

OPTIMAL DECISION ALTERNATIVES for ACQUIRING BRED BEEF HEIFERS

by MONTANA PRODUCERS

by

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

Montana cattlemen traditionally replace beef cows culled from the breeding herd with bred heifers produced from within their ranching operations. The purpose of this study is to compare alternative methods commercial cow-calf operators could utilize to acquire bred beef heifers. It is hypothesized that Montana ranchers could decrease the expected present cost of obtaining bred heifers by choosing acquisition alternatives appropriate to the current and lagged state of the cattle and feed market.

A behavioral price model for Montana bred beef heifers is theoretically developed and estimated. Time series equations that predict bred heifer price and feeder heifer price are estimated. Montana barley and alfalfa price series are combined in a weighted sum to form a price series used to construct a time series equation that will predict feed ration price per day for beef heifers. These time series equations are combined to form a system of three equations that jointly predict bred heifer, feeder heifer, and feed ration price.

Dynamic programming is utilized to compare the expected present cost and determine an optimal decision rule among twelve distinct strategies for acquiring bred beef heifers. Stochastic state variables are current feeder heifer price, lagged feeder heifer price, and current feed ration price. Transition probabilities for the stochastic variables are calculated from the system of three time series equations. Expected immediate costs for producing bred heifers include feeding, breeding, pregnancy testing and marketing costs.

The conclusion drawn from the dynamic programming model results is Montana ranchers can decrease the expected present cost of acquiring bred heifers. When feeder heifer price is decreasing the decision to sell heifer calves immediately after weaning and purchase bred heifers prior to calving is optimal. If feeder heifer price is relatively unchanged or slightly increasing retaining heifers over the winter, selling them in the spring, and purchasing bred heifers is the optimal decision policy. Raising bred heifers is optimal only when feeder heifer price is sharply increasing.

CHAPTER 1

INTRODUCTION

The beef cattle industry has been a major component of Montana's economy since the middle 1800's. Beef business began at the end of the 1850's when Granville Stuart gathered lame oxen from the Idaho region, fatten them on the abundant grasslands of Montana Territory, and sold the cattle at the bustling mining camps. In 1866 Nelson Story completed the first Texas cattle drive over the Bozeman Trail with 600 head of longhorns. The vast grasslands of Montana had begun to attract entrepreneurs of the cattle industry into the region and by 1883 there were 600,000 head of cattle on the range. Montana's cattle industry has continued to experience periods expansion or contraction of its cattle numbers, high or low fluctuations of its market prices, and financial gains or losses incurred by its cattlemen.

Today, beef cattle ranches remain an important sector of Montana's agricultural community. Fifty percent of the 1988 cash receipts for Montana agricultural commodities were generated from the sales of beef cattle and calves. The industry is primarily composed of commercial cow-calf operations which replace approximately twenty percent of their breeding herd annually. In 1988 Montana's cattlemen

utilized 255,000 head of beef cows for replacement purposes. The production of these replacement cows is a major input cost incurred by the beef industry. Production methods and decisions for selecting beef cow replacements are important to individual commercial ranching operations and the cattle industry as a whole.

Common practice for ranch management in Montana is to cull unproductive cows from the breeding herd; this occurs in the fall of the year after the calves have been weaned. The cows are replaced the following spring with two year old heifers ready to calve.

Montana ranchers traditionally raise the bred heifers used to replace the cows culled from the breeding herd. The production process of raising bred heifers is integrated into their cattle operations. The process begins at weaning with the selection of replacement heifer calves from among the calves produced by cows within the breeding herd. A feed ration, designed to meet specific nutrient levels, is fed throughout the winter to prepare heifers for the upcoming spring breeding season. The heifers are bred and turned out onto summer pasture until fall. Following pregnancy testing cull heifers, open heifers and bred heifers that did not grow and mature as anticipated, are sold. The remaining bred heifers enter the breeding herd the following spring, one and a half years after the initial stage of the production process.

An alternative method for acquiring bred heifers not commonly practiced in Montana is to sell all heifer calves after weaning and purchase bred heifers each spring prior to calving. This eliminates management of the production phase required to develop bred heifers. Of course, other options exist, heifer calves might be retained until spring and bred heifers purchased one year later, etc.

These examples of alternatives for bred heifer acquisition characterize the uncertainty involved in the decision making process. Production costs of bred heifers produced from within the herd will depend on feed prices, feed rations, disease prevention, death loss and rates of conception. A change in one input factor could create a significant variation in the final production cost of the bred heifer. The economic cost of bred heifers obtained outside the breeding herd from an auction market or by direct purchases (private treaties) will include purchase price, transaction costs, transportation costs and information costs. Incomplete information regarding the quality of the bred heifer and its potential offspring often exists. The cattlemen whom choose to raise bred heifers rather than purchase them, should consider the uncertainty introduced by both changing production costs and the lack of information available regarding a heifers quality.

Price change is a important source of uncertainty in future events. While a rancher has control over choosing

production techniques, i.e., the feed ration, implementing a better disease prevention program, and selecting a management plan to improve conception rates; the individual operator has no control over the changes in feed or cattle prices. Ideally, whether implicitly or explicitly, in the fall after weaning a rancher compares alternatives for acquiring bred beef heifers. For example, when raising a bred heifer is compared to the alternative of purchasing a bred heifer (each spring prior to calving) the current value of replacement heifer plus the expected production costs is contrasted with the expected purchase price of two year old heifers in a year and a half. When information costs are excluded, these risks of choosing one production alternative over another are predominately influenced by price changes.

Obtaining bred heifers as replacements in the breeding herd is a dynamic process. Assuming perfect information, the most efficient procedure to acquire bred heifers would be to select the alternative which minimized cost. However, since information is not perfect selecting the optimal policy may depend upon the evaluation of alternatives as new information becomes available at various stages throughout the decision process. For the efficient rancher this cost comparison strategy will usually occur successively over monthly, semi-annual or annual intervals. For example, in the fall a rancher may decide that relative costs and prices are such that it is cost efficient to retain heifer calves for

development and future use in the breeding herd. However, the next spring feed prices and/or bred heifer prices may have changed so that selling the yearling heifers and purchasing bred heifers the second spring prior to calving is the optimal choice.

Problem Statement and Assumptions

The economic objective for a ranch/firm is presumed to be profit maximization. Since the replacement cow is a capital input into the firm, the decision that minimizes the input's cost will contribute to maximizing the ranch's profits (Beattie).

The objective of this study is to determine the sets of decision rules, the decision policy, which minimizes the bred heifer input cost. The set of assumptions that formulate and explicitly define the problem are: 1) a bred heifer is produced in four stages which begin in the fall of year t (F_t), progress through the spring of $t+1$ (S_{t+1}), fall of $t+1$ (F_{t+1}) and end in the spring of $t+2$ (S_{t+2}). Spring and fall are from January 1st to June 30th and from July 1st to December 31st, respectively. ; 2) a constant breeding herd size is maintained from year to year; 3) by January 1st of each year a fixed number of bred heifers are to be available for use as replacement stock; 4) each fall calves will be weaned November 1st. On November 1st a decision will be made whether to retain heifer calves from the herd to initiate the

production of bred heifers or to sell the calves immediately which necessitates the purchase of replacements at a future date. It is presumed the quality level of a heifer purchased will be at least as good as the quality level of the one raised; 5) heifers raised on the ranch will be fed a ration composed of fixed portions of alfalfa hay and feed barley; 6) retained heifers will be bred May 1st, the spring of their yearling year; 7) a predetermined conception rate will be assumed and additional heifers will be retained and bred to compensate for open and/or other cull heifers. 8) In the fall after the heifers are bred the heifers will be pregnancy tested November 1st. Open and/or cull heifers will be sold as feeder heifers and the select bred heifers will be retained as replacements for cows in the breeding herd. 9) The bred heifers will calve on average about February 9th. Figure 1 presents the decision nodes, days between each node, and the semi-annual time periods of the decision problem.

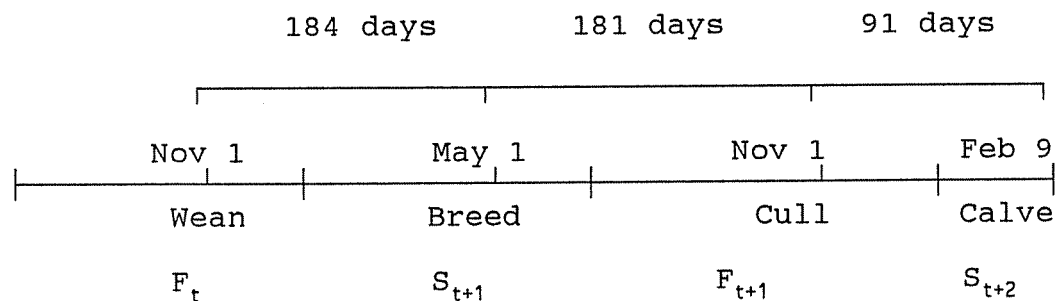


Figure 1. Bred heifer production calendar.

Semi-annual time periods assumed under (1) correspond to the intervals when a rancher has a natural opportunity to make decisions concerning the enterprise. Also, the time units are compatible with the available price observations on bred heifers. Since the problem is focused on replacement heifer acquisition alternatives and not the stocking rate problem, (2) and (3) are assumed. The chosen criterion for comparing decision alternative's is production cost minimization (without adjustment for possible quality differences among heifers), so (4) is presumed. The feed ration is composed of fixed portions, suggested in (5), in order to concentrate on feeder heifer and bred heifer price fluctuations and not the management problems subject to the nutritional requirements of beef heifers. The assumptions (6), (7), (8) and (9) are consistent with current ranching practices in Montana.

Purpose and Objective of Study

The purpose of this study is to formulate a decision model that will provide an optimal decision policy for the bred heifer acquisition problem. The policy minimizes the expected cost of obtaining a bred heifer to be utilized as a replacement cow. Initially, information from the analysis of the price models is used to construct the decision model. The optimal decision policy will then be interpreted from the

empirical data supplied by the model. The objectives are :

- 1) Develop a behavioral model of bred heifer price.
- 2) Construct time series models for bred heifer price, feeder heifer price and feed ration price.
- 3) Deduce the probability distributions from each time series model.
- 4) Construct a finite stage stochastic dynamic programming model and interpret the optimal decision alternative(s).

Procedure

In Chapter 2 the theoretical concepts of a stochastic dynamic programming problem are described. Decision making under uncertainty is combined with dynamic programming to construct a stochastic decision model which determines the least cost alternative of obtaining a bred heifer. The bred heifer problem is introduced in the context of dynamic programming. Chapter 3 focuses on the econometric estimation of price prediction models. First a bred heifer price behavioral model is presented. Next, a system of time series equations consisting of bred heifer, feeder heifer, and feed ration prices is specifically developed for implementation into the dynamic programming problem. Chapter 4 is devoted to the assumptions and structure of the empirical problem. Empirical results are presented and discussed. Summary and concluding remarks are presented in Chapter 5.

CHAPTER 2

DECISION MODEL

The objective of this chapter is to develop a decision model for the bred heifer acquisition problem. Stochastic decision processes are integrated into the dynamic programming problem to construct a finite stage stochastic dynamic programming model.

Decision Making Under Uncertainty

Decision processes can be either deterministic or non-deterministic. A process is deterministic when the outcome associated with each possible course of action is uniquely determined by the choice of the decision. A process is non-deterministic when a decision is made under conditions of uncertainty, i.e., for each possible course of action there is a number of possible outcomes. Stochastic processes are an important class of non-deterministic processes in which the result of a decision is to determine a probability distribution of outcomes (Bellman).

Because the probability distribution of outcomes can be generated from a system of prediction equations stochastic decision models are practical and common. Independent variables of the forecasting equations describe the current

state of the system and the dependent variables depict the predicted outcome. The probability distribution of outcomes of the prediction equations will be the joint probability density function (PDF) of the dependent variables. A PDF has a distribution equivalent to that of the system's estimated error terms if the estimated error structure is classical or autocorrelated (Johnston).

Uncertainty in the Bred Heifer Problem

Once calves are weaned, a rancher can either sell heifers as feeders or retain them for possible use in a breeding program. When heifers are retained, future input costs into their development are incurred. If heifers are sold, a decision will be made at a later date to repurchase a heifer or purchase a bred heifer.

A rancher's choice among the different alternatives is an example of decision making under uncertainty. Feed price changes, death losses, conception rates and variations of rates of gain can affect the cost of producing a bred heifer. All these variables as well as cattle price changes add to uncertainty of the future value of the bred heifer.

Dynamic Programming Theory

Dynamic programming provides a systematic procedure for determining the combination of decisions that is optimal in a decision process. The basic features that characterize

dynamic programming problems are presented below.

Stages. Decision problems are partitioned into stages. Each stage is a point where decisions must be made.

States. State variables describe the condition of the decision process. A state is defined by values of the state variables. The number of possible states may vary across stages.

Decision Alternatives. For each stage and state there is a set of decision alternatives. The decision made in a state at a particular stage will be one of the forces acting to translate the state as the process changes to the next stage.

State Transitions. The transition, the state taken on as the process moves from one stage to the next, may be deterministic or non-deterministic. A transition is deterministic when the state of the process at the next stage is exactly determined by the state and decision alternative of the current stage. Stochastic transitions differ from deterministic in that the state at the next stage is not exactly determined by the state and decision alternative at the current stage. Rather, the state and decision alternative at the current stage determine the probability distribution of outcomes across the possible states at the next stage.

Transition Probabilities. A stochastic dynamic programming problem must be formulated to meet the Markov condition. This condition requires that the optimal policy of a sequential decision process depend only on the state in the current

stage and not on the state in any previous stage(s). A conditional probability density function which satisfies the Markov requirement and describes the transition probabilities can be constructed from the probability distribution of outcomes generated by the forecasting equation of the stochastic decision procedure.

Expected Immediate Costs. These are costs incurred at the current stage in a particular state and for a specific decision alternative.

Discount Factor. The discount rate accounts for the opportunity cost of capital.

Recursive Relationship. The optimal decision policy is determined by solving the mathematical recurrence relation:

$$V_i(n) = \min_K \{ C_i^K(n) + b \sum_{j=1}^M p_{ij}^K(n) * V_j(n-1) \}. \quad (2.1)$$

Where:

- | | |
|------------|--|
| $V_i(n)$ | is the optimal value of the recurrence relation if it is in state i with n stages remaining. |
| K | is the set of decision alternatives. |
| n | is the stage, counting from the final stage $n=1$. |
| $C_i^K(n)$ | is the expected immediate costs (returns) for the i^{th} state, K^{th} decision alternative and n^{th} stage. |
| b | is the discount factor, $1/(1+r)$, and r is the discount rate. |

- $V_j(n-1)$ is the optimal value of the recurrence relation if it is in state j with $n-1$ stages remaining.
- $V_j(0)$ is the terminal value for being in the j^{th} state at the end of the decision process.
- M is the number of possible states in stage $n-1$.
- $p_{ij}^K(n)$ is the transition probability for being in the j^{th} state in stage $n-1$ given the process is in the i^{th} state and the K^{th} decision is made in stage n .

Dynamic programming problems worked out through the backward solution of the mathematical recurrence relation ease the burden on computational hardware because all possible policies or strategies do not have to be evaluated. The backwards solution technique follows from the principle of optimality which requires whatever the initial state and initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (Bellman).

Bred Heifer Acquisition Problem

The optimal policy for obtaining a bred heifer can be determined under the construct of a dynamic program. The components of the problem in the framework of dynamic programming follow.

The stages for this problem will be time periods separated into semi-annual intervals of spring (January 1st to June 30th) and fall (July 1st to December 31st). This is a finite stage problem with four stages that begins in the fall

of time period t (F_t) progresses through the spring of $t+1$ (S_{t+1}), the fall of $t+1$ (F_{t+1}), and ends in the spring of $t+2$ (S_{t+2}).

There are many factors which affect bred heifer production cost or bred heifer price. The major production cost is feed. Market price for bred heifers is dependent on input costs and conditions (feed prices and pasture conditions), output prices (calf prices), the volume of bred heifers available, and the number of beef cows in the national breeding herd. Each of the above factors (including lagged values) may influence the condition of the decision process and are candidates for state variables. In Chapter 3 the selection procedure for the appropriate state variables of the dynamic program is described and analyzed.

The decision nodes were selected to occur at a particular date in each stage in order to coincide with the natural time at which a rancher is likely to make a decision. Weaning will occur November 1st in stage F_t , breeding in S_{t+1} on May 1st, and pregnancy testing November 1st of stage F_{t+1} . At each of these nodes a decision will be made whether to keep or sell a heifer retained from the previous stage or to purchase a heifer/bred heifer when none was held at the previous stage. At the final decision node, January 1st of S_{t+2} , where the structure of the problem specification requires that the rancher have a bred heifer, the decision alternatives are either to hold the bred heifer produced

through retained ownership, sell cull heifers, or purchase a bred heifer. The structure of the decision alternatives available is shown in Figure 2. The decision tree does not include the stochastic elements of the decision problem, but presents the possible decisions accessible under the problem specification.

Expected immediate costs differ among the stages and states. Production costs incurred when heifers are retained are feed costs in each stage except S_{t+2} , breeding costs in S_{t+1} , pregnancy testing costs in F_{t+1} , and transaction costs for selling heifers in any stages. The discount factor, b , in the recursion relationship is determined by the discount rate r which is the real interest rate. A sickness and death loss rate is incorporated separately to isolate its affect in the model from other risk generating factors such as rate of conception.

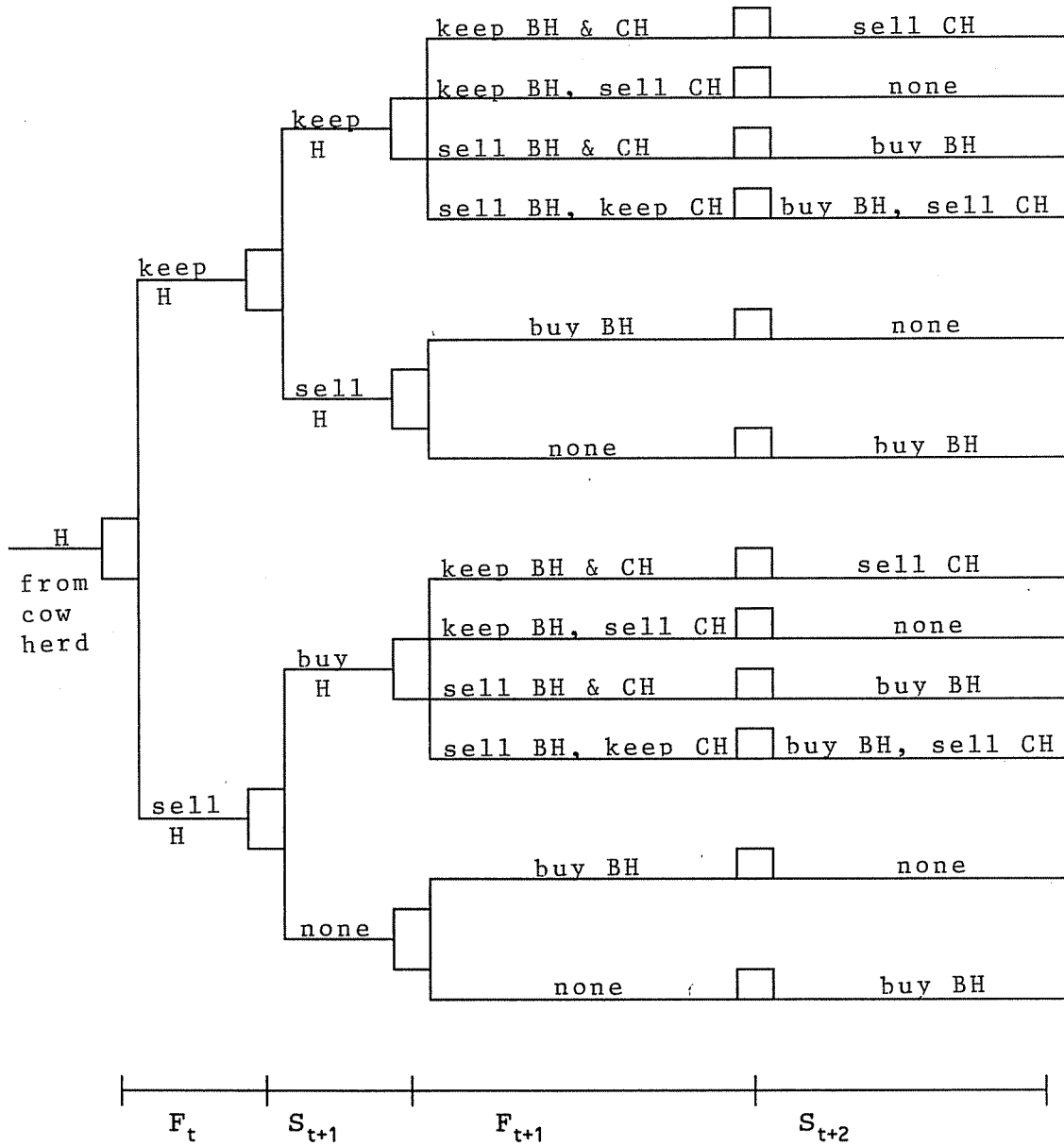


Figure 2. Decision tree. Twelve possible alternatives are available. The notation is simply H-heifer, BH-bred heifer, CH-cull heifer, and none-continue status of last period.

CHAPTER 3

PRICE MODELS

A behavioral model for bred heifer price is theoretically developed and estimated. Time series equations estimated for bred heifer price, feeder heifer price, and feed ration price are combined to form a system of price prediction equations. The error structure of this system is carefully analyzed and presented.

Demand for Bred Heifers

The demand for bred heifers is a derived demand for a capital asset in beef production (Jarvis). It is based upon the primary demand for the final output in the marketing system, retail beef products, which are processed from the offspring of breeding cattle. Primary demand (quantity demanded) is derived from constrained utility maximization. For tastes and preferences held constant, it is assumed to be a function of own price, substitute prices and income (Varian).

To develop a price behavioral model for bred heifers, an inverse derived demand relationship is hypothesized with bred heifer price specified as the dependent variable. This price is hypothesized to be a function of variables specific to the

cow-calf market level, i.e., input costs, range conditions, output prices, and quantities of beef cows and replacement heifers in the national beef cow herd.

Input levels determine the development of beef heifers. A replacement heifer in Montana may meet its nutritional requirements on pasture or on feed in a drylot facility. Pasture conditions are represented in the bred heifer price model by the Montana pasture and range index. The level of this index measures pasture availability and is hypothesized to affect derived demand. The feedlot ration which consists of eleven pounds of alfalfa hay and four pounds of feed barley was formulated to enable heifer calves to gain an average of 1.25 lbs/hd/day. Because feed is a major input cost of bred heifer production, feed ration price is proxied in the model by combining the Montana feed barley and alfalfa hay price series. Other factors constant, changes in the input variables pasture and range index and feed price should have a positive and negative effect on bred heifer demand, respectively.

Feeder heifers are both an output to be produced by a bred heifer and an input into producing a bred heifer. Therefore, feeder heifer price should be entered into the price model. This output price assesses the value of heifers marketed as commercial feeders and also proxies the cost to a producer buying heifers for breeding purposes. The price is a derived market price since it has its final linkage with

the primary demand end of the market. As an output price, an increase (decrease) in feeder heifer price would be expected to increase (decrease) the derived demand for bred heifers.

The volume of bred heifers marketed (representing a supply component in the market) would be expected to have a negative influence on bred heifer price and should be included in the model. In an effort to account for the cyclical effects of the cattle cycle on bred heifer price, the number of beef cows and heifers in the U.S. (that have calved) should be specified in the model.

Data

The bred heifer price model was based upon semi-annual units of observation. Weekly price data sets for bred cows and feeder heifers from 1976-1990 were collected from the USDA Agricultural Market News Service in Billings, Montana. Each weekly observation for both the bred cows and feeder heifers was given in a price range. Since sales volume data were not available a simple average of the price range, rather than a weighted average was used as the point estimate observation.

The data set for bred heifer price (two year old bred cows) contained only sporadic information. To construct a complete set of semi-annual observations from 1976-1990, the weekly prices for bred heifers and bred young cows (three to four year old bred cows) were averaged together in each of

the six month periods. This introduced bred young cow price into the dependent variable, bred heifer price. The error resulting from this averaging process is included in the error term of the regression equation. Since the regression of bred heifer price on bred young cow price implied that the two series were highly correlated, the use of bred young cow data was not expected to have a significant affect on the estimated error terms of the behavioral or time series models.

The USDA grade classification for bred heifers, bred young cows and feeder heifers changed during the sample period. In the data series the grade classification from 1976-1979 was choice, from 1980-1986 it was medium frame #1, and from 1986-1990 the grade was medium-large frame #1. Feeder heifer prices were those for the 500-700 pound category.

The range and pasture condition index, barley prices and alfalfa hay prices were obtained from the Montana Department of Agricultural and Statistical Reporting Service - USDA: Volumes XVII - XXVI. Because data on the number of bred heifers marketed in Montana were not available the number of replacement heifers in the U.S. (500 lbs. and over) were incorporated in the model. The national heifer replacement numbers and beef cow numbers reported for January 1 and July 1 were obtained from the Economic Research Service - USDA: Livestock and Meat Statistics, 1976-1990.

Behavioral Price Model

The bred heifer price model is hypothesized to exhibit dynamics through distributed lags on the independent variables. These lags represent past information producers incorporate to formulate current decisions regarding bred heifers. No a priori restrictions are imposed on the model via adjustment processes (i.e., adaptive expectations or partial adjustment hypothesis), rather the coefficients of the lagged variables (dynamics) are determined by the market data. This is possible when little information exists about the true nature of the dynamic adjustment processes and/or expected future price levels (Rucker, Burt and LaFrance). The model is initially estimated with contemporaneous and lagged variables given in the following:

$$P_t^{bh} = f(P_{t-j}^{bh}, P_{t-j}^{fh}, P_{t-j}^{bly}, P_{t-j}^{hay}, R_{t-j}, Q_{t-j}^{nhr}, Q_{t-j}^{nbh}, D, u_t) \quad (3.1)$$

where:

P_t^{bh}	is the bred heifer price, dollars per head.
P_t^{fh}	is the feeder heifer price, cents per pound.
P_t^{bly}	is the barley price, dollars per bushel.
P_t^{hay}	is the alfalfa price, dollars per ton.
R_t	is the Montana range and pasture index which ranges from 1-100, with the lower end representing poor conditions and upper end normal conditions.
Q_t^{nhr}	is the national replacement (500 lbs and over) inventory, thousands of head.

- Q^{nbh}_t is the national beef cow and heifer (that have calved) inventory, thousands of head.
- D is the dummy (binary) variable designed to capture seasonal effects of the spring price.
- u_t is a random error term, assumed to have mean zero, serial independence, and constant variance.
- j is equal to 0, 1, and 2 for contemporaneous, first order lagged, and second order lagged variables respectively.

All contemporaneous right hand side variables are assumed to be exogenous, i.e., uncorrelated with the error term u_t . The price variables were deflated by the Consumer Price Index (CPI) with base year 1967. The model was assumed to be linear in parameters and was estimated by Ordinary Least Squares (OLS).

The OLS estimates of the final equation (following) provided the best adjusted R^2 , smallest standard error of the estimate and the best predictive performance (based on the root mean square errors) relative to the initial model (3.1). Based on omitting variables not statistically significant at the five percent level, a large number of the independent variables and their lag structures were subsequently truncated. The final estimated equation for the behavioral bred heifer price model, with t-values in parentheses, is given as:

$$P^{bh}_t = 36.51 + 837.37 P^{fh}_t + .61 R_t - .016 Q^{nhr}_{t-1} \quad (3.2)$$

(1.03) (14.42) (2.63) (-2.71)

$$R^2 = .8959, S_y = 15.83, S_y/T = .0859, DW = 2.200$$

where R^2 is the adjusted R-square, S_y is the standard error of the estimate, S_y/T is the ratio of the S_y to the mean of the dependent variable ($T=184.25$) and DW is the Durbin-Watson statistic. Testing of the residuals failed to discern nonspherical properties, i.e., autocorrelation of the disturbance structure.

The hypothesized economic variables in regression equation (3.1), feed barley and alfalfa hay prices, beef cow and heifer numbers and seasonal variation were not statistically significant (at the 5% level) and were therefore omitted. Parameter stability of the model was tested by truncating the beginning and ending sample data. The results indicated no significant change in parameter values. A classification shift of bred cows and feeder heifers occurred in 1979 from choice to medium frame #1 and a classification expansion occurred in 1986 from medium frame #1 to medium-large frame #1. Structural change was tested by comparing the parameter estimates between the restricted and the unrestricted model. Again, the results indicated no significant change in the parameter values.

The independent variables of the price prediction model determine the state variables of the dynamic program. In order to retain models that are efficient predictors and yet do not complicate the structure of the dynamic program, a

simplification of the price forecasting equations is beneficial. For example in equation (3.2) the state variables required are P^{fh}_t , R_t , and Q^{nhr}_{t-1} . But since the first two variables are contemporaneous with P^{bh}_t additional prediction equations, transitions from $t-1$ to t would be required in the decision model.

Price Prediction Models

Simplification of the forecasting equations is accomplished by utilizing simple time series predictors. After testing many specifications a simple system of three time series equations representing bred heifer price, feeder heifer price, and feed ration price was found. The system was not only the simplest but retained most all the information of any corresponding system of structural equations. The respective OLS estimates of the time series equations are:

$$P^{bh}_t = -10.45 + 844.4 P^{fh}_t \quad (3.3)$$

(-.65) (12.46)

$$R^2 = .8557, S_y = 18.63, S_y/T = .1011, DW = 2.006$$

$$P^{fh}_t = .0333 + 1.49 P^{fh}_{t-1} - .7026 P^{fh}_{t-2} + .0333 D \quad (3.4)$$

(1.90) (10.2) (-4.83) (4.12)

$$R^2 = .8709, S_y = .0194, S_y/T = .0840, Dh = .2215$$

$$P^f_t = .0528 + .698 P^f_{t-1} \quad (3.5)$$

(2.03 5.14)

$$R^2 = .4941, S_y = .0276, S_y/T = .1504, Dh = .5957$$

where Dh is the Durbin-h statistic (the other statistics were previously defined). The t-ratios are given in parentheses.

Each of the above equations (3.3), (3.4), and (3.5) were estimated individually by OLS because the structure of the estimated error terms appeared to be well behaved, i.e., there was no discernable serial correlation or cross correlation of error terms. The hypothesis of no serial correlation was not rejected because of the values of the Durbin-Watson and Durbin-h test statistics. The estimated error terms of the three models were regressed on one another with no cross correlation of the error terms found. In addition the system was estimated using the Generalized Least Squares (GLS) procedure in SHAZAM, Version 6.1 (White et al., 1988). The three stage least squares algorithm converged in four iterations with no significant difference between the GLS parameter estimates and the OLS estimates of each equation. The results from the tests of the error structure indicate that the probability distributions of three equations in the system are pairwise independent.

Normality of the residual distribution was not rejected for equations (3.3), (3.4) or (3.5) (at the five percent

significance level) when utilizing the chi-square goodness of fit test. Visual inspection of the estimated probability density plot for each model reinforced this conclusion. Residual plots for (3.3), (3.4), and (3.5) are presented in Figures 3, 4, and 5. For each plot the residual values were divided into fifteen groups. The groups uniformly divided the residual axis.

The independent and normality properties of the error terms reduce the complexity and computational effort of the decision model. Since the probability distributions are pairwise independent, each distribution can be calculated separately rather than calculating a joint probability distribution of multivariates. Because the error terms are normally distributed, etc., OLS is the appropriate estimation procedure.

The system of time series functions form a set of first and second order autoregressive functions (equations (3.5) and (3.4), respectively) and a contemporaneous price transmission relationship (equation (3.3)). Diagnostic checking on equations (3.4) and (3.5) provided optimal lag length. Estimated feeder heifer price from equation (3.4) enters recursively into the bred heifer price of equation (3.3). Overall, the system is designed to perform period by period forecasts needed in the stochastic decision process of the dynamic programming algorithm.

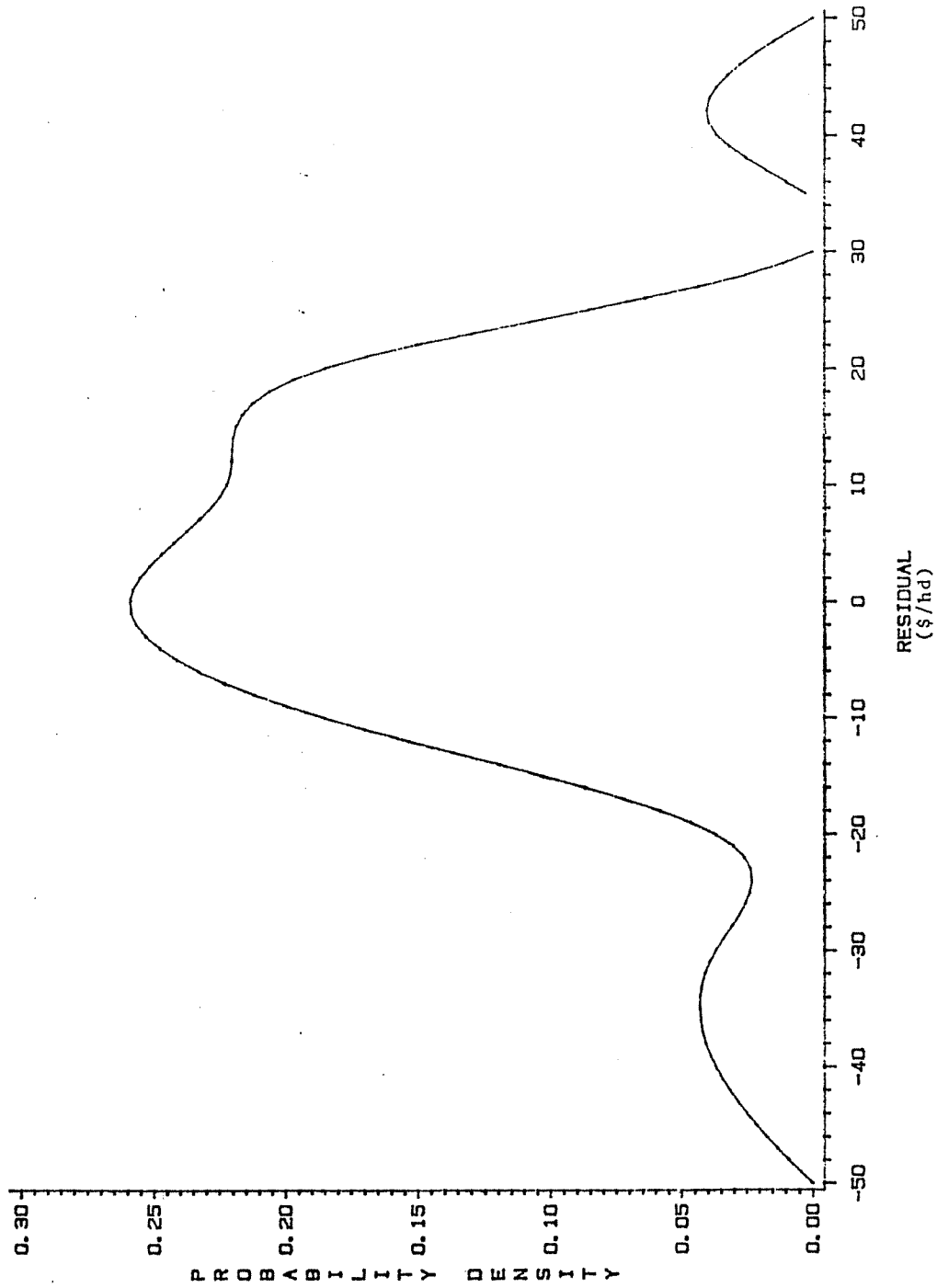


Figure 3. Residual plot for time series equation (3.3), bred heifer price.

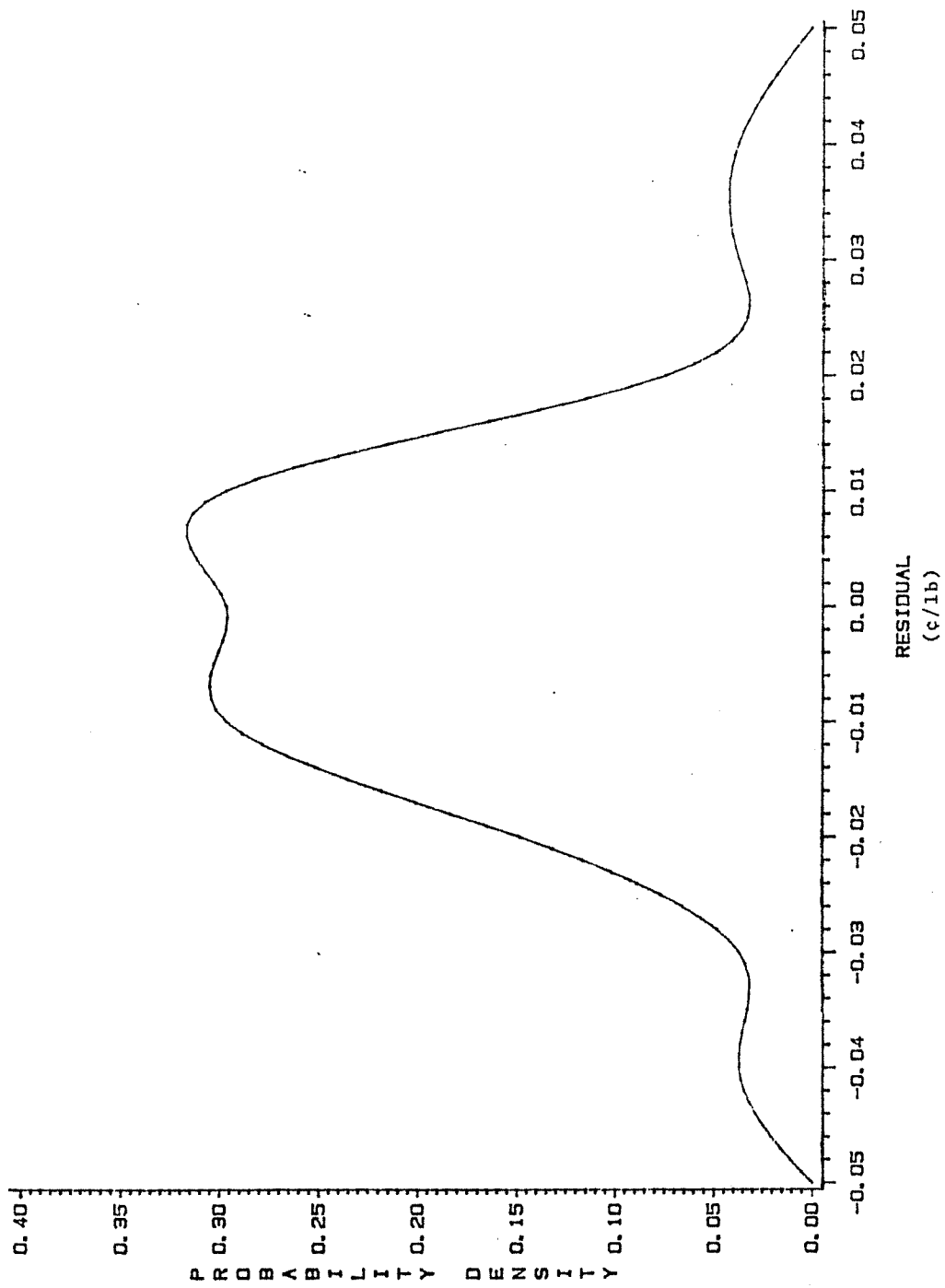


Figure 4. Residual plot for time series equation (3.4), feeder heifer price.

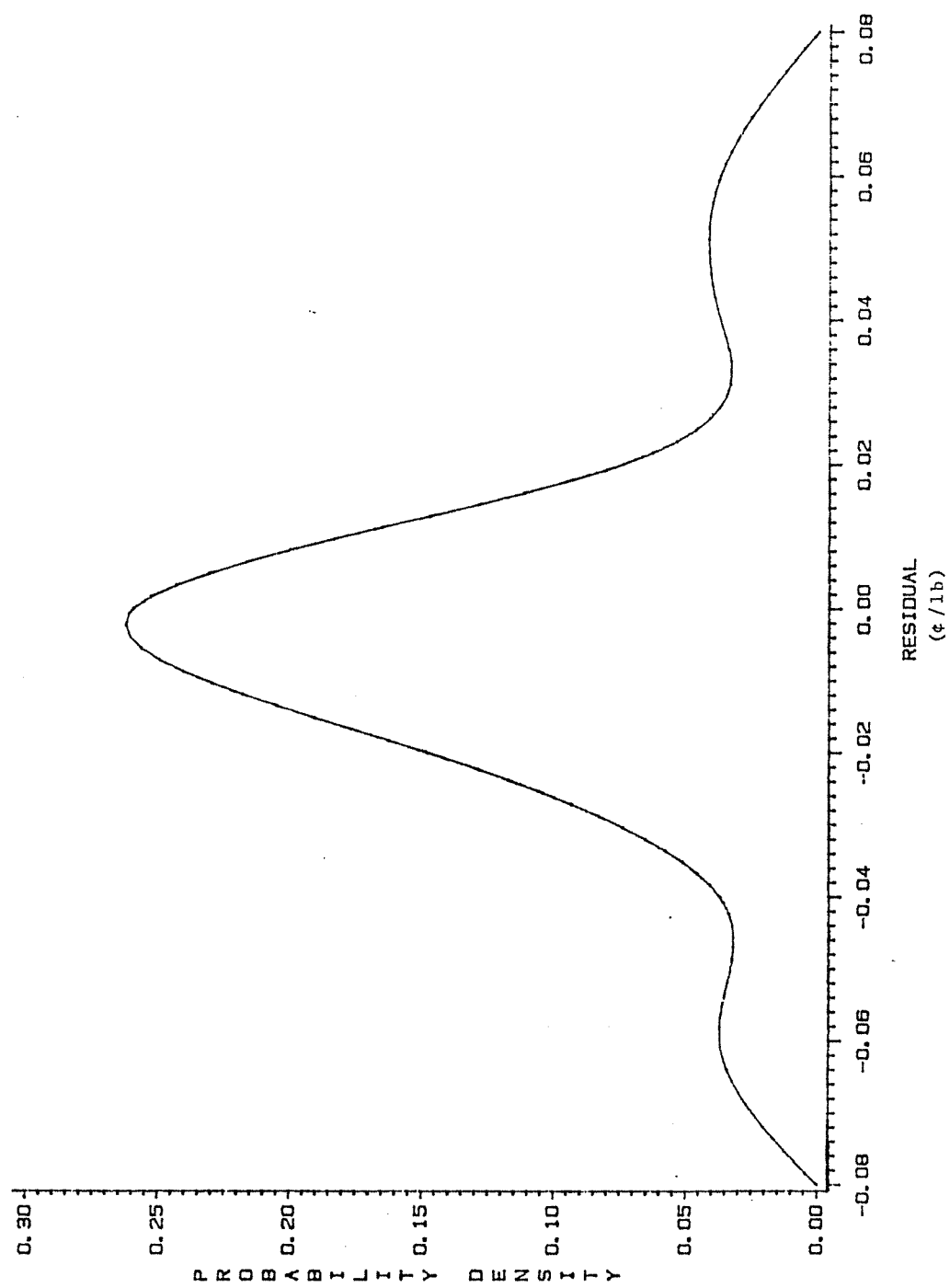


Figure 5. Residual plot for time series equation (3.5), feed ration price.

Discussion

A comparison of equation (3.2) to (3.3) has interesting consequences. Information is lost by deleting from equation (3.2) two independent variables, R_t and Q^{nhr}_{t-1} . But the information loss is small relative to the considerable reduction in the computational burden of the dynamic program, i.e., including R_t and Q^{nhr}_{t-1} as state variables in the dynamic program would increase the total number of states by a multiplicative factor of 139 (if each variable was discretized into 13 states). Suppose the total number of states of the dynamic program was 6877, then the addition of two state variables of 13 states each would increase the number of total states to 1,161,213.

The dependent variable of the feed price equation (3.5) is a weighted sum of alfalfa and barley price per pound and yields a feed ration price/day. The formulated wintering ration for retained heifer calves was eleven pounds of alfalfa and four pounds of barley. The standard units for alfalfa price is dollars per ton (2000 lbs per ton) and for barley dollars per bushel (48 lbs per bushel). The weighted sum then is

$$P^f_t = .0055 * P^{hay}_t + .0833 * P^{bly}_t. \quad (3.6)$$

Because the weighted sum combines alfalfa and barley price into a single feed ration price the number of state variables

is decreased by one. The combination reduces the computational burden when solving the decision problem, but still retains information of relative price changes for alfalfa and feed barley.

Summary

First, a behavioral model for bred heifer price was specified and parameter estimates obtained. Then to investigate the trade off between complicating the dynamic program and information loss, a system of two time series equations (3.3) and (3.4) modeling bred heifer price was specified and parameter estimates obtained. The results gave a relatively simple structure to the dynamic program yet provided adequate information.

Three state variables P_t^{fh} , P_{t-1}^{fh} , and P_t^f were extracted from the system of equations. Because bred heifer price in period t is expressed directly as a function of feeder heifer price in t (equation (3.3)) it was not necessary to include bred heifer price as a state variable in the dynamic program. Though it was not used as a state variable bred heifer price will be determined at each stage in the dynamic program by calculating equation (3.3). Equation (3.4), a second order autoregressive structure, captured the essential information for feeder heifer price. Equation (3.5), a first order autoregressive structure, provided the feed ration price per day for a beef heifer.

The next chapter uses the system discussed above to form the bases of a decision model via dynamic programming. Equations (3.4) and (3.5) provide the state variables, state transitions, and transition probabilities needed to construct the decision model.

CHAPTER 4

EMPIRICAL PROBLEM AND RESULTS

The empirical problem is formulated utilizing a finite stage stochastic dynamic programming model to determine the optimal policy for acquiring bred heifers; expected cost minimization is the decision criterion. The numerical results of the decision model with the corresponding decision policies are presented and discussed. Expected present cost of the optimal policy determined by the decision model is compared with that of the traditional policy of raising bred heifers, which is implemented by the majority of commercial beef cow producers. Each price quotation in this chapter is a real price with base year 1967.

Model Formulation

In this section the components of the dynamic programming problem are defined in terms of the empirical problem.

Stages. The four stages F_t , S_{t+1} , F_{t+1} , and S_{t+2} remain as previously defined. Decision nodes for F_t occur November 1st, S_{t+1} on May 1st, F_{t+1} on November 1st and S_{t+2} falling on January 1st. Since the dynamic program is solved using a backward induction process the time periods immediately preceding and

following stage N , $N > 2$, are $N+1$ and $N-1$ respectively with $N=1$ as the final stage.

States. The condition of the decision process in stage N is described by the values of the state variables. The variables found necessary to describe the important dimensions of the bred heifer production problem were feeder heifer price lagged one time period ($P_i^{fh}(N)$), contemporaneous feeder heifer price ($P_j^{fh}(N)$), and contemporaneous feed ration price ($P_k^f(N)$). Observed feeder heifer prices ranged from \$.17 to \$.39 per pound while observed feed ration prices ranged from \$.11 to \$.25 per pound. Feeder heifer price, both current and lagged, was discretized into the grid .13, .15, ..., .43 utilizing \$.02 intervals. Feed ration price was also discretized into \$.02 intervals to form the grid .08, .10, ..., .30. The grid for each state variable included additional values beyond the observed end points to compensate for truncation problems which occur because of the finite discretization process.

State Transitions. The dynamic programming problem in a specific state at each stage N is transformed to its future state in stage $N-1$ by the state transition functions. Feeder heifer price in stage $N-1$ is conditional upon $P_i^{fh}(N)$ and $P_j^{fh}(N)$ and is predicted by equation (3.4). Feed ration price is conditional only upon $P_k^f(N)$ and is predicted by equation (3.5). While both of the above transitions are stochastic in nature the transformation of current feeder heifer price in

stage N to lagged feeder heifer price in stage N-1 is deterministic and defined by the transition equation

$$P^{fh}_i(N-1) = P^{fh}_j(N). \quad (4.1)$$

Decision Alternatives. For each stage the decision alternatives available to the commercial operator are presented along with further assumptions of the model. On the first of November (F_t) weanling heifer calves will weigh 500 lbs. The producer has two options:

- 1) sell heifer calves.
- 2) retain heifer calves and place them on a feed ration (eleven pounds of alfalfa and four pounds of barley) designed for 1.25 lbs of gain per hd per day.

The following May 1st of S_{t+1} heifers will weigh 726.25 lbs. Yearling heifers that are retained in this stage will be a fed a ration of twenty pounds of alfalfa per head (designed for one lb of gain per day) and entered into the breeding program. The possible alternatives are:

- 1) keep heifers which were retained in F_t .
- 2) sell heifers which were retained in F_t .
- 3) buy heifers if none were retained in F_t .
- 4) take no action if heifers were not retained in F_t .

By F_{t+1} (on November 1st) heifers will weigh 910.25 lbs. All heifers retained from S_{t+1} will be pregnancy tested. Any bred heifers retained or purchased will be fed a ration consisting of twenty pounds of alfalfa per head per day. Cull heifers are heifers exposed at breeding but were open at

pregnancy testing or heifers that did not grow and mature as anticipated. The available decision alternatives are:

- 1) keep bred heifers and cull heifers.
- 2) keep bred heifers and sell cull heifers.
- 3) sell bred heifers and keep cull heifers.
- 4) sell bred heifers and cull heifers.
- 5) buy bred heifers if none were retained in S_{t+1} .
- 6) take no action if heifers were not retained in S_{t+1} .

The state of the dynamic program is explicitly defined by the three stochastic variables and implicitly defined by a set of deterministic variables. The deterministic states are defined by the deterministic variables of whether or not a heifer/bred heifer was retained, purchased, or sold in the previous stage.

Terminal Values. On January first in S_{t+2} the model is forced either to buy bred heifers (if none were retained from the previous stage), sell cull heifers not sold in F_{t+1} , or take no action if only bred heifers were retained in F_{t+1} . The terminal values range from \$143.55 to \$329.32 (\$/hd) for bred heifer price and from \$.17 to \$.30 (\$/lb) for 900 pound feeder heifers.

Discount Factor. The annual real opportunity cost of capital (before tax), r , was assumed to be 6 percent. The probability of sickness and death loss was assumed to be 1 percent during each stage. Hence, the combined value of the semi-annual discount factor is $1/((1+.06/2)*.99)$ or .9807.

Expected Immediate Costs. The production costs included in the model were feed, breeding, pregnancy testing, and transaction costs. Feed price is stochastic and is dependent upon the state of the system. The cost of breeding a heifer is subjective and specific to each particular ranching operation. Cost for this study was estimated under the assumption the heifers were artificially inseminated (A.I.) at five dollars per head (1967 dollars). This included costs for synchronization, semen, and A.I. technician. Pregnancy testing was valued at forty-two cents per head (1967 dollars). While it is difficult to place any value on the marketing cost of selling or purchasing heifers, it was assumed that a transaction cost of two percent was incurred for any heifers sold. The cost of purchasing heifers was a positive value and included into the expected costs. Revenue from any sales of heifers was a negative value and added to the expected costs.

Transition Probabilities. Transition probabilities transform the system from state i in stage n to state j in stage $n-1$. The probability distributions for feeder heifer prices and feed ration prices can be derived from the probability density functions of (3.4) and (3.5). General form equations for (3.4) and (3.5) are:

$$P^{fh}_j(N-1) = f_1(P^{fh}_i(N), P^{fh}_j(N), u_1) \quad (4.2)$$

and

$$P^f_k(N-1) = f_2(P^f_k(N), u_2) \quad (4.3)$$

respectively. Given results of Chapter 3, u_1 and u_2 are independent and distributed normally with mean zero and variance equal to the square of the standard error of the estimate of the respective regression equations. Hence, conditional probability functions for both feeder heifer price and feed ration price can be constructed to form transition probabilities;

$$\Pr(P^{fh}_j(N-1) | P^{fh}_i(N), P^{fh}_j(N)) \quad (4.4)$$

which is

$$N(.03 + 1.5 * P^{fh}_j(N) - .7 * P^{fh}_i(N), (.0194)^2) \quad (4.5)$$

and

$$\Pr(P^f_k(N-1) | P^f_k(N)) \quad (4.6)$$

which is

$$N(.05 + .7 * P^f_k(N), (.0276)^2). \quad (4.7)$$

Recursion Relation. The discrete model of the mathematical recursion relationship is presented below:

$$V_N(i, j, k) = \min_{(K)} \{C^K(P_k^f(N)) \quad (4.8)$$

$$+ .9807 \sum_{i=1}^{m_i} \sum_{j=1}^{m_j} \sum_{k=1}^{m_k} \Pr(j) \Pr(k) V_{N-1}(i, j, k) / \sum_{i=1}^{m_i} \sum_{j=1}^{m_j} \sum_{k=1}^{m_k} \Pr(j) \Pr(k)\}.$$

Where m_i , m_j , and m_k are the number of states at each stage for lagged feeder heifer price, current feeder heifer price, and feed ration price respectively. Also :

$$V_N(i, j, k) = V(P_i^{fh}(N), P_j^{fh}(N), P_k^f(N)) \quad (4.9)$$

$$\Pr(j) = \Pr(P_j^{fh}(N-1) | P_i^{fh}(N), P_j^{fh}(N)) \quad (4.10)$$

$$\Pr(k) = \Pr(P_k^f(N-1) | P_k^f(N)). \quad (4.11)$$

This model compares decision alternatives for each state in a particular stage. The alternative with the smallest expected present value is considered the optimal choice for that state and becomes a member of the sequence of optimal choices which constitutes the optimal policy of the dynamic program (via its backward recursion solution technique).

Optimal Decision Rules and Expected Costs

One characteristic of a dynamic program is the myriad of data computed; each of the $m_i \cdot m_j \cdot m_k$ states at each stage is described by the values of the three state variables, the value of the recursion relation, and whether or not a heifer was retained, sold, or purchased in the previous stage. Perhaps the clearest method to present the results is to plot

a set of two-dimensional graphs for a select number of subsets of the data. The first graph of each subset of the data output is a contour plot of the optimal decision policies for a fixed feed ration price per head. For the representative results three feed ration prices were selected; low (\$.11), mean (\$.18), and high (\$.25). Domain variables, current and lagged feeder heifer price, were chosen because the optimal decision policies were more sensitive to changes in feeder heifer price than feed ration price. The contour curves map out the values of the domain at which policy changes occur. For each contour plot there are three contour lines separating four individual regions each of which contains a distinct optimal policy. Only three lines, four regions are necessary since only four of the twelve possible strategies were found to be elements of the optimal set of decision rule. The optimal decision strategies that define the four regions are described in Table 1.

In the second graph of each result subset the expected present cost of the optimal policy is plotted versus current feeder heifer price for three levels of lagged feeder heifer price; low (\$.17), mean (\$.23), and high (\$.39). Each graph is still conditional to the respective fixed feed ration price.

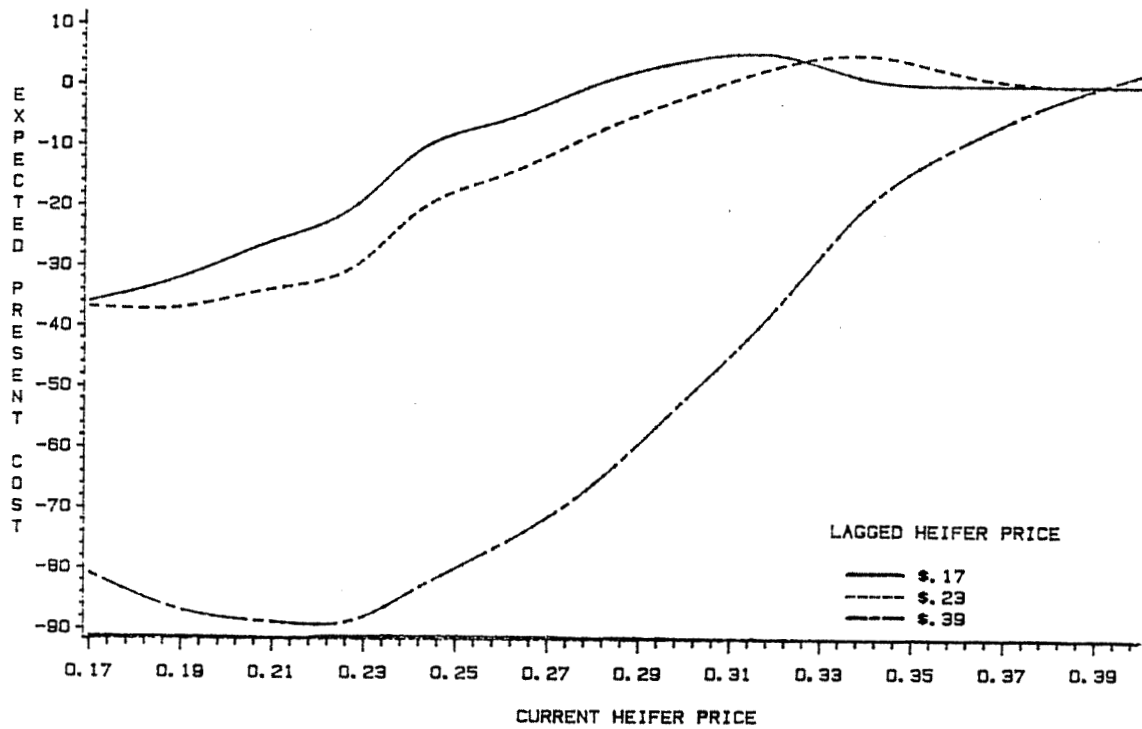
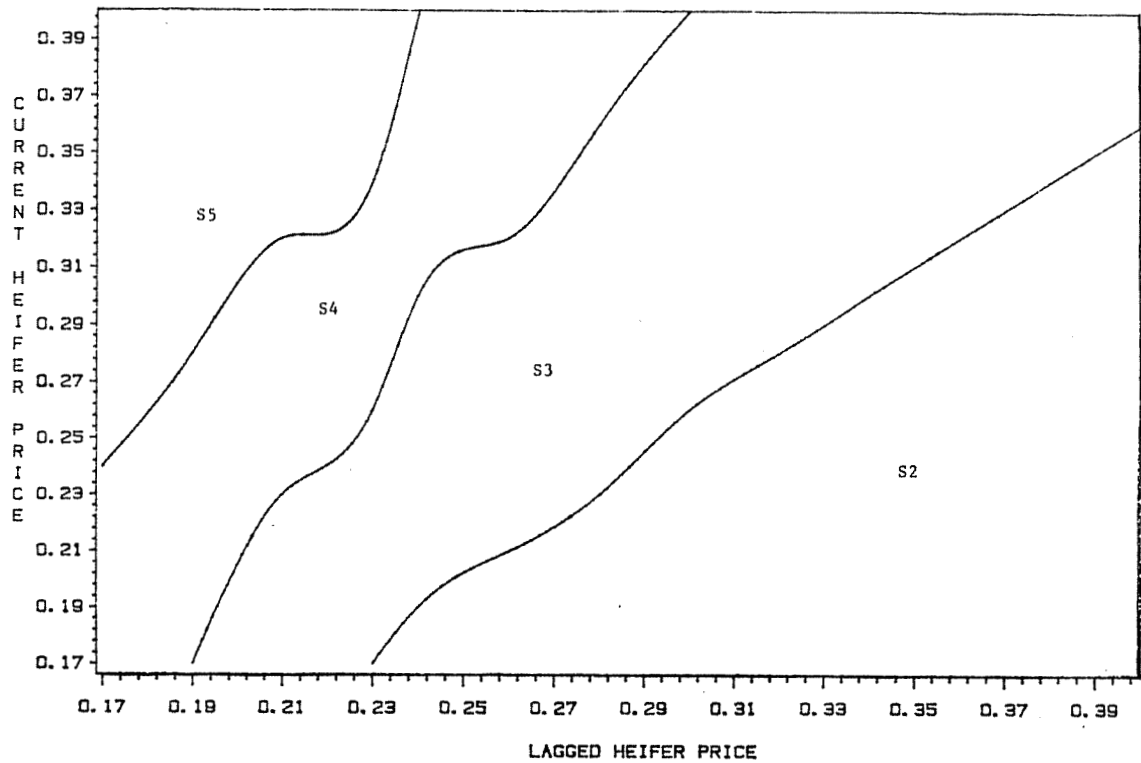


Figure 6. Model results 1, optimal decision policies and expected costs when feed ration price is \$.11/day.

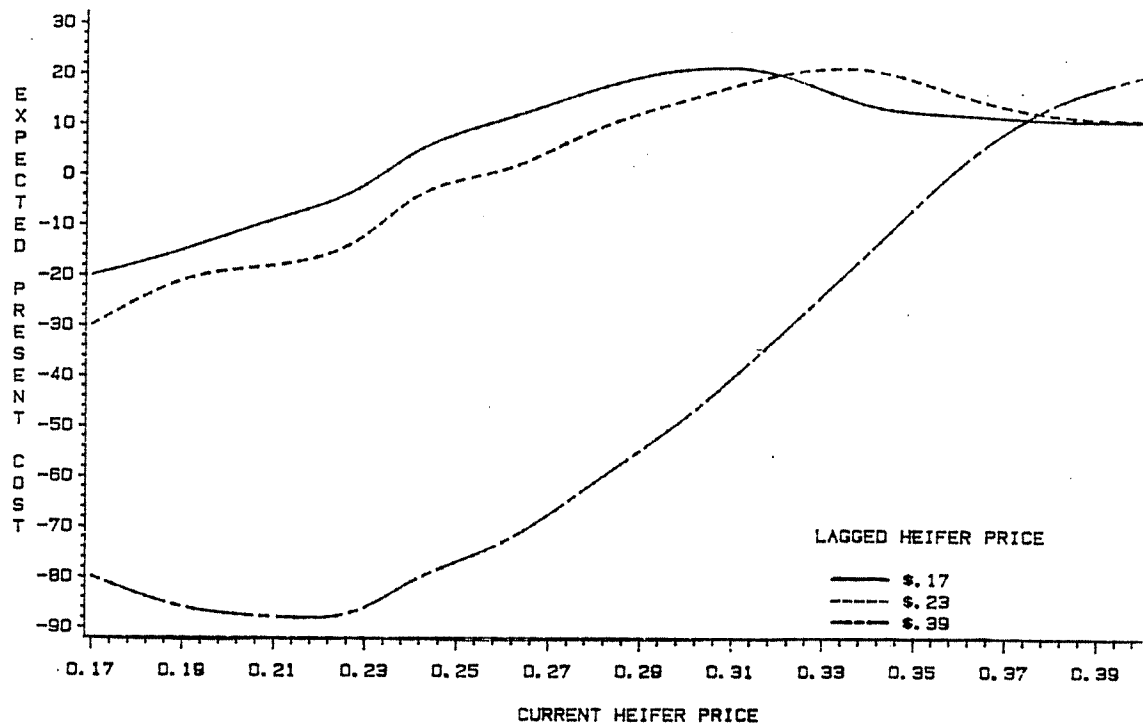
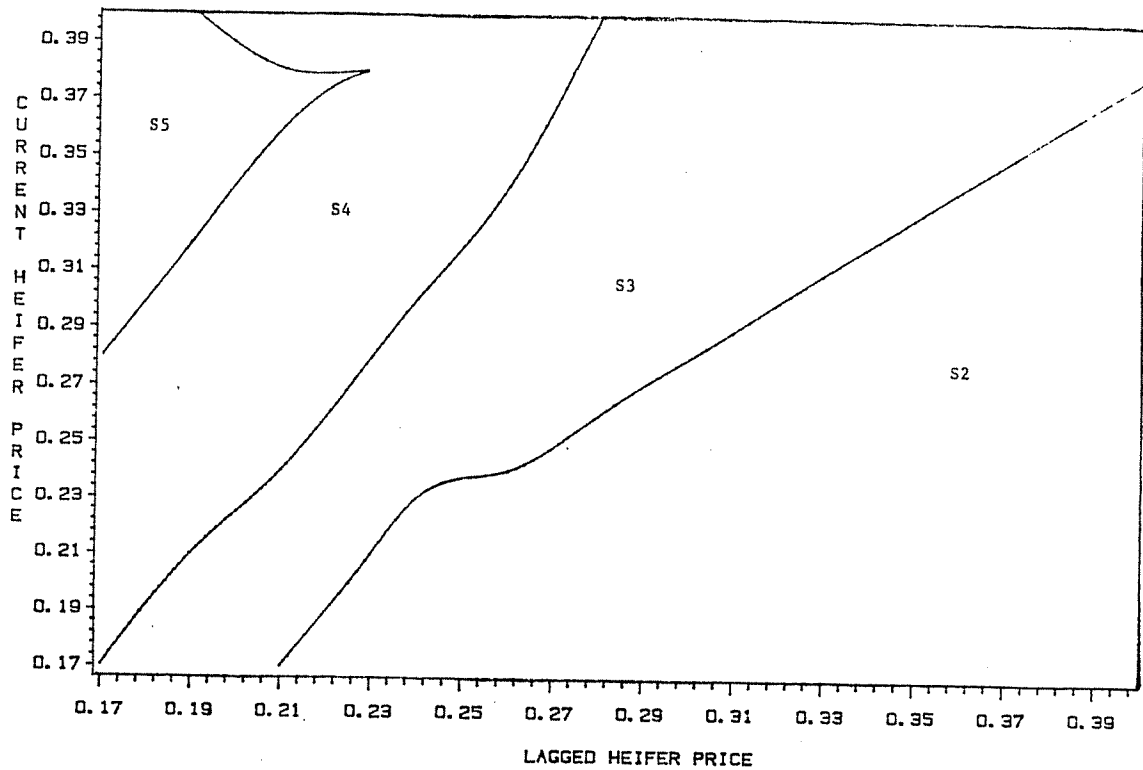


Figure 7. Model results 2, optimal decision policies and expected costs when feed ration price is \$.18/day.

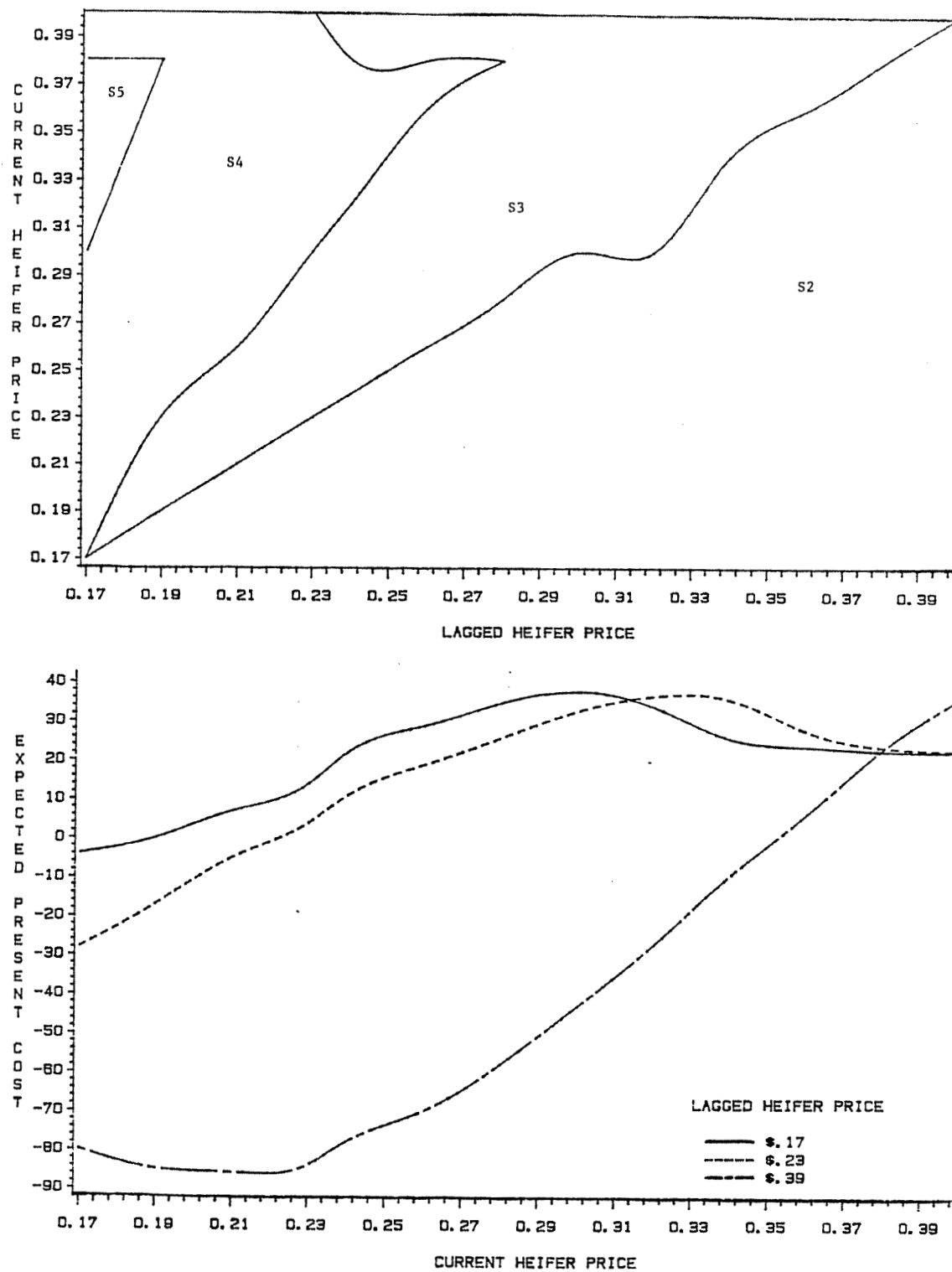


Figure 8. Model results 3, optimal decision policies and expected costs when feed ration price is \$.25/day.

Table 1. Decision strategies optimal in the empirical model.

Strategies	F_t	S_{t+1}	F_{t+1}	S_{t+2}
S2	sell H	none	none	buy BH
S3	keep H	sell H	none	buy BH
S4	keep H	sell H	buy Bh	none
S5	keep H	keep H	keep BH & CH	sell CH

Implication of Results

Change in the optimal policy and expected present cost as feed ration price progresses from low (\$.11), to mean (\$.18), and to high (\$.25) price levels will be addressed first. A discussion of the results at the specific feed levels will follow.

Compare the contour plots of the optimal decision policy (top plots of Figures 6, 7 and 8) at the three feed ration price levels. There is a natural progression in the structure of the decision policies to phase out alternatives S5 and S4 as feed ration price rises, while S3 and S2 are phased in. The change in optimal policy is intuitively reasonable from a ranchers point of view because as feed ration price rises alternative methods of producing a bred heifer (versus raising a bred heifer) become more economical.

The curves representing the value of the expected discounted present cost (bottom of Figures 6, 7, and 8) of the dynamic program were not structurally altered as feed

ration price was increased. Only an upward shift in the direction of increasing production cost was displayed. The decision model exhibits minimum expected present cost when contemporaneous feeder heifer price is low. The optimal policies that occur in this portion of feeder heifer price domain are S2, S3, and S4. When current feeder heifer price is relatively high expected present cost curves are at a maximum and the optimal decision policy is S5.

Figure 6 is conditioned by the feed ration price fixed at \$.11. In the region marked S2, below the right hand contour curve, lagged feeder heifer price (P_i^{fh}) is generally four cents higher than current feeder heifer price (P_j^{fh}). In this sector of the domain there is a high probability of a downward trend of feeder heifer price (hence, bred heifer price via equations (3.3) and (3.4)). It is feasible for a rancher to sell weanling heifers in F_t and purchase bred heifers in S_{t+2} because production costs are higher than the added value weanling heifer attains as it develops into a bred heifer, i.e., the bred heifer can be (with high probability) purchased cheaper than it can be produced on the ranch. Between the middle and right hand contour is the area where policy S3 is implemented. In this region P_i^{fh} and P_j^{fh} are relatively equal. Because of the seasonal increase of feeder heifer price and the weight gained by heifer calves over winter the optimal decision policy for F_t is to retain heifer calves, feed them out, and sell as yearling heifers in

S_{t+1} . Since the probability bred heifer price will increase significantly is low the optimal decision is then to purchase a bred heifer in S_{t+2} . Strategy S4 is optimal in the region between the left hand and middle contour curves, P_j^{fh} is somewhat larger than P_i^{fh} . If current feeder heifer price is rising it is optimal to retain the heifer calves over the winter and sell in the spring. But because the probability is higher that a significant increase in feeder heifer price could occur in F_{t+1} and expand into S_{t+2} (consequently higher bred heifer price) it is optimal to purchase a bred heifer in F_{t+1} . Where S5 is the decision policy P_j^{fh} is significantly greater than P_i^{fh} . There is a strong probability that both feeder and bred heifer price will increase at such a rate that production cost to raise a bred heifer is less than the added value a weanling heifer calf attains as it matures into a bred heifer. The structure of the policy regions in Figures 7 and 8 are essentially equivalent to Figure 6 except the contour curves are shifted to the left because of higher feed costs. The explanation of the optimal policies of a each sector is unchanged.

The bottom graph of Figures 6, 7, and 8 display expected present cost of the optimal decision policy plotted against current feeder heifer price with feed ration price fixed at \$.18 and for lagged feeder heifer price set at \$.17, \$.23, and \$.39. For each value of current feeder heifer price (given one of the three fixed values of lagged feeder heifer

price and fixed feed ration price) the expected present cost computed by the decision model can be read from the plot. The optimal decision policy can be determined by looking directly above to the contour plot. For example in Figure 6, when P_i^{fh} is \$.39 and P_j^{fh} is \$.22 the approximate expected present cost of obtaining a bred heifer is -\$88 utilizing decision policy S2. If P_i^{fh} is \$.17 and P_j^{fh} is \$.22 the expected present cost is close to -\$24 under policy S4.

The interpretation of these plots is that given a set of current and lagged feeder heifer prices (along with current feed ration price) the expected present cost of obtaining a bred heifer can be significantly reduced by implementing the optimal decision policy of the model instead of the traditional method of raising bred heifers, S5. Specifically, when feeder heifer price is decreasing sharply S2 offers a much smaller production cost of obtaining a bred heifer. If feeder heifer price is neither increasing nor decreasing significantly S3 and S4 provided lower cost options. In the next section a direct comparison of the expected present cost of the optimal policy from the decision model with that of S5, raising bred heifers, is made.

Expected Present Cost Comparisons

In the preceding section the output of the dynamic programming model, optimal policy and value of the recursion relation, was presented and discussed. This satisfied the

primary purpose of the study. But because the optimal policy differed from the strategy, S5, most often observed among Montana producers (for the majority of states) a comparison of expected present costs is presented.

The expected present cost output of the dynamic program is graphed against the present discounted value of strategy S5. The three graphs presented in Figures 9, 10, and 11 have lagged feeder heifer price set at \$.17, \$.23, and \$.39 respectively. For each graph feed ration price is fixed at its mean value of \$.18. The plots all suggest the traditional method, S5, is significantly more costly on the average. Alternatively, they indicate if a rancher follows the optimal decision policies production costs can be reduced significantly in a commercial cow calf operation.

Most ranchers favor raising bred heifers from within their breeding herd because there is a lack of information regarding the genetics, health, potential offspring, etc. of purchased bred heifers. The additional information on a raised heifer has a value; a rancher would be willing to incur a higher cost to raise a bred heifer than he would to buy an "unknown" bred heifer. However, notice in Figure 11 when lagged feeder heifer price is very high (\$.39) and current feeder heifer price is at the mean (\$.23) there are expected costs of -\$80 when the optimal decision rule, S2, is implemented and \$110 for S5. Thus, a rancher would lose the difference of \$190 by raising bred heifer. This \$190 then is

the amount the rancher is paying for the additional information he knows about his own heifer relative to a bred heifer offered in the market. The difference between the traditional strategy (S5) and the optimal policy indicated by the dynamic program (DP) yields the information cost for all initial states. These comparisons are presented in Figures 9, 10, and 11.

While the \$190 per bred heifer discussed above is extreme, the cost difference between the optimal rule of the dynamic program and raising bred heifers declines as feeder heifer prices began to increase. But under the specifications of this problem, if a rancher would follow the optimal decision rule of the DP the expected cost of acquiring a bred heifer will always be less than that of raising a bred heifer.

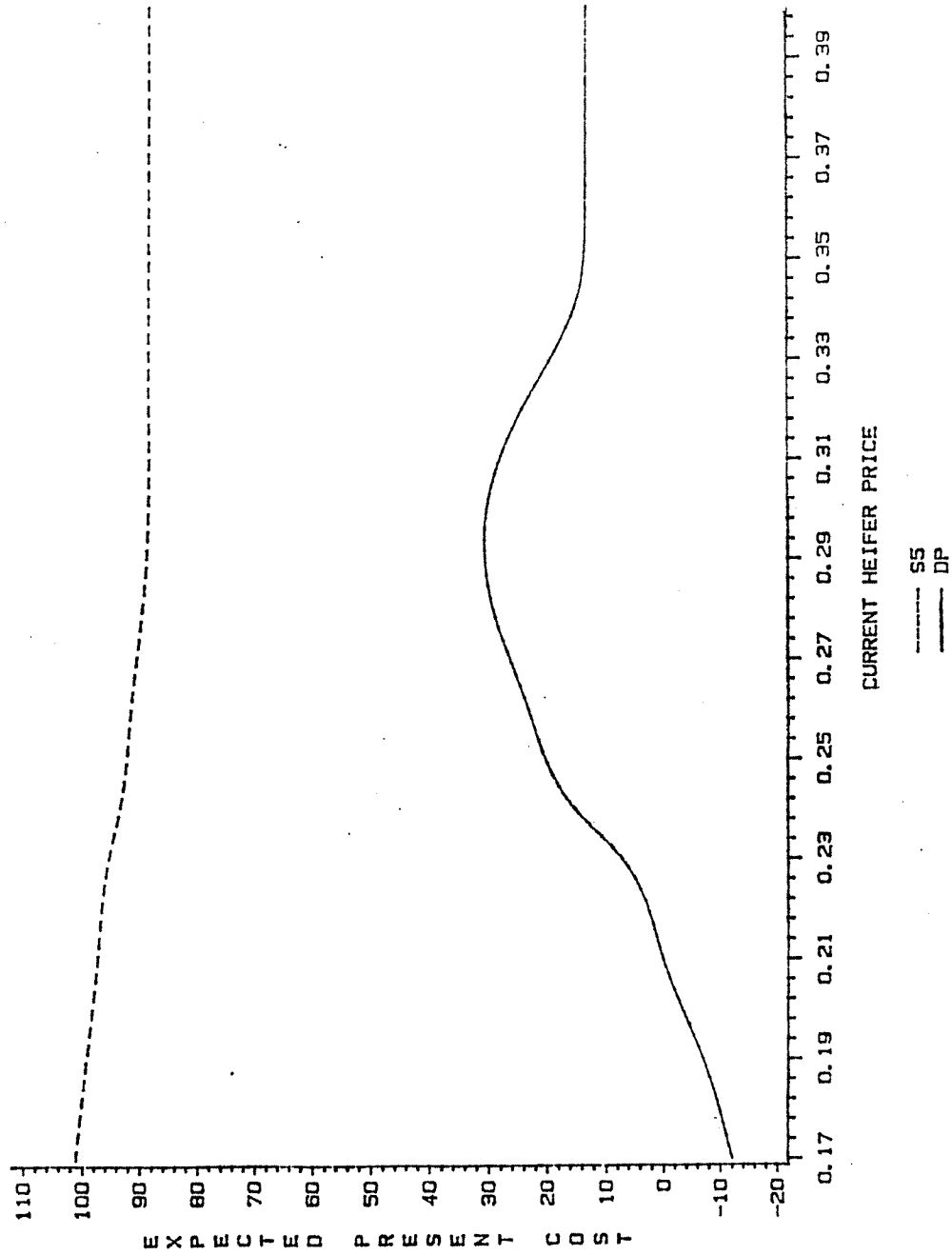


Figure 9. Expected cost comparison 1, between DP and S5 when lagged feeder heifer price is \$.17.

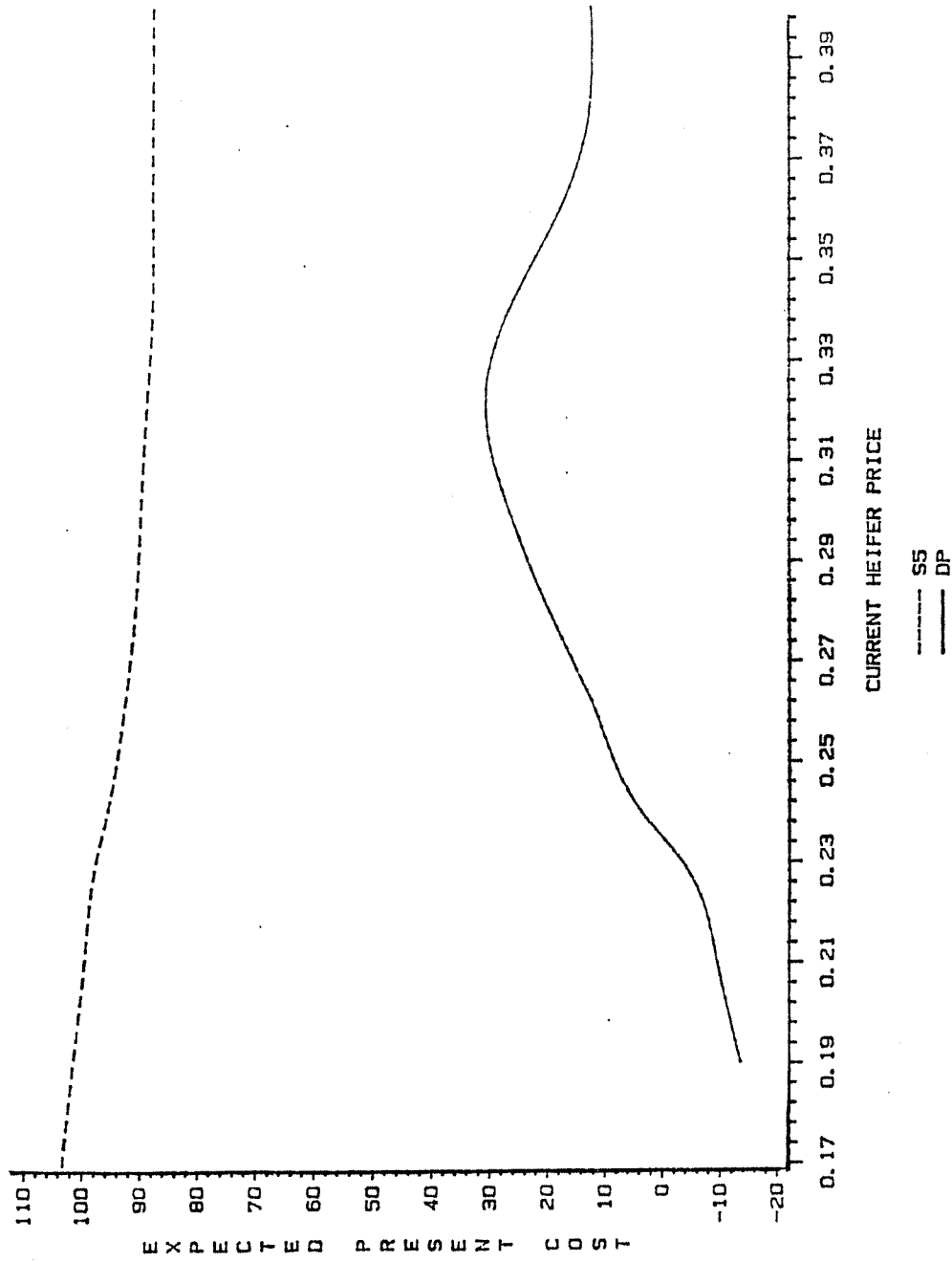


Figure 10. Expected cost comparison 2, between DP and S5 when lagged feeder heifer price is \$.23.

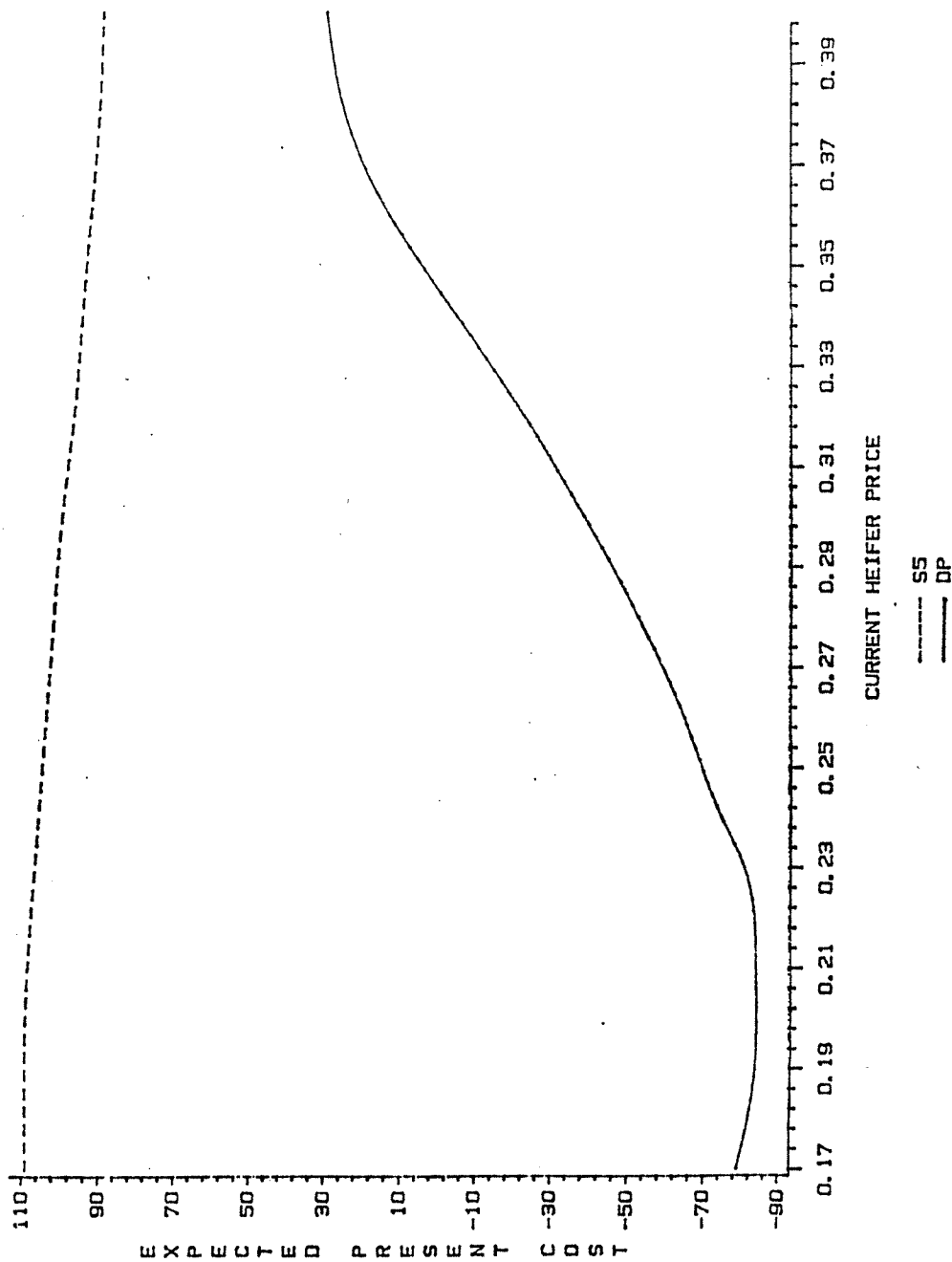


Figure 11. Expected cost comparison 3, between DP and S5 when lagged feeder heifer price is \$.39.

Strategy Comparisons Utilizing Observed Values

To complete this study, examples utilizing observed values from three time periods are constructed to compare strategies S2, S3, S4 and S5. The observed prices were obtained from Billings feeder and bred heifer price series. Input prices for the feed ration price were taken from Montana alfalfa hay and feed barley price series. During the first time period (fall of 1980 to spring of 1982) feeder heifer price declined, the next period (fall of 1983 to spring of 1985) the price of feeder heifers remained approximately constant and in the last time period (fall of 1986 to spring of 1988) feeder heifer price increased. The cost incurred with each strategy is computed under the constraints forced onto the decision model. The observed values are given in Table 2.

The computed costs presented in Table 3 have a \$ 119.31, 63.34, and 30.51 spread between the maximum and minimum values of the four strategies in the first, second, and third time periods, respectively. In the first two time periods S5 has the highest expected present cost, S3 is the least cost alternative in the second and third time periods, and S2 has the lowest cost of time period one.

These example results are consistent with and thus lend credibility to the results obtained from the decision model. The optimal policy is S2 when feeder heifer price declines

significantly (or is predicted to decline) and S3 is the optimal policy when feeder heifer price is expected to be constant or increase. From examination of the spread values it is apparent that significant savings on production cost can be realized when feeder heifer price decreases, but this savings is reduced as feeder heifer price increases.

Table 2. Observed Montana prices and feed ration cost.

Date	Feeder Heifer (\$/lb)	Bred Heifer (\$/hd)	Alfalfa (\$/ton)	Barley (\$/bu)	Feed Ration (\$/hd/day)
1980-2	.28	217.48	26.16	1.00	.23
1981-1*	.24	195.49	20.86	1.10	.21
-2**	.20	163.96	18.36	.85	.17
1982-1	.21	124.12	16.30	.81	.16
1983-2	.19	154.49	19.82	.77	.17
1984-1	.19	172.23	24.56	.83	.20
-2	.18	137.65	24.29	.81	.20
1985-1	.19	147.98	25.47	.68	.20
1986-2	.18	153.94	16.01	.48	.13
1987-1	.20	162.92	13.96	.49	.12
-2	.22	188.29	13.65	.57	.12
1988-1	.23	174.54	13.11	.48	.11

* First half of the year January 1 - June 31.

** Second half of the year July 1 - December 31.

Table 3. Cost comparisons among strategies.

Strategy	1980-2 to 1982-1	1983-2 to 1985-1	1986-2 to 1988-1
S5	77.09	87.48	44.86
S4	21.70	33.43	54.02
S3	-29.70	24.14	26.49
S2	-42.22	27.97	57.00
Optimal Strategy	S2	S3	S3
Spread	119.31	63.34	30.51

CHAPTER 5

SUMMARY AND CONCLUSION

The purpose of this study was to determine an optimal decision policy for the acquisition of bred beef heifers by Montana producers. A finite stage stochastic dynamic programming model was constructed to meet the explicit assumptions presented in the first chapter.

To describe the stochastic nature of the decision model a system of time series equations was developed. First, a behavioral model for bred heifer price was estimated in order to determine the economic variables that significantly affect bred heifer price. From this equation and the Montana feeder heifer price series (medium frame #1, 500-700 lbs) a pair of inter-related price prediction, equations (3.3) and (3.4), evolved for bred heifer price and feeder heifer price. A feed ration price prediction equation (3.5) was constructed from the Montana alfalfa and barley price series. The ration was designed to meet the daily nutritional requirements of maturing beef heifers. The stochastic state variables for the decision model were current feeder heifer price, lagged feeder heifer price, and current feed ration price. Because current bred heifer price was estimated as a linear function of contemporaneous feeder heifer price it was not necessary

to use it as a state variable. The combination of equations (3.4) and (3.5) provided a system of price forecasting equations from which the transition probabilities for the decision model were generated.

The inference from results of the decision model is that Montana ranchers can use available information to decrease the expected present cost of acquiring bred beef heifers. When feeder heifer price (hence, bred heifer price) is sharply decreasing, significant returns can occur if heifer calves are sold immediately after weaning and bred heifers purchased prior to calving. In times of relatively unchanged feeder heifer price the optimal decision policy is to retain heifer calves over the winter, sell them in the spring, and then purchase a bred heifer at a later date. The traditional method of raising bred heifers is optimal only when there is a very strong increase in feeder heifer price and hence, bred heifer price.

The model is limited by the number of stochastic state variables used to describe the decision process. The choice of the appropriate number is a compromise between computational expense and accuracy of larger problems versus some information loss due to reducing the number of state variables. Also, because the production techniques of raising bred heifers for each commercial cattle operations are very subjective only the basic production costs were assumed and implemented. The cost difference exhibited between the

optimal strategies of the dynamic program and the traditional method can be viewed as the cost of information. But this cost can vary significantly from operation to operation.

Additional work in the interpretation of information costs that exist when purchasing bred heifers may well be beneficial. Further research for determining the optimal number of replacement cows required for a particular breeding herd size is needed. Investigation comparing the value of a bred heifer to a feeder heifer should be considered. The comparison would involve estimating the value of the expected offspring of a bred heifer.

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APPENDICES

APPENDIX A

PRICE AND VOLUME DATA

Table 4. Price Data.

Year	Feeder Heifer (\$/cwt)	Alfalfa (\$/ton)	Range Index	Bred Heifer (\$/hd)	Barley (\$/bu)
1976-01	42.68	42.58	87	277.50	2.05
-02	37.56	46.83	85	232.50	2.01
1977-01	39.92	51.58	72	235.36	2.03
-02	43.27	55.92	67	293.13	1.56
1978-01	55.18	57.25	84	352.50	1.83
-02	69.29	42.67	96	595.00	1.73
1979-01	88.11	49.08	85	699.17	1.70
-02	87.38	56.08	70	561.00	2.11
1980-01	85.89	56.25	68	599.50	2.11
-02	78.88	66.17	65	550.00	2.54
1981-01	73.78	55.5	71	520.00	2.93
-02	65.69	51.17	84	457.50	2.36
1982-01	68.22	46.50	83	354.00	2.32
-02	66.29	49.5	96	448.75	1.96
1983-01	72.43	50.50	86	585.00	2.12
-02	67.50	59.83	80	466.25	2.33
1984-01	68.71	75.67	75	530.63	2.55
-02	66.20	76.33	61	432.50	2.55
1985-01	70.25	81.33	62	472.50	2.16
-02	63.09	83	46	432.50	2.11
1986-01	63.75	85.67	85	520.00	1.94
-02	62.97	52.83	85	507.84	1.60
1987-01	70.88	47	84	548.57	1.65
-02	75.44	47	83	647.92	1.97
1988-01	83.55	45.83	63	610.36	1.70
-02	82.13	85.33	37	612.50	2.88
1989-01	89.22	96.83	79	688.75	2.78
-02	89.91	69.17	86	701.79	2.22
1990-01	88.50	68.67	82	725.83	2.20

Table 5. Volume Data.

Year	Heifer Inventory (thou hd)	Beef Cow Inventory (thou hd)
1976-01	6527	42915
-02	6527	41443
1977-01	5842	41244
-02	5858	38738
1978-01	5385	37652
-02	5527	37062
1979-01	5770	37115
-02	5942	37107
1980-01	5837	39161
-02	6137	38773
1981-01	6233	40054
-02	6607	39230
1982-01	6120	38990
-02	6336	37940
1983-01	5800	38500
-02	6181	37484
1984-01	5500	37700
-02	5548	35406
1985-01	4900	35250
-02	5165	33753
1986-01	4800	34150
-02	5214	33945
1987-01	4800	34000
-02	5249	33183
1988-01	4800	34050
-02	5477	33515
1989-01	5000	34200
-02	5474	33705
1990-01	5200	34300

APPENDIX B

DYNAMIC PROGRAM CODE

Figure 12. Dynamic Program Code.

```

c      This is a mainline program designed to solve a finite
c      stage stochastic dynamic programming problem.
c
c      Initialize variables.
c
c
c      real*8 bc,ptc,cr,mv(2),IS(2),sigma(2),nu0,nul,nv0,nv1
c      real*8 PrFH(30),PFH(30),PrF(30),PF(30),PBH,NPFH
c      real*8 u00(30),u11(30),sumf,sumfh,EPV(20,20),nuv,alpha
c      real*8 v0(30,30,30),v1(30,30,30),EPVfH,EPVf,fnb,fnc,fncb
c      real*8 wt(4),beta,w0,w1,w00,w01,w10,w11,TD(27000,13),ntv
c      real*8 tv(30,30,30),PrFH2(30),sumfh2
c      real*8 sv(30,30,30),nsv,gamma,prob,rho
c      integer nc,nb,i,j,k,h,l,m(4),nfh,nf
c      integer ns,mk,D(27000,14),gf,gfh
c      integer n1,n2,ii,jj,kk
c
c      Open output file.
c
c      open(67,file='policy1.data',status='unknown')
c      open(65,file='policy2.data',status='unknown')
c      open(68,file='policy3.data',status='unknown')
c
c      Initialize data.
c
c      Discount rate.
c
c      beta=1/(1.03*.99)
c
c      Heifer weight.
c
c      wt(1)=910.25
c      wt(2)=910.25
c      wt(3)=726.25
c      wt(4)=500.00
c
c      Breeding cost/hd.
c
c      bc=15.0
c
c      Preg testing cost/hd.
c
c      ptc=1.25
c
c      Conception rate.
c
c      cr=.9
c
c      Number of bred heifers in St+2.

```

```

c
c   fnb=1.0
c
c   Additional heifers needed wrt conception rate.
c   fnc=fnb/cr-fnb
c
c   print*, ' Enter bred heifer scale for stage 1 : '
c   accept*,alpha
c
c   print*, ' Enter bred heifer scale for stage 2 : '
c   accept*,rho
c
c   print*, ' Enter feeder heifer scale for stage 3 : '
c   accept*,gamma
c
c   Initial data.
c   print*, ' Enter # states for feeder heifers: '
c   accept*,m(1)
c   print*, ' Enter # states for feed: '
c   accept*,m(2)
c   mv(1)=.23058
c   mv(2)=.18333
c   sigma(1)=.019369
c   sigma(2)=.027576
c   IS(1)=.02
c   IS(2)=.02
c
c   Initial prices.
c
c   call prb(mv(1),m(1),mv(1),sigma(1),IS(1),PrFH,sumfh)
c   call price(m(1),mv(1),sigma(1),IS(1),PrFH,PFH)
c
c   do i=1,m(1)
c     write(68,400) PFH(i),PrFH(i)
c   end do
c
c   call prb(mv(2),m(2),mv(2),sigma(2),IS(2),PrF,sumf)
c   call price(m(2),mv(2),sigma(2),IS(2),PrF,PF)
c
c   do i=1,m(2)
c     write(68,400) PF(i),PrF(i)
c   end do
c
c   Stage 1, terminal values.
c
c   fncb=fnc+fnb
c   do j=1,m(1)
c     PBH=844.0*PFH(j)*alpha-10.45
c     NPFH=.064+.6*PFH(j)
c     u00(j)=-.98*(NPFH*fnc*wt(1))
c     u11(j)=PBH*fnb

```

```

end do
c
c Stages 2-3-4.
c
n1=m(1)-1
n2=m(1)
do ns=2,4
mk=0
do i=1,m(1)
do j=1,m(1)
ufh=.03333+1.4875*PFH(j)-.70259*PFH(i)
if((ns.eq.2).or.(ns.eq.4))ufh=ufh+.03322
call nustate(m(1),ufh,PFH,IS(1),nfh,gfh,EPVFH)
PBH=844.0*PFH(j)*rho-10.45
do k=1,m(2)
D(mk+k,6)=int(PFH(i)*100.)
D(mk+k,7)=int(PFH(j)*100.)
D(mk+k,8)=int(PF(k)*100.)
D(mk+k,14)=gfh
uf=.05+.69798*PF(k)
call nustate(m(2),uf,PF,IS(2),nf,gf,EPVF)
c
call tranprb(m(1),ufh,sigma(1),IS(1),PrFH,PFH,sumfh)
call tranprb(m(2),uf,sigma(2),IS(2),PrF,PF,sumf)
c
c Expected discounted present value.
c
if(ns.eq.2)then
nu0=0.0
nu1=0.0
do ii=1,m(1)
nu0=nu0+PrFH(ii)*u00(ii)
nu1=nu1+PrFH(ii)*u11(ii)
end do
nu0=nu0/sumfh
nu1=nu1/sumfh
else
nv0=0.0
nv1=0.0
nsv=0.0
ntv=0.0
prob=0.0
do ii=1,m(1)
do jj=1,m(1)
do kk=1,m(2)
prob=prob+PrFH(jj)*PrF(kk)
nv0=nv0+PrFH(jj)*PrF(kk)*v0(ii,jj,kk)
nv1=nv1+PrFH(jj)*PrF(kk)*v1(ii,jj,kk)
nsv=nsv+PrFH(jj)*PrF(kk)*sv(ii,jj,kk)
ntv=ntv+PrFH(jj)*PrF(kk)*tv(ii,jj,kk)
end do

```

```

      end do
      end do
      nv0=nv0/prob
      nv1=nv1/prob
      nsv=nsv/prob
      ntv=ntv/prob
      end if

```

```

c
c
c

```

Did not have a heifer in the previous stage.

```

      if(ns.eq.2) then
        w0=PF(k)*fncb*184*1.17+PBH*fncb
        w1=beta*nul
        sv(i,j,k)=w1
        v1(i,j,k)=w0
        D(mk+k,2)=-11
        if(w0.gt.w1) then
          v1(i,j,k)=w1
          D(mk+k,2)=10
        end if
        D(mk+k,12)=int(v1(i,j,k))
      else if(ns.eq.3) then
        w0=PF(k)*fncb*186+gamma*PFH(j)*fncb*wt(3)+bc*fncb
        w0=w0+beta*nv0
        w1=beta*nv1
        sv(i,j,k)=beta*nsv
        v1(i,j,k)=w0
        D(mk+k,4)=-11
        if(w0.gt.w1) then
          v1(i,j,k)=w1
          D(mk+k,4)=10
        end if
        D(mk+k,10)=int(v1(i,j,k))
      else
        end if

```

```

c
c
c

```

Had a heifer in the previous stage.

```

      if(ns.eq.2) then
        NPFH=.064+.6*PFH(j)
        w00=PF(k)*fncb*184*1.17+beta*nu0
        w01=PF(k)*fncb*184*1.17-.98*(NPFH*fnc*wt(2))
        w10=PF(k)*fnc*184*1.17-.98*(PBH*fncb)+beta*(nul+nu0)
        w11=-PBH*fncb*.98-NPFH*fnc*wt(2)*.98+beta*nul
        v0(i,j,k)=w00
        D(mk+k,1)=200
        tv(i,j,k)=w01+ptc*fncb
        if(w00.gt.w01) then
          v0(i,j,k)=w01
          D(mk+k,1)=201
        end if

```

```

      if(v0(i,j,k).gt.w10) then
        v0(i,j,k)=w10
        D(mk+k,1)=210
      end if
      if(v0(i,j,k).gt.w11) then
        v0(i,j,k)=w11
        D(mk+k,1)=211
      end if
      v0(i,j,k)=v0(i,j,k)+ptc*fncb
      D(mk+k,13)=int(v0(i,j,k))
      else if(ns.eq.3) then
        w00=PF(k)*fncb*186+bc*fncb
      tv(i,j,k)=w00+beta*ntv
        w00=w00+beta*nv0
        w11=-.98*(gamma*PFH(j)*fncb*wt(3))+beta*nv1
        v0(i,j,k)=w00
        D(mk+k,3)=20
        if(w00.gt.w11) then
          v0(i,j,k)=w11
          D(mk+k,3)=21
        end if
      D(mk+k,11)=int(v0(i,j,k))
      else
        NPFH=1.08*PFH(j)
        w00=PF(k)*fncb*61
      TD(mk+k,1)=w00+beta*ntv
        w00=w00+beta*nv0
        w11=-.98*NPFH*fncb*wt(4)
      TD(mk+k,2)=w11+beta*nsv
        w11=w11+beta*nv1
        D(mk+k,5)=0
        D(mk+k,9)=int(w00)
        if(w00.gt.w11) then
          D(mk+k,5)=1
          D(mk+k,9)=int(w11)
        end if
      end if
    end do
  end do
  mk=mk+m(2)
end do

c
c   Output solution.
c
nc=0
nb=m(1)*m(1)
do l=1,m(2)
  do i=1,nb
    k=(i-1)*m(2)+1
    nc=1+nc
  
```

```

        write(65,100) nc,(D(k,j),j=1,9)
        write(68,150) nc,(D(k,j),j=6,13)
        write(67,175) nc,D(k,9),(TD(k,j),j=1,2)
    end do
end do
c
100 format(I6,3x,8I6,I9)
150 format(I6,3I5,5I9,I4)
175 format(I6,I9,3f10.2)
200     format(3x,f6.4,8x,f4.2)
300     format(4f8.3)
400 format(3f12.4)
450 format(I2,3f5.2,4f10.2)
c
    end

c *****
c SUBROUTINE PROGRAMS.
c *****
c *****
c subroutine nustate(m,u,EP,IS,nu,grid,EPV)
c
c     real*8 DIF(30),EP(30),EPV,D,a,b,IS,u
c     integer m,nu,i,grid
c
c     do i=1,m
c         DIF(i)=abs(u-EP(i))
c     end do
c
c     D=DIF(1)
c     nu=1
c
c     do i=2,m
c         if(D.gt.DIF(i))then
c             nu=i
c             D=DIF(i)
c         else
c             end if
c     end do
c
c     EPV=EP(nu)
c
c     b=float(m-1)/2.*IS+IS/2.
c     a=-b
c     b=EP((m-1)/2+1)+b
c     a=EP((m-1)/2+1)+a
c
c     if(u.gt.b)then
c         EPV=EP(m)
c         grid=1

```

```

      nu=m
    else if(u.lt.a)then
      EPV=EP(1)
      grid=-1
      nu=1
    else
      grid=0
    end if
c
    return
  end
c
c *****
c subroutine price(m,mv,sigma,IS,Pr,EP)
c
c   real*8 mv,IS,a,b,EP(30),dn,sigma,Pr(30),pi,ak,bk
c   integer i,m,n
c
c   a=IS/2.+mv
c   pi=3.1415927
c   n=(m-1)/2+2
c
c   do i=n,m
c     b=a+IS
c     ak=(a-mv)/sigma
c     bk=(b-mv)/sigma
c     dn=(Pr(i)*sqrt(2.*pi))
c     EP(i)=(exp(-((ak)**2.)/2.)-exp(-((bk)**2.)/2.))/dn
c     EP(i)=mv+sigma*EP(i)
c     a=b
c   end do
c
c   b=IS/2.+mv
c   n=(m-1)/2+1
c
c   do i=1,n
c     a=b-IS
c     ak=(a-mv)/sigma
c     bk=(b-mv)/sigma
c     EP(n-i+1)=exp(-((ak)**2.)/2.)-exp(-((bk)**2.)/2.)
c     EP(n-i+1)=EP(n-i+1)/(Pr(n-i+1)*sqrt(2.*pi))
c     EP(n-i+1)=mv+sigma*EP(n-i+1)
c     b=a
c   end do
c
c   return
  end
c
c *****
c subroutine prb(mv,m,u,sigma,IS,Pr,sum)
c

```

```

real*8 h,a,b,NC,IS,aj,bj,Pr(30),pi,sum
real*8 mv,u,sigma,ak,bk,NCC
integer i,j,n,nn,m

c
pi=3.1415927
h=.0002
a=mv
n=(m-1)/2+1

c
do i=1,n
  b=a+IS
  if(i.eq.1)b=a+IS/2.
  NC=0.0
  aj=a
  bj=a+h
  nn=nint(IS/h)
  if(i.eq.1)nn=nint(IS/(2.*h))
  do j=1,nn
    ak=(aj-u)/sigma
    bk=(bj-u)/sigma
    NCC=h*(exp(-((ak)**2.)/2.)+exp(-((bk)**2.)/2.))
    NC=NCC/(sigma*sqrt(8.*pi))+NC
    aj=bj
    bj=bj+h
  end do
  if(i.eq.1)then
    Pr((m-1)/2+1)=NC*2.
  else
    Pr(n-i+1)=NC
    Pr(n+i-1)=NC
  end if
  a=b
end do

c
sum=0.0
do i=1,m
  sum=Pr(i)+sum
end do

c
return
end

c
*****
subroutine tranprb(m,u,sigma,IS,Pr,P,sum)

c
real*8 h,a,b,NC,NCC,IS,aj,bj,Pr(30),pi,sum
real*8 u,sigma,ak,bk,P(30),ga,gb
integer i,j,n,m

c
pi=3.1415927
h=.0002
ga=P((m-1)/2+1)-float(m-1)/2.*IS-IS/2.

```

```

gb=P((m-1)/2+1)+float(m-1)/2.*IS+IS/2.
a=ga
sum=0.0

```

```

do i=1,m
  b=a+IS
  NC=0.0
  aj=a
  bj=a+h
  n=nint(IS/h)
  do j=1,n
    ak=(aj-u)/sigma
    bk=(bj-u)/sigma
    NCC=h*(exp(-(ak)**2.)/2.)+exp(-(bk)**2.)/2.)
    NC=NCC/(sigma*sqrt(8.*pi))+NC
    aj=bj
    bj=bj+h
  end do
  Pr(i)=NC
  sum=NC+sum
  a=b
end do

```

```

c
  if(u.le.P((m-1)/2+1)) then
    Pr(1)=Pr(1)+1-sum
  else
    Pr(m)=Pr(m)+1-sum
  end if

```

```

c
  sum=0.0
  do i=1,m
    sum=sum+Pr(i)
  end do

```

```

c
  return
end

```