

BIO-INSPIRED PROPULSION: VORTEX RING FORMATION FROM VARIED
FLEXIBILITY NOZZLES AND MODAL DYNAMICS OF PLUNGING TAPERED
HYDROFOILS

by

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ABSTRACT

Many aquatic animals and flying insects rely on flexible bodies and appendages for efficient locomotion. The flexibility inherent in these organisms has been linked to increased thrust and efficiency across diverse operating conditions. In this work, two bio-inspired propulsion mechanisms are examined: the first investigates pulses of fluid ejected through nozzles of varying flexibility, and the second explores a tapered flexible foil subjected to varied heaving amplitudes.

Jellyfish, squid, and siphonophores move by periodically contracting and expanding their bodies to expel and refill fluid from a flexible orifice, producing a series of vortex rings and stopping vortices. In this study, different volumes of fluid are ejected into a quiescent water tank through nozzles of varied stiffness to create vortex rings. Particle Image Velocimetry (PIV) is used to quantify thrust, and Finite-time Lyapunov Exponent (FTLE) fields are calculated to determine vortex ring pinch-off. It is found that the flexible nozzle stores and imparts elastic energy to the fluid, increasing the impulse of the ejected vortex ring. Impulse per unit volume is maximized at an optimal nozzle stiffness, wherein the damped natural frequency of the nozzle matches the fluid acceleration time. When the fluid decelerates, the more compliant nozzles collapse, suppressing unfavorable negative pressure regions from forming and instead eject additional fluid. Upon reopening, beneficial stopping vortices form within the nozzle, with circulation increasing with nozzle stiffness, indicating a second optimal stiffness criterion for a full expulsion-refill cycle analysis of this propulsion mechanism.

The flexural rigidity observed in insect wings and fish fins is commonly nonuniform and typically decreases from leading to trailing edge. This variation in stiffness encourages the propagation of traveling waves along the structure, increasing propulsive performance over a wider frequency range. In this study, rectangular foils undergo a sinusoidal heaving motion with varied amplitude and frequency. 2D mode shapes and resonant frequencies are measured using a scanning laser vibrometer. PIV is used to quantify the thrust from the wake of the foil. The traveling wave behavior is quantified and correlated with the thrust measured in the foil wake.

INTRODUCTION

Motivation

Fluid-structure interactions (FSI) play a crucial role in natural and engineered systems where a fluid interacts with a flexible structure to produce various effects including enhanced propulsive forces and higher energy efficiency. With advances in material science, engineering structures with flexible materials has gained interest with more durable and readily available flexible materials. Many researchers have used a “bio-inspired” engineering approach by examining natural systems, such as aquatic animals and insects, to gain insights into potential mechanisms to improve engineered systems for locomotion. Two discrete types of bio-inspired propulsion mechanisms are studied in this thesis work. First, a study of single pulses of fluid from varied stiffness nozzles based on a pulsed jet locomotor is examined. Second, a heaving foil with tapered stiffness is studied and the strength of the wake generated near and beyond resonance is quantified.

Pulsed Jet Propulsion Mechanisms:

Vortex rings are ubiquitous across many scales, playing key roles in systems such as starting jets for propulsion [47], aquatic locomotion in organisms such as fish [1], squid [2, 6, 33], salps [54], and jellyfish [14, 16, 31, 39]. They are also seen in respiratory flows such as coughing [76], and cardiac flows in the left ventricle within the heart [70]. A distinguishing feature of many of these systems is that fluid is pushed out of a flexible structure which is linked to high efficiency due to the interaction of the flexible structure with the surrounding fluid [83]. Dabiri [16] proposed that flexible living structures, such as those found in marine animals and the heart, operate under optimal conditions in terms of vortex formation, making

them highly efficient. Therefore, understanding the dynamics and formation of isolated vortex rings generated from flexible structures could provide further insight into engineering higher efficiency systems that leverage this phenomenon.

The vortex ring formation process is a well studied field of research as vortex ring flows are among the most efficient propulsion and mass transport mechanisms due to large amounts of fluid entrainment related to forming a coherent vortex structure [7, 8, 16, 48, 62, 63, 75]. Typically, hydrodynamic impulse is used to quantify the force generated by a vortex ring, which is the summation of fluid impulse from the change in fluid inertia and fluid pressure [73]. The total thrust generated by a starting jet is primarily governed by momentum flux, with pressure changes at the nozzle exit playing a secondary role. The momentum flux is directly influenced by the vortex generator, calculated as the time integration of $\rho_f U_e^2$, where U_e is the fluid velocity distributed across the nozzle exit being produced by the pump or propeller, and ρ_f is the fluid density. For a rigid nozzle, the momentum flux cannot be easily altered without changing the input to the system from the vortex generator. However, for a flexible nozzle, it has been shown that the momentum flux can be altered for the acceleration phase of a starting jet due to the fluid-structure interaction with the passively flexible nozzle [10]. Prior to uncovering the benefits of implementing a flexible nozzle, researchers were focused on maximizing the impulse generated from rigid nozzles, which led to studies examining ways to maximize the pressure impulse contribution to hydrodynamic impulse. The pressure impulse contribution, $p - p_\infty$, where p_∞ is the ambient fluid pressure and p is the near pressure field, is influenced by various factors, including fluid acceleration, nozzle design, and the duration of the expelled flow [28, 45, 47]. It has been shown that these factors, which are directly related to the vortex ring formation process, can maximize the thrust per unit volume, normalized by momentum flux, by increasing the pressure impulse [47]. The vortex ring formation process is characterized by a non-dimensional fluid slug length, or formation number, of $\frac{L}{D} \approx 4$, where L is the fluid

slug length and D is the nozzle exit, below which only a single vortex ring will form [32]. This has been identified as an operating limit for ejected volume, beyond which the efficiency of the propulsion mechanism would be expected to decrease. Thus, finding ways to increase the strength of a single vortex ring for propulsion has gained interest.

In the context of aquatic vehicles, pulsed jet propulsion has been shown to achieve significantly higher efficiency compared to steady-state jets of the same volume. This increased efficiency arises from the added mass and entrainment effects associated with the formation of multiple vortex rings, which are influenced by factors such as formation number, fluid acceleration, and Reynolds number [7, 8, 16, 48, 62, 63, 75]. The enhanced efficiency linked to the generation of vortex ring sequences results from both an increase in pressure impulse per pulse and the complex interactions among the pulses themselves [48].

Research by Ninh et al. [64] revealed that the interactions between vortices in a pulsed jet can lead to circulation changes ranging from a 10% increase to a 20% decrease, depending on the spacing of the vortex rings. For example, Xu and Dabiri [87] demonstrated that jellyfish could triple their swimming speed by using micro-controllers to regulate contraction frequencies, resulting in only a twofold increase in cost of transport—likely due to advantageous vortex interactions. Overall, jet-powered aquatic robots have significant potential because the impulsive nature of a starting jet can provide almost instantaneous thrust, unlike traditional propellers, which can take several seconds to reach full thrust [45]. This propulsion method allows for low-speed maneuvers, such as sideways movement and zero-radius turns, since it relies on internal jetting without external fluid manipulation [45]. Despite advancements in jet-propelled robotics, research on the use of flexible structures to enhance efficiency and maximize thrust per pulse remains limited. With this in mind, the formation process of single vortex rings generated with a flexible nozzle is studied in Chapter 4, to optimize the hydrodynamic impulse generated by a single pulse of fluid.

Oscillating Foil Propulsion Mechanisms:

Many aquatic animals and flying insects utilize flapping appendages for locomotion [23, 56, 79]. The flexibility found in both fish fins [49] and insect wings [12] has inspired a range of studies that have shown that a specific amount of flexibility enhances thrust generation or efficiency in a flapping wing or fin model [21, 24, 43, 51, 52, 58, 59, 67]. The inherent ability to deform under fluid loading generally enhances the performance of flexible foils through alignment of shed vortices and specific surface bending deformations. It has been suggested that resonant behavior resulting from the interaction of the fluid and structure (FSI) is a key factor behind the performance benefits [4].

Previous research has predominately focused on propulsors with uniform bending stiffness. However, many biological structures, such as fins or wings, exhibit varying mechanical properties along their chord length which can introduce traveling wave behavior and acoustic black hole effects to prevent the reflection of waves [13, 22]. This aspect of research has recently gained attention, despite its long known relevance to biological systems. Traveling wave behavior is indicative of the kinematics that certain types of species of fish (anguilliform i.e. eels) primarily use for transport, while the majority of fish utilize some varied amount of standing and traveling wave kinematics for locomotion [34]. Cui et al. [15] found that in living fish standing-wave based propulsion can generate substantial thrust, whereas traveling wave style propulsion is generally more efficient across a wider range of frequencies. In a lab setting, studies have indicated that structures with a stiffer leading edge can outperform those with uniform bending stiffness in terms of both thrust and efficiency [42, 88]. As previously noted, a nonuniform distribution of stiffness can introduce traveling wave behavior which is beneficial for the propulsive performance of a heaving foil relative to an equally stiff foil with uniform stiffness, primarily through increasing efficiency [22, 51, 77]. The use of a tapered foil generally creates a wider range of high efficiency heaving frequencies, especially post resonance where traveling wave behavior is likely to occur

[51, 88]. Traveling waves can alternatively be produced through high heaving amplitudes on a submerged uniform stiffness beam where nonlinear fluid damping begins to affect the kinematics of the beam [77, 78]. In these cases, a propulsive benefit of higher output velocity was found when traveling waves were produced along the uniform structure, and the mean fluid velocity became independent of tip velocity and became dependent on traveling waves for higher frequency modes.

Despite many advantages in the study of flexible foils and stiffness tapering, the effect of heaving amplitude on the structural dynamics of a tapered flexible foil has not been experimentally studied up to this point to the best of the author’s knowledge. For larger heaving amplitudes, the damping force on the foil becomes larger and likely will generate more nonlinear effects which could encourage traveling wave behavior as seen in uniform thickness beams and foils [77]. Based on this, there is still a need for a deeper understanding of how heaving amplitude affects traveling wave behavior and thrust generated by a heaving tapered flexible hydrofoil.

Objectives

Two projects were completed and are included in this Masters Thesis. The first project, *Formation of Multiple Vortex Rings from Passively Flexible Nozzles*, studied the effect of nozzle flexibility on hydrodynamic impulse generation for single pulses of fluid into a quiescent tank. The primary objectives of this project were to (1) provide background on the ideal fluid structure interaction for optimizing nozzle expulsion, (2) characterize the flow field generated and nozzle deformation, (3) relate these aspects to one another, and lastly (4) examine ideal structural characteristics for nozzle refilling, which is a largely understudied, yet important, aspect in generating thrust from a pulsed jet bioinspired propulsion mechanism.

The second project, *Modal Dynamics of Plunging Tapered Hydrofoils*, studied the effect

of foil heaving amplitude and frequency on the structural dynamics of the foil and related the structural dynamics to the wake produced. The primary objectives of this project were to (1) quantify the resonant frequencies and mode shapes of a tapered hydrofoil heaving at different amplitudes (varied damping), (2) examine how the structural dynamics, and traveling wave behavior, was introduced or altered at different heaving amplitudes, (3) quantify the momentum flux coefficient of the wake to estimate a propulsion force generated for each combination of heaving amplitude and frequency, and finally (4) relate the momentum produced by the wake to the measured modal dynamics and how the foil deformation may be altering the wake beneficially or negatively.

Thesis Outline

The rest of this thesis is organized as follows. First, we review current literature relevant to flexible structure design and optimizing thrust from different types of propulsors in Chapter 2. Specifically, discussion of studies of starting jets through nozzles and orifices, and heaving flexible foil dynamics with varied flexibility and their applications. Then in Chapter 3, the methods used for two separate projects that are both relevant to fluid structure interaction in maximizing propulsion will be outlined, the first being focused on vortex rings formed from flexible nozzles, and the second characterizing the modal dynamics and wake characteristics from a heaving tapered flexible foil. A short theory section will be contained in the literature review chapter for the vortex ring project, explaining how quantities such as vorticity, circulation, and impulse are derived. Next, a manuscript recently submitted to *Journal of Fluid Mechanics*, titled *Formation of Multiple Vortex Rings from Passively Flexible Nozzles*, will be included in Chapter 4. Lastly, the structural dynamics and wake characteristics from the secondary flapping foil project will be provided and examined in Chapter 5. Lastly, a conclusion and recommendations for future work are commented on in Chapter 6.

LITERATURE REVIEW

Vorticity, Circulation, and Hydrodynamic Impulse:

To understand the dynamics of rotational fluid movements, like vortex rings or vortices from a heaving foil, vorticity and circulation need to be defined. Vorticity and circulation are commonly used quantities to describe fluid rotation. Vorticity, ω (1/sec), is the curl of a vector field, $\boldsymbol{\omega} = \nabla \times \mathbf{v}$, and can be thought of as the pseudo-vector measure of the spinning motion of a vector field. If we expand the cross product in cartesian coordinates we get:

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \quad (2.1)$$

Next, if we are calculating ω in a 2D flow where the velocity is independent of the z direction, or $v_z = 0$, ω can be simplified to:

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z} \quad (2.2)$$

We are only left with the $\hat{z}$ component of vorticity, or a scalar field of vorticity values multiplied by the $\hat{z}$ direction. This is the calculation used for our Particle Image Velocimetry datasets, where we have taken a 2D cross-section of a 3D flow upon an axis of symmetry where there is no out-of-plane motion or $v_z = 0$. Positive and negative vorticity then indicate the direction of rotation, where a counter-clockwise rotation is positive by the right-hand rule. In general, vorticity is the microscopic measure of fluid rotation, as it is a vector field which can provide a measure of rotation at any singular point in the fluid. Circulation, Γ , is the macroscopic scalar measure of fluid rotation and can be represented as the line integral of the velocity on a closed curve C [73]:

$$\Gamma = \oint_C \vec{u} \cdot d\vec{\ell} \quad (2.3)$$

If we say that the curve C is reducible and can be continuously shrunk to a point without going outside the fluid, we can use Stokes Theorem, which shows that circulation around a reducible curve is equal to the flux of vorticity through an open surface A bounded by curve C . In layman's terms, if we take the line integral of the velocity field around C we will get the same result as the area integral of vorticity within A , as the integration elements, or point values of vorticity, within the bounding area will essentially cancel to only leave the exterior velocities. From stokes theorem [73]:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_A (\nabla \times \vec{F}) \cdot d\vec{A} \quad (2.4)$$

Next, drawing the direct parallel to calculation circulation and using ω , we arrive at:

$$\Gamma = \oint \vec{u} \cdot d\vec{\ell} = \iint_A \vec{\omega} \cdot \hat{n} dA \quad (2.5)$$

Although Γ can be defined as the line integral of velocity around a surface, conceptually, it is easier to understand the relationship between circulation and vorticity to think of circulation as the area integral of vorticity. For a vortex ring, circulation is often used to define vortex strength, or the amount of vorticity contained, across a 2D cut through the center plane of the vortex.

The derivation of hydrodynamic impulse is significantly more involved and is not shown here, but is provided from Saffman [73]:

$$I_h = \frac{1}{2}\rho_f \int_C x \times \omega dV \quad (2.6)$$

Vortex Ring Literature Review:

The following literature review for *Formation of Multiple Vortex Rings from Passively Flexible Nozzles* is found below; this is also available in Section 1 of the submitted paper to *Journal of Fluid Mechanics*.

Vortex rings are ubiquitous across many scales, playing key roles in systems such as starting jets for propulsion [47], aquatic locomotion in organisms such as fish [1], squid [2, 6, 33], salps [54], and jellyfish [14, 16, 31, 39]. They are also seen in respiratory flows such as coughing [76], and cardiac flows in the left ventricle [70]. A distinguishing feature of many of these systems is that fluid is pushed out of a flexible structure which is linked to high efficiency due to the interaction of the flexible structure with the surrounding fluid [83]. Dabiri [16] proposed that flexible living structures, such as those found in marine animals and the heart, operate under optimal conditions of vortex formation, making them highly efficient. Therefore, understanding the dynamics and formation of isolated vortex rings generated from flexible structures could provide further insight into engineering higher efficiency systems that leverage this phenomenon.

Vortex rings are typically generated in experiments by expelling fluid through a sharp-edged nozzle or orifice into another larger body of fluid using a piston. The formation process of vortex rings from rigid nozzles has been well studied. Didden [25] identified several key parameters governing vortex ring formation, including the nozzle diameter (D), the piston stroke length (L), and the velocity history of the piston motion $U_p(t)$. For short piston stroke to diameter ratios $\frac{L}{D} = \frac{U_p(t)t}{D}$, a single vortex ring forms, entraining all the expelled fluid given an impulsive piston motion. [32] identified a limiting stroke ratio ($\frac{L}{D}$) ranging from 3.6 to 4.5 at which a single vortex ring is formed, and any larger stroke ratio results in additional expelled fluid forming a trailing jet behind the vortex ring, making it a limited process. Further studies have shown that different nozzle geometries and velocity flow profiles can

alter the critical $\frac{L}{D}$ value at which a vortex ring will pinch-off and no longer gain circulation [18, 32, 46], Dabiri and Gharib [18] demonstrated that by forcing a circular nozzle exit to reduce in area as fluid was being expelled, vortex ring pinch-off could be delayed until $\frac{L}{D} \approx 8$, due to changes in the output velocity and shear layer development. Krieg and Mohseni [46] found that pinch-off is determined by the characteristic velocity of the vortex ring relative to the feeding velocity of the liquid. Similar to Dabiri and Gharib [18], Krieg and Mohseni [46], showed that vortex ring pinch-off could be delayed up to a value of $t^* \approx 8$ for a continually accelerating feeding velocity so that the vortex ring could not outpace the feeding source. Their work provided valuable insights into predicting pinch-off timing under time-varying output velocity programs.

Hydrodynamic impulse can be used to quantify the fluid thrust generated by a vortex ring, which is the summation of fluid impulse from the change in fluid inertia and fluid pressure [73]. The total thrust supplied by a starting jet is primarily controlled by the momentum flux, and secondarily by the pressure change at the nozzle exit. The change in momentum flux is directly affected by the vortex generator as it is the integration of $\rho_f U_e^2$, where U_e is the fluid velocity distributed across the nozzle exit being produced by the pump or propeller. For a rigid nozzle, this cannot be easily altered without changing the input to the system from the vortex generator. However, the pressure change, $p - p_\infty$, where p_∞ is the ambient fluid pressure, is affected by several factors including the fluid acceleration, nozzle geometry, and duration of the velocity being expelled [28, 45, 47]. These factors are all closely related to the vortex ring formation process and help describe the increase in impulse by forming a single vortex ring. It was determined that the maximum thrust normalized by momentum flux could be achieved at $t^* \approx 4$, for an impulsively started piston, due to the increase in pressure rise at the nozzle exit when a vortex ring pinches off [47]. Notably, thrust could be increased with a faster fluid acceleration for the same t^* value, as would be expected based on the increased fluid momentum [47]. Gao et al. [28] stated that the total

impulse depends on the fluid deceleration as well, which can create a stopping vortex near the nozzle exit which favorably contributes to the pressure impulse. Additionally, Yin and el Hak [89] demonstrated that for a pumping jet-style propeller, refilling the body creates positive momentum and a stopping vortex within the body cavity.

When applied to aquatic vehicles, it has been demonstrated that pulsed jet propulsion can achieve a much higher efficiency compared to steady-state jets of equivalent volume. The benefits stem from the added mass and entrainment effects from forming a series of vortex rings, which depends on formation number, fluid acceleration, and Reynolds number [7, 8, 16, 48, 62, 63, 75]. The increase in efficiency associated with creating sequences of vortex rings is due to a combination of increased pressure impulse per pulse as well as the interactions of each pulse with one another [48]. However, Ninh et al. [64] showed the vortex-vortex interactions of a pulsed jet could either increase circulation by as much as 10% or reduce it by 20% depending on the spacing of each vortex ring with one another. For instance, Xu and Dabiri [87] showed that controlling contraction frequencies in jellyfish with micro-controllers could triple their swimming speed, while only creating a twofold increase in the cost of transport, likely due to beneficial vortex interactions. In general, jet-powered aquatic robotics show promise in that the impulsive motion of a starting jet can provide thrust almost instantaneously compared to several seconds for a traditional propeller to reach the desired thrust [45]. This type of propulsion can enable low-speed maneuvering such as sideways translation, and zero radius turns (yaw), all because there is no external fluid manipulator simply an internal jetting cavity ([45]). Although there are many achievements in terms of jet-propelled robotics, there is limited research into the effect of utilizing flexible structures to increase efficiency and maximize thrust per pulse from these types of aquatic vehicles.

Jellyfish and squid have been shown to be among the most efficient swimmers, attributed to their flexible, deformable bodies creating desirable movement kinematics [14]. Jellyfish locomotion encompasses three parts: suction thrust during contraction, passive energy

recapture during relaxation, and a wall effect created by the interaction with counter-rotating vortices left in their wake [31]. Squid exhibit similar swimming kinematics as well. [3] showed that squid utilize two distinct modes of locomotion, alternating between small bursts of fluid that create individual vortex rings, and larger jets that generate multiple vortex structures at the expense of efficiency for additional force. Xiaobo et al. [86] numerically demonstrated that in the case of a soft bodied jet propulsor, with geometry similar to a squid, sucking liquid into the body has the same effect as pushing water out by increasing positive momentum within the body, which is likely related to a squid’s swimming efficiency. Numerical simulations of jellyfish revealed that synchronizing contractions with the natural wave speed of the bell margin results in a maximum thrust efficiency, related to the resonant frequency of varied stiffness bodies [38].

The impact of employing flexible flaps and hinges to manipulate the flow produced by starting jets has been studied as well. Many nature-inspired researchers have shown that a specific amount of flexibility enhances thrust generation or efficiency in terms of a flapping wing or fin model [21, 24, 43, 51, 52, 58, 59, 67]. All of these studies indicate that a specific level of flexibility increases thrust production or power efficiency in flapping wing or fin models, attributed to the alignment of shed vortices and surface deformations. However, the number of studies relating flexibility to thrust generation from a starting jet is still somewhat lacking, with most findings showing that indefinitely decreasing stiffness increases thrust efficiency. Das et al. [20] showed that flexible flaps installed at the outlet of a rectangular channel could amplify fluid impulse by a factor of two while utilizing the same energy input, due to rearrangement of vorticity of the jet. Jung et al. [41] demonstrated that by using everting flexible elastic sheets at a nozzle exit, the fluid impulse of a jet could be enhanced by as much as 14 times, with increased bending rigidity of the sheets.

Studies of flow within a flexible tube have been investigated but with limited application to jet or vortex flows. Many studies focus on flexible tubes with rigid circular supports at

either end [44, 53, 68]. Through the use of shell theory coupled with hydrodynamic equations, the natural frequency and vibration mode of a submerged cylindrical shell can be predicted [44]. The hydrodynamics and inertial effects within the thin shell significantly alter the frequency and mode shapes resulting from the thin shell, but this can be accounted for using a mass and speed ratio relation of the shell and the fluid [68]. Additionally, the length to radius ratio (L/R) and wall thickness to radius ratio (h/R) have a large impact on the stability of the resulting deformations relative to the velocity of the fluid being expelled [65]. Specifically, Paak et al. [65] showed that for clamped-free boundary conditions, the output velocity can excite periodic, multi-mode, or chaotic vibration depending on the velocity ratio $U = u_b/c_s$ where u_b is the bulk fluid velocity and c_s is the wave speed on the cylinder surface. Choi and Park [10] found that there is an optimal stiffness for a circular nozzle to maximize thrust for jet flows, rather than indefinitely increased thrust for a decreased stiffness. Their work derived an optimal flexibility condition, relating the stiffness of the nozzle to the acceleration of the fluid being expelled. In a subsequent study, their work showed that these concepts could be applied with single pulsed jet flows [11]. However, these concepts have yet to be analyzed with respect to the wake formation of single vortex rings to maximize pressure impulse and thus ejected volume. Additionally, the effect of stopping vortices has not yet been accounted for in these models in terms of finding an optimal stiffness relative to the fluid generation apparatus.

Plunging Tapered Hydrofoil Literature Review:

Many aquatic animals and flying insects use flapping appendages for movement [23, 56, 79]. Research on the flexibility of fish fins [49] and insect wings [12] has highlighted how this flexibility can improve the performance of thrust-generating structures, specifically enhancing efficiency and total thrust production. The ability of flexible foils to deform under fluid forces typically leads to performance gains by facilitating the alignment of shed

vortices and promoting specific bending patterns on the surface of the foil or fin. Studies have demonstrated that a certain degree of flexibility can enhance the efficiency of flapping wing and fin models beyond which the efficiency of the system decreases [21, 24, 43, 51, 52, 58, 59, 67]. The advantage of a flexible propulsor depends on many factors including the kinematics of the motion, the plan-form shape, and structure stiffness [19, 43]. More recently, the study of tapered foils has begun to gain interest provided that the mechanical properties (stiffness) of both fish fins and insect wings are typically varied along the chord or span [13]. Computational and experimental studies both indicate that the use of a structure with a stiffer leading edge and more flexible trailing edge is more efficient and can produce more thrust than those with a constant bending stiffness [22, 36, 42, 51, 77, 88]. These tapered structures also have the significant benefit of having a high efficiency and propulsive regime over a much wider band of frequencies, especially above resonance, compared to a constant bending stiffness counterpart which typically only see a peak in performance close to resonance [51, 88]. Regardless of taper, it has been proposed that resonant interactions between the fluid and the structure, known as fluid-structure interaction (FSI), play a critical role in these performance improvements for both tapered and nontapered flexible foil, but traveling wave behavior can play an equivalently large role when nonlinear effects dominate the FSI [4, 22, 77, 78].

Several active and passive mechanisms exist for generating traveling waves on a finite length structure. Typically, if a wave starts to form on a plate-like structure it will travel to the end of the plate and get reflected creating a backward wave. When the forward and backward waves interact with one another, it creates a standing wave [22]. Due to the boundary conditions provided to beam-like structures, such as a wing or foil, they do not inherently produce traveling waves, and adjustments in boundary conditions need to be made to suppress standing wave generation [78]. Loh and Ro [55] actively introduced pure traveling waves to a uniform beam in air by using a two-mode excitation method with two

forces positions symmetrically along the beam length, with identical forcing amplitude but with a 90° phase difference. They were able to control the direction of the traveling wave by modulating the phase difference between $\pm 90^\circ$ and found that the traveling wave behavior was maximized when the beam was operated between two resonant frequencies, particularly in the higher modes. In a passive approach, dissipation can dampen the propagation of an incident traveling wave which suppresses wave reflection and encourages traveling wave formation [72]. In this case, dissipation is related to the nonlinear transverse drag force inherently imposed on a submerged foil heaving in water. Consequently, this allows for the production of traveling waves by imparting a single excitation to a structure such as a cantilever beam subjected to base motion at large enough amplitude where dissipation forces take effect [72].

Another passive mechanism to creating traveling waves in a finite length structure is through the use of a structure with gradually decreased thickness, such as a tapered beam [60]. As the beam thickness reduces along the length and approaches zero, the wave speed of the structure goes to zero as well. This results in a region where waves are trapped and is known as an acoustic black hole [22, 60]. The acoustic black hole effect is used in vibration absorption applications [27], but can also be used to generate traveling waves on a flexible hydrofoil to promote high-efficiency kinematics.

Several recent studies have focused on heaving and pitching foils with a stepwise reduction in stiffness along the chord to encourage traveling wave behavior. This style foil is selected over a fully tapered foil due to the ability to actively modulate the amount of flexibility in the foil by implementing an adjustable length rigid leading edge with a highly flexible trailing edge [36]. Lucas et al. [57] showed that having a stepwise decrease in material stiffness at $2/3$ of the length along the chord of a fin from leading edge to trailing edge could result in higher thrust coefficients and lower cost of transport. Interestingly, it was shown that through the use of this stepwise change in stiffness and a 0° angle of attack pitching

program, they could achieve foil kinematics very similar to freely swimming fish which was related to their thrust performance. However, traveling wave behavior was not quantified in this study. A similar style foil installed on a bio-inspired fish robot with a stepwise change in stiffness was found to produce the largest thrust when the stiffness of the fin scaled with the heaving and pitching frequency, implying resonance [90]. Han et al. [36] used a similar style foil, which showed that nonuniform flexible foils outperform their rigid and uniformly flexible counterparts.

In terms of optimal conditions for stiffness, a variety of options have been proposed related to both pitching and heaving foils. Moored et al. [61] proposed that in a free stream the optimal thrust conditions occur when the driving frequency of a flexible flapping (pitching and heaving) fin matches the resonant frequency of the wake, or the vortex shedding frequency. It was then shown that a flexible flapping foil was capable of suppressing asymmetric vortex shedding through compliant deformation, thus inhibiting deflection of the propulsive jet in unfavorable directions [91]. Dewey et al. [24] demonstrated that thrust efficiency and force generation of a flexible pitching panel in a free stream are maximized when two conditions are met. First, the Strouhal number ($St = \frac{f_p A_p}{u_\infty}$), is in the range of $0.25 < St < 0.35$, where f_p is pitching frequency, A_p is pitching amplitude, and u_∞ is the free stream velocity, which also happens to include the St range fish tend to swim at [57]. Second, the panel needs to be pitched at the structure's natural frequency, which could achieve $\approx 100\%$ increase in thrust production and efficiency compared to a rigid panel. Park et al. [67] described the optimal flexibility for thrust generation in terms of the phase lag ($= \pi/2$) between the pitching angle of the foil, and the rotation of the trailing edge due to flexibility. Li et al. [52] additionally showed that flow-induced oscillations of a flexible membrane enhances lift generation by extending the leading edge vortices and a transition of vibrational modes related to the frequency lock-in between the natural structural frequency of the membrane and the vortex-shedding frequency. Han et al. [36] proposed that to maximize thrust from a

pitching stepwise flexible hydrofoil, it should (1) oscillate above the first natural frequency (2) have a flexion ratio $0.4 < \lambda < 0.7$ where $\lambda = \frac{c_r}{c}$, where c_r is the rigid section chord length and c is the flexible section chord length (3) have a low non-dimensional structural rigidity. Although (2) and (3) do not apply to a truly tapered foil with a consistent change in thickness, and thus stiffness and rigidity, (1) becomes interesting as this transition between the first and second bending modes of a beam is advantageous for thrust, and this transition regime is associated with traveling wave behavior [78].

Leroy-Calatayud et al. [51] showed that the traveling wave index (I_{TW}), or ratio of real to complex modes from a complex orthogonal decomposition of foil displacement, could be used to help describe the optimal efficiency of heaving tapered foils peaking at a frequency slightly beyond the first natural frequency of the flexible foils. However, these foils were driven at a quite large heaving amplitude ($0.1A = 9.5mm$) where nonlinear behavior is much more likely to take effect and is discussed in the subsequent paragraph. It was noted that maximal thrust occurred at resonance for both a tapered and uniform foil, but the efficiency of a tapered foil was greater for a much larger spectrum relative to a uniform thickness foil and began to have a second peak in efficiency beyond the first resonance. Interestingly, Leroy-Calatayud et al. [50] provided the relationship between I_{TW} and efficiency directly as the frequency was increased, which demonstrated that the traveling wave index created a 'zig-zag' pattern decreasing for a short period in relation to efficiency, and then increasing again as the input frequency continued to increase. It was noted that the second bending mode was beyond the capabilities of their experimental setup and that the relationship in the efficiency vs. traveling wave index could be insightful when approaching the second bending mode. In a recent 3D computational study, Demirer et al. [22] described traveling wave behavior in a tapered flexible foil as an acoustic black hole, where at the trailing edge the propulsor slows down and attenuates flexural waves, reducing wave reflection and thus enhancing traveling wave propulsion. Similar to previous work, it was shown that

tapered propulsors outperform uniform ones, especially for post-resonance frequencies, and is directly linked to the ability of the tapered propulsor to generate traveling waves with a large amplitude trailing edge displacement. Skriptyan et al. [77] showed that the first mode shape in a cantilever beam submerged in water induced more fluid velocity than the second mode shape, however they also described traveling waves creating efficient locomotion indicated by higher normalized (fluid-to-beam tip) velocities regardless of bending mode. As discussed in Skriptyan et al. [78], clear traveling wave phenomena were observed approaching the second resonant frequency of a 2D beam, but with a node indicative of the second bending mode of a beam which would reduce fluid entrainment.

Foil heaving amplitude has also been investigated for flexible but constant chord-wise mechanical property wings wherein nonlinearity begins to arise at larger heaving amplitudes. Quinn et al. [71] showed that the time averaged net force generated by a flexible heaving panel increased with heaving frequency, but with local maximums near resonant frequencies of the flexible panels where the trailing edge amplitude was increased. It was later shown that changing the heaving amplitude adds a significant nonlinearity to the forces acting upon the foil, accounted for in model predictions through the application of a quadratic damping term from the transverse drag force on the heaving wing [66]. Paraz et al. [66] noted that the nonlinear transverse drag force acting upon the foil was crucial in model predictions, even for "small" deflections ($A = 4\text{mm}$). Note that "small" is in quotations here as what defines a "small" heaving amplitude is widely varied between studies. Andres Goza and Rowley [4] noted that similar to a simple spring-mass damper system, increasing heaving amplitude created wider, less poignant peaks in the frequency response function for a flexible foil. It was proposed that in small heaving amplitude regimes ($\frac{A}{C} = 0.001$) resonance dominates the propulsive efficiency and thrust, while at larger heaving amplitudes ($\frac{A}{C} = 0.1$) nonlinear damping creates a larger effect, where resonant and non-resonant mechanisms play a role [4]. Unfortunately, no physical distances were described in this study, where nondimensional

numbers such as the heaving ratio divided by chord length were provided.

Several methods exist for quantifying the amount of traveling wave behavior present on a structure. The first proposed method is the Complex Orthogonal Decomposition (COD) [26]. This method is identical to a Proper Orthogonal Decomposition (POD) where an input signal, displacement, or velocity, matrix is decomposed into 3 sub-matrices comprised of a mode matrix, an energy matrix representing the strength of each mode, and a transposed vector matrix, but with complex modes included in the analysis. In POD, no imaginary numbers are included and as a result, cannot decompose temporal modes. Alternatively in COD, both the real and imaginary parts of the data are included in the algorithm. The real part for example being the centerline foil displacement over time, and the imaginary part being the Hilbert transform of the real signal. To then distinguish between standing or traveling wave dominated behavior in a complex mode from the COD, one can use the traveling index (I_{TW}) which is defined by the inverse of the condition number of the matrix whose two columns are the real and imaginary parts of the entire mode. A pure traveling wave will yield $I_{TW} = 1$, as the real and imaginary parts are orthogonal and of the same magnitude. Alternatively, for a standing wave $I_{TW} = 0$, will be comprised of a mode with its two components parallel or of differing magnitudes [26].

Previous work has examined how to quantify thrust from a heaving, or flapping, foil by measuring wake parameters with PIV. This is an area of interest as some applications, especially studies of living animals, cannot easily implement a load cell for force measurement, while a wake survey method can be much easier to implement to estimate lift, drag, and thrust. One commonly used model to estimate lift (L) from a wing wake is the Kutta-Joukowski (KJ) theorem, $L_v(t) = \rho U b \Gamma(t)$, which commonly underpredicts the lift amplitude [35]. Here, U is the free-stream velocity, b is the wing span, and $\Gamma(t)$ is the circulation measured at the Trefftz plane or center of the span. Wang et al. [84] later showed that the KJ theorem only recovers the lift generated by vortex structures, but neglects the added

mass effect from a deflected wake. It was shown that through adding the vortex lift L_v to $L_{acc} = -\rho k \cdot \int_{V_f} \frac{\partial u}{\partial t} dV - \rho k \cdot \int_{\partial B} \mathbf{u} \cdot \frac{\mathbf{u}}{2} n dS$, an accurate estimate for the lift generation could be estimated. However, these applications primarily apply to lift generation from a tilted foil in a cross flow, which does not have direct application to a heaving foil in quiescent water.

Alternatively, for a heaving foil with zero free-stream velocity Heathcote et al. [37] proposed that the thrust force could be estimated through estimation of the time averaged x-momentum $M_x = \rho_f \int \bar{U}^2 dy$, where $\bar{U}$ is the time average X-direction velocity. More recently, Park et al. [67] verified that this calculation for M_x showed a clear linear relationship with the thrust force measured from a force sensor for a pitching panel in quiescent fluid. They also proposed that the ratio of $\frac{M_x}{M_y}$, where M_y is calculated analogously to M_x in the transverse direction, was useful in quantifying the amount of the wake directed in the positive thrust direction. In contrast, they noted that the strength (Γ) of the trailing edge vortices was not directly related to the thrust produced by different flexibility foils, but rather a representation of the total amount of displaced fluid without any sign to the direction the fluid was pushed.

Despite all of the great achievements in regard to quantifying and studying uniform and nonuniform flexible foils, there is still a need for an experimental investigation into the relationship between heaving amplitude and traveling wave behavior. Although, there have been computational studies into relating the effect of heaving amplitude on a uniform stiffness flexible beam and output fluid thrust [4], effects of pure structural dynamics on heaving amplitude for a submerged beam [77], the effect of the heaving amplitude and traveling wave dynamics have still not been experimentally related to one another. Therefore, in 5 we discuss the results from such a study where the heaving amplitude of a flexible foil is varied, related to the amount of traveling wave behavior, and the effect on output thrust is quantified.

METHODS

Methods: Vortex Ring

The following description of experimental methods for *Formation of Multiple Vortex Rings from Passively Flexible Nozzles* is found below; this is also available in Section 2 of the submitted paper to *Journal of Fluid Mechanics*.

Experimental Setup

Experiments were conducted in a 46 cm $\times$ 46 cm $\times$ 46 cm free-surface water tank filled with water to a height of 41 cm. An Aladdin NE-1000 syringe pump was used to generate the vortex flows. A 140 mL Monoject syringe was connected to a 1" SCH40 PVC pipe (Inner Diameter = 2.66 cm) with flexible tubing. Nozzles with diameter ($D = 2.55$ cm) and length ($L_{nozzle} = 11.5$ cm) were installed at the end of the pipe to eject fluid downward in the center of the tank, 14 cm below the water surface. Further description of the nozzle adapters will be discussed in Section 3. A schematic of the experimental setup is shown in figure 3.1. The origin of the coordinate system used for calculations and defining coordinate directions is defined by the center of the nozzle exit with the positive Y direction oriented vertically downward in the flow direction, as shown in figure 3.1 nozzle detail.

The syringe pump velocity profiles were defined using the Windows Command Prompt Program, in combination with an Excel spreadsheet to format the input data, to control the velocity (U_{piston}) and duration (t_{cycle}) of the syringe piston motion. Three syringe piston velocity programs were selected corresponding to formation lengths, $\frac{L}{D} = \int_0^{t_{cycle}} \frac{U_{piston}(t)}{D} dt = 1, 2, 4$, ensuring the rigid nozzle would produce a single vortex ring with negligible trailing wake at the maximal ejected volume [32]. Each profile had an impulsively started piston motion measured at the syringe pump. The input diameter-based Reynolds Number ($Re_D = \frac{U_{piston} D_{piston}}{\nu}$), where ν is the dynamic viscosity of room temperature water, was held constant

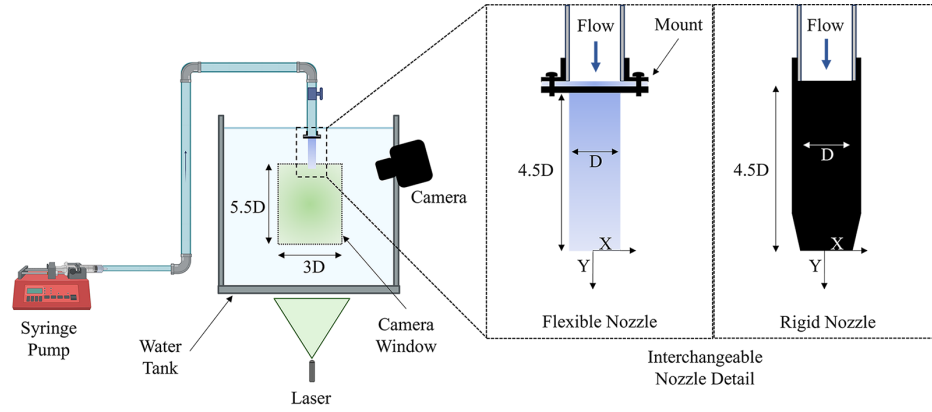


Figure 3.1: Schematic of the experimental setup used for generating the vortex flows with NE-1000 Aladdin pump. A 1mm thick laser sheet is used to illuminate a 2D plane of the tank. Interchangeable nozzle detail is also shown for mounting the rigid and flexible nozzles.

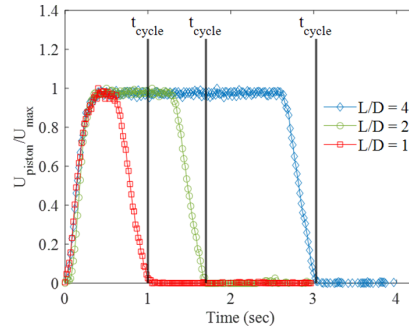


Figure 3.2: Measured syringe pump piston velocity by syringe motion normalized by the programmed maximum velocity for the pump (U_{max}).

at 1000. The programmed piston velocity profiles were verified by imaging the rear edge of the piston and tracking the displacement over time using an in-house MATLAB script. In brief, the tracking was accomplished by converting the videos to images, reducing each image to the area of interest, passing a Canny Edge detection filter over the images, and binarizing the images using a threshold value [9]. Each piston velocity profile was imaged 5 times to ensure repeatability and was averaged over these trials and is plotted in figure 3.2. The mean steady state piston velocity data for each L/D is 1.68 cm/sec with a 95% confidence interval of ± 0.01 cm/sec from a one-sample t-test.

Nozzle Construction and Characterization

A rigid nozzle was 3D printed out of PLA plastic with a layer height of .03mm. The rigid nozzle was manufactured to be the same length ($L_{nozzle} = 11.5$ cm) as the flexible nozzles, with the only difference being a beveled edge of 25 degrees at the exit. This nozzle design was selected to be consistent with previous studies in vortex ring literature [18, 32, 47].

The flexible nozzles were molded from SmoothOn SortaClear 40A two-part liquid silicone. The selected nozzle construction method was adopted from Choi and Park [10], using a pour mold method over a 2.55 cm diameter (D) aluminum rod press fit into a 3D printed base to create the mounting surface for the flexible nozzles. Figure 3.1 shows a complete flexible nozzle with the integrated mounting surface, and figure 3.3(a) shows an isometric view of the flexible nozzle geometry. After curing, the nozzles were cut to a length (L_{nozzle}) of 11.5 cm measured from the top of the nozzle base to the edge of the nozzle exit, resulting in an aspect ratio ($\frac{L_{nozzle}}{D}$) of 4.55. This aspect ratio was chosen to ensure all of the fluid ejected was initially contained within the nozzle. This is in contrast to Choi and Park [10, 11], where an aspect ratio of 2 was used (ejected fluid slug not initially fully contained within flexible nozzle). The stiffness of the flexible nozzles was controlled by adding different mass percentages ($\%ma = 15\% - 50\%$) of SmoothOn Silicone Thinner relative to the base liquid silicone mixture.

The flexible nozzles were mounted to the experimental setup using a 3D printed part that clamped onto the base of the nozzle using bolts, shown in figure 3.1. This setup was used to reduce stress concentrations from forming at the edges of the nozzle which could potentially affect the nozzle deformation behavior. Additionally, the mount created a consistent rigid boundary condition between the different nozzles to ensure the repeatability of the experiment. After a nozzle was clamped onto the mount, it was installed onto the outlet pipe using an interference fit. The rigid nozzle was similarly installed with an interference fit on the outlet pipe.

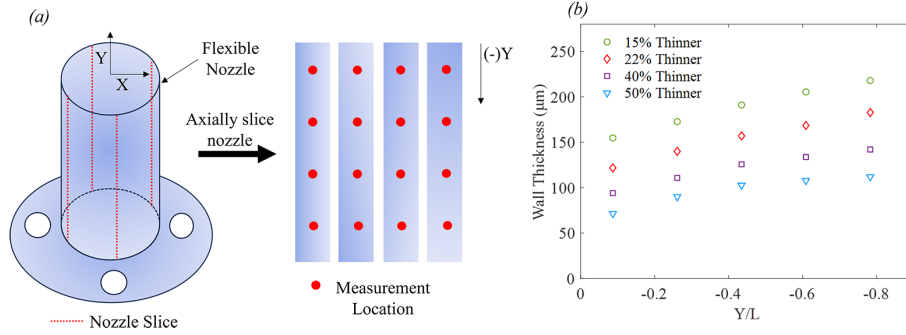


Figure 3.3: Nozzle wall thickness measurement schematic and variation in wall thickness. (a) Schematic indicating the cut sections used for measuring nozzle wall thickness. The nozzles were cut along their streamwise direction to create 4 strips that were measured at 4 streamwise locations along the nozzle’s length. (b) Distribution of measured nozzle thickness along the length (L) of the nozzles. The labeled % values in the legend correspond to the %ma of silicone thinner added to the mixture to mold each nozzle.

For each nozzle, a representative nozzle was made to be sliced into strips to measure the mean nozzle wall thickness (h) using an Olympus BX60 microscope with a $10\times$ power lens. The wall thickness was measured at a total of 16 locations for each nozzle, at 4 streamwise locations 2.5 cm apart and 4 equally spaced azimuthal locations around the nozzle circumference at each stream-wise height as shown in figure 3.3(a). The wall thickness variation was negligible with a maximum azimuthal variation of 4.1% for the manufactured nozzles. The wall thickness varied approximately linearly along each nozzle’s length due to the pour molding manufacturing method. The spatial distribution of the wall thickness along the nozzle length (L) is plotted in figure 3.3(b). The thickness decreased at an average rate of $17.7 \mu\text{m}/D$ along the streamwise direction of the nozzle. The total change in thickness was approximately constant between the different nozzles, thus the thickness was averaged over the measured locations for each nozzle. Mean wall thickness (h) varied from $188.4\mu\text{m}$ to $96.8\mu\text{m}$ across the different nozzles.

Young’s Modulus was calculated by completing tensile tests on 5mm thick dog bone samples prepared with cross sectional geometry as described in ASTM D412-16 Standard

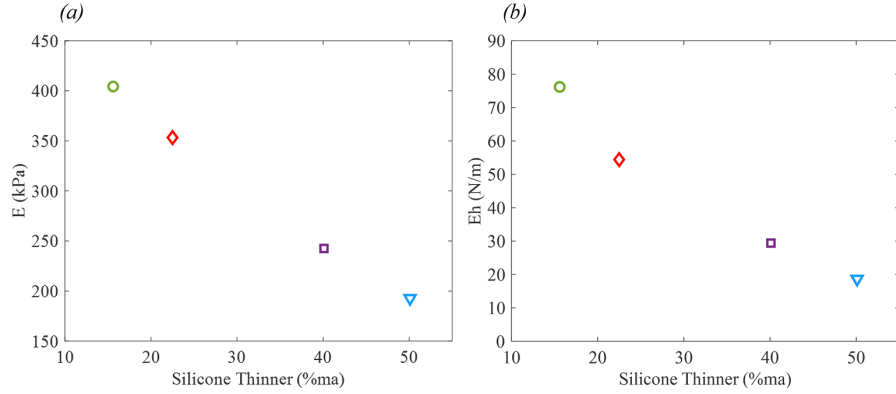


Figure 3.4: Material properties measured with varied %ma thinner (a) Young's Modulus (E) averaged over 8 samples from each %ma of silicone thinner added to the nozzle mixtures. (b) E multiplied by the mean wall thickness (h), defining the characteristic stiffness of the varied %ma thinner nozzles.

Test Methods for Vulcanized Rubber and Thermoplastic Elastomers—Tension [5]. The tests were completed on an Instron 312 Series Frame with a 25 kN load cell. For each nozzle, 8 tensile tests were completed from two different batches of silicone, one batch from the silicone used for the experimental nozzle, and the second batch from the silicone used for the representative nozzle. The maximum variation in E between the two sets was negligible, with the maximum being 4.2%. Young's Modulus (E) varied from 404 kPa to 193 kPa as shown in figure 3.4(a).

The representative characteristic stiffness parameter was chosen to be Young's Modulus (E) multiplied by the mean wall thickness (h), which is plotted in figure 3.4(b). Overall, a set of 5 nozzles were tested with characteristic stiffness parameter $Eh = 19$ N/m, 29 N/m, 54 N/m, 78 N/m, and $Eh = \infty$ N/m (rigid). Table 3.1 summarizes the parameters studied in this experiment.

Velocity Field Measurements

Particle Image Velocimetry (PIV) was conducted to quantify the velocity fields beneath the nozzles. Images were sampled at 60 Hz with a resolution of 1080×1920 pixels. The

Parameter	Value
Nozzle Diameter (D) (mm)	25.5
Nozzle Length (L_{nozzle}) (mm)	115.0
Characteristic Stiffness (Eh) (N/m)	19, 29, 54, 76, ∞ (rigid)
Formation Length ($t^* = \frac{L}{D}$)	1, 2, 4
Re_D	1000

Table 3.1: Vortex Ring Experimental Parameters

camera was triggered to start filming simultaneously with the initiation of the pump using an in-house LabVIEW program via a MyDAQ (National Instruments). Hollow glass sphere particles with mean diameter of 10 μm , and average density of 1.10 g/mL were used to seed the tank (Potters' Industries Spherichel, 110P8). A 532nm continuous wavelength laser and a convex cylindrical lens were mounted underneath the tank to illuminate a 1 mm vertical sheet passing through the center cross-section of the nozzle in either the XY or XZ plane as defined in figure 3.1, with the Z direction being out of the page. Cross-correlation of the image pairs was completed using a multiple pass interrogation method with successive reductions in window sizes using the open-source software PIVlab [81, 82]. For a field of view of $5.5D \times 3D$, a final interrogation window size of 64×64 pixels with 50% overlap was chosen. This resulted in a vector space of 59×32 vectors with a vector spacing of $.098D$. The time interval between image pairs, Δt , was .0167 seconds and the particle motion between image pairs was limited to approximately 1/4 of the window size to ensure the accuracy of the cross correlation [85]. As the flexible nozzles were found to deform about a preferential axis, PIV was performed both perpendicular (XY plane), and parallel (XZ plane), to this preferential axis to ensure axisymmetry in our results. For each nozzle, 10 trials were completed, with 5 trials from each plane. In subsequent plots, the average of these 10 trials is plotted unless

otherwise noted. The mean maximum spatially averaged velocity measured $0.019D$ below the rigid nozzle outlet was calculated to be 3.97 cm/sec across all ejected volumes, with a 95% confidence interval of ± 0.1 cm/sec based on a one-sample t-test. All reported confidence intervals hereon represent the 95% confidence interval based on a one-sample t-test. The flexible nozzles altered the measured exit velocity, which is discussed in the results, where the $Eh = 29$ N/m nozzle had the largest variation, with a mean maximum spatial averaged velocity of 6.40 cm/sec and confidence interval ± 0.3 cm/sec.

Nozzle Deformation Measurements

To track the side deformation of the flexible nozzles, the particles were removed from the tank, and the laser sheet was used to illuminate the longitudinal center cross section of the nozzle in the XY and XZ planes. The camera imaged the center plane deformations, which were used to track the nozzle deformation using an in-house MATLAB edge detection code. The same image processing steps used to determine the syringe pump velocity profiles were applied to these images for nozzle tracking. Two time sets were considered, forced vibration when the pump was active, and free vibration when the pump had stopped moving and the nozzle was freely oscillating. The bottom millimeter of the nozzle was used as a representative point to track the position over time. Fast Fourier Transforms (FFTs) were completed in MATLAB to find the repeated oscillation frequencies from the free vibration positional datasets. The single-side amplitude spectrum of the position data, or the occurrence of each frequency in the positive domain, was calculated using a sampling period of .0167 sec and a sample length of 600 samples (10 seconds). The single sided amplitude spectrum was plotted to find a peak in frequency occurrences to estimate the damped natural frequency of (ω_d) of the nozzles. The deformation viewed from beneath the nozzle (YZ plane) was also imaged.

FTLE Fields and Ridges

The predicted material boundaries of the generated vortex rings were identified by detecting ridges within the Finite-Time Lyapunov Exponent (FTLE) field using the LCS Matlab Kit V2 by the Biological Propulsion Laboratory at the California Institute of Technology [17, 69, 74]. The FTLE fields are directly derived from the velocity fields generated from PIV by either initiating fluid particle tracking forward or backward in time. This process yields both positive (forward) pFTLE and negative (backward) nFTLE fields. The FTLE field is computed by:

$$\sigma_T^t(\mathbf{x}) = \frac{1}{|T|} \ln \left| \frac{d\phi_t^{t+T}(\mathbf{x})}{d\mathbf{x}} \right| \quad (3.1)$$

where $\sigma_T^t(\mathbf{x})$ signifies the flow map of particles from their location at $\mathbf{x}$ at time t to $t + T$. $|T|$ represents the integration time used to track the particle motion, with $T < 0$, indicating backward time, and $T > 0$, corresponding to forward time. After obtaining the FTLE fields, regions of local maxima, or ridges, were identified to pinpoint regions where fluid transport is restricted or intensified. These ridges are known as Lagrangian Coherent Structures (LCS), which are valuable for analyzing vortex ring pinch-off. Negative LCS (nLCS), found from the ridges within a nFTLE field, indicate attractive material lines in the flow field, while positive LCS (pLCS), found from ridges within a pFTLE field, indicate repelling material lines. When a nLCS and pLCS combine to form a closed loop, it signifies the formation of a pinched-off vortex ring [74]. The presence of a pLCS on a vortex ring's rear edge suggests that further fluid entry into the enclosed structure of the vortex ring from the vortex generator is unlikely, and the vortex ring will not gain additional vorticity. The integration length $|T|$ chosen for the data analyzed within this paper corresponds to 1 second or 60 image frames. A step size of 1 and a time step of 0.0167 seconds per frame were used.

Methods: Plunging Foil

Experimental Setup

Experiments were conducted in a $43\text{cm} \times 45\text{cm} \times 90\text{cm}$ tank filled to a height of 36cm with water. An extruded aluminum superstructure was constructed around the tank and used as a mounting platform to mount the foil-driving assembly shown in 3.5. The assembly was mounted to put the foil in the center of the tank width, located 60cm from the tank end, or $2/3$ of the length distance. This was done to prevent wall effects from taking place when the foil began to produce a net positive thrust and high velocity vectors in the downstream direction. A 7lb shaker with a built-in amplifier was used to oscillate a flexible foil in a linear heaving motion (The Modal Shop, K2007E01). The shaker armature was threaded to a machined aluminum interface part with threaded inserts on both sides. On the opposing side of the interface part, a $1/4''$ rod was installed which was supported in a block of high-density polyethylene through a $1/4''$ hole, as shown in detail in 3.6(a) and (b). The supporting block was implemented to prevent excessive bending on the shaker by supporting the vertical load from the foil assembly. To prevent rotation of the foil and mounting rod, two pieces of laser cut acrylic were mounted beneath the shaker to create a guide slot for the mounting rod. The interface part was bolted onto a 3D printed part where a mounting rod could be installed with a set screw. The flexible foil was installed on the mounting rod, such that the motion of the shaker would translate the foil linearly back and forth, with the foil perpendicular to the heaving motion. A 35A73 Modal Shop accelerometer was stuck onto a flat surface on a 3D printed part with wax, which was installed on the top of the interference part to measure the input acceleration provided to the flexible foils. Two 532nm lasers were installed on the sides of the tank to illuminate a 2D horizontal plane located in the center of the span of the foils. A Chronos CR21-HD high speed camera was mounted underneath the tank to image the wake dynamics and vorticity generated by the

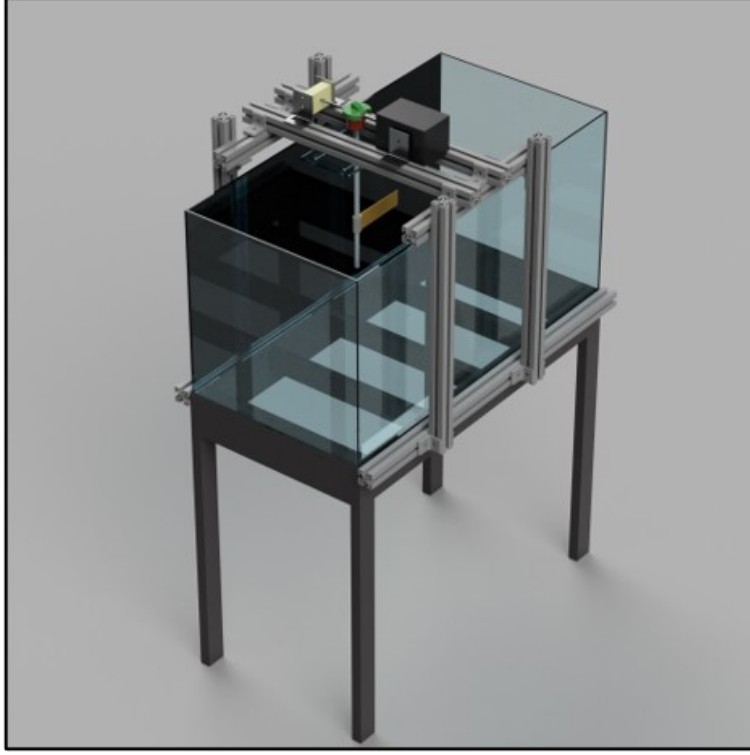


Figure 3.5: Experimental Setup used to drive the wing in the tank. Here the wing is shown as yellow for image clarity.

different heaving dynamics imposed on the flexible foils. A Polytec PSV-400 Scanning Laser Head was used to measure the output response (velocity) of the flexible foils as they were driven in varied heaving motions.

The mounting rod was made out of a 36cm long 1.27cm (1/2 in) thick hollow aluminum rod with a wall thickness of 0.158cm (1/16 in). This shaft was selected to have a high stiffness relative to that of the wing so that the shaft did not compound the measured vibrational effects of the wing at higher frequencies. Approximating the mounting rod as a cantilever beam, the first bending mode of the shaft in air was expected to be at $\omega_1 = 88.7\text{Hz}$ using the solution to free vibration of a continuous cantilever beam, $\omega_{n1} = (1.875)^2 \sqrt{\frac{EI}{mL^4}}$. Given that the mounting rod was submerged in water this solution would not be exactly correct, yet it provided a design guide for having enough structural rigidity to prevent any

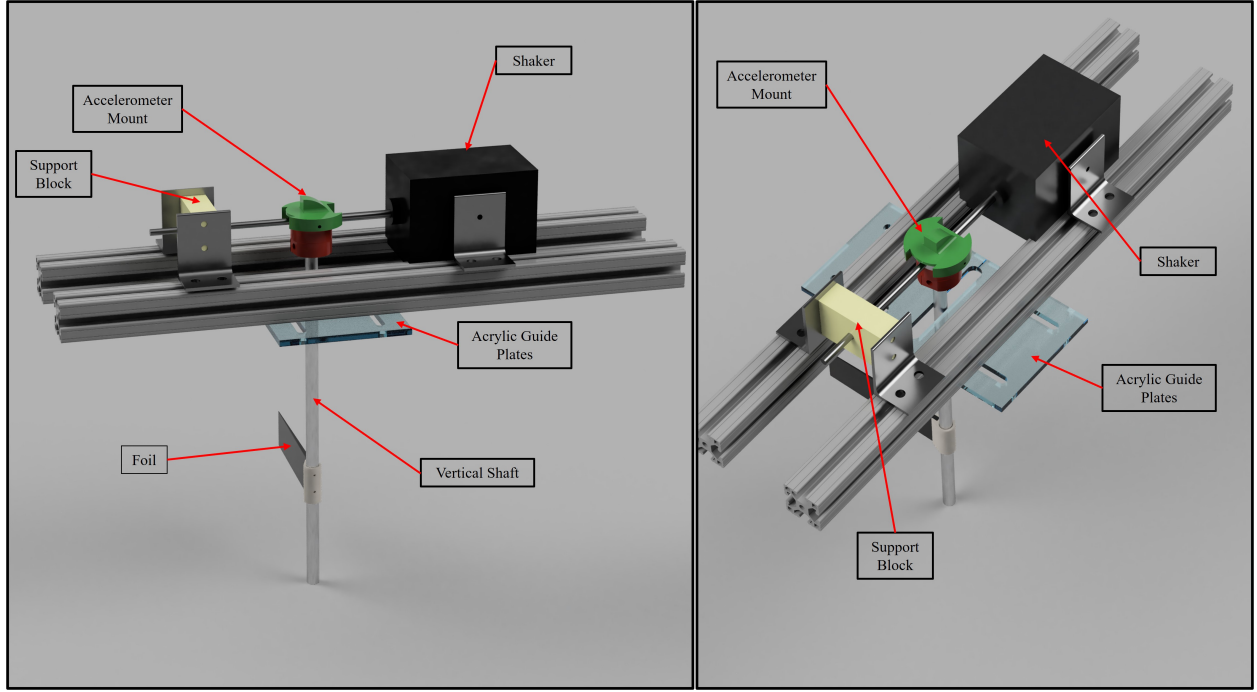


Figure 3.6: Shaker assembly with components labeled. (a) View angle 1 (b) View angle 2

compounding effects. From the experiments described below, the first resonant frequency of the rod partially submerged in water was $\omega_{n1} = 69\text{Hz}$. With the added drag damping on the rod, a downward shift in the prediction for ω_{n1} was expected. The rod and foil were additionally scanned in air and ω_{n1} was measured to be 72Hz with the added mass of the foil in air and lack of fluid damping. This rod was selected to have the first natural frequency approximately an order of magnitude greater than the frequencies studied for the heaving wing.

Foil Design and Construction

The flexible foils were printed out of TPE-SEBS 1300 95A flexible filament with reported $E = 93 \text{ MPa}$ [40]. The foils were printed such that the vertical step layers were along the chord of the wing to ensure as consistent span wise material properties as possible. 3 shells, or exterior solid fill lines, were used in the printing process, such that the front and back

of the wing would have constant bending stiffness. The infill pattern used was MakerBot Linear, which created a pattern of cross-hatched 'X' patterns within the wing itself. The geometry of the wing was designed to have a tapered thickness from the leading edge thickness $t_{LE} = 4.5$ mm to a trailing edge thickness of $t_{TE} = 0.6$ mm, or a taper ratio $\alpha = 1 - \frac{t_{LE}}{t_{TE}}$ of 0.87. The wing chord length (C) was 0.137m, with a span (S) of 0.0381m. The trailing edge thickness was selected based on 3D printing limitations while getting as close to a truly tapered geometry as possible to promote traveling wave dynamics [60]. Thinner trailing edge thicknesses would result in nonuniform layer deposition and a large inward step at the tip of the wing, whereas the 0.6mm thick trailing edge resulted in a 3-layer wide tip, to prevent this undesirable step-decrease in thickness. The foils were epoxied into a 3D printed PLA part with a female connection to connect the foils to the mounting rod with two set screws, a schematic of the wing setup is shown in 3.7. For testing, the wings were mounted 10cm from the end of the mounting rod measured from the base of the span.

Dynamic Vibration Analysis

The frequency response function (FRF) was found for the flexible foil at a variety of heaving amplitudes, and thus varied damping. The FRF is the ratio between input (shaker acceleration) and output (measured foil velocity) in the frequency domain. Peaks in the FRF typically represent a given structure's resonant frequencies, or the frequencies at which the output response is magnified relative to the input force. The input force (acceleration) was measured using the aforementioned accelerometer, and the output response (velocity) was measured using a PSV 400 Scanning Laser Head, both of which are discussed below. A dynamic signal analyzer (DSA) was used to generate signals to power the shaker and accelerometer. The DSA also was used to simultaneously record the acceleration data from the accelerometer, and the output velocity data from the scanning head when performing an experiment. The DSA used the PSV acquisition software to convert the time domain

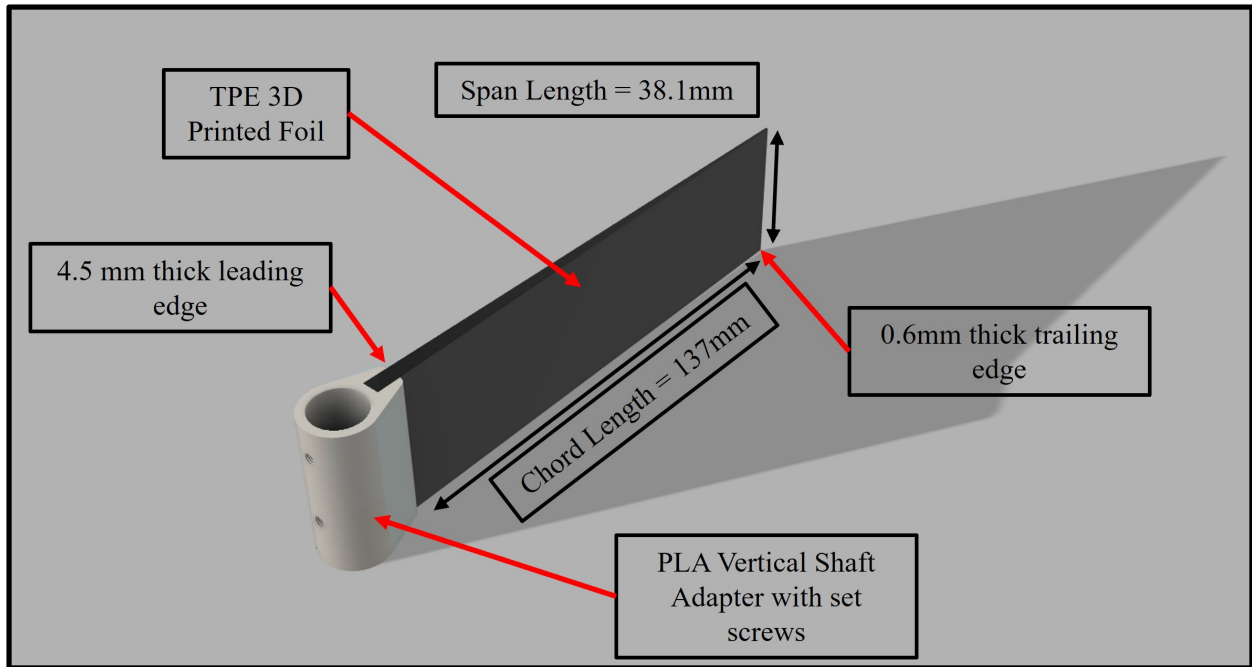


Figure 3.7: 3D printed TPE Foil geometry and setup with 3D printed PLA shaft connection part

data to the frequency domain with Fast Fourier Transforms (FFTs) and calculated the FRF by dividing the frequency domain velocity response by the frequency domain acceleration input. A summary of this process is shown in 3.8(a) and (b). For scans used to determine FRFs, the DSA provided the shaker with a "periodic chirp" sinusoidal voltage signal that linearly increased in oscillation frequency from 1 to 20Hz over 16 seconds. 800 FFT lines were used resulting in a frequency domain resolution of 0.125 Hz. The resonant frequencies were identified within the FRF by importing the data into MATLAB and applying a peak-picking peak method to find local maxima in the FRF. A 10-point smooth was applied to the data to increase the accuracy of the peak-picking algorithm. The damping ratios and Q criterion were directly measured from the DSA, which used a modal curve fitting toolbox.

The PSV 400 scanning laser head vibrometer directly measured the velocity of the wing at 9 point locations. The points were defined by an evenly spaced 3×3 grid distributed across

the wing. For each scan location, the frequency spectrum was found by complex averaging of 5 sweeps of a given signal range. The laser head was mounted on a tripod so that it could be maneuvered to be perpendicular to the wing with $\approx 61\text{cm}$ of horizontal separation. The laser was individually focused on each scan location prior to recording any data. After scanning all of the points, the average velocity data gathered across the entire grid was used to find the FRF for the foil at the driving amplitude with the above described method with the DSA. Single point data was also examined to ensure an accurate representation of the resonant frequencies at the foil tip as the scan location had an effect on measured resonant peak locations. The scanning head was also used to find the FRF for the mounting rod alone, where the frequency range was increased to range from 20-100Hz. In this case, 7 scan locations were equally spaced along across the shaft's 36cm length, and the same procedure was applied in measuring the output velocity and normalizing by input acceleration. This was done to ensure the partially submerged shaft did not have any resonant peaks within the range studied for the wing. The hollow rod was found to have a first resonant peak at 69Hz,

or approximately one order of magnitude greater than the first resonant frequency measured for the wing, which would have a negligible effect on the wing's motion at the lower driving frequencies.

The DSA PSV Presentation software was used to visualize 3D and 2D mode shapes of the foil from the 9 scanned data points distributed across the foil. The presentation software has a built in function to illustrate the velocity and acceleration data with a colorplot animation which was used to illustrate the first and second bending modes of the foil, as well as bending shapes between the first and second natural frequency where varied amounts of combined bending modes were visualized.

Due to having an open loop system where only an input voltage is provided to the shaker, the heaving amplitude of the shaker varied with frequency. An example curve of physical distance traveled for a constant voltage is shown to show the variation in amplitude due to changes in frequency from the shaker in figure 3.9(a) with the solid horizontal black lines representing the desired heaving amplitude. This was inevitable for the current configuration as forces generated by the foil would alter the final displacement of the shaker. As shown, only a 0.5Hz change in frequency results in a change in heaving amplitude from $\approx 1.5cm$ to $\approx 1.75cm$. To combat this issue a constant heaving amplitude was confirmed through image tracking with the PIV image sets using an in-house MATLAB edge detection script to ensure constant displacement as the frequency was varied. A point within 2.0 cm of the LE of the foil was used for tracking as no significant amount of bending was occurring at this chord wise location and would be representative of the motion imposed on the foil. For this analysis, constant sine signals were provided to the shaker with a single input voltage and frequency. At each driving frequency, a range of input voltages were provided to the shaker to iteratively reach a constant shaker displacement based on image tracking as shown in 3.9(b). Note only 3 frequencies at one heaving amplitude are shown for clarity, but this process was completed for the entire parameter space. The variation in voltage used to

achieve constant heaving amplitude ranged from $\pm 7.5mV$ for $A = 0.5mm$, up to $\pm 12.0mV$ for the $A = 1.5$ and $3mm$ for the range of frequencies from 1.5-6Hz. Across all trials, the 1Hz voltage needed to be substantially increased ($\approx 30 - 50mV$) to achieve constant heaving amplitude. This may have been due to either the shaker primarily being designed to operate at higher frequencies with low mass loading, or having to overcome static friction within the support block and acrylic guide plates at the low frequency.

Velocity was found for each run by numerically differentiating the positional data using a first-order finite difference scheme to ensure equal positive and negative velocity peaks. An example position and velocity dataset for $A = 3mm$ at $f = 4$ Hz is shown in figure 3.10. Note that given the finite difference scheme is sensitive to slight variations in position, this velocity data was only used as a gut check to confirm equal positive and negative velocities where the scanning laser vibrometer was used to measure the output velocity otherwise. This was only of concern for the constant driving frequencies provided to the foils for the fluid dynamics analysis as the FRFs were normalized by the input acceleration data.

Heaving Foil Particle Image Velocimetry

PIV was performed on the oscillating foils using the shaker setup as described in section 3. A Chronos 21-HD camera was mounted underneath the tank on extruded aluminum rails and recorded images at 250Hz with a resolution of 1080×1920 pixels. The DSA was used to drive the shaker for PIV at different input sine wave voltages with a single frequency for each run. The DSA function generator was used instead of a traditional independent function generator to eliminate any sources of error in input voltages, and thus physical distances, when correlating the mode shapes and resonant frequencies measured with the scanning laser relative to the fluid wake structures measured in PIV. Hollow glass sphere particles with mean diameter of $10 \mu m$, and average density of 1.10 g/mL were used to seed the tank (Potters' Industries Spherichel, 110P8). Two 532nm continuous wavelength

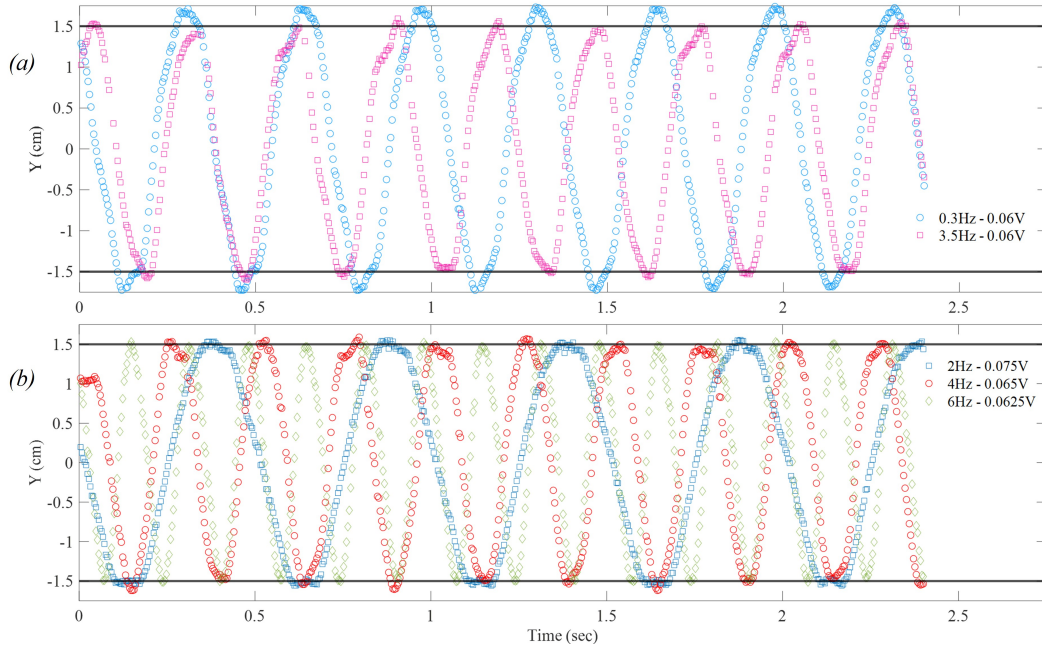


Figure 3.9: Shaker displacement tracking at the foil leading edge. Solid black lines represent the desired heaving amplitude. (a) Constant voltage provided to the shaker (b) Voltage varied systematically to get constant heaving amplitude at the foil leading edge.

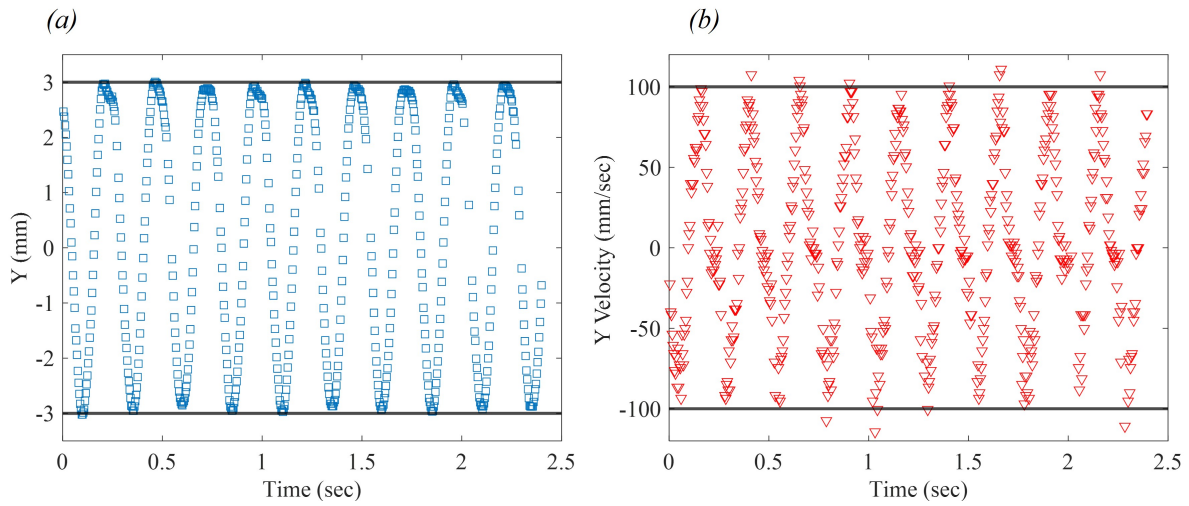


Figure 3.10: Shaker displacement tracking at the foil leading edge. (a) Positional data. (b) Velocity data from the first-order finite difference of position.

lasers were mounted on the sides of the tank to illuminate a 1.5 mm horizontal sheet passing through the center of the foil's span. Two lasers were used to provide additional light to the high-speed camera which had a short exposure time and to illuminate both sides of the foil to prevent any dark regions where particles would not be illuminated on one side of the foil, which would be problematic with a one-sided light source. Cross-correlation of the image pairs was completed using a multiple pass interrogation method with successive reductions in window sizes using the open-source software PIVlab [81, 82]. A region of interest of 1300×1080 pixels was used to narrow the field of view to the wake of the foil and eliminate regions without fluid velocity to reduce computation time. For a total physical field of view of $10.1\text{cm} \times 8.42\text{cm}$, a final interrogation window size of 32×32 pixels with 50% overlap was chosen. This resulted in vector spacing of 1.25mm or $0.032S$. The time interval between image pairs, Δt , was 0.004 seconds and the particle motion between image pairs was limited to approximately $1/4$ of the window size to ensure the accuracy of the cross correlation [85]. The frame rate and window size were selected based on the particle motion of the highest velocity trials, i.e. higher frequency and amplitude, to ensure all of the fluid motion would be accurately captured with the current PIV settings.

The foils were oscillated for 10 seconds before recording image sets to allow for the wake to reach quasi-steady state dynamics. For each combination of heaving amplitude and frequency, one trial of 20 seconds was completed. The time average mean wake dynamics were found by averaging 4 cycles of motion based on the driving frequency (f).

The input frequencies (f) were varied from 1-6Hz in 0.5Hz increments, and the heaving amplitudes were 0.5, 1.5, and 3mm. Reynolds Number was calculated using $Re = \frac{\rho_f(2\pi f A)C}{\mu}$ and ranged from 411-14810 across the parameter space. Provided that the foil motion was occurring within the quiescent tank with no cross flow the characteristic velocity chosen was $(2\pi f A)$. A summary of the parameter space for this study is shown in 3.2, as well as relevant length scales used for the wing design.

Parameter	Value
Heaving Amplitude (A) (mm)	0.5, 1.5, 3
Input Frequency (f) (Hz)	1-6 by 0.5 increments
Chord Length (C) (m)	0.138
Span Length (S) (m)	0.038
Taper Ratio (α) (m)	0.83
$Re = \frac{\rho_f(2\pi f A)C}{\mu}$	411-14810

Table 3.2: Plunging Foil Experimental Parameters

Traveling Wave Quantification:

The traveling wave index (I) was calculated for the varied heaving amplitudes to quantify the deformation as either standing ($I = 0$) or traveling ($I = 1$) waves. The traveling wave index was calculated using complex orthogonal decomposition (COD) as outlined in [26]. In short, we transform positional and time data into the complex domain with a Fast Fourier Transform (FFT), transpose the complex signal, and find the correlation matrix R . To complete the COD we take the eigenvectors and eigenvalues from the correlation matrix to gain insight into the complex modes present on the structure, where the eigenvalues represent the prevalence of the corresponding eigenvectors. Then we find the real and imaginary parts of the first column of eigenvectors, assuming that only the first mode is prevalent in the foil behavior. Lastly, we take the inverse of the condition number of the real and imaginary parts of the first mode to get the traveling wave index (I) [26].

In order to find I , several distinct positions on the wing needed to be tracked in time. Adapting the code used to verify the shaker movements, a small window is additionally looped over to track small portions of the wing chord. After tracking one complete oscillation cycle, the window was moved down the wing chord, and the time series data was looped over

until the entire chord was tracked. Initially, 26 evenly spaced points were tracked, and there was a 0.01 change in calculated I in reducing the measurement count to 13 points. As such, for subsequent trials, 13 points were tracked along the wing chord to reduce computational time. Additionally, it was found that there was a negligible difference (± 0.04) in I when running just over one complete oscillation compared to multiple periods of oscillation through the code. As such, I was quantified with displacement data from just over one complete oscillation period to reduce the computation time, particularly in the image tracking step.

FORMATION OF MULTIPLE VORTEX RINGS
FROM PASSIVELY FLEXIBLE NOZZLES

Contribution of Authors and Co-Authors

Author: Brysen J. Mitchell

Contributions: Performed research presented in the following manuscript. Helped design experimental setup. Manufactured nozzles. Quantified nozzle material properties. Measured flow fields using PIV. Processed PIV data. Generated plots and interpreted data. Wrote manuscript.

Author: Sarah E. Morris

Contributions: Oversaw and provided mentorship in all project components. Funded project. Edited and submitted the current manuscript.

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Abstract

This study examines the effect of nozzle flexibility on vortex ring formation at $Re = 1000$. The flexible nozzles impart elastic energy to the flow, increasing the hydrodynamic impulse of the vortex ring dependent on the input fluid acceleration and the initial nozzle tip deflection (predicted by the measured nozzle damped natural frequency). When these timescales are synchronized, the output velocity and hydrodynamic impulse of the vortex ring are maximized. Vortex ring pinch-off is predicted using the output velocity for each nozzle and is confirmed with closed FTLE contours. The lowest tested input formation number, $L/D = 1$, generates a greater increase in impulse than $L/D = 2$ and $L/D = 4$, due to a higher relative increase in total ejected volume and by remaining in the single vortex formation regime. At $L/D = 2$ and $L/D = 4$, multiple vortex structures are observed due to the interplay of the counter-flow generated by the nozzles re-expanding and the steady input flow. At the end of the pumping cycle, during fluid deceleration, the flexible nozzles collapse. This helps in suppressing unfavorable negative pressure regions from forming within the nozzle, instead expelling additional fluid from the nozzle. Upon reopening, beneficial stopping vortices form within the nozzles, with circulation correlated to nozzle stiffness. This highlights a secondary optimal stiffness criterion that must be considered in a full-cycle analysis: the nozzle must be compliant enough to collapse during deceleration, yet remain as stiff as possible to reopen quickly to maximize efficiency in refilling.

Introduction

Vortex rings are ubiquitous across many scales, playing key roles in systems such as starting jets for propulsion [47], aquatic locomotion in organisms such as fish [1], squid [2, 6, 33], salps [54], and jellyfish [14, 16, 31, 39]. They are also seen in respiratory flows such as coughing [76], and cardiac flows in the left ventricle [70]. A distinguishing feature

of many of these systems is that fluid is pushed out of a flexible structure which is linked to high efficiency due to the interaction of the flexible structure with the surrounding fluid [83]. Dabiri [16] proposed that flexible living structures, such as those found in marine animals and the heart, operate under optimal conditions of vortex formation, making them highly efficient. Therefore, understanding the dynamics and formation of isolated vortex rings generated from flexible structures could provide further insight into engineering higher efficiency systems that leverage this phenomenon.

Vortex rings are typically generated in experiments by expelling fluid through a sharp-edged nozzle or orifice into another larger body of fluid using a piston. The formation process of vortex rings from rigid nozzles has been well studied. Didden [25] identified several key parameters governing vortex ring formation, including the nozzle diameter (D), the piston stroke length (L), and the velocity history of the piston motion $U_p(t)$. For short piston stroke to diameter ratios $\frac{L}{D} = \frac{U_p(t)t}{D}$, a single vortex ring forms, entraining all the expelled fluid given an impulsive piston motion. [32] identified a limiting stroke ratio ($\frac{L}{D}$) ranging from 3.6 to 4.5 at which a single vortex ring is formed, and any larger stroke ratio results in additional expelled fluid forming a trailing jet behind the vortex ring, making it a limited process. Further studies have shown that different nozzle geometries and velocity flow profiles can alter the critical $\frac{L}{D}$ value at which a vortex ring will pinch-off and no longer gain circulation [18, 32, 46], Dabiri and Gharib [18] demonstrated that by forcing a circular nozzle exit to reduce in area as fluid was being expelled, vortex ring pinch-off could be delayed until $\frac{L}{D} \approx 8$, due to changes in the output velocity and shear layer development. Krieg and Mohseni [46] found that pinch-off is determined by the characteristic velocity of the vortex ring relative to the feeding velocity of the liquid. Similar to Dabiri and Gharib [18], Krieg and Mohseni [46] showed that vortex ring pinch-off could be delayed up to a value of $t^* \approx 8$ for a continually accelerating feeding velocity so that the vortex ring could not outpace the feeding source. Their work provided valuable insights into predicting pinch-off timing under time varying

output velocity programs.

Hydrodynamic impulse can be used to quantify the fluid thrust generated by a vortex ring, which is the summation of fluid impulse from the change in fluid inertia and fluid pressure [73]. The total thrust supplied by a starting jet is primarily controlled by the momentum flux, and secondarily by the pressure change at the nozzle exit. The change in momentum flux is directly affected by the vortex generator as it is the integration of $\rho_f U_e^2$, where U_e is the fluid velocity distributed across the nozzle exit being produced by the pump or propeller. For a rigid nozzle, this cannot be easily altered without changing the input to the system from the vortex generator. However, the pressure change, $p - p_\infty$, where p_∞ is the ambient fluid pressure, is affected by several factors including the fluid acceleration, nozzle geometry, and duration of the velocity being expelled [28, 45, 47]. These factors are all closely related to the vortex ring formation process and help describe the increase in impulse by forming a single vortex ring. It was determined that the maximum thrust normalized by momentum flux could be achieved at $t^* \approx 4$, for an impulsively started piston, due to the increase in pressure rise at the nozzle exit when a vortex ring pinches off [47]. Notably, thrust could be increased with a faster fluid acceleration for the same t^* value, as would be expected based on the increased fluid momentum [47]. Gao et al. [28] stated that the total impulse depends on the fluid deceleration as well, which can create a stopping vortex near the nozzle exit which favorably contributes to the pressure impulse. Additionally, Yin and el Hak [89] demonstrated that for a pumping jet style propeller, refilling the body creates positive momentum and a stopping vortex within the body cavity.

When applied to aquatic vehicles, it has been demonstrated that pulsed jet propulsion can achieve a much higher efficiency compared to steady-state jets of equivalent volume. The benefits stem from the added mass and entrainment effects from forming a series of vortex rings, which depends on formation number, fluid acceleration, and Reynolds number [7, 8, 16, 48, 62, 63, 75]. The increase in efficiency associated with creating sequences of

vortex rings is due to a combination of increased pressure impulse per pulse as well as the interactions of each pulse with one another [48]. However, Ninh et al. [64] showed the vortex-vortex interactions of a pulsed jet could either increase circulation by as much as 10% or reduce it by 20% depending on the spacing of each vortex ring with one another. For instance, Xu and Dabiri [87] showed that controlling contraction frequencies in jellyfish with micro-controllers could triple their swimming speed, while only creating a twofold increase in the cost of transport, likely due to beneficial vortex interactions. In general, jet-powered aquatic robotics show promise in that the impulsive motion of a starting jet can provide thrust almost instantaneously compared to several seconds for a traditional propeller to reach the desired thrust [45]. This type of propulsion can enable low-speed maneuvering such as sideways translation, and zero radius turns (yaw), all because there is no external fluid manipulator simply an internal jetting cavity ([45]). Although there are many achievements in terms of jet propelled robotics, there is limited research into the effect of utilizing flexible structures to increase efficiency and maximize thrust per pulse from these types of aquatic vehicles.

Jellyfish and squid have been shown to be among the most efficient swimmers, attributed to their flexible, deformable bodies creating desirable movement kinematics [14]. Jellyfish locomotion encompasses three parts: suction thrust during contraction, passive energy recapture during relaxation, and a wall effect created by the interaction with counter-rotating vortices left in their wake [31]. Squid exhibit similar swimming kinematics as well. [3] showed that squid utilize two distinct modes of locomotion, alternating between small bursts of fluid that create individual vortex rings, and larger jets that generate multiple vortex structures at the expense of efficiency for additional force. Xiaobo et al. [86] numerically demonstrated that in the case of a soft bodied jet propulsor, with geometry similar to a squid, sucking liquid into the body has the same effect as pushing water out by increasing positive momentum within the body, which is likely related to a squid's swimming efficiency. Numerical simulations of jellyfish revealed that synchronizing contractions with the natural wave speed of the bell

margin results in a maximum thrust efficiency, related to the resonant frequency of varied stiffness bodies [38].

The impact of employing flexible flaps and hinges to manipulate the flow produced by starting jets has been studied as well. Many nature-inspired researchers have shown that a specific amount of flexibility enhances thrust generation or efficiency in terms of a flapping wing or fin model [21, 24, 43, 51, 52, 58, 59, 67]. All of these studies indicate that a specific level of flexibility increases thrust production or power efficiency in flapping wing or fin models, attributed to the alignment of shed vortices and surface deformations. However, the number of studies relating flexibility to thrust generation from a starting jet is still somewhat lacking, with most findings showing that indefinitely decreasing stiffness increases thrust efficiency. Das et al. [20] showed that flexible flaps installed at the outlet of a rectangular channel could amplify fluid impulse by a factor of two while utilizing the same energy input, due to rearrangement of vorticity of the jet. Jung et al. [41] demonstrated that by using evertting flexible elastic sheets at a nozzle exit, the fluid impulse of a jet could be enhanced by as much as 14 times, with increased bending rigidity of the sheets.

Studies of flow within a flexible tube have been investigated but with very limited application to jet or vortex flows. The majority of studies focus on flexible tubes with rigid circular supports at either end [44, 53, 68]. Through the use of shell theory coupled with hydrodynamic equations, the natural frequency and vibration mode of a submerged cylindrical shell can be predicted [44]. The hydrodynamics and inertial effects within the thin shell significantly alter the frequency and mode shapes resulting from the thin shell, but this can be accounted for using a mass and speed ratio relation of the shell and the fluid [68]. Additionally, the length to radius ratio (L/R) and wall thickness to radius ratio (h/R) have a large impact on the stability of the resulting deformations relative to the velocity of the fluid being expelled [65]. Specifically, Paak et al. [65] showed that for clamped-free boundary conditions, the output velocity can excite periodic, multi-mode, or chaotic

vibration depending on the velocity ratio $U = u_b/c_s$ where u_b is the bulk fluid velocity and c_s is the wave speed on the cylinder surface. Choi and Park [10] found that there is an optimal stiffness for a circular nozzle to maximize thrust for jet flows, rather than indefinitely increased thrust for a decreased stiffness. Their work derived an optimal flexibility condition, relating the stiffness of the nozzle to the acceleration of the fluid being expelled. In a subsequent study, their work showed that these concepts could be applied with single pulsed jet flows [11]. However, these concepts have yet to be analyzed with respect to the wake formation of single vortex rings to maximize pressure impulse and thus ejected volume. Additionally, the effect of stopping vortices has not yet been accounted for in these models in terms of finding an optimal stiffness relative to the fluid generation apparatus.

In this study, we experimentally investigate vortex ring formation through orifices of varied flexibility, aiming to maximize thrust (circulation and impulse) with a fixed kinematic input to the nozzle. An alternative optimal flexibility condition is proposed based on measured material properties for the flexible nozzles and their corresponding damped natural frequencies relative to the fluid acceleration in Sections 4 and 4. The measured circulation is related to the effective formation length ($\frac{L}{D_{eff}}$) from the different nozzles of varied stiffness and compared with findings of previous literature in Section 4. In Section 4, the vortices generated by the different flexible nozzles are studied using Finite Time Lyapunov Exponent Fields (FTLEs) to experimentally determine primary vortex ring pinch-off and verified by the predicted pinch-off time from the work of Krieg and Mohseni [46]. The vorticity distribution between the primary vortex ring is delineated from the rest of the flow and used to further understand the optimal conditions for the current flow characteristics. Lastly in Section 4, particle image velocimetry is performed within the flexible nozzles to quantify the stopping vortex formed at the end of the nozzle deformations, which provides a fuller picture of the performance of these nozzles in a pulsed jet framework.

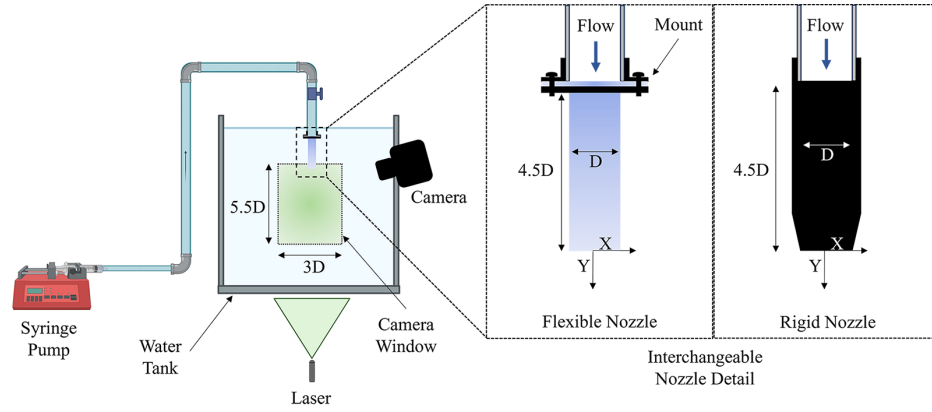


Figure 4.1: Schematic of the experimental setup used for generating the vortex flows with NE-1000 Aladdin pump. A 1mm thick laser sheet is used to illuminate a 2D plane of the tank. Interchangeable nozzle detail is also shown for mounting the rigid and flexible nozzles.

Methods

Experimental Setup

Experiments were conducted in a $46 \text{ cm} \times 46 \text{ cm} \times 46 \text{ cm}$ free-surface water tank filled with water to a height of 41 cm. An Aladdin NE-1000 syringe pump was used to generate the vortex flows. A 140 mL Monoject syringe was connected to a 1" SCH40 PVC pipe with inner diameter of 2.66 cm with flexible tubing. Nozzles with diameter ($D = 2.55 \text{ cm}$) and length ($L_{nozzle} = 11.5 \text{ cm}$) were installed at the end of the pipe to eject fluid downward in the center of the tank, 14 cm below the water surface. Further description of the nozzle adapters will be discussed in Section 4. A schematic of the experimental setup is shown in figure 4.1. The origin of the coordinate system used for calculations and defining coordinate directions is defined by the center of the nozzle exit with the positive Y direction oriented vertically downward in the flow direction, as shown in figure 4.1 nozzle detail.

The syringe pump velocity profiles were defined using the Windows Command Prompt Program, in combination with an Excel spreadsheet to format the input data, to control the velocity (U_{piston}) and duration (t_{cycle}) of the syringe piston motion. Three syringe piston

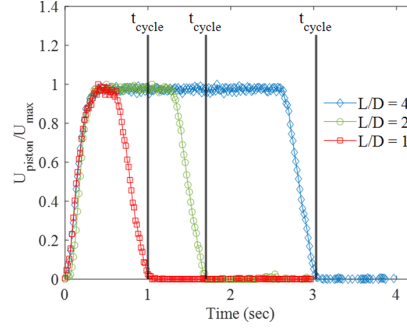


Figure 4.2: Measured syringe pump piston velocity by syringe motion normalized by the programmed maximum velocity for the pump (U_{max}).

velocity programs were selected corresponding to formation lengths, $\frac{L}{D} = \int_0^{t_{cycle}} \frac{U_{piston}(t)}{D} dt = 1, 2, 4$, ensuring the rigid nozzle would produce a single vortex ring with negligible trailing wake at the maximal ejected volume [32]. Each profile had an impulsively started piston motion measured at the syringe pump. The input diameter-based Reynolds Number ($Re_D = \frac{U_{piston} D_{piston}}{\nu}$), where ν is the kinematic viscosity of room temperature water, was held constant at 1000. The programmed piston velocity profiles were verified by imaging the rear edge of the piston and tracking the displacement over time using an in-house MATLAB script. In brief, the tracking was accomplished by converting the videos to images, reducing each image to the area of interest, passing a Canny Edge detection filter over the images, and binarizing the images using a threshold value [9]. Each piston velocity profile was imaged 5 times to ensure repeatability and was averaged over these trials and is plotted in figure 4.2. The mean steady state piston velocity data for each $\frac{L}{D}$ is 1.68 cm/sec with a 95% confidence interval of ± 0.01 cm/sec from a one-sample t-test.

Nozzle Construction and Characterization

A rigid nozzle was 3D printed out of PLA plastic with a layer height of .03mm. The rigid nozzle was manufactured to be the same length ($L_{nozzle} = 11.5$ cm) as the flexible nozzles, with the only difference being a beveled edge of 25 degrees at the exit. This nozzle design

was selected to be consistent with previous studies in vortex ring literature [18, 32, 47].

The flexible nozzles were molded from SmoothOn SortaClear 40A two-part liquid silicone. The selected nozzle construction method was adopted from Choi and Park [10], using a pour mold method over a 2.55 cm diameter (D) aluminum rod press fit into a 3D printed base to create the mounting surface for the flexible nozzles. Figure 4.1 shows a complete flexible nozzle with the integrated mounting surface, and figure 4.3(a) shows an isometric view of the flexible nozzle geometry. After curing, the nozzles were cut to a length (L_{nozzle}) of 11.5 cm measured from the top of the nozzle base to the edge of the nozzle exit, resulting in an aspect ratio ($\frac{L_{nozzle}}{D}$) of 4.55. This aspect ratio was chosen to ensure all of the fluid ejected was initially contained within the nozzle. This is in contrast to Choi and Park [10, 11], where an aspect ratio of 2 was used (ejected fluid slug not initially fully contained within flexible nozzle). The stiffness of the flexible nozzles was controlled by adding different mass percentages ($\%ma = 15\% - 50\%$) of SmoothOn Silicone Thinner relative to the base liquid silicone mixture.

The flexible nozzles were mounted to the experimental setup using a 3D printed part that clamped onto the base of the nozzle using bolts, shown in figure 4.1. This setup was used to reduce stress concentrations from forming at the edges of the nozzle which could potentially affect the nozzle deformation behavior. Additionally, the mount created a consistent rigid boundary condition between the different nozzles to ensure the repeatability of the experiment. After a nozzle was clamped onto the mount, it was installed onto the outlet pipe using an interference fit. The rigid nozzle was similarly installed with an interference fit on the outlet pipe.

For each nozzle, a representative nozzle was made to be sliced into strips to measure the mean nozzle wall thickness (h) using an Olympus BX60 microscope with a $10\times$ power lens. The wall thickness was measured at a total of 16 locations for each nozzle, at 4 stream-wise locations 2.5 cm apart and 4 equally spaced azimuthal locations around the nozzle

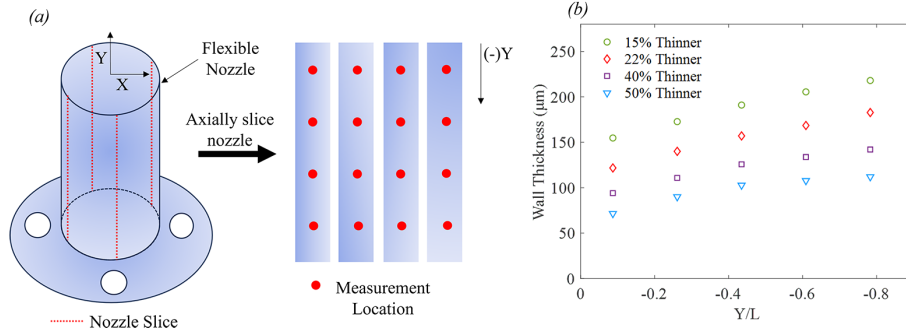


Figure 4.3: Nozzle wall thickness measurement schematic and variation in wall thickness. (a) Schematic indicating the cut sections used for measuring nozzle wall thickness. The nozzles were cut along their streamwise direction to create 4 strips that were measured at 4 streamwise locations along the nozzle’s length. (b) Distribution of measured nozzle thickness along the length (L) of the nozzles. The labeled % values in the legend correspond to the %ma of silicone thinner added to the mixture to mold each nozzle.

circumference at each stream-wise height as shown in figure 4.3(a). The wall thickness variation was negligible with a maximum azimuthal variation of 4.1% for the manufactured nozzles. The wall thickness varied approximately linearly along each nozzle’s length due to the pour molding manufacturing method. The spatial distribution of the wall thickness along the nozzle length (L_{nozzle}) is plotted in figure 4.3(b). The thickness decreased at an average rate of $17.7 \mu\text{m}/D$ along the streamwise direction of the nozzle. The total change in thickness was approximately constant between the different nozzles, thus the thickness was averaged over the measured locations for each nozzle. Mean wall thickness (h) varied from $188.4\mu\text{m}$ to $96.8\mu\text{m}$ across the different nozzles.

Young’s Modulus (E) was calculated by completing tensile tests on 5mm thick dog bone samples prepared with cross sectional geometry as described in ASTM D412-16 Standard Test Methods for Vulcanized Rubber and Thermoplastic Elastomers—Tension [5]. The tests were completed on an Instron 312 Series Frame with a 25 kN load cell. For each nozzle, 8 tensile tests were completed from two different batches of silicone, one batch from the silicone used for the experimental nozzle, and a second batch from the silicone used for the

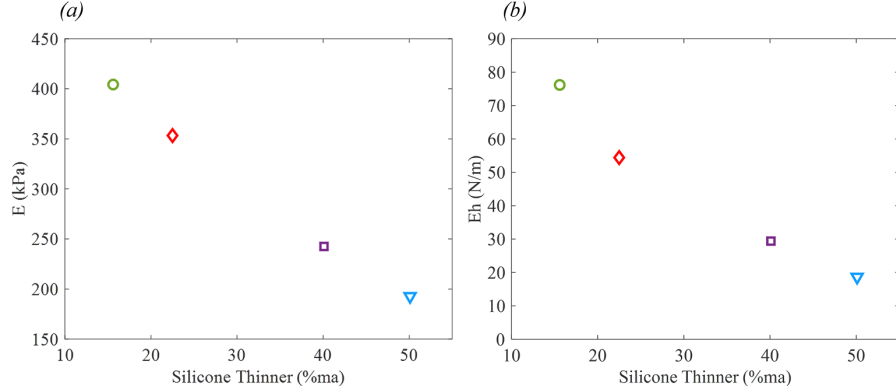


Figure 4.4: Material properties measured with varied %ma thinner (a) Young's Modulus (E) averaged over 8 samples from each %ma of silicone thinner added to the nozzle mixtures. (b) E multiplied by the mean wall thickness (h), defining the characteristic stiffness of the varied %ma thinner nozzles.

representative nozzle. The maximum variation in E between the two sets was negligible, with the maximum being 4.2%. Young's Modulus (E) varied from 404 kPa to 193 kPa as shown in figure 4.4(a).

The representative characteristic stiffness parameter was chosen to be Young's Modulus (E) multiplied by the mean wall thickness (h), which is plotted in figure 4.4(b). Overall, a set of 5 nozzles were tested with characteristic stiffness parameter $Eh = 19$ N/m, 29 N/m, 54 N/m, 78 N/m, and $Eh = \infty$ N/m (rigid). Table 4.1 summarizes the parameters studied in this experiment.

Velocity Field Measurements

Particle Image Velocimetry (PIV) was conducted to quantify the velocity vector fields beneath the nozzles. Images were sampled at 60 Hz with a resolution of 1080×1920 pixels. The camera was triggered to start simultaneously with the initiation of the pump using an in-house LabVIEW program via a MyDAQ (National Instruments). Hollow glass sphere particles with mean diameter of $10 \mu\text{m}$, and average density of 1.10 g/mL were used to seed the tank (Potters' Industries Spherichel, 110P8). A 532nm continuous wavelength laser and

Parameter	Value
Nozzle Diameter (D) (mm)	25.5
Nozzle Length (L_{nozzle}) (mm)	115.0
Characteristic Stiffness ($ Eh$) (N/m)	19, 29, 54, 76, ∞ (rigid)
Formation Length ($t^* = \frac{L}{D}$)	1, 2, 4
Re_D	1000

Table 4.1: Experimental Parameters

a convex cylindrical lens were mounted underneath the tank to illuminate a 1 mm vertical sheet passing through the center cross-section of the nozzle in either the XY or XZ plane as defined in figure 4.1, with the Z direction being out of the page. Cross-correlation of the image pairs was completed using a multiple pass interrogation method with successive reductions in window sizes using the open-source software PIVlab [81, 82]. For a field of view of $5.5D \times 3D$, a final interrogation window size of 64×64 pixels with 50% overlap was chosen. This resulted in a vector space of 59×32 vectors with a vector spacing of $.098D$. The time interval between image pairs, Δt , was .0167 seconds and the particle motion between image pairs was limited to approximately 1/4 of the window size to ensure the accuracy of the cross correlation [85]. As the flexible nozzles were found to deform about a preferential axis, PIV was performed both perpendicular (XY plane), and parallel (XZ plane), to this preferential axis to ensure axisymmetry in our results (see Appendix 4 for further details). For each nozzle, 10 trials were completed, with 5 trials from each plane. In subsequent plots, the average of these 10 trials is plotted unless otherwise noted. The mean maximum spatially averaged velocity measured $0.019D$ below the rigid nozzle outlet was calculated to be 3.97 cm/sec across all ejected volumes, with a 95% confidence interval of ± 0.1 cm/sec based on a one-sample t-test. All reported confidence intervals hereon represent the 95% confidence

interval based on a one-sample t-test. The flexible nozzles altered the measured exit velocity, which is discussed in Section 4, where the $Eh = 29$ N/m nozzle had the largest variation, with a mean maximum spatial averaged velocity of 6.40 cm/sec and confidence interval ± 0.3 cm/sec.

Nozzle Deformation Measurements

To track the side deformation of the flexible nozzles, the particles were removed from the tank, and the laser sheet was used to illuminate the longitudinal center cross section of the nozzle in the XY and XZ planes. The camera imaged the center plane deformations, which were used for nozzle tracking with an in-house MATLAB edge detection code. The same image processing steps were applied to these images as was used to determine the syringe pump velocity profiles. Two time sets were considered, forced vibration when the pump was active, and free vibration when the pump had stopped moving and the nozzle was freely oscillating. The bottom millimeter of the nozzle was used as a representative point to track the position over time. Fast Fourier Transforms (FFTs) were completed in MATLAB to find the repeated oscillation frequencies from the free vibration positional datasets. The single-side amplitude spectrum of the position data, or the occurrence of each frequency in the positive domain, was calculated using a sampling period of .0167 sec and sample length of 600 samples (10 seconds). The single sided amplitude spectrum was plotted to find a peak in frequency occurrences to estimate the damped natural frequency of (ω_d) of the nozzles. The deformation viewed from beneath the nozzle (YZ plane) was also imaged.

FTLE Fields and Ridges

The predicted material boundaries of the generated vortex rings were identified by detecting ridges within the Finite-Time Lyapunov Exponent (FTLE) field using the LCS Matlab Kit V2 by the Biological Propulsion Laboratory at the California Institute of Technology [17, 69, 74]. The FTLE fields are directly derived from the velocity fields

generated from PIV by either initiating fluid particle tracking forward or backward in time. This process yields both positive (forward) pFTLE and negative (backward) nFTLE fields. The FTLE field is computed by:

$$\sigma_T^t(\mathbf{x}) = \frac{1}{|T|} \ln \left| \frac{d\phi_t^{t+T}(\mathbf{x})}{d\mathbf{x}} \right| \quad (4.1)$$

where $\sigma_T^t(\mathbf{x})$ signifies the flow map of particles from their location at $\mathbf{x}$ at time t to $t + T$. $|T|$ represents the integration time used to track the particle motion, with $T < 0$, indicating backward time, and $T > 0$, corresponding to forward time. After obtaining the FTLE fields, regions of local maxima, or ridges, were identified to pinpoint regions where fluid transport is restricted or intensified. These ridges are known as Lagrangian Coherent Structures (LCS), which are valuable for analyzing vortex ring pinch-off. Negative LCS (nLCS), found from the ridges within a nFTLE field, indicate attractive material lines in the flow field, while positive LCS (pLCS), found from ridges within a pFTLE field, indicate repelling material lines. When a nLCS and pLCS combine to form a closed loop, it signifies the formation of a pinched-off vortex ring [74]. The presence of a pLCS on a vortex ring's rear edge suggests that further fluid entry into the enclosed structure of the vortex ring from the vortex generator is unlikely, and the vortex ring will not gain additional vorticity. The integration length $|T|$ chosen for the data analyzed within this paper corresponds to 1 second or 60 image frames. A step size of 1 and a time step of 0.0167 seconds per frame were used.

Results and Discussion

Changes in Flow Characteristics Due to Nozzle Flexibility

First, we examine how the development of the vortical structures in the flow fields is influenced by the fluid-structure interaction as the characteristic stiffness parameter (Eh) is varied to investigate whether an optimal flexibility condition exists for our given input flow

parameters. Figure 4.5(a) illustrates the flow development from the $Eh = \infty$ N/m (rigid) nozzle for $\frac{L}{D} = 2$ from (i) $t/t_{cycle} = 0.41$ to (iv) $t/t_{cycle} = 3$, where t_{cycle} is the time when the pump stops for each $\frac{L}{D}$. The nozzle position relative to the flow field is shown as the black trapezoid at the top of each frame. As the fluid is expelled from the nozzle, the shear layer rolls up, forming a single primary vortex ring (PV) as it enters the quiescent tank. The plots show a cross section cut across the middle of the ring, which is displayed as a single counter-rotating vortex pair of opposite sign vorticity. This is consistent with Gharib et al. [32], wherein the ejected fluid from impulsive piston motion will form a single vortex ring for $\frac{L}{D} < 4$. However, this behavior changes when Eh is sufficiently low enough (flexible enough) to perturb the input flow conditions.

Figure 4.5(b-e; i - iv) shows the flexible nozzle ($Eh = 76, 54, 29, 19$ N/m) flow fields, under the same input kinematic flow conditions from the pump ($\frac{L}{D} < 2$). As Eh decreases (becomes more flexible) the flow structures begin to deviate from the rigid case. For $Eh = 76$ N/m the vortex cores are slightly elongated, but become approximately circular after $t/t_{cycle} = 1.5$. It is understood that the alteration is due to small scale oscillations ($\approx 0.02D$) of the flexible nozzle. As Eh is lowered further, the deformation of the nozzles becomes an order of magnitude more pronounced (up to $\approx 0.19D$), multiple distinct vortex structures form, and the PV begins to significantly differ from the rigid case. For $Eh = 54$ N/m in figure 4.5(c), two cores form, where the secondary vorticity eventually leapfrogs through the PV core, but the two cores do not separate from one another. The formation of this second core of vorticity corresponds to the nozzle oscillations and rapid collapse at $t = t_{cycle}$, which is examined further below. For $Eh = 29$ N/m in figure 4.5(d), the PV separates from the nozzle prior to $t/t_{cycle} = 1$. A weak secondary vortex structure (SV) forms, and separates from the nozzle at $t/t_{cycle} = 1.5$. Notably the PV travels significantly further than the rigid case reaching $3.4D$ compared to $2.2D$ at $t/t_{cycle} = 2.5$, defined by the averaged location of peak vorticity between the positive and negative cores. For the most flexible

nozzle, $Eh = 19$ N/m, two separate vortex structures also form, but the SV is slightly larger than that produced by the $Eh = 29$ N/m nozzle. Interestingly, in this case, the PV travels a shorter distance of $3.1D$ below the nozzle at $t/t_{cycle} = 2.5$, compared to the slightly stiffer $Eh = 29$ N/m nozzle, implying an optimal stiffness condition greater than $Eh = 19$ N/m. This will be discussed further in Section 4. Similar trends are observed at the other ejected volumes ($\frac{L}{D} = 1, 4$), see Supplementary Material 1.

To compare the vortex rings produced by the different nozzles, we analyze cross-sectional cuts of the vortex rings generated by the $Eh = \infty$ N/m (rigid) nozzle and the $Eh = 29$ N/m nozzle, at $t/t_{cycle} = 2$ for $\frac{L}{D} = 1$. This simplifies the comparison by focusing on single PV structures as shown in the vorticity plots in figures 4.6(a) and (b). Figures 4.6(c) and (d) display the spatial distribution of vorticity and velocity components measured along the cut lines. These cut lines are defined by the vertical (Y) location of peak vorticity for each trial. As inferred by the distance traveled by the PV produced from the $Eh = 29$ N/m nozzle, the peak velocity is 2.5 times higher, and the peak vorticity is 2.8 times higher than the rigid nozzle case. Additionally, the wider spatial distribution velocity and vorticity suggest that the $Eh = 29$ N/m nozzle not only produces a stronger PV, but a larger PV compared to the rigid nozzle.

Next, we analyze the temporal variation of vortex spacing (b) and core diameter (a) for each nozzle, as plotted in figure 4.7(a) and (b) for $\frac{L}{D} = 2$. The primary vortex ring is tracked for this analysis, with b defined as the distance between the peak vorticity values of each core, and a is defined by the distance between the two points bounding 10% of the maximum vorticity, averaged between the positive and negative cores. As shown in figure 4.7(a), the flexible nozzles generate vortex rings with wider spacing, with each ring converging to an approximately constant spacing of $1.41 \pm 0.15 \frac{b}{D}$, compared to $1.19 \pm 0.13 \frac{b}{D}$ for the $Eh = \infty$ N/m (rigid) nozzle at $t = 3t_{cycle}$. The vortex ring core diameter (a) varies slightly with Eh , with the flexible nozzles all creating larger cores, as shown in figure 4.7(b). The $Eh = 54$,

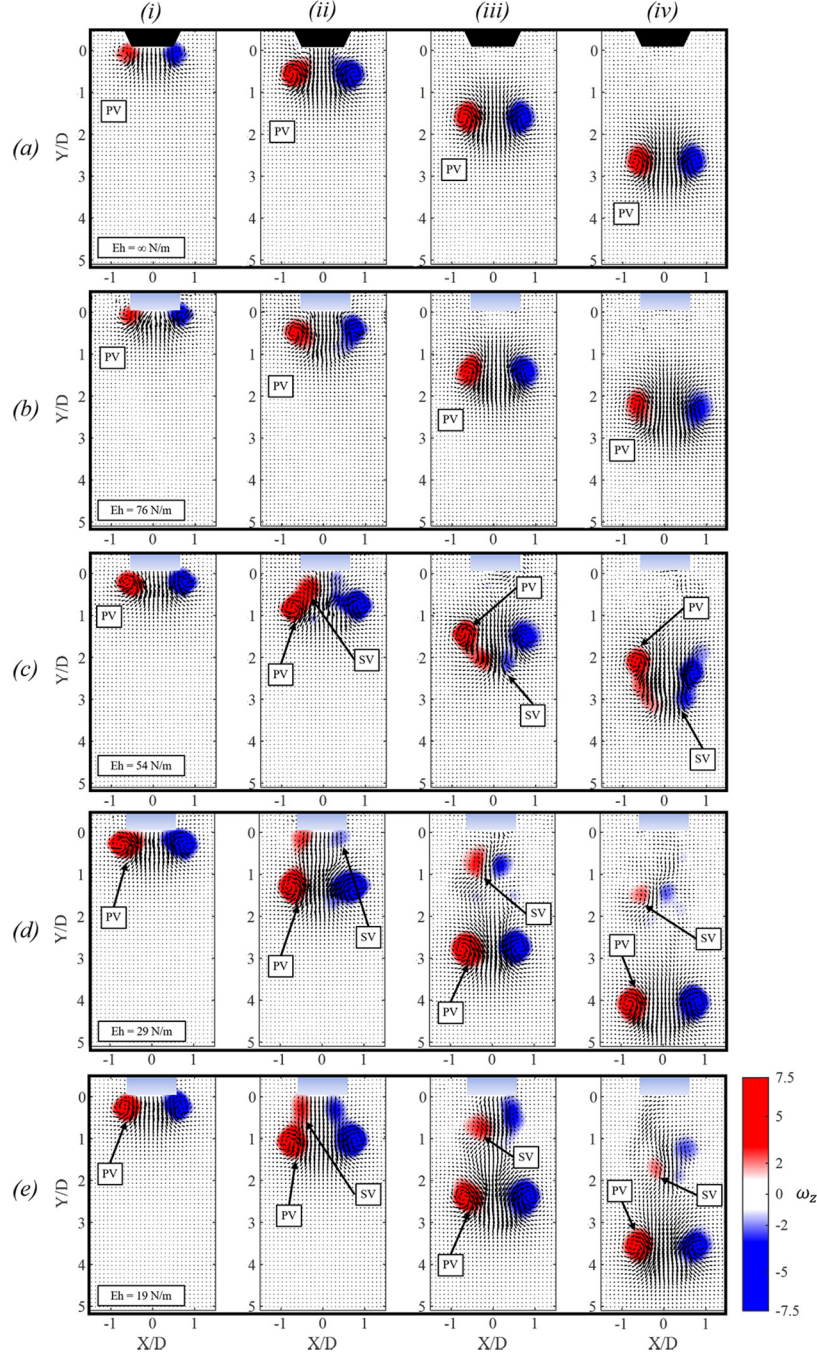


Figure 4.5: Vorticity and vector fields measured for (i) $t/t_{cycle} = 0.41$, (ii) $t/t_{cycle} = 1$, (iii) $t/t_{cycle} = 2$, and (iv) $t/t_{cycle} = 3$ for each nozzle given the same kinematic input from the pump for $\frac{L}{D} = 2$ (a) Rigid nozzle ($Eh = \infty$ N/m) (b) $Eh = 76$ N/m; (c) $Eh = 54$ N/m; (d) $Eh = 29$ N/m; (e) $Eh = 19$ N/m

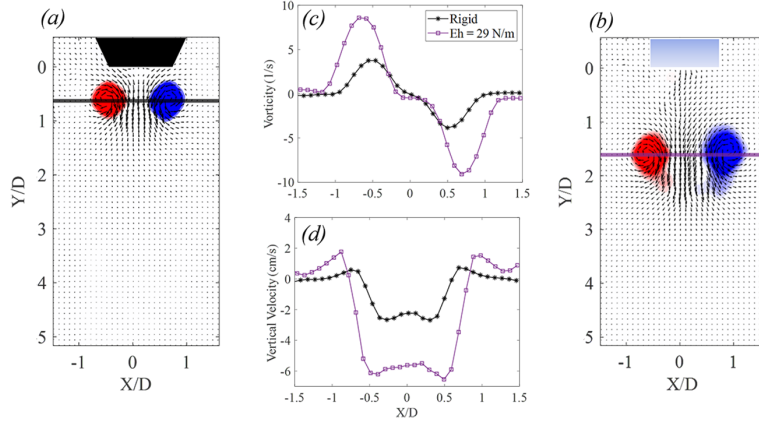


Figure 4.6: (a) Rigid nozzle PIV vector plot and vorticity at $t/t_{cycle} = 2$ for $\frac{L}{D} = 1$. The solid line represents a cut section of the vortex ring and corresponds to the centered plots of vorticity and velocity. (b) $Eh = 29$ N/m nozzle PIV vector plot and vorticity, with cut line selected by the point of highest vorticity. (c) Vorticity across the vortex ring cut plane for the rigid and $Eh = 29$ N/m nozzles. (d) Vertical velocity across the vortex ring cut plane for the rigid and $Eh = 29$ N/m nozzles. Legend is the same for (c) and (d).

29, and 19 N/m nozzles converge to a core diameter of $1.13 \pm 0.09 \frac{a}{D}$, and the $Eh = 76$ N/m nozzle converges to $1.01 \pm 0.09 \frac{a}{D}$, compared to $0.82 \pm 0.06 \frac{a}{D}$ for $Eh = \infty$ N/m at $t = 3t_{cycle}$. Here, the bounds are defined by the 95% confidence interval for $\frac{L}{D} = 2$. The other ejected volumes follow the same trends, but with smaller core diameter and spacing for $\frac{L}{D} = 1$, and larger for $\frac{L}{D} = 4$. Given the larger (b, a) , stronger (ω_z) , and faster moving vortex rings generated by the flexible nozzles, the velocity produced by these nozzles should scale accordingly.

The vertical velocity spatially averaged across the nozzle exit (U_e), measured $0.19D$ below the nozzles, is plotted for $\frac{L}{D} = 1$, in figure 4.8(a). As expected by the distance traveled by the PV in the vorticity plots in figure 4.5(a-e), U_e increases as Eh is reduced to a maximum at $Eh = 29$ N/m, and declines for $Eh = 19$ N/m. Furthermore, the time to reach the maximum U_e becomes delayed with decreased Eh . The $Eh = 19$ N/m nozzle does not accelerate to its peak velocity until $t/t_{cycle} = 1.0$, whereas the $Eh = 76$ N/m nozzle reaches its peak much quicker at $t/t_{cycle} = 0.36$.

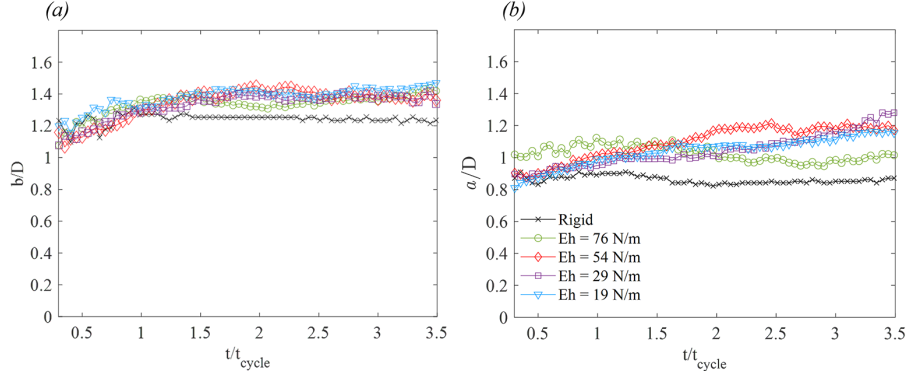


Figure 4.7: Primary vortex ring spatial parameters measured for $\frac{L}{D} = 2$. Points are plotted every .083sec for clarity. (a) Vortex spacing (b), normalized by the nozzle diameter (D) over dimensionless time normalized by t_{cycle} . (b) Vortex core diameter (a), normalized with the nozzle diameter (D), measured over time normalized by t_{cycle} . Legend is the same for (a) and (b).

To further examine the input flow conditions perturbed by the flexible nozzles, the output velocity from the rigid nozzle ($Eh = \infty$ N/m) is plotted for all $\frac{L}{D}$ values in figure 4.8(b). To compare the timescales of nozzle deformation with the input fluid flow, we define the time for the rigid nozzle to accelerate to 90% of U_e as $t_{acc} \approx 0.71$ sec, as shown in figure 4.8(b). Next, we examine how the nozzle deformations vary with Eh to understand the kinematics affecting the flow fields.

Generally, each of the less stiff (more flexible) nozzles, ($Eh = 54, 29, 19$ N/m), exhibited similar nozzle deformation kinematics. The deformation is characterized by 3 important time phases: 1) initial tip deflection, 2) periodic oscillations, and 3) nozzle collapse and return to original state. These three stages are plotted with overlaid time-colored plot images of the $Eh = 54$ N/m nozzle for $\frac{L}{D} = 4$ in figure 4.9(a-e). See Supplementary Material 2 for similar plots for each of the flexible nozzles. During phase 1, the initiation of deformation, a traveling wave, or series of traveling waves, propagates down the nozzle. In turn, the first oscillation inward generates the largest deflection at the nozzle tip while the pump is running, defined as t_{maxdef} . This motion is shown for the $Eh = 54$ N/m nozzle in figure 4.9(a) and (d). In

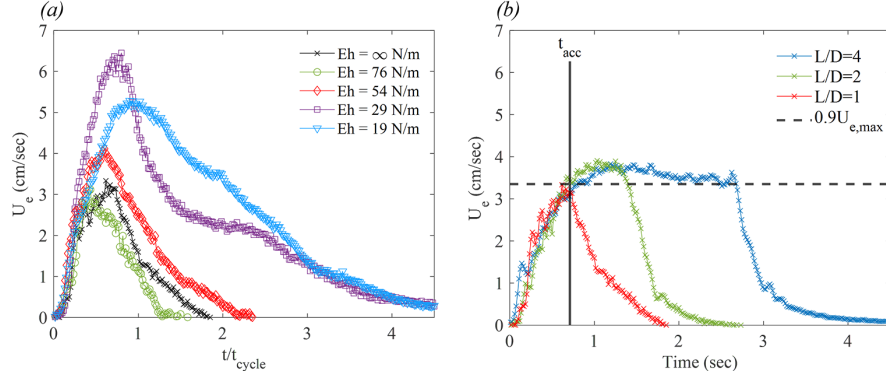


Figure 4.8: Temporal variation in vertical velocity spatially averaged across the nozzle exit (U_e) measured $.19D$ below each nozzle. (a) U_e for $\frac{L}{D} = 1$ across all Eh values for $(t/t_{cycle} = 0 - 4.5)$. (b) U_e for $Eh \approx \infty$ N/m (rigid), for all $\frac{L}{D}$ values. t_{acc} is defined as the time needed for the rigid nozzle to accelerate to 90% of its maximum velocity.

phase 2, $t_{maxdef} < t < t_{cycle}$, the nozzles return to their initial position but with an over correction, expanding into the opposite plane of the initial tip deflection. These oscillations continue at varying frequencies depending on Eh as shown in figure 4.9(b) and (e), until the pump turns off at $t/t_{cycle} = 1$ marking the end of phase 2. In phase 3, $t > t_{cycle}$, each less stiff nozzle ($Eh = 54, 29, 19$ N/m) collapses asymmetrically inward due to the decreased pressure from the quick decrease in velocity as the pump shuts off as shown from $t_{cycle} < t < 2t_{cycle}$ in figure 4.9(c) and (f). The $Eh = 54$ N/m nozzle tip immediately reopens after partially collapsing. In contrast, the $Eh = 29$ and 19 N/m nozzles fully collapse and remain in the collapsed position for 9 and 25 seconds respectively, before reopening and oscillating back to their original position. Each nozzle consistently collapsed on one preferential axis for every trial.

The $Eh = 76$ N/m nozzle was observed to have different kinematics than the less stiff nozzles. While the pump was moving, small scale ($\approx 0.02D$) axisymmetric oscillations moved the nozzle slightly inward and outward until the pump stopped. Similar to the other nozzles, the pressure change at the nozzle exit created a small peak in deflection ($\approx 0.04D$) but the $Eh = 76$ N/m nozzle did not exhibit any collapse. Afterward, it oscillated back and forth

until the viscous forces from the surrounding water damped the nozzle motion to a halt.

The bottom millimeter of the flexible nozzles was also quantitatively tracked. As expected, reducing the nozzle stiffness resulted in greater maximum deformation magnitudes at $t = t_{maxdef}$, ranging from $0.02D$ to $0.19D$ for the $Eh = 76$ and 29 N/m nozzles respectively. Although the $Eh = 19$ N/m nozzle is the least stiff, the peak deformation was lower at only $0.15D$. This will be discussed further in Section 4. The $Eh = 54$ N/m tip deformation is shown in figure 4.10(a), illustrating the previously described deformation of the nozzle's bottom millimeter. For reference, a plot illustrating the tip deformation of all the flexible nozzles is shown in figure 4.11(a). In figure 4.10(a), phases 1, 2, and 3 are delineated by orange, blue, and green shading. After the pump turns off, the oscillation behavior can be modeled as an underdamped free oscillation since no external forces are acting on the nozzle, other than the initial condition of starting in a collapsed state. This is highlighted with yellow shading, in figure 4.10(a and b). The natural frequency of the flexible nozzles is found by transforming the positional periodic motion of the free oscillations into the frequency domain using Fast Fourier Transforms (FFTs). The data used for this frequency analysis begins when the nozzle returns to its original position ($X/D = 0$) after collapsing due to the change in pressure. The data used for the FFT analysis was obtained from views from both the XY and XZ planes.

For each nozzle, the frequency spectrum reveals a clear peak, indicating resonance at these specific frequencies under free oscillation, with an example frequency spectrum shown in figure 4.10(c). This resonant peak corresponds to the damped natural frequency (ω_d) for each nozzle. A summary of the variation of ω_d with Eh is shown in figure 4.10(d), which is intuitive in that the stiffer nozzles have higher ω_d values as stiffness scales with natural frequency. These ω_d values can be incredibly useful in that they can be used to predict the timing of t_{maxdef} . The maximum variation in measured ω_d , was for the $Eh = 54$ N/m nozzle where $\omega_d = 0.737 \pm 0.013$ Hz based on the 95% confidence interval.

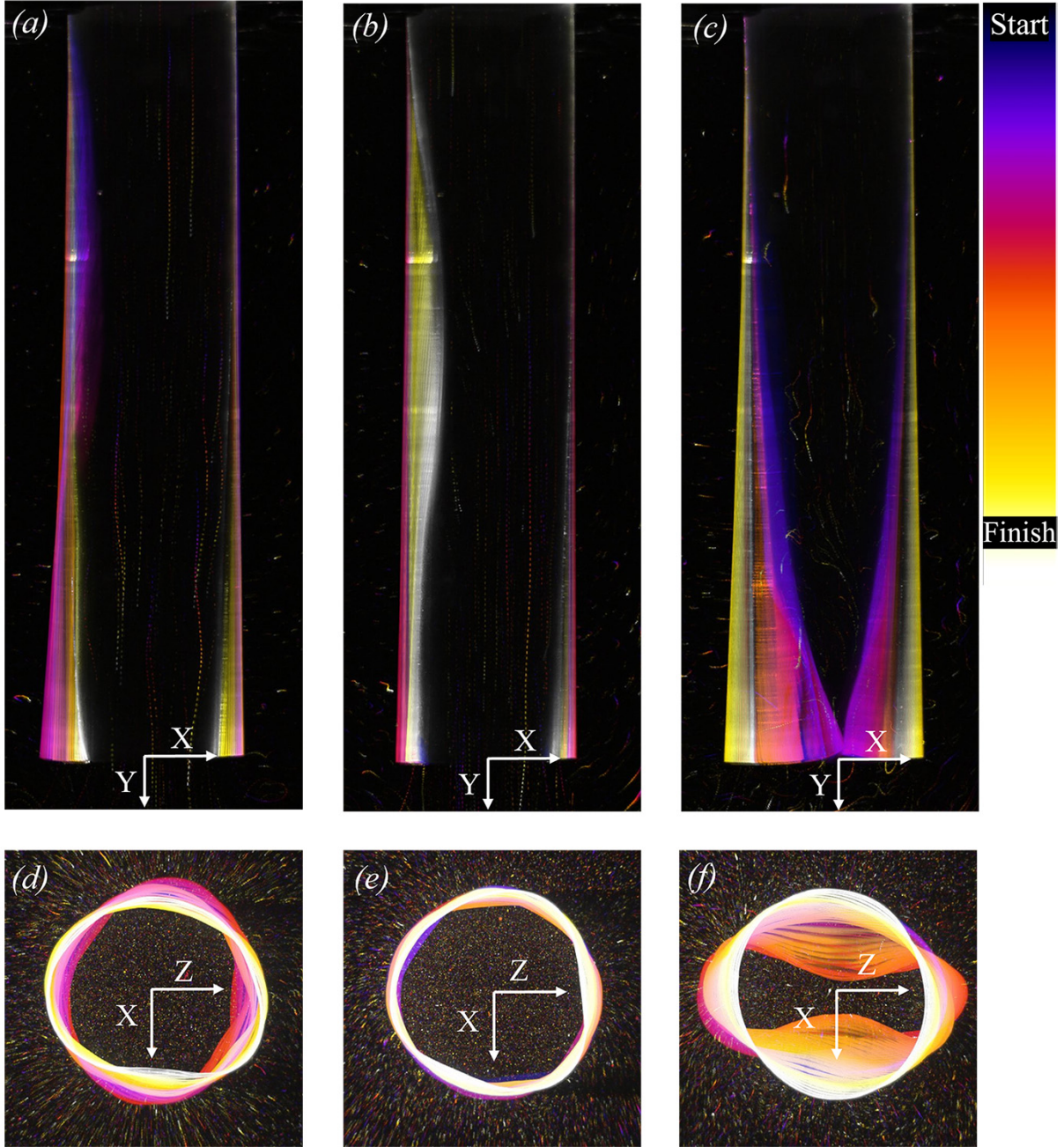


Figure 4.9: Imaging of the $Eh = 54 \text{ N/m}$ nozzle for $\frac{L}{D} = 4$. The color plot scale bar applies to each image, with the Start and Finish defined as follows in the sub figure descriptions. (a,d) Nozzle deformation for $t = 0 - t_{maxdef}$. (b,e) Nozzle deformation for $t = t_{maxdef} - t_{cycle}$. (c,f) Nozzle deformation for $t = t_{cycle} - 2t_{cycle}$.

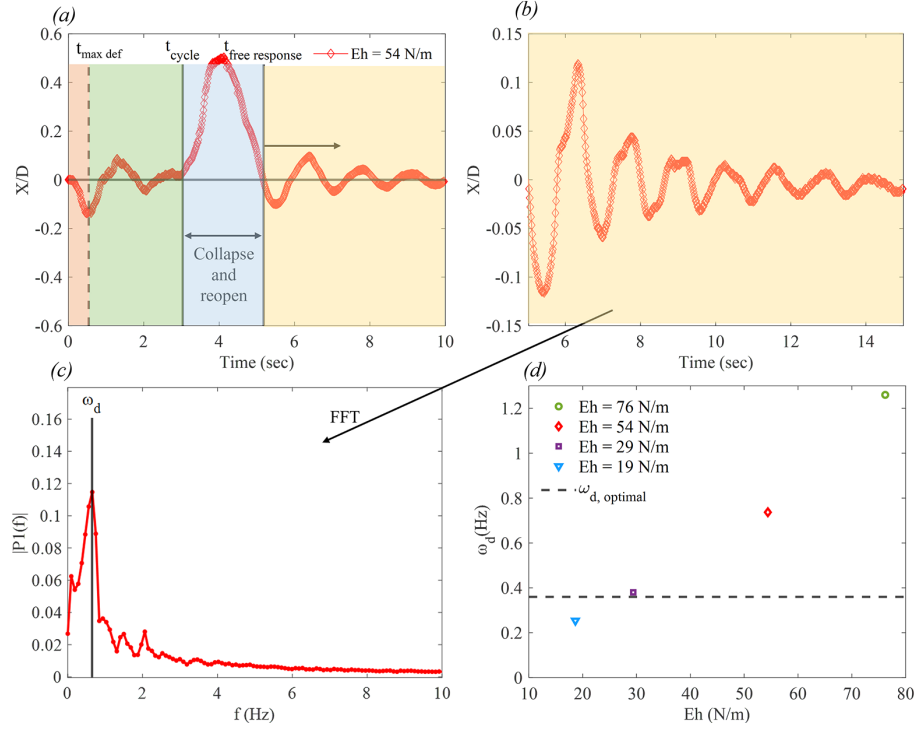


Figure 4.10: (a) Temporal variation of position (X/D) for the $Uh = 54$ N/m nozzle tip. The $t_{\text{freereponse}}$ line indicates the start of the data used to find the frequency spectrum. (b) Tracked nozzle deflections used for FFT to quantify the frequency spectrum. The start is initiated when the nozzle tip passes its original position ($X/D = 0$), after undergoing collapse due to the negative pressure gradient. (c) Frequency spectrum obtained from the positional data. ω_d is identified as the most prevalent frequency in the frequency domain. (d) ω_d for the different Uh nozzles found using the same process as outlined here.

Choi and Park [10, 11] proposed that synchronizing the maximum deformation with the input fluid acceleration, i.e. timing the release of elastic energy to the fluid, creates the optimal conditions for thrust from a starting jet. Based on this, we hypothesize that the optimal flow conditions can be predicted using the damped natural frequency of the nozzle, such that the optimal condition will occur when $t_{acc} = t_{maxdef}$, where t_{acc} is the time to reach peak fluid acceleration from the rigid nozzle, and t_{maxdef} is the time of maximum nozzle deformation while the pump is running. Using the damped natural frequency, ω_d , of each nozzle we predict t_{maxdef} to occur at $\frac{1}{4}$ of an oscillatory cycle, i.e. $t_{maxdef} = \frac{1}{4\omega_d}$. If $t_{acc} = t_{maxdef} = \frac{1}{4\omega_d}$, we can estimate the optimal nozzle damped frequency, $\omega_{d,optimal}$, for the present flow conditions (measured t_{acc}). Under the current flow conditions, $\omega_{d,optimal} = .36$ Hz. For the tested nozzles, the $Eh = 29$ N/m nozzle with $\omega_d = 0.38 \pm 0.008$ Hz is closest to $\omega_{d,optimal}$, shown in figure 4.10(d). This is consistent with our observations that the $Eh = 29$ N/m nozzle produces the strongest (optimal) vortex rings for the current input flow conditions.

Optimal Conditions, Fluid Impulse, and Circulation

To further investigate the relationship between t_{maxdef} and t_{acc} , we examine the measured output velocities from the $Eh = 54, 29$, and 19 N/m nozzles, all of which have different t_{maxdef} based on their respective ω_d . Three different timing scenarios can occur ($t_{acc} > t_{maxdef}$, $t_{acc} < t_{maxdef}$, $t_{acc} = t_{maxdef}$), which is summarized by the nozzle tip deflection plot in figure 4.11(a) and the output velocity profiles from the $Eh = 54, 29$ and 19 N/m nozzles in figure 4.11(b-d). The $Eh = 76$ N/m nozzle velocity is not plotted as it matches the same timing scenario as the $Eh = 54$ N/m nozzle but with a much lower magnitude. For reference, t_{maxdef} can be interpreted from figure 4.11(a) as the time to reach the first big peak in inward deflection.

In the first scenario, where $t_{acc} > t_{maxdef}$, the nozzle tip reaches maximum inward

deformation prior to the end of input fluid acceleration of the unperturbed flow. This condition is met by the $Eh = 54$ N/m nozzle, where there is a slight enhancement in peak output velocity shown in figure 4.11(b). Here the nozzle contraction occurs at $t_{maxdef} = 0.65t_{acc}$, after which the velocity decreases, as elastic energy is no longer being supplied to the flow after t_{maxdef} . In contrast, after t_{maxdef} the nozzle extracts energy back from the fluid to re-expand to its original position. In the re-expansion process, a suction velocity is generated that counters the input flow direction and causes a decline in output velocity as the input fluid flow continues to accelerate until t_{acc} . This mismatch in timing results in the output velocity profile reaching the maximum velocity at time t_{maxvel} , where $t_{maxdef} < t_{maxvel} < t_{acc}$, as the nozzle re-expansion temporarily impedes the output velocity growth after t_{maxdef} .

The second scenario ($t_{acc} < t_{maxdef}$) is met by the $Eh = 19$ N/m nozzle where the nozzle contraction occurs at $t_{maxdef} = 1.4t_{acc}$ shown in figure 4.11(d). In this case, elastic energy supplied by the nozzle is transferred to the fluid throughout all of t_{acc} , which supplements the velocity growth for a much longer period resulting in higher peak velocities compared to $Eh = 54$ N/m. However, the response of the nozzle lags behind the input acceleration of the fluid, so a portion of the elastic energy stored within the nozzle has not been fully transferred to the flow before t_{acc} . As the input acceleration ends at t_{acc} , there is a decrease in the growth rate of velocity. However, as the nozzle is still supplying elastic energy to the fluid, the velocity continues to grow until time t_{maxvel} where $t_{acc} < t_{maxvel} < t_{maxdef}$. Due to the delay in t_{maxdef} , the acceleration is slower, and the peak velocity is lower compared to the optimal case.

For the optimal conditions, the $Eh = 29$ N/m nozzle tip reaches peak inward deflection at the same exact time as the end of the input fluid acceleration, $t_{maxdef} = t_{acc}$. This results in all of the elastic energy stored in the nozzle being imparted to the flow within t_{acc} as shown in figure 4.11(c), as well as a greater tip deflection shown in figure 4.11(a). In this case, all of the elastic energy from the nozzle is supplied to the fluid within the same

duration in which the input flow is accelerating, maximizing the growth in the fluid velocity. Additionally, it is understood that the pressure change from acceleration to steady velocity input flow and the formation of a vortex ring creates a low pressure region at the nozzle tip at the same time as t_{maxdef} , creating a positive feedback loop that magnifies the inward tip deflection and output [28]. This behavior is seen as a slight inflection point for all of the flexible nozzles in 4.11(a) at $t \approx t_{acc}$, which results in the $Eh = 29$ N/m nozzle deflecting an additional $0.027\frac{X}{D}$. Importantly, the nozzle deformation behavior is consistent across all $\frac{L}{D}$ values during the fluid acceleration period, ensuring that this optimal timing is effective regardless of ejected volume.

For the $Eh = 54, 29$, and 19 N/m nozzles with $\frac{L}{D} = 2$ and 4 , there is a decline in output velocity after t_{maxdef} , or t_{acc} , depending on which occurs later, despite the input fluid velocity remaining constant. This decrease is attributed to the nozzle re-expanding back to its original position after t_{maxdef} , creating a counter flow opposing the input flow conditions and extracting energy back from the fluid to reopen. As the nozzles return to their original position, the piston motion continues and then accelerates the fluid back to the input velocity until t_{cycle} . In contrast, for $\frac{L}{D} = 1$, t_{cycle} occurs before the nozzle can completely return to its original position, negating the need to overcome the entire counter flow generated by the nozzle reopening, maximizing the relative flow output. The effect of $\frac{L}{D}$ is explored further in Section 4.

To compare the vorticity generated as Eh is varied, the total circulation (Γ) is calculated using:

$$\Gamma = 0.5 \int_C |\omega_z| dA \quad (4.2)$$

The 90% vorticity contour is chosen as the integration area, C , within the field of view defined by $-1.5 < x/D < 1.5$, and $-.5 < y/D < 5.5$. Here, $|\omega_z|$ is the vorticity perpendicular to the measurement plane, as the flow is axisymmetric ($\omega_x = \omega_y = 0$). Circulation is

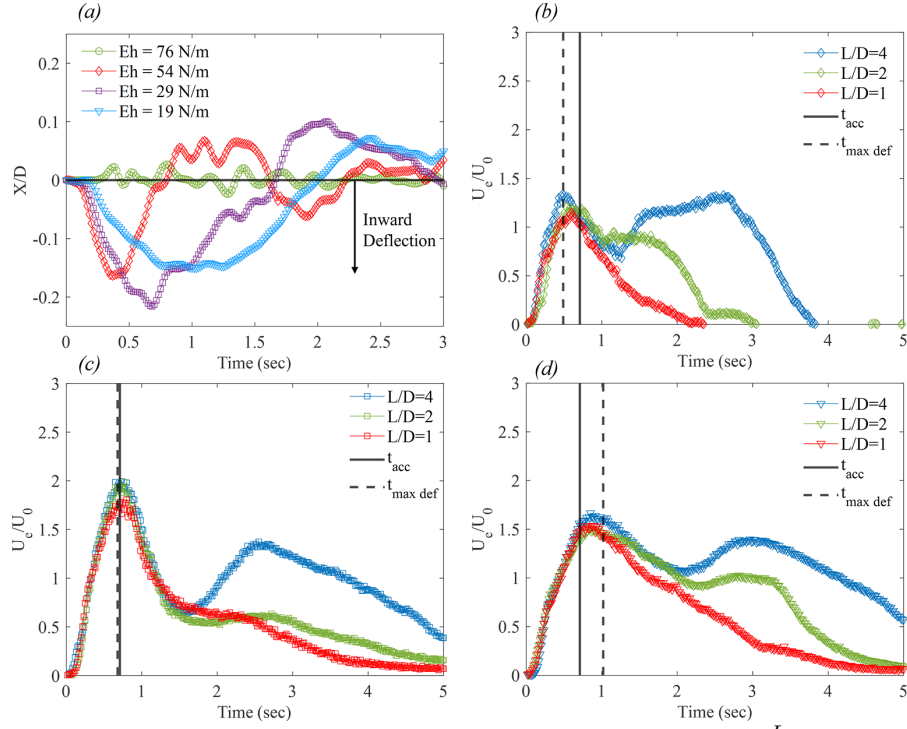


Figure 4.11: (a) Temporal variation in nozzle tip deflection for $\frac{L}{D} = 4$. (b-c) Spatially averaged velocity measured $0.19D$ beneath the nozzle exit (U_e) for $Eh = 54, 29, 19$ N/m for all $\frac{L}{D}$ normalized by the steady state rigid nozzle velocity (U_0) measured for each respective $\frac{L}{D}$. (b) U_e/U_0 for $Eh = 54$ N/m. (c) U_e/U_0 for $Eh = 29$ N/m. (d) U_e/U_0 for $Eh = 19$ N/m.

nondimensionalized by dividing the temporal data by the circulation generated by the rigid nozzle (Γ_0) at $t/t_{cycle} = 1$. Figure 4.12(a) shows the temporal variation of total circulation ($\frac{\Gamma}{\Gamma_0}|_{t/t_{cycle}=1}$) as Eh is varied for $\frac{L}{D} = 1$. Similar to the trend observed to this point, the least stiff nozzles, $Eh = 19$ N/m, and $Eh = 29$ N/m, produced the largest relative circulation, $\frac{\Gamma}{\Gamma_0} = 2.87$ and 2.91 respectively. However, the $Eh = 19$ N/m, nozzle requires nearly twice as much time ($t/t_{cycle} = 2$) to reach peak circulation compared to the $Eh = 29$ N/m nozzle ($t/t_{cycle} = 1.1$). It is noted that Γ is found to be equal in both the parallel (XY) and perpendicular (XZ) planes to the preferential axis of nozzle collapse for all experiments. See Appendix 4 for circulation plots comparing the two measurement planes.

To quantify the effect of nozzle stiffness on the output thrust, hydrodynamic impulse (I_h) is calculated using Saffman [73]:

$$I_h = \frac{1}{2} \rho_f \int_C \mathbf{x} \times \boldsymbol{\omega} \, dV \quad (4.3)$$

where cylindrical coordinates are used with position vector $\mathbf{x} = (r, \theta, y)$ with the origin located at the center of the nozzle exit, with positive y in the flow direction. The vorticity vector is simplified and given as ω_z for consistency. Considering $dV = 2\pi r \, dA$, hydrodynamic impulse can be calculated as $I_h = \pi \rho_f \int_C r^2 |\omega_z| \, dA$. Here, ρ_f is the fluid density at room temperature and r is the radial distance to each integration element from the nozzle center line. Hereon, hydrodynamic impulse is referred to as I . In figure 4.12(b), the temporal distribution of impulse ($\frac{I}{I_0}|_{t/t_{cycle}=1}$) is plotted, where I_0 is the rigid nozzle impulse measured at $t/t_{cycle} = 1$. The measured hydrodynamic impulse increases as Eh declines (becomes less stiff), up to $Eh = 29$ N/m. Similar to circulation, the impulse generated by the least stiff nozzle ($Eh = 19$ N/m) eventually achieves similar amounts of hydrodynamic impulse to the $Eh = 29$ N/m nozzle, but not until $t/t_{cycle} = 2.75$, compared to $t/t_{cycle} = 0.8$, which is predicted to result in much lower force production which scales with impulse generated

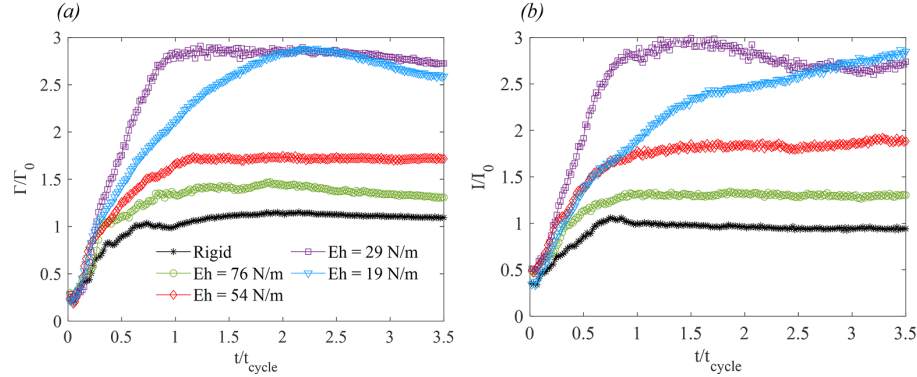


Figure 4.12: Temporal variation in total circulation and impulse for $\frac{L}{D} = 1$. (a) Total circulation (Γ) normalized by the rigid nozzle ($Eh = \infty$) circulation at $t/t_{\text{cycle}} = 1$. (b) Total impulse (I), normalized by the rigid nozzle ($Eh = \infty$ N/m) impulse at $t/t_{\text{cycle}} = 1$. Legend is the same for (a) and (b).

per time [73]. As previously discussed in Section 4, the flexible nozzles collapse under the negative pressure gradient formed upon fluid deceleration. Importantly, the hydrodynamic impulse generated from the flexible nozzles has no dip at $t = t_{\text{cycle}}$, which would indicate a loss in the pressure impulse [28]. In contrast, the unfavorable pressure gradient formed within the rigid nozzle results in a clear loss in hydrodynamic impulse at $t = t_{\text{cycle}}$ due to the generation of negative pressure impulse, as seen in figure 4.12(b). Through the nozzle collapse mechanism, the negative pressure gradient is instead suppressed as shown in figure 4.12 (b), resulting in no additional losses in hydrodynamic impulse. A summary of the measured impulse at t_{cycle} across all Eh and $\frac{L}{D}$ is presented in figure 4.13(a). The results indicate the $Eh = 29$ N/m nozzle produces the largest impulse and any increase or decrease in ω_d , results in a reduction in output.

Interestingly, as a result of these differing measured output velocities with nozzle stiffness (despite the same kinematic input), the fluid slug length expelled also differs with flexibility. Specifically, the flexible nozzle deformations and collapse after $t/t_{\text{cycle}} = 1$ expel additional fluid relative to the rigid case, increasing the total ejected volume. As such, the effective expelled fluid slug length needs to be derived by integrating the U_e profiles and

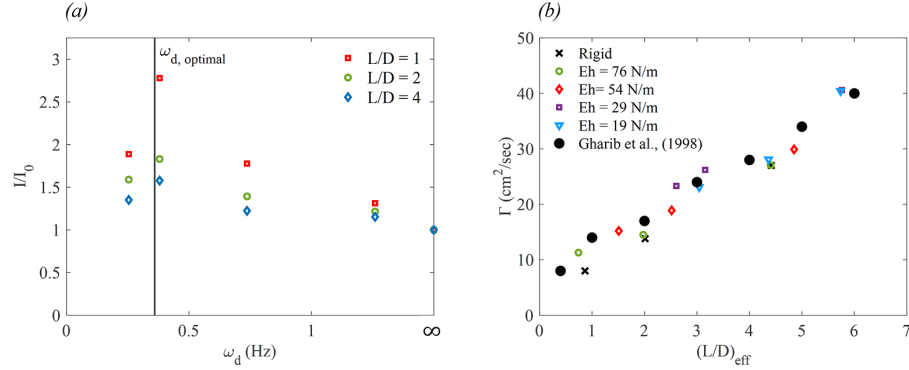


Figure 4.13: Summary of the total measured impulse and circulation from each nozzle over all $\frac{L}{D}$ values. (a) Normalized impulse plotted versus damped natural frequency ω_d , with the predicted optimal $\omega_{d,optimal}$ condition shown. (b) Measured total circulation versus effective ejected volume $((\frac{L}{D})_{eff})$. Data obtained from [32] is plotted to compare our findings with.

dividing by the nozzle diameter to yield $(\frac{L}{D})_{eff} = \frac{1}{D} \int U_e(t) dt$. The circulation produced by the different nozzles should scale linearly with increasing $(\frac{L}{D})_{eff}$ [25]. This is confirmed in figure 4.13(b), where data obtained from Gharib et al. [32] is plotted and matches the results of our study using $(\frac{L}{D})_{eff}$ well.

The lowest ejected volume, $\frac{L}{D} = 1$, resulted in the highest normalized impulse compared to the rigid nozzle data, as shown in figure 4.13(a). This is contributed to four aspects: initial increase of fluid velocity from the nozzle elastic energy, no loss in pressure impulse due to fluid deceleration, high relative increase in $(\frac{L}{D})_{eff}$, and formation of a single coherent vortex structure. The first aspect is related to the flexible nozzles imparting elastic energy when $t < t_{maxdef}$, which occurs early in the fluid expulsion process. As the deformation behavior of the nozzles, and thus elastic energy supplied, is approximately constant across all $\frac{L}{D}$, the relative gain of this energy is maximized for the lowest ejected volume. Additionally, after t_{maxdef} the flexible nozzles extract energy back from the flow to re-expand, which creates a suction velocity that counters the input flow direction. For $\frac{L}{D} = 1$, the pump turns off when the nozzle re-expansion process is taking place, whereas the larger ejected volumes must completely overcome this suction velocity to continue to expel additional fluid.

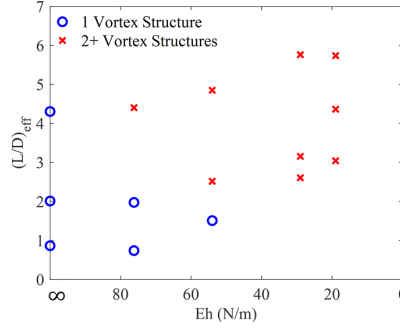


Figure 4.14: Measured number of vortex structures for all measured $(\frac{L}{D})_{eff}$ and Eh

The second aspect, the suppression of the negative pressure gradient from nozzle collapse, is amplified at lower ejected volumes. For lower ejected volumes, a slight loss of hydrodynamic impulse due to negative pressure impulse has a proportionally larger effect on the total produced. The flexible nozzles collapse inward when this unfavorable pressure region is formed, and in turn not only suppresses the negative impacts of the negative pressure region but also expel additional fluid volume.

The third aspect, the relative increase in effective ejected volume, $(\frac{L}{D})_{eff}$, is maximized at lower ejected volumes. The added volume from the nozzle deformation is approximately constant across all $\frac{L}{D}$ for a given Eh . Consequently, a small increase in ejected volume can significantly enhance the normalized impulse and circulation when a small $\frac{L}{D}$ is ejected from the pump's perspective.

The fourth aspect is attributed to the formation of a single coherent vortex structure which maximizes the pressure impulse created per unit volume [47]. The impulse gain from pressure impulse can be a quite substantial contribution of the total impulse when forming an isolated vortex ring, thus can be significantly leveraged to maximize thrust efficiency per unit volume from this type of propulsor. This aspect is studied in Section 4.

Vortex Ring Formation and Relative Vorticity Contributions

As shown in Section 4 and Supplementary Material 1, larger ejected volumes from the more flexible nozzles result in multiple vortex structures, even when $(\frac{L}{D})_{eff} < \approx 4$. This is summarized in figure 4.14, where the number of vortex structures and measured $(\frac{L}{D})_{eff}$ is plotted. Krieg and Mohseni [46] showed that vortex ring pinch-off can be predicted using the output velocity (U_e) and the characteristic velocity (U_c) of the vortex ring. They showed that a vortex ring will pinch-off and stop gaining vorticity when the ring exceeds its feeding velocity, which is $(2U_e)$ for fully developed Poiseuille, or pipe, flow. For the current flow conditions, it is assumed that by the end of fluid acceleration, the flow is fully developed into Poiseuille flow allowing for this subsequent pinch-off prediction to be used. The U_c of the ring is derived in [46], and is as shown here:

$$U_c = \sqrt{\frac{\rho_f \pi \Gamma^3}{4I}} \geq 2U_e \quad (4.4)$$

Here all variables are defined as described in Section 4. Figure 4.15(a) shows U_c , compared with $2U_e$. As shown in (a), the primary ring is expected to pinch-off at $t_{pinch} = 1sec$, when U_c surpasses $2U_e$. We confirm this prediction by calculating the pLCS and nLCS ridges in the FTLE field in this time frame to show material boundaries indicative of pinch-off [74]. In figure 4.16(a-c), a closed LCS loop from overlaying the nFTLE and pFTLE fields is shown, indicating a pinched off vortex ring after the predicted t_{pinch} . As vortex ring pinch-off is not an instantaneous process, additional time steps are shown at $t = 1.5t_{pinch}$ and $t = 3.75t_{pinch}$ in figure 4.16(b) and (c) to provide additional insight that a single vortex structure is formed with a closed material boundary from the LCS. As time progresses for $\frac{L}{D} = 4$, additional vorticity is produced and secondary vortex structures form in the wake of the primary vortex (PV). However, the primary closed loop of vorticity continues to travel downward without gaining additional fluid from the trailing jet formed in its wake, indicating

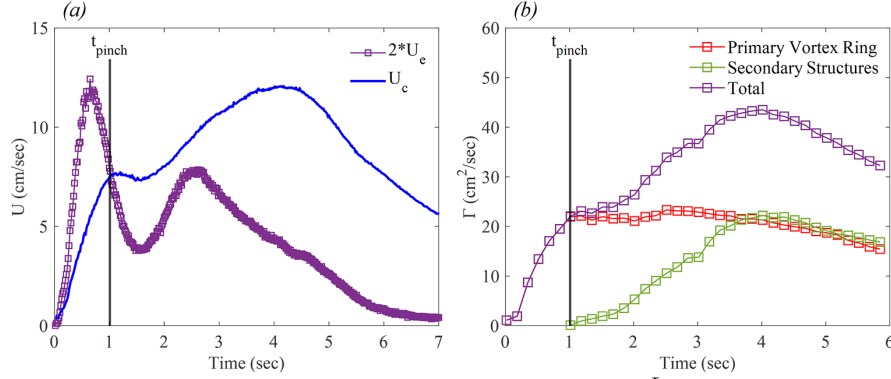


Figure 4.15: Predicting PV pinch-off for $Eh = 29$ N/m, $\frac{L}{D} = 4$. (a) Temporal evolution of U_c and $2U_e$ to predict PV pinch-off when $U_c > 2U_e$ ($t_{pinch} \approx 1.0$ sec) (b) Circulation contributions from PV and SV, calculated using the vorticity contours informed by the nLCS and pLCS.

an accurate prediction of t_{pinch} .

Through examining FTLE fields for each case within the studied parameter space, we can confirm closed off structures within the flow field. With this information, the vorticity fields measured directly from PIV can be used to separate the total circulation produced by each nozzle into the primary vortex ring and the trailing jet formed in its wake. Figure 4.16(d-e) shows vorticity contours at the same instants as the FTLE fields in 4.16(a-c), which are used to delineate circulation into separate parts of the flow, as informed by closed LCS contours in the FTLE field.

Figure 4.15(b) shows the circulation contributions from the PV and SV structures and their sum of total circulation for $Eh = 29$ N/m. By informing the vorticity calculation through the use of the FTLE fields, the circulation from separate parts of the flow was confidently calculated without question as to whether or not vorticity was included from the trailing jet when calculating circulation for the PV and vice versa. As expected, the PV does not gain additional vorticity after the predicted t_{pinch} . Interestingly, the PV contains approximately 50% of the total circulation produced, while the piston has only moved $\frac{1}{3}$ of its total prescribed motion for $\frac{L}{D} = 4$. Additionally, for the $Eh = 29$ N/m nozzle $\frac{L}{D} = 1$,

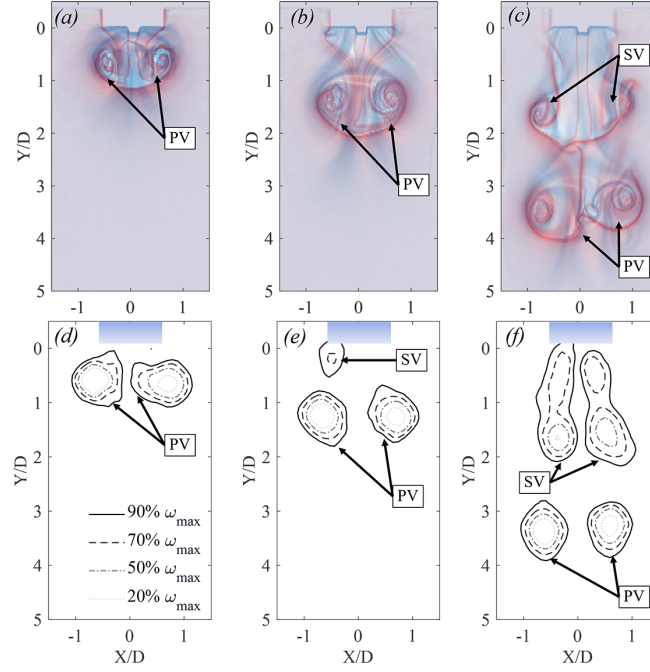


Figure 4.16: FTLE fields and vorticity plots from $Eh = 29$ N/m nozzle with $\frac{L}{D} = 4$. The red contours show attractive pLCS, and the blue contours show repelling nLCS. (a) nLCS and pLCS for $t = t_{pinch}$. (b) nLCS and pLCS for $t = 1.5t_{pinch}$. (c) nLCS and pLCS for $t = 3.75t_{pinch}$. (d-e) Vorticity contours corresponding to the same time steps as the FTLE plots directly above each frame.

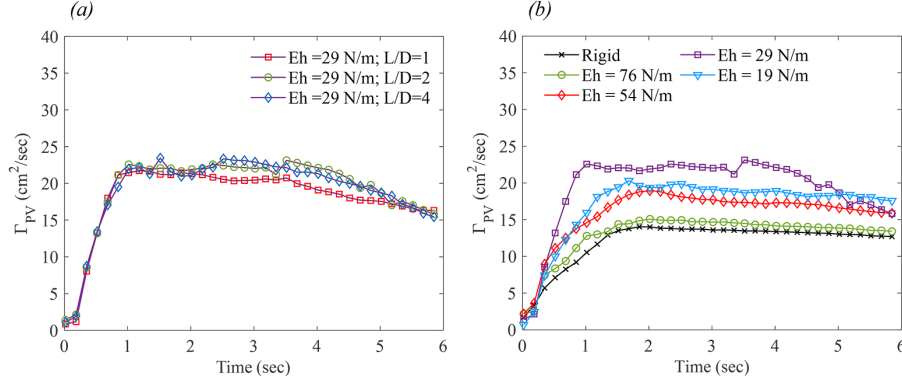


Figure 4.17: Primary vortex circulation. (a) Temporal development of only the primary vortex circulation for all $\frac{L}{D}$ values generated with the $Eh = 29$ N/m nozzle. (b) Temporal development of primary vortex circulation from each Eh nozzle for $\frac{L}{D} = 2$.

$t_{pinch} \approx t_{cycle}$, indicating that the pump turns off when the PV exceeds the feeding velocity. In turn, this optimizes the ejected volume by entraining the maximum vorticity within the PV, while minimizing the piston motion. In figure 4.17(a) we compare the PV circulation measured from each ejected volume for the $Eh = 29$ N/m nozzle and despite multiple vortex structures forming for $\frac{L}{D} = 2, 4$, we confirm that the PV strength (Γ_{PV}) does not grow with additional ejected volume beyond $\frac{L}{D} = 1$. Given the exit (escape) velocity (U_e) measured for each nozzle is approximately constant for $t/t_{pinch} < 1$ we expect to find similar pinch-off results from varied $\frac{L}{D}$.

This process was repeated for each nozzle and the vorticity contained within the PV as Eh is varied follows the same trend as total circulation as shown in figure 4.13(b). The temporal variation of circulation measured for each of the PV cores produced by each nozzle from $\frac{L}{D} = 2$ is plotted in figure 4.17(b). One point to note is that the $Eh = 19$ N/m PV pinches off significantly later than $Eh = 29$ N/m, as shown by the prolonged increase in circulation; however, the circulation contained within the PV is lower due to the lack of momentum flux (exit velocity) compared to the $Eh = 29$ N/m nozzle. The $Eh = 29$ N/m nozzle creates the strongest PV and further explains why the high nondimensionalized

impulse is reached for $\frac{L}{D} = 1$, as the pump turns off immediately following PV pinch-off maximizing the gain from pressure impulse. However, there is a very weak secondary structure formed for $Eh = 29 \text{ N/m}$, $\frac{L}{D} = 1$, when the nozzle collapses after t_{cycle} , yet the strength of this structure is very small and almost the entirety of the ejected fluid is contained within the PV.

Stopping Vortex Formation and Implications for Bio-Propulsors

To provide a complete picture of an entire cycle of the movement of the flexible nozzles, we investigate the flow structure produced by the nozzles reopening after collapse. Given that the nozzles were manufactured using clear silicone, we can perform PIV within the structure of the flexible nozzles [10]. As described in Section 4, the less stiff nozzles ($Eh = 54, 29, 19 \text{ N/m}$) collapse due to a pressure change at the outlet after the pump stops moving at t_{cycle} . This is in contrast to the low pressure region formed within rigid nozzles upon fluid deceleration causing a decrease in pressure impulse [28]. The flexible nozzles compliantly deform when this negative pressure gradient is formed and as a result see no loss in impulse as would be characterized by a slight dip in hydrodynamic impulse [48]. After collapse, the nozzles reopen at time t_{open} causing a suction velocity, which creates a stopping vortex of varied strength within the nozzle itself, similar to stopping vortices found in rigid nozzles that positively contribute to pressure impulse [28]. The time to reach t_{open} is 0.95 seconds after t_{cycle} for $Eh = 54 \text{ N/m}$, whereas t_{open} does not occur until 9 seconds and 25 seconds after t_{cycle} for $Eh = 29$ and 19 N/m respectively. Additionally, the time for the nozzles to fully refill (t_{refill}) (the time at which the circulation of the stopping vortex is the highest), is delayed with decreased nozzle stiffness. A summary of these timescales is shown in table 4.2. Given that the $Eh = 76 \text{ N/m}$ nozzle does not collapse, it does not produce a strong stopping vortex within the nozzle interior and is not included in this stopping vortex analysis.

Figure 4.18(a-c), shows stopping vortices forming in the interior of the flexible $Eh =$

Nozzle Stiffness	$t_{\text{open}} - t_{\text{cycle}}$	$t_{\text{refill}} - t_{\text{open}}$
$Eh = 54 \text{ N/m}$	0.95 seconds	0.87 seconds
$Eh = 29 \text{ N/m}$	9 seconds	2.16 seconds
$Eh = 19 \text{ N/m}$	25 seconds	4.88 seconds

Table 4.2: Nozzle Reopening Timescales

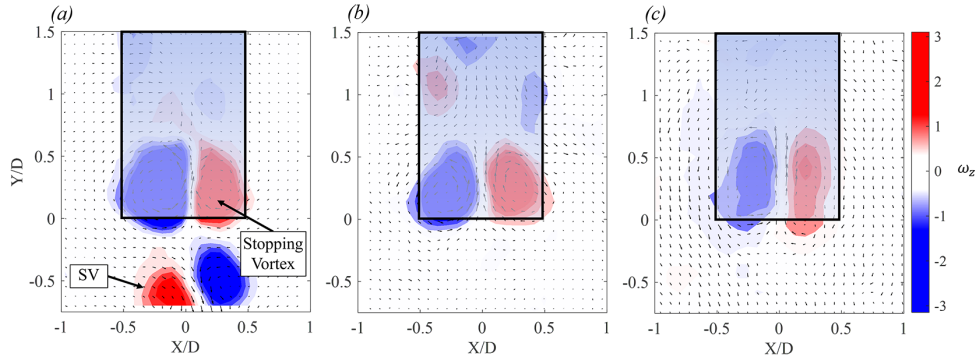


Figure 4.18: Stopping vortices produced varied Eh nozzles at $t = 0.75(t_{\text{refill}} - t_{\text{open}})$ (a) $Eh = 54 \text{ N/m}$. Note that the $Eh = 54 \text{ N/m}$ nozzle immediately reopens, and thus there is secondary vorticity labeled in the frame, separate from the stopping vortex. Only vorticity contained within the stopping vortex is included in the circulation calculation. (b) $Eh = 29 \text{ N/m}$ (c) $Eh = 54 \text{ N/m}$

54, 29, and 19 N/m nozzles at $t = 0.75(t_{refill} - t_{open})$, or the time at which the stopping vortex has reached 75% of the maximum stopping vortex circulation. The strength of these stopping vortices (Γ_{stop}) is calculated using the circulation formulation described in Section 4 and the temporal variation is plotted in figure 4.19(a and b). In figure 4.19(a) $\frac{\Gamma_{stop}}{\Gamma_0}|_{t/t_{cycle}=1}$ is plotted, where Γ_0 is the circulation produced by the rigid nozzle throughout fluid expulsion for $\frac{L}{D} = 1$. This normalization is selected to illustrate the importance of accounting for these stopping vortices relative to the circulation generated by the rigid nozzle. In figure 4.19(b) $\frac{\Gamma_{stop}}{\Gamma_x}$, is plotted where Γ_x is the maximum circulation achieved by each respective flexible nozzle in expelling fluid for $\frac{L}{D} = 1$. Note, that the bounding area for the stopping vortex calculation was altered such that vorticity more than $0.1D$ below the nozzle tip was not included in the calculation to avoid including vorticity produced in the jetting phase for the 54 N/m nozzle as this nozzle starts to form stopping vortices rapidly after jetting. Additionally, measurements were cut off at $t - t_{open} = 3$ seconds, for $Eh = 29$ and 54 N/m, due to the earlier occurrence of maximum Γ_{stop} . Alternatively, measurements were continued to $t - t_{open} = 6$ seconds for the much slower reopening $Eh = 19$ N/m nozzle. The strength of the stopping vortex scales with the stiffness of the collapsing flexible nozzles with the $Eh = 54$ N/m nozzle creating the strongest and $Eh = 19$ N/m generating the weakest, as the stiffer nozzle reopens much faster, i.e. shorter t_{refill} , generating a stronger stopping vortex. The strength of the vortex is not affected by $\frac{L}{D}$ as the same initial condition is provided to each nozzle at the end of flow generation with the same deceleration of fluid, or pressure change resulting in a collapsed nozzle shape. In terms of total momentum produced, forming stronger stopping vortices within the nozzle can increase the thrust of a bio-inspired propulsor, due to increased forward momentum within the nozzle itself [86]. It has been demonstrated that for the swimming performance of siphonophores, animals that propel themselves with cyclical jets with refill cycles, the refill phase augments their distance traveled by 17% over theoretical results which only generate thrust from the jet phase [80].

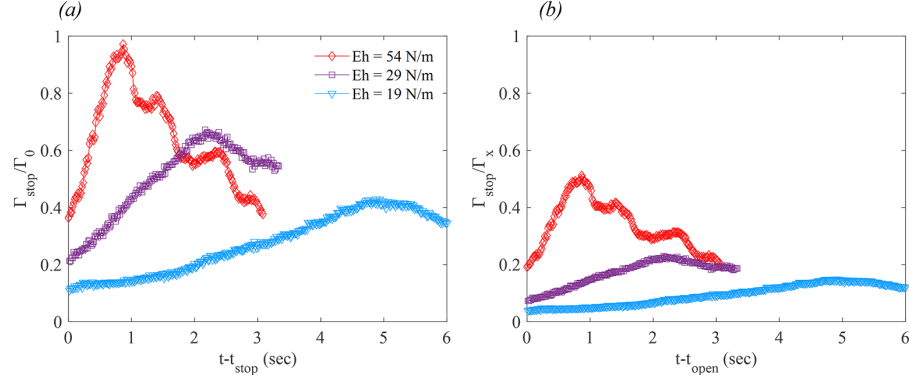


Figure 4.19: Circulation produced by the stopping vortices formed within the $Eh = 54, 29$, and 19 N/m nozzles after t_{open} . (a) $\frac{\Gamma_{\text{stop}}}{\Gamma_0} \big|_{t/t_{\text{cycle}}=1}$ (b) $\frac{\Gamma_{\text{stop}}}{\Gamma_x}$. Legend is the same for (a) and (b).

As the fluid accelerates back into the siphonophore, or in our case the nozzle, it generates a high pressure region that should augment thrust. Thus, quantifying this stopping vortex is imperative, in terms of summing the total thrust produced by our varied flexibility nozzles. Returning to jet propelled aquatic animals such as jellyfish and squid, they primarily produce forward momentum in the expelling cycle of their motion, but their swimming efficiency is linked to the generation of positive momentum in the refilling cycle known as passive energy recapture [30]. The locomotion of jellyfish particularly is significantly augmented by the presence of a stopping vortex which can account for up to 50% of the jellyfish's travel per cycle [29].

To summarize, the present study, in alignment with previous studies [10, 11], indicates that the optimal stiffness nozzle in the jetting phase should have peak tip deformation synchronized with the input fluid acceleration for maximal momentum transfer. However, we also observe that it is equally important that the nozzle reopens as quickly as possible during the refilling cycle, which adds a second condition to define the optimal nozzle stiffness for a complete expulsion-refill cycle. This secondary aspect of nozzle stiffness is characterized by a nozzle that is compliant enough to collapse from fluid deceleration, yet remains as stiff

as possible to reopen quickly. Despite forming a stopping vortex, this is expected to benefit propulsion by creating net positive momentum and a favorable pressure gradient. This suggests that tuning the system (t_{acc}) to maximize the jetting conditions for the stiffest nozzle capable of collapse would generate the largest net thrust over a full expulsion-refill cycle by additionally forming the strongest stopping vortices.

Conclusions

In this work, the relative impulse generated by a flexible nozzle was found to be increased by up to 2.8 times compared to that of a rigid case with the same kinematic input to the nozzle. Similar to previous studies we find that decreasing stiffness past an optimal value results in a decline in measured output. The most flexible nozzle tested in this study was found to reduce the maximum measured circulation and impulse due to the nozzle's damped frequency becoming too slow relative to the input fluid acceleration. Synchronizing the timing at which the nozzle contracts with the end of the unperturbed fluid acceleration resulted in the optimal thrust conditions by timing the release of elastic energy. When these timescales align, the entirety of the elastic energy from the nozzle is imparted while the input fluid is accelerating achieving peak output velocities, resulting in the largest increase in circulation and impulse. It is additionally observed that when these timescales are synchronized, the slight negative pressure from the change in input velocity and vortex formation generates a positive feedback loop wherein the optimal stiffness nozzle deformation is magnified. Alternatively, if the nozzle contracts too quickly or too slow relative to the input flow, there is only a modest increase in output velocity and impulse.

The lowest ejected volume, $\frac{L}{D} = 1$, generated the largest relative impulse due to the initial peak increase in fluid velocity from the nozzle elastic energy, minimizing the negative pressure gradient in deceleration via nozzle collapse, a higher relative increase in $(\frac{L}{D})_{eff}$, and the formation of a single coherent vortex structure. The combination of these factors

results in the greatest measured hydrodynamic impulse by increasing the momentum flux (output velocity) and the pressure contribution through the formation of a single vortex ring while simultaneously suppressing the negative deceleration pressure gradient through nozzle collapse. For example, for a larger ejected volume of $\frac{L}{D} = 4$ and the optimal stiffness $Eh = 29$ N/m nozzle, the secondary structures contribute approximately half of the total circulation, while consuming 2/3 of the piston motion measured at the pump. It is also noted, that after the nozzle provides elastic energy to the flow, it extracts energy back from the fluid to reopen, decreasing the output velocity. For the lowest ejected volume, the pump turns off when the nozzle begins to re-expand, negating the need to overcome the counter-flow velocity generated in the re-expansion process, where higher ejected volumes continue to eject fluid to overcome this counter flow.

In this study, we predict the vortex pinch-off time for each nozzle and find that the primary vortex (PV) pinch-off is relatively unaffected by different ejected volumes for the optimal stiffness nozzle. This is due to a peak in output velocity which feeds the PV for a short time, but as the output velocity decreases due to the nozzle re-expanding, the PV outpaces the feeding velocity. The predicted pinch-off for the optimal material property nozzle occurs at almost the exact time at which the pump stops moving for $\frac{L}{D} = 1$ as measured at the pump. In turn, this generates the strongest possible single vortex ring for the provided input flow conditions which maximizes the pressure contribution to hydrodynamic impulse. This is similar to previous work which has shown that hydrodynamic impulse from a rigid nozzle is maximized per unit volume via the pressure impulse contribution up to $\frac{L}{D} \approx 4$ when a single vortex ring without a trailing jet is formed [47].

For the most flexible nozzles ($Eh = 54, 29, 19$ N/m), the fluid deceleration creates a negative pressure gradient causing varying nozzle collapse after t_{cycle} . When the nozzles reopen, they create a stopping vortex within the nozzle walls, which should theoretically positively contribute to the overall momentum of a free-bodied system [86]. The stopping

vortex was maximized from the $Eh = 54$ N/m nozzle, due to relatively higher nozzle stiffness allowing for a much quicker reopening, yet it was still flexible enough to collapse under the negative pressure gradient upon fluid deceleration. Due to the extremely low stiffness of the $Eh = 29$ and 19 N/m nozzles, they remain collapsed for 9 and 29 seconds respectively, which would become significant in terms of pulsed jet flow. Further examination into a pulsed flow may result in the reduction of the propulsive benefit from these nozzles as they would be starting with a closed initial configuration which is a lower energetic state, as the elastic potential energy from the nozzle is not available as an initial condition. Overall, selecting a fluid acceleration timescale that matches the elastic properties of a flexible nozzle while still maintaining the most stiffness possible in the nozzle should theoretically create the maximum thrust per pulse based on the current study, to generate the most momentum impulse in the jetting phase and the strongest stopping vortex in the refilling phase.

Back-Matter Provided for Submission

Supplementary Data

Supplementary material are available, and are shown at the end of this document.

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Declaration of Interests

The authors report no conflict of interest.

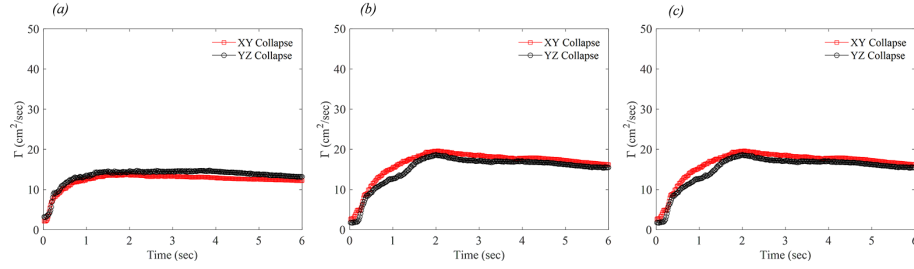


Figure 4.20: Circulation measured in the XY and XZ planes for the $Eh = 54$ N/m nozzle. (a) $\frac{L}{D} = 1$ (b) $\frac{L}{D} = 2$ (c) $\frac{L}{D} = 4$

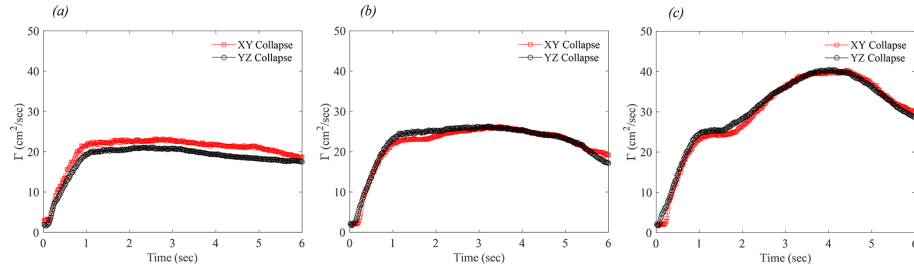


Figure 4.21: Circulation measured in the XY and XZ planes for the $Eh = 29$ N/m nozzle. (a) $\frac{L}{D} = 1$ (b) $\frac{L}{D} = 2$ (c) $\frac{L}{D} = 4$

Comparison of Circulation Generated in XY and YZ Planes

The following figures demonstrate the formation of a closed constant loop of vorticity from each of the flexible nozzles despite loss of axisymmetric shape upon nozzle collapse for the $Eh = 54, 29$, and 19 N/m nozzles. The rigid nozzle and $Eh = 76$ N/m nozzle are not shown as there was no preferential axis of nozzle collapse for these nozzles. In Figures 4.20, 4.21, 4.22, the circulation measured in the perpendicular XY and XZ planes are shown for the $Eh = 54, 29$, and 19 N/m nozzles respectively.

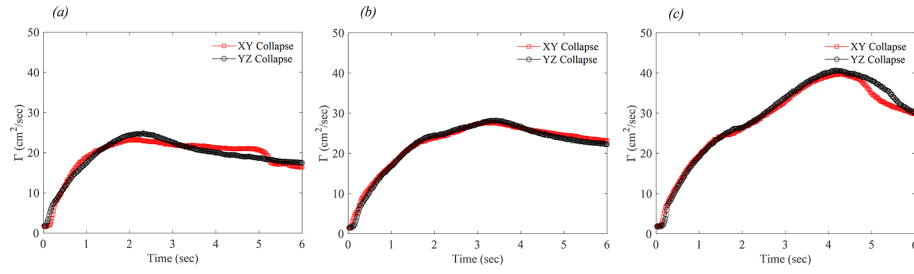


Figure 4.22: Circulation measured in the XY and XZ planes for the $Eh = 19$ N/m nozzle.
 (a) $\frac{L}{D} = 1$ (b) $\frac{L}{D} = 2$ (c) $\frac{L}{D} = 4$

As shown in 4.20, 4.21, 4.22, the vorticity distributed within the vortex rings is constant regardless of viewing plane.

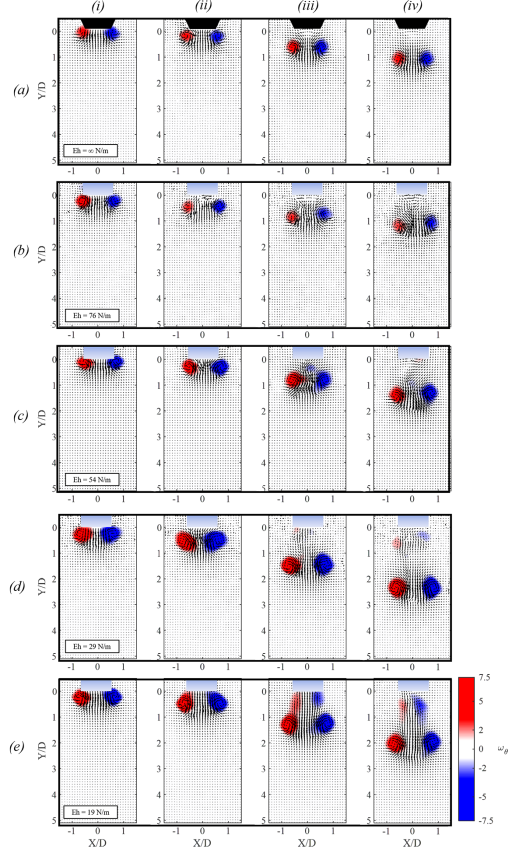
Supplementary Material 1: Vorticity and Vector Plots for all L/D 

Figure S1.A: Vorticity and vector fields measured for $(i)t/t_{cycle} = 0.7$, $(ii)t/t_{cycle} = 1$, $(iii)t/t_{cycle} = 2$, and $(iv)t/t_{cycle} = 3$ for each nozzle given the same kinematic input from the pump for $\frac{L}{D} = 1$ (a) Rigid nozzle ($Eh = \infty$ N/m) (b) $Eh = 76$ N/m; (c) $Eh = 54$ N/m; (d) $Eh = 29$ N/m; (e) $Eh = 19$ N/m

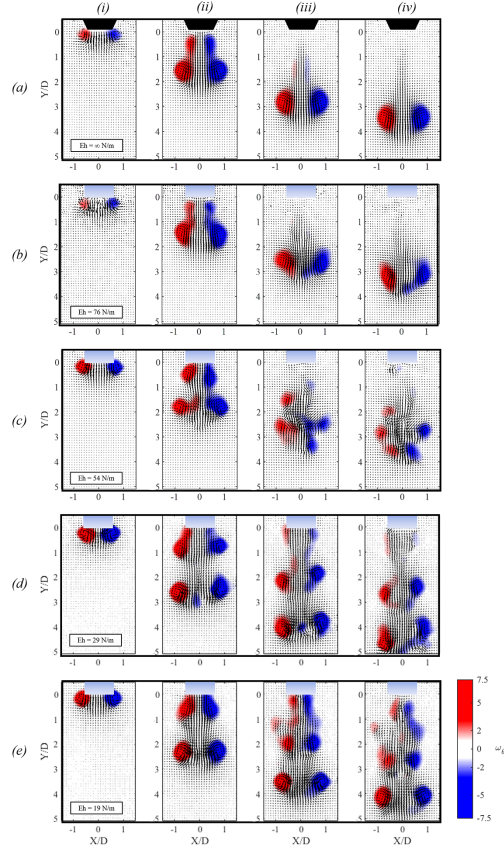


Figure S1.B: Vorticity and vector fields measured for (i) $t/t_{cycle} = 0.23$, (ii) $t/t_{cycle} = 1$, (iii) $t/t_{cycle} = 1.5$, and (iv) $t/t_{cycle} = 1.75$ for each nozzle given the same kinematic input from the pump for $\frac{L}{D} = 4$ (a) Rigid nozzle ($Eh = \infty$ N/m) (b) $Eh = 76$ N/m; (c) $Eh = 54$ N/m; (d) $Eh = 29$ N/m; (e) $Eh = 19$ N/m

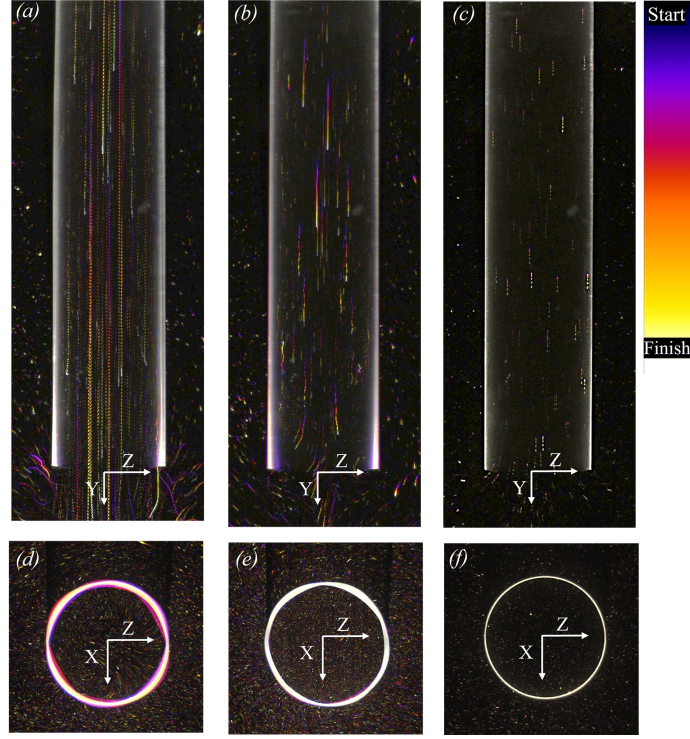
Supplementary Material 2: Nozzle Deformation Behavior for all Eh 

Figure S2.A: Imaging of the $Eh = 76 \text{ N/m}$ nozzle for $\frac{L}{D} = 4$. The colorplot scale bar applies to each image, with the Start and Finish defined as follows in the sub figure descriptions. (a,d) Nozzle deformation for $t = 0 - t_{maxdef}$. (b,e) Nozzle deformation for $t = t_{maxdef} - t_{cycle}$. (c,f) Nozzle deformation for $t = t_{cycle} - 2t_{cycle}$.

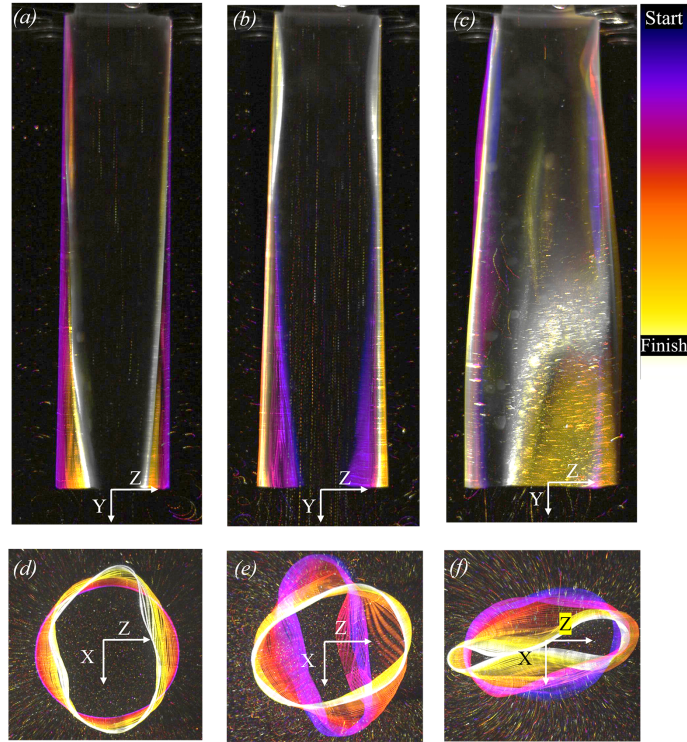


Figure S2.B: Imaging of the $Eh = 29 \text{ N/m}$ nozzle for $\frac{L}{D} = 4$. The colorplot scale bar applies to each image, with the Start and Finish defined as follows in the sub figure descriptions. (a,d) Nozzle deformation for $t = 0 - t_{maxdef}$. (b,e) Nozzle deformation for $t = t_{maxdef} - t_{cycle}$. (c,f) Nozzle deformation for $t = t_{cycle} - 2t_{cycle}$.

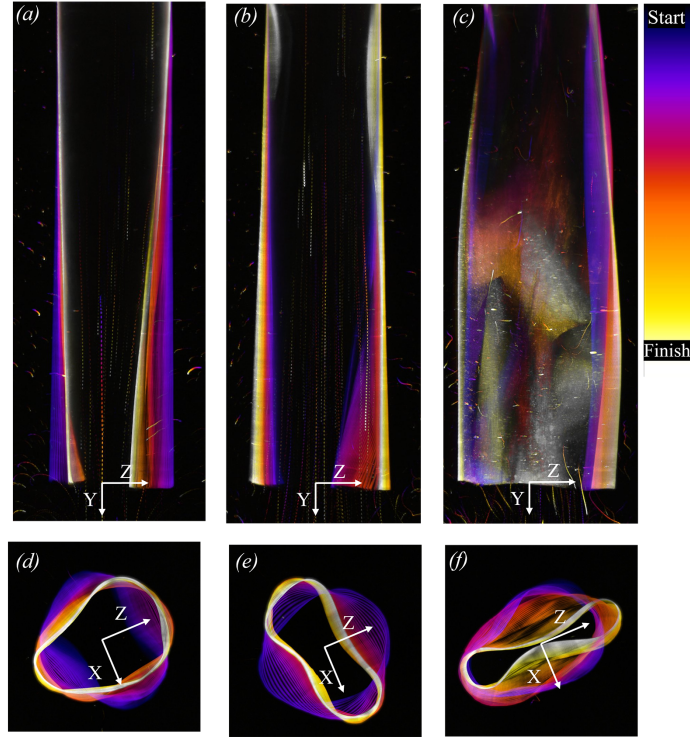


Figure S2.C: Imaging of the $Uh = 19$ N/m nozzle for $\frac{L}{D} = 4$. The colorplot scale bar applies to each image, with the Start and Finish defined as follows in the sub figure descriptions. (a,d) Nozzle deformation for $t = 0 - t_{maxdef}$. (b,e) Nozzle deformation for $t = t_{maxdef} - t_{cycle}$. (c,f) Nozzle deformation for $t = t_{cycle} - 2t_{cycle}$.

PLUNGING TAPERED HYDROFOIL

Frequency Response Functions

The frequency response function was found for frequency sweeps of 1-20Hz with the input shaker voltage varied from 10-120mV and is shown in figure 5.1. This range of voltages was selected as this voltage range encompassed the desired physical heaving amplitudes of $A = 0.5, 1$, and 3 mm. The driving frequency range, $f = 1 - 20$ Hz was used as it included the first and second resonant peaks in the foil motion. One item to note is that the 120mV trial was only completed over a range of $1 - 10$ Hz, as this larger heaving amplitude was out of range for the vibrometer voltage sensitivity for higher frequencies. This truncation was acceptable as the first bending mode was characterized for this trial and the driving frequencies used for PIV ($f = 1 - 6$ Hz) were well within this range. Although a constant input voltage did not consistently result in a constant heaving amplitude (± 0.5 mm) due to the interaction of the foil with the shaker, the frequency response functions gathered are still relevant to the foil resonant peaks at varied heaving amplitude given the acceleration normalization. It would have been preferred to have a closed loop system with constantly varied voltage to actively adjust the input voltage based on feedback from a linear displacement sensor, which may be an addition to the setup for future work. However, given that we normalize by the input accelerometer data, these curves are still valid as the accelerometer measured any change in heaving amplitudes and thus acceleration and is accounted for. As noted in 3, the variation in voltage used for PIV to achieve constant heaving amplitude ranged from ± 7.5 mV for $A = 0.5$ mm, up to ± 12.0 mV for the $A = 1.5$ and 3 mm. This variation in input corresponds to a potential change in the first resonant frequency, $\omega_{n1} = \pm 0.25$ Hz for both the $A = 0.5$ and 1.5 mm trials, and $\omega_{n1} = \pm 0$ Hz, for the $A = 3$ mm, based on the FRF curves shown. As shown in 5.1(b), the three FRF curves closest to the physical heaving amplitudes used for PIV are shown with the frequency domain parameter space labeled.

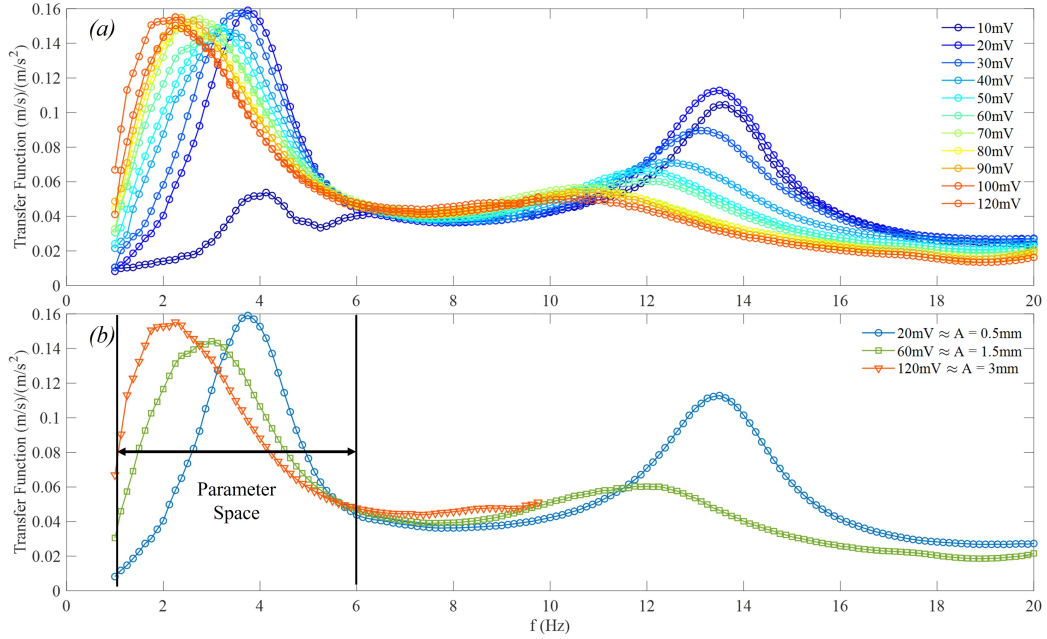


Figure 5.1: Frequency response functions. (a) FRF for input voltage varied across parameter space for 1-20Hz. (b) FRF for parameter space studied further.

The resonant peaks clearly begin to shift down and broaden as heaving amplitude (voltage), and transverse damping, is increased. The resonant peaks for the dataset were found through the use of a peak-picking algorithm on the 10-point smoothed data in MATLAB and are plotted in 5.2(a). The damping ratios for each resonant peak (ζ_1 and ζ_2) were measured directly from the scanning laser vibrometer using the modal curve fit toolbox, and are shown in 5.2(b). As shown, the damping ratios grow proportionally as the input voltage was increased. This was expected as the transverse damping on the foil was directly proportional to the transverse velocity of the foil, which is related to the heaving amplitude and frequency. It should be noted that the higher input frequencies resulted in lower ζ_2 values. This is likely due to the much lower velocity magnitude at this resonant peak, shown in the FRF. Lastly, the Q factor was found for each resonant peak ($Q = \frac{1}{2\zeta}$), providing an alternative expression for the amount of damping present measured in the foil's response. As shown in 5.2(c), the Q factor decreases approximately exponentially with

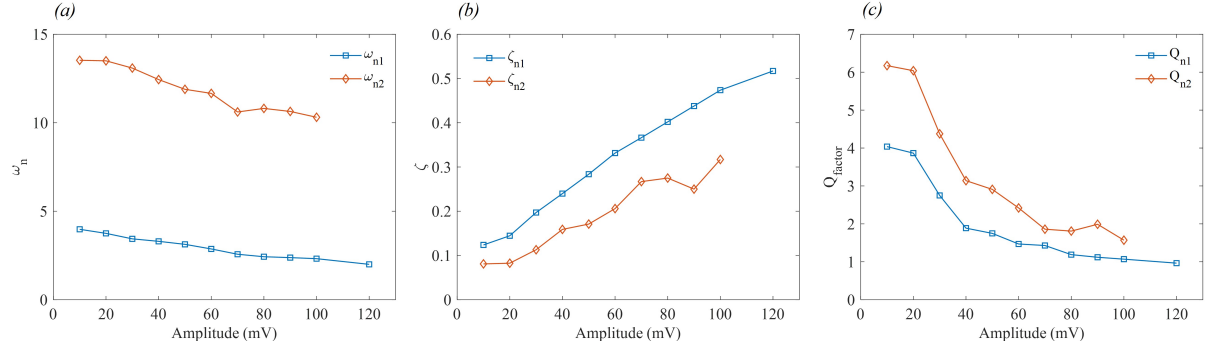


Figure 5.2: Vibration parameters (a) First (ω_{n1}) and second (ω_{n2}), resonant frequencies measured at each input voltage. (b) Damping ratios (ζ_1 and ζ_2) for each resonant peak (c) Q factor ($\frac{1}{2\zeta}$) for each resonant peak

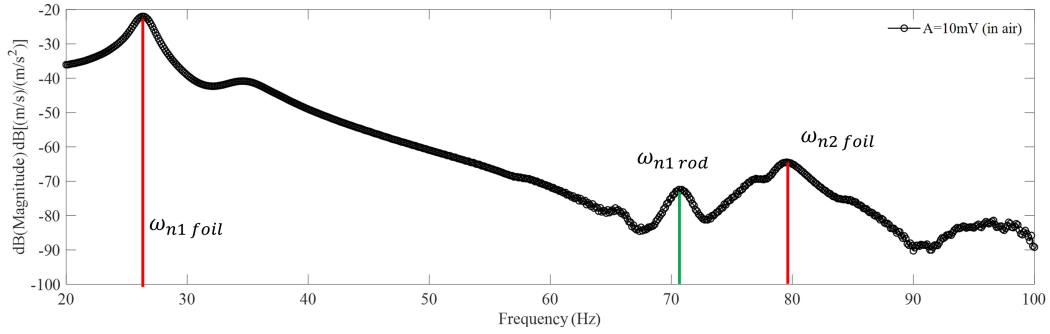


Figure 5.3: FRF for the foil scanned in air dB scale, with first and second resonant frequencies labeled. Note that the vertical mounting shaft had a resonant peak at 72.5 Hz with the added mass of the foil.

increased heaving amplitude, roughly scaling with transverse drag on the foil.

The wing was additionally scanned in air, which is shown in 5.3, note the logarithmic scaling of the FRF. Here the resonant peaks are shifted significantly higher in the frequency domain with the lack of damping applied to the structure. The resonant peak for the mounting rod the foil was mounted to was clear in the FRF and is labeled in 5.3. The rod geometry and material were selected to have resonance approximately an order of magnitude greater than the range studied in liquid.

Mode Shapes

Next, we examine mode shapes formed from the heaving foil at different frequencies and amplitudes. The first and second bending modes are clear at resonant peaks in the FRF and appear the same regardless of heaving amplitude. 5.4(*i-vi*) the mode shapes generated for the first and second resonance peaks of the foil for $A = 1.5\text{mm}$ with time series representing one half of an oscillation cycle ($\frac{1}{2}T$), where T is equal to the period of motion, or time to complete one oscillation. The color plot corresponds to output velocity, normalized by the acceleration of the input. The green arrows indicate the direction and magnitude of the foil motion, with the small arrows representing either end of the peak displacement driven by the shaker. Both the first and second modes are indicative of the shapes generated by an Euler-Bernoulli continuous cantilever beam, with the second bending mode showing a node formed at approximately $\frac{2}{3}C$. However, as shown in 5.1(b), the operating range of frequencies at which we will be driving the foil primarily corresponds to the first bending mode.

Below the first resonance, very little bending occurs within the foil, where the foil is almost purely translating, regardless of heaving amplitude. However, given the lower resonant peaks as the heaving amplitude is increased, bending starts to occur at lower input frequencies for the higher heaving amplitude cases. Beyond resonance, there are some interesting deformation mechanics at play, particularly at higher heaving amplitudes. Similar to previous studies [51, 88], we find that between the first and second resonant peaks in the FRF, there is a mixture of the first and second bending mode that may be generating traveling waves.

The deformed shapes generated by two frequencies ($f = 3.75, 6\text{Hz}$) above resonance ($\omega_{n1} = 3.25\text{Hz}$) for $A = 1.5\text{mm}$ are shown in 5.5(*i-viii*). If we first examine 5.5(*i-iv*), the general behavior appears similar to the first bending mode at resonance. However, the length

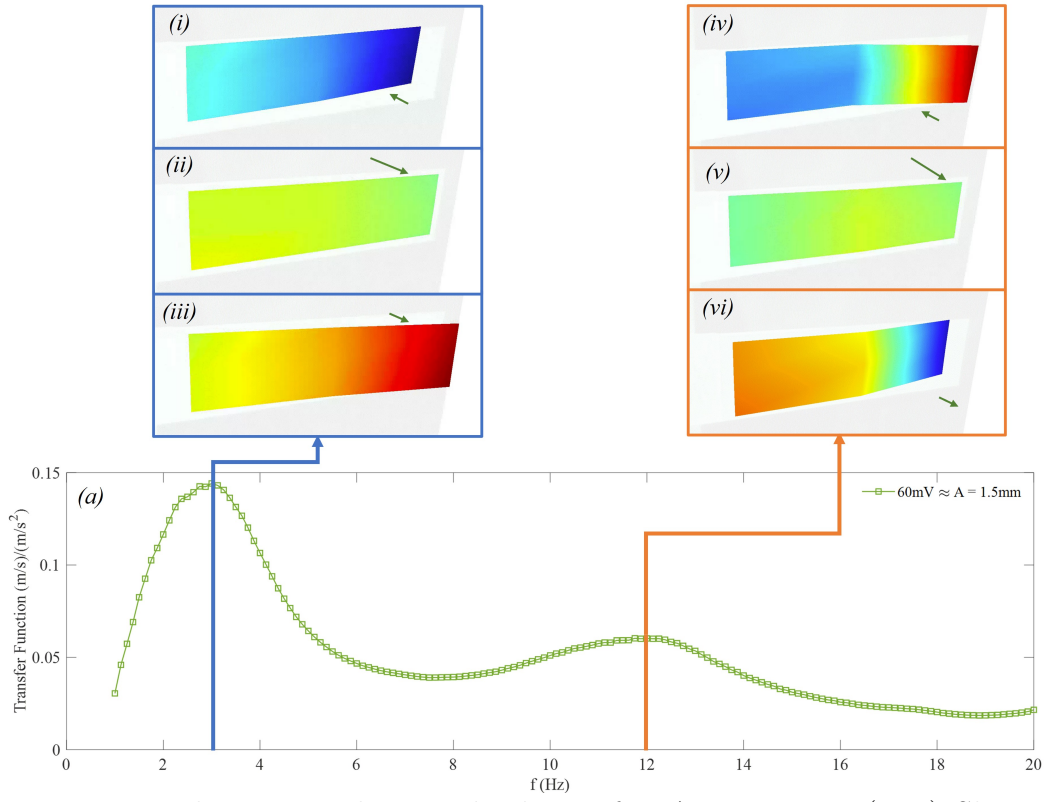


Figure 5.4: FRF and corresponding mode shapes for $A = 1.5\text{mm}$. (i-iii) Show the first bending mode of the foil for $t = 0T, t = \frac{1}{4}T, t = \frac{1}{2}T$. (iv-vi) show the second bending mode with the same time steps.

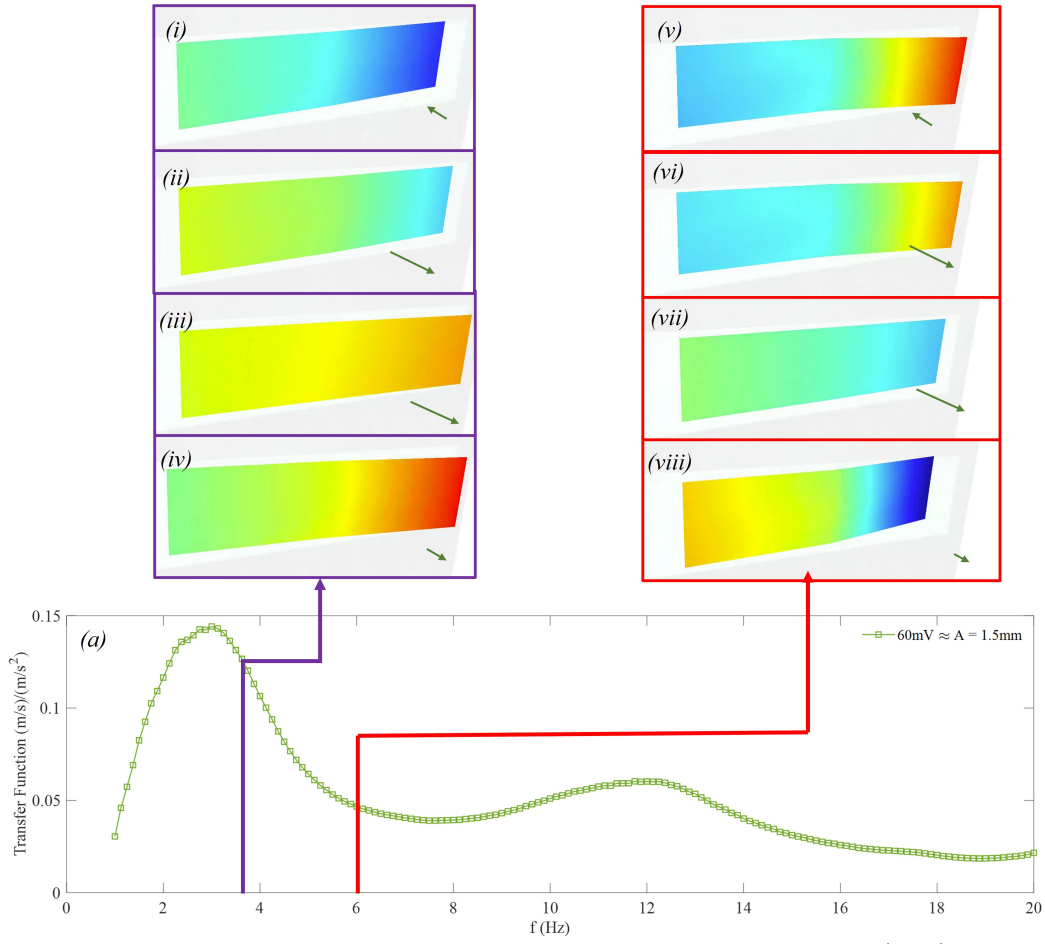


Figure 5.5: FRF and corresponding bending shapes for $A = 1.5\text{mm}$. $(i-iv)$ correspond to $f = 3.75\text{Hz}$, at times $t = 0T, \frac{1}{8}T, \frac{1}{4}T, \frac{3}{8}T, \frac{1}{2}T$. $(v-viii)$ correspond to $f = 6\text{Hz}$, at the same time steps.

along the chord that approaches the high magnitude state is somewhat diminished, where less of the middle portion of the chord is deflecting. Additionally, if we compare 5.5(*i*) to (*ii*) it can be seen that a region of higher magnitude (cyan) appears to be propagating down the chord of the structure, indicative of traveling wave dynamics.

Next, if we examine 5.5(*v-viii*), it appears somewhat similar to the second bending mode, with a smaller region of peak deflection. It is a bit challenging to see in the time series, but there appears to be a node starting to form at $\frac{2}{3}C$, similar to the shape characterized by the second bending mode of a beam. Beyond the developing node, regions of high magnitude propagate down the structure, again demonstrating that there could be some amount of traveling wave behavior occurring. This can be seen through the comparison of 5.5(*v*) to (*vi*), where the yellow region is moving along the chord of the foil in time. Additionally, one notable difference between these bending shapes is the phase of the leading and trailing edge. At $f = 6\text{Hz}$ the trailing and leading edges are almost exactly out of phase (i.e. trailing edge is in red when the leading edge is in blue). In contrast, the $f = 3.75\text{Hz}$ case has the trailing edge closer to being in phase with the leading edge. Based on previous studies of constant thickness foils, we expect to see more nonlinearity at higher heaving amplitude from the transverse damping [4]. Next, we generate a similar plot, but for $A = 3\text{mm}$, shown in 5.6(*i-viii*).

We again see similar deformed shapes to the $A = 1.5\text{mm}$ case, with each frequency appearing somewhat similar to the first and second bending modes of a beam. If we compare 5.6(*i*) to (*ii*), we can see the cyan region propagating down the length of the foil. It should additionally be noted that the phase of the leading and trailing edge are somewhat close to being in phase for this case as shown by both the leading and trailing edge being either blue or red at the end of the shaker motion. Next, if we inspect 5.6(*v-viii*) we can see a node forming at $\frac{2}{3}C$, with what appears to be traveling wave behavior beyond the node, similar to the lower amplitude case above.

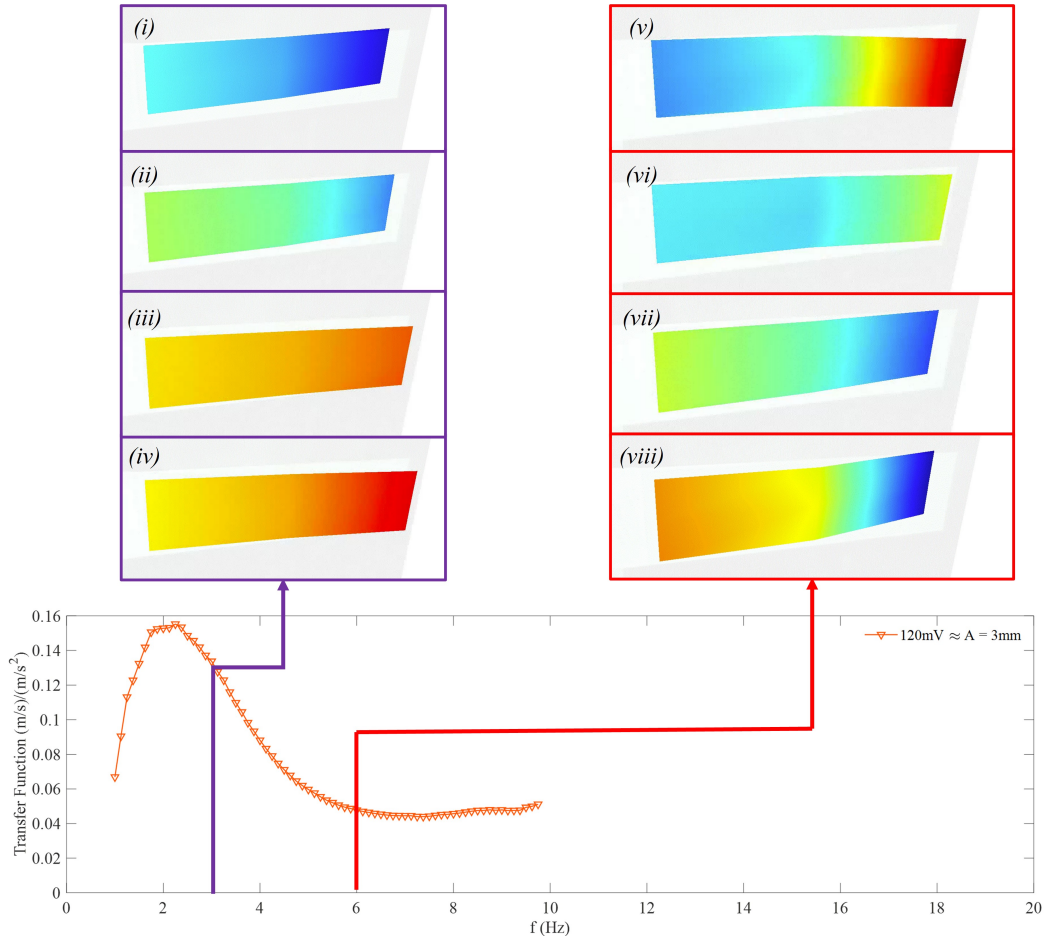


Figure 5.6: FRF and corresponding mode shapes for $A = 3\text{mm}$. (i-iv) correspond to $f = 3\text{Hz}$, at times $t = 0T, \frac{1}{8}T, \frac{1}{4}T, \frac{3}{8}T, \frac{1}{2}T$. (v-viii) correspond to $f = 6\text{Hz}$, at the same time steps.

With knowledge of the resonant frequencies and oscillation behavior, we now examine the wakes generated from the flexible foil when driven below, at, and beyond resonance for varied heaving amplitudes.

Wake Characteristics:

In terms of fluid dynamics, we first examine instantaneous vorticity plots from the $A = 1.5mm$ heaving amplitude case, with frequency varying from $\frac{f}{\omega_{n1}} = 0.76, 1.07, 1.84$, shown in 5.7(a-c; *i-iv*). The panels *i-iv* are indicative of $t \approx 0 - 1.0T$, or one period of oscillation for the frequency shown in the given row, with each separated in time by one-quarter period. The color plot represents positive (red CCW-rotating), and negative (blue CW-rotating) vortices, with the color scale being $\frac{\omega_z}{f}$, or vorticity normalized by heaving frequency. The green arrows indicate the direction of foil travel in the corresponding plot, with the size of the arrow corresponding to the velocity of the foil. The larger arrows indicate the foil is translating close to its maximum velocity, whereas the smaller arrows indicate the foil is changing direction and beginning to travel in the opposing direction. The velocity vectors were chosen to be removed from this plot to allow for better clarity of the vortex structures. We choose to analyze $\frac{f}{\omega_{n1}} = 1.07$, as we later find that operating slightly above the first natural frequency results in better thrust performance, similar to previous work [36].

The wake of the heaving foil at quasi-steady state (i.e. was allowed to oscillate for ≈ 10 seconds before measurement) is characterized by vortices rolling up and shedding off of the trailing edge of the foil. The roll up process occurs when the foil is translating in one direction, then the vortices are shed when the direction of oscillation changes. Given that the tank is quiescent, there is no continuous flow to carry these vortices away from the structure. As a result, the vortices produced in a subsequent oscillation cycle have a great effect on the previous, resulting in a fairly turbulent wake. Additionally, operating

at relatively high frequencies (6Hz) resulted in a more turbulent wake, especially when comparing the present study to previous work in a quiescent tank. Previous studies typically operate in relatively low-frequency regimes $f = 0.5 - 3.5\text{Hz}$, with foil stiffness approximately an order of magnitude lower $E \approx 1\text{MPa}$ (silicone) [51, 67].

At a glance, it can be seen that the input frequency close to resonance is generating the strongest wake structures as shown by the size of the positive and negative vortices being rolled up on the foil when comparing between 5.7(a-c). At this heaving amplitude resonance was measured at $\omega_{n1} = 3.25\text{Hz}$, based on the FRF. From qualitatively examining this plot, there is some evidence of resonance behavior, or larger stronger vortices being formed close to ω_{n1} , which would be closest to corresponding to 5.7(b).

It appears when close to resonance, the previously formed core of vorticity has slightly less of an impact on the subsequently formed one compared to beyond resonance, resulting in the stronger cores surrounding the foil. This can be seen when comparing 5.7(b) *ii* and *iv* to 5.7(c) *ii* and *iv*, or times when the foil direction is changing (small green arrow). When operating close to resonance two opposite sign cores of vorticity are allowed to persist longer at the ends of the stroke, whereas the frequency above resonance dissipates the previously formed core resulting in only positive or negative vorticity on the edge of the foil. This likely has to do with the phase associated with approaching the second bending mode of a beam, where the leading and trailing edges are out of phase with one another for the higher frequency case. As such, the foil leading edge starts to travel in the opposite direction of the trailing edge likely reducing entrainment to the vorticity being formed.

Next, we examine the time-averaged vector and vorticity fields produced by these instantaneous plots to gain more insight into the wake characteristics as frequency is changed. 5.8(a-c) shows the 4 cycle time averaged magnitude of velocity (U_{total}) for $A = 1.5\text{mm}$, and $\frac{f}{\omega_{n1}} = 0.76, 1.07, 1.84$. Note the different scale bar in (a). It can be seen that below resonance, in (a) the time average velocity is far lower than at or beyond resonance shown in (b) and

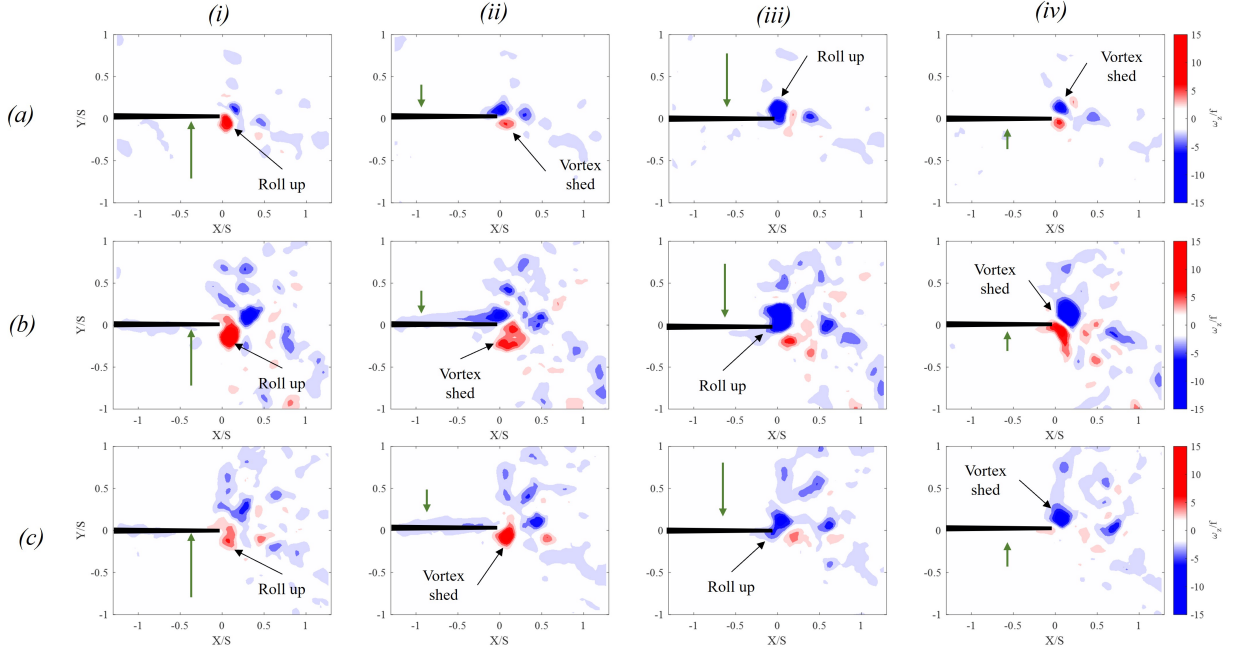


Figure 5.7: Wake characteristics produced by $A = 1.5\text{mm}$ for varied f (a) $\frac{f}{\omega_{n1}} = 0.76$. (b) $\frac{f}{\omega_{n1}} = 1.07$. (c) $\frac{f}{\omega_{n1}} = 1.84$. (i) $t = \frac{0}{T}$. (ii) $t = \frac{0.25}{T}$. (iii) $t = \frac{0.5}{T}$. (iv) $t = \frac{0.75}{T}$

(c). This plot is shown for reference for peak velocity values and to provide an idea of what the physical velocities the foil is making.

5.9(a-c; i-iii), shows the time averaged normalized vorticity ($\omega_z^* = \omega_z / (f)$ where ω_z is vorticity and f is input frequency, normalized X-direction velocity $U^* = U / (fA)$, and velocity magnitude $U_t^* = U_{total} / (fA)$, for $\frac{f}{\omega_{n1}} = 0.76, 1.07, 1.84$.

Here, it can be seen that as heaving frequency is increased, U^* , and U_t^* are reduced when below or beyond resonance with the current normalization when comparing 5.9(c), to (b), or (a). The normalization does play a factor in making 5.9(c; i-iii) color plots appear weaker, in that they were produced with a relatively higher input frequency, $f = 6\text{Hz}$. Note the magnitude of 5.9(b) and (c) are similar as shown in 5.8(b) and (c). The normalization was chosen based on the characteristic velocity for the quiescent experiment (fA), based on that used in previous literature [37, 67].

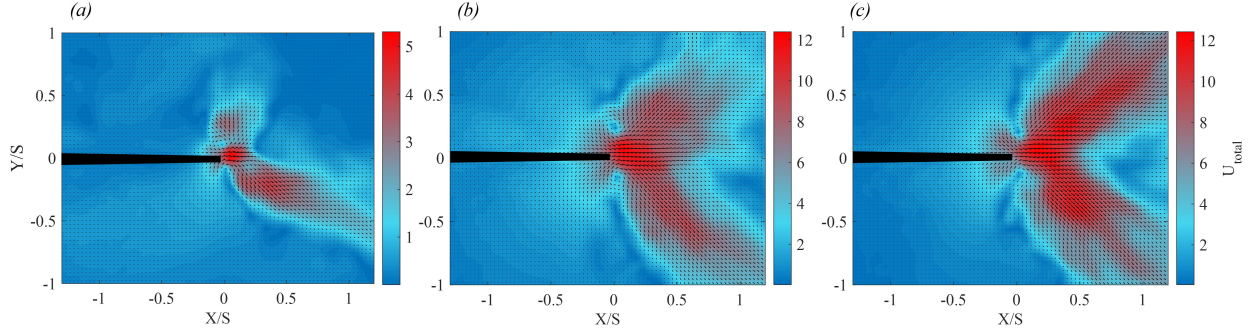


Figure 5.8: Velocity magnitude U_{total} in (cm/sec). a) $\frac{f}{\omega_{n1}} = 0.76$. (b) $\frac{f}{\omega_{n1}} = 1.07$. (c) $\frac{f}{\omega_{n1}} = 1.84$

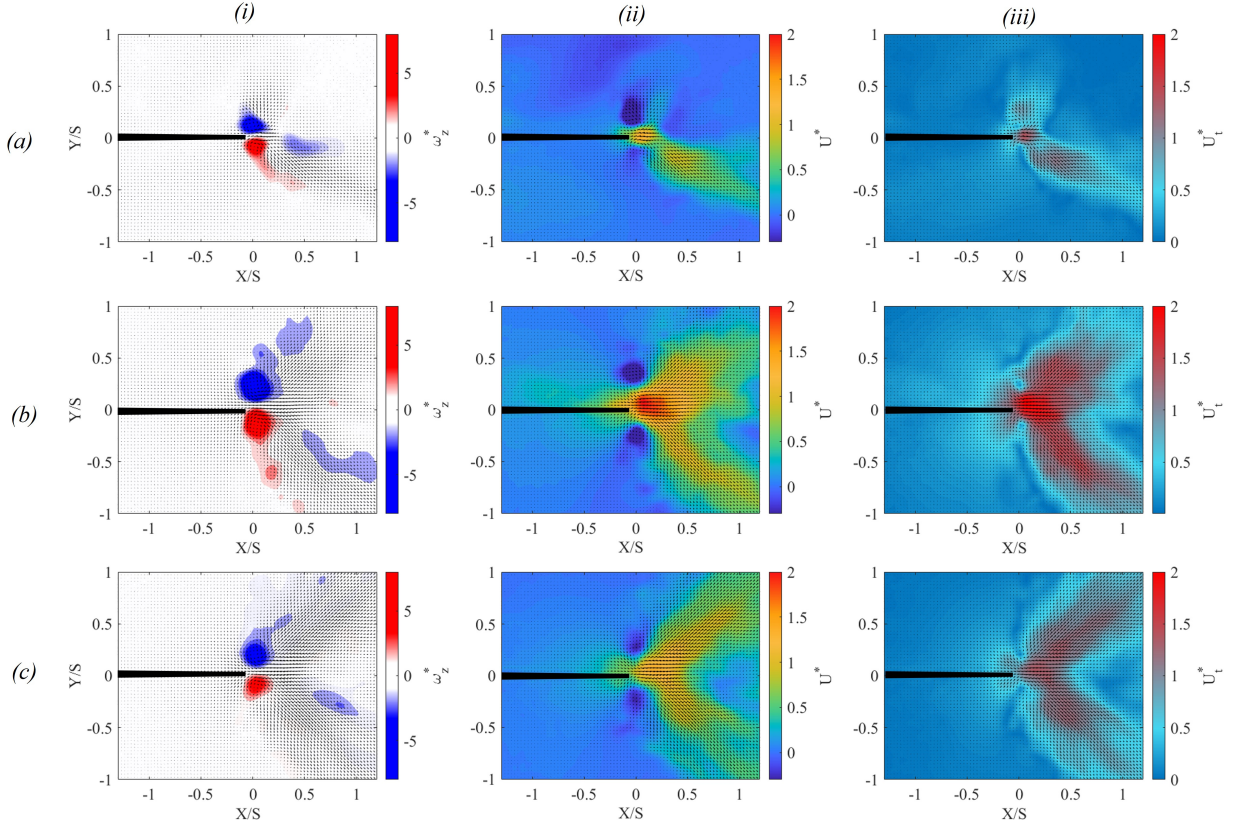


Figure 5.9: Mean time averaged wake characteristics from a time corresponding to 4 complete oscillations of the foil for $A = 1.5\text{mm}$. (i) Vorticity ω_z^* . (ii) x-velocity U^* . (iii) velocity magnitude U_t^* . (a) $\frac{f}{\omega_{n1}} = 0.76$. (b) $\frac{f}{\omega_{n1}} = 1.07$. (c) $\frac{f}{\omega_{n1}} = 1.84$

The wake width of the U^* component is relatively similar between $\frac{f}{\omega_{n1}} = 1.07$ and 1.84 case as shown in 5.9(b) and (c), but with $\frac{f}{\omega_{n1}} = 1.07$ having slightly more liquid being forced in the streamwise direction. The higher frequency case has a slightly larger teal 'v' shaped region separating the higher velocity fluid wakes generated from heaving in each direction. For $\frac{f}{\omega_{n1}} = 0.76$, one side of the wake is directed almost entirely in the positive Y-direction while the other side is pushed in the streamwise velocity with a similar trajectory to the higher frequency cases. This is discussed in the subsequent paragraph, but the direction of this wake seems somewhat random and is sensitive to convective currents within the tank. The maximum measured velocity for this case was only 5cm/sec at the foil tip, compared to 12cm/sec at the foil tip for resonance and beyond. Generally, below resonance, the heaving foil behaves more rigidly and directs more fluid in the transverse direction than the higher frequency cases.

It should also be noted that the low-frequency oscillation has a clearly biased wake toward the (-Y) direction in 5.9(a). This was found for the majority of the low-frequency oscillation trials, however, the direction of the bias was somewhat random. Once a preferential wake direction was selected in reaching quasi-steady state, the foil would continue to push fluid in this direction. However, between trials and frequencies, the direction of the bias varied between the positive and negative Y directions. It is understood that this bias is partially due to the randomness involved with the lower Re flow, and potential background convection currents in the tank.

Up to this point, we have only examined vector and vorticity plots for the moderate heaving amplitude $A = 1.5\text{mm}$. It is found that the other heaving amplitudes follow similar general trends outlined here when comparing velocity and vorticity plots in terms of $\frac{f}{\omega_{n1}}$. The lower heaving amplitude ($A = 0.5\text{mm}$) exhibits identical behavior but at higher input frequencies provided the higher measured resonant peak for this amplitude. In contrast, the high heaving amplitude ($A = 3\text{mm}$) behaves similarly but at relatively lower input

frequencies provided the lower measured resonant peak for this amplitude.

Quantifying Circulation and Momentum

To compare the vorticity generated as A and f was varied, the instantaneous positive circulation $|\Gamma|^+$, and negative circulation $|\Gamma|^-$, were calculated using [73]:

$$|\Gamma|^\pm = 0.5 \int_C |\omega_z|^\pm dA \quad (5.1)$$

The 90% vorticity contour was chosen as the integration area, C , within the field of view defined by $-1.3 < X/S < 1.3$, and $-1.0 < Y/S < 1.0$. Here, $|\omega_z|$ is the vorticity perpendicular to the measurement plane, as we assume the flow is approximately axisymmetric measured at the center span of the foil ($\omega_x = \omega_y = 0$). A plot of instantaneous $|\Gamma|^\pm$ is shown for $A = 1.5\text{mm}$, $f = 3.5\text{Hz}$ in 5.10(a). As shown, the positive and negative circulation oscillates at the same frequency as the input, shown via time normalization with the input period. In turn, this matches the vortex shedding and roll-up ideas previously described, wherein the increases and decreases in $|\Gamma|^\pm$ correlate to the opposite sign vorticity periodically forming on either side of the foil. The solid lines represent the time average value measured for 4 complete heaving cycles of the foil for the $|\Gamma|^+$ and $|\Gamma|^-$ vorticity contributions. The average of these ($|\Gamma|^\pm = \frac{|\Gamma|^+ + |\Gamma|^-}{2}$) was calculated and used to create 5.10(b), where the time average circulation is plotted for the varied heaving amplitudes and frequencies. Here, $|\Gamma|^* = \frac{|\Gamma|^\pm}{(2\pi f A)C}$.

Interestingly, at the lowest heaving amplitude ($A = 0.5\text{mm}$), there is a clear peak in $|\Gamma|^*$ close to resonance $\frac{f}{\omega_{n1}} \approx 1$. In contrast, the larger heaving amplitudes ($A = 1.5, 3\text{mm}$), generate maximal circulation beyond resonance, approximately at $1.3\frac{f}{\omega_{n1}}$. Given the higher damping resulting in more non-linearity, the output may be more dominated by non-resonant effects compared to the low heaving amplitude trial. The frequency range of high $|\Gamma|^*$ is wider for the higher heaving amplitudes, which is to be expected provided the wider

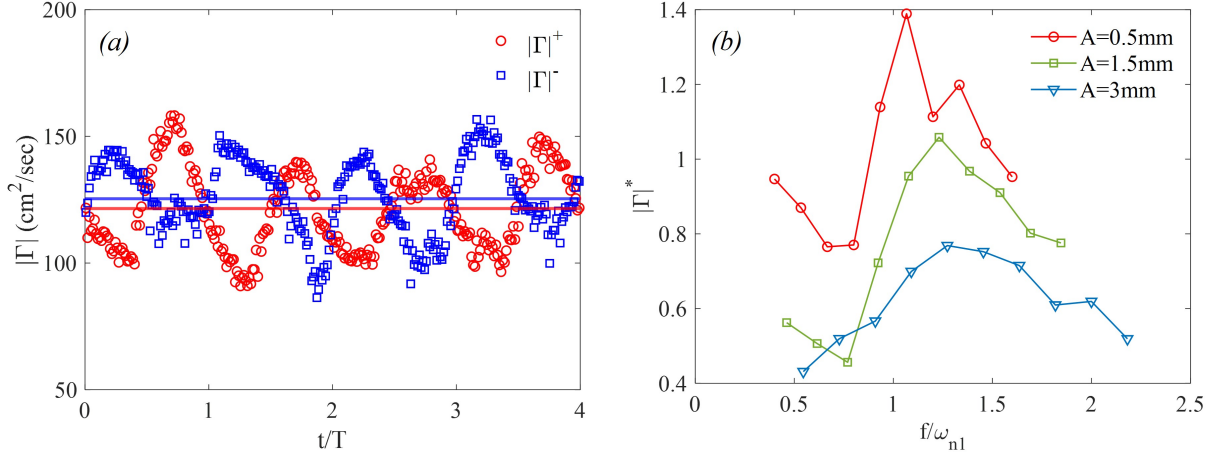


Figure 5.10: Instantaneous and time-averaged circulation. (a) $|\Gamma|^\pm$ measured for $f = 3.5\text{Hz}$. Solid lines indicate the 4 period average of the instantaneous values. (b) $|\Gamma|^*$

peaks in the FRF. It would be interesting to see how $|\Gamma|^*$ is affected beyond the current parameter space at higher $\frac{f}{\omega_{n1}}$. It should be noted that $|\Gamma|^*$ is not necessarily an accurate representation of the thrust force generated by the different driving conditions. $|\Gamma|^*$ is more accurately interpreted as the total amount of fluid being pushed by the foil, with no sign associated with the direction of displaced fluid [67].

To gain a quantitative measure similar to thrust force, we compare the amount of wake directed in the transverse (Y) and streamwise (X) directions based on the X and Y momentum flux. Here, the momentum flux per unit span is calculated by [37]:

$$M_x = \rho_f \int \bar{U}^2 dy \quad (5.2)$$

$$M_y = \rho_f \int \bar{V}^2 dx \quad (5.3)$$

Here $\bar{U}$ and $\bar{V}$ represent the time-averaged X-direction and Y-direction velocities respectively. For this analysis, each frequency and amplitude combination was averaged over

4 complete cycles to obtain $\bar{U}$ and $\bar{V}$. The downstream integration location for calculating M_x was varied based on the heaving amplitude and frequency and was chosen to be the location of maximum M_x for each case. This was done because the high velocity region close to the foil was extended and reduced as A was varied, which would bias the resulting M_x values as the wake spread with increased distance from the foil's trailing edge. M_y was calculated above and below the foil and was summed between these two values, to ultimately estimate the amount of fluid propagating in the transverse direction at $Y/S = \pm 0.33SA$. This was done as the smaller heaving amplitude generated a wake that was dissipated at varied distances from the foil edge. For example, the low heaving amplitude resulted in velocity fields concentrated close to the foil so the integration line was shifted closer to the foil to capture the changes in M_y and M_x between frequencies. It should be noted that the outcome of this calculation is affected by the measurement location, and the chosen locations used here were selected based on those selected in previous work [67]. Although the integration line location does affect the magnitude calculated for M_x , the trends between points appeared to be relatively unaffected by changing integration location.

5.11(a) and (b) show the momentum flux calculated for each heaving amplitude and input frequency. As expected, both M_x and M_y grow with respect to increased heaving amplitude and frequency. There is a small frequency range at which $M_x(A = 1.5mm)$ is greater than $M_x(A = 3mm)$, but to account for this different the $M_y(A = 3mm)$ are significantly greater than $M_y(A = 1.5mm)$ indicative of much more displaced fluid. This likely has to do with the set integration point for M_y . There appear to potentially be local minimum and maximum in both M_x and M_y , requiring a plot of the unitless frequency. Additionally, given the somewhat nonphysical units of the momentum flux $\frac{kg}{sec^2}$ we calculate the momentum flux coefficient.

Next, we calculate a nondimensional momentum flux coefficient, for both the X and Y directions using [37, 67]:

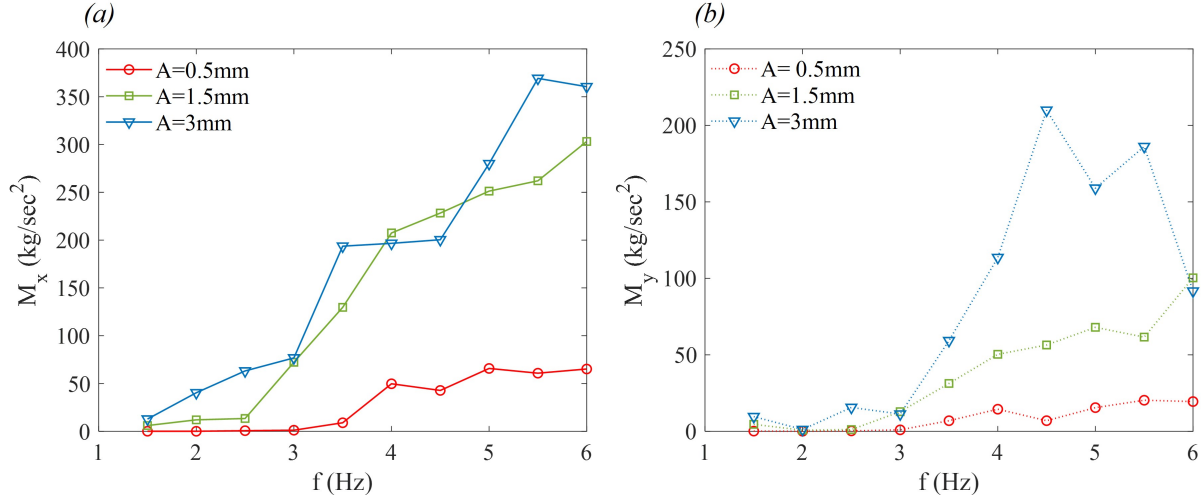


Figure 5.11: X and Y momentum flux for each heaving amplitude: (a) $A = 0.5$ mm, (b) $A = 1.5$ mm, (c) $A = 3$ mm

$$M_x^* = \frac{\rho_f \int \bar{U}^2 dy}{0.5 \rho_f (2 * \pi * f * A)^2 * C} \quad (5.4)$$

$$M_y^* = \frac{\rho_f \int \bar{V}^2 dx}{0.5 \rho_f (2 * \pi * f * A)^2 * C} \quad (5.5)$$

5.12(a-c) shows the momentum flux coefficients M_x^* and M_y^* for each heaving amplitude. For all input heaving amplitudes, we see peaks in both M_x^* and M_y^* but at different points slightly beyond resonance as shown in 5.12(a) and (b). It is challenging to see with the relative scaling, but below resonance, M_x^* and M_y^* are of similar magnitude, indicating that approximately equal amounts of fluid are being directed in the streamwise direction relative to the amount being pushed in the transverse direction. It can additionally be noted that as heaving amplitude is increased to $A = 3$ mm the point beyond resonance is slightly increased 5.13(a). This is similar to the findings of Han et al. [36], in that optimal propulsive performance for a foil with a stepwise reduction in stiffness is found beyond the first natural frequency. In general, across all amplitudes the relationship of M_x^* and M_y^* is somewhat

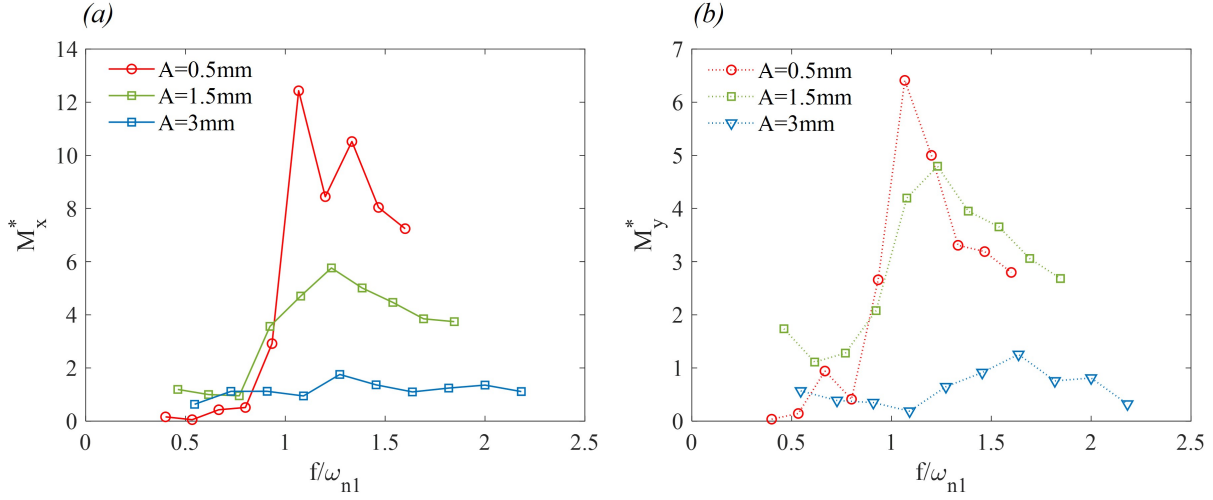


Figure 5.12: X and Y momentum for each heaving amplitude: (a) $A = 0.5\text{mm}$, (b) $A = 1.5\text{mm}$, (c) $A = 3\text{mm}$

coupled in that as M_x^* increases, M_y^* increases as well, but by significantly varied magnitudes. We see good qualitative agreement in peak values when comparing the total momentum flux coefficient $(M_x^* + M_y^*)$ to $|\Gamma|^*$, as would be expected (i.e. more total momentum flux should correlate to increased circulation). The magnitude found for M_x^* was highly affected by the normalization by the square of the input characteristic velocity $(2\pi fA)^2$, where $A = 0.5\text{mm}$ resulted in normalizing by relatively small values, which generated large values of M_x^* . Next, we investigate the amount of fluid entrained in both directions by comparing the momentum flux coefficients $\frac{M_x^*}{M_y^*}$, in 5.13.

We see a clear peak in $\frac{M_x^*}{M_y^*}$ for $A = 3\text{mm}$, but for the other heaving amplitudes there is not as clear of a peak, as shown in 5.13. $A = 1.5\text{mm}$ and $A = 0.5\text{mm}$ have the lowest ratios for the majority of frequencies. However, this plot is highly sensitive to the relatively small values of M_y^* , especially when the M_x^* is relatively large, which can greatly alter the location of peak values in this plot. As such, the magnitude of this plot should be interpreted carefully, and M_x^* is likely a better representation of the best thrust-producing operating conditions for each heaving amplitude. However, this could be thought of as a representation of efficiency,

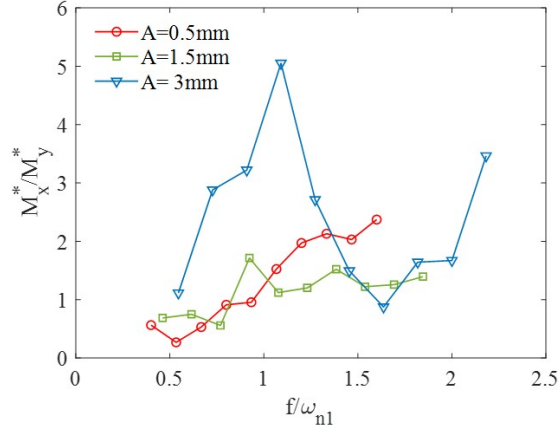


Figure 5.13: Momentum flux coefficients, $\frac{M_x^*}{M_y^*}$, for each heaving amplitude

where a lower ratio represents more fluid being "wasted" in the transverse direction. This should not be thought of exactly as efficiency, but provides some idea of how much fluid is being pushed in the thrust producing direction. Interestingly, the $A = 3\text{mm}$ momentum coefficient ratio starts to increase beyond resonance. This may be due to increased traveling wave behavior which is linked to thrust efficiency [51]. In contrast, the momentum flux ratio for $A = 0.5, 1.5\text{mm}$ appear to maintain a relatively level trend after after resonance, when compared with $A = 3\text{mm}$. However, given the limited parameter space it is challenging to make any solid conclusions as to what will happen further beyond resonance.

We still do not have a clear explanation or relation between the deflection mode shapes to the fluid dynamics we are measuring from PIV other than seeing peaks in output parameters close to or above resonance. We can observe that the leading and trailing edge of the foil begins to get out of phase with one another far beyond resonance, which is known to be negative for thrust performance. However, it has also been shown that for a tapered foil at high heaving amplitude secondary peaks in propulsion performance may occur due to traveling wave behavior [51, 88]. To gain further quantitative insight into the fluid-structure interactions at play, we perform 2D image tracking along the length of the foil chord. This

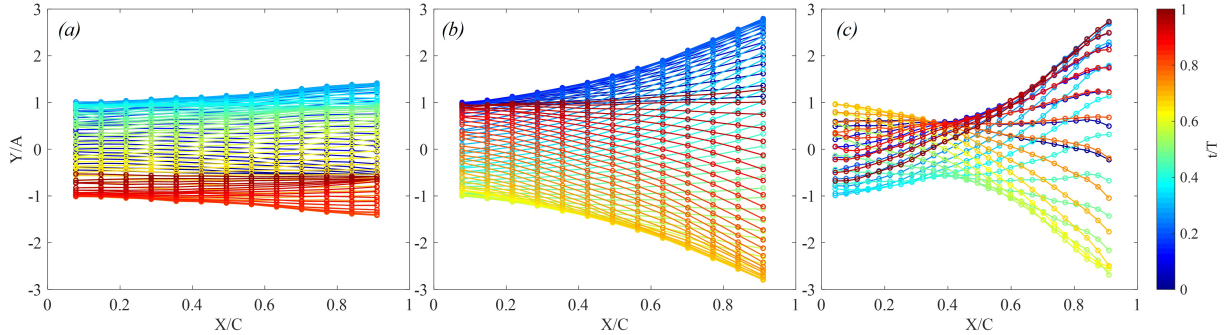


Figure 5.14: Foil deformation envelope for $A = 1.5\text{mm}$ (a) $\frac{f}{\omega_{n1}} = 0.67$. (b) $\frac{f}{\omega_{n1}} = 1.07$. (c) $\frac{f}{\omega_{n1}} = 1.84$

data then allows for the determination of the peak tip displacement beyond input amplitude ($a - A$), traveling wave index (I_{TW}), and phase lag (Φ) of the leading edge a_{LE} to the trailing edge a_{TE} .

Tip Deflection and Traveling Waves

The foil was tracked along its chord across 13 distributed points from the PIV to determine the peak tip displacements, deflection over time, the traveling wave index (I_{TW}), and phase lag (Φ) to provide additional quantitative information about the foil displacement beyond the mode shapes gathered from the laser vibrometer. This analysis is particularly useful in directly identifying the foil motion for a given fluid output, as the displacement data was obtained using the same image sets as those used for PIV. The deflection patterns for the $A = 1.5\text{mm}$, and $\frac{f}{\omega_{n1}} = 0.67, 1.07, 1.84$ are shown in 5.14(a-c), where the color of the lines indicate time.

In 5.14(a), there is very little bending occurring, as shown by deflection shapes on the DSA, but the tip still oscillates slightly beyond the base heaving amplitude. As shown in 5.14(b) and (c), the peak tip displacement for both $\frac{f}{\omega_{n1}} = 1.07$ and 1.84 , is approximately constant at $\frac{a}{A} = \pm 2.75$. A notable difference between these panels is that when farther

beyond resonance, as shown in 5.14(c), a node is beginning to form at $\frac{X}{C} = 0.4$, showing the behavior noted in 5.5. Similar plots were found for the different heaving amplitudes, but with shifted $\frac{f}{\omega_{n1}}$. It is challenging to see the time series bending mechanics shown in these plots, but these plots are still useful in illustrating the total deformation range for a set frequency and amplitude. In turn, this provides a partial explanation for previous findings up to this point, that a larger region (X/C) of tip deflection slightly above resonance is likely the cause of generating the largest M_x^* and $|\Gamma|^*$, for this heaving frequency. Another observation that can be made from these plots is that in 5.14(b) the leading and trailing edge of the foil appear to be some amount out of phase, but not an extreme amount. In 5.14(c) it can clearly be seen that the phase lag of the trailing edge is approximately 180° behind the leading edge. This is likely part of the loss in M_x^* seen for the higher heaving frequencies, where the leading and trailing portions of the foil are working against each other in terms of fluid entrainment. However, this type of behavior may be beneficial in increasing the ratio of $\frac{M_x^*}{M_y^*}$.

An alternative way to plot this same information is with a 2D color plot, with location on the structure on the x-axis and time on the y, with the color plot representing deformation magnitude. This style of plot allows for a clearer understanding of the time series bending mechanics but makes the total bending behavior a bit more convoluted, which is why it is presented second. For reference, MATLAB-generated behavior for pure standing and traveling waves on an imaginary foil are shown in 5.15(a) and (b), to become familiarized with this style of plot. The pure standing wave in 5.15(a) is shown by regions of high amplitude oscillating up and down in the center of the foil chord. Currently, $1/2$ of a sine wave is modeled on the structure, resulting in little to no deformation at the leading and trailing edge of the imaginary foil. The pure traveling wave in 5.15(b) is characterized by slanted regions of high deformation, which correspond to a region of high deformation translating down the chord of the imaginary foil.

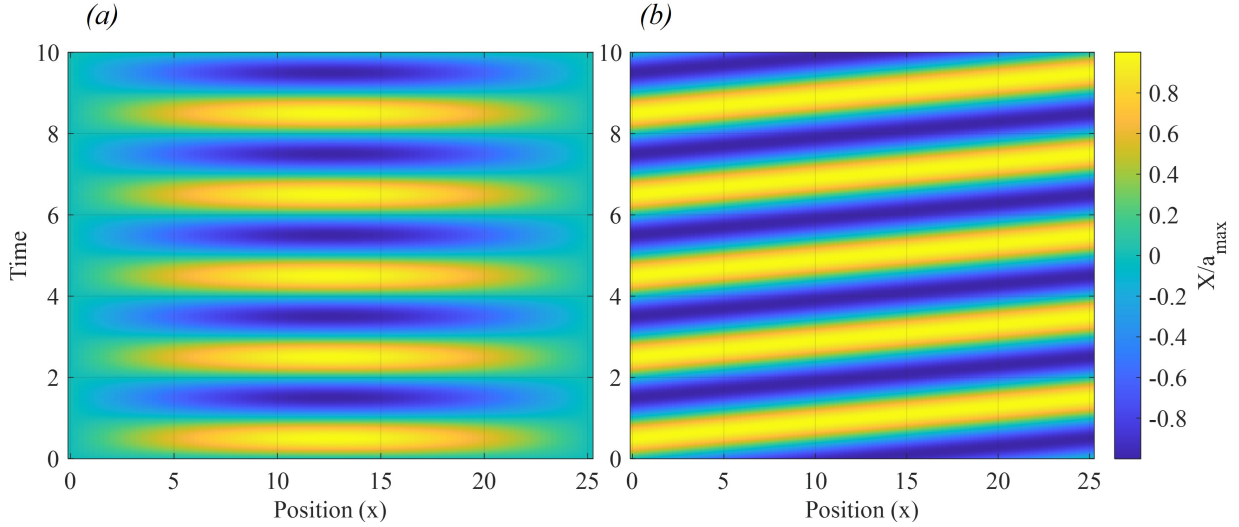


Figure 5.15: Deformation color plot for pure standing and traveling waves on an imaginary foil. (a) Standing wave, modeled by $y = \sin(\omega t)\sin(kx)$, where ω is the oscillation frequency and k is the wave number. (b) Traveling wave modeled by $y = \sin(\omega t - kx)$

Next, we can examine a plot of this style for measured data from the tracked foil shown in 5.16(a-c), corresponding to the same data shown in 5.14(a-c). Note, that the color bar is normalized by the maximum tip deflection measured in each plot, making the color bar scale from $(-1) - (1)$. Only one oscillation period is shown, but as the time would progress further along the Y-axis, the deformation behavior can be thought of as looping back to $t/T = 0$.

In 5.16(a), almost pure heaving motion with no bending is shown by the colorplot as time progresses in the Y-direction for $\frac{f}{\omega_{n1}} = 0.67$, as previously noted. There is some oscillating tip deflection independent of the heaving base shown by a slightly more yellow region at $t/T \approx 0.4$ towards the end of the wing chord, however, this is fairly minimal. As the frequency is approximately increased to resonance shown in 5.16(b), $\frac{f}{\omega_{n1}} = 1.07$, the regions of high deflection begin to tilt diagonally, indicative of traveling wave behavior [78]. Essentially, as the Y location, or time, is increased, regions of high deflection propagate down the structure in the X direction, reaching maximum deflection at the foil tip. The region of high deflection (bright yellow, or dark blue), corresponds to a significant portion of the

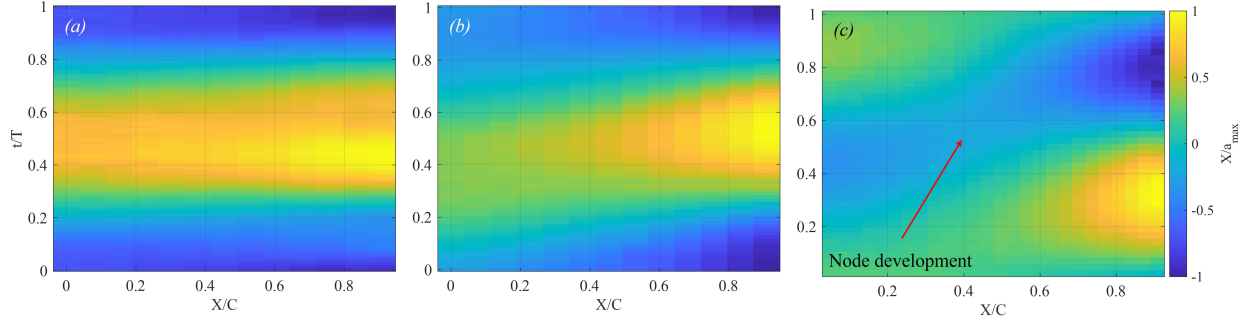


Figure 5.16: Foil deformation colorplot for $A = 1.5\text{mm}$. Time progresses in Y , and X represents the location along the foil's chord. (a) $\frac{f}{\omega_{n1}} = 0.67$. (b) $\frac{f}{\omega_{n1}} = 1.07$. (c) $\frac{f}{\omega_{n1}} = 1.84$

foil chord, ($\approx 0.55\frac{X}{C}$) when the high deformation region is propagating down the chord at $\frac{t}{T} = 0.4 - 0.6$. As the input frequency is increased further shown in 5.16(c), $\frac{f}{\omega_{n1}} = 1.6$, the regions of high deformation tilt further, perhaps indicating a greater amount of traveling wave kinematics. However, as previously noted, a node is beginning to form, similar to the second bending mode of a beam. This can be seen by the slightly darker blue and brighter yellow regions at the base of the foil compared to where the node is labeled in the figure. So instead of regions of high deformation propagating down the foil, a point develops at which the foil is actually moving less than the leading edge of the wing. In turn, the length of the region of high deflection across the chord is significantly reduced and would displace less water than the slightly lower frequency closer to resonance. In other words, there may be more traveling wave kinematics for the higher frequency case, but these waves are broken into two parts by the node development so the propulsive benefits may not be completely realized.

A similar set of plots are examined for $A = 3\text{mm}$ for qualitative examination of the amount of traveling wave behavior and deformation in time shown in 5.17(a-c). Here, we show $\frac{f}{\omega_{n1}} = 0.72, 1.23, 2.18$ in (a-c) respectively. Here, we can see similar behavior where below resonance we see approximate heaving behavior, and above resonance some amount of traveling wave behavior. Again, for the high-frequency case, we can see a node developing,

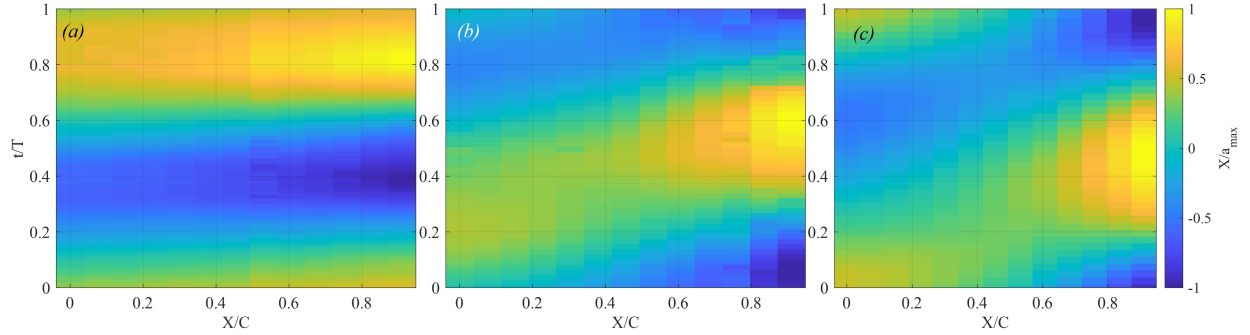


Figure 5.17: Foil deformation color plot for $A = 3\text{mm}$. (a) $\frac{f}{\omega_{n1}} = 0.72$. (b) $\frac{f}{\omega_{n1}} = 1.23$. (c) $\frac{f}{\omega_{n1}} = 2.18$

which prevents the high deformation region from entirely traveling down the foil, hindering propulsive performance in terms of M_x^* . Notably, for this higher heaving amplitude case, we see that the regions of high deformation begin to tilt further than the $A = 1.5\text{mm}$ cases, indicative of traveling wave kinematics.

To quantify the amount of traveling wave behavior, we calculate the traveling wave index (I_{TW}) from the COD of the positional data measured from the foil tracking. The method used here is described in 3, and is adopted from [26]. The traveling wave index found for the parameter space is shown in 5.18(a). From this plot, it can be gathered that we see more traveling wave kinematics as we increase heaving amplitude and frequency, which is to be expected [78]. For $A = 3\text{mm}$, I_{TW} is seemingly continuing to increase within the parameter space. In contrast, for the lower heaving amplitudes I_{TW} appears to be leveling off, or perhaps decreasing, yet it is hard to make any claim without further experimentation. As a side note, provided that $A = 0.5\text{mm}$ had a higher resonant frequency, it does not start to exhibit traveling wave behavior until much higher input f when compared to $A = 3\text{mm}$, despite having similar values in terms of nondimensional frequency. This is a logical result, however, we do not see a clear relationship between the traveling wave index and measured M_x^* which was described in [51]. The I_{TW} values measured here appear to continue to increase

as we increase input frequency, while Leroy-Calatayud et al. [51] found a clear peak indicating a maximum in traveling wave behavior of their much more flexible foils that were oscillated at higher amplitudes ($A = 9.5\text{mm}$). As a result, we may see a clearer relationship between I_{TW} and M_x^* if we continued the experiment to higher frequencies and amplitudes with the relatively stiffer foil studied in the present experiment. There is likely some relationship between I_{TW} and the increase in $\frac{M_x^*}{M_y^*}$ measured for the $A = 3\text{mm}$ heaving amplitude at the highest driving frequencies shown in 5.13. In this plot for $A = 3\text{mm}$, $\frac{M_x^*}{M_y^*}$ appears to be growing significantly when compared to the lower heaving amplitude cases when operated beyond resonance. Additionally, the relative amount that M_x^* is reduced beyond resonance for both $A = 3$ and 1.5mm is relatively lower than the $A = 0.5\text{mm}$ case, perhaps related to increased I_{TW} . For the present parameter space, we likely are seeing resonant behavior being the driver of propulsive performance, provided that we are in a relatively low heaving amplitude regime, and do not see a peak in I_{TW} . Further examination of higher frequency ranges for the current parameter space might show peaks in I_{TW} that could correspond to second peaks in M_x^* or $\frac{M_x^*}{M_y^*}$.

Alternatively, we examine the tip displacement amplitude beyond the input A of the different heaving amplitudes ($a - A$) in 5.18(b). The peaks in tip displacement do not exactly correspond to the resonance values we found based on the FRF analysis, and consistently are beyond resonance. This is possibly due to the variation in input voltage provided to the foils to achieve constant LE displacement, or because we measured the FRF in terms of velocity normalized by acceleration, not position normalized by velocity. The locations of peak tip displacement do not directly correspond to increases in input voltage to obtain constant displacement, likely ruling out this possibility. Regardless, the peak tip displacement values do occur close to the peaks measured for M_x^* , particularly for $A = 1.5$, and 3mm . Additionally, the width of the peaks in displacement is representative of the FRF, where at higher heaving amplitudes wider peaks of high magnitude were found. For

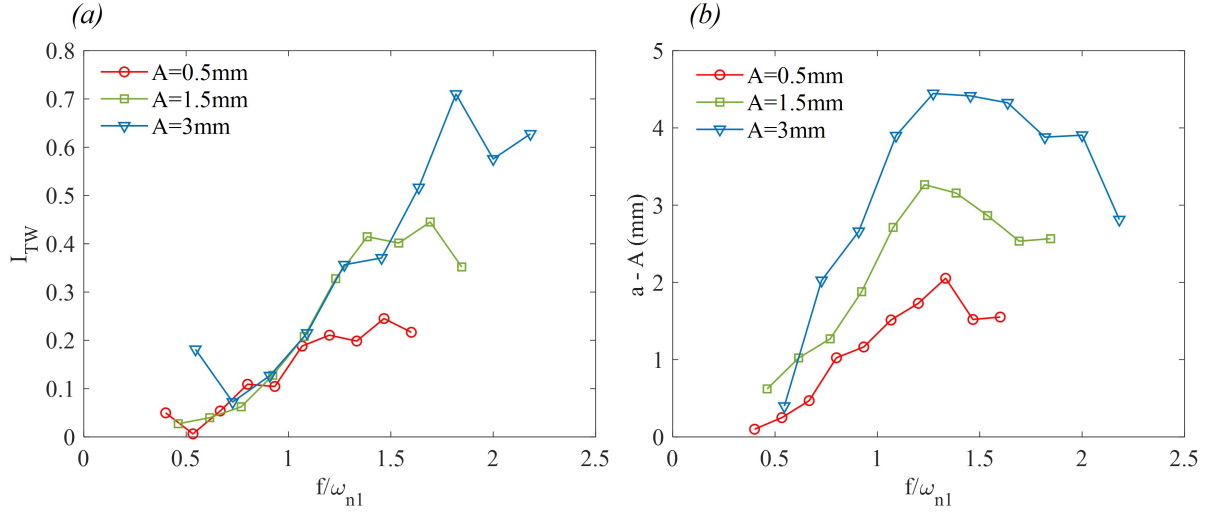


Figure 5.18: Foil Displacement Parameters: (a) Traveling wave index I_{TW} . (b) Tip displacement beyond the input amplitude, $a - A$.

the present experiment, we may be operating in a relatively linear regime where resonance behavior is controlling the fluid dynamics we are measuring as described by [4]. However, the wide peak in tip displacement for $A = 3\text{mm}$ is related to more nonlinear behavior due to the nonlinear transverse damping force. Lastly, we check the phase difference (Φ) between the leading and trailing edge displacement as this was described to be an optimal thrust condition for a uniform pitching flexible foil in quiescent water [67]. We define Φ as the change in phase from the leading edge, a_{LE} , to the trailing edge, a_{TE} , as being positive despite the trailing edge lagging behind the leading edge for all cases examined. Shown in 5.19(a-c) we compare the phase lag (Φ) between the tracked trailing and leading edge of the foil as the input frequency is varied. As shown, it can be seen that as f is reduced from 6 to 2Hz, the phase lag between the leading and trailing edge of the foil approaches zero.

We find Φ for the entire parameter space and plot it in 5.20(a), and compare the times at which $\Phi = 90^\circ$ based on the optimal relation for a pitching flexible foil described by [67]. Here we similarly find relatively good agreement between peaks in M_x^* and when $\Phi = 90^\circ$ produced by a heaving flexible foil as shown in 5.20(b). Essentially, for a $\Phi = 90^\circ$ phase

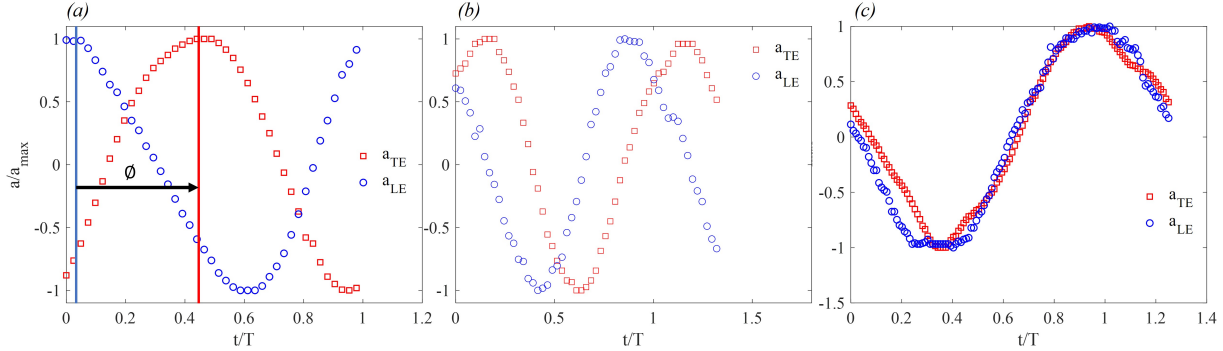


Figure 5.19: Phase lag measured for $A = 1.5\text{mm}$ (a) $f = 6\text{Hz}$. (b) $f = 4.5\text{Hz}$, (c) $f = 2\text{Hz}$

lag, the foil has the maximum bending occurring when the heaving velocity is approaching its maximum (center of the shaker stroke). As a result, the most possible fluid would be entrained into one heaving oscillation and in turn, be pushed to generate a net positive momentum flux. If the phase lag is too slow, (i.e. operated at too high of a frequency), the leading and trailing edge get out of phase from one another. This can be seen in 5.17(c), where at certain time steps (y-values) the trailing edge is highly deformed in the positive direction, while the leading edge is deformed in the negative direction. This would result in less fluid entrainment, and thus less fluid being pushed to generate momentum. As noted by the deformation mode shape analysis, a node is beginning to form on the foil, and a portion of the foil is moving less than the forced leading edge, which would additionally reduce entrainment. Lastly, if the foil has too low of a phase lag, (i.e. operated at too low of a frequency) a relatively larger portion of fluid gets pushed in the transverse direction as indicated in 5.13. At this range, with a low phase lag and low driving frequency, there is little to no bending occurring within the foil and it approaches the behavior of a completely rigid foil.

Although the phase lag provides a partial explanation for the increased M_x^* measured close to resonance, it does not explain the differences in M_x^* , or $\frac{M_x^*}{M_y^*}$ beyond resonance at the different heaving amplitudes. At higher ends of the current parameter space, we know

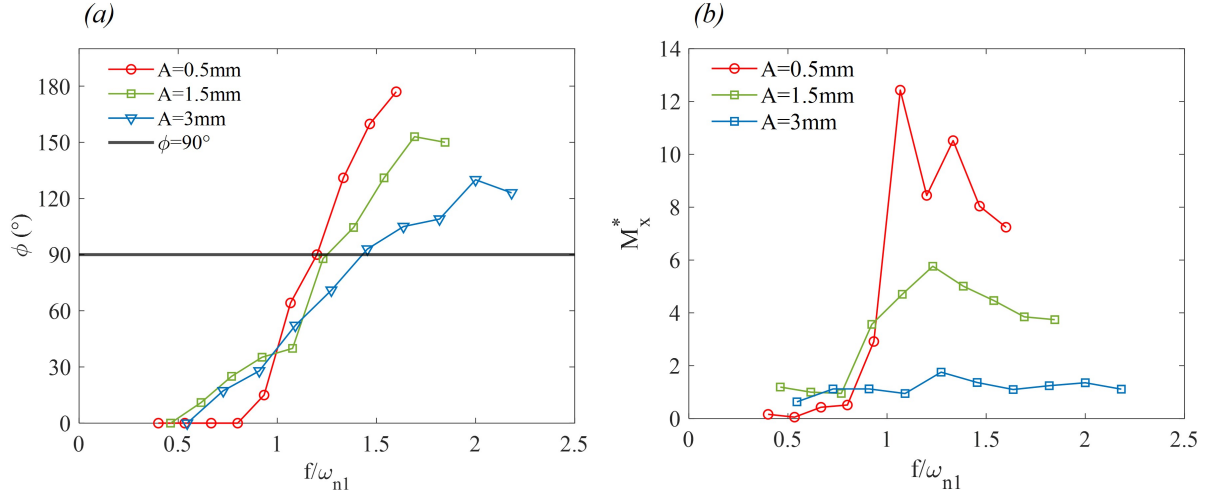


Figure 5.20: Phase lag effects. (a) Φ for the entire parameter space. Black line indicates $\frac{\pi}{2} = 90^\circ$ phase lag. (b) M_x^* shown due to relevant effects of Φ

more traveling wave behavior is occurring from the traveling wave index. There may be an interplay here, where if we examined higher frequencies, we may see additional traveling wave behavior and thus secondary peaks in propulsive performance. It could be possible that for a tapered foil, traveling waves may increase propulsive performance between resonant peaks. Alternatively, when driven near resonance the tip deflection and phase lag may be more relevant in describing the optimal conditions.

Conclusions

In the present study, we quantify resonant frequencies of a heaving tapered foil through identification of peaks in the frequency response function (FRF) measured with a scanning laser vibrometer. We find that as heaving frequency and amplitudes are increased, the resonant peaks in the FRF get wider and less prominent characterized by larger amounts of damping. The damping ratios and Q-factors are quantified for each resonant peak indicating that increased foil velocity results in larger damping forces from a transverse drag force. The mode shapes of the foil are identified at different driving frequencies and it is shown that the

first and second bending modes remain approximately constant, in terms of shape, regardless of the heaving amplitudes studied. Between the first and second resonant peaks traveling wave behavior arises, and becomes more prominent at higher heaving amplitudes due to the nonlinear damping force.

The wake of the foil is characterized and the thrust generated by the foil is quantified with the X-direction momentum flux coefficient from the time-averaged foil wake. Peaks in momentum flux coefficient are found slightly beyond resonance for all heaving amplitudes. The amount beyond resonance was found to increase with heaving amplitude, perhaps due to more nonlinear foil kinematics from quadratic damping and less clear resonant peaks in the FRF.

After characterizing the wake, we find that foil resonance and the phase lag of the trailing edge of the foil play a crucial role in determining the output momentum flux coefficient measured from the varied amplitude heaving foil in a quiescent fluid. When the foil is close to resonance the foil trailing edge begins to deform significantly, and the deformation is quantified by tracking the length of the foil for the varied heaving amplitudes and frequencies. The peaks in the momentum flux coefficient approximately correlate to peaks measured for the largest tip displacements, which in itself would entrain additional fluid. Additionally, we find that when the trailing edge of the foil has a $\approx 90^\circ$ phase lag, which occurs just beyond resonance, the foil bends to its peak deflection while the heaving velocity of the shaker is approaching its maximum (center stroke). In turn, the largest amount of fluid gets entrained into the foil wake and can then generate a greater net momentum flux. When the foil is driven too quickly, the phase lag between the leading and trailing edge increases and eventually becomes almost completely out of phase (177°). This results in less fluid entrainment and less fluid being pushed to generate momentum. If the foil is operated at too low of a frequency, i.e. too low of a phase lag, the majority of fluid gets pushed in the transverse direction and the foil does not bend substantially.

We also quantify the traveling wave behavior of the foil with the traveling wave index (I_{TW}) but do not find a direct relation between I_{TW} and the output measured M_x^* for the current parameter space. We do find that more traveling wave behavior occurs for a higher heaving amplitude as predicted by previous computational and experimental work [4, 77, 78]. The peaks in M_x^* broaden at higher heaving amplitudes and do not decrease as rapidly as the low heaving amplitude, which could be related to I_{TW} . Additionally, at the edge of the parameter space at the largest heaving amplitude and frequency combinations, we may be seeing a second peak in $\frac{M_x^*}{M_y^*}$ forming where the most traveling wave behavior is occurring. The current parameter space may not be operating in a high enough heaving amplitude and frequency combination regime relative to the foil stiffness to show a peak in I_{TW} , and thus its effects are not completely understood. One possible explanation is that we are generating higher I_{TW} primarily through the mixing of the first and second bending mode and secondarily through the nonlinear transverse drag force. At some combination of heaving amplitude (nonlinear damping) and structural stiffness, it is possible that traveling wave dynamics become more significant than resonance and the phase lag condition, most likely between resonant peaks.

In future work, it would be interesting to examine: (1) higher heaving amplitudes, (2) higher frequencies, and (3) a uniform foil control. Through increasing the amplitude and frequency, we expect greater amounts of nonlinear effects and may approach a traveling wave dominated optimal condition. In studying a uniform control foil we may gain a more clear understanding of the performance of the tapered foil beyond resonance.

CONCLUSION

Two studies were completed in fulfillment of this master's thesis.

The first studied vortex ring formation from varied flexibility nozzles. An optimal stiffness condition was identified for a flexible nozzle in the jetting phase that maximized hydrodynamic impulse per unit volume related to the damped natural frequency of the nozzle and fluid acceleration. The pinch off time for vortex rings produced by the different nozzles was predicted and confirmed, to further explain increases in hydrodynamic impulse from varied ejected volumes. It was also found that in deceleration the more compliant nozzles collapsed due to a negative pressure region within the nozzle that resulted in additional expelled fluid. Upon reopening, the nozzle would refill itself at varied rates which resulted in beneficial stopping vortices within the nozzle walls that increased in strength with increased nozzle stiffness. This identified a secondary optimal stiffness condition that needs to be considered for the implementation of a flexible nozzle propulsion mechanism.

Future work from this study could consider further optimization of the refilling cycle by altering the deceleration rate while also considering the optimal stiffness criteria for jetting. By tuning the pump deceleration for the nozzle collapse and refilling condition relative to the nozzle stiffness, even higher nondimensional impulse and circulation may be achieved. Additionally, it would be interesting to see this type of propulsion mechanism studied on a free-swimming robot. It could be useful to quantify the difference in distance traveled over time based on the optimal conditions found here for flexible nozzles and compare those to a rigid nozzle outlet.

The second study examined a tapered flexible foil propulsion mechanism. Resonant frequencies of the foil were found for varied heaving amplitudes (varied damping). It was found that the foils generated the greatest X-direction momentum flux when the trailing edge of the foil was phase lagged by approximately 90° behind the trailing edge. The amount of

traveling wave behavior present within the foil was quantified with the traveling wave index value. It was found that increasing heaving amplitude and frequency both increased the amount of traveling wave behavior on the foil. The upper limit to the traveling wave index was not found, but higher traveling wave indices were identified further into the nonlinear regime, indicating some dependence on the amount of traveling wave behavior with the amount of nonlinear damping.

Future work from this study could consider several distinct aspects to increase the amount of nonlinear behavior present in the structure, which may further encourage traveling wave behavior. Several options include: (1) input higher heaving amplitudes, (2) higher frequencies, and (3) implement a more compliant foil to reduce resonant peaks. (1) and (2) in particular could compound to drive greater amounts of nonlinear damping, which may result in a traveling wave dominated optimal condition for thrust. Lastly, comparing the results of this experiment to a uniform foil would provide a clearer baseline as to how much of the foil behavior can be attributed to the foil tapering versus the heaving amplitude.

In both studies, the design of flexible propulsion mechanisms is highly dependent on the interaction between the flexible structure and the working fluid, which in turn influences the performance of the system. In the first study, optimal nozzle stiffness was found when the structural properties of the nozzle matched the fluid's acceleration, while in the second study, the heaving amplitude of the flexible foil directly affected the frequencies that generated the largest momentum flux. This underlines the importance of fluid-structure coupling in the design of a flexible propulsion mechanism, and the importance of the operating conditions. In the first study, it was shown that a flexible nozzle could compliantly deform when unfavorable pressure regions formed within the nozzle, resulting in a propulsive benefit. This is similar to trends previously observed in studies of flexible foils, where the inherent ability to deform under fluid loading generally results in better propulsive performance by aligning shed vortices and allowing beneficial surface deformations. Based on the second

study, it would be interesting to explore if the amount of tapering on a flexible nozzle, similar to the tapered foil, could induce additional traveling wave behavior, potentially improving hydrodynamic impulse for a wider range of input accelerations. Interestingly, jellyfish exhibit a tapered stiffness that encourages traveling waves along their bell, which likely is a factor in their highly efficient locomotion. As such, the idea of tapering is likely important to consider in the design of any flexible propulsion mechanism, regardless of whether the structure is a foil or nozzle. Understanding how these types of flexible structures interact with the surrounding fluid is key to optimizing their propulsive performance, highlighting the importance of considering the operating conditions when designing a flexible propulsion mechanism.

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