the standard error of the estimate to the mean of the dependent variable.

BA.⁸

Both of these models can be viewed as second order nonhomogeneous, nonstochastic difference equations. The characteristics of model BM implied by the estimated difference equation, as well as other characteristics of this model, will be analyzed and discussed in detail. Model BA will not be analyzed and discussed in detail, since a number of indications led to the belief that BM was a superior model. A cursory inspection of the summary statistics reported in Table 4 would appear to indicate that the opposite might be the case. Model BA has a higher adjusted multiple correlation coefficient ($\bar{R}^2$), a lower standard error of the estimate, a lower standard error of prediction, and

⁸ A third specification of the breeding herd model looked promising but was overlooked until this thesis was almost completed. This model differed from model BM only in that it had a second order lag on the beef to corn price ratio and a first order lag on the Montana calf price variable. The estimation results of this model were

$$\begin{aligned}
 y_t = & -139270 + .1766HP_{t-1} - .1016HP_{t-2} + 7030B/C_{t-1} - 3537B/C_{t-2} \\
 & \quad (13.4) \quad (5.4) \quad (17.1) \quad (11.4) \\
 & + 1192MC\$_{t-1} + 1.702E(y_{t-1}) - .8453E(y_{t-2}) - .8418u_{t-1} \\
 & \quad (3.9) \quad (61.4) \quad (54.3) \quad (5.6) \\
 & - .5498u_{t-2} \\
 & \quad (3.7)
 \end{aligned}$$

$$\bar{R}^2 = .9963 \quad S.E.E. = 21,900 \quad S.E.P. = 23,850$$

Further testing of the properties of this model might yield interesting results.

a lower ratio between the standard error of the estimate and the mean of the dependent variable. However, there were numerous other results that contradicted these statistics. Included among these results were:

- the precision on the estimated coefficients on the primary price variables (AF\$'s and MC\$'s) is considerably less in model BA.
- BA is a more complex model than BM, which might lead to the suspicion that the specification of the model may not be very stable as the sample period changes. This suspicion was borne out.
- BA did not predict as well outside of the sample period as model BM. Several simplified and altered versions of BA were tested for prediction capabilities, but none of them did as well as model BM.
- there were numerous indications that model BA was little more than an 'empirical model,' in the sense that it fit the data well, but the results did not lend themselves to intuitive explanation.

Elasticity estimates, results of predictions, effects of changing the sample period, etc. for model BA will all be reported in tables. Most of these results will be mentioned in passing (if not in the body of this thesis, then in footnotes) so that the reader may judge for himself as to the relative merits of model BA. Beyond this, however, BA will not be analyzed or discussed in any detail.

Model BM, presented in Table 4, can be reduced to the following

second order difference equation,⁹

$$(1) \quad y(t) = 1.7102y(t-1) - .83200y(t-2) + K$$

$$\text{where } K = 1118,470 + .16888E(HP_{t-1}) + \dots -2146.6E(MC\$_{t-2}) \\ - .69622E(u_{t-1}) - .39377E(u_{t-2})$$

If it is assumed that the explanatory variables are nonstochastic and if the error term is replaced by its mean of zero, then

$$K = -118,470 + .16888\overline{HP}_{t-1} + \dots -2146.6\overline{MC\$}_{t-2}$$

For simplicity, evaluate the difference equation (1) at the means of the explanatory variables. Then,

$$K = -118,470 + .16888\overline{HP}_{t-1} + \dots -2146.6\overline{MC\$}_{t-2} = 186,427.5$$

It can be shown that the general solution to the nonhomogeneous difference equation (1), is [Chiang, pp. 583-4]

$$(2) \quad y_t = (.9121)^t (e \cos Qt + f \sin Qt) + 1,530,603.4$$

where e and f are determined jointly by the initial conditions of the system and the value of K (recall that K is dependent on the values of the independent variables at which the difference equation is evaluated). The solution to this model indicates that the system is cyclical

⁹This is actually a nonstochastic difference equation and could formally be written as,

$$E(y_t) = 1.7102E(y_{t-1}) - .83200E(y_{t-2}) + k$$

The expectation operators have been dropped in the body of the paper for notational simplicity.

in nature. The value of Q is determined by the values of the parameters of the homogeneous portion of the difference equation in (1), and in turn determines the length of the cycle implied by the model. For this particular system, it can be shown that a cycle with a period between seventeen and eighteen years is implied.¹⁰ When interpreting this result, it should be kept in mind that the difference equation (1), is necessarily analyzed under unrealistic *ceteris paribus* assumptions with respect to the explanatory variables, i.e., prices and other variables are treated as fixed. In reality, prices will be changing constantly, providing incentives for ranchers to react quickly, thus making the "real-world" cycle shorter than indicated above.

The solution shown in (2) also indicates that the system is stable. To see this, suppose that initially the system is in equilibrium and is then submitted to a one-time exogenous disturbance. As time passes, i.e., as t increases, $e \cos Qt + f \sin Qt$ will simply repeat itself and $(.9121)^t$ will approach zero asymptotically. The system will therefore approach a stable position as t increases. In the system represented by (2), the stationary value is 1,530,603.4. As indicated above, the

¹⁰The term $e \cos Qt + f \sin Qt$ will repeat itself every 360° . It can be shown that, for this model, $Q \approx 20.4^\circ$. Thus $y(t)$ will undergo a cycle of length, $\frac{360^\circ}{20.4^\circ} = 17.7$ years.

time path followed by the system after the occurrence of such a disturbance will be one of dampening oscillations about the stationary value.

Referring again to Table 4, it can be seen that the short run (about three months) effect of a \$1.00 change in the average of October through December Montana calf prices in year t will be to increase breeding herds by about 3,200 head on January 1 of year $t+1$. The short run price elasticity (calculated at the sample means) is thus .0573. In other words, a 1 percent change in Montana calf prices in year t results in (*ceteris paribus*) a .06 percent change in breeding herd sizes on January 1 of year $t+1$. This implies that breeding herd sizes are relatively unresponsive to changes in calf prices in the short run. This would, of course, be expected considering the biological constraints inherent in the cattle industry.

To calculate the long run price elasticity, imagine that the system is initially in equilibrium at price $MC\$^*$ and herd size y^* . Since this is an equilibrium position, it will be the case that,

$$y_t = y_{t-1} = y_{t-2} = \dots = y^*$$

and

$$MC\$_t = MC\$_{t-1} = MC\$_{t-2} = \dots = MC\$^*$$

In equilibrium, the regression equation can thus be viewed as,

$$(3) \quad y^* = \alpha + 3213.7MC\$^* - 2146.6MC\$^* + 1.7102y^* - .83200y^*$$

$$\text{where } \alpha = -118,470 + .16888HP^* - .10935HP^* + 3580(B/C)^*.$$

Rearranging (3),

$$(3') \quad y^* = \alpha' + \left(\frac{3213.7 - 2146.6}{1 - 1.7102 + .832} \right) \text{MC\$}^*$$

$$= \alpha' + 8761.1 \text{MC\$}^*$$

$$\text{where } \alpha' = \frac{\alpha}{1 - 1.7102 + .832}$$

From the above it is apparent that the long run effect of a \$1.00 change in Montana calf prices will be to increase the equilibrium herd size by 8,761 head. The long run price elasticity would thus be .1520. The response of ranchers to a given change in prices is considerably larger if they are given an indefinite period of time to react than if they are only given a few months (or years). This is, of course, consistent with general economic theory.¹¹

Table 5 presents the above elasticities, as well as the elasticities with respect to each of the other explanatory variables for four different lengths of time. Notice that for Montana calf prices (as well as for all the other variables in both of the breeding herd models), the 10 year elasticity is greater than the long run elasticity. This phenomenon seems to be due to the cyclical nature of the adjustment

¹¹ In particular, this is consistent with LeChatelier's Principle, which (paraphrased slightly) states that the response in an economic process to a change in a relevant variable will be greater, the fewer the constraints on the system. Thus, as time passes, more of the inputs into the cattle production process become variable, i.e., the constraints on the system are reduced, and the response to a change in a given explanatory variable will be greater (for further elaboration on this subject, see Silberberg, 1978).

TABLE 5

ELASTICITIES*

MODEL	<u>Hay Production</u>				<u>Beef Price/Corn Price</u>				<u>Price**</u>			
	One Year	Two Year	Ten Year	Long Run	One Year	Two Year	Ten Year	Long Run	One Year	Two Year	Ten Year	Long Run
BA	.3342	.7419	1.373	1.137	.1160	.2669	.5723	.4589	.0438	.0492	.2113	.1520
BM	.3625	.7478	1.326	1.049	.0464	.1258	.5246	.3811	.0573	.1169	.1956	.1561
TA	.3334	.6248	.9172	.3743	.0648	.1825	1.418	1.024	.0114	.0321	.2493	.1800
TM	.3502	.6606	.9003	.4564	.0479	.1336	.9278	.6569	.0232	.0649	.4504	.3189

* All elasticities are calculated at the sample means of the linear model.

** Elasticities with respect to prices received by Montana ranchers for their calves in models BM and TM; and with respect to the average annual Kansas City Feeder Prices in models BA and TA.

process in these models. Intuitively, it implies that, *ceteris paribus*, ranchers have a tendency to 'overadjust' their herd sizes in response to changes in the explanatory variables - when they discover this overadjustment, they compensate, change their herd sizes in the other direction, overadjust again, compensate again, etc. Hence the adjustment process implied by this model (and all three of the others) is one of dampening oscillations about the new equilibrium herd size. This differs from the geometrically declining type of an approach to the new equilibrium implied by many currently popular economic models.

When analyzing the other elasticities presented in Table 5, care must be taken not to interpret the elasticities with respect to the beef to corn price ratio as price elasticities. This variable is thought to be serving as a proxy for the profitability of cattle feeding and can be interpreted as an important variable in the formation of beef producers' future expectations of calf or feeder steer prices. The elasticity estimates of Table 5 are therefore saying that a one percent increase in the beef to corn price ratio¹² in year $t-1$ will result in a .05 percent increase in herd sizes in year t (or a .13

¹² An increase in this price ratio proxies for an increase in the profitability of feedlot operations. There is no reason, however, that a one percent increase in this ratio will necessarily imply a one percent increase (or any other given percentage change) in that profitability. It is not possible, therefore, to use these models to say that a given percentage increase in the profitability of cattle feeding will result in a given percentage change in herd sizes.

percent change over two years, etc.).

Interpretation of the elasticities on the hay production variable are straightforward. Other factors constant, a one percent increase in hay production in year $t-1$ will lead to a .36 percent increase in herd sizes in year t , a .75 percent increase over two years, and a 1.05 percent increase after an indefinite period of time.

Consideration of some of the characteristics of the beef producing industry leads to hypotheses about possible forms that the distributed lag patterns on the independent variables may take. First, consider the effects of an increase in hay production in some year t . An increase in hay production means that ranchers will have more feed available for the winter. This will cause them to carry more cattle through the winter than might otherwise be the case. The breeding herds will be affected in two ways. First, ranchers will keep more of the marginally productive cows already in the herd, and second, they will keep more of the heifer calves born in the spring of year t . These calves, which are approximately nine months old, will not be counted as part of the breeding herd on January 1 of year t , since the breeding herd is defined to include only beef cows and heifers one to two years old. An increase in hay production in t thus only affects breeding herds to the extent that it induces ranchers to keep more older cows. On January 1 of year $t + 2$, the calves kept because of the increase in hay production in year t will be about 20 months old and

will be counted as part of the breeding herd. Since a large portion of the increase in breeding herds comes about through keeping more calves, it might be hypothesized that the effect of an increase in hay production in year t on breeding herds in $t + 2$ would be relatively large. Beyond year $t + 2$, it would seem that the only effects of changing hay production in year t will be to the extent that calves added to the breeding herd in year $t + 1$ and their offspring will remain in the breeding herd for several years.

Analogous arguments can be made to formulate hypotheses that the effect of a change in the beef to corn price ratio or Montana calf price variables in year t on breeding herds in year $t + 2$ should be relatively large.

Distributed Lag Patterns

Table 6 and the graphs of the estimated distributed lag patterns in Appendix B give an indication of the shapes of the distributed lag patterns on the independent variables implied by model BM (and also model BA). The above hypotheses seem to be confirmed, since for all three of the explanatory variables the two year partial derivatives are relatively large.¹³ Notice that for hay production, the lag pattern

¹³ Look at the lag pattern on annual feeder prices in model BA. The two year response to a change in this variable is very small. The general shape of the lag pattern simply does not seem to lend itself to any intuitive explanation. In defense of this model, though, it

TABLE 6
PARTIAL DERIVATIVES

Model BM

Time For Response*

	1 Year	2 Years	3 Years	4 Years	5 Years	6 Years	7 Years	8 Years
Hay Production	.1689	.1795	.1664	.1353	.0929	.0463	.0019	-.0352
Beef Price								
Corn Price	3581	6124	7494	7721	6969	5495	3600	1584
Montana Calf Prices	3214	3350	3055	2437	1626	754	-64	-736

Model BA

Time For Response

	1 Year	2 Years	3 Years	4 Years	5 Years	6 Years	7 Years	8 Years
Hay Production	.1557	.1899	.1922	.1662	.1194	.0614	.0019	-.0502
Beef Price								
Corn Price	8952	11,640	12,290	11,070	8399	4853	1059	-2399
Annual Feeder Prices	2456	482	1312	1844	2040	1914	1526	967

* The values in this table indicate the effects of a change in the value of an independent variable in year t on breeding herds i years in the future (i = 1,2,...8). For example, a \$1.00 change in feeder prices in year t will result in a change in breeding herd sizes of 1312 in year t+3. The cumulative effect of a change in an independent variable can be calculated by summing the individual entries.

peaks and then falls off quite rapidly, while for the beef to corn price ratio, it peaks later. This might be occurring because the latter variable is affecting ranchers expectations and these expectations change gradually and might be affected by prices several years back. This effect, in addition to effects due to calves remaining in the breeding herd for several periods, might explain the difference in the shapes of the lag patterns in the hay production and B/C variables.¹⁴

It was initially thought that there may be some problem with simultaneous equations bias in model BM because of a joint dependence between calf prices and the size of breeding herds. However, since

¹³(cont.) should be pointed out that the B/C and AF\$ variables are correlated and have a joint effect on cattle numbers. Looking at the effects of either of these variables in isolation may yield puzzling results.

¹⁴To this point, a comparison of BA and BM would indicate that judging by the conventional summary statistics presented in Table 4, a second-order lag on calf prices and a first-order lag on beef to corn ratio seems to have about the same explanatory power as a third-order lag on annual feeder prices and a second-order lag on the beef to corn price ratio. This would indicate that the Montana calf price variable is doing a more direct job of measuring the way in which producers form their price expectations. This makes sense since the Kansas City prices are not net of transportation costs and buyers fees, and are thus a rather muddled measure of what ranchers receive for their cattle.

calves are not included in breeding herds, this concern was allayed.¹⁵ One problem with using Montana calf prices as an explanatory variable instead of feeder prices is that the estimates of calf prices probably contain more error than the estimates of feeder prices. This is because feeder price averages are calculated from widely reported and (presumably) highly accurate central market price data, while calf prices are estimated by (hopefully random) sampling of calves within the state. This latter technique would seem to be considerably more susceptible to error than the former. However, since there does not seem to be any reason that calf prices should be contemporaneously correlated with the error term, asymptotic properties of the estimated coefficients should be retained (Kmenta, 1971, pp. 297-304).

Model Predictions

The models were subjected to a number of tests to see how well they would predict outside the sample period.¹⁶ Two basic criteria.

¹⁵ If a joint dependence problem existed, then it would be the case that an increase in breeding herds caused by an increase in the number of calves retained by ranchers (holding other factors constant) would lead to an increase in calf prices. Since a change in the number of calves kept by ranchers does not change the number of animals in the breeding herd category (until the calves are older), there cannot be a joint dependency problem in this model.

¹⁶ No attempts were made at predicting future values of cattle numbers. The "prediction tests" that were run were conducted by truncating the sample period at different points, using the actual values of the independent variables (in the models estimated using the truncated samples) to obtain "predicted" herd sizes, and comparing these predictions with actual herd sizes.

were used. The first was to compare the residuals (the difference between the observed and predicted values of the dependent variable) with the estimated standard errors of prediction. If the residual was larger than two times the standard error of prediction, it was taken as an indication that the model did a poor job of predicting. The second criterion used was to look at the root mean squared errors (RMSE's) of prediction.¹⁷ This statistic gives some indication of the average amount by which the values predicted by the model differ from the observed values of the dependent variable. By comparing the RMSE's for the models, an indication of which model did a better job of predicting was obtained.

Tables 7 and 8 show the comparisons between the residuals and the standard errors of prediction. From Table 7, it can be seen that, judging by this criterion, model BM did not do a very good job of predicting outside of the sample period. It failed the test for predicting 1979 breeding herd sizes both one and two years outside of the sample period. It also had definite problems predicting 1976, 1977,

¹⁷This statistic was calculated using the formula,

$$RMSE_j = \sqrt{\frac{1}{n} \sum_{i=1}^n \hat{e}_i^2}$$

where j is the number of years beyond the end of the sample for which the prediction is being made; n is the number of predictions that were made ' j ' years outside of the sample; and $\hat{e}$ is the difference between observed and predicted values of the dependent variable.

TABLE 7

PREDICTION RESULTS - MODEL BM

Residuals and Standard Errors of Prediction¹

Number of Years Beyond End of Sample Sample Period	1	2	3	4
1949-1978	156,000 (94,000) **			
1949-1977	37,000 (82,000)	203,000 (153,000) **		
1949-1976	-14,600 (90,000)	16,000 (147,000)	174,000 (233,000)	
1949-1975	-96,000 ** (74,000)	-159,000 ** (128,000)	-189,000 ** (181,000)	-95,500 (229,000)
1949-1974	-33,000 (64,000)	-132,000 ** (104,500)	-213,000 ** (167,000)	-263,000 ** (222,000)
1949-1973	45,000 * (57,000)	-1,900 (85,000)	-81,000 (133,000)	-150,000 (201,000)
1949-1972	-26,000 (57,000)	25,000 (75,000)	-38,000 (108,000)	127,000 * (159,000)
1949-1971	38,000 (61,000)	8,200 (87,000)	73,000 (118,000)	26,000 (173,000)
1949-1970	-24,000 (66,500)	9,000 (96,500)	-28,000 (132,000)	25,000 (168,000)
1949-1969	3,000 (82,000)	-21,000 (129,000)	14,000 (183,000)	-22,000 (239,000)

¹ Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

* Indicates that the residual is greater than 1.5 times the standard error of prediction.

** Indicates that the residual is greater than twice the standard error of prediction.

TABLE 8

PREDICTION RESULTS - MODEL BA

Residuals and Standard Errors of Prediction¹

Number of Years Beyond End of Sample Sample Period	1	2	3	4
1949-1978	107,000 (77,000) **			
1949-1977	2,900 (70,500)	111,000 (131,000) *		
1949-1976	16,000 (88,000)	26,500 (153,000)	142,000 (229,000)	
1949-1975	-68,000 (114,000)	-112,000 (243,000)	-167,000 (378,000)	-98,000 (470,000)
1949-1974	49,500 (102,000)	41,000 (267,000)	85,000 (522,000)	117,000 (788,000)
1949-1973	78,000 * (85,000)	165,000 * (180,000)	242,000 (364,000)	393,000 (655,000)
1949-1972	12,000 (108,000)	101,000 (242,000)	210,000 (446,000)	316,000 (759,000)
1949-1971	—	—	—	—
1949-1970	—	—	—	—
1949-1969	43,000 (147,000)	-42,000 (258,000)	-71,000 (431,000)	-77,000 (797,000)

¹ Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

* Indicates that the residual is greater than 1.5 times the standard error of prediction.

** Indicates that the residual is greater than twice the standard error of prediction.

and 1978 herd sizes. The fact that this model was unable to predict 1979 herd sizes is not really very surprising since the graph of breeding herds (Figure A-1 in Appendix A) indicates that 1979 may have been a turning point in the cycle. It does not seem unusual that a model would have trouble predicting such a situation. Notice that if the sample period was terminated prior to 1974, the model predicted quite well up to three, four or even more years outside of the sample period.

Comparing Tables 7 and 8, it would initially appear that BM does a considerably poorer job of predicting than BA.¹⁸ But these results

¹⁸ According to Table 8, BA also has problems predicting 1979 herd sizes, but did fairly well (by this criterion) elsewhere. The blank entries in this table indicate the inability of the computer program to converge on a solution. The apparent cause of this inability to converge was that the model was overparameterized (several of the parameters were so highly correlated with each other that the model was not able to isolate the individual effects of their respective variables). Two simplified versions of this model were therefore constructed and tested to see how well they predicted.

$$y_t = \alpha_0 + \beta_1 HP_{t-1} + \beta_2 HP_{t-2} + \beta_3 (B/C)_{t-1} + \beta_4 AF\$_{t-1} + \beta_5 AF\$_{t-2} \\ + \lambda_1 E(y_{t-1}) + \lambda_2 E(y_{t-2}) + \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \epsilon_t$$

The justification for this specification is that the two variables that were deleted from BA to form the model ((B/C)_{t-2} and AF\$_{t-3}) were the ones whose parameters seemed to be causing most of the parameter correlation problems.

The second modified version of BA (model BA(iii)) was specified as,

$$y_t = \alpha'_0 + \beta'_1 HP_{t-1} + \beta'_2 HP_{t-2} + \beta'_3 (B/C)_{t-1} + \beta'_4 (B/C)_{t-2} + \beta'_5 AF\$_{t-1} \\ + \lambda'_1 E(y_{t-1}) + \lambda'_2 E(y_{t-2}) + \rho'_1 u_{t-1} + \rho'_2 u_{t-2} + \epsilon_t$$

are deceptive. Notice that the standard errors of prediction for model BM are generally considerably smaller than BA. This means it requires a much smaller prediction error for BM to fail this test. If the standard errors of prediction for BM had been as large as those of BA, there would be very little difference in the models as far as their performance in meeting this first criterion is concerned.

Tables 9 and 10 show the comparison between the residuals and standard errors of prediction for two altered versions of model BA, referred to as models BA(ii) and BA(iii) (see footnote 18 for a discussion of these models). Table 9 indicates that when the sample period was extended beyond 1972, model BA(ii) did a very poor job of prediction. Otherwise, the model did quite well. Table 10 indicates that model BA(iii) did a poor job of predicting one year outside of the sample in four out of the ten years for which it was tested. A model that consistently cannot predict one year beyond the end of the sample is probably not specified correctly.

To this point, the models have been tested to see how well they met the first criterion. The results of these tests are somewhat

¹⁸(cont.) The justification for specifying this model was that when an F-test was run on the joint significance of $AF\$_{t-2}$ and $AF\$_{t-3}$ in model BA the results were inconclusive. When these two variables were dropped from the model, the third-order serial correlation parameter also became insignificant.

TABLE 9

PREDICTION RESULTS - MODEL BA(ii)Residuals and Standard Errors of Prediction¹

Sample Period	Number of Years Beyond End of Sample			
	1	2	3	4
1949-1975	-143,000 ** (57,000)	-262,000 ** (99,000)	-404,000 ** (162,000)	-392,000 ** (214,000)
1949-1974	42 (58,000)	-143,000 ** (82,000)	-262,000 ** (129,000)	-404,000 ** (194,000)
1949-1973	54,000 * (58,000)	44,000 (88,000)	-66,000 (123,000)	-163,000 * (178,000)
1949-1972	-22,000 (64,000)	30,000 (94,000)	5,100 (132,000)	-115,000 (179,000)
1949-1971	29,000 (72,000)	15,000 (124,000)	85,000 (180,000)	74,000 (255,000)
1949-1970	-43,000 (78,000)	-38,000 (132,000)	-81,000 (199,000)	-46,000 (264,000)
1949-1969	47,000 (196,000)	37,000 (187,000)	77,000 (295,000)	76,000 (415,000)

¹ Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

* Indicates that the residual is greater than 1.5 times the standard error of prediction.

** Indicates that the residual is greater than twice the standard error of prediction.

TABLE 10

PREDICTION RESULTS - MODEL BA(iii)Residuals and Standard Errors of Prediction¹

Number of Years Beyond End of Sample Sample Period	1	2	3	4
1949-1978	100,000 (89,000) **			
1949-1977	-2,600 (74,000)	97,000 (129,000)		
1949-1976	-41,000 (82,000)	-54,000 (118,000)	33,000 (166,000)	
1949-1975	-77,000 * (87,000)	-142,000 * (145,000)	-186,000 * (195,000)	-119,000 (225,000)
1949-1974	-12,000 (86,000)	-96,000 (162,000)	-169,000 (257,000)	-221,000 (333,000)
1949-1973	87,000 ** (72,000)	94,000 (140,000)	105,000 (262,000)	117,000 (421,000)
1949-1972	-7,500 (74,000)	76,000 (127,000)	74,000 (229,000)	72,000 (395,000)
1949-1971	118,000 ** (107,500)	210,000 * (261,000)	471,000 * (541,000)	767,000 (1,042,500)
1949-1970	-35,000 (68,000)	48,000 (151,000)	92,000 (290,000)	272,000 (514,000)
1949-1969	-43,000 (79,000)	-109,000 (151,000)	-67,000 (253,000)	62,000 (397,000)

¹ Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

* Indicates that the residual is greater than 1.5 times the standard error of prediction.

** Indicates that the residual is greater than twice the standard error of prediction.

deceiving and inconclusive. The second criterion was, once again, to compare the RMSE's of prediction for these models. Table 11 presents the results of this test.¹⁹

Using the RMSE's calculated from the differences between observed and predicted values of the dependent variables, it can be seen that overall BM is clearly the best model. The 'combination model' appears to do a better job of prediction, on average, than either BA or BA(iii). It also appears to do a slightly better job of predicting one or two years outside of the sample than model BM, but beyond that, the combination model is inferior to BM.²⁰

¹⁹The 'combination model' in Table 11 uses the original specification of model BA when the sample period is terminated after 1971, and the specification of BA(ii) when the sample is ended on or before 1971. There were two reasons for combining the model in this fashion. First, as already pointed out, it was not possible to find a solution using model BA when the sample terminated in 1970 and 1971 (when the sample terminated in 1969, the results were nonsense, since the calculated serial correlation parameters indicated an unstable system), so the results of BA(ii) were used in these instances. Second, the specification of model BA appeared to differ for different sample periods. If a researcher were constructing a model similar to BA and only had data up through 1970 or 1971, he would probably use a specification more like that of BA(ii) than BA.

²⁰It is of interest to note that model BA(iii), one of the simplified versions of BA, does not do as well at predicting as the more complex BA. Also, BA(ii) does a poor job of predicting when recent years are included in the sample. This indicates that if central market prices are going to be used as explanatory variables, then the fairly complex lag structure of model BA seems to be necessary for proper specification of the system.

Table 11 has another set of statistics which should be discussed and considered. The bracketed terms in the table are the root mean squared errors of prediction calculated with the 'residuals' defined as the difference between the 'expected' value and the observed value rather than as the difference between the predicted and observed values. To see the difference this makes, consider the following general model

$$(4) \quad y_t = \alpha_0 + \beta_1 X_t + \lambda_1 E(y_{t-1}) + u_t$$

$$\text{where } u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$$

$$E(u_t) = E(\varepsilon_t) = 0$$

$$\text{and } E(\varepsilon_t \varepsilon_s) = 0 \quad \forall t \neq s \quad t = 1, 2, \dots, T$$

Define the "expected value" of y_t as²¹

$$(5) \quad \begin{aligned} \bar{y}_t &= \hat{\alpha}_0 + \hat{\beta}_1 X_t + \hat{\lambda}_1 \bar{y}_{t-1} + \hat{\rho}_1 E(u_{t-1}) + \hat{\rho}_2 E(u_{t-2}) \\ &= \hat{\alpha}_0 + \hat{\beta}_1 X_t + \hat{\lambda}_1 \bar{y}_{t-1} \end{aligned}$$

Where $\hat{\alpha}_0$, $\hat{\beta}_1$, $\hat{\lambda}_1$, $\hat{\rho}_1$ and $\hat{\rho}_2$ are the least squares (or maximum likelihood) estimates of α_0 , β_1 , λ_1 , ρ_1 and ρ_2 respectively. $\bar{y}_{t-1}$ can be thought of as the empirical analog of $E(y_{t-1})$.

²¹In this context, 'expected value' refers to the best estimate of the systematic portion of the variation in the dependent variable. This differs from the earlier definition of the expected value in which, $E(y_t) = \alpha_0 + \beta_1 X_t + \lambda_1 E(y_{t-1})$.

TABLE 11

PREDICTION RESULTSRoot Mean Squared Errors*

	RMSE ₁	RMSE ₂	RMSE ₃	RMSE ₄	RMSE ₅	RMSE ₆	RMSE ₇
Model BM	64,102 [60,380]	97,387 [69,861]	125,139 [76,438]	130,174 [67,769]	140,056 [61,474]	144,947 [75,695]	152,328 [73,931]
Model BA	57,796 [48,563]	97,415 [56,210]	164,855 [67,651]	238,017 [83,799]	384,384 [141,794]	599,082 [202,590]	718,511 [229,444]
Model BA(iii)	65,146 [61,914]	112,977 [71,210]	198,625 [110,220]	327,019 [185,693]	538,636 [275,174]	899,935 [466,801]	1,434,776 [712,024]
Combination BA & BA(ii)	54,584 [46,761]	86,765 [48,391]	149,060 [66,391]	203,717 [68,195]	321,494 [124,642]	478,972 [168,082]	529,340 [176,790]

* RMSE_j is the root mean squared error for predictions 'j' years beyond the end of the sample.

Unbracketed numbers are RMSE's calculated using the differences between observed and predicted values of the dependent variable. Bracketed values are calculated using the differences between observed and expected values.

Define the predicted value of y_t as

$$(6) \quad \hat{y}_t = \hat{\alpha}_0 + \hat{\beta}_1 x_t + \hat{\lambda}_1 \bar{y}_{t-1} + \hat{\rho}_1 \hat{u}_{t-1} + \hat{\rho}_2 \hat{u}_{t-2}$$

$$\text{where} \quad \hat{u}_{t-1} = y_{t-1} - \bar{y}_{t-1}$$

If estimates of y_t outside the sample period are desired, they might be obtained in two different ways. First the "expected values" could be used, resulting in

$$\bar{y}_{t+1} = \hat{\alpha}_0 + \hat{\beta}_1 x_{t+1} + \hat{\lambda}_1 \bar{y}_t$$

$$\bar{y}_{t+2} = \hat{\alpha}_0 + \hat{\beta}_1 x_{t+2} + \hat{\lambda}_1 \bar{y}_{t+1}$$

•
•
•
•

$$\bar{y}_{t+j} = \hat{\alpha}_0 + \hat{\beta}_1 x_{t+j} + \hat{\lambda}_1 \bar{y}_{t+j-1}$$

From this, it can be seen that the value of $\bar{y}_{t+j}$ will be partially determined by all previous forecasts of $\bar{y}$.

The second method of obtaining estimates of y outside the sample would be to use the "predicted values," which results in

$$\begin{aligned} \hat{y}_{t+1} &= \hat{\alpha}_0 + \hat{\beta}_1 x_{t+1} + \hat{\lambda}_1 \bar{y}_t + \hat{\rho}_1 \hat{u}_t + \hat{\rho}_2 \hat{u}_{t-1} \\ &= \hat{\alpha}_0 + \hat{\beta}_1 x_{t+1} + \hat{\lambda}_1 \bar{y}_t + \hat{\rho}_1 (y_t - \bar{y}_t) + \hat{\rho}_2 (y_{t-1} - \bar{y}_{t-1}) \end{aligned}$$

$$\begin{aligned}
\hat{y}_{t+2} &= \hat{\alpha}_0 + \hat{\beta}_1 X_{t+2} + \hat{\lambda}_1 \bar{y}_{t+1} + \hat{\rho}_1 \hat{u}_{t+1} + \hat{\rho}_2 \hat{u}_t \\
&= \hat{\alpha}_0 + \hat{\beta}_1 X_{t+2} + \hat{\lambda}_1 \bar{y}_{t+1} + \hat{\rho}_1 (y_{t+1} - \bar{y}_{t+1}) + \hat{\rho}_2 (y_t - \bar{y}_t) \\
&\vdots \\
\hat{y}_{t+j} &= \hat{\alpha}_0 + \hat{\beta}_1 X_{t+j} + \hat{\lambda}_1 \bar{y}_{t+j-1} + \hat{\rho}_1 \hat{u}_{t+j-1} + \hat{\rho}_2 \hat{u}_{t+j-2} \\
&= \hat{\alpha}_0 + \hat{\beta}_1 X_{t+j} + \hat{\lambda}_1 \bar{y}_{t+j-1} + \hat{\rho}_1 (y_{t+j} - \bar{y}_{t+j-1}) + \hat{\rho}_2 \\
&\quad (y_{t+j-2} - \bar{y}_{t+j-2})
\end{aligned}$$

From the preceding, it can be seen that the only difference between 'expected' and 'predicted' values is that the latter uses the estimated serial correlation structure of the error term to obtain the forecasted y values. If the residuals from these two methods differ, it must therefore be due to the inclusion of this serial correlation structure. Table II indicates that the RMSE's of the predicted values are considerably larger than the RMSE's of the expected values. This is probably an indication that the estimated structure of the error term is not appropriate outside the sample period. In other words, the serial correlation structure seems to change as the sample period changes, implying that attempts to use this structure to help forecast will probably make the forecasts worse rather than better. The reason for these problems

with estimation of the structure of the error term might be either that the structure is non-stationary (in which case there isn't much hope of correcting the problem) or that the sample size simply is not large enough to accurately estimate the structure.

Using the difference between the observed and 'expected' values to calculate the RMSE's, the 'combination' model probably does the best job for predictions up to three or four years outside of the sample (although if a formal test were run, it is doubtful that this model would do significantly better than model BA). For predictions beyond four years outside of the sample, model BM is definitely superior to any of the others.

Specification Tests

Informal checks were run to see whether the basic models (BA and BM) were sensitive to the specification of the sample period. If the parameter values jumped around and the general structure of the model seemed to change considerably when the end points of the sample period were changed, this would indicate specification error in the models. Tables 12-15 show the results of these runs. Table 12 shows the effects on model BM when years at the beginning of the sample period are cut off. As can be seen, the fit of the models, the parameter values, the standard errors of the estimate, etc., all stay relatively constant when the sample period is changed in this way. The largest

TABLE 12

MODEL BM

EFFECTS OF CUTTING INITIAL YEARS FROM SAMPLE PERIOD

	Sample Period				
	1947- 1979	1948- 1979	1949- 1979	1950- 1979	1951- 1979
R^2	.9976	.9974	.9972	.9969	.9967
$\bar{R}^2$	.9964	.9960	.9955	.9950	.9946
S.S.E.	111935x10 ⁵	111338x10 ⁵	110707x10 ⁵	110141x10 ⁵	101312x10 ⁵
S.E.E.	23,100	23,600	24,100	24,700	24,400
S.E.P.	30,700	31,400	32,200	33,100	33,000
Parameters					
Constant	-125,000	-121,000	-118,000	-117,000	-119,000
HP _{t-1}	.169 (11.5)	.169 (11.2)	.169 (10.9)	.167 (9.98)	.160 (9.07)
HP _{t-1}	-.104 (5.09)	-.107 (4.78)	-.109 (4.56)	-.109 (4.40)	-.101 (3.97)
(B/C) _{t-1}	3430 (11.5)	3500 (9.52)	3580 (7.84)	3690 (6.35)	4200 (5.74)
MCS _{t-1}	3350 (9.96)	3290 (8.35)	3210 (7.00)	3130 (5.83)	2840 (4.72)
MCS _{t-2}	-2130 (8.96)	-2130 (8.77)	-2150 (8.45)	-2160 (8.15)	-2270 (8.04)
E(y _{t-1})	1.70 (64.5)	1.71 (55.4)	1.71 (50.7)	1.71 (48.0)	1.72 (48.1)
E(y _{t-2})	-.832 (45.7)	-.832 (44.7)	-.832 (43.5)	-.833 (41.5)	-.837 (41.3)
ρ_1	-.674 (4.17)	-.691 (4.24)	-.696 (4.22)	-.697 (4.15)	-.674 (3.90)
ρ_2	-.369 (2.28)	-.390 (2.39)	-.394 (2.39)	-.391 (2.32)	-.362 (2.09)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

variation in estimated parameter values appears to occur when the sample is started in 1951 rather than in 1950. A formal statistical test developed by Gregory C. Chow (1960) for determining if the specification of a model is the same for different sample sets was used to test for stability of the model.²² This test indicated that there was no structural shift between 1950 and 1951. Further tests using this Chow test also indicated that there were no structural shifts between any of the other years tested.

Table 13 shows the effects on model BM of cutting recent years from the sample period. The first thing one notices when looking at this table is that the addition of 1979 data seems to have a very noticeable effect on the model. The adjusted multiple correlation coefficient, the standard errors of the estimate, the standard error of prediction, and several of the parameter estimates all change considerably. The Chow test, when used to test for the stability of the model²³ between sample periods 1949-1978 and 1949-1979, indicates that the 1979 observation does not come from the same population as earlier observations, i.e. the proper specification of the model is different for these two sample periods. The results were the same when the model with a 1949-1979 sample period was compared with the 1949-1977

²² Actually, the Chow test is only approximately valid in this context since it was designed for linear regressions, while the model in this paper is non-linear in the parameters.

²³ Unless otherwise stated, all tests were run at an α level of 5%.

TABLE 13

MODEL BM

EFFECTS OF CUTTING RECENT YEARS FROM SAMPLE PERIOD

	Sample Period									
	1949- 1979	1949- 1978	1949- 1977	1949- 1976	1949- 1975	1949- 1974	1949- 1973	1949- 1972	1949- 1971	1949- 1970
R^2	.9972	.9982	.9983	.9982	.9986	.9986	.9986	.9984	.9984	.9982
$\overline{R}^2$	.9955	.9971	.9971	.9970	.9977	.9975	.9973	.9970	.9968	.9962
S.S.E.	110707x10 ⁵	69360x10 ⁵	66500x10 ⁵	66246x10 ⁵	47721x10 ⁵	44764x10 ⁵	38222x10 ⁵	35932x10 ⁵	31720x10 ⁵	30160x10 ⁵
S.E.E.	24,100	19,600	19,800	20,300	17,800	17,900	17,100	17,300	17,000	17,400
S.E.P.	32,200	26,600	27,100	27,800	23,600	23,000	22,500	22,600	22,400	23,300
Parameters										
Constant	-118,000	-89,500	-84,900	-87,000	-112,000	-123,000	-110,000	-119,000	-110,000	-112,000
HP _{t-1}	.169 (10.9)	.133 (7.79)	.129 (6.93)	.131 (7.01)	.131 (8.69)	.128 (8.47)	.122 (7.96)	.124 (7.49)	.117 (6.49)	.121 (6.60)
HP _{t-2}	-.109 (4.56)	-.106 (5.45)	-.107 (5.33)	-.107 (5.29)	-.089 (4.98)	-.079 (3.94)	-.081 (3.96)	-.079 (3.76)	-.075 (3.37)	-.081 (3.50)
B/C _{t-1}	3580 (7.84)	4300 (10.2)	4350 (9.76)	4330 (9.80)	4110 (11.4)	3960 (9.89)	4360 (9.08)	4110 (6.99)	4520 (6.34)	4250 (5.39)
MC\$ _{t-1}	3210 (7.00)	3050 (8.25)	3210 (7.71)	3090 (5.67)	2140 (3.67)	1920 (3.09)	1790 (2.93)	1730 (2.72)	1800 (2.65)	1790 (2.63)
MC\$ _{t-2}	-2150 (8.45)	-2250 (10.7)	-2400 (8.67)	-2290 (5.06)	-1070 (1.83)	-641 (.878)	-862 (1.15)	-572 (.682)	-978 (1.01)	-751 (.732)
E(y _{t-1})	1.71 (50.7)	1.82 (43.1)	1.84 (36.4)	1.83 (32.7)	1.75 (33.7)	1.72 (29.0)	1.76 (27.3)	1.73 (23.3)	1.77 (20.8)	1.76 (20.2)
E(y _{t-2})	-.832 (43.5)	-.893 (36.8)	-.906 (30.1)	-.898 (25.6)	-.843 (24.9)	-.827 (21.2)	-.852 (20.3)	-.830 (16.1)	-.867 (13.9)	-.845 (12.3)
ρ_1	-.696 (4.22)	-.818 (6.00)	-.834 (5.72)	-.873 (6.10)	-.938 (6.10)	-.896 (6.01)	-.823 (6.10)	-.758 (5.49)	-.672 (4.43)	-.714 (4.62)
ρ_2	-.394 (2.39)	-.664 (4.87)	-.619 (4.25)	-.652 (4.55)	-.601 (3.91)	-.650 (4.37)	-.738 (5.47)	-.737 (5.34)	-.686 (4.52)	-.690 (4.47)
E(y ₁₉₄₈)	925,000 (86.4)	935,000 (131)	938,000 (120)	937,000 (115)	932,000 (127)	932,000 (128)	933,000 (138)	932,000 (129)	934,000 (119)	933,000 (118)
E(y ₁₉₄₇)	910,000 (44.6)	919,000 (69.8)	922,000 (64.9)	920,000 (61.5)	913,000 (65.3)	915,000 (65.5)	912,000 (69.7)	911,000 (66.0)	915,000 (63.1)	911,000 (58.0)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

and 1949-1976 model.²⁴

Another point at which the model seems to change significantly is when 1976 data is added to the sample period. The Chow test indicates that the model with sample period 1949-1975 is significantly different from the model with sample period 1949-1976. However, in this case, if more data than 1976 alone are added to the sample, i.e. if 1975 and 1976 or 1976 and 1977 data are added simultaneously, then the structure of the system does not change significantly (according to the Chow test). Table 13 also indicates that the addition of 1970 and 1971 to the data sample may result in significant changes in the specification of the model. In this instance, however, the results of the

²⁴The results of estimating this model with the addition of data on 1980 herd sizes were

$$\begin{aligned}
 y_t = & -116,000 + .167HP_{t-1} - .111HP_{t-2} + 3640B/C_{t-1} + 3170MC\$_{t-1} \\
 & \quad (12.5) \quad (4.84) \quad (10.1) \quad (7.9) \\
 & -2140MC\$_{t-2} + 1.72E(y_{t-1}) - .834E(y_{t-2}) - .712u_{t-1} \\
 & \quad (8.7) \quad (92.6) \quad (52.2) \quad (4.41) \\
 & -.406u_{t-2} \\
 & \quad (2.51)
 \end{aligned}$$

$$\bar{R}^2 = .9957 \quad \text{S.S.E.} = 110991 \times 10^5$$

$$\text{S.E.E.} = 23,600 \quad \text{S.E.P.} = 32,100$$

These results indicate that the 1980 observation probably came from the same population as the 1979 observation.

Chow test show that the structure did not change significantly during this period.

These results indicate that model BM definitely had problems when 1979 data was added to the sample period. Other than this, the structure of the system appeared to be stable and model BM appears to do well at explaining the variation in the dependent variable.²⁵

Table 13 indicates one other result worth noting. The t-values on the estimated parameter values indicate that, with one exception, the precision on the parameter estimates yielded by model BM was quite high. This exception is that the second-order lag on Montana calf prices loses its significance when the data sample is terminated prior to 1976. One possible explanation for this can be obtained from looking at the graph of real Montana calf prices over time. This graph shows that, starting in 1972 or 1973, the variation in this variable

²⁵ Tables 14 and 15 show the effect on model BA of changing the sample period. Tests on the stability of this model when years were cut off the early part of the model indicate that the structure of the system is quite stable (None of the F-statistics calculated using the Chow test were even close to 1.0). Table 15 indicates that this model also had problems when 1979 data were added to the sample. Indeed, when the Chow test was run to compare the models with samples 1949-1979 and 1949-1978, the results indicated that there was a structural shift in the system. However, unlike model BM, if 1978 and 1979, or 1977-1979 are added simultaneously, the Chow test indicates that the structure of the system has not changed. A survey of the parameter values in Table 15 indicates that they do quite a bit of jumping around as the sample period is changed. Chow tests indicate, however, that all of the other data sets belong to the same population, i.e., the structure of the system is stable.

TABLE 14

MODEL BA

EFFECTS OF CUTTING INITIAL YEARS FROM SAMPLE PERIOD

	Sample Period				
	1947- 1979	1948- 1979	1949- 1979	1950- 1979	1951- 1979
R^2	.9988	.9987	.9986	.9984	.9983
$\overline{R}^2$	.9978	.9976	.9973	.9969	.9965
S.S.E.	51729x10 ⁵	49871x10 ⁵	49671x10 ⁵	48331x10 ⁵	47189x10 ⁵
S.E.E.	17,400	17,700	18,200	18,600	19,100
S.E.P.	24,000	24,200	25,000	25,800	26,600
Parameters					
Constant	-183,000	-178,000	-178,000	-177,000	-178,000
HP	.158	.156	.156	.154	.150
t_{-1}	(15.5)	(15.4)	(14.7)	(13.4)	(10.5)
HP	-.076	-.079	-.079	-.078	-.075
t_{-2}	(4.93)	(5.13)	(5.00)	(4.80)	(4.17)
(B/C)	8960	8960	8950	9030	9110
t_{-1}	(10.3)	(10.6)	(10.1)	(9.86)	(9.70)
(B/C)	-3880	-3810	-3810	-3820	-3740
t_{-2}	(4.81)	(4.84)	(4.68)	(4.57)	(4.30)
AF\$	2380	2450	2460	2430	2560
t_{-1}	(2.23)	(2.36)	(2.27)	(2.16)	(2.24)
AF\$	-3510	-3760	-3760	-3900	-4390
t_{-2}	(2.18)	(2.37)	(2.30)	(2.25)	(2.30)
AF\$	2490	2620	2620	2730	3020
t_{-3}	(2.99)	(3.17)	(3.08)	(2.99)	(2.98)
E(y_{t-1})	1.72	1.73	1.73	1.73	1.73
	(99.8)	(90.3)	(83.3)	(79.2)	(76.8)
E(y_{t-2})	-.870	-.871	-.871	-.871	-.871
	(66.4)	(68.1)	(66.0)	(64.2)	(62.5)
ρ_1	-1.23	-1.32	-1.33	-1.30	-1.30
	(7.65)	(8.60)	(8.54)	(8.01)	(7.86)
ρ_2	-1.05	-1.17	-1.17	-1.13	-1.12
	(5.31)	(6.16)	(6.10)	(5.64)	(5.44)
ρ_3	-.409	-.519	-.524	-.490	-.483
	(2.54)	(3.38)	(3.37)	(3.03)	(2.92)

All abbreviations are as defined in previous tables or in the body of the paper.

TABLE 15

MODEL BA

EFFECTS OF CUTTING RECENT YEARS FROM SAMPLE PERIOD

	Sample Period									
	1949- 1979	1949- 1978	1949- 1977	1949- 1976	1949- 1975	1949- 1974	1949- 1973	1949- 1972	1949- 1971	1949- 1970
R^2	.9986	.9991	.9991	.9991	.9991	.9990	.9991	.9990		.9995
$\bar{R}^2$	.9973	.9982	.9981	.9980	.9979	.9977	.9978	.9972		.9983
S.S.E.	49671x10 ⁵	31840x10 ⁵	31336x10 ⁵	31605x10 ⁵	29090x10 ⁵	27270x10 ⁵	20798x10 ⁵	20749x10 ⁵		5773x10 ⁵
S.E.E.	18,200	15,100	15,600	16,200	16,300	16,500	15,200	16,100		10,700
S.E.P.	25,000	20,500	21,700	22,900	22,900	22,800	21,300	23,000		15,100
Parameters										
Constant	-178,000	-156,000	-155,000	-157,000	-149,000	-143,000	-115,000	-107,000		-111,000
HP _{t-1}	.156 (14.7)	.141 (13.3)	.141 (12.2)	.140 (11.5)	.134 (10.2)	.144 (8.97)	.150 (9.93)	.151 (9.50)		.164 (18.0)
HP _{t-2}	-.079 (5.00)	-.086 (6.36)	-.086 (6.08)	-.085 (5.72)	-.081 (5.26)	-.091 (5.02)	-.109 (5.50)	-.113 (4.14)		-.129 (2.46)
(B/C) _{t-1}	8950 (10.1)	8380 (10.8)	8370 (10.3)	8550 (8.80)	6860 (3.77)	9150 (3.01)	11,000 (3.80)	11,500 (3.15)		13,200 (2.01)
(B/C) _{t-2}	-3810 (4.68)	-3040 (4.10)	-3040 (3.90)	-3170 (3.50)	-1630 (.977)	-3810 (1.34)	-5700 (2.07)	-6180 (1.76)		-7930 (1.25)
AFS _{t-1}	2460 (2.27)	3420 (3.51)	3450 (3.09)	3610 (2.86)	3310 (2.50)	2890 (2.21)	1300 (.839)	968 (.428)		487 (.102)
AFS _{t-2}	-3760 (2.30)	-4960 (3.43)	-5000 (3.09)	-5310 (2.92)	-4270 (1.98)	-4430 (2.15)	-2600 (1.17)	-2220 (.771)		-1990 (.379)
AFS _{t-3}	2620 (3.08)	2810 (3.88)	2820 (3.63)	3010 (3.20)	2140 (1.70)	2630 (1.98)	1700 (1.32)	1550 (.937)		1910 (.589)
E(y _{t-1})	1.73 (83.3)	1.78 (67.5)	1.78 (56.3)	1.78 (49.3)	1.76 (38.7)	1.80 (28.2)	1.86 (27.2)	1.88 (16.4)		1.90 (6.22)
E(y _{t-2})	-.871 (66.0)	-.893 (63.4)	-.894 (50.8)	-.899 (38.5)	-.866 (22.0)	-.910 (14.8)	-.964 (15.4)	-.980 (9.90)		-.991 (3.76)
ρ_1	-1.33 (8.54)	-1.34 (8.16)	-1.34 (7.97)	-1.32 (7.74)	-1.33 (8.22)	-1.47 (9.77)	-1.45 (10.1)	-1.49 (10.6)		-2.02 (4.67)
ρ_2	-1.17 (6.10)	-1.23 (6.43)	-1.22 (6.22)	-1.21 (6.14)	-1.19 (6.01)	-1.30 (6.41)	-1.46 (10.4)	-1.50 (10.9)		-2.09 (3.72)
ρ_3	-.524 (3.37)	-.463 (2.81)	-.457 (2.72)	-.466 (2.73)	-.565 (5.49)	-.661 (4.40)	-.710 (4.94)	-.737 (5.23)		-1.21 (2.28)
E(y ₁₉₄₈)	939,000 (128)	944,000 (150)	944,000 (124)	946,000 (100)	939,000 (72.8)	944,000 (68.0)	945,000 (82.6)	946,000 (79.9)		948,000 (82.0)
E(y ₁₉₄₇)	920,000 (67.2)	928,000 (80.2)	928,000 (67.8)	930,000 (57.1)	922,000 (44.8)	925,000 (49.4)	920,000 (58.9)	920,000 (57.7)		917,000 (117)

S.S.E. is the sum of squared errors of the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

is greater than at any other time in the sample period. This implies that the reason the second order lag on calf prices doesn't enter the model with a significant coefficient prior to 1975 (note that for 1975 data on breeding herds, $MC\$_{t-2}$ is calf prices) is that the variation in this variable was not large enough for the model to be able to distinguish between the effects of first and second order lagged calf prices. Another possible explanation for this reduced significance on $MC\$_{t-2}$ is that cattle producers became more sophisticated in formulating their expectations about the future price and profitability of their industry during this time. This explanation is somewhat trite in the opinion of this writer and seems to give a disproportionately small amount of credit to the intelligence and sophistication of past producers relative to present producers.²⁶

To summarize the breeding herd models, both BA and BM appear to be solid models. If anything, BA has a slight edge, judging by the usual summary statistics and the stability of the model as measured by the

²⁶ A look at the precision on the parameter estimates of model BA, as shown in Table 15, indicates that the estimates of all the second or higher order lags on the price variables [$(B/C)_{t-2}$, $AF\$_{t-2}$, $AF\$_{t-3}$] lose their significance around 1974 or 1975. An explanation similar to the explanation given as to why $MC\$_{t-2}$ loses its significance seems to be appropriate. The graphs of annual feeder prices and the beef to corn price ratio (especially the graph of the latter variable) indicate that prior to 1973 there simply may not have been enough variation in these variables to isolate the effects of the individual lags on the variables.

Chow test. However, BM has a distinct advantage in its long range prediction abilities. From a mechanical standpoint, BM (unlike BA) does not run into convergence problems when recent years are removed from the sample. Also, the coefficients on the $AF\$_{t-j}$ variables in the BA model show a strong tendency to shrink to zero and lose significance as recent years are cut from the sample. Primarily because of these last three factors (and secondarily because of the greater precision on the price variable in BM), it is felt that overall, BM is a superior model.

Total Cattle Numbers Models

The sample period used for the primary total cattle numbers models was 1950-1979. The reason for starting this sample period around 1950 was the same as for the breeding herd models. As in the case of the breeding herd models, there was no particular reason for starting with 1950 rather than with 1949 or 1951.

Table 16 presents the two total cattle numbers models. The only difference between these models is that TA uses annual average Kansas City feeder steer prices and TM uses Montana calf prices as an explanatory variable. Each model has a first order lag on the respective price variable, a second order lag on hay production, a first order lag on beef to corn price ratio, and second order serial correlation.

As with the breeding herd models, both of these models can be viewed as second order nonhomogeneous difference equations. Both of

TABLE 16

SUMMARY OF TOTAL CATTLE NUMBERS MODELSMODEL

TA $w_t = -111,530 + .25472HP_{t-1} - .23661HP_{t-2} + 8185.0B/C_{t-1} + 1134.8AF\$_{t-1}$
 (8.20) (10.50) (7.273) (22.59) (2.174)

$+ 1.8139E(w_{t-1}) - .87723E(w_{t-2}) - .49147u_{t-1} - .65668u_{t-2}$
 (119.9) (53.89) (3.569) (4.769)

$R^2 = .9970$ $\bar{R}^2 = .9955$ S.E.E. = 31,193 S.E.P. = 41,893

 d.f. = 19 Sample Period: 1950 - 1979 S/M = .01197

TM $w_t = -97,864 + .26759HP_{t-1} - .24217HP_{t-2} + 6043.2B/C_{t-1} + 2157.1MC\$_{t-1}$
 (16.72) (12.94) (8.877) (6.964) (3.148)

$+ 1.7912E(w_{t-1}) - .86409E(w_{t-2}) - .56587u_{t-1} - .68649u_{t-2}$
 (111.1) (64.48) (4.263) (5.171)

$R^2 = .9974$ $\bar{R}^2 = .9961$ S.E.E. = 29,076 S.E.P. = 39,067

 d.f. = 19 Sample Period: 1950 - 1979 S/M = .01116

Numbers in parentheses are estimated t-values. S.E.E. is the standard error of the estimate; S.E.P. is the approximate standard error of prediction; d.f. is the number of degrees of freedom; and S/M is the ratio of the standard error of the estimate to the mean of the dependent variable.

these models will be analyzed and discussed in detail, since neither of them appeared to be superior.

Model TA can be reduced to the following second order difference equation,

$$(7) \quad w(t) = 1.8139 w(t-1) - .87723 w(t-2) + K$$

where

$$\begin{aligned} K &= -111,530 + .25472 \overline{HP}_{t-1} + \dots + 11348 \overline{AF\$}_{t-1} \\ &= 148,916.7 \end{aligned}$$

The general solution to this second order difference equation is

$$w(t) = (.93665)^t (e \cos Qt + f \sin Qt) + 2,351,440$$

Thus, the model implies that the system is cyclical in nature and that, if the system is disturbed from an equilibrium position by an exogenous shock, the system will eventually return to a stable position. The time path of this adjustment process will be one of dampened oscillations around the stationary position, 2,351,440. The length of the cycle implied by the model is about 25 years. Again, keep in mind that the cycle is longer than the cycle observed in the real world, since the model is analyzed under the restrictive assumption that prices (and other explanatory variables) are constant.

From Table 16, it can be seen that the short run effect of a \$1 change in average annual feeder prices in year $t-1$ will be to increase total cattle numbers by about 1135 head on January 1 of year t . The short run price elasticity is therefore .0114. Note that this price

elasticity is much smaller than the short run elasticity of breeding herds with respect to feeder steer prices. Comparing Tables 6 and 17, it can be seen that the short run effect of a change in feeder prices on breeding herds is larger than the effect on total cattle numbers (2456 vs. 1135). This at first seems inconsistent with some of the earlier analyses on the breeding herd. Recall that an increase in feeder prices in year t leads ranchers to increase their expectations about future prices. Because of this, they will want to increase their herd sizes. They do this by culling fewer of their older cows and by keeping more of their heifer calves. The increase in cull cows will result in an increase in both breeding herds and total cattle numbers. The increase in heifer calves, however, will only increase total cattle numbers since the breeding herd classification does not include heifer calves less than one year old. According to this, the effect of a change in feeder prices on total cattle numbers should be greater than the effect on breeding herds. But this is the opposite of the results given by the models. A possible explanation of this might be the following: the total cattle numbers category includes the breeding herd, as well as calves (less than one year old), heifers kept for purposes other than breeding, and steers and bulls more than a year old. When feeder prices increase, there are two opposing forces acting on cattle producers. First, future prices will be expected to increase putting pressure on ranchers to increase their herd sizes, and

TABLE 17
PARTIAL DERIVATIVES

MODEL TA

Time for Response*

	1 Year	2 Years	3 Years	4 Years	5 Years	6 Years	7 Years	8 Years
Hay								
Production	.2547	.2254	.1855	.1386	.0888	.0395	-.0063	-.0461
Beef Price								
Corn Price	8185	14,850	19,750	22,800	24,030	23,590	21,710	18,690
Annual								
Feeder Prices	1135	2058	2738	3161	3332	3271	3010	2591

MODEL TM

Time for Response

	1 Year	2 Years	3 Years	4 Years	5 Years	6 Years	7 Years	8 Years
Hay								
Production	.2676	.2371	.1935	.1418	.0867	.0328	-.0162	-.0573
Beef Price								
Corn Price	6043	10,830	14,170	16,020	16,460	15,640	13,780	11,180
Montana								
Calf Prices	2157	3864	5057	5719	5875	5581	4920	3991

*The values in this table indicate the effects of a change in the value of an independent variable in year t on total cattle numbers i years in the future (i = 1,2,...,8). For example, a \$1.00 change in feeder prices in year t will result in a change in breeding herd sizes of 2738 in year t+3. The cumulative effect of a change in an independent variable can be calculated by summing the individual entries.

second, as prices rise there is an incentive to sell more cattle immediately. Thus, when prices rise, total cattle numbers increase since more heifer calves and cull cows are kept in response to increased expectations, but more steer calves and steers older than one year are sold to feedlots (rather than being fed through the winter by the rancher) in direct response to the increase in feeder prices. If the decrease in cattle numbers due to selling more steers is greater than the increase due to keeping more heifer calves, then one would get the result that breeding herds increase by more than total cattle numbers in response to an increase in feeder prices. Skipping ahead slightly, a similar story can be told to explain why the short run response to a change in Montana calf prices is greater for breeding herds than for total cattle numbers.

The long run effect of a \$1.00 increase in annual feeder prices implied by this model is an increase in total cattle numbers of 17,919 (or $\frac{1134.8}{1-1.8139 + .87723}$). The long-run price elasticity would thus be .1800.

Table 5 presents the above elasticities as well as the elasticities for the other independent variables for several different lengths of time. Notice that, as with the breeding herd models, for all the explanatory variables in both models, the ten-year elasticities are larger than the long-run elasticities. Again, this is probably due to

the cyclical nature of the system and the resulting overadjustment by cattle producers. Once again, the interpretation of the partial derivatives and elasticities with respect to hay production are straightforward, but care should be taken when interpreting these for the beef to corn price ratio.

Model TM (see Table 16) can be reduced to the following second order difference equation,

$$(8) \quad w(t) = 1.7912 w(t-1) - .86409w(t-2) + K$$

where

$$K = -97,864 + .26759 \overline{HP}_{t-1} + \dots + 2157.1 MC\$_{t-1} = 174,154.7$$

The general solution to (8) is

$$w(t) = (.9296)^t (e \cos Qt + f \sin Qt) + 2,389,281$$

This model, like all of the previous models, implies a system with a stable, cyclical adjustment process. The length of the cycle implied by this model is slightly over twenty three years. The stationary position of the system is 2,389,281 head of cattle. Tables 5 and 17 present the elasticities and the responses in the dependent variable in year $t + j$ to a change in the independent variables in year t .

To this point, the two total cattle numbers models seem quite comparable. The summary statistics of Table 16 indicate that model TM has a higher R^2 and $\bar{R}^2$ and lower standard error of the estimate and standard error of prediction. The difference, however, is very small.

Model Predictions

To compare these models further, they were tested to see how well they would predict outside the sample period. Here, as with breeding herds, two criteria were used to measure how well the models did in these tests. The first was to compare the residuals with the standard errors of prediction (multiplied by 2). The results of this test are shown in Tables 18 and 19. Both models do quite well except when the sample is terminated in 1971 or 1972. Neither model is able to predict 1973 cattle numbers, either one or two years outside of the sample.²⁷

Model TA does a poor job predicting four years outside of the sample when the sample stops in both 1971 and 1972, while model TM does a poor job of predicting one, two, three and four years ahead when the sample period is 1950-1972. On the basis of this first criterion, neither model seemed superior.

The second method of testing the models for their prediction capabilities was to calculate and compare their root mean squared errors (RMSE's). Table 20 shows the results of these calculations. A comparison of the unbracketed terms in this table indicates that Model TM has

²⁷ A possible explanation of why the models had trouble predicting around 1973 is that corn prices, as well as feeder and fat cattle prices, started changing dramatically about this time. It may be that the model simply could not handle these fluctuations for the first year or two.

TABLE 18

PREDICTION RESULTS - MODEL TA

RESIDUALS AND STANDARD ERRORS OF PREDICTION¹

Number of Years Beyond End of Sample Sample Period	1	2	3	4
1950-1978	17,000 (142,000)			
1950-1977	-77,000 (133,000)	-96,000 (237,000)		
1950-1976	6,000 (132,000)	-68,000 (231,000)	-81,000 (399,000)	
1950-1975	-31,000 (118,000)	-31,000 (188,000)	-125,000 (306,000)	-170,000 (493,000)
1950-1974	-3,000 (121,000)	-35,000 (171,000)	-35,000 (261,000)	-132,000 (394,000)
1950-1973	70,000 (136,000)	73,000 (212,000)	75,000 (332,000)	130,000 (507,000)
1950-1972	-122,000 (100,000)**	-86,000 (136,000)	-117,000 (172,000)	-188,000 (210,000)*
1950-1971	-36,000 (105,000)	-157,000 (139,000)**	-135,000 (182,000)	-176,000 (223,000)*
1950-1970	2,700 (128,000)	-33,000 (173,000)	-153,000 (232,000)	-129,000 (288,000)
1950-1969	-46,000 (167,000)	-68,000 (390,000)	-132,000 (526,000)	-286,000 (661,000)

¹Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

** Indicates that the residual is greater than twice the standard error of prediction.

* indicates that the residual is greater than 1.5 times the standard error of prediction.

TABLE 19

PREDICTION RESULTS - MODEL TM

RESIDUALS AND STANDARD ERRORS OF PREDICTION

Number of Years Beyond End of Sample Sample Period	1	2	3	4
1950-1978	10,500 (130,000)			
1950-1977	-42,000 (125,000)	-51,000 (224,000)		
1950-1976	29,000 (109,000)	-8,000 (190,000)	11,000 (342,000)	
1950-1975	-14,000 (99,000)	-17,000 (138,000)	-25,000 (226,000)	-16,000 (388,000)
1950-1974	-4,600 (108,000)	-17,00 (133,000)	13,000 (174,000)	-30,000 (258,000)
1950-1973	29,000 (121,000)	22,000 (163,000)	14,000 (211,000)	55,000 (263,000)
1950-1972	-119,000 ** (95,000)	-106,000 * (132,000)	-127,000 * (167,000)	-172,000 * (191,000)
1950-1971	-21,000 (103,000)	-138,000 * (139,000)	-132,000 (186,000)	-159,000 (232,000)
1950-1970	13,000 (127,000)	-500 (177,000)	-109,000 (240,000)	-93,000 (299,000)
1950-1969	-30,000 (167,000)	-29,000 (284,000)	-67,000 (512,000)	-199,000 (561,000)

¹ Numbers in parentheses are two times the estimated standard errors of prediction. Unenclosed numbers are the residuals.

**Indicates that the residual is greater than twice the standard error of prediction.

*Indicates that the residual is greater than 1.5 times the standard error of prediction.

TABLE 20

PREDICTION RESULTSRoot Mean Squared Errors*

	RMSE ₁	RMSE ₂	RMSE ₃	RMSE ₄	RMSE ₅	RMSE ₆	RMSE ₇
Model TA	55,181 [57,504]	81,051 [70,785]	112,711 [69,945]	180,526 [122,483]	213,776 [124,630]	292,636 [158,417]	427,200 [253,208]
Model TM	44,096 [50,996]	62,160 [57,624]	79,853 [51,112]	123,859 [87,096]	146,611 [78,386]	189,761 [89,673]	251,356 [142,611]

* RMSE_j is the root mean squared error for predictions 'j' years beyond the end of the sample.

Unbracketed numbers are RMSE's calculated using the differences between observed and predicted values of the dependent variable. Bracketed values are calculated using the differences between observed and expected values.

a fairly distinct advantage, especially for predictions beyond two years outside of the sample. As was discussed in the analysis of the breeding herd models, when serial correlation is present, better estimates of values of the dependent variable outside of the sample may be obtained by using 'expected values' rather than 'predicted values' of the dependent variable.²⁸ The RMSE's using this definition of the residual (observed value minus expected value) are the bracketed figures in Table 20. A comparison of these values shows that, first, the bracketed RMSE's are smaller than the unbracketed RMSE's for both models in all cases except for predictions one year outside of the model, and second, that model TM again yields distinctly smaller RMSE's.

²⁸ Here again, the 'expected value' of the dependent variable is defined as,

$$w_t = \hat{\alpha}_0 + \hat{\beta}_1 HP_{t-1} + \hat{\beta}_2 HP_{t-2} + \hat{\beta}_3 (B/C)_{t-1} + \hat{\beta}_4 MCS_{t-1} \\ + \hat{\lambda}_1 w_{t-1} + \hat{\lambda}_2 w_{t-2}$$

while the 'predicted value' is defined as,

$$w_t = \hat{\alpha}_0 + \hat{\beta}_1 HP_{t-1} + \hat{\beta}_2 HP_{t-2} + \hat{\beta}_3 (B/C)_{t-1} + \hat{\beta}_4 MCS_{t-1} \\ + \hat{\lambda}_1^w w_{t-1} + \hat{\lambda}_2^w w_{t-2} + \hat{\rho}_1 \hat{u}_{t-1} + \hat{\rho}_2 \hat{u}_{t-2}$$

Specification Tests

The stability of these models was checked by estimating the same specification of the models for different sample periods. As with the breeding herd models, the Chow test was used as a formal test of the stability of the model. Tables 21 - 24 show the estimated models for the different sample periods tested. Notice that the testing of the effects of cutting initial years from the sample period were more extensive for these two models than for the breeding herd models. Initially, the same tests were run for both groups of models, but the stability of the total cattle numbers models appeared to be more questionable, so further runs were made.

Table 21 shows the effects on model TA of changing the year in which the sample period is started. The parameter estimates seem to do quite a bit of jumping around as the initial year of the sample is changed, but the Chow test indicates that the only instance in which the structure of the system appears to change significantly is when the sample period is changed from 1949-1979 to 1950-1979. There is no other combination of sample periods for which this test indicates that the change in the structure is even close to significant.²⁹ These

²⁹ Even for the sample period 1949-1979 and 1950-1979, the change was just barely significant at an α level of 5%. The calculated F-statistic was 4.39 and the critical F-value was 4.38.

TABLE 21
MODEL TA
EFFECTS OF CUTTING INITIAL YEARS FROM SAMPLE PERIOD

	Sample Period									
	1947- 1979	1948- 1979	1949- 1979	1950- 1979	1951- 1979	1952- 1979	1953- 1979	1954- 1979	1955- 1979	1956- 1979
R^2	.9969	.9968	.9966	.9970	.9966	.9964	.9964	.9963	.9962	.9966
$\bar{R}^2$	.9955	.9953	.9949	.9955	.9948	.9943	.9941	.9938	.9935	.9940
S.S.E.	24029x10 ⁶	22760x10 ⁶	22760x10 ⁶	18487x10 ⁶	18332x10 ⁶	17262x10 ⁶	16285x10 ⁶	15469x10 ⁶	15281x10 ⁶	13354x10 ⁶
S.E.E.	33,000	32,900	33,700	31,200	31,900	31,900	31,900	32,100	33,000	32,100
C.E.P.	44,100	44,000	45,200	41,900	43,100	43,200	43,400	43,800	45,200	44,000
Parameters										
Constant	-131,000	-124,000	-125,000	-112,000	-108,000	-113,000	-115,000	-113,000	-110,000	- 96,200
HP _{t-1}	.254 (10.5)	.262 (10.8)	.262 (10.4)	.255 (10.5)	.256 (10.4)	.276 (9.33)	.298 (9.63)	.305 (9.55)	.307 (9.15)	.320 (10.1)
HP _{t-2}	-.224 (7.28)	-.238 (7.52)	-.238 (7.05)	-.237 (7.27)	-.241 (7.24)	-.260 (7.11)	-.285 (7.68)	-.295 (7.60)	-.300 (7.13)	-.328 (7.84)
B/C _{t-1}	7760 (25.9)	7830 (26.1)	7820 (24.3)	8190 (22.6)	8320 (17.0)	7700 (10.1)	6920 (7.08)	6380 (5.53)	6100 (4.46)	4980 (3.37)
AFS _{t-1}	2020 (8.75)	1800 (6.09)	1800 (4.45)	1130 (2.17)	916 (1.24)	1590 (1.61)	2310 (2.03)	2760 (2.22)	2970 (2.15)	3800 (2.69)
E(w _{t-1})	1.81 (119)	1.81 (120)	1.81 (117)	1.81 (120)	1.81 (117)	1.81 (114)	1.81 (119)	1.81 (114)	1.82 (99.6)	1.83 (91.7)
E(w _{t-2})	-.884 (55.6)	-.878 (54.7)	-.878 (51.6)	-.877 (53.9)	-.874 (51.5)	-.871 (51.0)	-.867 (53.4)	-.869 (53.6)	-.869 (51.5)	-.865 (53.4)
ρ_1	-.472 (3.53)	-.532 (4.17)	-.532 (4.10)	-.491 (3.57)	-.524 (3.76)	-.546 (3.56)	-.670 (4.42)	-.684 (4.73)	-.679 (4.51)	-.771 (5.06)
ρ_2	-.641 (4.80)	-.692 (5.43)	-.692 (5.34)	-.657 (4.77)	-.661 (4.74)	-.585 (3.82)	-.614 (4.05)	-.676 (4.68)	-.658 (4.37)	-.666 (4.37)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

TABLE 22

MODEL TA

EFFECTS OF CUTTING RECENT YEARS FROM SAMPLE PERIOD

	Sample Period										
	1950- 1979	1950- 1978	1950- 1977	1950- 1976	1950- 1975-	1950- 1974	1950- 1973	1950- 1972	1950- 1971	1950- 1970	1950- 1969
R^2	.9970	.9970	.9972	.9972	.9971	.9967	.9964	.9970	.9967	.9961	.9956
$\bar{R}^2$	.9955	.9954	.9956	.9954	.9951	.9944	.9936	.9945	.9937	.9921	.9906
S.S.E.	18487x10 ⁶	18434x10 ⁶	17202x10 ⁶	17193x10 ⁶	16922x10 ⁶	16919x10 ⁶	15975x10 ⁶	11635x10 ⁶	11187x10 ⁶	11185x10 ⁶	10871x10 ⁶
S.E.E.	31,200	32,000	31,800	32,800	33,600	34,800	35,100	31,100	31,900	33,400	34,800
S.E.P.	41,900	43,400	42,900	43,700	44,900	46,500	46,400	39,900	42,100	45,300	47,400
Parameters											
Constant	-112,000	-112,000	-115,000	-114,000	-123,000	-125,000	-90,200	-158,000	-169,000	-168,000	-183,000
HP _{t-1}	.255 (10.5)	.251 (8.77)	.263 (8.85)	.261 (7.91)	.266 (7.63)	.266 (7.31)	.247 (5.89)	.304 (8.99)	.312 (8.99)	.311 (8.27)	.320 (8.08)
HP _{t-2}	-.237 (7.27)	-.235 (6.83)	-.230 (7.03)	-.231 (6.48)	-.222 (5.64)	-.221 (4.96)	-.232 (4.24)	-.237 (5.82)	-.243 (5.88)	-.242 (4.97)	-.268 (4.22)
(B/C) _{t-1}	8190 (22.6)	8260 (16.4)	7610 (10.2)	7690 (6.47)	7190 (4.66)	7140 (3.91)	8480 (3.43)	5840 (3.21)	5600 (3.11)	5600 (2.95)	5860 (2.82)
AFS _{t-1}	1130 (2.17)	1140 (2.13)	1270 (2.45)	1230 (1.92)	1520 (1.73)	1560 (1.32)	406 (.229)	2550 (1.86)	2850 (2.06)	2830 (1.94)	2980 (1.97)
E(w _{t-1})	1.81 (120)	1.82 (71.7)	1.79 (46.9)	1.79 (30.3)	1.77 (23.5)	1.76 (20.2)	1.82 (15.6)	1.70 (19.7)	1.69 (19.7)	1.69 (18.6)	1.71 (16.5)
E(w _{t-2})	-.877 (53.9)	-.881 (39.9)	-.862 (31.8)	-.865 (24.0)	-.853 (19.9)	-.852 (17.7)	-.837 (14.1)	-.800 (16.3)	-.789 (15.7)	-.789 (14.8)	-.783 (14.1)
ρ_1	-.491 (3.57)	-.489 (3.52)	-.474 (3.51)	-.469 (3.41)	-.453 (3.10)	-.451 (3.03)	-.343 (2.17)	-.515 (3.14)	-.545 (3.33)	-.540 (3.22)	-.597 (3.47)
ρ_2	-.657 (4.77)	-.663 (4.77)	-.698 (5.16)	-.698 (5.07)	-.666 (4.56)	-.669 (4.51)	-.631 (3.99)	-.616 (3.75)	-.639 (3.90)	-.638 (3.80)	-.639 (3.71)
E(w ₁₉₄₉)	1,857,000 (119)	1,857,000 (116)	1,856,000 (119)	1,856,000 (115)	1,855,000 (108)	1,856,000 (104)	1,846,000 (92.0)	1,866,000 (109)	1,870,000 (109)	1,370,000 (101)	1,875,000 (96.9)
E(w ₁₉₅₀)	1,807,000 (71.7)	1,809,000 (67.6)	1,799,000 (64.3)	1,800,000 (60.5)	1,798,000 (56.9)	1,798,000 (54.1)	1,783,000 (47.1)	1,800,000 (55.5)	1,805,000 (55.7)	1,804,000 (52.9)	1,807,000 (51.7)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

TABLE 23

MODEL TM

EFFECTS OF CUTTING INITIAL YEARS FROM SAMPLE PERIOD

	Sample Period									
	1947- 1979	1948- 1979	1949- 1979	1950- 1979	1951- 1979	1952- 1979	1953- 1979	1954- 1979	1955- 1979	1956- 1979
R^2	.9975	.9974	.9972	.9974	.9971	.9969	.9967	.9965	.9964	.9971
$\bar{R}^2$	.9964	.9962	.9958	.9961	.9954	.9951	.9947	.9942	.9938	.9948
S.S.E.	19159x10 ⁶	18655x10 ⁶	18491x10 ⁶	16063x10 ⁶	15931x10 ⁶	14978x10 ⁶	14680x10 ⁶	14609x10 ⁶	14562x10 ⁶	11539x10 ⁶
S.E.E.	29,500	29,800	30,400	29,100	29,800	29,700	30,300	31,200	32,300	29,800
S.E.P.	39,500	39,900	40,800	39,100	40,200	40,200	41,000	42,200	43,700	40,400
Parameters										
Constant	-101,000	-99,400	-100,000	-97,900	-97,500	-93,500	-89,700	-87,900	-85,900	-58,200
	.273	.277	.277	.268	.267	.286	.294	.294	.294	.307
HP _{t-1}	(14.0)	(14.0)	(13.6)	(12.9)	(12.3)	(11.0)	(11.0)	(10.6)	(10.1)	(11.9)
	-.240	-.248	-.246	-.242	-.243	-.263	-.274	-.275	-.277	-.305
HP _{t-2}	(9.77)	(9.60)	(9.03)	(8.88)	(8.71)	(8.34)	(8.32)	(7.88)	(7.43)	(8.84)
	5060	5280	5120	6040	6280	5460	5130	5040	4910	2890
(B/C) _{t-1}	(12.8)	(10.8)	(8.05)	(6.96)	(5.34)	(3.92)	(3.53)	(3.27)	(2.86)	(1.62)
	2930	2740	2880	2160	1980	2480	2630	2660	2730	3810
MCS _{t-1}	(11.1)	(7.54)	(5.73)	(3.15)	(2.20)	(2.45)	(2.57)	(2.50)	(2.39)	(3.37)
	1.78	1.78	1.78	1.79	1.79	1.79	1.79	1.79	1.79	1.79
E(w _{t-1})	(130)	(126)	(120)	(111)	(103)	(98.2)	(98.8)	(93.8)	(87.9)	(94.2)
	-.862	-.860	-.860	-.864	-.864	-.856	-.353	-.853	-.853	-.841
E(w _{t-2})	(66.8)	(65.9)	(64.4)	(64.5)	(62.8)	(58.2)	(57.7)	(55.9)	(53.6)	(57.1)
	-.575	-.611	-.612	-.566	-.585	-.623	-.687	-.683	-.676	-.791
ρ_1	(4.62)	(5.06)	(4.93)	(4.26)	(4.35)	(4.25)	(4.75)	(4.70)	(4.50)	(5.04)
	-.699	-.730	-.722	-.686	-.690	-.632	-.659	-.673	-.659	-.340
ρ_2	(5.61)	(6.05)	(5.82)	(5.17)	(5.14)	(4.31)	(4.55)	(4.63)	(4.38)	(4.08)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

TABLE 24

MODEL TM

EFFECTS OF CUTTING RECENT YEARS FROM SAMPLE PERIOD

	Sample Period										
	1950- 1979	1950- 1978	1950- 1977	1950- 1976	1950- 1975	1950- 1974	1950- 1973	1950- 1972	1950- 1971	1950- 1970	1950- 1969
R ²	.9974	.9974	.9975	.9975	.9973	.9970	.9966	.9973	.9970	.9964	.9959
$\bar{R}^2$	.9961	.9960	.9960	.9959	.9956	.9949	.9939	.9951	.9942	.9928	.9913
S.S.E.	16063x10 ⁶	16041x10 ⁶	15637x10 ⁶	15386x10 ⁶	15312x10 ⁶	15305x10 ⁶	15147x10 ⁶	10422x10 ⁶	10275x10 ⁶	10208x10 ⁶	10074x10 ⁶
S.E.E.	29,100	29,900	30,300	31,000	32,000	33,100	34,100	29,500	30,600	32,000	33,500
S.E.P.	39,100	40,300	39,800	39,900	42,000	44,000	44,400	37,900	40,600	43,400	45,800
Parameters											
Constant	-97,900	-97,900	-98,200	-96,900	-98,300	-98,900	-94,600	-111,000	-113,000	-111,000	-117,000
	.268	.266	.272	.265	.266	.266	.260	.298	.300	.297	.300
HP _{t-1}	(12.9)	(10.9)	(10.1)	(8.77)	(8.39)	(8.12)	(7.35)	(10.1)	(9.63)	(8.85)	(8.36)
	-.242	-.241	-.241	-.241	-.238	-.238	-.235	-.268	-.273	-.265	-.282
HP _{t-2}	(8.88)	(8.40)	(8.40)	(7.95)	(7.49)	(7.08)	(6.46)	(8.70)	(7.90)	(6.06)	(4.61)
	6040	6090	5780	6210	5970	5910	6500	4690	4700	4650	4910
(B/C) _{t-1}	(6.96)	(6.50)	(5.50)	(4.50)	(3.57)	(3.08)	(2.72)	(2.55)	(2.48)	(2.27)	(2.11)
	2160	2160	2160	2030	2150	2200	1770	3030	3070	3050	3060
MCS _{t-1}	(3.15)	(3.07)	(3.05)	(2.61)	(2.30)	(1.91)	(1.14)	(2.50)	(2.45)	(2.29)	(2.18)
	1.79	1.79	1.78	1.79	1.79	1.79	1.80	1.75	1.76	1.75	1.77
E(w _{t-1})	(111)	(74.6)	(50.2)	(36.2)	(31.6)	(29.5)	(26.1)	(33.0)	(31.7)	(28.5)	(21.9)
	-.864	-.866	-.856	-.866	-.863	-.862	-.871	-.826	-.824	-.827	-.826
E(w _{t-2})	(64.5)	(46.9)	(34.6)	(26.9)	(24.2)	(22.9)	(20.5)	(23.7)	(22.6)	(21.5)	(20.2)
	-.566	-.565	-.552	-.528	-.518	-.516	-.456	-.613	-.620	-.595	-.619
c ₁	(4.26)	(4.22)	(4.12)	(3.91)	(3.68)	(3.62)	(3.04)	(3.87)	(3.88)	(3.61)	(3.66)
	-.686	-.692	-.706	-.712	-.696	-.701	-.680	-.651	-.661	-.655	-.654
c ₂	(5.17)	(5.16)	(5.27)	(5.27)	(4.94)	(4.91)	(4.55)	(4.11)	(4.13)	(3.97)	(3.86)
E(w ₁₉₄₉)	1,865,000	1,865,000	1,864,000	1,864,000	1,863,000	1,864,000	1,859,000	1,875,000	1,876,000	1,874,000	1,876,000
	(134)	(130)	(128)	(124)	(118)	(113)	(101)	(124)	(120)	(111)	(105)
E(w ₁₉₄₈)	1,819,000	1,820,000	1,812,000	1,816,000	1,816,000	1,816,000	1,809,000	1,823,000	1,824,000	1,823,000	1,823,000
	(83.1)	(78.4)	(68.6)	(63.8)	(60.8)	(57.1)	(49.6)	(59.5)	(58.0)	(54.5)	(52.2)

S.S.E. is the sum of squared errors or the residual sum of squares. All other abbreviations are as defined in previous tables or in the body of the paper.

results indicate that when it is desired to check the stability of a system, it may not be enough to compare the estimated parameter values for different sample periods. Estimates of parameter values in a model may be highly correlated, either pairwise or groupwise. In this case, very different combinations of parameter values may yield approximately the same fit. The Chow test is based on a comparison of the residual sum of squares for different sample periods, and thus is not concerned directly with whether or not the parameter values jump around. This explains why the parameter values in these models can jump around as they do, and yet a formal test indicates that the structure of the system does not change.³⁰

Table 22 shows the effects on model TA of changing the years in which the sample period is ended. Again the parameter estimates change quite a bit, but in no case is the change in the structure significant. In fact, the only situation in which the Chow test indicates that the change is even close to being significant is when the sample period is changed from 1950-1972 to 1950-1973. The approximate t-values in Table 22 show that the precision on the feeder price parameter estimates is

³⁰ An informal check on the stability of the system might be based on a comparison of the distributed lag pattern on the independent variables. For example, the parameter estimates on HP_{t-1} , HP_{t-2} , $E(w_{t-1})$ and $E(w_{t-2})$ may jump around quite a bit between two sample periods but maintain a relationship such that the distributed lag pattern on this variable does not change significantly. The two models are therefore saying that the effects of changes in hay production on the dependent variable are the same in the short and long runs.

poor for many of these sample periods. The beef to corn price ratio variable fades in significance as recent years are cut from the sample. This might be due to the increased variation in this variable after about 1973. The precision on the other variables in the model is very good for all of the sample periods.

Table 23 shows the effects on model TM of changing the initial years of the sample period. For this model, it was found that the structure of the system was the same for all of the sample periods. When Chow tests were run using the results shown in Table 24, it was found that only in the case in which the sample period was changed from 1950-1972 to 1950-1973 did the structure of the system change. The precision of the parameter estimates on the Montana calf price variable faded out as the sample period was changed, but not as badly as the feeder price parameter in model TA. Also, like model TA, the coefficient on the beef to corn price ratio systematically loses its significance as recent years are cut from the sample period.

To this point in the analysis, it appears that model TM did as well or better than model TA in all of the tests to which the models were subjected. Based on this analysis it appears that model TM would probably be chosen as the superior model. However, there is a theoretical problem that seems to be avoided in model TA, but not in TM.

In model TM, Montana calf prices and total cattle numbers may very well be jointly dependent. When calf prices rise, farmers will be induced to keep more heifers to build up their breeding herds. This results in an increase in total cattle numbers. But an increase in total cattle numbers means fewer calves are marketed which may drive up calf prices in Montana. The extent of bias from this simultaneity problem may be small because of the strong link between the Montana and national markets. This problem, whatever its magnitude, is avoided in model TA since the number of calves sold to Kansas City is such a small portion of total transactions that a change in the number of cattle marketed from Montana would not be expected to affect the price in Kansas City. Primarily because of this potential joint dependence problem in model TM (and the accompanying possibility of bias and inconsistency in the parameter estimates), it was decided to report and analyze both of the models.

Further Testing of the Final Models

A number of further tests were run on the final models with several different objectives in mind, one of which was to get a better idea of the roles being played by the different variables. In this section several of these tests will be described. The reasons for running the variations on the models will be discussed, the results will be presented or described, and then the implications of these results will be examined.

The Role of Hay Production

Of the variables in the models, the role of the hay production variable was the most puzzling. It was hypothesized that this variable might be serving as a proxy for weather, since it seemed reasonable to expect the level of rainfall (or precipitation) to be correlated with hay production. Several runs were made to try to test this hypothesis. The basic technique in most of these runs was to exclude the first-order lag on hay production from the calculation of the lagged expectation of the dependent variable. The effect of this is to remove the influence of the hay variable from the dynamics of the equation. To understand the reasoning behind this technique, consider the following.

The theoretical justification for using the expected value of the lagged dependent variable is that ranchers will base their production decisions for the current year on the level of the dependent variable in preceding years, net of any random disturbances. In other words, producers are intelligent enough to realize that some portion of the previous levels of the dependent variable was strictly due to chance and should not be counted on to occur again, so that the effects of these chance happenings should not enter into any current or future decisions. Only the systematic portion of the variation in lagged values of the dependent variables should be used to make current production decisions. Weather variations are an example of random

occurrences that may affect the current level of the dependent variable, but which should be netted out of any future influence on the dependent variable. Suppose hay production is acting as a proxy for weather. Then including hay production as an independent variable will increase the explanatory power of the model, but there would be no reason to include its effect as a component of the lagged value of the expectation of the dependent variable since its effects do not carry over to future years. If it is the case that hay production is proxying for weather, then the random influences of weather fluctuations should be largely felt within one year. That is, a year marked by an exceptional level of rainfall (say year t) will result in a bumper hay crop, which induces ranchers to hold more cattle through the winter (increase in herd sizes on January 1 of year $t + 1$.) The effects of the weather variation in year t would not be expected to affect hay crops in year $t + 1$. Thus, whatever effects hay production in year t has on cattle numbers in year $t + 2$ must be due to this variable playing some role other than that of a proxy for weather. This argument is the basis for excluding the first-order lag and including the second-order lag on hay production in the lagged expectation of the dependent variable.

Using the preceding as a justification, a model was tested in which the first-order lag on hay production was removed from the lagged expectations of the dependent variable. In addition, the

model was specified with first, second and third-order lags on the hay production variable. The third-order lag was included to allow the distributed lag pattern on hay production more flexibility in the shape it assumed.³¹ Suppose this model turned out to be inferior to the basic model. This would imply that the first-order lag on hay production has a systematic effect and should be included within the nonstochastic difference variables. This would imply that hay production is not acting primarily as a weather surrogate. Suppose, on the other hand, that the model was not significantly hurt by removing hay production from the lagged expectation of the dependent variable. This indicates that hay production is not playing a systematic role in the system, i.e., it is acting, at least to some degree, as a proxy for weather.

When this test was actually run on a revised version of model

³¹ Suppose the model was specified with only first and second-order lags on hay production. Then, when the first-order lag was excluded from the lagged expectation of the dependent variable, the rational distributed lag on hay production within these nonstochastic terms would have only one element in the numerator. This implies that the shape of the lag is dictated solely by the difference equation parameters. However, if the numerator has more than one term (second and third-order lags on hay production) then the shape of the lag pattern is determined jointly by the parameters in the numerator and those in the denominator.

TA,³² the results indicated that the exclusion of first-order hay production from the lagged expectations of the dependent variable was definitely detrimental to the model. It resulted in a model with a lower multiple (and adjusted) correlation coefficient, higher standard error of the estimate, higher standard error of prediction and reduced precision on the estimates of the coefficient on the price variables. There were also indications of serious specification error in the model. These results were taken as a fairly strong indication that the hay production variable was not serving solely as a proxy for weather conditions.

The graph of the levels of hay production over the last thirty years indicates that the level of this variable increased continuously during the sample period used. This led to the suspicion that hay production might be acting as a trend variable. To test this, models

³²The specification of this particular model was

$$\begin{aligned}
 w_t = & \alpha_0 + \beta_1 HP_{t-1} + \beta_2 HP_{t-2} + \beta_3 HP_{t-3} + \beta_4 (B/C)_{t-1} \\
 & + \beta_5 AF\$_{t-2} + \beta_6 AF\$_{t-3} + \beta_7 AF\$_{t-4} + \lambda_1 E(w_{t-1}) \\
 & + \frac{\lambda}{2} E(w_{t-2}) + \delta_1^u t-1 + \delta_2^u t-2 + \delta_3^u t-3 + \epsilon_t
 \end{aligned}$$

The basic difference between this model and the standard specification of model TA is that $AF\$_{t-1}$ has been replaced by $AF\$_{t-2}$, $AF\$_{t-3}$, and $AF\$_{t-4}$, and the third order serial correlation parameter has been added. The reason for running this particular specification of the model is discussed later in this section. For present purposes, it is most important to point out that the explanatory power of this model was not any less than that of the standard specification of the model.

BM and TA were run with a trend variable in place of the first and second-order lags on hay production. The idea here was that if hay production is serving primarily as a trend variable, the fit of the model should not be significantly decreased by making such a substitution. These runs resulted in dramatically inferior models. From this, it was concluded that hay production is not primarily picking up trend effects.

Tests for Consistency with Theoretical Models

In the preceding subsection, the analysis of the four final models revealed that all of them implied cyclical adjustment patterns. In the theoretical literature, the justification for using distributed lag models in applied work comes from either a partial adjustment or an adaptive expectations framework. Neither of these frameworks (or a combination of the two) allows for a model with imaginary roots. This means that there is no formal justification from economic theory for a model that implies cyclical adjustment patterns. Thus, attempts were made to reconcile the models developed in this study with models consistent with the current state of economic theory. To do this, certain restrictions were placed on the parameters in the stochastic difference equations so the models could not assume a cyclical form. The results of these restricted models were then compared with the

unrestricted models to see if they differed significantly. An explanation of the restrictions placed on the models and a discussion of the results follows.

In the analysis of second-order difference equations, the characteristics roots can be of three types: (1) real and distinct, (2) real and equal, and (3) complex. The case in which the roots are real and equal is the boundary between the region in which the roots are real, and the region in which the roots are complex (Griliches, 1967). A series of tests was run to see how detrimental it would be to the model to constrain the roots in such a way that they were forced to be real and equal. It can be shown that the model that should be estimated is of the form

$$(9) \quad Y_t = \alpha_0' + \beta_1' X_t + \lambda_1 E(y_{t-1}) + \lambda_2 E(y_{t-2}) + \varepsilon_t'$$

where $\lambda_1 = 2\lambda$ and $\lambda_2 = -\lambda^2$. Rewriting (9) with

these constraints the model becomes,³³

$$(10) \quad Y_t = \alpha'_0 + \beta'_1 X_{1t} + 2\lambda E(y_{t-1}) - \lambda^2 E(y_{t-2}) + \varepsilon_t$$

Models with these restrictions on the parameters on the difference equation can be derived from a system in which it is assumed that the weights on the lag distribution of the independent variables come from a distribution known as the Pascal distribution (see Solow, Griliches, 1967, pp. 25-27 and Kmenta, pp. 487-491). Notice that in (10) there is one less parameter to be estimated than previously (only λ must be estimated now, whereas before λ_1 and λ_2 had to be estimated) implying that this model has an additional degree of freedom. Models with the preceding restrictions on the parameters on the difference equation will be referred to as "Pascal models."

It was felt that forcing the model to conform to the above restrictions might not be seriously detrimental since the estimated

³³To see that the roots of this difference equation are real and equal, solve the homogeneous difference equation,

$$(i) \quad Y_t - 2\lambda Y_{t-1} + \lambda^2 Y_{t-2} = 0$$

as a solution, try $Y_t = Ab^t$. The result is

$$(ii) \quad Ab^t - 2\lambda Ab^{t-1} + \lambda^2 Ab^{t-2} = 0$$

factoring out an Ab^{t-2} term, the result is

$$b^2 - 2\lambda b + \lambda^2 = 0$$

The solutions to this quadratic equation are, $b_1 = b_2 = \lambda$.

values of the parameters in the unconstrained models did not appear to grossly violate the Pascal constraints (see Tables 4 and 21). Three of the basic models (BM, BA, and TM) were run using this constraint. Table 25 shows the results of these runs. It is readily apparent from the summary statistics in this table that the Pascal models are inferior to the unconstrained models. In all three cases, the constrained models have lower multiple (and adjusted multiple) correlation coefficients, higher standard errors of the estimate and standard errors of prediction, and in almost all cases, lower precision on the parameter estimates. Also, in both of the breeding herd models, the maximum likelihood estimates of the Y_{1947} value are considerably different from the actual value of that variable. This may be an indication of specification error in these models, since it implies that the actual value of 1947 breeding herds did not fit as well with the rest of the model as did an artificially obtained value for that variable (that value that results in the lowest residual sum of squares). In the constrained model, these artificially obtained initial condition values were much more different from the actual values than in the unconstrained models. Finally, an F-test was run to compare the models and see if they differed significantly. In all three cases, the Pascal models differed from the basic models at a 99 percent level of significance. In other words, constraining the difference equation parameters

TABLE 25

COMPARISON OF BASIC MODELS WITH 'PASCAL MODELS'

MODEL BM			MODEL BA			MODEL TM		
	BM	BM-P		BA	BA-P		TM	TM-P
R^2	.9972	.9952	R^2	.9986	.9955	R^2	.9974	.9953
$\bar{R}^2$	.9955	.9929	$\bar{R}^2$	.9973	.9918	$\bar{R}^2$	.9961	.9933
S.S.E.	110707x10 ⁵	186007x10 ⁵	S.S.E.	49671x10 ⁵	159324x10 ⁵	S.S.E.	16063x10 ⁶	29034x10 ⁶
S.E.E.	24,100	30,500	S.E.E.	18,200	31,600	S.E.E.	29,100	38,100
S.E.P.	32,200	40,900	S.E.P.	25,000	43,800	S.E.P.	39,100	51,300
Parameters								
Constant	-118,000	-39,500	Constant	-178,000	-69,200	Constant	-97,900	-76,800
HP _{t-1}	.169 (10.9)	.170 (6.97)	HP _{t-1}	.156 (14.7)	.158 (5.23)	HP _{t-1}	.268 (12.9)	.289 (9.45)
HP _{t-2}	-.109 (4.56)	-.186 (8.34)	HP _{t-2}	-.079 (5.00)	-.175 (6.31)	HP _{t-2}	-.242 (8.88)	-.323 (11.2)
(BS/CS) _{t-1}	3580 (7.84)	4760 (7.85)	(BS/CS) _{t-1}	8950 (10.1)	6520 (2.35)	(BS/CS) _{t-1}	6040 (6.96)	9560 (10.2)
MCS _{t-1}	3210 (7.00)	2380 (3.28)	(BS/CS) _{t-2}	-3810 (4.68)	-877 (.360)	MCS _{t-1}	2160 (3.15)	-390 (.487)
MCS _{t-2}	-2150 (8.45)	-2420 (5.14)	AFS _{t-1}	2460 (2.27)	5250 (1.56)	E(w _{t-1})	1.79 (111)	.923 (79.1)
E(y _{t-1})	1.71 (50.7)	.909 (52.7)	AFS _{t-2}	-3760 (2.30)	-8740 (1.84)	E(w _{t-2})	-.864 (64.5)	-.442 (2.90)
E(v _{t-2})	-.832 (43.5)		AFS _{t-3}	2620 (3.08)	4110 (1.64)	ρ_1	-.566 (4.26)	-.442 (2.90)
ρ_1	-.696 (4.22)	-.585 (3.48)	E(y _{t-1})	1.73 (83.3)	.912 (42.9)	ρ_2	-.686 (5.17)	-.548 (3.59)
ρ_2	-.394 (2.39)	-.351 (2.09)	E(v _{t-2})	-.871 (66.0)		E(w ₁₉₄₉)	1,865,000 (134)	1,858,000 (83.6)
E(y ₁₉₄₈)	925,000 (86.4)	922,000 (59.7)	ρ_1	-1.33 (8.54)	-.794 (4.48)	E(w ₁₉₄₈)	1,819,000 (83.1)	1,773,000 (50.8)
E(y ₁₉₄₇)	910,000 (44.6)	866,000 (30.5)	ρ_2	-1.17 (6.10)	-.626 (3.16)			
			ρ_3	-.524 (3.37)	-.242 (1.36)			
			E(y ₁₉₄₈)	939,000 (128)	929,000 (38.4)			
			E(v ₁₉₄₇)	920,000 (67.2)	872,000 (19.2)			

BM-P, BA-P, and TM-P are the "Pascal versions" of BM, BA, and TM respectively. To give some idea of the accuracy of the estimates of the initial condition parameters, the actual value of the dependent variables in the 'initial condition years' were: $y_{1948} = 932,000$; $y_{1947} = 916,000$, $w_{1949} = 1,939,000$, $w_{1948} = 1,847,000$.

so that the characteristic roots were forced to be real and equal did have a significantly detrimental effect on the models.

A combination partial adjustment, adaptive expectations model can be formulated that results in the same general specification as model BM but with certain constraints among the parameters.³⁴ It can be shown that the roots of the second-order difference equation derived from this "theoretical" base cannot be complex.³⁵ Runs were made with

³⁴ Models could have also been formulated to coincide with the other basic specifications, but the computer programming was difficult enough, and the results obtained with model BM were unpromising enough, that it was decided that the cost of pursuing this line of research further would outweigh the benefits.

³⁵ Proof: Suppose we have a combination adaptive expectations, partial adjustment model of the form

$$(a) \quad y_t^* = \beta X_t^* + \epsilon_t$$

where

$$(b) \quad y_t - y_{t-1} = \lambda(y_t^* - y_{t-1})$$

and

$$(c) \quad X_t^* - X_{t-1}^* = \gamma(X_t - X_{t-1}^*)$$

Solving (b) and (c) for y_t^* and X_t^* respectively, we get

$$(b') \quad y_t^* = \frac{1-\theta L}{1-\theta} y_t$$

$$(c') \quad X_t^* = \frac{1-\sigma}{1-\sigma L} X_t$$

where $\theta = 1-\lambda$, $\sigma = 1-\gamma$, and L is the lag operator.

the constraints implied by this "theoretical model" to see if constraining the model to have real roots would have a significant effect.

This theoretical formulation of the model resulted in a system that was considerably more nonlinear in the parameters than the unconstrained model. To assure that the theoretical model did not converge to a local minimum residual sum of squares, the system was given several different sets of starting values. In all cases, the model converged to the same values for all of the summary statistics, parameter estimates, etc., (indicating that the model was converging on a global minimum). Results of the estimation of this model were very poor, which implies that constraining the model to conform to the partial adjustments, adaptive expectations form had extremely detrimental effects.

³⁵(cont.) Substituting into (a) for y_t^* and X_t^* , we get

$$(d) \quad \left(\frac{1-\theta L}{1-\theta}\right)y_t = \beta\left(\frac{1-\sigma}{1-\sigma L}\right)X_t + \varepsilon_t$$

or

$$(d') \quad y_t = \beta(1-\sigma)(1-\theta)X_t + (\theta+\sigma)y_{t-1} - \theta\sigma y_{t-2} + (1-\theta)(\varepsilon_t - \sigma\varepsilon_{t-1})$$

The solution to this second order difference equation will have complex roots, i.e., the adjustment process of the system will be cyclical, if $(\theta+\sigma)^2 - 4\theta\sigma < 0$. But $(\theta+\sigma)^2 - 4\theta\sigma = \theta^2 + 2\theta\sigma + \sigma^2 - 4\theta\sigma = \theta^2 - 2\theta\sigma + \sigma^2 = (\theta-\sigma)^2 > 0$. Thus, the combination adaptive expectations, partial adjustment model cannot have complex roots.

Apparently, the cyclical nature of cattle numbers is such an integral part of the system that a model that does not allow for this feature is of little practical use.

Chapter 5

CONCLUSIONS AND LIMITATIONS

In this chapter, a brief discussion of the possible implications of cattle inventory response to changes in the independent variables and an interpretation of the error structure of the final models will be presented. The accomplishments of this thesis will be summarized and potential extensions of this work will be discussed. Some possible limitations on the usefulness of the models developed will also be examined.

In the final section of the preceding chapter, the results of tests for determining the role played by the hay production variable were described. It was determined that this variable did not appear to be acting as a proxy for weather conditions or as a trend variable. The question remains as to what role this variable is playing. Recall that the results of the tests on this variable indicated that its effects did play an important part in determining the level of the systematic component of the lagged dependent variables. The most plausible interpretation of the role played by the hay production variable is that its level is an indication of investments or commitments of resources to the cattle production process. If ranchers decide they want to increase the size of their herds, it is likely they will want to increase hay production as well, so they will have winter feed available for the additional cattle. Thus an increase

in hay production in a given year might indicate that ranchers plan to enlarge their herds in future years. This interpretation of the meaning of the hay variable is consistent with the distributed lag structure found in the final models and with the results of the tests described earlier. The importance of the distributed lag and the protracted response to hay production suggests that ranchers as a group are committed to changes in cattle numbers in association with changes in hay production for several years, possibly because hay is a perennial crop.

There were three price variables in the final models: the price of Montana calves, the Kansas City price of feeder steers, and the beef to corn price ratio. In one respect, the interpretation of these variables is straightforward. They appear to be acting as proxies for future price expectations. The form of the lag distribution implied by the estimated coefficients might be used to draw inferences about the way in which expectations are formed. In another respect their interpretation is not so clear. Earlier, two types of effects of price changes were described: a "short-run" effect that influenced the tendencies of ranchers to sell their cattle immediately, and a second effect on their expectations about future price levels. Conjectures about which of the variables in the models is playing which of these roles are highly speculative. It is apparent that in each of the models, the beef to corn price

ratio variables contain some information not included in the other price variables. It seems quite plausible that this price ratio variable (and its movements) might be serving as an indicator of the current and future profitability of the cattle feedlot business from which the demand for Montana feeders is derived. Ranchers may be using this information to form their price expectations for several years in the future.¹ That is, if feedlot operations have been getting more and more profitable in recent years, ranchers might expect favorable prices to prevail for sometime into the future. Trends in the other price variables might be interpreted as providing information about short term future price movements. The effects of movements in the most recent values of these variables may contain the influences that make ranchers want to sell their stock immediately in response to high prices. Once again, this discussion about the distinction between the roles played by the price variables has not

¹The question might be raised as to whether ranchers actually keep track of the relationship between the price of corn and the prices of slaughter cattle and use this information to form future expectations. It seems likely that many ranchers are not aware of these specific relationships. An alternative explanation of the role of this variable is that it is acting as an indication of activity by buyers for feedlots. That is, when feedlot operations become profitable (this should generally be indicated by an increase in the beef to corn price ratio), the purchasers for these feedlots should become more active. Ranchers may be using this information on buyer activity (rather than direct observation of the relationship between beef and corn prices) to form their future expectations.

been verified by any empirical evidence and are thus only speculation.

(It is not clear how these conjectures might be tested empirically.)

In all three of the final models presented, the estimated structures on the error terms were second order autoregressive processes of the form

$$(1) \quad u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \varepsilon_t$$

where ε_t is assumed to have the classical properties. In all of the models, the algebraic signs on the estimates of ϕ_1 and ϕ_2 were negative. To determine what these estimated structures might imply about the dynamics of the error components of the models, the autocorrelation functions were estimated and rough plots of the implied correlograms were sketched.² These showed that the "adjustment process" to a nonzero error term in a given year was cyclical in nature. For example, a large positive error term in a given year implied that the error terms in the following two years would be negative. For all of the models, the implied dynamic systems were stable. A possible intuitive interpretation of the meaning of these estimated error structures might be the following.

Suppose that in a given year, ranchers cull their older cows at an exceptionally low rate. This might occur because utility cow

²For a discussion of these concepts see Nelson (1973, Chapter 3).

prices are exceptionally low, or because range conditions are very good at the start of the winter, or because ranchers believe the forthcoming winter will be unusually mild. Cattle numbers in this year will be larger than predicted by the model, implying a positive error term. In the following year or two, ranchers will be left with disproportionately high ratios of older cows. To adjust for this, they will cull at unusually high rates. Actual cattle numbers will therefore be lower than predicted by the model, i.e. negative error terms will be observed. Conversely, exceptionally high rates of culling in a given year (negative error terms) lead to unusually low numbers of older cattle in later years, implying low rates of culling (positive error terms) in those years.

An alternative explanation of these patterns in the residuals (which does not necessarily contradict the preceeding one) might be based on either habit or capacity constraints faced by ranchers. Suppose that, in a given year, the influence of some variable not included in our models leads ranchers to keep unusually large numbers of heifers to expand their breeding herds. In this case, the models underpredict and positive error terms are observed. In the following year or two ranchers look at their herds and decide they are too large, possibly because of capacity constraints (e.g. there isn't enough pasture land for the unusually large herds) or because they

decide they have attempted to expand their herds at faster than "optimal" rates (the optimal rate of expansion may be consistent with a certain age distribution of the herd which has been violated by keeping such large numbers of replacement heifers). Thus, herds are cut back and the models overpredict, i.e. negative error terms are observed. These interpretations of the meanings of the error structures are highly speculative and it seems as though it would be extremely difficult to develop empirical tests of their validity.

The attempts to develop models for estimating Montana cattle numbers seem to have been quite successful. The final models explain very high proportions of the variations in herd sizes over time and the variables in the empirical models are consistent with those in the theoretical model. In addition the models are not particularly sensitive to the specification of the sample period and they predict well beyond the end of the sample. The models indicate that ranchers' decisions regarding herd sizes are strongly influenced by the prices they have received for their cattle and by the availability of feed in recent periods. They also imply that rigidities in the cattle production process (as indicated by recent levels of cattle numbers) play a strong role in determining herd sizes. All of the models imply cyclical patterns of adjustment in cattle numbers. Tests indicated that these cyclical reactions were a very important

component of the system. From a strictly empirical point of view, these cyclical patterns have probably been imposed on the models by the historically cyclical movements in herd sizes and cattle prices. Contemplation of different characteristics of the industry and the nature of the economic decision making process indicate that these cycles are probably largely attributable to slow adjustments in expectations and various biological constraints in the cattle production process.

It is hoped that the estimation of these models using state-level data will provide useful information and insights for the estimation of similar national models. If attempts to estimate national models are even close to being as successful as the attempts on the state level, then they should provide a significant improvement over previous work in this area. Improved inventory equations should lead to improved estimates and predictions of quantities that depend on estimates of cattle numbers (e.g. slaughter and hence retail meat supplies).

One important characteristic of the models estimated in this thesis was that the results were markedly better when the model was estimated as a nonstochastic difference equation than when it was estimated in its stochastic form. Although a number of different interpretations of the role of these lagged values of the dependent

variable are possible, one of the most plausible is that they are acting as measures of the aggregate rigidities and inflexibilities in the cattle production process. Advantages of specifying the model as a nonstochastic difference equation (in addition to the fact that it explains a considerably larger proportion of the variation in the dependent variable) include the assurance of consistency of the parameter estimates, and the clear delineation between the random and systematic components of the system that results with this specification.

One caveat concerning potential limitations in the usefulness of this model should be mentioned. The entire structure of the cattle industry and of the models estimated in this project depends heavily on the nature of the fluctuations in the relevant price variables. Over the past thirty years these fluctuations have been distinctly cyclical. Thus, the models in this paper might be expected to explain and predict well under conditions in which these cyclical variations continue. If these cyclical movements ceased (say, for example, because prices were officially frozen), it is unlikely that the models would be as useful for predicting future cattle numbers. The problem in such a situation is analogous to the problem of predicting the level of a dependent variable when values of the independent variables fall outside of the range used for

constucting the model. In that case the precision of the forecasts is expected to be poor. If cyclical price fluctuations cease, a "second-order" version of this prediction problem will be encountered. The models will be forced to try to predict values of the dependent variables using independent variables that follow a path different from any path followed by those variables during the period used to construct the model. The resulting predictions would be expected to be imprecise.

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APPENDICES

APPENDIX A

The Data

The purpose of this appendix is to describe the data used in the development of the models presented in the body of this thesis. In particular, this appendix includes the following,

- concise definitions of the variables used and the sources from which the series were obtained.
- discussions of problems that were encountered in constructing the different data series and descriptions of the methods used to circumvent these problems.
- the data series actually used to estimate our empirical models.
- statistics on the transformed variables that entered into the final models.
- graphs of the time paths of the variables that entered into the final models.

Construction of the Data Series

Total Cattle Numbers

This variable is a measure of the total number of cattle and calves (both beef and dairy) in the state at the beginning of each year. The source of this data was Montana Agricultural Statistics a publication of the Montana Department of Agriculture and the Montana Crop and Livestock Reporting Service. No problems were encountered in constructing this data series. Recent values for this variable (and several of the other variables) were obtained directly from the Montana Crop and Livestock Reporting Service.

Breeding Herds

The breeding herd variable included all cows (two years and older) and heifers (one to two years old) that were not counted as milk animals. A problem was encountered in constructing this series since the categories (as reported in the Montana Agricultural Statistics) into which the animals were divided were changed in 1970. After 1970, our data series was obtained by finding the sum of beef cows, beef cow replacements, and "other" heifers 500 pounds and over. It was difficult to tell how much effect this change in the definition of the variables had on the models. To test for the effects of this change, a dummy variable was included in some early specifications of the model (this variable was assigned the value of "1" for all years after 1970), but it did not enter with any significance. An alternative way of testing for the influence of the change in the definition of this variable is to use the Chow test to compare the estimated models from the sample periods 1949-1970 and 1949-1971. As reported in the body of this paper, this test indicated that these periods came from the same underlying population, i.e., the change in the definition of this variable did not appear to make a significant difference.

Hay Production

This variable is the total annual production of all types of hay (in tons) in the state. It includes both irrigated and non-irrigated

hay production. The source of this data series was the Montana Agricultural Statistics and revised estimates for the last five or six years were obtained directly from the Montana Crop and Livestock Reporting Service. No problems were encountered in constructing this data series.

Annual Feeder Steer Prices

This data series reports the prices of feeder steers in Kansas City. These prices are averages of all weights and grades shipped. The average annual prices were calculated using a simple average of the monthly prices. This data was obtained from the Livestock and Meat Statistics, a publication of Department of Agriculture. No problems were encountered in constructing this data series.

Montana Calf Prices

This variable is a measure of the average prices received by Montana ranchers for their calves in the months of October, November and December. These months were used because, on the average, about 84% of all the calves marketed out of state were sold during these months. The weight assigned to each month was the proportion of the total number of calves sold during these three months that was sold in the month in question (for example, if 1 million calves were sold in October through December of a given year, and 300,000 of those were

sold during October, then the weight assigned to October for that year would be .30). A problem was encountered in constructing this data series. There were only thirteen years (1958-1970) in which the data on out-of-state cattle shipments were reported in the Montana Agricultural Statistics. For these years, the monthly weights were calculated from the reported figures, but it was necessary to estimate the monthly weights for the remaining years in the sample. Using the data that was available on out-of-state shipments, it was found that there was no significant trend in the proportion of cattle that was sold during these three months. Using this result, it seemed reasonable to use the average proportion during the thirteen years on which there was data as a weight for the remaining years. For example, the average proportion of the total number of calves sold (during the months of October through December) in November during the years 1958-1970 was .413. This number was used as the weight on the November price for the other years in the sample.

Beef to Corn Price Ratio

This variable is a measure of the ratio of the price of choice beef steers to the price of #2 yellow corn. The corn prices used were October thru September weighted averages of the price of a bushel of corn at Omaha. Prices through 1970 were obtained from Agricultural

Statistics while more recent prices were obtained from Feed Situation.

The only problem encountered in constructing this series was that the price series changed from #3 corn prices to #2 prices around 1950.

This did not really present a problem though, since for the five years that the two series overlapped, the prices of the two types of corn were identical. Thus, the actual prices of #3 corn prior to 1950 were used, i.e., no adjustments were made in that series. Since the months used to calculate these average corn prices did not coincide with the calendar year, a decision on which calendar year to assign them, had to be made. It was decided to consider the corn price average from October of year $t-1$ to September of year t to be the average price of corn in year t . For example, the average price of corn from October of 1949 to September of 1950 was assigned to the calendar year of 1950.

The beef price used in constructing the data series for this variable was the average wholesale price of beef steers at Omaha ("choice" grade). For the years between 1950 and 1974, the price data was obtained from Agricultural Statistics. For the most recent years, the data was obtained from the Commodity Year Book and Livestock and Meat Statistics. A small problem was encountered in constructing this series since the prices were not reported before 1950. The problem was "small" because the data series used in the final model

only used data from 1949-1979. A price series similar to the Omaha series existed for Chicago steer prices (this series was not used because it ceased in 1969). Data for the Chicago series was available prior to 1950. Thus, to generate early estimates for the Omaha series, a regression equation between the Omaha and Chicago series was estimated and then the Chicago prices were used to obtain the estimated Omaha prices.

Price Index

The price index used to deflate the price variables in our models was an index of prices paid by farmers for commodities, services, interest, taxes and wage rates. This data series was obtained either from Agricultural Prices or directly from the Economics, Statistics and Cooperatives Service, a branch of the U.S. Department of Agriculture. No problems were encountered in constructing this series.

The remainder of this appendix consists of tables and figures displaying the data used to estimate our models.

TABLE A-1
DATA ON VARIABLES USED IN FINAL MODELS

Year	Total Cattle Numbers	Breeding Herds	Hay Production	Annual Feeder Prices	Montana Calf Prices	Choice Beef Steer Prices	Price of Corn	Price Index
1945	1,925,000	962,000	2,681,000	13.07	12.24	16.15	1.07	.56
1946	1,925,000	941,000	2,395,000	15.87	15.19	19.14	1.38	.61
1947	1,829,000	916,000	2,441,000	20.87	20.09	29.73	1.65	.70
1948	1,847,000	932,000	2,704,000	25.54	22.11	34.41	2.34	.76
1949	1,939,000	973,000	2,151,000	21.34	20.83	27.70	1.30	.73
1950	1,726,000	963,000	2,730,000	26.67	28.16	28.88	1.21	.75
1951	1,881,000	1,060,000	2,399,000	32.61	33.83	34.92	1.59	.82
1952	2,064,000	1,163,000	2,804,000	25.55	21.85	32.37	1.77	.84
1953	2,152,000	1,240,000	3,114,000	17.35	14.63	22.77	1.53	.81
1954	2,303,000	1,291,000	2,921,000	18.97	17.34	23.45	1.46	.85
1955	2,395,000	1,353,000	3,054,000	18.60	17.29	22.16	1.46	.81
1956	2,443,000	1,390,000	2,695,000	17.37	16.76	20.99	1.38	.81
1957	2,273,000	1,272,000	3,050,000	20.33	22.06	22.61	1.30	.84
1958	2,174,000	1,258,000	3,042,000	25.56	30.20	26.39	1.12	.86
1959	2,201,000	1,291,000	2,985,000	25.61	26.41	26.93	1.09	.87
1960	2,215,000	1,318,000	2,990,000	22.43	24.09	25.18	1.07	.88
1961	2,155,000	1,334,000	2,328,000	23.10	26.24	23.79	1.01	.88
1962	2,113,000	1,352,000	3,348,000	24.53	29.84	26.45	1.03	.90
1963	2,325,000	1,452,000	3,600,000	22.45	25.97	23.21	1.13	.91
1964	2,627,000	1,596,000	3,380,000	19.79	21.72	22.21	1.17	.92
1965	2,756,000	1,713,000	3,821,000	22.50	24.88	25.12	1.26	.94
1966	2,841,000	1,730,000	3,287,000	25.41	27.88	25.69	1.29	.99
1967	2,862,000	1,787,000	3,864,000	24.67	28.31	25.27	1.30	1.00
1968	2,984,000	1,839,000	3,603,000	25.89	29.32	26.83	1.14	1.03
1969	2,984,000	1,839,000	3,854,000	29.30	32.93	29.66	1.20	1.08
1970	3,014,000	1,893,000	4,081,000	30.15	35.38	29.34	1.28	1.12
1971	3,165,000	1,944,000	3,872,000	32.09	36.57	32.42	1.44	1.18
1972	3,165,000	2,034,000	4,227,000	38.89	49.24	35.83	1.23	1.25
1973	3,187,000	2,073,000	4,012,000	49.13	58.72	43.89	1.80	1.44
1974	3,380,000	2,204,000	4,047,000	36.49	27.27	41.82	2.79	1.64
1975	3,340,000	2,132,000	4,248,000	33.42	31.37	39.76	3.05	1.80
1976	3,150,000	1,989,000	3,978,000	37.65	35.05	39.11	2.66	1.91
1977	2,980,000	1,892,000	3,907,000	38.74	41.96	40.38	2.15	2.02
1978	2,680,000	1,720,000	4,576,000	56.39	73.17	52.34	2.69	2.19
1979	2,607,000	1,801,000	4,291,000	NA*	89.81	67.67	2.28	2.50
1980	2,645,000	1,817,000	NA*	NA*	NA*	NA*	NA*	NA*

* Data was not available at the time the final runs were made.

TABLE A-2

STATISTICS ON TRANSFORMED VARIABLES IN THE FINAL MODELS*

VARIABLE	RANGE		MEAN	STANDARD DEVIATION
	HIGH	LOW		
Total Cattle Numbers	3,380,000	1,726,000	2,605,700	463,900
Breeding Herds	2,209,000	963,000	1,578,000	360,650
Hay Production	4,576,000	2,151,000	3,387,500	626,280
Beef to Corn Price Ratio	29.130	14.626	20.453	4.0614
Montana Calf Prices	41.26	16.63	28.12	6.75
Kansas City Feeder Prices	39.79	18.57	26.18	4.84

*Price variables have been deflated.

Number of Cattle
(Millions)

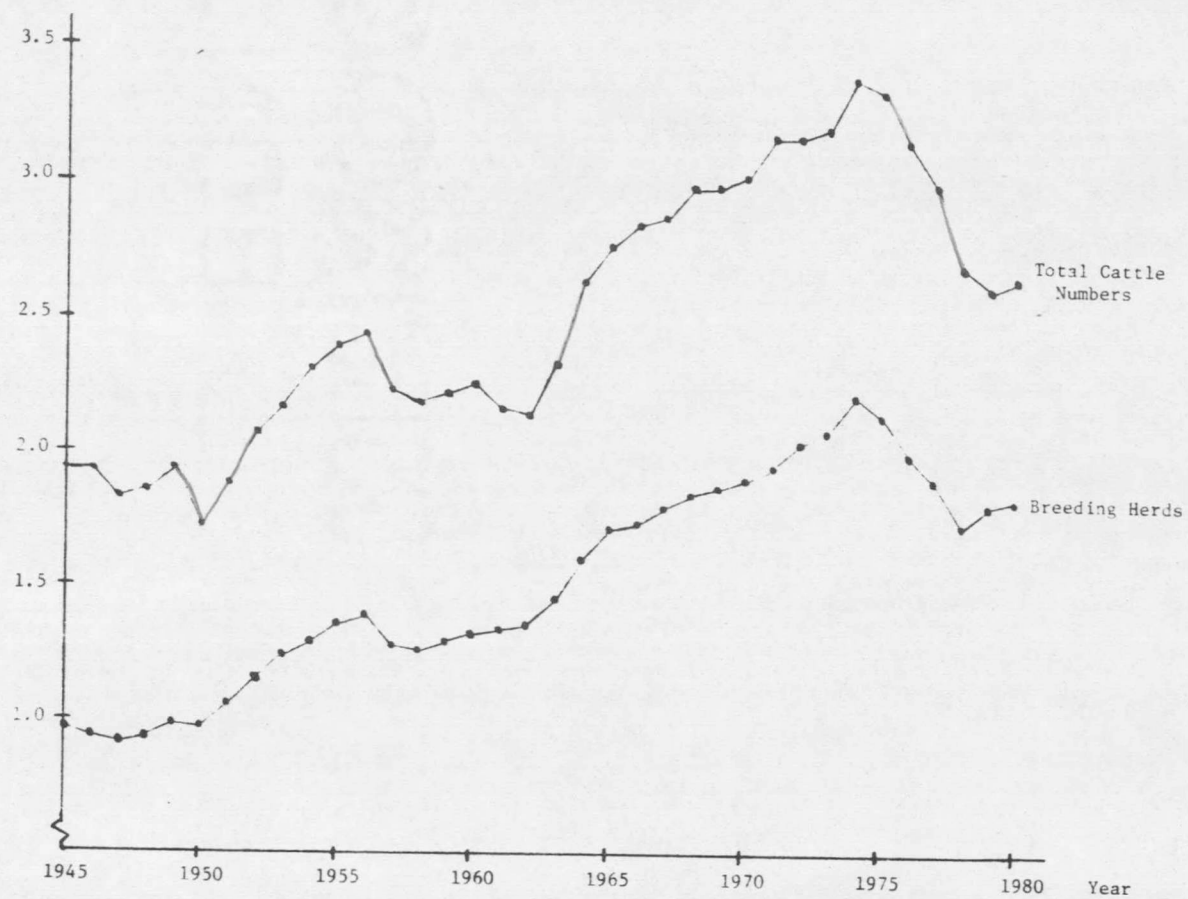


FIGURE A-1. MONTANA CATTLE HERD SIZES, 1945-1980

Hay Production
(Millions of tons)

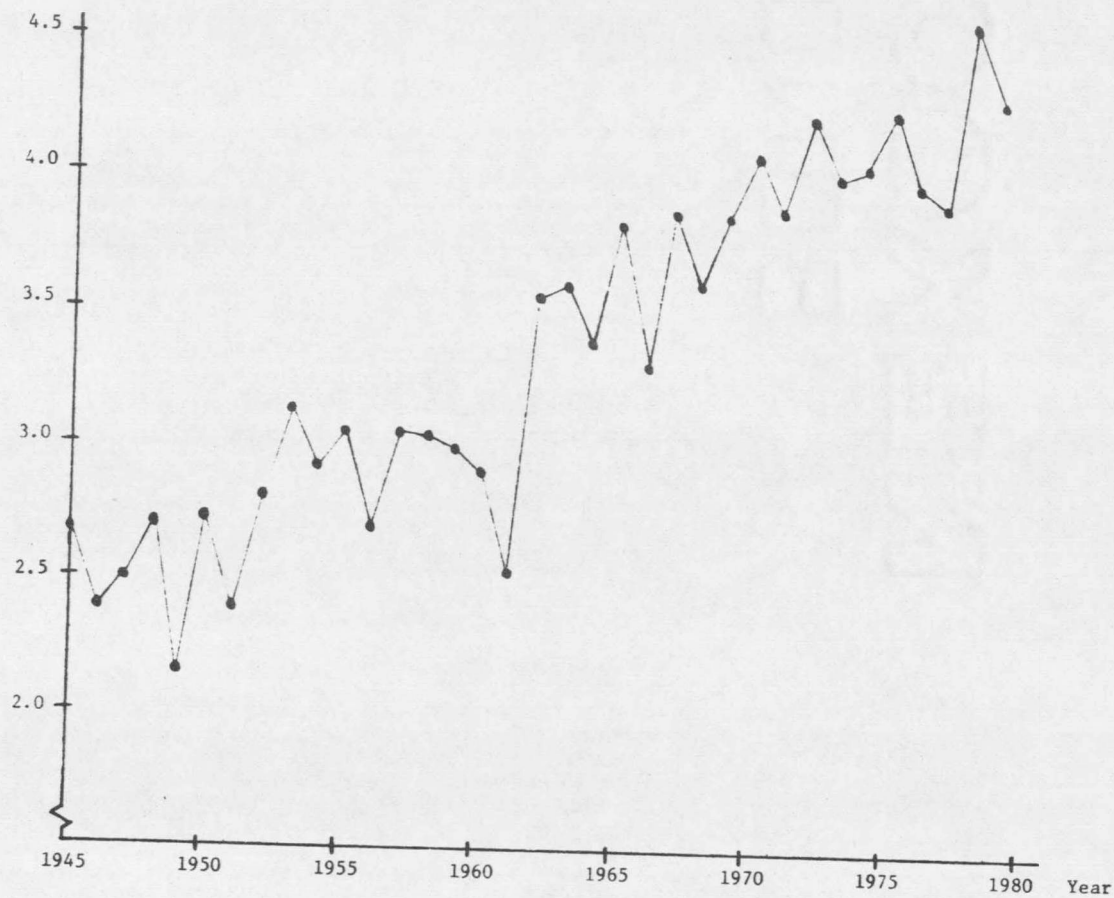


FIGURE A-2. MONTANA HAY PRODUCTION, 1945-1979

Beef to Corn
Price Ratio

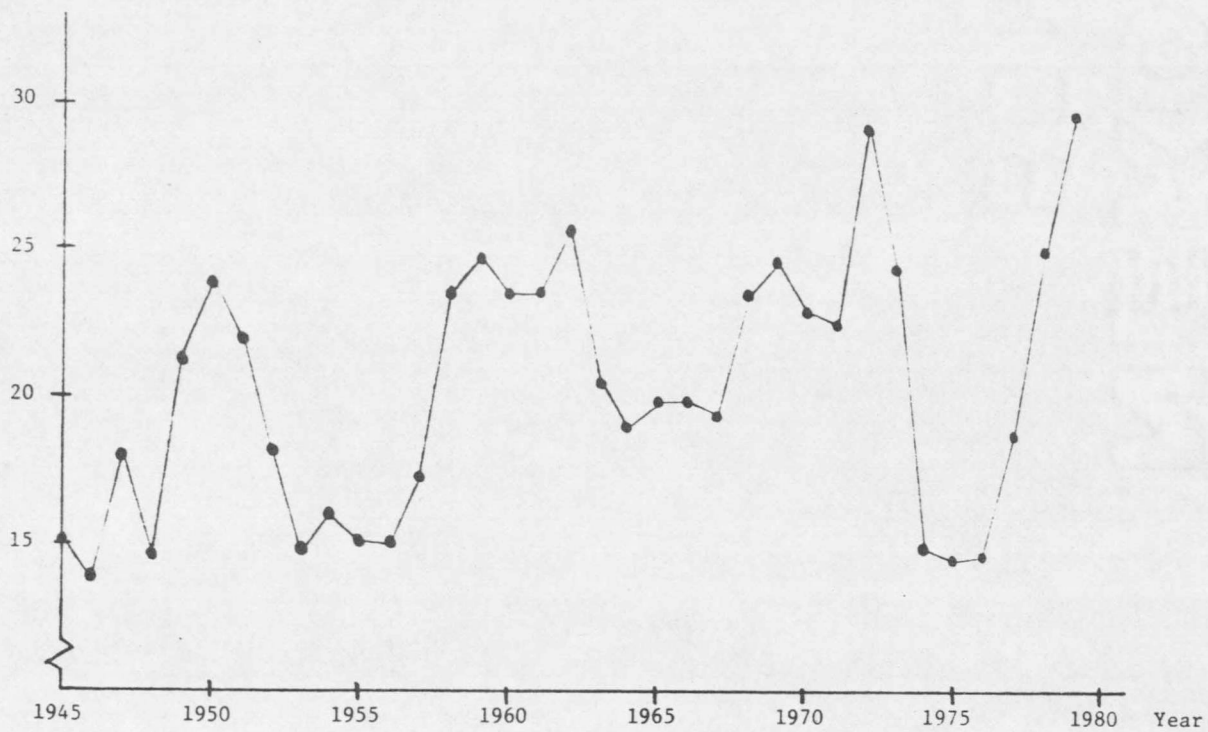


FIGURE A-3. RATIO OF THE PRICE OF CHOICE BEEF STEERS TO THE PRICE OF CORN, 1945-1979

Dollars Per
Hundred Weight

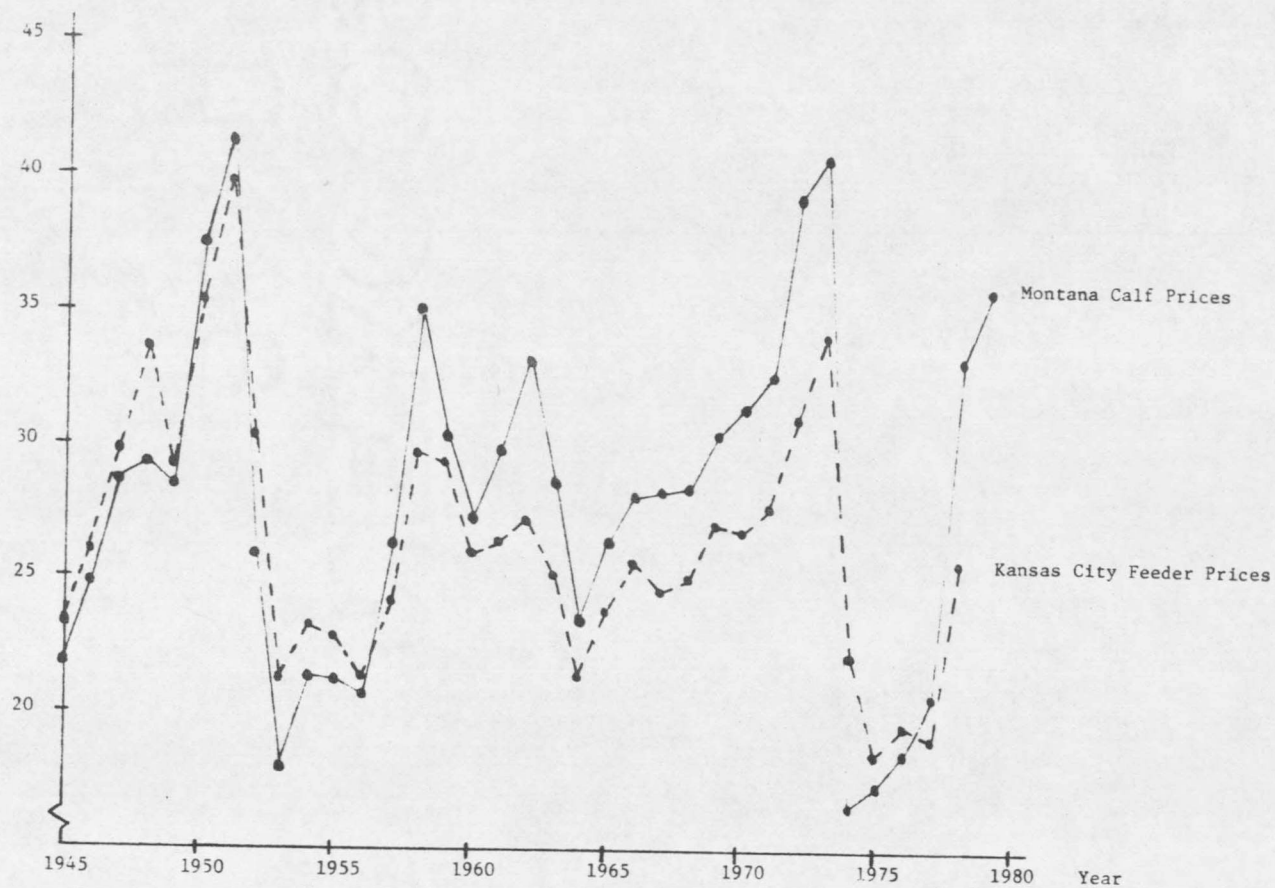


FIGURE A-4. DEFLATED MONTANA CALF AND KANSAS CITY FEEDER PRICES,
1945-1980

APPENDIX B

Estimated Distributed Lag Patterns

This appendix contains plots of the estimated distributed lag patterns on the explanatory variables in the final models. The numbers on the horizontal axis, when multiplied by ten, represent the number of years that have passed since the occurrence of an exogenous change in the level of the relevant independent variable. For example, "2" indicates 20 years, "4" indicates 40 years, and so forth.

These lag patterns can be interpreted as "normalized" lag patterns, in the sense that the partial effects of the changes in the variables in each year have been divided by the total effect (the summation of all the partial effects over an infinite period) of a change in that variable on the dependent variable. For example, suppose the partial effect in year 10 of a \$1 change in calf prices (in year 0) is 70 head of cattle and that the total effect on herd sizes of this change in calf prices is 1400 head. Then the height of the normalized lag distribution in year 10 would be $\frac{70}{1400} = .05$ (to read the graphs in this way, the numbers on the vertical axes must be multiplied by .1). If the lag pattern in a given model is restricted to nonnegative values, then the normalized lag distribution can be interpreted in a manner analogous to a probability density function in statistical analysis. This is a useful way of obtaining an intuitive interpretation of the graphs in this appendix (keeping

in mind that the analogy is not complete when the lag pattern is allowed to assume negative values, since probabilities are nonnegative).

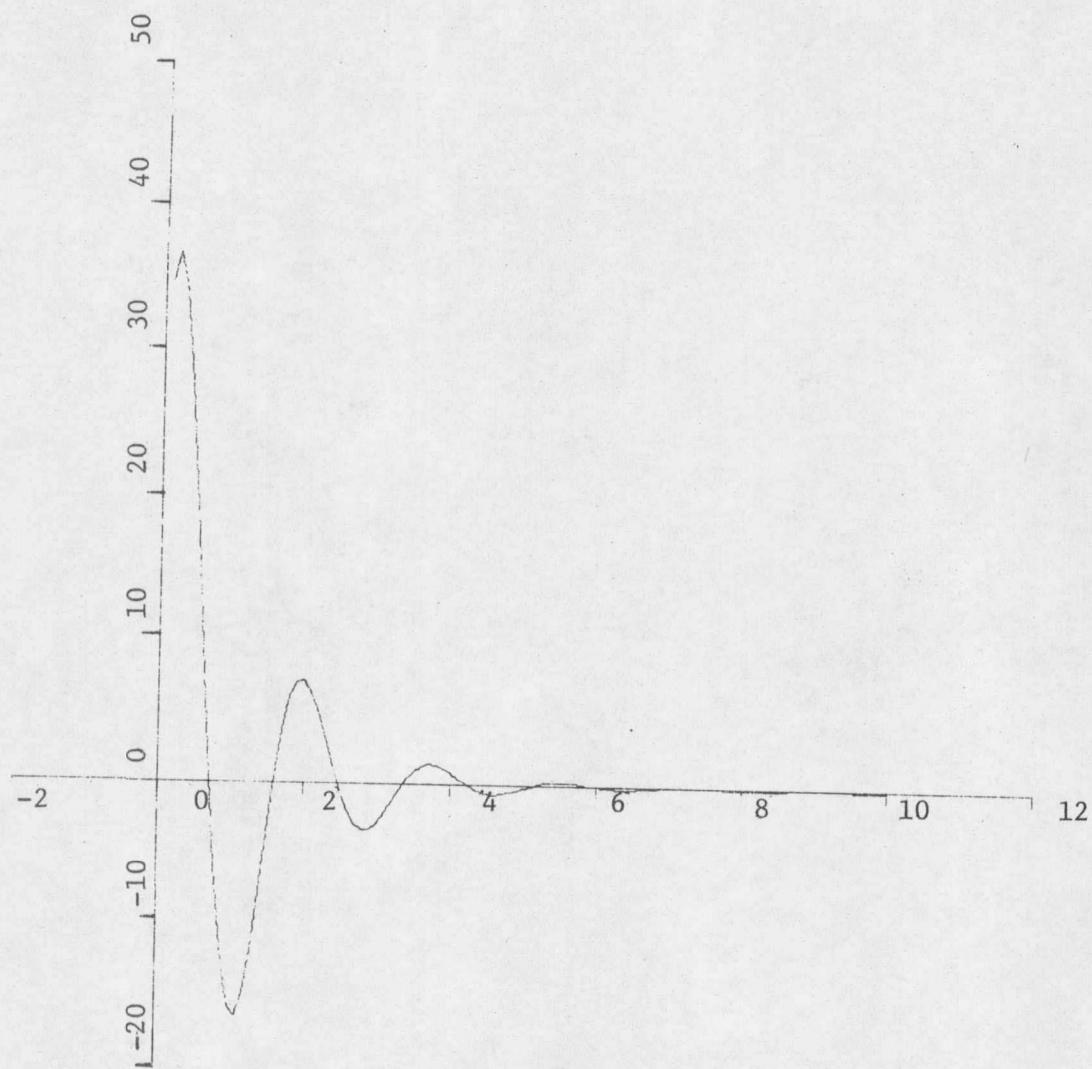


FIGURE B-1. ESTIMATED DISTRIBUTED LAG PATTERN ON HAY PRODUCTION:
MODEL BM

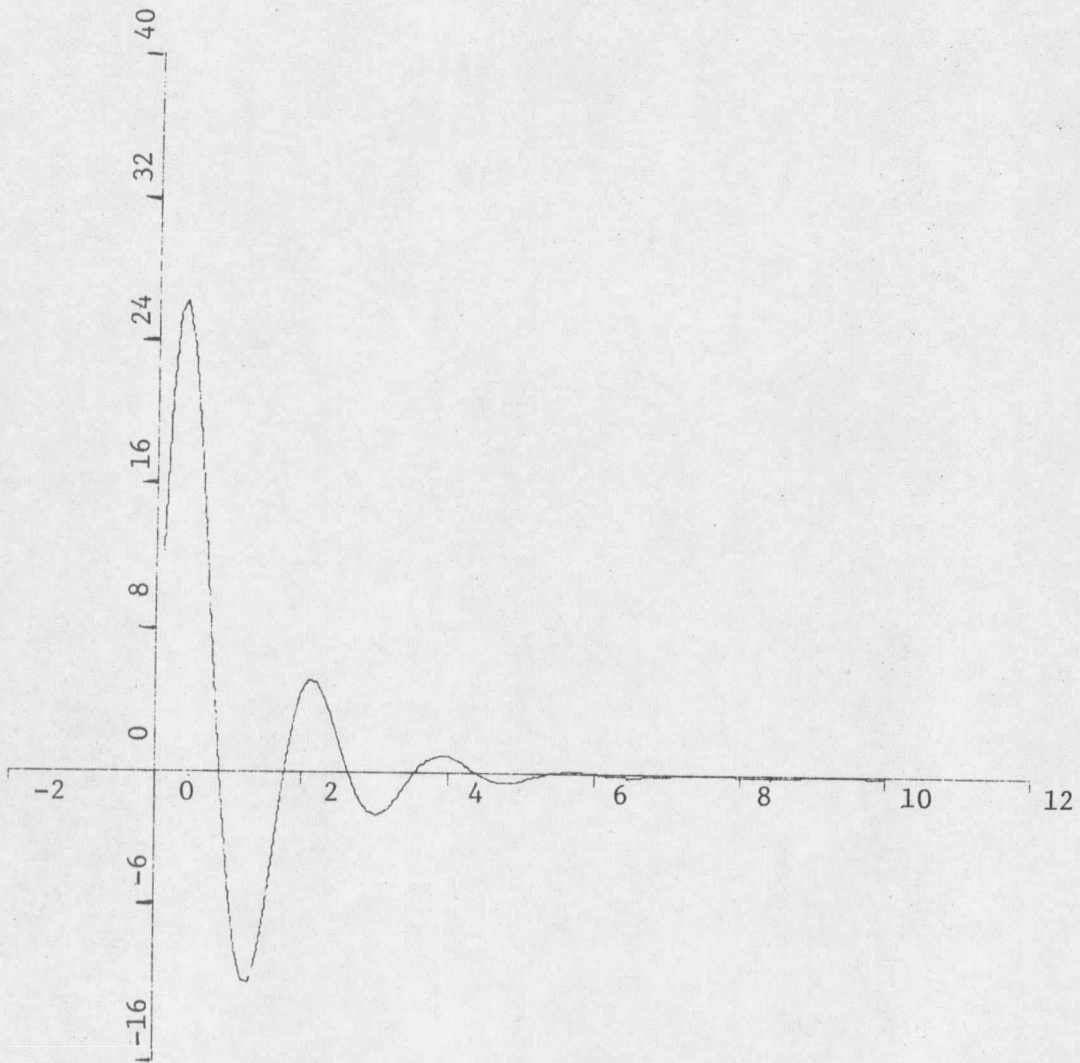


FIGURE B-2. ESTIMATED DISTRIBUTED LAG PATTERN ON BEEF TO CORN PRICE RATIO: MODEL BM

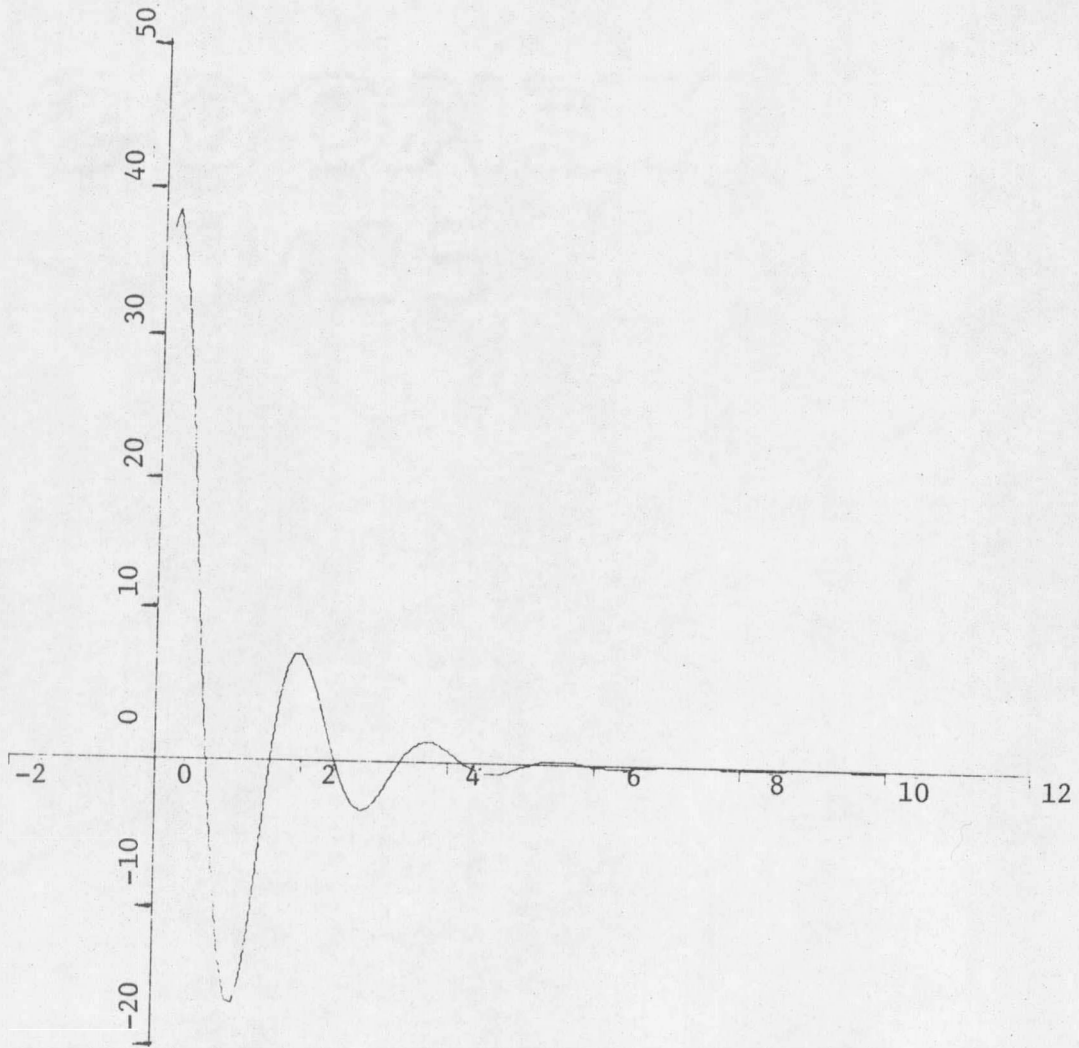


FIGURE B-3. ESTIMATED DISTRIBUTED LAG PATTERN ON MONTANA CALF PRICES:
MODEL BM

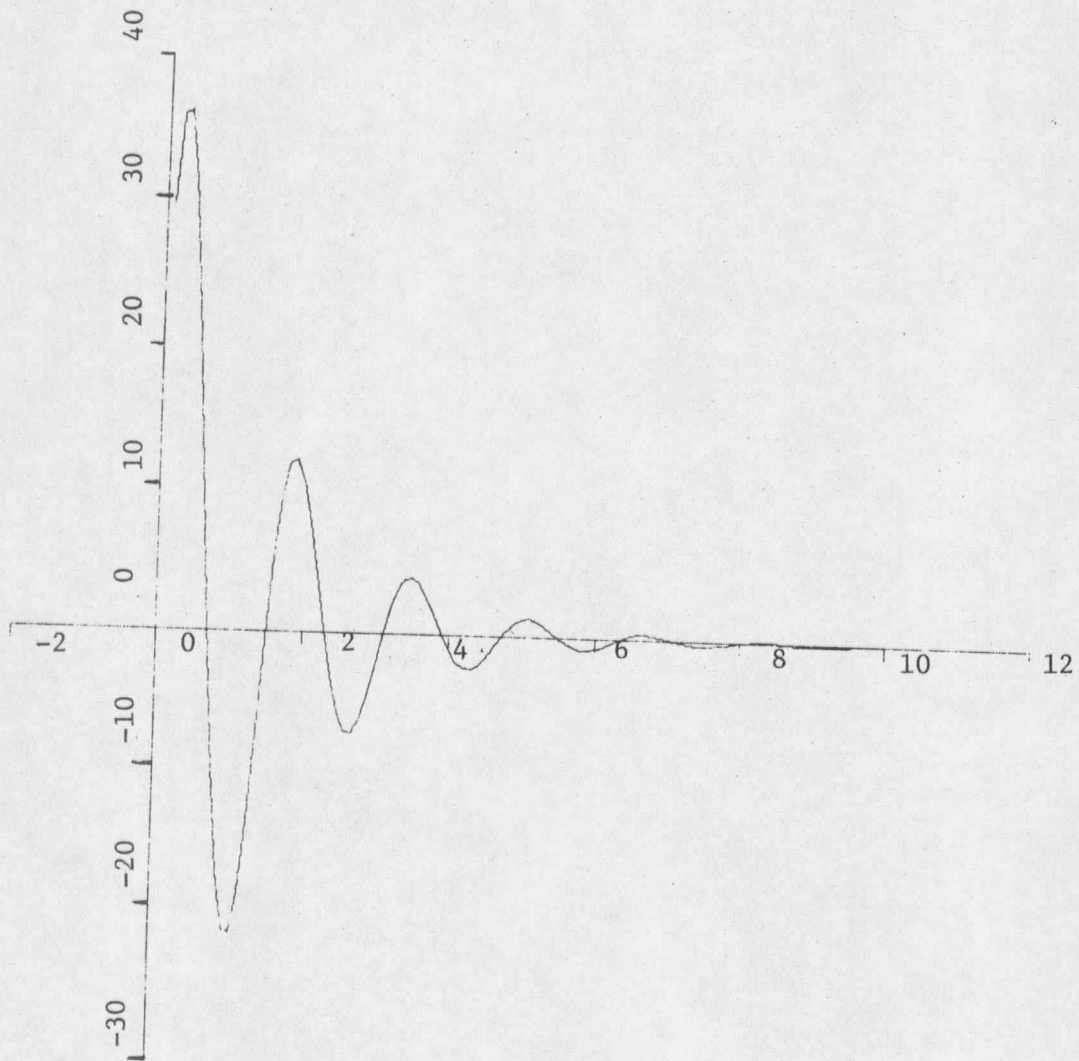


FIGURE B-4. ESTIMATED DISTRIBUTED LAG PATTERN ON HAY PRODUCTION:
MODEL BA

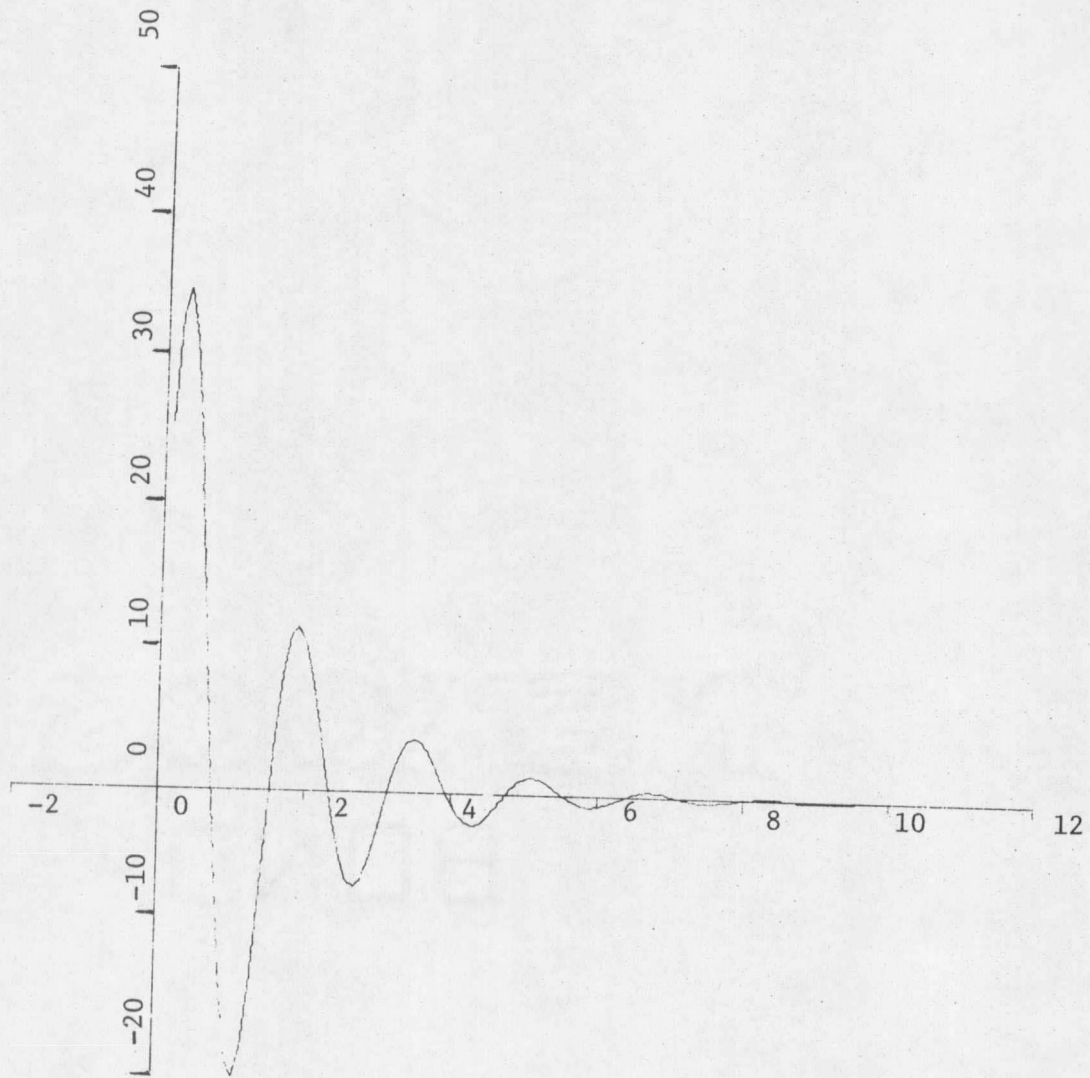


FIGURE B-5. ESTIMATED DISTRIBUTED LAG PATTERN ON BEEF TO CORN PRICE RATIO: MODEL BA

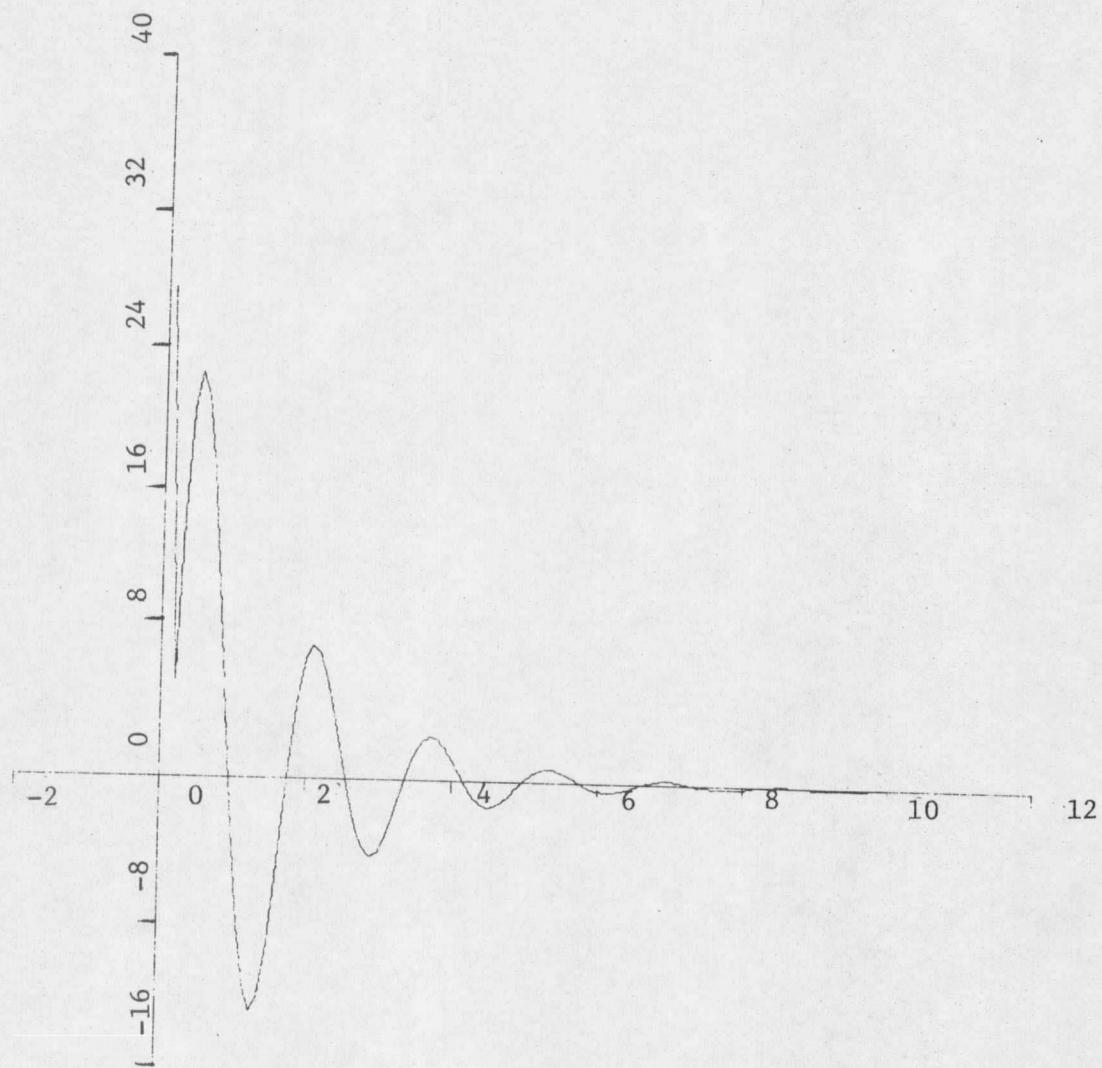


FIGURE B-6. ESTIMATED DISTRIBUTED LAG PATTERN ON KANSAS CITY FEEDER PRICE: MODEL BA

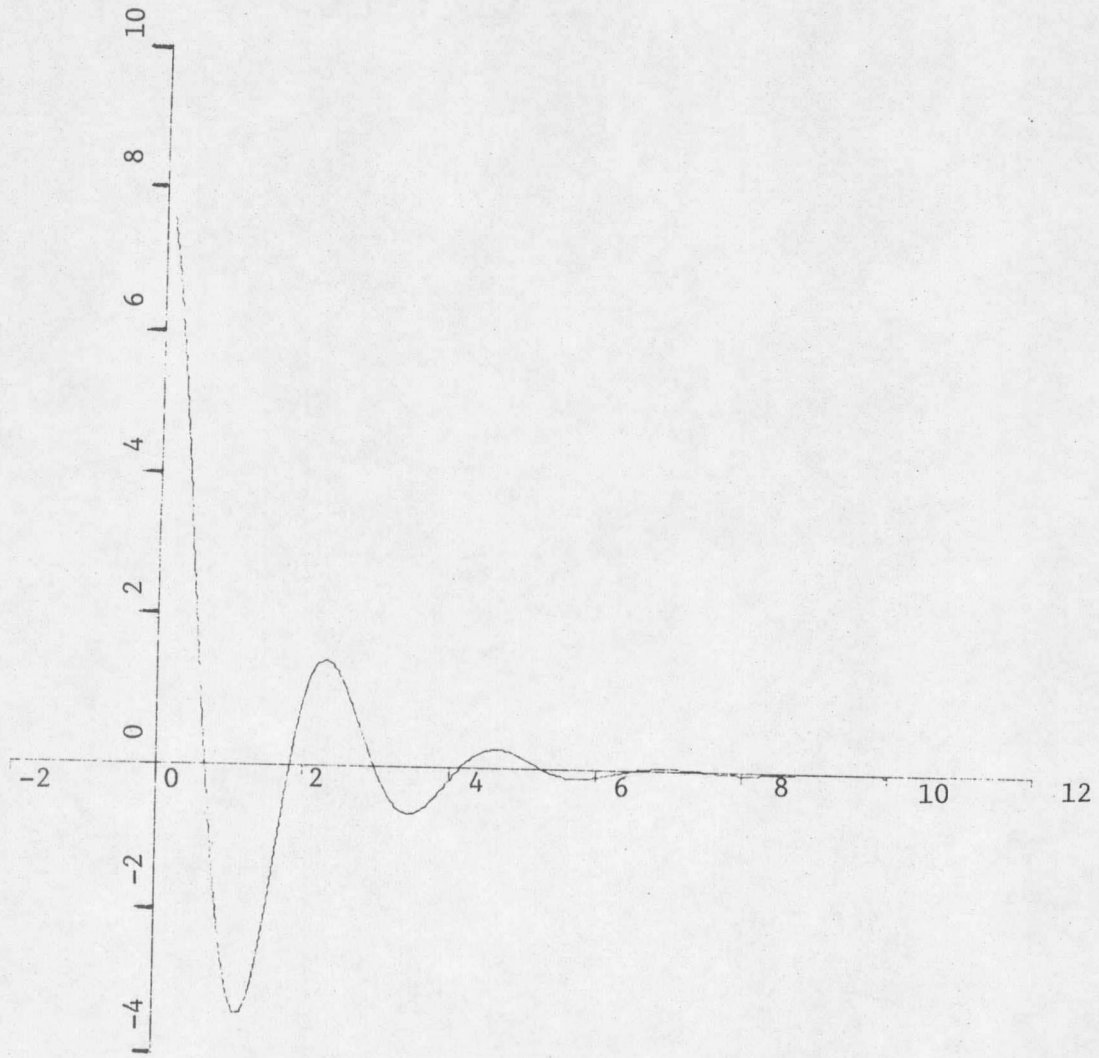


FIGURE B-7. ESTIMATED DISTRIBUTED LAG PATTERN ON HAY PRODUCTION:
MODEL TM

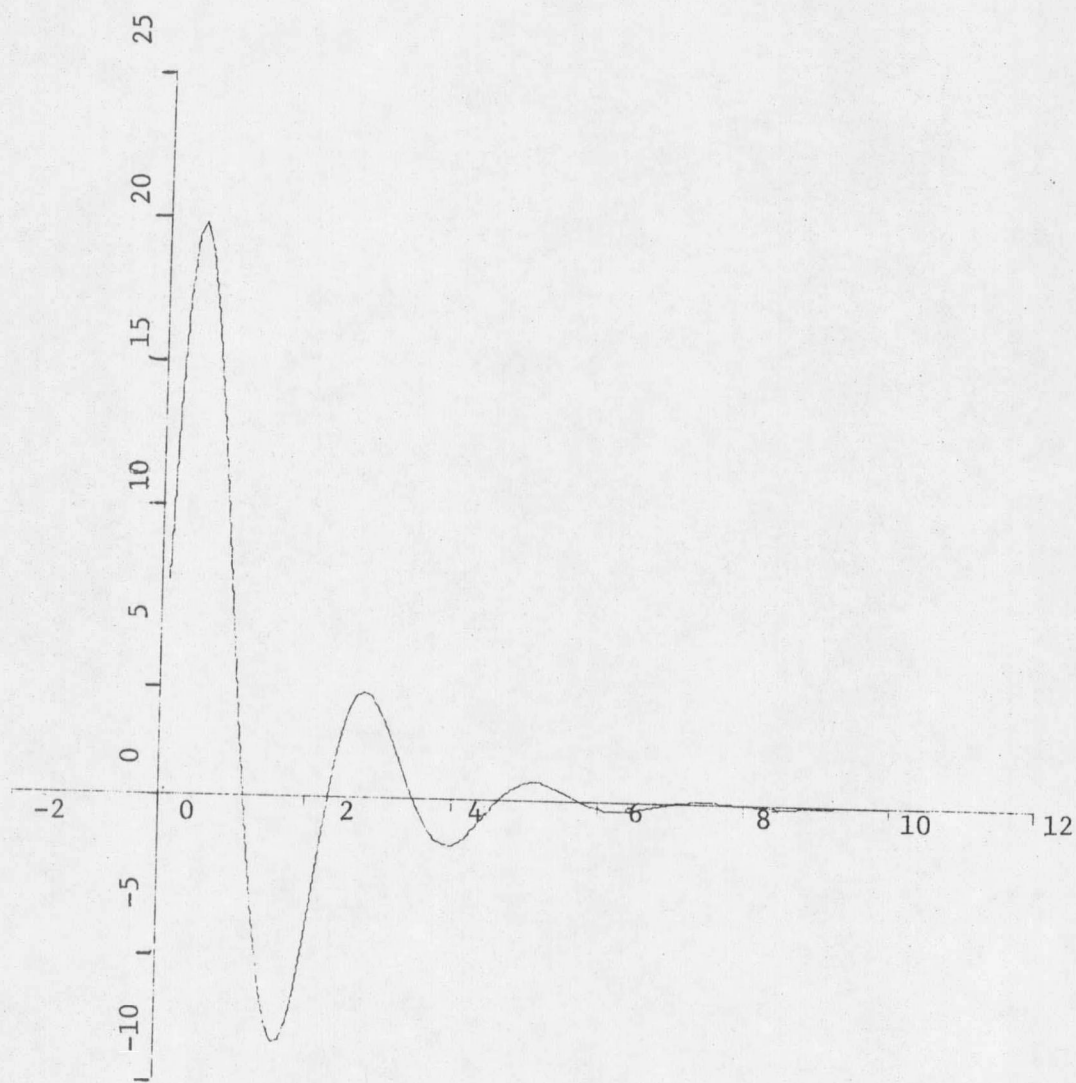


FIGURE B-8. ESTIMATED DISTRIBUTED LAG PATTERN ON BEEF TO CORN PRICE RATIO: MODEL TM

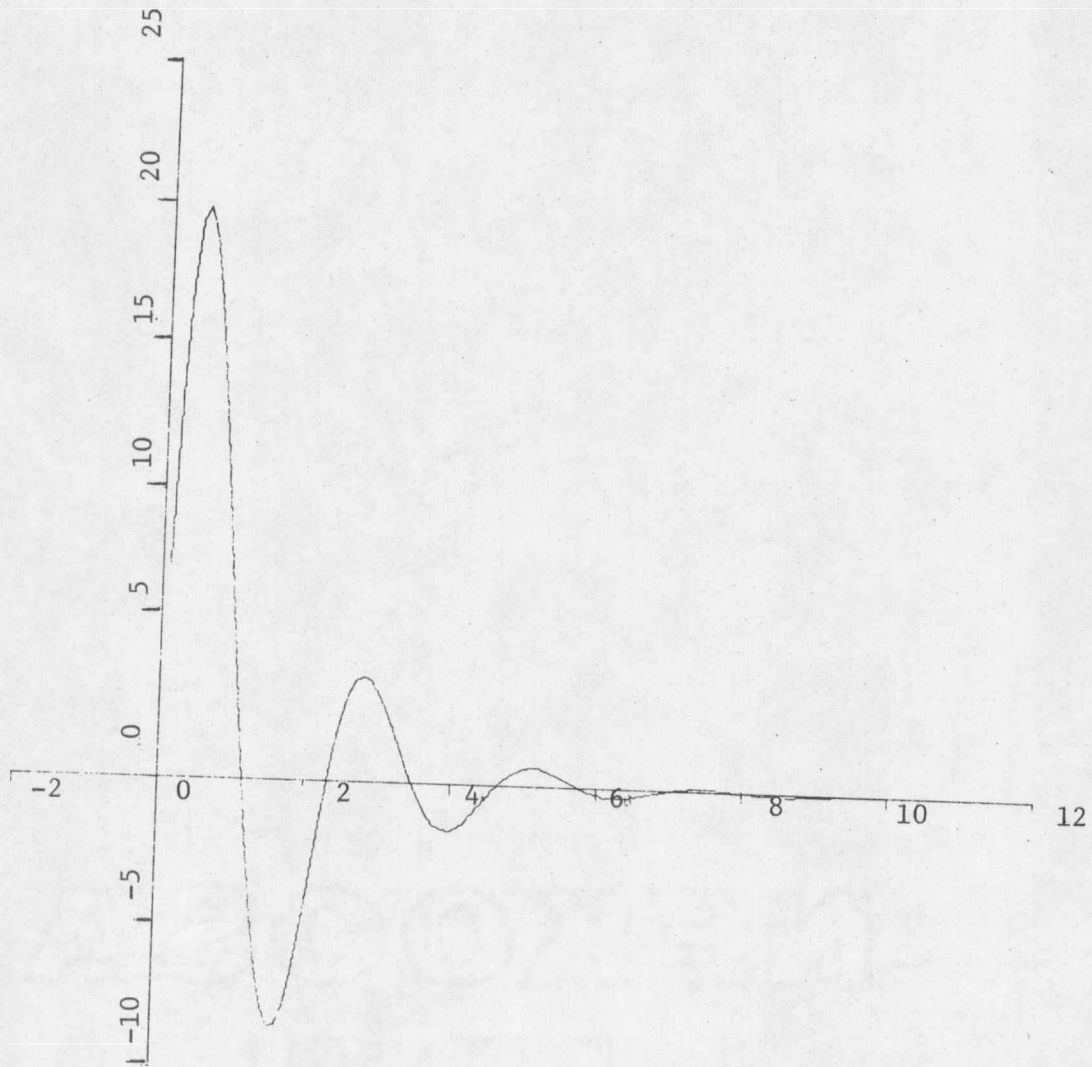


FIGURE B-9. ESTIMATED DISTRIBUTED LAG PATTERN ON MONTANA CALF PRICES:
MODEL TM

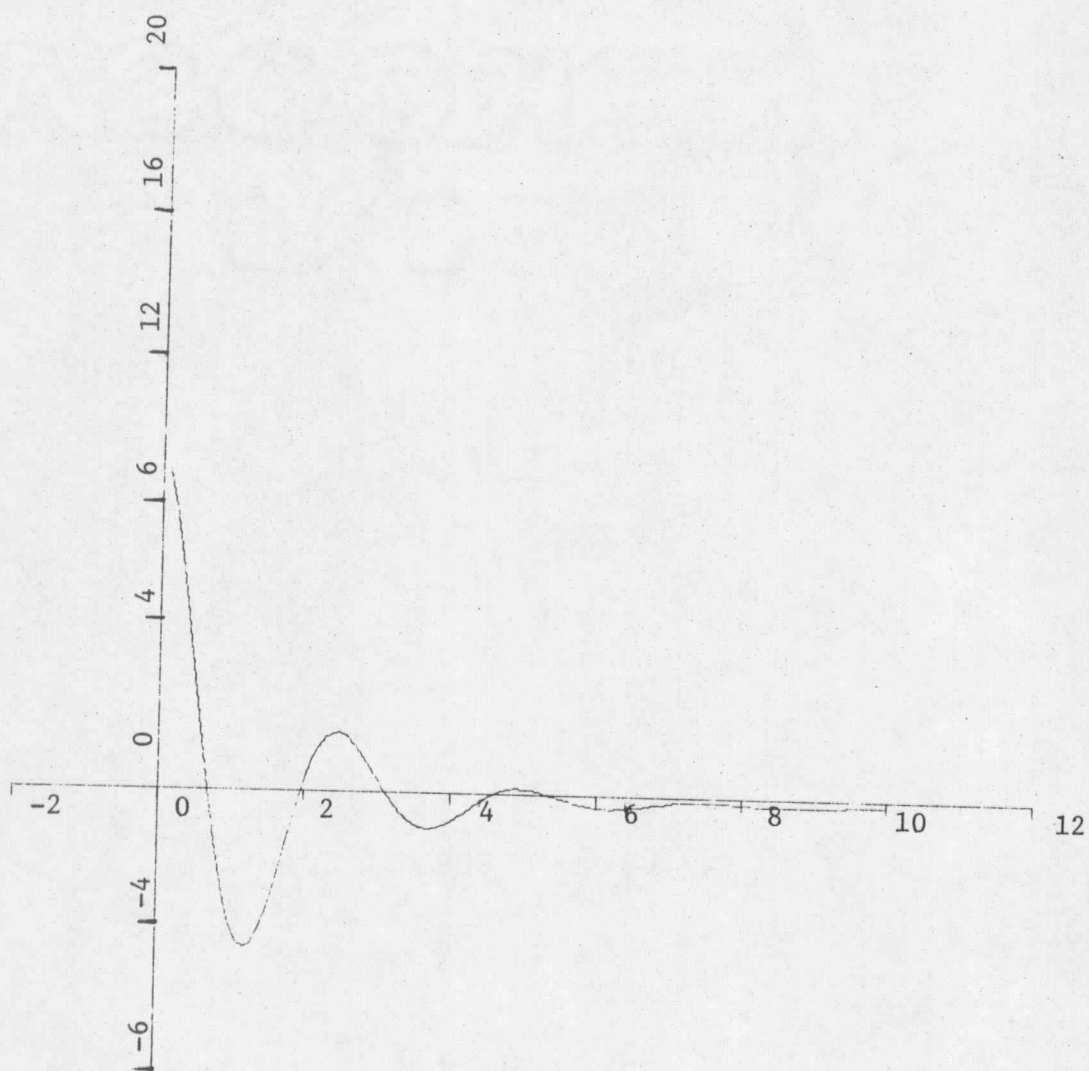


FIGURE B-10. ESTIMATED DISTRIBUTED LAG PATTERN ON HAY PRODUCTION:
MODEL TA

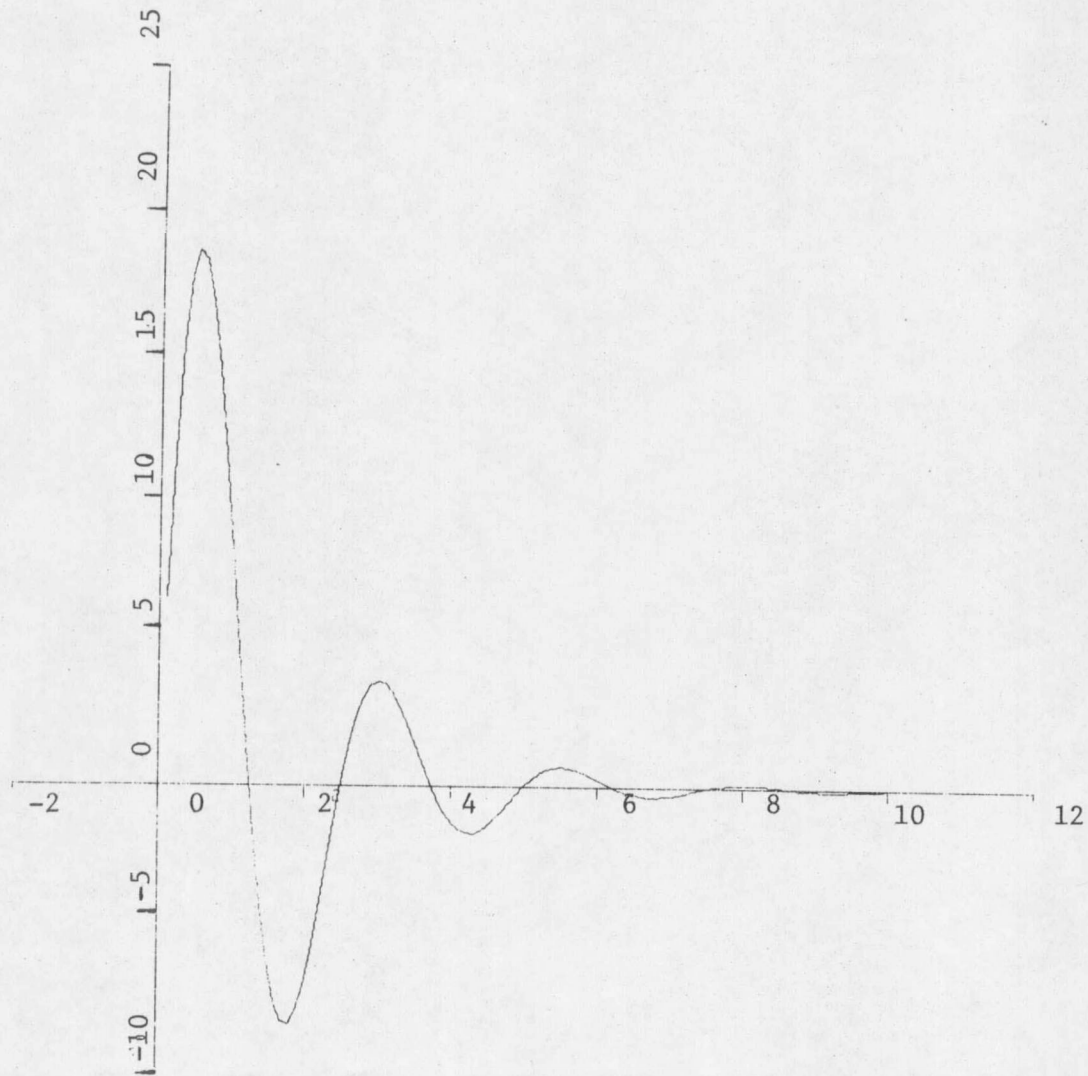


FIGURE B-11. ESTIMATED DISTRIBUTED LAG PATTERN ON BEEF TO CORN PRICE RATIO: MODEL TA

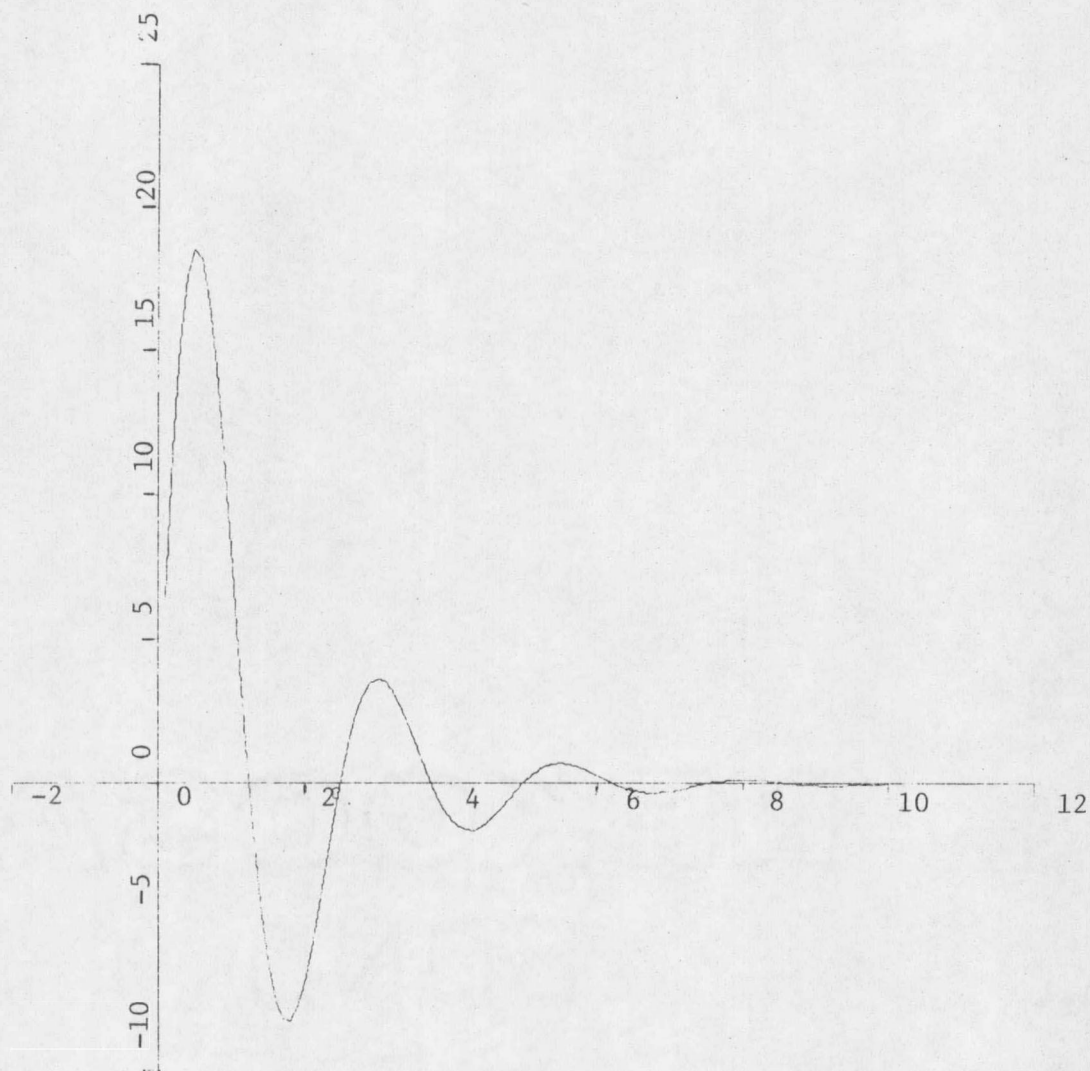


FIGURE B-12. ESTIMATED DISTRIBUTED LAG PATTERN ON KANSAS CITY
FEEDER PRICES: MODEL TA

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