

INQUIRY TO ENHANCE MATH CURRICULUM AND
ENGAGE STUDENTS IN ACTIVE LEARNING

by

Rebecca Jo Kennedy

A professional paper submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Science Education

MONTANA STATE UNIVERSITY
Bozeman, Montana

July 2026

©COPYRIGHT

by

Rebecca Jo Kennedy

2026

All Rights Reserved

DEDICATION

This research is dedicated to my family who saw me through every night and weekend of homework. They gave selflessly so that I could finish the work I set out to do. I would not have completed this project without them. To my husband, thank you for doing the dishes and keeping the kids safe and sane. We would be eating out of bags and boxes without you. Also, thank you for reading this paper over and over and helping me think through the challenges. I also dedicate this research to my students. They are math warriors and make me proud every day.

ACKNOWLEDGEMENTS

To my instructors, Dr. Christina Pavlovich and Dr. John Graves, many thanks for keeping me going, your understanding, and your grace. Through the many hours of change due to circumstances outside of our control, your support and guidance were invaluable. To my science reader, Dr. Nicholas Lux, you are a great example of what it means to be a teacher and I am so thankful I had you along on my journey. A special thank you to Mary Alice Carlson for stepping in when I needed to secure a second science reader. Your support and guidance were extremely vital for ensuring the completion of this project.

TABLE OF CONTENTS

1. INTRODUCTION AND BACKGROUND.....	1
Context of the Study	1
Purpose of Study	5
Focus Statement	5
2. CONCEPTUAL FRAMEWORK.....	6
Introduction.....	6
Challenges in Secondary Math Classrooms.....	6
Traditional Instruction vs Active Engagement.....	8
Cognitive Development and Representation in Mathematics.....	10
Active Learning as a Theoretical Approach to Instruction	13
Engagement and Inquiry in Secondary Mathematics	15
3. INSTRUCTIONAL DESIGN FOR ACTIVE LEARNING IN SECONDARY MATHEMATICS	19
Overview of Instructional Approach.....	19
Instructional Sequence and Structure.....	20
Daily Lesson Structure.....	20
Integration of Instructional Components	24
Core Instructional Resources	25
Concrete, Representational, and Abstract Strategies	25
Inquiry-Based Learning Tasks	28
Collaborative Learning Structures	31
Extended Learning: Project-Based Applications	33
Instructional Design Rationale.....	35
Using These Resources in Practice	36
Appendices Overview	37
4. VALUE: REASONING AND REFLECTION	39
Instructional Significance	39
Conceptual Implications	40
Implications for Classroom Practice.....	41
Implications for Student Learning	43
Professional Reflection	44
Future Directions	45
Conclusion	46
REFERENCES CITED.....	48

APPENDICES	53
APPENDIX A: CRA MAT	54
APPENDIX B: INVESTIGATION CARDS	56
APPENDIX C: EXTENSTION CARDS	58
APPENDIX D: PROJECT: SLOPE AND WATERSHEDS 5E LESSON PLAN	60
APPENDIX E: CHATGPT TRANSCRIPT	66
APPENDIX F: GROUP WORK RUBRIC	68
APPENDIX G: PBL RUBRIC	70

LIST OF TABLES

Table	Page
1. Table 1. Traditional Mathematics Classroom Routine.....	20
2. Table 2. Redesigned Active Learning 45-Minute Classroom Routine	21
3. Table 3. Redesigned Active Learning 90-Minute Classroom Routine	23
4. Table 4. Redesigned Active Learning Two-Day Classroom Routine	24

LIST OF FIGURES

Figure	Page
1. Figure 1. An image of colored algebra tiles for building equations.....	26
2. Figure 2. A photo of algebra tiles being used to create and solve equations	26
3. Figure 3. A CRA Mat students use to construct the concrete, representation, and abstract Algebra equation.	28
4. Figure 4. Investigation Card 3 (front and back) used for inquiry-based learning in Algebra 1.....	30
5. Figure 5. Extension Card 3 (front and back of card) used for inquiry-based learning in Algebra 1.....	31

ABSTRACT

In today's high school mathematics classrooms, instruction is traditionally teacher-centered in nature where students are simply passive recipients of knowledge. Research suggests that for students to learn, they need to be active participants in their own learning. The desire to change the traditionally taught math class to a student-centered approach guides this educational scholarship. It involves an integration of a revised classroom routine and opportunities to engage students in active learning through inquiry-based opportunities. This approach involves using teaching strategies that bring students into the learning, guiding them to be constructors of their own knowledge. The materials and resources that were created provides a way for active learning to be possible in a secondary math class.

CHAPTER ONE

INTRODUCTION AND BACKGROUND

Context of the Study

Being required to read an unchosen and undesirable text can be difficult. It can be agonizing for a person to engage with the text; to truly absorb the information or story when the book does not pique their interest or capture their attention. It isn't that one cannot read in this situation; the book just doesn't ignite the mind. It causes a lack of motivation and interest. Now translate that to the typical math classroom. Lessons are predictable and teacher-centered. The teacher gives the lesson through examples from a book. Students write what the teacher says and complete guided practice problems together. Students then finish the rest of the problems independently to conclude the class. Day in and day out, this is the math classroom that follows a school-mandated curriculum and pacing guide. The nature of the curriculum requires students to complete the lessons and pass the tests. Math becomes mind-numbing. This traditional, lecture-based routine is not compelling and students are disinterested and unmotivated.

A recent study by Schwartz, Bozick, Diliberti, and Ohls (2025) on middle and high school math students, found that about 50% of these students reported losing interest during their math lessons about half or more of the time they were in class (Schwartz et al., 2025). The study also found that students who are bored in math class prefer face to face activities that apply to real-world scenarios. Another finding of their study revealed that students have not fully recovered from the learning loss of the COVID-19 pandemic. Students are performing worse than they were pre-pandemic on national assessments. Student attendance has also suffered since

then, with students now absent from school at significantly higher rates (Schwartz et al., 2025). So, not only are teachers tasked with making up this learning loss, and promoting positive attendance shifts to families and students, they are also facing an increase in boredom, inattentiveness, and limited focus.

As students report a loss of interest in the math classroom, more and more school districts have adopted curricula that teachers are required to teach which can create disconnectedness between students and math. Traditional curricula have been picked apart for their overemphasizing of computational procedures, content that does not connect to knowledge and skills, and how it rarely encourages evidence-based practices (The IRIS Center, 2026). More districts have moved to curricula that is aligned with the Common Core State Standards (CCSS) to ensure that students are college and career ready (The IRIS Center, 2026). CCSS-aligned curricula are more diverse in terms of being informed by instructional practices and are evidence-based. It is a rigorous approach and requires students to apply knowledge through higher-order thinking skills (The IRIS Center, 2026). In mathematics, curriculum is widely accepted and encouraged because it is a subject that is heavily linked to the textbook (Remillard, 2005). Having high-quality curriculum in schools can be good for students (Steiner, 2024). However, there are not a lot of built-in authentic inquiry opportunities, leading to a heavy use of curriculum materials, such as the textbook, and digital resources which are not always accessible to all districts. Furthermore, teachers who feel comfortable supplementing resources and active learning opportunities into other subjects tend to rely mostly on the provided curriculum for math (Remillard, 2005). Janine Remillard states in her examination of teachers' use of mathematics curricula that the reasons teachers choose to use the math curricula more frequently are the views

around the nature of math, how it is learned, and how comfortable teachers are with the subject in general (Remillard, 2005).

With the curriculum also comes pacing guides. It is the school districts' way of keeping teachers on pace with the standards that are to be covered within the grade level to prepare students for mandated state testing. Pacing guides are usually written by district instructional coaches with guidance provided by curriculum representatives (Hemmler et al., 2023). Pacing guides determine the scope and sequence that should be taught to prepare students to meet standards and testing expectations (Hemmler et al., 2023). The pacing guide dictates what lessons are taught, when, and for how long. Although they are helpful to guide teachers to reach district expectations, they usually do not leave room for teacher autonomy. Pacing guides typically move through a lot of material in shorter amounts of time which lead to frustration in teachers and students (Hemmler et al., 2023). Hemmler and her associates found that teachers feel "...pacing guides perpetuate myopic views on acceptable ways of teaching and timing of topic coverage that leads to 'curricular privileging'" (Hemmler et al., 2023). In other words, pacing guides are a one-size-fits-all approach that can hinder an equal and fair education for all. The question then is this: does a strict adherence to curriculum and pacing guides serve all students?

To complete the picture of today's math classroom, one must look also to the learner. Through this research, the question of why students are so unmotivated to learn now, more than ever, kept resurfacing. People are motivated to learn when three of their psychological needs are met. Those needs are autonomy, competence, and relatedness (Ryan & Deci, 2000). Historically, students went to school to learn. Teachers taught the lesson and the students worked through the

lesson. There was no thought towards student-centered classrooms. According Deci and Ryan motivation is valued because of its consequences; motivation produces (Ryan & Deci, 2000). Generally, students are extrinsically motivated to do math by the school, their parents, their teachers, and their grades. Students who are authentically motivated through opportunities for self-direction are more interested, excited, and confident which improves performance, persistence, and creativity (Ryan & Deci, 2000). Teachers who support student autonomy through student-led learning opportunities instead of creating a teacher-centered learning environment facilitate their students' intrinsic motivation, curiosity, and eagerness for challenge (Ryan & Deci, 2000).

Focusing mostly on inquiry-based learning to facilitate active learning makes a lot of sense in a high school math class. Students need to be excited about learning math for themselves to truly be motivated and engaged. Teacher-centered lessons have been shown to not create excitement and interest in students during math class (Schwartz et al., 2025). To carry this out, understanding that curriculum and pacing guides are mandated in most districts, a system of integration was designed to provide an active learning approach. Inquiry-based learning is defined as a student-centered instructional practice that prioritizes questioning, active exploration, and problem-solving. Active learning boosts engagement by engaging students in discussions, collaborative group work, problem-solving and reflection. It requires students to do and think about the problems rather than just passively receiving knowledge from the textbook or the teacher. The goal of this research was to prove that active learning is possible and can work with curriculum instead of against it, or be made to feel like a time-consuming side quest.

Purpose of Study

The purpose of this Educational Scholarship Capstone is to encourage an active learning environment in a secondary mathematics classroom by including opportunities for inquiry and exploration. The active learning approach utilizes teaching strategies such as project-based learning and inquiry-based learning, both of which provide opportunities for student agency and encourages students to become active participants in the classroom. This approach also connects to understanding how students learn and creating learning opportunities to support diverse learning styles.

When teachers have to adhere to district mandated curriculum and pacing guides, there is limited room or opportunity for exploration, projects, and practice in mathematics classrooms. This leads to boredom and lack of interest from the students (Hopkins, 2024). A shift is needed to ensure learning and authentic, student engagement in math. One way to achieve this is to implement inquiry-based learning, an approach that is often used as a strategy for teaching science. Tying scientific inquiry to mathematical reasoning across various lessons helps bridge the gap between these two content areas. Through this interdisciplinary application, students are able to think about math in new ways which fosters engagement and real-world approaches to mathematics while also creating an active learning environment for students.

Focus Statement

The focus of this project is to present a revised classroom routine with inquiry- and exploration-based learning opportunities aligned with existing standards to promote active learning in a secondary mathematics classroom.

CHAPTER TWO

CONCEPTUAL FRAMEWORK

Introduction

The United States, according to the 2022 PISA (Program for International Student Assessment) worldwide study, scored lower than the other OECD (Organization for Economic Co-operation and Development) countries of which there are 38 on student performance in math (Education GPS, 2026). Couple this with a passive learning environment and it creates the ‘perfect storm’ for our nation’s low performance in mathematics. With students’ inconsistent performance in mathematics due to math anxiety, diffidence, and skill gaps, teaching mathematics can be a difficult challenge to navigate (Wen & Dubé, 2022). A critical factor is Math Avoidance Behavior (MAB) which causes students to disengage, have a lack of motivation, and hinders the learning process (Wen & Dubé, 2022). It is imperative to find a way to get students excited about math and provide engaging opportunities for them to learn. To get to the bottom of student challenges in mathematics, one must take a closer look at students’ motivation, cognitive development, and learning through inquiry and projects where instruction shifts to become student-focused.

Challenges in Secondary Math Classrooms

There are multiple factors that are affecting students’ academic potential in today’s classrooms. These factors have led to a significant decline in motivation, academic performance, and poor attitudes in regards to learning math in secondary institutions (Wen & Dubé, 2022). One leading factor is the aforementioned Math Avoidance Behavior (MAB). MAB is when an

individual engages in avoidance behaviors by taking less math-related classes and pursuing fewer STEM-related occupations due to experiencing math anxiety or negative feelings in regards to math (Choe et al., 2019). A study of math avoidance behavior by Choe et al. (2019) set out to prove that it isn't just an individual's math performance that contributes to avoidance but that anxiety about math also contributes to this behavior. Their study confirmed that anxiety and performance play a huge part in why individuals avoid math in general. In the classroom, this can look like absenteeism (skipping class), shutting down (sleeping or heads down), or refusal to complete work. This lends to the theory that students have poor attitudes towards math due to both anxiety and performance. The study also confirms that student inability is not the only issue affecting student outcomes and suggests that a shift to student-focused instruction is a viable solution (Choe et al., 2019). While MAB is a behavioral factor, the cognitive development of the students also plays a critical role.

Along with MAB, students are reported to lose interest in mathematics at higher rates as opposed to other subjects (Schwartz et al., 2025). In the Schwartz, Bozick, Diliberti, and Ohls (2025) study, students are disinterested in math, students' learning loss is significant and has not fully recovered post COVID-19, and high absenteeism is still on an upward trend (Schwartz et al., 2025). When students are not interested in a subject, when they have no buy-in, or it is difficult, boredom is prevalent and motivation suffers. Students who are not interested, bored, and missing important class time usually do not perform well academically, and math performance takes a hit.

Traditional Instruction vs Active Engagement

To fully understand the subject of this educational scholarship, one must understand the pedagogical differences between traditional norms and active learning experiences. Teaching approaches to mathematics can significantly impact student understanding and academic achievement (Islam et al., 2024). Traditional teaching practices involve teacher-centered, direct instruction with little room for student-driven questioning or investigation. This creates a passive learning experience where students are simply recipients of knowledge rather than an active contributor to their own learning. Students in this setting learn through lectures and heavily rely on textbooks for knowledge. Peer discussions are almost nonexistent due to the time constraints of following a curriculum. This creates a lack of space for “math talk” or dialogues involving peer questioning and answers which in turn provoke deeper thought and construction of knowledge.

Teachers ordinarily follow a teacher-centered approach in mathematics because it can be easier, and they perhaps feel safer following a set curriculum scope and sequence with provided student practice problems. This teacher-centered approach is the norm in secondary mathematics classrooms. In a study conducted by Elizabeth Lake with the UCL Institute of Education in the United Kingdom, she found that mathematics teachers are wary of taking risks when giving instruction in math (Lake, 2019). Using curriculum takes the cognitive load off of the teacher and reduces the event that a student may ask a question for which the teacher does not have an answer (Lake, 2019). Using high-quality curriculum mitigates this concern, but lacks in teacher autonomy and limits opportunities for active learning.

Transitioning from a wholly passive learning environment to a more active learning environment is complicated. It requires a shift from comfortable teaching and learning practices to a more immersive, thought-provoking learning experience. Through this transition, the role of the teacher changes from directing all of the decisions in the learning process to being a facilitator of learning, guiding students as they make decisions and explore possible solutions.

Active learning is an approach to teaching that encompasses multiple teaching strategies in which learners are engaged, purposefully participating throughout the instruction (University of Minnesota, 2025). Active learning classrooms are student-led in nature. According to Nurbavliyev, Kaymak, and Sydykov (2022), active learning consists of but is not limited to project-based learning, peer reviews, hands-on activities, inquiry-based learning, and the integration of immersive technology. In this structure, the students move from the role of a passive learner into the role of an active learner.

Yet, challenges persist. Students are pushed out of their comfort zones where think-aloud, discussions, and discovery are obstacles that are tough to overcome. When students are not used to pushing themselves through this productive struggle, encouraging them through the hard stuff can seem a lost cause at times. It is an opposition to change. Students and teachers alike are challenged to be thinkers and constructors of their own knowledge versus having knowledge handed to them and hoping for the best. However, this adjustment of teaching and learning should happen for long-term retention and positioning students to become critical thinkers in a world where drill and kill is not working (Brown et al., 2014).

According to Dr. Karyne Ang, who has conducted research in teaching and learning delivery, studies regarding transitioning from passive to active learning environments are scarce.

The impacts of different delivery techniques to improve student learning are yet to be explored in depth (Ang et al., 2021). Through her research, Dr. Ang designed a workshop format to help facilitators and participants transition from a passive to an active learning model. This format includes four propositions for assisting the transition. First, Dr. Ang suggests teachers should help students prepare to shift their thinking towards self-management and responsibility. This requires a move away from being dependent to independent thinkers, much like a child growing more independent from their parents as they develop and grow. Her second proposition conveys that student engagement should be flexible and multidimensional. Providing a variety of learning activities such as discussions, simulations, mini-lessons, videos, projects, and quizzes allows students to engage with various forms of content which appeals to their unique learning styles. The third proposition Dr. Ang provides is teaching students to collaborate with peers. This helps students be motivated to engage in social learning by fostering peer discussion, practice, production, and inquiry. Her fourth and final proposition is to provide the scaffolding students require to transition, which becomes a “learning to learn” opportunity. Students need scaffolding to accept responsibility, to acquire knowledge, and to construct meaning and understanding. (Ang et al., 2021). To achieve this transition, one must understand cognitive development, diverse learning styles, and ways students’ progress through representations of their learning.

Cognitive Development and Representation in Mathematics

Cognitive theory suggests that the brain processes, stores, and retrieves information which improves comprehension, retention, and problem-solving skills (Feder, 2024).

Psychologist Jean Piaget developed this theory, Stages of Cognitive Development, in his work with young children and focused on how people gain knowledge (Feder, 2024). Piaget theorized

that children develop cognitively based on assimilation and accommodation, a “learn and change as you grow” approach (Madanagopal, 2020). Piaget believed that cognitive development progressed through four distinct phases: sensorimotor (object permanence), preoperational (symbolic thought), concrete operational (operational thought), and formal operational (abstract concepts). These stages are age-specific and happen in the same order for every child. The last stage, formal operational where children develop the ability to think abstractly, was believed to happen at around age 11 and beyond (Madanagopal, 2020). This is where a breakdown in the theory occurs as no child is identical to another and age does not determine ability. Students at any age need intentionally designed learning experiences to progress from sensorimotor, object permanence to the formal operational, or abstract concepts. These learning experiences should be active in nature, helping students move from concrete materials to abstract thinking.

This cognitive mismatch in age and ability does not transfer to a secondary mathematics class. Math is commonly abstract at the high school level. In other words, the math problems are usually theoretical, intangible, and complex, without any physical characteristics. While being taught higher order mathematics, students sit in lecture-based, teacher-centered classrooms with paper and pencil as the only form of engagement. There are very little opportunities for students to actively engage with different materials or media in many high school math classrooms. As a rule, curriculum is followed to fidelity and a textbook is the students’ Bible. The textbook has been said to be used more than in other content areas such as Science and English (Ferguson, 2010). The same I Do, We Do, You Do sequence is the norm in every lesson, every day. There are generally no inquiry opportunities and pencil/paper tasks, which are abstract in nature, rule the day. Students of any age who have not moved through certain stages of development will

struggle with an abstract, or symbolic view (McLeod, 2024). When students reach high school, and they have not been able to progress through these stages of development successfully, abstract problems can be an insurmountable challenge.

To be able to understand the abstract nature of Algebra 1 for example, students must be able to connect the different representations of algebraic expressions and equations. According to Jerome Bruner's theory, expanding on and slightly differing from Piaget, he theorized that children construct knowledge and meaning through active experience (McLeod, 2024). Bruner believed that there are three modes of representation: enactive (action-based/manipulative/concrete), iconic (pictorial/representational), and symbolic (language-based/abstract). When students of any age are given new lessons or units to learn, Bruner believed that it is most effective for students to progress through each of these modes sequentially before moving to the next mode of representation (McLeod, 2024). Students in secondary math classes are represented at each of Bruner's three modes of representation with understanding and abilities scattered across a broad range of development and mathematical understanding (Payne et al., 2026). Some students are still at the enactive and iconic stages while others have had the opportunities to progress to the symbolic mode of representation. With this diversity in learning and cognitive development, a shift in teaching practice is essential to benefit all students.

Another reason students may struggle in math to be motivated and perform well academically is that they are not engaged in multiple ways, the content is typically given in one format, and students lack the opportunity to demonstrate in their unique ways what they have learned. The Universal Design for Learning (UDL) framework addresses this issue. The UDL

framework was designed to improve teaching and learning by implementing three main principles into instruction (CAST, 2024). These principles are: 1) multiple means of engagement, 2) multiple means of representation, and 3) multiple means of action and expression (CAST, 2024). In the math classroom, students should have all three of these principles included in instruction. This can provide students with distinct and individualized approaches which help them to be motivated to learn and perform better academically. Together, Piaget's Cognitive Development Theory, Bruner's Theory of Cognitive Development, and the use of UDL suggest that learning is constructed through multiple modes of representation and individual experience. This work supports a shift from a passive, one-size-fits-all approach to an individualized, multimodal, active learning approach to instruction in mathematics.

Active Learning as a Theoretical Approach to Instruction

Active learning has been described as engaging, interactive, and productive (Nurbavliyev et al., 2022). It is a method of instruction that allows and encourages students to actively participate in their learning. It encourages students to do more than just interact with a textbook and memorize rules and steps for solutions. It is based on the Constructivist Learning Theory which was developed through the works of John Dewey (1929), Jerome Bruner (1961), Lev Vygotsky (1962), Jean Piaget (1980), Ernst von Glasersfeld (1995), and others. Constructivism suggests that learning is not to just be delivered by teachers, but that students need to actively construct their own knowledge through their own experiences (Bada & Olusegun, 2015). Nurbavliyev, Kaymak, and Sydykov's study (2022) investigated the effect of the active learning teaching method on students' achievement, attitude, and motivation in mathematics with 10th grade students from Nurorda School in Nur-Sultan, Kazakhstan. What they found was that

students who were given the ability to actively construct their own knowledge through activity-based learning noticeably increased their academic performance, with slight improvements in attitude and motivation (Nurbavliyev et al., 2022).

A look at how active learning happens is described in the book *Make it Stick* by Brown, Roediger, & McDaniel (2014). They describe a pilot sitting in class day in and day out listening to lectures about the plane he is flying and common situations that may occur while flying. The pilot tries very hard to pay attention but finds it hard to focus on the content being discussed, even though it is completely relevant in his life. They go on to describe what happens when one day, during a lecture, the instructor asks if anyone had ever had a specific fuel light come on during a flight. Someone raised their hand and the instructor asked that pilot to tell the class what happened. All of a sudden, the lecture became interesting as another pilot explained their thoughts and reactions in a high-risk situation. This peer interpretation of a real-life situation helped other pilots to begin thinking “what if it was me”. From there the pilot instruction course changed to a mix of classroom and simulations where pilots were able to apply their knowledge and think through problems (Brown et al., 2014). This shows how students, when given the opportunity, and asked the right questions begin to think for themselves and connect to the content in a meaningful, real way. Had the pilots been given strictly lecture-based instruction, they may not have been able to think critically when a high-risk situation presented itself. Incorporating active learning addressed the pilots’ specific needs, their different learning styles, and presented the lessons in different ways to stimulate their minds and fully engage them through lived experiences. As in the pilot’s situation where they had to learn by doing, asking questions, and thinking through solutions, so do mathematics students. Math isn’t just a set of

rules to memorize. Math is a system of logic that must be explored, questioned, and thought about to make it stick.

Sadly, mathematics classrooms keep teacher-centered, lecture-based practices as the norm, with the rare investigation or project as the exception. Engaging in active learning both as the teacher and for the students can be a positive and much needed change to a low-risk, monotonous teacher-centered approach. Active learning inspires students to think and explore beyond a set of problems on a worksheet. Therefore, using inquiry-based strategies rather than memorization of facts and textbooks could provide a meaningful, active learning approach to mathematics.

Engagement and Inquiry in Secondary Mathematics

Active learning requires engagement. Engagement declines when students' attitudes towards math is negative. Some of these attitudes stem from lack of confidence in math, students' self-perceptions, enjoyment, and math anxiety (Wen & Dubé, 2022). Student confidence and self-perceptions relate to how students believe they can manage the difficult nature of mathematics and how they perceive themselves in the math classroom (Wen & Dubé, 2022). If a student feels as if they cannot handle the math or they view themselves as “not good at math”, engagement with math will be low. If they are confident and see themselves as mathematicians, engagement is higher. Enjoyment and math anxiety are also factors that affect engagement. Studies have explored the correlation between enjoyment and math achievement. Through these studies it was found that enjoyment in mathematics has a positive interrelation to math achievement. On the flip side, anxiety towards mathematics can negatively affect student performance and achievement (Wen & Dubé, 2022). Because attitude affects engagement

directly, it is important to help students address their confidence, self-perceptions, enjoyment, and anxiety. This can be done by applying active learning as an approach to mathematics through inquiry and project-based learning.

Inquiry-based learning (IBL) is active learning that is multi-faceted (Ernst et al., 2017). IBL helps students develop critical thinking and problem-solving skills. It also helps students learn how to collaborate with others through group work, building social-emotional development and the ability of students to adapt. This development in thinking, problem-solving, collaborating, social-emotional learning, and construction of knowledge is what can build student engagement in the classroom. IBL supports a meaningful link between content and the real-world helping students to understand that they will, in fact, use math in real life. IBL invites students to actively investigate driving questions rather than passively receiving facts from the teacher. In IBL, the teacher is a facilitator of learning and the students are the drivers.

Ordinarily, math curriculum has pacing guides with strict adherence to covering a set amount of material within a set amount of time. Jean-Luc Dorier and Francisco Javier Garcia conducted a study analyzing the constraints which could favor or hinder the implementation of inquiry-based mathematics across 12 European countries (Dorier & Garcia, 2013). They determined that society, school, pedagogy, and discipline (science and mathematics) played major roles in the success or failure of IBL as an approach to teaching and learning. They note that using IBL requires time as it is student-driven and should allow for students to ask questions, test results, make mistakes, analyze errors, build their own understanding of the material being presented, and conduct self-evaluations, or peer reviews, on their inquiry projects (Dorier & Garcia, 2013).

Although IBL can be challenging to implement it has been shown to increase the understanding of mathematics in students in upper-division, proof-based mathematics, calculus, and elementary mathematics (Ernst et al., 2017). As students work together in groups, learning to make sense of mathematics through IBL, their understanding and ability to problem-solve expands and engagement within the discipline increases (Dorier & Garcia, 2013). From here, students can really delve deeply into the math through projects which are focused on student interests and relevance using the inquiry skills they acquired through student-driven investigations.

Through projects, students are provided the opportunity to solve problems in real-world contexts. This approach provides relevance whether culturally or personally and helps students build confidence and have positive self-perceptions. Projects, or project-based learning (PBL), has been around since Confucius and Aristotle who were advocates of learning by doing (Boss, 2011). In the 1800's, philosopher John Dewey continued this thought when he disagreed with the view that students were only passively learning. He believed that "education is not preparation for life; education is life itself" (Boss, 2011). Dewey's idea was later expanded on by the physician Maria Montessori and the developmental psychologist Jean Piaget, who both believed that experiences, not just listening to words, is what helps us to learn (Boss, 2011).

The project-based learning approach works in all disciplines and highlights active, student-directed learning. Projects are based on a driving question that requires investigation, exploration, authenticity, and testing of solutions (Laverdiere, 2025). PBL is student-centered because the focus is on what the students are doing and can last for as long as the students show interest in the subject. An example of PBL should be a project that is meaningful and pull from

students' personal lives connecting content and context. For instance, Native American students and/or those learning about tribes can study the mathematics of blood quantum, fractional and exponential mathematics that the United States government and tribes use to calculate the perceived amount of "Indian blood" a person may have based on ancestry (Carson et al., 2024). This inquiry into blood quantum can help students understand the connection between math and a current social issue directly related to them as tribal members/descendants and schools implementing Indian Education for All (IEFA).

When learning is centered around inquiry and projects that grab student interest, students gain a deeper understanding of the concepts being taught. Using these approaches, students construct their own knowledge through active investigations. Inquiry and projects insist that students do and think for themselves, creating the necessary shift from passive to active learning. Learning then becomes effective because students must construct the knowledge for themselves, becoming creators of meaning through their own experiences and reflection.

CHAPTER THREE

INSTRUCTIONAL DESIGN FOR ACTIVE LEARNING IN SECONDARY MATHEMATICS

Overview of Instructional Approach

Integrating the active learning strategies into a math classroom can be demanding. This shift requires teachers and students alike to change their perceptions of teaching and learning mathematics from a teacher-centered, passive-learning approach, to an instructional design that takes time and practice (Ang et al., 2021). However, once implemented, this instructional design has the potential to positively change learning and engagement.

The modified instructional design presented in this chapter includes not only applying inquiry, projects, and active learning opportunities, but also provides a way for teachers to implement this design within their existing curricula. Instructional design strategies that are seamlessly integrated are necessary to help teachers meet strict district pacing guides (Remillard, 2005) (Hemmler et al., 2024). To make the transition a little bit smoother, it may be necessary for a teacher to adjust their classroom routine/schedule. A redesigned classroom routine for a 45-minute class period can allow teachers to implement this approach in a curriculum-guided structure. Daily lessons revolve around this schedule, using lessons from the curriculum and including inquiry investigations to engage the student in constructing their own knowledge of the concept. The classroom routine follows the 5E lesson plan, without having to create 5E lesson plans for every lesson using an active learning design. Support for Concrete, Representational, and Abstract (CRA) as an instructional application is provided through a physical CRA mat with

instructions for its use and implementation. Inquiry investigation cards and extension cards are available to work with the lesson presented in the curriculum. Investigation cards are used for individual and group explorations to improve student engagement and understanding. As a culminating study of the concept being taught, a project is provided for students to ask questions about and explore with a real-world application. With this instructional approach, active learning is feasible in the secondary mathematics classroom.

Instructional Sequence and Structure

Daily Lesson Structure

A typical math classroom schedule consists of a bell ringer, the lesson, guided practice, independent practice, and an exit ticket. This traditional routine is shown in Table 1.

Table 1. Traditional Mathematics Classroom Routine

Daily Schedule (45-minute class period)	
Bell Ringer (Practice of previously learned skill)	5 minutes
Lesson (Introduction of concept being taught-I DO)	20 minutes
Guided Practice (Checks for understanding-WE DO)	5 minutes
Independent Practice (Students practice new concept-YOU DO)	10 minutes
Exit Ticket (Formative Assessment)	5 minutes

As shown, this traditional routine is not a student-centered approach given the amounts of time students are bearing the cognitive weight of their own learning. According to this design students are actively engaged for about 20 minutes of a 45-minute class period. One could argue that this doesn't represent active learning when students are only engaged with the textbook

during the guided and independent practice parts of the routine. Most of the doing is done by the teacher and this design lacks student agency. This schedule is typical in elementary, middle, and high school mathematics classrooms. Most curricula closely follow this design.

This redesigned active learning routine provides time for carrying out projects, inquiry, and group work (Table 2). The routine also provides a way for mathematics teachers to include active learning opportunities while still following their district-mandated curricula.

Table 2. Redesigned Active Learning 45-Minute Classroom Routine

Daily Schedule (45-minute class period)	
Bell Ringer (Practice of previously learned skill)	3 minutes
Hook: Engage (Introduce the question/phenomenon)	2 minutes
Inquiry/Project: Explore (Students actively investigate the topic)	10 minutes
Mini-Lesson: Explain (Introduce the concept/skill)	8 minutes
Group Work/Project: Extend (Students work together applying new skill)	10 minutes
Independent Study: Extend (Practice applying knowledge to problems in workbook)	10 minutes
Wrap-Up: Evaluate (Formative Assessment/Quick Checks)	2 minutes

This schedule is structured using the 5E lesson plan model. This lesson plan model was created by Rodger Bybee et al., for the Biological Science Curriculum Study (BSCS) in 1987. The phases are set up in a way as to support and have the ability to apply this model to units within curricula (Bybee et al., 2006). The 5E lesson plan model consists of five components that encourage an active-based learning structure. The 5E's that are covered in this model are engage,

explore, explain, extend and evaluate (Bybee et al., 2006). The 5E framework is used in this teacher-created active learning classroom routine (Table 2) and each of the 5E's are represented as phases.

In the Engage phase teachers can compose a hook to grab students' attention which is designed to engage and spark interest in the lesson. Hooks are usually brief and meant to engage the learners immediately. Teachers present the hook which could be anything from "I have a mystery envelope here for any student who can..." (insert something that relates to what they are learning about) to a demonstration of a marble rolling down a wooden plane. The Explore phase of the routine allows for students to explore the phenomenon, a problem, or a scenario. At this time, students can ask questions, notice patterns, and test solutions. This prompts students to think about the concept that will be introduced. Teachers provide support and guiding questions. During the Explain phase, the teacher provides a brief mini-lesson, to shed light on concepts that became apparent during the exploration and to introduce the new concept. Instructional resources, such as the textbook, can be used to support this process. In the Extend phase, students apply their understanding through structured group work, using teacher-designed extensions that reinforce and deepen the targeted concept. Teachers provide support during this phase by helping students construct their own knowledge through higher-order questioning. Students are given time for independent study; a time where they engage with their textbooks or practice the new skill in other ways. This time can also be used to differentiate and scaffold for students who need extra support. Finally, in the Evaluate phase, students are presented with an exit ticket or quick check to determine mastery of the skill they learned about. With this structure, it is feasible to include active learning teaching strategies that can fit within the structured curriculum of a high

school mathematics classroom. This set routine allows the teacher to continue with the curriculum without having too many independent modules. If needed, this routine can cover a 90-minute block as provided in Table 3.

Table 3. Redesigned Active Learning 90-Minute Classroom Routine

Daily Schedule (90-minute class period)	
Bell Ringer (Practice of previously learned skill)	5 minutes
Hook: Engage (Introduce the question/phenomenon)	5 minutes
Inquiry/Project: Explore (Students actively investigate the topic)	20 minutes
Mini-Lesson: Explain (Introduce the concept/skill)	15 minutes
Group Work/Project: Extend (Students work together applying new skill)	20 minutes
Independent Study: Extend (Practice applying knowledge to problems in workbook)	20 minutes
Wrap-Up: Evaluate (Formative Assessment/Quick Checks)	5 minutes

Within some curricula, specific lessons require two days depending on the complexity of the concept students are learning. A two-day routine is provided for this scenario. This routine may also come in handy if the teacher feels the need, due to student understanding or mastery, to take two days to cover a more complex lesson and can do so while following their district curriculum and pacing guides.

Table 4. Redesigned Active Learning Two-Day Classroom Routine

Day 1: Engage & Explore (45-minute class period)	
Bell Ringer (Practice of previously learned skill)	3 minutes
Hook: Engage (Introduce the question/phenomenon)	7 minutes
Inquiry/Project: Explore (Students actively investigate the topic)	30 minutes
Initial Wrap-up: Evaluate (Quick Check of Exploration)	5 minutes
Day 2: Explain & Extend/Apply (45-minute class period)	
Review: (Revisit findings from previous day's inquiry)	5 minutes
Mini-Lesson: Explain (Introduce the concept/skill)	15 minutes
Group Work/Project: Extend (Students work together applying new skill) -AND/OR- Independent Study: Extend (Practice applying knowledge to problems in workbook)	20 minutes
Wrap-Up: Evaluate (Formative Assessment/Quick Checks)	5 minutes

Integration of Instructional Components

Utilizing this daily lesson structure facilitates needed time to create an active learning environment using CRA (Concrete, Representational, Abstract) materials and strategies, inquiry, and collaboration. These components all fit within the restructured daily lesson routine and can be used in conjunction with curriculum and textbooks. All of the strategies provided in this framework allow students to engage with UDL (Universal Design for Learning) strategies which

include multiple means of engagement and representation to portray their understanding through inquiry as individuals and as a group.

Using CRA enables students to interact with manipulatives allowing them to thoroughly explore the concept by progressing through concrete, pictorial (representational), and abstract structures. This ties directly to the constructivist theory that suggests students' individual development is tied to active exploration (McLeod, 2024). Inquiry-based learning opportunities are implemented through mini-investigation scenarios where students can explore problems in-depth and try to solve it on their own before the teacher explains the concept. Along with the mini-investigations, group investigations during the extension phase empowers students to work together to extend on the knowledge they have constructed. Interchanging inquiry and projects in the explore and extend phases allows the teacher to implement a project-based learning opportunity at the end of a unit of study as a culminating application of acquired skills and knowledge. As reflected in the active learning classroom routines, creating time each day for students to explore and practice the concepts they are learning allows teachers to continue to use the curriculum and engage students in active, meaningful ways.

Core Instructional Resources

Concrete, Representational, and Abstract Strategies

Using the CRA strategy in mathematics provides much needed scaffolds for building problem-solving skills which students can use to solve real-world problems. This strategy can be used during the Explore or Extend phase of the 5E model active learning classroom routine depending on student needs. The CRA strategy begins with introducing a concept by representing it in a concrete form through the use of manipulatives and movement. Students use

concrete objects to “do” or model math problems. These objects can be blocks, colored chips, pattern blocks, fraction bars, and algebra tiles. The second stage of CRA is representational. This is the “seeing” form of CRA where students transform the concrete model into a semi-concrete model. This is usually represented by pictures or drawings of the concept being learned. The final stage of CRA is the abstract stage. This is the symbolic representation of the concept being taught. In math, students move from the pictorial to the abstract by modeling problems using symbols. An example of this would be the operational symbols (+, -, ×, ÷) common in math (Methods, 2024).

To implement this strategy, teachers may use such manipulatives such as algebra tiles and geoboards. The use of algebra tiles should be explicitly modeled to students. Algebra tiles are used to construct the concrete representation of an algebraic equation (Figure 1). Students use the tiles as a way to physically manipulate the equation to help with understanding (Figure 2).



Figure 1. An image of colored algebra tiles for building equations

$$\begin{array}{l}
 3x + 2 = x - 6 \\
 \begin{array}{c} \text{3 green rectangles} \\ \text{2 orange squares} \end{array} = \begin{array}{c} \text{1 green rectangle} \\ \text{6 orange squares} \end{array} \\
 3x - x + 2 = x - x - 6 \\
 \begin{array}{c} \text{3 green rectangles} \\ \text{2 orange squares} \end{array} = \begin{array}{c} \text{1 green rectangle} \\ \text{6 orange squares} \end{array} \\
 2x + 2 - 2 = -6 - 2 \\
 \begin{array}{c} \text{2 green rectangles} \\ \text{2 orange squares} \end{array} = \begin{array}{c} \text{6 orange squares} \\ \text{2 orange squares} \end{array} \\
 2x = -8 \\
 \frac{2x}{2} = \frac{-8}{2} \\
 x = -4 \\
 \begin{array}{c} \text{1 green rectangle} \\ \text{4 orange squares} \end{array} = \begin{array}{c} \text{4 orange squares} \end{array}
 \end{array}$$

Figure 2. A photo of algebra tiles being used to create and solve equations

Algebra tiles are made up of three shapes: a small square, a rectangle, and a large square. Each shape has a green/blue side which are positive and a red/yellow side which are negative. The small square represents the number one. The rectangle represents the variable. The large square represents the variable squared. Students match the tiles to the expression or equation they are working with as shown in Figure 2. A key concept in using algebra tiles is the zero pairs concept. Zero pairs occurs when a positive tile is paired with a negative tile of the same shape and size. These tiles cancel each other out and are removed from the workspace. Students manipulate the tiles according to the order of operations until they have only one variable tile on one side of the equal sign and whatever is left from solving on the other side of the equal sign.

Once students have mastered the equations at this level, they move into the next stage which is pictorial representation. For this stage, students draw pictures that may be similar to the tiles they used during the concrete period to continue to construct their knowledge of the concept. Students may also draw tape diagrams, dots, squares, and/or balance squares which are a t-shape that visually represents the equal sign in algebra. As students' progress through this stage, they are able to visualize the manipulatives become more abstract in nature through drawings and figures which represent the expression or equation.

The abstract stage is where the student has the ability to see or read the problem and solve it by using mathematical symbols and systems of equations. Figure 3 shows a teacher-created CRA mat which is similar to a place-value mat used in elementary classrooms (Appendix A). This mat helps transfer the concrete and pictorial representations to an abstract equation by structuring all three stages in the same way, in the same place so students can make the connections they need to build their understanding. The CRA Mat can be used to support

students understanding of Algebra 1 concepts such as solving equations of linear and non-linear functions. They also assist in building understanding at each stage of Bruner's theory of cognitive development.

CRA MAT		
Write your equation in each of the boxes:		
		
Build the problem using manipulatives	Draw a visual model of the problem	Solve the problem using numbers and mathematical symbols
C	R	A

Figure 3. A CRA Mat students use to construct the concrete, representation, and abstract Algebra equation.

Some students need the extra support that CRA provides to help them construct the knowledge they need to progress to the abstract skills typically found in Algebra 1. Being able to solve abstract problems helps students develop strong problem-solving skills needed for projects and inquiry.

Inquiry-Based Learning Tasks

Inquiry-based learning, or IBL, is a key component for obtaining an active learning approach. IBL encourages active investigation rather than simply receiving facts about the topic. Students are able to work independently or in groups to explore a phenomenon or a problem. During this inquiry routine, students ask questions, test results, make mistakes, analyze errors, and build their conceptual understanding.

To incorporate IBL into curriculum, teachers can input these tasks into the Explore and Extend phases within the daily lesson routine (Table 2). This facilitates supplementing lessons with inquiry opportunities. Because most teachers have curriculum that is district/school mandated, they need to be able to still teach from the curriculum and use materials such as student workbooks. Some curricula provide inquiry opportunities, but typically it is digital and students are only able to explore with digital graphs. To provide students with more opportunities to actively analyze and evaluate information by using more than just their iPad, teachers can use inquiry/scenario cards for variety and a more active learning experience.

Inquiry cards, or what the teacher termed Investigation/Extension Cards for use in Algebra 1, are tools that can prompt thought, ignite curiosity, and guide problem-solving. Canva was used to create the cards shown in Figure 4 starting with a blank canvas. To generate the examples below (Figure 4 and Figure 5), ChatGPT was used by the teacher to guide the structure of the cards since there is little reference for inquiry to follow within curriculum. Through discourse with ChatGPT-generated text, one can learn a lot about how to structure a solid Inquiry card. Inquiry (Investigation/Extension) cards should encourage students to think on their own and should contain a scenario, data or a visual, and thinking prompts (OpenAI, 2026; see Appendix E for the full transcript). The teacher aligned the investigation and extension cards with three lessons in a unit within Algebra 1 district curriculum.

Investigation Card 3 Lesson 4.1 Intercepts	
Title: Quarantean You have a \$50 gift card for Quarantean, the local coffee shop. Each drink costs \$5.	
Drinks Purchased	Balance Remaining
0	?
2	\$40
?	\$25
?	\$0
Questions: 1.If you haven't bought any drinks yet, how much money is on the card? What point would this be on the y-axis? 2.How many drinks can you buy before the card is completely empty? Where would this point be on the x-axis?	
Materials you will need to complete the tasks: • Paper/Whiteboard to record your answers • Graph board or graph paper • markers/highlighters • pencil	

Figure 4. Investigation Card 3 (front and back) used for inquiry-based learning in Algebra 1

The investigation and extension scenario cards in this project cover graphing linear functions, rate of change and slope, and slope-intercept form as taught in Algebra 1. Two investigation opportunities per lesson are provided, as most lessons introduce two concepts or strategies. There are eight investigation cards (Appendix B) in total covering linear and non-linear functions through three different lessons. Using the investigation cards, students read the prompt and answer the questions. They can ask their own questions, formulate responses, and represent them in multiple ways. These ways consist of using paper/pencil, whiteboard/marker, or record their investigation by video. Teachers support this Explore phase by circulating around the students, encouraging thought, and supporting outcomes.

Each investigation card is paired with an extension card (Figure 6). The extension cards (Appendix C) are used for group work during the Extend phase. Extension cards broaden or advance the concept that students investigated during the Explore phase. Extension cards can be omitted if there is not enough class time. They could be used as a unique way to assess understanding as a formative assessment.

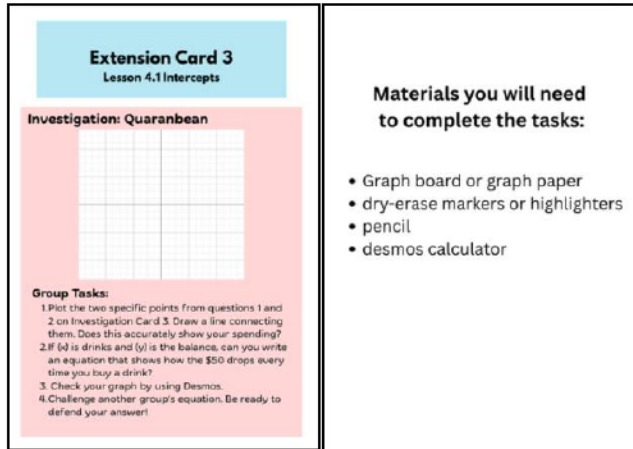


Figure 5. Extension Card 3 (front and back of card) used for inquiry-based learning in Algebra 1

Pairing the extension cards with an investigation card increases time for exploration of the specific skill they are covering. These extension cards firm up what students learn through their own investigations and through the lesson the teacher presents during the Explain phase. They are meant to extend the concept in a way that encourages students to work together and apply what they have learned. A list of materials that the students will need to complete the extension is listed on the back of each card. While students work together in groups to extend their understanding and apply new concepts, teachers act as facilitators. Teachers move between groups challenging student thinking and ensuring students are applying their newly gained knowledge in different contexts.

Collaborative Learning Structures

Students working together in groups fosters teamwork by encouraging learning through active exploration. In this setting, students have discussions, share knowledge, and engage in peer learning and teaching. The students learn to apply their knowledge to real-world problems, build deeper understanding through collaborative conversations, and share accountability.

This collaboration is structured in small groups of 2-4. Small groups help students stay focused on the task and limit distractions. Each person in the group is assigned a role: recorder, questioner, presenter, etc. The recorder ensures that the discussion and work are recorded in some way whether in writing, typing, or video recording. The recorder, or thinker, communicates their ideas to the doers, or builders of the solution. The questioner is in charge of asking questions such as, “Why does that work?”, “Can you explain that step to all of us again?” or, “What does that number/letter represent?”. This helps the group to verbalize their mathematical reasoning and all students become stakeholders in the process. The presenter is the one who builds the solution from input from all members of the group as they discuss, guide, and verbalize their mathematical process. The presenter then presents the group’s investigation to another group, the teacher, or to the rest of the class. A rotation of roles can ensure that all students learn the math and be able to think mathematically. This teaches them to be supportive and understand that everyone is responsible and accountable for the work of the group as any one of them could be called on to explain their work.

When introducing group work, the teacher sets clear expectations for behavior, responsibility of workloads, the desired outcomes (work samples, etc.), and group accountability. These expectations may need to be repeated often in the beginning, until students are able to self-direct the work of the group, meeting the expected standard. A rubric (Appendix F) is available to assist groups in meeting the expectations. The rubric guides students by helping them to stay on task and reach the desired outcomes.

The collaborative learning structure, or group work, supports learning by providing a space and time for students to discuss their investigations, suggest possible solutions, build

speaking and listening skills, develop critical thinking, and the ability to justify the group's answers. By mastering these collaborative learning routines, students develop academic stamina, interpersonal skills, and shared responsibility that are required to navigate the student-directed inquiry of project-based learning.

Extended Learning: Project-Based Applications

Project-based applications can be implemented during the Explore and Extend phases of the daily lesson structure, but may require more than the time allotted for these two phases. Because project-based learning (PBL) can take more time than a single class period, it is recommended that the project replaces the daily routine as PBL encompasses all phases of the 5E model. This ensures the framework is still being met, but allows more time in the schedule to complete the project. It is advised to use up to two to three days for a well-rounded PBL in which students can actively engage, learn, and apply the concepts they have mastered. For a unit on linear and nonlinear functions, multiple project-based learning opportunities are available for teachers to introduce and have students investigate.

A project (PBL) investigating slope with a local watershed is provided in this instructional design (Appendix D). The project is written using a 5E lesson model. The project is designed to be the closing of a unit focused on Linear and Nonlinear Functions, specifically, the topics of rate of change and slope. The core idea covered in this lesson is measuring how one quantity changes relative to another. Before beginning the project (possibly the day before), the teacher should provide a background lesson about watersheds (Calvo, 2014). This is important as students may not know what a watershed is and what it impacts. This is also where a cross-curricular approach connects science to math.

At the beginning of the project, students are provided with a phenomenon and a driving question. As students explore, the teacher facilitates the learning by providing materials and opportunities for students to discuss and record their observations, helping students to stay focused and engaged. Students interact with the phenomenon of water staying in place or running off an object. The teacher provides topographical maps of a local watershed and students select three different areas of the creek to calculate the slope for each area (Creek Connections, n.d.).

The teacher then facilitates the Explain phase where a whole group discussion helps to clear up any misconceptions about the observed phenomenon and topographical maps. The teacher introduces any new concepts or ideas that have an impact on the project. The teacher also provides guiding questions and suggestions to help students think about whether the watershed is linear or nonlinear, connecting the project to the overall unit (Bassage, 2019).

The 5E lesson model had the Extend phase after the Evaluate phase. This was adjusted by moving the Extend phase to come before the Evaluate phase. The justification for this is due to students needing to produce a product of their learning so the teacher can use the project as a summative assessment for the overall unit. The Extend phase directs students to create a 3D model of the local watershed they studied during the Explore and Explain phases of the project.

Students form into the same groups they were in during the Explore phase. The teacher provides a rubric (Appendix G), materials, parameters, and definitions of representations in the model. Students use the rubric to guide their construction of the 3D model. Students trace and cut cardboard to recreate the watershed from the map as a 3D representation. Students use layers to show rate of change and slope within the watershed. Students label their 3D representation with

elevation markers, calculated slopes, and ridgelines to designate the watershed boundary. The teacher checks throughout to ensure students are following measurements and calculations as they build. The teacher rotates through the groups asking guiding questions and challenging designs (Creek Connections, n.d.) (Bassage, 2019).

When students have completed their 3D models, teachers use the PBL rubric (Appendix G) to evaluate students' group project. The evaluation assesses group problem-solving, math (equations and graphs), speaking and listening, and the final product (3D model). This acts as a summative assessment of the content covered in the linear and nonlinear functions unit.

Instructional Design Rationale

The rationale for this instructional design comes from the understanding that there are many approaches and strategies available to create an active learning environment in a high school math classroom. One of the strategies featured in Jerome Bruner's theory is the use of concrete, representational, and abstract (CRA) models used in teaching and learning in kindergarten through 12th grade math classrooms (Methods, 2024). According to the Pennsylvania Training and Technical Assistance Network (Methods, 2024), Jerome Bruner's theory of learning and cognitive development states that learners construct their own knowledge through active experiences (McLeod, 2024). He believed that every learner progresses through three stages of learning, concrete, representational (pictorial), and abstract (CRA). The CRA mat and manipulatives address this progression with an individual approach, scaffolding students to reach the abstract goals of Algebra 1 content.

Other approaches used in this instructional design is inquiry-based learning (IBL) and project-based learning (PBL). Students who are given time to work through inquiry and projects

associated with the real world increase their understanding of mathematics (Ernst et al, 2017). The opportunities for IBL and PBL provided in this design are integrated within the existing secondary mathematics classroom system which uses curriculum and follows a pacing guide.

Through practice, the restructured daily routine is designed to work with multiple approaches (CRA, IBL, PBL) and curriculum in tandem to maximize active learning. Utilizing this framework can serve as a starting point for student engagement, motivation, and improved academic performance.

Using These Resources in Practice

To manage the schedule so that the teacher does not fall behind in curriculum or pacing, it is important for teachers and students to start slow by learning one routine at a time. Teachers must set the expectation of each phase of the daily routine and follow it with opportunities to practice. With practice, each routine has the potential to become easier and flow smoothly. A suggestion would be to use timers until the flow of the schedule becomes natural. It is an adjustment and may take time to implement. It may be helpful to use the first two weeks of school, when schedules are more flexible to practice this routine. During this practice phase, inquiry opportunities can be found through a quick AI query using Google Gemini or ChatGPT. It is always important to check AI for errors in content, but this is a quick and simple way to implement tasks for practice purposes.

After practicing the schedule routine, teachers should give deep thought to group structure. Suggestions for this include thinking about the students and their mixed abilities. Groups can be structured based on heterogeneous (mixed abilities) to support learning styles and math ability. There are many reasons for specific grouping but, teachers should ensure that every

student in the group is contributing and learning. This creates a space where every student is recognized, supported, and given every opportunity to learn. The structure of groups is vital, should be monitored throughout, and changes made when needed.

To incorporate an active learning classroom environment, teachers should secure the materials needed beforehand. The teacher should plan for the unit ahead of time and determine what manipulatives, digital tools, and/or task cards will be utilized. A set of resources for a unit on linear and non-linear functions is provided in this instructional design. However, it only covers one unit and is meant to provide an example of the project and some resources for teachers to try. Beginning and continuing an active learning-based classroom, requires more resources. The template for the Investigation/Extension cards are provided (Appendices B & C) and are able to be edited for other lessons. A 5E PBL example is also provided in Appendix D and can be used as a guide for different PBL lessons.

No matter how much prep and planning goes into lessons, students invariably ask unexpected questions. Being flexible is essential in this design. It is okay to take time and acknowledge these questions as students bring a different set of logic to the discussions. Knowing when to “circle back” or return to the topic at hand is key to staying on track.

Appendices Overview

The appendices provide the framework of the active learning, inquiry-based design discussed in this paper. Appendix A contains the teacher-created CRA mat for implementing concrete, representational, and abstract scaffolding to support students. Appendix B includes the Investigation cards and a link for editing/copying purposes used during the Explore phase of the daily structure for an active learning classroom. These are used for individual or small group

inquiry opportunities. Appendix C provides the Extension cards and a link for editing/copying purposes. These cards are used during the Extend phase of the active learning environment daily structure. The cards guide group work and extend the concept students learned during the Explore phase. Appendix D provides the project-based learning 5E lesson plan on Slopes and Watersheds. This lesson is to be utilized as a stand-in for a few days in the daily structure as it covers each specific area of the structure. Appendix E includes a link to the ChatGPT transcript that was used to facilitate the design and creation of the Investigation and Extension cards. Appendix F contains the Group Work Rubric used during the Extend phase of the daily structure to guide students in productive groupwork. Appendix G provides the PBL Rubric for the 5E lesson plan on Slopes and Watersheds and is used to guide the teacher's assessment of the project from beginning to end.

CHAPTER FOUR

VALUE: REASONING AND REFLECTION

Instructional Significance

The 5E/CRA instructional design produced in this educational scholarship is an active learning model. Rather than introducing an additional curriculum, this instructional framework partners with a district mandated instructional unit within Algebra 1. It adjusts the passive standard and has the potential to make math approachable, fostering student connection to the content. Teachers can encourage students who exhibit Math Avoidance Behavior (MAB) such as negative self-talk, to explore math within their differing learning abilities and representations. The use of CRA strategies like engaging with manipulatives and drawings mitigates cognitive barriers by providing students with scaffolds to help them move to an abstract concept. Inquiry-driven investigations shift student learning by engaging with a concept prior to instruction. This provides a point of reference for them, or background knowledge, for the concept, activating the following instruction with concrete investigations. This design facilitates confidence and accountability for learning. Student agency is fostered as students take responsibility for their progress, expound on mathematical ideas, acquire cognitive processes, and build collaborative relationships. This instructional design has the capacity to increase engagement as it allows students to: discuss with their peers in a collaborative way, physically manipulate data inputs and outputs on a graph, and think about questions and answers as a means of learning. MAB can be reduced and a growth mindset can be strengthened through active learning.

Conceptual Implications

This instructional design explicitly implements constructivism, a theory that suggests learning happens when humans construct their own knowledge through experiences (Bada & Olusegun, 2015). Modeled after this theory, the instructional design presented in this project brings theory into practice by providing authentic, inquiry-based learning opportunities that actively engages students. When students are able to manipulate and discuss versus sitting and listening, they understand the concept in a deeper way. Active learning can be interpreted as math talk happening in various places in the classroom with questions, debate, and possible answers. It can take the form of multimodal representations such as students building, drawing, and manipulating graphs to arrive at a conclusion. Textbooks become a resource instead of the primary objective of their learning experience. Realigning the daily classroom routine allows for active learning while keeping pace with district curriculum requirements. This realignment is necessary to include room for students to ask questions, make connections, investigate, and arrive at solutions. With representation in the concrete, representational, abstract (CRA) strategy and the 5E lesson plan model, to inquiry and connection to real-world investigations, this design opens the door for the teacher to be a facilitator of learning in their classrooms versus being the primary leader. It also provides a space for students to build their own knowledge instead of just being the receiver of knowledge.

The framework of the design is demonstrated through investigation cards like the coffee shop gift card task (Figure 4), which requires students to explore the decreasing value of a gift card as it was used to purchase coffee at a local coffee shop. Rather than passively learning a formula for slope, students evaluate the information and respond to guided prompts that

promotes cognitive expansion beyond the given data. Students learn to use critical thinking skills by extending the investigations using extension cards (Figure 5). By manipulating a Desmos graph of x - and y -coordinates, students ascertain algebraic equations by pattern recognition and problem solving. Students connect what they are learning about abstract linear intercepts to a real-world mathematical phenomenon. The progression between the investigations and extensions help mitigate the cognitive mismatch between the concrete and abstract modes of representation.

Implications for Classroom Practice

To create an active learning environment, the teacher should prioritize time to explicitly teach the routine and the expectations of each phase of the 5E model. Teachers should also explicitly model expectations for collaborative work such as positive, meaningful contributions to discussions, and group responsibilities. While the provided materials cover a single three-lesson unit, the framework serves as a blueprint for teachers to model their own inquiry investigations and projects after the design across other curricular units. Parts of the design can be applied without any new resources, such as the 10-minute mini-lesson and the use of the CRA Mats. The materials are designed to align with Algebra 1 curriculum and pacing guides.

Active learning frameworks utilize manipulatives or visual tools, problem-solving activities, and group collaboration for investigative tasks (Nurbavliyev, et.al. 2022). Using any of the suggested strategies (CRA, IBL, PBL) and collaborative work can be incorporated independently of each other. This allows teacher and students to adapt slowly to new routines. The instructional principles practiced in the framework include: 1) reviews via formative bell ringers and extension tasks), 2) presenting material in targeted mini-lessons, 3) scaffolded

application using investigation cards, 4) continuous formative assessment during collaboration using rubrics and 5) summative evaluation through project-based learning. Active learning encourages students to become problem solvers and critical thinkers in and outside of the math classroom by teaching them to persevere through challenges, engage in challenging dialogue, and learn through experiences.

A key assumption in the design of this instructional resource is that inquiry works best when students are provided time to explore a concept individually or in small groups. In practice, students examine problems, ask questions, build models, and construct responses which allows for student agency and independence. Asking questions guides students as they think through a particular math problem. Building models with manipulatives helps students to physically construct cognitive processes. This sensory (enactive) interaction bridges the gap from concrete manipulation to pictorial (iconic) representation. Once students have solidified their understanding with visuals like coordinate planes and data tables represented within the inquiry cards, they have the scaffolding required to move into abstract (symbolic) representation and manipulation. This entire process can be accomplished by simply providing students with time to engage with and understand the problem. The CRA, IBL, and PBL strategies are independent variables and therefore can be implemented independent of one another. This design provides a way for teachers to slowly move away from the passive standard of learning without inciting major disruptions to current classroom routines. Once new routines are solidly implemented, teachers may choose to include other strategies to encourage and promote a more holistic active learning environment.

Implications for Student Learning

When students construct their own knowledge through active manipulation of materials like algebra tiles and explore through inquiry-based investigations/extensions, they have the ability to transform from a fixed mindset to a growth mindset. This type of transformation has the potential to move them away from “I’m bad at math” to “I can investigate math and learn”. This metamorphosis aims to facilitate reducing Math Avoidance Behavior (MAB), a behavior that resembles disengagement, cognitive withdrawal, and chronic absenteeism (Bada & Olusegun, 2015).

Students conceptual understanding has the potential to deepen as they learn the why and how of math. For example, the inquiry component of the framework features a physical investigation where students explore slope by measuring the *rise* and *run* of the staircases in a gym. This serves as a primer for students to understand the two variables that determine the slope of a line and how these variables change for different staircases before confronting slopes of lines on a coordinate grid. Through this investigation, students have the opportunity to notice that the steepness of the staircase changes depending on how high and how far the stairs extend. Through inquiry and multiple representations this active learning strategy offers a way to meet students’ individual learning needs. Along with measuring gym stairs (concrete), students are given the opportunity to draw the stairs with the determined *rise* and *run* on a graph (representational). Students can represent the *rise* and *run* as slope through an equation (abstract) by computing the two variables. This student-centered approach assists in guiding students through the math which replaces the passive, traditional, direct instruction typical in math classrooms.

Students also actively explore new concepts through collaborative efforts. Students assist each other in measuring, talk through how to draw the graph according to their measurements, and ask questions as they create their final equations. Thoughts and ideas find support, or are challenged in a group setting. This collaborative effort elaborates on the content being studied as students share their own knowledge of measurement, the importance of accuracy, and the mathematical operations needed to find the answer. Through this type of collaboration, students can learn to apply their knowledge in different contexts, building pathways to deeper understanding.

Professional Reflection

The creation of this framework developed from an observed identification of MAB and exposed a one-way information framework. A shift needed to take place in instruction to improve teaching practices of secondary mathematics so that students had a chance to learn in a way that was meaningful and appropriate to their individual needs. This highlighted the necessity of aligning what was determined by constructivism as ways students learn to an instructional design that works with what teachers are already doing. This alignment of routines, Bruner's Enactive-Iconic-Symbolic framework (McLeod, 2024), and inquiry-based investigations ensured that the lesson becomes a bridge supported on all sides by student exploration.

The discrepancy between the curriculum and students' cognitive readiness, and other factors, like MAB, did not support engagement, interest, and motivation. Math required learning and doing, and that was missing in the traditional routine. Students needed something else, something that worked with them and supported their individual needs. That is when an active learning approach became the mission.

This instructional design prompted a fundamental shift from a teacher-centered, passive learning environment to a student-centered, active learning environment. This transition shifted the role of the teacher from a single presenter of information to a facilitator of inquiry. Through this facilitation, opportunities were provided for students to explore the math, make mistakes, talk about them, discuss ideas and solutions with peers, and use various tools to support cognitive connections. Rather than passive compliance, the classroom became dynamic with students as the primary knowledge constructors. The design challenged pedagogical thought in regards to the traditionally taught math class with rote memorization and disconnected practice. The philosophy of the teacher changed through this process understanding that rote memorization and practicing concepts that were not understood did very little for student success.

From this endeavor, new possibilities surfaced. Ideas for inquiry and implementation into the current curriculum beyond one unit became an obtainable goal. Including more kinesthetic movement into an abstract world brought about solid ideas such as the staircase investigation thwarting the passive nature of abstract algebraic models. Time adjustments to the 45-minute routine were adjusted to fit the needs of diverse classroom settings. Future versions of this framework must be adaptable ensuring that active learning becomes the standard instead of just an intervention.

Future Directions

While this framework focused exclusively on linear functions, future development entails expanding on this design across 10-12-unit Algebra 1 curriculum. New investigation and extension cards will be created to support all units in Algebra 1 as well as more unit ending

project-based learning opportunities. To connect abstract problems and real-world phenomenon further, the framework will include place-based activities (field opportunities) to explore mathematics in the outside world. A specific midterm opportunity involves an investigation of Indigenous tipi (lodge) ring sites within prairie regions. Students will gather data by mapping tipi rings to determine the number of camps, study the geometry and symmetry of the camp design, analyze rocks from a single tipi ring to determine the size of the lodge and graph to compare number and varying sizes of lodges within the camp.

Going forward refining the inquiry and extension cards are the crucial next step to ensure that the mathematics follows the rigor of the standard each investigation explores. The cards should be refined as student and teacher exploration determines inconsistencies, or redesign them for optimizing performance of the tasks to streamline the inquiry process and maximize the time allowed.

Conclusion

Although traditional models of direct instruction have been the way to teach secondary mathematics in the past, newer research suggests that passively gaining knowledge is not sustainable for student motivation or deep conceptual understanding (Wen & Dubé, 2022). Cognitive theory and constructivists have researched how people construct knowledge through active exploration (McLeod, 2024). Strategies, designs, and approaches to active learning have existed for some time, and still there are far too many teacher-centered, passive learning classrooms lacking opportunities for exploration and allowing students to be constructors of their own knowledge. This comes from the push to get through the content, master the practice, and pass the test. This design shows that there can be both exploration and curriculum. Educators can

allow students to build their own understanding and still teach the content they need to teach. It is possible and all students deserve the right to deeply engage with math. Math avoidance behaviors (MAB) can be superseded when educators provide scaffolded instructional designs, recognize diverse cognitive abilities, and allow students to explore and connect with mathematics in ways that promote interest and student-driven learning.

REFERENCES CITED

- Ang, K. C., Afzal, F., & Crawford, L.H. (2021). Transitioning from passive to active learning: Preparing future project leaders. Project Leadership Society, 2, 100016.
<https://doi.org/10.1016/j.plas.2021.100016>
- Bada, S. O. & Olusegun, S. (2015). Constructivism learning theory: A paradigm for teaching and learning. Journal of Research & Method in Education, 5(6), 66-70.
[https://vulms.vu.edu.pk/courses/edu201/downloads/edu%20201%20\(assignments%20\).pdf](https://vulms.vu.edu.pk/courses/edu201/downloads/edu%20201%20(assignments%20).pdf)
- Bassage, S. (2019). Earth's features: Teacher's guide sample. BOCES 4 Science.
<https://www.boces4science.org/Downloads/TG%20Sample%20Earth's%20Features.pdf?v=19#:~:text=The%20main%20topics%20covered%20in%20this%20unit,land%20and%20water%20features%20in%20an%20area>
- Boss, Suzie. (2011). Project-based learning: a short history. Edutopia. George Lucas Educational Foundation. <https://www.edutopia.org/project-based-learning-history>
- Brown, P.C., Roediger III, H. L., & McDaniel, M. A. (2014). Make it stick: The science of successful learning. Harvard University Press.
<https://books.google.com/books?hl=en&lr=&id=oneWAwAAQBAJ&oi=fnd&pg=PR7&dq=make+it+stick+brown&ots=rrR-7g2vcg&sig=uEMJNDtkwUFS6UC2QQ4FdOeQXX8#v=onepage&q=make%20it%20stick%20brown&f=false>
- Calvo, L. M. (2014). Project PORTS curriculum and activity guide. Haskin Shellfish Research Laboratory, Rutgers, The State University of New Jersey. https://hsrl.rutgers.edu/wp-content/uploads/2026/01/PORTS_1-1_An_Estuary_Nearby_remediated.pdf
- CAST. (2024). CAST Universal design for learning guidelines version 3.0.
<https://udlguidelines.cast.org>
- Choe, K. W., Jenifer, J. B., Rozek, C. S., Berman, M. G., Beilock, S. L. (2019). Calculated avoidance: Math anxiety predicts math avoidance in effort-based decision-making. Science Advances, 5(11), eaay1062. <https://doi.org/10.1126/sciadv.aay1062>
- CRA Methods. (2024). Concrete-Representational-Abstract (CRA): Instructional sequence for mathematics. PaTTAN. Pennsylvania Training and Technical Assistance Network. Commonwealth of Pennsylvania. https://www.pattan.net/getmedia/9059e5f0-7edc-4391-8c8e-ebaf8c3c95d6/CRA_Methods0117
- Creek Connections. (n.d.). Delineation revised [PDF]. Allegheny College.
<https://sitesmedia.s3.amazonaws.com/creekconnections/files/2014/03/DelineationRevised.pdf#:~:text=To%20determine%20which%20direction%20water%20will%20flow,with%20a%20cross%20hair%20indicating%20the%20hilltop>

- Cornerstone University. (2018). What are multiple intelligences and how do they affect learning. Cornerstone University. Lifelong Learning Matters. <https://www.cornerstone.edu/blog-post/what-are-multiple-intelligences-and-how-do-they-affect-learning/>
- Dorier, J-L., Garcia, F. J. (2013). Challenges and opportunities for the implantation of inquiry-based learning in day-to-day teaching. ZDM Mathematics Education., 45(837-849). <https://rdcu.be/e2SrD>
- Education GPS, OECD. (2026). PISA 2022 Results (Volume I): The state of learning and equity in education. <http://gpseducation.oecd.org>
- Ernst, D. C., Hodge, A., Yoshinobu, S. (2017). What is inquiry-based learning. Doceamus: Notices of the AMS., 64(6). <https://www.ams.org/journals/notices/201706/rnoti-p570.pdf?adat=June/July%2006&trk=1536&cat=none&type=.pdf>
- Feder, M. (2024). What is cognitive learning theory. University of Phoenix. https://www.phoenix.edu/articles/education/what-is-cognitive-learning-theory.html#:~:text=Cognitive%20learning%20theory%20can%20help%20educators:%20*,which%20shares%20many%20concepts%20with%20cognitive%20learning.
- Ferguson, K. (2010). Inquiry based mathematics instruction versus traditional mathematics instruction: the effect on student understanding and comprehension in an eighth-grade pre-algebra classroom. Cedarville University: Cedar Commons. <https://files01.core.ac.uk/download/pdf/301479298.pdf>
- Hemmler, V. L., Kenney, A. W., Dulong Langley, S., Gubbins, E. J., Callahan, C. M., & Siegle, D. (2024). Elementary teachers' perceptions and reported enactment of autonomy from prescribed pacing guides. Journal of Education., 204(3), 562-575. <https://doi.org/10.1177/00220574231190935>
- Hopkins, A. (2024). Examining the impact of implementation fidelity mandates on instructional practices in k-8 settings: a critical qualitative study. The Graduate School: The University of Maine. <https://digitalcommons.library.umaine.edu/cgi/viewcontent.cgi?article=5005&context=etd>
- Islam, M. S., Sarker, M. S., Abir, S. S., Kamal, F., & Roy, B. K. (2024). Effect of student-centered teaching approach on academic performance in mathematics at the secondary school level. IJORER: International Journal of Recent Educational Research, 5(6), 1492-1505. https://scholar.google.com/scholar?q=Effect+of+Student-Centered+Teaching+Approach+on+Academic+Performance+in+Mathematics+at+the+Secondary+School+Level+Md+Sefatul+Islam*,+Md+Shakil+Sarker,+Shahriar+Shajid+Abir,+Fairuz+Kamal,+Bijon+Kumar+Roy+Khulna+University,+Khulna,+Bangladesh&hl=en&as_sdt=0,27#

- Khan Academy. (2026). Khanmigo rubric generator. [Large language model].
<https://www.khanacademy.org>
- Lake, E. (2019). 'Playing it safe' or 'throwing caution to the wind': Risk-taking and emotions in a mathematics classroom. *Lumat: International Journal of Math, Science and Technology Education*, 7(2), 50-64. <https://doi.org/10.31129/LUMAT.7.2.335>
- Laverdiere, M. (2025). Incorporating project-based learning in elementary and secondary schools. Marymount University: News. <https://marymount.edu/blog/incorporating-project-based-learning-in-elementary-and-secondary-schools/#:~:text=Inquiry%20drives%20the%20PBL%20process,might%20approach%20future%20challenges%20differently>.
- Madanagopal, D. (2020). Piaget's theory and stages of cognitive development-An overview. *Scholars Journal of Applied Medicines*, 8(9), 2152-2157.
https://saspublishers.com/media/articles/SJAMS_89_2152-2157.pdf
- McLeod, S. (2024). Jerome Bruner's theory of learning and cognitive development. *Simply Psychology: Psychology: Child Psychology*.
<https://www.simplypsychology.org/bruner.html>
- National Council of Teachers of Mathematics. (2023). Equitable integration of technology for mathematics learning: a position of the national council of teachers of mathematics. NCTM. <https://www.nctm.org/standards-and-positions/equitable-integration-of-technology/#:~:text=Technological%20tools%20can%20provide%20access,Yadav%20%20Lachney%2C%202022>)
- Nurbavliyev, O., Kaymak, S., & Sydykov, B. (2022). The effect of active learning method on students' academic success, motivation and attitude towards mathematics. *Journal of Language and Linguistic Studies*, 18(2).
<https://www.jlls.org/index.php/jlls/article/view/4409>
- OpenAI. (2026). ChatGPT (Mar 6 version) [Large language model].
<https://chatgpt.com/share/69c34084-fbc4-83e8-a136-870774af5612>
- Payne, S.B., Powell, S.R., & Fry, E.C. (2026). A synthesis of mathematics interventions for high school students with mathematics difficulties. *Journal of Learning Disabilities*, 59(2), 75-93. <https://doi.org/10.1177/00222194251385204>
- Remillard, J. T. (2005). Examining key concepts in research on teacher's use of mathematics curricula. *Review of Education Research*, 75(2), 211-246. <https://journals-sagepub-com.proxybz.lib.montana.edu/doi/pdf/10.3102/00346543075002211>

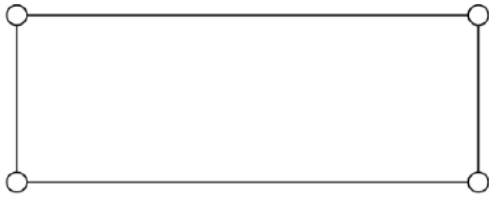
- Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social-development, and well-being. *American Psychologist.*, 55(1), 68-78. <https://doi.org/10.1037/0003-066X.55.1.68>
- Schwartz, H. L., Bozick, R., Diliberti, M. K., & Ohls, S. (2025). Students Lose Interest in Math: Findings from the American Youth Panel. Research Report. RR-A3988-1. RAND Corporation. https://www.rand.org/pubs/research_reports/RR-A3988-1.html
- Steiner, D. (2024). The unrealized promise of high-quality instructional materials. *State Education Standard*, 24(1), n1. <https://www.nasbe.org/the-unrealized-promise-of-high-quality-instructional-materials/>
- The IRIS Center. (2026). High-quality mathematics instruction: What teachers should know. Vanderbilt University. <https://iris.peabody.vanderbilt.edu/module/math/cresource/q1/p02/#content>
- University of Minnesota. (2025). Center for Educational Innovation. *Active Learning*. <https://cei.umn.edu/teaching-resources/active-learning#:~:text=Active%20learning%20refers%20to%20a,individual%20work%20and/or%20reflection>
- Wen, R. & Dubé, A.K. (2022). A systematic review of secondary students' attitudes towards mathematics and its relations with mathematics achievement. *Journal of Numerical Cognition*, 8(2), Article e7937. <https://doi.org/10.5964/jnc.7937>

APPENDICES

APPENDIX A: CRA MAT

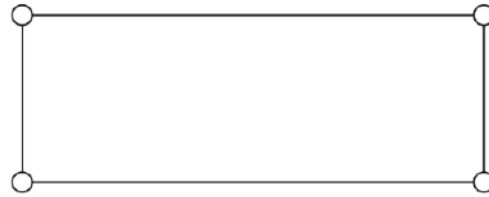
CRA MAT

Write your equation in each of the boxes:



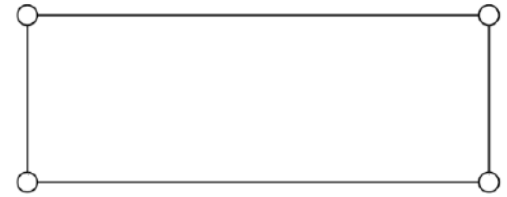
**Build the problem
using
manipulatives**

C



**Draw a visual
model of the
problem**

R



**Solve the problem
using numbers and
mathematical
symbols**

A

APPENDIX B: INVESTIGATION CARDS

Investigation Card 1
Lesson 4.1 Tables

Title: Grades
Your grade in math begins at 0 at the beginning of the semester. Every day you turn in one daily math assignment.

Day	Math assignment
0	0
1	8
2	16
3	24

Questions:
1 What pattern do you notice?
2 What does the starting value represent?
3 Draw a graph and plot the points.

Investigation Card 2
Lesson 4.2 Tables as a Line

Title: Mystery Graph
Here is the graph of the linear function related to this situation:

Questions:
1 What do you notice about the points?
2 What information do the points give you?
3 Write the equation for this graph.
4 Explain the graph in Desmos.

Investigation Card 3
Lesson 4.3 Interpreting

Title: Question
You have a \$20 gift card for Quiksilver. The local coffee shop sells coffee for \$3.

Cups purchased	Amount
0	7
2	14
4	21
6	28

Questions:
1 If you haven't bought any coffee yet, how much money is on the card? What point does this represent?
2 How many dollars can you buy before the card is completely empty? Where would this point be on the x-axis?

Investigation Card 4
Lesson 4.2 Tables as a Change

Title: Mystery Data
Rita about the road show. Every Monday there is a table where we are doing things. For each hour, another thing changes. Height (feet).

Hour	Height
0	0
1	10
2	20
3	30
4	40
5	50

Questions:
1 Can you create a table from your graph?
2 Does your table show a steady change?
3 Can you graph it?

Investigation Card 5
Lesson 4.3 Tables

Title: Making Investments
Dana is a farmer. She grows corn and wheat. She has a table showing how much she has planted. Based on your time and the distance at which you stopped.

Time	Wheat	Corn
0	0	0
1	10	10
2	20	20
3	30	30

Questions:
1 How does the table create a table from your graph?
2 What does the table tell you about the relationship between wheat and corn?
3 How does distance change each second?
4 What is happening in the relationship? The farmer you work?

Investigation Card 6
Lesson 4.3 Steps-Intercept

Title: Motion
A car starts at a speed of 0 and goes to 60 mph in 10 seconds. It then goes to 120 mph in 20 seconds. It then goes to 180 mph in 30 seconds. It then goes to 240 mph in 40 seconds. It then goes to 300 mph in 50 seconds. It then goes to 360 mph in 60 seconds. It then goes to 420 mph in 70 seconds. It then goes to 480 mph in 80 seconds. It then goes to 540 mph in 90 seconds. It then goes to 600 mph in 100 seconds.

Time	Speed
0	0
10	60
20	120
30	180
40	240
50	300
60	360
70	420
80	480
90	540
100	600

Questions:
1 How does the table create a table from your graph?
2 What does the table tell you about the relationship between speed and time?
3 How does speed change each second?
4 What is happening in the relationship? The car you work?

1 - IC1 Front

2 - IC1 Back

3 - IC2 Front

4 - IC2 Back

5 - IC3 Front

6 - IC3 Back

Link to File: [Investigation Cards](#)

APPENDIX C: EXTENSTION CARDS

⊖ 1 - EC1 Front

⊖ 2 - EC1 Back

⊖ 3 - EC2 Front

⊖ 4 - EC2 Back

⊖ 5 - EC3 Front

⊖ 6 - EC3 Back

Extension Card 1
Lesson 4: Tables

Investigation: Compare Two Students

1. Create a table for each student, starting at 0.
Your classroom starts with 10 points but earns 2 ballfinger points every day.

→

Group Task:

1. Create a table for each student.
2. Plot the equation for each student on a graph.
3. Plot the points on a graph for each student.
4. Discuss the graphs and what is different about each graph?

Extension Card 2
Lesson 4: Points on a Line

Investigation: Point Plotting

Group Task:

1. Look at the other points that could lie on the line.
2. Plot these points on the graph.
3. Discuss the points and what is different about each point.
4. Challenge another group's solution!

Extension Card 3
Lesson 4: Inequalities

Investigation: Quasi-Number

Group Task:

1. Plot the inequality $y > x + 1$ on the graph.
2. Plot the inequality $y < x + 1$ on the graph.
3. Plot the inequality $y = x + 1$ on the graph.
4. Discuss the graphs and what is different about each graph.

Extension Card 4
Lesson 4: Slope of Change

Investigation: Mystery Data

Group Task:

1. Create a table for each student.
2. Plot the equation for each student on a graph.
3. Plot the points on a graph for each student.
4. Discuss the graphs and what is different about each graph?

Extension Card 5
Lesson 4: Slope

Investigation: Walking Investigation

Group Task:

1. Create a table for each student.
2. Plot the equation for each student on a graph.
3. Plot the points on a graph for each student.
4. Discuss the graphs and what is different about each graph?

Extension Card 6
Lesson 4: Slope Strategy

Investigation: Student Roles

1. Create a table for each student.
2. Plot the equation for each student on a graph.
3. Plot the points on a graph for each student.
4. Discuss the graphs and what is different about each graph?

Materials you will need to complete the tasks:

- Graph board or graph paper
- dry-erase markers or highlighters
- pencil

Link to File: [Extension Cards](#)

APPENDIX D: PROJECT: SLOPE AND WATERSHEDS 5E
LESSON PLAN

5E AND INQUIRY LESSON PLAN

Unit Topic: Topographic Mapping of a Watershed

Lesson Math Content: Rate of Change; Slope and Slope-intercept

Length: Two-Three 45-minute class periods

Performance Expectation: Determine slope percentages of a local creek. Use a model to illustrate slope (steep/flat) and map a watershed boundary.

Materials Needed:

Teacher: Slightly rounded plate, water, cup; Cardboard for troughs; Topographic maps of local watershed ([USGS Watershed Maps How To](#)) ([National Map Viewer](#)); Computer; Smart board or Smart TV; Rubric

Student Materials: Paper/Whiteboard; Pencil/Dry-Erase Marker; Calculator, Ruler; Cardboard; Markers/Paint; Highlighters

Background: [Background Investigation of a Watershed](#) (Texas State University, 2026). This background lesson may be helpful for students who have not been exposed to mapping. Contour lines on a map depict four basic slope types: uniform gentle, uniform steep, concave, and convex. Topographic Influence (Slope) describes the flow of water from high to low elevations. Boundaries and ridges are defined by high points (elevation). Elevation is the vertical distance (height) of a specific point on the Earth's surface, such as a mountain, city, or hiking trail, measured relative to a fixed reference point, most commonly meaning sea level. Scale relates to how small watersheds can exist within larger watersheds. Understanding mapping basics helps us identify boundaries and understand elevations. Steepness determines whether water will soak into the ground or continue over a surface.

A watershed is an area of land where all the water—whether from rain, snowmelt, or springs—drains into a common water body, such as a river, lake, or ocean. Watersheds vary in size, from small drainage basins for local streams to massive systems like the Columbia River Basin, which covers multiple states. The boundaries of a watershed are defined by natural land formations, such as mountains and hills, which direct the flow of water (Department of Ecology, 2026).

Engage

Opening: This lesson is designed to be the closing of a unit focused on Linear and Nonlinear Functions. Specifically, the topics of **rate of change** and **slope**. The core idea covered in this lesson is *Measuring How One Quantity Changes Relative to Another*. As students enter class, the teacher will present two photos, one of a steep mountain and one of a flat plain to students for the anchoring phenomenon of the unit. Students will not be told what the pictures represent. Teacher Note: The two photos should be found prior to teaching the lesson and should be of areas near to where the students reside. This connects the lesson to place. Teachers should elicit student experiences with the following questions (5 minutes).

1. *If you had to climb 500 feet, would you rather do it over a distance of 1,000 feet or 5,000 feet? Why?*

Driving Question: *How does the **rate of change** in elevation determine where our local water flows?*

As you are having this conversation on the phenomenon, place your plate on a table and use a cup to pour water on the plate. The water will pool at the bottom of the plate. Then, tip the plate and use the cup to pour more water on the plate. The water runs off the plate. Turn the plate over to where it is upside down. Use the cup to pour water onto the plate. The water will run down the convex sides of the plate. Repeat the entire process a couple of times. Finally, pour water onto the concave side of the plate and tip the plate at varying degrees. The water will pour out of the plate at different rates of speed. Tell students to record observations about the phenomena in their notebooks using all five senses (5 minutes).

Potential Observations: Plate on a table. Water is pouring into the plate. It sounds like a waterfall. Water is gathering at the bottom of the plate. Plate is turned upside down. Water is poured on the plate. Water runs off the plate. Plate is now turned right side up. Water is pouring into the plate. Plate is tipping slowly, and the water is falling off of the plate. It sounds like rain. Plate is tipping more and the water is falling off faster. Plate is almost vertical and all the water has poured off.

Explanation of Phenomenon: When water is poured into the plate, it gathers at the bottom. The plate is in its concave form, allowing the water to pool at the bottom of the plate. When the plate is turned over, it is in its convex state. When the water is poured onto the plate in this state, the water does not gather and runs off the sides. When water is poured back into the plate in its concave form, and then the plate is tipped slightly, the water runs off slowly. As the plate tips more and more, more water pours off at a faster rate. When the plate is vertical, all the water pours off the plate and water is no longer on the plate.

Explore

Debrief with students on the results of the Plate and Water Phenomenon. Consensus observations should include: When the plate is flat and concave, the water stays on the plate. When the plate is convex or tipped at different degrees, water runs off the plate (5 minutes).

Revisit the question: *How does the **rate of change** in elevation determine where our local water flows?* At this point, most students will respond with discussions about the plate tipping and water running off of the plate. Students will draw on their past experiences with the flow of water. They may bring up waterfalls, lakes, rivers, rain, etc. Before students investigate ask them: *At what angle or **slope** does the water stop pooling and start moving?*

Let students investigate how water flows. Divide students into 5 lab groups. Each group will need a plate, a cardboard trough, water, and a way to record their observations. Have groups experiment with pouring water on the plate and then on the trough. Teacher Note: Have the students perform the experiment over small plastic bins so that the workspace remains safe and clear of water. Have students record their observations of the flow of water in their math notebooks. Prompt them specifically to describe the relationship between the steepness of the trough and the speed of the water. Students should see that the speed of the water depends on the steepness of the plate or trough. Provide students with USGS topographic maps of a local creek. Students select three distinct segments of the creek (15 minutes).

- Segment A: High elevation/mountains
- Segment B: Foothills
- Segment C: Flat plains

Explain

After students made observations and recorded questions about the Water Flow Phenomenon in their Math Notebooks, and using their personal map investigations as a guide, have students whiteboard why they think a higher rate of change ($m > 1$) result in faster water velocity. Introduce the formula for finding *rate of change*. The changes in y (dependent variable) are divided by the changes in x (independent variable).

$$\frac{y_2 - y_1}{x_2 - x_1}$$

(Where y is the elevation, and x is the horizontal distance)

Formula: Elevation A – Elevation B/Distance A – Distance B

Students use the maps scale and contour intervals to calculate the slope (m) for each creek segment that they chose previously.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Tell students to represent the flow of water by drawing a watershed. Allow students to do a quick gallery walk once their boards are done to see other groups' work. Have students take a picture of their group's whiteboard and submit it to your Learning Management Site when done (10-15 minutes).

Closing: When done whiteboarding, discuss the flow of water in both the opening phenomenon (the water and plate) and their experiment. Have a concluding discussion on how

the creek follows what happened in the phenomenon, following the contours of the land and changing velocity as the elevation decreased by varying slopes. Also discuss linear vs. nonlinear. Ask students to look at the creek as a whole. Ask: Is the slope constant? Introduce the idea that real-world landscapes are rarely perfectly linear, but we use "average rate of change" to model them (5 min).

Extend

(This phase has been moved to before the Evaluate phase, as this part of the lesson should be included before evaluating student understanding.)

In this lesson, students will build a 3D Rate of Change Model. Students will construct their models using either cardboard or clay depending on what the teacher has available. This construction will be done in layers. Each layer of cardboard represents a specific step in elevation.

Teacher Role: Provide a standard cardboard thickness (e.g., 1/8 in.) and define what the “rise” (y) will represent in real-world feet (e.g., one layer = 40ft elevation). Before students glue pieces together, check their “run” (x). Ensure they are spacing the layers according to their map measurements. Rotate through the groups and ask, *if your slope calculation was steeper here, should these cardboard edges be closer together or further apart?*

Student Role: Students trace the contour lines from their topographic map onto cardboard sheets. Each contour line becomes a layer. Students stack the layers to represent the change in elevation. Each layer added is a constant vertical “rise” (y) (conversations may erupt about linear vs nonlinear). Students must ensure that the horizontal distance between the edges of the cardboard layers matches the map’s scale. Students label each cardboard layer with its elevation and the calculated slope of that specific layer. This ensures the math is visible on the final product.

1. Steep Slope ($m > 1$): Cardboard edges are nearly on top of each other.
2. Gentle Slope ($m \approx 0$): Cardboard edges are spread far apart.

Students identify the highest points where the “rise” (y) stops going up and begins going down (the peak or “zero slope” point), where the rate of change goes from positive to negative. Students mark this ridgeline with a designated color to show the watershed boundary.

Evaluate

Before students begin the extension and commence building their 3D models of a watershed, they will be given a copy of the rubric provided in Appendix G to help guide their work. During this lesson, the teacher will visit groups around the room while students are discussing. The teacher should be asking probing questions to keep students on task and challenge them to apply concepts as well as math vocabulary learned in previous lessons. To promote crosstalk, the teacher should redirect some questions directed to the instructor to another member in the lab group so that they share knowledge with one another. Each 3D model and

group dynamics will be graded using the rubric (Appendix G). This is the culminating summative assessment for this unit on linear and nonlinear functions, with an emphasis on rate of change and slope.

APPENDIX E: CHATGPT TRANSCRIPT

Link to File: [ChatGPT Transcript for Inquiry Card Discussion](#)

APPENDIX F: GROUP WORK RUBRIC

Rubric - Group Work

Criterion	Emerging	Progressing	Excelling
Problem-Solving	- The group identifies the main question or problem to be solved - The group lists at least one possible approach or solution	- The group identifies the main question or problem to be solved - The group lists at least two possible approaches or solutions - The group explains the reasoning behind their chosen approach - The group documents steps taken to investigate the problem	- The group identifies the main question or problem to be solved - The group lists multiple possible approaches or solutions - The group explains the reasoning behind their chosen approach - The group documents steps taken to investigate the problem - The group evaluates the effectiveness of their solution and considers alternative strategies - The group justifies their final answer with evidence from their investigation
Math: Equations, Graphs, etc.	- The group includes at least one relevant equation or calculation in their work - The group presents a basic graph or table related to the problem	- The group includes multiple relevant equations or calculations - The group presents a graph or table that accurately represents data or relationships - The group explains how the math supports their solution	- The group includes multiple relevant equations or calculations, showing all steps - The group presents a graph or table that accurately represents data or relationships, with clear labels and units - The group explains how the math supports their solution - The group uses mathematical reasoning to justify their answer and connects their math work to real-world implications or applications
Speaking and Listening	- The group presents their solution to the class or teacher - At least one group member speaks during the presentation	- The group presents their solution to the class or teacher - Multiple group members contribute to the presentation - The group responds to at least one question or comment from peers or teacher	- The group presents their solution to the class or teacher, with all members participating - The group responds thoughtfully to questions or comments from peers or teacher, providing evidence or clarification - The group asks questions or seeks feedback from others to improve their work - The group demonstrates active listening by referencing ideas or suggestions from peers during their presentation or discussion

APPENDIX G: PBL RUBRIC

Rubric - PBL Collaboration

Criterion	Emerging	Progressing	Excelling
Problem-Solving	- Work identifies the main problem or question - Project includes at least one proposed solution or approach	- Work identifies the main problem or question - Project includes at least one proposed solution or approach - Project describes reasoning behind the chosen solution or approach - Work includes evidence of considering alternative solutions	- Work identifies the main problem or question - Project includes at least one proposed solution or approach - Project describes reasoning behind the chosen solution or approach - Work includes evidence of considering alternative solutions - Project includes a comparison of multiple solutions, with justification for the final choice based on evidence or data
Math: Equations, Graphs, etc.	- Work includes at least one relevant equation, calculation, or graph related to the project - Mathematical work is connected to the main problem	- Work includes at least one relevant equation, calculation, or graph related to the project - Mathematical work is connected to the main problem - Project includes explanations of how the math supports the solution - Work includes correct use of mathematical notation and labels	- Work includes at least one relevant equation, calculation, or graph related to the project - Mathematical work is connected to the main problem - Project includes explanations of how the math supports the solution - Work includes correct use of mathematical notation and labels - Project includes multiple representations (e.g., equations, graphs, tables) and connects them to justify the solution
Speaking and Listening	- Work includes evidence that group members participated in discussion (e.g., meeting notes, audio/video clips, or written summaries) - Project includes at least one instance of group members sharing ideas	- Work includes evidence that group members participated in discussion (e.g., meeting notes, audio/video clips, or written summaries) - Project includes at least one instance of group members sharing ideas - Project includes documentation of group members responding to each other's ideas (e.g., questions, clarifications, or feedback) - Work includes evidence of group members listening and incorporating feedback	- Work includes evidence that group members participated in discussion (e.g., meeting notes, audio/video clips, or written summaries) - Project includes at least one instance of group members sharing ideas - Project includes documentation of group members responding to each other's ideas (e.g., questions, clarifications, or feedback) - Work includes evidence of group members listening and incorporating feedback - Project includes a summary of how group communication contributed to the final solution or product, with examples of effective collaboration
Product	- Work includes a completed 3D model that addresses the main question or problem - Product includes a basic explanation of the solution	- Work includes a completed 3D model that addresses the main question or problem - Product includes a basic explanation of the solution - Product includes supporting evidence (e.g., data, calculations, research) for the solution - Product includes a justification for the solution	- Work includes a completed 3D model that addresses the main question or problem - Product includes a basic explanation of the solution - Product includes supporting evidence (e.g., data, calculations, research) for the solution - Product includes a justification for the solution - Product includes a reflection on the process, limitations, and possible improvements, and connects the solution to broader concepts or real-world applications