Unit hydrographs developed for selected drainage basins in northcentral and northwestern Montana
by Ola Kaarstad

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

Abstract:
Unit hydrographs for 12 watersheds in northcentral and northwestern Montana were derived.

The data used were obtained from the June 1964 rainstorm and following floods.

A computer program was used to derive the unit hydrograph of "best fit" using a unit time of 1 hour for 8 and a unit time of 20 min. for 4 of the watersheds. For convenience of manipulating the shape of the unit hydrographs to obtain a "best fit", the equation defining the instantaneous unit hydrograph was used in the program.

The results were divided into 3 groups based upon the "goodness of fit" to peaks and magnitudes of peaks between the computed and the actual complex hydrographs, and upon overall goodness of fit.

The results were: Group 1; Excellent results.

Waterton River near Waterton Park, Alberta Sullivan Creek near Hungry Horse South Fork Flathead River near Hungry Horse Sun River at Gibson Reservoir near Augusta Group 2; Good results. . - South Fork Milk River near Babb South Fork Milk River at Del Bonita Cut Bank Creek at Cut Bank Dearborn River near Craig Lone Man Coulee near Valier Group 3; Poor results.

Waterton River near International Boundary Badger Creek near Browning South Fork Judith River near Utica The unit hydrographs obtained were compared with unit hydrographs derived from 3 synthetic unit hydrograph formulas developed by other geographic areas. Snyder’s formulas appeared to give the best results, although as might be expected, none of the 3 gave good agreement.

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Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

June, 1969

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the faculty of Civil Engineering and Mechanical Engineering and to Dr. Tom Hanson of Agricultural Engineering for their assistance, and in particular to Dr. Eldon R. Dodge for his patient assistance and guidance in preparing this thesis.

Special thanks are extended to W.K. Kellogg Foundation for the scholarship that made it possible for the author to do his work toward the M.S. degree at Montana State University.

Gratitude and thanks are extended to the author's wife, Mrs. Hildegun Kaarstad, for her patience and understanding during the course of this study.

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ABSTRACT

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Badger Creek near Browning
South Fork Judith River near Utica

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CHAPTER I

INTRODUCTION

The prediction of flood peaks and flood hydrographs is important in many areas of water resource and hydraulic engineering practice. If the maximum flood which may occur during a particular period of time can be accurately estimated, levees can be built to required elevation, rip-rap of adequate size can be laid out, dam spillways can be adequately designed, bridge openings and piers can be designed properly and culverts of adequate but economical size can be chosen.

In most geographic areas rain is probably the most important flood producing factor, especially for smaller watersheds, 1000 sq. mi. or less. It is therefore of great value to be able to predict the flood-flow which will be caused by a certain volume of rain distributed in time in a certain way.

There are several methods available for flood prediction. Some are for rain only, while others are for floods produced from snowmelt alone or in combination with rain. The following is a listing of the most commonly used methods for flood prediction today.

1. Frequency analysis (flood peaks)
   a) Annual floods
   b) Partial duration series or floods above a base

2. Comparisons of historical floods in a drainage area basin with maximum flood experience in the immediate area as well as in other areas
   a) Envelope curves (Creager equation)
   b) Maximum flood equations (Meyer formula)

3. Unit hydrograph approach
   a) Empirically derived

b) Synthetically derived
c) Digital similation (Stanford Watershed Model(1))

Of the above mentioned methods the unit hydrograph approach is most used. In this method the characteristics of the watershed are expressed in the shape of the unit hydrograph, which represents the time distribution of the surface runoff produced by a storm of short duration. These characteristics are those that affect overland flow, since the definition of the unit hydrograph includes only overland flow. In order to apply a unit hydrograph to a given rain, however, it is also necessary to know the magnitude of the "losses" of rain on a watershed, such as infiltration, wetting and detention storage, so excess rain can be estimated closely.

There appear to have been few if any, unit hydrographs derived for any Montana drainage areas. The objective of this study is to derive some unit hydrographs for selected drainage areas in Montana. In order to do so the author searched records of isolated rainstorms with corresponding runoff data. No suitable storms of this kind were found. In the search for data the extensive rainstorm of June 1964 appeared to be one that had sufficient rainfall and runoff data. Although the 1964 storm was not the kind preferred for the derivation of unit hydrographs, it was decided to use it as a basis for an attempt to derive some unit hydrographs for the area of Montana covered by this storm. Because of the complexity of the storm a trial and error procedure was necessary, and the instantaneous unit hydrograph method was decided upon, modified so as to use rainfall of 20 minutes or 1 hour's length as unit time instead of instantaneous rainfall. Unit hydrographs have been derived for 12 drainage areas. The unit hydrographs derived are compared with synthetic unit hydrographs derived for the same drainage areas by the formulas of Snyder, Soil Conservation Service and Gray.

CHAPTER II

UNIT HYDROGRAPH DEVELOPMENT

Although Folse (2) as early as 1929 started to work with separation of base flow and overland flow, rain loss due to variable infiltration, depression storage and wetting, and also derived physical constants which in effect were the successive ordinates for a 24 hour unit hydrograph, it is Sherman's name that is connected to the development of the unit hydrograph theory.

Sherman (3) proposed his theory of the unit graph in 1932, defined as the hydrograph resulting from 1 inch of excess rain occurring during a unit time period (of 1 day, 12 hours, 6 hours, ...) and evenly distributed over the drainage basin. Definitions in connection with rainfall and runoff are given below.

1. Excess rainfall is defined as that rain in excess of wetting the ground and the plants, evaporation, depression storage and infiltration, which reaches the streams as overland flow or surface runoff.

2. Excess rainfall rate is defined as excess rainfall in inches per hour.

3. Surface or overland runoff is defined as that part of stream flow which reached the stream channel over the surface of the land.

4. Interflow is defined as water flowing between the surface and the groundwatertable toward the streams.

5. Baseflow is defined as water flowing to the streams from the groundwater.

Sherman defined his well-known theory of unit hydrograph for surface runoff only.

When the unit hydrograph for one unit time is known, a flood resulting from a complex rainstorm can be computed.

Some basic assumptions are associated with the unit hydrograph idea, Chow (4).

1. The excess rainfall is uniformly distributed within its duration.

2. The excess rainfall is uniformly distributed throughout the whole area of the drainage basin.

3. The base time or duration of hydrograph of direct runoff due to excess rainfall of unit duration is constant.

4. The ordinates of the excess runoff hydrograph of a common base time are directly proportional to the total amount of direct runoff.

5. For a given drainage basin the hydrograph of runoff due to a given period of rainfall reflects all the combined physical characteristics of the basin.

Regarding assumption 1. it is well-known that rainfall is more likely to have a nonuniform than a uniform distribution, Bernard (5). For large drainage basins in particular the rainfall distribution over the area is usually not uniform. Regarding assumption 5. the physical characteristics will change with season and man-made adjustments. From the above comments about the basic assumptions, it will be seen that they probably never can be met perfectly under natural conditions. However, by careful selection of data so as to meet the assumptions closely, the results although subject to a certain degree of error, are acceptable for engineering purposes.

Unit Hydrograph Analysis

For the derivation of the unit hydrograph it is necessary to have a recorded flood hydrograph and a record of the associated hourly recorded rainfall. Chow (4) points out that care should be taken to make sure that the above mentioned assumptions, especially 1. (uniform excess rainfall distribution throughout the unit time period) and 2. (uniform excess rainfall distribution over the drainage area) are met as closely as possible, and states:

> A hydrograph resulting from an isolated, intense, short duration storm of nearly uniform distribution in space and time is the most desireable. If such single peaked, well-defined hydrographs are not available, it is necessary to derive the unit hydrograph from a complex storm.

Bernard (5) states that the runoff following the storm must have been uninterrupted by low temperatures, ice and snowmelt.

Linsley et al. (6) states that a unit hydrograph is best derived from the hydrograph of a storm of reasonable uniform intensity and with a runoff volume close to an inch or above.

The derivation of a unit hydrograph from a single simple storm is quite straightforward. After the base flow and the interflow are separated, leaving a surface runoff hydrograph, the amount of surface runoff is represented by the area under the resulting surface runoff hydrograph. If each ordinate is divided by the total amount of surface runoff, the resulting hydrograph represents a one-inch volume of runoff. If possible the unit hydrograph should be computed from several storms and an average unit hydrograph derived.

If a complex storm must be used, the method of Collins (7) can be used. It includes 4 steps: 1. assume a unit hydrograph and apply it to all excess rainfall blocks except the largest; 2. subtract the resulting hydrograph from the actual hydrograph, and reduce the residual to unit hydrograph terms; 3. compute a weighted average of the assumed unit hydrograph and the residual unit hydrograph, and use it as the revised approximation for the next trial; 4. repeat the previous 3 steps until the residual unit hydrograph does not differ by more than a permissible amount from the assumed hydrograph.

A concept nearly associated with the unit hydrograph theory is the S-hydrograph, Morgan and Hullinghorse (8). The S-hydrograph is defined as the direct runoff from a continuous uniform excess rain during a long period of time. From a S-hydrograph the unit hydrograph of an excess rain of any unit time can be derived. Fig. 1 shows how the S-hydrograph is formed and how to derive a unit hydrograph from it. The procedure is to construct the S-hydrograph from an assumed uniform continuous rainfall, then offset the S-hydrograph for the desired duration of excess rain t_0. By taking the differences between the original and the offset S-hydrographs, and dividing the differences by rainfall rate times t_0, the desired unit hydrograph is obtained.

Instantaneous Unit Hydrograph

When the duration of excess rainfall becomes infinitesimally small, the resulting unit hydrograph is called the instantaneous unit hydrograph, Chow (4).

Several investigators have proposed models that with or without their knowledge build upon the principles of the instantaneous unit hydrograph. A committee report by the Boston Society of Civil Engineers in 1930 (9), indicated that an instantaneous storm could give information about the watershed characteristics. Clark (10) connected this idea to the unit hydrograph idea in 1945 for the first time.

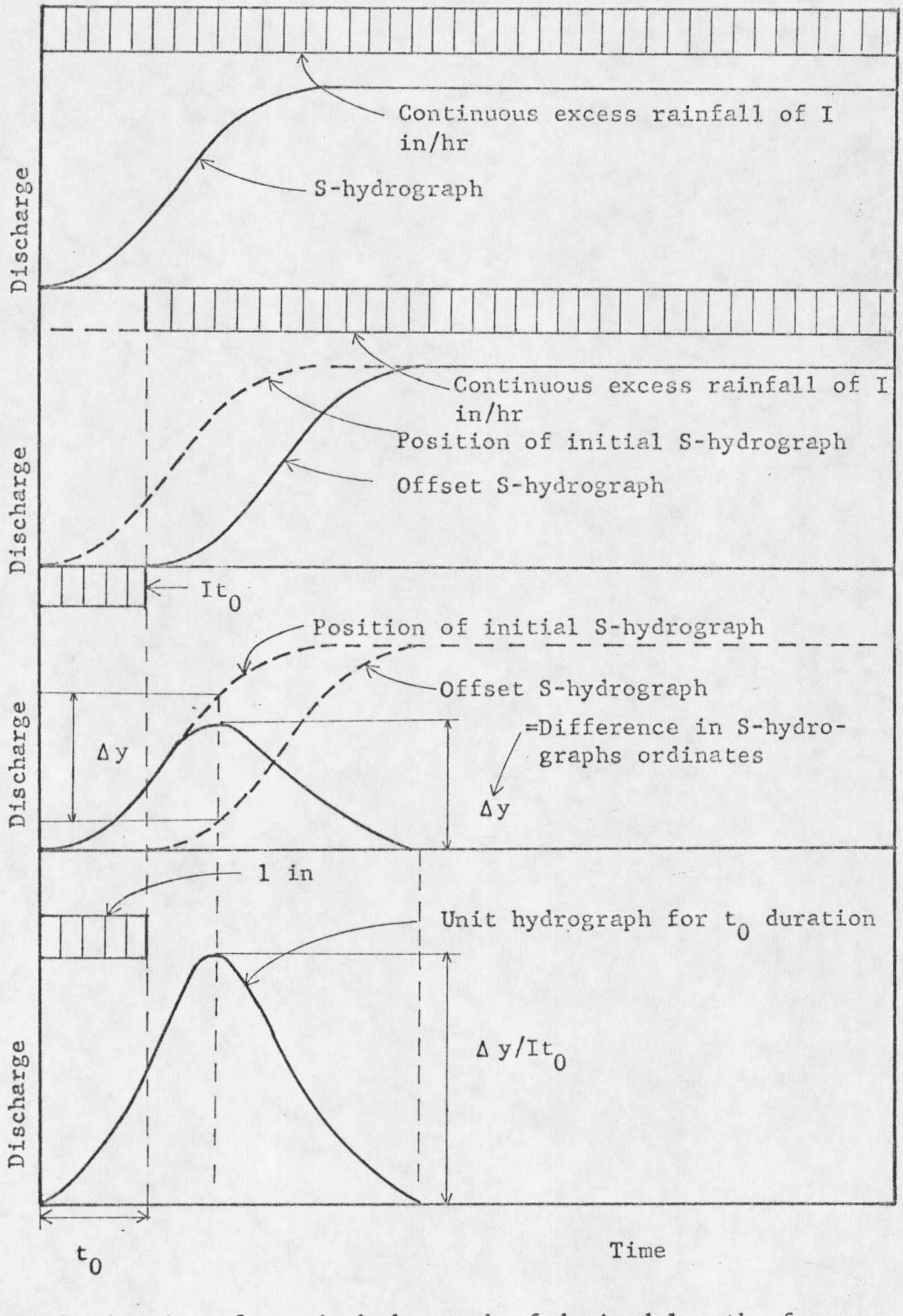

FIGURE 1. - Derivation of a unit hydrograph of desired length of excess rain duration from a S-hydrograph.

Edson's approach. In 1951 Edson (11) developed a two-parameter equation for the unit hydrograph but he did not connect it to the instantaneous unit hydrograph. He dealt with a theory about isochrones representing equal travel time from each local part of the drainage area, Fig. 2.,

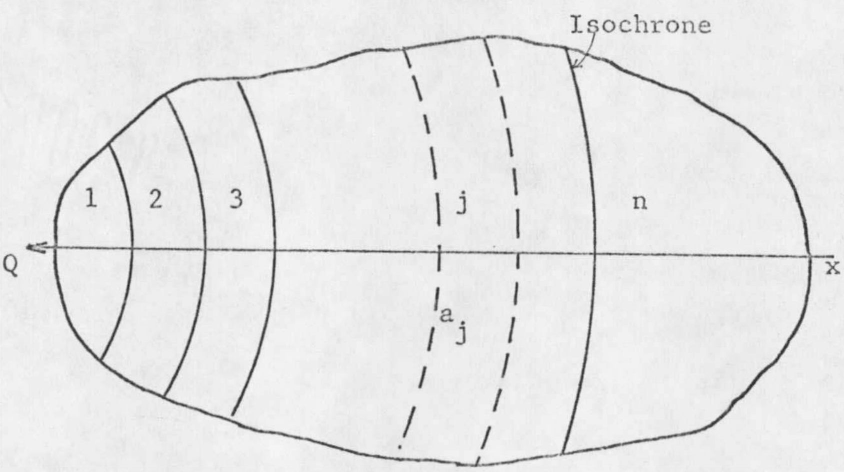

FIGURE 2.- Development of area-time diagram. Drainage basin simulated by a linear channel.

the discharge from each local part becoming:

$$Q \propto t^{w} \qquad w > 1 \qquad (1)$$

where w is a parameter that has to be greater than 1, see below.

Edson further reasoned:

> When runoff occurs simultaneously over the whole drainage area the time of travel required for each component is so affected by the presence of the other components that the hypothetical isochrones are invalidated. It is reasonable to regard the consequent delay in delivery as the result of valley storage. In a certain sense therefore the valley

of the watershed acts as a reservoir, the discharge of which is known to decrease exponentially with time:

$$Q \propto e^{-yt} \qquad y > 0 \qquad (2)$$

where y is a parameter that has to be greater than 0.

By combining (1) and (2) the resulting hydrograph would follow a curve described by the relationship:

$$Q \propto t^{w} e^{-yt} \qquad (3)$$

He then adopted the following equation for the unit hydrograph:

$$u = B\, t^{w} e^{-yt} \qquad (4a)$$

where u is the ordinate for the unit hydrograph at time t. And since all the water is discharged as the unit hydrograph he set up the following identity:

$$\int_0^{\infty} u dt = C\, A \text{ cfs-days} \qquad (4b)$$

where C is a conversion constant equal to 242/9 when A is in sq. miles, dt in hr and u in in/hr. He then set $w = (n-1)$ and $yt = z$. Integrating Eq. (4a) one obtains:

$$\int_0^{\infty} u dt = B\, y^{-n} \int_0^{\infty} z^{n-1} e^{-z} dz = B\, y^{-n}\, \Gamma(n)\, e^{-z} \text{ cfs-days} \qquad (4c)$$

where $\Gamma(n)$ is the gammafunction of n, which for whole numbers are equal to $(n-1)!$ By equating (4b) to (4c) B is found to be:

$$B = \frac{CA\, (yt)^{w}\, e^{-yt}}{\Gamma(w+1)} \qquad (5)$$

whence (4a) becomes:

$$u = \frac{CA\ (yt)^{w}\ e^{-yt}}{\Gamma(w+1)} \tag{6}$$

That the value of w > 1 is evident for the following reason. If Eq. (4a) is differentiated with respect to time and equated to zero u must have maximum or minimum value:

$$\frac{du}{dt} = B\ t^{w-1}(w-yt)e^{-yt} = 0$$

which is satisfied for any of the following values of t;

$$t = 0 \text{ provided } w > 1$$

$$t = w/y$$

$$t = \infty$$

if $w > 1$ $dQ/dt = \infty$ when $t = 0$ which is impossible with a constant rate of excess rainfall.

Nash's model. - In 1957 Nash (12) introduced his model for the instantaneous unit hydrograph. He considered the drainage basin as n identical linear reservoirs each having the storage characteristics of the form:

$$V = k\ Q \tag{7}$$

where k is storage coefficient and V is storage volume. The outflow from one reservoir now becomes the inflow to the next.

When an instantaneous inflow takes place to the first reservoir its level is raised to accommodate for the increased storage and the outflow raises momentarily from zero to V/k, and diminishes with time according to the equation:

$$Q_1 = \frac{V\ e^{-t/k}}{k} \qquad (8)$$

where V is the volume under the hydrograph, Q_1 is discharge and t is time. Since Q_1 becomes the inflow to the second reservoir it has an outflow according to the equation:

$$Q_2 = \int_0^t \frac{1}{k}\ e^{-\tau/k}\ \frac{V}{k}\ e^{-(t-\tau)/k} d\tau$$

$$Q_2 = \frac{1}{k} \int_0^t e^{-t/k}\ \frac{V}{k}\ d\tau = \frac{V}{k^2} e^{-t/k}\ \tau \Big|_0^t = \frac{V}{k^2}\ t\ e^{-t/k}$$

By using a similar procedure for the 3rd, 4th, ..., (n-1)th reservoirs the outflow from the nth reservoir is given by:

$$Q_n = \frac{V}{k\ (n-1)!}\ (t/k)^{n-1}\ e^{-t/k} \qquad (9a)$$

To allow for fractional values of n, equation (a) takes the form of

$$Q_n = \frac{V}{k\ \Gamma(n)}\ (t/k)^{n-1}\ e^{-t/k} \qquad (9b)$$

By comparing this with Edson's formula (6) it is seen that by replacing y with 1/k and w with n-1 equation (6) becomes:

$$u = \frac{C\ A}{k\ \Gamma(n)} (t/k)^{n-1}\ e^{-t/k}$$

which becomes identical to (9b) by setting C A = V.

Figure 3a shows the theoretical assumptions for Nash's linear model, and Figure 3b shows the resulting hydrographs after the flow has gone through 1, 2, 3, ..., n reservoirs.

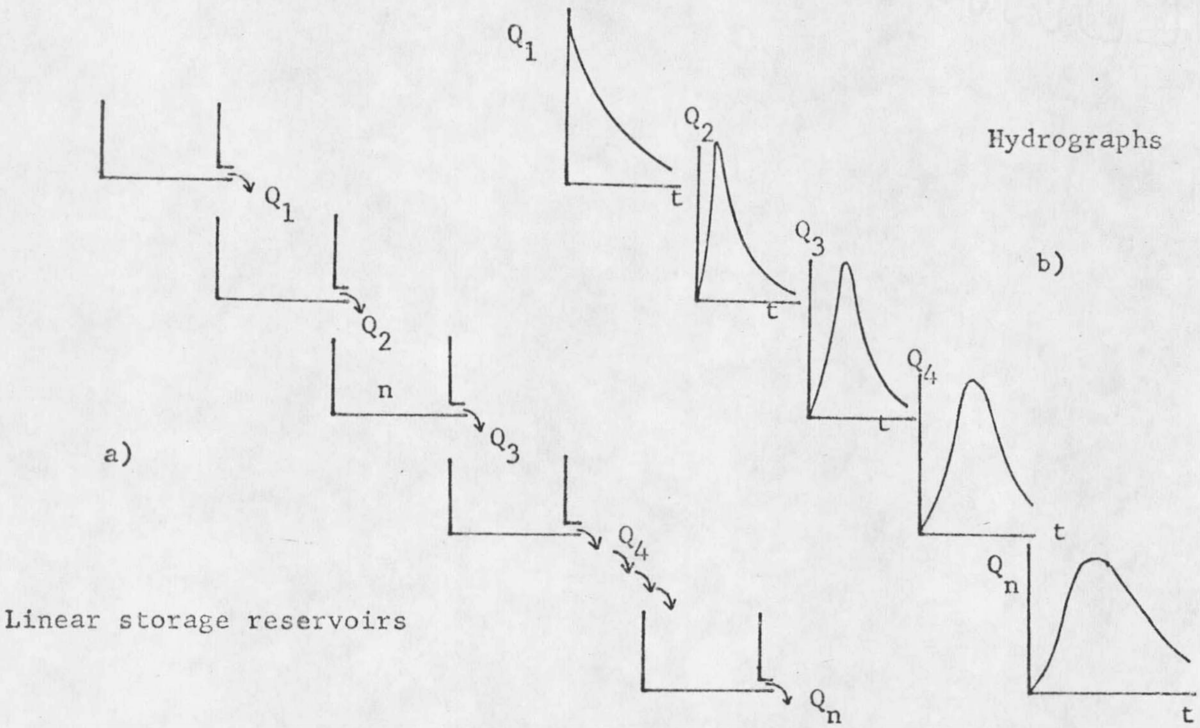

FIGURE 3. - Routing of instantaneous inflow through a series of linear storage reservoirs. (Nash's model.)

Differentiation of (9a) with respect to time gives:

$$\frac{dQ}{dt} = \frac{V(n-1)}{k\,\Gamma(n)}(t/k)^{n-2}e^{-t/k} - \frac{V}{k^2\,\Gamma(n)}(t/k)^{n-1}e^{-t/k}$$

By using the fact that $dQ/dt = 0$ when $t = t_p$ we get

$$t_p = (n-1)\,k \qquad (10)$$

where t_p is time to peak of the hydrograph. If V is taken as volume of runoff in acre-inches the following relation is valid:

$$V = 640\ A\ R$$

where A is drainage area in sq. mi. and R is surface runoff in inches. Eq. (9a) becomes the equation for the unit hydrograph for a given drainage basin when R equals 1 inch of runoff.

$$u(t) = \frac{640\ A}{k\ \Gamma(n)}\ t/k^{n-1} e^{-t/k} \qquad (11)$$

The Computation of a Complex Hydrograph from an Instantaneous Unit Hydrograph

When an excess rainfall of function $I(\tau)$ with duration t_0 occurs, Fig. 4a, infinitesimal element of this excess rainfall will produce a runoff hydrograph equal to the product of $I(\tau)$ and the instantaneous unit hydrograph $u(t-\tau)$, Fig. 4b. If then the principle of superposition in the unit hydrograph theory is applied, the complex hydrograph is obtained, Fig. 4c. The ordinate of the complex hydrograph at time t is:

$$Q(t) = \int_0^{t' \leq t_0} u(t-\tau)\ I(\tau) d\tau \qquad (12)$$

This is called the convolution integral or Duhamel integral; $U(t-\tau)$ is a kernel function. $I(\tau)$ is the input function, and $t' = t$ when $t < t_0$ and $t' = t_0$ when $t \geq t_0$. τ is lag time or time from beginning of surface runoff, t_0 is time when excess rain ceases.

While Eq. (11) from the standpoint of its rational development is in

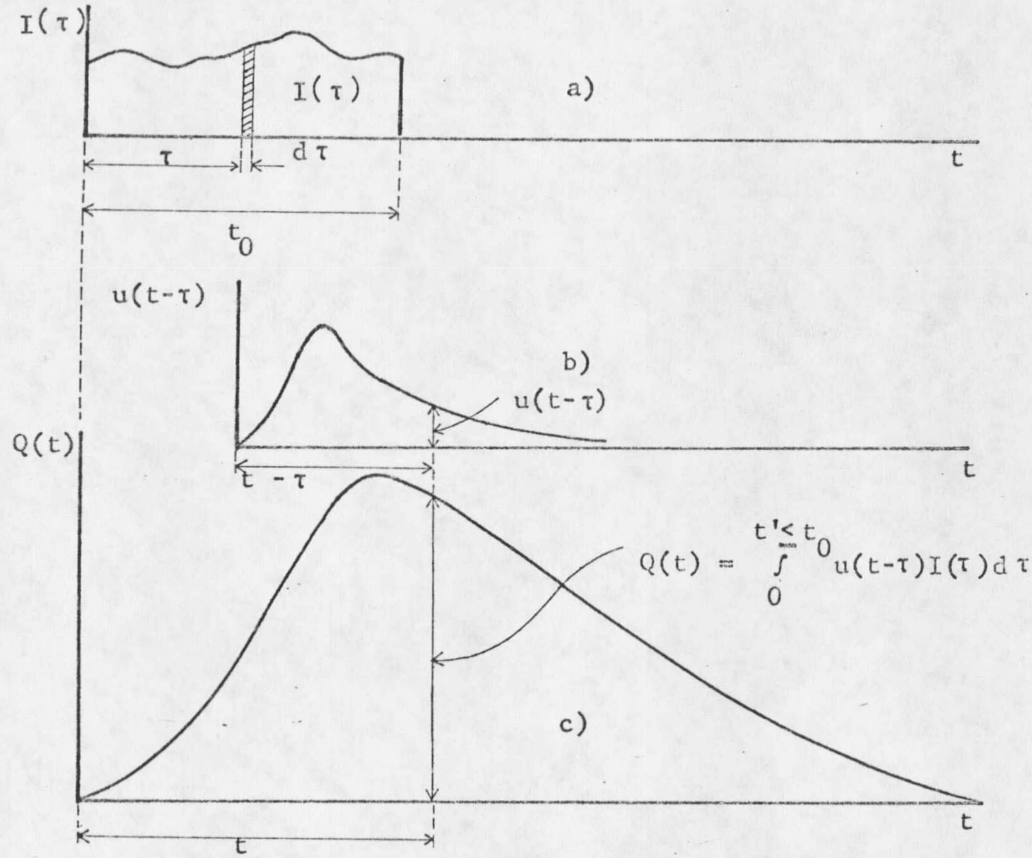

FIGURE 4. - Convolution of I(τ) and instantaneous unit hydrograph.

theory only applicable to rainfall of infinitesimal duration, it does provide a convenient mathematical model for an ordinary unit hydrograph. By choosing different values of t_p and n, from which different values of k can be computed, Eq. (10), it is possible to produce a unit hydrograph with any desired time to peak, magnitude of peak or time duration (time from beginning of runoff to the time runoff ceases) for a particular watershed.

Regarding Eq. (11) in this perspective one may use a finite summation form of the convolution integral thereby introducing an error which hopefully is negligible for $\Delta\tau \leq 1$ hour.

$$Q(t) = \sum_{i=1}^{t' \leq t} \overset{o}{u}(t-\tau)I(\tau)\,\Delta\tau \qquad (13)$$

t is equal to i t where i is equal to the number of time increments Δt.

Some Characteristics of the Instantaneous Unit Hydrograph

By expressing rainfall and runoff in the convolution integral in the same units, the ordinates of the instantaneous unit hydrograph has the dimension of time $^{-1}$. The form of the instantaneous unit hydrograph looks like a single peaked hydrograph.

A large n in Eq. (11) causes a large peak and a short time duration, while a small n results in a small peak and a long time duration for the same t_p. It is possible to get the same magnitude of peak for several combinations of t_p and n by increasing n as t_p is increased.

The following properties are valid:

$0 < u(t)$	a positive peak value	for $t > 0$
$u(t) = 0$		for $t = 0$
$u(t) \to 0$		for $t \to \infty$
$\int_0^\infty u(t)dt = 1.0$		

CHAPTER III

SYNTHETIC UNIT HYDROGRAPHS

Since the unit hydrograph contains only surface runoff, several of the watershed properties such as soil characteristics and hence infiltration do not affect the form of the hydrograph. Several investigators have tried to correlate some of the other important watershed characteristics to the empirically derived hydrographs. For instance, efforts have been made to correlate length of main stream, land slope, and area with such hydrograph characteristics as time to peak, magnitude of peak and base time.

Snyder

In 1938 Snyder (13) derived empirical formulas that related some of the watershed characteristics with certain properties of the unit hydrograph. He derived his relationships from several watersheds in the Appalachian Highlands which varied in size from 10 to 10,000 sq. mi.

He gave the following relationship for time to peak:

$$t_p = C_t(L_D L_{ca})^{0.3} + t_R/2 \qquad (14)$$

where t_p is time to peak in hours, L_D is length of main stream from outlet to divide in miles, L_{ca} is length along main stream in miles, from outlet to the point that is nearest the centroid of the area of the watershed, t_R is the length of unit rainfall in hours, C_t is a coefficient that ranged from 1.8 to 2.2 for Snyder's study. For the discharge he found:

$$Q_p = 640\ C_p A/t_p \qquad (15)$$

Where Q_p is peak discharge of the unit hydrograph in cfs, A is the area in sq. mi., and C_p another coefficient which ranged from 0.56 to 0.69.

Linsley et al. (6) gave the following modified equation for t_p:

$$t_p = C_t(L_D L_{ca}/\sqrt{S_D})^{0.36} \div t_R/2 \tag{14a}$$

where S_D is the weighted geometric slope of the channel in percent and C_t has different values for mountain areas, foothill areas, and valley areas.

<u>Soil Conservation Service (SCS)</u>

The U.S. Soil Conservation Service has derived a dimensionless unit hydrograph with ordinate values expressed by the ratio Q_t/Q_p and abcissa values as the dimensionless ratio t/t_p, where Q_t is discharge at any time t. These results were obtained from studies on a large number of watersheds that varied widely in size and location (14).

T_c, time of concentration, defined as the travel time for water from the most remote point on the divide to the outlet, was used to find t_p. T_c is calculated from the following formula:

$$T_c = (11.9\ L^3_D/H)^{0.385} \tag{16}$$

where H is the elevation difference between the outlet and the divide in feet, t_p is then calculated as:

$$t_p = 0.6T_c + t_R/2 \tag{17}$$

Eventually Q_p is found as:

$$Q_p = 484\ A\ V/t_p \tag{18}$$

where V is volume of surface runoff in inches.

Gray

In Gray's study (15) 42 watersheds in Illinois, Iowa, Nebraska, Ohio and Wisconsin were investigated. He shows that L_M (length of main stream in miles) defined as length of solid blue or dotted line on U.S. Geological Survey topographic map, and L_{ca} usually are highly correlated, and both L_M and L_{ca} are also correlated to the size of the area. For the data studied he found:

$$L_M = 1.40\ A^{0.568} \tag{20}$$

$$L_{ca} = 0.54\ L_M^{0.96} \tag{21a}$$

This leads to

$$L_{ca} = 0.73\ A^{0.55} \tag{21b}$$

Langbein and others (16) made a study of 340 drainage areas in the North-Eastern United States and found:

$$L_{ca} = 0.90\ A^{0.56} \tag{21c}$$

This shows that the watershed characteristics can vary from one location to another.

Gray's further development builds upon Edson's and Nash's models. He used a dimensionless graph with two linearly related parameters γ' and q

$$Q_{(t/t_p)} = \frac{25.0(\gamma')^q}{\Gamma(q)}\ e^{-\gamma'/t_p}\ (t/t_p)^{q-1} \tag{22}$$

where γ' and q have the linear relationship

$$q = \gamma' + 1 \tag{23}$$

The ratio t_p/γ' measures the storage characteristics of the basin. Gray made a grouping of watersheds into geographic locations when correlating the storage characteristics of the basin to the length and average geometric slope of the main stream, using the following formula:

$$t_p/\gamma' = x(L_M/\sqrt{S_c})^m \tag{24}$$

with x varying from 7.40 to 11.40 and m from 0.498 to 0.562. He also found a relation between time to peak and γ'

$$t_p/\gamma' = \frac{1}{\frac{2.676}{t_p} + 0.0139} \tag{25}$$

From (24) and (25) t_p and γ' can be computed and then q can be computed from (23).

McSparran

A more involved investigation at least as far as drainage characteristics are concerned, was made by McSparran in Pennsylvania (17). It will be mentioned here, but it is beyond the scope of this study to take up and use the same principles. McSparran used 6 different watershed characteristics.

1. Area, A
2. Length of main stream from outlet to divide, L_D
3. Average geometric slope of main stream, S_D
4. Drainage density, D_d, defined as the ratio of the total

length of all streams in the basin to the area, in miles per square mile.

5. The percentage of wooded area, W_a; 5 is used for W_a if the actual value is 5 or less.

6. Watershed shape factor, F, defined as the ratio of the length L_D, to the diameter of a circle with an area equal to the drainage area. This shape factor was first used by Wu (18) in a study in Indiana.

By using many combinations of n and t_p, McSparran programmed an electronic computer to synthesize hydrographs by using the summation form of the convolution integral, Eq. (13), see page 16. He found that different values for n (number of linear reservoir) and t_p (time to peak) applied for different storms for the same watershed. The n and t_p that gave best fit for the largest storm were selected.

Setting 640A = V in Eq. (11) and solving for k in Eq. (10)

$$k = \frac{t_p}{n-1}$$

Eq. (11) can be expressed as

$$Q(t) = \frac{V(n-1)^n}{\Gamma(n)\ t_p} (t/t_p\ e^{-t/t_p})^{n-1} \tag{11a}$$

by using the time to peak as the basis for dimensionless ratio. This formula gives dimensionless hydrographs of the form shown on Fig. 6a. Each n gives a different hydrograph.

Wu (19) made a study on the basis that n could be correlated with recession curves of dimensionless hydrographs derived from Eq. (11a). For a linear storage relationship the recession curve will plot as a straight

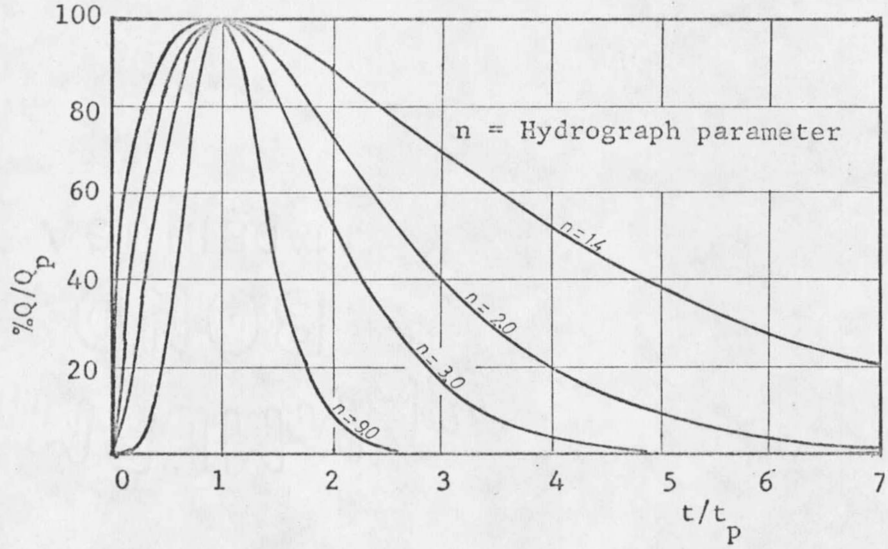

FIGURE 5. - Dimensionless hydrographs

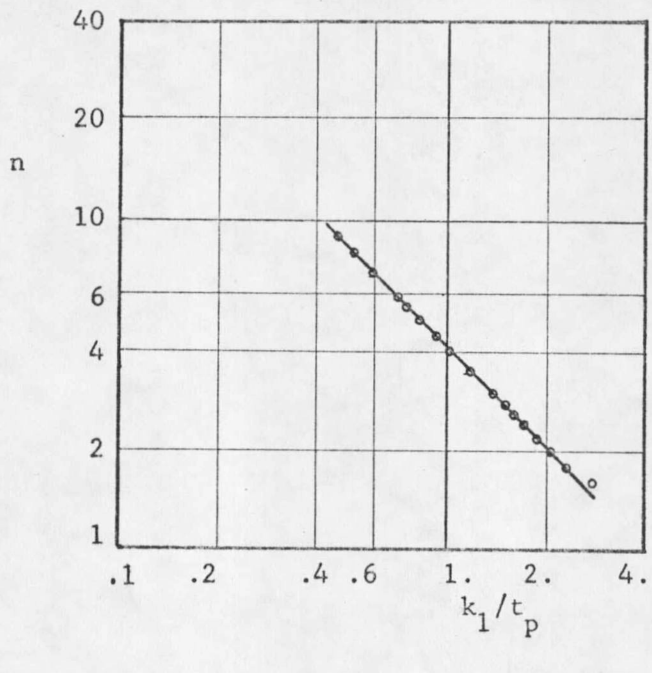

FIGURE 6. - The curve shows a log-log plot of the relationship between k_1/t_p and n. The equation for the line is: $n = 4.1/(k_1/\ p)$, (eq. 26, page 23).

line on semi-log paper. Wu found the following relationship

$$k_1/t_p = \frac{(t_1-t_0)/t_p}{2.3 \log \frac{q_0/q}{q/q_p}}$$

where k_1 is the recession storage coefficient from q_0 to q; q_0, q and q_p are flow at time t_0, t_1 and t_p respectively. By plotting the recession curve of the dimensionless hydrographs for different values of n as shown on Fig. 5, on semi-log paper, he found values for k_1/t_p for each n-defined curve. Fig. 6 shows the graphical relationship between n and k_1/t_p as found from the semi-log plots when plotted on log-log paper. The slope of the line is -1 for n-values greater than 2; it can therefore be expressed as an hyberbola

$$n = 4.1/(k_1/t_p) \qquad \text{for } n > 2 \qquad (26)$$

McSparran solved for k_1 in Eq. (26) after finding n and t_p as described earlier. By using multiple correlation he then derived the following formula for time to peak:

$$t_p = \frac{8.51\ A^{0.0732} W^{0.186}}{S_D^{0.699} D_d^{0.417}} \qquad (27)$$

He found it necessary to divide Pennsylvania into two regions when finding k_1:

Region I, east, north and west Pennsylvania

$$k_1 = \frac{8.90\ A^{0.0131} W_a^{\ 0.407}}{S_D^{\ 0.949} D_d^{\ 0.752}} \tag{28a}$$

Region II, south-central Pennsylvania

$$k_1 = \frac{3.65\ A^{0.490}}{D_d^{\ 0.588} W_a^{\ 0.325}} \tag{28b}$$

He then substituted (27) and (28a) or (27) and (28b) into (26) in order to find equations for n

Region I

$$n = \frac{3.91\ A^{0.0601} D_d^{\ 0.335}}{S_D^{\ 0.250} W_a^{\ 0.221}} \tag{29a}$$

Region II

$$n = \frac{9.57\ D_d^{\ 0.171} W_a^{\ 0.511}}{S_D^{\ 0.699} A_a^{\ 0.417}} \tag{29b}$$

CHAPTER IV

STORM AND FLOODS OF JUNE 1964 IN NORTHCENTRAL AND NORTHWESTERN MONTANA

On June 7th and 8th, 1964 an extensive area of about 17,000 sq. mi. on both sides of the Continental Divide in northcentral and northwestern Montana experienced a rainstorm that for most of the area produced the most disastrous flood ever recorded. Up to 13 inches of rainfall was actually measured, and at higher elevations rainfall of about 16 inches during a 36 hour period was estimated. The floods that followed caused an estimated damage of about 55 million dollars and 30 people lost their lives.

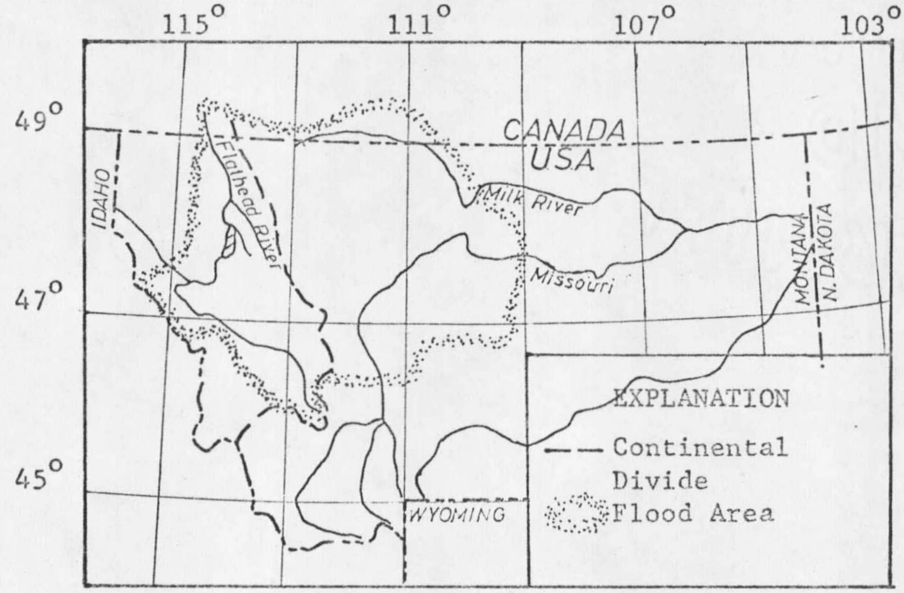

FIGURE 7. - Area covered by heavy rain. Fig. taken from U.S.G.S. Water-Supply Paper 1840-B.

Fig. 7 shows the area covered by the rainstorm. In 1967 a complete report was published with the title: "Floods in Montana, June 1964", U.S.G.S. Water-Supply Paper 1840-B (20). It covered in detail antecedent conditions, the meteorology of the storm and a description of the floods.

Antecedent Hydrologic Conditions

Prior to the heavy rainfall the rivers were at high stages due to snowmelt and precipitation in May and early June. Fig. 8 shows antecedent precipitation for 4 of the stations in the area from May 1st to June 6th.

Because of cold weather in March, April and early May snowmelt was

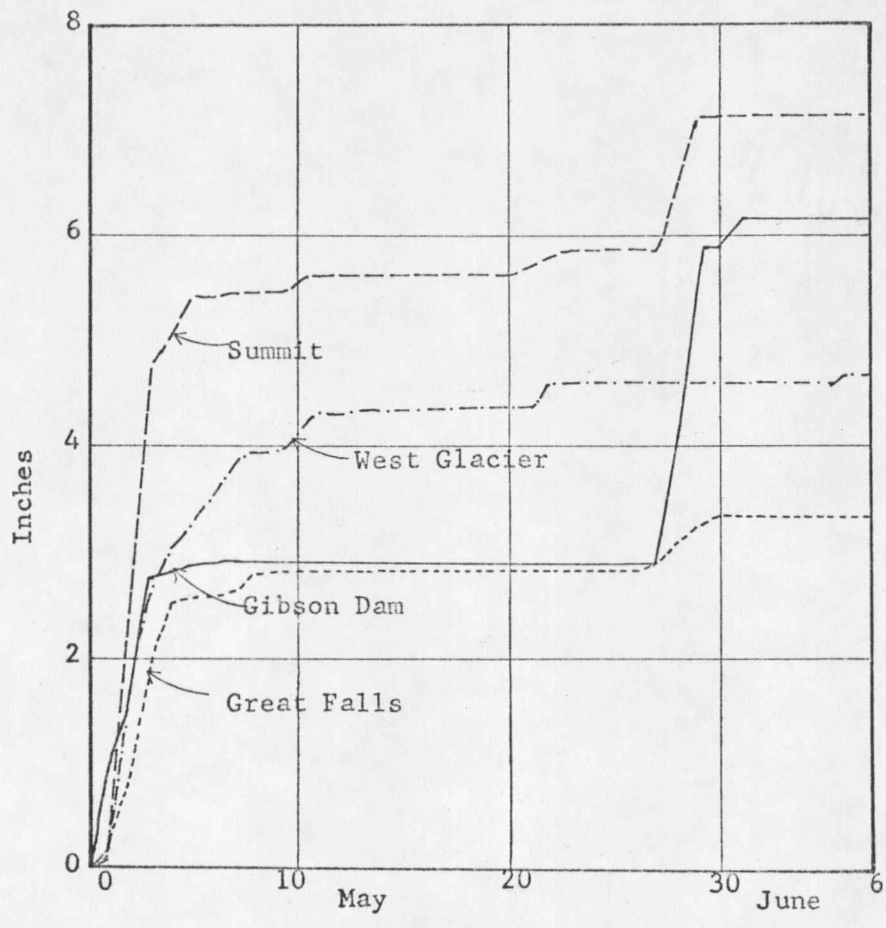

FIGURE 8. - Accumulation of rain at four weather stations, May 1st to June 6th, 1964.

delayed. When warmer weather started in the second half of May it therefore caused a greater rise in the rivers than normal. When the rainstorm of June 7-8 started a considerable amount of snow was left a higher elevations and at somewhat lower elevations in well sheltered areas. Measurements made after the flood indicated that snowmelt from higher elevations may not have had much effect on the total runoff, but that snowmelt at lower elevations may have been significant in some of the areas. The report further states that the soil moisture was at field capacity in recently snowcovered areas, and near field capacity in foothill areas due to above normal rain and cold weather.

The Meteorology of the Storm

A rainy season generally occurs in Montana from late May to the end of June. By this time the air is warm enough to carry considerable amounts of moisture from south to north. In 1964 several rain producing factors worked together to cause the heavy rain. There was a low pressure over the above mentioned area, causing large air masses to move in, mostly from the Gulf of Mexico. The air masses were forced to rise by the mountains as they moved toward the Continental Divide. The dewpoints on the ground surface were very high. In the later part of the storm a cold front moved southward from Canada and caused additional lifting of the moist air masses.

Fig. 9 is an isohyetal map showing the precipitation in different areas and Fig. 10 shows the accumulated rainfall for some of the stations.

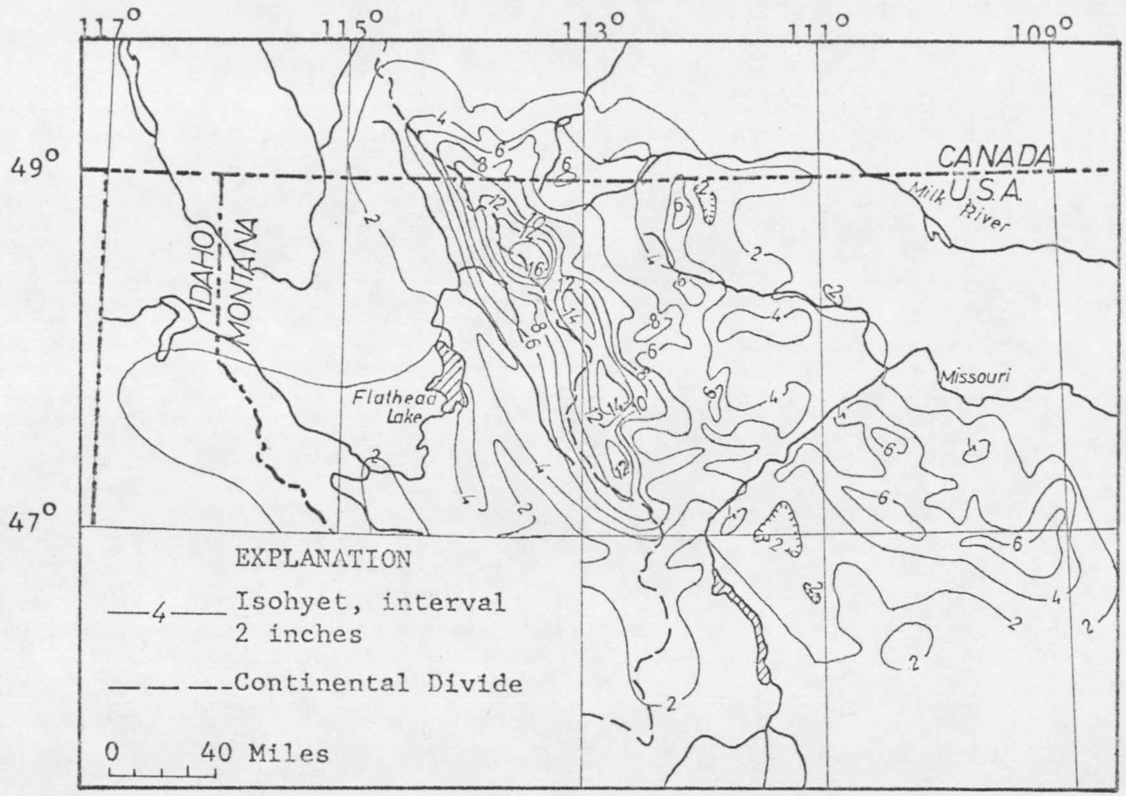

FIGURE 9. - Total precipitation for June 7 - 8, 1964. Highest centers estimated because of lack of measurements in mountains. Fig. taken from U.S.G.S. Water-Supply Paper 1840-B.

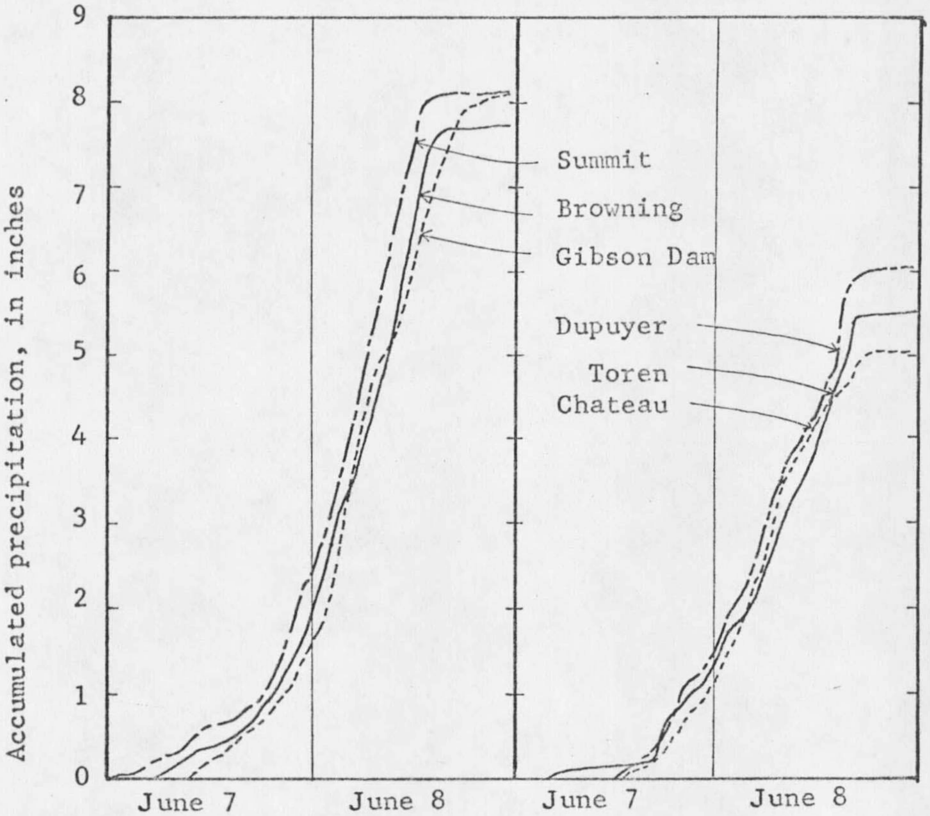

FIGURE 10. - Mass curves of accumulation of precipitation with time from recording rain gages in (or on edge of) storm area. June 7-8, 1964.

CHAPTER V

OUTLINE OF THIS STUDY

The purpose of this study was to determine some unit hydrographs for a few selected drainage areas in Montana which have experienced severe rain-produced floods. It was decided to derive such unit hydrographs from historical flood-rainfall data. The necessary data was found to be very difficult to obtain. It appeared that recorded rainfall data from heavy storms usually were not accompanied by sufficient floodflow data. In several instances the opposite had happened; a significant flood was measured at one or more streamgaging stations, but adequate rainfall data were not available.

Personnel of the U.S. Geological Survey and U.S. Weather Bureau in Helena suggested the big rainstorm of 1964 and the resulting floods as a possible source for data. Because of the extensive area covered by the storm, there were many recording raingages and streamgages within the storm's limits. One big disadvantage is that no correlated rainfall-runoff data appears to be available for the same area from other storms.

Because it was felt that any derived unit hydrographs even though quite inaccurate, would represent a contribution, it was decided to carry out the study, and in order to do so the big storm in 1964 was selected to give the necessary data. Since the rainfall from this storm was fairly evenly distributed over a large area, see Fig. 10, rainfall data would hopefully be representative even though obtained a considerable distance away from the watershed under study.

Selected Drainage Areas

U.S.G.S. Water-Supply Paper 1840-B lists a large number of drainage areas and gives the average daily stages and discharges for May-June 1964. For several of the river-stations the report also gives the stages and discharges for shorter intervals, down to hourly steps during the flood on the 7th, 8th and 9th of June. Eleven of these drainage areas were selected for the purpose of deriving unit hydrographs from the 1964 flood. The streams were selected because they gave the necessary time-discharge data. Table 1 gives a listing of the selected drainage areas with the basin characteristics used in this study. In addition to drainage areas selected from the report, the data for Lone Man Coulee were made available by Prof. Williams (21) and used in the same way as the others. Fig. 11 shows the locations for the gages for the different streams and also the locations of the recording raingages used for this study. Both the raingages and the streamgages are identified by numbers as shown in Tables 1 and 2. Table 2 lists the raingages used, together with location, start and end of rainfall, total precipitation and length of rainstorm.

The Thiessen Polygon method was used to derive the average hourly rainfall for each drainage area.

The Reliability of the Data

Discharge data. - Since this flood exceeded every previously recorded flood for most of the area, in some cases by a factor of 10 or more, reliable stage-discharge records were not available. The flood flows also changed the stream beds radically in many cases, producing severe erosion and/or debris deposition. Such deposits sometimes caused partial blocking and

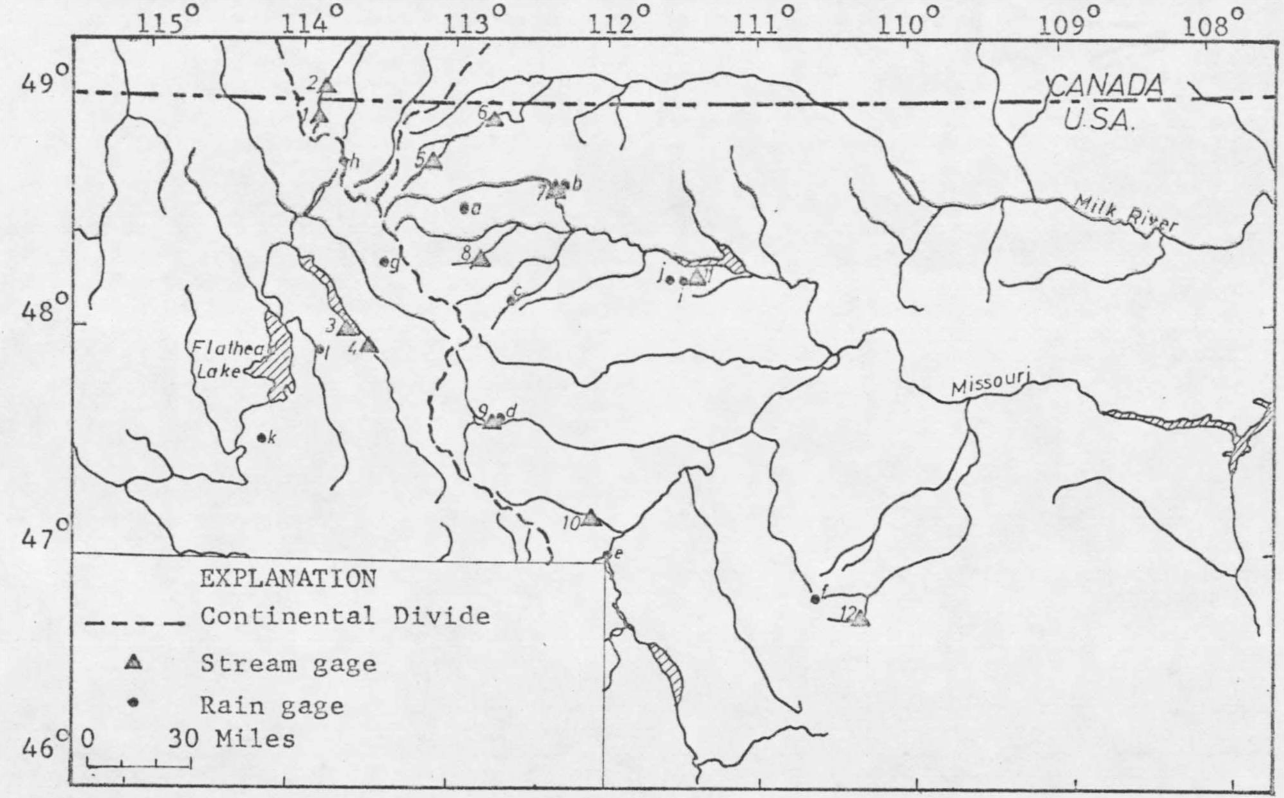

FIGURE 11. - Location of stream gages and rain gages for drainage basins studied in this report. Numbers conform to those in Table 1 and 2.

TABLE 1. - Listing of Drainage Areas with most important Characteristics

Area No.	Name of Drainage Area	Area A sq.mi.	Length of Main Stream: to Divide L_D mi.	Length of Main Stream: as Showing on Map L_M mi.	Length of Main Stream: to Centre of Area L_{ca} mi.	Ave. Geometric slope: to Divide S_D per cent	Ave. Geometric slope: of Main Stream S_M per cent	Elev. Diff. betw. outlet and divide H ft.
1.	Waterton River n. Int. Bound.	61.0	11.0	10.0	4.5	1.65	1.17	2600
2.	Waterton River n. Watert. Park	238.0	Not available					
3.	Sullivan Creek n. Hungry Horse	71.3	14.0	13.5	3.5	2.11	1.99	3180
4.	S.F. Flathead Riv. n. Hungry Horse	958.0	57.5	55.5	32.0	0.53	0.56	4100
5.	S.F. Milk River n. Babb	68.6	16.0	15.5	9.0	1.60	1.47	3500
6.	S.F. Milk River at Del Bonita	325.0	40.5	40.0	21.0	0.73	0.66	4350
7.	Cut Bank Creek at Cut Bank	1065.0	62.0	61.0	28.0	0.55	0.51	4800
8.	Badger Creek n. Browning	133.0	24.0	23.0	12.0	1.15	1.08	2700
9.	Sun River at Gibson Reservoir	575.0	31.0	30.0	8.0	1.10	1.01	3500
10.	Dearborn River n. Craig	325.0	44.0	42.0	22.0	0.86	0.83	3800
11.	Lone Man Coulee near Valier	14.0	7.2	6.8	4.6	0.76	0.71	410
12.	S.F. Judith River n. Utica	58.7	11.0	9.5	4.0	2.32	1.99	1500

TABLE 2. - Recording Raingages used in this study for Thiessen Polygon

No.	Name	Location	Start rainfall		End rainfall		Tot.prec.	Duration
			Date	Hour	Date	Hour	Inches	Hours
a.	Browning	Glacier County	6/7	6 AM	6/8	4 PM	7.68	35
b.	Cut Bank Airport	Glacier County	6/7	2 PM	6/8	3 PM	3.11	26
c.	Dupuyer	Pondera County	6/7	1 PM	6/8	7 PM	6.03	31
d.	Gibson Dam	Lewis and Clark County	6/7	10 AM	6/8	10 PM	8.09	36
e.	Holter Dam	Lewis and Clark County	6/7	8 AM	6/8	11 PM	2.16	39
f.	Kings Hill	Cascade County	6/7	7 AM	6/8	11 PM	2.13	41
g.	Summit	Flathead County	6/7	2 AM	6/8	3 PM	8.09	38
h.	Grinnel	Glacier County	6/7	5 AM	6/8	2 PM	5.50	34
i.	Geiger	Pondera County	6/7	5 AM	6/8	5 PM	4.81	28
j.	Toren	Pondera County	6/7	5 AM	6/8	6 PM	5.34	29
k.	Round Butte	Lake County	6/7	2 AM	6/8	12 AM	2.40	34
l.	Swan Lake	Lake County	6/7	1 AM	6/8	10 PM	3.51	45

hence may have altered former stage-discharge relations. Paper 1840-B attempts to take all this into account, but states:

> The stages and associated discharges should not be used in preparation of a stage-discharge relation (rating curve) for use outside the flood period. For many stations the relation used to compute the discharge was shifted from the basic rating for various reasons, such as backwater from debris, blockage or other changes in control condition.

The discharges were computed by using stage-discharge relationships for discharges not exceeding values measured earlier, and by the slope-area method for greater discharges. In some cases log-log plots were extended above the established range of data. Although these data cannot be expected to be very precise, it appears that every step possible has been taken to make them as reliable as possible. Since the purpose of this study is to find a time-discharge relationship, not a stage-discharge relation, the data for discharge can hopefully be used with confidence.

Rainfall data. - The rainfall data used in this study are all obtained from recording raingages and are believed to be reliable. It should be mentioned however, that the data from Grinnel seems to be a little out of order. It is given to 0.1 inch only, compared with 0.01 inch for all the other stations used in this study. The rainfall at Grinnel is also suspiciously uniform compared to the rainfall pattern recorded elsewhere during the same storm. It is well known that rainfall can vary considerably at spots only a few miles apart. This problem is taken up under the discussion later in the report.

Runoff Separation

As mentioned earlier, the unit hydrograph theory includes only overland runoff, hence baseflow and interflow are separated from total runoff to produce the overland runoff ordinates.

It is almost impossible to determine when overland flow actually ceases because it takes place so gradually. There does not seem to be a technique for separation of baseflow and interflow from overland flow that can be said to be the only correct one. Investigators feel that consistency is more important than the method used, Wisler and Brater (22). Barnes (23) has shown that the recession curve plotted on semilog paper gives well defined changes in slope, called breakpoints. He defines the first as the point where overland flow ceases and interflow plus baseflow contribute all the discharge. The second breakpoint is where interflow ceases and baseflow dominates the runoff. This study makes use of these assumptions. The runoff is cut off at the first breakpoint and a straight line is drawn from that point to the beginning of rise of the hydrograph. Fig. 12a shows a semilog plot of a recession curve, and Fig. 12b shows how the separation of overland flow from interflow, baseflow and possible snowmelt takes place. In order to find the breakpoints, all the recession curves were plotted on semilog paper. Each graph gave a well defined breakpoint. However, the South Fork Flathead River had a discharge at the breakpoint below the value corresponding to the start of rise. For this reason a slightly higher point on the recession limb was used as cut off point.

It appears that some snowmelt is involved in all or most of the flood hydrographs. Although this most likely did not affect the total amount of flow significantly, it would probably be a very important factor in the

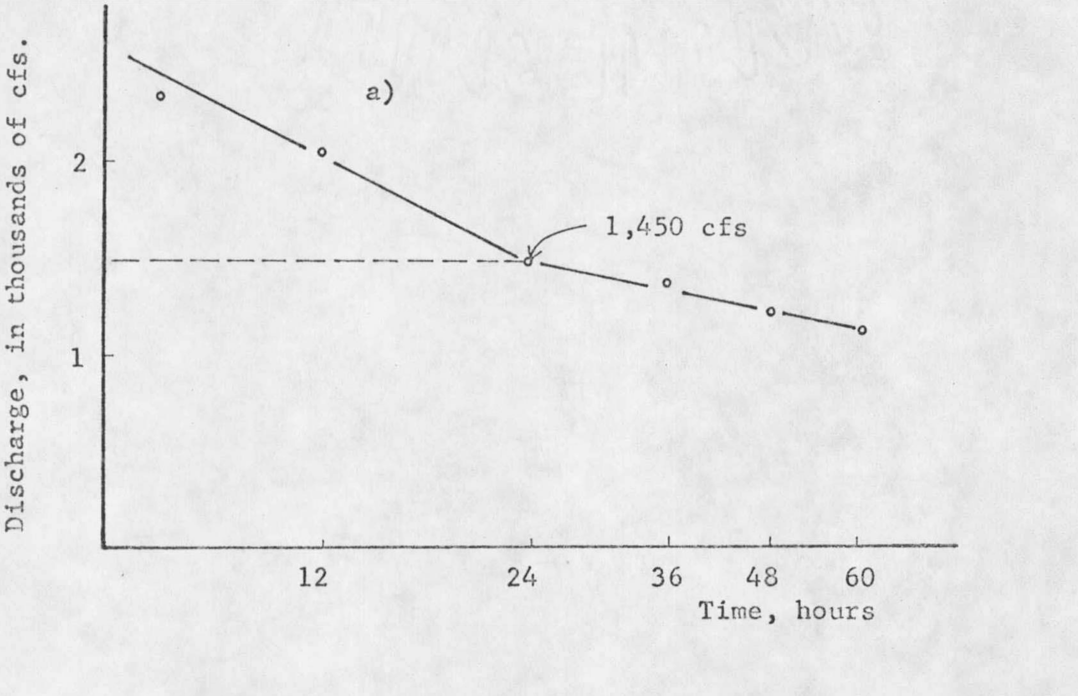

FIGURE 12. - Separation of baseflow, interflow and possible snowmelt from overland flow. a) Semilog plot to find the breakpoint between overland flow and interflow. b) Total runoff hydrograph divided as shown on Fig. 12b.

baseflow. The runoff from snowmelt could move the breakpoint one way or the other. Some of the runoff from snowmelt may remain above the separation line and some below. Since the separation line is taken from the beginning of rise of the hydrograph, and since snowmelt is already involved at this point, it is hoped that the snowmelt will thus be classified with ordinary baseflow and interflow. Since rain is a poor "snowmelter," Leber (24), compared to heat transfer by direct sunshine, it is not likely that the snowmelt increased during the rainy, cloudy period.

If after the main rainstorm has stopped, delayed rain occurs that disturbs the recession curve, it is necessary to estimate the recession which would have occurred without the later rain. Fig. 13 shows how this may be done.

The rainstorm of June 7-8, 1964 had a trend that showed increasing intensity as time went on, see Fig. 10, page 29, and it ceased rather abruptly. For this reason it was not found necessary to draw any estimated recession curves for this study.

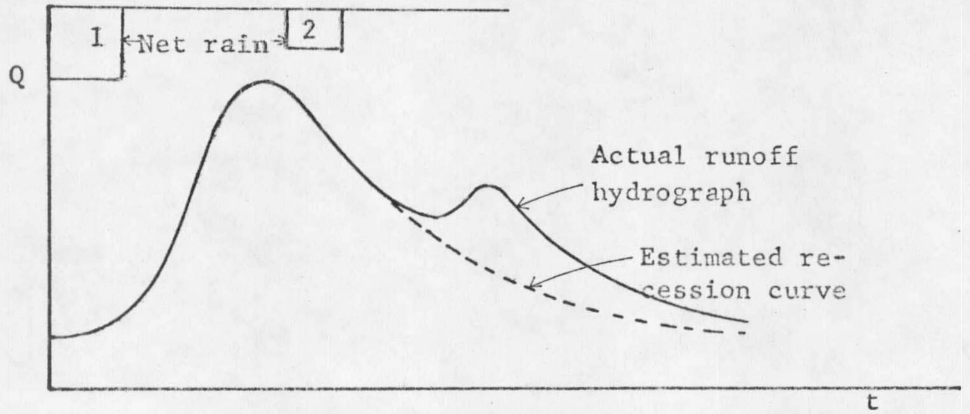

FIGURE 13. - How to draw an estimated recession curve to eliminate effect of second net rain.

Separation of Excess Rain

After the overland flow was separated, the volume under the curve was measured with a planimeter and also computed by summing ordinates. This volume was then set equal to excess rain, and a horizontal line was drawn on the hydrograph of the hourly rainfall computed from Thiessen polygon method (25), so that the volume above the line was equal to excess rain. McSparran tried both this method (constant infiltration) and variable infiltration and found the latter best. The variable infiltration method is beyond the scope of this study, and doubts can be raised if it would be more applicable for the following reasons. As mentioned earlier the soil moisture varied from field capacity to almost so at the start of the rainstorm. Since the rain started with a moderate intensity, the ground became even more saturated before the heavy rain started. This would tend to produce a uniform infiltration rate during the period of excess rainfall. In addition the raingages do not cover the different drainage areas too well, so the actual rainfall could have varied considerably from what is obtained from the Thiessen polygon. A constant rate of loss is therefore assumed to be as accurate as the data warrant.

The Thiessen polygon method was not strictly followed. It turned out that Swan Lake alone gave better results (see discussion) for S.F. Flathead River than <u>Gibson Reservoir and Swan Lake</u> together. Also Cut Bank and Grinnel gave better results for S.F. Milk River at Del Bonita than <u>Browning and Grinnel</u>. Dupuyer and Summit gave better results for Badger Creek than <u>Browning and Summit</u>. Although the underlined combinations were those given by the Thiessen Polygons, the others have been used. For the rest of the watersheds the Polygon method has been followed.

Method Used to Find Unit Hydrograph

Since this is a complex rainstorm with excess rainfall distributed over a long period of time, the only way to fit a computed hydrograph to the actual measured hydrograph is by a trial and error procedure. The same approach was taken as outlined when discussing McSparran's study, see page 20. A computer program was developed that made use of the instantaneous unit hydrograph idea. The computer Sigma 7, Fortran IV, was given many combinations of t_p (time to peak) and n (numbers of linear reservoirs) for each basin. Time to peak was varied from 40 min. up to 7 hours with 20 min. intervals, and from 5 hours up to 23 hours with 1 hour intervals. The value of n was varied from 1.6 up to 9.8 with 0.1 intervals at the lower range increasing to 0.3 intervals at the upper range. The 20 min. intervals were only used for watersheds that had t_p equal to 5 hours after the hourly intervals were tried. Values of k were found from Eq. (11), see page 14. The unit hydrograph was then computed from Eq. (12), see page 14. In order to use the computer finite differences of dt had to be used. Therefore the summation form of the convolution integral, Eq. (13), see page 16, was used to obtain a complex hydrograph. By using the summation form of the convolution integral, a complex hydrograph was obtained for each watershed for each combination of t_p and n. When 20 min. intervals were used for t_p, the ordinates for the unit hydrograph were taken at 20 min. intervals. The 20 min. ordinates were also computed for the complex hydrograph in these cases. This was done so that one did not lose the peak discharges.

For each complex computed hydrograph, obtained for different combinations of t_p and n used in the unit hydrograph, the ordinates were compared with the corresponding ordinates for the actual surface runoff hydrograph, computing for each case

$$E^2 = \sum_{i=1}^{m} (Y(i)_a - Y(i)_c)^2 \tag{30}$$

where $Y(i)_a$ is the i-th ordinate of actual hydrograph, $Y(i)_c$ is the corresponding ordinate for the computed hydrograph, and both values are taken at 1 hour intervals throughout the duration (m hours) of the computed hydrograph.

When all combinations of t_p and n were tried, the unit hydrograph corresponding to the least value of E^2 from Eq. (30) was selected as the "best fit" unit hydrograph for the given watershed. The values for t_p and n used in this unit hydrograph are listed for all the drainage basins in Table 3. Values of k_1 computed from Eq. (26) are also listed in Table 3.

Two different relative time scales were used when the comparisons were made. The first method described above let the computed hydrograph start at the same time as start of rise of the actual hydrograph. It was found that sometimes the computed and actual hydrograph had about the same magnitude of peak and approximately the same form, but the peaks were offset with respect to each other on the time axis. In the second method the peak of the computed hydrograph was shifted so it and the peak of the actual hydrograph occurred at the same time. For both methods the ordinates of the hydrographs corresponding to the t_p and n producing the least value of E^2 were computed. The values for t_p and n in the best unit hydrograph for this method are also listed in Table 3. The computer program used can be found in Appendix C in the copy belonging to the Dept. of Civil Engineering.

Synthetic Methods

To determine how well formulas derived for prediction of synthetic unit hydrographs agree with the unit hydrographs for the areas studied, Snyder's, SCS's and Gray's methods were used to calculate values of t_p and Q_p (time and discharge at peak). These values were compared with the corresponding values for the best fitted computed unit hydrograph obtained as described on page 40. It was felt that if these two characteristics agreed fairly well, the remainder of the unit hydrograph would also agree. For all methods and for all watersheds 1 inch of rainfall was applied during a unit time of 1 hour. Table 4 lists the values computed for t_p and Q_p for each method. The corresponding values for the best unit hydrograph are also given. Appendix B gives examples of the computations used to find values for t_p and Q_p for each method.

Check on Computer Program

The computer program used became fairly involved. It was therefore desirable to check it on a fairly simple model. There were two reasons for this, a) it would be easier to find out if all the excess rain was used, if the excess rain was distributed properly with time and if the summation form of the convolution integral Eq. (13), page 16, was working as it should, b) it would give an estimate of how well the unit hydrograph method used could reproduce a hydrograph calculated from the theoretical open channel shallow-water equations.

TABLE 3. - Values for t_p, n and k_1 for the unit hydrograph that gave best fit for the different watersheds

Area No.	Name of drainage area	Computed hydrograph and actual hydrograph start at the same time			Computed hydrograph shifted so t_p coincide with t_p of actual hydrg.		
		t_p, hr	n	k_1	t_p, hr	n	k_1
1.	Waterton River n. Int. Bound.	6.0	2.6	9.5	12.0	4.8	10.3
2.	Waterton River n. Watrt. Park	12.0	2.0	24.6	17.0	2.0	34.8
3.	Sullivan Creek n. Hungry Horse	19.0	4.5	17.3	9.0	2.0	18.5
4.	S.F. Flathead River n. Hungry Horse	20.0	3.4	24.0	12.0	2.0	24.6
5.	S.F. Milk River at Babb	2.0	2.0	4.1	2.3	2.2	4.3
6.	S.F. Milk River at Del Bonita	6.7	7.3	3.8	2.7	2.7	4.2
7.	Cut Bank Creek at Cut Bank	22.0	5.5	16.4	17.0	3.3	21.1
8.	Badger Creek n. Browning	0.7	3.9	0.7	1.7	9.8	0.7
9.	Sun River at Gibson Reservoir	6.0	1.9	12.9	11.0	3.0	15.1
10.	Dearborn River n. Craig	15.0	2.0	30.7	23.0	3.0	31.5
11.	Lone Man Coulee n. Valier	2.0	2.4	3.4	2.0	2.4	3.4
12.	S.F. Judith River n. Utica	8.0	4.5	7.3	7.0	2.2	13.0

TABLE 4. - Time to peak and discharge at peak for computed instantaneous unit hydrograph and by using 3 different synthetic unit hydrograph set of formulas

Area No.	Name of drainage area	Calc. unit hydrograph: Calc. and act. hydr. start at the same time		Calc. unit hydrograph: Comp. shifted so t_p agrees w/t_p of act.		Snyder		SCS		Gray	
		t_p hr.	Q_p cfs	t_p hr.	Q_p cfs	t_p hr.	Q_p cfs	t_p hr.	Q_p cfs	t_p hr.	Q_p cfs
1.	Waterton River n. Int. Bound.	6.0	3119	12.0	2475	4.9	5609	1.6	18018	1.3	13640
2.	Waterton River n. Watert. Park										
3.	Sullivan Creek n. Hungry Horse	19.0	1736	9.0	1865	4.6	6850	2.0	17218	1.5	25524
4.	S.F.Flathead Ri. n. Hungry Horse	20.0	18304	12.0	18796	21.5	18207	7.4	62793	26.1	61981
5.	S.F.Milk River n. Babb	2.0	8076	2.3	7682	5.4	5120	2.2	15424	2.0	29367
6.	S.F.Milk River at Del Bonita	6.7	30831	2.7	37708	12.7	10690	5.1	30888	8.6	32608
7.	Cut Bank Creek at Cut Bank	22.0	25539	17.0	23400	13.3	30330	8.0	64350	39.4	38000
8.	Badger Creek n. Browning	0.7	84294	1.7	59871	9.7	5790	3.5	18354	3.0	34738
9.	Sun River at Gibson Resv.	6.0	21224	11.0	18110	4.2	62150	4.9	56748	4.3	105405
10.	Dearborn River n. Craig	15.0	5105	23.0	4896	12.9	10480	5.8	27025	7.0	40110
11.	Lone Man Coulee n. Valier	2.0	2009	2.0	2009	3.2	2105	2.0	3410	1.5	79049
12.	S.F. Judith Riv. n. Utica	8.0	3395	7.0	2190	3.7	7390	2.0	14300	1.1	36200

Prof. Givler, oral communication, has a theoretical model with a tilted plane of 10 sq. mi. All precipitation on the plane is equated to excess rain. The surface runoff from the plane is routed through a rectangular channel. The flow on the plane and through the channel are described by the shallow-water equations, Stoker (26). The rainfall used to produce a two-peaked hydrograph and the ordinates of the hydrograph obtained by Givler, were used as input data for the computer program. Fig. 14 shows the theoretical hydrograph calculated from the shallow-water equations and the hydrograph obtained using the best unit hydrograph on the same rainfall data. The last mentioned hydrograph was calculated using the same procedure as described on page 40, with time to peak varied from 12 min. to 72 min. with 6 min. intervals.

The overall fit is good. It is seen that the hydrograph computed from the instantaneous unit hydrograph starts to rise a little earlier and peaks slightly lower than the hydrograph obtained by Givler. This result confirms that the computer program is working satisfactorily. It seems to select a unit hydrograph that can reproduce a given complex hydrograph closely.

<u>Check on Lone Man Coulee</u>

To be considered reliable for flood prediction for a given watershed, a unit hydrograph is preferred which has been derived from more than one rainstorm for which adequate rainfall and runoff data exist, rather than only one as in this study. This appears impossible for most of Montana. However, Lone Man Coulee, a relatively small watershed has been gaged and has been instrumented with two recording raingages for several years (22).

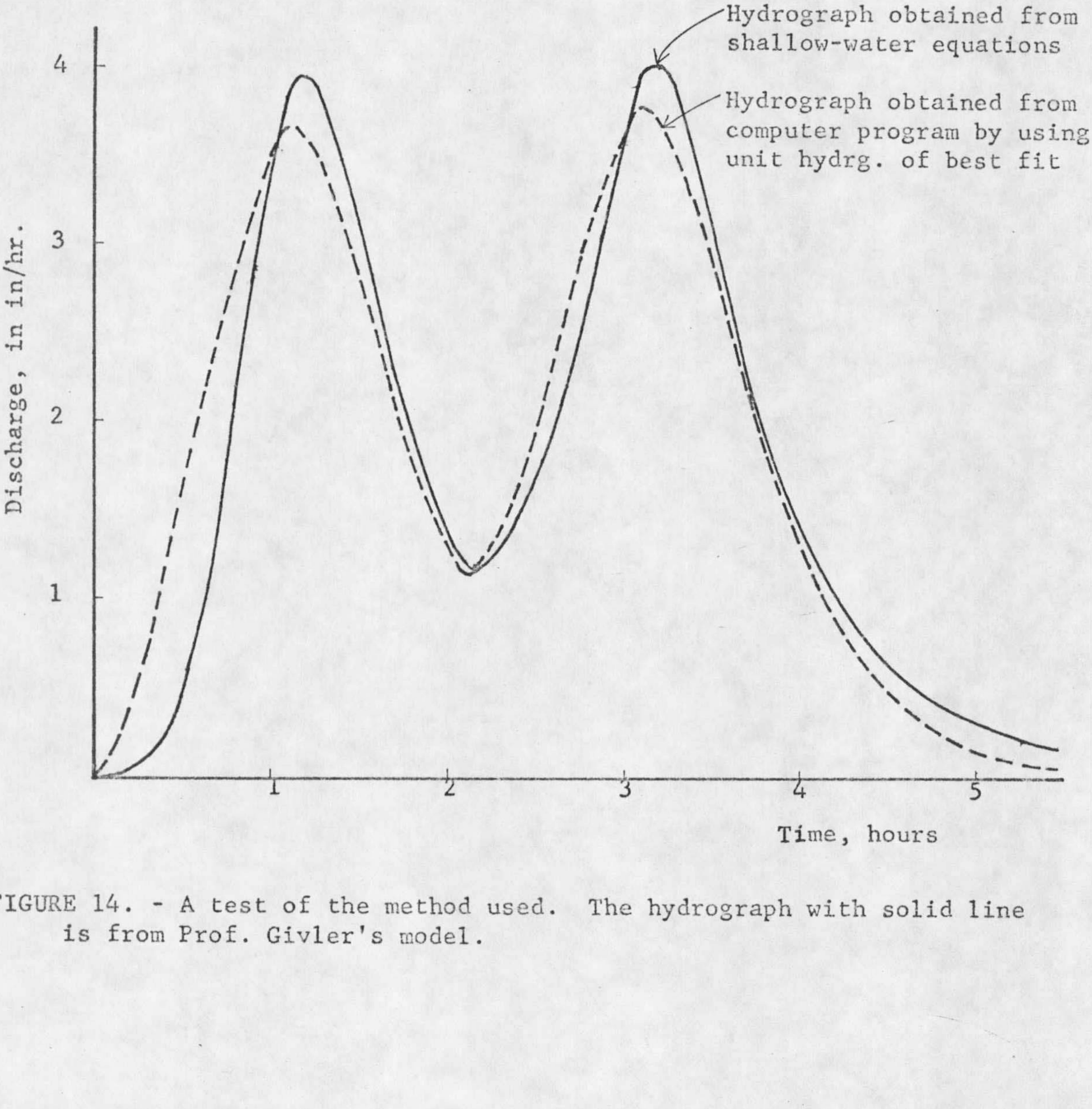

FIGURE 14. - A test of the method used. The hydrograph with solid line is from Prof. Givler's model.

During this period 3 rainstorms other than the June 7-8, 1964 event produced peaks of significance. Unit hydrographs have been derived for runoff of these storms in exactly the same manner as for the June 7-8, 1964 storm. The results are shown in Fig. 15. It will be seen that the unit hydrograph derived for the June 7-8, 1964 storm has the biggest peak. Since that rainstorm also was the most intense in in/hr, of the 4 studied, the unit hydrograph, which results, may be assumed to give best results for intense rainstorms. The parameters for Lone Man Coulee given in Table 3 are for the June 7-8, 1964 event.

The May 3, 1964 and the June 17, 1965 unit hydrographs are quite different from the other two. No simple explanation has been found for this. Fig. 15b shows the excess rainfall for each of the storms. The low intensity and long duration of excess rain for the two above mentioned storms probably causes some of the discrepancy, Linsley et al. (6), see page 6.

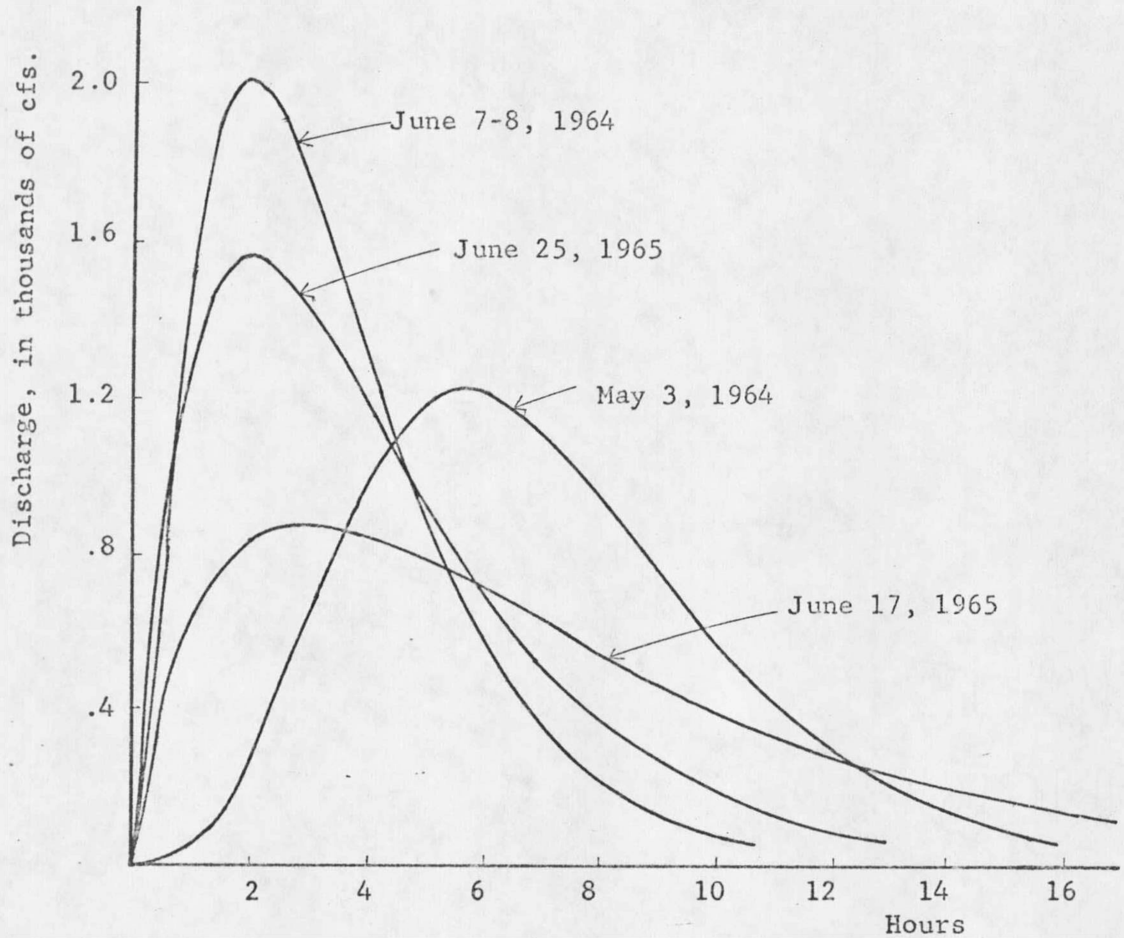

FIGURE 15a. - Four unit hydrographs derived from 4 different storms at Lone Man Coulee developed by program.

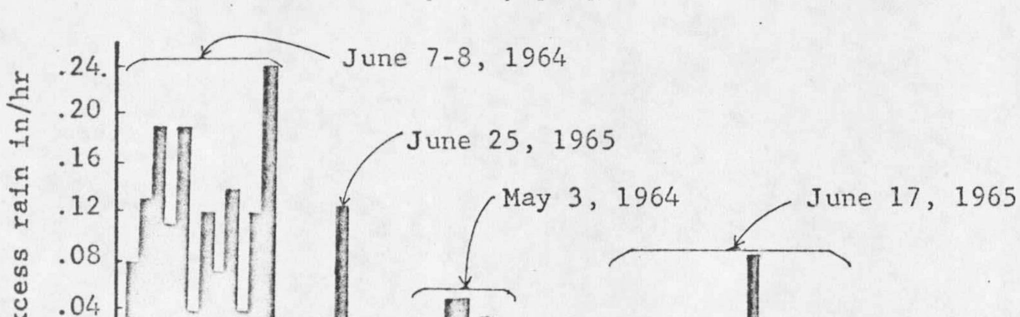

FIGURE 15b. - Lone Man Coulee, excess rainfall for 4 different storms.

CHAPTER VI

DISCUSSION

In order to discuss the results it is convenient to classify the computed hydrographs into different groups depending upon the comparisons between selected characteristics of the computed and actual hydrographs. Time to peak, discharge at peak and the general "goodness of fit" of the computed hydrograph to the actual hydrograph were chosen as the characteristics to be compared. On the basis of this the drainage areas were divided into three groups as follows:

Group 1; excellent results

a) Time to peak $|t_{pa} - t_{pc}| < 20\%$

b) Discharge at peak $|Q_{pa} - Q_{pc}| < 20\%$

c) Excellent overall goodness of fit of the computed hydrograph to the actual graph.

Group 2; good results

a) Time to peak $20\% \leq |t_{pa} - t_{pc}| < 30\%$

b) Discharge at peak $20\% \leq |Q_{pa} - Q_{pc}| < 30\%$

c) Fairly overall goodness of fit of the computed hydrograph to the actual graph.

Group 3; poor results

a) Time to peak $|t_{pa} - t_{pc}| \geq 30\%$

b) Discharge at peak $|Q_{pa} - Q_{pc}| \geq 30\%$

c) Poor overall fit of the computed hydrograph to the actual graph.

In the listing above t_{pa} means time to peak of actual hydrograph, t_{pc}

is time to peak of computed hydrograph, Q_{pa} is discharge at peak of actual hydrograph and Q_{pc} is discharge at peak of computed hydrograph. Comparison c) makes the classifying process somewhat subjective.

Table 5 shows in which group the different drainage basins were placed and gives a listing of t_{pa}, t_{pc}, Q_{pc}, $(t_{pa} - t_{pc})/t_{pa} \times 100$ and $(Q_{pa} - Q_{pc})/Q_{pa} \times 100$. The different values are given for both methods of computing the complex hydrograph, see page 41 . The graphs of actual surface runoff and of computed surface runoff after the two methods are given in the following order; Group 1, Figs. 16a - d; Group 2, Figs. 17a - e; Group 3, Figs. 18a - c.

S.F. Milk River near Babb and Dearborn River meet the requirements for Group 1 for comparison a) and b), however, the overall fitness does not justify placing them in Group 1, see Figs. 17a and 17d, they are placed in Group 2.

Lone Man Coulee was placed in Group 2 although it did not meet the requirement for comparison b) for this group, but since t_{pa} and t_{pc} were the same, see Table 5, and the overall fitness of the computed hydrographs to the actual were good, see Fig. 17e, it was placed in Group 2.

The difference for t_{pa} and t_{pc} in percent for translated hydrographs will, of course, be zero. Area numbers used in Table 5 are the same as on Fig. 11, see page 32.

The watersheds in the different groups do not seem to be located in specific areas. All the drainage basins in Group 1, however, are located either on the west side of the Continental Divide or next to it on the east

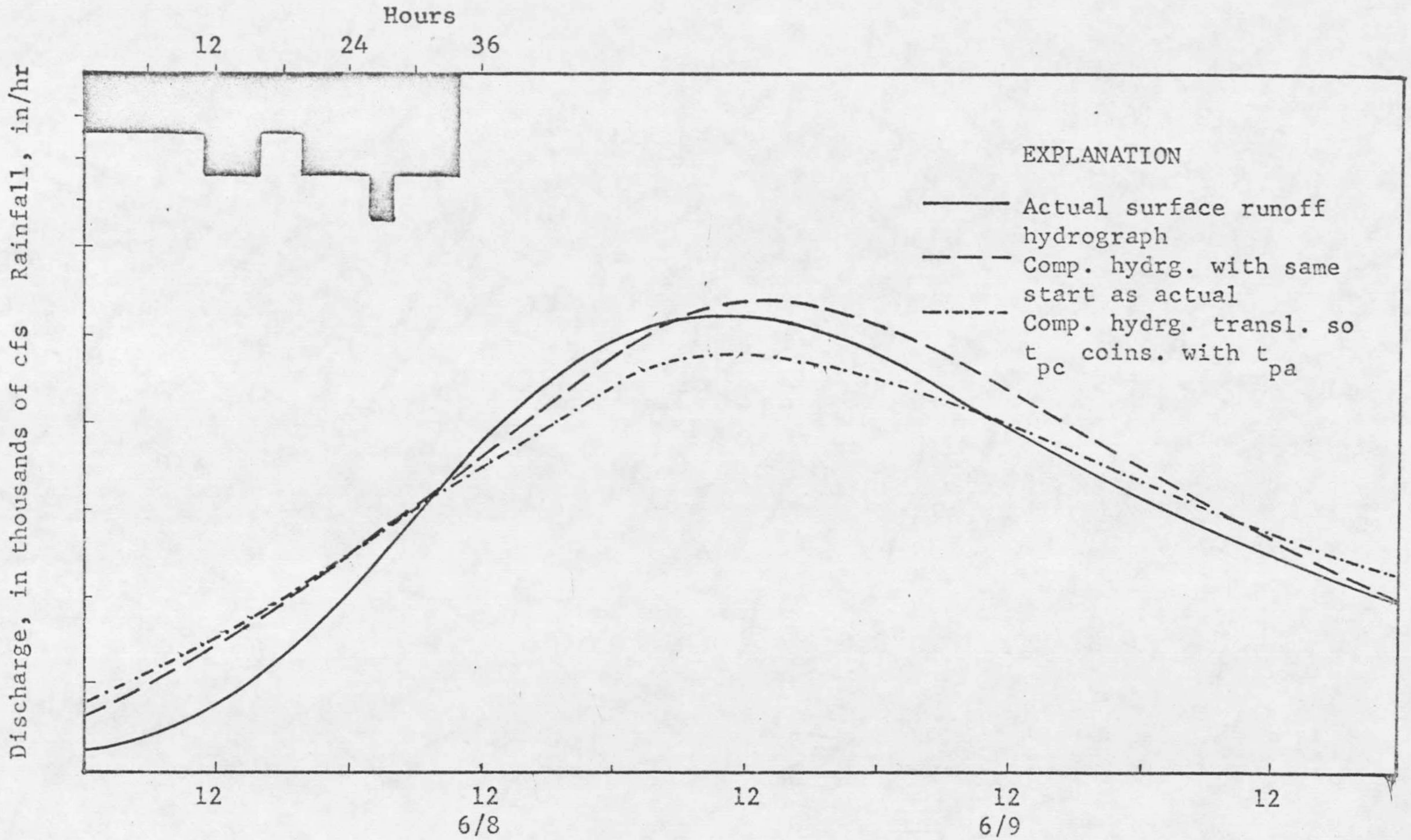

FIGURE 16a. - Hydrographs for Waterton River near Waterton Park, Alberta

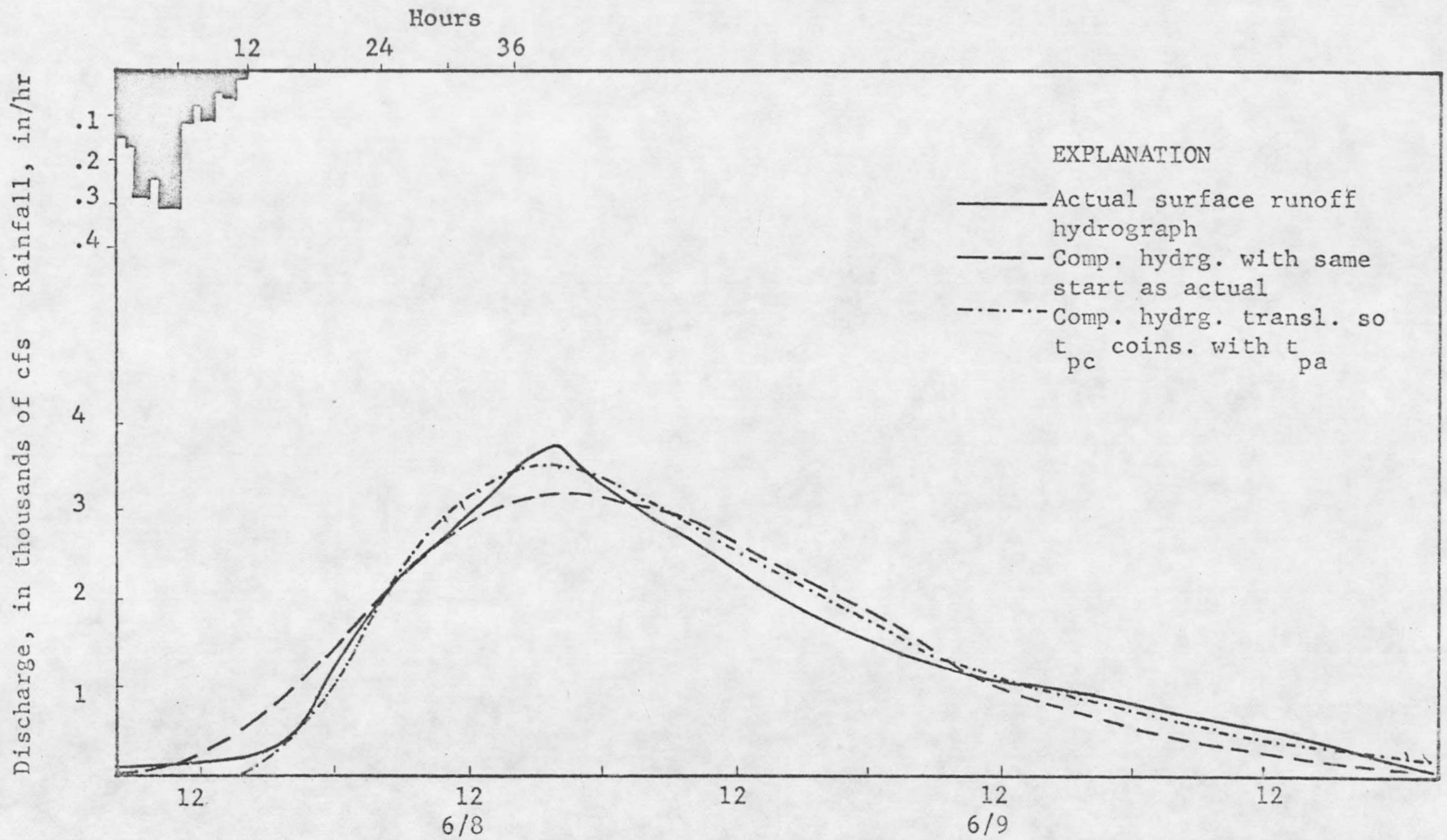

FIGURE 16b. - Hydrographs for Sullivan Creek near Hungry Horse

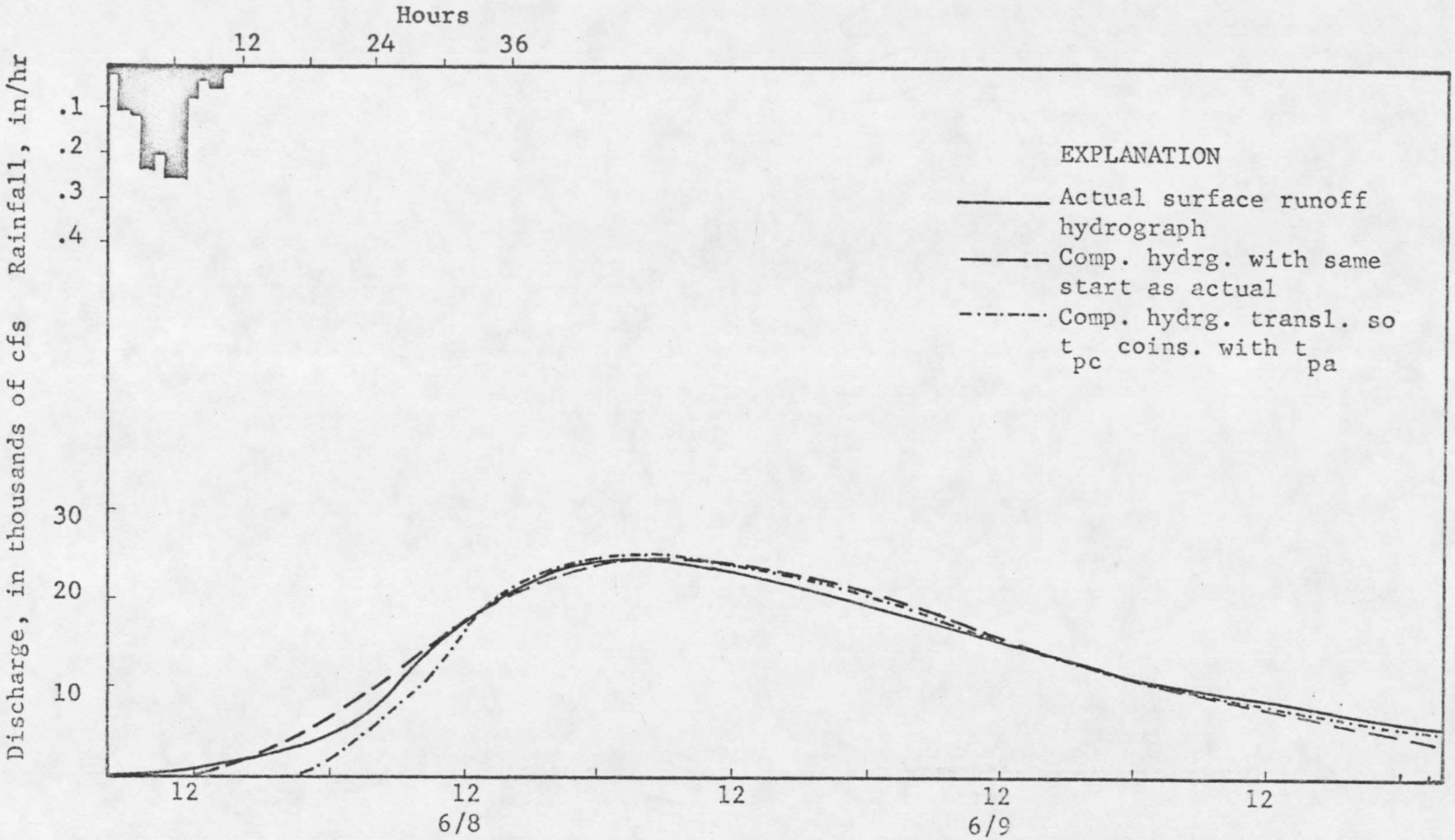

FIGURE 16c. - Hydrographs for South Fork Flathead River near Hungry Horse

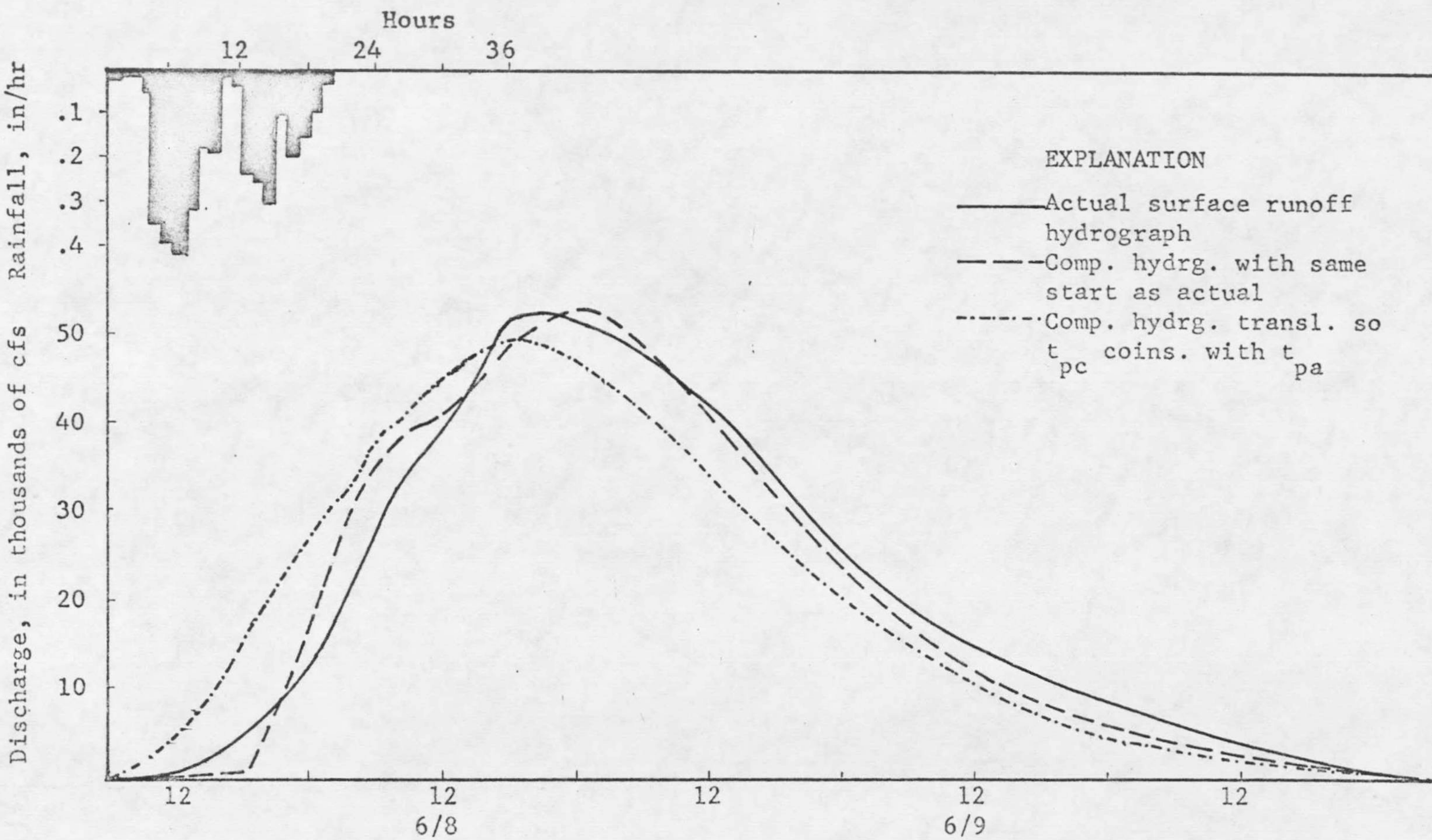

FIGURE 16d. - Hydrographs for Sun River at Gibson Reservoir near Augusta

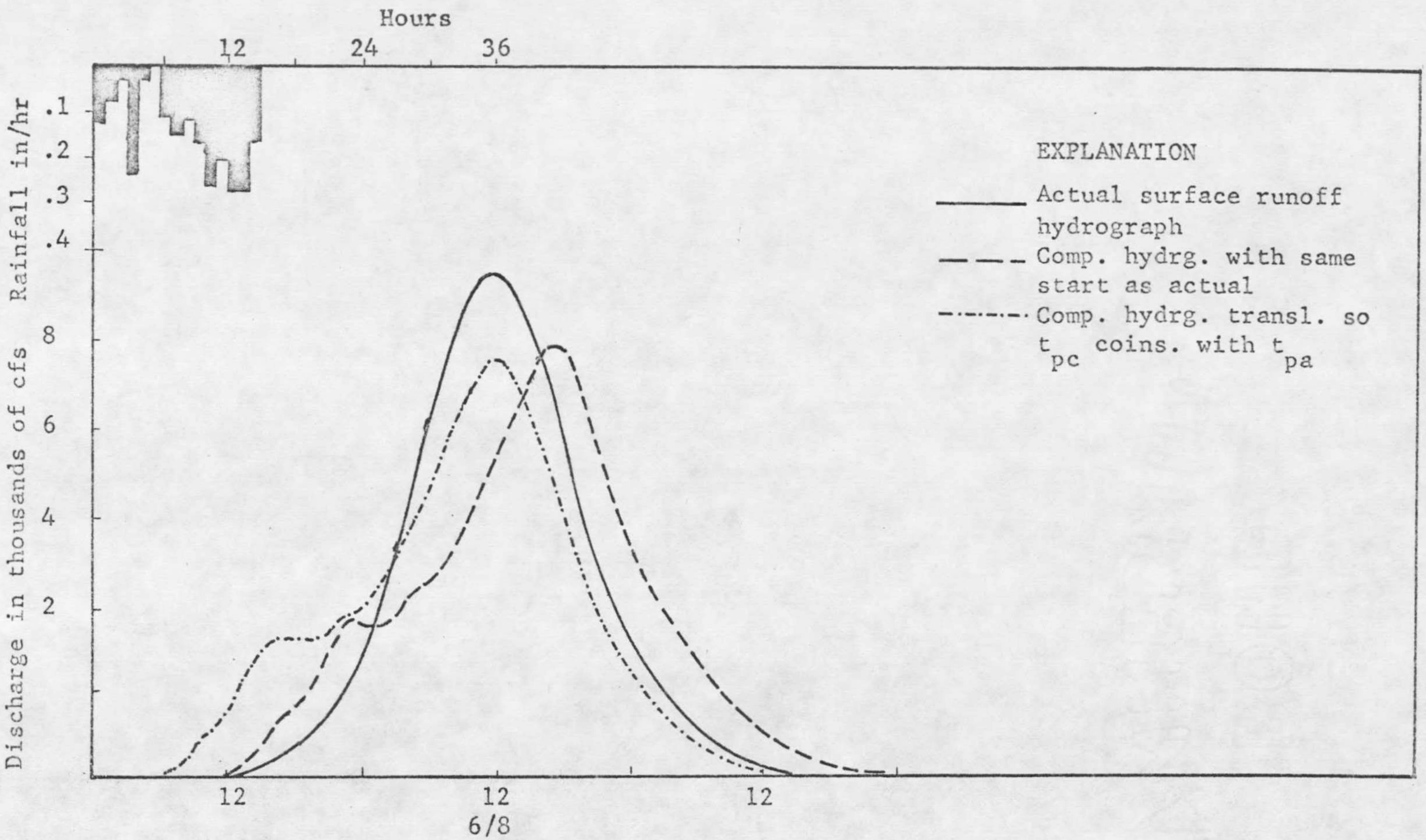

FIGURE 17a. - Hydrograph for South Fork Milk River near Babb

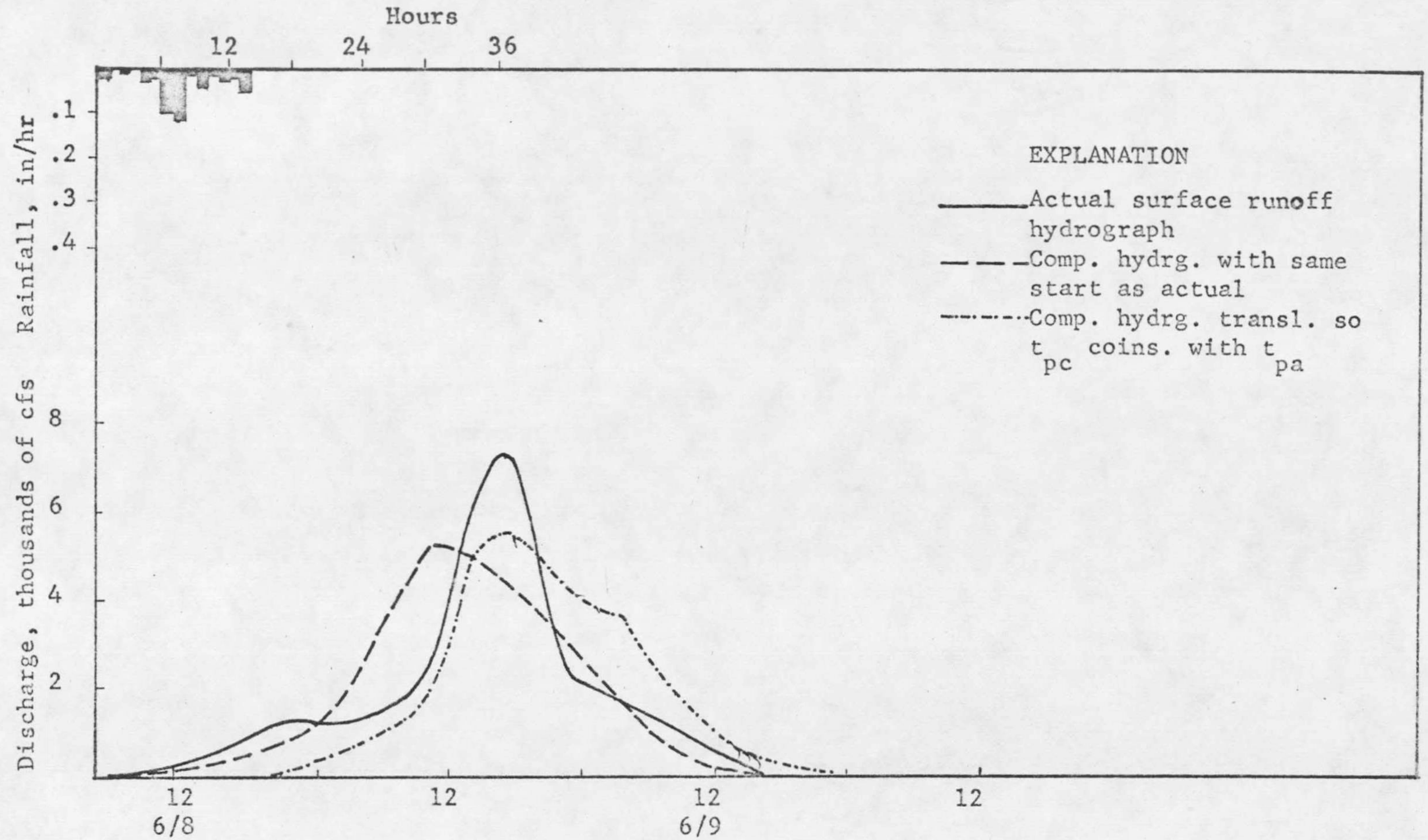

FIGURE 17b. - Hydrograph for South Fork Milk River at Del Bonita

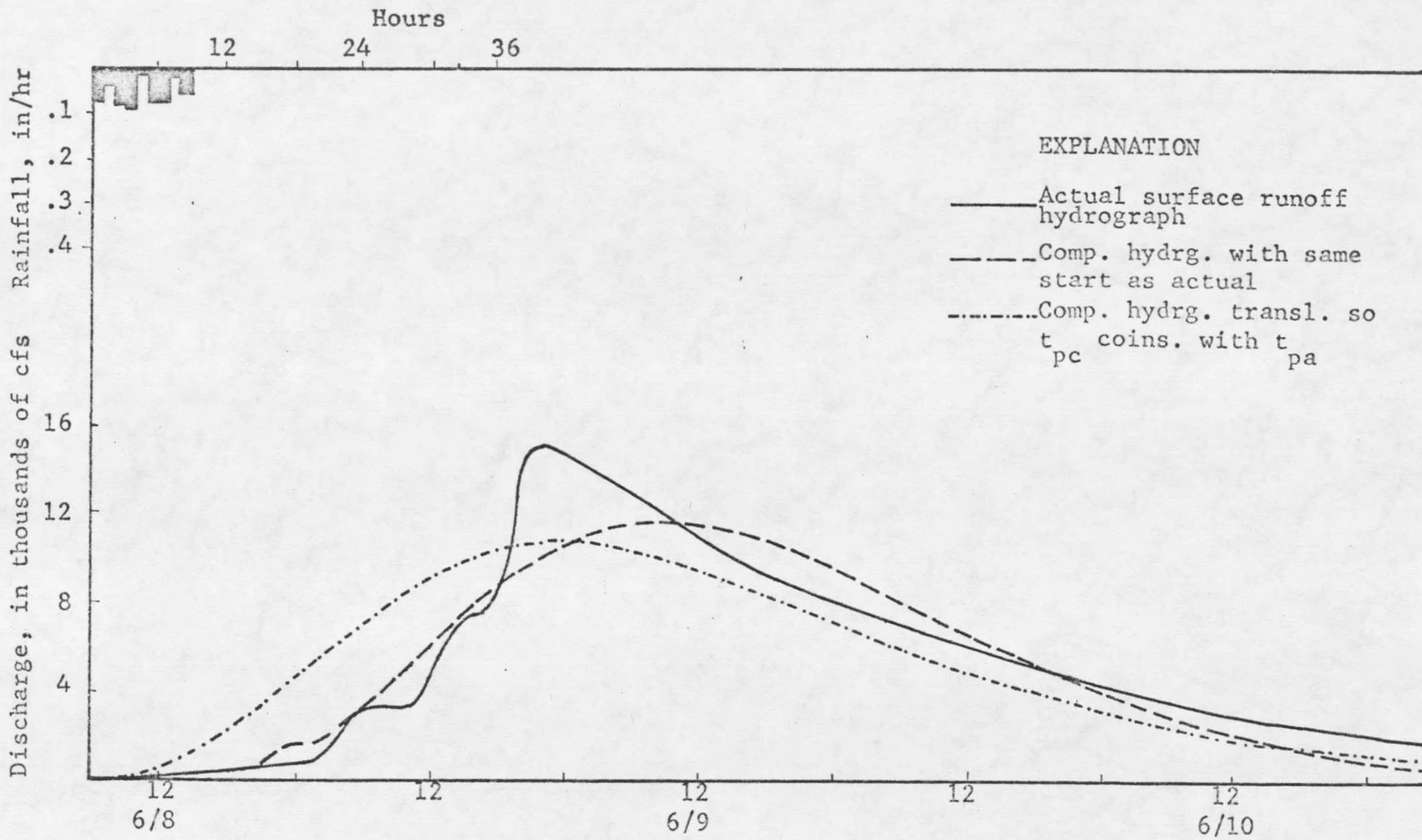

FIGURE 17c. - Hydrographs for Cut Bank Creek at Cut Bank

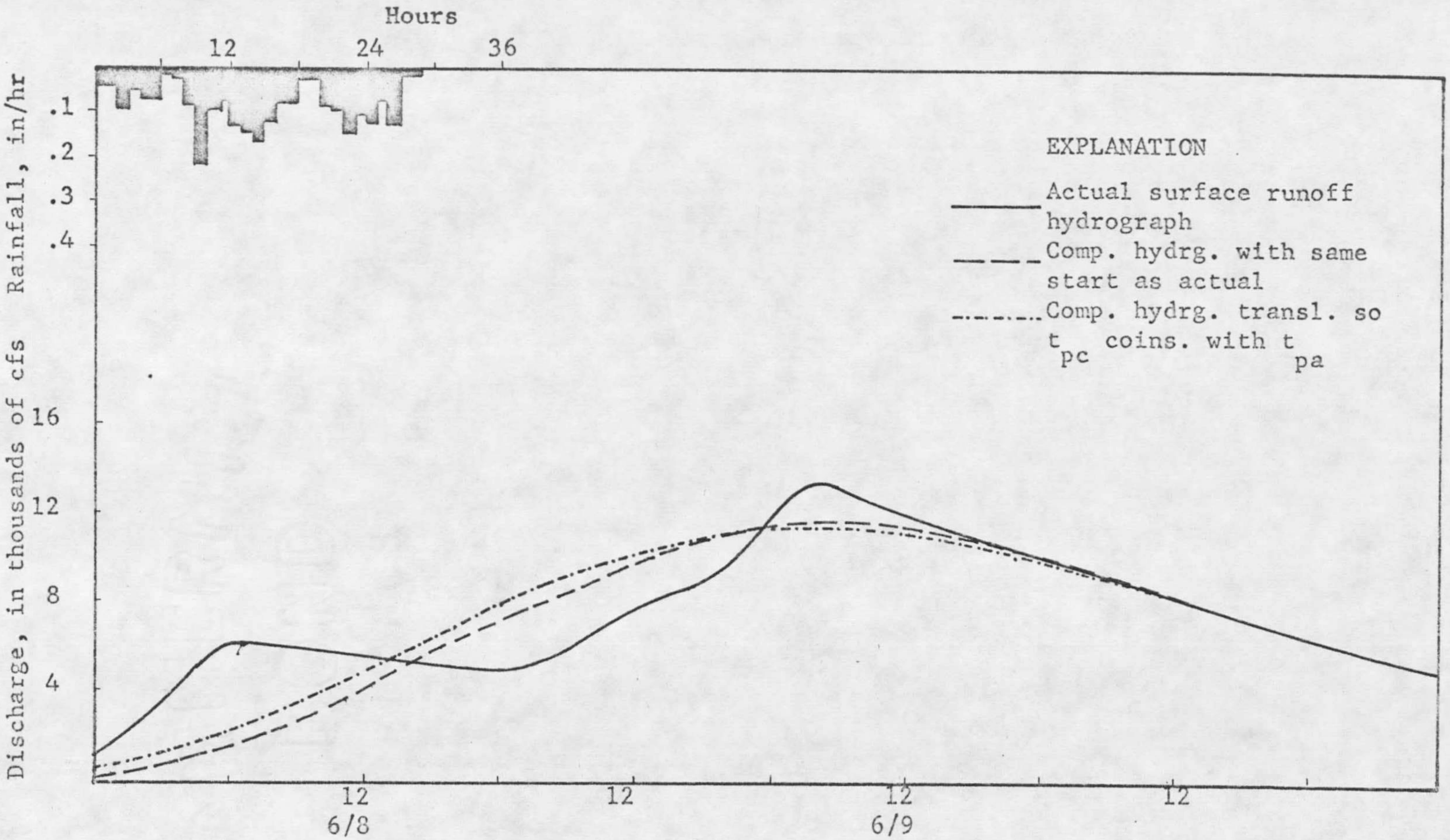

FIGURE 17d. - Hydrographs for Dearborn River near Craig

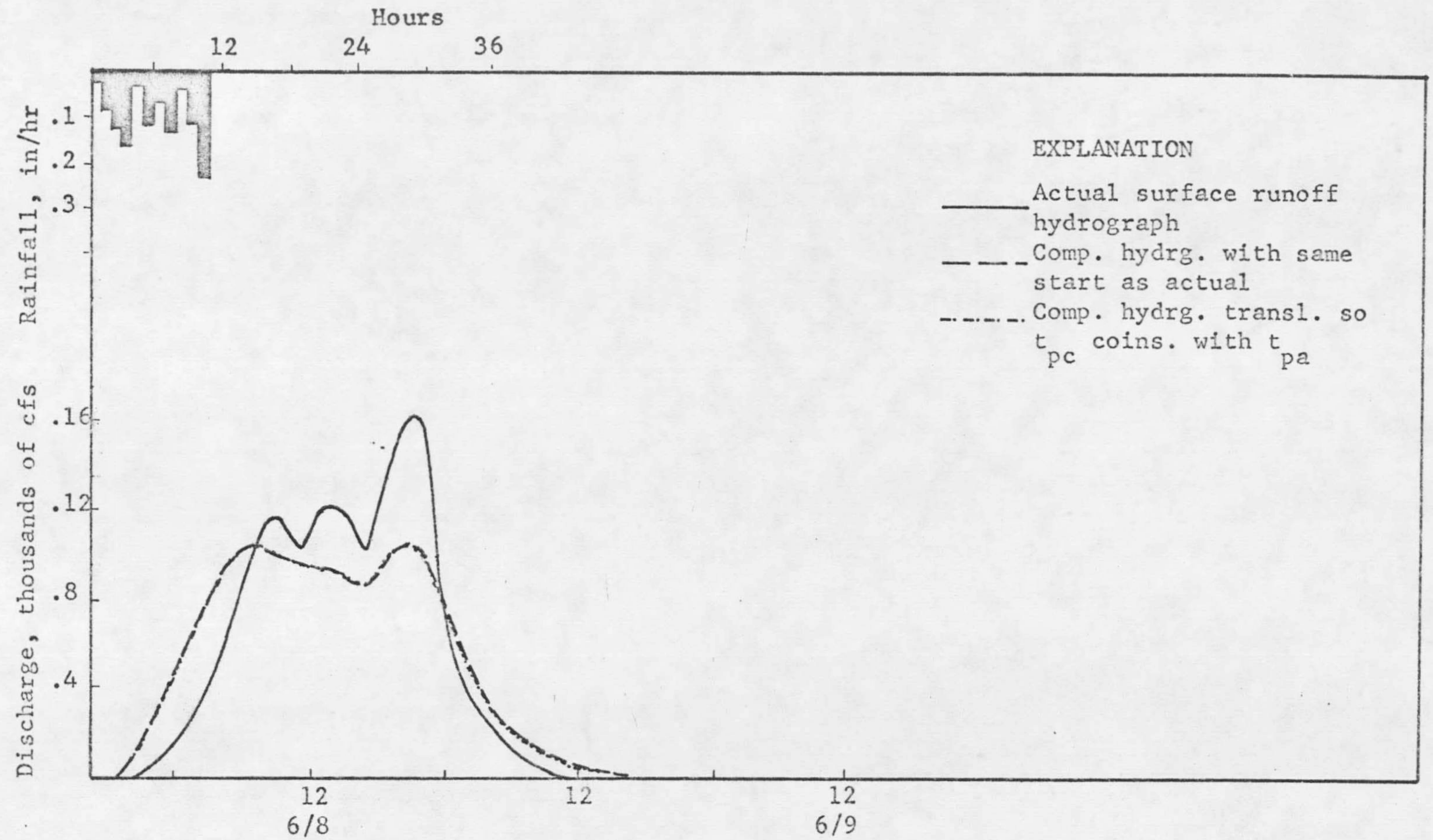

FIGURE 17e. - Hydrographs for Lone Man Coulee near Valier

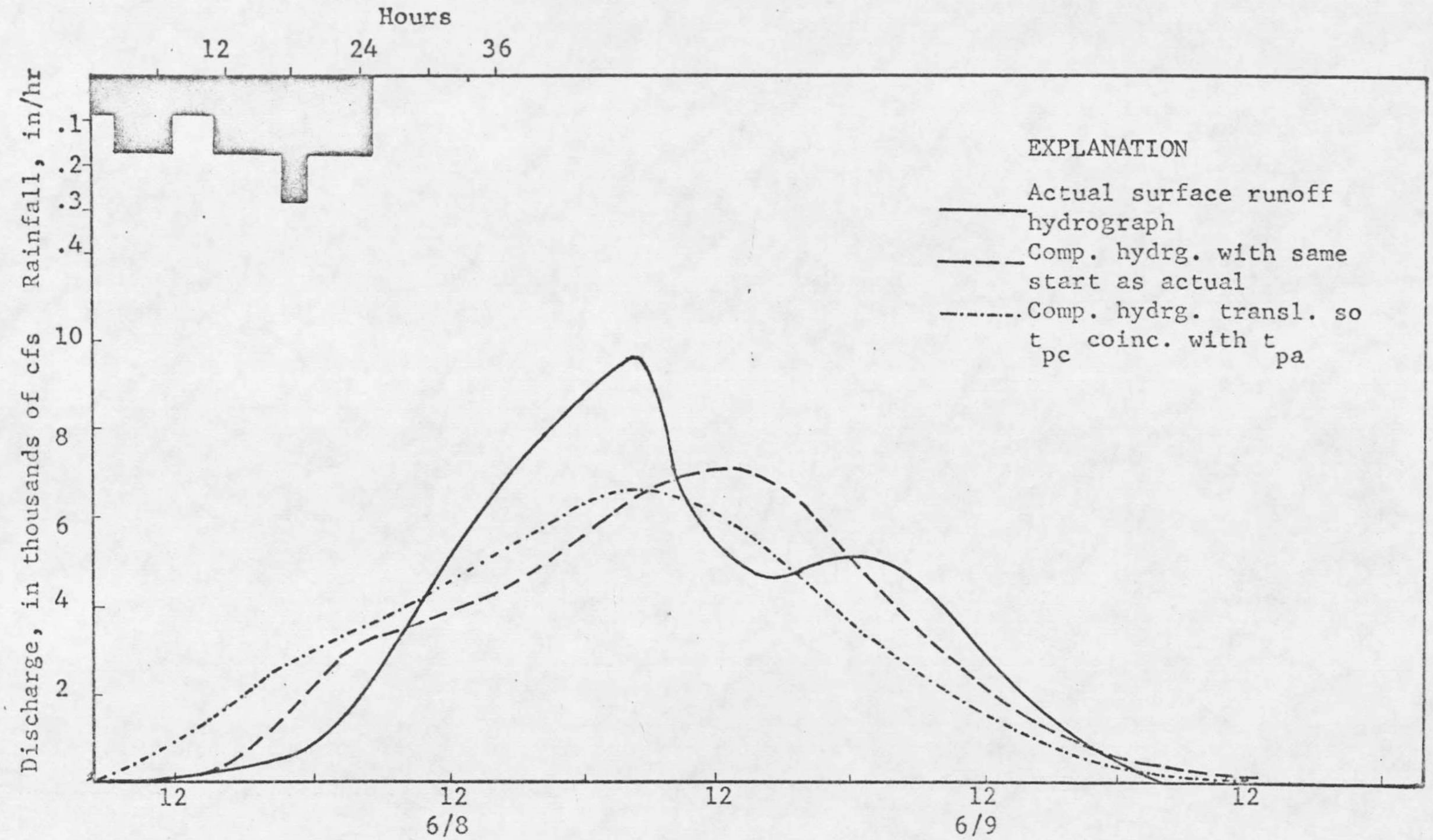

FIGURE 18a. - Hydrographs for Waterton River near International Boundary

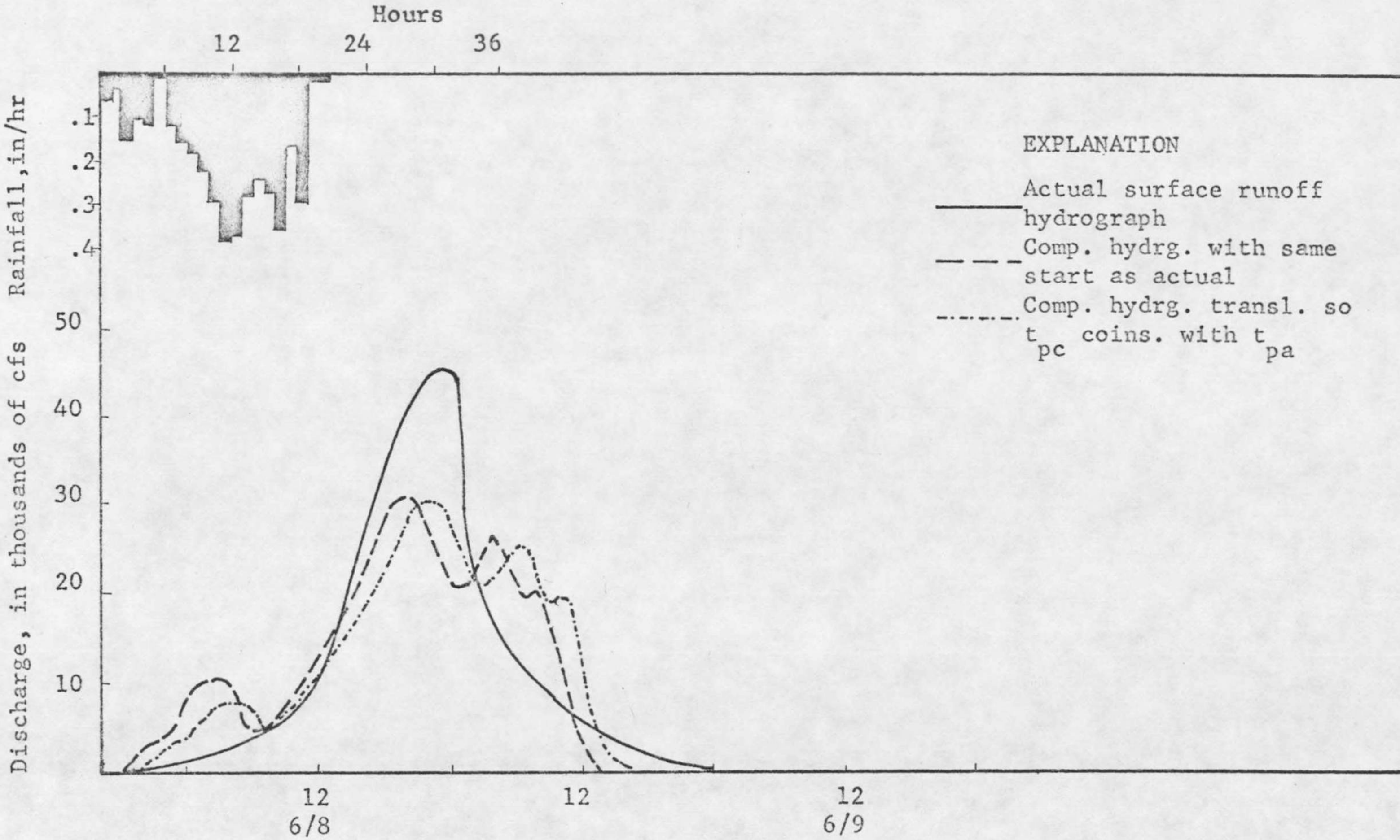

FIGURE 18b. - Hydrographs for Badger Creek near Browning

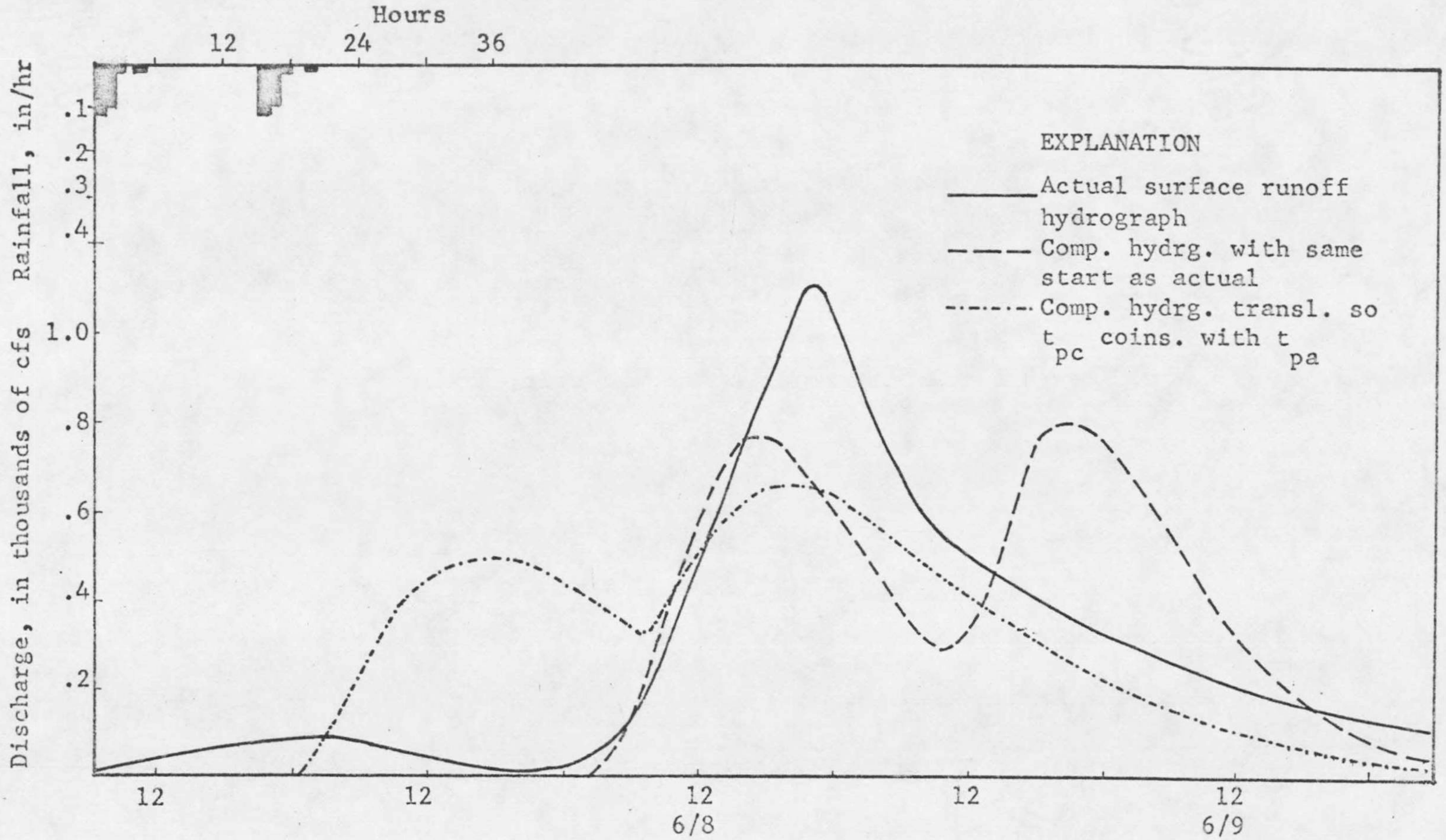

FIGURE 18c. - Hydrographs for South Fork Judith River near Utica

TABLE 5. - Time to peak and peak discharge for actual and computed hydrographs

Area no.	Name of drainage area	Actual graph		Computed graphs				$\frac{(t_{pa} - t_{pc})100}{t_{pa}}$		$\frac{(Q_{pa} - Q_{pc})100}{Q_{pa}}$	
				S.S.		T.S.					
		t_{pa} hr.	Q_{pa} cfs	t_{pc} hr.	Q_{pc} cfs	t_{pc} hr.	Q_{pc} cfs	S.S. %	T.S. %	S.S. %	T.S. %
Group 1											
2.	Waterton Riv. N Water. P.	37	21,000	37	24,150	33	18,900	0.0	0.0	-15.0	10.0
3.	Sullivan Cr. n. Hungry Hor.	22	3,700	23	3,200	22	3,400	-4.5	0.0	13.5	8.1
4.	S.F. Flathead Riv.n.H.Hor.	23	24,000	25	23,850	23	24,450	-8.7	0.0	0.6	-1.9
9.	Sun Riv. at Gibson Reserv.	16	53,000	19	53,100	16	49,400	-18.7	0.0	-0.2	6.8
Group 2											
5.	S.F. Milk Riv. n. Babb	13	11,400	15	9,500	13	9,400	-15.4	0.0	17.0	17.8
6.	S.F.Milk Riv. at D.Bonita	16	14,500	14	10,250	16	11,000	12.5	0.0	29.3	23.1
7.	Cut Bank Cr. at Cut Bank	21	15,200	26	15,550	21	10,650	-23.3	0.0	2.2	30.0
10.	Dearborn Riv. n. Craig	35	13,400	35	11,750	35	11,450	0.0	0.0	12.3	14.6
11.	Lone Man Coulee	14	1,660	14	1,037	14	1,037	0.0	0.0	37.4	37.4
Group 3											
1.	Watert. Riv.n.Int. Bound.	21	9,750	25	7,150	21	6,660	-23.8	0.0	32.0	33.3
8.	Badger Cr. n. Browning	14	46,800	13	31,700	14	30,700	7.7	0.0	32.2	34.4
12.	S.F.Judith Riv.n. Utica	10	1,105	23	800	10	635	-130.0	0.0	38.2	48.9

Area no. in table 6 corresponds to area number on Fig. 11, see page 32.

S.S. refers to same start as actual, and T.S. refers to translated start on the time-axis so $t_{pa} = t_{pc}$

side, see Fig. 11.

There is a decreasing trend with respect to the sizes of the drainage areas in the different groups such that Group 1 has bigger average size than Group 2, see Table 6, which has bigger average size than Group 3.

TABLE 6. - Average size of the drainage areas and average storage coefficients for the recession curves for the different groups

Group	Drainage area, average size, sq. mi.	Storage coefficients for recession curve, average	
		Comp. and act. start same time k_1	Comp. transl. on time-axis k_1
1	461	22.2	23.2
2	359	11.7	12.9
3	84	5.8	8.0

The average recession storage coefficients, k_1 see Eq. (26), page 23, for the different groups are given in Table 6. It shows a decreasing trend of k_1 from Group 1 through 3. A large k_1 gives a smooth hydrograph because the drainage area does not respond quickly to rainfall. Hence the actual excess rainfall pattern can vary considerably from the applied, and still the computed and the actual hydrographs do not have to differ significantly from each other. As k_1 decreases the applied rainfall pattern must correspond more closely to the actual if the computed graph shall reproduce the actual hydrograph fairly well.

Table 7 gives a listing of the raingages from which the rainfall data

TABLE 7. - Location of raingages with respect to watersheds

Area no.	Drainage area	Recording raingages used		Location of raingages with respect to the watershed	
		No.	Name	Distance from boundary, miles	Distance from centroid of area, miles
	Group 1				
2.	Waterton River near Wt. Park	h	Grinnel	10	Not available
3.	Sullivan Creek n. Hungry Horse	1	Swan Lake	4	6
4.	S.F. Flathead River n. H.H.	1	Swan Lake	8	24
9.	Sun River at Gibson Reservoir	d	Gibson Reserv.	0	9
	Group 2				
5.	S.F. Milk River n. Babb	a	Browning	12	15
		h	Grinnel	14	20
6.	S.F. Milk River at Del Bonita	b	Cut Bank	27	37
		h	Grinnel	14	29
7.	Cut Bank Creek at Cut Bank	a	Browning	-6	14
		b	Cut Bank	0	23
		g	Summit	16	41
		h	Grinnel	13	41
10.	Dearborn Riv. n. Craig	d	Gibson Reserv.	18	31
		e	Holter Dam	14	25
11.	Lone Man Coulee n. Valier	i	Geiger	-0	3
		j	Toren	-1	2
	Group 3				
1.	Waterton River n. Int. Boundary	h	Grinnel	10	16
8.	Badger Creek n. Browning	c	Dupuyer	19	27
		g	Summit	11	18
12.	S.F. Judith River n. Utica	f	Kings Hill	11	15

is obtained for the different watersheds. The area no. for watersheds and the no. for raingages are as found on Fig. 11, page 32. The distances to the boundary and to the centroid of area from the different raingages are also given. A minus sign indicates the raingage is inside the boundaries of the given watershed. No general trend can be ascertained from this table. The drainage areas in Group 1 cannot be said to have significantly better coverage of raingages than the drainage areas in Groups 2 and 3.

Discussion of the Individual Hydrographs

Several factors will affect the hydrographs; seven of them are listed below.

1. Actual rainfall pattern different from what is applied.

2. Initial loss can vary considerably from what is assumed, due to depressions in the ground that can be fairly large and hence cause the actual runoff to start much later than estimated. This will also give a different rainfall pattern from what is used.

3. Debris could have caused more blocking of stream channel than accounted for, thereby producing incorrect discharge estimates.

4. Variable amount of snowmelt.

5. In some cases retardation of the flow by the snow pack could have caused a delay in the runoff at the start of rise or throughout the hydrograph.

6. Tributaries could sometimes have affected the general shape of the hydrograph.

7. The assumption of a constant infiltration rate may cause a larger error than estimated.

Group 1. - The main reason for the good agreement in this Group is apparently the great storage capacity of these drainage basins, see Table 7. Sullivan Creek has the smallest k_1 in this group, see Table 3, page 43.

The rainfall at Swan Lake apparently was similar to the rainfall on Sullivan Creek watershed. A large storage coefficient will cause that variation in excess rainfall with time will not be reflected as smaller disturbances on the runoff hydrograph. Factors 2., 5. and 7. can probably explain the small irregularities that exist at the start of rise of the hydrographs. Figs. D1 - D3, Appendix D pages 85-87 show maps of the drainage areas in the Group together with streamgages and raingages. The map for Waterton River near Waterton Park is not available.

Group 2. - South Fork Milk River near Babb does not have good coverage of raingages, factor 1. The small peak at the beginning of the computed hydrographs could probably have been avoided if variable infiltration had been used. Factor 2. could also have helped to smooth out the actual hydrograph at the start.

South Fork Milk River at Del Bonita suffers from the long distances to the raingages, see Table 7. But the sharp peak of the actual hydrograph about midway between start and stop of surface runoff, could indicate blocking of the stream for a certain period of time, factor 3. The sharp peak could also be caused by large differences in rainfall intensity at the upper and lower parts of the watershed, the sharp peak at Del Bonita comes about 3 hours later than the peak near Babb. See Figs. 17a and 17b.

Tributaries, factor 6. seem to have had some effect on the actual runoff hydrograph of Cut Bank Creek. This shows up as irregularities on the rising limb of the actual hydrograph. This is reasonable when looking at the map of the watershed, Fig. D6, Appendix D, page 90. It shows 2 relatively large contributaries close to the outlet. Because of the size of the watershed, over 1000 sq. mi., factor 1. is also likely to have had some effect.

The same factors mentioned for Cut Bank Creek could also have affected Dearborn River. If it had not been for the good fit of the recession curves, this watershed could not have been listed in Group 2. Figs. D4 to D8 shows maps of the drainage basins for this Group. They are found in Appendix D, pages 88-92.

Lone Man Coulee has good coverage of raingages, however, the control, a culvert, was washed out so the biggest peak is computed using the slope-area method. This can sometimes be quite inaccurate and is probably part of the reason why the peak of the actual graph is that much bigger than the peaks of the computed. Part of the reason might be that the excess rainfall distribution applied does not correspond to the actual.

Group 3. - Waterton River is only covered by the rainfall data from Grinnel which is quite a distance away, see Table 7, page 65, and as previously mentioned also has a more uniform trend in the rainfall pattern than the other raingages in the area, hence the most important reason for not getting better fit for the computed graphs are found in factor 1. from the listing above.

Badger Creek's computed hydrographs have without doubt been affected by factor 1. The very steep recession curve of the actual graph also indicates that there could have been some blocking of the stream, factor 3. Factor 2 could probably have eliminated the first little peak on the actual graph that shows up on the computed graphs.

The excess rain applied to South Fork Judith River is taken from Kings Hill which is situated about 15 miles from the centroid of the area. The data are clearly not representative. The actual hydrograph is smooth, so it is not likely that factor 3. has helped to eliminate the double peak the applied rainfall data gives. Factor 2. and 7. could cause some of the discrepancy. If much of the excess rain used for computing the first peak was lost as infiltration and depression storage and this had been taken into account when separating excess rain, the actual graph could probably have been reproduced more closely by the computed graphs. But again, to determine these variables are beyond the scope of this study. Figs. D9 to D11, Appendix D, pages 93-95 show maps of the drainage areas for this Group.

A factor listed above, but not mentioned under the discussion of the drainage basins is 4. variable snowmelt. It is assumed that snowmelt was fairly constant throughout the flood period and has been included in base-flow and interflow when they were separated from surface runoff. It is recognized that this estimation may be wrong so that snowmelt in some cases seriously alters the surface runoff hydrograph, however, there is probably no way of determining the extent of such effects.

Concerning factor 5., Leber (24) states that the net effect of water draining through the snowpack is a time delay of runoff of 3 to 4 hours for moderately deep packs. For areas that had a significant amount of snowcover this factor could help to explain why many of the actual hydrographs start rising very slowly. The differences on this point between computed and actual hydrographs can also partly be explained by the behavior of the applied unit hydrograph. It has a fairly steep slope from the beginning. The hydrographs from Givler's model show the same pattern Fig. 14, page 46.

The Comparisons with Synthetic Methods

Table 3, see page 43, shows fairly big differences for the derived unit hydrographs, and for the unit hydrographs computed after Snyder's, SCS's and Gray's methods for both time to peak, t_p, and discharge at peak, Q_p, which could be expected since none of the formulas have been derived using data from this particular region. The general pattern shows considerably less time to peak for the synthetic methods and a much greater value for discharge at peak. Of the three methods Snyder's seems to give the best agreement for these drainage basins. It appears that Snyder's formulas might be used with a certain degree of confidence in this region. Time to peak for Snyder is computed after Eq. (14a), see page 18, and Appendix B.

Comment on Gray's Derivations

Gray grouped the watersheds he studied into different regions, see page 20, with different values for x and m, see Appendix A, in each region. In this study Gray's formulas were first tried with x = 9.27 and m = 0.562 which were the values Gray found for Central-Iowa-Missouri-Illinois-Wisconsin, which gave negative values for time to peak for some of the watersheds. In order to get positive values for t_p for all of the watersheds the values of x and m found by Gray for Nebraska-Western Iowa was used (x = 7.40, m = 0.498). Eq. 24 has the form

$$t_p/\gamma' = x(L_M/\sqrt{S_M})^m$$

but since

$$S_M = h'/L_M$$

Eq. (24) can be written

$$t_p/\gamma' = x\left(\frac{L_M^{3/2}}{\sqrt{h'}}\right)^m$$

See Appendix A for all the variables and coefficients.

Since S_M is derived by using L_M, L_M and S_M can hardly be said to be independent of each other. It is therefore not surprising that the formulas can give negative values for time to peak, when used outside the region or for other watersheds than from which they were derived.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Unit hydrographs are derived for 12 drainage areas in northwestern and northcentral Montana. The data were selected from U.S.G.S. Water-Supply Paper 1840-B (23), which is a report on the big flood of June 7-8, 1964 in the above mentioned region. The empirical derivation of the unit hydrograph is done by using a trial and error procedure to determine the best fit between a hydrograph computed from an assumed unit hydrograph and the corresponding actual surface runoff hydrograph. For 4 of the watersheds, the best computed hydrograph shows excellent fit with the actual, 5 watersheds show good fit and 3 fit poorly.

Two different fitting methods were used, one with start of the computed hydrograph at the same time as the start of the actual surface runoff hydrograph, and one that moved the computed hydrograph so that the peaks occurred at the same time. The results for the two methods are not significantly different.

Time to peak and discharge at peak were computed using 3 different synthetic methods; Snyder's, SCS's and Gray's. None of the methods compared well with the same values for the empirically derived unit hydrographs. Snyder's formulas produced less errors than the others.

The program was checked on a hydrograph produced by Prof. Givler's model. The result of the comparison was good, and the unit hydrograph was assumed to be capable of reproducing such a hydrograph closely, given the excess rainfall distribution.

Four unit hydrographs were derived from 4 different rainstorms for

Lone Man Coulee. They had quite different form, the more intense rain is believed to give the more reliable results for prediction of floods from heavy rainfall.

The values for t_p and n listed in Table 3 for Group 1, are to be used in Eqs. (10) and (11). This gives the unit hydrographs for Group 1. These unit hydrographs can be assumed to be reliable for intense rain, especially when the antecedent conditions are similar to those of early June 1964.

The values for t_p and n listed for watersheds in Group 2 should be used with care. Some adjustments are probably necessary. The results obtained for Group 3 are not recommended for use. Further studies are needed for these watersheds.

If it should become possible sometime in the future, a study should be taken to find out what kind of initial losses, depression storage and infiltration rates these areas have in order to be able to distribute the excess rain properly with time. This would certainly help to get better results for t_p and n. It is also suggested that an approach similar to McSparran's (18), should be used if synthetic formulas were to be derived for this region.

APPENDIX

APPENDIX A

LIST OF SYMBOLS

A - area

Δ - fractional part of an area

B - a variable used by Edson

C - a coefficient used by Edson

C_p - a coefficient used by Snyder

C_t - a coefficient used by Snyder

D_d - drainage density; ratio of total length of all streams in drainage basin to area. Streams measured as all solid or dotted blue lines found on a topographic map.

e - base number of natural logaritms

F - watershed shape factor, defined as the ratio of the length of main stream from outlet to divide to the diameter of a circle with an area equal to the drainage area.

I - rainfall rate

k - storage coefficient for the drainage basin

k_1 - recession storage coefficient for the drainage basin

L_{ca} - length of main stream from outlet to point nearest the centroid of the watershed.

L_D - length of main stream from outlet to divide

L_M - length of main stream as found on topographic map as solid or dotted blue line.

m - coefficient used by Gray

n - number of linear reservoir as defined by Nash

P_R - period of rise of unit hydrograph

Q - discharge rate

q - parameter used by Gray

R - runoff in inches

S_M - average geometric slope of L_M

S_D - average geometric slope to divide of L_D

t - length of time elapsed after start of surface runoff

t_0 - length of total excess rain

t' - length of rainfall of influence shorter than length of total rain

t_p - time to peak of unit hydrograph

t_R - length of unit rain

u - ordinate of unit hydrograph

V - volume of runoff

v - volume of unit hydrograph

W_a - wooded area of watershed in per cent

w - parameter used by Edson

x - coefficient used by Gray

y - inverse storage coefficient used by Edson

z - coefficient used by Edson equal to yt

T_c - time of concentration, travel time of water from most remote point on divide to outlet.

Γ - gamma function; for whole numbers: $\Gamma(n) = (n-1)!$

γ' - parameter used by Gray

τ - time elapsed after start of excess rainfall

Fig. A1 shows how the different watershed characteristics were obtained.

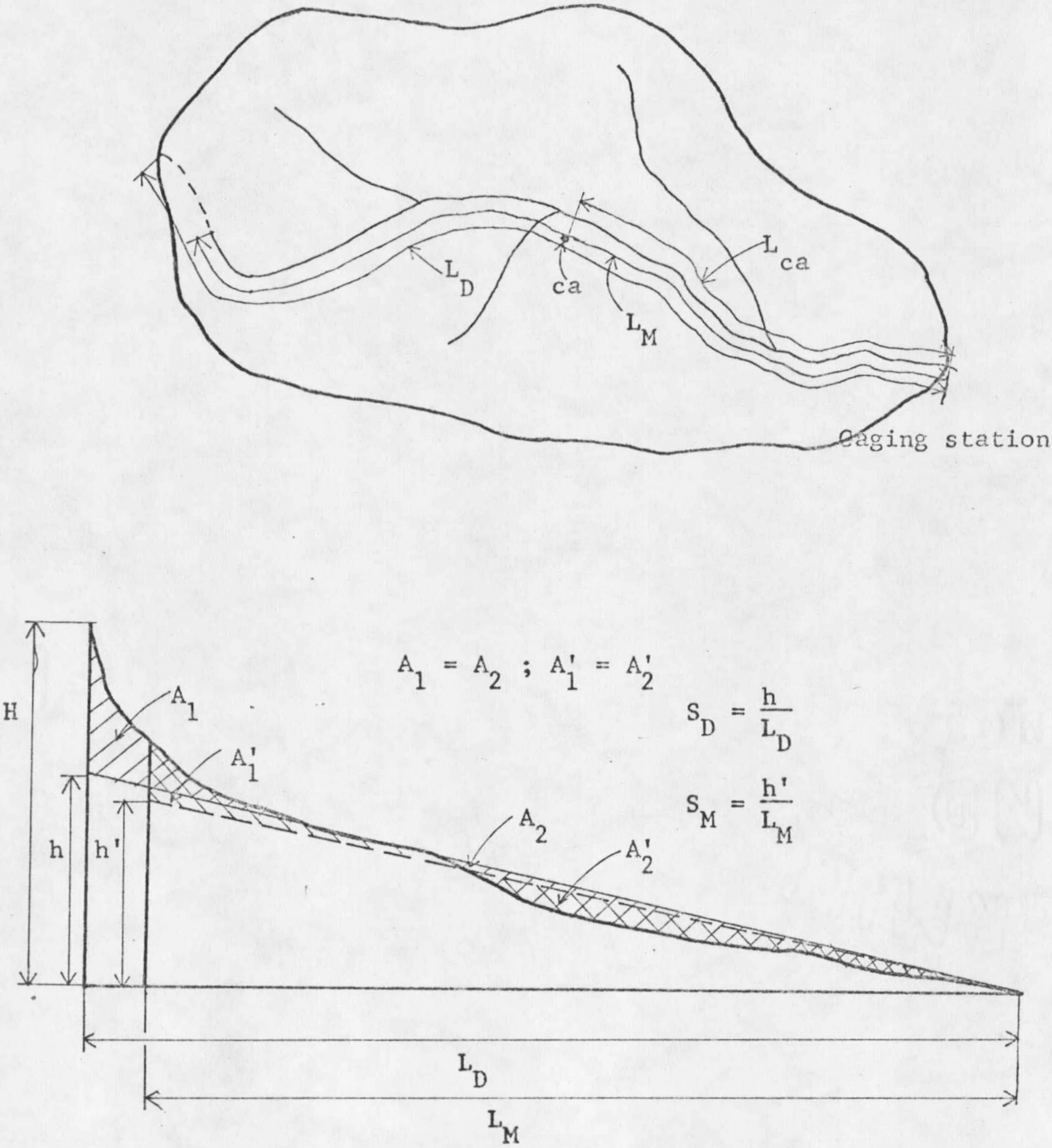

FIGURE A1. - How the different watershed characteristics used in this study have been obtained.

APPENDIX B

Sample calculations in order to find time to peak and discharge at peak for the three synthetic methods used.

Given

Elevation difference between outlet and divide	$H = 370$
Area	$A = 15$ sq. mi.
Length of main stream to divide	$L_D = 10$ mi.
Length of main stream	$L_M = 9$ mi.
Length of main stream to center of area	$L = 4.5$ mi.
Slope to divide	$S_D = 0.49$
Slope of main stream	$S_M = 0.42$
Snyder coefficient modified by Linsley	$C_t = 1.72$
Snyder coefficient	$C_p = 0.625$
Gray coefficient	$x = 7.40$
Gray coefficient	$m = 0.498$
Length of rainfall	$t_R = 1.0$ hour
Volume of rainfall	$v = 1.0$ inch

a) Snyder

$$t_p = t_R/2 + C_t\left(\frac{L_D L_{ca}}{\sqrt{S_D}}\right)^{0.32} \qquad \text{Eq. (14a)}$$

$$t_p = 0.5 + 1.72\left(\frac{10x4.5}{\sqrt{0.49}}\right)^{0.32} \qquad = 7.0 \text{ hours}$$

$$Q_p = 640A\,\frac{C_p}{t_p} \qquad \text{Eq. (15)}$$

$$Q_p = 640 \times 15x\,\frac{0.625}{7.0} \qquad = 858 \text{ cfs}$$

b) Soil Conservation Service

$$T_C = \left(11.9\frac{L_D^3}{H}\right)^{0.385} \qquad \text{Eq. (16)}$$

$$T_C = \left(11.9\,\frac{10^3}{370}\right)^{0.385} \qquad = 3.7 \text{ hours}$$

$$t_p = t_R/2 + 0.6\ T_C \qquad \text{Eq. (17)}$$

$$t_p = 0.5 + 0.6 \times 3.7 \qquad = 2.7 \text{ hours}$$

$$Q_p = 484Av/t_p \qquad \text{Eq. (18)}$$

$$Q_p = 484 \times 15 \times 1/2.7 \qquad = 2{,}690 \text{ cfs}$$

c) Gray

$$t_p/\gamma' = x(L_M/\sqrt{S_M})^m \qquad \text{Eq. (24)}$$

$$t_p/\gamma' = 7.40(9/\sqrt{0.42})^{0.498} \qquad = 27.5$$

$$t_p/\gamma' = 27.5 = \frac{1}{2.676/t_p + 0.0.139} \qquad \text{Eq. (25)}$$

This give

$$t_p = \frac{27.5 \times 2.676}{1 - 0.382} \qquad \fallingdotseq 118 \text{ min.}$$

$$q = 1 + \gamma' \qquad \text{Eq. (23)}$$

$$q = 1 + t_p/(t_p/\gamma') = \frac{118}{27.5} \qquad = 5.3$$

$$\gamma' = q - 1 = 5.3 - 1 \qquad = 4.3$$

$$Q(t_p/t_p) = \frac{25.0\ (\gamma')^q\ e^{-\gamma'}\ (t_p/t_p)^{(q-1)}}{\Gamma(5.3)} \qquad \text{Eq. (22)}$$

$$Q_{(1)} = \frac{25.0 \times 4.3^{5.3} \times e^{-4.3}}{\Gamma(5.3)} \qquad = 22.5\%$$

Since the dimensionless graph is defined as follows:

$$Q_{(t/t_p)} = (\%\ \text{Flow}/(0.25 t_p))$$

where flow is the volume under the dimensionless graph we get for Flow

$$\text{Flow} = \frac{ft^3}{sec}\ 0.25\ t_p$$

or

$$\text{Flow} = \frac{ft^3}{sec}\ 0.25 \times 118 \times 60 \qquad = 1{,}785\ \frac{ft^3}{sec} - sec$$

Volume of unit hydrograph

$$V = \frac{A\ (5.280)^2\ (1)}{12} = 34,700,000 \text{ cfs}$$

Hence

$$Q_p = \frac{ft^3}{sec} = \frac{34,700,000 \times 0,225}{1,785} = 4,380 \text{ cfs}$$

APPENDIX C

THE COMPUTER PROGRAM

A listing of the computer program is found in the Civil Engineering Department's copy of the thesis.

APPENDIX D

Maps of Drainage Areas

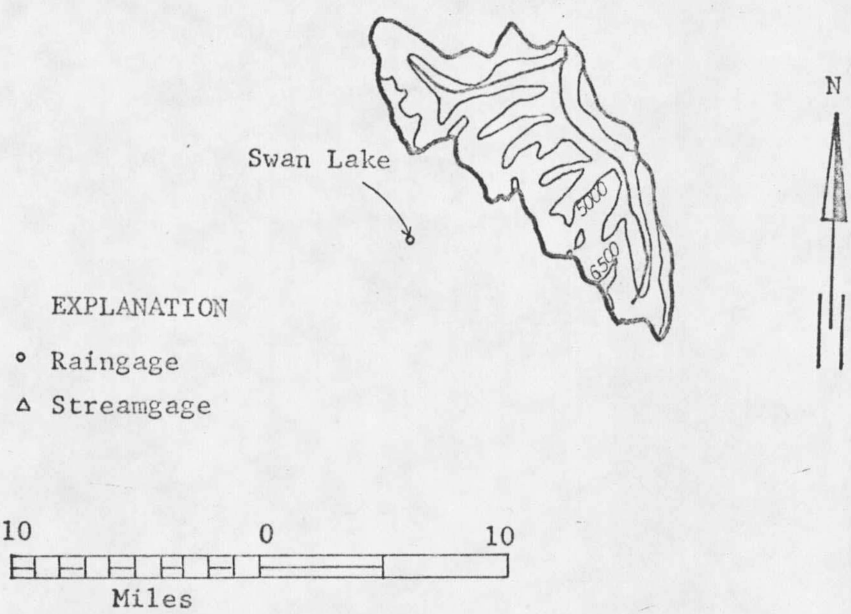

FIGURE D1. - Sullivan Creek's drainage basin near Hungry Horse, Montana. Location of gaging station; lat 48°01'45'', long 113°42'10''.

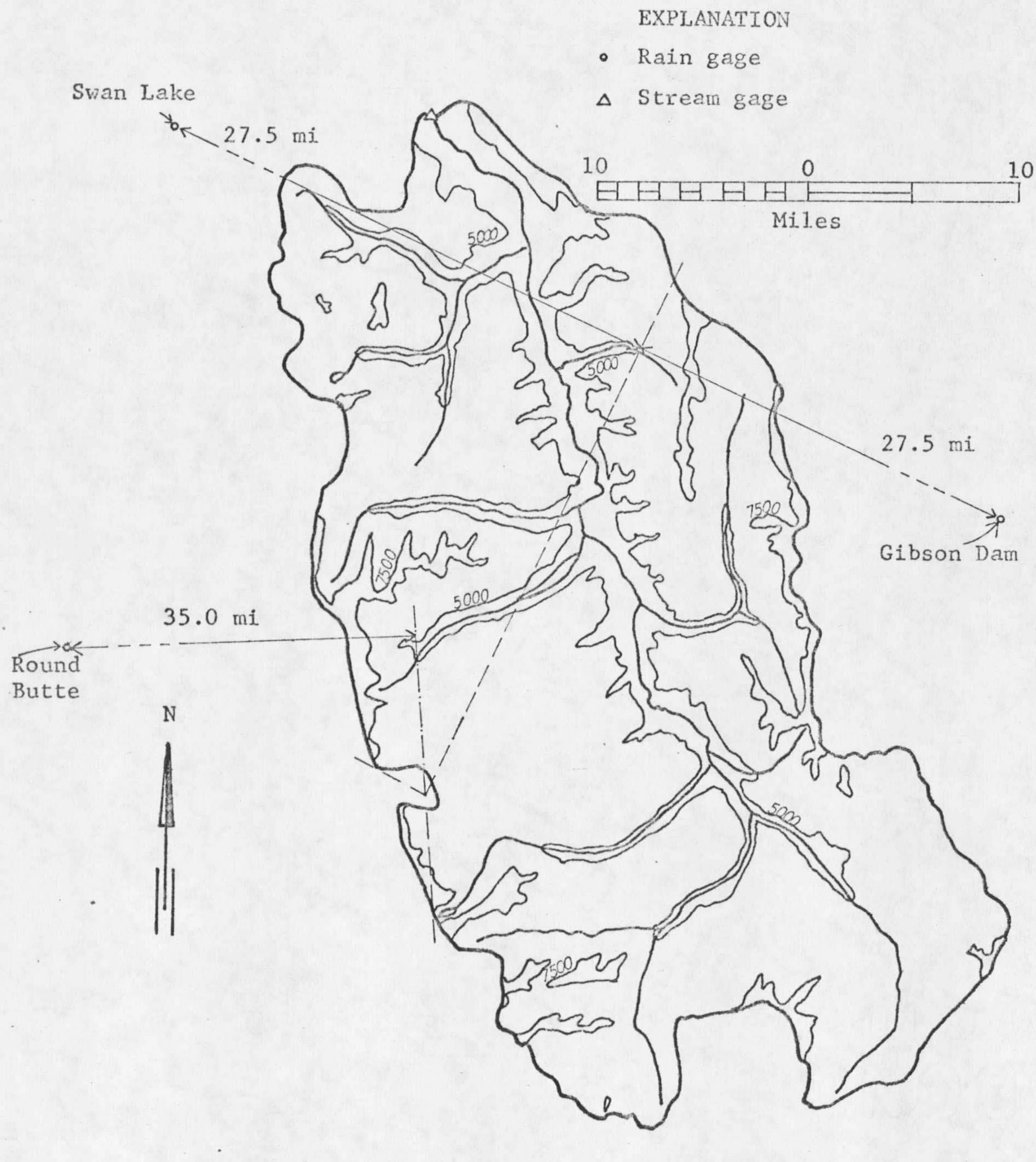

FIGURE D2. - South Fork Flathead's drainage basin at Spotted Bear Ranger Station near Hungry Horse, Montana. Location of gaging station; lat 47°55'20'', long 113°31'25''.

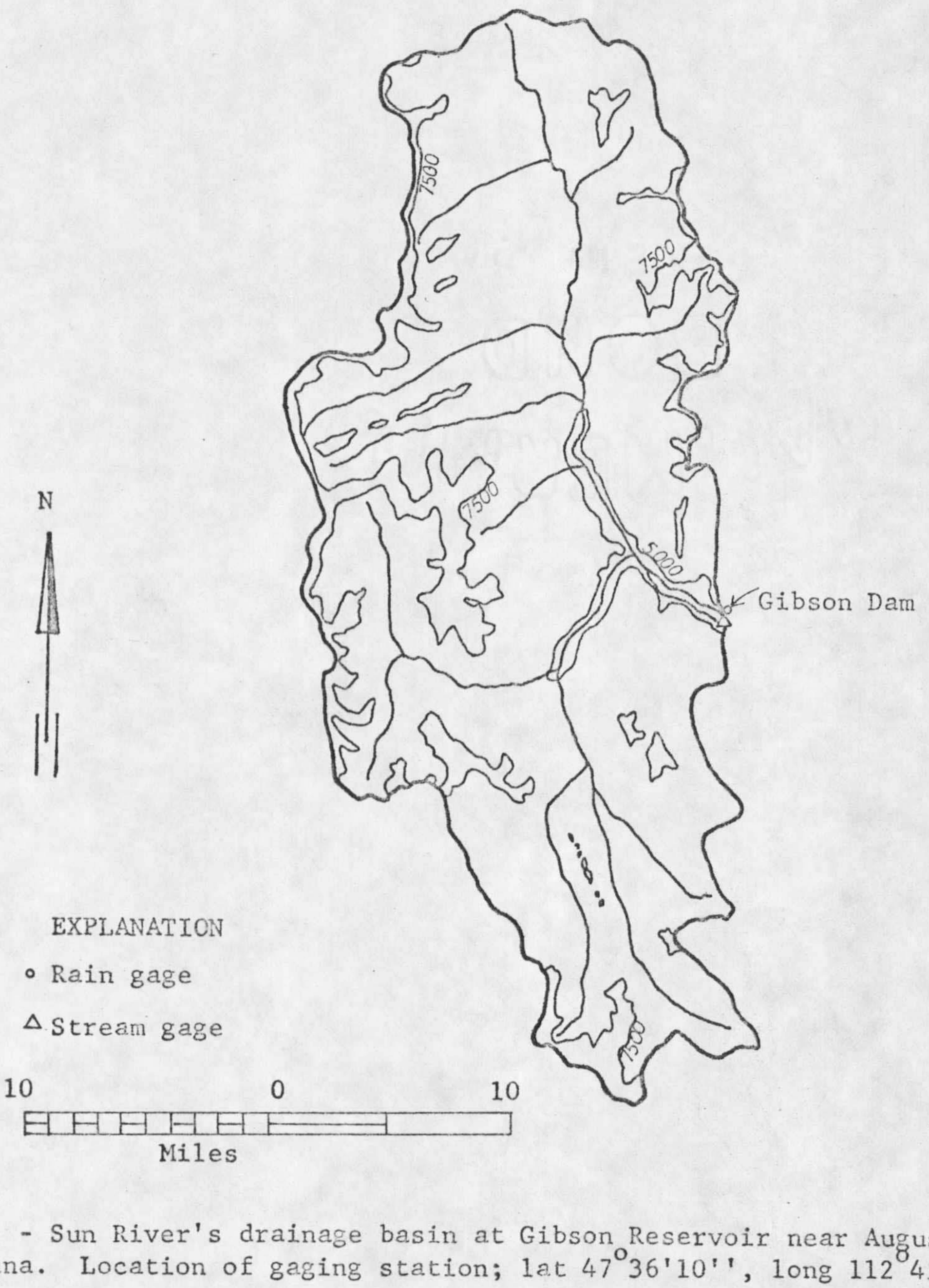

FIGURE D3. - Sun River's drainage basin at Gibson Reservoir near Augusta, Montana. Location of gaging station; lat 47°36'10'', long 112°45'40''!

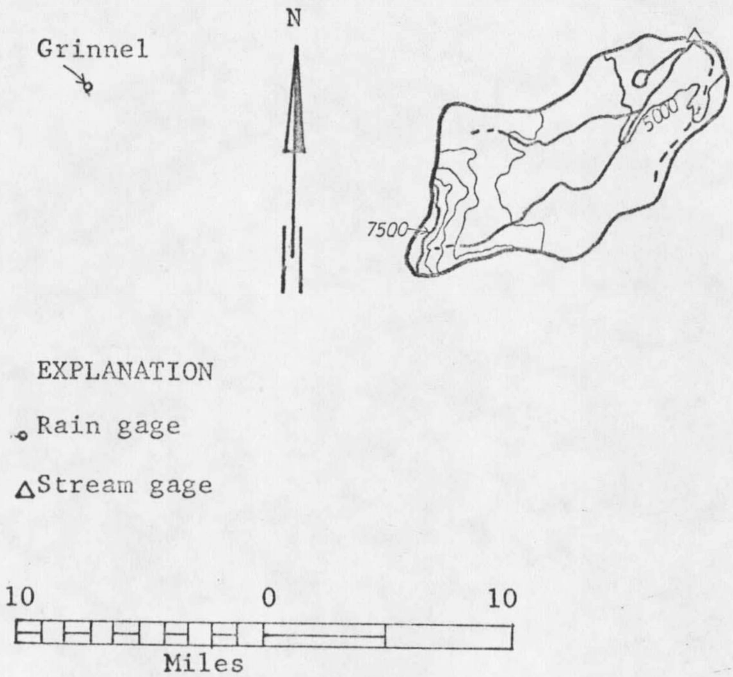

FIGURE D4. - South Fork Milk River's drainage basin near Babb, Montana. Location of gaging station; lat 48°45'00'', long 113°10'00''.

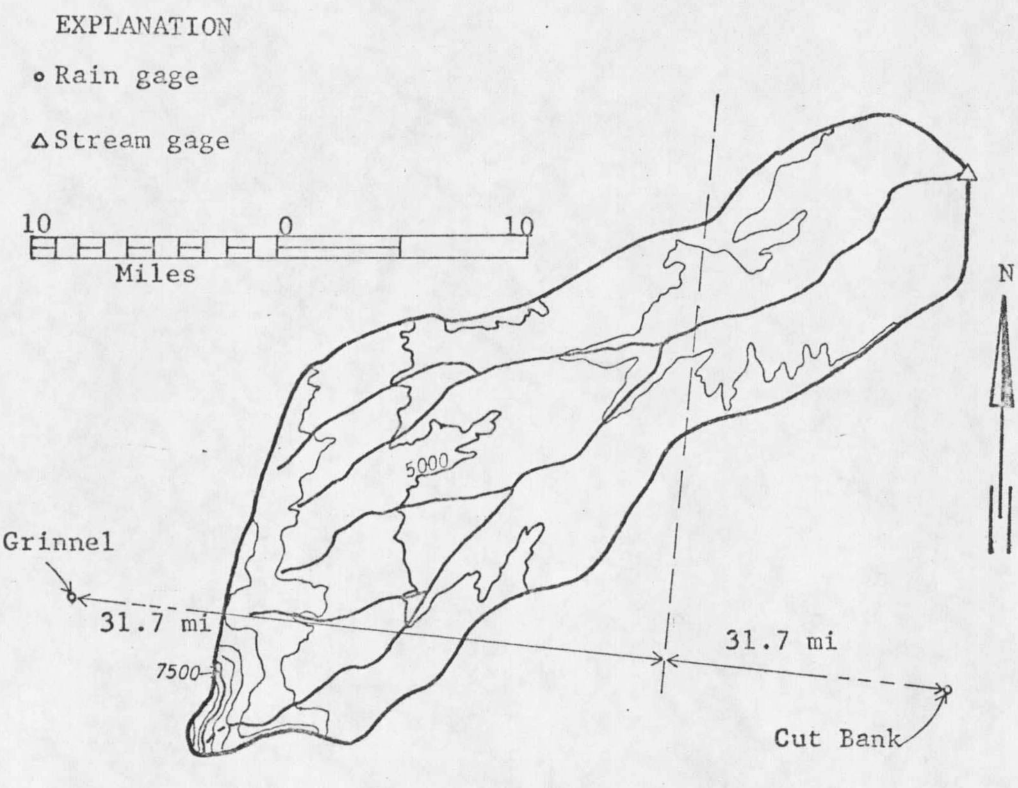

FIGURE D5. - Milk River's drainage basin near Del Bonita, Montana. Location of gaging station; lat 48°57', long 112°45'.

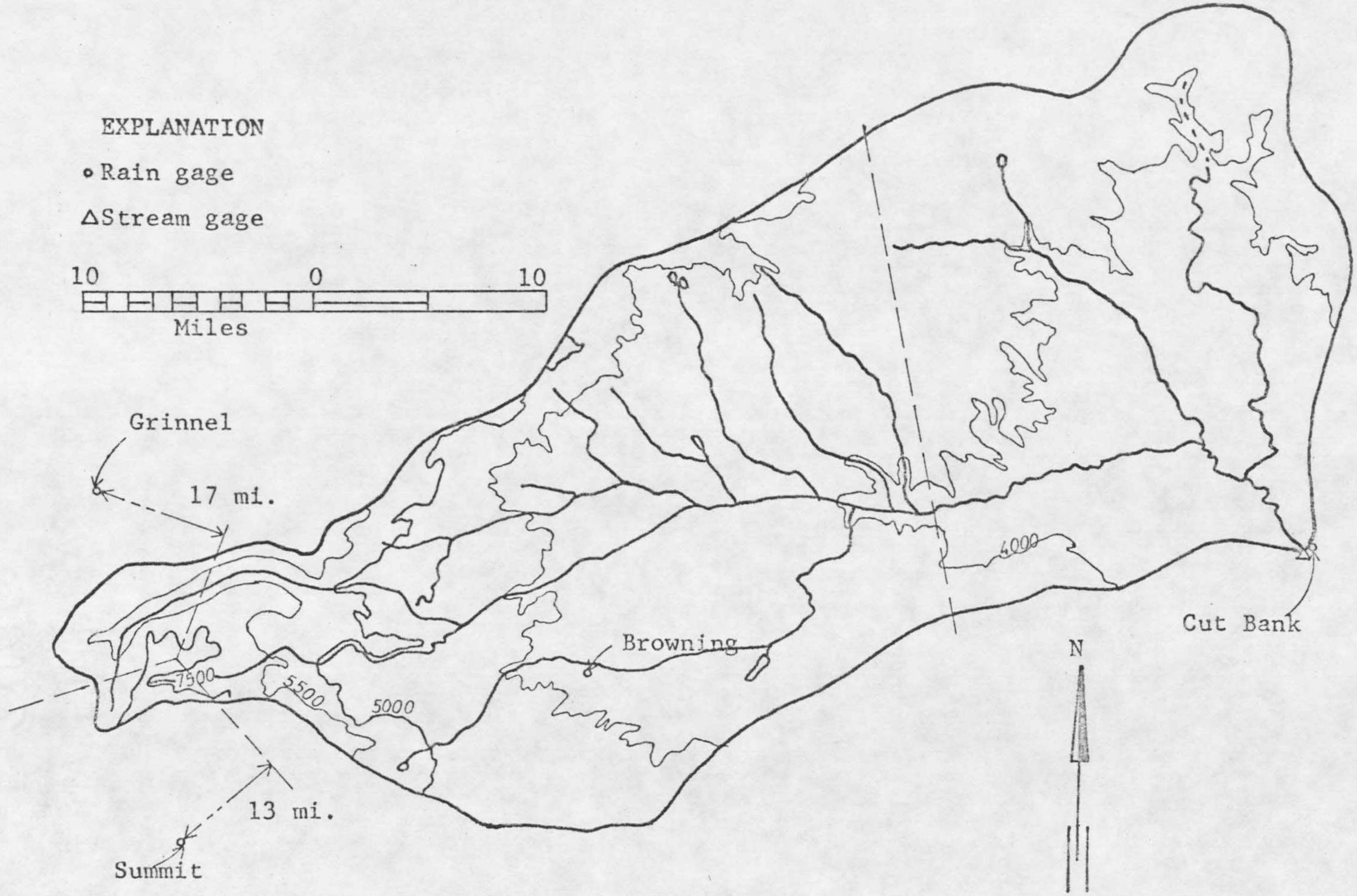

FIGURE D6. - Cut Bank Creek's drainage basin at Cut Bank, Montana. Location of gaging station; lat 48°38'00'', long 112°20'40''.

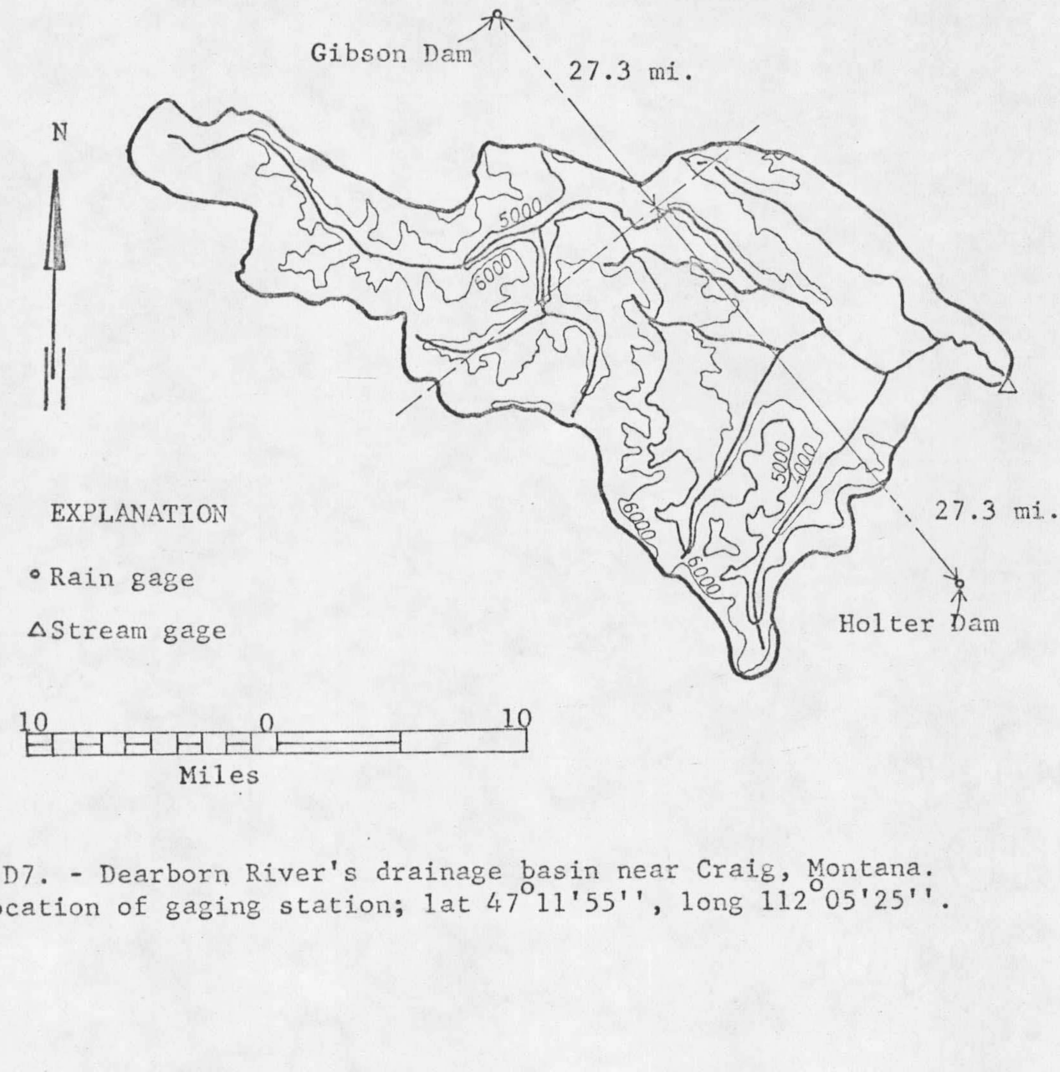

FIGURE D7. - Dearborn River's drainage basin near Craig, Montana. Location of gaging station; lat 47°11'55'', long 112°05'25''.

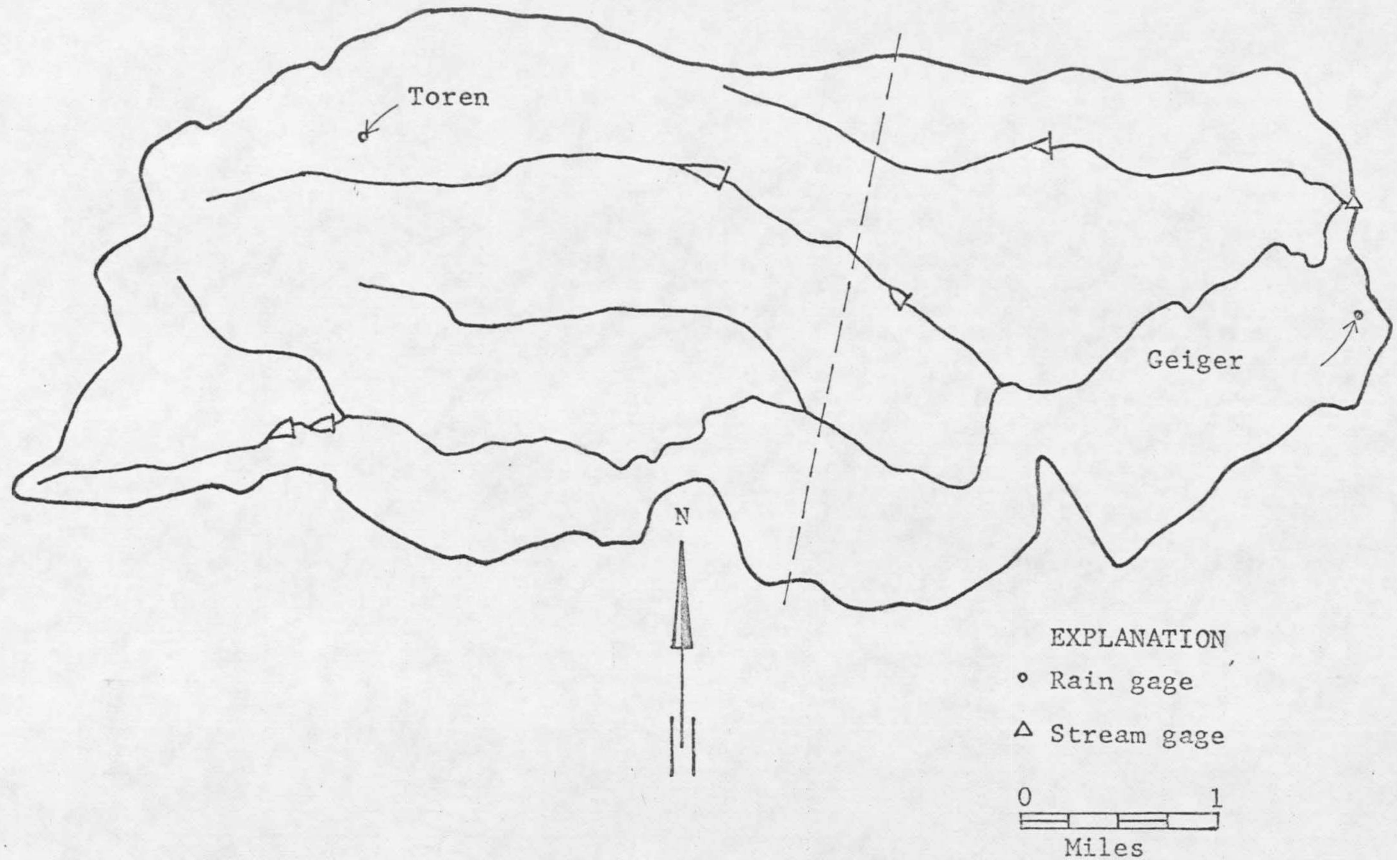

FIGURE D8. - Lone Man Coulee's drainage basin near Valier, Montana. Location of the gaging station on the corner of R-2-W, R1-W and T-29-N, T-30-N.

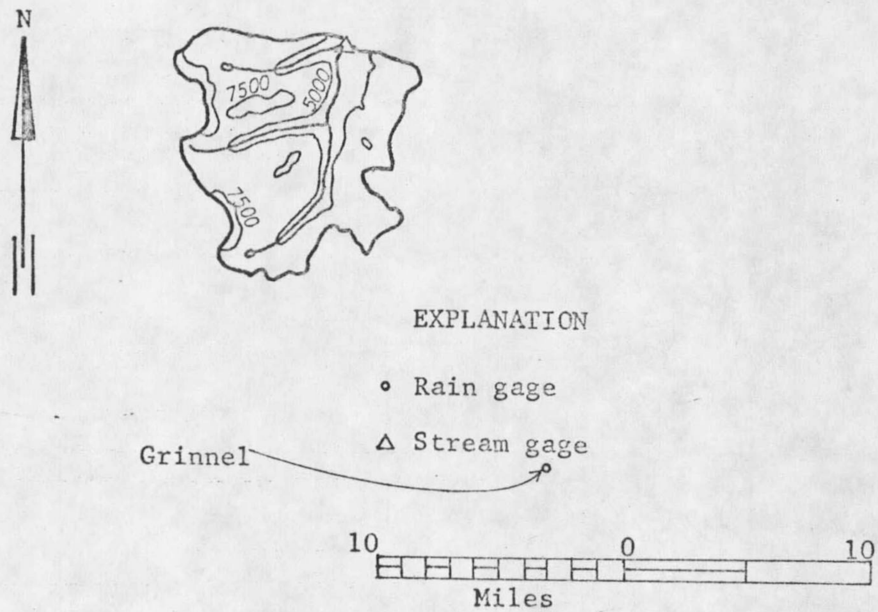

FIGURE D9. - Waterton River's drainage basin near International Boundary. Location of gaging station; lat 48°57'20'', long 113°54'00''.

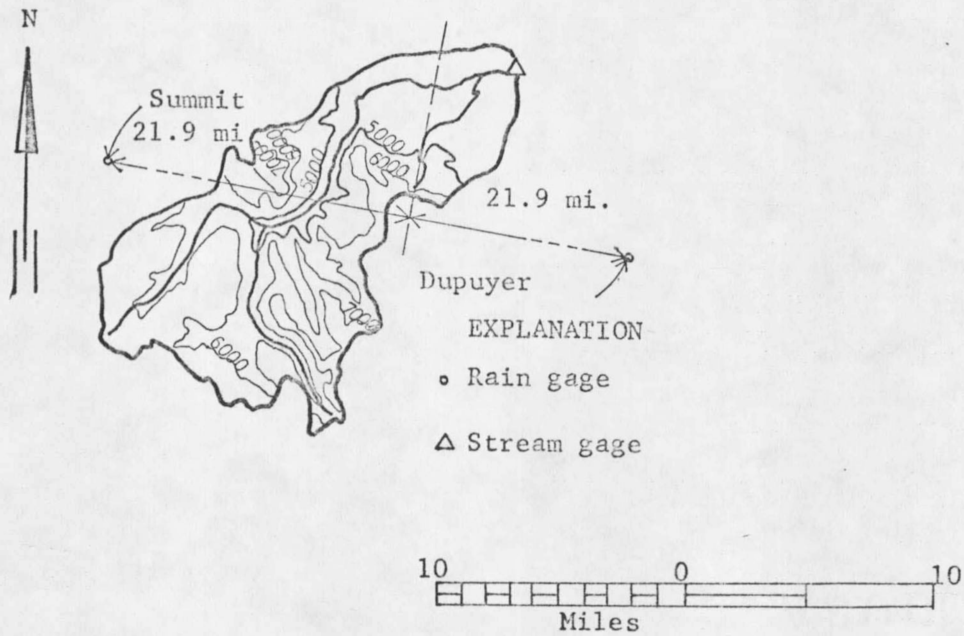

FIGURE D10. - Badger Creek's drainage basin near Browning, Montana. Location of gaging station; lat 48°21'00'', long 112°50'20''.

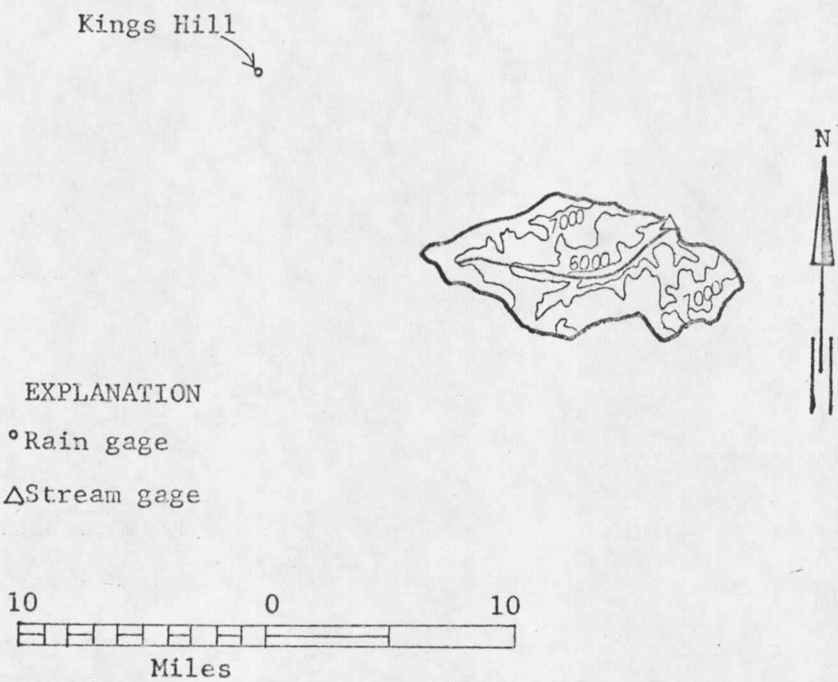

FIGURE D11. - South Fork Judith River's drainage basin near Utica, Montana. Location of gaging station; lat 46°45', long 110°19'.

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