Parameter investigation for low frequency vibration of the forearm
by Roger Edward Matz

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

Abstract:
The need to be able to predict the response of portions of the human body to dynamic loading has been well established, and it is the purpose of this study to experimentally investigate the effects of several independent parameters as they relate to the frequency response of a biological system. Specifically, the forearm of volunteer subjects was excited by a very small amplitude, sinusoidal forcing function at a position near the styloid process, and the response of the arm was measured with an accelerometer located near the olecranon process. The input forcing function frequency was automatically swept through a frequency range of 80 to 600 Hz, and the resultant output frequency response was recorded as a power spectral density curve by means of a wave analyzer. The natural frequencies of the arm were considered to be the frequencies at which relative peak amplitudes occur in the power spectral density curve. For each subject, four independent parameters, as well as the dependent variable (natural frequency), were measured. These four independent parameters were r (1) arm length as measured from elbow to wrist, (2) bone size as indicated by wrist size, (3) muscle development as indicated by the per cent increase in upper arm circumference from unflexed to flexed position, and (4-) fleshiness as indicated by the ratio of unflexed upper arm circumference to wrist width. The subjects utilized were approximately 250 college-age males.

The significance of the effects of each of the foregoing parameters, and of the interactions between these parameters, on the dependent variable was examined via a 2n fixed factor, factorial experiment. The experimental results indicated all of the four factors significantly affected the resonant frequency at the 99% level of confidence. It was further determined that each of these factors had significant interactions, with all two way interactions except arm length-bone size and muscle development-fleshiness significant. All three way interactions were significant.

No general trend was determined for any factor except bone size, for which an increase in level tended to cause a decrease in natural frequency.

It was concluded that although other parameters may be found which affect the resonant frequency, these four were the ones of most importance.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The need to be able to predict the response of portions of the human body to dynamic loading has been well established, and it is the purpose of this study to experimentally investigate the effects of several independent parameters as they relate to the frequency response of a biological system. Specifically, the forearm of volunteer subjects was excited by a very small amplitude, sinusoidal forcing function at a position near the styloid process, and the response of the arm was measured with an accelerometer located near the olecranon process. The input forcing function frequency was automatically swept through a frequency range of 80 to 600 Hz, and the resultant output frequency response was recorded as a power spectral density curve by means of a wave analyzer. The natural frequencies of the arm were considered to be the frequencies at which relative peak amplitudes occur in the power spectral density curve. For each subject, four independent parameters, as well as the dependent variable (natural frequency), were measured. These four independent parameters were: (1) arm length as measured from elbow to wrist, (2) bone size as indicated by wrist size, (3) muscle development as indicated by the per cent increase in upper arm circumference from unflexed to flexed position, and (4) fleshiness as indicated by the ratio of unflexed upper arm circumference to wrist width. The subjects utilized were approximately 250 college-age males.

The significance of the effects of each of the foregoing parameters, and of the interactions between these parameters, on the dependent variable was examined via a 2^n fixed factor, factorial experiment. The experimental results indicated all of the four factors significantly affected the resonant frequency at the 99% level of confidence. It was further determined that each of these factors had significant interactions, with all two way interactions except arm length-bone size and muscle development-fleshiness significant. All three way interactions were significant.

No general trend was determined for any factor except bone size, for which an increase in level tended to cause a decrease in natural frequency.

It was concluded that although other parameters may be found which affect the resonant frequency, these four were the ones of most importance.

Chapter 1

INTRODUCTION

Previous investigations have dealt with in vivo measurement of resonant frequencies of the bone-flesh structure of the forearm (1,2)* Dr. Jurist (2) attempted to correlate several physical factors of the subjects with the determined resonant frequency, and the ulna length. The investigator found in a study of children that the product of resonant frequency and ulna length was significantly correlated with height, weight, upper arm circumference, and bone mineral content. He also concluded that the resonant frequency was not significantly correlated with ulna length for boys of age 6-11 years.

The intent of this study was to explicitly define several physical factors, including some investigated by Jurist and to determine if these factors significantly affect one resonant frequency of the forearm. The parameters studied were:

A) Ulna length
B) Bone size--as indicated by wrist width

*Numbers in parentheses refer to corresponding entry in Bibliography.

C) Muscle development--defined by per cent increase in upper arm circumference between unflexed and flexed position

D) Fleshiness--as indicated by ratio of unflexed upper arm circumference to wrist width.

The investigation of the effects of these factors was accomplished by the theory of experimental design (3,4,5). Experimental design is a widely used industrial tool (5), allowing efficient collection, organization, and analysis of experimental data.

Chapter 2

METHOD OF ANALYSIS

2.1 Experimental Design

The experiment was designed and the data analyzed using the principles of experimental design.

Experimental design gives one reasonable assurance that the results of an experiment will provide definitive answers to the questions under investigation.

The method includes preplanning of the experimental method, provides the manner or order in which the experimental data is to be collected, and provides the means of analyzing the data and drawing valid conclusions.

2.2 2^n Design

There are many choices of possible experimental designs using the four factors presented as independent variables and resonant frequency as the response variable (3,4). The nature of the current interest in these factors, however, indicated the likely choice to be a 2^n factorial design. A 2^n factorial design (Appendix A) is used as a screening experiment--as a prelude to detailed study of a particular situation. This design allows one to selectively determine which of the chosen independent variables significantly affect the response variable, and also whether possibly the independent variables affect the response variable differently for different level combinations of the other independent variables (interaction effects). This allows screening of those factors which need not be included in the detailed study.

For the 2^n design two levels are selected for each of the n factors (usually chosen at the extremes of possible levels.) Thus, for the experimental conditions all possible combinations of the levels of the n factors are used, hence a factorial design. For this 2^4 design there are 16 possible experimental conditions. If the extremes of the factors are chosen as follows:

	Factor	Levels
A:	arm length	long arm, short arm
B:	bone size	big wrist, small wrist
C:	muscle development	large muscle increase, small muscle increase
D:	fleshiness	very fleshy, slightly fleshy

then one possible experimental condition would be a resonant frequency determination for a person fitting in the category of short arms, big wrists, large increase in muscles and very fleshy (bcd). Similarly there are 15 other possible combinations. These levels would be specified at appropriate numerical values, based on a survey of possible variations.

In general, the experiment is replicated (performed for each of the 16 possible conditions more than once) in order to increase the degrees of freedom for experimental error. With two replicates there are a total of 32 experimental determinations.

2.3 Desired Results

The desired results from the 2^n design were:

1) justifiable conclusions as to whether:

   a) the selected independent factors significantly affect the response variable,
   b) there is interaction between the independent variables; that is, whether the factors affect the resonant frequency differently at different levels of other factors,

2) general trends in the effect on the resonant frequency of a change in the level of a certain factor.

The 2^n design does not allow a determination explicitly of the nature of the factor effect on the response variable, i.e., linear, quadratic, cubic, etc., but does allow a postulation of trends.

3) a postulate as to whether or not all significant factors have been included in the experiment.

Number 3 may be possible by some consideration of the experimental error.

2.4 Methodology

For this experiment the collected data were analyzed by the application of analysis of variance techniques. In the calculation of any variance, the analysis begins by finding the total sum of squares of the deviations of each experimental value from the overall mean. This total sum of squares of the deviations then is segmented into the sums of squares of deviations due to the effects of each of the factors, the sums of squares of deviations due to the interaction effects, and the sum of squares of deviations due to random error. (Appendix B)

Dividing each of these sums of squares by its appropriate degrees of freedom yields unbiased estimates of the experimental variance. If the null hypothesis is true, that the factors or interactions do not significantly affect the response variable, then each of these segmented variances is a chi-square distributed, unbiased estimate of the variance due to random error σ_e^2. The critical region of an F distribution formed by taking the ratio of the various segmented variances to the sample error variance is then used to test the null hypothesis at a chosen level of confidence, α, where α x 100 represents the per cent rejection of a true hypothesis. For this investigation α was chosen at .01.

The critical region is generally taken as the upper tail of the F distribution, (Figure 4) rejecting the null hypothesis in any test if $F > F_{1-\alpha}$, where α is the area of the upper tail.

Having determined the significance of the various factors and interactions, the experimental data are then plotted in a manner which may indicate general trends.

Chapter 3

EXPERIMENTAL PROCEDURE

3.1 Instrumentation

The instrumentation utilized to introduce and detect the vibrations in the arm was required to have the following properties:

1) The transducers must be sufficiently small such that they do not significantly change the response of the system.

2) Instrumentation error must be small in the low frequency range.

3) The excitation source must allow a sweep through a wide frequency range in search of resonant frequencies, and yield visual representation of the system response.

4) The amplitudes of vibration must be relatively small and thus the input and pickup transducers must be relatively sensitive.

5) The transducers must be positioned on the arm in nearly identical positions for each of the subjects.

The input device developed was a modified 0.1 watt one-inch Calectro speaker, illustrated in Figure 1. The piston, fixed in the speaker cone, allowed a

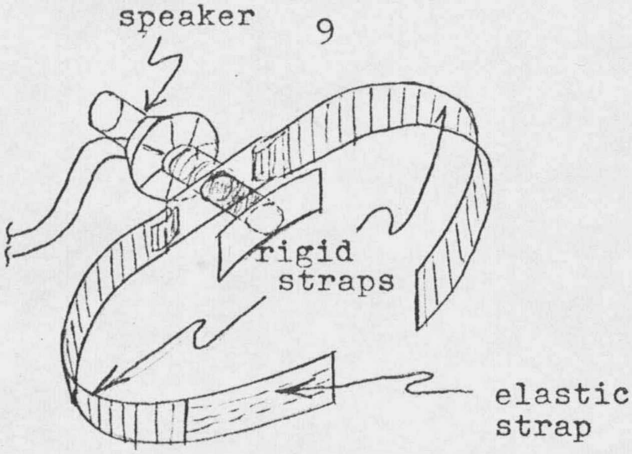

Figure 1. Vibration Input Device

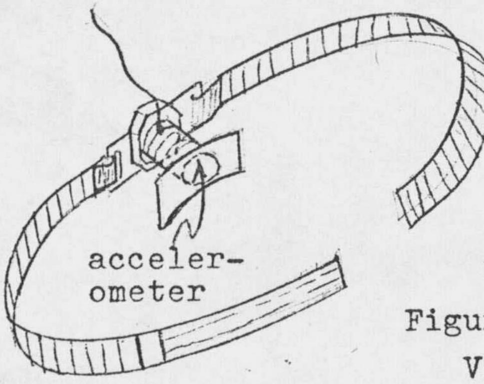

Figure 2.
Vibration pickup device

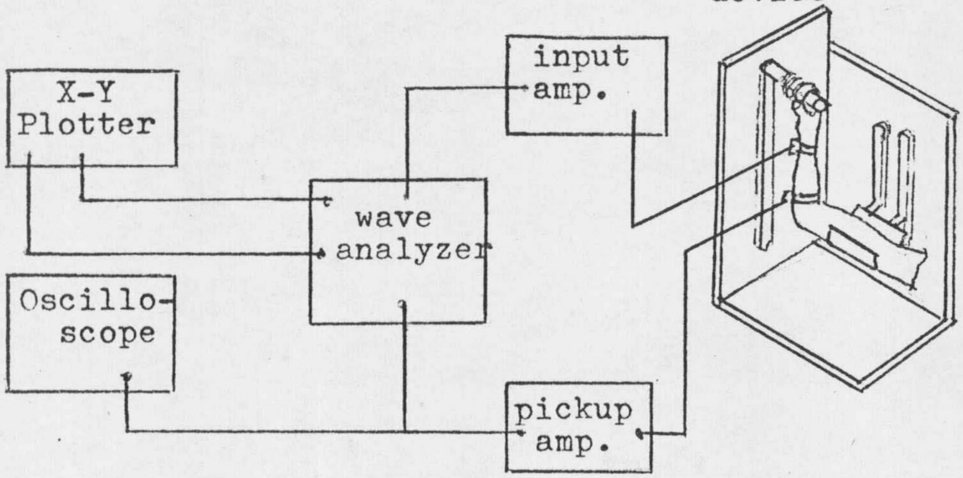

Figure 3. Experimental Schematic

small-area contact for signal input. The particular strap arrangement chosen allowed approximately constant contact pressure. Early work indicated pressure variations would significantly contribute to experimental error.

The pickup device, Figure 2, was a modified accelerometer, with the same strap arrangement as for the speaker.

The input device was strapped to the arm at a distance of 1/8 of the total ulna length above the styloid process, and the pickup at 5/16 of the ulna length from the olecranon process tip. These particular locations were chosen because of the proximity of the ulna to the flesh surface, allowing more nearly direct input to and pickup from the bone itself, which should be the major system component.

The apparatus used in the experiment is presented schematically in Figure 3. The wave analyzer acted both as a source for the input signal and as a means of comparing the input and pickup signals in the arm. It automatically swept through the desired frequency range with approximately constant input power. The recorder allowed plotting the spectral density curve for use in determining natural frequencies (Figure 9, Appendix C). The arm support device was constructed to allow nearly repeatable positioning of the subjects' arms and the transducers attached to the arms.

3.2 Procedure of Testing

Tests were run on approximately 250 volunteer, male college students in the age range of 19-24 years.

For each subject the quantities recorded were:

a) arm length
b) wrist width
c) flexed and unflexed upper arm circumference
d) height
e) weight
f) age
g) previous history of illnesses which might affect vibration response
h) spectral density curve.

Each subject was categorized according to the defined 2^n experimental conditions to determine his fit within any category. Since, as mentioned, the factor levels were defined at the extremes for the 2^n design, the major portion of the subjects did not properly fit within the defined categories and were not useable. The maximum number of replicates was limited by the minimum number of subjects in any one of the 16 categories, since for the analysis it is desirable to have the same number in each group. This number was two.

Having recorded the natural frequencies for these conditions, an analysis of the results by standard methods of analysis of variance was possible (3,4).

3.3 Equipment Error Analysis

The possible error contributed to the experimental results by the various pieces of equipment is as follows:

Calibrating sine wave generator: ±1%
Wave analyzer: ±1%
Vibration meter: ±1%

Since the amplitude response was relative, the amplitude error is not considered in this analysis.

Chapter 4

DISCUSSION OF RESULTS

4.1 Analysis of Variance

The experimental data and calculations of the various sums of squares are presented in Appendix C. The resulting F tests are summarized by an Analysis of Variance table, Table 1.

At the 99% level of confidence the corresponding F statistic is $F_{1,15} = 8.68$. F values of Table 1 which lie in the rejection region indicated in Figure 4, imply that that particular factor or interaction truly does affect the resonant frequency. It is noted that, except for the AB interaction, the CD interaction, the ABCD interaction, and the replication effect, all other effects are significant at the .01 level.

Higher numbers for these F values indicate greater significance, hence, some of the main factor effects are more significant than are the others. Bone size and fleshiness are much more significant than arm length in affecting the natural frequency, and are also more significant than muscle development. Although

Table 1 Analysis of Variance

Source	degrees freedom	Sums of Squares	F
A: arm length	1	1875.78	8.695
B: bone size	1	16882.03	78.26
C: muscle devel.	1	8417.53	39.02
D: fleshiness	1	15532.03	72.0
2-way interactions			
AB	1	810.03	3.75
AC	1	15708.78	72.82
AD	1	42122.53	195.26
BC	1	8547.78	39.62
BD	1	3180.03	14.74
CD	1	979.03	4.54
3-way interactions			
ABC	1	21580.03	100.03
ABD	1	15975.78	74.05
ACD	1	20757.03	96.22
BCD	1	18963.78	87.91
4-way interaction			
ABCD	1	33.5	<1
Replicates	1	344.5	1.60
Random error	15	3235.97	------

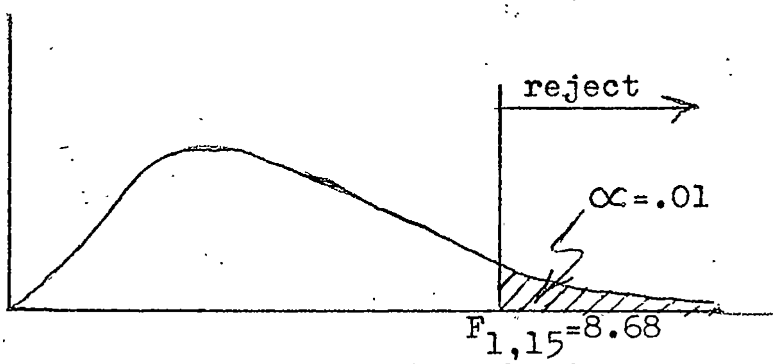

Figure 4. F Distribution

the arm length is just slightly significant at the .01 level, the importance in considering this factor will be seen later in considerations of interactions.

Before proceeding to discussion of interaction effects, some interpretations of interactions may be in order.

A significant two factor interaction indicates that the effect of a particular factor on the response variable is different at different levels of the other factor, where the effect of a factor may be defined as the change in response variable produced by a change in the factor level. These two-way interactions will become clear in the plots presented in section 4.2.

The fact that AD interaction is significant means that the change in resonant frequency between levels of arm length is different for different levels of fleshiness. Similar conclusions are made for arm length-muscle development, bone size-muscle development, and bone size-fleshiness interactions--AC,BC, and BD, respectively. Again the fact that some of these F statistics are much larger than the others indicates that those interactions are much more pronounced than the others.

Similar conclusions may be drawn for the three-way interactions, where three-way interactions indicate that the interaction of two factors is different at different levels of a third factor.

As mentioned before, although the main effect of arm length is only slightly significant, the importance of considering variations in this factor is shown in the significance of many of the interactions of the other parameters with arm length. These factors affect the resonant frequency differently for different levels of arm length.

4.2 Interaction Plots

Figures 5 through 8 are plots of cell totals used in calculating the various two-way interactions versus one of the factors, for each plot.

A given line represents the variation in response variable with the change in level of the abscissa variable at the constant level of the factor indicated on the line. Thus there will be two lines for each of the three remaining factors for each plot. For a given line the effects of the other two factors have been effectively averaged at each endpoint. The straight

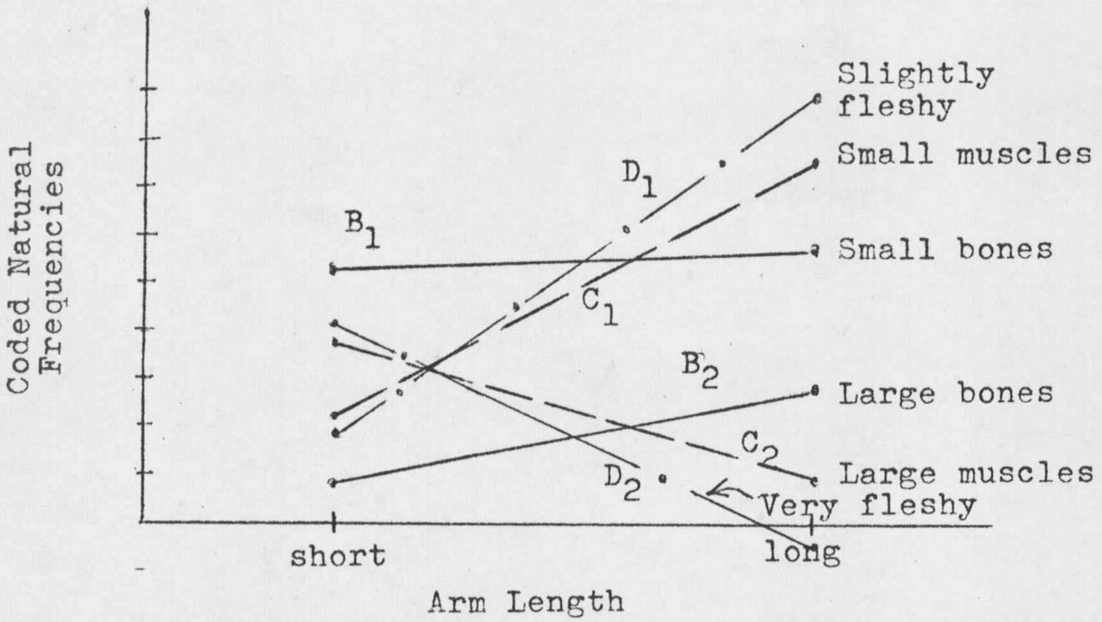

Figure 5. Arm Length
Interaction

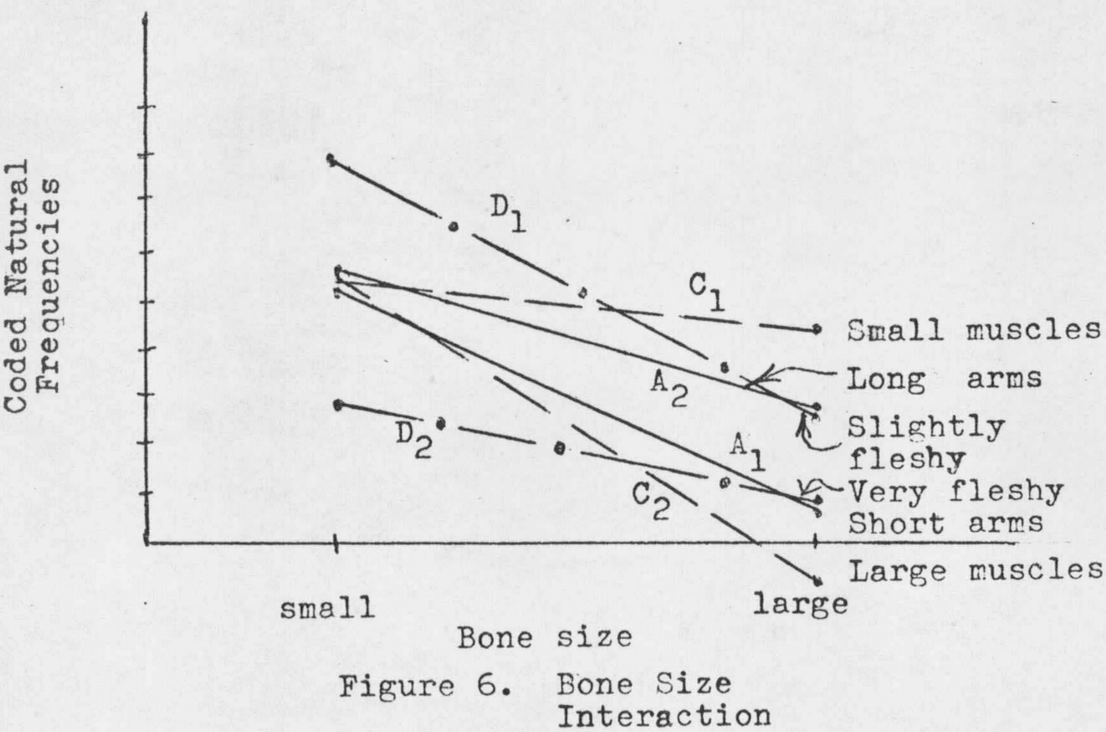

Figure 6. Bone Size
Interaction

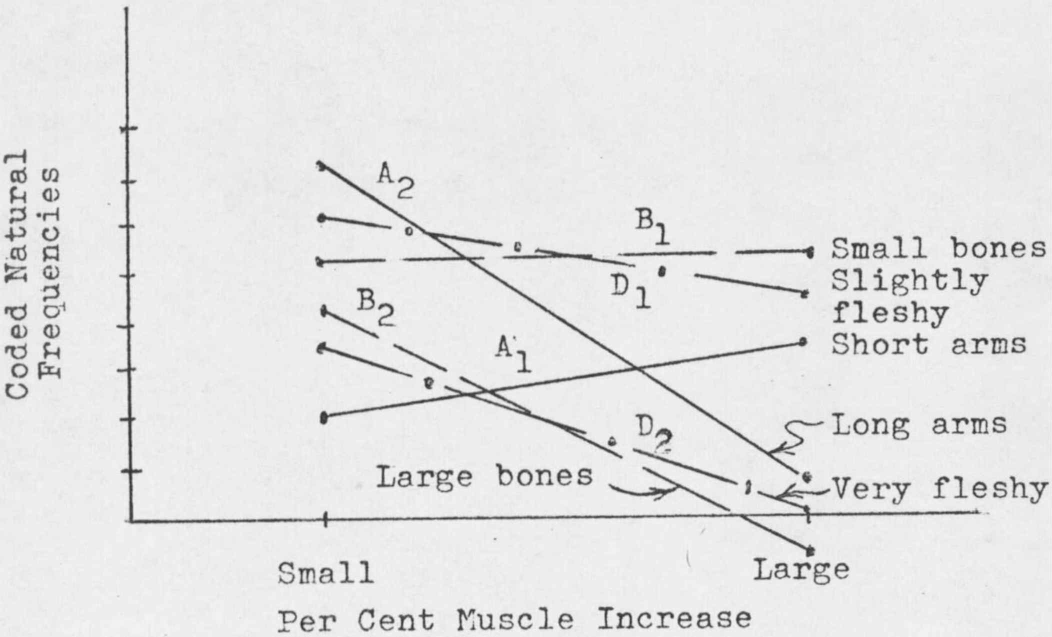

Figure 7. Muscle Development Interaction

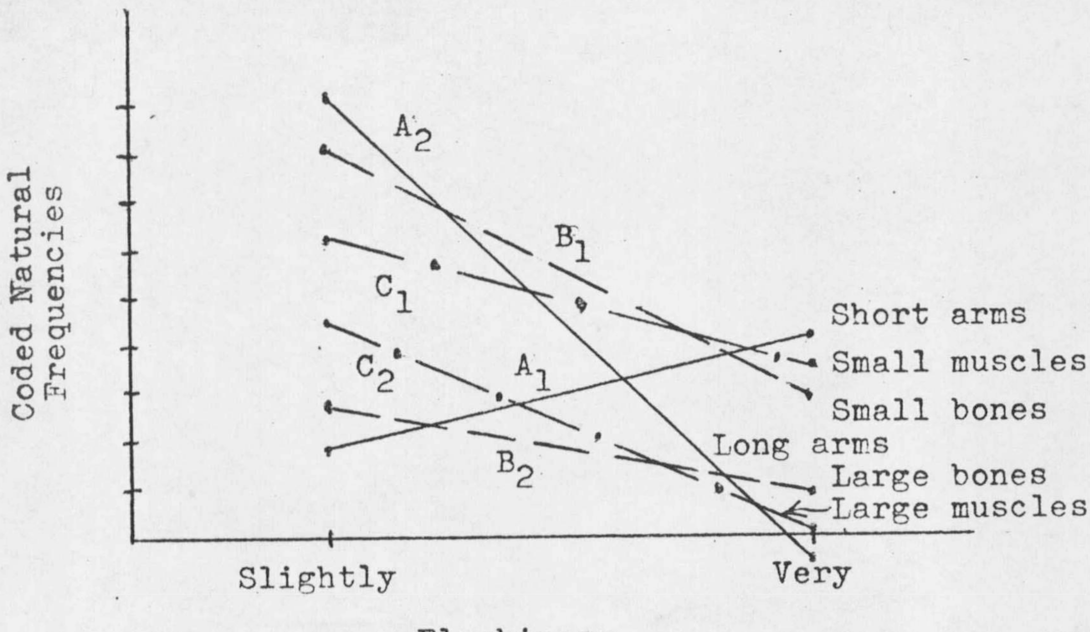

Figure 8. Fleshiness Interaction

line drawn between two endpoints is not meant to indicate a linear relationship, but only a general trend, since, the nature of the variation cannot be explicitly determined without further experimentation. If the two lines for the two levels of a factor are nearly parallel, the conclusion is that no interaction exists between this factor and the abscissa factor, since the abscissa factor affects the response variable in the same manner at both levels of the other factor. If, further, the two lines are spaced closely together, the conclusion is that this factor has little effect on the response variable, since different levels of the factor produce nearly the same results. If the two curves are widely separated, the factor does affect the resonant frequency, and it may also be possible to obtain some insight as to the nature of the effect. It should be noted, however, that although a plot in which a given factor is not the abscissa factor may indicate a certain type of variation of response variable with change in level of the factor, the plot with this factor as the abscissa variable should be considered before making generalizations as to the nature of the variation. This will be seen shortly.

Thus, plots such as Figures 5 through 8 serve many useful purposes. They afford a graphical representation of the interaction effects and main effects, and in addition to indicating possibly whether or not these effects are significant, provide insight into the nature of the effect of a particular factor.

The conclusions resulting from a study of these figures are as follows:

Figure 5: It is apparent that there is no fixed variation of natural frequency with arm length. Although an increase in arm length causes an increase in resonant frequency in some cases, it causes a decrease in resonant frequency in other cases, illustrating the interaction of arm length with the other factors.

For the two curves for factor B--wrist size--the wide separation of these curves indicates the significant effect of wrist size on resonant frequency, and their parallel nature indicates very slight interaction between arm length and bone size. In addition the curves imply that increasing the wrist size tends to decrease the resonant frequency, and Figure 6 indicates that this is the case.

The difference in the effect of arm length for different levels of fleshiness--D--is easily seen by the difference in curves D_1 and D_2 (AD interaction.) Similarly one can see the large interaction between arm length and muscle development. (AC)

The general nature of the effects of muscle development and fleshiness on resonant frequency are difficult to determine from Figure 5. It does appear that the effects on the resonant frequency of changing levels of muscle development or fleshiness are less significant for shorter arm people than those with longer arms. This is indicated by the difference in the spacing of the end points of the respective curves for the two levels of arm length. These conclusions are supported by the difference in slopes of the two arm length curves in Figure 7 and in Figure 8. For long arms an increase in the level of either muscle development or fleshiness tends to decrease the natural frequency, while the opposite effect is observed for short arms.

Figure 6: The most obvious observation from Figure 6 is the general effect of a change in level of wrist size. As concluded before, an increase in bone size tends to cause a decrease in natural frequency for all levels of the other factors.

Again the lack of interaction in arm length and wrist size can be seen by the approximately parallel curves A_1 and A_2, and the small effect of arm length is indicated by the relatively close spacing of these curves.

The large effect of fleshiness is seen in the separation of D_1 and D_2 curves. From these D curves it appears that as the level of fleshiness increases, the resonant frequency decreases; however, Figure 8 shows that this is not always the case. Also, there is little interaction between bone size and fleshiness, although, the effect of a change in level in fleshiness is less for larger wrist sizes than for smaller wrist sizes.

The effect of a change in level of muscle development is greater at the higher level of wrist size than at the lower level. These conclusions are supported by the nature of the B curves in Figures 7 and 8. The effect of an increase in muscle development is to decrease the natural frequency at the larger wrist size, while having little effect for the small wrist size, as seen in the B curves in Figure 7. This indicates the significant bone size-muscle development interaction.

<u>Figure 7</u>: There seems to be no general trend

for the effect of muscle development on natural frequency since the curves slope upwards in some cases and downwards in the others.

There is a large interaction effect between arm length and muscle development, with the effect of arm length greater for the small muscle development than for the large muscle development.

Again, the effect of a change in wrist size is observed by the separation of B_1 and B_2, and the nature of the effect is the same as indicated before. The conclusion is that the effect of a change in bone size is less at lower levels of muscle development than at the upper levels of muscle development, and the nature of curves B_1 and B_2 again indicates the significant interaction of bone size and muscle development.

Noting the separation of the D_1 and D_2 curves, the large effect of fleshiness is again obvious. Here again an increase in fleshiness appears to cause a decrease in natural frequency, although this is not true in every case. These curves indicate little interaction between fleshiness and muscle development. (CD)

<u>Figure 8</u>: The general trend in Figure 8 is a decrease in natural frequency with an increase in

fleshiness, except at the lower level of arm length, A_1, where the opposite effect is observed. The pitfall of considering one set of these curves alone is shown by this example. Both Figures 6 and 7 indicated that an increase in D caused a decrease in natural frequency. Similarly, although the separation of the C_1 and C_2 curves indicates the large muscle development effect, Figure 7 shows that an increase in muscle development does not always produce a decrease in natural frequency.

The interaction between fleshiness and bone size is more easily observed in this figure, with the bone size effect as noted previously, and also the large interaction effect of arm length and fleshiness is obvious.

Many of the previous observations and conclusions may again be made using Figure 8.

Chapter 5

CONCLUSIONS AND SUMMARY

In summary, the effects of four physical parameters of a human subject on a resonant frequency of the forearm have been investigated. These parameters were:

A: Arm length
B: Bone size
C: Muscle development
D: Fleshiness

The subjects utilized were college-age males.

The experimental investigation was designed and analyzed using the methods of Experimental design, via a 2^n fixed factor factorial design.

It was determined that all of these four factors significantly affected the resonant frequency (at the .01 level.) Of the four, arm length was found to be the least significant, and bone size and fleshiness the most significant. It was further determined that many of the two-way interactions and all of the three-way interactions were significant. The conclusion drawn from these significant interactions is that the various factors affect the resonant frequency response of the forearm differently at different level combinations of the other factors.

Although arm length was found to be only slightly significant at the .01 level, the importance of considering this factor lies in the large interaction effects between it and the other factors.

There appeared to be no general trends for the nature of the effects of arm length, muscle development, and fleshiness on the resonant frequency; however, the effect of an increase in bone size was a decrease in resonant frequency.

Examining the experimental error term of Table 1, one possible conclusion is that other physical factors may be found which could reduce the size of this term (since they were unaccounted for in this investigation and would thus appear as random variation.) However, since this term is much smaller than the main factor effects, it would seem reasonable to assume that these four factors would be the important ones which need be used in determining the nature of the resonant frequency response of the forearm.

APPENDIX

Appendix A

2^n EXPERIMENTAL DESIGN

The 2^n experimental design combines n factors each at two levels in a factorial manner to yield the experimental conditions; thus, one has 2^n experimental conditions.

The method, then, provides categories within which the factor levels must fall in order that a particular test be used as one of the experimental values.

A particular type of notation is standard (3,4) for this type of design. For the factors A,B,C, and D let the lower case letter represent the upper level of a factor; i.e.,

a: long arms
b: large wrists
c: large per cent muscle increase
d: very fleshy

and let a 1 represent the lower level of the factor.

Thus the various combinations of the lower case letters and ones represent the experimental conditions. For the 2^4 design the possible conditions are then: 1,a, b,ab,c,ac,bc,abc,d,ad,bd,abd,cd,acd,bcd, and abcd, where 1 represents the condition of all factors at their lower level (an individual with short arms, small wrists, small muscles, and not fleshy), a--all factors except A at low levels, and so on.

With two replicates, one then has 32 total data values.

Appendix B

MATHEMATICAL MODEL AND ANALYSIS

The mathematical model for the four factor experiment is

$$X_{ijklm} = \mu + A_i + B_j + (AB)_{ij} + C_k + (AC)_{ik} + (BC)_{jk} + (ABC)_{ijk} + D_l + (AD)_{il} + (BD)_{jl} + (CD)_{kl} + (ABD)_{ijl} + (ACD)_{ikl} + (BCD)_{jkl} + (ABCD)_{ijkl} + \epsilon_{m(ijkl)}$$

where

X_{ijklm} = a particular experimental value

μ = population mean

A_i = effect of factor A

B_j = effect of factor B

C_k = effect of factor C

D_l = effect of factor D

$\epsilon_{m(ijkl)}$ = random error

and the terms in parentheses are the various factor interactions.

This equation attempts to explain each reading in terms of the population mean, the possible factor effects the interaction effects, and the random error.

Using this equation the sum of squares equation can be derived as:

$$\sum_i \sum_j \sum_k \sum_l \sum_m (X_{ijklm} - \bar{X}_{.....})^2 = \sum_i \sum_j \sum_k \sum_l \sum_m \Big[(\bar{X}_{i....} - \bar{X}_{.....})^2 + (\bar{X}_{.j...} - \bar{X}_{.....})^2 + (\bar{X}_{..k..} - \bar{X}_{.....}) + (\bar{X}_{...l.} - \bar{X}_{.....})^2 + \text{Interaction terms} + (X_{ijklm} - \bar{X}_{ijkl.})^2 \Big]$$

with the dot notation indicating that that subscript has been summed over. The first term on the right represents sum of squares due to the effect of factor A (SS_A), the second SS_B, the third SS_C, the fourth SS_D, the next the various interaction sums of squares, and finally the sum of squares due to random error, SS_E.

It is seen that if the null hypothesis is correct, that is, if factor A does not affect the resonant frequency, and if factor B does not affect the resonant frequency, and so on, then each of these sums of squares when divided by their respective degrees of freedom will be chi-square distributed, unbiased estimates of σ_e^2--the error variance, with all terms on the right separately yielding independent estimates of σ_e^2.

Thus, the concept of the F distribution can be used to test the null hypothesis--testing these sample variance estimates against the sample error variance, SS_E/df_{error}. Comparing these calculated F statistics with the appropriate tabulated F value at a prechosen confidence level, conclusions can be made concerning whether these variances truly are estimates of the error only, or whether they differ significantly from the error estimate, indicating the factors or interactions do have an effect.

The various sums of squares are generally calculated by short-cut methods such as the Method of Yates (3,4), considerably shortening the analysis procedure. This method, limited to 2^n designs, will be illustrated in Appendix C.

Appendix C

RESULTS

The 32 treatment values may be listed in standard order as in Table 2 (coded by subtracting 300 from each resonant frequency reading.)

The resonant frequencies were selected as the lower peak in a typical power spectral density curve, Figure 9.

The following sums of squares are found by standard methods.

$SS_{total} = 194,950.97$

$SS_{replicates} = 344.53$

$SS_{treatments} = 191,370.47$

$SS_{error} = 3,235.97$

To determine the main factor effect sums of squares and the interaction sums of squares the Method of Yates is used. (3,4)

For this method a listing of the treatment conditions in standard order is made and in an adjacent column the treatment totals are placed. (Table 3) The calculations are performed in the next 4 columns--or in general the next n columns. The numbers in each column are obtained by following the same procedure.

Table 2 Coded Experimental Results

Treatment	Replicate 1	Replicate 2	Totals
1	30	35	65
a	105	110	215
b	15	42	57
ab	165	120	285
c	60	30	90
ac	200	230	430
bc	-15	-12	-27
abc	-30	-12	-42
d	108	90	198
ad	45	20	65
bd	-40	-55	-95
abd	100	90	190
cd	90	80	170
acd	-65	-80	-145
bcd	70	70	140
abcd	-65	-90	-155
Totals	773	668	1441

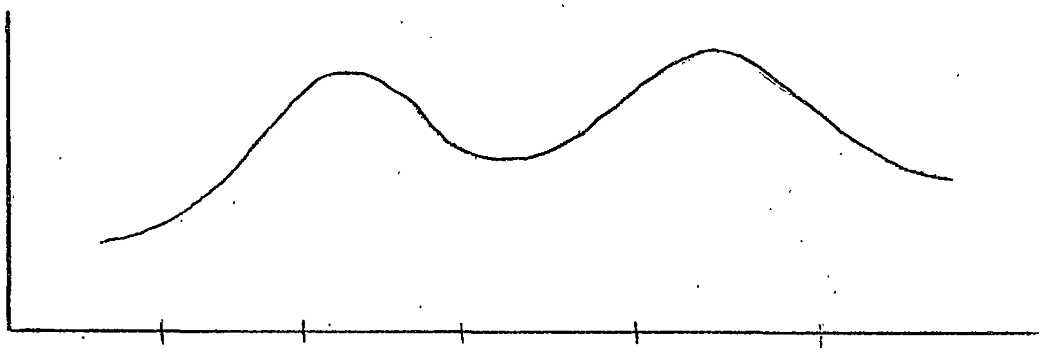

Figure 9. Typical Power Spectral Density Curve

Table 3 Yates Method Analysis

Treatment	Total	1	2	3	4	SS
1	65	280	622	1073	1441	64890.03
a	215	342	451	368	245	1875.78
b	57	520	358	703	-735	16882.03
ab	285	-69	10	-458	161	810.03
c	90	263	378	-527	-519	8417.53
ac	430	95	325	-208	-709	15708.78
bc	-27	25	152	-277	-523	8547.78
abc	-42	-15	-610	438	-837	21580.03
d	198	150	62	-171	-705	15532.03
ad	65	228	-589	-348	-1161	42122.53
bd	-95	340	-168	53	319	3180.03
abd	190	-15	-40	-762	715	15975.78
cd	170	-133	78	-651	-177	979.03
acd	-145	285	-355	128	-815	20757.03
bcd	140	-315	418	-433	-779	18963.78
abcd	-155	-295	20	-398	35	33.5

Table 4 Summary of Sums of Squares

Source	degrees freedom	Sum of Squares	Mean Square
A	1	1,875.78	↑
B	1	16,882.03	Same as
C	1	8,417.53	
D	1	15,532.03	Sum
AB	1	810.03	
AC	1	15,708.78	of
AD	1	42,122.53	
BC	1	8,547.78	Squares
BD	1	3,180.03	
CD	1	979.03	
ABC	1	21,580.03	
ABD	1	15,975.78	
ACD	1	20,757.03	
BCD	1	18,963.78	
ABCD	1	33.5	↓
ERROR	15	3,235.97	215.73

For column 1, for example, in the upper half of the table the numbers are obtained by adding successive pairs of numbers in the previous column, from top to bottom. The second half of the table, then, is obtained by subtracting successive pairs of the previous column, and in each case the first number is subtracted from the second; for example, 215 - 65= 150, and so on.

The final column is obtained by squaring the values in column 4--n--and dividing by $r \times 2^n$, where r is the number of replicates. This column contains the sums of squares of factors and interactions.

The results of the calculations are summarized in Table 4.

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