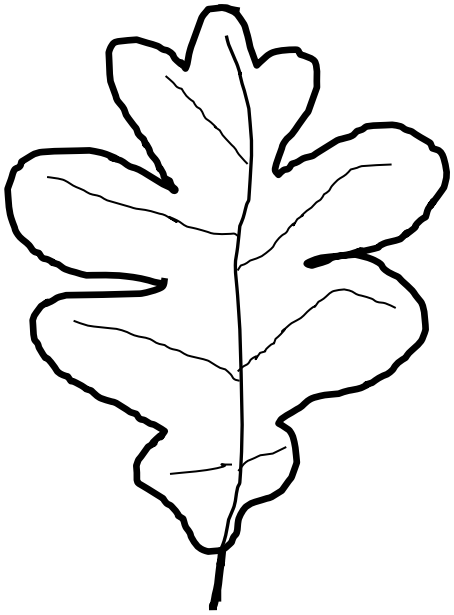
$x^2 - 5x = 24$

$$f(x) = \begin{cases} -x + 2 & if \ x \le 1 \\ (x-3)^2 & if \ x > 1 \end{cases}$$

X=± 1/4

M121Q: *College Algebra*
Structured Notes Workbook
for
Fall 2025 Sections 1 – 25



Acknowledgements

There are many people who deserve thanks for making this structured note packet / workbook possible. In particular, I am indebted to Megan Wickstrom for her wisdom, listening-ear, encouragement and support. It is such a joy to have her as a course supervisor for M121Q: *College Algebra* at Montana State University (MSU)!

Several others contributed in significant ways to making this packet and the transition to OER a reality for M121Q: *College Algebra*.

- Tom Hayes, Mary Ann Sodja and Jocelyn Reid were the first to create a structured note packet / workbook for use with *College Algebra* instruction at MSU many years ago. The packet has undergone complete transformation(s) since then, but the structure and philosophy are largely inherited from them – perhaps even a few items from their work remain here and there in the packet.
- Brian Rossman and Christina Trunnell and the Montana State University (MSU) Library OER Initiative provided funding support to transition the workbook from connection to a traditionally published textbook to being supported by publicly available materials. Jacqueline Frank (Accessibility Librarian) provided guidance on making the packet accessible and the library reviewed the packet for accessibility. Furthermore, the MSU Library OER Initiative provided a means of publishing the workbook under a Creative Commons license.
- David Lippman and the *MyOpenMath* community deserve many thanks for providing an excellent free online mathematics homework platform and problems that can be used with M121Q: *College Algebra* at MSU.
- Many people / entities deserve thanks for creating and making publicly available online materials that serve as reference materials for this note packet. Definitions used in the packet frequently mirror those in reference materials and examples often utilize problems that are "sisters" to ones in reference materials. In order of frequency of use as reference material:
    - OpenStax: *Intermediate Algebra, 2e,* Lynn Marecek and Andrea Honeycutt Mathis
    - OpenStax: *College Algebra, 2e,* Jay Abramson
    - Scottsdale Community College Mathematics Division OER Materials Page: *College Algebra: An Investigation of Functions, 4e*, primary authors David Lippman, Melonie Rasmussen and Jay Abramson
    - OpenStax: *Elementary Algebra, 2e,* Lynn Marecek, Mary-Anne Anthony Smith and Andrea Honeycutt Mathis
    - Khan Academy | Math
    - OpenStax: *College Algebra 2e with Corequisite Support,* Jay Abramson and Sharon North
    - OpenTextBookStore: *Precalculus / College Algebra 3e*, Carl Stitz and Jeff Zeager

Heidi Staebler

To Students

Please bring this note packet / workbook to class with you every day. We will use it for instruction and it will provide you with a well-organized set of class notes.

Sections that are labeled (R) should be review material for you. They are topics that you should already be skilled in OR be able to quickly get back your skills by doing some review. We will teach *some* of these sections in class. Other sections will not be taught in class, but lecture videos that match the note pages will be provided with **required** sectional online homework assignments. In addition, a small number of the sections are considered **optional** – to be utilized if you personally need to review the topics. Lecture videos for optional sections will be provided with associated optional online homework assignments.

Course Expectations

This course requires a significant amount of time outside of class. The general expectation for a university-level course is that you put in 2 to 3 hours of work outside of class for every 50-minute class session. That means, it is normal / expected for students to work on M121Q: *College Algebra* 6 – 9 hours per week OUTSIDE OF CLASS. You may need to spend more time on this course if you struggle with algebra.

Specific Expectations:

- Attend EVERY class session and ACTIVELY participate. Mathematics is not a spectator sport. The only way to learn mathematics is to DO mathematics.
- Do ALL online homework assignments and complete them ON TIME.
- GET HELP when you need it. There are many support resources available. Please make use of them.
- STUDY for quizzes, not just for exams. In particular, WORK THROUGH the problems in weekly quiz preparation handouts.
- Prior to an exam, WORK THROUGH the set of exam review problems that will be provided to you in Brightspace D2L. That is probably the best way to prepare for exams in this course.
- REGULARLY check announcements within Brightspace D2L – at least every other day. Important information will be posted there.
- BE IN GOOD COMMUNICATION with your instructor. Let them know when you are sick or are experiencing significant issues that are preventing class attendance. Seek out their help and advice. They truly care about you and want you to be successful at MSU and in this course.

Best Wishes for a Good Semester!

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Preliminaries Chapter

Sections P.1 – P.4

Section P.1: Solving Linear Equations (R)

Part 1: Solving Linear Equations with Whole Numbers & Decimals

Reference Materials:

Open Stax: *Elementary Algebra 2e* by Maracek, Anthony-Smith & Honeycutt Mathis
Chapter 2 Section 2.5: Solve Equations with Fractions or Decimals

Objectives:

- Review how to solve linear equations that involve whole numbers and decimals

Important Vocabulary Words:

Expression, Equation, Solve

Basic Linear Equations in One Variable

What is the major difference between an equation and an expression?

Expression Example: Equation Example:

What does it mean to solve an equation (in one variable)?

Example 1: Solve the equation for x. $6x-(x+2)=3(x-4)-8$

Example 2: Solve the equation for x. $4x + 0.05(3x - 2) = 0.2(6x + 5)$

Part 2: Solving Linear Equations with Fractions

Reference Materials:

Open Stax: *Elementary Algebra 2e* by Maracek, Anthony-Smith & Honeycutt Mathis
Chapter 1 Section 1.6: Add and Subtract Fractions
Chapter 2 Section 2.5: Solve Equations with Fractions or Decimals

Objectives:

- Review how to solve linear equations that involve fractions

Important Vocabulary Words:

Expression, Equation, Solve, "Eliminate the Fractions" Strategy, Least Common Multiple (LCM)

Solving Linear Equations with Fractions – Simplify and Eliminate the Fractions

Least Common Multiple (LCM) of a Set of Numbers:

Simplify By Adding / Subtracting Fractions to Combine Terms:

Determine the LCM of the denominators and multiply each term by "1" in the form $\frac{n}{n}$ to create a common denominator = LCM of the denominators. Then add / subtract to combine terms.

Eliminate the Fractions Strategy:

Multiply both sides of the equation by the LCM of the denominators and use the LCM to cancel out the denominators.

Example 1: Solve for x. $\frac{1}{5}x-\frac{1}{3}(x+2)=4x$

Example 2: Solve for x. $2-\frac{x-3}{6}=\frac{4x+1}{10}$

You Try! Example 3: Solve for x. $\frac{x-6}{3}-\frac{3x+2}{4}=\frac{x}{12}$

Section P.2: Radicals and Power Equations (R)

Part 1: Evaluating Radicals

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 8 Section 8.1: Simplify Expressions with Roots

Khan Academy Resources. All Khan Academy content is available for free at www.khanacademy.org .

- YouTube Video: Introduction to Square Roots
- YouTube Video: Introduction to Cube Roots

Objectives:

Review / learn how to work with simple radical expressions

Important Vocabulary Words:

Radical, (Principle) Root, Absolute Value

Introduction to Radicals

What is the square root of 9, i.e. $\sqrt{9}$?

What is the cube root of 8, i.e. $\sqrt[3]{8}$?

*Note: For even roots, if you are given a radical you must assume that we want only the *positive* (principal) root.

Can we take even roots of negative numbers? ($\sqrt{-16}$)

What about odd roots of negative numbers? ($\sqrt[3]{-8}$) Why?

Example 1: Evaluate / simplify each of the following.

A. $\sqrt{64}$ =_____ B. $\sqrt[3]{125}$ = _____ C. $\sqrt[4]{81}$ = _____

Another way to think:

Working with a Variable:

Recall **Absolute Value**:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \leftrightarrow \text{ if a is 0 or a positive number} \\ -a & \text{if } a < 0 \leftrightarrow \text{if a is a negative number} \end{cases}$$

Examples:

$|5| = 5$ $|23.7| = 23.7$ $|0| = 0$

But:

$|-8| = -(-8) = 8$ $|-46.72| = -(-46.72) = 46.72$

Rule: When n is even:	When n is odd:
$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[n]{x^n} = x$

Example 2: Simplify each of the following.

A. $\sqrt{x^2}$ = _____ B. $\sqrt[3]{x^3}$ = _____

Question: Is $\sqrt{4+9} = \sqrt{4} + \sqrt{9}$?

Example 3: For $x > 0,$ simplify each of the following or state that it cannot be simplified.
Recall: If $x > 0$, then $|x| = x$.

A. $\sqrt{x^2 + 9}$

B. $\sqrt[3]{x^3 - 8}$

IMPORTANT!!

You Try! Example 4: For $x > 0$, evaluate / simplify each of the following or state that it cannot be simplified.

A. $\sqrt[3]{-27} =$

B. $\sqrt{25x^2} =$

C. $\sqrt{49 + x^2} =$

Part 2: Solving Power Equations

Reference Materials:

Math Class Rocks Resources: https://mathclassrocks.com

- YouTube Video: Solve Equations Using Square and Cube Roots

Objectives:

Review / learn how to solve power equations

Example 1: Solve each of the following equations.

A. $x^2 = 36$

B. $x^3 = -125$

C. $x^4 = 81$

D. $x^2 = -49$

Rule: Solve $x^n = c$, where c is a real number.

Section P.3: The Rectangular Coordinate System and Graphs of Equations (R)

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.1: Graph Linear Equations in Two Variables

Objectives:

- Be able to plot (x, y) coordinate pairs
- Be able to use a T-chart to plot graphs of equations in x and y
- Understand what x- and y-intercepts are

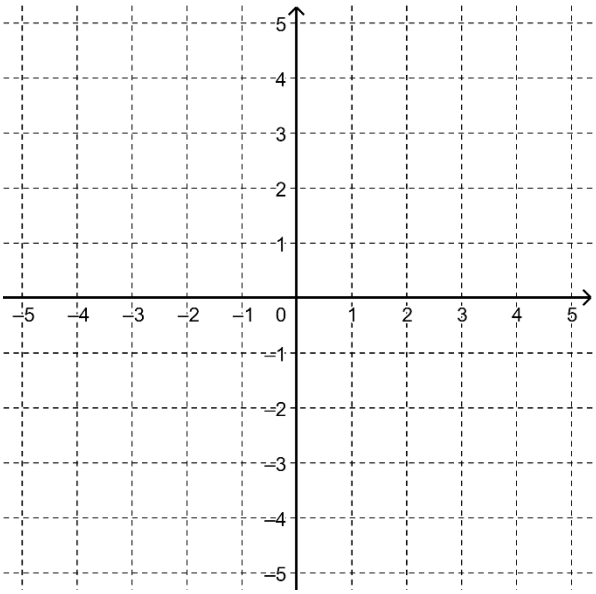
Important Vocabulary Words:

Solution, Rectangular (Cartesian) Coordinate System, Graph, x-axis, y-axis, x-intercept, y-intercept

---

Plotting Solutions to an Equation in the Rectangular (Cartesian) Coordinate System

We can represent an ordered pair (x, y) using the coordinate (Cartesian) plane.



Example 1: Plot and label the following ordered pairs in the rectangular (Cartesian) coordinate plane.

A. $(4, 2)$

B. $(-1, 3)$

C. $(1, -5)$

D. $(-3, -4)$

Definition: Solution of an Equation Involving x and y

An ordered pair (x, y) is a solution to an equation involving x and y if the equation is a true statement when the x- and y- values of the ordered pair are substituted into the equation.

Example 2: Find some solutions to $y = 3x - 2$. List solutions as ordered pairs in the form (x, y).

Graphing Equations

Definition: The **graph** of an equation in x and y is the set of all points (x, y) that satisfy the equation.

Graph of a Line

The graph of an equation of the form $y = mx + b$, where m and b are real numbers, is a line.

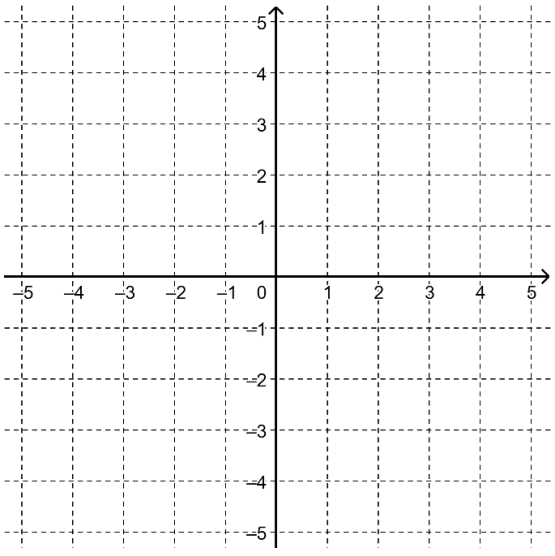
- Every point on the line is a solution to the equation.
- Every solution to the equation is a point on the line.

Basic Equation Graphing Method:

1. Make a T-chart or table of solutions to the equation by choosing values of x and using the equation to calculate corresponding values of y.
2. Plot enough solutions (x, y) to know what the overall graph looks like and then connect the points with a smooth curve or line.

Example 3: Use the basic equation graphing method to draw the graph of $y = -2x + 4$.

x	y
0	
1	
2	
3	
4	



Definitions: X-intercept & Y-intercept

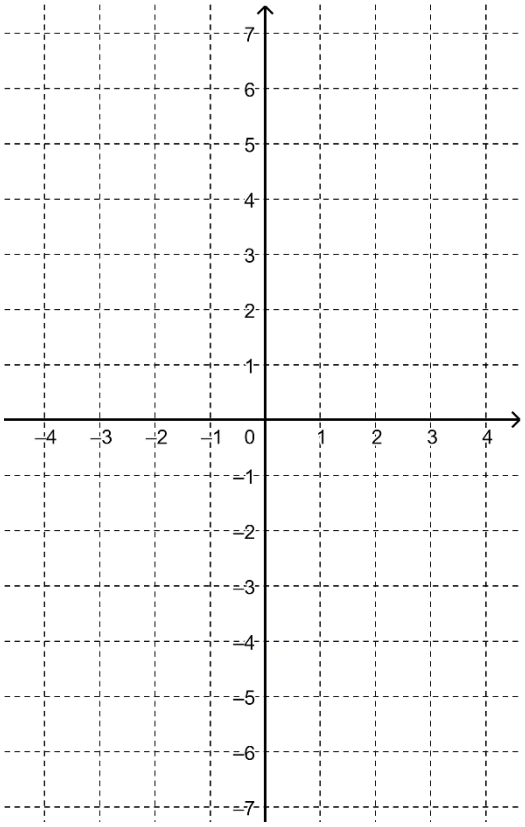
The **x-intercept** of a graph is the point $(x, 0)$ where the graph crosses the x-axis.

The **y-intercept** of a graph is the point $(0, y)$ where the graph crosses the y-axis.

Example 4: What are the x- and y-intercepts for the graph of $y = -2x + 4$?

x-intercept:_____ y-intercept: _____

Example 5: Use the basic equation graphing method to draw the graph of $y = 2x^2 - 2$ and determine the x- and y-intercepts of the graph.



Section P.4: Slopes and Lines (R)

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.2: Slope of a Line & Section 3.3 Find the Equation of a Line

Objectives:

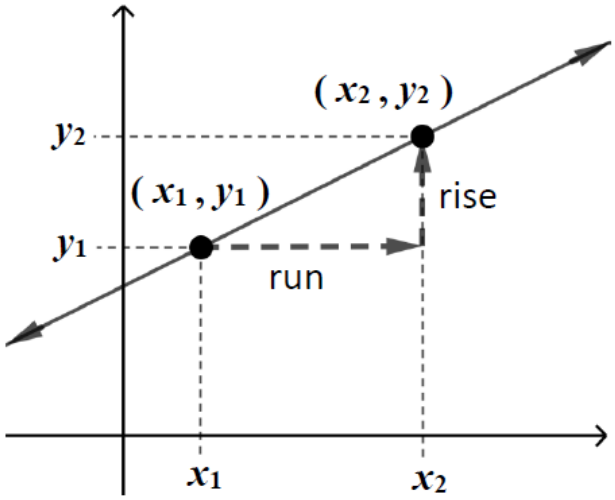
- Develop the slope formula and the slope-intercept form of the equation of a line
- Use the slope formula and slope-intercept form of the equation of a line to determine the equations of lines and to graph lines when an equation is provided.

Important Vocabulary Words:

Slope, Slope-Intercept Form

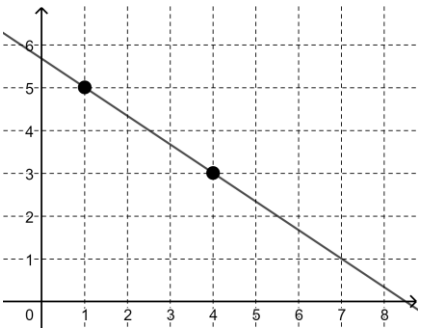
Formula for Slope Given Two Points on a Line:

$$\text{Slope = m = } \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

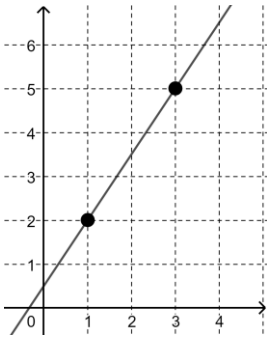


Example 1: Determine the slopes of the lines shown below.

A.

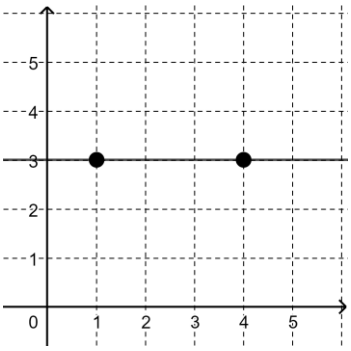


B.

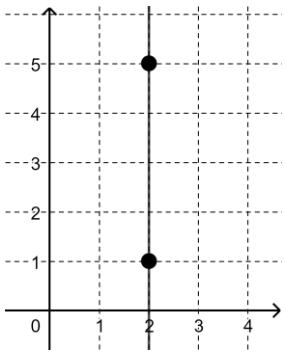


Example 2: Determine the slopes of the following lines.

A.



B.



Summary: Line Direction ↔Slope

Positive Slope ↔

Negative Slope ↔

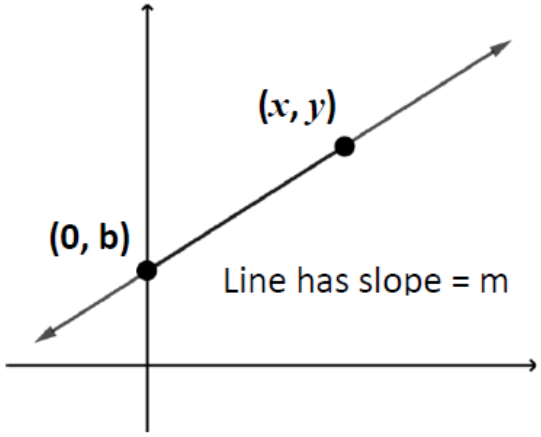
Horizontal Line: **Vertical Line:**

Slope-Intercept Form of the Equation of a Line:

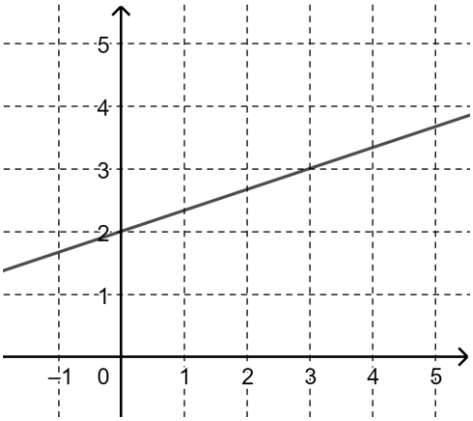
Slope-Intercept Form of the Equation of a Line

$y = mx + b$ m = slope of the line

$(0, b)$ is the y-intercept of the line



Example 3: Determine the equation of the line whose graph is shown.



Example 4: Determine the equation of the line whose graph passes through $(3,-1)$ and $(2,4)$.

Chapter 1
Functions

Sections 1.1 – 1.8

Section 1.1: Relations and Functions

Part 1: Relations

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.5: Relations and Functions and Section 3.6 Graphs of Functions

Objectives:

- Become familiar with various means of visualizing data and how they represent a relation
- Learn and be able to apply the terms domain and range
- Be able to write a relation as a set of ordered pairs

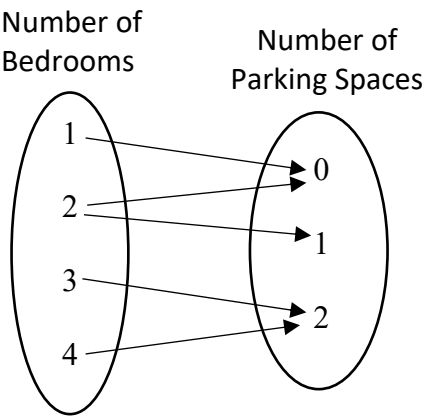
Important Vocabulary Words:

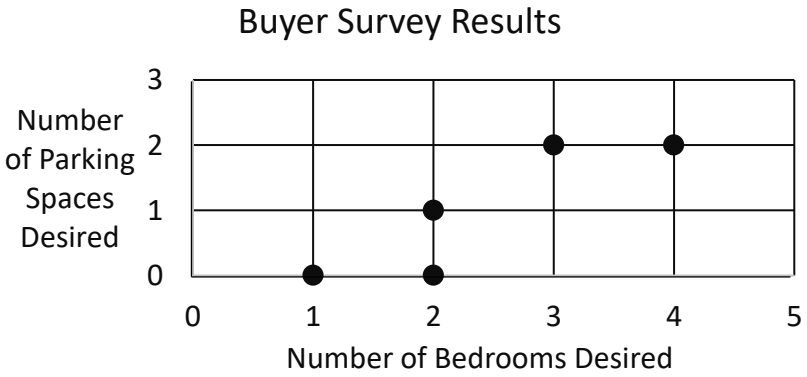
Relation, Domain, Range

Example 1: A developer contracted with a realtor to survey buyers with a strong interest in purchasing a condo in the location and price per square foot that the developer wishes to build. The developer wanted to know the number of bedrooms and the number of covered parking spaces potential buyers would want. Results from the first five buyers surveyed are shown below.

Number of bedrooms	1	2	2	3	4
Number of parking spaces	0	0	1	2	2

$$\{(1,0),(2,0),(2,1),(3,2),(4,2)\}$$





Definitions:

Relation: Any set of ordered pairs.
Input ↔ first value in the ordered pair
Output ↔ second value in the ordered pair

Domain: The set of all input values.

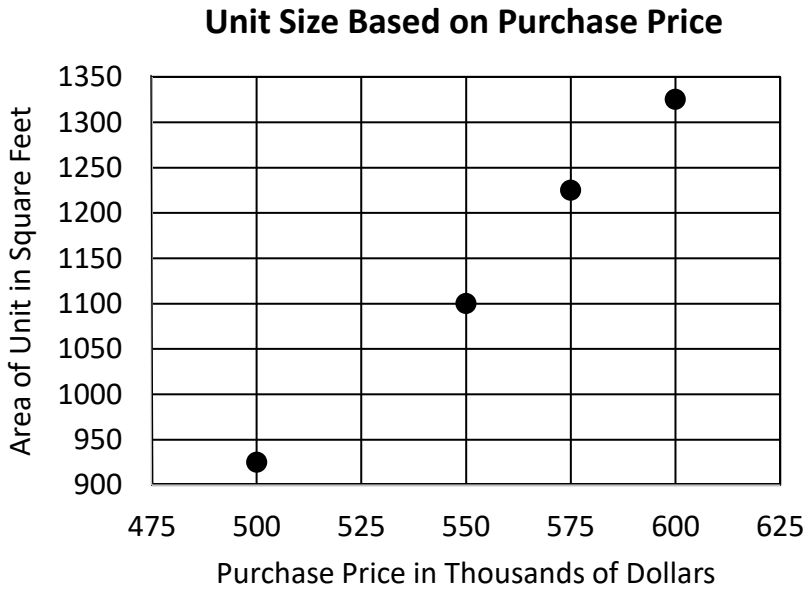
Range: The set of all output values

For Example 1:

Domain:

Range:

You Try! Example 2: The graph below illustrates the area in square feet of each style of unit in the proposed condominium development based on the purchase price for it in thousands of dollars.



A. How many square feet are in a unit that is priced at $600,000?

B. How much would a unit with 1100 square feet cost?

C. Provide the set of ordered pairs for the relation that is shown in the graph.

B. State the domain and range of the relation. Write your answers using set notation.

Domain:

Range:

Part 2: Introduction to Functions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.5: Relations and Functions

Objectives:

- Be able to determine if a relation is a function based on the definition of function
- Be able to use the vertical line test to determine if a graph represents a function

Important Vocabulary Words:

Function, Vertical Line Test, Function Notation

Compare Part 1 Example 1 and Example 2. Besides the contexts and the number values, what is different about the two relations?

Definition of Function

A **function** is a relation that assigns to each element in its domain exactly one element in its range.

In other words, each "input value" corresponds to exactly one "output value."

Example 1: Function? Yes / No

x	y
1	3
2	7
3	1
4	4

Example 2: { (1, 4), (2, 5), (2, 6), (3, 7) } Function? Yes / No

Example 3: Function? Yes / No

Input	2	4	6	8
Output	3	5	3	9

You Try! Example 4: Determine if the following relations represent functions. Use the definition of function to support your answers.

A.

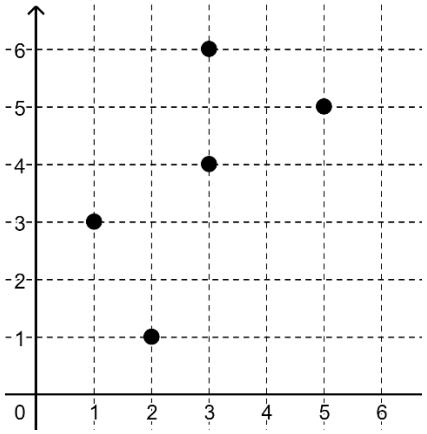
Fall Semester Year	**Required Minimum Math SAT Score For M121Q**
2015	540
2016	570
2017	570
2018	570
2019	570
2021	560

B.

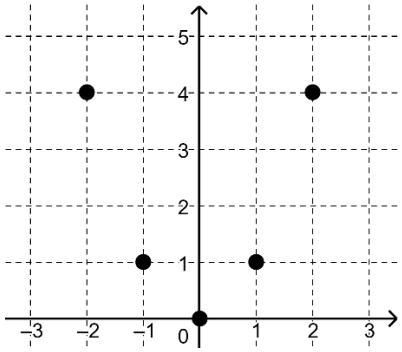
Number of People in Household	**Number of Bedrooms in Home**
2	1
4	4
3	2
4	3
2	2
5	4

You Try! Example 5: Determine if the following graphs represent functions. Use the definition of function to support your answers.

A.



B.



Vertical Line Test:

If _____ vertical line intersects a graph _____,

then the graph DOES represent a function.

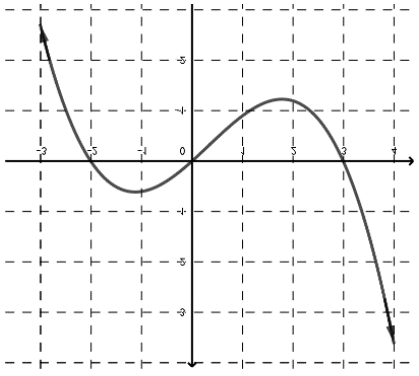
However,

If _____ vertical line intersects a graph _____,

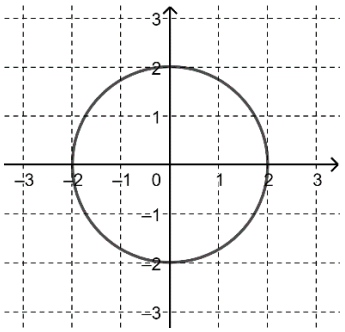
then the graph does NOT represent a function.

Example 6: Use the Vertical Line Test to determine if the following graphs represent functions.

A.



B.



Pull It All Together You Try! Problems for Section 1.1 Part 2

1. A relation is a **function** if _____.

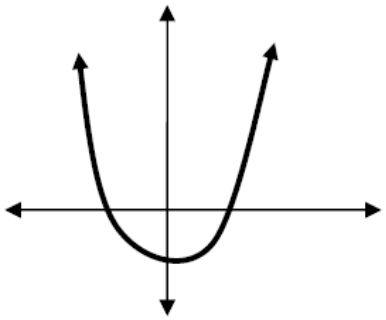
2. Determine whether each relation / graph represents a function. Respond with "Yes" or "No."

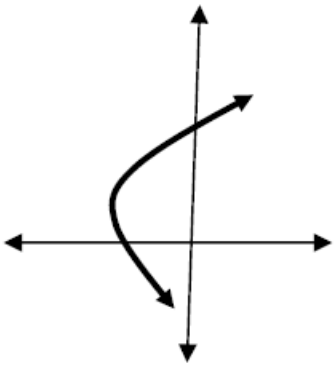
A. { (-2,3), (-1,3), (0,3), (1,4), (2,5) } _____

B. { (-2,-1), (-2,0), (0,1), (1,2), (2, 3) } _____

C. _____

D. _____





Section 1.2: Function Notation and Net Change

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.5: Relations and Functions

Objectives:

- Be able to use and interpret function notation.
- Understand the phrases: independent variable, dependent variable, ___ is a function of ___
- Be able to determine net change and interpret its meaning in context

Important Vocabulary Words:

Function Notation, independent variable, dependent variable, ____ is a function of _____, Net Change

---

New Way to Think About Functions
A **function** f is a rule that assigns exactly one output to each input.

Equation Form		Function Notation Form
$y = x^2 + 4$	$\leftrightarrow$	$f(x) = x^2 + 4$

Find $f(2)$. What does $f(2)$ represent?

Example 1: Let $g(x) = -x^2 + 3x$. Evaluate each of the following.

A. $g(1)$ B. $g(-2)$ C. $g(a)$

You Try! Example 2: Let $g(x) = 2(x-3)^2$.

A. What is the name of the function? _____ B. What letter represents the input? _____

C. What represents the output? D. Determine $g(5)$ and $g(k)$.

Dependent & Independent Variables

The *input variable* is referred to as the **independent variable** and the *output variable* is referred to as the **dependent variable**.

↔ What comes out of a function depends upon what goes into it.

____ is a function of ___

We say: The **output** variable **"is a function of"** the **input** variable

Example 3: According to Cosmic Pizza's online menu, the cost, C, of an 18-inch pizza with t toppings is given by: $C(t) = 22.5 + 2.5t$. (Cosmic Pizza, 2022)

A. Describe the input and output variables.

Input Variable = _____ = _____

Output Variable = _____ = _____

We say: _____ **is a function of** _____

B. Determine $C(3)$ and write a sentence, using everyday English, that interprets the meaning of your answer in the given context.

C. Determine $C(5) - C(3)$ and write a sentence, using everyday English, that interprets the meaning of your answer in the given context.

Net Change in a Function

The **net change** in a function f from input a to input b = $f(b) - f(a)$

$\leftrightarrow$ represents the increase (+) or decrease (-) in the function outputs

You Try! Example 4: Assuming normal air density, the amount of power generated by a wind turbine depends upon the type of turbine and the windspeed. For windspeeds between 4 m/s and 12 m/s, the power P, in kilowatts (kW), generated by a particular type of wind turbine can be approximated by $P(w) = 0.955w^3$, where w is the windspeed in meters per second (m/s).

Informed by a Sciencing website (K. Lee, 2020) and Wind farm Bop website (F. Miceli, 2012)

A. Fill in the blanks:

Independent Variable: _____ Dependent Variable: _____

We say: _____ is a function of _____

Input Variable: _____ = _____
Word Description

Output Variable: _____ = _____
Word Description

B. Determine the net change in P from $w = 6$ to $w = 10$. Include appropriate units with your answer.

C. Write a complete sentence, using everyday English, that interprets the meaning of your answer to B in the given context.

Section 1.3: Interval Notation and Basic / Toolkit Functions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 2 Section 2.5: Solve Linear Inequalities
Chapter 3 Section 3.6: Graphs of Functions

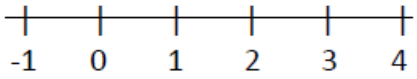
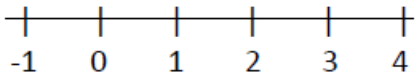
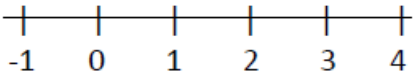
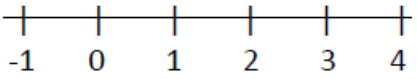
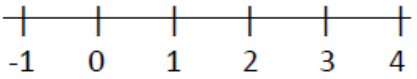
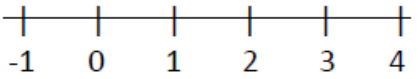
Objectives:

- Be able to represent an interval using interval notation
- Be able to graph functions by plotting points
- Know shapes & key points, domain and range for 6 basic / toolkit types of functions

Important Vocabulary Words:

Interval Notation, Domain, Range, Basic / Toolkit Functions

Interval Notation:

Inequality	Number Line Graph 1	Number Line Graph 2
$1 \le x < 3$		
	Interval Notation:	
$x \le 2$		
	Interval Notation:	
$x > 1$		
	Interval Notation:	

You Try! Example 1: Express each of the following using interval notation.

A. $x < 4$
Interval Notation:

B. $-3 < x \le 5$
Interval Notation:

C. $x \ge -1$
Interval Notation:

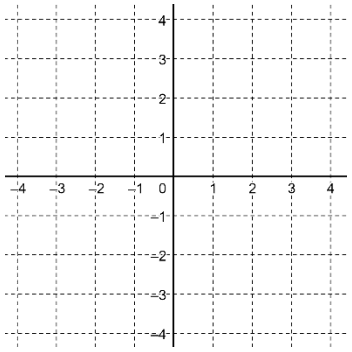
Basic Graphing Strategy

When graphing functions, x-coordinates ↔ input values and y-coordinates ↔ output values. Hence, we talk about the graph of "$y = f(x)$." A basic technique is to create a T-chart. Choose x-values and use the algebraic expression of the function to calculate corresponding y-values. The graph is the set of all such (x, y) points. Plot enough points to be able to determine the overall shape of the graph.

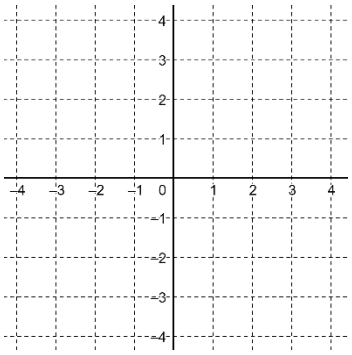
Six Basic / Toolkit Functions

Example 2: Graph the following functions by plotting points. Use interval notation to state the domain (x values) and range (y values) of each function. **Learn the shapes of all six of the basic / toolkit function types.**

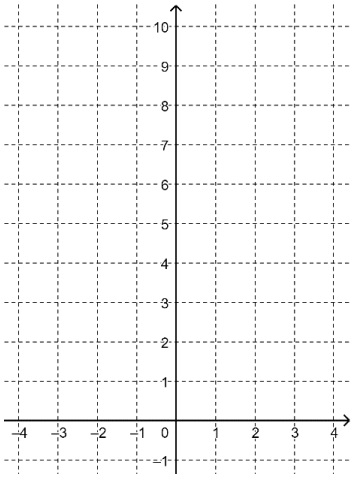
A. Constant functions: $f(x) = \#$
Example: $f(x) = 3$



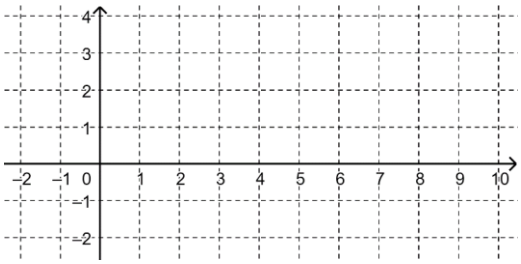
B. Linear Functions: $f(x) = mx + b$
Example: $f(x) = 2x - 1$



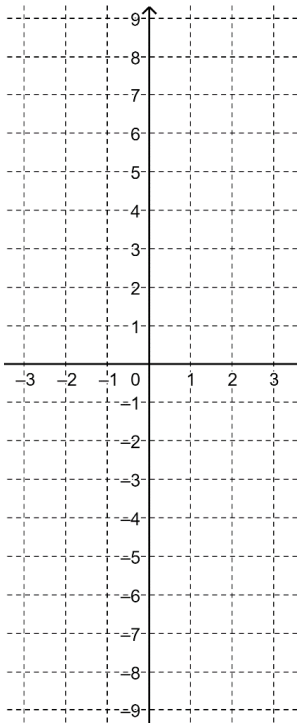
C. The square function: $f(x) = x^2$



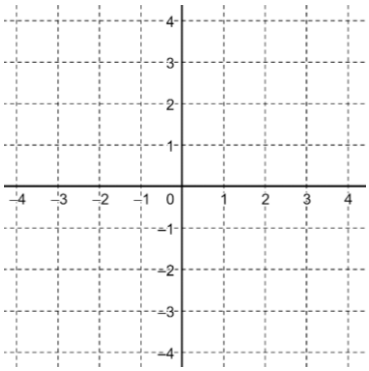
D. The square root function $f(x) = \sqrt{x}$



E. The cube function: $f(x) = x^3$



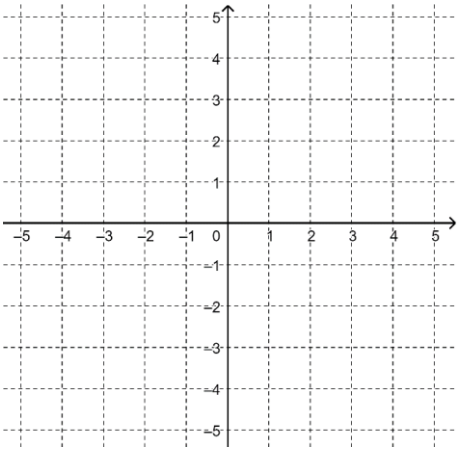
F. The absolute value function $f(x) = |x|$



Graphing Simple Transformations of Basic / Toolkit Functions

Strategy: Set up a "T-chart" to get several points to plot and also use what you know about the shapes of the six basic / toolkit functions.

You Try! Example 3: Sketch the graph of $f(x) = -(x-2)^2$ and determine its domain and range.



Section 1.4: Getting Information from a Graph

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.6: Graphs of Functions

Objectives:

Based on the graph of a function, be able to:

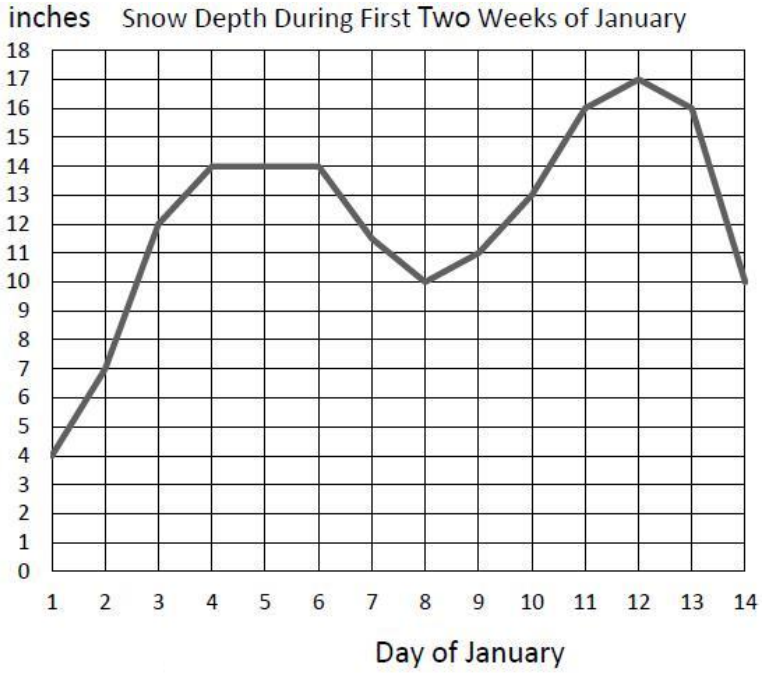
- Determine the domain and range
- Evaluate the function for a given input value and solve equations of the form $f(x) = \#$
- Determine x- and y-intercepts

Important Vocabulary Words:

Interval Notation, Domain, Range, X-intercept, Y-intercept

Introductory Example

The graph of $y = d(x)$ is shown, where $d(x)$ is the snow depth, in inches, on the x th day of January.



A. What is the domain of d ?

B. What is the range of d ?

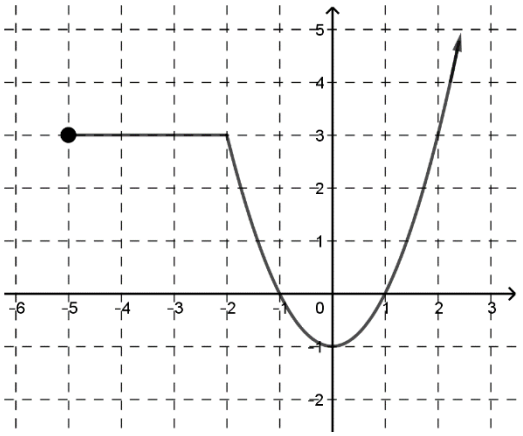
C. How deep was the snow on January 9?
↔ Determine $d(9)$.

D. When were there 16 inches of snow on the ground?
↔ Find the value(s) of x for which $d(x) = 16$.

E. When was the snow depth 7 inches or less?
↔ Find the value(s) of x for which $d(x) \le 7$.

Piecewise-defined Function Examples:

Example 1: The graph of $y = f(x)$ is shown.



Use the graph to determine the following.
Use interval notation and coordinate pair notation whenever appropriate.

Domain of f: _____

Range of f: _____

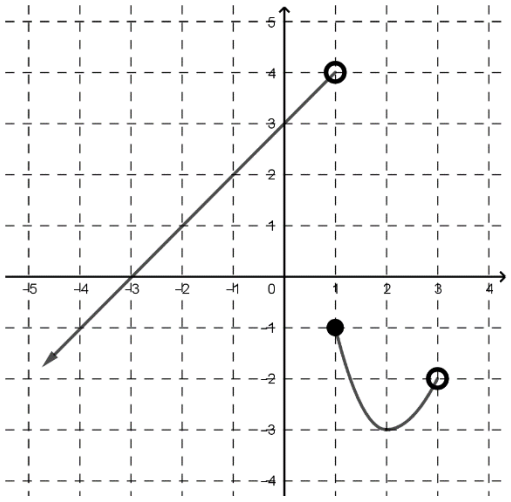
$f(2) =$ _____

Value(s) of x for which $f(x) = 0$: _____

Values of x for which $f(x) < 0$: _____

x-intercept(s): _____ y-intercept: _____

You Try! Example 2: The graph of $y = g(x)$ is shown.



Use the graph to determine the following.
Use interval notation and coordinate pair notation whenever appropriate.

Domain of g: _____

Range of g: _____

$g(-1) =$ _____

$g(x) = -1$ for $x =$ _____

Value(s) of x for which $g(x) > 0$: _____

Value(s) of x for which $g(x) \geq 2$: _____

x-intercept(s): _____ y-intercept: _____

Section 1.5: Domains of Functions Presented Algebraically

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.2: Domain and Range

Objectives:

- Be able to determine the domain of a given function (assuming real number output values) and write it using interval notation

Important Vocabulary Words:

Domain, Interval Notation, Polynomial Function, Rational Function, Root / Radical Function

---

Introductory Example

Let $f(x)=\dfrac{\sqrt{x}}{x-4}$. Find each of the following:

A. $f(5)=$_____

B. $f(0)=$_____

C. $f(4)=$_____

D. $f(-9)=$ _____

F. Domain of f so that it outputs only real numbers:

To determine the domain of a function that is presented algebraically so that it outputs only real numbers, there are <u>two issues to consider</u>:

1.

2.

How do we determine the domain of a polynomial function?

Examples: $f(x) = 4x^3 - 3x^2 + 5x - 12$ Domain:

$g(x) = 3x^2 - 5, \quad -2 \le x \le 10$ Domain:

How do we determine the domain of a rational (fractional) function?

BIG IDEA:

Example: $f(x) = \dfrac{3x+15}{x-4}$ Domain:

How do we determine the domain of a root/radical function so that it outputs only real numbers?

BIG IDEA: $\sqrt[n]{negative\ number}$ is "okay" when n is ODD,
BUT, is "not a real number" when n is EVEN.

→ For ODD roots, one does NOT need to be concerned about restricting x in order to avoid a negative number under the root symbol.

→ For EVEN roots, one MUST consider the necessity of restricting x in order to avoid a negative number under the root symbol.

Example 1: Determine the domains of the following functions so that they output only real numbers. Write answers using interval notation.

A. $f(x) = \sqrt{7-x}$

B. $g(x) = \sqrt[3]{2x+8}$

Domain:

Domain:

Example 2: Determine the domains of the following functions so that they output only real numbers. Write answers using interval notation.

A. $h(x)=\dfrac{\sqrt{x+5}}{x-2}$

Domain:

B. $k(x)=\dfrac{3}{\sqrt{x-1}}$

Domain:

You Try! Example 3: Determine the domains of the following functions so that they output only real numbers. Write answers using interval notation.

A. $f(x)=\dfrac{x-5}{3x+21}$

Domain:

B. $g(x)=\dfrac{x^3-4x}{3}$

Domain:

C. $h(x)=\sqrt[3]{5x+2}$

Domain:

D. $i(x)=\sqrt{12-2x}$

Domain:

E. $j(x)=\dfrac{\sqrt{x-3}}{x-7}$

Domain:

F. $k(x)=\dfrac{x-7}{\sqrt{x-3}}$

Domain:

Section 1.6: Piecewise Defined Functions

Part 1: Introduction to Piecewise Functions

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.2: Domain and Range

Objectives:

- Understand the notation for piecewise defined functions and be able to evaluate such a function when given an input value
- Be able to use piecewise defined functions to answer application questions

Important Vocabulary Words:

Piecewise Defined Functions

Example 1: $h(x) = \begin{cases} x^2 & if \quad x < 1 \\ x+1 & if \quad x \geq 1 \end{cases}$

Find each of the following:

$h(-3)$

$h(0)$

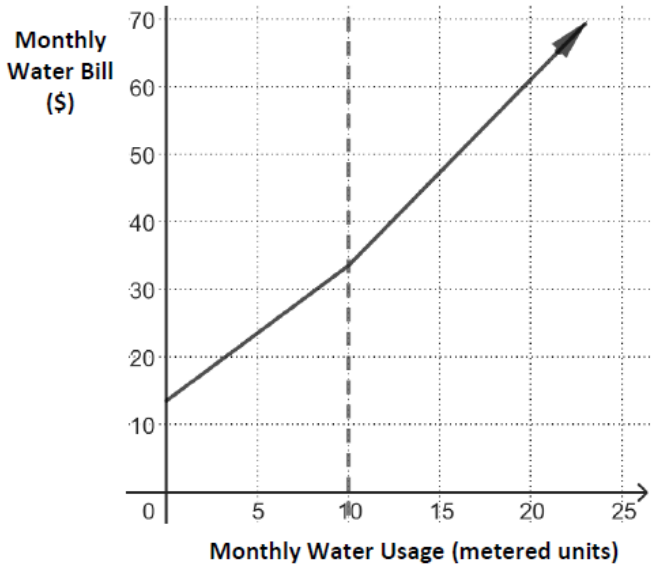
$h(3)$

$h(1)$

You Try! Example 2: The monthly water bill B in dollars for a residence in a city is given by $B(x)$, where x is the number of metered units of water used during the month.
(1 metered unit = 100 cubic feet of water.)

Informed by Salt Lake City Utilities website (Salt Lake City, 2023)

$$B(x)=\begin{cases}13.5+2x & \text{if } 0\le x\le 10\\ 6+2.75x & \text{if } x>10\end{cases}$$



A. Find $B(5)$ and $B(20)$. Include appropriate units with your answers.

B. How many cubic feet of water was used if the monthly water bill is $30?
Set up and solve an equation to answer the question.

Strategy to Determine an Algebraic Expression for a Piecewise Function in an Application

1. Determine what the input and output variables represent.
2. Determine "boundary" input value(s) and also pick an input value from each of regions created by the boundary value(s). Write down EVERY arithmetic step used to determine corresponding output values.
3. To create algebraic expressions for each region, mimic the arithmetic steps used in #2. I.e. replace the input numbers with "x" and follow the same arithmetic processes.

Example 3: Suppose a state taxed income as follows:

- 5% tax for taxable income up to $30,000
- 7% tax on *additional* taxable income over $30,000

Then the amount of tax a person owes, T, is a piecewise function of a person's taxable income x.

A. Definitions (words): Input: $x =$ _____

Output: $T(x) =$ _____

B. Determine $T(20,000)$, $T(30,000)$ and $T(40,000)$. Write down EVERY mathematical calculation!

C. Express T as a piecewise defined function:

$$T(x) = \begin{cases} _____ & if \ _____ \\ _____ & if \ _____ \end{cases}$$

Part 2: Graphing Piecewise Functions

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.2: Domain and Range

Objectives:

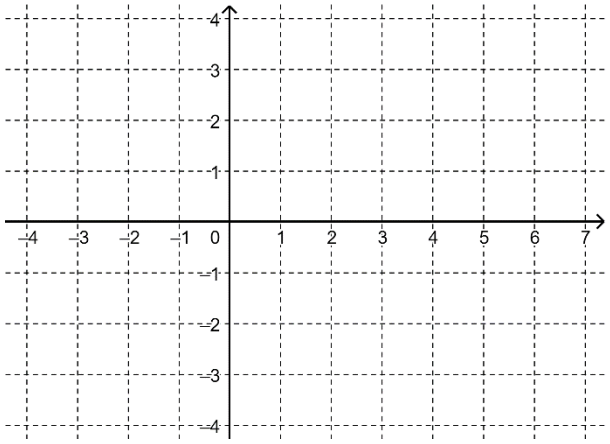
- Be able to graph piecewise defined functions by plotting points and by using knowledge about the graphs of the six basic / toolkit functions

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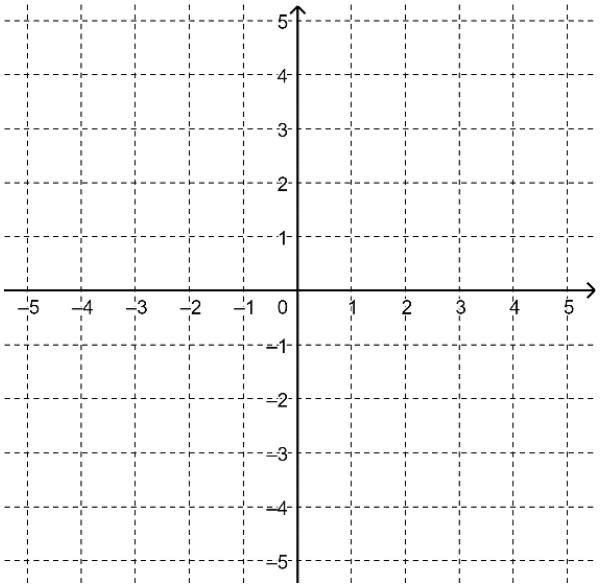
Graphing Piecewise Defined Functions

1. **Getting Set Up:** Determine the "boundaries" for the rules and indicate them on graph. (NOTE: These are not actually part of the graph, but they serve as guides.) Label the regions determined by the boundaries and indicate which rule is to be graphed in each of the regions.
2. **Plot Graph within Each Region:** For each region, look at the corresponding rule and determine "shape" information about what a graph that corresponds to that rule should look like. (Is it a line? If so, what is the slope? Is it a one of the basic / toolkit functions or a simple transformation of one of those functions? Etc.) For each region, create a T-chart of points to plot and use what you know about the basic shape to help sketch the graph **in that region**. **BE CAREFUL! Do not let graphs extend beyond their regions.**
3. **Points at Boundaries:** Use the rules to plot points on the boundaries of the regions to which they correspond. Based on whether or not a boundary is actually part of the region for a rule, use a "closed" or "open" dot. Make sure your graphs "connect" to the points on the boundary line(s).

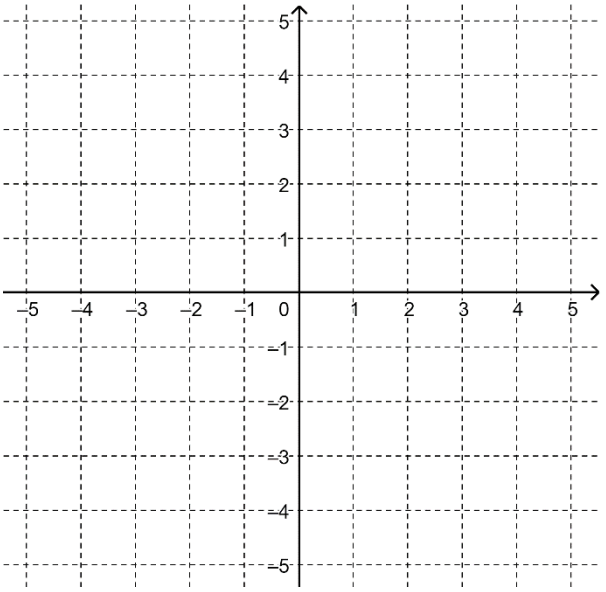
Example 1: Sketch the graph of $f(x)=\begin{cases}\sqrt{2-x} & \text{if} \quad x\le 2\\ \frac{1}{2}x-3 & \text{if} \quad 2<x\le 6\end{cases}$



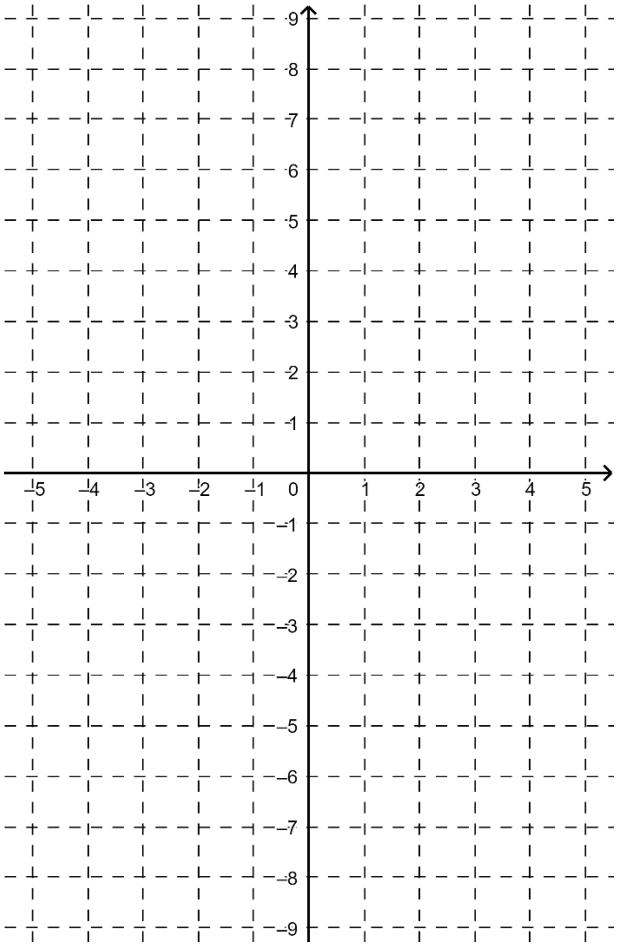
You Try! Example 2: Sketch the graph of $f(x)=\begin{cases} -3 & if \quad x<0 \\ -2x+1 & if \quad x\geq 0 \end{cases}$



You Try! Example 3: Sketch the graph of $f(x)=\begin{cases} x+5 & if \quad -5\leq x<-2 \\ |x|-1 & if \quad x\geq -2 \end{cases}$



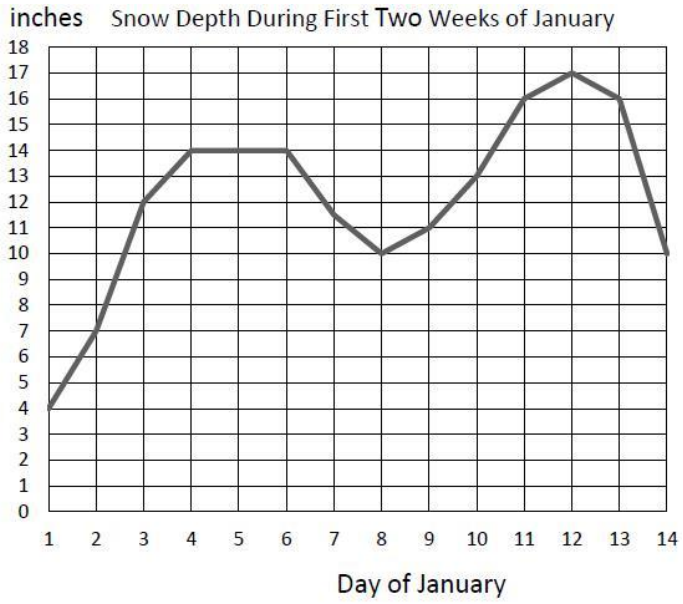
You Try! Example 4: Sketch the graph of $f(x)=\begin{cases} x^3 & if \quad -2<x\le 2 \\ \frac{3}{2}x-5 & if \quad x>2 \end{cases}$



Section 1.7: Rates of Change and Behavior of Graphs

Introductory Example – A Continuation

The graph of $y = d(x)$ is shown, where $d(x)$ is the snow depth, in inches, on the x th day of January.



A. When was the snow depth increasing, decreasing, constant?
↔ Find the interval(s) on which d is increasing, decreasing, constant.

Increasing: Decreasing: Constant:

B. What was the maximum snow depth?
↔ Find the maximum value of d.

On which day was the snow the deepest?
↔ Find the *location* of the maximum value of d.

C. On average, did the snow depth increase more quickly between Day 3 and Day 4 or between Day 8 and Day 12? How can you tell?

Part 1: Average Rate of Change

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.3: Rates of Change and Behavior of Graphs

Objectives:

- Understand the definition of average rate of change and how it relates to the graph of a function
- Be able to apply average rate of change to a variety of function settings
- Be able to write a sentence, using everyday English, that interprets the meaning of average rate of change in a given context

Important Vocabulary Words:

Average Rate of Change, Secant Line

Definition of Average Rate of Change

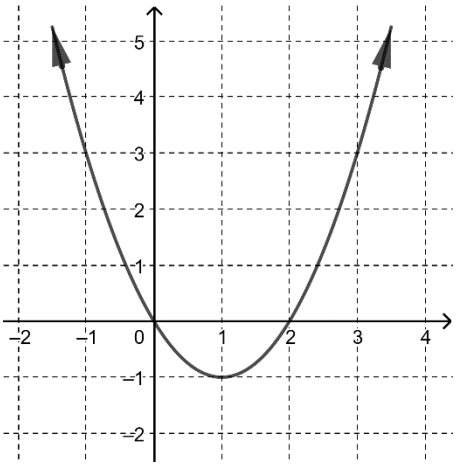
The **average rate of change** in a function f between two input values x_1 and x_2 **describes *how quickly*, on average, the function increases or decreases** over the interval $[x_1, x_2]$. It is the total change of the function values (output values) divided by the change in the input values.

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \text{Slope of the secant line between } (x_1, y_1) \text{ and } (x_2, y_2)$$

$$\text{where } y_1 = f(x_1) \text{ and } y_2 = f(x_2)$$

Example 1: Let $g(x) = x^2 - 2x$.



A. Determine the average rate of change in g between $x = 1$ and $x = 3$.

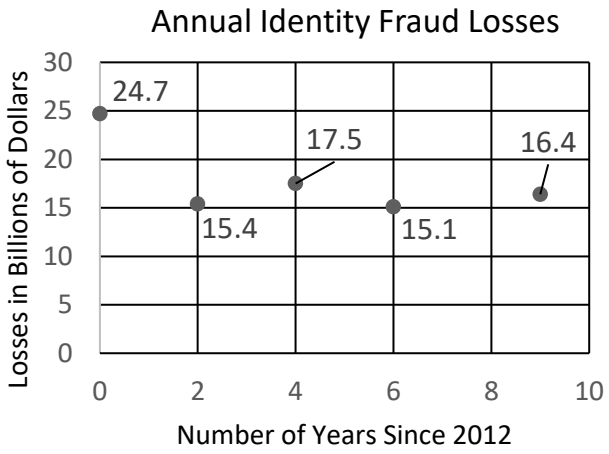
B. **You Try!** Determine the average rate of change in g over the given intervals. Use function notation in your work.

i.) Between $x = -1$ and $x = 0$

ii.) Between $x = -1$ and $x = 3$

C. Why is the answer to A positive while the answer to B part i) is negative?

Example 2: The graph below shows the annual amount of loss, L, in billions of dollars, in the U.S. due to identity fraud between 2012 and 2021, where x is the number of years since 2012.



A. Determine the average rate of change in L between $x = 0$ and $x = 6$. Include appropriate units with your answer.

B. Write a complete sentence that interprets your answer from A in the given context.

Informed by Bureau of Justice Statistics Bulletins for 2012 (E. Harrell & L. Langton, 2013), 2014 (E. Harrell, 2015), 2016 (E. Harrell, 2019), 2018 (E. Harrell, 2021), 2021 (E. Harrell & A. Thompson, 2023)

C. Determine the net change in L between $x = 0$ and $x = 6$.
Include appropriate units with your answer.

D. Write a complete sentence that interprets your answer from C in the given context.

Part 2: Behavior of Graphs

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.3: Rates of Change and Behavior of Graphs

Objectives:

- Be able to determine where a function is increasing, decreasing and constant from its graph
- Be able to determine local minimum and local maximum values and where they occur

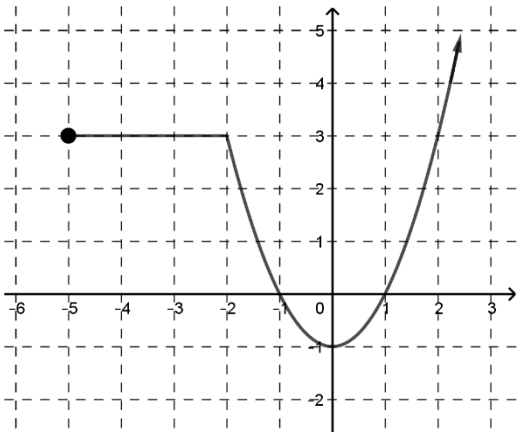
Important Vocabulary Words:

Increasing, Decreasing, Constant, Local Minimum, Local Maximum

Increasing, Decreasing, Constant

A function is **increasing** over an interval (a,b) if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any two values x_1 and x_2 in (a,b).
A function is **decreasing** over an interval (a,b) if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any two values x_1 and x_2 in (a,b).

Example 1: The graph of $y = f(x)$ is shown. Use the graph to determine where f is increasing, decreasing and constant. Write your answers using interval notation.

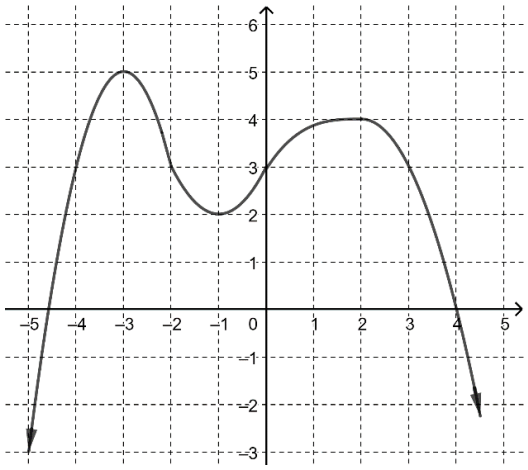


Increasing: _____

Decreasing: _____

Constant: _____

You Try! Example 2: The graph of $y = g(x)$ is shown. Use the graph to determine where g is increasing and decreasing. Write your answers using interval notation.



Increasing: _____

Decreasing: _____

Example 3: For the graph of g shown in Example 2, determine the **local maximum value(s)** and **local minimum values(s)** and where they occur.

_____ is a local maximum value and it occurs at _____

_____ is a local _____ value and it occurs at _____

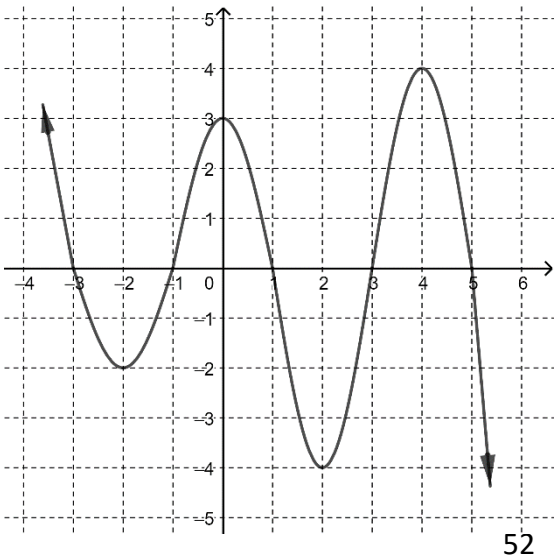
_____ is a local _____ value and it occurs at _____

Local Maximum and Local Minimum Values

$y = f(c)$ is a **local maximum value** occurring at $x = c$ if there is an interval (a,b) containing c such that $f(c) \geq f(x)$ for all values x in (a,b).

$y = f(c)$ is a **local minimum value** occurring at $x = c$ if there is an interval (a,b) containing c such that $f(c) \leq f(x)$ for all values x in (a,b).

You Try! Example 4: The graph of $y = h(x)$ is shown. Determine the following:



Local maximum value(s) of h: _____

Value(s) of x at which they occur: _____

Local minimum value(s) of h: _____

Value(s) of x at which they occur: _____

Section 1.8: Models and Applications

Part 1: Working with Formulas

Reference Materials:

Open Stax: *College Algebra 2e with Corequisite Support* by Jay Abramson and Sharon North
Chapter 2 Section 2.3: Models and Applications

Objectives:

- Be able to solve a formula for a specified variable

Important Vocabulary Words:

Formula

What is a Formula?

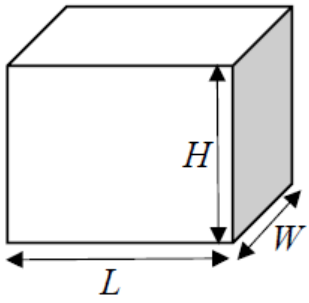
A formula is an equation involving multiple variables / quantities which illustrates relationships between the quantities that always hold true.

Example 1: The formula $F = \frac{mv^2}{r}$ calculates the centripetal force F felt by an object of mass m traveling at a velocity v on a circular path with resistance r. (Example: The force pushing a car off the road on a curve, where resistance corresponds to friction between the tires and the road.)

Solve the formula for v.

Example 2: A fully enclosed rectangular box is shown.

A. Find a formula for the surface area of the box.



B. Solve the formula above for H.

You Try! Example 3: Simple Interest Formula: $A = P + Prt$

A = amount after t years t = time in years P = amount invested
r = annual interest rate as a decimal

A. Solve the formula for r.

B. Solve the formula for P.

Part 2: Geometric Applications

Reference Materials:

Open Stax: *College Algebra 2e with Corequisite Support* by Jay Abramson and Sharon North
Chapter 2 Section 2.3: Models and Applications

Scottsdale Community College – Math Blog: *College Algebra, An Investigation of Functions*, primary authors David Lippman, Melonie Rasmussen and Jay Abramson
Chapter 3 Section 3.2: Quadratic Functions Example 1 - page 173 of the pdf file (printed page 165)

Objectives:

- Be able to create a mathematical model for a geometric scenario and use the model to answer questions about the scenario.

Important Vocabulary Words:

Mathematical Model

Definition of Mathematical Model

A **mathematical model** is a mathematical representation (such as an equation or function) of a real-world situation.

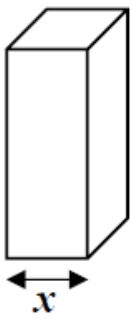
Useful Modeling Strategies:

1. Choose variables to use.
2. Draw and label a diagram.
3. Use simple cases to explore the situation numerically.
4. Describe relationships verbally first, and then represent them algebraically.
5. Check the model with simple cases to make sure it makes sense.

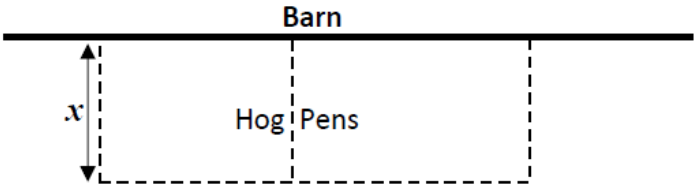
Example 1: A company wants to package its beverages in a rectangular carton with a square base They want the carton to be two and a half times as tall as it is wide.

A. Determine an expression for the volume of the carton as a function of the width, x, of its base.

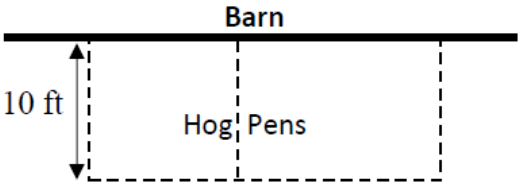
B. What dimensions should the company use for the carton if they want it to hold 1280 milliliters? NOTE: 1 milliliter = 1 cm^3



Example 2: A farmer has 180 feet of fencing and needs to enclose a rectangular area divided by a fence into two pens for his hogs. The farmer intends to build the pen up against the side of a long barn so that he will not have to use fencing along that side. His goal is to create as big of an enclosure as he can with the amount of fencing that he has.



A. Start by experimenting with a number to get a sense of how things work. Determine the total area of the enclosure when x is 10 feet.



B. Label the total length of the enclosure as L. Then use x and L to create a fencing "constraint" equation involving 180.

Constraint Equation:

C. Determine an expression for the total area of the two hog pens **as a function of only x**, where x is the width of the hog pens as shown in the diagram. HINT: Solve the constraint equation for L.

Area = $A(x) =$ _____

Use Graphing Technology for D: (You will learn a "pencil & paper" algebraic strategy in Chapter 3.)

D. Based upon the graph of $A(x)$, what width should the farmer use for the rectangular enclosure in order to provide maximum area for the hogs within it?

You Try! Example 3: The length of a rectangular room is 5 feet less than twice its width w.



A. Determine an expression for the perimeter P of the room as a function of its width w.

B. What is the width of the room if it has a perimeter of 80 feet?

C. What is the area of the room if it has a perimeter of 80 feet?

Chapter 2
Linear Functions

Sections 2.1 – 2.2

Section 2.1: Introduction to Linear Functions

Part 1: Definition of Linear Function

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 4 Section 4.1: Linear Functions

Objectives:

- Know the definition of a linear function and be able to apply it when information is presented in table form, in graph form and in word form.

Important Vocabulary Words:

Linear Function, Form of a Linear Function, Rate of Change, Slope, Initial Value, Y-intercept, Units

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Definition of Linear Function

A function f is a **linear function** if the graph of f is a line. This occurs when f has constant average rate of change; that is, its graph has constant slope.

Form of a Linear Function

The ***form*** of a linear function matches the slope-intercept form of a line:

$$f(x) = mx + b$$

$m =$ constant rate of change = slope

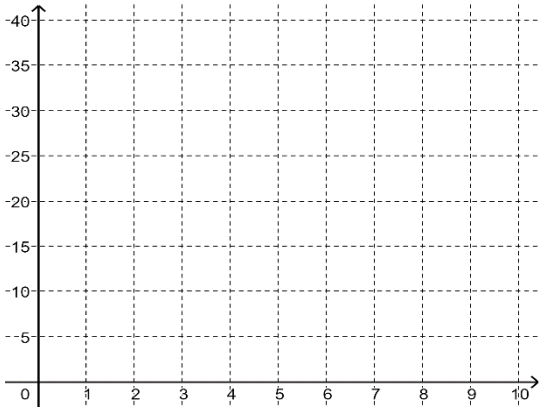
$b =$ initial or starting value (when $x = 0$) $\leftrightarrow$ y-intercept: $(0, b)$

Example 1: To make money for school, Sue sells snow cones during the summer. She started a work shift with \$10 in her cash box. With each customer, Sue kept track of the total number of snow cones sold and the total amount in the cash box as shown in the table below.

Total Number of Snow Cones Sold	0	3	7	9
Amount in Cash Box	\$10	\$18.25	\$29.25	\$34.75

The amount of money in the cash box, A, is a function of the total number of snow cones sold, n.
Is $A(n)$ a linear function?

A. Draw and label a graph of the data.
Does it appear to be linear?

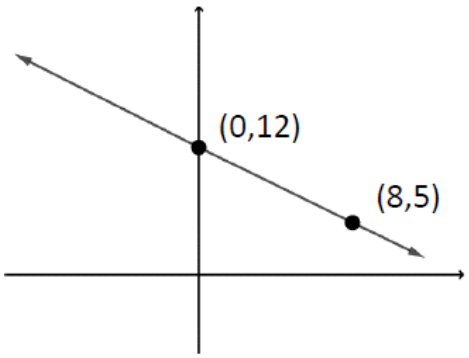


B. Sometimes points *appear* to lie on a line, but actually do not. To truly determine if $A(n)$ is a linear function, calculate the average rates of change ↔ slopes to see if they are constant. Include units in your work and your answers.

C. Is $A(n)$ a linear function? If so, write an algebraic expression for $A(n)$.

Circle One: Yes No $A(n) =$ _____

You Try! Example 2: The graph of a linear function f is shown below. Determine an algebraic expression for $f(x)$.



$f(x) =$ _____

Example 3: At sea level the boiling point of water is 100 °C. But as one climbs in elevation, the boiling point of water decreases by *approximately* 0.0033 ° per meter in elevation. Find a linear function $B(x)$ that *approximates* the boiling point of water at an elevation of x meters.

Informed by online altitude / pressure / boiling point calculator (B. Szyk, 2024)

Part 2: Interpreting Slope and Y-intercept in an Applied Scenario

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 4 Section 4.1: Linear Functions

Objectives:

- Be able to write complete sentences, using everyday English, that interpret the meanings of the slope and the y-intercept for a given linear model in an applied setting.

Important Vocabulary Words:

Rate of Change, Slope, Initial Value, Y-intercept, Units

Example 1: The speed of sound, S, changes with the air temperature, t. For temperatures between 0°C (32°F) and 50°C (122°F), the relationship between the two can be *approximated* by the following:

$$S(t) = 13t + 741$$

where $S(t)$ is the speed of sound in miles per hour and t is the temperature in degrees Celsius.

Informed by a speed of sound calculator on a National Weather Service website (T. Brice and T. Hall)

A. Draw a rough sketch of the graph of $y = S(t)$.
   i. Label the y-intercept with the appropriate number.
   ii. Label the horizontal and vertical axes with appropriate *words* and *units*.
   iii. Indicate 0 on the horizontal axis.
   iv. Draw a slope triangle: Label the "run" as 1 and the "rise" with the unit increase from the slope. In addition, label the rise and run with the appropriate units.



B. Write a complete sentence, using everyday English, which interprets the meaning of 741 in the given context.

C. Write a complete sentence, using everyday English, which interprets the meaning of 13 in the given context.

Section 2.2: Using Point-Slope Form to Create Linear Models

Part 1: Point-Slope Form of a Line

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 3 Section 3.3: Find the Equation of a Line

Objectives:

- Know and understand the point-slope form of the equation a line
- Be able to use the point-slope form to create the equation of a line through two given points

Important Vocabulary Words:

Slope-Intercept Form of a Line, Point-Slope Form of a Line

Slope Between Two Points on a Line

$$\text{Slope} = \text{m} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

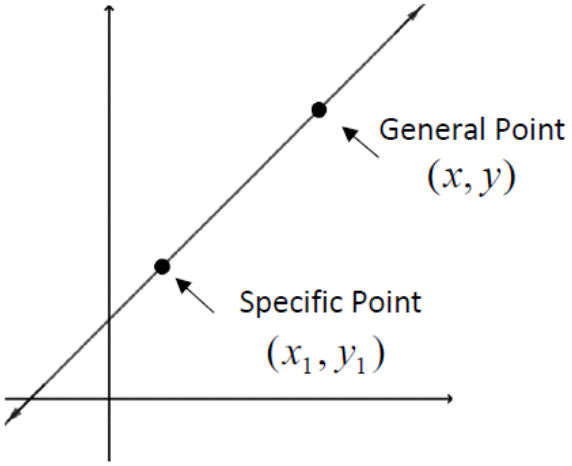
where (x_1, y_1) and (x_2, y_2) are two points on the line

Slope-Intercept Form of the Equation of a Line

$$y = mx + b$$

where $(0, b)$ is the *y-intercept*
m = *slope* of the line

Development of the Point-Slope Form of the Equation of a Line



Point-Slope Form of the Equation of a Line

where (x_1, y_1) is a point on the line and m = slope of the line

Example 1:

A. Use **point-slope** form to find the equation of the line that has slope = 3 and that passes through the point $(4,10)$.

B. Rewrite the equation in **slope-intercept** form.

You Try! Example 2: Use the point-slope form of the equation of a line to determine the equation of the line that passes through (5, -2) and (-1, 13). Then convert the equation to slope-intercept form.

Part 2: Creating a Linear Model Based on Two Data Points

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 4 Section 4.1: Linear Functions

Objectives:

- Be able to create a linear function model when given two data points in an applied scenario
- Be able to write complete sentences, using everyday English, that interpret the meanings of the slope and the y-intercept for a given linear model in an applied setting.

Important Vocabulary Words:

Point-Slope Form of the Equation of a Line

Example 1: Cigarette smoking used to be much more popular than it is now. In 1964 the U.S. Surgeon General issued a first report on the dangers of smoking. Then, in January 1971 smoking ads became banned on television and radio. As a result of these efforts and others, cigarette smoking has been in steady decline in the United States. 37.1% of US adults smoked cigarettes in 1974. By 2004 that figured had dropped to 20.9% of adults. (See references at end of problem)

A. Find a linear function $S(t)$ that models the percent of US adults who smoked cigarettes t years after 1964. If needed, round decimal numbers to two decimal places.

Linear Model: _____

B. Draw and label a rough sketch of the graph of $y = S(t)$. In particular, label the y-intercept and draw and label a "slope triangle."



C. For the linear model found in Part A, write complete sentences, using everyday English, that interpret the meanings of the b and the m values in the given context.

Meaning of b:

Meaning of m:

D. According to the model, what percent of U.S. adults smoked in 2014?
See how your answer compares with *data provided by the American Lung Association!

E. If the model continued to be valid, in which year does it predict that only 10% of US adults smoke cigarettes?

Informed by an online PBS News article (Ruble, 2014), an online article in the New Yorker (Whiteside, 1970) and *data found on American Lung Association Website (American Lung Association)

Chapter 3
Quadratic and Polynomial Functions

Sections 3.1 – 3.7

Section 3.1: Factoring Foundations (R)

Part 1: Factoring Out the Greatest Common Factor

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 6 Section 6.1: Greatest Common Factor and Factor by Grouping

Objectives:

- Find the greatest common factor for two of more monomial expressions
- Factor the greatest common factor from a polynomial

Important Vocabulary Words:

Factors, Product, Greatest Common Factor, Factor

Factors, Product and Greatest Common Factor

$2 \cdot 6 = 12$ $\leftrightarrow$ 2 and 6 are **factors** of 12 and 12 is referred to as the **product** of 2 and 6

Similarly, by the Distributive Property of Multiplication Across Addition or Subtraction:

$3x(x+5) = 3x^2 + 15x$ $\leftrightarrow$ $3x$ and $x+5$ are **factors** of $3x^2 + 15x$

$3x^2 + 15x$ is the **product** of $3x$ and $x+5$

When comparing two or more numbers, the **greatest common factor** of the numbers is the largest number that is a factor of all of the numbers. For example, 6 is the greatest common factor of 12, 24 and 30. The numbers 2, 3 and 6 are all common factors of 12, 24 and 30, but 6 is the largest common factor. This idea can be extended to algebraic expressions.

Definition of Greatest Common Factor
The greatest common factor (GCF) of two or more expressions is the largest expression that is a factor of all of the expressions.

Example 1: Find the greatest common factor of $10x^2$, $18x^3$ and $4x^4$.

Factoring a Polynomial Expression By Un-distributing The Greatest Common Factor

BIG IDEA: Factoring a polynomial expressing means "**reversing" the distributive property of multiplication across addition or subtraction**.

Applying the Distributive Property: $3x(x+5) \rightarrow 3x^2+15x$

Factoring – Reverse the Distributive Property: $3x^2+15x \rightarrow 3x(x+5)$

Process for Factoring By Un-distributing the GCF

1. Find the GCF of all of the terms of the polynomial.
2. Rewrite each term as a product using the GCF.
3. Un-distribute, i.e. the "reverse" Distributive Property to factor the expression.
4. Check by using the Distributive Property to multiply the two factors.

Example 2: Factor each of the following polynomial expressions by factoring out the greatest common factor.

A. $8x^4+28x^3$

B. $50x^3+30x^2+60x$

You Try! Example 3: Factor each of the following polynomial expressions by factoring out the greatest common factor.

A. $2x^3+14x^2-26x$

B. $5x^4-35x^2$

Part 2: Factoring Trinomials With x^2 As the Leading Term

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 6 Section 6.2: Factor Trinomials

Objectives:

- Factor trinomials of the form ax^2+bx+c by using trial and error

Important Vocabulary Words:

Prime

BIG IDEA: We will utilize patterns that occur when you multiply factors of the form $(x+a)$ or $(x-a)$ where a is a positive whole number.

Factoring Trinomials of the Form $x^2 + bx + c$ and $x^2 - bx + c$

Example 1: Use the Distributive Property to multiply $(x+3)(x+5)$, $(x-3)(x-5)$ and notice a useful pattern for factoring the resulting products.

Example 2: Factor the following:

A. $x^2+11x+18$

B. $x^2-9x+20$

Factoring Trinomials of the Form $x^2 - bx + c$ and $x^2 - bx - c$

Example 3: Use the Distributive Property to multiply $(x-3)(x+5)$, $(x+3)(x-5)$ and notice a useful pattern for factoring the resulting products.

Example 4: Factor the following:

A. $x^2 - 7x - 30$ B. $x^2 + 6x - 16$ C. $x^2 + 6x - 8$

Put It All Together

You Try! Example 5: Factor the following or state that it is prime.

A. $x^2 - 10x + 21$ B. $x^2 + x - 3$ C. $x^2 + 3x - 18$

Part 3: Factoring A Difference of Squares

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 6 Section 6.3: Factor Special Products

Objectives:

- Factor expressions of the form $a^2 - b^2$
- Understand that expressions of the form $a^2 + b^2$ generally cannot be factored

Important Vocabulary Words:

Difference of Squares, Prime

Example 1: Multiply each of the following. What do you notice?

A. $(2x+3)(2x+3)$ B. $(2x-3)(2x+3)$ C. $(2x-3)(2x-3)$

Factoring a Difference of Squares

If a and b are real numbers, then $a^2 - b^2 = (a+b)(a-b)$

Example 2: Factor the following:

A. $x^2 - 36$ B. $16x^2 - 1$ C. $x^2 + 4$

You Try! Example 3: Factor the following or state that it is prime.

A. $x^2 - 81$ B. $9x^2 + 25$ C. $9x^2 - 25$

Section 3.2: Solving Quadratic Equations (R)

What is a quadratic equation?

A quadratic equation is an equation in which the variable is squared and no exponent larger than 2 is used. The following are examples of quadratic equations:

$$2x^2 - 32 = 0 \qquad x^2 + x - 12 = 0 \qquad 3x^2 - 13x = 10 \qquad (n+2)^2 - 9 = 0$$

The last example can be more-clearly seen as being quadratic when it is simplified.

$$(n+2)^2 - 9 = 0 \;\rightarrow\; (n+2)(n+2) - 9 = 0 \;\rightarrow\; n^2 + 2n + 2n + 4 - 9 = 0 \;\rightarrow\; n^2 + 4n - 5 = 0$$

Part 1: Factoring Method for Solving Quadratic Equations

Reference Materials:

Open Stax: *Elementary Algebra 2e* by Maracek, Anthony-Smith & Honeycutt Mathis
Chapter 7 Section 7.6: Quadratic Equations

Objectives:

- Be able to solve quadratic equations by using factoring and the Zero Product Property

Important Vocabulary Words:

Zero Product Property

Zero Product Property

If $a \cdot b = 0$, then $a = 0$ or $b = 0$ (or both)

Example 1: Solve the following equations.

A. $(x+2)(x-5) = 0$

B. $x^2 - 7x + 12 = 0$

Example 2: Solve the following equations.

A. $3x^2 + 15x = 0$

B. $x^2 + 2x = 35$

You Try! Example 4: Solve the following equations.

A. $x^2 - 6x = 0$

B. $x^2 + 7x + 6 = 0$

C. $x^2 - 7x = 18$

It is possible to use this technique to solve more-complex quadratic equations, such as the example below. However, most students find the quadratic formula method (Part 3) to be easier to use when solving equations of the form $ax^2 + bx + c = 0$ and $a \neq 1$.

Solve: $3x^2 - 13x - 10 = 0 \rightarrow (3x + 2)(x - 5) = 0 \rightarrow$

$3x + 2 = 0$ or $x - 5 = 0$

$-2 \quad -2$ $+5 \quad +5$

$3x = -2$ $x = 5$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-2}{3}$

$x = -2/3$

Part 2: Square Root Method for Solving Quadratic Equations

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 9 Section 9.1: Solve Quadratic Equations Using the Square Root Property

Objectives:

- Be able to solve quadratic equations by the square root property

Important Vocabulary Words:

Square Root Property

Square Root Property

Solutions to $x^2 = a$ are $x = \sqrt{a}$ and $x = -\sqrt{a}$ $\leftrightarrow$ $x = \pm\sqrt{a}$

Square Root Method For Solving Quadratic Equations

1. Completely isolate the squared part of the equation.
2. Square root both sides and apply the Square Root Property
   → **Do not forget to use ± !** There are two solutions.

Example 1: Solve the following equations.

A. $2x^2 - 10 = 0$ B. $3x^2 + 12 = 0$ C. $5(x-1)^2 - 10 = 70$

Something Important to Notice: In all of the above problems **x is only found in the squared part of the equation.** When that is not the case, other strategies for solving quadratic equations are easier to use. For example, the square root method is not the best method for solving $3(x+4)^2 + 2x = 0$ because there is an x outside of $(x+4)^2$.

You Try! Example 2: Solve the following equations using the square root method OR the factoring method.

A. $3x^2 - 75 = 0$ B. $x^2 - 4x = 0$ C. $4(x+3)^2 + 2 = 30$

Part 3: Using the Quadratic Formula to Solve Quadratic Equations

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 9 Section 9.3: Solve Quadratic Equations Using the Quadratic Formula

Objectives:

- Know the quadratic formula
- Be able to use the quadratic formula to solve quadratic equations

Important Vocabulary Words:

Quadratic Formula

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ for $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You are expected to memorize the quadratic formula. There are songs / rhymes / stories available to aid memorization. YouTube is a good place to look for them.

Example 1: Solve the following equation. $3x^2 - 13x - 10 = 0$

Example 2: Solve the following equation. $2x^2 + 5x = 1$

You Try! Example 3: Solve the following equation. $5x^2 - 2x = 7$

Section 3.3: Transformations of x^2

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 9 Section 9.7: Graph Quadratic Functions Using Transformations

Objectives:

- Be able to use the ideas of vertical and horizontal shifts, vertical reflection and vertical stretch / compression to graph quadratic functions of the form $f(x) = a(x+h)^2 + k$
- Be able to create an algebraic expression for a quadratic function from its graph based upon the vertex and a point on the graph that is one horizontal unit away from the vertex.

Important Vocabulary Words:

Parabola, Vertical Shift, Horizontal Shift, Vertex, Axis of Symmetry, Vertical Reflection, Vertical Stretch / Compression Factor

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The Overall Idea

One way to think about quadratic functions is as transformations of the basic / toolkit function $y = x^2$. You graphed some transformations of basic / toolkit functions in the Section 1.3 homework assignment and likely noticed some patterns. In this section you will use a Desmos activity site to work through an exploration of those patterns.

Desmos Exploration

1. Turn on the first quadratic function, **f(x)** $= x^2 + k$. Then, vary the *k* slider to see how the parameter *k* alters the graph.

   Write down your observations below. Be sure to discuss the difference between when *k* is negative and when *k* is positive.

2. Turn off **f(x)** and turn on **g(x)** $= (x+h)^2$**.** Vary the *h* slider to see how the parameter *h* alters the graph.

   Write down your observations below. Be sure to discuss the difference between when *h* is positive and when *h* is negative. Does the graph behave the way you had expected?!

3. Turn off **g(x)** and turn on **i(x)** $= ax^2$ AND **x = {0 <y <a: 1}** AND **x={a<y<0: 1}** AND **y={0<x<1: 0}**.
   Vary the *a* slider to see how the parameter *a* alters the graph.
   NOTE: When *a* = 0 the graph disappears! Why?

   Write down your observations below.

   A. What is the difference between when a is positive and when a is negative?

   B. What is the difference between when $a > 1$ or $a < -1$ and when $-1 < a < 1$ (assuming $a \neq 0$)?

   C. How long is the blue line? Does its length ever change?

   D. How long is the orange line? Be sure to compare it to the value of the a slider!

   a is referred to as the ***vertical stretch / compression factor***. (It is NOT slope!!!)

   Space for notes from discussion with instructor:

4. Turn off **i(x)** AND **x = {0 <y <a: 1}** AND **x={a<y<0: 1}**. Then turn on **j(x)** $= a(x+h)^2 + k$ AND **x = {k < y < a+k: 1-h}** AND **x={a+k<y<k: 1-h}** AND **y={-h<x<1-h: k}**.

   Vary the *h*, *k* and *a* sliders to get a sense of how they work together.
   Write down any new observations.

Then, set $h = 3$, $k = -1$ and $a = -4 \leftrightarrow \quad j(x) = -4(x+3)^2 - 1$

A. What is the ***vertex*** of the resulting graph? B. What is the ***axis of symmetry*** of the graph?

C. Which direction does the ***parabola*** open? Why does it open that way?

D. Has the graph been stretched or compressed when compared with the graph of $y = x^2$?
   What is the length of the orange line?

Summary:

To shift the graph of $y = x^2$ **up** by a #: _____

To shift the graph of $y = x^2$ **down** by a #: _____

To shift the graph of $y = x^2$ **right** by a #: _____

To shift the graph of $y = x^2$ **left** by a #: _____

To ***vertically stretch / compress***
the graph of $y = f(x)$ # times as tall: _____

To ***vertically reflect*** the graph of $y = x^2$ across
the horizontal line that passes through its vertex: _____

Honing Your Skills:

Determine the values for *h*, *k* and *a* that accomplish the following transformations of $y = x^2$ and write down the matching algebraic expression for j(x).

1. Shift down 4 and right 2 $j(x) =$ _____

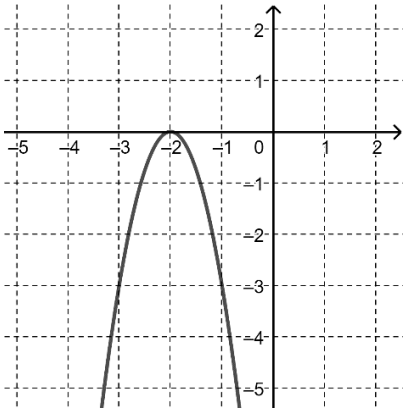
2. Reflect vertically and shift left 5 $j(x) =$ _____

3. Stretch 2 times as tall and shift up 3.5 $j(x) =$ _____

4. Reflect vertically, compress 1/4 as tall, shift down 3 and right 4 $j(x) =$ _____

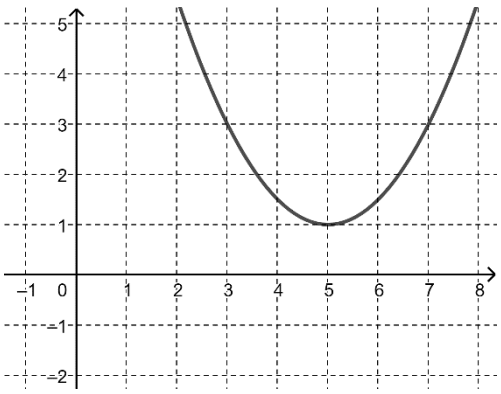
Determine algebraic expressions to match the following graphs

5.



$j(x) =$ _____

6.



$j(x) =$ _____

Section 3.4: Three Forms of a Quadratic Function

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 5 Section 5.1: Quadratic Functions

OpenTextBookStore: *Intermediate Algebra* – College of Redwoods Department of Mathematics
Chapter 5 Section 5.3: Zeros of the Quadratic pages 465 - 466

Objectives:

- Know what a quadratic function is and three primary forms in which it can be written
- Know characteristics of each form and how they relate to the graph of a quadratic function
- Be able to identify the following features of the a graph of a quadratic function: vertex, axis of symmetry, stretch / compression factor, x-intercept(s), y-intercept

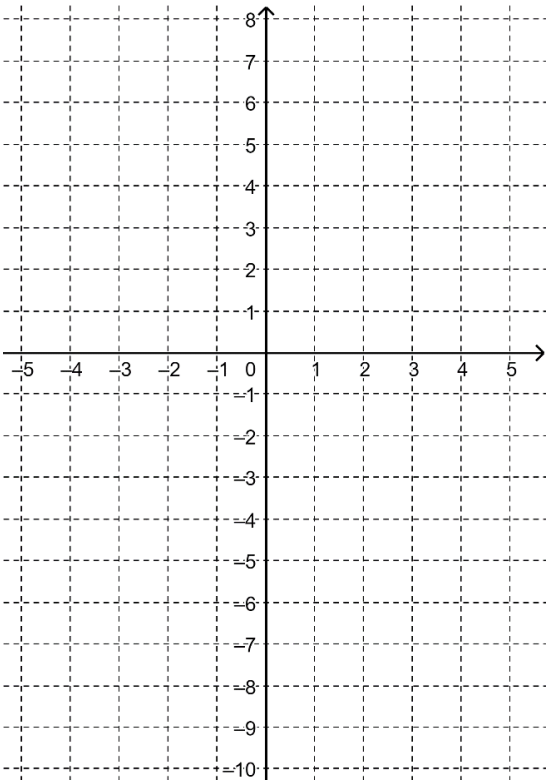
Important Vocabulary Words:

Standard Form, General Form, Factored Form, Vertex, Axis of Symmetry,
Stretch / Compression Factor, X-intercept, Y-intercept

Introduction and Overview

Example 1: Let $f(x) = 4(x+2)^2 - 9$, $g(x) = 4x^2 + 16x + 7$, $h(x) = 4(x+0.5)(x+3.5)$.

A. Use graphing technology such as the Desmos Calculator to graph all three functions and sketch their graphs on the axes below. What do you notice?!



B. Determine the following and label them in the graph:

Vertex: _____

Axis of Symmetry: _____

Stretch Factor: _____

x-intercepts: _____

y-intercept: _____

Overview of the Three Forms of a Quadratic Function

The graph of a quadratic function is always a parabola.

General Form of a Quadratic Function:

$f(x) = ax^2 + bx + c$; where $a,\ b,\ c$ are real numbers. If $a > 0$: $a < 0$:

y-intercept:

Factored Form of a Quadratic Function:

$f(x) = a(x-m)(x-n)$; where $m,\ n$ are real numbers. If $a > 0$: $a < 0$:

x-intercepts:

Standard Form of a Quadratic Function:

$f(x) = a(x-h)^2 + k$; where $a,\ h,\ k$ are real numbers. If $a > 0$: $a < 0$:

Vertex:

Focus on Standard and General Form

You Try! Example 2: Let $f(x) = -2(x-1)^2 + 8$.

A. Determine the coordinates of the vertex.

B. The graph of $y = f(x)$ is a parabola. Does the parabola open up or down?

C. Determine the y-intercept of the graph.

D. Express the function in general form.

Determining X-intercepts and Y-intercepts

To find the x-intercept(s) of the graph of f:	To find the y-intercept of the graph of f:
Solve for x: $f(x) = 0$	Determine: $y = f(0)$

E. Determine the x-intercepts of the graph of $y = f(x)$. **There are three ways to do this**!

First Way:

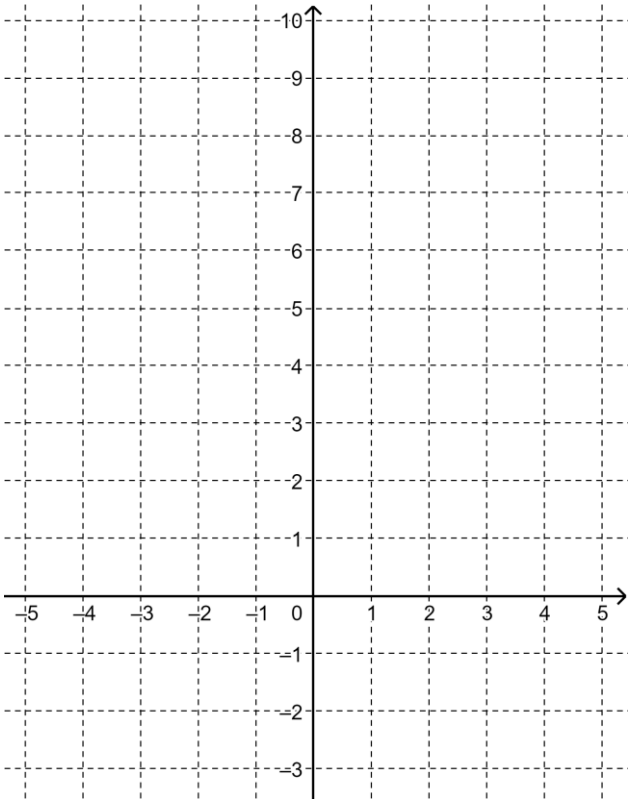
Second Way:

Third Way:

Quadratic Formula:

The solutions for $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. **You MUST know this!**

F. Sketch the graph of $y = f(x)$.



Section 3.5: Modeling with Quadratic Functions

Part 1: General Form - A Method for Finding the Vertex

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 5 Section 5.1: Quadratic Functions

Objectives:

- Be able to determine the vertex for a quadratic function presented in general form
- Be able to answer maximum / minimum value questions for a quadratic function in general form

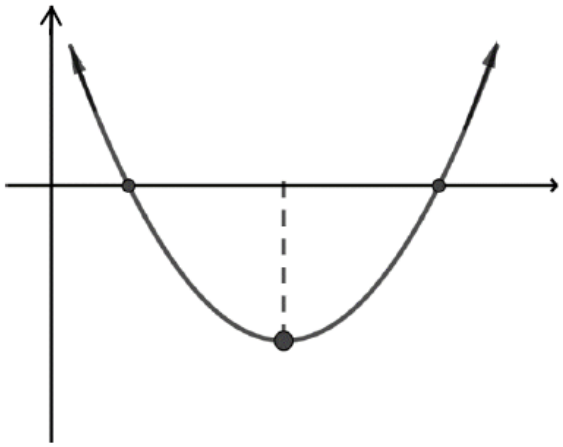
Important Vocabulary Words:

General Form, Vertex, Maximum Value, Minimum Value

Development of a Formula / Procedure to Find the Vertex

BIG IDEA: The x-coordinate of the vertex of a quadratic function occurs exactly half-way between the x-intercepts of the graph of that function.

Graph of $f(x) = ax^2 + bx + c$



Formula / Procedure to Find the Vertex of a Quadratic Function in General Form:

If $f(x) = ax^2 + bx + c$, then the vertex of the graph of $y = f(x)$ is at: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Application of the Formula / Procedure

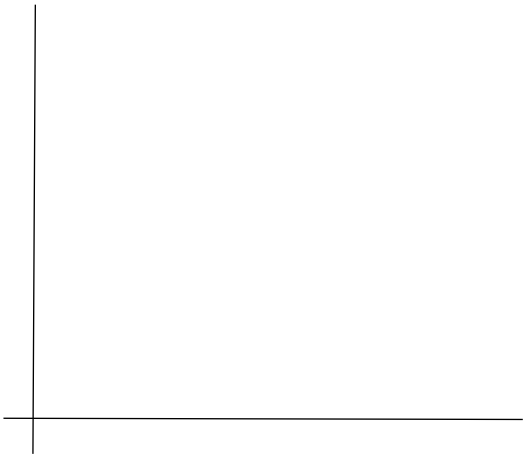
Example 1: Let $f(x) = -\frac{x^2}{3} + 2x + 7$.

A. What is the vertex of the graph of f?

B. Does f have a maximum or minimum value?
State the maximum or minimum value of f and the value of x at which it occurs.

You Try! Example 2: A volleyball flies into the air after a player successfully reacts to a strong serve. The height above the ground, in feet, of the volleyball t seconds after the player received it is given by $h(t) = -16t^2 + 48t + 1$. What is the maximum height attained by the ball, and when did it occur?

A. Draw a rough sketch of the graph by plotting and labeling the y-intercept and using what you know about the direction the graph opens. Label the axes with words.



B. Determine the maximum height of the ball and when it occurred.

Part 2: Quadratic Modeling

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 5 Section 5.1: Quadratic Functions

Objectives:

- Be able to create quadratic function models to represent applied situations
- Be able to use the formula / procedure from Part 1 to answer maximum / minimum value questions
- Be able to determine an output value corresponding to a given input value for a quadratic model
- Be able to use the quadratic formula to determine input values corresponding to a given output value for a quadratic model
- Be able to determine whether a modeling question is asking about the vertex, input value(s) or an output value for a quadratic model

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Optimization Word Problems – Strategy Recommendations

1 First read through the entire problem to get an "overview" perspective.

2. Find the question or task sentence and use it to:
   - Determine what is to be maximized or minimized. That is what you need to create a function / model for if one is not already provided.
   - Determine the input variable for the output quantity that is to be maximized or minimized.

3. Strategies to help you create a function:
   - Draw and label a picture;
   - Calculate output values for some simple numerical input values in order to help you discover and understand mathematical relationships;
   - Write down mathematical relationships in words BEFORE trying to express them algebraically;
   - Test your function / model with simple cases to see if it makes sense.

4. Draw a rough sketch of the graph of your function / model.

5. Use $x = -b/2a$ to determine the input-coordinate of the vertex of the graph.

6. Use $y = f(-b/2a)$ to determine the output-coordinate of the vertex of the graph.

Business Terms

Price: What **consumers** pay for a product.

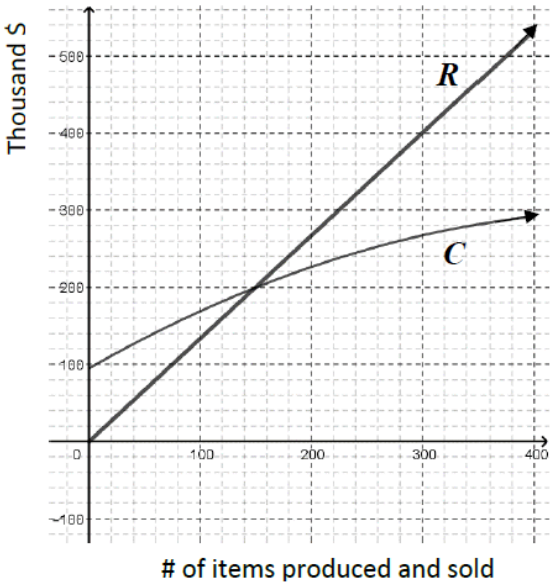
Cost: Business expenses to make and sell a product.

Revenue: Money coming in
Revenue = Price x Quantity Sold

Profit: Amount kept after subtracting business expenses
Profit = Revenue – Cost

Understanding Business Terms & Their Relationships

The graphs of monthly revenue, $R(x)$, and cost, $C(x)$, in thousands of dollars from producing and selling *x* items are shown below.



A. What production levels result in the company making a profit?

B. How many items must the company produce and sell to break even?

C. What production levels result in the company taking a monthly loss?

Examples

Example 1: During the summer months, Bozeman Bike & Ski found that they could rent an average of 8 bicycles per day at a rate of $30 per day. But, if they decreased their price to $25, they would rent 18 bicycles each day. Furthermore, BB&S noticed that there is a linear relationship between the price they charge to rent a bicycle and the number of bicycles they could expect to rent out each day. **BB&S wants to know what to charge for a bicycle rental in order to have the largest daily revenue.**

A. Determine an algebraic expression for Bozeman Bike & Ski's daily revenue from bicycle rentals when they charge x dollars per day to rent a bicycle.

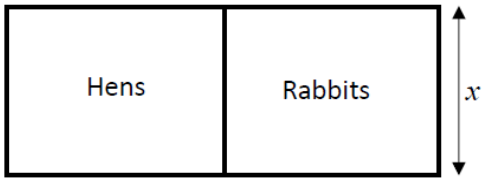
$R(x) =$_____

From Part A: $R(x) =$ _____ where $x =$ _____

B. Determine Bozeman Bike & Ski's largest possible daily revenue from bicycle rentals and the amount they should charge in order to obtain the maximum revenue. Round your answers to the nearest cent.

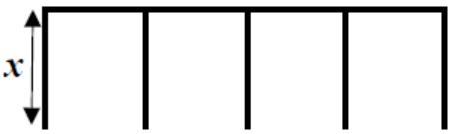
C. Bozeman Bike and Ski estimates that its average daily cost to provide bike rentals is $200. To show appreciation to the local community, a couple of days a year they offer bike rentals to people who live in Bozeman at a very low rate. What is the lowest price that BB&S can charge and still break even for the day, i.e. have enough revenue to meet their expenses? Round your answer to the nearest cent.

Example 2: Betty has 72 linear feet of chicken-wire fencing and wants to use it to make a portable rectangular frame with a divider down the middle (as shown in the diagram) in order to provide separate outdoor exercise spaces for her laying hens and pet rabbits. What dimensions should Betty use to make the rectangular frame if she wants to give her animals as much space to roam as possible?



You Try! Example 3: A landscaping business wants to build a series of four open-front bins for storing various types of mulch as shown in the diagram below. They have enough materials to build 130 linear feet of walls.

A. Determine a function for the total area of the set of bins in terms of their depth, x.



From Part A: $A(x) =$ _____ where $x =$ _____

B. What is the largest total area that can be enclosed in the bin set and what overall dimensions should be used to build the bin set in order to achieve it?

C. If the landscaping business wants each bin to enclose 150 ft^2, what dimensions should be used to build the set of bins?

Section 3.6: X-intercepts of a Quadratic Function

Part 1: Factored Form of a Quadratic Function

Reference Materials:

See reference material provided in D2L.

Objectives:

- Know connections between the factored form of a quadratic function and its graph
- Be able to sketch the graph of a quadratic function that is presented in factored form
- Be able to determine the factored form of a quadratic function based on the x-intercepts of its graph and another point on its graph

Important Vocabulary Words:

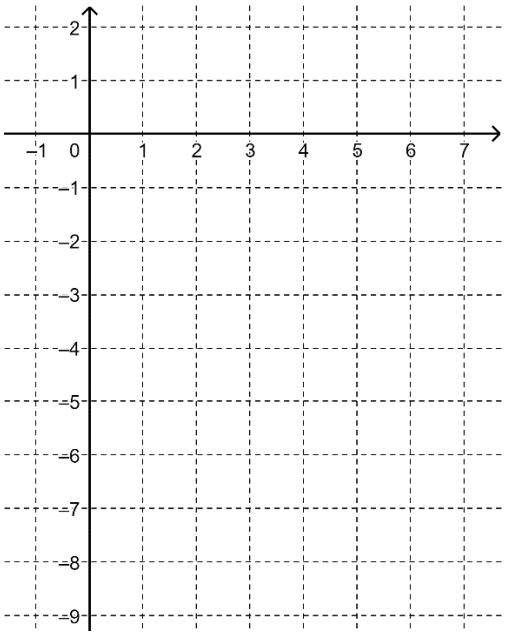
Factored Form, X-intercepts

Factored Form of a Quadratic Function:

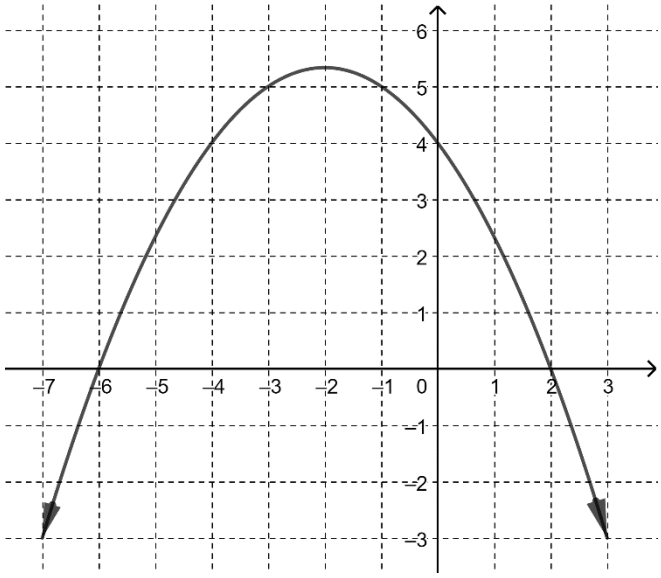
$f(x) = a(x-m)(x-n)$; where a, m, n are real numbers. $a > 0$: opens up $a < 0$: opens down

x-intercepts: $(m,0), (n,0)$; x-coordinate of vertex $= \dfrac{m+n}{2}$; y-coordinate of vertex $= f\left(\dfrac{m+n}{2}\right)$

Example 1: Determine the x-intercepts and the vertex of graph of $f(x) = 2(x-1)(x-5)$ and use them to sketch the graph.



Example 2: Determine the algebraic expression in factored form of the quadratic function whose graph is shown.



You Try! Example 3: The graph of a quadratic function f has x-intercepts at $(-9,0)$ and $(5,0)$ and passes through the point $(1,8)$.

A. Determine an algebraic expression for $f(x)$.

$f(x) =$ _____

B. Determine the vertex of the graph of $y = f(x)$.

Vertex: _____

REVIEW - You Try! Example 4: Let $g(x) = -x^2 + 4x - 7$.

A. Find the x- and y-intercepts for the graph of $y = g(x)$.
Write your answers using *coordinate pair notation*.

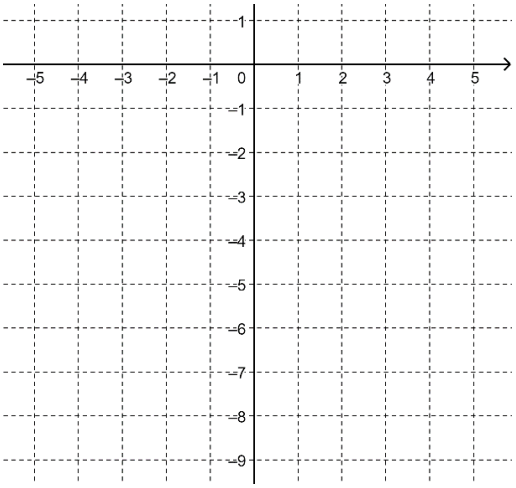
x-intercept: _____

y-intercept: _____

B. Determine the vertex for the graph of $y = g(x)$.

Vertex: _____

C. Sketch the graph of $y = g(x)$.



Part 2: Using the Quadratic Formula to Factor

Reference Materials:

See reference material provided in D2L.

Objectives:

- Be able to convert a quadratic function from general form to factored form; especially by using the quadratic formula as "bridge" in the process

Important Vocabulary Words:

Factored Form, Quadratic Formula

Example 1: Find the x-intercepts for the graph of $h(x) = 2x^2 - 18x + 36$. Write your answers using coordinate pair notation.

Example 2: Use the quadratic formula to find the x-intercepts for $f(x) = 3x^2 - 144x - 5184$. Then rewrite $f(x)$ in factored form: $f(x) = a(x-m)(x-n)$.

Factored form: $f(x) =$ _____

You Try! Example 3: Use the quadratic formula to rewrite $g(x) = 5x^2 - 37x - 24$ in factored form: $f(x) = a(x-m)(x-n)$.

Factored form: $g(x) =$ _____

Part 3: The Discriminant

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 9 Section 9.6: Graph Quadratic Functions Using Properties

Objectives:

- Be able to use the discriminant to determine the number of real solutions to a quadratic equation and hence use it to determine the number of x-intercepts for the graph of a quadratic function

Important Vocabulary Words:

Discriminant

The Discriminant:

The **Discriminant** of the quadratic equation $ax^2+bx+c=0$ is $\boldsymbol{D=b^2-4ac}$;
i.e. the part under the square root in the Quadratic Formula.

- If $D<0$ (negative), then the equation has no real solutions.
- If $D=0$, then the equation has exactly one real solution.
- If $D>0$ (positive), then the equation has two real solutions.

Example Graphs:

Example 1: Use the discriminant to determine the **number of** x-intercepts for the graphs of the following quadratic functions:

A. $f(x)=x^2+2x+5$ B. $g(x)=4x^2-4x+1$ C. $h(x)=2x^2-8x-3$

Section 3.7: Polynomial Functions

Reference Materials:

Khan Academy Resources. All Khan Academy content is available for free at www.khanacademy.org .

- YouTube Video: Polynomials intro
- YouTube Video: Zeros of Polynomials: plotting zeros
- YouTube Video: Zeros of Polynomials (with factoring): common factor
- Exposition / Lesson: Positive & negative intervals of polynomials
- YouTube Video: Positive and negative intervals of polynomials

Objectives:

- Know what polynomial functions are, polynomial vocabulary terms and what the graphs of polynomial functions look like
- Be able to determine the zeros / x-intercepts of a simple polynomial function by using factoring and the Zero Product Property
- Be able to determine intervals where a polynomial function is positive and negative and make graphical connections

Important Vocabulary Words:

Polynomial, Degree, Leading Term, Leading Coefficient, Zeros, X-intercepts, Zero Product Property

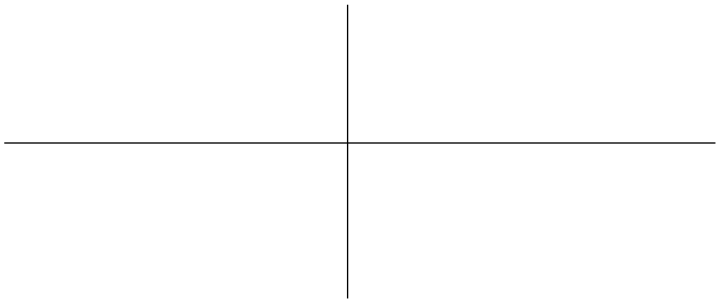
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Definition of a Polynomial Function:

A **polynomial function** is a function of the form: $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$;
where $a_0, a_1, \ldots, a_n$ are real numbers and *n* is a positive whole number.
n (the largest exponent) is called the **degree** of the polynomial.
$a_n x^n$ is called the **leading term** of the polynomial.
a_n is called the **leading coefficient** of the polynomial.

Examples: **Non-Examples**

Graphs of polynomials look like smooth curves with no jumps or breaks.



Zeros and X-intercepts

The **zeros** of a polynomial $p(x)$ are the solutions to $p(x) = 0$

If q is a zero of $p(x)$, then $p(q) = 0$ and $(q, 0)$ is an x-intercept for the graph of $y = p(x)$.

Fact: A polynomial of degree *n* can have up to *n* zeros / x-intercepts.

Finding Zeros / X-intercepts of a Polynomial of Degree > 3

Solve $p(x) = 0$ by factoring $p(x)$ and using the Zero Product Property.

Zero Product Property

$a \cdot b = 0$ if and only if $a = 0$ or $b = 0$

Example 1: Given polynomial: $p(x) = -2x^3 + 16x^2 - 30x$

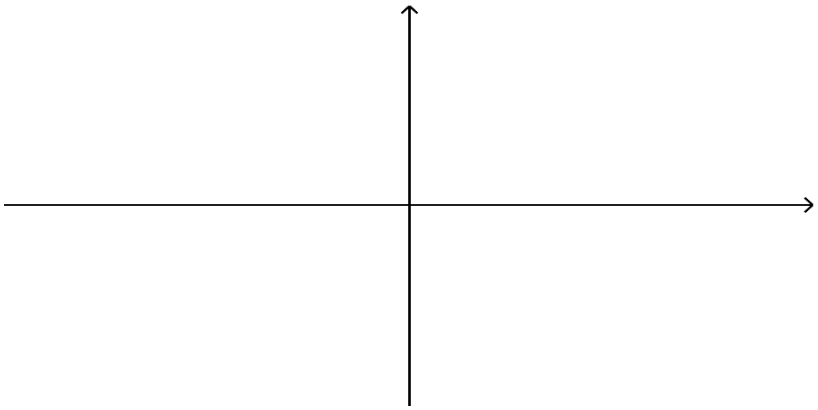
A. Determine the zeros of $p(x)$ and the x-intercepts of the graph of $y = p(x)$. Use coordinate pair notation to write the x-intercepts.

A function $f(x)$ is **positive over an interval** if $f(x) > 0$ for all values of x in the interval.

A function $f(x)$ is **negative over an interval** if $f(x) < 0$ for all values of x in the interval.

B. Determine the intervals over which $p(x)$ is positive and negative.

C. Draw a rough sketch of the graph of $y = p(x)$.

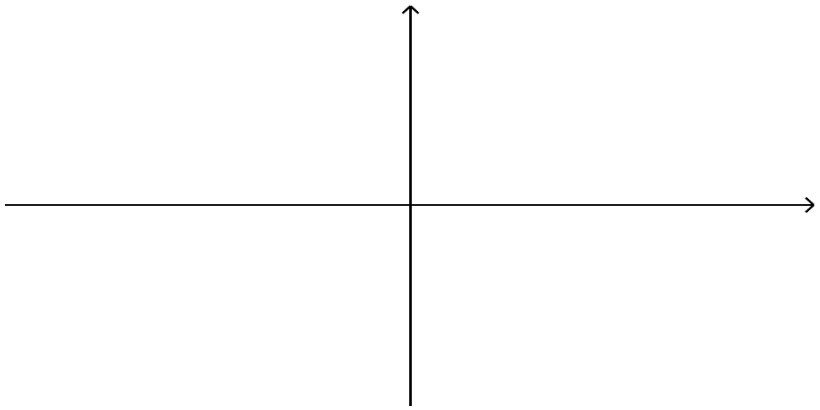


You Try! Example 2: Given polynomial: $q(x) = 3x^3 - 75x^2$

A. Determine the x-intercepts of the graph of $y = q(x)$. Write your answers using coordinate pair notation.

B. Determine the intervals over which $q(x)$ is positive and negative.

C. Draw a rough sketch of the graph of $y = q(x)$.



You Try! Example 3: Find the zeros of $r(x) = -7x^5 + 21x^4 + 7x^3$.

Chapter 4

Rational Functions, Combining Functions & Function Notation - Beyond the Basics

Sections 4.1 – 4.6

Section 4.1: Rational Expressions

Part 1: Introduction to Rational Expressions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 7 Section 7.1: Multiply and Divide Rational Expressions

Objectives:

- Know what is meant by a rational expression
- Be able to state the restrictions of a rational expression
- Be able to simplify rational expressions

Important Vocabulary Words:

Rational Expression, Simplify

Definition of a Rational Expression:

A rational expression is an expression of the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. To avoid $q(x) = 0$, the value of x may need to be restricted.

Example 1: What are restrictions on x for $\frac{3x+7}{x^2-16}$?

Like fractions, rational expressions can be **simplified**.

Example 2: A. Simplify: $\frac{20}{15}$ B. Simplify: $\frac{x^2-5x}{x^2-3x-10}$ Restrictions: _____

Part 2: Multiply & Divide Rational Expressions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 7 Section 7.1: Multiply and Divide Rational Expressions

Objectives:

- Be able to multiply and divide rational expressions and write the answer in simplest form

---

Because they are essentially algebraic fractions, multiplication and division of rational expressions mirror multiplication and division of fractions.

Example 1: A. Multiply and simplify: $\frac{6}{5}\cdot\frac{7}{9}$ B. Multiply and simplify: $\frac{4x}{x+3}\cdot\frac{x^2-9}{2x^2-10x}$

Example 2: A. Divide and simplify: $\frac{10}{3}\div\frac{20}{21}$ B. Divide and simplify: $\frac{x-2}{x^2+6x+5}\div\frac{2x-x^2}{x+5}$

You Try! Example 3: Divide and simplify: $\dfrac{x^2}{x^2-x-6} \div \dfrac{x^2+5x}{3-x}$

Part 3: Add and Subtract Rational Expressions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 7 Section 7.2: Add and Subtract Rational Expressions

Objectives:

- Be able to add and subtract rational expressions and write the answer in simplest form

---

Addition and subtraction of rational expressions mirror addition and subtraction of fractions ↔ need to utilize a common denominator!

Example 1: Perform the operations and simplify your answers.

A. $\dfrac{2}{3}+\dfrac{4}{5}$

B. $\dfrac{x}{x+3}+\dfrac{2}{x}$

Example 2: Perform the operations and simplify your answers.

A. $\dfrac{7}{15}-\dfrac{1}{6}$

B. $\dfrac{5}{x^2-2x}-\dfrac{3}{x^2-4}$

You Try! Example 3: Perform the operations and simplify your answers.

A. $5-\frac{x}{x+4}$ HINT: $5=\frac{5}{1}$ B. $\frac{x}{x-5}+\frac{1}{x^2-2x-15}$

Section 4.2: Rational Equations

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 7 Section 7.4: Solve Rational Equations

Objectives:

- Be able to solve equations and formulas that involve rational expressions

Important Vocabulary Words:

Clear / eliminate the fractions

Eliminate the Fractions Strategy:

Multiply both sides of the equation by the LCM of the denominators. Then undo the division on both sides of the equation by using the LCM to cancel out the denominators.

Example 1: Let $f(x)=\frac{1-x}{5x+2}$.

Solve for x: $f(x)=\frac{4}{3}$

Are there any restrictions on x to consider?

You Try! Example 2: Let $g(x)=\frac{6}{(x+7)^2}$.

Solve for x: $g(x)=2$

Are there any restrictions on x to consider?

Example 3: Solve for x: $\frac{3}{x+2}+\frac{1}{x-2}=1$. Are there any restrictions on x to consider?

You Try! Example 4: Solve for x: $\frac{5}{x-3}-\frac{1}{x}=0$. Are there any restrictions on x to consider?

You Try! Example 5: Two resistors with resistances A and B are connected in parallel leading to a combined resistance C. The relationship between A, B and C is given by $C = \frac{AB}{A+B}$.

Solve the formula for B.

Section 4.3: Rational Functions

Part 1: Introduction to Rational Functions

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website
College Algebra, An Investigation of Functions by David Lippman, Melonie Rasmussen & Jay Abramson
Chapter 3 Section 3.4: Rational Functions pages 201 – 208 of the pdf-file (printed pages 193 – 200)
NOTE: We are not using rational functions content on pages beyond those stated above.

Objectives:

- Know what rational functions, vertical asymptotes and horizontal asymptotes are
- Be able to identify the domain, vertical asymptote(s) and horizontal asymptote from the graph of a rational function
- Be able to determine the domain, vertical asymptote(s) and x- and y-intercepts of a rational function when given its algebraic expression

Important Vocabulary Words:

Rational Function, Vertical Asymptote, Horizontal Asymptote, X-intercept, Y-intercept

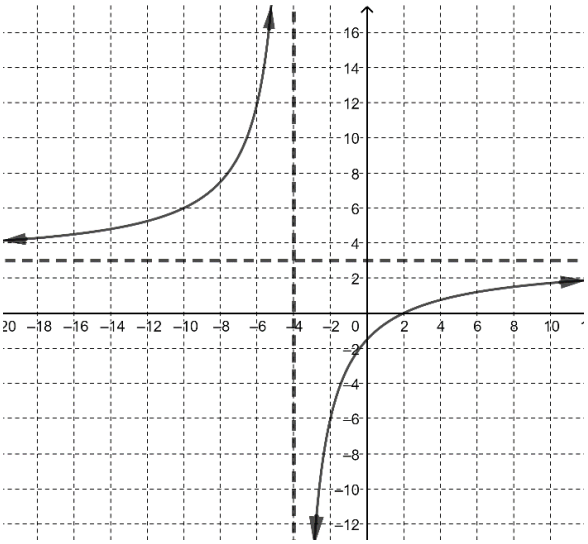
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Features of Rational Functions

Rational Function:

A function of the form $f(x)=\dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial functions.

Example 1: The graph of $f(x)=\dfrac{3x-6}{x+4}$ is shown below. What do you notice?



Domain: _____

x	$y=f(x)$
–4.1	183
–4.01	1803
–4.001	18003

x	$y=f(x)$
–3.9	–177
–3.99	–1797
–3.999	–17997

x	$y=f(x)$
–100	3.1875
–1,000	3.0181
–10,000	3.0018

x	$y=f(x)$
100	2.8269
1,000	2.9821
10,000	2.9982

Vertical Asymptote:

A vertical line $x = a$ where the output values tend towards positive or negative infinity as the input values get closer and closer to a. (Function graphs **never cross** a vertical asymptote.)

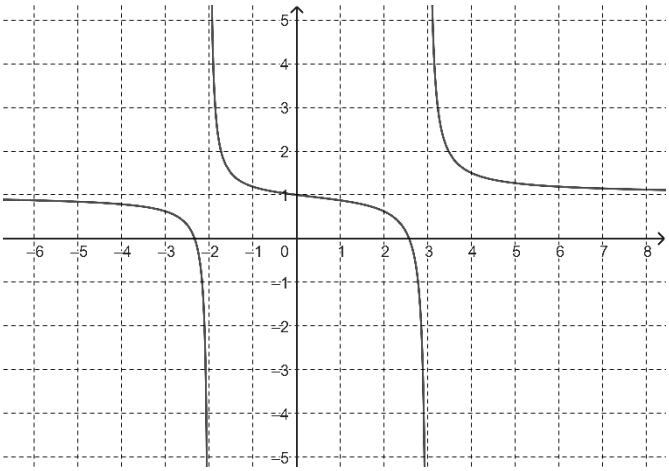
A function $f(x)$ has a vertical asymptote at $x = a$ if _____ as _____ .

Horizontal Asymptote:

A horizontal line $y = b$ where the graph gets closer and closer to the line as the input values get larger and larger in either the positive or negative direction; i.e. on the far right or far left sides of the graph. (Function graphs **can cross** a horizontal asymptote.)

A function $f(x)$ has a horizontal asymptote at $y = b$ if _____ as _____ .

You Try! Example 2: Determine the domain, vertical asymptote(s) and horizontal asymptote, y-intercept and x-intercept(s) for the rational function whose graph is shown below.



Domain: Vertical Asymptote(s):

Horizontal Asymptote: y-intercept:

Approximate the x-intercept(s):

Example 3: Find the following for $f(x) = \dfrac{x^2 - 4x - 5}{x - 2}$.

A. Domain

B. Vertical Asymptote(s)

C. Y-intercept

D. X-intercept(s)

To find the vertical asymptote(s) of a rational function:

To find the y-intercept of a rational function:

To find the x-intercept(s) of a rational function:

Unusual Cases

Example: Find the vertical asymptote(s) for $f(x)=\dfrac{x+3}{x^2+2x-3}$.

Work: Look for when denominator = 0 $\leftrightarrow$ Solve for *x*: $x^2+2x-3=0$

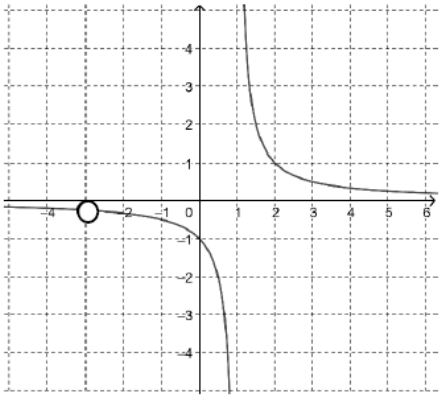
$$\rightarrow (x+3)(x-1)=0$$

$$\rightarrow x+3=0 \text{ or } x-1=0$$

$$\rightarrow x=-3 \text{ or } x=1$$

So, one expects the graph of $y=f(x)$ to have two vertical asymptotes.

BUT, the graph of $y=f(x)$ is:



What is happening?!

Simplify: $f(x)=\dfrac{x+3}{x^2+2x-3}=\dfrac{\cancel{x+3}}{\cancel{(x+3)}(x-1)}=\dfrac{1}{x-1}$ for $x\neq -3$.

So, the graph of $f(x)=\dfrac{x+3}{x^2+2x-3}$ is the same as the graph of $g(x)=\dfrac{1}{x-1}$ except that the graph of $y=f(x)$ has a hole at $(-3,-1/4)$.

** It is necessary to simplify rational functions BEFORE finding vertical asymptotes.

GOOD NEWS: In this course we will only ask you to work with rational functions that are already simplified. We will not expect you to recognize when graphs have holes.

Practice

You Try! Example 4 A & B: Find the domain, vertical asymptote(s), y-intercept and x-intercept(s) for each of the following rational functions. [You will find it easier if you first factor whenever possible.]

A. $f(x)=\dfrac{2x+3}{4x+1}$

B. $g(x)=\dfrac{-x+1}{x^2-5x+6}$

Domain: Vertical Asymptote(s):	Domain: Vertical Asymptote(s):
Y-intercept:	Y-intercept:
X-intercept(s):	X-intercept(s):

Part 2: Horizontal Asymptotes

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website
College Algebra: An Investigation of Functions by David Lippman, Melonie Rasmussen & Jay Abramson
Chapter 3 Section 3.4: Rational Functions pages 201 – 208 of the pdf-file (printed pages 193 – 200)
NOTE: We are not using rational functions content on pages beyond those stated above.

Objectives:

- Be able to determine the horizontal asymptote of the graph of a rational function when it exists based on the algebraic structure of the function
- Be able to determine when the graph of a rational function does not have a horizontal asymptote based on the algebraic structure of the function

Important Vocabulary Words:

Horizontal Asymptote, Degree

Example 1:

- Examine the graphs of the functions in *Desmos* in order to determine the horizontal asymptotes.
- Look at the algebraic structures of the functions to see why those horizontal asymptotes are occurring.

A. $f(x) = \dfrac{2x}{x-1}$

B. $g(x) = \dfrac{3x}{x^2-1}$

C. $h(x) = \dfrac{4x^2}{x-3}$

Summary: How To Determine a Horizontal Asymptote

Pattern based on the degrees of the numerator and the denominator:

Case 1:

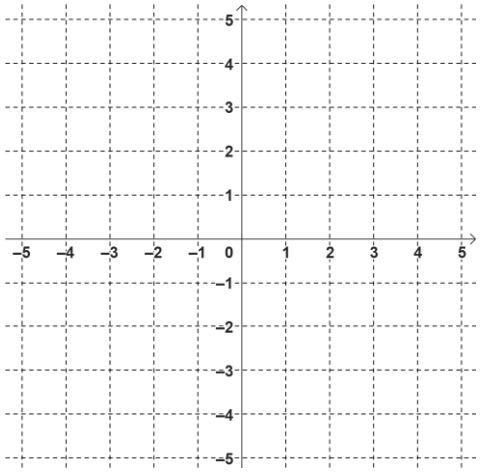
Case 2:

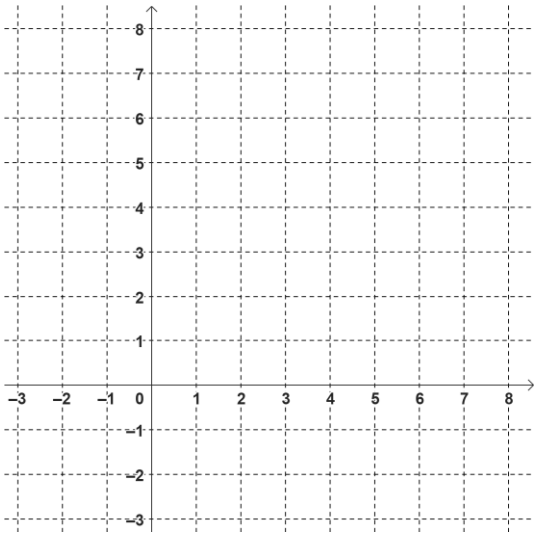
Case 3:

Example 2 (Continuation of Example 4 A & B from Part 1):
Find the horizontal asymptotes for the following functions and then sketch their graphs.

A. $f(x)=\dfrac{2x+3}{4x+1}$

B. $g(x)=\dfrac{-x+1}{x^2-5x+6}$





Part 3: Putting Everything Together

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website
College Algebra: An Investigation of Functions by David Lippman, Melonie Rasmussen & Jay Abramson
Chapter 3 Section 3.4: Rational Functions pages 201 – 208 of the pdf-file (printed pages 193 – 200)
NOTE: We are not using rational functions content on pages beyond those stated above.

Objectives:

- When they exist, be able to determine the domain, vertical asymptote(s), y-intercept, x-intercept(s) and horizontal asymptote of a rational function based on its algebraic expression

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Example 1: Let $h(x)=\frac{2x^3-18x}{x^2+5}=\frac{2x(x^2-9)}{x^2+5}=\frac{2x(x+3)(x-3)}{x^2+5}$. When they exist, determine the x-intercept(s), y-intercept, vertical asymptote(s) and horizontal asymptote for the graph of $y=h(x)$. Write intercepts using coordinate pair notations and asymptotes as equations of lines.

X-intercepts:	Y-intercepts:
Vertical Asymptote(s):	Horizontal Asymptote:

You Try! Example 2: Let $h(x) = \dfrac{5x^2 + 80}{x^2 + 10x - 200}$. When they exist, determine the x-intercept(s), y-intercept, vertical asymptote(s) and horizontal asymptote for the graph of $y = h(x)$. Write intercepts using coordinate pair notations and asymptotes as equations of lines. Show all work.

Factorization of $h(x) =$

X-intercepts:	Y-intercepts:
Vertical Asymptote(s):	Horizontal Asymptote:

Section 4.4: Arithmetic of Functions

Part 1: Function Arithmetic Notation

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.4: Composition of Functions

Khan Academy Resources. All Khan Academy content is available for free at www.khanacademy.org .

- Exposition / Lesson: Intro to combining functions
- Exposition / Lesson: Adding and subtracting functions
- Exposition / Lesson: Multiplying and dividing functions

Objectives:

- Know and be able to use notation for the arithmetic of functions when functions are presented algebraically and graphically

---

Introductory Example

Bozeman Bike & Ski (BB&S) found that it could rent an average of 8 bicycles per day at a rate of $30 per day. But, for each $1 decrease in the rental price, they expect to rent 2 more bicycles each day.

Let x = price decrease in dollars.

Then, the *price* of renting a bicycle for a day is given by: $p(x) = 30 - x$.
And, the quantity of bicycles that BB&S expects to rent per day is given by: $q(x) = 8 + 2x$.

Let $R(x) =$ BB&S's daily revenue from bicycle rentals when the price is decreased by x dollars.
Since $revenue = price \times quantity$, $R(x) = p(x) \cdot q(x)$. Another way to write this is $R = p \cdot q$ or pq where $R(x) = (pq)(x) = p(x)q(x) = (30 - x)(8 + 2x)$.

Notation System

Function Arithmetic Notation

$$(f+g)(x) = f(x) + g(x) \qquad (f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example 1: Let $f(x) = x - 3$, $g(x) = \sqrt{x+1}$, $h(x) = \sqrt{4-x}$ and $k(x) = x^2$.

Find each of the following functions.

$(f+k)(x) =$ $\qquad$ $(g-h)(x) =$

$(fg)(x) =$ $\qquad$ $\left(\frac{g}{f}\right)(x) =$

Example 2: Let $f(x)=\frac{x+2}{6}$ and $g(x)=3\sqrt{x}$. Determine each of the following.

A. $(f+g)(1)$

B. $\left(\frac{f}{g}\right)(4)$

You Try! Example 3: Let $h(x)=2x+3$ and $k(x)=\frac{x+1}{x}$.

A. Find and simplify $\left(\frac{h}{k}\right)(x)$.

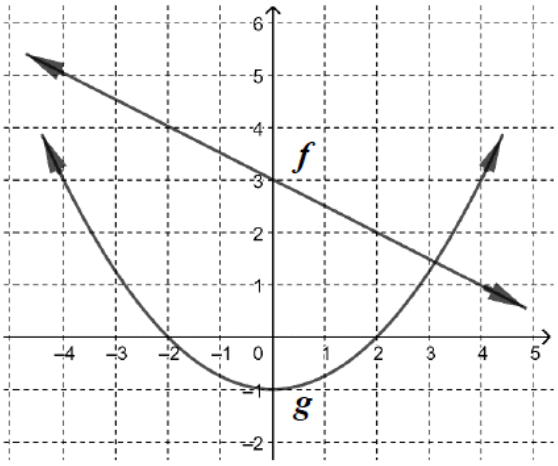
B. Determine $(hk)(2)$.

Example 4: The graphs of f (line) and g (parabola) are shown below. Determine each of the following.

A. $(f-g)(0)=$

B. $(fg)(-4)=$

C. **You Try!** $(f+g)(4)=$



D. **You Try!** $\left(\frac{f}{g}\right)(-2)=$

Part 2: Applications

Reference Materials:

Resources available on the Khan Academy website:

- Exposition / Lesson: Adding and subtracting functions
- Exposition / Lesson: Multiplying and dividing functions

Objectives:

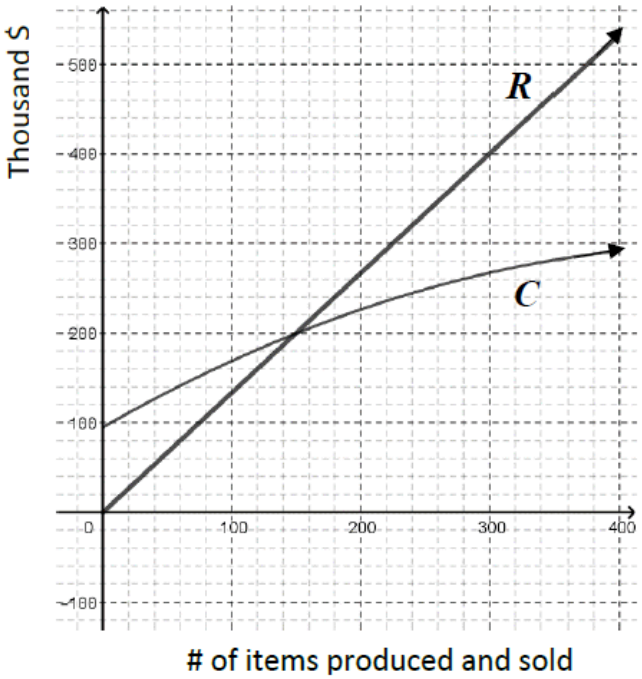
- Be able to use arithmetic of functions notation as a tool when modeling applied situations

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Example 1: Let $W(t)$ be the annual number of deer in Wisconsin that died due to wasting disease t years after 2015 and $P(t)$ be the total deer population in Wisconsin t years after 2015. What does $\left(\frac{W}{P}\right)(4)$ represent?

You Try! Example 2: Let $K(m)$ be the number of kilowatt-hours of electricity that a household used during month m last year and $p(m)$ be the price per kilowatt-hour during month m last year. What does $(Kp)(2)$ represent?

Example 3: The graphs of monthly revenue, $R(x)$, and cost, $C(x)$, in thousands of dollars from producing and selling x items are shown below.



Let $P(x) = (R - C)(x) = R(x) - C(x)$ be the profit from producing and selling x items in a month.

A. Complete the table and use it to sketch in the graph of $y = P(x)$.

x	0	100	200	300	400
$R(x)$	0	135	265	400	540
$C(x)$	100	170	225	265	295
$P(x)$					

B. What relationships do you notice between the graphs of $R(x)$, $C(x)$ and $P(x)$?

Section 4.5: Function Composition

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.4: Composition of Functions

Objectives:

- Know and be able to use notation for function composition when functions are presented in table, algebraic and graphic forms
- Be able to decompose a function into component functions

Important Vocabulary Words:

Function Composition

Example 1: For windspeeds between 4 m/s and 12 m/s, the power P, in kilowatts (kW), generated by a particular type of wind turbine can be approximated by $P(w) = 0.955w^3$, where w is the windspeed in meters per second (m/s). Suppose the function $W(t)$, shown in the table below, gives windspeeds in meters per second recorded at the location of such a turbine on a spring day, where t is the time of day in hours (1 pm ↔ hour 13).

Informed by a Sciencing website (K. Lee, 2020) and Wind farm Bop website (F. Miceli, 2012)

t = time in hours	8	10	12	14	16	18
$W(t)$ = wind speed (m/s)	5.1	4.8	4.5	6.3	8.2	8.4

How much power was the wind turbine generating at 12 noon?

What is $P(W(18))$? What does it tell you?

Function Composition

When the output of a function g is used as the input of a function f, then the entire process is referred to as a **composite function**: $f(g(x))$.

IDEA: $x \to \boxed{g} \to g(x) \to \boxed{f} \to f(g(x))$

The domain of the composite function consists of all values x in the domain of g for which $g(x)$ is also in the domain of f.

Example 2: Let $f(x)=\frac{15x}{x+2}$, $g(x)=\sqrt{x+6}$, and $h(x)=2x-1$. Determine the following.

A. $g(h(x))=$

B. $f(g(x))=$

C. $g(h(10))=$

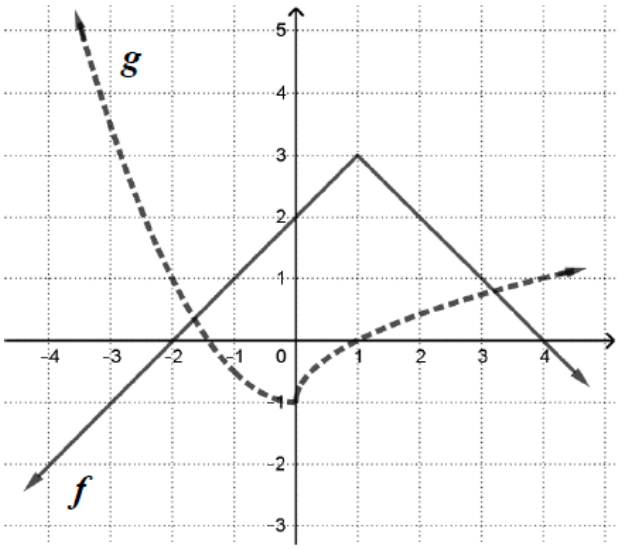
D. $h(f(1))=$

You Try! Example 3: Let $f(x)=x-2$ and $g(x)=x^2-3x$.

A. Find and simplify $g(f(x))$.

B. Determine $f(g(-2))$.

Example 4: The graphs of f (solid) and g (dashed) are shown below. Use the graphs to evaluate the following composite functions.



A. $f(g(1))$

B. $f(f(-1))$

You Try! C. $g(f(-4))$

You Try! D. $g(g(4))$

Example 5: Write $h(x)=\sqrt[3]{4x-5}$ as the composition of two functions.
Determine nontrivial functions f and g such that $h(x)=f(g(x))$. [Nontrivial: $f(x)\neq x$ and $g(x)\neq x$]

You Try! Example 6: Determine nontrivial functions f and g such that $f(g(x))=\dfrac{4}{(x+7)^2}$.

Section 4.6: Function Notation – Beyond the Basics

Part 1: Function Notation

Reference Materials:

OpenTextBookStore: *Precalculus / College Algebra, 3rd Edition* by Carl Stitz and Jeff Zeager
Chapter 1 Section 1.4: Function Notation – Example 1.4.2 on pages 66 – 67 of the pdf file (printed pages 56 – 57)

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.1 Functions and Function Notation – Example 6

Objective:

- Understand and be able to evaluate a variety of algebraic expressions that involve function notation

---

Example 1: Let $f(x)=5x+\sqrt{x}$. Evaluate the following.

A. $f(4)$

B. $f(a+2)$

C. $f(x-9)$

D. $3f(w)-1$

Example 2: Given $f(x)=x^2+4x-1$, determine the following. Do NOT simplify your answers.

A. $f(x)+h$

B. $f(x+h)$

You Try! Example 3: Let $f(x)=\dfrac{2x}{x-3}$. Evaluate each of the following.

Do NOT simplify your answers for B, C, D, E and F.

A. $f(6)=$

B. $f(b-5)=$

C. $f(x)+h=$

D. $f(8a)=$

E. $f(x+h)=$

Part 2: Function Notation in a Graph Setting and the Difference Quotient

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.1 Functions and Function Notation – Example 6

Objectives:

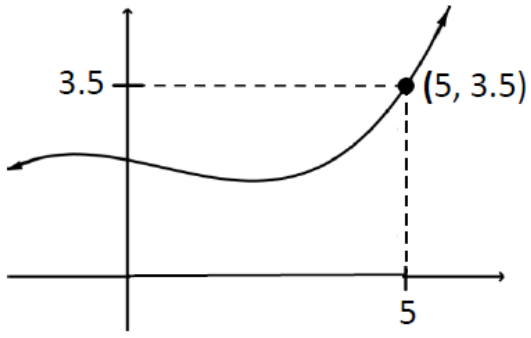
- Understand and be able to use function notation in a graph setting
- Be able to set up and simplify the difference quotient for linear and quadratic functions

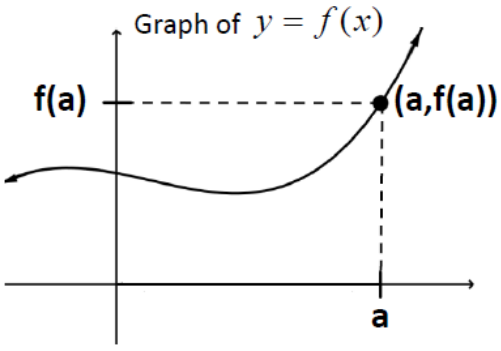
Important Vocabulary:

Difference Quotient

Function Notation in a Graph Setting

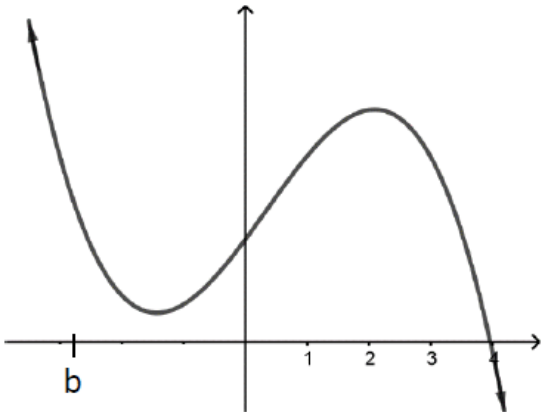
Function notation can be used to label y-coordinates of points on a graph and values on the y-axis.





Example 1: Use the graph to address tasks A - D.

Graph of $y = f(x)$



A. Plot and label the point $(2, f(2))$ on the graph.

B. Label the appropriate "tick mark" on the y-axis with $f(2)$.

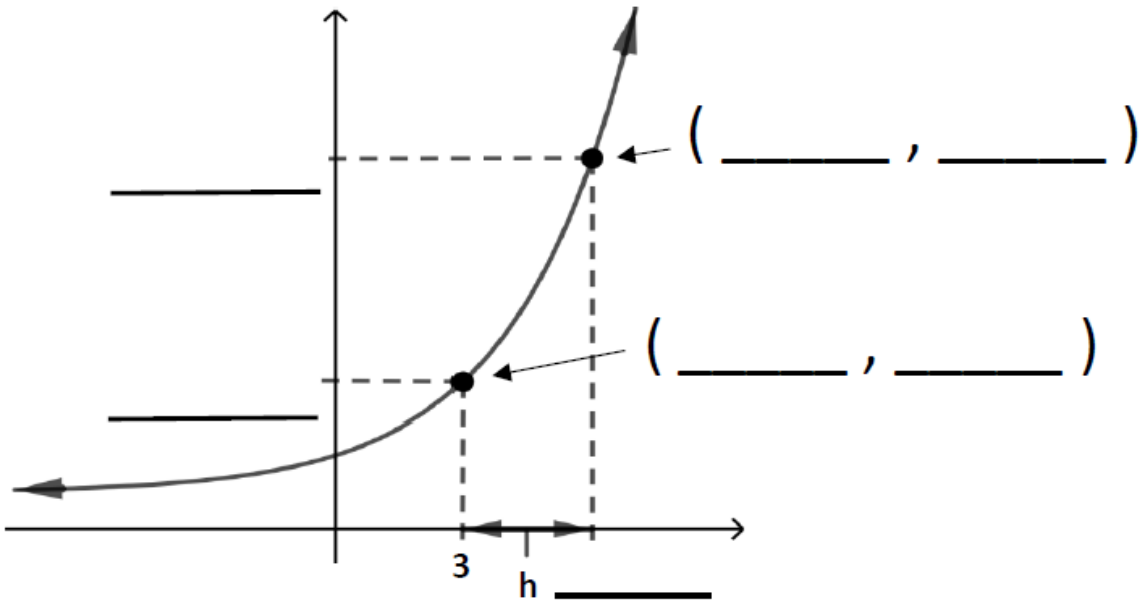
You Try!

C. Plot and label the point $(b, f(b))$ on the graph.

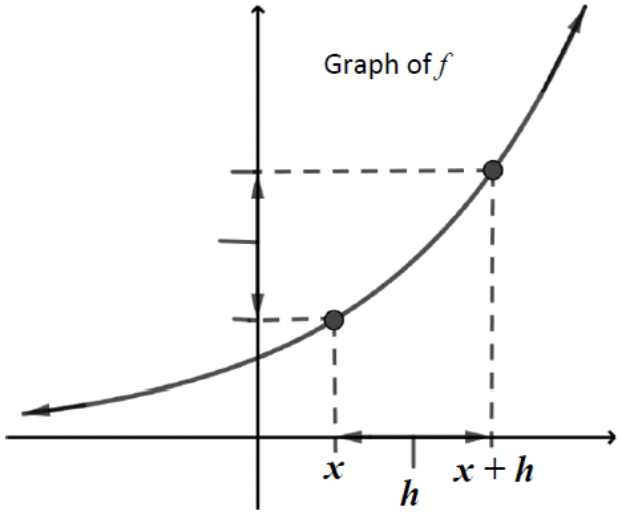
D. Label the appropriate "tick mark" on the y-axis with $f(b)$.

Example 2: Fill in the blanks with appropriate notation.

Graph of $y = f(x)$



The Difference Quotient



Difference Quotient

= Average Rate of Change in f on $[x, x+h]$

$= \dfrac{rise}{run} =$

Example 3: Given $f(x) = 4x - 3$, find AND SIMPLIFY $\dfrac{f(x+h) - f(x)}{h}$ for $h \neq 0$.

Step 1: Find and simplify $f(x+h) =$

Step 2: Find and simplify $f(x+h) - f(x) =$

Step 3: Find and simplify $\dfrac{f(x+h) - f(x)}{h} =$

Example 4: Given $f(x) = 2x^2 - x$, find AND SIMPLIFY $\frac{f(x+h) - f(x)}{h}$ for $h \neq 0$.

Step 1: Find and simplify $f(x+h) =$

Step 2: Find and simplify $f(x+h) - f(x) =$

Step 3: Find and simplify $\frac{f(x+h) - f(x)}{h} =$

You Try! Example 5: Let $f(x) = 4x^2 - 3x + 2$.

Use the three-step process to find and simplify $\frac{f(x+h) - f(x)}{h}$ for $h \neq 0$.

Label your work at each step with appropriate function notation.

Step 1:

Step 2:

Step 3:

Chapter 5
Exponential Functions

Sections 5.1 – 5.5

Section 5.1: Exponent Properties (R)

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 5 Section 5.2: Properties of Exponents and Scientific Notation

Objectives:

- Be able to simplify algebraic expressions involving integer exponents

---

Exponent Properties

Each horizontal pair of cells contains equivalent statements		Examples
$a^m a^n$		
$\frac{a^m}{a^n}$		
a^0		
a^{-n}		
$\frac{1}{a^{-n}}$		
$\left(a^m\right)^n$		
$(ab)^n$		
$\left(\frac{a}{b}\right)^n$		

Avoid common errors: $(a + b)^n$ and $(a – b)^n$

Example 1: Simplify each expression using positive exponents only.

A. $\left(3x^2x^{-3}x^0\right)^4$

B. $\left(\dfrac{5x^{-4}}{x+2}\right)^2$

C. $\dfrac{6x^8y^2}{15x^5y^{-4}}$

You Try! Example 2: Simplify each expression using positive exponents only.

A. $\dfrac{(2x^2)^3}{x^9}$

B. $8\left(\dfrac{a^{-2}}{b^4}\right)^{-3}$

C. $\dfrac{9x^{-2}y}{24x^4y^{-8}}$

Section 5.2: Radicals and Rational Exponents

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 1 Section 1.3: Radicals and Rational Exponents

Objectives:

- Be able to simplify radical expressions
- Be able to rationalize denominators involving square roots

Important Vocabulary Words

Rationalize, Conjugate

Radicals are really an alternate notation for fractional exponents: $\sqrt[n]{a} = a^{\frac{1}{n}}$. Hence, they have the same properties as exponents have.

Radical Properties

Each horizontal pair of cells contains equivalent statements		Examples
$\sqrt[n]{a}$		
$\sqrt[n]{a^m}$		
$\sqrt[n]{a^n}$		
$\sqrt[n]{ab}$		
$\sqrt[n]{\frac{a}{b}}$		
$\sqrt[n]{a+b}$		
$\sqrt[n]{a-b}$		

Example 1: Rewrite each radical expression as an expression involving a rational exponent.

A. $\sqrt[6]{x}$ B. $\sqrt[3]{x^4}$ **You Try!** C. $\sqrt[5]{x^2}$

Example 2: Rewrite $x^{7/2}$ as a radical expression.

Example 3: Use the properties of radicals to simplify each expression. Assume that all variables represent **positive** real numbers.

A. $\sqrt{56x^3}$ B. $\sqrt{45x^4}$

D. $\sqrt[3]{250x^4}$ C. $\sqrt[3]{x^3-8}$

Addition, Subtraction & Multiplication with Radicals

Use the following facts to add, subtract and multiply expressions involving radicals:

$$a\sqrt[n]{c}+b\sqrt[n]{c}=(a+b)\sqrt[n]{c}, \quad a\sqrt[n]{c}-b\sqrt[n]{c}=(a-b)\sqrt[n]{c}$$

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}, \quad a\sqrt[n]{c}\cdot b\sqrt[n]{d}=(a\cdot b)\sqrt[n]{c\cdot d}$$

Example 4: Perform the operations and simplify.

A. $\left(4+6\sqrt{2}\right)-\left(1-\sqrt{2}\right)$ B. $\left(4+6\sqrt{2}\right)\left(1-\sqrt{2}\right)$ C. $\left(5+2\sqrt{3}\right)\left(5-2\sqrt{3}\right)$

Rationalizing a Denominator That Involves a Square Root

Rationalizing is the process of rewriting a fractional expression so that there no longer is a square root in the denominator.

- If there is a single square root and no addition or subtraction in the denominator, multiply both the numerator and the denominator by the square root term and simplify.

  Example: $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$

- If the denominator involves addition or subtraction, multiply both the numerator and the denominator by the **conjugate** of the denominator ("same" thing with the opposite sign) and simplify:

  Example:

$$\frac{5}{3+\sqrt{2}} = \frac{5}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{5\left(3-\sqrt{2}\right)}{\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)} = \frac{15-5\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-\sqrt{4}} = \frac{15-5\sqrt{2}}{9-2} = \frac{15-5\sqrt{2}}{7}$$

Example 5: Simplify, if possible, and then rationalize the denominator.

A. $\sqrt{\frac{4}{5}}$

You Try! B. $\sqrt{\frac{5}{3}}$

B. $\frac{-1}{4+\sqrt{3}}$

You Try! C. $\frac{3}{2-\sqrt{5}}$

Section 5.3: Using Like Bases to Solve Exponential Equations

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 6 Section 6.6 Exponential and Logarithmic Equations through TRY IT #3

Objectives:

- Be able to use the one-to-one property of exponential functions to solve exponential equations
- Be able to use exponent properties to simplify exponential expressions, including when there is variable is in the exponent position
- Be able to convert radical notation to exponent notation

Important Vocabulary Words

One-to-One Property of Exponential Functions

Using the One-to-One Property of Exponential Functions to Solve Exponential Equations

One-to-One Property of Exponential Functions:
Let b be a positive real number. Then $b^S = b^T$ if and only if $S = T$.

Exponential Equation Solving Method:

1. Use exponent rules to rewrite and simplify both sides of an exponential equation to get it in the form $b^S = b^T$.
2. Then, by the One-to-One Property of Exponential functions, set $S = T$.
3. Solve the resulting equation for the unknown variable.

Exponent Rules:

#1. $b^n b^m = b^{n+m}$
Example: $b^2 b^5 = b^7$

#2. $\frac{b^n}{b^m} = b^{n-m}$ or $\frac{1}{b^{m-n}}$
Example: $\frac{b^2}{b^5} = b^{-3}$ or $\frac{1}{b^3}$

#3. $\left(b^n\right)^m = b^{nm}$
Example: $\left(b^5\right)^2 = b^{10}$

#4. $b^{-n} = \frac{1}{b^n}$ *and* $\frac{1}{b^{-n}} = b^n$
Examples: $b^{-4} = \frac{1}{b^4}$ and $\frac{1}{b^{-3}} = b^3$

#5. $(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Examples: $(ab)^3 = a^3 b^3$ and $\left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$

#6. $\sqrt[n]{b} = b^{1/n}$ Example: $\sqrt[3]{b} = b^{1/3}$

Example 1: Use the one-to-one property of exponential functions to solve the following equations:

A. $5^{2x+7} = 5^3$

B. $3^{x-6} = 9$

C. $2^{x+5} = \frac{1}{8^x}$

D. $\sqrt{b} = b^3 b^{5x}$

E. $\frac{e^{x^2}}{e^{15}} = \left(e^x\right)^2$

F. $\sqrt[3]{7^x} = 49^{x+1}$

You Try! Example 2: Use the one-to-one property of exponential functions to solve the following equations:

A. $e^{2x-1} = \left(e^{x+2}\right)^3$

B. $2^{2x+1}2^{x-3} = 2^{x^2}$

(More Challenging) Example 3: Use the one-to-one property of exponential functions to solve the following equations:

A. $\left(\frac{1}{5}\right)^x = 125^{x+2}$

B. $2(16^{x+1}) = \frac{1}{\sqrt{8}}$

You Try! Example 4: Use the one-to-one property of exponential functions to solve the following equation: $\frac{1}{27} = \sqrt{3^{4x+8}}$.

Section 5.4: Exponential Growth and Decay

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website - *College Algebra, An Investigation of Functions*, primary authors David Lippman, Melonie Rasmussen & Jay Abramson Chapter 4 Section 4.1: Exponential Functions pages 229 – 239 of the pdf-file (printed pages 221 – 231)

Objectives:

- Be able to tell the difference between a linear situation and an exponential situation
- Know what is meant by the growth / decay factor and the percent growth / decay rate and the relationship between the two, and be able to calculate them when exponential data is separated by one unit of time.
- Be able to create an exponential model based on initial value and growth / decay factor or percent growth / decay rate information and use the model answer questions.
- Know what the graphs of exponential functions look like in growth and decay situations

Important Vocabulary Words

Initial Value, Exponential Function, Growth / Decay Factor, Percent Growth / Decay Rate

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Example 1: Jordan and Casey both received $10,000 when they graduated from college; with instructions to invest the money in order to save for a down payment on a future home.

Jordan invested in a start-up business that promised to pay $550 each year that Jordan lets them use the $10,000. NOTE: $550 = 5.5% of $10,000.

Casey invested the money in a money market account that earns 5% *compounded annually* – meaning that at the end of each year the interest that Casey earned that year becomes part of the base amount (principle) that earns interest the next year.

A. Complete the given tables for Jordan and Casey and use them to determine functions $J(x)$ and $C(x)$ for the amount each person's investment is worth after x years.

Jordan's Investment

Year	Work Space	Total Amount
0		
1		
2		
3		
15		

$J(x) =$ _____

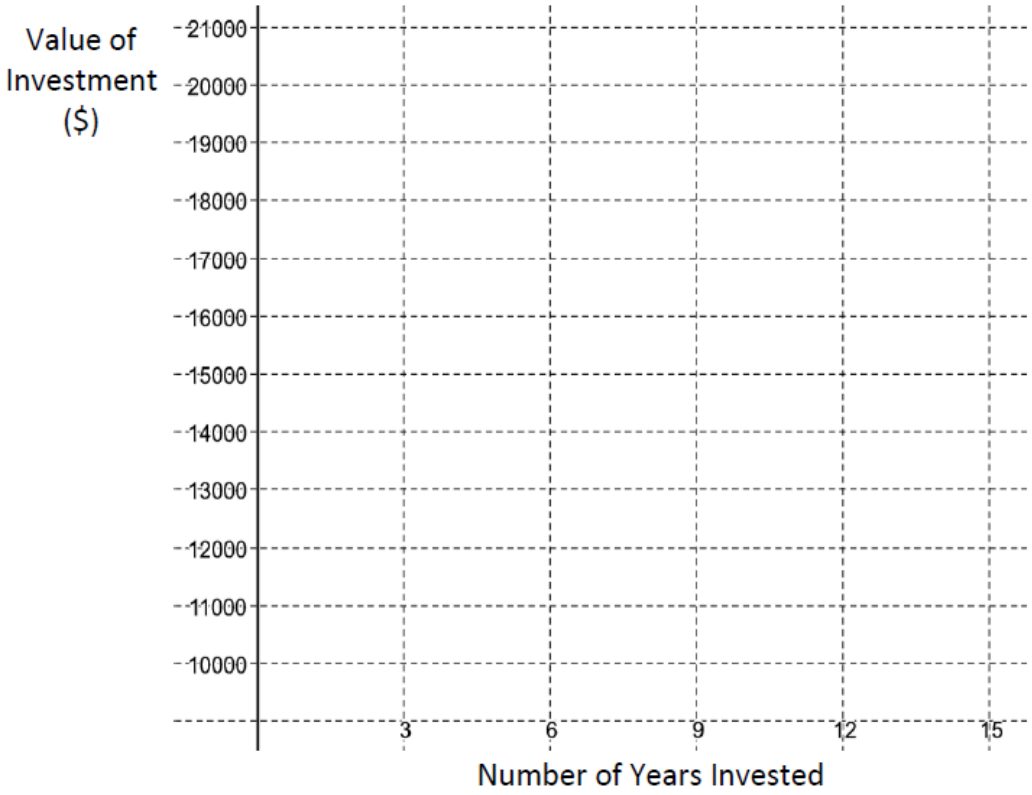
Work Space:

Casey's Investment

Year	Work Space	Total Amount
0		
1		
2		
3		
15		

$C(x) =$ _____

B. Draw the graphs of $y = J(x)$ and $y = C(x)$ below.



Percent Increase / Decrease

$$\frac{\text{End Value} - \text{Start Value}}{\text{Start Value}} \times 100$$

C. By what percent did Jordan's \$10,000 increase over 15 years?
Is your answer equivalent to $15 \times 5.5\%$?

D. By what percent did Casey's \$10,000 increase over 15 years?
Is your answer equivalent to $15 \times 5\%$?

E. **Important!** Compare your answers to C and D. What can you do with percent growth in a linear situation that you can**not** do in an exponential situation?

Example 2: The radioactive isotope Iodine-123 decays quickly and is used in medical imaging. The table below illustrates how quickly a 20 g sample of I-123 would decay over a few hours.

Time (hours)	0	1	2
Amount of I-123 Remaining (g)	20	18.98	18.01

Based on WikipediA information for Iodine - 123 (Wikipedia contributors, 2024)

A. Use the amounts shown in the table to determine the percent decrease in the amount of I-123 remaining over each 1-hour time period. Round your answers to one decimal place.

Between 0 – 1: Decreased by _____ % over hour 1

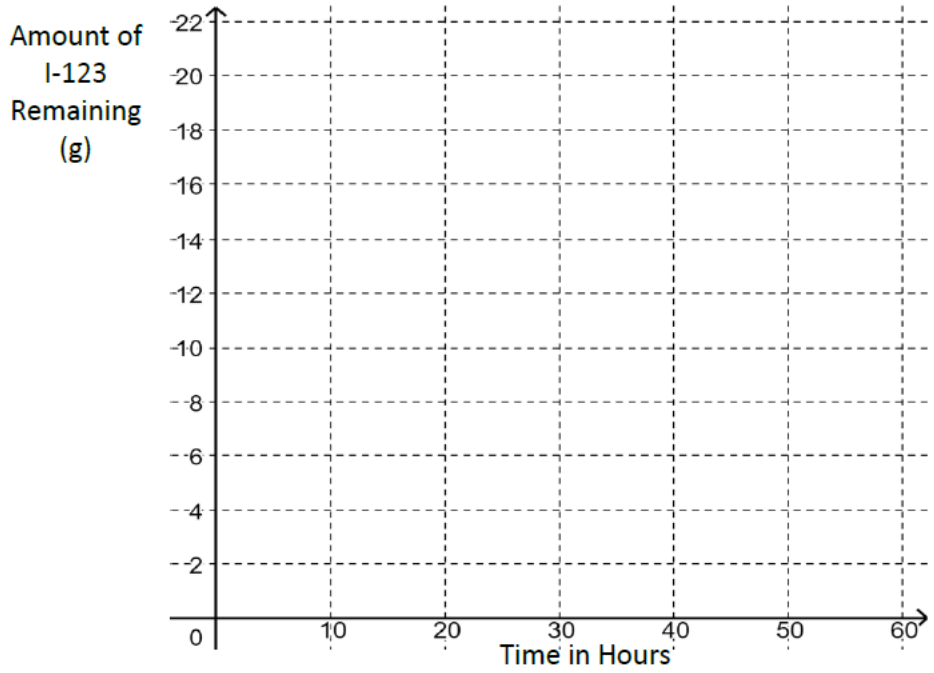
Between 1 – 2: Decreased by _____ % over hour 2

B. Create a function $I(x)$ for the number of grams of I-123 remaining after x hours.

Time (hours)	**Work Space**	**Amount of I-123 Remaining (g)**
0		20
1		18.98
2		18.01
10		

$I(x) =$ _____

C. Calculate amounts at 30 hours and 60 hours and draw the graph of $y = I(x)$ below.



Exponential Growth / Decay Functions

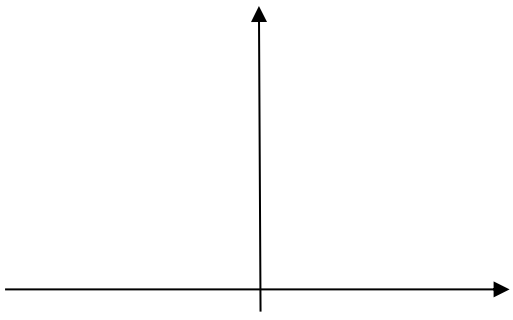
Exponential growth / decay functions correspond to the situation of **constant percent growth / decay for each 1-unit increase in x** and are of the form:

$$f(x) = a(1+r)^x \qquad \text{OR} \qquad f(x) = ab^x \text{ where } b = 1 + r$$

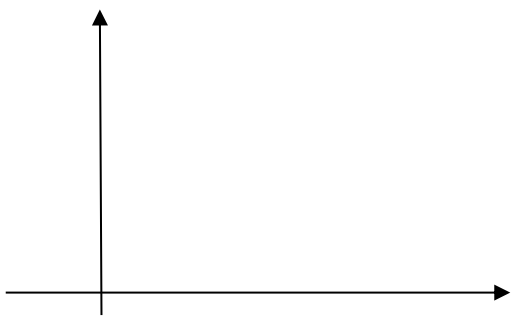
Where:

- a is the initial value ↔ the value when $x = 0$
- r is the **percent growth / decay rate**, written in decimal form
  r is positive ↔ growth; r is negative ↔ decay
- b is the **growth / decay factor or growth / decay multiplier**
  b is always positive; $b > 1$ ↔ growth; $0 < b < 1$ ↔ decay

Exponential Growth Graph



Exponential Decay Graph



You Try! Example 3: Greenville had a population of 5,000 in 2020. For each of the following scenarios, create an appropriate function $P(t)$ to model the population of Greenville t years after 2020.

A. The population decreased by 120 people each year.

$P(t) =$ _____

B. The population increased by 12% each year.

$P(t) =$ _____

C. The population decreased by 12% each year.

$P(t) =$ _____

Example 4: A local bank is currently offering a 6-month certificate of deposit (CD) at an annual rate of 4.8% *compounded monthly* – meaning that the investment earns $4.8\%/12 = 0.4\%$ per month and the interest earned one month is included in the base amount (principle) that earns interest the next month. If you invest $15,000 in the CD, how much money will you gain over the 6 months? Round your answer to two decimal places.

You Try! Example 5: How much money would you need to invest at an annual rate of 4.8% *compounded quarterly* to have $25,000 in 10 years? Round your answer to the nearest dollar.

Section 5.5: Finding Equations of Exponential Functions

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website - *College Algebra, An Investigation of Functions*, primary authors David Lippman, Melonie Rasmussen & Jay Abramson Chapter 4 Section 4.1: Exponential Functions pages 233 – 234 of the pdf-file (printed pages 225 – 226)

Objectives:

- Be able to create an algebraic expression for an exponential function when given the y-intercept and another point on the graph of the function or corresponding information
- Be able to interpret all parts of an exponential model equation in an applied situation
- Know what the graphs of exponential growth and decay functions look like over the domain of all real numbers

Important Vocabulary Words

Exponential Function, Growth / Decay Factor, Percent Growth / Decay Rate, Initial Value

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Example 1: Some hobby farmers noticed 6 ground squirrels in their pasture, but did nothing. Four years later they counted 47 of them in the pasture!

A. Assuming that the ground squirrel population grows exponentially, create a model for the number of ground squirrels living in the pasture t years after the farmers first noticed them. Round b in your model to four decimal places.

B. According to your model, what is the percent growth rate for the ground squirrel population?

C. By what percent did the ground squirrel population grow over the four years?

D. Does dividing your answer in C by 4 result in the same answer as that in B?

You Try! Example 2: The Scripps Institute of Oceanography has been measuring the concentration of CO_2 in the earth's atmosphere since 1958. In that year, the concentration peaked at approximately 318 parts per million (ppm). The May 2024 seasonal peak measurement was approximately 427 ppm.

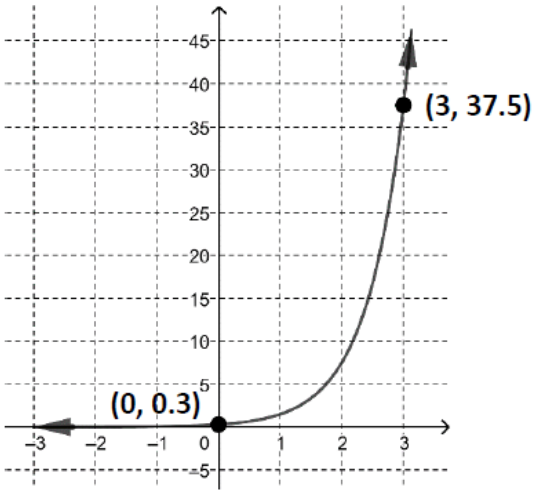
Informed by a June 2024 National Oceanic and Atmospheric Administration news release (T. Stein, 2024) and an online Keeling Curve – full record graph (Monroe, R.)

A. Create an exponential model for the seasonal peak concentration of carbon dioxide (in ppm) in the atmosphere t years after 1958. Round b in your model to 4 decimal places.

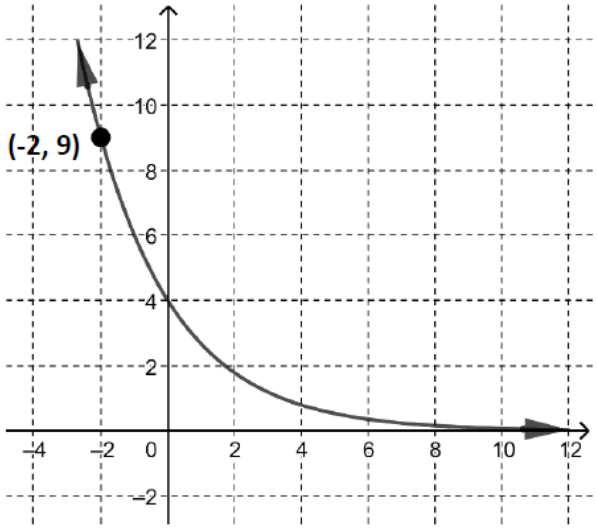
B. According to your model, by what percent per year has the $C0_2$ concentration in the atmosphere been rising?

You Try! Example 3: Determine algebraic expressions for the functions whose graphs are shown.

A.



B.



Chapter 6
Logarithms and Base e Exponential Models

Sections 6.1 – 6.5

Section 6.1: Inverse Functions

Part 1: Introduction to Inverse Functions

Reference Materials:

Scottsdale Community College Mathematics Division OER Resources Website
College Algebra, An Investigation of Functions by David Lippman, Melonie Rasmussen & Jay Abramson
Chapter 1 Section 1.6: Inverse Functions pages 99 – 102 of the pdf-file (printed pages 91 – 94)

Objectives:

- Know what an inverse function is and be able to appropriately use it in an applied setting
- Know the notation for the inverse of a function
- Be able to evaluate an inverse function when the original function is presented numerically or graphically

Important Vocabulary Words:

Inverse Function

Example 1: For windspeeds between 4 m/s and 12 m/s, the power P, in kilowatts (kW), generated by a particular type of wind turbine can be approximated by $P(w) = 0.955w^3$, where w is the windspeed in meters per second (m/s).

Informed by a Sciencing website (K. Lee, 2020) and Wind farm Bop website (F. Miceli, 2012)

A. If the wind turbine is producing 400 kW of power, at what speed is the wind blowing? Round your answer to one decimal place.

B. Determine a function $W(p)$ that provides the windspeed in m/s associated with a power output of p kilowatts from the wind turbine.

Definition of Inverse of a Function

A function g is the inverse of a function f if $g(y) = x$ whenever $f(x) = y$ for all values of x in the domain of f. The standard notation for "inverse of f" is f^{-1}.

$$f^{-1}(y) = x \leftrightarrow f(x) = y$$

A is the domain of f and B is the range $f \leftrightarrow$ B is the domain of f^{-1} and A is the range of f^{-1}

Example 2: If $f(4) = -3$, Then $f^{-1}(_____) = _____$. ← Fill in the blanks.

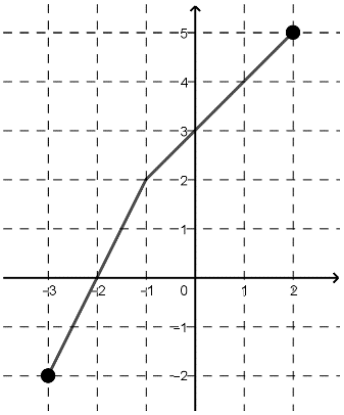
Example 3: A table of values is provided for the function g. Determine the following:

x	$g(x)$
2	3
5	8
7	5
12	2
20	15

A. $g^{-1}(8) =$ B. $g(5) =$ **You Try!** C. $g^{-1}(2) =$

You Try! D. $g(2) =$ **You Try!** E. $g(g^{-1}(5)) =$

Example 4: The graph of $y = f(x)$ is shown below. Determine the following:



A. $f(2) =$ B. $f^{-1}(5) =$ C. $f^{-1}(2) =$

You Try! D. $f^{-1}(4) =$ **You Try!** E. $f^{-1}(-2) =$

You Try! F. $f^{-1}(f(1)) =$

Composition Property of Inverse Functions

$f^{-1}(f(x)) = x$ for all x in the domain of f and $f(f^{-1}(y)) = y$ for all y in the range of f

OR

g is the inverse of f if and only if:

$g(f(x)) = x$ for all x in the domain of f and $f(g(y)) = y$ for all y in the range of f

Part 2: When Is an Inverse Relation a Function?

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.1: Functions and Function Notation (See "Determining Whether a Function is One-to-One" and "Using the Horizontal Line Test")

Scottsdale Community College Mathematics Division OER Resources Website
College Algebra, An Investigation of Functions by David Lippman, Melonie Rasmussen & Jay Abramson
Chapter 1 Section 1.6: Inverse Functions page 103 of the pdf-file (printed page 95)

Objectives:

- Know the definition of one-to-one and be able to determine if a function that is presented numerically is one-to-one
- Be able to use the horizontal line test to determine if a function that is presented graphically is one-to-one
- Understand the requirement that a function be one-to-one in order for its inverse to be a function

Important Vocabulary Words:

One-to-One, Horizontal Line Test

Recall Definitions:

A **relation** is a set of ordered pairs.

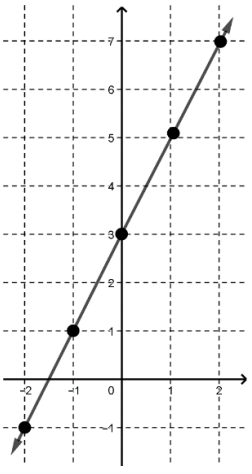
A **function** is a relation that assigns to each element in its domain exactly one element in its range.

Example 1:

x	$y = f(x) = 2x + 3$
-2	-1
-1	1
0	3
1	5
2	7

y	$x = f^{-1}(y)$

Graph of $y = f(x)$



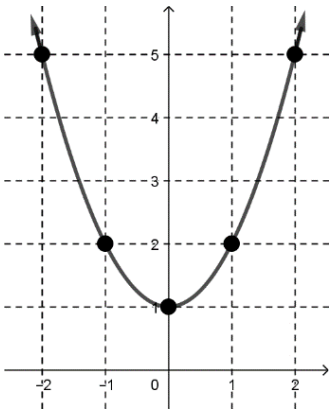
Is f^{-1} a function?

Example 2:

Graph of $y = g(x)$

x	$y = g(x) = x^2 + 1$
-2	5
-1	2
0	1
1	2
2	5

y	$x = g^{-1}(y)$



Is g^{-1} a function?

Definition of One-to-One Function

A function f is **one-to-one** if each output value corresponds with exactly one input value, that is, if: $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.

When f^{-1} Is a Function

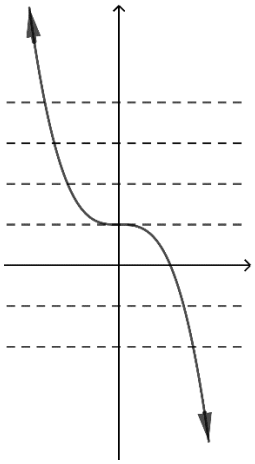
f^{-1} is a function $\leftrightarrow$ the function f is one-to-one

Horizontal Line Test

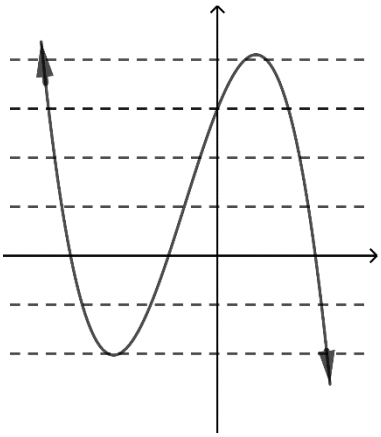
If _____ horizontal line intersects the graph of $y = f(x)$ _____, then f IS one-to-one.

If _____ horizontal line intersects the graph of $y = f(x)$ _____, then f is NOT one-to-one.

Function **Passes** the Horizontal Line Test



Function **Does Not Pass** the Horizontal Line Test



You Try! Example 3: Fill in the blanks.

A. f^{-1} is a function if and only if f is _____ .

B. A relation f is a **function** if each _____ corresponds to exactly one

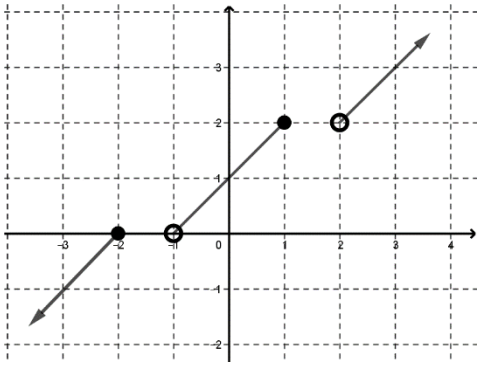
_____.

C. A function f is **one-to-one** if each _____ corresponds to exactly one

_____.

You Try! Example 4: Determine whether or not each function is one-to-one.

x	0	1	2	3	4
$g(x)$	4	7	9	4	8



Part 3: Finding and Verifying Inverse Functions

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 3 Section 3.7: Inverse Functions (See "Verifying That Two Functions Are Inverse Functions" through Example 3 and "Finding Inverses of Functions Represented by Formulas" through Example 8)

Objectives:

- Be able to use the composition property to determine if a function is the inverse of a given function
- Be able to determine an algebraic expression for the inverse of a one-to-one function that is presented algebraically

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Composition Property of Inverse Functions

$f^{-1}(f(x)) = x$ for all x in the domain of f and $f(f^{-1}(y)) = y$ for all y in the range of f

OR

g is the inverse of f if and only if:

$g(f(x)) = x$ for all x in the domain of f and $f(g(y)) = y$ for all y in the range of f

Example 1: Let $f(x) = 5(x+1)$ and $g(x) = \frac{x}{5} - 1$. Use the *Composition Property of Inverse Functions* to test whether $g = f^{-1}$.

Example 2: Let $f(x) = \frac{x+3}{2x}$ and $g(x) = \frac{2x}{x+3}$. Use the *Composition Property of Inverse Functions* to test whether $g = f^{-1}$.

Example 3: What is f^{-1} for $f(x)=\dfrac{x+3}{2x}$?

Steps for Determining the Inverse of a One-to-One Function *f*

1. Write the function in equation form. That is, replace $f(x)$ with y.
2. Solve for x in terms of y.
3. Replace x with $f^{-1}(y)$.

You Try! Example 4: Find $f^{-1}(y)$ for $f(x)=\dfrac{\sqrt[3]{x+5}}{4}$. Then use the *Composition Property of Inverse Functions* to verify your answer.

Section 6.2: Logarithmic Functions

Part 1: Definition of Logarithm

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 10 Section 10.3: Evaluate and Graph Logarithmic Functions

Objectives:

- Know the definition of the function $\log_a(x)$ and what is meant by the common logarithm and the natural logarithm
- Be able to use the definition of logarithm to go back and forth between exponential and logarithm forms of an equation
- Be able to use the definition of logarithm to evaluate logarithms and solve simple logarithm equations

Important Vocabulary Words:

Logarithm Base a, Common Logarithm, Natural Logarithm, Logarithm Form of an Equation, Exponential Form of an Equation

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Example 1: Find $f^{-1}(x)$ for $f(x)=2^x$.

Definition of Logarithm

For $a>0,\ a\neq 1$, logarithm base a is the inverse of a base a exponential function:

$x=\log_a(y)$ is equivalent to $y=a^x$

Hence: $f(x)=a^x \ \leftrightarrow \ f^{-1}(x)=\log_a(x)$ where $x>0$

Common and Natural Logarithms

Common Logarithm: Base 10 $\leftrightarrow$ $\log(x)=\log_{10}(x)$

Natural Logarithm: Base e $\leftrightarrow$ $\ln(x)=\log_e(x)$

Like π, the natural number shows up so frequently that it is given its own symbol: "e." In addition, like π, it has infinitely many decimal places and there is no pattern in it.

e = 2.718281828100459 . . . ≈2.718

Example 2: Use the definition of logarithm to express each exponential equation in logarithm form.

A. $5^2 = 25$ B. $10^{-1} = 0.1$ C. $e^0 = 1$

Example 3: Use the definition of logarithm to express each logarithm equation in exponential form.

A. $\log_3(81) = 4$ B. $\log(100) = 2$ C. $\ln(5) \approx 1.61$

You Try! Example 4: If $f(x) = 8^x$, then $f^{-1}(x) =$ _____ .

You Try! Example 5: Use the definition of logarithm to express each equation in logarithm form.

A. $9^{1/2} = 3 \leftrightarrow$ _____ B. $e^{t+1} = w \leftrightarrow$ _____

You Try! Example 6: Use the definition of logarithm to express each equation in exponential form.

A. $\log_T p = m \leftrightarrow$ _____ B. $\log 5 = k \leftrightarrow$ _____

You Try! Example 7: Use a scientific calculator to evaluate each of the following rounded to four decimal places:

A. $3e^{-1.5} \approx$ B. $10 + 2e^{4/3} \approx$

You Try! Example 8: The only logarithm keys on the face of a scientific calculator are the common logarithm "log" and the natural logarithm "ln." Use them to evaluate the following rounded to four decimal places.

A. $\log(7) \approx$ B. $\ln(8) \approx$

Example 9: Use the definition of logarithm to evaluate the given logarithms.
HINT: Label each expression as "x," rewrite in exponential form and solve for x.

A. $\log_4 16$

B. $\log_3\left(\frac{1}{27}\right)$

Example 10: Use the definition of logarithm to solve for x.

A. $\log_x 16 = 2$

B. $\log_5(x+1) = 2$

You Try! Example 11: Use the definition of logarithm to evaluate the given logarithms.

A. $\log_2(\sqrt{2})$

B. $\log(0.1)$

You Try! Example 12: Use the Definition of Logarithm to solve for x.

A. $\log_9(7x) = \frac{1}{2}$

B. $\log_{x+1}(8) = -3$

Part 2: Graphs of Logarithm Functions

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 10 Section 10.3: Evaluate and Graph Logarithmic Functions

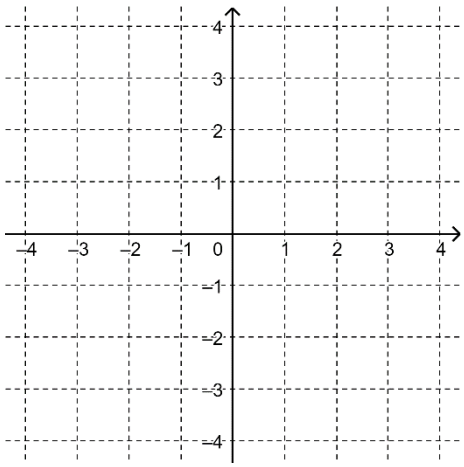
Objectives:

- Be able to sketch the graph of $y = \log_a(x)$ for a given value of a
- Know what the domain and range of a logarithm function are

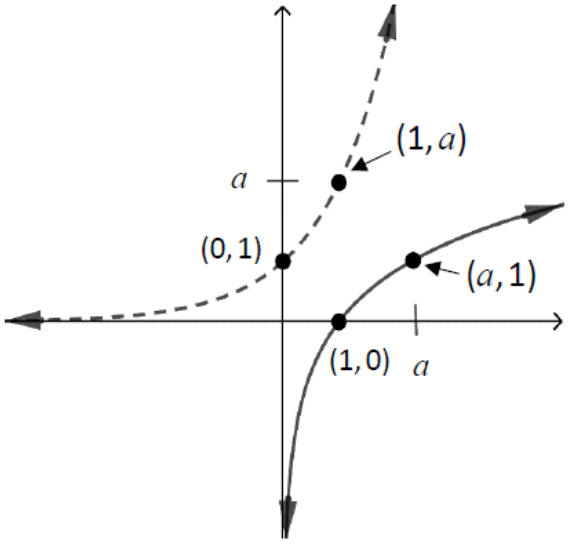
Example 1: Complete the table for $f(x) = 2^x$ and sketch its graph. Then complete the table for $f^{-1}(x)$ and sketch its graph on the same axes.

x	$f(x) = 2^x$
-2	
-1	
0	
1	
2	

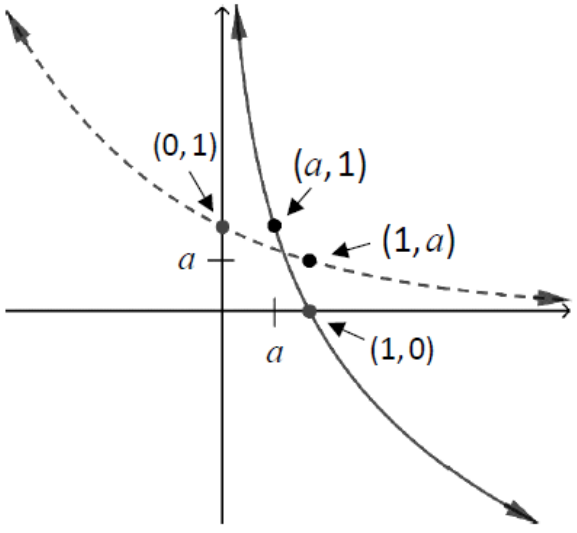
x	$f^{-1}(x) =$



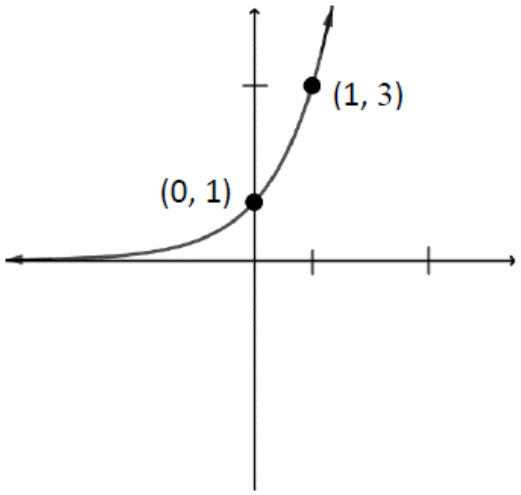
Graph of $f(x) = a^x$ (dashed) and $f^{-1}(x) = \log_a(x)$ (solid) for $a > 1$



Graph of $f(x) = a^x$ (dashed) and $f^{-1}(x) = \log_a(x)$ (solid) for $0 < a < 1$



You Try! Example 2: The graph of $f(x) = 3^x$ is shown with two key points labeled.



A. On the same axes, sketch the graph of $f^{-1}(x) = \log_3(x)$ and label the corresponding two key points.

B. What are the domain and range of $f^{-1}(x) = \log_3(x)$? Domain: _____

Range: _____

You Try! Example 3: Name two points that the graph of $g(x) = \log(x)$ passes through.

Point #1: Point #2:

You Try! Example 4: Use a scientific calculator to evaluate the following. Round your answers to four decimal places. Notice anything? WHY?!

A. $\log(-2) \approx$

B. $\ln(0) \approx$

Section 6.3: Logarithm Properties

Part 1: Evaluation Properties

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 10 Section 10.4: Use the Properties of Logarithms

Objectives:

- Know evaluation and inverse properties of logarithms and be able to use them to evaluate expressions involving logarithms

Important Vocabulary Words:

Inverse Properties

Example 1: Evaluate the following logarithmic expressions.

A. $\log_7 1$

B. $\log_5 5$

Example 2: Let $f(x) = a^x$. Then $f^{-1}(x) = \log_a(x)$. Determine $f^{-1}\left(f(x)\right)$ and $f\left(f^{-1}(x)\right)$.

Recall Composition Property of Inverse Functions: $f^{-1}\left(f(x)\right) = f\left(f^{-1}(x)\right) =$ _____

Logarithm Evaluation Properties

1. $\log_a 1 = 0$ because
2. $\log_a a = 1$ because

Inverse Properties

3. $\log_a\left(a^x\right) = x$ from Example 2
4. $a^{\log_a x} = x$ from Example 2

Example 3: Use the logarithm evaluation properties to evaluate the following expressions.

A. $\log_2 16 =$

B. $\log 10$

C. $\log_4 \sqrt[3]{16}$

C. $e^{\ln 8}$

You Try! Example 4: Use the logarithm evaluation properties to evaluate the following expressions.

A. $\log 1000$

B. $10^{\log 17}$

C. $\log_5 \left({}^{1}\!/_{125} \right)$

Part 2: Expand / Condense Properties

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 10 Section 10.4: Use the Properties of Logarithms

Objectives:

- Know the expand / condense properties of logarithms and be able to use them to expand and condense logarithmic expressions

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Expand / Condense Logarithm Properties

Expand / condense logarithm properties mirror exponent rules! Assume $a > 0,\ a \neq 1$.

Properties of Exponents		**Properties of Logarithms**	
Product Property:	$a^M \cdot a^N = a^{M+N}$	Product Property:	$\log_a(M \cdot N) = \log_a(M) + \log_a(N)$
Quotient Property:	$\frac{a^M}{a^N} = a^{M-N}$	Quotient Property:	$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$
Power Property:	$\left(a^M\right)^p = a^{pM}$	Power Property:	$\log_a\left(M^p\right) = p \cdot \log_a(M)$

Example 1: Use the *Expand / Condense Logarithm Properties* to **fully expand** each expression. In addition, use the *Logarithm Evaluation Properties* to evaluate logarithms whenever possible without the use of a calculator.

A. $\log\left(\frac{10}{x}\right)$

B. $\ln(5e^2)$

C. $\log_2\left(\frac{7\sqrt{x}}{8}\right)$

You Try! D. $\log\left(1000x^2\right)$

You Try! E. $\log_3\left(\frac{9x}{y^4}\right)$

F. $\log\left(xy^3\sqrt{z+1}\right)$

G. $\ln\sqrt{\frac{5x}{e^3}}$

You Try! H. $\log\left(\frac{x-2}{x^4(x+3)^5}\right)$

You Try! I. $\ln\sqrt[3]{xe^{4x}}$

Example 2: Use the *Expand / Condense Logarithm Properties* to **condense into a single logarithm**. Do not use fractional or negative exponents in your answer.

A. $\ln(x+2)-3\ln(x)$

B. $\frac{1}{2}\log x+4\log(x-5)$

You Try! C. $2\ln(x-3)-\frac{1}{3}\ln(x+4)$

You Try! D. $\ln(x-6)-3\ln(x+1)+\frac{1}{2}\ln x$

Self-Quiz!

1. Use the Definition of Logarithm to rewrite each equation in its equivalent exponential form.

A. $\log_b x = k$ → _____ B. $\ln y = x$ → _____

2. Use the Definition of Logarithm to rewrite each equation in its equivalent logarithm form.

A. $b^M = 3x$ → _____ B. $e^x = 3$ → _____

3. Evaluate <u>without</u> the use of a calculator. (Your answer should be an integer or a fraction.)

$$\log_2 8 =$$

4. Use properties of logarithms to **<u>fully</u> expand** the following logarithms, **evaluating wherever possible** without the use of a calculator.

A. $\log\left(10{,}000\ \sqrt[3]{x-1}\right)$

B. $\ln\left(\dfrac{e^3}{x^4}\right)$

5. Use properties of logarithms to **condense** the expression to a single logarithm. Do not use fractional or negative exponents in your answer.

$$\log z + \frac{1}{2}\log y - 5\log x$$

Self-Quiz! Solutions

1. Use the Definition of Logarithm to rewrite each equation in its equivalent exponential form.

A. $\log_b x = k \rightarrow \boxed{b^k = x}$ B. $\ln y = x \rightarrow \log_e y = x \rightarrow \boxed{e^x = y}$

2. Use the Definition of Logarithm to rewrite each equation in its equivalent logarithm form.

A. $b^M = 3x \rightarrow \boxed{\log_b(3x) = M}$ B. $e^x = 3 \rightarrow \log_e(3) = x \rightarrow \boxed{\ln(3) = x}$

3. Evaluate <u>without</u> the use of a calculator. (Your answer should be an integer or a fraction.)

$$\log_2 8 = x \rightarrow 2^x = 8 \rightarrow 2^x = 2^3 \rightarrow x = 3 \rightarrow \log_2 8 = \boxed{3}$$

OR

$$\log_2 8 = \log_2(2^3) = \boxed{3}$$

4. Use properties of logarithms to **<u>fully</u> expand** the following logarithms, **evaluating wherever possible** without the use of a calculator.

A. $$\log\left(10{,}000\sqrt[3]{x-1}\right) = \log(10{,}000) + \log(x-1)^{1/3}$$
$$= \log\left(10^4\right) + \frac{1}{3}\log(x-1)$$
$$= \boxed{4 + \frac{1}{3}\log(x-1)}$$

B. $$\ln\frac{e^3}{x^4} = \ln e^3 - \ln x^4$$
$$= \boxed{3 - 4\ln x}$$

5. Use properties of logarithms to **condense** the expression to a single logarithm. Do not use fractional or negative exponents in your answer.

$$\log z + \frac{1}{2}\log y - 5\log x = \log z + \log y^{1/2} - \log x^5$$
$$= \log\left(zy^{1/2}\right) - \log x^5$$
$$= \boxed{\log\left(\frac{zy^{1/2}}{x^5}\right) \text{ or } \log\left(\frac{z\sqrt{y}}{x^5}\right)}$$

Section 6.4: Using Logarithms to Solve Exponential Equations

Reference Materials:

Open Stax: *Intermediate Algebra 2e* by Maracek & Honeycutt Mathis
Chapter 10 Section 10.5 Solve Exponential and Logarithm Equations – Solve Exponential Equations Using Logarithms

Objectives:

- Be able to use $\log$ or $\ln$ to solve exponential equations with one exponential term.

Example 1: If you invest $1,000 at an annual rate of 5% compounded monthly, how long would it take for your money to grow to $1,500? Round your answer to the nearest month.

Technique to Solve an Equation with One Base b Exponential Term

You Try! Example 2: The average annual return on investment in the S&P 500 stock index between the beginning of 1957 and the end of 2023 was 10.32% per year. If you were to invest $10,000 at that annual percentage growth rate, how long would it take for your money to double? Round your answer to one decimal place. Informed by S&P 500 data (I. Webster, 2024) on officialdata.org website

Example 3: Solve each equation for x. Round your answer to three decimal places.

A. $20e^{-0.3x} = 15$

You Try! B. $5e^{x+3} + 8 = 20$

Technique to Solve an Equation with One Base e Exponential Term

Section 6.5: Base e Exponential Models

Reference Materials:

Open Stax: *College Algebra 2e* by Jay Abramson
Chapter 6 Section 6.7: Exponential and Logarithmic Models

Objectives:

- Be able to model exponential growth and decay situations using base e exponential models
- Know what is meant by the ***instantaneous*** **percent growth / decay rate** in such models

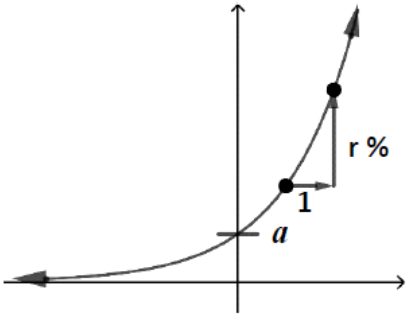
Important Vocabulary Words:

Instantaneous Percent Growth / Decay Rate

Periodic Growth / Decay Exponential Model (Base b)

The exponential function $f(t) = a \cdot b^t$ models exponential growth if $b > 1 \leftrightarrow r > 0$; decay if $0 < b < 1 \leftrightarrow r < 0$. Where:

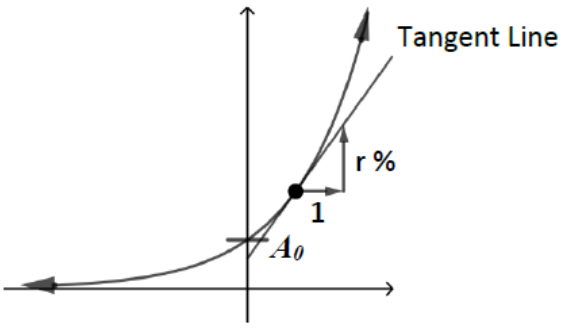
$t =$ number of time periods that have gone by
$a = f(0) =$ initial value when $t = 0$; $b = 1 + r$
$r =$ **periodic percent growth or decay rate** expressed as a decimal



Instantaneous Exponential Growth / Decay Model (Base e)

The exponential function $f(t) = A_0 e^{rt}$ models exponential growth if $r > 0$ and exponential decay if $r < 0$. Where:

$t =$ number of units of time that have gone by
$A_0 = f(0) =$ initial value when $t = 0$
$r =$ **instantaneous percent growth or decay rate** expressed as a decimal



Example 1: The population of Bozeman can be modeled by $P(t) = 37,280e^{0.0357t}$, where t is the number of years since 2010.

Informed by a U.S. Census Bureau website (United States Census Bureau, 2024)

A. According to the model, what was the population of Bozeman in 2010?

B. According to the model, has the population of Bozeman been increasing or decreasing? How can you tell?

C. Find the **instantaneous** percent growth or decay rate. Express your answer as a percent per unit of time.

D. According to the model, what will be the population of Bozeman in 2030?

E. According to the model, when will the population of Bozeman reach 100,000 people?

Example 2: Cesium 137 is a biproduct of nuclear fission. It is a radioactive contaminate of concern from nuclear accidents and weapons, but also has medical and industrial uses. Given that Cs-137 has a half-life of 30.17 years, create an exponential model of the form $A(t) = A_0 e^{rt}$ for the amount of Cs-137 remaining from a 5g sample as a function of time. Round r in your model to three decimal places.

Informed by a CDC Radiation Emergencies website (U.S. Centers for Disease Control and Prevention, 2024)

You Try! Example 3: Inspectors for the U.S. Department of Agriculture test some ground beef for the bacterium Escherichia coli. A sample taken from the meat at 9 am is found to have a bacterium count of 150 colony forming units (CFU). The meat is kept at a temperature of 100°F and another sample is taken at 9:30 am which is found to have a bacterium count of 425 CFU.

Informed by a USDA Food Safety and Inspection Service website (USD Food Safety and Inspection Service, 2017)

A. Determine an exponential model $A(t) = A_0 e^{rt}$ for the expected bacterium count from a sample taken t minutes after 9 am if the meat is kept at 100°F. Round r in your model to four decimal places.

B. According to your model, how long does it take E. coli bacterium to double in number if it is kept at 100°F?

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