EXPLORING THE IMPACT OF STANDARDS-BASED LEARNING ON LESSON PLANNING IN AN UNDERGRADUATE MATH METHODS COURSE

by

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DEDICATION

This study is dedicated to my daughters, their friends, and the elementary-aged children in the United States who deserve to be provided with opportunities to explore meaningful mathematics in ways that develop their critical thinking skills and support them in becoming informed, engaged citizens with the power to shape their world.

ACKNOWLEDGEMENTS

First and foremost, I wish to thank the staff and students at Chief Dull Knife College in Lame Deer, Montana, where I began my teaching career in mathematics and learned that *doing* mathematics well and *teaching* mathematics well require different skill sets. Although I now teach elsewhere, I first learned the importance of student-centered instruction during my four years at the tribal college.

I also thank my committee members for their critical review of numerous iterations of this study, which changed shape several times from the point of inception to data collection to analysis—and all for the better thanks to their detailed feedback. The process of feedback, reflection, and revision that they facilitated reminded me of what it is like to be on the receiving end of constructive criticism, which in turn has helped me to develop empathy with my pre-service teachers, who I guide through similar revision cycles in my methods courses.

Finally, I would like to thank my mother, who despite living at a distance did her best to support me during my doctoral program by visiting during key junctures such as comprehensive exams, and by supporting me financially with my daughters' education during pandemic years, when public school was not a reliable option. Even though my dreams have often made her nervous, this was a safe one that she could stand firmly behind. I also relied on the help of several other adults in my life to watch my girls during weekends and evenings when I took classes to complete my initial doctoral coursework. It definitely takes a village!

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GLOSSARY

Educator Preparation Program (EPP): A program of study designed to support pre-service teachers in acquiring the skills and knowledge needed to demonstrate competency and acquire licensure in K-12 education.

Formative Assessment: Informal assessment designed to gauge student progress toward desired learning outcomes; takes many forms, including hands-on tasks, one-on-one or group conversations, doodling, journaling, and written reflections, among others.

In-service Teacher: A practicing teacher in charge of his or her own classroom.

Mathematical Knowledge for Teaching (MKT): The knowledge needed to effectively teach mathematics, including an understanding of the underlying meaning behind mathematical procedures as well as familiarity with the various ways in which students conceptualize and approach mathematics.

Pre-service Teacher (PST): A teacher in training enrolled in an educator preparation program.

Standards-Based Learning (SBL): Instruction and assessment characterized by ongoing formative assessment in which teachers support students in understanding and improving their level of performance with respect to pre-established learning outcomes.

Standards-Based Instruction: Mathematics instruction in line with the principles of the Association of Mathematics Teacher Educators *Standards for Preparing Teachers of Mathematics*.

Student-Centered Instruction: Instruction characterized by open-ended problem solving in which educators respond flexibly to student thinking, acting as guides for their students as they work to make sense of the mathematics presented to them and to make connections across concepts; foundational to *Standards*-based instruction.

Summative Assessment: Formal assessment designed to gauge student achievement with respect to desired learning outcomes, typically in the form of quizzes, tests, and written papers.

Traditional Instruction and Assessment: Instruction and assessment characterized by teacher-driven modeling in which teachers provide students with step-by-step instructions for learning and evaluate student performance via summative assessments.

ABSTRACT

The study examines the performance of pre-service K-8 mathematics teachers on lesson planning assignments using the Association of Mathematics Teacher Educators (AMTE) 2017 *Standards for Preparing Teachers of Mathematics* as a reference for evaluation. In addition to contributing to the literature on pre-service mathematics teachers' pedagogical content knowledge, the study aims to evaluate the impact of standards-based learning (SBL) on student growth in a higher education setting, where SBL has not been widely implemented or studied. A case study research design is utilized to identify strengths and weaknesses in planning for mathematics instruction in a cohort of 21 PSTs enrolled in a math methods course at a small public university, with comparisons made between first and final drafts of two lesson-planning assignments on which PSTs received extensive instructor feedback. Interviews were also conducted with four participants who student taught during the subsequent semester. Findings indicated growth in performance between drafts of the lesson-planning assignments in terms of designing student-centered math lessons but also revealed gaps in PSTs' mathematical content knowledge. Specifically, most PSTs lacked an ability to link procedures to their underlying concepts, resulting in less than proficient performance on mathematics lesson planning. Contributions of the study include the development of an AMTE-aligned tiered rubric for evaluating both PST and mathematics teacher educator (MTE) performance in mathematics instruction and for potential use in framing and evaluating *Standards*-based practicum experiences in mathematics teacher preparation.

CHAPTER ONE

INTRODUCTION

Background

> The future mathematical success of our nation's children is largely dependent on the teachers of mathematics they encounter from prekindergarten to Grade 12 […] Those involved in preparing teachers of mathematics must ensure that all their candidates have the knowledge, skills, and dispositions to provide all students access to meaningful experiences with mathematics. (AMTE, 2017, xii).

Mathematical literacy is becoming increasingly vital for informed participation in contemporary society. Algorithms and data streams pervade our lives, underpinning everything from credit scores, to selection processes for coveted jobs and college admittances, to the posts we see from our friends on social media (O'Neil, 2017). Despite decades of reform efforts to improve mathematics achievement in the United States, the achievement gap between lower performing and higher performing students remains among the highest in the world (National Center for Educational Statistics, May 2021).

As articulated by the National Council of Teachers of Mathematics (NCTM) in its 2000 publication *Principles and Standards for School Mathematics*, which codified over a decade's worth of research aimed at improving mathematics instruction, raising achievement for all groups of students

> requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence (p. 3).

NCTM's vision involves positioning students as "autonomous learners" who "become confident in their ability to tackle difficult problems" (p. 21). To support students in this endeavor, teachers "must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (p. 17).

Problem Statement

Unfortunately, as expounded in Chapter 2, research on both in-service and pre-service mathematics teachers has revealed significant gaps in the pedagogical content knowledge of many elementary-school educators, making it difficult to realize NCTM's vision. As a mathematics teacher educator (MTE), I have a role to play in supporting NCTM's ambitious goals for mathematics instruction in the United States by helping to prepare teachers who "understand the big ideas of mathematics" and can support students in tackling complex problems from multiple angles (NCTM, 2000, p. 17). In 2017, the Association of Mathematics Teachers Educators (AMTE)—with the help of MTEs and stakeholders from across the country, including representatives of NCTM—published *Standards for Preparing Teachers of Mathematics* (hereafter referred to as the *Standards*), which serves as a guide for MTEs and educator preparation programs (EPPs) in designing and assessing curricula for pre-service mathematics educators.

Interested in determining whether my pre-service teachers are meeting AMTE's *Standards*—which are intended to support NCTM's vision—I designed the present study to document the level of pedagogical content knowledge possessed by my pre-service teachers (PSTs) and to describe the ways in which this knowledge grows via the intervention of standards-based learning. I also sought to identify themes in my PSTs' work that would lend insight into the reasons why PSTs struggle to obtain full proficiency with respect to the

Standards. I employ Situated Learning theory to examine the interactions between PSTs, MTEs, and performance that result from utilizing standards-based assessment within a master-apprentice relationship. The findings from the study will inform continued improvements to my practice and to the field of mathematics teacher education as MTEs seek to support our PSTs in becoming effective mathematics educators capable of fostering a mathematically literate society.

Intervention

In Spring 2019, I implemented standards-based learning (SBL) for the first time in my K-8 math methods course. My transition from traditional instruction and assessment to standards-based learning was motivated by a desire to improve my PSTs' performance on lesson planning assignments, which I felt did not live up to my PSTs' potential. Although I always offered my PSTs the opportunity to revise and resubmit assignments, few ever did, citing busy schedules and impending due dates in other courses. By building feedback and revision cycles directly into my course structure, I hoped to ensure my PSTs would have the opportunity to learn from my feedback and revise their lesson plans accordingly before being assigned final grades in the course. A secondary motivation for implementing standards-based learning was to model this type of assessment for my PSTs, who as future K-8 teachers, will likely be charged with implementing SBL in their future classrooms.

In standards-based learning, students are evaluated against specific learning criteria (standards) rather than against arbitrary point totals. Assessments are used to gauge students' proficiency with respect to the standards at various points during a course of study to inform next steps in instruction as teachers and students work collaboratively to support student mastery of course concepts. Ongoing formative assessment, timely instructor feedback, and multiple opportunities for students to demonstrate mastery are hallmarks of SBL, which is becoming increasingly common in K-12 schools across the country (Heflebower et al., 2019).

Standards-based learning emphasizes assessment *for* learning, whereas traditional assessment emphasizes assessment *of* learning (Boaler, 2016). In SBL, assessments and feedback are utilized to advance students towards mastery of course objectives; students are expected to learn and grow from the feedback they receive by acting on that feedback with the support of their instructors (Heflebower et al., 2019). In contrast, traditional instruction and assessment is characterized by the administration of summative assignments such as essays, quizzes and tests in which students receive numerical scores based on a ratio of the number of points earned to the total number of points possible. Traditional assessment is simple for instructors to administer and track, making it nearly ubiquitous in post-secondary education. However, it harbors inherent flaws that make it difficult to gauge students' true level of learning with respect to course outcomes. For instance, by conflating academic performance with non-academic factors such as attendance, students can be assigned passing grades despite never mastering course content, or they can earn low grades despite demonstrating mastery (Guskey, 2015; Marzano, 2000; Townsley & Varga, 2018). Furthermore, grades based on a hundred-point scale are highly unreliable, with scores varying widely from instructor to instructor (Guskey, 2015). Finally, traditional assessment is normative in nature, giving more information about students' relative ranks in class than about the knowledge and skills they have acquired, contributing to unnecessary competitiveness among peers (Guskey, 2015; Sadler, 1987).

Standards-based learning has its roots in mastery learning, developed over 50 years ago by the educational researchers John Carroll and Benjamin Bloom. Carroll posited that rather than being a function of a student's absolute capacity to learn, aptitude is instead a function of what a student can learn within a given amount of time (Block, 1971a; Carroll, 1971). Some students require more time and practice than others to comprehend concepts and to master skills, so when performance is evaluated based on a limited number of assessments that cannot be revisited and revised, students' absolute capacity to learn is not being measured—only their capacity to learn within the time constraints of the course. In mastery learning—as in standards-based learning—students are afforded additional time as necessary to achieve proficiency. Instructor feedback, supplemental study sessions, tutoring sessions, readings, exercises, and/or audio or visual supports are provided above and beyond the required course materials to help students who need additional support to master key concepts (Block, 1971b).

Mastery learning has at its core the assumption that "under appropriate instructional conditions virtually all students can learn well what they are taught" (Guskey, 1980, p. 105) and that "the normal curve is not sacred," meaning that the final distribution of grades in a course should not form a bell curve but should instead be skewed to the left if instruction in the course is to be deemed effective (Bloom, 1971, p. 49; Guskey, 2015). According to early research summarized by Block (1971a), 75-90% of students in courses utilizing mastery learning achieved the same level of performance as the top 25% of students in courses utilizing traditional instruction, suggesting that mastery learning could be a promising approach to supporting students in achieving proficiency in their coursework.

In adopting standards-based learning in my K-8 math methods course, I moved up the due dates for the main lesson planning assignments to allow time for revisions. I committed myself to providing detailed feedback on each assignment utilizing a course rubric aligned with national standards documents. The culminating project in the course became a final portfolio in which my PSTs submitted revisions to their major assignments along with a written reflection on key course concepts. Because both the homework and classwork during the last two weeks of the semester were dedicated to preparing the portfolios, my PSTs had built-in time to work in their revisions. My hope in making these changes was to support improved performance among my PSTs on their mathematics lesson plans, the subject of the present study.

Positionality

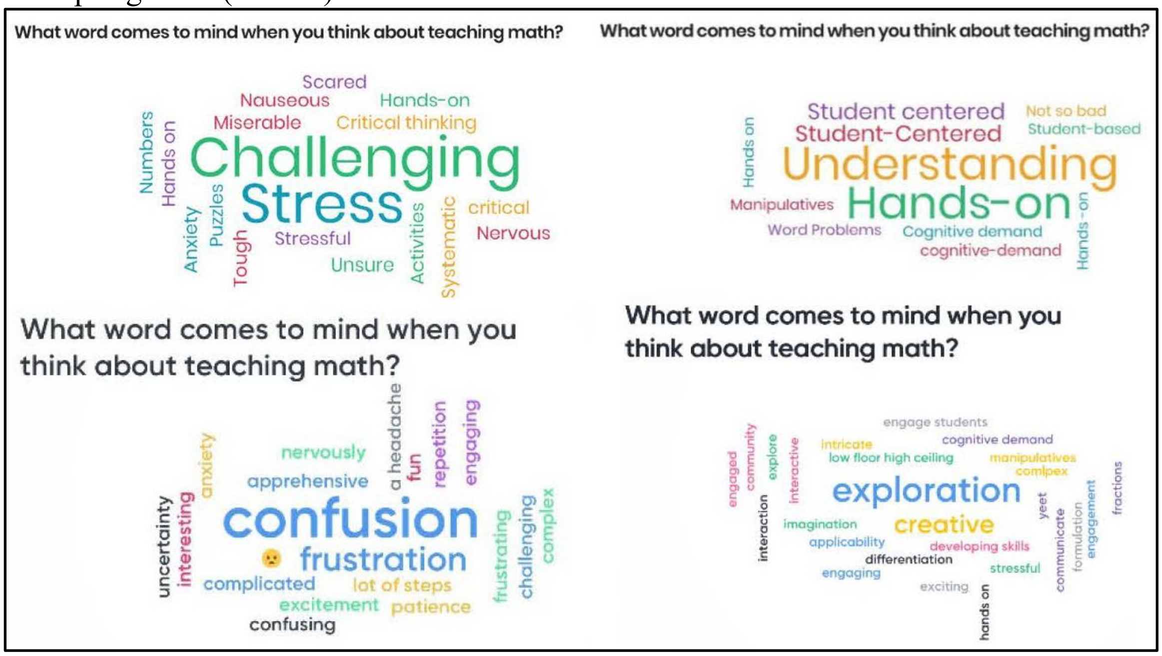
In my role as a teacher educator specializing in K-12 mathematics instruction, I endeavor to model for my PSTs best practices in instruction and assessment. My methods courses emphasize collaborative, open-ended problem solving aimed at supporting students in making important mathematical connections as advocated by AMTE (2017) and NCTM (2000; 2014), and I encourage my pre-service teachers to design lesson and unit plans that incorporate these features. To support them in doing so, I assign readings and videos that feature mathematics instruction in line with AMTE's *Standards* (hereafter referred to as *Standards*-based instruction); I have them explore and solve problems with a variety of mathematical manipulatives; I provide exemplars of well-designed lesson plans; and I arrange real-world teaching experiences followed by opportunities for reflection and discussion.

My commitment to *Standards*-based mathematics instruction stems from my early career experiences teaching mathematics to urban youth pursuing entrance to the Community College of Philadelphia and to Indigenous students at one of my current state's tribal colleges. It was eye-opening for me—a White, middle-class female who enjoyed the opportunity to attend high-quality public schools—to learn that many students in our nation graduate from high school without fundamental skills in mathematics. I quickly became passionate about finding ways to support my students' college dreams by helping them to experience success in the developmental math courses I taught, which I discovered required deviating from the traditional step-by-step instruction that had always worked well for me. I realized that they, and surely millions of other students, needed a different approach to mathematics instruction that would enable them to make sense of the mathematics for themselves and to connect it to their culture, which was different from mine.

Knowing that early experiences with mathematics are essential for developing a robust understanding of mathematics as an adolescent and adult (Aubrey et al., 2006; Jordan et al., 2009), I became interested in teaching the teachers who would one day work with the children of my students. As a fixed-term faculty member in one of the educator preparation programs in my state, I again found myself surprised that even among my mostly White, middle- and working-class female students, math skills were limited, prompting me to realize that our nation's math woes were far greater than I had fathomed. Many of the PSTs pursuing K-8 licensure in my department enter my methods course having developed unproductive, if not harmful, beliefs about mathematics and math instruction during their prior schooling due to the same step-by-step, disconnected mathematics instruction that my urban and reservation students had experienced. Many of my PSTs feel anxious about teaching mathematics and harbor fixed mindsets about math achievement, assuming the widespread belief that only those born with a natural gift for mathematics can perform well in the subject (Boaler, 2016).

Figure 1 shows a collage of the responses shared by PSTs in my K-8 math methods course to a question I ask on the first and last days of class: “What word comes to mind when you think about teaching math?” I administer the question through an interactive presentation using Mentimeter, gathering students’ anonymously reported feelings about the subject. As the first-day word clouds reveal, sentiments towards math are mostly negative and traditional. Fortunately, by the last day of class, sentiments become primarily positive and reform oriented, suggesting that my course design appears to contribute to improved attitudes towards mathematics. However, I have not always felt as successful in improving my PSTs’ competency to generate *Standards*-based math lessons. By the end of each semester, I have often found that many of my students still struggle to translate their newfound enthusiasm for mathematics into coherent, cognitively demanding math lessons for elementary and middle school students, and I end up awarding A’s for work that I do not consider to adequately exemplify *Standards*-based instruction.

Figure 1. Pre-service elementary teachers' attitudes towards math instruction on the first day of their math methods class (left) and last day of their math methods class (right) in Spring 2018 (top) and Spring 2019 (bottom).



In an effort to establish and maintain a set of higher expectations for my pre-service teachers—and to support them in achieving those expectations—I switched my assessment approach from traditional to standards-based in Spring 2019. As noted above, the switch was also motivated by my desire to model best practices for my students, many of whom will move on to teach in school districts that utilize SBL, including the school district closest to my university. As Carless (2002), articulates, "the teacher education context provides a particular incentive to model assessment for learning, in view of the hope that in future, trainee teachers can themselves implement good practices in assessment" (p. 353). Noting, anecdotally, an improvement in the quality of students' lessons and unit plans after switching my assessment regime, I embarked on a mission to systematically study my PSTS' learning outcomes with respect to the *Standards.* I endeavored to determine whether, and in what ways, SBL might be leading to improvements in my PSTs' ability to plan for effective mathematics instruction as promoted by AMTE and NCTM.

Study Purpose and Research Questions

The study is motivated by the following research questions:

- In what ways does the opportunity to receive timely and specific feedback and to revise lesson plans accordingly help PSTs to better develop and apply their knowledge and skills with respect to AMTE's 2017 *Standards*?
- In what ways are PSTs meeting and not meeting the *Standards* in their lesson planning assignments?
- What themes emerge from PSTs' work on lesson planning for K-8 mathematics?

The study addresses a gap in the literature on standards-based learning in mathematics teacher education specifically and in higher education more broadly by focusing on the student learning outcomes associated with the feedback and revision cycles characteristic of SBL. Existing studies on standards-based learning have focused on other aspects of SBL, mostly in contexts outside of mathematics teacher education. In some studies, only final learning outcomes were analyzed, as measured by final exam scores or final course grades (Bloom, 1971; Guskey & Monsaas, 1979; Mevarech & Werner, 1985; Stange, 2018). In other studies, student opinions of SBL (Beatty, 2013; Buckmiller et al., 2017; Carless, 2002; Goos & Moni, 2001; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020) and instructor experiences with SBL (Beatty, 2013; Carless, 2002; Goos & Moni, 2001; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020; Stange, 2018; Weir, 2020) were documented. In one study, alignment between student-generated self-assessment scores and teacher-generated scores was examined (Orsmond et al., 2002). Studies have not yet examined differences in student performance on first and final drafts of assignments on which students have had the opportunity to make revisions after receiving instructor feedback.

The study specifically analyzes SBL from a perspective of Situated Learning theory and "legitimate peripheral participation" (Lave and Wenger, 1991), in which apprentices work closely with masters to gain the knowledge and skills necessary for autonomous work in a profession. In their tome, Lave and Wenger expound upon four domains of full professional participation: access, achievement, identity, and power. This study examines the first two, access and achievement, in seeking to document the ways in which instructor feedback informs PST performance.

Organization

Chapter 2 summarizes the literature on effective assessment and effective mathematics instruction. It also summarizes literature describing PSTs' tenuous relationship with mathematics. Chapter 3 outlines the theoretical framework of Situated Learning in more detail and links it to the analytical framework utilized for measuring PST performance. It also describes the study setting, participants, and research methods. Chapter 4 shares the findings from data analysis while Chapter 5 discusses the findings' implications for the field of mathematics teacher education.

CHAPTER TWO

LITERATURE REVIEW

Overview

In this chapter, I describe research demonstrating the effectiveness of the type of mathematics instruction advocated by NCTM and AMTE and built into the learning outcomes of my course. I also share research that underscores PSTs' fragile relationship with mathematics, leaving many of them ill-equipped to implement this type of mathematics instruction in their future careers. I conclude by sharing research that positions standards-based learning as a potentially effective intervention for increasing student performance in post-secondary education.

Standards-Based Mathematics Instruction

The AMTE *Standards* (2017) were designed in response to a legacy of research in mathematics education that demonstrates the effectiveness of student-centered instruction in improving student performance and reducing achievement gaps between different groups of students. Student-centered instruction is characterized by open-ended problem solving in which educators respond flexibly to student thinking, acting as guides for their students as they work to make sense of the mathematics presented to them and to make connections across concepts (Day, 2021).

A review of NCTM's *Principles and Standards for School Mathematics* (2000) and its updated *Principles to Actions* (2014)—both of which informed the type of teaching advocated in

AMTE's *Standards*—as well as the *Standards* document itself indicates several influential researchers and research groups in mathematics education whose work is summarized in this section. The outcomes of their studies, described below, suggest that the ideas advocated by NCTM and AMTE are not only feasible for young learners but also more promising than traditional instruction, which is characterized by teacher-centered learning in which knowledge and skills are passed on from informed teacher to uninformed student rather than developed collaboratively through hands-on engagement with mathematical concepts and representations.

Fuson and Briars (1990) were among the earliest researchers to document the benefits of hands-on tools in improving learning outcomes among children from a variety of socio-economic backgrounds. In their study, they explored the use of base-ten blocks to teach multi-digit addition and subtraction to first- and second-grade students in an urban school district. The researchers found that second graders and high-ability first graders demonstrated skill levels higher than typical third graders when engaged in instruction emphasizing the conceptual underpinnings of standard algorithms rather than focusing on algorithms alone.

In a study of her own fourth-grade classroom, Lampert (1986) illustrated her students' abilities to engage in sense making about multi-digit multiplication by exploring and justifying number decomposition strategies devised by the students themselves with support from Lampert. Citing Lampert's research and the need for further investigation into the potential benefits of instruction that is designed to support and extend students' ways of thinking and understanding mathematics, Carpenter et al. (1989) conducted a quasi-experimental study to explore the student learning outcomes associated with instruction implemented by teachers who had received explicit training in student thinking. Students of teachers in experimental groups outperformed students of teachers in control groups on two of four assessment measures and exhibited more productive beliefs about mathematics. The project, deemed Cognitively Guided Instruction (CGI), became highly influential in ensuing efforts to promote student-centered instruction. For example, the 1997 volume *Making Sense: Teaching and Learning Mathematics with Understanding* (Hiebert et al., 1997) included work on CGI and also cited Fuson and Briars (1990) research on ten-structured thinking.

Further advancing the findings on student-centered instruction, Cobb et al. (1991) compared the mathematical knowledge and attitudes of students in second-grade classrooms implementing a problem-solving curriculum against those in control classrooms implementing a traditional curriculum. The researchers found that students in the experimental classrooms demonstrated stronger conceptual understanding and more productive dispositions toward mathematics. Similar outcomes resulted from research conducted by Boaler (1998), Boaler and Staples (2008), Boaler and Sengupta-Irving (2016), and Stein et al. (2009), in which students working collaboratively to solve open-ended mathematical tasks featuring high levels of cognitive demand outperformed students in control groups characterized by traditional instruction. As Stein et al. (2009) summarized, “student learning gains were greatest in classrooms in which instructional tasks consistently encouraged high-level student thinking and reasoning and least in classrooms in which instructional tasks were consistently procedural in nature” (p. xviii). Not only were learning gains greatest among students in the experimental groups in many of these studies, but achievement gaps narrowed and/or disappeared (Boaler, 1998; Boaler, 2011; Boaler and Staples, 2008).

Elementary Pre-service Teachers and Mathematics

Implementing the type of mathematics instruction described above "requires not just pedagogical skills but also content-specific knowledge, skills, and dispositions" (AMTE, 2017, p. 2). Unfortunately, many PSTs majoring in elementary education exhibit noticeable gaps in mathematical content knowledge as well as unproductive beliefs about the subject and their ability to teach it well, a dilemma not unique to the United States (Hart et al., 2016; Ingram et al., 2018; Lutovac & Kaasila, 2014; Setra, 2017; Swars, 2005). Many PSTs enter their educator preparation programs with negative "mathematics-related teacher identities" due to negative experiences and/or to poor content knowledge in mathematics (Lutovac & Kaasila, 2018).

In a 2015 study, Shirvani found that among 87 PSTs administered a Texas assessment of sixth-grade content knowledge in mathematics, the average percent of correct items was in the low 80s for three of six domains; was only 77% for statistics and probability; and only 58% for measurement. In a 2011 study comparing performance on fraction-related assessment items, PSTs from the United States scored an average of 58.9% whereas their Taiwanese counterparts scored an average of 84.3% (Luo et al., 2011). As Chinnappan and Forrester (2014) found, "pre-service teachers come into teacher education programs with knowledge of fractions and fraction operations that is mainly about procedures with limited appreciation of the conceptual basis of these concepts and operations" (p. 894). In a study on PSTs' rehearsed explanations of problem solving, numerous weaknesses were likewise identified in PSTs' conceptions of proportional reasoning, including an inability to adequately explain procedures or to depict them visually based on the underlying concepts (Inoue, 2009). Even on presumably simpler concepts taught in lower primary grades, PSTs have struggled to demonstrate a deep knowledge of content: Zilkova et al. (2015) found that “pre-service teachers can identify planar shapes only in standard position and they do not have an accurate understanding of the properties of the shapes or they are uncertain in terminology” (p. 34).

Encouragingly, researchers have shown that positive experiences during pre-service teacher training can improve PSTs’ relationships with mathematics and motivate them to adopt more positive, optimistic mathematics-related teacher identities. Engaging in student-centered mathematics coursework, including open-ended problem solving and the use of manipulatives, as well as reflection on past experiences with mathematics seems to support PSTs in improving their attitudes towards math and math teaching (Hart et al., 2016; Ingram et al., 2018; Leavy & Hourigan, 2018; Lutovac & Kaasila, 2014; Setra, 2017).

Intertwined with the concept of mathematics-related teacher identity is a person’s sense of self-efficacy. PSTs with negative mathematics-related teacher identities also tend to exhibit low self-efficacy, expressing fear and anxiety when asked to engage with the subject or to envision themselves as future teachers of mathematics (Lutovac & Kaasila, 2014; Swars, 2005). PSTs with low self-efficacy express a lack of confidence in their abilities to teach math effectively to their future students, whereas PSTs with high self-efficacy express a confidence in their ability to overcome their gaps in knowledge and to continually grow as math educators. Such PSTs often begin their programs with the same negative attitudes as their peers but are able to overcome them through participation in their educator preparation programs (EPPs). As with PSTs’ mathematics-related teacher identities, PSTs’ sense of self-efficacy can be improved through positive experiences in educator preparation, including opportunities to increase their content knowledge (Lutovac & Kaasila, 2014; Setra, 2017; Swars, 2005).

In the next section, I provide an overview of the research base in standards-based learning, a promising approach for improving student learning outcomes and attitudes in post-secondary education.

Standards-Based Learning

As described in Chapter 1, a transformative proposition underlies the theory of mastery learning originally advanced by Benjamin Bloom and integrated into contemporary approaches to standards-based learning (SBL): student learning outcomes associated with schooling should not follow the normal distribution or infamous "bell curve," with a certain percentage of students falling far below the mean, others clustering at various distances around the mean, and others falling far above the mean. As Bloom articulated,

> The normal curve is not sacred. It describes the outcome of a random process. Since education is a purposeful activity in which we seek to have students learn what we teach, the achievement distribution should be very different from the normal curve if our instruction is effective. In fact, our educational efforts may be said to be unsuccessful to the extent that student achievement is normally distributed." [emphasis original] (Bloom, 1971, p. 49)

Bloom's argument, which built on the work and theories of John Carroll, broke with long-held traditional approaches to instruction and assessment, in which final grades reflected the assumption that some students would excel, most would be middling, and some would struggle or fail. Instead, mastery learning assumed that almost every student, if not all students, in a classroom should be reaching the learning objectives of a course of study. It is the job of educators, as advocated by NCTM and AMTE, to figure out ways to help *all* students achieve course objectives if we wish to claim that our instruction is indeed "a purposeful activity in which we seek to have our students learn what we teach" (Bloom, 1971, p. 49).

Three key features of mastery learning include: the specification and sharing of standards so that students know the objectives towards which they are striving; the provision of targeted formative feedback so that students are aware of their performance with respect to the standards at various time points during their course of study; and opportunities for students to enhance their performance before final summative assessments are administered (Bloom, 1971). Each of these features is shared with the contemporary standards-based learning movement and are described in the sections below.

Specifying Standards

As Bloom wrote in 1971, "one necessary precondition [to mastery learning] is the specification of the objectives and content of instruction" (p. 56). Sadler, writing in 1987, echoed Bloom's call for more transparency in establishing learning criteria:

> One of the conditions necessary for the intelligent use of feedback is that learners know not only their own levels of performance but also the level or standard aspired to or expected. Without an appreciation of that, students' efforts in production are likely to contain elements of random trial and error." (p. 196)

Establishing learning standards up front and sharing them with students diverges from traditional approaches to instruction and assessment, in which "the standards are … more or less inaccessible to lay persons, forming as they do part of the knowledge structures of experts" (Sadler, 1987, p. 199). Sadler (1987) defines the term *standard* as "a definite level of excellence or attainment, or a definite degree of any quality viewed as a prescribed object of endeavour or as the recognized measure of what is adequate for some purpose, so established by authority, custom, or consensus" (p. 194). Advocates of present-day SBL underscore the importance of sharing standards with students and of utilizing sample student work, or exemplars, to illuminate the standards so that students can better understand and envision the quality of work expected of them (Heflebower et al., 2019). For the purposes of this paper, the terms *learning standards* and *learning objectives* are used interchangeably.

In addition to making the objectives of a course of study more transparent to students, setting standards up front enables educators to evaluate student performance against a specific set of desired outcomes as opposed to comparing students against one another. Under traditional, normative grading, the most capable students often earn A's regardless of whether they have demonstrated true mastery of the course content, while other students earn grades relative to the grades of the strongest students in the class. Such grading systems are demoralizing to students who are not in the top tier of learners; whether they achieve the primary course objectives or not, their grades will always be below those of other students (Block, 1971b). Such conditions promote a fixed mindset towards mathematics achievement, with large percentages of students assuming that they are simply not "math people" (Boaler, 2016).

Sadler (1987) elaborates on the complexities of establishing appropriate learning standards for any given course of study. Not only must mastery criteria be established but also descriptors to distinguish between students who achieve full mastery versus those who achieve varying degrees of partial mastery. The issue of grain size further complicates matters: if standards are too numerous and fine-grained, they become unwieldy to work with; if standards are too few and too broad, then students' final grades stand to suffer significantly if their competency levels are misjudged on one or more of the standards. Additionally, verbal descriptions of competency can be interpreted differently by different instructors and students. To mitigate the complications involved with establishing and interpreting mastery learning standards, Sadler recommends the use of exemplars, or "key examples chosen so as to be typical of designated levels of quality or competence," to complement standards so that instructors and students have a lens through which to properly interpret the standards (1987, p. 200).

Writing in 2006, Marzano likewise acknowledges the complexities involved in setting specific—but not too specific—learning standards. The issue that contemporary educators face is converting overbearing national and state standards documents that contain an "inordinate amount of content" into more manageable sets of standards for teachers and students to utilize in everyday instruction (p. 13). Marzano recommends identifying the "essential elements" embedded in standards documents and organizing them into categories of related information and skills (p. 19). For example, number sense might be one essential element in a mathematics course, encompassing number systems (such as base-10), the relationships between number operations (addition, subtraction, multiplication, and division), and the ability to estimate (p. 24). Not only does this process narrow down the list of standards, it also highlights the interconnectedness among the seemingly countless skills and concepts embedded in national and state standards documents such as the Common Core State Standards and the AMTE *Standards*.

To distinguish between varying levels of mastery with respect to standards, Marzano recommends utilizing four-point scales that include half-point increments for performance that straddles two proficiency descriptors. In the context of standards-based learning, the established standards, including the descriptors for varying levels of proficiency, are referred to as "proficiency scales" (Heflebower et al., 2019, p. 8). The practice of evaluating students work against a pre-established set of standards has been shown to produce more reliable evaluations of student performance as compared to traditional, points-based grading (Marzano, 2006).

Formative Assessment and Feedback

Formative assessment and feedback are the cornerstone of standards-based learning. As Black and Wiliam (1998) report in their landmark literature review on formative assessment,

> The research reported here shows conclusively that formative assessment does improve learning. The gains in achievement appear to be quite considerable, and as noted earlier, amongst the largest ever reported for educational interventions. As an illustration of just how big these gains are, an effect size of 0.7, if it could be achieved on a nationwide scale, would be equivalent to raising the mathematics attainment score of an 'average' country like England, New Zealand or the United States into the 'top five' after the Pacific rim countries of Singapore, Korea, Japan and Hong Kong. (p. 61)

Black and Wiliam (1998) define formative assessment as, "encompassing all those activities undertaken by teachers, and/or by their students, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged" (pp. 7-8). The aim of formative assessment is to pinpoint areas of strength and weakness with respect to the standards and to identify clear paths towards mastery. To be effective in this regard, formative assessment and feedback must be frequent, timely, and connected to the standards established for the course of study (Airasian, 1971; Alonzo et al., 2019; Carroll, 1971; Heflebower et al., 2019). As Airasian (1971) points out, “If learners must wait one or two or more months to discover that they have not mastered a concept introduced in the first week of instruction, being informed of such errors benefits them little” (p. 79).

One of the most common mechanisms of formative assessment is teacher observation since “almost all teachers gather what can be termed formative evidence during their instruction. Teachers implicitly select and respond to cues from their pupils” (Airasian, 1971, p. 80). Additional avenues for formative assessment include conversations with students, student journals and notebooks, homework, quizzes, essays, projects, and oral and written reports (Arisian, 1971; Carroll, 1971; Guskey, 2007; Heflebower et al., 2019; Marzano, 2006).

To function as formative assessment in a standards-based learning environment, the use of these tools must be purposeful: it “requires the teacher to deliberately watch a student for specific actions required by the language on a proficiency scale” (Heflebower et al., 2019, p. 76). Since an aim of SBL is for students to achieve mastery of educational outcomes that include higher-order thinking skills, then formative assessment items must invoke such skills in order to evaluate students’ proficiency with respect to higher-demand tasks. If formative assessment items comprise solely multiple-choice, true-false, or single-response questions but a final summative evaluation will involve synthesizing information learned in a course, then formative assessment will not serve its intended purpose of identifying areas of strength and weakness with respect to the learning objectives (Airasian, 1971; Black & Wiliam, 1998; Guskey, 2007; Marzano, 2006).

Coupled with formative *assessment* must be formative *feedback*. As Sadler (1998) articulates, “Incorporating feedback is surely as fundamental a characteristic of responsible and responsive learning systems as having a teacher at all" (p. 79). Stefani (1998) defines feedback as, “information that provides the performer with direct usable insights into his/her current performance, based on tangible differences between current performance and the learner’s hoped for performance” (p. 348). “Direct usable insights” include specific suggestions for remedial actions or “correctives” to be taken by individual students and/or the teacher to improve students’ performance vis-à-vis the standards (Airasian, 1971; Stefani, 1998). In Black and Wiliam’s (1998) research review, a key finding from the literature on formative assessment is that feedback focused on the correctness of answers or final products can have negative impacts on student learning outcomes, whereas feedback emphasizing students' processes for achieving final answers and putting together final products moves students towards their learning goals.

In order for teachers to generate useful feedback for their students, formative assessments must be evaluated with respect to the standards rather than with respect to a point total, which is characteristic of traditional instruction and assessment. "For the purposes of formative assessment, a single score hides more than it reveals" (Airasian, 1971, p. 84). Two students earning the same number of points on an assignment may in fact be mastering different objectives. For scores to be useful, they should be linked to the relevant learning standards incorporated into an assessment so that students know which standards they have mastered and which standards require further attention (Airasian, 1971; Marzano, 2006). Better still, formative feedback should comprise comments alone without a score or grade in order to encourage students to develop a growth mindset about their work (Airasian, 1971; Black & Wiliam, 1998; Boaler, 2016; Carroll, 1971).

Formative feedback can also be generated by students through processes of peer- and self-assessment. Research evidence summarized by Black and Wiliam (1998) and Dochy, Segers, and Sluijsmans (1999) suggests that peer- and self-assessment can be beneficial for students because it contributes to an increase in metacognitive skills such as reflection and problem solving. To be effective, students must be trained in peer- and self-evaluation and provided with specific structures within which to generate useful feedback (Black & Wiliam, 1998). Students might assess their peers using formats such as "two stars and a wish," which requires students to provide both praise as well as constructive criticism to their peers (Boaler, 2016, p. 157). They might also generate and answer their own or their peers' questions about the topic of study (Black & Wiliam, 1998; Boaler, 2016). Other options include evaluating their own or their peers' work against teacher-generated checklists or against the standards themselves; responding to targeted reflection questions; teaching one another the concepts under study; and even generating sketches to encapsulate their present knowledge of the subject (Boaler, 2016).

Opportunities for Improvement and Enrichment

When formative assessment indicates that students have not yet achieved mastery of one or more of the intended learning objectives, opportunities for improvement should be provided to students so that they can work towards proficiency. Such opportunities may include participating in supplemental study sessions or tutoring sessions; reading supplemental material or re-reading key passages of previously assigned material; completing additional exercises or tasks; engaging in academic games or puzzles; listening to or watching selected audio or visual supplements; and re-teaching, among other strategies (Block, 1971b). As Guskey (2007) underscores,

> Because of individual differences among students, no single method of instruction works best for all. To help every student learn well, therefore, teachers must differentiate their instruction, both in the initial teaching and especially through the corrective activities (Bloom, 1976). In other words, to decrease variation in results, teachers must increase variation in their teaching. (p. 16)

These additional opportunities are a method for allowing students more *time* to meet the stated standards. As related in the Introduction, Carroll identified pacing as an inhibiting factor for many students' "opportunity to learn" (Carroll, 1971, p. 32). Although students may be asked to engage in supplemental tasks outside of class, Guskey (2007) recommends providing class time for students to engage in corrective activities:

> Teachers who ask students to complete correctives outside of class as a homework assignment or during special study sessions held before or after school rarely experience success. Instead, they quickly discover that those students who could benefit most from the corrective process are the least likely to take part. (p. 18)

For teachers who worry that allotting class time for correctives will force them to cut other content from their courses, Guskey (2007) points out that time spent on correctives early on in a course will save time in reviewing and re-teaching concepts later on.

For students who demonstrate mastery of the course objectives on a given formative assessment, opportunities for enrichment can be provided to them during time when other students are revisiting un-mastered material. These might be small projects or more challenging versions of tasks already assigned, or extensions to such tasks. Alternatively, higher-aptitude students can be incorporated into cooperative learning groups along with lower-aptitude students to assist the latter as they work towards proficiency. Here again, training must be provided so that the higher-aptitude students understand the need to help draw out the necessary skills in their peers as opposed to taking tasks over for them (Boaler, 2016; Guskey, 2007).

Standards-Based Learning in Higher Education

Standards-based learning has taken root in K-12 schools across the United States (Scarlett, 2018). While SBL has gained traction in higher education in recent years, it is not yet widely used (Alonzo et al., 2019; Klecker & Chapman, 2008; Wong & Kang, 2012). As noted by Wong and Kang (2012), "while ML [mastery learning] has been around in the education landscape for over half a century, there is still a lack of literature that investigates the suitability and effectiveness of adopting ML to higher education" (p. 207). Among the studies that have been published within higher education, the following aspects of mastery learning and standards-based learning (which are defined similarly in the literature) have been examined: student learning outcomes as indicated by final exam scores and/or final course grades (Bloom, 1971; Guskey & Monsaas, 1979; Mevarech & Werner, 1985; Stange, 2018); students' use of rubrics (Hendry et al., 2012; Orsmond et al., 2002); student opinions of standards-based learning (Beatty, 2013; Buckmiller et al., 2017; Carless, 2002; Goos & Moni, 2001; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020); and instructor experiences implementing standards-based learning (Beatty, 2013; Carless, 2002; Goos & Moni, 2001; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020; Stange, 2018; Weir, 2020). Of the studies identified, five arose from teacher education contexts: Buckmiller et al., 2017; Carless, 2002; Goos and Moni, 2001; Prasad, 2020; and Scarlett, 2018. Only Prasad 2020 dealt specifically with pre-service elementary mathematics teachers.

Summary of Research Findings

Studies examining student learning outcomes have reported primarily positive results associated with mastery learning. In an early study reported by Bloom (1971), statistically significant differences were found between the mean performance of students in a mastery learning section of a course and students in a non-mastery section, with 80% of students in the mastery section earning A's on the final examination and only 20% of students in the traditional section earning A's. In a quasi-experimental study conducted across the City Colleges of Chicago, Guskey and Monsaas (1979) found that students in mastery learning sections of courses ranging from English to mathematics earned higher final exam scores and course grades than their peers in non-mastery learning sections taught by the same instructors. The mastery learning sections also demonstrated greater student retention, with lower percentages of students dropping their courses before the end of the semester. Mevarech and Werner (1985) found that students in mastery sections of an introductory gerontology course outperformed their peers in a non-mastery section of the course on final exam questions involving higher-mental processes and problem-solving skills.

Other teacher-researchers sensed improvements in their students' performance in their courses but did not specifically compare student work across or between semesters (Carless, 2002; Prasad, 2020; Weir, 2020). Several pointed out the benefit of upholding higher standards for students such that students could not earn a passing grade in their courses simply by accumulating a minimal number of points without necessarily demonstrating mastery of the concepts (Prasad, 2020; Selbach-Allen et al., 2020; Weir, 2020). For example, Selbach-Allen et al. (2020) commented, "Before incorporating standards-based grading, we all had students who found ways to earn enough points to pass our courses without developing crucial skills" (p. 1115-1116). They also noted that fears about students taking advantage of the opportunity to revise by submitting low-quality work, knowing that it could be redone at a later date, did not play out in their courses: "many students quickly figure out after the first assignment that revisions are more difficult if they did not start from a good first draft" (p. 1119).

In contrast to the above findings, Stange (2018) found that students performed more poorly on the final exam than would have been predicted by their grades on the formative assessments offered throughout the course, suggesting that the instructor's approach to standards-based learning (referred to as standards-based grading in her study) were not as effective as one might expect based on the results of the research cited above. Although the instructor (also the author and researcher) provided near-daily opportunities for formative assessment and quickly addressed gaps in proficiency, the researcher's approach to formative assessment failed to incorporate higher-order skills such as synthesizing information across standards:

> The nature of the badges system left no room for questions that tested some synthesis of their topics. This made all the topics appear disjointed and separate, and, I fear, unmotivated. An alternative system may be to allow some synthesis problems combining an understanding of different badges, which are then graded with respect to more than one standard. (p. 806)

Indeed, providing formative assessment problems aligned with higher-order skills is among the recommendations put forth by proponents of mastery learning and standards-based learning (Airasian, 1971; Black & Wiliam, 1998; Guskey, 2007; Marzano, 2006).

Orsmond et al. (2002) reported the findings of a third in a series of studies designed to identify conditions that support students' understanding of the standards against which their performance in courses is judged. They found that instructor feedback coupled with the use of exemplars, as recommended in earlier research (Sadler, 1987), increased alignment between instructor-generated scores and student-generated scores on a significant course assignment (in this case, a scientific poster on the topic of histology), underscoring the importance of exemplars in illuminating the standards for students. Hendry et al. (2012) likewise found that intentional use of exemplars supported students in understanding the criteria outlined in course rubrics.

Post-secondary students, by and large, have been found to view mastery learning favorably. Early research studies on mastery learning indicated that "mastery learning students … exhibited markedly greater interest in and attitudes toward the subject learned compared to non-mastery learning students" (Block, 1971a, p. 9). Recent research corroborates these results.

In their respective studies, Beatty (2013), Buckmiller et al. (2017), Carless (2002), Goos & Moni (2001), Scarlett (2018), and Selbach-Allen et al. (2020) all found that student feedback on the use of standards-based learning strategies was primarily positive. Students appreciated receiving feedback on early quizzes or drafts of assignments coupled with opportunities to improve their final scores (Beatty, 2013; Carless, 2002; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020). They also appreciated knowing up front the objectives towards which they should be striving and against which their work would be evaluated, which helped them to identify their strengths and weaknesses with respect to the course content (Beatty, 2013; Buckmiller et al., 2017; Goos & Moni, 2001; Scarlett, 2018). As Buckmiller et al. noted, "a well written rubric, for example, gives students a chance to demonstrate proficiency without negotiating the impossible task of reading an instructor's mind" (2017, p. 156).

Negative comments from students mostly reflected initial confusion about a mastery learning or standards-based grading system (Beatty, 2013; Buckmiller et al., 2017; Prasad, 2020; Scarlett, 2018; Selbach-Allen et al., 2020) and feeling conflicted about completing corrective tasks that are not graded but are designed to prepare students for exam retakes when assignments for other courses *are* going to be graded, hence demanding more immediate attention (Beatty, 2013). As one student opined,

> The primary issue I have is that other classes don't have this mentality. So there are a large number of assignments due, that are required in order to get good grades. […] I hope you understand how hard it is to put off something that's being graded, with the potential of not completing the assignment, or completing it at a sub-standard level, in order to study/work on something that isn't going to "directly" impact my grade. (Beatty, 2013, pp. 17-18)

Prasad (2020) accounted for this type of dilemma in her course by assigning a final portfolio—which forms part of each student's grade—comprising revisions to earlier assignments that did not initially meet the learning targets.

Difficulties faced by instructors included the significant up-front time investment in generating standards and associated assessments, although the up-front investment resulted in time saved when grading assignments against clearer criteria (Beatty, 2013; Selbach-Allen et al., 2020). Prasad (2020) noted the significant time investment in grading final portfolios at the end of every semester; however, by having students submit all of their revisions during finals week rather than at varying times throughout the term, time was saved on continuously grading revisions at earlier points during the semester. For all these teacher-researchers, the positive outcomes associated with SBL were considered well worth the time.

Approaches to Standards-Based Learning

There are almost as many approach to SBL in higher education as there are practitioners and researchers implementing the approach. As Alonzo et al. (2019) note, "Although there are agreements about the definition of SBA [standards-based assessment], the concept is interpreted in different ways by academics, which consequently leads to different SBA practices to the extent that some practices are not fully aligned to the principles described" (p. 637) Indeed, as discussed above, Stange (2018) ultimately found that her approach to SBL did not meet the full criteria for effective practice because she did not include higher-order thinking skills in her formative assessments. The teacher-researchers whose studies are summarized in this section all varied in their approaches to SBL and in some cases utilized different terminology, such as standards-based assessment or mastery learning. However, they all shared the same underlying assumption that SBL must, at the very least, involve setting and sharing learning standards with students and must incorporate ongoing formative assessment of student learning outcomes with opportunities for students to improve their performance based on feedback.

Methods of developing standards, of aligning assessments to standards, of providing feedback and correctives, and of calculating final course grades varied from teacher-researcher to teacher-researcher, although there were several commonalities among them. Scarlett (2018), who integrated mastery learning into a sophomore-level assessment course for K-12 teaching majors, began the process of delineating specific learning standards for his course by composing ten broad statements reflecting what teacher candidates should know and be able to do. The statements were translated into specific learning targets—or product goals—aligned with the course content using language borrowed from Bloom's Taxonomy to ensure a range of cognitive skills were reflected. A separate set of process goals was developed to enable attendance, class participation, and assignment completion to be incorporated into the final course grade, underscoring the importance of these habits for individuals who aim to work as professional educators. To distinguish between varying levels of performance, Scarlett developed a five-point rubric for each process and product goal. The separation of process goals from product goals enabled both of these elements to factor into a final grade calculation without amalgamating the two into a single point total. Students would be able to see a separate score for each category within the set of standards so that they would know the specific areas in which they failed or succeeded with respect to the course objectives.

In developing standards for a college physics course, Beatty (2013) consulted the course textbook to identify specific skills and competencies that could be translated into learning objectives, of which over 100 were developed and assessed. After this quantity proved to be a logistical nightmare in the first course of a two-course sequence, Beatty pared down the number of objectives to 25 in the second course. These objectives were broader than the objectives in the first course, which were too fine-grained to evaluate effectively or efficiently. Weir (2020) similarly found that she needed to adjust her course objectives as the semester progressed and ultimately determined that "20 to 30 objectives can be reasonably tested during a semester" (p. 1001). Like Beatty (2013), Stange (2018) found that establishing standards incorporating too many distinct facts and skills proved to be unwieldy and caused difficulties in attempting to assess higher-order skills such as problem solving and critical thinking. Stange used a one-point scale to assess each standard, with half of a point reflecting partial mastery. Neither Beatty (2013) nor Stange (2018) incorporated process goals into their final grades calculations, which were limited to performance on quizzes and exams (including re-takes on exams after engaging in corrective action).

Teacher-researchers were careful to develop assessment questions and/or tasks designed to provide students with opportunities to demonstrate achievement of the stated learning objectives. Beatty (2013) and Scarlett (2018) both discussed the benefit of engaging in this alignment exercise in terms of ensuring coherency in their courses. While Beatty (2013) and Stange (2018) both developed new assessment instruments to ensure alignment between standards and assessments, Scarlett (2018) utilized existing assessments but in a more intentional manner, paying closer attention to the strength of evidence demonstrated by students with respect to the various learning standards. Carless (2002), who wrote about applying standards-based learning to a particular project, likewise did not alter the nature of the final product required, only the process of arriving at the final product and grade.

Each of the teacher-researchers took different approaches to providing feedback and opportunities for improvement. Carless (2002) embedded opportunities for feedback and improvement directly into the course, providing class time for students to engage in brainstorming for their project, to present and receive feedback on initial drafts, and to revise their work. Stange (2018) also provided class time for re-assessment in the form of daily quizzes, on which standards-based targets would be assessed repeatedly until the class as a whole demonstrated mastery. Beatty (2013) incorporated a mix of in-class and out-of-class corrective and reassessment opportunities. Homework sets designed to prepare students for pre-tests, exams, and re-tests were to be completed outside of class. Pre-tests and exams were conducted during class time, with re-tests completed outside of normal class hours. Feedback was provided on the homework sets and exams, although the exact nature of this feedback was not discussed.

None of the instructors in the studies discussed above incorporated early assignments in their final grade calculations, including only assignments that demonstrated the highest level of mastery achieved by each student. In one study, the researchers reported assigning grades based on the competency demonstrated only on final projects and assignments at the end of the term such that a student who had previously demonstrated mastery but could not maintain it on a summative assessment would not earn a mastery score for the relevant item on the course rubric (Buckmiller et al., 2017). Stange (2018) assigned final grades based on a combination of traditional and mastery grading; she calculated 30% of the course grade based on a traditionally scored final exam and 70% of the course grade based on the highest quiz scores attained with respect to each learning standard. The other instructors converted standards-based rubric scores to percentage grades, although it is unclear how they determined which letter grades would correlate with which percentages. Stange (2018) established the following criteria but does not articulate a rationale: “80% and above was some flavour of A; cutoffs for B, C, D were, respectively, 65%, 54%, and 50%” (p. 803).

Several considerations emerge from the teacher-researchers’ various approaches to implementing standards-based learning in their courses, including an affirmation of the need to carefully delineate standards such that they are comprehensive yet not overly fine grained, as suggested in earlier work by Sadler (1987). To develop standards, the teacher-researchers all worked backwards by first identifying the learning targets associated with their courses, a process that included consulting externally validated resources such as established taxonomies, as in the case of Scarlett (2018), and textbooks, as in the case of Beatty (2013). Also important is the process of aligning standards to assessments, whether those assessments are generated from scratch or adapted from existing assessments. Formative assessments should not be factored into final course grades since they reflect progress towards course objectives and not a student’s summative level of performance. Finally, according to Guskey (2007), providing at least some in-class time for correctives and reassessment is best for students who are most likely to struggle, which was a practice adopted by most of the teacher-researchers cited above.

Areas for Continued Research

While the instructors and students represented in the studies above reported primarily favorable experiences with standards-based learning along with improved scores on final examinations, the quality of student work was not examined. Carless (2002) "had the impression that as a whole, the body of work was superior to that produced by other groups when such a systematic feedback process had not been carried out," but he did not systematically compare and contrast the work submitted by students across different time points or different semesters (p. 359). He asks, "to what extent are our learners […] making improvements in learning in accordance with the feedback which they have received?" (p. 360). Scarlett (2018) echoes Carless' question: "If the standards-based grading approach is meant to improve learning, do we know this is really happening?" (p. 68).

In a literature review of action research (AR) in higher education, Gibbs et al. (2017) note,

> In the bulk of the literature, AR is centred primarily on description of the reflective process rather than any detailed critical evaluation of the intervention/innovation and methodology. Such accounts rely heavily on personal teacher and student reflection; although this is an integral part of the AR process, a more mixed methods approach could widen the impact and scrutiny of the research. (p. 9)

All the action research studies on standards-based learning cited above took this reflective approach. While the teacher-researchers relayed recommendations for implementing SBL based on their experiences and those of their students, these experiences were not systematically analyzed. There is a clear need for action researchers studying standards-based learning to apply more rigorous research methods to evaluate their students' learning outcomes. Hence this study attempts to answer Carless' question above by evaluating student work against a set of nationally accepted standards integrated into a specific analytical framework delineated in the subsequent chapter.

Summary

Professional bodies in the field of mathematics education are striving to change the ways in which mathematics is taught to K-12 students such that all students can experience success in the subject. In order to be prepared to teach using *Standards*-based practices, PSTs must be provided with experiences that foster positive mathematics-related teacher identities and increase their sense of self-efficacy.

Based on the existing literature, SBL appears to be a promising approach to increasing the learning outcomes among students in higher education, including PSTs. Students and instructors largely view standards-based learning favorably and studies indicate that students' knowledge and skills can improve under SBL as compared to traditional instruction and assessment. However, to date, exam scores and final course grades are the only measures of student learning outcomes reported in the literature, suggesting a need to explore the quality of student work associated with standards-based learning. While action research is common in the literature on SBL, there is a need to approach action research more systematically, with clearly articulated methods designed to enhance credibility, transferability, and confirmability. In the following chapter, I describe my methods for the present study.

CHAPTER THREE

RESEARCH METHODS

Overview

In this chapter, I describe my research approach for the present study. I utilized a case study design implemented via action research, in which I served as both the course instructor and primary researcher. A case study design was chosen because I was seeking to "provide an in-depth understanding" of "a clearly identifiable case with boundaries" (Creswell & Poth, 2018, p. 100). Similar to Berk and Hiebert (2009), my aim was to engage in systematic inquiry on the effectiveness of my math methods course with an eye toward generating knowledge useful for other MTEs by evaluating my PSTs' learning outcomes with respect to broadly accepted standards for pre-service mathematics teachers, an aim that is "compatible with the notion of teacher as a researcher" (p. 339).

The study involved 21 students enrolled in a single section of the K-8 math methods course that I teach at a small, urban, public university in the northwestern United States. I gathered and analyzed first and final drafts of lesson and unit plans submitted by the Fall 2019 cohort and conducted observations and interviews with four participants who student taught during the subsequent semester. My data analysis process involved multiple cycles of qualitative coding in which I evaluated lesson and unit plans with respect to AMTE's *Standards* (2017) and the Danielson Framework (2007) and also searched for emergent themes in my PSTs' work.

I begin the chapter by framing the research within the theoretical framework of Situated Learning before delineating the analytical framework I used for analyzing lesson plans and unit plans. I then describe the study setting, participants, and procedures in more detail, including measures taken to ensure credibility, reliability, and transferability.

Theoretical Framework: Situated Learning Theory

Pre-service teachers enrolled in educator preparation programs are in many ways like traditional apprentices who work alongside a master, gradually taking over greater control of the primary responsibilities of a profession. PSTs begin by learning the basic language and philosophy of education from veterans in the field while engaging in brief field experiences supervised by in-service teachers. After experiencing initial tastes of classroom life, they engage in more extended and independent opportunities to teach while learning to complete complex tasks such as lesson planning and unit planning.

In their seminal work *Situated Learning: Legitimate Peripheral Participation*, Lave and Wenger (1991) describe the conditions associated with successful and unsuccessful apprenticeships, outlining contexts that support and inhibit apprentices' acquisition of the skills and knowledge necessary for autonomous work in their fields. Apprenticeships that enable newcomers to observe masters engaging in their work and to incrementally take on greater responsibility while receiving direct feedback from their masters are more successful than apprenticeships in which newcomers are relegated to limited, and often more menial, aspects of the profession. Successful apprenticeships provide *access* to the knowledge and skills needed for *achievement* in the profession. They also help apprentices to develop an *identity* appropriate to the profession that lends legitimacy and *power* to become active agents of advancement and change within their communities of practice. These four domains of *legitimate peripheral* *participation*—access, achievement, identity, and power—are key to sustaining professions as new generations gradually replace old generations in a continuous cycle of renewal.

The mathematics teacher educator Rochelle Gutierrez (2012) has adapted Lave and Wenger's work to develop what she deems the axes of equity in mathematics education. Access and achievement are associated with the dominant axis of equity for their prominence in traditional public schooling, where grades and standardized test scores are often emphasized over other outcomes of education, including identity and agency (power). These latter two outcomes form the critical axis of equity for their role in enabling legitimate participation in both mathematics as well as society at large. Gutierrez argues that access and achievement are only part of the equation as our society works towards greater equity. Women, people of color, and others who are traditionally excluded from participation and/or success in mathematics must be given access to meaningful mathematics experiences and supported in achieving mastery of the subject matter—but they must also be supported in identifying themselves as competent mathematicians and in developing a sense of agency within the discipline, able not just to participate in it but to influence it.

Like Lave and Wenger (1991), Gutierrez (2012) argues that learning contexts matter for the success of teacher-learner, master-apprentice relationships:

> Contexts have always mattered to me. Perhaps it is because I was raised to believe that communities shape and support individuals into the beings they become. Some contexts bring out the best in me, while others hide my strengths. Considering my worldview, it makes sense that my research would pay particular attention to contexts. (p. 17)

Along with Boaler and Staples (2008), Boaler and Sengupta-Irving (2016), Stone and Hamann (2012), and Wilson et al. (2019), Gutierrez (2012) has found that contexts play a key role in student success. Commonalities across contexts that support success in mathematics include high expectations that are made explicit to students along with collaborative, open-ended tasks characterized by high levels of cognitive demand. Like the apprentices of tailors, midwives, and quartermasters—all of whom engaged newcomers in the full breadth of the actual work of each trade—the students of teachers who provide opportunities to think and act like mathematicians exhibit the greatest achievement and most positive attitudes towards mathematics.

In the present study, I focus on the first two domains of legitimate participation in a community of practice: access and achievement. The content of my mathematics methods course is designed to provide access to the field of mathematics education to my PSTs. The activities I engage them in enable them to practice the actual work of masters in a supported environment in which expectations are high but stakes are low since the PSTs are not yet held accountable for the learning outcomes of real K-8 students.[1] As Lave and Wenger (1991) expound, legitimate access

> might include who is involved; what they do; what everyday life is like; how masters talk, walk, work, and generally conduct their lives; how people who are not part of the community of practice interact with it; what other learners are doing; and what learners need to learn to become full practitioners. It includes an increasing understanding of how, when, and about what old-timers collaborate, collude, and collide, and what they enjoy, dislike, respect, and admire. In particular, it offers exemplars (which are grounds and motivation for learning activity), including masters, finished products, and more advanced apprentices in the process of becoming full practitioners. (p. 95)

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[1] My PSTs do engage in two field experiences in my methods course, but they are evaluated on their ability to learn and grow from the experiences rather than their ability to have the students they work with meet specific learning objectives.

The tasks, readings, videos, discussions, exemplars, teaching experiences, instructor modeling, and instructor feedback included in my course are intended to offer PSTs the *access* outlined in the above excerpt. The intervention of SBL is intended to support PSTs' *achievement* of the knowledge and skills necessary to implement *Standards*-based instruction, as described previously in Chapter 2. While identity and power are also crucial to the future success of my PSTs as *Standards*-based mathematics educators—fostering a sense of belonging and empowerment to change math education for the better—incorporating these two domains into the present study would have made the scope of the undertaking too unwieldy for a single project; hence these domains are recommended as areas for future research.

In the next section, I delineate the analytical framework used to evaluate my PSTs' achievement with respect to AMTE's *Standards* (2017) as well as the Danielson Framework (2007), which is widely used for evaluating teacher performance across the United States.

Analytical Framework: AMTE-Danielson Course Rubric

To evaluate my PSTs' achievement with respect to planning *Standards*-based mathematics lessons and units, I developed a detailed rubric delineating skills and knowledge relevant to lesson planning as set forth in the *Standards*. The rubric, shown below in Table 1, is also aligned with the Danielson Framework (2007), which has been adopted by my department for purposes of both teacher educator and pre-service teacher evaluation. As Berk and Hiebert (2009) advised,

> A first step in addressing the challenge of preparing prospective teachers and developing shareable knowledge for improving teacher preparation is to select an explicit, specific, and targeted set of learning goals that are shared among the teacher educators and are

> committed to for the long run. Doing so allows the teacher educators to treat the course as an object of study, thereby generating cumulating and shareable knowledge. (p. 341)

By developing learning targets based on two sets of widely accepted standards for pre-service and in-service teacher evaluation, I can ensure that the outcomes I report will serve as a useful reference for other MTEs.

Overview of the Standards Documents

In this section, I provide an overview of the two sets of standards integrated into the course rubric outlined in Table 1: the Danielson Framework (2007) and the Association of Mathematics Teacher Educators *Standards for Preparing Teachers of Mathematics* (2017).

The Danielson Framework.

The Danielson Framework, designed to evaluate the performance of in-service educators, is motivated by constructivist theories of learning and an assumption that "it is important for students—all students—to acquire deep and flexible understanding of complex content, to be able to formulate and test hypotheses, to analyze information, and to be able to relate one part of their learning to another" (Danielson, 2007, p. 15). Like NCTM and AMTE, the Danielson Framework identifies equity as a priority. Of seven common themes identified in the framework, equity is the first:

> Implicit in the entire framework, particularly those domains relating to interaction with students … is a commitment to equity. In an environment of respect and rapport, *all* students feel valued. When students are engaged in a discussion of a concepts, *all* students are invited and encouraged to participate. When feedback is provided to students on their learning, it is provided to *all* students. (p. 32)

The Framework also recognizes that context matters: "no one approach is a 'one size fits all.' But some approaches will be better suited to certain purposes than others. Making good and defensible choices is the hallmark of a professional educator" (p. 25). However, "beneath the unique features of each situation are powerful commonalties. It is these commonalties that the framework addresses" (p. 22).

Danielson attempts to capture the commonalties of high-quality teaching and learning by organizing standards into four domains: planning and preparation (Domain 1); the classroom environment (Domain 2); instruction (Domain 3); and professional responsibilities (Domain 4). Under planning and preparation, the importance of high expectations and incorporating student needs, interests, and choice undergird each individual component. Under classroom environment, productive and respectful collaboration among students is a strong theme. Student authority and autonomy is evident under instruction, with students being held accountable to both participating in and shaping classroom discourse and activities, which should be challenging, engaging, and of interest to students. The domain of professional responsibilities holds teachers accountable to continually improving their practice through reflection and collaboration, including collaboration with students and their families.

AMTE *Standards for Preparing Teachers of Mathematics*. Whereas the Danielson Framework is intended to be applicable across disciplines and is designed with in-service teachers in mind, the AMTE *Standards* are specifically targeted to pre-service K-12 math teachers. Because the *Standards* are specific to mathematics instruction and to pre-service teachers, my research questions emphasize achievement with respect to the *Standards*, whereas the Danielson Framework served as a guide for my word choice in the rubric due to my PSTs' familiarity with the Framework. Like Danielson, the *Standards* emphasize equity, collaboration, problem solving, and student and family involvement in setting goals and planning for instruction. Furthermore, the *Standards* describe the depth and extent of mathematical knowledge for teaching that effective pre-service educators should hold, including an ability to make connections across various mathematical topics, representations, and grade levels. Four overarching standards are incorporated into the document: mathematics concepts, practices, and curriculum; pedagogical knowledge and practices for teaching mathematics; students as learners of mathematics; and social contexts of mathematics teaching and learning.

The *Standards* explicitly address the four dimensions of equity as outlined by Gutierrez (2012). In addition to a generic goal to “promote equitable teaching,” the standards include specific indicators aligned to Gutierrez’s framework. Indicator C.4.1, “provide access and advancement,” is aligned with the components of the dominant axis of equity: access and achievement. Indicators C.4.2 and C.4.4, “cultivate positive mathematical identities” and “understand power and privilege in the history of mathematics education” are aligned with the components of the critical axis of equity: identity and power. The written descriptions that elaborate these indicators incorporate a strong emphasis on context, demanding that pre-service math educators know how to draw on and bolster students’ unique cultural and mathematical strengths. Teachers should be able to connect classroom contexts to students’ local contexts and bring the two together in ways that enable students to simultaneously develop both strong mathematical capabilities and strong mathematical identities.

Delineating Rubric Domains, Components, and Criteria

The rubric used for this study and shown in Table 1 emphasizes aspects of Danielson and the AMTE *Standards* most relevant to lesson planning. Aspects of the standards related to implementing lessons are not included in the rubric—either for the purposes of this study or for my course syllabus—because I am unable to individually evaluate each of my PSTs during the limited field experiences included in my course, when PSTs all teach in different classrooms at the same time. Aspects related to family and community involvement are also not included because they are beyond the scope of the course.

Since other teacher educators in my department utilize the Danielson Framework in their courses, most of the domain and component names in my rubric are borrowed directly from those of the Danielson Framework. For the purposes of this study, I use the term "domain" to refer to the broader categories in the course rubric (e.g., *demonstrating knowledge of content*). I use the term "component" to refer to the subcategories within each domain (e.g., *mathematical concepts* and *significance*). The names of the four proficiency levels are also borrowed directly from Danielson, whereas the criteria for each proficiency level reflect an amalgamation of elements of Danielson and the *Standards*. Like Danielson, the rubric utilizes a 4.0 scale, as recommended by Marzano (2006) and Guskey (2015), to maximize validity and reliability. Smaller scales do not allow for sufficient differentiation between proficiency levels whereas larger scales result in less reliability. For the specific standards from Danielson and AMTE incorporated into the rubric, refer to the codebook in Appendix B, which lists the standards according to the relevant domains and components.

Since the AMTE *Standards* are not delineated into specific proficiency levels, only describing the knowledge and skills of a proficient pre-service math educator, I first established the criteria for proficient performance in my rubric and then worked backwards to generate criteria for each of the other three levels. As I did so, I referred to both Danielson—which delineates criteria for the four levels of proficiency, although the levels are not specific to mathematics education—as well as to the Task Analysis Guide in Stein et al. (2009, p. 6). The Task Analysis Guide describes four levels of cognitive demand associated with mathematics tasks: *doing mathematics*, *procedures with connections*, *procedures without connections*, and *memorization*.

Stein et al.'s (2009) criteria for *doing mathematics*—"complex and nonalgorithmic thinking" that builds on prior knowledge and makes connections across concepts (p. 6)—directly informs the type of math tasks advocated in the *Standards* and hence informed my criteria for proficient performance in my rubric. For example, the *Standards* recommends selecting "meaningful tasks to motivate student learning, develop new mathematical knowledge, and build connections between conceptual and procedural understanding" and "that promote reasoning and problem solving, provide multiple entry points, have high ceilings to offer challenges, and support varied solution strategies" (p. 14). My criteria for proficiency in *nature of tasks* and *use of resources* reflect these goals: "tasks position students as the primary doers and feature a high level of cognitive demand, permitting non-algorithmic thinking and multiple solution strategies and fostering conceptual as well as procedural understanding" and "resources are utilized to support student exploration and sense making and to illuminate important mathematical connections."

The criteria for nearing proficient performance in my rubric mostly reflects proficient performance but with a need for further clarification or additional details. For example, the criteria for nearing proficient for *nature of tasks* and *use of resources* repeats the criteria for proficient performance "but require[s] further development to be fully effective." For other categories, minor oversights distinguish nearing proficient performance from proficient performance.

I defined the criteria for basic performance to align with traditional notions of mathematics instruction, which Goldsmith and Shifter (1997) describe as, "grounded in the belief that students learn by receiving clear, comprehensible, and correct information about mathematical procedures […] Classroom instruction is organized around the transfer of information from knowledgeable teacher to uninformed student" (pp. 22-23). This type of instruction is characteristic of Stein et al.'s (2009) *procedures without connections* for its "limited cognitive demand," its focus "on producing correct answers rather than developing mathematical understanding," and on "explanations that focus solely on describing the procedure that was used" (p. 6). For instance, my criteria for basic performance in *nature of tasks* is, "tasks feature student engagement but the level of cognitive demand is low, emphasizing prescribed routines or procedures without meaningful connections to concepts." Although there is a role for rote practice within mathematics instruction, lesson plans based solely on procedural fluency would not meet the *Standards*.

I defined unsatisfactory performance as lesson planning that exhibited substantial gaps in understanding or outright misconceptions; incorporated mathematically inappropriate or culturally disrespectful content; focused on memorization only; and/or overlooked the associated component of a lesson plan (such as forgetting to include prompts or describe how manipulatives would be utilized). My criteria for unsatisfactory performance in *nature of tasks*, for example, is "tasks are entirely teacher led with no opportunity for active student engagement, and/or tasks emphasize memorization only." My criteria for unsatisfactory performance in *use of resources* is, "resources are utilized incorrectly or inappropriately, or manner of use cannot be determined from the lesson."

Table 1 shows the criteria for each proficiency level of each component of my rubric.

Table 1. AMTE-Danielson Rubric for Pre-Service Mathematics Educators

	Proficient for Pre-Service Expectations	**Nearing Proficient for Pre-Service Expectations**	**Basic**	**Unsatisfactory**
Demonstrating Knowledge of Content (Danielson 1a; AMTE C.1.1, C.1.4-5, C.2.2, C.3.1-2) *Mathematical concepts* *Significance* *Prerequisite knowledge and skills* *Student thinking*	Mathematical Concepts			
	Demonstrates a solid understanding of mathematical concepts and their relationship to procedures; tasks and prompts help student to link procedures and concepts.	Demonstrates sufficient understanding of mathematical concepts and their relationships to procedures but exhibits some gaps in knowledge; tasks and prompts help students to connect procedures to concepts but PST overlooks an aspect of the procedure or concept.	Demonstrates limited understanding of mathematical concepts and struggles to connect procedures and concepts; tasks and prompts focus on answers.	Demonstrates significant gaps and misunderstandings in mathematical knowledge; procedures and concepts are misunderstood or inappropriately linked.
	Significance			
	Incorporates mathematical and/or real-world scenarios that aptly illuminate mathematical concepts and naturally lend themselves to modeling.	Incorporates mathematical and/or real-world scenarios that are relevant and lend themselves to modeling but are not fully developed.	Incorporates mathematical and/or real-world scenarios that are tangential or contrived, or that are disconnected from the rest of the lesson.	Incorporates mathematical and/or real-world scenarios that are inappropriate, or does not provide a purpose for the mathematical activity.
	Prerequisite Knowledge and Skills			
	Invokes key background knowledge and skills in lesson plans, providing a solid foundation for building new knowledge and skills.	Invokes relevant background knowledge in lesson plans but overlooks a relevant concept or skill, or incorporation of prior knowledge or skills requires further development to provide a strong foundation for building new knowledge and skills.	Identifies relevant background knowledge but does not attempt to use it as a foundation to build new knowledge and skills; or identifies and incorporates some relevant knowledge but overlooks a fundamental concept.	Does not identify or incorporate relevant background knowledge or skills, or incorporation of knowledge or skills is inappropriate.
	Student Thinking			
	Anticipates an appropriate range of potential solution strategies and responses and anticipates potential points of confusion.	Anticipates potential solution strategies and responses but overlooks a key strategy or response; may not anticipate potential points of confusion.	Expects a single specific solution strategy and response; does not anticipate potential points of struggle.	Does not indicate expected or anticipated solution strategies or responses; does not anticipate potential points of confusion.

Table 1 Continued

Setting Instructional Outcomes and Assessment (Danielson 1c/e/f, 3c-d; AMTE C.2.2) *Alignment between learning objectives and learning standards* *Alignments between learning objectives and assessments* *Level of cognitive demand of objectives and assessments* *Developmental progression and pacing*	Alignment Between Learning Objectives and Learning Standards			
	Learning objectives align with an appropriate grade-level standard in a clear, specific, and measurable way.	Learning objectives align with an appropriate grade-level standard in a way that is either clear, specific, or measurable, but not all three; or learning objectives align with a standard that is similar to the standard indicated in the lesson in a clear, specific, and measurable way but the actual standard that the lesson addresses is a different standard than the one indicated in the lesson.	Learning objectives appear to align with an appropriate grade-level standard but are not fully clear, specific, or measurable.	Learning objectives do not align with an appropriate grade-level standard or are too unclear to decipher.
	Alignment Between Learning Objectives and Assessments			
	Objectives and assessments align in terms of both content and process and are supported by the lesson's main activities; examples of satisfactory student work and explanations that would meet the objectives are provided.	Objectives and assessments are mostly aligned in terms of both content and process and are supported by the lesson's main activities but may require further development and/or examples of satisfactory student work and explanations that would meet the objectives are not provided.	Objectives and assessments align in terms of content or process but not both, or objectives and assessments align but are not supported by the main activities in the lesson.	Objectives and assessments are unaligned and/or inappropriate, or alignment cannot be discerned due to lack of clarity.
	Level of Cognitive Demand of Objectives and Assessments			
	Objectives and assessments reflect a high level of cognitive demand and are designed to elicit information about both conceptual and procedural understanding; opportunities are provided for student self-evaluation.	Objectives and assessments reflect a high level of cognitive demand and are designed to elicit information about both conceptual and procedural understanding but are not fully developed and/or opportunities are not provided for student self-evaluation.	Objectives and assessments reflect a low level of cognitive demand and primarily emphasize procedural understanding.	Objectives and assessments emphasize memorization only.
	Developmental Progression and Pacing			
	Learning objectives reflect appropriate attention to the development progression of a concept, supporting procedural fluency by first developing conceptual understanding.	Learning objectives reflect a slightly rushed or prolonged developmental progression of a concept but afford some opportunity to develop conceptual understanding.	Learning objectives proceed far too quickly or slowly through the developmental progression of a concept, rushing or overlooking concepts to focus on procedures.	Learning objectives do not follow a logical developmental progression.
Demonstrating Knowledge of Resources (Danielson 1d, 3c; AMTE C.1.4/6, C.2.3) *Suitability of resources* *Use of resources*	Suitability of Resources			
	Selected resources are optimally suited to the mathematical content intended to be conveyed in a lesson or unit.	Selected resources are suited to the mathematical content intended to be conveyed in a lesson or unit but a particularly salient resource has been overlooked.	Selected resources may be used for the mathematical content intended to be conveyed in a lesson or unit but other resources would be better suited to the content and/or selected resources are unnecessarily limited.	Selected resources are ill suited to the mathematical content intended to be conveyed in a lesson or unit.
	Use of Resources			
	Resources are utilized to support student exploration and sense making and to illuminate important mathematical connections; students are given choice in how resources are utilized whenever appropriate.	Resources are utilized to support student exploration and sense making and to illuminate important mathematical connections but efforts require further development to be fully effective.	Resources are utilized in a prescriptive or non-mathematical manner with little or no opportunity for exploration, sense making, or illuminating important mathematical connections.	Resources are utilized incorrectly or inappropriately, or manner of use cannot be determined from the lesson.

Table 1 Continued

Instructional Strategies (Danielson 1e, 3b-d; AMTE C.1.4, C.2.2-3) *Coherence* *Nature of tasks* *Nature of prompts*	Coherence			
	Lessons and units feature a well-defined, unifying theme and a logical flow, with opportunities for reflection and closure.	Lessons and units feature a unifying theme, a mostly logical flow, and opportunities for reflection and closure, but one or more of these elements requires further development.	Lessons and units feature a mostly logical flow but include abrupt transitions, do not include a discernible theme, and/or opportunities for reflection and closure are absent.	Lessons and units feature an inappropriate theme and/or the flow of the lesson or unit is not logical.
	Nature of Tasks			
	Tasks position students as the primary doers and feature a high level of cognitive demand, permitting non-algorithmic thinking and multiple solution strategies and fostering conceptual as well as procedural understanding.	Tasks foster conceptual as well as procedural understanding but are teacher directed rather than student driven, or tasks feature the characteristics of proficiency but require further development to be fully effective.	Tasks feature student engagement but the level of cognitive demand is low, emphasizing prescribed routines or procedures without meaningful connections to concepts.	Tasks are entirely teacher led with no opportunity for active student engagement, and/or tasks emphasize memorization only.
	Nature of Prompts			
	Prompts are designed to elicit information about conceptual and procedural understanding and to encourage reflection, justification, and connections.	Prompts are designed to elicit information about conceptual and procedural understanding but do not encourage reflection, justification, and/or connections, or prompts incorporate the features of proficiency but more such prompts are needed for the lesson to be fully effective.	Prompts emphasize answer and procedures only.	Prompts are inappropriate or absent.
Advocacy and Equity (Danielson 1e-f; AMTE C.2.1, C.4.1-4) *Differentiation* *Perspectives*	Differentiation			
	Lessons and units feature multiple entry points and specific supports for struggling students as well as appropriate extensions for student ready for a challenge.	Lesson and units feature either multiple entry points and specific supports for struggling students or appropriate extensions for students ready for a challenge but not both, or lessons and units incorporate the features of proficiency but require further development to be fully effective.	Lessons and units suggest plans for differentiation, but plans are vague or are not linked to the concept under study.	Lessons and units do not incorporate plans for differentiation, or plans for differentiation are inappropriate or alter the level of cognitive demand for different learners.
	Perspectives			
	Incorporates multiple perspectives and contexts into lesson and unit plans, including those of non-dominant and historically marginalized groups, in a way that challenges stereotypes and predominant paradigms.	Incorporates multiple perspectives and contexts into lesson and unit plans, including those of non-dominant and historically marginalized groups, but these require further development to effectively challenge stereotypes and predominant paradigms.	Lesson appears to be framed from the perspective of the dominant group without due diligence to verify whether commonly held assumptions are shared by other groups.	Incorporates inaccurate, offensive, or disrespectful information into lessons and unit plans such that negative stereotypes are reinforced.

Once I delineated the criteria for each component at each proficiency level, I was ready to begin analyzing my data. Before describing my data analysis procedures, I first provide background on the study context and participants in the following section.

Context and Participants

As stated in the chapter overview, I utilized a case study design for my research, which enabled me to take a deep dive into the lesson plans submitted by a select cohort of my PSTs. I chose to analyze the lesson plans from the students in my Fall 2019 section of K-8 math methods, as Fall 2019 was the second semester during which I had implemented standards-based learning in my course. After my pilot in Spring 2019, I conducted some informal analysis of my students' first and final drafts of assignments and developed a desire to know more specifically in what ways my PSTs were learning (and not learning) from the feedback and revision cycles I had embedded into the course. Hence, in Fall 2019, I obtained permission from my university's institutional review board and from my PSTs to analyze their work and conduct follow-up interviews with four of them who would be moving on to student teaching during the subsequent semester.

Context

The study analyzed data from lesson plans submitted as part of my K-8 math methods course, which PSTs take in their third or fourth year of their educator preparation programs. The prerequisites for the course are successful completion (earning a C or higher) in two math content courses taken during the freshman and/or sophomore years. The course includes a review of mathematical manipulatives introduced in the content courses as well as extensive instructor modeling of *Standards*-based mathematics instruction in which PSTs play the role of K-8 students. It also includes readings, videos, class discussions, and written reflections designed to further illuminate the features of *Standards*-based instruction and to provide a research base for its tenets. The primary textbook from the course is Jo Boaler's *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching* (2016). Readings are also drawn from *Making Sense: Teaching and Learning Mathematics with Understanding* (Hiebert et al., 1997).

Before writing independent lesson plans during the final third of the semester, PSTs work in groups of three to four to design lessons for students in real classrooms at a local elementary school, where they teach two separate lessons on topics assigned by the cooperating teachers. The field experiences provide PSTs with some insight into student thinking and learning trajectories, which helps to inform their lesson plans later in the semester.

Prior to Spring 2019, when I utilized traditional assessment in the course, the unit plan served as the culminating project for my PSTs and was handed in during final exam week; hence, only limited feedback was offered to PSTs as they worked on their unit plans during the final week of class. Other assignments received detailed feedback but were not expected to be resubmitted unless PSTs chose to do so to improve their scores (few ever did). During Fall 2019, after implementing SBL, the due dates of the two major independent lesson planning assignments in the course—an Indian Education for All (IEFA)[2] lesson plan and the unit plan—were moved up such that detailed feedback could be provided on both assignments enough in advance of finals week that they could be revised and resubmitted by PSTs in the form of a final portfolio along with a reflective essay summarizing key course concepts.

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[2] The state in which my institution is located requires that all teachers in public schools incorporate instruction on the historical and contemporary experiences and contributions of the state's Indigenous groups; the legislation mandating this instruction is referred to as Indian Education for All, or IEFA.

The rubric I used to evaluate student work during Fall 2019—the semester during which I gathered data—was a more limited version of the rubric shown in Table 1, which was the rubric utilized for this study. For a copy of my original rubric, which represented my first attempt at delineating standards for my course, see Appendix A. I shared this rubric with my PSTs on the first day of class and had them self-evaluate against the rubric at various points during the semester. I also provided feedback on PSTs' assignment drafts using a digital version of the rubric embedded in the online course shell. Hereafter I refer to it as my "original" or "initial" rubric.

Since I was unaware of ATME's *Standards* (2017) when I created my initial rubric, I drew from Danielson (2007) and NCTM's *Principles and Standards for School Mathematics* (2000). The rubric, though helpful as an initial tool for evaluating student work against a specific set of criteria that could be shared with my PSTS, omitted some crucial criteria for effective lesson planning, such as providing opportunities for reflection and closure on the key concepts in a lesson. It also did not emphasize equity as clearly as in the *Standards* and in my updated rubric used for the study (see Table 1).

Participants

The participants were 21 pre-service teachers enrolled in a single section of the K-8 math methods course that I teach at a small, urban, public university in the northwestern United States. Most students in the course were elementary teaching majors in their junior or senior years of college; two of the students were graduate students; only one of the students was male.

As related in my positionality statement, the pre-service teachers in my K-8 methods course often harbor negative attitudes about mathematics and mathematics instruction, and this cohort was no different. As one of the PSTs noted in a written reflection completed early in the semester, “I was one of those kids that felt I wasn't good at math.” Another wrote, “I always dreaded going to math class in high school because I had a fixed mindset that I wouldn’t do good in class.” However, as indicated by other comments in their reflections, my PSTs were eager to learn how they could approach math differently in their future classrooms so that their students would enjoy more positive experiences. For example, one PST commented that she was “excited to learn there are strategies and efforts in place to curb this trend.” Another stated, “It makes so much more sense to me why we are trying to change math teaching, because obviously something needs to change so that students aren’t traumatized.”

About half of the students in the course had taken one or both of their mathematics content courses with me and thus had already been exposed to *Standards*-based instruction prior to taking K-8 math methods; the other half had taken those courses with one or more adjunct instructors who demonstrate varying degrees of proficiency in modeling the *Standards*. Regardless of their prior exposure, as noted above, most of the teacher candidates in the course quickly embraced the tenets of *Standards*-based instruction even though they initially struggled to translate those tenets into appropriate lesson plans for K-8 students.

In addition to analyzing all of PSTs’ learning outcomes with respect to their lesson planning assignments in the course, I also conducted observations and interviews with four of the PSTs as they student taught the following semester. All four of these PSTs were female (the only male student in the Fall 2019 section of my methods course was not student teaching during

Spring 2020). Three of the PSTs were White and one of them was biracial, having both White and Indigenous ancestry. Two of the PSTs earned the same grade in the course (an A), demonstrating a mix of *nearing proficient* and *proficient* performance, despite having very different prior experiences with mathematics. One of them, who identified math as her weakest subject, took both of the math content courses with me; the other PST—the only one of the four who identified math as her strongest subject—took neither of those courses with me and described the content courses she had taken as very procedural in nature. Another PST, who earned a B+ by demonstrating mostly *nearing proficient* performance in the course, is a special education double major; she regularly expressed frustration that her special education coursework seemed to indicate a much different approach to math instruction than I was teaching her and that while she was compelled by the research base supporting *Standards*-based math instruction, she felt nervous about trying to teach this way because she had never seen it modeled in an actual K-8 classroom. The fourth PST earned a B- in the course, demonstrating mostly *basic* and *nearing proficient* performance, and expressed much greater confidence in math instruction than her actual performance on assignments would have suggested. I purposefully invited these four PSTs to participate from among seven who student taught during Spring 2020 because their experiences with mathematics reflected a range of attitudes and approaches to mathematics, from lack of confidence to confident and from purely procedural to more open ended.

Data Sources

To answer my research questions, I saved for analysis all the first and final drafts submitted by PSTs for the two independent lesson planning assignments in the course, the IEFA lesson plan and the unit plan. I also observed and interviewed four of the participants who student taught during the subsequent semester. Additionally, I kept a detailed log of researcher memos and also recorded annotations on the codes I assigned during data analysis. Each of these data sources informed my findings and is described in more detail below.

Lesson Planning Assignments

Although effective lesson planning does not necessarily equate to effective *Standards*-based teaching, as Morris and Hiebert (2017) assert, "the evidence suggests that how teachers plan lessons is related to how they teach lessons" (p. 536). Hence, lesson plans were chosen as a measure of my PSTs performance with respect to the *Standards* given my inability to individually evaluate their teaching during the brief field experiences incorporated into my methods course.

I chose to analyze both the IEFA lesson and the unit plan because together they enabled me to evaluate the full breadth of skills and knowledge incorporated into my AMTE-Danielson rubric (see Table 1). The IEFA lesson plan incorporated aspects of the *advocacy and equity* domain that were not directly incorporated into the unit plan, whereas the unit plan incorporated aspects of *setting instructional outcomes and assessment* that are not adequately captured in a stand-alone IEFA lesson. Furthermore, the IEFA lessons captured a broader range of grade levels and skills than the unit plans, with more PSTs attempting higher-level mathematics in their IEFA lessons than in their unit plans.

A copy of my unit plan instructions and of my lesson plan template are included in Appendix C. The unit plan comprised three lesson plans designed to introduce a Common Core standard to K-8 students; PSTs could select a standard and grade level of their choice. The lesson plan template was utilized for both the unit plan as well as the individual IEFA lesson. To support my students in envisioning the features of an effective IEFA lesson, I modeled an example lesson for them during class prior to discussing the assignment. In my modeling, I emphasized the importance of making the lesson both culturally and mathematically rich by sharing historical information about the tribe I had chosen to feature; by engaging my PSTs in a "Notice and Wonder" (NCTM, n.d.) about a video of a traditional game being played by contemporary members of the tribe; and having my PSTs engage in a simulation of the game while discussing the mathematics involved in the possible outcomes. I was careful to stress with my students that simply performing mathematics on an Indigenous artifact is not sufficient to satisfy the *Essential Understandings Regarding Montana Indians* (Montana Office of Public Instruction, 2019), which include criteria such as showcasing the diversity among tribes and identifying the unique contributions of individual tribes to the state's history, culture, economy, etc. For their own lessons, PSTs were instructed to choose a tribe or reservation of interest from the list of state-recognized tribes and tribal territories, conduct independent research on the tribe or reservation, and generate a math lesson linking the information they found to a relevant K-8 Common Core math standard as well as one or more of the "Essential Understandings," the state-level standards established for IEFA instruction.

<u>Observations and Interviews</u>

In order to learn more about the impact of standards-based learning on my PSTs, I also conducted observations and interviews with four of the PSTs who student taught during the subsequent semester. For each of the four PSTs, I conducted one teaching observation as well as a separate 30-minute interview. The interviews were conducted via WebEx, my university's video conferencing platform, since they occurred during Spring 2020 after all schools in my state closed due to the COVID-19 pandemic (I was able to conduct the teaching observations before the PSTs' student teaching was cut short by the pandemic). All teaching observations and interviews were audio recorded and each interview was transcribed in full. The teaching observations were not transcribed since they served primarily to provide context for the interviews, during which PSTs regularly referred to their cooperating teachers and/or student teaching experiences. Because my PSTs were all in classrooms in which traditional instruction was the norm, and because one of my PSTs was actively discouraged by her cooperating teacher from utilizing *Standards*-based instruction during her lessons, I did not evaluate my PSTs' student teaching performance against my AMTE-Danielson rubric.

Each one-on-one interview was semi-structured, guided by the following questions: What word comes to mind when you think about teaching math? How would you describe your relationship with mathematics and with math teaching? What kind of math teacher do you hope to be? How confident do you feel in your ability to achieve that vision? How do you feel the assignments and feedback in EDU 397C impacted you as an emerging educator? The questions were deliberately open ended to encourage sincere reflections on my PSTs' sense of achievement with respect to the *Standards*. They were also conducted well after final grades for the methods course had posted so that the participants would not worry that their responses might impact their grade in my course.

Researcher Memos and Annotations

When I began forming my ideas for this study, I recorded my developing thoughts in a Word document that I extended throughout my research process. I labeled each entry by date as well as by the aspect of the study the entry pertained to (for example, literature review, data analysis, findings). The memos enabled me to reflect on and adjust my research process as necessary and to keep track of emerging themes in my PSTs' work. I also recorded annotations on the codes I assigned during my first cycle of coding, described in the subsequent section, in order to document my rationales for assigning each code. My annotations informed revisions to my rubric during the early stages of my study and also provided data for my second cycle of coding.

Data Analysis

To evaluate my data, I engaged in multiple cycles of coding, each of which is described in turn below.

First Cycle Coding

I began by uploading all my PSTs' IEFA lessons and unit plans (both first and final drafts) to NVivo, in which I established an a priori coding structure based on my AMTE and

Danielson-aligned rubric (see Table 1). To minimize potential bias in my analysis of the assignments, I first replaced all PSTs' names with numbers utilizing a random number generator such that the "teacher" name on each lesson plan became Student 1 or Student 2 rather than Samantha or Natasha (I use fictitious names here to protect the anonymity of my students). While it was impossible for me to forget the authors of several assignments that had stood out to me during Fall 2019, I was able to disassociate most of the assignments from their authors during data analysis.

Each code in the a priori scheme I utilized represented a particular proficiency level within a particular component of a particular domain in the rubric, for example *basic* performance within the component of *mathematical concepts* within the domain of *Demonstrating Knowledge of Content*. I used the drag and drop feature in NVivo to code each lesson such as by highlighting a lesson objective and dragging it to the appropriate proficiency level within *Alignment Between Learning Objectives and Learning Standards* or highlighting a task description and dragging it to the appropriate proficiency level within *nature of tasks*. As I coded each lesson planning assignment, I recorded annotations to document my rationales for each code. For example, on a basic code in the component of *prerequisite knowledge and skills*, I commented, "Identifies knowledge of skip counting and addition but does not invoke in lessons."

Using my rubric and the a priori codes as a guide, I recorded at least one code (proficiency level) for each component in the rubric. In most cases, the objectives, tasks, prompts, etcetera, across an entire lesson or unit were similar in nature, enabling me to select single passages of text to serve as representatives for the PSTs' overall performance in each component. For example, rather than code every single prompt in a lesson in which most or all prompts were basic in nature, I selected a representative prompt and coded it to the basic proficiency level within *nature of prompts*. In some cases, the nature of a particular component varied enough across a lesson or unit plan that I selected two or three representative portions of text for the assignment. For example, in some of the unit plans, the nature of tasks in one or two of the lessons were nearing proficient but in another one or two of the lessons they were basic and/or unsatisfactory; hence the tasks in one lesson were coded to *nearing proficient* in *nature of tasks* but the tasks in another lesson were coded to *basic* and/or *unsatisfactory*. However, every single task in a lesson or unit plan was not coded, as this would have made the codes difficult to interpret. For instance, if 63 *basic* codes were recorded under *nature of tasks*, it could indicate that every PST included a few basic tasks in their lessons and units, or it could mean that a few PSTs included a lot of basic tasks in their lessons and units. Hence, only one task that was representative of all other tasks at the same proficiency level was coded.

I began by coding the unit plan first drafts since these would include a greater number of instances of each rubric component (since there were three lessons in each unit plan) and hence give me greater insight into the ways in which my PSTs' work would align or not align with my rubric criteria. As I coded the drafts, I noticed that I sometimes struggled to determine which code to assign due to a lack of clarity between proficiency levels, which would have made it difficult for another MTE to replicate my study and might have allowed bias to creep into my findings. For example, if I struggled to decide between basic and nearing proficient, I might choose nearing proficient for a student I knew to be stronger in mathematics and basic for a student I knew to be weaker in mathematics rather than making my decision based purely on the evidence in front of me. Hence, I began recording notes in my research memos on aspects of the rubric that required clarification and/or elaboration so that I could accurately assess my PSTs' work with the rubric. For example, on May 11, 2021, I wrote,

> I realized that the category "Real-World Significance" does not capture lessons that may not include any real-world significance at all, which is true of some of the lessons; I also realize, as I'm reading *Mathematician's Lament* [Lockhart, 2009], that a good math lesson does not need to include real-world significance: it can simply be an exploration of a mathematical phenomenon. However, there should indeed be a *purpose* for engaging with the mathematics, whether it be to understand how mathematics can be useful in the real world or to understand underlying mathematical principles.

To be sure, the *Standards* do not require that every single math problem be embedded in a real-world context. This realization prompted me to adjust both my original criteria for the component of *significance* as well as its name, which was originally titled *real-world significance*. Instead of requiring that lessons "incorporate interdisciplinary and/or real-world scenarios that aptly illuminate mathematical concepts and naturally lend themselves to modeling," I adjusted the criteria to "incorporates mathematical or real-world scenarios that aptly illuminate concepts and naturally lend themselves to modeling."

After coding all the unit plan first drafts, I revised my rubric in accordance with my memos and developed a codebook to illuminate each proficiency level of each component (see Appendix B). The codebook includes a column that aligns each component of the rubric with specific outcomes from the *Standards* and the Danielson Framework; a column that describes the forms of "access" to master knowledge and skills that was provided to PSTs with respect to each component (e.g., specific readings or activities); and two columns providing examples for each code adapted from my PSTs' actual lesson plans. For each component, I selected a specific example directly from a PST's lesson, which reflected a particular proficiency level; to illustrate each of the other three proficiency levels, I described in italics ways in which the example would have had to be different to meet the criteria for each of the other three levels. This level of detail enabled me to evaluate my PSTs' lesson plans with greater accuracy and consistency, thereby reducing the potential for bias to influence my analysis.

After revising my rubric and developing the associated codebook, I started my coding process again from the beginning, recoding the unit plan first drafts, then coding the unit plan final drafts and finally coding the IEFA lesson plan first and final drafts according to my refined delineations. The rubric included in Table 1 is the final, refined version of the rubric that aligns with the codebook and that was utilized to complete this first cycle of coding.

Second Cycle Coding

After coding my PSTs' IEFA lessons and unit plans to gain a sense of their performance with respect to the *Standards*, I engaged in a second cycle of coding to search for emergent themes in their work. My findings from the first cycle of coding informed this second cycle of coding by raising a key question in my mind: Why did so many of my PSTs achieve *nearing proficient* performance but not *proficient* performance on final drafts of their lesson planning assignments even after engaging in feedback and revision cycles? What seem to be hurdles for them in achieving proficient performance with respect to the *Standards*?

Since the *nearing proficient* criteria in my rubric were the most vague of the three proficiency levels—primarily referring to features of proficient performance that "require further development"—I used my second cycle of coding to explore the reasons why my PSTs' lessons required further development. What was missing or not fully adequate about their lesson plans? To answer this question, I read through the annotations I had recorded on *nearing proficient* codes for the final drafts of each assignment during my first cycle of coding. As I did so, I jotted down by hand the reasons I had indicated in my annotations for not assigning full proficiency. For example, in an annotation on a unit plan introducing the concept of perimeter, I wrote, “More thought needs to be put into second lesson task with composite shapes to avoid confusion between internal sides and external perimeter.” This comment prompted me to jot down “concepts not clearly distinguished from related concepts” as an emergent theme. On another unit plan I noted, “Moves to expanded form by LP3 [lesson plan 3], which is a bit rushed considering LP1 is a first introduction to the concept of 100,” prompting me to add “concept development rushed” as an emergent theme. On an IEFA lesson I recorded, “A couple of possible procedures are mentioned for doing conversions and the concept of proportional reasoning is invoked, which undergirds unit conversions, but the procedures and concept aren’t clearly linked in an explicit way,” which links to the theme “connections between procedures and concepts not adequately addressed.”

As I read through my annotations, I utilized check marks to indicate themes that appeared repeatedly across the annotations. Once I developed a hand-written list of reasons why I felt my PSTs were *nearing proficient* as opposed to *proficient*, I clarified the language in these themes and created a new coding scheme in NVivo to formally code all my *nearing proficient* annotations, enabling me to generate a tally for each of the emergent themes. I then organized these themes into three overarching themes, as described in Chapter 4, where I report my findings.

Interview Data

After completing my first and second cycles of coding and organizing my emergent themes, I revisited the transcripts from my interviews with the four PSTs who student taught during the semester following their enrollment in my K-8 math methods course. In rereading the transcripts, I searched for instances of the emergent themes within the data, highlighting passages of text that related to the themes. I did not systematically code the interview data, using it instead as a form of triangulation to illuminate the findings from my lesson plan analysis. For instance, during their interviews several of the PSTs expressed confidence in teaching math at lower grade levels but still felt uneasy about teaching upper-elementary math topics, bolstering one of the overarching themes identified in the lesson planning data (lack of mathematical content knowledge).

Methods for Ensuring Credibility, Transferability, and Confirmability

In their 2009 article, Berk and Hiebert lament the fact that "most [teacher preparation] programs, perhaps even some highly effective ones, have not captured the local knowledge they are acquiring so it can be shared with others" (pp. 337-338). In response to their call for more shareable and systematic approaches to research on the outcomes associated with mathematics teacher preparation, I have designed this study with credibility, transferability, and confirmability in mind. Most importantly, I utilized a clearly delineated rubric aligned to two widely accepted sets of standards for pre-service and in-service educators. The rubric utilizes a four-point scale, determined by researchers to be a point range that maximizes interrater reliability (Guskey, 2015). A codebook was developed to illustrate the use of the rubric using specific examples from students’ lesson plans as well as to demonstrate the rubric’s alignment to the two sets of standards. Multiple data sources were utilized for triangulation, and member checks were conducted with the student teachers to verify interpretations of the interview data. In the following chapter, my findings are described in detail such that the reader should feel confident in the conclusions and areas for future research drawn from the data.

CHAPTER FOUR

FINDINGS

Overview

In this chapter I present my research findings. In the first section, I share the outcomes from my first cycle of coding, in which I coded the data according to the proficiency levels delineated in the AMTE-Danielson rubric (see Table 1) and codebook (see Appendix B) to answer my first two research questions: In what ways does the opportunity to receive timely and specific feedback and to revise lesson plans accordingly help PSTs to better develop and apply their knowledge and skills with respect to AMTE's 2017 *Standards*? In what ways are PSTs meeting and not meeting the *Standards* in their lesson planning assignments? These findings are organized based on the rubric components, with components in which notable improvement was demonstrated being described first (areas exhibiting notable growth), followed by components in which performance stayed roughly the same (areas exhibiting little growth), and lastly by components in which performance declined (areas exhibiting decline in performance).

In the second section, I describe the themes that emerged from my second cycle of coding, in which I analyzed my first-cycle coding notes on all lesson plans coded as nearing proficient to identify the reasons why so many lesson plans were coded as nearing proficient rather than proficient. These findings answer my second research question: What themes emerge from PSTs' work on lesson planning for K-8 mathematics? To further illuminate these findings, I conclude the chapter by sharing data from interviews with the four participants who student taught during the subsequent semester.

First Cycle Coding Results

Overview

Table 2 provides a summary of coding outcomes from the first cycle of coding, during which the first and final drafts of two different lesson-planning assignments (the Indian Education for All lesson plan and the unit plan) were coded using the Danielson-AMTE rubric and codebook. The table conveys PSTs' strengths and weaknesses with respect to the various domains and components incorporated into the rubric.

Table 2. Tally of Coded References for Drafts and Final Submissions of Two Lesson Planning Assignments

		IEFA draft	IEFA final	UP draft	UP final
Demonstrating Knowledge of Content	Mathematical Concepts				
	Proficient	1	1	4	4
	Nearing proficient	8	11	12	13
	Basic	7	7	4	3
	Unsatisfactory	5	3	1	2
	Significance				
	Proficient	3	4	6	8
	Nearing proficient	11	10	13	12
	Basic	6	6	3	2
	Unsatisfactory	1	1	6	4
	Prerequisite Knowledge and Skills				
	Proficient	0	0	4	5
	Nearing proficient	2	5	4	3
	Basic	12	13	11	11
	Unsatisfactory	7	3	3	2

Table 2 Continued

Demonstrating Knowledge of Content	Student Thinking				
	Proficient	1	2	4	4
	Nearing proficient	5	8	5	10
	Basic	4	6	2	2
	Unsatisfactory	11	5	10	5
Setting Instructional Outcomes and Assessment	Alignment Between Learning Objectives and Learning Standards				
	Proficient	0	5	12	15
	Nearing proficient	6	7	9	9
	Basic	6	4	0	0
	Unsatisfactory	9	5	3	3
	Alignment Between Learning Objectives and Assessments				
	Proficient	1	7	7	7
	Nearing proficient	15	9	13	13
	Basic	3	2	1	1
	Unsatisfactory	5	4	3	1
	Level of Cognitive Demand of Objectives and Assessments				
	Proficient	1	2	5	6
	Nearing proficient	10	13	6	6
	Basic	10	7	12	11
	Unsatisfactory	1	0	0	0
	Developmental Progression and Pacing				
	Proficient	2	4	5	6
	Nearing proficient	4	5	9	11
	Basic	12	11	5	3
	Unsatisfactory	3	1	2	1

Table 2 Continued

Demonstrating Knowledge of Resources	Suitability of Resources				
	Proficient	0	3	7	9
	Nearing proficient	13	13	6	8
	Basic	8	5	9	4
	Unsatisfactory	0	0	3	1
	Use of Resources				
	Proficient	1	1	5	6
	Nearing proficient	6	9	11	10
	Basic	10	9	6	6
	Unsatisfactory	4	2	2	2
Instructional Strategies	Coherence				
	Proficient	1	3	3	6
	Nearing proficient	15	13	12	9
	Basic	5	5	4	4
	Unsatisfactory	1	0	2	2
	Nature of Tasks				
	Proficient	3	3	5	7
	Nearing proficient	8	8	12	10
	Basic	11	10	6	5
	Unsatisfactory	0	0	1	1
	Nature of Prompts				
	Proficient	0	1	4	5
	Nearing proficient	4	8	14	13
	Basic	6	10	4	3
	Unsatisfactory	11	2	2	1

Table 2 Continued

Advocacy and Equity	Differentiation				
	Proficient	2	3	3	4
	Nearing proficient	6	11	6	7
	Basic	7	4	12	10
	Unsatisfactory	6	3	5	3
	Perspectives				
	Proficient	0	1	0	0
	Nearing proficient	9	9	1	0
	Basic	10	9	20	21
	Unsatisfactory	3	3	0	0

For most components in the rubric there are a total of 21 tallies—one for each participant—for each draft of each assignment. In some cases, a student's work could not be adequately captured by a single code, leading to greater than 21 tallies for certain components. For example, in a few of the unit plans, lesson objectives were split close to evenly between two different proficiency levels for *alignment between learning objectives and assessments.* In one of the IEFA lessons, the PST incorporated a theme with great potential to challenge a dominant paradigm, but she perpetuated another dominant paradigm (that all Native American groups are essentially the same) by repeatedly using the generic term "Native American" throughout her lesson; hence her lesson was coded at both the nearing proficient and basic levels of performance under the category of *perspectives*. For most drafts, however, the bulk of the work met the criteria for one specific code.

Areas Exhibiting Notable Growth

For seven of the rubric components, PSTs made notable improvements in performance between drafts of each assignment and/or between the first independent lesson planning assignment (the IEFA lesson plan) and the final independent lesson planning assignment (unit plan). I determined that improvement was “notable” if Table 2 showed that the proficiency level at which the most assignments were coded increased by at least one level between drafts or between assignments. For example, notable improvement was demonstrated in *student thinking* because most assignments were coded as unsatisfactory on both the IEFA and unit plan first drafts, whereas most were coded as nearing proficient on both the IEFA and unit plan final drafts. There was also notable improvement in *developmental progression and pacing* because most assignments were coded as basic on both IEFA drafts (first and final) but as nearing proficient on both unit plan drafts. When referring to this type of between-assignment growth in the sections that follow, I simply note that growth was observed between the IEFA lesson and the unit plan, meaning that both drafts (first and final) of the unit plan demonstrated greater proficiency than both drafts of the IEFA lesson.

Table 3 shows the components in which notable growth was demonstrated, including *student thinking*, *alignment between learning objectives and learning standards*, *developmental progression and pacing*, *suitability of resources*, *use of resources*, *nature of tasks*, and *nature of prompts*. It is a reproduction of the salient components from Table 2 for easy reference. In the sections below, I expound upon each of these components in turn.

Table 3. Tally of Coded References for Drafts and Final Submissions of Assignments Exhibiting Notable Growth

		IEFA draft	**IEFA final**	**UP draft**	**UP final**
Demonstrating Knowledge of Content	Student Thinking				
	Proficient	1	2	4	4
	Nearing proficient	5	8	5	10
	Basic	4	6	2	2
	Unsatisfactory	11	5	10	5
Setting Instructional Outcomes and Assessment	Alignment Between Learning Objectives and Learning Standards				
	Proficient	0	5	12	15
	Nearing proficient	6	7	9	9
	Basic	6	4	0	0
	Unsatisfactory	9	5	3	3
	Developmental Progression and Pacing				
	Proficient	2	4	5	6
	Nearing proficient	4	5	9	11
	Basic	12	11	5	3
	Unsatisfactory	3	1	2	1
Demonstrating Knowledge of Resources	Suitability of Resources				
	Proficient	0	3	7	9
	Nearing proficient	13	13	6	8
	Basic	8	5	9	4
	Unsatisfactory	0	0	3	1

Table 3 Continued

	Use of Resources				
Demonstrating Knowledge of Resources	Proficient	1	1	5	6
	Nearing proficient	6	9	11	10
	Basic	10	9	6	6
	Unsatisfactory	4	2	2	2
Instructional Strategies	Nature of Tasks				
	Proficient	3	3	5	7
	Nearing proficient	8	8	12	10
	Basic	11	10	6	5
	Unsatisfactory	0	0	1	1
	Nature of Prompts				
	Proficient	0	1	4	5
	Nearing proficient	4	8	14	13
	Basic	6	10	4	3
	Unsatisfactory	11	2	2	1

<u>Student Thinking.</u> PSTs made notable improvements in the category of *student thinking* between first drafts and final drafts of each assignment as shown in Table 2. This stands in contrast to the other components in the domain *demonstrating knowledge of content*, for which PSTs showed little growth. Much of the growth in *student thinking*—for which a PST must demonstrate an ability to anticipate an appropriate range of potential solution strategies as well as potential points of confusion—was due to the fact that PSTs simply did not including any references to student thinking within their first drafts of their lessons; once PSTs were reminded

to include anticipated solution strategies and potential points of confusion within their lesson plans, performance improved from unsatisfactory to either basic or nearing proficient performance, with more final drafts being coded as nearing proficient than any other proficiency level.

Alignment Between Learning Objectives and Learning Standards. PSTs also demonstrated noticeable improvement in *alignment between learning objectives and learning standards*—the only rubric component on which a majority of PSTs demonstrated proficient performance by the end of the semester—between their first and final drafts of their IEFA lessons as well as between the IEFA lesson and the unit plan, with most PSTs achieving proficient performance on both of their unit plan drafts but lower levels of proficiency on their IEFA lesson drafts. There was almost no improvement between the two drafts of the unit plan, but since most of the first drafts were already proficient or nearing proficient, there was less room for improvement than on the IEFA drafts.

To be proficient in this component, PSTs must establish learning objectives that align with an appropriate grade-level standard in a clear, specific, and measurable way. For the IEFA lessons, it appeared that many PSTs came up with a lesson idea and then looked for a relevant standard to link it to rather than building a lesson up from a chosen standard, as most appeared to do for the unit plan; this resulted in a lack of alignment between the standard and the lesson objectives for many of the lessons. For example, one PST found information about the meaning of circles to various Indigenous tribes and linked it to a fourth-grade standard about symmetry even though symmetry was only mentioned briefly within the lesson and not mentioned at all within the stated objectives. Another PST came up with an idea for measuring a Chippewa canoe and converting the measurements between inches and centimeters, pegging it to a fourth-grade standard on knowing the relative sizes of measurements *within* a given measurement system (such as inches to feet) rather than *between* measurement systems. Once PSTs were made aware of the lack of alignment between their chosen standards and objectives, many adjusted either their standards, their objectives, or both to create better alignment.

Developmental Progression and Pacing. PSTs demonstrated improvement on this component between their IEFA lessons and their unit plans, with most IEFA lesson drafts (both first and final) being coded as basic or unsatisfactory but most unit plan drafts being coded as nearing proficient or proficient. However, there was minimal improvement between the first and final drafts of each individual assignment despite feedback from me with respect to this component.

To achieve proficiency in this component, PSTs must develop learning objectives that reflect appropriate attention to the development progression of a concept, supporting procedural fluency by first developing conceptual understanding. In many of the IEFA lessons, despite setting challenging goals in their lesson objectives, PSTs covered concepts only very briefly or not at all, emphasizing procedures and/or answers over underlying meanings. For example, one PST—whose lesson objectives appeared to reflect a high level of cognitive demand by requiring students to explain *why* a given number would be rounded in a particular way—only addressed the explicit rule for rounding within the lesson, with no discussion of why this is the rule or why rounding is useful. For other lessons, PSTs moved too quickly from conceptual development to procedural application, such as a PST who included an excellent activity on dividing up grids to illustrate patterns associated with multiplying and dividing by powers of 10 but then quickly moved on to a lengthy packet in which these skills were applied.

Overall, PSTs' unit plans progressed at more appropriate paces and included more targeted efforts to incorporate conceptual development. For most of the lessons coded as nearing proficient, I noted that PSTs were trying to fit a bit too much into a three-day unit, but not as much as those coded as basic. For example, one unit plan coded as nearing proficient on the topic of solving problems with coins included too many activities, but most of the activities focused on a similar theme (finding given totals using various combinations of coins) and allowed for some conceptual development. In a unit plan on a similar topic that was coded as basic, students were expected to learn to identify coins on the first day, add coins on the second day, and subtract them on the third day, with no discussion of methods students might use to add and subtract the coin values.

Suitability of Resources. Notable growth was observed between the IEFA lesson and the unit plan as well as between drafts of the unit plan in terms of *suitability of resources*, with many PSTs making adjustments to their drafts in response to feedback given on this component. To achieve proficiency, PSTs must select resources that are optimally suited to the mathematical content intended to be conveyed in a lesson or unit. The primary reason for PSTs performing at lower levels of proficiency on earlier assignments and earlier drafts with respect to *suitability of resources* appears to be oversight of relevant tools for addressing the key concepts embedded in a lesson or unit. Drafts coded as nearing proficient incorporated a variety of appropriate tools but overlooked a particularly salient tool. For example, in a unit plan on missing addends, the PST planned to encourage the use of counters, pictures, and number bonds but overlooked number lines as an additional resource for supporting students in identifying missing addends.

Drafts that incorporated tools that were not ideal (but not entirely inappropriate) and/or that did not incorporate a variety of tools were coded as basic. For example, in a unit introducing three-digit numbers, the PST planned to have students utilize a place-value mat with simple counters to represent each place value, whereas base-10 blocks would be more appropriate. In an IEFA lesson on solving multiplication word problems within cultural contexts, the PST only mentioned white boards and dry erase markers as resources for solving the problems, whereas counters, base-10 blocks, and/or multiplication charts would be helpful tools as well. Of the first drafts coded as basic, eight included a limited number of resources, five included resources that were not ideal for the math content, and four exhibited both of these shortcomings.

Use of Resources. PSTs performed less proficiently in *use of resources* than in *suitability of resources*, with fewer achieving proficiency and nearing proficiency. Nonetheless, notable growth was still observed between the IEFA lesson and unit plan. To be proficient in *use of resources*, PSTs must utilize resources in support of student exploration and sense making and to illuminate important mathematical connections while allowing students choice in how resources are utilized when appropriate.

For drafts coded as basic, resources were not utilized to draw out or discuss important mathematical concepts—they were simply used to solve problems, with an emphasis on answers over conceptual development. In several of the IEFA drafts, PSTs planned to have students utilize beads to model addition but did not incorporate discussion that would help students to link the beads to the concept of addition or to the specific values in an equation. On another draft, the PST mentioned the use of various manipulatives for solving multiplication word problems in a cultural context but did not embed any questions designed to draw connections between the manipulatives, the scenarios in the word problems, and the concept of multiplication. On a unit plan draft, a PST planned to have students utilize counters to solve addition problems, but all the embedded prompts within the unit focused on final answers rather than concepts; for example, “if you take that group of color cubes and the other group of color cubes and put them together how many will you have?”

For drafts coded as nearing proficient, PSTs either made attempts at concept development but their attempts were insufficient to support the intended connections, or they utilized resources for concept development in a portion of the lesson but did not develop other concepts embedded in the lesson. For example, one PST set students up nicely to discover the conversion rate between inches and feet through the use of rulers but then provided no indication of how she would support students in discovering or understanding the multiplicative process necessary to determine how many total inches are in a 21-foot canoe. A few PSTs utilized the resources effectively for concept development but did not draw out connections among the various resources used. For instance, in a unit plan on equivalent fractions, the PST used a different tool in each lesson—and used each tool effectively to develop conceptual understanding—but did not incorporate any discussion on how the tools were related or why all of them could be used to model the same concept in different ways.

Nature of Tasks. PSTs also demonstrated notable growth in *nature of tasks* between their IEFA lesson plans and their unit plans, with a majority of PSTs performing at the basic level on their IEFA lessons and a majority performing at nearing proficient on their unit plans; however, little improvement was made between drafts of each individual assignments. To demonstrate proficiency, PSTs must design or select tasks that feature a high level of cognitive demand; position students as the primary doers in the activity; encourage non-algorithmic thinking and multiple solution strategies; and foster conceptual as well as procedural understanding.

In their IEFA lessons, many PSTs posed mathematical tasks that were purely procedural in nature. While these tasks reflected worthwhile elements of Native American culture or data, the mathematics was rudimentary, with little to no connection to concepts. For example, several PSTs created lessons in which students generated beadwork and wrote number sentences for their creations, but the tasks were framed as prescriptive step-by-step procedures as opposed to opportunities for discovery, discussion, and/or reflection on key mathematical and cultural concepts. Other PSTs simply planned to have students perform calculations based on data sets, such as rounding population data to the nearest thousand or converting population data from fractions to decimals to percents.

In their unit plan drafts, a greater number of PSTs incorporated more open-ended tasks into their lessons. Students were asked to find various coin combinations to equal given totals, decompose numbers in multiple ways, figure out the missing facts in fact families, and compare and contrast shapes, among other cognitively challenging activities. PSTs who demonstrated proficiency in this component designed tasks that fostered clear connections between conceptual and procedural understanding, such as a PST who had students utilize graph paper to determine the surface area of various cubes and prisms and then develop formulas based on their thought processes. For PSTs who demonstrated nearing proficient performance, a variety of small oversights and inconsistencies resulted in this designation, in many cases related to the *nature of prompts*, which I discuss next. For example, many PSTs posed a conceptually rich problem but did not couple it with meaningful discussion that would facilitate connection between the procedures utilized to solve the problem and the underlying mathematical concepts. Other PSTs began their units with conceptually oriented tasks but did not refer back to these tasks or build upon them in later lessons in their units, such as a PST who launched a unit on prime and composite numbers with a hands-on factoring activity using beans but did not refer back to this activity—or the use of manipulatives—in the remaining lessons of his first draft.

<u>Nature of Prompts.</u> PSTs demonstrated notable improvements in *nature of prompts* between their IEFA lessons and their unit plans as well as between their first and final drafts of their IEFA lessons. It seems at first PSTs may have overlooked my lesson planning criteria to incorporate specific prompts to solicit student thinking and generate discussion, as nearly half of the PSTs did not include any prompts at all in their first IEFA drafts but responded to feedback reminding them to include specific questions they could ask students during their lessons. For this component, PSTs need to incorporate prompts that are designed to elicit information about conceptual and procedural understanding and that encourage reflection, justification, and connections.

Areas Exhibiting Little Growth

For five of the rubric components, little growth between drafts or between assignments is evident in the data summarized in Table 4 below, which reproduces the salient components from Table 2 for ease of reference. For these components, a few of the final drafts of each assignment improved in performance but most assignments remained at the same proficiency level between drafts of each assignment as well as between the IEFA lesson and the unit plan. These components include *mathematical concepts*, *significance*, *prerequisite knowledge and skills*, *alignment between learning objectives and assessments*, and *coherence* as shown in Table 4. Three of these five components fall within the domain of *mathematical concepts*. Below, I describe the findings associated with each of these components in further detail.

Table 4. Tally of Coded References for Drafts and Final Submissions of Assignments Exhibiting Little Growth

		IEFA draft	IEFA final	UP draft	UP final
Demonstrating Knowledge of Content	Mathematical Concepts				
	Proficient	1	1	4	4
	Nearing proficient	8	11	12	13
	Basic	7	7	4	3
	Unsatisfactory	5	3	1	2
	Significance				
	Proficient	3	4	6	8
	Nearing proficient	11	10	13	12
	Basic	6	6	3	2
	Unsatisfactory	1	1	6	4

Table 4 Continued

	Prerequisite Knowledge and Skills				
Demonstrating Knowledge of Content	Proficient	0	0	4	5
	Nearing proficient	2	5	4	3
	Basic	12	13	11	11
	Unsatisfactory	7	3	3	2
	Alignment Between Learning Objectives and Assessments				
Setting Instructional Outcomes and Assessment	Proficient	1	7	7	7
	Nearing proficient	15	9	13	13
	Basic	3	2	1	1
	Unsatisfactory	5	4	3	1
	Coherence				
Instructional Strategies	Proficient	1	3	3	6
	Nearing proficient	15	13	12	9
	Basic	5	5	4	4
	Unsatisfactory	1	0	2	2

Mathematical Concepts. For *mathematical concepts*, more students were coded at nearing proficient performance across the various drafts than were coded at any of the other three levels, with almost no change in codes between the first and final drafts of the two assignments. A few drafts moved up a level or two in proficiency but most stayed the same between drafts. To be deemed proficient, PSTs must demonstrate a solid understanding of mathematical concepts and their relationships to procedures within their lesson plans. For drafts receiving a code of nearing proficient, PSTs incorporated a conceptually oriented activity but did not adequately draw out the concepts via appropriate questioning and discussion and/or included discussion about one concept but not another, missing opportunities to make important connections and/or distinctions. For example, in an IEFA lesson on probability, the PST did not clearly distinguish between theoretical and experimental probability and the different procedures for determining each of these values, although the general concept of probability was discussed in terms of a real-world activity. In many of the unit plans on coins, PSTs supported student exploration and discussion around various coin combinations that would result in given totals but did not include discussion about methods used for calculating the totals, an important link between procedures and concepts.

Significance. As with *mathematical concepts*, more students were coded at nearing proficient performance for *significance* across the various drafts than were coded at any of the other three levels of proficiency. To achieve proficiency, PSTs must incorporate mathematical and/or real-world scenarios that aptly illuminate mathematical concepts and naturally lend themselves to modeling. For many of these lessons, rudimentary mathematical modeling was incorporated (such as by modeling number sentences with beads, modeling multiplicative comparison scenarios with number bars, and modeling factors and multiples with beans) but the modeling activities required further development to draw out key concepts, making this component highly intertwined with mathematical concepts. For example, in an IEFA lesson on Northern Cheyenne beadwork, a PST planned to have students model addition with colored beads to practice applying various addition strategies taught in first grade (such as associating to make a ten and using doubles facts). The main instruction for the activity, however, was to create a number sentence that reflects the beads chosen for a bracelet, with no requirement to choose bead combinations that are ideal for particular strategies or to explain which strategy would be ideal for a particular combination of beads (such as 7 red beads and 7 blue beads representing a doubles fact). The nature of the task was basic (featuring a low level of cognitive demand and emphasizing procedures over concepts), but the *significance* of the task was nearing proficient for its modeling potential.

Other PSTs simply included incomplete or unclear instructions for their modeling activities. For example, in the IEFA lesson on probability, the PST did not indicate how many rounds of the game would be played or what students should be doing during play to ensure that the appropriate mathematical data would be gathered.

Prerequisite Knowledge and Skills. For prerequisite knowledge and skills, most PSTs demonstrated basic performance, which indicates that a PST either "identifies relevant background knowledge but does not use it as a foundation to build new knowledge and skills" or "identifies and incorporates some relevant knowledge but overlooks a fundamental concept." In rereading the notes I recorded during my first cycle of coding for each final draft of an IEFA lesson or unit plan, I found that 16 of the PSTs mentioned relevant prerequisite knowledge in the front matter of their lesson plans but did not explicitly incorporate or draw upon this knowledge within the lessons, a requirement for proficiency. These PSTs were familiar with the concepts and skills that formed a foundation for those addressed in their lessons but did not draw upon them effectively within the lessons.

In 10 of the final drafts, PSTs incorporated relevant prior knowledge in their lessons but overlooked a fundamental concept. For example, in a lesson featuring scaled bar graphs, the PST invoked prior knowledge of gathering and displaying data but overlooked prior knowledge of division, a fundamental skill for creating scaled graphs. In another lesson, the PST incorporated prior knowledge of measurement in a lesson on converting feet to inches but failed to mention or incorporate knowledge of either addition or multiplication as a necessary prerequisite to being able to calculate the conversions.

Coherence. For this component, most assignments were coded as nearing proficient across the various drafts. There are several possible reasons why a draft could be coded as nearing proficient according to the rubric criteria for this component: the theme *or* the flow *or* the closure (or any combination of these) needed further development. Full proficiency is defined as lessons and units that feature a well-defined, unifying theme and a logical flow, with opportunities for reflection and closure. For the 27 drafts coded as nearing proficient, the need for a more targeted closure was the most common reason for not demonstrating full proficiency, with 17 of the drafts being annotated with comments related to closure.

Issues with flow were another factor in being coded as nearing proficient rather than proficient, with 11 of the 27 nearing proficient drafts being annotated with comments related to flow, such as having a random activity that is not directly connected to the other activities in a lesson or unit or ordering activities in a manner that is less than ideal, such as using a video that defines shapes to open a lesson in which students are expected to come up with their own definitions. Issues with overall theme were less common, with 5 of the 27 nearing proficient drafts requiring further development of a theme that was evident but not thoroughly integrated across all activities in a lesson or unit.

Areas Exhibiting Decline in Performance

For three of the rubric components, a decline in performance is evident in the data summarized in Table 5, a reproduction of the salient components from Table 2 for easy referencing. For these components, more assignments were coded at lower levels of performance on the unit plan drafts (the final independent lesson planning assignment) than on the IEFA plan drafts (the first independent lesson planning assignment). These three categories are *level of cognitive demand of objectives and assessments*, *differentiation*, and *perspectives*, as shown in Table 5. The latter two components comprise the *advocacy and equity* domain in the rubric, suggesting that this is an area in particular need of attention. I describe the results associated with each of the three components below.

Table 5. Tally of Coded References for Drafts and Final Submissions of Assignments Exhibiting Decline in Performance

		IEFA draft	IEFA final	UP draft	UP final
Setting Instructional Outcomes and Assessment	Level of Cognitive Demand of Objectives and Assessments				
	Proficient	1	2	5	6
	Nearing proficient	10	13	6	6
	Basic	10	7	12	11
	Unsatisfactory	1	0	0	0
Advocacy and Equity	Differentiation				
	Proficient	2	3	3	4
	Nearing proficient	6	11	6	7
	Basic	7	4	12	10
	Unsatisfactory	6	3	5	3
	Perspectives				
	Proficient	0	1	0	0
	Nearing proficient	9	9	1	0
	Basic	10	9	20	21
	Unsatisfactory	3	3	0	0

<u>Level of Cognitive Demand of Objectives and Assessments.</u> For this component, PST performance appeared to decline between the first and last independent lesson planning assignments, with half or more of the IEFA drafts being coded as nearing proficient but most unit plan drafts being coded as basic. However, a greater number of the unit plan drafts were coded as proficient than were the IEFA drafts, indicating improvement for some PSTs. That being said, six of the IEFA final drafts and only one of the unit plan final drafts were coded as

nearing proficient due solely to a lack of opportunity for student self-evaluation, with the objectives and assessments otherwise meeting the criteria for proficient performance, indicating that with the inclusion of an opportunity for student self-evaluation, quite a few more IEFA drafts would have been proficient as well. To demonstrate proficiency, objectives and assessments must reflect a high level of cognitive demand and be designed to elicit information about both conceptual and procedural understanding, with opportunities provided for student self-evaluation.

Overall, PSTs incorporated more ambitious mathematics into their IEFA lessons than into their unit plans, resulting in a higher level of cognitive demand in the objectives and assessments. In their efforts to showcase various Native American cultures, they aimed to have students analyze and create patterns, solve problems involving feeding large groups of people, figure out how much hay would be needed to sustain multiple horses over multiple days, and graph data, among other challenging goals (however, as discussed with respect to other components of the rubric, this higher level of cognitive demand was not successfully maintained in the lessons' tasks).

In contrast, in their unit plans most PSTs emphasized recognition and identification over conceptual understanding. Many of the drafts of those who attempted to incorporate explanation and justification into their objectives and assessments required only statements of fact or procedures from students, suggesting that some PSTs may be conflating conceptual understanding with verbal or written recitation of basic knowledge and processes. On four of the drafts, I recorded notes such as, "Explain objectives emphasize 'explain THAT' rather than 'explain WHY.'" For example, one PST included the objective, "The students will be able to explain the value of each coin and identify each coin correctly." Another PST included the objective, "Students will be able to explain what a fact family is," with an anticipated answer being, "A fact family is a group of 3 numbers that are used in related equations." Neither of these objectives requires students to understand significant mathematics, only to repeat information learned in class.

Differentiation. Six PSTs improved their performance on *differentiation* between their first and final drafts of their IEFA lessons in response to feedback, demonstrating improvement between drafts of the IEFA lesson; however, performance declined between the IEFA lesson and unit plan assignment. To demonstrate proficiency, PSTs needed to design lessons and units that feature multiple entry points and specific supports for struggling students as well as appropriate extensions for students ready for a challenge.

On the unit plans, PSTs' differentiation plans were often too vague, resulting in a basic code for this component. Despite prompting to incorporate more specific differentiation into their units, most PSTs failed to do so on their final drafts, leaving differentiation plans nonspecific, such as mentioning that students who finish early would be given "more challenging problems" or "larger numbers" but not indicating what those problems or numbers would be. Differentiation was typically the last aspect of their drafts that I commented on in my feedback due to the organization of my original rubric (see Appendix A), so if PSTs were addressing my feedback in the order they were reading it, they may have run out of time to incorporate this feedback into their revisions; indeed, as related in the third part of this chapter, one of the PSTs I interviewed during the subsequent semester admitted to running out of time to address all of the feedback offered on her drafts, and it is likely that other PSTs experienced the same setback.

Perspectives. PSTs demonstrated their weakest performance overall in this component, with only one PST achieving proficiency on their IEFA lesson and only one PST achieving nearing proficiency on their unit plan, with all other students performing at the basic level on their unit plans. To demonstrate proficiency, PSTs must incorporate multiple perspectives and contexts into lesson and unit plans, including those of non-dominant and historically marginalized groups, in a way that challenge stereotypes and predominant paradigms.

Given that there was no requirement during Fall 2019 to incorporate multiple perspectives into unit plans, it is not surprising that PSTs' unit plans overwhelming reflected the dominant cultural perspective. IEFA lessons, on the other hand, were designed to explicitly incorporate the perspectives of historically marginalized groups. For IEFA lessons coded as nearing proficient, I mostly commented in my coding notes that the IEFA component of the lessons needed to be better linked to the mathematics and/or the IEFA component needed to be revisited throughout the lessons and not just introduced in the beginning. For drafts coded as basic, my coding notes commented on a lack of background or information on the Native American data or artifacts utilized in the lessons; a tendency to lump all tribes together by referring to data or artifacts as "Native American" rather than belonging to a specific tribe; and the incorporation of unsubstantiated assumptions about a tribal group or groups, such as assuming that a tribe would use Eagle feathers in jewelry without verifying whether this would indeed be the case.

For drafts coded as unsatisfactory, PSTs incorporated inaccurate information, in one case utilizing color meanings from the wrong tribe and in another misinterpreting the rules of a Native game. A third PST framed her lesson around the notion that "Natives were put onto tribal lands by the American government," which is an inaccurate interpretation of the treaty process that established reservations.

Summary

In approximately half of the components of the AMTE-Danielson rubric utilized to evaluate and code the lesson plan data, PSTs demonstrated improved performance between drafts of their assignments and/or between their first independent lesson planning assignment (the IEFA lesson) and their final independent lesson planning assignment (their unit plan). In many components, PST performance remained roughly the same between the two assignments as well as between the first and final drafts of each individual assignment, and on three components, PST performance declined. In only one component—*alignment between learning objectives and learning standards*—did a majority of PSTs demonstrate proficient performance by the final assignment of the semester, with nearing proficient performance being the most common proficiency level obtained by PSTs across most rubric components. In the subsequent section, I share the findings from my second cycle of coding, in which I analyzed my annotations on all nearing proficient codes to search for themes that might help to explain why so many of my PSTs were able to achieve nearing proficient performance but not fully proficient performance on their lesson planning assignments.

Second Cycle Coding Results

Overview

In this section, I discuss the results of my second cycle of coding, in which I examined my annotations on all nearing proficient codes of the final drafts of the lesson planning assignments (IEFA lesson and unit plan). I chose to focus on final drafts because they represented the best efforts of my PSTs by the conclusion of the Fall 2019 semester and I wanted to know why, even after receiving feedback, PSTs still seemed to be struggling to achieve full proficiency on their assignments. To illustrate and support my findings, I also share some of the feedback I offered to PSTs on these components and compare it to the revisions that were made on their drafts.

Table 6 identifies the themes that emerged from my efforts to look for possible patterns in the reasons why so many components were coded as nearing proficient rather than proficient. For each theme, the number of drafts in which the theme appeared is tallied in the column labeled as “file” and the number of instances of the theme appearing across the various drafts is tallied in the column labeled as “references.” For several of the drafts, a given theme appeared more than once within the draft, hence the number of references is often larger than the number of files.

In most cases, a theme appearing twice within a draft meant that the same issue was linked to multiple components on the rubric. For example, in one IEFA lesson, the PST failed to distinguish between experimental and theoretical probability in her modeling activity and simply addressed “probability,” an oversight that was tied to elements of *mathematical concepts* (not fully illuminating the concept), *significance* (modeling experimental probability without distinguishing it from theoretical probability), and *use of resources* (not utilizing the resources to highlight the difference between theoretical probability and experimental probability).

Table 6. Tally of References Coded to Emergent Themes Based on a Review of Nearing Proficient References

	Files	**References**
Not fully developed from start to finish	9	11
Expected methods or responses not specified	23	46
Adjustment to flow needed	11	12
Adjustment to numerical values needed	3	5
Adjustment to wording needed	5	5
Concept development rushed	14	19
Connections between procedures and concepts not adequately addressed	31	85
Connections between various methods or representations not adequately addressed	8	13
Concepts not clearly distinguished from related concepts	6	14
Tangential tasks or discussions included	5	6
Aspects of concepts or procedures overlooked	17	45
Useful tools, representations, or formats overlooked	21	26

There was a total of 269 nearing proficient codes across the 42 final drafts of the two assignments (21 final drafts per assignment). There is a total of 292 references in Table 6, indicating that for 23 of the nearing proficient codes, there was more than one theme evident in my coding notes. Below I describe the themes from Table 6 as well as their relationships to three overarching themes that emerged from the data during second cycle coding.

Emergent Themes

In this section I describe each of the themes listed in Table 6 using details from my coding notes and the final drafts to which they refer.

Not Fully Developed from Start to Finish. In nine of the final drafts, I noted that the component could not be coded as proficient because a task was not fully developed or described. The task included enough detail for me to feel confident that it was appropriate to the mathematics intended to be conveyed but not in enough detail for me to know how the PST planned to implement the task from start to finish. For instance, in one lesson plan a worksheet was mentioned but not included in the submission, so I could only evaluate the sample problems that were explicitly mentioned within the lesson plan; while these sample problems were fully appropriate for the stated lesson objectives, I had no way of determining whether the rest of the worksheet would be appropriate (*alignment between learning objectives and learning assessment*). In another draft, the PST utilized rulers well to support student discovery of the conversion rate between inches and feet but did not develop a full plan for the next step of the lesson on determining the number of inches in a 21-foot canoe (*use of resources*). For three of the IEFA lessons, the modeling activities chosen were relevant and meaningful (as opposed to tangential or contrived) but were not described in full enough detail for me to know exactly how the PST planned to implement them (*significance*).

Expected Methods or Responses Not Specified. In 23 of the final drafts, PSTs did not specify the solution methods or responses expected from students in their tasks, discussions, and/or assessments. For a handful of the drafts, I coded *mathematical concepts* as nearing proficient because the tasks, prompts, and other aspects of the lesson suggested a thorough understanding of the concepts—however, without the inclusion of anticipated solution strategies or responses, I could not be absolutely sure.

For most of the nearing proficient codes that I associated with this theme, PSTs either did not include example solutions or responses to the planned assessments (*alignment between learning objectives and learning assessments*) or did not anticipate potential points of struggle or confusion in their lessons (*student thinking*). In some cases, PSTs provided expected solutions or responses to some of the assessments but not all of them while in other cases PSTs completely overlooked the lesson plan criteria to include example solutions expected for each planned assessment. In a few cases, PSTs overlooked potential points of struggle that would have been obvious had the PSTs taken the time to solve their planned problems themselves; for instance, in a lesson on division, the PST planned to have students divide 288 by 9 with base-10 blocks, which would be incredibly tedious given the fact that none of the place values can be split equally among 9 groups, meaning that students would need to take steps such as trading eight tens for 88 ones and distribute these equally.

<u>Adjustment to Flow Needed.</u> Drafts coded to this theme included an activity that would be better placed at another point in the lesson or unit or that would be best removed from the lesson or unit because it did not follow along with the flow of the other activities. As described in my data summary for *coherence* in the first part of this chapter, I made comments related to flow on 11 of the final drafts. For example, in a kindergarten unit on shapes, I recommended

moving the Jack Hartmann videos to later in the lessons so that the PST's activities involving sorting and defining shapes would not be pre-empted by the information in the video.

Adjustment to Numerical Values Needed. In drafts coded to this theme, a numerical value utilized for one of the assigned problems needed to be adjusted to better accommodate the intended strategies. One of these drafts was the lesson mentioned above on division (in which $288 \div 9$ would be too tedious to solve with base-10 blocks), and another was an IEFA lesson involving decimal addition in which students were asked to find the total price of 10 of a particular item, which would be more appropriate for a lesson on multiplication.

Adjustment to Wording Needed. For a handful of drafts, the wording of a task or learning objective needed to be adjusted in order to clarify the intended outcome. For four of these five drafts, assessment problems were worded in a way that did not actually require the use of the intended representation or tool. In two of the IEFA lessons, PSTs asked questions that were intended to encourage students to consult graphs they had created, but the questions actually asked for data that would only be evident from the students' tally sheets and not the graphs. For example, in one of the lessons, the PST instructed students to create graphs showing the total points each student in a group had earned while playing a game in which various point outcomes were possible depending on the distance of each of five tosses. Her assessment question designed to gauge students' ability to interpret their graphs asked students to determine the most common score from among the five possible outcomes, which would be disguised in a graph of total points earned. In two of the unit plans, PSTs asked questions intended to encourage the use of specific coins to solve problems but were worded in such a way that knowledge of individual coin values would not be required. For instance, in one of the unit plans, the PST posed the following problem: “Brady has 50 cents to spend at the bake sale. If he bought three cookies for 10 cents each and a cupcake for 15 cents, how much money will he have left over?" For the fifth draft coded to this theme, some of the lesson objectives were worded as activities rather than outcomes; for example, “students will explore the relationship between area and perimeter” rather than “students will be able to describe the relationship between area and perimeter.”

Concept Development Rushed. For many drafts, appropriate conceptual development was incorporated into a lesson or unit but was touched on too briefly to ensure that students would fully grasp the concept. Most of the 19 components of the 14 drafts coded to this node were related to *development progression and pacing*. For instance, one PST planned an IEFA lesson involving decimal division by powers of 10 that began with an excellent activity on splitting a whole into tenths using grid paper. Her questioning was on target with prompts such as, “What pattern do you see? Is your answer bigger or smaller than the dividend?” However, with just one example of dividing by a quantity less than one, students would not have the opportunity to discern a specific pattern before moving on to a worksheet involving the application of the intended pattern (that the decimal point moves to the right when dividing by a power of 10 that is less than one and to the left when dividing by a power of 10 greater than 1).

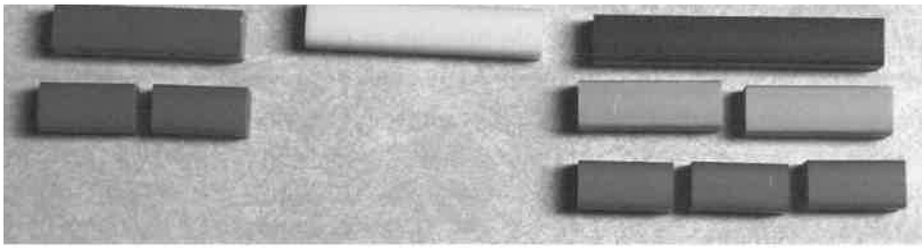
In many of the unit plans, PSTs aimed to cover too much ground too quickly. In an introductory unit plan on solving word problems with coins, for example, the PST incorporated conceptually rich activities and prompts related to making purchases in a school store but moved from problems involving coin totals (addition) to getting change (subtraction) within a span of two lessons without providing time for further practice with each separate operation. In a unit plan on three-digit numbers, the PST planned to introduce expanded form in the third lesson after the concept of 100 was just introduced in the first lesson.

Connections Between Procedures and Concepts Not Adequately Addressed. This was the most significant theme that emerged from the second cycle of coding, as evidenced by a total of 85 components from 31 of the 42 final drafts being coded to this theme. Components coded to this theme reflected efforts on the part of PSTs to link procedures to concepts, but the efforts were ultimately insufficient to support robust connections. For many of these components, the "why" piece was the missing link in the lessons. For example, in my annotations to my first round of coding, I typed numerous comments such as, "suggests some appropriate division strategies but doesn't adequately plan to discuss why these strategies work." PSTs designed conceptually rich tasks and *some* conceptually rich prompts, but the tasks and/or prompts fell short of supporting a strong connection between procedures and concepts.

For example, in an IEFA lesson involving division word problems, the PST challenged students to figure out how many people would go on each of four buses if there were 96 people attending a field trip. She included several prompts designed to encourage students to consider the underlying concepts (e.g., "How many groups is the number being broken down into?") but she did not include any questions designed to link the concept of division to the specific strategies that she anticipated within the lesson. For example, in anticipating that students might show four buses with 10 + 10 + 4 inside each bus, she might have asked, "Why do you have two tens and four ones inside each bus? What process did you use to arrive at that solution? How did you know to divide? How does your solution process relate to the meaning of division?" My coding notes on the draft of another PST's IEFA lesson on a similar topic reflected similar concerns: "Level of cognitive demand of tasks is high since students need to search for information to solve word problems but still needs further development to draw out connection between procedures and concepts."

In a unit plan on prime and composite numbers, the PST successfully set students up to understand the difference between prime and composite numbers by having them utilize Cuisenaire rods to show the factors for the numbers 3, 4, 5, and 6. The PST then has students factor the numbers by hand (presumably using a factor tree, although the expected method is not specified) and writes, "The teacher will ask the class to take time and see what they notice between the Cuisenaire rod groups and each number's factors; students should notice that the Cuisenaire rod groups that they found are each numbers factors." However, the PST does not plan any prompts to ensure that students notice these connections, which may not be fully clear to them, especially if they did not find all possible ways to decompose each number with the rods (or recall all possible factors when factoring with pencil and paper). Indeed, the PST's own example solution, shown below, does not include the units cubes that could be used to show 1 as a factor of each number; this is also an example of *aspects of concepts or procedures overlooked*.

Figure 2. PST's Visual Representation of Factors for 4, 5, and 6



In a unit plan on addition, the PST included relevant story problems such as six birds sitting on a tree and three more coming along, as well as a variety of pictures and manipulatives for students to solve the problems, all of which hint at the link between the concept of addition and the counting procedures expected for finding solutions in kindergarten. However, the PST never explicitly draws out the connection between the two. Most of her prompts in the lesson focus on answers, with no prompts such as, "How did you know to count all of these together? Why is this problem a putting together problem?" This PST's final unit plan draft increased in proficiency from her first draft in the component of *mathematical concepts*, which was listed as basic rather than nearing proficient because her initial lessons did not incorporate story problems, only straightforward addition problems such as 5 + 3, which would convey little sense of the meaning of addition to kindergarteners.

<u>Connections Between Various Methods or Representations Not Adequately Addressed.</u> For the 13 components from 8 final drafts coded to this theme, PSTs did not plan to explicitly draw out the connections between various solution methods or representations reflected in their lessons. While the methods and/or representations were conceptually meaningful and often discussed adequately independent of one another, they were not linked together. For instance, in

a unit plan on equivalent fractions, the PST incorporated activities involving graham crackers, pizzas, and fraction strips but did not link the three representations—each of which are the focus of a separate lesson in the unit—with questions such as, "How are the fraction strips similar to and different from the graham crackers and pizzas? Why can we use all three of these items to represent fractions?" In another unit plan, the PST had students create nets for rectangular prisms in one lesson and then wrap packages in another but did not incorporate any discussion linking the two, such as "How might your work with nets be useful in helping you think about how you can wrap your package as efficiently as possible?" or "How is the process of creating a net similar to wrapping a package?"

Concepts Not Clearly Distinguished from Related Concepts. A total of 14 components from 6 final drafts were coded to this theme due to lack of precision in defining and discussing closely related concepts. For example, in a unit plan on shapes, the PST's anticipated criteria for defining a circle—round with no corners or straight sides—would not distinguish it from an oval. In another unit plan on perimeter, the PST had students generate a composite shape and identify its perimeter but did not address the possibility that students might think all sides within the composite shape would be part of the shape's perimeter.

Tangential Tasks or Discussions Included. In five of the first drafts, activities or prompts were included in the lessons that were not directly connected to the main objectives of the lesson. For instance, in the unit plan on perimeter, the PST included a discussion about whether it was easier to create composite shapes with even numbers of sides or odd numbers of sides, but the focus of the lesson was on the impact of the total number of sides and individual side length on the overall perimeter. A much more useful discussion would have been on the impact of using the longest side of the trapezoid (the only piece in the activity with a side length of more than one unit) as an exterior side of the composite shape versus an interior side.

Aspects of Concepts or Procedures Overlooked. A total of 45 components from 17 final drafts were coded to this theme. Many PSTs planned well for one aspect of a task—such as discussing the features and purpose of a scaled bar graph—but failed to address another aspect of the task, such as methods students might use to determine an appropriate scale for their graphs. A number of PSTs failed to account for the process of addition in their unit plans on solving problems with coins; while they often planned for rich discussions about, for instance, how and why different combinations of coins can be used to represent equivalent values, they did not give any indication of methods they expected students to use to add the coins together to arrive at these values.

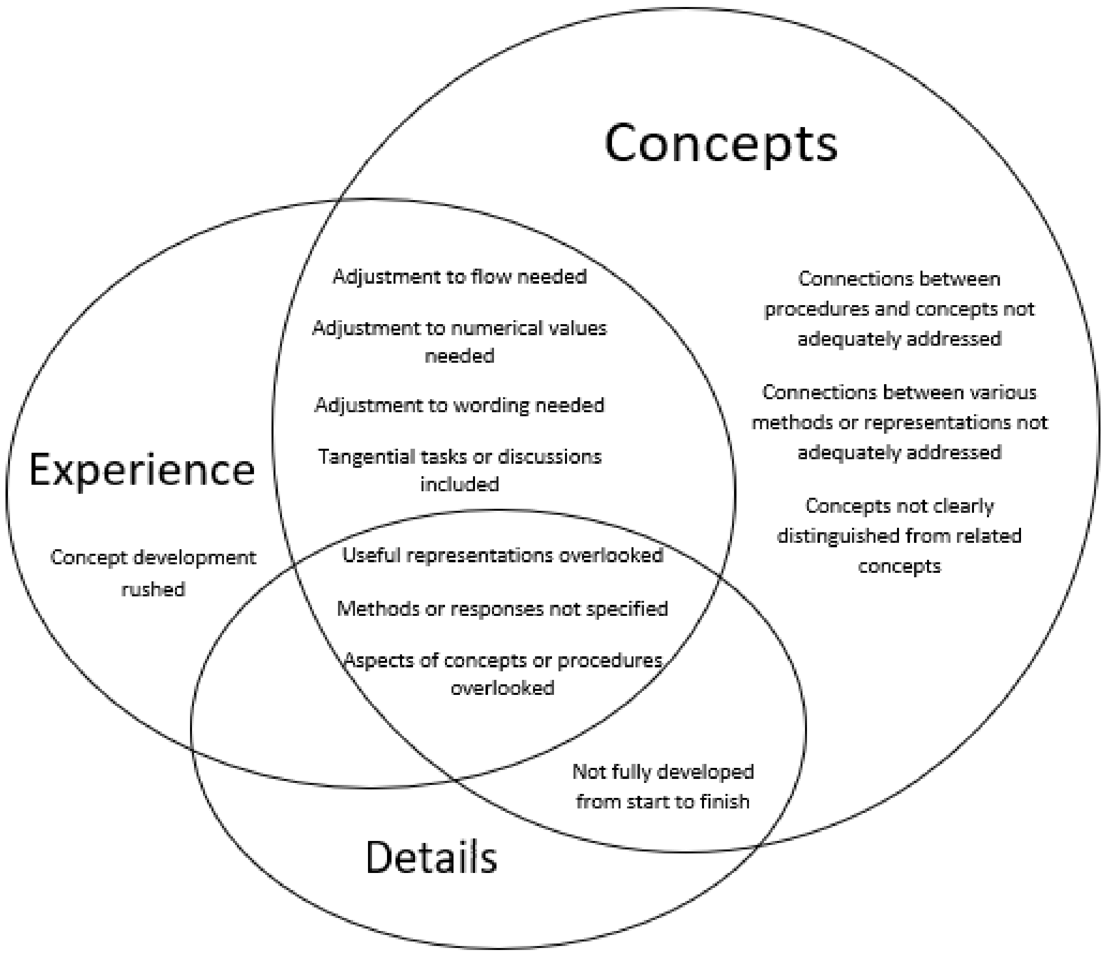
Useful Tools, Representations, or Formats Overlooked. On half of the final drafts—21 files—PSTs overlooked a useful tool, representation, or format that would have benefitted their lessons. In most of the unit plans on coins, PSTs did not mention any tools that could be used to help students find coin totals (due to overlooking expected methods for finding these totals). While the other resources utilized in their lesson plans—fake coins, shopping lists, concept maps, and catchy videos—were suitable for the units, they overlooked tools such as base-10 blocks and hundreds charts that could be used to support students in adding coin values together.

In the unit plan on prime and composite numbers, the PST overlooked multiplication charts as a useful tool for supporting students in identifying factors for numbers that would be too large to model with Cuisenaire rods. In two IEFA lessons on symmetry, students would have benefited from being provided with individual images or cutouts that could be folded as they discuss and test for symmetry rather than just worksheets with multiple images per page.

Overarching Themes

As I examined the themes described above and considered the ways in which they might be interrelated, three overarching themes emerged that I felt helped to explain the variety of gaps I identified in my PSTs' IEFA lessons and unit plans: underdeveloped mathematical content knowledge, lack of teaching experience, and oversight of lesson planning details. Figure 3 depicts these themes and their relationships to the emergent themes discussed above.

Figure 3. Overarching Themes and Their Relationships to Subthemes



Mathematical Concepts. The subthemes embedded in the circle labeled "Concepts" all suggest potential gaps in PSTs' mathematical knowledge. For example, if connections between procedures and concepts are not adequately addressed in a lesson, it may be that the PST does not possess a strong enough grasp of the mathematics to do so. This overarching theme occupies the most space in Figure 3 because it is associated both with the greatest number of subthemes as well as the greatest number of codes in Table 6.

The three subthemes situated solely within "Concepts" are areas in which PSTs would not be likely to improve performance simply by having more teaching experience and/or putting forth greater effort on lesson planning assignments. Indeed, many of the lessons achieving near proficiency on the final drafts were quite meticulously planned but nevertheless overlooked key concepts, connections, and/or distinctions. These oversights are closely related to the following rubric components: *mathematical concepts* (proficiency requires linking procedures and concepts); *use of resources* (proficiency requires illuminating important mathematical connections), *nature of tasks* (proficiency requires fostering conceptual as well as procedural understanding), and *nature of prompts* (proficiency requires eliciting information about conceptual and procedural understanding and encouraging connections). All four of these components follow similar patterns in Table 2, with greater performance on the unit planning assignments than on the IEFA lessons and with little improvement between drafts of the unit plan.

Other rubric components frequently coded to one or more of the three subthemes associated with "Concepts" included *coherence*—due to a lack of closure on the relationship between procedures and concepts—and *level of cognitive demand of objectives and assessments*. For the latter component, PSTs made clear efforts to require explanation and justification in their lesson plans but the emphasis was on explaining how procedures work rather than *why* they work. For instance, in a unit plan on solving problems with coins, students were expected to explain that they could use different combinations of coins to arrive at a given total but were not expected to explain why this could be done based on mathematical principles such as decomposition and equivalence.

To illustrate the nature of the oversights as well as PSTs' efforts to address them, Table 7 describes several of the first and final drafts of the IEFA lesson and unit planning assignments in relation to the feedback I provided to PSTs in Fall 2019. As exhibited in Table 7, final drafts of assignments often did not adequately address the feedback provided on first drafts, resulting in most drafts remaining at similar levels of performance. While some growth is evident in the table by comparing the descriptions of PSTs' original drafts to the revisions, improvements were ultimately insufficient to meet the criteria for proficient performance. In the case of one of the IEFA lessons, the PST chose to switch topics as opposed to revise her original topic (fraction multiplication), a strong indication that this PST lacked sufficient content knowledge to address the feedback provided on her first draft. One other PST (not referenced in Table 7) likewise switched topics on her IEFA lesson as opposed to revising her original topic based on the feedback provided, adjusting her lesson from a focus on scaled bar graphs to a focus on rounding to the nearest thousand. Copies of the first and final drafts of the lessons referenced in Table 7 are included in Appendix D. Each lesson or unit was drafted by a different PST such that no PST's work is included as both an example IEFA lesson and an example unit plan; this was done intentionally to present a broad representation of the study cohort.

Table 7. Unit Plan and IEFA Lesson Components with Insufficient Revisions Based on Feedback

Lesson, topic, and grade level	Rubric component	First draft		Feedback	Final draft	
		Code	Description of Draft		Description of Revisions	Code
IEFA lesson on multiplying fractions (changed to division with whole numbers) (4th)	Math concepts	B	The lesson appropriately links fraction multiplication to scaling up a recipe, but no indication is given of correct solutions or of the methods students will use to solve the problems; although manipulatives are mentioned, given the size of the numbers (feeding Indian tacos to 10,000 people at Crow Fair) and the repeated use of the term "calculations," it is assumed students are expected to use the standard algorithm to scale up the recipe.	"Are you expecting a procedural solution? How does one explain multiplying fractions by whole numbers? What explanation are you looking for? Cuisenaire rods are an excellent tool for multiplying fractions but won't work for large numbers. That doesn't mean they can't be used to help students understand the concepts in a way that can direct them towards a solution. For example, they could use the Cuisenaire rods to figure out how many groups of ¾ are needed to make a whole number (¾ + ¾ + ¾ + ¾ = 3) and then use this information to solve for larger numbers of people—although this will get complicated because your recipe is based on 2 pieces of fry bread. To differentiate for struggling learners, you could stick with smaller numbers such as creating enough fry bread to feed 12 people. In reality, students are not expected to be able to multiply such large numbers by fractions."	Rather than address the feedback provided on fraction multiplication, the PST completely revamped her lesson to focus on division with whole numbers, posing a scenario where 96 students need to split up evenly across 4 buses to ride to the Crow Fair. Students are expected to use base-10 blocks or drawings to solve the problem in a way that makes sense to them, but one of the two anticipated solutions makes little sense (writing "2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, and 2" within each of the four buses); it is unclear how or why the PST would anticipate this solution, whereas the other solution proposed (writing 10 + 10 + 4 within each bus) makes sense given the provided tools.	NP
	Use of resources	U	The use of manipulatives (Cuisenaire rods) cannot be determined from the lesson since no explanation or examples of their expected use is provided.		Base-10 blocks and drawings are utilized to support student independence in solving the problem but no examples of how the base-10 blocks are expected to be used are provided; expectations can be surmised from one of the anticipated solutions but the other solution is unclear, as noted above.	NP
	Nature of tasks	NP	The task positions students as the primary doers by tasking them with scaling up a given recipe with no explicit teacher direction, but it is unclear what methods students are expected to use in approaching the task.		The main worksheet includes four word problems appropriate to the context and grade level, but some of the numbers would be better adjusted to accommodate the use of base-10 blocks (e.g., 288 ÷ 9 would be very tedious to solve with these tools).	NP
	Nature of prompts	U	The few prompts included in the lesson are irrelevant to the important mathematics in the lesson (e.g., "Are there large differences between the amounts of the ingredients? Are the amounts surprising to you?").	"I'd like to see more prompts in your lesson plan that clearly target the mathematics. For example, asking if they notice anything interesting about the amounts doesn't necessarily focus in on key concepts in fraction multiplication. Consider what questions you will need to ask so that students are sure to discuss and understand key steps in the solution process of these problems. I suggest solving some of the problems for yourself to give you an idea of where students might struggle and how to help them work through those struggles with appropriate questions and prompts."	Several prompts that would facilitate reflection and justification are included in the lesson such as, "How many groups is the number being broken down to? How do you know?" However, prompts are needed to facilitate connections between procedures and concepts, for example, "Why can we break apart a number by place value and distribute each place value one at a time the way [student name] did?"	NP

Table 7 Continued

IEFA lesson on decimal addition (5^{th})	Math concepts	B	Students practice adding and subtracting decimals by finding price totals and differences for various parts of a bison. Emphasis is on lining up the decimals properly with no discussion of why this is the procedure and how it relates to underlying base-10 concepts even though the chosen standard is to "add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings."	"Can you incorporate some discussion questions to get students to explain why we want to line the decimal up when adding or subtracting decimals? They should be encouraged to invoke their relevant background knowledge on base-10 place values and fractions (not lining the decimals up could result in adding decimals with unlike denominators: for example, not correctly lining up 3.14 and .54 could result in adding 4/100 + 1/10). How can you make this type of reasoning a more explicit focus of the math lesson so that students understand WHY we add decimals the way we do? I think you would also want to have play money on hand for struggling students to be able to work with. This could help them make the connection between the need to line up the decimals since the dollar bills reflect quantities on one side of the decimal point and the coins reflect quantities on the other side of the decimal point."	The emphasis on the final draft remains procedural, with students expected to use the standard algorithm to add and subtract decimals. However, the PST added a few additional discussion questions designed to help students recognize that the decimals must be lined up because of place value, although place values are not modeled or discussed in further detail once this answer is provided.	NP
	Use of resources	B	Resources are limited to images of various bison parts labeled with their prices and do not help to illuminate connections between procedures and concepts.		Resources are still limited to images of various bison parts and their prices and do not help to illuminate connections between procedures and concepts.	B
	Nature of tasks	B	Tasks are limited to finding totals and differences using the standard algorithms for adding and subtracting decimals.		Tasks are still limited to finding totals and differences using the standard algorithms for adding and subtracting decimals.	B
	Nature of prompts	B	Prompts emphasize answers and procedures ("What if we line it up this way? What is 5 + 5…okay now what do we do next?").		PST added more conceptually oriented prompts including, "Why did you line up the problem this way? Why is it important to line up the decimals? If we don't line up the decimals what do you think would happen?" and "What do you remember about place value?" More are still needed to facilitate connections between concrete models/drawings and the standard algorithms.	NP

Table 7 Continued

IEFA lesson on unit conversions between measurement systems (6th)	Math concepts	NP	Students are given accurate conversion rates between inches and centimeters and inches and feet and are tasked with converting the average heights of various people groups from centimeters to feet to discover that Plains Indians were the tallest people on record in the late 1800s. Students are expected to consider how their knowledge of ratios can help them figure out the conversions (as opposed to being given a particular method), but it isn't clear what method(s) the PST is anticipating students might use.	"If you can anticipate more than one way students might solve the conversion problems, how can you draw this out in discussion so students can compare and contrast multiple solution methods? For example, some students might recognize that since 1 in. = 2.5 cm., then every 10 inches = 25 centimeters, so they might use a 10:25 ratio to solve the problems instead of a 1:2.5 ratio. Others might recognize that 1:2.5 is the same as 2:5 and use that ratio to solve the problems. I suggest working through the problem yourself in a couple of different ways to help you both with the anticipation component (anticipating potential misconceptions or points of confusion) as well as with the discussion prompts you might include in your lesson plan."	PST added one anticipated solution strategy for the very first conversion in the lesson (converting 10 centimeters to inches) by explaining, "Students might use different strategies for division such as dividing 10 lines into groups of 2.5 dashes." No further strategies were anticipated even though she planned to have the students share the different strategies that they came up with. Cross multiplying mentioned as prior knowledge in the beginning of lesson but not linked to the strategy suggested above.	NP
	Use of resources	NP	Conversion tables are used to support student exploration and discovery of methods for converting between measurement units but there is no prompting to support students in connecting the information in the tables to their knowledge of ratios *or to other resources that might be helpful such as rulers or bar models.*		Prompting (see below) and *use of supporting resources* is still insufficient to support students in connecting the information in the tables to ratio and proportional reasoning concepts.	NP
	Nature of tasks	NP	Students are given freedom to discover their own methods for converting the measurements but there is no guidance via questioning *or supporting resources (such as rulers or bar models)* to ensure that students are able to do this successfully.		Prompting (see below) and *use of supporting resources* is still insufficient to support students in discovering their own methods for converting the measurements.	NP
	Nature of prompts	U	No specific prompts are included in the lesson, although a few can be surmised from the lesson description (e.g., "I will remind students of their prior knowledge about ratios to prompt thinking").	"Reminding [students] of prior work on ratios and proportions is one good prompt. How else might you prompt them if that reminder doesn't help them? What might you remind them about ratios and proportions to jog their memory more? In developing appropriate prompts, you'll want to think about the ways that students might solve this problem."	PST added that she would also remind students of cross-multiplying, which inadequately addresses the need for prompting that will help students to recognize that they can use their knowledge of ratios to help them convert measurement units. Other prompts added to the lesson plan would facilitate critical thinking and connections (e.g., "What do you already know about converting that might be useful here? How did you convert 10 centimeters to inches? Can you use that here?"), but without key prompts designed to support students in their very first conversion in the lesson (10 centimeters to inches), these prompts will be insufficient.	NP

Table 7 Continued

Unit plan on shapes (K)	Math concepts	NP	Unit includes several activities that will help children to link definitions and concepts, such as sorting and distinguishing shapes and identifying shapes in the classroom (e.g., a door is a rectangle). Circles are not explicitly defined within the unit. Rectangles are inaccurately defined as having two long sides and two short sides.	"For lesson 1, how would you expect students to describe a circle? What would an accurate response sound like from a kindergartener? You mention that rectangles have different side lengths but this isn't always true since a square is actually a special type of rectangle."	PST added potential student descriptions of circles ("round," "looks like a ball") and removed the definition of a rectangle as two long sides and two short sides but did not replace it with any other description, so it is unclear whether the PST knows how to define a rectangle.	NP
	Use of resources	P/B	Resources are used to support non-algorithmic thinking and sense-making in sorting and modeling activities but are used in prescriptive and non-mathematical ways in art projects, where students are simply instructed to use specified shapes to make "an art project" or "drawing."	"Art projects are fun and important in kindergarten, but they take up the bulk of the time in two of your lessons. If you are going to use art projects, then there should be a clearer mathematical focus of the art projects. Can you make the instructions such that students really have to prove their knowledge of the shapes and apply them in a more meaningful way? Can you connect the art projects to discussions about the similarities and differences among the shapes and the ways in which they are used in the real world? Where do we see these shapes? What kind of real-world items could we 'model' with triangles and circles and why are the triangles and circles appropriate for this real-world item?"	One art project was unrevised (students are still instructed simply to create a drawing using 3 circles, 3 triangles, and 3 squares); to the other project (in which students are simply instructed to "make an art project") the PST added the explanation, "I will be including multiple other shapes on the traceable page. This way the students have to really think about the shapes that we have been learning about and pick out which ones they are." After students finish this project, PST added questions that should instead form part of a discussion prior to the art project such that students use the shapes for more purposeful modeling (e.g., "Now that you know what a triangle is and what it looks like, can you tell me things that you have seen that are triangles? Why do you think that they are in this shape instead of another shape?").	P/B
	Nature of tasks	P/B	Similar to use of resources, sorting task and modeling task (in which students create triangles by linking pretzels and marshmallows) feature a high level of cognitive demand but art projects do not meaningfully connect to concepts.			P/B
	Nature of prompts	NP	Several prompts included that encourage reflection, justification, and connections such as "which objects around the classroom are circles" and "explain what they built and why it is a triangle" but more prompts are needed, especially to support connections and reflection with respect to the art projects.	"I'd like to see more questions in your lessons focused on important mathematical ideas. I like your question about how a triangle and a circle are different. What other conceptual questions can you ask students in your various lessons to get them thinking about and articulating key ideas about shapes as opposed to simply gluing shapes together to make a random creation?"	PST added several prompts such as "Can you tell me things you have seen that are circles? Why do you think that they are in this shape instead of another shape?" but still overlooking a few crucial prompts such as "What makes the shapes in this column different from the shapes in this column?" during the sorting activity.	NP

Table 7 Continued

Unit Plan on Fact Families (1^{st})	Math concepts	NP	Unit sets students up to discover and define fact families for themselves by having students "notice and wonder" about related facts but does not discuss why there are always four facts in each fact family.	"It will be important for students to be able to explain WHY there are four equations in a fact family and to explore and explain the relationships between the addition and subtraction facts within each family."	PST added two prompts to lesson 2 designed to discuss why there are four facts in every family ("Do fact families always have 2 addition and 2 subtraction problems? Why or why not?") but gave no indication of the answer to these questions or supporting prompts that could be used to guide students toward the answer (e.g., "Is there any other number sentence you could create with these three numbers?").	NP
	Use of resources	NP/U	Students are prompted to model fact families in ways that make sense to them without explicit instruction from the teacher but anticipated use of manipulatives is not depicted or described.	"You mention the use of manipulatives but do not give any indication of how students might use these manipulatives to explore and explain the relationships within each fact family. How would the manipulatives be used to illustrate these relationships? Try using the manipulatives to do this yourself and see what you notice."	Anticipated use of manipulatives is still not depicted or described.	NP/U
	Nature of tasks	NP	Tasks position students as the primary doers (noticing and wondering, discerning missing facts, creating own fact families) but are not fully developed, especially in terms of the modeling tasks as indicated under use of resources. *In some cases, fact families to be modeled and/or discussed are not specified (e.g., "I will write a couple of problems on the board").*	"What manipulatives or tools would be best [for your modeling tasks]? How should they be used to best highlight the fact family relationships? How can you be prepared to guide students to use them appropriately so they don't end up confused? These should probably be used on day 1 to answer key questions such as why there are four facts in the opening activity fact family."	PST added that students would be able to come up to the projector and show their models and discuss the methods they think "would work best" but still no indication of what the models would look like or how the PST could guide them during exploration. *Some fact families to be modeled and/or discussed are still not specified.*	NP
	Nature of prompts	NP	Prompts are used throughout the unit to stimulate student thinking and discussion but are sometimes vague ("What do you already know that might be helpful here? What assumptions might we make?") and do not include prompts designed to foster understanding of why there are always four facts in a fact family.	"Your lessons could use more specific discussion prompts geared toward guiding struggling students and towards drawing out key ideas, such as WHY there are four facts in each fact family. How do we know this and how do we show this?"	No additional prompts added to guide struggling students (these prompts are still vague). PST did add two prompts designed to discuss why there are four facts in every family as noted above with respect to math concepts.	NP

Table 7 Continued

Unit plan on prime and composite numbers (4th)	Math concepts	NP	PST accurately defines prime and composite numbers and incorporates a conceptually oriented activity involving using counting beans to factor several numbers into equal groups (although he mistakenly uses the term "even" groups) to classify them as prime or composite. Remaining tasks rely on pencil-and-paper factoring (presumably using factor trees, although this is not stated) without making connections to the opening task with beans.	"It is unclear how you expect students to factor the given numbers. There is a strong emphasis on answers and not so much on how those answers were achieved. What methods were used? How do we show factors? How do we know for sure these are the factors? What does it even mean to factor? While I like your attempt to have students explore prime and composite numbers in lesson 1 and to notice the differences between them on their own, this activity is not set up appropriately. For one, it doesn't draw on students' supposed prior knowledge of factoring. It COULD draw on this if specific discussion questions were included to help students make appropriate connections between splitting bears into equal groups and what this has to do with factoring. Alternatively, Cuisenaire rods could be used or even grid paper. These types of tools should be incorporated more meaningfully throughout the unit to help all students develop a concrete understanding of the difference between prime and composite numbers and to provide students with a referent for their conversations about these types of numbers."	PST updated the instructions for the conceptually oriented factoring activity to find how many equal groups can be formed with the given numbers and switched counting beans for Cuisenaire rods; PST also added example solutions for showing the factors of 3, 4, 5, and 6 but did not use the unit cubes to show the factor of 1 for each value. Cuisenaire rods were also incorporated into another activity as well as the centers in lesson 2, although some of the values they are to be used for are quite large (13, 21, 37, and 49). A number of the expected solutions within the unit still imply a lack of connection to the tools such as "We think it's prime because it can only be divided by itself and one" rather than "We think it's prime because the only other Cuisenaire rod that can divide it equally is the unit cube." *One task still a timed prime vs. composite activity (technically final rubric shows P and B designations).*	NP
	Use of resources	NP	Beans are used to make connections between physical representations of prime and composite numbers and the written definitions but their use is insufficiently described, with no examples of expected solutions. Physical tools are not mentioned in the rest of the unit but would benefit several of the center activities described in which students sort and justify prime and composite numbers.			NP
	Nature of tasks	NP/U	Bean task requires further elaboration and use of proper terms (equal rather than even) as well as connections to other tasks in unit; *one task involves timing students' ability to determine whether a number is prime or composite (U).*			NP/U
	Nature of prompts	NP	Prompts encourage justification and connections (e.g., "How did your pod decide that 3 is a prime number?") but are few and far between.	"What guiding questions can you ask to help them understand key concepts?"	PST added several more prompts to the unit, including "Why did you/your group decide this number was primer or composite? How does its factors/factor pairs inform this conclusion? Did any groups get a different answer?" However, more prompts are still needed for the opening activity with Cuisenaire rods where the PST simply states, "students should notice that the Cuisenaire rod groups that they found are each number's factors" without asking any questions that would help to facilitate this connection.	NP

Note. Portions in italics represent elements of a unit plan that were included in my coding annotations as reasons for not assigning full proficiency but were not explicitly addressed in my feedback to students during Fall 2019.

As revealed in Table 7, PSTs made concerted efforts to improve their assignments based on feedback, in most cases making changes that improved the quality of their lessons. However, these improvements were not sufficient to satisfy the criteria for proficiency in the rubric components most closely associated with mathematical content knowledge. Understanding why and how mathematical procedures work seems to be one of the biggest challenges for PSTs. In most of the IEFA lessons and unit plans referenced in Table 7, PSTs were unable to incorporate satisfactory links between procedures and concepts, defaulting to incomplete explanations that relied heavily on previously learned rules and definitions. For example, in the unit plan on prime and composite numbers, the PST developed incomplete models using Cuisenaire rods (omitting the units cubes for a factor of 1) and relied on previously learned definitions to generate expected justifications for prime and composite numbers rather than relying on explanations that would arise from proficient use of the tools (e.g., “it’s prime because it can only be divided by itself and one” rather than “it’s prime because the only other piece that will divide the number into equal groups is the unit cube”). In the unit plan on fact families, the PST did not even attempt to develop visual models for the fact families and only added prompts—but no anticipated responses—as to why there appears to be four equations in a fact family.

In the IEFA lesson on measurement conversions, the PST struggled to come up with anticipated prompts and solutions that would clearly link knowledge of ratios and proportional reasoning to measurement conversions, instead vaguely mentioning “cross multiplying” along with one potential solution strategy that did not involve discussion of how or why splitting a 10-centimeter segment into equal groups of 2.5 relates to the ratio 2.5 centimeters to 1 inch. In the IEFA lesson on fraction multiplication, rather than attempt to address the feedback provided to her, the PST abandoned her original lesson plan and opted to focus on whole-number division instead.

Although it is possible that my feedback was misunderstood, I would submit that for most PSTs the lack of adequate revision is more likely indicative of weaknesses in content knowledge. As shown in Table 2, PSTs were able to demonstrate significant growth and proficiency in response to feedback on rubric components that require minimal, if any, content knowledge (e.g., *alignment between learning objectives and learning standards* and *suitability of resources*), indicating an ability to comprehend and apply feedback. It is possible to write objectives that align with given standards simply by replicating the language within the standards themselves without understanding the mathematics incorporated into them. Likewise, a PST can easily choose a relevant resource—especially when I provide explicit feedback recommending particular tools—whereas using a resource appropriately requires a deep understanding of the content.

Another indication that lack of mathematical content knowledge is at play is the fact that PSTs performed less well overall on their IEFA lessons in the rubric components of *mathematical concepts*, *use of resources*, *nature of tasks*, and *nature of prompts*, as exhibited in Table 2. On their IEFA lessons, PSTs attempted more ambitious mathematics, choosing higher grade level content and more advanced concepts such as fraction multiplication and probability but achieved lower proficiency levels than on their unit plans. On the unit plan assignment, more than half of PSTs (13 out of 21) chose grade-level standards from grades K-2; only five PSTs chose grade-level standards above third grade. In contrast, on the IEFA lesson, only six PSTs chose grade-level standards from grades K-2 and 13 PSTs chose grade-level standards above third grade.

Teaching Experience. Not all gaps in PSTs' lesson planning assignments are linked to mathematical knowledge alone. The subthemes associated with the circle labeled "Experience" are areas in which I would expect PSTs to perform significantly better with more teaching experience. Most of these subthemes fall at the intersection of "Concepts" and "Experience" because they will likely be addressed through a combination of both improved content knowledge as well as greater teaching experience. For instance, recognizing that certain numerical values will result in tedious work comes both from witnessing their pitfalls as students struggle with them in real time as well as from gaining a stronger grasp of the mathematical material (*adjustment to numerical values needed*). As someone with a deep knowledge of division, I can instantly recognize that 288 ÷ 9 would be a tedious problem to solve using base-10 blocks without ever having taught division to third-grade students. I know that none of the partial addends (200, 80, or 8) can be split evenly into nine groups, which will result in repeatedly regrouping the base-10 blocks such as by trading in 8 tens for 80 ones and then distributing all of those ones into 9 separate piles. Because of my content knowledge, I do not need to watch students spend hours working through this problem to know the outcome—but I would quickly learn to think more carefully about the numbers I choose if I were in a classroom where I assigned this problem but did not at first possess the requisite level of content knowledge.

Likewise, recognizing that the wording of a problem will not produce the intended outcomes or methods is something that would result from a combination of experience and increased content knowledge. In a real classroom with real students, I would eventually come to realize that asking my students to identify the most common outcome (as one of my PSTs asked in her IEFA lesson) from a graph that only displays the sum of outcomes for each group rather than each individual outcome would result in my students consulting their tally sheets rather than their graphs. If I did not possess the requisite knowledge of graphs to recognize this when I planned my lesson, experience would fill in the gap.

The subtheme *concept development rushed*—associated almost exclusively with the rubric component *developmental progression and pacing*—stands alone within the broader theme of "Experience" rather than at the intersection of "Concepts." Knowing how to appropriately pace the introduction and development of a new concept is a skill that can only be learned through experience working with real children and adolescents. While PSTs can study learning trajectories, this knowledge has more to do with understanding child development than with understanding mathematical content. Since my PSTs are accustomed to taking college courses in which new topics are often introduced in each subsequent class period they attend, they tend to assume children will likewise quickly move from one topic to the next, such as learning the coin values on one day, adding them the next day, and then subtracting them on the third day, as one of my PSTs anticipated in her unit plan draft. In response to feedback she received from me on indicating that she was moving too quickly through the material, the PST adjusted her final draft to assume prior knowledge of coin values but still proceeded through both adding and subtracting coins within three days, overlooking the need to slow down and discuss methods students would use for finding sums and differences of coins, seemingly assuming students would automatically transfer their knowledge of two-digit addition and subtraction to do so. While this PST's content knowledge of coins and their use was sufficient as indicated by the questions she asked and the scenarios she posed in her unit plan, her knowledge of children's developmental needs was not. With an opportunity to test her plan in a second-grade classroom, this PST would quickly realize that her unit was too ambitious and would need to proceed at a slower pace.

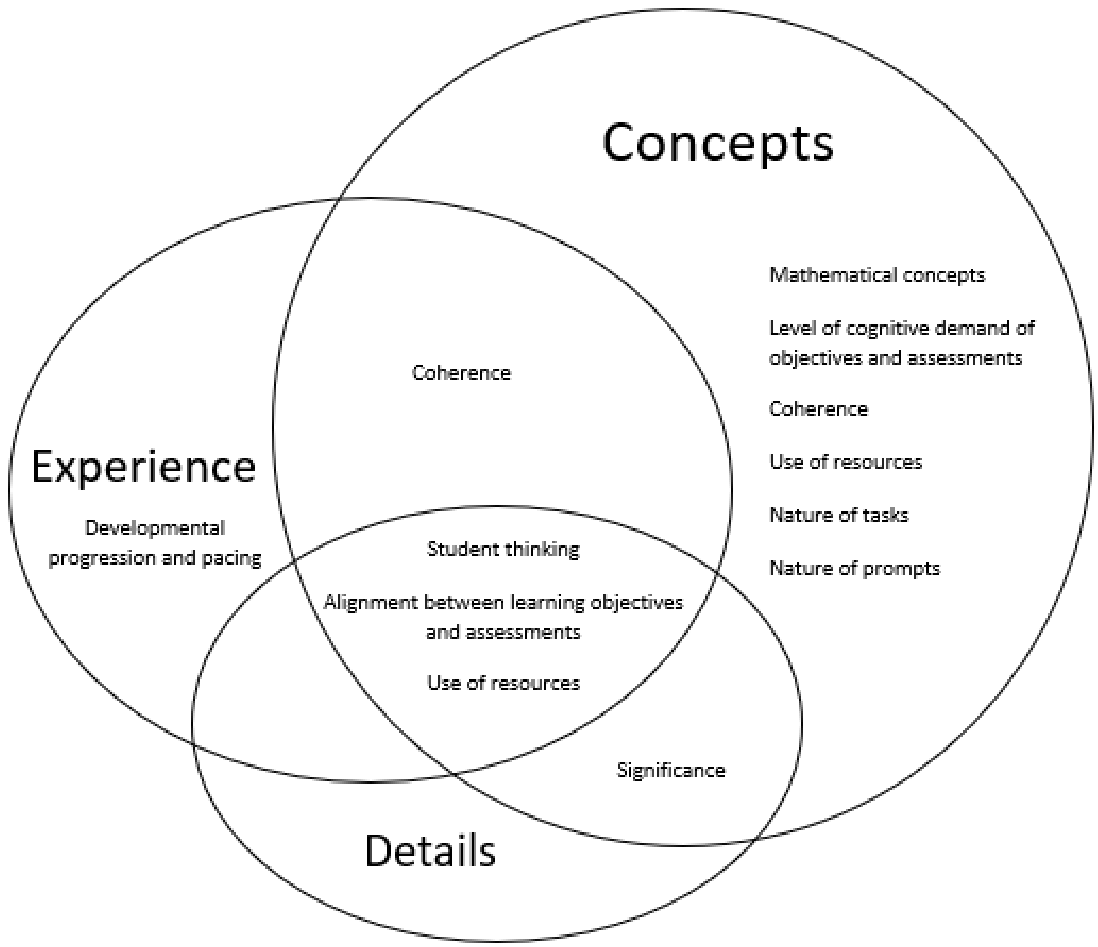
Details. This circle encompasses subthemes that indicate a lack of sufficient detail in PSTs' lessons and unit plans. Since a lack of sufficient detail in describing the implementation of a task and/or the concepts and procedures associated with it might also indicate that PSTs did not possess the requisite content knowledge to adequately plan the task, *not fully developed from start to finish* falls at the intersection of "Details" and "Concepts." The subthemes at the intersection of all three circles—*aspects of concepts or procedures overlooked, useful tools, representations, or formats overlooked,* and *expected methods or responses not specified*—suggest either that PSTs were not thorough in planning out their drafts ("Details"); did not possess the requisite content knowledge to incorporate all key tools, representations, formats, and/or anticipated responses ("Concepts"); or did not have enough teaching experience to be fully prepared for these aspects of lesson planning ("Experience").

Most of the lesson plan elements coded to the subthemes at the intersection of all three overarching themes were associated with the rubric components *student thinking*, *use of resources*, and *alignment between learning objectives and assessments*. Each of these components require providing examples of anticipated solutions and responses, which many PSTs did not include in their lesson plans.

Although there are no subthemes that fall solely within the broader theme of details, my experience working with PSTs suggests that lack of diligence in completing assignments according to the full set of criteria is likely a significant factor—along with insufficient content knowledge and lack of teaching experience—in PSTs not achieving full proficiency; hence, it is included as a separate, albeit smaller, theme within Figure 3 despite the fact that no subthemes are uniquely associated with it. Indeed, as related below on my findings from the one-on-one interviews with PSTs who student taught during the subsequent semester, my students do not always have or set aside sufficient time to complete their assignments to the requested level of detail.

Relationships Between Overarching Themes and Rubric Components. Figure 4 summarizes the rubric components most closely associated with each of the overarching themes. According to my annotations on all *nearing proficient* codes from my first cycle of coding, *mathematical concepts* and *nature of prompts* appeared to be the most salient components among those associated solely with "Concepts." The reasons why my PSTs were unable to demonstrate *proficient* performance in the other four components associated with "Concepts' most frequently had to do with the fact that PSTs did not possess deep enough content knowledge to ask questions that would establish high-level learning objectives; ensure closure on key mathematical ideas (embedded in *coherence*); connect procedures to concepts via the use of resources; and draw out underlying concepts from tasks. These other components were all solidly on the right track in terms of setting students up for meaningful mathematical learning but needed to be better supported by more targeted questioning. For example, in the IEFA lesson included in Table 7, the PST designed an excellent task for students to convert measurement units, with the intention that students would come up with their own strategies based on prior knowledge, but the PST did not seem to clearly understand how one would apply a basic ratio (1 inch = 2.5 centimeters) to convert other centimeter lengths to inches using methods other than cross multiplying, meaning she could not develop appropriate questioning to guide student thinking and discussions. Without this questioning, the implementation of the task was not fully clear and the closure did not successfully tie together multiple potential strategies.

Figure 4. Overarching Themes and Their Relationship to Rubric Components



The rubric components related to "Experience" (both individually and at the intersection of "Concepts") were primarily concerned with lesson pacing and sequencing. Without much experiencing teaching mathematics to K-8 students, my PSTs often proceeded too quickly through concepts, assuming a single lesson, task, or discussion would result in sufficient understanding on the part of students. They also sometimes sequenced lessons in ways that would not optimize concept development, such as by inserting tangential tasks or discussions into otherwise coherent lessons.

Rubric components at the intersection of all three overarching themes almost exclusively had to do with PSTs not providing adequate indications of expected solution strategies or responses. While assessments may have been designed well and resources chosen appropriately, examples of satisfactory work and explanations were often overlooked, either due to lack of attention to details, lack of experience with student thinking, and/or lack of sufficient understanding of the underlying mathematical concepts.

Insufficient Feedback

It is important to note that a lack of full proficiency on lesson and unit plans is not solely a result of gaps or inadequacies in my PSTs' knowledge and experience. As indicated by the italicized elements in Table 7, in some cases I failed to provide feedback on lesson components that would have benefitted from revision according to the annotations I made on PSTs' assignments during my first cycle of coding. Because the rubric I utilized for this study was more comprehensive than the rubric I utilized during Fall 2019—having updated my original rubric to incorporate the AMTE *Standards* to help me answer my research questions—I overlooked certain elements of PSTs' lesson plans in the feedback I provided to them. One student in particular would have likely achieved proficiency in nearly all aspects of her unit plan had I provided feedback on her first draft encouraging her to support students in making connections across the various representations included in her unit plan on fraction equivalence. She included a variety of appropriate formats in her unit plan (graham crackers, fraction strips, pies, and pizzas) and utilized all of them accurately but did not incorporate any discussion relating the various formats to one another, an oversight that resulted in a designation of nearing proficient performance in *use of resources* and *nature of prompts*.

On about half of the drafts, I failed to provide feedback related to prerequisite knowledge and skills that PSTs overlooked, likely because this component was embedded in the overall domain of *demonstrating knowledge of content*, for which there were often more pressing issues I needed to address in my feedback (such as assuming a scaled bar graph implied rounding the data to the nearest thousand rather than using a scale other than 1 on the y-axis). Since my domains were not partitioned into distinct components in my original rubric, I sometimes overlooked elements of the domain that were less pressing than others. However, given that incorporating prerequisite knowledge and skills effectively into a lesson or unit requires strong content knowledge, I suspect that even with greater feedback on this component, most PSTs would not have achieved full proficiency.

One rubric component—*coherence*—included criteria that was not in my original rubric: "opportunities for reflection and closure." Not including this criterion for lesson planning in my original rubric was clearly a major oversight on my part, and because it was not included I did not leave feedback related to closure on my PSTs' drafts. It is possible that more PSTs may have achieved greater proficiency in *coherence* had this not been the case. I provided no feedback related to closure during Fall 2019, yet for nine of the IEFA final drafts and two of the unit plan final drafts, this was the only reason that PSTs did not achieve proficiency in *coherence*. However, given my PSTs' struggles to adequately link procedures and concepts in their lesson plans, it is likely that even with feedback indicating their oversight of sufficient closure, many would have still struggled to incorporate satisfactory closure on the mathematical concepts incorporated into their lesson plans.

Supplementary Data: Interviews

The data from the one-on-one interviews conducted with four of the PSTs from the Fall 2019 cohort bolsters the overarching themes identified above. Three of the four PSTs identified themselves as needing to develop stronger content knowledge in mathematics (mathematical concepts); all of them indicated that they felt they would learn more about math instruction once they were in their own classrooms (teaching experience); and one of them admitted to not adequately revising an assignment because she ran out of time (details). None of the data contradicted these themes or suggested other notable themes.

Mathematical concepts. During each interview, I asked each PST how confident they felt about teaching math in their future classrooms. In their responses, three of the PSTs admitted to feeling weaker in math than in other subjects (the fourth PST identified math as her strongest subject). For instance, one of the PSTs acknowledged, “overall math can be a little intimidating for me, and um, more so at those higher grade levels.” Another PST reflected, “I don’t think of myself as being fully trained in math; I still have a lot to learn. But I’m a lot less scared of it now.” Even the PST who felt that math was her strongest subject admitted to gaps in her knowledge with respect to *use of resources*: “I feel confident in the story problems and I feel confident in, you know, being positive. But manipulatives can sometimes—truthfully the first time I worked with manipulatives as much as I did was in your class. […] It will take a little bit of getting used to."

When asked about the feedback that was provided to them on their assignments, none of the PSTs indicated that they did not understand the feedback, lending support to my hypothesis that gaps in content knowledge—as opposed to not understanding feedback—was the most likely contributor, along with lack of teaching experience and lack of attention to details (discussed below), to less-than-proficient performance on final drafts of my PSTs' assignments. The PSTs I interviewed made statements such as,

> I was frustrated when I got the feedback. I was like, "Ah, are you kidding me? I thought this was great!" And then I spent a long time fixing it. And after—while I was doing it—no one likes to do all that work, so you know? But *now* I feel pretty thankful that I put in all the work because I feel like I learned a lot from it.

Another PST echoed the reflection above:

> I feel like it was good because those are the questions that we need to ask ourselves and it's good to have somebody ask those questions before we have a lesson flop because now, I mean, I really haven't had a lot of lessons flop and I think it's because I was prepared so much with those detailed lesson plans. But it just sort of, you know, once you get into actually teaching and making lesson plans for the whole day and every subject, you're like, "That was a lot of work" that, you know, you want to say, "That was so unnecessary." But at the same time, we learned from it, you know? So your feedback was great because it gave us—it made us think about, "Oh, maybe I really *am* missing something." And you know, it'll make me think about it when I'm teaching in my classroom. Like *don't* forget this. You know? It could be those little nuggets that I'll always think about, like is my objective measurable? Is it exact? Can somebody come in and see exactly what they need to do to meet the objective?"

Two of the PSTs mentioned how helpful the level of detail in my feedback was compared to the feedback they have received from other instructors. For instance,

> A lot of teachers in the ed program—you turn things in and they kind of just, "Good job, you did it!" Kind of. I mean, they don't quite go into the depth of feedback that you give, and I think that might be half the reason why a lot of people were surprised by your

> feedback because a lot of other teachers don't put in that type of work. But I think that if you wouldn't have done that I wouldn't have gotten as much out of that course as I did."

Another PST reflected,

> It was always helpful because some teachers would say "Great job!" and give you a three out of four or gave you a four out of four and you always have room for growth, [but] they never said more. And if you would have done that then we wouldn't have known what we needed to change.

The same PST did note, however, that my feedback was sometimes difficult to break down because "seeing a whole paragraph is kind of intimidating to me." She recommended using bulleted lists or somehow organizing the feedback into more discrete chunks, which I have done in my updated rubric by breaking each domain into discrete components.

<u>Teaching Experience.</u> All four of the PSTs made comments indicating that they felt they would learn much more about math instruction once they were in their own classrooms. For instance, one PST reflected, "I thought it was going to be direct instruction and a lot of 'I need to just know what I'm doing.' […] But at the same time, like I've kind of already said, I'm going to be learning along the way," and "we're kind of learning together because like my fifth graders that I student-taught with, they were teaching me things that I didn't even know." She discussed how she had learned new types of solution strategies from her students that she had never been exposed to before, which relates to the subtheme *useful tools, representations, or formats overlooked*. In her case, greater experience enhanced her knowledge in this area.

Another PST discussed how she would need to learn with experience how to best integrate what she had studied in her special education courses (which mostly emphasized direct instruction) and what she had studied in my classes (which mostly emphasized teacher-guided discovery of concepts) when working with special education students in her future classrooms since she believed that the more hands-on approach she had learned in my courses would be beneficial for special education students. One of the other PSTs alluded to the impact of experience when she stated, "I feel more confident in the lower grades just because that's where I have experience."

Related to teaching experience is the ability to observe master teachers practicing their craft, as discussed in Chapter 3 on Situated Learning. While my PSTs felt that they ultimately needed more classroom experience to strengthen their knowledge and skills, they all commented on the impact that my modeling of *Standards*-based tasks had on their learning. Three of the four PSTs cited instructor modeling as the number one factor contributing to their learning and growth in the course while the other PST cited it as the second-most impactful factor (with the course textbook being the number one factor). One PST reflected,

> When we had to do the hands-on activities in class was very helpful because then I got to practice using the manipulatives and seeing how they worked and then going back to that "don't give answers" and look more at the strategies that they're using […] and having us practice going up and explaining our strategies and seeing my peers' different ideas from my own was all very helpful because then I saw how that works and what that looks like, and for me I need to do to learn.

Another PST commented,

> I think that you teaching us as if *we* were the elementary students and kind of modeling what it's like […] but also seeing you model that instruction over a period of time rather than just like doing it one time was really helpful.

Unfortunately, while my PSTs appreciated the opportunity to experience *Standards*-based tasks in my methods course as if they were K-12 students and I were their K-12 teacher, they did not get to observe *Standards*-based instruction in real K-12 classrooms. None of the four PSTs were able to be placed in classrooms with educators who utilize reform instruction, at least not as their primary approach to day-to-day teaching. One PST was particularly frustrated with her cooperating teacher's (CT's) approach to instruction and described how she primarily learned from her CT what *not* to do to effectively teach mathematics. She reflected,

> I think that understanding where math comes from is, like, really important, and trying to then relate that to the kids, so making the story problems personal is really important and I think that that's for sure something that I want to implement [...] My [CT] never really did story problems. She would actually cross them off. But it was helpful because now I know how important that is.

Another PST made similar comments about her CT, noting that she used "a lot of worksheets" and "step-by-step" instruction, which the PST said was "nice to see" because it modeled for her how *not* to engage students in mathematical exploration and sense making. While the PSTs mentioned learning from their student teaching experiences, none of them specifically cited their CTs as a source of that learning.

<u>Details.</u> As suggested earlier in this chapter, a probable contributing factor to less-than-proficient performance on lesson planning assignments is a lack of attention to detail, which for many PSTs is likely due at least in part to insufficient hours in a day to attend to all their classes and personal responsibilities (about one-fourth of the students at my university are non-traditional students, and many are parents). Although I did not explicitly inquire about it, one of the PSTs admitted,

> I know there were a couple of assignments where I did get a lot of feedback but I don't have time to fix all of these things—like I have six other classes, and I have to work, and I think I did the best I could on those. But if I *would* have fixed all of the things in your feedback, I think I probably would have learned a lot more than I did, even though I still feel like I learned a lot in class.

Summary

The themes that emerged from my second cycle of coding suggest that a lack of strong mathematical content knowledge may be limiting PSTs' ability to achieve full proficiency on their lesson planning assignments. As suggested by Figure 3, a lack of classroom teaching experience and/or a lack of attention to detail are also hypothesized to be contributing factors to the gaps and oversights in my PSTs' lesson planning assignments. Data from interviews with four of the PSTs during the subsequent semester support these themes. In some cases, a lack of sufficiently comprehensive feedback from me resulted in less than proficient performance on assignments, although this was not a major contributing factor in PSTs' overall performance. Ultimately, as discussed in the next chapter, more research is needed to tease out the reasons why PSTs achieved nearing proficiency rather than full proficiency on so many components of the AMTE *Standards*-aligned course rubric.

CHAPTER FIVE

DISCUSSION

Overview

Through this study, I aimed to better understand my PSTs' performance on lesson planning assignments for K-8 mathematics. After first implementing standards-based learning in my K-8 math methods course during Spring 2019 in hopes of improving my students' learning outcomes, I took a deep dive into their lesson plans from Fall 2019 to identify areas of growth and continuing weakness resulting from the feedback and revision cycles embedded into the course. Specifically, I addressed the following research questions:

- In what ways does the opportunity to receive timely and specific feedback and to revise lesson plans accordingly help PSTs to better develop and apply their knowledge and skills with respect to AMTE's 2017 *Standards*?
- In what ways are PSTs meeting and not meeting AMTE's 2017 *Standards* in their lesson planning assignments?
- What themes emerge from PSTs' work on lesson planning for K-8 mathematics?

In this chapter I situate the findings presented in Chapter 4 within the theoretical framework of Situated Learning and its interrelated arms of *access* and *achievement* as well as within the broader literature on mathematics teacher education. I also discuss implications of the findings for mathematics teacher educators (MTEs) and educator preparation programs (EPPs) and make suggestions for further use and experimentation with the analytical framework developed for the study.

Understanding PST Achievement: Limitations on Growth

In this section, I discuss the insight that my findings from Chapter 4 contribute to the existing literature on pre-service teacher performance in K-8 mathematics. As related in my theoretical framework (see Chapter 3), masters of a profession must concern themselves with the breadth and quality of the knowledge and skills that newcomers to their field acquire during apprenticeships. In the field of mathematics teacher education, a comprehensive set of standards for pre-service mathematics teachers has been developed to gauge PSTs' readiness for autonomous mathematics instruction, yet these standards have not been specifically applied to research studies on PST performance. As such, my second research question has not yet been addressed in the literature, while findings associated with my first and third research questions complement existing findings related to PSTs' mathematical content knowledge, as discussed below.

Mathematical Concepts.

As related in Chapter 2, existing research has shown that PSTs struggle to fully comprehend the concepts that underlie K-8 mathematics (Chinnappan & Forrester, 2014; Inoue, 2009; Luo et al., 2011; Shirvani, 2015; Zilkova et al., 2015). However, the relationship between PSTs' mathematical content knowledge and their lesson planning capabilities does not appear to have been extensively studied. We know PSTs struggle with fractions (Chinnappan & Forrester, 2014; Luo et al., 2011), proportional reasoning (Inoue, 2009), and measurement (Shirvani, 2015), and even two-dimensional shapes (Zilkova et al., 2015) but we do not know how this lack of content knowledge impacts lesson planning on these topics. In one study by Morris and

Hiebert (2017), the strength of the association between overall scores on a lesson planning assignment and the extent of coverage of the associated math topics in the participants' math content courses was explored, with findings indicating that more in-depth content coverage resulted in stronger lesson plans. However, the study participants were already in their second and third years of teaching at the time of the study, leaving a gap in the literature as to the impact of mathematical content knowledge on PST lesson planning, which the present study addresses.

In analyzing the reasons why many of my PSTs were able to achieve nearing proficient performance but not proficient performance on their lesson planning assignments, which was the focus of my second cycle of coding, I found that most of the reasons had to do with an insufficient understanding of mathematical concepts (see Chapter 4, Figure 3), which aligns with the findings from Morris and Hiebert (2017). While PSTs demonstrated growth in several areas of my AMTE *Standards*-aligned course rubric and made strides in presenting material in ways that would engage students in relevant, exploratory, hands-on activities, they struggled to adequately link procedures and concepts in a way that demonstrated the "deep and flexible knowledge of mathematics" called for by AMTE (2017, p. 2), limiting their ability to design robust mathematics lessons.

As Figures 6-7 illustrate, my PSTs demonstrated the strongest performance and the most growth in areas that did not require a deep knowledge of mathematical content (Figure 5), whereas they demonstrated weaker performance and less growth in areas requiring stronger content knowledge (Figure 6). The sole component of the *Standards*-aligned course rubric in which a majority of PSTs achieved proficient performance was *alignment between learning objectives and learning standards*. While being able to align standards and objectives is an essential aspect of lesson planning, doing so requires minimal content knowledge since PSTs, once directed to the appropriate standards via feedback, can simply replicate the language of the standards to write clear and measurable objectives. Other components in which PSTs demonstrated the greatest improvements—although not full proficiency—included *student thinking, developmental progression and pacing, suitability of resources, use of resources, nature of tasks,* and *nature of prompts*. For two of these components—*student thinking* and *nature of prompts*—improvements resulted mostly from adding required details that had been overlooked in the first draft of an assignment (such as incorporating specific prompts within a lesson plan along with expected student responses to those prompts). As with *alignment between learning objectives and learning standards*, improving performance in *suitability of resources* requires attention to feedback more so than knowledge of content, given that once appropriate tools are indicated, PSTs need only include them in their lists of resources for their lessons.

Figure 5. PST Growth in Components Not Requiring Strong Content Knowledge

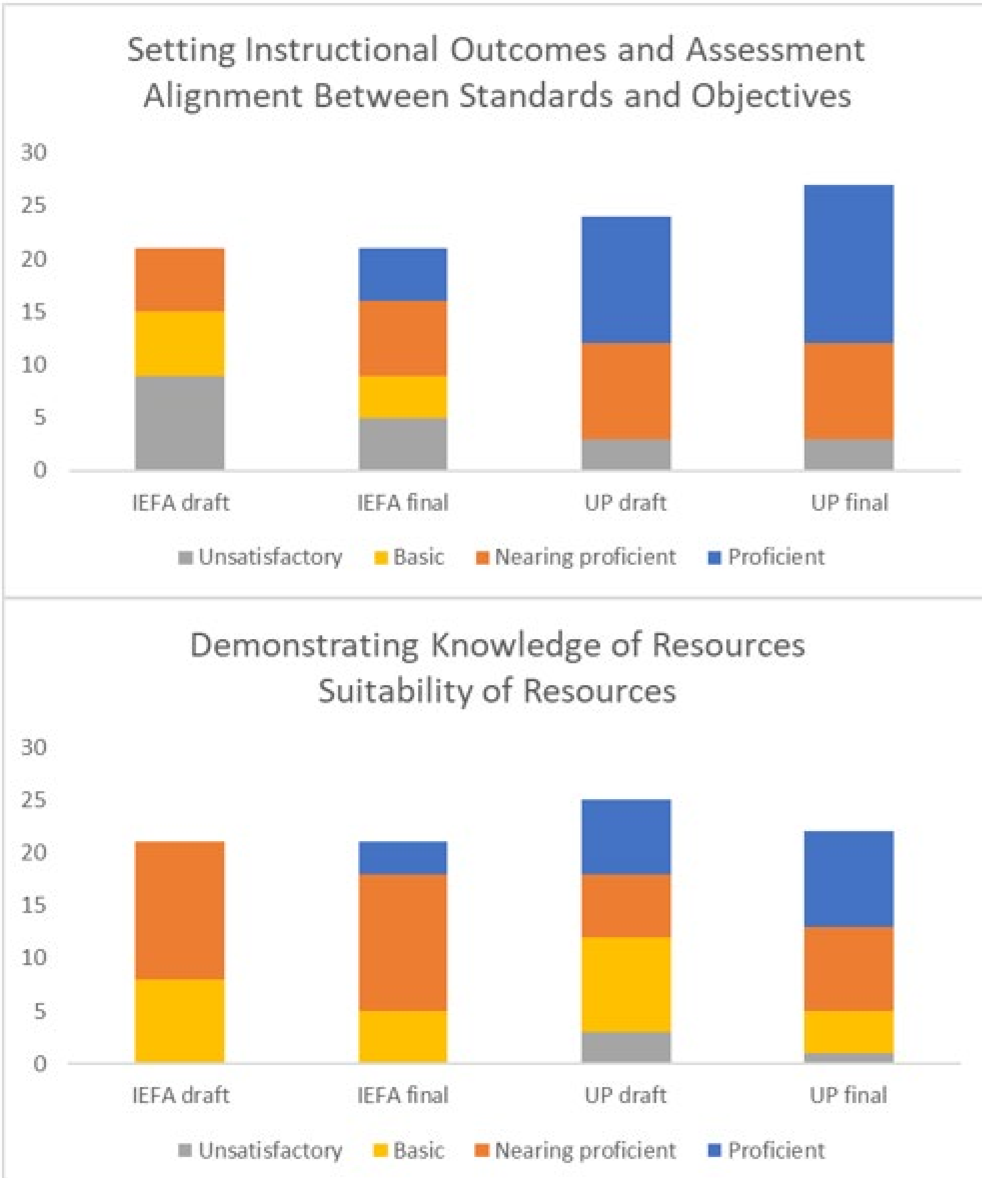
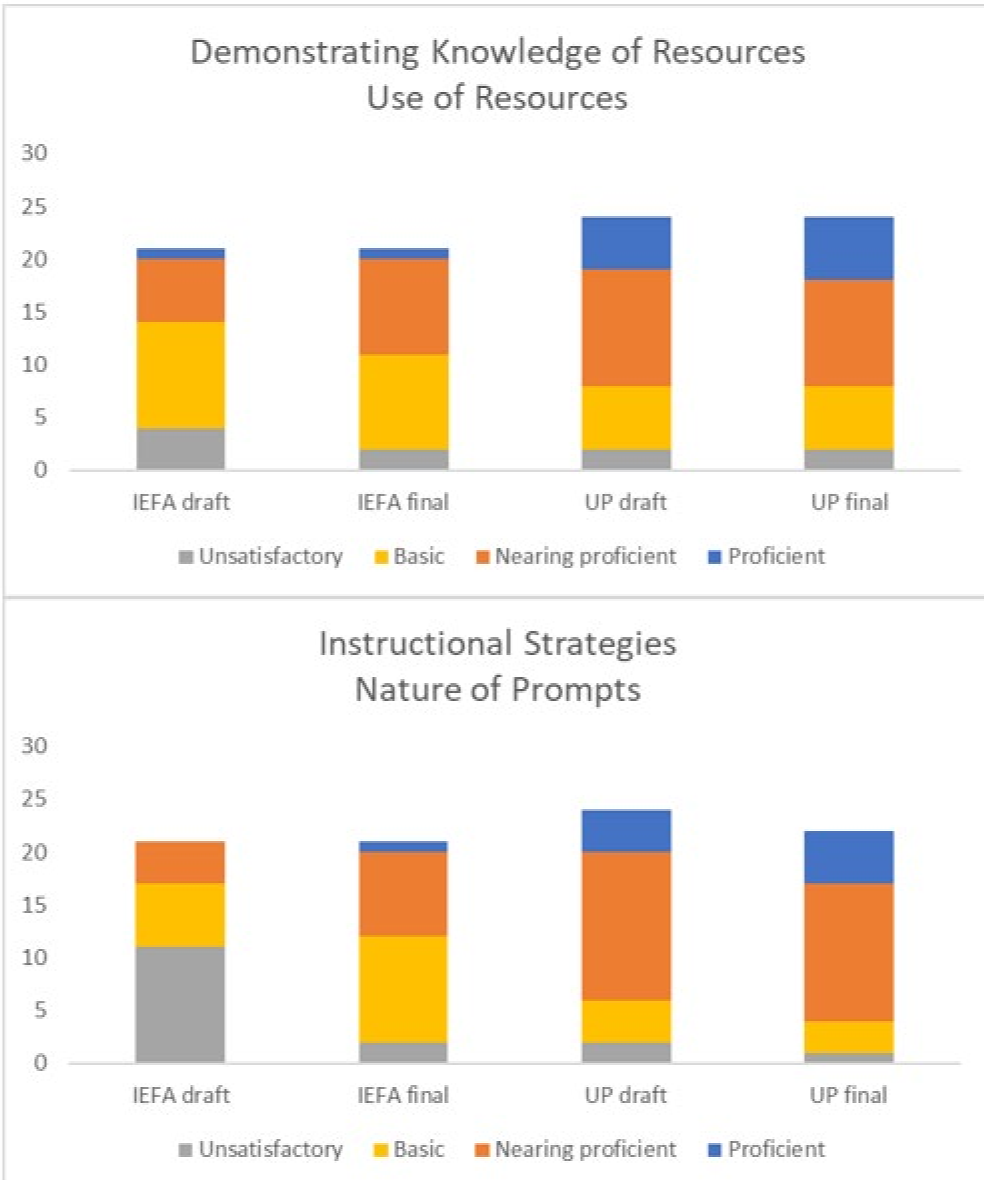


Figure 6. PST Growth in Components Requiring Strong Content Knowledge



To the contrary, *use of resources* does require knowledge of content. To use a tool appropriately necessitates an understanding of the physical representation of a concept. PSTs demonstrated growth in this component by clarifying and adjusting their use of tools across the semester to support more exploratory, conceptually based activity as opposed to merely using tools to arrive at answers. Likewise, they developed their ability to present tasks (*nature of tasks*) and pose questions (*nature of prompts*) that would engage students in higher-order thinking skills and not just rote application of procedures. In many of the IEFA lessons—the first independent lesson planning assignment—PSTs incorporated rich scenarios but only required students to perform basic mathematical skills, such as adding and subtracting decimals using the standard algorithm, rounding large numbers, and converting fractions to decimals and percents. In their unit plans—the final independent lesson planning assignment—PSTs incorporated more open-ended problem solving such as identifying and justifying equivalent fractions, decomposing numbers in more than one way, and sorting and distinguishing among shapes. Feedback received on the IEFA lesson—which was provided to PSTs the week they began drafting their unit plans—along with my continual modeling and discussion of open-ended math tasks during class likely played a role in the improved use of resources in PSTs' unit plans.

Limitations on Growth. Despite showing overall improvement in *use of resources*, *nature of tasks*, and *nature of prompts*, PSTs struggled to achieve full proficiency in these areas due to their inability to adequately utilize tasks, prompts, and tools to link procedures and concepts. In many cases, PSTs could pose a meaningful problem, choose an appropriate tool, and develop a mostly accurate model of a concept and/or procedure using the tool, but they could not use the tool to foster an understanding of why a procedure works based on the underlying concepts. For example, in a unit on fact families, the PST referenced an appropriate tool (snap cubes) and asked higher-order questions to encourage students to discover that there are four facts in each family, but she failed to adequately address feedback encouraging her to ensure that students would walk away from the unit understanding *why* there are four facts in each family.

Many PSTs demonstrated an exceptional ability to pose mathematical and real-world scenarios that lend themselves to modeling and illuminate important mathematical concepts (*significance*). Most PSTs were able to develop appropriate word problems, choose relevant data, identify suitable games, and otherwise set students up to explore and represent important mathematics. PSTs were often quite creative in establishing scenarios for their students to engage with, such as figuring out how much hay would be needed to feed a team of horses at an Indian relay, converting heights from a European document (given in centimeters) to more familiar units of measurement, and determining the various ways in which a graham cracker can be broken up along its scoring lines such that two people get the same amount. But as with *use of resources, nature of tasks*, and *nature of prompts*, PSTs struggled to utilize these scenarios to adequately link procedures and concepts. PSTs often knew which concepts were invoked by which scenarios but not how those concepts informed the procedures. For example, the PST who developed the lesson plan on converting metric-system measurements to English-system measurements knew that ratios are involved in converting measurement units but could not explain how or why procedures such as “cross multiplying” work.

PSTs were often on the cusp of developing exactly the type of math lessons envisioned in the *Standards*—relevant, hands on, and student centered—but were unable to fully bridge the gap between procedures and concepts. As related in Chapter 4, this shortcoming often resulted from an inability on the part of PSTs to support their tasks and resources with appropriate prompts designed to draw out key concepts. For instance, in the lesson on measurement conversions, the PST was unable to come up with questions such as, "How does the strategy of segmenting 10 centimeters into 4 groups of 2.5 relate to the strategy of using the equivalent fractions $1/(2.5) = 2/5 = 4/10$ to figure out how many inches are in every 10 centimeters? How can these two strategies help us to understand how 'cross-multiplying' works?" Given the weaknesses in PSTs' content knowledge identified by both the literature and this study, it is likely that most PSTs are not aware of the connections among various procedures and representations and hence do not know when and how to incorporate such prompts into their lesson plans.

In a dissertation study analyzing the development of PSTs' mathematical knowledge for teaching (MKT) over the course of the senior year of their EPP—during which they completed their math methods course (fall) and student teaching (spring)—Johnson (2011) similarly found that the participants struggled to demonstrate competency in the "connection" domain of the analytical framework utilized for the study, which included making connections between procedures and concepts and anticipating appropriate models, prompts, etc., to support students in developing a deep understanding of the mathematics. As Johnson reflected,

> Perhaps connection is an especially challenging category to enact because its dimensions draw on three realms of knowledge that are in and of themselves challenging: understanding the connected nature of mathematics, having content knowledge that is connected across procedures and concepts, and understanding children's thinking. It was not possible to determine whether the relative difficulty associated with the connection category for these preservice teachers was an issue of awareness or translation to practice. Either way, it seems that these preservice teachers may have especially benefited from additional awareness of and experiences with dimensions in the connection category. (p. 328)

In the subsequent section, I draw from my findings as well as those of Johnson (2011) and others to make recommendations for improving PSTs' MKT.

Improving Access: Enhancing PSTs' MKT

According to Situated Learning theory, apprentices must be granted sufficient and appropriate *access* to the knowledge and skills of masters to demonstrate the levels of *achievement* required to become autonomous participants in their respective fields. In *Situated Learning: Legitimate Peripheral Participation*, Lave and Wenger (1991) document key features of successful apprenticeships, including opportunities to both observe and participate in the work of skilled masters in such a way that apprentices can learn "how masters talk, walk, work, and generally conduct their lives" (p. 95). In this section, I couple my research findings with those from the literature to recommend ways in which PSTs can be granted greater access to the skills and knowledge of master teachers and MTEs such that PSTs can improve their MKT and hence their proficiency with respect to AMTE's *Standards*.

Increasing Access to Content Knowledge

A key finding from this study is that PSTs can develop their ability to design compelling mathematical tasks within the time span of a semester—coming up with creative, open-ended activities that lend themselves to appropriate mathematical modeling—but the effectiveness of these tasks is underdeveloped due to PSTs' weak MKT, which limits their ability to ask critical questions and bring about closure linking concepts to procedures. Strengthening PSTs' MKT is imperative to supporting their ability to design and implement math lessons that meet AMTE's *Standards*.

Skilled math educators and MTEs "understand and solve problems in more than one way, explain the meanings of key concepts, and explain the mathematical rationales underlying key procedures" (AMTE, 2017, p. 8). For PSTs to acquire this robust knowledge of content—which Johnson (2011) and others have referred to as *mathematical knowledge for teaching* or MKT (Ball et al., 2008; Hill et al., 2008) to distinguish it from the more ordinary content knowledge required for everyday problem solving—they must have opportunities to participate in the type of thinking that teachers who are well-versed in MKT engage in. As identified in my findings, a lack of understanding as to *why* procedures work acted as a ceiling for many of my PSTs, preventing them from advancing their lessons from nearing proficient performance to fully proficient performance. Given the link between greater MKT and more effective mathematics instruction (Hill et al., 2005; Hill et al., 2008; Morris & Hiebert, 2017) as well as links between MKT and confidence in teaching identified in this study as well as others (Ingram et al., 2018), interventions aimed at increasing pre-service teachers' MKT stand to have a significant impact on the quality of mathematics instruction in our nation's classrooms.

<u>Tailoring Math Content Courses.</u> Mathematics courses designed specifically for pre-service teachers have been shown to improve PSTs' content knowledge to a greater degree than general mathematics courses (Matthews et al., 2010). Courses incorporating activities that emphasize informal proof writing and argumentation are particularly valuable in improving pre-service teachers' MKT, particularly with respect to understanding the connections between procedures and concepts. For example, PSTs might be tasked with utilizing base-10 blocks to model the standard long-division algorithm and explain how and why it works rather than simply

modeling division word problems with the tool. This type of “unpacked” content knowledge, in which PSTs gain experience in linking procedures to concepts, is associated with more effective lesson planning once PSTs are in their own classrooms. In a study of 27 graduates of a single teacher education program, Morris and Hiebert (2017) found that teachers were more capable of developing robust lesson plans on content they had covered in its “unpacked” form in their math content courses than on content they had not covered in their content courses. Given that the most notable limitation to PSTs’ growth in the present study was an inability to connect procedures and concepts, approaching mathematics in this unpacked form stands to bring about significant gains in PST achievement with respect to lesson planning. With a deeper understanding of the underlying concepts, the study participants would have been able to respond more appropriately to feedback encouraging them to consider and connect multiple solution strategies, ask and answer “why” questions relating to the mathematics, and provide closure on key concepts.

Also valuable would be opportunities for PSTs to rehearse in-depth explanations of mathematical procedures and concepts in front of their MTEs and peers before being placed in real K-8 classrooms. As Inoue (2009) suggests,

> what is necessary for improving novice teachers’ [pedagogical content knowledge] and the quality of their explanations could be to conduct a series of evaluations of their explanations of the contents and give them detailed feedback on their instructional explanations. In fact, similar types of pre-service training are often used in other fields such as acting, where actors need to practice their scripts through a number of rehearsals before the performance, and music, where musicians rehearse their parts individually before playing with an orchestra in a concert hall. Similarly, it is reasonable to consider that pre-service teachers need to rehearse providing effective instructional explanations before actually teaching in the classroom setting. (p. 49)

In a study of 34 PSTs' instructional rehearsals, Inoue found that even PSTs with strong mathematics backgrounds struggled to adequately convey procedures and concepts in ways that would support K-8 students in making connections between the two, indicating a need for PSTs to have more practice in explaining the meaning behind procedures with the opportunity to receive constructive feedback on their explanations. Inoue is careful to point out,

> this is not to say that we should go back to conventional teaching characterised by the absorption model of teaching. Quite the contrary, what is suggested here is that teachers' explanations serve as an essential pedagogical component in scaffolding the students' reflective construction of conceptual knowledge. (p. 47)

The above comment relates to the study findings regarding PSTs' shortcomings in developing appropriate prompts to guide and support student thinking as they complete tasks. Part of scaffolding students' construction of knowledge involves asking critical questions as students work through tasks, prompting them to notice salient features and connections. In order to develop such prompts, PSTs must first enhance their ability to understand and articulate the salient connections themselves. Rehearsing explanations for why and how procedures work could be a fruitful early exercise in math content and/or methods courses to prepare PSTs for more robust lesson planning later on. If each PST in a cohort is assigned a different procedure to justify publicly in front of their peers, exposure to a variety of procedures and their rationales could be fostered.

Content courses that emphasize "connections or relations between mental representations of a concept" have also proven fruitful in helping PSTs to link procedures and concepts (Chinnappan & Forrester, 2014, p. 874). In their study of 114 PSTs enrolled in a first-year mathematics content and pedagogy course, PSTs who engaged in representational reasoning about fractions were able to improve their conceptual understanding of fraction operations. As the researchers concluded,

> Pre-service teachers can be supported, within an existing teacher education program, to construct conceptually and procedurally robust content knowledge through the development of appropriate representations of fraction concepts and operations. Further, our experience showed that the use of representationally rich instruction could enhance the building of the fraction procedural-conceptual knowledge nexus. (p. 894)

Similarly, Reid and Reid (2017) found that PSTs who engaged in coursework emphasizing open-ended problem solving featuring multiple representations and solutions strategies improved their conceptual and procedural understanding of mathematics. Given the study participants' oversights in suggesting potential solutions strategies and responses for the mathematical tasks and prompts incorporated into their lesson plans, greater exposure to a variety of methods and representations would bolster their lesson planning.

<u>Increasing Math Content Courses.</u> In some cases, PSTs may be engaging in the types of rich and rigorous mathematical experiences described above but are not required to take a sufficient number of content credits to account for the large gaps in knowledge that they often exhibit. At my university, PSTs in elementary education are required to take six credits of mathematical content prior to the methods course, whereas at other institutions in my state they must take nine credits. Increasing the number of required content courses may support PSTs in better meeting the *Standards* upon graduation and initial licensure. As found by Blomekea et al. (2012) in a large-scale international study on PSTs' mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK, akin to MKT), the extent of exposure to

various mathematics topics during schooling was the most significant predictor of PST performance in both MCK and MPCK.

In programs where PSTs demonstrate a wide range of abilities, a differentiated approach to improving content knowledge may be effective, as demonstrated by Ingram et al. (2018), whose educator preparation program provides supplemental content courses and modules for PSTs who need greater practice with key mathematical concepts. In their study, the number of PSTs demonstrating mastery of content knowledge improved from 40% upon entry to the program to 88% after three years. Reid and Reid (2017) similarly recommend a differentiated approach to improving mathematics content knowledge for PSTs who perform far below established standards.

Increasing Access to Teaching Experience

In Situated Learning, observation of skilled masters is critical to successful apprenticeships. Apprentices who are isolated from the work of masters, with few opportunities to observe and participate in the actual work of practitioners in their fields, end up with a more limited set of skills and knowledge (Lave & Wenger, 1991, p. 78). One of three overarching themes identified in Chapter 4 is teaching experience, which overlaps with mathematical content knowledge in key aspects of lesson planning such as anticipating possible solution strategies. Indeed, MKT relates to the specialized knowledge of a math teacher and not just the basic knowledge required to solve a math problem in a person's preferred manner. Several of the oversights in PSTs lesson plans, such as overlooking potential solution methods and moving through material too quickly, were likely due at least in part to a lack of time spent observing and instructing mathematics in K-8 classrooms.

Both mathematical content knowledge and teaching experience interact synergistically to produce more robust MKT. While the quality of a teacher's MKT has been found to be among the strongest predictors of *Standards*-based mathematics instruction, influencing "the quality of the modifications made to curriculum materials, the goals for student learning, and even beliefs about what mathematics *is*" (Hill et al., 2008, p. 497), experience working with real children is also important for developing pre-service teachers' MKT. Sherin (2002) found that teachers' MKT—referred to as pedagogical content knowledge in Sherin's study—is further developed as they gain experience negotiating unique and/or novel student thinking. As Sherin observed, "In brief, I find that novel student comments or methods are a critical factor in provoking teachers to move away from the constraints of their content knowledge complexes" (p. 145). Furthermore,

> The idea is that pedagogical reasoning acts on subject matter knowledge to produce pedagogical content knowledge. Thus, the teacher starts with an understanding of some particular content and, by considering how to present the content to students, transforms this knowledge into pedagogical content knowledge. (p. 146)

By observing students as they work through math problems and paying attention to the strategies they are using—which are often different from the strategies an adult teacher or PST might choose to utilize—teachers and PSTs expand both their repertoire of solution methods as well as their understanding of the content via the process of considering how and why students' strategies work.

In the EPP program at my university, PSTs participate in 120 hours of field experiences prior to student teaching. These hours encompass both observation and practice; however, only two of those hours (in which students teach two math lessons at a local elementary school as part of my method course) are required to focus on mathematics. The content areas that PSTs end up teaching during the rest of their practicum hours is largely dependent upon the times at which they can arrange to be in the classrooms, which in turn depends on the desires of their cooperating teachers, their class schedules, work schedules, and other responsibilities. Many of my PSTs have expressed avoiding math instruction to the extent possible during their field experiences because they feel less comfortable with mathematics than with other content areas such as English Language Arts.

Adjusting the Nature of Field Experiences. More opportunities to observe and work with K-8 teachers and students specifically on mathematics—giving PSTs the chance to witness students' learning progressions and problem solving in real time—would support PSTs in developing their MKT. However, increasing the quantity of field experiences without considering the *quality* of field experiences is unlikely to make a significant impact on PSTs' abilities to meet AMTE's *Standards*. For example, in a longitudinal study of a pre-service teacher who transitioned from a constructivist EPP into a student teaching experience dominated by direct instruction, the PST in question felt constrained and frustrated by her circumstances, without an opportunity to practice and develop the instructional strategies she had studied at her university (Smagorinsky et al., 2004). As Reid and Reid (2017) pointed out in a study involving a practicum component of their EPP, "one cannot assume that teaching math automatically improves math content knowledge (MCK), math knowledge for teaching (MKT), and/or confidence in teaching math" (p. 868). As with the PST in Smagorinsky et al. (2004), the PSTs in their study who were placed with teachers who utilized traditional instruction felt frustrated by the pressure to implement instruction that went against the best practices they had learned about in their EPP.

Similar to the PSTs in the studies cited above, all four of my PSTs who student taught during Spring 2020 were placed with educators who primarily utilized direct instruction. While they acknowledged learning valuable lessons from observing how *not* to implement the *Standards*, experiences observing *Standards*-based instruction would have provided them with richer opportunities to develop their knowledge and skills under the guidance of a master. Fortunately, the PSTs in my study continued to express constructivist views about math instruction despite their non-constructivist placements, but this is not always the case: as Grootenboer (2005) found in a study of 29 PSTs in New Zealand, positive attitudes developed in math methods courses can be easily overturned by practicum experiences that align with more traditional approaches to math instruction. According to Grootenboer,

> It seems that the practicum has the perceived power of being a real experience and hence relegates the experiences and learning of the tertiary course to being interesting but not applicable to the real world. […] While the initial changes of pre-service teachers' mathematical views seem to be important and to some degree achievable, the more apparent issue is the sustaining of those positive changes. The study clearly showed that experiences like the school-based practicum can "wash-out" the positive gains of the tertiary course. (p. 29)

Invoking Lave and Wenger (1991), Grootenboer points out that the practicum experience serves to enculturate PSTs into the in-service teacher community most closely tied to the practicum. Given that PSTs struggle to fully transition away from traditional views of mathematics even after engaging in constructivist coursework during their EPPs (Grootenboer, 2005; Hart et al., 2016; Schram et al., 1988), experiences observing and implementing masters of *Standards*-based instruction are crucial to supporting PSTs in sustaining their newfound views of mathematics.

One way to maintain greater control over the type of instruction that PSTs gain experience with is for MTEs to manage their field experiences themselves. At my university, PSTs are required to enroll in a reading clinic during which they work with a specific group of students across the course of a semester under the supervision of our English Language Arts educator to build the students' reading skills, but there is no similar requirement related to mathematics. As of this writing, I am piloting a math clinic in which PSTs work under my supervision to tutor K-5 children at an afterschool program in our community. The focus of the clinic is strengths-based learning and growth as emphasized in the recent publication *Strengths-Based Teaching and Learning in Mathematics: Teaching Turnarounds for Grades K-6* (Kobett & Karp, 2020), which functions as the primary text for the three-credit course. The clinic occurs once each week and is integrated into an already existing afterschool program, making the logistics simple enough for me to integrate into my teaching load. During the clinic, I am able to reinforce conceptual and representational approaches to mathematics and to support my PSTs in pressing for justification from the students they work with as they support the students in developing their problem-solving skills.

In the case of field placements with cooperating teachers (CTs), Johnson (2011) recommends developing observation forms specific to mathematics and providing training to CTs in the use of the forms, which should be aligned with key features of *Standards*-based instruction. In many schools of education, generic observation forms are utilized across the various content areas, resulting in

> a feedback loop in which if the preservice teachers receive generic feedback from their supervisors and cooperating teachers then it follows that they will place more of their attention in subsequent lessons on things like classroom management instead of dimensions of MKT. (Johnson, 2011, p. 349)

Johnson further recommends emphasizing gradual implementation of *Standards*-based instruction during field experiences such that PSTs focus on one or two elements at a time, with opportunities to receive feedback and reflect on their developing capabilities as math educators. Rather than emphasizing evaluative observations in which PSTs are expected to demonstrate competent performance—which likewise encourages them to focus on classroom management and on procedural approaches to instruction—observations should be structured to support PSTs in experimenting with one or two *Standards*-based practices at a time. Given the fact that several of the PSTs I interviewed for this study felt overwhelmed by the initial feedback they received on their lesson planning assignments (which covered every component of my rubric), a more gradual, focused approach that tackled one or two components at a time might help to build PSTs' skills and confidence without risking discouragement. For example, PSTs might begin with a focus on *significance* and *nature of tasks*—areas in which they demonstrated strengths in developing compelling mathematical scenarios—and move on to more challenging components such as *coherence* and *nature of prompts*, in which they struggled to draw adequate connections between procedures and concepts. In this way, initial success could build the confidence needed to tackle more difficult aspects of lesson planning and implementation. The rubric developed for this study could be utilized directly and/or adapted for the purpose of observing and reflecting on these elements of PSTs' math instruction during field experiences.

In a comparative study between two practicum experiences—one traditional and one in which PSTs, cooperating teachers, and university supervisors worked collaboratively to tackle problems of practice based on theory studied in the EPP—PSTs in the practicum emphasizing theory-practice connections and reflection demonstrated a stronger ability to link theory and practice in their final practicum portfolios (Stenberg et al. 2016). The study supports Johnson's recommendation to emphasize experimentation and reflection over evaluation in practicum experiences.

In the absence of cooperating teachers who are willing and able to link theory and practice during practicum experiences, Grootenboer (2005) suggests that MTEs find ways to "work alongside pre-service teachers on or after practicum to help them reflect upon their unmediated mathematical experiences and discuss their developing affective views" (p. 28). It is imperative that PSTs engage in critical reflection with a practitioner who can help them make sense of their experiences in light of the theory taught in their EPPs such that the PSTs are able to understand why, although common, traditional approaches to mathematics instruction are ultimately less effective than *Standards*-based practices (p. 29).

Instructor Modeling. Since observation, not just practice, is a critical component of successful apprenticeships, MTEs can also help to improve their PSTs' skills by modeling *Standards*-based mathematics instruction in their content and methods courses. As identified by Hart et al. (2016) in their review of studies on PSTs in elementary mathematics content courses, "the studies showed the use of problem-based instruction affording [PST]s opportunities to invent their own solution strategies and engage in productive struggle, in addition to an emphasis on peer interactions, were efficacious" in improving PSTs attitudes towards mathematics as well as their content knowledge (p. 739). Reid and Reid (2017) found similar results in their study of PSTs engaged in a reformed teacher preparation program, in which PSTs reported changing their mindsets about mathematics after engaging in open-ended, hands-on problem solving guided by their MTEs. Likewise, in their one-on-one interviews, all four of my PSTs who student taught during the subsequent semester commented on the impact that my modeling of *Standards*-based tasks had on their learning. Observing the way I initiated and implemented open-ended tasks enabled them to envision the principles of effective mathematics instruction that we read about and discussed in the course.

As an example of this type of instructor modeling, on the first day of each semester, I have my PSTs solve The Fencing Task (Stein et al., 2009, p. xvii) as shown in Figure 7. I model how to adjust the task to accommodate a "low floor," "open middle," and "high ceiling" as discussed in our textbook (Boaler, 2016, pp. 77-85) by providing a variety of tools (inch tiles, toothpicks, and grid paper) and adding extension questions such as, "if the pen did not have to be rectangular, is there another shape that would maximize the area inside the pen even further?" After completing the task, we discuss the deeper engagement and learning that arises from a task such as this one in comparison to a more straightforward task such as "Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?" (Stein et al., 2009, p. xvi). A task such as the latter creates little opportunity for the critical thinking and collaborative problem solving championed in our textbook and advocated by the *Standards*, whereas the Fencing Task fosters lengthy, detailed discussions on the relationships between area and perimeter.

Figure 7. The Fencing Task as Presented in Stein et al., 2009

The Fencing Task
Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits. a. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be? b. How long would each of the sides of the pen be if they had only 16 feet of fencing? c. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Importantly, as noted by one of my PSTs in her interview, I model this type of instruction at various points throughout the semester and not just once so that my PSTs gain repeated exposure to an approach to mathematics instruction that is for most PSTs far different from their experiences with math instruction in K-12. *Standards*-based instruction is also modeled in the two content courses that PSTs take at my university, although with varying degrees of fidelity due to the employment of adjuncts to teach approximately half of the sections of these courses. As Hart et al. (2016) noted in their literature review, "letting go of traditional perspectives on mathematics pedagogy learned over many years as students in classrooms was an arduous process and fraught with resistance" (p. 739), indicating that extended experiences with constructivist approaches to mathematics are critically important. Given that PSTs in my program mostly demonstrate nearing proficient performance with respect to AMTE's *Standards*—including PSTs who took both of their content courses with me and not with adjuncts—nine credits (six content credits plus three methods credits) of constructivist-oriented math content and pedagogy appears to be insufficient for supporting PSTs in meeting the *Standards*, suggesting that greater exposure is needed.

Increasing Access to the Specialized Knowledge of a Master

As noted above, for apprentices to acquire the skills and knowledge needed for successful participation in their professions, they must be provided with opportunities to learn "how masters talk, walk, work, and generally conduct their lives" (Lave & Wenger, 1991, p. 95). To acquire and demonstrate such an intimate level of knowledge and skill, apprentices must be granted access to the ways in which masters *think*. How does a math teacher educator—who already possesses strong MKT—approach a task? What goes through her mind as she develops a lesson plan? What types of questions does she ask to draw out the important mathematical concepts and link them to possible procedures?

Among the matters that masters must keep in mind as they guide apprentices toward autonomous entry into a profession are the criteria for proficient performance in their fields. What does it look like design a lesson plan exhibiting strong MKT? What standards must be met? In this section, I discuss the role of instructor feedback in granting PSTs access to the ways in which a master mathematics educators apply their MKT to lesson planning, as well as the role of evaluation criteria (course rubrics) in upholding the *Standards* and ensuring that appropriate and relevant feedback is provided.

Instructor Feedback. As related in Chapter 2, timely and specific feedback from instructors plays a key role in supporting student learning and growth. (Carroll, 1971; Heflebower et al., 2019). Feedback that is aligned to a specific set of learning standards is particularly useful in advancing students towards proficiency (Airasian, 1971).

In the case of PSTs, instructor feedback provides a window into the minds of MTEs—the "masters" of mathematics instruction—tasked with guiding PSTs towards mastery in the profession. As one of my PSTs noted in her one-on-one interview,

> I feel like [your feedback] was good because those are the questions that we need to ask ourselves and it's good to have somebody ask those questions before we have a lesson flop […] It'll make me think about it when I'm teaching in my classroom. Like *don't* forget this. You know? It could be those little nuggets that I'll always think about, like is my objective measurable? Is it exact? Can somebody come in and see exactly what they need to do to meet the objective?

The rhetorical questions I posed in my feedback—which were designed to encourage PSTs to strive for proficiency with respect to the *Standards* incorporated into the course rubric—gave this PST insight into the mental checklist I run through when planning my own math lessons, which she then applied to her own math lessons during student teaching. All four of the PSTs interviewed for this study attributed their learning and growth at least in part to the feedback provided to them on drafts of their assignments. They emphasized the importance of my feedback being detailed and specific. As one PST reflected,

> Some teachers would say "Great job!" and give you a three out of four […] and you always have room for growth [but] they never said more. And if you would have done that then we wouldn't have known what we needed to change.

Importantly, instructor feedback helped to reduce achievement gaps among my PSTs. With fewer PSTs performing at unsatisfactory and basic levels and more performing at nearing proficient and proficient levels on final drafts and on later assignments than on first drafts and on earlier assignments, performance grew more concentrated toward the upper end of the proficiency scale across the semester, as can be observed in Figures 6-7 above. As Bloom admonished over 50 years ago, "since education is a purposeful activity in which we seek to have students learn what we teach, the achievement distribution should be very different from the normal curve if our instruction is effective" (Bloom, 1971, p. 49).

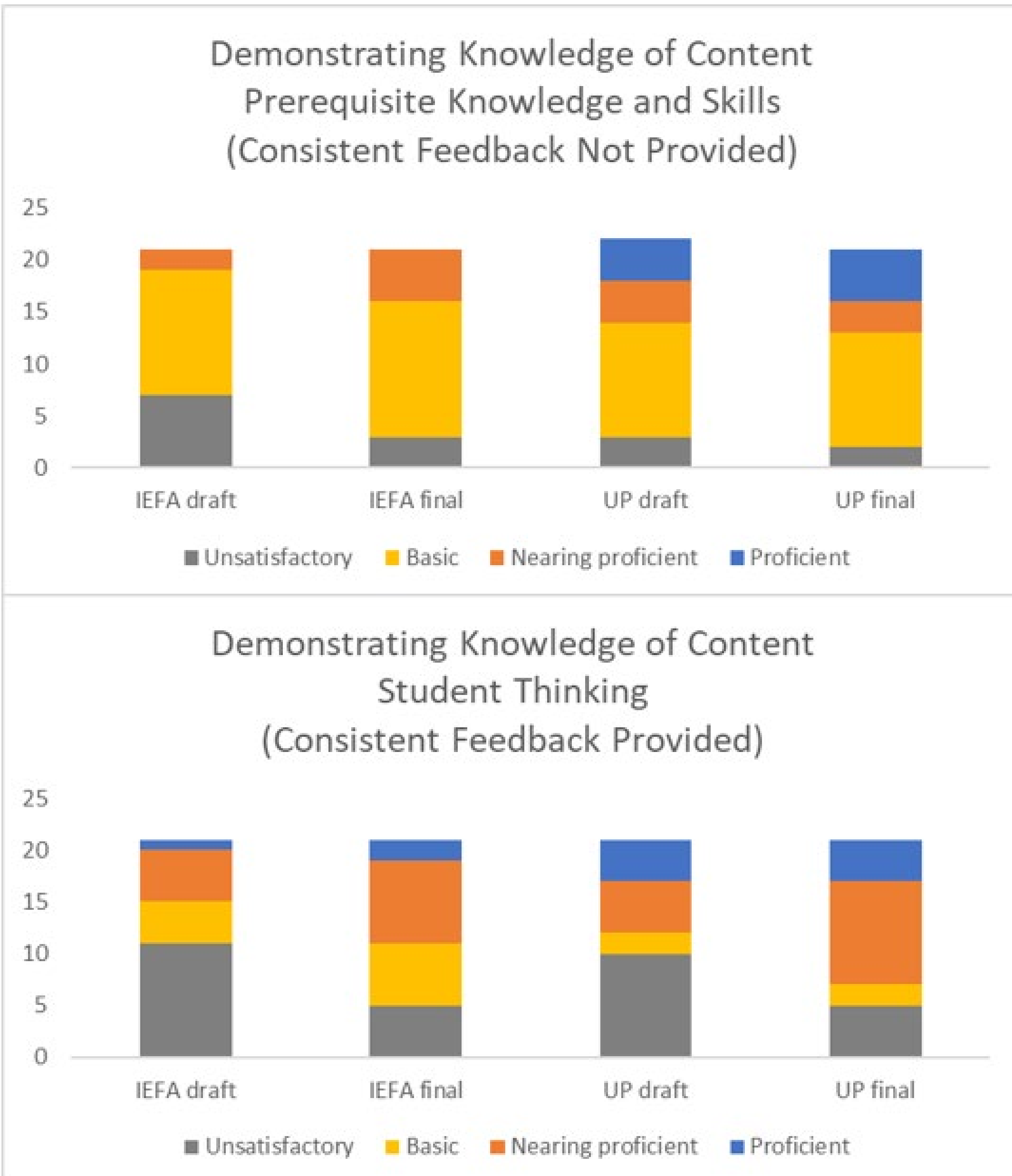
As evidenced in Chapter 4, PSTs demonstrated notable growth with respect to several components of effective lesson planning as outlined in AMTE's *Standards* and in the Danielson Framework, against which the course rubric is aligned. In some cases, growth resulted simply from being reminded of information that had been overlooked in first drafts, such as example student solutions and responses (*student thinking*) and pre-planned prompts (*nature of prompts*), and from having the opportunity to add this information into the final drafts. In other cases, PSTs relied on feedback to learn that the information they had included in their first drafts was inaccurate or insufficient, as was the case for *alignment between learning objectives and learning standards* and *alignment between learning objectives and assessments*.

Even when the most growth was evident between assignments rather than between drafts such as for the components of *developmental progression and pacing* and *use of resources*, I believe feedback played a role in the improvements. During the Fall 2019 semester, PSTs received feedback on their IEFA lesson drafts during the week they began working on their unit plan drafts. As such, my feedback on their IEFA drafts was fresh in their minds upon developing their unit plan drafts, which exhibited stronger performance overall compared to the IEFA drafts. Between the two assignments, there was no new instruction in the course other than to discuss the parameters of the unit plan, leading me to believe that my feedback on the IEFA drafts contributed to the stronger overall performance on the unit plan drafts.

Further evidence of the role of feedback on PST performance comes from the data shared in Table 7 in Chapter 4. Even though Table 2 in Chapter 4 indicates that for several components of the course rubric there was little change in the totals associated with each proficiency level between drafts—implying that feedback had no impact at all on most PSTs for these components—Table 7 describes ways in which PSTs made improvements to their lesson plans based on feedback even though gaps in their content knowledge prevented them from achieving full proficiency. For example, in the unit plan on prime and composite numbers, the PST clarified and expanded on an appropriate use of manipulatives in the unit, a substantial improvement over the initial plan to use them only for a brief opening activity that was disconnected from the rest of the unit. However, because the PST's use of the tool was incomplete and not adequately linked to the definitions used in the unit for prime and composite numbers, the draft did not improve in proficiency between the first and final draft, remaining at nearing proficient performance for both drafts.

Another indication that feedback plays a role in supporting PST performance is that PSTs were held back in areas in which I failed to provide feedback. As discussed in Chapter 4, I did not provide feedback related to *prerequisite knowledge and skills* on approximately half of the first drafts of each assignment. This component ended up being one of the few in which most PSTs remained at only a basic level of performance on the final drafts of both assignments. Figure 8 shows the contrast in performance between this component for which I failed to provide adequate feedback and another related component, *student thinking*, on which I did provide consistent feedback. I will discuss the issue of insufficient feedback in the next section on the role of course rubrics.

Figure 8. Contrast in Performance Based on Differences in Instructor Feedback



Course Rubrics. Rubrics serve the purpose of elucidating the standards for performance in a course, aiding both instructors (masters) and students (apprentices) in recognizing when mastery has been achieved and, when it has not, areas in which improvement is still needed. The results of this study have greatly illuminated the role of rubrics in PST performance. However, unexpectedly, rather than having a direct impact on PST performance by enabling them to have a clear picture of the objectives toward which they were striving and against which they could self-evaluate, my course rubric had a more indirect impact on PSTs by informing *my* instruction and *my* feedback *to* my PSTs. Since my PSTs expressed mixed feelings toward my course rubric—with some of them feeling confused and/or overwhelmed by it and others eventually coming to make sense of it as the semester progressed—it became evident that the rubric itself was not especially helpful to PSTs. It has, however, been extremely helpful as a guide for me in identifying strengths and weaknesses in my own feedback and instruction, which in turn impacts PST performance.

Importantly, I use my course rubric as a guide when planning my instruction to ensure that I am providing my PSTs with the information and experiences they need to meet the standards for proficiency in my course. Furthermore, I tend not to comment on criteria that is not incorporated into my course rubric since the rubric serves as my primary reference when I evaluate assignments. This means that if my rubric does not meet national standards then nor will my instruction or my feedback to my students. As discussed in Chapter 4, two of the components of my *Standards*-based rubric in which PSTs demonstrated the weakest overall performance—*differentiation* and *perspectives*—are both linked to the domain of *advocacy and equity*, which was nonexistent in my original rubric (recall from Chapter 3 that I drafted the rubric used during Fall 2019 before learning of the *Standards*). Although differentiation was embedded in my criteria for *instructional strategies* in my original rubric, *perspectives* was not included at all, meaning I gave absolutely no feedback related to this domain on PSTs' unit plans, of which all 21 were coded at a basic level of performance on the final draft—the lowest performance overall across the rubric components. Due to the nature of the IEFA assignment, which required incorporating the perspectives of one or more of our state's Indigenous groups, I did provide feedback relevant to *perspectives* on the IEFA drafts, generally couching it within the domain *knowledge of content* since knowledge of the chosen Indigenous group(s) formed part of the content of those lessons.

I also completely overlooked the importance of planning for adequate closure in a lesson because it was not a criterion in my original rubric; hence I failed to provide feedback related to closure on my PSTs lesson planning drafts. For nine of my PSTs, this lack of feedback (and the opportunity to revise based on that feedback) resulted in less-than-proficient performance on my revised rubric in the component of *coherence*, for which these PSTs otherwise met the criteria for proficiency. As noted in the previous section, I also failed to provide feedback on approximately half of the drafts with respect to *prerequisite knowledge and skills*, which was embedded in *knowledge of content* in my original rubric and not separated into its own distinct set of criteria. This represented another component in which PSTs demonstrated weaker performance overall.

By updating my course rubric to align with AMTE's *Standards* and using the updated version to evaluate my PSTs assignments for this study, I was able to identify gaps in my own instruction and feedback that, once filled, will enable me to better support my PSTs' success with respect to the *Standards*. I have already begun to address the gaps in *advocacy and equity* by adapting some of the math tasks that I model for my PSTs to incorporate diverse perspectives. Fostering skills in linking math tasks to compelling and relevant cultural information forms part of the MKT aimed for in the *Standards*, as expressed in the equity principle, and is imperative for MTEs to address. For example, I have built upon the concepts of area and perimeter introduced during The Fencing Task by developing a subsequent task in which my PSTs are challenged to design a community center for our city's Hispanic population. After watching a video about the annual Mexican Fiesta that takes place at a city park, I task my PSTs with designing a culturally appropriate community center given certain parameters such as an open, multi-use space large enough for both basketball games and large gatherings. I also encourage them to identify potential vacant buildings from the city's realtor listings that could be renovated for this purpose. Additionally, I have added a folder to my online course resources that includes links to websites with information about the experiences of various non-dominant groups, including BIPOC, low SES, and LGBTQ+, and I require my PSTs to utilize this information for at least one aspect of their unit plan (for example, generating a word problem or a "Notice and Wonder" activity that affirms the identity of members of one of the non-dominant groups).

Other areas for improvement include better supporting my PSTs in comprehending and utilizing the course rubric for purposes of self-evaluation. When one of my PSTs noted in her interview that simply assigning a three out of four in a category is not helpful (as her other instructors tend to do), it occurred to me that many PSTs likely do not know how to effectively interpret the information in a rubric so that they can figure out for themselves where and how they need to improve upon their work to advance from nearing proficient to proficient performance. This supposition is consistent with findings in the literature. As Hendry et al. (2012) noted, “as recent research has shown, many students find descriptions of standards difficult to understand” (p. 150). In their study, first-year law students enrolled in a section of a course in which the instructor facilitated guided discussion of three different assignment exemplars—each at different levels of proficiency—with respect to the assignment rubric outperformed students in other sections of the course. They concluded,

> Overall most students found it initially difficult to identify the poor and good assignments, until they engaged in teacher-led class discussion about the rationale for grading, or were eventually told. Although students in classes where the teacher led an interactive discussion about exemplars rated their understanding of the marking sheet higher, and were more likely to feel they would be marked fairly, all students still rated the marking sheet as less useful than exemplars for guiding them in completing their letter assignment. These results highlight the critical role of teachers’ facilitation skills in helping students understand the standards of work expected which are embedded in exemplars and summarised in marking sheets or rubrics. (p. 158)

Orsmond et al. (2002) likewise found that purposeful use of exemplars coupled with formative feedback supported students in their ability to self-evaluate more effectively against course objectives. The researchers found that purposeful use of exemplars had an even greater impact on students’ ability to self-evaluate against course criteria than involving students in the creation of the criteria. These findings suggest that a more intentional use of exemplars in my methods course could improve my PSTs’ understanding of the course rubric. More recently, I have begun providing exemplars reflecting multiple levels of performance (as opposed to just proficient performance) to my PSTs and engaged them in discussions about the differences among them; anecdotally, I have noticed higher scores on initial drafts of assignments than in Fall 2019, when I merely supplied a single exemplar of a lesson meeting the criteria for proficiency. I have not, however, systematically analyzed these outcomes, which would be an area for fruitful future research given that a greater ability on the part of students to comprehend and apply rubrics to their work would reduce the amount of additional written feedback required of instructors in order for their PSTs to recognize the revisions needed to achieve proficiency.

Adjusting Course Timelines

The final overarching theme identified in the data shared in Chapter 4—lack of attention to detail—suggests that some PSTs struggle to find time to adequately attend to all components of a lesson planning assignment regardless of the depth and breadth of the feedback offered to them (and/or they do not pay close enough attention to said detail). Indeed, many of my PSTs work part-time or even full-time jobs while attending school; many of them also care for children at home, a significant responsibility for anyone working towards a college degree and teacher licensure. As one of my PSTs admitted during her interview, she simply did not have enough time to attend to all the feedback I offered to her on drafts of her assignments.

Even with two weeks of class time to work on revisions, it is evident that at least some PSTs still did not have sufficient time to address all the feedback they received. As MTEs, we must plan carefully for how we structure our courses and the associated workload. Providing built-in opportunities for revision are certainly helpful (I suspect there would have been even greater evidence of lack of attention to detail had I not included two weeks of class time for my PSTs to prepare their portfolios) but we may need to device other creative ways to ensure that our students are able to invest the time necessary to make proficient revisions, which may include cutting back on other assignments and content. As John Caroll observed decades ago, achievement is in part a function of adequate time to learn, meaning that if MTEs do not provide PSTs with sufficient time to fully digest and implement feedback, we limit their growth (Caroll, 1971).

In the next section I offer some reflections on the breadth and depth of the *Standards* and suggest ways in which MTEs might address the most compelling aspects of the *Standards* raised by the study's findings without overwhelming their PSTs while possibly leaving other *Standards* to be addressed during our students' first years in their own classrooms. I also reflect on the feasibility of implementing standards-based learning (SBL) for MTEs.

Reflections on Feasibility: Addressing the AMTE *Standards* Via SBL

The AMTE *Standards for Preparing Teachers of Mathematics* is an ambitious document, delineating "the specific knowledge, skills, and dispositions that well-prepared mathematics teacher candidates at all levels will know and be able to do upon completion of an initial preparation program" (2017, p. 5). The goals embraced by the *Standards* are laudable and worthy, aiming to transform mathematics instruction for the benefit of each and every student in our nation's school systems. Whether the goals are *achievable* within the four years of a typical educator preparation program, however, requires serious consideration, especially given the deficits in mathematical knowledge and dispositions exhibited by so many K-8 PSTs upon entering their programs. In this section, I reflect on the *Standards* with respect to the study's findings and also offer recommendations related to the feasibility of implementing standards-based learning more broadly across mathematics teacher education.

Reflections on the Attainability of the *Standards* for EPPs

AMTE's *Standards* encompass 19 indicators tied to four overarching standards: mathematics concepts, practices, and curriculum; pedagogical knowledge and practices for teaching mathematics; students as learners of mathematics; and social contexts of mathematics teaching and learning. The rubric I utilized for this study addressed 14 of the 19 indicators across each of the four standards. Yet even without attempting to cover all the indicators, my students and I struggled to achieve full proficiency in most aspects of the *Standards*. As an MTE, the outcomes of my study prompt the following questions regarding the *Standards*:

- By attempting to incorporate as many of the indicators as possible into my course, am I covering each indicator too thinly to be effective?
- Would covering fewer indicators result in greater achievement for my PSTs? If so, which indicators should I emphasize?
- Are there indicators that are better left for PSTs' first years in their own classrooms? If so, how can I set up my PSTs to naturally achieve these standards once they become autonomous educators?

These wonderings raise important questions for future research, which I discuss in a later section of this chapter. Although I do not have answers to these questions, there are certain indicators within the *Standards* that seem to me to be more relevant to classroom teaching than to pre-service preparation. For example, indicators C.2.5 (enhance teaching through collaboration with colleagues, families, and community members) and C.4.5 (enact ethical practice for advocacy) both emphasize interactions beyond MTE-PST relationships and even beyond PST-student relationships. While many PSTs will have opportunities to interact with co-teachers, caregivers, and community members during their EPPs, I wonder about the feasibility (and even reasonability) of assessing their competency in these interactions given their stature as apprentices, who are not yet the teachers of record ultimately responsible for the students they work with. To be sure, these are objectives incorporated into the domain *professional responsibilities* within the Danielson Framework (2007), suggesting they may be more appropriate indicators for in-service teachers rather than pre-service teachers. As such, they might be moved to a separate section of the *Standards* that describe future outcomes of high-quality preparation programs and could be recommended as measures of the effectiveness of other indicators that *are* addressed within an EPP (such as indicator C.4.4, understanding power and privilege in the history of mathematics education).

It also occurs to me that given the fragility of many K-8 PSTs' mathematics-related teacher identities (Hart et al., 2016; Ingram et al., 2018; Lutovac & Kaasila, 2014; Lutovac & Kaasila, 2018; Setra, 2017; Swars, 2005), tackling too many indicators within a course of study could easily discourage PSTs from embracing *Standards*-based instruction, leading them to revert to the direct instruction most familiar to them (Grootenboer, 2005; Hart et al., 2016; Schram et al., 1988). Indeed, the four PSTs interviewed for this study all expressed feeling overwhelmed each time they received feedback on an assignment, with so many standards to improve upon. Simplifying their lesson planning assignments and limiting the number of indicators I attempt to address might ultimately be the better route to increasing their confidence and competency in *Standards*-based instruction.

Based on the outcomes of this study, emphasizing *mathematical concepts*, *significance*, *nature of tasks*, and *nature of prompts* without worrying as much about the wording of their objectives, alignment to the correct standards, and some of the other aspects of my rubric could prove fruitful in building both skills and self-assurance. PSTs demonstrated strong abilities to select appropriate scenarios (*significance*) and set students up with meaningful tasks (*nature of tasks*) but needed to strengthen their understanding of the underlying concepts (*mathematical concepts*) so that they could ask more critical questions (*nature of prompts*) to ensure their tasks would be fully effective in making key connections and drawing out key mathematical ideas. By focusing on these components within their lesson planning assignments, PSTs could develop enough confidence based on their strengths in the former (*significance* and *nature of tasks*) to feel comfortable tackling the latter areas in which they require more improvement.

Reflections on the Feasibility of SBL

Implementing standards-based learning required a significant time investment on my part as an instructor. As other SBL researchers have noted, it takes considerable time to develop appropriate and measurable standards for a course of study; however, once standards were developed, it became easier to grade assignments given the clarity in the criteria used for student evaluation (Beatty, 2013; Scarlett, 2018; Selbach-Allen et al., 2020).

The most significant time investment came from providing written feedback to each of my PSTs on drafts of their lesson plans, which took me anywhere from 15 minutes to 30 minutes per draft. With small class sizes, this was manageable for me but might not be manageable for MTEs with larger numbers of students (at least in the absence of TAs, which we do not have at my university). Strategies I utilized to afford myself the time to provide detailed feedback included not requiring revisions to be made until the last weeks of the semester since the final drafts would form part of PSTs' summative portfolios, which granted me a longer turnaround time to provide the feedback. I also moved class online during days when drafts were submitted, which added an extra 90 minutes to my week. During that 90 minutes, my PSTs were responsible for providing peer feedback on one another's drafts via the discussion feature on our online course management system.

Given my discovery—via the interviews I conducted with four of the study participants—that my PSTs struggled to understand the rubric itself and mostly relied on my written feedback, implementing strategies to increase my PSTs' ability to comprehend and apply the course rubric on their own stands to save even more time in providing feedback on drafts and to make SBL more feasible for faculty members at larger institutions. As Hendry et al. (2012) and Orsmond et al. (2002) have found, coupling rubrics with intentional use of exemplars can improve students' understanding of rubric criteria. While the researchers did not state whether the improvements in understanding resulted in an ability of students to rely solely on rubrics without the need for written feedback, further exploration in this area would prove highly fruitful in assuaging concerns that SBL requires more time than higher education faculty possess.

In the subsequent section, I posit questions that MTEs might explore in the interest of making SBL and the AMTE *Standards* more manageable, among other areas for future research that emerge from the study's findings.

Areas for Future Research

While the findings from the present study suggest promising approaches to improving pre-service teachers' MKT with respect to lesson planning, more research is ultimately needed to discern and describe PST and MTE performance with respect to AMTE's *Standards* as well as to define the parameters of effective interventions. Below, I recommend several areas for continued research.

<u>Improving PSTs' Achievement: MKT</u>

The findings from this study raise compelling questions about pre-service mathematics teacher education. Principal among them is how to best support PSTs in better comprehending the mathematics they will be charged with teaching once they become autonomous practitioners in the field. While findings from this study and from the literature suggest that greater exposure to "unpacked" content knowledge (Morris & Hiebert, 2017), MTE modeling of *Standards*-based mathematics instruction (Hart et al., 2016; Reid & Reid, 2017), and more opportunities to engage with real students about their mathematical thinking (Sherin, 2002) can strengthen pre-service teachers' MKT, many questions remain unanswered. For example, what number of content credit hours is ideal for K-8 PSTs? Do all PSTs require the same number of credit hours or can this aspect of EPPs be differentiated based on prior knowledge and skills? What topics are most essential to cover? Is there a transfer of skills from one topic to another if not all topics can be adequately covered within the time span of an EPP? Are there ways to integrate content courses, methods courses, and field experiences in ways that synergistically improve pre-service teachers' MKT? As Ingram et al. (2018) noted, "from our review of the relevant literature, there has surprisingly been only a small body of research on what interventions can improve pre-service teachers' mathematical content knowledge" (p. 45).

Improving PST Access: Instructor Feedback and Field Experiences

As discussed above, timely and detailed instructor feedback was correlated with improved performance on various components of the *Standards*-aligned course rubric, whereas lack of instructor feedback on certain components was correlated with more stagnant performance. Hence, there is evidence that instructor feedback provides access to knowledge and skills of a master move PSTs closer to the desired level of performance in a profession. Does the use of *Standards*-based rubrics such as the one outlined in this study help to focus instructor feedback such that it advances PSTs at other institutions towards proficiency as well? Can even further progress be made when PSTs are trained to utilize rubrics for self-evaluation purposes? While existing research has demonstrated the utility of exemplars in supporting students' understanding of rubric criteria (Hendry et al., 2012; Orsmond et al., 2002), more research is needed to identify the language, structure, and training needed to enable PSTs to understand the contents of course rubrics such that they can make accurate self-assessments about their work and so that instructors do not need to leave extensive written feedback for PSTs to be able to make sense of the standards, which is a time-consuming endeavor. As Hendry et al. (2012) point out, "Further research is needed on how students with different typical approaches to study might use exemplars in different contexts" (p. 159).

Also needed is further experimentation with field experiences. A common dilemma in the field of mathematics teacher education is finding cooperating teachers (CTs) who model the *Standards* in their instruction. With traditional, teacher-centered instruction still dominant in most classrooms, PSTs often do not have the opportunity to observe *Standards*-based instruction in K-8 classrooms. Indeed, none of the four PSTs who student taught following my methods course were able to be placed with teachers who utilized student-centered instruction. Although for three of the four PSTs, their CTs granted them the freedom to teach in whatever manner they chose to, the PSTs were unable to observe *Standards*-based instruction in action or to receive targeted support from their CTs when they attempted to implement it themselves. While the PSTs were able to learn through observation how they *did not* want to teach, not having the opportunity to observe exemplary modeling of how one *does* want to teach and to ask questions and seek feedback from a master is limiting. Identifying and developing field experiences for PSTs in which they can observe and participate in *Standards*-based instruction and reflect on the relationship between theory and practice is imperative for facilitating PSTs transition into the profession in a way that supports, rather than undermines, the *Standards*. In what ways can MTEs support PSTs and their cooperating teachers in linking theory and practice during their field experiences? Could the rubric developed for this study serve as a tool for focusing experimentation, feedback, and reflection around AMTE's *Standards*? As Grootenboer (2005) notes, "This problematic issue requires research and development with both pre-service and in-service teachers, and the community at large" (p. 28).

In the absence of qualified CTs, MTEs can model *Standards*-based instruction in their content and methods courses. But to what extent does this play a role in moving PSTs towards proficiency with respect to the *Standards*? Is MTE modeling sufficient for PSTs to learn to adequately implement *Standards*-based instruction? How often PSTs must experience this type of modeling for them to break away from long-held beliefs about traditional mathematics instruction?

<u>Gauging PSTs' Access and Achievement with Respect to the *Standards*</u>

While the present study found that PSTs exhibited the greatest weaknesses with respect to aspects of the *Standards* requiring a strong understanding of the links between procedures and concepts, further research is needed to further illuminate PST performance specifically with respect to AMTE's *Standards*. Such research would help to guide innovations in the field targeted at *Standards*-based evaluations of PST preparedness to implement the kind of robust mathematics instruction called for by AMTE. Further experimentation with the rubric would also help to illuminate its utility as a tool for self-evaluation on the part of MTEs and PSTs alike as well as its potential usefulness in guiding and evaluating practicum experiences. Can the tool be utilized to help focus planning and feedback sessions between PSTs, CTs, and MTEs around best practices? In what ways might it need to be adapted for such a purpose?

The codebook developed to accompany the rubric (see Appendix B) illuminates each proficiency level in the rubric with specific examples drawn from actual PST work, contributing a tiered progression from unsatisfactory to proficient performance with respect to the *Standards* heretofore undeveloped. The rubric can be utilized as a tool by other MTEs for gauging PST performance, reflecting on their instruction, encouraging PST self-reflection, and, potentially, developing content-specific observation forms and guidelines for evaluating field experiences in mathematics instruction as recommended by Johnson (2011).

Also interesting to explore would be areas in which MTEs are underperforming with respect to the *Standards*. As I discovered through the course of this study, I was not providing my PSTs with sufficient *access* to certain content, limiting their potential for growth in those areas. In what domains do MTEs and/or their EPPs tend to struggle to provide adequate content and learning experiences? The answer to this question would help to inform professional development opportunities, indicating areas for conference strands and further research to support MTEs in teaching to the *Standards*.

Exploring Knowledge and Skill Transfer Within the *Standards*

Given the extraordinary depth and breadth of the *Standards* and the struggle that MTEs and EPPs such as mine will experience in attempting to address every aspect of the standards during the four years of a typical EPP, it is worth exploring the ways in which knowledge and skills might transfer from one standard to another within the *Standards* such that MTEs could be tasked with a more focused, manageable set of goals. The rubric I utilized for this study incorporated 14 of the 19 indicators in the *Standards*, and even without attempting to address each and every standard, my PSTs and I struggled to meet full proficiency. Particularly valuable would be to explore links between knowledge and skills developed during EPPs and those developed during a candidates' first years of teaching in their own classrooms. For example, are PSTs who demonstrate proficiency in relevant mathematical content (indicator C.1.1) and mathematical practices and processes (C.1.2) able to successfully analyze the mathematical content of curriculum (C.1.4), analyze and attend to students' mathematical thinking (C.1.5 and C.3.1), and draw on students' mathematical strengths (C.4.3) once they are in their own classrooms without explicitly focusing on these indicators during their EPPs? Are PSTs who understand power and privilege in the history of mathematics education (C.4.4) able to successfully enact ethical practice for advocacy (C.4.5) without specifically implementing the latter during their time in college? The answers to these questions could inform future iterations of the *Standards* and/or practical guides for implementing the *Standards* when time and resources are limited.

<u>Gauging PSTs' Sense of Identity and Power in Mathematics Education</u>

The present study addressed two of four domains of full participation in a profession as discussed by Lave and Wenger (1991). Access and achievement were studied in terms of PSTs' ability to comprehend and act on master feedback (access) as they worked toward proficiency (achievement) in lesson planning for mathematics instruction. Also worth studying would be PSTs' sense of identity and power within the field of mathematics education and the impact that SBL has on these other two domains of professional participation. As other researchers have underscored, teachers' sense of identity and self-efficacy impact the ways in which they instruct and interact with their students (Cooney & Shealy, 1997; Cross Francis, 2015; Cross Francis et al., 2015; Povey, 1997; Stipek et al., 2001). Does standards-based learning help PSTs to build positive mathematical identities that make them feel capable of promulgating change in the way mathematics is taught in their schools and districts—or does it simply overwhelm them? How can the *Standards* best be addressed within EPPs to gradually build PSTs' skills and confidence without discouraging them? What types of experiences are needed to support PSTs in feeling both motivated and competent in implementing the vision set forth by AMTE (2017) and NCTM (2000; 2014)?

<u>Improving Learning Outcomes in My Course</u>

As Alonzo et al. (2019) underscore in their review of SBL in higher education, "Academics need to reflect on student evaluation and feedback and their overall teaching and assessment experience, and identify key areas in course design that need improvement and modify them accordingly" (p. 641). Several questions emerged for me from this study specific to my own students and courses. One question is the extent to which my PSTs' performance will improve in subsequent semesters now that I have aligned my course rubric with AMTE's *Standards* and updated my instruction accordingly. Will my PSTs in Fall 2021 demonstrate greater proficiency by the end of the semester in comparison with students from Fall 2019? How will my efforts to improve course content relevant to *perspectives* impact their performance in this component? I aim to investigate these questions in a follow-up study with students in my Fall 2021 cohort, when I updated my content to encompass the *perspectives* category of my rubric.

Another question is the extent to which a lack of attention to detail—the third overarching theme identified in Chapter 4—factors into my PSTs' performance. How many of my PSTs, like the one I interviewed in Spring 2020, simply do not have or set aside enough time to fully address my feedback? How can I adjust my course schedule and workload to ensure my PSTs have sufficient time to address the feedback they receive from me on assignments without feeling overwhelmed?

Limitations to the Study

The findings from this study are associated with a single cohort of pre-service teachers at one university in the United States. The participants were primarily female and White, with one male student and with one student self-identifying as biracial. The data analyzed were limited to lesson plans from 21 students and to one-on-one interviews with four of those students. As such, the findings are applicable to a limited context and would benefit from replication in other settings with more diverse groups of pre-service teachers. Given the alignment of many of the findings with relevant literature in mathematics education, replication by other MTEs in other contexts would be highly valuable in adding insight to our knowledge of the impact of standards-based learning on PST performance in lesson planning for mathematics instruction.

As a teacher-researcher, it is also possible that my own biases influenced my interpretation of the data. While I made every effort possible to view the data through a researcher's lens, as described in Chapter 3, the participants were ultimately my students, with whom I spent a full semester in a master-apprentice relationship.

Another limitation of the study is the fact that PSTs can pass my K-8 methods course by achieving nearing proficient performance on the rubric utilized for evaluating their lesson plans. As suggested by the *Details* theme discussed in Chapter 4, it is possible that PSTs did not submit lesson plans that reflected their best capabilities due to time constraints, competition from other coursework, and/or feeling satisfied with a score that would be sufficient to pass the course and move forward in their educator preparation programs even if it would not earn them fully proficient performance or A's (PSTs in my program can pass a class with a C or higher). Although I do not believe lack of attention to detail played as significant a role in PST performance as lack of content knowledge and lack of teaching experience—as indicated by its relative size in Figure 3, Chapter 4—it is impossible to know the extent to which it did impact PST performance on their lesson planning assignments.

Finally, while I made my best efforts to write clear, comprehensible feedback to my PSTs on their lesson planning assignments, it is possible that some of my feedback was misinterpreted or misunderstood. While there is evidence in the findings shared in Chapter 4 that PSTs were able to improve performance based on my feedback—suggesting that they understood at least some of it—it may be that not all of my feedback was comprehensible to my PSTs.

Conclusions

In this study I set out to determine the ways in which feedback and revision cycles embedded in standards-based learning enable PSTs to better develop and apply their knowledge and skills with respect to AMTE's *Standards for Preparing Teachers of Mathematics*. I also wanted to know in what ways my PSTs are meeting and not meeting the *Standards* in their lesson planning assignments as well as what themes emerged from their work that would provide clues to improving their performance. Since pre-service teachers engage in a process akin to apprenticeship as they proceed through their educator preparation programs, I framed the study using Lave and Wenger's (1991) notion of *legitimate peripheral participation*, in which apprentices learn from masters the skills and knowledge needed to become independent practitioners in a profession.

In this section, I summarize the findings associated with each of my three research questions:

- In what ways does the opportunity to receive timely and specific feedback and to revise lesson plans accordingly help PSTs to better develop and apply their knowledge and skills with respect to AMTE's 2017 *Standards*?
- In what ways are PSTs meeting and not meeting the *Standards* in their lesson planning assignments?
- What themes emerge from PSTs' work on lesson planning for K-8 mathematics?

The Role of Feedback in Improving PST Performance

As evidenced in Chapter 4, detailed feedback from a master in the profession of mathematics education can help to improve PST performance on lesson planning assignments with respect to national standards. The opportunity to receive and respond to instructor feedback resulted in notable improvement on the part of PSTs across several components of the course rubric. Feedback on an earlier assignment even impacted performance on a later assignment. Despite extensive feedback, however, PSTs still struggled to achieve full proficiency in most components of the rubric.

PST Performance with Respect to the *Standards*

Areas in which PSTs demonstrated the most growth following feedback included anticipating student thinking; aligning learning standards, objectives, and assessments; planning for a more appropriate pacing of new content; selection and utilization of resources; the design of mathematical tasks; and posing questions to prompt student thinking and discussion. While PSTs made notable gains in these areas, most PSTs demonstrated nearing proficient performance rather than proficient performance by the end of the semester. In only two components of the course rubric did more PSTs achieve proficient performance than any other level of performance: *alignment between learning objectives and learning standards* and *suitability of resources*. In three components, most PSTs achieved only basic performance: *differentiation, prerequisite knowledge and skills*, and *perspectives*. For the latter two components, instructor oversight in these areas was partly to blame for the lower performance, underscoring the important role of instructors in upholding and bolstering student performance.

Emergent Themes in PSTs' Lesson Planning Assignments

Limits to the impact of instructor feedback on PST performance were linked to a lack of sufficient content knowledge and a lack of classroom teaching experience. While PSTs demonstrated growth in many areas, many struggled to move beyond nearing proficient performance due to an inability to adequately explain procedures based on an understanding of concepts. While PSTs exhibited knowledge of procedures and a basic understanding of concepts, they could not link the two together in such a way that they could "explain the mathematical rationales underlying key procedures" (AMTE, 2017, p. 8), limiting their ability to achieve proficiency with respect to the *Standards*. PSTs also lacked experience in real classrooms, having little opportunity to witness and participate in student learning trajectories, student thinking, and other aspects of mathematics instruction required for effective lesson planning. Greater opportunities to engage in proof-like work in mathematics content courses, in which PSTs have the opportunity to explain and justify mathematical procedures based on the underlying concepts, and to observe and participate in classroom instruction alongside

Standards-based educators are recommended to support PSTs in meeting the *Standards* by the time they graduate from their apprenticeships and become autonomous teachers in their own classrooms.

Moving Forward

More research is needed to identify successful approaches to deepening pre-service teachers' content knowledge in K-8 mathematics and to including them in *Standards*-based instructional experiences. More work is also needed to support PSTs in understanding and applying rubrics for purposes of self-assessment. The field of mathematics teacher education would benefit from replication of this study by other MTEs to gain a broader understanding of the ways in which pre-service math teacher educators are meeting and not meeting AMTE's *Standards*. Where are the greatest weaknesses in our field? The greatest strengths? What might we as MTEs be overlooking in our coursework and in our feedback to students that is limiting their ability to achieve full proficiency with respect to the *Standards*? What *Standards* should we be emphasizing most heavily within the limited time span of an educator preparation program? These are crucial questions in an era when mathematical literacy is increasingly paramount to individual and societal success. By aligning our instruction to the most crucial elements of the *Standards*, which stive for mathematical literacy for *all* students, MTEs can play a part in creating a more informed and equitable society.

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APPENDICES

APPENDIX A

INITIAL RUBRIC UTILIZED FOR EVALUATION PURPOSES DURING FALL 2019

	Demonstrating Knowledge of Content (Domain 1a)	Setting Instructional Outcomes and Assessment (Domain 1c, f)	Demonstrating Knowledge of Resources (Domain 1d-e)	Instructional Strategies (Domain 1e; 3a-e)	Reflection (Domain 4a, e)	Communication (Domain 4d)	Professionalism (Domain 4f)
Proficient for Pre-Service Expectations	Demonstrates understanding of mathematical concepts and some connections between concepts and between math and other subjects. Invokes relevant background knowledge in lesson planning and attempts to anticipate potential misconceptions. Explains solutions to mathematical tasks and embeds appropriate mathematics in lessons and unit plans.	Establishes learning objectives that align with state standards and are appropriate for the age group. Objectives are specific and measurable, articulating what students will be able to *do, demonstrate, explain, justify*, etc., upon successful completion of a lesson. Assessments align with objectives and are designed to elicit information about both conceptual and procedural understanding. Opportunities are provided for student self-evaluation that promotes a growth mindset.	Uses a variety of relevant resources in planning for instruction, including course readings, curriculum guides, templates, online resources, technology, manipulatives, and visual aids. Selected resources are relevant to the learning objectives and are intended to support student exploration and sense making. Resources are utilized with an attempt to maintain high levels of cognitive demand during instruction.	Designs instruction with an obvious aim to engage all students in sense making. Lessons include specific and appropriate prompts to generate student thinking and discussion and to enable student choice in problem solving methods whenever appropriate. Plans for differentiation are relevant to the given topic and reflect an attempt to maintain a high level of cognitive demand for all students.	Reflects on personal strengths and weaknesses as well as those of others. Takes seriously all opportunities to learn from first-hand experience and the literature. Articulates lessons learned in a detailed manner, citing specific evidence that has supported personal growth. Continually improves performance based on lessons learned.	Takes a leadership role in class and group discussions, engaging with peers in a polite and cooperative manner and encouraging productive interaction among others. Listens to others' ideas and responds in ways that demonstrates appreciation for others' input. Articulates well-founded ideas about math and math instruction.	Comes fully prepared for scheduled classes and meetings. Plans ahead for absences, submitting all required work on time. Communicates promptly if situations arise that may prevent unintended delays. Work is clear, concise, complete, well organized, and free of errors.
Near Proficient for Pre-Service Expectations	Demonstrates understanding of mathematical concepts but may struggle to make connections between concepts and between math and other subjects. Attempts are made to invoke relevant background knowledge in lesson planning and to anticipate potential misconceptions, but attempts are incomplete. Explanations for solutions to mathematical tasks and the mathematics embedded in lessons move beyond procedures but are not fully developed.	Establishes learning objectives that align with state standards and are mostly appropriate for the age group. Objectives attempt to articulate what students will be able to *do, demonstrate, explain, justify*, etc., upon successful completion of a lesson, but are not always fully developed. Assessments mostly align with objectives and are mostly designed to elicit information about both conceptual and procedural understanding. Attempts are made to incorporate student self-evaluation in a manner that promotes a growth mindset, but attempts are not always fully developed.	Uses several relevant resources in planning for instruction, but variety is limited. Selected resources are mostly relevant to the learning objectives and efforts are made to use them to support student exploration and sense making, but efforts are not always fully developed. Resources are utilized with an attempt to maintain high levels of cognitive demand during instruction, but attempts are not always fully developed.	Designs instruction with an aim to engage all students in sense making. Lessons include some prompts to generate student thinking and discussion, but prompts are often insufficient. Endeavors to maintain a high level of cognitive demand for all students in plans for differentiation but attempts are not always fully developed.	Reflects on personal strengths and weaknesses as well as those of others. Takes most opportunities to learn from first-hand experience and the literature. Incorporates some details and supporting evidence when articulating lessons learned but some details and evidence are missing. Makes efforts to improve based on lessons learned.	Participates in class and group discussions, engaging with peers in a polite and cooperative manner. Listens to others' ideas and responds in ways that demonstrates appreciation for others' input. Articulates well-founded ideas about mathematics and mathematics instruction, but ideas often rely on the wording used by others or are not always clear or complete.	Comes prepared for most scheduled classes and meetings. Submits most required documents on time. Communicates about absences or delays but does not always plan ahead. Work is mostly clear and complete but is not always concise, well organized, or free of errors.

	Demonstrating Knowledge of Content (1a)	Setting Instructional Outcomes and Assessment (1c, f)	Demonstrating Knowledge of Resources (1d-e)	Instructional Strategies (1e; 3a-e)	Reflection (4a, e)	Communication (4d)	Professionalism (4f)
Basic	Demonstrates limited understanding of mathematical concepts and struggles to make connections between concepts and between math and other subjects. Attempts are infrequently made to invoke relevant background knowledge in lesson planning or to anticipate potential misconceptions. Explanations of solutions to mathematical tasks and the mathematics embedded in lessons are mostly procedural.	Learning objectives only articulate what students will be able to *do* and *demonstrate* but not to *explain* and *justify* and/or are often unclear or incomplete. Assessments emphasize procedural understanding only or do not align well with objectives. Opportunities are rarely provided for student self-evaluation.	Uses a limited number of resources without making an effort to use them in support of student exploration or sense making. Use of resources is teacher-centered with limited or no efforts to maintain a high level of cognitive demand.	Designs instruction with an aim to engage all students in class but emphasis is on procedures rather than on sense making. Appropriate prompts are absent from lessons or emphasize answers rather than processes. Plans for differentiation alter the level of cognitive demand for different learners.	Reflects on personal strengths and weaknesses but reflections lack detail or specific evidence that has supported personal growth. Struggles to articulate lessons learned based on the research literature.	Participates minimally in class and group discussions. Sits quietly while others speak but does not demonstrate having listened to others' ideas. Articulates ideas about math or math instruction that are based mostly on personal opinion.	Misses several scheduled classes and meetings and/or submits several required documents late. Communicates about absences or delays but doesn't plan ahead. Work is not always clear or complete and contains frequent errors.
Unsatisfactory	Demonstrates many gaps and misunderstandings in mathematical knowledge. Cannot explain solutions to mathematical tasks and/or the mathematics embedded in lessons is inappropriate.	Learning objectives are absent from lessons and/or objectives and assessments are unaligned or inappropriate.	Uses resources incorrectly or inappropriately or does not attempt to utilize relevant resources discussed in class.	Lessons are entirely teacher-led with no efforts to actively engage students. No efforts are made to differentiate instruction.	Reflections do not demonstrate a willingness to be self-critical. Blame is often placed on other individuals or on factors outside the self. Efforts to improve based on lessons learned are minimal or non-existent.	Does not participate in class or group discussions or participation is disrespectful and inappropriate. Does not articulate clear ideas about math or math instruction, or ideas are inappropriate.	Misses many scheduled classes and meetings and/or submits many required documents late without communicating about absences or delays or without acceptable reasons. Work is difficult to comprehend and contains many errors.

APPENDIX B

CODEBOOK

P = Proficient, NP = Nearing Proficient, B = Basic, U = Unsatisfactory

Non-italicized examples describe actual student lessons and unit plans; italicized examples represent researcher's descriptions of modifications to the student lessons and unit plans that would place them at a different level of proficiency.

AMTE & Danielson Standards	Forms of Access to Professional Skills & Knowledge	Definitions	Example 1	Example 2
Code: Demonstrating Knowledge of Content/Mathematical Concepts				
AMTE C.1.1 Explain the meanings of key concepts and explain the mathematical rationales underlying key procedures. AMTE C.1.5 Recognize that engaging in mathematics is more than finding an answer. Danielson 1a Know which questions sit on the fringes of what is known and which questions are likely to interest students, yield greater understanding, or represent conceptual dead ends.	Prior coursework (Math for Elementary Teachers I & II)	*P: Demonstrates solid understanding of mathematical concepts and their relationships to procedures; tasks and prompts help students to link procedures and concepts.* *NP: Demonstrates sufficient understanding of mathematical concepts and their relationships to procedures but exhibits some gaps in knowledge; tasks and prompts help students to connect procedures to concepts but PST overlooks an aspect of the procedure or concept.*	P: *Unit would need to begin with modeling and discussing the concept of a "bundle" of ten tens equaling 100 before moving on to represent three-digit numbers with the blocks.* NP: In an introductory unit on three-digit numbers, PST plans to have students use base-10 blocks to represent three-digit numbers and to ask students what each block represents and how they know this; however, PST overlooks the importance of first building up the concept of a "bundle" of ten tens equaling 100 before modeling three-digit numbers with the blocks.	P: *Unit would ensure that students can explain their use of counters, what they represent, and what it means to add numbers.* NP: *PST would have students use counters to add single-digit numbers, with an emphasis on ensuring students can explain why they are using the counters and how the counters represent the values they are adding; however, PST would overlook the importance of having students explain what it means to add numbers.*

		B: Demonstrates limited understanding of mathematical concepts and struggles to connect procedures and concepts; tasks and prompts focus on answers. *U: Demonstrates significant gaps and misunderstandings in mathematical knowledge; procedures and concepts are misunderstood or inappropriately linked.*	B: *Unit would utilize the blocks to represent three-digit numbers but would not incorporate any discussion of why each block is used to represent each place value; students would simply follow teacher's lead in using designated blocks for each place value.* U: *Unit would incorporate inaccurate mathematics, such as stating that each place value is 10 more (rather than 10 groups more) than the previous place value.*	B: In an introductory unit on addition, PST plans to have students use counters to add single-digit numbers and to ask how many counters they have in all when they put them together. U: *Unit would state that the word "more" automatically indicates addition without acknowledging that "more" can also appear in subtraction problems such as, "Jill has 3 apples and Juán has 2. How many more apples does Jill have?"*
Code: Demonstrating Knowledge of Content/ Significance				
AMTE C.1.2 Apply mathematical knowledge to real-world situations by using mathematical modeling to solve problems. AMTE C.1.5 Make connections across disciplines in ways that	Prior coursework (Math for Elementary Teachers I & II) Course textbook (*Mathematical Mindsets*) Instructor modeling of Notice & Wonder and 3-Act Tasks	*P: Incorporates mathematical or real-world scenarios that aptly illuminate mathematical concepts and naturally lend themselves to modeling.* *NP: Incorporates mathematical or real-world scenarios that are*	P: In an introductory unit on surface area, PST plans to have students determine the exact amount of wrapping paper needed to wrap a package with no gaps or overlaps so that no paper is wasted. NP: *PST would have students determine the surface area of*	P: *In addition to having students write down the class schedule and show the appropriate times, PST would also ask students why it is important to have a schedule and to be able to recognize the times on the schedule.* NP: In an introductory unit on telling and writing time, PST plans to

illuminate mathematical ideas. Danielson 1a Identify connections with other aspects of the discipline or with other disciplines.		*relevant and lend themselves to modeling but are not fully developed.* *B: Incorporates mathematical or real-world scenarios that are tangential or contrived, or that are disconnected from the rest of the lesson.* *U: Incorporates mathematical or real-world scenarios that are inappropriate and/or do not suggest a purpose for the mathematical activity.*	*household objects but would not discuss why anyone would need to know the surface area of these objects.* B: *PST would mention that surface area is important for things like wrapping packages but would have students practice finding surface area via a standard worksheet.* U: *PST would have students find the surface area of a piece of paper, which could lead students to conflate 2D area with 3D surface area (inappropriate); or PST would have students complete a standard worksheet with no discussion as to how or why surface area calculations can be applied (mathematical scenarios do not suggest a purpose*).*	ask students to write down the class schedule and show the appropriate times on analog and digital clocks, but students are not asked to consider why schedules and telling time are useful. B: *PST would make up several random story problems such as, "Ana sees that it is 4 o'clock. What does her clock look like?"* U: *PST would have students use their clocks to show the times on a military schedule (inappropriate).*
*NOTE: *The NP example for this category also does not explicitly discuss purpose; however, this example lends itself to modeling and merely requires the addition of a discussion to move the lesson to P, whereas the U example does* not *lend itself to modeling* or *suggest a purpose, indicating that the lesson requires significant further development to advance toward proficiency.*				
Code: Demonstrating Knowledge of Content/Prerequisite Knowledge and Skills				
AMTE C.1.4	Prior coursework	*P: Invokes key background*	P: *In addition to having students*	P: In an introductory unit on fraction

Make important connections among the mathematics taught in the grades or units before and after what they are teaching. AMTE C.2.2 Consider the prior knowledge and experiences students bring to a lesson. AMTE C.3.1 Know the mathematics that comes before and after a given topic. Danielson 1a Understand prerequisite relationships among topics and concepts and links to necessary cognitive structures by students to ensure understanding.	(Math for Elementary Teachers I & II) Montana Common Core Standards for Mathematical Practice & Content Course textbook (*Mathematical Mindsets*) Excerpts from *Making Sense: Teaching and Learning Mathematics with Understanding*	*knowledge and skills in lesson plans, providing a solid foundation for building new knowledge and skills.* *NP: Invokes relevant background knowledge in lesson plans but overlooks a relevant concept or skill, or incorporation of prior knowledge or skills requires further development to provide a strong foundation for building new knowledge and skills.* *B: Identifies relevant background knowledge but does not attempt to use it as a foundation to build*	*utilize prior knowledge of skip-counting to help them find sums of nickels and sums of dimes, PST would ask students to recall and apply strategies they have learned for adding two-digit numbers before having them find sums of various coin combinations.* NP: *In an introductory unit on solving problems with coins, PST would identify and incorporate prior knowledge of adding two-digit numbers but would overlook relevant knowledge of skip-counting, which is useful but not essential.* B: In an introductory unit on solving problems with coins, PST identifies and incorporates prior knowledge of skip-counting into lessons but overlooks prior	equivalence, PST plans to have students split a graham cracker equally among four group members and to identify the fractional name of each member's piece; then she will pass out another graham cracker, this time split into 8 pieces, and ask the students to share the pieces equally and determine whether they will end up with the same amount of graham cracker as before. NP: *PST would have students warm up by naming several fractions pictured as pies then would provide a definition of equivalent fractions for them before moving on to having students use their fraction strips to identify several fractions equivalent to ½; PST would ensure that prior knowledge of naming fractions is intact, but this knowledge would not be used to lead to a discovery of fraction equivalence.* B: *PST would acknowledge that*

		new knowledge and skills; or identifies and incorporates some relevant knowledge but overlooks a fundamental concept. *U: Does not identify or incorporate relevant background knowledge or skills, or incorporation of knowledge or skills is inappropriate.*	knowledge of two-digit addition, simply expecting students to be able to find sums of various coin combinations with no reference to previously learned strategies for multi-digit addition. U: *PST would not mention or include prior knowledge that would be necessary for finding sums of various coin combinations.*	*students should be able to recognize and name proper fractions before learning about equivalent fractions but would not invoke or utilize this prior knowledge within the lesson.* U: *PST would begin the lesson by incorrectly describing the numerator as determining the size of a fraction—the more pieces, the bigger the fraction.*
Code: Demonstrating Knowledge of Content/Students' Thinking				
AMTE C.3.1 Try to see mathematical situations through students' eyes. AMTE C.3.2 Anticipate students' diverse solution strategies.	Prior coursework (Math for Elementary Teachers I & II) Montana Common Core Standards for Mathematical Practice & Content Course textbook (*Mathematical Mindsets*) Excerpts from *Making Sense: Teaching and Learning Mathematics*	*P: Anticipates an appropriate range of potential solution strategies and responses; anticipates potential points of confusion.*	P: In an introductory unit on multiplication, PST anticipates a range of strategies, including pictures, counters, skip counting, and arrays; a template is provided for arranging equal groups for students who may struggle to organize their work, and guiding questions are incorporated to help students make the intended connections, for example, "How many-wheels does each car have	P: *PST would expect students to determine whether numbers are prime or composite either by factoring the numbers using lists or factor trees and/or by using Cuisenaire rods to determine whether the numbers can be divided into equal parts using rods other than just the unit rods; PST would provide multiplication tables as an aid and would plan guiding questions such as, "Can you find this number on the*

	with Understanding Teaching Channel & Inside Mathematics videos	*NP: Anticipates potential solution strategies and responses but overlooks a key strategy or response; may not anticipate potential points of confusion.* *B: Expects a single specific solution strategy and response; does not anticipate potential points of confusion.*	individually? Does each car have an equal number of tires?” NP: *PST would anticipate a more limited number of strategies such as pictures and counters only, overlooking skip counting and/or arrays; or PST would not anticipate potential points of struggle such as students struggling to organize their equal groups or not recognizing how parts of cars relate to equal groups.* B: *PST would focus on one strategy only, such as the use of counters to show equal groups, and would not incorporate plans for addressing potential points of struggle.*	*multiplication table? What do you think it means if you can find numbers close to it but not this exact number?” and, “I see you listed 9 as prime, can you explain your reasoning? Which rods did you try to fit into 9? Did you try all of them to see if there’s one that will divide it equally?”* NP: *PST would expect students to factor each number to determine whether it is prime or composite by listing out known multiplication facts or using a factor tree and would provide multiplication tables as a helpful tool for students who may struggle with multiplication facts.* B: In an introductory unit on prime and composite numbers, PST expects students to list out the factors of each number to determine whether it is prime or composite without any indication of other strategies that might be used or of potential points of

		U: Does not indicate expected or anticipated solution strategies or responses; does not anticipate potential points of confusion.	U: *PST would merely state that students would be solving multiplication problems, with no indication of expected solution strategies or potential points of struggle.*	struggle in identifying factors. U: *PST would merely state that students would be identifying prime and composite numbers, with no indication of how they would be expected to do so or in what ways they might struggle.*
Code: Setting Instructional Outcomes and Assessment/Alignment Between Learning Objectives and Learning Standards				
AMTE C.2.2 Articulate and clarify mathematics learning goals. Danielson 1c Establish clear instructional outcomes that relate to content standards and are appropriate for students.	Prior coursework (Curriculum Theory & Design) Montana Common Core Standards for Mathematical Practice & Content Supplemental reading on writing effective learning objectives Instructor exemplars	*P: Learning objectives align with an appropriate grade-level standard in a clear, specific, and measurable way.* *NP: Learning objectives align with an appropriate grade-level in a way that is either clear, specific, or measurable, but not all three; or learning objectives align with a standard that is similar to the*	P: *PST would write the following objective for the kindergarten standard on decomposing numbers less than or equal to 10 into pairs in more than one way: "Students will be able to decompose the number 10 into at least two different pairs using counters and a tens frame."* NP: *PST would write the following objective, but align it with the kindergarten standard on finding the number that makes ten when given a number 1-9 instead of the standard on decomposing 10 into pairs:*	P: *PST would write an objective such as, "Students will be able to explain verbally or in writing that values such as 10 and -10 are on opposite sides of zero within the context of real-world scenarios such as sea level or temperature."* NP: *PST would write an objective such as, "Students will understand that values such as 10 and -10 are the same distance from zero on the number line but are on opposite sides."*

		standard indicated in the lesson in a clear, specific, and measurable way but the actual standard that the lesson addresses is a different standard than the one indicated in the lesson. *B: Learning objectives appear to align with an appropriate grade-level standard but are not fully clear, specific, or measurable.* *U: Learning objectives do not align with an appropriate grade-level standard or are too unclear to decipher.*	*"Students will be able to decompose the number 10 into at least two different pairs using counters and a tens frame."* B: PST writes the following objective for a kindergarten lesson on decomposing numbers less than or equal to 10 into pairs: "Students will be able to write the numbers they are decomposing." U: *PST would write an objective that is so vague that alignment could not be determined, for example, "Students will be able to decompose," with no indication of whether students will be decomposing numbers or shapes, or what types of numbers or shapes they would be working with.*	B: *PST would write an objective such as, "Students will learn that positives and negatives are opposites."* U: PST writes the following objective for a 6th grade lesson on understanding that positive and negative numbers are used together to describe quantities having opposite directions or values even though addition and subtraction of integers is not introduced until 7th grade: "Students can show me that subtracting a negative number from a negative number uses the process of 'adding the opposite.'"
Code: Setting Instructional Outcomes and Assessment/Alignment Between Learning Objectives and Assessments				
AMTE C.2.4 Elicit and use evidence of student learning and engagement to analyze teaching. Danielson 1c	Prior coursework (Assessment in Education) Montana Common Core Standards for Mathematical	*P: Objectives and assessments align in terms of both content and process and are supported by the lesson's main activities; examples of*	P: *PST would clarify in the objective that students are explaining the place values of three-digit numbers and would provide an example number such as 243*	P: *PST would plan to assess the objective as described below and would include relevant discussion during the lesson; PST would also provide an example*

Establish outcomes that permit viable methods of assessment. Danielson 1f Design assessments that are appropriate to different types of outcomes and establish standards of performance.	Practice & Content Course textbook (*Mathematical Mindsets*) Teaching Channel & Inside Mathematics videos Instructor exemplars	*satisfactory student work and explanations that would meet the objectives are provided.* *NP: Objectives and assessments are mostly aligned in terms of both content and process and are supported by the lesson's main activities but may require further development and/or examples of satisfactory student work and explanations that would meet the objectives are not provided.* *B: Objectives and assessments align in terms of content or process but not both, or objectives*	*along with an example student response such as, "The 2 means 200, the 4 means 4 tens, and the 3 means 3 ones."* NP: In a 2nd grade lesson on three-digit numbers, PST includes the objective, "Students will be able to explain the different place values of digits in a given number," and plans to assess the objective by having students write an exit ticket "in which they have to explain the different place values of digits in a given number" after using base-10 blocks to represent and discuss three-digit numbers during the lesson; however, the specific number to be used in the exit ticket nor an example response are provided, and it's unclear from the objectives the number of place values being studied in the lesson. B: *PST would plan to assess the objective by having students show a three-digit number*	*student response such as, "Inches are related to feet because 12 inches are in a foot, but centimeters are smaller than inches and are not part of the same measurement system."* NP: *PST would plan to assess the objective by having students explain the difference between inches, centimeters, and feet either verbally or in writing and would support the objective by including explicit attention to these three units as students measure objects with a ruler.* B: *PST would plan to assess the objective by having students explain the differences between inches, centimeters, and feet either verbally or in writing, which would align with the objective, but would not support this objective via the lesson activities, which focus on measuring objects with a ruler without any explicit attention to these three different units.*

		and assessments align but are not supported by the main activities in the lesson. *U: Objectives and assessments are unaligned and/or inappropriate, or aligned cannot be discerned due to lack of clarity.*	*using base-10 blocks without requiring explanation.* U: *PST would plan to assess the objective by having students find the sum of two three-digit numbers.*	U: In a 2nd grade lesson on measuring objects with rulers, PST includes the objective, "Students will be able to differentiate inches, centimeters, and feet," but plans to evaluate the objective via an unspecified math notebook entry at the conclusion of a lesson that emphasizes measurement with rulers but does not include explicit use or discussion of these three different measurement units.
Code: Setting Instructional Outcomes and Assessment/Level of Cognitive Demand of Objectives and Assessments				
AMTE C.2.2 Elicit evidence of students' progress toward the intended mathematics learning goals using a repertoire of strategies. AMTE C.2.4 Gather evidence on students' multiple mathematical knowledge bases. AMTE C.4.2	Prior coursework (Assessment in Education) Course textbook (*Mathematical Mindsets*) Teaching Channel & Inside Mathematics videos Instructor exemplars	*P: Objectives and assessments reflect a high level of cognitive demand and are designed to elicit information about both conceptual and procedural understanding; opportunities are provided for student self-evaluation.*	P: *PST would make the objective more measurable by specifying that students will describe the relationship between area and perimeter and would provide more targeted prompts designed to elicit their findings about this relationship, for example, "What do you notice about the areas of your rectangles? Are they the same as the perimeter? Does the area change or stay*	P: *PST would include questions designed to prompt student self-evaluation such as, "How comfortable do you feel with decomposing numbers? What strategies did you find helpful in decomposing numbers?"* NP: In a 1st grade lesson on decomposing numbers, PST aims for students to demonstrate understanding that

Develop positive mathematical identities by focusing on robust goals for what is important to know and be able to do in mathematics, including doing mathematics for one's own sake. Danielson 1c Establish instructional outcomes that represent important learning, high expectations for students, and intellectual rigor. Danielson 3d Involves students in frequently assessing and monitoring the quality of their own work against the assessment criteria and performance standards.		*NP: Objectives and assessments reflect a high level of cognitive demand and are designed to elicit information about both conceptual and procedural understanding but are not fully developed and/or opportunities are not provided for student self-evaluation.* *B: Objectives and assessments reflect a low level of cognitive demand and primarily emphasize*	*the same? Do you notice a pattern in the way the area changes?"* NP: In a 4th grade unit on area and perimeter, PST includes a lesson with the objective for students to explore the relationship between area and perimeter by creating various rectangles with a fixed perimeter using grid paper; the objective is assessed through observation and targeted questioning, but questions emphasize strategies for finding area and perimeter rather than the relationship between them; opportunities are provided for student self-evaluation but emphasize perimeter only: "What strategies have you learned today that can help you find side lengths and perimeter?" B: *The objective for the lesson would be for students to find the areas and perimeters of various rectangles and would assess*	numbers can be decomposed in multiple ways and plans to assess the objective by having students model different ways to split the number 10 into groups using counters and asking questions such as, "Did everyone's groups look the same? What do you think this means?" However, no opportunities are provided for student self-evaluation. B: *The objective for the lesson would be for students to decompose numbers and PST would plan to assess the objective by looking for correct decompositions from each student, without any discussion as to the*

		procedural understanding. *U: Objectives and assessments emphasize memorization only.*	*the objective by looking for correct calculations.* U: *The objective of the lesson would be for students to memorize the formulas for area and perimeter.*	*significance of decomposition.* U: *PST would plan to have students memorize pairs of numbers that add to 10.*
Code: Setting Instructional Outcomes and Assessment/Developmental Progression and Pacing				
AMTE C.1.4 Make decisions about the sequencing and time required to teach content in depth. AMTE C.2.2 Understand mathematics learning goals for lessons and units and how these goals fit within a developmental progression of student learning. Danielson 1e Design lessons and units in which the development of concepts from simpler to more complex is clear. Danielson 3c Pacing is appropriate to the students and	Prior coursework (Curriculum Theory & Design) Montana Common Core Standards for Mathematical Practice & Content Excerpts from *Making Sense: Teaching and Learning Mathematics with Understanding* Teaching Channel & Inside Mathematics videos Instructor exemplars	*P: Learning objectives reflect appropriate attention to the developmental progression of a concept, supporting procedural fluency by first developing conceptual understanding.* *NP: Learning objectives reflect a slightly rushed or prolonged developmental progression of a concept but afford some opportunity to develop conceptual understanding.*	P: *PST would aim to have students explain what it means to add in the context of multiple real-world scenarios through the use of various tools such as counters, pictures, and number lines and would encourage students to relate these tools to representations involving symbols and equations.* NP: *PST would aim to have students be able to explain what it means to add numbers together and why this is useful in the context of real-world scenarios; however, emphasis would still shift too quickly to solving problems in the format of equations without explicit attention to how symbolic*	P: *PST would aim to have students measure objects to the nearest inch and to be able to explain the relevance of measurement in the context of real-world scenarios, including objects that measure exactly to an inch and those that must be estimated to the nearest inch.* NP: *PST would aim to have students measure objects to the nearest inch and to be able to explain the relevance of measurement in the context of real-world scenarios but would spend the entire introductory unit measuring objects that align perfectly with the inch marks on a ruler without moving on to estimate objects to*

to the content; students do not feel rushed and time does not drag.		*B: Learning objectives proceed far too quickly or slowly through the developmental progression of a concept, rushing through concepts to focus on procedures.* *U: Learning objectives overlook conceptual understanding and/or do not follow a logical developmental progression.*	*representations relate to earlier tactile and visual strategies.* B: In a three-day kindergarten unit introducing addition, PST aims to have students combine colored cubes to find sums within 5 on day 1, learn how to use the + and = signs for sums within 5 on day 2, and solve addition equations within 10 on day 3, with an emphasis on arriving at correct sums as opposed to understanding what it means to add or how addition is relevant to real-world problem solving. U: *Learning objectives would emphasize solving addition equations from day 1, with no explicit introduction to or discussion of what the + or = sign mean.*	*the nearest inch that do not align perfectly with inch marks.* B: *PST would aim to have students measure to the nearest inch on day 1, the nearest foot on day 2, and the nearest yard on day 3, with no discussion of why measurement is a relevant skill.* U: In a 2nd grade unit introducing measurement with rulers, PST aims to have students measure objects in the classroom with no discussion of how to measure to the nearest inch or what to do when objects don't line up perfectly with one of the inch marks on a ruler.
Code: Demonstrating Knowledge of Resources/Suitability of Resources				
AMTE C.1.4 Decide whether to replace or adapt materials to better address the content and	Prior coursework (Math for Elementary Teachers I & II)	*P: Selected resources are optimally suited to the mathematical content intended to be conveyed in a lesson or unit.*	P: *PST would incorporate the use of tens frames in conjunction with counters to support students in solving*	P: *PST would incorporate Cuisenaire rods and/or inch tiles in the unit, with the intention of having students model*

process expectations. AMTE C.1.6 Know when and how to use digital and physical tools to explore mathematical and statistical ideas and to build conceptual understanding of these; recognize the insights to be gained and possible limitations of different tools. Danielson 1d Know a wide range of resources available for enhancing content, including district, professional, and community resources, and utilize these resources flexibly. Danielson 3c Choose resources and instructional materials that are suitable for the students and applicable to the instructional outcomes.	Prior coursework (Instructional Technology) Montana Common Core Standards for Mathematical Practice & Content Sample *Go Math!* curriculum guides Teaching Channel & Inside Mathematics videos Instructor modeling In-class practice with mathematical manipulatives	*NP: Selected resources are suited to the mathematical content intended to be conveyed in a lesson or unit but a particularly salient resource has been overlooked.* *B: Selected resources are unnecessarily limited and/or selected resources may be used for the mathematical content intended to be conveyed in a lesson or unit but other resources would be better suited to the content.* *U: Selected resources are ill suited to the mathematical content intended to be conveyed in a lesson or unit.*	*for missing addends when the sum is 10.* NP: In a kindergarten lesson on solving for missing addends when the sum is 10, PST plans to have students draw pictures and use counters to solve word problems such as, "How many cookies did Sarah get at the bakery if she had 8 cookies but now has 10?" as well as play Phase 10; however, PST overlooks tens frames as a particularly salient tool. B: *PST would plan to have students use only one resource for solving missing addend problems, such as counters OR pictures but not both.* U: *PST would use flash cards such as 7 + __ = 10 in the unit but no other supporting manipulatives or resources.*	*prime and composite numbers with the rods and/or via rectangles (e.g., a rectangular area of 12 units can be composed with more than one L x W combination, making it composite, whereas a rectangular area of 7 units cannot, so it is prime).* NP: *PST would plan to use beans and would also include factor rainbows and multiplication tables in the unit but would overlook Cuisenaire rods or inch tiles as particularly salient tools.* B: In a 4th grade lesson introducing the difference between prime and composite numbers, PST plans to have students use beans to attempt to create equal groups out of the numbers 3, 4, 5, and 6. U: *PST would plan to have students use a number line for identifying prime and composite numbers.*

Code: Demonstrating Knowledge of Resources/Use of Resources				
AMTE C.1.4 Decide whether to replace or adapt materials to better address the content and process expectations. AMTE C.1.6 Know when and how to use digital and physical tools to explore mathematical and statistical ideas and to build conceptual understanding of these; recognize the insights to be gained and possible limitations of different tools. AMTE C.2.3 Weave a mathematical idea across many representations in ways that help students see connections among representations and see affordances of different representations. Danielson 1d	Prior coursework (Math for Elementary Teachers I & II) Prior coursework (Instructional Technology) Course textbook (*Mathematical Mindsets*) Excerpts from *Making Sense: Teaching and Learning Mathematics with Understanding* Teaching Channel & Inside Mathematics videos Instructor modeling In-class practice with mathematical manipulatives	*P: Resources are utilized to support student exploration and sense making and to illuminate important mathematical connections; students are given choice in how resources are utilized whenever appropriate.* *NP: Resources are utilized to support student exploration and sense making and to illuminate important mathematical*	P: In a 3rd grade lesson on equal groups, PST plans to show a video of Hot Wheels cars and to have students notice the equal groups in the video (groupings of cars and groupings of items on the cars such as sets of tires) then to have students use manipulatives or drawings to model other equal groupings of items on a set of cars while responding to prompts such as, "How do you know you have equal groups?" PST then plans to have students compare and contrast an image of a pickup truck and a semi to distinguish between equal groups and non-equal groups (for example, pickups and semis do not have equal groups of tires). NP: *PST would plan to have students notice and discuss equal groups in the video but not model them with manipulatives, or PST might plan to*	P: *PST would plan to have students create a picture using triangles, circles, and squares, with explicit instructions to use these shapes to model objects they see in the world around them; students would be expected to justify their use of specific shapes to model specific objects based on the properties of the shapes, for example using circles to represent the wheels on a car because wheels are round like circles.* NP: *PST would plan to have students create a picture using triangles, circles, and squares, with explicit instructions to use the shapes to model*

Know a wide range of resources available for enhancing content, including district, professional, and community resources, and utilize these resources flexibly. Danielson 3c Engage students mentally through the use of instructional resources and support student choice, adaptation, or creation of materials to enhance learning.		*connections but efforts require further development to be fully effective.* *B: Resources are utilized in a prescriptive or non-mathematical manner with little or no opportunity for exploration, sense making, or illuminating important mathematical connections.* *U: Resources are utilized incorrectly or inappropriately or, manner of use cannot be determined from the lesson.*	*have students model and discuss equal vs. unequal groups devoid of a relatable context.* B: *PST would plan to have students follow the teacher's lead in drawing equal groups, counting totals, and writing the associated number sentences such as 5 + 5 + 5 = 15 without discussion about the significance of what they are doing or why they are doing it.* U: *PST would have students model word problems involving equal groups by instructing them to identify the multiplication sentence and replace the numbers with counters; for example, 3 x 2 = ●●● x ●●.*	*objects they see in the world around them but with no discussion of why they chose to model specific objects with specific shapes.* B: In a kindergarten lesson on shapes, PST plans to have students draw a picture of themselves or of an animal using at least 3 triangles, 3 circles, and 3 squares, with no effort to encourage students to utilize the shapes to model real-world objects that might be more appropriate to the shapes or to discuss why they might use particular shapes to represent particular real-world objects. U: *PST would plan to have students make a tessellation using cutouts of triangles, circles, and squares, which is not possible to do.*
Code: Instructional Strategies/Coherence				
AMTE C.1.4 Make decisions about the sequencing and time required to teach the content.	Prior coursework (Curriculum Theory & Design) Excerpts from *Making Sense:*	*P: Lessons and units feature a well-defined, unifying theme and a logical flow, with opportunities for reflection and closure.*	P: In a 6th grade lesson introducing surface area, PST plans to have students notice and wonder about a set of three-dimensional shapes	P: *PST would have students begin the lesson by discussing why it is helpful to know how to add coins together and would encourage them to each think of*

Danielson 1e Design lesson and unit plans that reflect a logical sequencing of activities and in which the instructional outcomes, activities, materials, methods, and grouping of students are all in alignment. Danielson 3c Design coherent lessons that allow for reflection and closure.	*Teaching and Learning Mathematics with Understanding* Teaching Channel & Inside Mathematics videos Instructor modeling Instructor exemplars	*NP: Lessons and units feature a unifying theme, a mostly logical flow, and opportunities for reflection and closure, but one or more of these elements requires further development.*	including a cube, a rectangular prism, and a triangular prism; students will then use graph paper to find the area of one of the faces of a cube and form a hypothesis about how to find the total of all of the faces and then do the same for a rectangular prism, with the triangular prism being an optional challenge; toward the end of the lesson, PST informs students that what they have found is called surface area and asks them to write down their new discoveries about surface area in their math journals. NP: *PST would plan for the above activities but the wrap-up might simply involve the teacher explaining to students that what they have found today is called surface area without students reflecting on their own discoveries, or PST might have students find all of the faces of a cube and rectangle without including explicit discussion*	*an item they'd like to save up to buy before having them engage in the piggy bank activity; PST would plan to have students share and discuss their strategies for finding their totals and to write down their favorite strategy they learned from a peer.* NP: *PST would follow the plan below (which includes one divergent activity) but would also include a closing discussion about why it is helpful to know how to find sums of coins.* B: In a 2nd grade lesson on finding sums of coins, PST plans to have students sing along to a song reviewing coin names and values then to fill up a piggy bank with 10 coins (determined by rolls of a dice with images of the coins) and find the total

		B: Lessons and units feature a mostly logical flow but include abrupt transitions, do not include a discernible theme, and/or opportunities for reflection and closure are absent. *U: Lessons and units feature an inappropriate theme and/or the flow of the lesson or unit is not logical.*	*comparing and contrasting the two.* B: *PST would have students find the area of a rectangle, then define surface area without any discussion of the connection between the two; students would find the sum of all the areas of the faces of a cube and rectangular prism, ending the lesson by having students hand in their work without any closure or student self-reflection.* U: *PST would define surface area then have students practice finding the areas of several 2D rectangles before ending the lesson by having students find the surface area of a cube, with potential confusion resulting from defining surface area immediately before an activity involving 2D area.*	value before moving on to an activity where they find different coin combinations that add up to the same total; lesson ends with students filling out a piggy bank worksheet where they find the totals of given sets of coins. U: *PST would plan to have students order the coins from smallest to largest in size to review their names and values (without taking time to discuss the discrepancies between size and value) then teach students the difference between the dollar and cents symbol before having students find different coin combinations that add up to the same total (with none of the totals requiring the use of the $ symbol) before concluding with the piggy bank worksheet.*
Code: Instructional Strategies/Nature of Tasks				
AMTE C.2.2 Select cognitively challenging mathematical	Prior coursework (Math for Elementary Teachers I & II)	*P: Tasks position students as the primary doers and feature a high level of cognitive*	P: *Rather than provide the worksheet, PST would verbalize the scenario (e.g.,*	P: *PST would have students explore integers using red and black chips on a balance,*

tasks that provide multiple entry points and varied solution strategies to motivate student learning, develop new mathematical knowledge, and build connections between conceptual and procedural understanding. AMTE C.2.3 Position students as authors of ideas. Danielson 1e Prepare activities and assignments that emphasize thinking and problem-based learning, permit students choice and initiative, and encourage depth rather than breadth. Danielson 3d Prepare activities and assignments that challenge students to think broadly and deeply, to solve a problem, or to otherwise engage in	Course textbook (*Mathematical Mindsets*) Excerpts from *Making Sense: Teaching and Learning Mathematics with Understanding* Teaching Channel & Inside Mathematics videos Supplemental resources on strategies to promote critical thinking such as Notice & Wonder, Which One Doesn't Belong, Same/Different, etc. Instructor modeling Instructor exemplars	*demand, permitting non-algorithmic thinking and multiple solution strategies and fostering conceptual as well as procedural understanding.* *NP: Tasks foster conceptual as well as procedural understanding but are teacher directed rather than student driven, or tasks feature the characteristics of proficiency but require further development to be fully effective.*	*"there were 6 lady bugs and then some more joined and now there are 8") and encourage students to draw their own pictures of the scenario, with the option to use number lines and counters as supports; students would compare and contrast their solution approaches.* NP: In a 1st grade lesson on solving for missing addends using pictures, PST plans to have students complete a worksheet with images of a certain number of bugs to the left of the plus sign, a blank box to the right, and another image of a larger number of bugs to the right of the equal sign; pairs of students are left to their own devices to figure out how to solve for the missing addends and are allowed the option of using manipulatives to help them, but the worksheet confines them to a particular format as opposed to leaving open the option of, for example, drawing 6	*encouraging them to come to their own conclusions about what happens when the number of red and black chips is the same and different, encouraging them to notice, for example, that when a negative value is added to a positive value, the balance moves closer to zero.* NP: *PST would initiate the lesson with a demonstration of 5 negative chips and 5 positive chips balancing each other on a scale to provide a conceptual underpinning for using red chips to "cancel" black chips and vice-versa but the remainder of the lesson would proceed as described below.*

nonroutine thinking.		*B: Tasks feature student engagement but the level of cognitive demand is low, emphasizing prescribed routines or procedures without meaningful connections to concepts.* *U: Tasks are entirely teacher led with no opportunity for active student engagement, and/or tasks emphasize memorization only.*	ladybugs and recognizing that 2 more ladybugs need to be added to the drawing to show a total of 8 ladybugs. B: *PST would plan to have students complete the worksheet by providing them with explicit instructions to cross out the number of lady bugs that is the same in both groups and then draw the remaining number of lady bugs that is not crossed out into the empty box in the middle, with no discussion as to why this method identifies the missing addend and with no opportunity for other methods.* U: *PST would walk through the worksheet on the overhead projector, filling in all the missing addends and expecting students to watch and listen.*	B: In a 6th grade lesson on adding and subtracting integers, PST plans to have students use red and black chips to solve problems based on rules such as "canceling out" and "adding the opposite" without any mention of the underlying conceptual foundation for the rules. U: *Teacher would solve integer addition and subtraction problems by displaying the red and black chips on the projector and expecting students to take notes on the process.*
Code: Instructional Strategies/Nature of Prompts				
AMTE C.2.2 Plan purposeful and meaningful questions to probe student thinking, make	Course textbook (*Mathematical Mindsets*)	*P: Prompts are designed to elicit information about conceptual and procedural understanding and*	P: In a 3rd grade lesson on skip-counting as a strategy for solving problems involving equal groups, PST	P: *PST would prompt students to consider why there appear to be two addition and two subtraction*

the mathematics visible for discussion, and encourage reflection and justification. AMTE C.2.3 Engage students in whole-class discussions to share, compare, and analyze strategies and solutions; position students as authors of ideas. Danielson 3b Pose high-quality questions that promote thinking by students, encourage them to make connections among previously believed, unrelated concepts or events and to arrive at new understanding of complex material.	Excerpts from *Making Sense: Teaching and Learning Mathematics with Understanding* Teaching Channel & Inside Mathematics videos Supplemental reading on effective questioning for inquiry-based lessons Instructor modeling Instructor exemplars	*to encourage reflection, justification, and connections.* *NP: Prompts are designed to elicit information about conceptual and procedural understanding but do not encourage reflection, justification, and/or connections, or prompts incorporate the features of proficiency but more such prompts are needed for the lesson to be fully effective.*	plans to prompt students with question such as the following: "How is skip counting related to arrays?" and "How is skip counting easier than counting out every counter or tick mark?" and "Besides ranching, are there examples in your life when you would use skip counting?" PST also plans to have students explain whether they think drawing a picture, using manipulatives, or using skip counting is more effective for solving problems involving equal groups. NP: *PST would ask students questions such as, "Can you tell me how you used skip counting to solve that problem?" and "Why can we use skip counting to solve problems involving equal groups?" but would not prompt students to consider instances when skip counting could be used in real life or how it might be linked to some of the other strategies*	*equations in every fact family. Why is this the case? Will there always be just two addition and two subtraction equations? Why or why not?* NP: In a 1st grade lesson introducing fact families, PST plans to ask students what they notice and wonder about a fact family written on the board and then to ask them what they notice when comparing two different fact families after determining the two missing number sentences in the second fact family; although PST sets students up to discover that there are two addition and two subtraction sentences in each fact family, she never prompts them

		B: Prompts emphasize answers and procedures only. *U: Prompts are inappropriate or absent.*	*students have been studying such as arrays.* B: *PST would only ask questions such as, "What answer did you get when you used skip counting?" and, "Can you skip count those numbers for me?"* U: *PST would not include any prompts in the lesson, or PST would include inappropriate prompts such as, "Solve 13 + 13 + 13 using skip counting."*	to consider why this is the case. B: *PST would only ask questions such as, "What is the missing number sentence in this fact family?" and, "How many addition and subtraction sentences are in every fact family?"* U: *PST would not include any prompts in the lesson, or PST would include inappropriate prompts for the age group such as, "What if we used negative numbers?"*
Code: Advocacy and Equity/Differentiation				
AMTE C.2.1 Consider students' individual needs, cultural experiences, and interests as well as prior mathematical knowledge when selecting tasks and planning for mathematics instruction. AMTE C.4.1 Realize that access is increased when students can approach a	Prior coursework (Introduction to Teaching Exceptional Learners) Course textbook (*Mathematical Mindsets*) Supplemental resources on strategies to promote critical thinking such as Notice & Wonder, Which One Doesn't Belong,	*P: Lessons and units feature multiple entry points and specific supports for struggling students as well as appropriate extensions for students ready for a challenge.* *NP: Lessons and units feature either multiple entry points and specific supports for struggling students or appropriate extensions for*	P: *PST would incorporate more specific supports for students who may struggle, such as providing a hundreds chart and/or including guiding questions such as, "What if you tried to use nickels or pennies instead of dimes?"* NP: In a 2nd grade lesson on showing two different ways to find the same total, PST plans to challenge exceling students to create each specified	P: *PST would include specific guiding questions that could support struggling students, such as, "How many of the unit cubes do you count inside the stick? How do you think this can help us decide which place value the stick can be used for? How many of the unit cubes do you count inside the flat square?"* NP: *PST would plan to have advanced students work on four-digit numbers*

problem from multiple routes and when they use curriculum materials that include high-quality, meaningful tasks that go beyond basic skills; realize that advancement is possible when opportunities are provided to go beyond grade-level expectations. Danielson 1e Prepares learning activities that are differentiated, as appropriate, for individual learners.	Same/Different, etc. Instructor modeling Instructor exemplars	*students ready for a challenge but not both, or lessons and units incorporate the features of proficiency but these require further development to be fully effective.* *B: Lessons and units suggest plans for differentiation, but plans are vague or not linked to the concept under study.* *U: Lessons and units do not incorporate plans for differentiation, or plans for differentiation are inappropriate or alter the level of cognitive demand for different learners.*	amount with the least amount of coins and the most amount of coins; however, for struggling students, PST only suggests pairing them with more advanced students, with no other specific supports planned in advance. B: *PST would state that they would provide more challenging totals to advanced students and easier totals to students who struggle without specifying the parameters that would make numbers easier or more challenging.* U: *PST would not include plans for differentiation, or would plan to have struggling students work on subtraction problems from last week.*	*and would support struggling students with guiding questions, but no specific questions would be suggested.* B: *PST would have students who finish quickly play a math game on the computer.* U: In a 2nd grade lesson on three-digit numbers, PST plans to have struggling students work on two-digit numbers instead, which changes the objective for these students.
		Code: Advocacy and Equity/Perspectives		
AMTE C.2.1 Attend to developing students' identities and agency so that students can see mathematics as components of	Course textbook (*Mathematical Mindsets*) Essential Understandings Regarding Montana	*P: Incorporates multiple perspectives and contexts into lesson and unit plans, including those of non-dominant and historically*	P: *PST would incorporate information about the distinguishing features of each tribes' regalia to better support the Indian Education for All learning*	P: *PST would identify each tribe by name and include information about the original lands occupied by tribes to compare and contrast against current reservation*

their cultures and see themselves in the mathematics. AMTE C.4.2 Analyze task selections and reflect on ways in which they may shape students' mathematical identities, attending to issues of context that may privilege or exclude particular groups of students. AMTE C.4.3 Value and draw upon students' funds of knowledge to build upon the cultural, linguistic, and unique ways of knowing of their students. AMTE C.4.4 Understand roles of power, privilege, and oppression in the history of mathematics education and implement practices to empower each	Indians (Montana Office of Public Instruction document) Selected videos on topics related to equity, bias, and Indian Education for All Instructor modeling Instructor exemplars	*marginalized groups, in a way that challenges stereotypes and predominant paradigms.* *NP: Incorporates multiple perspectives and contexts into lesson and unit plans, including those of non-dominant and historically marginalized groups, but these require further development to effectively challenge stereotypes and predominant paradigms.* *B: Lesson appears to be framed from*	*standard specifying that "each tribe has a distinct and unique cultural heritage," which counters popular assumptions that most native people are very much alike.* NP: In a 1st grade lesson on addition and subtraction with beads, PST plans to have students watch a video of a powwow on the Northern Cheyenne Reservation and to share the Northern Cheyenne words for the various colors featured in Cheyenne beadwork along with other tribe-specific information; PST accurately indicates that various tribes are present at the powwow and that there are observable differences in their regalia but does not do the necessary research to be able to point out distinguishing features among the various tribes' regalia. B: *PST would show a video of a "Native American" powwow, without identifying any of the specific tribal*	*boundaries and would encourage students to debate the fairness of the treaties; or, PST might focus on contemporary information about tribal members living on and off reservations (for example, showing clips of urban powwows and drum groups) to counter the dominant perspective that Native Americans are historical figures without relevance in their contemporary communities.* NP: *PST would identify each tribe by name rather than referring to the tribes collectively as "Native Americans" and would include some basic information about the formation of the reservations, such as describing some of the historical treaties that were signed by the tribes.* B: In a 7th grade lesson on percent, PST plans to have students calculate the percent of Native Americans living on and off various reservations

and every student. Danielson 1f Makes a concerted effort to challenge negative attitudes or practices to ensure that all students, particularly those traditionally underserved, are honored in the school.		*the perspective of the dominant group without due diligence to verify whether commonly held assumptions are shared by other groups.* *U: Incorporates inaccurate, offensive, or disrespectful information into lessons and unit plans such that negative stereotypes are reinforced.*	*groups whose regalia is observable in the video, and would proceed to have students solve problems involving "Native American" beadwork.* U: *PST would show a video of an "Indian" powwow and describe it using offensive language such as "pagan" or "demonic."*	and to have students "realize that Native Americans are just like them" because they, too, move from place to place; the only tribe-specific information in the lesson is the population data, with no information about the various tribes or how the formation of their reservations differed due to different historical contexts and relationships with the federal government; PST utilizes the generic term "Native Americans" throughout the lesson, never referring to any of the Montana tribes by name. U: *PST would state that the federal government gave free land to the tribes, which is an inaccurate depiction of the treaty process and perpetuates the stereotype that Native Americans are lazy because they receive free benefits from the government.*

APPENDIX C

UNIT PLAN ASSIGNMENT INSTRUCTIONS AND LESSON PLAN TEMPLATE

For your mini unit plan, you will create a cohesive sequence of three lessons designed to introduce a new topic to your students. The mini unit should introduce one of the content standards in the Montana Common Core Standards for Mathematical Practice and Content in a way that draws on students' prior knowledge and enables them to use that prior knowledge to begin building new understandings. [Notice that I used the word *begin* to build new understandings: the mini unit should introduce a topic but not attempt to teach every aspect of the standard at once.]

The contents of your plan should include the following:

1. A cover page with the unit title, unit standard(s), unit goal, and your name;
2. An assessment rubric designed to capture advanced proficient, proficient, near proficient, and emergent understandings;
3. Three lesson plans completed using the lesson plan template (see below);
4. A written narrative addressing the following components of your plan:
    a. Unit Goal
        i. Explain the overall goal of your introductory unit. What do you hope your students will accomplish by the time you've spent three lessons introducing the topic you've chosen? What essential questions will get students started down the path of engaging with the content standard you have selected?
    b. Unit Sequence
        i. Explain the rationale behind your sequencing of the lessons. Why did you choose this particular order? How do the lessons build upon one another? What aspects of your standard have you chosen to focus on and why? What aspects are you leaving for later and why? How does your introductory unit gradually build students' conceptual and/or procedural understanding of the topic under study and hook them for continued learning?
    c. Assessment Rubric
        i. Describe the evidence of learning that you will be looking for and listening for during these three lessons. What, specifically, will you be looking for students to be able to do and say each day to demonstrate progress towards the unit goal? How will you distinguish between students who are comprehending the concept and those who are not?
    d. Mathematical Practice Standards
        i. How does the unit plan support at least TWO of the mathematical *practice* standards identified in the Montana Common Core Standards for Math?
    e. References
        i. What resources did you consult in forming your unit plan? Please include links and full citations for any electronic and/or print resources you utilized.

FAQ

How should I title my unit plan?
The unit title should be straightforward and reflect the fact that the unit is an introduction to the content standard under study.

How is the unit goal different from the lesson plan objectives?
The unit *goal* should be a statement reflecting the overarching aim of the introductory unit. Each of the *objectives* within your individual lesson plans form part of the progression towards achieving this ultimate goal.

Can I include an activity or project that will take more than one math period?
Individual lesson plans may span more than one day as necessary to accommodate projects or extended tasks. If you anticipate a lesson spanning two or three days, make sure to indicate this in your plan and to clearly describe the portions of the lesson that should be completed on each day. *Incorporating multi-day lessons does not excuse you from the requirement to include three lesson plans in your unit.*

What if I can't fit everything into three days?
You will not be able to cover everything you need to cover to fully address a standard within three days. Instead, you are introducing students to the standard and priming them for continued learning. For example, you will not be able to teach students how to divide fractions in just three days because dividing fractions involves dividing fractions by whole numbers, dividing whole numbers by fractions, and dividing fractions by fractions. You CAN, however, introduce students to *one* of these types of fraction division and lay the groundwork for conceptual understanding and continued learning, as described in my sample assessment rubric. Even then, the topic will likely need to be addressed again in the future to account for the variety of real-world and mathematical scenarios that involve dividing whole numbers by fractions. Your goal should be to give students a solid foundation in a small portion of the standard you've chosen and not to cover every single possible problem type associated with the standard.

How should I organize my unit plan for submission?
Please organize your unit plan materials in the order in which they are outlined on the previous page. Separate each of your lessons using tabs and label each tab according to its sequence in the unit (R for rubric, LP1 for lesson plan 1, LP2, LP3, N for narrative). You may use actual tab dividers or affix mini Post-It notes to the first page of each portion of the unit. Make sure to include any handouts, worksheets, visual aids, etc., that you plan to use in your lessons and include them within the tab of the lesson plan they accompany. Please do not place your unit plan in a binder. Simply use a binder clip to hold your materials together.

COE Lesson Plan

Lesson Teacher: Your name

Date: Month/t/2016

Lesson Grade Level:

Timeframe: -0:Length of lesson to -:-

Content Area: Math

Grouping Strategy: Select One

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Your response to this question should indicate that you've examined the content standards for the grade level being taught as well as previous grade levels to gain a sense of students' prior knowledge and current capabilities with respect to this lesson. Cite relevant Common Core standards in your response. Also briefly describe the solution methods you anticipate students using during the lesson and any potential misconceptions or mistakes they may make.

2. **How will you design the lesson to meet the needs of all learners in your classroom?** How will you ensure that learners of various skill levels and capabilities can engage constructively with the lesson? What assistance will you give or what questions will you ask students who become frustrated and request guidance in solving the task? (Keep in mind that you don't want to reduce the level of cognitive demand by giving away answers.) What will you do if students finish the task early? How will you extend the task so as to provide additional challenge?

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Should be concise and to the point.</td></tr>
<tr><td colspan="2">Common Core and/or State Standard: Include the domain number and description along with the wording of the specific CONTENT standard within the chosen domain cluster. Also list one or more of the mathematical PRACTICE standards listed at the beginning of the Common Core document.</td></tr>
<tr><td>Lesson Objective: A concise statement that explains what students will be able to do upon successful completion of the lesson. Should be SPECIFIC and MEASURABLE/observable.</td><td>Assessment of Learning: Explain your method(s) for assessing whether students achieve the lesson objective to the left. What specific formative assessment strategies will you use? What responses will demonstrate evidence of understanding?</td></tr>
<tr><td>Lesson Objective: One lesson objective might focus on skills students will be able to DEMONSTRATE from the lesson while the second lesson objective might emphasize concepts they should be able to EXPLAIN/justify.</td><td>Assessment of Learning: Explain your method(s) for assessing whether students achieve the lesson objective to the left. What specific formative assessment strategies will you use? What responses will demonstrate evidence of understanding?</td></tr>
<tr><td colspan="2">Based on the lesson objectives, select an appropriate teaching model other, please describe 5Es</td></tr>
<tr><td colspan="2">Indian Education For All (IEFA) ☐ No ☐ Yes. If yes, please describe For the IEFA lesson, describe how the lesson addresses one or more of the Essential Understandings.</td></tr>
</table>

Lesson Procedures/Activities	Materials	Classroom Management Needs
ENGAGE: How will you initiate the lesson? How will you focus students' attention? How you will set up today's task such that its importance and relevance is apparent to students?	List all materials you will need to have ready to implement this lesson.	Describe classroom management issues that may arise and how you will handle them as well as any preparations you will need to make in advance.
EXPLORE: How will students begin to explore the main question or problem? What questions might you ask to prompt students if they hesitate or struggle to get started? What questions might you ask to monitor students' thinking as they work?		
EXPLAIN: How will students be expected to share and discuss their discoveries as they explore? What questions will you ask to prompt meaningful explanations of student thinking?		
ELABORATE: How will students dig deeper into today's question, problem, or topic? How will they make connections between today's task and important mathematical concepts and/or other related subjects? What questions or prompts will set them off to make these connections?		
EVALUATE: How will students assess the discoveries they made or the skills they developed during this lesson? What questions can you ask to encourage student self-reflection with respect to the learning outcomes? What questions can you ask to guide students towards an understanding of the value of what they learned?		
Note that all of the 5Es are often embedded in a single, well-designed task--they do not have to be separate, distinct activities. However, it is important to describe how each of these components will be addressed within the task.		
If you plan to utilize any worksheets or templates during the lesson, please INCLUDE copies with your lesson plan. Also please INCLUDE copies of any figures or images you plan to sketch or display during the lesson. Prepare this lesson as if you were going to be handing it to a substitute teacher who needs to be able to envision exactly how the lesson should play out.		

Evidence of Lesson Effectiveness/Student Learning: You do not need to complete this portion.

Reflection and Recommendations for Next Time: You do not need to complete this portion.

Attachments, if required.

APPENDIX D

LESSON PLAN DRAFTS REFERENCED IN TABLE 7

IEFA Lesson (4th)

First Draft

COE Lesson Plan Template

Teacher: Student 12 **Lesson Date:** November 14, 2019

Grade Level: 4 **Timeframe:** 9:00 to 9:50

Content Area: Mathematics **Grouping Strategy:** Small Group, Large Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

Students have prior knowledge on how to multiply with a whole number. Students have begun to learn about multiplying whole numbers with fractions. In previous lessons, students have been using cuisenaire rods to help find solutions to written problems. This lesson will focus on the students previous understanding to solve a real-world problem.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

Students will be provided with cuisenaire rods to help solve their fraction problems if they are having difficulty visualizing the problem. Students will also be encouraged to discuss in their small groups to come up with an answer. They will be encouraged to share strategies with each other. If groups are moving through the problems quickly, I will provide more difficult fractions. If groups or the class moves through the lesson quickly, they will be provided an extension worksheet with the ingredient prices.

Lesson Plan Development

Lesson Title: Crow Fair Exploration
Common Core and/or State Standard(s): **(4.NF.4)** Solve word problems within cultural contexts, including those of Montana American Indians, involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef and there will be five people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? As a contemporary American Indian example, for family/cultural gatherings, the Canadian and Montana Cree bake bannock made from flour, salt, grease, and baking soda, in addition to 3/4 cup water per pan. When making four pans, how much water will be needed? **4.MP.3**. Construct viable arguments and critique the reasoning of others. **4.MP.4.** Model with mathematics. **4.MP.6.** Attend to precision.

<table>
<tr><td>Lesson Objective: Students will be able to accurately estimate the amount of 8 ingredients found in one Indian Taco and the amount of 8 ingredients needed for the Crow Fair.</td><td>Assessment of Learning: Students will be assessed on their worksheets that will display their estimations. Students will receive a “-” (working toward) if they accurately estimate <8 ingredients in both categories, “✓” (met) if the accurately estimate 8 ingredients in both categories, and a “+” (exceeds) if they accurately estimate >8 ingredients in both categories.</td></tr>
<tr><td>Lesson Objective: Students will be able to explain how they got their estimations through multiplication of whole numbers and fractions.</td><td>Assessment of Learning: Students will be assessed through teacher observation during the whole group discussions.</td></tr>
<tr><td colspan="2">Relevant Vocabulary:
Crow Fair- An annual event that takes place during the third week of August on the Crow Reservation in Montana. It involves the celebration of Crow culture, reunion of family groups, powwow, rodeo, horse racing, and commercial vendors.
Crow Pow Wow- A Crow dance celebration that is is the most fundamental form of celebration.
Indian Taco- Fry bread topped with various items that are commonly found in tacos.</td></tr>
<tr><td colspan="2">Teaching Model: 5 E’s</td></tr>
<tr><td colspan="2">Indian Education for All (IEFA) __ No _x_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Essential Understanding 3: The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs. Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories predate the “discovery” of North America.</td></tr>
</table>

Lesson Procedures/Activities:

ENGAGE: To open the lesson, I will start by showing the students the following video: https://www.youtube.com/watch?v=gukQD7O3p5A. This video shows a portion of a pow-wow at the Crow Fair in 2012. I will then ask the students what they witnessed in the video. Examples of questions I will ask will include: What did you see? What did you hear? If you were there, what would you touch? Smell? or taste? After we have discussed our senses, I will ask if any students have been to the Crow fair, and if so to please share. I also ask the students if any of them belong in the Crow tribe or have family members in the Crow tribe. We will then start to discuss the history of the Crow Fair (http://plainshumanities.unl.edu/encyclopedia/doc/egp.sr.012) I will then inform the students that the Crow Fair can attract more then 50,000 spectators each year.

EXPLORE:After discussing the history of the Crow Fair, we will then dive into exploration with the students. I will inform the students that a common food that can be found at the Crow Fair would be an Indian Taco. I will ask the students if anybody has ever had an Indian Taco, and if so, if they liked it. I will again inform the students, again, that the Crow Fair can attract more than 50,000 spectators. In their groups, I want them to discuss how many people they believe will have an Indian Taco(s). After a group has a guess on how many Indian Tacos they believe will be sold, I will pass the group a list of ingredients that are found in an Indian Taco. On the ingredient list will be a fraction amount of the ingredient found in 2 Indian Tacos. The students will be asked to work on the first ingredient of the list in their small groups.

EXPLAIN: After each group has had the opportunity to finish or attempt to finish the first ingredient calculation I will move the groups into discussion. I will have each groups speaker explain their estimations to the class. They will also explain the amount of ingredient they will need. Students will be encouraged to explain how they got their answers through their mathematical thinking.

ELABORATE: After the large group discussions, I will move the students back to small group discussions. Each small group will then begin working on the rest of the ingredients. The groups must then calculate the total amount of all the ingredients the Crow Fair will need. I will ask questions throughout their mathematical thinking such as, "Do you notice anything interesting about the amounts?" or "Are the amounts surprising you?"

EVALUATE: After students have finished their calculations, or have come close to finishing, I will pull the group together again for a large group discussion. I will ask them to share their total ingredient estimations with the class. Are there large differences between the amount of ingredients? Small difference? Why do they believe those differences are there? Did they solve their answers similar to each other? To gauge what individual students learned during the lesson, I will provide each with an exit ticket. On the exit ticket, it will ask the students 3 things they learned today, 2 things they found interesting, and 1 question they have.

Lesson Materials:

- Manipulatives
- Indian Taco Ingredients Worksheets
- Exit Ticket
- (Extension) Indian Taco Prices Worksheets

Classroom Management Needs: The students will be working in small groups and discussing in a large groups. For each small group there will be a speaker, note taker, time manager, and devils-advocate. When students are working in small groups, I will need to circulate around the room to make sure each group is on topic. I will also circulate to help with any confusion.

Extension: Make this a two day lesson by introducing the prices of the ingredients to the students. An example of the extension worksheet is attached to the lesson.

Sources:

Billings Gazette. (2012, August 23). Crow Fair Powwow 2012. Retrieved November 13, 2019, from https://www.youtube.com/watch?v=gukQD7O3p5A.

Boaler, J. (2016). Mathematical mindsets: unleashing students potential through creative math, inspiring messages, and innovative teaching. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.

Heidenreich, C. A. (n.d.). Crow Fair. Retrieved from http://plainshumanities.unl.edu/encyclopedia/doc/egp.sr.012.

Reed, L. D. (2018, January 1). Indian Fry Bread Tacos. Retrieved November 13, 2019, from https://www.tasteofhome.com/recipes/indian-fry-bread-tacos/.

Name:

Indian Taco Ingredients

Estimated amount of Indian Tacos needed for the Crow Fair:

____

Indian Taco Ingredients per 2 Indian Tacos:	Indian Taco Ingredients per 1 taco:	Indian Taco Ingredients per the whole Crow Fair:
3/4 cup all-purpose flour		
1/2 teaspoon baking powder		
1/4 teaspoon salt		
2/3 cup water		

1/2 pound lean ground beef		
2 tablespoons taco seasoning		
2 tablespoons chopped lettuce		
2 tablespoons chopped tomato		
2 tablespoons salsa		
2 tablespoons sour cream		

Name:

Exit Ticket

3 things I learned today…	2 things I found interesting…	1 question I have…

Name:

Indian Taco Ingredients Prices

Indian Taco Ingredients per the Whole Crow Fair	Estimations of the cost of Ingredients	Estimations of the cost of Ingredients for the Crow Fair
All-Purpose Flour:	1 cup - $0.10	
Baking Powder:	1 cup - $1.80	
Salt:	1 tsp. - $0.004	
Water:	1 cup - $0.50	

Lean Ground Beef:	1 lb - $3.80	
Taco Seasoning:	1 tbsp. - $0.90	
Chopped Lettuce:	1 tbsp. - $0.10	
Chopped Tomato:	1 tbsp. - $0.15	
Salsa:	1 tbsp. - $0.25	
Sour Cream:	1 tbsp. - $0.75	

IEFA Lesson (4th)

Final Draft

COE Lesson Plan Template

Teacher: Student 12

Lesson Date: November 14, 2019

Grade Level: 4

Timeframe: 9:00 to 9:50

Content Area: Mathematics

Grouping Strategy: Small Group, Large Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

In previous lessons, students have begun using modeling to solve division problems. They have been using base 10 blocks and drawings to find their solutions. In the third grade, students learned division within the number 100 (3.OA.7). This will be the first planned lesson that includes four-digit dividends with the students. I anticipate that students may struggle with the four-digit dividends in the lesson's problems. I will need to monitor student conversations to understand if they will need more modeling from me or a peer on dividing four-digit dividends.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

Students will be provided with base 10 blocks to help solve their division problems if they are having difficulty visualizing the problem. Students will also be encouraged to discuss with their elbow partners to come up with an answer. They will be encouraged to share strategies with each other. If a student or group of students solve the problems on the worksheet quickly, they will be given a multi-step problem to work on. An example of a multi-step problem given to them would be, "The Crow Fair attracts 50,000 spectators. If 41,600 spectators leave by the third day, how many spectators will be left on the fourth day? With the remaining spectators, how many can equally fit in 8 booths?"

Lesson Plan Development

Lesson Title: Crow Fair Exploration
Common Core and/or State Standard(s): **(4.NBT.6)** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.MP.3.** Construct viable arguments and critique the reasoning of others. **4.MP.4.** Model with mathematics. **4.MP.6.** Attend to precision.

Lesson Objective: Students will be able to model a division problem through a drawing. specify quantities working w/	**Assessment of Learning:** Teacher observation during student explanation and elaboration. Students should be partitioning the dividends into equal groups for the correct amount of divisors.
Lesson Objective: Students will be able to justify their solutions by totaling up their groupings to equal the dividend. this should be more emphasized in lessons	**Assessment of Learning:** During student explanation and elaboration, I will look for students to justify their solution by adding their groupings to equal the given dividend.

Relevant Vocabulary:
Crow Fair- An annual event that takes place during the third week of August on the Crow Reservation in Montana. It involves the celebration of Crow culture, the reunion of family groups, powwow, rodeo, horse racing, and commercial vendors.
Crow Pow Wow- A Crow dance celebration that is is the most fundamental form of celebration.
Indian Taco- Fry bread topped with various items that are commonly found in tacos.

Teaching Model: 5 E's

Indian Education for All (IEFA) __ No _x_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Essential Understanding 3: The ideologies of Native traditional beliefs and spirituality persist into modern-day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs. Additionally, each tribe has its own oral histories, which are as valid as written histories. These histories predate the "discovery" of North America.

Lesson Procedures/Activities:

ENGAGE: To open the lesson, I will start by showing the students the following video: https://www.youtube.com/watch?v=gukQD7O3p5A. This video shows a portion of a pow-wow at the Crow Fair in 2012. I will then ask the students what they witnessed in the video. Examples of questions I will ask will include: What did you see? What did you hear? If you were there, what would you touch? Smell? or taste? After we have discussed our senses, I will ask if any students have been to the Crow fair and if so to please share. I also ask the students if any of them belong in the Crow tribe or have family members in the Crow tribe. We will then start to discuss the history of the Crow Fair: "The Crow Fair started in 1904 as a way to encourage and support Crow tribe farmers. It also provided the opportunity for tribal members to showcase their culture. This includes native foods, clothing, and handicrafts. Over the years, the Crow Fair has turned into one of the largest Native American events in North America. It can track more than 50,000 spectators each year. The Crow Fair lasts for four days and consists of the celebration of Crow culture, the reunion of family groups, powwow, rodeo, horse racing, and commercial vendors". (http://plainshumanities.unl.edu/encyclopedia/doc/egp.sr.012). I will then inform the students that we are going to help plan for the Crow Fair using our division strategy of modeling. To invoke prior knowledge of modeling, I will have a student demonstrate the strategy with the problem 40 divided by 4. (Example: they could draw 4 circles and write 10 in each circle)

EXPLORE: After discussing the history of the Crow Fair and recapping modeling, we will then dive into exploration. I will pass out a Crow Fair Division worksheet to each student. With their elbow partner, they will begin working on the first problem (*On a Saturday, 96 fourth-grade students will be attending the Crow Fair. There are only 4 buses to seat all of the students. If students are organized equally, how many students will there be to each bus?*). Base 10 blocks will be available for students to use if they would like a physical manipulative. If students struggle to get started, I will break down the problem for them. For example, I will ask them what they know about the problem? What number is being broken down? How many groups is the number being broken down to? How do you know? How can you represent the groups?

EXPLAIN: After each group has had the opportunity to finish or attempt to finish the first problem, I will move the groups into a whole-class discussion. The goal of the discussion is to have students come to a consensus on how to accurately represent the division problem using modeling. I will begin by asking one student to share how they used modeling to solve their division problem. Whether they used a drawing or base 10 blocks. I will then ask students to put their thumb on their chest if their answer looks similar to the students. If not, to raise their hand. Students who raise their hand will be asked to share their solutions. Some differentiation I am expecting is students drawing buses that have the numbers 16, 10, and 4 or 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, and 2. Throughout the student's explanations, I will be asking questions such as, "Does each bus have the same amount?" and "Do the buses all together equal 96?"

ELABORATE: After the large group discussions, I will move the students back to their worksheet. Each pair will then begin working on the rest of their worksheets. As students proceed through the worksheet, I will ask questions along the way to guide their thinking. Questions will include, "Do you notice anything about creating the models? Are you able to start your models with larger numbers? Why do you think that is the case?"

EVALUATE: After students have had time to work on their worksheets, I will pull the students together again for another large group discussion. I will ask them to share the observations they have made in their solutions. Were there any problems that they found difficult to solve? Why do they think they were difficult? Did they find any problems easy to solve? Why do they believe they were easy to solve? To gauge what individual students learned during the lesson, I will provide each with a reflection sheet. On the worksheet, they will be asked to reflect on the following questions: "What was the big idea of the lesson?", "What good ideas did you have today?", "What questions do you have about today's work?" and "How is math helpful in real-world situations?"

Lesson Materials:

- Base 10 blocks
- Crow Fair Division Word Problems
- Exit Ticket
- Writing utensils
- Overhead projector for students to show their solutions

Classroom Management Needs:
Students will be working in small groups throughout the lesson. They will also be working as a whole group for discussions. Because of this, I will

need to circulate around the classroom to ensure that groups are on task. I will need to make sure they are not digressing in unrelated conversations or using their tools inappropriately.

Sources:

Billings Gazette. (2012, August 23). Crow Fair Powwow 2012. Retrieved November 13, 2019, from https://www.youtube.com/watch?v=gukQD7O3p5A.

Boaler, J. (2016). Mathematical mindsets: unleashing students potential through creative math, inspiring messages, and innovative teaching. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.

Heidenreich, C. A. (n.d.). Crow Fair. Retrieved from http://plainshumanities.unl.edu/encyclopedia/doc/egp.sr.012.

Name:____________________________

Crow Fair Division Word Problems

For the following problems, use modeling to find the solutions. Draw pictures to display how you got your answer.

1. On a Saturday, 96 fourth-grade students will be attending the Crow Fair. There are only 4 buses to seat all of the students. If students are organized equally, how many students will there be to each bus?

2. During the parade, the paraders will throw candy to the children. Suppose there are 288 bags of candy and 9 paraders who will be throwing the candy. How many bags of candy will each parader receive?

3. At the Crow Fair, there can be about 1,000 tipis that families can stay in. If the tipis are organized equally into 8 groups, how many tipis will be in each group?

4. On the first day of the Crow Fair, 5,565 cars will be driving in. There will be 5 parking lots for the guests to park in. How many cars can fit into each parking spot equally?

Name:________________________

Lesson Reflection

1. What was the big idea of the lesson?

2. What good ideas did you have today?

3. What questions do you have about today's work?

4. How is math helpful in real-world situations?

IEFA Lesson (5th)

First Draft

COE Lesson Plan Template

Teacher: Student 5	**Lesson Date:** 11/14/19
Grade Level: 5	**Timeframe:** 60 mins
Content Area: Math	**Grouping Strategy:** Whole Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

The students are currently learning about the Plains Indians in social studies. The focus this week revolved around the significance of bison to the Native Americans and how the European settlers almost killed bison into extinction in the late 1800's. The students have already been working on adding and subtracting decimals and have been introduced to multiplying decimals. Most of the students are still unfamiliar with multiplying decimals. About 80% of the students are nearing proficiency in adding and subtracting decimals; but the class as a whole could benefit from additional practice.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

For struggling learners I will have peers that are understanding the concept explain and demonstrate their thought process using the smart board to help others understand and give them other perspectives, engage the students by having them talk through their own thought process if they are struggling and guide them to correct errors as necessary, and pull small groups for further instruction during independent work time if they are struggling to complete tasks independently.

For advanced learners, challenge the learner by having them discover if they are adding 3 of the same number, they can multiply the number by 3 instead of adding it 3 times. I would do this by reviewing the repeated addition and how it links to multiplication. I could ask the students "So, your adding 3 bison robes at 3.50 each (3.50+3.50+3.50) what's another method we can use to solve this instead of addition?" I would give them more challenging questions that required them to add and subtract larger numbers and add and subtract 3-4 numbers at a time.

Lesson Plan Development

Lesson Title: Bison Values
Common Core and/or State Standard(s): Numbers and Operations in Base Ten (NBT) Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(5.NBT.7)** **Practice: 5.MP.2 Reason abstractly and quantitatively:** Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical

representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. **Essential Understanding #3:** The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs. **Essential Understanding #5:** There were many federal policies put into place throughout American history that have affected Indian people in the past and continue to shape who they are today. Many of these policies conflicted with one another. Much of Indian history can be related through several major federal policy periods: Colonization/Colonial Period, 1492 - 1800s Treaty-Making and Removal Period, 1778 – 1871 Reservation Period - Allotment and Assimilation, 1887 - 1934 Tribal Reorganization Period, 1934 - 1953 Termination and Relocation Period, 1953 - 1968 Self-Determination Period, 1975 - Present	
Lesson Objective: The students will gain further understanding of the significance of the bison to the Plains Indian Tribes and how the slaughter of the bison by the European settlers affected the tribes. Students will be able to add and subtract decimal amounts represented by money value for various parts of the bison.	**Assessment of Learning:** The students will complete an exit journal at the end of the lesson stating 3 ways the bison were important to the Native American Plains Tribes and 3 ways the slaughter of the bison affected the tribes.
Lesson Objective: By the end of the lesson students will be able to add and subtract decimals independently with 80% accuracy.	**Assessment of Learning:** The students will complete an activity worksheet in which they have to add and subtract decimals. The teacher will collect and grade the worksheet to determine if the students met the 80% accuracy goal.
Relevant Vocabulary: Bison-humpbacked shaggy-haired wild ox native to North America and Europe Decimals-a system of numbers and arithmetic based on the number ten, tenth parts, and powers of ten Addition-the process or skill of calculating the total of tow or more numbers or amounts Subtraction-the process or skill of taking one number or amount away from another Multiplication-the process of combining matrices, vectors, or other quantities under specific rules ot obtain their product	
Teaching Model: Scaffold Instruction with the 'I Do', 'We Do', 'You Do' concept in which the teacher will model how to perform the task, the teacher and students will do the task together, then the students with do the task independently.	
Indian Education for All (IEFA) __ No _X_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:	

I would link this math lesson into a social studies unit on Plains Indians and their way of life. I would link this to Essential Understandings 3 and 5 because in social studies they would be learning about the significance of the bison to Native American people and will watch the Smithsonian documentary: Native Americans Saw Buffalo as More Than Just Food. I would review how the Native Americans viewed bison as sacred creatures and how they used bison as their primary food source, clothing, housing, bedding and blankets (robes), storage and water bags, tools, utensils, weapons, and in ceremonial practices. I would lead a class discussion on how the European settlers nearly annihilated the entire bison species from 60-70 million bison roaming the plains during the 1700's to the early 1800's to less than a 1,000 bison left by 1889; and how European settlers killed bison as a tactic to starve the Native Americans, destroy their way of life, and attempt to get rid of Native Americans altogether; and how Europeans used bison make money by selling the robes, hide, meat, tongues, horns, hooves, and bones. Next, I will introduce the bison price guide that shows the value of bison parts in the 1872 and the equivalent value in today's standards and start the math activity.
Lesson Procedures/Activities: ENGAGE: The teacher will handout the bison uses guide and lead the students in a discussion "For the past couple weeks we have been learning about the Plains Indian Tribes in social studies; and this week we focused on the significance of the bison. What do you remember about the significance of the bison to Native American culture? How did the various tribes view the bison? What did they use the bison for? Outside of their basic needs like housing, clothing, utensils, and tools, what else was the bison used for? How about ceremonies, were the bison used in their ceremonial practices, if so how?" The teacher and the students would have a 5 min discussion to review key points learned about the significance of the bison in Native American culture. Next, the teacher will lead the students in a discussion (10 mins-this information will be previously introduced in social studies as well) about how the European settlers nearly exterminated all the bison and how this affected the Native American tribes: "Let's look at this timeline, in the 1700's and early 1800's there was an estimated 60-70 million bison roaming the Great Plains, but how many were left by 1889?" Why do you think the Europeans settlers killed off so many bison? What do you think they did with the bison when they killed them? Do you think they made money off of the bison?" Once the students come to the conclusion (with guidance if needed) that the Europeans killed the bison to destroyed Native American life and to sell the bison parts the teacher will introduce the 'Bison Value' sheet that lists how much the bison parts sold for in the 1800's and how much that would be in today's standards. EXPLORE: The teacher will ask "How much do you think a bison sold for in the 1800's? What parts of the bison do you think European settlers sold? How much do you think they got for each part?" The teacher will record the student responses on the board. Next, the teacher will ask "How much do you think (name listed item that the students came up with from the first question) would sell for today? Why do you think it would be (more or less depending on what the students respond)?"The teacher will record the students' responses again and compare them to the first set of responses. Then, the teacher will pass out precut bison parts (hide, robe, tongues, horns, hooves, bones, and meat, multiple of each piece, all labeled with their value-1872 value on one side and 2019 equivalent value on the other side) and have the students look over the pieces and their values. The students will be able to use the cutouts for visual representation to help them solve the problems during guided practice and independent practice. Next, the teacher will ask the students "How much money do you think the settlers receive for a whole bison? How do you know?" Next, the teacher will handout the activity worksheet and review the directions with the students.

EXPLAIN: The teacher will read through the first question with the class and ask the students "How do you think we calculate the value of 2 bison tongues in 1872?" The teacher would call a student up to work through the problem on the smart board. Once the student completes the problem the teacher will tell the students to give a thumbs up if they agree, a thumbs down if they disagree, or a sideways thumb if they're not sure. The teacher will call on students and have them explain why they agree or disagree with the answer and have a couple students come to the board to show their reasoning. The teacher will repeat this activity a few times over the course of 10 mins. The teacher will guide the solving process without giving the direct solution if students are struggling to solve the problem. The teacher may do this by asking leading questions, modeling an example, or showing how to line the problem up to prepare to solve, and reminding students to double check their answers. After, guided practice the teacher will have the students complete the remainder of the worksheet (last 5 questions) independently. The teacher will walk around the room and ask the student to explain their reasoning on how they came up with the answer they came up with. The teacher will help guide a student to recognize mistakes and make the necessary corrections by asking leading questions or say things like "What if we line it up this way? Or what is 5+5…okay now what do we do next?"

ELABORATE: Once the students have completed their worksheet the teacher will have a couple challenge questions written on the board. 'If the average bison produced 200 lbs. of meat how much would the whole bison sell for?', 'If a settler sold 4 bison robes how much money would he have in today's standards?' 'How much money would you need to buy 2 bison tongues, 1 bison robe, 3 lb. of meat, and 2 hides in 1872? How much money would you need today?' 'If you had 30 lbs. of bison meat in 1872 and you sold 20 lbs. how much would the value of the remaining meat be?'The students would solve these questions at their own pace on blank paper and move on to the next question once the completed the previous one. The teacher would walk around as before to ask students to explain how they came up with their answers and help them work through any procedural errors without giving the direct answer. If a student is getting through the questions quickly and accurately the teacher will give additional challenge questions to the student that involves having to multiple. For example, 'Instead of adding, what other method could you use to determine 'How much money would you need to buy 15 pounds of bison meat in 1872?' The student should identify multiplication and use multiplication to solve the problem.

EVALUATE: The teacher will collect the worksheets and the challenge papers. The teacher will call on students to come up and show the class how they solved the challenge questions. The teacher will once again have students give a thumbs up, thumbs down, or sideways thumb to show whether they agree, disagree, or are not sure. The teacher will make notes as to which concepts the students are understanding, and which concepts need further instruction and/or review. The teacher will also review the worksheets to determine individual student progress and assess strengths and emerging skills.

Lesson Materials:
Bison Uses Sheet
Bison Value Sheet
Bison Parts Cutouts
Bison Work Sheet
Smart Board
Pencils

Classroom Management Needs:
The teacher will need to keep student engagement throughout the lesson by having hand-on and student lead activities and participation. The teacher will need to assess student strengths and emerging skills and provide support as necessary by providing the student with the tools and skills they need to be successful; this may vary from student to student. The teacher will walk around the room during independent practice to continue to engage students and assess their though process and procedural knowledge.

IEFA Lesson (5th)

Final Draft

COE Lesson Plan Template

Teacher: Student 5 | **Lesson Date:** 11/14/19

Grade Level: 5 | **Timeframe:** 60 mins

Content Area: Math | **Grouping Strategy:** Whole Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

The students are currently learning about the Plains Indians in social studies. The focus this week revolved around the significance of bison to the Native Americans and how the European settlers almost killed bison into extinction in the late 1800's. The students have already been working on adding and subtracting decimals and have been introduced to multiplying decimals. Most of the students are still unfamiliar with multiplying decimals. About 80% of the students are nearing proficiency in adding and subtracting decimals; but the class as a whole could benefit from additional practice.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

For struggling learners I will have peers that are understanding the concept explain and demonstrate their thought process using the smart board to help others understand and give them other perspectives, engage the students by having them talk through their own thought process if they are struggling and guide them to correct errors as necessary, and pull small groups for further instruction during independent work time if they are struggling to complete tasks independently.

For advanced learners, challenge the learner by having them discover if they are adding 3 of the same number, they can multiply the number by 3 instead of adding it 3 times. I would do this by reviewing the repeated addition and how it links to multiplication. I could ask the students "So, your adding 3 bison robes at 3.50 each (3.50+3.50+3.S0) what's another method we can use to solve this instead of addition?" I would give them more challenging questions that required them to add and subtract larger numbers and add and subtract 3-4 numbers at a time.

Lesson Plan Development

Lesson Title: Bison Values
Common Core and/or State Standard(s): Numbers and Operations in Base Ten (NBT) Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings within cultural contexts, including those of Montana American Indians, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(5.NBT.7)** **Practice: 5.MP.2 Reason abstractly and quantitatively:** Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical

<table>
<tr><td colspan="2">representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

Essential Understanding #3: The ideologies of Native traditional beliefs and spirituality persist into modern day life as tribal cultures, traditions, and languages are still practiced by many American Indian people and are incorporated into how tribes govern and manage their affairs.
Essential Understanding #5: There were many federal policies put into place throughout American history that have affected Indian people in the past and continue to shape who they are today. Many of these policies conflicted with one another. Much of Indian history can be related through several major federal policy periods:
Colonization/Colonial Period, 1492 - 1800s
Treaty-Making and Removal Period, 1778-1871
Reservation Period - Allotment and Assimilation, 1887 - 1934
Tribal Reorganization Period, 1934 - 1953
Termination and Relocation Period, 1953 - 1968
Self-Determination Period, 1975 - Present</td></tr>
<tr><td>Lesson Objective: The students will gain further understanding of the significance of the bison to the Plains Indian Tribes and how the slaughter of the bison by the European settlers affected the tribes.
Students will be able to add and subtract decimal amounts represented by money value for various parts of the bison. Modification: Students will also be able to justify their answers by explaining how they came up with their answer and explaining their thought process/the steps they used to solve. Students will also explain the important of lining up the decimals and the importance of place value when adding and subtracting decimals.
S At' ()_t I f\
—, — ' '</td><td>Assessment of Learning:
The students will complete an exit journal at the end of the lesson stating 3 ways the bison were important to the Crow and Sioux tribes and 3 ways the slaughter of the bison affected the tribes.

Modification: The teacher will check in with students while they are working and have them verbally explain how they solved the problems and talk through their thought process. The teacher will also have the students explain how to line up addition and/or subtraction problems and why it's important to line it up in that manner. (The goal would be for students to make the connection to place value and how not lining up the placevalue correctly will lead to an incorrect answer.)</td></tr>
<tr><td>Lesson Objective:
By the end of the lesson students will be able to add and subtract decimals independently with 80% accuracy.</td><td>Assessment of Learning: The students will complete an activity worksheet in which they have to add and subtract decimals. The teacher will collect and grade the worksheet to determine if the students met the 80% accuracy goal.</td></tr>
<tr><td colspan="2">Relevant Vocabulary:
Bison-humpbacked shaggy-haired wild ox native to North America and Europe
Decimals-a system of numbers and arithmetic based on the number ten, tenth parts, and powers of ten
Addition-the process or skill of calculating the total of tow or more numbers or amounts
Subtraction-the process or skill of taking one number or amount away from another</td></tr>
</table>

Multiplication-the process of combining matrices, vectors, or other quantities under specific rules to obtain their product

Teaching Model: Scaffold Instruction with the 'I Do', 'We Do', 'You Do' concept in which the teacher will model how to perform the task, the teacher and students will do the task together, then the students with do the task independently.

Indian Education for All (IEFA) _ No _X_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

I would link this math lesson into a social studies unit on Plains Indians and their way of life. I would link this to Essential Understandings 3 and 5 because in social studies they would be learning about the significance of the bison to Native American people and will watch the Smithsonian documentary: Native Americans Saw Buffalo as More Than Just Food. I would review how the Native Americans viewed bison as sacred creatures and how they used bison as their primary food source, clothing, housing, bedding and blankets (robes), storage and water bags, tools, utensils, weapons, and in ceremonial practices. I would lead a class discussion on how the European settlers nearly annihilated the entire bison species from 60-70 million bison roaming the plains during the 1700's to the early 1800's to less than a 1,000 bison left by 1889; and how European settlers killed bison as a tactic to starve the Native Americans, destroy their way of life, and attempt to get rid of Native Americans altogether; and how Europeans used bison make money by selling the robes, hide, meat, tongues, horns, hooves, and bones. Next, I will introduce the bison price guide that shows the value of bison parts in the 1872 and the equivalent value in today's standards and start the math activity.

Lesson Procedures/Activities:

ENGAGE: Modification: The teacher will handout the bison uses guide and lead the students in a discussion "For the past couple weeks we have been learning about the Plains Indian Tribes primarily the Sioux and Crow tribes, in social studies; and this week we focused on the significance of the bison. What do you remember about the significance of the bison to Sioux and Crow culture? How did these tribes view the bison? What did they use the bison for? Outside of their basic needs like housing clothing, utensils, and tools, what else was the bison used for? How about ceremonies, were the bison used in their ceremonial practices, if so how?" The teacher and the students would have a 5 min discussion to review key points learned about the significance of the bison in Sioux and Crow culture. Next, the teacher will lead the students in a discussion (10 mins-this information will be previously introduced in social studies as well) about how the European settlers nearly exterminated all the bison and how this affected the Native American tribes: "Let's look at this timeline, in the 1700's and early 1800's there was an estimated 60-70 million bison roaming the Great Plains, but how many were left by 1889?" Why do you think the Europeans settlers killed off so many bison? What do you think they did with the bison when they killed them? Do you think they made money off the bison?" Once the students come to the conclusion (with guidance if needed) that the Europeans killed the bison to destroyed Native American life and to sell the bison parts the teacher will introduce the 'Bison Value' sheet that lists how much the bison parts sold for in the 1800's and how much that would be in today's standards.

EXPLORE: The teacher will ask "How much do you think a bison sold for in the 1800's? What parts of the bison do you think European settlers sold? How much do you think they got for each part?" The teacher will record the student responses on the board. Next, the teacher will ask "How much do you think (name listed item that the students came up with from the first question) would sell for today? Why do you think it would be ..(more or less depending on what the students respond)?"The teacher will record the students' responses again and compare them to the first set of responses. Then, the teacher will pass out precut bison parts (hide, robe, tongues, horns, hooves, bones, and meat, multiple of each piece, all labeled with their value-1872 value on one side and 2019 equivalent value on the other side) and have the students look over the pieces and their values. The students will be able to use the cutouts for visual representation to help them solve the problems during guided practice and independent practice. Next, the teacher will ask the students "How much money do you think the settlers receive for a whole bison? How do you know?" Next, the teacher will handout the activity worksheet and review the directions with the students.

EXPLAIN: The teacher will read through the first question with the class and ask the students "How do you think we calculate the value of 2 bison tongues in 1872?" The teacher would call a student up to work through the problem on the smart board. Modification: Once the student completes the problem the teacher will give the students a couple mins to look over the problem and think things over. Next, the teacher will tell the students to give a thumbs up if they agree, a thumbs down if they disagree, or a sideways thumb if they're not sure. The teacher will call on students and have them explain why they agree or disagree with the answer and have a couple students come to the board to show their reasoning explaining to the group how they came up with the answer. Modification: The teacher will also ask, "Why did you line up the problem this way? Why is it important to line up the decimals? If we don't line up the decimals what do you think would happen? Would we get the same answer?" The teacher will repeat this activity a few times over the course of 10 mins. The teacher will guide the solving process without giving the direct solution if students are struggling to solve the problem. The teacher may do this by asking leading questions, modeling an example, or showing how to line the problem up to prepare to solve, and reminding students to double check their answers. After, guided practice the teacher will have the students complete the remainder of the worksheet (last 5 questions) independently. The teacher will walk around the room and ask the student to explain their reasoning on how they came up with the answer they came up with. The teacher will help guide a student to recognize mistakes and make the necessary corrections by asking leading questions or say things like "What if we line it up this way? Or what is 5+5...okay now what do we do next?"

ELABORATE: Once the students have completed their worksheet the teacher will have a couple challenge questions written on the board. 'If the average bison produced 200 lbs. of meat how much would the whole bison sell for?', 'If a settler sold 4 bison robes how much money would he have in today's standards?' 'How much money would you need to buy 2 bison tongues, 1 bison robe, 3 lb. of meat, and 2 hides in 1872? How much money would you need today?' 'If you had 30 lbs. of bison meat in 1872 and you sold 20 lbs. how much would the value of the remaining meat be?" The teacher would make sure the students are lining up the problems correctly. If they are not lining up the decimals the teacher will say, "What do you remember about place value? How would you line up the decimals?" The students would solve these questions at their own pace on blank paper and move on to the next question once the completed the previous one. The teacher would walk around as before to ask students to explain how they came up with their answers and their thought process. The teacher will help them work through any procedural errors without giving the direct answer. If a student is getting through the questions quickly and accurately the teacher will give additional challenge questions to the student that involves having to multiply. For example, 'Instead of adding, what other method could you use to determine 'How much money would you need to buy 15 pounds of bison meat in 1872?' The student should identify multiplication and use multiplication to solve the problem.

EVALUATE: The teacher will collect the worksheets and the challenge papers. The teacher will call on students to come up and show the class how they solved the challenge questions. The teacher will once again have students give a thumbs up, thumbs down, or sideways thumb to show whether they agree, disagree, or are not sure. The teacher will make notes as to which concepts the students are understanding, and which concepts need further instruction and/or review. The teacher will also review the worksheets to determine individual student progress and assess strengths and emerging skills.

Lesson Materials:
Bison Uses Sheet
Bison Value Sheet
Bison Parts Cutouts
Bison Work Sheet
Smart Board
Pencils

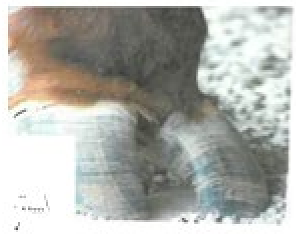
play money!

Classroom Management Needs:
The teacher will need to keep student engagement throughout the lesson by having hand-on and student lead activities and participation. The teacher will need to assess student strengths and emerging skills and provide support as necessary by providing the student with the tools and skills they need to be successful; this may vary from student to student. The teacher will walk around the room during independent practice to continue to engage students and assess their though process and procedural knowledge.

1872 Bison Parts Value	2019 Bison Parts Equivalent Value
Robe-$3.50	Robe-$73.83
Hide-$1.25	Hide-$26.37
Tongue-$2.50	Tongue-$52.73
Meat (per pound)-$0.25	Meat (per pound)-$5.27
Bones-$2.00	Bones-$42.19
Horns-$0.25	Horns-$5.27
Hooves-$0.25	Hooves-$5.27

Cut Outs for Visuals - IEFA Lesson

Robe



Hooves

Tongue

lorns

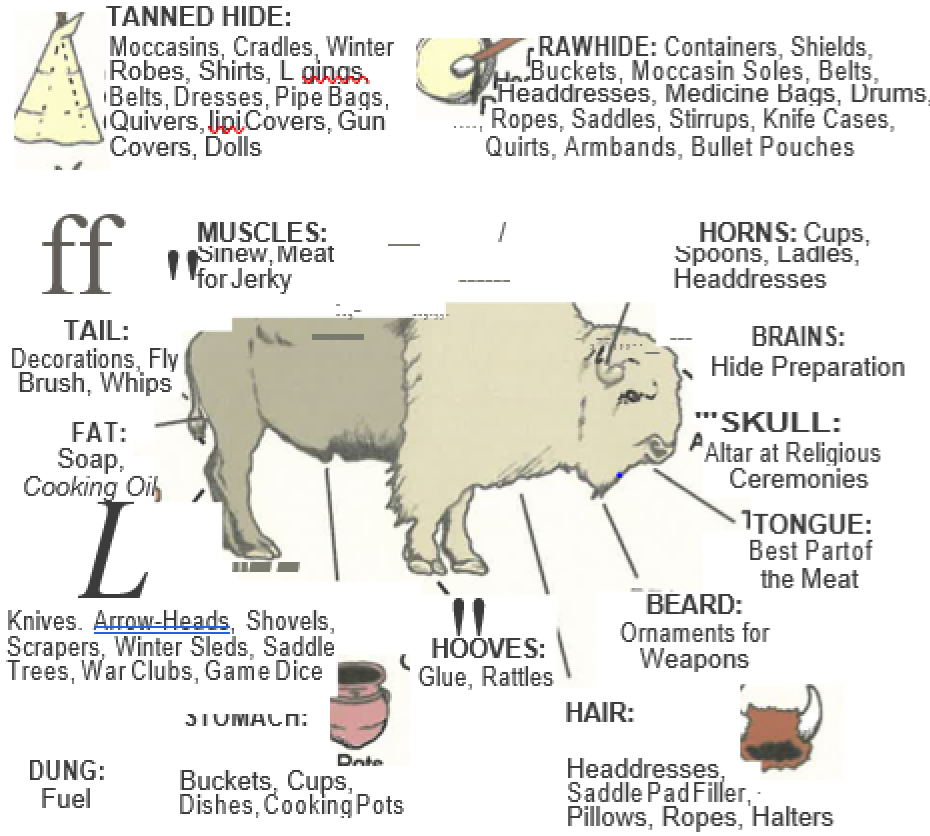
Whole Bison

Meat

Cut Outs for Visuals

Bones

Hide



for discussion - Bison Uses

Chart for discussion – Bison Use

IEFA Lesson (6th)

First Draft

COE Lesson Plan Template

Teacher: Student 9 **Lesson Date:**

Grade Level: 6th **Timeframe:** 1 hour

Content Area: Mathematics/Health/Social Studies

Grouping Strategy: Independent, whole group, and small group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

This lesson will follow the previous day's math lesson about rounding to the nearest 10th and 100th place. This lesson will build on the students rounding skills by requiring them to round to the nearest 100th. They have mastered rounding, but need more practice on differentiating between the 10th and 100th place. They have practiced converting whole numbers in many conversion units, but have not practiced converting decimal numbers and fractions of a whole. Their prior knowledge of ratios and whole numbers will be applied to exploring the conversion of decimals and more challenging measurements.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

This lesson starts out with the teacher reading and using guided practice to actively read the article. Students practice active reading independently after. Then they work in small groups to share ideas about the text. They get to work in the small groups to explore converting measurements. I will start asking them to convert 10 centimeters to inches so that they will master a less challenging conversion before moving on to the larger conversions. I will remind them of their knowledge about ratios as they explore converting. Students who struggle will get to that extra support from their peers by working in small groups. Students that find converting easy will get to explain it to their peers which develops their understanding even more. The students will get to work independently on the last conversion to make sure that they fully understand the concept. I will walk around to check for understanding so that all learners are successful.

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Converting Heights of Plains Indians</td></tr>
<tr><td colspan="2">Common Core and/or State Standard(s):

Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3-Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Practice Standards:
6.MP.1. Make sense of problems and persevere in solving them.
6.MP.2. Reason abstractly and quantitatively.
6.MP.4. Model with mathematics.
6.MP.6. Attend to precision.
6.MP.8. Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Lesson Objective:

Students will be able to convert measurements of centimeters to inches and inches to feet.</td><td>Assessment of Learning:

Students will create a conversion table that shows the 4 heights in centimeters (2nd column), in inches (3rd column), and in feet (4th column). Table template is attached.</td></tr>
<tr><td>Lesson Objective:

Students will explore the characteristics of a healthy diet and how the healthy diet of the Plains Indians contributed to their growth.</td><td>Assessment of Learning:

Students will identify availability of food and healthy eating habits of the Plains Indians by answering a question below the conversion table they created. The question is:
In 2-3 sentences, provide evidence from the reading and from the class discussion that suggests why the Plains Indians were the tallest people in the world in the 1800's.</td></tr>
<tr><td colspan="2">Relevant Vocabulary:

Centimeters
Inches
Feet
Convert
Conversion table
Plains Indians</td></tr>
</table>

Tribe
Teaching Model: 5 E's
Indian Education for All (IEFA) __ No _x_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians: 1. There is great diversity among the twelve sovereign tribes of Montana in their languages, cultures, histories, and governments. Each tribe has a distinct and unique cultural heritage that contributes to modern Montana
Lesson Procedures/Activities: ENGAGE: What is meant by "Plains Indians"? **Native American tribes that lived on the flat plains.** Does anyone know what tribes of Native Americans that live on the plains in Montana? **Cheyenne, Sioux, Blackfeet, Crow Comanche** What kinds of foods do you think were available on the plains for plains Indian tribes in Montana? **Buffalo, bugs, fish, bird, corn, ect.** When you think of a healthy meal, what kind of food comes to mind? What kinds of healthy foods do you like to eat? EXPLORE: As a class, we will active read the first two paragraphs of "Plains Indians Were Tallest, Healthiest People In World". **Highlight together:** American Plains Late 1800s Tallest people in the world Well-nourished Sickly victims European disease Students will independently read the rest of the article. I will encourage students to highlight, underline, or note anything they find interesting or important. Students will be put into groups of 4-5. They will discuss the article and share anything they found interesting or important. We will have a whole group discussion. I will ask each group to share something interesting they read. I will ask: What does the article say about why they were the tallest people in the world? Do you think how we eat today impacts our growth? What year or years were they known to be the tallest people?

If students note the heights listed in paragraph 5, I will note the heights on the board as they say them. If they do not mention them, I will ask:
About how tall were the tallest Plains Indians?
What other countries are known at this time to have the tallest people in the world and how tall were they?

EXPLAIN:
I will provide the class with the conversion of inches to centimeters on the board: **1 inch = 2.54 centimeters.** As a class, we will talk about rounding to the nearest 10^{th}. I will call on a student to round 2.54 to the nearest 10^{th}. I will ask if all students agree. Once all students understand rounding (which they have been working on previously), I will remind them by pointing to the board that 1 inch= 2.5 centimeters.

In small groups, they will convert 10 centimeters to inches. I will walk around checking understanding. I will remind students of their prior knowledge about ratios to prompt thinking. As a whole group, we will discuss their findings.

I will hand out a conversion table that I have created that only shows the measurement of the Plains Indians in centimeters rounded to the nearest whole number (see attached). I will write the conversion of centimeters to inches on the board: **1 inch = 2.5 centimeters.**
In their groups they will think about how to convert the height of the Plains Indians (173 centimeters) to inches.

They will fill in the table with the other heights provided in the article: Australian men 172 centimeters and European men 171 centimeters. I will

ELABORATE:

Once I notice groups are finishing up the 3 conversions, I will ask them to convert the inches to feet and provide the conversion on the board: **1 foot= 12 inches.**

When students have completed their table, we will have a whole group discussion about the data.

Students will share the numbers they found with the class and I will ask students if they agree or disagree.
I will ask:
Which one was easier to convert? Centimeters to inches or inches to feet?

EVALUATE:

Why is important to know how to convert measurements? **One answer I hope they share: Some measurements make more sense than others. It is easier for us to understand feet rather than centimeters in this instance.**

I will write the height of the shortest man known to the world and they will write it in their table: 55 centimeters. I will use the tape measurer to show them visually how tall this is and we will discuss it. In the remaining space on their conversion table, I will ask the students to independently convert the height of the shortest person in the world from centimeters to inches, and to feet. There is also a question they will answer below the table on the sheet of paper. I will walk around to check for understanding. They will turn the finished conversion chart in.
Lesson Materials: Copies of the article "Plains Indians Were Tallest, Healthiest People In the World" Pencil Highlighter Paper Copies of conversion chart
Classroom Management Needs: Students will work in small groups to discuss how to convert measurements. They will contribute ideas and build on their communication and collaboration skills. Some students need to read independently to understand a text, so they will all active read the article independently. They will also get the chance to convert the measurements of the shortest man independently. I will walk around the room during work time to ask prompting questions and check for understanding.

Answer Key:

	Height in Centimeters	**Inches**	**Feet**
Plains Indian	173 cm	68.11 in	5.68 ft
Australian	172 cm	67.72 in	5.64 ft
European	171 cm	67.32 in	5.61 ft
Shortest man	55 cm	21.65 in	1.8 ft

Table for students to fill out:

	Height in Centimeters	Inches	Feet
Plains Indian	173 cm		
Australian			
European			
Shortest man			

In 2-3 sentences, provide evidence from the reading and from the class discussion that suggests why the Plains Indians were the tallest people in the world in the 1800's.

IEFA Lesson (6^{th})

Final Draft

COE Lesson Plan Template

Teacher: Student 9 **Lesson Date:**

Grade Level: 6^{th} **Timeframe:** 1 hour

Content Area: Mathematics/Health/Social Studies

Grouping Strategy: Independent, whole group, and small group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

This lesson will follow the previous day's math lesson about rounding to the nearest 10^{th} and 100^{th} place. This lesson will build on the students rounding skills by requiring them to round to the nearest 100^{th}. They have mastered rounding, but need more practice on differentiating between the 10^{th} and 100^{th} place. They have practiced converting whole numbers in many conversion units, but have not practiced converting decimal numbers and fractions of a whole. Their prior knowledge of ratios and whole numbers will be applied to exploring the conversion of decimals and more challenging measurements.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

This lesson starts out with the teacher reading and using guided practice to actively read the article. Students practice active reading independently after. Then they work in small groups to share ideas about the text. They get to work in the small groups to explore converting measurements. I will start asking them to convert 10 centimeters to inches so that they will master a less challenging conversion before moving on to the larger conversions. I will remind them of their knowledge about ratios as they explore converting. Students who struggle will get to that extra support from their peers by working in small groups. Students that find converting easy will get to explain it to their peers which develops their understanding even more. The students will get to work independently on the last conversion to make sure that they fully understand the concept. I will walk around to check for understanding so that all learners are successful.

Lesson Plan Development

Lesson Title: Converting Heights of Plains Indians	
Common Core and/or State Standard(s): Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. 6.RP.3-Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. Practice Standards: 6.MP.1. Make sense of problems and persevere in solving them. 6.MP.2. Reason abstractly and quantitatively. 6.MP.4. Model with mathematics. 6.MP.6. Attend to precision. 6.MP.8. Look for and express regularity in repeated reasoning.	
Lesson Objective: Students will develop a method of converting measurements of centimeters to inches and inches to feet using their previous knowledge of ratios, proportions, and division involving whole numbers.	**Assessment of Learning:** Students will create a conversion table that shows the 4 heights in centimeters (2nd column), in inches (3rd column), and in feet (4th column). Table template is attached.
Lesson Objective: Students will be able to explain how the healthy diet of the Plains Indians contributed to their growth.	**Assessment of Learning:** Students will identify availability of food and healthy eating habits of the Plains Indians by answering a question below the conversion table they created. The question is: **In 2-3 sentences, provide evidence from the reading and from the class discussion that suggests why the Plains Indians were the tallest people in the world in the 1800's.**
Relevant Vocabulary: Centimeters Inches Feet Convert	

Conversion table Plains Indians Tribe
Teaching Model: 5 E's
Indian Education for All (IEFA) __ No _x_ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians: 1. There is great diversity among the twelve sovereign tribes of Montana in their languages, cultures, histories, and governments. Each tribe has a distinct and unique cultural heritage that contributes to modern Montana
Lesson Procedures/Activities: ENGAGE: What is meant by "Plains Indians"? **Native American tribes that lived on the flat plains.** Does anyone know what tribes of Native Americans that live on the plains in Montana? **Cheyenne, Sioux, Blackfeet, Crow Comanche** What kinds of foods do you think were available on the plains for plains Indian tribes in Montana? **Buffalo, bugs, fish, bird, corn, ect.** When you think of a healthy meal, what kind of food comes to mind? What kinds of healthy foods do you like to eat? EXPLORE: As a class, we will active read the first two paragraphs of "Plains Indians Were Tallest, Healthiest People In World". **Highlight together:** American Plains Late 1800s Tallest people in the world Well-nourished Sickly victims European disease Students will independently read the rest of the article. I will encourage students to highlight, underline, or note anything they find interesting or important. Students will be put into groups of 4-5. They will discuss the article and share anything they found interesting or important. We will have a whole group discussion. I will ask each group to share something interesting they read. I will ask: What does the article say about why they were the tallest people in the world? Do you think how we eat today impacts our growth?

What year or years were they known to be the tallest people?

If students note the heights listed in paragraph 5, I will note the heights on the board as they say them. If they do not mention them, I will ask:
About how tall were the tallest Plains Indians?
What other countries are known at this time to have the tallest people in the world and how tall were they?

EXPLAIN:
I will provide the class with the conversion of inches to centimeters on the board: **1 inch = 2.54 centimeters.** As a class, we will talk about rounding to the nearest 10^{th}. I will call on a student to round 2.54 to the nearest 10^{th}. I will ask if all students agree. Once all students understand rounding (which they have been working on previously), I will remind them by pointing to the board that 1 inch= 2.5 centimeters.

In small groups, they will convert 10 centimeters to inches (4 inches). I will walk around checking understanding. I will remind students of their prior knowledge about ratios and cross-multiplying to prompt thinking. I will remind students about strategies for dividing. Students might use different strategies for division such as dividing 10 lines into groups of 2.5 dashes. As a whole group, we will discuss their findings. If students are struggling, I will have other students who found the answer ask the struggling students prompting questions:
What have you found out so far?
What strategy are you using? Here is what I did, but they will not finish to their solution.

I will hand out a conversion table that I have created that only shows the measurement of the Plains Indians in centimeters rounded to the nearest whole number (see attached). I will write the conversion of centimeters to inches on the board: **1 inch = 2.5 centimeters.**
In their groups they will think about how to convert the height of the Plains Indians (173 centimeters) to inches.

They will fill in the table with the other heights provided in the article: Australian men 172 centimeters and European men 171 centimeters.

Prompting questions:
What do you already know about converting that might be useful here? How did you conver 10 centimeters to inches?
Can you use that here?
What are some other strategies your classmates shared that you understood?

ELABORATE:

Once I notice groups are finishing up the 3 conversions, I will ask them to convert the inches to feet and provide the conversion on the board: **1 foot= 12 inches.**

When students have completed their table, we will have a whole group discussion about the data.

Students will share the numbers they found with the class and I will ask students if they agree or disagree.
I will ask:
Which one was easier to convert? Centimeters to inches or inches to feet?

EVALUATE:

Why is important to know how to convert measurements? **One answer I hope they share: Some measurements make more sense than others. It is easier for us to understand feet rather than centimeters in this instance.**

I will write the height of the shortest man known to the world and they will write it in their table: 55 centimeters. I will use the tape measurer to show them visually how tall this is and we will discuss it.

In the remaining space on their conversion table, I will ask the students to independently convert the height of the shortest person in the world from centimeters to inches, and to feet. There is also a question they will answer below the table on the sheet of paper. I will walk around to check for understanding. They will turn the finished conversion chart in.

Lesson Materials:
Copies of the article "Plains Indians Were Tallest, Healthiest People In the World"
Pencil
Highlighter
Paper
Copies of conversion chart

Classroom Management Needs:

Students will work in small groups to discuss how to convert measurements. They will contribute ideas and build on their communication and collaboration skills. Some students need to read independently to understand a text, so they will all active read the article independently. They will also get the chance to convert the measurements of the shortest man independently. I will walk around the room during work time to ask prompting questions and check for understanding.

Answer Key:

	Height in Centimeters	Inches	Feet

Plains Indian	173 cm	68.11 in	5.68 ft
Australian	172 cm	67.72 in	5.64 ft
European	171 cm	67.32 in	5.61 ft
Shortest man	55 cm	21.65 in	1.8 ft

Table for students to fill out:

	Height in Centimeters	**Inches**	**Feet**
Plains Indian	173 cm		
Australian			
European			
Shortest man			

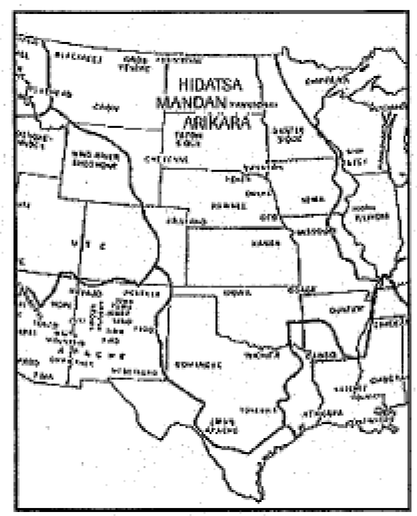
In 2-3 sentences, provide evidence from the reading and from the class discussion that suggests why the Plains Indians were the tallest people in the world in the 1800's.

Plains Indians Were Tallest, Healthiest People In World

Average height is a good way of measuring health in populations

The Native tribes on the American Plains in the late 1800s were the tallest people in the world, suggesting that they were surprisingly well-nourished, a new study found.

These results contradict the modern image of American Indians as being sickly victims succumbing to European disease, said Richard Steckel, co-author of the study and professor of economics and anthropology at Ohio State University.



"The Plains Indians had a remarkable record of nutritional and health success, despite the enormous pressures they were under," Steckel said

"What these height data show is the ingenuity and adaptability of the equestrian Plains tribes in the face of remarkable stress from disease and hardship," Steckel said.

"Plains tribes were widely spread out and very mobile, meaning they didn't live in one area long enough to accumulate the wastes and parasites that could become a threat to public health."

The average adult male Plains Indian stood 172.6 centimeters tall -- about 5 feet 8 inches. The next tallest people in the world at that time were Australian men, who averaged 172 centimeters. European American men of the time averaged 171 centimeters tall, and men living in European countries were typically several centimeters shorter.

Steckel conducted the study with Joseph Prince, an anthropologist at the University of Tennessee at Knoxville. Their results were published in a recent issue of The American Economic Review.

Steckel and Prince used recently discovered data collected by Franz Boas, one of the founders of American anthropology. Boas collected and analyzed data from several thousand Native Americans during the late 1800s as part of research he was doing for the Columbian Exposition, a fair held in Chicago from May to October 1893.

The researchers used data from 1,123 Natives from eight Plains tribes, including the Cheyenne, Sioux, Blackfeet and Comanche.

Steckel has conducted a variety of studies using stature as an alternative way of measuring the standard of living and overall health. Average height is a good way of measuring health in populations, he said, especially nutritional status, as determined by diet minus claims on the diet made by work and by disease. Genetic differences that are important in the heights of individuals approximately cancel in determining the average heights of entire populations.

This study shows that despite the many technological advantages that the European-American settlers had over the American Indians, the Plains tribes enjoyed better health, at least nutritionally.

"The modern perception that Native Americans were hapless and in poor health probably comes from the era at the turn of the century when Indians were put on reservations," Steckel said. "Native Americans often did suffer high rates of tuberculosis and other manifestations of poor health on the reservations, but they weren't always that way."

While the reasons for the general good health of the Plains Indians, compared to whites, has not been extensively studied, Steckel said several plausible theories exist. For one, the Plains Indians ate a varied diet that included a variety of native plants, as well as buffalo and other game that typically roamed the Great Plains, Steckel said.

Moreover, the Plains tribes were widely spread out and very mobile, meaning they didn't live in one area long enough to accumulate the wastes and parasites that could become a threat to public health, Steckel said.

American Indians did suffer from devastating epidemics such as smallpox that killed significant numbers of people, Steckel said. But the tribes took steps to minimize the effects of these epidemics, such as splitting up the tribe when the illnesses started, which helped stop the spread. Also, they were adept at reorganizing their small bands following deaths from epidemics.

Steckel said American Indians also lived in egalitarian societies that provided a strong safety net for the disadvantaged in their tribes, meaning that no one went hungry or uncared for.

In contrast to these advantages that American Indians enjoyed, many whites living in cities -- particularly the poor -- couldn't afford food for a healthy and complete diet. These large cities and towns were densely packed and lacked modern sanitary practices, meaning they were breeding grounds for disease. In addition, many poor had no safety net to help them in times of need and suffered from a lack of proper nutrition and medical care, Steckel said.

"The Plains Indians had a remarkable record of nutritional and health success, despite the enormous pressures they were under," Steckel said. "They developed a healthy lifestyle that the white Americans couldn't match, even with all of their technological advantages."

Unit Plan (K)

First Draft

COE Lesson Plan Template

Teacher: **Lesson Date:** 11/26/19

Grade Level: Kindergarten **Timeframe:** 10:00-10:20

Content Area: Math **Grouping Strategy:** Whole Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

- Students have prior knowledge of describing what objects and their positions to other things. They learned this in the standard before this (k.G.1). They are just now being introduced to two-dimensional shapes and proper names of each. Some of the students may struggle with the explain part of this concept because they are shy. They may also struggle picking the shapes out of other random shapes when they are different sizes.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

- For this lesson I will be sure to read all of the words to the children that are on the board and in the directions. I have created a lesson where all of the students will be up and moving so they don't get bored just sitting there. If the students are getting bored, I will suggest that they try and name their shapes if they aren't already circles.

Lesson Plan Development

Lesson Title: Exploring Circles	
Common Core and/or State Standard(s): - (K.G.A.2) Correctly name shapes regardless of their orientations or overall size. - (K.MP.6.) Attend to Precision- As kindergartners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.	
Lesson Objective: - Students will be able to demonstrate their knowledge of circles by coloring in just the circles out of a page of random shapes.	**Assessment of Learning:** - Students will be given an exit ticket. On this exit ticket students will have to color in all of the circles on the page, none of the other shapes.
Lesson Objective: - Students will be able to describe why they placed the circles on the circle side	**Assessment of Learning:** - The teacher will be observing whether or not they put the shape on the right side of the board and whether or not they can

and why they placed the other shapes on the side of non-circles.	explain why they put their shape where they did.

Relevant Vocabulary:
- Circles- round shapes that do not have straight parts.

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:
- To open this lesson, I will ask the students what a circle is and what it looks like.
- Let them answer
- Ask students if they see any circles around the classroom.
- Pick on students to answer.

EXPLORE:
- After going over what a circle is, and which objects are circles around the classroom I will pass out a random shape to each student. These shapes will include; circles, squares, rectangles, triangles, and hexagons.
- On the board there will be two columns, one labeled "circles" and the other labeled "other."
- I will pick students one after the other to come up to the front of the class and tape their shape to whichever side they think it belongs

EXPLAIN:
- Students will be explaining to the class why they put their shape where they did.
- Once they have placed their shape on the board, I will ask the class if they are right.
- If a student disagrees, I will have them explain why they think it should be on the other side.

ELABORATE:
- When all of the students have gone, I will ask them if they notice anything different about some of the circles. (some will be big, and some will be small)
- Ask students if they know what some of the shapes that they put on the "other" side are
- Have students explain to the class what the other shapes are if they know.

EVALUATE:

- When we have finished discussing ask the students what a circle is again and how they know.
- Hand out an exit ticket to each student
- This is a page with a few random shapes on it and they will be coloring in all of the shapes that are circles.
- This will show you if they can identify which shapes are circles out of a series of different shapes.

Lesson Materials:

- Shapes for each student
- Exit Ticket
- Tape
- Pencils

Classroom Management Needs:

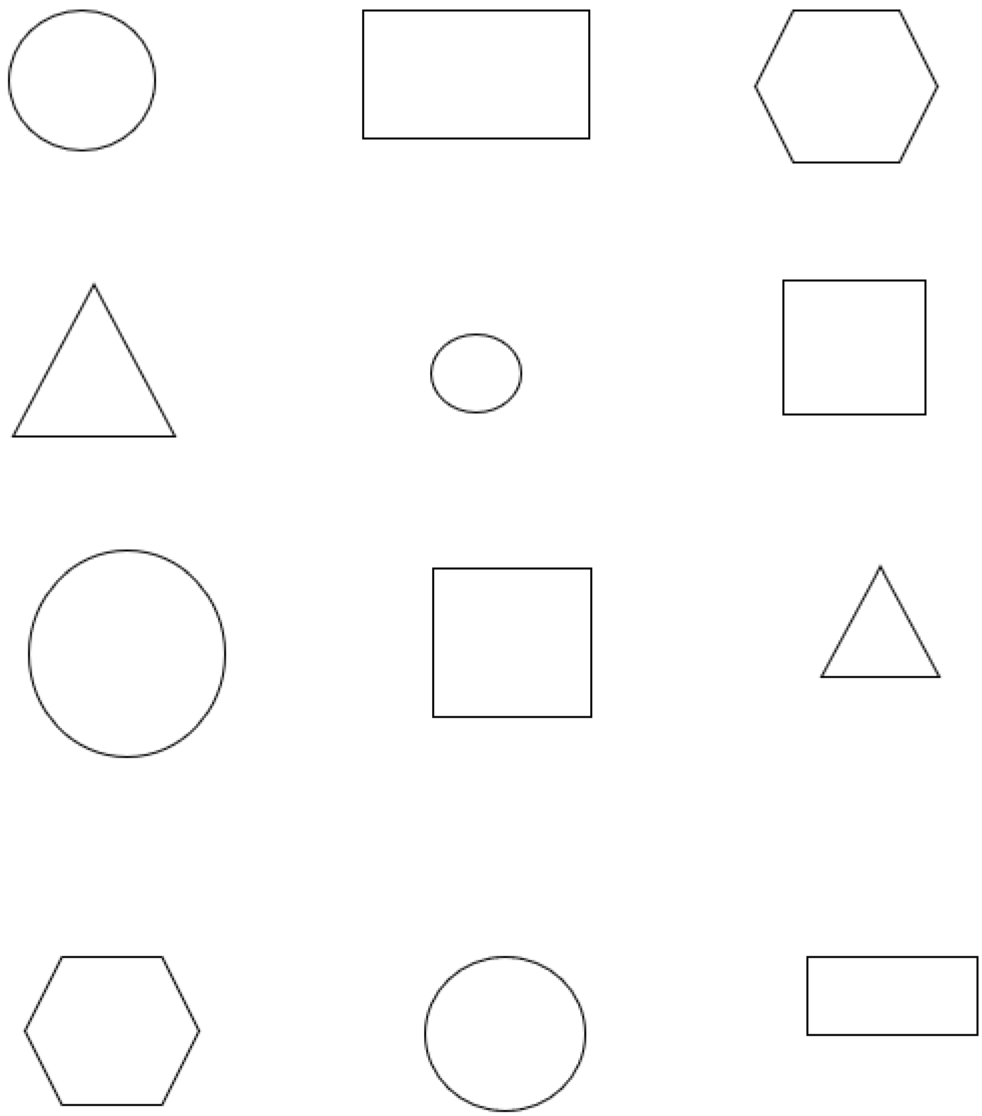
- Students will be working as a whole group. I will be calling students to come up and put their shapes up. With this I will need to be sure that the rest of the class has zero voices so we can say if we agree or not. I will also be sure that students aren't rude to each other if one gets it wrong.

Circles	Other

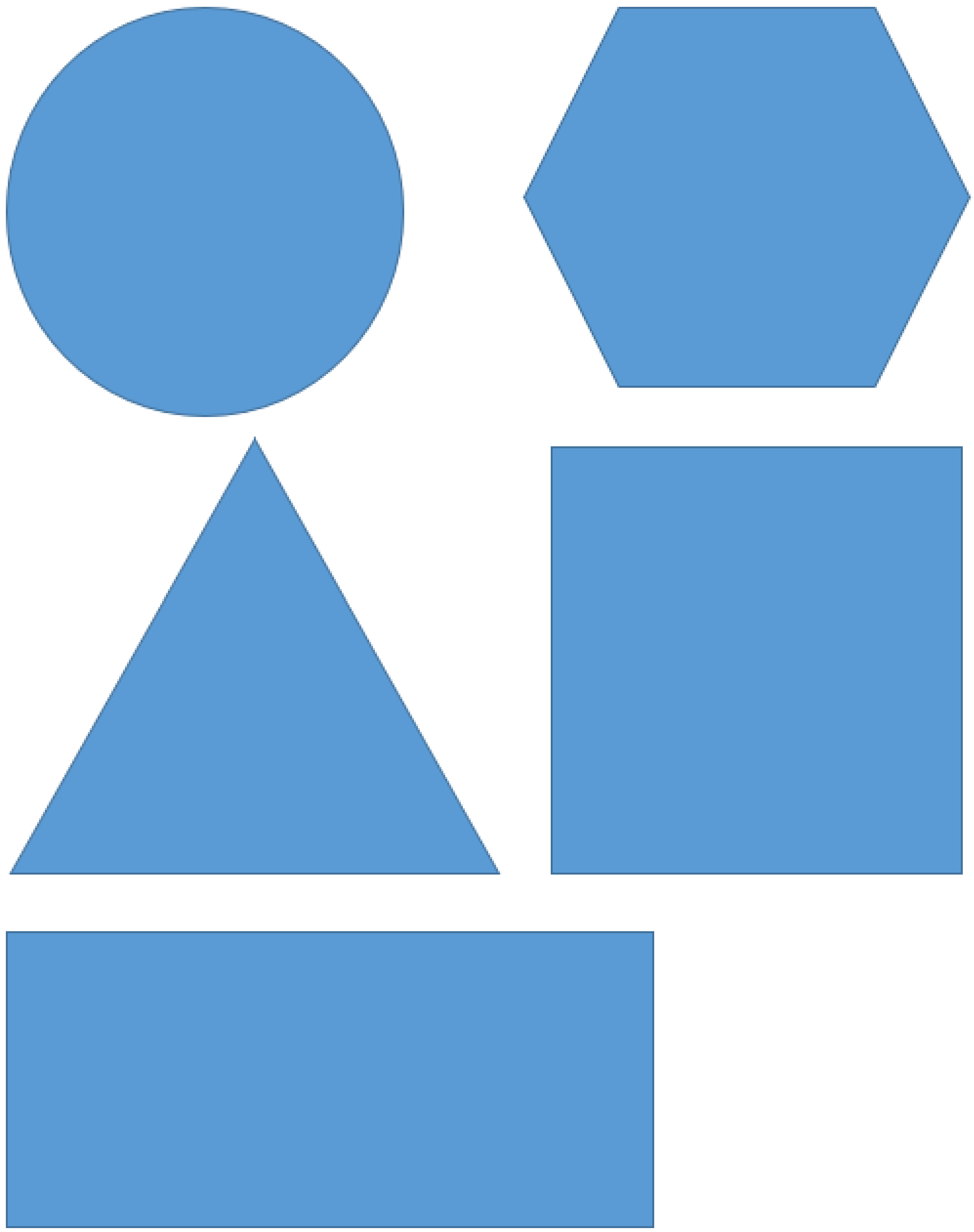
This is the chart that needs to be drawn on the board.

Exit Ticket

Directions: Color in all of the circles.



Shapes for Group Activity- Print off enough for each student to have one shape.



COE Lesson Plan Template

Teacher: **Lesson Date:** 11/26/19

Grade Level: Kindergarten **Timeframe:** 10:00- 10:20

Content Area: Math **Grouping Strategy:** Whole group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

- In the previous lesson students learned about what circles are and how to go about explaining what they look like. They are not familiar with how to look for and explain different key features of the shapes.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

- In this lesson I will be connecting it to things that they have eaten so that they stay engaged. They will be using different sized triangles so that they will know that a triangle is a triangle no matter how big or small. I will also be reviewing what we did yesterday (circles) so that they don't forget about what they have already learned. I have provided each student with pattern shapes that they can trace when they are doing the art project.

Lesson Plan Development

Lesson Title: Exploring Triangles	
Common Core and/or State Standard(s): - (K.G.A.2) Correctly name shapes regardless of their orientations or overall size. - K.MP.4. model with mathematics.	
Lesson Objective: - Students will be able to model and explain what a triangle looks like.	**Assessment of Learning:** - When they present what they made I will be noting whether or not they made a triangle and if they can explain why it is a triangle (three sides).
Lesson Objective: - Students will be able to use their prior knowledge of circles and triangles to create an art project using just those two shapes.	**Assessment of Learning:** - I will be assessing the students on whether they used just triangles and circles like they were supposed to or if they used other shapes.
Relevant Vocabulary: - Triangles- a shape with three sides that are all the same size. - Two-dimensional- a shape that has a width and a length, but no height.	

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:
- To start this lesson, I will have different two-dimensional shapes drawn on the board.
- I will go over the shapes with the students, asking them if they can tell me what the shapes are before I tell them.
- Ask students which one is a triangle and how they can tell it is a triangle. (three sides)
- Ask students if they can name any thing that they eat that is in the shape of a triangle.
- Let students answer.

EXPLORE:
- We are going to be making our own triangles today.
- Hand out pretzel-sticks and marshmallows to group.
- Each student should use these materials to make a triangle.
- They will have 10 minutes to build their triangles.

EXPLAIN:
- When everyone is finished building, I will have the students show the class what they built.
- Have students explain what they built and why it is a triangle
- If students didn't build a triangle, I ask them what a triangle is again, and they will have to say what they did wrong.

ELABORATE:
- When everyone is finished, I will ask if they can tell me the shape we learned about yesterday. (circles)
- I will ask them how a triangle is different from a circle.
- Hand out different colored paper to each group
- Have student trace and cut out different colored triangles and circles to make an art project.
- I will have an example of the project on the board and will be roaming around helping the students if they need it.
- Remind them that triangles can be all different sizes but have to have three straight sides.

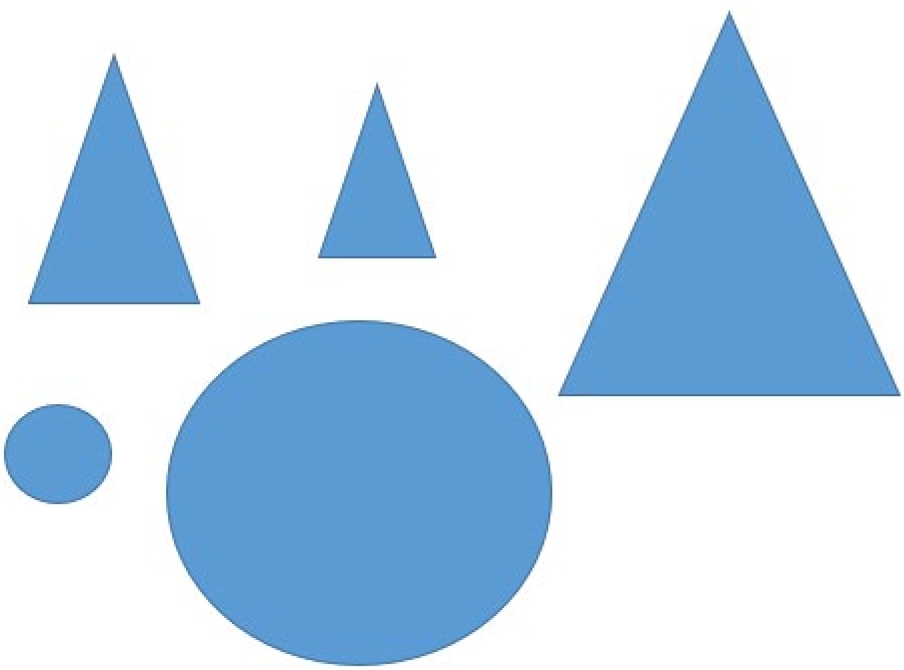
EVALUATE:
- After students have finished their art projects have them tell you what a triangle is and how many sides it has.
- Ask the students which shapes they used on their project (circles and triangles)
- I will look at each project and make sure that they just used circles and triangles to make sure they understand what those two shapes are.

Lesson Materials: - Pretzels - Marshmallows - Colored construction paper - Traceable triangles and circles - Glue - Drawn shapes on whiteboard
Classroom Management Needs: - During this lesson I will be roaming around the room while they are making their triangles out of pretzels so that they stay on track. I will be monitoring their voice levels because when students are working on projects, they tend to get a bit louder.

Marshmallow project: Students will be doing something like this but making **triangles** instead.

Triangle art project example

Traceable triangles and circles- hand out a set to each student



COE Lesson Plan Template

Teacher: **Lesson Date:** 11/26/19

Grade Level: Kindergarten **Timeframe:** 10:00-10:30

Content Area: Math **Grouping Strategy:** Whole Group/Partners

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

- Previously in this unit, students have learned about and how to explain the looks of two shapes; circles and triangles. They are able to learn and describe the different shapes, although they struggle with explaining them. We will continue to work on this skill.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

- For this lesson I will be letting the students use as many of the three shapes we have gone over as they want as long as its more than three. I will also have the shape on the bingo caller card in case students don't remember what they are or need a refresher. In the bingo game I have included a hexagon to get the kids thinking about what we have learned. Also, I have included the other two shapes into this lesson to serve as a refresher of the unit so far.

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Exploring Squares</td></tr>
<tr><td colspan="2">Common Core and/or State Standard(s):
- (K.G.A.2) Correctly name shapes regardless of their orientations or overall size.
- K.MP.4. Model with mathematics.</td></tr>
<tr><td>Lesson Objective:
- Students will be able to explain which shapes they used in their drawings and why.</td><td>Assessment of Learning:
- The teacher will be looking over the drawings to see if they have used the correct shapes.</td></tr>
<tr><td>Lesson Objective:
- Students will be able to demonstrate their knowledge of circles, triangles, and squares by participating in a bingo game where they have the shapes and the teacher is just saying which shape it is.</td><td>Assessment of Learning:
- The teacher will be walking around while calling out the shapes to see if the students are crossing out the correct shape when its name is called.</td></tr>
</table>

Relevant Vocabulary:

- Square- a shape with all of the same size sides that make 90-degree angles.
- Rectangle- a shape with two short ends and two long sides.

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:

- Ask students what a square looks like
- Have one student come up to the board and draw it
- Ask students how many sides it has.
- Ask students if they know the difference between a square and a rectangle
- Have students point out squares around the room. If they point out rectangles such as the door tell them that a rectangles have two long sides and two short ends. Squares have all of the same size sides.

EXPLORE:

- Ask students if all squares are the same size
- Let them answer
- Squares can be either big or small as long as it has four equal sides.
- Have students draw a picture either of themselves or of an animal
- When drawing they can only use squares, circles, and triangles.
- They also have to have at least 3 triangles, 3 circles, and 3 squares in their drawing.

EXPLAIN:

- When everyone is done with their drawing, have them switch their drawings with the person across from them.
- Have the students count how many of each shape their partner used
- Have students stand up one by one and explain the different shapes their partner used.
- If you see a shape that isn't one of the three, we were working on, ask them what shape it is and why they used it.

ELABORATE:

- Hand out bingo boards and chips to put on top to each group.
- Tell students that you are going to be just saying the name of the shape and they have to find one of them on their board to cover up.
- Play the game until students at least 4 students get a bingo.

EVALUATE:

- Ask students what a square is
- Let them answer
- Ask them what the shapes are that we have learned so far are.
- Let them answer
- What is the difference between a square and a rectangle?
- Let students explain.

Lesson Materials:

- Drawing paper
- Pencils
- Colored pencils
- Bingo board
- Bingo chips

Classroom Management Needs:

- For this lesson I will be walking around the room making sure all of the students are staying on task and that their voices don't get too loud. When they are sharing about their friends' projects, I will make sure that they are saying nice things. Also, when we are playing the bingo game, I will be walking around making sure that none of the students are cheating,

Shapes Bingo Caller Card

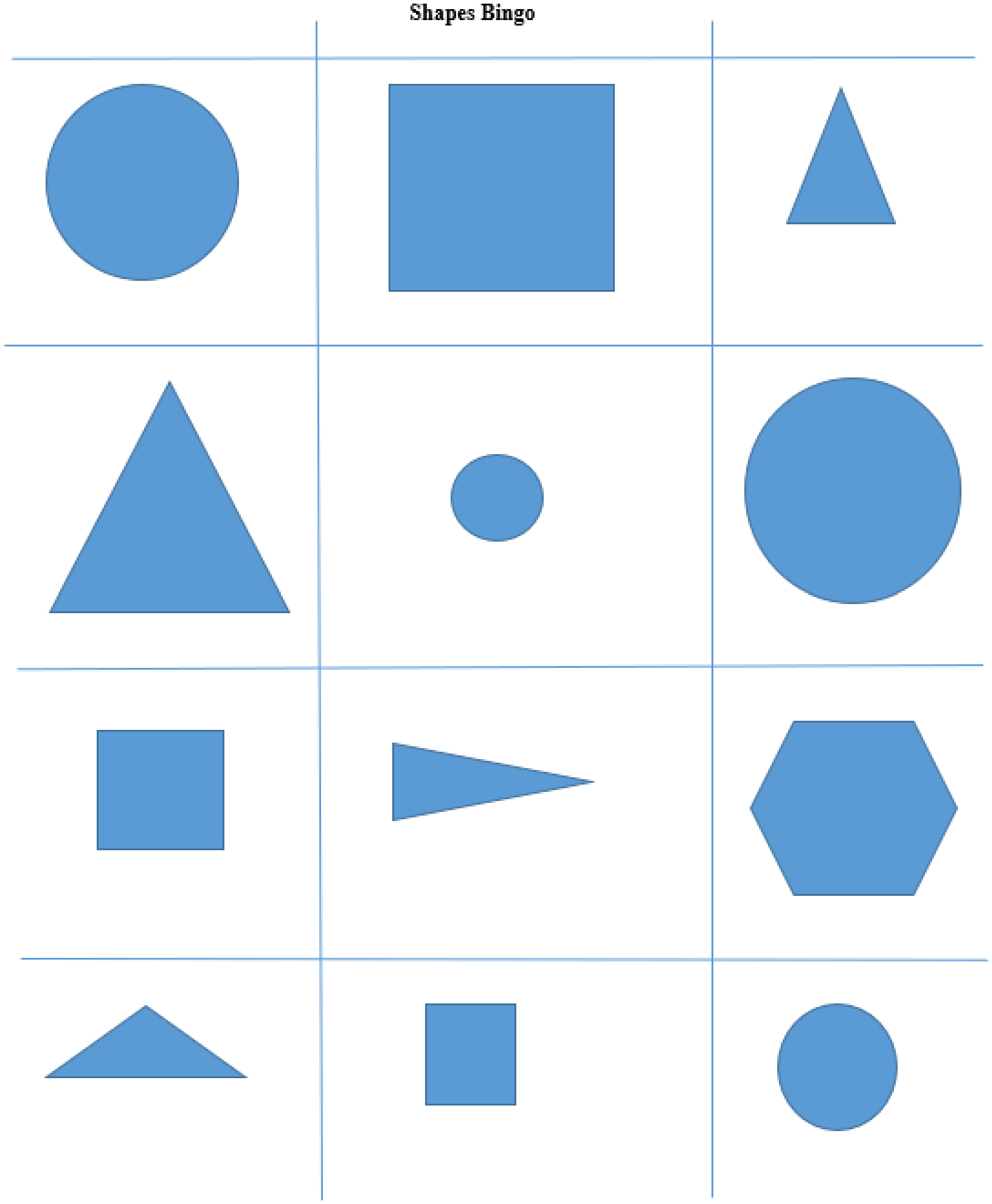
Triangle

Circle

Rectangle

Square

Shapes Bingo



Unit Plan (K)

Final Draft

Exploring Shapes

(K.G.A.2) Correctly name shapes regardless of their orientations or overall size.

(K.MP.6.) Attend to Precision

(K.MP.4.) Model with Mathematics

In three days, students will be able to understand, identify, and recognize different two-dimensional shapes.

Student 18

COE Lesson Plan Template

Teacher:

Lesson Date: 11/26/19

Grade Level: Kindergarten

Timeframe: 10:00-10:20

Content Area: Math

Grouping Strategy: Whole Group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

Students have prior knowledge of describing what objects and their positions to other things. They learned this in the standard before this (k.G.I). They are just now being introduced to two-dimensional shapes and proper names of each. Some of the students may struggle with the explain part of this concept because they are shy. They may also struggle picking the shapes out of other random shapes when they are different sizes.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

For this lesson I will be sure to read all of the words to the children that are on the board and in the directions. I have created a lesson where all of the students will be up and moving so they don't get bored just sitting there. If the students are getting bored I will suggest that they try and name their shapes if they aren't already circles. If students are struggling, I will take a step back and go over again what a circle is and what it looks like. I will also try and relate it to them as best as I can. This would be pointing out things that they have that are that shape. (maybe a chair, watch, or coin)

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Exploring Circles</td></tr>
<tr><td colspan="2">Common Core and/or State Standard(s):
- (K.G.A.2) Correctly name shapes regardless of their orientations or overall size.
- (K.MP.6.} Attend to Precision-As kindergartners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.</td></tr>
<tr><td>Lesson Objective:
- Students will be able to demonstrate their knowledge of circles by coloring in just the circles out of a page of random shapes.</td><td>Assessment of Learning:
- Students will be given an exit ticket. On this exit ticket students will have to color in all of the circles on the page, none of the other shapes.</td></tr>
<tr><td>Lesson Objective:</td><td>Assessment of Learning:</td></tr>
</table>

- Students will be able to explain why they placed the circles on the circle side and why they placed the other shapes on the side of non-circles.	- The teacher will be observing whether or not they put the shape on the right side of the board and whether or not they can explain why they put their shape where they did.

Relevant Vocabulary:

- Circles- round shapes that do not have straight parts.

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:

- To open this lesson, I will ask the students what a circle is and what it looks like.
- Let them answer (I will be expecting the students to say that it is round and that it only has one side. They may even say that it looks like a ball. Some of the students may just draw it in the air because they don't know how to describe it.)
- Ask students if they see any circles around the classroom.
- Pick on students to answer.

EXPLORE:

- After going over what a circle is, and which objects are circles around the classroom I will pass out a random shape to each student. These shapes will include; circles, squares, rectangles, triangles, and hexagons.
- On the board there will be two columns, one labeled "circles" and the other labeled "other."
- I will pick students one after the other to come up to the front of the class and tape their shape to whichever side they think it belongs
- I will then ask the students a few questions before getting started. Are there any sides on a circle? Do circles have any corners?

EXPLAIN:

- Students will be explaining to the class why they put their shape where they did.
- Once they have placed their shape on the board, I will ask the class if they are right.
- If a student disagrees, I will have them explain why they think it should be on the other side.

ELABORATE:

- When all of the students have gone, I will ask them if they notice anything different about some of the circles. (some will be big, and some will be small)
- Ask students if they know what some of the shapes that they put on the "other" side are
- Have students explain to the class what the other shapes are if they know. — How are circl[illegible]

EVALUATE:

- When we have finished discussing ask the students what a circle is again and how they know.
- Now that you know what a circle is and what it looks like, can you tell me things that you have seen that are circles? Why do you think that they are in this shape instead of another shape? Can you think of any thing that is in the shape of a circle that you don't think should be? Explain why you think this. (These answers may be all over the place, but at least they are explaining why they think what they do.)
- Hand out an exit ticket to each student
- This is a page with a few random shapes on it and they will be coloring in all of the shapes that are circles.
- This will show you if they can identify which shapes are circles out of a series of different shapes.

Lesson Materials:

- Shapes for each student
- Exit Ticket
- Tape
- Pencils

Classroom Management Needs:

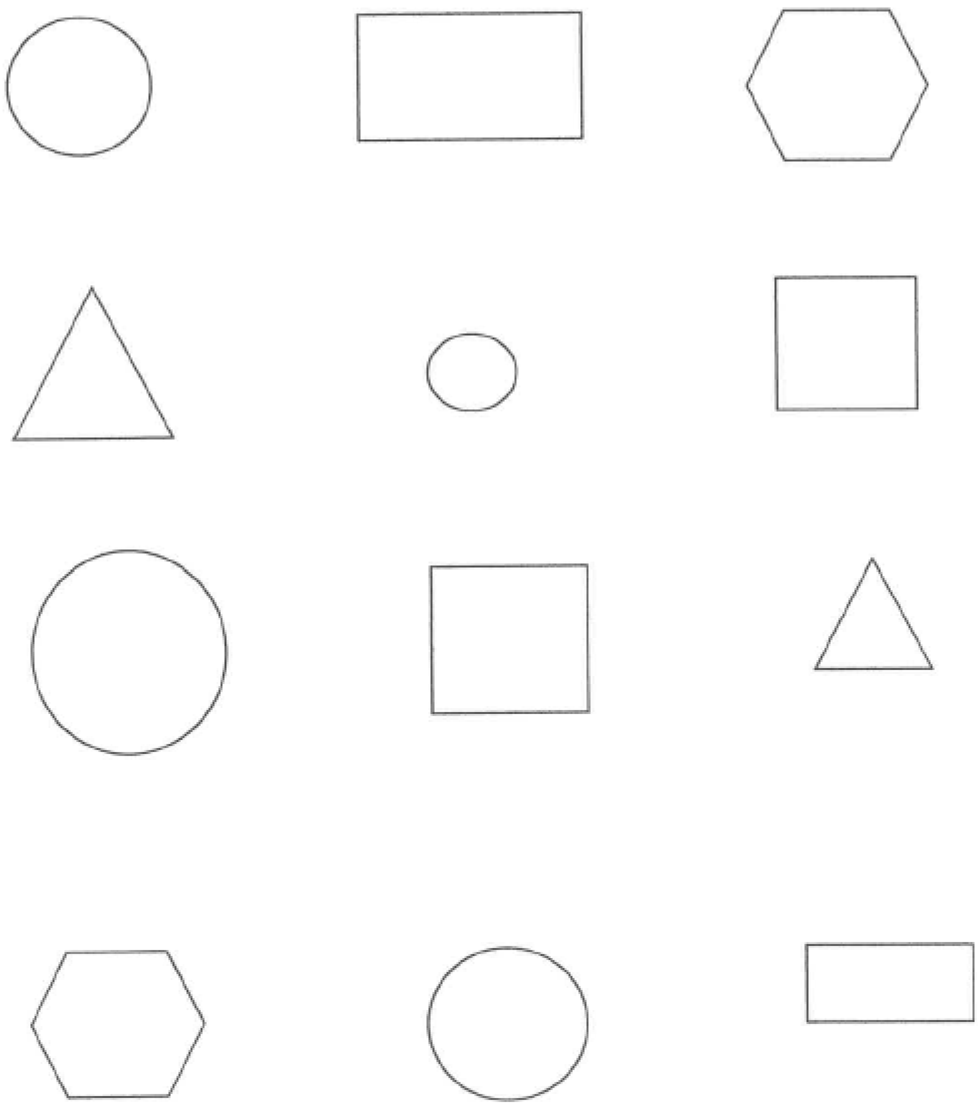
- Students will be working as a whole group. I will be calling students to come up and put their shapes up. With this I will need to be sure that the rest of the class has zero voices so we can say if we agree or not. I will also be sure that students aren't rude to each other if one gets it wrong.

Circles	Other

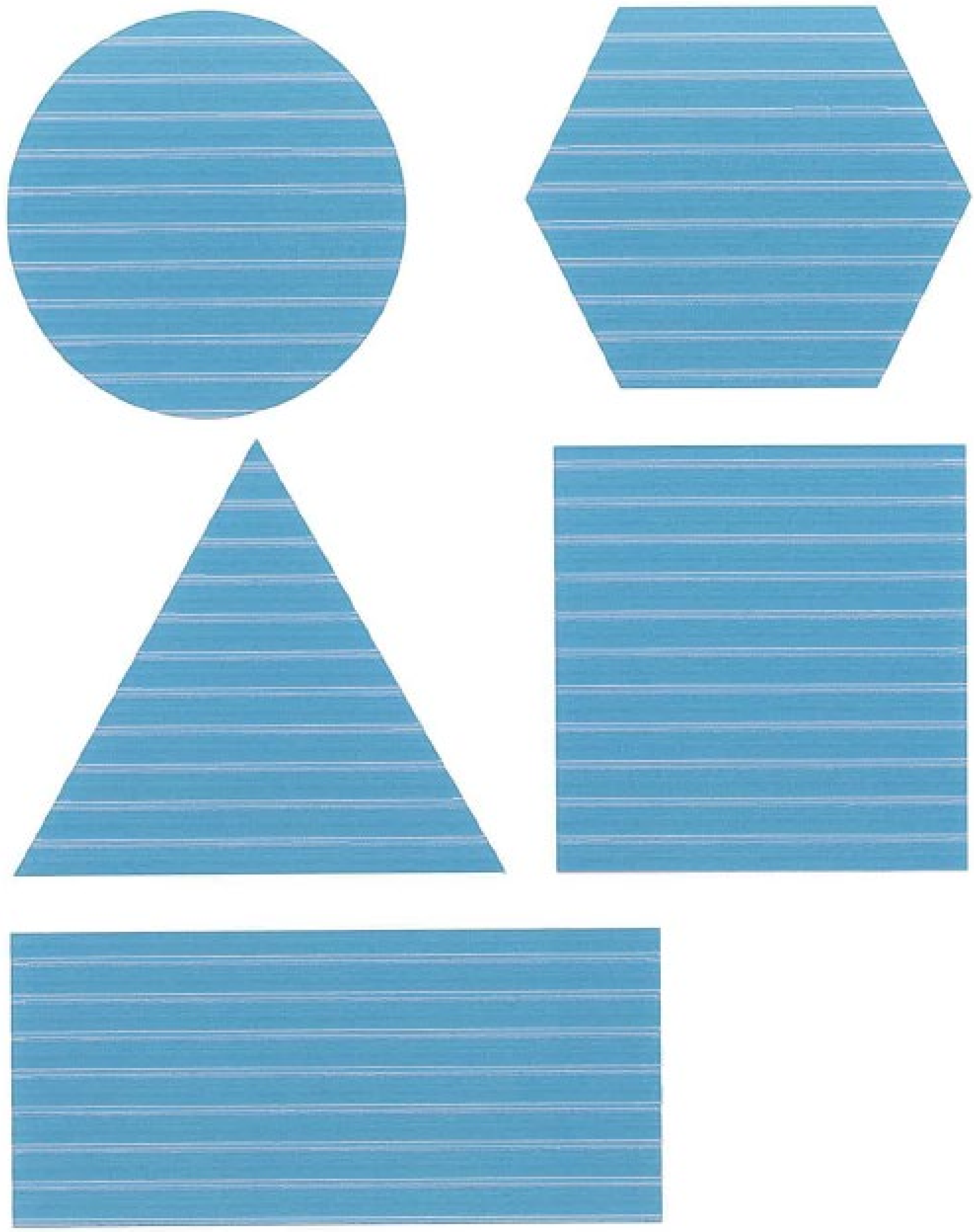
This is the chart that needs to be drawn on the board.

Exit Ticket

Directions: Color in all of the circles.



Shapes for Group Activity- Print off enough for each student to have one shape.



COE Lesson Plan Template

Teacher:

Lesson Date: 11/26/19

Grade Level: Kindergarten

Timeframe: 10:00- 10:20

Content Area: Math

Grouping Strategy: Whole group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

In the previous lesson students learned about what circles are and how to go about explaining what they look like. They are not familiar with how to look for and explain different key features of the shapes.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In this lesson I will be connecting it to things that they have eaten so that they stay engaged. They will be using different sized triangles so that they will know that a triangle is a triangle no matter how big or small. I will go over with the students a little bit of what sides are so that they will be able to understand what I mean when I ask how many sides a triangle has. I will also be reviewing what we did yesterday (circles) so that they don't forget about what they have already learned. I have provided each student with pattern shapes that they can trace when they are doing the art project. If students are struggling with this content, I will be pairing them up with a partner to help explain the concept to them. I will also have them think back to the objects the class pointed out in the classroom that are that certain shape, this might trigger their memory. If students already know their shapes, I will be asking them more questions as to where you might find them and why you think the people made those things that shape?

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Exploring Triangles</td></tr>
<tr><td colspan="2">Common Core and/or State Standard(s):
- (K.G.A.2) Correctly name shapes regardless of their orientations or overall size.
- K.MP.4. model with mathematics.</td></tr>
<tr><td>Lesson Objective:
- Students will be able to model and explain what a triangle looks like.</td><td>Assessment of Learning:
- When they present what they made I will be noting whether or not they made a triangle and if they can explain why it is a triangle (three sides).</td></tr>
<tr><td>Lesson Objective:</td><td>Assessment of Learning:
- I will be assessing the students on whether they used just triangles and</td></tr>
</table>

- Students will be able <u>explain</u> ~~what shapes~~ they ~~used and why they used them where they did~~. use shapes realistically to model real world	circles like they were supposed to or if they used other shapes.

Relevant Vocabulary:

- Triangles- a shape with three sides that are all the same size.
- Two-dimensional- a shape that has a width and a length, but no height.
- Side- The different lines on a two-dimensional shape are sides.

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:

- To start this lesson, I will have different two-dimensional shapes drawn on the board.
- I will go over the shapes with the students, asking them if they can tell me what the shapes are before I tell them.
- Ask students which one is a triangle and how they can tell it is a triangle. (three sides)
- What is a side? How would we explain how may sides a triangle has? (Explain this to the students after they attempt an answer)
- Ask students if they can name any thing that they eat that is in the shape of a triangle.
- Let students answer.

EXPLORE:

- We are going to be making our own triangles today.
- Hand out pretzel-sticks and marshmallows to group.
- Each student should use these materials to make a triangle.
- They will have 10 minutes to build their triangles.

EXPLAIN:

- When everyone is finished building, I will have the students show the class what they built.
- Have students explain what they built and why it is a triangle
- If students didn't build a triangle, I ask them what a triangle is again, and they will have to say what they did wrong.
- What other things could you make out of these materials? What other objects have you seen maybe at home or outside that are made of triangles?

ELABORATE:

- When everyone is finished, I will ask if they can tell me the shape we learned about yesterday. (circles)

- I will ask them how a triangle is different from a circle.
- Hand out different colored paper to each group
- Have student trace and cut out different colored triangles and circles to make an art project.
- I will be including multiple other shapes on the traceable page. This way the students have to really think about the shapes that we have been learning about and pick out which ones they are.
- I will have an example of the project on the board and will be roaming around helping the students if they need it.
- Remind them that triangles can be all different sizes but have to have three straight sides.

EVALUATE:

- After students have finished their art projects have them tell you what a triangle is and how many sides it has. How can you tell this is a triangle?
- Ask the students which shapes they used on their project and why they used them (circles and triangles)
- Can you think of any other animals that might have triangles and circles on their body? Which ones? Where can you find these shapes in nature?
- Now that you know what a triangle is and what it looks like, can you tell me things that you have seen that are triangles? Why do you think that they are in this shape instead of another shape? Can you think of anything that is in the shape of a triangle that you don't think should be? Explain why you think this. Can you think of anything that has both triangles and circles? (pizza)
- I will look at each project and make sure that they just used circles and triangles to make sure they understand what those two shapes are.

Lesson Materials:

- Pretzels
- Marshmallows
- Colored construction paper
- Traceable triangles and circles
- Glue
- Drawn shapes on whiteboard

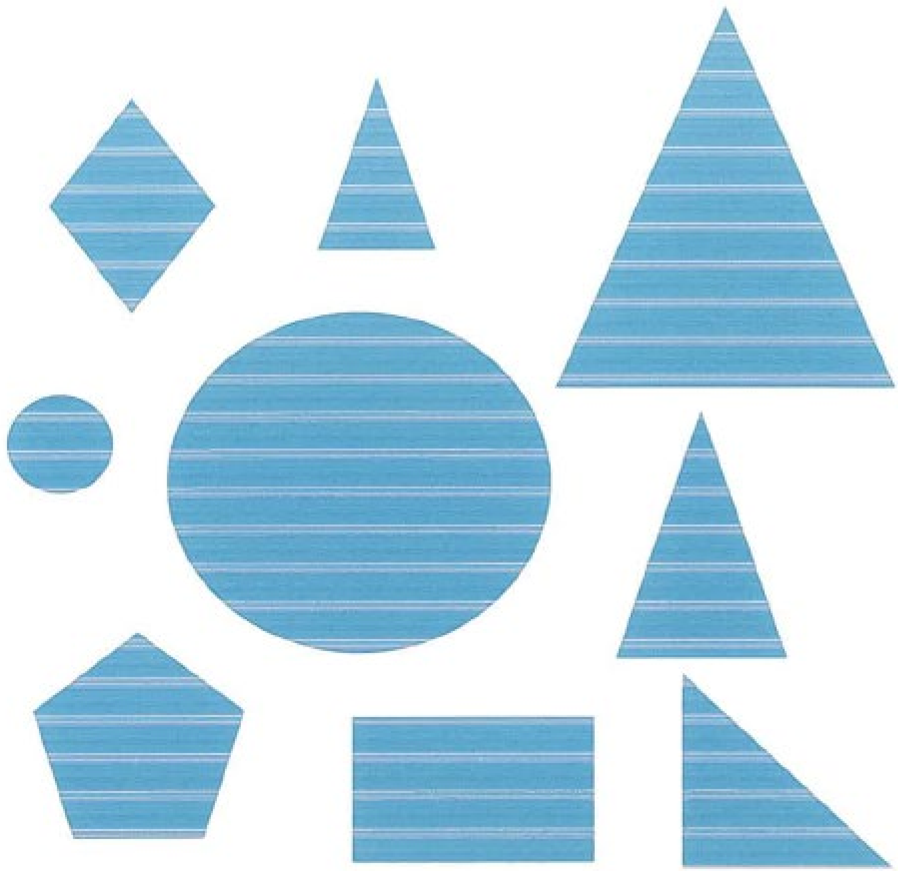
Classroom Management Needs:

- During this lesson I will be roaming around the room while they are making their triangles out of pretzels so that they stay on track. I will be monitoring their voice levels because when students are working on projects, they tend to get a bit louder.

Marshmallow project: Students will be doing something like this but making **triangles** instead.

Triangle art project example

Traceable triangles and circles- hand out a set to each student



COE Lesson Plan Template

Teacher:

Lesson Date: 11/26/19

Grade Level: Kindergarten

Timeframe: 10:00-10:30

Content Area: Math

Grouping Strategy: Whole Group/Partners

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

Previously in this unit, students have learned about and how to explain the looks of two shapes; circles and triangles. They are able to learn and describe the different shapes, although they struggle with explaining them. We will continue to work on this skill.

How will you differentiate the lesson to meet the needs of arr learners in your dassroom?

For this lesson I will be letting the students use as many of the three shapes we have gone over as they want as long as its more than three. I will also have the shape on the bingo caller card in case students don't remember what they are or need a refresher. In the bingo game I have included a hexagon to get the kids thinking about what we have learned. Also, I have included the other two shapes into this lesson to serve as a refresher of the unit so far. I will be going over what sides are again today to help students who have forgotten. If students are struggling, I will have them sit next to someone who is really getting it when we are doing the Bingo game. This will help them because if they can't locate the shape by just hearing the name, their partner may be able to help them out. If I have students who are ahead of everyone, I will be asking them more questions to get them thinking. (these questions are located at the end of the lesson)

Lesson Plan Development

Lesson Title: Exploring Rectangles	
Common Core and/or State Standard(s): - (K.G.A.2) Correctly name shapes regardless of their orientations or overall size. - K.MP.4. Moder with mathematics.	
Lesson Objective: - Students will be able to explain which shapes they used in their drawings and why.	**Assessment of Learning:** - The teacher will be looking over the drawings to see if they have used the correct shapes.

Lesson Objective:	Assessment of Learning:
- Students will be able to identify the different shapes we have learned about that are all different sizes out of number of other shapes.	- The teacher will be walking around while calling out the shapes to see if the students are crossing out the correct shape when its name is called.

Relevant Vocabulary:

- Rectangle- a shape with two short ends and two long sides.
- Side- The different lines on a two-dimensional shape are sides.

Teaching Model: 5 E's

Indian Education for All (IEFA) _x_ No __ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE:

- Ask students what a rectangle looks like
- Have one student come up to the board and draw it
- Do you remember what a side is? Explain how many sides a rectangle has.
- Have students point out rectangles around the room. Ask if they know of anything that is in the shape of a rectangle that is in the school? (the school my be rectangular, gym, or classrooms)

EXPLORE:

- Ask students if all rectangles are the same size
- Let them answer
- Have students draw a picture either of themselves or of an animal
- When drawing they can only use rectangles, circles, and triangles.
- They also have to have at least 3 triangles, 3 circles, and 3 rectangles in their drawing.

EXPLAIN:

- When everyone is done with their drawing, have them switch their drawings with the person across from them.
- Have the students count how many of each shape their partner used
- Have students stand up one by one and explain the different shapes their partner used.
- If you see a shape that isn't one of the three, we were working on, ask them what shape it is and why they used it.

ELABORATE:

- Hand out bingo boards and chips to put on top to each group.
- Tell students that you are going to be just saying the name of the shape and they have to find one of them on their board to cover up.
- Ask students what shapes they recognize on the board.
- Play the game until students at least 4 students get a bingo.

EVALUATE:

- Ask students what a rectangle is? How can you tell?
- Let them answer
- Ask them what the shapes are that we have learned so far are.
- Let them answer
- Now that you know what ~~a square~~ rect. is and what it looks like, can you tell me things that you have seen that are ~~squares~~? Why do you think that they are in this shape instead of another shape? Can you think of anything that is in the shape of a square that you don't think should be? Explain why you think this. Can you tell me anything that has all three shapes we have learned so far in it? (Pizza in a box) ✓

Lesson Materials:

- Drawing paper
- Pencils
- Colored pencils
- Bingo board
- Bingo chips

Classroom Management Needs:

- For this lesson I will be walking around the room making sure all of the students are staying on task and that their voices don't get too loud. When they are sharing about their friends' projects, I will make sure that they are saying nice things. Also, when we are playing the bingo game, I will be walking around making sure that none of the students are cheating,

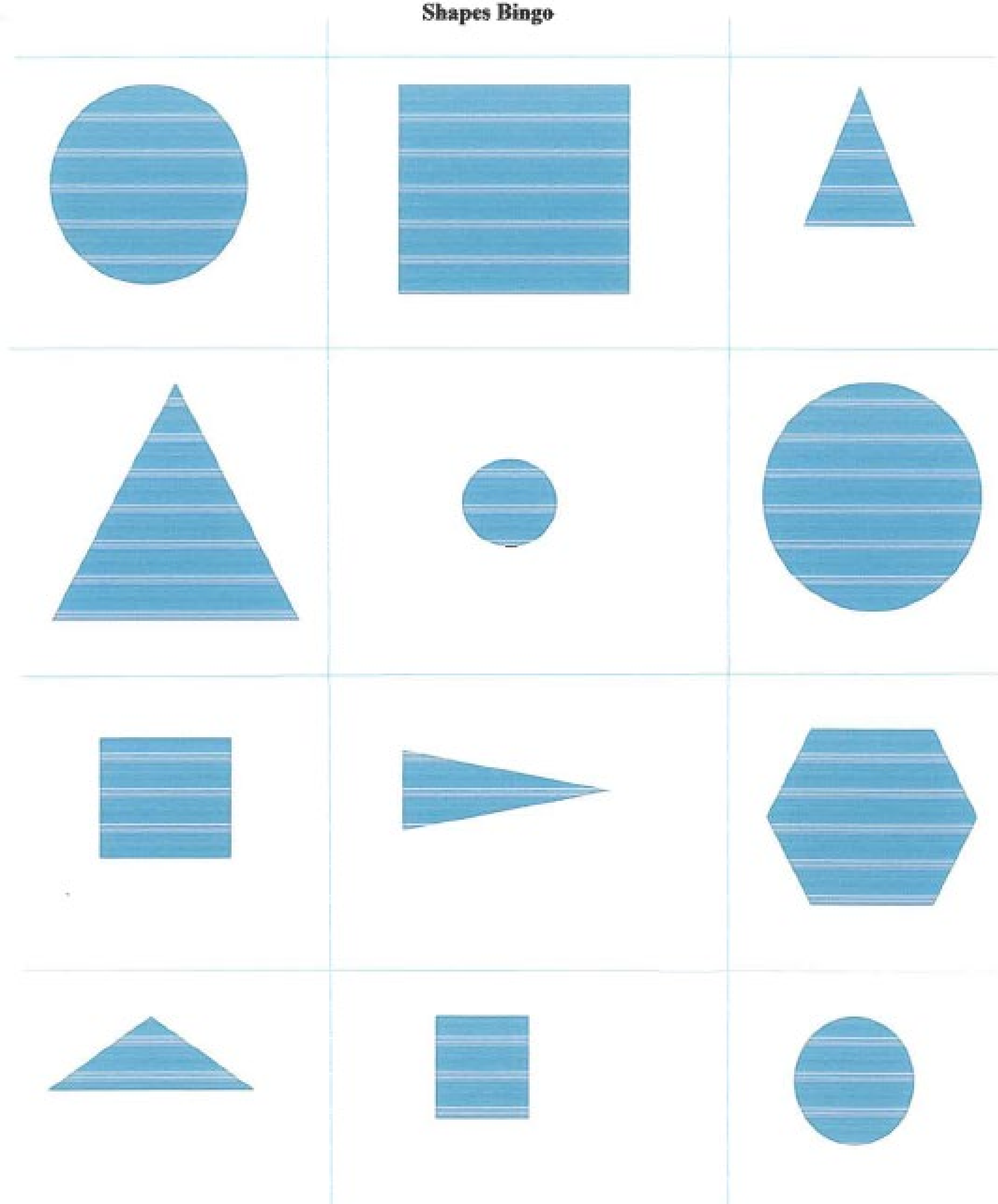
Shapes Bingo Caller Card

Triangle

Circle

Rectangle **Square**

Shapes Bingo



	I can do this, but I need more practice	I'm almost there	I did it!	I went above and beyond
Application of Concepts	With assistance, I can successfully name one of the shapes we have learned.	I can successfully name two of the shapes we have learned.	I can successfully name all three of the shapes we have learned (circles, triangles, and rectangles) when they are all the same size.	I can successfully name all three of the shapes we have learned (circles, triangles, and rectangles) when they are all different sizes.
Explanation of Concepts	I can generally explain what the three shapes (circles, triangles, and rectangles) look like when someone is prompting my thoughts.	I can explain what in detail what one of the shapes looks like.	I can explain in detail what two of the shapes look like.	I can clearly explain in detail what all three of the shapes look like and how many sides each have.
Mathematical Processes	I can count the numbers of sides of one of the shapes if someone helps me.	I can count the numbers of the sides on two of the shapes, but still need some help.	I can successfully count the numbers of sides on two of the shapes by myself.	I can successfully count the numbers of sides on all three of the shapes we have learned about without any help.

Narrative

Unit Goal

- My overall goal for this lesson is that in three days, students will be able to understand, identify, and recognize different two-dimensional shapes. My hope is that students will be able to learn and name the three shapes I have chosen for them. I will need a few more days to teach the rest of the main shapes to the students. I will be asking the students to tell me what they know about the shapes that we are learning about before we get started. I will also be asking them to explain what they do to me so I can see if they understand or not.

Unit Sequence

- I chose this sequence in my lessons so that the students are learning shapes that are easier first. First, they will be learning about the circle which has one side. Second, they will be learning about a triangle which has three sides. Third, they will be learning about a square which has four sides. The lessons build on one another by progressively getting more details about them. I will also be focusing on reviewing the shapes we have previously learned at the end of each lesson. They will usually be tied into one of the projects. I have done this so that the students don't forget what they have learned in the previous days. I will be leaving the harder shapes like rectangles and hexagons for the next couple days because they are harder to understand than theses ones I have talked about. I have designed these lessons so that the students are doing activities or crafts while learning about the shapes. I have don't this so that the students are staying involved and being hands on. This may help eliminate poor behavior and students getting bored.

Assessment Rubric

- The evidence of learning that I will be looking for in these three lessons is that students are able to name which shape is which and explain why what the shape looks like. I will also be looking for the students to remember the shapes we have learned about the previous days while we are doing our refresher. I will be wanting the students to be able to identify the shape even when it is a different size. I will be looking for the students to point out which shape is which when they are among other shapes. Also, I will be looking for the students to describe specific features of each shape like how many sides it has and the length of the sides. I will distinguish between the students who are comprehending the concepts and those who are not by how well they are able to tell the shapes apart. I will also be able to tell when students are either able or not able to name the shape when it is a different size.

Mathematical Practice Standards

- This unit supports the mathematical practice standards (K.MP.6.) Attend to Precision and (K.MP.4.) Model with Mathematics. The unit supports these two standards by having the students represent their shapes using different materials such as with pretzels and drawings. This unit is also having the students use their communication skills to explain what the shape is and what it looks like.

References

Office of Public Instruction. (2011). *K-12 Content Standards & Revisions- Kindergarten Mathematics*. Retrieved from http://opi.mt.gov/Educators/Teaching-Learning/K-12-Content-Standards-Revision

Pinterest (2019) *Kindergarten Math Activities*. Retrieved from https://www.pinterest.com/search/pins/?rs=ac&len=2&q=kindergarten%20math%20activities&eq=kindergarten

Unit Plan (1st)

First Draft

COE Lesson Plan

Lesson Teacher:

Date: 26 November 2019

Lesson Grade Level: 1st

Timeframe: 10:00-10:30

0Content Area: Math

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have prior knowledge of adding and subtracting within 20. This will be the student's first introduction to the concept of fact families. Students seem to be doing well with adding within 20, but there are several students who have been struggling with subtraction.

2. **How will you design the lesson to meet the needs of all learners in your classroom?** Various manipulatives will be provided for students to help them visually see how fact families are represented. I anticipate most students will need to use these as this is the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (i.e. 9-____=4 or ____+12=15)

Lesson Plan Development

Lesson Title: Fact Families	
Common Core and/or State Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6) **Practice Standard:** 1.MP.1. Make sense of problems and persevere solving them. **Practice Standard:** 1.MP.5. Use appropriate tools strategically. **Practice Standard:** 1.MP.7. Look for and make use of structure.	
Lesson Objective: Students will identify missing portions of a problem set.	**Assessment of Learning:** Students will be given an exit ticket that have half a problem set. Students will be asked to add the missing half of the problems.
Lesson Objective: Students will explain how many subtraction and addition problems problem sets usually contain.	**Assessment of Learning:** Teacher observation during lesson. I will be looking for an answer such as 'I notice that problem sets usually have 2 subtraction and 2 addition problems.'

Based on the lesson objectives, select an appropriate teaching model: 5 E's		
Indian Education For All (IEFA) No Yes. If yes, please describe		
Lesson Procedures/Activities	**Materials**	**Classroom Management Needs**
Engage/ Explore: For this portion of the lesson, I will write a fact family on the board (such as 3+8=11, 8+3=11, 11-8=3, and 11-3=8). I will then ask students to think about the problems for a few minutes (1-2). After the students think about the problems individually, students will be asked to turn to their elbow partners and discuss with them for another 1-2 minutes what they think the problems on the board have in common.	Whiteboard Sample Problems	Students should initially be seated at their desks, thinking about the problems on the board. Students may then talk quietly to their elbow partners about what the problems on the board have in common. The teacher will circulate amongst students to make sure they are not digressing into conversations about unrelated topics.
Explain: After students are given the opportunity to discuss the relationship between the facts on the board, I will call students' attention back to me and steer students toward a group discussion about what they think the problems on the board have in common. If I notice that students are struggling to come up with ideas about what the problems on the board have in common, I will ask students what they noticed and wondered about them. If this doesn't prompt any further answers, I will ask students what assumptions we could make about the problems.	Whiteboard Sample Problems	Students should be seated at their desks, raising their hands to answer questions asked by the teacher and waiting until they are called on to share their ideas.
Elaborate: After we discuss the problems that are on the whiteboard, I will write half a set of problems on the board. I will ask students to complete to think about the missing 2 problems and what they could be. I will ask students to give me a thumbs up against their chests when they have an answer. If I notice that students are struggling to come up with the missing problems, I will ask them the question "Where have you seen something like this before?" When all students have an answer, I will ask one student to come up to the board and write one of the missing problems and a second students to come up to the board and write the second missing problem. After the missing problems are written on the board, I will ask students if they all agree with the answers that were given. Why or why not? If someone says one of the answers is incorrect, I will ask the student what his/her answer would be and what method they used to come up with that answer. This will help me hear any discrepancies that students may have about the problems and help me correct them before we move into working with more fact families in the next lesson. After the problem set	Whiteboard Expo Marker	Students will be asked to sit at their desks while thinking about what the answers to the missing portions of the problem set on the board might be. Students should give the teacher a thumbs up against their chests when they have an answer and when all students are ready to share an answer, students should raise their hands if they wish to come write the answers of the board.

on the board is complete, I will ask students what they notice about the problem sets. I am looking for an answer such as "I notice that the problem sets all have 2 addition and 2 subtraction problems.'		
Evaluate: Using an exit ticket during the last 5 or so minutes of the lesson, students will be asked to fill in the missing portions of a problem set on an exit ticket so that the teacher can get an idea of whether or not students did or did not meet the objective of the lesson. Students will also be asked to rate themselves on how they are feeling about problem sets: red, meaning they don't get it at all, yellow meaning they sort of get the concept, and green meaning they are comfortable with the concept and think they are ready to move. I would expect a mixture of yellow and red ratings.	Exit Ticket Pencil Offices (if needed)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.
Evidence of Lesson Effectiveness/Student Learning: n/a **Reflection and Recommendations for Next Time:** n/a Attachments, if required.		

Exit Ticket

Name ______________________

Please complete the missing portions of the problem set below.

4+8=12

12-4=8

How do you feel about the concept of problem sets?

RED: I don't get this at all	YELLOW: I sort of get this	GREEN: I understand the concept and am ready to move on

COE Lesson Plan

Lesson Teacher:

Date: 26 Nov. 2019

Lesson Grade Level: 1st

Timeframe: 10:00-10:30

Content Area: Math

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have prior knowledge of adding and subtracting within 20. Students seem to be doing well with adding within 20, but there are several students who have been struggling with subtraction. In the previous lesson, students were introduced to the idea of fact families (called problems sets in the previous lesson), and most students rated themselves as red or yellow on their exit tickets.

2. **How will you design the lesson to meet the needs of all learners in your classroom?** Various manipulatives will be provided for students to help them visually see how fact families are represented. I anticipate most students will need to use these as this is the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (i.e. 9-____=4 or ____+12=15)

Lesson Plan Development

Lesson Title: Fact Families	
Common Core and/or State Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6) **Practice Standard:** 1.MP.1. Make sense of problems and persevere solving them. **Practice Standard:** 1.MP.5. Use appropriate tools strategically. **Practice Standard:** 1.MP.7. Look for and make use of structure.	
Lesson Objective: Students will use manipulatives to model 2 fact families of their choosing.	**Assessment of Learning:** Teacher observation during lesson
Lesson Objective: Students will explain what a fact family is.	**Assessment of Learning:** Students will be given an exit ticket at the end of the lesson to respond to the question: what is a fact family? An acceptable answer would be something like 'A fact family is a group of 3 numbers that are used in related equations.'

Based on the lesson objectives, select an appropriate teaching model: 5 E's		
Indian Education For All (IEFA) No Yes. If yes, please describe		
Lesson Procedures/Activities	**Materials**	**Classroom Management Needs**
Engage: For this portion of the lesson, we will do a brief review from yesterday's lesson with the intention of invoking prior knowledge. I will write half a problem set on the board and ask students to complete it. I will give students a couple of moments to ponder the answer to the question and then ask students to share their answers.	Whiteboard Expo Marker	Students should be seated at their desks thinking about the problem on the board in front of them. Students should signal to the teacher that they are ready to share an answer by putting a thumbs up against their chest. When everyone is ready to share an answer, the teacher will call on 2 students to tell the teacher the missing equation (1 student per equation).
Explore: Students will be given some manipulatives and will then be given 3-4 minutes to play with them and become familiar with the manipulatives. While students are becoming familiar with their manipulatives, I will write a couple of problems on the board. Attention will be called back to the board and students will then be asked to think of a way to model the equations on the board with their manipulatives. Students will be given a couple of minutes to do this individually and then may turn to their elbow partners and have a quite conversation about how they chose to model the equations.	Manipulatives (Base 10 blocks, snap cubes, etc.) White board Expo Marker	Students should be seated at their desks becoming familiar with their manipulatives. Once attention is called back to the board and instructions are given, students should work quietly for a few minutes to try and model a fact family by themselves, and then they may talk to their elbow partners using inside voices.
Explain: After giving students the opportunity to discuss how they modeled their problems with the manipulatives, I will steer students into a discussion about the different ways that were presented. After we have discussed the different ways that students have discovered, I will write the phrases 'fact family' and 'problem set' on the board. I will ask students to think about each of these terms for a brief moment. I anticipate students will not know what the term fact family means, so I will ask them questions such as 'What do you already know that might be helpful here?' and 'What assumptions might we make?' After allowing students a few minutes to think about these phrases individually, I will allow students to turn and talk to their elbow partners briefly. I will		Students should be seated at their desks, unless they are explaining how they used their manipulatives to model a fact family. If they are modeling a fact family, they may come show it on the projector. After the class discussion, students should be

call attention back to me and ask students what they think a fact family might be. I'm hoping to hear ideas from students similar to 'A fact family is a set of numbers that helps us with addition and subtraction facts.' I imagine getting to this point will take some prompting, so I will ask students questions such as 'What is the same and what is different?' or 'This is just a special case of what?'		thinking about the phrases on the board, raising their hands if they have questions.
Elaborate: After having our group discussion in the previous section, I will write the same fact family under each phrase on the board. I will then ask students to model the fact family using their manipulatives. After giving students about 5 minutes to complete this task, I will call attention back to me and ask students what they have noticed while they were creating their model of the fact family. If students are struggling to find something similar between the 2 models, I will ask them questions such as 'Where have you seen something like this before?' and 'What is the same and what is different between these 2?'. It is my hope that while students were creating their models, they discovered that a problem set and a fact family are the same thing.	Manipulatives White board Expo Marker	Students should be working independently on their model of the fact families. Once we come back to our group discussion, students should raise their hands to share their ideas and wait until they are called on to share them with the class.
Evaluate: In the last 5 minutes of the lesson, students will be given an exit ticket and asked to define in their own words what a fact family is. I would expect an answer such as 'A fact family is a group of numbers that helps us figure out addition and subtraction facts' or something similar from a first grader.	Exit Ticket Pencil Offices (if needed)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.
Evidence of Lesson Effectiveness/Student Learning: n/a **Reflection and Recommendations for Next Time:** n/a Attachments, if required.		

Exit Ticket

Name ______________________

Using your own words, tell me what a fact family is.

____
____
____
____
____
____
____
____

COE Lesson Plan

Lesson Teacher:

Date: 26 Nov. 2019

Lesson Grade Level: 1st

Timeframe: 10:00-10:30

Content Area: Math

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have spent the last 2 days working with fact families. Students appear to have grown fairly comfortable with the idea of fact families. Students also have prior knowledge of adding and subtracting within 20.

2. **How will you design the lesson to meet the needs of all learners in your classroom?** Various manipulatives will be provided for students to help them visually see how fact families are represented. I anticipate most students will need to use these as this is the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (i.e. 9-____=4 or ____+12=15)

Lesson Plan Development

<table>
<tr><td colspan="2">Lesson Title: Fact Families</td></tr>
<tr><td colspan="2">Common Core and/or State Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)

Practice Standard: 1.MP.1. Make sense of problems and persevere solving them.
Practice Standard: 1.MP.5. Use appropriate tools strategically.
Practice Standard: 1.MP.7. Look for and make use of structure.</td></tr>
<tr><td>Lesson Objective: Students will create their own fact family.</td><td>Assessment of Learning: Students will be given an exit ticket at the end of the lesson. Students will be expected to pick a number of 3-20 and write the complete fact family for it.</td></tr>
<tr><td>Lesson Objective: Students will be asked to explain how a number can be broken up.</td><td>Assessment of Learning: Teacher observation during lesson. During our group discussion about how to break up the number 13, I would expect to hear answers such as '13 can be broken in 9+4 or 8+5'.</td></tr>
<tr><td colspan="2">Based on the lesson objectives, select an appropriate teaching model: 5 E's</td></tr>
<tr><td colspan="2">Indian Education For All (IEFA) No Yes. If yes, please describe</td></tr>
</table>

Lesson Procedures/Activities	Materials	Classroom Management Needs
Engage: For this portion of the lesson, I will draw on students' prior knowledge of fact families by asking them to explain what a fact family is. I will also ask for a student volunteer or two to help show me how to represent a fact family using the manipulatives we worked with in the previous lesson.	Manipulatives Projection Device	Students should be seated at their desks thinking about how to describe a fact family. Those who wish to share an answer need to raise their hands and wait until they are called on to share their answers.
Explore: I will write number on the board. Students will be asked to think about how this number can be broken up. (e.g. 13, can be broken into 10+3 or 4+9) I will ask students to think about this individually for a few moments and then turn around and talk to their elbow partners about how they broke up the number. Allowing students to talk to their elbows partners is done with the intention of students helping each other come up with various ways to break up the number 13, rather than the teacher telling them.	Whiteboard Expo Marker	Students should be seated at their desks thinking about how they can break up the number 13. Once released to talk to their elbow partners, students should be chatting using their indoor voices. The teacher will circulate around the room a couple of times to make sure that the students are not digressing into conversations about unrelated topics.
Explain: After students have been given the opportunity to discuss how they broke up the number, I will call attention back to the number on the board and ask students to share how they broke the number up. As students are sharing their ideas, I will write each idea on the board. If I notice students who are struggling, I will prompt them with questions such as 'How could you simplify this problem?' or 'Where might you have seen something like this before?' There are multiple ways to break up the number 13, so for the sake of time, I would stop students after we had 6-7 different ways of breaking up 13.	Whiteboard Expo Marker	Students should be seated at their desks, raising their hands to ask questions or share how they broke up the number 13.
Elaborate: After our group discussion, we will have multiple ways of breaking up the number 13. Students will be asked to pick two of the possible ways and use these to create fact families. If students are struggling, they will be encouraged to use the available manipulatives and I will also prompt them with questions such as 'What is known and what is unknown?' and 'What do you already know that might be useful here?' Students will be given a few minutes to complete the task individually and then they may turn to their elbow partners to discuss the fact family they have chosen to create. This will hopefully allow the students who have not completed their fact families to get some ideas from	Manipulatives (if needed) Expo Marker Whiteboard	Students should be seated at their desks working individually until the teacher tells them to turn and talk with their elbow partners. Once elbow partner chats are allowed to begin, students should be using their indoor voices. One we begin

their peers about how they could possibly complete it. After 5-6 minutes, I will call attention back to the board and go through and complete the fact family for each broken up number on the board. Students will be asked to supply the answers for this portion, as this will help me hear any discrepancies in the answers or errors in how the students are thinking that I can correct.		our group discussion, students will be asked to raise their hands to share their thoughts/ideas and will be asked to wait to speak until they are called on.
Evaluate: In the last 5 minutes of the lesson, students will be given an exit ticket to respond to the following prompt: Please pick a number between 3 and 20 and create a fact family for it. I would expect an answer such as 15. 8+7=15, 7+8=15, 15-8=7, 15-7=8	Exit ticket Pencil Offices (if necessary)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.
Evidence of Lesson Effectiveness/Student Learning: n/a **Reflection and Recommendations for Next Time:** n/a Attachments, if required.		

Exit Ticket

Name ____________________

Please pick a number between 3 and 20. Using the number you picked, create a fact family for it.

Unit Plan (1^{st})

Final Draft

Student 8

Fact Families

Unit Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., **8** + 6 = **8** + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 +4 = 12, one knows 12 - **8** = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6)

Unit Goal: The overall goal of this unit is to help students become familiar with fact families. This concept is an essential building block to many different mathematical approaches, and if students do not have a strong foundation with this particular idea, these other approaches may be very difficult for students to understand. My hope is that by the time we are done with these 3 lessons, the students have completely grasped the concept of fact families and understand how they can be used to help students find unknown addition and subtraction equations as we continue on through the chapter. The most essential question I want students to be able to answer by the time we are done with this unit is "How can fact families help me with my addition and subtraction facts?" Students who have a good understanding of this concept will also have a strong foundation to build on when multiplication and division concepts are introduced later in 3^{rd} grade.

A. Unit Goal

The overall goal ofthis unit is to help students become familiar with fact families. This concept is an essential building block to many different mathematical approaches, and if students do not have a strong foundation with this particular idea, these other approaches may be very difficult for students to understand. My hope is that by the time we are done with these 3 lessons, the students have completely grasped the concept of fact families and understand how they can be used to help students find unknown addition and subtraction equations as we continue on through the chapter. The most essential question I want students to be able to answer by the time we are done with this unit is "How can fact families help me with my addition and subtraction facts?" Students who have a good understanding of this concept will also have a strong foundation to build on when multiplication and division concepts are introduced later in 3rd grade.

B. Unit Sequence

In my first lesson, students will become familiar with the idea of fact families and what they usually look like. However, the term 'fact family' will not be introduced to students until the second lesson. This is done intentionally so that students do not become too overwhelmed on the first day of the lesson. During the second lesson, we will continue working with fact families so that students are as familiar with fact families as possible. The second lesson will also introduce manipulatives for students to practice modeling fact families with. During the third lesson, students will be asked to demonstrate their knowledge of fact families by using manipulatives and also by writing them out. I structured the lessons like this because I feel like the first lesson is the most basic concept/building block of the 3 lessons, so to me, it made sense to start here. I didn't want to overwhelm students with too much terminology the first day, so that's why I waited until the second day to introduce the term 'fact families'. In doing this, this also helps students put a name to what they were learning the first day, instead of just introducing some random term that they had no prior knowledge of and would have just dismissed had I told them this term at the beginning of the first lesson.

C. Assessment Rubric

	I can do this, but I need more practice	**I'm almost there**	**I did it!**	**I went above and beyond**
Application of Concepts	With assistance, I can successfully use fact families to solve addition and subtraction	With very little assistance, I can successfully use fact families to solve addition and	With no assistance, I can successfully use fact families to solve addition and	I can successfully use fact families to help me solve problems that are missing numbers

	problems.	subtraction problems.	subtraction families.	(i.e. using a fact family to find the answer to 8-_=3)
Explanation of Concepts	I can explain how to use fact families to solve addition and subtraction problems when someone prompts me to begin and helps me finish my thoughts.	I can start an explanation of using fact families to solve addition and subtraction problems but need someone else's help to completely articulate my thoughts.	I can clearly and completely explain that a fact family is a set of 3 numbers that are used to make 4 mathematics facts.	I can clearly and completely explain that a fact family is a set of 3 numbers that are used to make 4 mathematics facts **AND** that these 4 facts are 2 addition facts and 2 subtraction facts.
Representation of Concepts	I can use fact families to solve addition and subtraction problems using my available manipulatives when someone helps me.	I can use fact families to solve addition and subtraction problems, but I need help choosing an appropriate manipulative to help me solve the problem.	I can accurately represent how to solve addition and subtraction problems using fact families using **one** manipulative. Base 10 blocks Snap Cubes Hand-drawn pictures Counters	I can accurately represent how to solve addition and subtraction problems using fact families using **two or more** manipulatives. Base 10 blocks Snap Cubes Hand-drawn pictures Counters
Mathematical Processes	I can solve and represent problems using fact families when someone helps me.	I can solve and represent problems using fact families but often make mistakes unless someone helps me.	I can solve and represent most problems using fact families with only a few occasional errors.	I can solve and represent all problems using fact families without any errors.

D. Mathematical Practice Standards

1. MP.I. Make sense of problems and persevere solving them.

Students should begin to recognize that doing math means solving various equations and problems and talking about how they solved a problem. Students should be willing to try different approaches to solving a problem if their initial approach resulted in an incorrect answer or didn't make sense to the student.

I.MP.5. Use appropriate tools strategically.

Students should begin to use the manipulatives/tools available to them to help solve the problem that they are working on. Students should also begin to determine what manipulatives are appropriate to use with the problem they are working on.

1. MP.7. Look for and make use of structure.

Students should begin to recognize different patterns and structures in math problems, meaning that if students see a fact family, they should recognize the pattern that fact families usually contain 2 addition problems and 2 subtraction problems.

E. References

1. Standards for Mathematical Practice: Grade 1 Explanations and Examples. (n.d.). Retrieved from https://opi.mt.gov/LinkClick.aspx?fileticket=flE0iJHneH4=&portalid=182.

COE Lesson Plan

Lesson Teacher:

Date: 26 November 2019

Lesson Grade Level: 1st

Timeframe: 10:00-10:30

0Content Area: Math

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have prior knowledge of adding and subtracting within 20. This will be the student's first introduction to the concept of fact families. Students seem to be doing well with adding within 20, but there are several students who have been struggling with subtraction. One issue I foresee is that students may write the fact families incorrectly, mixing up where the numbers go (i.e. 3-12=9). Therefore, I will need to make sure that I tell students that the largest number of the fact family needs to be on the righthand side' of the equal sign.

2. **How will you design the lesson to meet the needs of all learners in your classroom?** Various manipulatives will be provided for student to help them visually se how fact famiies are represented. I anticipate most students will need to use these as this 1s the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (i.e. 9-___=4 or_____+12=15)

Lesson Plan Development

Lesson Title: Fact Families	
Common Core and/or State Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 4 = 13 - 3 - 1 = 10 - 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6) **Practice Standard:** 1.MP.1. Make sense of problems and persevere solving them. **Practice Standard:** 1.MP.5. Use appropriate tools strategically. **Practice Standard:** 1.MP.7. Look for and make use of structure.	
Lesson Objective: Students will identify the missing equations of a fact family.	**Assessment of Learning:** Students will be given an exit ticket that have half a fact family. Students will be asked to add the missing half of the problems.
Lesson Objective: Students will explain how many subtraction and addition problems fact families usually contain.	**Assessment of Learning:** Teacher observation during lesson. I will be looking for an answer such as 'I notice that fact families usually have 2 subtraction and 2 addition problems.,'

Based on the lesson objectives, select an appropriate teaching model: 5 E's

'ndian Education For All (IEFA) No Yes. If yes, please describe

Lesson Procedures/Activities	Materials	Classroom M anag ent Ne ds
Engage/ Explore: For this portion of the lesson, I will write a For this portion of the lesson, I will write a fact family on the board (such as 3+8=11, 8+3=11, 11-8=3, and 11-3=8). I will then ask students to think about the problems for a few minutes (1-2). After the students think about the problems individually, students will be asked to turn to their elbow partners and discuss with them for another 1-2 minutes what they think the problems on the board have in common.	Whiteboard Sample Problems	Students should initially be seated at their desks, thinking about the problems on the board. Students may then talk quietly to their elbow partners about what the problems on the board have in common. The teacher will circulate amongst students to make sure they are not digressing into conversations about unrelated topics.
Explain: After students are given the opportunity to discuss the relationship between the facts on the board, I will call students' attention back to me and steer students toward a group discussion about what they think the problems on the board have in common. If I notice that students are struggling to come up with ideas about what the problems on the board have in common, I will ask students what they noticed and wondered about them. If this doesn't prompt any further answers, I will ask students what assumptions we could make about the problems.	Whiteboard Sample Problems	Students should be seated at their desks, raising their hands to answer questions asked by the teacher and waiting until they are called on to share t heir ideas.
Elaborate: After we discuss the problems that are on the whiteboard, I will write half a set of problems on the board. I will ask students to complete to think about the missing 2 problems and what they could be. I will ask students to give me a thumbs up against their chests when they have an answer. If I notice that students are struggling to come up with the missing problems, I will ask them the question "Where have you seen something like this before?" When all students have an answer, I will ask one student to come up to the board and write one of the missing problems and a second student to come up to the board and write the second missing problem. After the missing problems are written on the board, I will ask students if they all agree with the answers that were given. Why or why not? If someone says one of the answers is incorrect, I **will** ask the student what his/her answer would be and what method they used to come up with that answer. This will help me hear any discrepancies that students may have about the problems and help me correct them before we move into working with more fact families in the next lesson. After the fact family on the board is complete, I will ask students what they notice about the fact family.: I am looking for an	Whiteboard Expo Marker	Students will be asked to sit at their desks while thinking about what the answers to the missing portions of the fact family on the board might be. Students should give the teacher a thumbs up against their chests when they have an answer and when all students are ready to share an answer, students should raise their hands if they wish to come write the answers of the board.

answer such as "I notice that the fact families all have 2 addition and 2 subtraction problems.'		
Evaluate: Using an exit ticket during the last 5 or so minutes of the lesson, students will be asked to fill in the missing portions of a fact family on an exit ticket so that the teacher can get an idea of whether or not students did or did not meet the objective of the lesson. Students will also be asked to rate themselves on how they are feeling about problem sets: red, meaning they don't get it at all, yellow meaning they sort of get the concept, and green meaning they are comfortable with the concept and think they are ready to move. I would expect a mixture of yellow and red ratings.	Exit Ticket Pencil Offices (if needed)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.
Evidence of Lesson Effectiveness/Student Learning: n/a **Reflection and Recommendations for Next Time:** n/a Attachments, if required.		

Exit Ticket

Name__________

Please complete the missing portions of the fact family below.

4+8=12

12-4=8

How do you feel about the concept of fact families?

RED: I don't get this at all

YELLOW: I sort of get this

GREEN: I understand the concept and am ready to move on

COE Lesson Plan

Lesson Teacher: Lesson Grade | **Date:** 26 Nov. 2019

Level: 1st **Content Area:** Math | **Timeframe:** 10:00-10:30

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have prior knowledge of adding and subtracting within 20. Students seem to be doing well with adding within 20, but there are several students who have been struggling with subtraction. In the previous lesson, students were introduced to the idea of fact families and most students rated themselves as red or yellow on their exit tickets.
2. **How will you design the lesson to meet the needs of all learners in your classroom?** Various manipulatives will be provided for students to help them visually see how fact families are represented. I anticipate most students will need to use these as this is the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (~~i.e.~~ 9-___ =4 or ___ +12=15)

Lesson Plan Development

Lesson Title: Fact Families	
Common Core and/or State Standard: Add and subtract within 20 demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on, making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14), decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9), using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4) and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (1.OA.6) **Practice Standard:** 1.MP.1. Make sense of problems and persevere solving ~~them~~. **Practice Standard:** 1.MP.5. Use appropriate tools strategically. **Practice Standard:** 1.MP.7. Look for and make use of structure.	
Lesson Objective: Students will use manipulatives to model 2 fact families of their choosing.	**Assessment of Learning:** Teacher observation during lesson
Lesson Objective: Students will explain what a fact family is.	**Assessment of Learning:** Students will be given an exit ticket at the end of the lesson to respond to the question: what is a fact family? An acceptable answer would be something like "a fact family is a family is a group of 3 numbers that are used in related equations.'

Based on the lesson objectives, select an appropriate teaching model: 5 E's		
Indian Education For All (IEFA) No Yes. If yes, please describe		
Lesson Procedures/Activities	**Materials**	**Classroom Management Need**
Engage: For this portion of the lesson, we will do a brief review from yesterday's lesson with the intention of invoking prior knowledge. I will write If a problem set on the board and ask students to complete it. I will give students a couple of moments to ponder the answer to the question and then ask students to share their answers.	Whiteboard Expo Marker	Students should be seated at their desks thinking about the problem on the board in front of them. Students should signal to the teacher that they are ready to share an answer by putting a thumbs up against their chest. When everyone is ready to share an answer, the teacher will call on 2 students to tell the teacher the missing equation (1 student per equation).
Explore: Students will be given some manipulatives (I think base 10 blocks or snap cubes would be best for this activity) and will then be given 3-4 minutes to play with them and become familiar with the manipulatives. While students are becoming familiar with their manipulatives, I will write a couple of problems on the board. Attention will be called back to the board and students will then be asked to think of a way to model the equations on the board with their manipulatives. Students will be given a couple of minutes to do this individually and then may turn to their elbow partners and have a quite conversation about how they chose to model the equations.	Manipulatives (Base 10 blocks, snap cubes, etc.) White board Expo Marker	Students should be seated at their desks becoming familiar with their manipulatives. Once attention is called back to the board and instructions are given, students should work quietly for a few minutes to try and model a fact family by themselves, and then they may talk to their elbow partners using inside voices.
Explain: After giving students the opportunity to discuss how they modeled their problems with the manipulatives, I will have students come up to the projector and have them show us how they chose to model their problems. After allowing about 5 minutes for this activity, I will steer students into a discussion about the different ways that were shown and if there is one way they think would work best. After we have discussed the different ways that students have discovered, I will write the phrase 'fact family' on the board. I will ask students to think about this term for a brief moment. I anticipate students will not know what the term fact family means, so I will ask them questions such as 'What do you already know that might be helpful here?' and 'What assumptions might we make?' After allowing students a few minutes to think about these phrases	Projection device	Students should be seated at their desks, unless they are explaining how they used their manipulatives to model a fact family. If they are modeling a fact family, they may come show it on the projector. After the class discussion, students should be thinking about the

individually, I will allow students to turn and talk to their elbow partners briefly. I will call attention back to me and ask students what they think a fact family might be. I'm hoping to hear ideas from students similar to 'A fact family is a set of numbers that helps us with addition and subtraction facts.' I imagine getting to this point will take some prompting, so I will ask students questions such as 'What is the same and what is different?' or 'This is just a special case of what?'		phrases on the board, raising their hands if they have questions.
Elaborate: After having our group discussion in the previous section, I will write 2 fact families on the board. I will then ask students to model the fact family using their manipulatives. After giving students about 5 minutes to complete this task, I will call attention back to me and ask students what they have noticed while they were creating their model of the fact family. If students are struggling to find something similar between the 2 models, I will ask them questions such as 'Where have you seen something like this before?' and 'What is the same and what is different between these 2?'. After giving students about 5 minutes to complete this activity, I will call attention back to the board and ask students to share how they modeled one of the fact families. I will have a second student tell me how the modeled the second fact family. I will then steer students toward a group discussion about what they noticed while modeling their fact families. I'm hoping to hear answers such as 'Fact families have 2 addition and 2 subtraction problems.', as this will help steer my next questions for students, which will include questions such as 'Do fact families always have 2 addition and 2 subtraction problems? Why or why not?' To help prompt answers to this question, I will write 2-3 more fact families on the board and have students model them with their manipulatives to see what they come up with.	Manipulatives White board Expo Marker How else could you use these #s?	Students should be working independently on their model of the fact families. Once we come back to our group discussion, students should raise their hands to share their ideas and wait until they are called on to share them with the class.
Evaluate: In the last 5 minutes of the lesson, students will be given an exit ticket and asked to define in their own words what a fact family is. I would expect an answer such as 'A fact family is a group of numbers that helps us figure out addition and subtraction facts' or something similar from a first grader.	Exit Ticket Pencil Offices (if needed)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.

Evidence of Lesson Effectiveness/Student Learning: n/a

Reflection and Recommendations for Next Time: n/a

Attachments, if required.

Name ____________________________

Exit Ticket

Using your own words, tell me what a fact family is.

COE Lesson Plan

Lesson Teacher: Lesson Grade Level: 1st **Content Area:** Math

Date: 26 Nov. 2019

Timeframe: 10:00-10:30

Grouping Strategy: Whole Group

Preparing for Lesson Development

1. **What does your pre-assessment observation indicate about your student's needs and current performance and educational needs?** Students have spent the last 2 days working with fact families. Students appear to have grown fairly comfortable with the idea of fact families. Students also have prior knowledge of adding and subtracting within 20.

2. How will you design the lesson to meet the needs of all learners in your classroom? Various manipulatives will be provided for students to help them visually see how fact families are represented. I anticipate most students will need to use these as this is the first time we will be working with fact families. For students who immediately grasp the concept, I will challenge students to solve problems that are missing numbers (i.e. 9-___ =4 or ___ +12=15)

Lesson Plan Development

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Practice Standard: I.MP.I. Make sense of problems and persevere solving them.
Practice Standard: I.MP.S. Use appropriate tools strategically.
Practice Standard: 1.MP.7. Look for and make use of structure.</td></tr>
<tr><td>Lesson Objective: Students will create their own fact family.</td><td>Assessment of Learning: Students will be given an exit ticket at the end of the lesson. Students will be expected to pick a number of 3-20 and write the complete fact family for it.</td></tr>
<tr><td>Lesson Objective: Students will be asked to explain how a number can be broken up.</td><td>Assessment of Learning: Teacher observation during lesson. During our group discussion about how to break up the number 13, I would expect to hear answers such as '13 can be broken in 9+4 or 8+5'.</td></tr>
<tr><td colspan="2">Based on the lesson objectives, select an appropriate teaching model: S E's</td></tr>
<tr><td colspan="2">Indian Education For All (IEFA) No Yes. If yes, please describe</td></tr>
<tr><td>Lesson Proct! ι es/Activities</td><td>Materials | Classroom</td></tr>
</table>

		Management Needs
Engage: For this portion of the lesson, I will draw on students' prior knowledge of fact families by asking them to explain what a fact family is. I will also ask for a student volunteer or two to help show me how to represent a fact family using the manipulatives we worked with in the previous lesson.	Manipulatives Projection Device	Students should be seated at their desks thinking about how to describe a fact family. Those who wish to share an answer need to raise their hands and wait until they are called on to share their answers.
Explore: I will write number on the board. Students will be asked to think about how this number can be broken up. (e.g. 13, can be broken into 10+3 or 4+9) I will ask students to think about this individually for a few moments and then turn around and talk to their elbow partners about how they broke up the number. Allowing students to talk to their elbows partners is done with the intention of students helping each other come up with various ways to break up the number 13, rather than the teacher telling them.	Whiteboard Expo Marker	Students should be seated at their desks thinking about how they can break up the number 13. Once released to talk to their elbow partners, students should be chatting using their indoor voices. The teacher will circulate around the room a couple of times to make sure that the students are not digressing into conversations about unrelated topics.
Explain: After students have been given the opportunity to discuss how they broke up the number, I will call attention back to the number on the board and ask students to share how they broke the number up. As students are sharing their ideas, I will write each idea on the board. If I notice students who are struggling, I will prompt them with questions such as 'How could you simplify this problem?' or 'Where might you have seen something like this before?' There are multiple ways to break up the number 13, so for the sake of time, I would stop students after we had 6-7 different ways of breaking up 13.	Whiteboard Expo Marker	Students should be seated at their desks, raising their hands to ask questions or share how they broke up the number 13.
Elaborate: After our group discussion, we will have multiple ways of breaking up the number 13. Students will be asked to pick two of the possible ways and use these to create fact families. If students are struggling, they will be encouraged to use the available manipulatives and I will also prompt them with questions such as 'What is known and what is unknown?' and 'What do you already know that might be useful here?' Students will be given a few minutes to complete the task individually and then they may turn to their elbow partners to discuss the fact family they have chosen to create. This will hopefully allow the students who have not completed their fact families to get some ideas from their peers about how they could possibly complete it. After 5-6 minutes, I will call attention back to the board and go through and complete the fact family for each broken up number on the board. Students	Manipulatives (if needed) Expo Marker Whiteboard	Students should be seated at their desks working individually until the teacher tells them to turn and talk with their elbow partners. Once elbow partner chats are allowed to begin, students should be using their indoor voices. One we begin our group discussion, students will be asked to raise their hands to

will be asked to supply the answers for this portion, as this will help me hear any discrepancies in the answers or errors in how the students are thinking that I can correct.		share their thoughts/ideas and will be asked to wait to speak until they are called on
Evaluate: In the last 5 minutes of the lesson, students will be given an exit ticket to respond to the following prompt: Please pick a number between 3 and 20 and create a fact family for it. I would expect an answer such as 15. 8+7=15, 7+8=15, 15-8=7, 15-7=8	Exit ticket Pencil Offices (if necessary)	Students will be asked to complete their exit ticket in silence. If necessary, students will be given offices to ensure that the data from the assessment is accurate.

Evidence of Lesson Effectiveness/Student Learning: n/a

Reflection and Recommendations for Next Time: n/a

Attachments, if required.

Exit Ticket

Name ____________

Please pick a number between 3 and 20. Using the number you picked, create a fact family for it.

Unit Plan (4^{th})

First Draft

Unit Lesson Plans

Teacher:	**Lesson Date:**
Grade Level: 4th grade	**Timeframe:** (Length of lesson)
Content Area: Math	**Grouping Strategy:** Small group/whole class

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.OA.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson higher learners will have the opportunity to work with larger numbers beyond the general class assignment. This will allow them to apply the foundation of the concept in a more challenging way. For lower floor learners the activity will have multiple points of peer interaction and instructor explanation and support.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Introduction/ 1-10	
Common Core and/or State Standard(s): 4.OA.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify numbers between 1-10 that are prime or composite.	**Assessment of Learning:** Students will demonstrate this by completing the attached Identification page and submitting it, with Blue Pen Corrections, if need be, as an exit ticket.

Lesson Objective: Students will be able to define the mathematical terms prime and composite.	**Assessment of Learning:** This will be assessed through informal assessment.
Relevant Vocabulary: Prime, Composite, Factors	
Teaching Model:	
Indian Education for All (IEFA) _X_ No___Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:	
Lesson Procedures/Activities: ENGAGE: To begin the lesson students will be asked to make even groups out of the numbers 3, 4, 5, and 6 using counter beans. This will be done individually, but students will be able to consult their pod-mates. (This will go on for 5 minutes.) As a whole group students will be posed with the question: "Was it possible to complete the task?" Followed by the question: "Why or why not?" Students will then be asked to factor each number out; this will be done in their pods. When each pod has completed their factoring each pod will share their factoring of a specific number. The teacher will write out the factors for each number as identified by each pod. This will result with each number being factored on the board. It will look similar to this: 3: 1, 3 4: 1, 2, 4 5: 1, 5 6: 1, 2, 3, 6 EXPLAIN: The students will be asked to discuss in their pods what they notice about these four numbers when they see them factored out. These noticings will then be shared and discussed in class. Students will have the opportunity to share their observations concerning these numbers and why only a couple of them could be equally divided as well as any similarities they notice between those that could be divided equally and those that could not be divided.	

ELABORATE: The teacher will introduce the terms Prime number and Composite number to the class. The teacher will ask the students to think, then attempt to define what each means. Students will then share their ideas as to what the terms my mean in light of the lessons earlier tasks. Students will then be provided the following definitions:

Prime number: A number, that is greater than one, who's only positive factors are one and itself.

Composite number: A number that can be factored into smaller numbers, that are not the number one.

EXPLORE: Students, in their pods, will then be asked to classify if 3, 4, 5, and 6 are prime or composite. During this time the teacher will roam among the pods asking probing questions to individual students, such as, "how did your pod decide that 3 is a prime number?" Pods will then share their classifications with the class and describe how they came to their classifications.

Then students will receive the Identification page activity and complete the first number as a class. Then students will complete the Identification page activity on their own. This activity will see students identify whether a number, between 1-10, is prime or composite as well as write the number's factors.

In order to provide a high ceiling a second page (page B) will be available for students that are displaying early mastery/understanding of the concept. As students are working on the Identification page, the teacher will roam the classroom asking students to defend/explain their answers.

EVALUATE: When all students have completed page A of the Identification page activity students will put their pencils away and retrieve their blue pen. Students will then date, using their pen, their paper. The class will then go through the Identification paper together with students marking corrections on their papers using their pen. Students will be asked to share their classifications as well as defend their thinking using a given number's factors as proof to their conclusion. To end the lesson students will submit their Identification page as an exit ticket.

Lesson Materials: Pencils, Identification page(s), scratch paper, counter beans
Classroom Management Needs: Students will need to have a whisper during their pod discussions. Students will need to be at a zero voice, unless called upon, during the whole group discussions, the Identification page work time, and the Blue Pen Correction time.

Name:______________________ **A**

Directions: Identify whether the given number is Prime or Composite. Write the factors of each given number.

Number	Prime	Composite	Factors
2			
7			
9			
10			

Name:______________________ **B**

Directions: Identify whether the given number is Prime or Composite. Write the factors of each given number.

Number	Prime	Composite	Factors
11			
12			
27			
29			

Unit Lesson Plans

Teacher:

Lesson Date:

Grade Level: 4th grade

Timeframe: (Length of lesson)

Content Area: Math

Grouping Strategy: Small group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.OA.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson higher learners will have the opportunity to work with larger numbers throughout the stations This will allow them to apply the foundation of the concept in a more challenging way. For lower floor learners there will be numbers available to meet them where they are and to challenge them as needed. There will also be plenty of time for them to ask questions of their peers and the teacher throughout stations.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Stations/ 1-50	
Common Core and/or State Standard(s): 4.OA.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify and classify numbers between 1-50 that are prime or composite.	**Assessment of Learning:** Students will demonstrate this by completing the Prime and Composite page at station 1 and the classification addition activity at station 3.

Lesson Objective: Students will be able to explain their reasoning for classifying numbers as either prime or composite.	Assessment of Learning: This will be expressed in their completion of the Number Journal activity at station 2 and the Define and Factor activity at station 4.

Relevant Vocabulary:
Prime, Composite, Factors

Teaching Model:

Indian Education for All (IEFA) _X_ No___Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE: To begin the lesson each pod will be tasked with determining whether or not a number is prime or composite; and be able to describe why they came to that conclusion using that number's factors. These numbers will be: 13, 21, 37, and 49.

As a whole group, pods will share their answers to the above task.

ELABORATE: The teacher will then ask the students, what is a prime number? What is a composite number? After several students have answered and the class is on the right track the teacher will review the definitions given the day prior.

Prime number: A number, that is greater than one, who's only positive factors are one and itself.

Composite number: A number that can be factored into smaller numbers, that are not the number one.

EXPLORE: The teacher will then explain to the class that they will be completing several stations for today's lesson, all of which dealing with Prime and Composite numbers. The students will complete the stations with their pod-mates. Each station will be 9 minutes long with a 1 minute clean-up and transition time between stations. The teacher will then explain the stations to the class.

- Station 1: Prime and Composite Page - at this station, students will cut and paste numbers into one of two categories Prime or Composite. This will work on their recognition and classification of prime and composite numbers.

- Station 2: Number Journal Activity - at this station, students will create a journal for a number pulled from a jar. In the journal students must address whether or not their number is prime or composite and why.
- Station 3: Classification Addition Activity - at this station, students will organize cards as either prime or composite based on their sums, (e.g. a card says 1+2=____ since the sum is 3 this card will be in the prime group. A card that says 47+2=____ would be in the composite as the sum is 49.)
- Station 4: Define and Factor Activity - at this station, students will define, in their own words, what Prime and Composite Numbers are. They will also provide 3 examples of each as well as the prime factors of each of their examples demonstrating their understanding of what prime and composite numbers are.

EXPLAIN: While completing the stations, students will be asked by the teacher to explain their thinking in their given station activity. The teacher will ask questions such as why did you place 13 in the prime category to students in station 1 or why did you choose 49 as one of your examples, in station 4.

EVALUATE: When all students have completed the stations they will submit their papers from stations 1, 2, and 4 as an exit ticket to the lesson.

Lesson Materials:

Pencils, Prime and Composite page, Number Journal page, addition cards, Define and Factor page, Prime and Composite Group titles, number cards

Classroom Management Needs:

Students will need to have a whisper during their opening pod discussion as well as during stations. Students will need to be at a zero voice, unless called upon, during the whole group discussion and the explanation of the stations.

Name:____________________ Prime and Composite Page

Prime	Composite

3	45	31	19
22	47	32	27

Name:________________ Number Journal

My Number:

Dear Journal

____
____
____
____
____
____
____

What my number looks like (be creative):

Name:________________ Define and Factor Page

Define what a Prime Number is:	Define what a Composite Number is
List 3 examples, include their factors: 1. 2. 3.	List 3 examples, include their factors: 1. 2. 3.

Unit Lesson Plans

Teacher: **Lesson Date:**

Grade Level: 4th grade **Timeframe:** (Length of lesson)

Content Area: Math **Grouping Strategy:** whole class

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.OA.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson lower floor learners will be provided more time to answer in the Number Circle activity and will be provided adequate time during the CLASSIFY activity to determine an answer. Higher level learners will be challenged to answer as fast as they can, attempting to beat their own time, during the Number Circle activity.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Review/ review activities	
Common Core and/or State Standard(s): 4.OA.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify numbers between 1-50 as either prime or composite.	**Assessment of Learning:** Students will demonstrate this by calling out either prime or composite when called upon in the Number Circle activity.

Lesson Objective: Students will demonstrate the ability to classify numbers between 1-50 as either prime or composite.	Assessment of Learning: This will be evaluated in the CLASSIFY! activity.
Relevant Vocabulary: Prime, Composite,	
Teaching Model:	
Indian Education for All (IEFA) _X_ No___Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:	

Lesson Procedures/Activities:

ENGAGE: To begin the lesson students will be asked to classify several numbers as either prime or composite on personal white boards. The numbers will be presented one at a time, with time allotted for students to write their answers. These numbers will include: 3, 10, 27, and 47.

EXPLAIN: The students will be asked to discuss, in their pods, what makes a number prime and what makes it composite.

ELABORATE: Students will then share their definitions. Students should provide definitions similar to those below, that were discussed the prior two lessons:

Prime number: A number, that is greater than one, who's only positive factors are one and itself.

Composite number: A number that can be factored into smaller numbers, that are not the number one.

EXPLORE: The teacher will then explain to the class that they will be participating in two activities during this class period.

1. CLASSIFY!: In this activity students will do as they did in the opening activity. The teacher will provide the class with a number and the student will classify it as either Prime or Composite.
2. Number Circle: In this activity the students will stand in a circle and be asked to identify whether a number is prime or composite, e.g. the teacher will point to Billy

and say 7, Billy will need to respond Prime within 5 seconds or will be out of the circle. The last person standing in the circle wins, but no prizes will be given out. EVALUATE: To complete the class period each student will be asked to pick a number and identify if it is Prime or Composite and why. This will be done verbally as students are dismissed to line up for specialists.
Lesson Materials: Personal white boards, expo markers, eraser rags, list of numbers (for each activity) between 1-50
Classroom Management Needs: During the opening activity students will need to be at a whisper. During CLASSIFY! Students will need to be at whisper to zero voice. For Number Circle students will need to be at a whisper or zero voice, as well.

Unit Plan (4th)

Final Draft

Prime or Composite Numbers

Student 13

Unit Standard(s):
4.0A.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Unit Goal:
During this unit my goal is that students will begin to understand the concept of prime and composite numbers. Essential questions to launch this unit will be what do you notice questions related to grouping and prime factorization.

Unit Goal:
During this unit my goal is that students will begin to understand the concept of prime and composite numbers. Essential questions to launch this unit will be what do you notice questions related to grouping and prime factorization.

Unit Sequence:
I have sequenced my lessons to start with laying the foundation of vocabulary and concept practice. This then builds with a day of stations focused on reviewing the concept and the concept vocabulary while also building on to the concept by widening the numbers in which students are asked to interact with. These build to concept review day in which students are asked to recall what they have learned during the previous two lessons and use them in fun and engaging ways. I have chosen to leave out the term Prime Factorization as well as word problems as these would be a part of a day four and five, if not farther, in the unit. If I were to continue the unit beyond the initial three day four would introduce real-world story problems to the students.

Assessment Rubric:
During these lessons I will be looking for students to be able to define what a prime and composite number are, how to determine if a number is prime or composite, and how to properly classify a number as prime or composite. This will include listening for students to explain that a number is prime because it's factors are only one and itself or that a number is composite because it has factors beyond one and itself. Through multiple informal formative discussions/questions during small groups as well as whole group times I will listen for these explanations. With the use of several turn-in components, used as exit tickets; I will evaluate students' understanding of concepts as well.

Mathematical Practice Standards:
During this unit I will address the mathematical practice standards 4.MP.3 Construct Viable Arguments and Critique the Reasoning of Others and 4.MP.8 Look for and Express Regularity in Repeated Reasoning. This (4.MP.3) will be accomplished through informal question, such as "How did you determine that? And Why did you classify 7 as a prime number?" 4.MP.8 will be addressed in a what do you notice sequence on day 1, an Identification page on day 1, a review of what prime and composite numbers are on day 2 (including examples to classify), as well as a Define and Factor Activity station on day 2.

References:

MT Math Standards:
http://opi.mt.gov/LinkClick.aspx?fileticket=YJmGdb 9KE8%3d&portalid-182

Bright Hub Education: Number Circle and CLASSIFY! (I made modifications to them to better serve 4th graders.):
https://www.brighthubeducation.com/middle-school-math-lessons/104807-prime-and-composite-number-interactive-activities/

Teachers Pay Teachers (Modified/inspired for my unit):
https://www.teacherspayteachers.com/Product/Prime-and-Composite-Numbers-Activi-249917 4

https://www.teacherspayteachers.com/Product/Factors-Multiples-Prime-and-Composite-Number-Review-336443

https://www.teacherspayteachers.com/FreeDownload/Writing-Journal-Template-and-Journal-Topic-Ideas-for-Kids-with-Autism-294574

The Prime Glossary- Definitions (Modified for 4th graders):
https://primes.utm.edu/glossary/page.php?sort=Composite

https://primes.utm.edu/glossary/page.php?sort=Prime

	Need Some More Practice	Almost There	Did It	Above & Beyond
Application of Concepts	With help I can identify and classify 1-10 as prime or composite numbers.	I can successfully identify and classify the numbers 1-10 as either prime or composite.	I can successfully identify and classify the numbers 1-50 as either prime or composite.	I can successfully identify and classify numbers 1-50+ as either prime or composite.
Explanation of Concepts	With help I can explain why a number is prime or composite based on its factors.	With limited help I can explain why a number is classified as prime or composite based on its factors.	I am able to clearly, without assistance, explain why a number is classified as prime or composite based upon its factors (e.g. I know that 7 is prime because the only 1 and 7 are its factors. I know 8 is a composite number because 1, 2, 4, & 8 are its factors.).	I am able to clearly, without assistance, explain why a number is classified as prime or composite based upon its factors using proper terms appropriately (e.g. I know that 47 is prime because the only numbers it is divisible by are 1 and 47 and numbers divisible only by itself and one are prime. I know that 68 is a composite number because it is divisible by 1, 2, 4, 17, 34, & 68; these are its factors.).
Representation of Concepts	I can identify, classify, and group prime and composite numbers, if someone helps me.	I can identity, classify, and group prime and composite single digit numbers, with limited assistance, using one of these methods: • Factor tree • Visual groups • Verbally	I can identity, classify, and group prime and composite single digit numbers using two of these methods: • Factor tree • Visual groups • Verbally	I can identity, classify, and group prime and composite single digit numbers using all of these methods: • Factor tree • Visual groups • Verbally
Mathematical Process	If someone helps me I can identify and classify prime and composite numbers.	With limited assistance I can identify and classify prime and composite numbers. Without assistance I make several errors.	I can identify and classify prime and composite numbers with very few errors.	I can identify and classify prime and composite numbers without errors.

Unit Lesson Plans

Teacher: **Lesson Date:**

Grade Level: 4th grade **Timeframe:** (Length of lesson)

Content Area: Math **Grouping Strategy:** Small group/whole class

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.0A.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson higher learners will have the opportunity to work with larger numbers beyond the general class assignment. This will allow them to apply the foundation of the concept in a more challenging way. For lower floor learners the activity will be able to use manipulatives in order to work on their activity as well as will have multiple points of peer interaction and instructor explanation and support.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Introduction/ 1-10	
Common Core and/or State Standard(s): 4.0A.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify numbers between 1-10 that are prime or composite.	**Assessment of Learning:** Students will demonstrate this by completing the attached Identification page and submitting it, with Blue Pen Corrections, if need be, as an exit ticket.

Lesson Objective:
Students will be able to define the mathematical terms prime and composite.

Assessment of Learning:
This will be assessed through informal formative assessment.

Relevant Vocabulary:
Prime, Composite, Factors

Teaching Model:
5 E's

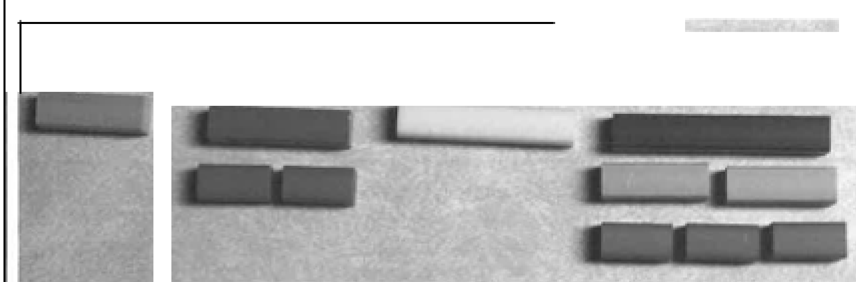
Indian Education for All (IEFA) _X_ No _ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE: To begin the lesson students will be asked how many equal groups can be made out of the numbers 3, 4, 5, and 6 using Cuisenaire rods; students should find that they can only break 3 and 5 one way and 4 and 6 multiple ways. This will be done individually, but students will be able to consult their pod-mates. (This will go on for 5 minutes.)

As a whole group students will be posed with the question: "How many groups were you able to make?" "What groups were they? And Can you show us one of your ways?"
The completed board will look similar to:

3 4 5 6



Students will then be asked to factor each number out; this will be done in their pods. When each pod has completed their factoring each pod will share their factoring of a specific number. The teacher will ask the class to take time and see what they notice between the Cuisenaire rod groups and each number's factors; students should notice that the

Cuisenaire rod groups that they found are each numbers factors. The class will then write out the factors for each number, in sequential order eliminating any repeated numbers. The board will look like:

3	4	5	6
1, 3	1, 2, 4	1, 5	1, 2, 3, 6

EXPLAIN: The students will be asked to discuss in their pods what they notice about these four numbers when they see them factored and grouped out. These noticings will then be shared and discussed in class. Students will be expected to identify that 3 and 5 can only be factored/divided by/to itself and one; demonstrated by students stating something like, "there is only one way to divide 3 (and 5)."

ELABORATE: The teacher will introduce the terms Prime number and Composite number to the class. Students will then be provided the following definitions:

Prime number: A number that can be divided only one way, one and itself.

Composite number: A number that can be divided more than one way.

EXPLORE: Students, in their pods, will then be asked to classify if 3, 4, 5, and 6 are prime or composite. During this time the teacher will roam among the pods asking probing questions to individual students, such as, "how did you/your pod decide that 3 is a prime number?" Pods will then share their classifications with the class and describe how they came to their classifications.

Then students will receive the Identification page activity and complete the first number as a class. During this class completion time Cuisenaire rods will be used to find the factor pairs for the number in order to determine whether it is a prime or composite number based upon its factors. Students will lead the teacher to solve and identify factor pairs for the number. The teacher will, at the students' direction, create factor pairs (displaying them using the classroom Elmo projector) asking probing questions, such as "Are we sure this is a pair?" How do we know these pairs are equal to our starting number?"

Students will then complete the Identification page activity on their own. This activity will see students identify whether a number, between 1-10, is prime or composite as well as write the number's factors.

In order to provide a high ceiling a second page (page B) will be available for students that are displaying early mastery/understanding of the concept. As students are working on the Identification page, the teacher will roam the classroom asking students to defend/explain their answers.

For those students that are struggling or need extra support, they will be able to use their Cuisenaire rods to assist them in visualizing the factor pairs/groups for each number. These students will also be able to draw out their pairs rather than write them out if need be. **EVALUATE:** When all students have completed page A of the Identification page activity students will put their pencils away and retrieve their blue pen. Students will then date, using their pen, their paper. The class will then go through the Identification paper together with students marking corrections on their papers using their pen. Students will be asked to share their classifications as well as defend their thinking using a given number's factors as proof to their conclusion. To end the lesson students will submit their Identification page as an exit ticket.
Lesson Materials: Pencils, Identification page(s), scratch paper, Cuisenaire rods
Classroom Management Needs: Students will need to have a whisper during their pod discussions. Students will need to be at a zero voice, unless called upon, during the whole group discussions, the Identification page work time, and the Blue Pen Correction time.

Name: _ _ _ _ _ _ _ _ _ **A**

Directions: Identify whether the given number is Prime or Composite. Write the factors of each given number.

Number	Prime	Composite	Factors
2			
7			
9			
10			

Name: _ _ _ _ _ _ _ _ _ **B**

Directions: Identify whether the given number is Prime or Composite. Write the factors of each given number.

Number	Prime	Composite	Factors
11			
12			
27			
29			

Unit Lesson Plans

Teacher: **Lesson Date:**

Grade Level: 4th grade **Timeframe:** (Length of lesson)

Content Area: Math **Grouping Strategy:** Small group

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate_ proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.0A.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson higher learners will have the opportunity to work with larger numbers throughout the stations This will allow them to apply the foundation of the concept in a more challenging way. For lower floor learners there will be cuisenaire rods, grid paper, and scratch paper available for students to use at multiple stations. Further, numbers will be available to meet them where they are and to challenge them as needed. Additionally, there will also be plenty of time for them to ask questions of their peers and the teacher throughout stations.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Stations/ 1-50	
Common Core and/or State Standard(s): 4.0A.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify and classify numbers between 1-50 that are prime or composite.	**Assessment of Learning:** Students will demonstrate this by completing the Prime and Composite page at station 1 and the classification addition activity at station 3.

Lesson Objective:	**Assessment of Learning:**
Students will be able to explain their reasoning for classifying numbers as either prime or composite.	This will be expressed in their completion of the Number Journal activity at station 2 and the Define and Factor activity at station 4.

Relevant Vocabulary:
Prime, Composite, Factors

expected

Teaching Model:
5 E's

Indian Education for All (IEFA) _x_ No _ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ELABORATE: The teacher will begin the lesson by asking the students, what is a prime number? What is a composite number? After several students have answered and the class is on the right track the teacher will review the definitions given the day prior. The teacher will listen for answers such as, "Prime numbers only have 1 and itself. Composite has more than that."

Prime number: A number that can be divided only one way, one and itself.

Composite number: A number that can be divided more than one way.

ENGAGE: To begin the lesson each pod will be tasked with factoring out and determining whether or not a number is prime or composite. Students will use Cuisenaire rods to determine the numbers' factor pairs, and will then mark the factor pairs on grid paper as well as writing the numbers factors below the factor pairs on their grid paper. These numbers will be: 13, 21, 37, and 49.

As a whole group, pods will share their answers to the above task. This will be done by pod representatives sharing their grid paper representations on the Elmo projector. Using each pod,s representations the class will discuss whether or not the given number is prime or composite based on its factor pairs and factors. The teacher will ask probing questions such as, "Why did you/your group decide this number was prime or composite? How does its factors/factor pairs inform this conclusion? Did any groups get a different answer, how?" The teacher will listen, and look at representations, for answers such as, " We decided it was prime because the only numbers that we could go into it are one and itself. We think it's prime because it can only be divided by itself and one."

EXPLORE: The teacher will then explain to the class that they will be completing several stations for today's lesson, all of which dealing with Prime and Composite numbers. The students will complete the stations with their pod-mates. Each station will be 9 minutes long with a 1 minute clean-up and transition time between stations. The teacher will then explain the stations to the class.

- Station 1: Prime and Composite Page - at this station, students will cut and paste numbers into one of two categories Prime or Composite. This will work on their recognition and classification of prime and composite numbers. There will be Cuisenaire rods available at this station for students to use if need be.
- Station 2: Number Journal Activity - at this station, students will create a journal for a number pulled from a jar. In the journal students must address whether or not their number is prime or composite and why.
- Station 3: Classification Addition Activity - at this station, students will organize cards as either prime or composite based on their sums, (e.g. a card says 1+2= _ since the sum is 3 this card will be in the prime group. A card that says 47+2=_ would be in the composite as the sum is 49.) Cuisenaire rods, grid paper, and scratch paper will be available at this station for students to use.
- Station 4: Define and Factor Activity - at this station, students will define, in their own words, what Prime and Composite Numbers are. They will also provide 3 examples of each as well as the prime factors of each of their examples demonstrating their understanding of what prime and composite numbers are. Cuisenaire rods, grid paper, and scratch paper will be available at this station for students to utilize.

EXPLAIN: While completing the stations, students will be asked by the teacher to explain their thinking in their given station activity. The teacher will ask questions such as why did you place 13 in the prime category to students in station 1 or why did you choose 49 as one of your examples, in station 4.

EVALUATE: When all students have completed the stations they will submit their papers, and any scratch & grid paper used, from stations 1, 2, and 4 as an exit ticket to the lesson.

Lesson Materials:
Pencils, Prime and Composite page, Number Journal page, addition cards, Define and Factor page, Prime and Composite Group titles, number cards, Cuisenaire rods, scratch paper, and grid paper

Classroom Management Needs:
Students will need to have a whisper during their opening pod discussion as well as during stations. Students will need to be at a zero voice, unless called upon, during the whole group discussion and the explanation of the stations.

Name:______________ Prime and Composite Page

Prime	Composite

3	45	31	19
22	47	32	27

Name:____________ Number Journal

My Number:

Dear Journal

____

____

____

____

____

____

____

What my number looks like (be creative):

Name:________ Define and Factor Page

Define what a Prime Number is:	Define what a Composite Number is
List 3 examples, include their factors: 1. 2. 3.	List 3 examples, include their factors: 1. 2. 3.

Unit Lesson Plans

Teacher: **Lesson Date:**

Grade Level: 4th grade **Timeframe:** (Length of lesson)

Content Area: Math **Grouping Strategy:** whole class

Preparing for Lesson Development

What do you know about your students' current performance and educational needs?

I know that my classroom is comprised of students at various levels of academic performance. All of them have been able to demonstrate proficiency at factoring numbers between 1-50, with the highest students able to comfortably factor numbers to 100. This is in completion of the first portion of standard 4.0A.4 which pertains to factoring whole numbers.

How will you differentiate the lesson to meet the needs of all learners in your classroom?

In order to differentiate the lesson lower floor learners will be provided more time to answer in the Number Circle activity and will be provided adequate time during the CLASSIFY activity to determine an answer. Higher level learners will be challenged to answer as fast as they can, attempting to beat their own time, during the Number Circle activity.

Lesson Plan Development

Lesson Title: Prime and Composite Numbers: Review/ review activities	
Common Core and/or State Standard(s): 4.0A.4: Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-1000 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	
Lesson Objective: Students will be able to identify numbers between 1-50 as either prime or composite.	**Assessment of Learning:** Students will demonstrate this by calling out either prime or composite when called upon in the Number Circle activity.
Lesson Objective: Students will demonstrate the ability to classify numbers between 1-50 as either prime or composite.	**Assessment of Learning:** This will be evaluated in the CLASSIFY! activity.

Relevant Vocabulary:
Prime, Composite,

Teaching Model:
5 E's

Indian Education for All (IEFA) _X_ No _ Yes. If yes, describe how the lesson addresses one or more of the Essential Understandings Regarding Montana Indians:

Lesson Procedures/Activities:

ENGAGE: To begin the lesson students will be asked to classify several numbers as either prime or composite on personal white boards. The numbers will be presented one at a time, with time allotted for students to write their answers. These numbers will include: 3, 10, 27, and 47.

EXPLAIN: The students will be asked to discuss, in their pods, what makes a number prime and what makes it composite.

ELABORATE: Students will then share their definitions. Students should provide definitions similar to those below, that were discussed the prior two lessons:

Prime number: A number, that is greater than one, who's only positive factors are one and itself.

Composite number: A number that can be factored into smaller numbers, that are not the number one.

EXPLORE: The teacher will then explain to the class that they will be participating in two activities during this class period.

1. CLASSIFY!: In this activity students will do as they did in the opening activity. The teacher will provide the class with a number and the student will classify it as either Prime or Composite.
2. Number Circle: In this activity the students will stand in a circle and be asked to identify whether a number is prime or composite, e.g. the teacher will point to Billy and say 7, Billy will need to respond Prime within 5 seconds or will be out of the circle. The last person standing in the circle wins, but no prizes will be given out.

EVALUATE: To complete the class period each student will be asked to pick a number and identify if it is Prime or Composite and why. This will be done verbally as students are dismissed to line up for specialists.

Lesson Materials: Personal white boards, expo markers, eraser rags, list of numbers (for each activity) between 1-50
Classroom Management Needs: During the opening activity students will need to be at a whisper. During CLASSIFY! Students will need to be at whisper to zero voice. For Number Circle students will need to be at a whisper or zero voice, as well.