



Dynamic balancing of irregular shafts
by Ralph C Challender

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering at Montana State College

Montana State University

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Abstract:

A production-line balancing machine, obtained through the War Assets Administration and received at Montana State College, is discussed in this paper. A discussion of the general problem of balancing along with the application of the principles of mechanics applying to balancing is given.

Some of the various types of balancing machines are described and the features of each are discussed. Apparatus, which can be used to determine unbalance and indicate the corrections to be made without removing the part from its location in a machine, is described.

Results obtained from the production-line balancing machine are included and discussed. The results obtained seem to indicate certain undesirable characteristics of the machine. An attempt is made to evaluate possible reasons for these characteristics.

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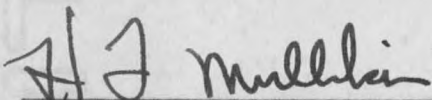
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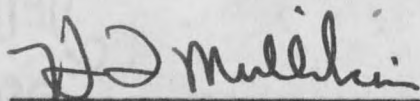
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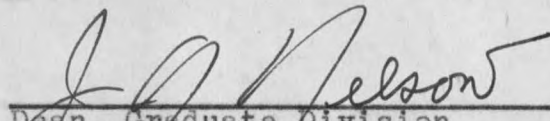
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Head, Major Department



Chairman, Examining Committee



Dean, Graduate Division

Bozeman, Montana

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TABLE OF CONTENTS

Acknowledgment	3
Abstract	4
Introduction	5
Balancing	7
Balancing Machines	17
The Surplus Machine	24
Analysis of Results	27
Discussion	38
Appendix	40
Literature Cited and Consulted	47

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ABSTRACT

A production-line balancing machine, obtained through the War Assets Administration and received at Montana State College, is discussed in this paper. A discussion of the general problem of balancing along with the application of the principles of mechanics applying to balancing is given.

Some of the various types of balancing machines are described and the features of each are discussed. Apparatus, which can be used to determine unbalance and indicate the corrections to be made without removing the part from its location in a machine, is described.

Results obtained from the production-line balancing machine are included and discussed. The results obtained seem to indicate certain undesirable characteristics of the machine. An attempt is made to evaluate possible reasons for these characteristics.

INTRODUCTION

The purpose of the research and work leading to this thesis was to install and place in operation a balancing machine received from war surplus. The machine was designed and built by a company engaged in the manufacture of aircraft engines. It was used in the actual production line to balance the crankshaft of a 12 cylinder, V-type engine.

In order to adapt the machine for laboratory experiments, a straight shaft was used. Six disks were made and placed along the shaft length and holes were drilled in these disks, parallel to the shaft axis at a uniform distance from the centers, to be used for the addition of weights.

After the preliminary work was finished and the shaft and disks were installed, the actual operation of the balancing machine was observed and studied to determine the characteristics of the machine. At this point certain results were questioned and an attempt to evaluate the results was made.

One of the primary purposes of a balancing machine is to determine the corrections needed to balance the unit in question in the most rapid and economical fashion possible. The balancing machine will accomplish the purpose of determining the required corrections, but the procedure involved is not of the easiest and most direct nature. The characteristics previously referred to are such that the magnitude

of the unbalance affects the angular position of the reading of the unbalance.

In the actual balancing of rotating units it is often not practicable or possible to completely remove or make allowance for the entire unbalance present. In this paper the procedure involved will be satisfactory for the particular case when the rotating part can be assumed completely rigid. When cases are involved where the elastic deflection of the parts should be considered the problem becomes more complex in nature and usually balancing can only be done for a particular speed or range of speeds.

Standard abbreviations are used for terms involved in the discussion. When a new abbreviation is introduced, a brief description follows the formula or equation where the symbol is used. Reference is made to the source of some of the statements and equations used. In the Balancing section, however, no specific references are made since the material is presented in a number of books dealing with the subject of vibrations.

BALANCING

Moving parts of a machine have in common, as a rule, either reciprocating motion or a motion of rotation such as that of the rotor of an electric motor. If the moving parts are not balanced, forces must be applied to produce the acceleration of the moving masses from the stationary parts of the machine, such as the bearings and the machine frame. It is in supplying these accelerating forces that serious trouble may arise, such as vibrations in the machine; heavy bearing pressure, which cause undue wear; defective work with grinding disks; and defective lubrication. It is, therefore, of great importance to balance or neutralize these forces in various types of machines.

In this paper, motions of rotation only will be considered. The rotating parts of a machine may be in static or dynamic balance. Static balance exists in a rotating part when the forces which act on the part, if the part is not running, are in equilibrium regardless of the position in which the part is placed. Dynamic balance exists in the system when the forces exerted by the masses of the moving system on the stationary parts of the machine are in equilibrium, and hence exert no varying resultant forces on the stationary parts of the machine.

From an analysis of a simple rotor containing only one unbalanced weight, the equation of the resulting force may

be determined. Assume that a single weight of magnitude, W , is located at a distance, r , from the center of rotation of a cylinder or disk. When the disk or cylinder is rotated, the unbalanced weight will have an acceleration toward the center of the body. In order to supply this centripetal acceleration the bearings supporting the rotating disk or cylinder and the associated shaft and journals will have to exert forces that will overcome or balance the resulting inertia or centrifugal force caused by the unbalanced weight. The centrifugal force may be represented by the equation $F = \frac{W}{g} r \omega^2$ where W is the unbalanced weight in pounds, g is the acceleration due to gravity expressed in feet per second per second, r is the distance from the center of rotation to the weight expressed in feet, and ω is the angular velocity in radians per second.

If a condition of this kind exists in a machine the forces required to resist the centrifugal force caused by the unbalanced weight will be transmitted from the rotating body to the frame of the machine. The force equation shows that the magnitude of the resulting forces will vary directly with the square of the speed of the rotating part. Figure 1 shows the variation of the unbalanced force with the speed of rotation for a unit unbalance of 1 ounce-inch. An ounce-inch is the standard unit used to measure unbalance and is defined as a weight of one ounce placed at a distance of one inch

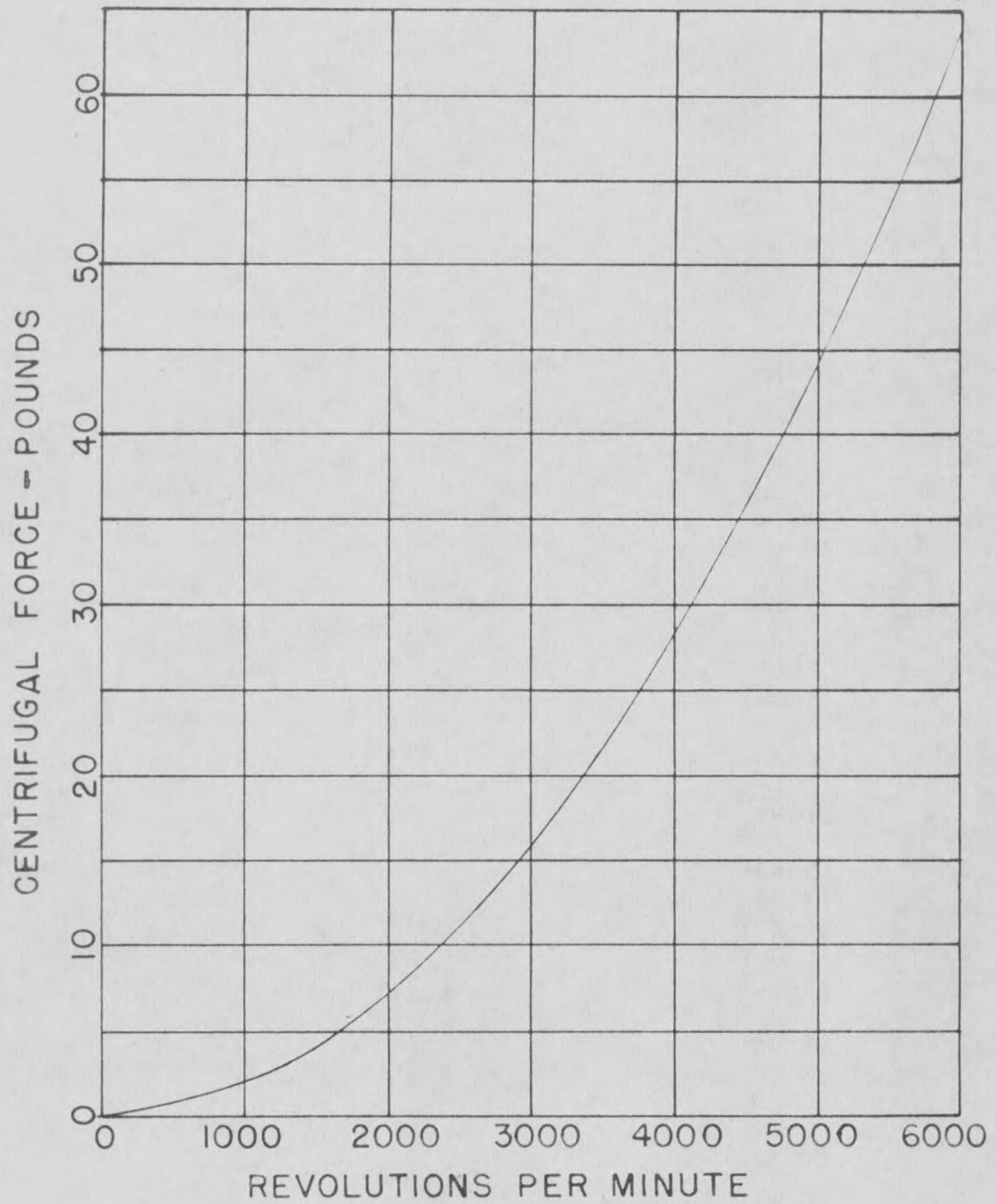


FIG. 1
VARIATION OF FORCE WITH SPEED

from the center of rotation.

Unbalance of this nature can be detected and corrected by simple means. By taking the rotating body and allowing it to rotate freely with its axis in a horizontal plane the heavy spot will come to rest at the low side indicating that weight should be removed at this location or that weight should be added diametrically opposite. The magnitude of the weight removed or the weight added should be of such a quantity that the equation $W_1 r_1 = W_2 r_2$ is fulfilled. In this equation W_1 represents the unbalanced weight; r_1 , the radial distance of the unbalanced weight; and W_2 and r_2 the weight and radial distance respectively of the balancing weight. This simple case is an example of static balancing since the unbalance can be detected by static means. Also the dynamic forces caused by the unbalanced weight in a rotating body will be balanced by the addition or removal of a single weight. This can be shown by a comparison of the equations involved in the two cases. In the case of rotating unbalance the correction is made by adding weight diametrically opposite the initial unbalanced weight or by removing weight at the same angular location. The value of the weight to be added or removed can be determined by equating the centrifugal force caused by the unbalanced weight to the centrifugal force caused by balancing weight. Since the balancing weight is added diametrically opposite, the forces are collinear and the

resultant is zero. Equating these forces, we obtain the relationship $\frac{W_1}{g} r_1 \omega^2 = \frac{W_2}{g} r_2 \omega^2$. Since the angular speed, ω , is the same for both weights it can be eliminated from the equation, leaving $W_1 r_1 = W_2 r_2$, which is the same equation as for the static balance case.

In the previous discussion, it was assumed that the removal or addition of the weight necessary to achieve the static balance was applied in the same plane, perpendicular to the shaft axis, as the unbalanced weight. If the method of static balancing is applied to rotating bodies that are of a considerable length, the position of the proper plane, along the shaft axis, for the addition of the correcting weight, will not be indicated. Assuming that the method is used for bodies of considerable length, the result may be more undesirable than the initial unbalance. If a weight of the proper magnitude and radial distance is added, unless the plane of the weight is the same plane as that of the unbalance, a dynamic couple will be introduced. Since it is not possible to determine the location of the proper plane to make corrections by static means, a different procedure is required for balancing long rotating bodies, even though the unbalance may be caused by a single weight.

In figure 2 the simple case of static unbalance is illustrated. Figure 3 shows the correction of this unbalance by the addition of a single weight placed diametrically

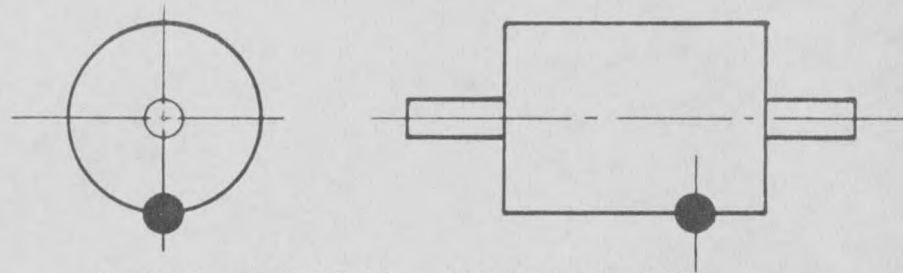


FIG. 2 STATIC UNBALANCE

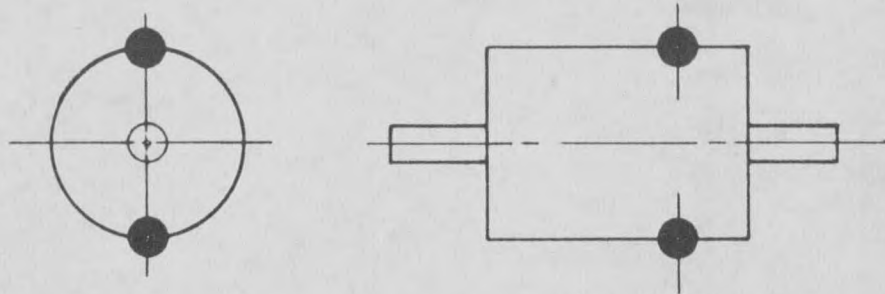


FIG. 3 STATICALLY BALANCED

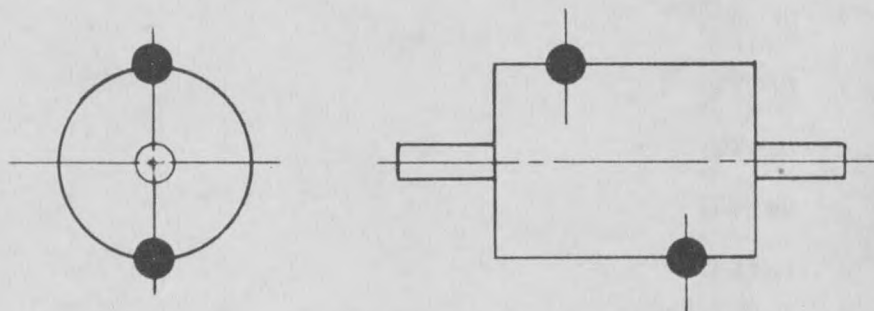


FIG. 4 STATIC BALANCE
WITH DYNAMIC UNBALANCE

opposite the unbalanced weight. Figure 4 shows the result of introducing a weight of correct magnitude and radial distance, but not in the same plane as the original unbalanced weight.

The general case, including both types of unbalance, will now be considered. Assume that a rotor contains various unbalanced weights as shown in figure 5. It is possible to determine if the rotor is in static balance by the method described previously. However, this approach will be bypassed and a method of correcting the unbalance will be described that determines the magnitude, angular position, and radial distance of single weights to be added in two different planes. First, choose planes I and II as the two correction planes for reasons of convenience in adding weights. Now, consider plane I as a reference plane. Taking moments with respect to plane I, find the weight to be added in plane II to make the resultant moment of the forces of the unbalanced weights and the weight in plane II zero. This step determines that the only unbalance remaining will have to be located in plane I. By considering plane II as the reference plane, the same procedure can be followed to find the balancing weight required in plane I. Thus, the system can be brought to a condition of dynamic balance with a weight placed in each of the correction or reference planes. In figure 6, this method is illustrated, using vectors to represent the moments of the

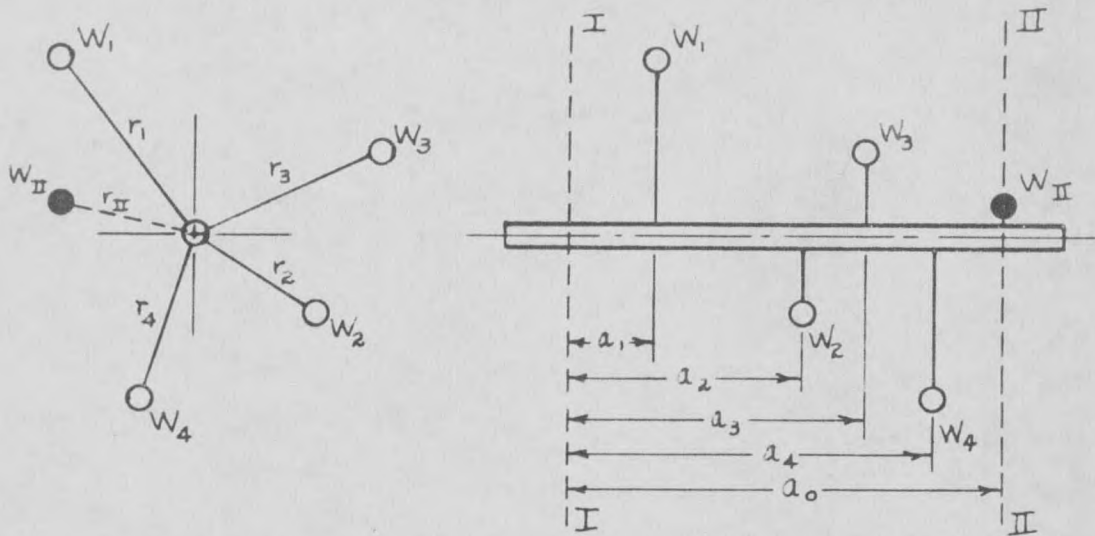


FIG. 5 GENERAL CASE OF UNBALANCE

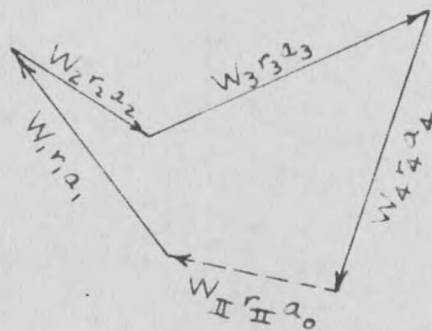


FIG. 6 MOMENTS WITH RESPECT TO PLANE I

forces with respect to the reference planes.

A different method, which will give identical results, is now described. Consider, as before, a rotating body containing various unbalanced weights and pick two reference planes, planes I and II. Add equal and opposite forces in plane I for each of the forces due to the unbalanced weights. This results in a force and a couple replacing each of the forces of unbalance. In other words, the original unbalanced forces have been replaced by a couple and a force in plane I. Next, find the resultant of the couples and add two forces, one in plane I and one in plane II, to form a couple that will be opposite to the resultant of the couples. Now, the system is in balance with the exception of the forces introduced in plane I. Find the resultant of the forces introduced in plane I and add a force in plane I to balance this resultant. Actually the only external changes made to the system have been the addition of a force in plane II to form part of the couple added to balance the original couples and the addition of a force in plane I to balance the forces added in plane I. These two forces are added by including weights of the correct magnitude, angular position, and radial distance in the reference planes.

Thus, the general case of unbalance in a rotating body can be balanced by the addition of a weight in each of two different reference planes. As stated before, it is assumed

that the rotating bodies are rigid and that elastic deflections can be neglected. The problem of balancing rigid rotating bodies is then one of finding the values of the weights, angles, and radial distances of weights to be added in two different planes perpendicular to the axis of rotation.

BALANCING MACHINES

A disk mounted on a rotating shaft which contains an unbalanced weight will vibrate and also cause rotating forces to be transmitted to the bearings. The vibration and the bearing forces can be made to disappear by attaching a small weight on the "light side" of the disk. The determination of the location of the correcting weight is a simple problem in statics. The shaft can be placed on parallel horizontal rails and the heavy side will then roll down. A trial correction weight can be added to the top side of the disk and the procedure continued by changing the weight until the shaft and disk are in apparent equilibrium. Then the shaft will remain in any position and the original unbalanced weight is approximately balanced by the correction weight. The shaft is placed on the parallel horizontal rails to eliminate as much as possible the frictional resistance to rotation.

The set of horizontal rails is the simplest static balancing machine in existence. For rotating bodies in which the main part of the mass is in disk form, static balance is the only balance that is necessary to insure vibration free operation at all speeds.

In case the rotating body is elongated and the mass or weight cannot be considered as concentrated in a thin disk, the condition of static balance is not sufficient to eliminate vibration. As shown in figure 4, a rotor may be in static

balance and yet exert rotating forces on the bearings when the shaft is revolving. A dynamic balancing machine or arrangement is required to determine the size and location of the unbalance.

A simple type of a dynamic balancing machine is now described. The rotor is placed in two bearings which are attached to a light table. The table is supported on springs at the four corners and can be made rotatable about either one of two axis, located in the planes that will be used as correction planes. The rotor is driven by a flexible shaft or belt, or can be driven by a motor rigidly mounted on the table. The latter scheme is not as desirable as the others since it increases the weight of the table which decreases the sensitivity of the machine. The balancing procedure is as follows. The table is held at one of the axis and released at the other and the rotor set in motion. After the unit comes to resonance, the amplitude of the vibration at the free end of the table can be measured and the position of the maximum amplitude determined. The position of the maximum amplitude will depend upon the vibration characteristics of the table and rotor. As we will show in the section devoted to an analysis of the surplus machine installed, the shaft will describe the heavy spot when it runs below the critical speed; the light spot when it runs above the critical speed; and a point which is 90 degrees behind the heavy spot when it

is run at exactly the critical speed. In actual practice this method is very inaccurate since the angle is dependent upon the speed of the rotor and varies through a maximum of 180 degrees.

A more satisfactory method¹ uses the basic arrangement described above, but does not depend upon the angle of the maximum amplitude. The method consists of observing the amplitude of vibration only and then making two additional runs with a small unbalance weight added at a convenient location for the first run and for the second run the unbalance weight is added in a diametrically opposite location. It can be shown that the three trial runs and the respective amplitudes will determine the unknown unbalance with a possibility of one ambiguity. This ambiguity can be removed by a fourth run which will also check the accuracy of the first three runs. If after the motions described above have been made, and the correction weight added, vibration is still present the procedure may be repeated once more. The vibration, if present, should be much less than the original vibration.

The procedure described for both of the above methods must be applied twice, once with the axis in plane I as the rotating axis and again with the axis in plane II as the

1. Den Hartog, J. P., 1947 MECHANICAL VIBRATIONS, pp. 296-297, McGraw-Hill Book Co., Inc., New York, N. Y.

rotating axis. Where large numbers of units have to be balanced as a requirement of production, the two methods described take too much time.

A similar arrangement is used in another type of balancing machine¹. The part to be balanced is supported and rotated on a pivoted carriage. One of the correction planes is located over the pivot axis and the unbalance is determined for this position of the unit and then with the specimen turned end for end, the unbalance in the other correction plane can be determined. The machine contains a compensating wheel rotating with the part in which both the magnitude and angular position of a balancing weight may be changed while the rotor and compensating wheel are running. By varying the angular position of the balancing weight or weights the operator can set the wheel to give the smallest amplitude of vibration. Then by varying the radial distance of the balancing weights the system can be brought into balance. The method does not require that the corrective weights be placed in the correction planes. Since the two unbalanced forces are determined and assuming the rotating unit is rigid, any two weights that will give the same external result may be added.

A method², which is simpler than the methods previously

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1. Thompson, W. T., 1948, MECHANICAL VIBRATIONS
p. 92, Prentice-Hall, Inc., New York, New York
 2. Den Hartog, J. P., 1947, pp. 300-302

described, can be used if means are available to measure the angle between the location of the unbalance in a rotor and the position of the maximum amplitude. Let the rotor be supported in two bearings, a and b, which are stiffly supported in the vertical direction and flexible in the horizontal direction. Two balancing planes will again be used and the balancing or correction planes taken at convenient locations for the addition or removal of weights. Imagine the rotor to be ideally balanced so that no vibration appears while it is rotating. Now a unit weight is added at correction plane I in a definite angular location. This will cause a vibration in both bearings and these vibrations are denoted as $\bar{\alpha}_{aI}$ and $\bar{\alpha}_{bI}$. The first subscript refers to the bearing where the vibration is measured and the second subscript refers to the location of the unbalance. For the general case, where damping is present, the numbers $\bar{\alpha}$ are complex because the damping and the speed of the rotor determine a phase angle between the unbalance force and the maximum amplitude.

In a similar manner the ideal rotor may be unbalanced with a unit weight in plane II, which will cause bearing vibrations $\bar{\alpha}_{aII}$ and $\bar{\alpha}_{bII}$. The four numbers $\bar{\alpha}$ are known as the complex dynamic influence numbers. These four influence numbers completely determine the inertia and elastic properties of the system for the speed at which they are deter-

mined. They are independent, however, of the amount of unbalance present.

Now suppose that an unbalance, which numerically as well as angularly differs from the unit unbalance, exists in plane I. Representing the unbalance by a complex number, $\bar{U}_I$, we see that the resulting vibration at bearing a will be represented by the product of two complex numbers, $\bar{\alpha}_{aI} \bar{U}_I$. The resulting vibration at bearing b will be represented by the product of two complex numbers $\bar{\alpha}_{bI} \bar{U}_I$. With these notations it is possible to write the equations for the bearing vibrations in terms of the four complex numbers $\bar{\alpha}$ and the complex numbers $\bar{U}_I$ and $\bar{U}_{II}$ representing the unbalance in planes I and II. Letting the complex numbers $\bar{V}_a$ and $\bar{V}_b$ represent the vibrations at bearings a and b respectively, we can write the following equations:

$$\bar{V}_a = \bar{\alpha}_{aI} \bar{U}_I + \bar{\alpha}_{aII} \bar{U}_{II}$$

$$\bar{V}_b = \bar{\alpha}_{bI} \bar{U}_I + \bar{\alpha}_{bII} \bar{U}_{II}$$

The bearing vibrations, $\bar{V}_a$ and $\bar{V}_b$, can be measured and the problem then requires the solution of the two equations containing the six known complex numbers for the two complex numbers $\bar{U}_I$ and $\bar{U}_{II}$ which will represent the equivalent unbalance in planes I and II.

Balancing apparatus is commercially available which utilizes the theory mentioned above for the purpose of balancing rotors without requiring them to be removed from

the bearings in which they normally operate. The vibrations at the two bearings of a machine can be measured with the use of an electrical circuit which enables the very small resulting vibrations of the machine frame to be amplified and the magnitudes measured. The angle of the maximum amplitude can be measured easily by employing a stroboscopic lamp which is made to flash at the high spot every revolution of the rotor. Various other methods can be employed to determine the high spot of the vibration.

THE SURPLUS MACHINE

The shaft to be balanced is set in bearings rigidly fastened to a table. The table is supported by four helical springs located at the corners. The shaft is driven externally through universal joints. The universal joints provide the flexibility necessary to allow the shaft and table to vibrate without restriction from the driving source. A mechanical linkage is connected to the table at each of the rear corners to measure the horizontal vibrations of the table. Through the linkage from each corner of the table mirrors are given small rotations. Light hitting the mirrors is reflected and the horizontal vibrations of the table cause the angles of the mirrors to change, thus changing the path of the reflected light. An autosyn motor connected to the driving shaft drives in turn an autosyn motor which is geared to a polished hexagonal shaft. The light reflected from the mirrors is in turn reflected from the revolving hexagonal shaft and strikes a target screen. The two light patterns on the target screen each have motions in two directions. The horizontal motion is caused by the horizontal vibration or movement of the table and the vertical motion is due to the revolving hexagonal mirror shaft and represents the rotation of the shaft. The target screen is divided in the vertical direction to represent one revolution of the shaft and in the horizontal direction to represent the

magnitude of the vibrations of the table.

Helical springs are attached to the table at both of the front corners and springs are effectively attached to the rear corners through the external linkages. A convenient means is provided by which the tension in the rear springs may be varied in order to position the light for each end of the table on the target screen. For convenience the light for the left end of the table is white and for the right end of the table is red.

A means of locking the table is provided to allow the shaft to be placed in the bearings and removed with the table rigidly held. A circuit is provided to release the table when the shaft is up to speed after starting and lock the table again when the balancing machine is turned off. A reversing arrangement is provided for the driving motor and is used as a brake to shorten the time required for the shaft and driving mechanism to come to rest.

A method is provided for effectively shifting the location of the observed vibrations with respect to the length of the table. The electrical controls are mainly contained in a control box at a rear corner of the machine and the start-stop switch is located at the front.

The balancing machine is made to permit simultaneous balancing in two planes since the table is free to vibrate at any or all lengthwise positions. However, the author

was unable, by considerable experimentation, to accomplish the simultaneous balancing procedure. But, by restricting the movement of one end of the table at a time, the shaft was approximately balanced. From a comparison of photographs, received from the manufacturer, of a similar machine and the actual machine, it is apparent that the makers have made certain changes in the design of the machine since the surplus machine was made. A few of the changes will be discussed in a later section of this paper.

ANALYSIS OF RESULTS

Consider a vertical shaft with a disk attached as shown in figure 7. The disk contains an unbalanced weight which moves the center of gravity, G, of the combined disk and weight away from the geometric center of the disk, S, toward the unbalance weight. Consider the disk to be located midway between two bearings and rotating with a constant angular speed. The shaft will have a deflection, when rotating, caused by the unbalance in the disk. Let the deflection, r , be the distance from the geometric center of the disk to the intersection of the center line of the bearings and the plane of the disk at O. The disk is under the action of two forces, the storing force of the shaft, kr , and the centrifugal force, $m\omega^2(r+e)$. The effect of gravity and friction is neglected. In order for these two forces to be in equilibrium they must be collinear, equal, and opposite. By equating the two forces¹ and solving for r , we obtain

$$r = \frac{m\omega^2 e}{k - m\omega^2} = \frac{\left(\frac{\omega}{\omega_n}\right)^2 e}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

where $\omega_n = \sqrt{k/m}$, the natural frequency of lateral vibration of the disk and shaft at zero speed.

The equation indicates that the critical speed of the shaft is equal to the natural frequency of lateral vibration

1. Thompson, W. T., 1948, p. 82-83

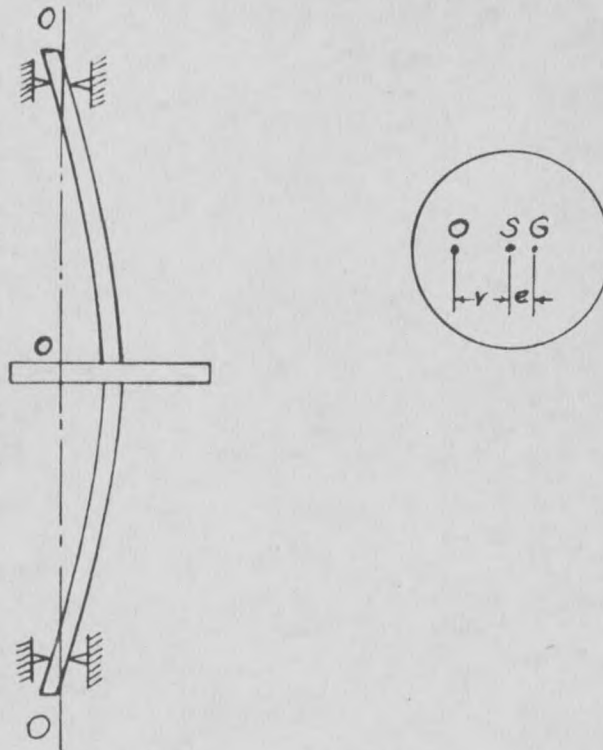


FIG. 7 ROTATING SHAFT

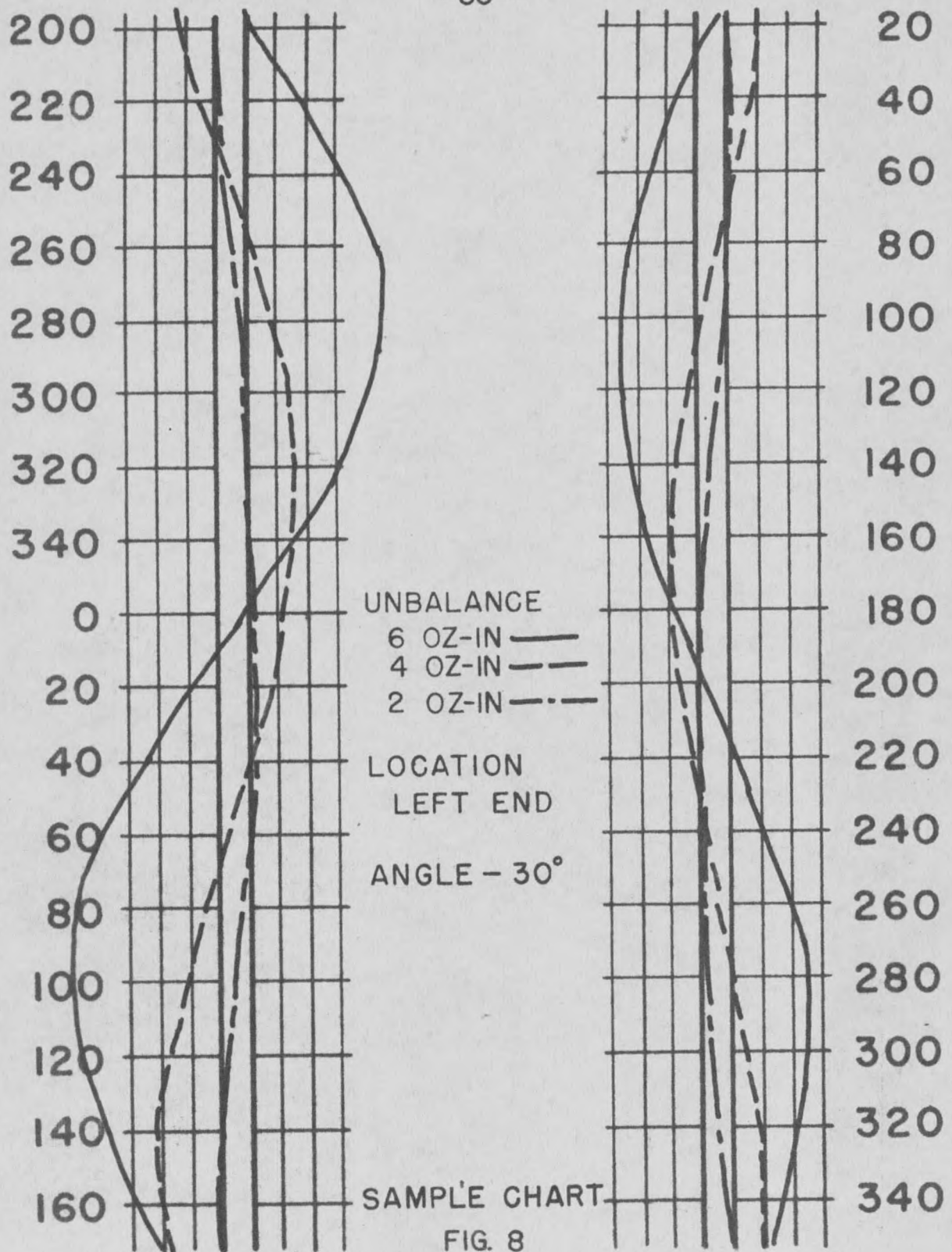
of the shaft and disk. For speeds below the critical speed the shaft and disk will vibrate with the heavy side out since r is positive. For speeds above the critical r is negative and the shaft will vibrate with the light side out.

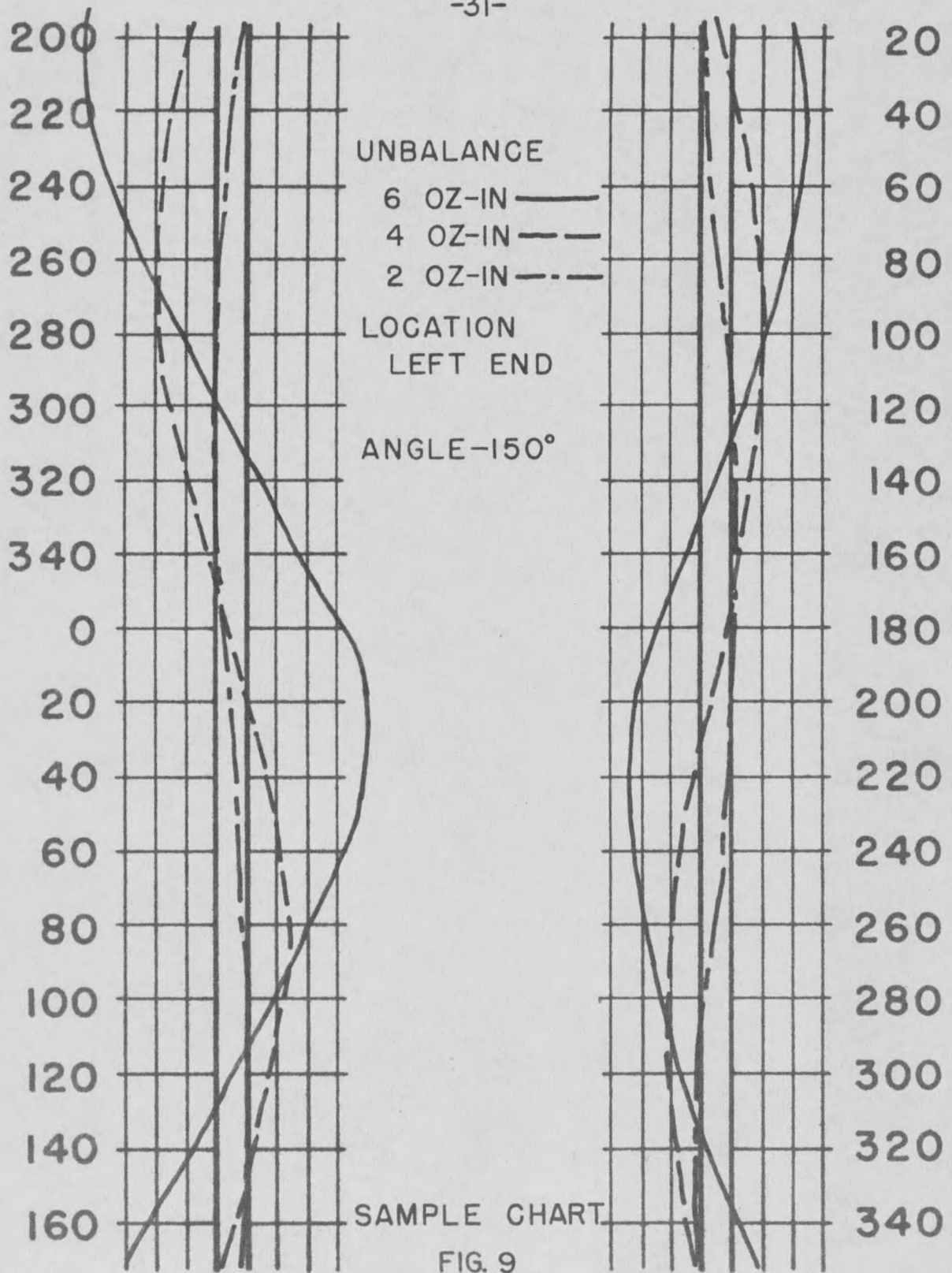
For a system, such as utilized in the balancing machine, where the unit is supported on springs, it can be shown¹ that the angle of the maximum amplitude will lag behind the unbalance force. The angle will vary between 0 and 180 degrees, depending on the damping present and the supporting spring constants. In the surplus balancing machine the table is free to move in both the horizontal and vertical directions. Thus, since the spring constants and the damping are different for the two directions, the amplitudes of the vibrations and the phase angles will be different for the two cases.

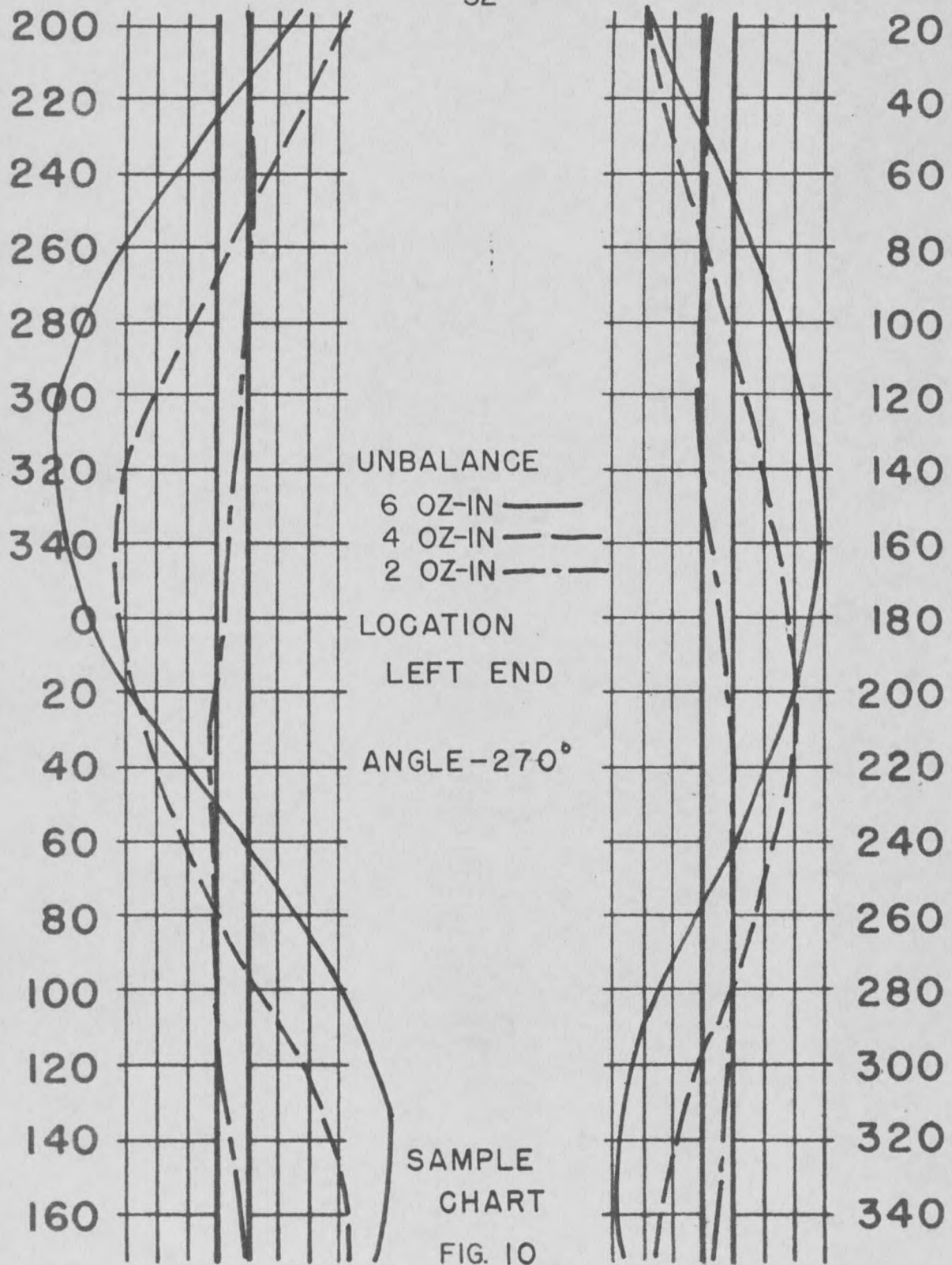
The previous discussion is included to present the basic difference between this balancing machine and the various types described in the section on Balancing Machines. In that section, the devices were assumed to have flexibility in only one direction, whereas the dynamic balancing machine investigated does not limit the vibrations to one direction.

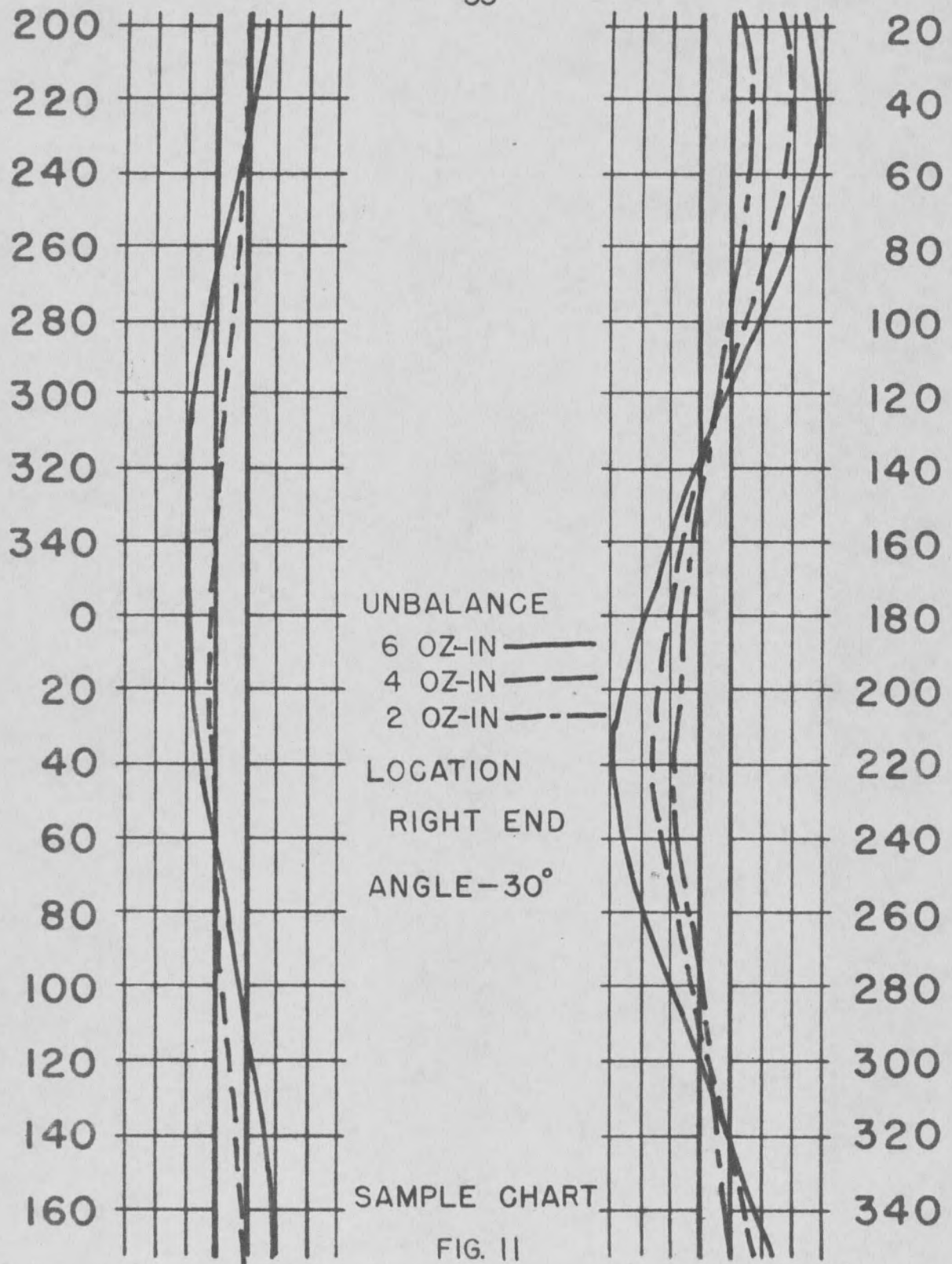
The results of the investigation are represented by six sample graphs reproduced from the patterns on the target screen for six different positions for the addition of known unbalance. The traces on the left side of the screen

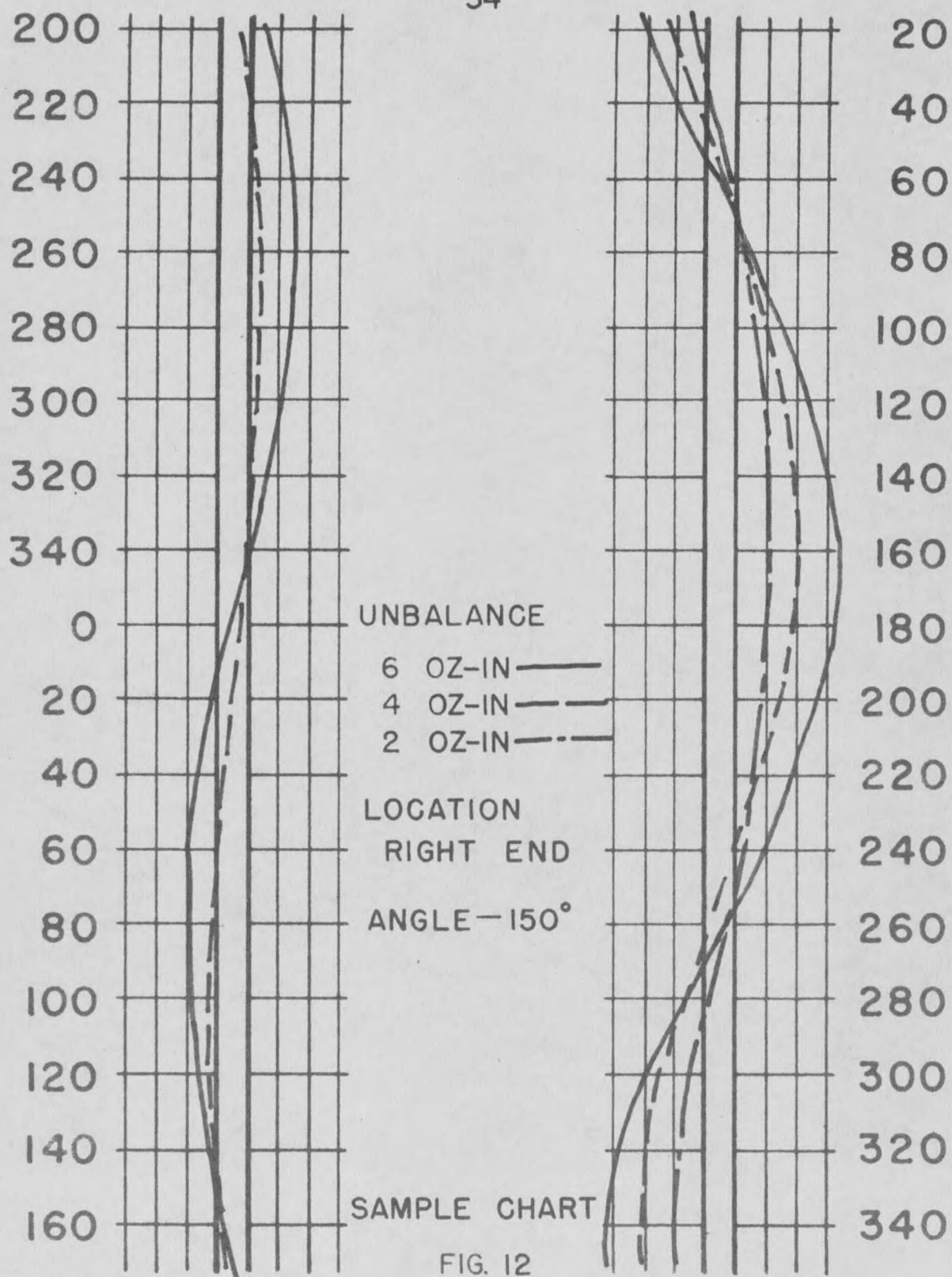
1. Thompson, W. T., 1948, pp. 72-75

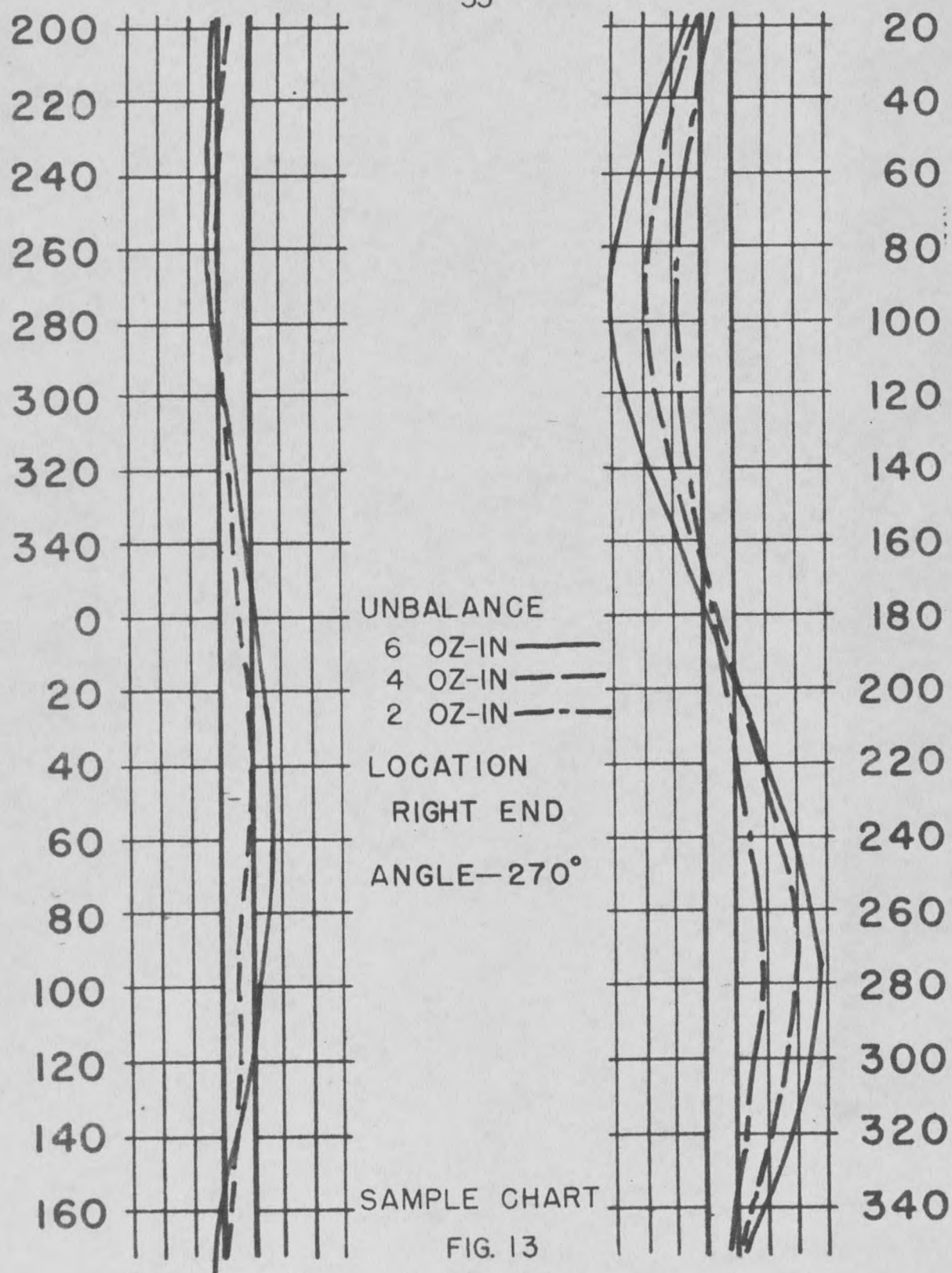












represent the vibration at the left end of the table and the traces on the right side indicate the vibrations at the right end of the table. Figure 8 shows the results of adding three different weights, one at a time to the left disk at the approximate location of the left linkage and at an angular location of 30° . Figure 9 shows the results for the same three weights added to the same disk at an angular location of 150° . Figure 10 represents the results obtained by adding the weights at 270° in the same disk. Figure 11 shows the readings obtained by the weights added at the right end of the shaft at a 30° angular location. Figure 12 shows the results with the weights added at the 150° angle on the disk at the right end. Figure 13 represents the readings with the weights added at the 270° location on the right disk.

Referring to figures 8 - 10, it will be seen that the magnitude of the weight added at any angular location on the left disk has a definite effect upon the angle of the maximum amplitude as well as upon the amplitude. The readings of the vibrations at the right end of the table caused by the addition of unbalance weights on the left disk are observed to be approximately 180° out of phase with the readings at the left end. The lagging phase angle of the vibrations at the left end, caused by unbalance in the left disk, is dependent upon the size of the unbalance. This discrepancy cannot be definitely explained by the author and possible

reasons for this fallacy are given in the Discussion.

Figures 11 - 13, representing the vibrations caused by unbalance in the disk located at the right end, show that the vibrations of the right end of the table vary only in amplitude for various values of unbalance at a definite angular location. This phenomena is in agreement with the material included for the case of simultaneous balancing in two planes and discussed in the section on Balancing Machines.

The author found that apparently only one of the four dynamic influence numbers α remains constant for this balancing machine. Numerous trials were made in attempting to vary the adjustments of the balancing machine in some manner that would give constant values for the dynamic influence numbers, but to no avail.

DISCUSSION

Two possible explanations for the non-constant dynamic influence numbers $\bar{\alpha}$ are made by the author.

The first reason is suggested by the method of support given the table. As was previously mentioned, the vibrations are not restricted to one direction and the resulting configuration of the vibration will depend upon the angular speed and the constants of the system in the two directions. As an example, consider the vibration in the vertical direction to have a phase angle between 90° and 180° . When the unbalance is in a vertical direction the resulting amplitude will be lagging and could possibly have a noticeable effect upon the amplitude in the horizontal direction at the particular instant. In the photographs mentioned previously, the method of supporting the table has been changed. The table is supported by rods about a foot long. The rods are so located that the table is in a stable position and the flexibility of the table in the vertical direction is greatly reduced in comparison with spring mounting. The flexibility in the horizontal direction is probably not decreased very much if at all.

The second explanation is suggested by the method used to connect the shaft to the driving head. A double universal joint was used to provide flexibility and it is possible that in some way this arrangement may introduce vibrations to the

shaft and table. If so, the vibration transmission could be balanced with the original balance and when various unbalances are introduced, the resulting motion of the table could cause forces to be transferred through the universal joints to the shaft and table.

APPENDIX

The results obtained by purposely unbalancing the shaft in the balancing machine seem to indicate that the unit is not usable for a laboratory experiment in actual balancing. The variation in the phase angle between the unbalance and the position of maximum amplitude destroys the main purpose of a balancing machine. However, the balancing machine can be used to illustrate the principles involved in dynamic balancing. A proposed laboratory experiment, using this machine, will be given. Photographs of the balancing machine are included as figures 14, 15, 16, and 17.

The author feels that the machine can be used for two different purposes. The first purpose, to illustrate principles of balancing to undergraduate students in a laboratory course, is given in this section. The second purpose, to supplement an advanced course in Mechanical Vibrations, is not described in this paper.

The laboratory experiment, proposed by the author, consists of two parts. The first part illustrates the introduction of dynamic unbalance by the improper addition of a weight in a plane not coincident with the plane of a single unbalanced weight. The second part shows the balancing of long rigid rotors by the addition of two weights in two different axial planes. It is assumed that the student will have completed the courses in Statics and Dynamics before this

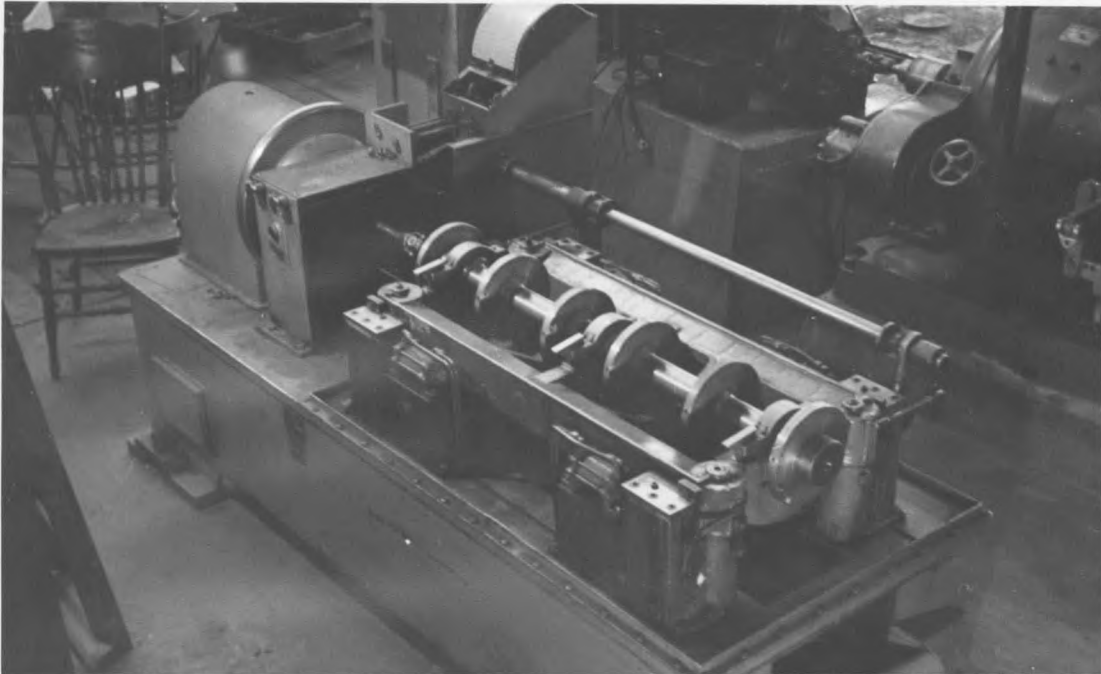


FIG. 14 RIGHT FRONT VIEW

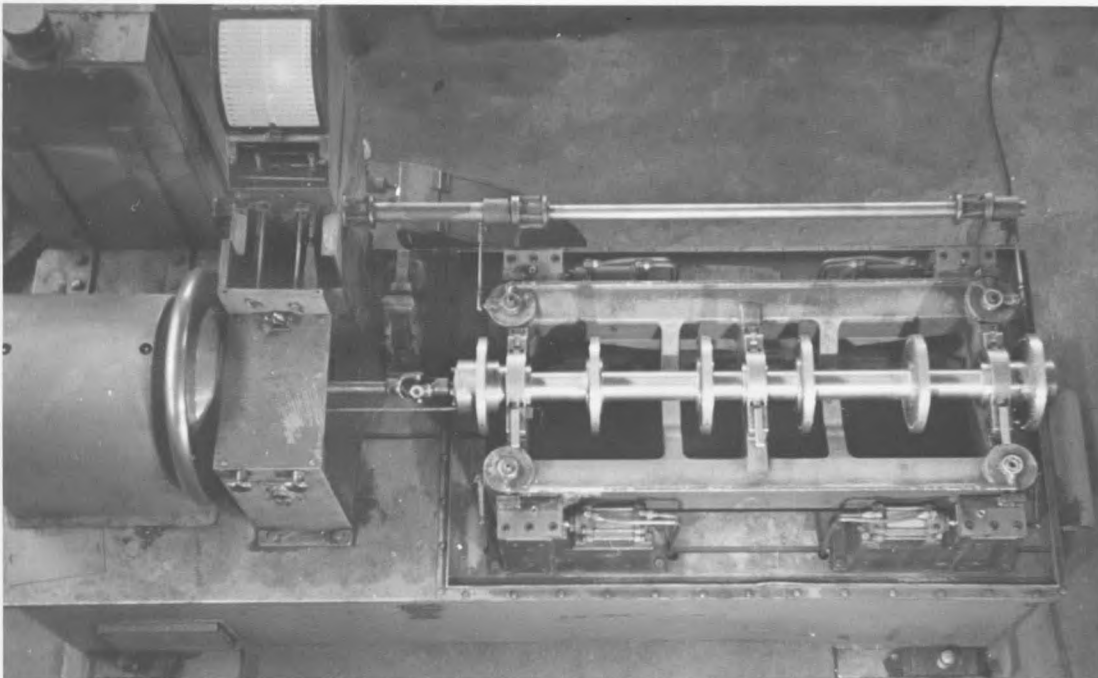


FIG. 15 TOP VIEW

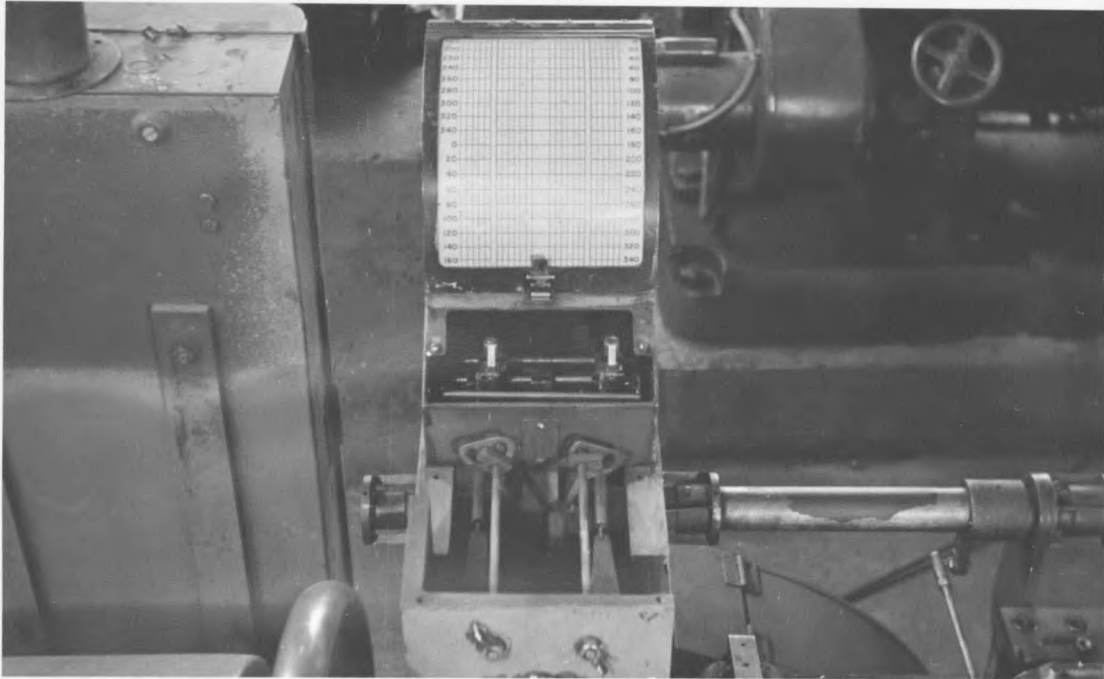


FIG. 16 VIEW OF INDICATOR CHART

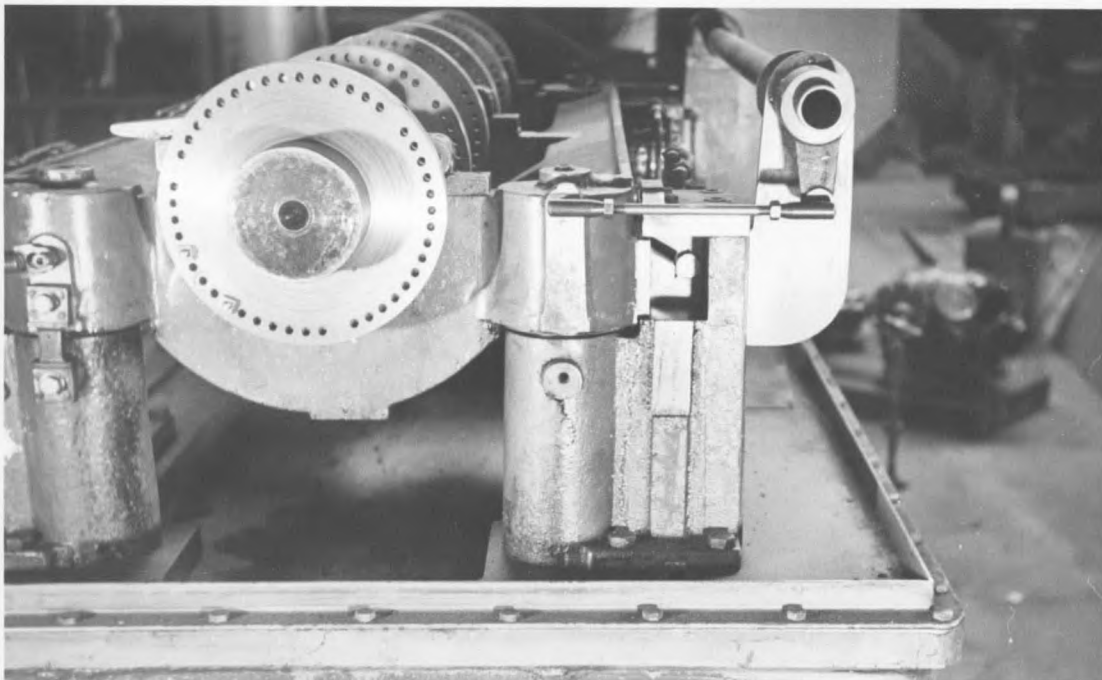


FIG. 17 RIGHT END VIEW

laboratory experiment is given.

Proposed Laboratory Experiment - Balancing

I. Preface

Unbalance in rotating bodies causes undesirable effects in machines such as heavy bearing pressure, undue wear, vibrations, defective work with grinding disks, and defective lubrication. Balancing of rotating bodies can be accomplished by several methods. For thin disks a condition of static balance is generally sufficient, whereas long rotors require an additional type of balancing, known as dynamic balance. Static balance refers to a condition of static equilibrium. Refer to the case of a thin disk mounted on a shaft. If the disk contains an unbalanced weight, and bearing friction is eliminated as much as possible, the heavy side of the disk will seek the lowest position when the axis of the shaft is horizontal. By repeated trials, the proper magnitude and radial distance of a balancing weight, placed diametrically opposite the heavy side, can be determined. The disk is now in a state of static balance. If the disk is rotated, the centrifugal force caused by the heavy side will be balanced by the centrifugal force caused by the balancing weight. Thus the disk is both statically and dynamically balanced.

In the previous discussion, we have assumed that the balancing weight was added in the same axial plane as the original unbalance weight. However, if this method is used

to balance a long rotor with a single unbalanced weight, the axial position of the unbalance will not be indicated. A weight of the proper value and radial distance and diametrically opposite the unbalance, added at any plane other than the plane of original unbalance, can lead to a condition far worse than the original unbalance. When rotating, if the centrifugal force of the balancing weight is not collinear with the centrifugal force of the unbalance weight, a dynamic couple is introduced. This dynamic couple, since it cannot be detected by static means, constitutes a dynamic unbalance. For rigid bodies, a condition of this kind can be corrected by adding weights to give an equal and opposite couple which will balance the first couple.

II. Procedure

The instructor will explain the balancing machine to be used in this experiment. The student should become familiar with the machine before proceeding. Caution must be exercised since the rotating shaft is not covered and the weights project from the disks. Select a nut and a bolt and a sufficient number of washers to give a weight of $1 \frac{1}{2}$ to 2 ounces. Select a second group to match the weight of the first.

Place one weight at a convenient location on the right disk. Start the machine and observe the unbalance readings. Next, add the second weight diametrically opposite the first

weight on the same disk and observe the readings when the machine is running. The second weight should nullify the unbalance caused by the first weight.

Remove the second weight from the right disk and add it to the left disk at the same angular location. When running the effect of the dynamic couple which has been introduced should be apparent. The preceding trials will illustrate the need for dynamic balancing.

The laboratory instructor will give the value and location of several weights to be added to the shaft disks. Remove the two weights used in the first part and place the stated weights at the indicated locations. Measure the distances between the disks and the radius of the centers of the holes drilled in the disks. Observe the unbalance. Two of the disks will be used as correction disks. Considering one of the disks as a reference plane, the moments of the added unbalance weights with respect to the reference plane can be taken. Calculate the value of a single weight to be added at the second correction disk to make the resultant of the moments of the weights, including the correction weight, equal to zero. Repeat the procedure to find a single weight to be added to the first correction disk. Add the two weights to the correction disks at the calculated locations. The shaft should now be balanced. The general case of unbalance in rigid rotating bodies is illustrated in the above

example.

The main purpose of a dynamic balancing machine is to determine the two corrections that must be made to balance rotating units. The methods used vary considerably and this machine was built to permit simultaneous balancing in two planes. Other types require two runs for balancing. The rotor is first balanced by the addition of a weight in one of the planes and then turned end for end to determine the weight needed for the second plane.

III. Results

The report should include the calculations made to determine the values of the two weights added to the shaft. Discuss the resulting balance obtained by the addition of the two weights.

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