



An analysis of interdependence in the Montana economy : an input-output study
by Theodore Austin Hoff

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY in Agricultural Economics
Montana State University
© Copyright by Theodore Austin Hoff (1969)

Abstract:

Scope and Method of Study The overall objective of this study was to analyze the interdependence that exists among and between Montana's economic sectors. The Leontief input-output model was used to carry out the analysis. The Montana model consisted of 12 endogenous sectors and 7 exogenous sectors. Secondary data was used to formulate the transactions matrix of the model. The transactions matrix presents the flow of goods and services, measured in dollar terms, between the sectors of the model during the calendar year 1963. The technical coefficients matrix of the endogenous sectors was then computed from the flow table. The technical coefficients matrix was inverted to get the matrix of interdependence coefficients. Sector output and income multipliers were derived from the interdependence coefficients of the model. Methods of adjusting the multipliers to handle special cases of output distribution were developed. A method of isolating the expansionary effects of an increase in personal income which results from increased production levels of the sectors was developed. A method of appraising the direction and degree of interdependence was also developed.

Findings and Conclusions Montana is dependent upon external markets for the sale of agricultural and manufacturing output. These sectors account for 30 percent of the total output of the state and 60 percent of their output is exported. These sectors also have the greatest economic impact with respect to a change in sector production, as indicated by the relative sizes of the sector output multipliers.

The Montana economy exhibits a very low degree of interdependence as compared to the national economy. The triangularized Montana transactions matrix shows that the remaining sectors of the economy are technically independent of the Livestock and Livestock Products sector and the Food and Kindred Products sector. One-way interdependence is also evident in the relationships of the remaining sectors. The amount of one-way interdependence shows that Montana is relatively much less developed than the national economy.

AN ANALYSIS OF INTERDEPENDENCE IN THE MONTANA ECONOMY:
AN INPUT-OUTPUT STUDY

by

THEODORE AUSTIN HOFF

A thesis submitted to the Graduate Faculty in partial
fulfillment of the requirements for the degree

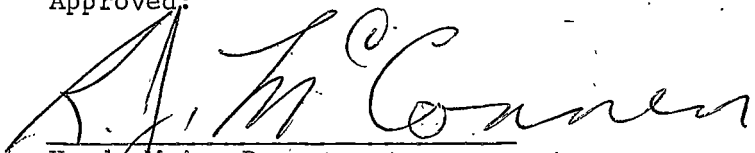
of

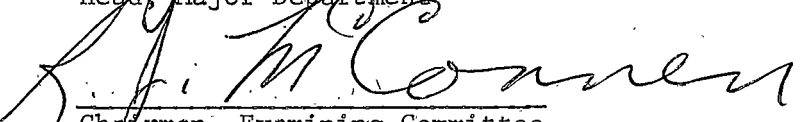
DOCTOR OF PHILOSOPHY

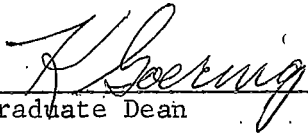
in

Agricultural Economics

Approved:


Head, Major Department


Chairman, Examining Committee


Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

March, 1969

ACKNOWLEDGMENT

The author wishes to express gratitude to the entire staff of the Department of Agricultural Economics, Montana State University, for the very rewarding experiences of the past five years. Special thanks is given to Professor Maurice C. Taylor, who not only has provided excellent professional guidance during my graduate studies, but has also been a warm personal friend. He has given freely of his time and energy in reviewing this manuscript and has provided most helpful comments and suggestions.

Special thanks also goes to Dr. R. J. McConnen, major advisor and Dr. Oscar Burt for the help they have provided during the preparation of this manuscript.

Last but not least I thank my wife, Joan. Her moral support and encouragement, not to mention her monetary support, has been the deciding factor in attaining this degree.

TABLE OF CONTENTS

	<u>Page</u>
CHAPTER I: INTRODUCTION.	1
The Problem	3
The Need for This Study	4
Objectives of This Study.	5
CHAPTER II: THE LEONTIEF INPUT-OUTPUT MODEL.	8
Review of the Literature.	8
Theoretical Explanation of the Model.	13
The Transaction Matrix.	13
The Technical Coefficients.	17
Interdependence Coefficients.	19
Output Multipliers.	21
Income Multipliers.	23
Resource Multipliers.	24
Assumptions of the Model.	24
CHAPTER III: A SURVEY OF THE MONTANA ECONOMY	27
Geographic and Economic Characteristics	27
The Human Resource.	30
Employment and Personal Income.	32
Education	39
CHAPTER IV: THE EMPIRICAL MODEL.	40
The Transactions Matrix	42
The Technical Coefficients.	45
The Interdependence Coefficients.	47
Sector Multipliers.	49
Conclusions.	59
CHAPTER V: INTERDEPENDENCE IN THE MONTANA ECONOMY.	61
Interdependence	61
Comparison of Two Economies	61
Comparability of Multipliers.	63
Comparison of Triangularized Matrices	66
The Four-Quadrant Model	75
Conclusions	78
CHAPTER VI: SUMMARY, IMPLICATIONS, AND LIMITATIONS	81
Summary	81
Implications.	85
Limitations	86

	<u>Page</u>
APPENDICES.	88
Appendix A: Statistical Appendix	89
Appendix B: Consolidation of the U. S. Matrix of Technical Coefficients.	145
LITERATURE CITED.	151

LIST OF TABLES

<u>Table</u>	<u>Page</u>
I DISTRIBUTION OF LAND OWNERSHIP IN MONTANA, 1960.	27
II TOTAL OUTPUT OF MONTANA, 1963, BY SECTOR	31
III EMPLOYMENT AND PERSONAL INCOME, 1963	33
IV GOVERNMENT TRANSFER PAYMENTS, 1963	37
V PERSONAL INCOME, 1958-1963	38
VI TRANSACTIONS TABLE OF THE MONTANA ECONOMY, IN THOUSANDS OF DOLLARS	44
VII TECHNICAL COEFFICIENTS OF THE ENDOGENOUS SECTORS OF THE MONTANA MODEL.	46
VIII INTERDEPENDENCE COEFFICIENTS	48
IX TABLE OF INDIRECT COEFFICIENTS	50
X EFFECTS OF A ONE DOLLAR CHANGE IN DEMAND OF THE FOOD AND KINDRED PRODUCTS SECTOR.	51
XI SECTOR OUTPUT AND INCOME MULTIPLIERS	53
XII COMPARISON OF THE OUTPUT MULTIPLIERS AND PURCHASES FROM THE ENDOGENOUS SECTORS	54
XIII ESTIMATED CHANGE IN OUTPUT AND PERSONAL INCOME THAT WOULD RESULT FROM AN INCREASE OF \$1,000,000 IN FINAL DEMAND FOR LUMBER AND WOOD PRODUCTS SECTOR OUTPUT	55
XIV SECTOR OUTPUT MULTIPLIERS: VALUE WITH HOUSEHOLDS EXCLUDED, HOUSEHOLDS INCLUDED, AND THE DIFFERENCE.	57
XV OUTPUT MULTIPLIERS	64
XVI TRIANGULARIZED MATRIX OF MONTANA TRANSACTIONS.	68
XVII THE TRIANGULARIZED MATRIX OF TRANSACTIONS DERIVED FROM THE U. S. MODEL.	70

TablePage

XVIII	COMPARISON OF SECTOR ORDER OF THE TRIANGULARIZED MATRIX AND THE RANK OF THE OUTPUT MULTIPLIERS	74
XIX	SECTOR PLACEMENT, BY QUADRANT, AND THE TYPE II MULTIPLIER	79

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Simple Transactions Table.	14
2	An Example of a Triangularized Matrix.	67
3	Four Quadrant Format of the Montana Sectors.	76

ABSTRACT

Scope and Method of Study

The overall objective of this study was to analyze the interdependence that exists among and between Montana's economic sectors. The Leontief input-output model was used to carry out the analysis. The Montana model consisted of 12 endogenous sectors and 7 exogenous sectors. Secondary data was used to formulate the transactions matrix of the model. The transactions matrix presents the flow of goods and services, measured in dollar terms, between the sectors of the model during the calendar year 1963. The technical coefficients matrix of the endogenous sectors was then computed from the flow table. The technical coefficients matrix was inverted to get the matrix of interdependence coefficients. Sector output and income multipliers were derived from the interdependence coefficients of the model. Methods of adjusting the multipliers to handle special cases of output distribution were developed. A method of isolating the expansionary effects of an increase in personal income which results from increased production levels of the sectors was developed. A method of appraising the direction and degree of interdependence was also developed.

Findings and Conclusions

Montana is dependent upon external markets for the sale of agricultural and manufacturing output. These sectors account for 30 percent of the total output of the state and 60 percent of their output is exported. These sectors also have the greatest economic impact with respect to a change in sector production, as indicated by the relative sizes of the sector output multipliers.

The Montana economy exhibits a very low degree of interdependence as compared to the national economy. The triangularized Montana transactions matrix shows that the remaining sectors of the economy are technically independent of the Livestock and Livestock Products sector and the Food and Kindred Products sector. One-way interdependence is also evident in the relationships of the remaining sectors. The amount of one-way interdependence shows that Montana is relatively much less developed than the national economy.

CHAPTER I--INTRODUCTION

Taylor has said that Montana is in the process of transition from a pre-industrial state to an industrial state. 1/ He describes the pre-industrial state as follows. It is characterized by a concentration of resources in "primary" industries such as agriculture and mining. The pre-industrial state's population is predominantly rural, with local government units designed to service the rural population. Interdependence of economic activities is also very low. He describes the industrial state as follows. It is characterized by a concentration of resources in secondary and tertiary industries, particularly manufacturing. The population is predominantly urban, and service industries develop to supply the needs of the urban population. Interdependence of economic activities reaches a high level, and government units are designed to serve the needs of an urban population.

The transitional economy lies somewhere between the pre-industrial and industrial economies. The primary industries are declining relative to the secondary and service industries, the population is changing from rural to urban centered, and government units are experiencing a change in the kind of government activity that must be provided.

Business and government leaders in Montana have recognized that Montana is in this transitional period of development. 2/ Organizations

-
- 1/ Maurice C. Taylor, "Montana in Transition," Montana Business Quarterly, Fall 1967, Bureau of Business and Economic Research, University of Montana, Missoula, 1967, p. 9.
- 2/ Montana Industrial Horizons, Vol. 5, No. 4, 1960; Vol. 5, No. 6, 1960; Vol. 8, No. 4, 1963; Vol. 9, No. 4, 1964, State Department of Planning and Economic Development.

have been established by government and businessmen with the goal of promoting economic development in an orderly and desirable pattern. For example, the Montana State Planning Board was originally set up in 1935, but remained in a state of limbo for many years. It was reactivated again on July 1, 1955, for the express purpose of promoting economic development. The State Planning Board was reorganized by action of the 1967 Montana Legislature, and the name changed to the "State Department of Planning and Economic Development," the name reflecting the increased interest in economic development.

City and county Planning Boards have been set up by local government units for the purpose of guiding the economic growth patterns of their localities. Planning institutes 3/ have been held so that the local planning boards can get together with the State Department of Planning and Economic Development to exchange viewpoints and ideas for promoting orderly growth.

Businessmen, through their local Chambers of Commerce, have expressed their opinions and desires concerning economic development. 4/ Montana Interests, Inc., was organized by Montana businessmen for the specific purpose of promoting economic growth in the state. 5/

3/ Montana Planning Institutes, co-sponsored by the Association of Montana Planning Boards and the Bureau of Government Research at the University of Montana. Proceedings have been published by the Bureau of Government Research, University of Montana, Missoula.

4/ "Chamber of Commerce Promotes Economic Development," Montana Industrial Horizons, Vol. 6, No. 1, 1961.

5/ Montana Industrial Horizons, Vol. 4, No. 1, 1968.

They have recognized that economic growth is vitally needed in Montana.

The Problem

Montana's rate of economic growth has been very low compared to the national average since World War II. 6/ Several people have expressed their opinions as to the reasons for the low growth rate and what actions the state can and should take to promote economic growth. 7/ Research projects have been undertaken for the purpose of analyzing the economy of the state to determine the reasons for the slow growth rate and to propose means of improving it. One of these projects, the Upper Midwest Economic Study, 8/ encompasses the entire area in the Ninth Federal Reserve District. The procedure of the Upper Midwest Economic Study was first to make a detailed quantitative description of the regions economy, followed by an analysis of change in the economic structure and relationships in the region. 9/

6/ Taylor, op. cit.

7/ Paul B. Blomgren, "The Montana Economy in Perspective," Montana Business Quarterly, Fall 1962; Maxine Johnson, "Problems of Economic Growth in Montana," Montana Business Quarterly, Winter 1963; and Maurice C. Taylor, op. cit.

8/ James M. Henderson and Anne O. Krueger, National Growth and Economic Change in the Upper Midwest, University of Minnesota Press, Minneapolis, 1965.

9/ John R. Borchert, "Regional Development and Local Planning," Proceedings of the Second Montana Planning Institute, Missoula, Montana, September 16 and 17, 1960, Bureau of Government Research, University of Montana, Missoula.

In August of 1967, the Governor and the State Department of Planning and Economic Development of Montana appointed the Bureau of Business and Economic Research, University of Montana, as the primary contractors for the state planning research and development program. ^{10/} In establishing this program, the state implicitly recognized that the problems of development that Montana is experiencing can be alleviated through proper use of planning. Planning, however, if it is to be of any use, must be based on knowledge of the economic, demographic, and sociological structure of the state. Thus, research in these areas is a prerequisite to formulation of effective development policy.

The Need for the Study

The model used in this study is designed to show the interdependence of economic activities within the state. It is concerned with the aggregate or macro-economic aspects of the economy, not with the micro-economic aspects of the firms that comprise the economy. The model is useful for indicating the present economic relationships and can be used to predict the effects of some types of economic changes in the economy, subject to the framework and assumptions of the model.

^{10/} State of Montana Planning Design Study, Bureau of Business and Economic Research, University of Montana, Missoula, 1968, p. 103.

The types of economic change that can be analyzed by the model are changes in the level of economic activity of the state's industries. As the level of activity changes, either by expansion or contraction of an industry's output, the effects of the change spread throughout the economy to a larger or smaller degree, depending upon the interdependency of the industries within the economy. The total effects of change can be divided into two categories: (1) the initial or direct effects; and (2) the secondary or indirect effects. For example, assume that a manufacturing firm decides to build a new plant in the state. The direct effects of this action include such things as the increase in employment and personal income, an increase in the tax base, and the new demand for intermediate inputs used by the plant and purchased from other firms in the economy. The indirect effects arise as the economy responds and adjusts to the direct effects. An increase in consumer spending will occur, due to the increase in personal income. Firms supplying intermediate inputs to the new plant will expand their activity, again increasing employment, intermediate input demand, etc. Thus, the initial impact of the change is multiplied as the economy adjusts to the new plant.

Objectives of This Study

The over-all goal of this study is to quantify the structural interdependence of the economic elements in the Montana economy, thus providing a means of analyzing the economic structure of the state.

The specific objectives of the study are:

- (1) To formulate an input-output model which will illustrate the structural interdependence of the economic enterprises within the State of Montana;
- (2) To estimate direct and indirect effects of changes in the economy under the assumptions of the model;
- (3) To appraise the degree of economic development exhibited by the state's economy as compared to the economy of the United States;
- (4) To compute the output and income multipliers for the economy;
- (5) To illustrate the usefulness of the model for evaluating economic change.

The Montana model in this study was constructed entirely from secondary data. The statistical appendix to this study, Appendix A, describes the methodology used to determine the elements of the transactions table along with the sources in which data was available for each sector. The model was then used to attain the objectives stated above.

Chapter II presents the review of the literature of input-output models, the theoretical explanation of the model, and the basic assumptions of the model. Chapter III describes the economy of Montana, the resources available to it, and the physical and demographic characteristics of the state. Chapter IV contains the empirical model of the state's economy and a discussion of the effects of changes in output of the state's industries. Chapter V compares the economy

of Montana with the economy of the United States, indicating the differences and similarities of the two economies. Chapter VI presents the summary, implications, and conclusions.

CHAPTER II--THE LEONTIEF INPUT-OUTPUT MODEL

Review of the Literature

Input-output type analysis was first conceived by Francois Quesnay with the publication of his Tableau Economique. Quesnay's Tableau stressed the economic interdependence of economic activities within the firm. 1/

About 100 years later, Leon Walras developed a system of simultaneous equations of an input-output nature. Equations representing the supply of and demand for the factors of production and commodity demand and supply were included. Prices were the variables in the system, thus solution of the system would give the equilibrium price of each and every item. The sheer size of the model precluded finding the solution.

Wassily Leontief is the principal developer of aggregate input-output analysis as we know it today. He formulated the first empirical interindustry model of the United States economy in 1936 2/ and subsequently published the first transactions table for the United States. 3/ This model consisted of 44 sectors based on the year 1929.

1/ William H. Miernyk, The Elements of Input-Output Analysis, Random House, New York, 1965, p. 4.

2/ Wassily Leontief, "Quantitative Input-Output Relations in the Economic System of the United States," The Review of Economics and Statistics, XVIII, August 1936, pp. 105-125.

3/ Wassily Leontief, The Structure of the American Economy, 1919-1939, Harvard University Press, Cambridge, Massachusetts, 1941.

The lack of high speed electronic computers made the mathematical calculations required to determine the inverse matrix impossible to complete, so the model was aggregated to a workable size, allowing the computation of the inverse matrix. As a check on the accuracy and reliability of the model, "reverse" projections were made to 1919, estimating the 1919 output. This estimate and the actual 1919 output compared very favorably, with the differences attributed mainly to resource changes.

The advent of the high speed electronic computer enabled input-output researchers to construct models that were disaggregated to a much larger extent. For example, a 450 sector model was constructed for 1947. 4/ This study was used as the basis for many regional studies. The Bureau of Labor Statistics constructed an 87 sector model for 1958. 5/ This is the most recently completed model of the United States economy. Present government plans call for construction of an input-output model every five years. 6/

Input-output analysis has been used extensively by many countries, principally by national planning and economic development committees. 7/

4/ Duane M. Evans and Marvin Hoffenberg, "The Interindustry Relations Study for 1947," The Review of Economics and Statistics, XXXIV, May 1952, pp. 99-142.

5/ M. R. Goldman, M. L. Marimont, and B. N. Vaccara, "The Inter-industry Structure of the United States--A Report on the 1958 Input-Output Study," Survey of Current Business, Vol. 44, No. 11.

6/ Ibid.

7/ Miernyk, op. cit., p. 79.

Its use has been generally limited to the developed economies where data is more readily available from secondary sources. Input-output analysis could be used to great benefit in analyzing under-developed economies to determine proper economic development policy. Lack of adequate data, however, usually prevents construction of empirical models.

Input-output analysis has become widely used for regional economic analysis since about 1951, starting with Walter Isards study. 8/ Studies have been conducted on county, multi-county, state, and multi-state areas. The Colorado River Basin Input-Output study 9/ is an example of the latter type, and is the largest regional study undertaken so far. At the present time, at least 54 regional studies have been completed with an additional 33 currently in progress. 10/

There are two avenues of approach when utilizing an input-output model in a regional study. The first entails applying the national

8/ Walter Isard, "Inter-Regional and Regional Input-Output Analysis: A Model of a Space Economy," Review of Economics and Statistics, XXXIII, November 1951, pp. 318-328.

9/ William H. Miernyk, Bernard Udis, and Clyde Stewart, The Colorado River Basin Input-Output Study, Sponsored by the U. S. Public Health Service.

10/ Philip J. Bourque and Gerald Hansen, An Inventory of Regional Input-Output Studies in the United States, Occasional Paper 17, Graduate School of Business, University of Washington, Seattle, 1967.

coefficients to the region, adjusting them to the economy of the region. Miernyk 11/ discusses one method, that of adjustment according to a location quotient. Other methods have also been used, some explained in rather vague terms. 12/

The other approach to formulating a regional model is to generate a transactions table from regional primary and secondary data. In some studies the data has been obtained via survey of businesses within the region. 13/ In others, data has been collected from secondary sources. 14/ Data collection via survey will likely yield the most accurate information with the added advantage of the data being in a directly usable form. The disadvantage of the survey method is that, considering the large number of business establishments involved, it is extremely costly and time consuming.

Utilizing secondary sources of data, on the other hand, is much less expensive. Bureau of the Census publications provide a wealth

11/ William H. Miernyk, "Long-Range Forecasting with a Regional Input-Output Model," Western Economic Journal, Vol. VI, No. 3, June 1968, pp. 165-176.

12/ Ibid., p. 166.

13/ Werner A. Hirsh, "Interindustry Relations of a Metropolitan Area," The Review of Economics and Statistics, XLI, November 1959, pp. 360-369.

14/ Frederick T. Moore and James W. Peterson, "Regional Analysis: An Interindustry Model of Utah," The Review of Economics and Statistics, XXXVIII, November 1955, pp. 368-381.

of information. There are also many other government publications of a statistical nature that will provide useful information. This method of data collection is relatively inexpensive, but there is a disadvantage in that the data is not given in exactly the form needed for that model, therefore it must sometimes be manipulated in order to put it in usable form. 15/

Input-output models can be used to analyze a particular industry in depth. This type of model would contain several sectors for the industry under study with a high degree of aggregation of the remainder of the economy. This has been done in several studies which emphasized agriculture. For example, Martin and Carter: 16/ emphasized the role of agriculture in the California economy. Their model contained 10 agricultural sectors and 27 non-agricultural sectors. On the other hand, the national model of 1958 contained only 2 agricultural sectors, but had 53 manufacturing sectors out of a total of 87 sectors. Other examples of input-output studies which emphasized agriculture are listed in the Literature Cited, references (59), (63), and (65).

15/ There has been some discussion among leading proponents of input-output analysis about government data publications. Government data collection agencies could go far toward alleviating data problems by collecting data in a form that would lend itself to input-output analysis, particularly in the various census publications.

16/ William E. Martin and Harold O. Carter, A California Interindustry Analysis Emphasizing Agriculture; Part I: The Input-Output Model; Part II: Statistical Supplement, University of California, Giannini Foundation Research Report No. 250, Berkeley, 1962.

Theoretical Explanation for the Input-Output Model

The input-output model consists of three basic parts: (1) the transactions or flow matrix; (2) the matrix of technical coefficients; and (3) the matrix of direct and indirect coefficients, called the matrix of interdependence coefficients. The transactions matrix is the heart of the model. The matrix of technical coefficients is derived from the transactions matrix, and the matrix of direct and indirect coefficients is derived from the matrix of technical coefficients.

The Transactions Matrix

Goods and services are produced for one of two purposes; they are either consumed by the public or they serve as an input for the production of other goods and services. The transactions matrix shows this distribution of goods and services among the sectors defined in the model.

As an example, assume a very simple economy with three producing sectors and one primary input. The transactions are expressed in dollar terms. The output of any one sector will either be used by that sector to produce more goods, serve as an input to one of the other producing sectors, or be purchased as a consumption item. These transactions can be represented by a system of equations:

$$X_1 = x_{11} + x_{12} + x_{13} + y_1$$

$$X_2 = x_{21} + x_{22} + x_{23} + y_2$$

$$X_3 = x_{31} + x_{32} + x_{33} + y_3$$

$$L_0 = L_{01} + L_{02} + L_{03} + y_0$$

where: X_i = gross output of the i^{th} sector;

L_0 = total primary input;

x_{ij} = output of the i^{th} sector purchased by the j^{th} sector and used to produce X_j ;

L_{0j} = primary resource purchased by the j^{th} sector to produce X_j ;

y_i = output of the i^{th} sector used to satisfy final demand;

y_0 = the amount of the primary resource purchased by final demand.

Figure 1 represents this system of equations in the form of a transactions table.

		Purchasing Sectors			Final Demand	Total Output
		1	2	3		
Producing Sectors	1	x_{11}	x_{12}	x_{13}	y_1	X_1
	2	x_{21}	x_{22}	x_{23}	y_2	X_2
	3	x_{31}	x_{32}	x_{33}	y_3	X_3
Primary Input		L_{01}	L_{02}	L_{03}	y_0	L_0
Total Input		X_1	X_2	X_3		

Figure 1. Simple Transactions Table.

Sectors 1, 2, and 3 comprise the processing or endogenous sectors, producing the goods and services for the economy. There are an equal number of rows and columns in the processing portion, with row i showing sector i sales and column i showing sector i purchases. The transactions table is similar to a double entry bookkeeping system inasmuch as the sum across the row will equal the sum across the column for each of the producing sectors. Elements within the processing sector show how much each sector purchases from every other sectors in order to produce its output. Alternatively, they show sales of each sector to the other sectors.

The primary input row shows the amount of primary inputs purchased by each sector. In this simple model, the primary input can be thought of as the human input. In an empirical model, primary inputs could be broken down into as many rows as desired. Typically, households, government, and imports comprise the primary input sectors.

The final demand sector shows how much output is purchased as a final demand item from each of the producing sectors. As with the primary input sector, the final demand sector can be expanded to include as much detail as desired. Typically, the final demand sectors include household or consumer demand, government demand, and exports.

The processing sectors are represented by both a row and a column, thus showing both input and output distribution. This need not be the case with the non-processing or exogenous sectors. The final demand sectors are independent of primary inputs in this model. They

are related inasmuch as payment received for primary inputs represents income which in turn is used to purchase final demand items, but there is not the direct relationship of the rows and columns as in the endogenous sectors.

When applying the input-output model to a specific research problem, the placement of each sector either in the endogenous or exogenous partition of the transaction matrix is determined by the researcher. In some cases he may wish to include a given sector in the endogenous partition. In others, he may put it in the exogenous section. The placement will be determined by the information the researcher wants to generate.

Endogenous sectors are in essence intermediate production activities responding to the demands of the final demand sector. As such, their production levels change only in response to stimuli from external sources. Exogenous sectors, on the other hand, are autonomous. They do not react to, but instead provide, the economic stimuli that causes change in the system. Thus the level of the production activities of the endogenous sectors are determined by the demands placed upon them by the exogenous sectors.

There are two types of input-output models, the "open" model and the "closed" model. The "open" model includes both endogenous and exogenous sectors while the "closed" model has only endogenous sectors. The "closed" model would imply a closed economic system with no external economic forces acting on it. The system is assumed to be in equilibrium and will remain in this state. An "open" model, however,

equilibrates to the exogenous final demand sectors. The exogenous sectors therefore determine the economic activity that takes place in the system in accordance with the assumptions of the model. 17/

Technical Coefficients

A technical coefficient a_{ij} is defined as the dollar amount of output of sector i required to produce one dollar of output in sector j . They are derived from the transactions table under the assumption that the output of a sector i is a linear function of the inputs purchased by sector i . Using the notation on page 8, the production function of the j^{th} sector can be written as

$$X_j = \sum_i x_{ij} + L_{0j}$$

The technical coefficients a_{ij} are derived by dividing both sides of the equation by X_j . Thus,

$$1 = \frac{X_j}{X_j} = \sum_i \frac{x_{ij}}{X_j} + \frac{L_{0j}}{X_j} = \sum_i a_{ij} + A_{0j}^*$$

where: $a_{ij} = \frac{x_{ij}}{X_j}$ and $A_{0j}^* = \frac{L_{0j}}{X_j}$. Also, $x_{ij} = a_{ij} X_j$.

Now substituting for x_{ij} in the system of equations on page 14, we can rewrite the equations as

$$X_i = \sum_j x_{ij} + y_i = \sum_j a_{ij} X_j + y_i$$

17/ The assumptions of the model are discussed on page 24 of this chapter.

The system of equations can be written in matrix form as

$$X = AX + Y$$

Where: X = the vector of gross output with elements X_i ,
 $i = 1, \dots, n$

A = the $n \times n$ matrix of a_{ij} , $i, j = 1, \dots, n$

Y = the vector of final demand with elements y_i ,
 $i = 1, \dots, n$

The equation $X = AX + Y$ can be rewritten in the form

$$Y = X - AX$$

The term by term expansion of this equation is

$$y_i = X_i - \sum_j a_{ij} X_j$$

In words, the output of sector i available for final demand y_i is equal to gross output (X_i) minus the output of sector i required by other sectors as intermediate inputs ($\sum_j a_{ij} X_j$).

Factoring the matrix equation $Y = X - AX$ results in the equation $Y = (I-A) X$. The square matrix $(I-A)$ is known as the "Leontief Matrix" and has some special properties. Recall that $a_{ij} = \frac{x_{ij}}{X_j}$, hence the a_{ij} 's are $0 \leq a_{ij} < 1$. Therefore, the elements of $(I-A)$, which we will denote as a'_{ij} , have the following properties:

- (1) $0 \leq a'_{ij} < 1$ for all $i = j$
- (2) $-1 < a'_{ij} \leq 0$ for all $i \neq j$
- (3) $0 \leq \left| \sum_i a'_{ij} \right| < 1$ for all j

these properties are sufficient to insure that the matrix $(I-A)^{-1}$ exists. ^{18/} Therefore the equation $Y = (I-A) X$ can be premultiplied by $(I-A)^{-1}$ to get the equation

$$X = (I-A)^{-1}Y$$

The matrix $(I-A)^{-1}$ is the matrix of interdependence coefficients. To aid the exposition, this matrix will later be referred to as the matrix B.

Interdependence Coefficients

The interdependence coefficients indicate the total change in input requirements of all sectors due to a unit change in demand for the output of any sector or group of sectors. Thus they indicate both direct and indirect effects. Direct effects are defined as the initial changes in gross output of each sector required to meet the change in final demand of a given sector. Indirect effects are the remaining changes that take place as the system adjusts to the initial change.

To illustrate the use of the interdependence matrix, consider the equation of the model in the form

$$X = (I-A)^{-1}Y$$

Assume a vector ΔY which shows a change in one or more of the elements of the final demand vector Y. The resultant system will then be

^{18/} G. Hadley, Linear Algebra, Addison-Wesley, Reading, Massachusetts, 1961, p. 118.

$$(X + \Delta X) = (I-A)^{-1} (Y + \Delta Y)$$

where ΔX is the vector of change in gross output due to the change in the final demand vector. Distributing the right side of the equation yields

$$X + \Delta X = (I-A)^{-1}Y + (I-A)^{-1}\Delta Y$$

Subtracting the original equation $X = (I-A)^{-1}Y$ from the above, we get $\Delta X = (I-A)^{-1}\Delta Y$. The elements of the vector ΔX are the changes that occur in the output of each sector of the model when there is a change in final demand in one or more sectors.

The $(I-A)^{-1}$ matrix is convergent, therefore it can be derived from a series expansion of $I-A^{-\infty}$. ^{19/} The definition of convergence is $\lim_{n \rightarrow \infty} A^n = 0$, A is a square matrix. If A is convergent, which is true of the matrix of a_{ij} 's of the Leontief input-output model, then the following equation is also true.

$$I = I-A^{-\infty} = (I-A)(I + A + A^2 + A^3 + \dots)$$

Premultiply both sides of this equation to get

$$(I-A)^{-1} = I + A + A^2 + A^3 + \dots$$

The series expansion can be substituted for the $(I-A)^{-1}$ matrix in the equation $\Delta X = (I-A)^{-1}\Delta Y$ to obtain the equation

^{19/} A detailed discussion of matrix convergence and the calculation of $(I-A)^{-1}$ via a series expansion is presented in Literature Cited references numbered 142, 146, 147, and 25.

$$\Delta X = (I + A + A^2 + A^3 + \dots + A^\infty)\Delta Y$$

Partially distributing the right side of this equation results in

$$\Delta X = (I + A)\Delta Y + (A^2 + A^3 + \dots + A^\infty)\Delta Y$$

In this equation, ΔX is shown as the sum of two components. The component $(I + A)\Delta Y = \Delta Y + A\Delta Y$ contains the initial change in final demand ΔY plus the initial adjustment required in the form of increased inputs from the remaining sectors. The remaining component $(A^2 + A^3 + \dots + A^\infty)\Delta Y$ contains the remaining rounds of adjustment before the system again reaches an equilibrium position. The changes represented by

$$\Delta Y + A\Delta Y$$

are called the direct effects, and the remainder are called the indirect effects. 20/

Output Multipliers

An output multiplier is defined as the ratio of change in gross output per dollar change in final demand. An output multiplier can be calculated for each sector, and an aggregate output multiplier would be a weighted sum of the sector multipliers.

20/ Hollis B. Chenery and Paul B. Clark, Interindustry Economics, John Wiley and Sons, New York, 1959, p. 52.

The output multipliers are derived from the $(I-A)^{-1}$ matrix.

In order to facilitate the discussion of the derivation, the following notation will be used:

- (1) ΔX is the vector showing change in gross output of each sector;
- (2) ΔY is the vector of change in final demand;
- (3) B is the matrix $(I-A)^{-1}$ with elements b_{ij} ;
- (4) B^i is the i^{th} column vector of B .

From the preceding section, we have the equation

$$(5) \quad \Delta X = (I-A)^{-1} \Delta Y = B \Delta Y$$

The term by term expansion of (5) is

$$(6) \quad \Delta X_i = \sum_j b_{ij} \Delta y_j$$

Assume that all y_j in Y are zero except $y_k \neq 0$. Then (6) becomes

$$(7) \quad \Delta X_i = b_{ik} \Delta y_k$$

since the remaining elements of the summation are zero. Using vector notation, this becomes

$$(8) \quad \Delta X = B^k \Delta Y = B^k \Delta y_k$$

The total change in gross output is the column sum of the vector X .

Therefore, summing the vectors of both sides of equation (8) we get

$$(9) \quad \sum_i \Delta X_i = \Delta y_k \sum_i b_{ik}$$

Letting $y_k = \$1$, the k^{th} sector output multiplier is $\sum_i b_{ik}$. To illustrate the use of the output multiplier, assume $\Delta y_k = \$15$, and $\sum_i b_{ik} = 2.3$. Then gross output, $\sum_i \Delta X_i$, would increase by $2.3 \times 15 = \$34.50$. The elements of ΔX show the increase in output of each sector.

Income Multipliers

Personal income is all income received by individuals in Montana. An income multiplier is defined as the ratio of change in total personal income per \$1 change in final demand for each sector. As in the case of the output multipliers, an income multiplier can be calculated for each sector. The derivation of the income multiplier is similar to the derivation of the output multiplier but involves one more matrix, P.

P is a square matrix where a diagonal element P_{ii} is the proportion of personal income derived per dollar of output of sector i. All off-diagonal elements are zero. Using the notation of the preceding section, we can set up the equations:

$$(10) \quad \Delta X = B \Delta Y$$

$$(11) \quad P \Delta X = P B \Delta Y$$

matrices P and B being of the same order. Let all elements $\Delta Y = 0$ except for the element $\Delta y_k \neq 0$. Following the procedure of (6), (7), (8), and (9), equation (11) reduces to

$$(12) \quad P \Delta X = y_k (PB)^k$$

where $(PB)^k$ is the k^{th} column vector of (PB) . Summing both sides of equation (12) we get

$$(13) \quad \sum_i (P\Delta X)_i = \Delta y_k \sum_i (Pb)_{ik}$$

The left side of equation (13) is the total change in personal income. Thus, the sum of the elements of $(PB)^k$ is the k^{th} sector personal income multiplier.

Resource Multipliers

Resources are utilized as inputs for production of each sector in combination with the inputs purchased from the remaining sectors. They may be explicitly stated in the model as in the case of labor, or they may be implicitly assumed. Water requirements are an example of the latter.

Resource multipliers are defined as the ratio of change in total resource requirements per \$1 change in final demand. The mathematics of the derivation of resource multipliers is exactly the same as for income multipliers, with the matrix P being replaced by the Matrix R , where r_{ii} is the amount of the resource required to produce a dollar of output in sector i , and the off-diagonal elements of R are zero.

Assumptions of the Leontief Input-Output Model

There are three basic assumptions of the Leontief model: 21/

- (1) Each commodity is supplied by a single industry of sector production;

21/ Ibid., p. 22.

- (2) The inputs purchased by each sector are a function only of the level of output of that sector;
- (3) The total effect of carrying on several types of production is the sum of the separate effects.

The first assumption rules out the possibility of purchasing a particular product from more than one sector. It also assumes that the output of a sector, when it produces two different types of goods, is always produced in the same ratio of the two goods. Thus each unit of output always contains the same proportion of the particular products produced by that sector.

The second assumption implies that a unit of output always requires the same proportions of inputs, thus eliminating the possibility of substitution of one input for another. The input coefficients are constant, and any expansion of output of a particular sector requires a proportionate increase in the inputs to that sector.

The third assumption rules out the possibility of external economies and diseconomies of scale. The second and third assumptions taken together insure that the production function is also linear.

The result of assumptions 2 and 3 above is the further assumption that the technical coefficients of the model are constant. The validity and reasonableness of this assumption has been argued extensively. Chenery and Clark discuss the pros and cons of this assumption and conclude that it is not unrealistic in the short run, periods of perhaps five years or less. 22/ Forecasts for more than

22/ Ibid., pp. 33-42.

five years ahead made on the basis of the model should be treated with caution. However, this is a fault common to most, if not all, predictive models.

The application of an input-output analysis for the purpose of analyzing the structure of an economy also requires the assumption that there are no errors of aggregation. That is, the correct intersectoral transactions are recorded in the transactions table.

Whether there are errors or not is extremely difficult to evaluate.

However, a testing procedure could be devised to determine the probable accuracy of the estimates. Quandt 23/ discusses the possibility of treating the a_{ij} 's as point estimates of the mean of a distribution, using standard statistical techniques to test the estimates. As yet there have not been any empirical input-output studies undertaken using this approach.

23/ Richard E. Quandt, "On the Solution of Probabilistic Leontief Systems," Naval Logistics Quarterly 6, No. 4, December 1959, and "Probabilistic Errors in Leontief Systems," Naval Logistics Quarterly 5, No. 2, June 1958.

CHAPTER III--A SURVEY OF THE MONTANA ECONOMY

An input-output model is a tabular description of the economic activities of the region under study. The degree of aggregation of the model and the composition of the sectors of the model will depend upon the characteristics of the economy. Thus, an overview of the Montana economy is needed to properly design the Montana input-output model and to evaluate the results.

Geographic and Economic Characteristics

There are approximately 93,600,000 acres in Montana. Table I shows the ownership distribution of the land resource as it was in 1960.

TABLE I. DISTRIBUTION OF LAND OWNERSHIP IN MONTANA, 1960.*

Ownership	Land	
	Acres	Percent
1. Private ownership	61,166,000	65.4
2. State ownership		
Grazing land	4,195,000	4.5
Agricultural land	487,000	.5
Timber & special grants	496,000	.5
Total State Land	5,178,000	5.5
3. Federal ownership		
Forest Service, forest	16,635,000	17.8
Forest Service, L. U.	1,936,000	2.1
Park Service	1,141,000	1.2
Bureau of Land Management	6,659,000	7.1
Other	885,000	.9
Total Federal Land	27,256,000	29.1
4. Total All Land Area	93,600,000	100.0

*Source: Montana Agriculture Basic Facts, Cooperative Extension Service and Agr. Expt. Sta., Bulletin 293, Montana State University, Bozeman, 1962, p. 13.

The topography of the land resource varies from the flatlands and rolling hills of eastern Montana to the tree-covered mountains of western Montana. Agricultural land includes about 13.5 million acres classed as tillable land with about 40 million acres of grazing land. 1/

The major share of the tillable land is used in dryland crop farming in the Great Plains Area of eastern and central Montana. Annual precipitation averages 14 inches, but has varied from a low of 8 inches to as much as 20 inches. 2/ The growing season, considered to run from the last frost in spring to the first frost in fall, is about 110 days on the average, but, like average precipitation, varies considerably from year to year and from location to location. 3/ Because of these climatic conditions, farmers in Montana are rather restricted as to the species of crops that can be produced economically. Wheat and barley are the principal cash crops, owing to their adaptability to the limiting climatic conditions.

The major livestock enterprise in the state is the range-based cow-calf operation, producing feeder calves for export. At the present

1/ State of Montana, Twenty-First Biennial Report of the Montana State Board of Equalization, Tribune Printing, Great Falls.

2/ Montana Agriculture Basic Facts, Cooperative Extension Service and Agr. Expt. Sta., Bulletin 293, Montana State University, Bozeman, 1962, p. 11.

3/ Montana State Department of Agriculture and the U. S. Department of Agriculture, Statistical Reporting Service, Montana Agricultural Statistics, Vol. X, Tribune Printing, Great Falls, 1965, p. 16.

time, there is relatively little cattle feeding in the state, with an average of 50,000 head on feed in 1963. 4/ Sheep and lamb enterprises provide the second largest share of livestock products, generating about 10 percent of livestock products cash receipts. As in the case of cattle, the sheep operations are range-based with the major product being feeder lambs.

Commercial forestland in the state is located mainly in western Montana. Altogether, commercial forest area amounts to 17.3 million acres of softwood saw timber with virtually no hardwood timber. 5/ Montana ranks second in value of lumber production in the Rocky Mountain States. 6/ The lumber industries provide considerable employment in Montana, and lumber products are one of the principal exports from the state.

There are extensive mineral operations in the state, with copper mining and petroleum extraction the main industries. Mineral industries, both mining and manufacturing, play very important roles in the economy

4/ U. S. Department of Agriculture, Statistical Reporting Service, Livestock and Meat Statistics, 1963, Supplement to Stat. Bul. 333, Washington, D. C., p. 14.

5/ Alvin K. Wilson and John S. Spencer, Jr., Timber Resources and Industries in the Rocky Mountain States, U. S. Forest Service Sta. Resources Bul. INT-7, 1967. Intermountain Forest and Range Expt. Sta., Ogden, Utah, p. 36.

6/ Ibid., p. 23.

of the state. 7/ Mineral leases on public lands produce in excess of \$5 million per year in Montana. 8/

Industrial activity is located primarily around Great Falls and Billings, with the major industrial activities consisting of primary metals refining and petroleum refining. 9/ Wood products plants are scattered throughout western Montana with the largest operations in Missoula County. 10/

Table II shows the total output of the state by sector. Manufacturing produces the greatest share of the total output, followed by agriculture and trade. Agriculture, mining, and manufacturing are the primary industries in the state, with the remaining sectors acting in supporting roles.

The Human Resource

The 1963 population in Montana was estimated to be 701,000 people, increasing at an average annual rate of .9 percent per year. 11/ The

7/ U. S. Department of Commerce, Bureau of the Census, Census of Mineral Industries, 1963, Area Statistics: Montana; U. S. Government Printing Office, Washington, D. C., 1967.

8/ U. S. Department of Interior, Bureau of Land Management, Public Land Statistics, 1963, Washington, D. C., 1964.

9/ U. S. Department of Commerce, Bureau of the Census, Census of Manufactures, 1963, Vol. III: Area Statistics.

10/ Ibid.

11/ U. S. Department of Commerce, Bureau of the Census, Statistical Abstract of the U. S., 1966, Washington, D. C., 1966.

TABLE II. TOTAL OUTPUT OF MONTANA, 1963, BY SECTOR.

Sector	Total Output a/	
	Thousand Dollars	Percent
Agriculture		
Livestock and livestock products	206,856	8.0
Crops	307,985	12.0
Manufacturing		
Food and kindred products	140,885	5.4
Lumber and wood products	168,504	6.6
Other manufacturing	365,533	14.3
Transportation and Public Warehousing	156,056	6.1
Communications and Public Utilities	132,840	5.2
Real Estate, Finance and Insurance	143,398	5.6
Mining	173,121	6.7
Services	206,777	8.1
Trade, Wholesale and Retail	374,734	14.6
Construction	189,503	7.4
Total	2,566,192	100.0

a/ Total output of each sector by product class is discussed in Appendix A.

population has shifted from predominantly rural in 1935 to over 50 percent urban in 1963. ^{12/} Along with the location shift, there has been a change in the age distribution of the population. In 1930, nearly 60 percent of the population was in the 18-64 year old age group. By 1960, this had decreased to 52 percent. During this period, total population increased by 25 percent, but the 18-64 age group increased by only 10 percent. ^{13/} This shift in age composition has increased the need for public facilities (schools, hospitals, etc.) beyond the need due simply to population growth.

Employment and Personal Income

Table III shows the distribution of employment and personal income in Montana in 1963. The trade sector, which includes all wholesale and retail establishments, employs the greatest number of people and also provides the greatest amount of personal income. There were 9,387 trade establishments in Montana in 1963, of which 1,864 had no payroll. The average number of employees on a payroll was five. Total wages and salaries in the trade sector amounted to \$151,716,000, proprietor income comprising the remainder of personal income earned in this sector.

Agriculture ranked second with respect to personal income but ranked seventh with respect to employment. The average size farm in

^{12/} Montana Agriculture Basic Facts, op. cit., p. 5.

^{13/} Ibid.

TABLE III. EMPLOYMENT AND PERSONAL INCOME, 1963.*

Sector	Employment a/ (Thousandths)	Personal Income b/	
		Thousand Dollars	Percent
Agriculture	13.0	201,008	12.7
Manufacturing	22.4	153,512	9.7
Transportation	12.1	92,148	5.8
Communications & Public Utilities	5.8	45,808	2.9
Real Estate, Finance & Insurance	6.8	49,565	3.1
Mining	7.1	58,377	3.7
Services	24.1	134,073	8.4
Trade	40.3	227,980	14.3
Construction	12.3	89,781	5.7
State and Local Government	23.4	131,000	8.2
Federal Government	10.0 <u>c/</u>	141,619 <u>d/</u>	8.9
Other	.6	108,147 <u>e/</u>	6.8
Total	163.6	1,433,118 <u>f/</u>	90.2
Federal and State Transfers <u>g/</u>		154,982	9.8
Total Personal Income		1,588,000	100.0

a/ Average number employed during the calendar year.

b/ Includes wages and salaries, proprietor income, property income, and transfers.

c/ Excludes military employment.

d/ Includes military payments.

e/ Includes rent payments, and miscellaneous income.

f/ Total earned income in Montana.

g/ Social Security, welfare, GI Bill, Farm Programs, etc.

*Source: Ninth Federal Reserve District Statistical Review

Montana was 2,436 acres in 1964. 14/ Since 1935 the trend has been toward larger and larger farms with fewer farm numbers. This has been particularly the case in the Great Plains dryland farming region of the state. The 1964 Census of Agriculture indicates a total of 27,020 farms, but only 21,044 had sales of farm products of \$2,500 or more. The size of farms and the type of crops and livestock produced lead to rather capital intensive farm operations with correspondingly low labor requirements.

Manufacturing industries ranked in third place both in employment and personal income. The lumber and wood products industry employed the greatest number of people, accounting for 36 percent of total manufacturing employment and 25 percent of total manufacturing output. 15/ The food and kindred products industry was second in manufacturing employment with 25 percent of the total, and food products output was about 20 percent of total manufacturing output. 16/ The primary metals industry, principally non-ferrous metal refining, ranked third in manufacturing employment with 16 percent of the total employment, but produced over 30 percent of the total output of the manufacturing sector. 17/ Similarly, the petroleum refining industry accounted for

14/ U. S. Department of Commerce, Bureau of the Census, Census of Agriculture, 1964, Statistics of the State and Counties.

15/ Census of Manufactures, op. cit.

16/ Ibid.

17/ Ibid.

only 4 percent of employment but over 15 percent of output. 18/ Both of the latter industries are capital intensive with relatively low labor requirements per unit of output. The remaining manufacturing industries in the state are relatively small, with combined employment equal to 19 percent of the total and output equal to only 10 percent of total manufacturing production.

The government sectors in Montana are very important sources of employment, providing jobs for over 20 percent of the civilian work force. State and local government employees included 12,043 people employed by educational institutions at all levels in 1962. 19/ Government wages and salaries, including military pay, accounted for 17.1 percent of personal income in the state.

Service industries, transportation, and construction each employed approximately the same number of people in 1963, although the service industries received a higher share of personal income than the others. This is to be expected, considering that professional incomes of doctors, lawyers, business consultants, consulting engineers, etc., are included in personal income of service industries.

The communications and public utilities sector employs about the same number of people as the real estate, finance, and insurance sector. Each of these sectors account for about 3 percent of total personal income.

18/ Ibid.

19/ U. S. Department of Commerce, Census of Government, 1962, Vol. III, No. 26, Government in Montana, Washington, D. C., 1964.

A rather surprising statistic is the large share, 9.8 percent, of personal income received as government transfer payments. Table IV shows the composition of transfer payments. State and local welfare payments include payments made under all programs administered by state and local agencies. Unemployment commission benefits were principally unemployment compensation claims. Agricultural programs payments include all payments associated with agriculture programs administered by the Montana State Office of the Agricultural Stabilization and Conservation Service. Social security benefits are self-explanatory. Veterans benefits include compensation and pensions, insurance and indemnities, vocational rehabilitation, education and training assistance, and miscellaneous benefits. Other government programs include ex-servicemen's unemployment payments, ex-federal employee unemployment payments, individual research grants, and the like.

Table V shows personal income of Montana for the period of 1953-1963. Total personal income increased almost 45 percent during the period, while per capita personal income increased only 26 percent. During the same period, personal income of the U. S. increased 62 percent and per capita income increased 36 percent. ^{20/} Montana ranked twenty-second in per capita income in 1953, but slipped to twenty-ninth by 1963. ^{21/} These figures indicate that Montana's rate of economic growth

^{20/} Survey of Current Business, Vol. 48, No. 4, pp. 13-14.

^{21/} Ibid.

TABLE IV. GOVERNMENT TRANSFER PAYMENTS IN MONTANA, 1963.

Source	Amount Thousand Dollars
State and Local	
Welfare payments <u>a/</u>	11,695 <u>a/</u>
Unemployment commission benefits <u>b/</u>	7,359 <u>b/</u>
Federal	
Agricultural programs payments <u>c/</u>	26,400 <u>c/</u>
Social Security benefits <u>d/</u>	61,689 <u>d/</u>
Veterans benefits	25,345
Other Government Programs	<u>22,494</u>
Total	<u>154,982</u>

- a/ Budget for the Biennium July 1, 1963 to June 30, 1965, Budget submitted by Tim Babcock, Governor, to the Montana Legislature, 1963 session, Literature Cited Reference 65, p. 155.
- b/ Montana Unemployment Compensation Commission, Twenty-eighth Annual Report, Reporter Printing and Supply Co., Helena, Montana, 1964, Literature Cited Reference 45.
- c/ Statistical Abstract of the United States, 1966, Government Printing Office, Washington, D. C., 1967, Literature Cited Reference 113, p. 288.
- d/ Ibid., p. 271.

TABLE V. PERSONAL INCOME, MONTANA, 1963-1967.*

Year	Total	Per Capita
	<u>Mil. Dol.</u>	<u>Dollars</u>
1953	1,096	1,779
1954	1,079	1,729
1955	1,178	1,852
1956	1,241	1,892
1957	1,297	1,944
1958	1,371	2,059
1959	1,345	2,010
1960	1,383	2,037
1961	1,371	1,973
1962	1,581	2,271
1963	1,588	2,266
1964	1,593	2,266
1965	1,712	2,436
1966	1,842	2,623
1967	1,934	2,759

*Source: Survey of Current Business, Vol. 48, No. 4, pp. 13-14.

was less than the national rate. By 1967, Montana had slipped down to thirty-first in per capita income. 22/

Education

The median educational level in Montana is somewhat higher than the national median. Forty-eight percent of the Montana population over 25 years old have completed high school as compared to only 41 percent of the national population. 23/ The median level of education in Montana is 11.6 years, while for the U. S. it is 10.6. 24/ College graduates as a proportion of the population is approximately the same for the state as for the nation. 25/ It appears, then, that the general educational level of the state is at least equal to the national average.

Comparison of the educational levels and money income of individuals indicates that education and income are positively correlated. The more education an individual has, the more probable that he is at the higher income levels. 26/ Yet in Montana, where the average level of education is higher than the national level, per capita income is less than the national average and has been decreasing with respect to the national average since 1950. This would seem to indicate that the Montana economy has not been able to fully utilize the human resources available to it.

22/ Ibid.

23/ U. S. Statistical Abstract, op. cit., p. 114.

24/ Ibid., p. 115.

25/ Ibid., p. 331.

26/ Ibid., p. 116.

CHAPTER IV--THE EMPIRICAL MODEL

Secondary data was used for this study. The base year of the model, 1963, was selected because of the availability of industrial statistics for that year from the quinquennial census of business, manufacturing, mineral industries, government, transportation, and the like. The statistical appendix to this study, Appendix A, presents a detailed discussion of data sources and estimating techniques used in the study.

The data used was presented for industries classified according to the Standard Industrial Classification Code. ^{1/} Therefore, sectors in the model were specified on this basis. There are 12 endogenous sectors in the model. Agriculture was divided into two sectors: Livestock and Livestock Products comprise one sector and Crops make up the other. This division allows the study of the two major types of agricultural enterprises in the state. Cow-calf range enterprises dominate the livestock sector, while wheat and barley production are the principal cash crop enterprises. Manufacturing enterprises were aggregated into three sectors: (1) Food and Kindred Products, (2) Lumber and Wood Products, and (3) Manufacturing. Meat products constitute the greatest share of output of the Food and Kindred Products sector, with flour and bakery products accounting for most of the remainder. Lumber and Wood products industries are included

^{1/} Bureau of the Budget, Office of Statistical Standards, Standard Industrial Classification Manual, U. S. Government Printing Office, 1967.

as a separate sector because of their great economic significance to the state. The remaining manufacturing activities are aggregated in the Manufacturing sector. Primary metals refining industries and petroleum refining account for the major share of Manufacturing output.

Mining was included as a separate sector because of the extensive mining operations in the state. Non-ferrous metal mining, particularly copper mining, constitute the major mining activities. Petroleum and natural gas extraction are also very important.

Service type industries were aggregated into five sectors: Transportation and Public Warehousing; Communications and Public Utilities; Real Estate, Finance, and Insurance; Services; and Trade. Construction activity was divided into two sectors; Maintenance Construction and New Construction.

With the exception of New Construction, the above sectors constitute the endogenous portion of the transactions matrix. The exogenous sectors include all final demand sectors. These sectors do not produce intermediate inputs for the endogenous sectors.

Along with New Construction, the exogenous sectors include government, households, and exports. The government activities were split into two sectors; State and Local Government, and Federal Government. With this division the economic interaction attributable to each can be shown.

Households constitute one sector of final demand, showing total consumption purchases of the Montana population. Households are

divided into three primary input rows, thus showing the labor, proprietor, and property income generated in each sector. These three sectors include all personal income received in the state.

The remaining exogenous sector is the Import-Export sector, which shows all imports and exports of the state.

A complete listing of the endogenous and exogenous sectors is shown below.

Endogenous

1. Livestock and Livestock Products
2. Crops
3. Food and Kindred Products
4. Lumber and Wood Products
5. Manufacturing
6. Transportation and Public Warehousing
7. Communications and Public Utilities
8. Real Estate, Finance, and Insurance
9. Mining
10. Services
11. Trade, Wholesale and Retail
12. Construction, Maintenance

Exogenous

13. Construction, New
14. State and Local Government
15. Federal Government
16. Households
 - Wages and Salaries
 - Proprietor Income
 - Property Income
17. Import-Export

The Transactions Table

The transactions table is the heart of the input-output model. This table shows the inter-sectoral flows of goods and services in the economy and provides the basis for calculation of the technical and interdependence coefficients.

Table VI shows the transactions table of the Montana Economy of 1963. Each row of the table indicates the output distribution of that sector, while each column shows the purchases made by that sector from the other sectors. For example, the first row of the table indicates the distribution of the products of the Livestock and Livestock Products sector. This sector retained \$9.4 million worth of output to be used in further production, sold \$35.0 million worth of output to the Food and Kindred Products sector, \$1.6 million to the Federal Government, \$6.6 million directly to consumers, and the remainder, \$154.2 million, was exported from the state. ^{2/}

Each column of the transactions table shows the amount purchased by a sector from the remaining sectors. The first column of the table shows the purchases of the Livestock and Livestock sector. The Crops sector supplies the greatest amount of input, \$67.3 million. It is interesting to note the large amount of input imported by the livestock sector. The principal import is livestock, amounting to over \$20 million, 95 percent of which were cattle imports. Other imports include veterinary supplies, specialized livestock equipment, motor vehicles, and the like.

The remaining sectors can be interpreted in a similar fashion. Each column vector is the production function of that sector, showing all of the inputs purchased to produce the indicated output.

^{2/} These figures, taken from the transactions table, were rounded off to the nearest tenth of a million dollars.

TABLE VI. TRANSACTIONS TABLE OF THE MONTANA ECONOMY, IN THOUSANDS OF DOLLARS.

	Purchasing Sectors	Livestock & Livestock Products	Crops	Food and Kindred Products	Lumber & Wood Products	Manufac- turing	Transporta- tion & Public Warehousing	Communica- tion & Public Utilities	Real Estate, Finance, & Insurance	Mining	Services	Trade, Whole- sale & Retail	Construc- tion & Maint.	Construc- tion-New	Exports	State & Local Govern- ment	Federal Govern- ment	Households
Producing Sectors		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Livestock & Livestock Products		9,410	0	35,029	0	0	0	0	0	0	0	0	0	0	154,228	0	1,609	6,580
2. Crops		67,341	9,385	28,028	1,694	0	0	0	0	0	0	0	0	0	186,475	0	11,750	1,312
3. Food & Kindred Products		3,688	0	13,413	0	0	0	0	0	0	0	0	0	0	1,641	949	1,051	116,615
4. Lumber & Wood Products		113	21	0	36,145	1,148	21	820	0	154	0	0	3,921	13,810	112,293	0	0	57
5. Manufacturing		1,033	20,479	612	1,512	7,187	9,310	549	803	1,418	762	6,462	3,441	15,486	238,270	3,038	103	53,068
6. Transportation & Public Warehousing		3,772	3,736	340	26,200	11,589	421	1,642	0	5,270	616	27,084	804	3,308	66,877	932	1,216	2,249
7. Communications & Public Utilities		2,271	3,126	868	4,491	14,845	1,837	1,792	2,868	5,259	1,271	22,647	116	759	1,201	12,821	4,909	43,399
8. Real Estate, Finance, and Insurance		9,111	19,309	1,879	2,780	3,931	2,060	1,873	1,792	2,868	2,275	11,919	568	2,554	0	2,579	9,313	68,587
9. Mining		49	3,720	0	214	159,417	0	2,178	0	2,475	0	0	354	3,471	0	1,243	0	0
10. Services		1,989	4,953	2,466	1,570	4,669	1,626	1,043	4,243	17,421	5,613	10,525	1,472	5,090	0	13,935	4,347	125,815
11. Trade		5,674	13,155	4,054	7,315	9,876	1,643	1,247	1,711	3,611	3,737	10,840	3,721	14,330	15,423	22,814	13,672	241,920
12. Construction, Maintenance		203	804	134	991	3,212	0	256	460	1,042	225	1,489	0	0	0	7,432	0	16,655
13. Construction, New		124	638	635	3,882	15,258	0	1,472	0	4,296	142	1,243	0	0	0	72,246	0	54,026
14. Imports		38,270	58,610	14,989	17,814	15,171	37,390	29,327	71,473	53,511	50,178	13,871	2,232	15,332	0	36,631	0	422,745
15. State & Local Government		13,858	17,313	5,157	4,603	22,308	7,412	17,636	3,512	5,862	1,930	19,247	169	511	0	24,020	0	422,745
16. Federal Government		1,004	674	4,179	4,943	26,870	2,188	18,139	8,973	11,057	5,955	17,899	2,062	6,221	1,755	0	0	132,871
17a. Wages & Salaries		10,804	17,952	23,292	44,314	56,524	76,752	32,641	31,186	45,107	87,778	164,571	15,214	67,318	0	131,000	141,619	39,432
17b. Proprietor Income		34,719	128,733	4,050	8,495	11,130	13,321	10,143	7,038	10,036	28,673	40,160	1,236	5,437	0	0	0	0
17c. Property Income		3,423	5,377	1,760	1,540	2,407	2,075	3,024	11,341	3,234	17,622	23,249	231	345	0	0	0	68,715
18. Total Inputs		206,856	307,958	140,885	168,504	365,533	156,056	132,840	143,398	173,121	206,777	374,734	35,541	153,962	329,640	189,589	1,516,170	

The export column and import row of Table VI is of special interest. It is evident after examination of these sectors, that Montana is highly dependent upon markets outside of the state, both for imports of inputs and exports of products. Total imports of the processing sectors amount to \$418 million, and constitute over 15 percent of the total inputs to these sectors. Exports of the two agricultural sectors and the three manufacturing sectors, totaling \$692 million, comprise almost 60 percent of the gross output of these five sectors, and over 30 percent of the total output of the state. The principal exports are feeder cattle, wheat and barley, wood products, non-ferrous metals, and gasoline. The prices of these products are established in national markets and Montana producers must accept the price. Because of the large amount of exports of this nature, any export market price fluctuations exert considerable influence on the economic well-being of producers and employees in the Montana economy, particularly in the exporting sectors.

Technical Coefficients

The technical coefficients, derived from the transaction table, are given in Table VII. They show the direct requirement of a sector from every other sector per dollar of gross output. Technical coefficients are relevant only for the endogenous sectors, therefore the final demand sectors are not included in the table.

The technical coefficients can be interpreted as follows. Assume an output of \$1,000 from the Mining sector. In order to produce this

TABLE VII. TECHNICAL COEFFICIENTS OF THE ENDOGENOUS SECTORS OF THE MONTANA MODEL.

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1. Livestock & Livestock Products	.04549	0	.24864	0	0	0	0	0	0	0	0	0
2. Crops	.32555	.03047	.19894	.01005	0	0	0	0	0	0	0	0
3. Food & Kindred Products	.01783	0	.09521	0	0	0	0	0	0	0	0	0
4. Lumber & Wood Products	.00055	.00007	0	.21451	.00314	.00013	.00617	0	.00089	0	0	.11032
5. Manufacturing	.00499	.06649	.00434	.00897	.01966	.05966	.00413	.00560	.00819	.00368	.01724	.09682
6. Transportation & Public Warehousing	.01823	.01213	.00241	.15549	.03171	.00270	.01236	0	.03044	.00298	.07227	.02262
7. Communications & Public Utilities	.01098	.01015	.00616	.02665	.04061	.01177	.08168	.00603	.03326	.00615	.06043	.00326
8. Real Estate, Finance, and Insurance	.04404	.06269	.01334	.01650	.01076	.01320	.01410	.01250	.01657	.01100	.03181	.01598
9. Mining	.00024	.01208	0	.00127	.43612	0	.01640	0	.01430	0	0	.00996
10. Services	.00962	.01608	.01750	.00932	.01277	.01042	.00785	.02959	.10063	.02714	.02809	.04142
11. Trade, Wholesale & Retail	.02743	.04271	.02878	.04341	.02699	.01053	.00939	.01193	.02086	.01807	.02893	.10469
12. Construction, Maintenance	.00098	.00261	.00095	.00588	.00879	0	.00193	.00321	.00602	.00109	.00397	0
13. Construction, New	.00060	.00207	.00451	.02304	.04174	0	.01108	0	.02481	.00069	.00332	0
14. Imports	.18501	.19030	.10639	.09385	.04150	.23960	.22077	.49842	.30910	.24267	.03703	.06280
15. State & Local Government	.06699	.05622	.03660	.03919	.06103	.04750	.13276	.02450	.03386	.00933	.05136	.00476
16. Federal Government	.00485	.00219	.02966	.02933	.07351	.01402	.13655	.06257	.06387	.02880	.04776	.05802
17a. Wages and Salaries	.05223	.05829	.16533	.26299	.15463	.49182	.24572	.21748	.26055	.42451	.43917	.42807
17b. Proprietor Income	.16784	.41799	.02875	.05041	.03045	.08536	.07635	.04908	.05797	.13867	.10717	.03478
17c. Property Income	.01655	.01746	.01249	.00914	.00659	.01329	.01329	.07909	.01868	.08522	.06204	.00650

output, the mining sector must purchase $\$1,000 \times .00089$ or 89 cents worth of inputs from the Lumber and Wood products sector; $\$1,000 \times .00819 = \$8.19$ from the Manufacturing sector; $\$1,000 \times .03044 = \$30.44$ from the Transportation and Public Warehousing sector; and so forth. The coefficients of the three household sectors show the proportion of Wages and Salaries, Proprietor Income, and Property Income generated by a dollars worth of output from the sector. Similarly, any increase in output will require a proportionate increase in inputs from each sector.

Technical coefficients are assumed to be constant over time. The assumption of constant coefficients implies, among other things, that technology and relative prices of inputs do not change. This may be a realistic assumption in the short run but is not realistic in the long run. New flow tables should be constructed regularly from the latest information so that any changes in coefficients will be recognized.

Interdependence Coefficients

Table VIII contains the interdependence coefficients. These coefficients include both direct and indirect changes in input requirements per dollar change in final demand. Table VIII is interpreted as follows. Assume that the final demand of, say, the Food and Kindred Products sector increases by $\$1,000$. In the process of satisfying this increase, the Food and Kindred Products sector will require additional inputs from the other sectors, who must in turn expand their production to supply the additional inputs. The final result, after all direct and indirect effects have been expended, is indicated

TABLE VIII. INTERDEPENDENCE COEFFICIENTS.

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	1.05320	.00014	.28955	.00017	.00012	.00004	.00003	.00004	.00007	.00005	.00282	.00033
2	.35803	1.03159	.32531	.01341	.00020	.00005	.00013	.00005	.00011	.00006	.00318	.00184
3	.02127	.00053	1.11154	.00066	.00045	.00015	.00013	.00014	.00027	.00021	.01081	.00127
4	.00152	.00118	.00098	1.27478	.00689	.00071	.00897	.00058	.00246	.00028	.00135	.14153
5	.03349	.07350	.03159	.02759	1.02974	.06214	.00649	.00667	.01249	.00478	.02441	.10715
6	.02963	.02058	.01837	.20584	.05221	1.00796	.01679	.00168	.03485	.00492	.07760	.05932
7	.02324	.02059	.02129	.04604	.06640	.01783	1.09150	.00824	.04065	.00865	.07125	.02382
8	.07335	.06966	.05213	.02851	.02297	.01553	.01697	1.01379	.02052	.01242	.03680	.02655
9	.01988	.04555	.01842	.01486	.45685	.02780	.02107	.00312	1.02078	.00227	.01208	.05814
10	.02352	.02689	.03435	.01996	.06440	.01556	.01232	.03188	.10814	1.02933	.03445	.05666
11	.04975	.05069	.05876	.06298	.04278	.01404	.01248	.01380	.02615	.01979	1.03446	.12105
12	.00291	.00411	.00306	.00832	.01229	.00088	.00246	.00344	.00664	.00131	.00472	1.00280

in the column of interdependence coefficients for the Food and Kindred Products sector. In this case, the required expansion by the Livestock and Livestock Products sector would be $\$1,000 \times .28955 = \$289.55$; the Crops sector, $\$1,000 \times .32531 = \$325.31$; and so forth. Notice that the interdependence coefficient of the Food and Kindred Products sector is 1.11154. This coefficient can be divided into two parts; 1.00000 and .11154. The first part shows the amount needed to satisfy the increase in final demand (i.e., $\$1,000 \times 1.00000 = \$1.00000$) and the second part shows the remaining expansion of the sector after adjustment to the increase in demand.

Table IX shows the indirect coefficients which are found by subtracting the direct requirements matrix (I+A) from the interdependence coefficients matrix $(I-A)^{-1}$. The direct or first round effects of the increase in demand is estimated by the (I+A) matrix times the increase, and the indirect effects are estimated by the indirect coefficients of Table IX times the increase. Table X shows the total, direct, and the indirect effects of a \$1 change in final demand of the Food and Kindred Products sector. The table indicates that for 8 of the 12 sectors, the indirect effects outweigh the direct effects.

Sector Multipliers

The theoretical bases of sector output, personal income, and resource multipliers were developed in Chapter II. The multipliers are used to estimate the change in total output (personal income, resource use) when there is an autonomous change in the level of final demand of a sector.

TABLE IX. TABLE OF INDIRECT COEFFICIENTS.

Sector	1	2	3	4	5	6	7	8	9	10	11	12
1	.00771	.00014	.04091	.00017	.00012	.00004	.00003	.00004	.00007	.00005	.00282	.00033
2	.03248	.00112	.12637	.00336	.00020	.00005	.00013	.00005	.00011	.00006	.00318	.00184
3	.00254	.00053	.01633	.00066	.00045	.00015	.00013	.00014	.00027	.00021	.00140	.00127
4	.00097	.00111	.00098	.06077	.00375	.00058	.00280	.00058	.00155	.00028	.00135	.03119
5	.02850	.00701	.02725	.01962	.01008	.00248	.00236	.00107	.00430	.00110	.00717	.01033
6	.01140	.00845	.01596	.05035	.02050	.00526	.00443	.00168	.00441	.00194	.00533	.03670
7	.01226	.01044	.01513	.01939	.02579	.00606	.00982	.00221	.00739	.00250	.01082	.01057
8	.02931	.00697	.03879	.01201	.01221	.00233	.00287	.00129	.00395	.00142	.00499	.01057
9	.01964	.03347	.01842	.01359	.02073	.02780	.00467	.00312	.00648	.00227	.01208	.04818
10	.01390	.01081	.01685	.01064	.05163	.00514	.00447	.00229	.00751	.00219	.00636	.01524
11	.02232	.00798	.02998	.01957	.01579	.00351	.00309	.00187	.00529	.00172	.00553	.01636
12	.00193	.00150	.00211	.00244	.00350	.00088	.00053	.00023	.00062	.00022	.00075	.00280

TABLE X. EFFECTS OF A ONE DOLLAR CHANGE IN DEMAND OF THE FOOD AND KINDRED PRODUCTS SECTOR.

Sector	Coefficients		Total Effect
	Direct	Indirect	
Livestock and livestock products	.24864	.04091	.28955
Crops	.19894	.12637	.32531
Food and kindred products	1.09521	.01633	1.11154
Manufacturing	.00434	.02725	.03159
Lumber and wood products		.00098	.00098
Transportation and public warehousing	.00241	.01596	.01837
Communications and public utilities	.000616	.01513	.02129
Real estate, finance, and insurance	.01334	.03879	.05213
Mining		.01842	.01842
Services	.01750	.01685	.03435
Trade	.02878	.02998	.05876
Construction, maintenance	.00095	.00211	.00306
Total	1.61627	.34908	1.96535

The i^{th} sector output multiplier is the column sum of the i^{th} column of the matrix of interdependent coefficients, $(A-I)^{-1}$. Table X of the preceding section illustrates the Food and Kindred Products output multiplier. It is the column sum of the elements of the "Total Effects" column of Table X. The sector output multipliers are the column sums of the matrix interdependence coefficients shown in Table VIII.

The sector personal income multipliers are derived by pre-multiplying the matrix of interdependence coefficients by the matrix P, where the diagonal element P_{ii} is the proportion of personal income per dollar of output of sector i , and all off-diagonal elements are zero. The elements of this matrix are the sums of the Wages and Salaries, Proprietor Income, and Property Income technical coefficients of each sector (Table VII). The elements of P are therefore $.23662_{11}$, $.49374_{22}$, $.20657_{33}$, $.32254_{44}$, $.19167_{55}$, $.59047_{66}$, $.34365_{77}$, $.34483_{77}$, $.34365_{88}$, $.33720_{99}$, $.64840_{1010}$, $.60838_{1111}$, and $.46935_{1212}$, with the remaining off-diagonal elements equal to zero.

The output and personal income multipliers are listed in Table XI. The largest output multipliers are those of the three manufacturing sectors, while the smallest are the Services sector and Real Estate, Finance, and Insurance sector multipliers.

The size of the output multiplier reflects the interaction of a sector with the remaining sectors. The sectors with higher proportions of internally purchased inputs will have, in general, higher multipliers. The sector output multipliers and the proportions of internally

TABLE XI. SECTOR OUTPUT AND INCOME MULTIPLIERS.

Sector	Multiplier	
	Output	Personal Income
Livestock and livestock products	1.69	.54159
Crops	1.35	.63271
Food and kindred products	1.97	.56688
Lumber and wood products	1.70	.62971
Manufacturing	1.75	.48906
Transportation and public warehousing	1.16	.64727
Communications & public utilities	1.19	.42017
Real estate, finance, and insurance	1.08	.38546
Mining	1.27	.47831
Services	1.08	.69209
Trade, wholesale and retail	1.31	.75059
Construction, maintenance	1.60	.72044

purchased inputs were ranked according to size. The results are shown in Table XII. The ranking is the same for both sets except for the Lumber and Wood Products sector and Livestock and Livestock Products sector, where the ranks are transposed. However, examination of the sizes of the multipliers and the proportions of internal purchases reveals that these two sectors have virtually the same size multiplier and proportions.

The income multipliers are useful for estimating the change in personal income that would result from changes in the level of production of the sectors. They can be used in conjunction with the output multipliers. An example of the application of the output and income multipliers is shown in Table XIII.

TABLE XIII. ESTIMATED CHANGE IN OUTPUT AND PERSONAL INCOME THAT WOULD RESULT FROM AN INCREASE OF \$1,000,000 IN FINAL DEMAND FOR LUMBER AND WOOD PRODUCTS.

Change In Final Demand	Multipliers		Estimated Total Changes	
	Output	Income	Output	Income
\$1,000,000	1.70	.62971	\$1,700,000	\$629,710

The output multipliers described above (hereafter called Type I multipliers) were derived with households placed as an exogenous sector, thus excluding the effects of changes in personal income. This will understate the true multiplier value, since any change in output of the economy will affect the amount of personal income received in the state, which in turn will affect personal consumption expenditures.

TABLE XII. COMPARISON OF THE OUTPUT MULTIPLIERS AND PURCHASES FROM THE ENDOGENOUS SECTORS.

Sector	Output Multiplier	Inputs Required from Endogenous Sectors <u>Percent</u>	Rank	
			Multiplier	Percent Inputs
Food & kindred prod.	1.97	61.6	1	1
Manufacturing	1.75	59.1	2	2
Lumber & wood prod.	1.70	49.0	3	4
Livestock & livestock products	1.69	50.6	4	3
Construction, maint.	1.60	40.5	5	5
Crops	1.35	25.5	6	6
Trade	1.31	25.2	7	7
Mining	1.27	23.1	8	8
Communications & public utilities	1.19	15.4	9	9
Transportation & public warehousing	1.16	10.8	10	10
Services	1.08	7.0	11	11
Real estate, finance, and insurance	1.08	6.9	11	12

Inclusion of the household sector as an endogenous sector results in higher values of the sector multipliers. Table XIV shows the sector multipliers with households included as an endogenous sector and the difference between the sizes of the multipliers with households included and excluded.

The multiplier derived with households included as an endogenous sector (hereafter called Type II multipliers) implies that any change in personal income will result in a proportional change in household purchases from the endogenous sectors. The assumption is that households react to any change in the economy in the same manner as the processing sectors, increasing its purchases from each sector proportionately. This assumption further implies that the income elasticity of demand for the products of each sector is the same, and that the Marginal Propensity to Consume is one. Thus, an increase in personal income will result in increased consumer expenditures, so the Type II multiplier will yield a higher estimate of total change than the Type I multiplier.

The relationship between the Type I and Type II multipliers is described as:

$$M_{iII} = M_{iI} + P_i M_h$$

where: M_{iII} = the Type II Multiplier of the i^{th} sector

M_{iI} = the Type I multiplier of the i^{th} sector

P_i = the income multiplier of the i^{th} sector

M_h = the household sector output multiplier

TABLE XIV. SECTOR OUTPUT MULTIPLIERS: VALUE WITH HOUSEHOLDS EXCLUDED,
HOUSEHOLDS INCLUDED, AND THE DIFFERENCE.

Sector	Multiplier Values		Difference Type II - Type I
	Households Included-Type II	Households Excluded-Type I	
Livestock & livestock products	2.56	1.69	.87
Crops	2.83	1.35	1.48
Food & kindred prod.	3.29	1.97	1.32
Lumber & wood prod.	3.19	1.70	1.49
Manufacturing	2.86	1.75	1.85
Transportation & public warehousing	2.68	1.16	1.53
Communications & public utilities	2.19	1.19	1.00
Real estate, finance and insurance	1.98	1.08	.90
Mining	2.32	1.27	1.05
Services	3.07	1.08	1.99
Trade	3.30	1.31	2.01
Construction, maintenance	3.27	1.60	1.67

Assuming that the Marginal Propensity to Consume is between zero and one, the true size of each sector output multiplier lies between the Type I and Type II multipliers. Using the notation above, the true multiplier M^t will be:

$$M_{iII} \geq M_i^t = M_{iI} + P_{ih} M_i (MPC)$$

where: (MPC) = the applicable Marginal Propensity to Consume

The source of a change in final demand must also be considered when using the multipliers to estimate total change in the economy. For example, consider the effect of a change in final demand of the Food and Kindred Products sector. If the change is \$1 million, application of the Type II multiplier would indicate an increase in the output of the economy of \$3.29 million. Part of this increase, however, is the result of an increase in the output of the Livestock and Livestock Products sector of \$248,000. This is the increase in intermediate inputs required by the Food and Kindred Products sector and supplied by the Livestock and Livestock Products sector. But this increase in intermediate inputs may be supplied out of livestock products exports. If so, then the Livestock and Livestock Products sector would not expand and the multiplier indicates a greater increase than would actually occur.

The output multiplier should be adjusted in this situation. The method of adjustment is to subtract the product $a_{ij} M_i$ from M_j , where a_{ij} is the technical coefficient, M is the multiplier, i is the

livestock sector, and j is the food products sector. Thus the adjusted Type II multiplier is $3.29 - (2.56 \times .24864) = .265$.

Conclusions

Agricultural, manufacturing, and mining enterprises constitute the primary sectors in the state from the standpoint of total output. The six sectors containing these enterprises account for 53 percent of total output from non-government economic activity. These sectors are highly dependent upon markets outside of the state for the sale of their products.

The remaining sectors, both private and government, operate in support of the primary sectors, providing intermediate inputs to the producing sectors and final consumption goods and services of the final demand sectors. The total output of these sectors is geared to the production levels of the primary sectors.

The output multipliers provide estimates of how output will change when the demand for the products of a given sector changes. They can be used to estimate which sectors have the greatest impact on the economy's total output.

The personal income multipliers indicate how personal income will change when a change in sector output occurs. They are more useful than the output multipliers when estimating how changes in sector output will affect the economic position of the human resource. For example, the largest Type I output multiplier is in the Food Products sector, but the largest personal income multiplier is in the Trade sector. Thus, given the same amount of increase in demand, the increase in the

trade sector will provide the greatest benefit to the human resource as measured in personal income.

CHAPTER V--INTERDEPENDENCE IN THE MONTANA ECONOMY

Interdependence

Sectoral interdependence of the economic activities that comprise an economy indicate the level of industrialization of that economy. The interdependence between two sectors may be unidirectional or bidirectional. A given sector may buy but not sell to another sector (unidirectional) or it may both buy and sell to another sector (bidirectional). The amount of purchases and/or sales, combined with the direction of transactions, indicates the degree of interdependence. A high degree of interdependence, indicated by extensive intersectoral flows in the transactions table, means that the economy is relatively developed. Conversely, a low degree of interdependence, indicated by a relatively large number of zero entries in the transactions table, is the mark of the less-developed economy. Thus a comparison of two transactions tables formulated from two different economies will indicate which of the two is relatively more developed. Moreover, if one of the economies to be considered exhibits the characteristics of the industrial state to a high degree, it can be used as a benchmark of the developed state.

Comparison of Two Economies

The United States economy is an excellent example of a developed state. The major share of its resources are allocated to secondary and tertiary enterprises. Over 90 percent of the United States National Income originated in these activities in 1963. 1/ The U. S.

1/ U. S. Department of Commerce, Bureau of the Census, Statistical Abstract of the United States, 1966, table 451, p. 320.

population is predominantly urban, with over 70 percent living in urban areas. 2/ Therefore, if the U.S. economy is used as a benchmark for comparison with the Montana economy, the degree of development of the Montana economy can be appraised.

In 1959 the National Accounts Review Committee set up at the request of the Bureau of the Budget to evaluate national accounts of the U.S., recommended that an input-output model of the U. S. economy be prepared at regular intervals.. 3/ The Office of Business Economics was assigned the responsibility for this work. The first model prepared by the Office of Business Economics was based on the year 1958. 4/ The model contained 87 endogenous sectors and 7 sectors of final demand. This model was used as the basis of comparison for evaluating the degree of development of the Montana economy.

In order to compare the Montana and U.S. models, the latter had to be reduced from 87 endogenous sectors to the 12 endogenous sectors of the Montana model. This was accomplished by consolidating the matrix of U. S. technical coefficients, then multiplying each of the resultant

2/ Ibid., table 12, p. 15.

3/ U. S. Department of Commerce, Office of Business Economics, Survey of Current Business, Vol. 44, No. 11, p. 10.

4/ The transactions table, technical coefficients, and interdependence coefficients were published in the Survey of Current Business, Vol. 44, No. 11, in the article "The Interindustry Structure of the United States." Several later articles have appeared in the Survey, including "Personal Consumption Expenditures in the 1958 Input-Output Study," (Vol. 45, No. 10) and "Additional Industry Detail for the 1958 Input-Output Study" (Vol. 46, No. 4).

column vectors by the Montana gross output of that sector to arrive at a transactions table. The consolidation procedure is the procedure used by Chenery and Clark, 5/ and is described in Appendix B.

Comparability of Multipliers

The size of the multiplier for a given sector is determined by the degree of interdependence existing between that sector and the remainder of the economy. Sector interdependence is indicated in two ways: (1) by the number of intersectoral transactions of the sector and (2) by the level of inputs purchased from endogenous sectors relative to total sector output. A high number of intersectoral transactions combined with a relatively high amount of endogenously purchased inputs will result in a relatively large multiplier value.

Sector output multipliers were calculated for the consolidated U.S. model and compared to the sector output multipliers of the Montana model. Both Type I and Type II multipliers were computed, and the results are shown in Table XV. Both the Type I and Type II multipliers derived from the Montana matrix were smaller than their counterparts derived from the consolidated U.S. matrix, showing that the Montana economy is less interdependent than indicated by the consolidated U.S. model.

The highest Type I output multiplier for both cases is found in the Food and Kindred Products sector. Examination of the input

5/ Chenery and Clark, op. cit., p. 36.

TABLE XV. OUTPUT MULTIPLIERS.

Sector	Type I		Type II	
	From Montana Model	From U. S. Model	From Montana Model	From U. S. Model
Livestock & livestock products	1.69	2.29	2.56	5.21
Crops	1.35	1.79	2.83	4.57
Food & kindred prod.	1.97	2.33	3.29	4.91
Lumber & wood prod.	1.70	1.88	3.19	4.06
Manufacturing	1.75	1.97	3.60	4.05
Transportation & public warehousing	1.16	1.50	2.81	4.19
Communications & public utilities	1.19	1.44	2.18	4.06
Real estate, finance, and insurance	1.08	1.56	1.98	4.46
Mining	1.27	1.52	2.32	3.89
Services	1.08	1.55	3.07	4.16
Trade, wholesale and retail	1.31	1.35	3.30	4.23
Construction, maint.	1.60	1.42	3.27	3.86

coefficients for this sector shows that, for both the Montana and U.S. models, a large share of the inputs are purchased from the endogenous sectors, as indicated by the size of the multipliers.

The relative rank of seven of the remaining sectors is approximately the same in the Type I model. For example, the Lumber and Wood Products sector has the third highest multiplier in the Montana model and the fourth highest in the U.S. model. Four of the sectors, however, show considerable change in rank. The Real Estate, Finance, and Insurance sector and the Services sector have the lowest multipliers in the Montana model, but rank sixth and seventh respectively among the consolidated U.S. model multipliers. Again, the explanation lies in the increased interdependence of the sectors of the consolidated U.S. model.

The Type II multipliers are derived by including Households as an endogenous sector. These multipliers are, in general, much higher than the Type I multipliers. This is a result of the increased interdependence brought about by the inclusion of Households. The difference between the values of the Type I and Type II multipliers indicates the effects of increased personal income and expenditures resulting from expansion of a given sectors output. For example, consider the Crops sector multipliers of the Montana model. The Type I multiplier is 1.35, the Type II multiplier is 2.83, and the difference is 1.48. Thus the expansion due to an increase in household expenditures, which is the result of increased household income, can be isolated.

Comparison of Triangularized Matrices

Chenery and Clark 6/ developed a procedure whereby the direction of interdependence sectors can be illustrated. This approach, called triangularization of the transactions matrix, results in an ordering of sectors according to the direction of interdependence between each.

Triangularization of a matrix is defined as the process of reducing the sum of the elements above the diagonal by rearranging the rows and columns of the matrix. The optimally triangularized matrix is the arrangement whereby the sum of elements above the diagonal is at a minimum.

The order of the rows in the optimally triangularized matrix can be used as an index of interdependence. The first row would be the sector of greatest one-way interdependence. If the optimum matrix contains only zeros above the diagonal, then any sector would not purchase from sectors above it and would not sell to sectors below it. Thus the triangularization process indicates the degree of interdependence within the economy. To quote Chenery and Clark:

"The extent to which an actual economy departs from this one-way interdependence is indicated by the proportion of transactions which fall above the diagonal in the optimum arrangement of sectors." 7/

Technical dependence is also illustrated by the triangularized matrix. Assume that the optimally triangularized matrix contains only zeros above the diagonal. Such a matrix is illustrated in

6/ Ibid., p. 143.

7/ Ibid., pp. 209-210.

Figure 2. Sector A purchases inputs from the remaining sectors but sells inputs only to itself. B purchases inputs from C but sells only to A and itself. C purchases only from itself but sells to both A and B. Thus, B and C are technically independent of A, since none of A's output is required in B's and C's production process. Similarly, C is technically independent of B. In this kind of situation, sector A could be eliminated from the economy and B and C could still continue their production. However, B and C are still economically dependent upon A, since elimination of A would reduce the market for the output of B and C. Thus the lower sectors of the triangularized matrix may be technically independent but economically dependent upon the upper sectors. If all off-diagonal elements were zero, all sectors would be both technically and economically independent of each other.

Buying Sectors	A	B	C
Selling Sectors			
A	*	0	0
B	*	*	0
C	*	*	*

* = a positive number

Figure 2. An Example of a Triangularized Matrix.

The Montana transactions table in optimally triangularized form is presented in Table XVI. This table contains only the endogenous sectors of the model. Food and Kindred Products is the least interdependent

TABLE XVI. TRIANGULARIZED MATRIX OF MONTANA TRANSACTIONS.

Purchasing Sector													
Selling Sector		3	1	12	4	2	5	9	11	6	8	10	7
3. Food & Kindred Products	13,413		3,688	0	0	0	0	0	0	0	0	0	0
1. Livestock & Livestock Prod.	35,029	9,410		0	0	0	0	0	0	0	0	0	0
2. Construction, Maintenance	134	203	0		991	804	3,212	1,042	1,489	0	460	225	256
4. Lumber & Wood Products	0	113	3,921	36,146		21	1,148	154	0	21	0	0	820
2. Crops	28,028	67,341	0	1,694	9,385		0	0	0	0	0	0	0
5. Manufacturing	612	1,033	3,441	1,512	20,479	7,187		1,418	6,462	9,310	803	762	549
9. Mining	49	354	214	3,720	159,417	2,475	0		0	0	0	0	2,178
11. Trade	4,054	5,674	3,721	7,315	13,155	9,876	3,611	10,840		1,643	1,711	3,737	1,247
6. Transportation & Public Warehousing	340	3,772	804	26,200	3,736	11,589	5,270	27,084	421		0	616	1,642
8. Real Estate, Finance, & Insurance	1,879	9,111	568	2,780	19,309	3,931	2,868	11,919	2,060	1,792		2,275	1,873
10. Services	2,466	1,989	1,472	1,570	4,953	4,669	17,421	10,525	1,626	4,243	5,613		1,043
7. Communications & Public Utilities	868	2,271	116	4,491	3,126	14,845	5,759	22,647	1,836	875	1,261	10,850	
Grand Total of Sector Output = \$2,412,230													
Total Above Diagonal = \$ 51,600													
Percent Above Diagonal = 2.14 %													

sector of the economy, selling only a small amount to the Livestock and Livestock Products sector. Examination of the products which comprise the output of the Food and Kindred Products sector (Appendix A) indicates that the Food and Kindred Products sector produces only consumer goods and livestock feed for sale within the economy, hides and skins being exported. Similarly, the livestock sector produces nothing that is used as an intermediate input except slaughter animals, some of which are purchased by the food sector. Since livestock purchases less from the food sector than vice-versa, the food sector is considered to be the least interdependent.

Table XVII contains the triangularized transactions matrix that was derived from the technical coefficients of the consolidated U.S. matrix. The ordering of sectors is similar to that of the Montana model. However, there are two minor variations in the sector order and one major variation.

The minor variations involve the interchange of two sets comprised of two sectors each. The first set contains the Transportation sector and the Trade sector. In the Montana matrix, the Trade sector is indicated to be less interdependent than Transportation and Public Warehousing, while the reverse is indicated in the U.S. matrix. The explanation for this lies in the geographic location of Montana within the U. S. Transportation expense as a percentage of total operating expense is determined by the distance transported and the type of commodity involved. Montana exports are of the bulky and

TABLE XVII. THE TRIANGULARIZED MATRIX OF TRANSACTIONS DERIVED FROM THE U.S. MODEL.

Selling Sector	Purchasing Sector											
	3	1	4	2	5	9	6	11	10	8	7	12
3. Food & Kindred Products	23,580	23,178	2	37	278	0	457	2,084	130	43	1	0
1. Livestock & Livestock Prod.	35,247	33,043	0	22,499	0	0	8	0	54	589	0	0
4. Lumber & Wood Products	8	14	47,783	22	1,148	154	42	229	50	16	11	880
2. Crops	10,376	55,735	4,051	9,320	22	0	158	0	0	912	0	0
5. Manufacturing	2,894	1,121	8,821	29,967	94,322	8,862	11,050	8,499	19,956	2,960	2,917	5,224
9. Mining	114	54	42	1,257	70,226	17,759	133	30	74	123	7,188	276
6. Transportation & Public Warehousing	5,675	4,096	8,563	3,847	11,591	5,379	9,942	1,559	1,421	1,162	1,642	628
11. Trade	5,200	7,356	7,314	13,545	9,866	3,686	4,593	6,224	8,616	1,711	1,320	2,904
10. Services	4,907	5,821	2,878	22,843	6,342	3,315	6,287	24,193	17,763	5,999	4,174	194
8. Real Estate, Finance, & Insurance	1,374	3,866	2,147	27,583	5,432	14,343	7,890	26,078	12,975	30,570	2,103	177
7. Communications & Public Utilities	1,113	1,136	1,525	3,218	7,705	3,040	2,694	11,523	6,801	2,273	15,405	91
12. Construction, Maintenance	504	1,839	308	4,965	355	118	5,712	3,050	1,427	4,544	3,561	2

Grand Total of Sector Output = \$2,412,230

Total Above Diagonal = \$ 154,097

Percent Above Diagonal = 6.39%

relatively low value per ton type, generally shipped a considerable distance. The commodities imported into the state, though generally of a higher value to weight ratio than exports, are also brought in from a substantial distance. Thus the higher transportation costs paid by the Montana economy, and in particular by the Trade sector, show up in the Montana model but not in the consolidated U.S. matrix.

The second minor variation is the transposition of the Services sector with the Real Estate, Finance, and Insurance sector. The Montana model indicates that the Real Estate, Finance, and Insurance sector is less interdependent than Services, while the U.S. matrix reverses the order of the two. As in the previous case, the order of the sectors is determined by the net of the transactions between the two. The Montana model therefore shows the Real Estate sector to be less interdependent.

The major difference in the order of the sectors of the two matrices lies in the position of the Maintenance Construction sector. This sector is third in the Montana model but appears last in the U.S. model, thus indicating greater interdependence in the latter. It appears, from this evidence, that Maintenance Construction is a much greater relative expense for the nation as a whole than for Montana. There are several possible explanations of this result. Two of the most plausible are:

- 1) The type of business activities occurring in Montana is such that less maintenance is required;
- 2) Plant and equipment used in Montana is newer and requires less maintenance at this point in time.

The first explanation applies to the agricultural, lumber and transportation sectors. For example, the Livestock and Livestock Products sector in Montana is range based, with a relatively small amount of capital invested in buildings relative to the U.S. sector. U.S. output includes a greater proportion of fed cattle, dairy products, hog production, etc., all of which require more capital invested in production facilities.

The second explanation is hypothesized on the fact that Montana is in the process of transition from a pre-industrial state to an industrial state. Much of Montana's industry is of fairly recent origin, hence plants have not depreciated as much as in the U. S. as a whole. If this is the case, maintenance expense would be less in Montana.

One-way interdependence occurs to a greater extent in the Montana matrix than in the U.S. model. The triangularized Montana matrix shows intersectoral transactions above the diagonal of \$51,600,000, which is 2.14 percent of the total output of the endogenous sectors. The consolidated U.S. matrix, on the other hand, indicates above diagonal intersectoral transactions of \$154,097,000, or 6.39 percent. If Chenery and Clark's criterion is applied, it would indicate that one-way interdependence is much more prevalent in Montana than would be indicated by the consolidated U.S. matrix. The comparable above diagonal transactions for national models are: Italy, 4.3 percent; Norway, 11.5 percent; Japan, 11.6 percent; and the U.S., 12.7 percent. 8/

8/ Ibid., p. 211, footnote 12.

The order of the sectors in the triangularized matrix indicates how the effects of changes in sector output are dispersed throughout the economy. In general, changes in output of the first sectors will have a greater impact than changes in output of the last sectors. A comparison of the order of the sectors in the triangularized matrix and their respective multipliers shows that, in general, the sectors with the higher multipliers are among the first sectors in the triangularized matrix. Table XVIII compares the order of the sectors of the triangularized matrix, the sector multipliers, and the relative rank of the multipliers. The first six sectors of the triangularized matrix have the largest multipliers, and the last six have the smallest. Thus, if general economic expansion is desired, a given expansion of the first sectors will produce greater total growth than will the same initial expansion of the last.

Examination of Table XVI also shows that the last 10 sectors are technically independent of the first two. The Food and Kindred Products sector and the Livestock and Livestock Products sector sell no intermediate inputs to the remaining sectors, hence elimination of these two sectors would not cause any technical production handicap to the remaining sectors. However, these two sectors do purchase \$129,937,000 worth of intermediate inputs from the remaining sectors, and account for 6.29 percent of their total sales. Thus there is some degree of economic dependency of the last 10 sectors on the first two.

TABLE XVIII. COMPARISON OF SECTOR ORDER OF THE TRIANGULARIZED MATRIX
AND THE RANK OF THE OUTPUT MULTIPLIERS.

Sector Order of the Triangularized Matrix	Multipliers	
	Value	Rank
Food and Kindred Products	1.97	1
Livestock and Livestock Products	1.69	4
Construction, Maintenance	1.60	5
Lumber and Wood Products	1.70	3
Crops	1.35	6
Manufacturing	1.75	2
Mining	1.27	8
Trade	1.31	7
Transportation and Public Warehousing	1.16	10
Real Estate, Finance, and Insurance	1.08	11
Services	1.08	11
Communications and Public Utilities	1.19	9

The Four Quadrant Model

Chenery and Clark 9/ have formulated a method of analyzing the pattern of trade exhibited by an economy. This method, called the "Four Quadrant Format," focuses on the relative importance of internal sales and purchases of the endogenous sectors. 10/

The relative importance of internal purchases can be expressed as the ratio of internal purchases to total purchases for each sector. Define the variable u_j such that

$$u_j = \sum_i a_{ij}$$

where the a_{ij} 's are the technical coefficients of the model and the index of summation includes only the endogenous sectors. Then u_j is the proportion of inputs purchased internally per dollar of output. If Households are included as an endogenous sector, inputs will include intermediate goods, services, and payment for primary inputs (labor, management, capital) supplied by the population of the economy. Payments for imported primary inputs will be excluded.

Similarly the variable w_i is defined as

$$w_i = \sum_j a_{ij} X_j \frac{1}{X_i}$$

where the a_{ij} 's are technical coefficients, X_j is the output of sector j ,

9/ Ibid., pp. 205-206.

10/ Internal sales are sales to other endogenous sectors. Similarly, internal purchases are purchases from endogenous sectors.

and the index of summation again includes only the endogenous sectors.

The ratio w_i is a measure of internal sales.

Define $\bar{u}_j$ and $\bar{w}_i$ as the respective arithmetic means of the variables u_j and w_i . The sectors of the model can then be arranged in a four quadrant format, each sector placed according to whether its u_j and w_i is above or below the mean. Figure 3 presents the sectors of the Montana model in such an arrangement.

Quadrant II			Quadrant I		
High Purchases: $u_j > \bar{u}_j = .705$			High Purchases: $u_j > \bar{u}_j$		
Low Sales: $w_i < \bar{w}_i = .653$			High Sales: $w_i > \bar{w}_i$		
Sector	u_j	w_i	Sector	u_j	w_i
Lvsk. & Lvsk. Prod.	.9010	1.247	Food & Kind. Prod.	.823	.949
Crops	.749	.356	Services	.719	.887
Lumber & Wood Prod.	.814	.252	Trade	.861	.823
Manufacturing	.782	.292	Construc., Maint.	.874	.717
Quadrant III			Quadrant IV		
Low Purchases: $u_j < \bar{u}_j$			Low Purchases: $u_j < \bar{u}_j$		
Low Sales: $w_i < \bar{w}_i$			High Sales: $w_i > \bar{w}_i$		
Sector	u_j	w_i	Sector	u_j	w_i
Transportation and Public Warehousing	.699	.534	Communications & Public Utilities	.499	.846
			Real Estate, Finance, & Insurance	.415	.906
			Mining	.568	.973
			Households	.446	.710

Figure 3. Four Quadrant Format of the Montana Sectors.

The quadrants of the format are described as follows. Quadrant I contains the sectors with high internal sales and purchases ($u_j > \bar{u}_j$, $w_i > \bar{w}_i$); Quadrant II, low sales and high purchases ($u_j > \bar{u}_j$, $w_i < \bar{w}_i$); Quadrant III, low sales and low purchases ($u_j < \bar{u}_j$, $w_i < \bar{w}_i$); and Quadrant IV, high sales and low purchases ($u_j < \bar{u}_j$, $w_i > \bar{w}_i$).

The sectors in Quadrant II are dependent upon external markets for the sale of the major share of their production but purchases most of their inputs internally. Stimulus for economic growth of these sectors must come through an increase in external demand, but the effect of growth in these sectors will be felt throughout the remaining sectors. These sectors will have relative high values of the multipliers.

The sectors in Quadrant IV, on the other hand, have relatively low multipliers. These sectors purchase most of their inputs from external sources. Expansion of these sectors will result when sectors in Quadrants I, II, and III expand, thus increasing the demand for intermediate inputs supplied by the sectors in Quadrant IV. It is interesting to note that Quadrant IV includes the Households sector, with only 44.6 percent of household expenditures going to endogenous sectors of the model. This fact emphasizes the small amount of consumer goods produced in Montana.

Only one sector, Transportation and Public Warehousing, is located in Quadrant III. This is somewhat of a misplacement because almost 70 percent of its inputs are internally purchased and over 53 percent of its sales are to the endogenous sectors. The placement of this sector in Quadrant III illustrates the shortcomings of using the mean

values of u_j and w_i as the indicators for placement of the sectors in the quadrants. It may be more realistic to arbitrarily establish a point, say 50 percent, as the basis for dividing the sectors, both for purchases and sales.

The sectors in Quadrant I purchase most of their inputs internally and sell the major share of their output internally. Economic growth of these sectors will provide a stimulus for expansion of the other sectors. These sectors will also react to expansion of the remaining sectors.

Table XIX shows the sectors located in each quadrant and their respective Type II multipliers. The nine sectors in Quadrants I, II, and III have the highest multiplier values, while the four sectors in Quadrant IV have the lowest multipliers. The relationship of the amount of endogenously purchased inputs and the size of the Type I multiplier for each sector was discussed previously. 11/

Conclusions

The degree of interdependence exhibited by an economy determines the total economic impact of changes in production levels of the sectors within that economy. The degree of interdependence can be appraised in several ways, one of which is through the use of the information provided in a Leontief input-output model. There is at present no cardinal means of measuring interdependence. The degree of interdependence of a given economy can only be measured relative to some other economy.

11/ Page 50 of this thesis.

TABLE XIX. SECTOR PLACEMENT, BY QUADRANT, AND THE TYPE II MULTIPLIERS.

Sector	Type II Multiplier
Quadrant I	
Food and Kindred Products	3.29
Services	3.07
Trade, Wholesale and Retail	3.30
Construction, Maintenance	3.27
Quadrant II	
Livestock and Livestock Products	2.56
Crops	2.83
Lumber and Wood Products	3.19
Manufacturing	3.60
Quadrant III	
Transportation and Public Warehousing	2.81
Quadrant IV	
Communications and Public Utilities	2.18
Real Estate, Finance, and Insurance	1.98
Mining	2.32
Households	2.38

The evidence indicates that the sectors of the Montana economy are less interdependent than their counterparts in the total U. S. economy. Thus the total effects of a change in a sectors output are less in Montana than the effect of the same magnitude of change in the U. S.

Comparison of the triangularized matrices of Montana and the U.S. shows that greater one-way interdependence exists in Montana than in the U.S. as a whole. The triangularized matrix also shows how economic changes are dispersed through the economy.

The four-quadrant format shows how the sectors of Montana depend upon external markets. The sectors in Quadrant II, which account for a substantial share of the economy's output, sell only a small portion of their output internally. The sectors in Quadrant IV, which supply inputs to the other endogenous sectors, must depend upon external sources for inputs.

CHAPTER VI--SUMMARY, IMPLICATIONS, AND LIMITATIONS

Summary

The general objective of this study was to examine the economic structure of the State of Montana through the use of a Leontief Input-Output model. The state's economy was divided into 19 sectors, 12 endogenous and 7 exogenous, that represent the principal economic sectors of the state. This division shows the economic importance of each sector and the interdependence that exists between these sectors.

Secondary data was used to formulate the empirical model. The results are presented in the transactions matrix, technical coefficients matrix, and interdependence coefficients matrix. The transactions matrix is the foundation of input-output analysis, and the other matrices are computed from it.

The transactions matrix represents the flow of goods and services, measured in dollars, between the sectors of the economy. It is a double-entry table, showing both sales and purchases of each sector.

The matrix of technical coefficients shows the amount of input required from each sector per dollar of output of each sector. The column vector for each sector is the aggregate production function of that sector. The technical coefficients measure the direct effects of changes in the levels of production of the sectors.

The matrix of interdependence coefficients show the combined direct and indirect effects on each sector due to a unit change in final demand

for each sector. These coefficients indicate the interrelationships that exist between the sectors of the economy.

The sector output multipliers estimate the change in total output that will result per dollar of increase in sector final demand. The largest Type I sector output multiplier is 1.97, the sector being Food and Kindred Products. Thus a change of output in this sector would generate a greater change in total output than would an identical change in output of any other sector. The sector with the second largest Type I multiplier is Manufacturing (1.75), followed by Lumber and Wood Products (1.70), Livestock and Livestock Products (1.69), Maintenance Construction (1.60), and Crops (1.35). With the exception of Maintenance Construction, these sectors also account for the largest proportions of total output produced in the state.

The Type II multipliers add the effects of changes in personal income to the Type I multipliers. They, therefore, provide estimates of the change in personal consumption expenditures. The largest Type II output multiplier, 3.30, is found in the Trade sector. This is attributable to the large proportion of personal income per dollar output of the Trade sector. The sector with the second largest multiplier is Food and Kindred Products (3.29), followed by Maintenance Construction (3.27), Lumber and Wood Products (3.19), and Services (3.07).

The Type II multiplier of the Households sector is 2.38. This multiplier is found by including Households as an endogenous sector and assuming a \$1 increase in personal income. The Type II multipliers,

including the Households multiplier, are based on the assumption that the Marginal Propensity to Consume is one, therefore the Type II multipliers may over-state the true multipliers.

The triangularized Montana transactions matrix indicates that, as compared to the U.S. economy, Montana is relatively less developed. The order of the sectors in the triangularized matrix is an index of the interdependence exhibited by each sector. In general, the effects of a change in production spread downward from the sector experiencing the initial change. The magnitude of the effects felt by the lower sectors will be greater than that felt by the higher sectors. The order of the sectors in the triangularized matrix is similar to the order of the sectors when ranked according to the size of the Type I multipliers, although there is some variation.

The four-quadrant format illustrates the flow of goods and services that exists between the sectors of the Montana economy. It also shows the relationship of the Montana sectors and the outside world. The format shows that the four sectors of Quadrant II (Livestock and Livestock Products, Crops, Lumber and Wood Products, and Manufacturing) depend upon external markets for the sale of much of their output. These sectors purchase intermediate inputs within the state but export the bulk of their products. These are also the sectors that account for the major share of the total output of the state.

The four sectors in Quadrant I (Food and Kindred Products, Services, Trade, and Maintenance Construction) are relatively contained by the

Montana economy. They buy most of their inputs and sell most of their products internally. The Transportation and Public Warehousing sector should perhaps also be placed in this quadrant since more than 50 percent of its purchases and sales are made internally.

Quadrant IV contains the remaining sectors (Communications and Public Utilities; Real Estate, Finance, and Insurance; Mining; and Households). These sectors sell most of their output internally but purchase inputs from external sources. Except for Households, the output of these sectors is relatively low. They can be classed as supporting sectors for the economic activity of the other sectors in the economy.

The general flow of goods and services used as intermediate inputs is from Quadrant IV through Quadrant I to Quadrant II. The output of Quadrant II sectors is exported to markets outside the state. Because of the direction of flow combined with the relative size of these sectors, they determine to a large extent the level of production of the remaining sectors.

The multipliers, the triangularized matrix, and the four-quadrant format all indicate that the most important sectors of the Montana economy are the agricultural and manufacturing sectors. Change in production of these sectors generate the greatest change in the state's economy. The remaining sectors play supporting roles, reacting in response to economic stimuli provided by the agricultural and manufacturing sectors.

Implications

At the start of this study it was hoped that the results would provide specific information that could be used as a guide to formulate development policy for Montana. At the conclusion of the study it is apparent that, while input-output analysis does provide useful information of a general nature, specific application of results must be made with great care. The input-output model shows what relationships and interactions exist, but does not show how to improve these relationships in a development context. However, the model does provide a better understanding of the structure of the Montana economy.

The major economic fact brought out by the study is that the Montana economy revolves very much around the extractive industries. Measured by their share of total output, the major industries in Montana are the agricultural and manufacturing industries, comprising five sectors in the model. The two agricultural sectors and the Lumber and Wood Products sector are directly extractive industries. The Food and Kindred Products sector and the Manufacturing sector draw the major share of their inputs from the extractive industries, agriculture and Mining respectively. The products of the major sectors are mostly exported, with the prices set in national markets. Therefore, if these sectors are to grow, they must improve their competitive position.

The implication of the preceding paragraph is that efforts to expand the remaining sectors will produce little effect of growth. They are supporting sectors, and the demand for their output is determined

by the major sectors. Therefore any efforts to expand the output of these supporting sectors in order to stimulate growth in the economy will produce relatively little effect.

Public policy aimed at promoting economic growth appears to require some knowledge of the marginal productivities of various resources in alternative employments. Such knowledge must include productivity estimates by sectors.

Limitations

The major limitation of Leontief Input-Output analysis is the assumption of constant technical coefficients. Technological advances and relative price changes bring about substitution of inputs, causing the technical coefficients to change overtime. Because of this, the model will lose its accuracy over time. If the model is in use over a period of time, it should be brought up to date as more accurate data becomes available.

A second limitation is that the analysis is based on an aggregate model. Each sector usually contains several industries, thus the sector technical coefficients do not represent either specific industries or firms within the sectors except by chance. The exception is if the model is disaggregated to the extent that each sector contains only one industry or one firm. By the same token, the model of a region should not be used as representative of a sub-region contained therein.

Use of the Leontief model is also limited by the availability of data. Collection of primary data sufficient to formulate a model is

very expensive, both in time and money. Secondary data is not usually given in the form required for use in an input-output model, hence errors may arise when interpreting and transcribing data. The data problem could be alleviated considerably if data collection agencies, in particular the Bureau of the Census, would modify their data collection process so as to obtain the information in a form suitable for input-output models. In this way, the time and effort spent in the formulation of an input-output model could be substantially reduced.

APPENDICES

APPENDIX A--STATISTICAL APPENDIX

The model is a standard Leontief-type input-output model, open, consisting of 13 endogenous sectors that are defined according to the Standard Industrial Classification Manual (8). Table I shows the composition of the endogenous sectors. There are six exogenous row vectors in the model including: (a) imports, (b) state and local government, (c) Federal government, (d) wages and salaries, (e) proprietor income, and (f) property income.

The import vector indicates purchases by each sector from outside the state. The State and Local Government vector shows the total amount of state and local taxes paid by each sector. The Federal Government vector shows both Federal taxes paid and resources purchased from the Federal Government. Examples of the latter are timber sales from public lands and mineral leases on public land. Federal taxes include corporation income taxes, payroll taxes, social security taxes, and federal excise taxes.

The remaining three row vectors can, as a group, be called the household sub-vectors. Wages and salaries are self-explanatory. Proprietor income reflects income earned by business proprietors.. Property income is income earned by property, real or personal, which accrues to households. This includes rent income, dividends, interest income, and personal capital gains.

Endogenous sectors have matching row and column vectors. Exogenous row vectors, on the other hand, need not have matching column vectors. In this model, the exogenous column vectors include: (a) exports,

TABLE A-1. INDUSTRIES INCLUDED IN THE ENDOGENOUS SECTORS.

Endogenous Sectors	Industries
1. Livestock & Livestock Products	a) Cattle & calves b) Sheep & lambs c) Hogs d) Dairy products e) Wool f) Poultry products g) Other livestock products
2. Crops	a) Wheat b) Barley c) Other feed grains d) Seed crops e) Other crops
3. Food & Kindred Products	a) Meat products b) Dairy products c) Grain mill products d) Sugar e) Beverages f) Miscellaneous
4. Lumber and Wood Products	a) Logging camps b) Saw mills and planing mills c) Mill work products
5. Other Manufacturing	a) Furniture & fixtures b) Paper & allied products c) Printing & publishing d) Chemicals & allied products e) Petroleum & Coal products f) Stone, clay & glass products g) Primary metal industries h) Fabricated metal products i) Machinery, except electrical j) Transportation equipment k) Miscellaneous manufacturing
6. Transportation & Public Warehousing	a) Rail carriers b) Truck carriers c) Airlines d) Miscellaneous transportation e) Public warehouses
7. Communications & Public Utilities	a) Public communications b) Electric & Gas services c) Radio & T.V. broadcasting
8. Real Estate, Finance, and Insurance	a) Real estate services b) Financial services c) Insurance except government
9. Mining	a) Primary metals mining b) Crude petroleum & natural gas c) Non-metallic minerals mining
10. Services	a) Public lodging b) Personal services c) Miscellaneous business services d) Auto services e) Miscellaneous repair services f) Amusement & recreation services g) Legal services h) Medical services i) Oil & gas field services
11. Trade	a) Wholesale trade b) Retail trade
12. New Construction	a) Residential construction b) Non-residential construction c) Non-building construction
13. Maintenance Construction	a) Residential maintenance b) Non-residential maintenance c) Non-building maintenance

(b) State and local government, purchases and transfers; (c) Federal government purchases; (d) Federal government transfers; and (e) households. The export vector indicates the value of sector output that is exported from the state. With the exception of the trade sector, exports are physical products. Exports are primarily raw materials, such as refined copper, cereal grains, and lumber. Raw materials exporting sectors are the two agricultural sectors, lumber and wood products, and manufacturing. Exports of the trade sector reflect the margins received by the wholesale trade industries, primarily farm products wholesalers, from sales outside the state. The amount exported from the communications and public utilities sector is the value of electricity generated in the state by privately owned utilities and then sold outside the state. Federal government exports reflect electricity generated in Montana by Federal Government utilities (Hungry Horse, etc.) and sold to utilities outside the state. The remaining exporting sector, Food and Kindred Products, exports primarily hog and beef carcasses that will be utilized in out-of-state plants as a processing input.

State and local government purchases and transfers show expenditures of state and local government. In contrast to the endogenous sectors, where total input equals total output, the exogenous sectors total input need not balance total output. In the case of state and local government, collections are greater than expenditures, the reason being a reduction in total debt (54, 99).

Federal Government Purchases and Federal Government Transfers are the column vectors that correspond to the federal government row vector. There was no source available from which to estimate total federal expenditures on imports. In the case of agriculture, federal purchases are in effect the subsidies paid on the basis of production (price support payments for wool, wheat, etc.). Federal government transfers, on the other hand, show the amount paid by the federal government that is not tied to production (ACP programs, social security payments, veteran's benefits, etc.).

The Households column vector corresponds to the three personal income row vectors of Wages and Salaries, Proprietor Income and Property Income. The difference between the sum of the three personal income vectors and total household expenditures is total personal saving.

General Estimating Procedure

The basic assumption of the model is that sector inputs would be obtained from endogenous sectors, if possible. For example, data from (68), (73), and (79) indicate that the crops sector purchased \$6,000,000 worth of fertilizer. Reference (81) indicated that \$5,231,000 worth of fertilizer was produced by the manufacturing sector. Reference (63) indicated a margin of approximately 15 percent between the two sectors. Therefore, fertilizer used by the crop sector equalled fertilizer produced in the state.

Output by product was obtained for all sectors. In many cases, output could be allocated on the basis of product. As an example

electric energy output is given as residential sales, commercial sales, etc. Combining output information of this type with input data resulted in fairly accurate estimates of intersectoral transactions.

Some other cases sample information was available, usually with respect to inputs. In these cases, the assumption was that the output (input) distribution of all firms in the industry was essentially the same as the output (input) distribution of the firm in the sample.

Finally, in those cases where little or no specific information was given with respect to an intersectoral transaction value, estimates were based on a combination of items: (1) national coefficients, (2) residual values, (3) professional opinion, and (4) evaluation of extraneous data.

A table has been constructed for each sector, Tables A-II through A-XVII, showing the inputs purchased by sector, the technical input coefficient, and the sources from which data concerning the intersectoral transactions were available.

A résumé of sector output and sector transactions follows each table. Sources of data used for this study will be referred to in the Appendix by the number which corresponds to the publications as listed in the Literature Cited.

TABLE A-II. LIVESTOCK AND LIVESTOCK PRODUCTS: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficients	Data Sources
Livestock & Lvsk. Prod.	9,410	.04549	47,82,86,92,94,96, 117
Crops	67,341	.32555	47,86,87,88
Food & Kindred Products	3,688	.01783	86,87,88,105
Lumber & Wood Products	113	.00055	105
Manufacturing	1,033	.00499	86,92,105
Transportation & Public Warehousing	3,772	.01823	86,92,95,143,121
Communications & Public Utilities	2,271	.01098	19,45,86,88,91
Real Estate, Finance, & Insurance	9,111	.04404	72,73,84,86,96, 117
Mining	49	.00024	86,110,96,81
Services	1,989	.00962	86,92,96,101,102, 81
Trade, Wholesale & Retail	5,674	.02743	81,86,92,96,97, 99,117
Construction, Maintenance	203	.00098	92,96,117,125, 126,128
Construction, New	124	.00060	92,96,117,125, 126,128
Imports	38,270	.18501	47,86,92,97,99
State & Local Government	13,858	.06699	49,74,86,96,117
Federal Government	1,004	.00485	86,92,117,128,130, 133,136
Wages and Salaries	10,804	.05223	86,92,117,128
Proprietor Income	34,719	.16784	86,92,117,128, 113,136
Property Income	3,423	.01655	86,92,128,136
Total Gross Inputs	206,856	1.00000	47,86,92,96,117

TABLE A-III. CROPS: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Crops	9,385	.03047	47,86,92,96,117
Lumber & Wood Products	21	.00007	86,105
Manufacturing	20,479	.06649	105,117,121
Transportation & Public Warehousing	3,736	.01213	85,86,96,117,121
Communications & Public Utilities	3,126	.01015	19,45,86,89,92,96
Real Estate, Finance, & Insurance	19,309	.06269	72,73,84,86,92,96
Mining	3,720	.01208	86,92,117
Services	4,953	.01608	81,86,92,101,117
Trade, Wholesale & Retail	13,155	.04271	81,86,92,97,99,117
Construction, Maintenance	804	.00207	86,92,96,125
Construction, New	638	.00261	86,92,96,125
Imports	58,610	.19030	46,86,92,96
State & Local Government	17,313	.05622	49,73,74,86
Federal Government	674	.00219	86,96,117,128,130,133
Wages and Salaries	17,952	.05829	86,96,117,128
Property Income	5,377	.01746	86,96,117,128
Proprietor Income	<u>128,733</u>	<u>.41799</u>	86,96,117,128,136
Total Gross Inputs	307,985	1.00000	

Resumé: Agricultural Sectors, Tables A-II and A-III.

Table A-IIa shows the composition of Agricultural output in Montana. Cattle and calves are the major livestock product, making up three-fourths of total livestock production. Data from (35) and (74) indicate that 896,000 head of cattle and calves were marketed exclusive of interfarm sales, and that 881,192 were shipped out of the state. Calves, heifers, and yearling steers made up 76 percent of out-of-state shipments. There were 98,000 fed cattle marketed during the year, with 112,000 cattle placed on feed. Thus, beef production in Montana is primarily the production of feeder stock with about 70 percent of cattle shipments going to the Midwest (34).

Of the remaining livestock production, sheep products were the most important, followed closely by dairy products. Hog and poultry production accounted for most of the remainder.

Considering the composition of Montana's livestock production, it is apparent that the livestock input vector is weighted heavily by range cattle production enterprises in the state.

Livestock output serving as livestock input was determined by subtracting Federal Purchases, Household Consumption and direct retail sales, exports, and input to the Food and Kindred Products sector from total livestock production.

The principal crop is wheat which accounted for over one-half of the total crop production in the state. Hay production is the second largest crop in the state, amounting to about one-fifth of the total crop output. Barley and sugar beets rank third and fourth, respectively

in crop production, and the remaining crop output consists of field seeds, potatoes, other feed grains, etc. Wheat and barley are the major crop products exported from the state.

TABLE A-IIa. AGRICULTURAL OUTPUT.

Product	Value
Crops:	
Wheat	168,236
Barley	32,157
Hay	66,600
Sugar Beets	15,314
Other Crops	<u>25,678</u>
Total Crops	307,985
Livestock:	
Cattle and Calves	155,926
Dairy Products	15,875
Sheep, Lambs and Wool	18,171
Hogs	9,202
Poultry Products	5,801
Other	<u>1,881</u>
Total Livestock	206,856
Total Agriculture	514,841

Reference (74) gives the on-farm disposition of cereal grains produced in the state, indicating the amount fed, amount used for seed, and the quantity sold. The total amount of crop output serving as a crop input (seed) was estimated from (74) and (35).

Crop input to the livestock sector was estimated with the assumption that crop output not utilized elsewhere was used in livestock production. The major crop input to livestock was hay, which accounted for about 90 percent of the total.

Food and Kindred Products output purchased by the livestock sector is given in (81), and was primarily prepared livestock feed. The remaining data sources supported this figure.

Purchases by livestock from the Lumber and Wood Products sector were indicated in (81), the major item being wooden fence posts.

Manufacturing output utilized by the livestock sector was estimated from (68), (83), and (81). Fuel purchases accounted for the major share. Manufacturing output purchased by the crop sector was estimated from (68), (83), and (81). Fuel was the major item purchased.

Transportation and Public Warehousing purchased by the livestock sector was estimated primarily from (125), supported by (95) and (97). This estimate reflects transportation purchased only by the livestock sector, and is not the total cost of transporting livestock and livestock products, most of which is paid by the purchasing sector (Food products, exports). Transportation purchased by the crop sector was given in (67), supported by (95) and (97). As in the case of

livestock, this figure reflects only transportation purchases by the crop sector, not the total cost of transportation of crop products.

Purchases from the Communications and Public Utilities sector was estimated from (15), (33), (74), and (100). Total purchases by Montana Agriculture was given. Sector allocation was based on the ratio of gross output of each sector. This figure overstates the input required for production purposes, due to the lack of a base for determining the personal consumption proportion of these purchases. Total purchases by agriculture of Real Estate, Finance, and Insurance was given in (56) and (66), supported by (68), (79), and (100). Finance input to agriculture was allocated on the basis of gross output of each agricultural sector. Insurance inputs were allocated in the same way except for crop insurance.

Services purchased by agriculture were estimated from (63), (68), and (87). Services not specifically indicated as being purchased by one of the agricultural sectors were allocated according to gross output of the agricultural sectors.

Wholesale and retail trade output purchased by agriculture was given by (83). As with services, any trade not directly indicated as going to a specific agricultural sector was allocated on the basis of agricultural sector gross output.

Construction statistics were given primarily in (108), and were supported by data from (73), (106), and (107).

Purchases from the State and Local Government are the state and local taxes paid by each sector. Reference (58) gives the amount of taxes collected by type of tax. Data from (73), (68), and (37) also provided information which aided in properly allocating state and local taxes between the two agricultural sectors.

Purchases from the Federal Government sector comprised both taxes and physical inputs. Tax data was given in (68), (100), (110), and (115). Agricultural use of Federal lands was found in (100) and (112).

Wages, proprietor income, and property income from the agricultural sectors was found in (68), (79), and (110). Reference (79) also indicated the sector allocation of wages and income which was supported by (68) and (110).

TABLE A-IV. FOOD AND KINDRED PRODUCTS: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Livestock & Livestock Prod.	35,029	.24864	82,92,47,95,105
Crops	28,028	.19894	47,92,105
Food & Kindred Products	13,413	.09521	105,106
Manufacturing	612	.00434	105,106
Transportation & Public Warehousing	340	.00241	48,105,112,113,114
Communications & Public Utilities	868	.00616	19,69,105,106
Real Estate, Finance, & Insurance	1,879	.01334	18,72,105
Services	2,466	.01750	101,105
Trade, Wholesale & Retail	4,054	.02878	67,99,105
Construction, Maintenance	134	.00095	18,105,122,125
Construction, New	635	.00451	105,106,125
Imports	14,989	.10639	105,106
State & Local Government	5,157	.03660	74
Federal Government	4,179	.02966	132,139
Wages and Salaries	23,292	.16533	51,105,128,131
Proprietor Income	4,050	.02875	128,138,139
Property Income	<u>1,760</u>	<u>.01249</u>	128,138,139
Total Gross Inputs	140,885	1.00000	

Resume: Food and Kindred Products

Table A-IVa shows the composition of Food and Kindred Products output given in (81). Meat products constitute the largest share of output, followed by sugar refinery products.

Agricultural inputs to the Food and Kindred Products sector are given in (81), substantiated by (35), (64), (74), and (77). Livestock products purchased by the Food sector were as follows:

Cattle and Calves	12,644,000
Sheep and Lambs	86,000
Hogs	9,022,000
Dairy Products	<u>13,277,000</u>
Total	35,029,000

Crop output purchased by the Food sector was as follows:

Wheat	12,609,000
Sugar Beets	14,272,000
Other Crops	<u>1,147,000</u>
Total	28,028,000

Purchases from all other endogenous sectors are indicated in (81), and the data from (81) is supported by the other data sources. State and local taxes were estimated from (58) and (121).

Federal taxes included corporation income taxes, excise taxes, and payroll taxes (Social Security, etc.). Data on Federal taxes by sector was available in (38), (110), and (115).

Data on Wages and Salaries, Proprietor Income, and Property Income was given in (38), (81), (110), (118), and (121).

TABLE A-IVa. FOOD AND KINDRED PRODUCTS OUTPUT. a/

Item	Output (\$1,000)
Beef Carcasses	16,704
Pork Carcasses	7,020
Processed Beef and Pork	9,609
Other Meat Products	<u>3,205</u>
Total Meat Output	36,538
Bottled Milk	15,465
Other Dairy Products	8,190
Intermediate Dairy Products	<u>5,216</u>
Total Dairy Output	28,871
Flour	17,753
Animal Feed	<u>3,688</u>
Grain Mill Output	21,441
Sugar Refining	31,068
Bread and Related Products	11,602
Beverages	7,461
Intermediate Food Products	<u>3,904</u>
	54,035
Total	140,885

a/ All output data are given in (81).

TABLE A-V. LUMBER AND WOOD PRODUCTS: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Crops	1,694	.01005	47,92
Lumber & Wood Products	36,146	.21451	8,105,106,60,144
Manufacturing	1,512	.00897	8,105,106
Transportation & Public Warehousing	26,200	.15549	8,105,113,144,145
Communications & Public Utilities	4,491	.02665	8,19,105,106
Real Estate, Finance, & Insurance	2,780	.01650	8,18,72
Mining	214	.00127	8,104,105,107
Services	1,570	.00932	8,101,102,104,105
Trade, Wholesale & Retail	7,315	.04341	8,104,105
Construction, Maintenance	991	.02304	8,104,105
Construction, New	3,882	.00588	8,104,105,126
Imports	15,814	.09385	8,60
State & Local Government	6,603	.03919	8,49,74
Federal Government	4,943	.02933	130,133,139
Wages and Salaries	44,314	.26299	8,51,128,131,138
Proprietor Income	8,495	.05041	8,128,138
Property Income	1,540	.00914	8,128,138
Total Gross Inputs	168,504	1.00000	

Resume: Lumber and Wood Products

The output of the lumber and wood products sector is comprised entirely of soft-wood products (126). Western Montana produces the greatest share of total production, accounting for approximately 84 percent (126,127).

Sources (34) and (79) indicate sales of \$1,694,000 worth of stumpage and saw logs from agriculture. It is assumed that this output was purchased by the lumber and wood products sector.

Saw logs and contract logging output from logging camps (SIC 241) was purchased by sawmills (SIC 242). This intermediate input is given in (81), (82), and (126).

Data on purchases from the remaining endogenous sectors is given primarily in (81), (82), and (7), and is supported for each sector by the data sources indicated. State and local taxes were estimated from (58), (7), and (121).

Transactions with the Federal Government included federal taxes and timber purchases. Timber sales from public lands are given in (112). Tax information is given in (115) and (121).

Wages and Salaries paid by this sector are given in (38), (81), and (110). Proprietor and Property Income generated by Lumber and Wood Products is given in (110) and (120).

TABLE A-Va. LUMBER AND WOOD PRODUCTS OUTPUT. a/

Item	Output
Logs and Bolts	20,741
Contract Logging	16,318
Rough Lumber	14,790
Dressed Lumber	62,975
Softwood Cut Stock	3,710
Custom Sawing	1,993
Plywood	14,251
Laminates	1,553
Miscellaneous Products	<u>32,173</u>
Total Output	168,504

a/ Data given in (81).

TABLE A-VI. MANUFACTURING: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	1,148	.00314	8,104,105
Manufacturing	7,187	.01966	104,105
Transportation & Public Warehousing	11,589	.03171	48,104,105,113
Communications & Public Utilities	14,845	.04061	19,21,69,104,105
Real Estate, Finance, & Insurance	3,931	.01076	72,104,105
Mining	159,417	.43612	104,105,121,122,123
Services	4,669	.01277	101,102,104,105
Trade, Wholesale & Retail	9,867	.02699	97,99,104,105
Construction, Maintenance	3,212	.00879	104,105,106,125
Construction, New	15,258	.04174	104,105,106,125
Imports	15,171	.04150	104,105,106
State & Local Government	22,308	.06103	49,74,105,104
Federal Government	26,870	.07351	51,104,128,131
Wages and Salaries	56,524	.15463	51,104,128,131
Proprietor Income	11,130	.03045	104,105,128
Property Income	2,407	.00659	104,105,128
Total Gross Inputs	365,533	1.00000	

Resume: Manufacturing

Two manufacturing industries account for more than 84 percent of total output of this sector while employing only 20 percent of total manufacturing employment. Primary metal industries account for the largest share, exporting almost 100 percent of their output. Petroleum refining is the other major manufacturing industry, exporting approximately one-third of their output (58). The output of the remaining manufacturing industries is utilized in the state. About 15 percent of total manufacturing output is in the form of Household consumption products.

Purchases from the mining sector account for about 44 percent of manufacturing inputs. The composition of mineral industries input, given in (80), (81), (90), (92), and (93) is:

Crude Petroleum	\$ 72,752,000
Crude Mineral Ore	82,172,000
Other Minerals	<u>4,493,000</u>
Total	\$159,417,000

Purchases from the remaining endogenous sectors were given in (81) and supported by the data sources indicated for each sector.

State and local taxes were estimated from (37), (58), (81), and (121). Federal taxes were estimated from (38), (81), (115), and (121).

Wages and Salaries, Proprietor and Property Income were estimated from (38), (80), (81), (110), and (113).

TABLE A-VIa. MANUFACTURING OUTPUT. a/

Item	Output (\$1,000)
Primary Metals	205,000
Fabricated Metal Products	7,618
Machinery	1,947
Transportation Equipment	2,439
Stone, Clay, and Glass Products	
Ready Mix Concrete	5,311
Other Concrete Products	3,191
Miscellaneous Products	10,261
Chemicals and Allied Products <u>b/</u>	5,231
Petroleum Products	103,094
Printing and Publishing	
Newspapers, Sales	5,316
Newspapers, Advertising	8,162
Commercial Printing	
Business Forms	1,913
Miscellaneous Printing	1,403
Miscellaneous Manufacturing <u>c/</u>	4,647
Total Manufacturing	365,533

a/ Data given in (80) and (81).b/ Fertilizer production only.c/ Apparel; furniture and fixtures; paper; rubber and plastic; leather; instruments; and miscellaneous manufacturing.

TABLE A-VII. TRANSPORTATION AND PUBLIC WAREHOUSING: PURCHASES,
TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	21	.00013	104,105,126
Manufacturing	9,310	.05966	104,105,112,126
Transportation & Public Warehousing	421	.00270	112,120,126
Communications & Public Utilities	1,837	.01177	19,21,112,120,126
Real Estate, Finance, & Insurance	2,060	.01320	72,113,120,126
Services	1,626	.01042	101,102,112,120,126
Trade, Wholesale & Retail	1,643	.01053	97,99,112,120,126
Imports	37,390	.23960	112,120,126
State & Local Government	7,412	.04750	49,74,112,126
Federal Government	2,188	.01402	112,120,126,133,139
Wages and Salaries	76,752	.49182	51,112,126,120
Proprietor Income	13,321	.08536	112,120,126
Property Income	2,075	.01329	112,120,126
Total Gross Inputs	156,056	1.00000	

Resume: Transportation and Public Warehousing

In the 1958 National Input-Output Study, transportation expenditures accounted for approximately 3.85 percent of total output (109). In this Montana Input-Output study, transportation accounts for about 6.11 percent of total output. The reasons for the relatively greater importance of transportation in Montana can be attributed to two major factors: (1) the distances involved in shipments to, from, and within the state; and (2) the make-up of commodities which are shipped from Montana.

Most of Montana's production is in the form of bulky and heavy but relatively low value products. Refined primary metals, cereal grains, livestock, and lumber, items that account for about 80 percent of Montana exports, have a relatively high ton-mile transportation expense. This factor, coupled with distance to the markets for these items, leads to transportation expense being a substantial amount.

Source (126) indicates that transportation expense is 20 to 25 percent of the value of final products of Montana lumber. Grain transportation to terminal markets (Portland or Minneapolis) is about 75 cents per hundredweight.

Livestock transportation costs are about \$2.00/Cwt for animals shipped to the midwest. These examples illustrate why transportation expense is relatively almost twice as large in the Montana economy as it is in the National economy.

Table A-VIIa shows the relative shares of transportation output generated by each sub-industry. Railroads are by far the

largest carrier in the state. The railroad revenue generated by shipments in the state is somewhat less than shown, due to commodities that are shipped through the state. The amount of this traffic will show up as part of the value of transportation output that is exported.

TABLE A-VIIa. TRANSPORTATION AND PUBLIC WAREHOUSING OUTPUT.

Item	Output (\$1,000)
Railroad Transportation <u>a/</u>	
Great Northern	58,568
Northern Pacific	52,872
Burlington	3,460
Soo Line	34
Union Pacific	2,078
Milwaukee Road	24,499
Total Railroad	147,511
Truck Freight Transportation <u>b/</u>	11,331
Passenger Transportation <u>c/</u>	2,249
Public Warehousing <u>d/</u>	965
Total	156,056

a/ Data submitted via telephone from Montana Railroad and Warehouse Commission, Helena, Montana.

b/ Data from (95) shows total Truck Revenue and Motor Freight Trucks for the Mountain Division, plus the number of Motor Freight Trucks in Montana. Montana Truck transportation revenue was estimated on the ratio of Montana Trucks/Mountain Division Trucks.

c/ Source (115) gives total Passenger excise tax. From this figure, Airline Passenger transportation expense was estimated at \$730,000. Other Passenger transportation was estimated from (94), (97), and (110).

d/ Public Warehousing expense in Montana was estimated from (97).

Estimating purchases of the transportation sector from the remaining sectors was extremely difficult as very little input data was available. The general estimating procedure used in this sector can be indicated as the following steps:

- 1) National transportation coefficients were condensed to the same sectors in the Montana model.
- 2) Dollar purchases from each sector were estimated from the national coefficients.
- 3) Output of a given sector was examined to see if that sector was able to provide the kind of product required by the transportation sector in the amount indicated.
- 4) If a given sector's output could supply the transportation sectors estimated requirements, it was assumed that it did so.
- 5) If the given sector was unable to supply the estimated requirements, the national coefficient was adjusted downward to fit the transaction.

Final estimations of the transportation input coefficients are shown in Table A-VIIb, along with the national coefficients. The state coefficients of the endogenous sectors are all less than the national coefficients. In one case, communications and public utilities, transportation expenditures were given.

The State and Local Government coefficient for Montana is larger than the national coefficient. This is explained by the relative amount of real property in the state owned by the railroads. Source (58) gives the amount of taxes levied against railroad property.

The Federal Government sector coefficient for the state is also larger than the national coefficient. The reason for this lies principally in that the state coefficient includes Federal Income taxes while the national coefficient does not.

TABLE A-VIIb. NATIONAL AND STATE INPUT COEFFICIENTS, TRANSPORTATION AND PUBLIC WAREHOUSING SECTOR.

Sector	Coefficient	
	State	National
Livestock & Livestock Products		.00005
Crops		.00101
Food & Kindred Products		.00293
Lumber & Wood Products	.00013	.00027
Manufacturing	.05966	.07081
Transportation & Public Warehousing	.00270	.06371
Communications & Public Utilities	.0117	.01226
Real Estate, Finance, & Insurance	.01320	.05056
Mining		.00085
Services	.01042	.04029
Trade	.01053	.02943
Construction, New		
Construction, Maintenance		.03660
Imports	.23383	.06414
State & Local Government	.04750	.02164
Households	.49182	
Proprietor Income	.0856	.60378
Property Income	.01329	

The national coefficient is not given for each of the remaining sectors, wages and salaries, proprietor income, and property income, but is given as a "value added" sector. Wages and salaries make up the dominant share of value added. Data on these three sectors were available from (110).

TABLE A-VIII. COMMUNICATIONS AND PUBLIC UTILITIES: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	820	.00617	19,20,21,89,105,116
Manufacturing	549	.00413	19,20,21,89,105,116
Transportation & Public Warehousing	1,642	.01236	19,20,21,89,105,116
Communications & Public Utilities	10,850	.08168	19,20,21,89,105,116
Real Estate, Finance, & Insurance	1,873	.01410	19,20,21,89,105,116
Mining	2,178	.01640	19,20,21,105,116
Services	1,043	.00785	19,20,21,89,105,116
Trade, Wholesale & Retail	1,247	.00939	19,20,21,89,105,116
Construction, Maintenance	256	.01108	19,20,21,89,105,116
Construction, New	1,472	.00193	19,20,21,116,125
Imports	29,327	.22077	19,20,21,116
State & Local Government	17,636	.13276	19,20,21,49,74
Federal Government	18,139	.13655	19,20,21,51,89,128
Wages and Salaries	32,641	.24572	19,20,21,51,89,128
Proprietor Income	10,143	.07635	19,20,21,89,128
Property Income	3,024	.02276	19,20,21,89,128
Total Gross Inputs	132,840	1.00000	

Resume: Communications and Public Utilities.

Sources (15), (16), and (17) provided most of the data for this sector. These sources indicated electric and natural gas output distribution and input data. Table A-VIIIa shows the total output of this sector and the output distribution of electricity and natural gas.

Communications output was obtained from the Montana Board of Railroad Commissioners. The distribution of Communications output, shown in Table A-VIIIa, was estimated by a representative of Mountain States Telephone and Telegraph Company. Mountain States output is about 95 percent of total communications output in the state.

Water supply utilities in Montana are operated by local governments. Total water utility revenue was obtained from (99). Operating expense and construction expenditures were also given in (99).

TABLE A-VIIIa. COMMUNICATIONS AND PUBLIC UTILITIES OUTPUT.

Item	Sales
Electricity <u>a</u> /:	61,821
a) Residential, farm & non-farm	23,179
b) Commercial	11,253
c) Industrial	11,936
d) Government	8,638
e) Railroads	985
f) Interdepartmental	102
g) For resale	5,728
Gas <u>b</u> /:	26,975
a) Residential	11,022
b) Commercial	4,879
c) Industrial	5,522
d) Government	2,452
e) Interdepartmental	61
f) For resale	3,039
Water Utilities <u>c</u> /:	5,900
Communications <u>d</u> /:	38,144
a) Residential	11,795
b) Business	20,899
c) Government	5,450
Total	132,840

a/ Data from (15), (16), (32), (58), (70).b/ Data from (33), (36), (53), (58).c/ Data from (89), (98), (99).d/ Data from (53), (110).

TABLE A-IX. REAL ESTATE, FINANCE, AND INSURANCE: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Manufacturing	803	.00560	18,79,117,126,139
Communications & Public Utilities	865	.00603	18,79,117,126,139
Real Estate, Finance, & Insurance	1,792	.01350	18,79,117,126,139
Services	4,243	.02959	18,79,117,126,139
Trade, Wholesale & Retail	1,711	.01193	18,79,117,126,139
Construction, Maintenance	460	.00321	18,79,117,126,139
Imports	71,473	.49842	18,79,117,126,139
State & Local Government	3,513	.02450	18,79,117,126,139
Federal Government	8,973	.06257	18,79,117,126,139
Wages and Salaries	31,186	.21748	18,51,126,139
Proprietor Income	7,038	.04908	18,126,127,128
Property Income	<u>11,341</u>	<u>.07909</u>	18,117,126,128
Total Gross Inputs	143,398	1.00000	

Resume: Real Estate, Finance, and Insurance

Output of financial agencies is calculated as the operating revenue of these agencies. The operating revenue of commercial banks in Montana is given in (14) by source of revenue (U. S. Government, securities, real estate loans, industrial loans, etc.). Operating expenses of commercial banks are also given in this source. Data on operating revenue and operating expenses of savings and loan agencies is given in (61) and (62).

Insurance output is assumed to be the difference between total insurance premiums collected and insurance claims paid. Source (56) gives data on premiums and claims by type of insurance (life, property, etc.). It was assumed that the insurance industry in Montana would not differ significantly from the national insurance industry with respect to inputs, therefore, national coefficients were used to estimate the distribution of inputs in the state industry.

Home and farm financing by insurance companies was estimated from (21) and (100). Insurance company financing is, therefore, underestimated to the extent of insurance company financing of business.

Production Credit Associations and Federal Land Banks finance only farm operations. Data on the output of PCA's and Land Banks is given in (66) and (73).

Real estate output was estimated by computing the ratio of employment in Montana Real estate to national employment in real estate and then multiplying national real estate output by this ratio. Real estate transfer tax stamps, given in (115), were used to calculate total

real estate sales. Estimated real estate sales commissions based on Federal transfer stamps supported the estimate based on employment.

The output of Finance agencies 1/ and miscellaneous finances 2/ were estimated by State/National employment ratios. Estimated output was supported by data from (110). Table A-IXa shows the output of this vector by agency.

Detailed operating expenses of commercial banks were given in (14). Inputs for the remaining agencies were estimated from (121).

Wages and Salaries, Proprietor and Property Income were given in (110). This data supported the estimates for these vectors derived from (121).

Purchases from the imports sector appear to be unreasonably high for this sector. However, a large part of this estimate is the payment for imported money capital and payment to companies outside the state. The latter is particularly true of insurance. Recurring premiums, usually sent by the policy holder directly to the home office of the insurance company, are represented here as moving through the state insurance sector.

1/ Agencies of this type include: Consumer Credit Corporation, Household Finance, Seaboard Finance, etc.

2/ Retail and wholesale credit, Credit Unions, etc.

TABLE A-IXa. REAL ESTATE, FINANCE, AND INSURANCE OUTPUT.

Agency	Production
Commercial Banking <u>a/</u>	50,498
Insurance <u>b/</u>	45,778
Savings and Loan Banks <u>c/</u>	9,060
Life Insurance Financing <u>d/</u>	3,246
Production Credit Associations <u>e/</u>	2,824
Federal Land Banks <u>f/</u>	4,756
Real Estate Services <u>g/</u>	9,950
Finance Agencies <u>h/</u>	8,390
Miscellaneous Financial <u>i/</u>	8,896
Total	143,398

a/ Source (14).b/ Source (56).c/ Sources (61) and (62).d/ Sources (21) and (100).e/ Sources (66) and (73).f/ Sources (66) and (73).g/ Estimated from (38), (51), (100), (110), (113), and (115).h/ Estimated from (38), (51), (100), (110), and (113).i/ Estimated from (38), (100), (110), and (113).

TABLE A-X. MINING: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	154	.00089	105,107,109,144
Manufacturing	1,418	.00819	105,107,109
Transportation & Public Warehousing	5,270	.03044	107,109,113
Communications & Public Utilities	5,759	.03326	19,21,107,109
Real Estate, Finance, & Insurance	2,868	.01657	18,72,107,109
Mining	2,475	.01430	107,109
Services	17,421	.10063	101,107,109,110
Trade, Wholesale & Retail	3,611	.02086	97,99,107,109
Construction, Maintenance	1,042	.00602	107,109,125
Construction, New	4,296	.02481	107,109,125
Imports	53,511	.30910	107,109
State & Local Government	5,862	.03386	49,74,107,109
Federal Government	11,057	.06387	107,109,130,133,139
Wages and Salaries	45,107	.26055	51,107,128,131,139
Proprietor Income	10,036	.05797	128,139
Property Income	3,234	.01868	128,139
Total Gross Inputs	173,121	1.00000	

Resume: Mining

The mining sector is dominated by two mining industries, Primary Metals, and Oil and Gas Extraction. Primary Metals account for 47 percent of mining output, while Oil and Gas production is 43 percent of the total. Table A-Xa shows the output of the mining sector.

Sources (90), (91), and (92) yielded considerable information on mining input requirements. Final estimates were made by combining data in these sources with the data in the sources indicated for each sector.

The large amount of purchases from the services sector are mainly specialized oil and gas field services. Oil and gas field services purchased in Montana amounted to \$14,595,000, (90). Oil and gas field services imported amounted to \$6,505,000, (91). Remaining imports included \$41,785,000 of specialized mining machinery.

TABLE A-Xa. MINING OUTPUTS. a/

Industry	Output (\$1,000)
Oil and Gas Extraction	
Crude Petroleum	72,752
Natural Gas	1,178
Metal Mining	
Copper Ores	71,040
Lead and Zinc	9,588
Other	1,544
Non-Metallic Minerals	
Sand and Gravel	2,942
Chemical and Fertilizer Minerals	5,848
Miscellaneous non-metallic	5,925
Miscellaneous Minerals	2,304
Total	173,121

a/ Data from (90).

TABLE A-XI. SERVICES SECTOR: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Manufacturing	762	.00368	101,102,105,108,126
Transportation & Public Warehousing	616	.00298	101,102,105,108,126
Communications & Public Utilities	1,271	.00615	19,21,101,102,108,126
Real Estate, Finance, & Insurance	2,275	.01100	18,72,101,102,108,126
Mining			
Services	5,613	.02714	101,102,108,126
Trade, Wholesale & Retail	3,737	.01807	97,99,101,108,126
Construction, Maintenance	225	.00069	101,102,108,126
Construction, New	142	.00109	101,102,108,126
Imports	50,178	.24267	101,102,108,126
State & Local Government	1,930	.00933	49,74,101,126
Federal Government	5,955	.02880	101,102,108,126,133
Wages and Salaries	87,778	.42451	51,128,131,139
Property Income	17,662	.08522	128,139
Proprietor Income	28,673	.13867	128,139
Total Gross Inputs	206,777	1.00000	

Resume: Services

Table A-XIa shows the composition of the Services sector output. Mining services output was given in (90). Medical services output was estimated from (100) and (109). Legal services output was estimated by multiplying the number of active lawyers by the average income of lawyers. Average income was obtained from the State Bar Association. Output of the remaining services was given in (88).

The Services sector inputs were estimated from (121) where business deductions for each type of service are given. It was assumed that the input structure of each service in Montana was similar to the national aggregation. Source (110) shows personal income generated by the services sector. This compared with the estimate derived from (121).

TABLE A-XIa. SERVICES OUTPUT.

Type of Service	Output (\$1,000)
Public Lodging	28,267
Personal Services	23,954
Business Services	11,766
Auto Repair	16,730
Other Repair	9,826
Amusement	12,120
Mining Services	14,595
Medical Services	77,987
Legal Services	11,532
Total	206,777

TABLE A-XII. WHOLESALE AND RETAIL TRADE: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Manufacturing	6,462	.01724	105,97-100,139
Transportation and Public Warehousing	27,084	.07227	105,97-100,139
Communications and Public Utilities	22,647	.06043	19,21,97-100,139
Real Estate, Finance, & Insurance	11,919	.03181	18,72,97-100,139
Services	10,525	.02809	105,97-100,139
Trade, Wholesale & Retail	10,840	.02893	.05,97-100,139
Construction, Maintenance	1,489	.00332	105,97-100,139
Construction, New	1,243	.00398	105,97-100,139
Imports	13,871	.03703	105,97-100,139
State & Local Government	19,247	.05136	105,97-100,49,74
Federal Government	17,899	.04776	105,97-100,133,139
Wages and Salaries	164,571	.43917	105,97-100,128,139
Proprietor Income	40,160	.10717	97-100,128,139
Property Income	23,249	.06204	97-100,128,139
Total Gross Inputs	374,734	1.00000	128,139

Resumé: Trade, Wholesale and Retail

The output of the trade sector is defined as "total receipts less the cost of goods sold." Total receipts of wholesale and retail trade are given in (86) and (84) respectively. "Cost of goods sold" was estimated from these sources and (121), where national aggregate receipts and "cost of goods sold" are given by "Kind of Business." It was assumed that the ratio of cost of goods sold/total receipts for each "Kind of Business" in Montana is similar to the national ratio. This method resulted in estimates for output as shown in Table A-XIIa.

TABLE A-XIIa.

Trade	Receipts	Cost of Goods Sold (\$1,000)	Output
Retail	965,734	719,139	246,595
Wholesale	<u>844,249</u>	<u>716,110</u>	<u>128,139</u>
Total	1,809,983	1,435,249	374,734

Input information for this sector was available from (83), (84), (85), (86), (110), and (121). These sources, combined with the sources listed by each sector, yielded the input estimates.

It is interesting to note the relatively low level of sector output serving as intrasector input. At first glance one might expect that most of the wholesale output would be purchased by retail. The reason this is not true in Montana is because of the kinds of wholesale business in the state. Farm products wholesaling is the biggest single wholesale business, accounting for about one-third of total wholesale receipts.

Lumber and Wood Products wholesaling accounts for about 10 percent more. These wholesalers act mainly in the capacity of exporters, thus their output is purchased outside the state. Much of the remaining wholesale output is direct wholesaling to consumers, particularly in the agricultural sectors, construction, and manufacturing.

TABLE A-XIII. CONSTRUCTION, NEW, SECTOR: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	13,810	.08970	105,124,125,126
Manufacturing	15,486	.10058	105,124,125,126
Transportation & Public Warehousing	3,308	.02147	112,113,105,126
Communications & Public Utilities	759	.00492	19,105,124,125,126
Real Estate, Finance, & Insurance	2,554	.01659	18,74,105,124,125,126
Mining	3,471	.02254	107,110,124,125
Services	5,090	.03305	101,105,125,126
Trade, Wholesale & Retail	14,330	.09306	97,99,105,125,126
Imports	15,322	.09958	105,124,125,126
State & Local Government	511	.00332	58,74,105,125
Federal Government	6,221	.04040	51,105,125
Wages and Salaries	67,318	.43724	105,124,125,126
Proprietor Income	5,437	.03531	105,124,125,126
Property Income	345	.00224	105,124,125,126
Total Gross Inputs	153,962	1.00000	105,124,125,126

TABLE A-XIV. CONSTRUCTION, MAINTENANCE, SECTOR: PURCHASES, TECHNICAL COEFFICIENT, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Lumber & Wood Products	3,921	.11032	105,124,125,126
Manufacturing	3,441	.09682	105,124,125,126
Transportation & Public Warehousing	804	.02262	112,113,105,126
Communications & Public Utilities	116	.00326	19,105,124,125,126
Real Estate, Finance, & Insurance	568	.01598	18,74,105,124,125,126
Mining	354	.00996	107,110,124,125
Services	1,472	.04142	101,105,125,126
Trade, Wholesale & Retail	3,721	.10469	97,99,105,125,126
Imports	2,232	.06280	105,124,125,126
State & Local Government	169	.00476	58,74,105,125
Federal Government	2,062	.05802	51,105,125
Wages and Salaries	15,214	.42807	51,105,125,126
Proprietor Income	1,236	.03478	105,124,125,126
Property Income	231	.00650	105,124,125,126
Total Gross Inputs	35,541	1.00000	105,124,125,126

Resume: Construction, New and Construction, Maintenance

Construction output data was obtained from sources (17) and (108). Residential building, non-residential building, and non-building construction figures were given for new construction. Maintenance construction was estimated by adding the maintenance construction data, given for all of the endogenous sectors and the government sector, to the estimated maintenance construction purchased by households. The latter was derived from (109), (100), (106), and (107).

Inputs to these two sectors were estimated from the national coefficients for these sectors. In those cases where the national coefficient indicated an amount of purchased input greater than the indicated sector could apply, the coefficient was adjusted downward.

Wages and Salaries, Proprietor Income, and Property Income for the construction sectors were given in (110).

TABLE A-XV. STATE AND LOCAL GOVERNMENT SECTOR: PURCHASES, TECHNICAL COEFFICIENT AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Food & Kindred Products	949	.00288	70,103,115,116
Manufacturing	3,038	.00922	70,103,115,116
Transportation & Public Warehousing	932	.00283	70,103,115,116
Communications & Public Utilities	13,821	.04193	19,21,70,115,116
Real Estate, Finance, & Insurance	2,579	.00782	18,70,72,115,116
Mining	1,243	.00377	70,103,115,116
Services	13,935	.04227	70,103,115,116
Trade, Wholesale & Retail	22,814	.06921	70,103,115,116
Construction, Maintenance	7,432	.21917	70,103,115,116
Construction, New	72,246	.02254	70,103,115,116
Imports	35,407	.10741	70,103,115,116
State & Local Government	24,020	.07287	70,103,115,116
Wages and Salaries	<u>131,224</u>	<u>.39808</u>	70,103,115,116
Total Gross Inputs	329,640	1.00000	70,103,115,116

Resume: State and Local Government

State and Local government is assumed to be measured by total revenue collected by state and local governments. Table A-XVa shows the breakdown of total government revenue by sector collections. The Property Tax is the primary revenue source for local governments, while license, income, excise, and miscellaneous taxes accrue mainly to the state government. Federal transfer payments to the State and Local Government sector for 1963 are given in (103), (116), and (117).

State and Local Government expenditures are given in (54), (89), (98), and (99). Where exact sector expenditures were not available, national coefficients were used to estimate sector expenditures. Wage and Salaries disbursements were the largest expenditures of this sector, with construction expenditure the second largest. These two expenditures accounted for more than 60 percent of total State and Local Government spending.

TABLE A-XVa. STATE AND LOCAL GOVERNMENT COLLECTIONS. a/

Sector	Total	Property Taxes	License, Excise, Income, Miscellaneous
Livestock	13,858	13,639	219
Crops	17,313	15,640	1,673
Food	5,157	1,239	3,918
Lumber	4,603	532	4,071
Manufacturing	22,308	4,419	17,889
Transport	7,412	6,181	1,231
Communications	17,636	14,784	2,852
Real Estate	3,513	1,918	1,595
Mining	5,862	1,495	4,367
Services	1,930	1,430	500
Trade	19,247	10,530	8,717
Construction, New	511	220	291
Construction, Maintenance	169	104	65
State & Local Government	24,020		
Federal Government	65,486 <u>b/</u>		
Households	<u>123,270</u>	<u>35,499</u>	<u>87,771</u>
Total	332,295	107,630	135,159

a/ Source (58).b/ The breakdown of Federal Grants is given in (100) and (116).

TABLE A-XVb. LICENSE AND EXCISE TAXES, BY SECTOR.

Sector	License Taxes	
	(\$1,000)	
<u>Communication and Public Utilities</u>		
Telephone Tax	398,860	} 1,130,012
Telegraph Tax	690	
Natural Gas	252,484	
Electric Energy	477,978	
Corporation License Tax	1,722,000	
Total	2,852,012	
<u>Transportation</u>		
Freight Line Tax	152,848	
Sleeping Car Tax	11,350	
Express Co. Tax	3,670	
Corporation License Tax	1,063,000	
Total	1,230,878	
<u>Mining</u>		
Oil Producers Tax	1,854,000	
Coal Mines	12,000	
Metal Mines	813,000	
Corporation License Tax	1,688,000	
Total	4,367,000	
<u>Manufacturing</u>		
Gypsum	82	
Vermiculite	7	
Gasoline Tax	16,589	
Corporation License Tax	1,211	
Total	17,889	
<u>Trade</u>		
Gas Imports	995	
Chain Store Tax	184	
Cigarette Tax	6,407	
Corporation License Tax	1,131	
Total	8,717	

TABLE A-XVc. HOUSEHOLD PAYMENTS TO STATE AND LOCAL GOVERNMENT.

Sector	Payments
<u>Households</u>	
Income Tax	14,209
Insurance Trust	20,462
Liquor Revenue	16,810
Natural Resources	3,667
Drivers Licenses	7,999
Parking	700
Special Assessments	3,123
Miscellaneous	<u>20,801</u>
Total	87,771

TABLE A-XVI. FEDERAL GOVERNMENT SECTOR: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Livestock & Livestock Prod.	1,609	.00849	83,92
Crops	11,750	.06198	83,92
Food & Kindred Products	1,051	.00554	83,92
Manufacturing	103	.00054	104,129
Transportation & Public Warehousing	1,216	.00641	113
Communications & Public Utilities	4,909	.02589	19,21
Real Estate, Finance, & Insurance	9,313	.04912	18
Services	4,347	.02293	101
State & Local Government	65,486 <u>a/</u>		116
Wages and Salaries	269,641 <u>b/</u>	.74699	51,117,120,134
Total Gross Inputs	383,097	1.00000 <u>c/</u>	

a/ Grants to State and Local Government.

b/ Wages and Salaries, 141,619; Federal Transfers, 128,022.

c/ Coefficients are based only on purchases of the government.

Resume: Federal Government, Purchases and Transfers

There is no single source that gives data on Federal receipts and expenditures in the state. Source (115) gave the detailed collections by the Internal Revenue Service. Source (112) listed Federal Revenue from public lands by source of revenue. Source (38) showed Social Security contributions and payments by sector. Source (16) indicated the sales of Electrical Energy from Federal Power Production sites giving the quantity purchased by each buyer. This information is combined in Table A-XVIa. Federal Corporate income taxes were the only Federal collections that had to be estimated. In two sectors, Communications and Public Utilities and Real Estate, Finance, and Insurance, corporate income taxes paid were given for the industries in those sectors producing the major share of output. Source (121) was used as the basis of estimating the corporate income tax of the other sectors.

Wages and Salaries and Transfers accounted for the major share of Federal expenditures in the state. This data was given in the sources indicated in Table A-XVI. Purchases from the agricultural sectors are in reality subsidy payments that were tied to production. For example, purchases from the livestock sector show \$1,609,000. This is the payment for wool and unshorn lambs, based on wool and lamb production. Federal Payments for such items as ACP practices which are not tied to production, are included in transfers to Wages and Salaries. In general, purchases from the remaining endogenous sectors were indicated in the sources shown for each sector.

TABLE A-XVla. . . FEDERAL COLLECTIONS.

Sector	Total	Social Security	Excise & Stamp Tax	Gift, Estate, Corporate Income, Personal Income	National Forests & Electricity Products
Livestock	1,004	131		169	704
Crops	674	254		210	210
Food	4,179	2,099	1,872	208	
Lumber	4,943	4,403		404	136
Manufacturing	26,870	4,198	21,480	1,192	
Transportation	2,188	1,792	74	322	
Communications	18,139	2,611	284	13,464	1,780
Real Estate	8,973	2,816	251	5,906	
Mining	11,057	3,840		1,331	5,886
Services	5,955	5,018	553	384	
Trade	17,899	16,384	952	563	
Const., New	6,221	5,734		487	
Const., Maint.	2,062	1,920		142	
Exports	1,755				1,755
Households	132,871			132,392	
Total	244,790	51,200	25,945	157,174	10,471

Government purchases understates the true amount of Federal expenditures attributed to Montana. Military expenditures are lacking entirely. This omission can be forgiven because of the manner in which military expenditures are made. Aside from wages and salaries (which are included) military expenditures would be for items not produced in Montana. Military procurement does not go through the state sectors. If total military procurement was shown in the Federal sector, it would appear as an increase in imports purchased by the Federal sector. Thus what at first glance would appear to be Federal spending in the state does not in reality contribute to the Montana economy.

Federal transfers and purchases combined are about one and one half times greater than Federal collections in the state. Federal payments to individuals (wages and transfers) comprise 17.2 percent of the personal income of the state. Thus, the Federal government contributes a considerable amount to the Montana economy.

TABLE A-XVII. HOUSEHOLDS SECTOR: PURCHASES, TECHNICAL COEFFICIENTS, AND DATA SOURCES.

Sector	Purchases (\$1,000)	Technical Coefficient	Data Sources
Livestock & Livestock Prod.	6,580	.00437	47
Crops	3,312	.00220	47
Food & Kindred Products	116,615	.07748	47,95,105
Lumber & Wood Products	57	.00004	8,105
Manufacturing	53,068	.03526	105,117
Transportation & Public Warehousing	2,249	.00149	111,114,117
Communications & Public Utilities	42,499	.02824	19,21
Real Estate, Finance, & Insurance	68,587	.04557	18,72,79,89
Services	125,815	.08360	107,126
Trade, Wholesale & Retail	228,544	.15185	97,99,117,126
Construction, Maintenance	16,655	.03590	22,125
Construction, New	54,026	.01107	117,125
Imports	422,745	.28089	
State & Local Government	123,270	.08190	74,103,116
Federal Government	132,871	.08828	51,133
Wages and Salaries	39,432	.02620	117,128
Property Income	68,715	.04566	117,128
Total Gross Inputs	1,516,170	1.00000	

Resume: Households

The household column sector shows personal consumption expenditures in the state. With some exceptions (see below) consumption expenditures by sector were determined as the residual of total sector production minus all sector purchases (except Households). The Households sector estimates and coefficients were then compared to data from (51), (100) and (109). The estimates were in general, smaller than indicated by data from the latter sources with the exception of imports. This is reasonable in view of the fact that Montana industries, particularly manufacturing, do not produce sufficient consumer goods for the Montana population.

Direct consumer purchases and consumption of agricultural output was given in (35) and (74). Residential purchases of electric power and natural gas were given in (15) and (17). Residential purchases of communications output was taken from Table A-VIIIa.

Output of commercial banks purchased by the Households sector was estimated from (14) and (100). Insurance purchases by Households was estimated from (56). Remaining purchases from this sector included Real Estate services, small loans, retail credit, etc. The estimate of total purchases from the Real Estate, Finance, and Insurance sector compared favorably with estimates derived from (50), (100), and (109). Construction expenditures by Households were given in (18) and (108).

Household purchases from the State and Local Government sector are the total taxes, license, etc., as shown in Table A-XVa. Similarly, purchases from the Federal Government sector are as shown in Table A-XVIa.

The Property Income estimate of Households purchases reflect rent paid for housing. This estimate was derived as the proportion of rented housing to owned housing times the total housing expense in Montana. Total housing expense was estimated on the assumption that housing expense as a percentage of personal income was the same in Montana as in the United States.

Wages and Salaries paid by Households are given in (110) as "Other Labor Income." Yard work, baby sitting, etc., are included in this category.

Total Household expenditures were determined by subtracting Personal Saving, given in (62), from Personal Income (110).

Resume: Import-Export

Estimates of imports purchased by each sector were determined as a residual of total output minus the sum of purchases from the other sectors. Part of the imports purchased by some sectors were given, such as in shipments of livestock (35) and industrial machinery (81). The imports of each sector appeared reasonable when compared to the known imports.

Nine sectors exported part of their production. Sector exports were treated as the residual of sector production minus the purchases of the other sectors in the model. In the case of the Federal Government exports, the amount exported (i.e., sold out of state) was given in (16).

The exports of the trade sector are mainly due to agricultural exports, particularly grain exports. Out-of-state purchasers of agricultural products pay for the services of Farm Products wholesalers (grain elevators, livestock wholesalers; etc.) when they purchase agricultural products from Montana.

APPENDIX B--CONSOLIDATION OF THE U. S. MATRIX OF TECHNICAL COEFFICIENTS

Each sector of the Montana model contains one or more j of the sectors of the U.S. model. Table B-1 lists the Montana sectors and the U.S. sectors contained in each. Ten of the sectors are common to both models. Of the remainder, the Montana manufacturing sector contains 17 sectors of the U.S. model; the Mining sector contains 5 U.S. sectors; Services, 6 sectors; Communications and Public Utilities, 3 sectors; Real Estate, Finance, and Insurance, 2 sectors. Industries in the remaining sectors of the U.S. model are not present in Montana, thus are included in the Import-Export sector of the Montana model.

The process used to consolidate the U.S. model to the size of the Montana model is described in Chenery and Clark. Their explanation involves only the aggregation of two sectors into one, but the process is applicable to n sectors.

The first step of the reduction process was to partition the U.S. matrix into blocks, each block containing the sectors to be consolidated. In order to accomplish this, the U.S. matrix sectors were rearranged into the order shown in Table B-1. The partitioning process then resulted in a square matrix, each partition of which was a submatrix of one or more positive elements. The submatrices along the diagonal of the partitioned matrix are necessarily square, while those off-diagonal submatrices are in general not square.

The next step was the column addition of the elements within each submatrix, thereby reducing each submatrix to the order $(1 \times n)$, where n is the number of U.S. sectors contained in the appropriate Montana sector. For example, n of the Manufacturing submatrix is 17.

TABLE B-1. CORRESPONDANCE OF MONTANA AND U.S. SECTORS.

Montana Sectors	U.S. Sectors
1. Livestock & Livestock Products	1.1 Livestock & Livestock Products
2. Crops	2.1 Other Agricultural Products
3. Food & Kindred Products	3.1 Food & Kindred Products
4. Lumber and Wood Products	4.1 Lumber & Wood Products, except containers
5. Manufacturing	5.1 Apparel
	5.2 Household Furniture
	5.3 Paper & Allied Products
	5.4 Printing & Publishing
	5.5 Chemicals & Selected Chemical Prod.
	5.6 Petroleum Refining & Related Prod.
	5.7 Rubber & Misc. Plastics Products
	5.8 Leather Tanning & Industrial Leather Products
	5.9 Stone & Clay Products
	5.10 Primary Nonferrous Metals Manufactures
	5.11 Heating, Plumbing & Structural Metal Products
	5.12 Other Fabricated Metal Products
	5.13 Construction, Mining & Oil Field Machinery
	5.14 Machine Shop Products
	5.15 Other Transportation Equipment
	5.16 Scientific & Controlling Instruments
	5.17 Miscellaneous Manufacturing
6. Transportation & Public Warehousing	6.1 Transportation & Public Warehousing
7. Communications & Public Utilities	7.1 Communications, except Radio & T.V.
	7.2 Radio & T.V. Broadcasting
	7.3 Electric, Gas, Water & Sanitary Services
8. Real Estate, Finance, and Insurance	8.1 Finance & Insurance
	8.2 Real Estate
9. Mining	9.1 Nonferrous Metal Ores Mining
	9.2 Coal Mining
	9.3 Crude Petroleum & Natural Gas
	9.4 Stone & Clay Mining & Quarrying
	9.5 Chemical & Fertilizer Mineral Mining
10. Services	10.1 Hotels; Personal & Repair Services, except Autos
	10.2 Business Services
	10.3 Research & Development Services
	10.4 Automobile Repair & Services
	10.5 Amusements
	10.6 Medical, Educational Services
11. Trade, Wholesale and Retail	11.1 Wholesale and Retail Trade
12. Construction, Maintenance	12.1 Construction, Maintenance
13. New Construction	13.1 New Construction
14. State and Local Government	14.1 State and Local Government
15. Federal Government	15.1 Federal Government
16. Imports	16.1 All sectors in the U.S. model that did not exist in the Montana economy. For example, Tobacco Manufactures.

The third step was to calculate the weighted average of the n elements in each submatrix. Recall that the elements of the technical coefficient matrix are $a_{ij} = \frac{x_{ij}}{X_j}$. The weight of each coefficient in the weighted average is therefore the proportion of each subsector's output relative to the total output of the consolidated sector. The consolidation equation is

$$a_{kl}^* = \sum_j^N a_j \frac{X_j}{\sum_j X_j}$$

where a_j is the j^{th} element of the $1 \times n$ submatrix, X_j is the output of the j^{th} sector of the submatrix, and kl locates the specific submatrix being aggregated. Thus the elements of each of the submatrices p_{kl} resulting from the partitioning process are reduced to the single element a_{kl}^* .

Note that, for the sectors common to both matrices, the submatrices contained only one element. Thus, for the set of sectors, $S = (1, 2, 3, 4, 6, 11, 12, 13, 14, \text{ \& } 15)$ there is the direct correspondence $p_{kl} = a_{kl} = a_{kl}^*$, $k, l \in S$.

To illustrate the reduction consolidation process, assume that a 4×4 matrix M_1 with sectors V_1, V_2, V_3 , and V_4 must be reduced to a 2×2 matrix M_2 with sectors V_2 and V_5 , V_5 being the aggregate of V_1, V_3 , and V_4 . The steps of the consolidation process, along with the changes in the matrix are as follows.

Step 1.--The original matrix is shown as Figure B-1. Rearrange the matrix so that the sectors to be aggregated (V_1 , V_3 , and V_4) will be contiguous, as shown in Figure B-2. Then partition the matrix so that each block contains the elements to be consolidated.

	V_1	V_2	V_3	V_4
V_1	.1	.2	.1	.2
V_2	.2	.1	.1	0
V_3	.4	.1	.1	.3
V_4	.1	.3	.2	.4

Figure B-1.

	V_2	V_1	V_3	V_4
V_2	.1	.2	.1	0
V_1	.2	.1	.1	.2
V_3	.1	.4	.1	.3
V_4	.3	.1	.2	.4

Figure B-2.

Step 2.--Add the column elements of the submatrices, thereby reducing all submatrices to the order $1 \times n$.

	V_2	V_1	V_3	V_4
V_2	.1	.2	.1	0
V_5	.6	.6	.4	.9

Step 3.--To calculate the weighted average, assume the output vector

$X = X_1 \ X_2 \ X_3 \ X_4 = 2 \ 4 \ 5 \ 3$. The weights are therefore

$$\frac{X_j}{\sum_j X_j}, \quad j = 1, 3, \text{ and } 4.$$

The elements a_{ij}^* of the final consolidated matrix are

Filmed as received
without page(s) 149.

UNIVERSITY MICROFILMS.

$$a_{11}^* = .1 \quad , \quad a_{21}^* = .6 \quad ,$$

$$a_{12}^* = (.2 \times .2) + (.1 \times .5) + (0 \times .3) = .09$$

$$a_{22}^* = (.6 \times .2) + (.4 \times .5) + (.9 \times .3) = .59$$

and the consolidation is completed.

LITERATURE CITED

- 1 American Bankers Association; Agricultural Committee. Farm Credit In Montana-1963. New York, 1964.
- 2 American Economic Association, Royal Economic Society. Surveys of Economic Theory, Vol. II: Growth and Development. St. Martins Press, New York, 1965.
- 3 Arrow, K. J., Samuel Karlin, and Patrick Suppes. Mathematical Methods in the Social Sciences, 1959. Stanford University Press, Stanford, California, 1960.
- 4 Barna, T. (ed.). The Structural Interdependence of the Economy. John Wiley & Sons, New York, 1956.
- 5 Baumol, W. J. "Activity Analysis in One Lesson." American Economic Review, December 1958, No. 48.
- 6 Economic Theory and Operations Analysis. Prentice Hall, Englewood Cliffs, New Jersey, 1963.
- 7 Blomgren, Paul B. "The Montana Economy in Perspective." Montana Business Quarterly, Fall 1962. Bureau of Business and Economic Research, University of Montana, Missoula, 1962.
- 8 Bolle, Arnold W., R. C. Harrington, and W. K. Gibson. The Economics of the Montana Forest Products Industry. Report prepared for the Upper Midwest Economic Study at the University of Montana, Missoula, 1964.
- 9 Borchert, John R. "Regional Development and Local Planning." Proceedings of the Second Montana Planning Institute, Missoula, Montana, September 16 and 17, 1960. Bureau of Government Research, University of Montana, Missoula.
- 10 Bouque, Philip J. and Gerald Hansen. An Inventory of Regional Input-Output Studies in the United States. Occasional Paper 17, Graduate School of Business, University of Washington, Seattle, 1967.
- 11 Bureau of the Budget, Office of Statistical Standards. Standard Industrial Classification Manual. U. S. Government Printing Office, Washington, D. C., 1967.
- 12 Charnes, A. "Consolidation of Leontief Matrices." As reported in "Discussion of the Role of Large Scale Computing in Economics." Econometrics XX, April 1953.

- 13 Charnes A., and W. W. Cooper. Management Models and Industrial Application of Linear Programming. Vol. I. John Wiley and Sons, New York, 1959.
- 14 Chenery, Hollis B. and Paul G. Clark. Inter-Industry Economics. John Wiley & Sons, New York, 1959
- 15 Davis, Harold T. The Theory of Econometrics. The Principia Press, Inc., Bloomington, Indiana, 1941.
- 16 Dorfman, Robert, Paul A. Samuelson, and Robert M. Solow. Linear Programming and Economic Analysis. McGraw Hill, New York, 1958. Chapters 8-12.
- 17 Evans, Duane, and Marvin Hoffenberg. "The Interindustry Relations Study for 1947." The Review of Economics and Statistics, XXXIV (May 1952).
- 18 Federal Deposit Insurance Corporation. Annual Report of the FDIC for the Year Ending December 31, 1963. U. S. Government Printing Office, Washington, D. C., 1964.
- 19 Federal Power Commission. Statistics of Electric Utilities in the United States, 1963. Privately Owned. U. S. Government Printing Office, Washington, D. C., 1964.
- 20 _____ . Statistics of Electric Utilities in the United States, 1963. Publically Owned. U. S. Government Printing Office, Washington, D. C., 1964.
- 21 _____ . Statistics of Natural Gas Utilities in the United States, 1963. Privately Owned. U. S. Government Printing Office, Washington, D. C., 1964.
- 22 F. W. Dodge Company, A Division of McGraw-Hill, Inc. Construction Statistics for 1963. Unpublished material transmitted from E. B. Krippner, 3109 Wilshire Boulevard, Los Angeles, Calif.
- 23 Goldman, M. R., M. L. Marimont, and B. N. Vaccara. "The Inter-Industry Structure of the United States--A Report of the 1958 Input-Output Study." Survey of Current Business, Vol. 44, No. 11, Department of Commerce, U. S. Government Printing Office, Washington, D. C., 1964.
- 24 Goodwin, R. "The Multiplier as a Matrix." Economic Journal, December 1959.

- 25 Hadley, G. Linear Algebra. Addison-Wesley, Reading, Massachusetts, 1961.
- 26 Henderson, James M. and Anne O. Krueger. National Growth and Economic Change in the Upper Midwest. University of Minnesota Press, Minneapolis, 1965.
- 27 Hirsh, Werner A. "Interindustry Relations of a Metropolitan Area." The Review of Economics and Statistics, XLI (November 1959).
- 28 Holley, J. L. "Notes on the Inversion of the Leontief Matrix." Econometrica 19, No. 3, July 1951.
- 29 Housing and Home Finance Agency. Seventeenth Annual Report. U. S. Government Printing Office, Washington, D. C., 1964.
- 30 "Input-Output Analysis: Its Use in Marketing." Printers' Ink. Decker Communications, Inc., New York, June 24, 1966.
- 31 Isard, Walter. "Inter-Regional and Regional Input-Output Analysis: A Model of a Space Economy." Review of Economics and Statistics, (November 1951).
- 32 Johnson, Maxine. "Problems of Economic Growth in Montana." Montana Business Quarterly, Winter 1963. Bureau of Business and Economic Research, University of Montana, Missoula, 1963.
- 33 Karlin, Samuel. Mathematical Methods and Theory in Games, Programming, and Economics. Vol. I. Addison-Wesley Publishing Co., Reading, Massachusetts, 1962.
- 34 Klein, Lawrence R. Econometrics. Row, Peterson, and Co., Evanston, Illinois, 1956.
- 35 Lancaster, Kelvin. Mathematical Economics. The Macmillan Co., New York, 1968.
- 36 Leontief, Wassily W. "Exports, Imports, Domestic Output, and Employment." Quarterly Journal of Economics, Vol. LX (February 1946).
- 37 _____. "Quantitative Input-Output Relations in the Economic System of the United States." The Review of Economics and Statistics, XVIII (August 1936).
- 38 _____. The Structure of the American Economy, 1919-1939. Harvard University Press, Cambridge, Massachusetts, 1941.

- 39 Maki, Wilbur R. Projections of Iowa's Economy and People in 1974. Ames; Iowa Agricultural Experiment Station Special Report 41, 1965.
- 40 Maki, Wilbur R. and Yien-l Tu. "Regional Growth Models for Rural Areas Development." Papers and Proceedings of the Regional Science Association. Vol. 9, 1962.
- 41 Martin, William E. and Harold O. Carter. A California Inter-Industry Analysis Emphasizing Agriculture. Part I: The Input-Output Model. Part II: Statistical Supplement. University of California, Giannini Foundation Research Report 250, Berkeley, 1962.
- 42 McConnen, R. J. (ed.). Montana Agricultural Basic Facts. Cooperative Extension Service and Agr. Expt. Sta. Bulletin 293, Montana State University, Bozeman, 1962.
- 43 Miernyk, William H. The Elements of Input-Output Analysis. Random House, New York, 1965.
- 44 _____. "Long-Range Forecasting with a Regional Input-Output Model." Western Economic Journal. Vol. VI, No. 3 (June 1968).
- 45 Montana Associated Utilities, Inc. Directory of Rural Electric and Telephone Cooperatives, 1965. Great Falls, Montana, 1965.
- 46 Montana-Dakota Utilities. Statistics of Natural Gas Sales, State of Montana. Letter transmitted by A. J. Mayer, Engineer, Montana-Dakota Utilities, Co., Minneapolis, Minnesota, May 3, 1968.
- 47 Montana Department of Agriculture and U. S. Department of Agriculture, Statistical Reporting Service. Montana Agricultural Statistics. Vol. X. Tribune Printing, Great Falls, Montana, 1965.
- 48 Montana Board of Railroad Commissioners. Railroad Freight Revenue: Ton-miles and Revenue Ton-miles. Unpublished material received from the Board of Railroad Commissioners.
- 49 Montana State Highway Commission, Gross Vehicle Weights Division. G.V.W. Trucks and Truck-Tractor, Trailer, and Semi-Trailer Fees. Unpublished information received from the Montana State Highway Commission.
- 50 Montana State Planning Board. Industrial Horizons. State of Montana. All volumes from 1957 to present.

- 51 Montana Unemployment Compensation Commission. Twenty-Eighth Annual Report. Reporter Printing and Supply Co., Helena, Montana, 1964.
- 52 Montana Department of Planning and Economic Development. Montana Directory of Manufacturers and Buyers Guide, 1967-68. Helena, Montana 1967.
- 53 Moore, Frederick T. and James W. Peterson. "Regional Analysis: An Inter-Industry Model of Utah." The Review of Economics and Statistics, XXXVII (November 1955).
- 54 Morgenstern, Oskar. (ed.): Economic Activity Analysis. John Wiley and Sons, New York, 1954.
- 55 Moses, L. N. "The Stability of Inter-Regional Trading Patterns and Input-Output Analysis." American Economic Review, No. 45 (December 1955).
- 56 National Bureau of Economic Research Studies in Income and Wealth. Input-Output Analysis: An Appraisal. Vol. 16, Princeton University Press, 1958.
- 57 Ninth Federal Reserve District. Statistical Review. Research Department, Federal Reserve Bank of Minneapolis, Minneapolis, Minnesota, 1966.
- 58 Nobus, Minabe. "Economic Growth and International Trade in a Simple Dynamic Leontief Model." Canadian Journal of Economics and Political Science. Vol. XXXII, No. 1 (February 1966).
- 59 Peterson, G. A. and Earl O. Heady. Application of Input-Output Analysis to a Simple Model Emphasizing Agriculture. Ames: Iowa Agricultural Experiment Station Bulletin 427, 1955.
- 60 Pissot, Henry J. and Harold E. Hanson. The Forest Resource of Western Montana. U. S. Forest Service Resource Bulletin INT-1, Ogden, Utah 1963.
- 61 Quandt, R. E. "On the Solution of Probabilistic Leontief Systems." Naval Logistics Quarterly 6, No. 4 (December 1959).
- 62 _____. "Probabilistic Errors in Leontief Systems." Naval Logistics Quarterly 5, No. 2 (June 1958).
- 63 Rao, Amanda S. and David J. Allee. An Application of Interindustry Analysis to San Benito County, California, University of California; Giannini Foundation Research Report No. 278, Berkeley, 1964.

- 64 Riley, V. and R. L. Allen. Inter-Industry Economic Studies.
The John Hopkins Press, Baltimore, 1960.
- 65 Scknitther, John A. and Earl O. Heady. Application of Input-
Output Analysis to a Regional Model Emphasizing Agriculture.
Ames: Iowa Agricultural Experiment Station Research
Bulletin 454, 1958.
- 66 Seligman, Benjamin B. Main Currents in Modern Economics. The
Free Press of Glencoe, Illinois, 1962.
- 67 Siman, Nancy W. "Personal Consumption Expenditures in the 1958
Input-Output Study." Survey of Current Business, Vol. 45,
No. 10, October 1965. U. S. Government Printing Office,
Washington, D. C., 1965.
- 68 Simpson, David and Jinkicki Tsubkui. "The Fundamental Structure
of Input-Output Tables. An International Comparison."
Review of Economics and Statistics, Vol. XLVII, No. 4
(November 1965).
- 69 State of Montana. Board of Railroad Commissioners, Natural Gas
Utilities Statistics. Letter transmitted from W. M. Johnson,
Auditor, Board of Railroad Commissioners, Montana, May 21,
1968.
- 70 _____ . Budget for the Biennium July 1, 1963 to June 30,
1965. Budget submitted by Tim Babcock, Governor, to the
Montana Legislature, 1964 session.
- 71 _____ . Budget for the Biennium July 1, 1965 to June 30,
1967. Budget submitted by Tim Babcock, Governor, to the
Montana Legislature, 1965 session.
- 72 _____ . Commissioner of Insurance. 1963 Production in
Montana by Stock Property, Casualty, and Life Insurers.
Unpublished information received from the Commissioner of
Insurance, State of Montana, 1968.
- 73 _____ . State Board of Hail Insurance. 47th Annual
Report of the State Board of Hail Insurance to Montana Grain
Growers. Reporter Printing and Supply Co., Helena, Montana,
1964.
- 74 _____ . State Board of Equalization. Twenty-First
Biennial Report of the Montana State Board of Equalization.
Tribune Printing and Supply Co., Helena, Montana, 1965.

- 75 _____ . Twenty-Second Biennial Report of the Montana State Board of Equalization. Tribune Printing and Supply Co., Helena, Montana, 1967..
- 76 Taylor, Maurice C. "Montana in Transition." Montana Business Quarterly, Fall 1967. Bureau of Business and Economic Research, University of Montana, Missoula, 1967.
- 77 Thiel, Henri, John C. G. Boot, and Teun Kloch. Operation Research and Quantitative Economics. McGraw-Hill Co., New York, 1965.
- 78 Tinbergen, Jan and Hendricus C. Boston. Mathematical Models of Economic Growth. McGraw-Hill Co., New York, 1962.
- 79 United Savings and Loan League. Savings and Loan Annals, 1964. United Savings and Loan League, 221 North LaSalle St., Chicago, Illinois, 1965.
- 80 _____ . Savings and Loan Fact Book, 1964. United Savings and Loan League, 221 North LaSalle St., Chicago, Illinois, 1965.
- 81 U. S. Department of Agriculture, Agricultural Marketing Service. Dollar Volume of Agriculture's Transactions with Industry. U. S. Government Printing Office, 1959.
- 82 _____ . Statistical Reporting Service, Economic Research Service. Livestock and Meat Statistics, 1963. Supplement for 1963 to Statistical Bulletin 333, U. S. Government Printing Office, 1965.
- 83 _____ . Agricultural Stabilization and Conservation Service. 1963 Annual Report: Montana. Bozeman, Montana, 1964.
- 84 _____ . Economic Research Service. Agricultural Finance Review, 1963. U. S. Government Printing Office, 1964.
- 85 _____ . Economic Research Service. An Analysis of Grain Transportation in the Northwest. U. S. Government Printing Office, 1964.
- 86 _____ . Economic Research Service. Farm Income: State Estimates, 1949-1966. Supplement to the July 1967 Farm Income Situation. U.S. Government Printing Office, 1967.
- 87 _____ . Economic Research Service. Feed Statistics Through 1966. Statistical Bulletin 410, U. S. Government Printing Office, 1964.

- 88 _____ . Economic Research Service. Supplement for 1964 to Livestock-Feed Relationships; 1909-1963. Statistical Bulletin 337. U. S. Government Printing Office, 1964.
- 89 _____ . Rural Electrification Administration. Rural Electrification Borrowers. 1963 Annual Statistical Report, REA Bulletin 1-1. U. S. Government Printing Office, 1964.
- 90 _____ . Rural Electrification Administration. 25th Annual Report of Energy Purchased by REA Borrowers, Fiscal Year Ended June 30, 1963. U. S. Government Printing Office, 1964.
- 91 _____ . Rural Electrification Administration. 26th Annual Report of Energy Purchased by REA Borrowers, Fiscal Year Ended June 30, 1964. U. S. Government Printing Office, 1965.
- 92 _____ . Statistical Reporting Service. Agricultural Statistics, 1964. U. S. Government Printing Office, 1964.
- 93 _____ . Statistical Reporting Service. Agricultural Statistics, 1965. U. S. Government Printing Office, 1965.
- 94 _____ . Statistical Reporting Service. Cattle and Calves and Sheep and Lambs on Feed, Selected States, 1959-1965. U. S. Government Printing Office, 1966.
- 95 _____ . Statistical Reporting Service. Livestock Slaughter, 1963. U. S. Government Printing Office, 1964.
- 96 U. S. Department of Commerce, Bureau of the Census. Census of Agriculture, 1964; Statistics of the State and Counties, Montana. U. S. Government Printing Office, 1966.
- 97 _____ . Bureau of the Census. Census of Business, 1963. Vol. I: Retail Trade, Summary Statistics. U. S. Government Printing Office, 1964.
- 98 _____ . Bureau of the Census. Census of Business, 1963. Vol. II: Retail Trade, Area Statistics. U. S. Government Printing Office, 1964.
- 99 _____ . Bureau of the Census. Census of Business, 1963. Vol. IV: Wholesale Trade, Summary Statistics. U. S. Government Printing Office, 1964.
- 100 _____ . Bureau of the Census. Census of Business, 1963. Vol. V: Wholesale Trade, Area Statistics. U. S. Government Printing Office, 1964.

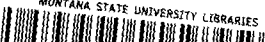
- 101 _____ . Bureau of the Census. Census of Business, 1963.
Vol. VI: Selected Services, Summary Statistics. U. S. Govern-
ment Printing Office, 1964.
- 102 _____ . Bureau of the Census. Census of Business, 1963.
Vol. VII: Selected Services, Area Statistics. U. S. Govern-
ment Printing Office, 1964.
- 103 _____ . Bureau of the Census. Census of Government, 1962.
Vol. VIII, No. 26, Government in Montana. U. S. Government
Printing Office, 1964.
- 104 _____ . Bureau of the Census. Census of Manufactures,
1963. Vol. I: Summary Statistics. U. S. Government Printing
Office, 1966.
- 105 _____ . Bureau of the Census. Census of Manufactures,
1963. Vol. II: Industry Statistics, Parts I and 2. U. S.
Government Printing Office, 1966.
- 106 _____ . Bureau of the Census. Census of Manufactures,
1963. Vol. III: Area Statistics. U. S. Government Printing
Office, 1967.
- 107 _____ . Bureau of the Census. Census of Mineral
Industries, 1963. Area Statistics: Montana. U. S. Govern-
ment Printing Office, 1967.
- 108 _____ . Bureau of the Census. Oil and Gas Field Services.
U. S. Government Printing Office, 1967.
- 109 _____ . Bureau of the Census. Petroleum and Natural Gas
Industries. U. S. Government Printing Office, 1967.
- 110 _____ . Bureau of the Census. Sand and Gravel. U. S.
Government Printing Office, 1967.
- 111 _____ . Bureau of the Census. Census of Transportation,
1963. Vol. I: Passenger Transportation Survey. U. S.
Government Printing Office, 1966.
- 112 _____ . Bureau of the Census. Census of Transportation,
1963. Vol. II: Truck Inventory and Use Survey. U. S.
Government Printing Office, 1966.
- 113 _____ . Bureau of the Census. Census of Transportation,
1963. Vol. III: Commodity Transportation Survey. U. S.
Government Printing Office, 1966.

- 114 _____ . Bureau of the Census. Census of Transportation, 1963. Vol. IV: Commodity Transportation Survey. U. S. Government Printing Office, 1966.
- 115 _____ . Bureau of the Census. Compendium of State Government Finances in 1963. U. S. Government Printing Office, 1964.
- 116 _____ . Bureau of the Census. Government Finances in 1963-64. U. S. Government Printing Office, 1965.
- 117 _____ . Bureau of the Census. Statistical Abstract of the United States, 1963. U. S. Government Printing Office, 1964.
- 118 _____ . Bureau of the Census. Statistical Abstract of the United States, 1964. U. S. Government Printing Office, 1965.
- 119 _____ . Bureau of the Census. Statistical Abstract of the United States, 1965. U. S. Government Printing Office, 1966.
- 120 _____ . Bureau of the Census. Statistical Abstract of the United States, 1966. U. S. Government Printing Office, 1967.
- 121 _____ . Bureau of Public Roads. Highway Statistics, 1963. U. S. Government Printing Office, 1965.
- 122 U. S. Department of Commerce and U. S. Department of Labor. Construction Review. Vol. VII, No. 12. U. S. Government Printing Office, 1961.
- 123 _____ . Construction Review. Vol. VIII, No. 9. U. S. Government Printing Office, 1962.
- 124 _____ . Construction Review. Vol. IX, No. 11. U. S. Government Printing Office, 1963.
- 125 U. S. Department of Commerce and Defense Services Administration. Construction Statistics: 1915-1964. U. S. Government Printing Office, 1965.
- 126 U. S. Department of Commerce, Office of Business Economics. Survey of Current Business. Vol. 44, No. 11. U. S. Government Printing Office, 1964.

- 127 _____ . Office of Business Economics. Survey of Current Business. Vol. 45, No. 4. U. S. Government Printing Office, 1965
- 128 _____ . Office of Business Economics. Survey of Current Business. Vol. 46, No. 8. U. S. Government Printing Office, 1966.
- 129 U. S. Department of Defense, Office of the Secretary. Military Prime Contract Awards. Release of March 1964. U. S. Government Printing Office, 1964.
- 130 U. S. Department of the Interior, Bureau of Land Management. Public Land Statistics, 1963. U. S. Government Printing Office, 1965.
- 131 U. S. Department of Labor, Bureau of Labor Statistics. Employment and Earnings Statistics for States and Areas, 1939-1963. U. S. Government Printing Office, 1964.
- 132 U. S. Department of Transportation, Bureau of Public Roads. Highway Statistics, Summary to 1965. U. S. Government Printing Office, 1967.
- 133 U. S. Treasury Department. 1963 Annual Report to the Commissioner of Internal Revenue. U. S. Government Printing Office, 1964.
- 134 _____ . Annual Report of the Secretary of the Treasury on the State of Finances, Fiscal Year 1963. U. S. Government Printing Office, 1964.
- 135 _____ . Annual Report of the Secretary of the Treasury on the State of Finances, Fiscal Year 1964. U. S. Government Printing Office, 1965.
- 136 _____ . Bureau of Internal Revenue. Statistics of Income, 1963. Corporation Income Tax Returns. U. S. Government Printing Office, 1964.
- 137 _____ . Farmers Cooperative Income Tax Returns. U. S. Government Printing Office, 1964.
- 138 _____ . Individual Income Tax Returns. U. S. Government Printing Office, 1964.
- 139 _____ . U. S. Business Income Tax Returns. U. S. Government Printing Office, 1964.

- 140 U. S. Veterans Administration. Annual Report, Administrator of Veterans Affairs. U. S. Government Printing Office, 1964.
- 141 Waldorf, William H. "Input-Output Analysis as a Tool in Agricultural Marketing Research." Agricultural Economics Research, Vol. XIV, No. 3. U. S. Department of Agriculture, Economic Research Service. U.S. Government Printing Office, 1962.
- 142 Waugh, Frederick. "Inversion of the Leontief Matrix." Econometrics, April 1950.
- 143 Western Highway Institute. Transportation in the Western Livestock Industry. Western Highway Institute, San Francisco, California, 1966.
- 144 Wilson, Alvin K. Output of Timber Products in Montana, 1962. U. S. Forest Service Research Paper INT-11, 1964.
- 145 Wilson, Alvin K. and John S. Spencer, Jr. Timber Resources and Industries in the Rocky Mountain States. U. S. Forest Service Resource Bulletin INT-7. Intermountain Forest and Range Experiment Station, Ogden, Utah, 1967.
- 146 Wong, Yi K. "An Elementary Treatment of the Input-Output System." Naval Research Logistics Quarterly 1 (December 1954).
- 147 _____ . "Quasi-Inverses Associated with Minkowski-Leontief Matrices." Econometrics 22, No. 3 (July 1954).

MONTANA STATE UNIVERSITY LIBRARIES



3 1762 10005629 8

D378	Hoff, Theodore Austin
H673	An analysis of inter-
cop. 2	dependence in the Montana
	economy

	NAME AND ADDRESS
X-60X	Kin Kradreger 220
	R. R. R. R.
	R. R. R. R.
1-2-71	hold.
	H. R.
OCT 27 1961	

D378

H673

cop. 2