New techniques in the analysis of homologous binal networks
by George Hampton Grenier

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
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Abstract:
Noting the general use of ever-higher frequencies in more compact equipment and at lower signal levels, as typified in missile-borne systems or resulting from the use of microelectronics, the author has considered it desirable to develop new techniques for use in the analysis of networks which are subject to interference. A singularly important subset of these is characterized by a symmetry about some reference in the physical and/or electrical sense.

Towards this end, performance criteria have been defined relating to the symmetry of these so called balanced networks and these embody a new set of balance factors involving the network's susceptibility to common-mode interference, A new list of definitions is offered in an effort to remove some of the ambiguities caused by the specialized usage of generic terms as well as applications inconsistent with presently existing professional standards. The networks under study are actually double ones and made up of two similar parts here called binary parts or biparts: the two biparts comprise the binal network or binet.

The concept of common-mode interference is enlarged to include paths of influx into all homologous node-pairs of a binet, to be driven through equivalent source impedances chosen to represent either the actual selfimpedances of an interfering wave, or variously, from worst to best case combinations of these.

An indefinite admittance matrix is used to simplify analyses of the resulting test-circuit configurations. A new method of simplifying pertinent cofactors has reduced calculations in a (2n)-node binet by a factor up to (2n-l)!/(n-1)! The simplification procedure is essentially dependent on the homology of the binet rather than its reciprocity, so is equally applicable to active networks. A new fault polynomial is proposed as an efficient mechanism to allow the choice of any order of simplifying approximation to be based on expected values of unbalance and the network parameters,, A new fault-minor expansion is offered as an alternative expression of the fault properties of the binet.

The proposed methods of analysis are particularly useful for calculating exact expressions representing the common-mode susceptibilities of large binets with relatively few unbalanced components involved at one time but with no limit to the magnitude of unbalance to be considered.

Those aspects of synthesis implicit in this approach to binets, are surveyed and objectives for future related studies are delineated.

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LIST OF SYMBOLS

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V_i	Voltage parameter	7
Z_{ij}	Impedance interconnecting terminals i and j	7
N	Number of independent nodal equations	6
J	Number of nodes	8
S	Number of separate parts in the network	8
s	The generalized frequency variable	10
E_o	Ideal voltage source parameter	11
$Z_5(s)$	Effective impedance of source, a function of the generalized frequency variable s	11 a
$\mathcal{E}(s)$	Electric field intensity, a function of s	12
$\mathcal{H}(s)$	Magnetic field intensity, a function of s	12
CMRF	Common mode rejection factor (See page 94)	13
K	Voltage amplification, a ratio of voltages	13
DF	Discrimination factor	14
L	Common mode to differential mode loss; a ratio of powers	16
G	Power gain of a network; a ratio of powers	16
RC	Abbreviation for the term "resistance-capacitance"	16
DM	Abbreviation for differential mode	16
CM	Abbreviation for common mode	17
RF	Abbreviation for radio frequency	19

y_{ij}	Element of admittance matrix in $i^{\underline{th}}$ row and $j^{\underline{th}}$ column position	25
Z_{jk}^{mn}	The open circuit transfer impedance between the $jk^{\underline{th}}$ and $mn^{\underline{th}}$ port	26
V_{mn}	Voltage drop from $m^{\underline{th}}$ to $n^{\underline{th}}$ node	26
I_{jk}	Current entering node j and leaving node k	26
$Y_{kl\ldots}^{ij\ldots}$	The signed minor of the indefinite admittance matrix Y with columns i, j (etc.) and rows k, l (etc.) removed	26
N_R	The total number of resistive elements in a network	28
N_S	The total number of resistive elements in equivalent network of the source	29
A	Voltage attenuation; a ratio of voltages	31
det	Abbreviation for determinant	32
A_1, A_2, A_3, A_4	Submatrices	33
Δ_{ij}	Unbalance admittance connected across nodes i and j	35
Y'	Y matrix including unbalanced values	36
$(Y_{CR}^{CR})'$	The signed minor of Y' with rows and columns C and R deleted	37
Y_Δ	The *leading* cofactor of the fault minor expansion, the totality of which includes *all* unbalance admittances and some of the binet's admittances	39
VCT	Abbreviation for voltage current transactor	43
$y(jk/mn)$	The ideal active element admittance in which current	

ABSTRACT

Noting the general use of ever-higher frequencies in more compact equipment and at lower signal levels, as typified in missile-borne systems or resulting from the use of microelectronics, the author has considered it desirable to develop new techniques for use in the analysis of networks which are subject to interference. A singularly important subset of these is characterized by a symmetry about some reference in the physical and/or electrical sense.

Towards this end, performance criteria have been defined relating to the symmetry of these so called *balanced networks* and these embody a new set of *balance factors* involving the network's susceptibility to *common-mode interference*. A *new* list of definitions is offered in an effort to remove some of the ambiguities caused by the specialized usage of generic terms as well as applications inconsistent with presently existing professional standards. The networks under study are actually double ones and made up of two similar parts here called *binary parts* or *biparts*: the two biparts comprise the *binal network* or *binet*.

The concept of common-mode interference is enlarged to include paths of influx into *all* homologous node-pairs of a binet, to be driven through equivalent source impedances chosen to represent either the actual self-impedances of an interfering wave, or variously, from worst to best case combinations of these.

An indefinite admittance matrix is used to simplify analyses of the resulting test-circuit configurations. A *new* method of simplifying pertinent cofactors has reduced calculations in a (2n)-node binet by a factor up to $(2n-1)!/(n-1)!$ The simplification procedure is essentially dependent on the homology of the binet rather than its reciprocity, so is equally applicable to active networks. A *new* fault polynomial is proposed as an efficient mechanism to allow the choice of any order of simplifying approximation to be based on expected values of unbalance and the network parameters. A *new* fault-minor expansion is offered as an alternative expression of the fault properties of the binet.

The proposed methods of analysis are particularly useful for calculating *exact* expressions representing the common-mode susceptibilities of *large* binets with relatively *few* unbalanced components involved at one time but with no limit to the *magnitude* of unbalance to be considered.

Those aspects of synthesis implicit in this approach to binets, are surveyed and objectives for future related studies are delineated.

I. A STATEMENT OF THE PROBLEM

1.0 Introduction. During the design and initial use of new electronic equipment, interference and noise in the signal or data portions of the electric circuits often cause some degradation of performance. Various government agencies, equipment manufacturers, and utility companies have published techniques for the measurement of interference in equipment and to a lesser extent the measurement of an equipment's susceptibility to the presence of such interference. Some of these are included in the literature cited in this chapter and listed at the end of the paper. It is to be expected that many of the susceptibility tests are peculiar to the specific kind of equipment under test. The first five on the list relate to a particular equipment's susceptibility to common mode (CM) interference. It is this latter kind which is of special interest here.

Interference waves entering electronic equipment may be induced by one or more of several mechanisms: Direct radiation from electric or magnetic fields, metallic conduction across common parasitic impedances such as in signal-ground networks, electromagnetic effects on control, input or output leads connected to the equipment or inadvertently, through an input transducer. The parasitic parameters involved are sometimes part of an intrasystem anamoly.

If the length of connecting wire leads is over a few feet and in an environment or at a signal level thought to require shielding in the first place, it is really not clear whether direct radiation, common coupling impedances or wire-conducted influx is more important. This particular problem is compounded by the fact that the addition of any kind of shields usually changes the ground-current configuration: so that interference in all three types of influx is simultaneously so modified. Still another complicating factor lies in the physical evidence that a degree of common-mode interference can exist across an asymmetrical (single-ended) input and indeed, may be present on more than just two conductors.

A significant part of this important problem lies in the fact that usually none of these paths of influx are readily measurable in an absolute sense. It appears to follow that for most practical configurations, conducted influx would be subject to easier measurement and that an attempt should be made to make these particular measurements more meaningful.

Suprisingly little work has been done in developing special analytical techniques for efficiently handling such problems. It is to be hoped that the concepts developed here would not only simplify the calculation of interference susceptibility, but would also lead to a new insight into the reduction of the susceptibility of networks to such kinds of interference.

1.1 THE "BALANCED" CIRCUIT

From the literature one might suspect that there are relatively few types of electrical circuits for which common mode interference has been known to be a serious problem. A singularly important catagory comes under the general heading of the "balanced" circuit.

Electronic circuits are often specially designed to be symmetrical with respect to a ground or reference terminal. In other words, one half the circuit is made to be the mirror image of the other. The literature has referred to these as balanced circuits and they are used to gain special advantages: such as those ordinarily realized in a push-pull arrangement - or to prevent interference as on low-level transducer leads or on telephone circuits. They are also used in broadcast transmission for the added reliability afforded.

Usually such configurations are easily recognizable - the term "balance" referring to like physical components symmetrically located about a center line typifying ground, or a reference terminal. Some consideration will show, however, that this particular concept of "balance" is rather limited and perhaps unrealistic in more ways than one. Clearly, each of the identical halves can be described by the same transfer function, say F(s), which can be realized by an infinitude of circuit configurations. Furthermore, due to inevitable, small, differences in the parasitic or distributed parameters involved in inter-connecting actual circuit components, as well as in-tolerance differences in the actual values of homologous components, good balance at all frequencies is often very difficult to attain. As a matter of fact, imbalance can be very large at (usually) higher frequencies so that the normal mode signal on just one side of the "balanced" circuit may be so attenuated as to approach that of the reference terminal.

On the other hand, at a particular port, an actual circuit designed to be asymmetrical or single-ended may appear to be "balanced" at one or more frequencies, because the port terminals can sustain identical voltages when driven by equivalent (effectively, low-) impedance sources. From a practical standpoint, therefore, the "balance" of a port or terminal-pair may refer to that port's ability to sustain the same voltage at discrete frequencies and with respect to two identical in-phase specified sources returned to a third reference terminal. Specifically involved of course is the common impedance of these sources compared with the values of the driving point impedances from each terminal to the reference terminal. The transfer characteristic is also subject to a similar but more involved definition of balance to be discussed later.

It is apparent that new terms are needed to distinguish between circuits so related. Longitudinal sections of network more naturally fall into the category of those portions formed by cutting off cascaded sections. On the other hand, transverse partitioning is not usually considered to be a cutting-off, but merely the result of identifying *two* parts symmetrically related to some physical line of reference. These parts will be called *binary parts* or *biparts* and the whole of such a network will be a *binal network* or a *binet*. If the network is designed to have equivalent elements symmetrically located about the reference, it will be called a *homologous binet*. If the network behaves electrically as a homologous network, this behavior will be referred to as *balance* and the network will be a *balanced binet* whether homologous or not.

Now an extremely important property of homologous binets is related to their ability to suppress or attenuate the effect of equivalent in-phase voltages applied between each terminal of any port and the chosen references:

This refers to the so-called "common mode rejection" of a circuit (See Definitions in Appendix) and is usually defined as a characteristic of a differential amplifier. When balance is considered in its broad sense, there is little reason even to limit the definition to homologous networks. (See Definitions.)

Telephone and utility people have long been aware of the problem of common mode interference on their relatively long lines (ex: Reference 1.1 a). Medical researchers making bioelectric measurements (exs: References 1.1 b, c, d) have introduced means of reducing the effects of common mode noise on the balanced input lines from their transducer instruments.

More recently, with the general use of ever higher frequencies in more compact equipment, it has been suggested that all wire-connected equipment be considered subject to possible common mode interference when installed in an unknown or otherwise questionable environment. (Reference 1.1 e)

It follows that an important insight into all such interference can be gained by the study of meaningful performance criteria related to the common-mode susceptibility of balanced circuits. An efficient means of analyzing such networks will contribute to optimum noise characteristics.

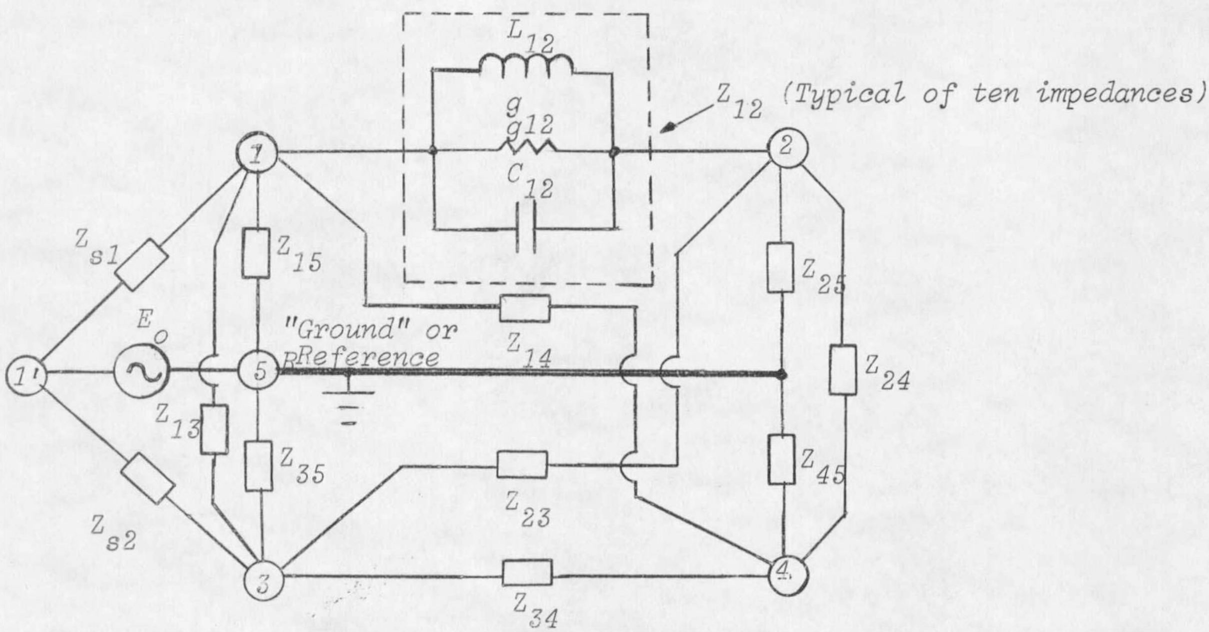
1.2 DISCUSSION AND DELINEATION

There have been substantial efforts to standardize test methods and to reduce ambiguity in the language used in describing interference problems.

(References 1.2 a - 1.2 i) Reference 1.2 a lists 102 references on the theory of grounding alone. In spite of this, it has been considered expedient to provide a list of definitions for possibly ambiguous terms used in this dissertation.

This immediately leads into a study of the various descriptors used in the analysis of homologous binets. Although R. D. Middlebrook's "Differential Amplifiers" (Reference 1.2 j) is applied largely to transistor d.c. amplifiers, the descriptors used therein and his Sequential Analysis method derived from LePage's bisection theorem (See References 1.2 k and 1.2 l) can readily be applied to most homologous binets. According to the above, practical circuits of interest in the interference problem can be subject to substantial values of imbalance. Unfortunately the Sequential Analysis method is difficult to apply when the imbalance is large.

In analyzing homologous binets, it appears that the toilsome duplication which prompted Middlebrook's initial efforts might also be obviated in the use of matrix descriptors, especially if computer time is to be saved. Then an immittance matrix can describe the balanced network configuration and the imbalance mechanisms can be represented by a *fault polynomial*. For example, assume the lumped, linear, finite passive bilateral network (LLFPB) of Figure 1.2 a illustrating any 5-terminal network, in which terminals 1 and 3 represent the input port and terminals 2 and 4 the output port with a fifth reference or ground node. It is at once clear that the (tested) network can be described by a 4th order node to datum admittance matrix since there will be a N=J-S=5-1=4 independant nodal equations. An *n* terminal network



$$\text{Balance Factor} = \frac{V_2 - V_4}{V_1' - V_5} = \frac{V_2 - V_4}{E_o}$$

E_o is applied through equivalent source impedances,

Z_{s1} and Z_{s2}.

Figure 1.2a A binet for analysis.

can be described by such a matrix, provided n-5 inactive nodes are suppressed by a convenient method, such as pivotal condensation. (Reference 1.2 m)

In Figure 1.2 a there are C_2^5 or 10 each of R L & C possible tested network components between all possible node pairs. For this homologous network operating in a balanced condition, only two nodal equations and seven each of R L & C components would actually be necessary. However computational difficulties arise when unbalance mechanisms require all four nodal equations and all ten of the components of R L & C.

The sequential method referenced above introduces an equivalent circuit for the normal mode as well as one for the common mode, each representable by a node to datum admittance matrix of order:

$$\left(\frac{J-1}{2} - S+1\right) = J' - S \qquad \text{Equation 1.2 a}$$

where S is the number of separate parts in the network. The unbalance of a component is accomodated by an added (shunt) current or (series) voltage generator depending on the imbalance mechanism.

Then the order of the admittance matrix will be increased by one since the equivalent circuits' effective number of nodes, J', will be increased by one. Now in solving the resultant matrix equation for say the node voltages, one is obliged to use currents under balanced conditions. The effort to get the currents in imbalance is substantial and it is evident that if these currents cannot be assumed equivalent, the method's simplicity is lost. This condition prevails when imbalances are not small with respect to respective component values.

Since *this* study will include *all* imbalances of whatever magnitude, a more advantageous set of analytical tools has been sought, using an indefinite admittance matrix. This approach should offer several advantages:

1. Initial identification of nodal-pairs and thus of the reference node is not necessary.
2. Matrix balance characteristics are maintained as much as possible.
3. Common mode nodes can be handled by merely adding admittance matrix elements while maintaining the equicofactor property.
4. The choice of reference node can be used to force the unbalance mechanisms onto the matrix diagonal for simplification purposes or for any other.

The latter two of these apparently involve new concepts and the usefulness of this equicofactor admittance matrix will be detailed in Chapter II, using a circuit similar to that of Figure 1.2 a.

1.3 TESTING CIRCUITS AND PERFORMANCE CRITERIA

As previously indicated, the selection of an effective testing configuration and measurement for susceptibility to common-mode interference is not a straightforward problem. The choice of source impedance to be used will determine to some extent the degree the test results will correlate with the actual interference. The actual "figures of merit" used to represent the balance of networks will determine the usefulness of the tests and the range of applicable networks. Furthermore, it is these "figures of merit" which will be optimized in future synthesis developments.

1.4 THE TESTING CIRCUIT SOURCE IMPEDANCE

The question of generator source impedance to be used as the common-mode generator (See Figure 1.4 a) actually raises many problems, some of which are presently without an exact, known solution.

In the introductory discussion, Section 1.0, it was suggested that interference influx by metallic conduction appearing as a common mode signal at two or more terminals of a circuit might result from common impedances (ex: ground straps in ground networks) from electromagnetic wave pickup by connecting leads (ex: induced voltages on transducer leads) and by the inadvertant application of a common mode voltage in the transducer. These concepts are illustrated in Figures 1.4 b and 1.4 c respectively. The equivalent common ground source illustrated in Figure 1.4 a and justified by the Substitution Theorem is shown with source impedance Z (s), a function of the generalized frequency variable s. Quasi-stationary conditions are therefore assumed thruout the ground network. Even with this simplification, however, the problem of specifying Z(s) is not straightforward. Although at low frequencies $Z(s) = R_g$ may be considered to be more troublesome from a practical standpoint (See note in Figure 1.4 b) inductive effects and ground network resonances are not uncommon at frequencies above 5 mc.

The equivalent induced voltage source as seen in Figure 1.4c would appear to be even more ambiguous. Here $Z_1(s)$ is the effective impedance of the interfering source driving the transducer shield. Since

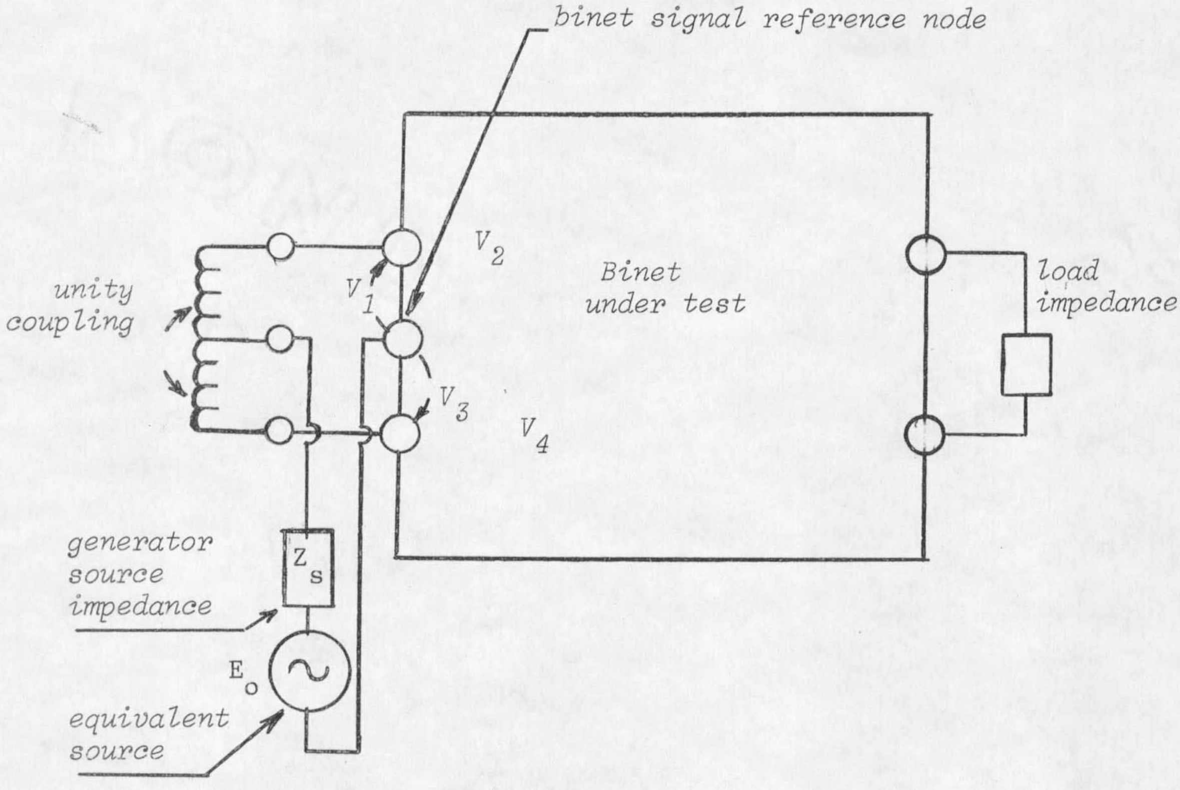


Figure 1.4a   A Common-mode testing Circuit connected to a binet.

Note: Most influx of this kind are low-frequency currents through common resistances. Here $Z_s = R_g$. (See Reference 1.2e, pgs. 2 through 14.)

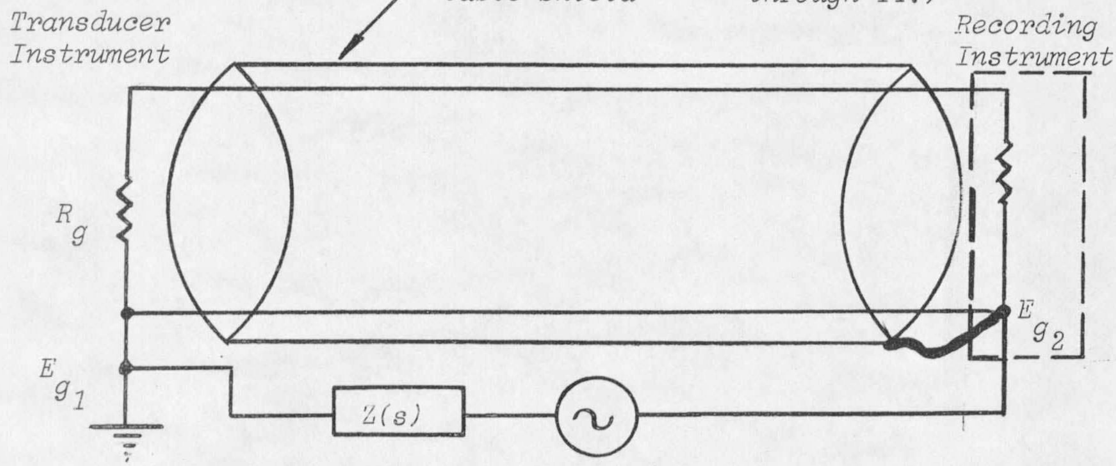


Figure 1.4b CM Interference from currents through common ground impedances.

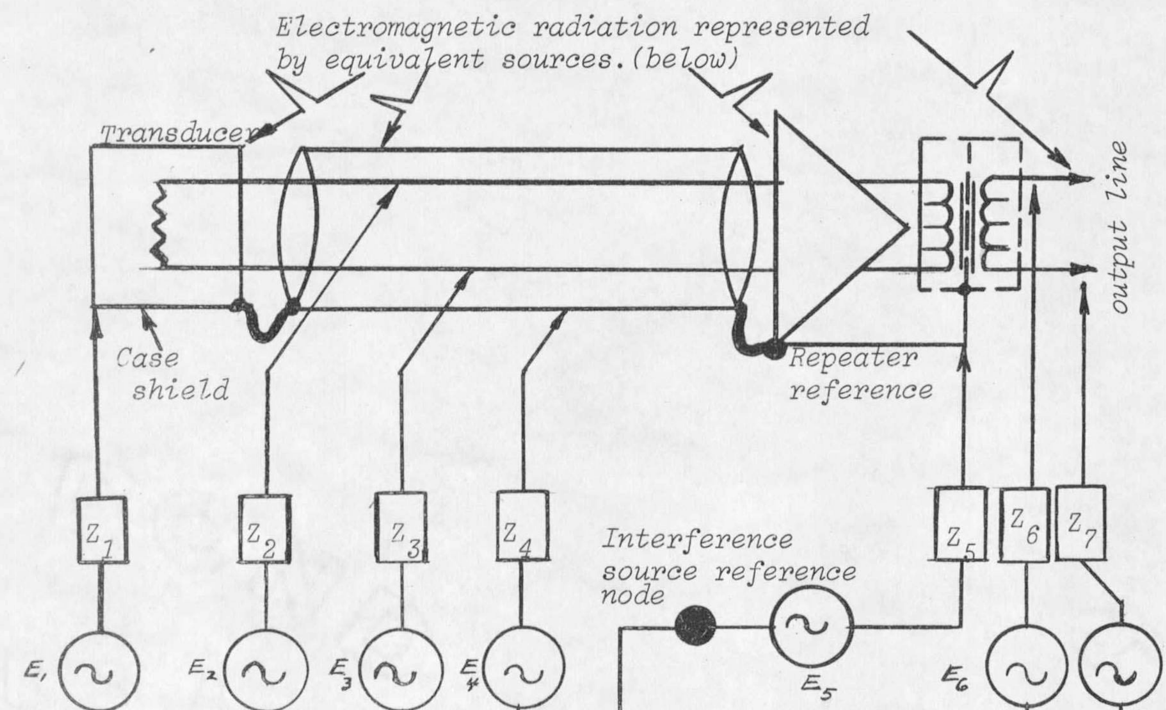


Figure 1.4c CM Interference from electromagnetic radiation.

it is assumed that the wavelength of this source is very much greater than any physical dimension encountered in the configuration, to a first approximation large shielding surfaces may be considered unipotential and such an equivalent source impedance may be assumed to exist. $Z_i(s)$, i=2, 3, 4, 5 and 6 are subject to the same arguments. Implicitly involved in each of these, may be the source impedance of the electromagnetic wave, which may be defined as $Z(s)=E(s)/H(s)$ where $E(s)$ and $H(s)$ are respectively electric and magnetic field intensities which are time and space dependent.

It is clear that when an interfering source can be sufficiently removed physically from the wire-connected equipment as to be considered a plane wave source, that the impedance of the electromagnetic wave in space will not affect the equivalent circuit of the "antenna" formed by the connecting leads. As already assumed, the signal wave length λ, will be much greater than the length of wire ℓ, connecting the equipment together. Then, for example, a 5 mc interfering signal received by a 30 foot (½" diameter) transducer lead would represent an equivalent generator impedance of about 10 ohms in series with a 72 μμfd capacitance. (See Hallén's curves; for example, Reference 1.4 a) This is rather high, about 440 ohms reactive unless lead and/or chassis inductances tend to complement this capacitive effect.

However, if the actual source is physically close to the connecting wires, the characteristic of the "near-field" interference will generally modify this equivalent circuit of the receiving antenna, so that quantitative results cannot be obtained readily, without making highly questionable assumptions about the current distributions in the "antenna". A closer study of the interference source impedance would be required in order to realize a true simulation in common mode measurements. Although some of this study is made in Chapter III, there is still much work to be done in this field.

Here one fortunate aspect of the source impedances to be used is the fact that for the three types of conducted common mode interference influx, the testing circuits' equivalent source configuration can apparently be the same. It is not immediately clear whether the balance factors should be the same, however.

1.5 *THE MEASURED QUANTITY - A FIGURE OF MERIT FOR BALANCE*

Another consideration for the measurements problem relates to the kind of "figure of merit" or balance factor which will be most useful in depicting balance.

Middlebrook suggested that the "common mode rejection factor" (CMRF) is perhaps the most important property of a differential amplifier. This factor is defined as the ratio of the common mode input voltage to the differential mode voltage which would produce the same voltage - or more briefly: the common mode input divided by the resulting transverse output referred to the input. This would not describe common mode interference susceptibility adequately, for the common-mode voltage source would swamp the effect of shunt unbalance at the input; furthermore the zero impedance source may make the circuit look unrealistically poor.

Still another "figure of merit" for the balanced circuit is the *"discrimination factor"* (DF). This is the ratio of the differential mode amplification of the network to the common mode amplification. However, it has been proven (Reference 1.1 b) that a high DF is not sufficient to ensure a high CMRF, and that the limiting case as DF becomes very large is a function of circuit unbalances only, and independent of the amplification ratio. Therefore, the DF does not appear to be as acceptable as even the CMRF.

The various telephone companies use a total of at least five methods in the commercial measurement of common mode balance. (Example, Reference 1.5 a) Commonly, though, the input terminals are differentially matched and the "figure of merit" of balance is simply the ratio of the differential mode (DM) or transverse output $(V_2 - V_4)$ referred to the input $(V_2 - V_4)/A$ to the common mode or longitudinally applied source voltage E_o (See Figure 1.5 a & b) or $(V_2 - V_4/E_oA)$ where A is the DM forward voltage amplification A, of the network:

$$A = \frac{V_2 - V_4}{V_1 - V_3} \qquad \text{Equation 1.5 a}$$

Typically, the telephone companies measure common mode rejection properties by means of Figure 1.5 a or b. A commonly-used longitudinal source impedance appears to be $Z_o/4$ where Z_o is the matching impedance for this port. Here, the value of $Z_o/4$ has become somewhat standard, because it is the lowest impedance available if a transverse matching split resistor is used. (Reference 1.5 b)

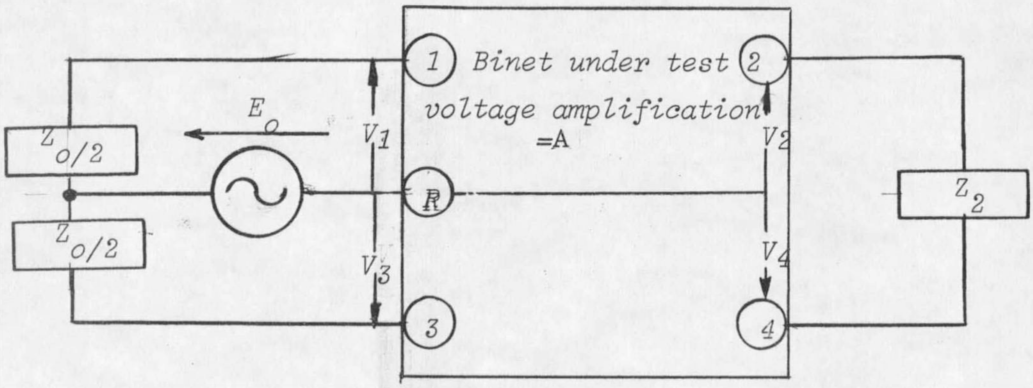


Figure 1.5a Longitudinal measurement using a split impedance; Z_o is usually a center-tapped resistor, R is a reference node.

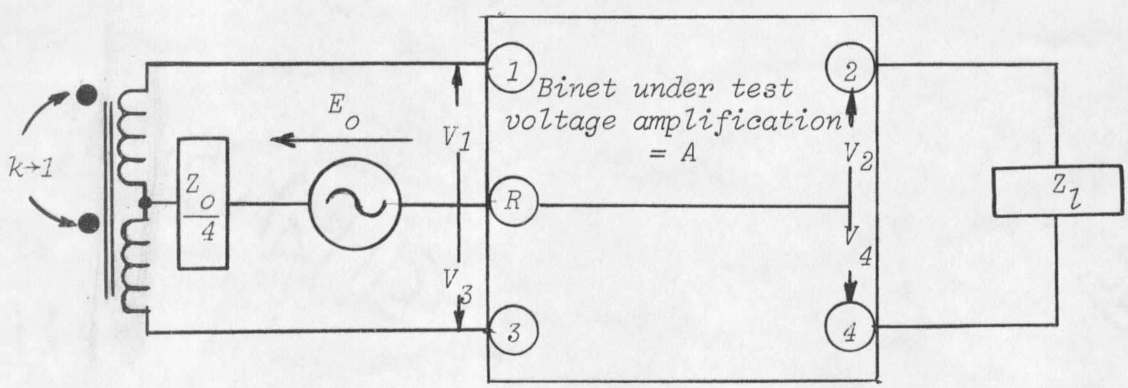


Figure 1.5b Longitudinal measurement using a split coil. Parasitic parameters of the coil are neglected.

Although this "figure of merit" would partially remove the two above noted objections to the CMRF, there apparently remain serious shortcomings which must be resolved. Under certain circumstances the length of the wire receiving the interference is much less than a wavelength, and the equivalent antenna impedance could be a series RC circuit with R near zero. Since the effective impedance of the ground leads, chassis and bonding straps seen by the "antenna" is largely inductive, an important case of common mode interference should arise at resulting resonant frequencies. This suggests that a more realistic interference "figure of merit" would appear to relate the amount of interference power in differential mode to the *available power* of the longitudinal source. "Available power" in this case is as defined in Reference 1.5 c that is, the load reactance complements that of the source as well as matches the source resistance. It is therefore suggested that this balance factor (L) be given as the ratio of the available power of the CM source to the DM power in the load referred to the input:

$$L = \frac{V_o^2}{4R_o/4} \div \frac{(V_2-V_4)^2}{Z_L G} \qquad \text{Equation 1.5 a}$$

Here G is the power gain of the network.

Several other properties of performance criteria are also of interest. The capability of combining the balance factor of one piece of equipment to the balance factor of another piece of equipment in cascade would be of obvious practical value. The choice of source impedance or other testing circuit parameter needed to give maximum change in test sensitivity for a given component change would also be of interest. A study of such a performance criterion - and implications of its properties in the interference susceptibility measurement problem will be treated in Chapter IV.

1.6 THE MEASURING CIRCUIT

Assuming that a reasonable figure of merit, a realistic source impedance and a practical method of realizing these can be defined for the various frequencies of interest, the remaining task would be simply to specify the testing circuit configurations needed to gain the desired information about a given tested network. This work is considered in Chapter II.

However, there is really no reason to limit the analysis and synthesis procedures to just one figure of merit, for the available representation is constrained in no such way. Therefore from *any* node-pair there can flow a differential mode power caused by a common mode source with specified impedance at that same node-pair or any other set of homologous pairs of terminals. Or from *any* node-pair or set of homologous pairs of terminals with a CM path to ground there can flow a common mode power caused by a differential mode source with specified impedance at that same node-pair or any other. It is of course true that binet imbalance will cause both kinds of power transfer. It is to be further noted that the suggested approach of using the network's indefinite admittance matrix allows that *any* node-pair whether or not designed originally as a port, can be looked upon as a balanced port with respect to an effectively low impedance interference source provided there be no limitation of the elements of the fault polynomial transforming the immittance matrix of the balanced network to one depicting the actual circuit. Thus a general method of analysis must be formulated to include the gamut of circumstances which can be encountered in practical interference situations. This is developed in Chapters II, III and IV.

1.7 BALANCED NETWORK SYNTHESIS

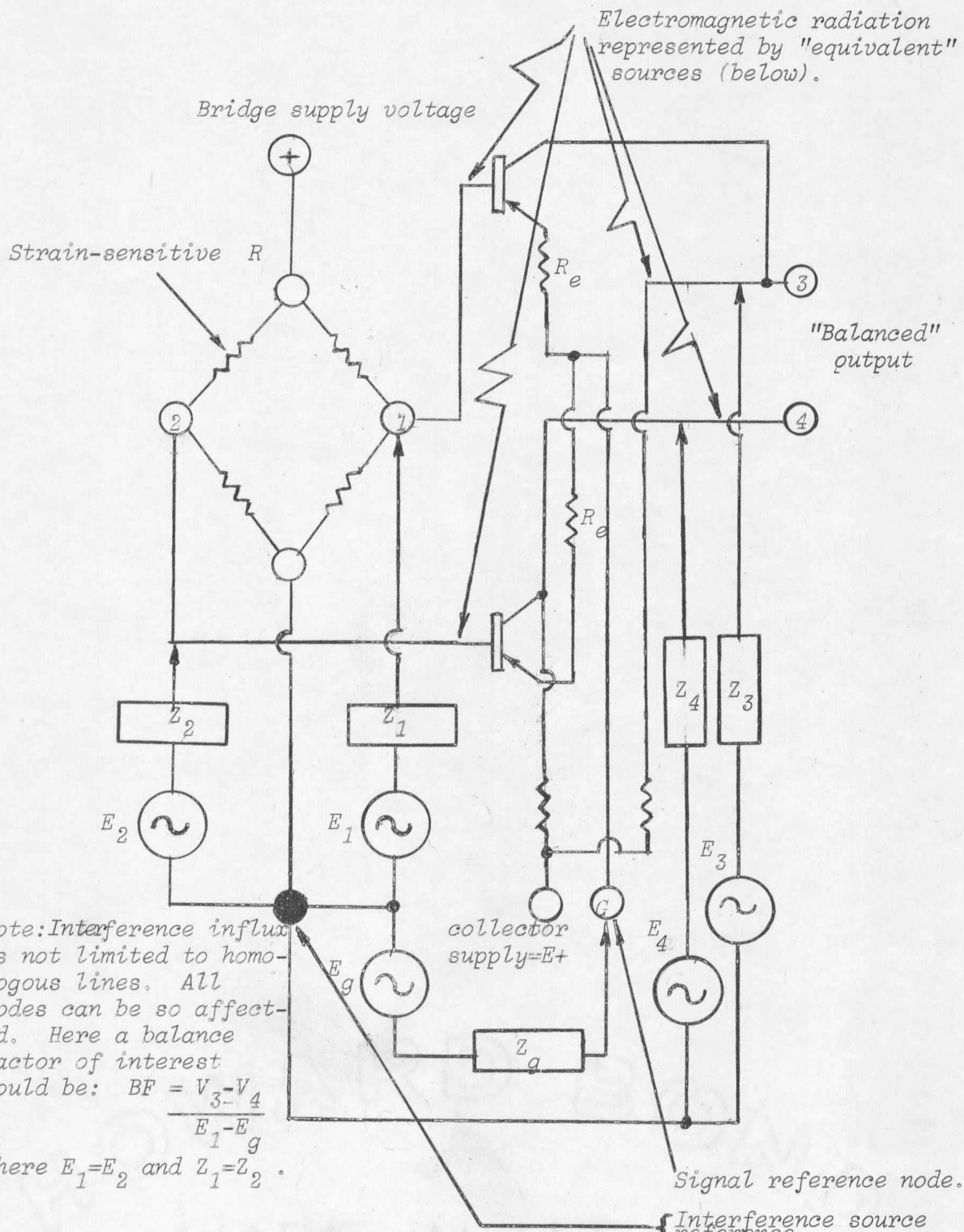
Chapters II, III and IV of this dissertation introduce several tools of analysis for the solution of binal network problems especially as related to the network's common-mode rejection properties. Various performance criteria are introduced not only to provide a means of appraising the common-mode interference characteristic of the circuit, but also to be able to minimize that susceptibility as a circuit synthesis criterion. The synthesis aspect is discussed further in Chapter VI.

II. ANALYSIS - 1

2.0 General Discussion. Before considering the several alternative measurement configurations for the determination of common mode susceptibility, it will be useful to effect meaningful ways of describing balance and an efficient method of analysing CM. - DM. problems in homologous networks. Initially, for definiteness, it will be well to restate such problems as they relate to a few important practical applications. Figures 2.0 a, b and c typify rather common phenomena causing interference in wire-connected equipment.

In Figure 2.0 a the equivalent voltage sources E_1, E_2, E_3, E_4 are those relating to *electromagnetic* pickup on the indicated wire leads, while E_g is that due to the coupled interference across a *common ground* impedance. The strain gage bridge of Figure 2.0 a shows two kinds of influx resulting in common mode interference. Figure 2.0 b illustrates *CM modulated RF interference* picked up on long output leads of an amplifier with pass band only including frequencies of the modulat*ing* signal. DM interference across the base and emitter saturates that lower level stage device causing detection of the modulating signal. In Figure 2.0 c, equivalent interference voltages can appear on an asymmetrical input giving rise to E_{DM} indicated.

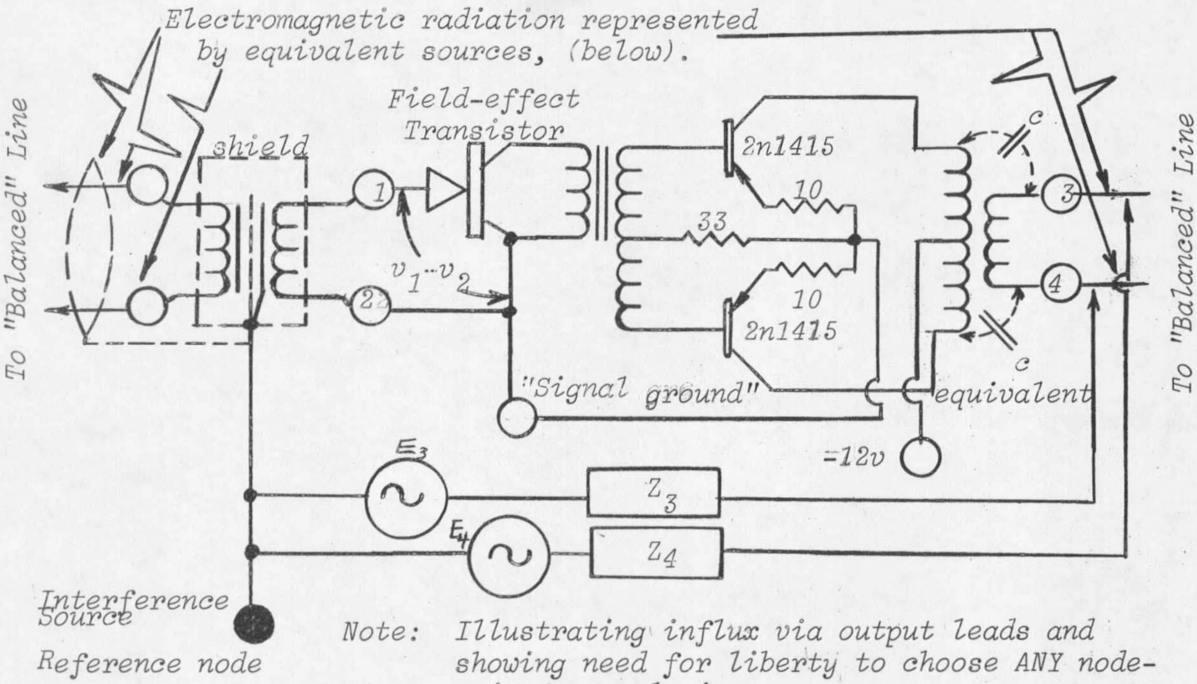
The general analysis of a circuit's susceptibility to such types of interference influx will require flexibility in the choice of node-pairs - sometimes three or more terminals representing the "common-mode input" with the DM. "output" chosen as any two nodes in the network. It seems clear



Note: Interference influx is not limited to homologous lines. All nodes can be so affected. Here a balance factor of interest would be: $BF = \dfrac{V_3 - V_4}{E_1 - E_g}$

where $E_1 = E_2$ *and* $Z_1 = Z_2$.

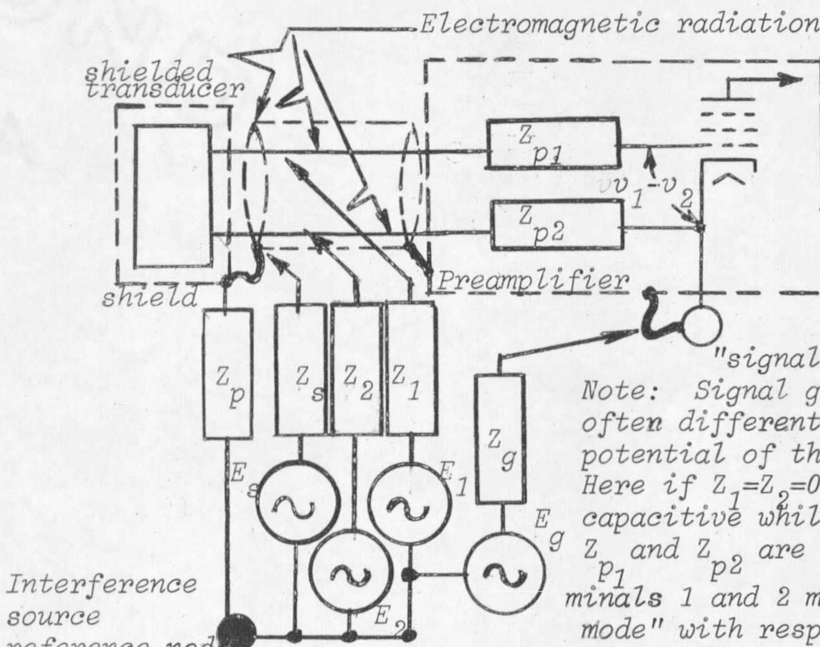
Figure 2.0a Strain gauge bridge.



Note: Illustrating influx via output leads and showing need for liberty to choose ANY node-pair for analysis.

A balance factor of interest is $BF = \dfrac{v_1 - v_2}{E_3} = \dfrac{E_{DM}}{E_3}$ where $E_3 = E_4$, $Z_3 = Z_4$.

Figure 2.0b Transformer-Coupled Amplifier.



Balance Factor

$$BF = \frac{v_1 - v_2}{E_1} = \frac{E_{DM}}{E_1}$$

where $E_1 = E_2 = E_s$ and $Z_1 = Z_2 = Z_s$

Note: Signal ground potential is often different than an assumed zero potential of the source reference node. Here if $Z_1 = Z_2 = 0$ (or if Z_1 and Z_2 are capacitive while parasitic impedances Z_{p_1} and Z_{p2} are inductive), then terminals 1 and 2 may sustain a "common mode" with respect to a source reference.

Figure 2.0c Non-homologous balance.

that such an analysis should involve finite, linear and lumped components.

As already indicated, much of this effort is centered about homologous binets. The consequent equivalence of (two pairs of) submatrixes of the immittance matrix will make it expedient to devise a means of simplification relating to this property rather than to that of reciprocity (symmetry about the main diagonal). Thus the way would be open to analyze active as well as passive networks.

2.1 THE INDEFINITE ADMITTANCE MATRIX

Shekel's indefinite admittance matrix (Reference 2.1 a) has an order one greater than for the ordinary immittance matrices, but otherwise has several useful properties: For simplification purposes, nodal suppression by a standard method of pivotal condensation (Reference 2.1 b) is readily applicable. Also if two nodes have only common-mode potentials, the pertinent indefinite admittance matrix is effected by simply adding the rows and columns affected. The current for the resultant node will be the sum of the currents of the original nodes. If the desired network transformation requires the formation of a node to datum matrix, the conversion is immediate by merely omitting the row and column corresponding to the desired reference node.

Since the indefinite admittance matrix is constructed from a reference node external to the network, a source connected as to simulate the interference would be returned to this node. In the analysis, the impedances

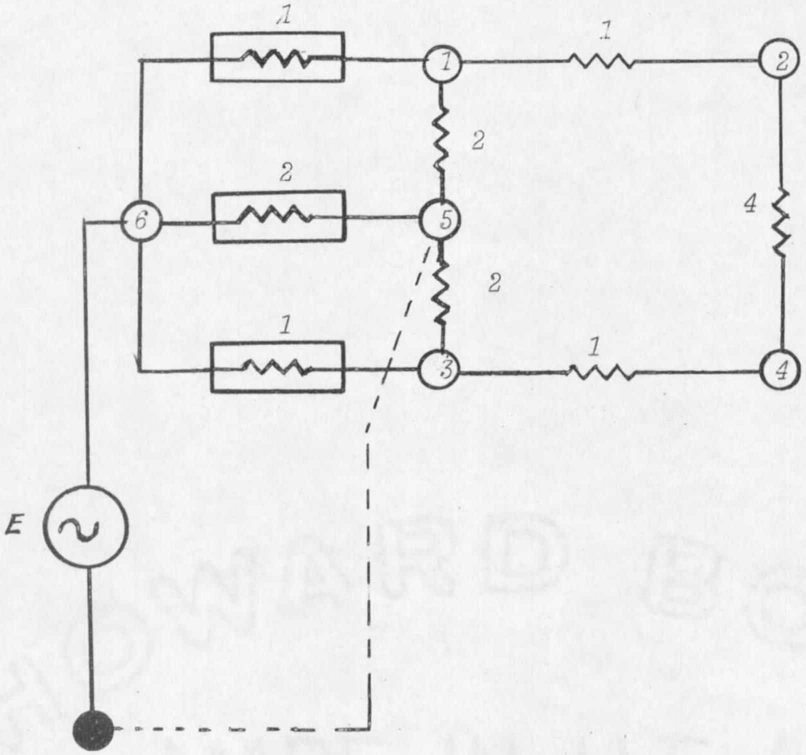


Figure 2.1a A simple binet for analysis.

of the simulating source and the testing circuit would then become part of the tested network itself. As can be seen in Figure 1.2 a, for example, the network reference or "ground" (terminal 5) cannot be considered an "external" node; it is obviously a part of the network.

The simple binet of Figure 2.1 a will have the indefinite admittance matrix:

$$[Y] = \begin{bmatrix} 4 & -1 & 0 & 0 & -2 & -1 \\ -1 & 5 & 0 & -4 & 0 & 0 \\ 0 & 0 & 4 & -1 & -2 & -1 \\ 0 & -4 & -1 & 5 & 0 & 0 \\ -2 & 0 & -2 & 0 & 6 & -2 \\ -1 & 0 & -1 & 0 & -2 & 4 \end{bmatrix} \qquad \text{Equation 2.1 a}$$

If a completely general homologous network were to be considered, each homologous portion would consist of n terminals, and the tested plus testing network, would include 2n+2 terminals as indicated in Figure 2.3 a. Of course the related indefinite admittance matrix would be of order 2(n+1) and can be written for this special purpose as:

$$[Y] = \begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1n} & y_{1c} & y_{1,n+1} & \cdots & y_{1,2n} & y_{1,2n+1} \\
y_{21} & y_{22} & \cdots & y_{2n} & y_{2c} & y_{2,n+1} & \cdots & y_{2,2n} & y_{2,2n+1} \\
\vdots & \vdots & & & & & & \vdots & \vdots \\
y_{n1} & y_{n2} & & y_{nn} & y_{nc} & y_{n,n+1} & & y_{n,2n} & y_{n,2n+1} \\
y_{c1} & y_{c2} & & y_{cn} & y_{cc} & y_{c,n+1} & & y_{c,2n} & y_{c,2n+1} \\
y_{n+1,1} & y_{n+1,2} & & y_{n+1,n} & y_{n+1,c} & y_{n+1,n+1} & & y_{n+1,2n} & y_{n+1,2n+1} \\
\vdots & \vdots & & & & & & \vdots & \vdots \\
y_{2n+1,1} & y_{2n+1,2} & \cdots & y_{2n+1,n} & y_{2n+1,c} & y_{2n+1,n+1} & \cdots & y_{2n+1,2n} & y_{2n+1,2n+1}
\end{bmatrix}$$

Equation 2.1 a

where $\sum_{i}^{2n+1} \bar{y}_{ij} = \sum_{j}^{2n+1} \bar{y}_{ij} = 0$ and where the ikth mutual element is $\bar{y}_{ik} = -g_{ik} + sC_{ik} - \frac{1}{sL_{ik}}$ and the main diagonal elements can be given by

$$\bar{y}_{ii} = \sum_{k=1}^{2n+1} g_{ik} + s(C_{ii} + \sum_{k=1}^{2n+1} C_{ik}) + \frac{1}{s}(\frac{1}{L_{ii}} + \sum_{k=1}^{2n+1} \frac{1}{L_{ik}})$$ Equation 2.1 b

If the currents are taken externally from a 2n+2nd node (c in the Figure) and all voltages are referenced to this node, in matrix notation, the currents are:

$$I] = [Y]\ V]$$ Equation 2.1 c

where I] and V] are again column vectors.

Now common mode signals have heretofore referred to an influx resulting in identical voltages on just *two* terminals of the network. However, there is no reason to assume that only two wires connected to the network will be so affected. Indeed it is completely possible and often likely that *all* wires into the network including the ground cable will carry related interference waves - often of different magnitudes and/or phases. If the source impedance is very low compared to the various impedance levels of the network, the interference wave at the several nodes will closely approximate that of a common mode.

2.2 ANALYSIS USING THE ZERO SUM MATRIX

The balance factor defined in Figures 1.2 a and 2.3 a can be readily determined by the solution of the matrix Equation 2.1 c. If I_{jk} represents a current flowing into terminal j and out of terminal k (See Figure 2.3 a) and all except the $m^{\underline{th}}$ and $n^{\underline{th}}$ terminals are opened, the open circuit transfer impedance between the $jk^{\underline{th}}$ and $mn^{\underline{th}}$ port will be:

$$Z_{jk}^{mn} = \frac{V_{mn}}{I_{jk}} = \frac{Y_{jk}^{mn}}{Y_k^n} \qquad \text{Equation 2.2 a}$$

where the latter term is the ratio of a second cofactor to a first cofactor as can be seen by Cramer's Rule.

In Section 2.3 an example will be given to demonstrate flexibility of the indefinite admittance matrix in the solution of the general homologous binet of Figure 2.3 a.

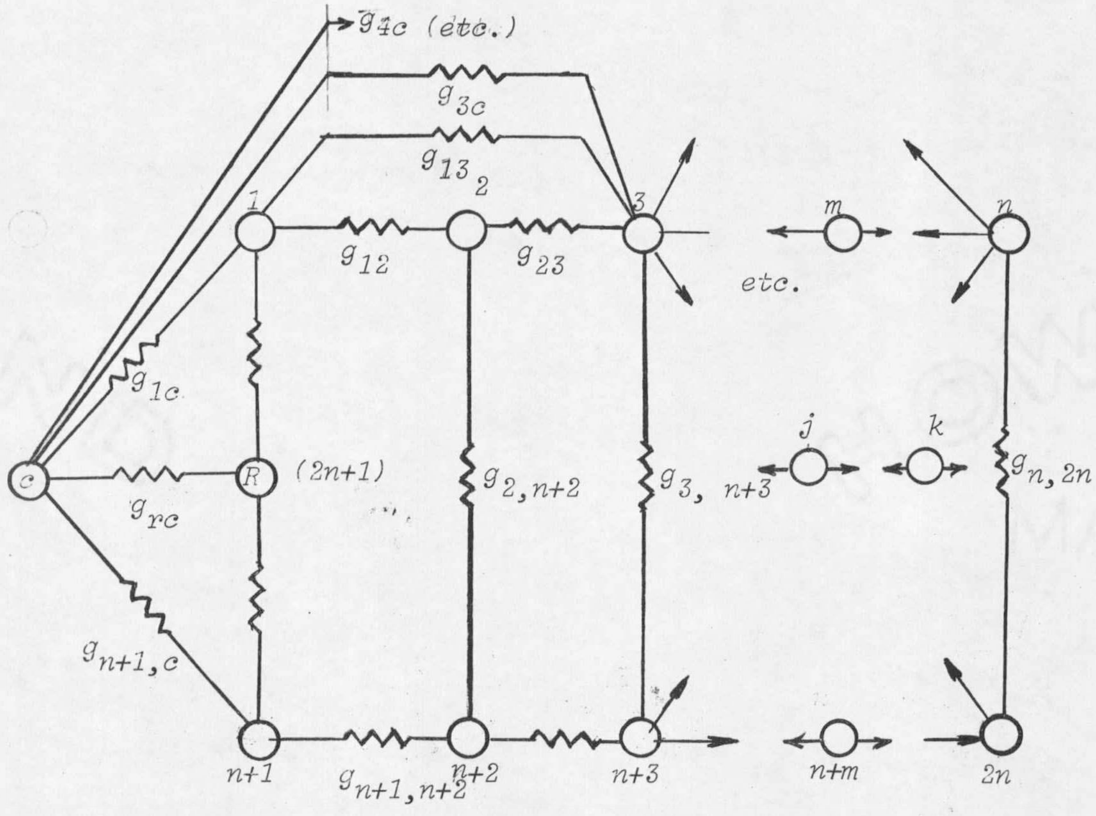


Figure 2.3a General Homologous passive binet.

2.3 THE GENERAL HOMOLOGOUS PASSIVE BINET

Figure 2.3 a shows a general (2n+1) terminal resistor network, homologous in the sense that $g_{12} = g_{n+1,\ n+2}$, $g_{23} = g_{n+2,\ n+3}$, etc. These are called series *intrabipart* elements because their nodes lie in the same binary part (See Definitions); there are also the shunt type connecting to reference $g_{1,\ 2n+1} = g_{n+1,\ 2n+1}$, $g_{2,\ 2n+1} = g_{n+2,\ 2n+1}$, etc. The total number of intrabipart conductances in each of the biparts is $C_2^{n+1} = \frac{(n+1)n}{2}$.

There is also a series type of *intrabipart* element such as $g_{1,\ n+2}$, $g_{2,\ n+1}$ which must be homologously equivalent, $g_{1,\ n+2} = g_{2,\ n+1}$ -------- $g_{2,\ n+3} = g_{3,\ n+2}$. It can be seen there are $C_2^{2n} - 2C_z^n - n$ elements of this type in the binet, since there are n shunt type interbipart elements such as $g_{1,\ n+1}$, $g_{2,\ n+2}$. These have no interelement constraints referring to balance and so their values have only a secondary effect on network balance.

The equivalent source conductances of Figure 2.3 a are shown and will be assumed homologous. Then $g_{1c} = g_{n+1,c}$, $g_{2c} = g_{n+2,\ c}$, etc. The total number of these possible is just 2n+1. In effect, this approach will enable the circuit synthesist and analyst alike, to predict interference susceptibility under more realistic conditions. The "metallic path" into the system can of course number (2n+1). Since the four types of elements are mutually exclusive of each other, their sum gives the total number of resistor elements N_R in the network:

$$N_R = 2C_2^{n+1} + C_2^{2n} - 2C_2^{n} - n + n$$

conductances in the network itself and $N_s = 2n+1$, for the number of possible equivalent source resistors. As to be expected, this total is equivalent to:

$$C_2^{2n+1} = \frac{(2n+1)2n}{2} = N_R$$

which is the total number of binet conductances possible when fully connected. The total number of binet *and* source conductances is:

$$N_R + N_s = (2n^2+n) + (2n+1) = 2n^2 + 3n+1 \qquad \text{Equation 2.3 b}$$

For n=3, $N_R + N_s = 18 + 9 + 1 = 28$ and the indefinite admittance matrix is of order $2n+2 = 8$. For n=5 still a relatively simple network, $N_R + N_s = 50 + 15 + 1 = 66$ elements, and the matrix of order $2n+2 = 12$. Since there may also be reactive elements as in equation 2.1 b, the total number of elements possible for n=5 will be $3(N_R+N_s) = 198$ when fully connected.

It is desired to completely define the conductive or "metallic" interference susceptibility of this general network introduced by the assumed source conductances. The indefinite admittance matrix is described in Equation 2.3 c:

$$[Y] = \begin{bmatrix} g_{11} & \cdots & g_{1n} & g_{1c} & g_{1,n+1} & \cdots & g_{1,2n} & g_{1,2n+1} \\ \vdots & & \vdots & & & & \vdots & \vdots \\ g_{n,1} & & g_{nn} & g_{nc} & g_{n,n+1} & & g_{n,2n} & g_{n,2n+1} \\ g_{c,1} & & g_{cn} & g_{cc} & g_{c,n+1} & & g_{c,2n} & g_{c,2n+1} \\ g_{n+1,1} & & g_{n+1,n} & g_{n+1,c} & g_{n+1,n+1} & & g_{n+1,2n} & g_{n+1,2n+1} \\ \vdots & & \vdots & & & & \vdots & \vdots \\ g_{2n+1,1} & & g_{2n+1,n} & g_{2n+1,c} & g_{2n+1,n+1} & & g_{2n+1,2n} & g_{2n+1,2n+1} \end{bmatrix}$$

Equation 2.3 c

where $g_{11} = g_{12} = (g_{1c}+\ldots+g_{1n})$. This is of order 2n+2 because the source-node C is also included.

The first n rows and columns largely describe the interconnection of the C_2^{n+1} homologous resistors in the upper portion of the network; although the principal diagonal elements must necessarily involve the n^2 homologous cross-connected elements and the n shunting elements. This submatrix will be called A_1. The (n+1) thru (2n) numbered rows and columns in like manner describe the C_2^{n+1} homologous resistors in the lower portion of the network. This is submatrix A_2. For conditions of balance, these submatrices are equal so that det A_1 = det A_2. Similarly the submatrix involving the (n+1) through 2n rows and 1 thru n columns (called B_1) is actually equal to the submatrix involving the (n+1) thru 2n columns and 1 thru n rows (called B_2). Then $B_1 = B_2$ and det B_1 = det B_2. It would be expected that such symmetry should lead to means of simplification in the calculation procedures to follow.

A *Common Mode Rejection Factor* was previously referenced on Page 13 of Chapter I and relates to a resultant "output" transverse voltage ratio $(V_n - V_{2n})$ referred to the input by

$$A_{DM} = \frac{V_n - V_{2n}}{V_1 - V_{n+1}} \qquad \text{Equation 2.3 d}$$

Voltage subscripts now refer to the nodes of Figure 2.3 a. Then the common mode attenuation factor would be

$$A_{CM} = \frac{V_n - V_{2n}}{A_T} \cdot \frac{1}{V_C - V_R} \qquad \text{Equation 2.3 e}$$

Using the earlier notation to describe the relevant second cofactors of Y from Equation 2.3 c there are:

$$A_T = \frac{V_n - V_{2n}}{V_1 - V_{n+1}} = \frac{Y^{n,\ 2n}_{1,\ n+1}}{Y^{1,\ n+1}_{1,\ n+1}} \qquad \text{Equation 2.3 f}$$

and

$$A_{CM} = \frac{1}{A_T}\,\frac{V_n - V_{2n}}{V_C - V_R} = \frac{1}{A_T}\,\frac{Y^{n,\ 2n}_{CR}}{Y^{CR}_{CR}} \qquad \text{Equation 2.3 g}$$

It is perhaps significant to mention here that the quantity $V_n - V_{2n}$ of Equation 2.3 d cannot cancel with $V_n - V_{2n}$ of Equation 2.3 e for they relate to different applied sources. The transverse attentuation without the equivalent interference source parameters will often be known, although a method of calculating A_{DM}, using the indefinite admittance matrix is indicated. *The calculation of* $Y^{n,2n}_{CR}$ *and* Y^{CR}_{CR} involve determinants of order 2n. The sign evaluation of the cofactor $Y^{n,2n}_{CR}$ is made by calculating the value of $(-1)^{n+2n+C+R}$ and multiplying this by the number $(-1)^k$, where k is the total number of interchanges necessary to put the cofactors' superscripts and subscripts in ascending order with respect to the row and column *positions* in $[Y]$ of Equation 2.3 c. Referring to that equation $Y_{CR}^{n,2n}$ will be:

$$Y_{CR}^{n,2n} = (-1)^{n+2n+C+R} \det \begin{bmatrix} g_{11} \cdots g_{1,n-1} & g_{1c} & g_{1,n+1} \cdots g_{1,2n-1} \cdots g_{1,2n+1} \\ \vdots \qquad \vdots & & \qquad\qquad\qquad\qquad \vdots \\ g_{1n} \quad g_{n-1,n} & g_{nc} & g_{n,n+1} \quad g_{n,2n-1} \quad g_{n,2n+1} \\ g_{1,n+1} \quad g_{n-1,n+1} & g_{c,n+1} & g_{n+1,n+1} \quad g_{n-1,2n-1} \quad g_{n+1,2n+1} \\ \vdots \qquad \vdots & & \qquad\qquad\qquad\qquad \vdots \\ g_{1,2n} \quad g_{n-1,2n} & g_{c,2n} & g_{n+1,2n} \cdots g_{2n-1,2n} \quad g_{2n,2n+1} \end{bmatrix}$$

Equation 2.3 h

where $g_{ik} = g_{ki}$ for a reciprocal network.

while Y_{CR}^{CR} will be:

$$Y_{CR}^{CR} = \det \begin{bmatrix} g_{11} & g_{12} \cdots & g_{1,n} & g_{1,n+1} \cdots & g_{1,2n} \\ g_{12} & g_{22} & g_{2,n} & g_{2,n+1} & g_{2,2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{1,n} & g_{2,n} & g_{nn} & g_{n,n+1} & g_{n,2n} \\ g_{1,n+1} & g_{2,n+1} & g_{n,n+1} & g_{n+1,n+1} & g_{n+1,2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{1,2n} & g_{2,2n} \cdots & g_{n,2n} & g_{n+1,2n} \cdots & g_{2n,2n} \end{bmatrix}$$

Equation 2.3 i

since $(-1)^{2C+2R}$ is always positive.

For Y_{CR}^{CR} representing a homologous reciprocal network, the simplification of Equation 2.3 i is straightforward, for then: adding the negative of row (n+1) to row 1, the negative of row (n+2) to 2, etc., and then adding column 1 to column (n+1), column 2 to (n+2), etc. and again noting that $(-1)^{2C+2R}$ is always positive will result in the simplification:

$$Y_{CR}^{CR} = \det \begin{bmatrix} (g_{11}+g_{1,n+1}) & (g_{n,n+1}-g_{1n}) & 0 & - - - - & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ (g_{1,2n}-g_{1n}) & (g_{n,2n}-g_{n,n}) & 0 & - - - - & 0 \\ g_{1,n+1} & g_{n,n+1} & (g_{n+1,n+1}-g_{1,n+1}) & & (g_{n+1,2n}-g_{n,n+1}) \\ \vdots & \vdots & & & \\ g_{1,2n} & g_{n,2n} & (g_{2n,n+1}-g_{1,2n}) & & (g_{2n,2n}-g_{n,2n}) \end{bmatrix}$$

Equation 2.3 j

Now for any $(2n)^{\underline{th}}$ order square matrix with $n^{\underline{th}}$ order submatrices: A_1, A_2, A_3 and A_4, if A_2 or A_3 is the null matrix, it is shown in Appendix C that:

$$\det \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} = \det\left[A_1\right], \det\left[A_4\right]$$

Equation C.2

and therefore directly:

$$Y_{CR}^{CR} = \det \begin{bmatrix} (g_{11}+g_{1,n+1}) & \cdots & (g_{n,n+1}-g_{1n}) \\ \vdots & & \vdots \\ (g_{1,2n}-g_{1n}) & \cdots & (g_{2,2n}-g_{n,n}) \end{bmatrix} \det \begin{bmatrix} (g_{n+1,n+1}-g_{1,n+1}) & \cdots & -g_{n+1,2n}-g_{n,n+1} \\ \vdots & & \vdots \\ (-g_{2n,n+1}-g_{1,2n}) & \cdots & g_{2n,2n}-g_{n,2n} \end{bmatrix}$$

Equation 2.3 k

So the chore of evaluating the determinant of a matrix of order *2n* is reduced to that of finding the product of the determinants of two $n^{\underline{th}}$ *order matrices*. The determination of $Y_{CR}^{n,2n}$ is performed in the same manner, where for the balanced case this must go to zero. Similarly, simplifications will be seen to be feasible in the open circuit transfer impedance expressions:

$$Z_{C,R}^{n,2n} = Y_{C,R}^{n,2n} \div Y_{R}^{2n}$$

where Y_R^{2n} is any one of the $(2n+2)^2$ equivalent first cofactors. Once Y_R^{2n} is found, it can be used to determine any driving point or transfer impedance, for then respectively, $Z_{ij}^{ij} = Y_{ij}^{ij}/Y_j^j$ and $Z_{kl}^{ij} = Y_{kl}^{ij}/Y_l^j$ where $Y_j^i = Y_l^j = Y_r^n$ which has been previously evaluated. Before continuing the common-mode susceptibility problem, it will be well to examine the form the simplification takes in the evaluation of one of the equicofactors.

From Equation 2.3 c it is evident that Y_R^R is a convenient first cofactor to evaluate. Repeating the procedure leading to Equation 2.3 k, one would arrive at:

$$Y_R^R = g_{cc} \; (Y_{CR}^{CR})'' \qquad \text{Equation 2.3 l}$$

where the second cofactors $(Y_{CR}^{CR})''$ is of the kind described by Equation C.2 and the double prime denotes the cofactor of a transformed matrix. With the exception of the final result, the development to arrive at Equation 2.3 l is identical to that to get Equation 2.3 k, and is therefore relegated to Appendix E.

It is apparent therefore that for any LLFPB homologous binet so tested, the first (equi)co-factor $Y_j^i = Y_R^R$ *of order (2n+1)* can be evaluated as the sum of the source conductances multiplied by the product of two co-factors *of order n.*

2.4 UNBALANCE OF THE BINET

Introducing unbalances or inequalities in any of the components intended to be homologous will of course make $Y^{n,2n}_{C,R}$ non-zero, for the balance factor $\frac{(V_n - V_{2n})}{(V_C - V_R)A_{DM}}$ will become non-zero. (A_{DM} is assumed finite) However such unbalanced components destroy the symmetry around reference which has led to the simplified $n^{\underline{th}}$ order evaluations of the $(2n)^{\underline{th}}$ order determinants. In Middlebrook's common mode equivalent network the unbalance mechanisms are handled essentially by assuming the unbalance is small. Here, however, there exists an opportunity to add the unbalance mechanisms of whatever magnitude and configuration without approximations, and this while retaining as many advantages of remaining homology as possible. For the balanced circuit of Figure 2.3 a, a decrease in conductance in the value of $g_{2,2n+1}$ for example, can be handled by adding one negative conductance $(-\Delta_{2,2n+1})$ across these terminals.

Now the expansion of the foregoing cofactors according to *row* elements and their cofactors, or, *column* elements and their cofactors is only one procedure of very many possible in expanding and evaluating these determinants.

The increased flexibility implied by the Cauchy expansions (Reference

2.4 a) will evidently be of use here. The Y matrix of Equation 2.3 c, with the unbalances noted above would be:

$$[Y]' = \begin{bmatrix} g_{11} & -g_{12} - - - - - - & -g_{1c} & -g_{1,2n+1} \\ -g_{21} & (g_{22}-\Delta_{2,2n+1}) - - - & -g_{2c} & (-g_{2,2n+1}+\Delta_{2,2n+1}) \\ \vdots & & \vdots & \vdots \\ -g_{c1} & -g_{c2} & \vdots & \vdots \\ \vdots & & \vdots & \vdots \\ -g_{2n+1,1} & (-g_{2n+2,2}+\Delta_{2,2n+1}) & -g_{2n+1,c} & +g_{2n+1,2n+1}-\Delta_{2,2n+1} \end{bmatrix}$$

Equation 2.4 a

And then $Y^{n,2n}_{C,R} = (-1)^{n+2n+C+R} \{\Delta_{2,2n+1}(Y^{n,2n,2n+1}_{C,R,2} - Y^{n,2n,2}_{C,R,2}) + Y^{n,2n}_{CR}\}$

Equation 2.4 b

But $Y^{n,2n}_{CR}$ is always zero, because it represents the transverse output of a balanced network. Clearly $\Delta_{2,2n+1}$, representing the shunt unbalance conductance will determine the transverse output directly as will the cofactors $Y^{n,2n,2}_{C,R,2}$ and $Y^{n,2n,2n+1}_{C,R,2}$ which depend on the location of the unbalance location and related balanced network elements.

Needless to say, the difference of cofactors should always be non-zero when the unbalance in question is located in a position to cause common mode to differential mode conversion. One case where it would be nearly zero would appear when an input unbalance is driven thru a source impedance approaching zero in magnitude.

In order to complete the general homologous example of Section 2.3 including the unbalance here, it will be necessary to determine the value of

Y_{CR}^{CR}. As before:

$$\left(Y_{CR}^{CR}\right)' = (+1)^{2(C+R)} \det \begin{bmatrix} g_{11} & -g_{21} & \cdots & -g_{1n} & -g_{1,n+1} & \cdots & -g_{2n} \\ -g_{12} & (g_{22}-\Delta_{2,2n+1}) & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ -g_{n1} & -g_{n2} & \cdots & +g_{nn} & -g_{n,n+1} & \cdots & -g_{n,2n} \\ -g_{n+1} & -g_{n+1,2} & \cdots & -g_{n+1,n} & +g_{n+1,n+1} & \cdots & -g_{n+1,2n} \\ & & & & \vdots & & \vdots \\ -g_{2n,1} & -g_{2n,2} & \cdots & -g_{2n,n} & -g_{2n,n+1} & \cdots & +g_{2n,2n} \end{bmatrix}$$

or

$$Y_{CR}^{CR} = \{Y_{CR}^{CR} - \Delta_{2,2n+1}\, Y_{C,R,2}^{C,R,2}\} \qquad \text{Equation 2.4 c}$$

It is to be expected that if $\Delta_{2,2n+1}$ is a very small part of the component it shunts, that the latter term in Equation 2.4 c may be completely negligible. In any event, Y_{CR}^{CR}, which is of order 2n, was found as the product of two $n^{\underline{th}}$ order cofactors in Equation 2.3 k. Again, $Y_{C,R,2}^{C,R,2}$ can be found by methods used to get Y_{CR}^{CR}.

If more than one component is unbalanced, an examination of Equation 2.4 a will show that as long as the third cofactor ($Y_{C,R,2}^{C,R,2}$ in the latter case) appears without unbalance components among its elements, that the second cofactor evaluation (of Y_{CR}^{CR} in this case) will contain just one term with unbalance components. When the third cofactor contains unbalance components, the third cofactor can be again expanded into a third cofactor and an unbalance component multiplied by a fourth cofactor, etc. This will of course involve a product of at least two unbalance elements. This process can be continued for unbalances in all n elements of the binary part, and a general expression would be similar to that of Equation B.3, Appendix B. Then, choosing an $i^{\underline{th}}$ row with u unbalances, the fault polynomial is:

$$(Y_{CR}^{CR})' = Y_{CR}^{CR} + \sum_{j=1}^{u} \Delta_{ij}\{Y_{CRi}^{CRj} + \sum_{k,l} \Delta_{kl}\left[Y_{CRik}^{CRjl} + \sum_{u,v} \Delta_{uv}(Y_{CRiku}^{CRjlv} + \ldots)\right]\}$$

$$+ \sum_{k,l} \Delta_{kl}\{Y_{CRk}^{CRl} + \sum_{r,s} \Delta_{rs}\left[Y_{CRkr}^{CRls} + \sum_{u,v} \Delta_{uv}(Y_{CRkru}^{CRlsv} + \ldots)\right]\}.$$

Equation 2.4 d

Equation 2.4 d is called the *fault polynomial* of the determinant, $(Y_{CR}^{CR})'$. The number of Δ's can be as high as the total number of such unbalance components possible in the network. If a Δ is added to *each* element of an $n^{\underline{th}}$ ordered matrix, it's determinant should consist of the sum of $2^n(n!)$ products.

As indicated in Appendix B, it is also possible to expand those rows or columns in a Laplace expansion. Then if only the $i^{\underline{th}}$ $j^{\underline{th}}$ and $2n^{\underline{th}}$ rows contain unbalance elements; for example, one can form a summation of minors containing the unbalance values where each of such *fault minors* is multiplied by its complementary signed minor, or:

$$(Y^{CR}_{CR})' = \sum_{k,l,m\,=\,1}^{2n} Y_{\Delta}\,\Delta Y^{C,R,k,l,m}_{C,R,i,j,2n} \qquad \text{Equation 2.4 e}$$

Here the summation includes all combinations of different k, l and m and for a 2n+1 order cofactor would have a total of C_3^{2n+1} terms. Equation 2.4e is called the *fault minor expansion of the determinant* $(Y^{CR}_{CR})'$.

2.5 DIAGONAL EXPANSIONS

When the ratio of voltages referenced to a particular $R^{\underline{th}}$ node is to be found, it will be of the form $\dfrac{Y^{iR}_{jR}}{Y^{jR}_{jR}}$ = a ratio of second cofactors.

The (equi)cofactor Y^R_R of Equation 2.3 1 is also of significance here. These have their $R^{\underline{th}}$ row and column suppressed and therefore all shunting unbalances to the $R^{\underline{th}}$ node will appear on the cofactors' principal diagonal. An expansion of these by diagonal elements is then convenient for, if:

$$Y = \qquad [Y]' = \begin{bmatrix} (y_{11}+\Delta_1) & -y_{12} & -y_{13} & \cdots & \\ -y_{21} & (y_{22}+\Delta_2) & -y_{23} & \cdots & \\ -y_{31} & -y_{32} & (y_{33}+\Delta_3) & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \\ & & & & (y_{nn}+\Delta_n) \end{bmatrix}$$

Then from Equation 2.4 d (or an inductive process) it is seen that:

$$
\begin{aligned}
\det(Y)' = {} & \Delta_1 \cdot \Delta_2 \text{---} \Delta_n + y_{11}\,\Delta_2 \text{---} \Delta_n + y_{22}\,\Delta_1\,\Delta_3 \text{---} \Delta_n + \text{---} y_{nn}\,\Delta_1 \text{---} \Delta_{n-1} \\
& + Y^{3\text{---}n}_{3\text{---}n}\,\Delta_3 \text{---} \Delta_n + Y^{1;4\text{---}n}_{1,4\text{---}n}\,\Delta_1\,\Delta_4 \text{---} \Delta_n + \Delta_1 \text{---} \Delta_{n-2}\,Y^{1\text{--},n-2}_{1\text{--},n-2} \\
& + Y^{4\text{---}n}_{4\text{---}n}\,\Delta_4 \text{---} \Delta_n + Y^{1;5\text{---}n}_{1,5\text{---}n}\,\Delta_1\,\Delta_5 \text{---} \Delta_n + \Delta_1 \text{---} \Delta_{n-3}\,Y^{1\text{--},n-3}_{1\text{--},n-3} \\
& + \text{---} \\
& + \Delta_{11}\,Y^1_1 + \text{---} + \Delta_{nn}\,Y^n_n \\
& + |Y|
\end{aligned}
$$

Equation 2.5 a

The unbalance polynomial of Equation 2.5 a offers another effective insight into the way unbalance anamolies can modify the quantity represented by the determinant.

2.6 *ANALYSIS - 1 IN RETROSPECT*

In chapter two a general ground-work has been laid for the analysis of common mode interference problems in lumped, linear, finite, passive and bilateral binets. The flexibility of Shekel's indefinite admittance matrix has been evident for such LLPFB networks: reference nodes can be defined at will; matrix symmetry is retained with its' simplifying consequences; and other properties of the matrix can be used to facilitate the solution of binet interference problems. The advantages of this zero-sum matrix are not limited to passive networks, however.

In the next chapter an idealized voltage-current or current-voltage transfer device will be used in an indefinite admittance matrix. It will be found that the simplifying procedures of Chapter II and the attendant useful properties of the zero sum matrix remain essentially intact. Then it will also be appropriate to more carefully examine the bases for such simplification.

3.0 Active Homologous Networks. Although passive parameters at the input of a network often play a key role in determining the network's susceptibility to conducted interference, the immediate necessity of raising low-level signals to less vulnerable values inextricably connects these parameters to some kind of active device. Furthermore, in the synthesis problem, it has been made clear that driving point (Reference 3.0 a) as well as transfer functions (References 3.0 b, 3.0 c, 3.0 d, 3.0 e, 3.0 f) can be made up of active R C networks. It seems mandatory therefore, that the general analysis be extended to at least include presently existing active devices. It will immediately appear, however, that there is no reason to limit consideration to already known devices. Network theorists can no longer enjoy the luxury of reasonably fixed quantities in the devices they describe in any general way. Therefore, two, three and four terminal devices should be amenable to the symbolism. Further more the active elements should be sufficiently idealized to allow representation of as many active devices as possible.

Now it may be argued that three-terminal active devices are more "fundamental" in that every four-terminal active device can be shown to be made up of a set of three-terminal elements. (Reference 3.0 g, for example) However, such realizations of a four-terminal device will include four three-terminal elements, generally, and at least if transistors are used, two of these must be 3-terminal negative transconductances. This kind of device has not yet materialized, so that such a "practical" breakdown appears to be inherently artificial. It seems that the representation of a four-terminal magnetic amplifier for example had better remain as a four-terminal active device, therefore, at least in the kind of analysis contemplated here. Now one might initially suppose that simplifications of analysis based on network balance may be forgone when reciprocity resulting in matrix symmetry around the main diagonal is given up. Before this is determined, an ideal active element must be considered which will be compatible with the development of Chapter II.

3.1 THE VOLTAGE-CURRENT TRANSACTOR

The transistor, tunnel diode, vacuum tube and other amplifying devices have been idealized by G. E. Sharpe, among others. This was done in his original work describing the voltage-controlled current source (Reference 3.1 a) and details of operation and properties of this "transactor" were expounded later in his Ph.D. dissertation of 1965. (See Reference 3.1 b) Figure 3.1 a shows a voltage-current transactor (V C T) of degree 4, the latter referring to the number of external nodes. The 4-terminal V C T is no more general in network analysis than the 3-terminal, and most practical applications either involve the latter or the two terminal elements, representing tunnel diodes, etc. The ideal active element is defined by $i_{jk} = y(jk/mn)V_{mn}$ where i_{jk} is the current entering j and leaving k while V_{mn} is the voltage drop from m to n.

Now given the task of fully representing an actual transistor using ideal elements, one would be obliged to include other parameters. One choice would include $C_{B'C}$, $C_{B'e}$, g_B and $(1-\alpha)g_e$ as indicated in Figure 3.1 b.

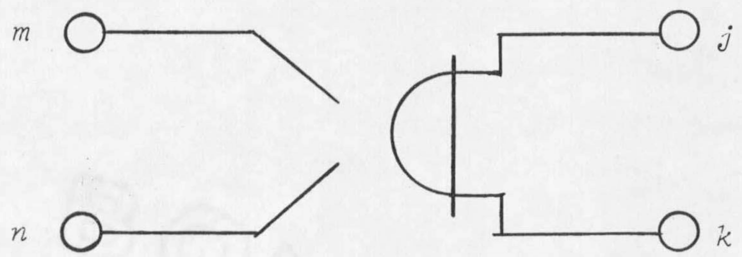


Figure 3.1a Voltage Current Transactor.

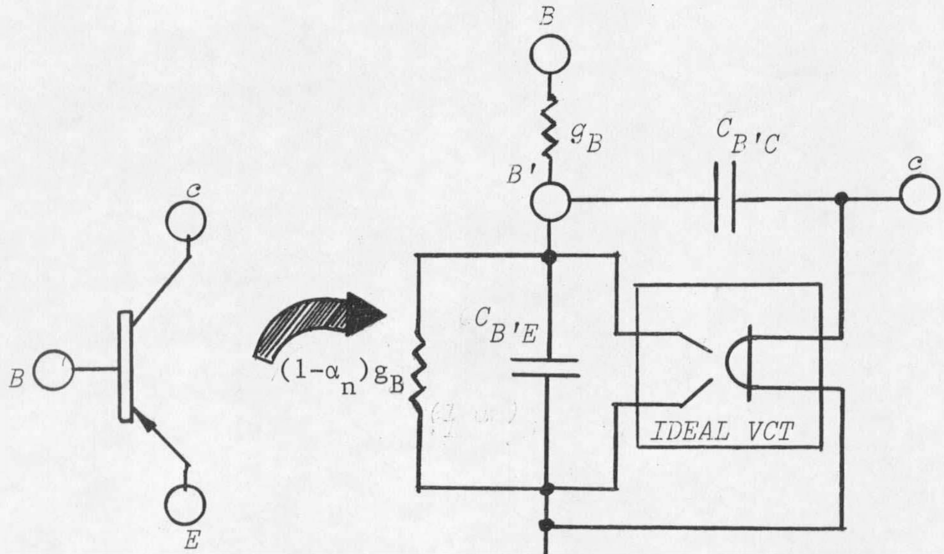


Figure 3.1b VCT representing an actual transistor

It is noted that these are connected in a way which creates an extra node in the network, (B'). The ideal transactor would obviously be defined without the set of ancillary parameters so that zero admittance would result across the voltage nodes of the ideal device with zero impedance across the current nodes.

The tunnel diode can be represented by a V C T degree 2 and would be defined by $i_{jk} = -y(jk/mn)V_{mn}$. Such a transactor and the tunnel diode with its conventional equivalent network is shown in Figure 3.1 c. It is also apparent that when j and k are shorted to m and n respectively that the element becomes a common passive admittance, $+y(jk/jk)$.

Before working out a practical example of the CM susceptibility of an active circuit, it will be well to investigate some of the properties of active elements in homologous binets, and to specify a systematic analytical procedure.

3.2 A NOTE ON THE SYNTHESIS OF THE (2n+2) TERMINAL ACTIVE BINET AND ITS CM MEASUREMENT CONFIGURATION.

Since it is assumed that the network's active elements will be limited to the (2n+1) nodes of the network itself, none of the transactors will be connected to terminal C of the general homologous network. (See Figure 2.3 a)

If it were assumed that each pair of terminals of a transactor can be

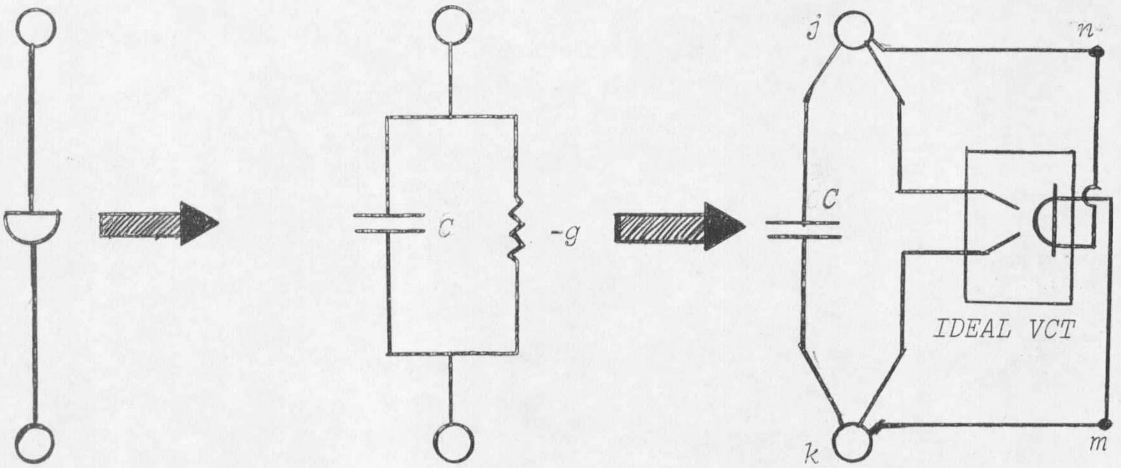


Figure 3.1c Tunnel diode representation by a VCT.

connected across *any* two different nodes, the total number of transactors in the (2n+1) terminal network would be $\left[\frac{(2n+1)(2n)}{2}\right]^2$ because each pair of terminals can be chosen in C_2^{2n+1} ways. The number of transactors of degree 2 for any (2n+1) terminal network would be expected to be $C_2^{2n+1} = 2n^2+n$. Similarly for transactors of degree three there is $P_3^{2n+1} = N_3 = \frac{2n+1}{2n-1}$. where the permutation is required since there are three distinct ways of orienting each combination of three terminals. The number of transactors of degree 4 would be $C_2^{2n+1} \cdot C_2^{2n-1} = 4n^4-4n^3-n^2+n$.

Since these are the only transactors possible, they should total N_N. But $N_N = \left[\frac{(2n+1)(2n)}{2}\right]^2 = 4n^4+4n^3+n^2$.
and $N_2+N_3+N_4 = (2n^2+n)+(8n^3-2n)+(4n^4-4n^3-n^2+n) = 4n^4+4n^3+n^2$.

It has been shown therefore that the total number of ways to connect (2n+1) nodes with two-branch elements (the voltage and current element) is the sum of the number of ways of connecting transactors of degrees four, three and two. Although a total of (2n+1) equivalent-source conductances is possible for such a network, a smaller number will usually be needed to represent the common-mode interference source. If this number is taken as N_s, then the indefinite admittance matrix describing the (N_s+N_N) element network will be of order (2n+2) and will be of rank (2n+1). Thus if the circuit has no homologous components and is non-reciprocal, there will be $(2n+1)^2$ independent elements of the admittance matrix of the circuit, and only (N_s+N_2) independent elements if reciprocal. For a balanced circuit, these quantities are $(\frac{2n+1}{2})^2$ and $\frac{N_s+N_2}{2}$ respectively.

These values, along with N_N, N_2, N_3 and N_4 are compared in Table 3.2 a with the number of *independent* elements based on matrix rank. It seems evident that the design of a balanced circuit with transactors of degrees three (or four) can lead to identical results for a surprisingly large number of different configurations when the performance is specified by an admittance matrix.

Table 3.2 a COMPARISON OF DISTINCT vs INDEPENDENT NUMBERS OF ELEMENTS (Non-reciprocal)

Number of homologous nodes in each part of binet	1	2	3	4	5	6
Total number distinct elements in binet	12	105	448	1305	3036	6097
Total number distinct elements of Degree 2	6	15	28	45	66	91
Total number distinct elements of Degree 3	6	60	210	504	990	1716
Total number distinct elements of Degree 4	0	30	210	756	2080	4290
Number of independent, non-reciprocal elements	9	25	49	81	121	169

Note: For a given matrix of rank (2n+1), the number of elements that can be chosen *independently* is only a small fraction of the number of distinct elements possible, in all but trivially small binets.

3.3 *FORMING THE Y MATRIX FOR A BALANCED ACTIVE NETWORK*

If active elements are added to the nodal configuration of the general homologous network of Figure 2.3 a, the (2n+2) x 1 current vector $I]$ and (2n+2) x 1 voltage vector $V]$ are related by a (2n+2) x (2n+2) equicofactor matrix $[Y]$:

$$I] = [Y]\, V]$$

Now from Percival (Reference 3.3a), a (2n+2)-terminal network is the summation of its constituent transactor elements expressed in matrix form. Such matrices are defined as the (2n+2)x(2n+2) transactor matrices:

$$t(jk/mn) = y(jk/mn)\ E(jk/mn) \qquad \text{Equation 3.3 a}$$

where $E(jk/mn) = (e_j - e_k)\ (e_m^T - e_n^T)$ with e_j the $j^{\underline{th}}$ unit vector, (Reference 3.3 b) and of order (2n+2) x 1:

$$e_j = (0,\text{------}0,\ 1,\ 0\text{------}0)^T.$$

The T superscript signifies the transpose matrix.

These transactor element matrices can be summed up to give the indefinite admittance matrix Y:

$$Y = \sum_{j,k,m,n} t(jk/mn) \qquad \text{Equation 3.3 b}$$

The number of terms in this series, N_3, will be determined by the network's connectiveness, of course, but it is clear that:

$0 \leq N_s \leq N_N$ if problem is non-trivial

$2n+1 \leq N_s \leq N_N$ if fully connected.

3.4 THE INDEPENDENT vs THE DISTINCT CHOICES OF Y ELEMENTS.

In considering the analysis of a homologous binet it is of interest to examine connectivity and component limitations which could be of some concern to the binet synthesist.

As seen from Table 3.2 a, the number of choices of *independent* elements is substantially less than the number of *distinct* elements when Degree 3 or Degree 4 devices are contemplated. Also it is apparent that even for the extreme case of a fully connected passive binet, there would still be a relatively large number (T) of active elements which can be chosen *independently:*

$$T = (2n+1)^2 - C_2^{2n+1} = 2n^2 + 3n + 1 \qquad \text{Equation 3.4 a}$$

Several statements can be made about the way which these active elements might be chosen:

(1.) The elements can be all of degree two (typifying tunnel diodes and other negative conductances), degree three (transistors, vacuum tubes, etc.) or degree four (magnetic amplifiers, saturable reactors, etc.) or any combination of these, making up the number of independent elements possible.

(2.) The choice is limited by the requirement of electrical balance in the binet. Of course this is not peculiar to active networks for, as previously indicated, the number of elements in a fully connected *passive* (2n+1)-terminal network would be $C_2^{2n+1} = 2n^2 + n$ while it is implicit that for a strictly homologous binet only a maximum of $\frac{C_2^{2n+1} - n}{2} + n = n^2 + n$

distinct choices can be made to establish element values.

(3.) In the case of transactors of degree three or four, the unbalancing tendency of a practical active element must be considered. Thus the infinite impedance of the voltage terminals cannot be depended upon in practice. See Figure 3.1 b for example.

These design constraints can be dealt with quite readily: It is obvious degree two and degree three transactors will usually be preferable, for reasons of economy. However, there are other considerations and it appears that the choice of transactor type should be left to initial specification, each transactor of whatever degree counting as one independent element in the makeup of the Y indefinite admittance matrix.

A disposition of the second question will assume that the designer may want to specify other than homologous components in the balanced binet. Therefore the choice of independent active elements will not depend on network homology. Finally, the unbalancing effects of the prevalent device (such as the transistor of Figure 3.1 b) might be minimized by confining all terminals of a transactor to the *same part* of the binet; while its counterpart is similarly confined to the other part. This has been the more typical manner in which active elements have been connected in a binet. The following discussion relates to this limitation.

According to this latter decision, only a small part of the possible number of transactors would thus be considered. For example, in a fully connected (2n+1)-terminal network only

$$\frac{(n+1)(n)(n-1)}{2(n+1)} = \frac{N_3}{2(n+1)} \qquad \text{Equation 3.4 b}$$

transactors of degree three will be needed to use up the total number of independent choices, for each part of the binet will be treated as isolated (n+1)-terminal networks with respect to active elements. In Equation 3.4 b, N_3 is the number of degree three transactors possible in an (n+1) terminal network. That this is the total number of transactors of degree three possible, can be seen by considering that in an (n+1)-node network, there are $n^2-C_2^{n+1} = \frac{n(n-1)}{2}$ independent elements possible after the C_2^{n+1} admittances are chosen. But it is to be noted that this

$$\frac{n(n-1)}{2} = \frac{(n+1)\ n\ (n-1)}{2(n+1)} = \frac{P_3^{n+1}}{2(n+1)} = \frac{N_3}{2(n+1)} .$$

Therefore, this is the total number possible.

3.5 SIMPLIFICATION OF THE Y MATRIX OF HOMOLOGOUS BINETS

With the limitations of statements on the previous pages, the general homologous binet of Figure 2.3 a can be assumed to include transactors of degrees two, three and/or four, but initially limited to a number not exceeding the number of independent elements according to the order of the networks indefinite admittance matrix. Of course it is understood that the synthesis procedure of a continuously equivalent type may call for a redundancy of elements beyond this number in order to realize desired performance criteria.

Since the active transactors have been assumed limited to the respective biparts of the binet, the number of independent choices of active elements for each (n+1)-node bipart, will be $n^2 - C_2^{n+1} = n^2 - n$ for the binet. Thus if only one pair of homologous nodes exists, (a 3 node binet plus an interference source node = 4 nodes) *no* active transactors will be possible under the assumed conditions. If only two homologous nodes exist (a 5 node binet plus an interference source node = 6 nodes) then *two* active transactors are possible in the binet, etc.

Since the form of the *general* n-node homologous binet and the simplification of several cofactors of its indefinite admittance matrix are already in evidence (Section 2.3), the use of a network with just two homologous nodes should allow simplicity and definitiveness in an illustration of the above statements.

The binet of Figure 1.2 a will be used with an additional node c for the equivalent source connection. Conductances will be used and terminal 5 will be called terminal g, representing the network's "signal ground". This is shown in Figure 3.5 a. It will be seen immediately that $2n+2 = 6$ nodes obtain and for a fully connected passive network (10 binet elements + 5 source elements) only $n^2 - n = 2$ active transactors can exist without redundancy, unless at least two of the binet's passive elements are eliminated.

For illustrative purposes, however, assume a redundancy and that four transactors are to be used having transfer admittances y(1g/24), y(3g/42), y(1g/12) and y(3g/34). For balance, y(1g/24) = y(2g/42) and y(1g/12) = y(3g/34). It is to be noted that transactor y(1g/24) for example, is not confined to a single binary part and therefore seemingly violates the disposition of statement 3 in Section 3.4. This is done deliberately, however, to emphasize that this is primarily a practical device limitation. When an *ideal* device is used, as is the case here, matrix symmetry can be maintained by adding another transactor in a homologous position (See y(3g/24) in Figure 3.5 a). There is the network's indefinite admittance matrix Y:

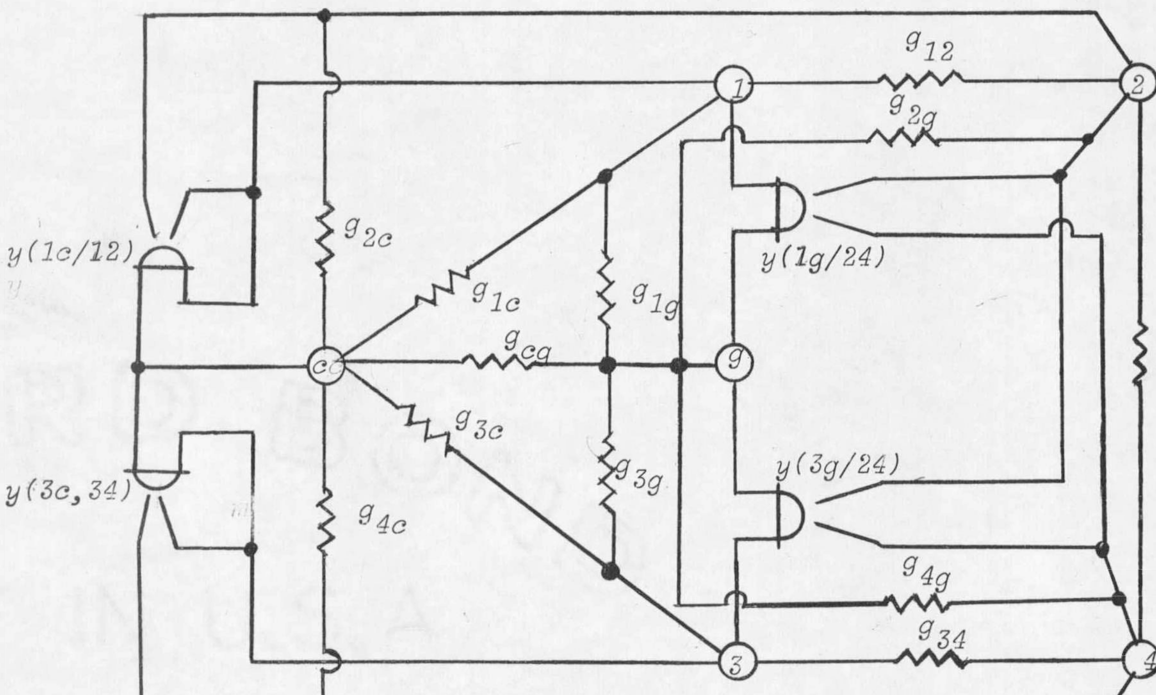


Figure 3.5a *A homologous binet with active elements.*

$$[Y] = \begin{bmatrix} g_{11}' & g_{12}' & -g_{1c} & -g_{13} & -g_{14}-y(1g/24) & -g_{1g} \\ -g_{21} & g_{22}' & -g_{2c} & -g_{23} & -g_{24} & -g_{2g} \\ -g_{1c} & -g_{2c} & g_{cc} & -g_{3c} & -g_{4c} & -g_{cg} \\ -g_{31} & -g_{32}-y(3g/42) & -g_{3c} & y(3g/34)+g_{23} & y(3g/42)-y(3g/34) & -g_{3g} \\ -g_{41} & -g_{42} & -g_{4c} & -g_{43} & g_{44}' & -g_{4g} \\ -g_{1g}-y(1g/12) & g_{g2}' & -g_{gc} & -g_{g3} & -g_{g4}+y(1g/24) & g_{gg}' \end{bmatrix}$$

Equation 3.5 a

where $g_{11}' = g_{12} + g_{13} + g_{14} + g_{1g} + g_{1c} + y(1g/12)$

and $g_{12}' = -g_{12} - y(1g/12) + y(1g/24)$

and $g_{22}' = g_{2c} + g_{21} + g_{2g} + g_{23} + g_{24}$

and $g_{g2}' = -g_{2g} - y(1g/24) + y(1g/12)$

and $g_{44}' = g_{41} + g_{42} + g_{4c} + g_{43} + g_{4g}$

and $g_{gg}' = g_{1g} + g_{2g} + g_{cg} + g_{3g} + g_{4g}$

Following the simplification procedure of Chapter II, the negative of row 4 is added to row 1; the negative of row 5 to row 2, etc. Then, since $g_{1c} = g_{3c}$ and $g_{2c} = g_{4c}$, adding column 1 to column 3 and column 2 to column4, etc., the determinant of the matrix Y is:

$$\det Y = \begin{vmatrix} (g_{11}'+g_{31}) & g_{12}'' & 0 & 0 & 0 & 0 \\ (-g_{21}+g_{41}) & (g_{22}+g_{42}) & 0 & 0 & 0 & 0 \\ -g_{1c} & -g_{2c} & +g_{cc} & 0 & 0 & -g_{cg} \\ -g_{31} & -y(3g/42)-g_{32} & -g_{3c} & g_{33}'' & -y(3g/34) & -g_{3g} \\ -g_{41} & -g_{42} & -g_{4c} & -g_{43}-g_{41} & g_{44}-g_{42} & -g_{4g} \\ -g_{1g}-y(1g/12) & g_{g2}' & -g_{cg} & g_{g3}'' & g_{g4}'' & +g_{gg} \end{vmatrix}$$

Equation 3.5 c

where $g_{12}'' = g_{12}' + y(3g/42) + g_{32}$

and $g_{33}'' = g_{31} + g_{33}' + y(3g/34)$

and $g_{g3}'' = -g_{3g} - g_{1g} - y(1g/12)$

and $g_{g4}'' = -g_{4g} - g_{2g} - g_{2g} + y(1g/12)$

Then second cofactor Y_{cg}^{cg} and equicofactor $Y_g^g = Y_j^i$ are of the forms of Equations 2.3 j and E.3 (See Appendix E) respectively and therefore subject to the same simplifying relations:

$$\det \begin{vmatrix} A_1 & A_2 \\ A_3 & A_4 \end{vmatrix} = \det A_1 \det A_4$$

Equation 3.5 d

whenever $[A_2] = [0]$ and/or $[A_3] = [0]$

3.6 *GENERALIZATION OF THE SIMPLIFICATION OF HOMOLOGOUS BINETS*

It is evident that a more general and perhaps rigorous statement on the simplification of homologous binets should be given. This will be done in two steps. First, an overall view can be afforded by the partitioning of the matrix of a convenient second cofactor or of a first cofactor of the equicofactor matrix. By multiplying this by the partitioned elementary matrices, which are indicated by the row and column transformations of Section 2.3, and by the use of Equation 3.5 d, the simplifying procedure will be made evident provided that certain of the original submatrices are equivalent. That this is indeed the case will be indicated then by the application of Percival's series. (Equation 3.3 b on page 49)

It is clearly evident that the simplification procedures of Section 2.3 will reduce to zero the elements of y_{ic} and y_{ci} for all $i \in c$ when these appear in the cofactor, for these are always chosen as homologous elements. The mechanism for reducing the remaining cofactors, for example Y_{CR}^{CR}, will be the above-named column and row transformations. For the matrix of Y_{CR}^{CR} the multiplication of row $(n + i)$ by (-1) and its addition to the row i for $i = 1$, ----n is equivalent to its premultiplication by the elementary transformation matrix $[P]$:

$$
[P] = \begin{array}{c}
\begin{array}{cccccccccccc}
 & & & & & & n^{\underline{th}}\ \text{column} & (n+1)^{\underline{th}}\ \text{column} & & & & \\
 & & & & & & \downarrow & \downarrow & & & &
\end{array} \\
\left[\begin{array}{cccccccccccc}
1 & 0 & \cdot & \cdot & \cdot & 0 & -1 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & 1 & 0 & \cdot & \cdot & \cdot & 0 & -1 & 0 & \cdot & \cdot & \cdot \\
\cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 \\
0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1
\end{array}\right]
\begin{array}{l}
\\ \\ \\ \\ \\
\leftarrow n^{th}\ \text{row} \\
\leftarrow (n+1)^{\underline{th}}\ \text{row} \\
\\ \\ \\ \\ \\
\end{array}
\end{array}
$$

The combination of multiplying column k by +1 and adding to column (n+k) for all k = 1....n can be accomplished by postmultiplying with the transformation matrix [Q]:

$$
[Q] = \begin{array}{c}
\begin{array}{cccccccccc}
 & & & & & (n+1)\ \text{column} & & & & \\
 & & & & & \downarrow & & & &
\end{array} \\
\left[\begin{array}{cccccccccc}
1 & 0 & \cdot & \cdot & 0 & 1 & 0 & \cdot & \cdot & 0 \\
0 & 1 & 0 & \cdot & \cdot & 0 & 1 & \cdot & \cdot & \cdot \\
\cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 \\
0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1
\end{array}\right]
\begin{array}{l}
\\ \\ \\ \\
\leftarrow n^{\underline{th}}\ \text{row} \\
\\ \\ \\ \\ \\
\end{array}
\end{array}
$$

For definiteness, a simple matrix is of use here, for an extension to the general case is immediate. Let there be an (n = 2) and a (2n + 2) = 6 node network having a $6^{\underline{th}}$ order indefinite admittance matrix and a fourth order Y_{CR}^{CR}, which will be made up of the four second-order submatrices A_1, A_2, B_1 and B_2. According to the above,

$$[Y]' = [P]\,[Y^{CR}_{CR}]\,[Q] \qquad \text{Equation 3.6 a}$$

will be:

$$[Y]' = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [A_1] & [B_1] \\ [B_2] & [A_2] \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then

$$[Y]' = \begin{bmatrix} [A_1-B_2] & [B_1-A_2] \\ [0+B_2] & [0+A_2] \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} [A_1-B_2] & [A_1-B_2+B_1-A_2] \\ [B_2] & [B_2+A_2] \end{bmatrix}$$

Because of binet homology, $[A_1] = [A_2]$. Because of the homology of some and the reciprocity of other (Passive) elements, $[B_1] = [B_2]$ so that:

$$\det [Y]' = \det \begin{bmatrix} A_1-B_2 & [0] \\ [B_2] & [B_2+A_2] \end{bmatrix} \qquad \text{Equation 3.6 b}$$

where $[0]$ is the second-order null matrix.

It is of interest to examine bases for the equivalences:

$$[A_1] = [A_2] \quad \text{and} \quad [B_1] = [B_2] \qquad \text{Equation 3.6 c}$$

It is only the shunt-connected elements across homologous nodes of the binet which cause dependence on *reciprocity* (ie here, $y_{31} = y_{13}$ and $y_{42} = y_{24}$). These will always appear on the principal diagonal of the submatrices. All other elements will be *homologous* in the binet, and the equivalences will depend only on that fact. It should be noted here that non-reciprocal elements connected across homologous nodes of the binet can be accounted for by a diagonal expansion as in Section 2.5, since again, all such elements appear only on the principal diagonals of the submatrices. Now, visualization of the way in which the transactors of various degrees adhere to these statements is by no means trivial, so that a more detailed description of the submatrices will be needed to maintain the identities of elements moved about by the partitioning operations.

It will now be convenient to break up the sum of matrices representing (A_1) into *inter*bipart passive elements and *intra*bipart voltage-current transactors of degrees two, three and four, called VCT_2, VCT_3 and VCT_4 respectively. Then for submatrix $\left[A_1\right]$, using Equation 3.3 b for the intrabipart elements, there is:

$$\left[A_1\right] = \sum_{i=1}^{n} e_i\, e_i{}'\, y_{ii} + \sum_{i=1}^{n-1} t(ip/ip) + \sum_{j=1}^{n-1} t(j1/q1) + \sum_{k=1}^{n-1} t(kr/ms) \qquad \text{Equation 3.6 d}$$

where e_i and e_i' are the column and row unit vectors respectively, with an n x n product and $p>i$ $q,l>j$ and $r,m>k$ respectively.

The last two terms of Equation 3.6 d refer to a set of intrabipart elements only and these will of course be representable by an equicofactor matrix of order n and rank (n-1). On the other hand, the interbipart elements including the transactors connected to the ground terminal 2n+1, will be representable as a sum of the values of such components in the particular row or column of the original indefinite admittance matrix $[Y]$. These sums will appear in $[A_1]$, quite naturally, only on the principal diagonal of the $[A_1]$ matrix, because they are connected to nodes out of the bipart under consideration.

Now it is also plain that $[A_2]$ for the other bipart can be represented as:

$$[A_2] = \sum_{i=n+1}^{2n} e_i\, e_i'\, y_{ii} + \sum_{i=n+1}^{2n-1} t(ip/ip) +$$

$$\sum_{j=n+1}^{2n-1} t(jl/ql) + \sum_{k=n+1}^{2n-1} t(kr/ms) \qquad \text{Equation 3.6 e}$$

where the inequalities of Equation 3.6 d still pertain. The first term of Equations 3.6 d and 3.6 e refers to equivalent values provided the "n" shunt-connected interbipart elements, as represented, obey reciprocity and the $\frac{2n\cdot(n-1)}{2}$ cross-connected and ground-return transactors are equivalent due to their homology. The second term on the right of Equations 3.6 d and e refer to the two terminal transactors, *positive and negative,* and these depend *only* on element homology. The latter is strictly an intra-bipart element.

The third terms represent transactors VCT_3 or CVT_3 but all four non-zero elements of the n x n matrix $t(j1/q1) = y(j1/q1)\, E(j1/q1)$ will be confined to the respective submatrices. A similar statement can be made for the VCT_4 or CVT_4.

The equivalence of the submatrices $[B_1]$ and $[B_2]$ is surprisingly similar in that reciprocity is required only along parts of the trace elements of the submatrices while all other elements are equivalent, due to their homology. Submatrices $[B_1]$ and $[B_2]$ can be expanded in a matric series as in Equations 3.6 d and e, but of course four-entry (four non-zero elements) transactors will not appear:

$$[B_1] = \sum_{i=1}^{n} \sum_{j=n+1}^{2n} e_i \, e_j' \; y(ij/ij)$$

$$[B_2] = \sum_{i=n+1}^{2n} \sum_{j=1}^{n} e_i \, e_j' \; y(ij/ij)$$

Reciprocity is required for some part of $y(ij/ij)$ when $i=j$ only.

It is seen therefore that the simplification effected by Equation E.5 is substantially dependent on the binet's homology - reciprocity being required only on n of the $(n-1)^2$ independent elements. Even this encumberance was not serious because such elements appear on the principal diagonal and can be handled along with ground return unbalances by the method of Section 2.5.

3.7 ANALYSIS - 2 IN RETROSPECT

Having repeated Chapter II for the case of active networks, and established that the means of simplification depend essentially on network homology rather than reciprocity, it will now be convenient to establish definitions for useful balance factors and means of their measurement. These, along with some practical applications will comprise the work of Chapters IV and V.

IV. THE SPECIFICATION AND MEASUREMENT OF CONDUCTED ELECTROMAGNETIC SUSCEPTIBILITY

4.0 Definition of the Problem. Government agencies and telephone and power utilities in several countries have been instrumental in establishing a number of common-mode interference specifications for electronic equipments and subsystems, and in general they have had similar objectives. However some differences in the the specified means of measurement have produced important and devious affects and therefore whenever possible individual engineers have tended to use tests which are suited in special ways to their own particular applications. Some of these appear explicitly in the Reference List such as Mil-I-6181-D and Mil-Std-826. The task of including the forementioned differences in a coherent general specification for CM measurements has proven to be difficult (See for example, Reference 4.0 a) and for at least homologous binets, some suggestions will be considered herein.

The underlying objective of measurements and tests of interest here will be to calculate, measure and thereby facilitate the reduction of unwanted noise voltage appearing across *any* number of nodes. Some evaluation or disposition will have to be made of the following in order to realize this objective:

1. Establishment of a basis for lumped equivalence.
2. The effective internal lumped-impedance of the interfering source.
3. The magnitudes of the more significant frequency components of

the interfering wave.

4. The effective impedance of the nodal configuration subject to the interference.

5. The amplification/gain or attenuation/loss of the network from source to normal mode nodal-pair.

6. The "figures of merit" of most general interest and utility to describe the quantity in (5) under the conditions of the previous four.

Another objective of the measurement problem is that of determining the actual conducted interference susceptiblity or balance factor of most significance for a given network. This will involve flexibility in the choice of nodes and a means of accounting for the influx of equivalent source power into various nodal configurations of the network.

This set of objectives may seem overmuch ambitious in the sense that a really satisfying disposition may not even be possible. However this is of the essence in the solution of a truly practical problem, where the questions to be answered appear as physical phenomena, rather than as academic exercises offered by the problem solver himself. In this paper, there will at least be an attempt to make inroads toward an eventual solution.

4.1 LUMPED EQUIVALENCE OF PHYSICAL CONFIGURATIONS

Although it is not always appreciated, distributed parameter systems were studied and important contributions made from 100 to 250 years ago by several of the great intellects such as Bernoulli, d'Alembert, Euler, Stokes, Helmholtz and Maxwell.

It was not until relatively recent times, however, that attempts were made to unify those principles variously applied to electric networks, fluid mechanics, heat conduction and the like. In the forties, G. Kron (References 4.1 a, 4.1 b, 4.1 c) made an attempt to replace several distributed parameter systems by lumped electrical networks. In his 1962 Ph-D dissertation, D. Pierre conducted a thorough survey of existing distributed-parameter literature especially pertinent to control applications and provided well over four-hundred related references. (See Reference 4.1 d) This substantial effort provided general methods for the transient analysis of distributed parameters systems in the time and generalized frequency domains.

Our interest here lies in the identification of certain physical situations with their effect on a given lumped parameter network. Toward this end several alternative paths are possible, depending on the allowable degree of approximation. When this level of approximation is to be kept low, results do not come easily for most practical configurations. (See Appendix *D*)

Practical alternatives to the foregoing comprise the more usual means of handling the equivalence problems. The more important distributed parameters are usually identified by either a previous experience, measurement, or by direct calculation and one of the requirements for the analysis or synthesis would involve the satisfaction of a performance specification thruout a range of expected values. Thus an anticipated electromagnetic disturbance may be known to be predominantly magnetic implying a low resistance equivalent source. The output of a transistor would have a known lumped collector capacitance. The parasitic impedances of a tunnel diode device are well defined, interelectrode capacitances of tubes, leakage reactances, interwinding and interturn capacitances of transformers and magnetic amplifiers are well known, resistors and capacitors are made with known values of inductance, etc. In fact, wherever the pass band and/or stability of a device is of critical importance, a range of stray parameter values is usually known, and often specified. Special configurations of wire leads have also been determined. (For example References 4.1 d, e, f, g, h). To illustrate, the latter reference gives an accepted equivalent circuit of a bond strap and for various bond strap dimensions and physical positions with respect to a ground plane, offers a table of values for use at *different* frequencies from 1 - 1000 mc/sec. (Figure 4.1 a)

It is seen therefore that a measurement of the common mode susceptibility of homologous binets will most likely involve lumped-equivalent parameters typifying stray effects as well as out of tolerance components. Unfortunately, the values of these lumped equivalents will not always be subject to checking calculations, so that the measurement must be made mindful of the possibility of practical interference conditions to which the tested network is more susceptible.

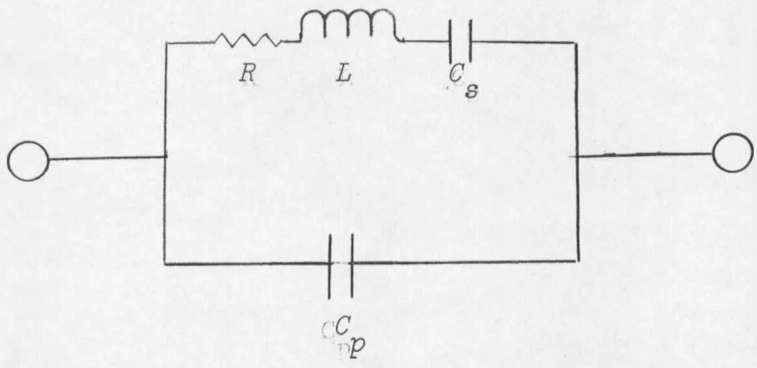
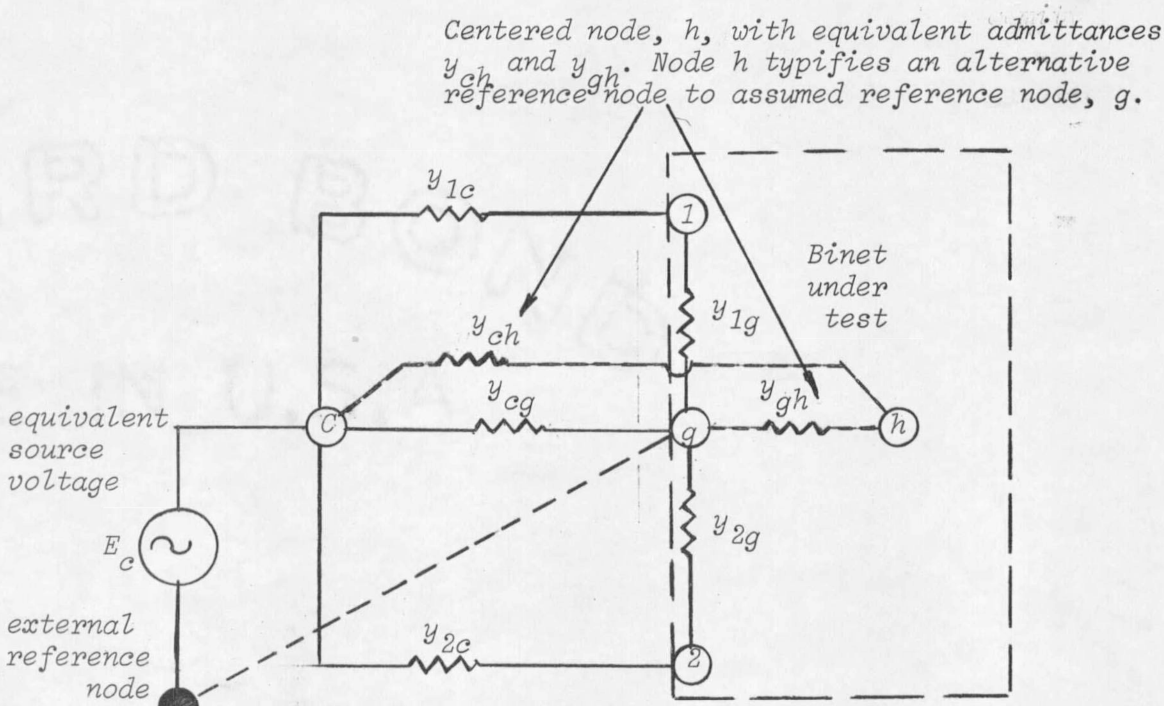


Figure 4.1a *Equivalent circuit of a bond strap*



Note: A Balance Factor $BF = \dfrac{V_1 - V_2}{V_C - V_g}$ *will determine the balance of driving point impedances* Z_{1g} *and* Z_{2g}*. Therefore it can be assumed that the binet can be replaced by components* y_{1g} *and* y_{2g}*.*

Figure 4.2a *Measuring the Balance Factor with a multiple source*

4.2 MULTIPATH INTERFERENCE SOURCES

In historical retrospect, common mode susceptibility *due to wire pickup* has always been given as a *ratio of voltages* and interpreted as a conversion from longitudinal to transverse voltage. There seems to be sound reason for such a balance factor based on the usual physical configurations under measurement as well as from the obvious fact that a power ratio based on equivalent impedance levels is specified by a voltage ratio.

The basic quantities involved in specifying common-mode wire pick-up susceptibility are the prevailing dynamic electromagnetic field causing the interference and the resulting unwanted voltage across a pair of terminals of the circuit interfered with. Although in many applications the results may prove to be the same, it seems apparent that there will be relatively few conditions where the CM to DM "loss of *available power*" (as opposed to the voltage attenuation depicting common mode susceptibility) will have more physical meaning. In the next section, it will be seen that these represent several important practical cases, however. In the case of interference waves conducted into equipment in a common mode, coupling between the spatial field thruout a relatively large physical volume and the particular pair of terminals in question will determine the susceptibility without respect to the electromagnetic energy of the field thruout the volume.

A ratio of voltages is considered a desirable measure of common-mode susceptibility. The single voltage source can appear as in Figure 4.2 a, with source impedances Z_{1c}, Z_{2c}, Z_{cg}, Z_{ch}, etc. These can be chosen to approximate known source impedances or merely to match impedances. Conceivably these source impedances might be chosen to produce either a best or a worst case testing configuration and the resulting "Balance Factor" would have yet another commensurate meaning. For simplicity only the driving point unbalance is measured in Figure 4.2 a.

The indicated balance factor or common-mode to normal-mode attenuation will be:

$$BF = \frac{V_1 - V_2}{V_c - V_g} = \frac{Y_{cg}^{12}}{Y_{cg}^{cg}} \qquad \text{Equation 4.2 a}$$

where the indefinite admittance matrix is

$$[Y] = \begin{bmatrix} y_{11} & -y_{12} & -y_{1c} & -y_{1g} \\ -y_{12} & y_{22} & -y_{2c} & -y_{2g} \\ -y_{1c} & -y_{2c} & y_{33} & -y_{cg} \\ -y_{1g} & -y_{2g} & -y_{cg} & y_{44} \end{bmatrix} \qquad \text{Equation 4.2 b}$$

where $y_{11} = y_{12} + y_{1g} + y_{1c}$

$y_{22} = y_{12} + y_{2g} + y_{2c}$

$y_{33} = y_{1c} + y_{2c} + y_{cg}$

$y_{44} = y_{1g} + y_{2g} + y_{cg}$

$y_{1c} + y_{2c}$ are the assumed equivalent source impedances and these are usually equal. Then *immediately:*

$$BF = \frac{y_{1c}y_{2g} - y_{1g}y_{2c}}{(y_{1g} + y_{1c})(y_{2g} + y_{2c}) + y_{12}^2} \qquad \text{Equation 4.2 c}$$

If a number (say k) of other power control or output leads were run in this same interference environment and connected to the tested network this physical situation could be introduced into the test by using k equivalent source impedances Z_{ic} $i=1---k$.

The resulting multifarious source configuration would be more meaningful than the customary one which relates to interference pickup on just a *single* pair of leads.

4.3 OTHER PATHS OF INFLUX

Of course there are other paths of influx which can result in CM interference and formally suggested methods of susceptibility measurement may have equally questionable results. Of these an important case is found when interfering and interfered networks share common impedances.

Suppose for example, that the impedance of a bonding strap is common to different circuits and that interference voltages appear across the strap due to the normal operation of one of the circuits. Depending on the kind of voltage wave existing across the strap, the equivalent circuit of this interfering source can be made to be a single impedance in series or shunt with a Thevinin's or Norton's source. The nature of this interfering wave and its source impedance driving such a bonding strap is indeed of significance here, and unfortunately one which has received too little attention in the literature. Its equivalent (open-circuit) impedance might be anything from that approaching zero to an open circuit.

In a digital system, consider the ground return current of a *degenerated pulse* caused by stray shunt-capacitances to a conducting chassis. The magnitude of an equivalent source impedance for such an interference source may approach the low value of the main power source of the system itself; and its frequency components would be of the order of the rise time of the digital system. On the other hand, the more easily determined ground-return *signal* currents will be largely fixed by the load impedances from which these currents are returning. The equivalent source impedance for this interference should be of a higher magnitude and the frequency components will be essentially those of the transmitted pulses. Evidently the measurement of the common mode rejection (or voltage attenuation) ratio as usually defined, will not necessarily involve the worst case of interference. However, a measurement referencing a transverse output to the *available power* would automatically be compared with the worst case.

Thus in Figure 4.3 a, the worst case for an equivalent Thevinin's impedance Z_{thev}, simulating that of the degenerated pulse source, would be a driving point impedance at terminals c and g which would compliment and

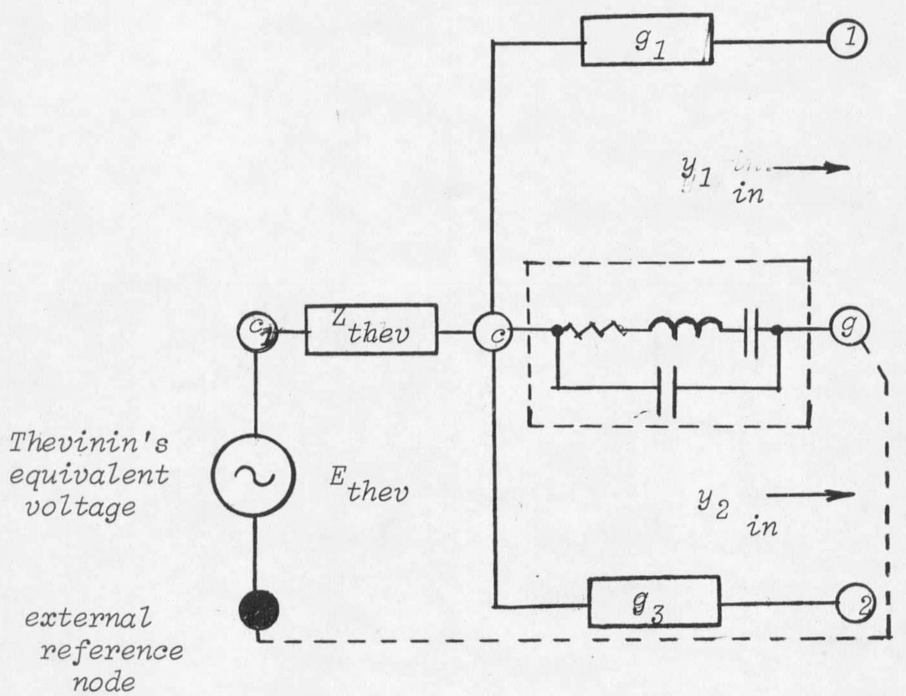


Figure 4.3a *Measurement of susceptiblity to common impedance interference.*

match the magnitude of the impedance of Z_{thev}. Here Z_{thev} would probably be low and capacitive at most pertinent frequencies, so the worst case would involve a low and inductive driving point impedance. A mere voltage attenuation ratio involving only E_{thev} and the resultant differential output would have much less quantitative significance. Figure 4.3 a is similar to 4.2 a except Y_g is the equivalent impedance of a bonding strap in particular (Reference 4.1 g) while another node C_1, has been added to accomodate the added source impedance Z_{thev}. Then the "available power" is $\frac{(E_{thev})^2}{4 \text{ Re } (Z_{thev})}$ and this related to the transverse power across terminals 1 and 2 would be a measure of the common mode susceptibility.

Still another path for conducted interference is that which has been called a transducer *common-mode voltage* (Reference 1.2 e) and is the result of a longitudinally applied bias. This may be the d.c. voltage applied to the reference of a pressure transducer or the inadvertent common-mode voltage arising from a temperature gradient along the length of a bonded thermocouple. The measurement is similar to that shown in Figure 4.2 a and it is of course possible that more than just two equivalent source impedances will be required.

According to the last named reference, the three types of common mode interference mentioned in this section and covered by Figures 4.1 a and 4.2 a are sufficiently broad to be representable of most applications. The next task in setting up a common-mode measuring circuit will be to investigate the source impedances to be used.

4.4 *EQUIVALENT SOURCE IMPEDANCES*

For purposes of measurement and calculation, an equivalent source can be chosen to provide one of the following:

i Simulate best (or worst) case interference conditions.

ii Simulate an anticipated interference source impedance.

iii Maximize sensitivity of the binet to an increment of unbalance.

iv Match the CM or DM driving point impedances.

As previously mentioned for single frequency sine wave testing into just two homologous nodes, a worst case should exist when the equivalent source impedances complement and match the driving point impedances of the network with respect to the reference, (node g in Figure 4.2 a) a condition tantamount to an increase in the common-mode equivalent generator voltage. Then for determining the $i^{\underline{th}}$ equivalent source, one would take for the driving point impedance, Z_i:

$$Z_i = \overline{Z}_{ig} = \frac{\overline{Y}^{ig}_{ig}}{\overline{Y}^{g}_{g}} \qquad \text{Equation 4.4 a}$$

where the bar represents a complex conjugate operation on the original matrix $[Y]$. Here $[\overline{Y}]$ is still double-centered (Reference 4.4 a) so that $\overline{Y}^g_g = \overline{Y}^i_j$ for i, j = 1---n (Reference 4.4 b). Therefore the determination of each equivalent source impedance would merely be that of finding the value of the second cofactor of Equation 4.4 a.

Simulation of a "worst case" when more than two homologous nodes are involved is not quite as simple, for there is no reason to assume that conjugate source impedances will now produce a worst case of interference susceptibility. Indeed, it is intuitively prospective that for every interference source and (2n + k)-node homologous binet, there will be (n + k) distinct values of source impedances which will give a highest susceptibility to common mode interference. Such a worst-case equivalent source configuration should be determinable by recognizing that a necessary condition for the existence of an extremum of a certain source configuration will be $\frac{df}{dZ_i} = 0$. A sufficient condition of a maximum or minimum relative to values of f at neighboring values of $y_i = \frac{1}{Z_i}$ is that $\frac{d^2f}{dy^2} < 0$ or $\frac{d^2f}{dy^2} > 0$ respectively, there. Now there are (n + k) source impedances possible for a homologous binet. For a stationary value of $f = \frac{Y^{n,2n}_{cr}}{Y^{cr}_{cr}}$ there is:

$$df = \frac{\partial f}{\partial y_1} dy_1 + \frac{\partial f}{\partial y_2} dy_2 --- \frac{\partial f}{\partial y_n} dy_n + --- + \frac{\partial f}{\partial y_{n+k}} y_{n+k} = 0 \qquad \text{Equation 4.4 b}$$

But the only constraints placed on the equivalent source impedances are that $y_{1c} = y_{n+1,c}$, etc. and since these constraints are removed by the simplification process, the y_i values are essentially independent so that Equation 4.4 b becomes:

$$\frac{\partial (Y_{c\ r}^{2\ 2n})}{\partial y_i} = \frac{Y_{c\ r}^{n\ 2n}}{Y_{cr}^{cr}} \frac{\partial Y_{cr}^{cr}}{\partial y_i} \qquad i=1 \text{ --- } (n+k) \qquad \text{Equation 4.4 c}$$

As to be expected, Equation 4.4 c is trivial unless an unbalance exists. For either a shunting or series unbalance, values of $(Y_{cr}^{cr})'$ and $(Y_{cr}^{n,2n})'$ are evidenced from Equations 2.4 d or e. Then for a shunting unbalance $(\Delta_{i,2n+1})$ in the $i^{\underline{th}}$ component,

$$(Y_{cr}^{cr})' = Y_{cr}^{cr} + \Delta_{i,2n+1} Y_{cri}^{cr\ 2n+1}$$

and

$$(Y_{cr}^{n,2n})' = \Delta_{i,2n+1} Y_{c,r,i}^{n,2n,2n+1}$$

since $Y_{cr}^{n,2n}$ must always be zero. Then a condition for the worst case is:

$$\frac{\partial^2 (Y_{cr}^{n,2n})}{\partial y_i^{\ 2}} Y_{cr}^{cr} < \frac{\partial^2 (Y_{cr}^{cr})}{\partial y_i^{\ 2}} Y_{c,r}^{n,2n} \qquad \text{Equation 4.4 d}$$

These calculations would be made to determine a set of n+k equations whose solutions for y_i would define the configuration of source impedances to which the binet would be most susceptible. It may be desirable to use these values in a simulation measurement. However it may be possible to use the best case where part of the source configuration is within the designer's control. Therefore as a tool of synthesis and further study:

$$\frac{\partial^2 Y_{cr}^{n,2n}}{\partial y_i^2} Y_{cr}^{cr} > \frac{\partial^2 Y_{cr}^{cr}}{\partial y_i^2} Y_{cr}^{n,2n} \qquad \text{Equation 4.4 e}$$

Thus if an influx *at discrete frequencies* is expected on an input nodal pair, it may be possible to couple the interference to *other* nodes in such a way that a given balance factor is improved. The basis for such an improvement might be the result of a continuously equivalent synthesis with optimization of the balance factor as a performance criterion.

ii As mentioned in Section 1.4 of Chapter I, *specification of a realistic source impedance* is by no means straightforward. When a voltage ratio (attenuation) balance factor is to be used, the source impedance configuration to use will not readily follow. A recent professional disposition (Reference 4.0 a) has recognized some of this problem by proposing that all susceptibility tests at least include the split-resistor configuration which matches (the magnitude of) the driven homologous node pair and that further tests be made when other source impedances are of interest.

The scope of interest in common mode susceptibility tests is actually not limited to any upper frequency in common use, but a survey of the literature would indicate that most serious interference occurs at frequencies below 10 mc. In particular, a revealing paper (Reference 4.4 c) on the electromagnetic environment of transport airplanes has reported that conducted broad band R.F. interference under airplane powered conditions is most serious between .2 and 2 megacycles. The shorter wave length is 150 meters so that in many important applications of interest, the lengths of wires 1, connected to equipment, are substantially less than a wavelength $\lambda >> 1$. Now if uniform current is assumed and the connecting wire is idealized as a monopole over a conducting ground plane, it has been shown (Reference 4.4 d) that the maximum effective aperture of the configuration is *independent* of the length of the wire and that this $A = \frac{3}{8\pi}\lambda^2$ although the effective value will be less than this depending on its termination and losses. In addition there seems to be little evidence that such a configuration can have a large resistive component. The radiation resistance of such a monopole has been calculated as $R_{rad} \doteq 400(\frac{h}{\lambda})^2$ (Reference 4.4 e for example). The derivation assumes a linear decrease in current to zero along the wire's length. For 2 mc plane wave and the above assumptions:

$$R_{rad} \doteq 400(\frac{50(.305)}{150})^2 \doteq 4 \text{ ohms.}$$

The result given by an induced - e.m.f. method for the above for $\frac{h}{\lambda} = .1$ is also 4 ohms. (Reference 4.4 f) In addition the reactance would vary from (-j 140) ohms with an effective wire diameter of about 6 inches to a reactance of (-j 430) ohms with an effective wire diameter of about one-half inch. The results of E. Hallén's work are illustrated on pages 482-485 of the latter reference and within the range of values of interest here, check with the induced emf method. Hallén's solution is that of an integral equation depicting current and is based on the known boundary conditions of the retarded potentials on the surface of the antenna. Thus it is seen that very low source impedances may be possible if the driving point impedances of the network are inductive at this frequency. For the example, inductances of from about 10 to 30 microhenries will produce resonance affects.

As straightforward as this development may seem to be, the hypotheses are subject to rather obvious grounds for criticism: The discussions have assumed an interception of plane waves: a condition which is not necessarily typical. Also a straight vertical monopole has been taken as the receiving mechanism for the interference. Perhaps most serious of all, the termination of the free end (the transducer end) has been assumed open-circuited. This is questionable in practice for a high frequency cable sheath is often shorted to a convenient ground perhaps through a bonding strap. (For example See Reference 4.4 g) Under these conditions the configuration more nearly approaches that of a high frequency transmission line of $l \lessapprox \frac{\lambda}{4}$ for which it is well known (See Reference 4.4 h) that:

$$Z_s = +j\, Z_0 \tan \beta l \doteq j(L/C)^{\frac{1}{2}} \tan\{\omega l(LC)^{\frac{1}{2}}\}$$

where L and C are the pertinent per unit length values of a high frequency line. Of course this is inductive, and the mechanism for conjugation will be a leading driving point impedance.

From the above, it seems that any general measurement procedure specified for *wire pickup* interference should not be expected to anticipate source impedances. However it is also evident that certain known configurations of source impedances do exist and that CM interference susceptibility of a network should be capable of describing isolation from *that* source-set when this latter is properly defined. This latter situation is especially true of susceptibility tests made to simulate CM transducer interference.

As indicated in Section 4.3, interference due to a common-impedance source will require a somewhat different approach and practical differentiation between the circuit under test and the interfering circuit may be difficult. In order to measure susceptibility to this kind of interference, it will be necessary to consider the ground configuration as part of the tested network, and distinct from the source. Therefore a lumped equivalent generator can be specified to reflect the physical make-up of the actual interference source. A meaningful susceptibility factor would be determined by the amount the configuration rejects the "available power" at the terminals of this equivalent generator.

iii One can define the sensitivity S of the balance factor (say BF) to a change in unbalance Δ as:

$$S = \frac{\partial(BF)}{\partial\Delta} \qquad \text{Equation 4.4 f}$$

In order to maximize this sensitivity with respect to a change in any of the testing circuit's components (Z_m) it would be necessary to find the value of Z_m defined by the extremum:

$$\frac{\partial S}{\partial Z_m} = \frac{\partial^2 (BF)}{\partial Z_m \partial \Delta} = 0 \qquad \text{Equation 4.4 g}$$

An example of the use of Equation 4.4 g is given in Reference 1.5 b.

iv The testing circuit is sometimes required to effect an impedance match on the input terminals of the binet under test. These conditions are illustrated in Figures 1.5 a and 1.5 b.

4.5 THE MAGNITUDE AND FREQUENCY OF THE INTERFERING WAVE

The Laplace Transform e(s) of an interference stimulus acting upon a *linear* system to produce a certain response w(s) through a system function h(s) can be written

$$w(s) = e(s)h(s) \text{ where } s = \sigma + j\omega = \text{generalized frequency} \qquad \text{Equation 4.5 a}$$

(Reference 4.5 a) This reference also shows that the transient response (in the time domain) can then be readily determined from either the Laplace Inversion Integral,

$$w(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} w(s) e^{st} ds \qquad \text{Equation 4.5 b}$$

or more directly from the convolution integral

$$w(t) = \int_0^t h(t-T)\ e(T)dT \qquad \text{Equation 4.5 c}$$

It is also readily seen that if the applied voltage e(s), is an impulse function then $e(s) = L(\delta(t)) = 1$ so that $w(s) = h(s)$. Therefore the transient response w(s) is simply the transform of the response to a unit impulse. This means that a steady-state sinueoidal response at all frequencies will contain *all* the information necessary to completely determine the transient response provided only that the transient excitation is Laplace Transformable. It is obvious however that this will require an analytical description of the common-mode susceptibility and that this may be very difficult to obtain. There have been papers written to make this work less toilsome (See Reference 4.5 b for example). Here the response function is written down as a Fourier series analysis of the square wave response:

$$w(t) \doteq A_o + \sum_{n=1}^{N} A_{2n-1} \sin \{(2n-1)\ \omega_1 t + \phi_n\} \qquad \text{Equation 4.5 d}$$

where $\omega_1 = 2nf_1 + f_1$ is the fundamental frequency of an applied square wave. The amplitude A_k and phases ϕ_k $k=1$ --- n can then be evaluated directly from the frequency response data.

Since the interest here is more of a fundamental nature, a variable-frequency steady-state sinusoidal source will be assumed. The only limit to its magnitude will be that indicated by the network's linearity capability.

4.6 *FIGURES OF MERIT DEPICTING BALANCE*

The definition of a most suitable balance factor to represent the common-mode susceptibility of a network may not be obvious from the foregoing. At the very least, further standardization is indicated in the area of common mode susceptibility measurements. It is expected that an initial effort would seek to provide definitions for the set of CM concepts which have appeared somewhat ambiguously in the literature and existing standards. Perhaps some of the definitions of Appendix A of this dissertation may be useful. In any event, the definitions must be made to be consistent with as many other authoritative standards as possible. It is also clear from the foregoing as well as from the literature (References 4.6 a-e) that the standards should be written for tests to be made in terms of power as well as voltage for conducted noise tests. Then each of these should be broken down into their basic components, explicitly giving not only formulas for use in measurements, but also means of most readily calculating the values involved from the commonly available information about the network - such as configuration and element value (or a node to datum admittance matrix for the network).

In brief, definitions and methods of measurement for common-mode susceptibility should prescribe:

i the source configuration used in the test(s)

ii the magnitude and frequency

iii the Balance Factor and an optimum means of calculating it

iv the circuit hookup where alternative connections can be made. (Should distinguish between the so-called "floating output common mode gain" and the "ground reference common mode gain" etc.)

v a categorization of all existing conductance tests and definitions so that these may have a disposition in new standards.

The Common Mode Rejection Factor often used to depict common-mode susceptibility, has several disadvantages mentioned in Chapter I: It does not alone account for driving point unbalance at the input and it is limited to just two nodes designated as the input pair. These are serious shortcomings, for all of a network's "unbalance" (or balance) characteristics can be lost by shorting the input leads together. Also since the gain is referenced back to the input node pair, the customary normal mode rejection does not allow for influx into more than just two nodes.

These will be obviated by the use of the method delineated in Chapter II. For definiteness in discussion, Figure 4.6 a is given to illustrate an n-homologous node-pair binet. Four nodes 1, 2, (n+1) and (n+2) are assumed to be connected to wire leads of sufficient length to be subject to interference through (homologous) impedances $Z_1 = Z_{n+1}$.

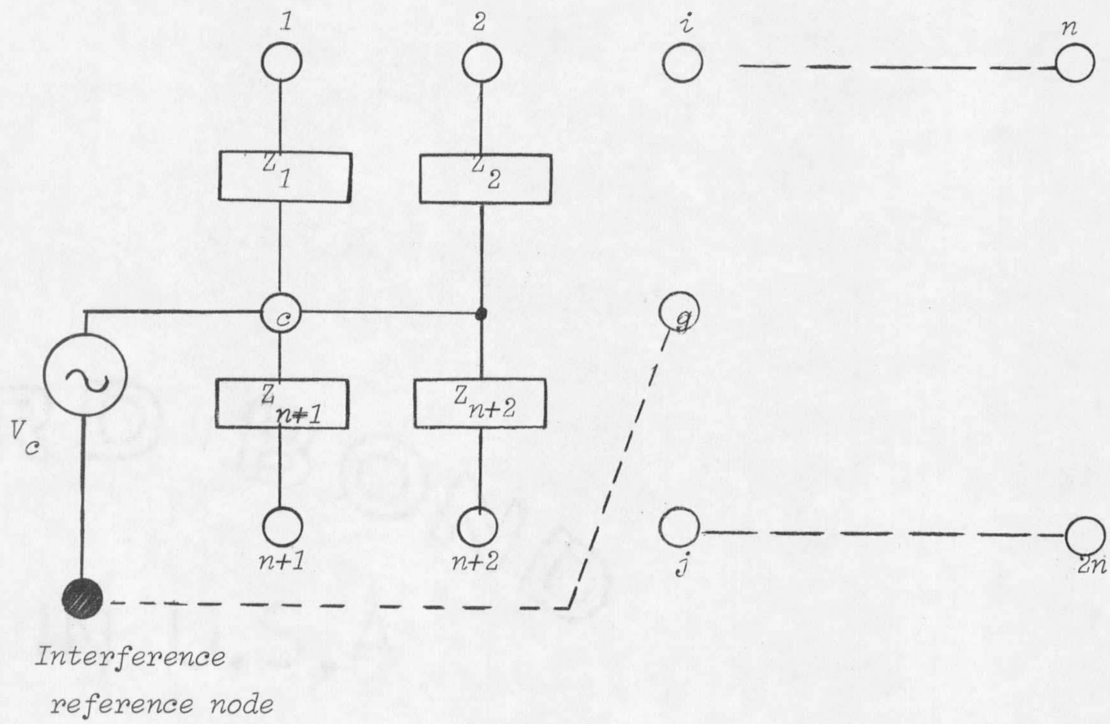


Figure 4.6a   Binet with n-homologous node-pairs.

and $Z_2 = Z_{n+2}$. Compared to V_c, the transverse voltage across nodes 1 and n+1 will be:

$$\frac{V_1 - V_{n+1}}{V_c - V_g} = \frac{Y_{cg}^{1,n+1}}{Y_g^g} \qquad \text{Equation 4.6 a}$$

Similarly for the second path,

$$\frac{V_2 - V_{n+2}}{V_c - V_g} = \frac{Y_{cg}^{2,n+2}}{Y_{cg}^{cg}} \qquad \text{Equation 4.6 b}$$

On the other hand, if interest were confined to a set of output terminals such as n, 2n then immediately:

$$\frac{V_n - V_{2n}}{V_c - V_g} = \frac{Y_{cg}^{n,2n}}{Y_{cg}^{cg}} \qquad \text{Equation 4.6 c}$$

The quantity of Equation 4.6 a depends on the equality of driving point impedances $Z_{1g} = Y_{1g}^{1g}/Y_g^g$ and $Z_{n+1,g} = Y_{n+1,g}^{n+1,g}/Y_g^g$ as does the quantity of Equation 4.6 b on driving point impedances $Z_{2g} = Y_{2g}^{2g}/Y_g^g$ and $Z_{n+2,g} = Y_{n+2,g}^{n+2,g}/Y_g^g$. The values of Equation 4.6 c on the other hand will depend on these driving point impedances, plus all other imbalances as well as the transverse gains (or losses) between each imbalance and the output.

i It is possible, by the enumeration of several alternative factors, to define a *Balance Loss Figure* (BLF) as the ratio of the interference power output of the network to the *noise* power output that would exist if the network were ideal (only thermal noise) and had a response just at the principal or desired frequency or band of frequencies.

This test would necessitate the assumption of a standard source configuration but would have the advantage of identifying the actual major noise source with the interference. Thus if the common-mode interference entered thru a pair of homologous nodes, but not the noise dependent "input pair", reference of the *noise* to the input pair would retain meaning. The Balance Figure and the nominally defined Noise Figure (For example see References 4.6 f, g) of the network would thus be related to the same "noiseless" or ideal amplifier. Evaluation of the Balance Figure from Equation 4.6 c is immediate. The Balance Figure could obviously be given in db: Balance Figure = 10 log P_I/P_N db; where P_I is the interference power output and P_N is the noise power output. A disadvantage of the BLF as defined is that it assumes that the noise level is pertinent.

Of course noise level may be much less than the expected interference. The latter may also have a maximum allowable value. Under such circumstances the Balance Sensitivity may be useful. This is discussed next.

ii Balance Sensitivity Attenuation Factor (BSAF) refers to the maximum allowable equivalent source voltage for a given source impedance configuration and given binet imbalance to maintain the interference level across a pair of homologous nodes below a prescribed limit. Thus for a given binet, source configuration, and maximum voltage allowed (V_m), the balance sensitivity in db would be:

Balance Sensitivity = 20 $\log_{10}(V_m/V_c-V_g)$ where V_m = V_i-V_j refers to the pair of homologous nodes (typically i & j) in Figure 4.5 a. This is 20 $\log_{10} Y^{ik}_{cg}/Y^{cg}_{cg}$ in terms of the second cofactors.

The figures of merit (i) and (ii) may describe the seriousness of a given interference source with respect to specific binets, but would not offer a convenient means of directly comparing two binets with respect to such a source. For this purpose, any of the following may be useful:

A Common Mode Rejection Factor (CMRF) has already been defined (Reference 4.6 h) as "the ratio of the CM input voltage to the DM input voltage that gives rise to the same output voltage". In that reference, Middlebrook divides the meaning of the CMRF into two categories: a split resistor output and a floating output. In this treatise, the conditions of zero and finite equivalent source impedances will be added so that there will be seven factors involved; each having its own specific application. The names here will be somewhat more explicit:

iii Driving Point CM Attenuation Factor (DPAF) refers to the transverse voltage across the driven nodes compared to the common mode source impedance configuration and binet imbalance. It is DPAF = 20 log V_n-V_{2n}/V_c-V_g = 20 log $Y^{1,n+1}_{cg}/Y^{cg}_{cg}$ for the homologous binet of Figure 4.6 a.

Of course the DPAF cannot involve CM - DM conversion occuring beyond the driven nodes and this will be handled exclusively by two transfer factors:

iv A CM Floating Transfer Attenuation Factor (FTAF) can be defined as the ratio of the CM input voltage divided into the DM input voltage that gives rise to the same *(floating)* output voltage. FTAF = 20 log

$20 \log (V_n - V_{2n})(V_c - V_g) K_f$ where K_f is the floating output transverse gain and the network terminals are driven by zero impedance sources.

v A CM split-load Transfer Attenuation Factor (STAF) can be defined as the ratio of the CM input voltage divided into the DM input voltage that gives rise to the same *(split-load)* output voltage. (STAF) = $20 \log V_n - V_{2n} / (V_c - V_g) A_s$. A_s is the split-load output transverse gain and the network terminals are driven by zero impedance sources.

vi A CM Floating Attenuation Factor (FAF) will include the driving point and transfer impedances of the network. It will be identical to FTAF except the network terminals are driven through finite equivalent source impedances.

vii A CM Split-Load Transfer Attenuation Factor (SAF) will include the driving point and transfer impedances of the network and will be identical to STAF except the network terminals are driven through finite equivalent source impedances.

It is of interest to note here that in calculating the exclusive transfer factors FTAF and STAF, there are two possible procedures to simplify the calculations: Since $V_1 = V_{n+1}$ when nodes 1 and n+1 are shorted together the (n+1) row and column can be added to the first row and column respectively, I_1 becoming $I_1 + I_{n+1}$ with row c and column c omitted. Of course the resulting admittance matrix will still be of an equifactor type. Instead of this procedure, the values of g_{ic} and g_{ck} in the $c^{\underline{th}}$ column and row can be set equal to zero and the simplification method of Chapter II applied immediately. In the first case, the factors will appear as a ratio of two third cofactors; in the latter case the factors will appear as a ratio of two second cofactors. There are apparently other figures of merit for depicting the balance of special configurations of balanced binets; those included here should adequately describe the CM interference susceptibility of several of the more important applications. Furthermore, each of these figures of merit imply a definite measurement configuration, the analysis of which has been simplified by the foregoing procedures.

Advantages have been evident in several respects:

i The balance factor calculation will relate an interference "output" at *any* homologous pair to an input at *any* centered pair of nodes.

ii This evident change in reference is immediate and without transformations which would otherwise be necessary.

iii The calculations involve the evaluation of first and second $(2n)^{\underline{th}}$ order cofactors by means of the simpler operation of determining the value of two-nth order cofactors. This involves the evaluation of 2 (n!) terms instead of (2n)! - a reduction of work by a factor of $\frac{(2n)!}{2(n!)} = \frac{(2n-1)!}{(n-1)!}$. For a relatively simple binet of just four homologous nodes, the number of terms involved will be cut by a factor of $\frac{7!}{3!} = 840$.

iv The number of paths of CM influx need not be limited to a single pair, for without increasing the order of the indefinite admittance matrix describing the binet under test, up to (n+1) homologous pairs can be driven by immittances representing the lumped-equivalent source.

v v A *discrete fault polynomial* and a *fault minor expansion* have been proposed (Appendix B) to place unbalance mechanisms in immediate evidence in the expression for interference susceptibility. In Chapter V, three examples are given to delineate the unbalance configurations leading to their respective usages.

V *THREE EXAMPLES*

5.0 THE SOLUTION OF A NUMERICAL PROBLEM. It was mentioned earlier that the analysis of a relatively simple homologous binet can involve a surprisingly large number of calculations, because each of the homologous elements appears twice in the binet. Although the advantage of the methods of Chapters II and III are even more evident when the number of binet nodes is large, there is still a saving in time in the analysis of simple binets.

In Figure 5.0 a, a simple resistor binet is shown with two of the "homologous" pairs unequal. The common mode rejection, CMR, is to be found. If this value were to be found by (a), a convenient but ordinary method and then (b), the methods of Chapter II, a direct comparison would be available. The solutions follow.

(a) An Ordinary Method. Since the binet of Figure 5.0 a would have five linearly independent nodal equations, the order of the node to datum admittance matrix would be five. This is:

$$[Y] = \begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}$$

where node 6 has been taken as reference. Then there is:

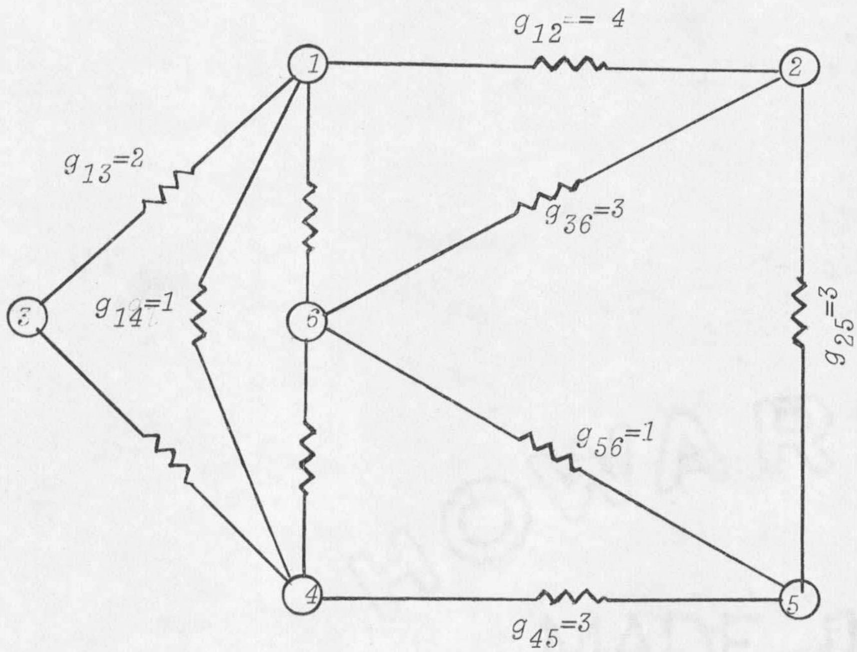


Figure 5.0a A resistor binet.

$$V_2 = \frac{\begin{bmatrix} 8 & 0 & -2 & -1 & 0 \\ -4 & 0 & 0 & 0 & -3 \\ -2 & I_c & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & 0 & 0 & -3 & 7 \end{bmatrix}}{\begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}} = \frac{A}{B} \quad \text{and} \quad V_5 = \frac{\begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & 0 \\ -2 & 0 & 4 & -2 & I_c \\ -1 & 0 & -2 & 7 & 0 \\ 0 & -3 & 0 & -3 & 0 \end{bmatrix}}{\begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}}$$

Also:

$$V_1 = \frac{\begin{bmatrix} 0 & -4 & -2 & -1 & 0 \\ 0 & 10 & 0 & 0 & -3 \\ I_c & 0 & 4 & -2 & 0 \\ 0 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}}{\begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}} = \frac{D}{B} \quad \text{and} \quad V_4 = \frac{\begin{bmatrix} 8 & -4 & -2 & 0 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & I_c & 0 \\ -1 & 0 & -2 & 0 & -3 \\ 0 & -3 & 0 & 0 & 7 \end{bmatrix}}{\begin{bmatrix} 8 & -4 & -2 & -1 & 0 \\ -4 & 10 & 0 & 0 & -3 \\ -2 & 0 & 4 & -2 & 0 \\ -1 & 0 & -2 & 7 & -3 \\ 0 & -3 & 0 & -3 & 7 \end{bmatrix}} = \frac{E}{B}.$$

Now the current I_s into the source node 3 is:

$$I_3 = (V_3 - V_1)\, g_{13} + (V_3 - V_4)\, g_{43}$$

Since $g_{13} = g_{43} = 2$: $I_3/2 + V_1 + V_4 = 2V_3$ and $I_3(\tfrac{1}{2} + \frac{E'}{B} + \frac{D'}{B}) = 2V_3$ where the prime indicates I_3 has been factored out of the cofactor. Then:

$$\text{CMR} = \frac{V_2 - V_5}{V_3 - V_5} = \frac{1}{V_3}\left(\frac{A}{B} - \frac{C}{B}\right) = \frac{I_3}{V_3}\left(\frac{A' - C'}{B}\right) = \text{CMR} = \frac{2(A' - C')}{\tfrac{1}{2}(B) + E' + D'}$$

Equation 5.0 a

Evaluating the cofactor A :

$$A = -I_3 \begin{vmatrix} 8 & -2 & -1 & 0 \\ -4 & 0 & 0 & -3 \\ -1 & -2 & 7 & -3 \\ 0 & 0 & -3 & 7 \end{vmatrix} = I_3 \left[-(-4) \begin{vmatrix} -2 & -1 & 0 \\ -2 & 7 & -3 \\ 0 & -3 & 7 \end{vmatrix} + (-3) \begin{vmatrix} 8 & -2 & -1 \\ -1 & -2 & 7 \\ 0 & 0 & -3 \end{vmatrix} \right]$$

$$= I_3 \left[4(-98+18+14) \quad -3(48+6) \right] = 256\, I_3$$

$$A' = 256.$$

A similar calculation is necessary to evaluate each of:

$C' = 636$, $\quad D' = 868$ and $E' = 946$.

The determination of B is more involved since this determinent is fifth-order:

$$B = -(-3)\begin{vmatrix} 8 & -2 & -1 & 0 \\ -4 & 0 & 0 & -3 \\ -2 & 4 & -2 & 0 \\ -1 & -2 & 7 & -3 \end{vmatrix} -(-3)\begin{vmatrix} 8 & -4 & -2 & 0 \\ -4 & 10 & 0 & -3 \\ -2 & 0 & 4 & 0 \\ -1 & 0 & -2 & -3 \end{vmatrix} +7\begin{vmatrix} 8 & -4 & -2 & -1 \\ -4 & 10 & 0 & 0 \\ -2 & 0 & 4 & -2 \\ -1 & 0 & -2 & 7 \end{vmatrix}$$

$$= +3\left[-(-4)\begin{vmatrix} -2 & -1 & 0 \\ 4 & -2 & 0 \\ -2 & 7 & -3 \end{vmatrix} -3\begin{vmatrix} 8 & -2 & -1 \\ -2 & 4 & -2 \\ -1 & -2 & 7 \end{vmatrix} -2\begin{vmatrix} -4 & -2 & 0 \\ 10 & 0 & -3 \\ 0 & -2 & -3 \end{vmatrix} +4\begin{vmatrix} 8 & -4 & 0 \\ -4 & 10 & -3 \\ -1 & 0 & -3 \end{vmatrix}\right]$$

$$+7\left[-(-4)\begin{vmatrix} -4 & -2 & -1 \\ 0 & 4 & -2 \\ 0 & -2 & 7 \end{vmatrix} +10\begin{vmatrix} 8 & -2 & -1 \\ -2 & 4 & -2 \\ -1 & -2 & 7 \end{vmatrix}\right]$$

$$\underline{\underline{B}} = 3(-96-436+72-816) + 7(-384+1520) = \underline{\underline{4064}}$$

Inserting these values into Equation 5.0 a:

$$CMR = \frac{2(538-636)}{\frac{1}{2}(4064)+946+868} = -.0508$$

(b) The New Method. Now the ideas of Chapter II can be used to determine the same CMR. The binet homology is recovered by defining unbalance elements $\Delta_1 = 1$ and $\Delta_2 = 2$ to shunt across node pairs 1, 2 and

2, 5 respectively so that:

$$g_{12}' = g_{12} + \Delta_1 = 3 + 1$$
$$g_{26}' = g_{26} + \Delta_2 = 1 + 2$$

The prime refers to the inclusion of unbalance values. The indefinite admittance matrix Y' is:

$$[Y]' = \begin{bmatrix} 7+\Delta_1 & -3-\Delta_1 & -2 & -1 & 0 & -1 \\ -3-\Delta_1 & \Delta_1-\Delta_2+7 & 0 & 0 & -3 & -1-\Delta_2 \\ -2 & 0 & 4 & -2 & 0 & 0 \\ -1 & 0 & -2 & 7 & -3 & -1 \\ 0 & -3 & 0 & -3 & 7 & -1 \\ -1 & -1-\Delta_2 & 0 & -1 & -1 & \Delta_2+4 \end{bmatrix} \qquad \text{Equation 5.0 a}$$

But $\text{CMR} = \dfrac{V_2-V_5}{V_3-V_5} = \dfrac{Y_{36}^{25'}}{Y_{36}^{36'}}$.

From Equation 2.4 d, the *fault polynomial* for $Y_{36}^{25'}$ is,

$$(Y_{36}^{25})' = Y_{36}^{25} + (-\Delta_1)(Y_{362}^{251}) + (-\Delta_2)(Y_{362}^{256} + \Delta_1 Y_{3621}^{2561})$$

$$= 0\ (+\Delta_1)(16) + (-\Delta_2)(48 + \Delta_1 6)$$

where Y^{25}_{36} must be zero; $$Y^{251}_{362} = - \begin{vmatrix} -2 & -1 & -1 \\ -2 & 7 & -1 \\ 0 & -3 & -1 \end{vmatrix} = -16;$$

$$Y^{256}_{362} = + \begin{vmatrix} 7 & -2 & -1 \\ -1 & -2 & 7 \\ 0 & 0 & -3 \end{vmatrix} = +48 \quad \text{and} \quad Y^{2561}_{3621} = +6$$

Also from Equation 2.4 d;

$$(Y^{36}_{36})' = Y^{36}_{36} + (-\Delta_1)\{Y^{361}_{362} + (-\Delta_1)Y^{3612}_{3621}\} + (\Delta_1+\Delta_2)\{Y^{362}_{362} + (+\Delta_1)Y^{3621}_{3621}\}$$

where the $\Delta_1{}^2$ terms are equal and opposite in sign giving.

$$(Y^{36}_{36})' = Y^{36}_{36} -\Delta_1(Y^{361}_{362} + Y^{362}_{362}) \quad -\Delta_2\{Y^{362}_{362} +\Delta_1 \; Y^{3621}_{3621}\}$$

Then from Equation 3.6 b:

$$Y^{36}_{36} = \begin{vmatrix} 7+1 & -3-6 & 0 & 0 \\ -3-0 & 7+3 & 0 & 0 \\ -1 & 0 & 6 & -3 \\ 0 & -3 & -3 & 4 \end{vmatrix} = (80-9)(24-9) = 1065$$

Also $Y_{362}^{361} = (-)\begin{vmatrix} -3 & -1 & 0 \\ 0 & 7 & -3 \\ -3 & -3 & 7 \end{vmatrix} = 129 \qquad Y_{362}^{362}\begin{vmatrix} 7 & -1 & 0 \\ -1 & 7 & -3 \\ 0 & -3 & 7 \end{vmatrix} = 273.$

So that:

$$CMR = \frac{V_2-V_5}{V_3-V_6} = \frac{16\Delta_1-\Delta_2(48+\Delta_1 6)}{1065+\Delta_1 144+\Delta_2 273+\Delta_1\Delta_2 40} \qquad \text{Equation 5.0 b}$$

If the values previously assumed for the Δ's are substituted in Equation 5.0 b, there is:

$$CMR = \frac{16-2(48+6)}{1065+144+546+80} = -.0508 \qquad \text{Equation 5.0 c}$$

This is exactly the same answer obtained in the nodal analysis of Part (a). Of course such agreement was expected since the solutions are both "exact". The common mode ratio of Equation 5.0 b is obtained as an explicit function of Δ_1 and Δ_2 and the advantages of such a result have already been discussed.

5.1 AN EXAMPLE OF THE USE OF THE DISCRETE FAULT POLYNOMIAL

The passive network juxtapositioned and connected with an active type input device will often determine the common mode susceptibility of an entire binet or system. In a recent paper, G. McDonald (Reference 5.1 a) offers a "typical transmission problem". (See Figure 5.1 a) Here an

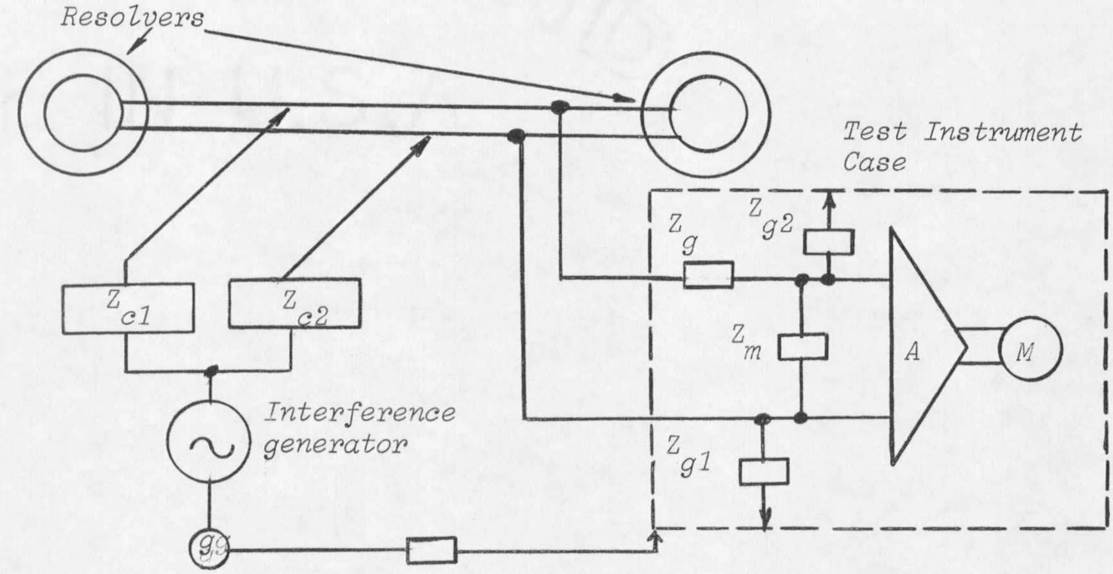


Figure 5.1a Resolver binet under measurement

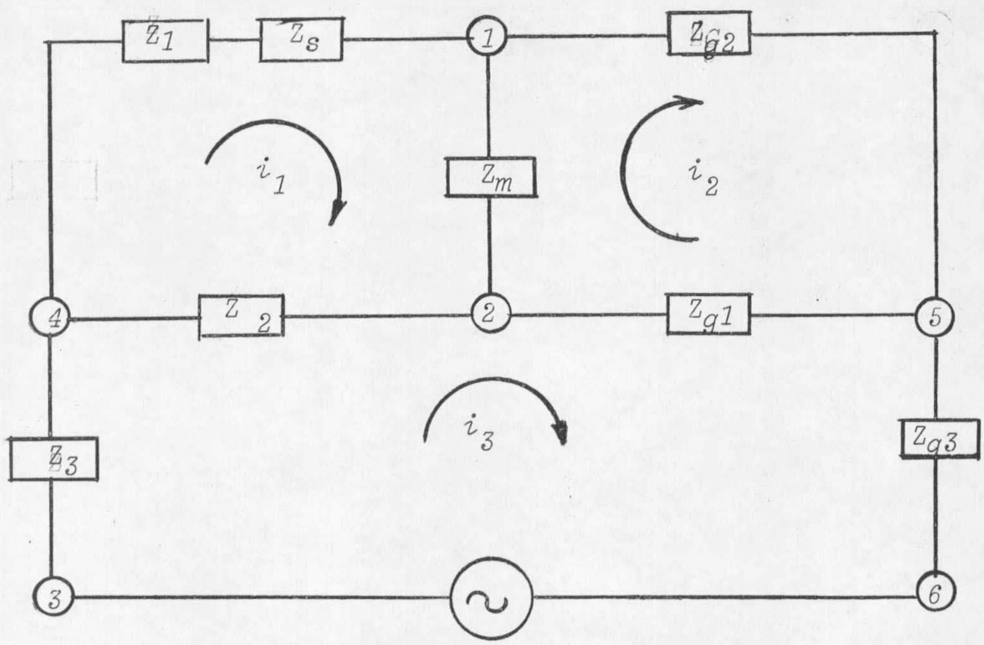


Figure 5.1b An equivalent of resolver binet of Fig. 5.1a with currents selected for the usual analysis.

asymmetric instrumentation circuit is shunted across a resolver transmission line, the latter in an interference environment. The author makes two important assumptions which could more typically modify results and it will be assumed that here, no experimental evidence is available to the contrary: (i) no current will be assumed to flow into the amplifier A, and (ii) the shunting returns to ground (Z_{g1} and Z_{g2} in Figure 5.1 a) are to be considered much more significant than those of the resolvers to ground, so that the entire return for the common mode interference source is through the paths provided by complex impedances Z_{g1} and Z_{g2}. Although these two simplifying assumptions seem limiting enough, still others are implicit. For example, the lumped equivalence of distributed parameters is not readily available at all frequencies, except by measurement. For purposes of comparison the two assumptions will be retained.

Nevertheless, it is apparent that a prolific amount of work is yet involved, using the conventional loop or nodal analysis. For purposes of simplification, one can initially use delta-wye transformations so that:

$$Z_1 = \frac{Z_r Z_c}{Z_r + 2Z_c}$$

$$Z_2 = \frac{Z_r Z_c}{Z_r + 2Z_c} \qquad \text{Equation 5.1 a}$$

$$Z_3 = \frac{Z_c^{\,2}}{Z_r + 2Z_c}$$

where $Z_r = \dfrac{\text{Resolver impedance}}{2}$

Here resolver line lengths are assumed to be equal, so that source impedance equivalents Z_{c1} and Z_{c2} are considered to be $Z_{c1} = Z_{c2} = Z_c$.

(a) The Results of a Common Method. Since each homologous pair involves two parameters implicitly, the solution of the loop equations is somewhat involved. Only the results will be given here, since the analysis is given in the referenced paper. From $V_{12} = (i_1-i_2)Z_m$,

$$\frac{V_{12}}{E_c} =$$

$$\frac{(Z_2Z_{g2} - Z_1Z_g - Z_sZ_{g1})Z_m}{Z_{g1}\left[Z_1Z_2+Z_1Z_3+Z_1Z_{g2}+Z_1Z_m+Z_1Z_{g1}+Z_2Z_3+Z_2Z_{g2}+Z_2Z_{g3}+Z_mZ_3+Z_mZ_{g2}+Z_mZ_{g3}+Z_2Z_s+Z_3Z_s +Z_{g2}Z_s+Z_mZ_s+Z_{g3}Z_s\right]}$$

$$+ Z_{g3}\left[Z_1Z_{g2}+Z_1Z_m+Z_2Z_{g2}+Z_2Z_m+Z_{g2}Z_m+Z_{g2}Z_s+Z_mZ_s\right]$$

$$+ Z_1\left[Z_2Z_{g2}+Z_2Z_m+Z_3Z_{g2}+Z_sZ_m +Z_3 \; Z_{g2}Z_m+Z_{g2}Z_s+Z_mZ_s\right]$$

$$+ Z_2\left[Z_3Z_{g2}+Z_3Z_m+Z_{g2}Z_m+Z_{g2}Z_s+Z_mZ_s\right]$$

Equation 5.1 b

Equation 5.1 b is even more involved than it appears due to the Equations 5.1 a.

If $\frac{Z_r}{Z_c} << 2$ then $Z_1 = Z_2 \doteq \frac{Z_r}{2}$ and $Z_3 \doteq \frac{Z_c}{2}$ Equation 5.1 c

Then $\frac{V_{de}}{E_c} =$

$$\frac{(Z_x Z_{g2} - Z_r - 2Z_s Z_{g1}) Z_{m/2}}{Z_{g1}\left[Z_r\left(\frac{Z_r}{4} + \frac{3Z_c}{4} + Z_{g2} + Z_m + Z_{g3}\right) + Z_m\left(\frac{3Z_c}{2} + 2Z_{g2}\right) + Z_s\left(\frac{Z_c}{2} + Z_{g2} + Z_{g3}\right)\right]}$$

$$+ Z_{g2}\left[Z_{g3}Z_r + Z_{g3}Z_m + Z_{g3}Z_s + Z_r^{\,2} + Z_c Z_r + \frac{Z_m Z_r}{2} + \frac{Z_s Z_r}{2}\right]$$

$$+ Z_{g3}\left[(Z_m Z_r + Z_m Z_s) + \frac{Z_m Z_r^{\,2}}{4} + Z_m \frac{Z_c}{2} Z_s + Z_c \frac{Z_m}{2} Z_r + Z_m Z_s \frac{Z_r}{2}\right]$$

Equation 5.1 d

Equation 5.1 b is Equation 10 of Reference 5.1 a and represents the terms involved in the third order node to datum impedance matrix of the homologous binet. Although some practical significance can be gleaned from this rather involved result, unbalancing mechanisms such as Z_s and differences in Z_{g1} and Z_{g2} are buried in the right hand side of Equation 5.1 b. In addition, the evaluation of this latter will be necessary to predict CM susceptibility, at least to the extent possible by a mere voltage ratio.

(b) The New Method. In the discussion to follow, instead of the single frequency impedance parameters $Z(j\omega)$ used above, admittances $y_{ik}(s)$ will serve. For then, using any (equivalent) interference excitation, E_c, the admittance $y_{ik}(s) = g_{ik} + sC_{ik} + \frac{1}{sL_{ik}}$ can be used to identify actual components.

Furthermore, and in line with Chapter II, all homologous components will be assumed equal, but with deviations made explicit in the matrix. Thus related to y_{g2} is $y_{g1} = y_{g2} + \Delta_1$ where Δ_1 is ordinarily complex. Similarly the series impedance Z_s of Figure 5.1 b will account for a decrease in admittance between terminal four and terminal one. Thus between nodes 2 and 4 (in the resulting Figure 5.1 c) will be y_2 and between nodes 1 and 4 will be $y_1 = y_2 - \Delta_1$. According to the above discussion, Δ_1 is assumed to be a positive quantity. Thus the simplification made possible by the homology of the binet will be retained as much as possible. According to the above and using the indefinite admittance matrix as in Chapter II, the network for analysis can be redrawn retaining the relevant subscripts of Figure 5.1 b.

From this latter figure, the indefinite admittance matrix for this binet is:

$$[Y]' = \begin{bmatrix} y_{11}-\Delta_1+\Delta_2 & -y_m & 0 & -y_2+\Delta_1 & -y_{g2}-\Delta_2 & 0 \\ -y_m & y_{22} & 0 & -y_2 & -y_{g2} & 0 \\ 0 & 0 & y_3 & -y_3 & 0 & 0 \\ -y_2+\Delta_1 & -y_2 & -y_3 & y_{44}-\Delta_1 & 0 & 0 \\ -y_{g2}-\Delta_2 & -y_{g2} & 0 & 0 & y_{55}+\Delta_2 & -y_{g3} \\ 0 & 0 & 0 & 0 & -y_{g3} & +y_{g3} \end{bmatrix}$$

Equation 5.1 e

where $y_{11} = y_2+y_m+y_{g2} = y_{22}$; $y_{44} = y_3+2y_2$ and $y_{55} = 2y_{g2}+y_{g3}$.

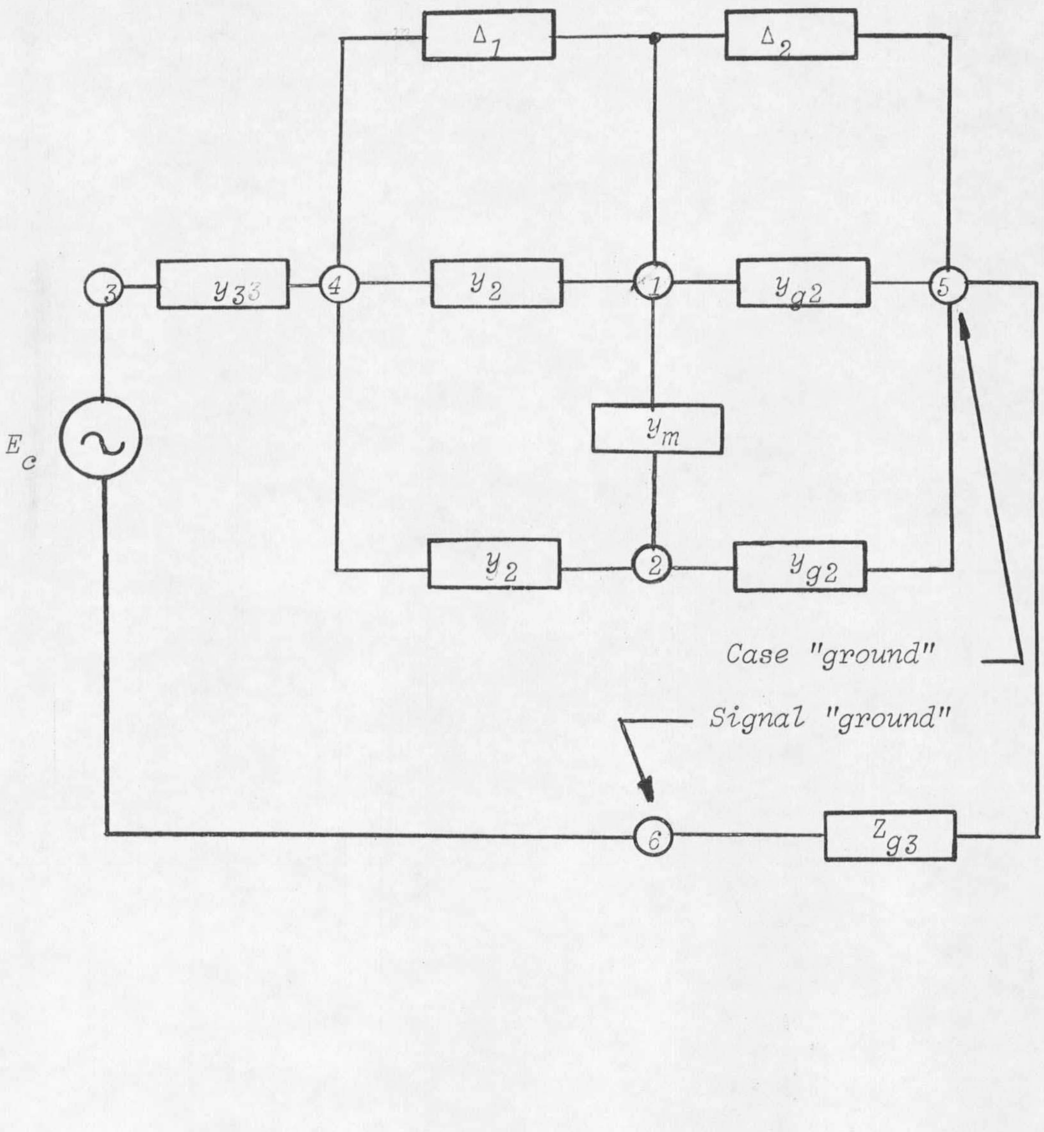


Figure 5.1c An equivalent representation of the binet of fig. 5.1a.

Then: $$\frac{V_1-V_2}{V_3-V_6} = \frac{(Y_{36}^{12})'}{(Y_{36}^{36})'} \qquad \text{Equation 5.1 f}$$

The fault polynomial in the numerator is immediate:

$$(Y_{36}^{12})' = +y_3 y_{g3}(y_{g2}\Delta_1 + y_2\Delta_2) \qquad \text{Equation 5.1 g}$$

In the denominator, there is:

$$(Y_{36}^{36})' =$$

$$Y_{36}^{36} + (\Delta_2 - \Delta_1)\left[Y_{361}^{361} + (-\Delta_1)Y_{3614}^{3614} + \Delta_2 Y_{3615}^{3615} + (-\Delta_1\Delta_2)Y_{36145}^{36145}\right]$$

$$+ \Delta_1\{Y_{361}^{364} + \Delta_1 Y_{3614}^{3641} + (-\Delta_2)Y_{3615}^{3641} + (\Delta_2)Y_{3615}^{3645} + \Delta_1\Delta_2 Y_{36415}^{36415}\}$$

$$+ (-\Delta_2)\{Y_{361}^{365} + \Delta_1 Y_{3614}^{3651} + (-\Delta_2)Y_{3615}^{3651} + (-\Delta_1)Y_{3614}^{3654} + \Delta_1\Delta_2 Y_{36154}^{36514}\}$$

$$+ \Delta_1\Delta_2 Y_{3645}^{3615} + \Delta_1\Delta_2 Y_{3645}^{3641}. \qquad \text{Equation 5.1 h}$$

When the Δ terms of Equation 5.1 h are collected:

$$(Y^{36}_{36})' = Y^{36}_{36} + \Delta_1\{Y^{364}_{361} - Y^{361}_{361}\} + \Delta_2\{Y^{361}_{361} - Y^{365}_{361}\}$$

$$-\Delta_2\Delta_1\{+Y^{3614}_{3614} + Y^{3615}_{3615} + Y^{3641}_{3615} + Y^{3651}_{3614} - Y^{3645}_{3615} - Y^{3654}_{3614} - Y^{3615}_{3645} - Y^{3641}_{3645}\}$$

$$+\Delta_2^{\,2}\{Y^{3615}_{3615} + Y^{3651}_{3615}\} + \Delta_1^{\,2}\{Y^{3614}_{3614} + Y^{3641}_{3614}\}$$

$$-\Delta_2^{\,2}\Delta_1\{Y^{36145}_{36145} + Y^{36514}_{36154}\} + \Delta_1^{\,2}\Delta_2\{Y^{36145}_{36145} + Y^{36415}_{36145}\}$$

The last four terms are each zero because the fourth and fifth cofactors in each pair of brackets are equal in magnitude and opposite in sign.

Therefore, the balance factor becomes:

$$\mathrm{CMR} = \frac{V_1 - V_2}{V_3 - V_6} = \frac{y_3 y_{g3}(y_{g2}\Delta_1 + y_2\Delta_2)}{Y^{36}_{36} + \Delta_2\{Y^{361}_{361} - Y^{365}_{361}\} + \Delta_1\{Y^{364}_{361} - Y^{361}_{361}\}}$$

$$-\Delta_2\Delta_1\{Y^{3614}_{3614} + Y^{3615}_{3615} + Y^{3641}_{3615} + Y^{3651}_{3614} - Y^{3645}_{3615} - Y^{3654}_{3614} - Y^{3615}_{3645} - Y^{3614}_{3645}\}$$

Equation 5.1 i

Here the cofactor Y^{36}_{36} can be simplified as in Section 5.0. By a trivial calculation, it is again evident that CMR = 0 when the Δ's are zero. $(Y^{36}_{36} \neq 0)$

In the problem above, the reference or "ground" for the interference source was taken as *signal* ground terminal 6. In no way has it been established that some other centered node would not represent a better approximation to the actual return potential of the interfering source. Since the indefinite admittance matrix is used in this new method, the reference node can be easily changed. Using terminal 5 (the instrument case) as the return for the interfering source, the rational function of fault polynomials is:

$$CMR = \frac{V_1 - V_2}{V_3 - V_5} = \frac{(Y_{35}^{12})'}{(Y_{35}^{35})'} = \frac{y_{g3} y_3 (y_{g2}\Delta_1 + y_2\Delta_2)}{Y_{35}^{35} + \Delta_1\{Y_{351}^{354} + Y_{354}^{351} - Y_{351}^{351} - Y_{354}^{354}\} + \Delta_2 Y_{351}^{351} - \Delta_1\Delta_2 Y_{3514}^{3514}} \qquad \text{Equation 5.1 j}$$

The ease with which the transfer voltage expressions can be derived for any interfering source reference and the advantages of the fault polynomial are in immediate evidence here. Because there are more centered nodes (terminals 3, 4, 5 and 6) than homologous node-pairs (terminals 1 and 2), the advantages of homology are not significant here.

In order to determine more of the effect of return impedance y_{g3} for either reference, it is of interest to determine driving point impedances. First, across terminals 3 and 6:

$$Z_{36} = \frac{Y_{36}^{36}}{Y_6^6} .$$

The equicofactor $Y_6^6 = Y_5^5$ and the latter will be more readily determined.

$$Y_5^5 = y_{g3} \begin{vmatrix} (y_2+y_m+y_g\ ) & -y_m & 0 & -y_2 \\ -y_m & y_2+y_m+y_{g2} & 0 & -y_2 \\ 0 & 0 & y_3 & -y_3 \\ -y_2 & -y_2 & -y_3 & y_3+2y_2 \end{vmatrix}$$

Using the simplification technique:

$$Y_5^5 = y_{g3} \begin{vmatrix} y_2+2y_m+y_{g2} & 0 & 0 & 0 \\ -y_m & y_2+y_{g2} & 0 & -y_2 \\ 0 & 0 & y_3 & -y_3 \\ -y_2 & -2y_2 & -y_3 & y_3+2y_2 \end{vmatrix} \quad \text{and}$$

$Y_5^5 = 2y_3y_{g3}y_2y_{g3}(y_2+2y_m+y_{g2})$. This relatively simple first cofactor is also the magnitude of each of the thirty-six first cofactors of Y. Then the driving point impedance under balanced conditions will be:

$$Z_{36} = \frac{\cancel{(y_2+2y_m+y_{g2})}\{y_{g2}(2y_{g2}(y_3+2y_2))+(2y_{g2}+y_{g3})\ y_2+y_{g2})(y_3+2y_2)-2y_2{}^2)\ \}}{2y_3y_{g2}\ y_2\cancel{(y_2+2y_m+y_{g2})}}$$

Equation 5.1 k

Then the fraction of voltage drop across y_{g3} will be:

$$\frac{V_{56}}{V_{36}} = \frac{2y_3y_{g2}y_2}{(2y_{g2}{}^2(y_3+2y_2) + (2y_{g2} + y_{g3})\{(y_2 + y_{g2})(y_3 +2y_2) -2y_2{}^2\}}$$

Equation 5.1 l

A calculations check can be made while again demonstrating the flexibility of the method, for the determination of the voltage ratio $\frac{V_{56}}{V_{36}}$ is also immediate. This can be evaluated from the cofactors since, $\frac{V_{56}}{V_{36}} = \frac{Y_{36}^{56}}{Y_{36}^{36}}$.

$$Y_{36}^{56} = -y_3 \begin{vmatrix} y_2+y_m+y_{g2} & -y_m & -y_2 \\ -y_m & y_2+y_m+y_{g2} & -y_2 \\ -y_{g2} & -y_{g2} & 0 \end{vmatrix}$$

and Y_{36}^{36} has already been found.

Y_{36}^{56} becomes:

$$-y \begin{vmatrix} (y_2+2y_m+y_{g2}) & 0 & 0 \\ -y_m & y_2+y_{g2} & -y_2 \\ -y_{g2} & -2y_{g2} & 0 \end{vmatrix}$$

$$= +2y_{g2}y_2y_3(y_2+2y_m+y_{g2})$$

so that,

$$\frac{Y_{36}^{56}}{Y_{36}^{36}} =$$

$$\frac{+2y_{g2}y_2y_3\cancel{(y_2+2y_m+y_{g2})}}{\cancel{(y_2+2y_m+y_{g2})}\left[2y_{g2}^{\ 2}(y_3+2y_2) + (2y_{g2}+y_{g3})\{(y_2+y_{g2})(y_3+2y_2) - 2y_2^{\ 2}\}\right]}$$

Equation 5.1 m

The expression of Equation 5.1 m is identical with that of Equation 5.1 l. Evidently the simplification procedures will enable a ready check of calculations and very often by hand.

5.2 AN EXAMPLE OF THE FAULT MINOR EXPANSION

The preceding examples have helped to illustrate several of the work-saving binet techniques suggested in earlier chapters. It was indicated that results were especially suited to further analysis and synthesis studies. The use of the *fault minor expansion* of Equation B.4 which describes the CM susceptibility of an *active* binet will complete this chapter.

The dc transistor differential amplifier of Figure 5.2 a is taken from an example in Reference 5.2 a; the nodes b_1' and b_2' are included in the figure to note those components necessary to improve the physical equivalence of the transactors. However, since they were not included in the reference they will be omitted here. Thus the network with homologous nodes b_1, c_1, and e_1, with centered nodes S and R will be analyzed. A ninth node c will be needed to accomodate the CM equivalent source. Although from a practical standpoint, it is clearly possible to have this interference source externally coupled to *all* nodes of the network, only two such paths will be shown, to correspond to the more usual treatment of differential amplifiers.

The indefinite admittance matrix, Y, for this network of Figure 5.2 a is of order 9 and rank 8. As is typical of circuits limited by economic consideration it is far from being fully connected with respect to its passive and more especially, its active elements. From the Table of 3.2 a, for three homologous nodes in each bipart, a total number of 448 *distinct*

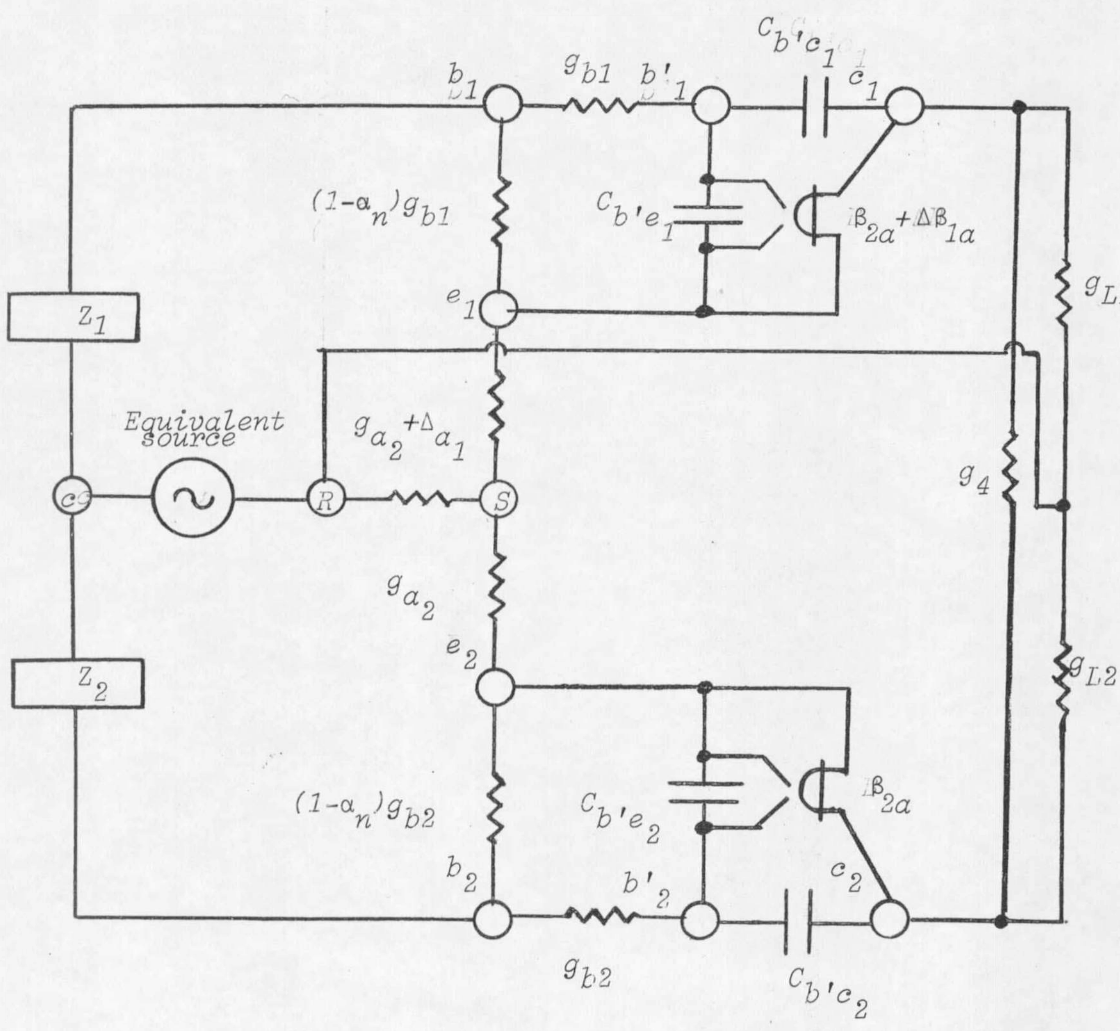


Figure 5.2a   *A dc transistor differential amplifier.*

elements would be possible with just one reference node R. Here there are two such nodes, R & S, with a consequent total of 784 distinct elements possible. Evidently, without the limitations imposed by cost, the designer would have substantial leaway in his choice of circuits and a correspondingly more difficult task of optimization. A voltage ratio of interest would be $\frac{V_{C_1}-V_{C_2}}{V_C - V_R}$ and possibly, $\frac{V_S-V_R}{V_C-V_R}$. These are:

$$\frac{V_{C_1C_2}}{V_{CR}} = \frac{(Y_{CR}^{C_1C_2})'}{(Y_{CR}^{CR})'} \quad \text{and} \quad \frac{V_{SR}}{V_{CR}} = \frac{(Y_{CR}^{SR})'}{(Y_{CR}^{CR})'} \qquad \text{Equation 5.2 a}$$

where the primes indicate the cofactors are unbalanced. These are the ratios of two seventh order cofactors.

It is immediately apparent that fault polynomials of the pertinent second cofactors should be calculable from one of the modified Laplace expansions as in Appendix B. Since the $R^{\underline{th}}$ row is omitted in the expressions above, one would expect the leading unsigned minors to be of no more than fourth order.

In $(Y_{C\ R}^{C_1C_2})'$ there will be four distinct rows in which the unbalance values will appear along with seven columns so that $C_4^7 = 35$ leading unsigned minors are possible. Of course many of these should be zero because of the limited connectiveness of the network. Then:

$$(Y_{C\,R}^{C_1C_2})' = \begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & 0 & y_{17} & 0 & 0 \\ y_{21} & y_{23} & 0 & 0 & 0 & 0 & y_{29}+\Delta_{29} \\ y_{31}+\Delta_{31} & y_{33}+\Delta_{33} & 0 & 0 & 0 & y_{31}-\Delta_{33} & 0 \\ 0 & 0 & y_{44} & y_{46} & y_{47} & 0 & 0 \\ 0 & 0 & y_{54} & y_{56} & 0 & 0 & y_{59} \\ 0 & 0 & y_{64} & y_{46} & 0 & y_{68} & 0 \\ 0 & y_{83}+\Delta_{83} & 0 & y_{86} & 0 & y_{88}+\Delta_{88} & y_{98} \end{vmatrix}$$

Equation 5.2b

where $\Delta_{33}' = \Delta_{33}+\Delta_{11}$, $\Delta_{31} = -\Delta_{11}$, $\Delta_{36} = -\Delta_{33}$, $\Delta_{27} = -\Delta_{22}$, $\Delta_{83} = -\Delta_{33}$, and $\Delta_{88} = +\Delta_{33}$.

The fault polynomial of the binet can be found by using the discrete fault polynomial Equation B.1 c *or* by using an expansion of fault minors as in Equation B.1 d. The number of terms in the discrete fault polynomial of Equation B.1 c becomes large when there are several unbalance elements off the row (or column) of leading elements, even though they may be in the same column (or row) as those of other leading elements. For example, in the polynomial expansion of $(Y_{C\,R}^{C_1C_2})'$ with leading element Δ_{36} there are four elements: Δ_{11}, Δ_{12}, Δ_{27} and Δ_{76} off the third row and sixth column. Therefore the number of terms involved with Δ_{36} will be $1 + 4 + C_2^4 + C_3^4 + C_4^4 = 15$. Of course several of these terms may be zero, like the product $\Delta_{36}(\Delta_{11}\Delta_{12}\, Y_{C\,R\,3\,11}^{C_1C_2\,6\,12})$ but nevertheless, the polynomial itself will contain a large number of terms. It is known that the numerator of the *homologous* cofactor $Y^{C_1C_2}_{C\,R}$ must be zero so that all the terms of this cofactor should sum to zero. If this fault minor expansion were used, the terms of $Y^{C_1C_2}_{C\,R}$ would be mixed with unbalance values.

Assuming therefore that the fault polynomial will be more convenient and, for simplicity, letting the *numerical* suffixes refer to the $7^{\underline{th}}$ order matrix of Equation 5.2 b:

$$
\begin{aligned}
(Y^{C_1C_2}_{C\,R})' = Y^{C_1C_2}_{C\,R} &+ \Delta_{31}\{Y^{C_1C_2\,1}_{C\,R\,3} + \Delta_{12}Y^{C_1C_2\,12}_{C\,R\,31} + \Delta_{27}Y^{C_1C_2\,17}_{C\,R\,32} + \Delta_{72}Y^{C_1C_2\,12}_{C\,R\,37} \\
&+ \Delta_{76}Y^{C_1C_2\,16}_{C\,R\,37} + \Delta_{12}\Delta_{27}Y^{C_1C_2\,127}_{C\,R\,312} + \Delta_{12}\Delta_{76}Y^{C_1C_2\,126}_{C\,R\,317} \\
&+ \Delta_{27}\Delta_{72}Y^{C_1C_2\,172}_{C\,R\,327} + \Delta_{27}\Delta_{76}Y^{C_1C_2\,176}_{C\,R\,327} + \Delta_{12}\Delta_{27}\Delta_{76}Y^{C_1C_2\,1276}_{C\,R\,3127}\} \\
&+ \Delta_{32}\{Y^{C_1C_2\,2}_{C\,R\,3} + \Delta_{11}Y^{C_1C_2\,21}_{C\,R\,31} + \Delta_{27}Y^{C_1C_2\,27}_{C\,R\,32} + \Delta_{76}Y^{C_1C_2\,26}_{C\,R\,37} + \Delta_{11}\Delta_{27}Y^{C_1C_2\,217}_{C\,R\,312} \\
&+ \Delta_{11}\Delta_{76}Y^{C_1C_2\,216}_{C\,R\,317} + \Delta_{27}\Delta_{76}Y^{C_1C_2\,276}_{C\,R\,327} + \Delta_{11}\Delta_{27}\Delta_{76}Y^{C_1C_2\,2176}_{C\,R\,3127}\}. \\
&+ \Delta_{36}\{Y^{C_1C_2\,6}_{C\,R\,3} + \Delta_{11}Y^{C_1C_2\,61}_{C\,R\,31} + \Delta_{12}Y^{C_1C_2\,62}_{C\,R\,31} + \Delta_{72}Y^{C_1C_2\,62}_{C\,R\,37} + \Delta_{27}Y^{C_1C_2\,67}_{C\,R\,32} + \Delta_{11}\Delta_{72}Y^{C_1C_2\,612}_{C\,R\,317} \\
&+ \Delta_{11}\Delta_{27}Y^{C_1C_2\,617}_{C\,R\,312} + \Delta_{12}\Delta_{27}Y^{C_1C_2\,627}_{C\,R\,312} + \Delta_{72}\Delta_{27}Y^{C_1C_2\,627}_{C\,R\,372} + \Delta_{11}\Delta_{72}\Delta_{27}Y^{C_1C_2\,6127}_{C\,R\,3172}\}.
\end{aligned}
$$

$$+ \Delta_{11} Y_{C\,R\,1}^{C_1C_2 1} + \Delta_{12} Y_{C\,R\,1}^{C_1C_2 2} + \Delta_{27} Y_{C\,R\,2}^{C_1C_2 7} + \Delta_{72} Y_{C\,R\,7}^{C_1C_2 2} + \Delta_{76} Y_{C\,R\,7}^{C_1C_2 6}$$

$$+ \Delta_{11}\Delta_{27} Y_{C\,R\,12}^{C_1C_2 17} + \Delta_{11}\Delta_{72} Y_{C\,R\,17}^{C_1C_2 12} + \Delta_{11}\Delta_{76} Y_{C\,R\,17}^{C_1C_2 16} + \Delta_{12}\Delta_{27} Y_{C\,R\,12}^{C_1C_2 27}$$

$$+ \Delta_{12}\Delta_{76} Y_{C\,R\,17}^{C_1C_2 26} + \Delta_{27}\Delta_{72} Y_{C\,R\,27}^{C_1C_2 72} + \Delta_{27}\Delta_{76} Y_{C\,R\,27}^{C_1C_2 76}$$

$$+ \Delta_{11}\Delta_{22}\Delta_{72} Y_{C\,R\,127}^{C_1C_2 172} + \Delta_{11}\Delta_{27}\Delta_{76} Y_{C\,R\,127}^{C_1C_2 176}.$$

Equation 5.2 c

Since $Y_{C\,R}^{C_1C_2}$ must be zero, it is obvious that $(Y_{C\,R}^{C_1C_2})'$ must also be zero when all the Δ's are zero, provided $(Y_{CR}^{CR})'$ is non-zero. The values of the cofactors $Y_{C\,R\,3}^{C_1C_2 1}$, $Y_{C\,R\,31}^{C_1C_2 12}$, etc. are found by getting the magnitudes indicated from Equation 5.2 c but the signs of these minors must come from its position in the *original* matrix of Equation 5.2 b. To see this for the second, third and fourth cofactors:

$$Y_{C\,R}^{C_1C_2} = 0 \qquad\qquad Y_{C\,R\,3}^{C_1C_2 1} = Y_{C\,R\,b_2}^{C_1C_2 b_1}$$

where the latter cofactor designation refers to the nodes of the network. Since the sign of $Y_{C\,R\,b_2}^{C_1C_2 b_1}$ must be derived from its position in the original

$(Y)'$ matrix, the cofactor $Y_{C\,R\,b_2}^{C_1C_2b_1}$ must be given in terms of that position:

$$Y_{C\,R\,3}^{C_1C_2 1} = Y_{C\,R\,b_2}^{C_1C_2b_1} \;=\!\!\bigcirc\!\!=\; Y_{794}^{251} = (+)\ abs\ (Y_{C\,R\,3}^{C_1C_2 1})$$

where *abs* means "the absolute value of".

Similarly, $Y_{CR\,31}^{C_1C_2 12} \;=\!\!\bigcirc\!\!=\; Y_{7931}^{2513} = (-)\ abs\ (Y_{C\,R\,31}^{C_1C_2 12})$

$$Y_{C\,R\,32}^{C_1C_2 17} \;=\!\!\bigcirc\!\!=\; Y_{7912}^{2519} = (+)\ abs\ (Y_{C\,R\,32}^{C_1C_2 17})$$

$$Y_{C\,R\,37}^{C_1C_2 12} \;=\!\!\bigcirc\!\!=\; Y_{7938}^{2513} = (+)\ abs\ (Y_{C\,R\,37}^{C_1C_2 12})$$

$$Y_{C\,R\,37}^{C_1C_2 16} \;=\!\!\bigcirc\!\!=\; Y_{7938}^{2515} = (+)\ abs\ (Y_{CR\,37}^{C_1C_2 16}) = 0$$

The latter cofactor is one of several which are zero.

The complete expression for $(Y_{C\,R}^{C_1C_2})'$ would be expected to be involved because of the number of unbalance values appearing in the original matrix, and because the expression is exact. The distinct advantage of the fault polynomial of Equation 5.2 c is that the simplifications can now be made on *actually anticipated unbalance values and* the weighting of the *particular* cofactor involved. Thus some, but not necessarily all "second order" and "third order" unbalances may be negligible and *any* desirable level of approximation can be assumed.

An advantage of the fault minor expansion of Equation B.1 d is that for an *exact* solution, the total work is sometimes less. Such cases can sometimes be identified by the number of zero elements in a given row. Since there are four rows and columns with unbalance entries, the fault minors will be fourth order and in evaluating Equation 5.2 c, as previously indicated, one would expect a maximum of $C_4^7 = 35$ minors in the expansion. However, in the evaluation of these minors, one notes that for row expansion along b_1, c_1, e_1, and s any minor including column b_2 will be zero. Therefore, the maximum number of non-zero minors will be $C_4^6 = 15$. Similarly for a column-wise expansion of fault minors along b_1, e_1, s, R any minor including row b_2 will be zero with the same number possible. It will be of interest here to see the row-wise expansion: There is:

$$
(Y_{CR}^{C_1C_2})' = \begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & 0 \\ y_{21} & y_{23} & 0 & 0 \\ y_{31}+\Delta_{11} & y_{33}+\Delta_{33} & 0 & y_{38}+\Delta_{38} \\ 0 & y_{83}+\Delta_{83} & y_{86} & y_{88}+\Delta_{88} \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_1e_2s}
$$

$$
+ \begin{vmatrix} y_{13}+\Delta_{11} & 0 & 0 & 0 \\ y_{23} & 0 & 0 & y_{29}-\Delta_{22} \\ y_{33}+\Delta_{33} & 0 & (y_{38}-\Delta_{33}) & 0 \\ y_{83}-\Delta_{33} & y_{86} & (y_{88}+\Delta_{33}) & y_{98} \end{vmatrix}' Y_{CR\ b_1C_1e_1s}^{C_1C_2e_1e_2s}
$$

$$
+ \begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & 0 \\ y_{21} & y_{23} & 0 & y_{29}-\Delta_{22} \\ y_{31}-\Delta_{11} & y_{33}+\Delta_{33} & 0 & 0 \\ 0 & y_{83}-\Delta_{33} & y_{86} & y_{98} \end{vmatrix}' Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_1e_2R}
$$

$$+\begin{vmatrix} y_{11}+\Delta_{11} & 0 & 0 & 0 \\ y_{21} & 0 & 0 & y_{29}-\Delta_{22} \\ y_{31}-\Delta_{11} & 0 & y_{31}-\Delta_{33} & 0 \\ 0 & y_{86} & y_{88}+\Delta_{33} & y_{78} \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_2}$$

$$+\begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & y_{17} \\ y_{21} & y_{23} & 0 & 0 \\ y_{31}-\Delta_{11} & y_{33}'+\Delta_{33} & 0 & 0 \\ 0 & y_{83}+\Delta_{83} & y_{86} & 0 \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_1e_2C}$$

$$+\begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & y_{17} & 0 \\ y_{21} & y_{23} & 0 & 0 \\ y_{31}-\Delta_{11} & y_{33}'+\Delta_{33} & 0 & y_{38}+\Delta_{38} \\ 0 & y_{83}+\Delta_{83} & 0 & y_{88}+\Delta_{88} \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_1C\ s}$$

$$+\begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & y_{17} & 0 \\ y_{21} & y_{23} & 0 & (y_{29}+\Delta_{29}) \\ (y_{31}-\Delta_{11}) & (y_{23}'+\Delta_{33}) & 0 & 0 \\ 0 & (y_{83}+\Delta_{83}) & 0 & y_{89} \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2b_1e_1C\ R}$$

$$+\begin{vmatrix} y_{13}-\Delta_{11} & 0 & y_{17} & 0 \\ y_{23} & 0 & 0 & 0 \\ y_{33}'+\Delta_{33} & 0 & 0 & y_{33}+\Delta_{38} \\ y_{83}+\Delta_{83} & y_{86} & 0 & y_{88}+\Delta_{88} \end{vmatrix} Y_{CR\ b_1C_1e_1s}^{C_1C_2e_1e_2C\ s}$$

Equation 5.2 d

Therefore in this particular example only eight of the thirty five minors possible actually materialize. This can be considered typical of the usual network.

It is rather obvious that the fault polynomial and the fault minor expansion will be ultimately termwise identical. Therefore in the expansion for $(Y_{CR}^{C_1C_2})'$ Equation 5.2 d, one would expect all terms without the Δ multiplier to sum up to be zero, and so these terms can be ignored in the calculations, or the evaluation may be used as a partial check.

It is apparent that when approximations are warranted, the fault polynomial will prove more practical than the fault minor expansion. In any event when the analysis is supposed to accomodate the interplay of several unbalance values, and when these are large in number, the amount of calculations increases substantially when an exact solution is required.

The calculation for the denominator of the balance factor of Equation 5.2 a is quite similar to the above except that (Y_{CR}^{CR}) will be finite, the value of which can be found by the method of Section 2.3. There is:

$$
(Y^{CR}_{CR})' = \begin{vmatrix}
y_{11}+\Delta_{11} & 0 & y_{13}-\Delta_{11} & 0 & 0 & 0 & 0 \\
y_{21} & y_{22}+\Delta_{22} & y_{23} & 0 & y_{15} & 0 & 0 \\
y_{31}-\Delta_{11} & 0 & y_{33}+\Delta_{33} & 0 & 0 & 0 & y_{38}-\Delta_{33} \\
0 & 0 & 0 & y_{44} & 0 & y_{46} & 0 \\
0 & y_{52} & 0 & y_{54} & y_{55} & y_{56} & 0 \\
y_{61} & 0 & 0 & y_{64} & 0 & y_{66} & y_{68} \\
0 & 0 & y_{83}-\Delta_{33} & 0 & 0 & y_{86} & y_{88}+\Delta_{33}
\end{vmatrix}
$$

Then: $(Y^{CR}_{CR})' =$

$$
\begin{vmatrix}
y_{11}+\Delta_{11} & 0 & y_{13}-\Delta_{11} & 0 \\
y_{21} & y_{22}+\Delta_{22} & y_{23} & 0 \\
y_{31}-\Delta_{11} & 0 & y_{33}+\Delta_{33} & 0 \\
0 & 0 & y_{83}-\Delta_{33} & y_{86}
\end{vmatrix}
Y^{CR\ b_1C_1e_1e_2}_{CR\ b_1C_1e_1s}
$$

$$
+ \begin{vmatrix}
y_{11}+\Delta_{11} & 0 & y_{13}-\Delta_{11} & 0 \\
y_{2} & y_{22}+\Delta_{22} & y_{23} & 0 \\
y_{31}-\Delta_{11} & 0 & y_{33}+\Delta_{33} & y_{38}-\Delta_{33} \\
0 & 0 & y_{83}-\Delta_{33} & y_{88}+\Delta_{33}
\end{vmatrix}
Y^{CR\ b_1C_1e_1s}_{CR\ b_1C_1e_1s}
$$

$$
+ \begin{vmatrix}
0 & y_{13}-\Delta_{11} & 0 & 0 \\
(y_{22}+\Delta_{22}) & y_{23} & y_{15} & 0 \\
0 & y_{33}+\Delta_{33} & 0 & y_{38}-\Delta_{33} \\
0 & y_{83}-\Delta_{33} & 0 & y_{88}+\Delta_{33}
\end{vmatrix}
Y^{CR\ C_1e_1C_2s}_{CR\ b_1C_1e_1s}
$$

$$+ \begin{vmatrix} y_{13}-\Delta_{11} & 0 & 0 & 0 \\ y_{23} & y_{15} & 0 & 0 \\ y_{33}+\Delta_{33} & 0 & 0 & y_{38}-\Delta_{33} \\ y_{83}-\Delta_{33} & 0 & y_{86} & y_{88}+\Delta_{33} \end{vmatrix}' \; Y^{CR\ e_1C_2e_2s}_{CR\ b_1C_1e_1s}$$

$$+ \begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & 0 \\ y_{21} & y_{23} & y_{15} & 0 \\ y_{31}-\Delta_{11} & y_{33}+\Delta_{33} & 0 & 0 \\ 0 & y_{83}-\Delta_{33} & 0 & y_{86} \end{vmatrix}' \; Y^{CR\ b_1e_1C_2e_2}_{CR\ b_1C_1e_1s}$$

$$+ \begin{vmatrix} y_{11}+\Delta_{11} & y_{13}-\Delta_{11} & 0 & 0 \\ y_{21} & y_{23} & y_{15} & 0 \\ y_{31}-\Delta_{11} & y_{33}+\Delta_{33} & 0 & y_{38}-\Delta_{33} \\ 0 & y_{83}-\Delta_{33} & 0 & y_{88}+\Delta_{33} \end{vmatrix}' \; Y^{CR\ b_1C_1e_2s}_{CR\ b_1C_1e_1s}$$

$$+ \begin{vmatrix} 0 & y_{13}-\Delta_{11} & 0 & 0 \\ y_{22}+\Delta_{22} & y_{23} & 0 & 0 \\ 0 & y_{33}+\Delta_{33} & 0 & y_{38}-\Delta_{33} \\ 0 & y_{83}-\Delta_{33} & y_{86} & y_{88}+\Delta_{33} \end{vmatrix}' \; Y^{CR\ C_1e_1e_2s}_{CR\ b_1C_1e_1s}$$

It is again apparent that the number of fault minors describing the desired cofactor is much less than the maximum number possible. Here that number is again $C_4^7 = 35$, but only seven of these minors (or their cofactor multiplicands) are non-zero.

In Chapter V certain of the means of analysis and simplification have been illustrated in two examples. Throughout this work, certain methods of synthesizing homologous binets have been implied, and it is in Chapter VI where this is made more explicit.

VI SYNTHESIS OF A BINET

6.0 DISCUSSION. According to a schedule of distinct vs independent elements possible, as in Table 3.2 a, a substantial number of alternative circuits is available to the binet synthesist. It is our purpose here to review the binet synthesis problem and to indicate approaches which show promise in reducing the time taken to specify binets with a low susceptibility to interference.

If an indefinite admittance matrix is used to specify the performance of a desired network, a realization of component values for the latter can be based on Percival's Equation (3.3 b). For the general (2n + 1)-node binet using elements of degrees two, three, and four:

$$Y = \sum_{i=1}^{2n+1} e_i e_i' \, y_{ii} + \sum_{i=1}^{2n} t(ip/ip) + \sum_{j=1}^{2n} t(jl/ql) + \sum_{k=1}^{2n} t(kr/ms)$$

Equation 6.0 a

where $p>i$, $q,l>j$, and $r,m>k$, and 1 is the reference for the degree 3 device. In the case of the *homologous* binet there will be approximately half the number of independent component choices indicated by Equation 6.0 a. Then this becomes:

$$Y = \sum_{i=1}^{n+1} e_i e_i' \, y_{ii} + \sum_{i=1}^{n} t(ip/ip) + \sum_{j=1}^{n} t(jl/ql) = \sum_{k=1}^{n} t(kr/ms)$$

Equation 6.0 b

with the same meaning of the indices. Now a realization schedule may be made up as.

made up having the same number of rows and columns as the original Y matrix. The active transactors can be placed in one section of the schedule, say above a main diagonal, with the passive components below. Limiting the transactor degree to two and three, the realization schedule elements can be shown (Reference 6.0 a) to be

$$y(j1/q1) = y_{jq} - y_{qj}$$

$$y(jq) = y(jq/jq) = -y_{qj}$$

$$y(1r) = -\sum_{s=1}^{r-1} (y_{sr} - y_{rs}) - y_{1r} \qquad \text{For } 1>r$$

$$= -\sum_{s=r+1}^{2n+1} (y_{rs} - y_{sr}) - y_{r1} \qquad \text{For } r>1$$

Equation 6.0 c

and from Equation 6.c, the elements can be found which make up the realization schedule for the binets:

0	$y(1l/2l)$ - - - -	$y(1l/nl)$	0	$y(1l/n+1,l)$ - - - -	$y(1l/2n,l)$
$y(12/12)$	0 - - - -	$y(2l/nl)$	0	$y(2l/n+1,l)$ - - - -	$y(2l/2n,l)$
$y(1n/1n)$	$y(2n/2n)$ - - - -	0	0	$y(nl/n+1,l)$	$y(nl/2n,l)$
$y(1l/1l)$	$y(2l/2l)$ - - - -	$y(nl/nl)$	0	0	0
$y(1,n+1/1,n+1)$	$y(2,n+1/2,n+1)$ - - -	$y(n,n+1/n,n+1)$	$y(l,n+1/l,n+1)$	0 - - - -	$y(2n,n+1/2n,l)$
$y(1,2n/1,2n)$	$y(2,2n/2,2n)$ - - -	$y(n,2n/n,2n)$	$y(l,2n/l,2n)$	$y(n+1,2/n+1,2\ )$ - - - -	0

TABLE 6.0 a

BINET REALIZATION SCHEDULE

It is plain that in the admittance matrix for the homologous binet, the suffixes n+1, n+2 can be replaced by suffixes 1,2. Similarly 1, n+2 by 2, n+1 etc; so in general,

$$y(j1/q1) = y_{jq} - y_{qj} = y_{n+j,\ n+q} - y_{n+q,\ n+j} = y(n+j,1/n+q,1)$$

$$y(jq) = -y_{qj} = -y_{n+q,\ n+j}$$

and

$$y(1r) = -\sum_{s=1}^{r-1} (y_{sr} - y_{rs}) - y_{1r} = -\sum_{s=1}^{n+r-1} (y_{s,\ r+n} - y_{r+n,\ s}) - y_{1,\ r+n}$$

$$= y(1,\ r+n) \qquad \text{For } 1>r+n$$

$$y(1r) = \sum_{s=r+1}^{2n+1} (y_{rs} - y_{sr}) - y_{r1} = -\sum_{s=r+1}^{2n+1} (y_{r+n,s} - y_{s,\ r+n}) - y_{r+n,\ 1}$$

$$= y(1,\ r+n) \qquad \text{For } 1<r+n$$

Equations 6.0 d

Table 6.0 a shows the number of the reference row and column (1), as one more than n, the number of homologous node-pairs. Of course this reference could have been placed anywhere in the indefinite admittance matrix so that the last equation of the 6.0 d equations would not have been necessary.

A realization of the active binet indicated by the admittance matrix is therefore direct and immediate. Of course if the homologous network can be realized with only matched pairs of components the susceptibility to common mode interference will be ideal. On the other hand unbalance of the various homologous pairs will ordinarily be unavoidable and it will be necessary to take steps to minimize the affects of such unbalance.

The hypothesis for the solution of such a minimization problem is simply that more than one physical realization will satisfy the performance requirements indicated by the binet's indefinite admittance matrix. It is possible to generate such classes of equivalent networks by a choice of active elements (Reference 6.0 b) or by a redundancy of passive elements. (Reference 6.0 c or d.)

There is of course a wide variety of synthesis procedures used to realize two-ports with stated transfer functions. As might be expected, properties of passive two-ports are deduced largely from the fact that energy stored or dissipated in such a network can never be negative. This statement can be formulated mathematically. If $[R]$, $[L]$ and $[D]$ are the component matrices of the node to datum impedance matrix: $[Z] = [R] + s[L] + \frac{[D]}{s}$ and with $I] = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ where I_1 and I_2 are loop currents:

$$\lfloor I^* \rfloor \; E] = \lfloor I^* \rfloor [R] \; I] + s \; I]^* \; [L] \; I] + \frac{1}{s} \; I]^* \; [D] \; I]$$

Equation 6.0 e

Equation 6.0 e has the physical significance of computing total power when $s = j\omega$ (and only then). For *any* excitation this can be written:

$$I^{*}E = F_o + sT_o + \frac{V_o}{s} = F(s) \qquad \text{Equation 6.0 f}$$

Because of restriction on *positive* dissipated power and positive energy stored, these latter quantities cannot be negative. F_o, T_o and V_o are the quadratic forms of I and since they can also be zero, they are considered to be positive semi-definite. Each one of the loop parameter matrices [R] [L] and [D] must be positive semi-definite. The resulting positive semi-definiteness of the quadratic forms of Equation 6.0 e should obviously be one of the basic requirements of all networks to be synthesized. Unfortunately, however, it is not as strong a requirement as one would like, for it does not automatically preclude negative matric elements. Historically this latter fact is apparently responsible (See for example, Reference 6.0 e) for the relatively slow development of at least one attractive method of synthesis; that proposed by Cauer (Reference 6.0 f) almost four decades ago. He suggested that the network matrix can be subjected to a linear transformation which would maintain as constant, desirable transfer or driving point characteristics. The transformed network matrix would then describe a particular circuit configuration, functionally acceptable, but with perhaps improved characteristics in other respects. A *congruance* transformation is sufficient to ensure preservation of the positive definiteness of the quadratic form of a network so that the transforming matrix A in $Y^1 = A^T YA$ need only be nonsingular. Here Y^1 is the transformed network matrix. Thus almost complete freedom exists to select the elements of A so that: *1)* desired transfer and/or driving point characteristics are held constant, *2)* negative elements of Y^1 are avoided and

3) an acceptable balance factor is maintained while *4)* other desirable characteristics are optimized.

This method of synthesis has some interesting properties which relate to the particular kind of synthesis problem under consideration. One initially notices that a degree of redundancy is required in the original circuit to be transformed, if the transformation is to reduce the number of components. On the other hand, if the original network already has a minimum number of components, the numbered tasks 3 or 4 (above) may require the *addition* of components.

Since such properties are completely compatible with the use of integrated circuits and thin film techniques and since the envisioned network transformations should be readily programmable for a computer solution, this method of synthesis is of interest here as a possible approach.

6.1 A CONTINUOUSLY EQUIVALENT SYNTHESIS

Here it will be convenient to delineate the present state of this art and indicate how the synthesis procedure can be applied to the common mode interference problem. Just recently there have been important contributions to this synthesis method. Some of these are listed under Reference Nos. 6.1 a - h. E. A. Guillamin in his "Passive Network Synthesis" initially noted the difficulty of maintaining passive elements in the transformed matrix and suggested that a transforming matrix made to be a function of an independent variable, might enable control. A few years later, in 1961, J. D. Schoeffler in his "Foundations of Equivalent Network Theory" (Reference 6.1 b) used this approach and so contributed to making the method more practical.

If the column vector of potential $V]$ is transformed to a new $V]'$ by $V]' = [P]^{-1} V]$ and if the new current vector $I]'$ is the transformed current equal to $I]' = [P]^T I]$, then for $I] = [Y] V]$ and $I]' = [Y]' V]'$, substitute: $I] = ([P]^T)^{-1} I]' = [Y] V] =$ $[Y][P][Y]'$. Therefore,

$$I]' = [P^T Y P] \; V]' \qquad \text{Equation 6.1 a}$$

and

$$[P^T Y P] = Y]' \qquad \text{Equation 6.1 b}$$

a congruent transformation provided $[P]$ is nonsingular. Now let $[P] =$ $[E] + \Delta x \; [B]$ so that:

$$[Y(x + \Delta x)] = \left[[E] + \Delta x [B]\right]^T [Y] \left[[E] + \Delta x [B]\right]$$

$$[Y(x + \Delta x)] = \left[[E] + \Delta x [B]^T\right] \left[[Y] + \Delta x [Y][B]\right]$$

$$[Y(x + \Delta x)] = |Y| + \Delta x [Y][B] + \Delta x [B^T Y] + (\Delta x)^2 [B^T Y B]$$

$$\frac{[Y(x + \Delta x)] - [Y(x)]}{\Delta x} = [B]^T [Y] + [Y][B] + \Delta x [B]^T [Y][B]$$

And in $\lim_{\Delta x \to 0}$: $\dfrac{d[Y]}{dx} = [B]^T [Y(x)] + [Y(x)] [B]$ Equation 6.1 c

Equation 6.1 c is a set of linear differential equations with initial conditions given by the elements of the original network. As in Chapter II, the Y matrix can be written:

$$[Y] = [G] + s[C] + \frac{1}{s}[L]^{-1},$$

a polynomial in s, and again with the generalized (complex scalar) s, relating to time t. $[G]$ is a matrix of conductances connecting every node-pair; $[C]$ a matrix of capacitances and $[L]^{-1}$ the inverse inductance elements. The $[G]$, $[C]$ and $[L]^{-1}$ matrices are of the same order as $[Y]$ which is: the number of nodal equations $= N = J - S$; J = number of junctions, and S is the number of separate parts of the network. Equations 6.1 c can be written as:

$$\frac{d}{dx}[G(x)] = [B]^T [G(x)] + [G(x)] [B]$$

$$\frac{d}{dx}[C(x)] = [B]^T [C(x)] + [C(x)] [B] \qquad \text{Equation 6.1 d}$$

$$\frac{d}{dx}[L^{-1}(x)] = [B]^T [L(x)]^{-1} + [L(x)]^{-1} [B]$$

These equations are 3N in number. Since each matrix, $[G(x)]$, $[C(x)]$ and $[L(x)]^{-1}$ consists of elements connecting all J nodes and assuming a connected network so that $S = 1$ then the number of elements is $C_2^J = C_2^{N+1} = \frac{(N+1)N}{2} = K$ elements of each kind.

Rearranging 6.1 d each component $g_1(x)$, $g_2(x)$, etc., can be expressed as a function of b so that:

$$\frac{d\ G(x)]}{dx} = [M]\ G(x)] \qquad \text{Equation 6.1 e}$$

where $G(x)]$ is now a column vector;

$$G(x)] = \begin{bmatrix} g_1(x) \\ \vdots \\ g_K(x) \end{bmatrix}$$ and $[M]$ is the matrix of the collected b_{ij}'s.

Define $|G(s')| = \int_0^\infty [G(x)]\ e^{-s'x} dx$ where s' relates now to the *independent variable x* with no relation to time t. Assuming here that $[M]$ is not a function of x: $s'\ [G(s)] - [G(0)] = [M]\ [G(s')]$ or

$$s'\ [E] - [M]\ [G(s')] = [G(0)].$$

Then $[G(s')] = s'\ [E] - [M]^{-1}\ [G(0)]$ where G(0) are the conductance values of the original network. Taking the inverse transform: $g_{ij}(x) = f(g_k(0), x, b_i s)$. There will be $K = C_2^{N+1}$ of these equations, although ordinarily a number of these will be zero - by chance or by design.

The choice of b_{ij}'s depends upong the driving point and/or transfer immittance desired to be kept constant, and upon the particular performance criteria chosen to guide the optimization. One such criterion is an acceptable balance factor.

6.2 THE CONTINUOUSLY EQUIVALENT SYNTHESIS OF A BINET

In the continuously equivalent synthesis procedure an "original" network must be designed as initially by standard synthesis methods. This would correspond to the realization of a (passive) binet as specified by Equations 6.0 d. Then equivalent circuits would be generated which would minimize susceptibility to common-mode interference. This is implemented as follows.

The admittance matrix os the binet is then transformed into a new network, according to the elements of the transforming matrix $[P] = [E] + \Delta x \; [B]$, which are selected to realize certain performance criteria. Now if some of the original network elements are desirable, and are to be held invariant in the transformation, this requirement can be introduced in one or more of the 3K Equations 6.1 e. If, for example, $C_3(x)$ and $g_3(x)$ are to be held constant in the synthesis,

$$\frac{dC_3(x)}{dx} = \frac{dg_3(x)}{dx} = 0$$

will apply these boundary conditions to the 3K component equations. It is understood of course that every constraint so imposed introduces less redundancy and places definite limits on the number and/or kind of realizable performance criteria available.

Performance criteria of interest at this point are twofold:

(i) to determine the elements of a binet having a specified balance factor and transfer function but with imposed (lumped-equivalent) distributed parameters.

(ii) to determine the elements of a binet with specified transfer function and acceptable balance factor, but with lowest sensitivity of network balance to changes of any number of specified components and/or the addition or change of parasitic elements.

With the exception of defining a meaningful balance factor, Problem i is straightforward: Define a $\xi_g(x)$ so that $\xi_g(x) = \Sigma_{k=1}^{K} [g_k(x) - g_k^D]^2$ where $\xi_g(x)$ is to be minimized, and $g_k(x)$ are the values of the continuously varying network conductances, as in the above. Also g_k^D are the parasitic elements the designer is obliged to work with. He is usually aware of such lumped-equivalent values and this much is assumed at this point. Then:

$$\frac{d\xi_g(x)}{dx} = 2\Sigma_{k=1}^{K} (g_k(x) - g_k^D) \frac{dg_k(x)}{dx} \qquad \text{Equation 6.2 a}$$

At x = 0 this simplifies to

$$\left.\frac{d\xi_g(x)}{dx}\right|_{x=0} = 2\Sigma_{k=1}^{K} g_k^o b_k \overset{\text{must be}}{<} 0 \qquad \text{Equation 6.2 b}$$

which will determine the sign of at least one of the b_{ij}'s.

Problem ii stated above is apparently much more involved. Although a transfer or driving point property of the network will be kept constant, we want also to minimize the change of balance with respect to changes in component values and changes or the addition of parasitic components.

The balance factor or balance figure of merit can relate to the "common mode attenuation" (See Definitions & Reference 1.5 b) comparing two voltages, "the common mode loss" comparing two powers, the difference between transverse and common mode attenuation, among several others.

There immediately arises a question of identifying the state of the circuit when the optimization is to take place. If the unbalance suggested on page 6 prevails, the effects of untoward deviations of the components from this limiting situation would have to be minimized. Suppose for example that the balance factor defined in Equation 1.5 a is chosen. Then the sensitivity of the common mode to differential mode loss (L) to a change in the i^{th} component can be defined as:

$$S_i = \frac{\frac{\Delta L}{L}}{\frac{\Delta e_i}{e_i}} = \frac{e_i}{L}\,\frac{\partial \ln L}{\partial \ln e_i} \qquad \text{Equation 6.2 c}$$

By the Chain Rule

$$\frac{dS_i}{dx} = \Sigma_{k=1}^{3K}\,\frac{\partial S_i}{\partial e_k}\,\frac{de_k}{dx} \qquad \text{Equation 6.2 d}$$

From Equation 6.2 c,

$$\frac{dS_i}{de_k} = \frac{de_i}{de_k}\frac{1}{L}\frac{\partial L}{\partial e_i} + \frac{e_i}{L}\frac{\partial^2 L}{\partial e_k \partial e_i}$$

$$\frac{dS_i}{de_k} = S_{ik}\frac{1}{L}\frac{\partial L}{\partial e_i} + \frac{e_i}{L}\frac{\partial^2 L}{\partial e_k \partial e_i}$$

Also $$\frac{dS_k}{de_i} = \frac{\partial e_k}{\partial e_i}\frac{1}{L}\frac{\partial L}{\partial e_i} + \frac{e_k}{L}\frac{\partial^2 L}{\partial e_i \partial e_k}$$

Assuming the second partials exist and therefore can be taken in either order

$$\left(\frac{dS_k}{de_i} - S_{ik}\frac{1}{L}\frac{\partial L}{\partial e_i}\right)\frac{L}{e_k} = \left(\frac{dS_i}{de_k} - \frac{S_{ik}}{L}\frac{\partial L}{\partial e_i}\right)\frac{L}{e_i}$$

Therefore: $$\frac{\partial S_k}{\partial e_i}\frac{e_i}{e_k} = \frac{\partial S_i}{\partial e_k}$$ Equation 6.2 e

And substituting in Equation 6.2 d have

$$\frac{dS_i}{dx} = \Sigma_{k=1}^{3K}\frac{e_i}{e_k}\frac{\partial S_k}{\partial e_i}\frac{de_k}{dx}$$

is a statement of intercomponent influence with respect to balance factor sensitivity. Of course at this point $\frac{\partial S_k}{\partial e_i}$ is still unknown and it is necessary to relate the longitudinal to transverse loss, L and consequently

G_p to the balance sensitivity.

The result of this should be a set of linear first order differential equations which should be readily solvable to yield the total of 3K sensitivities, S_k. Of course some of the possible components will ordinarily be zero, automatically making that S_k vanish.

Future work should include consideration of the various alternatives of sensitivity criteria that can be derived from these S_k. For example, a particular set of distributed parameters may be difficult to predict - or control. Therefore their sensitivities might be considered of greater importance and should be weighted above the sensitivities of the more predictable (and usually lumped) components when totaling a particular circuit's overall sensitivity criterion.

6.3 TRANSFORMATION CHARACTERISTICS OF A NETWORK AND ITS CM INTERFERENCE PROPERTIES

The network transforming matrix P defines a congruence transformation with certain important mathematical characteristics:

(i) If $[B] = [P]^T [A] [P]$, $\det[B] = (\det[P])^2 \det[A]$. (This describes the degree of change of immittance determinants in a transformation. Since, in practical cases, $[P]$ is usually unimodular, the immittance determinant is often invariant.)

(ii) For real [P], the sign of the determinant is invariant (Concerns the maintenance of algebraic signs of currents and voltages.)

(iii) Every symmetrical matrix [A] of rank r is congruent to a diagonal matrix whose first r diagonal elements are non-zero while all other elements are zero (concerns immittance matrices of reciprocal networks).

(iv) If a congruence class contains a symmetric matrix, then all matrices in this class are symmetric. (If *one* continuously equivalent network is reciprocal, they *all* are. Therefore networks containing negative elements should remain bilateral.)

(v) For elementary transformations of the first kind

$[P] = [E] + \eta\,[E^i_j]$, $\det [P] = \pm 1$ where η is a scalar and $[E^i_j]$ is the matrix with 0 as element in all except a one in the $j^{\underline{th}}$ row and $i^{\underline{th}}$ column.

(vi) If [P] is the permutation matrix associated with the permutation π then $|P| = \begin{cases} +1 \text{ if } \pi \text{ is even} \\ -1 \text{ if } \pi \text{ is odd} \end{cases}$

(vii) Elementary transformations of the third kind always have:

$\det [P] =$ product of λ_i's.

6.4 AN EXAMPLE

An example of how mathematical properties of the congruence transformation can be used in the study of common mode interference is evidenced in the practical case of alternating current resistance in grounding circuits. Assume the circuit of Figure 6.4 a in which the parasitic a.c. resistance is shown as R_p. Assuming all voltages are initially referred to node 1 and that for calculating purposes one would like to make node 2 the reference. The newly referenced voltages and the corresponding impedance matrix are shown primed. Referring to Figure 6.4 a, the transformed voltages primed, there is:

$$
\begin{aligned}
V_1' &= V_1 - V_7 \\
V_2' &= V_2 - V_7 \\
V_3' &= V_3 - V_7 \\
V_4' &= V_4 - V_7 \\
V_5' &= V_5 - V_7 \\
V_6' &= V_6 - V_7 \\
V_7' &= V_7 - V_7
\end{aligned}
$$

and $V_1'\rfloor = [P]\ V\rfloor$, or

$$
\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_5' \\ V_6' \\ V_7' \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}
$$

The permutation matrix $[P]$ is of the first kind and univodular (det $[P] = \pm 1$).

Therefore: $\det [P] = \det [P]^T = -1$

$$\det[P] = \det[P]^T = -1 .$$

This transformation requires a new impedance matrix $[Z']$ equal to:

$$[Z'] = [P]\ [Z]\ [P]^T \qquad \text{Equation 6.4 b}$$

So det $[Z']$ = det $[Z]$ and there is the theorem:

Theorem The determinant of the immittance matrix of a binet does not depend on the reference terminal chosen for the representation.

Implicitly involved here, $I] = [P]^T\ I']$ so that the new currents are readily found. Also in relating common mode properties of a system of cascaded networks, references for the terminal ports are seen to be easily related.

It is of interest here to note that the impedance matrix Z of Equation 6.4 b is of the node to datum type as distinguished from the previously indicated indefinite admittance kind. It has been assumed in Section 6.1 that the immittance matrices of interest were not equifactor either. It is of special interest also to note that the zero-sum property is not invarient in the congruence transformation of an equifactor matrix.

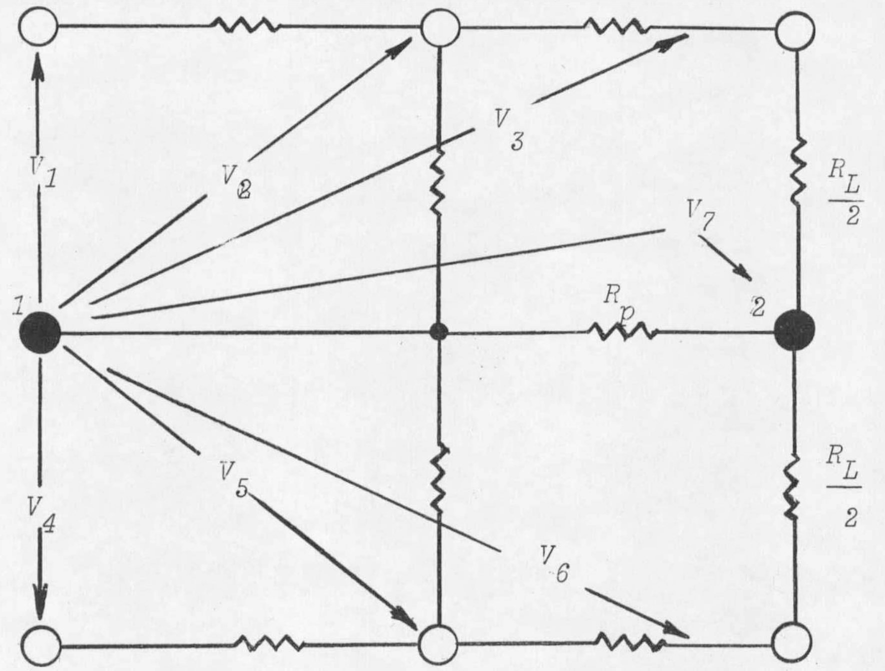


Figure 6.4a Resistance in ground lead: node 1 to node 2.

VII CONCLUSIONS AND RESULTS

Several new techniques have been developed to make the analyses of a common mode interference problem easier and more meaningful. Special attention has been given to homologous binal networks with *any* magnitude of unbalance and of any number of components.

A common-mode interference susceptibility test configuration was developed which was convenient to use from the simple two-wire input to the general case where all nodes of the binet are subject to interference influx. With Shekel's indefinite admittance matrix applied to this test-circuit configuration, quantities significant to the CM interference problem were shown to be readily calculated. These included the driving point impedance across *any* two nodes of the binet, the transfer impedance from *any* two nodes to *any* other two nodes, and related transfer voltage ratios.

New terms were introduced to state the CM interference problem: these included the *binal network*, the *homologous binal network*, the *balanced binal network* and their respective contractions. *New* balance factors were also introduced in order to cover typical applications of interest. These included the balance sensitivity attenuation factor (BSAF), the driving point CM attenuation factor (DPAF), a CM floating transfer attenuation factor (FTAF), a CM split load transfer attenuation factor (STAF), a CM floating attenuation factor (FAF) and a CM split load attenuation factor (SAF). It was shown that the first and second cofactors used in the CM analysis can be reduced in order by the number of terminals upon which the common mode voltage is impressed. Of greater significance, it was also shown that the *homology* of the 2n-node binet allowed a reduction of the number of terms in the cofactor expansion by a factor equal to the ratio of the number of terms in a (2n)th order cofactor to that of two $n^{\underline{th}}$ order cofactors. If this latter product is that of two numbers, the ratio is equal to $\frac{(2n-1)!}{(n-1)!}$, a very substantial simplification for even simple binets. The only requirement imposed on the binet is that each unilateral component connected across homologous nodes be matched exactly by an equivalent component conversely connected across the same nodes. Restricting the active elements to respective biparts of the binet is a practical means of accomplishing this. The limitation is not considered serious for essentially all homologous active networks are already designed this way to make balance more feasible.

A *new* fault polynomial has been proposed in which the unbalance values and their products are considered variables, with coefficients made up of the cofactors of the balanced binet. The fault polynomial allows simplifying approximations to be made at any level depending on anticipated values of unbalance as well as the homologous binets' cofactors. Further analyses are made possible by this explicit means of representing unbalance. A *new* fault-minor expansion is offered as an alternative means of expressing imbalance properties of the binet. In certain binets of low connectivity the expansion is simpler than the fault polynomial. The proposed methods of analysis are particularly useful for calculating *exact* expressions for balance, denoting the common-mode susceptibilities of *large* binets (Say three or more homologous nodes) with relatively *few* values of imbalance involved at one time and with *any* magnitude of imbalance.

The testing configuration and the analytical tools developed here lead quite naturally to a realization schedule for homologous binets. This physical realization can then be optimized according to interference susceptibility requirements and a prospective procedure appears to be that of the continuously equivalent type.

Future research should include such a synthesis of homologous binets. This should supply important insight into the common mode interference problem. Further work is also needed to learn more of the effect of interference on multiple pairs of homologous nodes through various permutations/combinations of source impedances.

APPENDIX A

DEFINITIONS

A set of definitions is given here for several reasons. Completely *new* terms have been defined and a ready reference is therefore necessary. Furthermore, in the contemporary literature, language pertinent to measurements of interest here is unfortunately pervaded by ambiguities and contradictions. For example, with reference to the meaning of gain, Professor W. Lynch (Reference A.1) comments: "...it is used somewhat indiscriminately to describe ratios of power, complex ratios of transmission quantities (ie. voltage or current) and the ratios of the magnitudes of these quantiiest The resulting ambiguity leaves the term almost meaningless in a general discussion of measurements."

To this state of affairs must be added the confusion peculiar to the common-mode interference specialty. It is of significance that much of the nomeclature has actually evolved from different industries. And sometimes the same quantity has distinct names that are not generally considered synonymous.

Nevertheless this paper does not purport to secure better standardization although it cannot help but note the need thereof. In the list to follow, wherever the term is new, or in the opinion of this writer contains a distinct elements of originality, it is identified by the suffix: (new). Again, the list of definitions was made complete only to meet the needs of this paper.

Balance (new). The term *balance* as used here refers only to relations of symmetry between *electrical* quantities and a reference such as ground. No physical configuration is necessarily implied here. It refers to a symmetry in electrical quantities *only*. Physical symmetry is indicated by the term homologous.

Balanced binal network (new). A binet displaying balanced characteristics.

Balance factor. Generic term relating to the class of number representing the susceptibility of a network to common mode interference.

Balanced line (new). A transmission line consisting of two conductors in the presence of a reference, *operated* in such a way that the voltage of the two conductors at identifiable points along the lines are equal in magnitude and opposite in polarity with respect to the reference. The currents in the two conductors are equal in magnitude and opposite in direction.

Balance loss figure - BLF (new). Is the ratio of the interference power output of a binal network, to the *noise* power output that would exist if the binet were *noiseless* and had a response only at the principal or desired frequency or band of frequencies.

Balance sensitivity attenuation factor - BSAF (new). Is the maximum allowable equivalent source voltage for a given source impedance configuration and given binet imbalance to maintain the interference level across a pair of homologous nodes below a prescribed limit.

Binal network (new). Is a network in two similar parts symmetrically related so that two nodes (with respect to a physical reference) nominally carry transverse and/or common-mode signal voltages. Physical symmetry is implied by prefixing with "homologous". Electrical symmetry is implied by prefixing with "balanced".

Binary. Consisting of, indicating, or involving two.

Binary part (new). One part of a binal network.

Bipart (new). A contraction of *binary part*.

Binet (new). A contraction of *binal network*.

Centered nodes (new). Are those terminals designed as electrical centers or neutrals with respect to homologous node pairs.

CM. Is an abbreviation for common mode.

Common mode gain. The ratio of common mode output power divided by the common mode input power.

Note: The term *common mode gain* is still used as a ratio of voltages (Reference A.2) although such usage has been deprecated for many years, [illegible] professional [illegible] (Reference A.[illegible]) [illegible] by several professional authorities (Notably, Reference A.3). For an excellent discussion of *gain* vs *amplification* see Reference A.1.

Common mode input. Is the ideal voltage source producing in-phase and equivalent components of voltage across homologous node-pair(s) of a binet. (See *longitudinal input.*)

Common mode resistance. Usually this term refers to the resistance between input (or output) signal leads and ground.

Note: It may be thought that this term would more suitably refer to the real part of the impedance seen by the interference source. However this meaning is here given the name *longitudinal driving point resistance.*

Common mode rejection - CMR. Is the ratio of the applied common mode input voltage to the equivalent normal-mode output signal it produces.

Common mode rejection factor - CMRF. The ratio of the CM input voltage to the DM input voltage that gives rise to the same output voltage.

Common mode voltage. The components of voltages which are equivalent across homologous nodes. This is often used as the maximum voltage which may be applied without breaking down insulation between the input (output) lines and ground. In this latter sense, common-mode voltage and common-mode resistance are of course unrelated to common mode rejection.

Common mode floating transfer attenuation factor - FTAF (new). Is the ratio of the CM input voltage divided into the DM input voltage that gives to the same (floating) output voltage.

Common mode floating attenuation factor - FAF (new). Is the same as FTAF except the binet terminals are driven through finite equivalent-source impedances.

Common mode split-load transfer attenuation factor - STAF (new). Is the ratio of the CM input voltage divided into the DM input voltage that gives rise to the same (split-load) output voltage.

Common mode split-load attenuation factor - SAF (new). Is the same as STAF except the binet terminals are driven through finite equivalent source impedances.

Conducted interference. Is that caused by the coupling effect of capacitance, resistance and inductance to the interference source. See metallic circuit.

DF. Is the abbreviated form of discrimination factor. It is the ratio of the differential mode amplification to the common mode amplification.

Differential amplifier. An amplifier whose input leads have identical driving point impedances with respect to a specified reference and which responds to differential mode signals.

Differential mode. Is the term used to describe the instantaneous difference in (signal) voltage between nodes. This can be considered to be the same as the transverse mode (signal) voltage.

DM. Is used as the abbreviation for differential mode.

Driving point common mode attenuation factor - DPAF (new). Refers to the transverse or differential mode voltage across the driven nodes compared to the longitudinal source of a certain source configuration.

Homologous. Corresponding to, as in relative position.

Homologous binal network (new). Is a binal network with n node-pairs of terminals physically (or with respect to components) symmetrical about a fixed reference.

Homologous nodes. Refers to a pair of nodes exclusive to each of the binary parts of a binet.

An interbipart element (new). Is the name given to that class of elements connecting nodes of different biparts.

An intrabipart element (new). Is the name given to that class of elements connecting nodes of one bipart only.

Longitudinal. (as used here) Refers to the lengthwise characteristics of a binal network, where the length is identically the path taken by the transverse signal passing through the network.

Longitudinal currents. Are the electrical duals of the common mode voltage and are the components of current which are equivalent flowing through homologous nodes in the direction of the transverse signal path.

At any instant the product of the longitudinal current and the common mode voltage is equal to the instantaneous power in the reference direction of the transverse signal path.

Longitudinal driving point impedance. Is the impedance function characterizing the impedance seen by the common mode input connected to reference and the homologous input nodes, the latter through equivalent source impedances.

Longitudinal input. (as used here) Refers to the single ideal voltage source driving either a centered impedance or one or more homologous node-pairs through any specified impedance configuration.

Metallic circuit. Is used mainly by telephone and utility companies to describe the wire-connected or signal portions of the network.

Normal mode. Is used mainly as an equivalent of transverse or differential mode. In special usage, normal mode may refer to the mode which is considered to be nominal; of course in these instances a common-mode signal and a transverse mode signal may both be required to make up a normal mode signal.

Transverse. Refers to the cross-wise characteristics of a binal network and is the complement of longitudinal.

APPENDIX B

DEVELOPMENT OF THE FAULT POLYNOMIAL

For purposes of binet analysis, it is desirable to express the balance factor ratio as a rational fault function; that is as a ratio of (fault) polynomials. A fault polynomial is defined here as a polynomial of unbalance values with coefficients made up of variously ordered cofactors of the matrix of the homologous binal network.

By means of an extension of the ideas of the Laplace Expansion for a determinant, it is possible to develop such a fault polynomial. Ordinarily this can be done before the cofactor simplifications are carried out, although the total computational work may sometimes be reduced by simplifying first.

If one takes any m rows of the $n^{\underline{th}}$ ordered matrix $[Y]$; no loss of generality will occur if it is the first m rows; ther can be formed a number of minors, N_m, each or order m. This number is equal to:

$$N_m = C_m^n = \frac{n!}{(m!)(n-m)!}$$

Multiplying each such minor by its *signed* minor of order $(N_m - m)$ one gets terms of the $[Y]$ determinant. It is clear that none of these duplicates any other, for each involves a different selection of rows and columns and therefore each term has a unique combination of elements. Adding up the total number of such terms this is:

$$N_m m!\ (n-m)! = \frac{n!}{m!\ (n-m)!}\ m!\ (n-m)! = n!$$

Therefore *all* of the terms of $|Y|$ will be so included and the expansion represents the value of the determinant of $[Y]$. Now it is obvious that if there are m rows containing the unbalances in the binet, the expansion by these m^{th} ordered minors would enable ready evaluation of the determinant. On the other hand if nearly all of the unbalance terms appear in a single row or column, a Laplace Expansion of that row or column would be convenient.

As an example one can consider the unbalanced matrix $[Y]'$ made up of the homologous matrix $[Y]$ representing an admittance matrix of a homologous binet and unbalance values, Δ added to the elements of $[Y]$ making up the elements of $[Y]'$. Evidently there is the identity:

$$(Y)' = \begin{vmatrix} y_{11}+\Delta_{11} & y_{12}+\Delta_{12} & y_{13}+\Delta_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{vmatrix} = y_{11}Y_1^1 + y_{12}Y_1^2 + y_{13}Y_1^3 + \Delta_{11}Y_1^2 + \Delta_{13}Y_1^3$$

$$(Y)' = Y + \Delta_{11}Y_1^1 + \Delta_{12}Y_1^2 + \Delta_{13}Y_1^3 \qquad \text{Equation B.1}$$

The fault polynomial of Equation B.1 is seen to have a "constant term", which is the admittance matrix of the homologous binet and its coefficients are the respective signed minors of the balanced binets. The simplicity of this expansion is a result of the Laplace expansion over the first row with all first-row cofactors Y_1^k k=1, 2, 3 having elements void of the unbalance values, Δ_{ik} $(i \neq 1)$. However it is to be expected that there will be unbalance entries in other places in the matrix say in the i, k=2, 3 position. It is possible to include this extra term in the expansion, provided all possible multiplications are carried out. Thus:

$$(Y)' = \begin{vmatrix} y_{11}+\Delta_{11} & y_{12}+\Delta_{12} & y_{13}+\Delta_{13} \\ y_{21} & y_{22} & y_{23}+\Delta_{23} \\ y_{31} & y_{32} & y_{33} \end{vmatrix}$$

$$= y_{11}(Y_1^1 + \Delta_{23}Y_{12}^{13}) + y_{12}(Y_1^2 + \Delta_{23}Y_{12}^{23})$$

$$+ y_{13}(Y_1^3 + \Delta_{23}Y_{12}^{33}) + \Delta_{11}(Y_1^1 + \Delta_{23}Y_{12}^{13})$$

$$+ \Delta_{12}(Y_1^2 + \Delta_{23}Y_{12}^{23}) + \Delta_{13}(Y_1^3 + \Delta_{23}Y_{12}^{33})$$

$$+ \Delta_{23}(Y_{21}^{31} + Y_{21}^{32})$$

where the last term is required to complete all possible products of the Laplace expansion. This can be rearranged as a fault polynomial:

$$(Y)' = |Y| + \Delta_{11}Y_1^1 + \Delta_{12}Y_1^2 + \Delta_{13}Y_1^3 + \Delta_{23}Y_2^3 + \Delta_{11}\Delta_{23}Y_{1\,2}^{1\,3} + \Delta_{12}\Delta_{23}Y_{1\,2}^{2\,3}$$

Equation B.2

where the terms $\Delta_{13}\Delta_{23}Y_{12}^{33}$ must be deleted since its unbalance values appear in the same column and thereby violate the Laplace expansion. It is seen that although a product of unbalances $\Delta_{ij}\Delta_{kl}$ may be a small number the particular cofactor may be very large so that the whole term $\Delta_{ij}\Delta_{kl}Y_{ik}^{jl}$ may be quite significant in the $(Y)'$ fault polynomial.

As is to be expected when several unbalance values appear off of the particular row(s) or column(s) of the Laplace Expansion, the number of terms in the fault polynomial becomes large. For example, if there are p unbalance values off of the unsigned minor portion of the expansion there will be p terms with single unbalances, $C_2^p = \frac{p(p-1)}{2}$ terms as the product of two unbalances; $C_3^p = \frac{p(p-1)(p-2)}{3!}$ terms as the product of three unbalances and of course a final term $(1 = C_p^p)$ as the product of all unbalances. Of course all products derived from at least two Δ's from the same row or column will be deleted as mentioned above.

Carrying out the Laplace Expansion *over an i^{th} row* for say m unbalances in an n^{th} order matrix, one arrives at the value of the determinant of the *unbalanced* Y matrix $(Y)'$ as:

$$(Y)' = Y + \sum_{j=1}^{n} \Delta_{ij}\{Y_i^j + \sum_{k,l}^{n} \Delta_{kl} Y_{ik}^{jl} + \sum_{r,s}^{n} \Delta_{rs}(Y_{ikr}^{jls} + \sum_{u,v} \Delta_{u,v}(Y_{ikru}^{jlsv} + \ldots)\}$$

$$+ \sum_{k,l} \Delta_{kl}\{Y_k^l + \sum_{r,s} \Delta_{rs} Y_{kr}^{ls} + \sum_{u,v} \Delta_{u,v}(Y_{kru}^{lsv} + \ldots)\}$$

Equation B.3

where each of the cofactors with two or more equal superscripts or two or mor equal subscripts is identically zero and every summation off the i^{th}

row is over all Δ's within thés *primed* cofactor identified by the cofactor immediately precedint the summation in question. Of course the first summation over the $i^{\underline{th}}$ row is over all Δ_{ij} in that row.

For an unbalance admittance added to every element of an $n^{\underline{th}}$ ordered matrix, there apparently would be $(2^n-1)n!$ terms including at least one Δ, and of course $n!$ terms without a Δ.

Equation B.3 will be called a *discrete fault polynomial* of the primed matrix $(Y)'$ and it defines a fault polynomial for unbalance elements throughout the matrix.

As indicated in Chapter II it is sometimes convenient to select the Δ's so they appear in only one bipart. Then one can choose the unsigned minors relating to this bipart as the basis for a Laplace Expansion. The evaluation of each unsigned minor can be made by means of Equation B.3 and each of these may then be multiplied by its respective *signed complementary minor*. In this way the unbalance values can be given directly, separated from the unprimed cofactors, and in their simplest form.

Thus in the general homologous binet of Figure 2.3 a, assuming only the $i^{\underline{th}}$, $j^{\underline{th}}$ and $(2n+1)^{\underline{th}}$ *rows* contain unbalance elements, one can form a summation of third order cofactors:

$$Y_\Delta Y^{k,l,m}_{i,j,2n+1} =$$

$$\begin{vmatrix} y_{ik}+\Delta_{ik} & y_{il}+\Delta_{il} & y_{im}+\Delta_{im} \\ y_{jk}+\Delta_{jk} & y_{jl}+\Delta_{jl} & y_{jm}+\Delta_{jm} \\ (y_{2n+1,k}+\Delta_{2n+1,k}) & (y_{2n+1,l}+\Delta_{2n+1,l}) & (y_{2n+1,m}+\Delta_{2n+1,m}) \end{vmatrix} Y^{k,l,m}_{i,j,2n+1}$$

$$(Y)' = \sum_{k,l,m=1}^{2n+1} Y_\Delta Y^{k,l,m}_{i,j,2n+1} \qquad \text{Equation B.4}$$

where the summation includes all combinations of *different* k,l, and m and for a (2n+1) order cofactor would have a total of $C_3^{2n+1} = \frac{4n^3-n}{3}$ terms. Equation B.4 will be called the *fault minor expansion* of the determinant. For purposes of illustration, two simple examples will be given which will demonstrate the methods implied by the use of Equation B.3 and B.4. An example of the use of Equation B.3 can be had from the evaluation of the determinant:

$$(Y)' = \begin{vmatrix} 1+\Delta_{11} & 1 & 1+\Delta_{13} & 2 \\ 1 & 1+\Delta_{22} & 1 & 1 \\ 0 & 1 & 1+\Delta_{33} & 1 \\ 1+\Delta_{41} & 0+\Delta_{42} & 0 & 1 \end{vmatrix}$$

where for checking purposes, values can be given to Δ_{ij}. Say $\Delta_{11} = \Delta_{13} = \Delta_{41} = 1$ and $\Delta_{22} = \Delta_{33} = 2$. With these values $(Y)' = 5$. From Equation B.3:

$$(Y)' = |Y| + \Delta_{11}(Y^{1}_{1} + \Delta_{22}Y^{1\ 2}_{1\ 2} + \Delta_{42}Y^{1\ 2}_{1\ 4} + \Delta_{22}\Delta_{33}Y^{1\ 2\ 3}_{1\ 2\ 3} + \Delta_{33}\Delta_{42}Y^{1\ 3\ 2}_{1\ 3\ 4})$$

$$+\Delta_{12}Y_2^2 + \Delta_{33}Y_3^3 + \Delta_{41}Y_4^1 + \Delta_{42}Y_4^2$$

$$+\Delta_{22}\Delta_{33}Y_{23}^{23} + \Delta_{22}\Delta_{41}Y_{24}^{21} + \Delta_{33}\Delta_{41}Y_{34}^{31} + \Delta_{33}\Delta_{42}Y_{34}^{32}$$

$$+\Delta_{22}\Delta_{33}\Delta_{41}Y_{2\,3\,4}^{2\,3\,1}$$

Equation B.5

Now

$$Y = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{vmatrix} = 0 \quad \text{because of the similar columns.}$$

Then noting the signs of the various cofactors (for example, $Y_{124}^{321} = -Y_{124}^{123} = +$ abs Y_{124}^{321})

$$\begin{aligned} (Y)' &= 0 + 1(0 + 2(1) + 2(1)) + 1(1 + 1(+0) + 2(-(-1)) + 1(-(1))) \\ &\quad + 2(0) + 2(-1) + 1(0) + 1(1) \\ &\quad + 2(2)(-1) + 2(1)(-1) + 2(1)(-1) + 2(-(-1) + 2(2)(1)(-(+2)) \\ &= -5 \end{aligned}$$

which is the correct value of $(Y)'$.

On the other hand, had the unbalance distribution been as indicated in Equation B.6, an unsigned minor expansion would have been more convenient. An example of the use of Equation B.4 can be had from evaluation of the determinant,

$$(Y)' = \begin{vmatrix} 1+\Delta_{11} & 1 & 1+\Delta_{13} & 1+\Delta_{14} \\ 1 & 3 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 1+\Delta_{41} & 1 & 1+\Delta_{43} & 0+\Delta_{44} \end{vmatrix}$$

Equation B.6

For the determinant of Example 1, values of $\Delta_{11} = \Delta_{13} = \Delta_{14} = \Delta_{41} = \Delta_{44} = 1$ is required with $\Delta_{43} = -1$.

Then:

$$(Y) = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

Then:

$$(Y)' = \sum_{k,l=1}^{4} Y_{\Delta} Y_{14}^{kl} = Y_{14}^{12}\Big[(1+\Delta_{11})1 - 1(1+\Delta_{41})\Big]$$

$$+ Y_{14}^{13}\Big[(1+\Delta_{11})(1+\Delta_{43}) - (1+\Delta_{41})(1+\Delta_{13})\Big]$$

$$+ \left[(1 + \Delta_{11})(0 + \Delta_{44}) - (1 + \Delta_{41})(1 + \Delta_{14})\right] Y_{14}^{14}$$
$$+ \left[1(1 + \Delta_{43}) - 1(1 + \Delta_{13})\right] Y_{14}^{23} + \left[1(0 + \Delta_{44}) - 1(1 + \Delta_{14})\right] Y_{14}^{24}$$
$$+ \left[(1 + \Delta_{13})(0 + \Delta_{44}) - (1 + \Delta_{43})(1 + \Delta_{14})\right] Y_{14}^{34}$$

$$(Y)' = -(\Delta_{11} - \Delta_{41})2 - \left[(\Delta_{11} - \Delta_{41}) + (\Delta_{43} - \Delta_{13}) + (\Delta_{11}\Delta_{43} - \Delta_{13}\Delta_{41})\right]2$$
$$+ 8\left[(\Delta_{11} - \Delta_{41}) + (\Delta_{44} - \Delta_{41}) + (\Delta_{11}\Delta_{44} - \Delta_{41}\Delta_{14})\right]$$
$$+ (\Delta_{43} - \Delta_{13}) - 3(\Delta_{44} - \Delta_{14} - 1)$$
$$+ \left[(-\Delta_{43}) + (\Delta_{44} - \Delta_{14}) + (\Delta_{13}\Delta_{44} - \Delta_{43}\Delta_{14}) - 1\right] 1$$

As a check, taking assumed values for the Δ's gives:

$$(Y)' = -(0)2 - (0) + (-2) + ((-1) - (1))\ 2 + 8\ 0\ -1\ -1$$
$$+ (-1)-1 \ - 3\ -1 \ + \ 1 + (1 + 1) - 1 \ \ 1$$
$$= -5, \text{ which checks with correct value.}$$

Now it may seem that the values selected for the Δ's were particularly fortuitous since they combine in Equation B.4 to yield rather simple results. However it is evident that the matrix $(Y)'$ is equicofactor with respect to all entries including the unbalance values, so that in Equation B.6, for example, one expects: $\Delta_{11} + \Delta_{41} = 0$ and $\Delta_{11} + \Delta_{13} + \Delta_{14} = 0$, etc. In the calculation of balance factors using first or second cofactors of $(Y)'$, there will usually remain some such vestige of the zero sum property in each of the cofactors, so that the fault polynomial will be simpler as a result.

A P P E N D I X C

SIMPLIFICATION PROOF

In the simplification procedures of Chapters II and III, it was necessary to assume that for the partitioned matrix Y:

$$Y = \begin{bmatrix} A_1 & B_1 \\ & \\ B_2 & A_2 \end{bmatrix}$$

the value of the determinant of (Y) was especially simple to calculate when *either* of the submatrices $[B_1]$ or $[B_2]$ was identically the null matrix, $[0]$ for then $\det[Y] = \det[A_1] \cdot \det[A_2]$

Aided by the Laplace Expansion of Minors as discussed in Appendix B, it is possible to prove this directly from a minor expansion about the rows or columns of $[A_1]$. Alternatively it is somewhat more involved to show that this is true by a direct row or column expansion of $\det[A_1]$.

By the minor expansion, with $[B_1] = [0]$, $\det[A_1]$ is the only non-zero minor in the expansion along rows of $[A_1]$ since in every other minor there are one or more column vectors identically zero. Then the entire Laplace Expansion of $\det[Y]$ is simply $\det[A_1]$ multiplied by its complementary signed minor, $+\det[A_2]$. When $[B_2] = [0]$, $\det[A_1]$ is the only non-zero minor in the expansion along the columns of $[A_1]$ since in every other minor there are one or more row vectors identically zero. Then the entire $\det[Y] = \det[A_1] \cdot \det[A_2]$.

APPENDIX D

LUMPED EQUIVALENCE OF DISTRIBUTED PARAMETER NETWORKS

Our ultimate interest here is the identification of certain physical configurations of conductors and their electrical effect on a given lumped parameter network. The immediate need is identified directly with the work of Trent (Reference D.1), Osterbrook (Reference D.2) and Tonning (References D.3 and D.4). These have attempted to develop systematic methods for the formation of an approximate discrete model for a given physical configuration. It follows that the *exact* formulation of the partial differential equations of a distributed parameter system will involve no subdivision of the system into smaller parts (components). In most practical cases of interest here, although the partial differential equations describing a system can be formulated, solutions are difficult if at all possible, and attention is usually given to approximations.

If reactive elements can be neglected and a homogeneous isotropic medium exists in the intervening space, with scalar constants ε and σ sufficient to represent its permittivity and conductivity, one can arrive at a reasonable value for a lumped-equivalent conductance of that distributed parameter quite readily. A tube of current can be assumed as in Figure D.1. The length of the tube is chosen small enough so that the capacitance between the ends of the element is a flat plate idealization:

$$C = \varepsilon \frac{da}{ds} \quad \text{farads}$$

with a conductance:

$$G = \sigma \frac{da}{ds} \text{ mhos.}$$

Here da and ds are the elements of area and the thickness between the ends, respectively. Then $\frac{G}{C} = \frac{\sigma}{\varepsilon}$ and since $G = \frac{I}{V}$ and $C = \frac{Q}{V}$:

$$\frac{Q}{\varepsilon} = \frac{I}{\sigma} \qquad \text{Equation D.1}$$

Therefore the voltage distribution around one of the wires due to its current and the medium's conductivity, can be related to the voltage distribution $V(r,\phi,Z)$ around a line due to a charge Q on it. For that case

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} = 0$$

assuming that a solution is:

$$V = K_1 \ln r + K_2$$

$$\frac{\partial V}{\partial r} = \frac{K_1}{r} \quad \text{and} \quad \frac{\partial^2 V}{\partial r^2} = \frac{K_1}{r^2}$$

The surface charge density $\rho_s = \frac{q}{2\pi R}$ and by Gauss Law, $D_n = \rho_s$ or,

$$E_n = \frac{\rho_s}{\varepsilon} = \frac{q}{2\pi R\varepsilon}$$

But $E_n = -\frac{\partial V}{\partial r} = -\frac{K_1}{r}$ and so at $r = R$: $E_n = -\frac{K_1}{R} = \frac{q}{2\pi R\varepsilon}$.

Therefore $K_1 = -\frac{q}{2\pi\varepsilon}$ or $E = \frac{q}{2\pi r\varepsilon} 1_r$ where 1_r is the unit vector and $V = -\frac{q}{2\pi\varepsilon} \ln r + K_2$.

For the above, therefore, $K_1 = \frac{I}{2\pi\sigma}$ where the reference K_2 is zero. If this line is considered the first, and a second line is imaged behind the conducting plane by an equal distance, it is apparent that $V_1 - V_2 = \frac{I}{2\pi\sigma}$ $(\ln \frac{2(d-R)}{R} - \ln \frac{R}{2(d-R)})$ where d is the distance of the wire from the conducting plane. Then the conductance between the wire and the conducting plane is simply: $\sigma = \frac{I}{V_1 - V_2} = \frac{2\pi\sigma}{\ln(\frac{d-R}{R})^2}$ mhos.

Matters become more complicated when reactive elements are involved. Then a reasonable approach would make use of a set of discrete subregions infolding the components of a resulting lumped parameter approximation of the distributed parameters physical system. A convenient shape for the subregion would be the volume element of a curvilinear coordinate system as shown in Figure D.2.

Then a linear graph can be associated with the set of subregions by specifying each vertex of the graph to represent the center of a respective element of volume, each linear graph edge corresponding to one of the six faces of the elements. Now in lumped parameter network analysis, the association of a voltage variable (scalar point function) with two points, suggests that a vector point function may be similarly identified if its line integral along one of the discrete edges is made significant by a pertinent property of the medium. In the limit as the differential dl approaches an infinitesimal:

$$v_{ab} = \int_a^b E \cdot dl \qquad \text{Equation D.2}$$

This equation relates the *vector-across* variable E (electric field intensity) to the *scalar across* variable, v_{ab} (the voltage drop from a to b). Similarly as da is made infinitesimally small:

$$I_{ab} = \int i \cdot da \qquad \text{Equation D.3}$$

Equation D.3 relates the *vector through* variable i (electric current density) to a *scalar through* variable I_{ab}. Vectors dl and da are indicated in Figure D.2. By the application of Stoke's theorem to Equation D.2, it is immediate that the conservative field property; $\nabla X E = 0$ corresponds to Kirchoff's voltage law applied to the linear graph edges. An application of the Divergence theorem to Equation D.3 similarly relates the principle of the conservation of charge, $\nabla \cdot i = \frac{\partial \rho}{\partial t}$ to Kirchoff's current law at each vertex. This, of course, establishes properties to be used in the selection of suitable *vector across* and *vector through* variables for *other* kinds of systems as well. (Reference D.2) However the interest here is especially related to electrical networks and a simple illustration may be helpful. Assume the cylindrically symmetric physical configuration of a long wire over a conducting plane, so that the quantities of Figure D.2 conveniently take over the values:

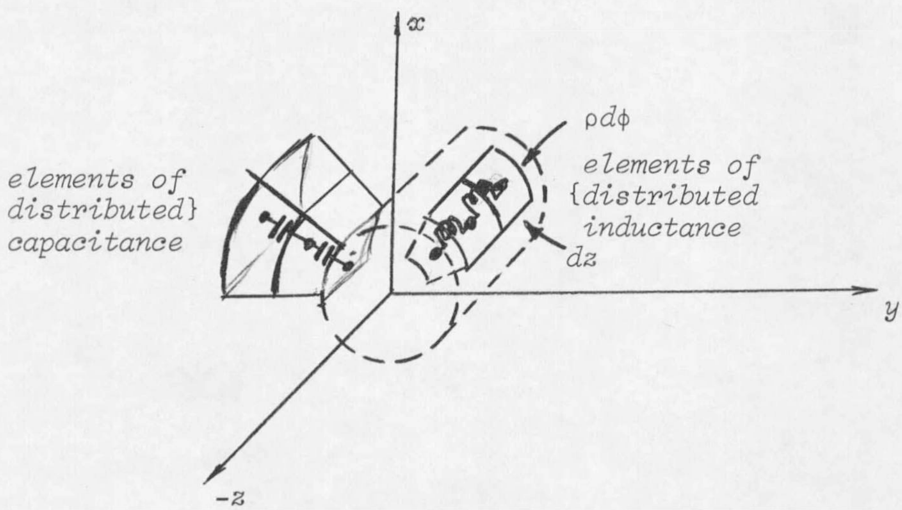


Figure D.3: Illustration of the subregional conversion

$(q_1, q_2, q_3) \equiv (\rho, \phi, Z)$ and $h_1 = 1$, $h_2 = r$, $h_3 = 1$.

An illustration of this appears in Figure D.3. If the wire conductor of radius "a" runs some "d" distance above a ground plane in the y = 0 plane, according to the above, the number of vertices needed would correspond to the degree of approximation required to keep the respective integrands of Equations D.2 and D.3 constant over the differential elements by the same amount. So in nearly all practical instances, the size of the finite-sized volume element approximating the infinitesimal element would be expected to be variable throughout the region. To the extent the simple geometry of a straight conductor over a conducting plane is intact, the scalar-across and scalar-through variables will be involved respectively with significant and calculable values of (per-unit) length capacitance and inductance. But as discontinuities occur in the coordinate symmetry, the task of solving the resulting differential equations will become essentially impractical. Unless guided by rather elaborate procedures, it is apparent that the discrete approximation of most practical problems will become too involved for routine analysis and synthesis.

On the other hand, one can approach this problem from purely a field point of view. Then from *Reference D.3,* A. Tonning suggests that whenever such a field problem is presented (in network terms) a reference system of vector functions will be necessary. This latter takes the form of a complete biothogonal set of vector functions which are tangent to the entire surface of conducting bodies. In the general case this will be an *infinite* biorthogonal set, and resulting matrices will be of infinite order. The completeness of the set allows that the tangential components of the fields on the surfaces bounding the region can be expanded in a series in one of the orthogonal sets provided only that the fields are square integrable over the surface. Tonning suggests that such a reference system will enable the determination of an impedance matrix (of infinite order in the general sense) along with corresponding terminal voltages and currents.

Consider a volume R bounded by a surface S in which the medium is linear, not a f(t) and contains no sources. (See Figure D.4) If $E_t(r) = 0$ for all d's on S, then the electric field throughout R must be zero except at certain frequencies which we will purposefully avoid. It can be readily proved that if we specify E and H throughout R at *t = 0 and* E_t all over the surface *then* E and H are uniquely determined *throughout R* for all t. (Reference D.4) This can also involve the specification of E_t on just one part of S and H_t on other part. Let $A_1(\bar{r})$, $A_2(\bar{r})$....and $B_1(\bar{r})$, $B_2(\bar{r})$...be an infinite set of real vector functions defined for every $\bar{r}$ on S such that: (1) $\bar{1}_n \cdot A_\alpha(\bar{r}) = \bar{1}_n \cdot B_\beta(\bar{r}) = 0$ (2) Set is orthonormal such that $\int_S A_\alpha(\bar{r})\, B_\beta(\bar{r})\, da = \delta_{\alpha\beta}$......$\beta,\alpha = 1, 2$. If we impress a field on S then $E_t = \sum_{m=1}^{\infty} V_m Z_m^{\frac{1}{2}} A_m(\bar{r})$ where $V_m = Z_m^{-\frac{1}{2}} \int_S A_m(\bar{r}) \cdot E_t\, da$.

If A_α and B_β is a *complete* biorthogonal set on S, then the tangential components of the fields may be expanded in a series using either A or B provided only that the fields are square integrable.

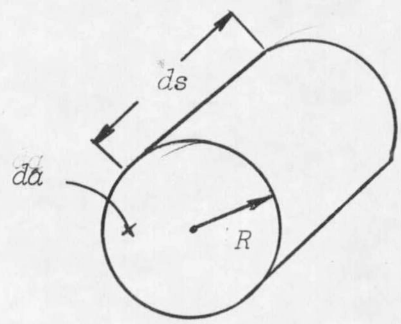
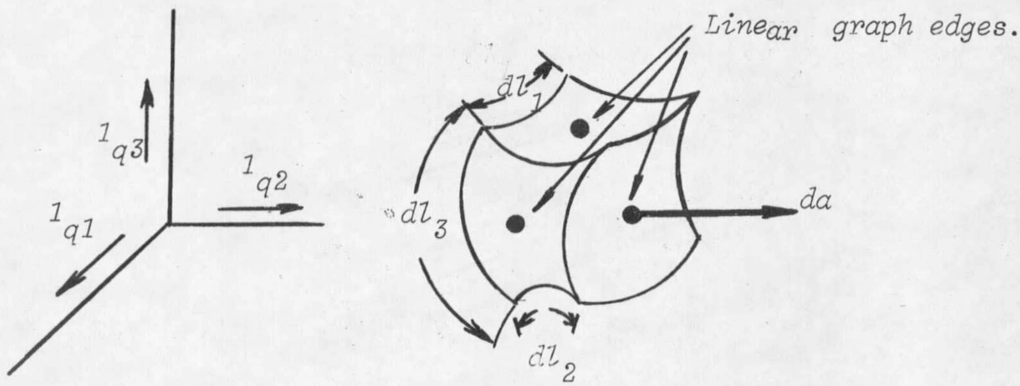


Figure D.1   A conducting element.



Elemental volume $d\tau = dl_1 dl_2 dl_3 = h_1 h_2 h_3 dq_1 dq_2 dq_3$ where h and q have the usual meanings.

Figure D.2   Subregional conversion to a linear graph.

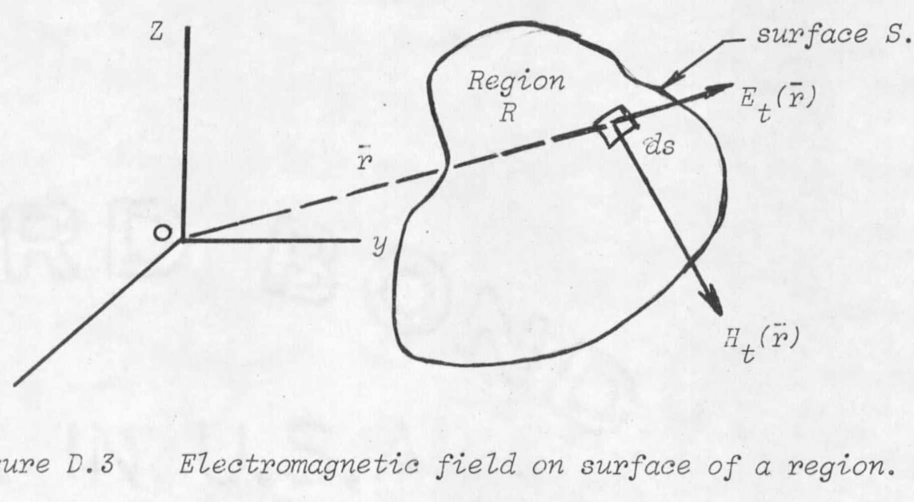


Figure D.3   Electromagnetic field on surface of a region.

The orthonormal system ϕ_n is said to be *complete* if the formula:

$$\int_a^b f(x)^2\, dx = \sum_k C_k^2$$

is true for all f, where the C's are the Fourier coefficients relative to ϕ_n such that:

$$C_k = \int_a^b f(x)\,\phi_n(x)\, dx$$

One can also expand in a series:

$$1_n \times H_t = \sum_{m=1}^{\infty} I_m Z_m^{-\frac{1}{2}} B_m(\overline{r})$$

where $I_m = Z^{\frac{1}{2}} \int_a^b (1_n \times H_t) \cdot A_m(\overline{r})\, da.$

We have assumed that E_t (along with H_t) is square integrable over S. If A_α is a complete set, then Perceval's *equality* holds:

$$\iint_S E_x(\overline{r})^2\, da = \sum_{k=1}^{\infty} C_k^2$$

is true for all $E_t(\overline{r})$ where C_1, C_2...are the Fourier coefficients of $E_t(\overline{r})$ relative to the orthogonal set A_α. But we have taken $C_k = V_k Z_k^{\frac{1}{2}}$. Therefore

$$\sum_{k=1}^{\infty} \left[V_k Z_k^{\frac{1}{2}}\right]^2 \quad \text{and} \quad \sum_{k=1}^{\infty} \left[I_k Z_k^{-\frac{1}{2}}\right]^2$$

converge to their respective integrals of $E_t(\bar{r})$ and $H_t(\bar{r})$. We then restrict Z_k so that

$$\sum_{k=1}^{\infty} |V_k|^2 \quad \text{and} \quad \sum_{k=1}^{\infty} |I_k|^2$$

are both convergent. The restriction is one of consequence since *absolute* convergence is stronger. See Reference D.4 for example. Here there is a linear boundary value problem with linear E - H relation. Therefore there is a linear relation between them and must have $I = \underset{\sim}{Y} V$ where I is column vector of I and $\underset{\sim}{Y}$ is the admittance matrix of the region R. [$\underset{\sim}{Y} = \underset{\sim}{Z}^{-1}$ provided $I = Y V$ has a unique solution.]

The region R is given the name "network" and if the medium is *linear* with electromagnetic properties *constant in time* then can refer to it as a *"linear stationary network"*.

Using the defined network concepts, it can easily be shown that if the total power dissipation in R be given by $P = -\text{Re}\, \frac{1}{2}\int_S (E_t \times H_t^*) \cdot \bar{1}_n da$ then $P = \frac{1}{2}\text{Re} \sum_{k,l=1}^{\infty} I_k^* Z_{kl} I_l$ or in matrix form $P = \text{Re}\frac{1}{2} [I^*] \underset{\sim}{Z} [I]$.

The network matrices $\underset{\sim}{Y}$ and $\underset{\sim}{Z}$ determine the response to an excitation but they are not *uniquely* determined by a given surface. According to Tonning, (Reference D.4) this arbitrariness can be removed by (1) changing the normalizing impedance and (2) defining a new complete orthogonal set. This is performed in three steps: (i) setting Z elements equal to unity

(ii) subject impedance matrix to a similarity transformation by a real orthogonal matrix and (iii) change normalizing impedances from 1 to Z_1', to Z_2' and 1 to Z_3', etc.

The problem of defining a new complete orthogonal set is a major one for even the simplest physical configurations anticipated here.

APPENDIX E

EVALUATION OF THE FIRST COFACTOR OF A MATRIX WITH HOMOLOGOUS BINET SYMMETRY

All first cofactors of an indefinite admittance matrix for a homologous binet have equal magnitudes and it is apparent that a most convenient one can be used to ascertain this value. Y_R^R is such a cofactor for this will preserve matrix symmetry.

Omitting the $R^{\underline{th}}$ row and column of Y in Equation 2.3 c there is:

$$Y_R^R = \det \begin{bmatrix} +g_{11} \cdots -g_{1n} & -g_{1c} & -g_{1,n+1} \cdots -g_{1,2n} \\ \vdots \qquad \vdots & & \vdots \qquad \vdots \\ -g_{1,n} \cdots +g_{n,n} & -g_{n,c} & -g_{n,n+1} \cdots -g_{n,2n} \\ -g_{1,c} \qquad -g_{n,c} & +g_{c,c} & -g_{c,n+1} \qquad -g_{c,2n} \\ -g_{1,n+1} \cdots -g_{n,n+1} & -g_{c,n+1} & +g_{n+1,n+1} \cdots -g_{n+1,2n} \\ \vdots \qquad \vdots & & \vdots \qquad \vdots \\ -g_{1,2n} \cdots -g_{n,2n} & -g_{c,2n} & -g_{n+1,2n} \cdots +g_{2n,2n} \end{bmatrix}$$

Equation E.1

Adding the negative of row (n+1) to row 1, the negative of row (n+2) to row 2 etc., will cause 0's to appear in the first n rows of the c column since $g_{n+1,c} = g_{1,c}$. Of course this assumes that the equivalent interfering source impedance to be applied to each wire of any pair of wires extending out of the network will be identical. This is based on the rather obvious presumption that, for any network designed to be balanced homologous lines will be run equivalently in the same noise environment. Reference return (longitudinal) conductances are not, of course, so constrained.

Then Y_R^R will appear as:

$$Y_R^R = \det \begin{bmatrix} g_{11}+g_{n+1,1} \cdots -g_{1n}+g_{n+1,n} & 0 & -g_{1,n+1}-g_{n+1,n+1} \cdots -g_{1,2n}+g_{n+1,2n} \\ \vdots \quad\quad \vdots & \vdots & \vdots \quad\quad \vdots \\ -g_{n1}+g_{2n,1} \cdots -g_{n+1,n}+g_{n+1,2n} & 0 & -g_{n,n+1}+g_{2n,n+1} \cdots -g_{n,2n}-g_{2n,2n} \\ -g_{1c} \quad\quad -g_{cr} & +g_{cc} & -g_{c,n+1} \quad\quad -g_{c,2n} \\ -g_{n+1,1} \cdots -g_{n+1,n} & -g_{n+1,c} & +g_{n+1,n+1} \cdots -g_{n+1,2n} \\ \vdots \quad\quad \vdots & \vdots & \vdots \quad\quad \vdots \\ -g_{2n,1} \cdots -g_{2n,n} & -g_{2n,c} & -g_{2n,n+1} \cdots +g_{2n,2n} \end{bmatrix}$$

Equation E.2

Again $g_{11} = g_{n+1,n+1}$ and etc., so that the first column added to the (n+1)st column will put zeros in the first n rows of the (n+1)st column. Then ½ the second column added to the (n+2)nd column will place zeros in the first n rows of the (n+2)nd column. This can be carried out until:

$$Y_R^R = \det \begin{bmatrix} g_{11}+g_{n+1,1} \cdots -g_{1n}+g_{n+1,n} & 0 & 0 & \cdots & 0 \\ \vdots \quad\quad \vdots & \vdots & \vdots & & \vdots \\ -g_n+g_{2n,1} \cdots g_{nn}+g_{2n,n} & 0 & 0 & \cdots & 0 \\ -g_{1c} \quad \cdots \quad 0 & g_{cc} & 0 & \cdots & 0 \\ -g_{n+1,1} \cdots -g_{n+1,n} & -g_{n+1,c} & g_{n+1,n+1}-g_{1,n+1} & \cdots & g_{n+1,2n}-g_{n+1,n} \\ \vdots \quad\quad \vdots & \vdots & \vdots & & \vdots \\ -g_{2n,1} \cdots -g_{2n,n} & -g_{2n,c} & -g_{2n,n+1} & \cdots & -g_{n,2n}-g_{2n,2n} \end{bmatrix}$$

Equation E.3

Then a Laplace expansion of column c gives:

$$Y_R^R = g_{cc}(Y_{RC}^{RC})'' - (-)\ g_{n+1,c}(Y_{R,n+1}^{RC})'' + g_{n+2,c}(Y_{R,n+2}^{RC})'' + \ldots (-1)^{2n-1}$$

$$g_{2n,c}(Y_{R2n}^{RC})''$$

Equation E.4

where the double prime refers to the transformed cofactor Y_R^R of Equation E.3. But all $(Y_{R,n+i}^{RC})''$ must be zero for i=1,...,n since the first row will contain all zeros in each case. Therefore $Y_R^R = g_{cc}(Y_{RC}^{RC})''$. The cofactor $(Y_{RC}^{RC})''$ is of the type

$$\begin{vmatrix} A_1 & 0 \\ A_3 & A_4 \end{vmatrix}$$

so that

$$\det \begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix} = \det[A_1]\det[A_4]$$

Equation E.5

It is apparent therefore that for any LLFPB balanced binet so tested *any* first (equi)cofactor Y_j^i *of order (2n+1)* is the product of the sum of the source impedance multiplied by the signed minor Y_{CR}^{CR}.

Cited Literature

1.1 a Ficchi, R. F. "Electrical Interference" Hayden Book Co. Inc. 1964.

1.1 b Parnum, D. H. "Biological Amplifiers" Wireless World, Nov. 1945 p. 337 - 340.

1.1 c Parnum, D. H. "Biological Amplifiers" Wireless World, Dec. 1945 p. 373 - 376.

1.1 d Johnston, D. L. "Electro-encephalograph Amplifier" Wireless Engineer, August 1947 p. 271 - 77.

1.1 e Grenier, G. H. "Interference Problems in Low Level Input Design" IEEE Trans. EMC. December 1965 p. 408 - 419.

1.2 a Blakely, C. E. et.al. "Bibliography on Radio Frequency Interference" IRE Trans. on RFI. February 1962.

1.2 b MIL-I-6181D. Military Specification "Interference Control Requirements - Aircraft Equipment" November 25, 1959.

1.2 c MIL-Std.-826 Military Standard (USAF) "Electromagnetic Interference Test Requirements & Test Methods" January 20, 1964.

1.2 d NASA NHB 5320.3 "Electromagnetic Compatibility Principles and

Practices" Apollo Program October 1965.

1.2 e Consolidated Systems Corp. "Instrumentation Grounding & Noise Minimization Handbook" Pomona, California AD 612027 January 1965.

1.2 f Morrison, R. "Shielding and Grounding for Instrumentation Systems" Dynamics Co., Monterey Park, California.

1.2 g Pearlston, C. B. "Case & Cable Shielding Bonding and Grounding Considerations in Electromagnetic Interference" IRE Trans. on RFI October 1962 p. 1 - 16.

1.2 h Craft, A. M. "Considerations in the Design of Bond Straps" IEEE Trans. on EMC. October 1964.

1.2 i Frederick Research Corp. "Handbook on Radio Frequency Interference" Vols. 1 - 4.

1.2 j Middlebrook, R. D. "Differential Amplifiers" J. Wiley & Sons; 1963.

1.2 k Angelo, E. J. "Electronic Circuits" McGraw Hill 1958 p. 283 - 287.

1.2 1 LePage and Seely "General Network Analysis" McGraw Hill 1952.

1.2 m Aitken, A. C. "Determinants and Matrices" Interscience Publishers 1954 p. 45.

1.4 a Jordan, E. C. "Electromagnetic Waves and Radiating Systems" Prentice Hall 1950 p. 482 - 3.

1.5 a Mayhead, L.V. "The Measurement of Small Signals in the presence of Common-Mode Interference" Electronic Engineering, Vol 34 July 1962 p. 483

1.5 b Grenier, G. H. "An Analysis of Common-Mode Measurements" IEEE Trans. of Task Force on Longitudinal Balance. March 1966, New York, N. Y.

1.5 c "IRE Standards on Audio Systems and Components - Methods of Measurement of Gain Amplification, Loss Attenuation and Amplitude Frequency Response." (1956) Proceedings IRE May 1956. p. 668.

2.1 a Shekel, J. "Analysis of Linear Networks" ScD Thesis. Massachusetts Institute of Technology; 1957.

2.1 b Aitken, A. C. "Determinants and Matrices" 9th Ed. Interscience Publishers, Inc. New York 1956 p. 74.

2.4 a Aitken, A. C. "Determinants and Matrices" 9th Ed. Interscience Publishers, Inc. New York 1956 p. 74.

3.0 a Kinariwala, B. K. "Synthesis of Active R. C. Networks" Bell Systems Technical Journal September 1959 p. 1269.

3.0 b Dietold, R. L. U. S. Patent #2,549,065. April 17, 1951.

3.0 c Bangert, J. T. "The Transistor as a Network Element" Bell Systems Technical Journal March, 1954 p. 329.

3.0 d Linville, J. G. "R C Active Filters" Proceedings IRE March 1954 p. 555.

3.0 e Horowitz, I. M. "Synthesis of Active R. C. Transfer Functions" Research Report R-507-56 PIB-437 Polytechnical Institute of Brooklyn November 1956.

3.0 f Yanagisawa, T. "RC Active Networks Using Current Inversion Type Negative Impedance Converters" Transactions Circuit Theory/IRE Sept. 1957 p. 140.

3.0 g Sharpe, G. E. "The Realization and Measurement of N-Terminal Active Networks" Journal of the Franklin Institute September 1965 p. 262.

3.1 a Sharpe, G. E. "Axioms on Transactors" IRE PGCT Vol. CT - 5 pgs. 189 - 196, 1958.

3.1 b Sharpe, G. E. "The Realization and Measurement of N-Terminal Active Networks" Ph-D dissertation University of London 1965.

3.3 a Percival, W. S. "On the Topology and Algebra of Linear Electrical Networks" Ph-D Thesis London University 1955.

3.3 b Bodewig, E. "Matrix Calculus" Interscience Publishers 1956 p. 9.

4.0 a Petrie, A. See Transactions: "Task Force on Longitudinal Measurements" IEEE Convention New York, N. Y. March 1966.

4.1 a Kron, G. "Equivalent Circuits of the Elastic Field" J. Applied Mechanics 1944 p. 149 - 161.

4.1 b Kron, G. "Equivalent Circuit of the Field Equations of Maxwell" Proc. IRE Vol. 32 1944 p. 289 - 299.

4.1 c Kron, G. "Equivalent Circuits of Compressible and Incompressible Fluid Flow Fields" J. Aeroscience Volume 12 1945 p. 221 - 231.

4.1 d Pierre, D. "Transient Analysis and Synthesis of Linear Control Systems Containing Distributed Parameter Elements" Ph-D dissertation University of Wisconsin 1962.

4.1 e Cockraft, J. D. "Skin Effect in Rectangular Conductos at High Frequencies" Proc. Royal Society (London) February 1929.

4.1 f Terman, F. E. "Radio Engineers Handbook" McGraw Hill 1947 p. 134.

4.1 g Trent, H. "Isomorphisms Between Linear Graphs and Lumped Physical Systems" Journal Acoustical Society of America Vol. 27 1955 p. 500 - 527.

4.1 h Craft, A. M. "Consideration in the Design of Bond Straps" IEEE Transactions on Electromagnetic Compatibility Vol. EMC - 6 p. 58 - 65 October 1964.

4.4 a Thomson, G. H. "The Factor Analysis of Human Ability" Houghton Miflin Co. Boston 1939.

4.4 b Sharpe, G. E. "Circuit Duality and the General Network Inverse" GPH Styan PGCT March 1965 pg. 22.

4.4 c Jorgensen, Goldman & Schulz "The Electromagnetic Environment of Transport Airplanes" Tenth Triservice Conference on Electromagnetic Compatibility November 1964.

4.4 d Kraus, J. D. "Antennas" McGraw Hill 1950 pg. 50.

4.4 e Jordan, E. "Electromagnetic Waves and Radiating Systems" Prentice Hall 1950 pg. 312.

4.4 f Jordan, E. "Electromagnetic Wave and Radiating Systems" Prentice Hall 1950 pg. 364.

4.4 g Schneider, D. "Designing for Low Level Inputs" Electronic Industries January 1961.

4.4 h Cheng, D. K. "Analysis of Linear System" Addison Wesley 1959.

4.5 a Gardner & Barns "Transients in Linear Systems" J. Wiley & Sons 1942.

4.5 b Bedford, A. V. & G. L. Fredendall "Analysis, Synthesis and Evaluation of Transient Response of Television Apparatus" Proc. IRE October 1942 pg. 440.

4.6 a Everett, Jr. W. W. & R. Powers "EMC A Position Paper" PGEMC Sept. 1966 p. 160.

4.6 b Brooks, J. L. "A Low Frequency Current Probe System for Making Conducted Noise Measurements" PGEMC June 1965 pg. 207: "conclusion".

4.6 c Grenier, G. H. "An Analysis of Common Mode Measurements" Task Force on Longitudinal Measurements. IEEE Convention New York, New York 1966.

4.6 d Cook, M. E. "RFI Instrumentation and Measurements" Tech Memo EMC TM-64-9 Rome Air Development Center, Griffiss AFB New York November 1964.

4.6 e Sproul, R. W. "The Future of EMC Instrumentation" PGEMC March 1965 pg. 35.

4.6 f Wend, M. "Handbook of Electronic Measurements" Vol. II Interscience Publishers 1957 pg. 11-3.

4.6 g "IRE Standard on Methods of Measuring Noise" Proc. IRE July 1953 pgs. 890-896.

4.6 h Middlebrook, R. D. "Differential Amplifiers" J. Wiley & Sons 1964 pg. 11.

5.1 a McDonald, G. M. "Instrumentation Problems Caused by Common-Mode Interference Conversion" Transaction on Electromagnetic Compatibility March 1966 pg. 17.

5.2 a Middlebrook, R. D. "Differential Amplifiers - Their Analysis and

and Their Applications in Transistor D.C. Amplifiers" J. Wiley & Sons 1964 pg. 94.

6.0 a Sharpe, G. E. "The Realization and Measurement of n-terminal Active Networks" Journal of the Franklin Institute September 1965 pgs. 263-266.

6.0 b Calahan, D. A. "Sensitivity Minimization in Active R-C Synthesis" P.G.C.T. March 1962 pgs. 38-42.

6.0 c Broome, P. W. and F. V. Young "The Selection of Circuit Components for Optimum Circuit Reproducibility" PGCT March 1962 pgs. 18-23.

6.0 d Calahan, D. A. "Modern Network Synthesis" Hayden Book Company, Inc. 1964.

6.0 e Darlington, S. "A Survey of Network Realization Techniques" IEEE Trans. Ckt. Theory December 1955.

6.0 f Cauer, W. "Vierpole" Elek. Mach. Tech. Vol. 6 pgs. 272-282 July 1929.

6.1 a Howitt, N. "Group Theory and the Electric Circuit" Physical Review Vol. 37 No. 12 1931 pgs. 1583-1595.

6.1 b Schoeffler, J. D. "Foundations of Equivalent Network Theory" Report 2 ONR Contract 1141 (10) December 7, 1961.

6.1 c Schoeffler, J. D. "Two and Three Element Kind Equivalent Networks" Report 3 ONR Contract 1141 (10) January 30, 1962.

6.1 d Waren, A. "Equivalence of Non-Reciprocal Networks" MS thesis Case Institute of Technology New York City.

6.1 e Guillemin, E. A. "Synthesis of Passive Networks" J. Wiley & Sons 1957 Chapter 5.

6.1 f Schoeffler, J. D. "Continuously Equivalent Networks and Their Application" Winter Meeting January 1963 IEEE.

6.1 g Calahan, D. A. "Modern Network Synthesis" p. 82-93 Hayden Book Co. 1964.

6.1 h Schoeffler, J. D. "Synthesis of Minimum Sensitivity Networks" PGCT June 1964 pgs. 271-76.

A.1 Lynch, W. (author) Wind, M. (editor) "Handbook of Electronic Measurements" Volume II pgs. 10-1 through 10-31 Interscience Publishers 1957.

A.2 Consolidated System Corporation "Instrumentation Grounding and Noise Minimization Handbook" Pomona, California AD 612 027 January 1965 page E-7.

D.1 Ramo & Whinnery "Fields and Waves in Modern Radio" pg. 452.

D.2 Spiegel "Advanced Calculus" Schaum Publications pg. 239.

D.3 Tonning, A. "A Contribution to the General Theory of Networks" National Defense Research Establishment Report #28 Oslo, Norway pg. 52.