

INTERFEROMETRIC METHODS
FOR SPATIAL-SPECTRAL HOLOGRAPHIC
SIGNAL PROCESSING APPLICATIONS

by

Owen Robert Wolfe

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DEDICATION

I dedicate this dissertation to my family, who have been supportive throughout my academic career. In particular, to my mother and father, who have always encouraged my creativity, to my sister, who has always been a faithful friend, and to my grandmother, Beverly Tucker, whose love has always been a source of strength and encouragement.

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VITA

Owen Robert Wolfe was born in Fairfax, Virginia, in 1994 and grew up in Reston, Virginia. In 2012 Owen graduated with an International Baccalaureate diploma from South Lakes High School. Owen then attended the College Of William & Mary in Williamsburg, Virginia, where he studied physics under the tutelage of Eugeniy E. Mikhailov and Irina Novikova. In 2016 Owen graduated, with honors, from the College of William & Mary, earning the degree of bachelor of science in physics with a minor in anthropology. Owen then chose to study at Montana State University for his graduate education in physics. He conducted his research under Dr. Wm. Randall Babbitt on spatial, spectral holographic signal processing at MSU: Spectrum Lab, earning the degree of master of science in optics and photonics in 2018.

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ABSTRACT

Spatial spectral holographic (SSH) systems have applications in signal processing including spectrum analysis to optical correlation. Beam splitters and material interactions can affect the phase relationships between fields in separate spots in the material (or fields bypassing the material). Interferometric methods allow for these phase shifts to be exploited for two primary uses: the isolation of fields generated by the material and the amplification of signals and the amplification of signals in the chirped readout of programmed spectra.

Stimulated photon echoes are used as the basis of SSH material-based optical systems and have been suggested as a protocol for optical quantum memory. These applications require the stimulated echo field to be isolated from a probe field. Current methods take an angular approach to isolating fields. An alternative approach uses interferometric methods to isolate fields in a way that can be implemented in a photonic integrated circuit. This dissertation will present a mathematical model for the isolation of stimulated photon echoes in both a 2-port Mach-Zehnder interferometer and a 4-port interferometer designed to emulate the isolation achieved via the angular "Box-Geometry" using interferometric methods. Experimental stimulated photon echo studies in a Mach-Zehnder interferometer built around a sample of 0.01% $Tm^{3+} : LiNbO_3$ will demonstrate the principle of interferometric isolation of stimulated photon echoes. Maxwell-Bloch simulations will be used to demonstrate the principle of echo isolation in the 4-port interferometer.

Use of the chirped interferometric readout of programmed spectra is demonstrated for SSH grating processing and spectrum analysis is shown in experiments in a sample of 0.01% $Tm^{3+} : LiNbO_3$. Improvements to signal-to-noise ratio over traditional readout are seen by using a strong local oscillator in both applications. In spectrum analysis applications, the recovery of the proper spectral line shape is demonstrated, and SNR improvements are shown for a given read power. In the grating experiments, a non-linear noise floor was shown that created a constraint on the usable read power for these experiments. In this case, the interferometric readout was able to produce an improvement in the signal-to-noise ratio that could not be achieved using a stronger read power.

BACKGROUND

Motivation

Spatial-spectral holographic (SSH) materials are a class of materials that can retain a record of the power spectra of light that has illuminated them. The power spectrum of the light used to illuminate the material is recorded in the absorption spectrum of the material (spectral) at the position where the illumination occurred (spatial). Furthermore, they can record interference patterns in both domains (holographic). These properties give these materials several applications in signal processing and optical computation. By recording and then reading out optical power spectra, SSH materials can be used as high bandwidth spectrum analyzers with a 100% probability of intercept [38, 40]. SSH spectrum analysis is closely linked to ranging applications, which can be done with SSH materials using either radar [23, 47] or lidar [41] signals. SSH materials can also be used for optical correlation [7, 11], which is useful in applications like pattern recognition and image processing. SSH processes like photon echoes have also been proposed as the mechanisms for optical quantum memories [20, 42].

Interferometers are devices that exploit the interference of two coherent optical fields. This interference is dependent on the relative phase difference between the optical fields. In SSH materials, the phase properties of the fields generated in the interaction with the material can be exploited in an interferometric scheme. In optical correlation experiments, interferometers can be used, like the angular methods used to isolate fields. Rather than isolating fields on different k-vectors, the interferometer can isolate fields on unique output ports. This configuration requires no free-space optics; therefore, this method may be a suitable technique for isolating fields in SSH applications in planar waveguides for

implementation in photonic integrated circuits. In readout contexts (such as spectrum analysis and ranging), the phase-sensitive interferometric signal can be amplified using a strong local oscillator field, and the amplitude spectrum can be recovered from the detected phase signal using a Hilbert transform. The amplification due to the local oscillator can provide avenues for improved SNR in SSH readout systems without increasing the power into the material.

A substantial amount of research has gone into the contemporary understanding of both interferometry and spatial-spectral holographic materials. This chapter will cover the basic theory around SSH materials and their applications to signal processing and will provide an overview of some of the formalism used in discussing interferometric methodology and the Mach-Zehnder interferometer, which is a widely used interferometric architecture that will figure prominently later in this thesis.

Introduction to Spatial Spectral Holographic Processing

Spatial Spectral Holographic Materials

Consider an ion in the ground state; when the ion is illuminated with light at the resonant frequency of the ion (ν_i), there is some probability that the ion will absorb a photon containing $h\nu_i$ of energy from the illuminating light and put the ion into the excited state. If the excited state is short-lived and the ion decays into another long-lived state (often called a bottleneck state). The ion will now no longer react to the illuminating field. If an ensemble of these ions is illuminated with a resonant optical field, the absorption rate will fall as the ions absorb light. When many of these ions are doped into a material, internal stresses and strains can shift the resonant frequency of the individual ions. The result is a statistical distribution of the resonant frequencies of the individual ions. Because the distribution of the ions is much broader than the line shape of the individual ions, the actual observed absorption spectrum of the material will show a much broader absorption line than would be expected for an

individual absorbing ion. This process is called inhomogeneous broadening [4]. The process of inhomogeneous broadening is illustrated in figure 1.1. The combined ensemble line shape

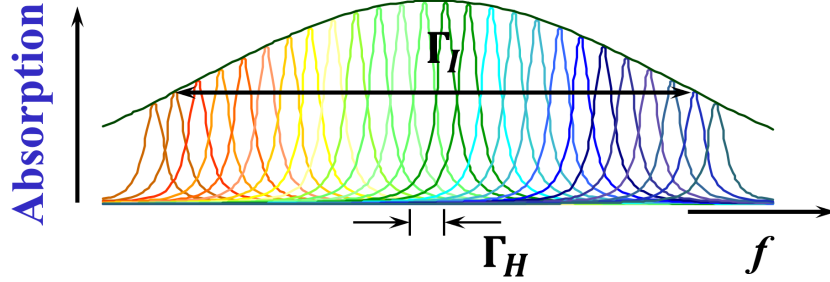


Figure 1.1: An illustration of inhomogeneous broadening. The large statistical distribution of the resonant frequencies of the ions leads to a much broader linewidth of the overall absorption feature than would be expected for the single ion resonance.

is called the inhomogeneous line shape (ℓ_I), and its width is referred to as the inhomogeneous linewidth (Γ_I). In SSH materials these bandwidths can be as high as 200 GHz . The line shape of the individual ions is referred to as the homogeneous line shape (ℓ_H), and its width is referred to as the homogeneous linewidth (Γ_H). The homogeneous line shape should be the same for all ions in the material, though displaced to different frequencies. Typically, in practical systems, the homogeneous linewidth can range from $10\text{ KHz} - 100\text{ KHz}$. Suppose the system is illuminated by a narrow single-frequency CW source. In that case, only those ions resonant at the frequency of that illumination will absorb a quantum of energy out of the illuminating field, become excited, and therefore lose the ability to absorb again. The result will be a local drop in the absorption spectrum of the ions in the material. The absorption spectrum now has a narrow “Spectral Hole” at the frequency of the illuminating field. This process is depicted in figure 1.2.

Materials with these inhomogeneously broadened absorbers distributed throughout them are referred to as spatial-spectral holographic (SSH) materials. This is because (as will be shown in some of the following sections) by storing information in the absorption

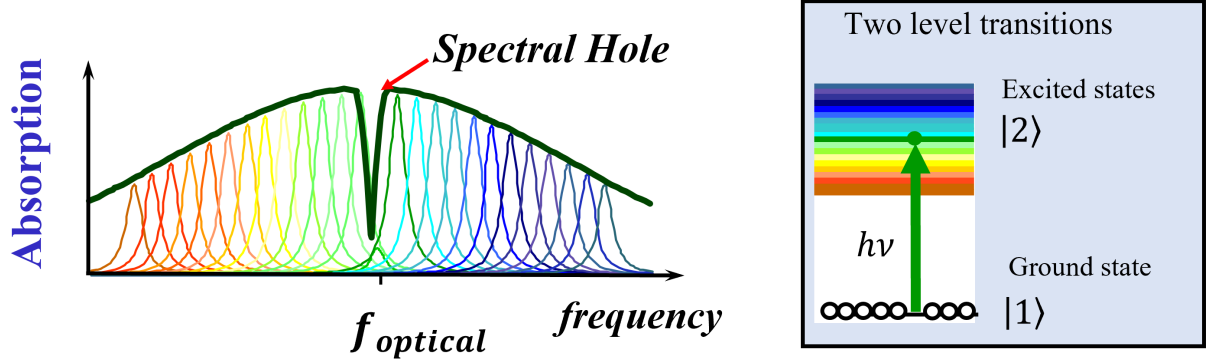


Figure 1.2: Diagram demonstrating the mechanism of spectral hole burning. Left: A spectral hole in an inhomogeneously broadened material. Right: Only resonant ions are excited into the excited state (no bottleneck state pictured here).

spectra distributed spatially through the material, these materials can record interference patterns or holograms, arising from either spatial interference or spectral interference hence the name spatial-spectral holographic materials.

Programming of Spatial Spectral Holographic Materials

A number of different methods can be used to program SSH materials. In order to mathematically describe some of these methods two mathematical models for the programming of SSH materials will be described.

Pulsed Programming of SSH Materials In some cases, it is useful to consider spectra programmed into a material by a signal with a finite duration. This is particularly of interest in cases where the material is programmed by an arbitrary signal with a finite duration (e.g. a brief pulse) and then considered over a time period of less than the material excited state lifetime T_1 . In these cases consider a pulse with an energy spectral density (units $\frac{J}{m^2 Hz}$) of $H(f, x, y)$. The material will see the energy flux spectral density as an effective energy spectral density ($H_{eff}(x, y, f)$) governed by the homogeneous lineshape ($\ell_H(f)$) of

the material according to equation 1.

$$H_{eff}(x, y, f) = (H(x, y, f) * \ell_H(f))(f) \quad (1.1)$$

After the material absorbs the pulse the absorption coefficient of the material will be altered. The new absorption coefficient can be expressed in terms of the energy spectral density, the initial programming coefficient α_0 , and a pulsed programming coefficient (C) with units $\frac{m^2 Hz}{J}$ according to equation 1.2.

$$\alpha(x, y, f) = \alpha_0 L [1 - C H_{eff}(x, y, f)] \quad (1.2)$$

The energy flux spectral density is related to the frequency domain electric field $\tilde{E}(x, y, f)$ (i.e. the Fourier transform of the time domain electric field) according to equation 1.3.

$$H(x, y, f) = \frac{1}{2} \epsilon_0 c n |\tilde{E}(x, y, f)|^2 \quad (1.3)$$

Where ϵ_0 is the vacuum permittivity of free space, c is the speed of light and n is the material refractive index. The modified absorption coefficient due to programming by a pulse within the excited state lifetime is then given by equation 1.4.

$$\alpha(x, y, f) = \alpha_0 L \left[1 - \frac{1}{2} \epsilon_0 c n C \left(|\tilde{E}(x, y, f)|^2 * L(f) \right) \right] \quad (1.4)$$

Constant Continuous Programming of SSH Materials The second programming model assumes that the material is illuminated for a long time by a field with a constant power spectrum. In these cases, it is assumed that holes are burned in a linear regime and the system is given enough time for the decay of the excited and bottleneck states to reach an equilibrium with the excitation of ground state ions by the programming field, this will

occur on a time scale much longer than the bottleneck or storage lifetime T_s . Assume that the optical field has a constant spectral intensity $J(x, y, f)$, which describes the density of optical intensity per unit frequency and can be expressed in units of $\frac{W}{m^2 Hz}$. This quantity can represent a field that varies rapidly (on time scales much less than T_s) so long as the power spectral density per unit area, averaged over the storage lifetime remains constant. From the perspective of the material, the effective spectral intensity ($J_{eff}(x, y, f)$) will be governed by $L(f)$ the normalized homogeneous lineshape of the material according to equation 1.5

$$J_{eff}(x, y, f) = (J(x, y, f) * \ell_H(f))(f) \quad (1.5)$$

After the system reaches equilibrium, the absorption coefficient can be described using a programming constant χ , with units $\frac{m^2 Hz}{W}$, which describes the depth of burned spectra per unit time per unit area per unit frequency, with equation 1.6. Practically, χ depends on various material parameters, including the material's upper state and storage lifetimes, the density of absorbers, the absorption length, the branching ratio, and the inhomogeneous lineshape.

$$\alpha(x, y, f) = \alpha_0 (1 - \chi J_{eff}(x, y, f)) \quad (1.6)$$

Causal Transmission Coefficients and the Hilbert Transform The transmission coefficient due to the modified absorption coefficient spectrum can be described in terms of its spatial and spectral dependence. If the material sample is continuously programmed and has a length L , then the intensity transmission coefficient (T) is given by equation 1.7.

$$T(x, y, f) = e^{-\alpha_0 L (1 - \chi J_{eff}(x, y, f))} \quad (1.7)$$

This coefficient can be used to estimate the optical power that will pass through the material; however, to calculate the temporal transmitted field, the phase response of the

continuously programmed material must also be considered. The magnitude of the field transmission coefficient ($\nu(f)$) is given by equation 1.8.

$$|\nu(f)| = e^{-\frac{1}{2}\alpha_0 L(1-\chi J_{eff}(x,y,f))} \quad (1.8)$$

To calculate the material's complete temporal response, the phase shifts due to the programmed spectrum must be considered. To preserve causality, the phase shifts must obey the Krammers-Kronig relations [32]. Enforcement of the Krammers-Kronig relations can be reduced to a simple procedure beginning with the non-causal frequency domain (amplitude) response implied as the square root of the magnitude response, inverse Fourier transforming the result to obtain the implied impulse response of the non-causal linear frequency response, removing the portion of the impulse response before time $t = 0$, multiplying by two for energy conservation, and then Fourier transforming back into frequency domain. The final result is a (complex) causal frequency domain amplitude response. In the time domain, this entire process can be expressed as a multiplication of the not-yet causal impulse response with a time domain factor $2U(t)$, where $U(t)$ is the Heaviside step function (defined as 0 before $t = 0$, and one after). Using the convolution theorem, the magnitude response can be used to calculate the causal response $\nu(f)$ by a convolution according to equation 1.9

$$\nu(f) = (|\nu(f)| * \mathcal{F}\{2U(t)\}(f))(f) \quad (1.9)$$

where $\mathcal{F}$ denotes the Fourier transform(see appendix A for more detail on integral transform conventions). The Fourier transform of 2 times the Heaviside step is given by 1.10

$$\mathcal{F}\{2U(t)\}(f) = \delta(f) + \frac{i}{\pi f} \quad (1.10)$$

The convolution then reduces to equation 1.11.

$$\nu(f) = |\nu(f)| + i \left(|\nu(f)| * \frac{1}{\pi f} \right) (f) \quad (1.11)$$

The real part of the response is the magnitude response, while the imaginary part is expressed as a convolution with the kernel $\frac{1}{\pi f}$. This convolution is known as the Hilbert transform (which will be denoted as $\mathcal{H}$, see appendix A for more detail on integral transform conventions) given in equation 1.12 [73].

$$\mathcal{H} \{G(f)\} = p.v. \int_{-\infty}^{\infty} \frac{G(f')}{f - f'} df' \quad (1.12)$$

Where p.v. denotes this is the principle value of the integral since, analytically, this integral will need to be calculated in the complex plane [32]. This integral can be computed numerically for practical purposes. Hilbert transforms are convolutions with odd functions, so the Hilbert transform will remove DC offsets. However, for the other terms, the Hilbert transform can be undone using the anti-involution property of the Hilbert transform (i.e., $\mathcal{H} \{ \mathcal{H} \{G(f)\} \} = -G(f)$ [73]). This now casual response can be multiplied by an input frequency domain field to determine the output field.

The product of any two causal frequency domain responses will also be causal. A frequency domain response of the form $e^{-\frac{1}{2}\alpha(f)L}$, the response can be written as a series given by equation 1.13.

$$e^{-\frac{1}{2}\alpha(f)L} = \sum_{j=0}^{\infty} \frac{\left(-\frac{1}{2}\alpha(f)L\right)^j}{j!} \quad (1.13)$$

This expansion is a power series in the absorption, and any product of two causal functions will be a new causal function; every power in the series will itself be causal. Therefore, enforcing causality on the absorption coefficient is sufficient to enforce causality on the entire system. The causal formulation of the field transmission coefficient is described

mathematically in equation 1.14.

$$\nu(f) = e^{-\frac{1}{2}(\alpha(f)L + i\mathcal{H}\{\alpha(f)L\})} \quad (1.14)$$

The real part of the causal argument of the exponential represents the magnitude response, while the imaginary part is a frequency-dependant phase that corresponds with that magnitude response. In the case of a spectrum programmed into an SSH material, the phase shift is given by equation 1.15.

$$\phi(x, y, f) = \mathcal{H} \left\{ -\frac{1}{2}\alpha_0 L (1 - \chi J_{eff}(x, y, f)) \right\} = -\frac{1}{2}\alpha_0 L \chi \mathcal{H} \{ J_{eff}(x, y, f) \} \quad (1.15)$$

Combining these phase shifts with the original spectrum, the causal transmission coefficient is given by equation 1.16

$$\nu(f) = e^{-\frac{1}{2}\alpha_0 L [1 - \chi (J_{eff}(x, y, f) + i\mathcal{H}\{J_{eff}(x, y, f)\})]} \quad (1.16)$$

This coefficient can now be used to compute transmission through the material by a frequency domain product, and the time domain response can be calculated as an inverse Fourier transform.

Programming and Interrogation of SSH Materials with Uniform Pencil Beams

Consider a continuous programming field with a separable spectral intensity that can be described in terms of the effective power spectral density (which is defined as a convolution of the power spectral density of the light with the inhomogeneous lineshape ($S_{eff}(f) = (S(f) * L(f))(f)$) (units $\frac{W}{Hz}$) with power evenly distributed across a circular top-hat beam of radius w_0 (and therefore an area of $A = \pi w_0^2$). The effective spectral intensity across the circular top-hat beam up to the beam radius is given by equation 1.17 and is presumed zero

otherwise.

$$J_{eff}(x, y, f) = \frac{S_{eff}(f)}{\pi w_0^2} \quad (1.17)$$

The programmed region of the material now has an intensity transmission coefficient given by 1.18.

$$|\nu(x, y, f)|^2 = e^{-\alpha_0 L \left(1 - \frac{\chi}{\pi w_0^2} S_{eff}(f)\right)} \quad (1.18)$$

If the material is then interrogated by another co-linear pencil beam of the same width with a power spectral density($S_{in}(f)$). The transmitted spectral intensity ($J_t(f)$) is given by equation 1.19.

$$J_t(f) = \frac{S_{in}(f)}{\pi w_0^2} e^{-\alpha_0 L \left(1 - \frac{\chi}{\pi w_0^2} S_{eff}(f)\right)} \approx \frac{S_{in}(f) e^{-\alpha_0 L}}{\pi w_0^2} \left(1 + \alpha_0 L \frac{\chi}{\pi w_0^2} S_{eff}\right) \quad (1.19)$$

The transmitted spectral intensity ($S_t(f)$) can be calculated by integrating over the top-hat beam area, leaving equation 1.20.

$$S_t(f) = \pi w_0^2 J_t(f) = S_{in}(f) e^{-\alpha_0 L} \left(1 + \alpha_0 L \frac{\chi}{\pi w_0^2} S_{eff}\right) \quad (1.20)$$

By introducing a new beam-specific parameter $\beta = \frac{\chi}{\pi w_0^2}$, the programming and interrogation of the material can be considered at the level of the total transmitted power in the beam. The changes to the absorption coefficient can then be expressed using equation 1.21.

$$\alpha(f) = \alpha_0(f) [1 - \beta S_{eff}(f)] \quad (1.21)$$

Readout Of Programmed Spectra

Transmission through an Inhomogeneously Broadened Material Using the simplified framework for describing continuously programmed spectral holes from equation 1.21, the field transmission coefficient of the material (with a programmed spectrum) of length L is

given by the equation 1.22.

$$\nu(f) = e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\})]} \quad (1.22)$$

This can be expressed approximately for weak spectra (i.e., $\beta S_{eff} \ll 1$) using equation 1.23.

$$\nu(f) = e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{1}{2}\alpha_0 L \beta (S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\}) \right] \quad (1.23)$$

If a complex electric field $E_i(t)$ with a frequency domain representation $\tilde{E}_i(f)$ (see appendix A for more detail on integral transform conventions) is transmitted through a continuously programmed SSH material, in the frequency domain, the electric field can be represented with equation 1.24.

$$\tilde{E}_t(f) = \tilde{E}_i(f) e^{-\frac{1}{2}\alpha_0 L} + \frac{\tilde{E}_i(f) e^{-\frac{1}{2}\alpha_0 L}}{2} \alpha_0 L \beta (S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\}) \quad (1.24)$$

The transmitted field consists of two terms. The first term represents the field transmitted through the material absent any programming (i.e., the same field that would be transmitted through an un-programmed material). The second term holds all the information about the interaction of the field with the continuously programmed spectrum.

Chirped Readout Of Spectral Holes A chirped optical field of chirp rate κ and a frequency f_0 at $t = 0$, like the one expressed in equation 1.25,

$$E_{chirp}(t) = E_0 e^{2\pi i (\frac{1}{2}\kappa t^2 + f_0 t)} \quad (1.25)$$

will have a linearly changing instantaneous frequency according to equation 1.26.

$$f(t) = \kappa t + f_0 \quad (1.26)$$

A chirp can then be used to interrogate a programmed SSH material. At low chirp rates (i.e., $\sqrt{\kappa} \ll \Gamma_H$), the chirp can be treated as doing a simple mapping of frequency domain to the time domain. In general, the frequency domain representation of the transmitted chirped field is given by equation 1.27.

$$\tilde{E}_t(f) = \tilde{E}_{chirp}(f)e^{-\frac{1}{2}\alpha_0 L} + \frac{\tilde{E}_{chirp}(f)e^{-\frac{1}{2}\alpha_0 L}}{2}\alpha_0 L\beta(S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\}) \quad (1.27)$$

where the frequency domain representation of the chirp is $\tilde{E}_{chirp} = E_0\sqrt{\frac{1}{\kappa}}e^{i\pi\frac{(f-f_L)^2}{\kappa}}e^{-i\frac{\pi}{4}}$. The time domain output field can be expressed using the convolution theorem. The transmitted field is expressed in equation 1.28

$$E_t(t) = E_{chirp}(t)e^{-\frac{1}{2}\alpha_0 L} + \frac{1}{2}\alpha_0 L\beta e^{-\frac{1}{2}\alpha_0 L} [E_{chirp}(t) * \mathcal{F}^{-1}\{S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\}\}](t) \quad (1.28)$$

The convolution integral can be converted into a Fourier transform into the instantaneous chirp frequency $f(t) = \kappa t + f_L$ because the quadratic phase of the chirp will contain cross terms, one of which becomes a rotating phase $e^{-2\pi(\kappa t + f_L)t'}$ where t' is the integration variable. The transmitted field can be written as 1.29

$$E_t(t) = E_{chirp}(t)e^{-\frac{1}{2}\alpha_0 L} \dots \dots + \frac{1}{2}\alpha_0 L\beta e^{-\frac{1}{2}\alpha_0 L} E_{chirp}(t)e^{\frac{i\pi}{4}} \left(e^{i\pi\kappa t^2} * (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \right) (t) \quad (1.29)$$

The signal can then be detected to measure the absorption spectrum. The simplest method of detection is direct detection, where the transmitted chirp can be thought of as interfering with the programmed spectrum-dependent term; this is done naturally when the transmitted field is detected because optical detectors measure optical power, which is related to the magnitude squared of the optical field. In the context of chirped readout, the detected signal is given up to 1st order in βS_{eff} in equation 1.30.

$$|E_t(t)|^2 \approx |E_{chirp}|^2 e^{-\alpha_0 L} \dots \dots + \alpha_0 L \beta |E_{chirp}|^2 e^{-\alpha_0 L} \text{Re} \left[e^{\frac{i\pi}{4}} \left(e^{i\pi \kappa t^2} * (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \right) (t) \right] \quad (1.30)$$

Typically, to remove the DC background from the transmitted chirp term and to reject common mode noise, the transmitted field is balanced detected against a chirped reference field that is also transmitted through the material so that at the detector $E_{ref} = E_{chirp} e^{-\frac{1}{2}\alpha_0 L}$. The difference in the powers (which are proportional to the magnitude squared of the fields) gives the differential power $P_\Delta = P_{chirp} - P_{ref}$, which is given by equation 1.31

$$P_\Delta = \alpha_0 L \beta P_{chirp} e^{-\alpha_0 L} \sqrt{\kappa} \text{Re} \left[e^{\frac{i\pi}{4}} \left(e^{i\pi \kappa t^2} * (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \right) (t) \right] \quad (1.31)$$

Slow Readout If the chirp rate is slow i.e.e, $\sqrt{\kappa} \ll \Gamma_H$, then the phase shifts incurred by the chirp due to the imaginary part of the signal have negligible effects on the magnitude of the signal, and the chirp will, therefore, simply map time domain into frequency domain according to equation 1.26. The differential power at the detector is given in equation 1.32.

$$P_\Delta(t) = P_{chirp} e^{-\alpha_0 L} \alpha_0 L \beta S_{eff}(\kappa t + f_L) \quad (1.32)$$

The detected signal is proportional to the power in the chirp field and the continuously programmed power spectrum.

Fast Readout At faster chirp rates, the phase shifts due to the dispersion from continuously programmed spectra will lead to distortions to the transmitted signal. Therefore, the convolution must be considered to describe the transmitted spectrum accurately. A spectral recovery technique was devised to extract the continuously programmed power spectrum from the measured signal [16, 72]. The spectral recovery algorithm works by convolving the

detector signal with a chirp $\Psi(t)$ that takes the form given in equation 1.33.

$$\Psi(t) = 2U(t)\sqrt{\kappa} \cos\left(\pi\kappa t^2 - \frac{\pi}{4}\right) \quad (1.33)$$

where $U(t)$ is the Heaviside step function. The recovered differential power $P_{\Delta,rec}$ is given in terms of the measured differential power by equation 1.34.

$$P_{\Delta,rec}(t) = (\Psi(t) * P_{\Delta}(t))(t) \quad (1.34)$$

This convolution becomes a frequency domain product via the convolution theorem. In the frequency domain, this chirp takes the form given in equation 1.35 .

$$\tilde{\Psi}(f) = e^{i \operatorname{sgn}(f) \frac{\pi f^2}{\kappa}} \quad (1.35)$$

where $\operatorname{sgn}(f)$ is the signum function, which returns the sign of the frequency. The frequency domain recovered spectrum can be written as equation 1.36.

$$\begin{aligned} \mathcal{F}\{P_{\Delta,rec}\} = & \\ \frac{1}{2}\alpha_0 L \beta P_{chirp} e^{-\alpha_0 L} & \left[e^{i(1+\operatorname{sgn}(f)) \frac{\pi f^2}{\kappa}} \mathcal{F}\{S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\} \right] \dots \\ \frac{1}{2}\alpha_0 L \beta P_{chirp} e^{-\alpha_0 L} & \left[e^{-i(1-\operatorname{sgn}(f)) \frac{\pi f^2}{\kappa}} \mathcal{F}\{S_{eff}(\kappa t + f_L) - i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\} \right] \end{aligned} \quad (1.36)$$

Because the power spectrum was made causal using the Hilbert transform, the Fourier transform $\mathcal{F}\{S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\}$ will only have negative frequency components and the Fourier transform $\mathcal{F}\{S_{eff}(\kappa t + f_L) - i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\}$ will only have positive frequency components. This fact can be exploited in conjunction with the signum functions

to reduce the frequency domain recovered differential power to equation 1.37.

$$\begin{aligned} \mathcal{F}\{P_{\Delta,rec}\} = & \\ \frac{1}{2}\alpha_0 L \beta P_{chirp} e^{-\alpha_0 L} \mathcal{F}\{S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\} + \dots & \quad (1.37) \\ \frac{1}{2}\alpha_0 L \beta P_{chirp} e^{-\alpha_0 L} \mathcal{F}\{S_{eff}(\kappa t + f_L) - i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}\} & \end{aligned}$$

Inverting this Fourier transform back into time domain gives the time domain recovered power shown in equation 1.38.

$$P_{Rec,\Delta}(t) = P_{chirp} e^{-\alpha_0 L} \alpha_0 L \beta S_{eff}(\kappa t + f_L) \quad (1.38)$$

Gratings and Spatial Spectral Holography

Spectral Gratings Assume the material is programmed by a pair of brief pulses, taking the spatial form of a pencil. The complex time domain electric field at the center of the pencil beam $E_1(t)$ will have a frequency domain representation $\tilde{E}_1(f)$. If that same field is time delayed by τ , the time-delayed version of the field ($E_1(t - \tau)$) will retain the same energy spectral density. However, the frequency domain representation will pick up a rotating phase related to the time delay ($\tilde{E}_1(f)e^{-2\pi i f \tau}$). This method can be exploited to encode time delays into an SSH material. Consider a brief pulse with field $E_1(t)$ and a second delayed brief pulse ($E_2(t - \tau_{21})$). The total time domain electric field used to program can be expressed as a sum of the two pulses according to equation 1.39.

$$E_T(t) = E_1(t) + E_2(t - \tau_{21}) \quad (1.39)$$

The frequency domain electric field of the two pulses is given by 1.40.

$$\tilde{E}_T(f) = \tilde{E}_1(f) + \tilde{E}_2(f)e^{-2\pi i f \tau_{21}} \quad (1.40)$$

The total energy flux spectral density at the center of the beam for the pair of pulses is then given by equation 1.41.

$$H(f) = \frac{1}{2}\epsilon_0 cn \left[|\tilde{E}_1(f)|^2 + |\tilde{E}_2(f)|^2 + 2 \operatorname{Re} \left(\tilde{E}_1^*(x, y, f) \tilde{E}_2(x, y, f) e^{-2\pi i f \tau_{21}} \right) \right] \quad (1.41)$$

Assuming the delay between the pulses is less than the coherence time of the material, this field will burn a sinusoidal grating into the absorption spectrum of the material with a spectral period τ_{21} [46, 50]. Assuming the pulses are weak enough that programming occurs in the linear regime, the causal material transmission coefficient for the programmed grating is given by equation 1.42 after enforcing causality using the Hilbert transform [9].

$$\nu(f) = e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{1}{2}\alpha_0 LC \left(\frac{1}{2}\epsilon_0 cn \left[|\tilde{E}_1(f)|^2 + |\tilde{E}_2(f)|^2 + \tilde{E}_1^*(f) \tilde{E}_2(f) e^{-2\pi i f \tau_{21}} \right] \right) \right] \quad (1.42)$$

This is illustrated pictorially in figure 1.3. Thus the material can record a time delay

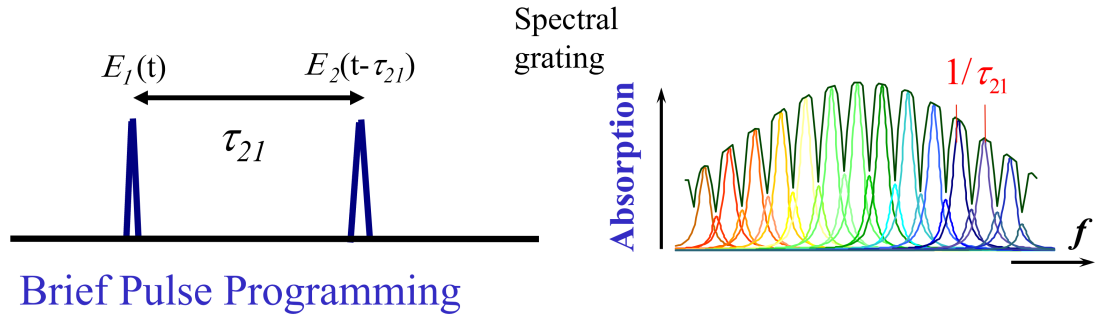


Figure 1.3: Left: A pulse pair used to program a spectral grating in an SSH material. Right: A spectral grating with a period τ_{21} programmed into the material.

in the transmission spectrum of the material. This is called spectral grating. It is a method of encoding time delays in an SSH material. Programming gratings with complex signals can result in modulated gratings known as spectral holograms. These holograms function

similarly to spatial holograms, as they store information through a spectral rather than a spatial interference pattern. [59]. The longest time delay that a material can encode is related to its linewidth. For a narrow linewidth in the kilohertz range, the material can encode time delays on the order of 10s of microseconds.

Stimulated Photon Echoes: Transmission Through a Spectral Grating When the spectral grating from equation 1.42 is interrogated by a third pulse with frequency domain representation $\tilde{E}_3(f)$, the transmitted field is given by equation 1.43.

$$\begin{aligned} \tilde{E}_t(f) = & \tilde{E}_3(f)e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{1}{4}\alpha_0 LC\epsilon_0 cn \left(|\tilde{E}_1(f)|^2 + |\tilde{E}_2(f)|^2 \right) \right] \dots \\ & \dots + \frac{1}{4}\alpha_0 LC\epsilon_0 cn \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \end{aligned} \quad (1.43)$$

The first term describes the transmission of the third pulse, while the 2nd term retains the time delay between the programming pulses encoded into the material. The result is that at a time delay of τ_{21} after the third pulse, there will be a new pulse with a frequency domain electric field given by equation 1.44 [11, 52] This new field is referred to as a stimulated photon echo [50].

$$\tilde{E}_{echo}(f) = \frac{1}{4}\alpha_0 LC\epsilon_0 cn \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \quad (1.44)$$

The resulting pulse can be thought of as the diffraction of the pulse off of the spectral grating. This third pulse is a product of the three constituent fields and thus can be thought of as a non-linear process. While most non-linear processes are due to non-linearities in time domain, SSH processes are non-linear in frequency domain. As such, virtually all SSH processes can be thought of as 3rd-order non-linear processes since the generated fields have a field dependence of order 3. Stimulated photon echoes are a means of recording and then recalling a time delay. While this derivation assumed a scheme of broadband brief pulses

for programming and readout(as depicted in figure 1.4), in principle, the programming fields need only to be broadband optical fields resonant with the material over a large portion of the inhomogeneous linewidth of the material, other spread-spectrum signals such as chirps or even noise could be used to program gratings [25, 41, 57].

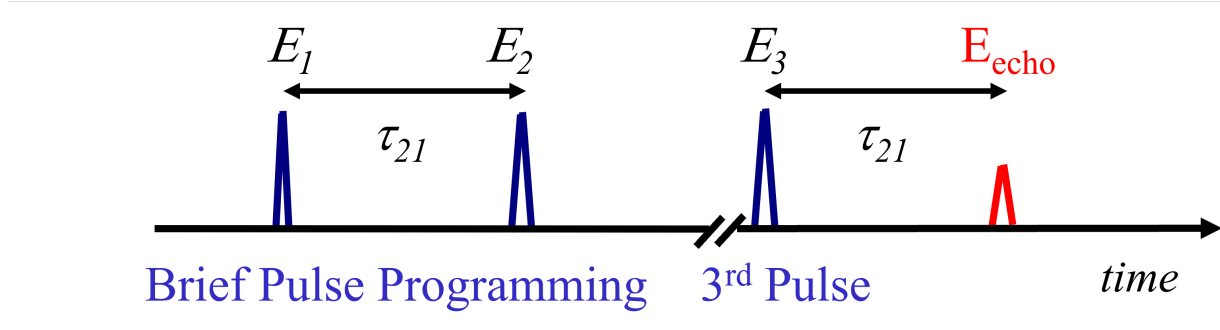


Figure 1.4: Stimulated photon echoes occur when a third pulse is used to interrogate a spectral grating. The result is that a portion of the third pulse is diffracted into the photon echo.

Spatial-Spectral Gratings in SSH Materials The derivation of stimulated photon echoes, as described thus far, assumed co-linear programming and readout of spectral gratings. This restriction is not a requirement for generating stimulated photon echoes. Instead, consider 3 electric fields (assume plane waves for simplicity), each with its own wave vector $\vec{k}$. Assuming that compared to the optical frequency, the pulse bandwidth is narrow, the wave vector of each field can be thought of as a single vector. This new spatial dependence can be described as using a spatially dependent phase $e^{i\vec{k}\cdot\vec{x}}$. Therefore, the grating term of the transmission spectrum for two fields with in-dependent wave vectors is given by equation 1.45.

$$\nu_{grating}(\vec{x}, f) = \frac{1}{4}\alpha_0 L C \epsilon_0 c n \tilde{E}_1^*(f) \tilde{E}_2(f) e^{i(\vec{k}_2 - \vec{k}_1) \cdot \vec{x}} e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \quad (1.45)$$

The spectral grating term now also has a spatial dependence of $e^{i(\vec{k}_2 - \vec{k}_1) \cdot \vec{x}}$. If the fields have the same wave vector magnitude and both have a component propagating in the $\hat{z}$

direction with different $\hat{x}$ components, with an angle θ in between them, then the phase factor that arises from spatial interference will be given by equation 1.46

$$e^{i(\vec{k}_2 - \vec{k}_1) \cdot \vec{x}} = e^{i|\vec{k}|x \cos \theta} \quad (1.46)$$

The total grating is, therefore, both a spatial and a spectral grating. This is why SSH materials are referred to as spatial-spectral holographic materials, the material can encode time delays and angles between pulses, and it can encode them simultaneously. The stimulated echo is then diffracted not only by the spectral grating but also by the spatial grating. The final diffracted echo field is given by equation 1.47.

$$\tilde{E}_{echo}(\vec{x}, f) = \frac{1}{4} \alpha_0 L C \epsilon_0 c n \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{i(\vec{k}_3 + \vec{k}_2 - \vec{k}_1) \cdot \vec{x}} e^{-\frac{1}{2} \alpha_0 L} e^{-2\pi i f \tau_{21}} \quad (1.47)$$

The diffracted photon echo will occur at a time τ_{21} after the third pulse, and the final pulse will have a wave vector $\vec{k}_{echo} = \vec{k}_3 + \vec{k}_2 - \vec{k}_1$. This constraint on the propagation direction of the stimulated echo can be used to isolate the echo field from the other fields spatially. One such scheme for doing this involves picking the three unit vector directions for wave vectors $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$ given in equations 1.48 through 1.50 to obtain the wave vector for the echo field in equation 1.51. This configuration is called the "Box Geometry" depicted in figure 1.5 [11, 71].

$$\hat{k}_1 = A\hat{z} + \frac{B}{\sqrt{2}}(\hat{x} + \hat{y}) \quad (1.48)$$

$$\hat{k}_2 = A\hat{z} + \frac{B}{\sqrt{2}}(\hat{x} - \hat{y}) \quad (1.49)$$

$$\hat{k}_3 = A\hat{z} + \frac{B}{\sqrt{2}}(-\hat{x} + \hat{y}) \quad (1.50)$$

$$\hat{k}_{echo} = \hat{k}_3 + \hat{k}_2 - \hat{k}_1 = A\hat{z} - \frac{B}{\sqrt{2}}(\hat{x} + \hat{y}) \quad (1.51)$$

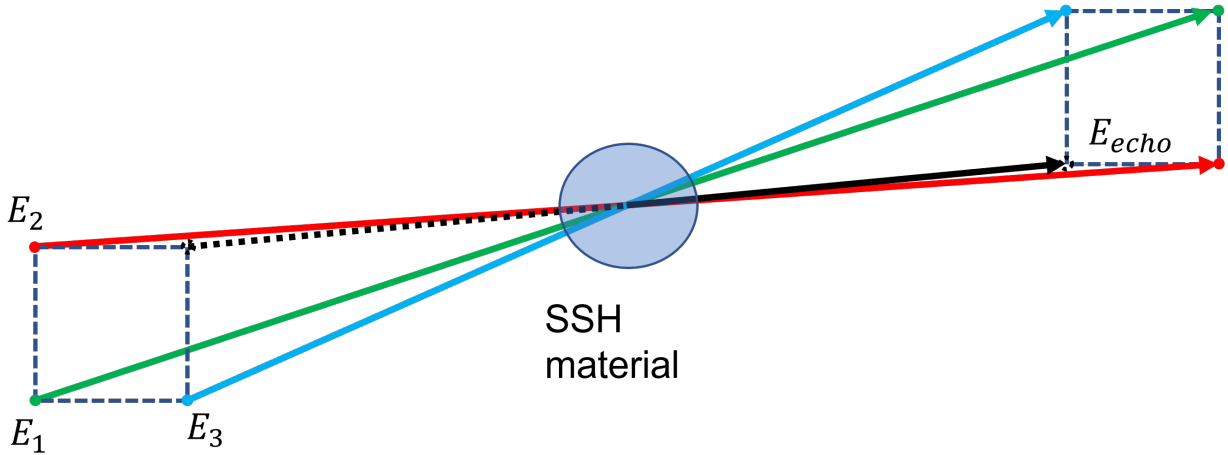


Figure 1.5: In the box geometry, the stimulated echo can be spatially isolated from the other fields by carefully choosing the wave vectors for injection into the material.

It is called the “Box Geometry” because it can be achieved by aligning the three fields parallel to each other onto three of the four corners of a square on a converging lens with its focal length at the center of an SSH material. Putting another lens one focal length after the center of the crystal will result in four parallel fields (the three generating fields and the photon echo) now forming the full square with the echo in what would have been the empty corner [11]. This is a good practice, especially when pulses are not the only fields of interest, as there may be no way to time gate echoes away from each other because more complicated spread spectrum signals are used to burn the gratings [25, 57].

Signal Processing Applications of SSH Materials

SSH Spectrum analysis (S2SA) Since SSH materials can be used to record power spectra, they can be used for spectrum analysis by using chirped readout as seen in equation 1.38 and working back to measure the programmed spectrum SSH materials can be used to measure the programmed power spectrum with a resolution bandwidth governed by the homogeneous linewidth of the material [38, 43].

SSH Materials have several properties that make them attractive for use in spectrum analysis. Because they record power spectra for up to the storage lifetime of the material (typically on the order of milliseconds), they will capture all signals present in that lifetime. Therefore, an SSH system has a 100% probability of intercepting a signal so long as the signal has enough SNR to be observed, while a traditional electronic spectrum analyzer requires that the local oscillator must scan over the signal's frequency while that signal is present in order for it to observe that signal. Therefore, a tone that turns on and off intermittently may not be observed by a traditional analyzer, while the SSH spectrum analyzer would observe that signal. Because SSH materials can be thought of as devices with only a frequency domain non-linearity, they do not contribute spurs into the analyzed spectrum.

SSH materials function as optical spectrum analyzers; however, by pairing the SSH material with an electro-optic modulator, RF signals can be modulated onto a laser field, and then the generated sidebands can be analyzed, allowing the device to function as an RF spectrum analyzer[38, 40].

The wide bandwidths of some SSH materials allowed for ultra-wideband spectrum analysis with bandwidths of up to 200GHz [63]. The resolution bandwidths of the material are governed by the homogeneous linewidth of the material, which can be as narrow as 10s of kHz [66].

SSH Grating Processing SSH materials can be used to store and process gratings that encode time delays due to time of flight in ranging applications. These could range from RF signals from different antennas modulated onto different optical fields [23, 47], to a pair of optical lidar signals [41, 60]. The delay encoded in the material can then be read out with an optical chirp. Based on the frequency of the signal that has been read out, the range of the return can be estimated.

Take, for example, a lidar scheme where the programming field consists of a broadband signal that is split in two; one part of that signal is sent to the target, and a portion is scattered back to the lidar system, where it is collected and then mixed with the other portion of the broadband signal that remains at a known power spectral density S_{ref} . The return has a power spectral density of S_{return} , which is unknown. The grating programmed into the material will take the form expressed in equation 1.52.

$$\nu_{grating}(f) = \frac{1}{2}\alpha_0 L \beta \sqrt{S_{ref} S_{return}} e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \quad (1.52)$$

As in the spectrum analysis case, this grating can be read out using a chirp. The differential power measured by the detector will be proportional to the geometric mean of the return and the reference power spectra and will have the form of the periodic grating. The voltage measured at the detector is then proportional to the same geometric mean and the grating (equation 1.53).

$$V_{det}(t) \propto \sqrt{S_{ref} S_{return}} \cos(2\pi \tau_{21} (\kappa t + f_0)) \quad (1.53)$$

Now looking at the detected voltage, the electrical power spectral density of the detector signal at the frequency $f = \kappa \tau_{21}$ will be proportional to the power spectral density of the reference field (which is known) and the power spectral density in the return (which is unknown). The electrical power spectral density of the detector signal is proportional to the

power spectrum of the return signal. Because the beat frequency of the detector signal is proportional to the programmed delay, the PSD of the detector signal gives a delay spectrum for the return signal.

As ranging processing systems, SSH materials can use a variety of broadband signals to the same effect, from single pulses to chirps [25, 57] to noise [41, 60]. Noise, in particular, has several advantages as a ranging signal; for example, since noise only auto correlates at zero delay, it removes range ambiguity.

Optical Correlation The fields of stimulated photon echoes depend on the product of three electrical fields in frequency domain (equation 1.54).

$$\tilde{E}_{echo}(f) \propto \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) \quad (1.54)$$

Using The convolution theorem, the time domain echo field will is a convolution of three fields (see equation 1.55).

$$E_{Echo}(t) \propto ((E_1^* * E_2) * E_3)(t - \tau_{21}) = ((E_1 \otimes E_2) * E_3)(t - \tau_{21}) \quad (1.55)$$

where $\otimes$ denotes cross-correlation and $*$ denotes the convolution between the fields. If one of the programming fields was a single broadband signal acting as a time reference and the other consisted of time-delayed versions of that reference. If the third field is a sequence of time delays, then the echo signal will indicate the correlation between the programmed delay sequence and the sequence used as the third field. The SSH material can therefore be used as a high bandwidth correlator [7, 11]. This can serve in applications ranging from header recognition [61] to image processing [8] to pattern recognition [24].

A Note on Signals of Interest In Spectrum Analysis and Grating Processing In both spectrum analysis and grating processing applications, signals of interest are extracted from

voltage signals that result from the chirped readout of programmed SSH spectra, but the definitions of the signals of interest are slightly different.

In spectrum analysis applications, the RF power (P_{RF}) put into materials is the signal of interest and the change to the absorption spectrum is directly proportional to detector voltage ($\alpha(\omega) \propto P_{RF}$). The detected voltage (V_{det}) using either traditional readout techniques or the interferometric readout technique that will be described later on are both proportional to the change to absorption spectrum, so the detector voltage will scale at 1dB per dB of RF programming power. Thus in spectrum analysis applications, the detector voltage is the signal of interest. In traditional readout detector voltage (V_{det}) is proportional to the read power so it will scale at 1 dB per dB of read power. In interferometric readout the detector voltage is proportional to the geometric mean of read power and LO power ($\sqrt{P_r P_{LO}}$), so interferometric spectrum analysis detector voltages will scale at $\frac{1}{2}dB$ per dB of either read or LO power (and at 1dB per dB under the condition that $P_{LO} = P_r$).

In grating processing applications, it is assumed that the grating was created by interference of a local copy of the broadband signal that has a fixed power, with a delayed copy of that signal the power of which is the signal of interest. The burned grating is proportional to the geometric mean of the powers of these two signals ($\alpha(\omega) \propto \sqrt{P_{local} P_{delayed}}$). In this case, the signal of interest is proportional to the voltage squared. In traditional readout of gratings, detector voltage is proportional to read power so the measured signal is proportional to the read power squared, and so signals scale at 2 dB per dB of read power. In interferometric readout, the detector voltage is proportional to the geometric mean of read and LO power, and therefore measured signals scale at 1dB per dB of either read or LO power. In the special case where the read and LO power are kept equal interferometric readout grating signals will scale at 2dB per dB of read/LO power.

The final case is time difference arrival applications where gratings are programmed by RF signals from two different antennas[10, 49], the power of these two return signals P_{return} are

expected to be the same so gratings are proportional to the return power ($\alpha(\omega) \propto P_{\text{return}}$). In this case, the detector voltage is proportional to the return signal. Thus in these applications detector voltage is the signal of interest.

The Maxwell Bloch Equations

The theoretical framework used to describe the propagation of electromagnetic fields was first developed over the 18th and 19th centuries culminating in James Clerk Maxwell's publication of *A Dynamical Theory of the Electromagnetic Field* in 1865 [45]. Maxwell's equations provide a theoretical framework for understanding light. However, more work would be needed to describe the interaction of light with inhomogeneously broadened systems. The next breakthrough would come from Felix Bloch in 1946, studying the dynamics of the magnetization of atomic nuclei [14]. The Bloch equations describe these dynamics and are widely used in the field of nuclear magnetic resonance. While developed for discussing nuclear magnetic phenomena, the Bloch equations turned out to be adaptable to other systems like optical interaction with absorbers. In their 1965 paper *Theory of Optical Maser Amplifiers* F. Arecchi and R. Bonifacio combined the material treatment of the Bloch equations, which govern the polarizations of the material, which can be used as source terms in Maxwell's equations to describe the propagation of the field through an inhomogeneously broadened medium [5]. The equations derived through this treatment are referred to as the Maxwell-Bloch or optical-Bloch equations. Now, the Maxwell-Bloch equations are widely used to describe what happens to optical fields as they interact with an inhomogeneously broadened media. They are a valuable tool for investigating the dynamics of SSH materials under an optical excitation. The following section lays out a derivation of the Maxwell-Bloch equations. In SSH materials, the Maxwell-Bloch equations provide useful insights into the mechanisms underlying spatial-spectral holography and describe the other coherent transient phenomena that occur in these systems.

The Bloch Equations

Density Matrices Wave functions provide an effective means of predicting the dynamics of a single quantum object. Still, they are unsuitable for describing quantum system ensembles in an incoherent statistical mixture. In an incoherent statistical mixture, each constituent of the ensemble may be in a different state. To describe these incoherent statistical mixtures, the preferred formalism is the density matrix formulation of quantum mechanics. The density matrix is an operator constructed from a weighted sum of outer products of the wave functions of the system's constituent elements [12] (note that these wave functions do not need to be orthogonal). The definition of a density matrix is given in equation 1.56

$$\hat{\rho} = \sum_n a_n |\Psi_n\rangle \langle \Psi_n| \quad (1.56)$$

where a_n is the probability that a constituent element of the system will be found in state $|\Psi_n\rangle$. In the density matrix formalism, expectation values are calculated according to equation 1.57.

$$\langle Q \rangle = tr \left(\hat{\rho} \hat{Q} \right) \quad (1.57)$$

If the system only has two accessible energy levels, the density matrix is a 2x2 matrix given in equation 1.58

$$\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \quad (1.58)$$

The diagonal elements ρ_{00} and ρ_{11} represent the probability of measuring the state in either the ground state $|0\rangle$ or the excited state $|1\rangle$. The sum of these two elements must always be one. Therefore, $Tr(\hat{\rho}) = 1$. The off-diagonal elements ρ_{01} and ρ_{10} are typically referred to as the coherences. The coherences describe the degree of superposition between the two states. This density matrix is capable of handling ensembles of quantum systems, which can be in either quantum state or some superposition of the two ($|\Psi\rangle = A|0\rangle + B|1\rangle$).

Absorbers in SSH materials can be treated as independent ensembles of two-level atoms at different resonant frequencies.

Bloch Vectors and The Pauli Spin Matrices To simplify the representation of many two-level systems, 2x2 hermitian matrices can be constructed from the Pauli spin matrices given by equation 1.59.

$$\begin{aligned}\hat{\sigma}_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \hat{\sigma}_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \hat{\sigma}_2 &= \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} & \hat{\sigma}_3 &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\end{aligned}\tag{1.59}$$

The Pauli spin matrices (σ_1, σ_2 and σ_3) have a well-known product relation given by equation 1.60

$$\hat{\sigma}_i \hat{\sigma}_j = \hat{\sigma}_0 \delta_{i,j} + i \epsilon_{i,j,k} \hat{\sigma}_k \tag{1.60}$$

where $\delta_{i,j}$ is the Kronecker delta, and $\epsilon_{i,j,k}$ is the Levi-Civita symbol. Since the Pauli spin matrices are traceless, a series of coefficients can be calculated for a 2x2 matrix operator $\hat{Q}$ can be calculated by calculating the trace of the product of the operator with the nth Pauli matrix as shown in equation 1.61.

$$q_n = \frac{1}{2} \text{Tr} \left(\hat{Q} \hat{\sigma}_n \right) \tag{1.61}$$

These real-valued coefficients are the coefficients needed to describe the operator as a sum of the Pauli matrices and the 2x2 identity matrix weighted by the coefficients calculated using 1.61. The original operator $\hat{Q}$ in terms of these coefficients is given by 1.62.

$$\hat{Q} = \sum_{n=0}^3 q_n \hat{\sigma}_n \tag{1.62}$$

The density matrix can also be described using this formalism by substituting the density matrix into equation 1.61 the decomposition coefficients for the density matrix can be calculated according to equation 1.63.

$$r_n = \frac{1}{2} \text{Tr} (\hat{\rho} \hat{\sigma}_n) \quad (1.63)$$

The constraint that the density matrix has a unit trace requires that $r_0 = \frac{1}{2}$; therefore, the density operator has only 3 unique degrees of freedom, which correspond to the 3 Pauli matrices. These three components can be described in a vector $\vec{r}$. This vector is known as the Bloch vector, which is a vector representation of the density matrix of the atomic ensemble. The components r_1 and r_2 correspond to the real and imaginary parts of the coherence, while the element r_3 , often called the inversion, describes the distribution of populations. All three values can range from -1 to +1, and the vector will have a maximum magnitude of 1. When the Bloch vector has a magnitude of 1, this corresponds to every constituent in the ensemble being in the same quantum state. If the coherences are zero (i.e., $r_1 = r_2 = 0$), the system lies along the r_3 axis, corresponding to a pure statistical mixture of the two accessible states. The density matrix can be described in terms of this vector. The density matrix can be described in terms of the inner product of the Bloch-vector with the pseudo-vector $\vec{\sigma}$, which is composed of the three Pauli matrices, according to equation 1.64.

$$\hat{\rho} = \frac{1}{2} (\hat{\sigma}_0 + \vec{r} \cdot \vec{\sigma}) \quad (1.64)$$

To calculate the commutator of an operator with the density matrix, the commutation relation of the Pauli spin matrices (equation 1.65) can be exploited.

$$[\hat{\sigma}_i, \hat{\sigma}_j] = 2i\epsilon_{i,j,k}\hat{\sigma}_k \quad (1.65)$$

The commutator of the density matrix with an operator $\hat{Q}$ can be expressed according to equation 1.66

$$[\hat{Q}, \hat{\rho}] = \sum_{i=1}^3 \sum_{j=1}^3 i q_i r_j \epsilon_{i,j,k} \sigma_k \quad (1.66)$$

Liouville's Theorem: Equations Of Motion for the Bloch Vector Assume that a plane wave electric field with a slowly varying amplitude envelope propagates through an SSH material. The field (at a single z-plane in the material) can be described in terms of a complex exponential at a frequency ω_L , and a slowly varying complex envelope $E(t)$, and a polarization unit vector $\hat{e}$. The total (vector) electric field is given by equation 1.67

$$\vec{E}(t) = E(t) \hat{e} e^{i(\omega_L t)} \quad (1.67)$$

The SSH material contains inhomogeneously broadened absorbers distributed according to the inhomogeneous lineshape around a nominal frequency ω_a (peak absorption frequency). At each detuning Δ away from the nominal transition frequency, there is an independent ensemble of absorbers. These ensembles can be described by density matrix $\hat{\rho}(\Delta)$. The base Hamiltonian operator for the ensemble at that frequency in a given z plane in the material is given by equation 1.68

$$\hat{H}_0(\Delta, z) = \begin{pmatrix} U_0 & 0 \\ 0 & U_0 + \hbar(\omega_a + \Delta) \end{pmatrix} \quad (1.68)$$

Where U_0 is the energy of the ground state. The interaction Hamiltonian of the system with the electric field is the scalar product of the electric field with the transition dipole between the ground state and excited state $\vec{\mu}$; the interaction Hamiltonian will be entirely off-diagonal because only superposition states of the ions in the SSH material have dipole

moments. The interaction Hamiltonian is given by equation 1.69.

$$\hat{H}_{int}(\Delta, t) = \begin{pmatrix} 0 & -(\vec{\mu} \cdot \hat{\epsilon}) E(t) e^{i(\omega_L t)} \\ -(\vec{\mu}^* \cdot \hat{\epsilon}^*) E^*(t) e^{-i(\omega_L t)} & 0 \end{pmatrix} \quad (1.69)$$

The total Hamiltonian is the sum of the base and interaction Hamiltonians given by equation 1.70.

$$\hat{H}_T(\Delta, t) = \begin{pmatrix} U_0 & -(\vec{\mu} \cdot \hat{\epsilon}) E(t) e^{i(\omega_L t)} \\ -(\vec{\mu}^* \cdot \hat{\epsilon}^*) E^*(t) e^{-i(\omega_L t)} & U_0 + \hbar(\omega_a + \Delta) \end{pmatrix} \quad (1.70)$$

The equations of motion for the density operator can be calculated as a commutator with the total Hamiltonian using Liouville's theorem[56] given in equation 1.71.

$$\dot{\hat{\rho}}(\Delta, t) = \frac{i}{\hbar} [\hat{\rho}(\Delta, t), \hat{H}_T(\Delta, t)] \quad (1.71)$$

The Hamiltonian can be decomposed into the Pauli spin matrix basis according to equation 1.72.

$$\begin{aligned} \hat{H}_T(\Delta, t) = & (U_0 + \frac{\hbar}{2}(\omega_a + \Delta)) \hat{\sigma}_0 + \frac{1}{2} \text{Re}((- \vec{\mu} \cdot \hat{\epsilon}) E(t) e^{i(\omega_L t)}) \hat{\sigma}_1 \dots \\ & \dots + \frac{1}{2} \text{Im}((- \vec{\mu} \cdot \hat{\epsilon}) E(t) e^{i(\omega_L t)}) \hat{\sigma}_2 + \frac{\hbar}{2}(\omega_a + \Delta) \hat{\sigma}_3 \end{aligned} \quad (1.72)$$

Using the commutation relations of the Bloch vector coordinates given by equation 1.66, the equations of motion for each Bloch vector component are given by equations 1.73, 1.74 and 1.75

$$\dot{r}_1 = -(\omega_a + \Delta) r_2 + \text{Im} \left(-\frac{(\vec{\mu} \cdot \hat{\epsilon}) E(t)}{2\hbar} e^{i(\omega_L t)} \right) r_3 \quad (1.73)$$

$$\dot{r}_2 = (\omega_a + \Delta) r_1 + \text{Re} \left(-\frac{(\vec{\mu} \cdot \hat{\epsilon}) E(t)}{2\hbar} e^{i(\omega_L t)} \right) r_3 \quad (1.74)$$

$$\dot{r}_3 = -\text{Re} \left(\frac{(-\vec{\mu} \cdot \hat{\epsilon}) E(t)}{2\hbar} e^{i(\omega_L t)} \right) r_2 + \text{Im} \left(\frac{(-\vec{\mu} \cdot \hat{\epsilon}) E(t)}{2\hbar} e^{i(\omega_L t)} \right) r_1 \quad (1.75)$$

The quantity $\frac{(\vec{\mu} \cdot \hat{\epsilon})E(t)}{2\hbar}$ has dimensions of frequency and can be described with a single quantity called the Rabi-frequency $\Omega(t) \equiv \frac{(\vec{\mu} \cdot \hat{\epsilon})E(t)}{2\hbar}$. The equations can then be simplified to equations 1.76, 1.77, and 1.78

$$\dot{r}_1 = -(\omega_a + \Delta) r_2 - \text{Im}(\Omega(t)e^{i(\omega_L t)}) r_3 \quad (1.76)$$

$$\dot{r}_2 = (\omega_a + \Delta) r_1 + \text{Re}(\Omega(t)e^{i(\omega_L t)}) r_3 \quad (1.77)$$

$$\dot{r}_3 = -\text{Re}(\Omega(t)e^{i(\omega_L t)}) r_2 + \text{Im}(\Omega(t)e^{i(\omega_L t)}) r_1 \quad (1.78)$$

The Rotating Wave Approximation The dependences on ω_a in equations 1.76 and 1.77 describe the tendency for the constant rotation of Bloch-Vectors about the population axis (i.e., constant precession of the coherence) at the optical frequency. To simplify these equations of motion, a frame of reference can be chosen such that the Bloch vector is rotating at the frequency of the resonance of the absorbers ω_a . In terms of The Bloch vector in the rotating frame ($\vec{r}'$), the true Bloch vector is given by equation 1.79.

$$\vec{r} = \begin{pmatrix} r'_1 \cos(\omega_a t) - r'_2 \sin(\omega_a t) \\ r'_2 \cos(\omega_a t) + r'_1 \sin(\omega_a t) \\ r'_3 \end{pmatrix} \quad (1.79)$$

The time derivatives of the Bloch vector in terms of the rotating frame Bloch coordinates are given by equations 1.80, 1.81, and 1.82

$$\dot{r}_1 = [\dot{r}'_1 \cos(\omega_a t) - \dot{r}'_2 \sin(\omega_a t)] - \omega_a r_2 \quad (1.80)$$

$$\dot{r}_2 = [\dot{r}'_2 \cos(\omega_a t) + \dot{r}'_1 \sin(\omega_a t)] + \omega_a r_1 \quad (1.81)$$

$$\dot{r}_3 = \dot{r}'_3 \quad (1.82)$$

In the rotating frame, the equations of motion for the Bloch vector become equations 1.83, 1.84, and 1.85

$$[\dot{r}'_1 \cos(\omega_a t) - \dot{r}'_2 \sin(\omega_a t)] = -r'_3 \operatorname{Im}(\Omega e^{i(\omega_L t)}) - [r'_2 \cos(\omega_a t) + r'_1 \sin(\omega_a t)] \Delta \quad (1.83)$$

$$[\dot{r}'_2 \cos(\omega_a t) + \dot{r}'_1 \sin(\omega_a t)] = r'_3 \operatorname{Re}(\Omega e^{i(\omega_L t)}) + [r'_1 \cos(\omega_a t) - r'_2 \sin(\omega_a t)] \Delta \quad (1.84)$$

$$\dot{r}'_3 = -[r'_2 \cos(\omega_a t) + r'_1 \sin(\omega_a t)] \operatorname{Re}(\Omega e^{i(\omega_L t)}) + [r'_1 \cos(\omega_a t) - r'_2 \sin(\omega_a t)] \operatorname{Im}(\Omega e^{i(\omega_L t)}) \quad (1.85)$$

Explicitly the real and imaginary parts of the fields are given by equations 1.86 and 1.87.

$$\operatorname{Re}(\Omega e^{i(\omega_L t)}) = \operatorname{Re}(\Omega(t)) \cos(\omega_L t) - \operatorname{Im}(\Omega(t)) \sin(\omega_L t) \quad (1.86)$$

$$\operatorname{Im}(\Omega e^{i(\omega_L t)}) = \operatorname{Im}(\Omega(t)) \cos(\omega_L t) + (\Omega(t)) \sin(\omega_L t) \quad (1.87)$$

If the envelope is varying slowly compared to the laser frequency (i.e., $\frac{1}{\Omega} \frac{\partial \Omega}{\partial t} \ll \omega_l$), then the sum frequency term at frequency $\omega_a + \omega_L$ will be at a very high frequency since these effects will be on the order of twice the optical frequency, which is exceptionally high compared to the bandwidth of the low bandwidth of the envelope. For simplicity, it can be assumed that the laser frequency and the nominal frequency of the absorbers are equal $\omega_L = \omega_a$ (using this formalism, lasers that are detuned from the nominal frequency can still be used by applying a rotating phase to the envelope) under which condition the equations of motion for the,

now rotating, Bloch vector coordinates are given by equations 1.88, 1.89 and 1.90.

$$\dot{r}'_1 = \Delta r'_2 - r'_3 \operatorname{Im}(\Omega(t)) \quad (1.88)$$

$$\dot{r}'_2 = -\Delta r'_1 + r'_3 \operatorname{Re}(\Omega(t)) \quad (1.89)$$

$$\dot{r}'_3 = -r'_2 \operatorname{Re}(\Omega(t)) + r_1 \operatorname{Im}(\Omega(t)) \quad (1.90)$$

These equations are referred to as the Bloch equations [14]; they are used to model the behavior of an ensemble of two-level systems to model the interaction of an ensemble of absorbers at detuning Δ will respond to a slowly varying field near the resonant frequency ω_a . Originally these equations were used to describe the dynamics of spin $\frac{1}{2}$ systems in a magnetic field, but these dynamics are very similar to those of optically resonant two-level systems under a coherent optical excitation.

Decay and Explicit Time Dependence The previous derivation assumed that the coherence and the excited state have infinite lifetimes. However, realistically this is not the case. Assume that the excited state has a lifetime T_1 and the coherences have a lifetime T_2 . The excited state and coherence lifetimes can be explicitly accounted for in the equations of motion for the Bloch vector in the rotating coordinates using equations 1.91, 1.92, and 1.93.

$$\dot{r}'_1 = \Delta r'_2 - r'_3 \operatorname{Im}(\Omega(t)) - \frac{r'_1}{T_2} \quad (1.91)$$

$$\dot{r}'_2 = -\Delta r'_1 + r'_3 \operatorname{Re}(\Omega(t)) - \frac{r'_2}{T_2} \quad (1.92)$$

$$\dot{r}'_3 = -r'_2 \operatorname{Re}(\Omega(t)) + r_1 \operatorname{Im}(\Omega(t)) - \frac{1 + r_3}{2T_1} \quad (1.93)$$

The time T_1 determines the duration over which spectra can be stored in an SSH material, while the coherence time T_2 governs the duration over which fields appear to be coherent to the material, it will also govern the homogeneous linewidth of the lorentzian lineshape according to 1.94.

$$\Gamma_H = \frac{1}{\pi T_2} \quad (1.94)$$

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The Bloch Equations The equations of motion of the rotating Bloch vectors are called the Bloch equations and can be represented using the vector product in equation 1.95

$$\dot{\vec{r}} = -\vec{r} \times \vec{\Omega} \quad (1.95)$$

The vector $\vec{\Omega}$ is called the Rabi-vector, which is given by equation 1.96.

$$\vec{\Omega} = \begin{pmatrix} \text{Re}(\Omega(t)) \\ \text{Im}(\Omega(t)) \\ -\Delta \end{pmatrix} \quad (1.96)$$

The real and imaginary parts of the Rabi frequency correspond to the in-phase and quadrature components of the envelope respectively. Assuming a field with a constant and entirely real Rabi-frequency (i.e., an in-phase amplitude envelope) Ω_0 interacting with an ensemble of ions at 0 detuning, initially in the ground state (i.e., $r_3 = -1$). The equations of motion for this system are given by equation 1.97

$$\dot{\vec{r}} = -\vec{r} \times \begin{pmatrix} \Omega_0 \\ 0 \\ 0 \end{pmatrix} \quad (1.97)$$

The solution in this situation can be calculated explicitly. The Bloch vector of the solution is given by 3

$$\vec{r}(t) = \begin{pmatrix} 0 \\ \sin(\Omega_0 t) \\ -\cos(\Omega_0 t) \end{pmatrix} \quad (1.98)$$

The result is that the vector will precess at the Rabi frequency; this process is called Rabi flopping and occurs at the Rabi frequency. The atomic populations cycle between the ground and excited states at the Rabi frequency. This effect was first described by I. I. Rabi in 1937 [54] in spin $\frac{1}{2}$ systems in a magnetic field.

A quantity called pulse area is useful to define for the Bloch equations since it will determine how a pulse will excite the material. In general, given a pulse with a time-dependent Rabi frequency $\Omega(t)$, the pulse area (Θ) is given by equation 1.99.

$$\Theta = \int_{-\infty}^{\infty} \Omega(t) dt \quad (1.99)$$

The inversion parameter can be estimated from the pulse area as $r_3 = -\cos(\Theta)$. Maximum inversion is therefore expected for a pulse of area π . In contrast, a $\pi/2$ is expected to put ions into a superposition state with equal contributions from the ground and excited states.

Material Polarizations To account for the time domain effects in an ensemble of inhomogeneously broadened absorbers, the lineshape of the absorption must be considered. The inhomogeneous line can be represented by a statistical distribution $g(\Delta)$ in terms of the detuning of the ions around the nominal transition frequency (ω_a). The width of this transition is governed by the inhomogeneous linewidth (Γ_H), which can be modeled for a Gaussian distribution according to equation 1.100

$$g(\Delta) = g_0 e^{-\frac{\Delta^2}{2\Gamma_I}} \quad (1.100)$$

where the inhomogeneous lineshape is normalized such that $\frac{1}{2\pi} \int_{-\infty}^{\infty} g(\Delta) d\Delta = 1$ (in which case $g_0 = \frac{\sqrt{2\pi}}{\Gamma_I}$). This function $g(\Delta)$ gives the probability that an absorber in the SSH material has a resonant frequency detuned by Δ from the nominal transition frequency ω_a . Electric polarizations in the material due to 2-level dynamics arise when the ensemble is in a superposition state because superposition states have electric dipoles. The overall material polarization is related to the coherence, so the macroscopic polarization of the material is given by equation 1.101.

$$\vec{P} = \frac{N\vec{\mu}}{2\pi} \int_{-\infty}^{\infty} g(\Delta) [(r'_1(\Delta) \cos(\omega_a t) - r'_2(\Delta) \sin(\omega_a t)) + i(r'_2(\Delta) \cos(\omega_a t) + r'_1(\Delta) \sin(\omega_a t))] d\Delta \quad (1.101)$$

Note that for a system in a superposition state, each ion will generate polarizations at its own resonant frequency ($\omega_a + \Delta$). Material polarization will generate new electric fields, which will interfere with the input field leading to changes to the field as it passes through the ensemble.

Spatial Propagation: The Maxwell Bloch Equations Consider an electric field interacting with a material. Until now, we were considering only a single z slice of the material; in order to consider propagation, the z position dependence of the field and therefore, the Rabi frequency must be considered, especially because the material will affect the Rabi frequency as the field passes through the material. The field can be described by a Rabi frequency with spatial and temporal dependence, $\Omega(z, t)$, a frequency $\omega_L = \omega_a$ propagating in the $\hat{z}$ direction with a wave-number $k = \frac{n\omega_a}{c}$, where n is the refractive index. Assuming that the electric field is polarized such that $\vec{\mu} \cdot \hat{\epsilon} = \|\vec{\mu}\|$, i.e., the electric field is polarized in the same direction as the dipole direction $\hat{\mu}$. The E field can be expressed in terms of Rabi frequency

according to equation 1.102.

$$\vec{E}(z, t) = \frac{2\hbar\hat{\mu}}{\|\vec{\mu}\|} \Omega \left(z, t - \frac{n}{c}z \right) e^{i(\omega_a t - kz)} \quad (1.102)$$

The sourced wave equation (equation 1.103) can be used to describe the propagation of fields [15].

$$\nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{P} \quad (1.103)$$

Substituting the E field in equation 1.102 into equation 1.103, a sourced wave equation can be expressed in terms of the Rabi envelope (equation 1.104).

$$\left(\nabla^2 - \frac{n}{c} \frac{\partial^2}{\partial t^2} \right) \hat{\mu} \Omega \left(z, t - \frac{n}{c}z \right) e^{i(\omega_a t - kz)} = \frac{\|\vec{\mu}\|}{2\hbar\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{P} \quad (1.104)$$

Since all of the spatial dependence is in the z direction, the sourced wave equation becomes 1.105.

$$\left(\frac{\partial^2}{\partial z^2} - \frac{n}{c} \frac{\partial^2}{\partial t^2} \right) \hat{\mu} \Omega \left(z, t - \frac{n}{c}z \right) e^{i(\omega_a t - kz)} = \frac{\|\vec{\mu}\|}{2\hbar\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{P} \quad (1.105)$$

To simplify calculations, the propagating wave can be observed in the frame of the propagating wave, i.e., $t' = t - \frac{n}{c}z$. After this coordinate change, the derivatives become $\frac{\partial}{\partial t'} \Omega = \frac{\partial}{\partial t} \Omega$ and $\frac{\partial}{\partial z} \Omega = \left(\frac{\partial}{\partial z} + \frac{n}{c} \frac{\partial}{\partial t'} \right) \Omega$. In this case, the sourced wave equation becomes equation 1.106.

$$\left[\frac{\partial^2}{\partial z^2} - \frac{2n}{c} \frac{\partial}{\partial t'} \frac{\partial}{\partial z} \right] \hat{\mu} \Omega(t') e^{i\omega_a t'} = \frac{\|\vec{\mu}\|}{\hbar\epsilon_0 c^2} \frac{\partial^2}{\partial t'^2} \vec{P} \quad (1.106)$$

The assumption that the envelope is slowly varying with respect to the carrier frequency can be expressed mathematically as requiring that $\frac{d}{dz} E \ll kE$ and $\frac{\partial}{\partial z} E \ll \omega_a E$. Therefore the term with the second derivative in z will be much less than the term containing the partial

time derivative. The equation simplifies to equation 1.107.

$$-\frac{2in\omega_a}{c}\frac{\partial}{\partial z}\hat{\mu}\Omega(z,t')e^{i\omega_at'} = \frac{\|\vec{\mu}\|}{\hbar\epsilon_0c^2}\frac{\partial^2}{\partial t'^2}\vec{P} \quad (1.107)$$

The spatial derivative of the Rabi frequency can be expressed in terms of the polarization.

$$\hat{\mu}\frac{\partial}{\partial z}\Omega(z,t')e^{i\omega_at'} = \frac{i\|\vec{\mu}\|}{2\hbar\epsilon_0nc\omega_a}\frac{\partial^2}{\partial t'^2}\vec{P} \quad (1.108)$$

The electric polarizations in the material due to interaction with the material are given by 1.101. These can be substituted into the wave equation, leaving equation 1.109.

$$\begin{aligned} \hat{\mu}\frac{\partial}{\partial z}\Omega(z,t')e^{i\omega_at'} = & \dots \\ & \dots \frac{-i\hat{\mu}N\|\vec{\mu}\|^2\omega_a}{4\pi\hbar\epsilon_0nc} \int_{-\infty}^{\infty} g(\Delta)[(r_1(\Delta)\cos(\omega_at') - r_2(\Delta)\sin(\omega_at')) \dots \\ & \dots + i(r_2(\Delta)\cos(\omega_at') + r_1(\Delta)\sin(\omega_at'))]d\Delta \end{aligned} \quad (1.109)$$

This can be decomposed into real and imaginary components to get equations 1.110 and 1.111.

$$\begin{aligned} (\cos(\omega_at)\frac{\partial}{\partial z}\text{Re}\{\Omega(z,t')\} - \sin(\omega_at)\frac{\partial}{\partial z}\text{Im}\{\Omega(z,t')\}) = & \dots \\ & \dots \frac{N\|\vec{\mu}\|^2\omega_a}{4\pi\hbar\epsilon_0nc} \int_{-\infty}^{\infty} g(\Delta)(r_2(\Delta)\cos(\omega_at') + r_1(\Delta)\sin(\omega_at'))d\Delta \end{aligned} \quad (1.110)$$

$$\begin{aligned} (\cos(\omega_at)\frac{\partial}{\partial z}\text{Im}\{\Omega(z,t')\} + \sin(\omega_at)\frac{\partial}{\partial z}\text{Re}\{\Omega(z,t')\}) = & \dots \\ & \dots \frac{N\|\vec{\mu}\|^2\omega_a}{4\pi\hbar\epsilon_0nc} \int_{-\infty}^{\infty} g(\Delta)(r_1(\Delta)\cos(\omega_at') - r_2(\Delta)\sin(\omega_at'))d\Delta \end{aligned} \quad (1.111)$$

Isolating the real and imaginary components of Ω gives equations 1.112 and 1.113.

$$\frac{\partial}{\partial z}\text{Re}\{\Omega(z,t')\} = \frac{N\|\vec{\mu}\|^2\omega_ag_0}{4\pi\hbar\epsilon_0nc} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2\Gamma_I}} r'_2(\Delta, z, t) d\Delta \quad (1.112)$$

$$\frac{\partial}{\partial z}\text{Im}\{\Omega(z,t')\} = -\frac{N\|\vec{\mu}\|^2\omega_ag_0}{4\pi\hbar\epsilon_0nc} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2\Gamma_I}} r'_1(\Delta, z, t) d\Delta \quad (1.113)$$

This can be expressed in terms of the absorption coefficient $\alpha_0 \equiv \frac{N\|\vec{\mu}\|^2\omega_a g_0}{2\hbar\epsilon_0 n c}$ using equations 1.114 and 1.115.

$$\frac{\partial}{\partial z} \text{Re}\{\Omega(z, t')\} = \frac{\alpha_0}{2\pi} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2T_1}} r'_2(\Delta, z, t) d\Delta \quad (1.114)$$

$$\frac{\partial}{\partial z} \text{Im}\{\Omega(z, t')\} = -\frac{\alpha_0}{2\pi} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2T_1}} r'_1(\Delta, z, t) d\Delta \quad (1.115)$$

All of the equations required to calculate how a material will propagate through an inhomogeneously broadened medium consisting of two-level absorbers are called the Maxwell-Bloch equations (or sometimes the optical Bloch equations)[15]. They are given in equations 1.116 through 1.120.

$$\dot{r}'_1(\Delta, z, t') = \Delta r'_2(\Delta, z, t') - r'_3(\Delta, z, t') \text{Im}(\Omega(z, t')) - \frac{r'_1(\Delta, z, t')}{T_2} \quad (1.116)$$

$$\dot{r}'_2(\Delta, z, t') = -\Delta r'_1(\Delta, z, t') + r'_3(\Delta, z, t') \text{Re}(\Omega(z, t')) - \frac{r'_2(\Delta, z, t')}{T_2} \quad (1.117)$$

$$\dot{r}'_3(\Delta, z, t') = -r'_2(\Delta, z, t') \text{Re}(\Omega(z, t')) + r_1(\Delta, z, t') \text{Im}(\Omega(z, t')) - \frac{1 + r_3(\Delta, z, t')}{2T_1} \quad (1.118)$$

$$\frac{\partial}{\partial z} \text{Re}\{\Omega(z, t')\} = \frac{\alpha_0}{2\pi} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2T_1}} r'_2(\Delta, z, t') d\Delta \quad (1.119)$$

$$\frac{\partial}{\partial z} \text{Im}\{\Omega(z, t')\} = -\frac{\alpha_0}{2\pi} \int_{-\infty}^{\infty} e^{\frac{\Delta^2}{2T_1}} r'_1(\Delta, z, t') d\Delta \quad (1.120)$$

The first three equations are the Bloch equations; they essentially represent rearrangements of the Bloch sphere due to the driving field, which will result in polarizations, which result in alterations to the field, which are described in the final two equations. These equations can be used to describe the dynamics of a material under illumination by a resonant electric field.

The Three-Level Rate Equations

Derivation Of the Three-Level Rate Equations

In planning and understanding experiments, it was important to understand how the material would react to continuous programming, so a model for the programming of the material, estimating material saturation, and choosing experimental powers is useful to determine how a programming field will rearrange the material populations on time scales exceeding the coherence time. In the past, others have presented models using the pumping and decay rates of the various states in a multi-level system to model the behavior of absorbers in an inhomogeneously broadened system [27, 81]. Here, such a model is presented for a 3-level system. In this system, there are two states, a ground state $|0\rangle$ and an excited state $|1\rangle$, which are coupled together by an optically accessible transition, and a bottle-neck state $|b\rangle$ into which the excited state will quickly decay but does not radiatively transition to the other two states. Populations in three states are represented by individual atomic populations ρ_{00} , ρ_{11} , and ρ_{bb} . Accounting for the absorbed quanta in the electric field due to absorption and then decay between states, the states' populations can be calculated for each state as a function of the number of photons illuminating those absorbers and time. Populations in the ground state are available for excitation, so changes to these populations will affect the material's absorption coefficient. Absorbers in the ground state can absorb a photon and become excited, while absorbers in the excited state can undergo stimulated emission; absorbers in the bottleneck state do not interact with the optical field. The absorption coefficient will increase with the ground state population and decrease with the excited state population. The effective absorption coefficient is described by equation 1.121

$$\alpha = \alpha_0 (\rho_{00} - \rho_{11}) \tag{1.121}$$

Absorption of Photons in an In-homogeneously Broadened Medium If the material is illuminated by a field with a spectral intensity $J_{eff}(f)$, then the field's effective photon flux (photons per unit time per unit area per unit frequency) can be calculated from the effective spectral intensity according to equation 1.122.

$$\Phi_{eff} = \frac{J_{eff}}{h\nu} \quad (1.122)$$

The rate at which absorbers are excited per unit area per unit frequency is therefore given by equation 1.123

$$\frac{dN_{abs}}{dt} = \Phi_{eff}(f) [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] \quad (1.123)$$

where N_{abs} is the number of absorbed photons per unit cross-sectional area per unit frequency ($\frac{\text{photons}}{\text{m}^2 \text{Hz}}$), area per unit frequency. The absorption of photons at a given frequency will excite absorbers into the excited state. The rate of absorption of photons can be translated into the rate of change of the ground and excited state populations given by equations 1.124 and 1.125

$$\dot{\rho}_{00} = -\frac{\Phi_{eff}(f)}{N_V G(f) L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] \quad (1.124)$$

$$\dot{\rho}_{11} = \frac{\Phi_{eff}(f)}{N_V G(f) L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] \quad (1.125)$$

Where N_V is the number density of absorbing ions per unit volume, and $G(f)$ is the inhomogeneous lineshape of the material. These expressions for rates represent the rates of change of populations absent any other effects.

Decay from Excited States In real systems, absorbers in the excited state will decay over time. The excited state population will undergo exponential decay at the excited state

decay rate (R_1) in the absence of any additional illumination according to equation 1.126.

$$\dot{\rho}_{11} = -R_1 \rho_{11} \quad (1.126)$$

R_1 is the total decay rate of the excited state, which is the inverse of the excited state lifetime ($R_1 = 1/T_1$). However, this does not indicate into which state the excited state will decay. In most spatial, spectral hole burning systems, there are two possible paths for the excited state to decay through; this means to calculate the rates of change in those other two levels, the rates of decay of the excited state into those states are needed. Assume that system has three levels: a ground state ($|0\rangle$), an excited state($|1\rangle$), and a bottleneck state($|B\rangle$). Decay rates from state j to state k can be represented by individual partial decay rates R_{jk} . The decay rates from the excited state to the ground state R_{10} and to the bottleneck state R_{1B} must sum together to the excited state decay rate ($R_1 = R_{10} + R_{1B}$). The decay rates to the ground and bottleneck states can be determined using a parameter called the branching ratio [31], which describes the fraction of absorbers that will end up in the bottleneck state as $t \rightarrow \infty$. Using this definition of the branching ratio ($\epsilon_{branching}$), the decay rate from the excited state to the bottleneck state is given by equation 1.127, and the decay rate directly to the ground state is given by equation 1.128

$$R_{1B} = \epsilon_{branching} R_1 \quad (1.127)$$

$$R_{10} = (1 - \epsilon_{branching}) R_1 \quad (1.128)$$

The bottleneck state will decay into the ground state according to a different rate R_{B0} , which is related to the storage lifetime ($R_{B0} = 1/T_{storage}$). Therefore, the exponential decay of each state leads to a system of differential equations that describe the decay of the excited state. This leads to a system of differential equations that describe the populations at each level

as the populations decay according to equations 1.129 to 1.131.

$$\dot{\rho}_{00} = R_{10}\rho_{11} + R_{B0}\rho_{BB} \quad (1.129)$$

$$\dot{\rho}_{11} = -R_{10}\rho_{11} \quad (1.130)$$

$$\dot{\rho}_{BB} = R_{1B}\rho_{11} - R_{B0}\rho_{BB} \quad (1.131)$$

The Three Level Rate Equations By combining the decay rate equations 1.129 to 1.131 and the absorption rate equations given in equations 1.124 and 1.125 give a system of ordinary differential equations that describe how the populations in a 3 level spectral hole burning system will change over time under illumination by an optical field of constant spectral intensity. The three-level rate equations are given by equations 1.132 to 1.134

$$\dot{\rho}_{00} = -\frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] + R_{10}\rho_{11} + R_{B0}\rho_{BB} \quad (1.132)$$

$$\dot{\rho}_{11} = \frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] - R_{10}\rho_{11} \quad (1.133)$$

$$\dot{\rho}_{BB} = R_{1B}\rho_{11} - R_{B0}\rho_{BB} \quad (1.134)$$

The populations, however, are constrained in that the sum of the populations is one, so the system of three equations can be reduced into a system of 2 equations (equations 1.135 and 1.136)

$$\dot{\rho}_{00} = -\frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] + (R_{10} - R_{B0})\rho_{11} - R_{B0}\rho_{00} + R_{B0} \quad (1.135)$$

$$\dot{\rho}_{11} = \frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{\alpha_0 L [\rho_{11}(f) - \rho_{00}(f)]}] - R_{10}\rho_{11} \quad (1.136)$$

The equilibrium solution (i.e., all rates go to zero) of this system of equations indicates what the proportion of absorbers will be in each state when holes are being burnt by a constant power spectrum. This will allow for a determination of the depth of burnt spectral

features given an incident power spectrum without the assumption that hole burning is linear (because the differential equation model allows for the depletion of absorbers). This model can therefore be used to determine the limitations of the linear burning and to estimate at what power spectral density holes will begin to saturate.

Analytical Equilibrium Solutions to the 3-Level Rate Equations When the system reaches equilibrium the populations will remain constant, so the differential equations become algebraic equations. These equations can be solved in the regime of weak hole burning (i.e., under the condition that the vast majority of absorbers are in the ground state or $\rho_{00} \approx 1$). Equations 1 and 1.138 can be solved to calculate the populations of each state at equilibrium.

$$-\frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{-\alpha_0 L}] + (R_{10} - R_{B0})\rho_{11} - R_{B0}\rho_{00} + R_{B0} = 0 \quad (1.137)$$

$$\frac{\Phi_{eff}(f)}{N_V G(f)L} [1 - e^{-\alpha_0 L}] - (R_{10} + R_{1B})\rho_{11} = 0 \quad (1.138)$$

Equation 1.138 can be solved to calculate the population in the excited state, which is given by equation 1.139.

$$\rho_{11} = \frac{\Phi_{eff}(f)}{N_V G(f)L} \frac{[1 - e^{-\alpha_0 L}]}{(R_{10} + R_{1B})} \quad (1.139)$$

Substituting this expression into equation allows us to solve for the ground state population.

The equilibrium population in the ground state is given by equation 1.140

$$\rho_{00} = 1 - \frac{\Phi_{eff}(f)}{N_V G(f)L} \frac{[1 - e^{-\alpha_0 L}]}{R_{B0}} \left(\frac{R_{B0} + R_{1B}}{R_{10} + R_{1B}} \right) \quad (1.140)$$

The expressions for ρ_{00} and ρ_{11} can be used to calculate ρ_{BB} , given in equation 1.141

$$\rho_{BB} = \frac{\Phi_{eff}(f)}{N_V G(f)L} \frac{[1 - e^{-\alpha_0 L}]}{(R_{10} + R_{1B})} \left(\frac{R_{1B}}{R_{B0}} \right) \quad (1.141)$$

These can be used to estimate the relative absorption through the material. The continuous burning parameter χ can be calculated in terms of ρ_{00} and ρ_{11} using equation 1.142

$$\chi = \frac{1 + \rho_{11} - \rho_{00}}{J_{eff}} \quad (1.142)$$

Substituting the expression for ρ_{11} from equation 1.139, the expression for ρ_{00} from equation 1.140 and the relationship between J_{eff} and Φ_{eff} an expression can be derived for χ in terms of material parameters. This expression is given in equation 1.143.

$$\chi = \frac{1 - e^{-\alpha_0 L}}{N_V G(f) L h \nu} \left(\frac{2 + \frac{R_{1B}}{R_{B0}}}{R_{10} + R_{1B}} \right) \quad (1.143)$$

Though at higher powers changes in the absorption factor ($e^{\alpha_0 L(\rho_{11} - \rho_{00})}$) will lead to saturation of the programmed spectra, causing the hole burning to become non-linear.

Numerical Solutions to the Three-Level Rate Equations

Simulation Details To solve for the steady-state distribution of ion populations, a simple $2^{nd}/3^{rd}$ order numerical integration routine in MATLAB was used to solve the 3-level rate equations. The code used to solve these equations numerically is given in appendix B This solution will give values for ρ_{00} and ρ_{11} ; ρ_{BB} can be calculated by enforcing the constraint that $\rho_{00} + \rho_{11} + \rho_{BB} = 1$. The simulations use an absorption length of 1.39, an excited state lifetime of $160\mu S$, a bottleneck lifetime of 4.5ms, and a branching ratio of 0.27. experiments are allowed to run for 100ms to ensure that the system is allowed to reach equilibrium.

Time Domain Results In the slow-burning regime, the photon flux density divided by the number of total absorbers per frequency per area is much less than the decay rate of the excited state lifetime. Specifically, $\frac{\Phi_{eff}}{N_v G(f) L} = 1 S^{-1}$ compared to a decay rate of $6.25 \times 10^3 S^{-1}$ In this case the system will instead slowly approach the steady state value monotonically as

seen in figure 1.6.

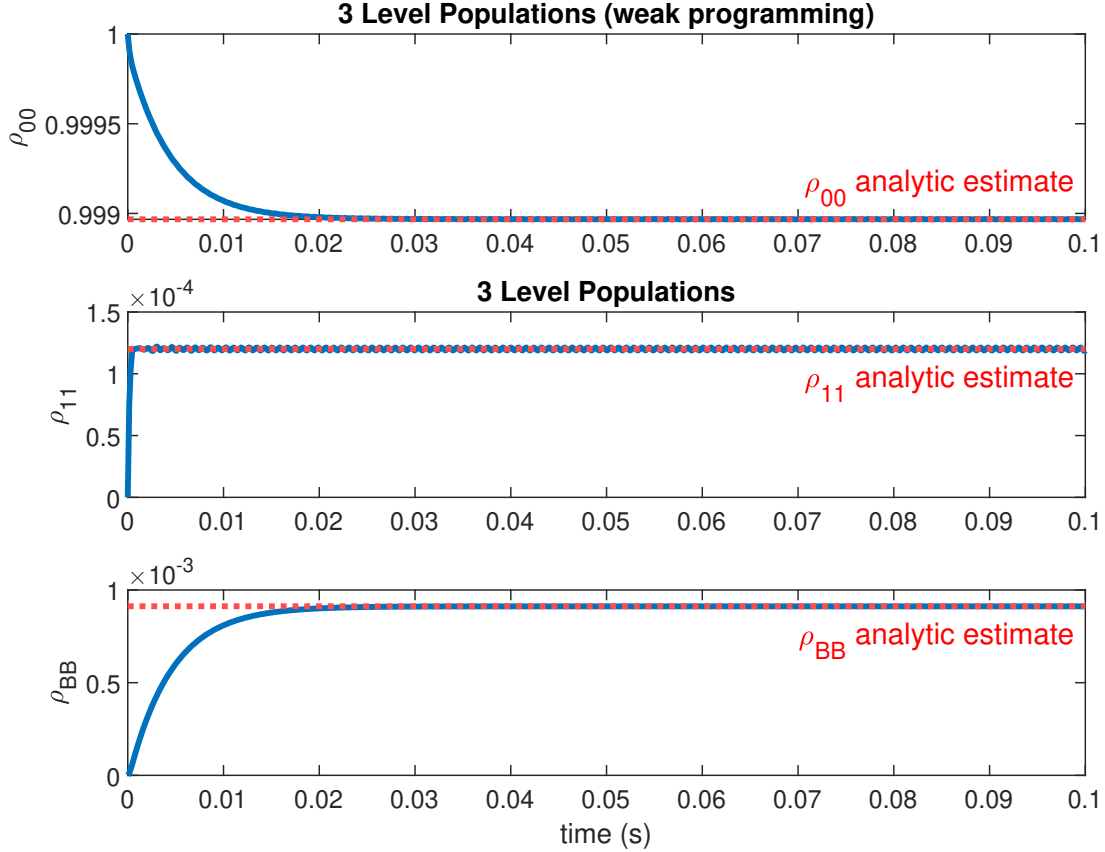


Figure 1.6: A simulation of rate equations for using relatively weak programming. In this case, the populations change monotonically. After around 20ms, the simulated populations approach the analytically calculated estimates of equilibrium populations

As expected, the populations all approach the final values monotonically rather than accumulating in one state after around 20ms. In this case, because the programming is weak, most ions remain in the ground state. Using a faster burn can cause the excited state population to spike before decaying to the bottleneck state before reaching equilibrium populations. as is shown in figure 1.7 In the case of fast burning, the photon flux density divided by the number of total absorbers per frequency per area is on the order of excited state decay rate ($\frac{\Phi_{eff}}{N_v G(f)L} = 10^3 S^{-1}$ compared to a decay rate of $6.25 \times 10^3 S^{-1}$).

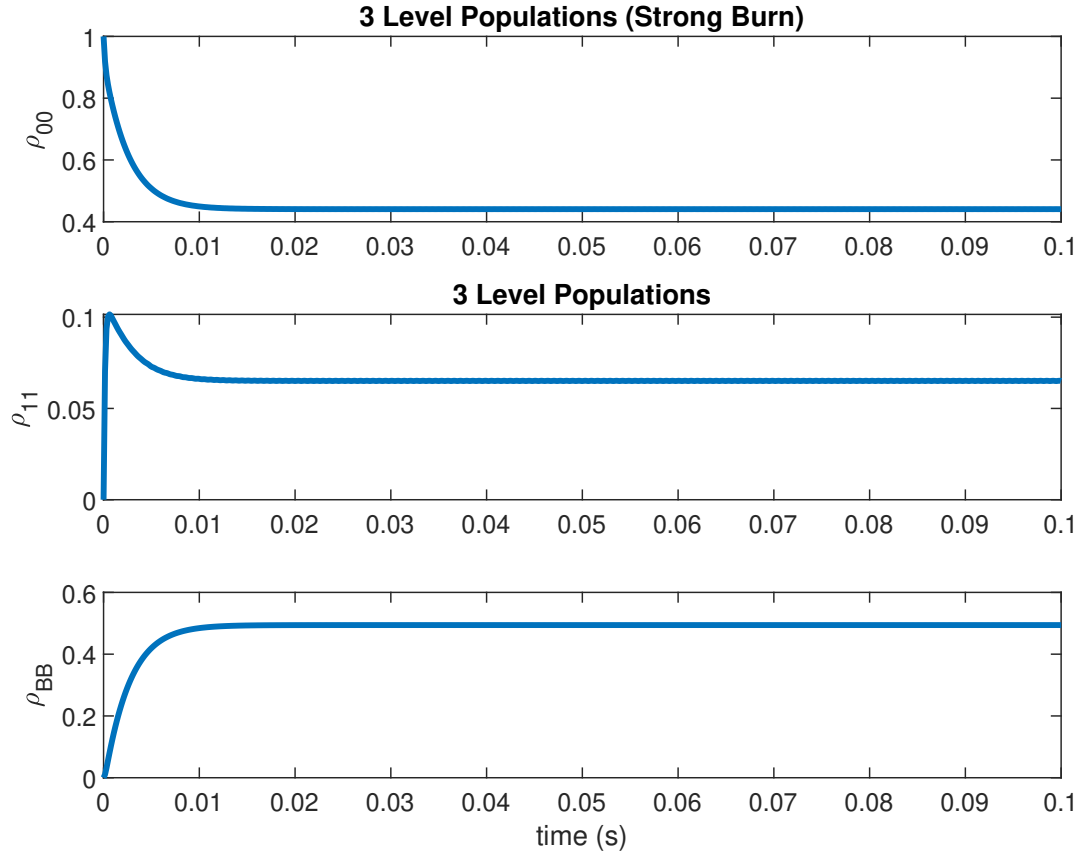


Figure 1.7: A simulation of rate equations using relatively weak programming. In this case, the system reaches equilibrium in well under 10ms; however, the equilibrium that the system reaches differs significantly from the equilibrium estimated for linear programming.

These time domain simulations will be used later in chapter 6 to estimate programming coefficients and compression levels for both continuous programming and programming with a fixed duration.

Interferometric Architectures

Interferometric Jones Vector Formalism

A collection of independent electric fields with the same polarization can be represented as a collection of scalar fields, which can be arranged into a vector with N elements. This vector essentially describes N different fields in N separate spatial channels. This vector can be used in analogy to the Jones vector formalism, which describes fields of different polarizations. In the interferometric formalism, each vector element represents the scalar electric field in each spatial channel. The interferometric vector does not describe the polarization vector of the field but is a description of the amplitudes of several different independent fields. The independent fields in each channel can be mixed using polarization-maintaining beam splitters, and phase shifts can be applied to fields on each channel using modulators or path length differences. With these elements, various manipulations can be applied to fields inside of the interferometric network. In the interferometric Jones-vector formalism, the individual polarization-maintaining elements, such as beam splitters and phase shifters, can be expressed with an operator $\hat{Q}$, which will take the form of an $N \times N$ matrix. The output scalar fields in N output ports can be described as a matrix-vector product of the operator representing the optics and the interferometric Jones vector describing electric fields in the N input ports of the device using equation 1.144.

$$\vec{E}_{out} = \hat{Q}\vec{E}_{in} \quad (1.144)$$

Furthermore, a system of M elements can be expressed as a product of M independent operators according to equation 1.145.

$$\vec{E}_{out} = \hat{Q}_M \dots \hat{Q}_3 \hat{Q}_2 \hat{Q}_1 \vec{E}_{in} \quad (1.145)$$

The number of channels required will govern the available elements for the system; however, typically, the form of these operators is similar regardless of the number of ports required for the device. The simplest nontrivial case is the 2-port case, which is a network consisting of two input ports and two output ports. The first of the operators that form the basis for 2-channel interferometric systems is the beam splitter given by equation 1.146. The real diagonal elements represent direct transmission through the splitter while the off-diagonal elements represent the $\pi/2$ phase shift incurred by reflected fields; this constraint on the phase of reflected fields is assumed in order to preserve time inversion symmetry and was described by G.G Stokes in 1849 [62].

$$\hat{Q}_{splitter} = \begin{pmatrix} \nu & ir \\ ir & \nu \end{pmatrix} \quad (1.146)$$

In equation 1.146, ν and r are the magnitudes of the field transmission and reflection coefficients, respectively. These coefficients are constrained such that $\nu^2 + r^2 = 1$. In the special case where $\nu = r = \frac{1}{\sqrt{2}}$, half of the optical power from an input channel will be diverted to two output channels equally. This case is referred to as a polarization maintaining 50-50 Beam splitter, and its matrix description is given in equation 1.147.

$$\hat{Q}_{splitter} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad (1.147)$$

Phase shifts can be expressed as a diagonal matrix with a complex exponential on each diagonal element. The element Q_{jj} represents the phase shift incurred by the field in channel j . Physically this can be implemented with a phase modulator or a slight length delay induced by a fiber stretcher or piezo-electric mounted mirror. The phase shifter matrix for 2x2 interferometric networks is shown in equation 1.148, where channels 1 and 2 each get

a unique phase shift ϕ_1 and ϕ_2 .

$$\hat{Q}_{Phase} = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix} \quad (1.148)$$

A third type of element can be described that changes where a field falls in the interferometric Jones vector. This can be thought of as swapping locations for fields artificially by physically moving a connection. Strictly speaking, this type of element is not necessary for this formalism; however, it may prove useful. It is given by equation 1.149

$$\hat{Q}_{Swap} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (1.149)$$

Together elements can be combined to form 2-port interferometric networks.

More complicated multi-port geometries (i.e., geometries involving more than two inputs and outputs) have been proposed as linear transformation devices for quantum optics and microwave photonics [19, 55]. Because networks of beam splitters and phase shifters can be used to realize any unitary operator [55], they provide a useful basis for the design of certain types of photonic integrated circuit design. Later, multi-port interferometers will be investigated for their applications in SSH systems.

The Mach Zehnder Interferometer

The Mach-Zehnder interferometer was first conceived for the measurement of the refractive indices of material samples by Ludwig Zehnder in 1891 [80]. It was improved by Ludwig Mach in 1892 [44]; both sought to create an interferometer with better spatial separation of fields within the interferometer than the existing Jamin interferometer first described by Jules Jamin in 1856 [33]. The Jamin interferometer uses two mirrors made of thick slabs of glass; beam splitting is achieved by partial reflection off the front face of the

mirror and then off of the back face of the mirror; the separation of the two paths in the Jamin interferometer is limited by the thickness of the glass, by using beam splitting prisms which directed fields in perpendicular directions, Mach and Zehnder were able to create a larger usable area for refractive index characterization. Mach also noticed that the fields at the output ports of the device showed inverted interference patterns. This latter property is of interest for interferometric SSH interferometers. The Mach-Zehnder interferometer, the Jamin interferometer, the Michelson interferometer, and the Fabry-Pérot interferometer all belong to a class of interferometers called $SU(2)$ interferometers [79] because they can be expressed in terms of the special unitary group of 2×2 matrices. These interferometers can interferometrically mix pairs of inputs to pairs of outputs. Of these interferometers, the Mach-Zehnder interferometer is the most interesting for SSH applications since all fields propagate in the same direction, and the two input ports and output ports are spatially separated.

The Mach-Zehnder interferometer has 2 input ports (IP1 and IP2) and two output ports (OP1 and OP2). The device consists of an input beam-splitter followed by a phase shifter and another beam-splitter. A Mach-Zehnder interferometer is depicted in figure 1.8. The

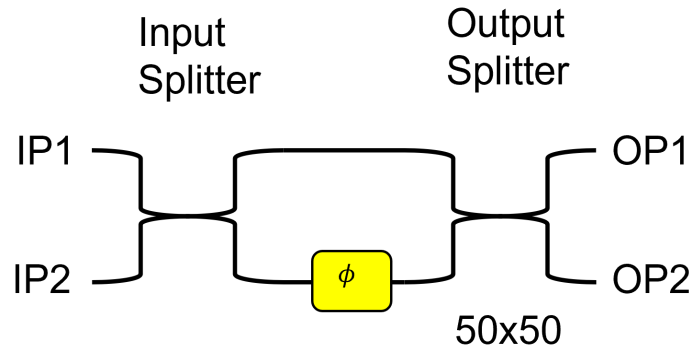


Figure 1.8: A Mach-Zehnder interferometer with an input splitter of arbitrary split ratio, a phase shifter, and a 50-50 output splitter

fields at each port can be expressed as a 2-element vector; the splitters and phase shifters can

be expressed as 2x2 matrices. The vector describing fields at the outputs can be expressed as a matrix product of the operators describing the input fields with a vector describing the input fields according to equation 1.150.

$$\begin{aligned} \begin{pmatrix} E_{OP1} \\ E_{OP2} \end{pmatrix} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_B} \end{pmatrix} \begin{pmatrix} \nu & ir \\ ir & \nu \end{pmatrix} \begin{pmatrix} E_{IP1} \\ E_{IP2} \end{pmatrix} \dots \\ &\dots = \frac{1}{\sqrt{2}} \begin{pmatrix} [\nu - re^{i\phi_B}] & i[r + \nu e^{i\phi_B}] \\ i[\nu + re^{i\phi_B}] & [\nu e^{i\phi_B} - r] \end{pmatrix} \begin{pmatrix} E_{IP1} \\ E_{IP2} \end{pmatrix} \end{aligned} \quad (1.150)$$

In the special case where $\nu = r = \frac{1}{\sqrt{2}}$ the expression reduces to

$$\begin{pmatrix} E_{OP1} \\ E_{OP2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} [1 - e^{i\phi_B}] & i[1 + e^{i\phi_B}] \\ i[1 + e^{i\phi_B}] & [e^{i\phi_B} - 1] \end{pmatrix} \begin{pmatrix} E_{IP1} \\ E_{IP2} \end{pmatrix} \quad (1.151)$$

The Mach-Zehnder interferometer has 4 key bias phase configurations. If $\phi_B = \pi$ or an odd multiple thereof, the expression reduces to a diagonal matrix. This condition is often referred to as “peak”, In this condition, fields at the input are mapped to the outputs with no mixing between them, and one field is phase shifted by π with respect to the other (see equation 1.152).

$$\hat{Q}_{MZ-Peak} = \frac{1}{2} \begin{pmatrix} [1 - e^{i\pi}] & i[1 + e^{i\pi}] \\ i[1 + e^{i\pi}] & [e^{i\pi} - 1] \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (1.152)$$

A related condition called the ”null” condition occurs when the bias phase is $\phi_B = 2\pi$ or a

multiple thereof, in which case the interferometer swaps the outputs (see equation 1.153).

$$\hat{Q}_{MZ-Null} = \frac{1}{2} \begin{pmatrix} [1 - e^{i2\pi}] & i [1 + e^{i2\pi}] \\ i [1 + e^{i2\pi}] & [e^{i2\pi} - 1] \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad (1.153)$$

The null condition Mach-Zehnder is essentially a phase-shifted version of the channel swap operator. In the peak and null conditions, the interferometer will map a field at the input port to a single output port. In an interferometer under either the peak or null condition, at the inputs and outputs, the fields are unique and separate, while inside the device, the fields are mixed; as will be shown later, the peak and null conditions can be used for isolating stimulated photon echo fields from certain input fields.

The other conditions are referred to as positive and negative quadrature. In quadrature, the phase bias has a value of an odd integer multiple of $\frac{\pi}{2}$. In these cases, equal amounts of optical power are distributed to each output port of the interferometer; however, small deviations to the bias phase will slightly perturb the relative power in each port. In the positive quadrature condition $\phi_B = \frac{\pi}{2} + 2\pi j + \delta\phi$ where j is an integer, and $\delta\phi$ is a slight deviation to the bias phase, the resulting operator describing the interferometer is given by 1.154

$$\hat{Q}_{MZ-+Quad} = \frac{1}{2} \begin{pmatrix} [1 - e^{i\frac{\pi}{2} + i\delta\phi}] & i [1 + e^{i\frac{\pi}{2} + i\delta\phi}] \\ i [1 + e^{i\frac{\pi}{2} + i\delta\phi}] & [e^{i\frac{\pi}{2} + i\delta\phi} - 1] \end{pmatrix} = \frac{1}{2} \begin{pmatrix} [1 - ie^{i\delta\phi}] & i [1 + ie^{i\delta\phi}] \\ i [1 + ie^{i\delta\phi}] & [ie^{i\delta\phi} - 1] \end{pmatrix} \quad (1.154)$$

If a field E_0 is injected into port IP1, then the output fields can be expressed according to equation 1.155.

$$\begin{pmatrix} E_{OP1} \\ E_{OP2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} E_0 [1 - ie^{i\delta\phi}] \\ iE_0 [1 + ie^{i\delta\phi}] \end{pmatrix} \quad (1.155)$$

The optical powers at the output ports can be expressed in terms of the power in the input

(P_0) according to equations 1.156 and 1.157.

$$P_{OP1} = \frac{P_0}{2} (1 + \sin(\delta\phi)) \quad (1.156)$$

$$P_{OP2} = \frac{P_0}{2} (1 - \sin(\delta\phi)) \quad (1.157)$$

In the negative quadrature condition, the bias phase is $\phi_B = -\frac{\pi}{2} + 2\pi J + \delta\phi$. The operator to describe the interferometer is given by 1.158.

$$\hat{Q}_{MZ--Quad} = \frac{1}{2} \begin{pmatrix} \begin{bmatrix} 1 - e^{i\frac{-\pi}{2} + i\delta\phi} \\ i \begin{bmatrix} 1 + e^{i\frac{-\pi}{2} + i\delta\phi} \end{bmatrix} \end{bmatrix} & i \begin{bmatrix} 1 + e^{i\frac{-\pi}{2} + i\delta\phi} \\ e^{i\frac{-\pi}{2} + i\delta\phi} - 1 \end{bmatrix} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \begin{bmatrix} 1 + ie^{i\delta\phi} & i \begin{bmatrix} 1 - ie^{i\delta\phi} \end{bmatrix} \\ i \begin{bmatrix} 1 - ie^{i\delta\phi} \end{bmatrix} & - \begin{bmatrix} ie^{i\delta\phi} + 1 \end{bmatrix} \end{bmatrix} \end{pmatrix} \quad (1.158)$$

If a field E_0 is injected into port IP1, then the output fields can be expressed according to equation 1.159.

$$\begin{pmatrix} E_{OP1} \\ E_{OP2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} E_0 \begin{bmatrix} 1 + ie^{i\delta\phi} \end{bmatrix} \\ iE_0 \begin{bmatrix} 1 - ie^{i\delta\phi} \end{bmatrix} \end{pmatrix} \quad (1.159)$$

The final powers at the output ports are then given by equations 1.160 and 1.161.

$$P_{OP1} = \frac{P_0}{2} (1 - \sin(\delta\phi)) \quad (1.160)$$

$$P_{OP2} = \frac{P_0}{2} (1 + \sin(\delta\phi)) \quad (1.161)$$

In both quadrature conditions, the relative power in each output is governed by deviations to the phase bias. Since the dependence is on the sine, for small perturbations to the phase, the output power distribution is linearly related to the bias phase of the interferometer. This configuration can be used for phase-sensitive interferometric detection of SSH spectra read out by chirps.

INTRODUCTION TO INTERFEROMETRIC SSH SIGNAL PROCESSING

As discussed in the preceding chapter, SSH materials can be used for various signal-processing applications. New architectures for implementing these processes can be created by inserting an SSH material into an interferometer and exploiting the phase properties of the SSH process to get the desired result. Two main applications of this are discussed. Firstly the isolation of stimulated photon echo fields in either a Mach-Zehnder or multi-port interferometer by interfering echoes generated in independent ensembles. The second major application is the improvement of SNR of chirped readout by using an interferometer to do phase-sensitive detection of the read-out spectra and using a local oscillator to provide gain to detected signals.

Interferometric Methods In SSH Systems

N inputs into an interferometric network can be described with an interferometric Jones vector $\vec{E}_0$. A system of M_1 elements (collectively defined as $\hat{Q}_{in}$) can be used to mix and phase shift the N input fields to generate a new set of fields that act as inputs to the material ($\vec{E}_{in}$ shown in equation 2.1).

$$\vec{E}_{in} = \hat{Q}_{in}\vec{E}_0 \quad (2.1)$$

Fields in the N separate channels will interact with an SSH material in N different spots. Interaction with the material will affect the field going through the material and cause whatever effects on the material (such as programming or some coherent effect) and modify the input field, creating an output field $\vec{E}'_{out}$, another set of M_2 elements ($\hat{Q}_{out}$) can then be used to mix and phase shift those elements for detection to achieve an interference regime that is beneficial for detection giving a new mixed output field $\vec{E}_{out}$. According to equation 2.2.

$$\vec{E}_{out} = \hat{Q}_{out} \vec{E}'_{out} \quad (2.2)$$

Schematically The entire model is described in figure 2.1

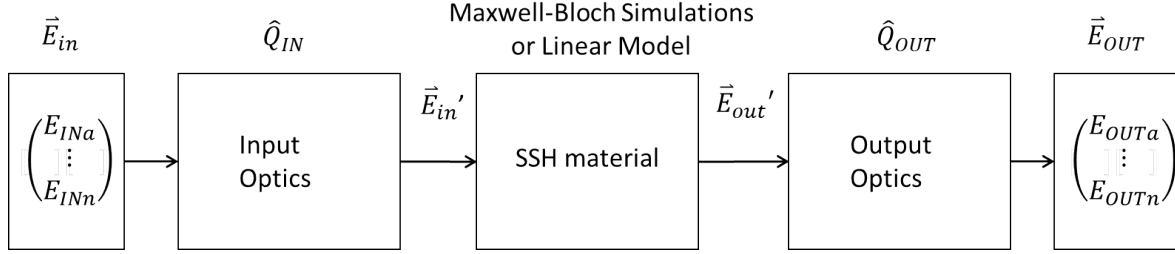


Figure 2.1: A summary of the general method for interferometric methods in SSH systems. A series of input fields are mixed and phase-shifted before interaction with an SSH material. The co-linear modified fields (which may be attenuated or contain echo fields) are mixed in a series of output elements to get the fields into some useful interference configuration for detection.

For the purpose of predictions and modeling, the interaction with the material can be accounted for using several methods, a linear response model, analytic models for echoes, incoherent programming models, or a full Maxwell-Bloch simulation.

This model is basically agnostic of the actual form that the individual optical elements take. For example, the beam splitters could be free space, fiber-coupled, or routed into a photonic integrated circuit. Phase shifts could be done electro-optically, or by using a physical length delay. The material could be multiple spots in a single material, single spots in multiple materials or even doped waveguides. any combination of these elements could be described using this formalism. Everything can be done with all co-linear fields and beam splitter networks which are entirely external to the material.

Two Port Interferometric Systems

Stimulated Echo Isolation In a Mach-Zehnder Interferometer Consider an ideal Mach-Zehnder interferometer (with a 50–50 input split ratio and a 50–50 output split ratio) biased

in peak, with both channels propagating through an SSH material as illustrated in figure 2.2. And assume this material is used to conduct stimulated photon echo experiments. The fields E_1 and E_3 are injected into port $IP1$, and the field E_2 is injected into port $IP2$. Going through the first splitter, the fields are mixed, so in the matrix notation, the interferometric Jones vectors for the 3 pulses are given in equation 2.3.

$$\vec{E}'_{In,1} = \frac{E_1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \vec{E}'_{In,2} = \frac{E_2}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} \quad \vec{E}'_{In,3} = \frac{E_3}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad (2.3)$$

The stimulated echo field in frequency domain is given by equation 1.54 and in this case, becomes an element-wise product of the three interferometric Jones vectors since the relative phases of the constituent fields vary by frequency-independent common phases induced by beam splitters. Just After it is generated in the material, the echo will have an interferometric Jones vector given by equation 2.4.

$$\vec{E}'_{out,echo} = \frac{E_{Echo}}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} \propto \frac{E_1^* E_2 E_3}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} \quad (2.4)$$

The vector component describes the relative phases between fields in each channel within the interferometer just after the crystal. This vector takes the same form as that of $\vec{E}'_{In,2}$. After the material, the echo passes through a phase shifter and the output splitter. The echo at the output of the device is given by equation 2.5

$$\vec{E}_{out,Echo} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_B} \end{pmatrix} \frac{E_{Echo}}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{E_{echo}}{2} \begin{pmatrix} i(1 + e^{i\phi_B}) \\ e^{i\phi_B} - 1 \end{pmatrix} \quad (2.5)$$

If the interferometer is biased in peak (i.e. $\phi_B = \pi$) then the output echo will take the form

given by equation 2.6

$$\vec{E}_{out,Echo} = \frac{E_{echo}}{2} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \propto \frac{E_1^* E_2 E_3}{2} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (2.6)$$

In this case the entirety of each field will be mapped entirely into only one port. The fields E_1 and E_3 will be directed out of output port OP1 while fields E_2 and E_{echo} are directed out of output port OP2. This scheme is depicted in figure 2.2. This configuration allows for the

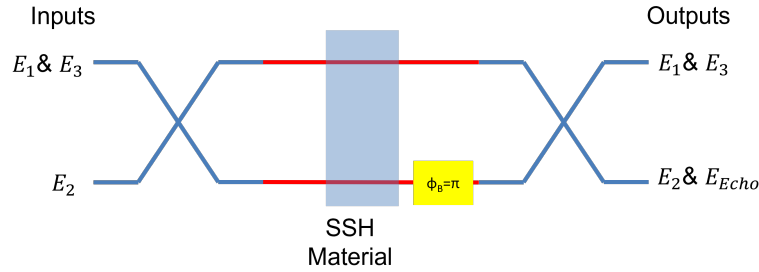


Figure 2.2: A Mach-Zehnder interferometer can be used to direct fields in stimulated photon echo experiments to different output ports; in this case, the echo field is isolated from fields E_1 and E_3 .

isolation of the echo field from fields E_1 and E_3 . A similar result would be obtained for a null bias ($\phi_B = 0$), though the output ports would be swapped for each field. A similar non-interferometric scheme can be arranged by injecting fields into the material along separate $\vec{k}$ -vectors as shown in figure 2.3. This scheme has long been used to isolate stimulated photon echoes from probe fields [6, 70]. This angular method for isolating the echo field from fields E_1 and E_3 requires that the experiment be conducted in free-space, while the interferometric method allows for this same isolation to be carried out not only in any network of beam splitters. These could be free space splitters; they could be fiber beam splitters or in a series of couplers in a photonic integrated circuit.

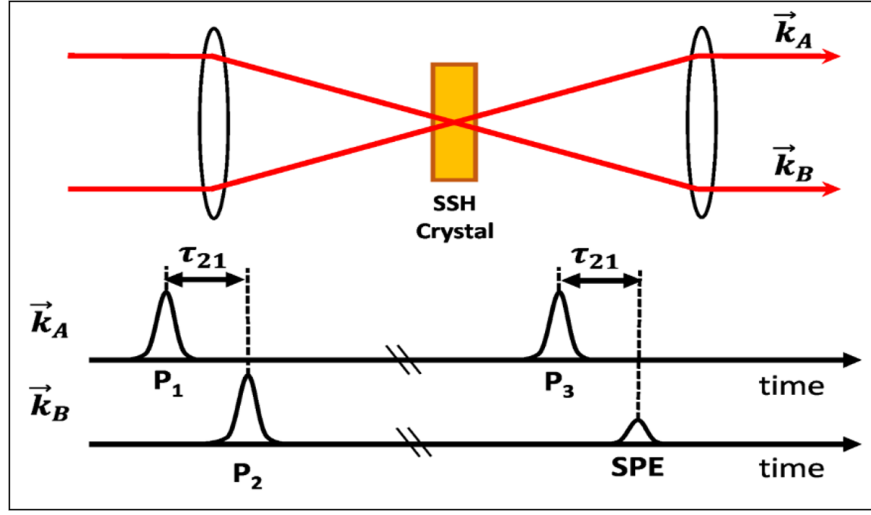


Figure 2.3: Using pulses injected along two separate $\vec{k}$ -vectors, the echo can be separated from fields E_1 and E_3 , a similar result to the 2 port interferometric echo scheme.

Interferometric Readout Since a Mach-Zehnder interferometer biased in quadrature can be used in phase-sensitive detection, the interferometric readout can be used to calculate the frequency-dependent phase shift due to spectra programmed in an SSH material. Consider an interferometric readout scheme as depicted in Figure 2.4. A splitter of arbitrary split

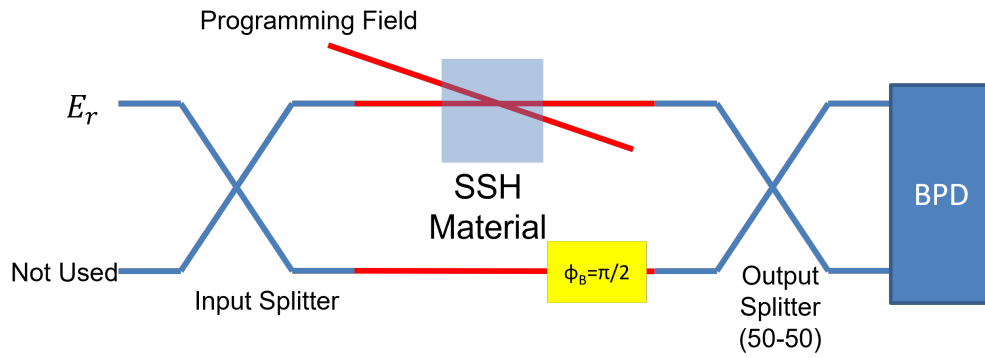


Figure 2.4: An interferometric readout scheme allows for a phase-sensitive readout of programmed spectra in SSH materials.

ratio will split the input chirp so that the read field components on each channel are given

by equation 2.7.

$$\vec{E}'_r = E_r \begin{pmatrix} \nu \\ ir \end{pmatrix} \quad (2.7)$$

The fields can now be thought of as the read field (going through the material) and a local oscillator (LO) field, which will go around the material and incurs a relative phase shift of $\phi_B = \frac{\pi}{2}$ to put the interferometer in quadrature; both fields are then coupled into a 50-50 splitter for coherent detection. The read field will be attenuated and phase shifted by the programmed SSH material spectrum. After the material and the phase shifter, the time domain fields can be expressed according to 2.8.

$$\vec{E}''_r(t) = E_r \begin{pmatrix} \nu e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(\kappa t + f_0) + i\mathcal{H}\{S_{eff}(\kappa t + f_0)\})]} \\ -r \end{pmatrix} \quad (2.8)$$

At the outputs, the fields are mixed by the 50 – 50 output splitter. The final output fields are given by equation 2.9.

$$\vec{E}_r(t) = \frac{E_r}{\sqrt{2}} \begin{pmatrix} \nu e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(\kappa t + f_0) + i\mathcal{H}\{S_{eff}(\kappa t + f_0)\})]} - ir \\ -r - i\nu e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(\kappa t + f_0) + i\mathcal{H}\{S_{eff}(\kappa t + f_0)\})]} \end{pmatrix} \quad (2.9)$$

The power in each output port is expressed in equations 2.10 and 2.11.

$$P_{OP1} = \frac{1}{2} \left(\nu^2 P_0 e^{-\frac{1}{2}\alpha_0 L (1 - \beta S_{eff}(\kappa t + f_0))} + r^2 P_0 - 2P_0 r \nu \operatorname{Im} \left[e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(\kappa t + f_0) + i\mathcal{H}\{S_{eff}(\kappa t + f_0)\})]} \right] \right) \quad (2.10)$$

$$P_{OP2} = \frac{1}{2} \left(\nu^2 P_0 e^{-\frac{1}{2}\alpha_0 L (1 - \beta S_{eff}(\kappa t + f_0))} + r^2 P_0 + 2P_0 r \nu \operatorname{Im} \left[e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(\kappa t + f_0) + i\mathcal{H}\{S_{eff}(\kappa t + f_0)\})]} \right] \right) \quad (2.11)$$

The differential power at the detector is the difference between the powers OP2 and OP1 given by equation 2.12.

$$P_{\Delta} = 2P_0 r \nu e^{-\frac{1}{2}\alpha_0 L} \sin \left(\frac{1}{2} \alpha_0 L \beta \mathcal{H} \{S_{eff}(\kappa t + f_0)\} \right) \quad (2.12)$$

Since the programmed spectra are weak (i.e., $\beta S_{eff} \ll 1$), the read field can then be approximated by equation 2.13.

$$P_{\Delta} = P_0 r \nu e^{-\frac{1}{2}\alpha_0 L} \alpha_0 L \beta \mathcal{H} \{S_{eff}(\kappa t + f_0)\} \quad (2.13)$$

This result is similar to the outcome for traditional readout schemes described in equation 1.32. However, the traditional readout signal does not depend on the r and ν splitter parameters and traditional readout measures the amplitude response rather than the phase response. As previously stated the two branches of the read chirp inside of the interferometer are a read field and an LO field. At the detection stage, the total power in the read field (i.e., the power contributions of the read field to the power at OP1 and OP2) total to $P_{read} = \nu^2 e^{-\alpha_0 L} P_0$ and the lo field will have a total power $P_{LO} = r^2 P_0$. The differential power can be described in terms of the read and LO power at the detector according to equation 2.14.

$$P_{\Delta} = \sqrt{P_r P_{LO}} \alpha_0 L \beta \mathcal{H} \{S_{eff}(\kappa t + f_0)\} \quad (2.14)$$

This expression describes the measured signals using interferometric readout. The interferometric signal can be restored to the form of the programmed spectra (rather than its Hilbert transform) by using a Hilbert transform (or, as will be discussed later, the spectral recovery algorithm for interferometric readout). In the case where the read and LO powers are the same (after post-processing interferometric results with a Hilbert transform), interferometric and traditional read give the same result. If, for the same read power, the LO is stronger than

the read power; the relative differential power will be greater for interferometric readout. Under certain circumstances, which will be discussed in detail later on, this can lead to improved sensitivity and signal-to-noise ratio (SNR) using interferometric readout.

A Note on Experimental Implementation of 2-port Interferometric Systems Later chapters of this dissertation discuss experimental results in 2-port interferometric SSH systems. The interferometric systems used in these studies use fiber-coupled beam splitters and fields launched into free space, where they pass through an SSH material (Tm^{3+} : $LiNbO_3$) mounted in a cryostat. The fields are then coupled back into optical fibers before the final beam splitter and detection.

Moving between fiber optics and free space introduces an extra degree of complexity in that some additional losses will be incurred when coupling back into fiber. In field separation applications these inefficiencies will have to be properly balanced in order to achieve high isolation of echo fields, though as will be seen in chapters 4 and 5. Tuning these coupling parameters allowed for some degree of compensation of the effects of imperfect beam splitter, albeit at a cost to signal power. In readout systems, these coupling losses only lead to losses in signal strength.

Four Port Interferometric Systems

Operators Interferometric architectures with 4 input ports and 4 output ports use interferometric elements to couple fields between 4 independent channels. Beam splitter elements in a 4x4 interferometric architecture can connect two independent channels j and k . These beam splitters are represented by 4x4 matrix operators $\hat{Q}_{Splitter:j,k}$. These operators are given in equation 2.15.

$$\begin{aligned}
\hat{Q}_{Splitter:1,2} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Splitter:2,3} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
\hat{Q}_{Splitter:3,4} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} & \hat{Q}_{Splitter:1,4} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \\
\hat{Q}_{Splitter:1,3} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Splitter:2,4} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}
\end{aligned} \tag{2.15}$$

Relative phase shifts can be applied in the four-port architecture as a diagonal matrix consisting of complex exponentials along the diagonal. This matrix operator, given in equation 2.16 will apply a phase shift to each channel independently.

$$\hat{Q}_{Phase}(\phi_1, \phi_2, \phi_3, \phi_4) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 & 0 \\ 0 & 0 & e^{i\phi_3} & 0 \\ 0 & 0 & 0 & e^{i\phi_4} \end{pmatrix} \tag{2.16}$$

These phase shifts can be due to a path length induced by PZT mirrors or a fiber stretcher, or they could be due to phase shifts from independent electro-optic phase modulators. These

phase shifts can be used to compensate for random phase shifts due to small length differences in the fibers that make up the interferometer. Finally, channels can be swapped; these operations are represented by the operators given in equation 2.17.

$$\begin{aligned}
\hat{Q}_{Swap:1,2} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Swap:2,3} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
\hat{Q}_{Swap:3,4} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \hat{Q}_{Swap:1,4} &= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} & (2.17) \\
\hat{Q}_{Swap:1,3} &= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Swap:2,4} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}
\end{aligned}$$

Using these operators, interferometric networks with 4 input ports and 4 output ports can be built to exploit the phase properties of fields in devices with 4 independent spots in an SSH.

Echo Isolation In a 4-port interferometer Consider a 4-port port interferometer consisting of a pair of ideal 50-50 splitters coupling fields from channels 1 and 2 and channels 3 and 4. A second pair of splitters then connect channels 2 and 3 and channels 1 and 4. The second pair of splitters will then be directed through an SSH material. Each of the

fields is then independently phase-shifted. The fields are directed through a network of splitters that is the mirror image of the input splitter network. The output splitter network consists of two splitters connecting channels 1 and 4 and channels 2 and 3, followed by two splitters connecting channels 1 and 2 and channels 3 and 4. The matrix operators for this interferometer are given in equations 2.18 and 2.19.

$$\hat{Q}_{in} = \hat{Q}_{splitter:1,4} \hat{Q}_{splitter:2,3} \hat{Q}_{splitter:1,2} \hat{Q}_{splitter:3,4} \quad (2.18)$$

$$\hat{Q}_{out} = \hat{Q}_{splitter:1,2} \hat{Q}_{splitter:3,4} \hat{Q}_{splitter:1,4} \hat{Q}_{splitter:2,3} \hat{Q}_{phase}(\phi_1, \phi_2, \phi_3, \phi_4) \quad (2.19)$$

This interferometer is shown in figure 2.5. This four-port interferometer design (ignoring material interaction) can map the field in an input port to a single output port depending on the bias phases (specifically under the condition that the relative phases are multiples of π between each other). This 4-port interferometer can be used to isolate stimulated echoes

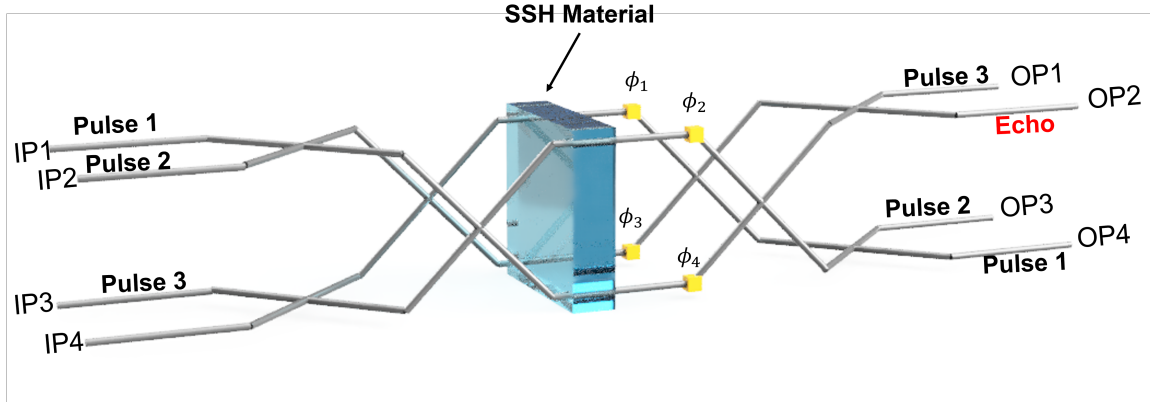


Figure 2.5: An example of a four-port interferometer with an SSH material. This Interferometric architecture can map input fields to unique ports at the output and match stimulated photon echoes into unique ports.

from the input fields in analog to the box geometry (depicted in figure 1.5 as it uniquely maps each input and the echo to its own unique port. This configuration will be described in more detail in chapter 4.

Other Multi-Port Interferometric Systems

Interferometric Programming and Readout in Entirely Interferometric Architectures

Interferometric Readout in a 3 Port Architecture: Interferometric readout and programming can be conducted entirely (programming and readout) in an interferometric architecture. A 3 port-all Interferometric SSH spectrum analyzer scheme can be contrived to illustrate the concept. This scheme is shown in figure 2.6.

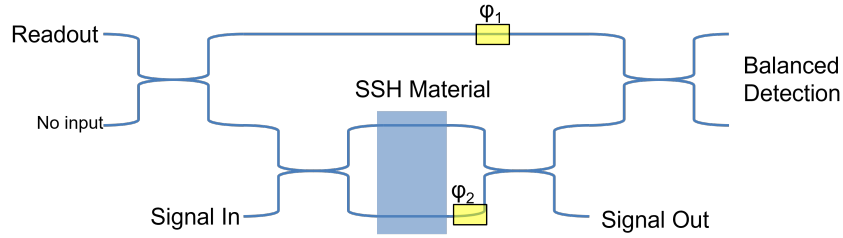


Figure 2.6: A 3-Port Scheme for all interferometric programming and readout of an SSH material. A Mach-Zehnder biased in peak is nested within an interferometer biased in quadrature.

The 3 port programming and readout scheme involves a peak-biased Mach-Zehnder interferometer nested onto one arm of a quadrature-biased Mach-Zehnder interferometer. Spectra are programmed and then read out co-linearly in 2 channels on a Mach Zehnder interferometer. The read field is then directed back into the Mach-Zehnder interferometer, while the programming signal is directed out of the device isolated from the read fields. The read field is then available for mixing with the LO field and then balanced detection so that the signals can be coherently detected. This 3-port readout scheme can be described using four-port operators by using a 4th port that is left idle. The input and output operators are given in equations 2.20 and 2.21

$$\hat{Q}_{in} = \hat{Q}_{splitter:2,3} \hat{Q}_{splitter:1,2} \quad (2.20)$$

$$\hat{Q}_{out} = \hat{Q}_{splitter:1,2} \hat{Q}_{splitter:2,3} \hat{Q}_{phase} \left(\frac{\pi}{2}, 0, \pi, 0 \right) \quad (2.21)$$

Interferometric Readout in a 4-Port Architecture: The interferometric readout can also be conducted in a 4-port architecture, as shown in figure 2.7. Functionally this architecture is very similar to the 3 port geometry described above however this system is symmetrical. This system can be biased for readout with balanced detection in a 4-port configuration using no additional angled beams for programming. The input and output

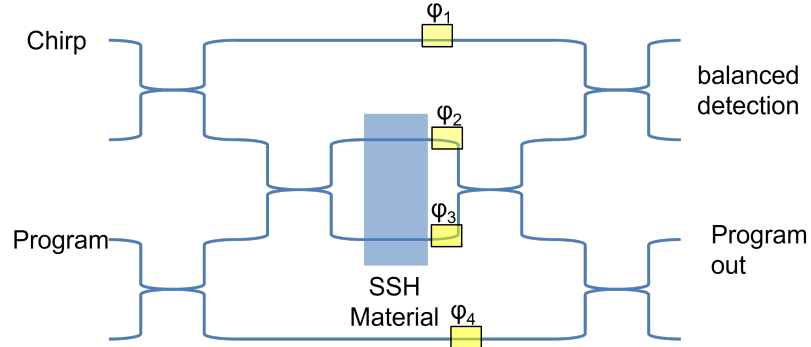


Figure 2.7: A 4 port interferometric configuration that could be used for interferometric readout.

matrices for this interferometer are given in equations 2.22 and 2.23.

$$\hat{Q}_{in} = \hat{Q}_{splitter:2,3} \hat{Q}_{splitter:3,4} \hat{Q}_{splitter:1,2} \quad (2.22)$$

$$\hat{Q}_{out} = \hat{Q}_{splitter:1,2} \hat{Q}_{splitter:3,4} \hat{Q}_{splitter:2,3} \hat{Q}_{phase} \left(\frac{\pi}{2}, 0, \pi, \pi \right) \quad (2.23)$$

This interferometric configuration allows the chirp to be isolated from programming fields using a Mach-Zehnder interferometer nested inside the device. Portions of the chirp and the read field can act as local oscillators for interference at the end. By biasing the internal

interferometer in peak, the two outer Mach-Zehnder interferometers can be independently biased to allow for balanced detection of the chirp and isolation of the output programming field into one port at the outputs. The chirp field will pass around the internal interferometer meaning extra programming power may be needed using this configuration.

MAXWELL-BLOCH SIMULATIONS

Numerical solutions to the Maxwell-Bloch equations provide a unique window into the dynamics of spatial-spectral holographic systems. The Maxwell- Bloch equations allow for the simulation of the fields resulting from an inhomogeneously broadened material driven by an arbitrary field. The Maxwell-Bloch equations are a time domain model of the behavior of SSH processes taking into account not only the effects of the process of interest but a full treatment of all the coherent effects that could arise in the system. Previously others have developed a wide variety of Maxwell-Bloch simulators. Maxwell-Bloch simulators capable of handling angled geometries [16, 17, 18] and a 2D Maxwell-Bloch simulator capable of handling the transverse beam profile of fields in the material [78]. We developed a new Maxwell- Bloch simulator to evaluate SSH systems because Spectrum-lab did not have access to a Maxwell-Bloch simulator for which we had source code. This new Maxwell-Bloch simulator was intended to be used with a GPU-enhanced machine to allow for enhanced parallel processing, so some attempts were made to improve the parallelized performance of the code. To study interferometric SSH systems, all that is required is a co-linear Maxwell-Bloch simulator capable of handling input fields with arbitrary fields. Several independent simulations can be used to simulate independent SSH interactions in multiple channels. The linear input and output optics can be handled with the matrix operators described in chapter 2.

Simulating The Bloch Equations

To simulate the Maxwell Bloch equations, propagation through an SSH material of absorption length $\alpha_0 L$ and bandwidth $\Delta\omega$ is divided into M_Δ detuning bins of width $\delta\Delta = \frac{\Delta\omega}{M_\Delta}$ and M_z slabs of absorption length $\frac{\alpha_0 L}{M_z}$. The Bloch equations will be solved in the first slab for all detunings to calculate a field correction to act as an input for the next slab; this

process is repeated sequentially to calculate the field as it propagates through the material.

The first step to simulating the Maxwell-Bloch equations is to solve the Bloch equations. For a given slice, the Bloch-equations must be solved for M_Δ independent frequency bins. For each frequency bin, three coupled differential equations must be solved (appendix C). The differential equation can be expressed as a function of this long vector. This system of differential equations can then be solved numerically simultaneously for all detunings. To solve the Bloch equations, the 4th/5th order Runge-Kutta routine ODE45 was used in MATLAB (see appendix C). The solutions of the Bloch equations from each slice are then used to describe corrections to the field as it propagates through the material.

Bloch Equation: Numerical Solutions

Single Frequency Bloch Example: Resonant Rabi Oscillations A simple test of the Bloch simulation is a test of the Rabi problem on resonance. This result is presented in equation . This was tested using a Rabi frequency of π MHz (assuming no decay), simulating material interaction for $5\mu s$ at 100 points within that interval. The results of this simulation are presented in figure 3.1. In figure 3.1, simulated the Bloch vector components r_1 and r_2 oscillate with a period of $2\mu s$ which corresponds to a Rabi frequency ($\Omega = \pi MHz$). The Bloch vectors oscillate as expected between +1 and -1, as expected given the initial conditions and Rabi frequency.

Single Frequency Bloch Example: Detuned Rabi Oscillations Rabi oscillations also occur when the atoms are not resonant at the oscillation frequency. In this case, all three Bloch equations are coupled with each other. Under these conditions, the Bloch vector components for ions of detuning Δ will oscillate at the magnitude of the Rabi vector, which is also known as the generalized Rabi frequency Ω_G (equation: 3.1).

$$\Omega_G = \sqrt{\Delta^2 + \Omega^2} \quad (3.1)$$

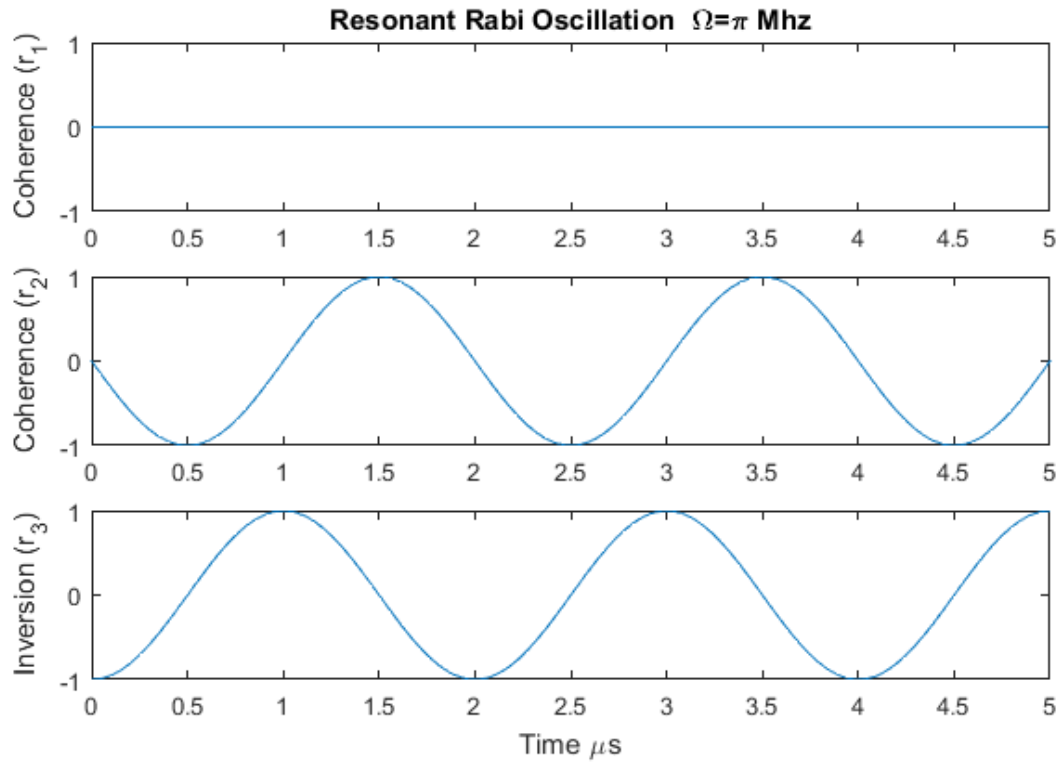


Figure 3.1: The behavior of the Bloch vector components while undergoing Rabi oscillations at resonance with $\Omega = \pi MHz$.

In the case of an on-resonance driving field, only two of the vector components. Driving off-resonance changes the plane in which the Bloch vector rotates, causing all three of the Bloch-vector components to vary in time. The solutions for all Bloch Vector components for a detuned Rabi oscillation are given in equations 3.2, 3.3 and 3.4.

$$r_1(t) = \frac{\Delta\Omega}{\Omega_g^2} (\cos(\Omega_g t) - 1) \quad (3.2)$$

$$r_2(t) = -\frac{\Omega}{\Omega_g} \sin(\Omega_g t) \quad (3.3)$$

$$r_3(t) = -\frac{\Delta^2}{\Omega_g^2} - \frac{\Omega^2}{\Omega_g^2} \cos(\Omega_g t) \quad (3.4)$$

To test the simulator off-resonance, Rabi oscillations were tested with a Rabi frequency of $\Omega = \pi MHz$ and a detuning $\Delta = \pi MHz$, which should yield a generalized Rabi frequency of $\Omega_g = \sqrt{2}\pi MHz$. The simulations were used to solve for Bloch-vector components over a duration of $5\mu s$ containing 100 points. Results are displayed in figure 3.2. All 3 Bloch vector components oscillate with a period of $1.41\mu s$, corresponding to the generalized Rabi frequency of $\sqrt{2}\pi MHz$. Given the Rabi frequency and detuning values, the expected solution for r_1 is $r_1(t) = \frac{1}{2} (\cos(\Omega_g t) - 1)$. The simulation results show that r_1 has oscillations that begin at their zenith at $t = 0$, with an amplitude of $1/2$ centered around $-\frac{1}{2}$. The expected solution for r_2 is $r_2 = -\frac{1}{\sqrt{2}} \sin(\Omega_g t)$, and as expected, r_2 is sinusoidal with a negative slope at $t=0$ and an amplitude of 0.707. The expected solution for r_3 is $r_3 = \frac{1}{2} (1 + \cos(\Omega_g t))$. The simulation result for r_3 takes the same form as the expected solution, a sinusoidal oscillation beginning at its nadir at $t = 0$ with an amplitude of $\frac{1}{2}$ centered at $-\frac{1}{2}$.

Simulating Propagation: The Maxwell Bloch Equations

As electric fields propagate through the material the fields interact with the material which generates fields in the material that will modify the total field as it propagates. These

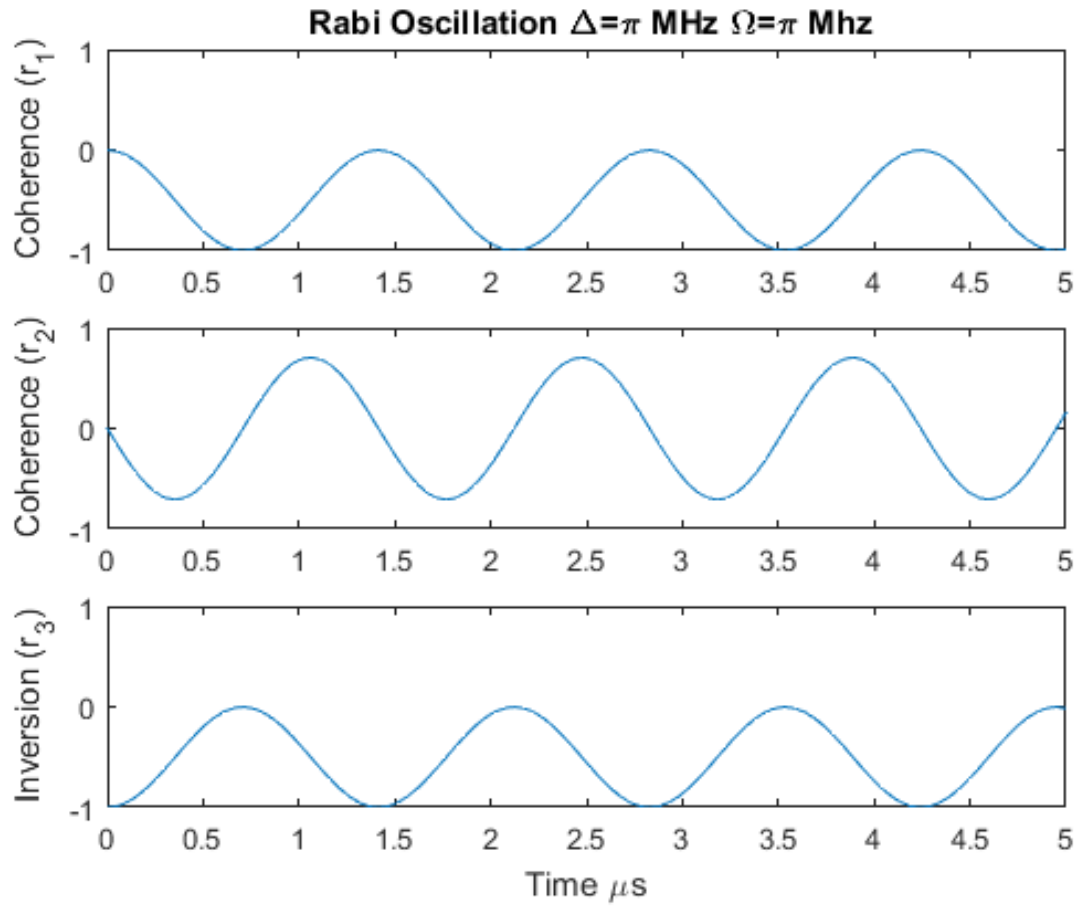


Figure 3.2: The behavior of the Bloch vector components while undergoing Rabi oscillations at a detuning of $\Delta = \pi \text{ MHz}$.

field corrections are calculated by solving the Bloch equations at M_Δ different detunings ($\Delta[m]$) and integrating the coherences r_1 and r_2 using a Riemann sum. In our simulations, the lineshape of the material is ignored because, for our simulations, the inhomogeneous linewidth was much larger than the bandwidth required for simulations so it was easier to use a uniform material lineshape. The correction to the Rabi frequency in z slice $j+1$ can be calculated based on the coherences in z slice j using equations 3.5 and 3.6

$$\text{Re}(\Delta\Omega(z[j+1], t)) = \frac{\alpha_0 L}{2\pi M_z} \sum_{m=1}^{M_\Delta} r'_2(\Delta[m], z[j], t) (\delta\Delta) \quad (3.5)$$

$$\text{Im}(\Delta\Omega(z[j+1], t)) = -\frac{\alpha_0 L}{2\pi M_z} \sum_{m=1}^{M_\Delta} r'_1(\Delta[m], z[j], t) (\delta\Delta) \quad (3.6)$$

where $\delta\Delta$ is the detuning step size. The correction is applied, and the modified Rabi frequency is used as the input field in the following slice. This process can be iterated through each slice of the material. This leads to the requirement that individual z-slices of the material are optically thin, i.e., $\frac{\alpha_0 L}{M_z} \ll 1$. The relative error in fields in each slab due to a non-infinitesimal absorption length should be $\frac{\alpha^2 L^2}{8M_z^2}$. This means the error is estimated to be about 2 parts in 1000 for $\frac{\alpha_0 L}{M_z} = 0.15$. Since ODE solving routines in MATLAB take function handles as inputs, the field corrections need to be interpolated so that the solver routine can call the field correction at intermediate steps between the steps where the solution is evaluated (as is required by Runge-Kutta methods). The procedure for Maxwell-Bloch equations is depicted in figure 3.3, and the MATLAB code is given in appendix C. This simulator can be used to simulate fields interacting with SSH materials in a wide variety of applications.

Additional Considerations For Maxwell-Bloch Simulations for SSH Systems

Vectorization vs. Parallelization in the Inhomogeneously Broadened Bloch Equations

The most important step of solving the Maxwell-Bloch equations is solving the in-

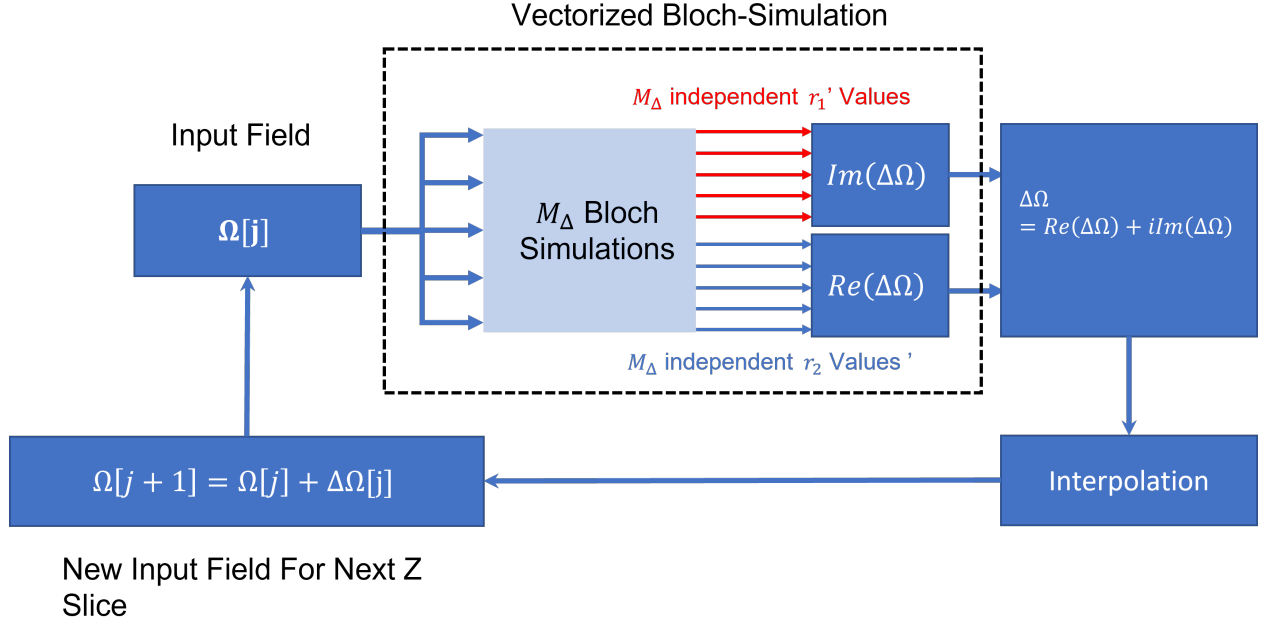


Figure 3.3: A block diagram showing the general method for the Maxwell-Bloch simulator. Input fields alter the Bloch vectors of M_Δ independent ensembles, from which field corrections can be calculated for the next material slice; this can be repeated until the field is propagated through the material.

homogeneously broadened Bloch equations. Conceptually, the simplest way to do this is to solve for the Bloch vector at each detuning value separately. Assume there is a 1-dimensional array of detuning values $\Delta[n]$. The individual Bloch equations to be solved are:

$$\dot{r}_1[n] = -\Delta[n]r_2[n] - Im(\Omega)r_3[n] \quad (3.7)$$

$$\dot{r}_2[n] = \Delta[n]r_1[n] + Re(\Omega)r_3[n] \quad (3.8)$$

$$\dot{r}_3[n] = Im(\Omega)r_1[n] - Re(\Omega)r_2[n] \quad (3.9)$$

Since each equation for each n value is independent, This seems to be a great candidate for parallelization at first glance. The simplest way to parallelize this in MATLAB is to execute each iteration in a parfor loop. The parfor loop is a control flow statement much like a for

loop, while the for loop runs sequentially, the parfor loop runs the loop in parallel (assuming that each iteration of the loop is independent of other iterations). However, MATLAB is strongly optimized for vectorized routines. Rather than executing an ODE solver algorithm for each detuning, it turns out to be faster to take a vectorized approach. To do so, a new 1d array is defined containing all of the components as:

$$R[3n - 2] = r_1[n] \quad (3.10)$$

$$R[3n - 1] = r_2[n] \quad (3.11)$$

$$R[3n] = r_3[n] \quad (3.12)$$

The new ODE system that is fed into the ODE solver routine is now a single system of $3N$ coupled ODEs rather than N systems of 3 coupled ODEs. The new system is as follows:

$$\dot{R}[3n - 2] = -\Delta[n]R[3n - 1] - Im(\Omega)R[3n] \quad (3.13)$$

$$\dot{R}[3n - 1] = \Delta[n]R[3n - 2] + Re(\Omega)R[3n] \quad (3.14)$$

$$\dot{R}[3n] = Im(\Omega)R[3n - 2] - Re(\Omega)R[3n - 1] \quad (3.15)$$

It appears that MATLAB already has some pre-optimization done for vectorized code, leading to the vectorized version being faster than solving for each detuning in parallel. In the initial development of the simulator, the vectorized method was used before parallelization was explored. When attempts were made to leverage parallel computing, tests were conducted using parfor however the vectorized version of the code ran around 5% faster than the explicitly parallelized version.

Reduction of the number of PP VAL/Anonymous Function Handle Calls Solving the Maxwell-Bloch equations requires the solution of the inhomogeneously broadened Bloch Equations to obtain a correction to the field. MATLAB's ODE solving routines need a function handle to describe the system of ordinary differential equations to be solved. The Maxwell-Bloch equations require knowledge of what the instantaneous electric field is. The electric field is modified by the internal polarizations created in the material simulated in the Bloch simulation. The modified field is the field used to drive the next slice of the simulation. This means the solution from horizontal slice m has to be converted into a function handle to serve as an input for slice $m+1$. Since solvers return a vector representing time series data for the real and imaginary parts of the field corrections, an interpolation routine can be used to create a description of the field for all times within the domain of interest (which is required for variable step methods that will use arbitrary time steps between the specified steps). To turn the interpolated correction into a function handle, the interpolation is used in an anonymous function (which can be defined on the fly). The first method tried was to create an anonymous function to describe the field corrections was to functionalize the field corrections, and then append them into an anonymous function describing the entire field. The exact implementation can be seen in the following code block.

The issue with this method is that both the `ppval` (which is how the interpolation is called) itself and anonymous function calls, in general, are very slow. Since they are virtually required for ODE solver routines, it is advantageous to reduce the number of times `ppval` and anonymous functions are called. The method shown requires that each time the appended field is called, it is also calling each individual interpolated correction from previous iterations that the number of actual anonymous function calls is now calling what could potentially be a large number of `ppvals` and anonymous function calls. A more efficient way to do this is to call the field function from the previous slice at each time step where the corrections are available. The corrections are added to the array of original field values. Now, the entire

```

##### Step 1: Calculate real and Imaginary Field Corrections
dOmega1=(Alpha/(2*pi))*R(:,2); % Real Field Corrections from R
values
dOmega2=-(Alpha/(2*pi))*R(:,1); % Imaginary Field Corrections
from R values

pp1=pchip(T,dOmega1); % Interpolate Real corrective terms
pp2=pchip(T,dOmega2); % Interpolate Imaginary corrective terms
correction=@(t) ppval(pp1,t)+i* ppval(pp2,t) % Combine real and
imaginary data to
create a complex field correction

##### Step 2: Apply Correction to Rabi Frequency
Omega=@(t) Omega(t)+dz*correction(t); % Apply Correction

```

Figure 3.4: implementation of the interpolation routine for calculating field corrections as a function handle.

vector is interpolated so that the corrected field can be defined as an anonymous function, only calling ppval once to function the entire field. This eliminates the requirement for calls to interpolations for previous slices or other nested anonymous functions, which will reduce the amount of time calling anonymous functions (which proved to be the bottleneck in the execution of the Maxwell-Bloch simulations.). This method is detailed in the code block in figure 3.4. The old method meant that after N material z slices, every time the field that has been modified by the previous slice is called, there is one anonymous function call to call the function itself, and N more calls to the interpolated correction from each previous slab, meaning that the function call for the Rabi frequency will require $N+2$ calls. Reinterpolating the entire Rabi frequency after corrections after propagating through each slab requires, that every time the Rabi frequency function is called, it requires only 1 anonymous function call. In a 4-slice simulation, a 12 % performance improvement was observed. In a 6-slice simulation, the improvement rose to 20%. The improved method for generating a function handle for the Rabi frequency speeds up the propagation of fields in Maxwell-Bloch

simulations; because of the requirement that the absorption length be small for individual Z slices, the ability to use more Z-slices allows for the extension of the simulator to more optically dense systems.

Output Curvature In Chirped Readout Simulations Maxwell-Bloch simulations of chirped optical fields interacting with an in-homogeneously broadened material were found to sometimes yield output fields with intensities displaying a concave curvature centered at the center of the simulated in-homogeneous line. Figure 3.5 shows an example of how this curvature manifested in a simulation. This curvature is undesirable as it adds background to signals, which reduces the effective dynamic range of the simulation, so it is preferable to control this curvature. A series of simulations were conducted to determine the origin of this curvature. It was apparent that the easiest way to reduce the magnitude of the curvature was to reduce the fractional bandwidth of the chirp used for the simulation compared to the overall material bandwidth used in the simulation.

The following sections will provide a brief justification for the assertion that this curvature is the result of the integration of the atomic coherence over a flat in-homogeneous line-shape.

A brief theoretical assessment of the origin of the curvature present on chirped fields out of an inhomogeneously broadened medium begins with the closed-form solution of the Bloch-Equations from Vega et al. (2018) [75].

$$r_2 = -\frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}} e^{\frac{-t}{T_2}} \sin\left(\sqrt{\Omega^2 + \Delta^2} t\right) \quad (3.16)$$

This solution describes how the atomic coherence of absorbers at detuning Δ will evolve under a constant, real electric field of Rabi frequency Ω (this field is assumed to be at zero detuning). While this solution assumes that the external driving field is at a single stable frequency, this solution can still be used to describe phenomenologically how a chirp might

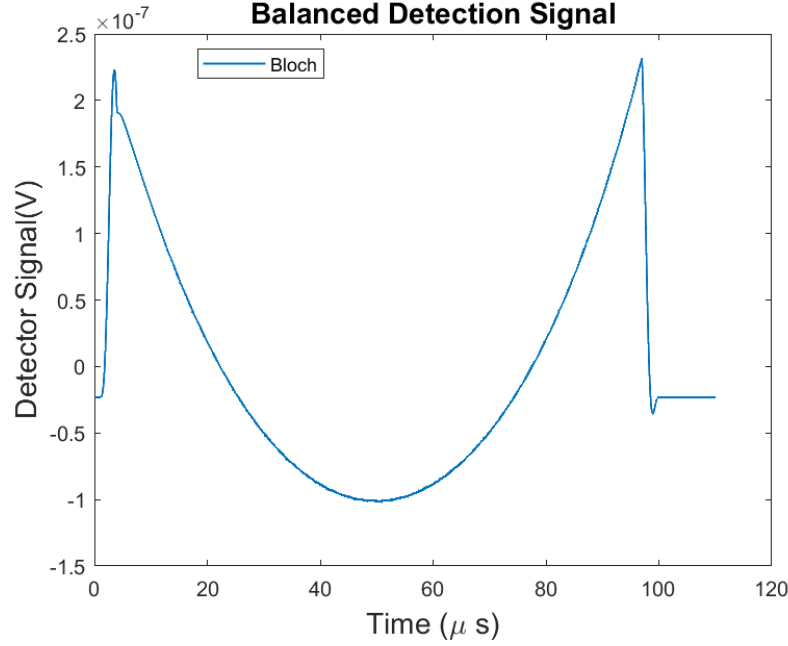


Figure 3.5: Curvature in the simulated balanced detection signal in a traditional readout simulation of a 50MHz wide chirp in a simulated material with 200MHz bandwidth, 2000 frequency bins, and a T_2 of $0.7958 \mu s$. The chirp matches zero-detuning at 50 seconds, so the curvature is centered around that time. These simulation results represent the difference in intensities between a chirped field that has passed through the inhomogeneously broadened material. This curvature is around 30dB down from the maximum signal that could be expected in this simulation.

interact with a material, given that the chirp's instantaneous frequency will vary linearly with time. A model that indicates changes to fields as a function of the drive frequency can become time domain features on a chirped field. To assess changes to the electric field, the atomic coherence (which is related to the material polarization and, therefore, the output field). The field of interest will therefore be:

$$\Omega(z = \delta z, t, \Delta_L) = \Omega(z = 0) - \frac{\alpha \delta z}{2\pi} \int_{\Delta_i}^{\Delta_f} \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}} e^{\frac{-t}{T_2}} \sin\left(\sqrt{\Omega^2 + \Delta^2} t\right) d\Delta \quad (3.17)$$

This model still admits only a Rabi field at zero detuning. To circumvent this, we will always remain in the laser frame in our rotating wave approximation. This can be achieved

by changing the bounds of integration (Δ_i and Δ_f) so that the laser frequency can still be a variable in this analysis. The new bounds in terms of the "true" laser detuning Δ_l are: $\Delta_i = -(\frac{\Delta_T}{2} + \Delta_l)$ and $\Delta_f = -\frac{\Delta_T}{2} - \Delta_l$ (Where Δ_T is the total material bandwidth).

$$\Omega(z = \delta z, t, \Delta_l) = \Omega(z = 0) - \frac{\alpha \delta z}{2\pi} \int_{-(\frac{\Delta_T}{2} + \Delta_l)}^{\frac{\Delta_T}{2} - \Delta_l} \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}} e^{\frac{-t}{T_2}} \sin\left(\sqrt{\Omega^2 + \Delta^2} t\right) d\Delta \quad (3.18)$$

Since the detuning only appears in the equation as $\sqrt{\Omega^2 + \Delta^2}$, the sine and the factor of $1/\sqrt{\Omega^2 + \Delta^2}$ combine to ensure that any expansion terms will all be even in Δ , which means that when those terms are integrated over the frequency band.

$$\int_{-(\frac{\Delta_T}{2} + \Delta_l)}^{\frac{\Delta_T}{2} - \Delta_l} \Delta^{2n} d\Delta = \frac{1}{2n} \left(\left(\frac{\Delta_T}{2} - \Delta_l \right)^{2n+1} - \left(- \left(\frac{\Delta_T}{2} + \Delta_l \right) \right)^{2n+1} \right) \quad (3.19)$$

These polynomials can be evaluated in terms of the binomial theorem:

$$\int_{-(\frac{\Delta_T}{2} + \Delta_l)}^{\frac{\Delta_T}{2} - \Delta_l} \Delta^{2n} d\Delta = \frac{1}{2n} \sum_{j=0}^{2n+1} \binom{2n+1}{j} \left(\frac{\Delta_T}{2} \right)^{2n-r+1} \Delta_l^j [(-1)^j - (-1)^{2n+1}] \quad (3.20)$$

or

$$\int_{-(\frac{\Delta_T}{2} + \Delta_l)}^{\frac{\Delta_T}{2} - \Delta_l} \Delta^{2n} d\Delta = \frac{1}{2n} \sum_{j=0}^{2n+1} \binom{2n+1}{j} \left(\frac{\Delta_T}{2} \right)^{2n-r+1} \Delta_l^j [1 - (-1)^{2n-j}] \quad (3.21)$$

This means that terms are only retained if $2n - j$ is even. Since $2n$ is always even, this means that this condition is only met when j is even, so the only Δ_l dependent terms remaining in the integral will always be an even power of Δ_l . This means that if we see any detuning dependence of the output fields, it will be symmetric around zero detuning which seems to conform well to the results obtained in Maxwell-Bloch simulations as seen in figure 3.5.

To evaluate the validity of the phenomenological model, the closed-form solution to the

Bloch equations can be integrated numerically over detuning and scaled according to the chirp rate to overlay onto the time domain results from the Maxwell-Bloch simulator. This will begin with field corrections obtained for any given laser frequency given in equation 3.18. Since the goal here is to determine how these effects manifest in a chirped context. Therefore the field will be calculated as a time average over the dwell time of the chirp, which is given by the chirp rate as $\sqrt{\kappa}$ so the average field, now only a function of Δ_l , is defined as

$$|\Omega_\mu(z = \delta z, \Delta_l)|^2 = \sqrt{\frac{1}{\kappa}} \int_0^{\sqrt{\kappa}} |\Omega(z = \delta z, t, \Delta_l)|^2 dt \quad (3.22)$$

To compare this result to the results for a chirped Maxwell-Bloch simulation, the instantaneous chirp frequency, and therefore time, needs to be connected to the chirp rate so: $\Delta_l = 2\pi(\kappa t + f_0)$ where f_0 is the start frequency of the chirp.

Using the presented phenomenological model, the time domain curvature due to a chirp can be estimated using the model from the preceding section. Figure 3.6, shows a comparison of simulation results to results calculated using the phenomenological model devised to describe this curvature. While the curves do not line up exactly, the results are alike in shape and within an order of magnitude. It is important to note that the phenomenological model presented differs from the full Maxwell-Bloch treatment in several ways. Hence, it is unsurprising that these models don't align.

When simulating the interaction of a chirped optical field interacting with an inhomogeneously broadened medium, a slight curvature can appear over the course of the chirp if the inhomogeneous broadening is simulated as a uniform distribution of absorbers over the in-homogeneous line-width. This curvature is largely due to changes in where the instantaneous field lies with respect to the frequency bounds of the homogeneous line. This curvature is a consequence of the Maxwell-Bloch equations themselves despite not necessarily

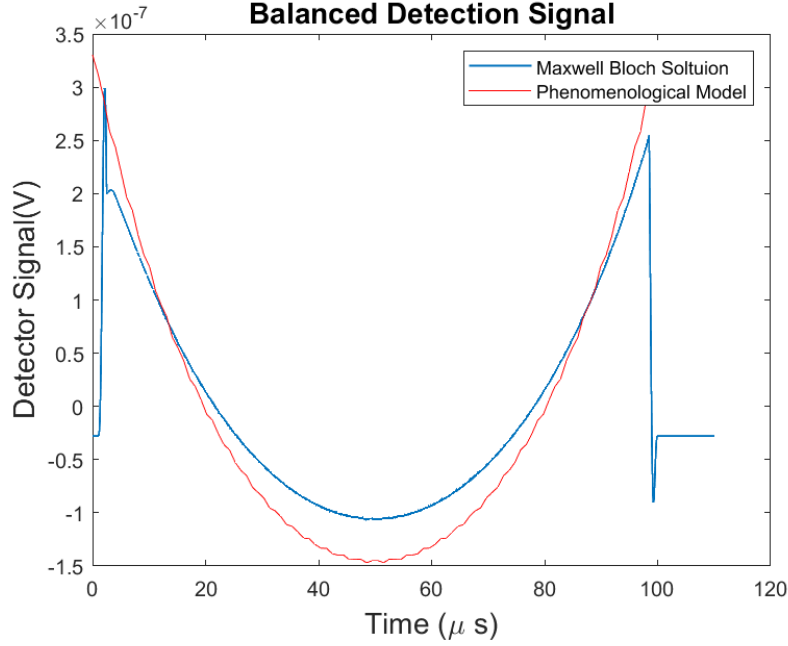


Figure 3.6: Curvature in the simulated balanced detection signal in a traditional readout simulation of a 50MHz wide chirp in a simulated material with 200MHz bandwidth, 2000 frequency bins, and a T_2 of $0.7958 \mu s$, The chirp matches zero-detuning at 50 seconds, so the curvature is centered around that time. These simulation results represent the difference in intensities between a chirped field that has passed through the in-homogeneously broadened material. The other curve (red) represents the estimated expected shape of the curvature around zero detuning expected according to the phenomenological model.

corresponding to any physical phenomenon. Practically this model can be used to put constraints on chirp bandwidth in comparison with the material bandwidth for Maxwell-Bloch simulations. Using the phenomenological model the full scale of the curvature (in Rabi frequency squared units) for a chirped Maxwell Bloch simulation can be estimated for a single slab using equation 3.23. Assuming the chirp bandwidth is smaller than the material bandwidth.

$$\Delta I = 4\alpha\delta z \left(\frac{\Omega\kappa\tau_{chirp}}{\Delta_T} \right) \quad (3.23)$$

The chirp bandwidth over the course of the simulation should be substantially less than the

bandwidth of the material. Functionally this means that most Maxwell-Bloch simulations will need to simulate more material bandwidth than chirp bandwidth.

Maxwell Bloch Simulation Results

2 Pulse Photon Echoes In SSH materials, a coherent interaction can occur when populations are brought into a superposition state by a $\frac{\pi}{2}$ pulse. The coherences are allowed to precess for a time τ . A π pulse then reverses the coherences. At a time τ after the π pulse, the material coherences will synchronize, and a pulse will be emitted [1, 22]. This process is called a photon echo (or often a two-pulse photon echo to distinguish it from the 3-pulse stimulated photon echo). Photon echoes are a useful diagnostic for Maxwell-Bloch Simulations.

Square Pulses The first pulse sequence used two hard-edged pulses (square pulses), generated using the function in appendix C, of an average Rabi-Frequency π MHz. The first($\frac{\pi}{2}$) pulse had a duration of $0.5\mu s$ after a duration of $2\mu s$, there was a second (π) pulse with a duration of $1\mu s$. The simulation was allowed to run for $10\mu s$ at a sample rate of 400 MSPS. The material bandwidth was 200 MHz with a spectral resolution of 0.05MHz, and a T_2 of $10\mu s$. The material with unit absorption length was subdivided into 4 material steps. The resulting time domain intensities (in this case, taking units of the magnitude squared of the Rabi frequency) in this simulation are shown in figure 3.7. As expected, a photon echo occurs at $2\mu s$ after the π pulse, matching the time delay between the leading edges of two input pulses. However, right on the leading edge of pulses, no material interaction occurs. Using a method better suited to handling instantaneous shocks (e.g., the MUSCL scheme [39, 74]) may better handle startup behavior. This issue can also be thought of as a result of finite material bandwidth, so the material cannot respond to the input pulse instantaneously. Within this simulation method, extra care should be taken to ensure that input pulse edges are rounded. This can be done in a variety of different ways. Raised cosines edges, gaussian

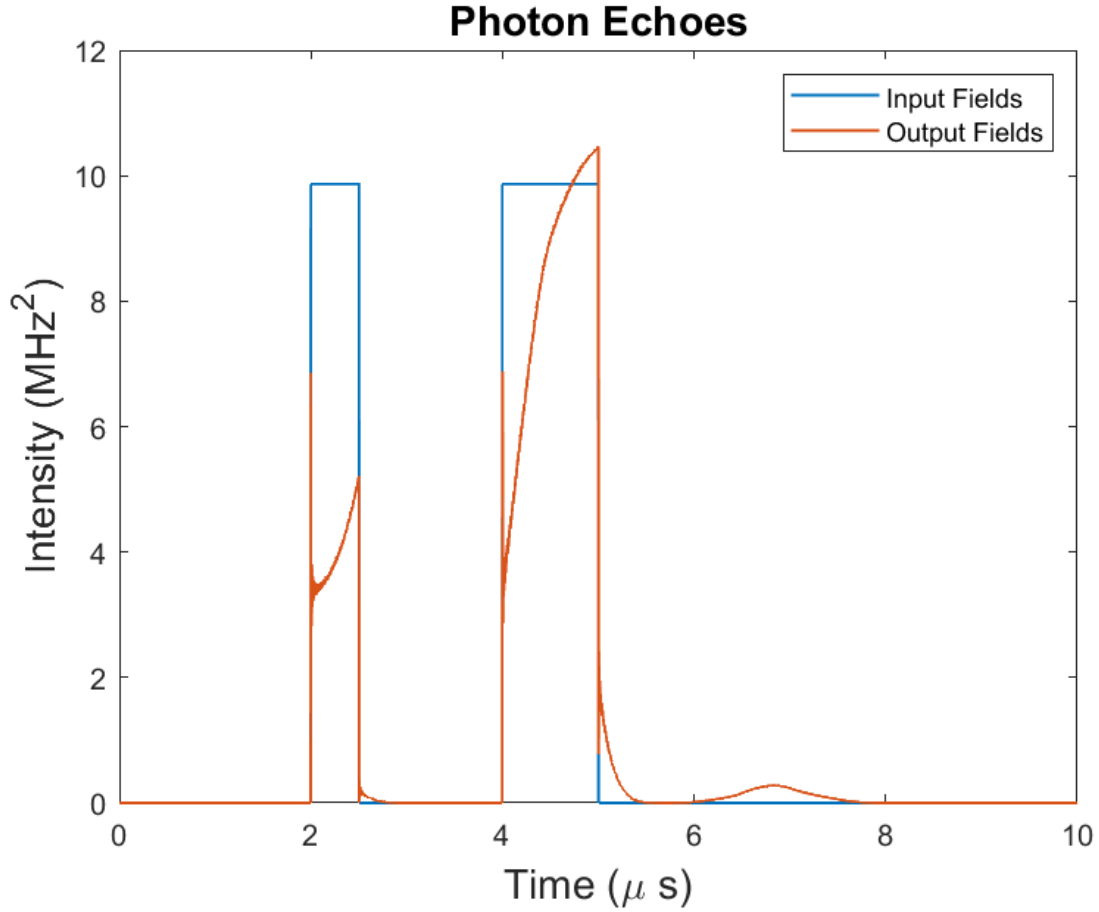


Figure 3.7: Output Intensity from a two-pulse echo simulation using sharp-edged input pulses.

pulses, and filtering are all viable routes to ensuring that the leading edges of pulses are well-behaved.

Filtered Pulses One choice of a filtering mechanism for this pulse pattern is to use a Gaussian in the frequency domain. Specifically, a Gaussian filter was applied to the spectrum of the read pulses. This yielded a filtered field with much smoother edges. The use of filtered pulses should improve the field behavior near the leading edges of the pulses after passing through the Maxwell-Bloch simulation. The smoother behavior near the edges of the pulse should help improve the ringing caused in the preceding section. The simulation results were

much closer to being in line with what is seen in experiments. These results are shown in figure 3.8.

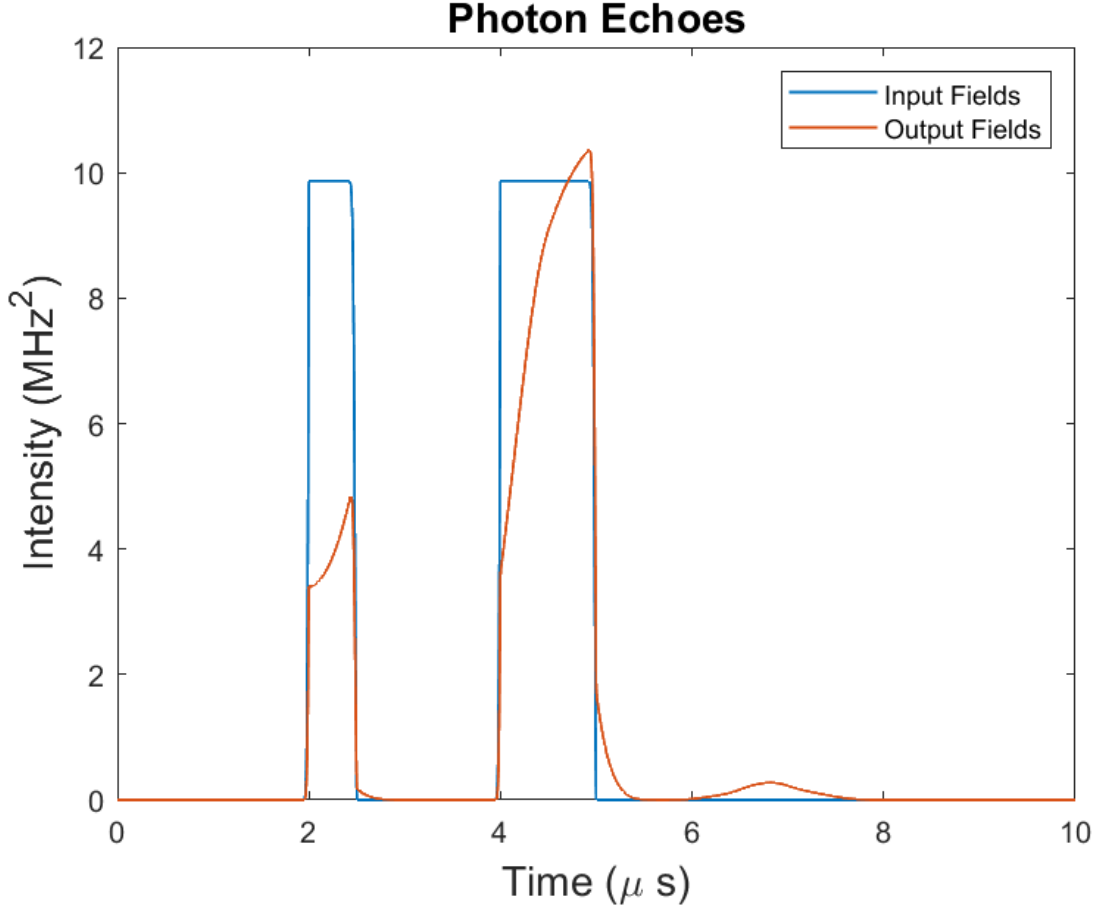


Figure 3.8: Output intensity for 2 pulse photon echoes using filtered pulses.

Filtering the pulses did help to suppress the sharp behavior near the leading edge of the pulse. The first pulse was strongly absorbed, and the echo occurred $2 \mu\text{s}$ after the second pulse. The code used to execute these sequences is given by appendix C.

Gaussian Pulses Another method is to use a smoother pulse shape. Gaussian pulses, for example, are a convenient pulse shape. Like square pulses, gaussian pulses extend infinitely in the frequency domain, however, the frequency roll-off of a Gaussian is much faster than the side lobe roll-off of the sinc squared. This means that the Gaussian is better spectrally

limited than the square pulse. In time domain, the Gaussian has an infinite temporal extent theoretically, so at $t=0$, there will be a small discontinuity as with square pulses. However, the Gaussian rolls off quickly, so the discontinuity at $t=0$ will be weak. More importantly, Gaussians do not exhibit the same extreme rise time given by square pulses. Gaussian pulses take a form given by equation 3.24

$$\Omega(t) = \Omega_0 \exp \left(-\frac{1}{2} \left(\frac{t - \tau}{\sigma} \right)^2 \right) \quad (3.24)$$

Where τ is the central time for the pulse, Ω_0 is the peak Rabi frequency, and σ is a measure of the duration. For such a Gaussian pulse, the pulse area is given by 3.25

$$\Theta = \int_{-\infty}^{\infty} \Omega(t) dt = \Omega_0 \sigma \sqrt{2\pi} \quad (3.25)$$

For a given peak Rabi frequency and desired pulse area, the required pulse width is given by equation 3.26.

$$\sigma = \frac{\Theta}{\Omega_0 \sqrt{2\pi}} \quad (3.26)$$

Using Gaussian pulses generated using the function in appendix C at $2\mu s$ and $4\mu s$, with a peak Rabi frequency of π MHz, a Maxwell Bloch Simulation was conducted with the same temporal, spatial, and spectral resolutions as in the previous 2 pulse echo simulations. The results of this simulation are given in figure 3.9. The output intensity is well-behaved when Gaussian Gaussian pulses are used. Again the echo occurs $2\mu s$ after the second pulse. Here we have the opportunity to analyze the absorption. The first pulse should have an intensity of $I_{out} = I_{in} \exp(-\alpha L)$ for $\alpha L = 1$ this means that $I_{out} = I_{in}/e$. The input Rabi frequency of πMHz yields a peak intensity of $\pi^2 MHz^2$ one should expect a peak output intensity of around $3.6 MHz^2$. This simulation shows a peak output intensity of around $3.9 MHz^2$. Since the peak of the Gaussian pulse occurs halfway through the duration of the pulse the

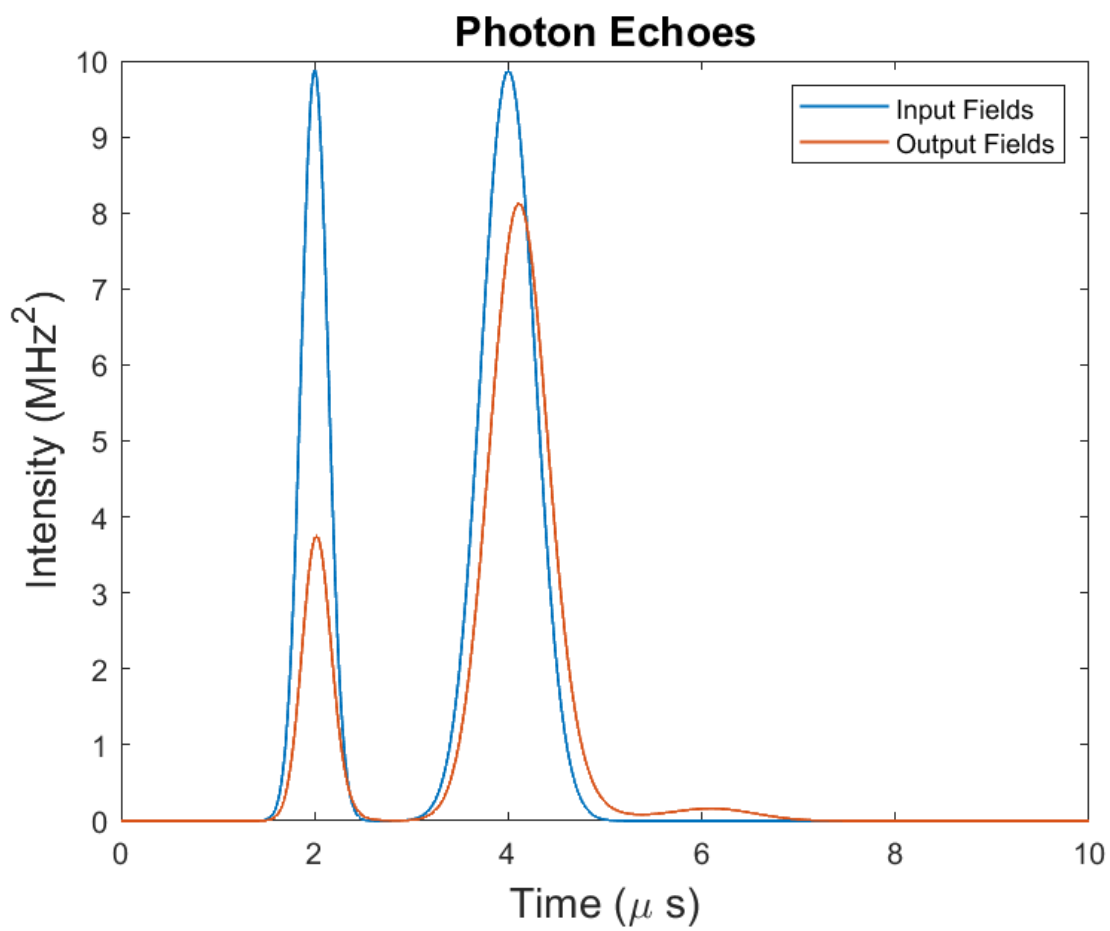


Figure 3.9: Output intensity for 2 pulse photon echoes using Gaussian pulses.

material has already been partially depopulated at the time of the peak intensity of the first input pulse, so the transmitted peak intensity of the first pulse will be slightly higher than the expected transmitted intensity for an unprogrammed material of $I_{in} \exp(-\alpha L)$.

Optical Nutation Optical nutation occurs when a field is suddenly applied to the SSH material [30]. When the field is applied, the ions in the medium undergo generalized Rabi oscillations. The fields produced by coherence induced in the medium lead to a Bessel function pattern in the output intensity [3]. Optical nutation is useful in a laboratory context since it can be used to measure absorption length or Rabi frequency. Using the Maxwell-Bloch simulator, optical nutation was simulated for a Rabi frequency of πMHz and an absorption length of $\alpha L = 1$. Switching on a field instantaneously has a sharp leading edge which could lead to strange sharp features, as seen in section 3. To circumvent this problem, a Gaussian frequency domain filter was used to smooth out the field's leading edge, making it easier for the ODE solver to handle the start-up behavior. The results of the simulation are shown in figure 3.10.

Figure 3.10 shows an optical nutation signal. Shortly after the field is switched on at $t=0$, the optical field is structured like a Bessel function, as expected. This figure features two horizontal lines marking Y_1 the intensity when the material begins to absorb the field (notice the little knee-like feature just after $t=0$) before the ions in the material are depopulated, and the absorption length drops. Y_{sat} marks the saturation intensity or the intensity that the field settles to at later times, after the material is fully saturated, which corresponds with the intensity absent any material absorption. By inverting the relationship between input and output intensity, equation 3.27, an expression for absorption length in terms of Y_1 and Y_{sat} can be derived using 3.27 [75]

$$\alpha L = -\ln\left(\frac{Y_1}{Y_{sat}}\right) \quad (3.27)$$

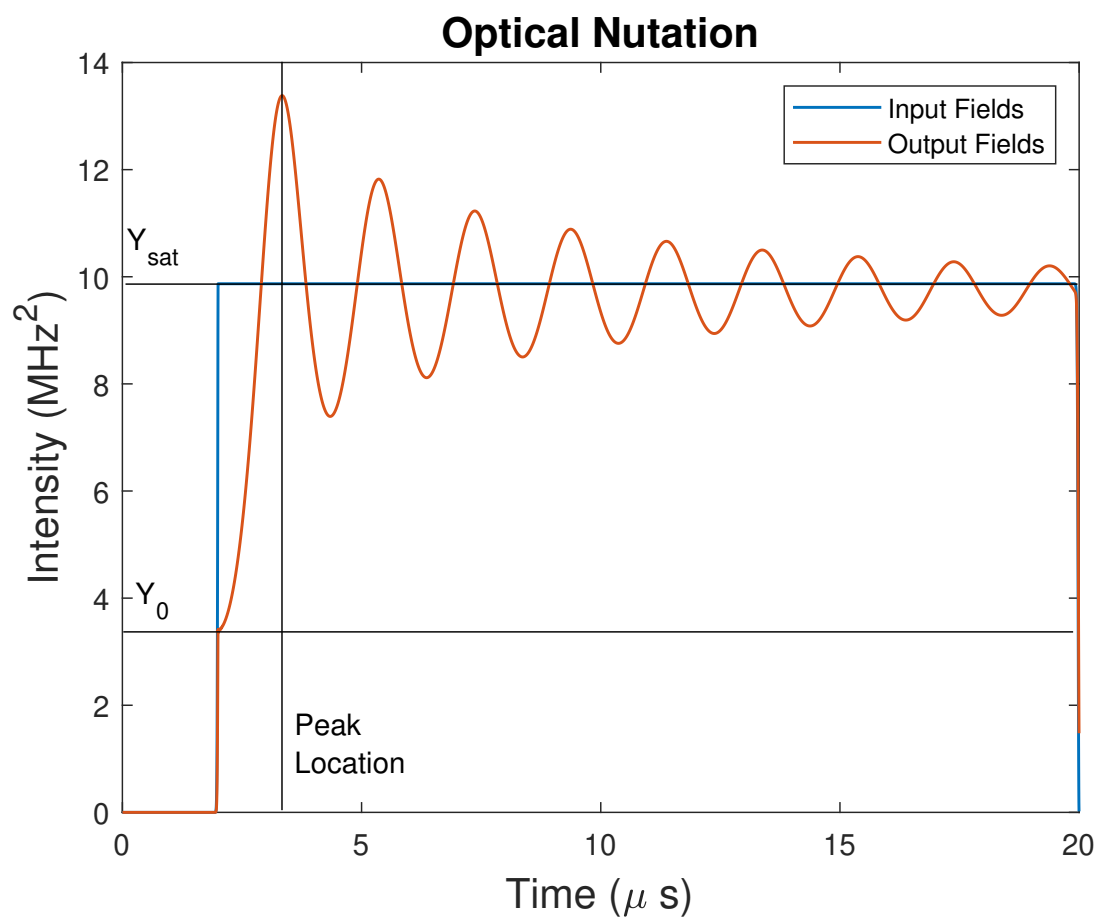


Figure 3.10: An optical nutation signal with horizontal lines indicated the important features needed to make measurements of αL .

Using equation 3.27 and the values from the simulation of $Y_1=3.8 \text{ MHz}^2$ and $Y_{sat}=10.1\text{MHz}^2$, the absorption length was calculated to be $\alpha L=0.977$ putting the results in the correct range for the absorption length. The simulation assumes a plane wave excitation of the material, the Bessel functional dependence for a plane wave excitation in an SSH material is $-J_0(\Omega t)$ [75] using this model, the expected maximum will occur at $\Omega t = 3.87$ for a plane wave. The maximum value occurs at $1.4\mu s$, which taking the results at face value, indicates a Rabi frequency of 2.7 MHz . This value differs from the used $\pi \text{ MHz}$, however, it has been shown that some degree of distortion to the maxima will occur due to longer absorption lengths [75]. More easily predictable results can be done by conducting this test at a lower absorption length ($\alpha_0 L = 0.1$). The results for this simulation are given in figure 3.11. At the lower absorption length, the estimate of the Rabi frequency from the time of maximum transmission improves substantially. The estimated absorption width of 0.098 corresponds well with the fixed absorption length of 0.1 .

Chirped Readout of a Spectrum By tweaking the initial conditions of the Bloch Vector (i.e., in the r_3 component as a function of frequency), spectra can be pre-programmed into a material so that the readout can be analyzed. This proved useful for determining the origin of nonlinear noise in readout experiments in chapter 8. As a demonstration of the Maxwell Bloch simulator, Lorentzian spectral holes, or sinusoidal gratings, were pre-programmed into the material and then read out by a chirp. In chirped readout experiments, it was found that the resolutions needed to be considered carefully, specifically the coherence decay needed to be short enough that given the homogeneous line could be well sampled by the frequency bins or $\delta\Delta \ll \Gamma_H$. Otherwise, coherent artifacts would be observed. Additionally, it was found that the material bandwidth needed to be substantially higher than the chirp bandwidth. Readout simulations were conducted lasting $40\mu s$ at an 800MSPS sample rate. The material bandwidth was 400MHz with a material spectral resolution of 0.025 MHz and a coherence

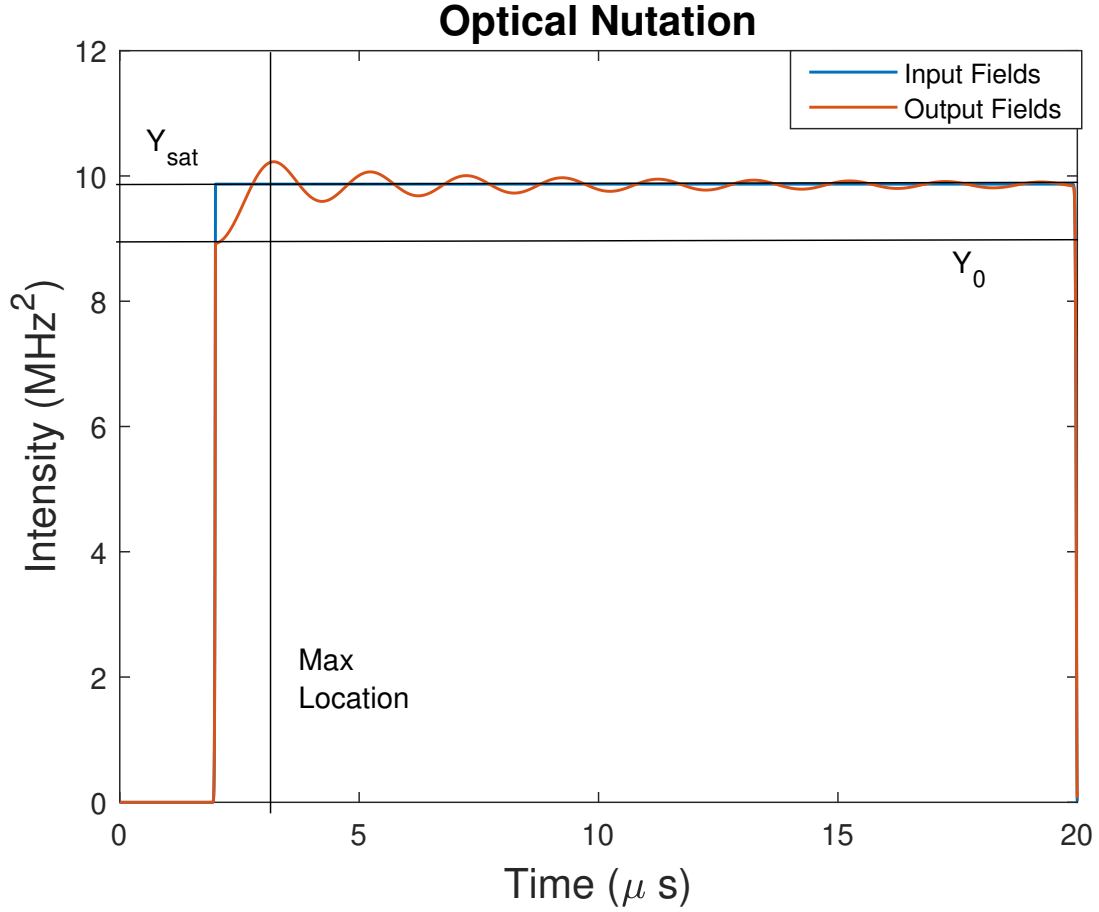


Figure 3.11: Maxwell-Bloch simulation for optical nutation in a thin medium ($\alpha_0 L = 0.1$) In This case, the maximum occurs at $1.22 \mu s$ for a plane wave. This corresponds to a Rabi frequency of 3.17 MHz, which corresponds very favorably to the used Rabi frequency of π MHz

time of $3.183 \mu s$. A hole with a relative depth of 0.5 (i.e., $r_3 = -0.5$) and width 0.1MHz was pre-programmed into the material and was then read out by a $5 \frac{MHz}{\mu s}$ chirp of Rabi frequency 0.25MHz was used to read out the field. The material had an absorption length of 1.39. The results are shown in figure 3.12. As expected, traditional readout gave a signal taking the form of a lorentzian hole after spectral recovery, despite the significant ring-down. The hole appeared in the correct location and was recovered to a height of roughly $0.1 MHz^2$. Similar simulations have been conducted previously. For example, [16] demonstrated the recovery of

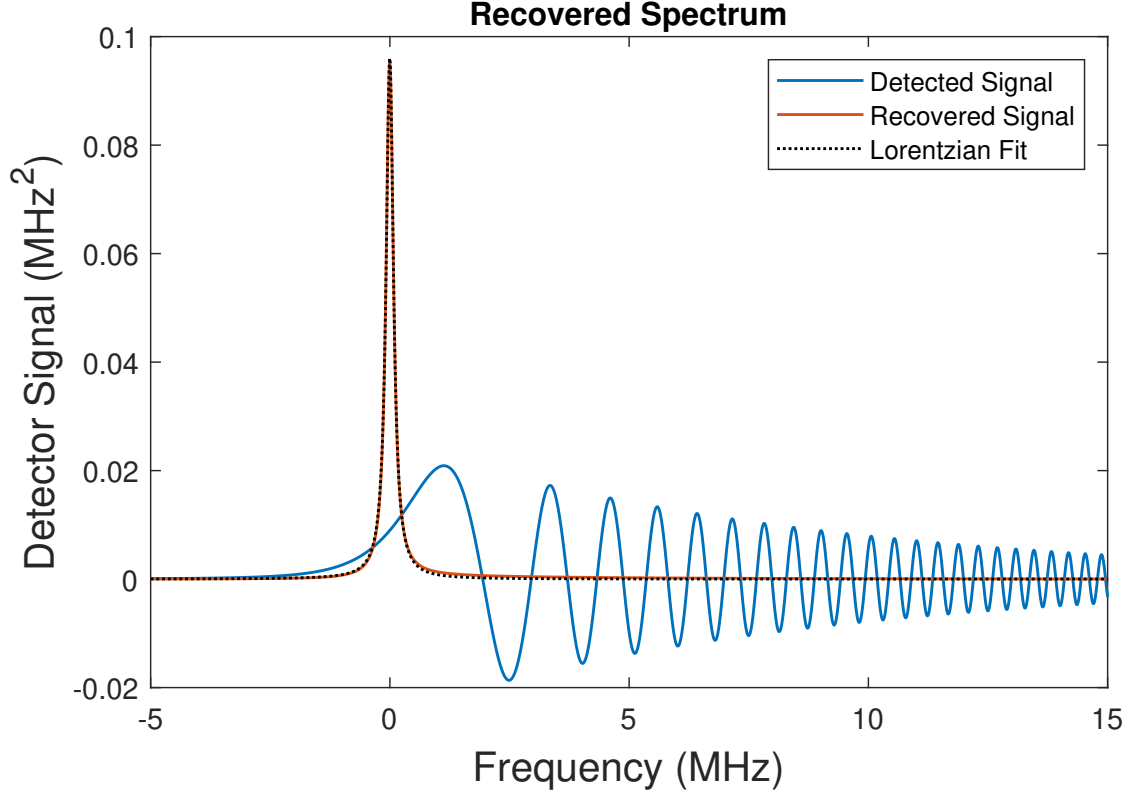


Figure 3.12: Simulated traditional readout traces. The traditional read signal is approximately Lorentzian after spectral recovery.

a 0.1MHz wide hole with a chirp of $1\frac{\text{MHz}}{\mu\text{s}}$, in that case, the ratio between chirp resolution and hole width ($\frac{\sqrt{\kappa}}{\Gamma_H}$) was 10, in this demonstration the ratio is about 22. Readout simulations are used in chapter 8 to demonstrate the power dependence of chirp dependant nonlinear noise when there is phase noise on the laser itself.

INTERFEROMETRIC SEPARATION OF STIMULATED PHOTON ECHOES

Introduction

Previous Work

Stimulated photon echoes (SPEs) are a phenomenon that occurs in spatial, spectral holographic systems [50]. Three broadband optical fields are required for the generation of stimulated photon echoes. First, a pair of broadband optical fields encode a time delay into the material by generating a spectral grating in the material's absorption spectrum. A third broadband field is then temporally diffracted off the spectral grating, leading to a new coherent signal that is delayed by the time delay encoded into the grating. This new coherent field is called a stimulated photon echo. This process and its close relatives have been explored for a variety of applications, from classical optical buffer memories [51] to quantum memories [42], to true-time-delay (TTD) for phased array radars [57], to optical correlation [7, 11].

Motivation

If stimulated photon echoes are generated in independent spots in a spatial, spectral holographic material by fields whose relative phases have been altered in each spot by mixing them through either one or a series of beam splitters. In that case, the stimulated echoes in each spot can be generated with a set of phases that allow them to interfere with each other after being combined with an output beam splitter (or a series of beam splitters). The echoes will constructively interfere at one output ports of the beam splitter(s) and destructively interfere on the others. The phases of the other fields can be contrived so that they interfere constructively on different ports than the echo. In other words, stimulated photon echoes can be isolated from the other fields needed to generate them interferometrically. Other

isolation schemes exist, but they require intersecting beams at different angles, making them unsuitable for implementation in 1D waveguides for use in photonic integrated circuits. The interferometric method can be conducted in all co-linear contexts allowing for the implementation of SPE-like processes in photonic integrated circuits.

Stimulated Photon Echoes

As a refresher to the reader, a summary of the background information on simulated photon echoes first shown in chapter 1 is presented here in a condensed form. A pair of broadband electric fields with a time delay of τ_{21} between them ($E_1(t)$ and $E_2(t + \tau_{21})$) have a joint energy spectral density $H(f)$ as shown in equation 4.1.

$$H(f) \propto |\tilde{E}_1(f) + \tilde{E}_2(f)e^{-2\pi if\tau_{21}}|^2 = |\tilde{E}_1(f)|^2 + |\tilde{E}_2(f)|^2 + \tilde{E}_1^*(f)\tilde{E}_2(f)e^{-2\pi if\tau_{21}} + c.c. \quad (4.1)$$

If the delay between fields is much less than the coherence time and the programming is sufficiently weak, then spectra programmed into an SSH material by a laser beam can be described according to equation 4.2

$$\alpha(f) = \alpha_0 (1 - CH(f)) \quad (4.2)$$

where C is a programming coefficient that is the ratio between the depth of burned spectra to the energy spectral density of the programming field. The magnitude of the field transmission coefficient, in the weak programming limit, is governed by the absorption coefficient and will take a form given by equation 4.3.

$$|\nu(f)| = e^{\frac{1}{2}\alpha_0 L(1-CH(f))} \approx e^{\frac{1}{2}\alpha_0 L} \left(1 + \frac{1}{2}\alpha_0 LCH(f) \right) \quad (4.3)$$

The phases of the transmission coefficient are constrained to ensure that the response

is causal. So when a third field $E_3(t + \tau_{21} + \tau_{32})$, which is delayed by τ_{32} after field E_2 , is transmitted through the grating, only the positive time-delayed copy will exist (since the other signal would occur before the time the third pulse hits the material). This diffracted portion is called the stimulated echo [50]. In frequency domain, the echo field can be expressed according to equation 4.4[11, 52]

$$\tilde{E}_{echo}(f) \propto \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{-2\pi i(2\tau_{21} + \tau_{32})f} \quad (4.4)$$

The frequency domain echo field is proportional to a product of the three constituent fields. And the echo signal is delayed to τ_{21} after the third field. The phase of the stimulated photon echo is governed by phases of the constituent fields.

Echo Isolation In a simple stimulated photon echo experiment using brief pulses, programming fields can be time gated. However, in correlation experiments, there may be an extended third field that takes the form of a spread spectrum signal. In these cases, there may be overlap between the probe field (E_3) and the stimulated echo (E_{echo}) because the echo field will be weaker than E_3 , the echo will be obscured by E_3 . Figure 4.1 depicts two cases of SPE experiments, one where the echo can be time-gated and the other where the echo cannot be time gated from the probe field. Angular schemes are traditionally employed in applications where the echo needs to be spatially isolated [6, 70]. The simplest of these schemes works by injecting fields into the SSH material at two different angles, as depicted in figure 2.3 in chapter 2. In a photon echo experiment, the wave vectors for each field ($\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$) will add together to give the echo wave vector, which is given by equation 4.5.

$$\vec{k}_{echo} = \vec{k}_3 + \vec{k}_2 - \vec{k}_1 \quad (4.5)$$

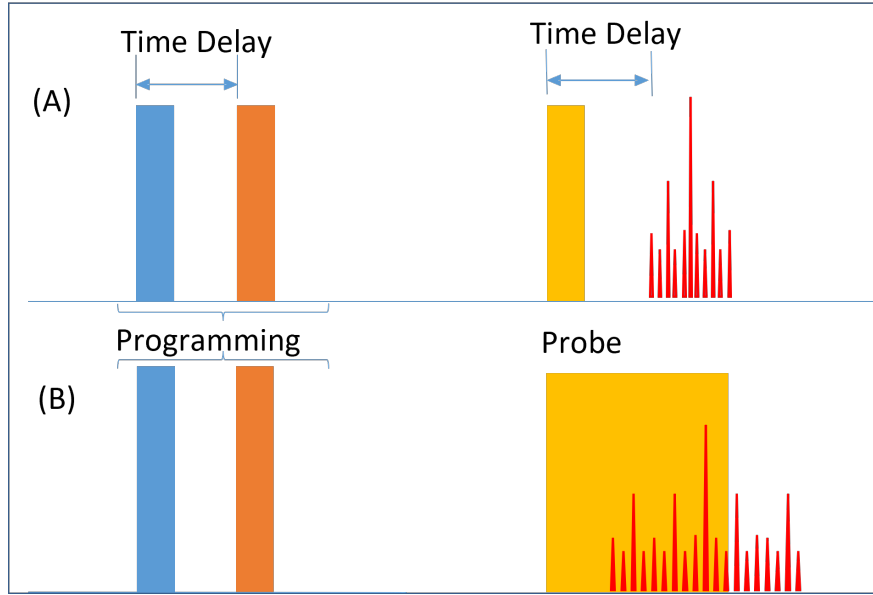


Figure 4.1: A: If the probe field is short enough, the echo can be time gated from the echo field. B: Longer probe fields may overlap echoes, in which case time gating is insufficient to isolate the echo field.

In this isolation scheme, E_1 and E_3 are injected into the material at the same angle, meaning that $\vec{k}_1 = \vec{k}_3$, while E_2 is injected at a different angle and has a wave vector $\vec{k}_2$. According to equation 4.5, the contributions to the echo wave vector from E_1 and E_3 will cancel out, leaving the echo on the same wave vector as E_2 ; therefore, the echo can be spatially isolated from E_3 . This method can be used for situations where E_1 and E_2 can be time-gated. A slightly more complicated method called the “box geometry” [11], shown in figure 1.5 in chapter 2, in which the three input fields and the echo each are directed out of the material at unique angles, making this method suitable for continuous program, continuous read optical correlation.

The following chapter will describe two new schemes to isolate stimulated photon echoes. These interferometric methods rely on the interference between stimulated echoes generated co-linearly in separate beams through the material. Two interferometric schemes are discussed in the chapter. The first is a 2-port scheme using a Mach-Zehnder interferometer

to isolate echoes analogously to the 2-angle method of stimulated photon echo isolation. The second method uses a more complicated configuration, with four input ports and four output ports to isolate the echo in an interferometric configuration that schematically resembles the “box geometry.”

Two Port Interferometric Stimulated Photon Echoes

Theoretical Consideration of Two-Port Interferometric Stimulated Photon Echo Contrast and Efficiency

Consider a realistic Mach-Zehnder interferometer with an input beam splitter with field transmission and reflection coefficients t_1 and r_1 (assuming no loss- $r_1 = \sqrt{1 - t_1^2}$). Fields from both outputs of the interferometer then pass through the SSH material, where they will be used to generate echoes. A phase control governs the relative phases between the fields in the interferometer’s two arms. The fields are then coupled into an output beam splitter with reflection and transmission coefficients r_2 and t_2 (assuming no loss $r_2 = \sqrt{1 - t_2^2}$.) The efficiencies of the fields coupling into two inputs of the output splitter are C_1 and C_2 . By injecting fields E_1 and E_3 into input port IP1 and E_2 into input port IP2, echoes are generated on the two arms of the interferometer; these echoes will interfere at the outputs in a manner governed by the split ratio, coupling efficiencies, and bias phase. Such an interferometer is shown schematically in figure 4.2. Under the condition that the bias phase is 0 and the splitters are ideal, and the coupling efficiencies are properly tuned, the echo will be directed out of port OP1 with E_2 , while the probe field E_3 (along with E_1) is directed out of port OP2, while the echo (along with E_2) is directed into OP1. However, slight variations in the split ratios and coupling efficiencies will cause some of the probe field to leak onto port OP1 and some of the echo to leak onto port OP2. In order to observe the echo at the output without it being obscured by leakage from E_3 , it is important to ensure the leakage

of E_3 onto OP2 is as small as possible. In addition, the echo should be maximized on OP1, so any leakage of the echo on OP2 is also detrimental. The following section will describe a method for optimizing the interferometer for echo isolation using coupling efficiencies. The

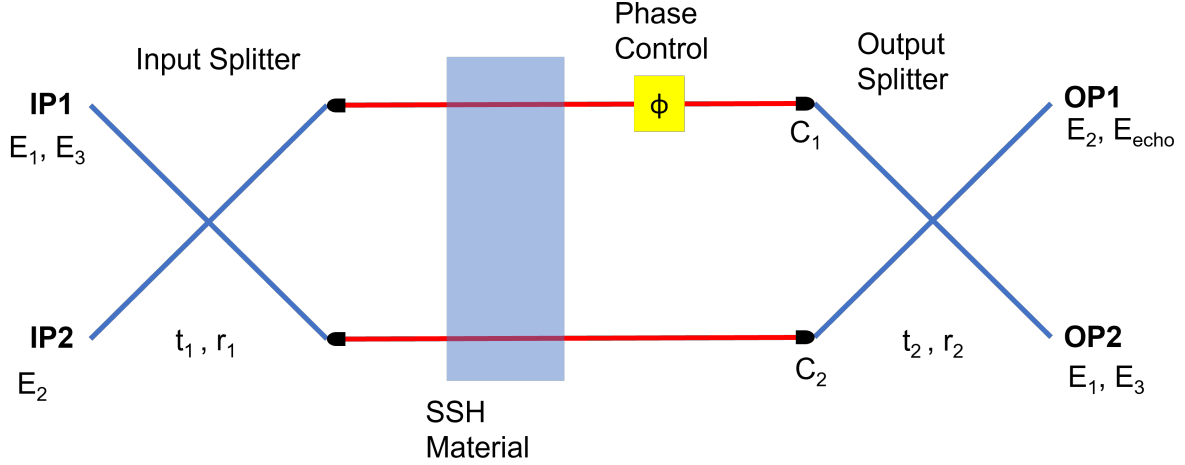


Figure 4.2: A Mach-Zehnder Interferometer with arbitrary losses and split ratios configured for stimulated photon echoes. Using a 50-50 splitter and a phase difference of 0 (i.e., null biased), and the coupling efficiencies are the same, the echo field E_{echo} can be isolated on OP1 from the probe field E_3 . Slight deviations to the split ratio will cause some leakage of fields into the wrong ports; this leakage can be managed by tuning the coupling efficiencies

interferometer can be broken up into a collection of input optics and a separate collection of output optics that can be expressed as matrix products. The input optics are just the beam splitter and is given by equation 4.6.

$$\hat{Q}_{in} = \begin{pmatrix} t_1 & ir_1 \\ ir_1 & t_1 \end{pmatrix} \quad (4.6)$$

The output optics encompasses the phase control, the coupling efficiency into the output splitter, and the output splitter itself. This operator is expressed as a product in equation

4.7.

$$\hat{Q}_{out} = \begin{pmatrix} t_2 & ir_2 \\ ir_2 & t_2 \end{pmatrix} \begin{pmatrix} \sqrt{C_1} & 0 \\ 0 & \sqrt{C_2} \end{pmatrix} \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{C_1}t_2e^{i\phi} & i\sqrt{C_2}r_2 \\ i\sqrt{C_1}r_2e^{i\phi} & t_2\sqrt{C_2} \end{pmatrix} \quad (4.7)$$

Using these two operators, and a model of the material interaction, the three constituent pulses and the stimulated echo can be accounted for using the interferometric Jones vector formalism.

Fields E_1 , E_2 and E_3 The constituent pulses can be accounted for by using only the linear optical model of the Mach-Zehnder and the transmission coefficient for the material ($\nu = e^{-\frac{1}{2}\alpha_0 L}$). The fields at the output can be described as a total operator for the interferometer as given in equation 4.8.

$$\hat{Q}_{total} = \hat{Q}_{out}e^{-\frac{1}{2}\alpha_0 L}\hat{Q}_{in} = e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} \sqrt{C_1}t_1t_2e^{i\phi} - \sqrt{C_2}r_1r_2 & i(\sqrt{C_1}r_1t_2e^{i\phi} + \sqrt{C_2}t_1r_2) \\ i(\sqrt{C_1}r_2t_1e^{i\phi} + \sqrt{C_2}t_2r_1) & -(\sqrt{C_1}r_1r_2e^{i\phi} - \sqrt{C_2}t_2t_1) \end{pmatrix} \quad (4.8)$$

When $\phi = 0$, this reduces to equation 4.9.

$$\hat{Q}_{total} = e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} \sqrt{C_1}t_1t_2 - \sqrt{C_2}r_1r_2 & i(\sqrt{C_1}r_1t_2 + \sqrt{C_2}t_1r_2) \\ i(\sqrt{C_1}r_2t_1 + \sqrt{C_2}t_2r_1) & -(\sqrt{C_1}r_1r_2 - \sqrt{C_2}t_2t_1) \end{pmatrix} \quad (4.9)$$

For an ideal interferometer (i.e., $C_1 = C_2 = 1$ and $r_1 = t_1 = r_2 = t_2 = \frac{1}{\sqrt{2}}$), the operator reduces to equation 4.10.

$$\hat{Q}_{total} = e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad (4.10)$$

Which is a phase-shifted and attenuated version of the 2x2 swap operator, meaning the input ports map to the opposite port at the output ports (i.e., IP1-OP2 and IP2-OP1).

However, considering possible imperfections, the Jones vector representation of each field. The representations of the output field Jones vectors of each of the three constituent fields are given by equation 4.11.

$$\begin{aligned} \vec{E}''_1 &= E_1 e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} \sqrt{C_1} t_1 t_2 - \sqrt{C_2} r_1 r_2 \\ i(\sqrt{C_1} r_2 t_1 + \sqrt{C_2} t_2 r_1) \end{pmatrix} & \vec{E}''_2 &= E_2 e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} i(\sqrt{C_1} r_1 t_2 + \sqrt{C_2} t_1 r_2) \\ -(\sqrt{C_1} r_1 r_2 - \sqrt{C_2} t_2 t_1) \end{pmatrix} \\ \vec{E}''_3 &= E_3 e^{-\frac{1}{2}\alpha_0 L} \begin{pmatrix} \sqrt{C_1} t_1 t_2 - \sqrt{C_2} r_1 r_2 \\ i(\sqrt{C_1} r_2 t_1 + \sqrt{C_2} t_2 r_1) \end{pmatrix} \end{aligned} \quad (4.11)$$

Assuming the splitter is nominally a 50-50 splitter, the subtractive terms will destructively interfere while the additive terms will constructively interfere; however, any imperfections in the splitters will result in the leakage of fields into the other output ports. Of most concern is the leakage from E_3 onto port OP1 since E_3 cannot necessarily be time gated from the echo; by tuning the coupling efficiencies so that $\sqrt{C_1} t_1 t_2 = \sqrt{C_2} r_1 r_2$, the leakage of E_3 can be eliminated.

The Echo Field

The echo field can be expressed as a product of the three constituent fields. Right before the material, the three constituent fields can be expressed according to equation 4.12.

$$\vec{E}'_1 = E_1 \begin{pmatrix} t_1 \\ i r_1 \end{pmatrix} \quad \vec{E}'_2 = E_2 \begin{pmatrix} i r_1 \\ t_1 \end{pmatrix} \quad \vec{E}'_3 = E_3 \begin{pmatrix} t_1 \\ i r_1 \end{pmatrix} \quad (4.12)$$

The echo field is proportional to the product of the three fields at the crystal and is given by equation 4.13.

$$\vec{E}_{echo} = \frac{1}{2}\alpha_0 LC\epsilon_0 cne^{-\frac{\alpha_0 L}{2}} E_1^* E_2 E_3 \begin{pmatrix} it_1^2 r_1 \\ r_1^2 t_1 \end{pmatrix} = \frac{1}{2}\alpha_0 LC\epsilon_0 cne^{-\frac{\alpha_0 L}{2}} r_1 t_1 E_1^* E_2 E_3 \begin{pmatrix} it_1 \\ r_1 \end{pmatrix} \quad (4.13)$$

The echo is then passed through the output optics, so the output interferometric Jones vector is given by equation 4.14.

$$\vec{E}''_{echo} = \frac{1}{2}\alpha_0 LC\epsilon_0 cne^{-\frac{\alpha_0 L}{2}} t_1 r_1 E_1^* E_2 E_3 \begin{pmatrix} i(\sqrt{C_1}t_1 t_2 + \sqrt{C_2}r_1 r_2) \\ -(\sqrt{C_1}t_1 r_2 - \sqrt{C_2}r_2 t_1) \end{pmatrix} \quad (4.14)$$

The echo field can be expressed in terms of the expected echo field from a co-linear echo experiment $E_{echo} = \frac{1}{2}\alpha_0 LC\epsilon_0 cne^{-\frac{\alpha_0 L}{2}} t_1 r_1 E_1^* E_2 E_3$. In this case, the final echo field Jones vector is given by 4.15.

$$\vec{E}''_{echo} = t_1 r_1 E_{echo} \begin{pmatrix} i(\sqrt{C_1}t_1 t_2 + \sqrt{C_2}r_1 r_2) \\ -(\sqrt{C_1}t_1 r_2 - \sqrt{C_2}r_2 t_1) \end{pmatrix} \quad (4.15)$$

The echo will constructively interfere on OP1 and destructively interfere on OP2. The echo is predominantly directed out of OP1, but imperfections will lead to leakage into OP2. The echo field energy in port 1 is undesirable because it corresponds to echo energy diverted away from detection.

Interferometer Contrast For an interferometer used to isolate fields, the contrast of the echo can be defined as the ratio of the power in the echo in the desired port (OP1) to the power leaked from these other fields into that port. In the case of 2 port interferometric stimulated echoes, E_1 and E_2 can be time-gated, so E_3 is the field that will temporally

overlap the echo, so the contrast of the echo field is best defined between the echo field and E_3 . The echo contrast will be defined between the powers of E_{echo} and E_3 on port 1 ($contrast_{echo}$) under the condition of no bias phase. To describe the contrast mathematically, the field dependence will be ignored because the field power of the signal at each output will be drastically different for the E_3 and E_{echo} , as the echo will be much weaker than E_3 . Mathematically the echo contrast is given in equation 4.16.

$$contrast_{echo} = \left(\frac{(\sqrt{C_1}t_1t_2 + \sqrt{C_2}r_1r_2)}{\sqrt{C_1}t_1t_2 - \sqrt{C_2}r_1r_2} \right)^2 \quad (4.16)$$

The introduction of coupling parameters to the interferometer gives an extra degree of freedom to attempt to improve the contrast ratio of the echo. Practically this parameter does not necessarily only refer to the coupling efficiency. Adding an attenuator to one of the interferometer paths would have the same effect. This contrast is influenced by the coupling efficiencies and the beam splitter split ratios. The relative coupling efficiency could represent a small attenuation in one of the fields and could offer a means of tweaking the contrast on one of the ports. Physically, this could be implemented with a variable optical attenuator or a neutral density filter. Additionally, a variable beam splitter can be used at the outputs to enforce equal transmission and reflection coefficients for either the input or output splitter.

Echo Efficiency The other figure of merit for interferometric stimulated photon experiments is Echo efficiency, which is a measure of the relative strength of the input echo signal. In this case, the efficiency of the echo can be defined as the ratio of the echo power in the desired port (OP1) of the real interferometer (which is assumed to have zero phase bias) to the echo power to the output of an ideal Mach-Zehnder interferometer. The

expression for the echo efficiency is given by equation 4.17

$$echo\ efficiency = 4(t_1 r_1)^2 \left(\sqrt{C_1} t_1 t_1 + \sqrt{C_2} r_1 r_2 \right)^2 \quad (4.17)$$

Like the echo contrast, the echo efficiency depends on the coupling efficiencies and beam splitter parameters. These two figures of merit can be explored to determine the preferred position of a nearly ideal beam splitter (i.e., the variable beam splitter tuned to be a 50-50 beam splitter) in the interferometer can be chosen by evaluating how the contrast and echo with one of the coupling efficiency parameters for different combinations of beam splitter parameters. The goal is to achieve the highest possible echo efficiency at maximum echo contrast.

Optimization With Imperfect Input and Output Split Ratios In a practical interferometer using fixed split ratio beam-splitters, both the input and output splitters may have imperfect split ratios. The introduction of coupling efficiency allows for the adjustment of the echo contrast by adjusting one of the coupling efficiencies at the output. Figure 4.3 shows how contrast and echo efficiency varies with the coupling coefficient C_1 with C_2 fixed at 0.7 (a plausible 70% coupling into the fiber). To demonstrate the effects of the poor coupling efficiencies, the results are displayed for an interferometer consisting of two 45-55 splitters; in this case, r_1 and r_2 are fixed at $\sqrt{0.45}$, and t_1 and t_2 are fixed at $\sqrt{0.55}$. By tweaking the coupling efficiencies the echo contrast can be maximized at a cost to efficiency. At the condition for maximal contrast, for this interferometer, the echo efficiency is predicted to be about -2.5 dB.

Contrast Optimization With Imperfect Output Split Ratio Using an adjustable beam splitter, it may be possible to improve the fidelity of the split ratio significantly. Still, these devices tend to be expensive, so it would be beneficial to determine if it would better to use a variable splitter as the first or second beam splitter. Figure 4.4 shows how contrast and

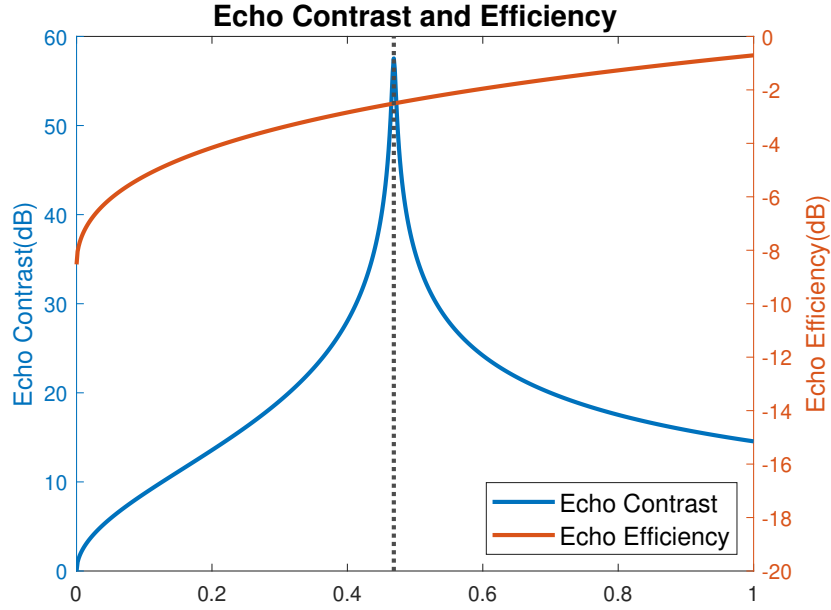


Figure 4.3: Echo contrast and efficiency for an interferometer with imperfect input and output beam splitters. The contrast can still be eliminated by removing probe field leakage at a cost to echo efficiency.

efficiency vary with the coupling coefficient C_1 with C_2 fixed at 0.7 (a plausible 70% coupling into the fiber). The ideal 50-50 input splitter has r_1 and t_1 set to $\frac{1}{\sqrt{2}}$. The imperfect splitter at the output has r_2 set to $\sqrt{0.45}$ and t_2 set to $\sqrt{0.55}$. Tuning of the free coupling efficiency allows for the maximization of echo contrast. In this case, at maximum contrast, the echo efficiency was -2.006dB.

Contrast Optimization With Imperfect Input Split Ratio The same procedure can be repeated for an interferometer with an ideal output splitter and an imperfect output splitter. Figure 4.5 shows how the contrast for each output port of the interferometer varies with coupling coefficient C_1 with C_2 fixed at 0.7. A perfect 50-50 splitter at the output puts r_2 and t_2 at $\frac{1}{\sqrt{2}}$. A 45-55 splitter at the output sets r_1 to $\sqrt{0.45}$ and t_1 to $\sqrt{0.55}$. In this case, tuning of the free coupling efficiency allows for the maximization of echo contrast. In this case, the echo efficiency was -2.055dB at maximum contrast. The efficiency at maximum

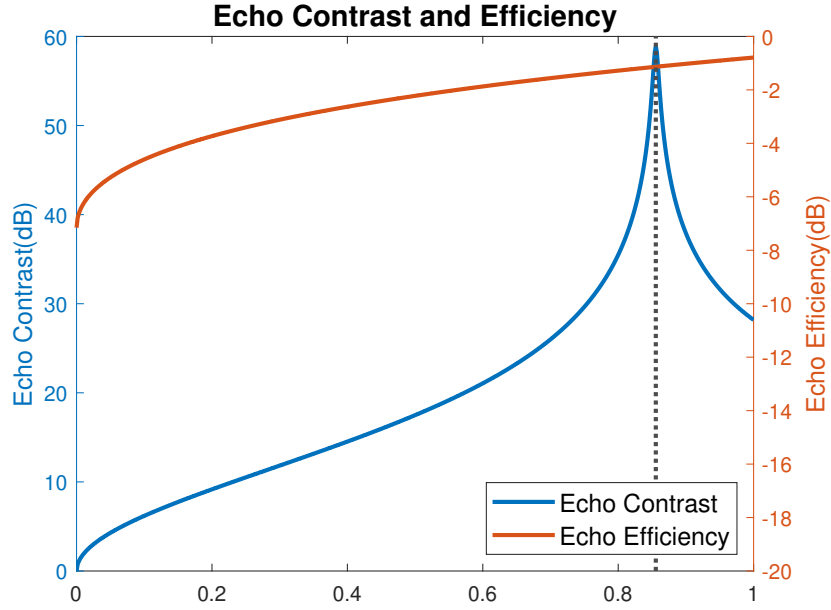


Figure 4.4: Echo contrast and efficiency as a function of coupling efficiency for an interferometer with a perfect input splitter and an imperfect output splitter.

contrast for an interferometer with an imperfect input is slightly lower than the efficiency at maximum contrast for an interferometer with an imperfect output splitter for imperfect splitters with the same split ratios. This indicates that the input splitter is slightly more important.

The slightly higher importance of the input splitter can be derived by directly comparing an interferometer with a perfect input splitter with an imperfect output splitter. The contrast is maximized when $\sqrt{C_1}t_1t_2 = \sqrt{C_2}r_1r_2$, so the maximum efficiency will always follow equation 4.18

$$efficiency = 16C_1t_1^4r_1^2t_2^2 \quad (4.18)$$

For the interferometer with a perfect input split ratio and an imperfect output split ratio, $r_1 = t_1 = \frac{1}{\sqrt{2}}$ and $t_2^2 = \frac{1}{2} + \delta T$, where δT is the deviation of the power transmission coefficient from an ideal transmission coefficient. For this interferometer, the echo efficiency is given by

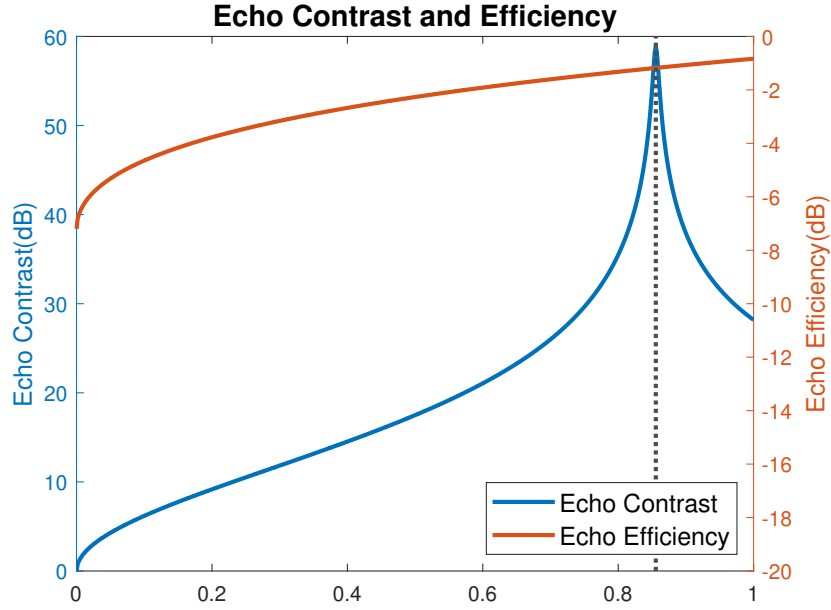


Figure 4.5: Echo contrast and efficiency as a function of coupling efficiency for an interferometer with a perfect output splitter and an imperfect input splitter.

4.19

$$efficiency_{ideal\ input} = C_1 (1 + 2\delta T) \quad (4.19)$$

For the interferometer with a perfect input split ratio and an imperfect output split ratio $r_2 = t_2 = \frac{1}{\sqrt{2}}$ and $t_1^2 = \frac{1}{2} + \delta T$. The efficiency is given by equation 4.20

$$\begin{aligned} efficiency_{ideal\ output} &= C_1 (1 + 2\delta T - 4\delta T^2 - 8\delta T^3) \dots \\ &\dots = efficiency_{ideal\ input} (1 - 4\delta T^2) \end{aligned} \quad (4.20)$$

Therefore the echo efficiency for an interferometer with an ideal input splitter will always be slightly higher than an interferometer with an ideal input splitter. The input splitter should be prioritized to have as close to an ideal split ratio as possible.

Experimental Demonstration of Two-Port Interferometric Stimulated Photon Echoes

An experimental demonstration of stimulated photon echo isolation was conducted, and the results were published in Optics letters as an article titled *Demonstration of stimulated photon echo isolation using interferometric cancellation* [77] which is presented in chapter 5.

Four Port Interferometric SPEs

Theoretical Description of Four-Port Interferometric Stimulated Photon Echoes

A 4 port interferometer can be contrived to mimic the "Box Geometry" (see figure 1.5)for isolating stimulated photon echoes. This interferometer consists of a pair of 50-50 splitters coupling fields from channels 1 and 2 and channels 3 and 4. A second pair of 50-50 splitters then couples channels 2 and 3 and channels 1 and 4. After the second pair of splitters, the fields will be directed through an SSH material. The fields on each port are then independently phase-shifted. The fields are then directed through an output splitter network. The output splitter network consists of two 50-50 splitters connecting channels 1 and 4 and channels 2 and 3, followed by two 50-50 splitters connecting channels 1 and 2 and channels 3 and 4. This interferometer was first described in chapter 2 and is shown again in figure 4.6 to refresh the reader. This interferometer design can uniquely map the fields at the input ports to the output ports depending on the bias phases (under the condition that the relative phases are multiples of π between each other). This 4-port interferometer can be described in terms of 4x4 matrix operators describing the required beam splitters given

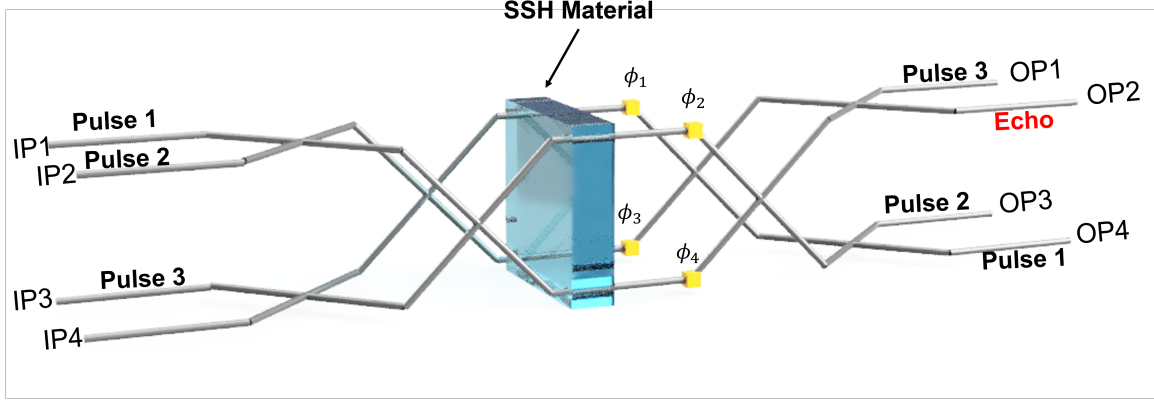


Figure 4.6: A refresher from chapter 2, This four-port interferometer for stimulated photon echo isolation will be discussed in greater detail in this chapter. This image assumes the bias phases are zero; this interferometer will map the pulses from input to output ports in a manner reminiscent of the “Box Geometry.”

in chapter 2 and repeated here as a refresher in equation 4.21

$$\begin{aligned}
 \hat{Q}_{Splitter:1,2} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Splitter:1,4} &= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{i}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \\
 \hat{Q}_{Splitter:2,3} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \hat{Q}_{Splitter:3,4} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ 0 & 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}
 \end{aligned} \tag{4.21}$$

Also required is the matrix describing independent phase shifts given in chapter 2 and

repeated here as a refresher in equation 4.22.

$$\hat{Q}_{Phase} = \begin{pmatrix} e^{i\phi_1} & 0 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 & 0 \\ 0 & 0 & e^{i\phi_3} & 0 \\ 0 & 0 & 0 & e^{i\phi_4} \end{pmatrix} \quad (4.22)$$

Assume this interferometer is used for stimulated photon echo experiments. In these experiments, input field E_1 is coupled into port IP 1, input field E_2 is coupled into port IP 2, and input field E_3 is coupled into port IP 3. The input optics can be described with an operator composed of the beam splitter operators; the final operator is given in equation 4.23.

$$\hat{Q}_{input} = \hat{Q}_{splitter,1,4} \hat{Q}_{splitter,2,3} \hat{Q}_{splitter,1,2} \hat{Q}_{splitter,3,4} = \frac{1}{2} \begin{pmatrix} 1 & i & -1 & i \\ i & 1 & i & -1 \\ -1 & i & 1 & i \\ i & -1 & i & 1 \end{pmatrix} \quad (4.23)$$

The fields at the material can be described as a matrix-vector product with the input operator $\vec{E}'_j = \hat{Q}_{input} \vec{E}_j$, which can further be described in terms of unit vectors $\vec{E}'_j = \hat{Q}_{input} E_j \hat{E}_j$. The interferometric Jones vectors for the input fields are given by equation 4.24.

$$E'_1 = \frac{E_1}{2} \begin{pmatrix} 1 \\ i \\ -1 \\ i \end{pmatrix} \quad E'_2 = \frac{E_2}{2} \begin{pmatrix} i \\ 1 \\ i \\ -1 \end{pmatrix} \quad E'_3 = \frac{E_3}{2} \begin{pmatrix} -1 \\ i \\ 1 \\ i \end{pmatrix} \quad (4.24)$$

The frequency domain description of the final echo field can be expressed explicitly according to equation 1.44 [7], which can be written in terms of a new fourth unit vector $\hat{E}'_4$ which represents the unit vector for a hypothetical field injected into IP 4 and mixed by the input network before entering the material, given by equation 4.25

$$\hat{E}'_4 = \frac{1}{2} \begin{pmatrix} i \\ -1 \\ i \\ 1 \end{pmatrix} \quad (4.25)$$

In terms of the new unit vector, the echo, the field can be written using equation 4.26.

$$\vec{E}_{echo} = -\frac{1}{4}\alpha_0 LC\epsilon_0 cn \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \hat{E}'_4 \quad (4.26)$$

The stimulated photon echo field will transform like a field originally directed into the input port IP4. The output splitter network can be expressed much as the input splitter network using equation 4.27. The stimulated photon echo field will transform like a field originally directed into input port IP4. Assuming all the bias phases are zero, the output splitter network can be expressed much as the input splitter network using equation 4.27.

$$\hat{Q}_{output} = \hat{Q}_{splitter,1,2} \hat{Q}_{splitter,3,4} \hat{Q}_{splitter,1,4} \hat{Q}_{splitter,2,3} = \frac{1}{2} \begin{pmatrix} 1 & i & -1 & i \\ i & 1 & i & -1 \\ -1 & i & 1 & i \\ i & -1 & i & 1 \end{pmatrix} \quad (4.27)$$

Output Fields can therefore be described in terms of fields at the material in the center of

the device ($\vec{E}'$) according to equation 4.28.

$$\vec{E}_{output} = \frac{1}{2} \begin{pmatrix} 1 & i & -1 & i \\ i & 1 & i & -1 \\ -1 & i & 1 & i \\ i & -1 & i & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_1} & 0 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 & 0 \\ 0 & 0 & e^{i\phi_3} & 0 \\ 0 & 0 & 0 & e^{i\phi_4} \end{pmatrix} \vec{E}' \quad (4.28)$$

Fields from input port IP1 are mapped to output port OP3, fields from input port IP2 are mapped to output port OP4, fields from input port IP3 are mapped to output port OP1 and fields from input port IP4 (and, therefore, the photon echo) are mapped to output port OP2. The frequency domain representation of the output fields can be written as a Jones vector for the output fields (equation 4.29).

$$\tilde{\vec{E}}_{out}(f) = \begin{pmatrix} -\tilde{E}_3(f) \\ \frac{1}{4}\alpha_0 L \beta \epsilon_0 c n \tilde{E}_1^*(f) \tilde{E}_2(f) \tilde{E}_3(f) e^{-\frac{1}{2}\alpha_0 L} e^{-2\pi i f \tau_{21}} \\ -\tilde{E}_1(f) \\ -\tilde{E}_2(f) \end{pmatrix} \quad (4.29)$$

Like the angular "box geometry" for stimulated echo experiments shown in figure 1.5, this interferometric configuration can spatially isolate fields in stimulated echo experiments.

The analysis above assumed zero bias phases, but if new bias phases are introduced, they can be chosen to direct the fields into any output port. Table 4.1 shows the resulting mappings for different bias combinations. SPE fields in the 4-port interferometer can be isolated into any port desired so long as the bias phases can be selected.

This could have been implemented in a photonic integrated circuit using doped waveguides as the SSH material and sections of electro-optic material as phase controls. Therefore processes like optical correlation can be performed in a photonic integrated circuit

ϕ_1	ϕ_2	ϕ_3	ϕ_4	OP1	OP2	OP3	OP4
0	0	0	0	E_3	E_{echo}	E_1	E_2
0	π	0	π	E_1	E_2	E_3	E_{echo}
0	0	π	π	E_2	E_1	E_{echo}	E_3
0	π	π	0	E_{echo}	E_3	E_2	E_1

Table 4.1: The bias control table for the 4-port interferometer describes where individual fields are directed under different interferometric bias conditions.

while still achieving the isolation of the echo field from the other fields in a way that is analogous to the box geometry though not requiring free-space optics for the non-co-linear beams needed for the box geometry. There will be a hit to the efficiency of the echo because the optical fields used to generate the echo are divided into four parts generating four separate echoes, each with one 64^{th} of the power of an echo generated by three co-linear fields with no splitters in a single material spot, the four echoes then constructively interfere at the output giving a single echo field at a single output with one 16^{th} the power of a fully co-linear echo.

Despite the apparent drop in echo power over the fully co-linear echo due to optical fields being spread to three locations in the interferometer, the interferometric configuration could, under some circumstances, yield stronger echoes. If the intensities of E_1, E_2 , and E_3 are constrained by saturation effects in the material, then, in the case of interferometric echos, the intensity in each spot in the material can be increased to the maximum intensity before saturation effects begin to appear in the case of a fully co-linear echo. This will produce 4 independent echoes with the intensity of the maximum co-linear echo. The result is that the maximum 4-port echo could be up to 4 times stronger than the maximum co-linear echo without introducing saturation effects.

Imperfections in the 4-Port Interferometer

The theoretical description of stimulated photon echoes in a four-port interferometer assumed ideal splitter parameters and phase biases. The inevitable introduction of imperfection to these parameters will lead to undesired leakage of fields into the echo port. As was the case in the two-port case, the echo contrast can be defined as the ratio of the power in the echo field on the echo port to the leakage power of the other fields. In the case of four port stimulated echoes, it may not be possible to time gate fields E_1 and E_2 , so rather than just the contrast between the echo field and the probe field, the echo contrast should be defined as the ratio of the power in the echo to the leakage power from all other fields as shown in equation 4.30.

$$contrast_{echo} = \frac{P_{echo}}{P_1 + P_2 + P_3} \quad (4.30)$$

As was the case with 2-port echoes, the echo contrast can be tuned by adding attenuation within the device. This will come at a cost to echo efficiency, which is the ratio of the echo power to the echo power, assuming an ideal interferometer (equation 4.31).

$$echo\ efficiency = \frac{P_{echo}}{P_{Echo,ideal}} \quad (4.31)$$

To simulate the operation of an imperfect 4-port interferometer, a matrix model of the imperfect interferometer is used to estimate the powers of each field on each port at the output. This imperfect interferometer model allows for imperfection in the split ratio ($\eta_i = \frac{t_i^2}{r_i^2}$) of every splitter and arbitrary phase shifts and attenuations at each critical juncture in the interferometer. For the purpose of this model, the phases are chosen so that (unless otherwise specified) $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$, and fields E_1 , E_2 , and E_3 are launched into port IP1, IP2, and IP3, respectively.

Split Ratio Imperfections Imperfections to the split ratios of the different splitters that form the 4-port interferometer can have a variety of effects on the performance of the interferometer. As a preliminary test of the effects of split ratio errors in an interferometer, we conducted a statistical study of the effects of errors in the split ratio. In this test, the split ratio of each splitter was simultaneously adjusted by adding a normally distributed random variable σ with a standard deviation of one, scaled by an error parameter to the ideal split ratio value of 1 ($\eta = 1 + \epsilon\sigma$), the other parameters that govern this device (i.e., phases and attenuations) were assumed to be ideal. This was repeated 10,000 times to calculate the mean of the relative power in each field as a function of the error parameter. The results are shown in decibels in figure 4.7 (in decibels, the error parameter is defined as $10\log_{10}(\epsilon)$). This plot shows the results for pulses 1 (but in principle, the results are similar for pulses 1 and 2). The leakage into other ports scales at 2dB/dB, indicating a quadratic dependence on the split ratio error. This indicates that using splitters with a split ratio tolerance of 1%, the leakage of pulse 1 into port OP2 (the echo port will be -52dB with respect to the power in pulse 1 (excluding loss due to absorption in the material)). At 0.1 % of split ratio error, the leakage into OP2 drops to -72dB. The leakage of pulse 1 into ports OP1 and OP2 is lower than the leakage into port OP4 by about 1.6 dB. This is because of differences in the paths that different fields take throughout the interferometer.

As was the case for the two-port interferometer, using attenuation in the system, some effects of splitter mismatch can be compensated for. However, unlike in the 2 port system, there are 16 places in the interferometer where attenuations can be altered within the device, so when dependences on attenuation are shown, the locations of these attenuations will be clearly stated. By exploring this space, we found that there are broadly six categories into which the splitters fall. Figure 4.8 on the following page is a color-coded 4-port interferometer indicating which splitters fall into which category.

In order to test the importance of each splitter, tests were done under the condition that

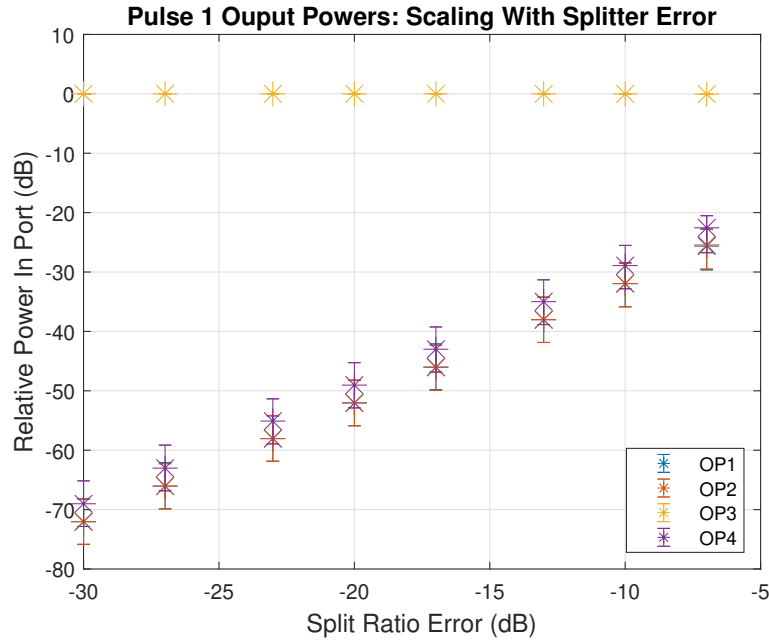


Figure 4.7: Power from pulse 1 in each output port as a function of the level of error in split ratio for splitters in the device.

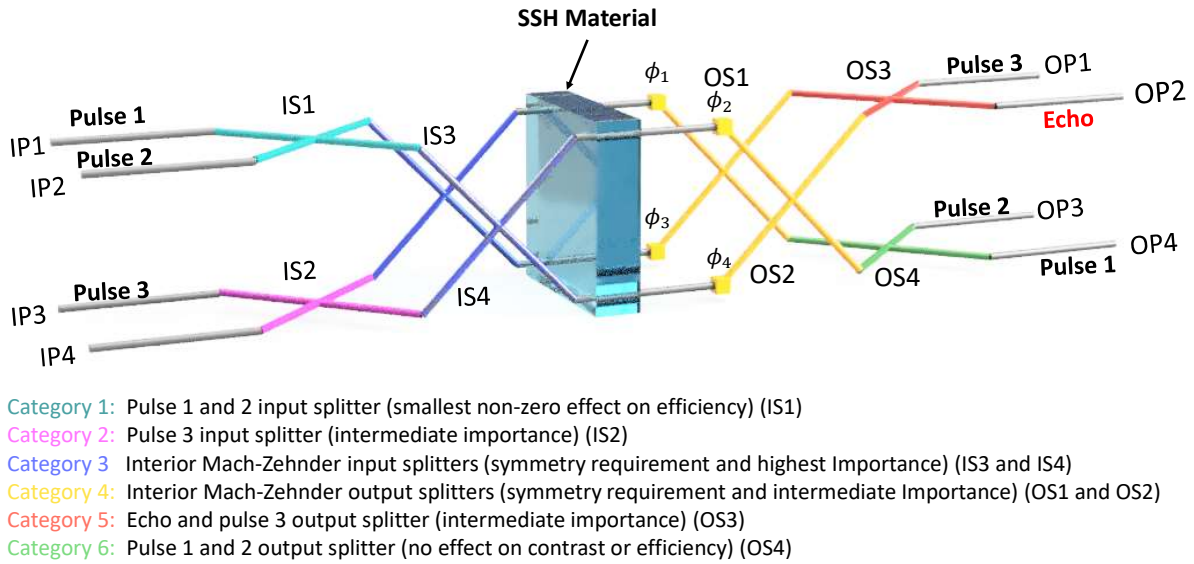


Figure 4.8: A color-coded and annotated 4-port interferometer that indicates the category to which each splitter belongs.

only the splitter(s) under test and the attenuations used to tune the device have non-ideal values. The other ports are assumed to have ideal parameters (split ratio=1 and no added attenuation). Some of the efficiency differences at maximum efficiency change only slightly between cases but there are differences that can help assign priority.

Category 1: Pulse 1 and 2 input splitter The first input splitter (IS1) connects the inputs of pulses 1 and 3 to the two internal Mach-Zehnders at the heart of the 4-port interferometer. The internal Mach Zehnders direct the entirety of these fields into channels 3 and 4 before the last output splitter. This means that deviations in this splitter will not necessarily lead to leakage into the echo port (OP2). Figure 4.9 shows how contrast and efficiency vary with attenuation on channel 1 after the beam splitter assuming that the first splitter after the input, which couples channel 1 and 2, has a split ratio of 0.9. After this splitter, the attenuation on channel 2 is 0.7.

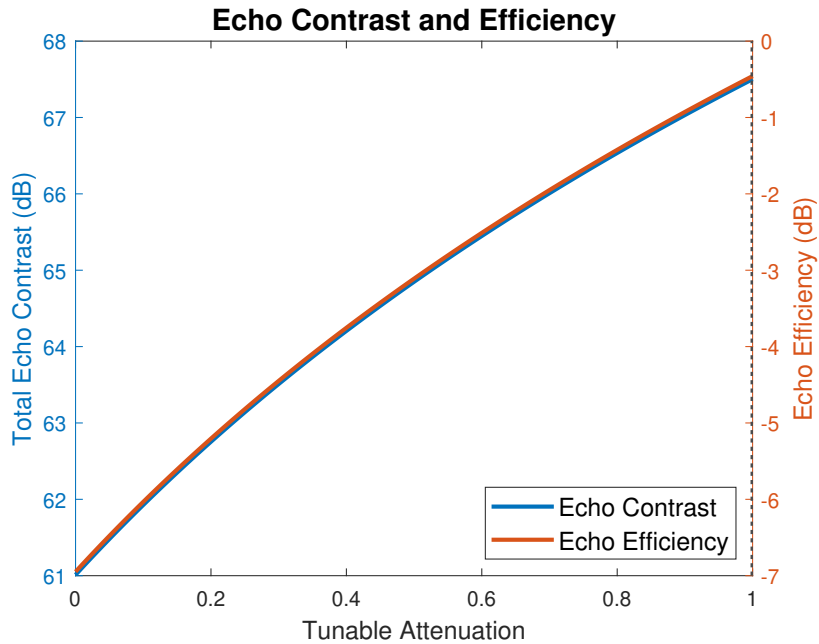


Figure 4.9: Echo contrast and efficiency of the attenuation of channel 1 after the beam splitter, which has a split ratio of 0.9. The attenuation on channel 2 after the splitter is set to 0.7.

Because split ratio errors in IS1 do not induce leakage onto the echo port, maximum contrast occurs with no attenuation. In the simulation, the echo efficiency at maximum contrast was -0.45dB. This splitter has only a very minor effect on the figures of merit for stimulated photon echo experiments.

Category 2: Pulse 3 input splitter Input splitter IS2 splits pulse 3 into the two internal Mach-Zehnders inside of the interferometer which then directs all of pulse 3 into output splitter OS3. At this last splitter, improper cancellation can lead to some leakage of pulse 3 into the echo port (OP2). Figure 4.10 shows contrast and efficiency as a function of the attenuation on channel 3 just after splitter IS2. The split ratio of IS2 is 0.9 and the attenuation on channel 4 was fixed to 0.7. The plot shows the contrasts between field 3 and the echo as a function of the attenuation on channel 3 after the splitter.

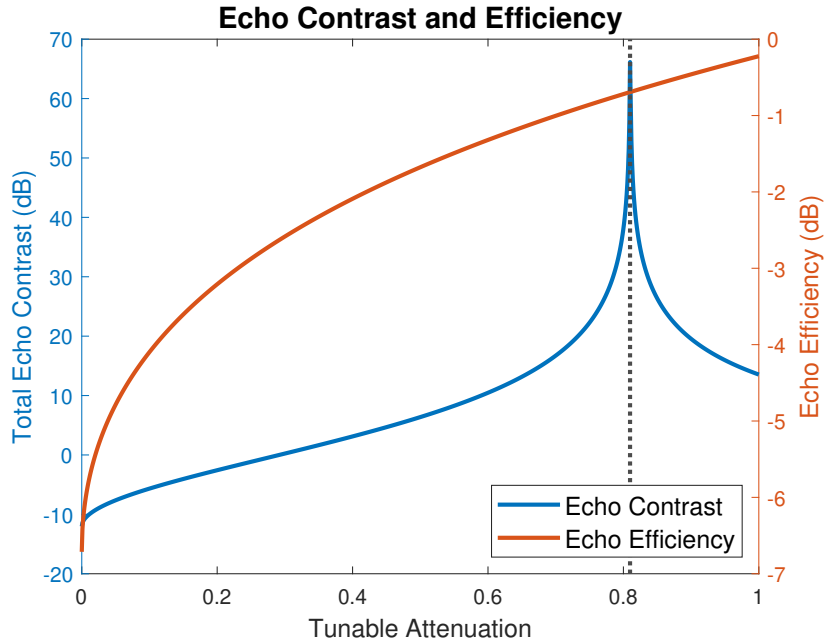


Figure 4.10: Contrasts between the echo and pulse 3 as a function of the attenuation of channel 3 after the beam splitter, which has a split ratio of 0.9. The attenuation on channel 4 after the splitter is set to 0.7.

The contrast can be optimized by eliminating the leakage into the echo port. At

maximum contrast, the echo efficiency was -0.6929 dB.

Category 3 Interior Mach-Zehnder Input splitters Splitters IS3 and IS4 are the input splitters for the two internal Mach-Zehnders inside of the four-port interferometer. It was found that the imperfections of these splitters could be compensated for, but this was only true under the condition that the imperfections (and therefore also the attenuation adjustments) were symmetric, that is, the split ratios and attenuations after the splitters were the same for both splitters. It is also important to note that another choice can be made as to whether to attenuate before or after the material. Attenuating before the material will induce losses which will scale non-linearly in the echo efficiency so it is better to attenuate after the material.

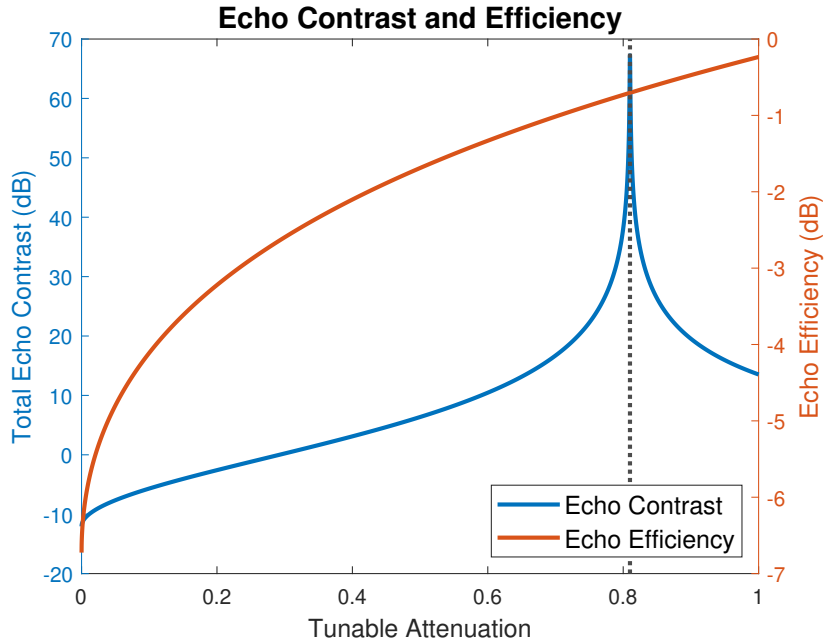


Figure 4.11: Echo contrasts and efficiency as a function of the attenuation of channels 1 and 3 after IS3 and IS4 which have a split ratio of 0.9, and the material. The attenuation on channels 2 and 4 after the material are set to 0.7

Figure 4.11 shows the contrast and efficiency in the case where the split ratios for IS3 and IS4 are 0.9, and the attenuations on channels 2 and 4 right after the material are 0.7.

The attenuations on channels 1 and 3 after the material are equal but allowed to vary. All other interferometer parameters are assumed to be ideal. At maximum contrast in this case the echo efficiency was -0.70443 dB, the lowest efficiency observed for a interferometer with an imperfect beam splitter of split ratio of 0.9 and a fixed coupling efficiency of 0.7 observed. Between the requirement that the two internal Mach-Zehnder interferometers be symmetric and the low efficiency at maximum contrast, these splitters have the most challenging requirements to meet.

Category 4: Interior Mach-Zehnder Output splitters As was the case for the splitters before the inputs to the material (IS3 and IS4), OS1 and OS2, the splitters right after the material, are part of the pair of parallel Mach-Zehnders at the heart of the 4-port interferometer. As a result, they have a similar symmetry requirement to the splitters that form the inputs to these Mach-Zehnders, that is in order to optimize contrast the split ratios of OS1 and OS2, as well as the attenuations before them must be the same (in order to make the two Mach-Zehnders symmetric). Figure 4.12 shows the contrast and efficiency in the case where the split ratios for OS1 and OS2 are 0.9, and the attenuations on channels 2 and 4 right after the material are 0.7. The attenuations on channels 1 and 3 after the material are equal but allowed to vary. All other interferometer parameters are assumed to be ideal. At maximum contrast in this case the echo efficiency was -0.69239 dB. This means the echo is slightly more efficient than the case where IS3 and IS4 are imperfect to the same degree. While OS1 and OS2 have the same symmetry requirement, the overall effect on efficiency is slightly less for the output splitters (much like the efficiency and contrast results for a 2-port Mach Zehnder).

Category 5: Echo and Pulse 3 Output Splitter Splitter OS3 (which couples channel 1 and 2 just before the output) serves to separate field 3 and the echo onto their respective output ports. Imperfections in this splitter can lead to leakage of pulse 3 onto OP2. Figure 4.13 shows the echo contrast and efficiency as a function of the attenuation on channel

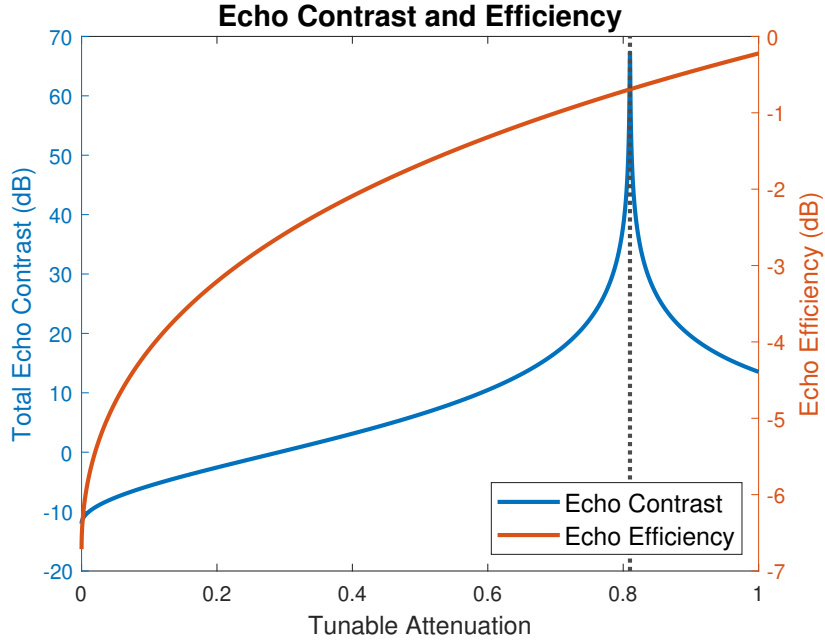


Figure 4.12: Echo contrasts and efficiency as a function of the attenuation of channels 1 and 3 just before OS1 and OS2, which have a split ratio of 0.9. The attenuations on channels 2 and 4 after the material are set to 0.7

1 right before splitter OS3, which has a split ratio of 0.9. In this example, the attenuation on channel 2 before OS3 is 0.7. At maximum contrast for this system, the echo efficiency is -0.69239 dB. Meaning the effects of deviations to this splitter are very similar to those of IS2.

Category 6: Pulse 1 and 2 output splitter Splitter OS4 connects channels 1 and 2 at the output; assuming the rest of the interferometer is ideal it does not affect the echo. The rest of the network before this splitter isolated fields 1 and 2 entirely into channels 3 and 4 and field 3 and the echo entirely into channels 1 and 2. Therefore this final splitter contains none of the echo fields and directs no fields into the echo port. This splitter will not affect either the efficiency or contrast of the echo. This splitter is, strictly speaking, not required for the isolation of the echo field. Instead, the function of this splitter is only to separate fields 1 and 2. In terms of optimization of echo isolation, this splitter is the least consequential.

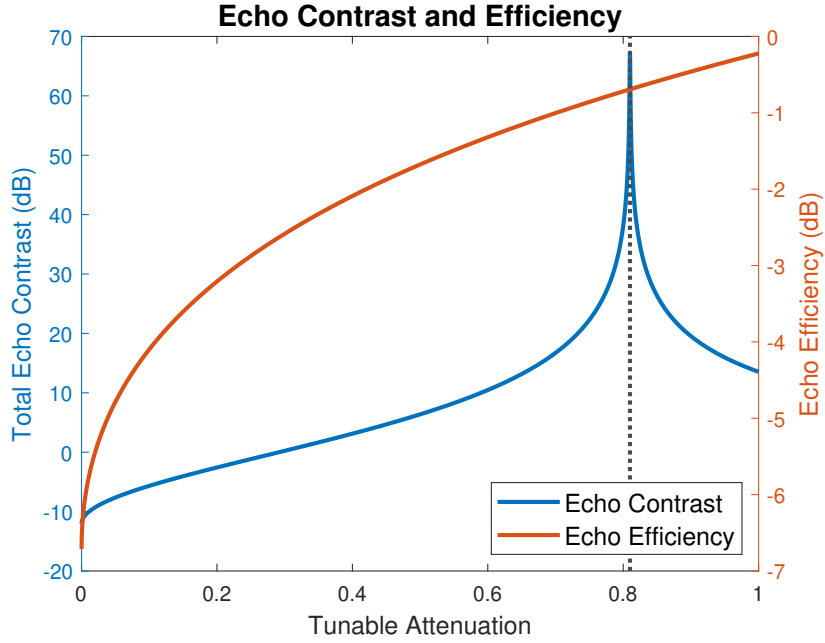


Figure 4.13: Echo contrast and efficiency of the attenuation of channel 1 before splitter OS3, which has a split ratio of 0.9. The attenuation on channel 2 after the splitter is set to 0.7.

Summary Studies of the echo contrast and efficiency in the 4-port interferometer indicate that there is a hierarchy of importance of the beam splitters that form the device. At the bottom of the hierarchy is splitter OS4, the splitter that separates pulses 1 and 2 at the output. This splitter has no bearing on echo isolation, and as such, very little attention must be paid to this splitter. Splitter IS1 was shown to have only a very minor effect on echo efficiency changes here do not induce leakage into the echo port but may degrade the echo efficiency. Splitters IS2 and OS3 had essentially the same effect on efficiency and contrast in that they had the same efficiency at maximum contrast for the same amount of deviation to split ratio. In the 2-port case, echo efficiency was more sensitive to changes in the split ratio of input splitters because of the nonlinear dependence on the input fields led to a nonlinear dependence on the input split ratio. In the four-port case, because only one field (E_3) is directed through port OS2, the echo efficiency depends linearly on the input split ratio, so the changes to these splitters are in a sense the same. Changes to the split

ratios of splitters OS1 and OS2 lead to the same level of change as the same changes to the split ratios of IS2 and OS3, but in order to optimize contrast the OS1 and OS2 have to be treated symmetrically, so although deviations to these splitters were similar to IS2 and OS3, this symmetry requirement makes these splitters slightly important. The most important splitters in the four-port interferometer are IS3 and IS4, not only did changes to these splitters lead to the largest degradation of echo efficiency, but they also have a symmetry requirement. These splitters should therefore be prioritized for split ratio fidelity. Though it should be noted that ultimately the variation in echo efficiency between different cases was very minor.

Phase Imperfections Interferometers are, by their very nature, sensitive to internal phase shifts. The two-port Mach-Zehnder only has a single relative phase that is of significance, while in the 4-port interferometer, at each junction between splitters, there is an opportunity for the phase shift to be non-ideal. To study the importance of phase shifts in each location, the relative intensity in each field on each port can be shown as a function of relative phase shift at a location in the interferometer can be shown. In the device, there are 6 possible locations for an interesting relative phase difference to arise between pairs of paths. These locations are marked in figure4.14. Only phase shifts that arise inside the device are of interest because relative phase differences between each field on different channels are what allow the required interference effects to arise. Phase shifts on fields before the device are common throughout the rest of the device. Only the relative shifts inside the device will have any bearing on the isolation of fields. For simplicity, the magnitude of each injected field is kept at 1, and the echo is treated as a simple product of fields (in the ideal case, the echo will be one-sixteenth the strength of the input fields). The different phase shifts in the 4 port interferometer can be broken down into three categories discussed below.

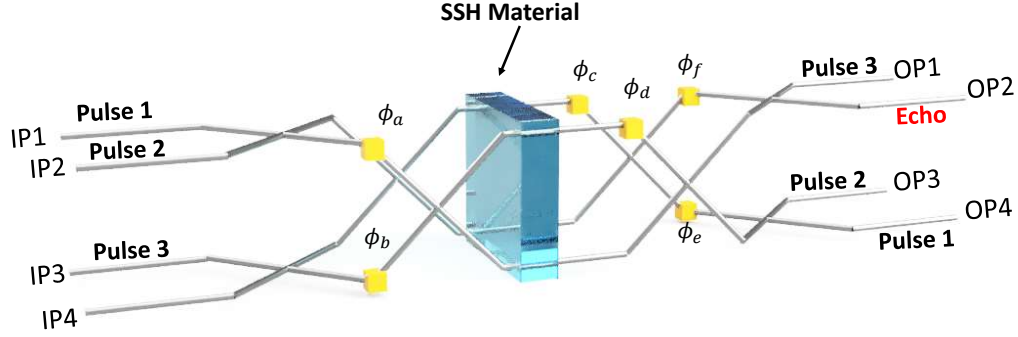


Figure 4.14: A four-port interferometer with phase locations marked to indicate the locations where interesting phase shifts can be introduced.

Phase shifts that only affect Pulses 1 and 2 A relative phase difference between channels 1 and 2 just after the splitter that first mixes pulses 1 and 2 (ϕ_a) or just before the splitter that separates pulses 1 and 2 at the output (ϕ_e) will only lead to effects to pulse 3 and the echo. Phase shifts in these locations will not lead to any fields leaking into the echo or leakage of the echo onto any other port. Figure 4.15 shows how the intensities in each field vary at each output port as a function of the relative phase shift on channel 1 just after the splitter that connects channels 1 and 2 at the input. Pulse 3 and the echo are entirely confined to OP 1 and OP2, respectively, while the relative portion of pulses 1 and 2 at those two output ports vary sinusoidally with the relative phase shift. This phase shift is, therefore, irrelevant to ensuring the isolation of the echo at the output.

Phase shifts that only affect Pulse 3 and the echo The splitters discussed in the previous paragraph described phase shifts that only affect pulse 1 and pulse 2 at the output, another class of phase shifts only affect pulse 3 and the echo at the output. Phase shifts just after the first splitter that splits pulse three (ϕ_b) or a phase shift just before the final splitter connecting channels 1 and 2 (ϕ_f) both belong to this class of phase shifts. Unlike the class of

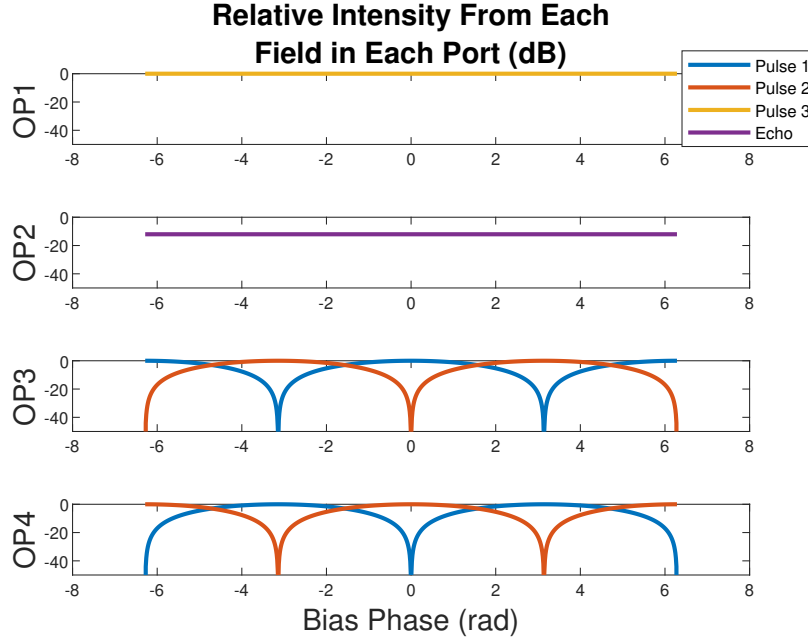


Figure 4.15: The relative intensity in each port from each field as a function of the bias phase on channel 1 right after the first input splitter.

phase shifts that only affect pulses 1 and 2, these phase shifts will cause issues because this phase shift will induce leakage of the echo into OP1 and leakage of the third pulse into the echo port. Figure 4.16 shows the intensities of each field on each output port as a function of the bias phase on channel 3 just after the first splitter that connects channels 3 and 4. By changing the phase, the entirety of the echo can be directed into either channel 1 or 2. The same is true with pulse 3.

Phase shifts that affect all fields Any phase shifts inside of the nested Mach-Zehnders at the heart of the 4-port interferometer (ϕ_c and ϕ_d) can lead to the mixing of all fields into all output ports. Here the phase shift could either be before or after the material, but it would not matter because the added common mode contributions to the echo phase would reduce to the original phase shift. Figure 4.17 shows the intensities of each field on each output port as a function of the bias phase on channel 1 inside one of the Mach-Zehnders inside the device. Unlike the previous cases where changes to the phase lead the entirety of

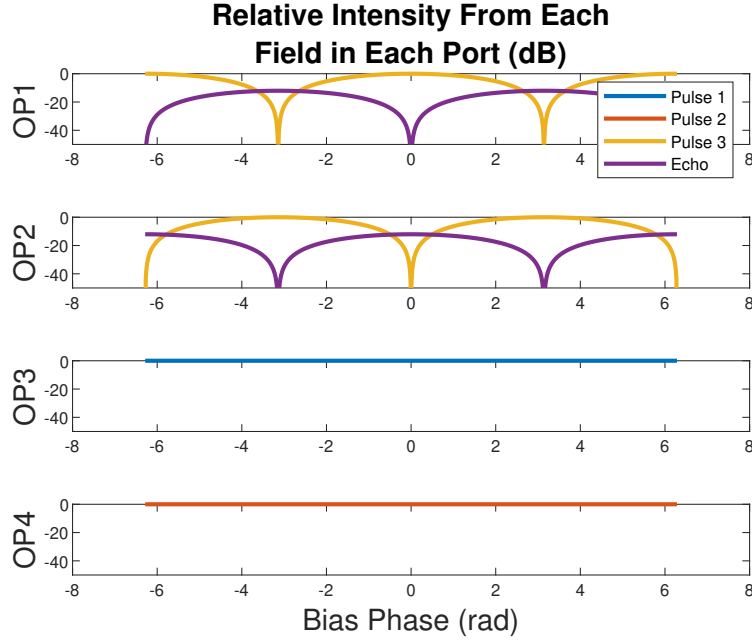


Figure 4.16: The relative intensity in each port from each field as a function of the bias phase on channel 3 right after the first input splitter.

the field to be entirely directed into another port, phase biases like this in the central portion of the device will lead to only partial redirection of fields. In this case, the peak leakage of each field is $1/4$ of the power in each field. In this condition, $1/4$ of the power in each field is on each port, in which case the outputs resemble the outputs of four co-linear SPE schemes.

Summary The effects of relative phase shifts in a 4 port interferometer, which could manifest as small path length deviations in an actual device, depend on where they occur in the device. Some phase shifts will affect the mixing of only two of the fields so long as one of those fields is not the echo. This is only an issue if those other fields must be separated from each other for some other reason. In cases where one of the two fields is the echo, this mixing is more detrimental because the echo is typically very weak, so leakage of other fields can obscure it. Phase differences within the two core Mach-Zehnders at the core of the device will lead to portions of each field being directed to each output port, including the

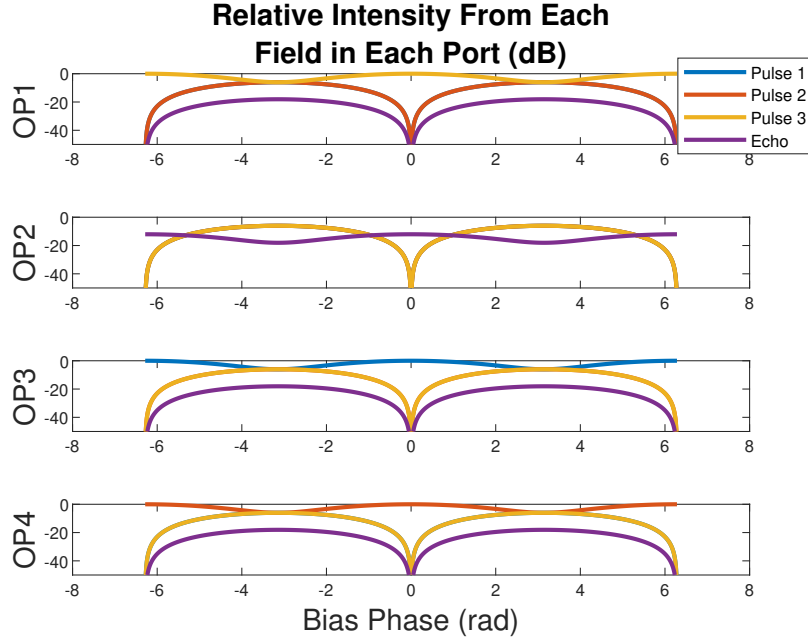


Figure 4.17: The relative intensity in each port from each field as a function of the bias phase on channel 1 right after the first input splitter.

echo port. Care must be taken to stabilize the phase biases within the device, which could lead to leakage of other fields into the echo port.

In experimental applications, these phase shifts would need to be accounted for. In free-space applications, it is likely that a corrective feedback scheme would be needed to control the phase. Integrated photonic applications systems tend to be more phase stable [35], so it may be less of an issue in integrated photonic systems.

Simulation of Four Port SPEs

The 4-port architecture for SPEs was simulated in an interferometric Maxwell-Bloch simulation. Simulations of both a simple echo sequence and optical correlation were conducted. Maxwell-Bloch Simulations allow for a full account of coherent transient effects in SSH materials.

Results: 4 Port Stimulated Photon Echoes Maxwell-Bloch simulations were conducted using the 4-port interferometer configuration with a peak Rabi frequency of π MHz. A pattern of 3 $\pi/2$ pulses was used (this meant the amplitude of the input fields to the interferometer had to be increased so that a $\pi/2$ pulse was directed into each material spot because by spreading the fields into four spots the intensity was decreased). Simulations were conducted over a duration of 30 μs at 140 MSPS. 70 MHz of bandwidth was used and divided into 0.01MHz bins with a decoherence time (T_2) of 10 μs . The $\frac{\pi}{2}$ pulses were injected into the interferometer at 2 μs , 4 μs , and 24 μs . Pulse 1 was injected into input port IP 1, pulse 2 was injected into port IP 2, and pulse three was injected into port IP 4. The interferometer was biased so that $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$. The results can be seen plainly when the resulting intensities are displayed on a linear scale. Figure 4.18 shows the three pulses and the echo, each isolated into a port as expected. When presented on a logarithmic scale, a few extra fields are present. These fields are the result of 2 pulse photon echoes between pulse pairs. Despite the extra fields, the echo is still well isolated. Figure 4.19 shows the results in the logarithmic scale. In the interferometric configuration, higher-order echoes have nowhere to go outside of the interferometer. Therefore, the echoes will end up somewhere within the interferometric network. In the “box-geometry”, this is not the case; the $\vec{k}$ relationships will require that 2-pulse echoes are directed onto $\vec{k}$ vectors that are not part of the detection scheme. The stimulated echo is not accompanied by higher-order echoes in the box geometry. Despite this limitation, even in the 4-port interferometer case in this simulation, only the SPE is directed out of port 3.

Results: 4 Port Correlation To demonstrate 4-port interferometric optical correlation, barker codes can be used as a test signal because they have a known auto-correlation. The barker code is a binary code that consists of signed bits in a specific pattern. Barker codes

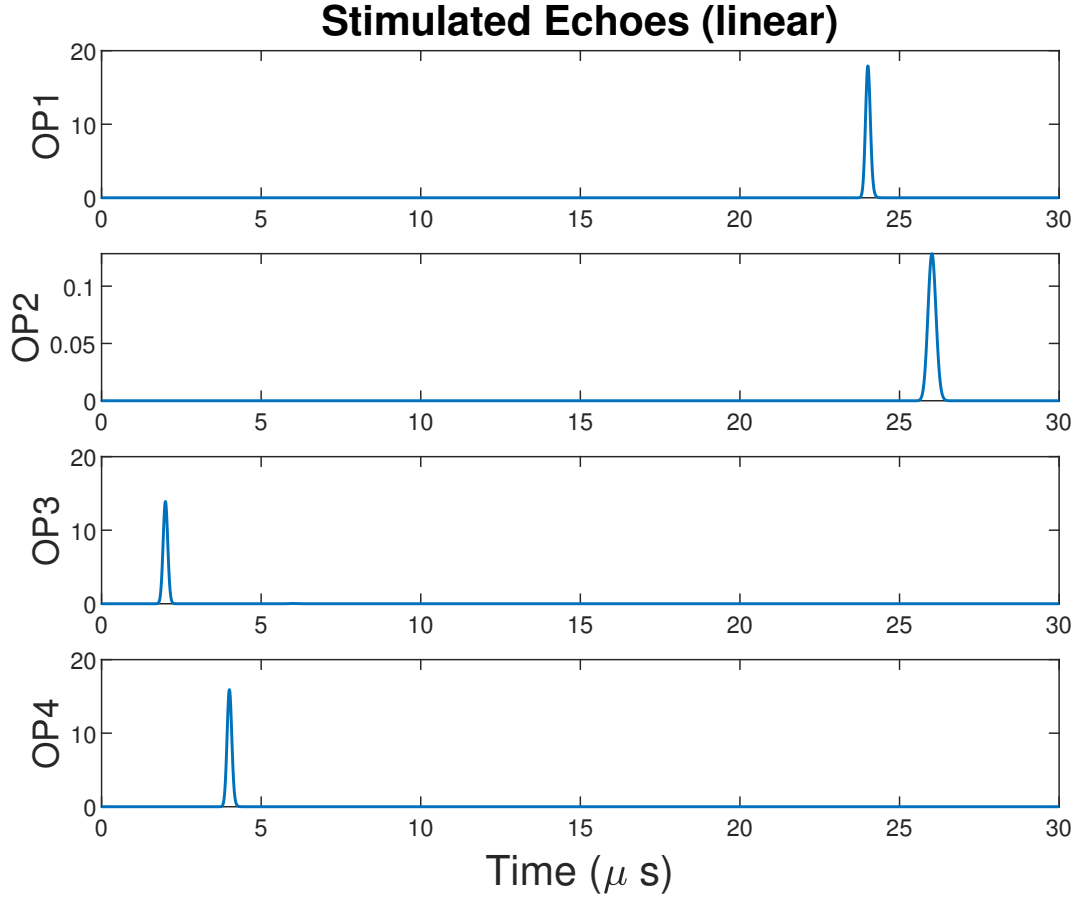


Figure 4.18: Stimulated echoes in a 4-port interferometer. The primary fields of interest are isolated in output ports

have a known auto-correlation, so they are a valuable diagnostic for correlation. A 5-bit barker code is shown in figure 4.20.

The barker codes for this simulation were generated as a series of 5 signed Gaussian pulses with a peak Rabi frequency of 0.1 MHz. These relatively weak pulses each had a nominal pulse area of $\frac{\pi}{20}$; the time delays between pulses in the barker codes were $0.5 \mu\text{s}$. One barker code was injected in the interferometer at $1 \mu\text{s}$ via port 1 to serve as a template for correlation. A Gaussian $\frac{\pi}{2}$ pulse of peak Rabi frequency 1 MHz. (acting as a time reference) was injected into port IP2 at $7 \mu\text{s}$. This sequence programmed the barker code

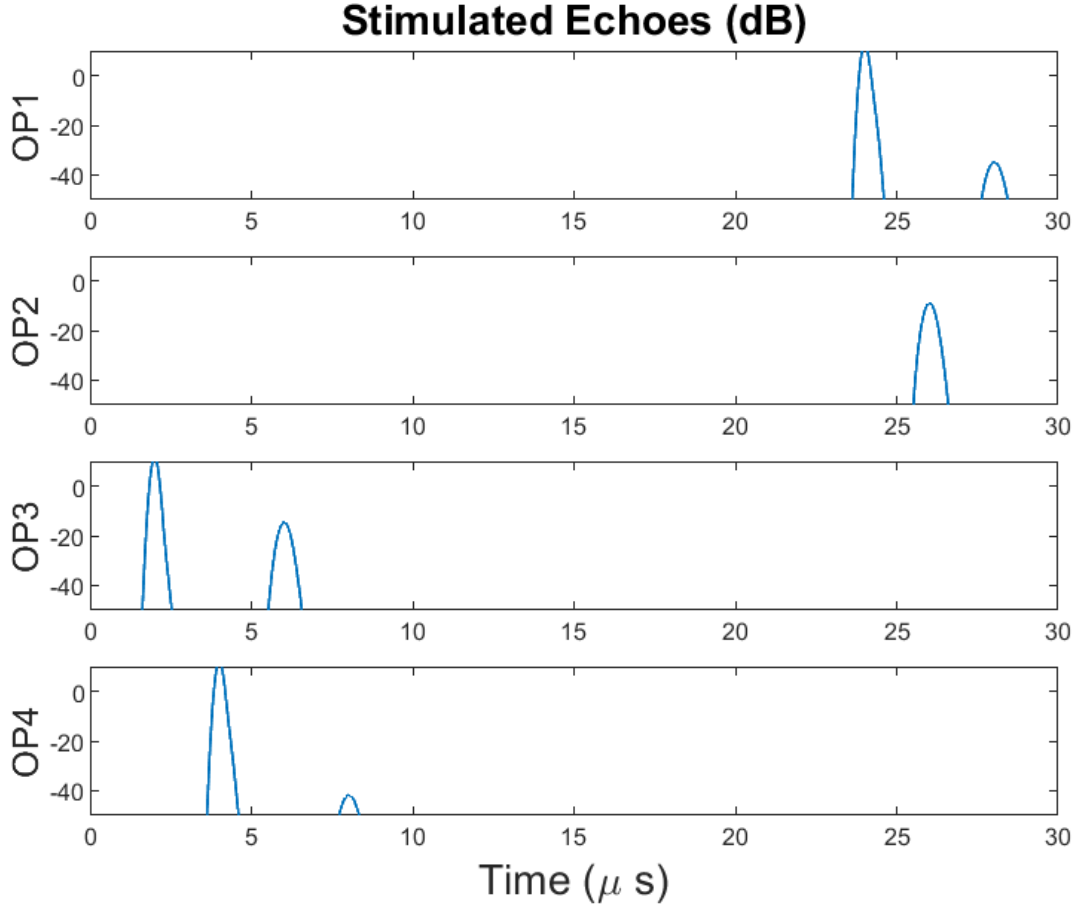


Figure 4.19: Fields in the output ports of a 4-port interferometer. The echo is directed out of port OP2 and is isolated; in addition to the SPE and the injected pulse pattern, several 2-pulse photon echoes between pulse pairs are still present.

delay sequence into the material as a series of gratings. The system was then interrogated with a “signal of interest” at $22\mu\text{s}$ when a second barker code was injected into port IP3. Figure 4.21 presents results in a logarithmic scale.

As in the case of the photon echoes, the correlation echo signal is directed out of a separate output port. This indicates that the correlation signal can be isolated into a unique port using the four-port interferometric architecture.

Barker Codes

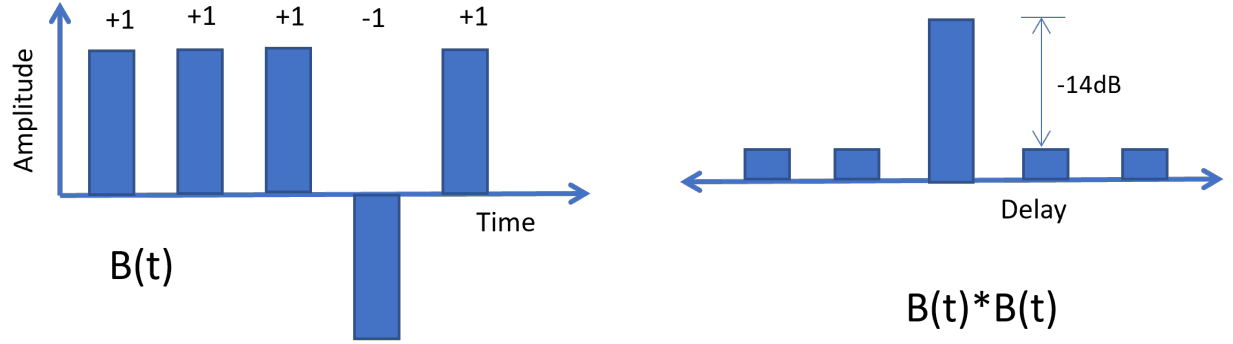


Figure 4.20: Left: The 5-bit barker code. Right: auto-correlation of the 5-bit barker code. It consists of a central lobe flanked by side lobes -14dB down from the central lobe.

Discussion and Conclusions

The relative optical phases of stimulated photon echoes generated in spatially separated ensembles can be set by the relative phases of the pulse sequence. The echo phase in each spot can be chosen to support constructive interference on one output port allowing for the isolation of the echo field. This process was demonstrated experimentally in a 2-port Mach Zehnder to isolate the echo pulse from the final probe pulse. A more complicated 4-port interferometer was designed to isolate the echo from the other three pulses. The operation of the 4-port method was demonstrated in Maxwell-Bloch simulations. Isolation of stimulated echoes can be helpful for applications where a stronger field obscures the relatively weak echo. Interferometric methods allow this separation to be conducted in co-linear geometries, which are suitable for applications in photonic integrated circuits.

Theoretical results indicated that the two-port echo scheme is susceptible to the interferometric tuning of the interferometer and the split ratio of the splitters used in

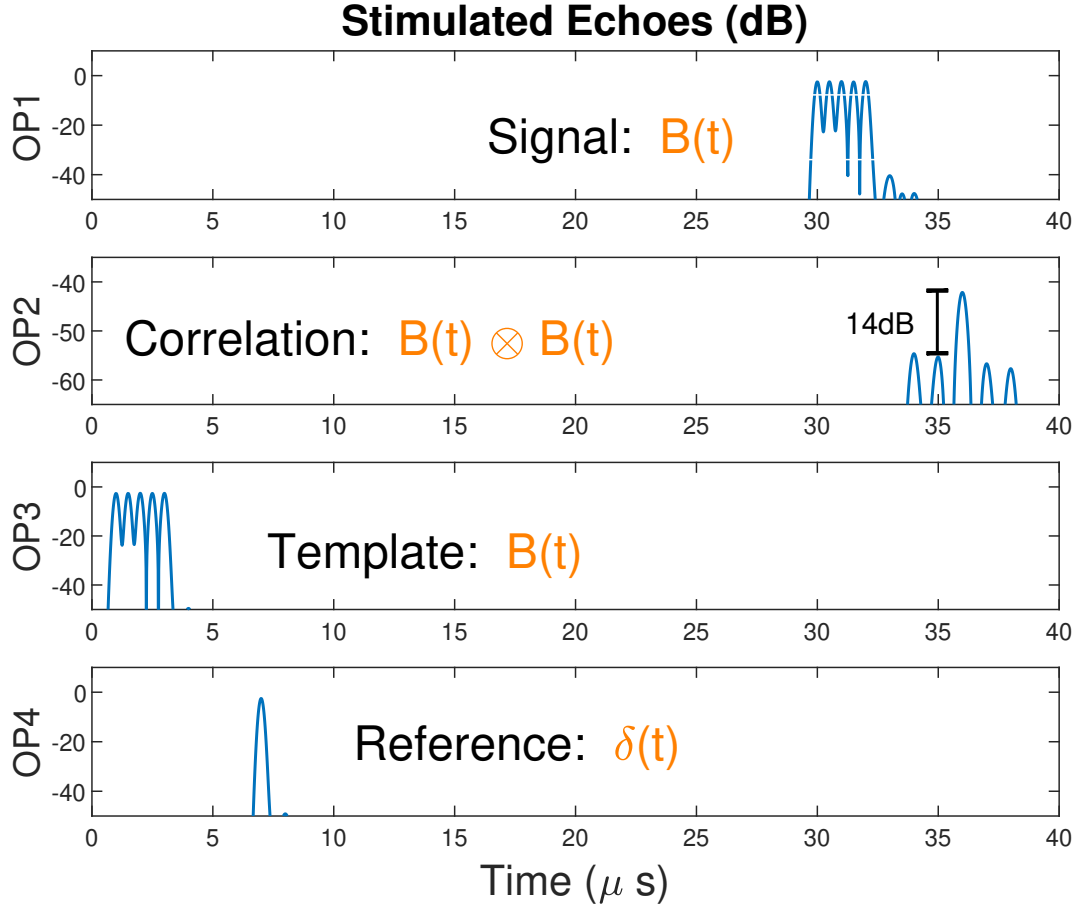


Figure 4.21: Optical correlation demonstration using a 5-bit barker code. The first barker code acts as a template for programming, and a brief Gaussian serves as a time reference; these two pulses program a grating that the signal can be correlated against. As expected, there is a central lobe with two side lobes on each side around 14dB below the central lobe.

the output splitter network. Similar results for four-port systems indicated that similar tuning methods could be used to optimize a four-port interferometer. Control systems must be developed to hold such a four-port interferometer in the correct phase bias configuration. These tolerances will likely be some of the most significant technical challenges to implementing interferometric networks in photonic integrated circuits.

Interferometric methods present a few other challenges. Firstly because SSH processes are nonlinear, dividing fields into multiple spots will result in lower efficiencies because the

process needs to be conducted independently in multiple locations meaning the echo fields will be weakened non-linearly; the higher the number of ports, the more substantial this effect will be. Secondly, because all of the fields are co-linear in the material, the wave vector of any coherent fields generated in the material interaction will be directed into the output optics of the interferometer so that extraneous echoes may be directed into directions within the interferometer. This was not observed in simulations. This could be an issue under certain circumstances. In contrast, in the “box geometry”, these extraneous fields are directed in directions not used for any of the primary fields of concern.

DEMONSTRATION OF STIMULATED PHOTON ECHO ISOLATION USING
INTERFEROMETRIC CANCELLATION

Contribution of Authors and Co-Authors

Author: Owen Wolfe

Contributions: Writing, Experiments, Theory.

Author: Tia Sharpe

Contributions: Experiments

Author: R. Krishna Mohan

Contributions: Theory

Author: Zeb W Barber

Contributions: Experiments and Theory

Owen was the primary author of this work conducting experiments, data processing, and developing the theory as well as conducting the primary writing. Tia Sharpe was instrumental in conducting the experiments. R Krishna Mohan provided invaluable insights in developing the theory and providing advice on the experiments. Zeb Barber helped in both developing the theory and the execution of experiments regarding this work.

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Demonstration of Stimulated Photon Echo Isolation Using Interferometric Cancellation

OWEN WOLFE^{1*}, TIA SHARPE¹, R. KRISHNA MOHAN¹, AND ZEB W. BARBER¹

¹Spectrum Lab, Montana State university, P.O. Box 173510 Bozeman, MT 59717-3510

*Corresponding author: owen.wolfe@montana.edu

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An alternative to phase matched angled beam spatial-spectral holographic grating geometries for separating stimulated photon echoes from the probing pulse is proposed and demonstrated. By use of a Mach-Zehnder geometry with inhomogeneously broadened medium in both paths, the stimulated photon echo can be interferometrically isolated from the generating probe pulse. This interferometric based technique is well-suited to waveguide geometries, which have benefits for future quantum and classical optical signal processing applications such as quantum memories, correlation, and efficient cryogenic microwave-to-optical conversion. Experimental demonstrations showing interferometric based isolation in Tm3+ doped LiNbO₃ at 3.2 K are performed and analyzed. © 2023 Optical Society of America

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Stimulated photon echoes (SPEs) in cryogenic rare earth spectral-hole burning materials have been proposed for a number of quantum and classical applications. In classical signal processing, high bandwidth real-time correlation [1, 2], true-time-delay for phased array beam steering (TTD) [3, 4], and time domain optical buffer memories [5] have been demonstrated. In quantum information processing, rare earth materials and SPE processes have been proposed for use as optical quantum memories and quantum repeaters including Atomic frequency comb (AFC) [6], Controlled Reversible Inhomogeneous Broadening (CRIB) [7], and for efficient cryogenic microwave-to-optical conversion [8]. The AFC protocol, in particular, is a straightforward SPE with spectral gratings specifically designed to provide high efficiency.

One major practical challenge for SPE processes arises when the photon echo delay is less than the length of the probe pulse, as can occur for non-Fourier limited structured pulses such as chirps or continuous signal streams [2, 9]. In these cases, the relatively small SPE is obscured by the generally much larger original pulse. To separate the probe pulse and the SPE, angled beam spatial-spectral holographic (SSH) techniques have been developed [1, 10]. These techniques use phase matching to generate the echo on a separate path from the probe (and/or

programming pulses) allowing independent detection of the SPE. However, these angled beam geometries require free space optics and bulk materials, which can be difficult to implement reliably, and more fundamentally, can constrain the light-material interaction because of optical diffraction. Methods suitable to waveguide geometries [11, 12] would allow enhanced sensitivity and efficiency due to extended light material interaction.

When two resonant optical pulses interfere in an inhomogeneously broadened medium, such as a cryogenic rare earth doped crystal, a grating is stored in the spatial domain and the spectral absorption structure of the material. Probing this grating with a third resonant pulse or signal results in the generation of a stimulated photon echo (SPE). The complex amplitude spectrum of the generated SPE at each point in the material, $\vec{r}$, is determined by the constituent pulses as

$$E_{echo}(\omega, \vec{r}) \propto E_1^*(\omega, \vec{r}) E_2(\omega, \vec{r}) E_3(\omega, \vec{r}), \quad (1)$$

where the fields E_1 , E_2 , and E_3 correspond to pulse 1, 2 and 3 respectively; and E_{echo} is the field of the stimulated echo. This formulation assumes that the sequential timing of the pulses are well-defined so that issues of causality are easily determined, otherwise more explicit time-causal forms are required.

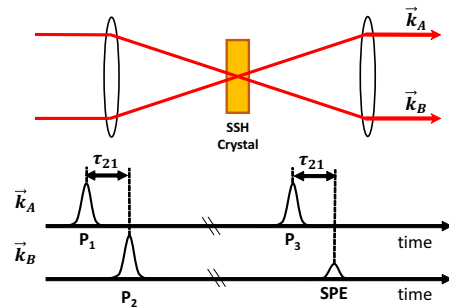


Fig. 1. In the angled beam geometry, pulses 1 and 3 are applied to the crystal at one angle, $\vec{k}_A$, while pulse 2 is applied at another, $\vec{k}_B$. From the phase matching condition Eq.2 the echo induced in the crystal propagates at the same angle as pulse 2.

For single beam SPE geometries the echo is emitted in the direction of the beam and is only separable from the other programming and probe pulses by time separation. For some applications the third pulse may be a pattern of pulses, which can temporally overlap with and obscure the resulting SPE, which in general is weak compared to pulse three. To isolate the echo from pulse three in space, the pulses can be applied to the medium at different angles. The summed Wave vector condition:

$$\vec{k}_{echo} = \vec{k}_2 + \vec{k}_3 - \vec{k}_1, \quad (2)$$

can be used to determine the direction the echo is emitted, which for isolation should be along a different spatial direction from pulse 3. For example, applying pulses 1 and 3 along one path $\vec{k}_A$ at a slight angle from the path of pulse 2, $\vec{k}_B$ will phase match the echo to propagate along $\vec{k}_B$, or in the same direction as pulse 2 separating the echo from pulse 3 spatially as shown in Fig. 1. If $\vec{k}_1, \vec{k}_2, \vec{k}_3$ are all along independent directions the echo can be emitted in an independent direction separable from all the previous programming and probe pulses [10]. While these angled beam geometries have been used successfully, they can be difficult to align and can severely constrain the combination of interaction length and optical intensity in the medium that together governs the strength of the light-matter interaction.

In this letter, we propose and demonstrate an interferometric technique to isolate the SPE from pulse 3, as an alternative to angled beam geometries. The technique generates two SSH gratings in the separate paths of a Mach-Zehnder interferometer (Fig. 2), which when probed generate echoes with a 180 degree relative phase difference so that they add constructively at the output of the interferometer, which is the dark port of pulse 3. The collinear geometry used in each path allows the use of single mode waveguides, and the interferometer geometry can be constructed using waveguide based 2x2 junctions raising the possibility of allowing highly integrated photon echo based quantum or optical processing methods in rare earth doped crystalline waveguides rather than bulk crystals. Interferometric techniques have been used to test the phase preservation in two pulse photon echo based memory approaches [13] and in atomic spectroscopy [14], however, the interferometric photon echo isolation technique discussed here has not been previously described or demonstrated.

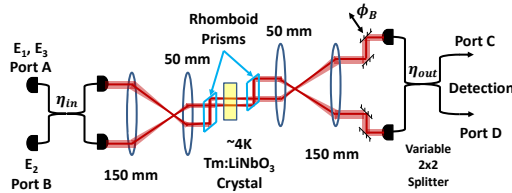


Fig. 2. A schematic of the Mach-Zehnder interferometer for stimulated photon echo experiments. Pulses are sent into the interferometer via input ports A and B to generate SPEs simultaneously on paths 1 and 2. A relative phase difference ϕ_B is introduced on path 1 to control how fields exit the interferometer on output ports C and D. In our experiments we used beams with a spot size around $200\mu\text{m}$ in the crystal and a separation of around 2mm .

The schematic for the technique and experiments is shown

in Fig. 2. Echoes are generated on both interferometric paths by sending pulse fields E_1 and E_3 into the interferometer via port A and pulse field E_2 into the interferometer via port B. The beam splitters in the interferometer introduce unknown phase shifts to transmitted and reflected fields, but without loss of generality we can describe these phase shifts as a single phase shift incurred by the reflected fields as ϕ_r (which is fundamentally constrained by symmetry to $\exp(i2\phi_r) = -1$ measured relative to the transmitted fields). For our analysis we have chosen the convention that $\phi_r = \pi/2$ or $\exp(i\phi_r) = i$. Fields on path 1 will incur an extra relative bias phase shift ϕ_B . Tracking the field E_3 through the interferometer and assuming perfect 50/50 beam splitters, the field is first split into the two interferometric paths as

$$E_{3,Path1} = \frac{i}{\sqrt{2}} E_3 \exp(i\phi_B) \quad (3)$$

$$E_{3,Path2} = \frac{1}{\sqrt{2}} E_3 \quad (4)$$

The fields on these two paths are then combined at the output ports of the interferometer as:

$$\begin{aligned} E_{3,C} &= \frac{1}{\sqrt{2}} [E_{3,Path2} + iE_{3,Path1}] \\ &= \frac{1}{2} E_3 [1 - \exp(i\phi_B)] \end{aligned} \quad (5)$$

Similarly

$$E_{3,D} = \frac{i}{2} E_3 [1 + \exp(i\phi_B)] \quad (6)$$

If $\phi_B = \pi$ then the entirety of the field E_3 will exit the interferometer via port C. To explain how the interferometer can separate the SPE from E_3 , we must develop the SPE fields through the interferometer. Using the bias phase to determine the phases of the fields in the two interferometric paths within the medium, the independent generation of stimulated photon echoes on each path is estimated from Eq. 1 giving echo fields of

$$\begin{aligned} E_{echo,path1} &\propto (i)(-i)2^{-3/2} E_1^* E_2 E_3 \exp(i\phi_B) \\ &\propto 2^{-3/2} E_1^* E_2 E_3 \exp(i\phi_B) \end{aligned} \quad (7)$$

for path 1 and

$$E_{echo,path2} \propto i2^{-3/2} E_1^* E_2 E_3 \quad (8)$$

for path 2. These fields are combined at the output ports of the interferometer as

$$\begin{aligned} E_{echo,C} &= \frac{1}{\sqrt{2}} [E_{echo,path2} + iE_{echo,path1}] \\ &\propto \frac{i}{4} E_1^* E_2 E_3 [1 + \exp(i\phi_B)] \end{aligned} \quad (9)$$

for port C, and

$$E_{echo,D} \propto \frac{1}{4} E_1^* E_2 E_3 [\exp(i\phi_B) - 1] \quad (10)$$

for port D. Under the condition $\phi_B = \pi$, the entirety of the echo is directed through port D, which is opposite that of pulse 3, regardless of the input phases of any of the pulses. This implies that pulse 3 and its echo(es) are complementary. If $\phi_B = \pi/2$, then half of the echo energy is emitted out of both ports and if $\phi_B = 0$, pulse 3 and echo ports are switched to D and C respectively. The interferometer phases at $\phi_B = 0$, and π

separates the SPE field from E_3 . This behavior is very similar to the angled beam geometry echo in 1, where the ports of the interferometer serve as the angled paths. Indeed, one sends pulses 1 and 3 through one input port of the interferometer and pulse 2 through the other, and the SPEs are observed out the same port as pulse 2 isolated from pulses 1 and 3.

The above analysis assumes ideal 50/50 beam splitters and perfectly matched paths in the interferometer. For the experimental, non-ideal case, we define the performance of the splitters as a power ratio, $\eta = R/T$, where T and R are the transmittance and reflectance of the beam splitter, or more commonly output power ratio when power is incident on one input of the splitter. Since the interferometer that we used for our experiments was comprised of a combination of free-space optics and fiber beam splitters, we also account for the relative coupling efficiencies into the fiber splitter at the output with coefficient C_1 and C_2 .

To maximize the isolation performance between pulse 3 and the echo, we need to determine how the performance depends on split ratios and coupling coefficients described above, to do this we define contrast ratios. The contrast ratio for one field at one port is the ratio of the maximized transmitted power on a port to the minimized transmitted power of the same field on the same port. By tracking fields through the interferometer and using the phase relations according to Eq.1 for the SPE fields, we find that the contrast ratio of pulse 3 and the echo on port D are the same and can be estimated as:

$$\text{Contrast}_1 \propto \left[\frac{1 + (\eta_{in}\eta_{out}C_2C_1^{-1})^{1/2}}{1 - (\eta_{in}\eta_{out}C_2C_1^{-1})^{1/2}} \right]^2 \quad (11)$$

On Port C the pulses again have the same contrast ratio which can be estimated as

$$\text{Contrast}_2 \propto \left[\frac{1 + (\eta_{in}\eta_{out}^{-1}C_2C_1^{-1})^{1/2}}{1 - (\eta_{in}\eta_{out}^{-1}C_2C_1^{-1})^{1/2}} \right]^2 \quad (12)$$

From these equations it can be seen that the contrast for both ports is the same in the special case $\eta_{out} = 1$ (i.e a 50/50 split ratio). When the contrast is maximized simultaneously on both ports, the entirety of the SPE will pass through port C and all of pulse 3 will pass through port D. A perfect output split ratio of 50/50, will therefore be a condition for optimal contrast. However, if a trade-off between echo intensity and echo isolation must be made it is preferable to prioritize echo isolation therefore maximizing the contrast ratio of pulse three on port D is preferred.

In the experiment, since most fixed fiber beam splitters can deviate significantly from the nominal split ratio, a variable splitter was used at the output, so that the split ratio could be tuned very close to a perfect 50/50. The coupling efficiency into the fibers also offers an additional experimental degree of freedom. Tuning the coupling efficiencies, by slightly misaligning one of the fiber coupler inputs, can compensate for an imperfect split ratio at the input splitter at the cost of reducing the output power in the echo, for better suppression of pulse 3.

Since the amplitude of the echo will be much less than the amplitudes of the constituent pulses, maximizing the power of the SPE will have major effects on the relative power between the SPE and residual pulse 3 in that port. One condition for maximum SPE amplitude is using excitation pulses with close to $\pi/2$ pulse area for programming the grating on each path. In

addition, since the peak amplitude of the SPE is proportional to the product of its constituent pulses (Eq.1), increasing the power of the pulses used to generate the SPE will result in higher power in the echo and therefore better contrast.

We conducted experiments to demonstrate the technique using a combination free-space / fiber Mach-Zehnder interferometer using a 1 cm long 0.001% Tm3+ doped LiNbO₃ crystal to generate the photon echoes. We used acousto-optic modulators (AOMs) to carve out the programming and probe pulses from CW 794.25nm light resonant with the $^3\text{H}_6$ to $^3\text{H}_4$ transition. Pulses 1 and 3 were coupled into one port of a 50/50 fiber beam splitter and pulse 2 was coupled into the other. The pulses combine in the splitter, and are sent to the two interferometer paths. Both paths pass through the Tm3+ doped LiNbO₃ crystal in a cryostat at 3.2 K using the same focusing and collecting optics, but do not overlap each other in the crystal. SPEs are generated on both paths (with a phase corresponding to Eq.1) and the paths are then fiber coupled and recombined using a 50/50 variable fiber splitter interfering coherently and forming the output ports of the interferometer.

The variable splitter and the fiber couplings were tuned to achieve maximal contrast on pulse 3. We built the interferometer with a piezoelectric mounted mirror to control the relative phase of the interferometer. This allows us to test the contrast of the interferometer by slowly modulating (< 1 Hz triangle wave) the path length difference (phase) while reiterating the 3-pulse echo sequence in the interferometer at a rate (> 10 Hz) much faster than the path length modulation. By saving successive oscilloscope traces containing signals from a single port (see Fig.3) we can track the contrast over the changing bias phases. Performing the measurement this way was easier than trying to hold (or stabilize) the phase of the interferometer constant as it drifts due to air currents and temperature drifts in the fiber.

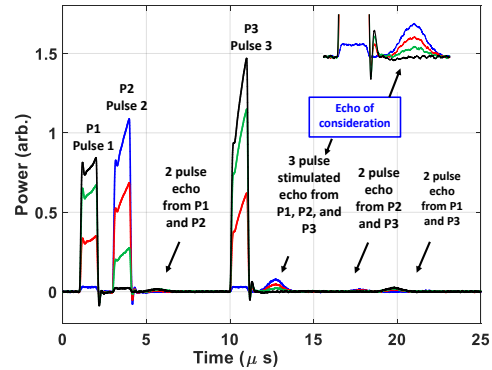


Fig. 3. Four successive traces of optical power at one of the output ports that contain the 3 pulse pattern and the SPE while the interferometer bias is being slowly modulated. Here all three pulses and the SPE are displayed as well as a series of other 2 pulse photon echoes corresponding to the different delays between the pulses.

As can be seen in Fig. 3, the output observed in one port of the interferometer, when pulses 1 and 3 are maximized due to the interferometer phase, pulse 2 and the SPE are minimized and vice versa. To determine the maximum contrast of the

interferometer, the integrated energy of each pulse (including the SPE) was measured and recorded for each sequence over many cycles of the path length modulation. This leads to a sinusoidal modulation of the pulse powers measured on a single port, which when plotted on a semilogarithmic axis allows the basic contrast of the different pulses to be easily read off. Figure 4 shows that the contrast for pulse 3 is > 23 dB. In addition, it can be seen that despite the energy efficiency of the SPE being only $\approx 2\%$, when the echo is maximized it is larger than the residual of pulse three by a factor of up to 4.

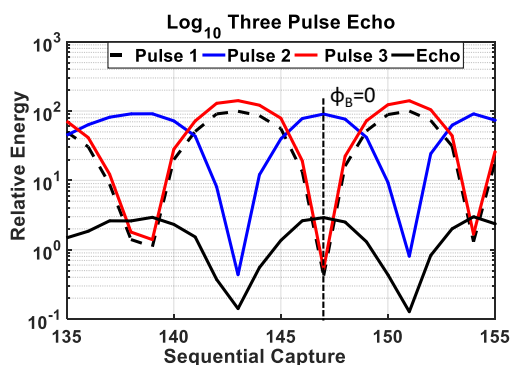


Fig. 4. The energy of each pulse and the SPE for each successive trace measured at one output port of the interferometer while the phase of the interferometric path is modulated with a slow triangle wave. Note that when the energy of the SPE is maximized the energy of pulse 3 minimized, importantly when the SPE is minimized its energy exceeds the energy of pulse 3.

The suppression of pulse 3 and therefore the signal to background ratio of the echo is strongly dependent on the balance and tuning of the interferometer. Measuring the contrast by looking at the minimum of a slow sinusoidal modulation limits the speed at which various parameters can be tuned to maximize the contrast. Using a servo to stabilize the interferometer would allow for more rapid iteration and tuning of the interferometer to further optimize the contrast. In addition, although polarization maintaining fibers were used, we found that polarizing the beams after the crystal to ensure only one polarization was launched into the final beam splitter was important for maximizing suppression and thereby contrast. Finally, despite low echo efficiency, we demonstrated a signal to background ratio, defined as the ratio between the maximized echo power and the minimized power of pulse 3, ranging between 3 dB and 6 dB.

Although the contrast still has room for improvement, we have demonstrated that interferometric methods show potential for isolating the stimulated photon echo from the generating probe pulse. This interferometric method can be used for waveguide photon echo geometries where angled beam geometries are not possible. Ultimately, one could imagine this technique could be applied directly using LiNbO₃ (or other material) photonic lightwave circuits with interferometer sections that are doped with Rare Earths for storage or processing.

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DESIGN CONSIDERATIONS FOR INTERFEROMETRIC SSH READOUT SYSTEMS

Systems for reading out programmed spectra in SSH materials are complicated and require consideration of a wide-ranging set of parameters that must be coordinated with each other to properly implement the programming and readout of SSH materials. Figure 6.1 gives a visual indication of the degree of complexity required of SSH readout experiments. This chapter will describe the subsystems used for interferometric SSH readout experiments and some of the design considerations required to set up these experiments.

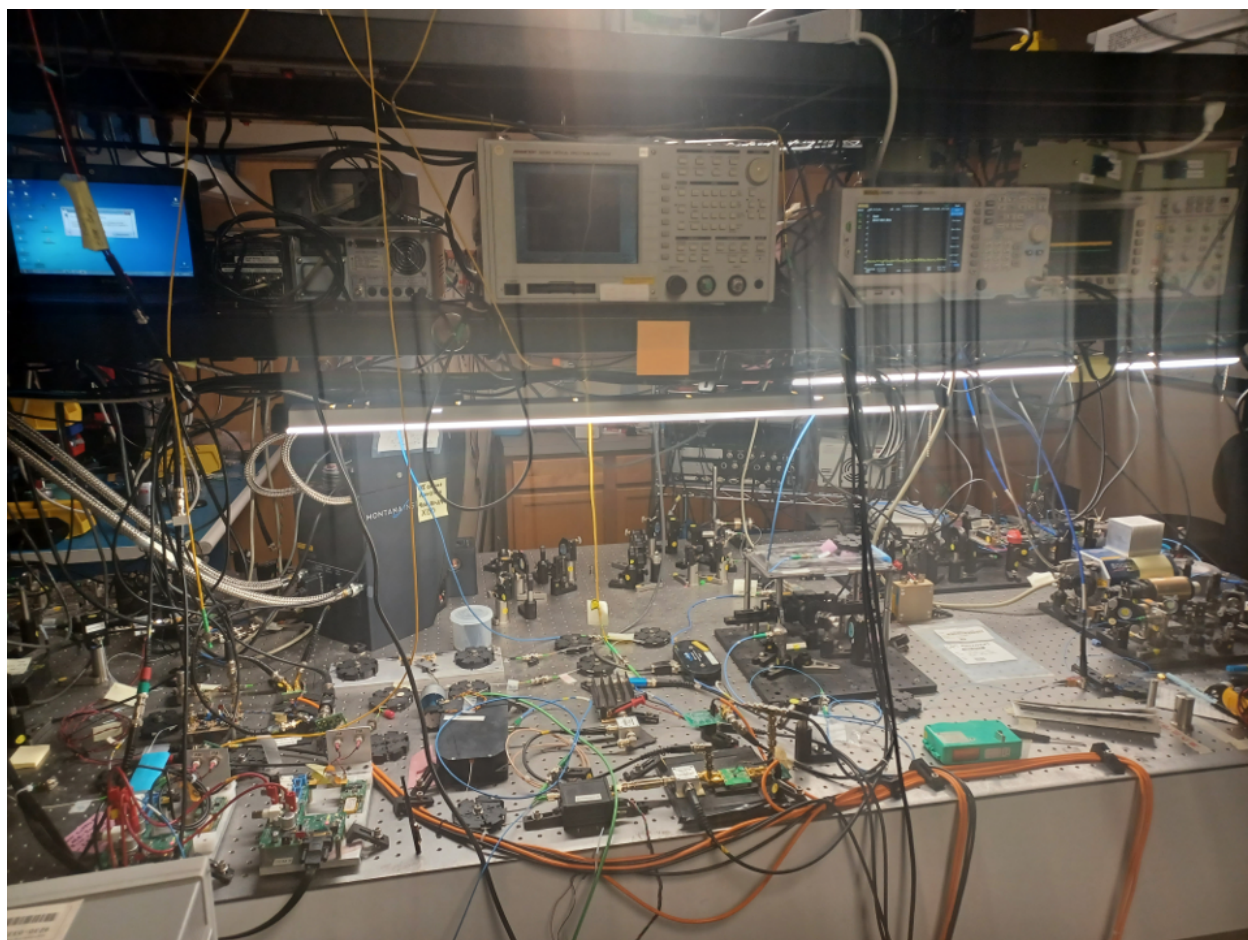


Figure 6.1: SSH readout systems are very complicated and contain several subsystems that need to be coordinated with each other.

Material Considerations

Many of the experimental parameters for SSH readout experiments are governed by the parameters of the SSH material, such as the coherence time (T_2), saturation conditions, and center wavelength. Therefore, the characterization of these parameters is a prerequisite for decision-making regarding other experimental parameters. In the conducted SSH readout experiments, a $10\text{mm} \times 10\text{mm} \times 9\text{mm}$ sample of $0.01\% \text{ Tm}^{3+} : \text{LiNbO}_3$ was used. The sample in question is pictured mounted in the Montana Instruments closed cycle cryostat used to hold the sample at 3K in figure 6.2.



Figure 6.2: A $10\text{mm} \times 10\text{mm} \times 9\text{mm}$ sample of $0.01\% \text{ Tm}^{3+} : \text{LiNbO}_3$ mounted in a Montana Instruments closed cycle cryostat

Long Pulse Measurements

As was discussed in chapter 3, when a long optical pulse is incident on an SSH material, the field will undergo optical nutation [3, 75]. First, the leading edge of the pulse is absorbed. The instantaneous power at the moment when the pulse is switched on represents the power attenuated to the maximum degree possible. The material interacting with the long pulse will undergo Rabi oscillations, resulting in a nutating electric field. For a Gaussian beam interacting with an optically thin material ($\alpha_0 L < 1$), this behavior can be modeled using equation 6.1 [64, 75].

$$P_{out}(t) = P_{in} \left(1 - 2\alpha_0 L e^{-\frac{t}{T_2}} \frac{J_1(\Omega_0 t)}{\Omega_0 t} \right) \quad (6.1)$$

Where P_{out} is the optical power out of the material, P_{in} is the power into the material, Ω_0 is the average Rabi frequency, and T_2 is the material coherence time, this model is a reasonable approximation for the behavior in an optically thin material however, it falls apart for optically thick materials [75]. However, the functional dependence of the nutation signal for thicker materials is still roughly similar, i.e., $P_{out} \propto \frac{J_1(\Omega_0 t)}{\Omega_0 t}$ for Gaussian beams [64] (the dependence differs from the dependence described in 3 since simulation results assumed plane waves, not gaussian beams), optical nutation can be used as a method for conducting measurements of two key material parameters, the absorption length, and the Rabi frequency.

Absorption Measurements

As was discussed in chapter 3. The absorption length of a material can be calculated using a long pulse measurement with equation 3.27 [75], and the rabi frequency can be estimated. Long pulse measurements were conducted using the 0.01% $Tm^{3+} : LiNbO_3$ sample used in S2SA and SSH grating processing experiments at various laser frequencies. Using equation 3.27, the absorption length could be calculated at each detuning. This allows for a mapping of the inhomogeneous line shape for this sample. Absorption measurements

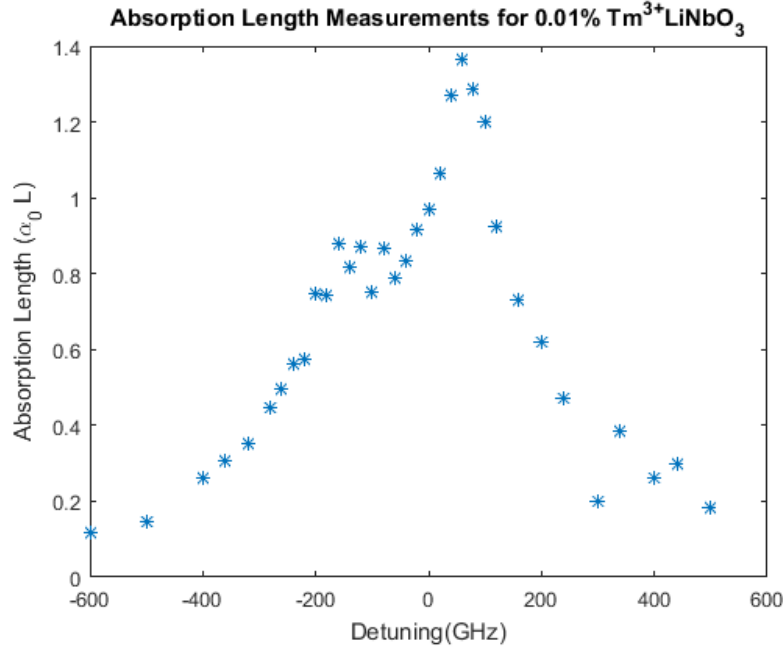


Figure 6.3: Absorption Length measurements for the sample of 0.01% $Tm^{3+}LiNbO_3$ were used for experiments.

indicated a peak absorption length of 1.39 (experiments were conducted within 2GHz of zero detuning). The lineshape corresponds well with other inhomogeneous lineshape measurements done of $Tm^{3+} : LiNbO_3$ [63].

Rabi Frequencies and Nutation Typically Rabi frequency measurements in SSH materials are conducted using optical nutation experiments by finding the point of maximum transmission in the nutation signal. Unfortunately, due to a poor material coherence time, it was not possible to use the simple peak measurement method. Looking at a nutation trace near the center of the material resonance frequency of the sample (see figure 6.3), the normal oscillatory start-up behavior of the nutation signal cannot be seen, so an alternative method of measurement is required. The Rabi frequency can be estimated by modeling fitting the start-up behavior of the nutation signal [64]. Fitting the data to the functional form of the nutation signal in the start-up behavior, a Rabi frequency of 1.5MHz was estimated, using

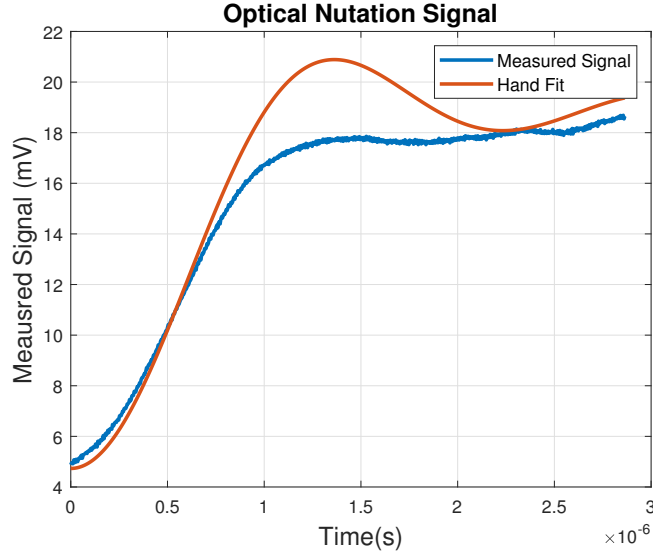


Figure 6.4: Nutation data from the material sample under consideration. Unfortunately, the coherence is washed out by a low T_2 , so the nutation signal was fit by attempting to replicate start-up behavior assuming the correct functional dependence.

15mW of optical power confined to a $300\mu m$ spot (see figure 6.4).

T2 Measurement

The material coherence time governs the homogeneous linewidth of the material and therefore controls spectral resolution. Initial T_2 measurements using 2 Pulse photon echo experiments indicated a coherence time of $1.6 \mu S$; however, later experiments did not correspond well with these results. New T_2 measurements were conducted using spectral gratings. The average electrical power measured in an SSH grating experiment will depend on the time delay (τ_{delay}) and the coherence time according to equation 6.2.

$$P_{RF}(\tau_{delay}) = P_{RF}(\tau_{delay} = 0)e^{\frac{-4\tau_{delay}}{T_2}} \quad (6.2)$$

The grating power at a series of different delays was measured for the sample, and using a linear fit in the logarithmic scale, the coherence time was measured to be roughly 260ns,

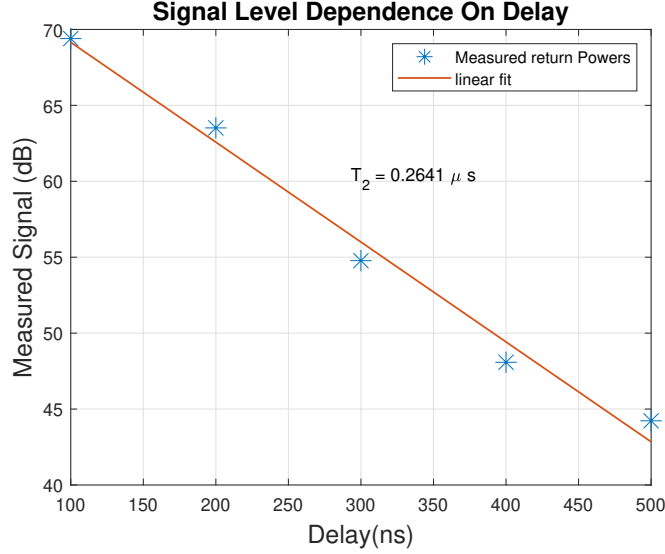


Figure 6.5: T_2 measurement for the sample of Tm^{3+} : $LiNbO_3$ had a coherence time of roughly 260 ns

as shown in figure 6.5. Attempts were made to improve the mounting of the sample into the material in the hopes that better thermal contact with the cryostat would decrease the material temperature and improve the coherence time. In the 3K range, the coherence time is susceptible to minor changes in the temperature. Remounting the sample led to over a 2-fold improvement in the coherence time was observed. The updated results are displayed in figure 6.6. The final coherence time of the material was around 600ns. It is important to note that the lower T_2 of 260ns was measured after spectrum analysis data was collected so data collected for SSH spectrum analysis assumes a 260ns coherence time while the later SSH lidar data corresponds to the 650ns data. The coherence time is related to the homogeneous linewidth by equation 6.3 [56].

$$\Gamma_H = \frac{1}{\pi T_2} \quad (6.3)$$

Therefore, the homogeneous linewidth for the system corresponds to $\Gamma_H = 1.2MHz$ for the crystal as mounted for spectrum analysis experiments and $\Gamma_H = 0.5MHz$ for grating processing experiments.

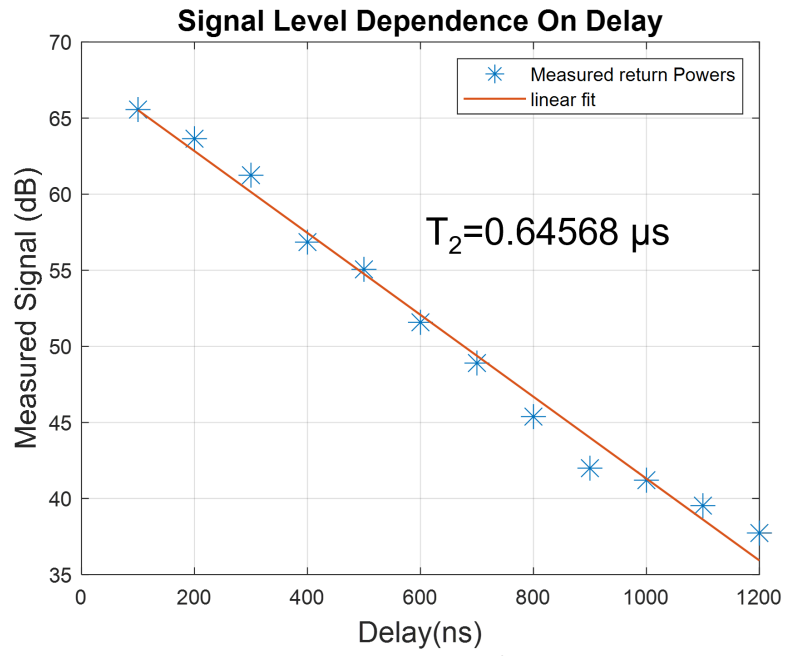


Figure 6.6: T_2 measurement for the sample of Tm^{3+} : $LiNbO_3$ had a coherence time of roughly 650ns

Typically the the laser linewidth and related phase noise would also play a role in the resolution of the material, however in this system, the measured laser linewidth (20KHz) was very narrow compared to the material linewidth so it has a negligible effect on the spectral resolution of the system. Phase noise will be discussed later in more detail in chapter 8.

Programming Considerations

Coherent Programming Programming considerations in the coherent limit are especially useful for understanding how the read chirp will coherently rearrange populations. The extent of these effects can be estimated using the Landau-Zener transition model. In the Landau-Zener transition model, the degree to which ions are excited by the read field is governed by a unit-less parameter called the Landau-Zener pump parameter. An optical field with a Rabi frequency Ω and a chirp rate κ will have a Landau-Zener pump parameter $\Phi = \frac{\Omega}{\sqrt{\kappa}}$ [25]. However, some calibration needs to be done to extrapolate the Rabi frequency from the experiment used to measure that quantity to apply that data to an arbitrary read field. Since Rabi frequency measurements were conducted at a specific spot size and optical power (150 μm radius spot at 15mW of optical power). The Rabi frequency can be estimated for a given spot size (w) and power (P) based on the measured Rabi frequency from the calibration experiment (Ω_{cal}), the calibration power P_{cal} and the calibration spot size (w_{cal}). The new Rabi frequency can be estimated using the fact that the ratio of the Rabi frequency to the root optical intensity is constant for a given sample (equation 6.4).

$$\frac{\Omega_{cal}}{\sqrt{\frac{P_{cal}}{\pi w_{cal}^2}}} = \frac{\Omega}{\sqrt{\frac{P}{\pi w^2}}} \quad (6.4)$$

Using this relation, the Landau-Zener parameter can be estimated for a chirp interacting with the material using equation 6.5[25].

$$\Phi = \sqrt{\left(\frac{P}{P_{cal}}\right) \left(\frac{\pi w_{cal}^2}{\pi w^2}\right) \frac{\Omega_{cal}}{\sqrt{\kappa}}} \quad (6.5)$$

From the Landau-Zener coefficient, the effect on the r_3 Bloch vector coordinate can be calculated using equation 6.6[25].

$$r_3(f) = 1 - 2e^{-\pi^2 \Phi^2} = 1 - 2e^{-\pi^2 \left(\frac{P}{P_{cal}}\right) \left(\frac{\pi w_{cal}^2}{\pi w^2}\right) \frac{\Omega_{cal}^2}{\kappa}} \quad (6.6)$$

In the limit where the programming is weak $\pi\Phi \ll 1$, this can be approximated using equation 6.7.

$$r_3(f) \approx -1 + 2\pi^2 \left(\frac{P}{P_{cal}}\right) \left(\frac{\pi w_{cal}^2}{\pi w^2}\right) \frac{\Omega_{cal}^2}{\kappa} \quad (6.7)$$

Assuming an initial condition of $r_3(f, t = 0) = -1$, relative changes to the absorption coefficient can be expressed in terms of changes to r_3 according to equation 6.8.

$$\frac{\Delta\alpha(f)}{\alpha_0} = -\Delta r_3(f) = -2\pi^2 \left(\frac{P}{P_{cal}}\right) \left(\frac{\pi w_{cal}^2}{\pi w^2}\right) \frac{\Omega_{cal}^2}{\kappa} \quad (6.8)$$

In the weak programming limit, this linear relation can be used to determine the effect of incidental programming due to the read chirp. Using the calibration values for the Rabi frequency, a read field chirp rate of $2MHz/\mu s$, and a programming spot size(radius) of 0.5 mm, the decrease to the absorption length for $200\mu W$ of optical power into the crystal corresponds to around 2.5%.

Incoherent Programming To estimate programming due to a long-duration programming pulse, numerical results from the 3-level rate equations can be used to determine the degree to which a field will program the material.

Experimental Parameters For Hole Burning Estimates Thulium ions in Lithium Niobate are resonant at 794nm via the $^3H_6 \rightarrow ^3H_4$ transition. The energy level diagram

for $Tm^{3+} : LiNbO_3$ is shown in figure 6.7. The excited state (3H_4) has a lifetime of $160\mu s$ with a branching ratio of 0.27 [67] into 3F_4 (the bottleneck state). The 3F_4 has a lifetime of $4.5ms$. The decay rates for the relevant transitions in $Tm^{3+} : LiNbO_3$ are summarized in table 6.1.

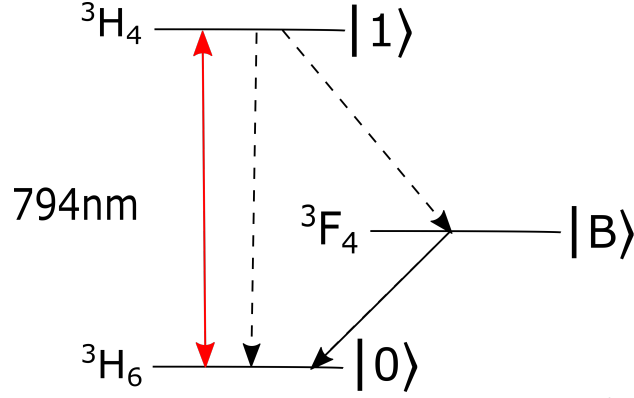


Figure 6.7: The relevant energy level diagram for $Tm^{3+} : LiNbO_3$

Quantity	Rate
R_1	$6250s^{-1}$
R_{10}	$1687.5s^{-1}$
R_{1B}	$4562.5s^{-1}$
R_{B0}	$222.22s^{-1}$

Table 6.1: Summary of total and partial decay rates of $Tm^{3+} : LiNbO_3$

The sample of $Tm^{3+} : LiNbO_3$ used for experiments has a concentration of 0.01% which corresponds to a number density of Tm^{3+} ions of $1.89 \times 10^{18}cm^{-3}$. The sample has physical

dimensions of $L = 10mm$, $W = 10mm$, and $H = 9mm$. The absorption length was around 1.39, giving an absorption constant of $\alpha_0 = 1.39cm^{-1}$.

The inhomogeneous line shape of a material is related to the absorption length given by equation 6.9.

$$\alpha(f)L = \sigma L N_v \Gamma_{hom} G(f) \quad (6.9)$$

where σ is the absorption cross-section, L is the length of the material, N_v is the number of absorbers, and $G(f)$ is the inhomogeneous line shape. Normalizing the absorption length curve allows for the calculation of the peak relative spectral density of Thulium Ions, which for this sample is about $0.002GHz^{-1}$, which corresponds to a volumetric number spectral density of $3.8 \times 10^{15} \frac{1}{cm^3 GHz}$.

Calculation of Programming Coefficient For programming by long pulses, the depth of programmed spectra can be estimated using the three-level rate equations described in chapter 2. This chapter will focus on programming coefficients in two key time regimes. Long time scales at which populations will stabilize, and the populations are no longer dependent on programming duration. This will give steady-state populations of each state as a function of incident spectral intensity. The second programming regime of interest is programming under the condition of a 3ms pulse (a condition that was used in readout experiments). Because $Tm^{3+} : LiNbO_3$ has a 4.5ms bottleneck lifetime this programming duration does not fall into the steady-state regime. The absorption coefficient can be calculated from the populations for a given effective spectral intensity (J_{eff}) and pulse duration τ_{pulse} using equation 6.10.

$$\alpha(J_{eff}, \tau_{pulse}) = \alpha_0 (\rho_{00}(J_{eff}, \tau_{pulse}) - \rho_{11}(J_{eff}, \tau_{pulse})) \quad (6.10)$$

The absorption coefficient can be approximated at low powers using the Taylor series given in equation 6.11.

$$\alpha(J_{eff}, \tau_{pulse}) \approx \alpha_0 + \frac{\partial \alpha(J_{eff}, \tau_{pulse})}{\partial J_{eff}} J_{eff} \quad (6.11)$$

In the linear regime, for a given programming pulse duration the constant $\chi(\tau_{pulse})$ determines how deep a power spectrum will burn itself into a material. In equation 6.12

$$\alpha = \alpha_0 (1 - \chi(\tau_{pulse}) J_{eff}) \quad (6.12)$$

where the coefficient χ is the burn parameter which is given by equation 6.13.

$$\chi(\tau_{pulse}) = -\frac{1}{\alpha_0} \frac{d\alpha}{dJ_{eff}} \quad (6.13)$$

The linear burn parameter can therefore be calculated by finding the slope of the quantity $1 - \frac{\alpha(J_{eff}, \tau_{pulse})}{\alpha_0}$, which is the change to the absorption coefficient due to programming, normalized to the unmodified absorption coefficient.

Continuous Programming Steady-state populations for various optical powers and normalized absorption coefficient calculated numerically using the expected rates for the previously described sample of 0.01% $Tm^{3+} : LiNbO_3$, and calculated with code shown in appendix B are shown in figure 6.8. The slope of the change to the normalized change to

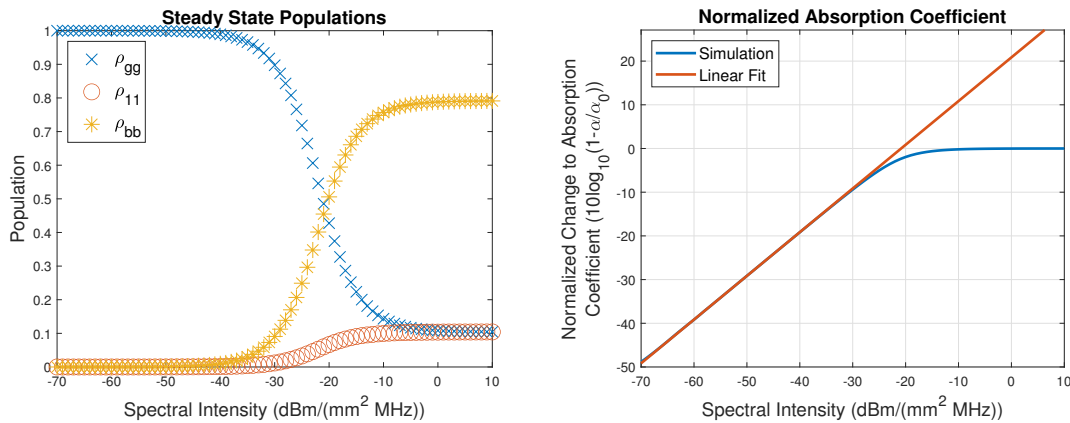


Figure 6.8: Left: Steady state populations for a 9mm long sample of 0.01% $Tm^{3+} : LiNbO_3$ Right: Changes to absorption coefficient due to programming in a 9mm long sample of 0.01% $Tm^{3+} : LiNbO_3$ due to programming, in the steady state programming limit.

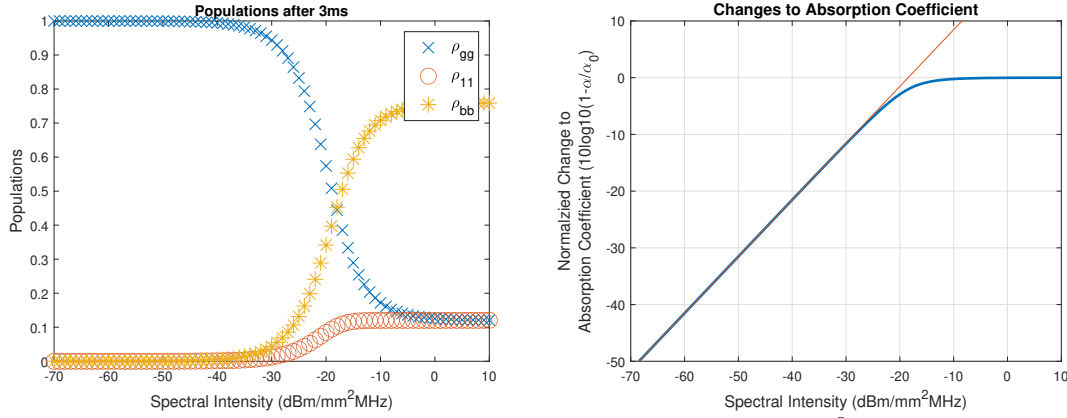


Figure 6.9: Left: Populations in a 9mm long sample of 0.01% $Tm^{3+} : LiNbO_3$ after a 3ms programming pulse. Right: Changes to absorption coefficient due to programming in a 9mm long sample of 0.01% $Tm^{3+} : LiNbO_3$ due to programming by a 3ms pulse.

the absorption coefficient was $\chi = 120 \frac{mm^2 MHz}{mW}$. The coefficient β (which will be used later to evaluate nonlinear noise) can be calculated for a given spot size as $\beta = \frac{120 \frac{mm^2 MHz}{mW}}{A_{spot}}$. The expected steady state programming coefficient for a 1mm diameter spot is $\beta = 160 \frac{MHz}{mW}$. 1dB Compression to the absorption coefficient occurs at roughly $0.0035 \frac{mW}{mm^2 MHz}$. For a 1 MHz wide spectral hole in a 1mm spot. Compression will occur at around -30 dBm of optical power.

3ms Programming Pulses Programming coefficients for a 3ms pulse can be estimated using simulations of the three-level rate equations for the duration of the pulse. Figure 6.9 shows the results of these simulations. Using these simulations the normalized change to the absorption coefficient was $\chi_{3ms} = 70 \frac{mm^2 MHz}{mW}$. The coefficient β can be calculated for a given spot size as $\beta_{3ms} = \frac{70 \frac{mm^2 MHz}{mW}}{A_{spot}}$. For a 3ms pulse, the programming coefficient for a 1mm diameter spot is $\beta_{3ms} = 89 \frac{MHz}{mW}$. From this system, it is estimated that 1dB of compression will occur at $0.0063 \frac{mW}{mm^2 MHz}$ or at around $-22 \frac{dBm}{mm^2 MHz}$ of optical power into the material.

RF Considerations In systems used for interferometric SSH readout experiments spectra are modulated onto the programming field using an electro-optic phase modulator. An electro-optic phase modulator will modulate the phase of an incident electric field (E_{in}) according to the drive voltage ($V(t)$) and the half wave voltage (V_{π}) as per equation 6.14.

$$E_{out}(t) = E_{in}(t)e^{i\frac{V(t)}{V_{\pi}}\pi} \quad (6.14)$$

Assuming a sinusoidal modulation $V(t) = V_0 \cos(\omega_{RF}t)$ the output field can be expanded using the Jacobi-Anger expansion. The output field, therefore, can be described in terms of a harmonic series with coefficients governed by the Bessel functions given in equation 6.15.

$$E_{out} = \sum_{n=-\infty}^{\infty} i^n J_n \left(\frac{V_0}{V_{\pi}} \pi \right) E_{in}(t) e^{in\omega_{RF}t} \quad (6.15)$$

Where J_n indicates the n^{th} Bessel function of the first kind. This harmonic series describes a signal with sidebands at integer multiples of the drive frequency. The relative strength of each sideband is governed by the ratio of the drive voltage to V_{π} . The relative power in the n th side band is given by equation 6.16.

$$P_n = J_n^2 \left(\frac{V_0 \pi}{V_{\pi}} \right) P_0 \quad (6.16)$$

At low RF voltages ($V_0 \ll V_{\pi}$), the Bessel function power dependence of the first sideband can be approximated according to 6.17.

$$P_1 \approx \frac{P_0}{4} \left(\frac{V_0 \pi}{V_{\pi}} \right)^2 \quad (6.17)$$

The power in the RF drive signal is $P_{RF} = \frac{V_0^2}{2Z}$ where Z is the impedance of the modulator. Therefore, in the low RF power limit, the power in the first side band will be linearly related

to the RF power driving the modulator according to equation 6.18.

$$P_{Optical} = \left(P_0 \frac{\pi^2 Z}{4V_\pi^2} \right) P_{RF} \quad (6.18)$$

This allows for conversion between RF power and optical power. At maximum, the first side band can contain up to 34% of the total optical power. This occurs at a drive voltage $V_0 = 0.58V_\pi$. This is the maximum drive power where the sideband power is monotonically increasing; this corresponds to RF drive saturation. In the linear response regime, for systems programmed with a phase modulator, an RF programming coefficient in analogy to the parameter β can be defined. this parameter is given (for a 3ms programming pulse) by equation 6.19.

$$\beta_{RF} = \beta_{3ms} \left(P_0 \frac{\pi^2 Z}{4V_\pi^2} \right) \quad (6.19)$$

Using 15mW of programming power and a 5V V_π , the RF programming efficiency is calculated to be $\beta_{RF} = 6.6 \frac{MHz}{mW}$ and 1dB of compression is expected to occur at about -9dBm of RF power.

Population Decay After programming fields are switched off, the bottleneck state will decay as a result programmed spectra will also begin to decay according to the bottleneck lifetime T_s . The absorption coefficient of a material programmed by a signal with an effective power spectrum ($S_{eff}(f)$) can be described as a function of time after the programming field is switched off according to equation 6.20 [71]

$$\alpha(f) = \alpha_0 \left(1 - \beta S_{eff}(f) e^{-\frac{t}{T_s}} \right) \quad (6.20)$$

In experiments, the timing was construed so that spectra were measured by a section of a chirp within $450\mu s$ of when the programming field was turned off. Given the 4.5ms bottleneck lifetime of $Tm^{3+} : LiNbO_3$, the decay of the programmed spectrum should lead to a change

to the hole depth of only 1dB.

Laser System for an SSH Readout System

Laser System Overview

Two separate optical fields are required to conduct SSH readout experiments. One field is required for programming spectra into the material. This field can take the form of any optical field resonant within the inhomogeneous line shape of the SSH material. The other needed field is a chirped field used to read out programmed spectra from the material. In order to preserve relationships between the frequency of the programming fields and the frequency of the chirp, the two lasers must be frequency stabilized to each other. An overview of the scheme used to do this in experiments is presented in figure 6.10.

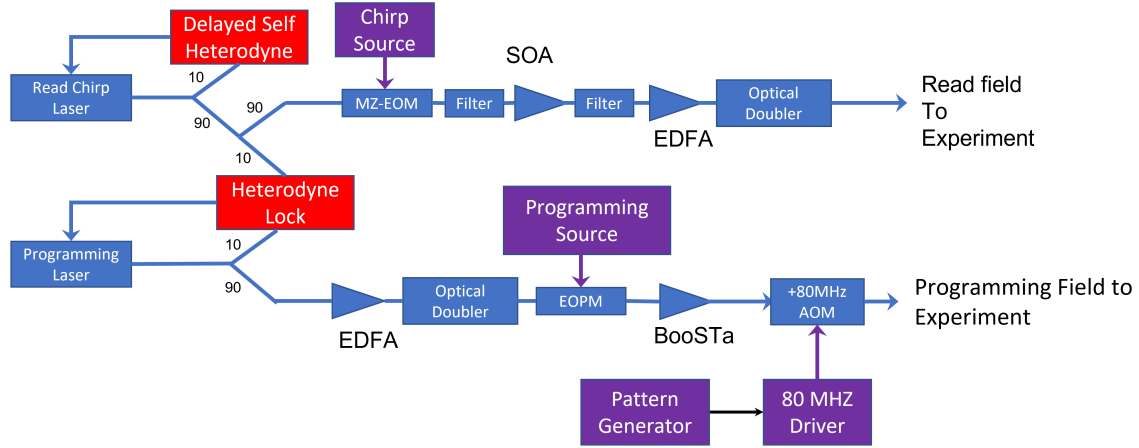


Figure 6.10: An overview of the laser system used to generate programming and read fields for SSH readout experiments.

The two lasers begin as 1588nm fields generated by a pair of Pure Photonics PPCL300 lasers. The read field acts as the primary field to which the programming field is stabilized. The read field is stabilized using a delayed self-heterodyne method, and the programming field is locked to the read field using a heterodyne lock. The now stabilized chirp field is

modulated using a push-pull Mach-Zehnder electro-optic modulator (MZ-EOM). A system of filters allows for the isolation of a single chirped side-band (the second upper side-band in this case). The chirped optical field is amplified using an erbium-doped fiber amplifier (EDFA) and then converted to 794nm by an optical doubler built by AdvR Inc. This field can now be used as a chirped readout field for SSH readout experiments. The programming field is amplified using an EDFA and doubled to 794nm. The 794nm programming field is then modulated using an electro-optic phase modulator (EOPM) to put RF side bands on the field, which will be programmed into the material. A BoosTA tapered semiconductor amplifier boosts the power in the programming field. The programming field is then sent through an acousto-optic modulator (AOM) to carve pulses out of the programming field to control the programming duty cycle.

Laser Stabilization

Each laser needs to be stabilized so frequency drifts do not alter the expected results in experiments. The stabilization methods for the programming and read fields were slightly different. The following section describes the stabilization methods for each field in detail. The locking schemes used for this readout system are shown in figure 6.11.

Read Field - Delayed Self-Heterodyne Locking The read field acts as the primary field for stabilization because the filters in the programming path need to be adjusted if the chirp start frequency is changed. This means that if the read field were stabilized to the programming field, every time the programming field was adjusted, the chirp field filters would need to be adjusted to align with the chirp band, which the programming has now altered. It is, therefore, advantageous to self-stabilize the read field and stabilize the programming field to the read field. Self-stabilization of the read field is achieved by delayed self-heterodyne. The delayed self-heterodyne method (depicted in figure 6.12, first described by Greiner et. al. in 1998[21]), splits the input field, half the power goes to a 50m fiber delay,

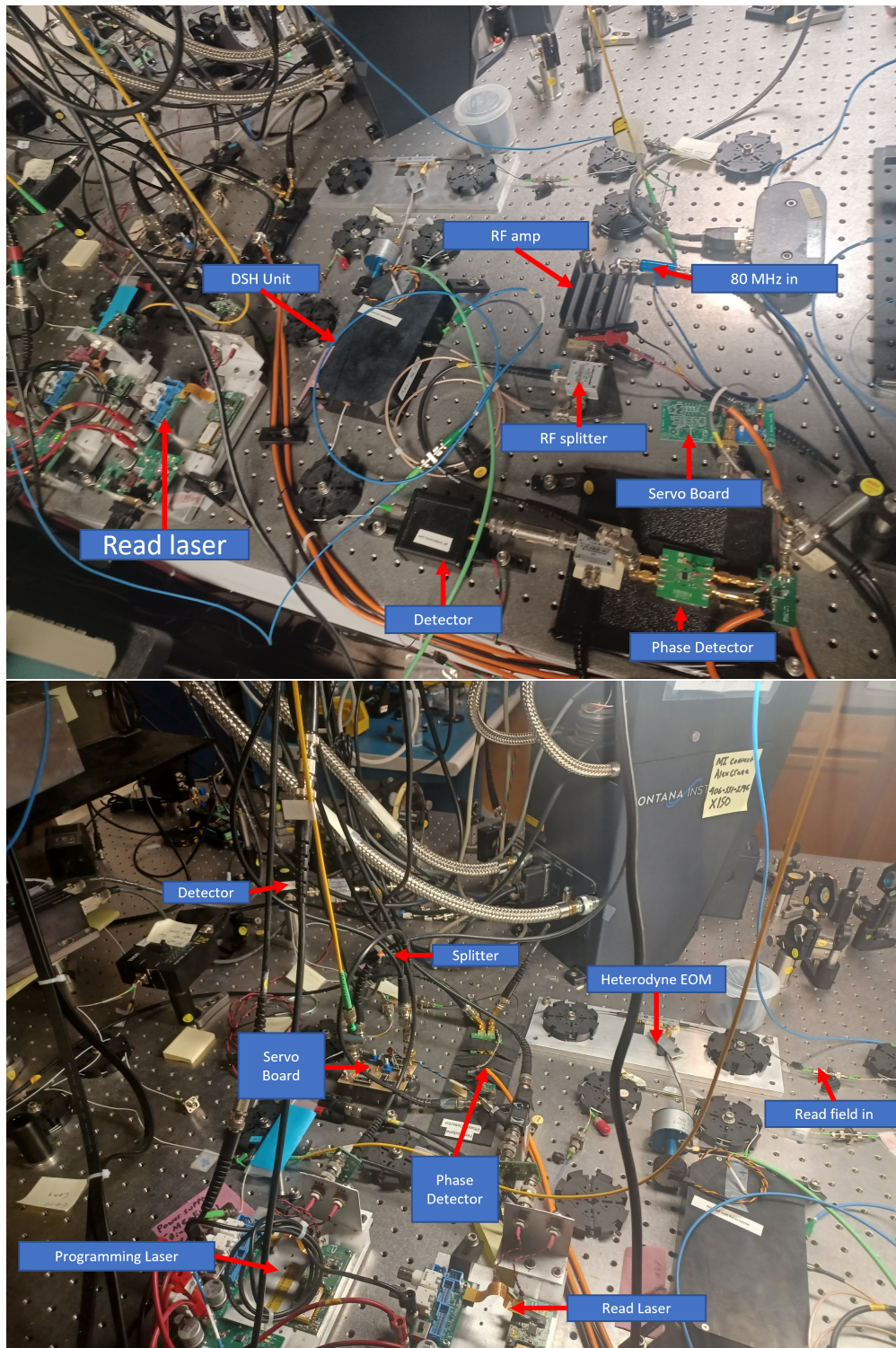


Figure 6.11: Top: A photo of the delayed self-heterodyne locking scheme used to stabilize the read field. Bottom: A photo of the heterodyne lock for the programming field with key components marked.

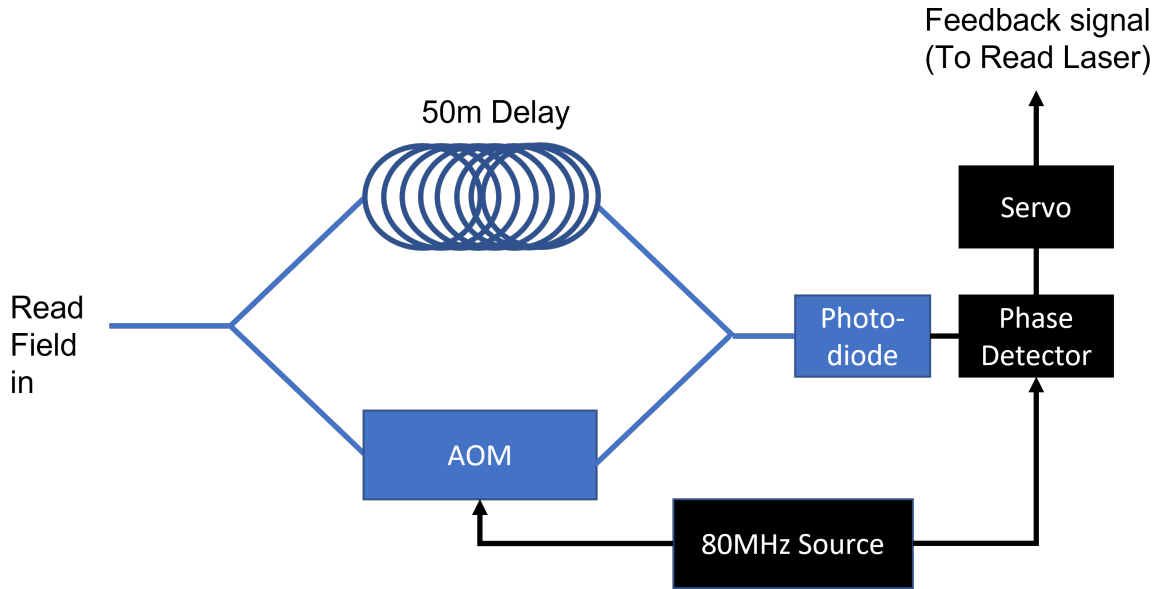


Figure 6.12: A delayed self-heterodyne locking system used for SSH readout experiments to stabilize the read field

and the other half goes to an 80MHz AOM to frequency shift the field in that arm. The frequency-shifted field and the time-delayed field are then combined and interfere with each other. The generated RF signal will be an 80MHz beat-note; a phase detector then senses deviations in the instantaneous phase of the beat-note with respect to the drive field. This phase measurement can be used as an error signal to drive a servo circuit that generates a feedback signal to stabilize the read field. Using this method, the read field was stabilized to a 10KHz linewidth at 1588nm, which corresponds to a 20KHz at 794nm (as was measured by a second delayed self-heterodyne system with a 25km fiber delay line).

Programming Field-Heterodyne Locking Since the read field is self-stabilized, it can be used to stabilize the programming field. The read field is stabilized by heterodyne locking to the read field. Heterodyne locking (see figure 6.13) works by modulating the read field at the desired frequency difference between the two fields. In this case, the 2nd sideband was used, so the drive frequency for the EOM used to generate sidebands was half the desired

frequency difference. The modulated read field is then mixed with the programming field

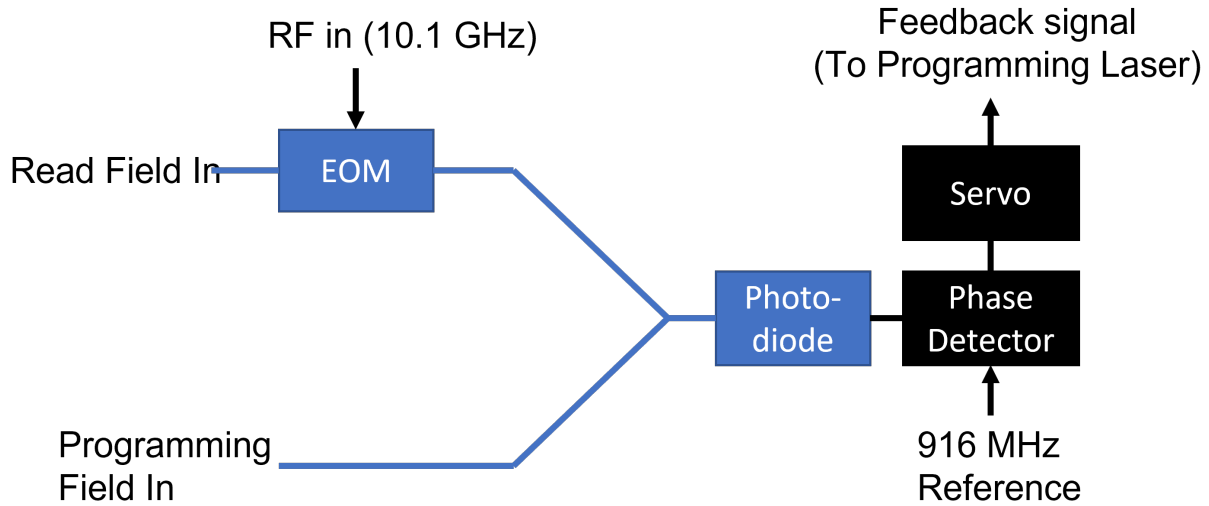


Figure 6.13: A Heterodyne locking scheme used to stabilize the frequency of the programming field for SSH readout experiments

to generate a beat note. The optical beat note is then measured with a photodiode. The photodiode signal is then phase detected against a known signal (in this case, at 914MHz). The phase detector gives input to a servo that can be used to feedback into the programming laser so that the beat note measured at the detector is fixed to 914MHz. The programming field, therefore, can be stabilized at a frequency offset with respect to the read field. Using this stabilization method, the programming field was stabilized to 10KHz at 1588nm and, therefore, 20KHz at 794nm (as was measured by another delayed self-heterodyne system with a 25km fiber delay line).

Chirp Generation

Chirp Rate Choice The experimental chirp rate will be important in determining other experimental parameters. In practice, the read chirp rate will be governed by the chirp

duration (τ_{chirp}) and the chirp bandwidth (Δf_{chirp}) according to equation 6.21.

$$\kappa = \frac{\Delta f_{chirp}}{\tau_{chirp}} \quad (6.21)$$

The duration of the chirp must be less than the material storage lifetime (for this case, 4.5ms). For this experiment, a 1ms chirp duration will work with the material lifetime without requiring high bandwidth detection. The chirp bandwidth is a free parameter. A read bandwidth of 2GHz was chosen for SSH experiments. The read field chirp rate was then determined to be $2 \frac{MHz}{\mu s}$. The general method for generating the required optical chirps is to generate a base-band optical chirp, use RF frequency multipliers to increase the bandwidth of the chirp, and then modulate the RF chirp onto the optical field and isolate the second side-band with filters to gain yet another bandwidth doubling of the bandwidth chirp. Beginning with a 1588nm optical field and frequency doubling using second harmonic generation to 794nm gives a further doubling in chirp bandwidth. This allows for the use of a digital-to-analog converter (DAC) of a sample rate in the range of less than 6 GSPS to generate high bandwidth optical chirps at 794nm of bandwidths up to 16GHz. Because the band of interest is the second sideband, care needs to be taken that other harmonics do not leak into the band of interest; this is done by constraining that the $2f_{max} < 3f_{min}$ so that there is no overlap between the upper part of the 2^{nd} side-band and the lower part of the 3rd side-band.

Chirp Generation System The RF chirp signal used to modulate the read field was generated by first synthesizing a chirp from 1.25 GHz to 1.75 GHz using a 6 GSPS analog devices AD9164 DAC evaluation board and an ADS7V2 FPGA evaluation board. These chirps can be multiplied up to a final RF chirp from 10-14GHz 6.14.

The 3 RF frequency doublers in the chirp generation scheme take a signal in the 1.25-1.75 GHz band and multiply them up 10-14 GHz band. The amplifiers and attenuators ensure that the powers are in the correct range for the frequency doublers. The filters

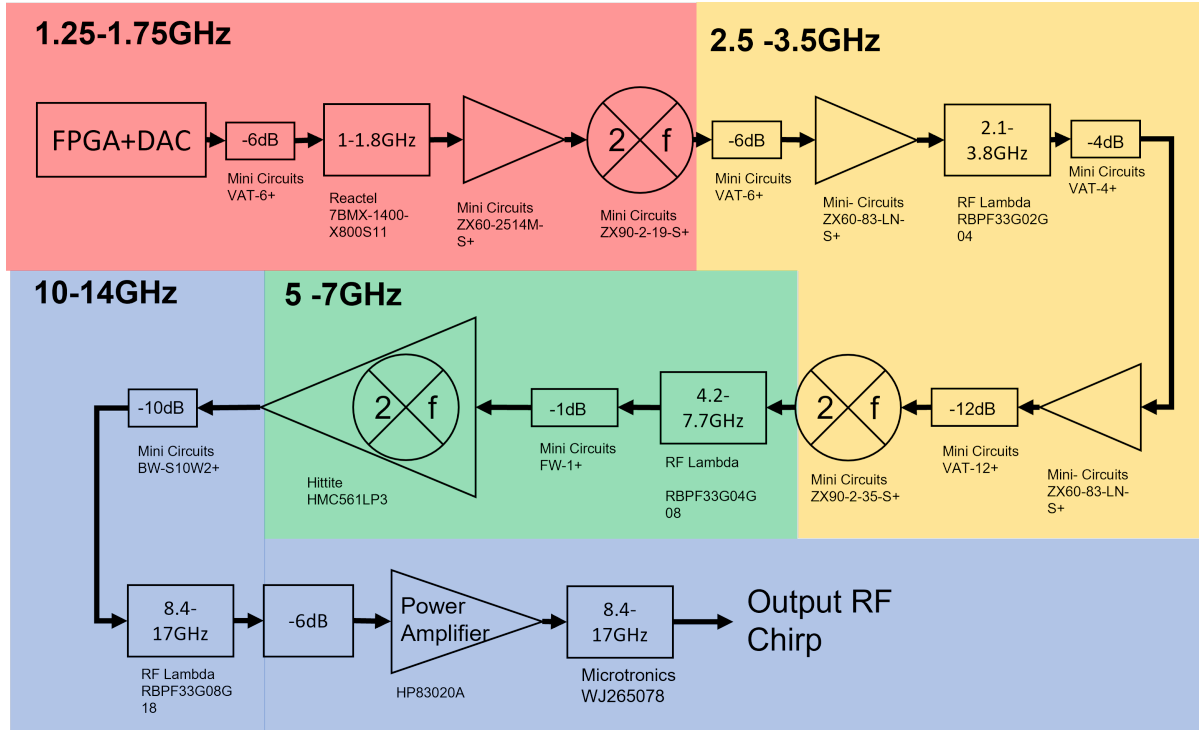


Figure 6.14: An RF chirp generation Scheme using an FPGA paired with a DAC to generate chirp waveforms in the 1.25GHz to 1.75GHz band, a system of frequency doublers and amplifiers increase the bandwidth of the chirp into the 10-14GHz range. Different colored regions indicate different frequency ranges.

remove unwanted harmonics leaving a well-isolated chirp in the 10-14 GHz band. This chirp is then amplified using a high bandwidth power amplifier to drive the MZ-EOM that generates chirped side bands on the programming field. The drive power and interferometric bias of the MZ-EOM were chosen to optimize the second upper side band, which gives the optical chirp an additional doubling of the bandwidth. A system of optical filters is then used to isolate the 2^{nd} upper sideband. The field is then optically doubled down to 794nm. By converting to 794nm, the optical bandwidth of the chirp is again doubled, meaning this system allows for the generation of a chirp with a bandwidth up to 16GHz. The RF and optical chirp generation components are shown photographically in figure 6.15.

While this system was designed and built for this experiment, there were severe

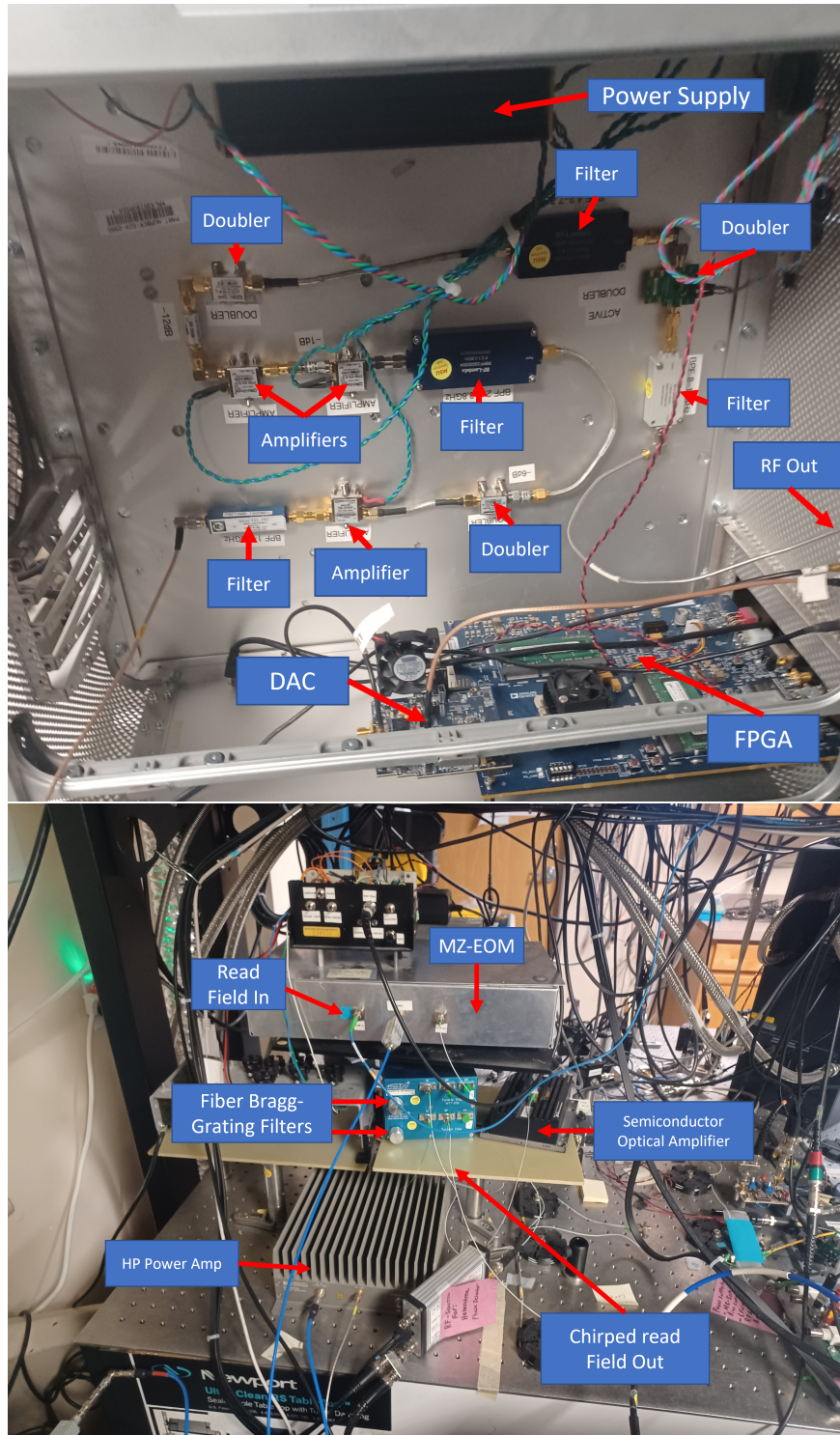


Figure 6.15: Top: A photograph of the RF chirper system with key components marked. Bottom: The optical portion of the chirp generation scheme with key components marked.

electronic issues with that system that led it to be too unstable to produce a consistent side band that could be used to seed the EDFA, so it had to be replaced with a chirp generated by a Tektronix AWG610 chirp which produced a chirp from 310 MHz to 372.5MHz which was mixed up into the desired base-band. Regardless of the source, this 62.5 MHz bandwidth chirp was multiplied up into a 500MHz bandwidth chirp using the multiplication chain depicted in figure 6.14. The final bandwidth at 794nm of chirps generated using this secondary method for chirp generation was 2GHz. This 2GHz (rather than the full 16GHz that can be achieved by the chirper) Bandwidth chirp was used in experiments.

Band Plan

In experiments, we used a 2GHz chirp (only one-eighth the potential bandwidth of the chirper). SSH readout experiments require that the readout chirp overlaps with the sidebands modulated onto the programming field so that they can be read out properly. Therefore, the relative frequencies of the read and reference field are not trivial and must be coordinated with the chirp band plan, and the frequency of RF signals modulated onto the RF signal. Figure 6.16 shows a band plan for an SSH readout experiment of frequencies that allows for a 2GHz span of optical spectrum analysis above the programming carrier. In readout experiments, a final chirp bandwidth of 2 GHz was achieved by generating a 62.5MHz bandwidth base-band chirp from 1.3851 GHz to 1.4476 GHz. This chirp is then sent through the multiplication chain to generate a chirp from 11.0812 GHz to 11.5812 GHz, which was used to drive the MZ-EOM. The 2nd sideband chirps modulated onto the carrier span from 22.1624 GHz to 23.1624 GHz from the carrier. The programming field carrier was placed roughly 21 GHz above the read field. The read field is stabilized with respect to the self-stabilized reference field frequency modulated at 10.1 GHz generating a side band at 20.2 GHz. The heterodyne locking scheme is then used to stabilize the programming field 916MHz above the second side-band of the read field, thus keeping the read field at a

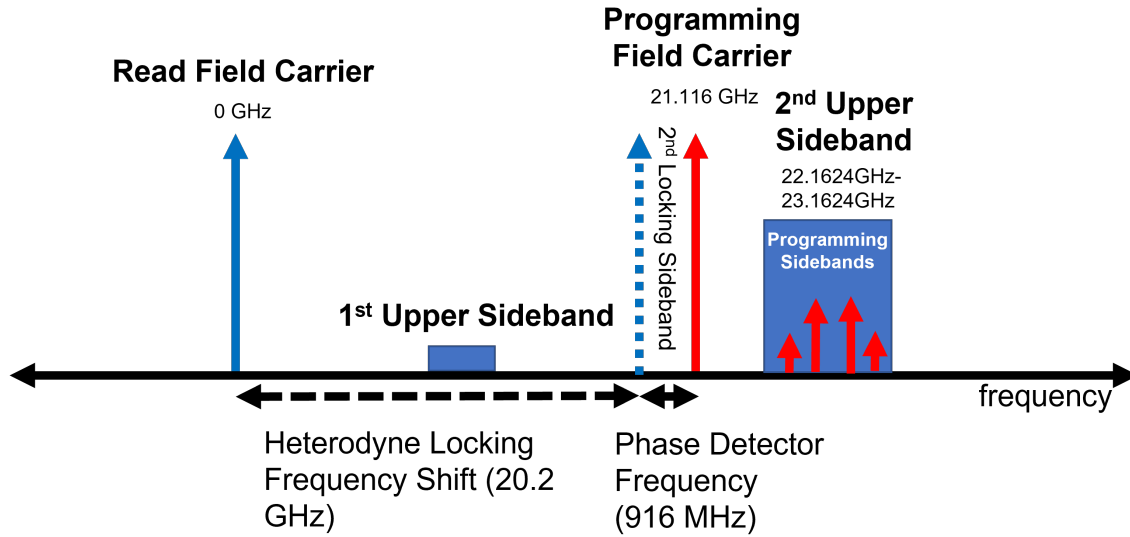


Figure 6.16: Band Plan for an SSH readout experiment indicating the location of important elements required for the frequency multiplication chain. This system allows for the programming and readout of RF spectra in the range of 2.1728GHz to 4.1728GHz.

constant 21.116 GHz above the carrier of the read field. Therefore, the optical chirp band is between 1.0464 GHz and 2.0464 GHz above the programming field carrier. Upon optical doubling, these differences are doubled meaning the chirp reads a range of 2.0928 GHz to 4.0928 GHz above the read field carrier. The 80 MHz AOM will shift the frequencies of the programming field by +80 MHz. Therefore, the system will analyze the RF power spectrum of signals within the range of .1728 GHz to 4.1728 GHz.

Design Considerations for a Mach Zehnder Interferometer for SSH Readout

The interferometer configuration used for interferometric SSH readout experiments is a Mach-Zehnder configuration. The optical system required for such readout experiments is shown in figure 6.17. The Mach-Zehnder interferometer for SSH readout requires a chirped read field to be split into a read path and an LO path. The actual split ratio of the splitter used to split the chirp is only relevant in that it can be used to control the power in the

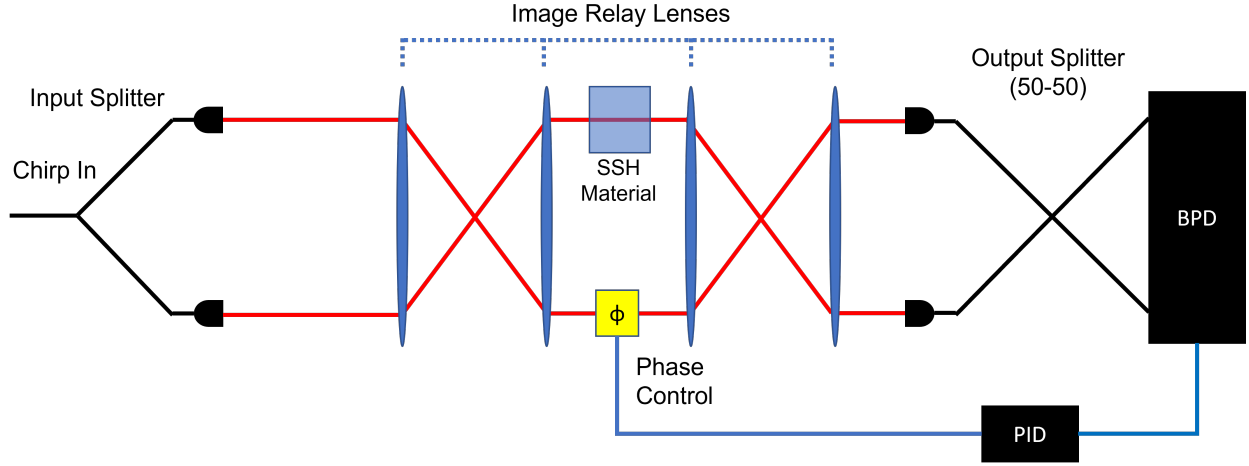


Figure 6.17: A rough schematic indicating some of the design considerations required for building an interferometer (not to scale).

read and LO fields. The fields then pass through a 4-f image relay system to ensure that the read field is imaged through the material and that the read field in the crystal is at a known spot size. The path length of the LO arm of the interferometer is modulated using a phase control device, which is used in a control loop to control the relative bias phase of the interferometer. The read field passes through the SSH material in the middle of the image relay system, where it will interrogate the material. After the end of the image relay system, the fields are coupled back into fiber and passed through a variable 50-50 beam splitter before balanced detection. This section will discuss some of the other practical considerations of interferometer design, such as path length tolerance, spot size, stabilization, and splitter fidelity must be evaluated.

Path Length Difference Considerations

The difference in path length between the two arms of the interferometer needs to be considered on two scales. The first is the gross path length difference, which has more to do with the total relative time delay between fields in each arm of the interferometer. The other scale is the relative phase shift between the two arms of the interferometer, which will

govern whether the interferometer will be in quadrature or the peak or null condition.

Gross Path-Length Tolerance The gross path length difference tolerance of an interferometer for SSH readout applications is governed by two factors, the interferometer bandwidth and the frequency-modulated continuous wave (FMCW) beat note due to the path length difference arising from the interference of the chirp with itself.

Interferometer Bandwidth The path length difference governs the bandwidth over which interference will occur in a Mach-Zehnder interferometer. Suppose, for example, a brief pulse goes through an interferometer where the path length difference is greater than the pulse duration. In that case, there will be no interference because the pulse does not interact with itself at the output. Since the duration of the pulse is inversely related to the bandwidth of the pulse, the less the path length difference, the higher the bandwidth of the interferometer. Specifically, the result is governed by equation 6.22.

$$\Delta f_{interferometer} = \frac{c}{\Delta \ell} \quad (6.22)$$

In SSH systems, the highest frequency signal that will need to be detected is $f_{Max} = \kappa T_2$, where κ is the chirp rate and T_2 is the coherence time of the material. In SSH applications, the interferometer bandwidth should be much greater than the required detection bandwidth for good cancellation. The order of magnitude of this constraint can be roughly estimated by equation 6.23.

$$\Delta \ell \ll \frac{c}{\kappa T_2} \quad (6.23)$$

For a 1 microsecond coherence time with a $2 \frac{MHz}{\mu S}$ chirp, this requires that the path length difference is much less than 150 meters (which is very easy to achieve). Because of the significant length difference allowed here, this will not be the governing constraint of the system.

FMCW Beat-Note When a chirped optical field interferes with a delayed copy, the result will be a beat note at a frequency given by equation 6.24.

$$f_{beat} = \frac{\kappa \Delta \ell}{c} \quad (6.24)$$

Typically optical chirps are confined to a specific duration (t_{chirp}), so the period of the beat note should be minimized reduced to as close to zero as possible so that the FMCW interference terms don't contribute to the background. This is done by ensuring that the period of the beat note is much less than the duration of the chirp ($\frac{1}{f_{beat}} \ll \tau_{chirp}$). This will be achieved when equation 6.25 is satisfied.

$$\Delta \ell \ll \frac{c}{\kappa t_{chirp}} \quad (6.25)$$

In a system with a 1ms chirp duration and a chirp rate of $2 \frac{MHz}{\mu S}$, the path length difference should be much less than 150cm, meaning this was the more serious though still reasonable constraint. The gross path length difference of the interferometer was balanced to within 11 millimeters using a translation stage upon which one of the fiber couplers was mounted. Initial length measurements were done using a 16GHz wide chirp and attempting to flatten the background signal measured on a balanced detector as much as possible. Beyond the resolution of the chirp, the path length difference was estimated by comparing the slope of the background signal in quadrature to the full dynamic range of the interferometer in peak and null.

Phase control Interferometric SSH readout applications require that the interferometer is biased in quadrature. In most laboratory applications, the interferometric bias cannot be chosen from the outset. Instead, the interferometer must be actively stabilized using a feedback control system such as a PID controller.

To use feedback control methods, an error signal is needed. In our experiments, we used the output signal from the balanced detector for interferometric readout and used that signal as the error signal, which could be used by the PID controller shown in figure 6.18 on page 170, to generate an error signal, which is then put into a PZT driver to generate a signal to drive a mirror mounted on a PZT to alter the relative length of one arm of the interferometer. The feedback will then work to hold the balanced detector signal to zero and therefore hold the interferometer in quadrature. The PID circuit shown in figure 6.18 on page 170 is built from a pair of operational amplifiers (op-amps). At the input to the device, an error signal is generated as the sum of the detector signal input and a set point, which is set by a potentiometer. This first op-amp serves as the PID controller itself, essentially providing proportional and integral gain simultaneously. The feedback path contains a capacitor in series with a potentiometer (which is itself being used as a variable resistor), this allows for a frequency-dependent integrator gain component and a frequency-independent proportional gain term. The potentiometer allows for the control of the proportional gain. The signal then passes through a second potentiometer which acts as a variable voltage divider allowing for control over the total gain of the signal after the first op-amp. These two controls are essentially coupled. These two degrees of freedom allow the user to tune the control loop to the desired system. The secondary feedback path (which can be toggled with a switch) provides low-frequency feedback while the user is tuning the loop. Removing this secondary feedback path would normally cause issues with the circuit as errors accumulate driving the circuit away from the desired state. However, when the system is very close to being stabilized the user can remove this additional feedback to provide strong gain at low frequencies, which can greatly help with the stabilization of the system. In this case, the system provides the feedback that allows the integrator to remain in a useful state under those conditions. The second op-amp is a toggleable inverter allowing the user to change the sign of feedback as needed.

Using the read field from the SSH readout to generate an error signal removes the complication of adding another field into the system for stabilization purposes but does create some other complications. The first complication is that the stabilization circuit will respond to fluctuations in the read field due to the readout spectra, suppressing the very signals that the system is supposed to observe. This can be mitigated by reducing the bandwidth of the PID controller to less than the inverse of the duration of the readout chirp, which was 1 ms. In order to prevent over-stabilization, the PID controller bandwidth was restricted to 1 KHz. This was done by setting the integrator RC constant to 1ms by using a $1K\Omega$ input resistor and a $1\mu F$ feedback capacitor. This low controller bandwidth led to stability issues in the interferometric background. These background fluctuations proved to be problematic in spectrum analysis experiments and had to be removed using a background subtraction technique. These fluctuations were less problematic in grating processing experiments because a range gate could be applied to ignore low-frequency fluctuations. In experiments, we found that heating occurs that leads to thermal broadening of the sample leading to reduced coherence times. To mitigate this, the duty cycle of the read field had to be regulated to prevent thermal broadening. This means that the PID controller needed to re-engage the locking system for every cycle (1s) at the beginning since no light was sent to the detector during the downtime. This caused problems since the feedback loop used an open loop integrator, meaning that during the time period when there was no light at the detector, the integrator would saturate because there was no corrective feedback. To prevent this, a relay driven by an Arduino microcontroller board connected to a “relay shield” (which allows for very easy control of a relay) allowed for the relay to be triggered synchronously with the experiment to short the leads on the capacitor used in the open loop integrator. Ensuring that until the programming signal turned back on, the integrator would not accumulate charge, causing the controller to drive the interferometer out of the desired range.

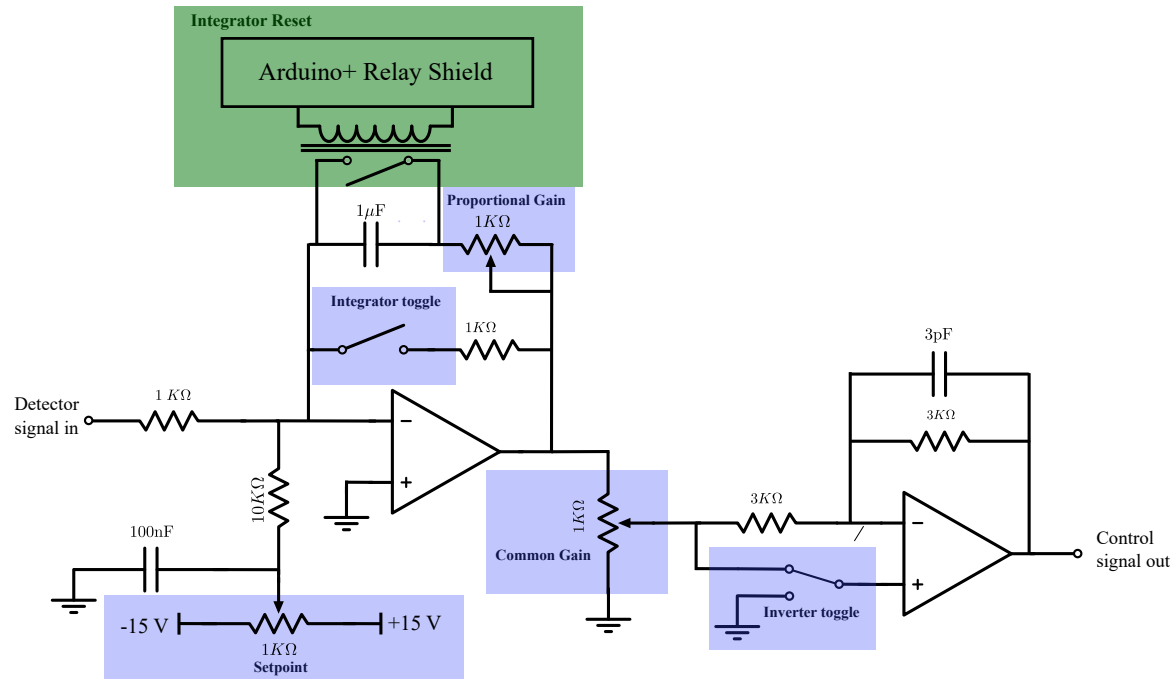


Figure 6.18: This circuit diagram depicts the PID controller used to stabilize the Interferometer in quadrature for interferometric SSH readout experiments. The Integrator reset portion is highlighted in green and the user-accessible controls are highlighted in blue.

Interferometer Power Matching

Full Description of Splitter Parameters Assume that light can be injected either way through the device (either into ports A and B or into ports X and Y) and that there are losses on each port of the device (perhaps due to kinks in the fiber or some other defects), These will be accounted for these losses using coefficients $\epsilon_A, \epsilon_B, \epsilon_X$, and ϵ_Y . Also, assume that there is some common loss through the device that can be described with a single efficiency coefficient ϵ_{com} . No assumption must be made that the split ratio (and therefore T and R coefficients) are the same regardless of which direction light propagates through the device. To describe this possible asymmetry using 2 sets of transmission and reflection coefficients. T (A to X or B to Y) and R (A to Y or B to X) describe fields from inputs A and B being split onto outputs X and Y, while T' (X to A or Y to B) and R' (X to B and Y to A) represent the splitter for fields propagating in the opposite direction. T and R coefficients are subject to the conditions that $T+R=1$ and $T'+R'=1$. A beam splitter with these parameters marked is depicted in figure 6.19.

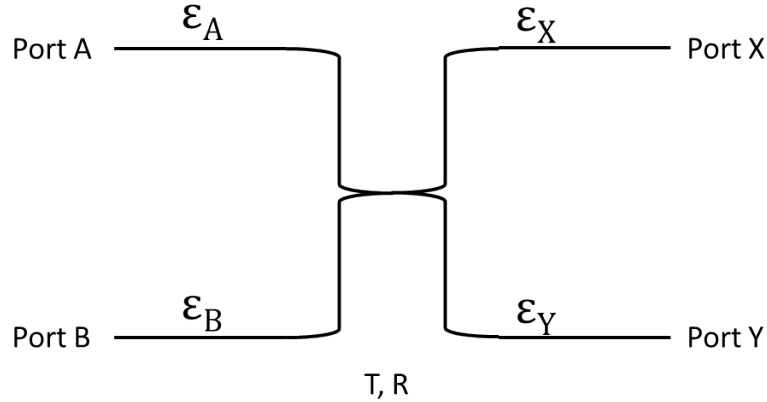


Figure 6.19: Output powers of a realistic beam-splitter will be affected by internal losses on each port in addition to the coupling between input ports and output ports.

There is an additional degree of asymmetry in fiber-coupled beam splitters because, at

the inputs, there can be some additional loss due to a poor fiber connection. Assume that these losses are incurred when light is injected into a specific port (but not when we measure it leaves the fiber for measurement), this loss will be taken into account with fiber coupling coefficients C_A, C_B, C_X and C_Y (it is important to note that these only come into play when each port is being used as an input). Assuming an input power in port i P_0 , the output power P_{ij} in port j can be calculated from the coefficients using 6.26.

$$\begin{aligned}
P_{AX} &= P_0 \epsilon_{com} C_A \epsilon_A \epsilon_X T & P_{BY} &= P_0 \epsilon_{com} C_B \epsilon_B \epsilon_Y T \\
P_{AY} &= P_0 \epsilon_{com} C_A \epsilon_A \epsilon_Y R & P_{BX} &= P_0 \epsilon_{com} C_B \epsilon_B \epsilon_X R \\
P_{XA} &= P_0 \epsilon_{com} C_X \epsilon_X \epsilon_A T' & P_{XB} &= P_0 \epsilon_{com} C_X \epsilon_X \epsilon_B R' \\
P_{YA} &= P_0 \epsilon_{com} C_Y \epsilon_Y \epsilon_A R' & P_{YB} &= P_0 \epsilon_{com} C_Y \epsilon_Y \epsilon_B T'
\end{aligned} \tag{6.26}$$

dividing by the input powers we can define a series of efficiencies given by 6.27.

$$\begin{aligned}
\eta_{AX} &= \epsilon_{com} C_A \epsilon_A \epsilon_X T & \eta_{BY} &= \epsilon_{com} C_B \epsilon_B \epsilon_Y T \\
\eta_{AY} &= \epsilon_{com} C_A \epsilon_A \epsilon_Y R & \eta_{BX} &= \epsilon_{com} C_B \epsilon_B \epsilon_X R \\
\eta_{XA} &= \epsilon_{com} C_X \epsilon_X \epsilon_A T' & \eta_{XB} &= \epsilon_{com} C_X \epsilon_X \epsilon_B R' \\
\eta_{YA} &= \epsilon_{com} C_Y \epsilon_Y \epsilon_A R' & \eta_{YB} &= \epsilon_{com} C_Y \epsilon_Y \epsilon_B T'
\end{aligned} \tag{6.27}$$

from these measured efficiencies, the ratios between transmission and reflection coefficients can be calculated according to equation 6.28.

$$\begin{aligned}
\frac{R}{T} &= \sqrt{\frac{\eta_{AY} \eta_{BX}}{\eta_{AX} \eta_{BY}}} \\
\frac{R'}{T'} &= \sqrt{\frac{\eta_{YA} \eta_{XB}}{\eta_{XA} \eta_{YB}}}
\end{aligned} \tag{6.28}$$

From these values, R values (and therefore T values) can be calculated explicitly according to equation 6.29.

$$\begin{aligned}
R &= \frac{1}{\sqrt{\frac{\eta_{AX}\eta_{BY}}{\eta_{AY}\eta_{BX}} + 1}} \\
R' &= \frac{1}{\sqrt{\frac{\eta_{XA}\eta_{YB}}{\eta_{YA}\eta_{XB}} + 1}}
\end{aligned} \tag{6.29}$$

Using the calculated T and R coefficients and ratios of the measured efficiencies (η_{jk}) to calculate ratios between the port efficiencies (ϵ_j) as shown in equation 6.30.

$$\begin{aligned}
\frac{\epsilon_A}{\epsilon_B} &= \frac{\eta_{XA}R}{\eta_{XB}T} \\
\frac{\epsilon_X}{\epsilon_Y} &= \frac{\eta_{AX}R'}{\eta_{AY}T'}
\end{aligned} \tag{6.30}$$

the ratios between the fiber coupling efficiencies (C_j) can also be calculated according to equation 6.31.

$$\begin{aligned}
\frac{C_A}{C_X} &= \frac{\eta_{AX}T'}{\eta_{XA}T} & \frac{C_B}{C_X} &= \frac{\eta_{BX}R'}{\eta_{XB}R} \\
\frac{C_A}{C_Y} &= \frac{\eta_{AY}R'}{\eta_{YA}R} & \frac{C_B}{C_Y} &= \frac{\eta_{BY}T'}{\eta_{YB}T}
\end{aligned} \tag{6.31}$$

Using these measurements, one cannot ascribe specific losses to each variable (hence the inclusion of the common efficiency) by enforcing that all of these variables are between 0 and 1. So three of these efficiencies can be set to one, and then the other efficiencies can be calculated with respect to those efficiencies. First of all, one can determine which of A-B efficiencies (ϵ_A and ϵ_B) is larger and then assign the higher efficiency to be one and calculate the other efficiency. One can then repeat this process for the X-Y efficiencies (ϵ_X and ϵ_Y) and set the higher efficiency to 1. With more effort and comparison, one can find which input connection has the highest coupling efficiency (C_j) and set that efficiency to one so that the other efficiencies can be calculated from this efficiency. With this convention for values of $\epsilon_A, \epsilon_B, \epsilon_X, \epsilon_Y, C_A, C_B, C_X$, and C_Y and the already calculated reflection and transmission coefficients can be used to calculate the common efficiency (ϵ_{com}) according to equation 6.32. In fact, one could calculate the common efficiency from every measured efficiency (η_{jk}) as the common efficiency can be calculated from η_{AX} .

$$\epsilon_{com} = \frac{\eta_{AX}}{C_A \epsilon_A \epsilon_X T} \quad (6.32)$$

Interferometric Effects of Beam Splitter Imbalance The interferometer's performance is dependent on the properties of the beam splitters. In readout applications, the performance is much more dependent on the output splitter than the input splitter because the output splitter functions in mixing the read and LO fields as well as distributing power to the two ports of the balanced detector equally. The input splitter only changes the relative power in the read and LO fields (which could be taken care of using an optical attenuator). Assuming that before the output splitter, the read and LO field are $\pi/2$ out of phase with each other at a power of P_{LO} and P_{read} , respectively, and are mixed in a beam splitter with parameters described in the preceding subsection. Then the output Powers at the interferometer will be given by equations 6.33 and 6.34.

$$P_X = \epsilon_{com} C_A \epsilon_A \epsilon_X T P_{read} + \epsilon_{com} C_B \epsilon_B \epsilon_X R P_{LO} \quad (6.33)$$

$$P_Y = \epsilon_{com} C_B \epsilon_B \epsilon_Y T P_{read} + \epsilon_{com} C_A \epsilon_A \epsilon_Y R P_{LO} \quad (6.34)$$

The differential power (measured by the BPD) will then be given by equation 6.35.

$$P_{\Delta} = C_B \epsilon_{com} \epsilon_B (\epsilon_Y T - \epsilon_X R) P_{LO} + C_A \epsilon_{com} \epsilon_A (\epsilon_Y R - \epsilon_X T) P_{read} \quad (6.35)$$

The structure of equation 6.35 indicates that the suppression of the read and LO fields are essentially individually suppressed. If the splitter is ideal, then both the read and LO fields will be suppressed at balanced detection; however, if there are any imperfections in the splitter, the system will no longer perfectly cancel the residual read and LO fields. If a variable split ratio splitter is used, the split ratio can be changed to eliminate one of the fields; however, the other field will not be canceled out. This imperfect cancellation will lead

to slight variations in the DC background of the detected signal and will result in imperfect cancellation of common mode noise from that field. If one of the fields is stronger, it will contribute more to common mode noise; therefore, the stronger field should be prioritized for cancellation. In interferometric readout experiments, the LO power is typically stronger than the read field, so the read field should be prioritized for cancellation. Of the available variable splitters, the best set of measured properties were: $\eta_{com} = 0.94$, $\eta_A = 0.99$, $\eta_B = 1$, $\eta_X = 1$, $\eta_Y = 0.97$, $C_A = 0.97$, and $C_B = 0.88$. Therefore if the LO field (into Port A) is canceled when $T = 0.51$ and $R = 0.49$ then the read field will still be canceled to within 2%, which means that up to 17dB of common mode rejection of the read field can be achieved despite imbalances. Noting again that in the strong LO field limit, noise contributions from the LO field will be much stronger compared to contributions to the read field. The background arising from the read field will be very small compared to the signals of interest in interferometric readout.

Image Relay System

A desired spot size of 1mm was chosen for S2SA experiments to allow for more optical power in the chirp fields without excessive material depopulation. Fiber launchers 5mm at 800nm were used to launch light into the free space portion of the interferometer. These 800nm launchers have a nominal output spot size of 0.86 mm, so the spot size is already in the right size range. To retain that spot size through the system, an image relay system using two 200mm lenses before the material (with an additional 200mm lens used to focus the programming field so that the read field was also part of the image relay system even though the read field was not able to be brought into the system until after the first image relay lens) and two 150mm lenses after the material. The lenses were spaced so that the focal plane of each lens overlapped with the back focal plane of the next. The spot size was measured using a turning mirror at the presumed location of the final image of 0.9mm,

meaning the spot was roughly at the desired size of 1mm.

Detection Considerations

Balanced Detection of Optical Signals

Balanced photodetectors (BPDs) are detectors that produce a time domain voltage signal that is proportional to the difference of the time domain powers incident on the two photodiodes that make up the detector. This allows for the subtraction of common mode background and common mode noise, which can greatly simplify ultrasensitive detection [28, 29]. At each of the two diodes, the amount of photo-current produced by the diode for a given incident power is governed by a quantity called the responsivity ($\mathcal{R}$), which is described in units of amperes per watt. Typically, a BPD has an additional circuit called a transimpedance amplifier, which converts the differential current into a stronger voltage signal. The transimpedance amplifier is typically described by a quantity called the transimpedance gain (G_T), which is described in volts per ampere. The end result is a voltage signal proportional to the difference in optical powers. Balanced photodetectors have a few major benefits. Firstly, balanced detectors can be used to remove some unwanted background and therefore free up dynamic range for signals of interest; secondly, they can be used in applications where the desired signal is the difference between the two powers (for example, in a quad-biased interferometer the difference of powers at each port is proportional to the bias phase) and thirdly by subtracting instantaneous powers common mode noise on the beams can be canceled out leading to lower noise floors.

Time Domain Optical Measurement Using a Balanced Photo Detector To model the behavior of a balanced photodetector, assume that two separate optical fields are incident on the two photodiodes that make up the balanced detector. The optical field incident on the first photodiode (PD 1) has a power of $P_1(t)$, and the field incident on the first photodiode

(PD 2) has a power of $P_2(t)$. Each photodiode will emit a photo-current according to the power at the diode and the responsivity ($\mathcal{R}$). This relation is given by equation 6.36.

$$i_{PD} = \mathcal{R}P_{PD} \quad (6.36)$$

The total currents from the two photodiodes are subtracted from each other to give the total balanced photo-current (equation 6.37).

$$i_{BPD}(t) = \mathcal{R} (P_1(t) - P_2(t)) \quad (6.37)$$

The voltage put out by the balanced detector is governed by the transimpedance amplifier in the device, which converts the photo-current to a voltage according to the transimpedance gain (G_T). This allows for the calculation of the voltage signal using equation 6.38.

$$V_{BPD}(t) = G_T \mathcal{R} (P_1(t) - P_2(t)) \quad (6.38)$$

This voltage signal is the signal of interest in experiments; typically, it is digitized and processed digitally.

Detector Noise Sources

Detector Thermal Noise and Noise Equivalent Power Specifications Thermal noise (Johnson-Nyquist Noise) inside the balanced photodiode circuitry is a major noise source in photodetectors. Typically this noise is intrinsic to the operation of the device and is typically cited in terms of a quantity called noise equivalent power (NEP/Q_{NEP}). This quantity is usually specified in $W/\sqrt{Hz}$. The NEP specifies the equivalent minimum detectable optical signal. Using the NEP and the detector bandwidth (Δf_{det}), the time domain RMS current

noise can be calculated using equation 6.39.

$$i_{RMS,NEP} = \mathcal{R}Q_{NEP}\sqrt{\Delta f_{det}} \quad (6.39)$$

From this RMS current, the expression for time domain RMS voltage can be calculated (equation 6.40).

$$V_{RMS,NEP} = G_T \mathcal{R}Q_{NEP}\sqrt{\Delta f_{det}} \quad (6.40)$$

Note that this does not depend on power, as it is an intrinsic property of the device.

Shot Noise In Balanced Detection The other primary noise source of interest arising at the balanced detector is the shot noise. Shot noise arises from Poisson statistics in the counting of quanta of electric current; of particular interest is the shot noise of measuring the photocurrents. Starting by counting photoelectrons, the Poisson error in counting photoelectrons is given by equation 6.41.

$$\langle N_{Photoelectrons} \rangle_\sigma = \sqrt{N_{Photoelectrons}} \quad (6.41)$$

The total error in the charge measured can be calculated by multiplying the photon counting error by the electron charge (q_e) given in equation 6.42.

$$\langle q_{Photo} \rangle_\sigma = q_e \sqrt{N_{Photo}} \quad (6.42)$$

Assume that the charge is accumulated from a photodiode that is emitting photoelectrons over the time resolution of the detector (which is in turn governed by the detector bandwidth $t_{res} = 1/2\Delta f_{det}$). In this case, the accumulated charge is given by equation 6.43.

$$\langle q_{Photo} \rangle_{\sigma} = q_e \sqrt{\frac{i_{det}}{2q_e \Delta f_{det}}} = \sqrt{\frac{q_e i_{det}}{2 \Delta f_{det}}} \quad (6.43)$$

Therefore, the photo-current error over the detection bandwidth is the error in the photo-current divided by the time interval, which is expressed in equation 6.44 [26].

$$\langle i_{Photo} \rangle_{\sigma} = \sqrt{2q_e i_{det} \Delta f_{det}} \quad (6.44)$$

The current fluctuations due to shot noise are independent on each diode and then are subtracted. This means that each diode's RMS shot noise currents must be added in quadrature to calculate the total current noise figure 6.45.

$$\langle i_{BPD} \rangle_{\sigma} = \sqrt{\langle i_{PD1} \rangle_{\sigma}^2 + \langle i_{PD2} \rangle_{\sigma}^2} = \sqrt{2q_e \mathcal{R}(P_1 + P_2) \Delta f_{det}} \quad (6.45)$$

This RMS current can be used to calculate the time domain RMS noise voltage (equation 6.46

$$\langle V_{BPD} \rangle_{\sigma} = G_T \sqrt{2q_e \mathcal{R}(P_1 + P_2) \Delta f_{det}} \quad (6.46)$$

Quantization Noise Additional noise is coupled into digital signals when they are converted into digital signals using an analog-to-digital converter (ADC). Round-off error in digital-to-analog conversion will lead to noise on the digitized signal. The maximum SNR of digitized signals is governed by the vertical resolution of the converter ($SNR_{MAX} = 6.02N_{bits} + 1.76dB$ [34]). The full scale of the detector governs the maximum signal power, so the equivalent electrical power of the quantization noise is given by equation 6.47.

$$P_{Quantization} = \frac{V_{FullScale}^2 10^{-\frac{6.02N_{bits} + 1.76dB}{10}}}{8Z} \quad (6.47)$$

From the noise equivalent electrical power, one can calculate the RMS fluctuations in

voltage noise (equation 6.48).

$$V_{RMS,Quantization} = \frac{V_{FullScale} 10^{-\frac{3.01N_{bits}+0.88dB}{10}}}{2\sqrt{2}} \quad (6.48)$$

Typically, especially at higher sampling rates, round-off error is not the primary cause of the ADC floor. Distortions and the thermal noise of the ADC circuitry contribute as well so the detector noise floor is specified in an effective number of bits (ENOBs) which can be used in place of N_{bits} to estimate the true noise floor of the ADC.

Balanced Detection For Analyzing Power Spectra

In a variety of applications, such as SSH grating processing experiments, measurement requires determining the spectral content of the time domain voltage signal from balanced detection.

General Treatment of Measurement of a Sinusoidal Differential Power Consider the case where the signal on each diode of the BPD contains both common mode terms (which will be canceled by the balanced detection) and a sinusoidally varying difference between the powers on each diode at frequency f . This sinusoidal variation could have a variety of causes, such as coherent interference between two chirps, interference between two CW fields at different frequencies, modulation on the chirp due to interaction with a spectral grating, or some external modulation. This power will be described as $\Delta P(t)$ (equation 6.49).

$$\Delta P(t) = P_{\Delta} \cos(2\pi ft) \quad (6.49)$$

The time domain voltage due to the balanced detection can be calculated according to equation 6.50.

$$V_{BPD}(t) = G_t \mathcal{R} P_{\Delta} \cos(2\pi ft) \quad (6.50)$$

The total electrical power dissipated in the circuit is therefore given by equation 6.51.

$$P_{electrical,measured} = \frac{V_{RMS}^2}{Z} = \frac{G_t^2 \mathcal{R}^2 P_\Delta^2}{2Z} \quad (6.51)$$

where Z is the impedance of the measurement device (typically the impedance of the ADC used to digitize the BPD signal, however, often in applications, the actual conversion to power units proper can be omitted, and signal levels can be calculated in terms of voltage squared units). Note that in this case, the electrical power is proportional to the optical power squared; this is because, ultimately, photodetectors produce a voltage based on the optical power rather than creating an electrical power proportional to the optical power.

Given the electrical power in the signal, one can estimate the electrical power spectral density of the voltage signal measured at the detector. Because the source is a sinusoidal signal, all of the signals will be confined to the resolution bandwidth around the positive and negative frequencies of the sinusoid (since this is a real signal). The resolution bandwidth is governed by the measurement time ($\Delta f_{res} = 1/t_{meas}$). In frequency domain, all of the power is confined into two peaks, one at $-f$ and one at $+f$, with half the total power distributed into each peak both of which have a width of the resolution bandwidth. Each peak will have an average power given by equation 6.52.

$$S_{signal,electrical} = \frac{t_{meas} G_t^2 \mathcal{R}^2 P_\Delta^2}{4Z} \quad (6.52)$$

The power spectrum is computed by digitizing the voltage signal and then using a discrete Fourier transform algorithm to calculate the power spectrum. The DFT will estimate the power per resolution bandwidth frequency bin. The computed power per bin is then half the total electrical power. The actual value given by the DFT algorithm is governed by the normalization used by the particular implementation of the DFT. MATLAB's fast Fourier transform uses no normalization for the FFT but a $1/N$ normalization for the inverse

transform. This means that the values from the Fourier transform will have a multiplicative factor of N_s , the number of samples, so the calculated value of the power spectra will get a factor of N_s^2 . The signal level given in MATLAB is given by equation 6.53.

$$U_{signal} = \frac{N_s^2 G_t^2 \mathcal{R}^2 P_\Delta^2}{4Z} \quad (6.53)$$

Coherent Detection To estimate the electrical signals in coherent detection applications, consider an optical signal from the coherent beat note between two fields, on each diode in the balanced detector. We will assume the relevant signals are the product of a delayed signal and a coherent reference copy of that signal given in equation 6.54.

$$P_{\pm} = \frac{1}{2} \left[P_{signal}^2 + P_{ref}^2 \pm 2\sqrt{P_{signal}P_{ref}} \cos(2\pi f_{beat}t) \right] \quad (6.54)$$

Therefore, the time domain voltage signal is given by equation 6.55.

$$V_{BPD}(t) = 2G_t \mathcal{R} \sqrt{P_{signal}P_{ref}} \cos(2\pi f_{beat}t) \quad (6.55)$$

The average electrical power measured at the detector is governed by the RMS voltage from the balanced detector and is given by equation 6.56.

$$P_{electrical,BPD} = \frac{V_{RMS}^2}{Z} = \frac{2G_t^2 \mathcal{R}^2 P_{signal}P_{ref}}{Z} \quad (6.56)$$

Sinusoidal Signals have a power spectrum governed by the resolution bandwidth of the measurement. The resolution bandwidth is the inverse measurement time in this case. Therefore the average power spectral density of the measured signal is given by equation 6.57.

$$S_{electrical,BPD} = \frac{V_{RMS}^2 t_{meas}}{Z} = \frac{G_t^2 \mathcal{R}^2 P_{signal}P_{ref} t_{meas}}{Z} \quad (6.57)$$

Therefore, the signal level for a coherent beat note is given by equation 6.58.

$$U_{signal} = \frac{N_s^2 G_T^2 \mathcal{R}^2 P_{signal} P_{ref}}{Z} \quad (6.58)$$

Noise In Power Spectra The spectral properties of electronic noise at the detector can be estimated with an RMS Noise figure $V_{Noise,RMS}$ measured on an ADC with impedance Z . Assuming that over the detector bandwidth, the noise level is flat, the power spectral density is given by equation 6.59.

$$S_{Noise} = \frac{|\langle V_{BPD} \rangle_\sigma|^2}{Z \Delta f_{det}} \quad (6.59)$$

For the digitized signal, the limiting bandwidth of the detection scheme (Δf_{det}) is either the detector bandwidth or the Nyquist bandwidth (assuming there is an anti-aliasing filter in place) which-ever is smaller. Assume the system is sampled at a sampling rate f_s and N_s samples are collected. Since FFTs are DFTs, the resulting spectrum will not be the true power spectral density per frequency bin. The DFT frequency resolution is given by $\Delta f_{bin} = 1/t_{meas}$ where t_{meas} is the duration of the sampled record of the signal which is also the quotient of the number of samples and the sample rate. Therefore, the noise power per bin is given by equation 6.60.

$$P_{noise,Bin} = S_{Noise} \frac{1}{t_{meas}} = \left(\frac{f_s}{N_s} \right) \frac{|\langle V_{BPD} \rangle_\sigma|^2}{Z \Delta f_{det}} \quad (6.60)$$

Using MATLAB's FFT (unless some additional normalization is applied), the FFT will add a factor of the number of samples to the equation. Therefore, when measuring a power spectrum, there is an additional factor of the number of samples squared in the measured

noise level, as shown in equation 6.61.

$$U_{Noise} = N_s^2 P_{noise, Bin} = f_s N_s \frac{|\langle V_{BPD} \rangle_\sigma|^2}{Z \Delta f_{det}} \quad (6.61)$$

Thermal Noise Floor Given the thermal RMS voltage noise from equation 6.40, the estimated noise floor using MATLAB's FFT will be given by equation 6.62.

$$U_{NEP} = \frac{f_s N_s G_T^2 \mathcal{R}^2 Q_{NEP}^2}{Z} \quad (6.62)$$

Shot Noise floor Given the RMS shot noise voltage from equation 6.46, the shot noise contribution to measured spectra using MATLAB's FFT is given by equation 6.63.

$$U_{Shot} = \frac{f_s N_s G_T^2 2q_e \mathcal{R} (P_1 + P_2)}{Z} \quad (6.63)$$

Quantization Noise In power spectra, the system's quantization noise will affect the power spectrum's noise floor. This contribution can be calculated beginning with the equivalent electrical power due to quantization given by equation 6.47. The quantization noise is spread across the entire signal spectrum (from negative to positive Nyquist) because it is an intrinsic property of detection. Hence, there is no way to filter it in the electrical domain; however, DSP techniques can be used to filter the signal digitally if required. Using MATLAB'S Normalization convention, the quantization noise floor can be expressed as equation 6.64.

$$U_{noise, quantization} = \frac{N_s V_{fs}^2 10^{-\frac{6.02 N_{bits} + 1.76 dB}{10}}}{8Z} \quad (6.64)$$

Note that the Noise floor only increases linearly with the number of samples. It is constant with the sampling rate, meaning that the effective floor will drop with the block size. In our experiments, quantization Noise was a weak contribution (i.e. less than the detector thermal

noise floor) so it will be omitted in most later discussions of noise contributions.

Signal-to-Noise Ratio In grating processing applications where signals are evaluated as power spectra, the typical metric used is the SNR as measured between the measured signal and the measured noise floor. Since these are power measurements, the noises can be added linearly. The total SNR is given by equation 6.65 (note detector noise is omitted, this is because in our experiments, it was found not to contribute).

$$SNR = \frac{U_{signal}}{U_{shot} + U_{NEP}} \quad (6.65)$$

The total SNR for this system is then given by equation 6.66.

$$SNR = \frac{\frac{1}{4}N_s \mathcal{R}P_{\Delta}}{f_s (\mathcal{R}Q_{NEP}^2 + 2q_e (P_1 + P_2))} \quad (6.66)$$

Window Functions Typically window functions are applied to signals before power spectrum estimations. These functions have various applications but are typically used to limit spectral leakage. Window functions will cause some attenuation to the signal, and the degree of this attenuation can be calculated by comparing the result with the expected result for a square window. Assume that a function will use a real-valued window function $F_w(t)$ for a signal of duration T. The integral of the window squared, given in equation 6.67 is the attenuation due to the window.

$$A_{window} = \frac{1}{T} \int_0^T F_w^2(t) dt \quad (6.67)$$

As an example, we will analyze the Hann window. The Hann window takes the form: $F_H(t) = \left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{T}\right)\right)$ so the attenuation is given in equation 6.68.

$$A_{Hann} = \frac{1}{T} \int_0^T \left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{T}\right)\right)^2 dt = \frac{3}{8} \quad (6.68)$$

Therefore the total attenuation to the signal due to using a Hann window is 3/8 or -4.26dB. If a Hann window is to be used, the calculated signal and noise levels in the power spectrum will be lower than expected by -4.26dB. This analysis can be repeated for other window functions.

Processing Gain The SNR can be rewritten to show that the dependence of signals depends on the ratio of the number of samples and the sample rate, as shown in equation 6.69.

$$SNR = \frac{N_s}{f_s} \frac{\frac{1}{4}\mathcal{R}P_\Delta}{\mathcal{R}Q_{NEP}^2 + 2q_e(P_1 + P_2)} \quad (6.69)$$

However, that ratio equals the measurement time, so the measured SNR is given by equation 6.70.

$$SNR = t_{measure} \frac{\frac{1}{4}\mathcal{R}P_\Delta}{\mathcal{R}Q_{NEP}^2 + 2q_e(P_1 + P_2)} \quad (6.70)$$

. This indicates that the SNR can be improved by increasing the duration of the measurement. This improvement is typically referred to as processing gain. In the case of digital signals, it is often quoted in terms of the block size (a convenient unit-less quantity); however, in this case, it is critical to remember that to get processing gain, the block size needs to be increased without changing the sample rate to get the expected improvements. The phenomenon of processing gain can be described as follows. The resolution bandwidth limits signal power spectral density. Therefore, all the signal power is confined to one resolution-limited spectral band. Regardless of the measurement duration, the signal level is constant. On the other hand, the noise has a constant spectral density, so as the resolution

bandwidth increases, the noise power per bin decreases. The net effect is that increasing the duration results in an improved SNR.

Suppression of Common Mode Noise

The previously discussed noise sources were noise sources arising from detection considerations. However, if there is noise on the optical fields, this noise will couple into detection unless it is common-mode noise, which, if the powers and path lengths are appropriately balanced, can be canceled out.

Path Length Difference Considerations for Balanced Detection Assume there is a noise source in the system that can be described in terms of the time domain power of the noise, a detector of bandwidth Δf_{det} and therefore a detector integration time $t_{det} = \frac{1}{2\Delta f_{det}}$ (which is a property of the detector). Suppose the optical noise power on one photo-diode is $P(t)$ and the signal on the other photo-diode is a time-delayed copy of that noise $P(t + t_{delay})$. In that case, the time domain voltage signal produced by the detector will be expressed by equation 6.71.

$$V(t) = \frac{G_t \mathcal{R}}{t_{det}} \int_{t-t_{det}}^t (P(t' + t_{delay}) - P(t')) dt' \quad (6.71)$$

Looking in frequency, the Fourier transform of the time domain voltage is given by equation 6.72.

$$\tilde{V}(f) = \frac{G_t \mathcal{R} \mathcal{F}\{P(t)\}}{2\pi f t_{det}} (e^{2\pi f t_{delay}} - 1) (1 - e^{-2\pi f t_{det}}) \quad (6.72)$$

the electrical power spectral density BPD voltage can be expressed with equation 6.73.

$$s(f) = 4 \frac{G_t \mathcal{R} \mathcal{F}\{P(t)\}}{z} \text{sinc}^2(f t_{det}) \sin^2(f t_{delay}) \quad (6.73)$$

The first factor is essentially the power spectrum of the noise if it were detected on only

one diode ($s_0(f)$)). So the final noise power spectral density is given by equation 6.74.

$$s(f) = 4s_0(f) \text{sinc}^2(ft_{res}) \sin^2(ft_{delay}) \quad (6.74)$$

If $t_{delay} \ll t_{res}$, then the power spectrum will be reduced essentially to zero; this is especially true if the noise is confined by some other process to the central peak of the detector response. So the condition for achieving good common mode rejection is that $t_{delay} \ll \frac{1}{2\Delta f_{det}}$. Assuming that the time delay arises from a path length difference ($t_{delay} = \frac{\Delta\ell}{c}$), the final requirement on the path length difference is given by equation 6.75.

$$\Delta\ell \ll \frac{c}{2\Delta f_{det}} \quad (6.75)$$

Therefore, the path length difference tolerance is governed by the required detection bandwidth. A 1 MHz bandwidth requires a path length difference of much less than 1.5 km while a 1GHz detection bandwidth of much less than 150cm, both of which are relatively easy conditions to meet. ThorLabs, the manufacturer of photodiodes used for experiments, indicated that at 4 MHz of bandwidth, 30dB of common mode rejection can be achieved with a path length difference on the order of half a meter of the path length difference [69]. The path lengths after the splitter are within about a centimeter, at which suppression could hypothetically be up to 60dB.

Common Mode Rejection Ratio If the read powers are well matched and the path length mismatch is sufficiently small, then the noise will be suppressed up to what is called the common mode rejection ratio (CMRR) of the BPD. This quantity specifies the degree to which a BPD can suppress common mode noise. This will be discussed in more detail in chapter 8.

Sample Noise Calculations

Using the math from the preceding sections, the noise properties of a particular detector/detection scheme can be predicted. These predictions will use a noise model considering thermal and photo-electronic shot noise. In experiments, a ThorLabs PDB450A balanced detector was used to detect 794nm optical signals. The signals were then digitized using a 50 Ω impedance digitizer. Using these parameters (along with the properties of the detector given in table 6.2), the RMS voltage noise and the power spectrum noise floor for various LO powers. Table 6.2 indicates the properties for the variable balanced detector at different bandwidths (note that the parameters vary for different bandwidth settings).

Bandwidth (MHz)	Responsivity (A/W)	Transimpedance Gain (V/A)	$Q_{NEP}(pW/\sqrt{Hz})$
0.1	0.53	0.5×10^7	1.4
0.3	0.53	0.5×10^6	1.1
4	0.53	0.5×10^5	3.3
45	0.53	0.5×10^4	28.9
150	0.53	0.5×10^3	123

Table 6.2: Table of detector properties for the ThorLabs PDB450a balanced detector.

Time Domain RMS Noise Given the system parameters, the noise level can be calculated at different detector gain/bandwidth settings as a function of LO power, assuming a 5 μW read field power. The thermal noise contribution can be calculated from equation 6.40, and the shot noise contribution per watt of incident electrical power can be calculated from equation 6.46. The voltage noise parameters for the detector are given by 6.3. These values can be added in quadrature to determine the total voltage noise in detection. Assuming a system with equal powers on each detector, the total noise voltage can be calculated as a function of LO power (see figure 6.20). From the calculated curves, it is clear

Bandwidth (MHz)	Transimpedance Gain (V/A)	Thermal Noise (mV)	Shot Noise $V/\sqrt{W}$
0.1	0.5×10^7	1.17	0.6530
0.3	0.5×10^6	0.16	0.1131
4	0.5×10^5	0.17	0.0413
45	0.5×10^4	0.51	0.0139
150	0.5×10^3	0.39	0.0025

Table 6.3: Table of noise properties for the ThorLabs PDB450a balanced detector at different settings.

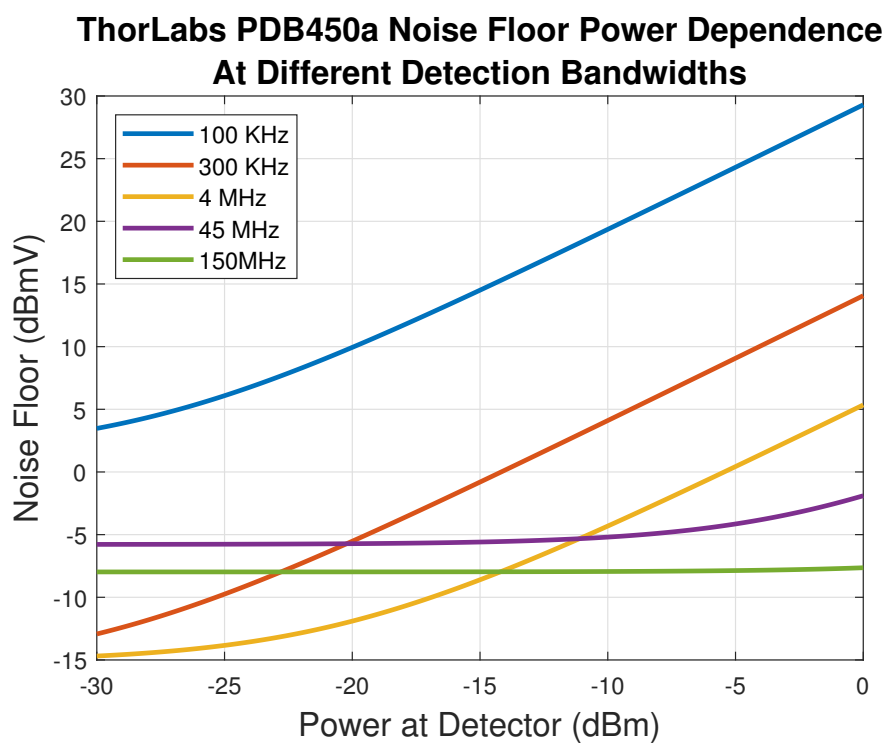


Figure 6.20: RMS Voltage Noise curves at different detection bandwidths for the PDB450a

that noise contributions are increased at higher bandwidths, especially because the signal gain is decreased at these higher bandwidths.

Power Spectrum Noise Floor In power spectra, the final noise floor will be governed by the detector parameters, the number of samples, and the sample rate. Given a sample rate of 20MSPS, a record length of 8000 samples, and assuming the detector was used in the 4MHz condition (these conditions were chosen to replicate experimental conditions), the use of a Hann window was emulated by subtracting 4.26dB from the final result. The expected noise floor observed in MATLAB's power spectra is shown in figure 6.21.

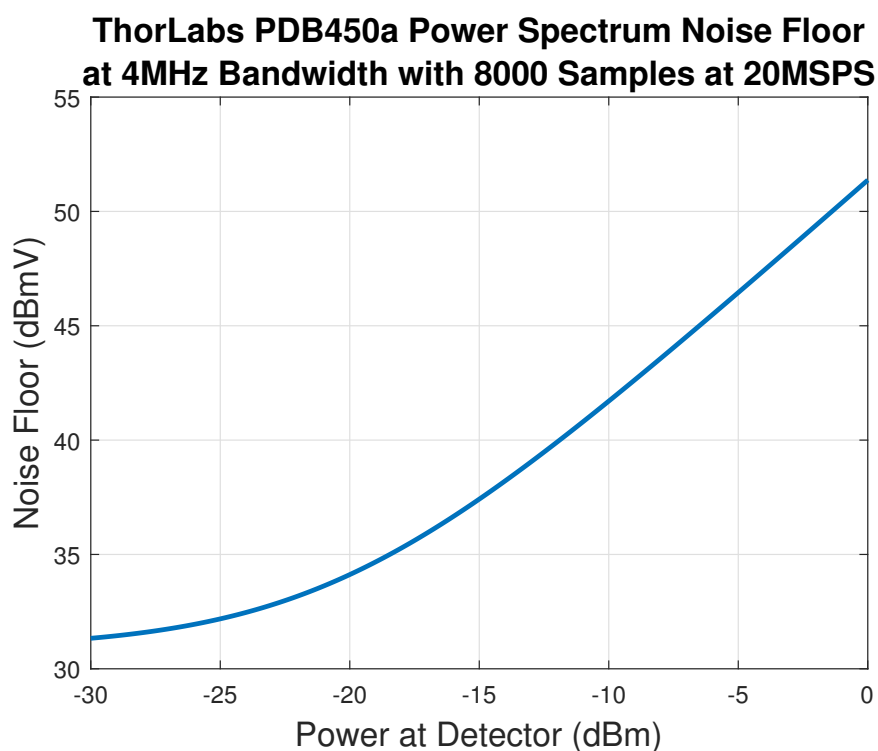


Figure 6.21: Projected noise levels in power spectra estimated for MATLAB's FFT using the PDB450a detector at 4MHz bandwidth, collecting 8000 samples at 20MSPS

Choosing Detection Bandwidth

One of the major parameters needed to pick a detector is bandwidth specification. This is relatively straightforward in cases where the goal is to measure sinusoidal signals; the bandwidth should be sufficient for detection. However, typically higher bandwidth detectors have lower sensitivity than lower bandwidth detectors, so some care needs to be taken in

picking a detector. In cases of observing pulses or other time domain features, the desired temporal resolution will govern the bandwidth.

Chirped Readout Of a Narrow Spectral Feature When using a chirped field to measure spectral holes, the detector bandwidth will be constrained by the spectral features that will be encountered. We will assume a Lorentzian feature in a hole-burning material with a linewidth of Γ_H being read out by a chirped optical field with chirp rate κ . The required bandwidth to readout will be given by equation 6.76.

$$\Delta f = \frac{\kappa}{\Gamma_H} \quad (6.76)$$

Therefore to measure a 1.2 MHz Spectral hole using a chirp field at $2 \frac{MHz}{\mu S}$, a minimum bandwidth of 1.6 MHz is needed to fully resolve the hole (roughly corresponding to the SSH spectrum analysis data). A 4 MHz bandwidth is required to fully resolve spectral holes of 0.5 MHz width.

SSH Grating Processing In SSH grating processing applications, the bandwidth governs the maximum range since the detected beat note is given by equation 6.77.

$$f_{beat} = \frac{2d\kappa}{c} = \Delta t_{max}\kappa \quad (6.77)$$

Where d is the distance, c is the speed of light, and κ is the chirp rate. In SSH lidar applications, this is governed by the homogeneous linewidth according to equation 6.78.

$$d_{max} = \frac{c}{4\Gamma_H} \quad (6.78)$$

Given a linewidth of 0.5 MHz, the maximum range given the material constraints is expected to be around 150m, corresponding to a delay of about $1\mu s$. The beat note of this maximum

range grating will be around 2MHz, meaning the minimum detector bandwidth for this system with spectral gratings is 2MHz.

Final Choice of Bandwidth Setting The applications using the material will require a 2 MHz detection bandwidth. The bandwidth setting on the detector with the lowest relative noise floor was the 4MHz bandwidth setting. Therefore in experiments, the 4MHz bandwidth setting was used, and a 5MHz low-pass anti-aliasing filter was installed after the detector.

Final System

Final Interferometer System Design

The final interferometer design is depicted as a schematic in figure 6.22. As a photograph in figure 6.23, this system should be able to analyze a 2GHz region in the absorption spectrum of the sample, which corresponds to a 2GHz bandwidth of RF signals on modulated onto the programming field. The system will have a spectral resolution of 1MHz (or 2 MHz, depending on when the experiment was conducted). Digital signals are captured at 20 MSPS, which is more than enough to observe signals captured by the 4 MHz detector. The interferometer is balanced to within 2 cm. Though the output splitter is imperfect, the imperfections should still allow common mode suppression to around 18dB (higher for the LO field).

Experiment Timing

Timing in SSH readout experiments is important. In this particular system, several timing elements need to be coordinated. The experiment's explicit timing is shown in figure 6.24. The timing sequence references a primary trigger generated by the chirper. The FPGA and DAC system (and the AWG used to replace it when it failed) generate triggers at the beginning of each chirp which occurs sequentially at 1ms intervals; since a longer timing duration was required for experiments, this duration was extended to 1 second to allow for

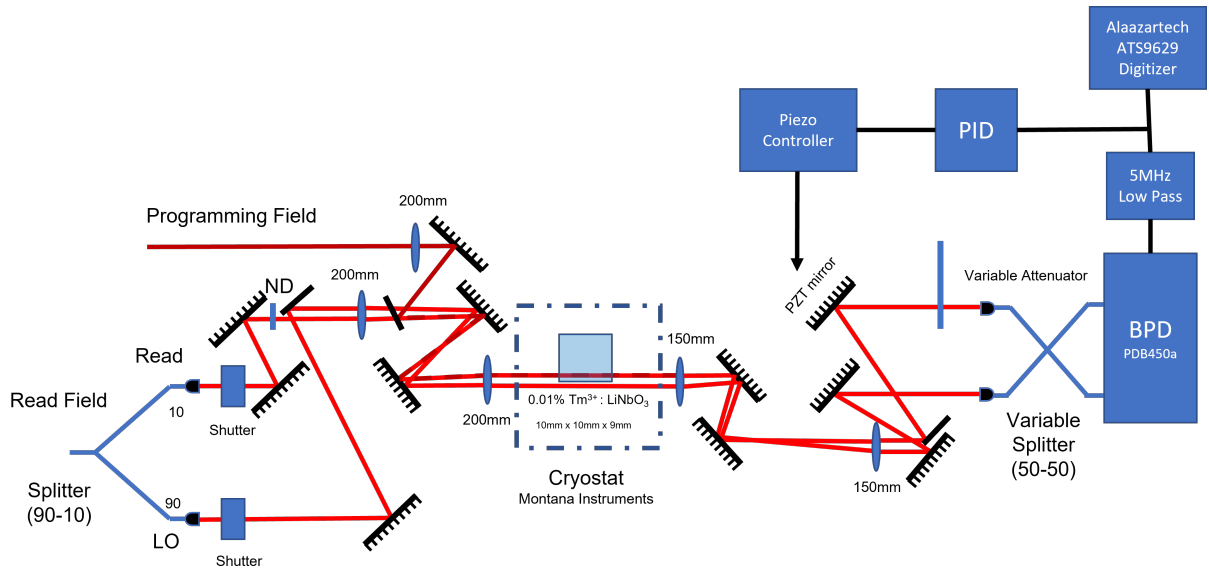


Figure 6.22: A diagram indicating the optical design considerations, including lenses used for an image relay and the rough configuration of turning mirrors.

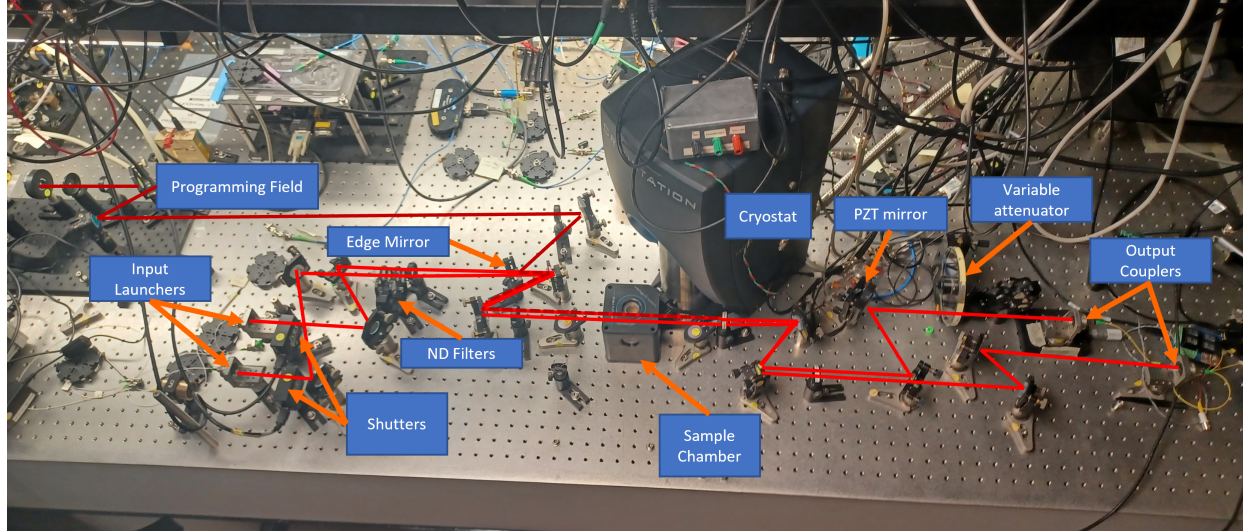


Figure 6.23: A photograph of the interferometer system used for interferometric readout experiments with fields and some key components marked

all of the required events to occur. To achieve this extended sequence duration, an SRS DG535 delay generator was used with a 1s sequence duration so that the timing system would only accept the primary timing trigger once every second. The subsequent timing was

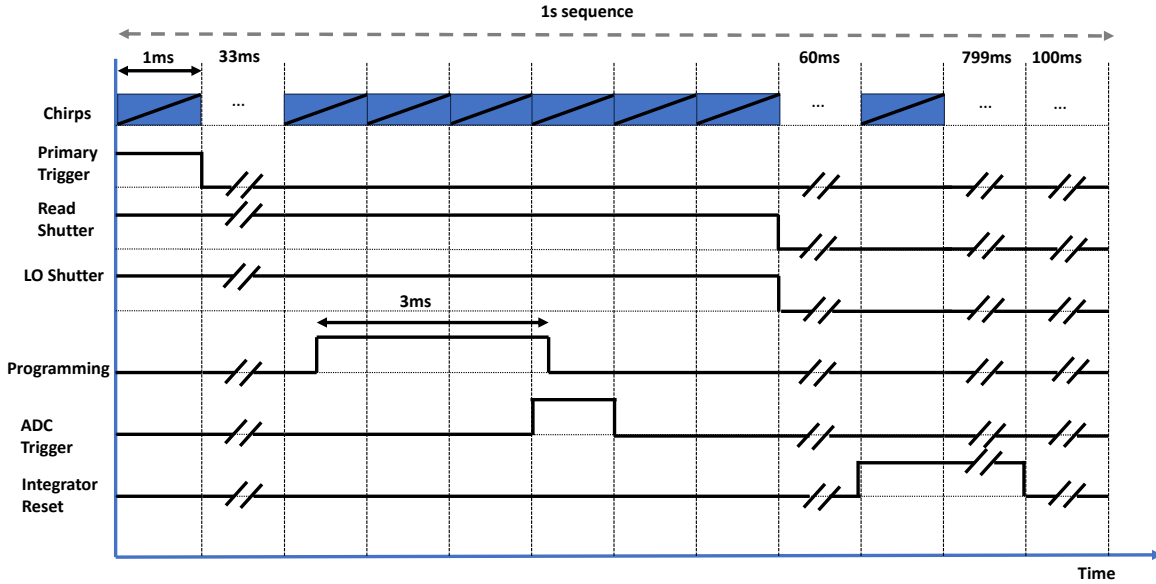


Figure 6.24: The timing diagram for interferometric SSH readout experiments. This 1-second sequence shows the timing between key elements in the interferometric SSH readout system.

controlled using an HP 8175a programmable delay generator. Just after the beginning of the sequence, the shutters that control the read and LO fields are switched on, these mechanical devices need a few milliseconds to open, and after around 10 ms, they are fully open. These shutters serve to control the duty cycle of the read field to avoid power broadening, which was observed before the implementation of the shutter system. The read and LO fields need to be on for around 30 ms before measurements are conducted to give the feedback control loop to stabilize the system. At 37ms, a trigger pulse is sent to the ATS9626 ADC to begin capture. 700s before the ADC trigger pulse, a pulse is sent to the AOM driver to generate a programming pulse of duration 3ms. This pulse programs the material and is timed so that while it overlaps the read chirp, the programming field is switched off before the captured read chirp can scan over any signals of interest. 3ms after the rising edge of the ADC trigger, the read and LO field shutters switch off. 60ms after the shutters are closed, a pulse is sent to the microcontroller driving the relay that shorted the integrator; this ensures that the

integrator is shorted for 800ms of downtime. 100ms after the end of this downtime, the cycle will begin again upon receiving another primary trigger signal.

INTERFEROMETRIC READOUT FOR SSH SPECTRUM ANALYSIS

Motivation

The portion of the optical power spectrum of an optical field that falls within the inhomogeneous absorption line of a spatial-spectral holographic (SSH) material will burn itself into the absorption spectrum of that material. The modified absorption spectrum can be extracted using another chirped optical field. Therefore, the optical power spectrum of the first field (within the inhomogeneous line width of the material) can be extracted [16]. By modulating an RF signal onto an optical field using an electro-optic modulator, the RF power spectrum of the signal can be analyzed using the material [40, 43]. Using this method, there is a 100% probability of intercept of signals [38] because the material will record the optical power spectrum of everything that illuminates it. Signal-to-noise ratio (SNR) in SSH spectrum analysis is a critical parameter because it will help determine the functional dynamic range of the device. Typically, a stronger read field in the material will improve the signal-to-noise ratio. Still, there can be limitations on read chirp power due to effects like thermal broadening and material depopulation effects. By using interferometric readout where the read field is mixed with a strong coherent local oscillator (LO) signal, the read signal can be amplified, which under many circumstances will lead to an improvement to signal to noise ratio (however, this is dependent on the behavior of the noise floor in addition to the behavior of the signals), allowing for a better signal-to-noise ratio using the same read power.

Spectral Programming of SSH materials

As was discussed in chapter 1, if a field of optical power spectral density $S(f)$ programs holes SSH material of initial absorption coefficient (α_0), the modified absorption coefficient

(α) due to depopulation from the programming field of the material can be described using equation 7.1

$$\alpha(f) = \alpha_0 (1 - \beta S_{eff}(f)) \quad (7.1)$$

where β is a parameter that governs the depth of spectral holes programmed into the material by a beam of a given area. Assume a programming field has an effective power spectral density $S_{eff}(f)$ The magnitude of the field transmission coefficient for the programmed material, as originally given in equation 1.8 in chapter 1, is reiterated here for pencil beams in equation 7.2)

$$|\nu(f)| = e^{-\frac{1}{2}\alpha(f)L} = e^{-\frac{1}{2}\alpha_0 L(1-\beta S_{eff}(f))} \quad (7.2)$$

where L is the length of the material. However, to complete the model, the phase of $\nu(f)$ needs to be selected to preserve causality using the Hilbert transform [32] (this was discussed in greater detail in chapter 1 on page 6) The causal field transmission spectrum of the material for programmed beams, originally given in equation 1.16 on page 9 in chapter 1 and is reiterated here in equation 7.3.

$$\nu(f) = e^{-\frac{1}{2}\alpha_0 L[1-\beta(S_{eff}(f)+i\mathcal{H}\{S_{eff}(f)\})]} \quad (7.3)$$

Traditional and Interferometric readout of Programmed SSH materials

Chirped Readout of Programmed Spectra

Chirped optical fields can be used to read out spectra programmed into SSH materials, as was discussed previously in chapter 1. Two readout methods will be explored, traditional readout, which was discussed extensively in chapter 1, and interferometric readout, which was briefly discussed in chapter 2. The following section describes both methods generally, so there will be some overlap with material previously presented. A chirp is a signal with a time-dependent quadratic phase. Chirps are useful for mapping the frequency domain onto

the time domain. A unit amplitude chirp of chirp rate κ and start frequency f_L is described by equation 7.4.

$$\Phi(t) = e^{2\pi i[\frac{1}{2}\kappa t^2 + f_L t]} \quad (7.4)$$

In the frequency domain, the same chirp can be represented as the Fourier transform of the time domain chirp (see equation 7.5).

$$\tilde{\Phi}(f) = \mathcal{F}\{\Phi(t)\} = \sqrt{\frac{1}{\kappa}} e^{\frac{-i\pi(f-f_L)^2}{\kappa}} e^{-i\frac{\pi}{4}} \quad (7.5)$$

The optical field will be defined as some field amplitude (E_0) times the unit chirp described in equation 7.5. The time domain optical field used to interrogate programmed SSH materials is then given by equation 7.6.

$$E_r(t) = E_0\Phi(t) = E_0 e^{2\pi i[\frac{1}{2}\kappa t^2 + f_L t]} \quad (7.6)$$

The read field transmitted through the material can be expressed in frequency domain as the product of the field transmission spectrum, and the frequency domain chirped field, which is expressed in equation 7.7.

$$\tilde{E}_{out}(f) = E_0 \tilde{\Phi}(f) e^{-\frac{1}{2}\alpha_0 L [1 - \beta(S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\})]} \quad (7.7)$$

In the linear programming regime (i.e., $\beta S_{eff} \ll 1$), the output field is given approximately by equation 7.8.

$$\tilde{E}_{out}(f) \approx E_0 \tilde{\Phi}(f) e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{1}{2}\alpha_0 L \beta (S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\}) \right] \quad (7.8)$$

The time domain transmitted read field can be calculated from the frequency domain read

field as given by equation 7.9.

$$E_{out}(t) = E_0 e^{-\frac{1}{2}\alpha_0 L} \left[\Phi(t) + \frac{1}{2}\alpha_0 L \beta \left(\mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S_{eff}(f) \right\} + i \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) \mathcal{H} \{ S_{eff}(f) \} \right\} \right) \right] \quad (7.9)$$

The transmitted time domain field contains 3 terms. The first term represents the transmitted chirp absent any effects other than simple absorption; the second and third terms represent the real and imaginary parts of the interaction with the programmed spectrum, respectively.

Slow Readout In the slow readout regime, the instantaneous chirp bandwidth ($\Delta f_{instantaneous} = \sqrt{\kappa}$) is much less than the homogeneous linewidth of the material ($\sqrt{\kappa} \ll \Gamma_H$). In the slow readout regime, the material responds to the chirp as if it had a different frequency at each time. The result is that the chirp essentially maps times to frequencies as $f_c(t) = \kappa t + f_L$ the time domain output field is then given by equation 7.10.

$$E_{out}(t) = E_0 e^{-\frac{1}{2}\alpha_0 L} \Phi(t) \left[1 + \frac{1}{2}\alpha_0 L \beta (S_{eff}(\kappa t + f_L) + i \mathcal{H} \{ S_{eff}(\kappa t + f_L) \}) \right] \quad (7.10)$$

In this case, the three terms in the time domain transmitted field: the transmitted chirp, a term related to the amplitude of the burned hole, and the imaginary term proportional to the frequency dependent phase shift incurred by the field, which is in turn proportional to the Hilbert transform of S_{eff} .

Fast Readout and the Spectral Recovery Algorithm

Fast Readout-Coherent Detection Signals If the instantaneous chirp bandwidth ($\sqrt{\kappa}$) is not much less than the homogeneous linewidth, more care must be taken to invert the

Fourier transform. The chirp will not simply map time onto frequency because the most narrow spectral features will distort the chirp. Using the convolution theorem, the readout spectrum is given by equation 7.11.

$$E_{out}(t) = ... \\ ...E_0 e^{-\frac{1}{2}\alpha_0 L} \left[\Phi(t) + \frac{1}{2}\alpha_0 L \beta \left[(\Phi(t) * \mathcal{F}^{-1} \{S_{eff}(f)\}) (t) + i (\Phi(t) * \mathcal{F}^{-1} \{\mathcal{H} \{S_{eff}(f)\}\}) (t) \right] \right] \quad (7.11)$$

The following sections will describe two coherent detection techniques. The first is the widely employed traditional method where the transmitted portion of the chirp interferes with the portion of the chirp that interacts with programmed spectra. The second is an interferometric readout where the portion of the chirp that interacts with programmed spectra interferes with a separate local oscillator. The detection signal for either method can be described as proportional to either the real or imaginary part of a generalized detection signal $F(t)$ (specifically, the traditional signal will be proportional to the real part of $F(t)$. In contrast, interferometric readout gives the imaginary part of $F(t)$) which is given in equation 7.12

$$F(t) = \Phi^*(t) (\Phi(t) * \mathcal{F}^{-1} \{S'(f)\}) (t) \quad (7.12)$$

where Φ is the chirp defined in equation 7.5, and $S'(f)$ is the causal version of the programmed spectrum ($S'(f) = S_{eff}(f) + i\mathcal{H} \{S_{eff}(f)\}$) by rearranging the arguments in the chirp within the convolution integral this convolution can be reduced to a Fourier transform for a frequency variable ($f = \kappa t + f_L$) shown in equation 7.13.

$$F(t) = \int_{-\infty}^{\infty} e^{2\pi i (\frac{1}{2}\kappa t'^2)} \mathcal{F}^{-1} \{S'(f)\} e^{-2\pi i (\kappa t + f_L)t'} dt' = \mathcal{F} \left\{ e^{2\pi i (\frac{1}{2}\kappa t'^2)} \mathcal{F}^{-1} \{S'(f)\} \right\} (\kappa t + f_L) \quad (7.13)$$

This Fourier transform maps frequency domain back into the time domain. Using the convolution theorem again, this can be recast as frequency domain convolution mapped into time domain as shown in equation 7.14.

$$F(t) = \sqrt{\frac{1}{\kappa}} e^{-i\frac{\pi}{4}} \left(e^{-i\pi \frac{f^2}{\kappa}} * S'(f) \right) (\kappa t + f_L) = \sqrt{\frac{1}{\kappa}} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} e^{-i\pi \frac{f'^2}{\kappa}} S'(\kappa t + f_L - f') df' \quad (7.14)$$

By substituting the integration variable $f' = \kappa t'$ and substituting the differential element $df' = \kappa dt'$, equation 7.15 is obtained.

$$F(t) = \sqrt{\kappa} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} e^{-2\pi i \left(\frac{1}{2} \kappa t'^2 \right)} S'(\kappa(t - t') + f_L) dt' \quad (7.15)$$

This can be explicitly expressed as a time domain convolution between the chirp and the causal power spectral density (S'), giving the final form of $F(t)$ in equation 7.16.

$$F(t) = \sqrt{\kappa} e^{-i\frac{\pi}{4}} \left(e^{-2\pi i \left(\frac{1}{2} \kappa t'^2 \right)} * S'(\kappa t + f_L) \right) (t) \quad (7.16)$$

By the same process, the complex conjugate $F^*(t)$ is given by equation 7.17.

$$F^*(t) = \sqrt{\kappa} e^{i\frac{\pi}{4}} \left(e^{2\pi i \left(\frac{1}{2} \kappa t'^2 \right)} * S'^*(\kappa t + f_L) \right) (t) \quad (7.17)$$

Fast Readout-Spectral Recovery algorithm The spectrum $S'(t)$ can be extracted from $F(t)$ and $F^*(t)$ by convolving them with a real-valued chirped recovery kernel $\Psi(t)$ given in equation 7.18 [16, 72].

$$\Psi(t) = 2\sqrt{\kappa} U(t) \cos \left(\frac{1}{2} \kappa t^2 + \frac{\pi}{4} + \phi_m \right) \quad (7.18)$$

where $U(t)$ is the Heaviside step. This recovery kernel is the same as the recovery kernel given by Chang et al. [16, 72], except that an arbitrary phase shift ϕ_m has been added.

The phase shift ϕ_m is a mixing angle that may arise in cases where the coherent detection signal contains mixed real and imaginary parts of F in the measured signal. The addition of the mixing angle allows the recovery algorithm to be applied to a broader class of problems, including interferometric readout. The original spectrum can be recovered by convolution of the coherent detection signals with the recovery kernel $\Psi(t)$. By the associative property of convolution, the convolution of the chirp with the recovery kernel can be done first, leaving the programmed spectrum $S'(f)$. The recovered elements from $F(t)$ and $F^*(t)$ are given by equations 7.19 and 7.20.

$$g_+ = (\Psi(t) * F(t))(t) = \sqrt{\kappa} e^{-i\frac{\pi}{4}} \left(\left(\Psi(t) * e^{-2\pi i(-\frac{1}{2}\kappa t'^2)} \right) * S'(\kappa t + f_L) \right)(t) \quad (7.19)$$

$$g_- = (\Psi(t) * F^*(t))(t) = \sqrt{\kappa} e^{i\frac{\pi}{4}} \left(\left(\Psi(t) * e^{2\pi i(\frac{1}{2}\kappa t'^2)} \right) * S'^*(\kappa t + f_L) \right)(t) \quad (7.20)$$

These convolutions can be expressed as a frequency domain product as shown in equations 7.21 and 7.22

$$\tilde{g}_+ = \tilde{\Psi} e^{\frac{i\pi f^2}{\kappa}} \mathcal{F} \{ S'(\kappa t + f_L) \} \quad (7.21)$$

$$\tilde{g}_- = \tilde{\Psi} e^{\frac{-i\pi f^2}{\kappa}} \mathcal{F} \{ S'^*(\kappa t + f_L) \} \quad (7.22)$$

The frequency domain form of the recovery kernel is given by equation 7.23.

$$\tilde{\Psi}(f) = e^{i \operatorname{sgn}(f) \left(\frac{\pi f^2}{\kappa} + \phi_m \right)} \quad (7.23)$$

The frequency domain expressions for $\tilde{g}_+$ and $\tilde{g}_-$ can be rewritten in frequency domain as equations 7.24 and 7.25.

$$\tilde{g}_+ = e^{-i(1+\text{sgn}(f))\frac{\pi f^2}{\kappa}} e^{i\text{sgn}(f)\phi_m} \mathcal{F}^{-1} \{S'(\kappa t + f_L)\} \quad (7.24)$$

$$\tilde{g}_- = e^{-i(1-\text{sgn}(f))\frac{\pi f^2}{\kappa}} e^{i\text{sgn}(f)\phi_m} \mathcal{F}^{-1} \{S'^*(\kappa t + f_L)\} \quad (7.25)$$

Because S' and S'^* have imaginary parts prescribed by the Hilbert transform, which was initially presented to remove the anomalous response from negative times. Because they were mapped into time domain by an earlier integral, the Fourier transform of these functions will be single-sided in frequency domain. Specifically, $S'(\kappa t + f_L)$ has only negative frequency components while $S'^*(\kappa t + f_L)$ has only positive frequency components. Therefore, the signum functions can be omitted. The final forms of $\tilde{g}_+$ and $\tilde{g}_-$ are given by equations 7.26 and 7.27

$$\tilde{g}_+(t) = e^{-i\phi_m} \mathcal{F}^{-1} \{S'(\kappa t + f_L)\} \quad (7.26)$$

$$\tilde{g}_-(t) = e^{i\phi_m} \mathcal{F}^{-1} \{S'^*(\kappa t + f_L)\} \quad (7.27)$$

The time domain forms of g_+ and g_- are related to S' and S'^* as seen in equations 7.28 and 7.29.

$$g_+(t) = e^{-i\phi_m} S'(\kappa t + f_L) \quad (7.28)$$

$$g_-(t) = e^{i\phi_m} S'^*(\kappa t + f_L) \quad (7.29)$$

If there is no mixing angle (i.e. $\phi_m = 0$) The real and imaginary components of the causal spectrum $S'(t)$ can be computed from g_+ and g_- . The results correspond to the recovery of the real and imaginary components of $F(t)$ as shown in equations 7.30 and 7.31

$$g_r = (\Psi(t) * \text{Re}[F(t)])(t) = \frac{1}{2}(g_+(t) + g_-(t)) = S_{eff}(\kappa t + f_L) \quad (7.30)$$

$$g_i = (\Psi(t) * \text{Im}[F(t)])(t) = \frac{1}{2i}(g_+(t) - g_-(t)) = \mathcal{H}\{S_{eff}(\kappa t + f_L)\} \quad (7.31)$$

This is the basis of how spectra are extracted from fast-chirped balanced detection signals in traditional readout.

If there is some mixing between the real and imaginary components that can be expressed as a function of the mixing angle ϕ_m , such as the mixing that occurs due to the interferometric phase in the interferometric readout, the mixed, measured signal $F''(t)$ is expressed by equation 7.32.

$$F''(t) = \text{Re}[F(t)e^{i\phi_m}] = \frac{1}{2}(F(t)e^{i\phi_m} + F^*(t)e^{-i\phi_m}) \quad (7.32)$$

If this mixing angle is known, it can be applied to the recovery kernel to cancel out the mixing, leaving equation 7.33.

$$g''(t) = (\Psi(t) * F''(t))(t) = S_{eff}(\kappa t + f_L) \quad (7.33)$$

In the case of traditional readout, the mixing angle is 0 and the extant spectral recovery algorithm described by Chang et al. [16, 72] can be used. In an ideal interferometric readout scheme, as shall be shown shortly, the mixing angle is $\pi/2$. If the recovery algorithm was applied not accounting for the mixing phase, then the recovered component would be the imaginary part. In order to fully recover the programmed spectrum, an extra Hilbert transform is required; however, accounting for the mixing phase properly in the recovery algorithm allows for the recovery of the programmed spectrum without the additional Hilbert transform. However, as was the case with the Hilbert transform, the DC component of the

recovered spectrum is not present, so a small amount of added effort will be required to recover the desired spectrum. In our experiments this was handled incidentally by the background subtraction algorithm.

Traditional Readout

The commonly used detection scheme for the readout of spectral features in an SSH material is to use a balanced detector to measure the difference between the power in the read field and a reference field (which has a power equal to the power of the read field at the detector, which can be achieved by having both fields propagate through the material or by attenuating the programming field to the correct power). This scheme is depicted in figure 7.1.

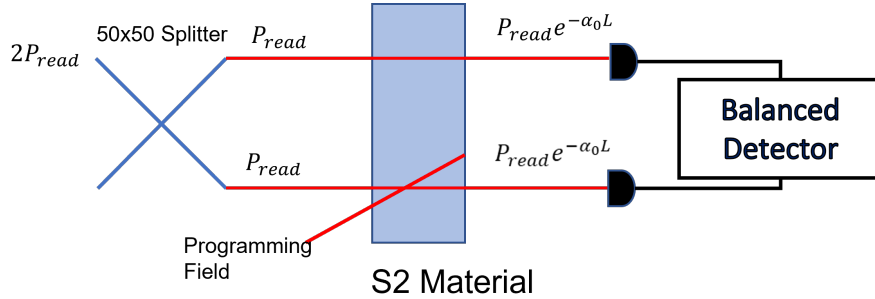


Figure 7.1: A Schematic of a traditional readout system with optical powers indicated at critical locations

Using the equation for balanced detector signals given in equation 1.31 from chapter 1. At the balanced detector, the differential power measured at the detector is then given by equation 7.34.

$$P_{\Delta} = P_{read} e^{-\alpha_0 L} \alpha_0 L \beta \operatorname{Re} \left[\Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} S_{eff}(f) \right\} + i \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} \mathcal{H} \{ S_{eff}(f) \} \right\} \right] \quad (7.34)$$

Given a balanced detector with responsivity $\mathcal{R}$ and transimpedance gain G_T the measured

voltage is then given by equation 7.35

$$V(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read} e^{-\alpha_0 L} \operatorname{Re} \left[\Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} S_{eff}(f) \right\} + i \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} \mathcal{H} \{ S_{eff}(f) \} \right\} \right] \quad (7.35)$$

If the power into the crystal is P_{read} , the power at the detector (absent any programming) is $P_{read,det} = P_{read} e^{-\alpha_0 L}$ so the measured voltage can be calculated in terms of the read power at the detector, which is given by 7.36. This will also account for any additional attenuation after the material.

$$V(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} \operatorname{Re} \left[\Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} S_{eff}(f) \right\} + i \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi} \mathcal{H} \{ S_{eff}(f) \} \right\} \right] \quad (7.36)$$

This balanced detection voltage can be used to recover the programming power spectrum either in the slow read limit or in the fast readout using the spectral recovery algorithm.

Slow Readout In the case of slow readout, the slow chirp will map frequency domain changes to the chirp amplitude changes in time, giving a detector voltage as seen in equation 7.37.

$$V(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} S_{eff}(\kappa t + f_L) \quad (7.37)$$

In this case, the amplitude of the programmed spectrum can be calculated from the balanced detector voltage.

Fast Readout In the fast readout limit, the time domain voltage is given by equation 7.38.

$$V(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} \operatorname{Re} \left[\Phi^* \left(\Phi * \mathcal{F}^{-1} \{ S_{eff}(f) \} \right) (t) + i \Phi^* \left(\Phi * \mathcal{F}^{-1} \{ \mathcal{H} \{ S_{eff}(f) \} \} \right) (t) \right] \quad (7.38)$$

In terms of the generic detector signal F , the time domain voltage of a traditional readout signal is given by equation 7.39.

$$V(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} \text{Re}[F(t)] \quad (7.39)$$

In the fast readout limit, the convolutions need to be confronted directly. This is done by using the spectral recovery algorithm. The recovered version of the voltage signal can be calculated by convolving the voltage signal with the recovery kernel with no mixing angle resulting in the recovered signal proportional to the effective programming spectrum shown in equation 7.40.

$$V_{rec}(t) = (\Psi(t) * V(t))(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} S_{eff}(\kappa t + f_L) \quad (7.40)$$

Interferometric Readout

The interferometric readout scheme coherently mixes the field out of the crystal with an optical local oscillator (LO) chirp in a 50-50 beam splitter. The mixed fields at the outputs are then balanced and detected. The interferometric readout scheme is depicted in figure 7.2.

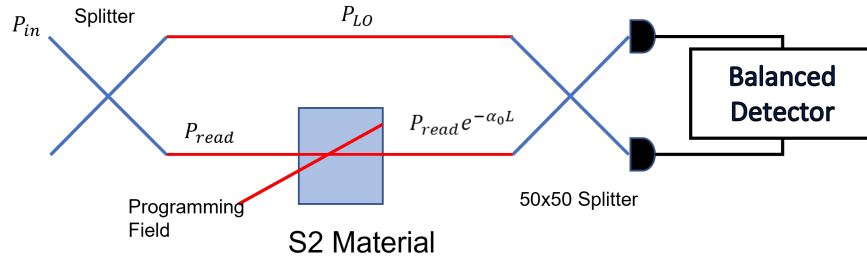


Figure 7.2: A schematic of an interferometric readout system with optical powers indicated at key locations

Before the 50-50 splitter, the LO field is given by equation 7.41.

$$E_{LO}(t) = iE_{LO,0}\Phi e^{i\phi_B} \quad (7.41)$$

Where $E_{LO,0}$ is the LO field amplitude, and ϕ_B is the interferometric bias that governs if the interferometer is in the quadrature, peak null, or some other condition. The additional factor of i reflects the phase shift at the input splitter, which in this case is not a 50-50 splitter since the power in the LO field is much greater than the power in the read field. The 50-50 beam splitter at the output of the interferometer mixes the read field (equation 7.9) and the local oscillator field 7.41 into two mixed fields given by equations 7.42 and 7.43.

$$E_1(t) = \frac{i}{\sqrt{2}} \left[E_0 e^{-\frac{1}{2}\alpha_0 L} \left[\Phi(t) + \frac{1}{2}\alpha_0 L \beta \left(\mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) \right] + E_{LO,0} \Phi e^{i\phi_B} \right] \quad (7.42)$$

$$E_2(t) = \frac{1}{\sqrt{2}} \left[E_0 e^{-\frac{1}{2}\alpha_0 L} \left(\Phi(t) + \frac{1}{2}\alpha_0 L \beta \left(\mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) - E_{LO,0} \Phi e^{i\phi_B} \right) \right] \quad (7.43)$$

The power out of each port of the interferometer is proportional to the magnitude squared of each field at each port of the interferometer. The powers at each output port are given by equations 7.44 and 7.45.

$$P_1(t) = \frac{1}{2} [P_{read} e^{-\alpha_0 L} + P_{LO} + \dots \dots \frac{1}{2} \sqrt{P_{LO} P_{read} e^{-\alpha_0 L}} e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) + C.C] \quad (7.44)$$

$$P_2(t) = \frac{1}{2} [P_{read} e^{-\alpha_0 L} + P_{LO} \dots \dots - \frac{1}{2} \sqrt{P_{LO} P_{read} e^{-\alpha_0 L}} e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) + C.C] \quad (7.45)$$

The differential power at the detector is then given by equation 7.46.

$$P_{\Delta}(t) = \sqrt{P_{LO} P_{read} e^{-\alpha_0 L}} Re \left[e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) \right] \quad (7.46)$$

The voltage at the detector is then given by equation 7.47.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read} e^{-\alpha_0 L}} \text{Re} \left[e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) \right] \quad (7.47)$$

The read power at both ports of the detector is $P_{read,det} = P_{read} e^{-\alpha_0 L}$, so the measured voltage can be calculated in terms of the read power at the detector, which is given by 7.48.

This will also account for any additional attenuation after the material.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \text{Re} \left[e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \mathcal{F}^{-1} \left\{ \tilde{\Phi}(f) S'(f) \right\} \right) \right] \quad (7.48)$$

The programming power spectral density can be calculated from the measured voltage either in the slow readout limit or in the fast readout regime with a recovery algorithm.

In interferometric readout, nonlinear terms, arising from the magnitude squared of the term dependent on the programmed spectrum, become common mode due to the output splitter. The result is that these terms are suppressed by balanced detection. In traditional readout, these terms are not common mode and can affect the measured power spectrum. Some have suggested methods to handle these terms in traditional readout [53]. This is not required when using interferometric readout because those terms are rejected.

Slow Readout In the slow readout regime, the convolutions go away and the chirp directly maps frequency domain onto time domain leaving a detector voltage given by equation 7.49.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \text{Re} \left[e^{-i\phi_B} \left(1 + \alpha_0 L \beta (S_{eff}(\kappa t + f_L) + i \mathcal{H} \{ S_{eff}(\kappa t + f_L) \}) \right) \right] \quad (7.49)$$

Recovery of the programmed spectrum will depend on the interferometer's phase bias. Ideally, the interferometer will be biased in quadrature, so the two cases that will be

considered are an ideally quadrature-biased interferometer and an interferometer where the phase is biased slightly out of quadrature.

Case 1: Ideal Quadrature Biased Interferometer If the interferometer is biased in quadrature, that is if the bias phase is set so that $\phi_B = \frac{\pi}{2}$. In quadrature, the detector voltage is given by equation 7.50

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \text{Im} [\alpha_0 L \beta (S_{eff}(\kappa t + f_L) + i \mathcal{H} \{S_{eff}(\kappa t + f_L)\})] \quad (7.50)$$

The measured voltage is then directly proportional to the Hilbert transform of the recovered spectrum as shown in equation 7.51.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \alpha_0 L \beta \mathcal{H} \{S_{eff}(\kappa t + f_L)\} \quad (7.51)$$

This voltage signal is proportional to the Hilbert transform of the programmed spectrum. In quadrature, interferometric readout gives the phase of the programmed spectrum. The amplitude spectrum can be recovered by taking an additional Hilbert transform according to equation 7.52.

$$V'(t) = -\mathcal{H} \{V(t)\} = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \alpha_0 L \beta S_{eff}(\kappa t + f_L) \quad (7.52)$$

Case 2: Interferometer with Slight Deviation from Quadrature Bias Assuming the interferometer is approximately in quadrature with a slight deviation such that $\phi_B = \frac{\pi}{2} + \delta\phi$, the detector voltage will be given by 7.53.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \text{Im} [e^{-i\delta\phi} (1 + \alpha_0 L \beta (S_{eff}(\kappa t + f_L) + i \mathcal{H} \{S_{eff}(\kappa t + f_L)\})))] \quad (7.53)$$

In this case, inverting the Hilbert transform will give only part of the solution this portion will be called V_r shown in equation 7.54

$$V_r = -\mathcal{H}\{V(t)\} = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} \text{Re} \left[e^{-i\delta\phi} (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \right] \quad (7.54)$$

The other portion can be calculated by performing another Hilbert transform (equation 7.55). Using the Hilbert transform here brings everything back to the imaginary part but removes the DC term.

$$V_i = \mathcal{H}\{V(t)\} = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} \text{Im} \left[e^{-i\delta\phi} (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \right] \quad (7.55)$$

The V_r and V_i can be combined to create a complex signal V_c given by equation 7.56 .

$$V_c(t) = V_r(t) + iV_i(t) = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} e^{-i\delta\phi} (S_{eff}(\kappa t + f_L) + i\mathcal{H}\{S_{eff}(\kappa t + f_L)\}) \quad (7.56)$$

The phase imbalance can now be compensated by multiplying by a phase factor $e^{i\delta\phi}$ taking the real part of the phase-corrected complex voltage signal gives the final measured signal given in equation 7.57.

$$V'(t) = \text{Re} \left[e^{i\delta\phi} V_c(t) \right] = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} S_{eff}(\kappa t + f_L) \quad (7.57)$$

Fast Readout In the fast readout regime, the convolutions of the chirp with the read field will not simply give a direct mapping of the frequency domain to the time domain, and the convolutions in the time domain voltage provided in equation 7.58 need to be undone using the recovery algorithm.

$$V(t) = G_T \mathcal{R} \sqrt{P_{LO} P_{read,det}} \operatorname{Re} \left[e^{-i\phi_B} \left(1 + \alpha_0 L \beta \Phi^* \left(\Phi * \mathcal{F}^{-1} \{S'(f)\} \right) (t) \right) \right] \quad (7.58)$$

When convolving with the recovery kernel, the DC component will go away since the chirp will always convolve to zero with the DC term, leaving the effective voltage given by equation 7.59 that can be considered.

$$V_{eff}(t) = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} \operatorname{Re} \left[e^{-i\phi_B} \left(\Phi^* \left(\Phi * \mathcal{F}^{-1} \{S'(f)\} \right) (t) \right) \right] \quad (7.59)$$

In terms of the generic detector signal, the relevant portion of the interferometric readout signal is given by equation 7.60.

$$V_{eff}(t) = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} \operatorname{Re} \left[e^{-i\phi_B} F(t) \right] \quad (7.60)$$

In the effective voltage, it can be seen that the bias phase is equivalent to the bias phase used in the recovery algorithm. Using a mixing angle $\phi_M = -\phi_B$, regardless of the interferometric bias (as long as it is known), the resulting recovered voltage signal will be given by equation 7.61.

$$V_{rec}(t) = (\Psi(t) * V(t)) (t) = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} S_{eff} (\kappa t + f_L) \quad (7.61)$$

While it is convenient to implement the phase shift in the recovery chirp to remove the mixing between real and imaginary parts, it is not required. Using a chirp with no mixing angle will lead to a recovered voltage equal to the slow readout signal given in equation 7.49. In this case, the Hilbert transform methods used for slow readout can be implemented to get the final programmed spectra.

Method Comparison

Regardless of whether the spectra were read out in the slow or fast readout limits, the processed interferometric and traditional read signals are given by equations 7.40 and 7.61 respectively. The measured signals in both methods take a very similar form; however, the power dependencies differ. Assuming all parameters are the same between both methods, the ratio of the measured voltages is given by equation 7.62

$$\frac{V_{rec,IF}}{V_{rec,Trad}} = \sqrt{\frac{P_{LO}}{P_{read,det}}} = \sqrt{\eta} \quad (7.62)$$

where $\eta \equiv \frac{P_{LO}}{P_{read,det}}$ is the LO gain parameter and essentially defines how much higher the measured interferometric voltage signal will be than the traditional read signal of the same power. This means that the signal can be much stronger in interferometric readout with a strong LO for the same power into the material. In cases where the noise contributions do not oppose this improvement, this will lead to improvements to the system's signal-to-noise ratio. Aside from the improved signal strength, the other difference between the two methods is that slightly more post-processing is required for slow readout in the interferometric case because the Hilbert transform needs to be inverted to get the programmed spectrum. In the fast readout case, the post-processing method is essentially the same for both methods since the mixing angle phase shift can be implemented in the recovery algorithm to disentangle the amplitude and phase of the programmed spectrum.

Special Case: A Single-Narrow Spectral Hole

Suppose the material is programmed with a single frequency CW optical field of power P_0 and frequency f_H with a laser line-width (Γ_L) much narrower than the material line-width (Γ_H). In that case, the power spectral density of the CW can be treated as a delta function, so the effective programmed power spectral density in the material will be given by equation

7.63.

$$S_{eff} = (P_0 \delta(f - f_0) * L(f))(f) = \frac{P_0}{\pi} \frac{\frac{1}{2}\Gamma_H}{\left(\frac{1}{2}\Gamma_H\right)^2 + (f - f_0)^2} \quad (7.63)$$

This power spectral density will correspond to an imaginary part related to the Hilbert transform of (S_{eff}) given by equation 7.64.

$$\mathcal{H}\{S_{eff}(f)\} = \frac{P_0}{\pi} \frac{f - f_0}{\left(\frac{1}{2}\Gamma_H\right)^2 + (f - f_0)^2} \quad (7.64)$$

This imaginary part of the causal lineshape is often called the "dispersive lineshape". The total causal power spectral density is given by equation 7.65.

$$S'(f) = S_{eff}(f) + i\mathcal{H}\{S_{eff}(f)\} = \frac{P_0}{\pi} \left[\frac{\frac{1}{2}\Gamma_H + i(f - f_H)}{\left(\frac{1}{2}\Gamma_H\right)^2 + (f - f_H)^2} \right] \quad (7.65)$$

The real and imaginary parts of $S'(f)$ are shown in figure 7.3.

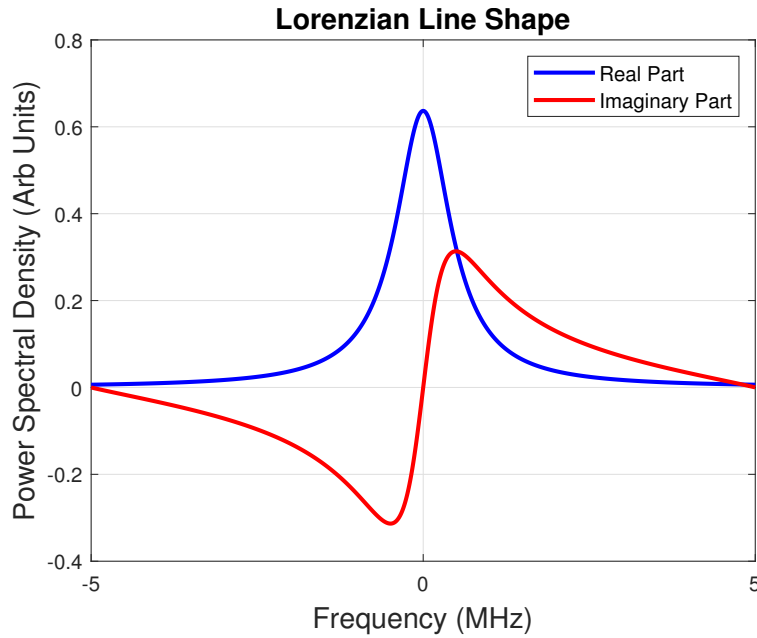


Figure 7.3: Real and imaginary parts of the causal power spectrum of a lorentzian spectral hole. The real part is the Lorentzian shape, while the imaginary part takes the dispersion line shape.

After a spectral hole is burned into the material, either the traditional or interferometric readout method can map this power spectrum into the time domain. Given the results from the preceding section and substituting in the effective power spectral density of a spectral hole, the voltage signals will be given by equations 7.66 and 7.67.

$$V_{Trad}(t) = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} P_0 \frac{1}{\pi} \frac{\frac{1}{2} \Gamma_H}{\left(\frac{1}{2} \Gamma_H\right)^2 + (\kappa t + f_L - f_0)^2} \quad (7.66)$$

$$V_{IF}(t) = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} P_0 \frac{1}{\pi} \frac{\frac{1}{2} \Gamma_H}{\left(\frac{1}{2} \Gamma_H\right)^2 + (\kappa t + f_L - f_0)^2} \quad (7.67)$$

Both signals maximize at the point when $\kappa t + f_L - f_0 = 0$. This gives a signal level proportional to the programming power. These voltage levels are given in equations 7.68 and 7.69.

$$V_{Trad,max} = G_T \mathcal{R} \alpha_0 L \beta P_{read,det} P_0 \frac{2}{\pi \Gamma_H} \quad (7.68)$$

$$V_{IF,max} = G_T \mathcal{R} \alpha_0 L \beta \sqrt{P_{LO} P_{read,det}} P_0 \frac{2}{\pi \Gamma_H} \quad (7.69)$$

Slow Readout In the slow chirp rate regime, the read signals for interferometric readout in quadrature bias and traditional readout directly map the imaginary and real parts of the programmed power spectral density into time domain. Interferometric readout, therefore, directly gives a signal proportional to the Hilbert transform of the programmed spectrum, so the Hilbert transform must be inverted to recover the programmed spectrum. Analytic results depicting traditional and idealized interferometric read results are shown in figure 7.4.

In the interferometric case, deviations from the quadrature bias phase can cause mixing between real and imaginary parts as is known the mixing can be undone as shown in figure 7.5.

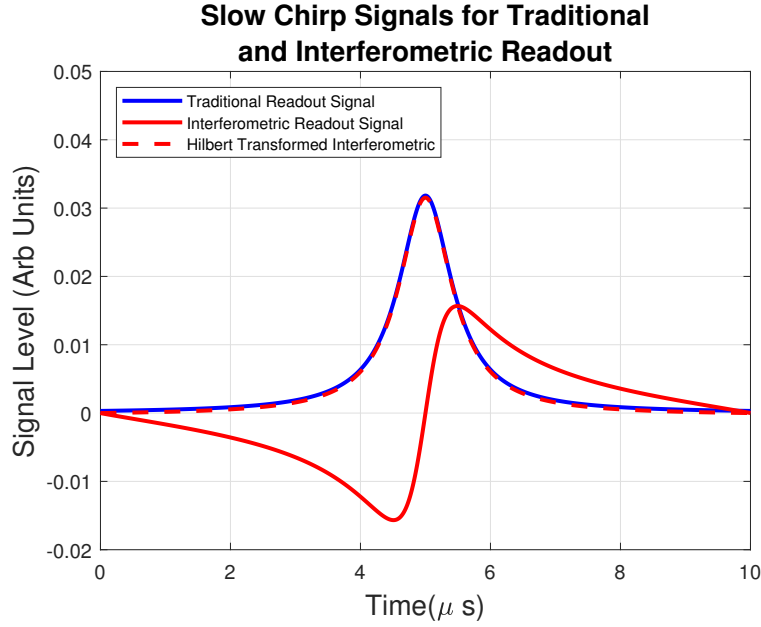


Figure 7.4: Comparison of analytically calculated results for traditional and interferometric readout. In the interferometric case, after inverting the Hilbert transform there is good agreement with traditional results after inverting the Hilbert transform.

Fast Readout In the fast readout regime, the convolutions cannot be easily written off and need to be considered. The convolutions will take the form of a ringing signal when the chirp frequency hits the resonant frequency of the hole. The recovery algorithm can take these ringing signals and recover the proper spectral lineshape. In the case of traditional readout, no phase needs to be applied to the readout chirp to recover the final spectrum. In the case of interferometric readout, if no phase shift is added, the recovered signal will take the form of a dispersive lineshape rather than the expected Lorentzian lineshape. If the interferometric bias of $\pi/2$ is considered, then the Lorentzian signal will be recovered. Analytically calculated examples of the raw and recovered signals are displayed in equation 7.6. The maximum amplitude of voltage signals in the un-recovered signals is lower than the final recovered amplitude. This is because information has been spread from throughout the duration of the ring down, but the recovery algorithm shifts that information back to within a single peak; this is referred to as processing gain, in analog to FFT measurements where

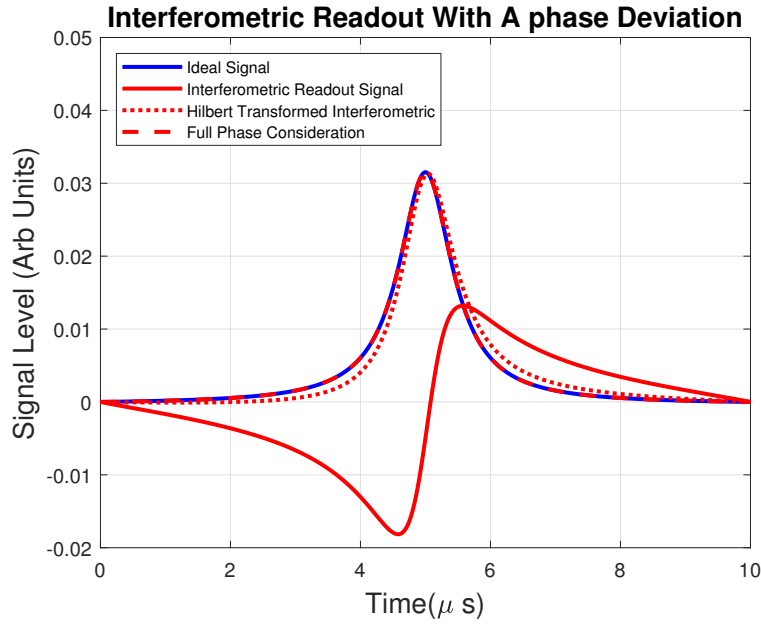


Figure 7.5: Comparison of analytically calculated results for traditional and interferometric readout (with a slight phase deviation). After inverting the Hilbert transform of the interferometric signal, there is a deviation between the interferometric and traditional readout; this deviation is corrected if the mixing is properly accounted for.

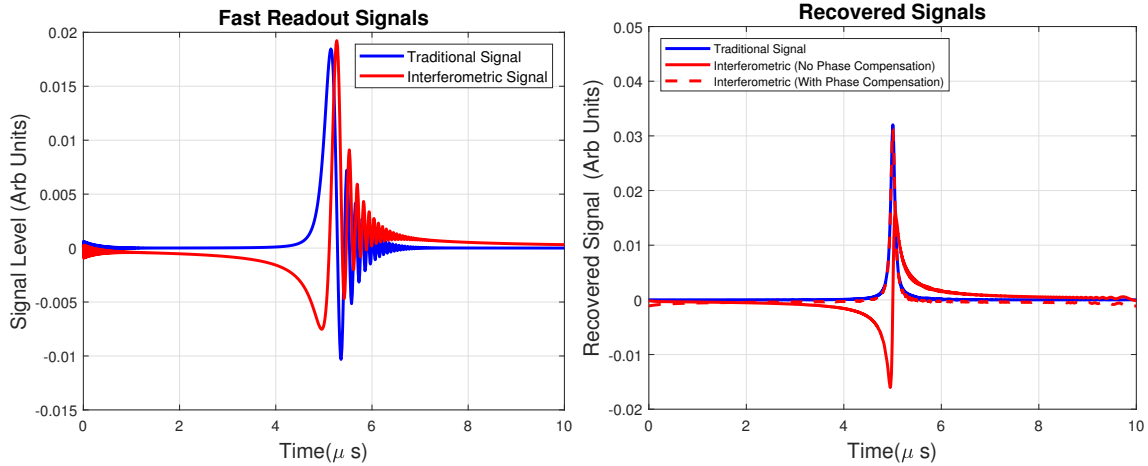


Figure 7.6: Left: Raw readout signals for interferometric and traditional readout. Right: Recovered signals for traditional and interferometric readout.

by using a large block size the signal to noise ratio can be improved.

Power Spectrum Calculation

Both Traditional and interferometric readout provide time-domain voltage signals proportional to the RF power spectrum. The power spectrum of the programming field for either a traditional or interferometric readout signal using equation 7.40 or 7.61 respectively. As was discussed in chapter 6 for a given programming power P_{prog} , burn parameter β , and EOM electrical parameters an RF programming coefficient β_{RF} can be used to calculate the burned hole spectrum due to a given input RF power spectrum. The RF power spectrum can be calculated from a recovered readout voltage from either a traditional or interferometric readout scheme using equations 7.70 or 7.71 respectively.

$$S_{eff,RF}(\kappa t + f_0) = \frac{V_{rec,trad}(t)}{P_{prog} G_T \mathcal{R} \alpha_0 L \beta_{RF} P_{read,det}} \quad (7.70)$$

$$S_{eff,RF}(\kappa t + f_0) = \frac{V_{rec,IF}(t)}{P_{prog} G_T \mathcal{R} \alpha_0 L \beta_{RF} \sqrt{P_{LO} P_{read,det}}} \quad (7.71)$$

Noise Sources and Signal-to-Noise Ratio in SSH Spectrum Analysis

Noise Sources in SSH Spectrum Analysis

In SSH spectrum analysis, time domain voltage noise will correspond to a power noise floor for measured signals. The noise floor in SSH spectrum analysis experiments the average estimated power in a region absent any signal. Because the measured RF power spectrum should always be positive, the absolute value of the voltage should be taken before the power spectrum is estimated. By doing this the properties of the voltage noise will be slightly changed. After taking the absolute value, the noise will be governed by the half-normal distribution (rather than the normal distribution). If the Gaussian voltage noise has an RMS voltage of $V_{RMS,Noise}$, the average noise floor of the voltage signal used for spectrum

analysis is given by equation 7.72 [13].

$$V_{Noise} = \sqrt{\frac{2}{\pi}} V_{RMS,Noise} \quad (7.72)$$

The noise contributions from different independent noise sources can be calculated by adding them in quadrature. The total noise floor due to N individual noises sources with individual RMS noise contributions $V_{RMS,n}$ can be estimated using equation 7.73.

$$V_{Noise,Total} = \sqrt{\frac{2}{\pi} \sum_{n=1}^N V_{RMS,n}^2} \quad (7.73)$$

Noise contributions from various processes can be estimated and combined to estimate the experimental noise floor.

Thermal Noise

As was discussed in chapter 6, thermal noise due to the detector electronics depends entirely on the detection parameters. The RMS voltage noise due to electronic thermal noise in the detector is given by equation 6.40 in chapter 6 in terms of the detector noise equivalent power (Q_{NEP}) [76], the detection bandwidth (Δf_{det}), the transimpedance gain and the responsivity. The contribution of the thermal noise to the average absolute value of the measured voltage is given by 7.74.

$$V_{thermal} = G_t \mathcal{R} Q_{NEP} \sqrt{4 \frac{\Delta f_{det}}{\pi}} \quad (7.74)$$

The thermal noise is not power dependent and is the minimum noise floor in all traditional and interferometric readout experiments.

Shot Noise

Poissonian fluctuations in detector voltage due to counting error in the photo-current are called shot noise. The voltage noise due to these fluctuations was given in equation 6.46 in chapter 6. The noise floor, in spectrum analysis due to shot noise, is then given by 7.75.

$$V_{shot} = G_T \sqrt{\frac{4q_e}{\pi} \mathcal{R} (P_1 + P_2) \Delta f_{det}} \quad (7.75)$$

The shot noise contribution to the noise floor depends on the incident power and, therefore, will present differently in interferometric and traditional readout.

Traditional Readout In traditional readout, the read and reference fields have the same power at the detector (i.e., $P_1 = P_2 = P_{r,det} = P_r e^{-\alpha_0 L}$), the shot noise contribution to the noise floor in interferometric readout. The voltage noise floor in traditional readout is given by equation 7.76.

$$V_{shot,trad} = G_T \sqrt{\frac{8q_e}{\pi} \mathcal{R} P_{read,det} \Delta f_{det}} \quad (7.76)$$

In traditional readout, the noise floor increases with square root read power but decreases with $\alpha_0 L$.

Interferometric Readout In the interferometric readout, the total power at the detector is the some of the read and LO fields at the detector. The shot noise floor is then given by equation 7.77.

$$V_{shot,IF} = G_T \sqrt{\frac{4q_e}{\pi} \mathcal{R} (P_{LO} + P_{read,det}) \Delta f_{det}} \quad (7.77)$$

The interferometric shot noise contribution depends on both the read and LO power at the detector. Under the condition where the read and LO fields have the same power at the detector, the noise will be equivalent to the traditional shot noise. In the limit where the LO is much stronger than the read field, the shot noise floor contribution is given by equation

7.78.

$$V_{shot,IF} = G_T \sqrt{\frac{4q_e}{\pi} \mathcal{R} P_{LO} \Delta f_{det}} \quad (7.78)$$

In the strong LO limit, the shot noise contribution to the noise floor will be dominated by the local oscillator shot noise.

Signal To Noise Ratio In SSH Spectrum Analysis

The signal-to-noise ratio (SNR) for a given signal in SSH spectrum analysis system looking at a single Lorentzian hole is the ratio of the peak voltage signal (given by equations 7.68 and 7.69) to the average voltage noise floor.

Traditional Readout In Traditional readout experiments, the expected SNR is given by equation 7.79.

$$SNR_{trad} = \frac{\alpha_0 L \beta P_{read,det} P_0 \frac{2}{\pi \Gamma_H}}{\sqrt{\frac{4\Delta f_{det}}{\pi} (Q_{NEP}^2 + \frac{2q_e}{\mathcal{R}} P_{read,det})}} \quad (7.79)$$

In the detector-limited regime, the SNR will increase linearly with read power, and in the shot-limited regime, the SNR will increase with square root read power.

Interferometric Readout In interferometric SSH spectrum analysis experiments, the expected SNR is given by equation 7.80.

$$SNR_{IF} = \frac{\alpha_0 L \beta \sqrt{P_{read,det} P_{LO}} P_0 \frac{2}{\pi \Gamma_H}}{\sqrt{\frac{4\Delta f_{det}}{\pi} (Q_{NEP}^2 + \frac{q_e}{\mathcal{R}} (P_{read,det} + P_{LO}))}} \quad (7.80)$$

In the limit where the LO field is much stronger than the read field, the SNR is given by equation 7.81.

$$SNR_{IF} = \frac{\alpha_0 L \beta \sqrt{P_{read,det} P_{LO}} P_0 \frac{2}{\pi \Gamma_H}}{\sqrt{\frac{4\Delta f_{det}}{\pi} (Q_{NEP}^2 + \frac{q_e}{\mathcal{R}} P_{LO})}} \quad (7.81)$$

In the detector thermal limit, the SNR will increase with root read power and with the square-root LO power, and in the shot noise limited regime, the SNR will increase with root read power.

Comparison The expressions for signal-to-noise ratio given for interferometric and traditional readout are very similar. In fact, under the condition that $P_{read,det} = P_{LO}$, the expressions for interferometric and traditional SNR are identical. Interferometric read, however, has an additional parameter with which to improve SNR in the LO power. For a given read power, in the detector limit, interferometric readout with a strong LO will have an SNR higher than the traditional SNR by a factor of $\sqrt{\frac{P_{LO}}{P_{read,det}}}$. In the shot limited regime, the SNR is a factor of $\sqrt{2}$ higher than for traditional readout; this is because, in the shot limit, the interferometric noise floor is dominated by the LO field, while in traditional readout, the read and reference fields contribute equally to the noise floor.

Experiment Overview for Interferometric SSH Readout

Experimental Apparatus

An SSH readout system was built to conduct experiments in either the traditional or interferometric configuration to compare interferometric and traditional readout for SSH spectrum analysis applications. A block diagram of the experimental apparatus is depicted in figure 7.7.

The system begins with a pair of stabilized $1.588\mu m$ laser fields that are doubled down to the required 794nm field required to interrogate the ${}^3H_6 \rightarrow {}^3H_4$ transition in a $10mm \times 10mm \times 9mm$ sample of 0.01% $Tm^{3+} : LiNbO_3$. One field is modulated with two CW RF side bands to program spectral holes into the material's absorption spectrum. An attenuator stack allows for the control of the RF power used to drive an electro-optic phase modulator (EOPM) with a half wave voltage V_π of 5 volts to modulate sidebands onto the

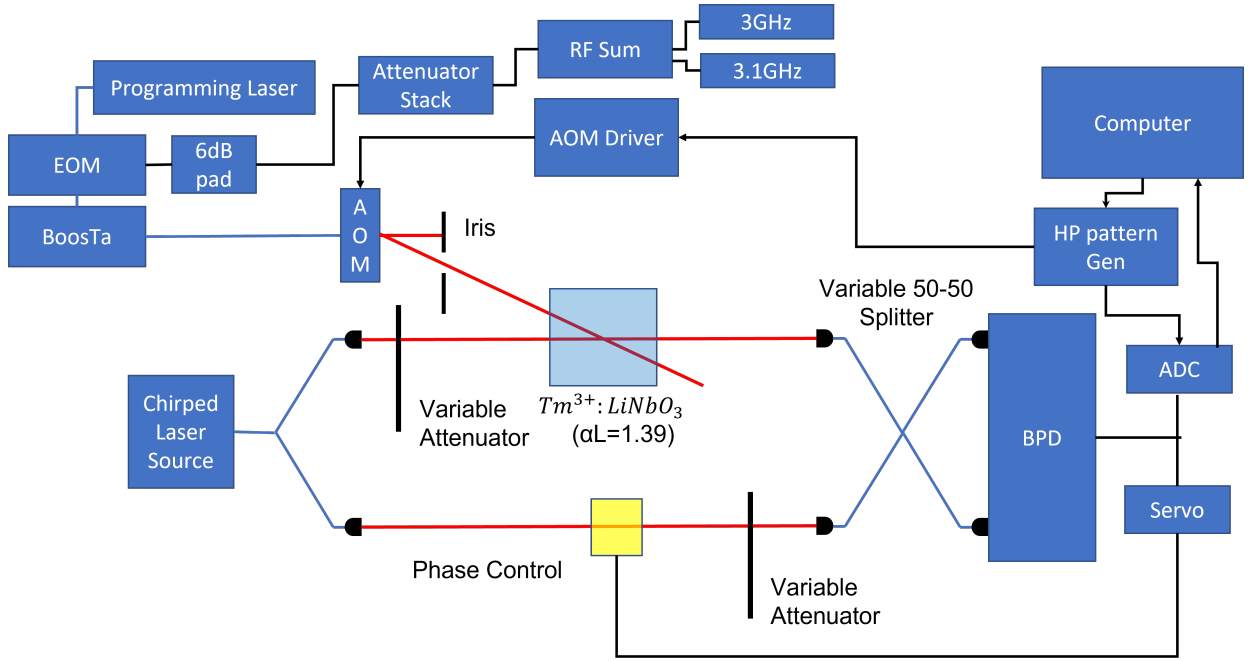


Figure 7.7: A block diagram of the readout system used to demonstrate both traditional and interferometric readout for SSH spectrum analysis applications. This apparatus is explained in greater detail in the text below.

programming field. The EOPM used to modulate the programming field was driven by two signal generators to create RF tones at 3 GHz and 3.1 GHz at a power of about 10 dBm each. A 12dB attenuator after the summing junction and losses in cables lead to a final RF Power into the EOPM of -3 dBm from each tone. An acousto-optic modulator was used to carve pulses out of the programming field. The programming field after the AOM has an optical power of 12mW and was directed into the $Tm^{3+}:LiNbO_3$ sample mounted in a Montana Instruments closed-cycle cryostat. The other field had a 2GHz bandwidth 1ms duration chirp modulated onto it, serving as the read field. The read field was split into two separate paths, one path going through the crystal overlapping the programming field. The other field goes around the cryostat to serve as a reference field in traditional readout experiments and as an LO field in interferometric readout experiments. Shutters carve 40ms pulses out of the two read fields to reduce the read field duty cycle to 0.4% to prevent thermal

broadening due to heating of the material. An optical attenuator before the crystal allowed for control of the optical read power. A 4f image relay system with unit magnification serves to ensure a 1mm spot size for fields inside the crystal. An additional attenuator wheel allows for control of the optical power of the LO Reference field. The two fields are then coupled into fiber. In the traditional configuration, the read and reference fields are directed to the two ports of a ThorLabs PDB450A balanced photo-detector set to the bandwidth = 4MHz, and transimpedance gain = $0.5 \times 10^5 V/A$ setting, and the read and reference field powers at the detector are matched. In the interferometric configuration, a variable 50-50 splitter is used to connect to the detector in place of the fiber patch cords. The 50-50 splitter is tuned to cancel out the LO field (since, in many of the experiments presented in this dissertation, it will be much greater than the read field). In interferometric readout, the signal can be used as an error signal for a PID controller that drives a PZT-modulated mirror to hold the interferometer in quadrature. The RF signal produced by the detector was then filtered with a 5MHz filter to prevent aliasing and then was digitized and captured using an AlazarTech ATS9626 16-bit digitizer [2] at a sample rate of 20 MSPS.

Processing Overview

Voltage signals in interferometric and traditional SSH spectrum analysis experiments can be processed to estimate the RF power spectrum used to drive the programming EOPM. The RF power in each tone and the noise in this measurement can be estimated from the estimated power spectrum of isolated CW RF tones. Processing is done using the following steps:

1. The bias phase is estimated by measuring the mean value of the signal in a region near the peak and then estimating the bias phase by calculating where the background falls within the range of differential powers (i.e., the full swing between estimated differential signals at peak and null). This estimate can be used to determine if the

device is appropriately biased by setting a threshold of phase bias (in this case a relatively loose tolerance of a 5 percent deviation from quadrature was used). If the interferometric bias exceeds the threshold, the system will repeat the measurement until a suitable trace is recorded.

2. The spectral recovery algorithm is applied to the trace, using either no phase shift (for traditional readout) or $\pi/2$ plus the phase deviation measured in step 1 for interferometric readout.
3. The recovered trace is cut down to the 1ms region of interest (i.e., a single chirp cycle).
4. The trace of interest is background subtracted using the method described in the following section to remove unwanted fluctuations.
5. The trace is then calibrated to an optical power spectrum using either equation 7.40 for traditional readout or equation 7.61 for interferometric readout. The value of the programming coefficient used for these calculations is $\beta_{3ms} = 89 \frac{MHz}{mW}$. The optical power can be calibrated to an RF power using the modulator's response in the linear region for this system. The expected value for the RF programming coefficient is $\beta_{RF} = 6.6 \frac{MHz}{mW}$. This was calculated using the three-level rate equations for a 3mS programming pulse because the long programming pulses are best modeled by incoherent programming.
6. The time axis is converted to a frequency axis by using $f(t) = \kappa t + f_0$.
7. The peak is searched for by finding the maximum value in a narrow region of 40 samples around the expected peak location. This value corresponds to the peak power spectral density.

8. The average noise power spectral density is calculated by finding the mean power spectral density in a region without any signals or spurs.
9. The estimated signal power and noise power are calculated by dividing by $\left(\frac{\pi\Gamma_H}{2}\right)$. For these experiments, the homogeneous linewidth for the sample is estimated to be 1.2 MHz.
10. The experiment is repeated 30 times, and the signal and noise values are averaged.

The post-processing steps applied to a single trace are depicted in figure 7.8

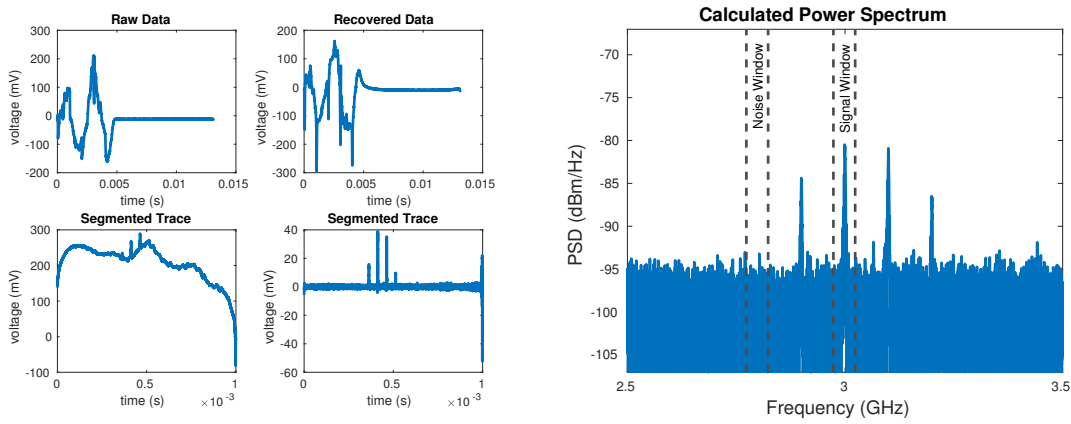


Figure 7.8: Left: Processing steps depicted for a single trace. Top right: Raw data. Top Left: Recovered full trace. Bottom Left: 1 chirp cycle pulled from a single trace. Bottom right: Background subtracted recovered trace. Right: calculated and calibrated power spectrum with noise and peak signal measurement regions indicated.

Background Subtraction

SSH Spectrum analysis is susceptible to fluctuating backgrounds. In SSH spectrum analysis, fields are presumed to be centered around a null background. Any slow background fluctuations will then appear as strong measurement floors, obscuring signals of interest. In most SSH spectrum analysis applications, some background subtraction is conducted. Typically these methods involve comparison to a signal-free trace and assuming that the

background fluctuations are inherent to the chirp and can therefore be subtracted out from each individual shot. This can be problematic in interferometric spectrum analysis because background fluctuations can be induced by slight changes to the interferometric phase. These fluctuations are independent and cannot be estimated by looking at a signal-free trace as can typically be done in traditional readout. These effects can be mitigated using a suitable feedback control loop. In the implemented system, this was complicated by the fact that the read signal was used as the error signal, so the PID response had to be less than the repetition rate of read chirps. This relatively slow response led to poor locking and often relatively strong background fluctuations. To combat these additional fluctuations in the detected signal, a background subtraction method had to be used to estimate and subtract this background from the captured traces to leave only noise and the readout spectrum. This algorithm is not a general background subtraction for an arbitrary spectrum but a specific algorithm for a multi-tone SNR study, where there is presumed to be a finite number of tones (and therefore also third-order spurs, which must also be considered) in known locations, these areas can be removed from the trace, which can then be high pass filtered to remove low-frequency oscillations, without inducing distortions to the line shapes of measured signals due to the filter. This filtered background trace lacks signal peaks and can be used as an estimate of the background. The steps to subtraction are as follows.

1. Regions of a radius of 100 samples (5 microseconds or 10MHz) around the known locations of programming peaks are removed from the trace.
2. To prevent distortions due to the application of a low pass filter to the lorentzian lineshape, regions around the known peaks are removed and regions are replaced with a linear interpolation of the edges of the region. The endpoints used are an average of the 10 points in the used portion of the trace before and after the removed region.
3. The trace with the removed peaks is low pass filtered using a bidirectional 2^{nd} order

digital butter-worth filter with a 150 kHz filter bandwidth as an estimation of the background. The filter is bidirectional so that the shape of the underlying background is not distorted or shifted due to group delays. A 2^{nd} order filter was chosen to reduce ringing due to steeper slopes observed around the edges of traces. The filter bandwidth of 150 kHz was picked by trial and error until the background was removed and was picked in conjunction with the windows around the known peak locations. The window size must be coordinated with the filter bandwidth so that the fluctuations in the removed regions are well above the filter cut-off, so only the low-frequency background is cut out, and fluctuations within the time scale of the window are preserved.

4. The filtered trace is then used as an estimated background and is subtracted from the original signal to get the background-free trace.

The background subtraction process is depicted in figure 7.9. This background subtraction

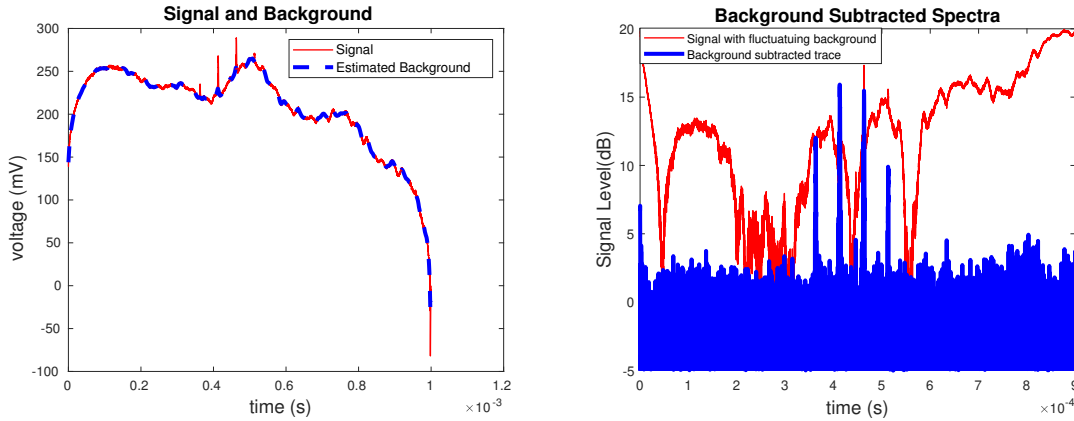


Figure 7.9: Left: The captured and recovered trace (in red) with the estimated background marked as a dashed blue line. Right: A comparison of measured SSH spectra with and without background subtraction. The fluctuations in the background lead to spectra with high background.

method is not an ideal general-purpose background subtraction but rather is a purpose specific for experiments using a finite number of tones. A better technique would be to

stabilize the interferometer using a different field that will not interact with the thulium ions. This other field could be used not only to stabilize the interferometer to a high bandwidth using feedback control but also to measure relative path length fluctuations, which could, in turn, be used to estimate the interferometric background. Existing background subtraction techniques employed for traditional readout could then be used to subtract the remaining background.

Experimental Results

Demonstration of Interferometric readout

Dispersive Lineshape and Recovery In interferometric readout experiments, the expected line shape is the dispersion line shape expected for a lorentzian(i.e., the Hilbert transform of a lorentzian). As expected (in the slow regime), the observed line shape in interferometric readout experiments showed a dispersive lineshape rather than the traditional Lorentzian lineshape. But using the new recovery algorithm, which can handle arbitrary mixing phases (including $\pi/2$ for interferometric readout), the expected lineshape can still be recovered. Figure 7.10 depicts a captured interferometric trace and the recovered lineshape.

Demonstration of SNR Improvement Using LO Gain The primary interesting feature of interferometric readout is the ability to achieve higher signal-to-noise ratios at a given read power using a strong LO. For example, a $1\mu W$ read power traditional analyzer configuration was compared with an interferometric analyzer with a $1\mu W$ read power and a $1000\mu W$ LO power. The measured spectra can be observed and compared. Both tested configurations were programmed by a pair of RF tones at 3 GHz and 3.1GHz at powers controlled by an attenuator stack. Figure 7.11 compares interferometric and traditional readout with an RF power in the 3GHz side-band of -3.05dBm. Interferometric readout with a strong LO substantially improves the observed SNR by about 8dB. In the interferometric case, the noise

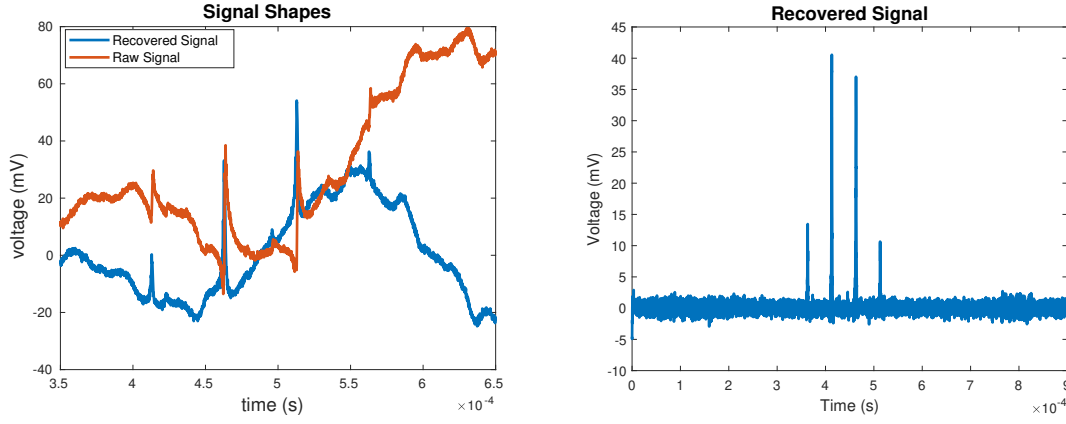


Figure 7.10: Left: An example of lineshapes from Interferometric readout experiments. The example trace shows 2 RF tones (as well as 3rd order spurs due to those signals). The red trace is the dispersive lineshape expected for interferometric readout, and the blue trace shows how recovery can be used to restore the expected Lorentzian lineshape. Right: The final recovered shapes signal after recovery and background subtraction. .

floor should be dominated by shot noise which has the same LO power dependence as signals. So while a 15dB amplification of signals is expected, this should only correspond to about 8dB of improvement to SNR. Figure 7.12 compares interferometric and traditional readout with an RF power in the 3GHz side-band of -21dBm. At the lower RF, power tones cannot be reliably observed in traditional readout but can be resolved in interferometric readout. This indicates that interferometric readout can offer better dynamic range and sensitivity to weak and a higher SNR for all signals for a given read power.

Demonstration: Two Independent Tones To demonstrate the ability of the interferometric readout scheme to interdependently measure two tones with independently varying powers, two RF tones one at 3GHz and one at 3.1GHz. The single tone had a power held at -13dBm, and the second tone had its power controlled by an attenuator stack and an interferometric SSH analyzer analyzed both with a $1\mu W$ read field and a $20\mu W$ LO field. The power was then measured at each tone. The results of this experiment are shown in figure 7.13. As expected, the analyzer can independently measure the power in each tone.

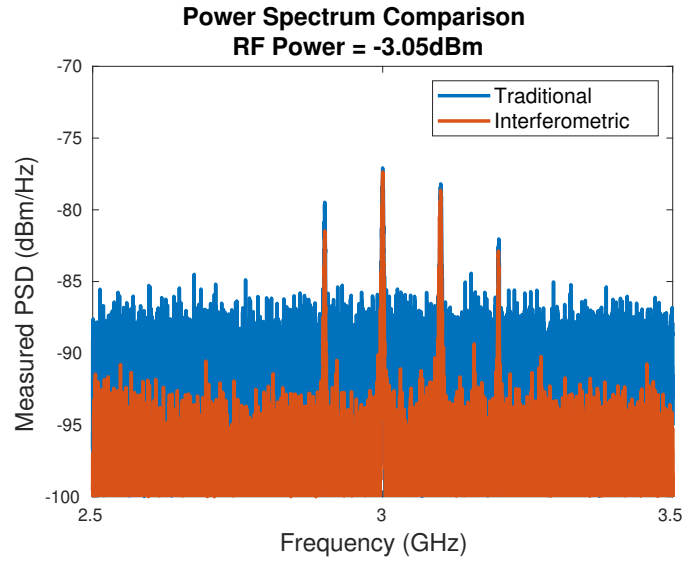


Figure 7.11: Measured RF spectrum using a $1 \mu W$ read power with -3.05dBm of RF power in the 3GHz tone. Interferometric read shows an improved SNR over the traditional method.

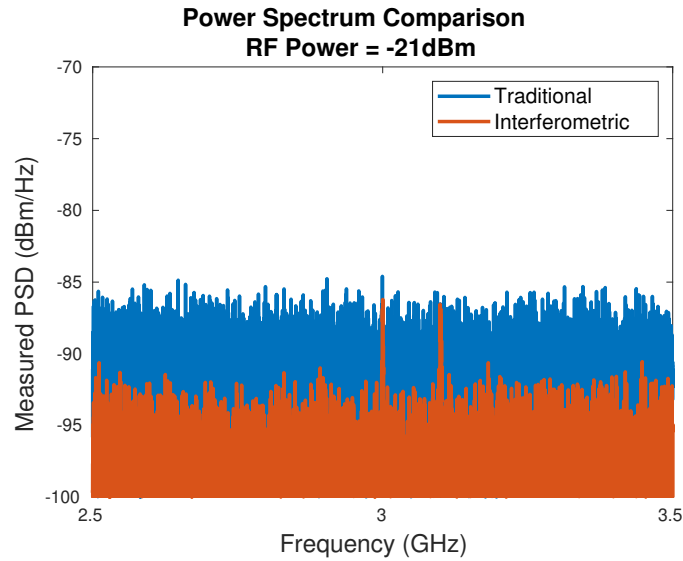


Figure 7.12: Measured RF spectrum using a $1 \mu W$ read power with -21dBm of RF power in the 3GHz tone. Interferometric readout shows an improved SNR over the traditional method. Tones observed in interferometric readout are not resolvable in traditional readout but are easily observable in interferometric readout.

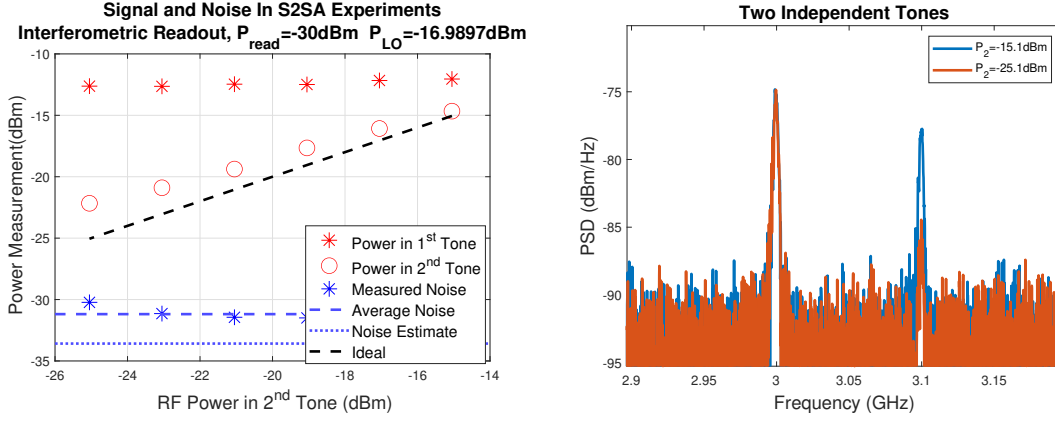


Figure 7.13: Left: Signal and noise measurements at different RF powers. The 1st (static) tone remains at the same power measured at just over -16 dBm, while the measured power in the 2nd (changing tone) changes with the RF power. Right: example traces at different tone 2 powers. In both examples, RF tone 1 is roughly the same strength while tone two changes between the two example powers (-15.1dBm and -25.1 dBm)

The signal corresponds within a few dB of the expected signal, while the noise was 2.5dB higher than the expected noise floor.

Signal And Noise Measurements

To explore the performance of interferometric readout systems in comparison to traditional readout systems, experiments were conducted where two RF tones were modulated onto a programming field, and the readout was used to measure the power in one of the tones. The power in the RF signal was then changed using a variable attenuation stack.

1 μ W (-30dBm) Read Power Signal and noise results for various analyzers using a 1 μ W-30dBm read field in either a traditional configuration or an interferometric configuration using a variety of LO powers are displayed in figure 7.14. The measured traditional signals and noises correspond well with the interferometric results for interferometric readout with a 1 μ W (-30dBm) LO field as expected. At lower RF powers the signal measurements trail off as they approach the noise floor. Because the signal measurement uses the maximum value

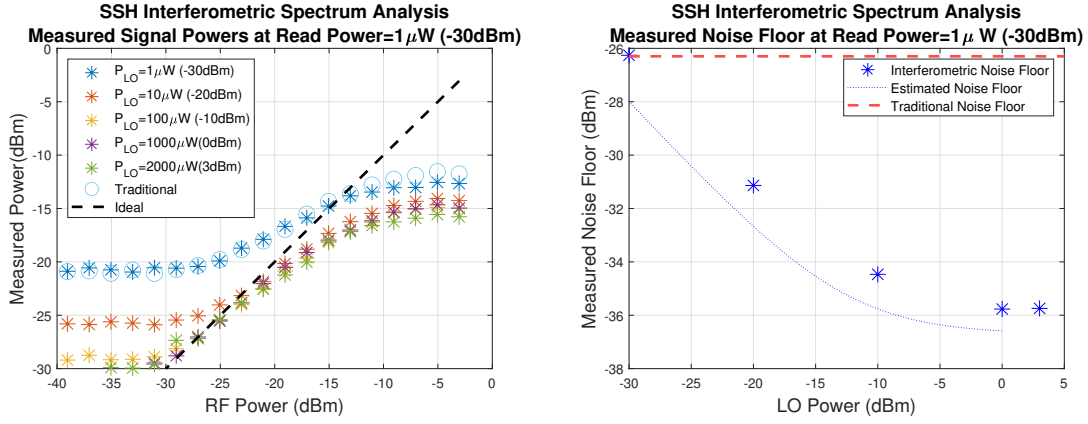


Figure 7.14: Measured signal and noise Using traditional and interferometric readout configurations with 1μ W of read power. Left: Measured signal Powers in Interferometric readout at different LO Powers and traditional readout. Different readout configurations are indicated with different colors and markers, The ideal analyzer response is marked with a black dashed line. Right: measured noise floors for traditional readout and interferometric readout at different LO powers.

in 40 sample windows, the window size will lead to around a 5dB difference between the expected noise floor and the average maximum signal meandered in the 40 sample window. Interferometric analyzers with higher read powers appear to be more sensitive to weaker fields and therefore fell into the noise at lower powers. The improvements in the sensitivity are due to the drop in the noise with increasing LO power. Due to this drop the noise floor was 10dB lower for interferometric readout at high LO (0dBm) powers than traditional readout at 1μ W/(-30dBm). The noise in these experiments proved to be consistently roughly 2dB lower than the measured noise floor though the shape of the expected trend appears consistent with the experimental data. The measured signals also show variation in the measured power of around 2db on either side however improvements to the sensitivity can still be observed. These improvements to sensitivity can be seen by slightly shifting one of the spectra to overlap in the saturation regime. Figure 7.15 depicts response curves for traditional readout with 1μ W read power and interferometric readout with 1μ W read power and 1000μ W LO power. The interferometric case shows a noise floor at around -32dBm and

signals start to fall into the noise at around 25dBm while the traditional read has a noise floor around -26dBm and the signal falls into the noise at around -19dBm, which indicates a 6dB improvement to the sensitivity.

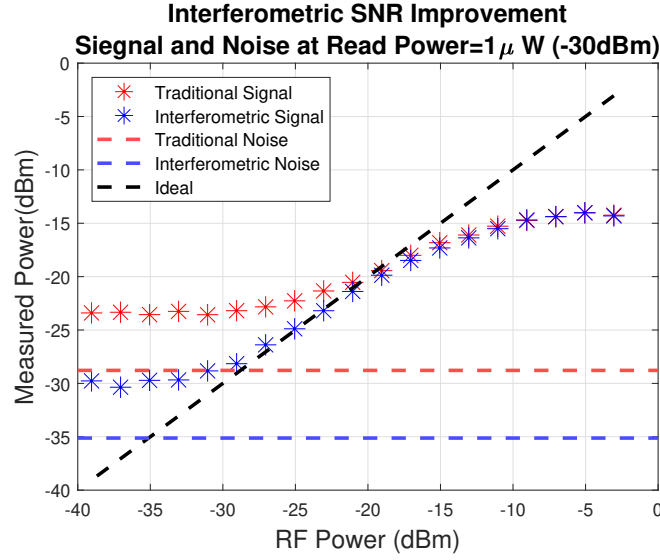


Figure 7.15: Traditional and interferometric ($P_{LO} = 1000\mu W$ or 0dBm) response curves at $1\mu W$ of read power, overlapped to clarify improvement to sensitivity. Interferometric readout shows a clear improvement to sensitivity and SNR (data shifted to overlap in the saturation regime.)

5.2μW (-22.8dBm) Read Power Similar experiments were conducted at $5.2\mu W$ - 22.8dBm of read power. The measured signal levels and noise floors for spectrum analyzers using this read power in interferometric and traditional readout configurations are displayed in the figure 7.16. As expected, traditional results are very similar to interferometric results using $5.2\mu W$ (-22.8dBm) LO field. At lower LO powers, the power measurements roll off to around 5dB above the measured noise floor. The noise floor again drops with increasing LO power until rolling off (this is expected to correspond with the shot noise regime). At the highest read powers, the interferometric noise floor is around 6dB below the traditional noise floor. This improvement corresponds well with the sensitivity (and SNR) improvement seen

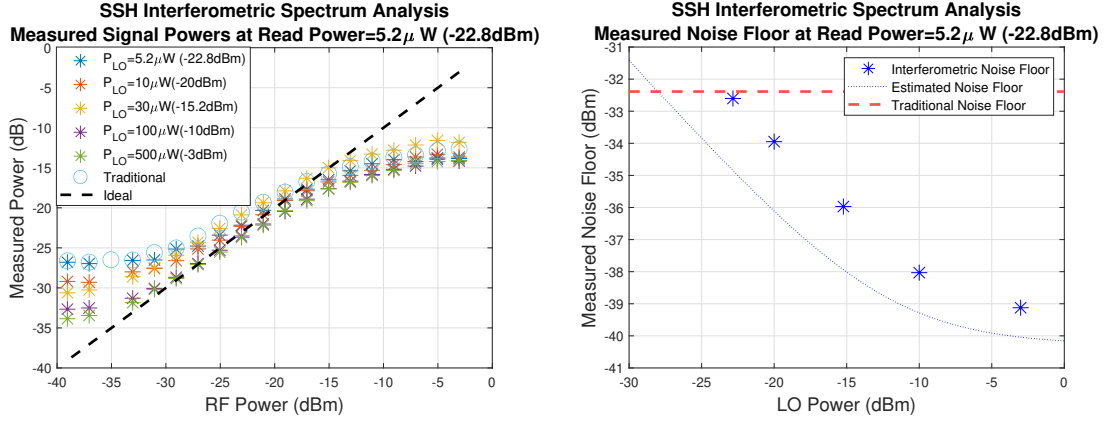


Figure 7.16: Signal and noise Measurements using traditional readout and interferometric readout at a variety of LO powers with $5.2\mu W$ of read power. Left: Measured signal powers in Interferometric readout at different LO powers and traditional Readout. Right: Measured noise floors for traditional readout and interferometric readout at different LO powers

in figure 7.17. The 6dB improvement to SNR appears to be paired with a 5dB improvement

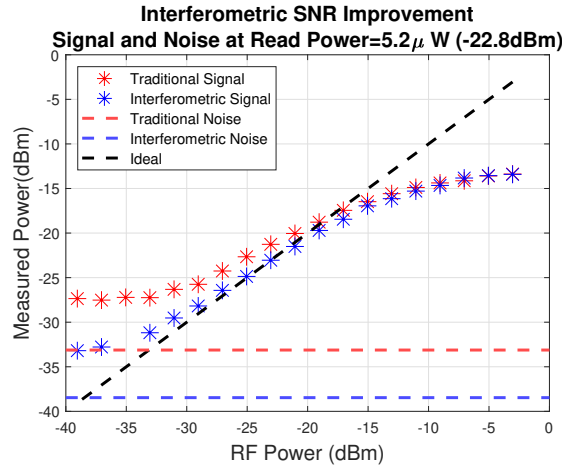


Figure 7.17: Traditional and interferometric ($P_{LO} = 1000\mu W$ or 0dBm) power response curves at $5.2\mu W$ of read power, overlapped in saturation to better compare sensitivity. Interferometric readout shows a clear improvement in sensitivity and SNR.

to the sensitivity. Again the noise floor appeared to be around 2dB below the estimated noise floor and a few dB of fluctuation to the measure saturation power for different analyzers.

$10\mu W$ (-20dBm) Read Power The results of 2 tone experiments using a $10\mu W$ read field in either a traditional readout or interferometric readout at several different LO power is shown 7.18. At a read power of $10\mu W$, signal and noise levels again correspond well between

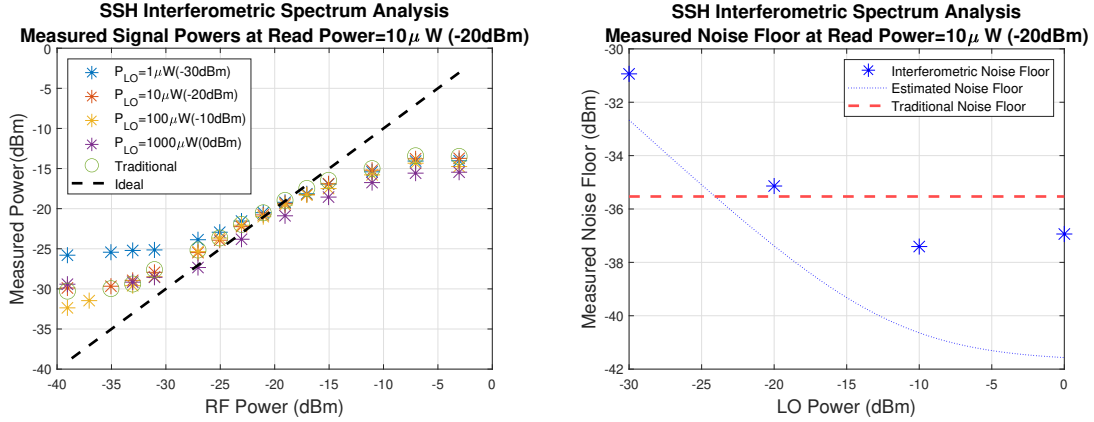


Figure 7.18: Measured signal and noise levels using different SSH spectrum analyzer configurations with $10\mu W$ of read power. Left: Measured signal powers in interferometric readout at different LO powers and traditional readout. Right: Measured noise floors for traditional readout and interferometric readout at different LO powers.

traditional readout and interferometric readout under the condition where $P_{read} = P_{LO}$. Improvements to sensitivity and noise floors are seen compared to the weak LO ($1\mu W$) for higher LO powers. However, this case shows inferior performance compared to the traditional read case, and for higher LO powers, the performance appears comparable to traditional readout. The noise initially shows the same 2dB difference with the expected noise, but at higher power, the difference between expected and observed noise floor grows; this is an indication that there is a noise source that contributes differently to the total noise than the expected thermal and shot noise. Figure 7.19 shows the response for traditional readout and readout with a stronger LO; however, no improvements are to be seen, and the noise floor does not change substantially.

This lack of improvement to the noise floor and sensitivity is likely the result of non-linear chirp-dependent noise (later chapters will discuss this noise floor in greater detail).

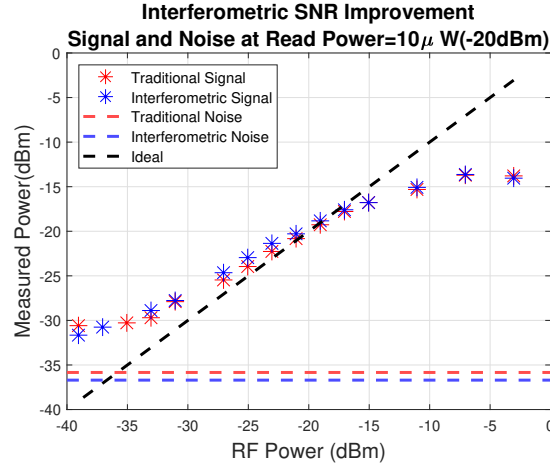


Figure 7.19: Traditional and interferometric ($P_{LO} = 1000\mu W$ or 0dBm) response curves at 10 μW of read power overlapped to better compare responses. Interferometric readout show similar results.

Conclusions

Interferometric readout of SSH materials was demonstrated to produce dispersive waveforms at detection, which could be recovered to recover the programmed spectrum, which can be amplified by the use of a strong local oscillator; the amplification of the signal was shown to amplify signals more than increases to the noise floor, allowing for improved signal to noise ratio and sensitivity compared to a traditional readout system with equal read powers.

This interferometric readout technique for SSH spectrum analysis was shown to achieve the same, or in many cases, better sensitivity and SNR than traditional readout. The modified spectral recovery algorithm used in interferometric readout proved its ability to recover interferometric readout signals into Lorentzian line shapes. Experimental results showed up to a 5dB improvement in sensitivity and up to a 6 dB decrease in the noise floor over a traditional SSH spectrum analyzer with the same read power, provided the LO power is much greater than the read power.

Improvements to the SNR or sensitivity can be achieved using a stronger read field. However, in cases where there is a limitation to the amount of read power that can be used, the extra gain given to signals by a strong LO field can be used to improve SNR and sensitivity without increasing power into the crystal.

At high read powers ($10\mu W/-20\text{dBm}$), noise appeared to limit observations of improvement to SNR and sensitivity. Later experiments on interferometric SSH grating processing experiments seemed to indicate that at higher read powers into the material, non-linear noise will quickly begin to dominate the noise floor. This likely was the cause of the lack of improvement seen at $10\mu W$ of read power. If the noise were thermal and shot noise limited, an improvement to the noise floor would be expected.

The experimental apparatus used to test interferometric readout was sufficient for measuring a power spectrum consisting of individual tones. However, the low-frequency background fluctuations due to fluctuations in the interferometric bias phase would cause problems in measuring an arbitrary signal or signals at unknown locations. These problems can be mitigated by using a locking scheme that uses a separate field, detuned far enough from the material resonance to lock the interferometer at a bandwidth sufficient to remove all background fluctuations.

INTERFEROMETRIC SSH GRATING PROCESSING

IntroductionMotivation

Time delays between high bandwidth signals result in spectral interference patterns that can be used to program spectral gratings into SSH materials. The period of the spectral grating depends on the time delay. By analyzing the programmed material spectrum the time delay can be extracted. When programmed with optical broadband lidar signals, SSH systems can process those lidar signals photonically. The correlative properties of SSH systems allow for the use of noise as a lidar waveform [41, 60]. When programmed with radio signals from different antennas, SSH systems can be used to process radar signals or to do time difference of arrival (TODA) measurements to geolocate RF emitters [10, 49]. The ability to identify weaker returns in these applications is dependent on the signal-to-noise ratio of the return. Under certain circumstances, using a strong local oscillator in an interferometric readout scheme, signals can be amplified while only slightly increasing the noise level, meaning that the signal-to-noise ratio can be improved, allowing for the identification of weaker returns.

Time Delay Storage in SSH Systems

Time delays are encoded in SSH materials as spectral gratings. Spectral gratings arise as interference terms from the joint power spectrum of a broadband signal and a time-delayed copy. Consider, for example, a lidar system where a broadband lidar signal is sent to the target, coupled back into the lidar system, and combined with the un-delayed local copy before being sent to the material. The combined electric field into the crystal is given by equation 8.1

$$E_{total}(t) = E_{ref}(t) + E_{return}(t + \tau_{21}) \quad (8.1)$$

where τ_{21} is the time delay due to the time of flight of the return field. In the joint power spectrum, the time delay corresponds to a sinusoidal grating given by equation 8.2.

$$S_{total}(f) = S_1(f) + S_2(f) + 2\sqrt{S_1(f)S_2(f)} \cos(2\pi f\tau_{21}) \quad (8.2)$$

This joint power spectrum will lead to a grating in the field transmission given by equation 8.3 (note that the cosine turns into a complex exponential due to the Hilbert transform)

$$\nu(f) \approx \left(1 - \frac{1}{2}\alpha_0 L\beta (S_1(f) + S_2(f)) - \alpha_0 L\beta \sqrt{S_1(f)S_2(f)} e^{i2\pi f\tau_{21}} \right) e^{-\frac{1}{2}\alpha_0 L} \quad (8.3)$$

The grating shows up in the transmission coefficient as a frequency-dependent rotating phase in the term $\alpha_0 L\beta \sqrt{S_1(f)S_2(f)} e^{i2\pi f\tau_{21}}$. This grating will again show up as a delay imparted on incident fields that pass through the programmed material.

Signal Levels in SSH Grating Readout

Read Out of Spectral Gratings

Overview of Processing Methods for Spectral Gratings Spectral gratings are evaluated in grating processing applications using a chirped field of the form $E_0 e^{2\pi i(\frac{1}{2}\kappa t^2 + f_0 t)}$ (as has been described earlier) to readout the spectral grating. This field will interact with the material giving an output time domain field that can be expressed using equation 8.4.

$$\begin{aligned} E_{out}(t) = & E_0 e^{-\frac{1}{2}\alpha_0 L} \left(1 - \frac{1}{2}\alpha_0 L\beta (S_1 + S_2) \right) e^{2\pi i(\frac{1}{2}\kappa t^2 + f_0 t)} \dots \\ & \dots - E_0 e^{-\frac{1}{2}\alpha_0 L} \alpha_0 L\beta \sqrt{S_1 S_2} e^{2\pi i(\frac{1}{2}\kappa(t-\tau_{21})^2 + f_0(t-\tau_{21}))} \end{aligned} \quad (8.4)$$

Using coherent detection, the second term can be extracted (either in traditional or interferometric detection). The resulting optical field will exhibit a beat note with a frequency dependent on the range. The detected voltage signal can be digitized and used to

estimate the optical return's full range spectrum by estimating the detected signal's electrical power spectrum as the magnitude squared of the fast Fourier transform (FFT).

Traditional Readout The traditional method for readout, described in chapter 1, will result in a total detector voltage given in equation 8.5.

$$V(t) = \mathcal{R}G_T \left(\left(\frac{1}{2} \alpha_0 L \beta (S_1 + S_2) P_r e^{-\alpha_0 L} \right) + 2P_r e^{-\alpha_0 L} \alpha_0 L \beta \sqrt{S_1 S_2} \cos(2\pi \kappa t \tau_{21}) \right) \quad (8.5)$$

where $\mathcal{R}$ is the detector responsivity and G_T is the transimpedance gain. Range data encoded as gratings will be represented by the AC signals, so the signal of interest is the AC signal given in equation 8.6.

$$V_{ac}(t) = \mathcal{R}G_T \left(2P_r e^{-\alpha_0 L} \alpha_0 L \beta \sqrt{S_1 S_2} \cos(2\pi \kappa t \tau_{21}) \right) \quad (8.6)$$

The read power at the detector is $P_{r,det} = P_r e^{-\alpha_0 L}$ so the differential voltage can be expressed in terms of the read power at the detector (equation 8.7).

$$V_{ac}(t) = \mathcal{R}G_T \left(2P_{r,det} \alpha_0 L \beta \sqrt{S_1 S_2} \cos(2\pi \kappa t \tau_{21}) \right) \quad (8.7)$$

This signal is then processed to calculate the range spectrum by taking the magnitude of the Fourier transform squared. The electrical power spectrum of the measured signal is given by equation 8.8.

$$S_{electrical,trad}(f) = 4P_{r,det}^2 \alpha_0^2 \beta^2 \mathcal{R}^2 G_T^2 S_1 S_2 |\mathcal{F}[\cos(2\pi \kappa t \tau_{21})]|^2 \quad (8.8)$$

The quantity $U(f)$ is the magnitude squared of the single-sided FFT as calculated by MATLAB and will serve as a power spectrum of interest, this quantity can be estimated

with equation 8.9.

$$U_{trad}(f) = N_{samples}^2 \alpha_0^2 L^2 \beta^2 \mathcal{R}^2 G_T^2 S_1 S_2 P_{r,det}^2 \quad (8.9)$$

Interferometric Readout Using the interferometric readout technique first described in chapter 2 and described in detail for spectrum analysis applications in chapter 7. The voltage on the balanced detector is given by equation 8.10.

$$V(t) = \mathcal{R} G_T \left(2\alpha_0 L \beta \sqrt{P_{LO} P_r} e^{-\alpha_0} \sqrt{S_1 S_2} \sin(2\pi \kappa t \tau_{21}) \right) \quad (8.10)$$

This can be written in terms of the read power at the detector using equation 8.11.

$$V(t) = \mathcal{R} G_T \left(2\alpha_0 L \beta \sqrt{P_{LO} P_{r,det}} \sqrt{S_1 S_2} \sin(2\pi \kappa t \tau_{21}) \right) \quad (8.11)$$

The signal level in interferometric readout schemes can be calculated by taking the magnitude squared of the Fourier transform of the signal to calculate the electrical power spectrum of the signal, which is given by equation 8.12.

$$S_{Electrical,IF}(f) = 4P_{LO} P_{r,det} \alpha_0^2 L^2 \beta^2 \mathcal{R}^2 G_T^2 S_1 S_2 |\mathcal{F}[\sin(2\pi \kappa t \tau_{21})]|^2 \quad (8.12)$$

Using MATLAB to calculate the power spectrum as the magnitude squared of the fast Fourier transform will result in a signal level that can be estimated by equation 8.13.

$$U_{IF}(f) = N_{Samples}^2 \alpha_0^2 L^2 \beta^2 \mathcal{R}^2 G_T^2 S_1 S_2 P_{LO} P_{r,det} \quad (8.13)$$

Comparison of Traditional and Interferometric Signals The expressions for interferometric and traditional readout signals are similar. We will define a ratio of interferometric to traditional readout signals to help judge relative performance. The LO gain parameter η

is given by equation 8.14.

$$\eta = \frac{U_{IF}}{U_{trad}} = \frac{P_{LO}}{P_{r,det}} \quad (8.14)$$

This indicates that the key to determining the relative strength of grating signals can be increased over the traditional limit by increasing the LO power beyond the read power. While improvements to the signal level can be seen due to a strong LO field are expected, the effect on the signal-to-noise ratio (SNR) will also depend on the behavior of noise contributions under those same circumstances.

Noise Sources in SSH Grating Readout

Overview

In SSH lidar applications, there are a variety of noise sources that contribute to the overall noise floor of the system. The different contributions will have different dependences on power and, therefore, will affect the power dependence of the signal-to-noise ratio. Table 8.1 describes how different noise contributions depend on read and LO power in interferometric readout (labeled IF- P_r and IF- P_{LO} respectively) and the dependence on read power in traditional readout (labeled Trad- P_r). SNR dependence for interferometric readout is assumed to be in the regime where the LO power is much greater than the read power.

Some noise sources are common mode and are therefore subject the common mode rejection ratio (CMRR) of the detector.

Detection Noise Contributions

Many of the noise contributions are inherent to the detection system. These properties are governed by the detector parameters and, in the case of photo-electron shot noise, the power incident on the detector. The detection noise contributions (Quantization noise, thermal noise, and shot noise) were described in more detail in chapter 6, so the following subsection will briefly summarize the key points about each detection noise source.

Category	Noise Source	Dependence on P_r and P_{LO}		SNR Dependence			Subject to CMRR
		Traditional	Interferometric	Trad- P_r	IF- P_r	IF- P_{LO}	
Detection	Quantization	Const.	Const.	P_r^2	P_r^1	P_{LO}^1	No
	Thermal \ NEP	Const.	Const.	P_r^2	P_r^1	P_{LO}^1	No
	Photo-electron Shot	$2P_r$	$P_r + P_{LO}$	P_r^1	P_r^1	P_{LO}^0	No
Source	RIN	P_r^2	P_{LO}^2	P_r^1	P_r^1	P_{LO}^{-1}	Yes
	Phase Noise	P_r^2	$P_r P_{LO}$	P_r^1	P_r^0	P_{LO}^0	Yes
Nonlinear holeburning	RIN (Holeburning)	P_r^4	$P_r^3 P_{LO}$	P_r^{-2}	P_r^{-2}	P_{LO}^0	No
	Phase (Holeburning)	P_r^4	$P_r^3 P_{LO}$	P_r^{-2}	P_r^{-2}	P_{LO}^0	No
	Chirp Rate Non-linearity	P_r^4	$P_r^3 P_{LO}$	P_r^{-2}	P_r^{-2}	P_{LO}^0	No

Table 8.1: A table of noise sources that can arise in SSH grating processing systems as well as their dependencies on read and LO power in interferometric and traditional readout schemes.

Quantization Noise Some noise will be added to the system via round-off error in analog to digital conversion. The properties of this noise will be governed by the properties of the analog-to-digital converter (ADC). Specifically, the noise will be dependent on the number of vertical resolution bits of the ADC (N_{Bits}) and the full-scale voltage of the converter [34]. The RMS noise voltage due to quantization will be given by equation 8.15.

$$V_{RMS,Quantization} = \frac{V_{FullScale} 10^{-\frac{3.01N_{bits}+0.88dB}{10}}}{2\sqrt{2}} \quad (8.15)$$

The noise due to quantization in the power signal that will be calculated via processing in MATLAB can be estimated from the RMS voltage quantization and the number of samples (equation 8.16).

$$U_{noise,quantization} = \frac{N_s V_{fs}^2 10^{-\frac{6.02N_{bits}+1.76dB}{10}}}{8} \quad (8.16)$$

This noise contribution will be independent of the sample rate, the power on the detector, or anything other than the number of ADC bits, the full-scale voltage of the ADC, and the

number of samples. It is important to note that while the ideal quantization noise is governed by nominal resolution. Typically the noise floor of a digitizer isn't just the quantization floor, there is likely noise and distortions that reduce the ADC's effective resolution. The effective resolution of the ADC can be described by the effective number of bits (ENOBs) which can be used in place of N_{bits} in equations 8.15 and 8.16.

Direct measurements of the ADC and detector Noise floor indicated that the detector noise floor (absent any signals) was substantially higher than the ADC noise floor. Noise models presented in the future will focus on noise from the detector and optical noise.

Thermal Noise \ Detector NEP Thermal electronic noise in the detector is an intrinsic property of the detector. Thermal noise is described in terms of the detector noise equivalent power (NEP or Q_{NEP}), which describes how much equivalent power the noise represents for a given bandwidth. The RMS detector noise floor can be described using the NEP, the detector bandwidth, the detector responsivity ($\mathcal{R}$), and the detector transimpedance gain (G_T) using equation 8.17 [69].

$$V_{RMS,Thermal} = G_T \mathcal{R} Q_{NEP} \sqrt{\Delta f_{det}} \quad (8.17)$$

From the thermal RMS noise voltage, the number of samples (N_s), and the sample rate (f_s), the thermal contribution to the noise floor in the power spectrum as calculated by MATLAB can be estimated using equation 8.18

$$U_{Thermal} = f_s N_s G_T^2 \mathcal{R}^2 Q_{NEP}^2 \quad (8.18)$$

Photoelectron Shot Noise At the detection stage, noise will couple into the system as counting error in the number of photoelectrons produced in the circuit. The RMS noise voltage due to the error in photon counting can be expressed in terms of the detector

bandwidth, the detector responsivity, the detector transimpedance gain, the electron charge (q), and the total power on each diode of the balanced detector using equation 8.19.

$$V_{RMS,BPD} = G_T \sqrt{2q_e \mathcal{R} (P_1 + P_2) \Delta f_{det}} \quad (8.19)$$

From the shot RMS noise voltage, the number of samples (N_s), and the sample rate (f_s), the shot noise contribution to the noise floor in power spectra calculated in MATLAB can be estimated using equation 8.20 [26].

$$U_{Shot} = 2f_s N_s G_T^2 q_e \mathcal{R} (P_1 + P_2) \quad (8.20)$$

Since the shot noise depends on the optical power incident on the detector, it will have slightly different forms in interferometric and traditional readout. In traditional readout, the power on each diode is identical to the read power ($P_1 + P_2 = 2P_r$), so the noise floor for traditional read is expressed by equation 8.21.

$$U_{Shot,trad} = 4f_s N_s G_T^2 q_e \mathcal{R} P_r \quad (8.21)$$

While in interferometric read, the total power on the two detectors is the sum of the read and LO powers ($P_1 + P_2 = P_r + P_{LO}$), so the noise floor for interferometric read is expressed by equation 8.22.

$$U_{Shot,IF} = 2f_s N_s G_T^2 q_e \mathcal{R} (P_r + P_{LO}) \quad (8.22)$$

In the case of a strong local oscillator, this means that nearly all of the shot noise will be due to the LO field, whereas the traditional read (and the equivalent interferometric read under the condition where $P_r = P_{LO}$) will have equivalent noise contributions from both the read and reference fields.

Source Noise Contributions

Some noise sources are the result of imperfections in the optical source. These imperfections could either arise from optical sources or from RF sources used in modulation. In most cases, the source noise contributions are common mode and, therefore subject to common mode rejection. When this occurs, typically the noise is not entirely canceled but instead is canceled up to the common mode rejection ratio (CMRR) is the absolute value of the ratio of the differential power under the condition that the power on each photodiode is the same to the power on one of those detectors as given by equation 8.23.

$$CMRR = \left| \frac{\Delta P_{measured}}{P} \right| \quad (8.23)$$

The detected noise after balanced detection ($Noise_{BPD}$) can be calculated in terms of the common mode noise expected for one beam ($Noise_{CM}$) and the CMRR according to equation 8.24.

$$Noise_{BPD} = CMRR \times Noise_{CM} \quad (8.24)$$

Relative Intensity Noise (RIN) Relative intensity noise (RIN) is a type of noise that consists of random fluctuations in the instantaneous intensity of an optical field. Typically the RIN is specified in either percentage or dBc. The RMS optical power fluctuations of a laser field are given in terms of the power (P_r) and the RIN figure (Q_{RIN}) by equation 8.25.

$$\delta P_{RIN,RMS} = P_r 10^{\frac{Q_{RIN}}{10}} \quad (8.25)$$

How this noise manifests in grating signals depends on the configuration used for readout. However, typically RIN contributions are kept fairly small in SSH readout systems, and balanced detection does a good job of suppressing contributions due to RIN.

Traditional Readout To model the effects of RIN in the interferometric readout, note that equations 8.26 and 8.27 give the powers on the two diodes

$$P_a = P_r \left(1 + \sigma(t) 10^{\frac{Q_{RIN}}{10}} \right) \quad (8.26)$$

$$P_b = P_r \left(1 + \sigma(t) 10^{\frac{Q_{RIN}}{10}} \right) \quad (8.27)$$

Where $\sigma(t)$ is a randomly distributed variable. In a balanced detection system, RIN is subject to common mode rejection and can be suppressed up to the common mode rejection ratio of the detector. Because the RIN is common mode, it will be subject to this subtraction. The measured electrical power spectrum due to RIN will then be given by equation 8.28

$$S_{RIN} = CMRR \frac{G_T^2 \mathcal{R} P_r^2 10^{\frac{2Q_{RIN}}{10}}}{\Delta f_{RIN}} \quad (8.28)$$

where (Δf_{RIN}) is the bandwidth of the RIN, which leads to a noise contribution given by equation 8.29.

$$U_{RIN} = CMRR \frac{f_s N_s G_T^2 \mathcal{R} P_r^2 10^{\frac{2Q_{RIN}}{10}}}{\Delta f_{RIN}} \quad (8.29)$$

In Interferometric Readout In interferometric readout, the powers at each port of the interferometer are given by equations 8.30 and 8.31.

$$P_a = \left(P_r + P_{LO} - 2\sqrt{P_r P_{LO}} \cos(\phi_B) \right) \left(1 + \sigma(t) 10^{\frac{Q_{RIN}}{10}} \right) \quad (8.30)$$

$$P_b = \left(P_r + P_{LO} + 2\sqrt{P_r P_{LO}} \cos(\phi_B) \right) \left(1 + \sigma(t) 10^{\frac{Q_{RIN}}{10}} \right) \quad (8.31)$$

Ideally, the interferometer is biased in quadrature (i.e., $\phi_B = \frac{\pi}{2}$), which will eliminate the cross terms. Deviating far from quadrature will lead to RIN cross terms which are not common mode and will not be suppressed by balanced detection. The limiting contribution to RIN will likely come from the LO since interferometric readout improvements are expected

at higher LO powers. In quadrature, using an LO much stronger than the read field, the RIN electrical power spectrum will be given by equation 8.32.

$$S_{RIN} = CMRR \frac{G_T^2 \mathcal{R} P_{LO}^2 10^{\frac{2Q_{RIN}}{10}}}{\Delta f_{RIN}} \quad (8.32)$$

This leads to a noise contribution given by equation 8.33.

$$U_{RIN} = CMRR \frac{f_s N_s G_T^2 \mathcal{R} P_{LO}^2 10^{\frac{2Q_{RIN}}{10}}}{\Delta f_{RIN}} \quad (8.33)$$

In interferometric readout, it may be the case that the output splitter is not ideal. Ideally, cancellation of the LO field should be prioritized in these cases because it will typically be stronger, meaning it will contribute more RIN.

Phase Noise The other major source of noise stemming from sources is phase noise. Phase noise essentially describes random fluctuations in the optical phase as it accumulates. Phase noise can be modeled as a random walk in the instantaneous phase of an oscillator due to Gaussian frequency noise. Assuming an oscillator phase of $\phi_{oscillator}$ (which is a time-dependent phase that could represent the linear instantaneous phase of a sinusoidal oscillator or the quadratic phase of a chirped field), the linewidth due to broadening from the phase noise is Γ_L and a random variable that describes the density of instantaneous phase fluctuations $\sigma(t)$ the time domain complex field is given by equation 8.34.

$$E(t) = E_0 e^{i\phi_{oscillator}(t) + i\sqrt{2\pi\Gamma_L} \int_0^t \sigma(t') dt'} \quad (8.34)$$

The result is a field where there are no variations in amplitude, and therefore the instantaneous power is constant. The power spectral density of the field, however, will be altered. That is to say, unless some other process can convert phase noise to some

form of amplitude noise, the phase noise will not appear in the noise floor. Rather, it will affect spectral readout because it will limit the instantaneous linewidth of the laser. If there are processes that cause phase-to-amplitude conversion on the read chirp before the detection process, these effects will be common mode. However, there is some possibility that interaction with a spectral hole, grating, or other feature within the readout scheme may lead to residual non-common mode optical noise on the signal. In interferometric readout, there may also be issues if the interferometer is severely unbalanced (e.g., the interferometer length mismatch is on the order of c/Γ_H). Phase-to-amplitude conversion in interferometric read can be mitigated by using a laser with a very narrow linewidth. In SSH applications, to get down to the material resolution, the laser linewidth should be substantially less than the read field (or programming field) linewidth ($\Gamma_H \gg \Gamma_L$). Therefore, any length mismatches in the interferometer that would contribute to the noise floor would perforce be much longer than the coherence length of the material (defined in analogy to a laser coherence length). The more relevant source of phase noise in SSH systems is likely phase-to-amplitude conversion due to a spectral hole or some other effect. Any uncanceled amplitude-converted phase noise would have the same read and LO power dependence as signals.

Nonlinear Hole Burning Noise

When reading the material's absorption spectrum, the chirp will alter the populations of the ions distributed in the material. Noise on the read chirp can lead to random fluctuations burned into the ion populations. In these cases, the depth of the features burned into the material will increase with the power of the read field. The chirp will also read out the programmed noise and will read out the programmed noise just as it would programmed gratings, yielding extra power-dependent factors (P_r^2 for traditional readout and $P_r P_{LO}$ for interferometric readout). The result is noise that grows non-linearly with the read power into the material. Because these noise contributions have a more aggressive dependence on

read power than signals, they will constrain the maximum signal-to-noise ratio. As will be shown later in traditional readout, these noise contributions rise as P_r^4 , meaning it will put a hard limit on SNR improvements due to a higher read power in traditional readout systems. In interferometric readout, as will be shown shortly, the nonlinear noise will rise as $P_r^3 P_{LO}$, so it still puts a limit on read power into the crystal. However, the dependence on LO power is the same linear dependence as expected for signals. The burned noise can come from a variety of noise sources, as described below.

General Description of Nonlinear Hole-Burning Noise A general model can be used to describe nonlinear hole-burning noise. This model will assume that there is some additional depopulation due to an imperfect chirped read field of power P_r and chirp rate κ . It will be assumed that this field has a flat intensity profile over a circular spot of radius w_0 . Burning in the material will be governed by an intensity burn parameter χ (recall from chapter 1 that $\beta = \frac{\chi}{2\pi r_0^2}$, χ is used for this description of nonlinear noise to explicitly indicate the role of spot size, and programming duration on the form of nonlinear noise). Assume that the noise that will be burned can be characterized by a parameter Ξ that determines the amount of noise that will be burned into the material per a given root noise bandwidth Δf . Note this bandwidth is referring to the bandwidth over which noise is able to program into the material. This could either be limited by the bandwidth of noise on the read chirp or the bandwidth of the material, whichever is more restrictive. Using these parameters, we can model the power spectrum with a randomly distributed variable with unit variance $\sigma(\Delta)$ to get equation 8.35, which is a model for the population with randomly distributed deviations to the populations at each detuning in the material.

$$r_3(\Delta) = -1 + \frac{\chi}{\kappa} \frac{P_r}{\pi w_0^2} + \Xi \sqrt{\Delta f} \sigma(\Delta) \frac{\chi}{\kappa} \frac{P_r}{\pi w_0^2} \quad (8.35)$$

Note that equation 8.35 has two read power dependent terms; term one has no variation and therefore represents DC bleaching of the material due to the read chirp (it is independent of imperfections of the chirp and is simply an effect of illuminating the material with a chirp). While this DC bleaching term can contribute to saturation, saturation is not considered in this mode. The second term (containing the random variable) represents the noise that is programmed into the material. The governing quantity Ξ is dependent on the properties of the chirp and the noise source, which could be one of a variety of sources, including relative intensity noise on the read field, phase noise on the read field, and instability of the instantaneous chirp rate of the read field. These alterations to the material populations will correspond to a modified field transmission coefficient for the material given by equation 8.36, assuming that the incidental programming due to the chirp remains in the linear regime. As has been discussed previously in chapter 1, care must be taken to ensure that the transmission spectrum's phase is correct to retain causality in the transmission spectrum. This is done using the Hilbert transform of the frequency-dependent components. Since all of the detuning dependence in the transmission spectrum is wrapped up in the random variable (because the chirp will translate time domain fluctuations to holes at different detunings), this can be done by calculating the imaginary part of the random variable $\sigma'(\Delta) = \sigma(\Delta) + i\mathcal{H}[\sigma(\Delta)]$.

$$\nu(\Delta) \approx e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{\alpha_0 L \chi}{2\kappa} \frac{P_r}{\pi w_0^2} + \Xi \sqrt{\Delta f} \sigma'(\Delta) \frac{\alpha_0 L \chi}{2\kappa} \frac{P_r}{\pi w_0^2} \right] \quad (8.36)$$

The read field is then transmitted through the material and is modified by the transmission coefficient. We will assume a slow readout where the chirp can map frequency domain onto time domain ($\Delta = \kappa t + f_0$). The output read field (with input field amplitude E_0) is given by 8.37

$$E_{out}(t) = E_0 e^{2\pi i(\frac{1}{2}\kappa t^2 + f_0 t)} e^{-\frac{1}{2}\alpha_0 L} \left[1 + \frac{\alpha_0 L \chi}{2\kappa} \frac{P_r}{\pi w_0^2} + \Xi \sqrt{\Delta f} \sigma'(\kappa t + f_0) \frac{\alpha_0 L \chi}{2\kappa} \frac{P_r}{\pi w_0^2} \right] \quad (8.37)$$

To determine how this modified field will be measured and thus correspond to a noise floor after detection, the readout method must be considered.

Traditional Readout of Incidentally Programmed Spectra In traditional readout, the power in the modified read field is balanced detected against a reference copy. The differential detection signal resulting from the incidental programming due to the read field is given by equation 8.38).

$$P_{out}(t) = P_r e^{-\alpha_0 L} + \frac{\alpha_0 L \chi P_r^2 e^{-\alpha_0 L}}{\pi w_0^2 \kappa} + Re[\sigma'(\kappa t + f_0)] \frac{2\Xi \sqrt{\Delta f} \alpha_0 L \chi P_r^2 e^{-\alpha_0 L}}{\pi w_0^2 \kappa} \quad (8.38)$$

The time domain voltage is proportional to the difference between the read power and the reference power, assuming that the reference field has the correct power to cancel out the dc terms leaving equation 8.39.

$$V_{out}(t) = Re[\sigma'(\kappa t + f_0)] \frac{2G_t \mathcal{R} \Xi \sqrt{\Delta f} \alpha_0 L \chi P_r^2 e^{-\alpha_0 L}}{\pi w_0^2 \kappa} \quad (8.39)$$

The RMS voltage of the nonlinear-hole burning noise as observed at detection is then given by equation 8.40.

$$V_{RMS} = \frac{2G_t \mathcal{R} \Xi \sqrt{\Delta f} \alpha_0 L \chi P_r^2 e^{-\alpha_0 L}}{\pi w_0^2 \kappa} \quad (8.40)$$

The electrical power spectral density (in voltage squared units) of that noise is assumed to be flat up to the noise bandwidth and is therefore expressed by equation 8.41.

$$S_{NL-HB,trad} = \frac{4G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^4 e^{-2\alpha_0 L}}{\pi^2 w_0^4 \kappa^2} \quad (8.41)$$

The noise floor calculated in MATLAB is then given by equation 8.42.

$$U_{NL-HB,trad} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^4 e^{-2\alpha_0 L}}{\pi^2 w_0^4 \kappa^2} \quad (8.42)$$

Interferometric Readout of Incidentally Programmed Spectra In interferometric readout, the modified electric field from equation 8.37 is mixed coherently with a local oscillator chirp (of arbitrary power but the same time dependence as the read field) that has some interferometric phase bias with respect to the read field (because they were split from some master chirp) in a 2x2 50-50 beam splitter. In interferometric readout the power difference is given in equation 8.43.

$$\Delta P(t) = Re \left[e^{i\phi_B} \sigma'(\kappa t + f_0) \right] \frac{2\Xi \sqrt{\Delta f} \alpha_0 L \chi \sqrt{P_r^3 e^{-\alpha_0 L} P_{LO}}}{\pi w_0^2 \kappa} \quad (8.43)$$

Assuming the interferometer is balanced in quadrature ($\phi_B = \frac{\pi}{2}$), the output voltage will be given by equation 8.44.

$$V(t) = Im \left[\sigma'(\kappa t + f_0) \right] \frac{2G_t \mathcal{R} \Xi \sqrt{\Delta f} \alpha_0 L \chi \sqrt{P_r^3 e^{-\alpha_0 L} P_{LO}}}{\pi w_0^2 \kappa} \quad (8.44)$$

This result is very similar to the result for traditional readout (equation 8.39). They primarily differ in that the interferometric readout depends on the square root of the read power cubed and the square root of the LO power ($\sqrt{P_r^3 P_{LO}}$). In contrast, traditional readout depends on the read power squared (P_r^2). The interferometer assesses the imaginary part of the random variable while traditional readout assesses the real part; this difference matters very little in calculating noise floors since the Hilbert transform of the random variable will have the same power spectral density as the random variable itself. The RMS voltage is, therefore, given by equation 8.45.

$$V_{RMS} = \frac{2G_t \mathcal{R} \Xi \sqrt{\Delta f} \alpha_0 L \chi \sqrt{P_r^3 e^{-\alpha_0 L} P_{LO}}}{\pi w_0^2 \kappa} \quad (8.45)$$

The electrical power spectral density (in voltage squared units) of the non-linear hole-burning noise is flat up to the noise bandwidth. Therefore, the interferometric readout can be expressed by equation 8.46.

$$S_{NL-HB,IF} = \frac{4G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^3 e^{-\alpha_0 L} P_{LO}}{\pi^2 w_0^4 \kappa^2} \quad (8.46)$$

The noise floor calculated in MATLAB is then given by equation 8.47.

$$U_{NL-HB,IF} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^3 e^{-\alpha_0 L} P_{LO}}{\pi^2 w_0^4 \kappa^2} \quad (8.47)$$

Recasting in Terms of Power At Detector. Currently, the powers in equations 8.42 and 8.47. are the powers before any attenuation. This assumption is fine for the local oscillator because the LO power can just be reckoned as the LO power at the detector. However, the read power (P_r) needs to be accounted for both at the detector and the material, so additional attenuation in between the crystal and detection must also be considered. We will assume the read field experiences some excess linear attenuation that can be described with a transmission coefficient T_{excess} . With this added excess attenuation equations 8.42 and 8.47 become equations 8.48 and 8.51.

$$U_{NL-HB,trad} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^4 T_{excess}^2 e^{-2\alpha_0 L}}{\pi^2 w_0^4 \kappa^2} \quad (8.48)$$

$$U_{NL-HB,IF} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_r^3 T_{excess}^2 e^{-\alpha_0 L} P_{LO}}{\pi^2 w_0^4 \kappa^2} \quad (8.49)$$

the nominal read power at the detector is $P_{r,det} = P_r T_{excess} e^{-\alpha_0 L}$ which can be arranged so that $P_r = \frac{P_{r,det}}{T_{excess} e^{-\alpha_0 L}}$. This can be simplified using a total transmission coefficient $T_{total} \equiv T_{excess} e^{-\alpha_0 L}$. This quantity describes the total transmission from the crystal to detection. In the case of interferometric read, the read power at the detector is the total read power on both arms of the splitter if the LO path is blocked. Equations 8.48 and 8.49 can be recast as equations 8.50 and 8.51.

$$U_{NL-HB,trad} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_{r,det}^4}{T_{total}^2 \pi^2 w_0^4 \kappa^2} \quad (8.50)$$

$$U_{NL-HB,IF} = \frac{4f_s N_s G_t^2 \mathcal{R}^2 \alpha^2 L^2 \chi^2 \Xi^2 P_{r,det}^3 P_{LO}}{T_{total}^2 \pi^2 w_0^4 \kappa^2} \quad (8.51)$$

Comparison Nonlinear Hole burning noise has similar forms for both readout methods. Equations 8.50 and 8.51 both depend on a series of system-dependent constants and a constant describing the properties of the burned noise, while the results from the two methods are different only in terms of the power dependence. Both traditional and interferometric noise floors depend on the parameter Ξ , which is related to the noise process; this parameter can be calculated for various noise sources.

Nonlinear Hole-burning Noise due to RIN

Fluctuations in the intensity of the read field, as described in equation 8.25, will lead to the programming of spectra into the material. The fluctuations of programming due to the RIN can be described using equation 8.52.

$$\delta r_{3,RIN}(\Delta) = 10^{Q_{RIN}/10} \sigma(\Delta) \frac{\chi}{\kappa} \frac{P_r}{\pi w_0^2} \quad (8.52)$$

Therefore, the noise hole burning parameter for RIN is given by equation 8.53.

$$\Xi_{RLN} = \frac{10^{\frac{Q_{RLN}}{10}}}{\sqrt{\Delta f}} \quad (8.53)$$

Non-linear Hole-burning Noise due to Chirp Rate Instability

Slight changes to the instantaneous chirp rate can result in noise programmed into the material absorption spectrum [68]. Deviations to the read chirp rate will lead to deviations to the instantaneous dwell time of the chirp ($t_{dwell} = \frac{1}{\sqrt{\kappa}}$). A deviation of the chirp rate $\delta\kappa$ will correspond to a fluctuation in the dwell time given by equation 8.54.

$$\delta t_{dwell} = \frac{\partial t_{dwell}}{\partial \kappa} \delta \kappa = -\frac{1}{2} \sqrt{\frac{1}{\kappa^3}} \delta \kappa \quad (8.54)$$

If this variation is written in terms of a relative variation $\bar{\kappa} \equiv \frac{\delta \kappa}{\kappa}$ the variation in dwell time is given by equation 8.55.

$$\delta t_{dwell} = -\frac{1}{2} \sqrt{\frac{1}{\kappa}} \bar{\kappa} \quad (8.55)$$

Since the population modification due to coherent programming from the chirp depends on the dwell time squared (Landau-Zener pump parameter squared [25]). The modification to the population is given by 8.56.

$$r_3(\Delta) = -1 + \frac{\chi}{\kappa} \frac{2P_r}{\pi w_0^2} e^{-\frac{2\rho^2}{w_0^2}} \left(1 - \frac{1}{2} \bar{\kappa} \sigma(\Delta)\right)^2 \approx -1 + \frac{\chi}{\kappa} \frac{2P_r}{\pi w_0^2} e^{-\frac{2\rho^2}{w_0^2}} (1 - \bar{\kappa} \sigma(\Delta)) \quad (8.56)$$

The nonlinear hole burning noise parameter due to chirp rate instabilities is given by equation 8.57.

$$\Xi_{\kappa} = \frac{\bar{\kappa}}{\sqrt{\Delta f}} \quad (8.57)$$

Nonlinear Hole-burning Noise due to Phase Noise

Phase noise on the read field will also lead to hole-burning noise. However, it was modeled using Maxwell-Bloch simulations rather than an analytic model describing the noise.

In these simulations, a random walk phase was added to the read chirp's instantaneous phase to model the read field's broadening due to phase noise.

A series of Maxwell-Bloch simulations were run, tracking the noise floor and changing different variables to track the noise floor's dependence on each variable. Dependence on sample rate and block size was as expected (equations 8.50 and 8.51). A linear dependence of the noise on laser linewidth (Γ_L) was observed as was an inverse cubic dependence on the chirp rate, which indicates an additional inverse dependence on the chirp rate and an inverse dependence on the material coherence time. These dependencies can be summarized using these parameters and another parameter (ξ_{phase}) that summarizes any additional parameters needed to summarize the noise programming (equation 8.58).

$$\Xi_{Phase} = \sqrt{\frac{\Gamma_L}{\kappa T_2}} \xi_{Phase} \quad (8.58)$$

Additional care must be taken to ensure that the results of the Maxwell-Bloch simulation accurately reflect results in a real system. This was done by calibrating the Rabi frequency to the Rabi frequency in the same system at a known power and a known spot size. This calibration constant will also help define the burn parameter χ . Assuming a Landau-Zener transition, the burn parameter is given by equation 8.59.

$$\chi = 2\pi^2 \Omega_{cal}^2 \frac{\pi w_{cal}^2}{P_{cal}} \quad (8.59)$$

Using a read power dependence simulation of a traditional read scheme, a linear fit was used to attempt to calculate the parameter ξ_{phase} . The simulation used a chirp of chirp rate $0.5 \text{ MHz}/\mu s$ (from -25MHz to +25MHz), a calibration power of 15mW, a calibration Rabi frequency of 1.5 MHz, a calibration spot size of 0.15mm, a simulation spot size of 0.5mm, and a laser linewidth of 20KHz. The simulated material has an inhomogeneous bandwidth of 300MHz divided into bins of width 0.1MHz, a coherence time of $0.79\mu s$, and an absorption

length of 1.39 divided into 4 segments. The chirp runs for $100 \mu s$. The number of samples in the final processing block is 57131 samples, and the sample rate is 600 MSPS. Results presented in figure 8.1 assumed a responsivity of $0.53 \frac{A}{W}$, a transimpedance gain of $0.5 \times 10^5 \frac{V}{A}$, and power units of $mV^2 \times Samples^2$.

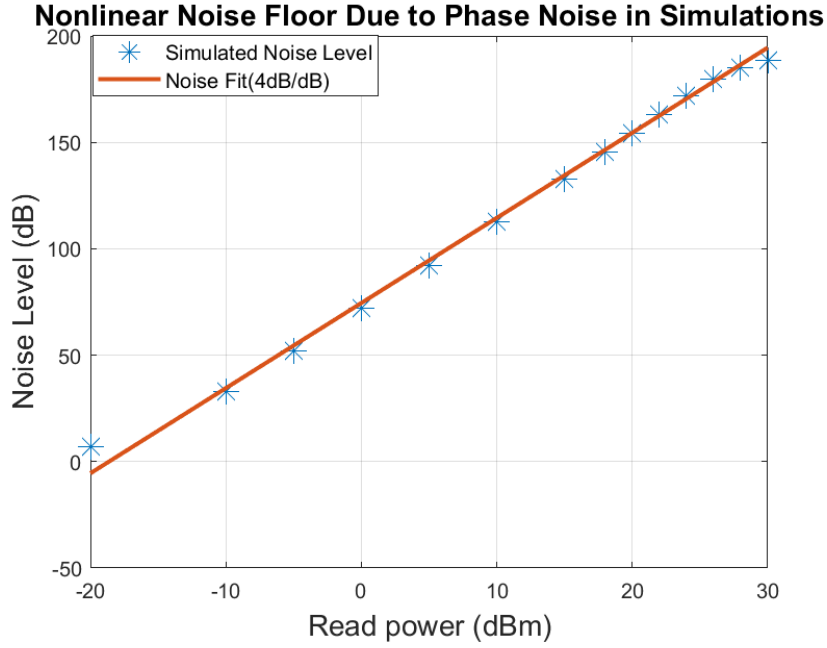


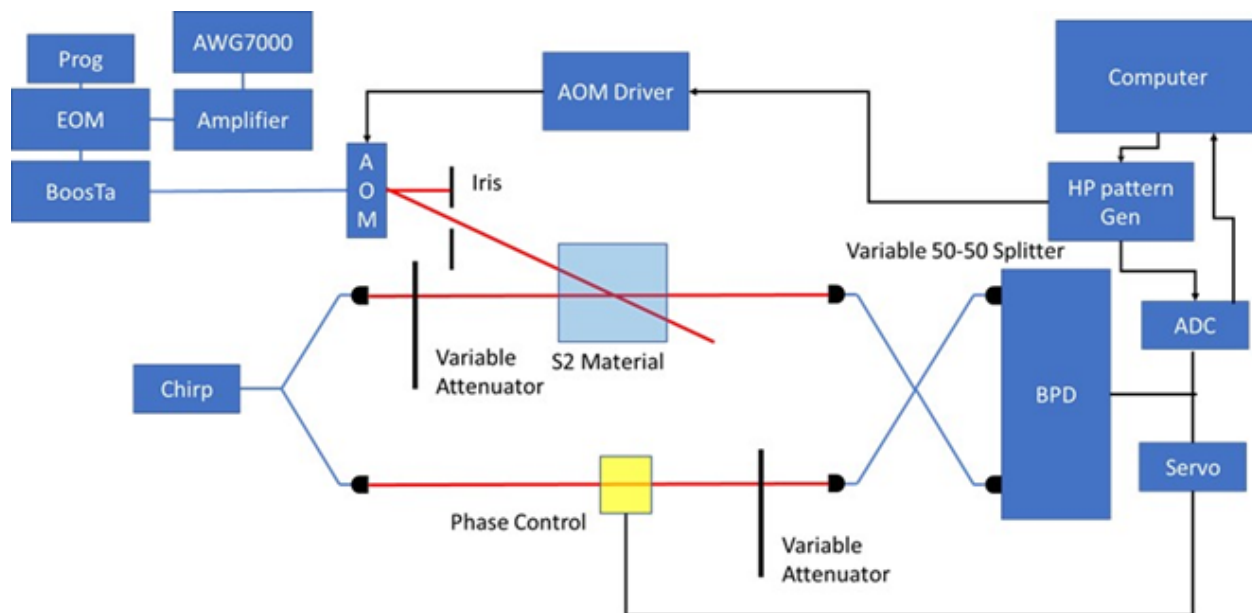
Figure 8.1: Maxwell-Bloch simulation results indicate a quartic dependence of the noise floor on read power for grating measurements made on a signal-free background by a read field with phase noise, as is expected given the model for the origin of nonlinear noise.

Given the parameters and a coherence time of $0.8 \mu s$, the fit on the noise level was used to calculate a value of $\Xi_{Phase,simulation} = 5.28 \times 10^{-17} Hz^{-1}$. From this, the value of $\xi_{Phase,simulation}$ was calculated to be $1.45 \times 10^{-7} \sqrt{Hz^{-1}}$.

Experiment Overview

Description of Experimental System

Interferometric readout can reliably be compared to traditional readout in the SSH readout system built to read out SSH spectra at MSU-Spectrum Lab. The system was built



field, that burns for a duration of $3ms$. The programming pulses are carved out using an AOM. The time delay is encoded onto the programming field using an electro-optic phase-modulator (EOPM) driven by a Tectronix AWG80000, producing a $0.01ms$ duration $1GHz$ chirp summed with its delayed counterpart. The RF power of this waveform is approximately -3 dBm. The spectral interference between the chirp and the delayed chirp generates a spectral grating in the material, which is read out by the read field. The read field and the reference/LO field are created by splitting a $1ms$ duration $2GHz$ bandwidth chirped optical field at $794nm$ that is spectrally overlapped with the lower chirped side-band of the programming fields. The read field is attenuated before going into the crystal so that power into the crystal can be controlled. The read field then passes through the programmed

region of the crystal. This field is modified by the crystal and is ready for detection. The other field, the reference/LO field, passes around the crystal and is reflected off of a piezoelectrically modulated mirror for phase control in the interferometric configuration, and then through a variable optical attenuation so that the power of the field can be controlled at the detection stage. The difference between the interferometric and traditional readout schemes is the configuration of the detection stage. In the traditional case, the read and reference fields are coupled into fiber optic patch cords and routed into the ThorLabs PB450a variable gain balanced detector, which at 50Ω of input impedance while using the 4MHz detection bandwidth has a transimpedance gain of $0.5 \times 10^5 \frac{V}{A}$ and a responsivity of $0.53 \frac{A}{W}$ and a NEP figure of $3.3 \frac{pW}{\sqrt{Hz}}$ [69]. A variable attenuator is used to balance the powers on each diode of the balanced detector; balancing the power allows for the cancellation of common mode noise and removes DC offset. In the interferometric configuration, the read and LO fields are coupled into a variable 50-50 splitter/coupler. The variable splitter is tuned to a 50-50 split ratio by blocking the read field and adjusting until the balanced detection signal generated by the split LO field is zeroed out to ensure that the system is tuned to remove the LO background. The outputs of the variable splitter are then detected at the BPD. The variable attenuator allows for control of the power in the LO at the detection stage. The detected signal in the interferometric case is used as an error signal for a low bandwidth PID controller to stabilize the interferometer's phase bias. By holding the detected signal around zero, the interferometer is held roughly in quadrature, which is the preferred condition for interferometric readout. The PID controller provides a drive signal for a PZT controller, which is used to drive the piezoelectric mirror, which is used to drive the optical phase in the stabilization loop. In both the interferometric and traditional cases, the signals are digitized using a 16Bit AlazarTech ATS9626 250MSPS ADC Card. The signals are then processed digitally.

Overview of Data Processing Method

Data captured using the experimental apparatus are captured on the digitizer card, and then the capturing and processing scheme can be described as follows:

1. The signal is read into the computer, and a pre-check is conducted to ensure that the interferometric bias is relatively low. The detected signal is measured and compared to the zero position of the interferometer (which is governed by the dc bias of the balanced detector). Using experimental parameters, the rough interferometric bias is estimated and is compared with a threshold to ensure the interferometer is within 5% of the maximum phase bias. If the signal is within these bounds, it is kept, saved, and processed, but if it is not, the computer will wait for another capture.
2. The digital signal is cut down to the relevant region. Specifically, the region of interest is from 0.3ms to 0.7ms for a 0.4ms duration. The digitizer card is sampling at 20 MS. This gives a full block size of 8000 samples.
3. The mean is removed from the signal, and a Hann window is applied to prevent spectral leakage.
4. The signal is then fast Fourier transformed to get the signal into frequency domain; this will also give the signal processing gain so that even if the signal is not immediately visible in the time domain, it can be pulled from the noise.
5. The Fourier transformed signal is magnitude squared so that the measured signal produced by MATLAB represents the power spectrum of delays.
6. Signals are measured by finding the local maximum around the location of the predicted beat note frequency based on the programmed delay (which is known *a priori*). In this experiment, the delay was fixed at 300ns. The power in the frequency bin corresponding to this delay is used as the signal value.

7. Noise is calculated by taking the average of the linear power spectrum in the region from 2.5MHz to 3.5MHz. (which was a region free of visible spurs). The average noise power per frequency bin in this region is used as the noise value.
8. The experiment is repeated 30 times, the signal and noise values are averaged, and the signal-to-noise ratio is computed.

Figure 8.3 is an example of a typical trace indicating the measured peak and the noise measurement region.

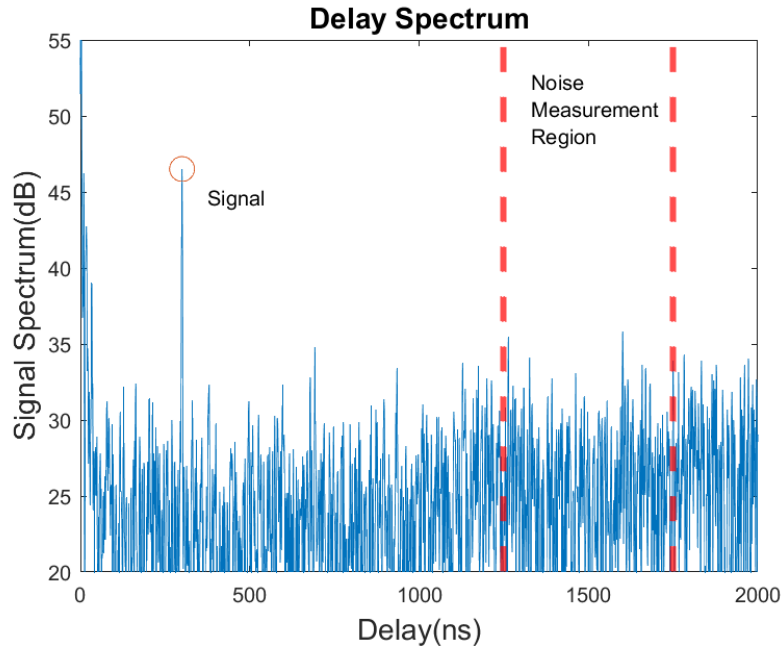


Figure 8.3: An example trace indicating the peak location and the region used to make noise measurements

Power Dependence Results

Programming Power Dependence

To test the programming power dependence of the interferometric read system, the system was held at 5uW of read power and 1mW LO power, and the RF attenuation before

the EOM was used to modulate the programming field which had 12mW of total power and with a -0.5mW RF modulation an instantaneous power in each sideband of 0.12mW or about -9.2dBm. The expected effect is that the signal strength should scale at 2dB/dB with RF attenuation (since the attenuator stack is attenuating both the programming chirp and the delayed programming chirp equally). Experimental results presented in figure 8.4 demonstrate the proper 2dB/dB scaling of signal levels and signal-to-noise ratios, with the programming power. As seen in the results, increasing programming power will result in an

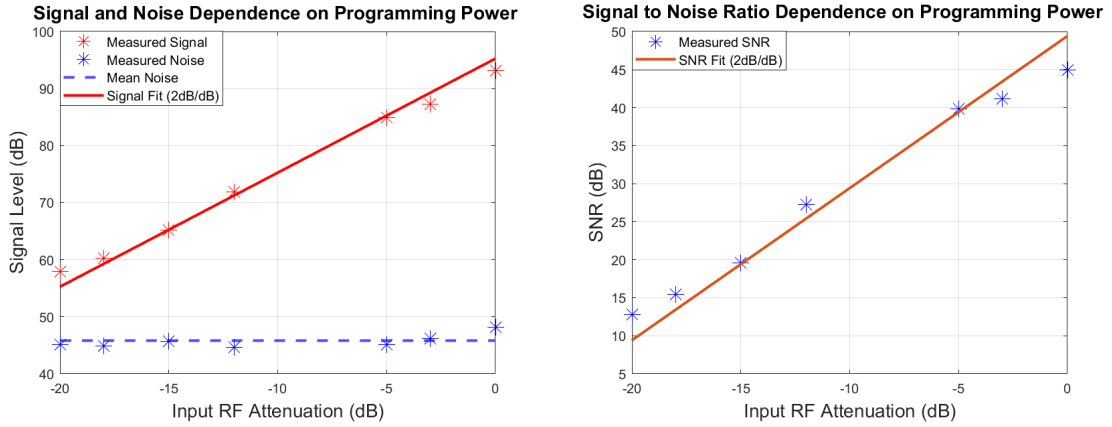


Figure 8.4: Left: Signal and Noise dependence on RF attenuation, Right: SNR dependence on programming power

increase in the signal without affecting the noise floor. This indicates that more programming power will provide a higher SNR until either RF or material saturation. In these experiments, gratings were programmed in the linear regime and grating saturation was not seen.

Read Power Dependence

In both traditional and interferometric readout, absent nonlinear hole-burning noise, increasing the read power should improve the SNR. To track this behavior, the signal and noise levels were measured in interferometric and traditional readout systems at a variety of different read power levels. Interferometric readout measurements were done in the condition where $P_r = P_{LO}$. In all of these experiments, the ratio between the power into the crystal

to the power at the detector was measured to be 20 (this accounts for both the material absorption and any excess attenuation). The results of these experiments are depicted in figure 8.5.

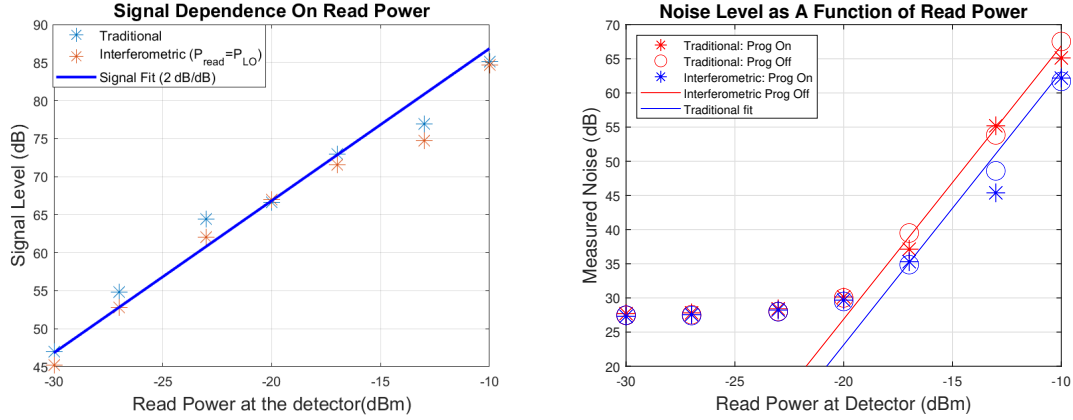


Figure 8.5: Left: Signal level dependence on read power in interferometric and traditional readout. Right: Read power dependence of noise level in interferometric and traditional readout schemes with and without a programming field present.

Signal levels in both interferometric and traditional readout show a 2dB/dB dependence on read power. The noise floors in both readout methods begin in the detector limit before growing non-linearly at around -20dBm ($10\mu W$) of read power. There is a small offset in between the phase noise in interferometric and traditional readout measurements which indicates that the noise floor is slightly lower in interferometric readout than interferometric readout even under conditions where the methods are expected to yield equivalent results. The noise can be estimated by doing a 4dB/dB fit on the nonlinear section of the noise floor measured in traditional readout experiments and using equation 8.50 and using equation 8.51 to fit the nonlinear noise figures for interferometric and traditional readout. For traditional readout, the non-linear noise figure is calculated to be $\Xi_{\text{trad}} = 9.6 \times 10^{-18} \text{Hz}^{-1}$ assuming that phase noise was the main source of nonlinear noise is phase noise and that the calculated value of ξ_{phase} , assuming a T_2 of $0.65\mu s$, and a laser line width of 20 KHz , of $\xi_{\text{phase,trad}} = 1.86 \times 10^{-7} \sqrt{\text{Hz}^{-1}}$. A programming coefficient of $\chi = 120 \frac{\text{mm}^2 \text{MHz}}{\text{mW}}$, was estimated for this

sample, for incoherent programming in the steady state limit. Using the Rabi frequency to estimate the programming coefficient yielded results that were orders of magnitude lower than the observed result, however in this regime, that assumption could reflect the fact that gratings are allowed to accumulate incoherently in the material over time scales longer than the coherence time because the chirps begin about 40ms before data is collected. In interferometric readout, this noise level is slightly lower, with values extracted from a fit of the noise data from interferometric experiments of $\Xi_{IF} = 3.8 \times 10^{-18} \text{Hz}^{-1}$ and $\xi_{\text{phase}, IF} = 1.17 \times 10^{-7} \sqrt{\text{Hz}^{-1}}$, which is slightly lower than the Maxwell-Bloch simulation result shown in figure 8.1 which was $\xi_{\text{phase}, \text{sim}} = 1.45 \times 10^{-7} \sqrt{\text{Hz}^{-1}}$. The simulation value is in the same range as the measured values for traditional and interferometric readout. This indicates that incidental programming due to phase noise on the read chirp is the source of non-linear noise in experiments.

The signal and noise levels are combined to calculate the signal-to-noise ratios at each power; this is depicted in figure 8.6

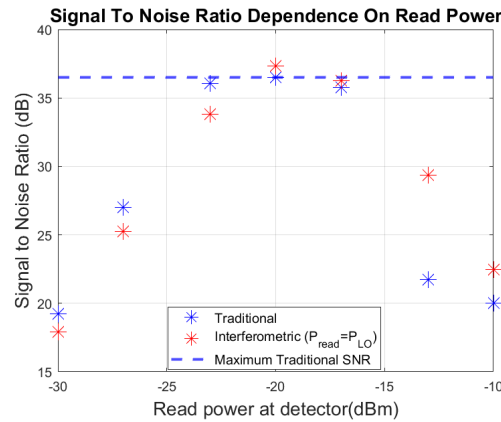


Figure 8.6: SNR as a function read power for traditional and interferometric readout

The non-linear noise rises faster with read power than the signal level, meaning that traditional read (and interferometric read under the condition $P_r = P_{LO}$). This system has a maximum SNR of 38dB at about (-20dBm or $10\mu W$).

Local Oscillator Power Dependence

To determine the effects of increasing the local oscillator power in interferometric readout, the dependence of the signal and noise levels were tracked in interferometric readout configurations with the same read power. In all of these experiments, the ratio between read power into the crystal to the power at the detector is 20 (this accounts for both the material absorption and any excess attenuation). In these experiments read power is measured at the detector.

Local Oscillator Power Dependence at $1\mu W$ (-30dBm) of Read Power At $1\mu W$ (-30dBm) of read power at the detector ($20\mu W$ / -17dBm at the material), the signal and noise levels follow the expected dependences (shown in figure 8.7). The signal level rises with LO power at 1dB/dB, and the noise floor agrees with the estimated noise floor arising from the thermal and shot noise. The signal, noise, and SNR show good agreement between traditional read and interferometric read at $P_r = P_{LO}$. The signal-to-noise ratio begins

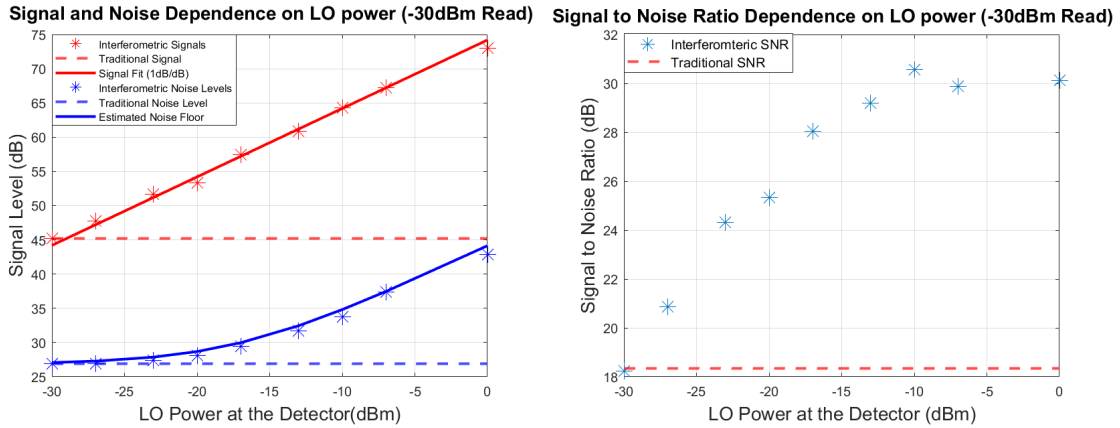


Figure 8.7: Left: Signal and Noise Levels in interferometric readout at $1\mu W$ (-30dBm), Right: Signal-to-noise ratio calculated from the data on the left.

at around 18dB at $P_{LO} = -30dBm$ and rises with LO power while the system is in the detector-limited regime. At about -20dBm of LO power, the shot noise begins to dominate

the noise floor, and the SNR rolls off to around 30dB. At this read power, LO gain was able to improve the signal-to-noise ratio by 12dB over traditional read.

Local Oscillator Power Dependence at $5\mu W$ (-23dBm) of Read Power At $5\mu W$ (-23 dBm) of read power at the detector ($100\mu W$ / -10dBm at the material), the signal and noise levels follow the expected dependencies (shown in figure 8.8). As expected signal level rises 1dB per dB of LO power. The noise floor agrees with the estimated noise floor arising from only thermal and shot noise. Again the performance of interferometric and traditional readout are very similar at $P_r = P_{LO}$ as is predicted by theory. At the lowest experimental LO

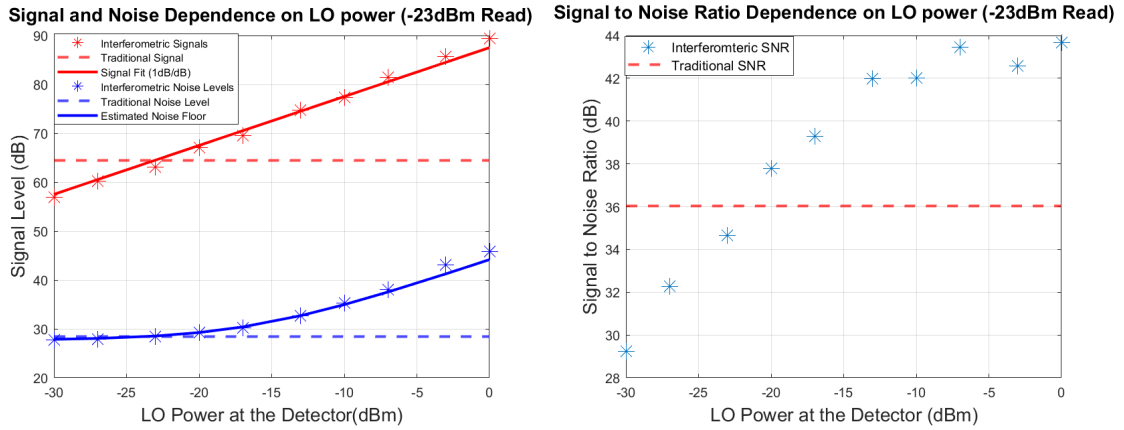


Figure 8.8: Left: Signal and noise levels in interferometric readout at $5\mu W$ (-23dBm). Right: Signal-to-noise ratio calculated from the data on the left.

power of -30 dBm, the SNR is 29dB and rises linearly with LO power in the detector-limited regime. At just over -23dBm of read power, the interferometric SNR intersects traditional SNR. Shot noise begins to contribute at around -23dBm of LO power, at which point the SNR begins to flatten. At the highest LO powers, the SNR is up to 43 dB. The overall SNR improvement using interferometric readout, observed at this read power, was 6dB.

Local Oscillator Power Dependence at $10\mu W$ (-20dBm) of Read Power At $10\mu W$ (-23dBm) of read power at the detector ($200\mu W$ / -7dBm at the material), the signal level,

again, rises with the expected one dB per dB of read power (shown in figure 8.9). A first attempt at a noise model using just the thermal and shot noise did not effectively model all of the noise behavior (see the solid line in figure 8.9). An additional term was added to the noise model that includes the contributions due to the nonlinear noise floor using the nonlinear noise figure of $\Xi_{trad} = 9.6 \times 10^{-18} Hz^{-1}$ calculated from noise floor power dependence data and using equation 8.51 to calculate noise in the experiment calculated from the read power dependence data. The adjusted estimate (the dashed line in figure 8.9) indicates that at -20dBm of read power, the nonlinear noise is starting to contribute to the noise floor.

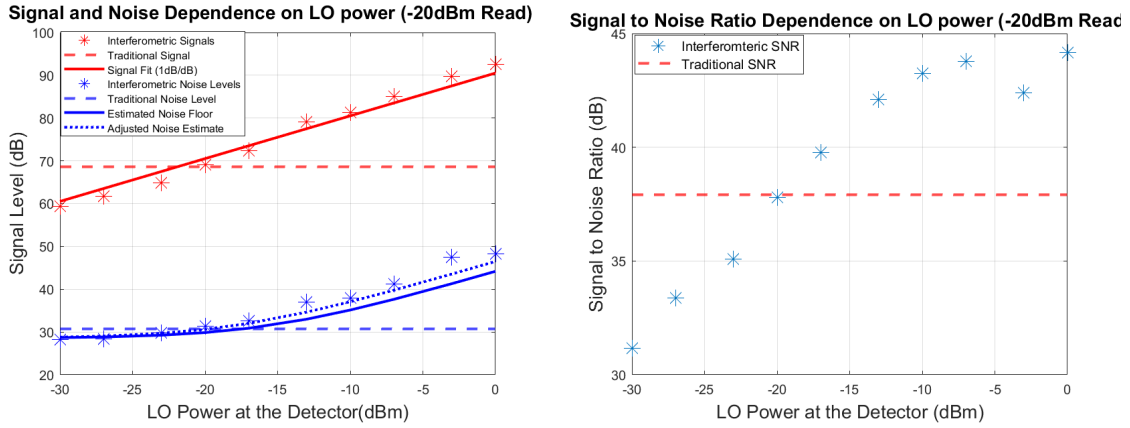


Figure 8.9: Left: Signal and noise levels in interferometric readout at $5 \mu W$ (-23dBm), Right: signal-to-noise ratio calculated from the data on the left.

At this read power, LO gain improved the signal-to-noise ratio by 6dB over traditional readout giving a maximum SNR of 44dB, which is similar to the result at -23dBm of read power (albeit with a slightly higher SNR). This is because these results occur near the maximum SNR (which is limited by nonlinear hole-burning noise) so locally, the SNR dependence on read power is fairly weak.

Demonstrations

Pulling Signals Out of Noise

Interferometric readout can be used to pull signals that would be buried in the noise in traditional readout above the noise floor using a strong LO. These improvements have been demonstrated in two cases: in the case of a weak read field and in the case of a weak programming signal. In both cases, it is demonstrated that interferometric readout can be used to pull previously unseen signals above the noise floor.

Weak Read To demonstrate the ability of the system to pull signals measured with a weak readout signal. The read beam was attenuated to 100nW (-40dBm) at the detector ($2\mu\text{W}$ / (-27dBm) at the crystal). First, the signal was measured in traditional readout. Then the measurement was repeated using interferometric readout with a 1mW (0dBm) LO (figure 8.10).

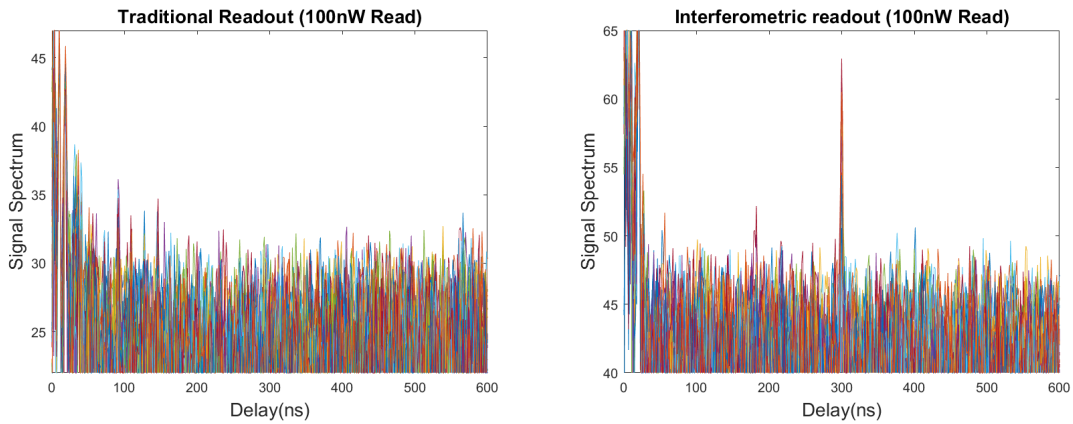


Figure 8.10: Pulling signals read out with a weak read field, out from the noise floor. Left: traditional readout with a 100nW read field. Right: interferometric readout of the same signal readout with 100nW of read power using a strong local oscillator.

In the traditional case, the weak read field cannot pull the programmed signal above the noise floor. In the interferometric case, the signal can be seen above the noise floor of the

detector. In fact, this signal that was not visible in the traditional case now has nearly 15dB of SNR. The noise model predicts a maximum interferometric improvement over traditional readout of about 22dB for this read power, 7dB of LO gain went to pulling the signal up to the noise floor.

Weak Signal The measurement of weak signals was done in the regime where the highest SNR was seen in the traditional case. Specifically, a $10\mu W$ read field at the detector ($200\mu W$ of read power at the crystal). The RF attenuation of RF signals driving the EOM was increased until the signals dropped below the noise floor (this occurred after 18dB of RF attenuation). The same experiment was then repeated with a $5\mu W$ read power, $1mW$ LO power interferometric read scheme (figure 8.11). In the traditional case, the weak signal

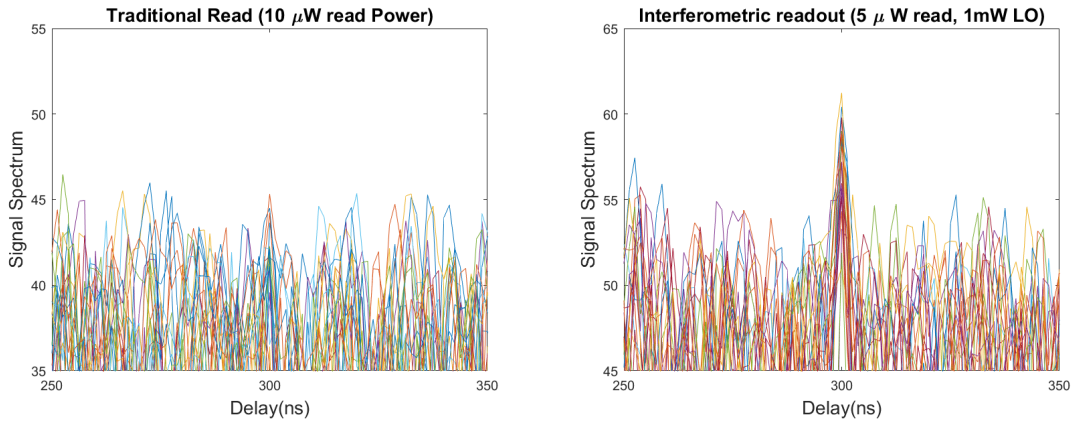


Figure 8.11: Pulling weak signals out from the noise floor. Left: traditional readout with a $10\mu W$ read field. Right: Interferometric readout of the same signal readout with a $5\mu W$ using a strong LO. Using the strong LO pulls the weak signal out of the noise floor and into a clear view.

cannot be seen in a single shot of the delay spectrum, even though the system is tuned to the conditions of maximum SNR. In contrast, the interferometric system can see the delay signal in most shots. This sensitivity improvement using interferometric readout using LO gain cannot be achieved using traditional readout with any read power. This sensitivity is

a true improvement to sensitivity improvement that can not be replicated using a stronger read field.

Simultaneous vs Delayed Readout

All of the previously displayed experimental data used a read scheme where the captured grating is measured after the end of the programming pulse. A demonstration was done to show that the interferometric readout technique can also handle simultaneous readout where the grating is read out concurrently with the programming pulse. In figure 8.12 spectra for simultaneous and delayed readout are very similar, this means that in this system, cross talk between the programming and readout fields do not affect the final spectra. If all of the previous experiments were done with simultaneous readout there would not have been much effect on the results.

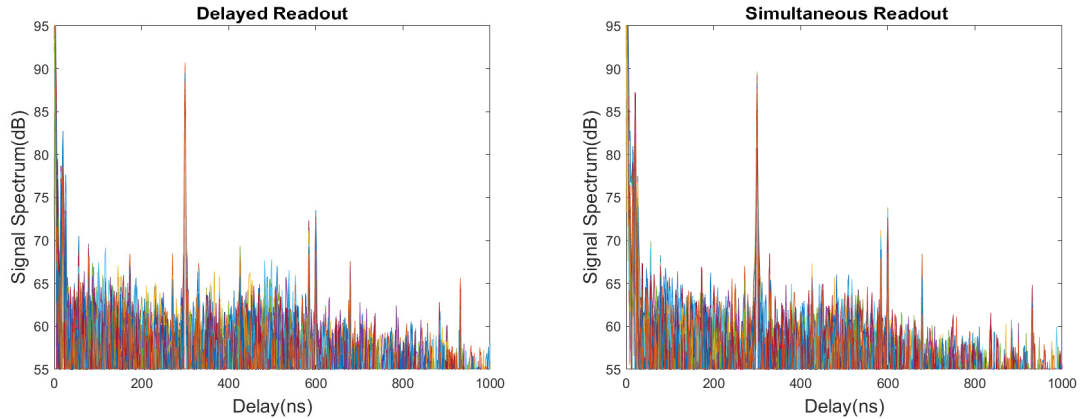


Figure 8.12: Interferometric delay spectra for simultaneous vs. delayed readout. Left: delayed readout. Right: simultaneous readout.

Correlated Noise Programmed Gratings As others have demonstrated[41, 60], an attractive property of SSH grating systems is that noise is an ideal broadband waveform for programming gratings. Since noise is only correlated at zero lag, there will be no range ambiguity if a noisy laser is used to program gratings. We were able to demonstrate

interferometric readout and processing of noise-programmed spectra. To program these spectra, a broadband noise signal using a random variable was filtered to create a band-limited spectral grating. The correlated noise grating signal was then read out using a $5\mu W$ read field and a $1mW$ LO. An example of a noise-programmed grating read out by an interferometric system is presented in figure 8.13, demonstrating this feature of SSH grating systems.

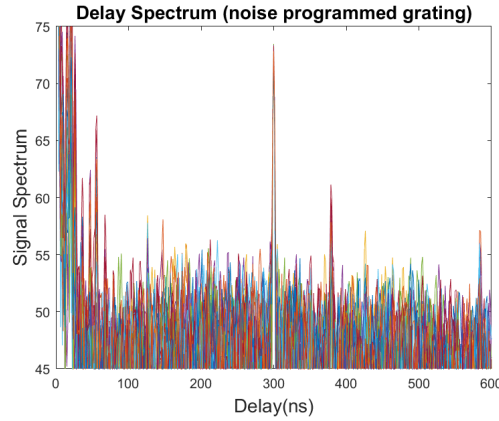
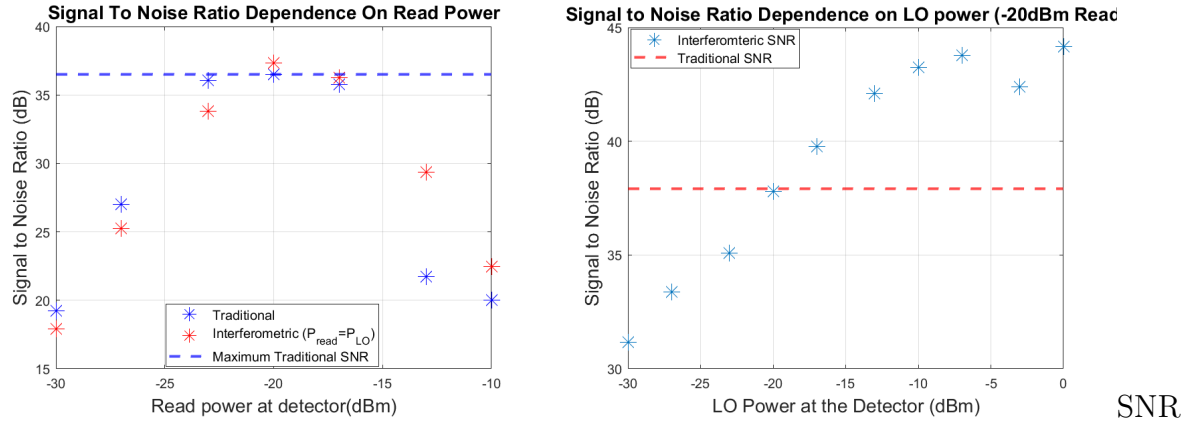


Figure 8.13: A grating signal is programmed into the material using noise and delayed noise, then read out using interferometric readout.

Discussion

True SNR Improvement Through LO Gain

The non-linear noise floor in traditional readout gave rise to a maximum in the SNR dependence on read power. Interferometric readout provides the system with another degree of freedom that can be used to improve SNR; in this case, we can see that the highest measured SNR in an LO dependence plot exceeds the highest estimated power in traditional readout experiments (see figure 8.14). In the interferometric readout, this other degree of freedom can bring the system into the shot noise-limited regime. This allows for higher SNR detection of gratings. The parameters line up well so that in this system, the maximum



dependence on LO power (10 microwatts)

Figure 8.14: Together, these two plots demonstrate the overall improvement of SNR using interferometric readout. Left: Traditional and comparable interferometric SNR dependence on Read power. There is a clear global maximum of around 38 dB. Right: Interferometric readout hits its maximum at around 44 dB. Therefore, using interferometric read can give around a 6dB improvement over this system's best possible traditional readout.

traditional SNR occurs before the system enters the LO shot noise limit; this means that using a strong LO can provide gain to bring the system into the shot noise limit. If this local maximum occurred while the traditional system was shot noise limited, the maximum SNR improvement that could be expected by using interferometric readout with a strong LO would be 2dB; however, because the system is right at the edge of the detector limited region, the LO can supply enough gain to bring signals far above the detector noise, This allows for the over SNR improvement of 6dB demonstrated experimentally.

The improvement over the maximum traditional SNR indicates a regime in which the improvements due to using interferometric readout are not only a contrived improvement that could be matched with an increase in read power. Interferometric readout can actually offer true improvements to SNR that cannot be made in a traditional readout system. Meaning the maximum achievable SNR for the interferometric system is higher than the maximum achievable SNR using a comparable traditional readout system.

However, a better study of the non-linear noise floor and saturation effects needs to be conducted to fully parameterize these improvements or the conditions under which they are applicable. Accounting for these dependencies will allow for the determination of the conditions under which a true LO improvement to SNR.

Prescriptions for SNR Improvement

In both interferometric and traditional readout of SSH gratings, non-linear hole-burning noise can limit the read power used to interrogate gratings. The non-linear noise is, therefore, a very important consideration in achieving the maximum signal-to-noise ratio for a given system.

In SSH grating readout systems, the main contributions to the noise floor are due to thermal noise, shot noise, and hole burning noise. In our experiments, source noise contributions were very weak and accounted for by common mode rejection, and the ADC noise floor was below the detector noise floor, so these contributions were ignored. The SNR for traditional readout and interferometric readout can therefore be modeled by equations 8.60 and 8.61.

$$SNR_{Trad} = \left(\frac{N_S}{f_s} \right) \frac{\alpha_0^2 L^2 \beta_P^2 S_1 S_2 P_r^2}{Q_{NEP}^2 + \frac{4q}{R} P_r + \frac{4\alpha^2 L^2 \chi^2 \Xi^2 P_r^4}{T_{total}^2 \pi^2 w_0^4 \kappa^2}} \quad (8.60)$$

$$SNR_{IF} = \left(\frac{N_S}{f_s} \right) \frac{\alpha_0^2 L^2 \beta_P^2 S_1 S_2 P_r P_{LO}}{Q_{NEP}^2 + \frac{2q}{R} (P_{LO} + P_r) + \frac{4\alpha^2 L^2 \chi^2 \Xi^2 P_r^3 P_{LO}}{T_{total}^2 \pi^2 w_0^4 \kappa^2}} \quad (8.61)$$

In the limit where the local oscillator is very strong compared to the read field (i.e., $P_{LO} \gg P_r$), and the read field is strong enough that the shot and non-linear noise terms dominate the power-independent thermal noise (i.e., $\frac{2qP_{LO}}{R} \gg Q_{NEP}^2$) the interferometric SNR reaches an asymptotic limit of maximal SNR given by equation 8.62.

$$SNR_{IF} = \left(\frac{N_S}{f_s} \right) \frac{\alpha_0^2 L^2 \beta_P^2 S_1 S_2 P_r}{\frac{2q}{R} + \frac{4\alpha^2 L^2 \chi^2 \Xi^2 P_r^3}{T_{total}^2 \pi^2 w_0^4 \kappa^2}} \quad (8.62)$$

The interferometric readout SNR for a given system is a function of 2 variables: The read power (P_r) and the LO power (P_{LO}). Figure 8.15 shows SNR estimated using these equations for traditional and interferometric readout using a strong LO. Red lines indicate traditional readout, and blue lines indicate interferometric readout using a strong LO. All models use theoretical estimates of shot and thermal noise. The solid lines are models that use a nonlinear noise model with the parameter ($\xi_{phase,trad} = 1.86 \times 10^{-7} \sqrt{Hz^{-1}}$) estimated from the linear fit of traditional noise shown in figure 8.6. While the dashed blue line indicates interferometric readout using a nonlinear noise parameter estimated from interferometric readout ($\xi_{phase,IF} = 1.17 \times 10^{-7} \sqrt{Hz^{-1}}$). The dotted lines represent readout without nonlinear noise (where SNR can always grow with read power). Experimental data is overlayed on the plot.

Using $\xi_{phase,trad}$ to describe nonlinear noise for both traditional and interferometric readout, the plot indicates separate maxima for both interferometric and traditional readout. The interferometric SNR is maximized at a read power a few dB lower than traditional read, but this global maximum is higher than the maximum traditional SNR. This indicates that to maximize SNR by using interferometric readout with a strong LO, a slightly lower read power should be used in interferometric readout with a strong LO. In the estimated plot, the maximum improvement is on the order of 4dB, which is slightly lower than the 6dB of improvement observed in experiments. The improvement can be raised to the observed 6dB of improvement when the value $\xi_{phase,IF}$ is used to model nonlinear noise in interferometric readout. This indicates there is some additional subtraction present in the interferometric readout. Most of this section will proceed assuming that the nonlinear noise coefficient is the same for both readout methods, however at the end of the section, the likely origins of this additional SNR improvement, which is seen in the different SNR curves using nonlinear noise figures from traditional or interferometric experiments, will be presented. Figure 8.15 indicates improvements to SNR by using interferometric readout with a strong local oscillator

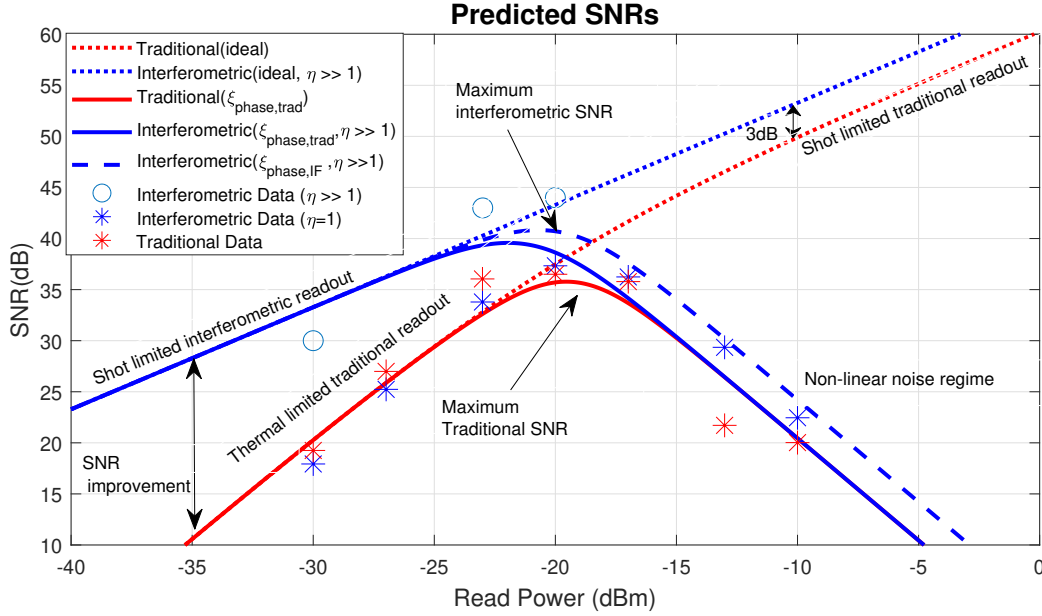


Figure 8.15: SNR estimates for interferometric and traditional readout overlaid with experimental data. Blue lines indicate interferometric readout, while red lines indicate traditional readout. The plot indicates several different SNR improvement regimes and the estimated maximal SNR conditions for both methods. For the solid lines, nonlinear noise parameters were estimated using the results from figure 8.5 using data to estimate the nonlinear hole-burning figure using traditional data. This value was $\xi_{\text{phase,trad}} = 1.86 \times 10^{-7} \sqrt{Hz^{-1}}$. Using this model an SNR improvement, using interferometric readout with a strong LO of about 4dB is expected at a read power of -23 dBm. Experimentally we observed about 6dB of SNR improvement at -20dBm of read power. This is better reflected by nonlinear hole-burning noise figure estimated from interferometric readout of $\xi_{\text{phase,IF}} = 1.17 \times 10^{-7} \sqrt{Hz^{-1}}$. Which is shown in the dashed blue lines, this may indicate some additional improvement using interferometric readout than was initially expected and better explains data shown in figure 8.9. The dotted lines indicate readout absent nonlinear noise. In this case, traditional read begins in the thermal limit before shifting to the shot limit. Ideal interferometric read with a strong LO is always shot noise limited. In the shot-limited regime, SNR in interferometric readout is 3dB higher.

can be made at a given read power and improvements to the maximal SNR that can be made by comparing the independently maximized SNRs for both traditional and interferometric readout.

The confluence of different noise sources primarily governs the degree of SNR improvement that can be expected. Therefore, to estimate the degree of improvement,

the SNRs of each method need to be compared in different noise regimes in order to assess the degree and type of improvement that can be expected. These regimes can be described essentially in terms of which noise parameters are relevant to estimating SNR in each limit. Table 8.2 summarizes the degree of SNR improvement that can be achieved using interferometric readout depending on which terms contribute to the noise floor. Each

Case	Noise Contributions						SNR Improvement $\left(\frac{SNR_{LF}}{SNR_{T_{rad}}}\right)$
	Traditional			Interferometric			
	Thermal	Shot	Non-linear	Thermal	Shot	Non-linear	
Case 1	✓	×	×	✓	×	×	$\frac{P_{LO}}{P_r} (> 3dB)\dagger$
Case 2	✓	×	×	×	✓	×	$\frac{Q_{NEP}^2 R}{2qP_r} (> 3dB)\dagger$
Case 3	×	✓	×	×	✓	×	$3dB\dagger$
Case 4	✓	×	✓	×	✓	✓	$\frac{4Q_{NEP}}{3}\sqrt[3]{\left(\frac{R}{2q}\right)\frac{\alpha_0 L\chi\Xi}{T_{total}\pi w_0^2\kappa}} (> 2dB)\ddagger$
Case 5	×	✓	✓	×	✓	✓	$2dB\ddagger$

† Indicates improvement to SNR given a specific Read Power

‡ Indicates an improvement to maximum SNR, assuming that both traditional and interferometric readout are using the correct optimized read power.

Table 8.2: This table lists 5 cases in which the improvement to SNR uses interferometric readout with a strong LO. The degree of improvement expected regimes. It is important to note that different regimes may have improvements defined differently. This is because only in cases 4 and 5 is the usable read power limited by non-linear noise, which allows for optimization of read power for the highest possible SNR. In cases 4 and 5 where non-linear noise limits SNR, the improvement factor is calculated by independently optimizing SNR for both interferometric and traditional readout and comparing the optimized SNRs. Cases 1 two and 3 do not consider non-linear noise and therefore increases to read power will presumably SNR. In these cases, improvements to SNR are cited for a given read power.

of the 5 cases represents a limit where a subset of the three considered noise sources are relevant to the estimation of SNR. In each of these regimes, the improvement to SNR can be calculated in a relatively closed form and represent possible conditions under which a readout system could operate and how the SNR could be improved by using interferometric

readout for those conditions. Broadly, there are two classes of cases, one where the non-linear noise is assumed not to contribute, which could be achieved using some combination of faster chirps, a cleaner laser or a very narrow material linewidth, and one where non-linear noise is an important consideration. The presence of non-linear noise leads to global maxima in SNR, which means that the read power can be selected to maximize SNR. In these cases, the SNR improvement is reckoned between the optimized traditional SNR and the optimized interferometric read. Absent non-linear noise, the SNR for both traditional and interferometric read will be improved by increasing read power, so in these cases, the SNR improvement is reckoned between the traditional and interferometric SNR at a given read power (which is assumed to be much less than the LO power in the interferometric case).

Cases 1, 2, and 3: No Nonlinear Noise In cases where a non-linear noise floor does not limit read power, there may be some external reason to limit read power, such as power broadening. In these cases, the most effective comparison between interferometric and traditional readout is at the same read power. However, still, the SNR improvements in this regime are governed by the noise conditions (e.g. thermal limited or shot noise limited) in both interferometric and traditional readout.

Case 1: Thermal Limited Detection: Suppose both traditional and interferometric readout schemes only show thermal contributions to the noise floor. In that case, noise floors are identical for both interferometric and traditional readout ($Q_{NEP} \gg \frac{2qP_{LO}}{R} \gg \frac{4qP_r}{R}$). In these cases, the only difference between the SNRs is signal level, meaning that the improvement goes as the LO gain parameter (equation 8.63).

$$\frac{SNR_{IF}}{SNR_{Trad}} = \frac{P_{LO}}{P_r} = \eta \quad (8.63)$$

This situation was observed experimentally in all of the LO dependence plots at low

read powers where the noise floor is still detector limited (see figures 8.7 and 8.8).

Case 2: Thermal Limited Traditional Read, Shot Limited Interferometric Read: In cases where traditional readout is limited by thermal but interferometric read is limited by shot noise (i.e. $\frac{2qP_{LO}}{R} \gg Q_{NEP} \gg \frac{4qP_r}{R}$), as was observed in the LO dependence study at $1\mu W$ (see figure 8.7) at high LO Powers after the SNR saturates. In these cases, the maximum SNR improvement at high read powers is given by equation 8.64.

$$\frac{SNR_{IF}}{SNR_{Trad}} = \frac{Q_{NEP}^2 \mathcal{R}}{2qP_r} \quad (8.64)$$

Given the detector, NEP $Q_{NEP} = 3.3 \frac{pW}{\sqrt{Hz}}$ and responsivity $\mathcal{R} = 0.53 \frac{A}{W}$, the SNR improvement observed expected in our system is then $\frac{18\mu W}{P_r}$. In the interferometric LO power dependence study with $1\mu W$ of read power, and given the system parameters, interferometric read with $1\mu W$ of read power is expected to show $\frac{18\mu W}{1\mu W} = 18 = 12.6dB$ more SNR than traditional readout at the same read power, which corresponds well with the 12dB of observed SNR improvement.

Case 3: Shot Limited Detection When both interferometric read and traditional are shot limited (i.e. $\frac{qP_{LO}}{R} \gg \frac{2qP_r}{R} \gg Q_{NEP}$), the signal to noise improvement is a factor of 2 (3dB) for interferometric read with a strong local oscillator over traditional read because in interferometric read the noise floor is dominated by the shot noise on the LO field (which will be much larger than the read shot noise contribution) while in the traditional readout scheme, there are equal contributions from the read and reference fields meaning that even in the shot noise limit for a given read power interferometric readout with a strong LO will give a 3dB SNR improvement over traditional readout.

Cases 4 and 5: Non-Linear Noise is Present In cases where the read power that can be used to read out spectra in the material is limited by the nonlinear noise floor (as described in section 8). The read power in each method can be adjusted to improve SNR. This procedure

can be done independently for interferometric readout in the limit where $P_{LO} \gg P_r$ and for traditional readout. In these regimes, the SNR improvement is the ratio of the maximized SNRs (rather than SNRs assuming the same read power). The degree of improvement observed will depend on the degree of noise contribution in each readout scheme. Two particular regimes are of interest. The first regime of interest is where traditional readout noise is due to thermal noise and nonlinear noise, while interferometric read is limited by shot noise and nonlinear noise. The second is the regime where both traditional and interferometric read are limited by a combination of shot and nonlinear noise.

Case 4: Thermal limited Traditional Readout, Shot Limited Interferometric Read, with Nonlinear Noise Limitation In the regime where traditional readout is limited by thermal noise and nonlinear hole-burning noise, the SNR can be approximated using equation 8.60 omitting the shot noise term in the denominator. By maximizing SNR in this regime, the maximizing traditional read power for this regime can be calculated using equation 8.65.

$$P_{r,Trad,max} = \sqrt{\frac{Q_{NEP}T_{total}\pi w_0^2\kappa}{2\alpha_0 L\chi\Xi}} \quad (8.65)$$

The same process can be performed for interferometric readout by maximizing 8.61 omitting the thermal term from the denominator. The maximizing interferometric read power is given by equation 8.66.

$$P_{r,IF,max} = \frac{1}{2} \sqrt[3]{\frac{2q}{\mathcal{R}} \left(\frac{T_{total}\pi w_0^2\kappa}{\alpha_0 L\chi\Xi} \right)^2} \quad (8.66)$$

Using the two maximizing read powers, the improvement due to interferometric readout with a strong local oscillator over traditional readout in this regime can be simplified to the expression given in equation 8.67.

$$\frac{SNR_{IF}}{SNR_{Trad}} = \frac{4Q_{NEP}}{3} \sqrt[3]{\left(\frac{\mathcal{R}}{2q} \right) \frac{\alpha_0 L\chi\Xi}{T_{total}\pi w_0^2\kappa}} \quad (8.67)$$

This regime was not seen in the experimental system as the shot floor, and nonlinear floor began contributing significantly around the same time, meaning that the maximal SNR traditional read had noise contributions from the thermal noise, shot noise, and nonlinear noise.

Case 5: Shot Limited Detection with Non-linear Noise When both read schemes are limited by shot noise and non-linear noise, the read powers can be optimized by ignoring the thermal terms in the denominators. The maximum read power in interferometric readout can be calculated using equation 8.66 and the maximum read power in traditional read can be calculated according to equation 8.68.

$$P_{r,trad,max} = \sqrt[3]{\frac{q}{2\mathcal{R}} \left(\frac{T_{total}\pi w_0^2\kappa}{\alpha_0 L\chi\Xi} \right)^2} \quad (8.68)$$

Substituting these expressions for maximizing read power into the expressions for signal-to-noise for traditional and interferometric readout, the maximum SNR for interferometric readout is a factor of $\sqrt[3]{4}$ (or 2dB) higher than the maximum traditional SNR in this regime. This regime was not observed experimentally. This regime represents a worst-case scenario where the lowest level of SNR improvement over traditional readout using interferometric readout can be expected.

SNR Improvement Beyond Expectations

The model for traditional and interferometric readout using the same nonlinear noise parameter ($\Xi_{IF} = \Xi_{trad}$) underestimated the experimentally observed SNR improvements. The upper dashed line in figure 8.15, indicates the estimated SNR using separate noise coefficients for interferometric and traditional readout ($\Xi_{IF} \neq \Xi_{trad}$), both estimated from noise data in figure 8.5. The data from that experiment was read power dependence data, which indicates that both interferometric readout and traditional readout had a 4dB/dB

nonlinear noise dependence at higher powers. However, for interferometric readout, the noise was consistently a few dB lower ($\frac{\Xi_{IF}}{\Xi_{trad}} = 0.4 \text{ or } -4\text{dB}$). The explanation for the nonlinear noise is that the chirp will both program noise into the spectrum and read it out, so at first glance, the noise coefficients for each method should be the same. One of the primary differences between interferometric and traditional readout is a beam splitter at the end of the system just before the BPD. The beam splitter makes other terms related to the field which has interacted with programmed spectra squared (such as the terms that Oberto et al. attempted to remove through post-processing [53]), common mode at the detector, and therefore, subject to common mode rejection. So if some higher order terms become common mode in interferometric readout but not in traditional readout, those terms could contribute to a higher noise floor in traditional readout as opposed to interferometric readout. This could explain why the SNR improvements seen in experiments are higher than those predicted by theory, as described in the preceding subsection.

Conclusion

The interferometric readout technique, used in SSH lidar applications, was demonstrated to amplify signals using a strong local oscillator, allowing for improved signal-to-noise ratios because the amplification of the signal was greater than increases to the noise floor induced by the use of the strong local oscillator. Not only are improvements to SNR expected for readout systems using a given read power, but the maximum SNR observed in interferometric readout was higher than the maximum observed SNR for traditional readout. In experimental studies, SNR improvements of 6dB were demonstrated using interferometric readout compared to the highest achievable SNR using the same system with traditional read. The level of improvement due to using interferometric read with a strong LO is highly dependent on the system parameters; analysis indicates that in the nonlinear noise limited regime, an SNR improvement of at least 2dB can be achieved, while for a given read power,

absent nonlinear noise, an SNR improvement of at least 3dB can be expected for a given read power. Additional common mode cancellation in interferometric readout may be the reason for increased SNR enhancement of 6dB in nonlinear noise-limited regimes.

Interferometric readout proved to be sensitive to low-frequency fluctuations in the interferometric phase that lead to clutter at lower frequencies. These fluctuations could be mitigated by a better interferometric stabilization method. In the presented experiments, stabilization is limited because locking was done using the read field to generate an error signal. A superior locking system would use a field detuned far enough from the inhomogeneously broadened transition so that phase stabilization could be done continuously using a CW field (without readout spectra riding on them).

CONCLUSION

Summary of Results

Echo Isolation

In interferometric architectures, The phases of photon echoes generated in independent, spatially separated ensembles can be controlled by injecting the required input pulse sequence into a network of beam splitters. The relative phases of the input pulses in each channel, when they enter the material, will result in different optical phases of the echo field. The echo fields can interfere at the output of a network of beam splitters so that the echo field will constructively interfere at an isolated output of the beam splitter network. The ability to produce and isolate stimulated photon echoes interferometrically was demonstrated using experiments and Maxwell-Bloch simulations.

The principle of interferometric stimulated photon echo isolation was demonstrated experimentally in a 2-port Mach-Zehnder interferometer. Echoes generated in two separate spots in the material interfered at the output ports so that the echo would appear on one port while the probe field would appear on the other. Experiments showed an echo field that was 6dB higher than the leaked portion of the probe field on that same port.

A theoretical framework for considering spatial, spectral holographic systems in interferometric networks with an arbitrary number of channels was developed to extend this principle to higher-order networks. This theoretical framework was used to design an architecture for a 4-port interferometer that could be used for echo isolation. The ability of this interferometric architecture to isolate stimulated echoes into a single unique output port, in an analog to the angular "Box Geometry", was demonstrated in Maxwell-Bloch simulations.

Interferometric Readout of SSH Spectra

SSH spectra can be read out interferometrically by using a chirped readout field passing through a programmed spot in the material and mixing that field with a strong local oscillator (LO) chirp in a 50-50 beam splitter. The interference signal can be measured using balanced detection on the splitter's outputs. The resulting signal can be used to estimate the phase response of the programmed material, from which the magnitude response can be calculated. Using a local oscillator chirp, which can bypass the material, allows for the amplification of signals. Amplifying signals allows for the improvement of the signal-to-noise ratio in specific noise regimes.

SSH readout experiments were conducted in a Mach-Zehnder interferometer for spectrum analysis applications. In this system, the dispersive lineshape expected for narrow spectral features was observed. A recovery script capable of recovering the standard Lorentzian spectral shape for interferometric readout was demonstrated. The additional LO power parameter allowed for the amplification of detected signals using a strong LO, which in turn allowed for improved SNR and sensitivities for a given read field power.

The same interferometric readout scheme was then used to process spectral gratings. In the grating processing study, improvements to SNR were again observed at a given read power. In this study, a noise floor with a nonlinear dependence on the noise floor was observed. This put limits on the read power that could be used in SSH experiments; this limitation meant that the LO power provided a means of SNR improvement that could not be recovered using a stronger read field. For this system, the highest observed interferometric SNR for any read power was nearly 8dB higher than the highest observed traditional SNR. Theoretical results predicted a 4dB SNR improvement using interferometric readout with a strong LO, but in experiments, an 8dB improvement was observed.

Interferometric readout with a strong LO was demonstrated to be able to improve SNR and sensitivity in SSH readout systems. The primary consideration for the degree of

improvement due to the use of interferometric readout with a strong LO is governed by the noise regime the system is in. Since detector thermal noise is constant, gain to signals due to a strong LO can yield massive improvements to SNR. In the shot noise-limited regime, for a given read power, improvements are expected to be more modest (1.5dB for spectrum analysis and 3dB for spectral gratings). True improvement (i.e., an improvement that could not be achieved by increasing read power) requires that the read power is limited in some way that the LO power isn't. Nonlinear noise observed in experiments provided one such improvement; the nonlinear noise put limits on the amount of power that could be put into the material, which led to improved SNR that could only be achieved in this system with interferometric readout.

Experimental Considerations

Splitter Fidelity

Theoretical studies of interferometric SSH systems indicated the importance of the fidelity of beam splitters. In the case of spectrum analysis systems, the output splitter had effects on SNR.

In interferometric readout applications, the input splitter primarily directs different amounts of power into the read and LO fields, so the split ratio for input splitters in these applications should be chosen to set the LO and read field powers. While the output splitter needs to match the power in The read and LO field components on each output port. Errors in the splitter will lead to incomplete cancellation of background and common-mode noise.

In echo isolation applications, non-ideal split ratios of 50-50 splitters anywhere in the system may be problematic, but by tuning internal attenuations of fields at different points in the interferometer, some of these issues can be accounted for. It was found that input splitters tended to have greater importance in achieving a higher echo efficiency at maximum contrast when the contrast was tuned by adjusting attenuations in the device.

In experiments, a variable 50-50 splitter was used as an output because the adjustability allowed it to be tuned to ensure good interferometric contrast, which corresponds to both better echo isolation and better common mode suppression. Any future work on interferometric SSH systems will require attention to the fidelity of the input splitters because imperfections to input splitters had slightly more effect on echo efficiency even when the contrast was maximized. In 2015 Miller et. al [48] suggested using a double Mach-Zehnder as a tunable splitter that can be used to create a perfect splitter using imperfect elements at the cost of overall efficiency. This may be a useful method for correcting beam-splitter imperfections in interferometric SSH architectures.

Interferometric Bias Control

Another critical consideration for interferometric SSH systems is the interferometric bias control. Holding interferometers in the proper bias condition is critical. In echo isolation applications, the interferometric bias has to be held in peak or null to ensure that the echoes are directed into the correct port and to ensure that there is no leakage of the other pulses into the echo port. In readout experiments, the interferometer needs to be held in quadrature; deviations can lead to mild distortions to the spectral shape and a fluctuating background.

In experiments, the bias control proved to be problematic. In the stimulated echo isolation experiment, a suitable phase control scheme was not devised. Instead, the interferometric bias was scanned periodically by slowly driving the PZT mirror with an external signal and observing echoes at different points along the slow modulation. This allowed for the observation of echoes at different points along the curve, where echo isolation was seen in the peak bias condition. Readout experiments used the balanced detection signal as an error signal to stabilize the interferometer in quadrature. This proved problematic because using the signal of interest as the stabilization signal meant that if the bandwidth of the PID controller was not sufficiently low, it would attempt to stabilize the observed

spectra. This led to a very low stabilization bandwidth, and some of the interferometric fluctuations remained above that bandwidth. These showed up as fluctuating backgrounds that changed from shot to shot. In SSH spectrum analysis experiments, this background fluctuation led to the necessity of single-shot background estimation. In the SSH grating processing experiments, these fluctuations could be frequency gated since they were still relatively low-frequency fluctuations.

Many of the added complications that were added by fluctuating bias could have been avoided with a better stabilization scheme. A better scheme could use a far-detuned field co-linear with the fields in the interferometer as a stabilization beam. This beam could be isolated through polarization or wavelength division multiplexing. This far-detuned field would not excite or interact with the material but would have a relatively close wavelength to the resonant fields. For example, at 743nm, a detuning of 1 THz represents a 0.2% change in wavelength. However, it would be sufficient to avoid serious material interaction while still remaining very close to the original wavelength, meaning the detuned interferometer mode should be able to stabilize the resonant interferometer mode. This field could be a simple CW field and would be able to stabilize the interferometer up to a relatively high frequency without the stabilization scheme trying to stabilize away signals. In this case, the normal control systems methods for stabilizing a Mach-Zehnder interferometer could be used.

A more elaborate control systems scheme would be needed to hold a 4-port (or even a higher order) interferometer at the correct interferometric biases; these higher-order networks will require more involved control systems because there are more degrees of freedom. Systems like these would likely also use detuned fields. Such a system will likely use differences of the detuned fields at different output ports as error signals for a series of dither locking systems in order to generate control signals to stabilize the phases at different points in the interferometer.

DC Bias In Interferometric SSH Spectrum Analysis

Processing of interferometric readout signals requires either a Hilbert transform or its equivalent to recover the desired spectrum. This results in a loss of the DC component of the recovered spectrum. This component will need to be restored somehow to achieve the desired spectrum. In experiments, this was done incidentally by the background subtraction algorithm; however, even if the background was not present, some means of restoring the DC component so that the measured signals are measured with respect to true zero power. This can be done in background subtraction or by using an unprogrammed region of the material spectrum as a reference.

Material Decoherence in Readout Experiments

In readout experiments, the coherence time of the material proved to be lower than expected (650 ns at the highest). It seems that poor thermal contact led to low coherence times in the material, as remounting led to an improved coherence time; however, even after several attempts at achieving better thermal contact with the cryostat, the coherence time was still relatively low at 650ns. For $Tm^{3+} : LiNbO_3$, the dependence of the coherence time is strong, so small improvements to thermal contact can have big effects on the coherence time. If the coherence time were improved, the spectral resolution of the analyzer would have been improved, and therefore a wider bandwidth of spectral gratings could have been observed.

Aberrations

In our experiments programming fields are injected into lenses slightly off-center so aberrations on the fields before they enter the material are expected. This could lead to issues with mode overlap inside of the material, which may have led to a loss in efficiency. This is not only of concern in interferometric methods, as both the traditional readout

geometry and the “box geometry” would be equally susceptible to these issues.

Future Avenues of Research

Higher Order Architectures

Experimental demonstrations in this dissertation were concerned with 2-port Mach-Zehnder interferometers. Future research considering interferometric SSH processing could experimentally test either of the 4-port interferometers described in the introduction. As previously mentioned, it would be critical for these studies to consider an interferometric bias control system for such a device. Higher-order interferometric methods also provide a wide field of study. Interferometric SSH networks with even more than 4 channels may provide interesting new applications for SSH processing such as all-optical neural networks.

Unified Architecture It may be possible to unify the designs for four-port interferometric readout and 4 port-stimulated echo isolation into a single architecture that could be used as a re-configurable device built into a photonic integrated circuit for use for spectrum analysis, SSH grating processing, or optical correlation, all in one device. If the phase controls can be electrically switched for different phase configurations and modulations for fields in different input ports can also be altered electrically for different types of programming or readout, then this whole device could be electrically reconfigured on the fly for different processing tasks. An example of such a unified architecture is displayed in figure 9.1. In the unified architecture, the additional Mach-Zehnders can be biased to reconfigure the device for different applications. This device can be reconfigured from the configuration for echo isolation shown in figure 2.5 and the 4 port configuration for interferometric readout shown in figure 2.7. These biases could also be used to control the amount of power used as a local oscillator field for interferometric readout. Additional Mach-Zehnders act as switches before the crystal so that some of the individual fields could be directed around the SSH material.

While the device uses 6 ports two of the inputs/outputs go unused. This is because the additional ports are merely required to introduce paths around the crystal. Increasing the number of ports allows for extended complexity. One could conceive of a wide variety of interferometric architectures for a number of processing and isolation tasks.

Interferometric SSH Interactions with Non-Classical Light

Interferometric methods with SSH materials may have a variety of applications with non-classical optical fields. Multi-port interferometric networks may be useful architectures for applying rare earth-doped materials as quantum memories in linear optical quantum computing, which is practically similar to the generalized interferometric methods described previously [36, 37, 65]. Interferometric detection with squeezed optical fields can be used to beat the standard shot noise limit for phase-sensitive detection [79]; this may apply to SSH readout, in which case an interferometric architecture may be a suitable candidate for a means of exploiting this property.

SSH processing in Waveguides for Photonic Integrated Circuits

A useful complementary study for this work would be a study of SSH processing in waveguides. Waveguides doped with rare earth ions would be interesting for creating a solid-state architecture for SSH processing in a photonic integrated circuit. The interferometric methods lend themselves well to application in photonic integrated circuits because phase differences between fields in different material spots can be interfered with to isolate fields. Angular approaches (such as the “box geometry”) require free-space injection of fields into the material, which cannot be achieved in photonic integrated circuits. So interferometric architectures could be an attractive alternative for SSH processing in photonic integrated circuits. Rare earth-doped waveguides are also an example of a situation in which optical fields are confined to a small space, and therefore, weak read fields would be needed to read out programmed spectra without inducing saturation effects or nonlinear noise. In these

cases, interferometric readout provides a means of achieving better signal-to-noise ratios without increasing intensity.

Photonic integrated circuits could be contrived using doped regions for SSH processing, a network of splitters to interfere fields could be routed into the material substrate, and active electro-optic regions to apply interferometric biases or modulation to fields inside of the photonic integrated circuit. The entire circuit could be held at a low temperature in a cryostat. In these cases, higher-order interferometric networks with multiple doped regions so that multi-stage SSH processing could be conducted where the resulting fields from an SSH process could be processed again using another SSH process. Interferometric networks could provide a general means of conducting complicated processing tasks using higher-order architectures.

Application of Interferometric methods to Other Optical Processes SSH processes are essentially non-linear processes. The output field can be described as a product of input fields in frequency domain. Most non-linear processes, such as second harmonic generation or four-wave mixing, typically arise as products of fields in time domain. The phase properties of some non-linear optical processes may be similar to the phase relations in SSH processing. The network methods for describing these types of interactions may also be suitable for implementation in interferometric architectures. These phase properties could be applied for the isolation or coherent amplification of fields in other non-linear systems. Interferometers may be usable for the isolation of certain harmonics or four-wave mixing terms from pump fields, all done in a collection of independent periodically poled materials to allow for the implementation of these non-linear processes in photonic integrated circuits.

The interferometric readout technique described and demonstrated may also be useful in studying the absorption spectra of other systems sensitive to the power injected into the sample. Limitations to input power could arise for several reasons, such as alteration of

spectra or physical damage to the sample. In either case, a strong LO field could be used to improve the signal-to-noise ratio in measurements of the absorption spectra.

Analysis of Read Power Limitations

The ability of interferometric readout to provide gain to signals allows for improvements to SNR and sensitivity of SSH readout systems. Neglecting limitations on read power into the material, any improvement to SNR gained by using a strong LO could be reclaimed using a stronger read field. There are, however, limitations on the read field that can be used for SSH experiments. The experimental study on the readout of spectral gratings showed that non-linear noise could arise at high read field powers. Other limitations exist on power into the material, for example, power-broadening effects can lead to decreased resolution, and there may be other sources of the observed non-linear noise.

Fluorescence is another source of noise that will scale with power into the material that warrants deeper analysis. The chirp fraction of absorbers excited by the chirp field that decay directly into the ground state will fluoresce in the wavelength range of the read field. Because fluorescence radiates spherically, the fluorescence noise will depend on the solid angle of the mode into which it must couple. In our system, the solid angle of the fiber is very low (4×10^{-7} steradians, which would correspond to a maximum coupling efficiency of -60dB) because the fluorescence propagates a long way through free-space optical elements, so noise due to fluorescence was not observed in experiments. This solid angle could be much larger in a photonic integrated circuit context because fluorescence occurs inside a waveguide. If the chirp rate is high enough, most of the fluorescent light may not interfere with the LO field within the detector band, in which case the LO will not amplify it. This would mean that this noise could be suppressed in interferometric readout.

A more thorough study of these power limitations can paint a clearer picture of the conditions under which improvements cannot be achieved with a stronger read power.

Conclusions

The phase properties of fields in SSH systems can be exploited using interferometric methods for several applications. The phase of an echo field can be exploited to spatially isolate it using beam splitters before and after the material. The phase shifts due to the Krammers-Kronig relations allow for phase-sensitive interferometric readout of spectra in SSH materials. This, in turn, allows for the property of introducing signal gain from a strong local oscillator. The phase dependence of interferometric networks paired with the phase properties of spectra programmed into SSH materials creates a space in which many potential applications and architectures exist. This dissertation explored the space of signal processing applications of these systems. The space is still wide and there are a variety of different avenues still left to explore both in the realm of SSH applications, and in adjacent domains.

APPENDICES

APPENDIX A

INTEGRAL TRANSFORM CONVENTIONS AND NOTATION

Fourier Transform Conventions

The Fourier transform ($\mathcal{F}$) of a time domain function $G(t)$ can be denoted as $\tilde{G}(f)$. The Fourier transform can be inverted by the inverse Fourier transform ($\mathcal{F}^{-1}$).

$$\tilde{G}(f) = \mathcal{F}\{G(t)\} = \int_{-\infty}^{\infty} G(t)e^{-2\pi ft} dt \Leftrightarrow G(t) = \mathcal{F}^{-1}\{\tilde{G}(f)\} = \int_{-\infty}^{\infty} \tilde{G}(f)e^{2\pi ft} df \quad (\text{A.1})$$

The Hilbert Transform

The Hilbert transform of a function is described in terms of the Cauchy principle value (*p.v.*) of a convolution integral. Within the scope of this dissertation the Hilbert transform is used in frequency domain to enforce the causal response of linear systems.

$$\mathcal{H}\{G(f)\} = p.v. \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{G(f')}{f - f'} df' \quad (\text{A.2})$$

Convolution

The convolution between any two functions of the variable, $F(x)$ and $G(x)$ is an integral transform denoted by an $*$. The result is a new function that is a function of the variable x .

$$(F(x) * G(x))(x) = \int_{-\infty}^{\infty} F(x - x')G(x')dx' \quad (\text{A.3})$$

The Convolution Theorem

The fourier transform of a product of two functions in time domain is equivalent to the convolution of the fourier transforms of those functions in frequency domain. Likewise, the inverse Fourier transform of a frequency domain product is equivalent to the convolution of the time domain functions.

$$\mathcal{F}\{F(t)G(t)\} = (\tilde{F}(f) * \tilde{G}(f))(f) \Leftrightarrow \mathcal{F}^{-1}\{\tilde{F}(f)\tilde{G}(f)\} = (F(t) * G(t))(t) \quad (\text{A.4})$$

Cross-Correlation

Cross-correlation is another integral transformation between two functions $F(x)$ and $G(x)$ similar to the convolution but denoted by a $\otimes$ and defined slightly differently.

$$(F(x) \otimes G(x))(x) = \int_{-\infty}^{\infty} F^*(x - x')G(x')dx' = \int_{-\infty}^{\infty} F^*(x')G(x' + x)dx' \quad (\text{A.5})$$

Cross-Correlation Theorem

Like convolution, cross-correlation in frequency domain can be written as the Fourier transform of a time domain product, and like wise the time domain cross-correlation can be expressed as an inverse Fourier transform of a frequency domain product.

$$\mathcal{F}\{F^*(t)G(t)\} = \left(\tilde{F} \otimes \tilde{G}(t)\right)(f) \Leftrightarrow \mathcal{F}^{-1}\left\{\tilde{F}^* \tilde{G}(f)\right\} = (F(t) \otimes G(t))(t) \quad (\text{A.6})$$

Where * indicates the complex conjugate. In this case it is critical to note that the conjugate $\tilde{F}^*$ indicates the complex conjugate of the frequency domain representation function $\tilde{F}$. That is $\tilde{F}^* = \mathcal{F}\{F\}^*$ and **NOT** the frequency representation of the complex conjugate ($\mathcal{F}\{F^*\}$).

APPENDIX B

3-LEVEL RATE EQUATIONS CODE

The following code was used to execute simulations of the 3-level rate equations.

Function for Simulating the 3-Level Rate Equations with a Constant Driving Field

This function can be used to estimate the populations in each state as a function of time.

```
function [T_out, pop_out] = Continuous_burn(burn_rate, T, alphaL, T1, ...
... Tb, BR, pop_0)
% Solve Rate Equations for a given burn rate
% (number of photons per frequency bin/number
% of absorbers per frequency bin)

% Step one: Calculate rates From decay times and Branching Ratios
L1=1/T1;
Lbg=1/Tb;
L1b=BR*L1;
L1g=L1-L1b;

% Step two: Define Differential Equation Model
Pop_dot=@(t, pop)[-burn_rate*(1-exp(alphaL*(pop(2,:)-pop(1,:)))) ...
...+L1g*pop(2,:)+Lbg*(1-pop(1,:)-pop(2,:)); ...
... burn_rate*(1-exp(alphaL*(pop(2,:)-pop(1,:))))-L1*pop(2,:)];
% Step Three: Solve Differential Equations
[T_out, pop_out]=ode23( Pop_dot, T, pop_0);
end
```

Script to Calculate Populations of each state as a function of time

This script can be used to generate plots showing the 3 level populations as a function of time under illumination by a field of constant intensity

```
%
close all
clear all
tic

T1=160*10^-6;
Tb=4.5*10^-3;
BR=0.27;
alphaL=1.39;
T=linspace(0,0.01,100000);
populations_0=[1;0];
%parameters to calculate burn rate
h=6.626*10^-34; % Planck's Constant
```

```

c=3*10^8; % Speed of light

% Programing beam parameters
R=165*10^-4; % Spot Radius
R=R*3;
wavelength=793*10^-9; % Wavelength
% Material Parameters
NO=1.89*10^18; % total Number of Absorbers
RSD=0.002*10^-9; % relative spectral density

S_eff=0.15*10^-3/(0.19*10^6);
L=1; % Material length
%calculated parameters
A=pi*R^2; % Beam Area
I1=S_eff/A; % Intensity
U_photon=h*c/wavelength; % Photon energy quantum
Phi1=I1/U_photon; % Incident photon flux density
burn_rate=10^4; % Burn rate
toc
[T_out, pop_out] = Continuous_burn( burn_rate, T, alphaL, T1, Tb, ...
... BR, populations_0);
figure(1)
subplot(3,1,1);
plot(T_out, pop_out(:,1))
title('3_Level_Populations')
ylabel('\rho_{gg}')
figure(1)
subplot(3,1,2)
plot(T_out, pop_out(:,2));
ylabel('\rho_{11}')
ylim([0,1])
figure(1)
subplot(3,1,3);
plot(T_out, 1-pop_out(:,1)-pop_out(:,2));
ylabel('\rho_{bb}')
xlabel('time(s)')
toc

```

Code to Calculate Programming Coefficient

This script can be used to estimate the programming coefficient and 1dB compression point for the material under programming by a constant field after a long duration.

```

close all
clear all

```

```

T1=160*10-6;
Tb=4.5*10-3;
BR=0.27;
alphaL=1.39;
T=linspace(0,1,100000); % create time vector
L=1;
populations_0=[1;0];
%parameters to calculate burn rate
h=6.626*10-34; % Planck's constant
c=3*108; % Speed of light
wavelength=793*10-9;
% Material Parameters
N0=1.89*1018; % total Number of Absorbers
RSD=0.002*10-9; % relative spectral density
Nvf=N0*RSD;
U_photon=h*c/wavelength;

S0=10-7; % 1 mW/(MHz*mm2) in W/(Hz*cm2).
           % This will be the Reference Spectral Intensity

Phi0=S0/U_photon;
LP0=Phi0/(Nvf*L);
atten=-70:10;
atten2=(-90:10);
Sat_depth=[];
toc

pop_l=[];
pop_g=[];

for k=1:length(atten);
    [ T_out, pop_out ] = Continuous_burn(LP0*10^(atten(k)/10),T,...
    ... alphaL,T1,Tb,BR,populations_0);

    Sat_depth(k)=pop_out(end,2)-pop_out(end,1);
    pop_l(k)=pop_out(end,2);
    pop_g(k)=pop_out(end,1);
    clear T_out
    clear pop_out
    toc
end
pop_b=1-pop_l-pop_g;
toc

```

```

transmission=10*log10(exp(alphaL*Sat_depth)-exp(-alphaL)) ...
...-10*log10(1-exp(-alphaL)); % for standard Hole readout (no Mixing)

%fitting the 1dB curve
fittype=@(b,x) x+b; % linear fit
param_guess=[22];
A=pi*0.05^2 %Beam Area
L=1;
N_channel=Nvf*A*L*0.2*10^6;
rho_shot=sqrt(N_channel)/N_channel;

delta_T_shot=10*log10(exp(alphaL*(2*rho_shot-1)) ...
...-exp(-alphaL))/(1-exp(-alphaL));
[fitobject,gof]=fit(atten(1:40)',transmission(1:40)',...
...fittype,'startpoint',param_guess);
figure(1)
plot(atten,transmission); hold on
title('Saturation_Curve_for_0.01%_Tm^{3+}:LiNbO_3')
xlabel('Spectral_Intensity_(dB_to_1_mW/mm^2_MHz)')
ylabel('Change_in_Transmission_(dBfs)')
hold off

figure(2)
plot(atten,pop_g,'x'); hold on
plot(atten,pop_l,'o'); hold on
plot(atten,pop_b,'*'); hold on
title('Steady_State_Populations')
xlabel('Spectral_Intensity_(dB_to_1_mW/mm^2_MHz)')
ylabel('Populations')
legend('\rho_{gg}','\rho_{11}','\rho_{bb}')
hold off

figure(3)
plot(atten,transmission); hold on
plot(atten2,fitobject(atten2));
plot(atten2,ones(size(atten2))*delta_T_shot)
title('Saturation_Curve_for_0.01%_Tm^{3+}:LiNbO_3')
xlabel('Spectral_Intensity_(dB_to_1_mW/mm^2_MHz)')
ylabel('Change_in_Transmission_(dB)')
legend('Relative_Transmission','Linear','Shot_Noise')
hold off

figure(4)
plot(atten,transmission-fitobject(atten'))); hold on
title('Compression')

```

```

xlabel('Spectral_Intensity_(dB_to_1_mW/mm^2_MHz)')
ylabel('compression(dB)')

hold off
%Compare results to linear model to identify 1dB compression:

L1=1/T1;
Lbg=1/Tb;
L1b=BR*L1;
L1g=L1-L1b;
RATES=10.^(atten/10)*LP0;

rho_11= RATES*(1-exp(-alphaL))./(L1);
rho_bb=(L1b/Lbg)*rho_11;
s_gg=rho_11+rho_bb;

R3=(rho_11+s_gg-1);

TR2=exp(alphaL*R3)-exp(-alphaL);

figure
plot(atten,10*log10(TR2)-10*log10(exp(1-alphaL))); hold on
plot(atten,transmission);
ylim([-50,0])
xlabel('Spectral_Intensity_(dB_to_1_mW/mm^2_MHz)')
ylabel('Change_in_Transmission_(dBfs)')
title('Comparison_with_Approximate_Solution')
legend('Aproximate_Solution','Numerical_Solution')
hold off

% Error Figures for Steady State Solutions
%Calculate all Rates
L1=1/T1;
Lbg=1/Tb;
L1b=BR*L1;
L1g=L1-L1b;
%Solve for population rate of change
burn_rate=(LP0*10.^(atten/10));
rho_gg_dot=-burn_rate.*(1-exp(alphaL*(pop_1-pop_g)))...
...+L1b*pop_1+Lbg*pop_b;
rho_11_dot=burn_rate.*(1-exp(alphaL*(pop_1-pop_g)))-L1*pop_1;
rho_bb_dot= L1b*pop_1-Lbg*pop_b;
% Plot population rates of change
figure
plot(atten,rho_gg_dot,'x'); hold on

```

```

plot(atten, rho_11_dot, '*')
plot(atten, rho_bb_dot, '+')
xlabel('Spectral Intensity (dB to 1 mW/mm2 MHz)')
ylabel('Population Rate of Change')
title('Steady State Rate of change')
hold off

```


APPENDIX C

MAXWELL BLOCH SIMULATION CODE

These functions and scripts are the elements used for Maxwell-Bloch simulations.

Core Simulator Elements

The core Maxwell Bloch Simulator consists of several functions that are used to execute Maxwell Bloch simulations.

Vectorized Bloch Equations

This function describes the Bloch equations for all detunings at once as a large vector.

```
function dr = bloch_eqs_vec(r, rabi, detuning, t, T1, T2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Bloch Eqs gives the time derivative of the Bloch Vector
% for ions resonant at some detuning from the central
% resonance, given a perturbing field
% rabi at time t and current Bloch vector r.
% This model also includes decoherence (T2) and
% Population Decay(T1)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Initialization Stuff
drdt = zeros(size(r));
rabi_eval = rabi(t);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 1.) First Bloch equation: includes coherence decay
drdt(1:3:end) = detuning.*r(2:3:end)-imag(rabi_eval).*r(3:3:end) ...
... -(1/T2)*r(1:3:end);
%% 2.) Second Bloch equation: includes coherence decay
drdt(2:3:end) = -detuning.*r(1:3:end)+real(rabi_eval).*r(3:3:end) ...
... -(1/T2)*r(2:3:end);
%% 3.) Third Bloch equation: includes population decay
drdt(3:3:end) = imag(rabi_eval).*r(1:3:end) ...
... -real(rabi_eval).*r(2:3:end)-(0.5/T1)*(r(3:3:end)-1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
dr = drdt; %% Define Final Output
end
```

Bloch Equation Simulator

This function will simulate the Bloch equations

```
function [T,R_out1, R_final] = bloch_sim(R0,TSPAN,FREQS,RABI,T1,T2)
%% [T,R_out1,l, R_final] = bloch_sim(R0,TSPAN,FREQS,RABI,BW)
%% Solves The bloch equations for an inhomogeneously broadened medium
%% with bandwidth BW perturbed by a field RABI, stimulating
frequencies
```

```

%%% FREQS for the times defined by TSPAN beginning with the Initial
      Bloch
%%% vector conditions R0
%%% R_final gives final Bloch vector positions for each detuning
      allowing
%%% for future instantiation of the simulator
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% NOTE:
%%% Previous Versions Contained Provisions for a non-flat lineshape
%%% I have left Some of the code in nessicary for this but it will
%%% be commented out
%%% so that the simulation can be executed Slightly Faster
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% 1.) Execute Solver
OPTIONS=odeset('RelTol', 1e-9, 'Abstol', 1e-9);

if size(R0) == [1, 3]
R0 = repmat(R0, length(FREQS), 1);
end

ODEFUN=@(t,x) bloch_eqs_vec(x, RAB1, FREQS', t, T1, T2);
%%% Define Function For Solver to execute

[T, R_gen]=ode45(ODEFUN,TSPAN,R0,OPTIONS);
% Use ODE Solver to Solve the Bloch-equations
%[T, R_gen]=ode23(ODEFUN,TSPAN,R0);
% Use ODE Solver to Solve the Bloch-equations
% At all detunings, currently set up to
% use ode45 as the solver method others
% should work well also (ODE23 is
% preferred in some circumstances)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Make changes here For Inhomogeneous Lineshape Code
%g=@(w)exp(-(w/BW).^2); % Simulate inhomogeneous broadening
%g=@(w) 1+0*w; % flat spectrum
%g_mat=repmat(g(FREQS)',length(TSPAN),1);
%%% match dimensions so we can multiply by the Bloch vector
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% 2.) Integrate Over Detunings
g=ones(size(FREQS));
%flat inhomogeneous lineshape (see above provisions for other methods)
g_mat = repmat(g, length(TSPAN), 1);
% match dimensions so we can multiply by the Bloch vector
F_step = abs(FREQS(2) - FREQS(1));
% define frequency increment

```

```

%% Execute Integration over lineshapes
%% Execute Solution sums for each Bloch vector component
%% i.) R1 Bloch sphere component (Imaginary coherence)
R1=sum(R_gen(:, 1:3:end) .* g_mat, 2) * F_step;
%% ii.) R2 Real Bloch sphere component (Real coherence)
R2=sum(R_gen(:, 2:3:end) .* g_mat, 2) * F_step;
%% iii.) R3 Bloch sphere component (Population)
R3=sum(R_gen(:, 3:3:end) .* g_mat, 2) * F_step;
% this Is the least useful one may want to strip out in the future
%% 3.) Shape everyhting into a 3 element array at the output
%% Detuning Integrated bloch sphere components
R_out(1, :) = R1;
R_out(2, :) = R2;
R_out(3, :) = R3;
R_out1 = R_out';
%% Final Populations at each detuning
R_final = R_gen(end, :);
end

```

Maxwell Bloch Simulator

This function is used to simulate the Maxwell-Bloch Equations

```

function [ T, E, R_final_spatial ] = maxwell_bloch_sim(R0, TSPAN,...
... FREQS,RABI, ALPHA, N_slices, L, T1, T2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Core Maxwell Bloch Simulation: Evaluates Field Corrections
%%due to bloch sphere effects
%% Simulates Maxwell Bloch Equations for an inhomogeneously
%%broadened material with Bandwidth BW, Absorbtion Alpha,
%%Length L, divided into N_slices, by simulating frequency
%%channels defined by Freqs, for times TSPAN beginning from
%%initial Bloch vector conditions R0
FREQS = -2 * pi * FREQS; % converting freqs from Hz to angular
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 1.) Define initial Parameters
%% a.) Initialize Function Handles for fields
F = @(t) 0; % Initialize function To describe Fields
% This anonymous Function defines field corrections due to the
% interaction with the Bloch sphere
% Before the first slice this should be set to zero
OMEGA = @(t) RABI(t); % Field is described as a rabi frequency
% This is a separate variable because the simulation
% will modify this function after each slice
%% b.) Misc
dz = L / N_slices; % create spatial horizontal slice size

```

```

U = zeros(N_slices , 3 * length(FREQS)); % Creates Dummy array
% (This variable will contain temporary population data)
%% 2.) Core simulation Loop
for j=1:N_slices % Iterate to find and apply corrective terms
    %% a.) Initialize bloch vector initial conditions on a
    %% per slice basis
    R_in = R0(:, j)'; % Define Bloch Sphere initial Conditions
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %% b.) Execute Bloch Equation Simulation to calculate Bloch
    %% Sphere coordinates as a function of time
    %% EXECUTE BLOCH SIMULATION HERE:
    [T, R, R_final] = bloch_sim(R_in, TSPAN, FREQS', OMEGA, T1, T2);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %% c.) Calculate field corrections
    %%% Step 1: Calculate real and Imaginary field Corrections
    % Real field corrections from R values
    dF1 = (ALPHA / (2 * pi)) * R(:, 2);
    % Imaginary field corrections from R values
    dF2 = -(ALPHA / (2 * pi)) * R(:, 1);
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %% d.) Apply field corrections
    % add corrections to omega as a vector
    OMEGA_VEC = OMEGA(T) + dz*(dF1 + i*dF2);
    pp1 = pchip(T, OMEGA_VEC); % interpolate the corrected field
    % create function handle for interpolated field
    OMEGA = @(t) ppval(pp1, t);
    U(j, :) = R_final; % store R_final for each slice
end
E = OMEGA(T); %Sample corrected field function
R_final_spatial = U;%Take final grating array
end

```

Pulse Shapes

These pulse shapes functions are useful for generating pulses in Maxwell-Bloch simulations

Hard Edged Pulses

This function describes hard-edged pulses.

```

function [ y ] = hard_edges (x, f, xmin, xmax )
%This function basically applies a hard window so that
%for values less than min and values greater than
%max the function is zero, if min<x<max
% The function will simply return f

```



```

        0, sqrt(2), 0;
        i, 0, 1];
Q3. Bias=@(x,y,z) [exp(i*x), 0, 0;
        0, exp(i*y), 0;
        0, 0, exp(i*z)];
Q3. Read_in=Q3.lower*Q3.upper;
Q3. Read_out= @(x,y,z) Q3.upper*Q3.lower*Q3.Bias(x,y,z);
Q3. Read_full= @(x,y,z) Q3. Read_out(x,y,z)*Q3. Read_in;

% Initializes Q4 a struct which contains 4x4 Matrix
% Operators to describe 4 port network
Q4. Splitter_upper=sqrt(1/2)*[1, i, 0, 0;
        i, 1, 0, 0;
        0, 0, sqrt(2), 0;
        0, 0, 0, sqrt(2)];
Q4. Splitter_lower=sqrt(1/2)*[sqrt(2), 0, 0, 0;
        0, sqrt(2), 0, 0;
        0, 0, 1, i;
        0, 0, i, 1];
Q4. Splitter_center=sqrt(1/2)*[sqrt(2), 0, 0, 0;
        0, 1, i, 0;
        0, i, 1, 0;
        0, 0, 0, sqrt(2)];
Q4. Splitter_outer=sqrt(1/2)*[1, 0, 0, i;
        0, sqrt(2), 0, 0;
        0, 0, sqrt(2), 0;
        i, 0, 0, 1];
Q4. bias=@(x,y,z) [1, 0, 0, 0;
        0, exp(i*x), 0, 0;
        0, 0, exp(i*y), 0;
        0, 0, 0, exp(i*z)];
Q4. if4port_first= Q4. Splitter_upper*Q4. Splitter_lower ...
...*Q4. Splitter_center*Q4. Splitter_outer;
Q4. if4port_second=Q4. Splitter_center*Q4. Splitter_outer*...
..Q4. Splitter_upper*Q4. Splitter_lower;
Q4. if4port= @(x,y,z) Q4. if4port_first*Q4. bias(x,y,z)*Q4. if4port_second;

```

Sample JSON File

The Maxwell-Bloch simulation routines use JSON files to define key parameters, most aspects of the simulations can be altered by changing one of these files. Here is an included JSON file, this particular file is used for chirped readout of a spectral hole.

```

{
    "time_frequency":{
        "frequency_resolution": 0.025,

```

```

    "bandwidth_material": 400,
    "safety_factor_frequency_resolution": 1,
    "safety_factor_bandwidth": 2,

    "t_dead": 1,
    "t_cooldown": 0,
    "t_start": 0,
    "t_duration": 40
},
"material": {
    "ALPHA": 1.39,
    "L": 1,
    "n_slices": 10,
    "T1": 10e6,
    "initial_populations": {
        "grating_delay": 0,
        "match_grating": false,
        "round_grating": false,
        "f_0_hole": 0,
        "spectrum_depth": 0.5,
        "spectrum_offset": 0,
        "r3_min": -1,
        "Raise_N": 20,
        "spectrum_type": "hole"
    }
},

"interferometer": {
    "reference_around_xtal": false,
    "insertion_port": "P1",
    "LO_gain_parameter": 1,
    "method": "traditional",
    "n_ports": 2,
    "total_attenuation": 0.05
},

"input_field": {
    "round_chirp": false,
    "calibrate_rabi": false,
    "read_chirp_rate": 5,
    "freq_0": -100,
    "power_level": [0],
    "base_power_level": 1e-3,
    "calibration_power": 15e-3,
    "calibration_rabi": 1.5,

```



```

    "calibration_spot": 0.3e-3,
    "experimental_spot": 1e-3,
    "samples_to_round": 300,
    "phase_noise_properties": {
        "seed_value": 1,
        "linewidth": 0.02,
        "simulate_readout_phase_noise": false
    }
},
"detector": {
    "model_detector": false,
    "responsivity": 0.53,
    "transimpedance_gain": 0.5e5,
    "R_load": 1,
    "Noise_floor_dB": 0,
    "ADC_units": 1e3
},
"processing": {
    "match_output_power": true,
    "subtract_mean": false,
    "process_fullblock": false,
    "simulate_detection_noise_floor": false,
    "simulate_shot_noise_floor": false,
    "simulate_added_attenuation": false,
    "show_total_grating": true,

    "apply_spectral_recovery": true,
    "subtract_background": false,
    "show_population_spectrum": true,
    "display_time_domain_anatomy": true,
    "window_on": false,
    "window_method": "hanning"
},
"options": {
    "quickrun": false,
    "save_data": false
}
}

```

Simulation Routines

These Simulation routines are used to simulate different types of SSH processes. These routines take JSON files as inputs in order to set key parameters and select key options

Single Port Simulator

This simulation is used to simulate echoes and nutation in a single co-linear path.

```
% Initialization stuff
close all
clear all
experiment_name='ECHO_DEMO_1';
experiment_file=[experiment_name, '.json'];
%check OS type then execute path dependent
%commands based on OS type
if ispc
% for Windows Machines
addpath('.\core_simulator');
addpath('.\auxillary_functions');
addpath('.\Pulse_shapes');
experiment = [ '\', experiment_file ] ;
base_directory='.\';
else
% for Linux/Unix/Mac Machines
addpath('./core_simulator');
addpath('./auxillary_functions');
addpath('./Pulse_shapes');
experiment = [ './', experiment_file ];
base_directory='./';
end
% Initialize Interferometric Operators
init_if_jones_operators
%%% Import json spec defining the experiment
fid = fopen(experiment);
json_raw = fread(fid,inf);
json_str = char(json_raw ');
fclose(fid);
experiment = jsondecode(json_str);
clear fid json_raw json_str
J=clock;
tic %start the clock
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% time-frequency setup %%%%%%%%%%
% Bandwidth and Resolution
% Frequency Resolution of material(MHz)
frequency_resolution = experiment.time_frequency.frequency_resolution;
% Bandwidth of the material (MHz)
% (Full width: positive and negative Detunings)
Bandwidth_material = experiment.time_frequency.bandwidth_material;
% Safety Factor for FFT limited Resolution
safety_factor_frequency_resolution =
experiment.time_frequency.safety_factor_frequency_resolution;
```



```

% Determine if detector properties are modeled
model_detector = experiment.detector.model_detector;
%Detector Rabi Responsivity A/MHz^2;
responsivity = experiment.detector.responsivity;
% Detector Transimpedence_gain V/A
transimpedence_gain = experiment.detector.transimpedence_gain;
% Load Impedence
R_load = experiment.detector.R_load; % Load Impedence
Noise_floor_dB = experiment.detector.Noise_floor_dB; % Noise Floor in dB
% ADC uses mV instead of Volts
ADC_units = experiment.detector.ADC_units;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Processing setup %%%%%%%%%%
% simulate a detector noise floor
simulate_detection_noise_floor = ...
... experiment.processing.simulate_detection_noise_floor;
%Simulate Shot noise at detector (Work in progress)
simulate_shot_noise_floor = experiment.processing.
    simulate_shot_noise_floor;
% calculate full depth grating: i.e. average gratings in all slices
show_total_grating = experiment.processing.show_total_grating;
show_population_spectrum = experiment.processing.
    show_population_spectrum;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%          Prepare For Core LOOP          %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate FFT resolution(and therefore Experiment Duration)
Resolution_FFT=frequency_resolution/safety_factor_frequency_resolution;
% calculate Nyquist
Nyquist_rate=(Bandwidth_material/2)*safety_factor_bandwidth;
% Sample Rate Based on Nyquist Frequency (samples per microsecond/MHz)
Sample_rate=2*Nyquist_rate;
t_duration_full=t_duration;
% Process parameters
t_start=0;
time_block_size=Sample_rate*t_duration_full; % calculate time block size
time_step=(1/Sample_rate); % time Step in Microseconds
t_stop=t_duration_full-time_step; % end time in Microseconds

t0=t_start:time_step:t_stop; % Time Vector (microseconds)
DELTA=(-Bandwidth_material/2:frequency_resolution:(Bandwidth_material
    /2-frequency_resolution))'; % Frequency Vector (detunings, MHz)
freqs=(-Nyquist_rate):Resolution_FFT:(Nyquist_rate-Resolution_FFT); %
    Frequency Vector (frequency Domain Representation, MHz)

```

```

%% 2: Material Parameters
alpha(1)=ALPHA; % Field 1 Sees the material
if calibrate_rabi
% Experimental mode area
    Experimental_area=pi*(Experimental_spot/2)^2;
    % Calibration mode Area
    Calibration_area=pi*(Calibration_spot/2)^2;
    % Calibrated Base Rabi (for 0dBm of optical Power)
    base_rabi=sqrt((Calibration_area/Experimental_area)*...
        ... (Base_power_level/(2*Calibration_power)))* calibration_rabi;
else
    % Starting Rabi condition is 1MHz If power
    % calibration Is not specified
    base_rabi = 1;
end
%% 3: Fields
switch lower(pulse_type)
case 'hard' %Hard Edge pulse
    f_0=@(x) 0.*x+1;
    pulse= @(t,tau,A) raw_rabi*hard_edges(t,f_0,tau,tau+A/raw_rabi);
case 'gaussian' % Gaussian Pulse
    pulse=...
        ...@(t,tau,A) raw_rabi*gaussian_pulse(t,tau,A/(raw_rabi*sqrt(2*pi)))
        );
otherwise
    error("invalid value for pulse type")
end

switch lower(pattern_type)
case 'echo' % 2 Pulse Echo
    OMEGA_0= @(t) pulse(t,TP1,pi/2)+pulse(t,TP2,pi)
case 'spe' % Stimulated Echo
    OMEGA_0= @(t) pulse(t,TP1,pi/2)+pulse(t,TP2,pi/2)+pulse(t,TP3,pi/2)
case 'nutatation' % Nutation
    f_0=@(x) 0.*x+1;
    OMEGA_0= @(t) raw_rabi*hard_edges(t,f_0,2,t_stop)
case 'barker' % Barker Correlation Pattern
    t_interval=0.5;
    patern= @(t,tau) pulse(t,tau,pi/2)+pulse(t,tau+t_interval,pi/2)+...
        ... pulse(t,tau+2*t_interval,pi/2)-pulse(t,tau+3*t_interval,pi/2)...
        ...+ pulse(t,tau+4*t_interval,pi/2);
    OMEGA_0= @(t) patern(t,TP1)+pulse(t,TP2,pi/2)+patern(t,TP3);
    spectrum_freq=fourier_frequencies(SampleRate, N);
    FILT_SIGNAL=abs(ifft(fft(F0(T_0)).*g(spectrum_freq)'));

```

```

otherwise
    error("invalid value for pattern type")
end

PORTS = eye(n_ports);
%% ii.) Edge rounding (option) [Highly Recomendted Option]

if round_chirp
    rounding_chirp = ...
    ... @(t) rounded_edge_with_dead_time(t_dead, t_duration, Sample_rate
    , ...
    ... samples_to_round, t_start, t'); % apply Rounding to front edge of
    chirp
else
    rounding_chirp = @(t) t'.^0;
end

if soften_pulse
    sigma=10;
    g=@(w)exp(-(w/(sigma)).^2);
    spectrum_freq=fourier_frequencies(Sample_rate, length(t0));
    % generate causal filter shape
    FILT_SIGNAL=ifft(fft(OMEGA0(t0)).*hilbert(g(spectrum_freq')));
    GP=pchip(t0, FILT_SIGNAL);
    OMEGA0 = @(t) ppval(GP, t);
end

initial_pop=r3_min*ones(size(DELTA));
% Generate Array of Start Points
bloch_vec_A=vertcat(zeros(size(initial_pop')), ...
... zeros(size(initial_pop')), initial_pop');
% Reshape Array of Startpoints
bloch_vec_B=reshape(bloch_vec_A, [(3*(length(DELTA))), 1]);
R00=[0;0;-1]; % Unprogramed Initialization Vector
% Programed Channel 1
VEC_ch1= repmat(bloch_vec_B, 1, n_slices);
% Unprogramed Channel 2
VEC_ch2=create_initial_array(R00, length(DELTA), n_slices);

R_all(1, :, :)=VEC_ch1; % Create Array for all Bloch Vectors For all
channels
R_all(2, :, :)=VEC_ch2; %

input_network=1;
Fr=@(t) OMEGA0(t)';

```

```

Fr2=@(t) input_network*Fr(t);
PORTS = eye(n_ports);
Pop_out=[];
Eb_out=[];

for n=1:n_ports % for loop to run Maxwell-Bloch on each channel
    toc
    OMEGA2= @(t) transpose(PORTS(n,:)*Fr2(t));
    R_in=reshape(R_all(n,:,:),[3*(length(DELTA)),n_slices]);
    [T, Eb, Rb_final_spatial] = maxwell_bloch_sim(R_in ,t0 , DELTA,
        OMEGA2,...
        ... alpha(n), n_slices , L, T1, T2);% Run simulation
    toc
    Eb_out(n,:)=Eb; % isolate Field out
    if show_total_grating
        % Isolate Grating
        Pop_out(n,:)=mean(Rb_final_spatial(:, 3:3:end), 1);
    else
        Pop_out(n,:)=Rb_final_spatial(1, 3:3:end);
    end
    toc
end
Grating_out=Pop_out(1,:);% Grating at the end of the simulation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plot Outputs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1); clf
plot(t0,abs(OMEGA0(t0)).^2,'LineWidth',2);
xlabel('Time_\mu s','FontSize',14)
ylabel('Input_Intensity_(MHz^2)','FontSize',14)
title('Input_Fields','FontSize',14)

figure(2); clf
plot(t0,abs(Eb_out).^2,'LineWidth',2);
xlabel('Time_\mu s','FontSize',14)
ylabel('Output_Intensity_(MHz^2)','FontSize',14)
title('Input_Fields','FontSize',14)

figure(3); clf
plot(t0,10*log10(abs(Eb_out).^2),'LineWidth',2);
xlabel('Time_\mu s','FontSize',14)
ylabel('Output_Intensity_(dB_MHz)','FontSize',14)
title('Input_Fields','FontSize',14)

```

```

figure(4);clf % Grating before and After
plot(DELTA,initial_pop,'LineWidth',1); hold on;
plot(DELTA,Grating_out,'LineWidth',1); hold off
legend('Initial_Population','Final_Population')
xlabel('Detuning(MHz)','FontSize',14)
ylabel('r_3');
title('Gratings','FontSize',14)
ylim([-1,1])

figure(5);clf
plot(t0,abs(OMEGA0(t0)).^2,'LineWidth',1); hold on
plot(t0,abs(Eb_out).^2,'LineWidth',1);hold off
xlabel('Time_\mu s','FontSize',14)
ylabel('Intensity_(MHz^2)','FontSize',14)
title('Photon_Echoes','FontSize',14)
legend({'Input_Fields','Output_Fields'})

```

Two Port Readout Simulator

This simulation routine can be used to simulate chirped readout of a pre-programmed spectrum either using the traditional balanced detection method or the interferometric method.

```

% Initialization stuff
close all
clear all
experiment_name='demo_readout_for_thesis_2';
experiment_file=[experiment_name,'.json'];
%check OS type then execute path dependent
%commands based on OS type
if ispc
% for Windows Machines
addpath('.\core_simulator');
addpath('.\auxillary_functions');
addpath('.\Pulse_shapes');
experiment = ['./',experiment_file] ;
base_directory='.';
else
% for Linux/Unix/Mac Machines
addpath('./core_simulator');
addpath('./auxillary_functions');
addpath('./Pulse_shapes');
experiment = ['./',experiment_file] ;

```



```

base_directory='./';
end
% Initialize Interferometric Operators
init_if_jones_operators
%%% Import json spec defining the experiment
fid = fopen(experiment);
json_raw = fread(fid,inf);
json_str = char(json_raw');
fclose(fid);
experiment = jsondecode(json_str);
clear fid json_raw json_str
J=clock;
tic %start the clock
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% time-frequency setup %%%%%%%%%%
% Bandwidth and Resolution
% Frequency Resolution of material(MHz)
frequency_resolution = experiment.time_frequency.frequency_resolution;
% Bandwidth of the material (MHz)
% (Full width: positive and negative Detunings)
Bandwidth_material = experiment.time_frequency.bandwidth_material;
% Safety Factor for FFT limited Resolution
safety_factor_frequency_resolution =
experiment.time_frequency.safety_factor_frequency_resolution;
% safety factor for Bandwidth (enforce that signal Nyquist is
% greater than material bandwidth)
safety_factor_bandwidth=...
...experiment.time_frequency.safety_factor_bandwidth;
t_duration = experiment.time_frequency.t_duration;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%material setup%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Minimum Value For the grating is -1
r3_min = experiment.material.initial_populations.r3_min;
%%% Note that the choice of spectrum type will affect
%%% the post-processing
spectrum_type = experiment.material.initial_populations.spectrum_type;
ALPHA = experiment.material.ALPHA; % absorption Length (cm-1)
L = experiment.material.L; % Sample length (cm)
n_slices = experiment.material.n_slices; % number of simulation Slices
T1 = experiment.material.T1; % Material population lifetime
% delay (for Grating)
grating_delay = experiment.material.initial_populations.grating_delay;
% hole location for a spectral hole;
f_0_hole = experiment.material.initial_populations.f_0_hole;
% grating depth(0 to 1)
spectrum_depth = experiment.material.initial_populations.spectrum_depth
;

```

```

% grating offset(0 to 1)
spectrum_offset = experiment.material.initial_populations .
    spectrum_offset;
% minimum value For the grating is -1
r3_min = experiment.material.initial_populations.r3_min;
Raise_N = experiment.material.initial_populations.Raise_N;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Interferometer Setup %%%%%%%%%%
% Take One Field around the Material without evaluating Maxwell Bloch
Reference_around_xtal=experiment.interferometer.reference_around_xtal;
METHOD = experiment.interferometer.method; % Configuration
n_ports = experiment.interferometer.n_ports;
insertion_port = experiment.interferometer.insertion_port;
% LO gain Parameter (only aplicable for interferometric readout)
LO_gain_parameter = experiment.interferometer.LO_gain_parameter;
total_attenuation = experiment.interferometer.total_attenuation;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% input_field Setup %%%%%%%%%%
% Chirp rounded using a function with a sine edge
round_chirp = experiment.input_field.round_chirp;
% Determine if rabi properties are simulated.
calibrate_rabi = experiment.input_field.calibrate_rabi;
% apply Phase noise to the chirp field
simulate_readout_phase_noise = experiment.input_field .
    phase_noise_properties.simulate_readout_phase_noise;
% Laser Linewidth (MHz)
linewidth = experiment.input_field.phase_noise_properties.linewidth;
%Read Rate in MHz/uS
read_chirp_rate = experiment.input_field.read_chirp_rate;
% start Frequency
freq_0 = experiment.input_field.freq_0;
% Experiment Power Level
power_level = experiment.input_field.power_level;

if calibrate_rabi
% Experimental mode area
    Experimental_area=pi*(Experimental_spot/2)^2;
    % Calibration mode Area
    Calibration_area=pi*(Calibration_spot/2)^2;
    % Calibrated Base Rabi (for 0dBm of optical Power)
    base_rabi=sqrt((Calibration_area/Experimental_area)*...
        ... (Base_power_level/(2*Calibration_power)))* calibration_rabi;
else
% Starting Rabi condition is 1MHz If power
% calibration Is not specified
    base_rabi = 1;
end

```

```

samples_to_round = experiment.input_field.samples_to_round;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% detectors Setup %%%%%%%%%%
% Determine if detector properties are modeled
model_detector = experiment.detector.model_detector;
%Detector Rabi Responisivity A/MHz^2;
responsivity = experiment.detector.responsivity;
% Detector Transimpedence_gain V/A
transimpedence_gain = experiment.detector.transimpedence_gain;
% Load Impedence
R_load = experiment.detector.R_load; % Load Impedence
Noise_floor_dB = experiment.detector.Noise_floor_dB;% Noise Floor in dB
% ADC uses mV instead of Volts
ADC_units = experiment.detector.ADC_units;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% processing Setup %%%%%%%%%%
% Match Field powers at the output
match_output_power = experiment.processing.match_output_power;
% subtract off the means of signals (recomended)
subtract_mean = experiment.processing.subtract_mean;
% Process the entire block of data (not recomended)
process_fullblock = experiment.processing.process_fullblock;
% Apply window of specified type to processing

% signal(recomended for gratings)
window_on = experiment.processing.window_on;
% simulate a detector noise floor
simulate_detection_noise_floor = experiment.processing.
    simulate_detection_noise_floor;
%Simulate Shot noise at detector (Work in progress)
simulate_shot_noise_floor = experiment.processing.
    simulate_shot_noise_floor;
% calculate full depth grating: i.e. average gratings in all slices
show_total_grating = experiment.processing.show_total_grating;
% apply spectral recovery algorithm
apply_spectral_recovery = experiment.processing.apply_spectral_recovery
;
% Apply background subtraction(relevant for time domain SNR calculation
)
subtract_background = experiment.processing.subtract_background;
window_method = eval("@"+experiment.processing.window_method);
show_population_spectrum = experiment.processing.
    show_population_spectrum;
Display_time_domain_anatomy =...
... experiment.processing.display_time_domain_anatomy;
simulate_added_attenuation = ...

```

```

... experiment.processing.simulate_added_attenuation;

%%% Calculated Parameters:
% Coherence time based on frequency resolution
T2=1/(4*pi*frequency_resolution);
% Calculate FFT resolution(and therefore Experiment Duration)
Resolution_FFT=frequency_resolution/safety_factor_frequency_resolution;
% calculate Nyquist
Nyquist_rate=(Bandwidth_material/2)*safety_factor_bandwidth;
% Sample Rate Based on Nyquist Frequency (samples per microsecond/MHz)
Sample_rate=2*Nyquist_rate;
%Experiment Duration Properties
% Experiment Duration Based on Resolution (microseconds)
t_duration_full=t_duration+t_dead+t_cooldown;
% Process parameters
time_block_size=Sample_rate*t_duration_full; % calculate time block size
time_step=(1/Sample_rate); % time Step in Microseconds
t_stop=t_start+t_duration_full-time_step; % end time in Microseconds

%%% Time and Frequency Vectors
t0=t_start:time_step:t_stop; % Time Vector (microseconds)
% Frequency Vector (detunings, MHz)
DELTA=...
(-Bandwidth_material/2:frequency_resolution ...
...:(Bandwidth_material/2-frequency_resolution))';
% Frequency Vector (frequency Domain Representation, MHz)
freqs=(-Nyquist_rate):Resolution_FFT:(Nyquist_rate-Resolution_FFT);

%%% 2: Material Parameters
if Reference_around_xtal==1
    alpha(1)=ALPHA; % Field 1 Sees the material
    alpha(2)=0;      % Field 2 does not
    %atten_field=[1,0;0,exp(-ALPHA*L/2)];
    atten_field=[1,0;0,1];
else
    alpha(1)=ALPHA;
    alpha(2)=ALPHA;
    atten_field=[1,0;0,1];
end

%%% 3: Interferometric Setup
%%% Define Ports
PORTS = eye(n_ports);
P1 = PORTS(:, 1);
P2 = PORTS(:, 2);
%%% Define Network Elements for Desired Configuration

```

```

insertion_port=eval(insertion_port);
if strcmp(METHOD,'Interferometric')
% Define Matrix Elements for a Mach-Zehnder Interferometer
input_network=Q2.splitter;% 50-50 Splitter at Input
% 50-50 Splitter at Input
% with interferometric bias
% (and controllable LO gain)
output_network=Q2.splitter*Q2.bias(pi/2)*...
...[1,0;0,sqrt(LO_gain_parameter)]*atten_field;
else
input_network=Q2.splitter;% 50-50 Splitter at Input
output_network=Q2.bias(0)*atten_field;% Pass Through
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 4.) Phase Noise Properties
if simulate_readout_phase_noise
% Calculate Phase Noise Coefficient
phase_noise_level=sqrt(2*pi*linewidth/Sample_rate);
% Generate Phase Noise Vector
phi_noise=phase_noise_level*cumsum(randn(1,length(t0)));
% Interpolate phase noise
Noise_spline=spline(t0,phi_noise);
% Generate function to describe the noise
phi_func=@(t)ppval(Noise_spline,t);
else
phi_func=@(t) 0*t.^0;% maintains the same shape as t
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 5: Field Parameters
% Chirp Field Properties: Universal
if calibrate_rabi
% Experimental mode area
Experimental_area=pi*(Experimental_spot/2)^2;
% Calibration mode Area
Calibration_area=pi*(Calibration_spot/2)^2;
% Calibrated Base Rabi (for 0dBm of optical Power)
base_rabi=sqrt((Calibration_area/Experimental_area)*...
...(Base_power_level/(2*Calibration_power)))*calibration_rabi;
else
% Starting Rabi condition is 1MHz If power
% calibration Is not specified
base_rabi = 1;
end
%% ii.) Edge rounding (option) [Highly Recomendaded Option]

```

```

%Raise_N=1;
if round_chirp
    % apply Rounding to front edge of chirp
    rounding_chirp = @(t) rounded_edge_with_dead_time(t_dead ,...
... t_duration , Sample_rate , samples_to_round , t_start , t ');
else
    rounding_chirp= @(t) t' .^ 0;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% 6: Initial Material Populations
%%% Select Parameters (Depth and Delay or location
%%%depending on type of signal) Grating parameters
%%%if match_grating
%% Spectral Hole Parameters
L0=1/(T2*pi); % Lorentzian Hole width
%% Common Parameters
%% Generate Populations
if round_grating
    N_raised_cosine_points=length(DELTA);
    PHLGRATING_dummy=linspace(-pi,pi,N_raised_cosine_points);
    Grating_round_off=((0.5*(1+cos(PHLGRATING_dummy)).^(1/Raise_N)))';
else
    Grating_round_off=1;
end
switch lower(spectrum_type)
case 'grating' % Spectral Grating
    initial_pop=spectrum_depth*Grating_round_off.*...
... (1+cos(2*pi*DELTA*grating_delay))+r3_min+(spectrum_offset*2);
case 'hole' % Spectral Hole
    initial_pop=spectrum_depth*Grating_round_off.*...
... (0.5*L0)^2./((0.5*L0)^2+(DELTA-f_0_hole).^2)+r3_min;
case 'flat' % Flat Spectrum
    initial_pop=r3_min*ones(size(DELTA))+spectrum_offset;
otherwise
    error("invalid value for ...
... material.initial_populations.spectrum_type")
end
%%% Generate Initial Populations for both ports
% Saturation Level Of Grating As a Percentage
saturation_level=100*0.5*(max(initial_pop)+1);
% Generate Array of Start Points
bloch_vec_A=vertcat(zeros(size(initial_pop')),...
... zeros(size(initial_pop'),initial_pop'));
% Reshape Array of Startpoints
bloch_vec_B=reshape(bloch_vec_A,[(3*(length(DELTA))),1]);

```

```

%% Loop Execution
%%% Initialization
if ~quickrun
    pump_parameter=zeros([1,1]);
    Signal_level=zeros([1,1]);
    SNR=zeros([1,1]);
    Noise_level=zeros([1,1]);
    all_signals=zeros([1,2,length(t0)]);
else
    power_levels=power_level(1);
    pump_parameter=zeros([1,length(power_levels)]);
    Signal_level=zeros([1,length(power_levels)]);
    SNR=zeros([1,length(power_levels)]);
    Noise_level=zeros([1,length(power_levels)]);
    all_signals=zeros([length(power_levels),2,length(t0)]);
end
Eb_out_all=zeros([length(power_levels),2,length(t0)]);
E_injected_all=zeros([length(power_levels),2,length(t0)]);
all_signals_modified=zeros([length(power_levels),2,length(t0)]);
%%% Main Loop (Over Read Powers)
for j=1:length(power_levels)
    %%% Things Are now on a Per Iteration Basis
    %% Generate Electric Field
        % Rabi Frequency For Read Field
        % Simulated Read Chirp (function, rabi units)
        read_rabi=sqrt(2)*base_rabi*sqrt(10^(power_levels(j)/10));
        Fr=@(t) read_rabi*(insertion_port.*rounding_chirp(t).*...
        ...exp(2*pi*1i*(0.5*read_chirp_rate*t'.^2+freq_0*t')...
        ...+(1/(2*pi))*phi_func(t')));
        Fr2=@(t) input_network*Fr(t);
        % calculate pump parameter
        pump_parameter(j)=...
        ...(1/sqrt(2))*read_rabi/sqrt(read_chirp_rate);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Maxwell Bloch Loop
%%% Initialization
% Initialize Needed arrays
% Initialize Matrix for output electric fields
    Ea_out=zeros([2,length(t0)]);
    Eb_out=zeros([2,length(t0)]);
% Initialize Matrix For Populations/Gratings
    Pop_out=[];

```

```

if Reference_around_xtal==1
    N_ports=1;
else
    N_ports=2;
end
for n=1:N_ports % for loop to run Maxwell-Bloch on each channel
    OMEGA2=@(t) transpose(PORTS(n,:)*Fr2(t));
    R_in=reshape(R_all(n,:,:),[3*(length(DELTA)),n_slices]);
    [T, Eb, Rb_final_spatial] = maxwell_bloch_sim(R_in ,t0, ...
    ...DELTA, OMEGA2, alpha(n), n_slices , L, T1, T2);% Run simulation
    Eb_out(n,:)=Eb; % isolate Field out
if show_total_grating
    % Isolate Grating
    Pop_out(n,:)=mean(Rb_final_spatial(:, 3:3:end), 1);
else
    % Isolate Grating
    Pop_out(n,:)=Rb_final_spatial(1, 3:3:end);
end
toc
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Breakout Desired Variables after execution
%% of maxwell bloch simulation
%% a.) Fields
%% STEP 1: Implied Ouputs:
%% (no material interaction Just pass through network)
E_injected=Fr2(t0');
% apply output splitter network to input fields
% (i.e. Implied output fields based on outputs)
E_0=output_network* E_injected;
E_1_0=E_0(1,:); % First Port (IMPLIED FIELDS)
E_2_0=E_0(2,:); % Second Port (IMPLIED FIELDS)

%% STEP 2: Build Full Field based on methods
if Reference_around_xtal==1;
    E_calc(1,:)=Eb_out(1,:); % First port(through the material)
    E_calc(2,:)=E_injected(2,:); % Second Port(through the material)
else
    E_calc=Eb_out; % (Both Fields through material)
end
%% STEP 3: Propagate Fiedld through Output network
% Shaping Output field with output network
E_total_final=output_network*E_calc;
%% STEP 4: Break Field into Port Based Parts
% Isolate Signals on Each Output port (port 1)

```



```

E1=E_total_final(1,:);
% Isolate Signals on Each Output port (port 2)
E2=E_total_final(2,:);
Eb_out_all(j, :, :)=Eb_out;
E_injected_all(j, :, :)=E_injected;
all_signals(j, :, :)=E_0;
all_signals_modified(j, :, :)=E_total_final;
%% b.) Populations
Grating_out=Pop_out(1,:);% Grating at the end of the simulation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Post Processing of Fields
%% STEP 1: Simulate Detection
%% a.) Power Measurements
P_1=abs(E1).^2; % Power at Detector 1
P_2=abs(E2).^2; % Power at Detector 2
P_1_0=abs(E_1_0).^2; % Power at Detector 1
P_2_0=abs(E_2_0).^2; % Power at Detector 2

if calibrate_rabi
% Power at Detector 1
P_1=(P_1/(calibration_rabi^2))*...
... (Experimental_area/Calibration_area)*2*Calibration_power;
% Power at Detector 2
P_2=(P_2/(calibration_rabi^2))*...
... (Experimental_area/Calibration_area)*2*Calibration_power;
% Power at Detector 1
P_1_0=(P_1_0/(calibration_rabi^2))*...
... (Experimental_area/Calibration_area)*2*Calibration_power;
% Power at Detector 2
P_2_0=(P_2_0/(calibration_rabi^2))*...
... (Experimental_area/Calibration_area)*2*Calibration_power;
end

if model_detector
% Voltage Signal Calculated by applying detector parameters
P_1=P_1*responsivity*transimpedance_gain*ADC_units;
% Voltage Signal Calculated by applying detector parameters
P_2=P_2*responsivity*transimpedance_gain*ADC_units;
%% Implied Fields
% Power at Detector 1
P_1_0=P_1_0*responsivity*transimpedance_gain*ADC_units;
% Power at Detector 2
P_2_0=P_2_0*responsivity*transimpedance_gain*ADC_units;
end

```

```

%% b. Power Level Matching
T_mean_start=t_dead+t_start+10*T2; % 10 t2s after begining of chirp
% 10ms before end of chirp
T_mean_stop=t_dead+t_start+t_duration-10*T2;
% Define Window To calculate means for calculation
mean_window=round(( [ T_mean_start , T_mean_stop]-t_start)*Sample_rate);
P_1_unaltered=P_1; % Retain an unaltered Copy of the signals before
    rescaling
P_2_unaltered=P_2; % Retain an unaltered Copy of the signals before
    rescaling

if match_output_power %% Match Powers of
%% Use these lines to match to mean
% Calculate Mean in the mean region
    P1_mean=mean(P_1(mean_window(1):mean_window(2)));
% Calculate Mean in the mean region
    P2_mean=mean(P_2(mean_window(1):mean_window(2)));
%% Use these lines to match to peak
% P1_mean=max(P_1(mean_window(1):end));
%% Location at which noise will be measured
% P2_mean=max(P_2(mean_window(1):end)); ion+4*peak_window;

%STEP 1: Pre-processing
    POWER_ratio=P1_mean/P2_mean;% Ratio to match power levels
    P_2=P_2*POWER_ratio; % Apply Ratio to match power Levels
end
%% c.) Simulate Balanced detection
if simulate_added_attenuation
    Excess_attenuation=exp(ALPHA*L)*total_attenuation;
else
    Excess_attenuation=1;
end
% Balanced Detection Signal
    SIGNAL=Excess_attenuation*(P_1-P_2);
if subtract_mean
    SIGNAL=SIGNAL-mean(SIGNAL);
end
if simulate_detection_noise_floor %% Apply Detector Noise floor
%% add In Simulated Noise floor
    SIGNAL=SIGNAL+responsivity*transimpedence_gain*...
        ... ( Noise_floor*(2*(rand(size(SIGNAL))-1/2)));
end
%% Calculate power Spectra

```

```

if process_fullblock
    if window_on==1
        window_vector=window(window_method,length(SIGNAL))';
        SIGNAL=SIGNAL.*window_vector;
    end
    % Power Spectrum of Balanced Detection Signal (Convert to Power)
    delay_spectrum=(1/R_load)*abs(fftshift(fft(SIGNAL))).^2;
    delays=freqs/read_chirp_rate; % delays (frequencies/ chirp rate) (us)
    ranges=delays*300;
    resolution_general=Resolution_FFT;
    SIGNAL_2=SIGNAL;
    T_segmented=t0;
    f_start_2=freq_0;
else
    % Begin Processing Window
    T_start_processing=t_start+t_dead+grating_delay+2*T2;
    % End Processing Window
    T_stop_processing=t_start+t_dead+t_duration-grating_delay-2*T2;
    % Amount of time Used in Processing Block
    T_block_processing=T_stop_processing-T_start_processing;
    % Amount of Unused block
    unused_block_size=round((t_duration_full-...
    ... T_block_processing)/Sample_rate);

    % Start Index For Processing Block
    i_start_processing=1+round((T_start_processing-t_start)*Sample_rate);
    % Start Index For Processing Block
    i_stop_processing=1+round((T_stop_processing-t_start)*Sample_rate);

    % Segmented time domain Signal
    SIGNAL_2=SIGNAL(i_start_processing:i_stop_processing);
    % Time Block for Time domain signal
    T_segmented=t0(i_start_processing:i_stop_processing);
    f_start_2=freq_0+t0(i_start_processing)*read_chirp_rate;
    % time That we expect to see the peak
    peak_location=(f_0_hole-f_start_2)/read_chirp_rate;
    % half window around which we will find the peak
    peak_window=(3/(T2*pi))/read_chirp_rate;
    % Convert to indicies
    peak_window_indicies=round([peak_location-peak_window,...
    ... peak_location+peak_window]*Sample_rate);
    if subtract_background
        [SIGNAL_2, correction] = compensated_detrending(SIGNAL_2,...
        ... peak_window_indicies);
    end
end

```

```

if window_on==1 %% Apply window to apply to Signal
    window_vector=window(window_method,length(SIGNAL_2))';
    SIGNAL_2=SIGNAL_2.*window_vector;
end
elseif subtract_mean %% Subtract means from signals
if window_on==1 %% Apply window to apply to Signal
    window_vector=window(window_method,length(SIGNAL_2))';
    SIGNAL_2=SIGNAL_2.*window_vector;
end
    SIGNAL_2=SIGNAL_2-mean(SIGNAL_2);
else
if window_on==1 %% Apply window to apply to Signal
    window_vector=window(window_method,length(SIGNAL_2))';
end
end

% Power Spectrum of Balanced Detection Signal
delay_spectrum=(1/R_load)*abs(fftshift(fft(SIGNAL_2))).^2;
resolution_new_block=1/(T_segmented(end)-T_segmented(1));
processing_block_size=length(SIGNAL_2); % New processing Block size
% Frequency Vector (frequency Domain Representation, MHz)
freqs=(-Nyquist_rate):resolution_new_block:(Nyquist_rate);
delays=freqs/read_chirp_rate; % delays (frequencies/ chirp rate) (us)
ranges=delays*300; % Ranges
resolution_general=resolution_new_block;% New block resolution

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% SNR CALCULATION Method depends on type of spectrum
%% (currently method only exists for gratings)
%% Will add time domain for holes later
switch lower(spectrum_type)
case 'grating' % Grating(Grating Domain SNR)
%% Step 1: Calculate Signal
% Delay Window (search Five times around the peak)
Signal_window_delay=(grating_delay+(5/(read_chirp_rate))*...
...[-frequency_resolution,frequency_resolution]);
% Convert Delay window to frequencies
signal_window_freqs=Nyquist_rate+Signal_window_delay*read_chirp_rate;
% Convert Delay window to Indices
signal_window_index=round(signal_window_freqs/resolution_general);
% Frequency Search window
Freqs_window=freqs(signal_window_index(1):signal_window_index(2));
[Signal_level(j),I]=...
...max(delay_spectrum(signal_window_index(1):signal_window_index(2)));

```

```

%% Step 2: Calculate Noise
half_noise_window=0.5;
Noise_location_delay=2;

Noise_window_delay=(Noise_location_delay+...
...[-half_noise_window,half_noise_window]);
Noise_window_freqs=Nyquist_rate+Noise_window_delay*read_chirp_rate;
noise_window_index=round(Noise_window_freqs/resolution_general);
Noise_level(j)=...
mean(delay_spectrum(noise_window_index(1):noise_window_index(2)));

%% Step 3: calculate SNR
SNR(j)=Signal_level(j)/Noise_level(j);
case 'hole' % Spectral Hole (Time Domain SNR)
%STEP 0: Parameters
% Location at which noise will be measured
noise_location=peak_location+4*peak_window;
% half window at which noise will be measured
noise_window=peak_window;
noise_window_indicies=round([noise_location-...
...noise_window,noise_location+noise_window]*Sample_rate);
%STEP 1: Pre-processing
%% b.) Recovery(optional)
if apply_spectral_recovery
    tmax=T_segmented(end)-T_segmented(1);
    SIGNAL_2 = -1*recovery8(length(SIGNAL_2),...
    ...tmax, read_chirp_rate, SIGNAL_2,'traditional');
end
%% c.) hilbert (mandatory for interferometric)
if strcmp(METHOD,'Interferometric')
    SIGNAL_2=-imag(hilbert(SIGNAL_2));
end
%STEP 2: Calculate Signal Level
% Calculate Signal Level
[Signal_level(j),I]=...
...max(SIGNAL_2(peak_window_indicies(1):peak_window_indicies(2)));
%STEP 3: Calculate Floor
% calculate Noise Level
Noise_level(j)=...
...std(SIGNAL_2(noise_window_indicies(1):noise_window_indicies(2)));
%STEP 4: Calculate SNR
    SNR(j)=Signal_level(j)/Noise_level(j);
case 'flat' % Flat spectrum (No Signal)
% NO SNR NEEDS TO BE CALCULATED
    Signal_level(j)=0;

```

```

    Noise_level(j)=0;
    SNR(j)=0;
end
%% PLOTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% a.) Generally Applicable Plots (1s-10s)
%%% Display Gratings
figure(1) % Grating before and After
plot(DELTA,initial_pop,'LineWidth',1); hold on;
plot(DELTA,Grating_out,'LineWidth',1); hold off
legend('Initial_Population','Final_Population')
xlabel('Detuning(MHz)','FontSize',14)
ylabel('r_3');
title('Gratings','FontSize',14)
ylim([-1,1])
%% Time Domain Signals
%% a.) At Each Port
figure(2) % Powers at Port 1
plot(t0,P_1_0,'LineWidth',1);hold on
plot(t0,P_1_unaltered,'LineWidth',1);hold off
if strcmp(METHOD,'Interferometric')
legend('Implied_Intensity_(No_Material)',...
... 'Output Intensity (through material)')
else
legend('Injected_Intensity','Output_Intensity')
end
xlabel('Time_(\mu s)','FontSize',14)
ylabel('Detector_Signal_(mV)','FontSize',14)
title('Power_Levels_at_Port_1_(Read)','FontSize',14)
figure(4) % Powers at port 2
plot(t0,P_2_0,'LineWidth',1);hold on
plot(t0,P_2_unaltered,'LineWidth',1);hold off
if strcmp(METHOD,'Interferometric')
legend('Implied_Intensity_(No_Material)',...
... 'Output Intensity (through material)')
else
legend('Injected_Intensity','Output_Intensity')
end
xlabel('Time_(\mu s)','FontSize',14)
ylabel('Detector_Signal_(mV)','FontSize',14)
title('Power_Levels_at_Port_2_(Ref)','FontSize',14)

%%% Power at Each Detector
figure(3) % Powers at both Detectors
plot(t0,P_1,'LineWidth',1);hold on

```

```

plot(t0,P_2,'LineWidth',1);hold off
legend('Port_1','Port_2')
xlabel('Time_\mu s','FontSize',14)
ylabel('Detector_Signal(mV)','FontSize',14)
title('Power_Levels_at_each_Detector','FontSize',14)
%% Raw BPD signal
figure(5) % BPD Signal
plot(t0,SIGNAL,'LineWidth',1);
xlabel('Time_\mu s','FontSize',14)
ylabel('Detector_Signal(mV)','FontSize',14)
title('Balanced_Detection_Signal','FontSize',14)
%% Truncated/ Processing Signal
figure(6) % BPD Signal
plot(T_segmented,SIGNAL_2,'LineWidth',1);
xlabel('Time_\mu s','FontSize',14)
ylabel('Detector_Signal(mV)','FontSize',14)
title('Tuncated_Balanced_Detection_Signal','FontSize',14)
%% b.) Type Dependant Plots(100s)
switch lower(spectrum_type)
case 'grating'
%% Frequency Spectrum
figure(101)
plot(freqs,10*log10(delay_spectrum),'LineWidth',1);hold on
plot(Freqs_window(I),10*log10(Signal_level(j))...
..., '*', 'MarkerSize',12); hold on
plot(freqs(noise_window_index),...
...10*log10(delay_spectrum(noise_window_index)), '0'...
..., 'MarkerSize',12);hold off
legend('Delay_Spectrum','Signal_Level','Noise_Window_Bounds')
xlabel('Frequency(MHz)','FontSize',14)
ylabel('Return_Power(dB)','FontSize',14)
title('Power_Spectrum_of_Balanced_detector_Signal','FontSize',14)
xlim([0,8*grating_delay*read_chirp_rate])
%% Delay Spectrum
figure(102)
plot(delays,10*log10(delay_spectrum),'LineWidth',1);
xlabel('Delay_\mu s','FontSize',14)
ylabel('Return(dB)','FontSize',14)
title('Delay_Spectrum','FontSize',14)
xlim([0,8*grating_delay])

case 'hole'
%% Time Domain Spectrum Showing info
figure(101) % BPD Signal
plot(T_segmented,SIGNAL_2,'LineWidth',1);hold on

```

```

plot(T_segmented(I+peak_window_indicies(1)),...
... Signal_level(j),'0','MarkerSize',12);hold on
plot(T_segmented(noise_window_indicies),...
... SIGNAL_2(noise_window_indicies),'0','MarkerSize',12);hold off
legend('Processing_Signal','Peak_Location','Noise_Window_Bounds')
xlabel('Time_\mu s','FontSize',14)
ylabel('Detector_Signal(mV)','FontSize',14)
title('Signal_used_for_Processing','FontSize',14)

case 'flat'
%%% Frequency Spectrum
figure(101)
plot(freqs,10*log10(delay_spectrum),'LineWidth',1);
xlabel('Delay_\mu s','FontSize',14)
ylabel('Return_dB','FontSize',14)
title('Delay_Spectrum','FontSize',14)
xlim([0,8*grating_delay])
end
%%% c.) Optional Plots (200s)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if show_population_spectrum
%%% Population Power spectrum
%%% Calculate Power Spectrum From Grating
f_start_g=-25; % population Start Frequency
f_stop_g=25; % Population Stop Frequency
D_Delta=DELTA(2)-DELTA(1);% Grating Frequency Step
% Calculate Start Index for Gratings
i_start=round((Bandwidth_material/2+f_start_g)/D_Delta);
% Calculate Stop Index for Gratings
i_stop=round((Bandwidth_material/2+f_stop_g)/D_Delta);
% Isolate the Relevant Spectral Region
DELTA_2=DELTA(i_start:i_stop);
% Calculate the Number of points in the grating
N_DELTA=length(DELTA_2);
% Calculate the effective sampling rate
%(Direct mapping of frequency to time via Chirp)
Sample_rate_grating=read_chirp_rate/D_Delta;
% Effective nyquist rate based on number of points in the grating
NYQUIST_GRATING= Sample_rate_grating/2;
df_grating=Sample_rate_grating*1/(N_DELTA); % Step size of grating
% Calculation of Hypothetical frequencies in delay spectrum domain
freqs_theoretical=NYQUIST_GRATING:df_grating:(NYQUIST_GRATING-
df_grating);
corrected_grating_initial=initial_pop(i_start:i_stop);% Pull Up grating
corrected_grating_initial= corrected_grating_initial-...

```



```

...mean( corrected_grating_initial);% Subtract mean off of grating
% apply window to grating
corrected_grating_initial=corrected_grating_initial.*...
...hanning(length(corrected_grating_initial));
% subtract mean again incase it is changed by windowing
corrected_grating_initial= corrected_grating_initial -...
...mean( corrected_grating_initial);
% Pull Up grating
corrected_grating_final=Grating_out(i_start:i_stop)';
% Subtract mean off of grating
corrected_grating_final=corrected_grating_final -...
...mean(corrected_grating_final);
% apply window to grating
corrected_grating_final=corrected_grating_final.*...
...hanning(length(corrected_grating_final));
% subtract mean again incase it is changed by windowing
corrected_grating_final=corrected_grating_final...
..-mean(corrected_grating_final);
% Calculate power levels
GRATING.SPECTRUM=(1/length(initial_pop))^2*...
...abs(fftsift(fftw(corrected_grating_initial)))^2;
GRATING.SPECTRUM_final=...
...(1/length(Grating_out))^2*...
...abs(fftsift(fftw(corrected_grating_final)))^2;%

%% PLOT GRATINGS
figure(201)
plot(freqs_theoretical,10*log10(GRATING.SPECTRUM),...
... 'LineWidth',1);hold on
plot(freqs_theoretical,10*log10(GRATING.SPECTRUM_final),...
... 'LineWidth',1); hold off
legend('Initial_Grating','Final_Grating')
xlabel('Frequency_(MHz)','FontSize',14)
ylabel('Grating_Power_(dB)','FontSize',14)
title('Initial_Grating_Spectrum','FontSize',14)
xlim([0,5*grating_delay*read_chirp_rate])

end
if Display_time_domain_anatomy
figure(202)
plot(t0,SIGNAL,'LineWidth',1); hold on
plot(T_segmented,SIGNAL_2,'LineWidth',1);
xline(T_segmented(1),'-',{ 'Start_processing', 'block'});
xline(T_segmented(end),'-',{ 'End_processing', 'block'});
if spectrum_type == "hole"

```

```

xline(T_segmented(peak_window_indicies(1)), '--')
xline(T_segmented(peak_window_indicies(2)), '--')
xline(T_segmented(noise_window_indicies(1)), ':')
xline(T_segmented(noise_window_indicies(2)), ':')
end
hold off
xlabel('Time_\mu s', 'FontSize', 14)
ylabel('Detector_Signal(mV)', 'FontSize', 14)
title('Anatomical_Plot_Showing_Processing_Signal')
legend('BPD_signal', 'Processing_Signal')
ylim([-3*max(abs(SIGNAL)), 3*max(abs(SIGNAL))])
end
end

if ~quickrun
correction_factor=0;
figure(301)
plot(power_levels, 10*log10(Signal_level)+correction_factor ...
..., '*', 'MarkerSize', 12)
xlabel('Read_power(dB)', 'FontSize', 14)
ylabel('Signal_level(dB)', 'FontSize', 14)
title('Signal_as_a_Function_of_Read_Power', 'FontSize', 14)
figure(302)
plot(power_levels, 10*log10(Noise_level)+correction_factor, '*' ...
..., 'MarkerSize', 12)
xlabel('Read_power(dB)', 'FontSize', 14)
ylabel('Noise_level(dB)', 'FontSize', 14)
title('Noise_as_a_Function_of_Read_Power', 'FontSize', 14)
figure(303)
plot(power_levels, 10*log10(SNR+correction_factor), '*', 'MarkerSize', 12)
xlabel('Read_power(dB)', 'FontSize', 14)
ylabel('Signal_to_Noise_Ratio(dB)', 'FontSize', 14)
title('SNR_as_a_Function_of_Read_Power', 'FontSize', 14)
figure(301); grid on
figure(302); grid on
figure(303); grid on
end

disp('Simulation_Completed_Successfully!')
if experiment.options.save_data
file_name_save=[base_directory, experiment_name];

Signal_level_save=Signal_level;
Noise_level_save=Noise_level;
SNR_save=SNR;

```

```

save(file_name_save ,...
't0' ,...
'power_levels' ,...
'Signal_level_save' ,...
'Noise_level_save' ,...
'SNR_save' , ...
'all_signals' ,...
'Eb_out_all' ,...
'E_injected_all' ,...
'all_signals_modified')
disp('Data Saved!')
%% Simulation over
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% KEY TO PLOT NUMBERING SCHEME
%% 1s-10s: Individual plots corresponding to every shot:
%% 100s: Optional Plots Corresponding on a per shot basis
%% 200s: Spectrum type dependant plots
%% 300s: Plots indicating signal and noise level dependances

```

Four Port Simulator

This simulation routine simulates stimulated photon echoes in a 4 port interferometer, it can be configured for simple SPEs or correlation.

```

close all
clear all
experiment_name='barker';
experiment_file=[experiment_name , '.json' ];
%check OS type then execute path dependent
%commands based on OS type
if ispc
% for Windows Machines
addpath('.\core_simulator');
addpath('.\auxillary_functions');
addpath('.\Pulse_shapes');
experiment = [ '\', experiment_file ] ;
base_directory='.';
else
% for Linux/Unix/Mac Machines
addpath('./core_simulator');
addpath('./auxillary_functions');
addpath('./Pulse_shapes');
experiment = [ './', experiment_file ];
base_directory='./';

```

end

```

init_if_jones_operators % Initialize Interferometric Operators
%%% Import json spec defining the experiment
fid = fopen(experiment);
json_raw = fread(fid,inf);
json_str = char(json_raw');
fclose(fid);
experiment = jsondecode(json_str);
clear fid json_raw json_str
J=clock;
tic %start the clock
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% time-frequency setup %%%%%%%%%%
% Bandwidth and Resolution
% Frequency Resolution of material(MHz)
frequency_resolution = experiment.time_frequency.frequency_resolution;
% Bandwidth of the material (MHz)(Full width: positive and Negative
    Detunings)
Bandwidth_material = experiment.time_frequency.bandwidth_material;
% Safety Factor for FFT limited Resolution
safety_factor_frequency_resolution =...
... experiment.time_frequency.safety_factor_frequency_resolution;
% safety factor for Bandwidth
% (Signal Nyquist is greater than material bandwidth)
safety_factor_bandwidth =experiment.time_frequency.
    safety_factor_bandwidth
t_duration = experiment.time_frequency.t_duration;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% material setup %%%%%%%%%%
ALPHA = experiment.material.ALPHA; % absorption Length (cm-1)
L = experiment.material.L; % Sample length (cm)
n_slices = experiment.material.n_slices; % number
T1 = experiment.material.T1; % Material population lifetime
T2= experiment.material.T2; % Material Coherence lifetime
% Minimum Value For the grating is -1
r3_min = experiment.material.initial_populations.r3_min;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Interferometer Setup %%%%%%%%%%
n_ports = experiment.interferometer.n_ports;
insertion_port = experiment.interferometer.insertion_port;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% input_field Setup %%%%%%%%%%
% Chirp rounded using a homeade function with a sine edge
round_chirp = experiment.input_field.round_chirp;
% Determine if rabi properties are simulated.
calibrate_rabi = experiment.input_field.calibrate_rabi;
% Experiment Power Level
power_level = experiment.input_field.power_level;

```



```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%                               Prepare For Core LOOP                               %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate FFT resolution (and therefore Experiment Duration)
Resolution_FFT=frequency_resolution/safety_factor_frequency_resolution;
% calculate Nyquist
Nyquist_rate=(Bandwidth_material/2)*safety_factor_bandwidth;
% Sample Rate Based on Nyquist Frequency (samples per microsecond/MHz)
Sample_rate=2*Nyquist_rate;
t_duration_full=t_duration;
% Process parameters
t_start=0;
time_block_size=Sample_rate*t_duration_full; % calculate time block size
time_step=(1/Sample_rate); % time Ste Spectral Gratingp in Microseconds
t_stop=t_duration_full-time_step; % end time in Microseconds
t0=t_start:time_step:t_stop; % Time Vector (microseconds)
% Frequency Vector (detunnings, MHz)
DELTA=(-Bandwidth_material/2:frequency_resolution:...
...(Bandwidth_material/2-frequency_resolution))';
% Frequency Vector (frequency Domain Representation, MHz)
freqs=(-Nyquist_rate):Resolution_FFT:(Nyquist_rate-Resolution_FFT);
%% 2: Material Parameters
alpha(1)=ALPHA; % Field 1 Sees the material
if calibrate_rabi
% Experimental mode area
    Experimental_area=pi*(Experimental_spot/2)^2;
% Calibration mode Area
    Calibration_area=pi*(Calibration_spot/2)^2;
% Calibrated Base Rabi (for 0dBm of optical Power)
    base_rabi=sqrt((Calibration_area/Experimental_area)*...
...(Base_power_level/(2*Calibration_power)))*calibration_rabi;
else
% Starting Rabi Condition is 1MHz If Power Calibration Is Not Specified
    base_rabi = 1;
end
%% 3: Fields
%% DEFINE PORT VECTORS
P1=[1;0;0;0];
P2=[0;1;0;0];
P3=[0;0;1;0];
P4=[0;0;0;1];

switch lower(pulse_type)
case 'hard' %Hard Edge pulse

```

```

    f_0=@(x) 0.*x+1;
    pulse=@(t,tau,A) raw_rabi*hard_edges(t,f_0,tau,tau+A/raw_rabi);
case 'gaussian' % Gaussian Pulse
    pulse=@(t,tau,A) raw_rabi*gaussian_pulse(t,tau,A/(raw_rabi...
...*sqrt(2*pi)));
otherwise
    error("invalid value for pulse type")
end

switch lower(pattern_type)
case 'echo' % 2 Pulse Echo
    OMEGA_0=@(t) P1*pulse(t,TP1,pi/2)+P2*pulse(t,TP2,pi)
case 'spe' % Stimulated Echo
    OMEGA_0=@(t) P1*pulse(t,TP1,pi/2)+P2*pulse(t,TP2,pi/2)+...
...P3*pulse(t,TP3,pi/2)
case 'nutatation' % Nutation
    f_0=@(x) 0.*x+1;
    OMEGA_0=@(t) P1*raw_rabi*hard_edges(t,f_0,2,t_stop)
case 'barker' % Barker Correlation Pattern
    t_interval=0.5;
    patern=@(t,tau)...
...0.5*pulse(t,tau,pi/2)+pulse(t,tau+t_interval,pi/2)...
...+pulse(t,tau+2*t_interval,pi/2)-...
...pulse(t,tau+3*t_interval,pi/2)+pulse(t,tau+4*t_interval,pi/2);
    OMEGA_0=@(t) P1*patern(t,TP1)+P2*pulse(t,TP2,pi/2)+...
...P3*patern(t,TP3);
otherwise
    error("invalid value for pattern type")
end
PORTS = eye(n_ports);
%% ii.) Edge rounding (option) [Highly Recomendad Option]
if round_chirp
% apply Rounding to front edge of chirp
    rounding_chirp = @(t) rounded_edge_with_dead_time(t_dead,t);
else
    rounding_chirp=@(t) t'.^0;
end

if soften_pulse
    sigma=10;
    g=@(w)exp(-(w/(sigma)).^2);
    spectrum_freq=fourier_frequencies(Sample_rate, length(t0));
    FILT_SIGNAL=ifft(fft(OMEGA_0(t0)).*hilbert(g(spectrum_freq')));
    GP=pchirp(t0,FILT_SIGNAL);
    OMEGA_0=@(t) ppval(GP,t);

```

end

```

initial_pop=r3_min*ones(size(DELTA));
% Generate Array of Start Points
bloch_vec_A=vertcat(zeros(size(initial_pop')),zeros(size(initial_pop')),
    ,initial_pop');
bloch_vec_B=reshape(bloch_vec_A,[(3*(length(DELTA))),1]); % Reshape
    Array of Startpoints
R00=[0;0;-1]; % Unprogramed Initialization Vector
VEC_ch1= repmat(bloch_vec_B,1,n_slices); % Programed Channel 1
VEC_ch2=create_initial_array(R00,length(DELTA),n_slices); %
    Unprogramed Channel 2
R_all(1,:,:)=VEC_ch1; % Create Array for all Bloch Vectors For all
    channels
R_all(2,:,:)=VEC_ch1; % Create Array for all Bloch Vectors For all
    channels
R_all(3,:,:)=VEC_ch1; % Create Array for all Bloch Vectors For all
    channels
R_all(4,:,:)=VEC_ch1; % Create Array for all Bloch Vectors For all
    channels

input_network=Q4.if4port_second;
output_network=Q4.if4port_first;
Fr=@(t) OMEGA0(t');
Fr2=@(t) (input_network*Fr(t));
PORTS = eye(n_ports);
Pop_out=[];
Eb_out=[];
Q4.if4port_second
for n=1:n_ports % for loop to run Maxwell-Bloch on each channel
toc
n
OMEGA2= @(t) transpose(PORTS(n,:)*Fr2(t));
R_in=reshape(R_all(n,:,:),[3*(length(DELTA)),n_slices]);
[T, Eb, Rb_final_spatial] = maxwell_bloch_sim(R_in,t0, DELTA,...
...OMEGA2, alpha, n_slices, L, T1, T2);% Run simulation
toc
Eb_out(n,:)=Eb; % isolate Field out
if show_total_grating
    % Isolate Grating
    Pop_out(n,:)=mean(Rb_final_spatial(:, 3:3:end), 1);
else
    Pop_out(n,:)=Rb_final_spatial(1, 3:3:end);
end
% me sure we delete unneeded older versions of the rabi vector

```



```

toc
end
Grating_out=Pop_out(1,:);% Grating at the end of the simulation
E_out=output_network*Eb_out;
E1=E_out(1,:);
E2=E_out(2,:);
E3=E_out(3,:);
E4=E_out(4,:);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plot Outputs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
INPUT_FIELDS=OMEGA_0(t0);
figure(1);clf
subplot(4,1,1)
plot(t0,abs(INPUT_FIELDS(1,:)).^2,'LineWidth',1);
ylabel('IP1','FontSize',12)
title('Input_Fields','FontSize',14)
subplot(4,1,2)
plot(t0,abs(INPUT_FIELDS(2,:)).^2,'LineWidth',1);
ylabel('IP2','FontSize',12)
subplot(4,1,3)
plot(t0,abs(INPUT_FIELDS(3,:)).^2,'LineWidth',1);
ylabel('IP3','FontSize',12)

subplot(4,1,4)
plot(t0,abs(INPUT_FIELDS(4,:)).^2,'LineWidth',1);
ylabel('IP4','FontSize',12)
xlabel('Time_(\mu s)','FontSize',14)

figure(2);clf
subplot(4,1,1)
plot(t0,abs(E1).^2,'LineWidth',1);
ylim([0,20])
xlim([0,30])
ylabel('OP1','FontSize',12)
title('Stimulated_Echoes(linear)','FontSize',14)
subplot(4,1,2)
plot(t0,abs(E2).^2,'LineWidth',1);
xlim([0,30])
ylabel('OP2','FontSize',12)
subplot(4,1,3)
plot(t0,abs(E3).^2,'LineWidth',1);

```

```

xlim([0,30])
ylim([0,20])
ylabel('OP3','FontSize',12)
subplot(4,1,4)
plot(t0,abs(E4).^2,'LineWidth',1);
xlim([0,30])
ylabel('OP4','FontSize',12)
ylim([0,20])
xlabel('Time_\mu s','FontSize',14)

```

```

figure(3);clf
subplot(4,1,1)
plot(t0,10*log10(abs(E1).^2),'LineWidth',1);
ylim([-20,16])
xlim([0,30])
ylabel('OP1','FontSize',12)
title('Stimulated_Echoes_(dB)','FontSize',14)
subplot(4,1,2)
plot(t0,10*log10(abs(E2).^2),'LineWidth',1);
ylim([-30,6])
xlim([0,30])
ylabel('OP2','FontSize',12)
subplot(4,1,3)
plot(t0,10*log10(abs(E3).^2),'LineWidth',1);
ylim([-20,16])
xlim([0,30])
ylabel('OP3','FontSize',12)
subplot(4,1,4)
plot(t0,10*log10(abs(E4).^2),'LineWidth',1);
ylim([-20,16])
xlim([0,30])
ylabel('OP4','FontSize',12)

```

```

figure(3);clf
subplot(4,1,1)
plot(t0,10*log10(abs(E1).^2),'LineWidth',1);
ylim([-50,10])
xlim([0,30])
ylabel('OP1','FontSize',12)
title('Stimulated_Echoes_(dB)','FontSize',14)
subplot(4,1,2)
plot(t0,10*log10(abs(E2).^2),'LineWidth',1);
ylim([-50,10])
xlim([0,30])

```

```

ylabel('OP2','FontSize',12)
subplot(4,1,3)
plot(t0,10*log10(abs(E3).^2),'LineWidth',1);
ylim([-50,10])
xlim([0,30])
ylabel('OP3','FontSize',12)
subplot(4,1,4)
plot(t0,10*log10(abs(E4).^2),'LineWidth',1);
ylim([-50,10])
xlim([0,30])
ylabel('OP4','FontSize',12)
xlabel('Time_\mu s','FontSize',14)

```

APPENDIX D

SSH READOUT CODE

These functions are used in SSH readout experiments.

Post Processing Functions

Especially in SSH spectrum analysis applications these functions are useful for post-processing spectra.

Interferometric Recovery

The interferometric recovery algorithm is a simple modification of existing spectral recovery algorithms. This algorithm can be used to recover interferometric or traditional readout signals based on the bias phase.

```
function out = recovery_mixed(N, tmax, kap, data, phase)
% N data points, length of sample time in seconds, Kappa
data=data(:);
dt = tmax/N; % time step size as defined by input data
% Frequency, N points long from -1/2 to (N-1)/2N
freq = (1/dt)*((0:N-1)-N/2)/N;
data = fft(data);
data = fftshift(data);
% Mixing Phasor (use 0 rad for traditional readout
% or pi/2 for idealized interfeormteric readout)
phasor=exp(1i*sign(freq)*phase);
% Recovery kernel defined
recovery = phasor.*exp(-1i*pi*(-sign(freq).*freq.^2)/kap);
data = data.* recovery(:); % Apply recovery
data = ifft(fftshift(data));% Return to time domain
out = real(data);
end
```

Background Subtraction

This routine is used to subtract backgrounds from an individual spectrum analysis trace. It works by lowpass filtering to create a background trace, omitting the known signal locations.

```
function [signal_out] =background_subtract_filtered(fc,fs,...
... signal,peak_locations,half_window,averaging_count)
% Background subtraction function for holes
% Generate Butterworth lowpass
[b1,a1] = butter(1,fc/(fs/2), 'Low');
window_indicies=1:(2*half_window-1); % Window edges
Bound_indicies=[0,(2*half_window)];

chunk1=signal(1:(peak_locations(1)-half_window));
chunk2=signal((peak_locations(1)+half_window):...
```

```

... ( peak_locations (2)-half_window ));
chunk3=signal (( peak_locations (2)+half_window ) :...
... ( peak_locations (3)-half_window ));
chunk4=signal (( peak_locations (3)+half_window ) :...
... ( peak_locations (4)-half_window ));
chunk5=signal (( peak_locations (4)+half_window ):end);
%% interpolate around peaks

peak1_interp=interp1 ( Bound_indicies , [ mean(chunk1 ((end-...
... averaging_count):end)) , mean(chunk2 (1:averaging_count)) ] ,...
... window_indicies , 'Linear' );

peak2_interp=interp1 ( Bound_indicies , [ mean(chunk2 ((end-...
... averaging_count):end)) , mean(chunk3 (1:averaging_count)) ] ,...
... window_indicies , 'Linear' );

peak3_interp=interp1 ( Bound_indicies , [ mean(chunk3 ((end-...
... averaging_count):end)) , mean(chunk4 (1:averaging_count)) ] ,...
... window_indicies , 'Linear' );

peak4_interp=interp1 ( Bound_indicies , [ mean(chunk4 ((end-...
... averaging_count):end)) , mean(chunk5 (1:averaging_count)) ] ,...
... window_indicies , 'Linear' );

masked_trace=[ chunk1 ' , peak1_interp , chunk2 ' ,...
... peak2_interp , chunk3 ' , peak3_interp , chunk4 ' ,...
... peak4_interp , chunk5 ' ] ' ;

background_individual=filtfilt ( b1 , a1 , masked_trace );
signal_out=signal-background_individual;
end

```

Trace Segmentation Function

This function automates cutting out the required component of the trace.

```

function [ trace_isolated , time_isolated ] =...
... isolate_trace ( trace_in , t_in , start_index , trace_length )
%%% [ trace_isolated , time_isolated ] =...
%%% ... isolate_trace ( trace_in , t_in , start_index , trace_length )
% A Simple function to isolate a single shot from
% a raw data sequence.
% This function will isolate the section of
% the input trace and will
% provide a commensurate time vector.
% (for use on time domain data);

```

```

trace_isolated= trace_in(start_index:start_index+trace_length-1);
time_isolated= ...
... t_in(start_index:start_index+trace_length-1)-t_in(start_index);
end

```

Processing Scripts

These Processing scripts are used to process interferometric data readout data.

SSH Spectrum Analysis Processing Script

This script was used to process interferometric SSH spectrum analysis data.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%                               %%
%%      Processing Script      %%
%%      For Interferometric S2SA      %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Preamble
addpath(' ./S2SA_functions')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% File Name Book Keeping & Relevant Powers
% define directory of interest
directory_name=' ./SNR_16-Dec-2021_interferometric_1uW_read_10uW_LO';
% load sys_config_file
load(horzcat(directory_name, '/sys_config.mat'));
base_filename='RF_attenuation_';
P_LO=10e-6; % LO power
P_read=1e-6; % read Power
% RF Attenuations
attenuations=[0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Options
phase_compensate=true;
Method='Interferometric'; % Choose method interferometric or trad
%Method='trad';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Experiment Parameters & Constants
%%% Experimental Paramters
alpha_L=1.39; % absorption length
beta=160e9; % Programming Coefficient
kap=2e12; % Chirp rate (Hz^2)
prog_power=15e-3; % Total Programming field power
RF_power_0=-3.05; % Nomional input power to EOM
Z=50; % EOPM Impedence
V_pi=5;

```

```

RF_pre_attenuation_log=-0;
RF_pre_atten_lin=10^(RF_pre_attenuation_log/10);
RF_beta=prog_power*(pi^2*Z/(4*V_pi^2))*RF_pre_atten_lin;

if analyze_RF_power
    beta=beta*RF_beta;
end

G_t=0.5e5; % Detector Transimpedence Gain (V/A)
responsivity=0.53;% Detector Responsivity (A/W)
ADC_units=1e3; % ADC uses mV as Primary Unit
T2=0.26e-6; % Material Coherence time
f_start=0; % Start Frequency
q=1.602e-19; % Electron Charge
Q_NEP=3.3e-12; % NEP in units of W/rtHz
det_BW=4e6; % Detection_Bandwidth
%%% Post Processing Parameters
decimation_factor=0.004;
%%% Filter Paramters: 1: Background Calculation Filter
fs=sys_config.sample_rate;
fc=1.5e5;%%% Background Filter Cutoff
peak_locations=[7265,8265,9265,10265];
half_window=35;
averaging_count=2;
%%% Filter Parameters: 2: Smoothing Filter
windowSize = 4; % Size of block smoothing filter window
b = (1/windowSize)*ones(1,window-size); %create filter window
a = 1; % a coefficient
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Calculated Parameters
RES_BW=0.5/T2; % Inhomogeneous Bandwidth
%%% RMS Noise Voltage
% Thermal Noise
V_RMS_NEP=ADC_units*G_t*responsivity*Q_NEP*sqrt(2*det_BW);
% Shot Noise
V_RMS_shot=ADC_units*G_t*sqrt(4*2*q*responsivity*det_BW*(P_LO+P_read));
% Total Noise
V_total =sqrt(V_RMS_NEP^2+V_RMS_shot^2);
% S2SA Noise PSD (calculated)
NOISE_PSD=V_total/(ADC_units*G_t*responsivity*alpha_L*beta*sqrt(P_LO*
    P_read));
Noise_power=NOISE_PSD*RES_BW;% S2SA Calculated Noise Power
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Windows & Critical Indices
% First Trace of Used Data

```



```

FIRST_SAMPLE=floor(5*10-5*sys_config.sample_rate);
% Duration of trace
TRACELENGTH=floor(1+1*10-3*sys_config.sample_rate);
new_window(1)=floor(4*10-4*sys_config.sample_rate);
new_window(2)=floor(4.25*10-4*sys_config.sample_rate);
noise_window(1)=floor(3.0*10-4*sys_config.sample_rate);
noise_window(2)=floor(3.25*10-4*sys_config.sample_rate);
PEAK_search_region(1)=floor(sys_config.sample_rate*320*10-6);
PEAK_search_region(2)=floor(sys_config.sample_rate*550*10-6);
blank_window=[150,350];
FIRST_SAMPLE_check=floor(5*10-5*sys_config.sample_rate);
TRACELENGTH_check=floor(1*10-3*sys_config.sample_rate);
%% Phase Calculation Parameters
[Data_BACKGROUND]=load(horzcat(directory_name,'/background.mat'));
MEASURED_BACKGROUND=Data_BACKGROUND.background_data;
mean_value_NO_LIGHT=mean(MEASURED_BACKGROUND(1000:6000));
MEAN_CHECK_REGION(1)=floor(sys_config.sample_rate*440*10-6);
MEAN_CHECK_REGION(2)=floor(sys_config.sample_rate*460*10-6);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Plot Parameters
%% Font, Line and Marker Sizes
single_shot_linewidth=1;
line_width=2;
Marker-Size=12;
title_size=14;
label_size=14;
legend_size1=12;
legend_size2=10;
if strcmp(Method,'Interferometric');
    line2_Individual_plots=...
    ['Interferometric_Readout,'...
    % P readout in dBm
    ... P_{read}=',num2str(10*log10(1e3*P_read))...
    ... , 'dBm', ' P_{LO}=',num2str(10*log10(1e3*P_LO)) , 'dBm' ];
else
    line2_Individual_plots=...
    ['Traditional_Readout,P_{read}=',num2str(10*log10(1e3*P_read)) , 'dBm' ];
end
lower_offset=10;
upper_offset=30;
noise_floor_est_dBm=10*log10(1e3*NOISE_PSD);
% XLIM vector for spectra
Spectrum_YLIM=[noise_floor_est_dBm -...
... lower_offset , noise_floor_est_dBm+upper_offset ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Loop Parameters & Initializing Loop Variables

```

```

signals=[];
signal_errorbar=[];
noises=[];
noises_2=[];
full_signals=[];
n_shots=30;
N_start=1;
N_stop=n_shots+N_start-1;
shots=[N_start:N_stop];
tic
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% LOOPS
for j=1:length(attenuations) %% Loop 1: Over attenuations
    signal_level=[]; % Initialize Signal Levels
    noise_level_1=[];
    noise_level=[];
    % create filename
    filename=horzcat(directory_name, '/', base_filename, ...
        ... num2str(attenuations(j)), 'dB_', num2str(1), '.mat');
    load(filename); % Load Data
    %% Measuring Phase & Selecting Processing Phase
    %% Step 1: Select Base Phase
    if strcmp(Method, 'Interferometric')
        Phi_base=pi/2; % 90 Degree Phase (IF)
    else
        Phi_base=0; % 0 Degree Phase (Trad)
    end
    %% Step 2: Calculate Phase Deviation
    if phase_compensate && strcmp(Method, 'Interferometric')
        [trace_isolated_temp, time_temp] = ...
            ... isolate_trace(Measured_data, time_data, ...
            ... FIRST_SAMPLE_check, TRACE_LENGTH_check);
        MEAN_IN_CHECK_REGION=abs(mean(trace_isolated_temp(...
            ... MEAN_CHECK_REGION(1):MEAN_CHECK_REGION(2))) - ...
            ... mean_value_NO_LIGHT)
        phase_deviation= 1*asin(MEAN_IN_CHECK_REGION/(ADC_units*...
            ... alpha_L*G_t*responsivity*sqrt(P_LO*P_read)));
    else
        phase_deviation=0;
    end
    %% Step 3: Calculate ( +/- Quad)
    sign_check=true;
    inversion_phase_0=1;
    inversion_phase=-inversion_phase_0;
    loop_counter=1;

```

```

while sign_check && (loop_counter < 3)
%% Step 4: Calculate Total Phase
% Final Phase Deviation
recovery_phase=inversion_phase*(Phi_base-phase_deviation);
%% Recovery & Trace Segmentation
%% Step 1: Execute Recovery
tmax2=time_data(end)-time_data(1);
N2=length(time_data);
recovered_signal=recovery_mixed(N2, tmax2, kap, Measured_data,
    recovery_phase); % apply traditional recovery
%% Step 2: Segment Trace
% Segment Trace
[recovered_signal_raw, time] =...
... isolate_trace(recovered_signal, time_data, FIRST_SAMPLE, TRACELENGTH);
% subtract mean
recovered_signal_raw=recovered_signal_raw-recovered_signal_raw(end);
% Calculate New phase
inversion_phase=inversion_phase*...
... sign(recovered_signal_raw(new_window(1)));
if ~(inversion_phase==inversion_phase_0)
    sign_check=false;
end
    loop_counter=loop_counter+1;
end
%% Background Subtraction
% Execute Background
[background_subtracted_trace]=...
background_subtract_filtered(fc, fs, recovered_signal_raw, ...
... peak_locations, half_window, averaging_count); Subtraction
%% Smoothing (optional)
if apply_smoothing
    tmpfilt = filter(b,a, background_subtracted_trace); %apply filter
    signal_filt = tmpfilt(windowSize/2:end); %remove transients
    timel=time(windowSize/2:end); %create new time vector
else
    timel=time;
    signal_filt=background_subtracted_trace;
end
%% Calibration
%% Step 1: Power Calibration
if strcmp(Method, 'Interferometric')
    Signal_PSD =...
    signal_filt / (ADC_units*G_t*responsivity *...
    ... alpha_L*beta*sqrt(P_LO*P_read));
else

```

```

    Signal_PSD= signal_filt/(ADC_units*G_t*responsivity ...
    ...*alpha_L*beta*P_read);
end
%%% Step 2: Frequency Calibration
f_calibrated=f_start+kap*timel;
%%% Signal and Noise Measurement
%%% Step 1: Signal Measurements
final_trace_3= Signal_PSD(new_window(1):new_window(2));
switch Signal_method
case 1
    [Ms,Is]=max(abs(final_trace_3));
case 2
    [Ms,Is]=max(abs(final_trace_3(240:300)));
case 3
    Is=265;
    Ms=abs(final_trace_3(Is));
case 4
    Is=265;
    Ms=(final_trace_3(Is));
case 5
    [Ms_dummy,Is]=max(abs(final_trace_3));
    Ms=sum(abs(final_trace_3))*mean(diff(f_calibrated))/(RES_BW);
end
signal_level(1)=Ms;
full_signals(j,1)=Ms;
%%% Step 2: Noise Measurements
final_noise_trace=Signal_PSD(noise_window(1):noise_window(2));
switch Noise_method
case 1
    noise_level(1)=std(final_noise_trace(240:300));
case 2
    noise_level(1)=std(final_noise_trace);
case 3
    noise_level(1)=mean(abs(final_noise_trace));
end
%%% Display Shot wise Results
%%% Multiple Plot Matrix for the Single Shot
if Single_shot_plots
figure(1); % Depicts Processing Steps
%%% Raw Data
subplot(2,2,1)
plot(time_data, Measured_data, 'LineWidth', single_shot_linewidth); ...
... hold on
ylabel('voltage_□(mV)')
xlabel('time_□(s)')

```

```

title('Raw_Data')
%%% Recovered Data
subplot(2,2,2)
plot(time_data,recovered_signal); hold on
ylabel('voltage_(mV)')
xlabel('time_(s)')
title('Recovered_Data')
%%% Segmented Trace
subplot(2,2,3)
plot(time,recovered_signal_raw); hold on
ylabel('voltage_(mV)')
xlabel('time_(s)')
title('Segmented_Trace')
%%% background Subtracted Trace (un-calibrated)
subplot(2,2,4)
plot(time1,signal_filt); hold on
ylabel('voltage_(mV)')
xlabel('time_(s)')
title('Segmented_Trace')
figure(100+j); % Final Trace
plot(f_calibrated,10*log10(1e3*abs(Signal_PSD))); hold on
ylabel('PSD_(dBm/Hz)','FontSize',label_size)
xlabel('Frequency_(Hz)','FontSize',label_size)
title(['Final_Power_Spectra:RF_attenuation=', num2str(attenuations(j))
    ],'dB' ],line2_Individual_plots},'FontSize',title_size)
ylim(Spectrum_YLIM)
figure(1000); % Depicts Peak finding
plot(final_trace_3,'LineWidth',line_width); hold on
plot(Is,Ms,'0','MarkerSize',Marker_Size); hold off
ylabel('Signal_(mV)','FontSize',label_size)
xlabel('Sample','FontSize',label_size)
title('Peak_Finding','FontSize',title_size)
end
end
%%% Post Processing
switch aggregation_method;
case 1
signals(j)=mean(abs(signal_level))*RES_BW;
case 2
signals(j)=mean(signal_level)*RES_BW;
case 3
signals(j)=sqrt(mean((signal_level).^2))*RES_BW;
end

signal_errorbar(j)=std(signal_level)*RES_BW;

```

```

switch noise_aggregation_method
case 1
noises(j)=mean(noise_level)*RES_BW;
case 2
noises(j)=sqrt(mean(noise_level.^2))*RES_BW;
case 3
noises(j)=median(noise_level)*RES_BW;
end

%% Display attenuation Level Results
if Single_shot_plots
figure(1);
subplot(2,2,1); hold off
subplot(2,2,2); hold off
subplot(2,2,3); hold off
subplot(2,2,4); hold off
figure(100+j);
if window_markers
xline(f_calibrated(noise_window),'--black')
xline(f_calibrated(new_window),'--black')
end
hold off
end
toc

%% "Finalized" Results
%% Plot Results
if Noise_method == 3
noise_est_offset=-10*log10(sqrt(2/pi));
else
noise_est_offset=0;
end
RF_POWER=(-1*attenuations+RF_power_0);
if analyze_RF_power
figure(3001)
plot(RF_POWER,10*log10(1e3*signals),'*r',...
... 'MarkerSize',Marker_Size); hold on
plot(RF_POWER,10*log10(1e3*noises),'*b',...
... 'MarkerSize',Marker_Size)
yline(10*log10(1e3*mean(noises)),'--b',...
... 'LineWidth',line_width);
yline(10*log10(1e3*Noise_power)-noise_est_offset,':b','LineWidth',
... line_width)
plot(RF_POWER,RF_POWER,'--k','LineWidth',...

```

```

... line_width); hold off
xlabel('RF Power (dBm)', 'FontSize', label_size)
ylabel('Power Measurement (dBm)', 'FontSize', label_size)
title({'Signal and Noise In S2SA Experiments', ...
... line2_Individual_plots}, 'FontSize', label_size) ...
legend({'Signal', 'Noise', 'Average Noise', 'Noise Estimate', ...
... 'Predicted Power Measurement'}, ...
'FontSize', legend_size1, 'Location', "northwest")
grid on

figure(3002)
plot(-attenuations, 10*log10(1e3*...
... (signal_errorbar+signals)), '* ', ...
... 'MarkerSize', 14); hold on;
plot(-attenuations, 10*log10(1e3*signals), ...
... '* ', 'MarkerSize', 14);
plot(-attenuations, 10*log10(1e3*(signals -...
... signal_errorbar)), '* ', 'MarkerSize', 14);
plot(-attenuations, 10*log10(1e3*noises), ...
... '* ', 'MarkerSize', 14)
yline(10*log10(1e3*mean(noises)), '--b', 'LineWidth', 2); hold off
xlabel('RF Power (dB)', 'FontSize', 14)
ylabel('Power Measurement (dBm)', 'FontSize', 14)
title({'Signal and Noise In S2SA Experiment', ...
... line2_Individual_plots}, 'FontSize', 14)
legend({'Signal Upper Bound', 'Noise', ...
... 'Signal Lower Bound', ...
... 'Average Noise'}, 'FontSize', ...
... legend_size1, 'Location', "northwest")
grid on
else

figure(3001)
plot(-attenuations, 10*log10(1e3*signals), '*r', ...
... 'MarkerSize', Marker_Size); hold on
plot(-attenuations, 10*log10(1e3*noises), ...
... '*b', 'MarkerSize', Marker_Size)
yline(10*log10(1e3*mean(noises)), '--b', ...
... 'LineWidth', line_width);
yline(10*log10(1e3*Noise_power)-noise_est_offset, ':b', ...
... 'LineWidth', line_width); hold off
xlabel('RF Attenuation (dB)', 'FontSize', label_size)
ylabel('Power Measurement (dBm)', 'FontSize', label_size)
title({'Signal and Noise In S2SA Experiments', ...
... line2_Individual_plots}, 'FontSize', label_size)

```

```

legend({'Signal','Noise','Average_Noise',...
... 'Noise Estimate'},'FontSize',...
... legend_size1,'Location','northwest')
grid on

figure(3002)
plot(-attenuations,10*log10(1e3*(signal_errorbar+signals)),'*',...
... 'MarkerSize',14);hold on;
plot(-attenuations,10*log10(1e3*signals),'*','MarkerSize',14);
plot(-attenuations,10*log10(1e3*(signals-signal_errorbar)),'*'...
..., 'MarkerSize',14);
plot(-attenuations,10*log10(1e3*noises),'*','MarkerSize',14)
yline(10*log10(1e3*mean(noises)),'--b','LineWidth',2); hold off
xlabel('RF_Attenuation_(dB)','FontSize',14)
ylabel('Power_Measurement_(dBm)','FontSize',14)
title({'Signal_and_Noise_In_S2SA_Experiment',line2_Individual_plots},'
FontSize',14)
legend({'Signal_Upper_Bound','Noise','Signal_Lower_Bound',...
... 'Average Noise'},'FontSize',legend_size1,'Location','northwest')
grid on
end

%% Save Data
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Step 1: Generate Output Variable Name
if save_data
disp('Saving_Data...')
if strcmp(Method,'Interferometric')
var_name=['Results_IF_',num2str(P_read*1e6),...
... 'uW_read_',num2str(P_LO*1e6),'uW_LO'];
else
var_name=['Results_trad_',num2str(P_read*1e6),...
... 'uW_read'];
end
struct_name = genvarname(var_name);
disp(['Data_Structure_Name:',struct_name])
%% Step 2: Generate File Name
Save_file_name=['./',var_name];
%% Step 3: Pick Values to Save
temp_struct.Attens=attenuations;
temp_struct.signals_complete=signals;
temp_struct.Signal_errorBars=signal_errorbar;
temp_struct.noises_complete=noises
temp_struct.noise_level_mean=mean(noises);
temp_struct.RF_power=RF_power_0;

```



```

if strcmp(Method,'Interferometric')      temp_struct.Calibration_coeff=
    ADC_units*G_t*responsivity*...
... alpha_L*beta*sqrt(P_LO*P_read);
else
temp_struct.Calibration_coeff=ADC_units*G_t*responsivity*...
... alpha_L*beta*P_read;
end
temp_struct.power_read=P_read;
temp_struct.power_LO=P_LO;
temp_struct.power_prog=prog_power;
temp_struct.configuration=sys_config;
temp_struct.resolution_bandwidth=RES_BW;

eval([struct_name,'=temp_struct']);
disp('...')
save(Save_file_name,'-struct',struct_name);
disp('Data Saved!')

```

SSH LIDAR Processing Script

This script was used to process interferometric SSH Lidar data.

```

close all
clear all
addpath('.\S2SA_functions')
Base_folder_name_IF=...
... '\ LO_gain_21-Jun-2022_interferometric_10uW_read_';
directory_trad=...
... '\ LO_gain2_21-Jun-2022_Traditional_10uW_read_';
file_base_IF=...
... 'Grating_dat_with_chirps_BW_1000MHz_delay_300ns_';
file_base_trad=...
... 'Grating_dat_with_chirps_BW_1000MHz_delay_300ns_';
load(horzcat(directory_trad,'\sys_config.mat'));
suffix='.mat';

```

%% System parameters

```

clear LO_powers
clear SNRS
clear Averaged_spectr
clear Noise_levels
clear signal_levels

```

%% Procesing Parameters

```

K=2*10^12; % Chirp Rate

```

```

read_power=10;
LO_powers=[1,2,5,10,20,50,100,200,500,1000];
programed_delay=300;
%% Timing Parameters
FIRST_SAMPLE_base=floor(305*10-5*sys_config.sample_rate);
% for A 1GHz Grating
FIRST_SAMPLE_grating=floor(1.6*10-4*sys_config.sample_rate);
FIRST_SAMPLE=FIRST_SAMPLE_base+FIRST_SAMPLE_grating;
TRACELENGTH=floor(0.4*10-3*sys_config.sample_rate);

MEAN_CHECK_REGION(1)=floor(sys_config.sample_rate*10*10-6);
MEAN_CHECK_REGION(2)=floor(sys_config.sample_rate*50*10-6);

%% filter parameters
Nyquist=sys_config.sample_rate/2;
Wpass=100*103/Nyquist;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Loops
% loop Preperation: Common prep
n_start=1;
n_stop=30;
averaging_trace=0;
window_type=@hanning;
%window_type=@rectwin;
averaging_trace=0;
SNRS=[];
Averaged_spectra=[];
Noise_levels=[];
signal_levels=[];
%% Primary Loop Execution (loop over LO powers)
for j=1:length(LO_powers);
    LO_power=LO_powers(j);
    full_filename_base_if=...
        ... horzcat(Base_folder_name_IF,num2str(LO_power),...
        ... 'uW_LO','\ ',file_base_IF);
    %% Secondary Loop (over shots)
    for l=n_start:n_stop;
        full_filename_if=...
            ... horzcat(full_filename_base_if,num2str(l),suffix);
        load(full_filename_if);
        %% Begin Simple Processing Routine
        % Time Domain
        Measured_data=highpass((Measured_data),Wpass);
        [filtered_signal, time] = ...
        isolate_trace(Measured_data,time_data,...

```

```

... FIRST_SAMPLE, TRACELENGTH );
filtered_signal=filtered_signal-mean( filtered_signal );
filtered_signal=filtered_signal.*window( window_type , ...
... length( filtered_signal ) )';
% frequency Domain
frequency_domain_signal=fftshift( fft( filtered_signal - ...
... mean( filtered_signal ) ) );
spectrum=abs( frequency_domain_signal ).^2;
dF=sys_config.sample_rate/length( frequency_domain_signal );
frequencies=(-Nyquist):dF:( Nyquist-dF );
averaging_trace=averaging_trace+filtered_signal;
[M,I]=max( spectrum.*sign( frequencies ) );
measured_delay=10^9*frequencies(I)/K;
Delta=programed_delay-measured_delay;

[ Signal_level_IF_all(1,j) , Noise_level_IF_all(1,j) , ...
... SNR_IF_all(1,j) ] = ...
... Frequency_domain_SNR_V2( spectrum , sys_config.sample_rate , ...
... 2e6,1e6,0.595e6,0.01e6 );
[ Signal_level_IF_all(1,j) , Noise_level_IF_all(1,j) , ...
... SNR_IF_all(1,j) ] = ...
... Frequency_domain_SNR_V2( spectrum , sys_config.sample_rate , ...
... 2.5e6,1e6,programed_delay*1e-9*K,4*dF );
    end
end
IF_signal_levels=mean( Signal_level_IF_all ,1 );
IF_signal_levels_std=std( Signal_level_IF_all ,1 );

IF_noise_levels=mean( Noise_level_IF_all ,1 );
IF_signal_noise_std=std( Noise_level_IF_all ,1 )
IF_SNR_levels_1=mean( SNR_IF_all ,1 );
IF_SNR_levels_2=IF_signal_levels ./ IF_noise_levels;
Method_diff=max( IF_SNR_levels_1-IF_SNR_levels_2 );

% Loop preperation for different LO Powers
%% Single Loop for Traditional Readout.
Signal_level_trad_all=[];
Noise_level_trad_all=[];
SNR_trad_all=[];

for n=n_start:n_stop;
    full_filename_trad = ..
    ... horzcat( directory_trad , '\', file_base_trad , num2str(n) , suffix );
    load( full_filename_trad )
    %% Begin Simple Processing Routine

```

```

% Time Domain
Measured_data=highpass(( Measured_data ),Wpass);
[ filtered_signal , time] = ...
... isolate_trace( Measured_data ,time_data ,FIRST_SAMPLE,...
..TRACELENGTH );
filtered_signal=filtered_signal-mean(filtered_signal);
filtered_signal=filtered_signal.*window(window_type,...
...length(filtered_signal))';
% frequency Domain
frequency_domain_signal=fftshift(fft(filtered_signal -...
...mean(filtered_signal)));
spectrum=abs(frequency_domain_signal).^2;
dF=sys_config.sample_rate/length(frequency_domain_signal);
frequencies=(-Nyquist):dF:(Nyquist-dF);

averaging_trace=averaging_trace+filtered_signal;
[M,I]=max(spectrum.*sign(frequencies));

measured_delay=10^9*frequencies(I)/K;
Delta=programed_delay-measured_delay;
[ Signal_level-IF_all(1,j), Noise_level-IF_all(1,j), SNR-IF_all(1,j)
]...
= Frequency_domain-SNR-V2(spectrum, sys_config.sample_rate,...
...2 e6,1 e6,0.595 e6,0.01 e6);
[ Signal_level-trad_all(n), Noise_level-trad_all(n), SNR-trad_all(n)
]...
...=Frequency_domain-SNR-V2(spectrum, sys_config.sample_rate,...
...2.5 e6,1 e6,programed_delay*1e-9*K,4*dF);
end

Signal_level-trad=mean(Signal_level-trad_all);
Noise_level-trad=mean(Noise_level-trad_all);
SNR_level-trad=mean(SNR-trad_all);
%%Plots:
LO_power_mW=LO_powers*10^-3;
%% Plot 1: Signal Level
figure(1)
IF_signals=plot(10*log10(LO_power_mW),10*log10(IF_signal_levels),...
... '*','MarkerSize',14); hold on

trad_signal=yline(10*log10(Signal_level-trad),'--r','LineWidth',2);
xlabel('LO_Power(dBm)','FontSize',14);
ylabel('Signal_Level(dB)','FontSize',14);
title('Signal_Level_vs_LO_Power','FontSize',14)
grid on;

```

```

legend([ IF_signals , trad_signal ], { 'Interferometric_Readout', 'Traditional
    _readout' })
hold off ;

```

```

%% Plot 2: Noise Level

```

```

figure(2)
IF_noise=plot(10*log10(LO_power_mW), 10*log10( IF_noise_levels ), '* ',
    MarkerSize', 14); hold on
trad_noise=yline(10*log10( Noise_level_trad ), '--r', 'LineWidth', 2);
xlabel('LO_Power(dBm)', 'FontSize', 14);
ylabel('Noise_Level(dB)', 'FontSize', 14);
title('Noise_Level_vs_LO_Power_for', 'FontSize', 14)
grid on;
legend([ IF_noise , trad_noise ], { 'Interferometric_Readout', 'Traditional_
    readout' })
hold off ;

```

```

%% Plot 3: SNR

```

```

figure(3)
IF_SNR=plot(10*log10(LO_power_mW), 10*log10( IF_SNR_levels_1 ), ...
    ... '* ', 'MarkerSize', 14); hold on
trad_SNR=yline(10*log10( SNR_level_trad ), '--r', 'LineWidth', 2);
xlabel('LO_Power(dBm)', 'FontSize', 14);
ylabel('Signal_to_Noise_Ratio(dB)', 'FontSize', 14);
title('Signal_to_noise_Ratio_vs_LO_Power', 'FontSize', 14)
grid on;
legend([ IF_SNR, trad_SNR ], { 'Interferometric_Readout', 'Traditional_
    readout' })
hold off ;

```

```

LO_power_mW=LO_powers*10^-3;
LO_Gain=LO_powers/read_power;
figure(201)
t = tiledlayout(1,1);
ax1 = axes(t);
IF_signals=plot(10*log10(LO_power_mW), 10*log10( IF_signal_levels ), ...
    ... '*b', 'MarkerSize', 14); hold on
xlabel('LO_Power(dBm)', 'FontSize', 12);
ax2 = axes(t);
IF_signals_2=plot(10*log10(LO_Gain), 10*log10( IF_signal_levels ), ...
    ... '*b', 'MarkerSize', 14);
ax2.XAxisLocation = 'top';
xline(0, ':', '\eta=1', 'LineWidth', 2)
xlabel('LO_gain_Parameter_\eta(dB)', 'FontSize', 12);
trad_signal=yline(10*log10( Signal_level_trad ), '--r', 'LineWidth', 2)

```

```

ylabel('Signal_Level(dB)', 'FontSize', 12);
title('Signal_Level_vs_LO_Power_(5\mu_W_read)', 'FontSize', 14)
grid on;
legend([ IF_signals , trad_signal ] , ...
...{ 'Interferometric_Readout', 'Traditional_readout' })
hold off ;

figure(202)
t = tiledlayout(1,1);
ax1 = axes(t);
IF_noise=plot(10*log10(LO_power_mW), 10*log10( IF_noise_levels ) , ...
... '*b', 'MarkerSize', 14); hold on
xlabel('LO_Power(dBm)', 'FontSize', 12);
ax2 = axes(t);
IF_noise=plot(10*log10(LO_Gain), 10*log10( IF_noise_levels ) , ...
... '*b', 'MarkerSize', 14); hold on;
ax2.XAxisLocation = 'top';

xlabel('LO_gain_Parameter_\eta_(dB)', 'FontSize', 12);
trad_noise=yline(10*log10( Noise_level_trad ), '--r', 'LineWidth', 2)
xline(0, ':', '\eta=1', 'LineWidth', 2)
ylabel('Noise_Level(dB)', 'FontSize', 12);
title('Noise_Level_vs_LO_Power_(5\mu_W_read)', 'FontSize', 14)
grid on;
legend([ IF_noise , trad_noise ] , ...
...{ 'Interferometric_Readout', 'Traditional_readout' })
hold off ;
%title('Noise Level vs LO Power for ', 'FontSize', 14)
figure(203)
t = tiledlayout(1,1);
ax1 = axes(t);
IF_SNR=plot(10*log10(LO_power_mW), 10*log10( IF_SNR_levels_1 ) , ...
... '*b', 'MarkerSize', 14); hold on
xlabel('LO_Power(dBm)', 'FontSize', 12);
ax2 = axes(t);
IF_SNR2=plot(10*log10(LO_Gain), 10*log10( IF_SNR_levels_1 ) , ...
... '*b', 'MarkerSize', 14); hold on
xline(0, ':', '\eta=1', 'LineWidth', 2)
ax2.XAxisLocation = 'top';
xlabel('LO_gain_Parameter_\eta_(dB)', 'FontSize', 12);
trad_SNR=yline(10*log10( SNR_level_trad ), '--r', 'LineWidth', 2);
ylabel('Signal_to_Noise_Ratio_(dB)', 'FontSize', 14);
title('Signal_to_noise_Ratio_vs_LO_Power_(5\mu_W_read)' ...
..., 'FontSize', 14)
grid on;

```

```

legend([IF_SNR,trad_SNR],...
...{'Interferometric_Readout','Traditional_Readout'})
hold off ;

figure(301);
IF_signals1=plot(10*log10(LO_power_mW) ,...
...10*log10(IF_signal_levels) ,...
... '*b', 'MarkerSize',14); hold on
trad_signal1=yline(10*log10(Signal_level_trad),'--b','LineWidth',2);
IF_noise1=plot(10*log10(LO_power_mW),10*log10(IF_noise_levels) ,...
... '*r', 'MarkerSize',14); hold on
trad_noise1=yline(10*log10(Noise_level_trad),'--r','LineWidth',2);
hold off
xlabel('LO_Power_(dBm)', 'FontSize',14');
ylabel('Measured_level_(dB)', 'FontSize',14')
title('Signal_And_Noise_Levels_vs_LO_Power_(1\mu_W_Read)', ...
... 'FontSize',14')
legend([IF_signals1,trad_signal1,IF_noise1,trad_noise1],...
...{'Interferometric_Signal','Traditional_Signal',...
... 'Interferetric Noise','Traditional_Noise'})
grid on
if true
[ Sig ,Sig_error ,Noise ,Noise_error ,SNR_check ,SNR_error_check] =...
...SNR_ERROR_dB( Signal_level_IF_all ,Noise_level_IF_all ,1,1,'dB' );

clear SNR_error_check
SNR_error_check=0*Noise_error+Sig_error;
figure(1001)
errorbar(10*log10(LO_power_mW) ,Sig ,Sig_error , '*'); hold on
errorbar(10*log10(LO_power_mW) ,Noise ,Noise_error , '*');hold off
figure(1002)
errorbar(10*log10(LO_power_mW) ,SNR_check ,SNR_error_check , '*');
end

```

APPENDIX E

SONG LYRICS

My Favorite Things (From The Sound of Physics)¹

Lyrics by Owen Wolfe

To the tune of *My Favorite Things*, from *The Sound of Music* [58] by Richard Rodgers (musical composition) and Oscar Hammerstein (libretto).

Verse 1:

Optical Fibers
and connectorized cables
old lab equipment and optical tables
complex experiments with lasers in rings
these are my favorite optical things!

Verse 2:

Equations and whiteboards
and a change of notation
Cauchy and Riemann help with complex
integration
approximations make everything springs
these are my favorite theoretical things!

Verse 3:

Black Holes and space-time
and tensor contractions
Euler-Lagrange helps extremize actions
tons of equations where Einstein's is king
these are my favorite gravitational things!

Hook:

When the code breaks!
When the error pings!
When my data is bad.
I simply remember my favorite things
and then I don't feel so mad!

Verse 4:

White dwarfs and quasars
and cosmic inflation
galactic spectra and pulsar rotation
orbital forces push dust into rings
these are my favorite astronomical things!

Verse 5:

Crystals in K-space,
topological defects
surface electrons cause plasmon effects,
quantized vibrations cause phonons to sing
these are my favorite condensed matter
things!

Verse 6:

Landau and Lifshitz,
and good explanations,
theorems that don't require painful
derivation,
interesting problems about a ball on a string
these are my favorite educational things!

Hook:

When the code breaks!
When the error pings!
When my data is bad.
I simply remember my favorite things
and then I don't feel so mad!

¹There is no larger work titled *the Sound of Physics*, this was added to the title to distinguish from the original song on which this song is based

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