



An economic evaluation of fertilization of Montana winter wheat
by Craig Edgar Simons

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
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Abstract:

Relationships between yield and protein response of Montana winter wheat to fertilizer application and various soil and climatic conditions were studied using data from 19 experiments conducted in southeastern Montana during the years 1976, 1977 and 1978. Relationships were estimated using a generalized non-linear algorithm. Explanatory variables were applied nitrogen, phosphorus and potassium, soil nitrate and soil water, both measured to a depth of four feet, soil phosphorus, measured to a depth of two feet, and April through July precipitation. The winter wheat variety used at all sites was Centurk.

Yield response was estimated using a second degree polynomial equation with an additive component error term. Variables important in explaining yield were applied phosphorus, nitrogen and potassium, and soil nitrate and phosphorus. Soil water and April through July precipitation were not statistically significant in explaining variations in yield and were combined into a total water variable. Results stemming from this combination were little different than considering soil water and precipitation separately. Consequently, water was eliminated from the analysis except through interaction variables with both applied nitrogen and phosphorus.

Variables important in explaining protein response were applied nitrogen, phosphorus, and potassium and soil nitrate. Water was only statistically significant as a component of the interaction term with applied nitrogen.

The estimated yield response equation was used to determine optimal rates of fertilizer application under varying fertilizer and wheat prices. Results indicated that potassium application was not economic, and should not be applied at its current price of 16 cents per pound unless the price of wheat was approximately \$5.70. Phosphorus was not economic when soil phosphorus was at high levels of 20-25 pounds per acre measured in the top two feet of soil. Economic applications of phosphorus become much higher as water and applied nitrogen are increased. Nitrogen applications have the biggest effect on winter wheat yields and therefore, on profits. Economic applications of nitrogen depend heavily upon water and soil nitrate. The highest recommendation for nitrogen application occur when water is high and soil nitrate is at a relatively low level.

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FERTILIZATION OF MONTANA WINTER WHEAT

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CRAIG EDGAR SIMONS

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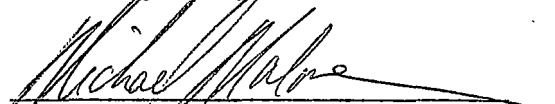
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ABSTRACT

Relationships between yield and protein response of Montana winter wheat to fertilizer application and various soil and climatic conditions were studied using data from 19 experiments conducted in southeastern Montana during the years 1976, 1977 and 1978. Relationships were estimated using a generalized non-linear algorithm. Explanatory variables were applied nitrogen, phosphorus and potassium, soil nitrate and soil water, both measured to a depth of four feet, soil phosphorus, measured to a depth of two feet, and April through July precipitation. The winter wheat variety used at all sites was Centurk.

Yield response was estimated using a second degree polynomial equation with an additive component error term. Variables important in explaining yield were applied phosphorus, nitrogen and potassium, and soil nitrate and phosphorus. Soil water and April through July precipitation were not statistically significant in explaining variations in yield and were combined into a total water variable. Results stemming from this combination were little different than considering soil water and precipitation separately. Consequently, water was eliminated from the analysis except through interaction variables with both applied nitrogen and phosphorus.

Variables important in explaining protein response were applied nitrogen, phosphorus, and potassium and soil nitrate. Water was only statistically significant as a component of the interaction term with applied nitrogen.

The estimated yield response equation was used to determine optimal rates of fertilizer application under varying fertilizer and wheat prices. Results indicated that potassium application was not economic, and should not be applied at its current price of 16 cents per pound unless the price of wheat was approximately \$5.70. Phosphorus was not economic when soil phosphorus was at high levels of 20-25 pounds per acre measured in the top two feet of soil. Economic applications of phosphorus become much higher as water and applied nitrogen are increased. Nitrogen applications have the biggest effect on winter wheat yields and therefore, on profits. Economic applications of nitrogen depend heavily upon water and soil nitrate. The highest recommendation for nitrogen application occur when water is high and soil nitrate is at a relatively low level.

Chapter 1

INTRODUCTION

PROBLEM STATEMENT

Nitrogen and phosphorus fertilizer use in Montana has increased substantially over the past several years. In 1970 Montana farmers used slightly less than seven pounds of actual nitrogen and phosphorus per acre. By 1976, this figure had more than doubled to over 14 pounds per acre.

During this same time period the prices of nitrogen and phosphorus fertilizer have fluctuated over a very wide range. Nitrogen varied from 10 to 30 cents per pound while the price of phosphorus was not as volatile, ranging from 21 to 53 cents per pound. Similarly, the price that farmers received for their wheat varied considerably over the same period. The average price farmers received for their wheat in 1978 was more than double the 1970 price. However, the 1978 price was only about 65 percent of the price in 1973. (Prices are not adjusted for inflation.)

As a consequence of these rapidly changing input and output prices, determining an optimal fertilizer policy is at best precarious. An uneconomic application of fertilizer could mean a significant loss of profits for the winter wheat producer.

Farmers' decision criteria for fertilizer application vary considerably. This is not difficult to understand as the yield and protein response to nitrogen and phosphorus are not well defined, especially

given variable soil and moisture conditions. In order for winter wheat producers to make economic applications of fertilizer, they must have two basic types of information. First, they must know the physical response of wheat to nitrogen and phosphorus at variable levels of application. This physical response must also be associated with variable soil and climatic conditions. Two types of physical response information are needed: (1) the producer needs good information about the incremental yields forthcoming from varying applications of nitrogen and phosphorus including marginal rates of technical substitution, and (2) marginal changes in the protein content associated with different levels of fertilizer application. In addition to physical response information, the producer also needs reliable estimates of fertilizer and winter wheat prices. Included in the winter wheat prices must be some reasonable estimate of the protein premium structure. Only when all this information is known can the producer expect to have a truly economically optimal fertilization program.

PURPOSE OF THE STUDY

The purpose of this study is to investigate and develop a decision criteria for profit maximization as related to fertilizer application on Montana winter wheat. Specifically, the objectives of this study are to:

- 1) Develop a winter wheat response (production) function for

yield and protein with respect to nitrogen and phosphorus fertilizer and other important soil and climatic conditions.

2) Determine optimal fertilizer application by considering the estimated production function, various winter wheat prices, fertilizer prices and protein premium structures.

3) Present the information from one and two above in a manner that helps winter wheat producers in Montana make economically rational decisions concerning their fertilizer programs.

Chapter 2

LITERATURE REVIEW

The ability to predict yield and protein response to fertilizer is necessary if the producer is to be able to determine optimal levels of fertilizer application. Research in this area is by no means a new or unique thing. Many factors thought to influence yield and/or protein content have been studied.

Increased yields and higher protein have often been attributed to nitrogen fertilizer (7, 9, 27). Other results indicate precipitation, soil moisture, phosphorus, potassium, soil type, temperature, varieties, row spacing and management practices influence yield and protein in grain (1, 4, 5, 26).

In very early work, Fisher (10) used a linear regression technique to examine the relationship between wheat yields and rainfall. Response functions were estimated by determining the change in yield resulting from an increase (or decrease) in average rainfall for a specific time of year. Similar work done in India (11) found that 75 percent of the total variation in yield was accounted for by variation in rainfall. The time of the growing season in which the rainfall was experienced was also important. More than normal rainfall one month before seeding time was found to be detrimental to wheat yields.

Research done on winter wheat in Kansas (30) indicated that the

influence of soil moisture at planting time and growing season precipitation affected yield and yield response to fertilizer to about the same degree. This, according to the study, differs from summer grown crops where soil moisture largely determines yield and yield response to fertilizer. Yield increases in this study were noted more often for nitrogen (58 percent) than for any other element. Phosphorus caused yield increases in 33 percent of the sites (most of which were very low in soil phosphorus). Potassium and sulfur had little effect on winter wheat yields.

The same study included analysis of protein response to fertilizer. Results showed almost no response to phosphorus. Nitrogen, however, increased protein .3 to .6 percent for each 20 pound increment.

In Australia (29), researchers found rainfall to be the major source of variation in yield. In this study, pre-sowing rainfall was found to be detrimental to yields. In all, rainfall accounted for about 80 percent of the variation in yield. Published research by Bauer, et al. (3) on yield experiments done on spring wheat and barley found water was the cause of 40 percent of the variation. Phosphorus was found to be of little use in predicting yields. Eck and Tucker (8) found soil moisture and precipitation to be critical in explaining variations in yield. Growing season temperature and organic matter were also found important in explaining variation in yield. Unfortunately, the relationships did not yield satisfactory prediction

equations. In a similar study (2), researchers found soil moisture was the best estimator of yield followed by minimum and maximum temperature. Rainfall was completely unsuitable for yield estimation in this research project.

Nebraska researchers (30) found a highly significant inverse protein-yield relationship for applied nitrogen. Nitrogen was found to increase yields on irrigated crops. At the same time, wheat had little yield response to nitrogen on dryland, but showed significant increases in protein content.

Results published by Smika, et al. (28) found grain yields positively correlated with soil water at seeding time. When soil water was included in the analysis, the largest yields were found with the lowest levels of soil nitrate. The exclusion of soil water from the analysis found no apparent relationship between soil nitrate and yield. Results also showed that yields were higher on mulched soil than bare soil (where $\text{NO}_3\text{-N}$ was available in sufficient quantities in both), but yield response to nitrogen fertilizer was greater on bare soil than mulched soil.

Jackson (16) used a stepwise multiple regression technique to predict grain yield and protein with and without nitrogen fertilizer additions. Estimations were also made of residual soil nitrate levels after harvest. In the first step, potential yield was estimated. Independent variables used were soil organic matter, growing season

precipitation, and evaporation rates. Soil nitrate, evaporation rates, potential yield, and available soil water were the variables then used to predict nitrogen fertilizer requirements. Variables used in predicting grain protein were potential yield, growing season rainfall, organic matter and soil nitrate.

When soil nitrate was higher than 120 kilograms per hectare, Olson, et al. (24) found little or no yield response to applied nitrogen. An additional 50 to 60 kilograms per hectare of applied nitrogen would, however, be needed to obtain maximum protein.

Researchers in Illinois (14) studied the effects of applied nitrogen on both hard and soft winter wheat. Results indicated increases in yield and protein percentage in all experiments. The favorable results from these experiments indicated that despite increased costs, split application of nitrogen fertilizer made late in the spring has merit in the interests of minimizing pollution.

Researchers in Montana (22), using five spring wheat varieties and five nitrogen treatments, found no difference in yield response due to variety. However, due to increasing nitrogen application rates, grain protein percent decreased as the grain to straw ratio increased for all varieties. Pendleton and Dungan (25) found that different varieties showed different yield response to nitrogen as well as different response to seeding rates. Similar research (19, 32) also showed varying yield response to different varieties.

Garnick, et al. (12), using data from 43 winter wheat experiments in Montana, estimated yield response using third degree polynomial equations. Explanatory variables included April through July precipitation, applied phosphorus, applied nitrogen and available soil nitrate and available soil water measured in the top four feet of the soil in early spring. Unfortunately, because of statistical insignificance, probably partially caused by measurement problems and variation across sites, soil water and applied phosphorus were excluded from the yield analysis. Results show that nitrogen and soil nitrate as well as precipitation increased yields while the nitrogen variables and applied phosphorus increased protein. Precipitation had a negative influence on protein.

In many of these studies there are conflicting results; Smika et al. (28) found soil nitrate important in explaining yield. Similarly, Bair and Robertson (2) found rainfall unsuitable for explaining yield, while others (11, 29) found it very significant. Variability in results from these studies in predicting protein and yield response may be attributable to variation in soil characteristics, management practices, distribution of nitrogen and phosphorus in the soil as well as many other factors.

It is hoped that the following research will contribute to a better understanding of the factors which influence yield and protein response in Montana winter wheat.

Chapter 3

ESTIMATION OF YIELD AND PROTEIN FUNCTIONS

SOURCE OF DATA

Data used in this research were collected from a series of yield/protein response experiments conducted under the auspices of Dr. Vincent A. Haby, assistant professor of soils, Montana State University at the Southern Agricultural Research Center, Huntley. Nineteen sets of data representing ten different sites are used in the final analysis. For each set of data there are 23 treatments, seven which are replicated twice for a total of 30 observations for each site, or 570 total data points. Site location and year are presented in Appendix A.

Data for the analysis were selected according to their ability to meet certain predetermined criteria. Experimental plots were located on summer fallow, with efforts made to insure each site was uniform in soil type, free of insects and weeds as well as any other damaging factors. A uniform stand of Centurk hard red winter wheat was seeded at all sites. As a result of these criteria, one site from the original data base of 21 sites was excluded because of hail damage and another was disqualified due to high soil salt content.

Independent variables recorded were soil nitrate ($\text{NO}_3\text{-N}$) and soil water, both measured to a depth of four feet in early spring, April through July precipitation, and soil phosphorus measured to a depth of two feet. Observations were also collected on the dependent variables,

yield and protein content, at harvest time.

Each fertilizer nutrient was applied at five different rates. Nitrogen was broadcast at rates ranging from 0 to 89 pounds in increments of 22.25 pounds per acre. Phosphorus was banded with the seed at rates varying from 0 to 35.6 pounds per acre of elemental phosphorus in increments of 8.9 pounds per acre. Potassium application rates ranged from 0 to 85.6 pounds of elemental K, in increments of 21.4 pounds per acre.

ALGEBRAIC SPECIFICATION OF THE RESPONSE FUNCTION

Selection of the algebraic specification of the protein and yield response function warrants careful consideration. The function must be able to describe the characteristics of the phenomenon being studied. No functional form will be an exact 'fit', but care must be taken to select a function with the most desirable characteristics.

In this study of the economic application of fertilizer, polynomial equations were selected for several major reasons. Consider a simple polynomial of the form

$$Y = \alpha + \beta_1 X + \beta_2 X^2$$

where:

Y = total output;

α = the value of the equation when the independent variable is

zero;

X = the units of the independent variable; and

β_1 and β_2 = the parameters of the equation.

Because we expect total output to reach a maximum and decline, we hypothesize that the parameter β_1 will be positive while β_2 is negative. Thus the equation lends itself to both positive and negative marginal products.

Extensions of the previous equation allow substantial flexibility to the production function being estimated. In the quadratic equation, the marginal product curve is linear. This restriction is easily overcome by the introduction of a cubic term.

It is also often true that influence of one explanatory variable depends upon the level of the other variables. Hence it may be necessary to incorporate interaction terms between two variables in the response function.

Previous studies (12, 16) indicate that polynomial equations have been quite useful in the study of yield and protein response to fertilizer application.

ADDITIVE ERROR MODEL

The first model used in the estimating yield and protein response was the following form:

$$(2.1) \quad Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7$$

$$\begin{aligned}
& + \beta_8 X_8 + \beta_9 X_9 + \beta_{10} X_{10} + \beta_{11} X_{11} + \beta_{12} X_{12} + \beta_{13} X_{13} \\
& + \beta_{14} X_{14} + \beta_{15} X_{15} + \beta_{16} X_{16} + \beta_{17} X_{17} + \beta_{18} X_{18} + \beta_{19} X_{19} \\
& + \beta_{20} X_{20} + \beta_{21} X_{21} + \beta_{22} X_{22} + W_{iLt} + U_L + V_t
\end{aligned}$$

where:

Y = estimated yield or protein;

β_i ($i = 0$ to 22) = unknown parameters to be estimated;

X_1 = applied nitrogen;

X_2 = applied nitrogen squared;

X_3 = applied phosphorus;

X_4 = applied phosphorus squared;

X_5 = precipitation;

X_6 = precipitation squared;

X_7 = soil water;

X_8 = soil water squared;

X_9 = soil nitrogen;

X_{10} = soil nitrogen squared;

X_{11} = soil phosphorus;

X_{12} = soil phosphorus squared;

X_{13} = applied nitrogen x applied phosphorus;

X_{14} = N x precipitation;

X_{15} = N x soil nitrogen;

X_{16} = precipitation x soil water;

X_{17} = precipitation x soil nitrogen;

X_{18} = soil water x soil nitrogen;

X_{19} = applied phosphorus x soil phosphorus;

X_{20} = applied phosphorus x precipitation;

X_{21} = potassium;

X_{22} = potassium squared;

W_{iLt} = the random variation due to experimental error associated
with location L, time t, and treatment i;

U_L = the random variation associated with location L; and

V_t = the random variation associated with year t.

Although the components of the error term cannot actually be separated in estimation, their distinction is vitally important statistically.

Define

$$\epsilon_{iLt} = W_{iLt} + U_L + V_t$$

where:

$$W_{iLt} \approx N(0, \sigma_w^2)$$

$$U_L \approx N(0, \sigma_u^2)$$

$$V_t \approx N(0, \sigma_v^2).$$

The assumption of homoskedasticity implies that

$$\text{Var}(\epsilon_{iLt}) = E(\epsilon_{iLt} - E(\epsilon_{iLt}))^2 = \sigma^2 = \sigma_u^2 + \sigma_w^2 + \sigma_v^2.$$

Further assumptions concerning the components include

- 1) $E(U_L V_t) = E(U_L W_{iLt}) = E(V_t W_{iLt}) = 0$
- 2) $E(U_L U_{L'}) = 0 \quad L \neq L'$
- 3) $E(V_t V_{t'}) = 0 \quad t \neq t'$
- 4) $E(W_{iLt} W_{i'L't'}) = E(W_{iLt} W_{iL't'}) = E(W_{iLt} W_{iLt'})$
 $= E(W_{iLt} W_{i'L't'}) = 0$ for $i \neq i', L \neq L', t \neq t'$.

The first assumption means statistical independence between the components. Assumptions 2, 3, 4 mean that there is an absence of serial correlation, and 4 also implies no correlation among treatments or across locations.

The coefficient of correlation between the disturbance at two different points in time, i.e., between ϵ_{iLt} and $\epsilon_{iLt'}$, $t \neq t'$ is

$$\frac{\text{COV}(\epsilon_{iLt}, \epsilon_{iLt'})}{[\text{Var}(\epsilon_{iLt}) \text{Var}(\epsilon_{iLt'})]^{1/2}} = \frac{E(\epsilon_{iLt} \epsilon_{iLt'})}{[E(\epsilon_{iLt}^2) E(\epsilon_{iLt'}^2)]^{1/2}} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2 + \sigma_w^2} = \frac{\sigma_u^2}{\sigma_\epsilon^2},$$

because of the assumptions of independence among components and independence within components over location, time, and treatment.

The coefficient of correlation between the disturbances of different cross-sectional units, i.e., between ϵ_{iLt} and $\epsilon_{iL't}$ ($L \neq L'$) is

$$\frac{\text{COV}(\epsilon_{iLt}, \epsilon_{iL't})}{[\text{Var}(\epsilon_{iLt})\text{Var}(\epsilon_{iL't})]^{1/2}} = \frac{E(\epsilon_{iLt}, \epsilon_{iL't})}{[E(\epsilon_{iLt}^2)E(\epsilon_{iL't}^2)]^{1/2}} = \frac{\sigma_v^2}{\sigma_u^2 + \sigma_v^2 + \sigma_w^2} = \frac{\sigma_v^2}{\sigma_\epsilon^2}.$$

Similarly the correlation coefficient between the disturbances of different treatments at a given time and location, i.e., between ϵ_{iLt} and $\epsilon_{i'L't}$ ($i \neq i'$) is

$$\frac{\text{COV}(\epsilon_{iLt}, \epsilon_{i'L't})}{[\text{Var}(\epsilon_{iLt})\text{Var}(\epsilon_{i'L't})]^{1/2}} = \frac{E(\epsilon_{iLt}, \epsilon_{i'L't})}{[E(\epsilon_{iLt}^2)E(\epsilon_{i'L't}^2)]^{1/2}} = \frac{\sigma_u^2 + \sigma_v^2}{\sigma_u^2 + \sigma_v^2 + \sigma_w^2} = \frac{\sigma_u^2 + \sigma_v^2}{\sigma_\epsilon^2}.$$

The final feature of the component error term is the coefficient of correlation between ϵ_{iLt} and $\epsilon_{i'L't'}$, ($i \neq i'$, $L \neq L'$, $t \neq t'$), where

$$\frac{\text{COV}(\epsilon_{iLt}, \epsilon_{i'L't'})}{[\text{Var}(\epsilon_{iLt})\text{Var}(\epsilon_{i'L't'})]^{1/2}} = \frac{E(\epsilon_{iLt}, \epsilon_{i'L't'})}{[E(\epsilon_{iLt}^2)E(\epsilon_{i'L't'}^2)]^{1/2}} = 0.$$

For a rigorous discussion of the component error term, the interested reader is referred to Wallace and Hussain (31) or Nerlove (23).

Due to the nature of the experiment, the three part error term previously outlined is not entirely correct for this study. These data as stated earlier, were obtained from nineteen experiments, representing ten sites. The duplicated (triplicated) sites are not actually on the same plots, but are only adjacent plots. As a result

of this, it is impossible to discern the cause of any variation due to time or location. Hence, for this analysis the variation due to location (U_L) and time (V_L) are combined leaving

$$Z_j = U_L + V_t.$$

The complete error term is then represented by the equation

$$\epsilon_{ij} = W_{ij} + Z_j$$

where $W_{ij} = W_{iLt}$ as previously defined.

The basic assumptions for the two components are

$$W_{ij} \approx N(0, \sigma_w^2)$$

$$Z_j \approx N(0, \sigma_z^2).$$

The assumption of homoskedasticity implies

$$\text{Var}(\epsilon_{ij}) = E(\epsilon_{ij} - E(\epsilon_{ij}))^2 = \sigma^2 = \sigma_w^2 + \sigma_z^2.$$

Other assumptions concerning the characteristics of the error term are

$$1) \quad E(W_{ij} Z_j) = 0$$

$$2) \quad E(W_{ij} W_{ij'}) = E(W_{ij} W_{i'j}) = E(W_{ij} W_{i'j'}) = 0$$

$$3) \quad E(Z_j Z_{j'}) = 0 \quad \text{where } i \neq i', j \neq j'.$$

Coefficients of correlation for c_{ij} can be generalized from the previous outline of the classical component error term.

MULTIPLICATIVE ERROR MODEL

A second model was considered which allows the component error term (sampling time and treatment specific errors), to be a proportion of the regression terms. This model specification would seem to make sense for agronomic data where standard errors of the estimate are proportional to the mean response function. The model is expressed as:

$$y = (\beta_0 + \beta_1 X_1 + \dots + \beta_{22} X_{22}) W_{ij} Z_j$$

where the variables maintain the same definition as in the previous model.

There is a slight change in the basic assumptions of the error term. In place of the assumption that $E(w_{ij}) = E(Z_j) = 0$ as in the additive error model, the assumption changes to $E(W_{ij}) = E(Z_j) = 1$ for the multiplicative case.

The multiplicative model is made estimable by taking the natural logarithm of both sides of the equation. This transformation leaves

$$\ln y = \ln(\beta_0 + \beta_1 X_1 + \dots + \beta_{22} X_{22}) + \ln W_{ij} + \ln Z_j$$

which is an intrinsically nonlinear function with respect to the

parameters as well as being nonlinear with respect to the variables.

ESTIMATION PROCEDURES

Estimations of the parameters in these models were derived utilizing maximum likelihood estimators. The generalized nonlinear regression program used in this research was written by Dr. Oscar R. Burt, Montana State University. The interested reader is referred to Malinvaud (21) Chapter 9 for a rigorous discussion of the statistical theory implicit in this program.

Although the method of maximum likelihood estimation is not a new statistical technique, an outline of the concept may be useful to the uninitiated reader.

Maximum likelihood estimation is based on the simple yet sound assumption that different populations yield different samples, and any given sample is more likely to come from one population than another. Specifically:

If a random variable X has a probability distribution $f(X)$ characterized by parameters $\theta_1, \theta_2, \dots, \theta_k$ and if we observe a sample $X_1, X_2, \dots, X_n$, then the maximum likelihood estimators of $\theta_1, \theta_2, \dots, \theta_k$ are those values of the parameters that would generate the observed sample most often (18).

The objective of finding the maximum likelihood estimators can be accomplished through the formulation of a likelihood function (denoted

ℓ). If the sample observations are independent, ℓ has the same formula as the joint probability function as defined

$$\ell = f(X_1, X_2, \dots, X_n)$$

or because of independence

$$\ell = f(X_1), f(X_2), \dots, f(X_n).$$

The maximum likelihood estimates are found by maximizing the likelihood function with respect to the parameters $\theta_1, \theta_2, \dots, \theta_k$. This is accomplished by setting the partial derivative of ℓ with respect to each of the unknown parameters equal to zero and solving simultaneously. This procedure (sometimes called the first order conditions) produces a relative maximum (or minimum) certain second order conditions have to be fulfilled.

To insure a relative maximum (minimum) the second derivative of ℓ with respect to the parameters $\theta_1, \theta_2, \dots, \theta_k$ must be negative (positive). If the second derivative equals zero, the test fails and other trial and error tests must be used.

STATISTICAL TESTS

Two tests were used to determine the statistical significance of the variables in the model.

The t-test was used to test the null hypothesis that the para-

meter β_k equals zero, implying no relationship between the independent variable X_k and the dependent variable Y . The appropriate test statistic is

$$-t_{n-k,p/2} \leq \frac{\hat{\beta}_k}{S_{\hat{\beta}_k}} \leq t_{n-2,p/2}$$

where:

n = number of observations

k = number of parameters

p = level of significance

$\hat{\beta}$ = estimated parameter

$S_{\hat{\beta}_k}$ = standard error of $\hat{\beta}_k$.

The second statistical test utilized is the likelihood ratio test.

This test is based on the assumption that the negative of twice the natural logarithm is asymptotically distributed as a χ^2 . The test statistic used is

$$n \ln \frac{D_S}{D_L} \approx \chi_q^2$$

where:

D = the determinant of the residual covariance matrix;

S = restricted regression;

L = general regression;

q = the difference in the number of degrees of freedom between the two models; and

n = number of observations.

A more intuitive explanation follows: If when comparing the plausibility of one value of β against another, given the sample $y_1, y_2, \dots, y_n$, that β which gives the likelihood function a larger value would be chosen. If an appreciably larger value cannot be found by searching different values of β other than those specified by the null hypothesis (H_0), then the most plausible value of β belongs to the set specified by H_0 . Under these conditions, H_0 would be accepted as true.

ESTIMATION PROCEDURES

Because of simplicity and lower cost, estimates were first made utilizing the additive error model. Table 1 displays the list of variables previously defined, their means, standard errors and adjusted coefficients of variation.

DETERMINING RELEVANT VARIABLES

Preliminary results implied an implausible behavior for the soil water and precipitation variables. Parameters for soil moisture showed increasing marginal response over most ranges of the data, but with incremental response negative for very low levels. Results also indicated increasing marginal returns for precipitation well beyond the

TABLE 1
VARIABLES USED TO EXPLAIN VARIATIONS IN YIELD AND
PERCENT PROTEIN IN WINTER WHEAT

Variable	Variable Description	Units	High	Low	Mean	Standard Deviation	Adjusted Coefficient of Variation
Y	Grain Yield	bu/A	76.16	13.00	40.55	12.88	.4675
X ₁	Applied Nitrogen (N)	lb/A	89.00	0.00	44.50	30.44	.6839
X ₂	(N) ²	lb/A	7921.00	0.00	2904.90	2849.20	.9808
X ₃	Applied phosphorus (P)	lb/A	35.70	0.00	17.83	12.21	.6851
X ₄	(P) ²	lb/A	1274.50	0.00	466.75	458.75	.9829
X ₅	Precipitation (Prec)	in.	18.68	5.15	9.69	3.97	.8725
X ₆	(Prec) ²	in.	348.94	26.52	109.72	91.09	1.1095
X ₇	Soil Water (SW)	in.	7.46	.24	2.98	1.53	.5575
X ₈	(SW) ²	in.	55.65	.06	11.24	12.21	1.0922
X ₉	Soil Nitrate (SN)	lb/A	291.50	12.00	62.46	49.62	.9833
X ₁₀	(SN) ²	lb/A	84972.00	144.00	6358.40	11547.00	1.8581
X ₁₁	Soil Phosphorus (SP)	lb/A	55.30	.70	14.02	6.93	.5205
X ₁₂	(SP) ²	lb/A	3058.00	.49	244.63	271.24	1.1110
X ₁₃	N X P		3177.30	0.00	793.47	877.00	1.1053
X ₁₄	N X Prec		1662.50	0.00	421.47	364.42	.8466
X ₁₅	N X SN		23389.00	0.00	2778.90	3296.50	1.1863
X ₁₆	Prec X SW		58.93	1.82	20.83	15.88	.8351
X ₁₇	Prec X SN		1997.20	82.92	589.70	439.04	.8663
X ₁₈	SW X SN		938.88	10.96	185.10	182.40	1.0474
X ₁₉	P X SP		1417.3	0.00	250.24	227.79	.9103
X ₂₀	P X Prec		666.88	0.00	172.85	146.17	.8456
X ₂₁	Potassium (K)	lb/A	85.20	0.00	42.57	29.11	.6837
X ₂₂	K ²	lb/A	7259.00	0.00	2658.30	2608.80	.9814

mean of 6.7 inches. Unlike soil moisture, precipitation had a positive marginal response at low levels.

Such results are difficult to understand, but are most likely due to serious errors in specification. Soil water has only one measurement corresponding to each site which seems reasonable considering the sites were selected to have uniform soil type and terrain. However, when considering different sites, soil water could be indirectly measuring certain site characteristics which by themselves could affect yield, such as permeability, terrain and other site specific characteristics. Similar problems exist with precipitation (e.g., did the moisture come all at once so that there was considerable runoff?).

Results from similar work done previously (6) indicated that combining soil water and precipitation may eliminate the dubious results of these resources. Combining these variables into a total water variable was completed in an effort to obtain more plausible results. It was hoped that this effort would also produce variables that were statistically significant. In the model displayed in Table 1, precipitation is redefined as total water, and soil water variables are excluded.

Unfortunately this tactic did not produce the desired results. Though long considered the most limiting input in wheat yield, results showing a negative marginal product at low levels of the water variable, followed by increasing marginal returns was implausible. Because of

these dubious results and lack of statistical significance according to the t-test variables, X_5, X_6, X_{17} were eliminated from the yield response estimation. The consequence of this action is to transfer the net effect of these variables into the error term. The exclusion of these variables did not affect the value of the other parameters. The interaction terms X_{14} and X_{20} were retained in the analysis because of very high statistical significance. The final model, with newly defined variables appears on Table 2.

MULTIPLICATIVE V.S. ADDITIVE ERROR

The hypothesis that the error components are proportional to the mean response function has some intuitive appeal for agronomic data. Because of the high cost of estimation with a multiplicative error model, the model was first estimated as previously described using the additive error model. A test (outlined by Just and Hallam, 15) was then used to determine the appropriateness of the multiplicative model.

The multiplicative error model can be expressed as

$$Y_{ij} = f(X_k)u_{ij}$$

where:

$$E(u_{ij}) = 1; \text{ and}$$

$$\text{Var}(u_{ij}) = E(u_{ij} - E(u_{ij}))^2 = \sigma_u^2 = \text{constant}.$$

TABLE 2
REGRESSION COEFFICIENTS FOR THE YIELD RESPONSE MODEL

Coefficient	Associated Variable	Estimate	t-Value
A	1	18.95400	6.28
b ₁	Applied Nitrogen (N)	.12431	4.90
b ₂	N ²	-.00131	-6.40
b ₃	Applied Phosphorus (P)	.25199	4.11
b ₄	P ²	-.00301	-2.38
b ₅	Soil Nitrate (SN)	.10236	3.96
b ₆	SN ²	-.00019	-2.29
b ₇	Soil Phosphorus (SP)	.58394	4.52
b ₈	SP ²	-.00873	-3.45
b ₉	N X P	.00088	2.26
b ₁₀	N X Water (W)	.01778	13.06
b ₁₁	N X SN	-.00088	-8.27
b ₁₂	P X SP	-.01129	-4.78
b ₁₃	W X P	.00958	2.46
b ₁₄	Potassium (K)	.02939	1.49
b ₁₅	K ²	-.00018	-.83

Standard Error of the Estimate = 10.98

N, P, SN, SP, K measured in lbs/acre.
W measured in inches.

Critical t-values;

10 percent level of significance - 1.64
5 percent level of significance - 1.96
1 percent level of significance - 2.58

Similarly, the additive error model can be expressed as

$$Y_{ij} = f(X_k) + \epsilon_{ij}$$

where:

$$E(\epsilon_{ij}) = 0; \text{ and}$$

$$\text{Var}(\epsilon_{ij}) = E(\epsilon_{ij})^2 = \sigma_\epsilon^2 = \text{constant}.$$

The test involves regressing the absolute value of the error term ($\hat{\epsilon}_{ij}$) from the additive error model (eq. 2.1) on the least squares estimate of the expectation of the dependent variable, i.e., on $\hat{Y}_{ij}$, the estimator for $E(Y_{ij})$. The estimated equation should take the form

$$|\hat{\epsilon}_{ij}| = a + bY_{ij}.$$

If the multiplicative error model is appropriate, the results from the regression should approximate

$$|\hat{\epsilon}_{ij}| = \sigma_u E(Y_{ij}).$$

σ_u can be approximated by $\hat{\sigma}_\epsilon / \bar{Y}$. Because the identity covariance is assumed, 'b' should not be statistically different from $\hat{\sigma}_\epsilon / \bar{Y}$, and 'a' should be close to zero.

An F-test was used to test the null hypothesis expressed as

$$H_0: \alpha = 0, \beta = \hat{\sigma}_\epsilon / \bar{Y}$$

where α and β are least squares estimates, and $\bar{Y}$ is the sample mean.

The results of the F-test at the .01 level indicated that the null hypothesis could not be accepted and the multiplicative disturbance model was discarded.

However, the estimated coefficient 'b' was positive and statistically significant at the .05 level when the t-test was used to test the null hypothesis expressed as

$$H_0: \beta = 0.$$

This result would seem to indicate the presence of serious errors in specification even with the additive error model.

ESTIMATED YIELD

The final yield equation from Table 2 was used to estimate expected yield for varying soil and moisture conditions, as well as varying fertilizer application rates.

Table 3 illustrates the yields to be expected when total water, soil phosphorus and applied potassium are at their mean levels, but with soil nitrate at the low level of 30 pounds per acre.

Increasing nitrogen up to 100 pounds per acre increases yields in all cases. The same is not true for phosphorus, which actually decreases yields when applied at a rate greater than 40 pounds per acre, especially if applied nitrogen is applied at low levels.

Table 4 illustrates the partial derivative of yield with respect

TABLE 3

ESTIMATED YIELD OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	29.2	30.8	31.8	32.2	32.1	31.3
10	31.9	33.6	34.7	35.2	35.1	34.3
20	34.3	36.0	37.2	37.8	37.8	37.2
30	36.4	38.3	39.5	40.2	40.3	39.7
40	38.2	40.2	41.6	42.3	42.5	42.0
50	39.8	41.9	43.3	44.2	44.4	44.1
60	41.2	43.3	44.8	45.8	46.1	45.8
70	42.3	44.5	46.1	47.1	47.6	47.4
80	43.1	45.4	47.1	48.2	48.7	48.6
90	43.6	46.0	47.8	49.0	49.6	49.6
100	43.9	46.4	48.3	49.6	50.3	50.4

Exogenous Variables

Water	10.1 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

TABLE 4

PARTIAL DERIVATIVE OF YIELD WITH RESPECT TO APPLIED NITROGEN
AT SPECIFIED LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	.28	.29	.30	.30	.31	.32
10	.25	.26	.27	.28	.29	.30
20	.23	.23	.25	.25	.26	.27
30	.20	.21	.22	.23	.23	.24
40	.17	.18	.20	.20	.21	.22
50	.15	.16	.17	.17	.18	.19
60	.12	.13	.14	.15	.16	.16
70	.09	.10	.12	.12	.13	.14
80	.07	.08	.09	.09	.10	.11
90	.04	.05	.06	.07	.08	.09
100	.02	.03	.04	.04	.05	.06

Exogenous Variables

Water	10.1 Inches
Soil Nitrate	30.3 Lbs./Acre
Soil Phosphorus	14.4 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

to applied nitrogen. This marginal response decreases as nitrogen is increased due to the negative coefficient associated with the nitrogen quadratic term. Similarly, the marginal response associated with applied nitrogen increases as applied phosphorus is increased. This is due to the positive interaction term for these two variables.

Tables 1-7 in the appendix display yield responses for other variables, including water and soil nitrate.

MARGINAL RATES OF SUBSTITUTION

When two inputs serve the same biological function for the winter wheat plant, one might expect the inputs to substitute for each other in a fixed proportion, regardless of the level of these or any other variables. If this were true, the winter wheat producer, in determining the quantity of nitrogen to apply, would need only to look at the level of soil nitrate, and then apply nitrogen fertilizer to reach the desired level for total nitrogen. However, constant marginal rates of substitution for soil nitrate and applied nitrogen as well as soil phosphorus and applied phosphorus, are not implied by the estimated response surface for winter wheat. Because of this, nutrient application rates based on the assumption of constant marginal rates of substitution may cause application rates to differ significantly from those suggested by the response function estimated in this study.

Table 5 displays the marginal rates of substitution of applied

TABLE 5

MARGINAL RATES OF SUBSTITUTION
APPLIED NITROGEN AND SOIL NITRATE

Soil Nitrate Lbs./Acre	Applied Nitrogen Lbs./Acre				
	0	20	40	60	80
0	3.12	3.16	3.21	3.30	3.50
20	3.18	3.23	3.32	3.47	3.85
40	3.25	3.32	3.45	3.70	4.50
60	3.30	3.44	3.63	4.07	6.19
80	3.43	3.58	3.88	4.71	--
100	3.55	3.77	4.25	6.15	--
120	3.68	4.03	4.87	--	--
140	3.89	4.40	6.12	--	--

Exogenous Variables

Water 10.1 Inches

Applied Phosphorus 17.8 Lbs./Acre

nitrogen for soil nitrate ($MP_N/MP_{SN} = MRS_{N \rightarrow SN}$), where the marginal product of nitrogen, for example, is equal to the partial change in yield with respect to a change in applied nitrogen, ($\partial Y / \partial N$). Marginal rates of substitution imply a trade-off in production. In Table 5, a marginal rate of substitution of 4.0 would indicate that one pound of nitrogen could be substituted for four pounds of soil nitrate, with no affect on yield.

Table 5 indicates that as nitrogen is increased for any given level of soil nitrate, the relative value of applied nitrogen is increased.

The marginal rates of substitution for different variables, including water and soil nitrate can be found in the appendix on Tables 8 and 9.

PROTEIN ESTIMATION

The final model used to predict yield was also employed in the protein response estimation. Several variables, found to be crucial in explaining yield were not statistically significant in the protein estimation. Linear and quadratic soil phosphorus, though important in explaining variation in yield, were not statistically significant at the .20 level according to the t-ratio, and were therefore excluded from the analysis. It is difficult to understand why soil phosphorus is not important in explaining protein content while applied phos-

phorus is. This may be the consequence of using soil phosphorus measurements to a depth of two feet while one foot may have been more appropriate.

Other variables excluded from the analysis due to statistical insignificance were quadratic soil nitrate, and the interaction terms, applied phosphorus, soil phosphorus and applied phosphorus water. No additional variables were included. The final variables, coefficients and t-ratios can be found in Table 6.

Applied nitrogen has a positive and increasing effect on protein content over most ranges of the data. The response function does indicate applied nitrogen has a negative influence on protein content when soil nitrate is at extremely high levels and applied nitrogen is very low. The most likely explanation for this phenomenon is a lack of precision in the polynomial response to observations on the fringes of the data. Soil nitrate had a similar effect on protein content, with the marginal response being positive over all ranges of the data. Soil nitrate did not, however, show increasing marginal response as its level increased.

Applied phosphorus exhibited a negative marginal response to protein content over most ranges of the data. At extremely high levels, phosphorus had a positive effect on protein when applied nitrogen was at its mean level. The affect of potassium was nearly the same, with a negative marginal response on protein well past the potassium

TABLE 6

REGRESSION COEFFICIENTS FOR PROTEIN ESTIMATION

Coefficient	Associated Variable	Estimate	t-value
b_0	1	9.60520	22.15
b_1	Applied Nitrogen (N)	.04025	6.82
b_2	N^2	.00011	2.20
b_3	Applied Phosphorus (P)	-.02652	-2.26
b_4	P^2	.00049	1.67
b_5	Soil Nitrate (SN)	.01884	7.40
b_6	N X P	-.00010	-1.07
b_7	N X Water (W)	-.00058	-1.82
b_8	N X SN	-.00018	-7.07
b_9	Applied Potassium (K)	-.01523	-3.31
b_{10}	K^2	.00014	2.66

Standard Error of the Estimate = 1.88

N, P, SN, SP, K measured in lbs/acre.
W measured in inches.

Critical t-values:

10 percent level of significance = 1.64
5 percent level of significance = 1.96
1 percent level of significance = 2.57

mean of 42 pounds per acre.

The t-ratios displayed in Table 6 indicate that all but three parameter estimates are significantly different from zero at the .05 level of significance. Of these three, two, applied phosphorus squared and the interaction term applied nitrogen-water were significant at the .1 level.

ESTIMATED PROTEIN

The estimated protein equation was used to estimate expected protein under varying soil and moisture conditions, as well as varying fertilizer application rates.

The results displayed in Table 7 show estimated protein content when the soil and climatic variables are at mean levels, except for soil nitrate which is at the low level of 30 pounds per acre.

Increasing nitrogen up to at least 100 pounds increases protein without exception. Phosphorus, however, has the opposite effect. In all cases, applied phosphorus decreases yield, except for application rates of over 30 pounds (see appendix Table 14). Table 8 displays the marginal response to changes in the nitrogen application rates, which are all positive.

Tables 10-16 in the appendix display protein responses for other variables with varying soil and climatic conditions.

TABLE 7

ESTIMATED PROTEIN OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	9.8	9.6	9.4	9.4	9.5	9.7
10	10.1	9.8	9.7	9.7	9.8	9.9
20	10.4	10.2	10.0	10.0	10.0	10.2
30	10.7	10.5	10.3	10.3	10.3	10.5
40	11.1	10.8	10.7	10.6	10.7	10.8
50	11.5	11.2	11.1	11.0	11.0	11.1
60	11.9	11.6	11.4	11.4	11.4	11.5
70	12.3	12.0	11.8	11.8	11.8	11.9
80	12.8	12.5	12.3	12.2	12.2	12.3
90	13.2	12.9	12.7	12.6	12.6	12.7
100	13.7	13.4	13.2	13.1	13.1	13.1

Exogenous Variables

Water	10.0 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

TABLE 8

PARTIAL DERIVATIVE OF PROTEIN WITH RESPECT TO APPLIED NITROGEN
AT SPECIFIED LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	.03	.03	.03	.03	.03	.02
10	.03	.03	.03	.03	.03	.03
20	.03	.03	.03	.03	.03	.03
30	.04	.03	.03	.03	.03	.03
40	.04	.04	.04	.03	.03	.03
50	.04	.04	.04	.04	.04	.03
60	.04	.04	.04	.04	.04	.04
70	.04	.04	.04	.04	.04	.04
80	.05	.04	.04	.04	.04	.04
90	.05	.05	.05	.05	.04	.04
100	.05	.05	.05	.05	.05	.05

Exogenous Variables

Water	10.1 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Chapter 4

PROFIT MAXIMIZING FERTILIZER POLICIES

Montana winter wheat producers must incorporate input (fertilizer) and output (wheat) prices with the yield equation (Table 2) to determine the profit maximizing fertilizer strategy. At first, it is assumed that producers can purchase all of the fertilizer necessary to attain this maximum profit, i.e., there are no effective borrowing or capital constraints faced by producers. Then capital constraints are considered.

UNLIMITED CAPITAL

Equation 4.1 below can be used to determine profit maximizing rates of fertilizer application. This is done by setting the first order conditions (equations 4.3, 4.4, 4.5) equal to zero and solving for the input variables nitrogen, phosphorus, and potassium. It is assumed that input and output prices are constant for each decision.

The profit equation is

$$4.1 \quad \Pi = P_y Y - P_n N - P_p P - P_k K$$

where:

Π = profit

P_y = price of wheat per bushel;

Y = yield of wheat per acre, as computed by the yield response equation;

P_n, P_p, P_k = price per pound of nitrogen, phosphorus, and potassium respectively; and

N, P, K = pounds per acre of nitrogen, phosphorus, and potassium, respectively.

P_y , price of wheat per bushel is considered exogenous when the protein premium is ignored in the decision process. When a protein premium structure is included, P_y is considered endogenous, as it depends upon the levels of nitrogen, phosphorus, and potassium. Both cases will be considered. All other prices are strictly exogenous.

The profit maximizing equation, after incorporating the expected yield estimation from Table 2 and considering P_y as exogenous is

$$4.2 \quad \Pi = P_y(b_o + b_1N + b_2N^2 + b_3P + b_4P^2 + b_5SN + b_6SN^2 + b_7SP + b_8SP^2 + b_9NP + b_{10}NW + b_{11}NSN + b_{12}PSP + b_{13}WP + b_{14}K + b_{15}K^2) - P_NN - P_PP - P_KK$$

where all variables and parameters are as previously defined. By taking the partial derivatives of equation (E) with respect to the input variables N, P , and K , we have the first-order conditions

$$4.3 \quad \frac{\partial \Pi}{\partial N} = P_y(b_1 + 2b_2N + b_9P + b_{10}W + b_{11}SN) - P_N;$$

$$4.4 \quad \frac{\partial \Pi}{\partial P} = P_y(b_3 + 2b_4P + b_9N + b_{12}SP + b_{13}W) - P_P;$$

$$4.5 \quad \frac{\partial \Pi}{\partial K} = P_y (b_{14} + 2b_{15}K) - P_K .$$

Since the variables W , SN , SP , and, in this case P_y , are exogenous, equations 4.3, 4.4, and 4.5 can be set equal to zero and solved for the variables N , P , and K . Second-order conditions $\frac{\partial^2 \Pi}{\partial N^2}$, $\frac{\partial^2 \Pi}{\partial P^2}$, $\frac{\partial^2 \Pi}{\partial K^2}$, must all be non-positive to ensure a maximization.

LIMITED CAPITAL

In the previous section, the decision making criterion ignored the distinct possibility that the producer may have an effective capital constraint. In the case of unlimited capital, the last dollar per acre spent on fertilizer yielded an equal value of wheat. If the producer cannot reach this point, a new decision rule must be implemented.

When confronted by a binding capital constraint, the last dollar spent on each input should produce an output valued at more than one dollar, and this output should be equal for all inputs. When the limited capital decision rule is used, it must be used in all facets of production, not just fertilizer.

The constrained maximization profit function can be expressed the Lagrangian form

$$4.6 \quad \Pi = P_y Y - P_N N - P_P P - P_K K + \lambda (P_N N + P_P P + P_K K - C)$$

where:

C = the capital constraint in dollars per acre; and

λ = the Lagrangian multiplier.

By substituting the yield equation from Table 2 into the above expression for Y , and taking the partial derivative with respect to the unknown variables, N , P , K , and λ , we obtain the first order conditions

$$4.7 \quad \frac{\partial \Pi}{\partial N} = P_y(b_1 + 2b_2N + b_9P + b_{10}W + b_{11}SN) - P_N + \lambda P_N$$

$$4.8 \quad \frac{\partial \Pi}{\partial P} = P_y(b_3 + 2b_4P + b_9N + b_{12}SP + b_{13}W) - P_P + \lambda P_P$$

$$4.9 \quad \frac{\partial \Pi}{\partial K} = P_y(b_{14} + 2b_{15}K) - P_K + \lambda P_K$$

$$4.10 \quad \frac{\partial \Pi}{\partial \lambda} = P_NN + P_PP + P_KK - C.$$

These first-order conditions are then set equal to zero and solved for the four unknowns.

FERTILIZER RECOMMENDATIONS WITHOUT PROTEIN RESPONSE

Table 9 displays the optimization results for both limited and unlimited capital, under various soil and moisture conditions. Exogenous variables relevant to the optimization are soil nitrate, measured to four feet in early spring, soil phosphorus, measured to two feet, and total water, which consists of soil water measured to four feet in early spring and April through July precipitation.

TABLE 9

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	21.97*	44.2	75.1	9.0	1.00	42.98	32.53
	15.00	41.7	66.4	0.6	1.21	35.28	20.28
	10.00	39.1	45.5	0.0	1.95	27.56	17.56
\$3.50	26.76*	45.7	81.0	14.6	1.00	55.33	28.57
	20.00	43.5	72.6	6.6	1.24	47.78	27.78
	15.00	41.6	66.4	0.6	1.41	41.16	26.16
	10.00	39.1	45.4	0.0	2.27	32.15	22.15
\$4.00	30.35*	46.7	85.5	18.9	1.00	67.08	36.73
	25.00	45.2	78.8	12.6	1.21	61.16	36.16
	20.00	43.5	72.6	6.6	1.41	54.60	34.60
	15.00	41.7	66.4	0.6	1.61	47.05	32.05
	10.00	39.1	45.5	0.0	2.60	36.74	26.74

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*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61¢/Lb.

Soil Nitrate 48.0 Lbs./Acre
Soil Phosphorus 14.0 Lbs./Acre
Water 10.0 inches

Potassium, (K), was eliminated from the analysis because it is not economic to apply potassium under current conditions. In order for potassium to be economic at its current price of 16¢ per pound, the price of wheat would have to be \$5.70 per bushel. Inversely, given a wheat price of \$4.00 per bushel, potassium would only be economic at a price slightly below 12¢ per pound.

Column 2 in Table 9 indicates the capital per acre spent on fertilizer for both limited and unlimited capital cases. Column 6 shows the revenue generated from the last dollar spent on fertilizer which is always equal to one dollar under the unlimited capital case and is always greater than one dollar in the limited capital case.

Tables 17-22 in the appendix contain optimization information under different soil and climatic conditions.

Figure 1 also represents nitrogen and phosphorus recommendations without a protein premium. This isocline-isoquant map does not indicate the actual quantity of these inputs to use, it only shows the appropriate input mix for these two inputs.

Isoquants for three different fertilizer yields (5 bu., 10 bu., 15 bu.) intersect the isoclines. The isoclines (the rays drawn from the axes to the maximum point) indicate the appropriate input mix for nitrogen and phosphorus, given their price ratios.

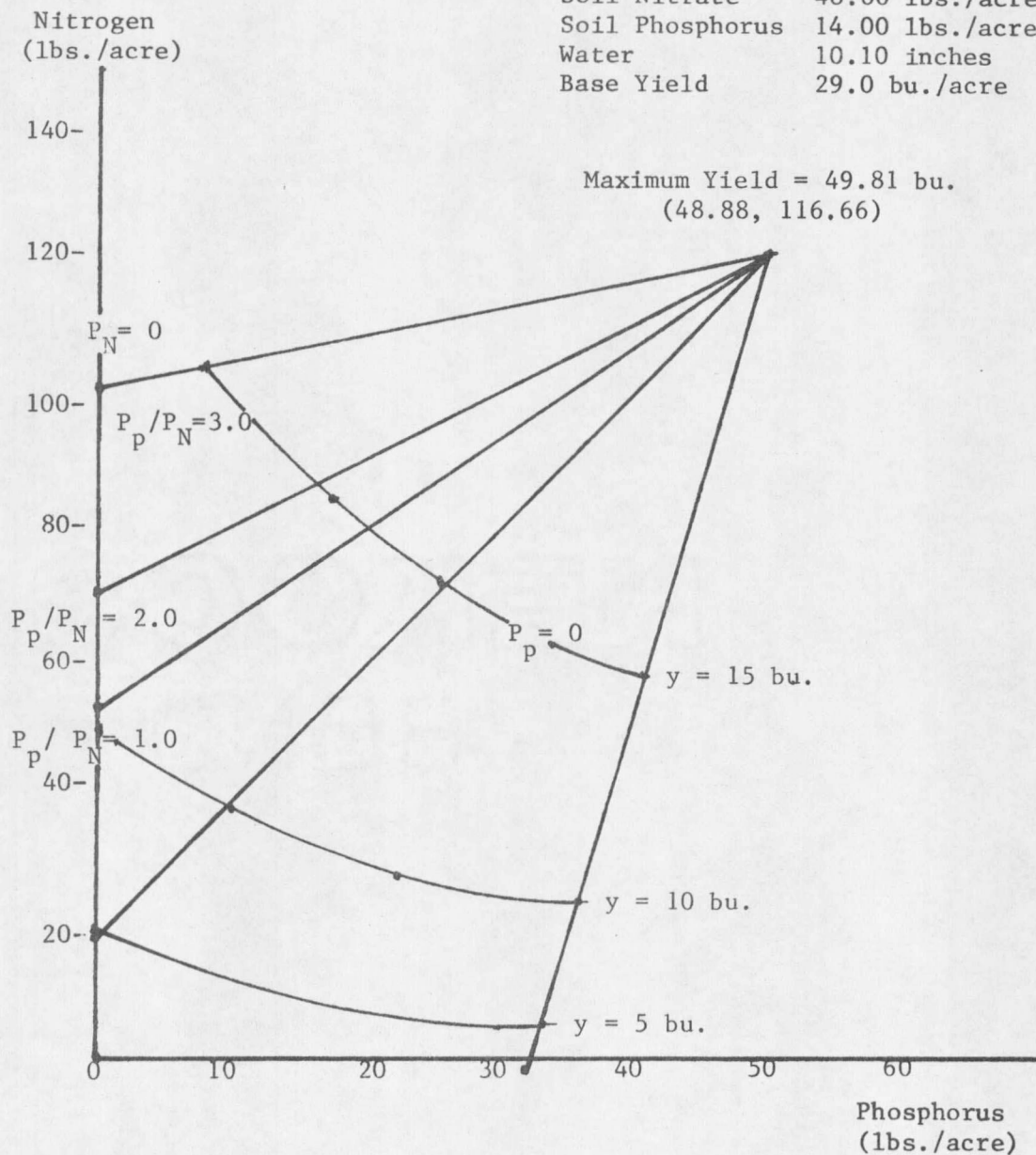
Exogenous variables displayed also include 'base yield'. This is the yield obtained with no fertilizer applications being made.

Figure 1

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED YIELD
INCREASES OF WINTER WHEAT AT SPECIFIED PRICE RATIOS
FOR PHOSPHORUS AND NITROGEN

Exogenous Variables

Soil Nitrate	48.00 lbs./acre
Soil Phosphorus	14.00 lbs./acre
Water	10.10 inches
Base Yield	29.0 bu./acre



Maximum yield is the highest yield attainable by applying fertilizer. Obtaining this yield would be uneconomic given current fertilizer and winter wheat prices.

Appendix figures 1-5 display isocline-isoquant maps for varying levels of the soil and water variables.

FERTILIZER PROFIT MAXIMIZATION WITH PROTEIN PREMIUM

When a protein premium is considered in the optimization strategy, the price of wheat is endogenous because it depends upon the level of protein. In turn, protein content depends upon the level of fertilizer (as well as various soil and climatic conditions, over which the producer has little control).

Utilizing a protein premium structure, P_y is substituted in equation 4.2 above with

$$4.11 \quad P_y = BP_y + \alpha_1 \%P + \alpha_2 \%P^2$$

Where: BP_y = base price for wheat when protein content is 10 percent or less;

$\%P$ = percent protein, derived from protein estimation from Table 6; and

α_1 and α_2 = parameters of the equation.

First-order conditions are then derived by taking the partial derivatives $\frac{\partial \Pi}{\partial N}$, $\frac{\partial \Pi}{\partial P}$ and $\frac{\partial \Pi}{\partial K}$. The resulting three equations are very

complicated, in that there are so many terms. As a result, the first-order conditions offer no intuitive explanation and are therefore left out.

In order to make the problem more intuitive, the first-order conditions are derived from profit equation 4.1. Note that P_y is endogenous.

$$4.12 \quad \frac{\partial \Pi}{\partial N} = P_y \frac{\partial Y}{\partial N} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial N} \right) Y - P_N ;$$

$$4.13 \quad \frac{\partial \Pi}{\partial P} = P_y \frac{\partial Y}{\partial P} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial P} \right) Y - P_P ; \text{ and}$$

$$4.14 \quad \frac{\partial \Pi}{\partial K} = P_y \frac{\partial Y}{\partial K} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial K} \right) Y - P_K ;$$

The first term on the right hand side of the equation concerns itself with the change in profits, caused by an increase in the fertilizer input. The second term measures the variation in profits as affected by the resulting change in price of wheat due to the change in protein content. As outlined in the section on protein estimation, each fertilizer input has a different effect upon protein content.

Second-order conditions must again be non-positive to insure a maximization.

When the producer is faced with a binding budget constraint, the profit equation becomes

$$\Pi = P_y P - P_N N = P_P P - P_K K + \lambda (P_N N + P_P P + P_K K - C)$$

where: λ is the lagrangian multiplier; and

C is the amount of capital available per acre.

The resulting first-order conditions are

$$\frac{\partial \Pi}{\partial N} = P_y \frac{\partial Y}{\partial N} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial N} \right) Y - P_N + \lambda P_N ;$$

$$\frac{\partial \Pi}{\partial P} = P_y \frac{\partial Y}{\partial P} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial P} \right) Y - P_P + \lambda P_P ;$$

$$\frac{\partial \Pi}{\partial K} = P_y \frac{\partial Y}{\partial K} + \left(\frac{\partial P_y}{\partial \%P} \cdot \frac{\partial \%P}{\partial K} \right) Y - P_K + \lambda P_K ; \text{ and}$$

$$\frac{\partial \Pi}{\partial \lambda} = P_N N + P_P P + P_K K - C .$$

Due to the fact that no algorithm was available to solve these first-order conditions, no results were derived utilizing a protein premium.

If results were obtained using a protein premium, optimal nitrogen recommendations would have been increased, due to the positive effect nitrogen has on protein. Similarly, recommended levels of phosphorus would have decreased, and potassium would not become economic at its current price of 16¢ per pound even given a wheat price of \$5.70, as in the optimization technique ignoring protein.

IMPLICATIONS OF DIMINISHING MARGINAL RATES OF SUBSTITUTION
OF APPLIED NITROGEN FOR SOIL NITRATE

MSU soil specialists currently utilize a fertilizer recommendation model for winter wheat which implies that the marginal rate of substitution of applied nitrogen for soil nitrate is constant and equal to one. These assumptions are contrary to the results of the yield response function estimated in this study. First, the estimated yield response relationship indicates diminishing marginal rates of substitution (DMRS) of applied nitrogen for soil nitrate. This means that the ability of applied nitrogen to replace soil nitrate diminishes as applied nitrogen is increased relative to soil nitrate while maintaining a specified level of yield. Also, the results of the estimated yield response function indicate that the marginal rate of substitution of applied nitrogen for soil nitrate is greater than one. For instance, a 40 bushel yield can be obtained with applied nitrogen-soil nitrate combinations varying from zero pounds of soil nitrate and 42.5 pounds of applied nitrogen to 140 pounds of soil nitrate and 2.8 pounds of applied nitrogen. Over this range of applied nitrogen-soil nitrate combinations, the marginal rate of substitution varies from 3.2 to 3.9. The results point out a rather significant discrepancy between the assumptions of the fertilizer recommendation model currently in use by soil specialists and the results of the estimated response function.

The critical comparison is how much discrepancy there is in the

economic outcomes of recommendations which arise from the use of the two models. The results of such a comparison are presented in Table 10

Basically, the constant rate of marginal substitution model (CMRS) computes applied nitrogen requirements by multiplying estimated wheat yield times two pounds of nitrogen per bushel and subtracting the amount of soil nitrate available in the top four feet of soil.

It is evident that the CRMS model consistently estimates the need for less applied nitrogen than the model based on the estimated yield relationship. The main factor operating here is the assumption in the CRMS model that a pound of applied nitrogen and a pound of soil nitrate are equivalent. The estimated economic consequences of the two models are quite different, especially in the mid-range of soil nitrate levels. Certainly, it is imperative that further research and study be directed at estimating marginal rates of substitution of applied nitrogen for soil nitrate.

TABLE 10

COMPARISON OF FERTILIZER RECOMMENDATIONS FOR CONSTANT MARGINAL RATE OF
SUBSTITUTION MODEL AND DIMINISHING MARGINAL RATE OF SUBSTITUTION MODEL

Item	Level of Soil Nitrate (lbs. in upper 4 foot)									
	30		50		70		110		180	
	CMRS	DMRS	CMRS	DMRS	CMRS	DMRS	CMRS	DMRS	CMRS	DMRS
Applied Nitro- gen (lbs./acre)	57.0	84.0	37.0	75.0	17.0	70.0	0	57.0	0	33.0
Yield (bu./acre)	41.2	43.7	38.9	43.2	36.4	43.4	35.3	43.1	38.7	42.2
Return Over Nitrogen Cost (ROVC) (\$/acre)*	29.24	32.13	19.36	26.22	9.60	22.57	0	14.70	0	4.99
Difference in ROVC (\$/acre)	2.88		3.86		12.97		14.70		4.99	

*Wheat valued at \$3.50 per bushel and nitrogen valued at \$.22 per pound.

Chapter 5

SUMMARY AND CONCLUSIONS

Data from 19 winter wheat experiments in south central Montana were used to estimate yield and protein response equations. These data were collected by Dr. Vincent Haby, soil scientist, Montana State University, of the Southern Agricultural Research Center, Huntley, during the years 1976, 1977, and 1978. These data consist of observations from the first three years of a five-year study. All experiments were conducted on dryland, summer fallow conditions. The winter wheat variety at all sites was Centurk. Observations were made on soil nitrate, and soil water, both to a depth of four feet; April through July precipitation, and soil phosphorus measured to a depth of two feet. Fertilizer application rates ranged from 0 to 89 pounds per acre for nitrogen; 0 to 35.7 pounds per acre for phosphorus; and 0 to 85.6 pounds per acre for actual potassium.

Yield response was estimated using a second degree polynomial equation with an additive component error term. The estimation technique utilized maximum likelihood estimators in a generalized nonlinear regression framework. Results from the estimation implied a behavior for soil water and precipitation which were contrary to the maintained hypothesis. Parameters for soil moisture showed increasing marginal response over most ranges of the data, but with incremental response negative for very low levels. Results also showed increasing

returns for precipitation well beyond the mean of 6.7 inches.

Results such as these are difficult to understand, but are likely due to serious errors in specification. Soil water, which has only one measurement for each site, may be indirectly measuring soil characteristics which vary across sites, such as permeability and terrain. Similar exist with precipitation. Precipitation falling in April certainly affects yield differently than precipitation occurring in July. Because of this, further research in this area should separate precipitation into different time periods.

Due to the dubious results concerning soil water and precipitation, these variables were combined into a total water variable. Results stemming from this combination were little different than considering soil water and precipitation separately. Consequently, water was eliminated from the analysis except as a component of interaction variables with both nitrogen and phosphorus.

The final equation estimating yield can be expressed as

$$\begin{aligned}
 Y = & 18.954 + .12343 X_1 - .00131 X_2 \\
 & + .25199 X_3 - .00301 X_4 + .10236 X_5 - .00019 X_6 \\
 & + .58394 X_7 - .00873 X_8 + .00088 X_9 \\
 & + .01778 X_{10} - .00088 X_{11} - .01129 X_{12} \\
 & + .00958 X_{13} - .02939 X_{14} - .00018 X_{15} \\
 & + W_{ij} + Z;
 \end{aligned}$$

Where:

Y = Estimated Yield;

X_1 = Applied Nitrogen (N);

$X_2 = N^2$;

X_3 = Phosphorus (P);

$X_4 = P^2$

X_5 = Soil Nitrate (SN);

$X_6 = SN^2$;

X_7 = Soil Phosphorus (SP);

$X_8 = SP^2$;

$X_9 = N \times P$;

$X_{10} = N \times \text{Water}$;

$X_{11} = N \times \text{SN}$;

$X_{12} = P \times \text{SP}$;

$X_{13} = P \times \text{Water}$;

$X_{14} = \text{Potassium (K)}$;

$X_{15} = K^2$;

W_{ij} = random variation associated with the i th treatment at
location and time j ; and

Z_j = random variation associated with different sites and/or
different years.

All resource inputs in this response function were characterized

by positive marginal products followed by maximum yield and negative marginal products. All parameter estimates were significant to at least the .05 level except X_{14} and X_{15} according to the t-statistic. Taken together, X_{14} and X_{15} were significant at the .01 level according to the likelihood ratio test.

The coefficient of correlation among treatments for a given year and location, (the ratio of the variance of the location-time specific effect (σ_z^2) to the total variance (σ^2)) was approximately .87. This implies that approximately 87 percent of the unexplained variation was due to heterogeneous sites and/or temporal effects. However, intermediate model specifications included an analysis limiting these data to sites which were repeated at least two years. This analysis also included dummy variables for location. Comparison of the variances and coefficients of correlation between the two models indicates that approximately 70 to 80 percent of the unexplained variation in the final model is caused by heterogeneous sites. This result indicates the importance of homogeneous locations in crop fertilizer response experiments.

An alternative model proposed for estimating yield response utilized a multiplicative component disturbance term. This model implies a direct proportional relationship between the component error term and the dependent variable, yield. A test used to measure the appropriateness of this model suggested that the additive distur-

bance was at least as appropriate as the multiplicative disturbance.

Fertilizer recommendations were developed using the yield response function. In this analysis, potassium was not economic to apply at current input and output prices and was therefore excluded from the analysis.

As expected, optimal rates of nitrogen application increased as water increased, due to the interaction between these two variables. Optimal nitrogen rates declined as soil nitrate increased. This result was also expected because both variables serve the same biological requirements for the winter wheat plant. High levels of soil phosphorus also reduced the optimal nitrogen application rates, although the relationship was not nearly as dramatic as between soil nitrate and nitrogen fertilizer.

Recommended application rates for phosphorus were heavily dependent upon the level of soil phosphorus. When soil phosphorus levels were high, optimal phosphorus application levels were at or near zero. The highest recommended rates for phosphorus occurred when soil phosphorus levels were low and water levels were very high.

In order for winter wheat producers to use these recommendations, they must have accurate measurements of soil phosphorus, soil nitrate and expected water. Soil water measurements and average April through July precipitation can be used to estimate expected water.

The model used to estimate yield was also employed to estimate

protein. The maintained hypothesis for this study was that any input which increases yield would have a negative affect on protein, with the exception of soil nitrate and applied nitrogen. The final model can be expressed as

$$Y = 9.6052 + .040246 X_1 + .000105 X_2 + .026521 X_3 \\ + .000493 X_4 + .018840 X_5 - .000097 X_6 - .000575 X_7 \\ - .000177 X_8 - .015227 X_9 + .000138 X_{10} + W_{ij} + Z_j$$

Where:

Y = percent protein;

$X_1 = N$;

$X_2 = N^2$;

$X_3 = P$;

$X_4 = P^2$;

$X_5 = SN$;

$X_6 = N \times P$;

$X_7 = N \times W$;

$X_8 = N \times SN$;

$X_9 = K$;

$X_{10} = K^2$;

W_{ij} = random variation associated with the i th treatment at location and time j ; and

Z_j = random variation associated with different sites and/or

different years.

In this protein response estimation, applied phosphorus had a negative influence on protein over most ranges of the data. However, when applied nitrogen was at the rate of 44.5 pounds per acre, applied phosphorus had a positive affect on protein when phosphorus application rates were above 31 pounds per acre. When nitrogen was at lower levels, applied phosphorus had a positive affect at slightly lower application levels. Similarly, potassium had a positive affect on protein when applied at rates greater than 55 pounds per acre.

Recommendations were not derived using a protein premium incorporated with the protein estimation due to the lack of a solution technique for the set of non-linear equations defining optimal input use.

If a protein premium had been included in the analysis, optimal rates of nitrogen application would be increased in the above analysis. Inversely, optimal phosphorus application rates would be diminished and potassium would require an even lower price to become economic.

Upon completion of Dr. Vincent Haby's five-year study, a complete economic analysis should be undertaken to help provide winter wheat producers with accurate information regarding fertilizer application rates. In this analysis, precipitation should be considered as different variables depending upon when the precipitation occurred during the growing season. Soil water may also require different specifi-

cations corresponding to different soil depths. Finally, although protein premiums may vary dramatically from year to year, the analysis should be completed with recommendations including a protein premium.

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APPENDICES

APPENDIX A

Site Location and Year

Site Location and Year

Site Location	Year
1. Art Hansen Farm, Melville, MT	1976
2. Don Herzog Farm, Rapelje, MT	1976
3. Don Holland Farm, Rosebud, MT	1976
4. Jim Larson Farm, Forsyth, MT	1976
5. Clinton McFarland Farm, Molt, MT	1976
6. Lloyd Mostal Farm, Rapelje, MT	1976
7. Floyd Warren Corporation, Hardin, MT	1976
8. Jim Larsen Farm, Forsyth, MT	1977
9. Ken Sire Farm, Ballantine, MT	1977
10. Harvey Warren Farm, Hardin, MT	1977
11. Southern Ag. Research Center, Huntley, MT	1977
12. Ken Hanson Farm, Melville, MT	1977
13. George Eastlick Farm, Molt, MT	1977
14. Stan Logan Farm, Huntley, MT	1978
15. Dennis and Charles Lee, Billings, MT	1978
16. Jim Larsen Farm, Forsyth, MT	1978
17. Dick Keller Farm, Custer, MT	1978
18. Ken Hanson Farm, Melville, MT	1978
19. George Eastlick, Molt, MT	1978

APPENDIX B

Physical Response and Economic Optimization

Appendix Table 1

ESTIMATED YIELD OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	31.7	31.5	31.8	32.2	32.5	32.9
10	33.4	34.1	34.8	35.5	36.2	36.9
20	35.4	36.4	37.5	38.6	39.6	40.6
30	37.1	38.5	39.9	41.3	42.7	44.2
40	38.6	40.3	42.1	43.9	45.6	47.4
50	39.8	41.9	44.0	46.1	48.3	50.4
60	40.7	43.2	45.7	48.2	50.6	53.1
70	41.4	44.2	47.1	49.9	52.7	55.6
80	41.8	45.0	48.2	51.4	54.6	57.8
90	42.0	45.5	49.1	52.6	56.2	59.2
100	41.9	45.8	49.7	53.6	57.5	61.4

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 2

PARTIAL DERIVATIVE OF YIELD WITH RESPECT
TO APPLIED NITROGEN

Applied Nitrogen Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	.24	.27	.31	.34	.38	.42
10	.21	.25	.28	.32	.35	.39
20	.19	.22	.26	.29	.33	.36
30	.16	.20	.23	.27	.30	.34
40	.13	.17	.20	.24	.28	.31
50	.11	.14	.18	.21	.25	.29
60	.08	.12	.15	.19	.22	.26
70	.06	.09	.13	.16	.20	.23
80	.03	.06	.10	.14	.17	.21
90	.00	.04	.07	.11	.15	.18
100	-.02	.01	.05	.08	.12	.15

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 3

ESTIMATED YIELD OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Nitrogen Lbs./Acre	Soil Nitrate Lbs./Acre					
	30	50	70	90	110	130
0	31.7	33.4	35.0	36.5	37.8	38.9
10	34.5	36.0	37.5	38.7	39.9	40.9
20	37.0	38.4	39.7	40.8	41.7	42.5
30	39.3	40.5	41.6	42.5	43.3	43.9
40	41.3	42.4	43.2	44.0	44.6	45.0
50	43.1	43.9	44.6	45.2	45.6	45.9
60	44.6	45.3	45.8	46.2	46.4	46.5
70	45.8	46.3	46.7	46.9	46.9	46.9
80	46.8	47.1	47.3	47.3	47.2	47.0
90	47.5	47.6	47.6	47.5	47.2	46.8
100	47.9	47.9	47.7	47.4	46.9	46.3

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Water	10.1 Inches
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 4

PARTIAL DERIVATIVE OF YIELD WITH RESPECT
TO APPLIED NITROGEN

Applied Nitrogen Lbs./Acre	Soil Nitrate Lbs./Acre					
	30	50	70	90	110	130
0	.29	.28	.26	.24	.22	.20
10	.27	.25	.23	.21	.20	.18
20	.24	.22	.21	.19	.17	.15
30	.21	.20	.18	.16	.14	.13
40	.19	.17	.15	.14	.12	.10
50	.16	.14	.13	.11	.09	.07
60	.14	.12	.10	.08	.07	.05
70	.11	.09	.07	.06	.04	.02
80	.08	.07	.05	.03	.01	.00
90	.06	.04	.02	.00	-.01	-.03
100	.03	.01	.00	-.02	-.04	-.06

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Water	10.1 Inches
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 5

PARTIAL DERIVATIVE OF YIELD WITH RESPECT
TO APPLIED PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	.19	.13	.07	.01	-.05	-.11
10	.20	.14	.08	.02	-.04	-.10
20	.21	.15	.09	.03	-.03	-.09
30	.22	.16	.10	.04	-.02	-.08
40	.23	.17	.11	.05	-.01	-.07
50	.23	.17	.11	.05	-.01	-.07
60	.24	.18	.12	.06	.00	-.06
70	.25	.19	.13	.07	.01	-.05
80	.26	.20	.14	.08	.02	-.04
90	.27	.21	.15	.09	.03	-.03
100	.28	.22	.16	.10	.04	-.02

Exogenous Variables

Water	10.1 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 6

ESTIMATED YIELD OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND PHOSPHORUS

Applied Phosphorus Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	36.5	38.1	39.7	41.3	42.9	44.5
5	37.5	39.1	40.8	42.5	44.2	45.9
10	38.2	40.0	41.8	43.6	45.3	47.1
15	38.9	40.7	42.6	44.5	46.3	48.2
20	39.3	41.3	43.3	45.3	47.2	49.2
25	39.7	41.7	43.8	45.9	47.9	50.0
30	39.8	42.0	44.2	46.3	48.5	50.6
35	39.9	41.1	44.4	46.6	48.9	51.1
40	39.7	42.1	44.5	46.8	49.1	51.5
45	39.5	41.9	44.4	46.8	49.2	51.7
50	39.0	41.6	44.1	46.7	49.2	51.8

Exogenous Variables

Applied Nitrogen	44.5 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 7

PARTIAL DERIVATIVE OF YIELD WITH RESPECT
TO APPLIED PHOSPHORUS

Applied Phosphorus Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	.20	.22	.24	.26	.28	.30
5	.17	.19	.21	.23	.25	.27
10	.14	.16	.18	.20	.22	.24
15	.11	.13	.15	.17	.19	.21
20	.08	.10	.12	.14	.16	.18
25	.05	.07	.09	.11	.13	.15
30	.02	.04	.06	.08	.10	.12
35	-.01	.01	.03	.05	.07	.09
40	-.04	-.02	.00	.02	.04	.06
45	-.07	-.05	-.03	-.01	.01	.03
50	-.01	0.08	-.06	-.04	-.02	.00

Exogenous Variables

Applied Nitrogen	44.5 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 8

MARGINAL RATES OF SUBSTITUTION
APPLIED NITROGEN FOR APPLIED PHOSPHORUS

Applied Phosphorus Lbs./Acre	Applied Nitrogen Lbs./Acre				
	0	20	40	60	80
0	1.45	1.08	.77	.49	.26
5	1.75	1.29	.90	.58	.31
10	2.19	1.58	1.09	.71	.38
15	2.89	2.02	1.37	.87	.48
20	4.19	2.76	1.80	1.12	.61
25	7.42	4.26	2.57	1.53	.81
30	--	8.99	4.36	2.32	1.17
35	--	--	--	4.55	1.95

Exogenous Variables

Water	10.0 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre

Appendix Table 9

MARGINAL RATES OF SUBSTITUTION
APPLIED PHOSPHORUS FOR SOIL PHOSPHORUS

Soil Phosphorus Lbs./Acre	Applied Phosphorus Lbs./Acre				
	0	10	20	30	40
0	.66	.70	.75	.85	1.11
5	.67	.71	.78	.96	2.02
10	.67	.73	.85	1.34	--
15	.68	.76	1.03	--	--
20	.69	.84	4.78	--	--
25	.72	1.33	--	--	--

Exogenous Variables

Applied Nitrogen 44.5 Lbs./Acre

Water 10.1 Inches

Appendix Table 10

ESTIMATED PROTEIN OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND TOTAL WATER

Applied Nitrogen Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	9.5	9.5	9.5	9.5	9.5	9.5
10	9.7	9.8	9.7	9.7	9.7	9.7
20	10.1	10.1	10.0	10.0	10.0	10.0
30	10.4	10.4	10.4	10.3	10.3	10.2
40	10.8	10.7	10.7	10.6	10.6	10.6
50	11.2	11.1	11.1	11.0	11.0	10.9
60	11.6	11.5	11.5	11.4	11.3	11.2
70	12.0	11.9	11.8	11.8	11.7	11.6
80	12.5	12.4	12.3	12.2	12.1	12.0
90	12.9	12.8	12.7	12.6	12.5	12.4
100	13.4	13.3	13.2	13.1	13.0	12.8

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 11

PARTIAL DERIVATIVE OF PROTEIN WITH RESPECT
TO APPLIED NITROGEN

Applied Nitrogen Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	.03	.03	.03	.03	.02	.02
10	.03	.03	.03	.03	.03	.03
20	.03	.03	.03	.03	.03	.03
30	.04	.03	.03	.03	.03	.03
40	.04	.04	.04	.03	.03	.03
50	.04	.04	.04	.04	.04	.03
60	.04	.04	.04	.04	.04	.04
70	.04	.04	.04	.04	.04	.04
80	.05	.04	.04	.04	.04	.04
90	.05	.05	.05	.04	.04	.04
100	.05	.05	.05	.05	.05	.04

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 12

ESTIMATED PROTEIN OF WINTER WHEAT AT SPECIFIED
LEVELS OF APPLIED NITROGEN AND SOIL NITRATE

Applied Nitrogen Lbs./Acre	Soil Nitrate Lbs./Acre					
	30	50	70	90	110	130
0	9.5	9.8	10.2	10.6	10.9	11.3
10	9.7	10.1	10.4	10.7	11.1	11.4
20	10.1	10.3	10.7	11.0	11.3	11.6
30	10.4	10.6	10.9	11.2	11.5	11.7
40	10.7	11.0	11.2	11.4	11.7	11.9
50	11.1	11.3	11.5	11.7	11.9	12.1
60	11.5	11.6	11.8	12.0	12.1	12.3
70	11.9	12.0	12.2	12.3	12.4	12.5
80	12.3	12.4	12.5	12.6	12.7	12.8
90	12.8	12.8	12.9	13.0	13.0	13.1
100	13.2	13.3	13.3	13.3	13.3	13.4

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Water	10.1 Inches
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 13

PARTIAL DERIVATIVE OF PROTEIN WITH RESPECT
TO APPLIED NITROGEN

Applied Nitrogen Lbs./Acre	Soil Nitrate Lbs./Acre					
	30	50	70	90	110	130
0	.03	.02	.02	.02	.01	.01
10	.03	.03	.02	.02	.02	.01
20	.03	.03	.02	.02	.02	.01
30	.03	.03	.03	.02	.02	.02
40	.04	.03	.03	.03	.02	.02
50	.04	.03	.02	.03	.02	.02
60	.04	.04	.03	.03	.03	.02
70	.04	.04	.03	.03	.03	.02
80	.04	.04	.03	.03	.03	.03
90	.05	.04	.04	.04	.03	.03
100	.05	.04	.04	.04	.03	.03

Exogenous Variables

Applied Phosphorus	17.8 Lbs./Acre
Water	10.1 Inches
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 14

PARTIAL DERIVATIVE OF PROTEIN WITH RESPECT
TO APPLIED PHOSPHORUS

Applied Nitrogen Lbs./Acre	Applied Phosphorus Lbs./Acre					
	0	10	20	30	40	50
0	-.03	-.02	-.01	.00	.01	.02
10	-.03	-.02	-.01	.00	.01	.02
20	-.03	-.02	-.01	.00	.01	.02
30	-.03	-.02	-.01	.00	.01	.02
40	-.03	-.02	-.01	.00	.01	.02
50	-.03	-.02	-.01	.00	.01	.02
60	-.03	-.02	-.01	.00	.01	.02
70	-.03	-.02	-.01	.00	.01	.02
80	-.03	-.02	-.01	.00	.01	.02
90	-.04	-.03	-.02	-.01	.00	.01
100	-.04	-.03	-.02	-.01	.00	.01

Exogenous Variables

Water	10.1 Inches
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 15

ESTIMATED PROTEIN OF WINTER WHEAT AT SPECIFIED
LEVELS OF PHOSPHORUS AND WATER

Applied Phosphorus Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	11.3	11.3	11.2	11.2	11.1	11.1
5	11.2	11.2	11.1	11.1	11.0	10.9
10	11.1	11.0	11.0	11.0	10.9	10.8
15	11.0	11.0	10.9	10.8	10.8	10.7
20	11.0	10.9	10.3	10.8	10.7	10.7
25	10.9	10.8	10.8	10.7	10.7	10.6
30	10.9	10.8	10.8	10.7	10.7	10.6
35	10.9	10.8	10.8	10.7	10.7	10.6
40	10.9	10.9	10.8	10.8	10.7	10.6
45	11.0	10.9	10.9	10.8	10.8	10.7
50	11.1	11.0	10.9	10.9	10.8	10.8

Exogenous Variables

Applied Nitrogen	44.5 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 16

PARTIAL DERIVATIVE OF PROTEIN WITH RESPECT
TO APPLIED PHOSPHORUS

Applied Phosphorus Lbs./Acre	Total Water Inches					
	7	9	11	13	15	17
0	-.03	-.03	-.03	-.03	-.03	-.03
5	-.03	-.03	-.03	-.03	-.03	-.03
10	-.02	-.02	-.02	-.02	-.02	-.02
15	-.02	-.02	-.02	-.02	-.02	-.02
20	-.01	-.01	-.01	-.01	-.01	-.01
25	-.01	-.01	-.01	-.01	-.01	-.01
30	.00	.00	.00	.00	.00	.00
35	.00	.00	.00	.00	.00	.00
40	.01	.01	.01	.01	.01	.01
45	.01	.01	.01	.01	.01	.01
50	.02	.02	.02	.02	.02	.02

Exogenous Variables

Applied Nitrogen	44.5 Lbs./Acre
Soil Nitrate	30.0 Lbs./Acre
Soil Phosphorus	14.0 Lbs./Acre
Applied Potassium	42.6 Lbs./Acre

Appendix Table 17

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	20.05*	51.9	91.1	0.0	1.00	52.58	32.53
	15.00	49.5	68.2	0.0	1.82	45.47	30.47
	10.00	45.8	45.4	0.0	2.63	34.26	24.36
\$3.50	20.93*	52.1	95.1	0.0	1.00	62.30	41.37
	15.00	49.5	68.2	0.0	2.12	53.05	38.05
	10.00	45.8	45.4	0.0	3.00	40.09	30.09
\$4.00	21.59*	52.3	98.1	0.0	1.00	71.91	50.32
	15.00	49.5	68.2	0.0	2.42	60.63	45.63
	10.00	45.8	45.4	0.0	3.50	45.81	35.81

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61¢/Lb.

Soil Nitrate 70.0 Lbs./Acre
Soil Phosphorus 25.0 Lbs./Acre
Water 14.0 Inches

Appendix Table 18

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	13.29*	40.2	60.4	0.0	1.00	27.59	14.30
	10.00	38.8	45.4	0.0	1.53	23.42	13.42
\$3.50	14.18*	40.5	64.6	0.0	1.00	33.15	18.97
	10.00	48.8	45.4	0.0	1.79	27.33	17.33
\$4.00	14.84*	40.6	67.4	0.0	1.00	38.59	23.75
	10.00	38.8	45.4	0.0	2.04	31.23	21.23

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61¢/Lb.

Soil Nitrate 30.0 Lbs./Acre
Soil Phosphorus 25.0 Lbs./Acre
Water 7.5 Inches

Appendix Table 19

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	35.10*	55.4	105.2	19.6	1.00	76.46	41.36
	25.00	51.5	93.7	7.6	1.30	64.84	39.84
	20.00	49.2	86.5	1.6	1.45	57.96	37.96
	15.00	46.4	68.2	0.0	2.08	49.45	34.45
	10.00	42.2	45.5	0.0	2.89	37.01	27.01
\$3.50	39.89*	56.9	111.2	25.3	1.00	94.39	54.50 ∞
	30.00	53.6	98.9	13.5	1.34	82.80	52.80
	20.00	49.2	86.5	1.6	1.69	67.32	47.62
	10.00	42.2	45.5	0.0	3.37	43.18	33.18
\$4.00	43.48*	57.8	115.6	29.6	1.00	111.73	68.25
	35.00	55.4	109.1	19.5	1.34	101.81	66.81
	25.00	51.5	93.7	7.6	1.74	86.45	61.45
	15.00	46.4	68.2	0.0	2.78	65.93	50.93

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
 Price of Phosphorus 61¢/Lb.

Soil Nitrate 48.0 Lbs./Acre
 Soil Phosphorus 14.0 Lbs./Acre
 Water 14.0 Inches

Appendix Table 20

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	43.85*	58.0	115.0	30.4	1.00	94.71	50.85
	35.00	54.6	104.0	19.9	1.26	84.68	49.68
	30.00	52.4	97.8	13.9	1.41	77.99	47.99
	25.00	49.9	91.6	8.0	1.56	70.55	45.55
	20.00	47.2	85.4	2.0	1.71	62.37	42.37
	10.00	39.5	45.5	0.0	3.11	39.18	29.18
\$3.50	48.65*	59.5	120.9	36.1	1.00	115.68	67.04
	40.00	56.6	110.2	25.8	1.30	105.74	65.74
	30.00	52.4	97.8	13.9	1.65	90.00	60.99
	20.00	47.2	85.4	2.0	2.00	72.76	52.76
	10.00	39.5	45.5	0.0	3.63	45.72	35.72
\$4.00	52.23*	60.4	125.4	40.0	1.00	136.05	83.82
	45.00	58.4	116.4	31.8	1.29	127.78	82.78
	35.00	54.6	104.0	19.9	1.69	112.91	77.91
	25.00	49.9	91.6	8.0	2.08	94.07	69.07
	10.00	39.5	45.5	0.0	4.15	52.25	42.25

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61¢/Lb.

Soil Nitrate 30.0 Lbs./Acre
Soil Phosphorus 9.0 Lbs./Acre
Water 14.0 Inches

Appendix Table 21

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	17.55*	38.8	50.5	10.6	1.00	27.11	9.57
	15.00	37.9	47.3	7.5	1.08	24.47	9.47
	10.00	36.0	41.1	1.6	1.23	18.72	8.72
\$3.50	22.34*	40.3	56.4	16.3	1.00	36.82	14.48
	15.00	37.9	47.3	7.5	1.26	28.55	13.55
	10.00	36.0	41.1	1.6	1.43	21.83	11.83
\$4.00	25.93*	41.2	60.9	20.6	1.00	45.93	20.00
	20.00	39.6	53.5	13.5	1.24	39.20	19.30
	15.00	37.9	47.3	7.5	1.42	32.62	17.62
	10.00	36.0	41.1	1.6	1.63	24.95	14.95

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61 ¢/Lb.

Soil Nitrate 70.0 Lbs./Acre
Soil Phosphorus 9.0 Lbs./Acre
Water 7.5 Inches

Appendix Table 22

ECONOMIC COMBINATIONS OF NITROGEN AND PHOSPHORUS AT SPECIFIED LEVELS OF
WHEAT PRICE, FERTILIZER PRICE, AND CAPITAL AVAILABILITY

Price of Wheat Per Bushel	Capital/ Acre	Yield (Bushels)	Nitrogen (Lbs./Acre)	Phosphorus (Lbs./Acre)	Return on Last \$	Fertilizer Revenue (\$)	Fertilizer Profit Per Acre
\$3.00	21.96*	38.9	64.7	12.7	1.00	37.64	15.68
	15.00	36.4	56.1	4.4	1.21	29.95	14.95
	10.00	34.2	45.4	0.0	1.53	23.42	13.42
\$3.50	26.75*	40.4	70.7	18.4	1.00	49.10	22.35
	20.00	38.3	62.3	10.3	1.24	41.56	21.56
	15.00	36.4	56.1	4.4	1.41	34.95	19.95
	10.00	34.2	45.4	0.0	1.79	27.33	17.33
\$4.00	30.34*	41.4	75.1	22.7	1.00	59.96	29.62
	25.00	39.9	68.5	16.3	1.21	54.05	29.05
	20.00	38.3	62.3	10.3	1.41	47.49	27.49
	15.00	36.4	56.1	4.4	1.61	39.94	24.94
	10.00	34.2	45.4	0.0	2.04	31.23	21.23

*Unlimited Capital

Exogenous Variables

Price of Nitrogen 22¢/Lb.
Price of Phosphorus 61¢/Lb.

Soil Nitrate 30.0 Lbs./Acre
Soil Phosphorus 9.0 Lbs./Acre
Water 7.5 Inches

APPENDIX C

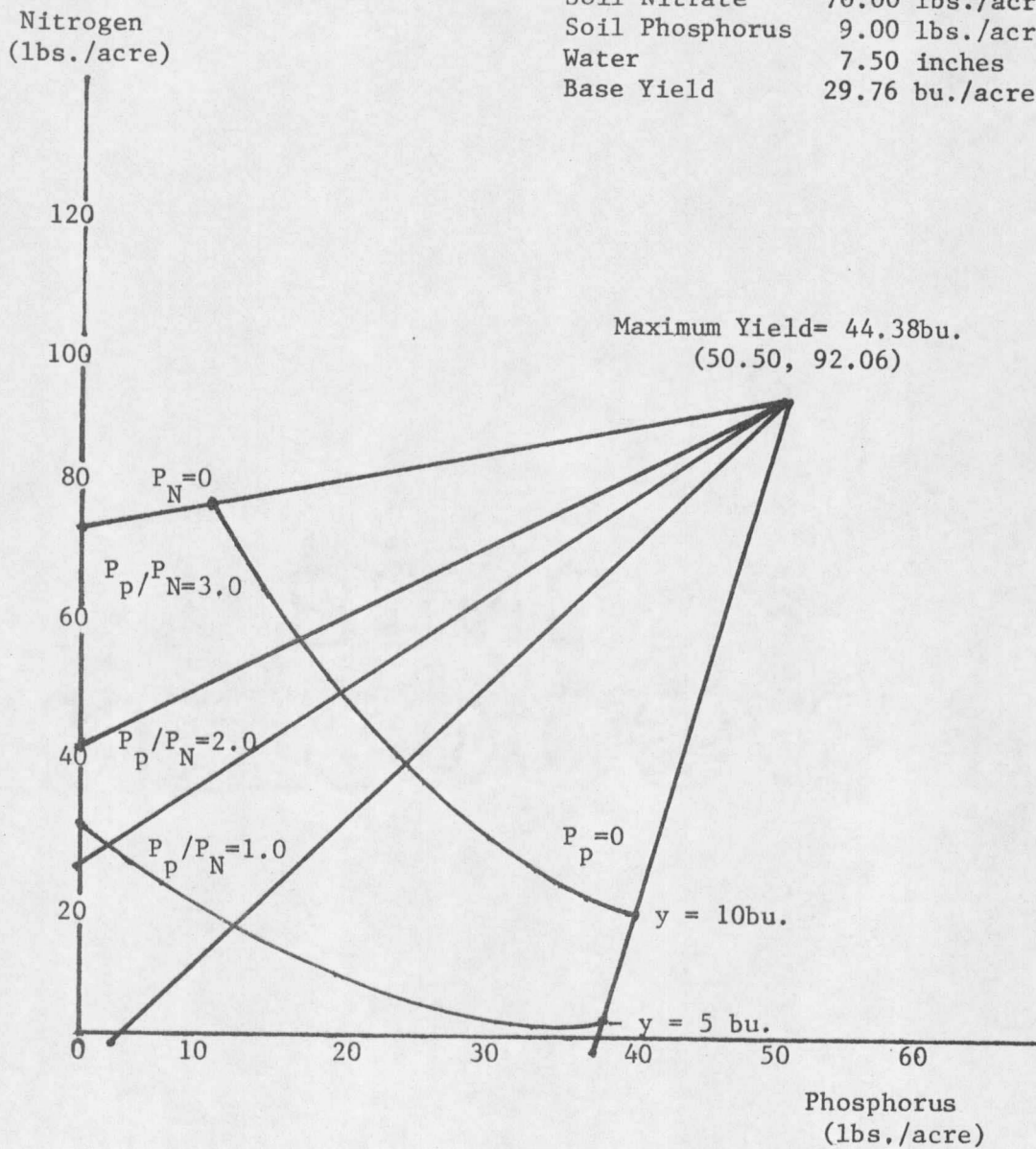
Physical Response Figures

Appendix Figure 1

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED
YIELD INCREASES ON WINTER WHEAT AT SPECIFIED
PRICE RATIOS FOR PHOSPHORUS AND NITROGEN

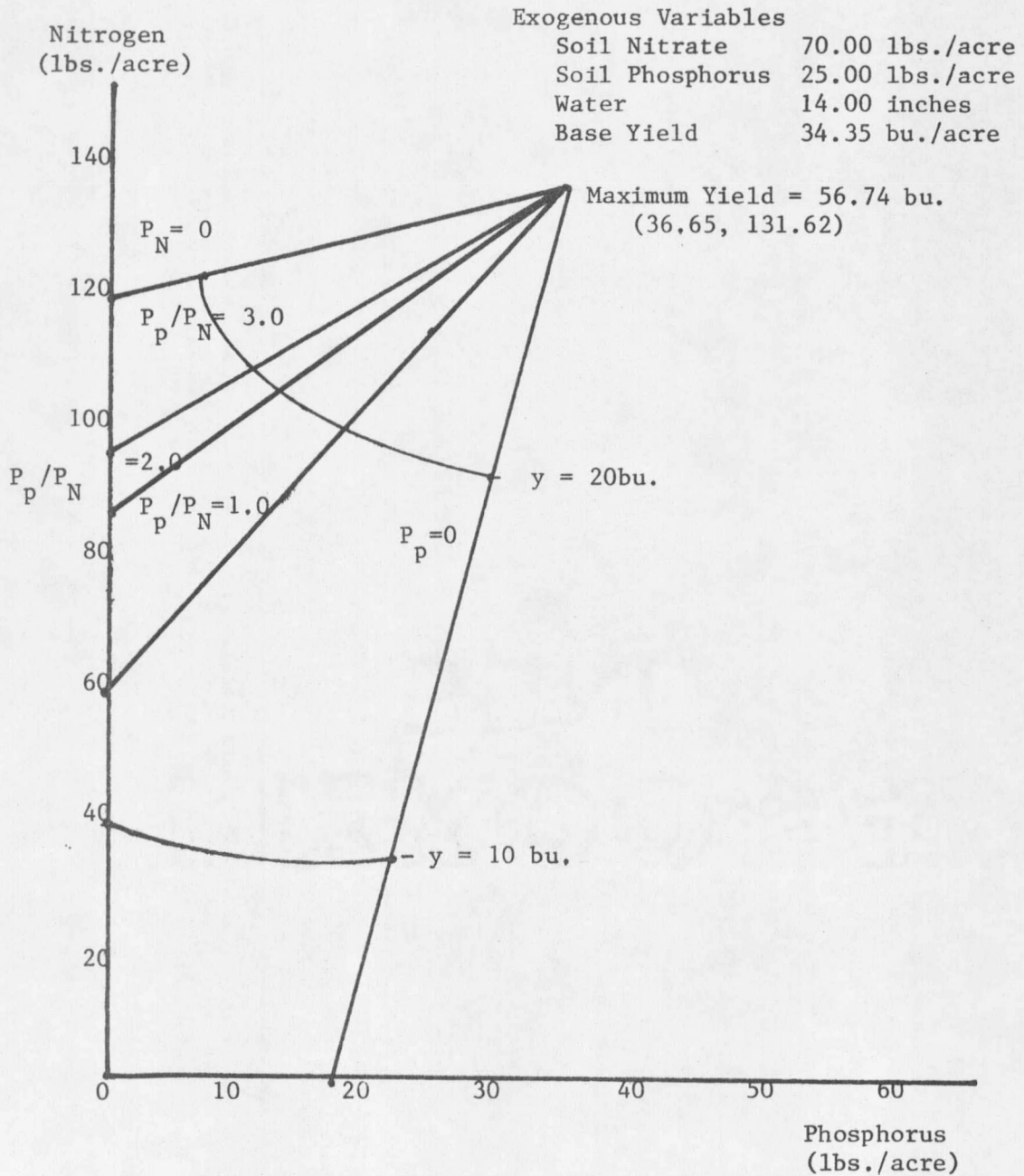
Exogenous Variables

Soil Nitrate	70.00 lbs./acre
Soil Phosphorus	9.00 lbs./acre
Water	7.50 inches
Base Yield	29.76 bu./acre



Appendix Figure 2

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED
YIELD INCREASES OF WINTER WHEAT AT SPECIFIED
PRICE RATIOS FOR PHOSPHORUS AND NITROGEN

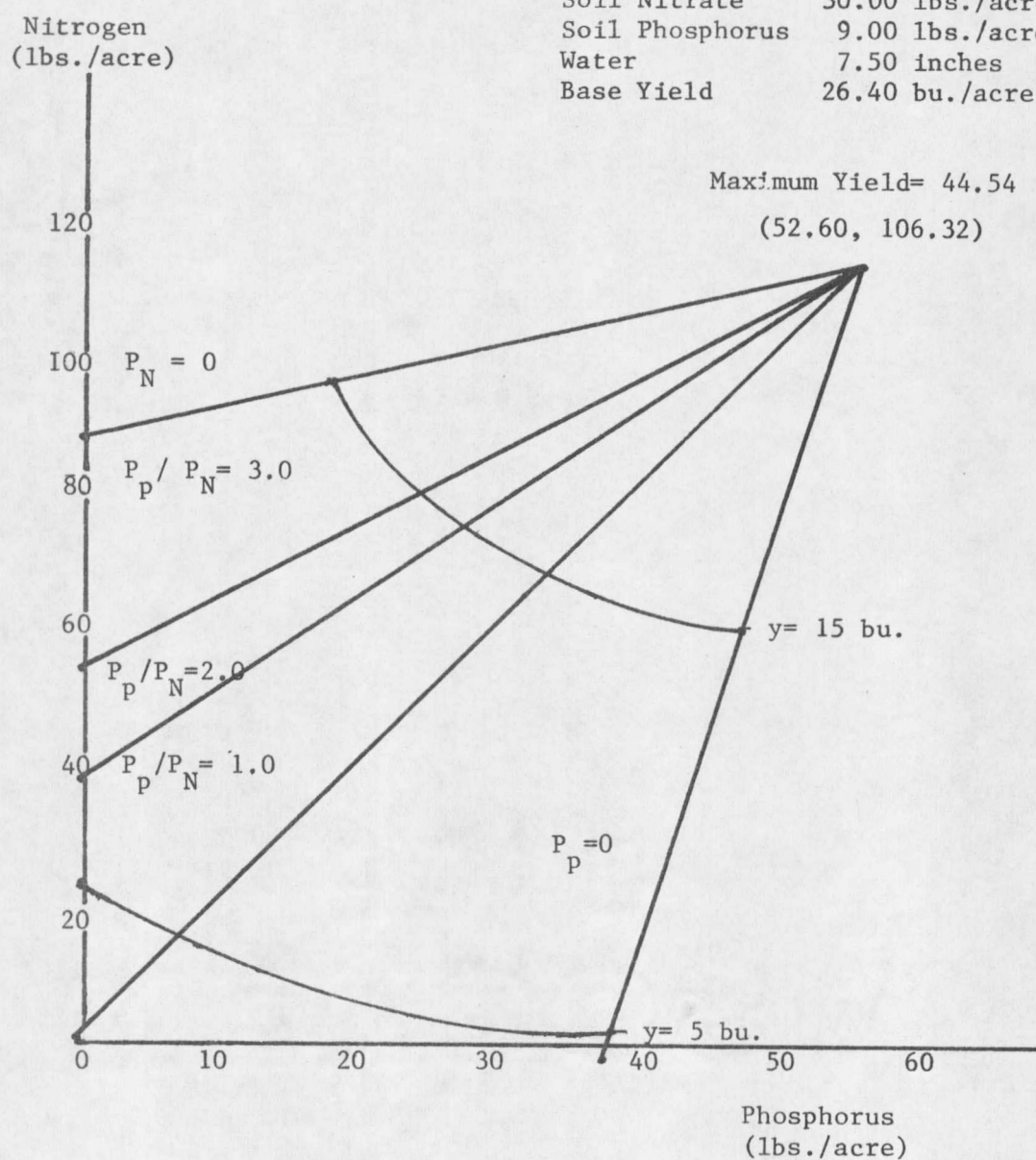


Appendix Figure 3

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED
YIELD INCREASES OF WINTER WHEAT AT SPECIFIED PRICE
RATIOS FOR PHOSPHORUS AND NITROGEN

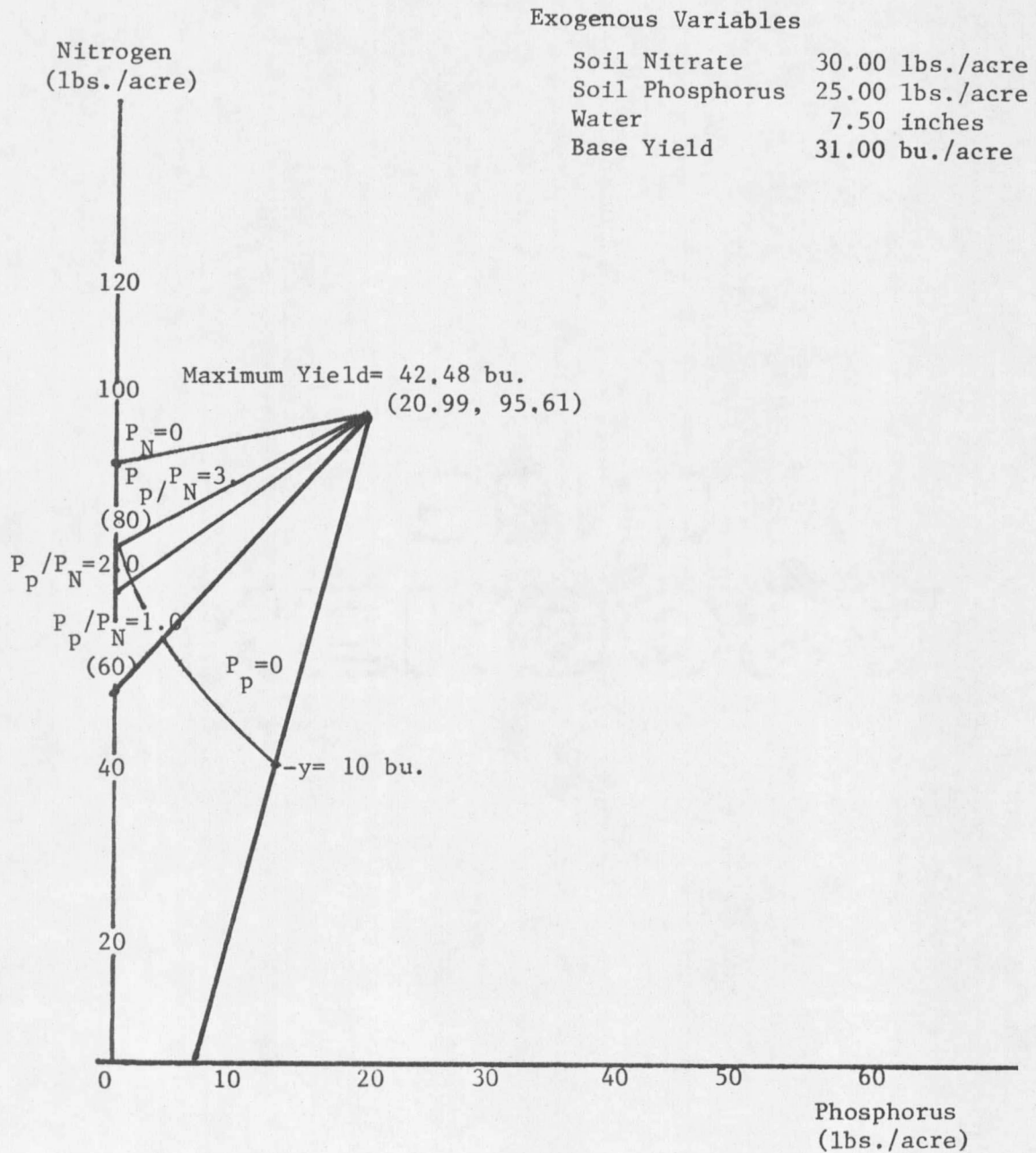
Exogenous Variables

Soil Nitrate	30.00 lbs./acre
Soil Phosphorus	9.00 lbs./acre
Water	7.50 inches
Base Yield	26.40 bu./acre



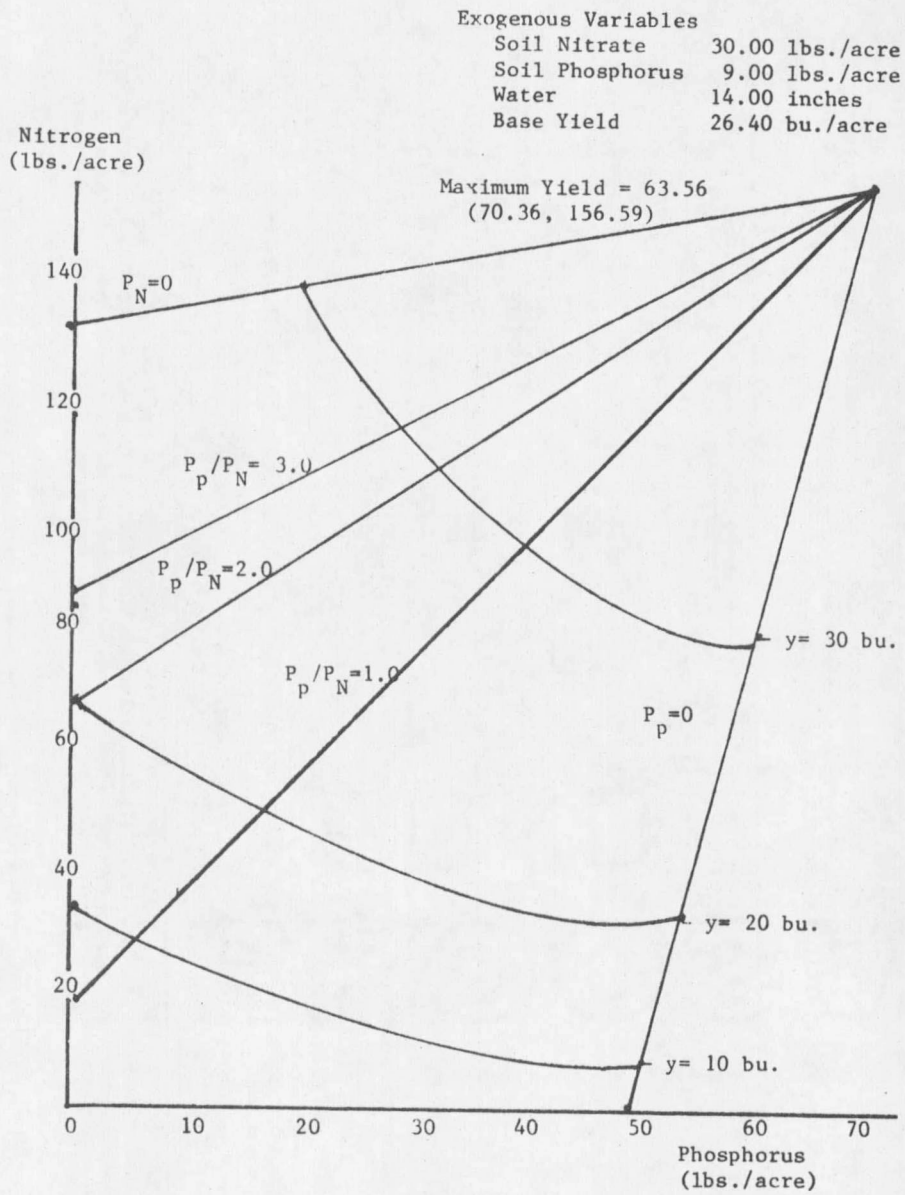
Appendix Figure 4

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED
YIELD INCREASES OF WINTER WHEAT AT SPECIFIED PRICE
RATIOS FOR PHOSPHORUS AND NITROGEN



Appendix Figure 5

COMBINATIONS OF NITROGEN AND PHOSPHORUS TO PRODUCE SELECTED
YIELD INCREASES OF WINTER WHEAT AT SPECIFIED PRICE
RATIOS FOR PHOSPHORUS AND NITROGEN



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