MONTANA STATE UNIVERSITY

A model for predicting ion concentrations in the Yellowstone River between Billings and Miles City, Montana : a management tool
by Richard William Karp

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Industrial and Management Engineering
Montana State University
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Abstract:
Regression equations were developed for the individual common ions versus electrical conductivity (EC), and the individual common ions versus EC and flow (Q). These regression equations relate the instantaneous concentration of common ions in Meq/L to the instantaneous EC and Q in µmhos/ cm @ 25°c and cfs, respectively. The regression equations relating the common ion concentrations to EC produced relatively good results. A cation-anion balance was used to determine the quality of the simulated common ion concentrations.

Water quantity and quality models based upon the principle of conservation of matter and employing the macro-to-micro philosophy were developed on an annual basis. A methodology for obtaining the quality model from the quantity model was developed. The quantity model simulates the amounts of flow associated with the various flow components; and the quality model simulates the EC associated with the various flow components. The use of EC in the quality model provides the necessary parameter for the use of the regression equations involving the common ion concentrations.

Both the quantity and quality models provide reasonable results that would be expected on an annual basis.

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Date November 30, 1976

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by

RICHARD WILLIAM KARP

A thesis submitted in partial fulfillment
of the requirements for the degree

of

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in

Industrial and Management Engineering

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TABLE OF CONTENTS

	Page
VITA	ii
ACKNOWLEDGMENT	iii
LIST OF TABLES	vi
LIST OF FIGURES	ix
ABSTRACT	x
CHAPTER I - Introduction	1
Scope and Objectives	3
Physical Description of the Study Area	4
CHAPTER II - Ion Concentration Versus Electrical Conductivity and Flow Relationships	7
Data Availability and Preparation	10
Regression Analysis	22
CHAPTER III - Large Scale Water Quantity and Quality Models	72
Quantity Model	73
Quality Model	84
CHAPTER IV - Concluding Remarks	101
Current Utility	101
Recommendations for Future Research	102
REFERENCES	105
APPENDICES	108
APPENDIX I - Raw Data	109
APPENDIX II - Chi-Square Goodness of Fit Test, Z-Test Statistic	141

Page

APPENDIX III - Quantity Model; 1944 to 1973 144

APPENDIX IV - Quality Model; 1944 to 1968, 1944 to 1973 . 154

LIST OF TABLES

Table		Page
1	Equivalent Weights and Conductance Factors for the Common Ions	8
2	Data Availability for the Four Sites	12
3	Summary of Hypothesis Testing for the Various Sites	20
4	Summary of Linear Regression Equations with log(ECM) as the Independent Variable for the Yellowstone River at Billings	23
5	Summary of Linear Regression Equations with log(ECM) as the Independent Variable for the Bighorn River at Bighorn	24
6	Summary of linear Regression Equations with log(ECM) as the Independent Variable for the Tongue River at Miles City	25
7	Summary of Linear Regression Equations with log(ECM) as the Independent Variable for the Yellowstone River at Miles City	26
8	Simulated Common Ion Data in Mg/L for the Yellowstone River at Billings	29
9	Simulated Common Ion Data in Mg/L for the Bighorn River at Bighorn	33
10	Simulated Common Ion Data in Mg/L for the Tongue River at Miles City	37
11	Simulated Common Ion Data in Mg/L for the Yellowstone River at Miles City	41
12	Log(ECM) Versus log(Q) Linear Regression Equations for the Sites	46
13	Log(ion) Versus log(ECM) and log(Q) Linear Regression Equations for the Yellowstone River at Billings	47

Table		Page
14	Log(ion) Versus log(ECM) and log(Q) Linear Regression Equations for the Bighorn River at Bighorn	48
15	Log(ion) Versus log(ECM) and log(Q) Linear Regression Equations for the Tongue River at Miles City	49
16	Log(ion) Versus log(ECM) and log(Q) Linear Regression Equations for the Yellowstone River at Miles City . . .	50
17	Simulated Common Ion Data for the Yellowstone River at Billings in Mg/L	52
18	Simulated Common Ion Data for the Bighorn River at Bighorn in Mg/L	56
19	Simulated Common Ion Data for the Tongue River at Miles City in Mg/L	60
20	Simulated Common Ion Data for the Yellowstone River at Miles City in Mg/L	64
21	Composite Data from the U. S. Geological Survey and Instantaneous Data from the Montana State W. Q. B. for the Tongue River at Miles City	67
22	Simulated Common Ion Data for Composite U. S. Geological Survey and Instantaneous Montana State W. Q. B. Data for the Tongue River at Miles City	69
23	Summary of Equations for Quantity Model	85
24	Summary of Equations for Quality Model	94
25	ECM Values for the Essential Parameters of the Quality Model, 1944 to 1968, Using L2 and L5 as Exogenous (units = μmhos/cm @ 25°c)	96
26	ECM Values for the Essential Parameters of the Quality Model, 1944 to 1968, Employing Perturbed Values of L5 (units - μmhos/cm @ 25°c)	98

Table		Page
A	Raw Data for the Yellowstone River at Billings	110
B	Raw Data for the Bighorn River at Bighorn	114
C	Raw Data for the Tongue River at Miles City	118
D	Raw Data for the Yellowstone River at Miles City	122
E	Common Ion Data for the Yellowstone River at Billings in Meq/L	124
F	Common Ion Data for the Bighorn River at Bighorn in Meq/L	128
G	Common Ion Data for the Tongue River at Miles City in Meq/L	132
H	Common Ion Data for the Yellowstone River at Miles City in Meq/L	136
I	Primary Flow Data 30 Years, 1944 - 1973	138
J	Simulated Values of ECM for 25 Years 1944 - 1968	140

LIST OF FIGURES

Table		Page
1	Schematic Map Showing Middle Yellowstone River Basin	5
2	Grouped ERR Data for the Yellowstone River at Billings . . .	16
3	Grouped ERR Data for the Bighorn River at Bighorn	17
4	Grouped ERR Data for the Tongue River at Miles City	18
5	Grouped ERR Data for the Yellowstone River at Miles City . .	19
6	Schematic of a Sub-basin for an Annual Flow Model	74
7	Schematic of Quantity Model Used in Sub-basin 42KJA	77
8	Schematic of Surface Water Storage	79
9	Schematic of Ground Water Storage	80
10	Schematic of Quality Model Used in Sub-basin 42KJA	88

ABSTRACT

Regression equations were developed for the individual common ions versus electrical conductivity (EC), and the individual common ions versus EC and flow (Q). These regression equations relate the instantaneous concentration of common ions in Meq/L to the instantaneous EC and Q in μmhos/cm @ 25°c and cfs, respectively. The regression equations relating the common ion concentrations to EC produced relatively good results. A cation-anion balance was used to determine the quality of the simulated common ion concentrations.

Water quantity and quality models based upon the principle of conservation of matter and employing the macro-to-micro philosophy were developed on an annual basis. A methodology for obtaining the quality model from the quantity model was developed. The quantity model simulates the amounts of flow associated with the various flow components; and the quality model simulates the EC associated with the various flow components. The use of EC in the quality model provides the necessary parameter for the use of the regression equations involving the common ion concentrations. Both the quantity and quality models provide reasonable results that would be expected on an annual basis.

CHAPTER I

Introduction

The passage of the Federal Water Pollution Control Act Amendments of 1972, Public Law 92-500, exemplifies a major concern for the protection of water quality in the nation's waters. Congress stated that the goals of the Act were to restore and maintain the chemical, physical, and biological integrity of these waters (Ref. 1). Thus, the Act implied the need to describe natural variation in the chemical, physical, and biological behavior of the water ecosystem. These descriptions are needed in order that rational management decisions can be made with respect to water quality.

In response to P. L. 92-500, many governmental agencies that are concerned with water resource management have been collecting water quality data on the nation's waterways. As water quality data increases in amount, it is necessary to analyze the data for relationships which can be used to determine what the natural variations of the various factors are in the ecosystem. These relationships can also provide predictions needed in water quality management. Further, such relationships may identify those parameters for which more data is needed and those parameters for which data collection can be discontinued.

Recent concern for the water quality in the Yellowstone River Basin in Montana has increased due to the increase in coal and energy related developments in the middle portion of the basin. An intent of this study was to analyze the available data for relations that would

aid in the water quality management of this area of the Yellowstone River Basin.

The inverse relationship between the concentration of dissolved constituents and flow (Q) has been well documented in the literature. This hyperbolic relationship has been pursued in a number of investigations on natural waters to determine regression equations. These regression equations are generally in terms of the logarithm of constituent concentration versus the logarithm of the flow (Ref. 7-11). Total dissolved solids (TDS) has received by far the most attention in its inverse relationship with Q (Ref. 8, 9). To date, these regression equations have involved the concentration of dissolved constituents in mass per volume units. In this study, however, the concentration of the common ions used in regression equations are in terms of milliequivalence per litre (Meq/L) which is a measure of the number of charged ion particles per volume. The regression equations are in terms of these concentrations for the common ions versus the measured electrical conductivity (ECM) and Q. In a recent investigation by Leonard Frost, Jr. (Ref. 5), linear regression analysis was performed on a number of dissolved constituents, including the common ions in mass per volume units of concentration, with ECM.

The majority of water quantity and quality models documented in the literature develop an extremely detailed model of a small incremental element of a basin. Often times, because of the extremely complex nature of the interacting components of a real world hydrologic

system, the model is either unsolvable, or because of generalizations and approximations does not give realistic results. In this study, the macro-to-micro philosophy of modeling of large scale systems was employed. This approach provides a number of advantages. First of all, it provides a working model of the system from the very beginning. Even though the model at the most macro level may not produce usable results, it provides a stepping stone toward the micro level. The formulation of relationships at the macro level provide structure that can be built upon at the next level of modeling. Thus, a systematic development of working models is produced which allows for feedback of information. The next level of modeling, toward the micro, provides a check on the model at the previous level. Another advantage to this method of modeling is that in many instances usable results can be obtained from the first few levels (Ref. 2).

Scope and Objectives

The scope and objectives of this study were:

1. To develop a system of regression equations for predicting the level (concentration) of selected water quality parameters at specific points along the Yellowstone River and its major tributaries between Billings and Miles City, Montana. Relationships were sought between the common ions - calcium (Ca), magnesium (Mg), sodium (Na), potassium (K), bicarbonate (HCO_3), carbonate (CO_3), sulfate (SO_4), and chloride (Cl) - and the measured electrical conductivity (ECM), and flow

(Q). The specific points on the Yellowstone River were the U. S. Geological Survey water quality and quantity stations at Billings and Miles City. The points on the major tributaries were the U. S. Geological Survey water quality and quantity stations on the Bighorn River near its confluence with the Yellowstone River at Bighorn, Montana, and on the Tongue River near its confluence with the Yellowstone River at Miles City.

2. To initialize an approach to the large scale modeling required to describe incremental changes in water quality parameters as a result of various water use categories in the Yellowstone River Basin. The Montana State Department of Natural Resources and Conservation's water planning model was calibrated on an annual basis for the Yellowstone River sub-basin 42KJA (Ref. 2, 3). The water planning model is a flow, hydrologic, or "quantity" model which has the potential of providing the flow component quantities required in modeling water quality. After the calibration of the water quantity model for sub-basin 42KJA on an annual basis, this model was employed in developing a water quality model.

Physical Description of the Study Area

Figure 1 shows a sketch map of the middle portion of the Yellowstone River Basin. This portion of the Yellowstone River Basin is on the western edge of the Great Plains. The basin is generally an area of rolling hills with gentle to moderate relief. The major surface water

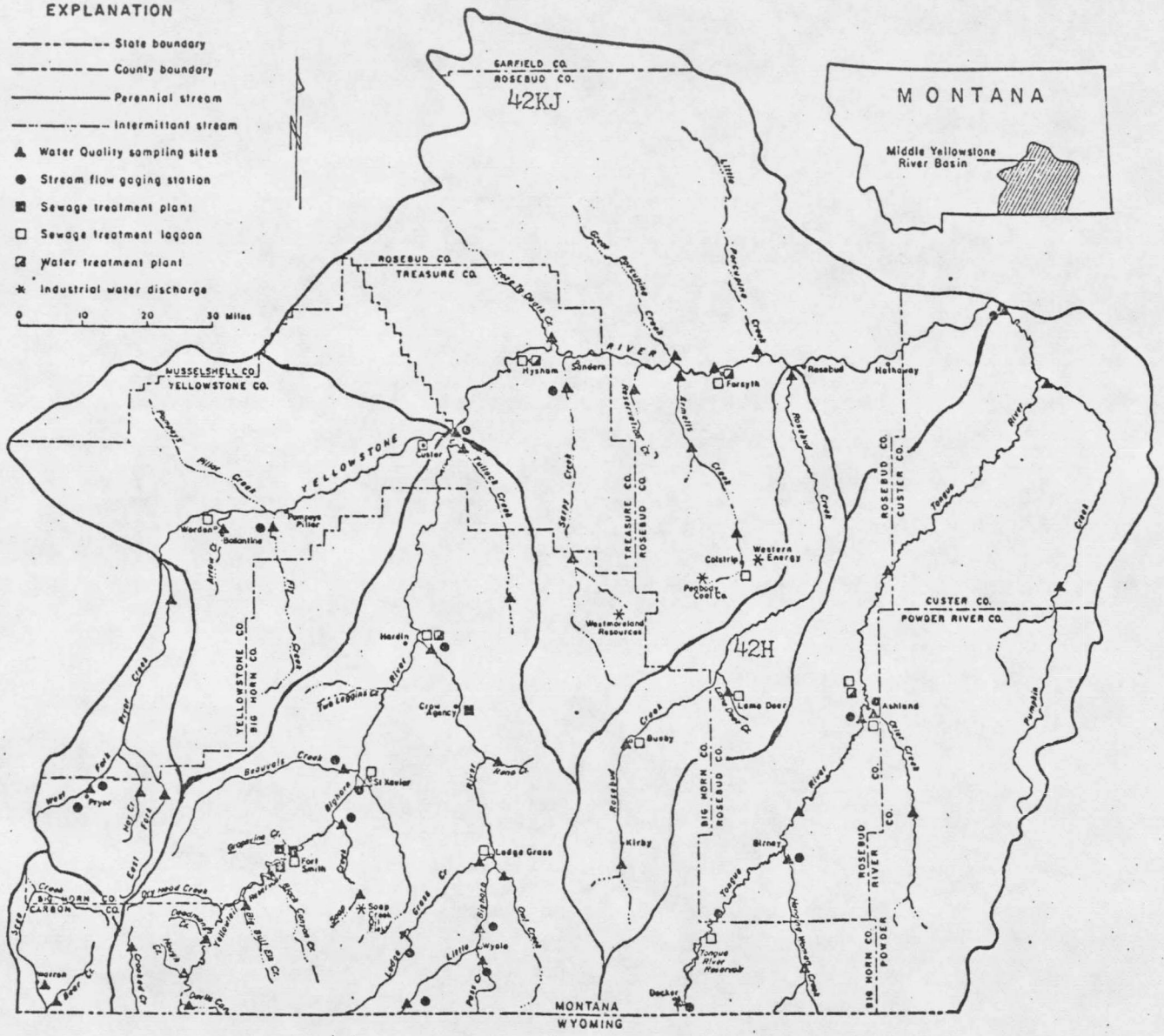


Figure 1 — Sketch map showing Middle Yellowstone River Basin
(Taken from Ref. 12)

inflows to the study area are the Yellowstone and Bighorn Rivers. The confluence of the Yellowstone and Tongue Rivers mark the eastern boundary of the study area. The Tongue River is not included as an inflow to sub-basin 42KJA.

Mountains to the west and southwest of the middle portion of the Yellowstone River Basin create a moisture shadow. Air masses moving toward the basin from these directions must rise to get over the mountains, and thus lose most of their moisture before reaching the basin. Average annual precipitation for the area ranges from 11 to 16 inches per year (Ref. 4).

About 75 percent of the land in the middle portion of the Yellowstone River Basin is used for pasture and range land in the production of livestock. Of the irrigated lands, about 70 percent is used in the production of hay, pasture and sugar beets. There is also a substantial amount of dryland farming for the production of grains. With the renewed interest in coal development, certain areas of this portion of the basin may, in the future, be supporting vast strip mines.

Agricultural irrigation is the primary water use in the basin. With the increase in coal and energy development expected to occur, the amount of water needed for industrial use will increase, but agricultural uses will probably still dominate because of existing water rights.

CHAPTER II

Ion Concentration Versus Electrical Conductivity and Flow Relationships

In this study, the dissolved constituents of interest are the common ions. The common ions represent the major cations and anions dissolved in natural waters. These ionic species are the positive (cations) and negative (anions) components of salts that have become dissolved in the water. These common ion constituents of natural waters are most likely derived from direct chemical solution of minerals in rocks and in soils in which the water comes into contact.

The common ion concentrations are generally expressed in milligrams per litre (Mg/L). This is a mass (weight) concentration. The mass of an ionic constituent divided by its equivalent combining mass (weight), termed equivalent weight, gives a measure of the number of ionic charges (positive or negative) of that specie in the water solution. The equivalent weights of the common ion, listed in Table 1, can be found in any general chemistry textbook. The result of the division of the mass concentration of an ionic specie in Mg/L by its equivalent weight has the units milli-equivalence per litre (Meq/L). In all natural water solutions there exists electrical neutrality, i.e. the number of positive charges equals the number of negative charges. Thus, by summing the concentration in Meq/L, of all the positively charged ions (cations) and subtracting the sum of all the negatively charged ions (anions) the result should be zero. Of course, there are many other

TABLE 1

Equivalent Weights and Conductance

Factors for the Common Ions

Common Ion (abbr.)	Equivalent Weight (Mg/Meq)	Conductance Factor (μmhos/cm @ 25°c/Meq/L)
Ca	20.040	52.0
Mg	12.156	46.6
Na	22.990	48.9
K	39.102	72.0
HCO_3	60.998	43.6
CO_3	29.995	84.6
SO_4	48.030	73.9
Cl	35.453	75.9

constituents dissolved in natural waters besides the common ions. Generally, the total amount of these other constituents is so small compared to the total amount of the common ions that they add little to the sum of either cations or anions and are well within the experimental error involved in measuring the common ions. Thus, in determining the sum of cations and anions in natural waters only the common ions are considered. The difference between the sum of cations and the sum of anions is termed the cation-anion balance. The cation-anion balance is generally used to indicate the quality of a chemical analysis performed on a sample of natural water.

Since the common ions exist in natural waters as charged species, natural waters are electrolytic; i.e., they are capable of carrying an electric current. The conductance of a water solution is related to the amount and type of dissolved ionic constituents. For example, a solution with a high concentration of common ions has a higher conductance than does a solution with a low concentration of common ions. Also, under identical conditions, potassium ions are better conductors than calcium ions. This is an innate feature of the individual ions. Thus, each ionic specie in a solution of natural water makes a contribution to the overall conductance of that solution. The conductance for strong electrolytes as the common ions are classified, approaches a definite value at infinite dillution. That is, when no other ions are present and the concentration of a particular ion is decreased, the conductivity due to that ion approaches a limiting value. This limiting value is

generally termed the conductance factor of the particular ion. Values of the conductance factor for the common ions are given in Table 1 in the units of micro-μmhos per centimeter at 25°c per Meq/L (μmhos/cm @ 25°c/Meq/L). The common unit of conductance reported in the analysis of natural waters is μmhos/cm @ 25°c. Of course, solutions of natural waters are not infinitely dilute and thus, an electrical conductivity calculated from the amounts of the various common ions (ECC) is generally higher in value than the actual measured electrical conductivity (ECM). This is primarily due to the perturbing effect the ions have on one another, which decreases their mobility. As a part of this study, an investigation was made to determine whether a statistical relationship existed between ECM and ECC, and the individual common ions. All relationships between common ions, electrical conductivity and flow rate employ Meq/L for common ion concentration, μmhos/cm @ 25°c for electrical conductivity, and cubic feet per second (cfs) for flow.

Data Availability and Preparation

For the common ion and ECM data available at the selected sites, there are two types, composite and instantaneous. Composite data are obtained by combining a number of samples together according to the value of the flow at the instant each sample was taken. Thus, a composite gives a flow weighted average over a particular time interval. Composite data for the selected sites are available from U. S. Geological Survey annual water resources reports (Ref. 6). Instantaneous data are obtained from individual samples taken at any instant in time. The term

grab sample data is often used to imply instantaneous data. Instantaneous data is available from the U. S. Geological Survey annual reports and from the Montana State Water Quality Bureau. Since one of the objectives of the study was to determine relationships between common ion concentration, ECM, and Q, the instantaneous data were preferred to the composite. Also, once the regression equations for the common ions were developed the composite data would possibly provide a check, much like the split record approach of validation in modeling. Data from the Water Quality Bureau was generally incomplete; i.e., missing certain values of common ion parameters or not having instantaneous flow values. The period of record for which data are available at the four sites is tabulated in Table 2. Only data predating Yellowtail Reservior (1967) were used for the site on the Bighorn River. The data is usually taken at monthly intervals. The earlier data are generally composite and the later data, instantaneous. Another reason for preferring the instantaneous data in the construction of the regression equations is that the most current data has the benefit of better technology.

As previously mentioned, the cation-anion balance is commonly used to determine the quality of a common ion analysis. This same idea was extended to determine the quality of the available instantaneous data. By using the cation-anion balance, a screening process was developed to exclude any sample data for which there was a high probability of error either in the chemical analysis or in the printing of the data for publication. A data point consists of the measured concentrations of

TABLE 2

Data Availability for the Four Sites

Site	Period of Collection	Type of Data
Yellowstone River at Billings	10/50 - 9/58	Composite
	7/63 - 12/69	Composite
	1/70 - Current	Instantaneous
Bighorn River at Bighorn	11/45 - 8/47	Composite
	3/49 - 12/50	Composite
	1/51 - 12/69	Composite
	1/70 - Current	Instantaneous
Tongue River at Miles City	9/48 - 9/49	Composite
	1/51 - 12/69	Composite
	1/70 - Current	Instantaneous
Yellowstone River at Miles City*	10/68 - Current	Composite/ Instantaneous

*For the site, Yellowstone River at Miles City, the flow is taken below the confluence with the Tongue River and the common ion and electrical conductivity data are taken above the confluence.

the various common ions in Meq/L, the measured electrical conductivity (ECM) in μmhos/cm @ 25°c and the measured flow (Q) in cfs. The cation-anion balance which was termed the difference between the sum of the cations and the sum of the anions is not in itself a good gage. For example, suppose that the sum of the cations and the sum of the anions were 2.00 and 1.50 Meq/L, respectively. Then the cation-anion balance would be .50 Meq/L. Now suppose for another sample the sum of the cations and the sum of the anions were 5.00 and 4.50 Meq/L respectively. Again, the cation-anion balance would be .50 Meq/L, but clearly the cation-anion balance in the former case is more seriously in error than in the latter case. In order to use the cation-anion balance for comparison to determine quality, the balance was normalized by dividing by the average of the sum of cations and anions. This normalized cation-anion balance was termed the error of balance (ERR) and is given algebraically by:

$$ERR = \frac{(\Sigma \text{ cations} - \Sigma \text{ anions})}{(\Sigma \text{ cations} - \Sigma \text{ anions})/2} \qquad (1)$$

Tables A through D in Appendix I are tabulations of the raw data for each of the four sites. The concentration of the common ions for this data is in Mg/L. Tables E through H in Appendix I give the concentration of the common ions in Meq/L, the value of ERR, the ERR sample mean ($\overline{ERR}$), and the ERR sample standard deviation (S_{ERR}) for each data point at the four sites.

For a given data point, the values of bicarbonate (HOC_3) and

carbonate (CO_3) together make up what is termed the alkalinity (ALK). In natural waters, especially in this region, HCO_3 is by far the major contributor to ALK. This is basically due to the fact that CO_3 can only exist in natural waters when the pH is greater than 8.3. At a pH of 8.3, any CO_3 present is chemically converted to HCO_3. HCO_3 also has another source besides the solution of minerals from rocks and soils, photosynthesis. Photosynthesis produces carbon dioxide (CO_2) which combines with water (H_2O) to produce HCO_3. CO_3 seems to be significant when present, but most often it is not present. ALK and HCO_3 were used in this study rather than HCO_3 and CO_3. When good regression equations exist for ALK and HCO_3, the concentration of CO_3 can be determined from the difference in ALK and HCO_3.

For the data points at each of the four sites, the ERR values were statistically tested for normal distribution with population mean (μ_{ERR}) equal to zero. A chi-square goodness of fit was employed to test this hypothesis. Rejection of the hypothesis could imply the following conditions:

1. The sample is not representative of one having an underlying normal distribution.

2. The population mean (μ_{ERR}) is not zero.

3. Both, μ_{ERR} is not equal to zero and the underlying distribution is not normal.

A rejection of the hypothesis because u_{ERR} is significantly different than zero could mean that the assumption of the common ions being the major

cations and anions in natural water solutions is not upheld. Neglecting a significant constituent in the determination of ERR would cause μ_{ERR} to be different than zero. Figures 2 through 5 show plots on normal probability paper of the ERR data associated with each site. If the ERR data falls along a straight line this indicates the possibility of an underlying normal distribution. The chi-square goodness of fit test for the ERR data for the Yellowstone River at Billings is shown in Appendix II. The chi-square goodness of fit tests for the other three sites were performed similarly. For each site, except the Yellowstone River at Miles City, the underlying distribution was determined to be normal with μ_{ERR} equal to zero. The level of significance (α) for the chi-square goodness of fit tests was set at .05. Since the hypothesis of a normal distribution with μ_{ERR} equal to zero was rejected for the Yellowstone River at Miles City, a hypothesis was set up to test whether or not μ_{ERR} was significantly different than zero. The z test statistic was employed for testing this hypothesis. The actual hypothesis test is shown in Appendix II. The results of these hypothesis tests are summarized in Table 3.

Using the normal distribution, upper and lower limits were determined on the ERR data for a 95 percent probability of occurrence.

$$Pr(L < ERR < u) = .95$$

$$Pr\left(\frac{L - \mu_{ERR}}{\sigma_{ERR}} < \frac{ERR - \mu_{ERR}}{\sigma_{ERR}} < \frac{u - \mu_{ERR}}{\sigma_{ERR}}\right) = .95$$

Thus, the lower limit, L, is given by:

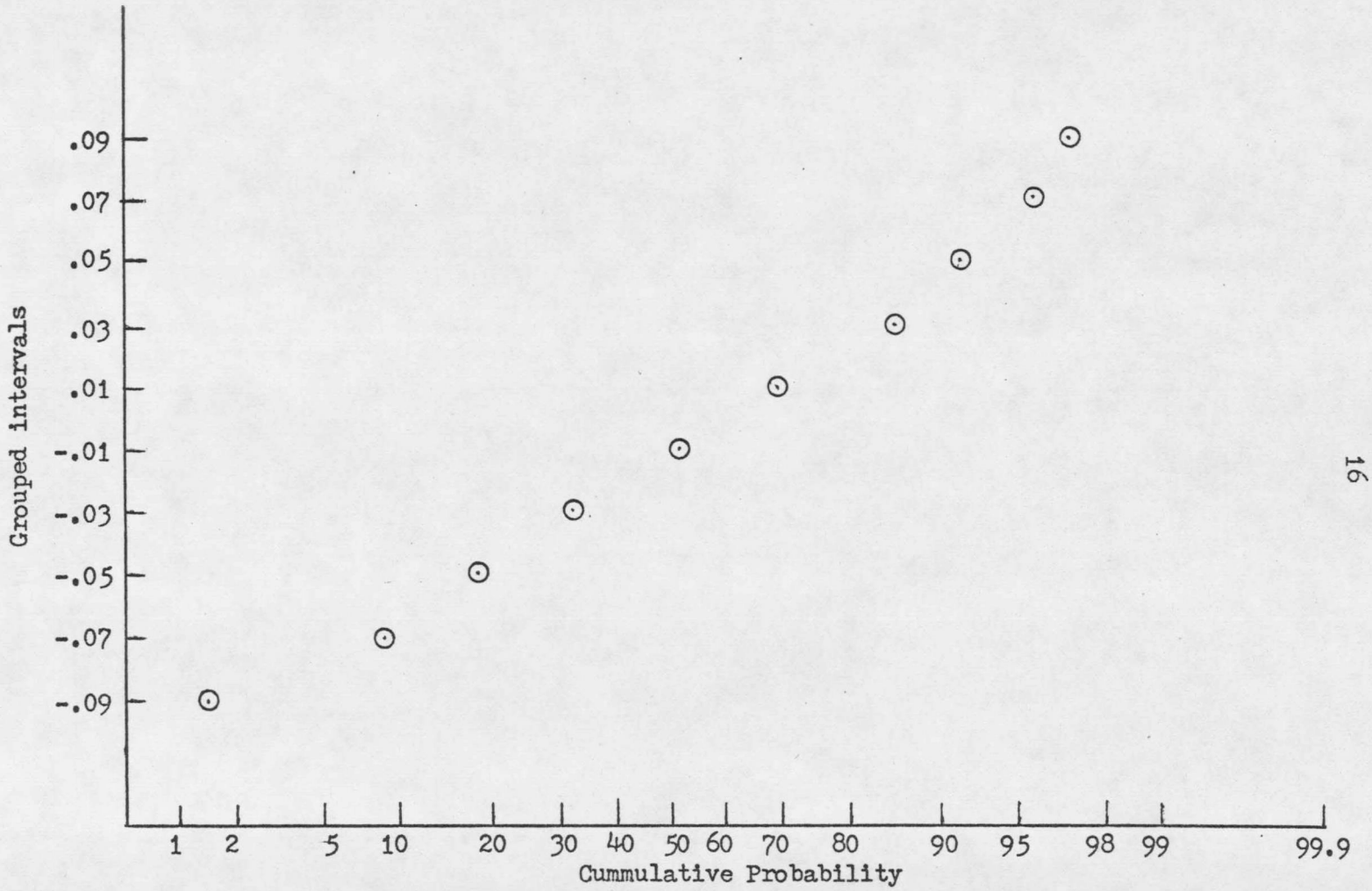


Figure 2. Grouped ERR Data for Yellowstone River at Billings

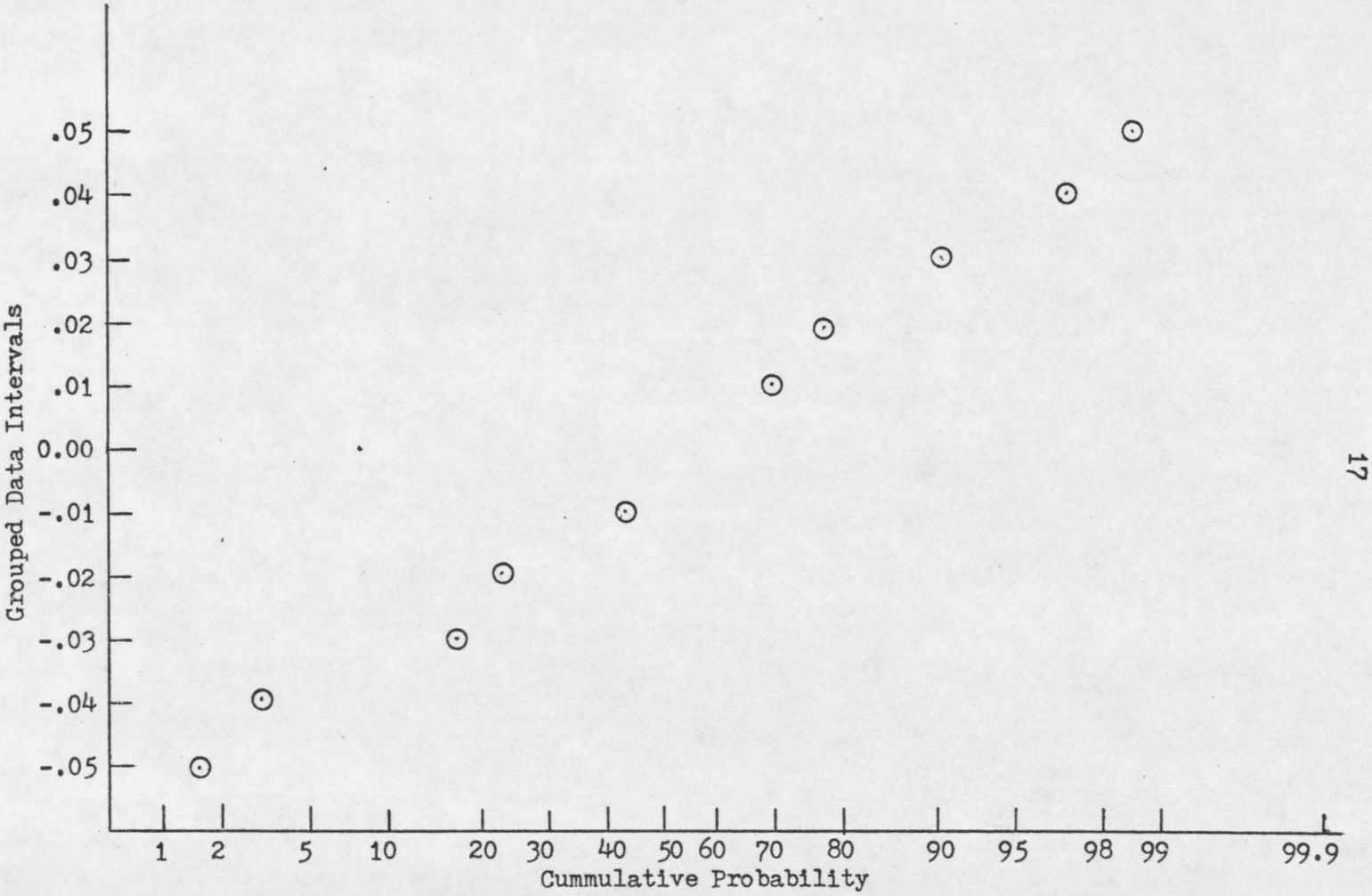


Figure 3. Group ERR Data for Bighorn at Bighorn

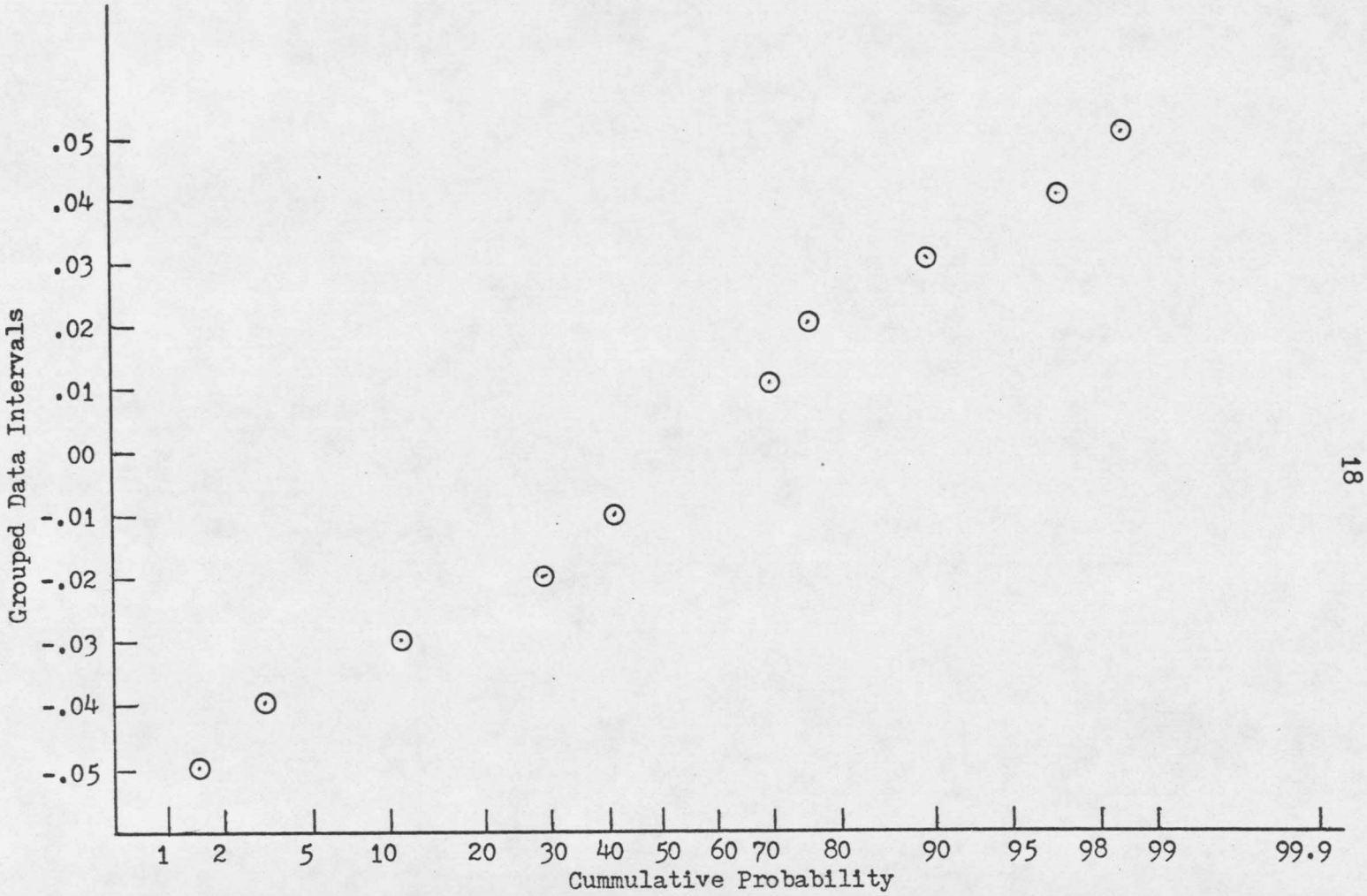


Figure 4. Grouped ERR Data for Tongue River at Miles City

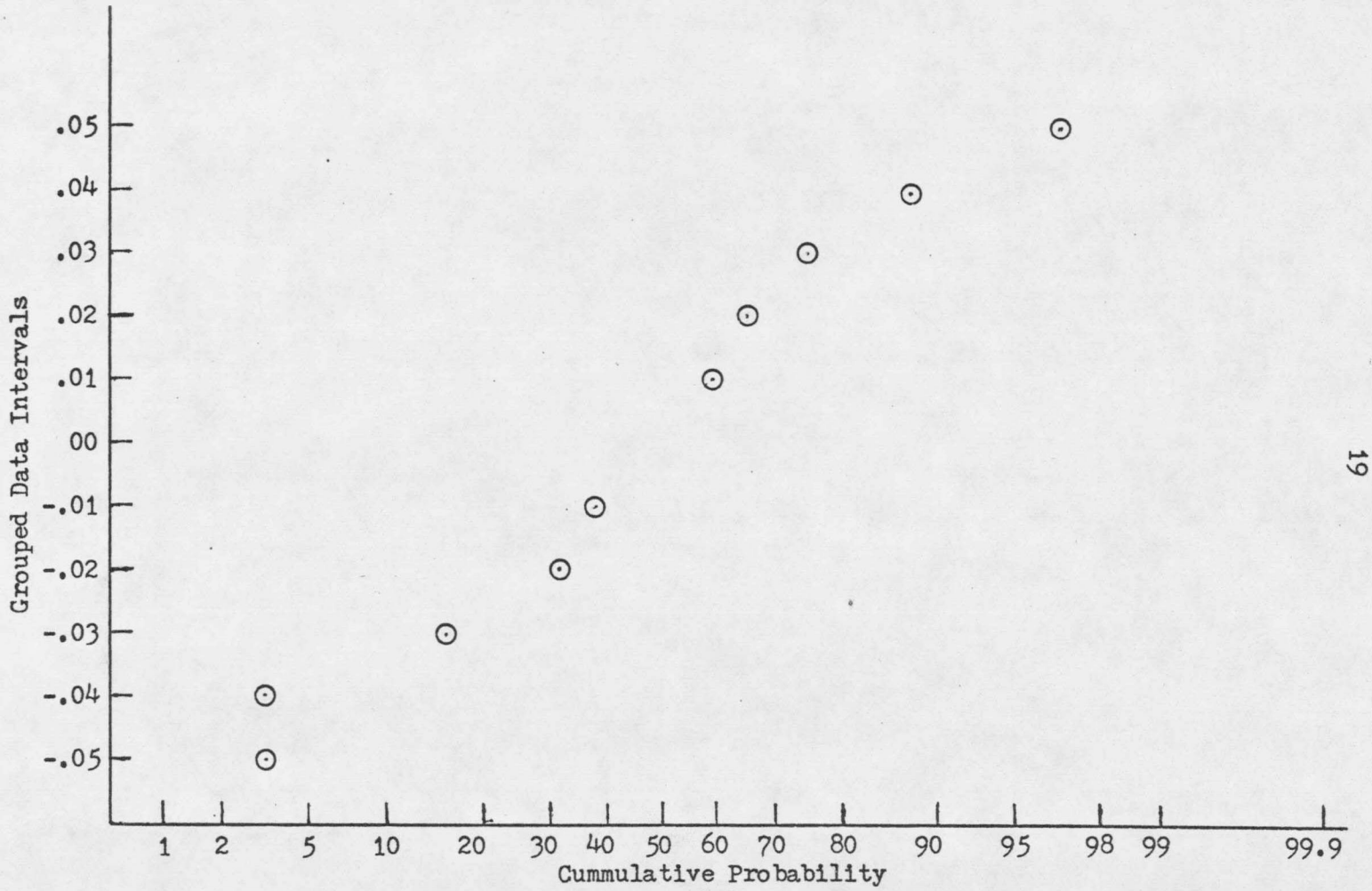


Figure 5. Grouped ERR Data for the Yellowstone River at Miles City

TABLE 3

Summary of Hypothesis Testing for the Various Sites

Site	Hypothesis Test	Test Statistic	Degrees of Freedom	Critical Test* Statistic Limit
Yellowstone River at Billings	Chi-square Goodness of Fit	$X^2 = 5.28$	5	$X^2_\alpha = 11.07$
Bighorn River at Bighorn	Chi-square Goodness of Fit	$X^2 = 2.90$	3	$X^2_\alpha = 7.81$
Tongue River at Miles City	Chi-square Goodness of Fit	$X^2 = 3.67$	3	$X^2_\alpha = 7.81$
Yellowstone River at Miles City	Chi-square Goodness of Fit	$X^2 = 17.77$	3	$X^2_\alpha = 7.81$
Yellowstone River at Miles City	Z Test for Means	$Z = 0.51$		$Z_{\alpha/2} \pm 1.96$

* α = .05 level of significance for all tests

$$L = -1.96 \cdot \sigma_{ERR}$$

and the upper limit, U, is given by

$$U = 1.96 \cdot \sigma_{ERR}$$

The sample standard deviation (S_{ERR}) was used as an approximation to σ_{ERR}. These upper and lower limits, termed acceptance limits, were used to screen the data. For example, consider a data point whose ERR value falls outside the interval created by the acceptance limits. Such a data point has a probability of occurrence of only .05. Since errors can be made in the chemical analysis of a water sample or in the tabulation and publishing of the data, there is a relatively high probability that some of the data for this data point is incorrect. By rejecting data points whose ERR values lie outside the interval, only good quality data is used in the development of the regression equations. One of the characteristics of least-square regression analysis is that minimum variance rather than minimum deviation is used. Thus, if erroneous data is used there is the possibility of the analysis showing little or no statistical correlation when in fact one does exist. Since the hypothesis of a normal distribution for ERR was rejected for the Yellowstone River at Miles City, no attempt was made to screen this data. This screening process resulted in the rejection of 3 data points from the 68 data points for the Yellowstone River at Billings, 3 data points from the 66 data points for the Bighorn River at Bighorn, and 2 data points from the 66 data points for the Tongue River at Miles City.

Regression Analysis

At each of the four sites, linear regression analyses were performed on the acceptable data. A value for the calculated electrical conductivity (ECC) was determined for each data point at each site using the conductance factors in Table 1.

$$ECC = \Sigma_i K_i (ion)_i \qquad (2)$$

Where K_i is the conductance factor for the i^{th} ion, and

$(ion)_i$ is the concentration in Meg/L of the i^{th} ion.

The ECC along with the common ions were regressed individually with the natural logarithm of the ECM as the independent variable. The results of the linear regression analyses for the four sites are shown in Tables 4 through 7. The general form of the regression equations is:

$$(ion)_i = bi \log(ECM) - ai \qquad (3)$$

Where:

$(ion)_i$ is concentration of the i^{th} ion in Meg/L or ECC

ai and bi are the regression constants for the i^{th} ion

ECM is in μmhos/cm @ 25°c

The regression equations have high R^2 values except for those of K and Cl. The R^2 values for these two ions are consistently low for each site. The t-values listed in Tables 4 through 7 refer to the significance of the coefficient bi on the independent variable. The .05 significance level (α) for the regression

TABLE 4

Summary of Linear Regression Equations with Log (ECM) as the Independent Variable for the Yellowstone River at Billings

Dependent Variable	Regression Equation	R^2	t Value
ECC	ECC = 359.68 · Log (ECM) - 1699.03	.959	38.60
Ca	Ca = 1.322 · Log (ECM) - 6.004	.923	27.45
Mg	Mg = 0.8547 · Log (ECM) - 4.022	.939	31.02
Na	Na = 0.9595 · Log (ECM) - 4.595	.891	22.68
K	K = .05645 · Log (ECM) - .2509	.561	8.97
ALK	ALK = 1.543 · Log (ECM) - 6.716	.963	40.41
HCO_3	HCO_3= 1.538 · Log (ECM) - 6.686	.962	39.65
SO_4	SO_4 = 1.454 · Log (ECM) - 7.153	.908	24.87
Cl	Cl = .1409 · Log (ECM) - .6565	.669	11.29

TABLE 5

Summary of Linear Regression Equations with Log (ECM) as the Independent Variable for the Bighorn River at Bighorn

Dependent Variable	Regression Equation	R^2	t Value
ECC	ECC = 1116 · Log (ECM) - 6479	.926	27.66
Ca	Ca = 3.047 · Log (ECM) - 16.91	.822	16.75
Mg	Mg = 2.420 · Log (ECM) - 14.17	.791	15.19
Na	Na = 4.194 · Log (ECM) - 24.90	.854	18.85
K	K = .07011 · Log (ECM) - .3750	.396	6.33
ALK	ALK = 2.132 · Log (ECM) - 11.04	.736	13.03
HCO_3	HCO_3= 2.092 · Log (ECM) - 10.78	.707	12.13
SO_4	SO_4 = 7.018 · Log (ECM) - 41.66	.892	22.50
Cl	Cl = .2846 · Log (ECM) - 1.635	.571	9.01

TABLE 6

Summary of Linear Regression Equations with Log (ECM) as the Independent Variable for the Tongue River at Miles City

Dependent Variable	Regression Equation	R^2	t Value
ECC	ECC = 1023 · Log (ECM) - 5813	.940	31.27
Ca	Ca = 2.278 · Log (ECM) - 12.15	.807	16.09
Mg	Mg = 3.756 · Log (ECM) - 21.64	.898	23.35
Na	Na = 3.067 · Log (ECM) - 17.84	.811	16.30
K	K = .09219 · Log (ECM) - .4877	.514	8.09
ALK	ALK = 3.448 · Log (ECM) - 18.56	.830	17.37
HCO_3	HCO_3= 3.411 · Log (ECM) - 18.33	.822	16.94
SO_4	SO_4 = 5.595 · Log (ECM) - 32.65	.898	23.38
Cl	Cl = .09498 · Log (ECM) - .5214	.535	8.45

TABLE 7

Summary of Linear Regression Equations with Log (ECM) as the Independent Variable for the Yellowstone River at Miles City

Dependent Variable	Regression Equation	R^2	t Value
ECC	ECC = 625.6 · Log (ECM) - 3272	.960	26.86
Ca	Ca = 1.834 · Log (ECM) - 9.130	.933	20.50
Mg	Mg = 1.355 · Log (ECM) - 7.070	.907	17.14
Na	Na = 2.207 · Log (ECM) - 11.83	.948	23.34
K	K = .0620 · Log (ECM) - .3069	.837	12.41
ALK	ALK = 1.660 · Log (ECM) - 7.816	.960	26.84
HCO_3	HCO_3= 1.637 · Log (ECM) - 7.688	.957	25.92
SO_4	SO_4 = 3.610 · Log (ECM) - 19.56	.927	19.57
Cl	Cl = .1926 · Log (ECM) - 1.006	.840	12.56

constant, bi, is ± 1.96. If the t-value is greater than 1.96 or less than -1.96, then the regression coefficient has statistical significance. Thus, the coefficients, bi, are all significant, even those for the regression equations of K and Cl. The low R^2 values for the K and Cl regression equations may suggest that the variations in K and Cl are not sufficiently measurable with variations in ECM. The magnitude of the K and Cl concentrations are relatively small in comparison to those of the other common ions. K and Cl concentrations are reported generally to only two significant figures. The current analytical methods for determining K and Cl concentrations can yield more accurate results than have been published. The publishing of K and Cl concentrations to only two significant figures is probably due to historical publishing and the fact that more accurate results have little effect on the cation-anion balance. By replacing the regression equations for K and Cl with the regression equation for ECC and the cation-anion balance equation, a better set of regressions might be produced. The procedure would be to calculate all the common ion concentrations, except for K and Cl, from their respective regression equations and then solve the ECC regression equation and the cation-anion balance equation simultaneously for K and Cl. In the ECC regression equation, ECC would be replaced by its expression from equation (2). The undesirable aspect of this system of equations is that the cation-anion balance equation is lost as a check on the predicted values of the common ions. The possibility of using the ECC regression equation to show consistency is also lost.

The form of the regression equation for ECC may be quite realistic, i.e. a physical regression equation as opposed to a purely statistical regression equation. For high ECM values the ECC values would tend to be quite a bit larger than the ECM values, whereas at low ECM values the ECC values would tend to be closer to the ECM values. Physically, this may be due to increased interference with ion mobility and the possible lack of full ionization of the salt molecules at high ECM values. The high R^2 values associated with the regression equations for the common ions may indicate that the relative proportions of the common ions in natural water solutions remain somewhat constant. For example, suppose that the fraction, $(ion)_i/ECC$, equaled a constant:

$$(ion)_i/ECC = C_i \qquad (4)$$

Then from the regression equation for ECC

$$ECC = B \log (ECM) - A$$

upon substituting for ECC from equation (4) and rearranging terms, the following relation is obtained:

$$(ion)_i = (C_i \cdot B) \log (ECM) - (C_i \cdot A)$$

This is the form of the regression equations for the common ions, equation (3) where $(C_i \cdot B)$ would be bi and $(C_i \cdot A)$ would be ai.

Using the regression equations in Tables 4 to 7, and the ECM data from which the regression equations were originally generated, values of the common ions were simulated. These simulated values for the common ions for the four sites are shown in Tables 8 through 11. The common ion concentrations have been converted back to Mg/L for comparison with

TABLE 8

Simulated Common ion Data in Mg/L for

the Yellowstone River at Billings

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO3 (Mg/L)	CO3 (Mg/L)	SO4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
37.9	13.2	26.1	3.4	152.7	.0	73.6	6.6	-.0046	391	393
37.7	13.1	25.9	3.4	151.7	.0	72.9	6.5	-.0048	387	389
38.3	13.3	26.4	3.4	153.8	.0	74.5	6.6	-.0044	396	398
42.4	14.9	29.8	3.7	168.4	.0	85.4	7.4	-.0021	459	465
40.8	14.3	28.5	3.6	162.8	.0	81.2	7.1	-.0029	434	438
43.2	15.2	30.5	3.8	171.2	.0	87.4	7.6	-.0018	473	479
38.7	13.5	26.8	3.4	155.5	.0	75.7	6.7	-.0041	402	405
32.1	10.9	21.3	2.9	131.9	.0	58.2	5.5	-.0090	316	315
6.1	.7	- .4	.7	39.7	.0	-10.4	.6	-.2120	123	118
18.0	5.3	9.5	1.7	81.9	.0	21.0	2.8	-.0332	190	185
31.1	10.5	20.5	2.8	128.6	.0	55.7	5.3	-.0098	306	304
34.5	11.8	23.3	3.1	140.4	.0	64.5	5.9	-.0070	345	345
37.5	13.0	25.7	3.3	151.0	.0	72.4	6.5	-.0049	384	386
38.3	13.3	26.5	3.4	154.1	.0	74.7	6.6	-.0043	397	399
37.5	13.0	25.7	3.3	151.0	.0	72.4	6.5	-.0049	384	386
34.1	11.7	22.9	3.1	139.1	.0	63.5	5.8	-.0073	340	340

Table 8 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO3 (Mg/L)	CO3 (Mg/L)	SO4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
37.9	13.1	26.1	3.4	152.4	.0	73.5	6.6	-.0046	390	392
40.9	14.3	28.6	3.6	163.3	.0	81.5	7.1	-.0029	436	440
40.3	14.1	28.1	3.6	160.9	.0	79.8	7.0	-.0032	425	429
23.2	7.4	13.8	2.1	100.3	.0	34.7	3.8	-.0204	229	225
14.3	3.9	6.5	1.4	68.9	.0	11.3	2.1	-.0492	166	161
17.1	5.0	8.8	1.6	78.8	.0	18.7	2.6	-.0363	184	179
27.3	9.0	17.3	2.5	115.0	.0	45.6	4.6	-.0141	266	263
32.3	11.0	21.5	2.9	132.8	.0	58.8	5.5	-.0087	319	318
33.0	11.2	22.0	3.0	135.1	.0	60.6	5.6	-.0082	327	326
36.5	12.6	24.9	3.3	147.5	.0	69.8	6.3	-.0055	371	372
38.3	13.3	26.4	3.4	153.8	.0	74.5	6.6	-.0044	396	398
43.5	15.4	30.8	3.8	172.4	.0	88.3	7.6	-.0016	478	485
40.3	14.1	28.1	3.6	160.9	.0	79.8	7.0	-.0032	425	429
39.8	13.9	27.7	3.5	159.3	.0	78.6	6.9	-.0035	419	422
40.9	14.3	28.6	3.6	163.0	.0	81.4	7.1	-.0029	435	439
46.8	16.6	33.5	41	184.0	.0	97.0	8.2	-.0002	539	549
22.2	7.0	13.0	2.1	96.9	.0	32.2	3.6	-.0223	221	217
16.8	4.9	8.5	1.6	77.8	.0	17.9	2.6	-.0374	182	177

Table 8 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
37.5	13.0	25.7	3.3	151.0	.0	72.4	6.5	-.0049	384	386
37.2	12.9	25.5	3.3	150.0	.0	71.6	6.4	-.0051	380	382
43.1	15.2	30.4	3.8	170.8	.0	87.2	7.5	-.0018	471	477
49.2	17.6	35.5	4.3	192.7	.0	103.4	8.7	.0007	588	602
39.0	13.6	27.0	3.5	156.4	.0	76.4	6.8	-.0040	406	409
41.2	14.4	28.8	3.6	164.1	.0	82.2	7.2	-.0028	439	444
41.3	14.5	28.9	3.7	164.5	.0	82.5	7.2	-.0027	441	446
44.3	15.7	31.4	3.9	175.1	.0	90.3	7.8	-.0013	492	499
23.2	7.4	13.8	2.1	100.3	.0	34.7	3.8	-.0204	229	225
18.5	5.6	10.0	1.8	84.0	.0	22.5	2.9	-.0314	194	189
28.6	9.5	18.3	2.6	119.5	.0	48.9	4.8	-.0125	279	276
36.1	12.5	24.6	3.2	146.2	.0	68.8	6.2	-.0058	366	367
38.4	13.4	26.5	3.4	154.3	.0	74.9	6.7	-.0043	398	400
38.5	13.4	26.6	3.4	154.8	.0	75.2	6.7	-.0042	400	402
41.9	14.7	29.5	3.7	166.8	.0	84.2	7.3	-.0024	452	457
19.9	6.1	11.1	1.9	88.8	.0	26.1	3.2	-.0275	204	199
40.8	14.3	28.5	3.6	162.8	.0	81.2	7.1	-.0029	434	438
40.7	14.3	28.4	3.6	162.4	.0	80.9	7.1	-.0030	432	436

Table 8 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
43.9	15.5	31.1	3.9	173.7	.0	89.3	7.7	-.0014	485	492
43.0	15.2	30.4	3.8	170.6	.0	87.0	7.5	-.0018	470	476
43.4	15.3	30.7	3.8	172.0	.0	88.0	7.6	-.0017	476	483
39.1	13.6	27.1	3.5	156.9	.0	76.8	6.8	-.0039	408	411
30.3	10.2	19.8	2.7	125.7	.0	53.6	5.1	-.0106	297	295
12.1	3.0	4.6	1.2	61.0	.0	5.4	1.7	-.0651	153	148
12.8	3.3	5.2	1.3	63.5	.0	7.3	1.8	-.0593	157	152
26.2	8.6	16.3	2.4	110.9	.0	42.6	4.3	-.0156	255	252
36.6	12.6	25.0	3.3	147.8	.0	70.0	6.3	-.0055	372	373
40.9	14.3	28.6	3.6	163.0	.0	81.4	7.1	-.0029	435	439
38.8	13.5	26.9	3.4	155.7	.0	75.9	6.7	-.0041	403	406
40.2	14.1	28.0	3.6	160.7	.0	79.6	7.0	-.0033	424	428
39.8	13.9	27.7	3.5	159.1	.0	78.4	6.9	-.0035	418	421
39.5	13.8	27.4	3.5	158.2	.0	77.8	6.9	-.0037	414	417
19.9	6.1	11.1	1.9	88.8	.0	26.1	3.2	-.0275	204	199
32.2	10.9	21.4	2.9	132.5	.0	58.6	5.5	-.0088	318	317

TABLE 9

Simulated Common ion Data in Mg/L for

the Bighorn River at Bighorn

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO3 (Mg/L)	CO3 (Mg/L)	SO4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
81.8	30.4	91.8	4.2	221.7	.0	321.4	11.5	.0021	982	982
76.9	28.1	84.1	4.0	211.4	.0	294.2	10.7	-.0001	906	906
82.2	30.6	92.4	4.2	222.5	.0	323.4	11.6	.0023	988	988
80.2	29.7	89.4	4.2	218.4	.0	312.7	11.3	.0015	957	957
79.8	29.4	88.6	4.1	217.5	.0	310.2	11.2	.0013	950	950
80.5	29.8	89.8	4.2	218.9	.0	314.1	11.3	.0016	961	961
81.1	30.1	90.7	4.2	220.1	.0	317.2	11.4	.0018	970	970
79.5	29.3	88.2	4.1	216.9	.0	308.8	11.2	.0011	946	946
75.8	27.5	82.4	4.0	209.1	.0	288.2	10.6	-.0007	890	890
72.3	25.8	76.8	3.8	201.7	.0	268.7	10.0	-.0026	841	840
64.5	22.1	64.6	3.4	185.6	.0	226.0	8.7	-.0077	741	740
49.7	14.9	41.1	2.8	154.5	.0	143.9	6.2	-.0237	581	580
77.2	28.2	84.5	4.0	212.0	.0	295.7	10.8	.0000	910	910
65.5	22.6	66.1	3.5	187.6	.0	231.4	8.9	-.0069	753	752
76.2	27.7	82.9	4.0	209.8	.0	290.1	10.6	-.0005	895	895
65.1	22.4	65.5	3.5	186.8	.0	229.2	8.8	-.0072	748	747

Table 9 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
72.0	25.7	76.3	3.8	201.1	.0	267.1	9.9	-.0027	837	836
71.0	25.2	74.8	3.7	199.1	.0	261.8	9.8	-.0033	824	823
76.6	27.9	83.7	4.0	210.8	.0	292.7	10.7	-.0002	902	902
77.9	28.5	85.7	4.0	213.5	.0	299.8	10.9	.0004	921	921
92.5	35.6	108.7	4.7	244.0	1.0	380.4	13.3	.0061	1173	1170
68.8	24.1	71.2	3.6	194.4	.0	249.3	9.4	-.0047	794	793
73.6	26.5	78.8	3.9	204.5	.0	275.9	10.2	-.0018	859	858
77.4	28.3	84.9	4.0	212.5	.0	297.2	10.8	.0002	914	914
71.0	25.2	74.8	3.7	199.1	.0	261.8	9.8	-.0033	824	823
75.0	27.1	81.1	3.9	207.4	.0	283.7	10.4	-.0011	879	878
69.8	24.6	72.8	3.7	196.5	.0	254.8	9.6	-.0041	807	806
73.6	26.5	78.9	3.9	204.6	.0	276.3	10.2	-.0018	860	859
66.3	23.0	67.4	3.5	189.3	.0	235.9	9.0	-.0064	763	762
78.2	28.7	86.2	4.1	214.2	.0	301.6	11.0	.0005	926	926
73.3	26.3	78.4	3.8	203.9	.0	274.3	10.1	-.0020	855	854
74.5	26.9	80.3	3.9	206.4	.0	281.0	10.3	-.0013	872	871
76.6	27.9	83.5	4.0	210.7	.0	292.4	10.7	-.0003	901	901
78.9	29.0	87.2	4.1	215.6	.0	305.2	11.1	.0008	936	936

Table 9 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
79.5	29.3	88.2	4.1	216.9	.0	308.8	11.2	.0011	946	946
78.0	28.6	85.9	4.1	213.8	.0	300.5	10.9	.0004	923	923
64.5	22.1	64.6	3.4	185.6	.0	226.0	8.7	-.0077	741	740
65.1	22.4	65.5	3.5	186.8	.0	229.2	8.8	-.0072	748	747
81.9	30.5	91.9	4.2	221.8	.0	321.7	11.6	.0022	983	983
79.3	29.2	87.9	4.1	216.5	.0	307.7	11.1	.0011	943	943
80.0	29.6	89.1	4.1	218.0	.0	311.6	11.3	.0014	954	954
78.2	28.7	86.2	4.1	214.2	.0	301.6	11.0	.0005	926	926
51.6	15.9	44.2	2.9	158.6	.0	154.8	6.6	-.0208	600	599
51.5	15.8	44.0	2.9	158.4	.0	154.2	6.5	-.0210	599	598
52.6	16.4	45.8	2.9	160.7	.0	160.3	6.7	-.0195	610	609
82.1	30.5	92.2	4.2	222.2	.0	322.8	11.6	.0022	986	986
82.6	30.8	93.0	4.3	223.2	.0	325.5	11.7	.0024	994	994
80.6	29.8	90.0	4.2	219.2	.0	314.8	11.4	.0016	963	963
83.5	31.3	94.6	4.3	225.3	1.0	330.9	11.8	.0028	1013	1010
93.0	35.8	109.6	4.7	245.1	1.0	383.3	13.4	.0062	1183	1180
85.9	32.4	98.3	4.4	230.2	1.0	344.0	12.2	.0038	1053	1050
78.7	28.9	86.9	4.1	215.1	.0	304.1	11.0	.0008	933	933

Table 9 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
76.4	27.8	83.3	4.0	210.4	.0	291.6	10.7	-.0003	899	899
74.2	26.8	79.8	3.9	205.8	.0	279.4	10.3	-.0015	868	867
66.2	22.9	67.3	3.5	189.1	.0	235.4	9.0	-.0064	762	761
76.5	27.9	83.4	4.0	210.6	.0	292.0	10.7	-.0003	900	900
77.4	28.3	84.9	4.0	212.5	.0	297.2	10.8	.0002	914	914
74.1	26.7	79.7	3.9	205.6	.0	279.0	10.3	-.0015	867	866
75.9	27.6	82.6	4.0	209.4	.0	289.0	10.6	-.0006	892	892
78.8	28.9	87.0	4.1	215.3	.0	304.5	11.0	.0008	934	934
75.0	27.1	81.1	3.9	207.4	.0	283.7	10.4	-.0011	879	878
63.1	21.4	62.3	3.4	182.6	.0	218.2	8.5	-.0088	724	723
92.0	35.3	107.9	4.7	242.9	1.0	377.5	13.2	.0059	1163	1160
89.8	34.3	104.5	4.6	238.5	1.0	365.7	12.9	.0052	1123	1120
55.4	17.7	50.1	3.0	166.4	.0	175.5	7.2	-.0162	638	637
87.1	32.9	100.1	4.5	232.6	1.0	350.3	12.4	.0042	1073	1070

TABLE 10

Simulated Common ion Data in Mg/L for

the Tongue River in Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
59.0	39.5	57.1	4.8	260.8	.0	212.6	3.8	-.0023	752	755
61.8	42.3	61.4	5.0	273.3	1.0	228.8	4.0	-.0018	801	802
77.0	57.5	84.9	6.2	342.9	1.0	318.6	5.2	.0001	1117	1120
81.3	61.8	91.5	6.6	362.4	1.0	343.8	5.5	.0005	1227	1230
71.3	51.8	76.1	5.8	316.8	1.0	284.9	4.7	-.0005	986	988
69.8	50.2	73.7	5.7	309.7	1.0	275.8	4.6	-.0007	953	955
77.8	58.3	86.2	6.3	346.5	1.0	323.4	5.2	.0002	1137	1140
67.8	48.2	70.6	5.5	300.6	1.0	264.0	4.5	-.0009	912	914
50.4	30.9	43.6	4.1	221.4	.0	161.8	3.2	-.0044	623	625
25.9	6.3	5.9	2.2	109.5	.0	17.3	1.4	-.0282	364	365
50.6	31.1	44.1	4.2	222.4	.0	163.1	3.2	-.0043	626	628
54.3	34.7	49.7	4.4	239.0	.0	184.5	3.5	-.0033	677	680
70.9	51.4	75.5	5.8	315.1	1.0	282.7	4.7	-.0005	978	980
64.4	44.9	65.4	5.2	285.2	1.0	244.1	4.2	-.0014	848	849
65.9	46.3	67.7	5.4	292.0	1.0	252.9	4.3	-.0012	875	877
73.2	53.7	79.0	5.9	325.4	1.0	296.1	4.9	-.0003	1028	1030

Table 10 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
77.4	57.9	85.5	6.3	344.7	1.0	321.0	5.2	.0001	1127	1130
74.1	54.6	80.4	6.0	329.4	1.0	301.3	4.9	-.0002	1048	1050
73.7	54.1	79.7	6.0	327.4	1.0	298.7	4.9	-.0002	1038	1040
79.0	59.5	88.0	6.4	352.0	1.0	330.3	5.3	.0003	1167	1170
66.3	46.8	68.4	5.4	294.1	1.0	255.6	4.4	-.0011	884	886
34.9	15.4	19.8	2.9	150.7	.0	70.5	2.0	-.0125	444	445
39.3	19.8	26.6	3.3	170.8	.0	96.4	2.4	-.0091	489	490
48.5	28.9	40.8	4.0	212.6	.0	150.4	3.0	-.0050	597	599
55.8	36.3	52.2	4.6	246.2	.0	193.8	3.6	-.0030	701	704
57.1	37.6	54.2	4.7	252.1	.0	201.3	3.7	-.0027	721	724
73.2	53.7	79.0	5.9	325.4	1.0	296.1	4.9	-.0003	1028	1030
75.4	55.9	82.4	6.1	335.3	1.0	308.8	5.0	-.0001	1077	1080
77.0	57.5	84.9	6.2	342.9	1.0	318.6	5.2	.0001	1117	1120
76.2	56.7	83.6	6.2	339.1	1.0	313.8	5.1	.0000	1097	1100
36.7	17.2	22.6	3.1	159.0	.0	81.2	2.2	-.0110	462	463
67.0	47.5	69.4	5.4	297.1	1.0	259.5	4.4	-.0010	897	899
76.2	56.7	83.6	6.2	339.1	1.0	313.8	5.1	.0000	1097	1100
43.3	23.8	32.8	3.6	189.1	.0	120.0	2.7	-.0070	533	535

Table 10 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
62.9	43.4	63.1	5.1	278.5	1.0	235.5	4.1	-.0016	821	822
67.2	47.6	69.7	5.5	297.8	1.0	260.4	4.4	-.0010	900	902
71.3	51.8	76.1	5.8	316.8	1.0	284.9	4.7	-.0005	986	988
70.3	50.8	74.5	5.7	312.1	1.0	278.8	4.7	-.0006	964	966
57.2	37.7	54.3	4.7	252.3	.0	201.7	3.7	-.0027	722	725
77.8	58.3	86.2	6.3	346.5	1.0	323.4	5.2	.0002	1137	1140
68.2	48.6	71.2	5.5	302.4	1.0	266.3	4.5	-.0009	920	922
37.1	17.6	23.2	3.1	160.8	.0	83.5	2.2	-.0106	466	467
52.4	32.9	46.9	4.3	230.6	.0	173.6	3.3	-.0038	651	653
66.1	46.6	68.0	5.4	292.9	1.0	254.1	4.3	-.0012	879	881
59.3	39.7	57.5	4.8	261.9	.0	214.0	3.8	-.0023	756	759
79.0	59.5	88.0	6.4	352.0	1.0	330.3	5.3	.0003	1167	1170
78.2	58.7	86.8	6.3	348.4	1.0	325.7	5.2	.0002	1147	1150
71.9	52.3	76.9	5.8	319.3	1.0	288.1	4.8	-.0004	998	1000
69.0	49.5	72.5	5.6	306.2	1.0	271.2	4.6	-.0008	937	939
69.9	50.4	73.9	5.7	310.3	1.0	276.6	4.6	-.0007	956	958
66.4	46.9	68.5	5.4	294.3	1.0	255.9	4.4	-.0011	885	887
40.5	21.0	28.5	3.4	176.2	.0	103.5	2.5	-.0084	502	503

Table 10 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
49.3	29.8	42.1	4.0	216.4	.0	155.3	3.1	-.0047	608	610
47.0	27.5	38.5	3.9	205.9	.0	141.7	2.9	-.0055	578	580
56.8	37.3	53.7	4.6	250.6	.0	199.5	3.7	-.0028	716	719
71.7	52.2	76.7	5.8	318.6	1.0	287.3	4.8	-.0004	995	997
68.3	48.7	71.4	5.5	302.8	1.0	266.9	4.5	-.0009	922	924
62.4	42.9	62.3	5.1	276.2	1.0	232.5	4.1	-.0017	812	813
64.9	45.4	66.2	5.3	287.6	1.0	247.3	4.3	-.0013	857	859
70.1	50.6	74.3	5.7	311.4	1.0	278.0	4.6	-.0006	961	963
53.4	33.8	48.4	4.4	235.0	.0	179.3	3.4	-.0036	664	667
50.6	31.0	44.0	4.1	222.1	.0	162.7	3.2	-.0043	625	627
72.8	53.2	78.3	5.9	323.4	1.0	293.5	4.8	-.0003	1018	1020
70.4	50.9	74.7	5.7	312.7	1.0	279.7	4.7	-.0006	967	969
59.3	39.7	57.5	4.8	261.9	.0	214.0	3.8	-.0023	756	759
67.1	47.5	69.5	5.5	297.3	1.0	259.8	4.4	-.0010	898	900

TABLE 11

Simulated Common ion Data in Mg/L for

the Yellowstone River at Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(μmho/cm @ 25°c) ECM'	ECM
56.8	21.5	59.0	3.8	182.5	1.0	191.7	8.9	.0021	681	681
60.9	23.4	64.7	4.1	193.7	1.0	211.1	9.6	.0019	762	762
64.0	24.7	68.9	4.3	202.0	1.0	225.5	10.2	.0017	827	828
65.2	25.3	70.7	4.4	205.5	1.0	231.5	10.4	.0016	856	857
62.7	24.2	67.2	4.2	198.6	1.0	219.6	10.0	.0018	800	800
50.0	18.5	49.6	3.4	164.0	1.0	159.6	7.6	.0026	566	566
33.9	11.2	27.4	2.3	120.2	.0	83.5	4.6	.0047	364	365
23.3	6.5	12.8	1.6	91.6	.0	33.8	2.7	.0084	274	274
35.8	12.1	30.1	2.4	125.5	.0	92.8	5.0	.0043	384	385
43.9	15.7	41.3	3.0	147.6	.0	131.0	6.5	.0031	478	480
55.6	21.0	57.4	3.7	179.4	1.0	186.2	8.7	.0021	660	660
63.8	24.6	68.7	4.3	201.6	1.0	224.7	10.2	.0017	824	824
56.3	21.3	58.3	3.8	181.0	1.0	189.1	8.8	.0021	671	671

Table 11 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	(µmho/cm @ 25°c) ECM'	ECM
36.2	12.3	30.6	2.5	126.6	.0	94.6	5.1	.0042	388	389
57.9	22.0	60.5	3.9	185.4	1.0	196.7	9.1	.0020	701	701
55.7	21.0	57.5	3.7	179.5	1.0	186.5	8.7	.0021	661	661
19.5	4.8	7.6	1.4	81.2	.0	15.8	2.0	.0115	247	247
43.6	15.6	40.7	2.9	146.5	.0	129.2	6.4	.0032	473	475
56.1	21.2	58.0	3.8	180.6	1.0	188.3	8.7	.0021	668	668
60.1	23.0	63.7	4.0	191.6	1.0	207.5	9.5	.0019	746	746
58.0	22.0	60.6	3.9	185.7	1.0	197.2	9.1	.0020	703	703
37.5	12.9	32.4	2.5	130.1	.0	100.7	5.3	.0040	402	403
47.9	17.5	46.7	3.2	158.2	.0	149.5	7.2	.0027	532	534
61.0	23.4	64.9	4.1	194.0	1.0	211.6	9.7	.0018	764	764
66.3	25.8	72.2	4.4	208.5	1.0	236.7	10.7	.0016	882	883
40.7	14.3	36.7	2.8	138.7	.0	115.5	5.9	.0035	438	439
53.6	20.1	54.7	3.6	173.9	1.0	176.8	8.3	.0023	625	625
50.3	18.6	50.1	3.4	164.9	1.0	161.1	7.7	.0025	571	571
57.9	22.0	60.6	3.9	185.5	1.0	196.9	9.1	.0020	702	702
60.2	23.1	63.8	4.0	191.9	1.0	207.9	9.5	.0019	748	748
19.8	4.9	8.0	1.4	82.0	.0	17.2	2.0	.0112	249	249
59.0	22.5	62.1	4.0	188.6	1.0	202.3	9.3	.0019	724	724

Tables A to D in Appendix I. Included in Tables 8 to 11 are the values of the ERR determined from the simulated values for the common ions. Also included in these Tables is a predicated value, ECM', of ECM and the actual value of ECM. ECM' was obtained by determining a value for ECC, ECC*, from the simulated results of the common ion concentrations (equation 2) and then by solving the regression equation for ECC (equation 4), for ECM':

$$ECC^* = \geq_i K_i (ion)_i \qquad (2)$$

$$ECC^* = B \log (ECM') - A \qquad (4)$$

$$ECM' = EXP((1/B) \cdot ECC^* + (A/B)) \qquad (5)$$

By substituting into equation (2) the regression equation expression for each of the common ions given by equation (3) and then substituting this into equation (5), the following expression is obtained:

$$ECM' = ((ECM)^{1/B(\geq K_i bi)}) (EXP (A/B) (EXP (-1/B (\geq K_i ai)) \qquad (6)$$

In order for ECM' to equal ECM in equation (6), the following must hold:

$$\geq_i K_i bi = B \qquad (7)$$

$$\geq_i K_i ai = A \qquad (8)$$

The closeness of the values of ECM' and ECM in Tables 8 to 11 further supports the conjecture that the relative proportions of the common ion concentrations remain somewhat constant in natural waters.

The Yellowstone River at Billings is the only site of the four above which there is no major regulation of flow. The Bighorn River at Bighorn is regulated quite heavily by Yellowtail Reservoir which is on the Bighorn River mainstem approximately 100 river miles upstream. The Tongue River at Miles City is regulated to some extent by the Tongue River

Reservoir, but not to the degree of the Bighorn River (Ref. 12). The site on the Yellowstone River at Miles City being above the confluence with the Tongue River is only influenced by Yellowtail Reservoir. Tables 8 to 11 may indicate that flow regulation reinforces or maintains this relative proportioning of the common ions. The simulated values in Tables 8 to 11, with the observed data in Tables A to D in Appendix I, compare favorably in most cases. The values of the ERR in Tables 8 to 11 give an indication of the quality of the simulated common ion concentrations. Of particular interest is the fact that in Tables 8 to 11, the largest absolute values of ERR occur consistently for the lowest ECM values. The relative proportions of the common ions break down or are altered at apparently low ECM conditions. Low ECM values are generally associated with relatively high flow conditions. To improve the regression equations at low ECM conditions, the addition of a flow dimension was considered.

As previously mentioned, as Q increases the concentration of dissolved constituents decreases. Regression equations involving the concentration of dissolved constituents with Q are generally developed on the basis of the logarithm of the concentration in Mg/L versus the logarithm of Q as the independent variable. The form of these equations is:

$$\log(\text{ion})_i = \log(ai) - bi \log(Q) \qquad (9)$$

These regression equations are then solved for the ion concentration to give:

$$(\text{ion})_i = ai(Q)^{-bi} \qquad (10)$$

Where:

$(ion)_i$ is the concentration of the i^{th} ion

Q is the flow

ai and bi are regression constants.

By altering the regression equations, the corresponding R^2 values are not necessarily applicable, as is especially true when the regression equations are more statistical than physical in character. As previously mentioned, TDS has received the most interest in its inverse relationship with Q. ECM and TDS correlate very strongly as might be expected since they essentially measure the same thing. Table 12 shows regression equations of the form:

$$\log (ECM) = \log (A) - B \log(Q) \qquad (11)$$

for the four sites. The extremely low R^2 values for the Bighorn and Tongue Rivers are most likely indicative of their upstream regulation. Note that the t-value for the Bighorn River at Bighorn indicates that the coefficient on the log(Q) term is not significantly different than zero. This indicates no appreciable relationship between ECM and Q. This same situation of low R^2 values was found for regressions involving the natural logarithm of the individual common ions, in Meq/L concentrations, with the natural logarithm of flow.

Tables 13 through 16 show sets of regression equations for each of the four sites. These regression equations involve the natural logarithms of ECM and Q as independent variables and the natural logarithm of the individual common ion concentrations as dependent variables. The form

TABLE 12

Log (ECM) vs. Log Q Linear Regression Equations for the Four Sites

Site	Regression Equation	R^2	t-Value
Yellowstone at Billings	log (ECM) = 9.501 - .4218 · log Q	.871	-20.66
Bighorn at Bighorn	log (ECM) = 7.175 - .04815 · log Q	.023	- 1.202
Tongue at Miles City	log (ECM) = 7.764 - .1763 · log Q	.322	- 5.423
Yellowstone at Miles City	log (ECM) = 10.57 - .4444 · log Q	.706	- 8.043

TABLE 13

Log (ion) vs. Log (ECM) & Log (Q) Regression Equations

for Yellowstone River at Billings

Dependent Variable	Regression Equations	R^2	t-Value
log (ECC)	log (ECC) = 1.044·log (ECM) - .01531·log (Q) - .03149	.992	31.29 - 1.016
log (Ca)	log (Ca) = .8129·log (ECM) - .05760·log (Q) - 3.760	.963	12.49 - 1.958
log (Mg)	log (Mg) = .9948·log (ECM) - .05535·log (Q) - 5.430	.983	19.43 - 2.393
log (Na)	log (Na) = 1.308·log (ECM) + .03793·log (Q) - 8.042	.964	15.50 .9949
log (K)	log (K) = .5696·log (ECM) - .1226· log (Q) - 4.859	.759	3.442 - 1.639
log (ALK)	log (ALK) = .8578·log (ECM) + .02842·log (Q) - 4.462	.978	20.21 1.482
log (HCO_3)	log (HCO_3) = .8529·log (ECM) + .02709·log (Q) - 4.423	.977	19.65 1.38
log (SO_4)	log (SO_4) = 1.450·log (ECM) - .02274·log (Q) - 8.098	.967	14.80 - .514
log (Cl)	log (Cl) = .5552·log (ECM) - .2867· log (Q) - 2.641	.798	2.663 - 3.043

TABLE 14

Log (ion) vs. Log (ECM) & Log (Q) Regression Equation

for Bighorn River at Bighorn

Dependent Variable	Regression Equations	R^2	t-Value
log (ECC)	log (ECC) = 1.096·log (ECM) - .001001·log (Q) - .4425	.947	32.45 .0936
log (Ca)	log (Ca) = .8956·log (ECM) + .03341 · log (Q) - 5.037	.842	17.87 2.106
log (Mg)	log (Mg) = 1.117·log (ECM) - .005305·log (Q) - 6.739	.844	17.77 - .267
log Na)	log (Na) = 1.274·log (ECM) - .04434 · log (Q) - 7.033	.867	19.12 - 2.103
log (K)	log (K) = .7306·log (ECM) + .05444 · log (Q) - 7.718	.431	6.713 1.581
log (ALK)	log (ALK) = .6712·log (ECM) + .03059 · log (Q) - 3.583	.772	14.23 2.05
log (HCO_3)	log (HCO_3) = .6612·log (ECM) + .03386 · log (Q) - 3.545	.746	13.29 2.15
log (SO_4)	log (SO_4) = 1.271·log (ECM) - .01324 · log (Q) - 6.747	.929	27.53 - .906
log (Cl)	log (Cl) = 1.117·log (ECM) + .04076 · log (Q) - 9.153	.606	9.597 1.107

TABLE 15

Log (ion) vs. Log (ECM) & Log (Q) Regression Equations

for Tongue River at Miles City

Dependent Variable	Regression Equation	R^2	t-Value
log (ECC)	log (ECC) = 1.124·log (ECM) - .001299·log (Q) - .6289	.977	42.06 - .16
log (Ca)	log (Ca) = .8215·log (ECM) + .007594·log (Q) - 4.433	.852	15.67 .466
log (Mg)	log (Mg) = 1.273·log (ECM) + .02187 · log (Q) - 7.458	.940	26.22 1.45
log (Na)	log (Na) = 1.207·log (ECM) - .06028 · log (Q) - 6.807	.902	17.73 - 2.85
log (K)	log (K) = .8247·log (ECM) + .02084 · log (Q) - 7.729	.635	8.87 .72
log (ALK)	log (ALK) = .7586·log (ECM) - .04305 · log (Q) - 3.344	.896	16.97 - 3.10
log (HCO_3)	log (HCO_3) = .7511·log (ECM) - .04319 · log (Q) - 3.297	.892	16.55 - 3.06
log (SO_4)	log (SO_4) = 1.403·log (ECM) + .02590 · log (Q) - 8.049	.958	31.87 1.89
log (Cl)	log (Cl) = .9806·log (ECM) - .001571·log (Q) - 8.787	.550	7.09 - .04

TABLE 16

Log (ion) vs. Log (ECM) & Log (Q) Regression Equations

for Yellowstone River at Miles City

Dependent Variable	Regression Equations	R^2	t-Value
log (ECC)	log (ECC) = 1.105·log (ECM) + .003845·log (Q) - .5665	.994	36.57 0.23
log (Ca)	log (Ca) = .9648·log (ECM) + .06474 · log (Q) - 5.860	.954	14.10 1.79
log (Mg)	log (Mg) = 1.012·log (ECM) - .05641 · log (Q) - 5.539	.935	9.65 - 1.02
log (Na)	log (Na) = 1.252·log (ECM) - .02602 · log (Q) - 7.016	.989	25.63 - 1.01
log (K)	log (K) = .8509·log (ECM) - .01299 · log (Q) - 7.779	.864	6.79 - .20
log (ALK)	log (ALK) = .6012·log (ECM) - .07205 · log (Q) - 2.165	.970	10.72 - 2.43
log (HCO_3)	log (HCO_3) = .6089·log (ECM) - .0624 · log (Q) - 2.310	.966	12.48 - 2.29
log (SO_4)	log (SO_4) = 1.468·log (ECM) + .05844 · log (Q) - 8.765	.981	21.20 1.60
log (Cl)	log (Cl) = 1.003·log (ECM) - .1048 · log (Q) - 6.977	.868	6.06 - 1.20

of these regression equations is given by:

$$\log(\text{ion})_i = \text{ai} \log(\text{ECM}) \pm \text{bi} \log(Q) - \log(\text{ci}) \qquad (12)$$

Where ai, bi and ci are regression constants. Comparing these sets of regression equations with those in Tables 4 to 11, it is apparent that the R^2 values are generally larger for the sets involving Q. Many of the t-values for the regression constants bi, which are the coefficients on the log(Q) term, indicate that bi is not significantly different than zero. These t-values are the lower values shown for each regression equation in Tables 13 to 16. These regression equations reduce to:

$$\log(\text{ion})_i = \text{ai} \log(\text{ECM}) - \log(\text{ci}) \qquad (13)$$

From which the following relation for $(\text{ion})_i$ is obtained:

$$(\text{ion})_i = 1/\text{ci}(\text{ECM})^{\text{ai}} \qquad (14)$$

Since the regression equations in Tables 13 to 16 must be altered to obtain values for the common ion concentrations, the corresponding R^2 values are not directly applicable. Although, these R^2 values should give an indication of the fit of the altered regression equations, especially if the equations involve some underlying physical relation.

Using the regression equations in Tables 13 to 16 and the ECM and Q data from which the regression equations were generated, values for the common ion concentrations were simulated. These simulated values for the common ions for the four sites are shown in Tables 17 through 20. Included in these Tables are values of ERR calculated from the simulated common ion concentrations. The values in Tables 17 to 20 are comparable to the values in Tables 8 to 11 and Tables A to D observed in Appendix

TABLE 17

Simulated Common ion Data for the

Yellowstone River at Billings in Mg/L

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
36.3	12.5	25.5	3.1	151.3	.0	69.3	5.7	-.0142	393	6090
36.6	12.6	24.9	3.2	148.9	.0	68.7	6.2	-.0054	389	4640
37.7	13.0	25.4	3.4	150.9	.0	71.3	6.7	-.0004	398	3700
44.2	15.7	30.5	3.9	169.9	.0	90.5	8.4	.0024	465	2180
41.3	14.5	28.6	3.6	162.8	.0	82.4	7.5	-.0011	438	2970
43.1	15.4	32.7	3.6	178.2	.0	92.7	6.8	-.0222	479	5000
38.3	13.3	26.0	3.4	153.1	.0	73.2	6.7	-.0015	405	3690
30.3	10.0	19.1	2.8	125.4	.0	50.2	5.0	-.0018	315	6230
12.2	3.4	5.7	1.2	57.1	.0	11.6	1.7	-.0456	118	41300
18.6	5.6	9.9	1.8	81.8	.0	22.7	2.8	-.0150	185	16500
29.2	9.6	18.3	2.7	122.0	.0	47.6	4.8	-.0037	304	7020
32.5	11.0	21.5	2.9	135.6	.0	57.2	5.2	-.0083	345	6510
36.2	12.5	24.7	3.2	148.2	.0	67.8	6.0	-.0067	386	4930
37.6	13.0	25.6	3.3	151.7	.0	71.4	6.4	-.0041	399	4180
36.6	12.6	24.5	3.2	147.5	.0	68.1	6.3	-.0017	386	4160
32.7	11.0	20.9	3.0	132.9	.0	56.4	5.6	.0018	340	4840

Table 17 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
37.7	13.0	24.7	3.4	148.3	.0	70.1	6.9	.0057	392	3110
41.5	14.6	28.7	3.7	163.4	.0	82.9	7.5	-.0012	440	2940
40.4	14.1	27.9	3.5	160.4	.0	79.7	7.2	-.0027	429	3330
22.0	6.9	12.7	2.1	96.1	.0	30.3	3.3	-.0140	225	13700
16.2	4.8	8.4	1.6	73.4	.0	18.4	2.3	-.0274	161	24400
17.6	5.3	9.6	1.7	80.6	.0	21.4	2.4	-.0337	179	27000
25.6	8.2	15.3	2.4	108.5	.0	38.3	4.1	-.0047	263	8890
30.2	10.0	19.5	2.7	137.0	.0	50.7	4.8	-.0080	318	7440
31.1	10.4	20.0	2.8	129.2	.0	52.7	5.1	-.0048	326	6460
35.0	11.9	23.6	3.1	143.9	.0	64.1	5.8	-.0069	372	5370
37.4	12.9	25.6	3.3	151.6	.0	71.0	6.3	-.0056	398	4440
45.7	16.4	32.2	4.0	176.0	.0	96.2	8.7	-.0005	485	2140
40.5	14.2	27.9	3.6	160.2	.0	79.8	7.3	-.0013	429	3160
39.7	13.9	27.4	3.5	158.3	.0	77.8	7.0	-.0027	422	3470
40.7	14.3	29.0	3.5	164.4	.0	82.1	6.9	-.0093	439	3970
46.7	17.2	39.9	3.6	203.1	.0	111.6	6.3	-.0469	549	8500
21.3	6.6	12.1	2.0	93.5	.0	28.7	3.2	-.0166	217	15200
17.8	5.3	9.4	1.7	79.0	.0	21.2	2.7	-.0183	177	18500
36.8	12.7	24.4	3.3	147.0	.0	68.3	6.6	.0021	386	3650

Table 17 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
35.5	12.2	24.5	3.1	147.6	.0	66.5	5.7	-.0118	382	5970
44.2	15.8	32.0	3.8	175.1	.0	93.2	7.8	-.0083	477	3030
56.0	20.8	42.0	4.8	208.9	.0	133.0	11.2	-.0087	602	1330
39.0	13.6	26.1	3.5	153.6	.0	74.5	7.2	.0034	409	3030
41.8	14.7	29.1	3.7	164.7	.0	84.0	7.5	-.0022	444	2970
41.5	14.6	29.4	3.6	166.1	.0	84.2	7.2	-.0075	446	3560
45.0	16.2	34.3	3.8	183.7	.0	98.7	7.3	-.0208	499	4240
21.3	6.7	12.9	1.9	97.6	.0	29.9	2.8	-.0353	225	24200
18.8	5.7	10.2	1.8	83.4	.0	23.4	2.8	-.0181	189	17600
26.6	8.6	16.3	2.4	113.1	.0	41.1	4.2	-.0073	276	9000
35.2	12.0	22.9	3.2	141.0	.0	63.3	6.3	.0036	367	3890
37.8	13.1	25.6	3.3	151.7	.0	71.8	6.6	-.0017	400	3830
38.0	13.2	25.7	3.4	152.2	.0	72.4	6.7	-.0012	402	3720
42.6	15.1	30.3	3.7	169.0	.0	87.5	7.5	-.0059	457	3140
18.9	5.8	11.2	1.7	88.7	.0	24.8	2.4	-.0446	199	33400
40.3	14.2	29.0	3.4	164.7	.0	81.6	6.6	-.0134	438	4610
40.2	14.1	28.8	3.5	163.9	.0	81.1	6.6	-.0121	436	4450
45.7	16.4	33.1	3.9	179.2	.0	97.7	8.2	-.0075	492	2680
44.4	15.9	31.7	3.9	174.3	.0	93.1	8.0	-.0053	476	2730

Table 17 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
45.1	16.1	32.3	3.9	176.2	.0	95.3	8.2	-.0047	483	2550
38.7	13.5	26.5	3.4	155.2	.0	74.7	6.7	-.0031	411	3770
28.7	9.4	17.5	2.6	118.7	.0	45.6	4.8	.0006	295	6440
14.8	4.3	7.6	1.4	69.2	.0	16.1	2.0	-.0451	148	38600
15.3	4.5	7.8	1.5	70.3	.0	16.8	2.1	-.0346	152	29800
24.7	7.9	14.5	2.3	104.7	.0	36.0	4.0	-.0041	252	9210
35.2	12.0	23.6	3.1	144.0	.0	64.5	5.9	-.0051	373	5040
40.7	14.3	29.0	3.5	164.4	.0	82.1	6.9	-.0092	439	3950
38.7	13.4	25.9	3.5	152.8	.0	73.7	7.0	.0028	406	3150
39.8	13.9	28.1	3.4	161.1	.0	79.1	6.7	-.0088	428	4160
39.4	13.8	27.4	3.4	158.5	.0	77.3	6.8	-.0054	421	3850
39.4	13.7	26.9	3.5	156.6	.0	76.5	7.0	-.0006	417	3320
18.9	5.8	11.2	1.7	88.7	.0	24.8	2.4	-.0446	199	33400
30.0	10.0	19.4	2.7	127.0	.0	50.4	4.7	-.0112	317	8250

TABLE 18

Simulated Common ion Data for the

Bighorn River at Bighorn in Mg/L

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
83.2	30.2	89.5	4.3	225.0	.0	319.4	11.8	-.0035	982	5970
77.4	27.6	80.8	4.0	213.1	.0	288.4	10.7	-.0056	906	5830
80.5	30.6	94.9	4.0	217.3	1.0	326.8	11.3	.0056	988	1900
79.9	29.4	88.7	4.1	217.2	.0	311.3	11.2	-.0001	957	3500
79.5	29.2	87.7	4.1	216.5	.0	308.2	11.1	-.0007	950	3650
81.1	29.5	87.8	4.2	220.4	.0	311.5	11.4	-.0027	961	4960
81.9	29.8	88.6	4.2	222.2	.0	315.0	11.5	-.0029	970	5230
80.9	29.0	84.7	4.2	220.6	.0	304.0	11.4	-.0057	946	6950
76.9	27.0	78.0	4.0	212.8	.0	280.8	10.7	-.0084	890	7900
72.5	25.4	73.2	3.8	203.2	.0	261.7	9.9	-.0085	840	6270
63.6	22.1	63.7	3.4	183.7	.0	224.3	8.4	-.0093	740	3740
51.0	16.8	46.9	2.8	155.9	.0	164.7	6.4	-.0202	580	3470
75.9	27.9	83.8	3.9	208.8	.0	292.7	10.5	-.0001	910	2900
64.3	22.5	65.4	3.4	184.9	.0	229.3	8.5	-.0077	752	3310
72.7	27.5	85.1	3.7	200.8	1.0	289.7	10.0	.0062	895	1280
64.4	22.3	64.1	3.4	185.7	.0	226.6	8.6	-.0100	747	4300

Table 18 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
71.2	25.3	74.0	3.7	200.0	.0	261.5	9.7	-.0058	836	4260
69.9	24.9	72.9	3.7	197.0	.0	256.8	9.5	-.0053	823	3740
75.6	27.6	82.5	3.9	208.3	.0	289.0	10.4	-.0012	902	3240
77.1	28.2	84.6	4.0	211.4	.0	296.7	10.7	-.0008	921	3320
98.0	36.7	110.9	4.9	254.2	.0	398.0	14.4	-.0013	1170	7220
69.2	23.8	67.6	3.7	196.6	.0	242.8	9.3	-.0116	793	7260
72.6	26.1	76.9	3.8	202.6	.0	270.7	9.9	-.0039	858	3760
74.5	28.1	86.8	3.8	204.7	1.0	297.0	10.3	.0055	914	1490
69.8	24.9	73.1	3.7	196.7	.0	256.9	9.5	-.0048	823	3540
75.1	26.7	77.9	3.9	208.3	.0	277.3	10.3	-.0061	878	5500
69.4	24.2	69.9	3.7	196.6	.0	248.9	9.4	-.0087	806	5280
72.4	26.1	77.4	3.8	202.0	.0	271.5	9.9	-.0030	859	3360
65.5	22.8	65.9	3.5	187.8	.0	232.6	8.7	-.0087	762	4060
77.9	28.4	84.5	4.0	213.5	.0	298.0	10.8	-.0021	926	4000
73.9	25.8	74.3	3.9	206.3	.0	266.8	10.1	-.0089	854	7070
75.2	26.4	76.2	4.0	209.1	.0	273.6	10.4	-.0083	871	7170
77.4	27.4	79.7	4.1	213.4	.0	285.8	10.7	-.0068	901	6750
77.7	28.8	87.1	4.0	212.3	.0	303.6	10.8	.0011	936	2760
78.6	29.1	88.0	4.0	214.4	.0	307.4	11.0	.0009	946	2970

Table 18 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
77.0	28.3	85.1	4.0	211.2	.0	297.8	10.7	-.0002	923	3100
63.7	22.1	63.6	3.4	184.0	.0	224.2	8.4	-.0097	740	3930
65.3	22.3	62.9	3.5	188.3	.0	225.3	8.7	-.0131	747	6500
82.7	30.3	90.4	4.2	223.5	.0	320.7	11.7	-.0019	983	4820
80.5	28.9	84.7	4.2	219.6	.0	303.1	11.3	-.0052	943	6420
81.3	29.2	86.0	4.2	221.2	.0	307.6	11.4	-.0047	954	6340
80.1	28.2	81.5	4.2	219.6	.0	294.7	11.2	-.0083	926	9160
53.5	17.4	47.5	3.0	162.5	.0	170.3	6.8	-.0231	599	6300
53.1	17.4	47.9	3.0	161.2	.0	170.4	6.7	-.0216	598	5100
53.0	17.8	50.2	2.9	160.3	.0	175.6	6.7	-.0165	609	3030
81.9	30.4	92.3	4.2	221.2	.0	323.5	11.5	.0010	986	3340
82.9	30.7	92.6	4.2	223.5	.0	326.2	11.7	.0000	994	3900
80.5	29.6	89.1	4.1	218.6	.0	313.5	11.3	-.0004	963	3720
84.9	31.2	93.4	4.3	227.9	.0	331.8	12.1	-.0016	1010	5040
94.6	37.3	118.6	4.6	245.0	1.0	409.2	13.8	.0087	1180	2040
86.2	32.7	100.6	4.3	229.4	1.0	351.1	12.3	.0037	1050	2880
79.3	28.5	84.1	4.1	217.0	.0	299.5	11.1	-.0044	933	5610
78.5	27.3	77.8	4.2	216.7	.0	283.1	10.9	-.0105	899	11100
72.8	26.4	78.6	3.8	202.6	.0	275.0	10.0	-.0020	867	3080

Table 18 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
63.9	22.9	67.8	3.4	183.3	.0	234.3	8.5	-.0034	761	2060
76.1	27.5	81.3	3.9	209.9	.0	287.2	10.5	-.0033	900	4230
77.6	27.9	82.3	4.0	213.3	.0	292.2	10.8	-.0042	914	5030
73.2	26.3	77.8	3.8	203.8	.0	273.9	10.0	-.0036	866	3750
76.3	27.1	79.3	4.0	210.9	.0	282.8	10.6	-.0060	892	5800
78.8	28.6	85.0	4.1	215.6	.0	300.8	11.0	-.0027	934	4500
77.2	26.6	75.0	4.1	214.4	.0	274.3	10.7	-.0122	878	12800
62.1	21.5	62.1	3.3	180.3	.0	218.0	8.2	-.0095	723	3420
91.8	36.7	118.3	4.5	238.6	1.0	402.7	13.3	.0122	1160	1310
91.4	35.1	109.3	4.5	239.4	1.0	381.2	13.2	.0051	1120	2860
55.0	18.7	53.5	3.0	164.4	.0	186.3	7.0	-.0131	637	2630
84.2	33.6	108.9	4.1	222.8	1.0	365.6	12.0	.0145	1070	840

TABLE 19

Simulated Common ion Data for the

Tongue River at Miles City in Mg/L

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
57.7	36.9	52.4	4.6	251.7	.0	196.0	3.6	-.0008	755	444
60.7	40.0	55.7	4.9	261.2	1.0	214.4	3.8	.0004	802	539
79.2	59.9	88.8	6.3	351.3	1.0	333.3	5.2	.0017	1120	188
85.7	67.7	98.5	6.8	374.2	1.0	381.8	5.8	.0011	1230	222
71.8	51.7	73.7	5.7	311.6	1.0	283.9	4.6	.0010	988	340
69.9	49.7	70.2	5.6	302.0	1.0	271.6	4.5	.0010	955	389
80.5	61.6	89.3	6.4	351.9	1.0	344.0	5.3	.0012	1140	245
67.6	47.4	65.0	5.4	287.5	1.0	257.9	4.3	.0011	914	569
49.6	29.4	40.2	4.0	212.7	.0	152.7	3.0	-.0014	625	815
32.0	15.0	20.4	2.6	139.2	.0	72.7	1.7	-.0143	365	1310
49.2	28.6	44.4	3.9	228.3	.0	147.7	3.0	-.0097	628	173
52.7	32.0	47.6	4.2	237.7	.0	167.1	3.2	-.0049	680	270
70.8	50.2	77.0	5.6	321.8	1.0	274.3	4.6	.0011	980	140
63.4	42.7	61.0	5.1	277.0	1.0	230.0	4.0	.0002	849	372
65.2	44.6	63.2	5.2	283.1	1.0	241.1	4.1	.0006	877	395
73.5	52.8	84.7	5.7	342.6	1.0	289.7	4.8	.0021	1030	78

Table 19 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
79.8	60.6	89.6	6.3	353.1	1.0	337.8	5.3	.0017	1130	195
75.3	55.6	80.4	6.0	329.5	1.0	307.3	4.9	.0012	1050	270
75.0	55.5	77.3	6.0	320.6	1.0	306.9	4.9	.0009	1040	430
82.1	63.2	93.9	6.5	363.8	1.0	353.9	5.5	.0018	1170	179
66.2	46.1	60.6	5.4	274.3	1.0	250.4	4.2	.0017	886	983
37.8	19.5	25.3	3.1	158.7	.0	97.0	2.1	-.0029	445	1950
40.5	21.4	30.7	3.2	180.2	.0	107.5	2.3	-.0110	490	554
47.5	27.2	41.0	3.8	216.6	.0	139.7	2.8	-.0091	599	257
54.4	33.6	48.8	4.3	241.0	.0	176.7	3.3	-.0028	704	361
55.7	34.9	50.1	4.5	244.7	.0	184.4	3.4	-.0018	724	410
74.3	54.5	77.5	5.9	321.5	1.0	301.0	4.8	.0010	1030	341
77.3	57.9	82.0	6.2	333.0	1.0	321.7	5.1	.0009	1080	345
79.0	59.4	90.9	6.2	357.2	1.0	330.0	5.3	.0024	1120	128
78.1	58.6	86.7	6.2	345.8	1.0	325.4	5.2	.0016	1100	198
39.1	20.6	26.0	3.2	161.0	.0	103.5	2.2	.0014	463	2800
67.1	47.2	60.8	5.4	274.6	1.0	257.1	4.2	.0019	899	1230
78.8	60.0	81.0	6.3	329.5	1.0	335.0	5.1	.0000	1100	604
43.9	24.5	31.9	3.6	183.6	.0	125.1	2.5	.0001	535	1660
61.6	40.7	59.9	4.9	274.2	1.0	218.0	3.9	-.0008	822	269

Table 19, (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
66.5	45.8	66.9	5.3	293.8	1.0	248.4	4.2	.0004	902	272
71.6	51.2	75.5	5.7	317.1	1.0	280.9	4.6	.0011	988	227
70.1	49.5	74.7	5.5	315.5	1.0	270.2	4.5	.0009	966	172
56.2	35.8	47.3	4.5	235.0	.0	189.4	3.4	.0017	725	1070
81.4	63.4	82.4	6.6	332.2	1.0	356.1	5.3	-.0012	1140	930
68.4	48.5	63.4	5.5	281.9	1.0	265.2	4.3	.0015	922	1040
39.3	20.7	26.9	3.2	165.0	.0	103.6	2.2	-.0021	467	1840
51.0	30.4	45.0	4.1	229.6	.0	158.3	3.1	-.0055	653	300
64.6	43.2	70.6	5.0	306.3	1.0	231.9	4.2	-.0015	881	69
57.9	37.2	52.7	4.6	252.6	.0	197.5	3.6	-.0007	759	447
81.9	62.7	96.0	6.5	369.4	1.0	350.6	5.5	.0026	1170	125
80.8	61.6	93.0	6.4	361.8	1.0	343.9	5.4	.0022	1150	150
72.4	52.2	75.9	5.8	317.9	1.0	286.9	4.7	.0011	1000	265
68.9	48.5	69.2	5.5	299.6	1.0	264.5	4.4	.0009	939	350
70.2	50.2	69.1	5.6	298.8	1.0	274.9	4.5	.0010	958	526
66.0	45.6	62.5	5.3	280.5	1.0	247.6	4.2	.0011	887	596
42.0	23.0	28.4	3.4	170.0	.0	116.8	2.4	.0044	503	3360
48.3	28.0	41.0	3.9	216.3	.0	144.5	2.9	-.0065	610	363
46.2	26.1	39.5	3.7	211.9	.0	133.3	2.8	-.0107	580	243

Table 19 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
55.1	34.2	51.3	4.4	249.4	.0	180.0	3.4	-.0040	719	235
72.2	52.0	75.6	5.7	317.0	1.0	285.7	4.7	.0011	997	267
67.7	46.9	70.2	5.4	303.2	1.0	254.9	4.3	.0004	924	200
60.9	39.8	60.4	4.8	276.4	1.0	212.5	3.8	-.0018	813	184
64.1	43.4	61.8	5.1	279.2	1.0	233.9	4.0	.0004	859	380
70.6	50.6	69.3	5.7	299.0	1.0	277.5	4.5	.0011	963	567
52.5	32.3	42.4	4.3	219.2	.0	169.3	3.1	.0017	667	1270
49.6	29.3	41.2	4.0	216.5	.0	152.0	3.0	-.0032	627	575
72.4	51.2	88.1	5.6	353.0	1.0	279.4	4.8	.0025	1020	33
70.7	50.5	71.6	5.6	305.9	1.0	276.9	4.5	.0010	969	373
57.5	36.3	56.3	4.5	264.8	.0	192.0	3.6	-.0041	759	150
65.6	44.1	73.7	5.1	315.1	1.0	237.2	4.2	-.0012	900	52

TABLE 20

Simulated Common ion Data for the

Yellowstone River at Miles City in Mg/L

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
55.1	21.3	57.7	3.8	184.4	1.0	182.1	9.0	.0013	681	7460
61.9	23.7	66.1	4.1	195.8	1.0	216.5	10.0	.0003	762	8530
66.8	25.9	73.5	4.4	207.0	1.0	243.4	10.9	-.0023	828	7880
69.1	26.7	76.7	4.6	211.1	1.0	256.2	11.3	-.0036	857	8000
65.0	24.8	70.3	4.3	201.5	1.0	232.7	10.4	-.0011	800	8680
48.2	17.0	44.9	3.2	157.8	.0	144.5	7.0	.0061	566	14800
33.7	10.3	25.3	2.2	113.5	.0	80.4	4.0	.0100	365	40200
26.1	7.5	17.5	1.7	93.1	.0	54.0	2.9	.0009	274	58400
33.9	11.3	27.5	2.3	122.5	.0	83.5	4.6	-.0022	385	19900
40.2	14.6	36.9	2.8	145.8	1.0	111.2	6.1	-.0022	480	10600
53.6	20.6	55.4	3.7	180.3	1.0	174.5	8.7	.0017	660	7860
65.9	25.9	73.3	4.4	208.1	1.0	239.8	11.0	-.0020	824	6900
55.0	20.7	56.3	3.7	180.6	1.0	180.2	8.7	.0024	671	9000
35.0	11.2	27.6	2.3	120.6	.0	86.6	4.5	.0059	389	28600
58.0	21.5	59.2	3.8	183.5	.0	194.1	8.9	.0027	701	10700
57.4	19.4	54.0	3.6	169.2	.0	185.7	7.8	.0070	661	22000

Table 20 (Cont.)

Ca	Mg	Na	K	HCO_3	CO_3	SO_4	Cl	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
23.4	6.9	15.4	1.5	88.5	.0	45.8	2.7	-.0148	247	48400
39.8	14.5	36.4	2.7	145.0	1.0	109.4	6.1	-.0030	475	10400
53.8	21.0	56.4	3.7	183.2	1.0	176.1	8.9	.0008	668	6840
60.5	23.2	64.5	4.1	193.9	1.0	209.2	9.8	.0006	746	8100
59.3	21.2	59.0	3.8	180.4	.0	198.3	8.7	.0036	703	14400
36.5	11.5	28.8	2.4	122.3	.0	91.8	4.6	.0096	403	32000
44.5	16.3	42.2	3.0	155.8	1.0	129.9	6.8	.0012	534	10300
61.2	24.0	66.7	4.1	198.9	1.0	214.4	10.2	-.0002	764	6800
71.4	27.5	79.5	4.7	214.1	1.0	268.9	11.5	-.0051	883	8600
39.6	12.6	32.1	2.5	129.1	.0	103.9	5.0	.0119	439	31100
50.3	19.6	51.9	3.5	176.3	1.0	159.4	8.4	-.0000	625	6610
47.6	17.4	45.8	3.2	161.9	1.0	143.6	7.3	.0030	571	10700
56.8	21.9	59.9	3.8	187.5	1.0	190.7	9.3	.0013	702	7640
61.4	23.0	64.4	4.0	191.9	1.0	212.4	9.6	.0010	748	9800
22.7	7.1	15.8	1.6	92.1	.0	44.9	2.9	-.0330	249	27400
58.1	22.7	62.4	4.0	192.3	1.0	198.3	9.7	.0006	724	6900

I. The ERR values in Tables 17 to 20 are generally smaller than those corresponding values in Tables 8 to 11, but the largest values of ERR still occur predominately at low ECM values. Although, the values of ERR for the simulated common ions in Tables 17 to 20 are not excessively large even at low ECM values. None of the ERR values for a particular site are larger in magnitude than the acceptance limits used in screening the original data. For example, the lowest ECM value for the Yellowstone River at Billings in Table 17 has a value of 118 umho/cm @ 25°c. The ERR corresponding to this data point is -.0456, which is about one-half the lower acceptance limit of -.0913 used to screen the data.

Table 21 shows a typical set of composite data collected by the U. S. Geological Survey and a set of instantaneous data collected by the Montana State Water Quality Bureau for the Tongue River at Miles City. Table 22 shows the simulated values of the common ions obtained from the set of regression equations in Table 15 and the values of ECM and Q from Table 21. The simulated data compare favorably with the measured values of the common ions and are quite adequate for the degree to which common ion data is essential in characterizing the water chemistry.

The ERR values indicate that no serious errors are occurring for the sets of regression equations at high ECM values, as occur at low ECM values. High ECM values generally occur at low flow conditions. The condition of low Q and high ECM is generally the most critical to the water eco-system, and is the condition at which the highest concentrations of dissolved constituents can occur.

TABLE 21

Composite Data from the U. S. Geological Survey and

Instantaneous Data from the Montana State W.Q.B. for

the Tongue River at Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
89	68	90	5.6	398	0	349	5.5	1200	143
61	57	85	5.5	311	0	307	8.5	1040	85.2
76	50	56	4.2	3.5	0	243		901	354
50	31	41	5.2	220	0	152	2.5	630	704
38	11	27	4.9	156	0	67	2.6	396	2800
35	14	13	2.0	148	0	50	1.5	327	1827
53	29	45	4.2	225	0	152	2.9	645	309
74	55	128	2.7	382	0	351	5.8	1240	21.7
100	77	146	7.9	440	0	508	8.4	1540	400
60	45	48	4.2	254	0	216	5.6	798	464
59	40	38	4.2	235	0	190	3.2	721	1139
38	15	13	2.7	153	0	56	2.0	361	3205
96	66	83	5.9	390	0	355	5.6	1200	151
48	26	38		199	2	139	1.7	572	300

Table 21 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	CO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
71	61	79		320	4	320	3.0	1060	245
37	17.8	22		151	0	88	1.5	420	1330
67	46	60		266	0	245	4.0	862	661
72	48	68		280	0	289	4.4	950	428
63	42	60		262	5	228	3.5	840	176
61	45.6	54	5.3	261	0	240	2.5	832	1510
33.7	15.2	18.0	3.3	137	0	70	0.6	362	4510
68	48.5	75	5.2	301	5	280	4.2	956	144
46.9	25.9	32.0	3.0	206	4	120	1.7	540	715
68	56	77	5.4	306	0	300	4.6	1069	218
67	67	101	5.4	321	0	370	7.0	1135	150
68	58	90	5.5	319	1	315	5.0	1104	185
89	72	155	8.6	434	0	475	7.8	1537	70

TABLE 22

Simulated Common ion Data for Composite U. S. G. S. and

Instantaneous W.Q.B. Data for the Tongue River at Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	ECM (μmho/cm @ 25°c)	Q (cfs)
82.7	65.0	98.1	6.6	376.8	364.8	5.6	-.0016	1200	143
73.3	53.5	85.1	5.8	345.7	294.4	4.9	-.0019	1040	85
65.8	46.0	65.7	5.3	291.6	249.8	4.2	-.0036	901	354
49.3	29.6	40.9	4.0	215.8	154.0	3.0	-.0067	630	704
34.0	16.9	21.5	2.8	143.0	83.2	1.9	-.0078	396	2800
29.0	13.1	17.5	2.4	125.9	62.9	1.6	-.0203	327	1827
50.0	30.0	44.2	4.0	227.6	155.8	3.1	-.0103	645	309
83.8	65.0	114.2	6.5	418.8	363.8	5.8	.0043	1240	22
100.6	86.8	143.1	7.9	481.0	500.8	7.2	.0039	1540	40
59.7	39.7	55.8	4.8	262.9	212.2	3.8	-.0044	798	464
55.3	35.5	46.8	4.5	234.1	188.4	3.4	-.0027	721	1139
31.6	15.1	19.1	2.6	132.5	73.3	1.7	-.0093	361	3205
82.8	65.0	97.7	6.6	376.0	365.3	5.6	-.0017	1200	151
45.3	25.7	38.3	3.6	208.1	131.5	2.7	-.0145	572	300
75.0	56.1	81.7	6.0	335.1	310.8	5.0	-.0028	1060	245
35.5	17.9	24.1	2.9	154.4	88.6	2.0	-.0136	420	1330

Table 22 (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	ERR	ECM (µmho/cm @ 25°c)	Q (cfs)
63.8	44.1	60.0	5.2	274.5	238.6	4.0	-.0032	862	661
68.9	49.4	69.2	5.6	301.1	270.4	4.5	-.0033	950	428
61.8	41.4	63.0	4.9	284.9	222.3	4.0	-.0055	840	176
62.4	43.0	54.5	5.1	257.3	232.3	3.9	-.0017	832	1590
31.7	15.2	18.7	2.6	130.8	74.3	1.7	-.0052	362	4510
68.7	48.6	74.5	5.5	317.1	265.2	4.5	-.0034	956	144
43.5	24.4	33.9	3.5	191.8	124.1	2.6	-.0106	540	715
75.5	56.6	83.1	6.1	339.0	313.6	5.0	-.0026	1069	218
79.1	60.6	91.4	6.3	360.5	337.8	5.3	-.0018	1135	150
77.4	58.7	87.3	6.2	349.8	326.7	5.2	-.0023	1104	185
100.9	87.6	138.0	8.0	468.9	506.7	7.2	.0007	1537	70

The inadequacy of the regression equations at low ECM values, which generally occur at high flows, is indicative of a change in the relative proportions of the common ions. At low ECM and high Q, the run off component contributes a greater proportion. The common ions in the run off component may also exist in some constant relative proportion which is quite different from the rest of the water making up the total flow. To improve further on the regression equations, i.e., provide better predictions at low ECM and high Q conditions, would require knowledge of the amounts of the different components of the flow and the ECM values associated with these various flow components. The flow factor is dealt with in Chapter III, where a first level of modeling incorporating flows is developed.

CHAPTER III

Large Scale Water Quality Model Employing Flow and Electrical Conductivity

As was mentioned at the end of Chapter II, the inadequacy of the set of regression equations in describing the common ion concentrations at low ECM and high Q has lead to the need to partition the flow into its various aggregate components such as run off, ground water discharge and instream flow. The set of regression equations are also somewhat static in character. That is, they represent the current situation but may not be applicable under a different set of circumstances, e.g. different water use policies. The question could be asked, "if a different water use policy was employed, how might this affect the regression coefficients?". To be able to predict ion concentrations for different water use policies will also require knowledge of the quantities of the different flow components. For example, suppose agricultural and industrial uses of water in sub-basin 42KJA increased to the point where irrigation return flows became a substantial contributor to the total flow in the Yellowstone River. Knowledge of how this might affect stream quality is of utmost importance, but the regression constants for the set of regression equations can be different under these circumstances. Another example would be the building of a reservoir on the Yellowstone River mainstem. The regression constants could be different if the Yellowstone River were regulated. To be able to predict these changes requires a large scale model that breaks the flow into its

various components as well as appropriating the amounts of the common ions associated with the various components.

A state water planning model developed by Boyd and Williams (Ref. 2) is currently being employed by the Montana State Department of Natural Resources and Conservation as a tool in managing water resources. This model represents the various components of flow and has been applied with relative success. The state water planning model is an application of the macro-to-micro philosophy of modeling large scale systems.

This modeling approach was employed in the Yellowstone River sub-basin, 42KJA. Sub-basin 42KJA is a hybrid sub-basin comprising 42KJ and 42A. 42KJ is a sub-basin of the Yellowstone River and 42A is the Rosebud Creek drainage (see Figure 1). Sub-basin 42KJ was previously calibrated (Ref. 3), but after a review of the calibration of 42KJ, 42A was included. The lack of data for 42A necessitated the change. An annual water quantity model was applied to sub-basin 42KJA. Once this model was calibrated, it was used to build an annual water quality model for 42KJA. Both of these models represent the first or most macro level of modeling.

Quantity Model

The quantity model is based upon balancing the volume of water that arises from the different flow components over a specified time. The model essentially employs the principle of the conservation of matter. Volumes of water are supplied to and demanded from the various sub-systems of the model. Figure 6 shows a schematic of the water quantity

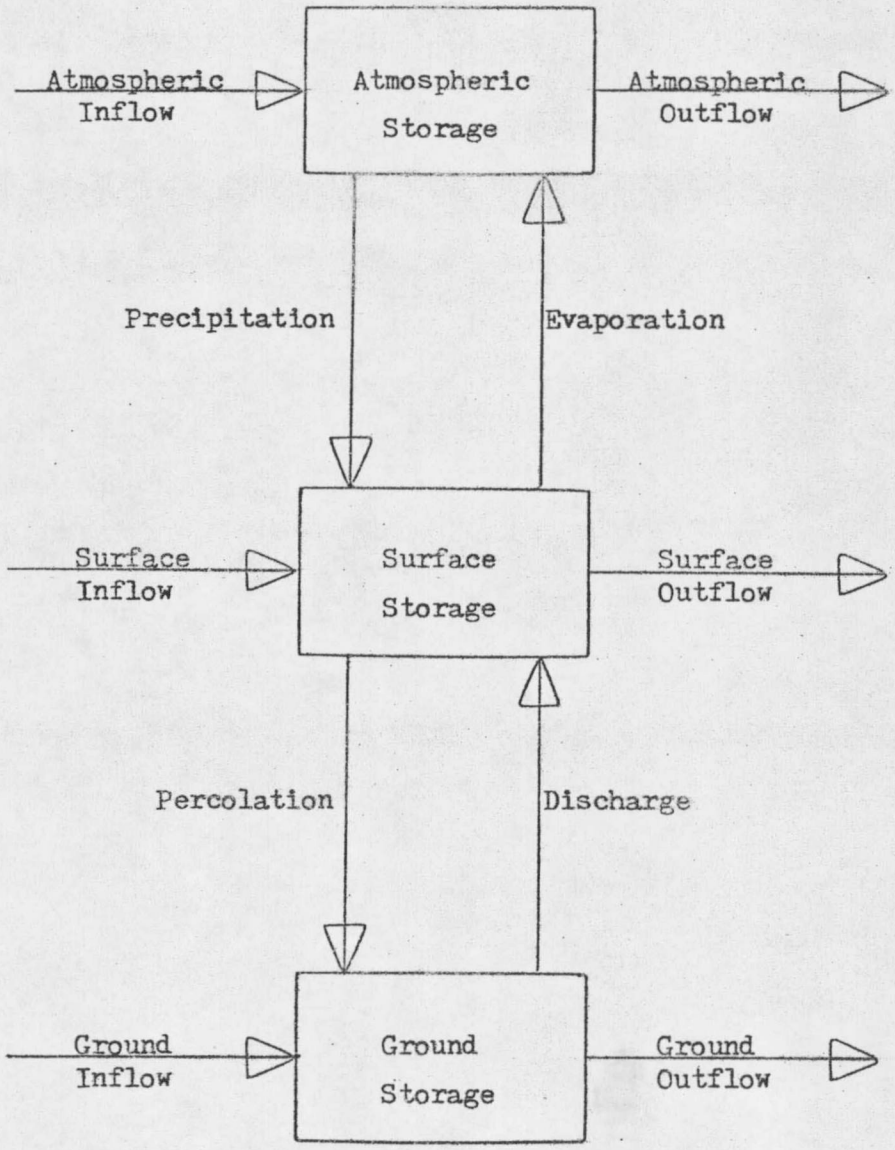


Figure 6. Schematic of a Sub-basin for an Annual Flow Model (Ref. 2)

model for a sub-basin at the annual level, depicting the various sub-systems. For any of the sub-systems, the supplies and demands must balance; i.e., the volume of water entering must equal the volume of water leaving, plus storage and loss. For example, consider the surface sub-system in Figure 6, surface inflow, precipitation, and ground water discharge represent volumes entering (supplies). Surface outflow, evaporation and percolation represent volumes leaving (demands). Since there can be changes in the amount of storage in the surface water sub-system, a change in storage can be either a supply or a demand. These components combine additively to produce a balance of supply and demand, which results in an equation termed a balance equation. Thus, a balance equation exists for every sub-system. When there are more unknown components than balance equations, regression equations must be supplied. Regression equations have the following form:

$$X_i = \Sigma_i \, C(k,j) X_j \qquad (15)$$

Where:

X_i is the parameter determined from the regression equation.

X_j's are the parameters comprising the independent variables.

$C(k,j)$'s are the coefficients of the X_j's.

k is the equation number of the sequence.*

Regression equations express the relationships that exist within the system. The coefficients of the regression equations are selected on the basis of a systems analysis. The systems analysis will often involve

*The numbering sequence is developed in the solution technique (Ref. 2).

expert opinion or most likely guesses. These regression equations can contain statistical and/or physical relationships. Physical regression equations are generally an aggregated form of a balance equation that will emerge when a sub-system at a more micro level of modeling is defined.

The quantity model applied to 42KJA was slightly modified from that in Figure 6. Only the surface water and ground water sub-systems along with the linkages between the surface water and atmospheric water sub-systems were considered. Figure 7 shows a schematic of this model. The parameters used in Figure 7 are defined as follows:

X1 = Volume of water that leaves the surface water sub-system as outflow in a one-year period.

X2 = Volume of water that enters the surface water sub-system as inflow in a one-year period.

X3 = Volume of water stored in the surface water sub-system at the beginning of the one-year period.

X4 = Volume of water stored in the surface water sub-system at the beginning of the one-year period.

X5 = Volume of water that enters the surface water sub-system as precipitation in a one-year period.

X6 = Volume of water that leaves the surface water sub-system as evaporation in a one-year period.

X7 = Volume of water that leaves the ground water sub-system as outflow in a one-year period.

X8 = Volume of water that enters the ground water sub-system as inflow in a one-year period.

X9 = Volume available for water storage in the ground water sub-system at the beginning of the one-year period.

X10= Volume available for water storage in the ground water sub-system at the end of the one-year period.

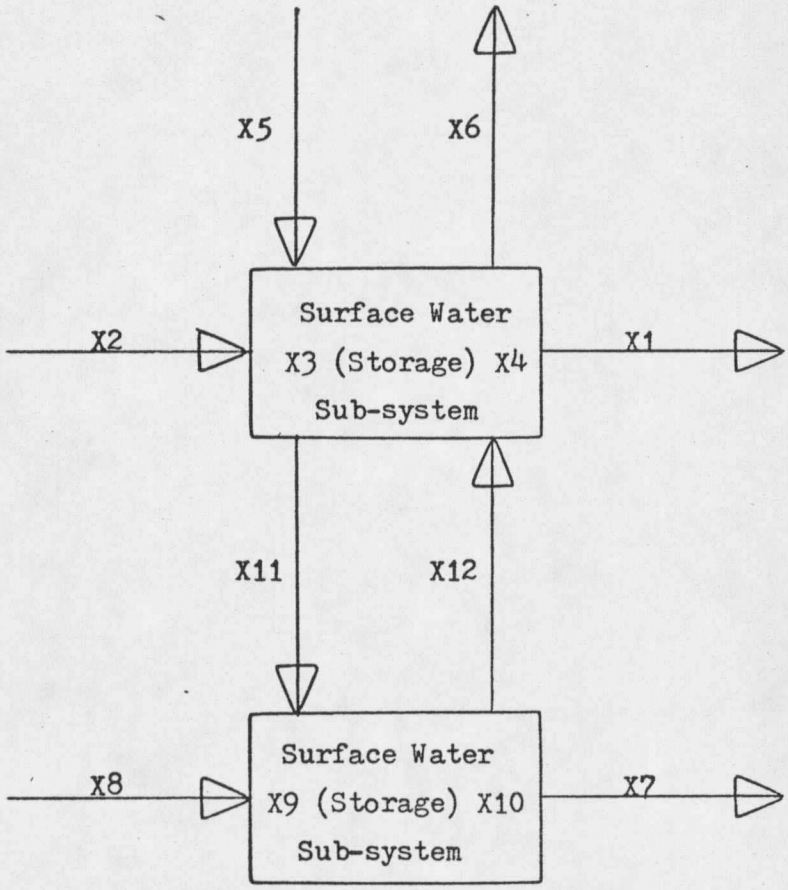


Figure 7. Schematic of Quantity Model used in Sub-basin 42KJA

X11 = Volume of water that leaves the surface water sub-system entering the ground water sub-system as percolation in a one-year period.

X12 = Volume of water that leaves the ground water sub-system entering the surface water sub-system as ground water discharge in a one-year period.

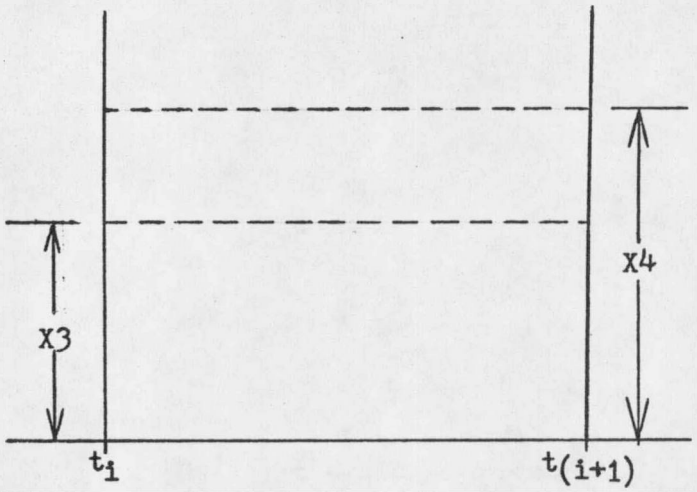
Applying the balancing condition for the two sub-systems yields the following two balance equations.

$$-X1 + X2 + X5 - X6 - X11 + X12 - (X4 - X3) = 0 \qquad (16)$$

$$-X7 + X8 + X11 - X12 + (X10 - X9) = 0 \qquad (17)$$

Equation (16) is the surface water sub-system balance equation and equation (17) is the ground water sub-system balance equation. The changes in the amounts of storage in the surface water and ground water sub-systems are explained in Figures 8 and 9 respectively. X10 and X9 of the ground water storage are much like X4 and X3 of the surface water storage, except that X4 and X3 measure volumes of water stored, and X10 and X9 measure the available volumes for storage of water. This seeming inconsistency in defining X10 and X9 was an attempt to facilitate their measurement as parameters; i.e., X10 and X9 as defined have the capability of being obtained from "specific yield" measurements.

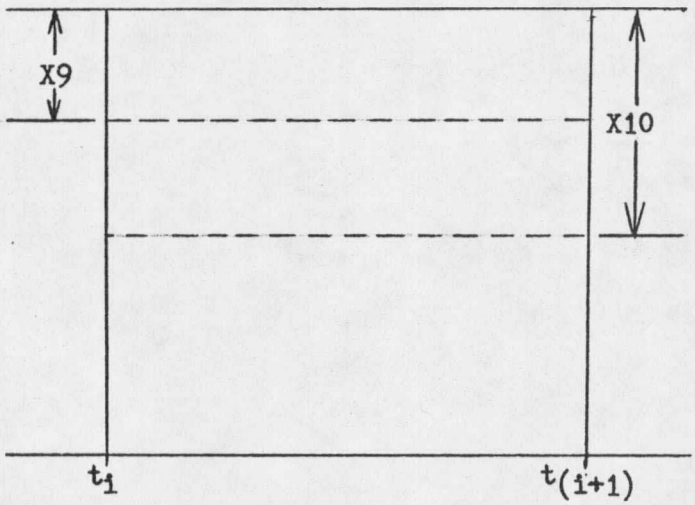
Equations (16) and (17) indicate that 12 parameters are essential to define the system. These parameters are classified into two groups, those for which data are available (primary parameters) and those for which no data are available (secondary parameters). Data were available for only 5 of the 12 essential parameters; X1, X2, X3, X4, and X5. These were the primary parameters. Annual data for X1, X2, and X5 for the 30



$X4 - X3 > 0$ Means the storage volume increased over the time interval, a demand.

$X4 - X3 < 0$ Means the storage volume decreased over the time interval, a supply.

Figure 8. Schematic of Surface Water Storage



X10 - X9 > 0: Means the available volume for storage increased over the time interval, a supply.

X10 - X9 < 0: Means the available volume for storage decreased over the time interval, a demand.

Figure 9. Schematic of Ground Water Storage

year period, 1944 to 1973 were provided by the Montana State Department of Natural Resources and Conservation. X3 and X4 were zero for each year. The remaining 7 parameters; X6, X7, X8, X9, X10, X11 and X12; were the secondary parameters. Since there are two balance equations, 2 of the 7 secondary parameters can be determined from these equations. This leaves five secondary parameters for which regression equations must be supplied. The regression coefficients were determined on a subjective basis using average annual values of the parameters as a guide (Ref. 2). The regression equations and coefficients used in the model for sub-basin 42KJA follow quite closely to those used in sub-basins 43Q and 42KJ (Ref. 2, 3).

The evaporation, X6, was determined from equation (16), the surface water sub-system balance equation.

The ground water inflow, X8, and the ground water outflow, X7, were assumed to follow the alluvium of the surface basin and to consist of two components: a statistical component correlated to the corresponding surface flow (X2 or X1) and a constant flow component. This resulted in the following two regression equations for X7 and X8 respectively:

$$X7 = C(2, 1)X1 + C(2, 13)X13 \qquad (18)$$

$$X8 = C(3, 2)X2 + C(3, 13)X13 \qquad (19)$$

Where X13 is an exogenous constant with unity value as required by the solution technique. The sum of the average annual ground water flows, $(\overline{X7} + \overline{X8})$ was assumed to be approximately 3 percent of the sum of the average annual surface water flows, $(\overline{X1} + \overline{X2})$. From Table I in Appendix I,

$$\overline{X1} = 8.317425 \text{ Million Acre-Feet (MAF)},$$

$$\overline{X2} = 8.223250 \text{ MAF.}$$

Thus,

$$(\overline{X7} + \overline{X8}) = .496220 \text{ MAF.}$$

Further, assuming $\overline{X7}$ equaled about 97 percent of X8,

$$\overline{X7} = (.97)\overline{X8}$$

then

$$\overline{X8} = .251888 \text{ MAF,}$$

$$\overline{X7} = .244332 \text{ MAF.}$$

The initial available ground water storage, X9, is equal to the final available ground water storage, X10, of the preceding year. The final available ground water storage, X10, is determined from equation (17), the ground water sub-system balance equations. A stable trend for the average annual available ground water storage was assumed, i.e., $\overline{X9}$ equals $\overline{X10}$. Thus, an initial value of $X9(X9_o)$ was specified for the 30-year period subject to $\overline{X9}$ equaling $\overline{X10}$. An approximation of the average annual final available ground water storage, $\overline{X10}$, was obtained from the sum of the average annual surface water inflow, $\overline{X2}$, for the entire Yellowstone basin and the average annual precipitation, $\overline{X5}$, for sub-basin 42KJA.

A proportion of this sum was allocated to sub-basin 42KJA based upon the ratio (.77) of the area of 42KJA to the area of the entire Yellowstone Basin.

$$\overline{X9} = \overline{X10} = .77(\overline{X2} \text{ entire basin}) + (\overline{X5} \text{ of } 42KJA)$$

$$\overline{X10} = 5.241101 \text{ MAF}.$$

$X9_0$ was determined subject to this average annual value.

The percolation was assumed to have the form of a physical regression equation.

$$X11 = (C(5,2)X2 + C(5,3,4)\cdot(X3 + X4)/2 + C(5,5)X5 + C(5,12)X12)SF \quad (20)$$

Where SF is a scaling factor such that on the average the water table remains stable. This is consistent with $\overline{X9}$ equaling $\overline{X10}$. The value of SF was determined from average annual values of $\overline{X2}$, $\overline{X3}$, $\overline{X4}$, $\overline{X5}$, $\overline{X11}$ and $\overline{X12}$. The following numerical values were used for the coefficients in the regression equation for X11:

$$C(5,2) = 1.0$$

$$C(5,3,4) = 0.5$$

$$C(5,5) = 2.0$$

$$C(5,12) = 0.5$$

The ground water discharge, X12, was also assumed to be given by a physical regression equation.

$$X12 = C(6,13)X13 - C(6;9,10)\cdot(X9 + X10)/2 \quad (21)$$

The average annual ground water discharge, $\overline{X12}$, was assumed to be approximately 7 percent of the average annual surface water outflow, $\overline{X1}$. Thus,

$$\overline{X12} = .582220 \text{ MAF}$$

C(6,13) was assumed to equal twice the value of $\overline{X12}$. Thus,

$$C(6,13) = 1.164439 \text{ MAF}$$

Then from equation (21) on an average annual basis:

$$C(6;9,10) = .111087$$

A value of $\overline{X11}$ was obtained from equation (17) on an average annual basis. This enabled a value for SF to be determined:

$$SF = .0360523$$

Table 23 summarizes the equations for determining the values of the secondary parameters for the quantity model.

The model was balanced (calibrated) by adjusting the value of $X9_o$ for the year 1944, until

$$\overline{X9} = \overline{X10} = 5.241101 \text{ MAF}$$

Appendix III gives a listing of the simulated secondary parameter values and the annual 42KJA computer program.

The validation method for the quantity model is outlined in Reference 2. A validation of the quantity model is not shown here since values of the various flow components are needed in the quality model, and a validation is presented for the quality model.

Quality Model

By multiplying the various flow components X_i by the density of water, the volume system could be changed to a mass system. The underlying principle would still be the conservation of matter. Suppose now, that the concentration of TDS (mass/volume) for the various components

TABLE 23

Summary of Equations for Quantity Model

Secondary Parameter	Equation
Evaporation X6	$X6 = -X1 + X2 + X3 - X4 + X5 - X11 + X12$
Ground water outflow X7	$X7 = .015X1 + .127127$
Ground water inflow X8	$X8 = .015X2 + .120983$
Final available ground water storage X10	$X10 = X7 - X8 + X9 - X11 + X12$
Percolation X11	$X11 = (X2 + .25(X3 + X4) + 2.0X5 + .5X12) \cdot .0360523$
Ground water discharge X12	$X12 = 1.164439 - .0555435(X9 + X10)$
Initial available ground water storage X9	$X9_i = X10_i - 1$

of flow were known. By multiplying the various volumes of flow for the time period by the average TDS concentration for the time period an average value of the mass of TDS associated with the various flow components over the time period would result. Let T_i represent the concentration of TDS associated with flow component X_i then,

$$L_i = X_i \cdot T_i \qquad (22)$$

Where

L_i is the mass of TDS associated with flow component X_i

L_i is generally termed the load. As an example, consider the balance equation for the surface water sub-system, equation (16). By multiplying the various X_i's by the appropriate T_i a balance equation is obtained for the L_i's.

$$-L1 + L2 + L5 - L6 - L11 + L12 - (L4 - L3) = 0 \qquad (23)$$

Equation (23) expresses the balancing (supply versus demand) of the mass of TDS arising from the various flow components in the surface water sub-system. Now if the regression equations for the quantity model were all physical regression equations, then the regression equations for the quality model would be:

$$L_i = \Sigma_i \, C(k,j) X_j T_j \qquad (24)$$

However, the regression equations for the quantity model are not all physical. In fact, only the regression equations for X11 and X12 are assumed to have physical character. Problems also arise with individual components. For example, X5 in the quantity model gives rise directly to run off. In the quality model the concentration that would be associated

with the precipitation and run off are quite different. Run off has come into contact with the soil and as a result becomes highly concentrated in TDS as compared to the concentration of TDS in the precipitation. The evaporation component (X6) has no associated concentration of TDS; i.e., T6 equals zero, and therefore, L6 equals zero. L3 and L4 are analogous to X3 and X4 of the quantity model but L9 and L10 differ.

$$L9 = X9' \cdot T9$$

$$L10 = X10' \cdot T10$$

Where X9' and X10' represent the volume of stored water in the ground water sub-system as opposed to X9 and X10 which represent the available volume for storage.

Figure 10 shows a schematic of the quality model for sub-basin 42KJA on an annual basis. The parameters used in Figure 10 are defined as follows:

L1 = Mass of TDS that leaves the surface water sub-system via the outflow, X1, in a one-year period.

L2 = Mass of TDS that enters the surface water sub-system via the inflow, X2, in a one-year period.

L3 = Mass of TDS stored in the surface water sub-system at the beginning of the period.

L4 = Mass of TDS stored in the surface water sub-system at the end of the period.

L5 = Mass of TDS that enters the surface water sub-system in one period.

L7 = Mass of TDS that leaves the ground water sub-system via outflow, X7, in one period.

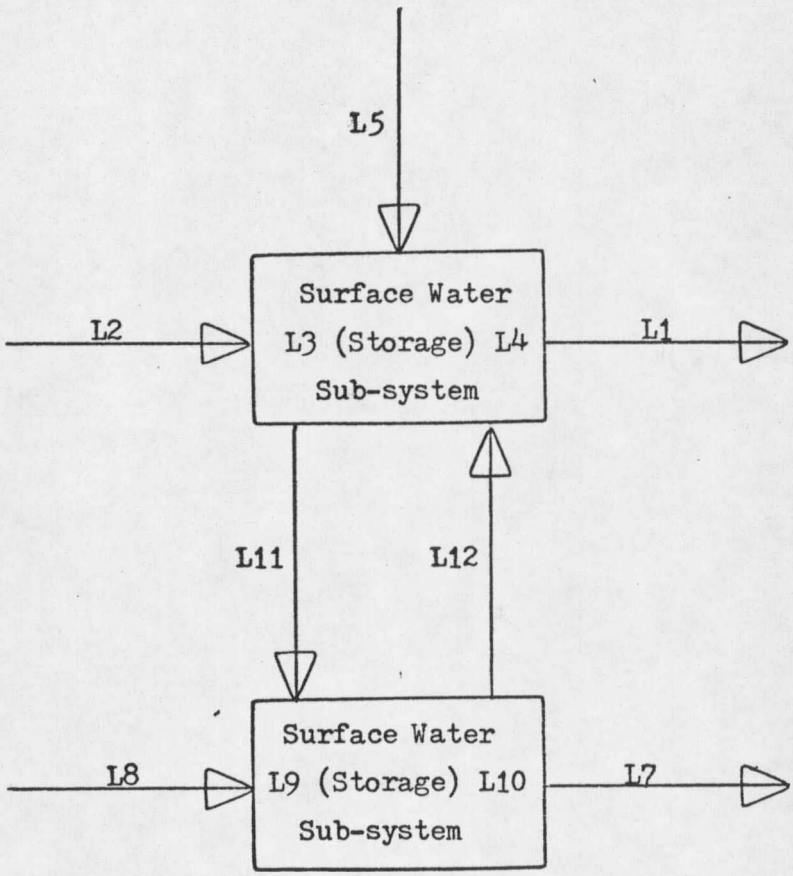


Figure 10. Schematic of Quality Model Used in Sub-basin 42KJA

L8 = Mass of TDS that enters the ground water sub-system via inflow, X8, in one period.

L9 = Mass of Tds stored in the ground water sub-system at the beginning of the period.

L10 = Mass of TDS stored in the ground water sub-system at the end of the period.

L11 = Mass of TDS that is transferred from the surface water to the ground water sub-systems in one period.

L12 = Mass of TDS that is transferred from the ground water to the surface water sub-systems in one period.

Analogous to the quantity model, there are two sub-systems shown in Figure 10 for the quality model. The balance equations for the two sub-systems are:

$$-L1 + L2 + L5 - L11 + L12 - (L4 - L3) = 0 \qquad (23)$$

$$-L7 + L8 + L11 - L12 - (L10 - L9) = 0 \qquad (25)$$

As has been previously mentioned ECM and TDS values correlate highly. That is, for any particular site it has been well established that:

$$ECM = \propto(TDS) \qquad (26)$$

Where:

$\propto$ is a proportionality constant (Ref. 7)

ECM values were used in place of TDS in the quality model. This substitution does not present any particular problem; the system is a pseudo mass balance, but the conservation of matter is still the underlying principle.

$$L_i = X_i \cdot E_i \qquad (27)$$

Where:

E_i is the ECM associated with flow component X_i

No units are explicitly expressed with the flow weighted ECM, $X_i \cdot E_i$, although it is implicitly understood that the values represent an amount of matter. More data were available in the form of ECM than TDS and the use of ECM provides the necessary parameter for the use of the regression equations for the common ions.

The primary parameters for the water quality model are L1, L2, L3 and L4. L3 and L4 are zero for each year, since X3 and X4 are zero for each year. Average annual values were available for ECM for the 5-year period, 1969 to 1973, from the U. S. Geological Survey (Ref. 6). Using the regression equations in Table 12 for the Yellowstone River at Billings and the Bighorn River at Bighorn, average annual values of ECM were calculated from $\bar{Q}$, the average annual value of Q. The regression equations in Table 12 relate the ECM to instantaneous Q values. The value of $\bar{Q}$ used for the Yellowstone River was increased in proportion to the amount of drainage area between Billings and the confluence with the Bighorn River. L2 was set equal to the sum of the annual ECM loads (flow weighted ECM) for the Yellowstone and Bighorn Rivers near their confluence. Using the regression equation in Table 12 for the Yellowstone River at Miles City, average annual values of ECM were calculated from the average annual values of Q. The values of Q for the Tongue River at Miles City were subtracted from the values of Q for the Yellowstone River at Miles City, since the values of Q for the Yellowstone River are taken below the

confluence with the Tongue River and the values of ECM are taken above the confluence. L1 was set equal to the annual ECM load for the Yellowstone River at Miles City. Regression values for both L1 and L2 were generated using equation (27) and the regression equations of Table 12, for the 25-year period, 1944 to 1968. The quality model was calibrated with these 25 years of data and then applied to the 5 years, 1969 to 1973, for which actual ECM data are available. The values of ECM for L1 and L2 are tabulated in Table J of Appendix I.

Two of the seven secondary parameters can be determined from the balance equations (23) and (25). L9 and L10 are assumed to be equal for each year; i.e., there is no yearly change in the mass of dissolved materials stored in the ground water sub-system. This assumption is based upon the extremely large volume of water stored in the ground water sub-system, the presumably small fluctuation that occurs in this volume from year to year, and the relatively constant value of ECM associated with ground water storage. So X9 minus X10 equals zero for each year. This leaves five secondary parameters for which regression equations must be supplied.

L5, the run off load is determined from the balance equation for the surface water sub-system, equation (23).

L7, the ground water sub-system outflow is determined from equation (25), the ground water sub-system balance equation.

More mass on the average is expected to enter the ground water sub-system via L11 than L8. The following proportion was assumed:

$$L8/(L8 + L11) = 1/4$$

leading to:

$$L8 = (1/3)L11 \qquad (28)$$

L11 was determined by employing the regression equation for X11 from the quantity model.

$$L11 = (L2 + 2.0L5 + .05L12)SF' \qquad (29)$$

Where SF' is again a scaling factor and is determined from averaging values of $\overline{L2}$, $\overline{L5}$, $\overline{L11}$ and $\overline{L12}$.

Since the definitions of L9 and L10 correspond to X9' and X10' rather than X9 and X10, the physical regression equation X12 in the quantity model was not applicable to the quality model. Instead the assumption was made that the annual average values of ECM associated with L7 and L12 were equal, i.e., E7 = E12 for each year. From this assumption the following regression equation was obtained:

$$L12 = (X12/X7) \cdot L7 \qquad (30)$$

The fraction (X12/X7) varies from year to year and is obtained from the quantity model. The ECM values of Rosebud Creek in the fall of the year range from 1500 to 2000 umhos/cm @ 25°c (Ref. 13). The water in Rosebud Creek at this time of the year was assumed to be primarily due to ground water discharge (X12). The concentration associated with this water was assumed to be representative of the concentration (E12) due to X12 for the entire sub-basin. The average annual volume of water from ground water discharge ($\overline{X12}$) for the 25 year period, 1944 to 1968, was obtained from the quantity model.

$\overline{X12}$ = .579220 MAF

This value of ground water discharge was multiplied by the range of ECM values for Rosebud Creek to get an estimate of the average annual load ($\overline{L12}$). $\overline{L12}$ ranged from 868.83 to 1158.44. From the quantity model, the average annual value of the fraction (X12/X7) was determined to be 2.281907. Using 1000.00 for the value of $\overline{L12}$, $\overline{L7}$ equaled 438.23. From equations (25) and (28) on an average annual basis $\overline{L11}$ and $\overline{L8}$ were determined to be 1078.6725 and 359.558 respectively. Using the values of $\overline{L1}$ and $\overline{L2}$ along with equation (23) on an average annual basis, $\overline{L5}$ was determined to be 1030.98, SF' was then determined to be .165170. An initial value of L8 ($L8_o$) is assumed such that $\overline{L8} = (1/3)\overline{L11}$. Table 24 summarizes the equations for determining the values of the secondary parameters for the quality model.

The quality model computer program and data output for the 25-year period, 1944-1968, are shown in Appendix IV. The model achieved a reasonable balance with $L8_o$ equal to 380 and SF' equal to .187. The data output gives ECM values in μmhos/cm @ 25°c for the various flow components. For L5, the associated run off flow component (X5') was assumed to be the difference between the precipitation, X5, and the percolation, X11 (X5' = X5 - X11).

The quality model was validated by interchanging the roles of L1 and L5; i.e., L5 was assumed to be an exogenous parameter and L1, endogenous, was simulated by the model. The data for the 25-year period, 1944 to 1968, were used (Appendix IV). The simulated values of the parameters

TABLE 24

Summary of Equations for the Quality Model

Secondary Parameter	Equation
Run off L5	$L5 = L1 - L2 + L11 - L12$
Ground water outflow L7	$L7 = L8 + L11 - L12$
Ground water inflow L8	$L8 = 1/3 L11$
Percolation L11	$L11 = (L2 + 2.065 + 0.5L12) \cdot 1651696$
Ground water discharge L12	$L12 = (X12/X7) \cdot L7$

using L2 and L5 as exogenous are shown in Table 25. The values of the parameters in Table 25 are comparable to the output data shown in Appendix IV. The favorable comparison of these values indicates how well the model reiterates the data, that is, its ability to reproduce historical record. With the model set up for L5 as an exogenous parameter, the values of L5 were perturbed. Increased coal mining and agricultural activities in the basin could possibly increase the concentration in the run off component. The values of L5 were increased by 25 percent. Using the perturbed values of L5, the values of the essential parameters simulated by the model are shown in Table 26. The values in Table 26 are comparable to those in the data output in Appendix IV. As would be expected to occur, the values of ECM associated with the outflow, E1, show an increase when the perturbed values of L5 were used as exogenous data. The average increase in E1 over the 25-year period was about 5 percent with a high of 9 percent and a low of 2 percent.

Using L1 and L2 as exogenous parameters, the quality model was applied to the 30-year period, 1944 to 1973, where available ECM values were used for the last five years. The data output for the 30-year period is also shown in Appendix IV. The simulated secondary parameter values of ECM for the last five years (1969 - 1973) are consistent with those for the previous 25 years (1944 - 1968).

The values of ECM for the secondary parameters generated by the quality model are reasonable, given the aggregating nature of annual averaging on the ECM values associated with the various flow components.

TABLE 25

ECM Values for the Essential Parameters of the Quality Model,

1944 to 1968, Using L2 and L5 as Exogenous

(units = μmhos/cm @ 25°c)

Year	E1	E2	E3	E4	E5	E6	E7	E8	E11	E12
1944	612.3	567.8	.0	.0	189.8	.0	2041.2	1485.6	1883.0	2041.2
1945	596.7	542.7	.0	.0	205.4	.0	1880.4	1958.4	2104.9	1880.4
1946	628.9	532.1	.0	.0	209.0	.0	1788.6	1684.6	1899.9	1788.6
1947	575.8	540.2	.0	.0	89.4	.0	1761.3	1439.7	1855.6	1761.3
1948	574.2	493.6	.0	.0	279.6	.0	1926.4	1505.1	2088.8	1926.4
1949	638.2	517.4	.0	.0	394.4	.0	1862.8	1920.9	2428.7	1862.8
1950	586.6	496.2	.0	.0	314.2	.0	1958.2	1510.9	2146.6	1958.2
1951	564.6	506.1	.0	.0	259.2	.0	2003.8	1653.1	2109.5	2003.8
1952	599.9	447.4	.0	.0	659.5	.0	2097.0	1789.8	2676.7	2097.0
1953	664.1	491.4	.0	.0	351.0	.0	1950.5	2063.4	2181.1	1950.5
1954	664.1	478.9	.0	.0	439.3	.0	1919.1	1732.9	2376.6	1919.1
1955	683.9	526.5	.0	.0	332.7	.0	1844.4	1831.6	2230.0	1844.4
1956	603.9	448.5	.0	.0	523.9	.0	2110.1	1451.6	2463.1	2110.1
1957	568.2	418.2	.0	.0	447.0	.0	2321.2	1715.5	2222.4	2321.2
1958	636.5	539.2	.0	.0	268.1	.0	1993.5	2177.6	2190.3	1993.5

Table 25 (Cont.)

Year	E1	E2	E3	E4	E5	E6	E7	E8	E11	E12
1959	639.2	466.4	.0	.0	428.1	.0	2017.2	1611.2	2312.1	2017.2
1960	723.5	501.3	.0	.0	559.6	.0	1903.6	2067.0	2724.7	1903.6
1961	775.9	490.3	.0	.0	453.2	.0	1816.0	1828.5	2425.9	1816.0
1962	584.1	476.7	.0	.0	311.6	.0	2127.2	1276.5	2049.9	2127.2
1963	611.6	512.9	.0	.0	198.8	.0	2067.4	1868.6	1860.6	2067.4
1964	590.7	522.1	.0	.0	196.0	.0	1994.8	1556.7	1941.9	1994.8
1965	543.6	497.7	.0	.0	184.7	.0	2082.5	1443.0	1908.1	2082.5
1966	705.2	439.3	.0	.0	623.5	.0	2039.5	2232.0	2718.1	2039.5
1967	560.9	464.7	.0	.0	314.6	.0	2136.6	1387.0	2060.4	2136.5
1968	563.9	486.9	.0	.0	226.7	.0	2142.5	1772.6	1900.5	2142.5

TABLE 26

ECM Values for the Essential Parameters of the Quality Model,

1944 to 1968, Employing Perturbed Values of L5

(units = μmho/cm @ 25°c)

Year	E1	E2	E3	E4	E5	E6	E7	E8	E11	E12
1944	634.8	567.8	.0	.0	237.2	.0	2152.0	1485.6	2012.1	2152.0
1945	614.6	542.7	.0	.0	256.7	.0	1987.4	2092.6	2214.6	1987.4
1946	654.9	532.1	.0	.0	261.2	.0	1909.1	1772.4	2037.9	1909.1
1947	584.6	540.2	.0	.0	111.7	.0	1827.4	1544.2	1904.0	1827.4
1948	596.5	493.6	.0	.0	349.5	.0	2042.7	1544.4	2236.1	2042.7
1949	669.1	517.4	.0	.0	493.0	.0	2009.3	2056.4	2627.4	2009.3
1950	615.1	496.2	.0	.0	392.7	.0	2119.6	1634.4	2323.9	2119.6
1951	585.1	506.1	.0	.0	324.0	.0	2140.1	1789.6	2242.8	2140.1
1952	639.3	447.4	.0	.0	824.4	.0	2298.9	1902.9	2963.4	2298.9
1953	709.3	491.4	.0	.0	438.7	.0	2159.1	2284.4	2414.2	2159.1
1954	711.6	478.9	.0	.0	549.1	.0	2131.6	1918.1	2642.7	2131.6
1955	725.0	526.5	.0	.0	415.9	.0	2029.4	2036.7	2444.2	2029.4
1956	646.4	448.5	.0	.0	654.9	.0	2340.0	1590.9	2739.7	2340.0
1957	609.0	418.2	.0	.0	558.7	.0	2583.6	1908.1	2474.0	2583.6
1958	664.4	539.2	.0	.0	335.1	.0	2161.2	2424.2	2345.9	2161.2

Table 26 (Cont.)

Year	E1	E2	E3	E4	E5	E6	E7	E8	E11	E12
1959	683.3	466.4	.0	.0	535.1	.0	2222.6	1725.6	2568.8	2222.6
1960	778.8	501.3	.0	.0	699.5	.0	2123.4	2296.5	3044.2	2123.4
1961	847.1	490.3	.0	.0	566.5	.0	2053.2	2042.9	2754.2	2053.2
1962	617.0	476.7	.0	.0	389.5	.0	2341.5	1449.3	2239.4	2341.5
1963	636.5	512.9	.0	.0	248.5	.0	2223.5	2041.4	1989.0	2223.5
1964	610.2	522.1	.0	.0	245.0	.0	2116.8	1664.1	2055.8	2116.8
1965	558.7	497.7	.0	.0	230.9	.0	2192.8	1527.7	2005.3	2192.2
1966	767.4	439.3	.0	.0	779.4	.0	2265.6	2345.7	3085.4	2265.6
1967	589.7	464.7	.0	.0	393.2	.0	2340.5	1574.4	2235.2	2340.5
1968	587.4	486.9	.0	.0	283.4	.0	2303.2	1923.1	2036.5	2303.2

At this macro level of modeling the regression equations for the common ions, which involve instantaneous ECM values, could be applied to get annual average concentrations. The assumption made is that for each flow component, the common ion regression equations have the same form (e.g. equation 3). The model must be extended further toward the micro to obtain an indication of how the ions are associated with the various flow components on a shorter time period. The Montana State Department of Natural Resources and Conservation has recently completed calibrating a monthly water quantity model for sub-basin 42KJA, which might also be used to generate flow for a monthly water quality model.

CHAPTER IV

Concluding Remarks

Current Utility

The regression equations for the common ions for this reach of the Yellowstone River are quite useful as they now stand. With the exception of high flow when ECM values will be low, excellent continuous values of the common ions can be simulated from continuous monitoring of ECM and Q. Devises for continuous monitoring of ECM and Q are currently available. Continuous flow recording devises have been employed for a number of years by the U. S. Geological Survey and the use of continuous recording ECM devises is currently increasing. The monitoring of ECM is of less cost than the analyzing of water samples for common ion concentrations. This aspect could release funds for sampling and analysis of parameters for which little data exists. Nutrients are an example of parameters for which little data are available. Analyses for nutrients cost considerably more than for common ions and the data which are available do not correlate well with either Q or ECM (Ref. 5, 10).

The regression analysis has also, pointed out the possible need for better measurements of K and Cl ion concentrations. Better techniques for analysis of K and Cl are available.

In trying to apply the regression equations to the concentration of other dissolved constituents, e.g. nutrients and metals, two problems are apparent. First, the ability to measure these other constituents is hampered by their relatively low concentrations. Thus, they may suffer

from the same problem as K and Cl. Secondly, some constituents, like nutrients are nonconservative; i.e., the ionic form of the constituent may change in the water solution. For example, bacterical action on ammonia (NH_3) can change it to nitrate (NO_3).

The regression equations for the common ions could also be used for quality control in the laboratory analyses. The values generated by the regression equations could provide a check on the chemical analyses.

Recommendation for Future Research

The regression equations involving the ion concentrations as expressed in terms of the natural logarithm of ECM seems to reveal some underlying physical character of natural waters. This logarithmic relationship needs to be tested further. Because data are collected in a periodical fashion at the four sites (e.g. monthly, daily), the distribution of the data over the range of flow values is biased. As a consequence of periodic sampling, more data are available at median flow values. This has a definite effect on the ability of the regression equations to predict ion concentrations at high and low flow conditions. The weakness of the ion regression equations at high Q, low ECM conditions indicates the need of including Q in the regression equations. The way in which Q was introduced into the regression analysis (equation 3) did not alleviate the problem at low ECM values. Further investigation is needed to determine how the Q dimension could be introduced. An example would be to let the regression coefficients of equation 3, bi, ai, be

linear functions of Q; e.g.

$$bi = ci \cdot Q + di \qquad (31)$$

$$ai = ei \cdot Q + fi \qquad (32)$$

More study is needed on other streams and different reaches of the same streams. Streams that cut through different geological strata may show different characteristic distributions for the common ion concentrations. Different reaches of the same stream may also show a different distribution of the common ion concentrations, even though the geological features are similar, due to different flow regimes. Investigating the application of the common ion regression equations to different streams and reaches of streams, and the various methods of introducing a flow dimension may not only increase their usefulness but may yield a better understanding of the physical phenomena involved. Even in situations where the given regression equation form is not applicable, further investigation would be useful.

An investigation as to how the regression constants change with flow regulation would be of value. For example, by studying a stream for which sufficient data exists before and after the implementation of a reservoir, the change in the regression coefficients due to regulation could be determined. Thus, an indication of how the distribution of common ions may change for a similar stream might be predicted.

In this study, the first level of a large-scale, water quality model was investigated as a possible method of obtaining information on the amounts of ECM associated with the various flow components. The

model produced reasonable results in the first level, but application of the common ion regression equations would yield only annual average common ion concentrations. Since the common ion regression equations involve instantaneous values of ECM and Q, they are at a much more micro level. By developing the model to a more micro level, the application of the common ion regression equations would yield more useful results. Since the monthly quantity model for 42KJA has been calibrated and is available, the development of a monthly quality model would be an appropriate next step. At the monthly level, the application of the regression equations for the common ion concentrations would yield monthly average values. Monthly average values of the common ion concentrations would be more useful to water quality planners and managers.

After the monthly water quality model for sub-basin 42KJA, the next more micro level of modeling might focus on a smaller geological unit. For example, the modeling of a drainage area of similar geologic features may be appropriate. This level of modeling might be coupled with an investigation of the common ion regression equations to the same drainage area.

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APPENDICES

APPENDIX I

Raw Data

Contents

Tables of raw data points for the Yellowstone River at Billings, the Bighorn River at Bighorn, the Tongue River at Miles City, and the Yellowstone River at Miles City. This data was obtained from U. S. Geological Survey annual report of quality of surface waters in Montana (Ref. 6).

Table of annual inflow and outflow and precipitation for sub-basin 42KJA.

Cross Reference

Pages iv, 13, 43, 44, and 51

TABLE A

Raw Data for the Yellowstone River at Billings

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
37.0	13.0	23.0	2.8	155	77	3.7	0.0	393	6090
37.0	12.0	23.0	2.5	150	68	5.0	0.0	389	4640
38.0	13.0	24.0	3.1	151	77	6.6	0.0	398	3700
45.0	15.0	26.0	3.2	171	89	10.0	0.0	465	2180
44.0	14.0	25.0	3.5	170	87	10.0	0.0	438	2970
45.0	14.0	35.0	3.5	172	97	8.6	0.0	479	5000
39.0	13.0	27.0	3.1	151	74	7.8	0.0	405	3690
30.0	9.9	19.0	3.6	118	51	5.7	0.0	315	6230
14.0	3.3	5.8	1.3	56	12	1.1	0.0	118	41300
18.0	6.0	11.0	1.9	79	24	2.7	0.0	185	16500
29.0	10.0	19.0	2.5	130	46	4.4	0.0	304	7020
31.0	11.0	20.0	4.2	138	58	4.5	0.0	345	6510
38.0	13.0	25.0	3.2	156	67	5.9	0.0	386	4930
37.0	13.0	28.0	3.1	153	76	6.4	0.0	399	4180
38.0	12.0	25.0	2.8	147	76	6.3	0.0	386	4160
36.0	10.0	25.0	4.1	134	49	6.4	0.0	340	4840
41.0	11.0	26.0	3.5	155	58	6.3	0.0	392	3110

Table A (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (umho/cm @ 25°c)	Q (cfs)
43.0	15.0	28.0	3.7	171	70	8.5	0.0	440	2940
43.0	14.0	30.0	3.6	160	88	7.9	0.0	429	3330
24.0	7.1	11.0	3.3	104	22	3.6	0.0	225	13700
16.0	5.0	7.3	1.2	74	19	2.0	0.0	161	24400
15.0	5.2	10.0	1.7	76	22	3.0	0.0	179	27000
24.0	8.3	16.0	2.4	119	39	3.7	0.0	263	8890
30.0	10.0	19.0	2.7	146	46	4.8	0.0	318	7440
30.0	11.0	20.0	2.6	128	59	5.7	0.0	326	6460
34.0	12.0	21.0	2.6	140	64	6.0	0.0	372	5370
38.0	13.0	25.0	2.8	151	77	6.0	0.0	398	4440
50.0	17.0	31.0	3.9	186	96	7.9	0.0	485	2140
46.0	13.0	25.0	3.5	161	84	7.9	0.0	429	3160
39.0	13.0	27.0	3.4	153	84	8.1	0.0	422	3470
41.0	14.0	29.0	3.2	162	79	6.6	0.0	439	3970
46.0	18.0	46.0	3.3	197	120	7.2	0.0	549	8500
21.0	6.5	13.0	1.8	96	33	3.7	0.0	217	15200
17.0	5.7	9.7	1.6	75	22	2.8	0.0	177	18500
35.0	13.0	26.0	3.0	138	62	5.7	0.0	386	3650
37.0	13.0	25.0	2.9	157	69	5.2	0.0	382	5970

Table A (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (umho/cm @ 25°c)	Q (cfs)
46.0	16.0	31.0	4.1	178	88	7.3	0.0	477	3030
50.0	20.0	40.0	6.1	202	118	9.0	0.0	602	1330
36.0	15.0	25.0	4.2	152	77	4.4	0.0	409	3030
39.0	14.0	31.0	4.5	158	86	10.0	0.0	444	2970
41.0	14.0	31.0	4.6	162	88	7.4	0.0	446	3560
38.0	16.0	37.0	7.7	174	97	8.8	0.0	499	4240
22.0	6.3	12.0	1.7	94	27	2.4	0.0	225	24200
18.0	5.7	11.0	2.1	78	24	7.9	0.0	189	17600
22.0	8.4	18.0	2.5	108	39	4.3	0.0	276	9000
36.0	13.0	26.0	3.6	147	68	6.6	0.0	367	3890
38.0	13.0	24.0	3.2	153	65	6.8	0.0	400	3830
37.0	12.0	26.0	3.6	146	74	7.2	0.0	402	3720
38.0	16.0	30.0	4.1	173	77	6.0	0.0	457	3140
21.0	5.5	10.0	1.8	94	24	2.1	0.0	199	33400
40.0	15.0	31.0	3.1	156	94	6.5	3.0	438	4610
43.0	15.0	29.0	3.2	167	70	7.6	0.0	436	4450
49.0	17.0	28.0	3.8	180	86	8.7	0.0	492	2680
48.0	16.0	30.0	3.6	171	98	7.6	0.0	476	2730
47.0	16.0	32.0	3.9	165	91	7.7	2.0	483	2550

Table A (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
39.0	13.0	28.0	3.4	149	78	6.4	0.0	411	3770
30.0	9.7	24.0	2.8	114	63	7.3	0.0	295	6440
14.0	4.1	6.8	1.4	67	13	1.8	0.0	148	38600
10.0	5.3	8.8	1.7	67	21	2.0	0.0	152	29800
25.0	8.1	16.0	2.5	103	42	3.5	0.0	252	9210
36.0	13.0	33.0	3.1	143	63	5.6	2.0	373	5040
41.0	16.0	27.0	2.7	169	79	6.0	0.0	439	3950
38.0	14.0	25.0	3.0	158	67	6.6	0.0	406	3150
38.0	15.0	30.0	3.2	164	75	5.6	0.0	428	4160
38.0	14.0	27.0	3.4	149	83	6.6	0.0	421	3850
40.0	13.0	27.0	2.8	162	76	6.6	0.0	417	3320
21.0	5.5	10.0	1.8	94	24	2.1	0.0	199	33400
29.0	10.0	19.0	2.4	131	49	4.5	0.0	317	8250

TABLE B

Raw Data for the Bighorn River at Bighorn

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
77	25	82	4.7	210	310	11.0	0	982	5970
77	27	81	3.6	207	290	11.0	0	906	5830
87	32	86	3.9	206	330	12.0	8	988	1900
92	26	95	3.8	211	320	14.0	1	957	3500
81	29	84	4.1	216	300	11.0	0	950	3650
75	28	82	3.9	218	290	11.0	0	961	4960
86	31	85	4.3	210	300	12.0	6	970	5230
85	30	81	3.9	231	300	12.0	0	946	6950
80	28	72	5.2	233	280	14.0	0	890	7900
77	27	66	3.3	214	250	11.0	0	840	6270
62	23	61	3.6	180	210	7.6	0	740	3740
48	17	46	3.0	149	160	6.1	0	580	3470
65	26	74	3.2	173	250	10.0	4	910	2900
63	22	66	3.3	183	230	9.3	0	752	3310
76	26	80	3.9	202	290	10.0	0	895	1280
72	25	80	3.7	200	260	11.0	0	747	4300
71	25	75	3.3	195	270	10.0	0	836	4260

Table B (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
73	24	73	3.5	203	270	10.0	0	823	3740
81	28	81	3.7	221	290	11.0	0	902	3240
80	29	85	3.3	223	280	10.0	0	921	3320
91	41	120	5.0	212	450	11.0	0	1170	7220
73	25	62	3.6	217	230	10.0	0	793	7260
73	26	71	3.6	216	260	12.0	0	858	3760
76	29	86	3.8	215	300	12.0	0	914	1490
70	24	72	4.0	195	260	10.0	0	823	3540
72	25	76	3.4	200	290	7.3	0	878	5500
64	22	76	3.1	185	250	8.8	0	806	5280
70	25	76	3.5	197	250	10.0	0	859	3360
68	22	70	3.5	183	240	9.1	0	762	4060
78	27	81	4.2	216	290	12.0	0	926	4000
75	25	71	4.3	204	270	11.0	0	854	7070
72	25	76	4.2	207	270	12.0	0	871	7170
77	26	81	4.0	202	280	12.0	0	901	6750
79	29	95	4.2	204	340	11.0	0	936	2760
77	27	94	3.3	215	320	11.0	0	946	2970
74	28	84	2.3	209	280	8.4	0	923	3100

Table B (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO3 (Mg/L)	SO4 (Mg/L)	Cl (Mg/L)	CO3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
70	24	69	3.9	191	250	8.1	0	740	3930
72	23	70	4.2	191	260	9.1	0	747	6500
83	32	88	3.9	226	300	12.0	0	983	4820
83	29	85	3.9	230	320	11.0	0	943	6420
85	30	81	6.1	226	290	13.0	0	954	6340
75	28	90	3.4	213	270	9.8	0	926	9160
58	22	33	3.1	176	150	7.6	0	599	6300
49	16	52	2.5	160	170	5.5	0	598	5100
50	18	52	3.0	160	180	6.9	0	609	3030
80	28	98	4.3	220	329	11.0	0	986	3340
75	32	93	4.7	226	326	11.0	0	994	3900
80	31	92	4.6	224	316	11.0	0	963	3720
83	31	100	5.1	235	334	13.0	0	1010	5040
88	45	123	5.5	279	418	10.0	0	1180	2040
84	34	110	5.7	236	357	10.0	0	1050	2880
88	31	81	3.7	246	302	7.4	0	933	5610
85	29	81	4.4	230	284	13.0	0	899	11100
71	28	84	3.9	216	277	11.0	0	867	3080
64	24	72	4.0	187	237	7.2	0	761	2060

Table B (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
73	26	84	3.6	212	288	13.0	0	900	4230
80	27	83	4.2	219	295	11.0	0	914	5030
70	24	84	4.1	188	284	9.4	0	866	3750
68	22	73	3.9	190	260	8.0	0	892	5800
78	29	87	4.1	229	301	11.0	0	934	4500
73	26	79	4.0	210	271	9.6	0	878	12800
60	21	64	3.4	172	220	7.7	0	723	3420
96	34	117	3.9	228	400	15.0	0	1160	1310
92	37	106	4.5	246	373	14.0	0	1120	2860
52	17	61	3.2	150	187	5.0	0	637	2630
87	35	100	4.4	229	349	14.0	0	1070	840

TABLE C

Raw Data for the Tongue River at Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
58	42	47	5.2	262	190	4.1	0	755	444
65	45	49	4.0	279	200	3.5	5	802	539
85	62	83	4.7	396	310	8.2	0	1120	188
95	69	88	6.1	448	350	6.0	0	1230	222
77	52	70	5.5	325	270	4.4	0	988	340
63	47	68	4.6	300	260	4.3	0	955	389
77	62	92	6.1	334	350	5.4	3	1140	245
71	50	55	5.1	280	280	5.0	0	914	569
46	30	40	3.6	208	160	2.6	0	625	815
33	16	17	1.9	139	68	1.4	0	365	1310
46	26	43	3.6	230	130	3.4	0	628	173
52	31	46	4.3	239	160	3.0	0	680	270
57	41	60	4.9	266	210	4.2	3	980	140
65	45	58	4.4	277	230	4.1	0	849	372
65	49	61	4.5	300	240	4.5	0	877	395
86	64	92	6.3	323	370	6.3	0	1030	78
76	54	77	5.1	372	280	4.8	0	1130	195

Table C (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
78	57	75	5.2	372	280	5.3	0	1050	270
64	48	90	5.5	299	320	4.9	0	1040	430
73	59	100	6.4	343	340	5.4	0	1170	179
68	51	54	4.8	282	260	4.4	0	886	983
36	20	26	2.8	157	89	2.8	0	445	1950
41	22	28	3.6	188	100	3.5	0	490	554
52	30	46	4.4	242	160	3.4	0	599	257
45	25	68	5.0	226	170	3.0	0	704	361
44	27	70	4.9	219	210	2.3	0	724	410
69	50	82	5.2	318	290	3.4	0	1030	341
75	55	88	5.3	334	330	5.3	0	1080	345
80	58	88	4.8	363	330	6.3	0	1120	128
84	59	78	5.5	388	310	5.2	0	1100	198
31	17	39	5.2	139	120	2.8	0	463	2800
70	47	64	6.0	263	270	4.7	0	899	1230
68	51	110	8.4	283	360	5.7	0	1100	604
45	25	30	3.2	169	130	2.4	0	535	1660
64	43	59	5.1	272	230	4.8	0	822	269
68	47	65	4.9	304	250	4.6	0	902	272

Table C (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
75	51	74	5.7	346	280	6.0	0	988	227
91	54	72	5.4	369	280	4.5	0	966	172
65	37	42	6.0	255	190	4.4	0	725	1070
78	57	94	8.2	293	360	4.4	0	1140	930
72	51	67	7.8	259	300	5.0	0	922	1040
35	20	33	3.0	152	110	5.4	0	467	1840
51	31	46	3.7	231	160	2.7	0	653	300
57	37	79	4.5	287	230	3.3	0	881	69
53	34	59	4.7	238	190	2.8	0	759	447
62	65	121	6.8	301	396	5.8	0	1170	125
85	70	87	6.3	389	337	4.2	0	1150	150
70	54	74	5.8	334	282	4.2	0	1000	265
68	49	74	6.0	315	260	4.6	0	939	350
73	53	71	6.2	310	278	4.2	0	958	526
64	47	66	6.6	268	265	4.2	0	887	596
49	23	25	3.3	196	114	0.8	0	503	3360
48	27	43	4.2	220	145	3.0	0	610	363
45	26	40	3.5	213	134	2.6	0	580	243
57	30	52	5.2	252	276	2.9	0	719	235

Table C (Cont.)

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (μmho/cm @ 25°c)	Q (cfs)
76	54	73	5.1	327	276	4.8	9	997	267
70	48	64	4.5	330	243	4.4	0	924	200
63	41	62	5.9	227	211	3.4	0	813	184
63	45	65	5.0	288	234	5.2	0	859	380
65	54	70	5.3	303	267	4.6	9	963	567
53	35	36	4.1	228	171	2.4	0	667	1270
53	31	34	3.5	237	143	2.2	0	627	575
76	54	100	9.9	339	345	5.0	0	1020	33
65	49	65	5.0	266	283	3.6	7	969	373
58	38	56	4.7	250	199	2.2	0	759	150
60	42	79	5.6	274	245	3.5	4	900	52

TABLE D

Raw Data for the Yellowstone River at Miles City

Ca (Mg/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
58	22.0	55	3.6	183	180	9.2	7	681	7460
64	23.0	63	3.5	193	210	10.0	0	762	8530
65	25.0	74	3.9	205	250	9.3	0	828	7880
72	27.0	76	4.3	203	250	12.0	2	857	8000
65	24.0	68	4.0	201	230	10.0	0	800	8680
53	18.0	43	3.1	162	140	7.5	0	566	14800
40	7.9	25	2.3	116	85	4.8	0	365	40200
28	7.6	17	1.7	90	49	3.8	0	274	58400
33	12.0	27	2.7	116	82	4.0	0	385	19900
38	16.0	39	2.7	153	110	5.8	0	480	10600
50	19.0	52	3.3	162	150	7.7	5	660	7860
66	25.0	70	3.7	200	220	11.0	0	824	6900
56	20.0	57	3.4	184	170	10.0	0	671	9000
33	12.0	28	2.1	119	81	5.6	0	389	28600
54	22.0	62	4.2	187	190	9.5	0	701	10700
52	19.0	60	4.2	163	200	9.1	0	661	22000
23	7.0	15	1.5	83	47	2.8	0	247	48400

Table D (Cont.)

Ca (g/L)	Mg (Mg/L)	Na (Mg/L)	K (Mg/L)	HCO_3 (Mg/L)	SO_4 (Mg/L)	Cl (Mg/L)	CO_3 (Mg/L)	ECM (µmho/cm @ 25°c)	Q (cfs)
38	15.0	37	2.7	151	110	5.8	0	475	10400
58	21.0	55	4.1	192	190	9.8	0	668	6840
64	30.0	62	4.4	189	230	7.9	0	746	8100
58	24.0	59	4.1	204	200	9.5	0	703	14400
36	12.0	29	1.9	123	98	3.9	0	403	32000
41	15.0	46	3.4	167	130	8.1	0	534	10300
61	25.0	69	4.4	196	223	9.0	0	764	6800
68	26.0	80	5.3	196	266	11.0	0	883	8600
39	14.0	30	2.6	142	91	3.2	0	439	31100
51	18.0	54	4.1	178	166	9.2	0	625	6610
50	16.0	45	2.9	160	152	7.0	0	571	10700
57	22.0	61	3.9	189	202	9.6	0	702	7640
63	22.0	64	4.5	196	220	11.0	0	748	9800
23	7.5	16	1.9	91	46	2.8	0	249	27400
59	23.0	70	4.3	196	210	6.8	0	724	6900

TABLE E

Common Ion Data for the Yellowstone River

at Billings in Meq/L

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
1.8463	1.0694	1.0004	.0716	2.5402	1.6031	.1044	-.0631
1.8463	.9872	1.0004	.0639	2.4583	1.4157	.1410	-.0296
1.8962	1.0694	1.0439	.0793	2.4746	1.6031	.1862	-.0419
2.2455	1.2340	1.1309	.0818	2.8024	1.8529	.2821	-.0509
2.1956	1.1517	1.0874	.0895	2.7860	1.8113	.2821	-.0755
2.2455	1.1517	1.5224	.0895	2.8188	2.0195	.2426	-.0142
1.9461	1.0694	1.1744	.0793	2.4746	1.5406	.2200	.0080
1.4970	.8144	.8264	.0921	1.9338	1.0618	.1608	.0230
.6986	.2715	.2523	.0332	.9177	.2498	.0310	.0464
.8982	.4936	.4785	.0486	1.2947	.4997	.0762	.0255
1.4471	.8226	.8264	.0639	2.1305	.9577	.1241	-.0164
1.5469	.9049	.8699	.1074	2.2616	1.2075	.1269	-.0475
1.8962	1.0694	1.0874	.0818	2.5566	1.3949	.1664	.0041
1.8463	1.0694	1.2179	.0793	2.5074	1.5823	.1805	-.0135
1.8962	.9872	1.0874	.0716	2.4091	1.5406	.1777	-.0208
1.7964	.8226	1.0874	.1049	2.1960	1.0202	.1805	.1150

Table E (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
2.0459	.9049	1.1309	.0895	2.5402	1.2075	.1777	.0607
2.1457	1.2340	1.2179	.0946	2.8024	1.4574	.2398	.0419
2.1457	1.1517	1.3049	.0921	2.6221	1.8321	.2228	.0037
1.1976	.5841	.4785	.0844	1.7044	.4580	.1015	.0350
.7984	.4113	.3175	.0307	1.2127	.3956	.0564	-.0663
.7485	.4278	.4350	.0435	1.2455	.4580	.0846	-.0775
1.1976	.6828	.6960	.0614	1.9502	.8120	.1044	-.0831
1.4970	.8226	.8264	.0691	2.3927	.9577	.1354	-.0808
1.4970	.9049	.8699	.0665	2.0977	1.2283	.1608	-.0435
1.6966	.9872	.9134	.0665	2.2944	1.3324	.1692	-.0355
1.8962	1.0694	1.0874	.0716	2.4746	1.6031	.1692	-.0292
2.4950	1.3985	1.3484	.0997	3.0482	1.9987	.2228	.0136
2.2954	1.0694	1.0874	.0895	2.6385	1.7488	.2228	-.0149
1.9461	1.0694	1.1744	.0870	2.5074	1.7488	.2285	-.0474
2.0459	1.1517	1.2614	.0818	2.6549	1.6447	.1862	.0122
2.2954	1.4808	2.0009	.0844	3.2285	2.4983	.2031	-.0116
1.0479	.5347	.5655	.0460	1.5733	.6870	.1044	-.0748
.8483	.4689	.4219	.0409	1.2291	.4580	.0790	.0078
1.7465	1.0694	1.1309	.0767	2.2616	1.2908	.1608	.0802
1.8463	1.0694	1.0874	.0742	2.5730	1.4365	.1467	-.0192

Table E (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
2.2954	1.3162	1.3484	.1049	2.9171	1.8321	.2059	.0219
2.4950	1.6453	1.7399	.1560	3.3104	2.4567	.2539	.0025
1.7964	1.2340	1.0874	.1074	2.4910	1.6031	.1241	.0017
1.9461	1.1517	1.3484	.1151	2.5894	1.7905	.2821	-.0218
2.0459	1.1517	1.3484	.1176	2.6549	1.8321	.2087	-.0069
1.8962	1.3162	1.6094	.1969	2.8516	2.0195	.2482	-.0198
1.0978	.5183	.5220	.0435	1.5405	.5621	.0677	.0051
.8982	.4689	.4785	.0537	1.2783	.4997	.2228	-.0521
1.0978	.6910	.7829	.0639	1.7699	.8120	.1213	-.0253
1.7964	1.0694	1.1309	.0921	2.4091	1.4157	.1862	.0192
1.8962	1.0694	1.0439	.0818	2.5074	1.3533	.1918	.0096
1.8463	.9872	1.1309	.0921	2.3927	1.5406	.2031	-.0195
1.8962	1.3162	1.3049	.1049	2.8352	1.6031	.1692	.0032
1.0479	.4525	.4350	.0460	1.5405	.4997	.0592	-.0579
1.9960	1.2340	1.3484	.0793	2.6566	1.9570	.1833	-.0295
2.1457	1.2340	1.2614	.0818	2.7369	1.4574	.2144	.0688
2.4451	1.3985	1.2179	.0972	2.9499	1.7905	.2454	.0341
2.3952	1.3162	1.3049	.0921	2.8024	2.0403	.2144	.0101
2.3453	1.3162	1.3919	.0997	2.7707	1.8946	.2172	.0539
1.9461	1.0694	1.2179	.0870	2.4419	1.6239	.1805	.0173

Table E (Cont.)

Ca	Mg	Na	K	ALK	SO4	Cl	ERR
1.4970	.7980	1.0439	.0716	1.8683	1.3116	.2059	.0073
.6986	.3373	.2958	.0358	1.0980	.2707	.0508	-.0373
.4990	.4360	.3828	.0435	1.0980	.4372	.0564	-.1560
1.2475	.6663	.6960	.0639	1.6880	.8744	.0987	.0047
1.7964	1.0694	1.4354	.0793	2.4102	1.3116	.1580	.1212
2.0459	1.3162	1.1744	.0691	2.7696	1.6447	.1692	.0048
1.8962	1.1517	1.0874	.0767	2.5894	1.3949	.1862	.0099
1.8962	1.2340	1.3049	.0818	2.6877	1.5615	.1580	.0246
1.8962	1.1517	1.1744	.0870	2.4419	1.7280	.1862	-.0108
1.9960	1.0694	1.1744	.0716	2.6549	1.5823	.1862	-.0256
1.0479	.4525	.4350	.0460	1.5405	.4997	.0592	-.0579
1.4471	.8226	.8264	.0614	2.1469	1.0202	.1269	-.0423

$\overline{ERR} = -.0092$

$S_{ERR} = .0466$

TABLE F

Common Ion Data for the Bighorn River

at Bighorn in Meq/L

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.8423	2.0566	4.0017	.1202	3.4416	6.4540	.3103	-.0183
3.8423	2.2211	3.5233	.0921	3.3924	6.0376	.3103	-.0063
4.3413	2.6324	3.7408	.0997	3.6426	6.8704	.3385	-.0034
4.5908	2.1389	4.1322	.0972	3.4913	6.6622	.3949	.0382
4.0419	2.3857	3.6538	.1049	3.5399	6.2458	.3103	.0089
3.7425	2.3034	3.5668	.0997	3.5727	6.0376	.3103	-.0212
4.2914	2.5502	3.6973	.1100	3.6415	6.2458	.3385	.0405
4.2415	2.4679	3.5233	.0997	3.7857	6.2458	.3385	-.0036
3.9920	2.3034	3.1318	.1330	3.8185	5.8294	.3949	-.0492
3.8423	2.2211	2.8708	.0844	3.5071	5.2049	.3103	-.0004
3.0938	1.8921	2.6533	.0921	2.9499	4.3721	.2144	.0255
2.3952	1.3985	2.0009	.0767	2.4419	3.3311	.1721	-.0125
3.2435	2.1389	3.2188	.0818	2.9685	5.2049	.2821	.0266
3.1437	1.8098	2.8708	.0844	2.9991	4.7885	.2623	-.0177
3.7924	2.1389	3.4798	.0997	3.3104	6.0376	.2821	-.0125
3.5928	2.0566	3.4798	.0946	3.2777	5.4131	.3103	.0245

Table F (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.5429	2.0566	3.2623	.0844	3.1957	5.6213	.2821	-.0169
3.6427	1.9743	3.1753	.0895	3.3268	5.6213	.2821	-.0385
4.0419	2.3034	3.5233	.0946	3.6218	6.0376	.3103	-.0007
3.9920	2.3857	3.6973	.0844	3.6546	5.8294	.2821	.0395
4.5409	3.3728	5.2197	.1279	3.4743	9.3688	.3103	.0082
3.6427	2.0566	2.6968	.0921	3.5563	4.7885	.2821	-.0162
3.6427	2.1389	3.0883	.0921	3.5399	5.4131	.3385	-.0361
3.7924	2.3857	3.7408	.0972	3.5235	6.2458	.3385	-.0091
3.4930	1.9743	3.1318	.1023	3.1957	5.4131	.2821	-.0215
3.5928	2.0566	3.3058	.0870	3.2777	6.0376	.2059	-.0516
3.1936	1.8098	3.3058	.0793	3.0318	5.2049	.2482	-.0114
3.4930	2.0566	3.3058	.0895	3.2285	5.2049	.2821	.0260
3.3932	1.8098	3.0448	.0895	2.9991	4.9967	.2567	.0102
3.8922	2.2211	3.5233	.1074	3.5399	6.0376	.3385	-.0175
3.7425	2.0566	3.0883	.1100	3.3432	5.6213	.3103	-.0304
3.5928	2.0566	3.3058	.1074	3.3924	5.6213	.3385	-.0314
3.8423	2.1389	3.5233	.1023	3.3104	5.8294	.3385	.0135
3.9421	2.3857	4.1322	.1074	3.3432	7.0786	.3103	-.0155
3.8423	2.2211	4.0887	.0844	3.5235	6.6622	.3103	-.0250

Table F (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.6926	2.3034	3.6538	.0588	3.4252	5.8294	.2369	.0226
3.4930	1.9743	3.0013	.0997	3.1302	5.2049	.2285	.0006
3.5928	1.8921	3.0448	.1074	3.1302	5.4131	.2567	-.0187
4.1417	2.6324	3.8278	.0997	3.7038	6.2458	.3385	.0394
4.1417	2.3857	3.6973	.0997	3.7693	6.6622	.3103	-.0396
4.2415	2.4679	3.5233	.1560	3.7038	6.0376	.3667	.0274
3.7425	2.3034	3.9147	.0870	3.4907	5.6213	.2764	.0678
2.8942	1.8098	1.4354	.0793	2.8843	3.1229	.2144	-.0005
2.4451	1.3162	2.2619	.0639	2.6221	3.5393	.1551	-.0370
2.4950	1.4808	2.2619	.0767	2.6221	3.7475	.1946	-.0388
3.9920	2.3034	4.2627	.1100	3.6054	6.8496	.3103	-.0091
3.7425	2.6324	4.0452	.1202	3.7038	6.7871	.3103	-.0244
3.9920	2.5502	4.0017	.1176	3.6710	6.5789	.3103	.0096
4.1417	2.5502	4.3497	.1304	3.8513	6.9537	.3667	.0000
4.3912	3.7019	5.3502	.1407	4.5723	8.7025	.2821	.0020
4.1916	2.7970	4.7847	.1458	3.8676	7.4325	.2821	.0287
4.3912	2.5502	3.5233	.0946	4.0315	6.2875	.2087	.0030
4.2415	2.3857	3.5233	.1125	3.7693	5.9127	.3667	.0211
3.5429	2.3034	3.6538	.0997	3.5399	5.7670	.3103	-.0018
3.1936	1.9743	3.1318	.1023	3.0646	4.9342	.2031	.0241

Table F (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.6427	2.1389	3.6538	.0921	3.4743	5.9960	.3667	-.0320
3.9920	2.2211	3.6103	.1074	3.5890	6.1417	.3103	-.0110
3.4930	1.9743	3.6538	.1049	3.0810	5.9127	.2651	-.0036
3.3932	1.8098	3.1753	.0997	3.1138	5.4131	.2257	-.0319
3.8922	2.3857	3.7843	.1049	3.7529	6.2667	.3103	-.0159
3.6427	2.1389	3.4363	.1023	3.4416	5.6421	.2708	-.0037
2.9940	1.7275	2.7838	.0870	2.8188	4.5803	.2172	-.0031
4.7904	2.7970	5.0892	.0997	3.7365	8.3278	.4231	.0229
4.5908	3.0438	4.6107	.1151	4.0315	7.7657	.3949	.0137
2.5948	1.3985	2.6533	.0818	2.4583	3.8932	.1410	.0357
4.3413	2.8792	4.3497	.1125	3.7529	7.2660	.3949	.0233

$\overline{ERR} = -.0020$

$S_{ERR} = .0251$

TABLE G

Common Ion Data for the Tongue River

at Miles City in Meq/L

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
2.8942	3.4551	2.0444	.1330	4.2937	3.9557	.1156	.0191
3.2435	3.7019	2.1314	.1023	4.7390	4.1639	.0987	.0195
4.2415	5.1004	3.6103	.2225	6.4898	6.4540	.2313	-.0000
4.7405	5.6762	3.8278	.1560	7.3420	7.2868	.1692	-.0272
3.8423	4.2777	3.0448	.1407	5.3262	5.6213	.1241	.0209
3.1437	3.8664	2.9578	.1176	4.9165	5.4131	.1213	-.0356
3.8423	5.1004	4.0017	.1560	5.5737	7.2868	.1523	.0067
3.5429	4.1132	2.3923	.1304	4.5887	5.8294	.1410	-.0367
2.2954	2.4679	1.7399	.0921	3.4088	3.3311	.0733	-.0325
1.6467	1.3162	.7395	.0486	2.2780	1.4157	.0395	.0048
2.2954	2.1389	1.8704	.0921	3.7693	2.7065	.0959	-.0270
2.5948	2.5502	2.0009	.1100	3.9168	3.3311	.0846	-.0105
2.8443	3.3728	2.6098	.1253	4.4593	4.3721	.1185	.0003
3.2435	3.7019	2.5228	.1125	4.5396	4.7885	.1156	.0144
3.2435	4.0309	2.6533	.1151	4.9165	4.9967	.1269	.0003
4.2914	5.2649	4.0017	.1611	5.2934	7.7032	.1777	.0405

Table G (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.7924	4.4423	3.3493	.1304	6.0965	5.8294	.1354	-.0292
3.8922	4.6890	3.2623	.1330	6.0965	5.8294	.1495	-.0082
3.1936	3.9487	3.9147	.1407	4.9001	6.6622	.1382	-.0439
3.6427	4.8536	4.3497	.1637	5.6212	7.0786	.1523	.0122
3.3932	4.1955	2.3488	.1228	4.6215	5.4131	.1241	-.0097
1.7964	1.6453	1.1309	.0716	2.5730	1.8529	.0790	.0305
2.0459	1.8098	1.2179	.0921	3.0810	2.0819	.0987	-.0184
2.5948	2.4679	2.0009	.1125	3.9660	3.3311	.0959	-.0298
2.2455	2.0566	2.9578	.1279	3.7038	3.5393	.0846	.0082
2.1956	2.2211	3.0448	.1253	3.5890	4.3721	.0649	-.0563
3.4431	4.1132	3.5668	.1330	5.2115	6.0376	.0959	-.0079
3.7425	4.5245	3.8278	.1355	5.4737	6.8704	.1495	-.0213
3.9920	4.7713	3.8278	.1228	5.9490	6.8704	.1777	-.0220
4.1916	4.8536	3.3928	.1407	6.3587	6.4540	.1467	-.0298
1.5469	1.3985	1.6964	.1330	2.2780	2.4983	.0790	-.0167
3.4930	3.8664	2.7838	.1534	4.3101	5.6213	.1326	.0229
3.3932	4.1955	4.7847	.2148	4.6379	7.4950	.1608	.0237
2.2455	2.0566	1.3049	.0818	2.7696	2.7065	.0677	.0258
3.1936	3.5373	2.5663	.1304	4.4576	4.7885	.1354	.0049

Table G (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
3.3932	3.8664	2.8273	.1253	4.9821	5.2049	.1297	-.0102
3.7425	4.1955	3.2188	.1458	5.6704	5.8294	.1692	-.0319
4.5409	4.4423	3.1318	.1381	6.0473	5.8294	.1269	.0206
3.2435	3.0438	1.8269	.1534	4.1790	3.9557	.1241	.0011
3.8922	4.6890	4.0887	.2097	4.8018	7.4950	.1241	.0363
3.5928	4.1955	2.9143	.1995	4.2446	6.2458	.1410	.0251
1.7465	1.6453	1.4354	.0767	2.4910	2.2901	.1523	-.0060
2.5449	2.5502	2.0009	.0946	3.7857	3.3311	.0762	-.0003
2.8443	3.0438	3.4363	.1151	4.7035	4.7885	.0931	-.0153
2.6447	2.7970	2.5663	.1202	3.9004	3.9557	.0790	.0240
3.0938	5.3472	5.2632	.1739	4.9329	8.2445	.1636	.0395
4.2415	5.7585	3.7843	.1611	6.3751	7.0162	.1185	.0317
3.4930	4.4423	3.2188	.1483	5.4737	5.8711	.1185	-.0141
3.3932	4.0309	3.2188	.1534	5.1623	5.4131	.1297	.0085
3.6427	4.3600	3.0883	.1586	5.0804	5.7878	.1185	.0236
3.1936	3.8664	2.8708	.1688	4.3921	5.5172	.1185	.0071
2.4451	1.8921	1.0874	.0844	3.2121	2.3734	.0226	-.0178
2.3952	2.2211	1.8704	.1074	3.6054	3.0188	.0846	-.0173
2.2455	2.1389	1.7399	.0895	3.4907	2.7898	.0733	-.0223

Table G (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
2.8443	2.4679	2.2619	.1330	4.1299	3.6642	.0818	-.0217
3.7924	4.4423	3.1753	.1304	5.6589	5.7462	.1354	-.0000
3.4930	3.9487	2.7838	.1151	5.4082	5.0591	.1241	-.0240
3.1437	3.3728	2.6968	.1509	3.7202	4.3929	.0959	.1315
3.1437	3.7019	2.8273	.1279	4.7198	4.8718	.1467	.0064
3.2435	4.4423	3.0448	.1355	5.2656	5.5588	.1297	-.0081
2.6447	2.8792	1.5659	.1049	3.7365	3.5601	.0677	-.0233
2.6447	2.5502	1.4789	.0895	3.8840	2.9772	.0621	-.0234
3.7924	4.4423	4.3497	.2532	5.5556	7.1827	.1410	-.0033
3.2435	4.0309	2.8273	.1279	4.5926	5.8919	.1015	-.0342
2.8942	3.1260	2.4358	.1202	4.0971	4.1431	.0621	.0325
2.9940	3.4551	3.4363	.1432	4.6237	5.1008	.0987	.0207

$\overline{ERR} = -.0011$

$S_{ERR} = .0282$

TABLE H

Common Ion Data for the Yellowstone River

at Miles City in Meq/L

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
2.8942	1.8098	2.3923	.0921	3.2324	3.7475	.2595	-.0071
3.1936	1.8921	2.7403	.0895	3.1629	4.3721	.2821	.0125
3.2435	2.0566	3.2188	.0997	3.3596	5.2049	.2623	-.0239
3.5928	2.2211	3.3058	.1100	3.3935	5.2049	.3385	.0322
3.2435	1.9743	2.9578	.1023	3.2941	4.7885	.2821	-.0104
2.6447	1.4808	1.8704	.0793	2.6549	2.9147	.2115	.0496
1.9960	.6499	1.0874	.0588	1.9010	1.7697	.1354	-.0037
1.3972	.6252	.7395	.0435	1.4750	1.0202	.1072	.0751
1.6467	.9872	1.1744	.0691	1.9010	1.7072	.1128	.0411
1.8962	1.3162	1.6964	.0691	2.5074	2.2901	.1636	.0034
2.4950	1.5630	2.2619	.0844	2.8215	3.1229	.2172	.0386
3.2934	2.0566	3.0448	.0946	3.2777	4.5803	.3103	.0386
2.7944	1.6453	2.4793	.0870	3.0155	3.5393	.2821	.0244
1.6467	.9872	1.2179	.0537	1.9502	1.6864	.1580	.0288
2.6946	1.8098	2.6968	.1074	3.0646	3.9557	.2680	.0028
2.5948	1.5630	2.6098	.1074	2.6713	4.1639	.2567	-.0310

Table H (Cont.)

Ca	Mg	Na	K	ALK	SO_4	Cl	ERR
1.1477	.5758	.6525	.0384	1.3602	.9785	.0790	-.0014
1.8962	1.2340	1.6094	.0691	2.4746	2.2901	.1636	-.0246
2.8942	1.7275	2.3923	.1049	3.1466	3.9557	.2764	-.0358
3.1936	2.4679	2.6968	.1125	3.0974	4.7885	.2228	.0437
2.8942	1.9743	2.5663	.1049	3.3432	4.1629	.2680	-.0307
1.7964	.9872	1.2614	.0486	2.0158	2.0403	.1100	-.0176
2.0459	1.2340	2.0009	.0870	2.7369	2.7065	.2285	-.0551
3.0439	2.0566	3.0013	.1125	3.2121	4.6427	.2539	.0129
3.3932	2.1389	3.4798	.1355	3.2121	5.5380	.3103	.0096
1.9461	1.1517	1.3049	.0665	2.3271	1.8946	.0903	.0358
2.5449	1.4808	2.3488	.1049	2.9171	3.4560	.2595	-.0234
2.4950	1.3162	1.9574	.0742	2.6221	3.1646	.1974	-.0239
2.8443	1.8098	2.6533	.0997	3.0974	4.2055	.2708	-.0222
3.1437	1.8098	2.7838	.1151	3.2121	4.5803	.3103	-.0314
1.1477	.6170	.6960	.0486	1.4913	.9577	.0790	-.0075
2.9441	1.8921	3.0448	.1100	3.2121	4.3721	.1918	.0273

$\overline{ERR}$ = .0019

S_{ERR} = .0211

TABLE I

Primary Flow Data 30 Years (1944 - 1973)

in Million Acre Feet (MAF)

Year	Yellowstone Bighorn	Bighorn Bighorn	Yellowstone Miles City	Precipitation
1944	4.429432	4.558000	9.662717	5.976344
1945	4.903299	3.401000	8.537485	3.412980
1946	4.536863	2.856000	7.488556	4.494581
1947	5.598576	3.983000	9.333501	3.853739
1948	6.032406	3.284000	9.396159	3.923335
1949	4.556255	2.600000	7.219993	2.763441
1950	5.628138	3.034000	8.911475	3.987130
1951	6.012505	3.543000	9.801787	3.752250
1952	5.988362	2.340000	8.426226	2.656153
1953	4.391895	2.030000	6.538806	3.876938
1954	4.567656	1.958000	6.538840	3.381083
1955	3.870799	2.178000	6.076215	3.439079
1956	5.892619	2.302000	8.287723	3.427480
1957	7.134494	2.435000	9.648042	4.378593
1958	4.337206	2.813000	7.267318	3.279590
1959	5.077166	2.098000	7.191112	3.702952
1960	3.639390	1.614000	5.280476	2.461868
1961	3.237737	1.175000	4.434866	3.241898
1962	6.013068	2.917000	9.004552	4.627968
1963	5.151859	3.000000	8.029416	4.593173
1964	5.400421	3.397000	8.755631	4.204611
1965	6.816616	3.967000	10.775285	4.419186
1966	4.327211	1.240000	5.629292	2.818538
1967	6.784331	3.184000	9.962166	4.410489

Table I (Cont.)

Year	Yellowstone Bighorn	Bighorn Bighorn	Yellowstone Miles City	Precipitation
1968	6.334088	3.359000	9.831044	4.857047
1969	5.623566	2.881000	8.635795	4.123416
1970	6.349800	2.851000	9.349343	4.384390
1971	7.077032	3.634000	10.880866	3.804442
1972	6.342625	3.667000	10.210567	4.857046
1973	5.163317	3.180000	8.417696	4.558376

TABLE J

Simulated Values of ECM for the 25 Years

1944 to 1968 Associated with L1 and L2

Year	EO	EI
1944	612.3	567.8
1945	596.7	542.7
1946	628.9	532.1
1947	575.8	540.2
1948	574.2	493.6
1949	638.2	517.4
1950	586.6	496.2
1951	564.6	506.1
1952	599.9	447.4
1953	664.1	491.4
1954	664.1	478.9
1955	683.9	526.5
1956	603.9	448.5
1957	568.2	418.2
1958	636.5	539.2
1959	639.2	466.4
1960	723.5	501.3
1961	775.9	490.3
1962	584.1	476.7
1963	611.6	512.9
1964	590.7	522.1
1965	543.6	497.7
1966	705.2	439.3
1967	560.9	464.7
1968	563.9	486.9

APPENDIX II

Contents

Chi-square goodness of Fit Tests for the error of balance, ERR, for common ion data for the specified sites: Yellowstone River at Billings, Bighorn River at Bighorn, Tongue River at Miles City, and Yellowstone River at Miles City.

Cross Reference

Pages iv and 15

X^2 Goodness of Fit for ERR for Yellowstone River at Billings

Hypothesis: $ERR \frown N(\mu, \sigma^2)$ with $\mu = 0$, $\sigma^2 = S^2_{ERR} = .0022$, $\alpha = .05$, $n = 68$

$$X^2_j = \sum_{j=1}^{r} \frac{(Obs_j - Exp_j)^2}{Exp_j}$$

Where:

r = number of intervals
Obs = observed frequency
Exp = expected frequency

$$Z = \frac{(ERR - \mu)}{\sigma_{ERR}}$$

ERR Interval	Z Interval	Obs_j	Exp_j	X^2_j
$-\infty \leftrightarrow -.07$	$- \leftrightarrow -1.502$	6	4.522	.483
$-.07 \leftrightarrow -.05$	$-1.502 \leftrightarrow -1.073$	6	5.107	.156
$-.05 \leftrightarrow -.03$	$-1.073 \leftrightarrow -.644$	9	8.036	.116
$-.03 \leftrightarrow -.01$	$-.644 \leftrightarrow -.215$	14	10.545	1.132
$-.01 \leftrightarrow .01$	$-.215 \longleftrightarrow .215$	12	11.580	.015
$.01 \leftrightarrow .03$	$.215 \longleftrightarrow .644$	11	10.545	.020
$.03 \leftrightarrow 00$	$.644 \longleftrightarrow 00$	10	17.665	3.326

k = the number of sample statistics used $X^2 = 5.248$

d.o.f. = r - k - 1 here $k = 1(S^2)$ so d.o.f. = 5

$X^2 = 11.07$ so $X^2 < X^2_\alpha$ and the hypothesis can not be rejected

Z-Test of μ_{ERR} for the Yellowstone River at Miles City

H_o: $\mu_{ERR} = 0$

H_1: $\mu_{ERR} \neq 0$

$\alpha = .05$

$\overline{ERR} = .0019$; $S_{ERR} = .0211$

Since $n = 32 > 30$ S_{ERR} was used to approximate σ_{ERR}

$$Z = \frac{\overline{ERR} - \mu_{ERR}}{\sigma_{ERR}/\sqrt{n}} = .509$$

$$Z_{\alpha/2} = \pm 1.96$$

$Z < Z_{\alpha/2}$ so the hypothesis can not be rejected

APPENDIX III

Quantity Model
1944 - 1973

Contents

Primary data and simulated secondary data output for the 30 year period 1944 - 1973 annual 42KJA computer program.

Cross Reference

Pages v and 84

```
C00.00**********************************************************************
C00.05'APPLICATION'*********************************************************
C     42KJASUB=BASIN, ANNUAL SOLUTION FOR YEARS 1944 THROUGH 1973.
C     SIMULATE SECONDARY DATA.
C     OUTPUT: PRIMARY AND SECONDARY DATA.
C00.10'DIMENSIONS'**********************************************************
C     NP = NUMBER OF PERIODS.
      NR=6;NC=13;NP=30
C     DIMENSION INSTRUCTIONS:D(NR*NC),Z(NC-NR),B(NR),K(NR),
C     W1(NR),W2(NR),Y(NR),A(NR*NR),L(NC),S(NC),C(NR,NC)
      DIMENSION D(78),Z(7),B(6),K(6),W1(6),W2(6),Y(6),A(36),L(13),S(13),
     *C(6,13),FY(30),FB(30)
      INTEGER P
C01.00**********************************************************************
C     INITIALIZE Z AND C.
      J1=NC-NR; J2=NC*NR
      DO 10 J=1,J1
   10 Z(J)=1.
      EQUIVALENCE (D,C)
      DO 15 J=1,J2
   15 D(J)=0.
C02.00**********************************************************************
C     INPUT SOLUTION SUBSCRIPTS, INITIAL EXOGENOUS VALUES, AND SYSTEM CONSTANTS.
      NAMELIST K,Z,C
      INPUT
      INPUT
      INPUT
C03.00**********************************************************************
C     ESTABLISH SOLUTION CODE VECTOR.
      CALL CODE(K,NR,NC,L)
C04.00**********************************************************************
C     ESTABLISH EXOGENOUS VECTOR.
      CALL EXOGENOUS(L,Z,NC,S)
C04.05'INITIAL VALUES'******************************************************
      SUM7=0.;SUM8=0.;SUM11=0.;SUM10=0.;SUM6=0.;SUM12=0.
   42 X9I=Z(6);S(9)=Z(6);HOLD=0;SF=.0360523
```

```
      C(5,11)=-1./SF
      DO 91 J=1,NP
C04.10'MAKE EXOGENOUS CHANGES'**********************************************
      READ(5,4100)FY(J),FB(J),S(1),S(5)
      S(2)=FY(J)+FB(J)
 4100 FORMAT(4X,F9.6,3F10.6)
      IF(J.GT.1) S(9)=Y(4)
      S(4)=0.;S(3)=HOLD;HOLD=S(4)
C05.00**********************************************************************
C     ESTABLISH ENDOGENOUS COEFCNT MATRIX.
      CALL DELETE COLUMN(D,L,NR,NC,A)
C06.00**********************************************************************
C     COMPUTE INVERSE OF THE COEFCNT MATRIX.
      CALL INVERSE(A,NR,W1,W2)
C07.00**********************************************************************
C     COMPUTE RIGHT-HAND-SIDE COLUMN VECTOR.
      CALL MULTIPLY(D,S,NR,NC,1,B)
      DO 70 JJ=1,NR
   70 B(JJ)=-B(JJ)
C08.00**********************************************************************
C     Y=A*B= ENDOGENOUS SYSTEM SOLUTION.
   80 CALL MULTIPLY(A,B,NR,NR,1,Y)
C08.05'TEST FOR OUTPUT'*****************************************************
C09.00**********************************************************************
C     ADJUST ARGUMENTS FOR DESIRED OUTPUT APPLICATION.
   90 CALL OUTPUT(Y,S,J)
C
C09.05'TEST FOR FINAL ITERATION '*******************************************
      SUM6=SUM6+Y(1);SUM7=SUM7+Y(2);SUM8=SUM8+Y(3);SUM10=SUM10+Y(4)
      SUM11=SUM11+Y(5);SUM12=SUM12+Y(6)
   91 CONTINUE
C09.15'FOOTNOTE REPORT'*****************************************************
   97  OUTPUT SUM6,SUM7/30.,SUM8/30.,SUM10/30.,SUM11/30.,SUM12/30.
      END
```

```
      SUBROUTINE CODE(K,NR,NC,L)
C     L=SOLUTION CODE VECTOR: '1'=EXOGENOUS, '0'=ENDOGENOUS.
      DIMENSION K(1),L(1)
      I=1
      DO 10 J=1,NC
      L(J)=1.
      IF(K(I).EQ.J) L(J)=0.
      IF(I.EQ.NR) GO TO 10
      IF(K(I).EQ.J) I=I+1
   10 CONTINUE
      RETURN
      END
      SUBROUTINE DELETE COLUMN(D,L,NR,NC,A)
C     A=ENDOGENOUS COEFCNT MATRIX OBTAINED BY DELETING EXOGENOUS COLUMNS.
      DIMENSION A(1),D(1),L(1)
      M=0;N=0
      DO 20 I=1,NC
      IF(L(I).NE.0.) GO TO 20
      DO 10 J=1,NR
      N=N+1
   10 A(N)=D(M+J)
   20 M=M+NR
      RETURN
      END
      SUBROUTINE EXOGENOUS(L,Z,NC,S)
C     FORMULATE S FROM L: 'ONES' REPLACED BY INITIAL EXOGENOUS VALUES.
C     S=EXOGENOUS VECTOR.
      DIMENSION L(1),S(1),Z(1)
      I=0
      DO 10 J=1,NC
      S(J)=L(J)
      IF(S(J).EQ.1.) I=I+1
      IF(S(J).EQ.1.) S(J)=Z(I)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INVERSE(A,N,L,M)                                        INVER  2
      REAL L,M                                                           INVER  3
      DIMENSION A(1),L(1),M(1)                                           INVER  4
      D=1.0                                                              INVER  5
      NK=-N                                                              INVER  6
      DO 80 K=1,N                                                        INVER  7
      NK=NK+N                                                            INVER  8
      L(K)=K                                                             INVER  9
      M(K)=K                                                             INVER 10
      KK=NK+K                                                            INVER 11
      BIGA=A(KK)                                                         INVER 12
      DO 20 J=K,N                                                        INVER 13
      IZ=N*(J-1)                                                         INVER 14
      DO 20 I=K,N                                                        INVER 15
      IJ=IZ+I                                                            INVER 16
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20                                INVER 17
   15 BIGA=A(IJ)                                                         INVER 18
      L(K)=I                                                             INVER 19
      M(K)=J                                                             INVER 20
   20 CONTINUE                                                           INVER 21
      J=L(K)                                                             INVER 22
      IF(J-K) 35,35,25                                                   INVER 23
   25 KI=K-N                                                             INVER 24
      DO 30 I=1,N                                                        INVER 25
      KI=KI+N                                                            INVER 26
      HOLD=-A(KI)                                                        INVER 27
      JI=KI-K+J                                                          INVER 28
      A(KI)=A(JI)                                                        INVER 29
   30 A(JI) =HOLD                                                        INVER 30
   35 I=M(K)                                                             INVER 31
      IF(I-K) 45,45,38                                                   INVER 32
   38 JP=N*(I-1)                                                         INVER 33
      DO 40 J=1,N                                                        INVER 34
      JK=NK+J                                                            INVER 35
      JI=JP+J
```

```
      HOLD=-A(JK)                                                       INVER 36
      A(JK)=A(JI)                                                       INVER 37
   40 A(JI) =HOLD                                                       INVER 38
   45 IF(BIGA) 48,46,48                                                 INVER 39
   46 D=0.0                                                             INVER 40
      WRITE(108,1000)
 1000 FORMAT( 'THE DETERMINANT OF''A''IS ZERO. THEREFORE EXECUTION WAS
     *HALTED. CHECK'/'FOR ALL ZERO ROW OR COLUMN AND FOR LINEAR DEPENDEN
     *CE BETWEEN EQUATIONS.')
      CALL EXIT
   48 DO 55 I=1,N                                                       INVER 42
      IF(I-K) 50,55,50                                                  INVER 43
   50 IK=NK+I                                                           INVER 44
      A(IK)=A(IK)/(-BIGA)                                               INVER 45
   55 CONTINUE                                                          INVER 46
      DO 65 I=1,N                                                       INVER 47
      IK=NK+I                                                           INVER 48
      IJ=I-N                                                            INVER 49
      DO 65 J=1,N                                                       INVER 50
      IJ=IJ+N                                                           INVER 51
      IF(I-K) 60,65,60                                                  INVER 52
   60 IF(J-K) 62,65,62                                                  INVER 53
   62 KJ=IJ-I+K                                                         INVER 54
      A(IJ)=A(IK)*A(KJ)+A(IJ)                                           INVER 55
   65 CONTINUE                                                          INVER 56
      KJ=K-N                                                            INVER 57
      DO 75 J=1,N                                                       INVER 58
      KJ=KJ+N                                                           INVER 59
      IF(J-K) 70,75,70                                                  INVER 60
   70 A(KJ)=A(KJ)/BIGA                                                  INVER 61
   75 CONTINUE                                                          INVER 62
C     D= VALUE OF THE DETERMINANT
      D=D*BIGA                                                          INVER 63
      A(KK)=1.0/BIGA                                                    INVER 64
   80 CONTINUE                                                          INVER 65
```

```
      K=N                                                          INVER 66
  100 K=(K-1)                                                      INVER 67
      IF(K) 150,150,105                                            INVER 68
  105 I=L(K)                                                       INVER 69
      IF(I-K) 120,120,108                                          INVER 70
  108 JQ=N*(K-1)                                                   INVER 71
      JR=N*(I-1)                                                   INVER 72
      DO 110 J=1,N                                                 INVER 73
      JK=JQ+J                                                      INVER 74
      HOLD=A(JK)                                                   INVER 75
      JI=JR+J                                                      INVER 76
      A(JK)=-A(JI)                                                 INVER 77
  110 A(JI) =HOLD                                                  INVER 78
  120 J=M(K)                                                       INVER 79
      IF(J-K) 100,100,125                                          INVER 80
  125 KI=K-N                                                       INVER 81
      DO 130 I=1,N                                                 INVER 82
      KI=KI+N                                                      INVER 83
      HOLD=A(KI)                                                   INVER 84
      JI=KI-K+J                                                    INVER 85
      A(KI)=-A(JI)                                                 INVER 86
  130 A(JI) =HOLD                                                  INVER 87
      GO TO 100                                                    INVER 88
  150 RETURN                                                       INVER 89
      END                                                          INVER 90
      SUBROUTINE MULTIPLY(A,B,N,M,L,R)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
      JI=J-N
      IB=IK
```

```
      R(IR)=0
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
      SUBROUTINE OUTPUT(Y,S,J)
C     OUTPUT PRIMARY AND SECONDARY VALUES.
      DIMENSION Y(1),S(1)
      M=1943+J
      WRITE(108,100)M,S(1),S(2),S(3),S(4),Y(1),S(6),Y(2),
     *S(8),S(9),S(10),Y(3),Y(4)
  100 FORMAT(I4,2F8.1,2F4.1,F8.1,F4.1,2F8.1,2F4.1,2F8.1)
      RETURN
      END
C
C
C
C
      SUBROUTINE CONDUCT(Q,S,Y)
C CALCULATION OF CONDUCTIVITIES
      DIMENSION S(1),Y(1),Q(1)
      Q(5)=Q(5)=Q(11)
      S(1)=S(1)/Q(1);S(2)=S(2)/Q(2);Y(1)=Y(1)/Q(5)
      Y(2)=Y(2)/Q(7);S(8)=S(8)/Q(8);Y(3)=Y(3)/Q(11);Y(4)=Y(4)/Q(12)
      RETURN
      END
```

Year	X1	X2	X3	X4	X5	X6
	X7	X8	X9	X10	X11	X12
1944	9.662717	8.987432	.000000	.000000	5.976344	5.155762
	.272068	.255794	4.958022	4.828998	.766131	.620834
1945	8.537485	8.304299	.000000	.000000	3.412980	3.246812
	.255189	.245547	4.828998	4.905658	.556724	.623742
1946	7.488556	7.392863	.000000	.000000	4.494581	4.415288
	.239455	.231876	4.905658	4.929637	.601753	.618152
1947	9.333501	9.581576	.000000	.000000	3.853739	4.084991
	.267130	.264707	4.929637	4.915237	.634443	.617620
1948	9.396159	9.316406	.000000	.000000	3.923335	3.832301
	.268069	.260729	4.915237	4.911296	.629920	.618639
1949	7.219993	7.156255	.000000	.000000	2.763441	2.842000
	.235427	.228327	4.911296	5.060693	.468262	.610560
1950	8.911475	8.662138	.000000	.000000	3.987130	3.729340
	.260799	.250915	5.060693	5.062124	.610635	.602182
1951	9.801787	9.555505	.000000	.000000	3.752250	3.482887
	.274154	.264316	5.062124	5.048882	.625919	.602838
1952	8.426226	8.328362	.000000	.000000	2.656153	2.653592
	.253520	.245908	5.048882	5.151797	.502555	.597858
1953	6.538306	6.421895	.000000	.000000	3.876938	3.826376
	.225209	.217311	5.151797	5.226043	.521669	.588017
1954	6.538840	6.525656	.000000	.000000	3.381083	3.457005
	.225210	.218868	5.226043	5.321490	.489486	.578592
1955	6.076215	6.048799	.000000	.000000	3.439079	3.503221
	.218270	.211715	5.321490	5.419604	.476283	.567841
1956	8.287723	8.194619	.000000	.000000	3.427480	3.343167
	.251443	.243902	5.419604	5.435936	.552693	.561484
1957	9.648042	9.569494	.000000	.000000	4.378593	4.195122
	.271848	.264525	5.435936	5.338334	.670922	.565998
1958	7.267318	7.150206	.000000	.000000	3.279590	3.225475
	.236137	.228236	5.338334	5.409232	.504484	.567481
1959	7.191112	7.175166	.000000	.000000	3.702952	3.712947
	.234994	.228610	5.409232	5.441556	.535807	.561748

1960	5.280476	5.253390	.000000	.000000	2.461868	2.607931
	.206334	.199784	5.441556	5.621256	.376823	.549972
1961	4.434866	4.412737	.000000	.000000	3.241898	3.349738
	.193650	.187174	5.621256	5.757701	.402442	.532412
1962	9.004552	8.930068	.000000	.000000	4.627968	4.420089
	.262195	.254934	5.757701	5.631567	.665234	.531839
1963	8.029416	8.151859	.000000	.000000	4.593173	4.624406
	.247568	.243261	5.631567	5.544664	.634882	.543672
1964	8.755631	8.797421	.000000	.000000	4.204611	4.168615
	.258461	.252944	5.544664	5.472395	.630299	.552513
1965	10.775235	10.783616	.000000	.000000	4.419186	4.274606
	.288756	.282737	5.472395	5.325504	.717597	.564686
1966	5.629292	5.567211	.000000	.000000	2.818538	2.906467
	.211566	.204491	5.325504	5.482589	.414109	.564120
1967	9.962166	9.968331	.000000	.000000	4.410489	4.291154
	.276559	.270508	5.482589	5.363141	.687529	.562029
1968	9.831044	9.693088	.000000	.000000	4.857047	4.584712
	.274593	.266379	5.363141	5.236975	.710051	.575672
1969	8.535795	8.504566	.000000	.000000	4.123416	3.961659
	.256664	.248551	5.236975	5.214559	.614452	.583924
1970	9.349343	9.200800	.000000	.000000	4.384390	4.165977
	.267367	.258995	5.214559	5.153062	.658455	.588585
1971	10.880866	10.711032	.000000	.000000	3.804442	3.559106
	.290340	.281648	5.153062	5.086252	.671213	.595712
1972	10.210567	10.009625	.000000	.000000	4.857046	4.539496
	.280286	.271127	5.086252	4.978802	.721998	.605391
1973	8.417696	8.343317	.000000	.000000	4.558376	4.455991
	.253392	.246133	4.978802	4.958056	.640517	.612511

APPENDIX IV

Quality Model
1944 - 1968
1944 - 1973

Contents

Primary data and secondary output for the 25 year period 1944-1968 and the 30 year period 1944-1973. Computer program for sub-basin 43KJA.

Cross Reference

Pages v, 93 and 95

```
C00.00*************************************************************************
C00.05'APPLICATION'************************************************************
C     42KJA SB=BASIN,LOAD MODEL
C     ANNUAL SOLUTION FOR THE YEARS 1944-1968
C     SIMULATE SECONDARY DATA.
C     OUTPUT: PRIMARY AND SECONDARY DATA.
C00.10'DIMENSIONS'*************************************************************
C     NP = NUMBER OF PERIODS.
      NR=4,NC=12,NP=30
C     DIMENSION INSTRUCTIONS:D(NR*NC),Z(NC-NR),B(NR),K(NR),
C     W1(NR),W2(NR),Y(NR),A(NR*NR),L(NC),S(NC),C(NR,NC)
      DIMENSION D(48),Z(8),B(4),K(4),W1(4),W2(4),Y(4),A(16),L(12),S(12),
     *C(4,12),EI(30),EO(30),Q(12)
      INTEGER P
C01.00*************************************************************************
C     INITIALIZE Z AND C.
      J1=NC-NR; J2=NC*NR
      DO 10 J=1,J1
   10 Z(J)=1.
      EQUIVALENCE (D,C)
      DO 15 J=1,J2
   15 D(J)=0.
C02.00*************************************************************************
C     INPUT SOLUTION SUBSCRIPTS, INITIAL EXOGENOUS VALUES, AND SYSTEM CONSTANTS.
      NAMELIST K,Z,C
      INPUT
      INPUT
      INPUT
C03.00*************************************************************************
C     ESTABLISH SOLUTION CODE VECTOR.
      CALL CODE(K,NR,NC,L)
C04.00*************************************************************************
C     ESTABLISH EXOGENOUS VECTOR.
      CALL EXOGENOUS(L,Z,NC,S)
C04.05'INITIAL VALUES'*********************************************************
      SF=.187
```

```
      SUMC=0.;SUM1=0.;SUM2=0.;SUM5=0.;SUM7=0.;SUM11=0.;SUM12=0.
      SUM8=0.
      C(3,11)=-1./SF;S(6)=0;S(9)=S(10)=0.
   42 X8I=Z(6);S(8)=Z(6);HOLD=0.
      DO 91 J=1,NP
C04.10'MAKE EXOGENOUS CHANGES'**********************************************
      READ(5,4100) (Q(I),I=1,12)
      READ(15,4105)EO(J),EI(J)
      S(1)=Q(1)*EO(J)
      S(2)=Q(2)*EI(J)
      C(4,7)=Q(12)/Q(7)
 4100 FORMAT(5X,6F12.6/5X,6F12.6)
 4105 FORMAT(2F7.1)
      IF(J.GT.1) S(8)=Y(3)/3
      S(4)=0.;S(3)=HOLD;HOLD=S(4)
C05.00**********************************************************************
C     ESTABLISH ENDOGENOJS CFFCNT MATRIX.
      CALL DELETE COLUMN(D,L,NR,NC,A)
C06.00**********************************************************************
C     COMPUTE INVERSE OF THE CFFCNT MATRIX.
      CALL INVERSE(A,NR,W1,W2)
C07.00**********************************************************************
C     COMPUTE RIGHT-HAND-SIDE COLUMN VECTOR.
      CALL MULTIPLY(D,S,NR,NC,1,B)
      DO 70 JJ=1,NR
   70 B(JJ)=-B(JJ)
C08.00**********************************************************************
C     Y=A*B= ENDOGENOUS SYSTEM SOLUTION.
   80 CALL MULTIPLY(A,B,NR,NR,1,Y)
C08.05'TEST FOR OUTPUT'*****************************************************
C09.00**********************************************************************
C     ADJUST ARGUMENTS FOR DESIRED OUTPUT APPLICATION.
      CALL CONDUCT(Q,S,Y)
   90 CALL OUTPUT(Y,S,J)
```

```
C09.05'TEST FOR FINAL ITERATION '**********************************************
      SUM5=SUM5+Y(1);SUM7=SUM7+Y(2);SUM11=SUM11+Y(3);SUM12=SUM12+Y(4)
      SUMC=SUMC+C(4,7);SJM1=SUM1+S(1);SUM2=SUM2+S(2);SJM8=SUM8+S(8)
      S(8)=S(8)*Q(8);Y(3)=Y(3)*Q(11)
   91 CONTINUE
C09.15'FOOTNOTE REPORT'*****************************************************
   97 OUTPUT SUM5/30.,SUM7/30.,SUM11/30.,SUM12/30.,SJM1/30.,
     *SUM2/30.,SUM8/30.
      END
```

Year	E1	E2	E3	E4	E5	E6	E7	E8			E11	E12
1944	612•3	567•8	•0	•0	189•8	•0	2041•2	1485•6	•0	•0	1882•9	2041•2
1945	596•7	542•7	•0	•0	205•4	•0	1880•3	1958•3	•0	•0	2104•8	1880•3
1946	628•9	532•1	•0	•0	209•0	•0	1788•5	1684•5	•0	•0	1899•8	1788•5
1947	575•8	540•2	•0	•0	89•4	•0	1761•3	1439•6	•0	•0	1855•5	1761•3
1948	574•2	493•6	•0	•0	279•6	•0	1926•3	1505•0	•0	•0	2088•7	1926•3
1949	638•2	517•4	•0	•0	394•4	•0	1862•7	1920•8	•0	•0	2428•6	1862•7
1950	586•6	496•2	•0	•0	314•2	•0	1958•2	1510•8	•0	•0	2146•6	1958•2
1951	564•6	506•1	•0	•0	259•2	•0	2003•8	1653•1	•0	•0	2109•5	2003•8
1952	599•9	447•4	•0	•0	659•5	•0	2097•0	1789•8	•0	•0	2676•7	2097•0
1953	664•1	491•4	•0	•0	351•0	•0	1950•4	2063•4	•0	•0	2181•0	1950•4
1954	664•1	478•9	•0	•0	439•3	•0	1919•1	1732•8	•0	•0	2376•6	1919•1
1955	683•9	526•5	•0	•0	332•7	•0	1844•3	1831•5	•0	•0	2229•9	1844•3
1956	603•9	448•5	•0	•0	523•9	•0	2110•2	1451•5	•0	•0	2463•2	2110•2
1957	568•2	418•2	•0	•0	447•0	•0	2321•2	1715•5	•0	•0	2222•4	2321•2
1958	636•5	539•2	•0	•0	268•1	•0	1993•5	2177•6	•0	•0	2190•3	1993•5
1959	639•2	466•4	•0	•0	428•1	•0	2017•1	1611•2	•0	•0	2312•0	2017•1
1960	723•5	501•3	•0	•0	559•6	•0	1903•5	2066•9	•0	•0	2724•6	1903•5
1961	775•9	490•3	•0	•0	453•2	•0	1816•0	1828•4	•0	•0	2425•9	1816•0
1962	584•1	476•7	•0	•0	311•6	•0	2127•3	1276•5	•0	•0	2050•0	2127•3
1963	611•6	512•9	•0	•0	198•8	•0	2067•4	1868•6	•0	•0	1860•6	2067•4
1964	590•7	522•1	•0	•0	196•0	•0	1994•8	1556•7	•0	•0	1942•0	1994•8
1965	543•6	497•7	•0	•0	184•7	•0	2082•5	1443•1	•0	•0	1908•2	2082•5
1966	705•2	439•3	•0	•0	623•5	•0	2039•5	2232•1	•0	•0	2718•1	2039•5
1967	560•9	464•7	•0	•0	314•6	•0	2136•6	1387•0	•0	•0	2060•3	2136•6
1968	563•9	486•9	•0	•0	226•7	•0	2142•6	1772•6	•0	•0	1900•6	2142•6

Year	E1	E2	E3	E4	E5	E6	E7	E8			E11	E12
1944	612.3	567.8	.0	.0	189.8	.0	2041.2	1485.6	.0	.0	1882.9	2041.2
1945	596.7	542.7	.0	.0	205.4	.0	1880.3	1958.3	.0	.0	2104.8	1880.3
1946	628.9	532.1	.0	.0	209.0	.0	1788.5	1684.5	.0	.0	1899.8	1788.5
1947	575.8	540.2	.0	.0	89.4	.0	1761.3	1439.6	.0	.0	1855.5	1761.3
1948	574.2	493.6	.0	.0	279.6	.0	1926.3	1505.0	.0	.0	2088.7	1926.3
1949	638.2	517.4	.0	.0	394.4	.0	1862.7	1920.8	.0	.0	2428.6	1862.7
1950	586.6	496.2	.0	.0	314.2	.0	1958.2	1510.8	.0	.0	2146.6	1958.2
1951	564.6	506.1	.0	.0	259.2	.0	2003.8	1653.1	.0	.0	2109.5	2003.8
1952	599.9	447.4	.0	.0	659.5	.0	2097.0	1789.8	.0	.0	2676.7	2097.0
1953	664.1	491.4	.0	.0	351.0	.0	1950.5	2063.4	.0	.0	2181.0	1950.5
1954	664.1	478.9	.0	.0	439.3	.0	1919.1	1732.8	.0	.0	2376.6	1919.1
1955	683.9	526.5	.0	.0	332.7	.0	1844.3	1831.5	.0	.0	2229.9	1844.3
1956	603.9	448.5	.0	.0	523.9	.0	2110.2	1451.5	.0	.0	2463.2	2110.2
1957	568.2	418.2	.0	.0	447.0	.0	2321.2	1715.5	.0	.0	2222.4	2321.2
1958	636.5	539.2	.0	.0	268.1	.0	1993.5	2177.6	.0	.0	2190.3	1993.5
1959	639.2	466.4	.0	.0	428.1	.0	2017.1	1611.2	.0	.0	2312.0	2017.1
1960	723.5	501.3	.0	.0	559.6	.0	1903.5	2066.9	.0	.0	2724.6	1903.5
1961	775.9	490.3	.0	.0	453.2	.0	1816.0	1828.4	.0	.0	2425.9	1816.0
1962	584.1	476.7	.0	.0	311.6	.0	2127.3	1276.5	.0	.0	2050.0	2127.3
1963	611.6	512.9	.0	.0	198.8	.0	2067.5	1868.6	.0	.0	1860.6	2067.5
1964	590.7	522.1	.0	.0	196.0	.0	1994.9	1556.7	.0	.0	1942.0	1994.9
1965	543.6	497.7	.0	.0	184.7	.0	2082.5	1443.1	.0	.0	1908.2	2082.5
1966	705.2	439.3	.0	.0	623.5	.0	2039.5	2232.1	.0	.0	2718.1	2039.5
1967	560.9	464.7	.0	.0	314.6	.0	2136.6	1387.0	.0	.0	2060.3	2136.6
1968	563.9	486.9	.0	.0	226.7	.0	2142.6	1772.6	.0	.0	1900.7	2142.6
1969	622.2	532.4	.0	.0	265.8	.0	2093.4	1809.9	.0	.0	2131.8	2093.4
1970	668.5	556.1	.0	.0	356.4	.0	2359.8	1685.8	.0	.0	2404.5	2359.8
1971	583.0	508.6	.0	.0	333.7	.0	2333.4	1873.8	.0	.0	2294.0	2333.4
1972	577.0	507.4	.0	.0	219.8	.0	2174.6	1893.0	.0	.0	1956.7	2174.6
1973	633.0	562.7	.0	.0	169.2	.0	1974.7	1913.3	.0	.0	1934.4	1974.7

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