



Boundary layer transition on the nozzle of a two-dimensional mach three wind tunnel
by Robert Lee Mueller

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
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Abstract:

Measurements were made to examine the progression of transition in the boundary layer growing on the nozzle of the Montana State University Supersonic Wind Tunnel. The effect on the transition process of several parameters such as unit Reynolds number, turbulence screens and position downstream from the sonic throat was considered. Pitot pressure fluctuations were measured and examined in terms of fluctuation intensity and fluctuation spectrum. Pitot pressure fluctuations were also examined in the time domain. The transition process was found to be composed of four regions; completely laminar, laminar unstable, unstable turbulent and fully turbulent. Of the four regions, the two unstable regions contained the highest intensity pitot pressure fluctuations. The transition process was found to be initialized via boundary layer instability not associated with surface roughness. Completion of the transition process from the onset to full turbulence was found to require a substantial portion of the nozzle length. Transition was found to occur non-uniformly around the perimeter of the tunnel cross-section.

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APPROVAL

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
C	-Sutherland's constant
C_f	-Skin friction coefficient
C_v	-Constant volume specific heat
kHz	-Kilocycles per second
M	-Mach number
MHz	-Megacycles per second
msec	-Millisecond
mVrms	-Root mean square millivolts
P	-Pressure
P'	-Fluctuating pitot pressure
psid	-Differential pounds per square inch
Re'	-Unit Reynolds number
Re_{tr}	-Transition Reynolds number
R_{air}	-Ideal gas constant for air
St	-Stanton number
T	-Temperature
u	-Velocity in the streamwise direction
x	-Streamwise coordinate axis
y	-Coordinate axis normal to nozzle
z	-Spanwise coordinate axis

NOMENCLATURE--ContinuedGreek Symbols

<u>Symbol</u>	<u>Description</u>
γ	-Ratio of specific heats
δ	-Boundary layer thickness
δ^*	-Displacement thickness
θ	-Momentum thickness
μ	-Viscosity
ν	-Kinematic viscosity

Subscripts

<u>Symbol</u>	<u>Description</u>
e	-Conditions at the edge of the boundary layer
o	-Stagnation conditions
STP	-Standard pressure and temperature (1 atmosphere and 32 °F)
∞	-Conditions based on tunnel core flow Mach number

ABSTRACT

Measurements were made to examine the progression of transition in the boundary layer growing on the nozzle of the Montana State University Supersonic Wind Tunnel. The effect on the transition process of several parameters such as unit Reynolds number, turbulence screens and position downstream from the sonic throat was considered. Pitot pressure fluctuations were measured and examined in terms of fluctuation intensity and fluctuation spectrum. Pitot pressure fluctuations were also examined in the time domain. The transition process was found to be composed of four regions; completely laminar, laminar unstable, unstable turbulent and fully turbulent. Of the four regions, the two unstable regions contained the highest intensity pitot pressure fluctuations. The transition process was found to be initialized via boundary layer instability not associated with surface roughness. Completion of the transition process from the onset to full turbulence was found to require a substantial portion of the nozzle length. Transition was found to occur non-uniformly around the perimeter of the tunnel cross-section.

INTRODUCTION

Freestream turbulence is commonly acknowledged to have adverse effects on boundary layer transition surveys of models in supersonic wind tunnels. Wind tunnels of conventional design are commonly plagued with high levels of freestream turbulence. Boundary layer values of Re_{tr} obtained in supersonic wind tunnels tend to be lower than those obtained in free flight due to the high levels of freestream noise encountered in wind tunnels. It is this characteristic of conventional supersonic wind tunnels that has prompted recent interest in the development of "quiet" wind tunnels. Such tunnels would be capable of operation with very low levels of freestream turbulence, more closely simulating the high altitude, free flight environment.

The condition of a boundary layer, laminar or turbulent, is an important consideration to the design of a supersonic vehicle. Laminar and turbulent boundary layers have considerable differences in terms of drag force and heat transfer characteristics. Another important design aspect is boundary layer separation. Laminar boundary layers separate more readily than turbulent boundary layers. This boundary layer feature has direct implications in the development of control surfaces for supersonic vehicles. Without accurate boundary layer transition information, the supersonic vehicle design process becomes extremely difficult if not impossible.

Two main sources of freestream turbulence in supersonic wind tunnels are convective and radiative disturbances. Convective disturbances originate in piping

upstream from the test section and are carried into the freestream along with the fluid flow. Radiative disturbances originate in turbulent boundary layers on the tunnel surfaces themselves and propagate into the freestream in the form of acoustical disturbances or pressure waves. There are other types of radiative disturbances, but of primary importance in the development of "quiet" wind tunnels is the type originating in turbulent tunnel boundary layers.

Premature transition generally develops in the following manner. First, there exists a transitional or turbulent boundary layer on the wind tunnel surfaces. The turbulent boundary layer leads to noise being radiated into the tunnel core flow. These pressure waves incident on a model in the tunnel core flow act as a significant input to an already unstable model boundary layer. The unstable model boundary layer and the freestream pressure fluctuations combine to induce premature boundary layer transition on the model.

Although a great deal of research has been accomplished to identify sources of freestream turbulence and their effects on boundary layer transition on models placed in wind tunnels, there exists only a limited understanding of the phenomenon actually producing the noise. For supersonic wind tunnels with $M_\infty \geq 2.5$ the pressure fluctuations arising from turbulent tunnel wall boundary layers become very large, and for $M_\infty > 3$, they completely dominate freestream noise¹. It was this information that prompted the current research. A detailed study was undertaken to examine the transition process of a boundary layer growing on the surfaces of a $M_\infty = 3$ wind tunnel.

BACKGROUND

Sources of Freestream Turbulence

Of the problems faced by transition research at supersonic speeds, freestream noise is perhaps the most troublesome to be overcome. Over the past 40 years a great deal of effort has been spent in achieving an understanding of freestream turbulence in wind tunnels. Initial research conducted by Laufer² and Morkovin³ was instrumental in identifying the phenomenon and clarifying its origins. More recent research conducted by Pate⁴ and Beckwith et al.⁵, to name a few examples, has served to verify earlier work and clarify the subject.

In terms of identifying and explaining sources of freestream turbulence, perhaps the most comprehensive treatment is given by Morkovin³. It was found that there are three types of disturbance modes responsible for freestream turbulence. The three modes are; entropy fluctuations (temperature spottiness), velocity fluctuations (vorticity), and pressure fluctuations. The three disturbance modes fall into two different categories of freestream noise, convective and radiative. Convective disturbances can only propagate along streamlines and develop in the region upstream from the tunnel throat. Radiative disturbances can travel across streamlines.

Entropy and velocity fluctuations belong to the convective category. Entropy fluctuations are generated by temperature gradients in the flow particularly where there is heating or cooling of the fluid. Entropy fluctuations are readily reducible

to negligibly small levels in adiabatic tunnels³. For unheated tunnels, entropy fluctuations are generally found to contribute little to the overall level of freestream turbulence.

Pressure fluctuations belong to the radiative category. In most wind tunnels, pressure fluctuations arise predominantly in the form of sound waves, although several other sources exist. The major source of pressure fluctuations (sound waves) is from turbulent boundary layers on surfaces comprising the test section of a wind tunnel. To a lesser degree they can also be found in stilling tanks upstream from the test section.

In research conducted by Laufer², a major point of interest arose. Laufer found that at $M_\infty > 2.5$, large increases in vorticity in the stilling tank had no effect on lowering the value of Re_{tr} on models, but at lower values of M_∞ increases in vorticity produced appreciable decreases in Re_{tr} . Since Laufer's initial investigation, it has generally been accepted that pressure fluctuations are the dominant mode of freestream turbulence above $M_\infty = 2.5$ to 3. Even in heated tunnels where the entropy mode becomes active, pressure fluctuations still represent a large portion of the total freestream disturbance level.

Morkovin³ concluded that both entropy and velocity fluctuations could be attenuated by proper stilling tank design, but little could be done to diminish pressure fluctuations. Morkovin further concluded that transition studies must be conducted with care and all inferences drawn with caution. Morkovin's conclusions of 1959 still represent a classical dilemma facing current transition research.

Freestream Turbulence Effects on Boundary Layer Stability and Transition

Current knowledge of boundary layer transition is fairly limited in terms of prediction and analysis. Pate⁴ presented a definition of transition as the portion of a boundary layer connecting the fully laminar and fully turbulent regions. This definition does not convey a great deal of information. Transition is a process occurring over a finite region of the boundary layer. It has generally been accepted that this process is composed of four fairly distinct regions.

1. The boundary layer is initially laminar and stable or "quiet".
2. The laminar boundary layer becomes unstable.
3. Turbulent bursts (intermittency) appear in the unstable boundary layer.
4. Intermittency becomes complete and the boundary layer is fully turbulent.

Due to the extremely complex nature of boundary layer stability the reader is referred to works by Schlichting⁶ and White⁷ for the development of stability theory. A detailed presentation of stability analysis and theory for compressible flow is beyond the scope of the present research.

The effects of freestream turbulence on a developing boundary layer have been investigated numerically by Mack⁸ in the development of his stability theory. Mack's results were then subject to experimental verification by Kendall⁹. Mack and Kendall found that around $M_\infty = 3$ to 4, freestream disturbances could be amplified by as much as an order of magnitude by a laminar boundary layer

growing on a model. They further discovered that amplification occurred regardless of the disturbance frequency. They found that the freestream pressure fluctuations acted as a forcing function on a laminar boundary layer. In a laminar boundary layer growing on a flat plate the amplification began at the leading edge and continued downstream until the point of transition. The overall concept of freestream turbulence as a cause of premature transition can be seen in figure 1.

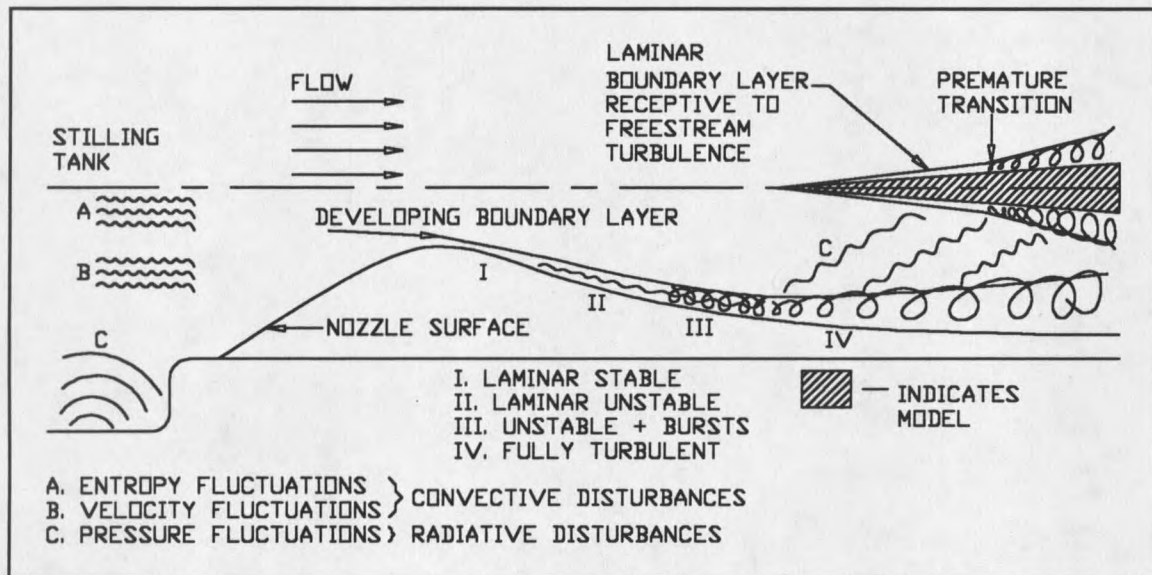


Figure 1.

Schematic of freestream turbulence modes, the boundary layer transition process, and premature transition.

There are several methods of transition prediction that have been developed over the years. One method is linear stability theory. Although this method offers one of the best possibilities for modeling boundary layer instability, the technique does not readily lend itself to use by the average research engineer. Due to the highly complex nature of the mathematical model and the intense computational requirements involved, the method is not easily applied. In addition, linear stability

only predicts the amplification of small disturbances, it does not explicitly predict boundary layer transition. It is necessary to correlate experimentally observed transition data with analytically calculated amplification rates for linear stability theory to yield transition information.

Another method of transition prediction is the formulation of correlations based on physical concepts and observed trends in data. The correlations are empirical and semi-empirical in nature. The general idea is to relate the onset of transition with certain physical quantities which can be measured and calculated. These types of correlations tend to be geometry specific pertaining to transition on flat plates, sharp cones and swept wings, to name a few examples. Even these relations have the drawback of becoming extremely complex when taking into account all of the parameters involved. They have the additional drawback of producing results that are not highly repeatable and with a large deviation. When considering the limited applications of these correlations, it can be seen that there is still a great deal of work yet to be accomplished. All of this effort has failed to produce even moderately reliable means of transition prediction¹⁰.

The condition of a boundary layer is a key element in the design of a supersonic vehicle, having a direct relationship to drag and heat transfer characteristics. Figures 2 and 3 show the difference between a laminar and turbulent boundary layer in terms of C_f and St . As can be seen, there is the possibility for the two parameters to increase by almost a factor of ten when changing from laminar to turbulent. At subsonic speeds C_f can account for as

much as 80 to 90% of total vehicle drag and roughly 50% at hypersonic speeds.

The heat transfer implications can be even more dramatic. It is the heat transfer rate that dictates the cooling and thermal insulation requirements which can ultimately determine the feasibility of a supersonic vehicle. A great deal of design work simply cannot be accomplished without accurate knowledge of transition.

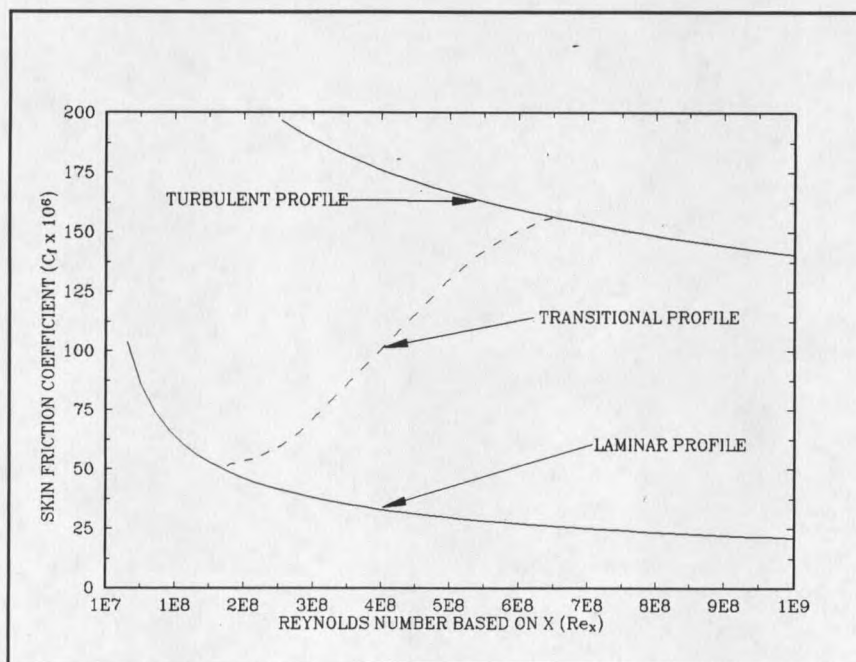


Figure 2.

Comparison of laminar and turbulent C_f profiles with the approximate transitional profile shown.

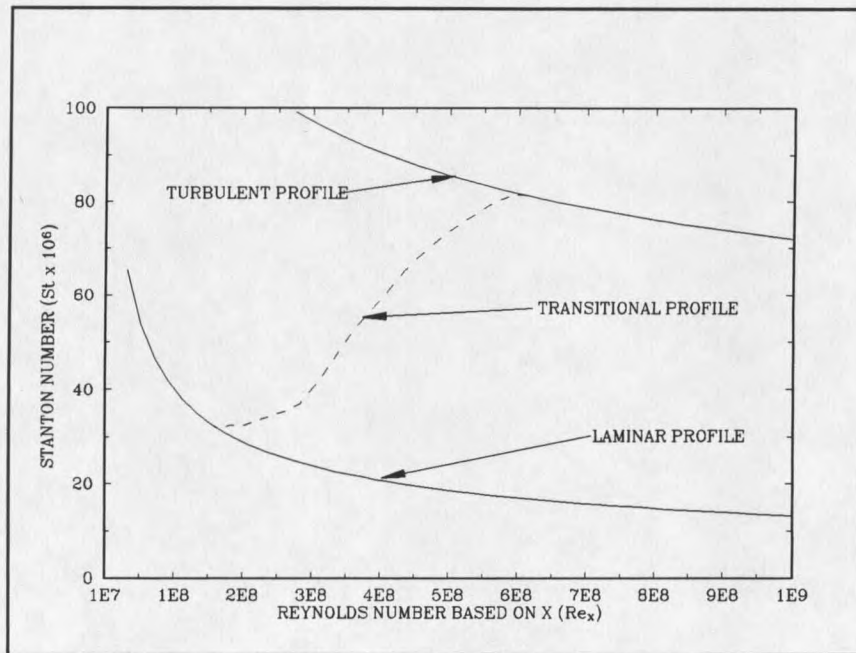


Figure 3.

Comparison of laminar and turbulent St profiles with the approximate transitional profile shown.

Although much effort has gone into researching freestream turbulence and its effects, there still remains one area that has received relatively little attention. Boundary layer transition on tunnel surfaces such as a nozzle is a subject where relatively little experimental work has been done.

One of the findings of Beckwith et. al.⁵ was that the highest levels of freestream noise occurred under transitional nozzle boundary layer conditions. The noise level in the freestream showed a significant decrease as the nozzle boundary layer went from transitional to fully turbulent. For many wind tunnels the transitional portion of a nozzle boundary layer can occupy a large region of the tunnel, and in many cases it cannot be avoided. With the higher levels of

freestream noise the effects on transition experiments can become much more pronounced. There is obviously some structure in the transitional boundary layer causing the increased levels of noise. The difficulty lies in detecting and identifying such a structure that could be capable of producing this effect.

In concept there are several basic mechanisms for developing turbulence in a laminar nozzle boundary layer. First, there are roughness effects, particularly near the throat where the boundary layer is thin and more susceptible to surface roughness. Such roughness effects should be evident in the form of small turbulent wake structures. This is a problem that no wind tunnel can be completely without. Even with highly polished nozzles, there is still the possibility of deposits forming on the surface due to impurities in the stream flow.

A second mechanism is the development of boundary layer instability. There are several types of boundary layer instability including Görtler vortices, Tollmien-Schlichting waves and cross flow instability. Görtler vortices develop a vortex structure with its axis aligned parallel to the direction of flow. The development of Görtler vortices requires an inflection point in the nozzle contour. Once the vortices appear, they persist throughout the boundary layer. Tollmien-Schlichting disturbances are two dimensional waves propagating in the direction of flow with the wave fronts aligned perpendicular or at an angle to the direction of flow.

In a study conducted by Harvey et. al.¹¹, the transition of a nozzle boundary layer was investigated at $M_\infty = 5$. Factors such as wall heating, wall roughness,

stilling tank screen configuration, nozzle contour and upstream piping were considered and their effects on nozzle boundary layer transition were investigated. It was concluded that transition on the nozzle was roughness dominated. This was indicated by an abrupt shift of the transition point from stations near the nozzle exit to a position at or near the nozzle throat with only a slight change in Re' .

The work of Harvey et. al.¹¹ also uncovered phenomena which remained unexplained at that time. It was found that under certain conditions the transition process was instigated in the boundary layer, but the boundary layer failed to become turbulent further downstream. In other cases the transition process required a large Reynolds number range for completion. It was stated that further investigation would be required to identify the mechanism leading to transition in this manner¹¹. Also noted was some discrepancy in transition Reynolds numbers measured by mean pitot boundary layer profiles and freestream noise measurements. Hot wire and fluctuating pitot pressure measurements of the freestream noise level were in general agreement indicating an abrupt rise to a peak while the mean pitot profiles still indicated a laminar boundary layer.

Research conducted thus far tends to favor boundary layer transition on a nozzle as a result of roughness in the throat region of a supersonic wind tunnel. However, certain unexplained phenomena have been observed which indicate the subject of transition on a nozzle still warrants further investigation. The possible existence of alternate mechanisms, such as instability, leading to transition of a

boundary layer on a nozzle cannot be entirely dismissed. It is this apparent gap in transition knowledge that has been the motivating factor behind the current research.

INITIAL PREPARATION

Goals

The goal of this research was to develop an understanding of the boundary layer transition process on the nozzle of the Montana State University Supersonic Wind Tunnel (MSU-SWT). Emphasis was placed on identification and observation of events occurring in the boundary layer prior to and immediately after the first appearance of turbulent bursts. The transition process was studied as a function of several parameters mainly tunnel surface, turbulence screens, and stagnation conditions.

Test Facility

All of the data was gathered in the MSU-SWT. The SWT is an open-circuit, continuous flow facility using air as the working fluid. The tunnel operates with a nominal test section Mach number of 3.0 and a Re' range of 0.58 to 1.7 million per inch. T_o can be varied from roughly 60 to 160 °F, and P_o can be varied from 600 to roughly 200 torr. The tunnel has the additional feature of a slot upstream from the nozzle facilitating the installation and removal of turbulence screens in the approach section. Screens can be installed and removed during tunnel operation eliminating the need for shut-down. Figure 4 illustrates the key tunnel features, and figure 5 provides a detailed drawing of the test section. The MSU-SWT is fully detailed in reference 12.

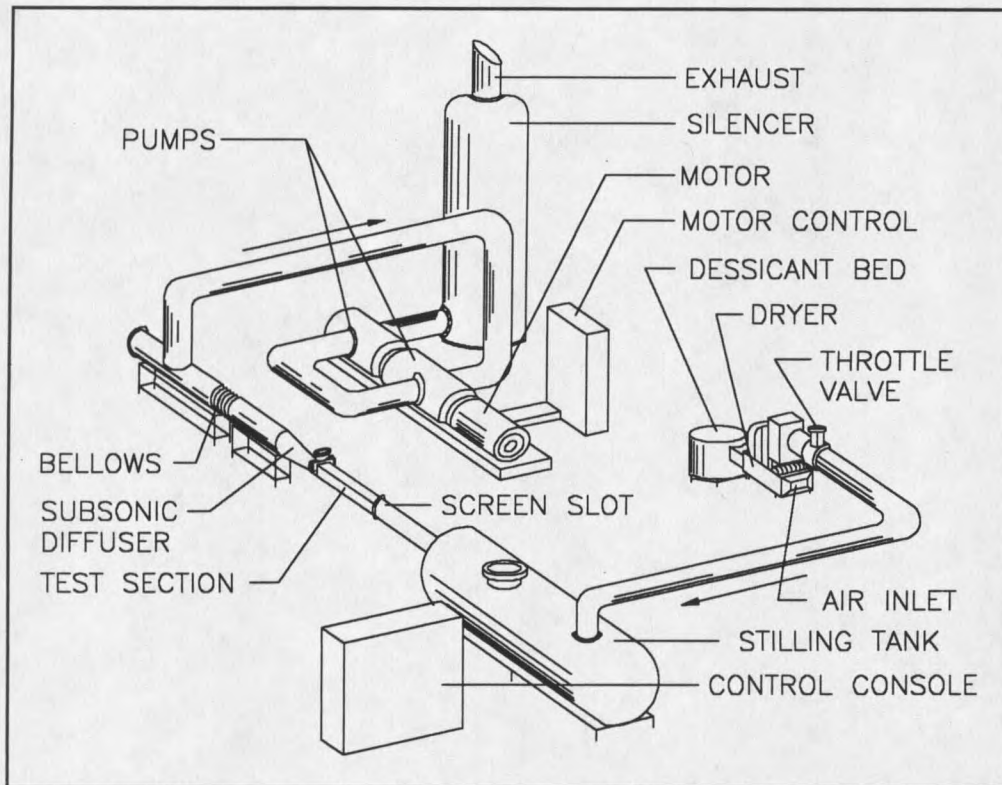


Figure 4.

Layout of the MSU-SWT indicating the key tunnel features and piping arrangement.

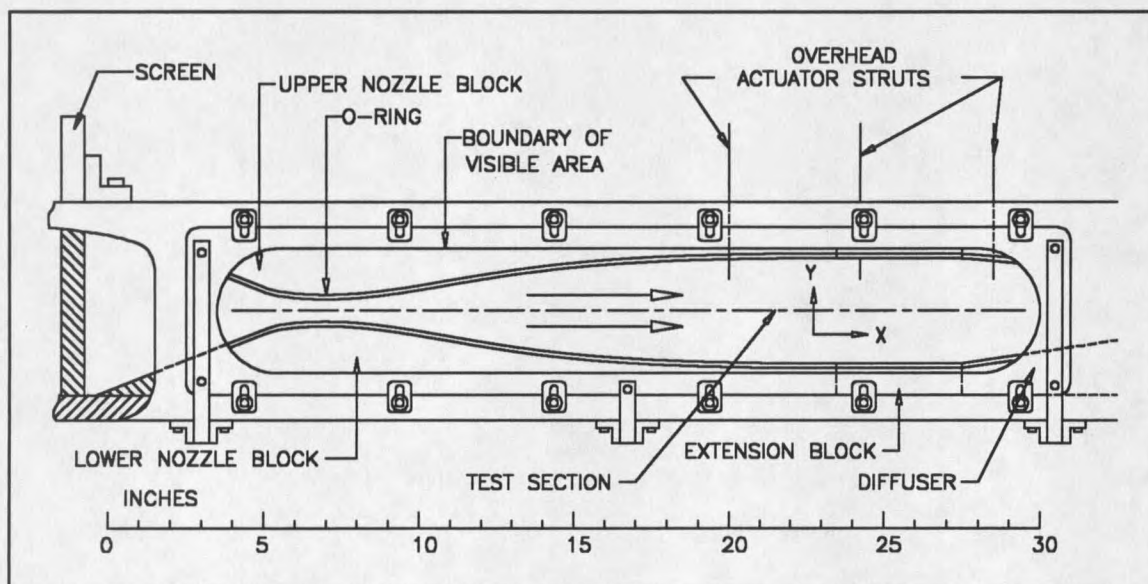


Figure 5.

Detailed drawing of MSU-SWT test section indicating the main nozzle components.

Core Flow Measurements

Initial tunnel preparation required maintenance work on several components of the tunnel test section, mainly the nozzle blocks. After removal and re-installation of the nozzle blocks, it was necessary to verify the core flow of the tunnel. These measurements were required to insure the proper Mach number was obtained in the test section. In addition to Mach number verification, these measurements revealed flow anomalies existing in the core flow and provided a means of tracking and elimination.

The core measurements were made with a pitot rake consisting of 10 pitot tubes and an external pressure transducer system fully described in reference 12. The rake was aligned parallel to the flow with the 10 pitot tubes spanning the width of the tunnel at equally spaced intervals, the tube tips forming a line perpendicular to the flow. With the tunnel operating at a constant stagnation pressure, the rake was traversed at a constant value of x from the upper to the lower nozzle surface while the pressure from a single pitot tube was measured. This process was then repeated for the remaining 9 tubes.

Typical results are shown in figure 6. Figure 6 shows the original test section Mach number of 3.0 was obtained with a maximum deviation of 0.03. There appeared to be no major shock waves present which could possibly disturb the core flow and invalidate future experiments.

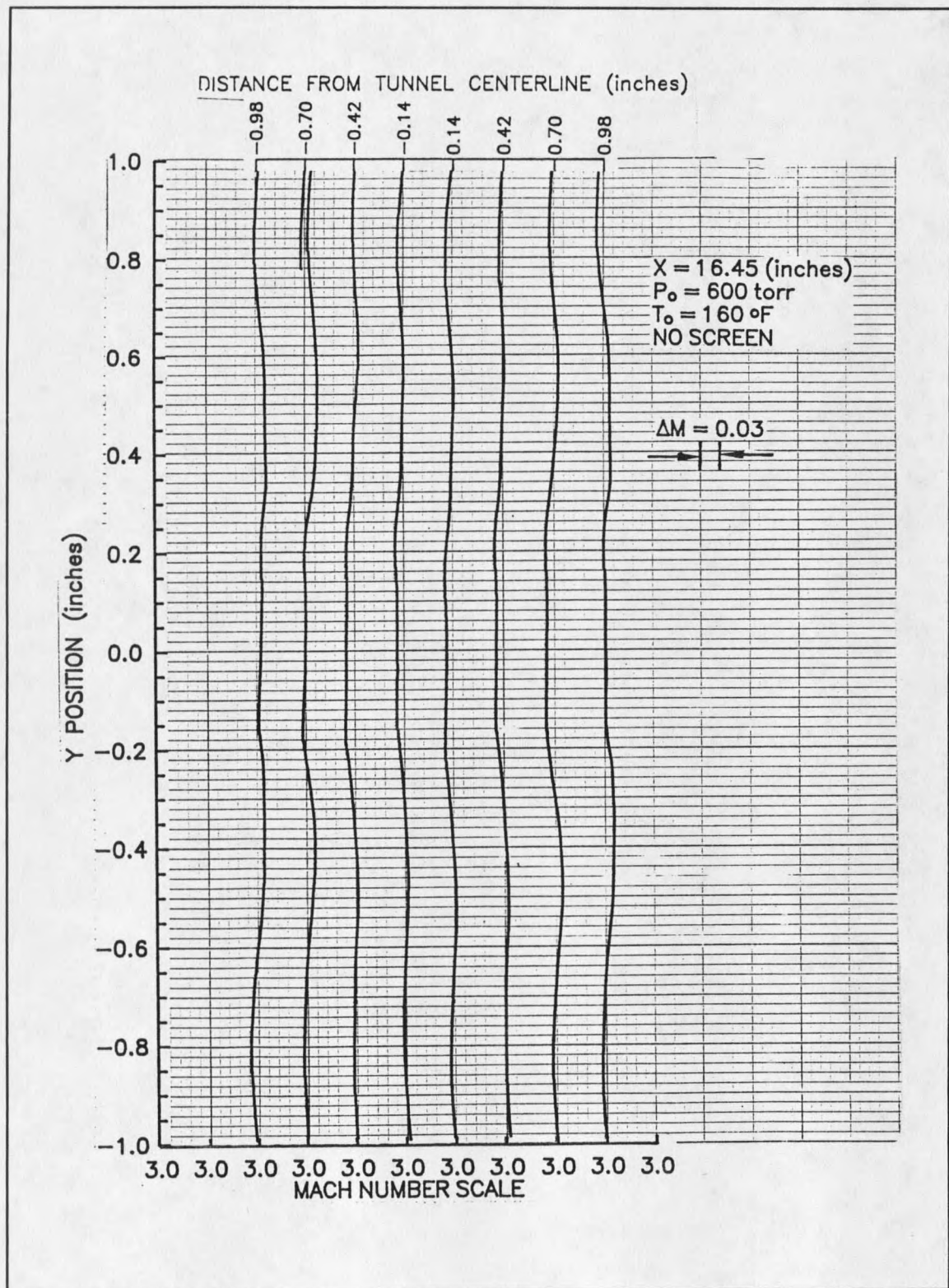


Figure 6.
Typical core flow measurement results.

The two outermost tubes, 1 and 10, were not included due to intrusion of the sidewall boundary layer. At $P_o = 600$ torr and $x = 16.14$ ", the sidewall boundary layer was thick enough that tubes 1 and 10 fell inside the sidewall boundary layer and yielded inappropriate results. The eight remaining tubes covered a 2" span of the 3.1" wide test section at 0.28" intervals. The total vertical traverse was limited to 2" of the 3.2" test section height due to the constriction of the diffuser section.

Boundary Layer Calculations

In order to validate experimental results it was necessary to characterize the MSU-SWT nozzle boundary layer as a "normal" boundary layer exhibiting normal growth patterns. This would allow the results of this investigation to be more readily applied to wind tunnels in other facilities. In addition, prior knowledge of the boundary layer properties would aid in the placement of probes in the boundary layer as well as provide some indication of the probe resolution in the boundary layer.

The approach used to characterize the boundary layer was to compare theoretical and experimental results for several of the boundary layer properties. In order to provide easily accessible theoretical results it was necessary to develop a method for the calculation of boundary layer properties in the MSU-SWT. Requirements of the method were that it be easily applied with minimal computational requirements and be able to provide reasonably accurate results.

The method chosen was the Kármán-Pohlhausen method with the application of the Holstein-Bohlen simplification. This method provided the greatest possibility for accurate results with a reasonably simple analytical model. The method is fully described in reference 13.

Initial assumptions inherent in the method itself are the following.

1. The Prandtl number of the fluid is equal to 1.
2. The heat transfer at the solid boundary is zero.
3. The viscosity is related linearly to the temperature.

The viscosity assumption is reasonable for flows with freestream pressure gradients due to the fact that the wall temperature at the solid boundary is constant as a consequence of assumptions 1 and 2. The above assumptions posed no great obstacles to obtaining reasonable solutions based on flow conditions in the MSU-SWT. The final assumption made was that the boundary layer had an initial thickness of zero at the sonic throat. The results of this method are valid only for laminar, 2-D boundary layers, although axis-symmetric solutions can be obtained through the transformation of Mangler¹³.

The solution process consisted mainly of developing a FORTRAN program which would implement the Kármán-Pohlhausen method with the modifications required to specifically fit the MSU-SWT. For a program listing see appendix.

The first step in the program was to calculate the Mach number variation down the length of the nozzle. This was accomplished with the use of the known nozzle coordinates, listed in reference 12, and the Mach number area ratio relationship. In order to increase the resolution of the solutions, the nozzle

coordinates downstream from the throat were curvefit for use in the program. The curvefit used is shown as equation 1. Use of a curvefit allowed for the solution of a greater number of points down the length of the nozzle. The curvefit, along with the first and second derivatives can be seen in figures 7 and 8. The curvefit derivatives provided additional insight as to the accuracy of the curvefit.

$$Y = \left[\frac{a + cX + eX^2}{1 + bX + dX^2 + fX^3} \right] \quad \text{Equation (1)}$$

Coefficients of equation 1.

$a = 0.352856$
 $b = 0.009602$
 $c = 0.009100$
 $d = 0.017426$
 $e = 0.044736$
 $f = 0.000360$

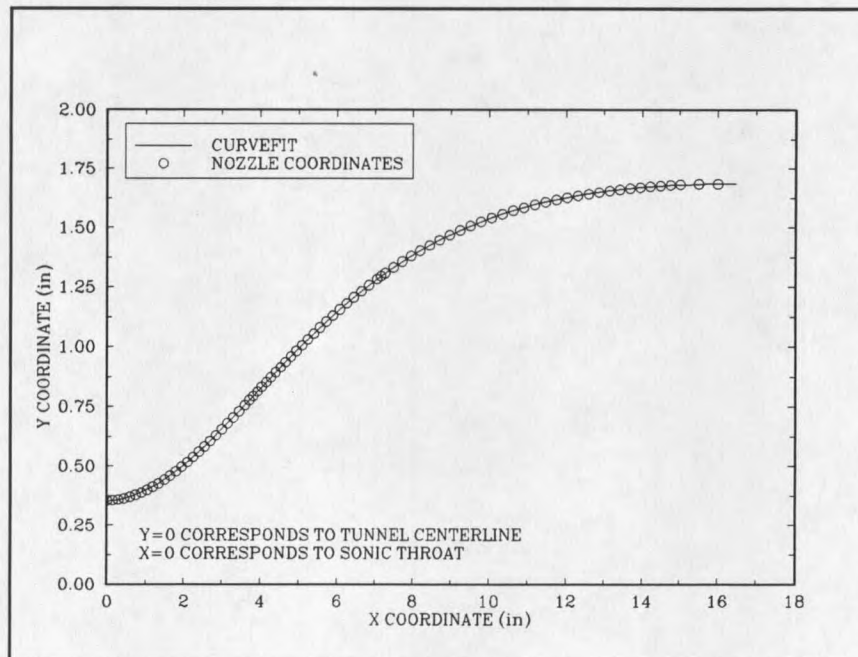


Figure 7.

Comparison of nozzle curvefit with the actual coordinates.

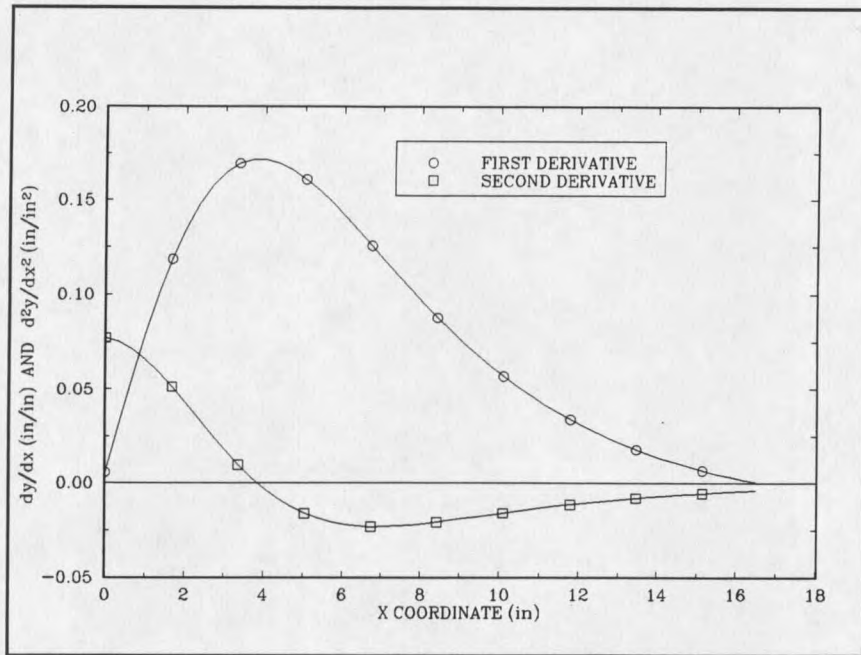


Figure 8.

Plot of the first and second derivatives of the nozzle curvefit.

Once the Mach number was obtained as a function of x and appropriate stagnation conditions provided, the remaining quantities required for a solution by the Kármán-Pohlhausen method could then be calculated.

The values of several constants required in the computations are listed below:

$$\begin{aligned}
 \gamma &= 1.4 \\
 R_{air} &= 287 \quad \text{J/(kg } ^\circ\text{K)} \\
 \mu_{STP} &= 16.8\text{E-6} \quad \text{Pa/sec} \\
 \nu_{STP} &= 13.0\text{E-6} \quad \text{m}^2/\text{sec} \\
 C &= 1
 \end{aligned}$$

Several boundary conditions were also required by the program in order to compute the desired results. At the throat M_e was assumed to be 1, and u_e was

assumed to be equal to the local speed of sound. At the throat du_e/dx was estimated to be roughly equivalent to the value immediately down stream from the throat. The third and final condition was that du_e/dx was equal to zero at the trailing edge of the nozzle.

Results

The most important result required for this investigation was the boundary layer thickness. This was the main parameter governing probe placement and resolution in the boundary layer. In figure 9 the analytical results for δ were compared to the experimental results of Berger¹⁴. The accuracy of the analytical approach appeared to be well within acceptable limits even with the coarse approximations required in developing the program.

In addition, δ^* and θ were also compared to the experimental results of Berger. As can be seen in figures 10 and 11, the analytical results show a greater discrepancy versus the experimental results. The greatest difference can be seen in figure 9 for δ^* .

In addition to experimental results, numerical results were also available for comparison. These results were adapted from work by King¹⁵. The analytical solution for δ compared favorably with the numerical results as did the results for θ . It is interesting to note that the computational results for θ were nearly identical even though both showed a significant deviation from the experimental results. Again the results for δ^* showed the greatest deviation from the numerical results.

Although the analytical approach appeared inadequate in predicting δ^* and θ in the SWT, it did provide an acceptable means of predicting δ . The method had the advantage of ease of application, while time requirements to obtain a solution were kept to a minimum.

In all cases, the experimental results of Berger compared favorably with the analytical and numerical results. This provided an indication that the nozzle boundary layer of the MSU-SWT could be classified as a normal boundary layer.

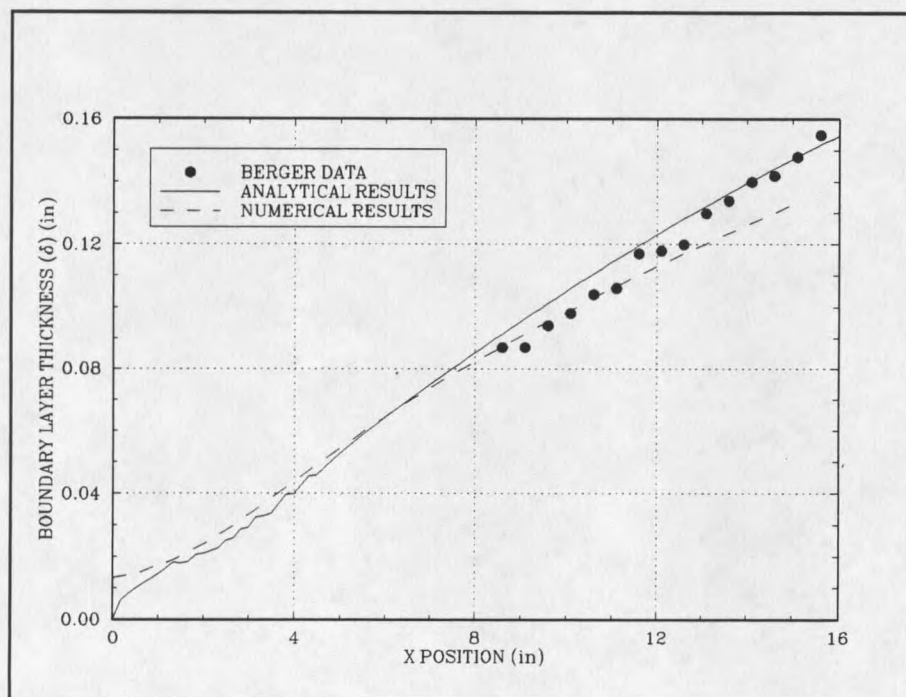


Figure 9.

Results from the analytical solution to the boundary layer thickness compared to experimental and numerical results

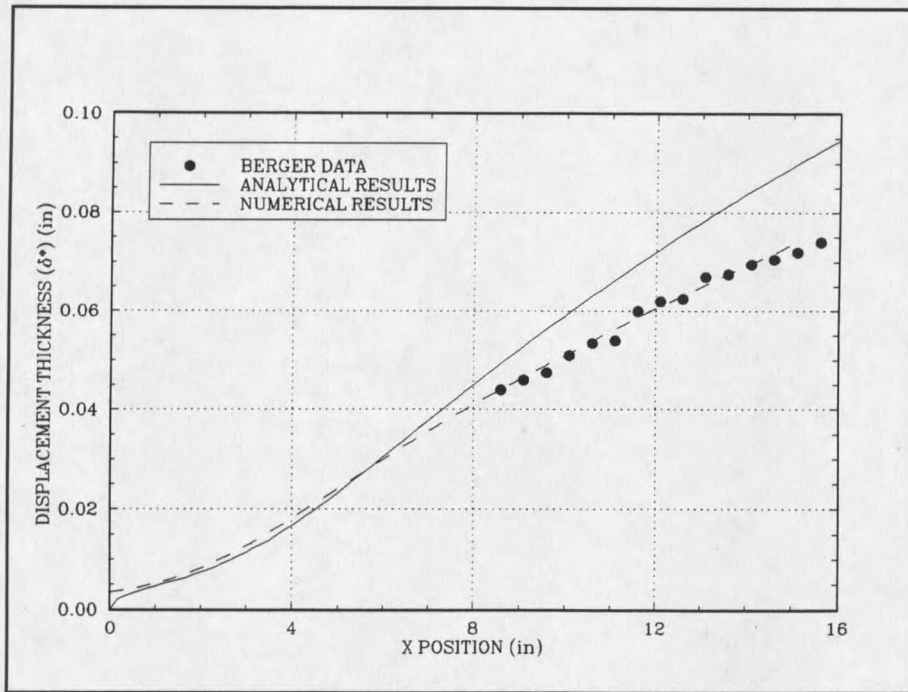


Figure 10.

Results from the analytical solution to the displacement thickness compared to experimental and numerical results.

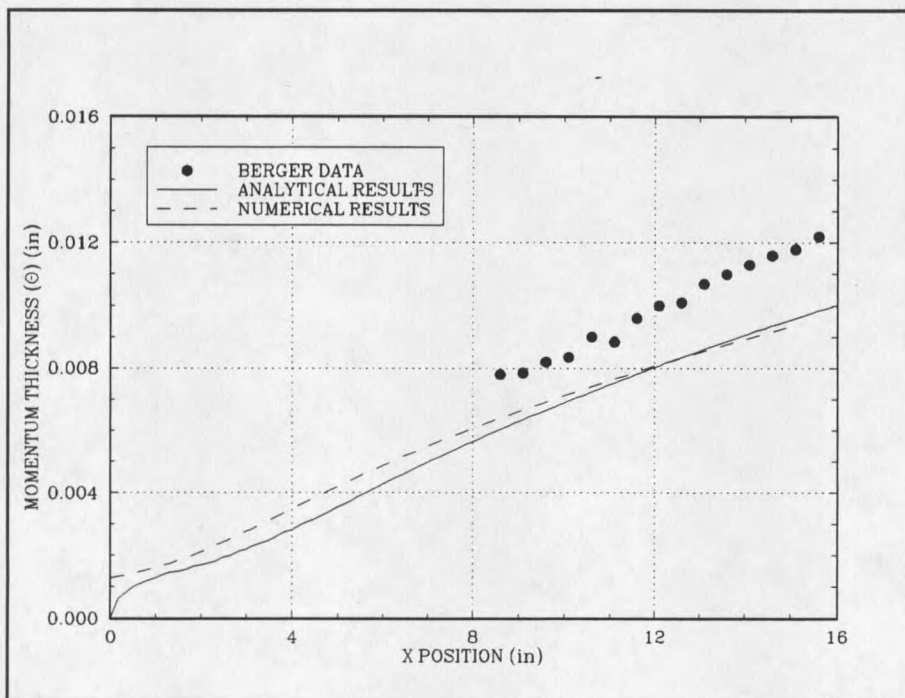


Figure 11.

Results from the analytical solution to the momentum thickness compared to experimental and numerical results.

EXPERIMENTAL INVESTIGATION

The Three Types of Experiments Conducted

The initial investigation of the tunnel boundary layer was to be conducted mainly as a function of P_o . For this class of experiments, the tunnel x position, T_o , screen configuration and surface would be fixed while P_o was swept continuously from the maximum attainable pressure to the breakdown of supersonic flow*. In this manner it would be possible to map out the transition process in the SWT as a function of the previously mentioned variables.

The primary objective of this type of investigation was concerned only with transition on the nozzle. This made the upper and lower nozzle blocks the main tunnel surfaces of interest. Several x positions downstream from the throat were considered, covering the majority of the length of both the upper and lower nozzle surfaces. Due to the 2-D nature of the tunnel it was hoped that these two surfaces would provide similar results indicating transition occurring in an identical manner on the upper and lower nozzle surfaces. If this result proved valid it would eliminate one of these surfaces from further investigation.

Due to the large number of screen types possible, the turbulence screen configuration was limited to two possibilities; no screen, and with screen. A 320

* The breakdown of supersonic flow was characterized by shock waves traveling forward into the test section from the diffuser eliminating supersonic flow in the test section. This condition resulted from insufficient supply pressure upstream from the test section for the pumps to maintain adequate flow through the test section.

mesh screen, the finest mesh readily available, was used during all measurements made with a screen in the tunnel. Although the use of two screen configurations limited the scope of the investigation, the primary interest was to first identify whether the screens had any effect on the transition process.

Due to the vast number of possible T_o values, only two values were chosen for the initial investigation; a low value of roughly 65 to 80 °F, and a high value of 160 °F. In order to further eliminate the dependence of the test results on T_o , the two parameters, T_o and P_o would be combined into the unit Reynolds number based on M_∞ during later reduction of the data. For actual tunnel run conditions, at constant P_o this variation in T_o corresponded to a Re'_∞ variation of approximately 20%. The entire test matrix can be seen in table I. As can be seen in table I, for a single x position, it was necessary to repeat the P_o sweep a total of eight times.

Table I.
Test matrix for a single x position.

x position	T_o	SCREEN	SURFACE
x"	LOW	YES	FLOOR
SAME	LOW	YES	CEILING
SAME	LOW	NO	FLOOR
SAME	LOW	NO	CEILING
SAME	HIGH	YES	FLOOR
SAME	HIGH	YES	CEILING
SAME	HIGH	NO	FLOOR
SAME	HIGH	NO	CEILING

In the second mode of investigation, P_o would remain at a fixed value while data was gathered at several x locations covering the majority of the nozzle length. The parameters of the investigation T_o , screen configuration and tunnel surface would remain fixed for a given value of P_o . Again the main aim of this type of investigation was to map out the transition process in the tunnel, but with the emphasis placed on the lengthwise variation. It was hoped that this method of investigation would provide information on transition as a region with definite boundaries i. e. an onset and ending point. This would provide qualitative information concerning a total transition length relative to the nozzle length, and also provide an onset location with respect to x and its variation with P_o . Knowledge of the onset location would greatly facilitate the investigation of events just prior to the onset of turbulence.

The third investigation conducted covered the nozzle surfaces as well as the sidewalls. The aim of this investigation was to determine if transition occurred uniformly along the perimeter of a cross-section of the tunnel. The information was gathered at fixed T_o and screen configurations while P_o was swept from maximum to the breakdown of supersonic flow. Data was taken under these conditions at several locations along the perimeter of the tunnel surfaces at a fixed x location. The process was then repeated for several x locations. In addition to the P_o sweeps boundary layer profiles obtained with the DPP were also used as a means to provide information concerning transition.

Transition Instrumentation

Due to the wide range of frequencies involved in the transition process, the selection of measuring devices played a key role in the investigation. The onset of turbulence involves the development of high frequency fluctuations. In addition to high frequencies the possibility of low frequency oscillations or instability also exists. It would be nearly impossible to determine the transition bandwidth prior to conducting experiments, therefore the bandwidth of the detection device must be broad enough to capture all events of interest. The work of Tritz¹⁶ indicated the full spectrum of frequencies expected in a fully turbulent boundary layer in the MSU-SWT extends as far as 1 MHz. A requirement of the detection device was that it would be capable of capturing a large portion of this spectrum, up to around 400 to 500 kHz.

The instrument chosen for boundary layer measurements was the dynamic pitot probe or DPP. The DPP consists of a micro-miniature pressure transducer fixed to the tip of a long strut. The transducer is a Kulite Semiconductor CQ-030-100 piezoresistive model which employs a four arm wheatstone bridge diffused onto a silicon diaphragm. The transducer has a sensitivity of 0.098 mV/psi/Vdc, a differential range of 0 to 100 psid and a bandwidth of 0 to 1.5 MHz. It forms a pitot probe with a tip diameter of 0.032" with a concentric active diameter of 0.01". A picture of the DPP is shown in figure 12.

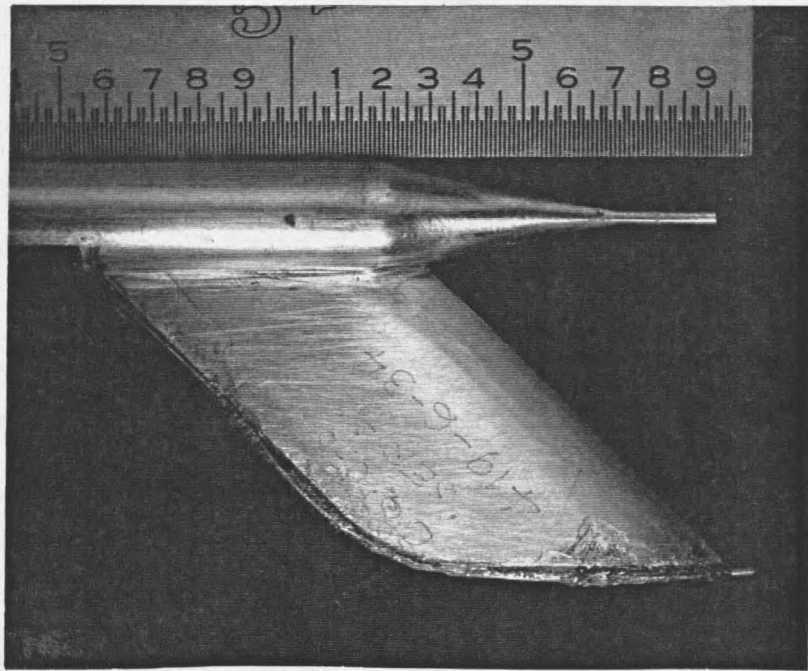


Figure 12.

DPP and mounting, actual transducer is shown in the lower right of picture.
Scale above probe reads in 1/10ths of an inch.

Data gathered during the investigation was recorded via a Honeywell, model 7600, 7 channel, reel to reel analog tape recorder. The recorded data would provide a complete library of the experiments conducted during the course of the investigation. It also provided a means for reducing the data gathering requirements of the investigation. The P_o sweeps were first recorded, and from this data it was possible to extract information concerning the second phase of the investigation, the transition variation with respect to x at a fixed P_o . The analog

system was capable of recording frequencies up to 400 kHz which was consistent with the bandwidth requirements of the experiments.

Due to the low level output of the DPP, it was necessary to amplify the signal for recording purposes. The amplifier used was a unit fabricated at MSU for use with the wind tunnel lab. It was an AC differential amplifier with a range of 30 Hz to 3 MHz and variable gain. The details of the complete recording circuit are shown in figure 13 along with several other peripheral devices used to monitor the DPP signal as it was recorded.

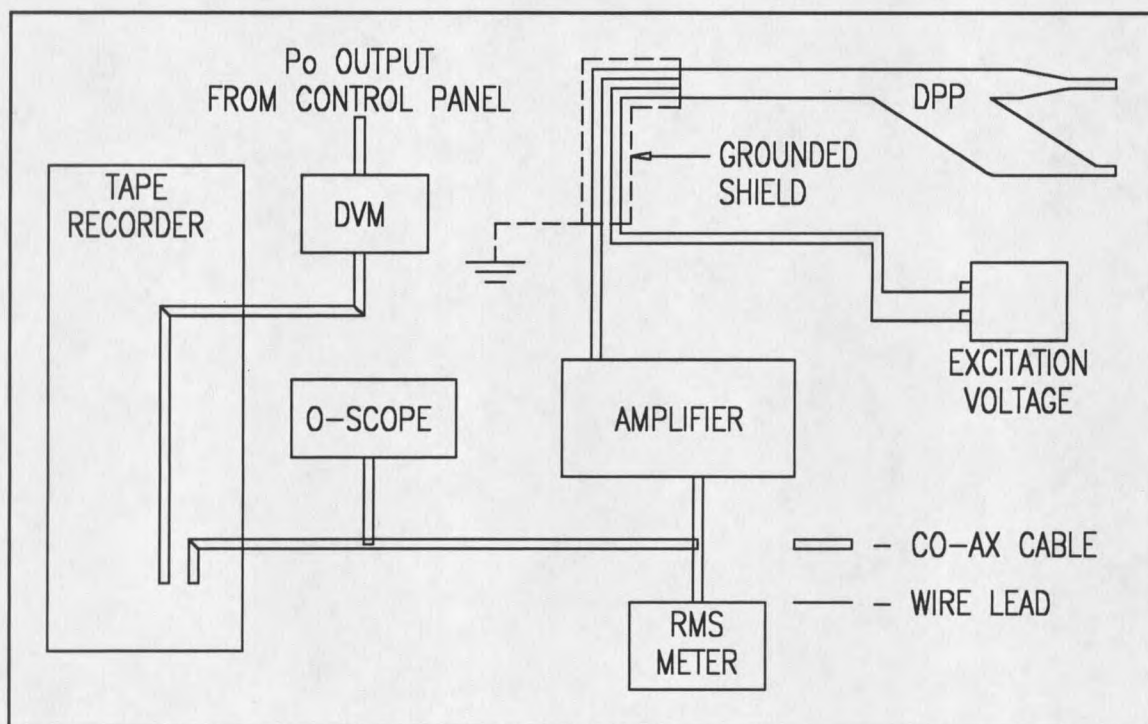


Figure 13.

Schematic diagram of data acquisition components and wiring.

A second data acquisition system was also used. This system was a Hewlett Packard digital data acquisition system (HP-DAS). The main components of the HP-DAS were a HP -3456A Digital Voltmeter, and a HP 3497A Data Acquisition/Control Unit. Both units were driven by a HP 9836 computer system. Due to the low digitization rate of this system, its primary use was to gather data for presentations purposes with limited applications for data reduction.

Data Reduction

The raw DPP data would provide only a qualitative evaluation of the pitot pressure fluctuations as a function of P_o and x . In order to obtain more useful information from the recorded data several data reduction schemes were employed.

In order to determine the onset of true turbulence, defined by high frequency fluctuations in the boundary layer, and to determine the possible existence of boundary layer instability, low frequency boundary layer fluctuations, the recorded P_o sweeps were examined under two different conditions. The first method was to examine the raw DPP vs. P_o signal including all frequencies. The second method was to examine the same data through a high pass filter allowing only frequencies greater than 150 kHz to pass*. It was felt that the 150 kHz cutoff would be high enough to eliminate any signal due to instability, but low enough to capture the first appearance of turbulent bursts. The filter used was a Krohn-Hite

* Unless otherwise stated this was the standard filtering scheme used throughout the investigation.

model 3202R dual channel filter.

In addition to the raw DPP data the frequency spectrum of the signal was examined as well as the signal in the time domain. The spectrum was obtained using a Rockland FFT SR/S Real Time Spectrum Analyzer, and the time domain was obtained using a Tektronix 2220, 60 MHz Digital Storage Oscilloscope.

Once the signal was reduced by the various methods, the resulting voltage outputs were recorded via the HP-DAS for conversion to useful quantities for evaluation and presentation.

EXPERIMENTAL SETUP

Recording Circuit Noise Measurements

During the initial phase of the investigation, the data recorded consisted of two parts. The primary data recorded was the raw DPP signal in the form of an AC voltage. The second type of data was the stagnation pressure which was recorded as a DC voltage with the calibration of one mV per torr. Both types of data were recorded simultaneously on two separate channels of the analog tape recorder.

During the course of the investigation, it was necessary to establish a minimum signal level from the DPP. The establishment of the minimum signal level was basically a measurement of the recording circuit noise level. The noise level was found to be approximately 41 mVrms, roughly 10% of the maximum signal level encountered in the boundary layer. This indicated that the recording circuit was sensitive enough for boundary layer measurements. However, during the course of measurements made in the freestream the signal level observed was identical to the previously measured noise level*. This result indicated that the recording circuit was not sensitive enough to measure fluctuations in the core flow of the tunnel.

* In all cases where the freestream signal level is mentioned, this signal level is equivalent to the recording circuit noise level.

Method of Probe Placement

One of the problems faced early in the experiment was finding a suitable method of positioning the probe in the boundary layer. For any type of accurate comparison to be made from all the data gathered, it was necessary to position the probe at roughly the same position in the boundary layer for all measurements. This problem was complicated by the fact that the probe, having a finite tip diameter, provided different resolutions with variations in the boundary layer thickness.

During the course of preliminary measurements, the DPP output was recorded while the probe was traversed vertically through the boundary layer. These traverses started from positions near the nozzle surface and continued out into the freestream. In all cases these traverses indicated that there was a definite signal peak occurring in the boundary layer some finite distance from the nozzle surface. It was decided that the location of the peak signal provided an acceptable means of positioning the probe in the boundary layer. This decision was based on the fact that the peak provided a method of repeatedly placing the probe in the boundary layer at nearly the same location over the entire nozzle length. The peak signal location also appeared to be the major point of interest in the boundary layer, indicating the region of maximum pressure fluctuations.

Since the DPP measured only P' , there was no attempt to accurately measure the boundary layer thickness with this probe. However, the y-profiles obtained with the probe did yield a qualitative feel for the boundary layer thickness

as well as a reasonable prediction as to the status of the boundary layer, i. e. laminar or turbulent. Shown in figure 14 are profiles obtained with the DPP indicating differences between laminar and turbulent profiles.

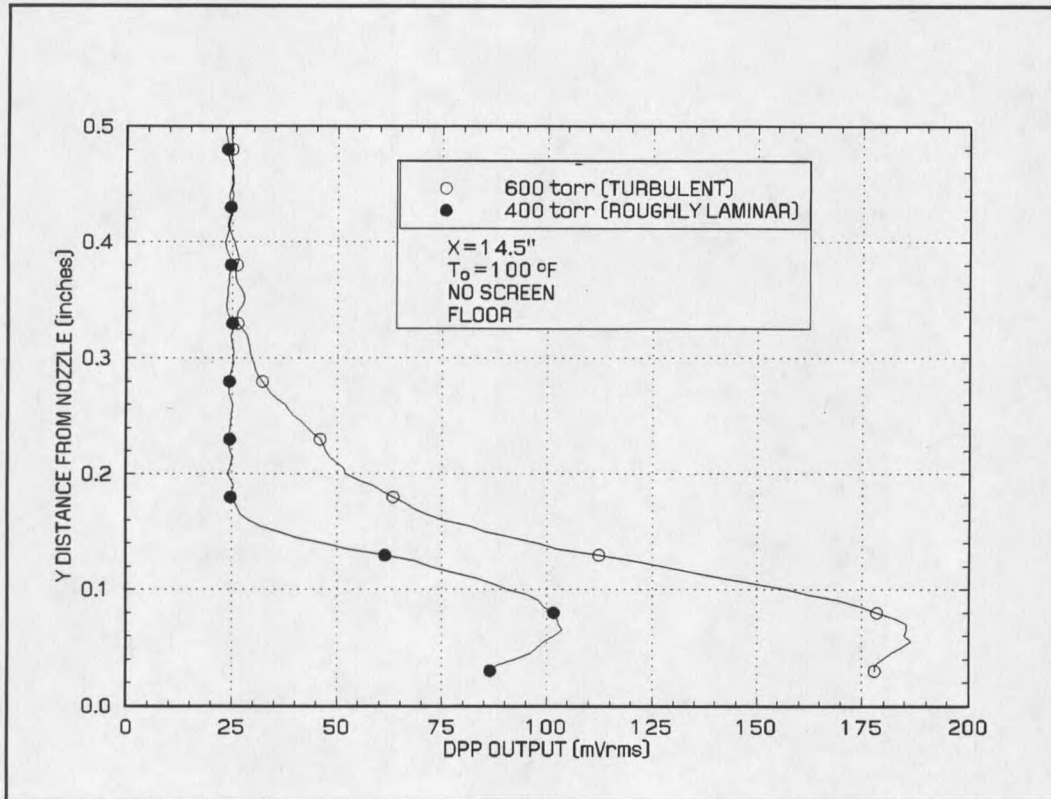


Figure 14.

Comparison of laminar and turbulent boundary layer profiles obtained with the DPP.

At low pressure (low Re'), the profile indicates a sharp, well defined signal peak around 0.06" with a sharp decrease in signal level to the freestream level occurring over a small y displacement. These features tend to indicate a laminar boundary layer. With increasing P_o , the profile retains the peak, but this feature becomes broader, and less pronounced. The return of the profile to the freestream

signal level occurs gradually over a larger y displacement with the final return to the freestream level occurring at a greater distance from the nozzle surface when compared to the trace taken at lower P_o . These features would generally be indicative of a turbulent boundary layer.

One point of interest noted in figure 14 was the existence of a peak in the 400 torr trace. In a completely laminar boundary layer zero signal level was expected across the entire boundary layer. However, in the 400 torr trace the DPP indicated fluctuations in the boundary layer. This signal was believed to be due to low frequency fluctuations which would indicate instability but not turbulence. In the absence of high frequency fluctuations a boundary layer is considered laminar even though low frequency fluctuations may exist.

It was during the course of this type of investigation that further problems with finding a suitable method for determining probe placement were encountered. While examining y -profiles at several values of P_o for a single x position, it was noted that there was a tendency for the peak to shift in the y direction. Under laminar boundary layer conditions, this peak shift was negligibly small to almost non-existent, posing no further problems to probe placement, but under turbulent conditions, there was the potential for a considerable shift. This can be seen in figure 15.

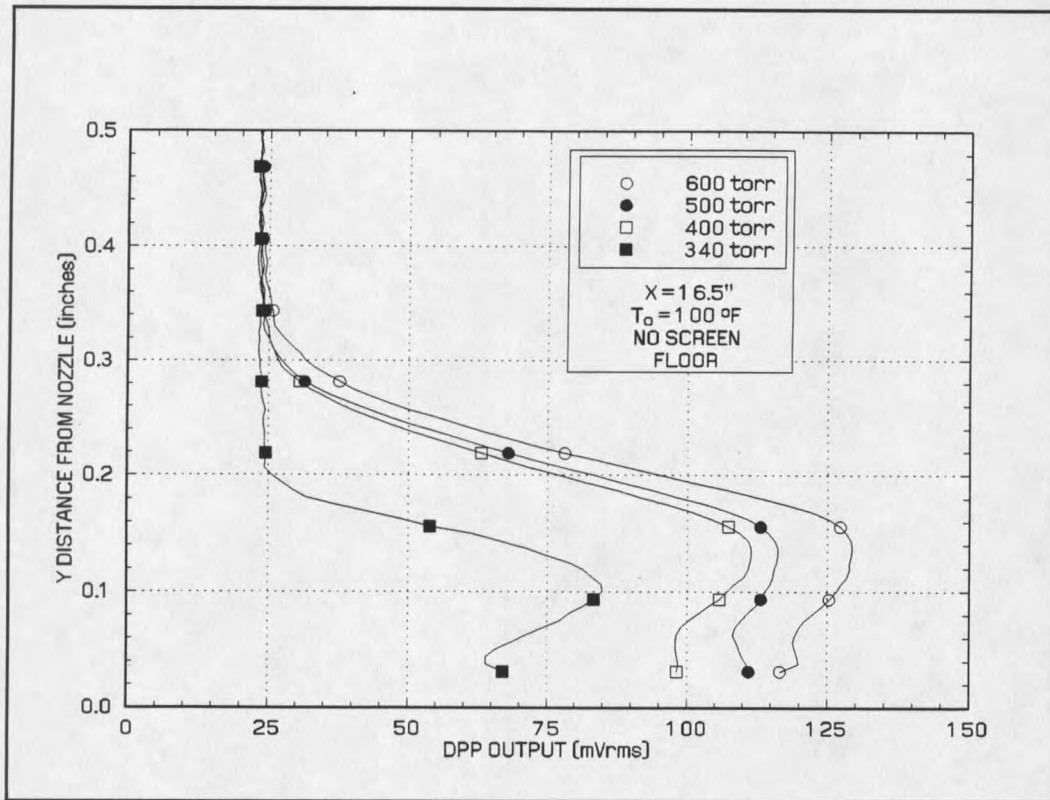


Figure 15.

Boundary layer profiles obtained with the DPP showing the peak signal level shifting with variation in stagnation pressure.

As can be seen in figure 15, the peak had a tendency to move outward from the nozzle surface with increasing P_o . It was observed under fully turbulent conditions that this shift could be of large enough magnitude for the probe to be directly at the peak at one pressure, and miss the peak entirely at another pressure. Under these circumstances, the only acceptable accommodation was to determine a y position that would remain at or near the peak over the intended range of P_o . With this method of probe placement, it was possible to obtain results with minimal skewing due to the shift in the peak location. Although the primary concern of the investigation was with events occurring in the boundary layer prior

to fully turbulent conditions, the fully turbulent boundary layer would still provide possible insight to occurrences in the transitional boundary layer.

Once the method of probe placement was resolved, the recording of the actual data was begun. The standard method was to obtain several y-profile traces at a fixed tunnel configuration but for varying pressures. The data acquisition circuit for obtaining the y-profiles can be seen in figure 16. From these profiles, the location for the probe was selected, and the DPP was positioned in the boundary layer at the desired location. P_o was then swept continuously from the maximum attainable pressure to the breakdown of supersonic flow while the DPP output and P_o were continuously recorded to analog tape. This procedure was then repeated for the entire test matrix as previously discussed. The nozzle length was covered from $x = 5.69$ " to 19.45 " in roughly 2 to 3 inch intervals with the probe positioned along the vertical center line of the nozzle.

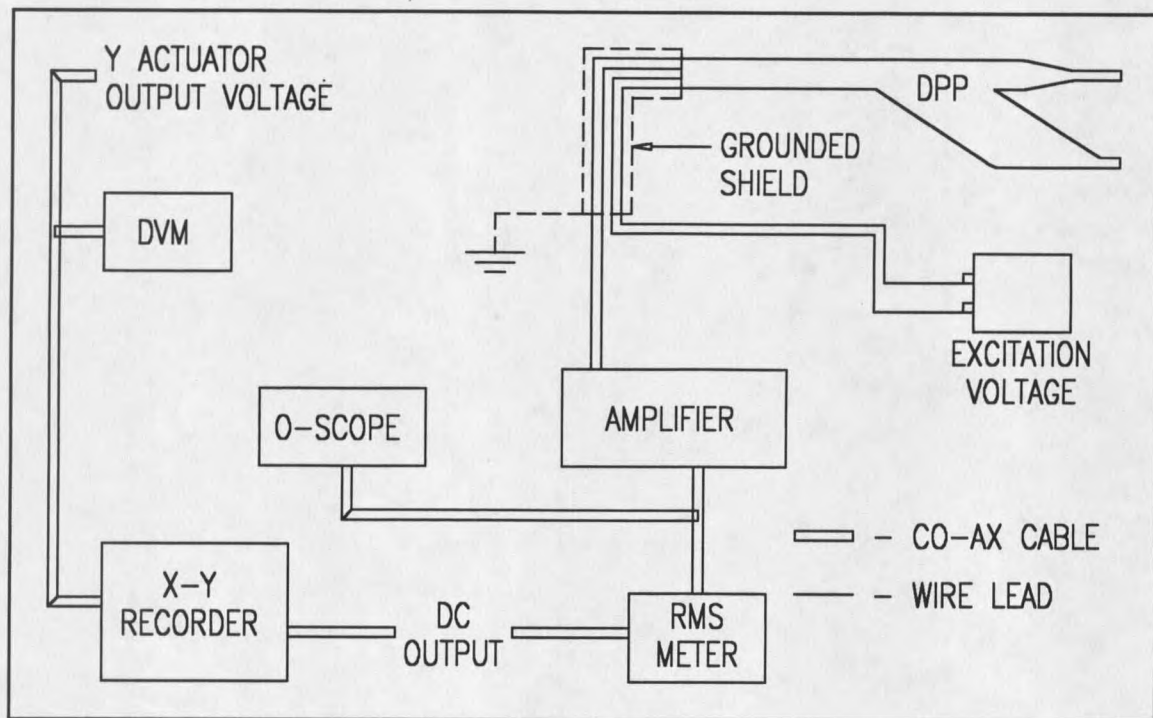


Figure 16.
Schematic diagram of y-profile recording components and wiring.

INITIAL DATA REDUCTION

Amplifier Noise Findings

One of the preliminary findings occurred during a comparison of freestream and laminar boundary layer frequency spectra. It was anticipated that the two spectra should be roughly identical indicating the recording noise spectrum of the data acquisition system. However, there appeared a strong signal peak in the neighborhood of 25 kHz. The peak was strong enough that it could possibly alter the spectra of the DPP data obtained in a boundary layer with a relatively low signal level. Initially the origin of the peak was not apparent and it was necessary to investigate the possible sources. For the following results it was necessary to clarify whether the peak was due to the boundary layer signal or noise in the recording circuit.

The investigation began by examining the DPP signal under several different tunnel conditions. The following is a listing of the configurations used:

- ▶ Normal recording circuit, DPP in free stream.
- ▶ Normal recording circuit, DPP in known laminar boundary layer.
- ▶ Same, DPP in free stream hidden behind a strut.
- ▶ Same, DPP in tunnel, tunnel not operating.
- ▶ Same, DPP outside tunnel, tunnel running.
- ▶ Same, DPP outside tunnel, tunnel not operating.
- ▶ Previous configurations repeated, but with the recording amplifier off.

The frequency spectra were examined for most of these cases, but after a cursory examination, it became clear that the source of the anomalous signal in the spectra was the recording amplifier.

Shown in figure 17 are several spectrum traces obtained from this examination. In all cases where the recording amplifier was operating the spectra show a distinct and significant peak in the 25 kHz region. There was no peak evident in the signal with the amplifier removed from the circuit.

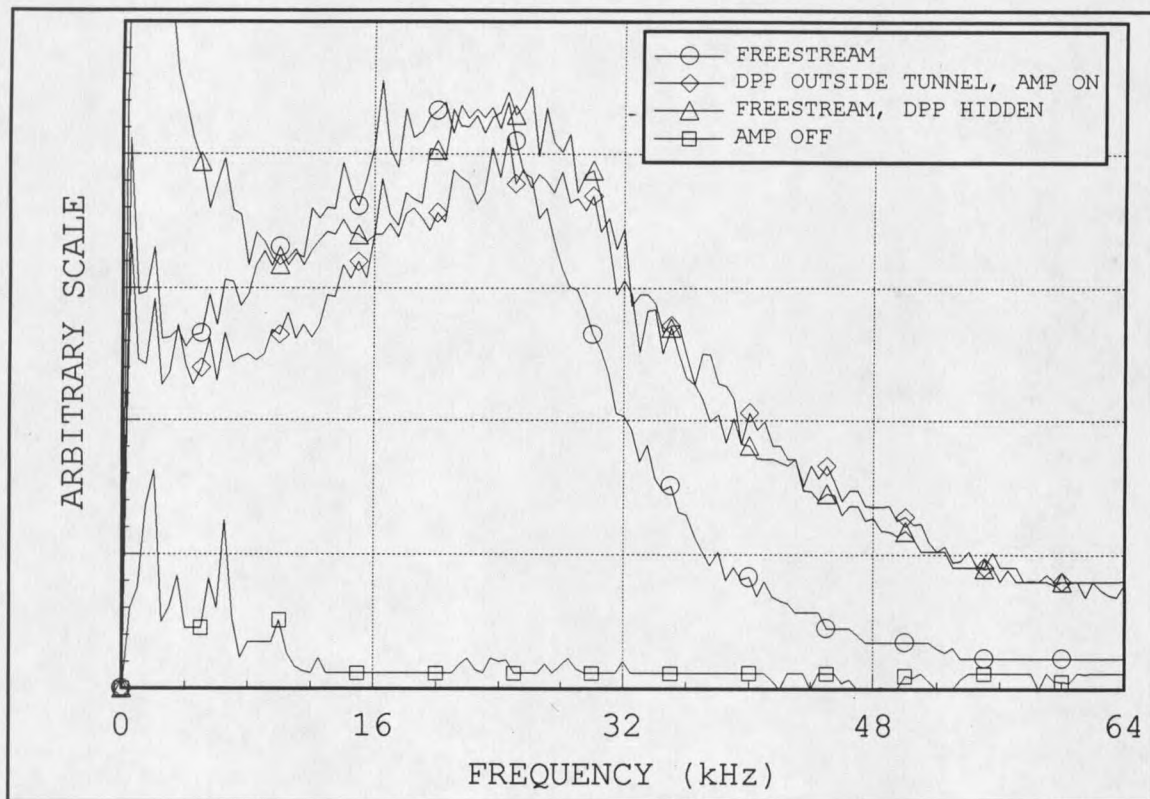


Figure 17.

Frequency spectrum results from the amplifier noise analysis.

Although unfortunate, it was necessary for the amplifier to remain in the recording circuit since this was the only means of boosting the signal to a suitable level for recording. No alternate amplifiers were available at the time. Under most circumstances, the peak was of sufficiently low amplitude compared to the signal from a transitional boundary layer that it posed little problem. However, under

laminar and pre-transitional boundary layer conditions it was possible for the peak to appear in the spectra or at least distort the results in the 25 kHz region.

Under laminar and pre-transitional conditions, it was necessary to identify and make note of the source of the exaggerated signal around 25 kHz. In addition, the spectra regions that were of greatest interest consisted of low frequencies, between 1 to 5 kHz, and high frequencies, roughly 150 kHz and above. The bulk of the amplifier noise peak remained outside of these critical regions.

DPP Versus Stagnation Pressure Results

Figures 18 to 25 show the results of plotting the DPP output, minus the freestream signal level, versus P_o on the floor of the SWT. Each trace on the graphs indicates a P_o sweep at fixed tunnel conditions for a single x position. Each graph shows the P_o sweeps at all x positions for a fixed tunnel configuration. In cases where the data was reduced through a high pass filter, the results provided a fair evaluation of the development of turbulence (high frequency fluctuations) in the boundary layer. The wide band results were more difficult to interpret due to the combined presence of high and low frequencies.

Figures 18 and 19 show the results from two similar tunnel configurations, the only difference being the value of T_o . As can be seen from the two figures, there was virtually no effect of temperature in altering the wide band P_o sweeps.

Figures 20 and 21 show the results of data gathered with screens in place.

In contrast to the data gathered without screens, there was a slight temperature effect noted in figures 20 and 21. Around 550 to 600 torr, the DPP output at low temperature tended to be higher than at high temperature. This result was reasonable given the fact that Re'_{∞} increases with decreasing temperature indicating that at low temperature transition should occur earlier than at high temperature.

Comparison of the results with screens and without screens indicated two points of interest. At positions near the throat, and at low values of P_o , the boundary layer appeared much quieter with screens. However, at higher pressure the DPP output with screens increased rapidly to a higher level than the output without screens. At positions toward the nozzle exit there was very little difference between the results both with and without screens.

Figures 22 to 25 show the same data as figures 18 to 21, but in this case the data has been reduced through a high pass filter. Comparison of figures 22 and 23 revealed a definite shift in the apparent onset of transition between high and low temperature results without screens. At $x = 14.75$ " the low temperature trace shows a significant amount of high frequency activity while the high temperature trace shows only moderate activity. For the results with screens, figures 24 and 25, the same trend was observed, but the trend was less evident due to the absence of a trace at 14.75 " for the case at low temperature.

With one exception there was no major difference between the results with screens and the results without screens. At high pressure, low temperature and

x positions near the throat, the results with screens indicated greater high frequency activity than the results without screens. This effect was less noticeable at high temperature.

The same process was repeated for all data recorded on the ceiling. The results for the ceiling data were nearly identical to the results presented from the floor. This result indicated that the progression of transition was roughly identical on the floor and ceiling of the SWT. Based on this result, the only surface considered during the bulk of the investigation was the floor.

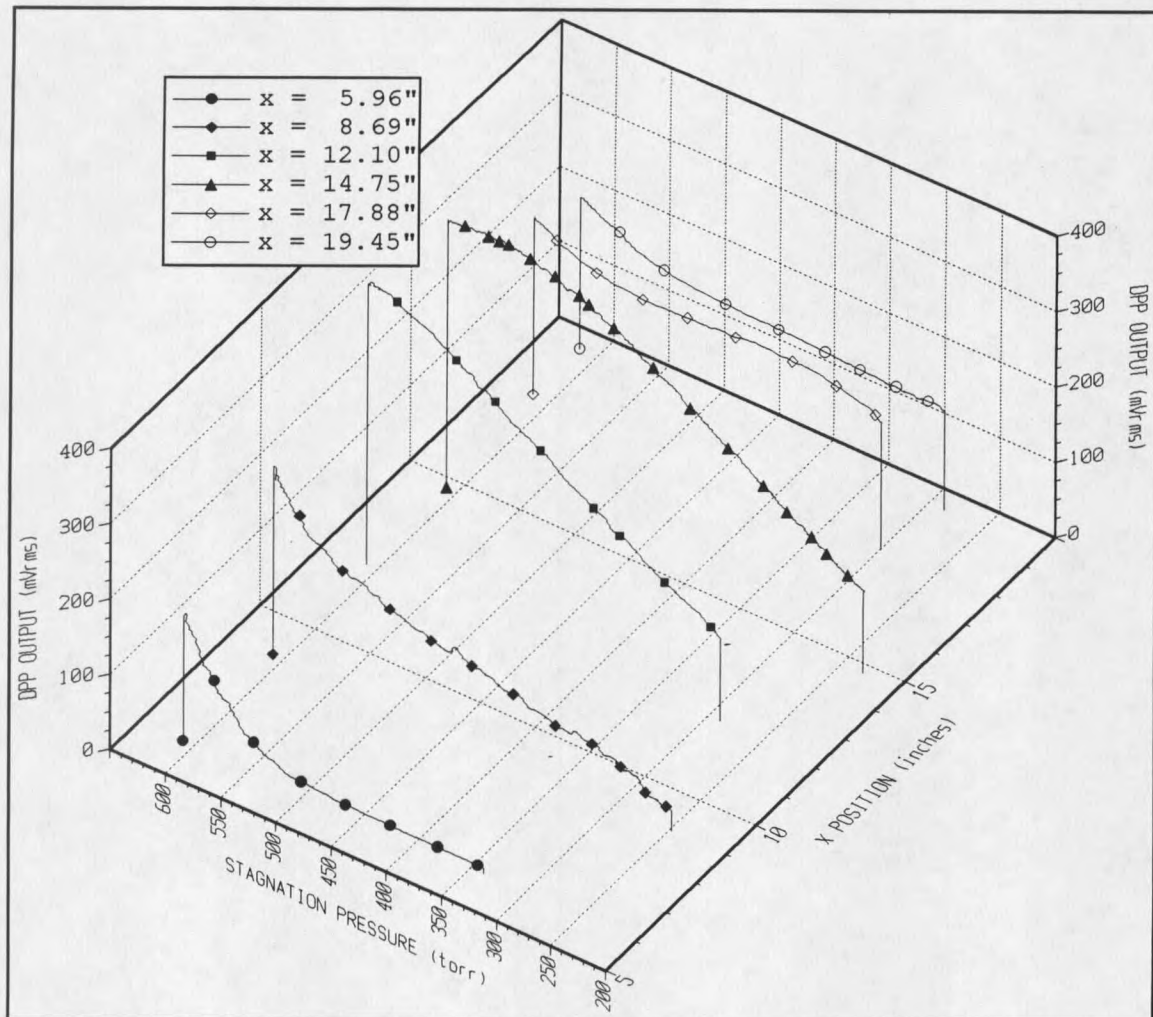


Figure 18.

DPP vs. P_0 plots for wide band, low temperature, no screen tunnel conditions.

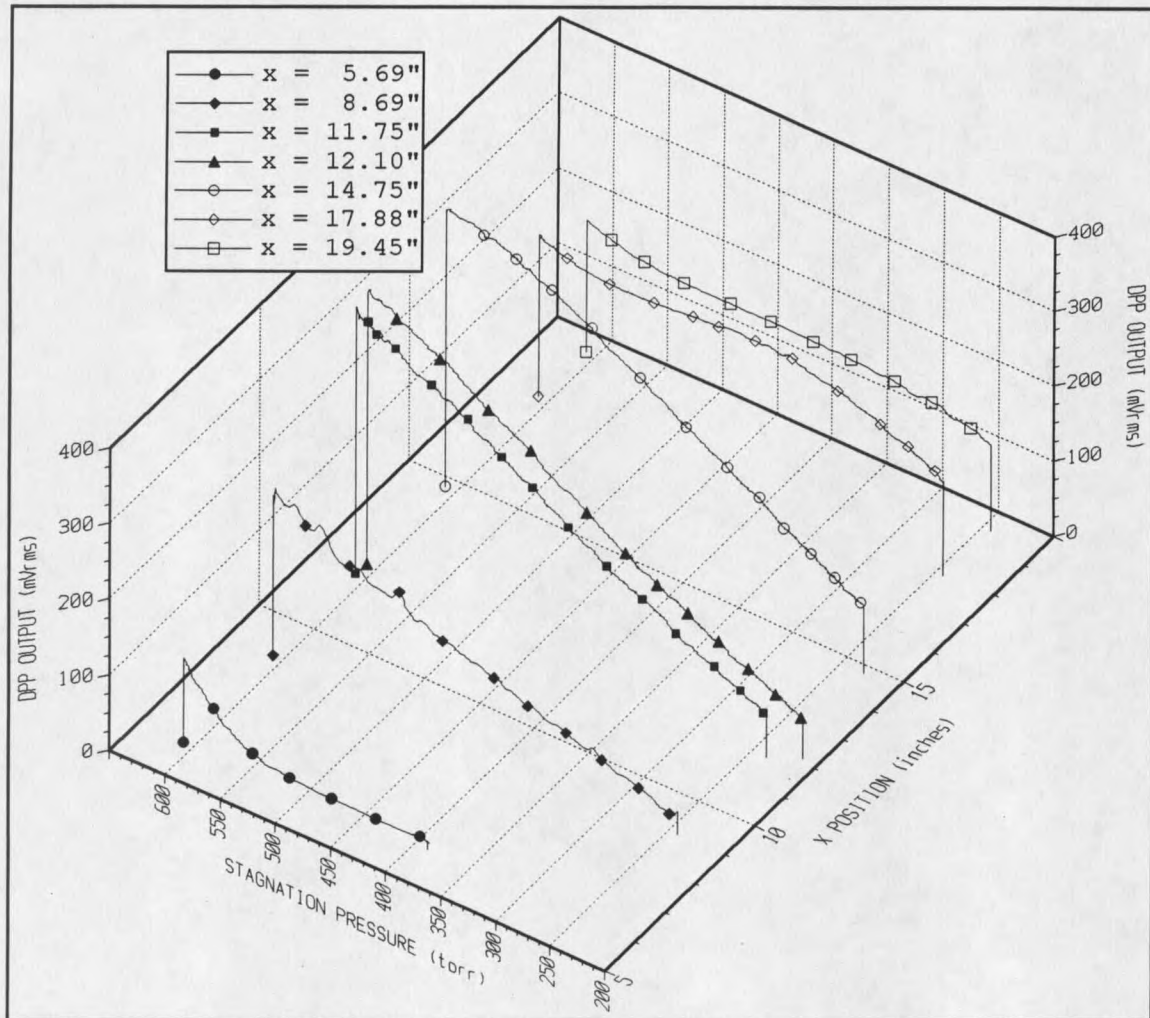


Figure 19.

DPP vs. P_o plots for wide band, high temperature, no screen tunnel conditions.

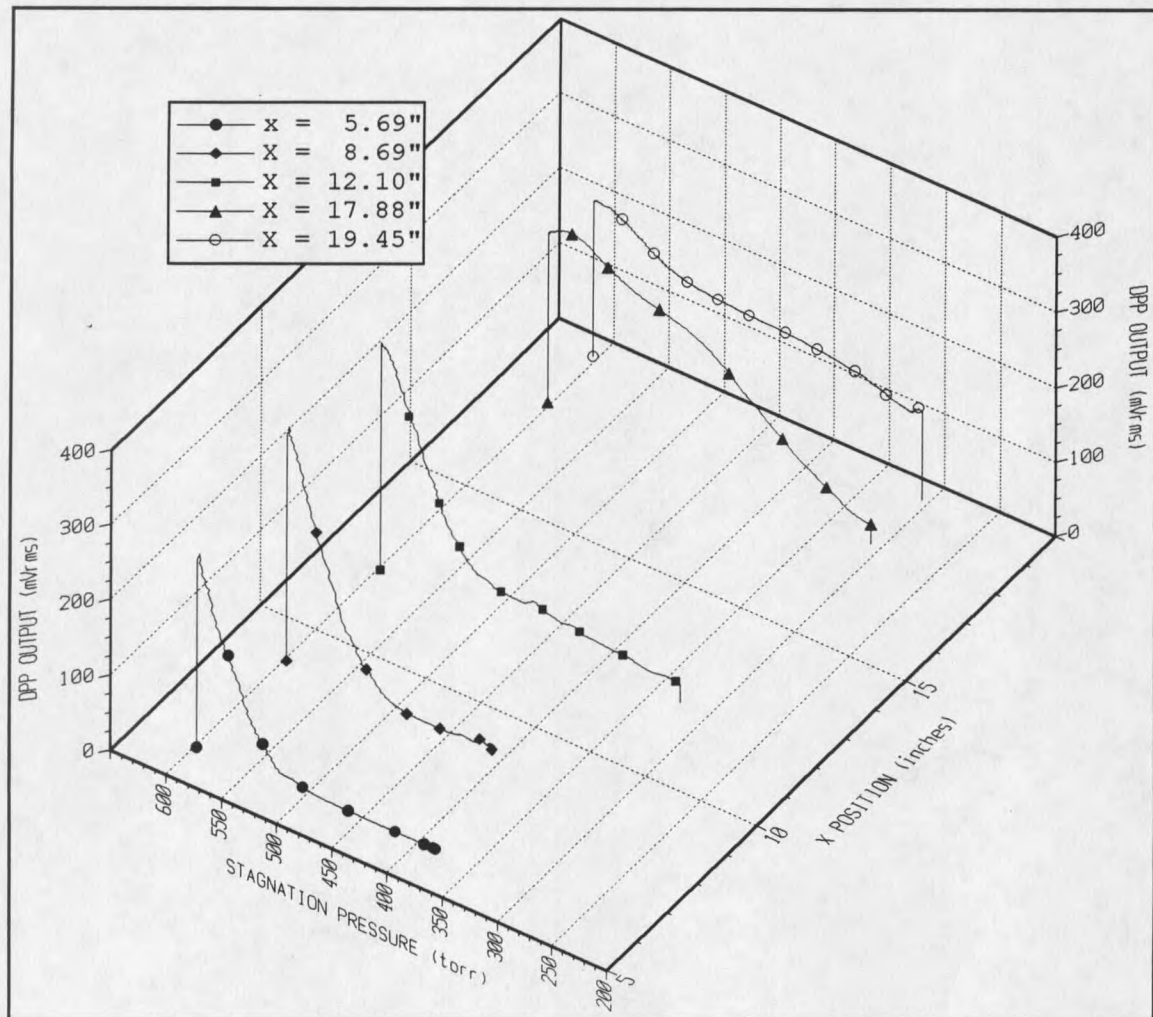


Figure 20.

DPP vs. P_0 plots for wide band, low temperature, with screen tunnel conditions.

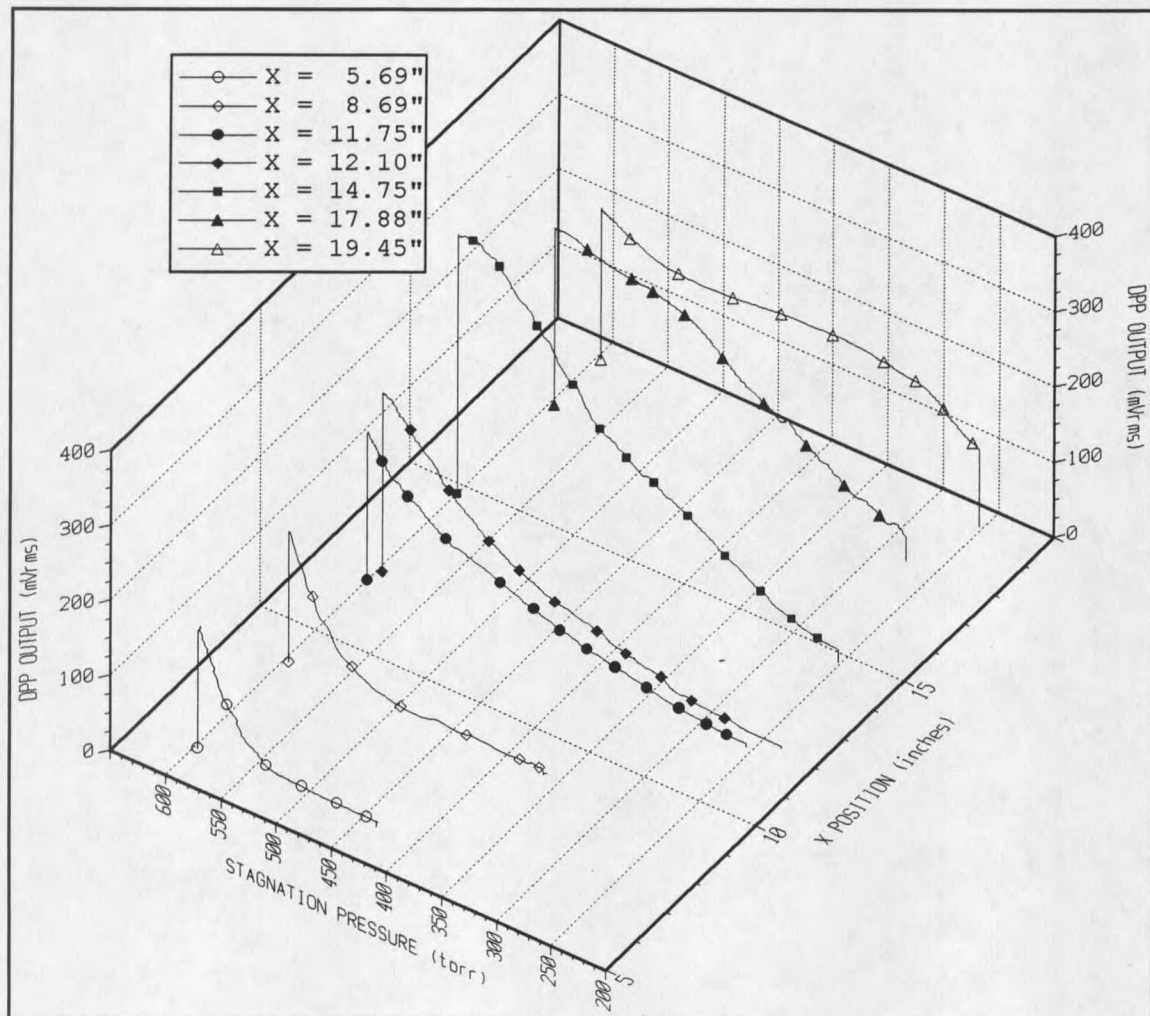


Figure 21.

DPP vs. P_0 plots for wide band, high temperature, with screen tunnel conditions.

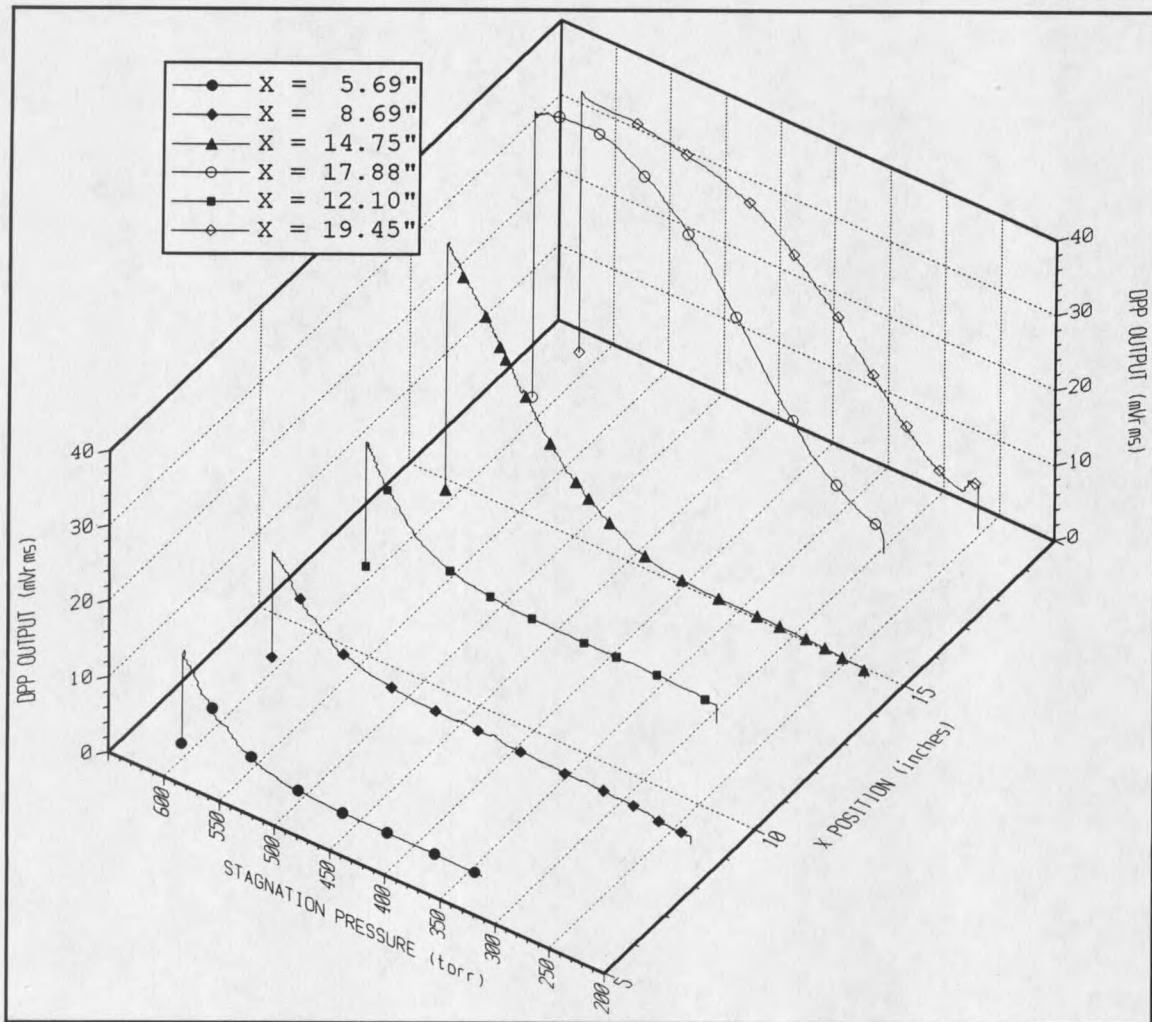


Figure 22.

DPP vs. P_0 plots for high pass, low temperature, no screen tunnel conditions.

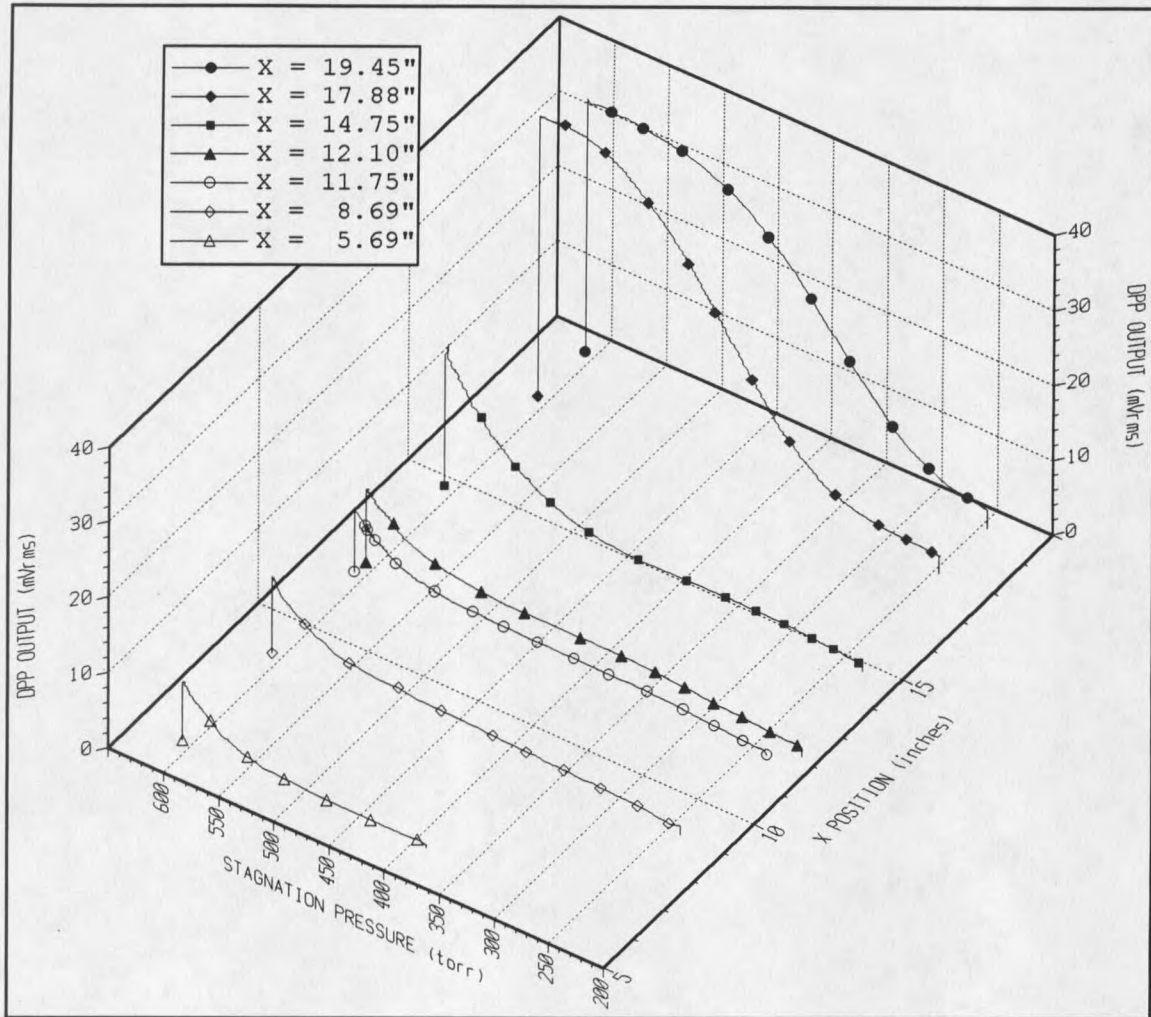


Figure 23.

DPP vs. P_o plots for high pass, high temperature, no screen tunnel conditions.

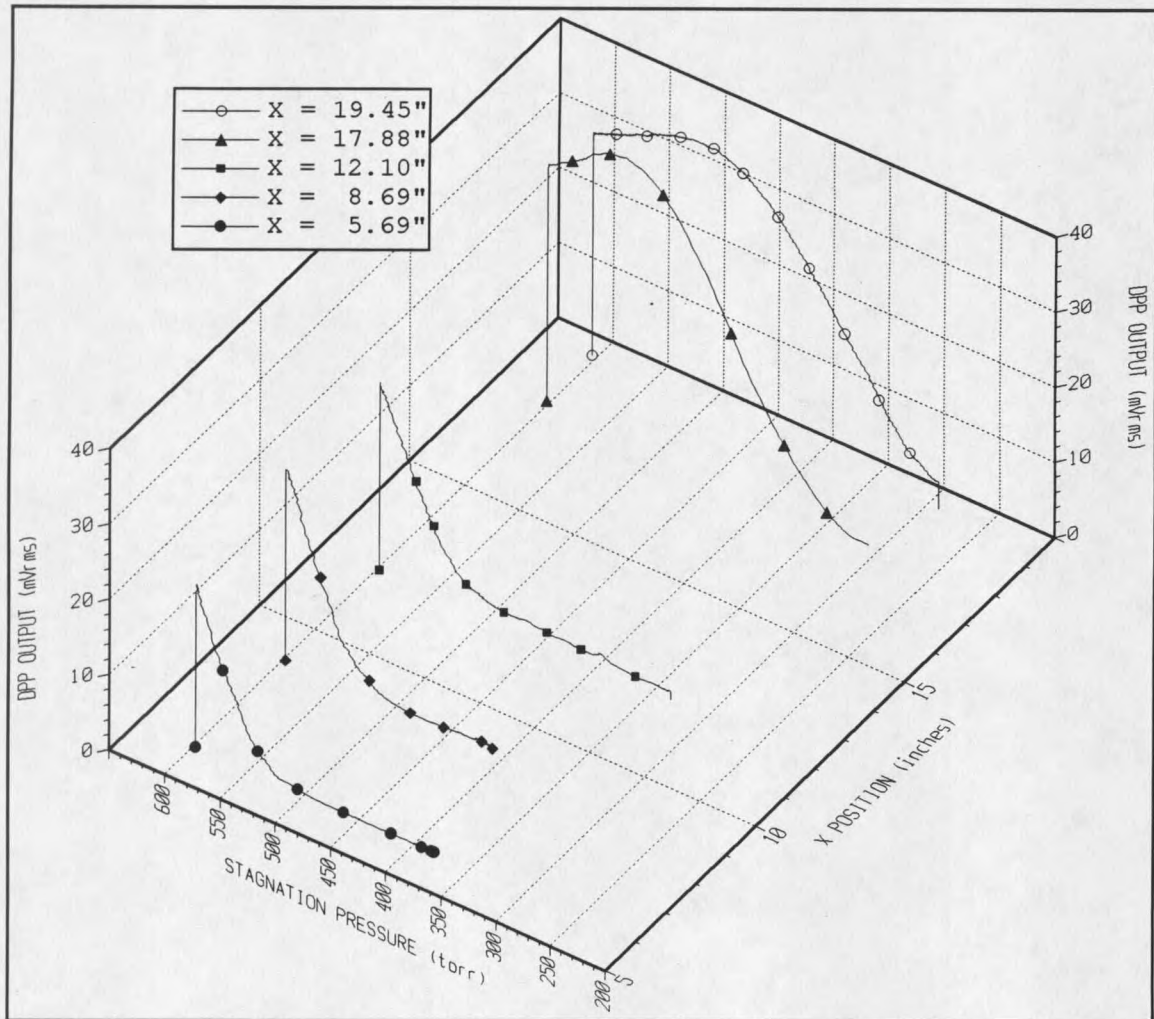


Figure 24.

DPP vs. P_0 plots for high pass, low temperature, with screen tunnel conditions.

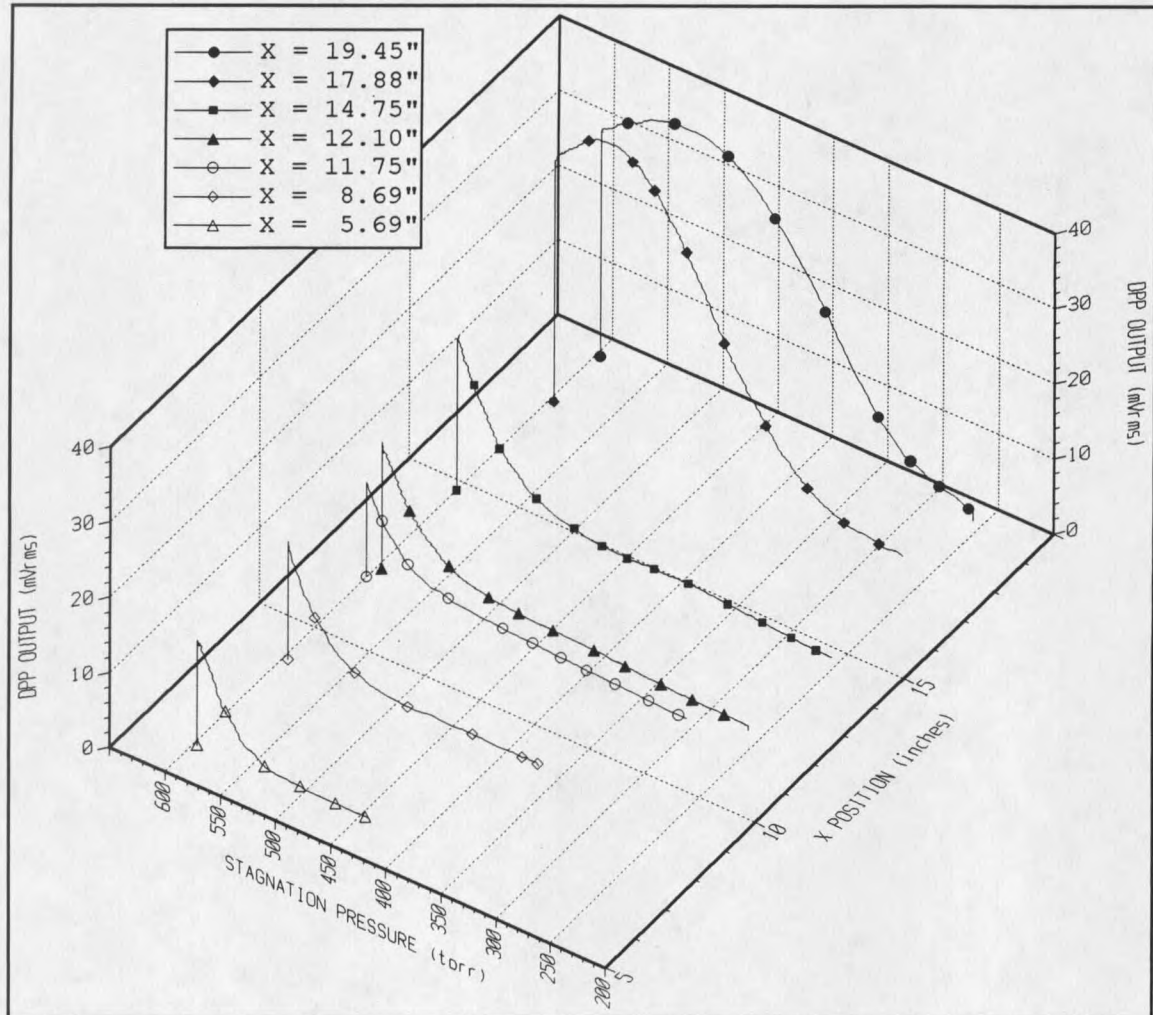


Figure 25.

DPP vs. P_0 plots for high pass, high temperature, with screen tunnel conditions.

Transition Maps

The results of the DPP vs. P_o sweeps provided the data used to generate the transition maps. The data was gathered in the following manner. First, for all traces where a signal "peak" was evident, a point was recorded by its x position and the value of P_o indicating the location of the peak. In the case of the wide band traces, the peaks implied a rather ambiguous signal maximum. However, the high pass peaks represented the maximum activity of high frequencies which was more readily correlated to the end point of transition.

Second, all points where the DPP signal first departed from the freestream signal level, or zero signal level, were recorded in the same manner as the peaks. Under wide band conditions the first departure from the freestream level indicated increased intensity of P' , however the nature of the increased signal remained unknown. The first departure in the high pass traces ideally signified the first appearance of turbulent bursts.

The third type of data gathered from the DPP vs. P_o traces were transitional points. For all traces which exhibited no signal peak or no return to the freestream level, the end points of the traces were recorded as transitional points. These points indicated neither a fully laminar nor fully turbulent boundary layer. They did however, indicate points where the boundary layer was known to be transitional and helped to define the regional boundaries on the maps being developed.

Once all of the data points were gathered, the first priority was to combine the high and low temperature data into a single data set. This was accomplished

by converting the scale from P_o to Re'_∞ which incorporated both P_o and T_o . The conversion was accomplished through the following relationships defined in [17].

$$\mu = 2.270 \frac{T^{\frac{3}{2}}}{T + 198.6} \times 10^{-8} \frac{lb \text{ sec}}{ft^2} \quad (2)$$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad (3)$$

$$\frac{Re_\infty}{l} = \frac{P_o M_\infty}{\mu_o} \sqrt{\frac{\gamma}{(\gamma - 1) C_v T_o}} \left(\frac{T_o}{T} \right)^{\frac{\gamma - 2}{\gamma - 1}} \frac{\frac{T}{T_o} + \frac{198.6}{T_o}}{1 + \frac{198.6}{T_o}} \quad (4)$$

Equation 2 defines the relationship between air viscosity and temperature based on Sutherland's formula. This approximation is valid for temperatures ranging from 180 to 3400 °R. Equation 3 relates the stagnation temperature to the local temperature based on the local Mach number. For the purposes of this investigation the test section Mach number, $M_\infty = 3.0$, was used. In a wind tunnel, if isentropic expansion is assumed from a total pressure of P_o and Eq. (2) is used for the variation of viscosity with temperature the Reynolds number per unit reference length is given by Eq. (4).

The value of C_v used was 4290 ft²/sec² °R which was consistent with the ideal gas approximation for the ratio of specific heats, $\gamma = 1.4$.

With known values for P_o and T_o it was then possible to convert all the data

points obtained from the DPP vs. P_o plots to points in the Re'_x domain. Once the transformation was complete it was possible to generate the maps shown in figures 26 and 27.

Detailed examination of figure 26 (no screens) indicated that the map could be roughly divided into four regions. The boundaries of the four regions are indicated by the solid lines. Zone I was observed to be an area where there was no measurable signal with the DPP. This zone was defined as "completely laminar" with little to no disturbance of the boundary layer.

Zone II defined a region where wide band activity was observed but with the absence of high frequency activity. This zone was defined as "laminar unstable" due to the absence of high frequencies (turbulence), but with activity at frequencies below 150 kHz.

Zone III exhibited activity in both the wide band and high pass signals. This region was defined as "unstable turbulent". The region exhibited turbulence based on the high frequency activity, while the low frequency signal approached a maximum.

Zone IV was defined as "fully turbulent". This was due to the fact that the high frequency activity had reached a maximum.

Maps were also created for the ceiling. These maps confirmed the earlier belief that the transition process progressed nearly identically on both the upper and lower nozzle surfaces.

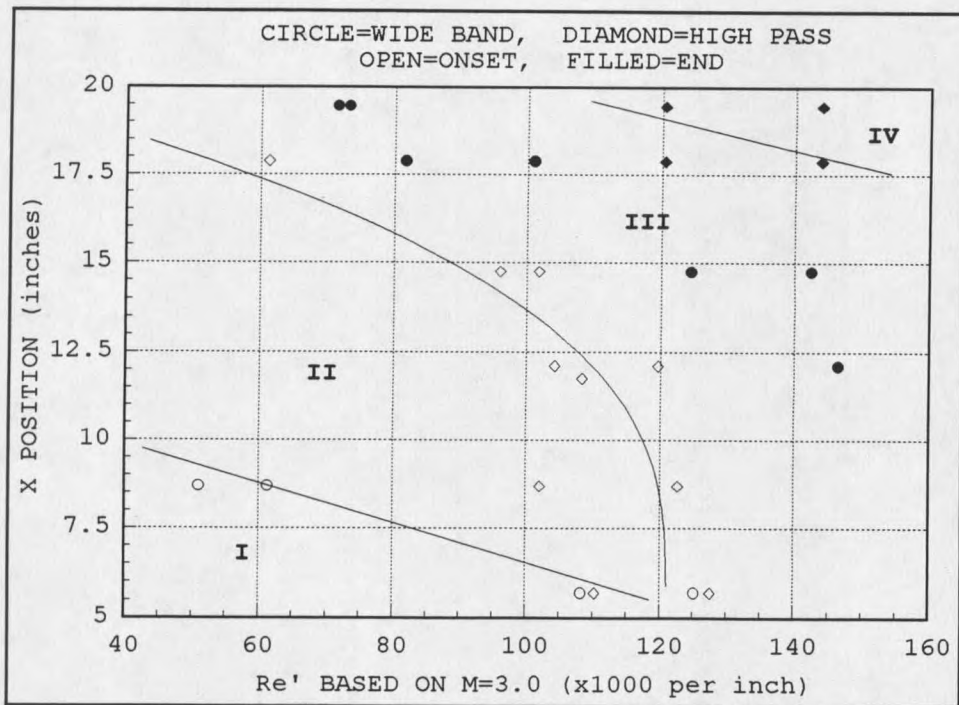


Figure 26.

Transition map on floor, no screen.

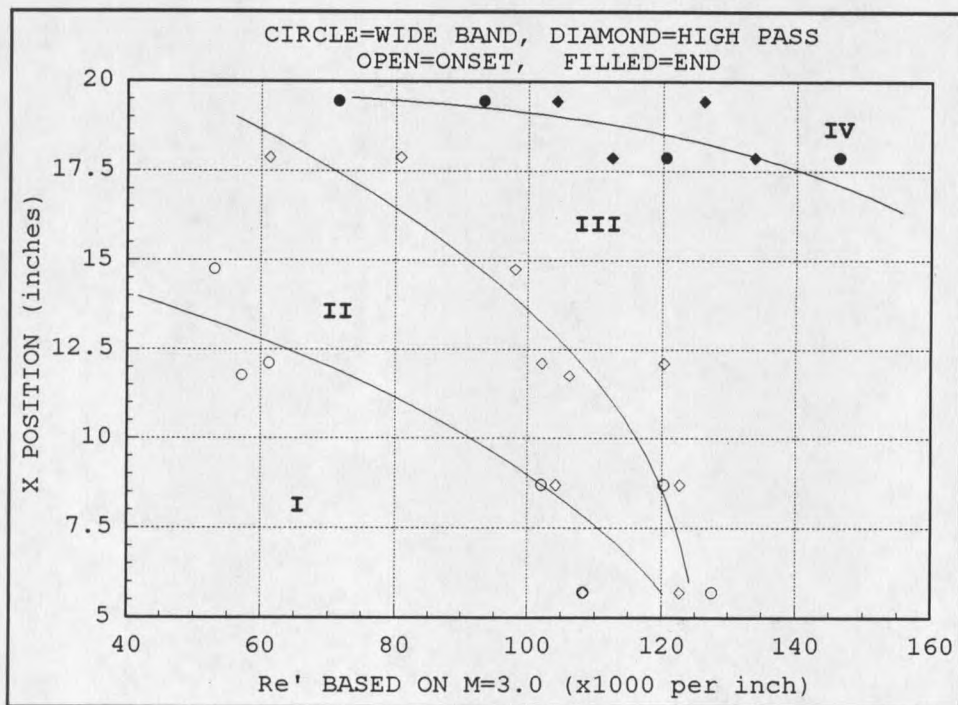


Figure 27.

Transition map on floor, with screen.

Figure 27 represents the same tunnel configuration as figure 26, but in the case of figure 27, the 320 mesh screen was installed in the tunnel. The primary difference observed between figures 26 and 27 was the apparent shifting of the line separating regions I and II. For the case with screens, the initial appearance of low frequency activity appeared to be delayed as compared to the case without screens. The net effect of the screens as seen in the maps was a compression of the unstable laminar region. Although the unstable region appeared compressed, the boundary defining the first appearance of turbulent bursts appeared in the same location as in the map without screens. These two results indicated that the screen inhibited the development of instability in the boundary layer, but also accelerated the development of turbulent bursts.

One final interesting feature was found on the transition map with screens. In the range $Re'_\infty = 100,000$ to $120,000$ per inch and an x range of 5 to 10 inches the region defining the first appearance of turbulent bursts appeared to coincide with the beginning of the unstable region. This observation indicated that turbulent bursts were occurring with little or no low frequency activity.

After consideration of the maps, an explanation was sought for the unusual results apparent in the map with screens. The investigation consisted mainly of examining the frequency spectra and time base wave forms from the data obtained in the region of interest. In addition, data was examined directly on an oscilloscope during tunnel operation while repeating y -traverses in the region of interest.

Investigation of Accelerated Burst Development

As was suspected, it was found that the phenomenon of turbulent bursts in the absence of low frequency activity was yet another problem with the placement of the probe in the boundary layer. For all cases in the recorded data where turbulent bursts appeared without any low frequency activity, it was found that there was indeed a low frequency wave form present. The absence of the low frequency wave forms in the recorded data was merely a consequence of poor probe placement in the boundary layer.

Upon repeating y-traverses for several of the test matrix configurations and observing the DPP output directly on an oscilloscope, the following observation was made. It is possible to position the probe in the boundary layer in such a manner as to observe bursts while completely missing the underlying low frequency waves. Only by lowering the probe further into the boundary layer would the low frequency activity become apparent. The y displacement required to move out of the low frequency region into the bursts only region was found to be as little as 10 to 25 thousands of an inch. This displacement was small enough to maintain probe placement at the signal peak of a y-traverse which explained why the error was not previously detected. Figure 28 shows a rough schematic of how the error occurs.

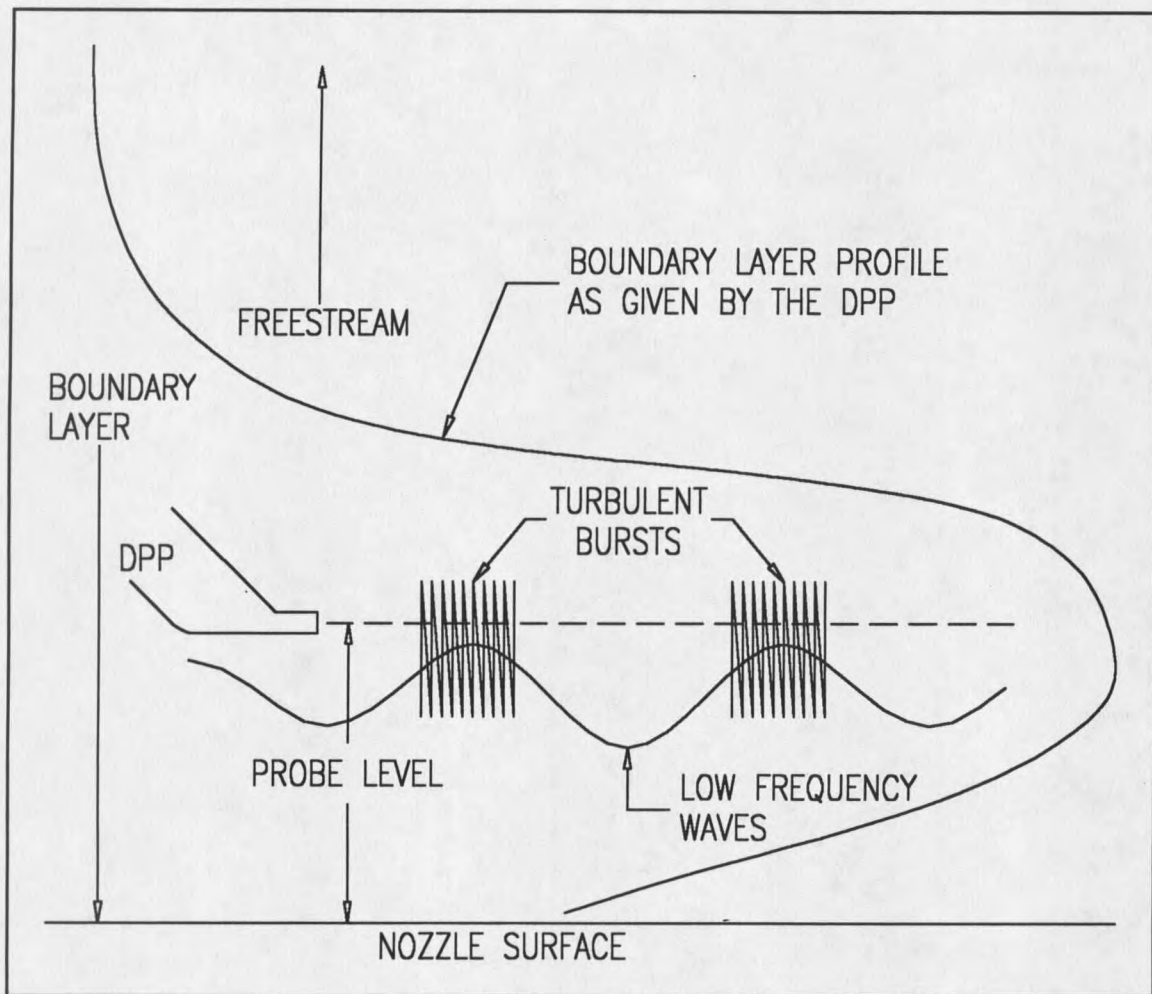


Figure 28.

Diagram of poor probe placement leading to the observation of turbulent bursts in the absence of low frequency activity.

Upon further examination of the recorded data, it was found that the improper probe placement was not limited to the data with screens in place. Although the majority of the erroneous data was recorded with the screens, some data recorded without screens also exhibited a similar phenomenon. Improper probe placement tended to occur mainly at x positions near the throat where the boundary layer was thinner, making the effect more pronounced.

Although this represented a rather serious error in the data acquisition procedures, several factors combined to prohibit the re-gathering of the data. An initial attempt was made to acquire the additional data that would play a key role in the stability analysis. It was during the initial phases of the data acquisition that both the primary and secondary DPP failed. Due to the amount of time required for a replacement DPP to arrive and existing time constraints it was not possible to acquire further data. The existing data eventually proved sufficient to complete the investigation, but it was not possible to conduct a boundary layer stability analysis.

X-WISE TRANSITION SURVEY

After completing and examining the transition maps from the first portion of the investigation the next step was to begin the x-wise transition survey. In this portion of the investigation the transition process was to be considered as a function of x at constant stagnation pressure. Flow both with and without screens was considered. The tunnel surface considered was the floor, and T_o was maintained at 160 °F.

Two values of P_o were selected for the investigation. These values were chosen from the transition maps as pressures that, over the x range considered, would cover all four of the transition regions. The high pressure, selected to cover events in the transitional to turbulent areas, was 600 torr. The low pressure, selected to cover laminar to transitional events, was 350 torr.

This portion of the investigation required no new data gathering, but instead utilized existing data stored on tape. This reduced the investigation to a matter of finding the DPP vs. P_o sweep corresponding to the desired x position and tunnel configuration, then examining the recorded data of that trace at 350 and 600 torr.

In order to obtain a useful amount of data from the tape it was necessary to examine the data at a reduced playback speed. This was due to the fact that the data was originally recorded as a continuous P_o sweep at a recording speed of 120 inches per second (ips). Normal playback speed, 120 ips, would yield only a few seconds of data at a single value of P_o . This time allotment was insufficient for data reduction purposes. Therefore, a playback speed of $3\frac{3}{4}$ ips was used for

this portion of the investigation. Although completion of some spectrum analyses still required more time than the playback speed reduction afforded, the variation of P_o during this interval did not exceed 1 to 5 torr.

One fortunate characteristic of the tape recorder was that there was little or no variation in the playback output from one playback speed to another. This condition was aided further by the fact that analysis of the data would be qualitative only.

DPP Output at Constant Pressure Results

The first means of examining the data was to plot the DPP output voltage at constant pressure versus x position. The P_o sweeps under the desired tunnel configuration were examined and for each x position available, the DPP signal level was recorded at specific pressures. The data at a specific pressure was then plotted vs. x position yielding curves of DPP signal level vs. x position at constant pressure. Figures 29 and 30 present the results for the wide band DPP output. Figures 31 and 32 present the high pass results.

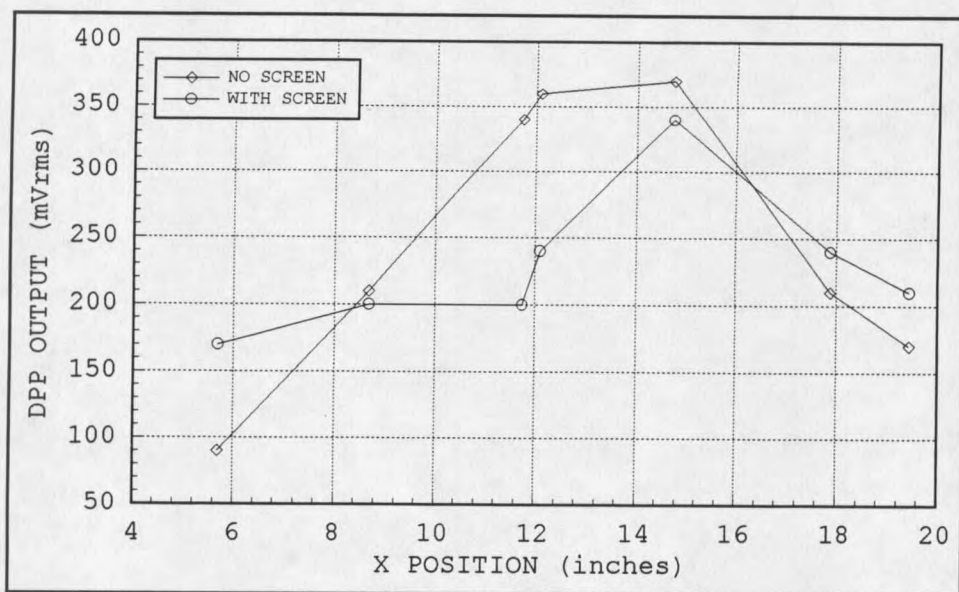


Figure 29.

DPP output vs. x position at 600 torr, wide band results.

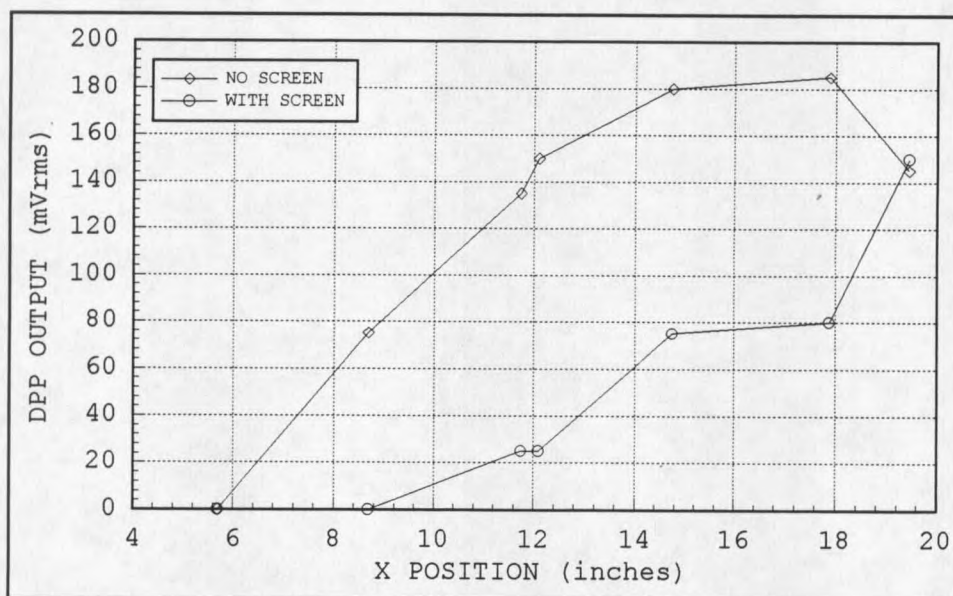


Figure 30.

DPP output vs. x position at 350 torr, wide band conditions.

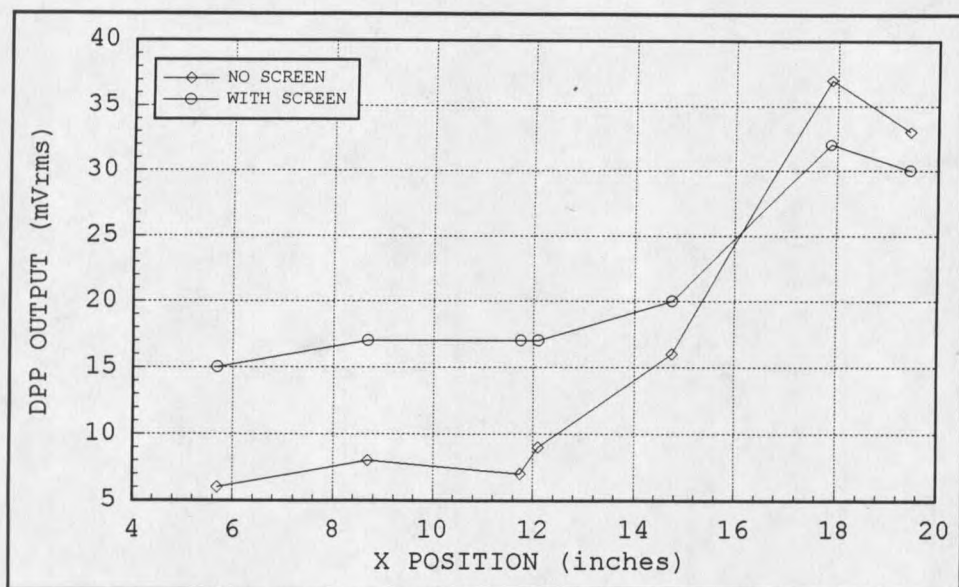


Figure 31.

DPP output vs. x position at 600 torr, high pass conditions.

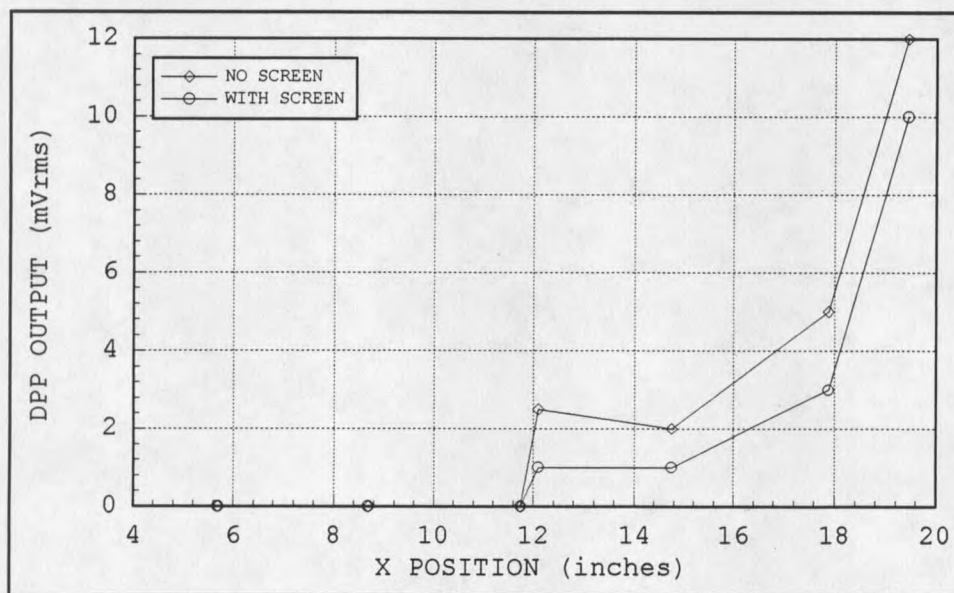


Figure 32.

DPP output vs. x position at 350 torr, high pass conditions.

At 600 torr, wide band conditions, the results with screens and without screens were similar. In both cases the DPP output showed an increasing trend with increasing pressure until the x position of 14.75". At 14.75" the signal level began to decrease. The only difference in the results occurred at the initial x positions. The signal with screens at $x = 5.69$ " was higher than the signal without screens, and increased at a more gradual rate.

At 350 torr, wide band conditions, there was a more notable difference. The results with screens indicated a gradual increase in signal level over the entire range of x positions. Without screens the signal rose sharply until 14.75". At 14.75" the signal leveled off then began to decrease at 17.88".

At 600 torr, high pass conditions, the results were similar for the case of screen versus no screen. Both signal levels remained fairly constant from 5.69" to 11.75" with the only difference being the magnitude of the signal. With screens the signal level was initially lower. At 12.10" the signal level showed a gradual increase for the case without screens and a sharp increase for the case with screens. In both cases the signal increase reached a maximum around 17.88" and decreased from 17.88" to 19.45".

The screen and no screen results at 350 torr, high pass conditions, were nearly identical. The signal level remained zero until 11.75". From 11.75" to the final x position both sets of data indicated identical increasing trends.

Frequency Spectra Results

Two different frequency ranges were used for the presentation of results; the ranges were 0 to 6.4 kHz and 0 to 160 kHz. Each spectra graph presents the spectrum traces at all x positions for a given pressure and frequency range. Each trace on the graphs corresponds to the spectrum at a single x position for the given conditions. In this manner it is possible to observe the development of the frequency spectra from one x position to the next.

The vertical scale of the graphs is not consistent at each pressure and frequency range. However, the scale is identical at a single pressure and frequency range for graphs both with and without screens. In all cases the amplifier noise spectrum was subtracted from the boundary layer spectrum.

Figures 33 and 34 present the low frequency results at 600 torr. For the case with no screen, figure 33, the spectra exhibited an increasing trend with increasing x . Starting at $x = 5.69''$, all frequencies increased in amplitude until the position $x = 14.75''$. The frequency range from 0 to 3 kHz showed the most growth. Between 14.75" and 17.88" the entire range of frequencies decreased sharply in amplitude then remained unchanged at 19.45".

In figure 34 (with screens) the spectra showed a nearly identical trend as in figure 33. The only difference was the magnitude of the increase in amplitude. The spectra for the case with screens exhibited only slight increases and decreases in amplitude.

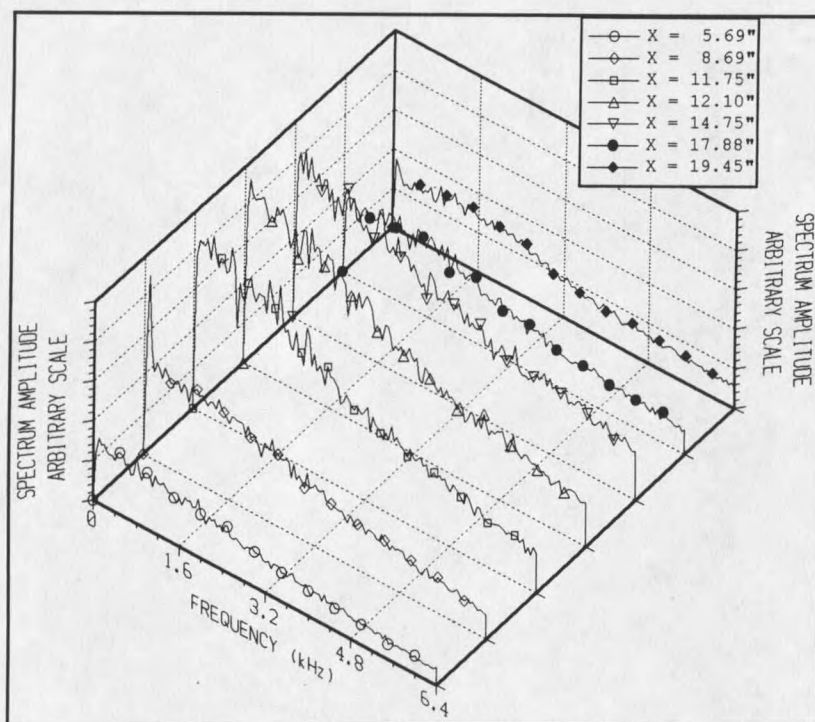


Figure 33.

Frequency spectra at 600 torr, no screen, low range.

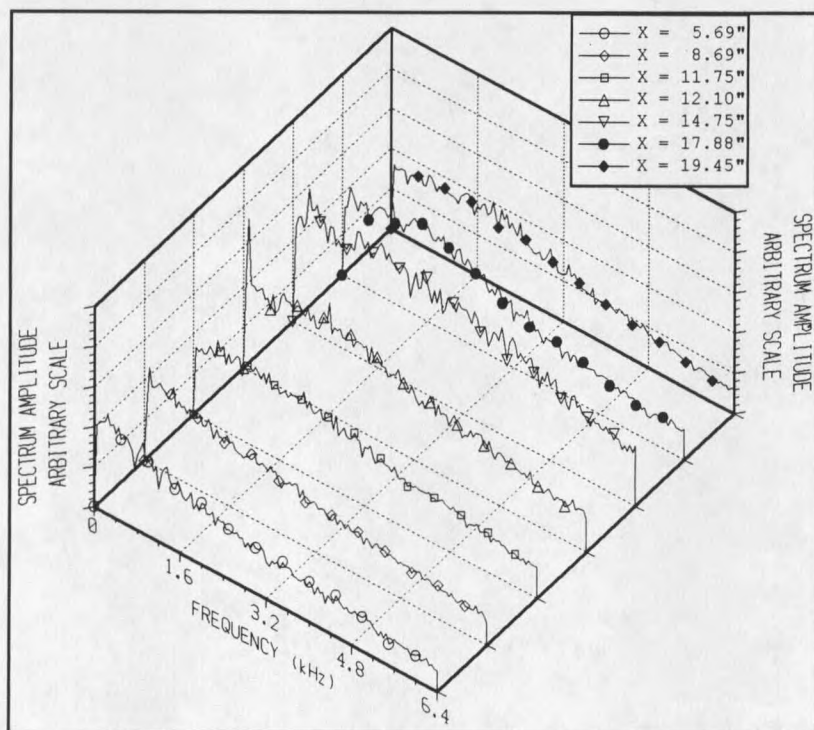


Figure 34.

Frequency spectra at 600 torr, with screen, low range.

Figures 35 and 36 present the results for the high frequency range at 600 torr. Without screens the spectra showed a slight increasing trend at all frequencies from $x = 5.69''$ to $12.10''$. At $14.75''$ there was a sharp increase in the amplitude of all frequencies. At $17.88''$ there was another increase in the range 20 to 80 kHz. At $19.45''$ there was a large decrease in the amplitude of frequencies between 80 to 160 kHz.

With screens the spectra exhibited similar behavior with two minor exceptions. At $5.69''$ the amplitude of frequencies in the range 40 to 160 kHz was slightly higher with screens. At $19.45''$ the signal decrease noted without screens was not present in the results with screens.

Figures 37 and 38 present the results for the low frequency range at 350 torr. Without screens the spectra showed a steadily increasing trend at all frequencies from $x = 5.69''$ to $17.88''$. The most rapid increases occurred in the range 0 to 3.2 kHz. At $19.45''$ there was a slight decrease in the amplitude of all frequencies.

With screens the spectra showed near zero signal levels at $5.69''$ and $8.69''$. At $11.75''$ there was limited activity in the range 0 to 3.2 kHz with the spectrum remaining unchanged at $12.10''$. The first notable increase in amplitude occurred at $14.75''$ with increases over the entire range of frequencies. At $17.88''$ there was a slight decrease in the range 0 to 1.6 kHz producing a nearly flat spectrum over the range 0 to 6.4 kHz. At $19.45''$ the spectrum remained flat, but at a slightly increased level.

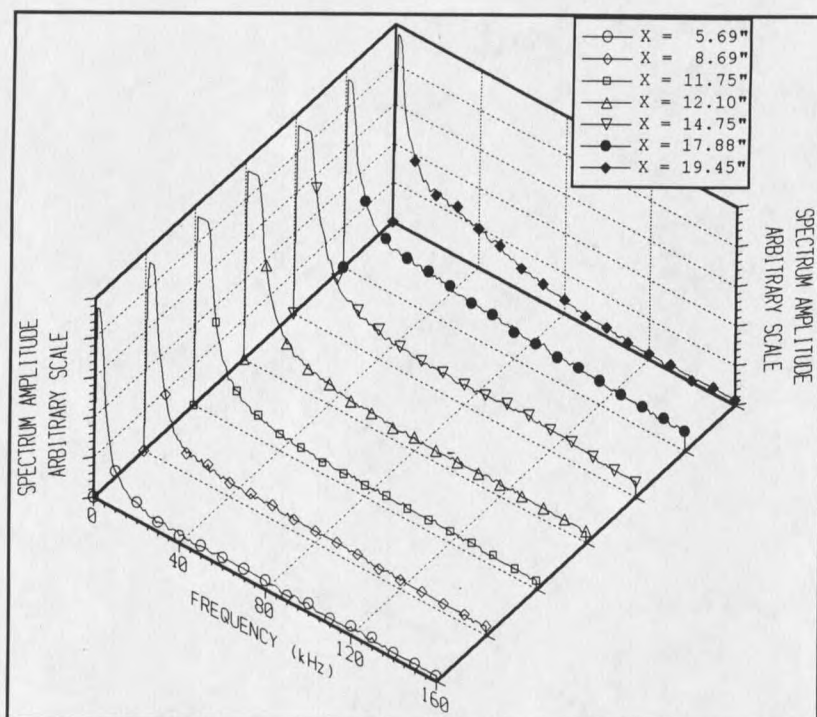


Figure 35.

Frequency spectra at 600 torr, no screen, high range.

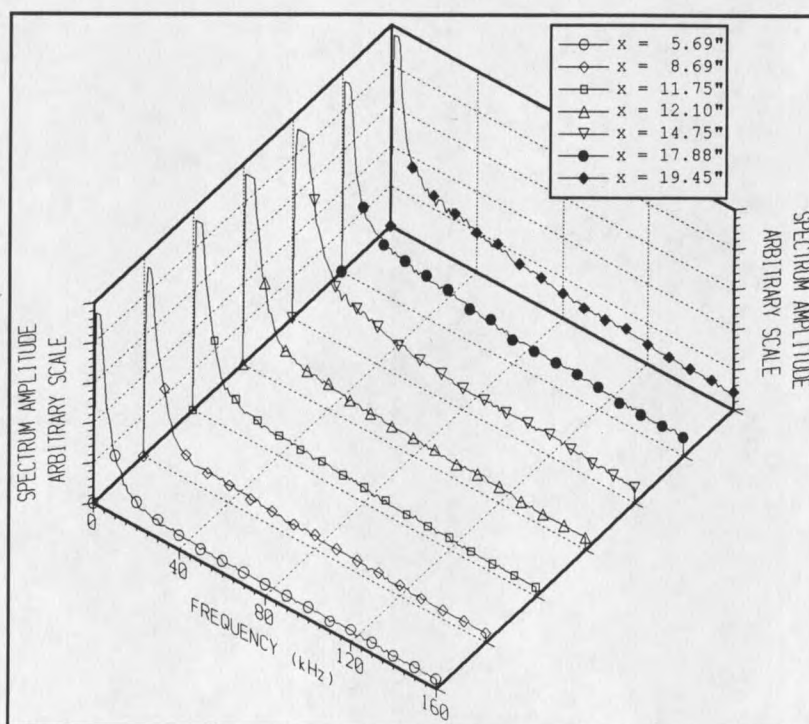


Figure 36.

Frequency spectra at 600 torr, with screen, high range.

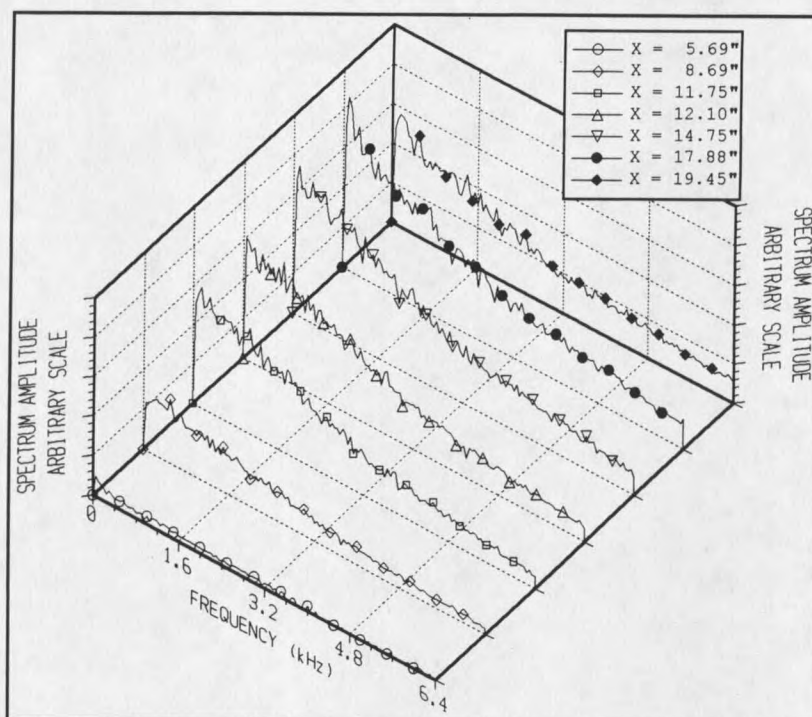


Figure 37.

Frequency spectra at 350 torr, no screen, low range.

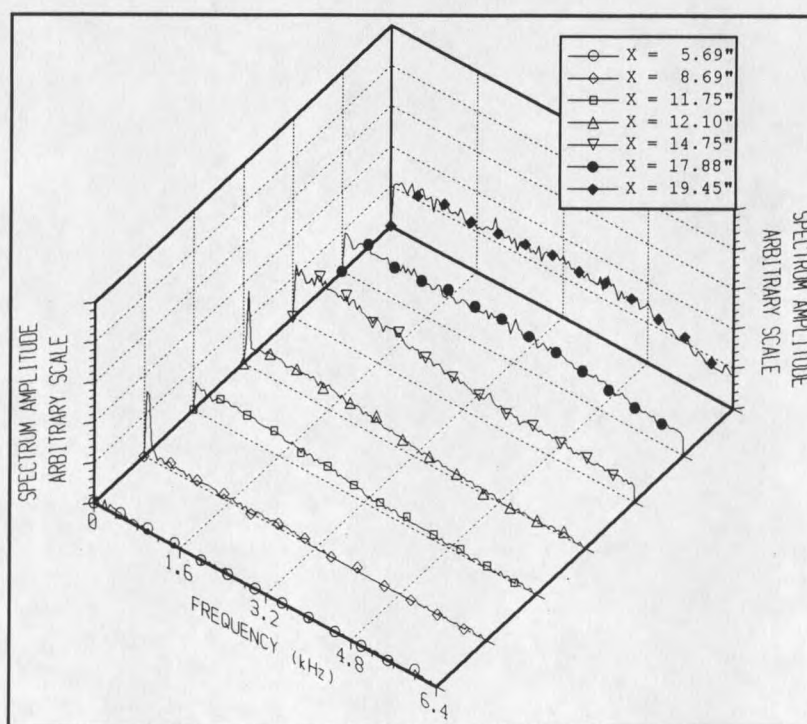


Figure 38.

Frequency spectra at 350 torr, with screen, low range.

Figures 39 and 40 present the results of the high frequency range at 350 torr. Without screens there was virtually no high frequency activity until 17.88". At 17.88" the spectrum without screens showed a significant increase in the 0 to 40 kHz range with additional increases in the 40 to 160 kHz range. At 19.45" there was a large increase in the 20 to 60 kHz range with a slight decrease in the 80 to 160 kHz range.

With screens the spectra showed no high frequency activity until 17.88". At 17.88" there was a sharp increase in the 20 to 60 kHz range with frequencies above 80 kHz remaining inactive. At 19.45" there was another major increase in the 20 to 60 kHz range with slight increases in the 80 to 120 kHz range.

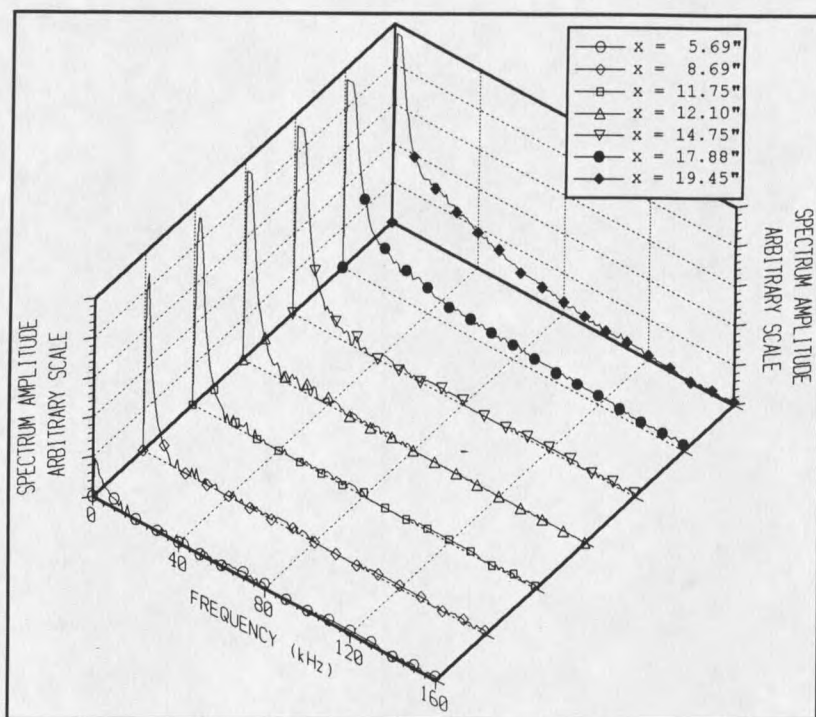


Figure 39.

Frequency spectra at 350 torr, no screen, high range.

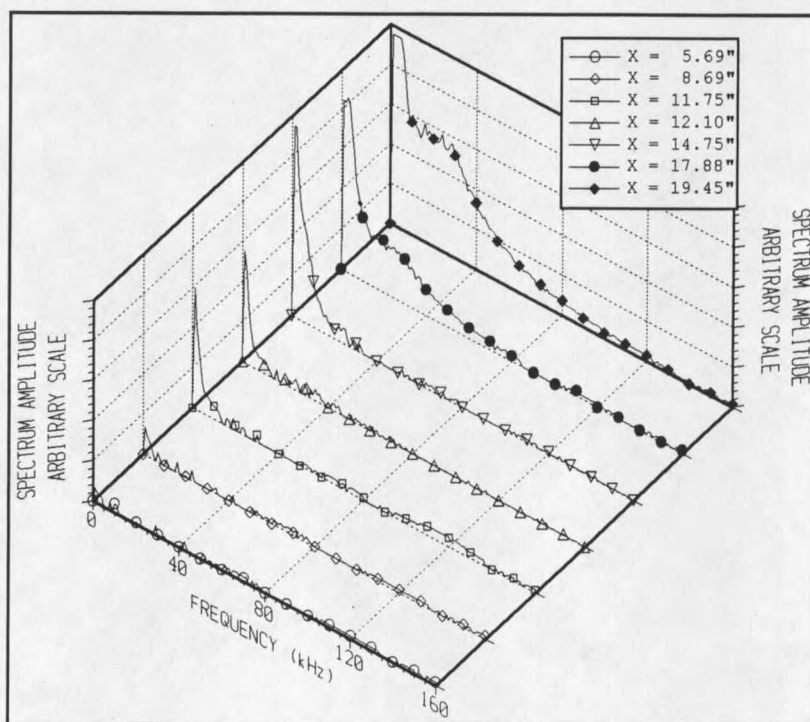


Figure 40.

Frequency spectra at 350 torr, with screen, high range.

Wave Form Results

The wave form figures each consist of two traces. The upper trace is the wide band DPP output, and the lower trace is the same DPP output reduced through a high pass filter. The filter was set to allow only frequencies greater than 150 kHz to pass. The high pass traces were intended to better illustrate the occurrence of turbulent bursts. All wave forms were obtained with the same oscilloscope settings.

It is important to note that the wave forms only indicate a snapshot in time of a continuous signal. During acquisition of the wave forms, it was attempted to obtain traces that were representative of the observed continuous signal.

Figure 41 shows the wide band wave form of the freestream signal. This trace is provided as a reference for comparison to the boundary layer traces.

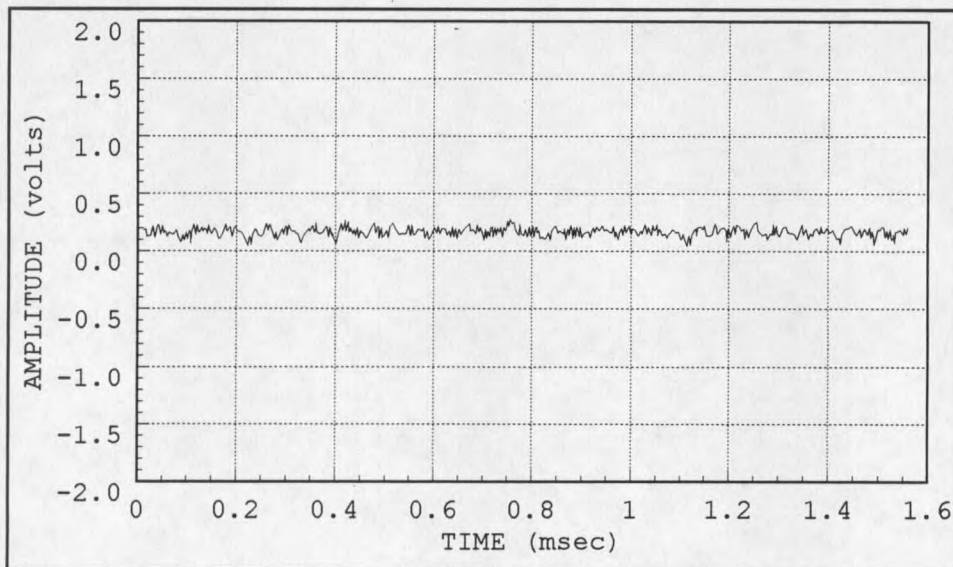


Figure 41.
Wave form of the freestream signal.

Figure 42 presents the results of the time domain analysis at 600 torr. At $x = 5.69$ " traces both with and without screens indicate the occurrence of turbulent bursts in the absence of low frequency activity. The traces at 5.69" provided good examples of the results of poor probe placement in the boundary layer. At $x = 5.69$ " the intermittency factor, as observed on the oscilloscope*, was relatively low.

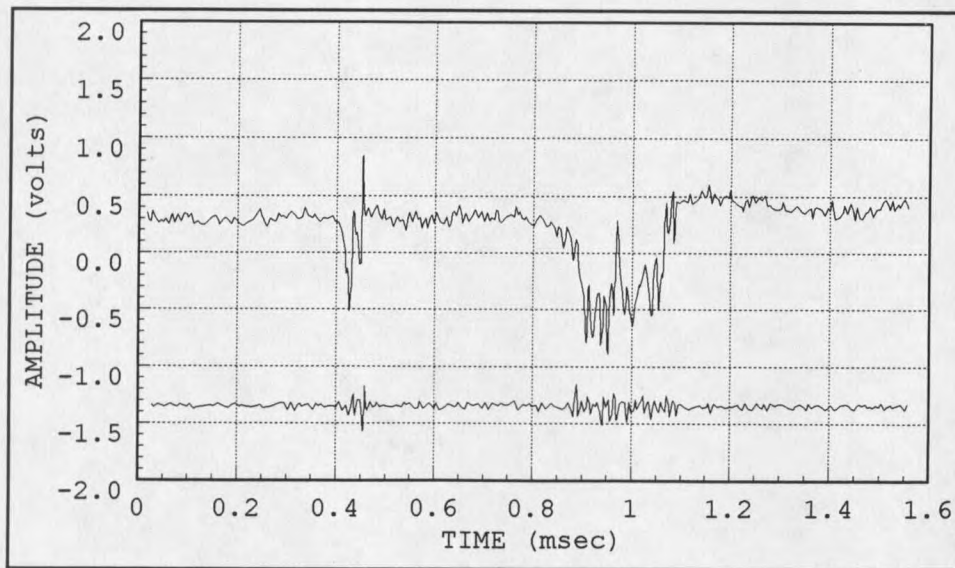
At $x = 8.69$ " the traces without screens indicated low frequency waves of relatively large amplitude along with the occurrence of turbulent bursts. With screens there was still no evidence of low frequency activity although the intermittency factor appeared to increase.

At $x = 11.75$ " the traces without screens indicated increasing intermittency with little change in the low frequency waves. The first appearance of low frequency waves occurred for the case with screens, while intermittency remained relatively unchanged.

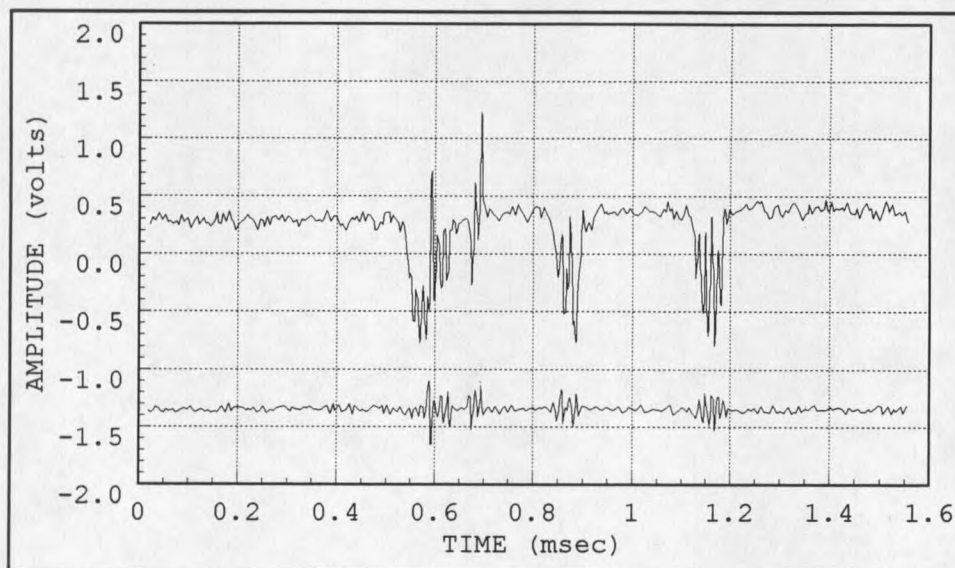
At $x = 12.10$ " there appeared to be a slight increase in intermittency for the case without screens, with the low frequency waves exhibiting a fairly regular pattern. The traces with screens showed a slight increase in amplitude of the low frequency waves with little change in intermittency.

* The intermittency factor describes the percentage of the signal composed of high frequencies. This was a highly subjective and qualitative observation based entirely on the observed oscilloscope signal.

Figure 42.
Time based wave forms at $P_o = 600$ torr.

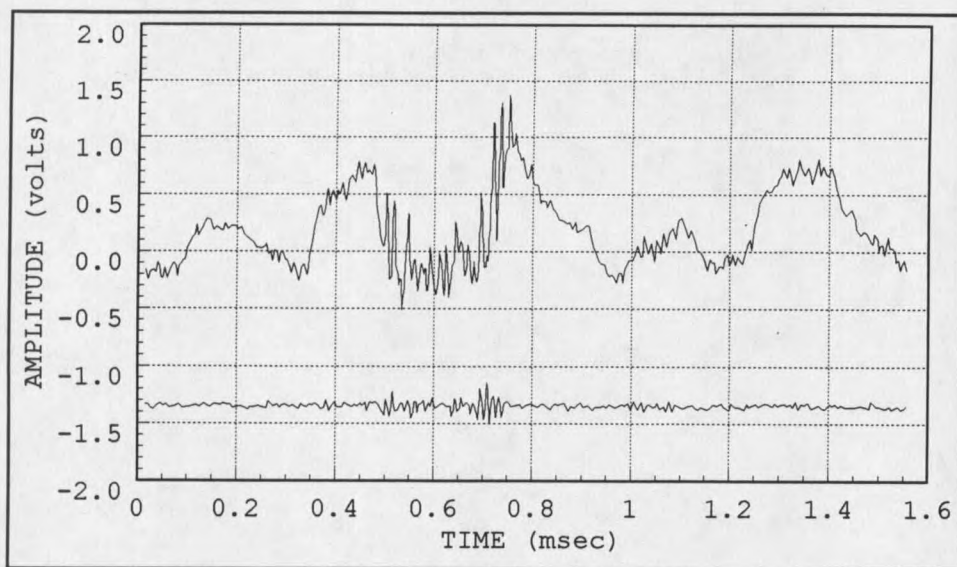


$x = 5.69''$, no screen

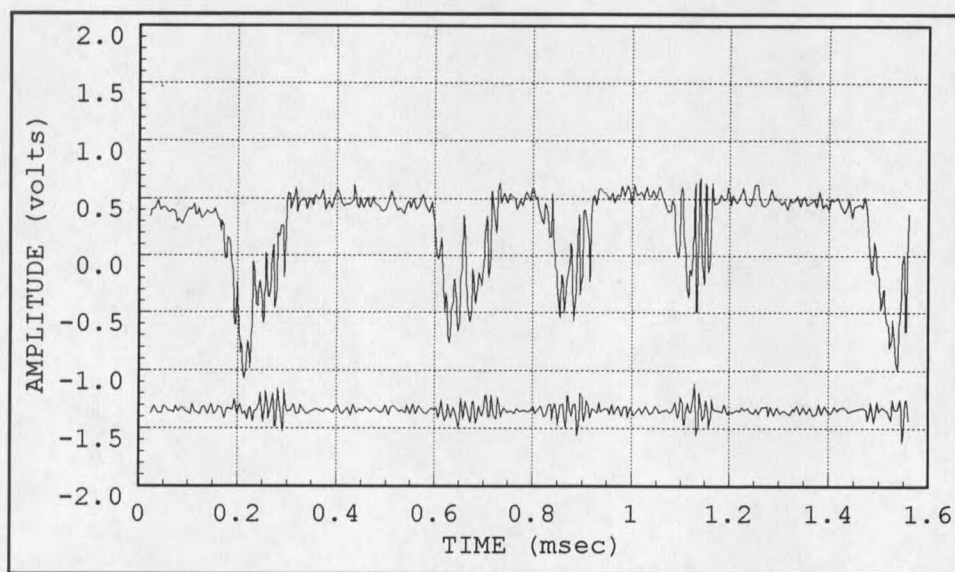


$x = 5.69''$, with screen

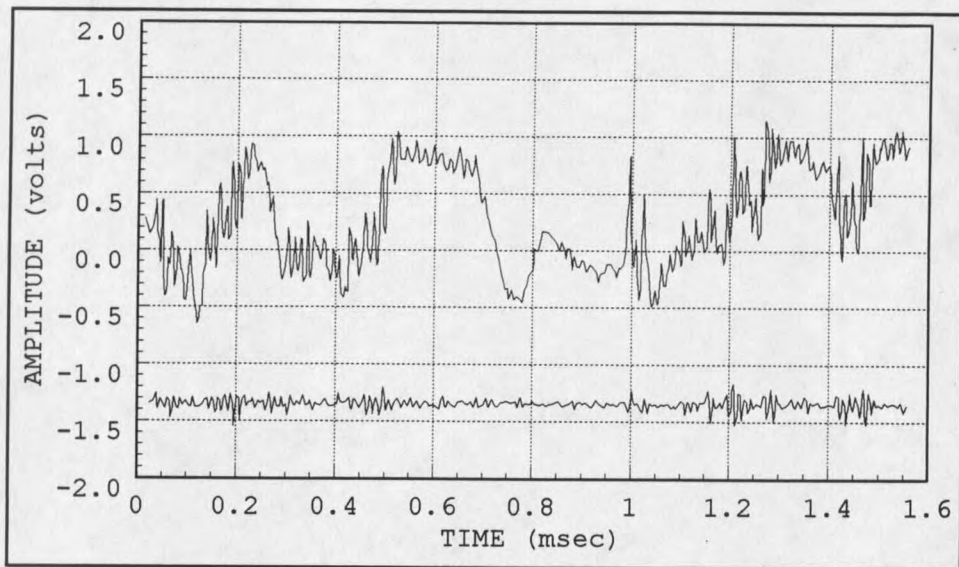
Figure 42.--Continued



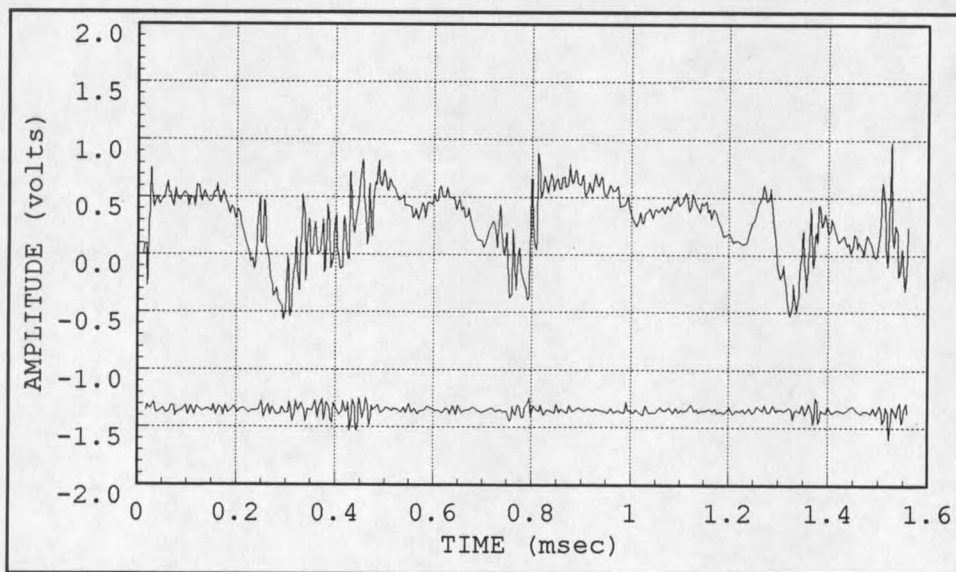
x = 8.69", no screen



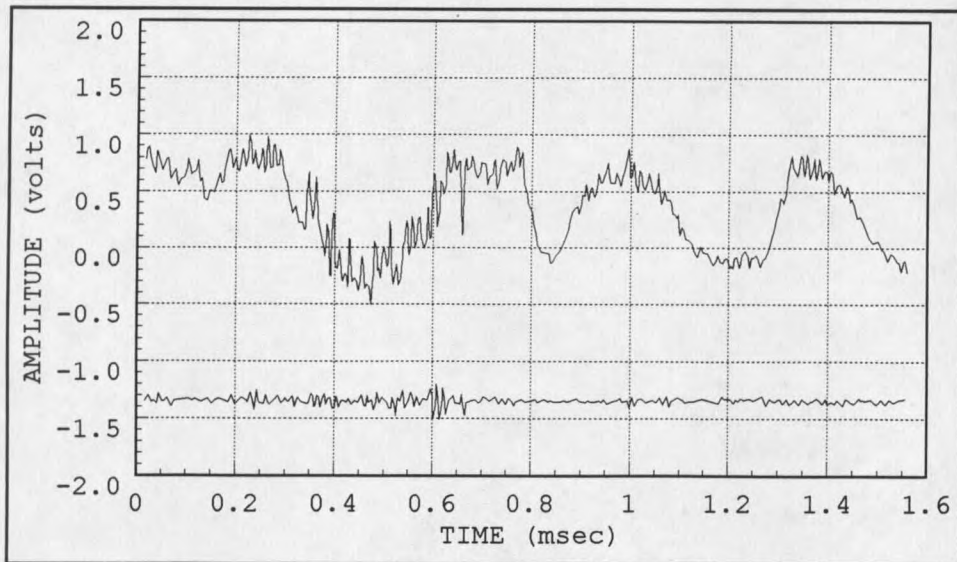
x = 8.69", with screen

Figure 42.--Continued

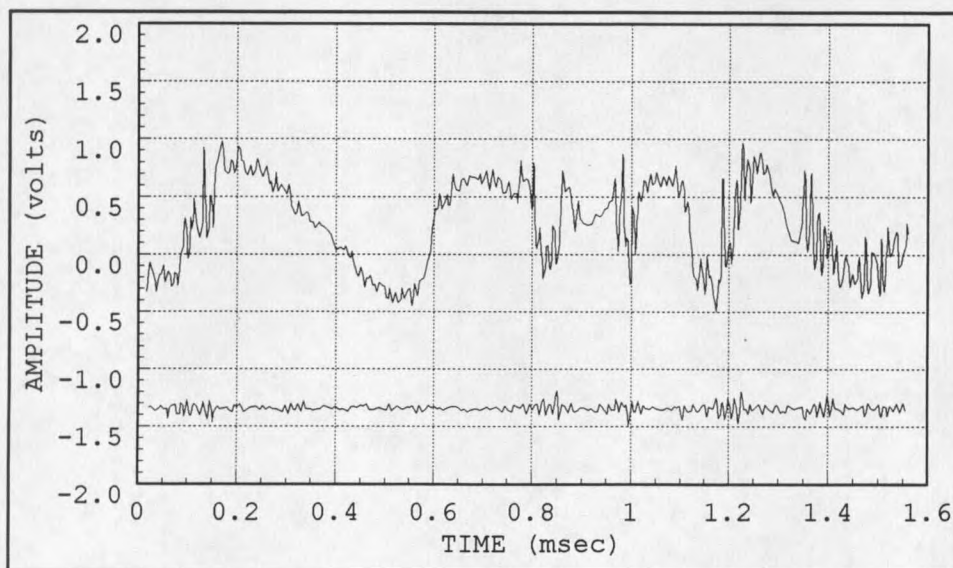
x = 11.75", no screen



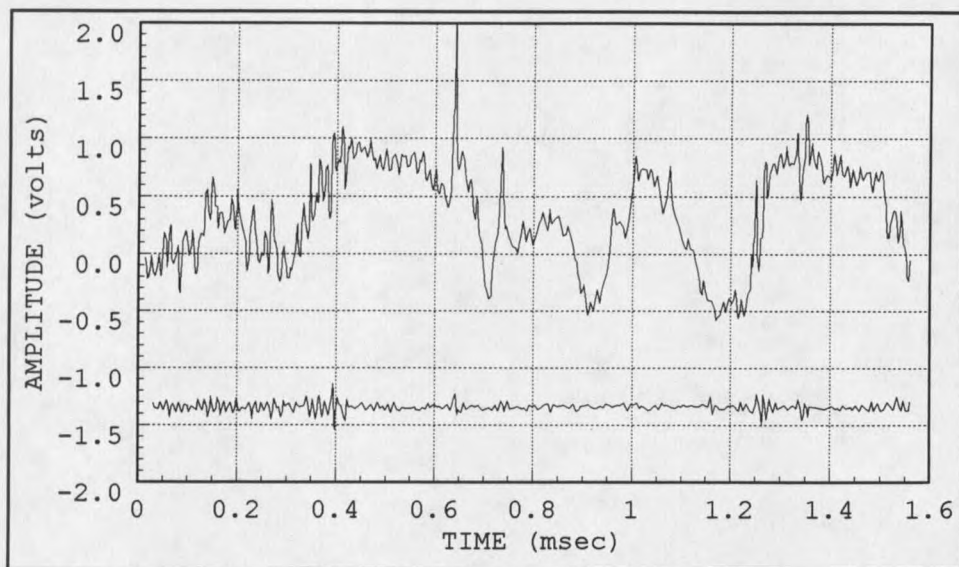
x = 11.75", with screen

Figure 42.--Continued

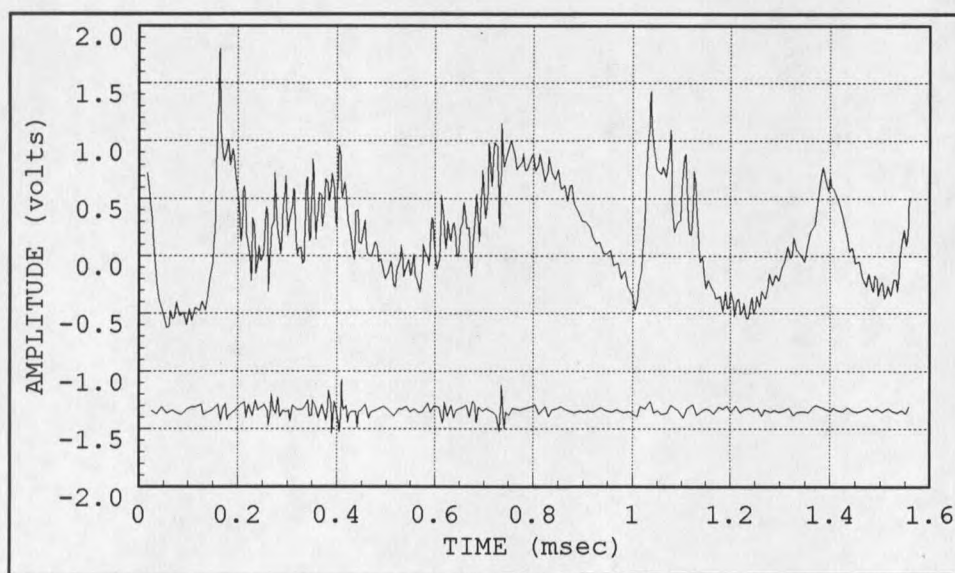
x = 12.10", no screen



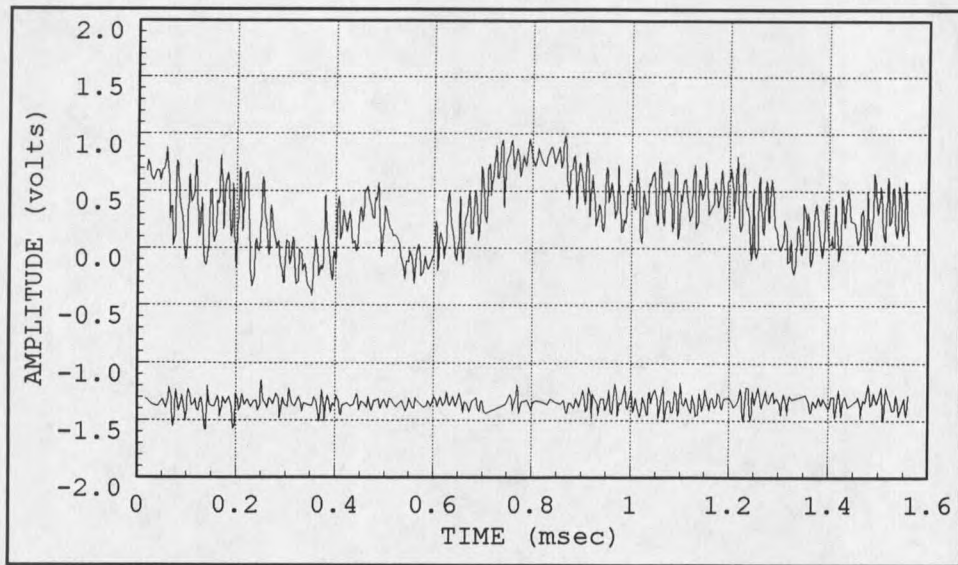
x = 12.10", with screen

Figure 42.--Continued

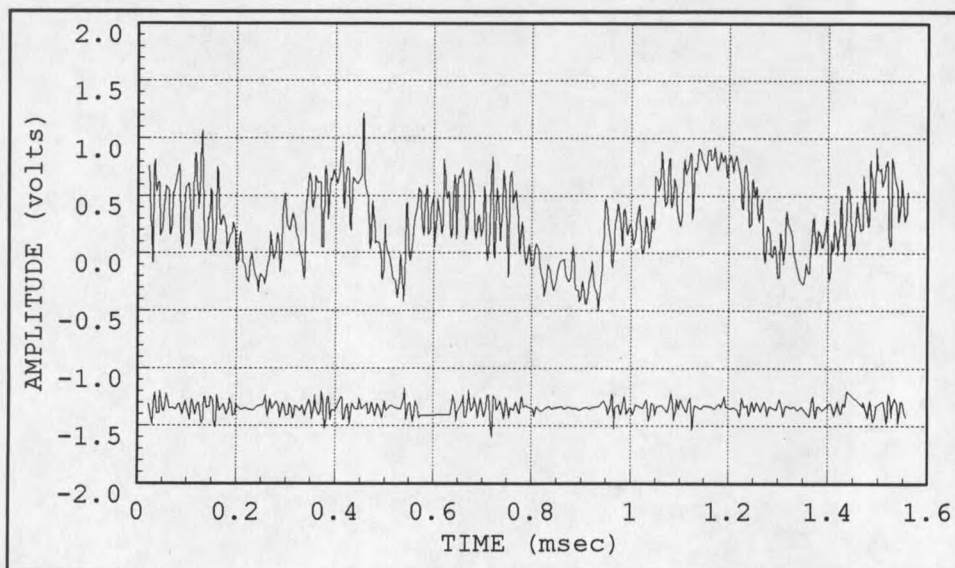
x = 14.75", no screen



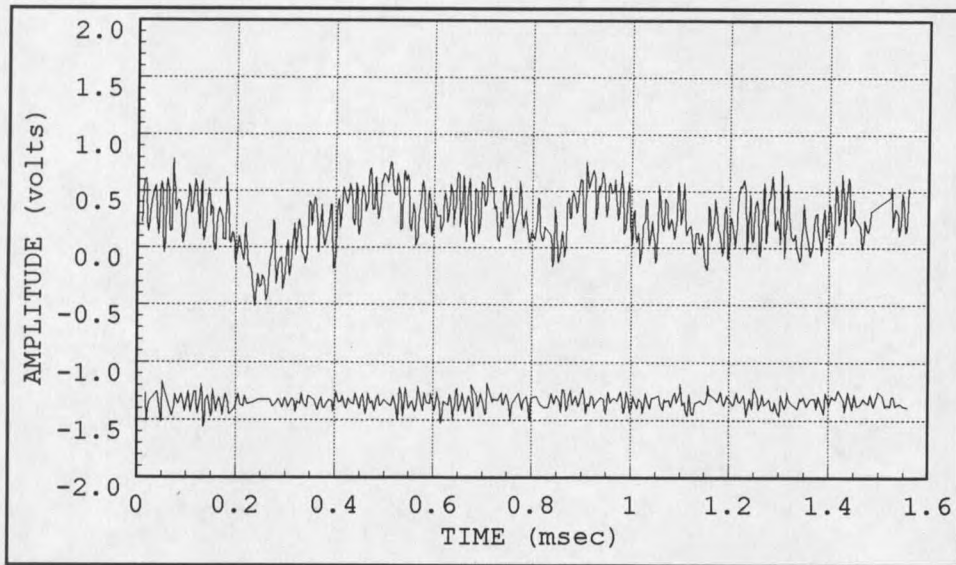
x = 14.75", with screen

Figure 42.--Continued

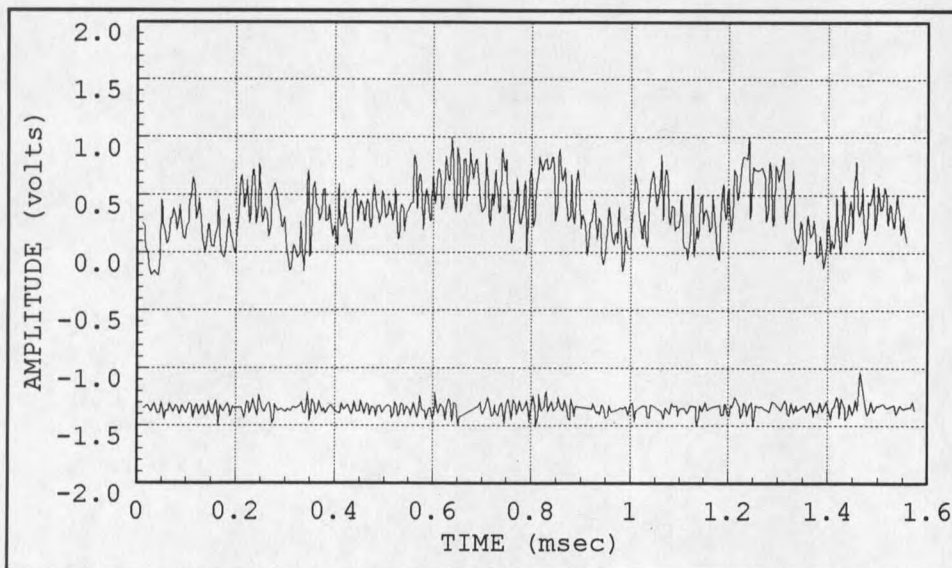
x = 17.88", no screen



x = 17.88". with screen

Figure 42.--Continued

x = 19.45", no screen



x = 19.45", with screen

At $x = 14.75''$ traces both with and without screens exhibited high amplitude, irregular, low frequency activity with the intermittency factor approaching 100%.

At $x = 17.88''$ and $19.45''$ the traces both with and without screens exhibited similar trends. At $17.88''$ intermittency reached 100% while the amplitude of the low frequency activity decreased. At $19.45''$ the low frequency activity showed a further decrease reaching almost negligibly small levels.

Figure 43 shows the wave form results at 350 torr. At $5.69''$ the traces both with and without screens indicated negligible activity with the wide band traces being virtually identical to the freestream trace.

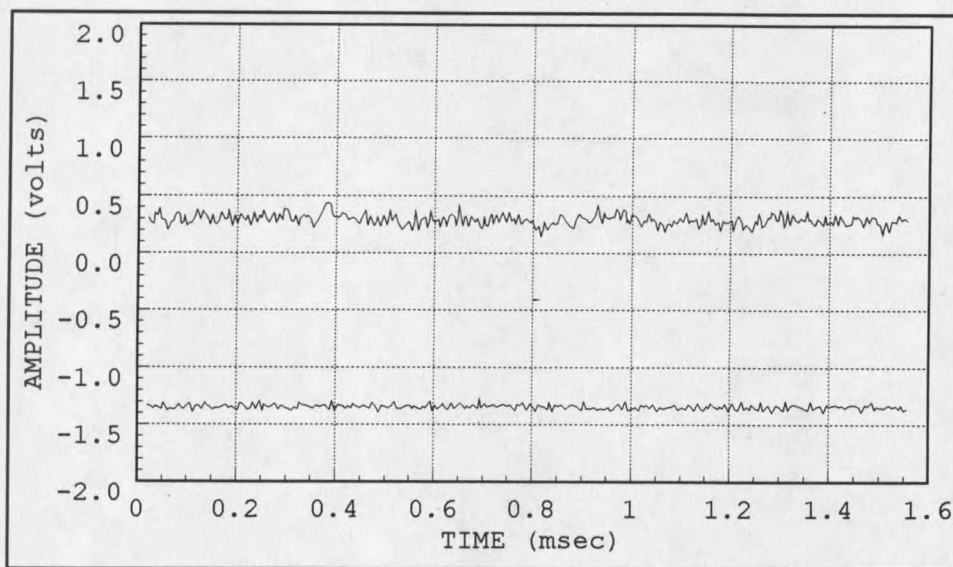
At $8.69''$ there was no change in the traces with screens. Without screens the traces indicated the development of low amplitude low frequency waves.

At $11.75''$ the low frequency waves increased in amplitude for the case without screens. With screens the traces revealed a slight amount of low frequency activity.

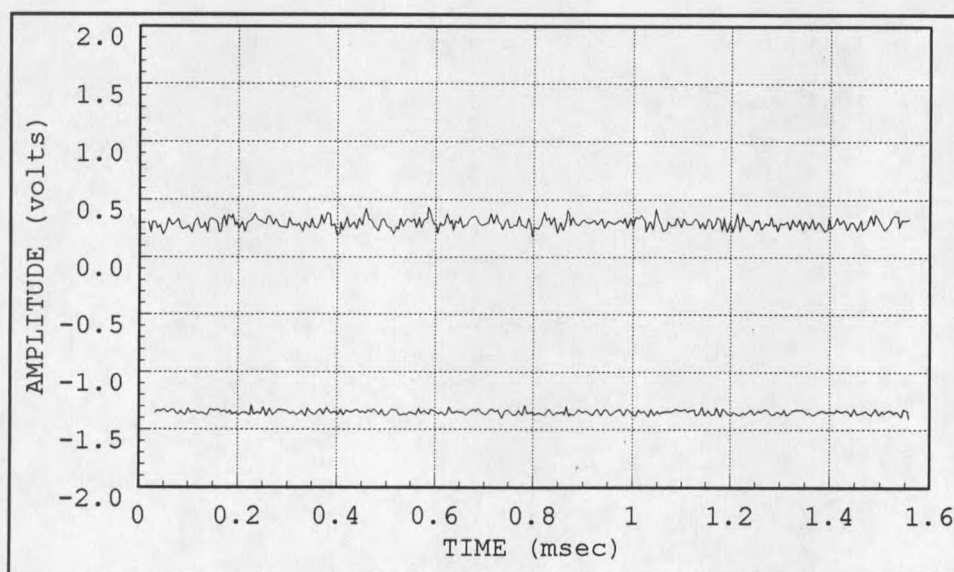
At $12.10''$ and $14.75''$ the traces both with and without screens indicated slight increases in the amplitude of the previously noted low frequency activity.

At $17.88''$ the first appearance of turbulent bursts was observed in both cases with and without screens. Without screens the low frequency waves remained relatively unchanged from the previous position. With screens, the low frequency waves appeared to completely die out.

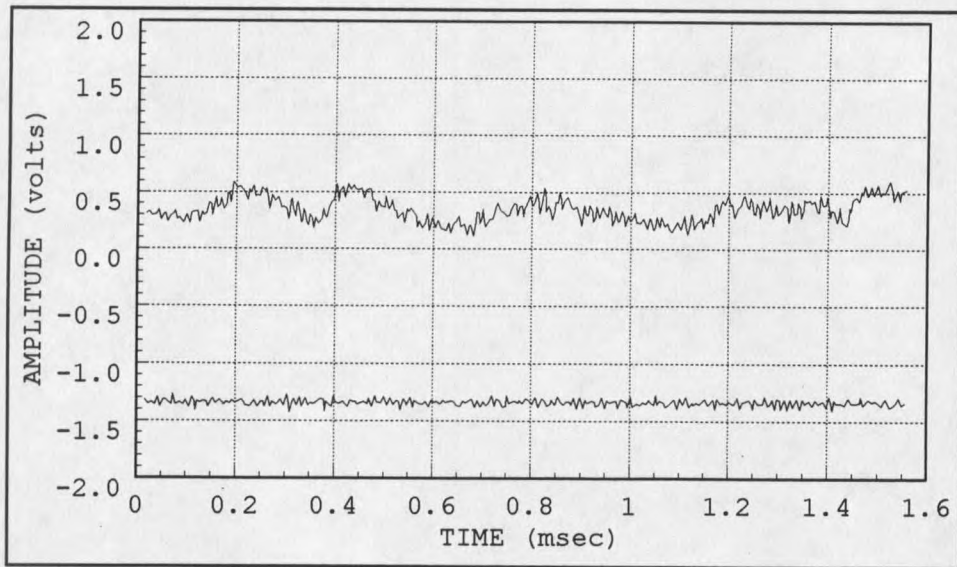
Figure 43.
Time based wave forms at $P_o = 350$ torr.



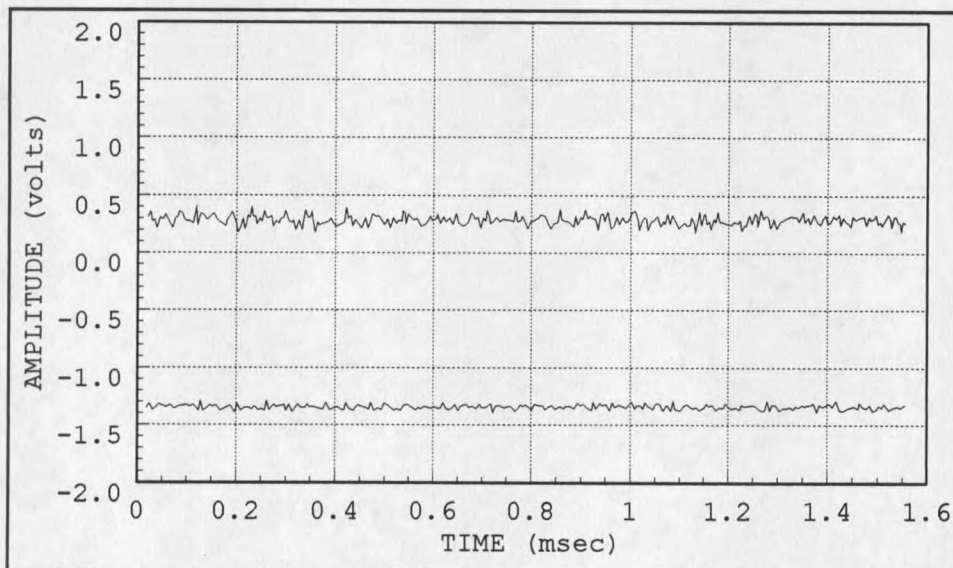
$x = 5.69''$, no screen



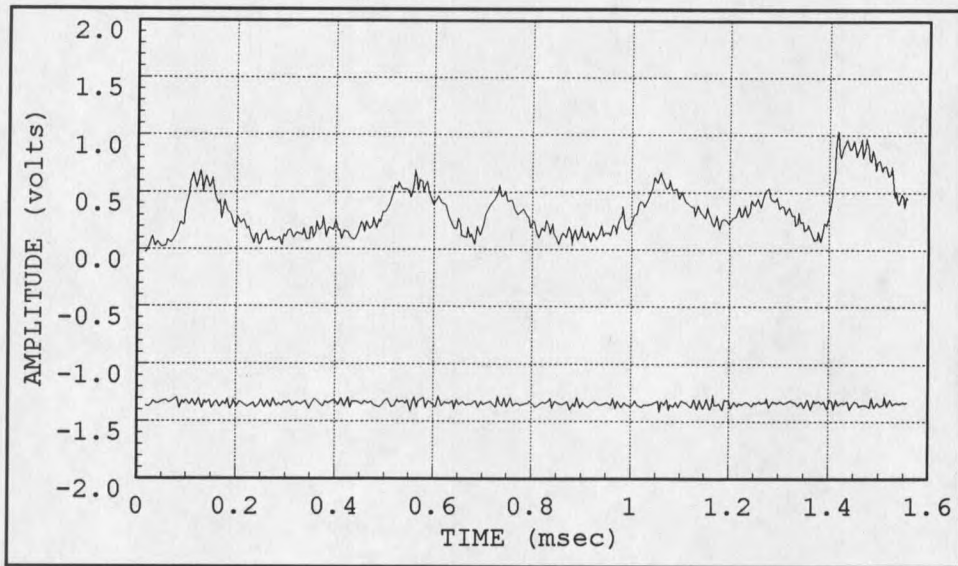
$x = 5.69''$, with screen

Figure 43.--Continued

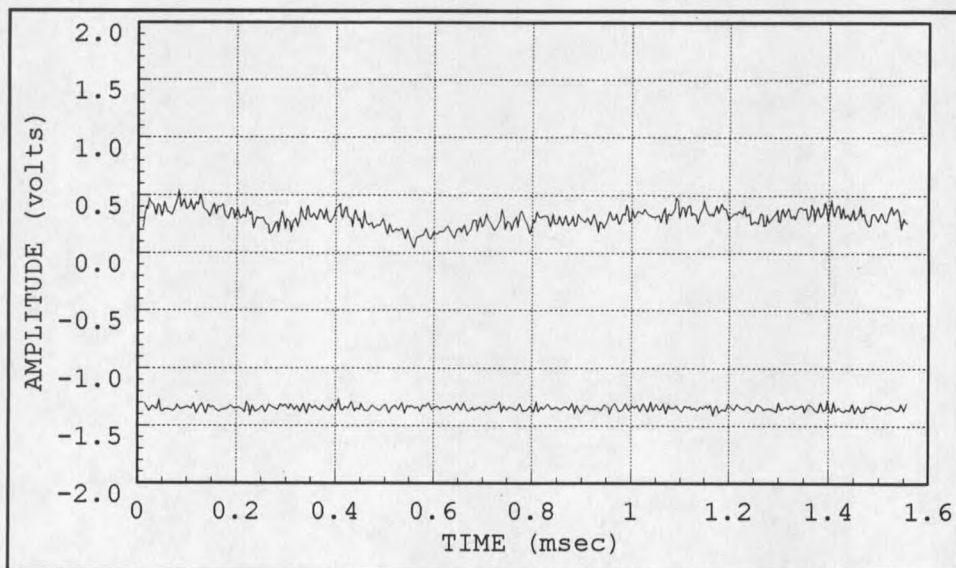
x = 8.69", no screen



x = 8.69", with screen

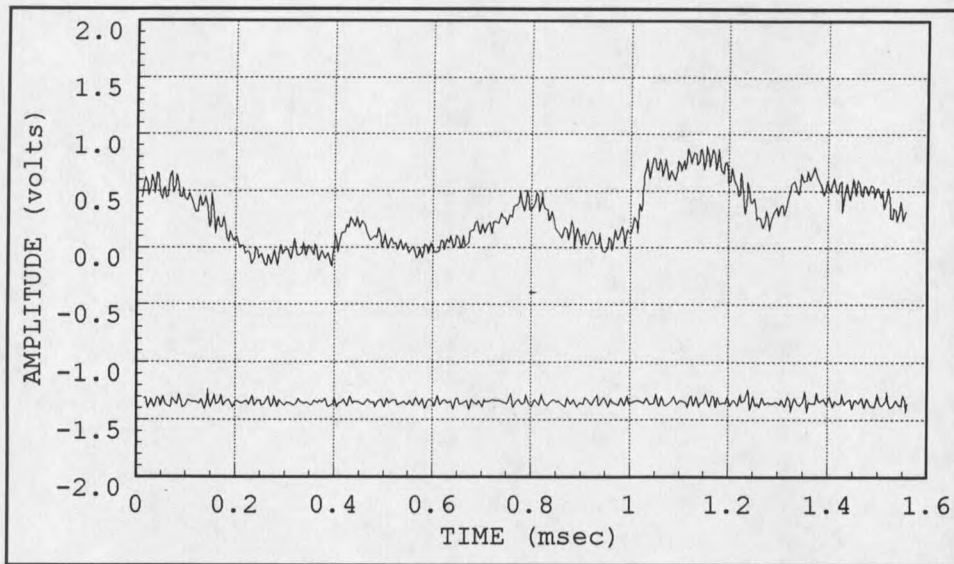
Figure 43.--Continued

x = 11.75", no screen

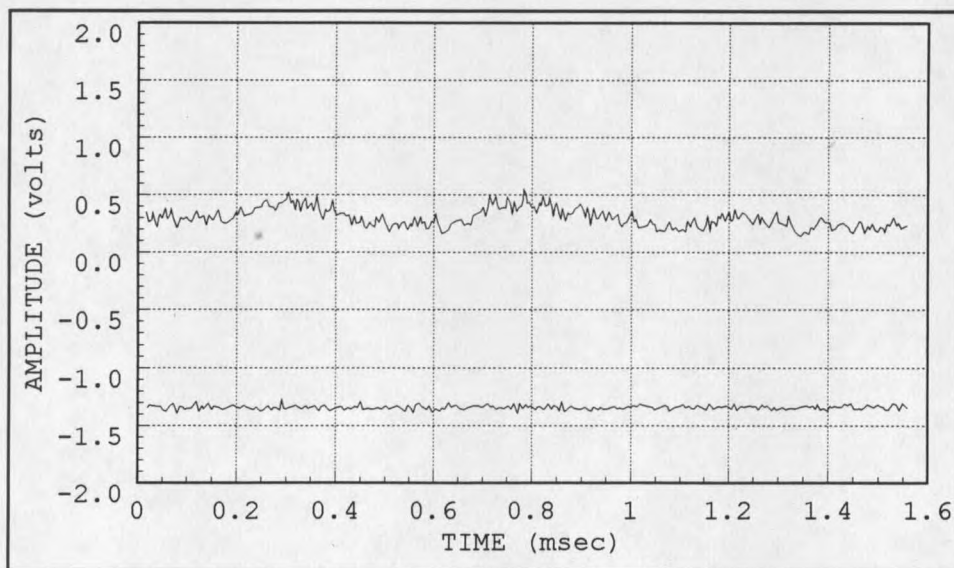


x = 11.75", with screen

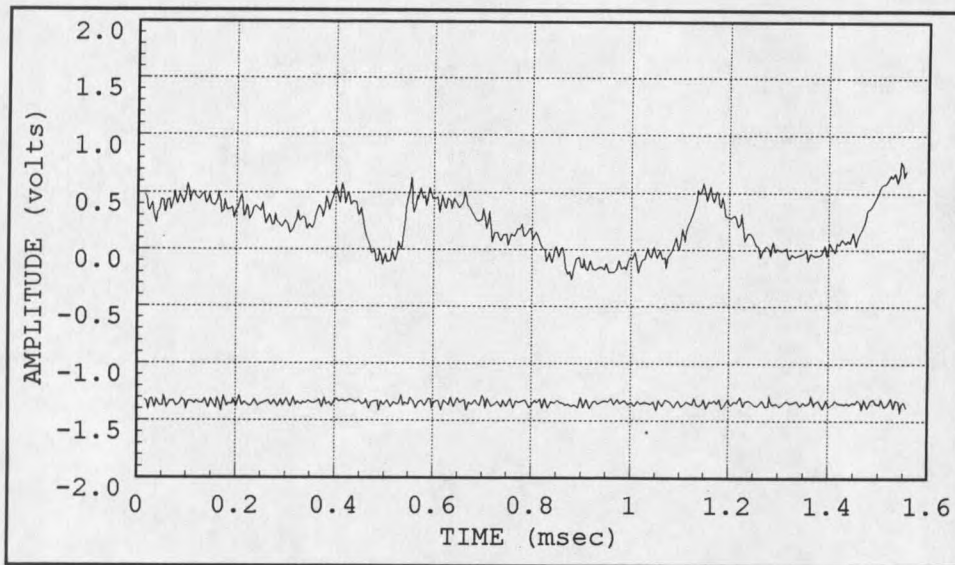
Figure 43.--Continued



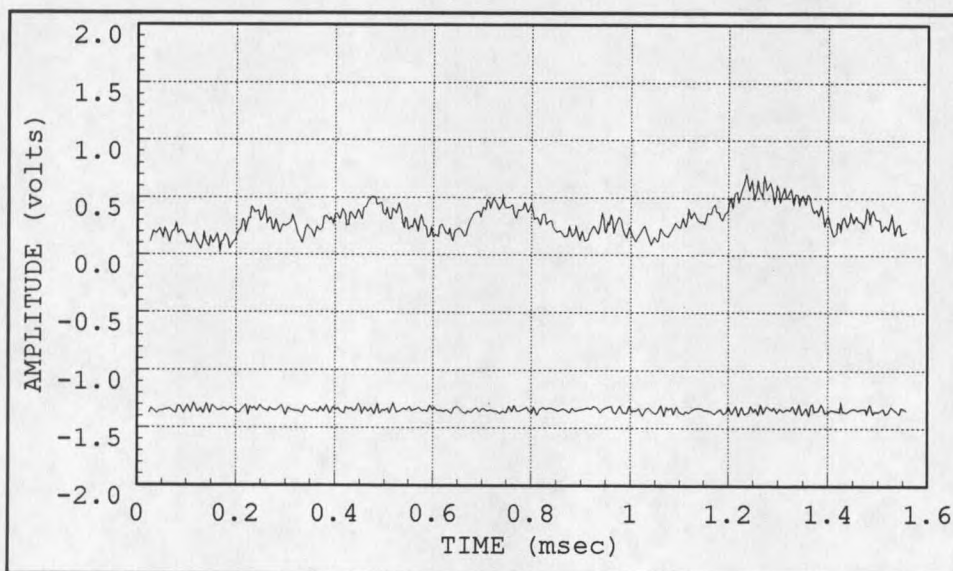
x = 12.10", no screen



x = 12.10", with screen

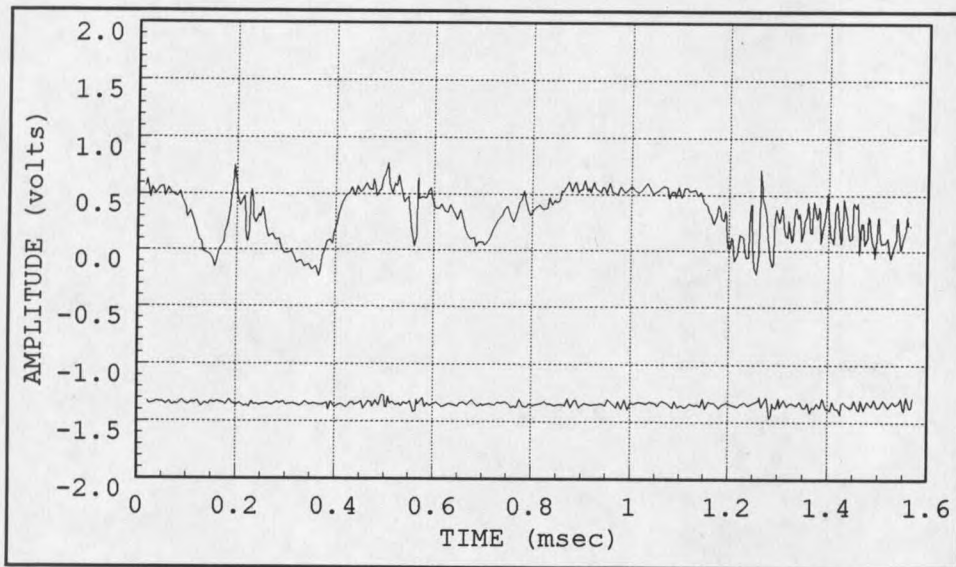
Figure 43.--Continued

x = 14.75", no screen

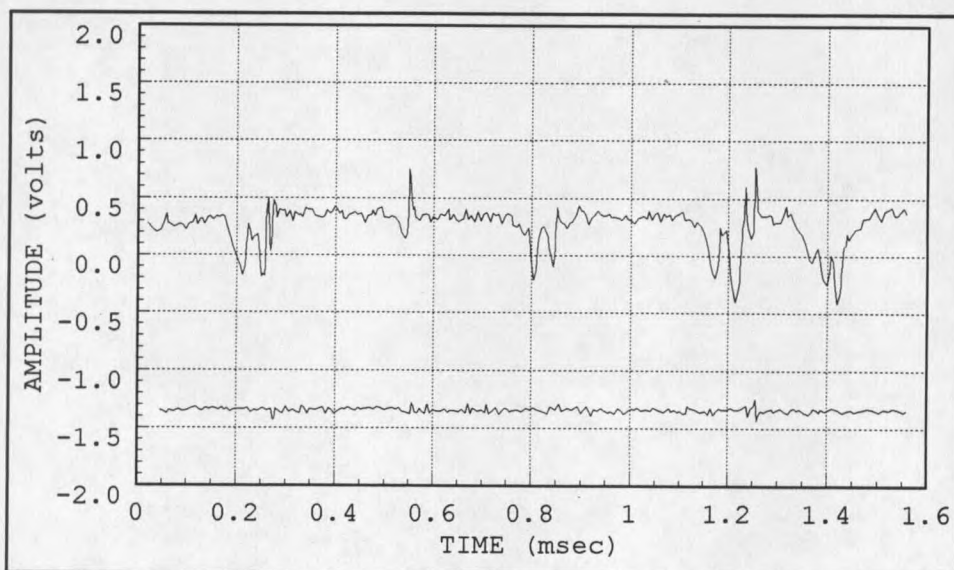


x = 14.75", with screen

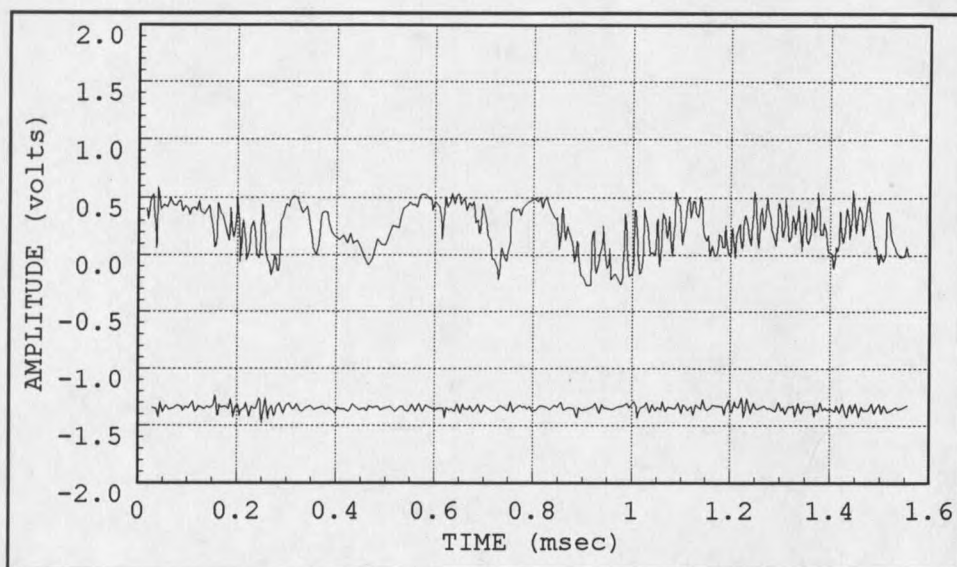
Figure 43.--Continued



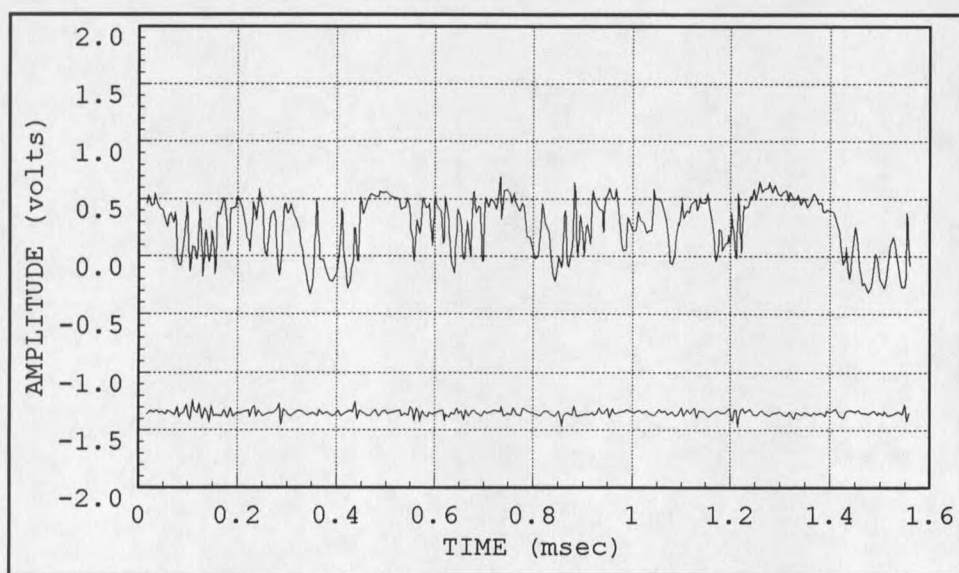
x = 17.88", no screen



x = 17.88", with screen

Figure 43.--Continued

x = 19.45", no screen



x = 19.45", with screen

In the traces with screens at 17.88" the frequency of the bursts appeared to be below 150 kHz. This observation was due to the observed lack of activity in the high pass trace which indicated a significant amount of high frequency activity below 150 kHz. This result provided a possible explanation for the unusual humps in the frequency spectra at 350 torr for the case with screens.

At 19.45" the traces without screens showed little change in the low frequency activity with a large increase in intermittency. With screens there appeared to be significant high frequency activity, but very little activity above the 150 kHz cutoff of the filter.

SPATIAL TRANSITION SURVEY

This portion of the investigation covered the lower half of the tunnel at three different x locations. Due to restricted actuator motion it was impossible to cover the entire span of either sidewall or the floor of the tunnel. Therefore, the survey was limited to the lower half of the wind tunnel.

In all cases T_o was maintained at 100 °F, no screens were used and there was no filtering of the recorded data. Two types of data were obtained. Profile data, taken as the DPP traversed the boundary layer, was the first type. The profiles were taken at a constant value of P_o . The second type of data recorded was the variation of the DPP signal with variation in P_o . This data was taken at fixed x , y and z coordinates above the surface of interest.

Again several problems were faced in determining a suitable location for probe placement during the P_o sweeps. The standard practice was to place the DPP at the location of the peak signal of the boundary layer profiles. This provided a suitable method on the nozzle surface where the profiles were similar in nature and there was little to no shifting of the peak. However, on the sidewalls the profiles did not exhibit this similarity. On the sidewalls the peaks shifted upwards by as much as 0.1" with increasing pressure. With the active diameter of the DPP being 0.01", there was no single location in the boundary layer where the probe remained at the signal peak over the entire range of P_o . In this case the DPP was generally positioned at the peak occurring closest to the surface of interest. In this manner there was less chance of the probe falling outside of the

boundary layer during the P_o sweeps.

Figure 44 shows the actual locations along the tunnel perimeter where the data was taken, at each of the three x locations selected for the survey. The horizontal and vertical scales in figure 44 are the actual tunnel width and height in inches. Positions along the sidewalls are measured with reference to the lower nozzle surface, and the positions along the floor are measured with reference to the north sidewall.

Figure 44.
Cross section of the tunnel showing locations along the perimeter where data was recorded.

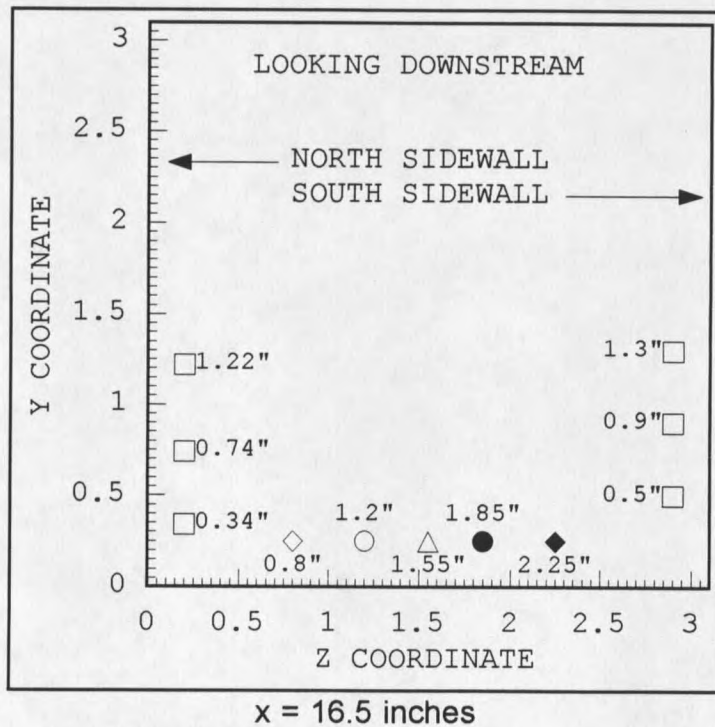
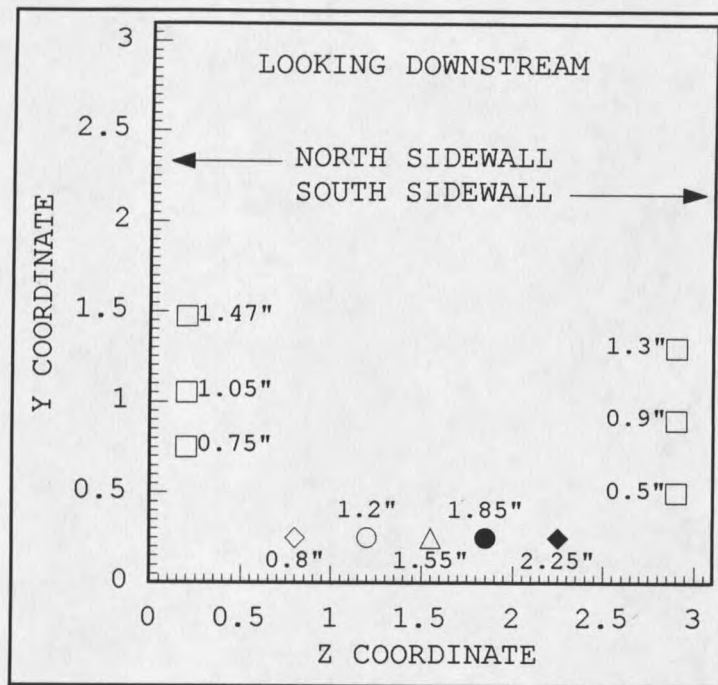
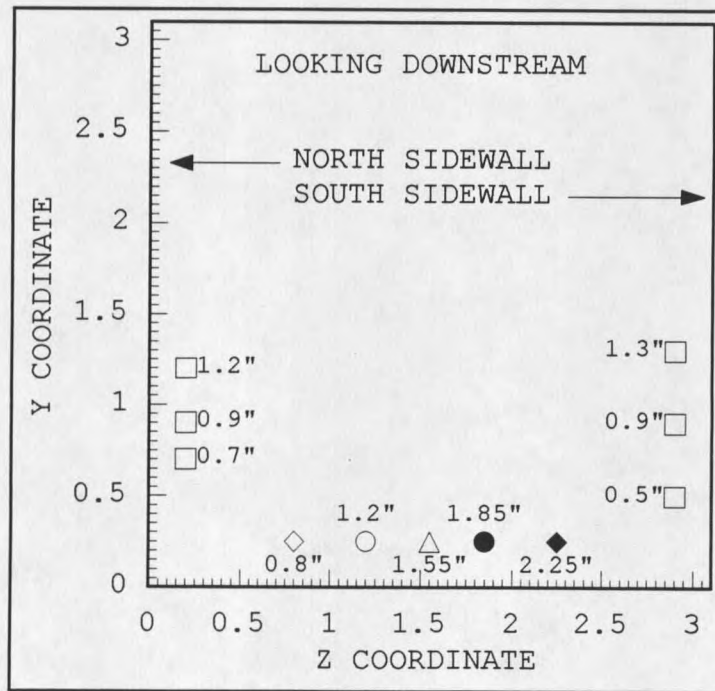


Figure 44.--Continued



x = 14.5 inches



x = 11 inches

Floor Results

The three x locations selected represented the maximum stroke obtainable with the x actuator. At each x position data was recorded at five different z positions across the width of the tunnel. At positions near the sidewalls there was a problem with choking of the tunnel, causing the breakdown of supersonic flow. Therefore, the positions furthest from the tunnel centerline were 0.8" from the sidewalls. The five z positions examined were 0.8", 1.2", 1.55" (centerline), 1.85" and 2.25". All z measurements were referenced from the north sidewall. Positions at $z = 1.85"$ and $2.25"$ corresponded to 0.8" and 1.2" from the south sidewall.

Figure 45 shows the boundary layer profiles on the floor at $x = 16.5"$. The profiles behaved in a similar manner across the width of the tunnel. At 600 torr the signal approached the freestream level around 0.3" from the floor at all z positions. At the three central z positions the signal approached the freestream level around 0.25" for the 500 and 400 torr traces. At the two positions nearest the sidewalls there was little change in the profiles over the three pressures.

At all z positions and all pressures the profiles exhibited what was considered a turbulent profile. These profiles were characterized by a gradual decrease in signal level from the peak to freestream level. This indicated a thick and poorly defined boundary layer.

Figure 45.
Boundary layer profiles on tunnel floor at $x = 16.5$ inches.

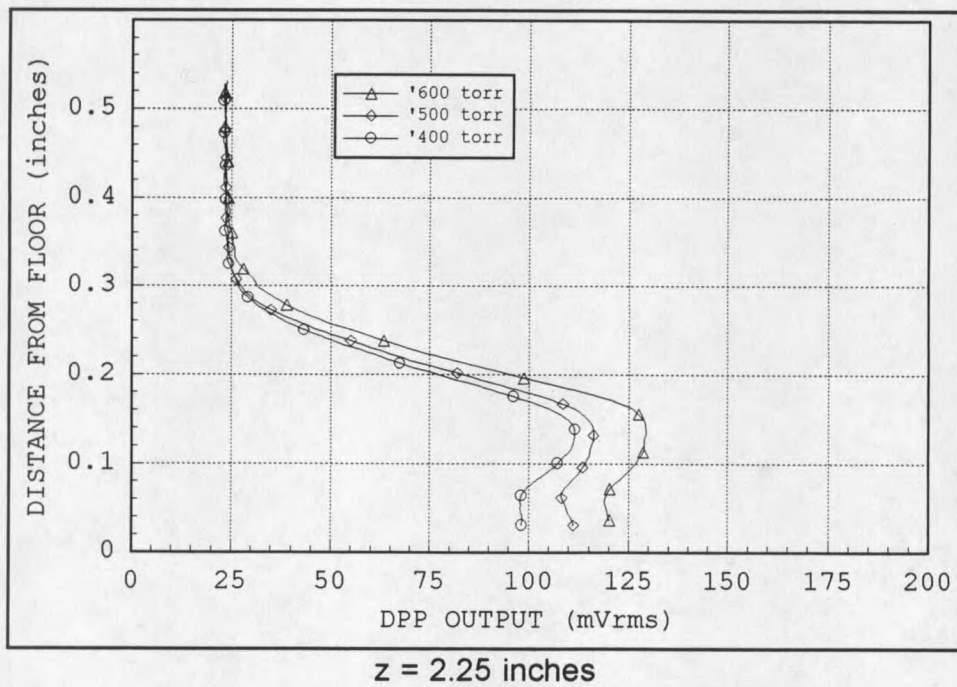
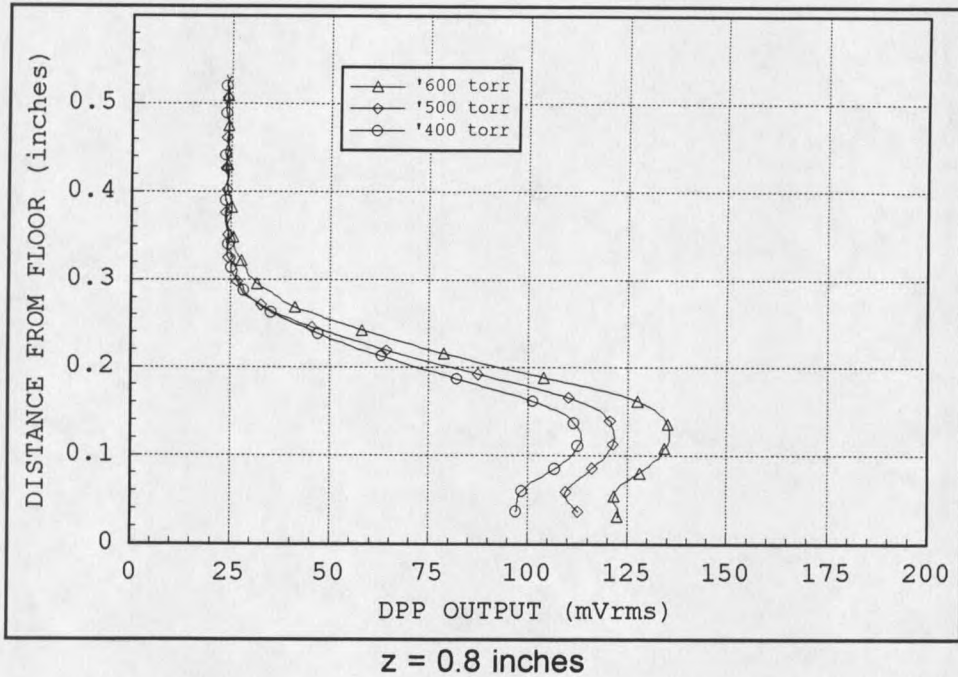


Figure 45.--Continued

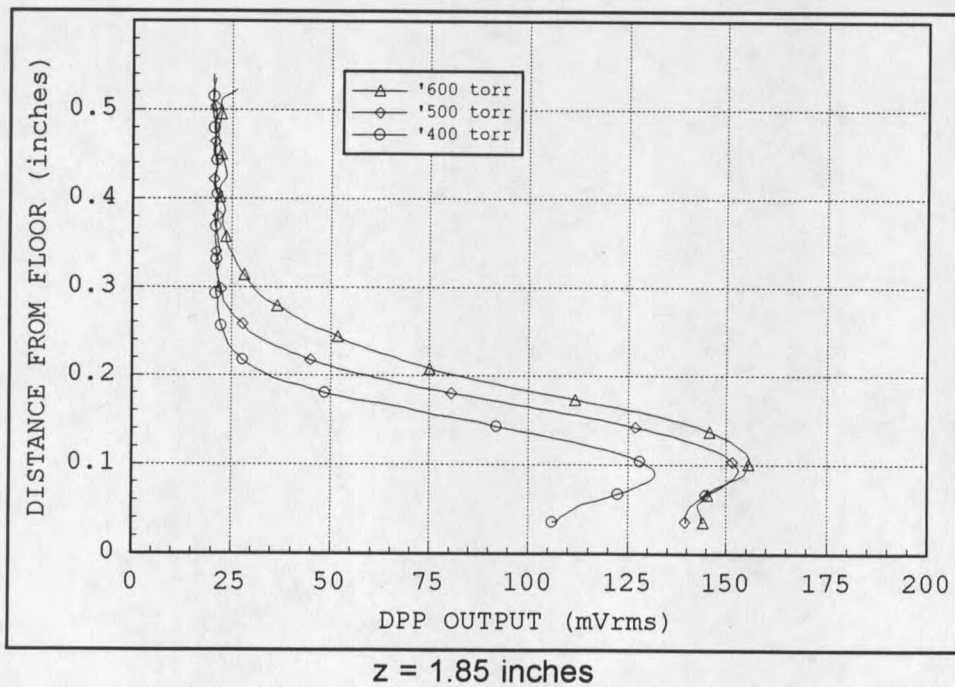
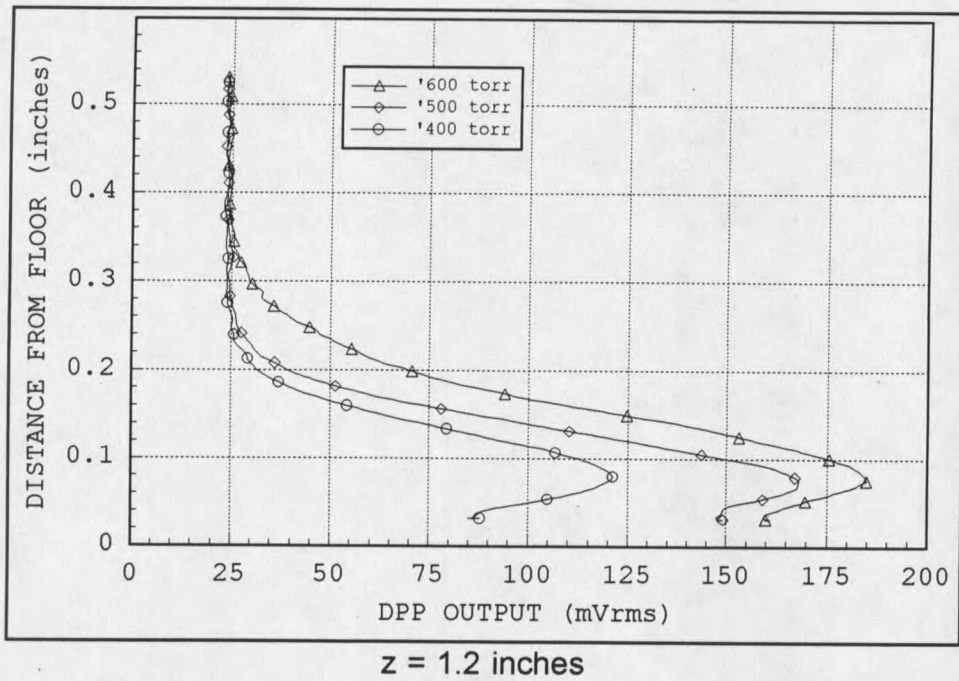
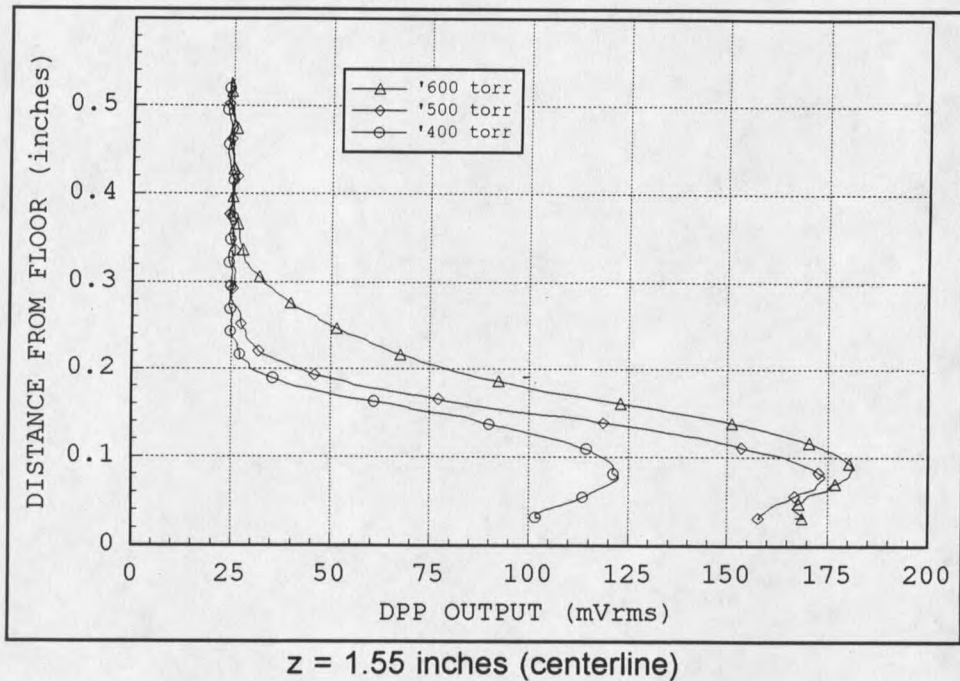


Figure 45.--Continued



The most notable trait at $x = 16.5''$ was the variation in the magnitude of the signal peaks. There appeared to be some non-uniformity across the width of the tunnel with the maximum signal occurring at the centerline and decreasing towards the sidewalls.

Figure 46 shows the P_o sweeps at $x = 16.5''$. The five traces showed no major differences in the transition process. At all z positions the signal level decreased steadily with decreasing pressure. The traces also indicated symmetry about the vertical centerline of the tunnel. The most notable trend observed was an abrupt signal decrease at the two locations nearest the sidewalls.

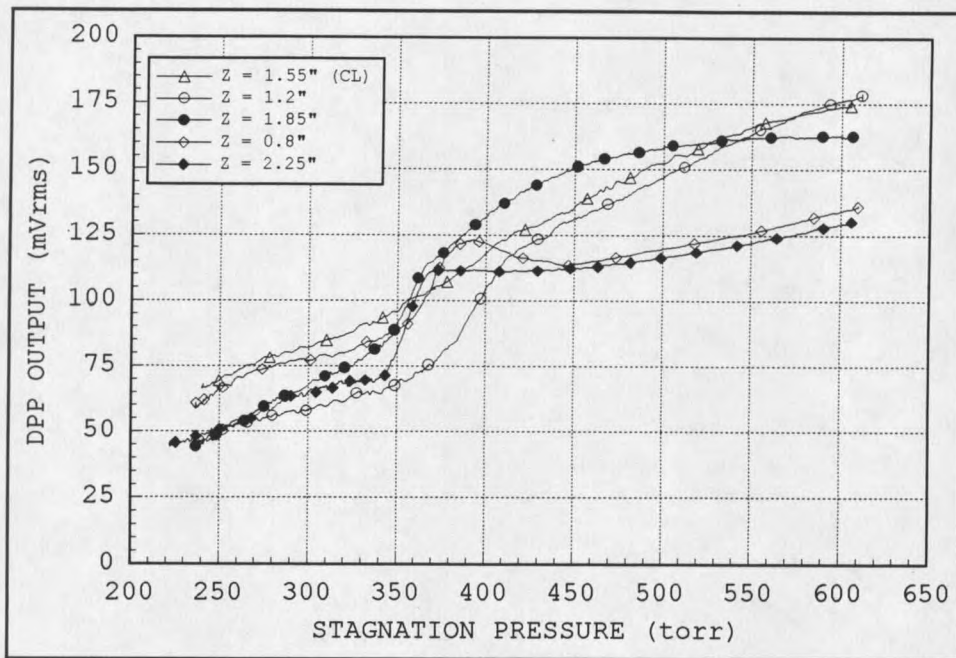


Figure 46.
DPP vs. P_o on floor at $x = 16.5$ inches.

Figure 47 presents the profiles at $x = 14.5$ ". At 14.5 " the profiles exhibited nearly identical tendencies as those at 16.5 " with two exceptions. For the three central z positions the profiles appeared to be transitional to laminar at 400 and 500 torr. But, profiles nearest the sidewalls still indicated turbulence at the two lower pressures. The second major difference was that the peak signal levels remained constant at the two positions nearest the sidewalls while the three central positions exhibited large decreases in signal level with each decrease in P_o . This provided evidence of transition near the center of the tunnel while the boundary layer near the sidewalls remained fully turbulent.

Figure 47.
Boundary layer profiles on floor at $x = 14.5$ inches.

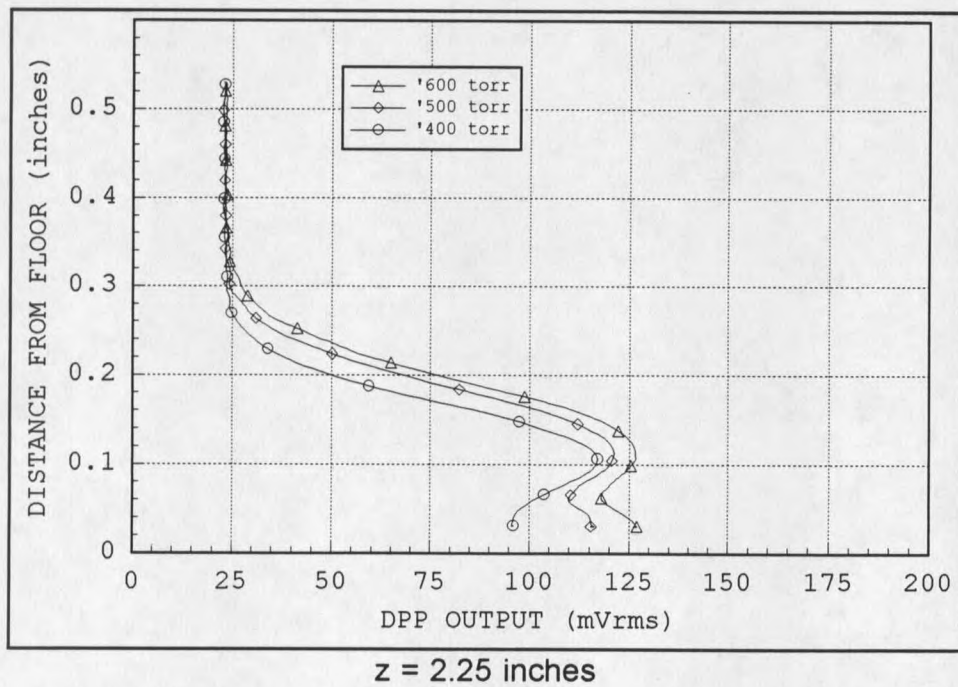
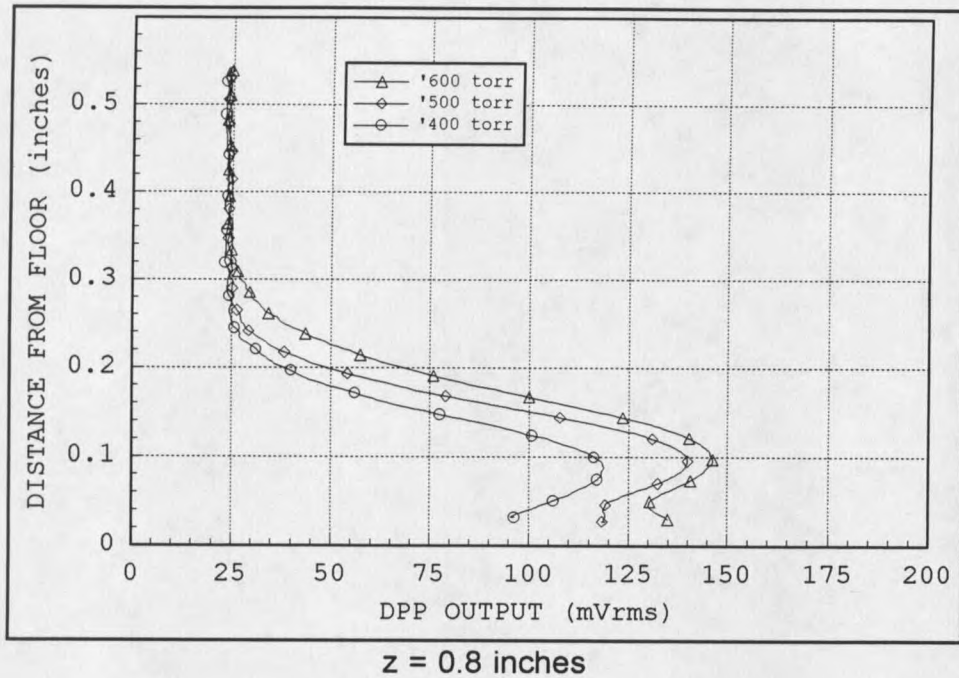


Figure 47.--Continued

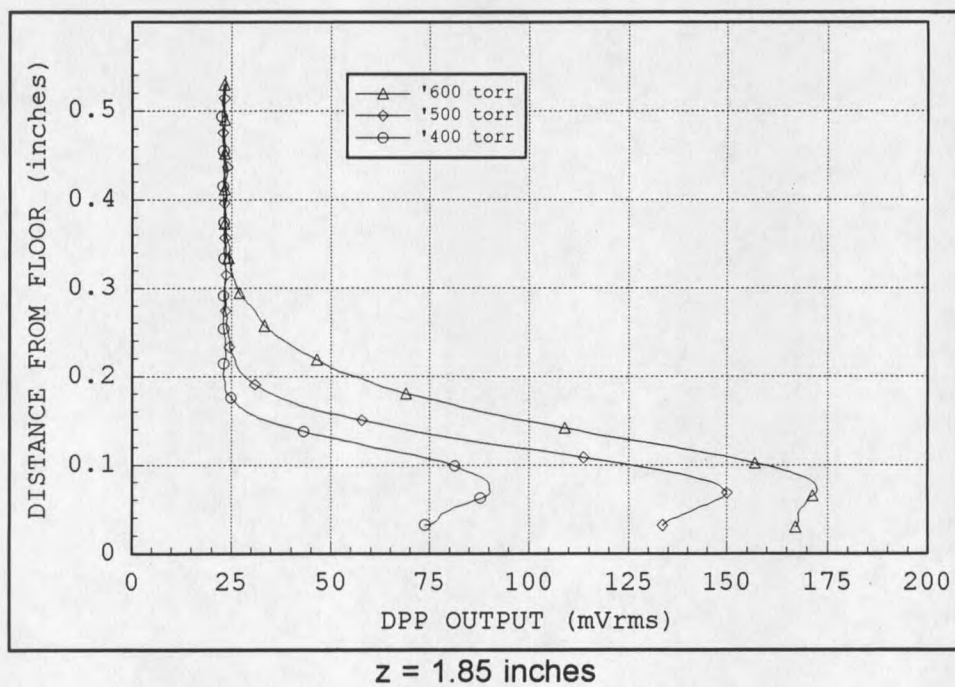
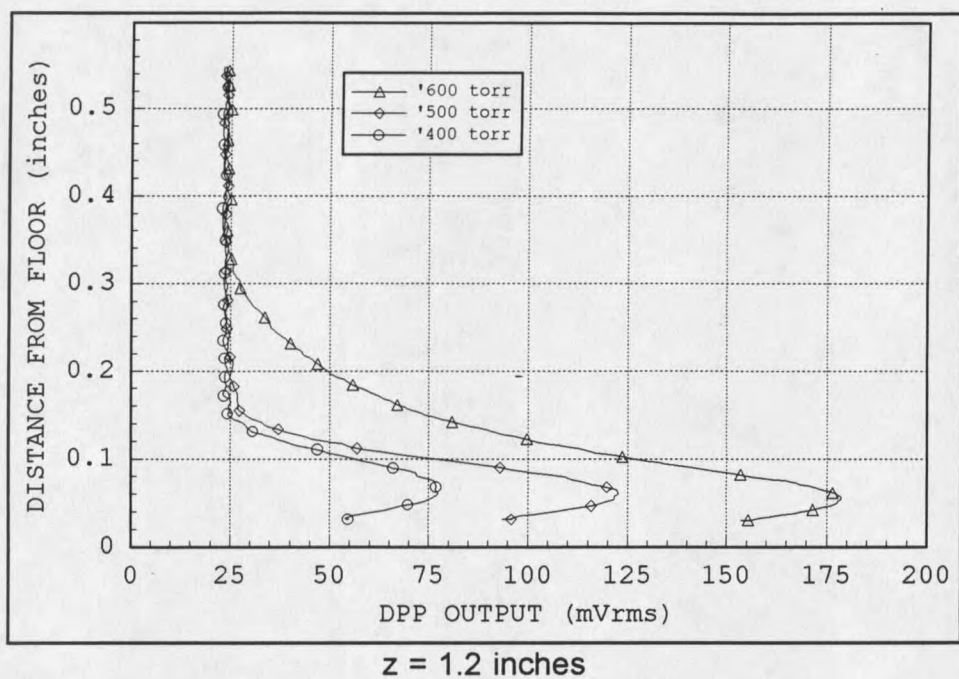
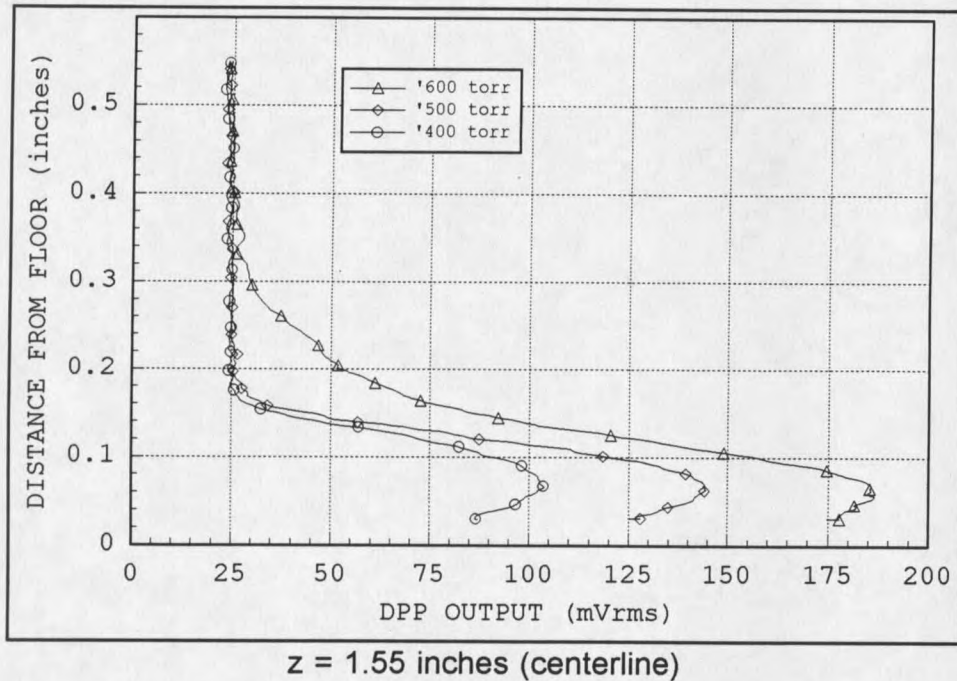


Figure 47.--Continued



The P_o sweeps of figure 48 showed no major discrepancies in transition across the width of the tunnel. The traces still indicated symmetry about the vertical centerline. The most notable feature observed in figure 48 was the abrupt signal decrease in the two traces nearest the sidewalls while the three central traces all exhibited uniform decreasing trends.

Figure 49 presents the profiles at $x = 11$ ". At 11" there appeared to be a reversal of a previously noted trend. The peak signal level was now a maximum at the two positions nearest the sidewalls and was a minimum at the two middle positions ($z = 1.2$ " and 1.85"). At all five z positions the profiles at 600 torr indicated a transitional boundary layer while the traces at 400 and 500 torr

indicated a laminar to transitional boundary layer.

The P_o sweeps of figure 50 all showed the same decreasing trend with the only difference being the overall signal levels. All traces again indicated symmetry about the vertical centerline.

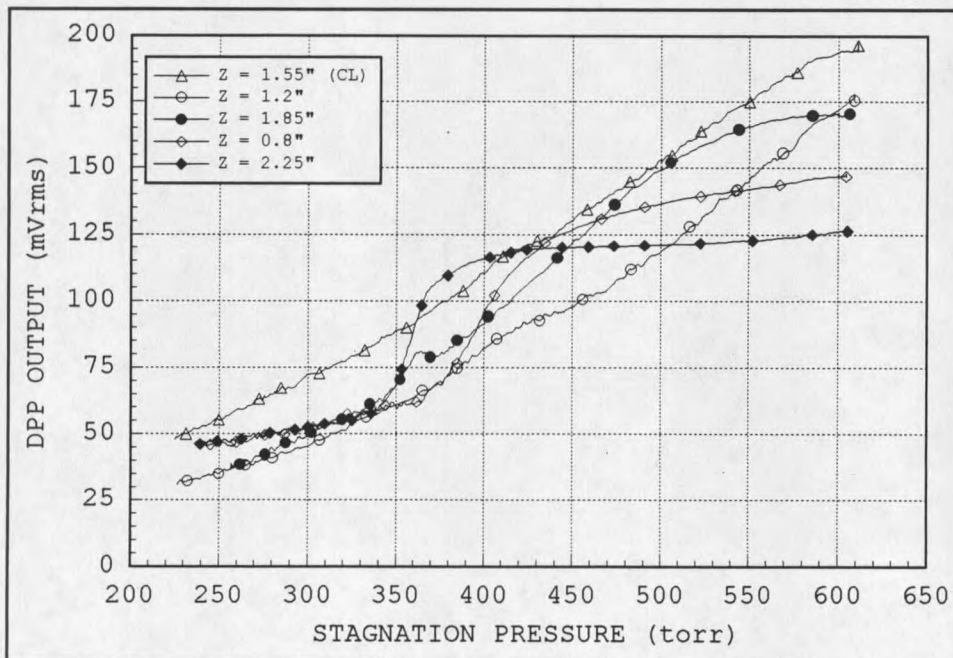


Figure 48.

DPP vs. P_o on floor at $x = 14.5$ inches.

Figure 49.
Boundary layer profiles on floor at $x = 11$ inches.

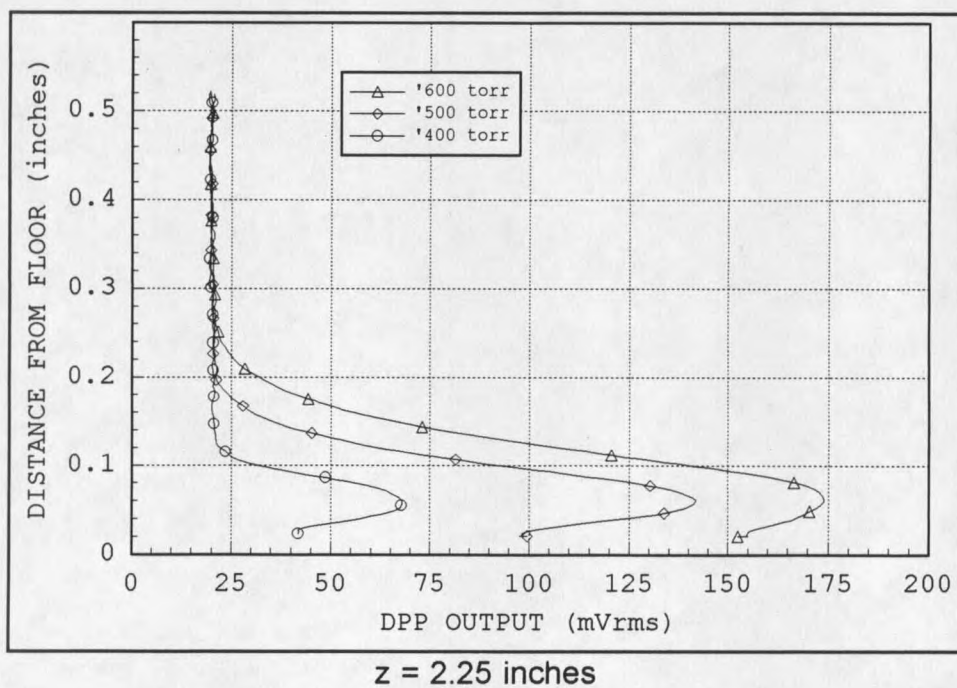
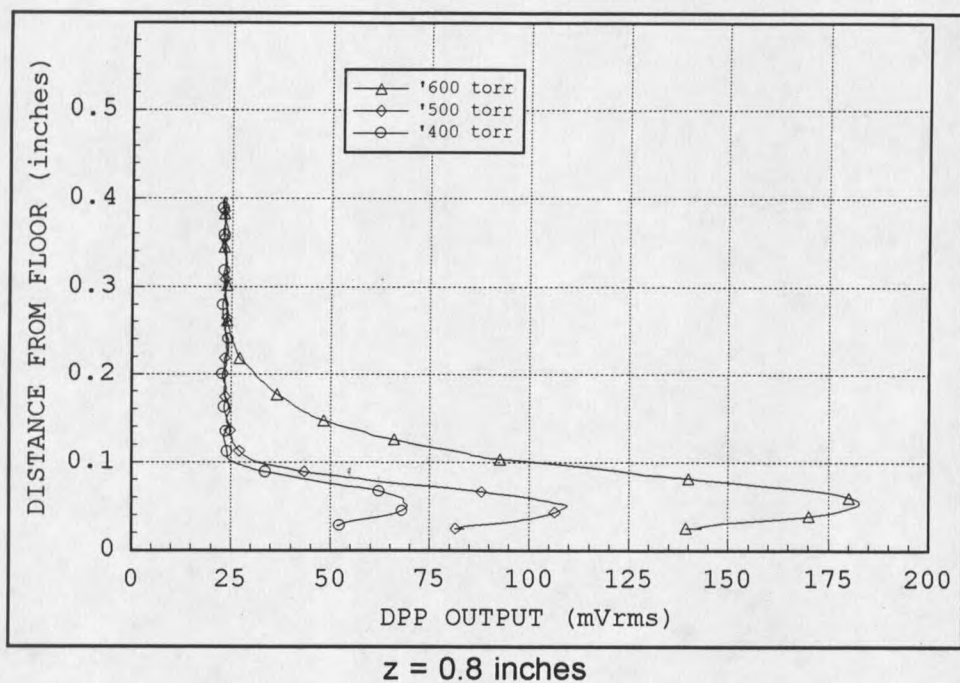


Figure 49.--Continued

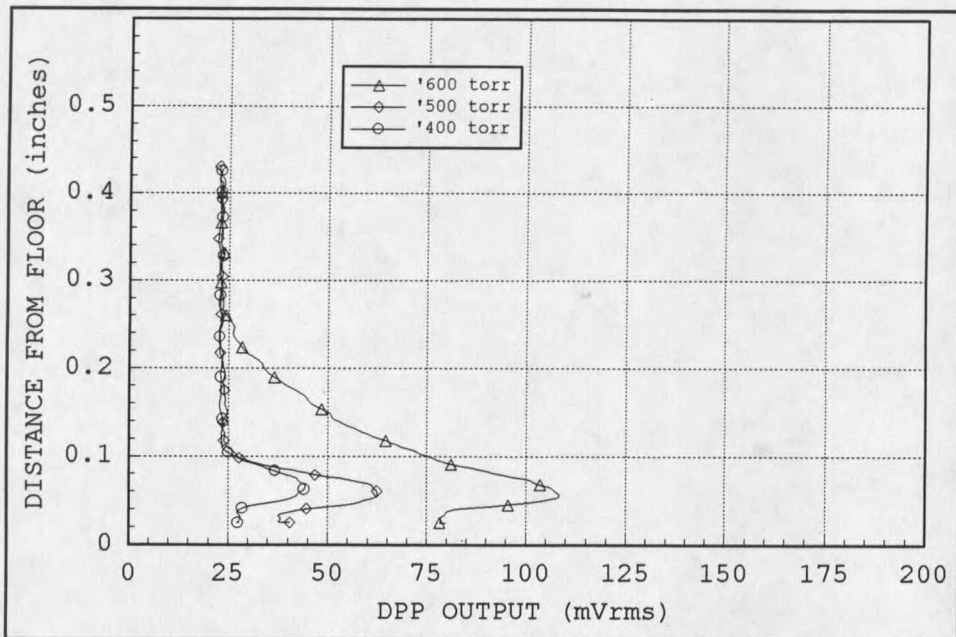
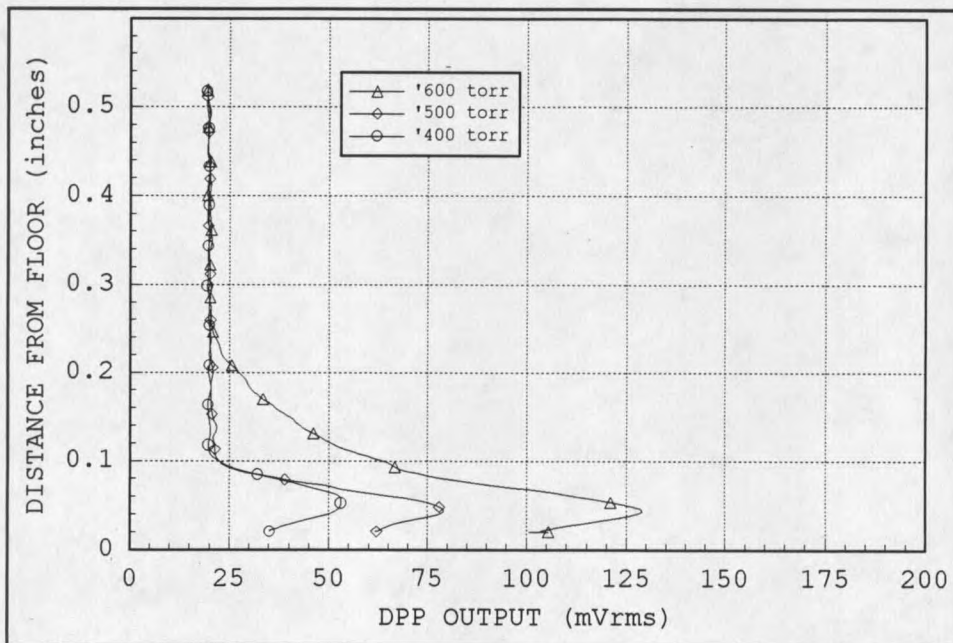
 $z = 1.2$ inches $z = 1.85$ inches

Figure 49.--Continued

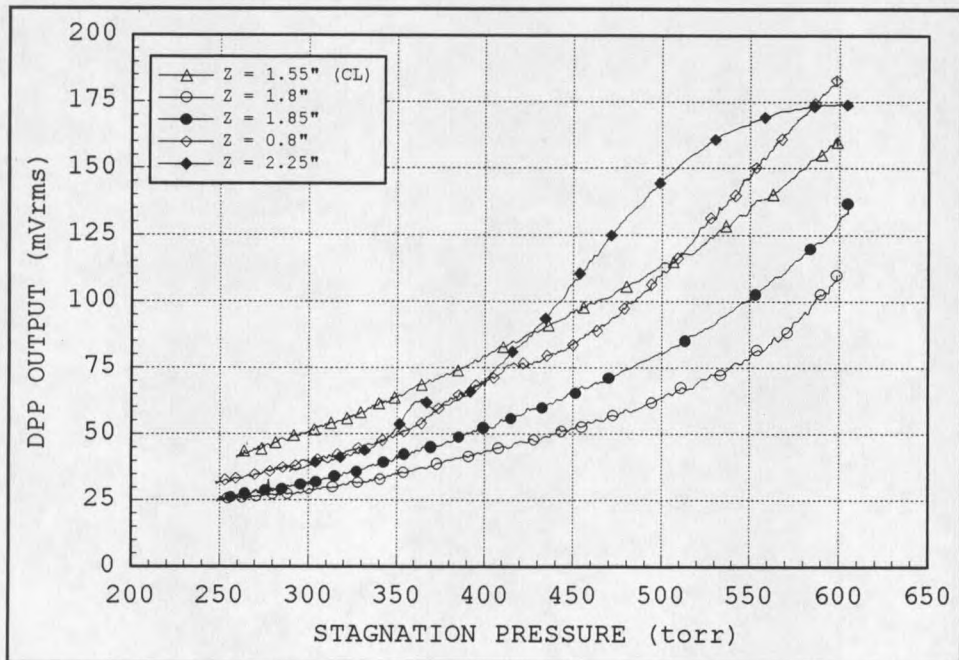
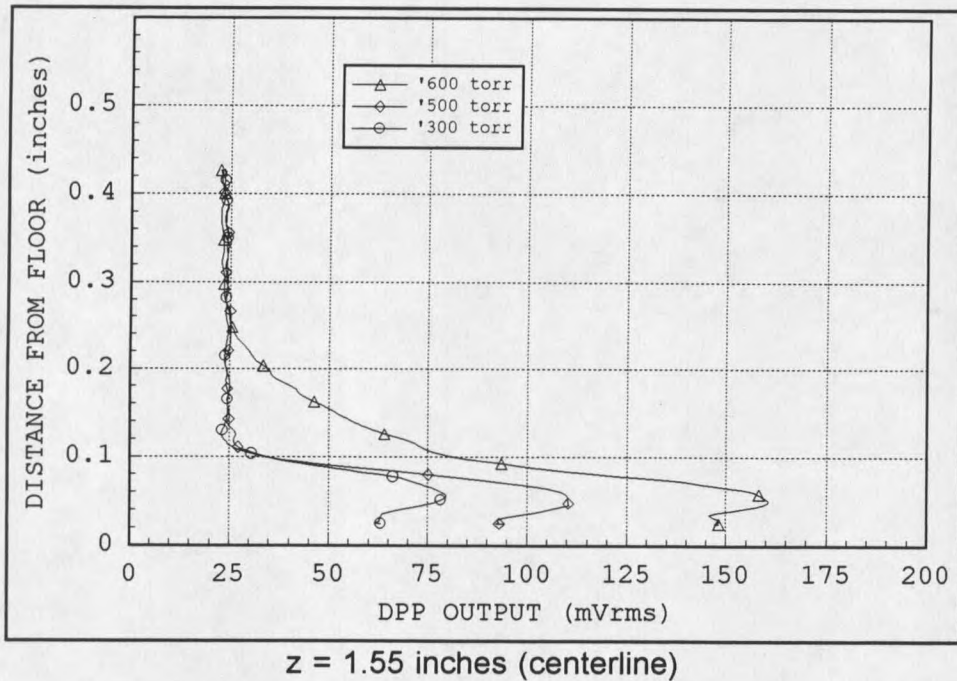


Figure 50.

DPP vs. P_o on floor at $x = 11$ inches.

Sidewall Results

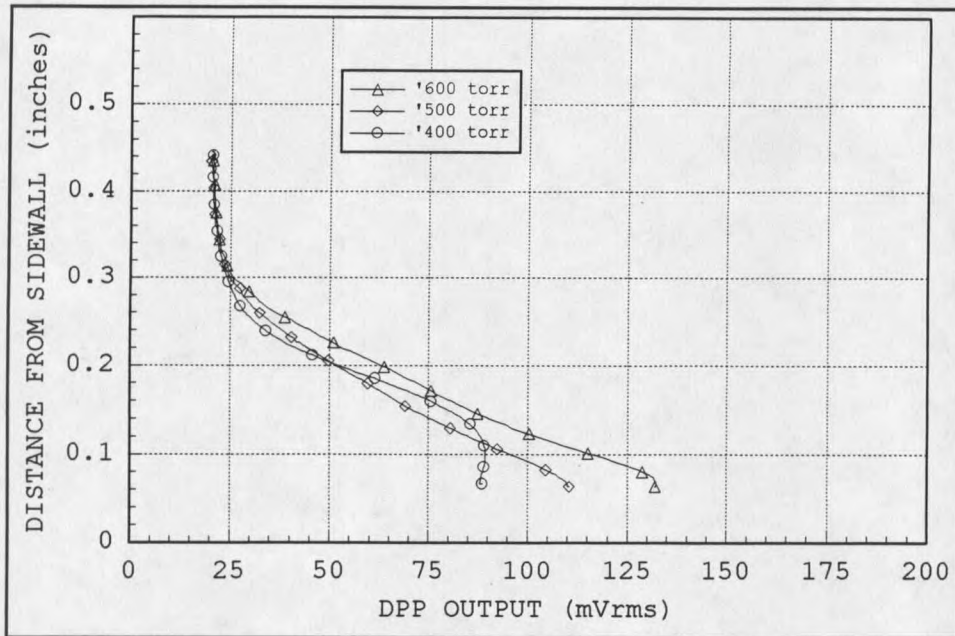
Figure 51 presents the profiles on both the north and south sidewalls at $x = 16.5$ ". As can be seen from the figure, the method of probe placement became extremely arbitrary with regard to the P_o sweeps on the sidewalls. The P_o sweeps on the sidewalls provided no useful information regarding transition and boundary layer behavior. This was due to the large and unpredictable shifts of the signal peak of the profiles on the sidewalls. The shifting of the signal peak tended to produce highly erratic results in the P_o sweeps which made interpretation of the P_o sweeps nearly impossible.

Interpretation of the sidewall boundary layer profiles was also open to much speculation. On the north sidewall at $x = 16.5$ " and $y = 0.34$ " it was believed the unusual nature of the profiles, as compared to the other y positions, was due to the combined effect of the floor and sidewall boundary layers. At all y positions the signal peaks were generally lower than the peaks observed on the floor. At positions near the floor the 500 and 600 torr profiles tended to be turbulent in nature with the 400 torr profiles appearing to be transitional to laminar. At the positions furthest from the floor profiles on both sidewalls appeared turbulent at all pressures.

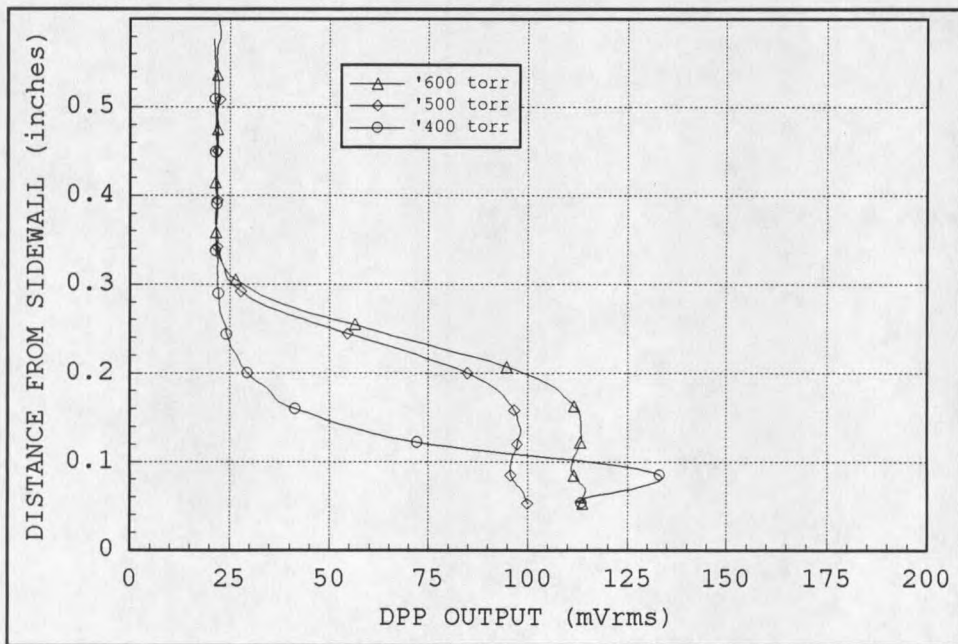
Although the y positions on both sidewalls did not match exactly, the y positions closest to each other on the north and south sidewalls did indicate some symmetry of the boundary layer behavior on the two sidewalls at the higher pressures.

Figure 51.

Boundary layer profiles on the north and south sidewalls at $x = 16.5$ inches.



north sidewall, $y = 0.34$ inches



south sidewall, $y = 0.50$ inches

Figure 51.--Continued

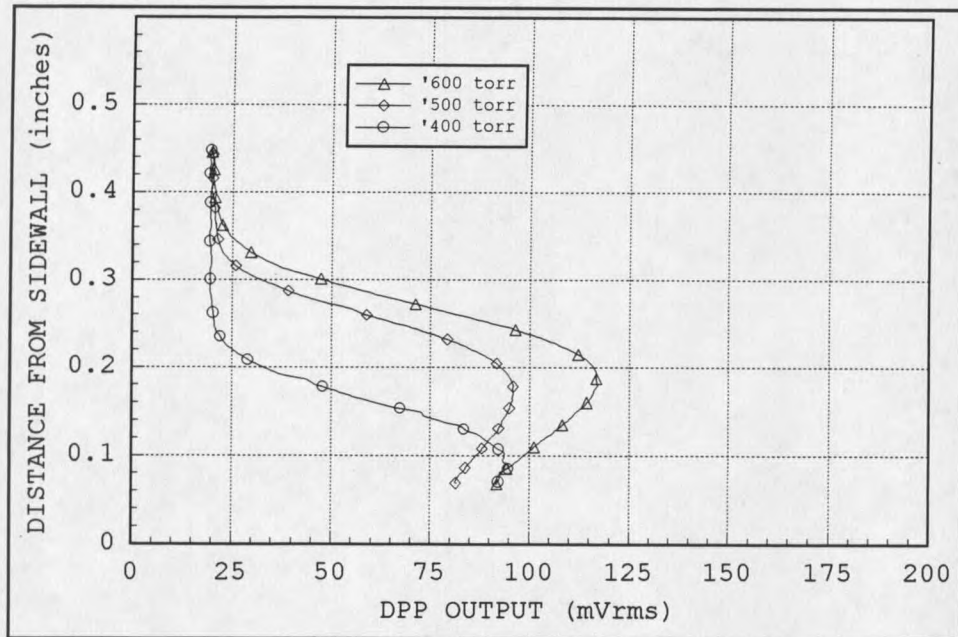
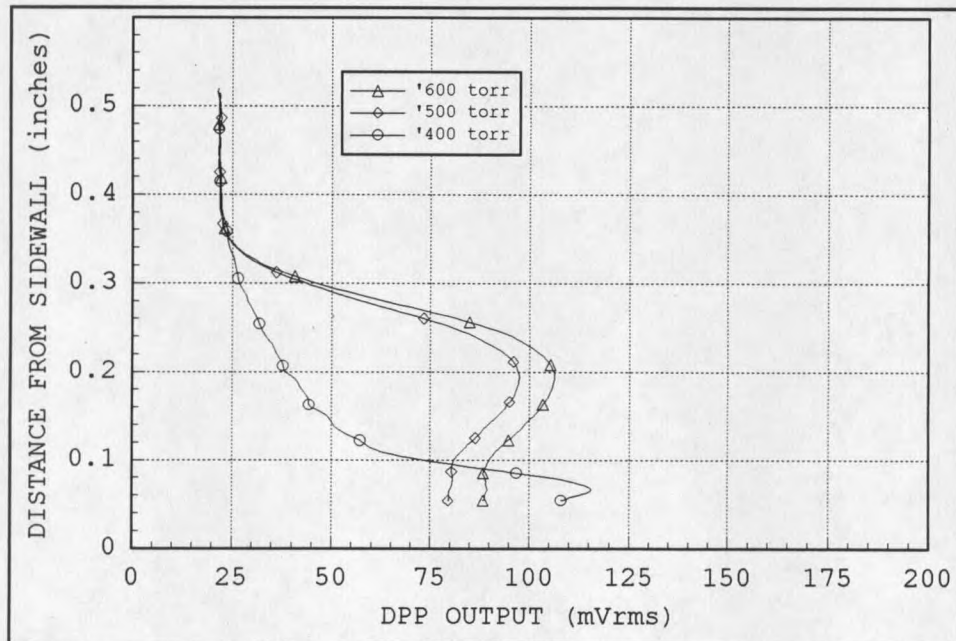
north sidewall, $y = 0.74$ inchessouth sidewall, $y = 0.90$ inches

Figure 51.--Continued

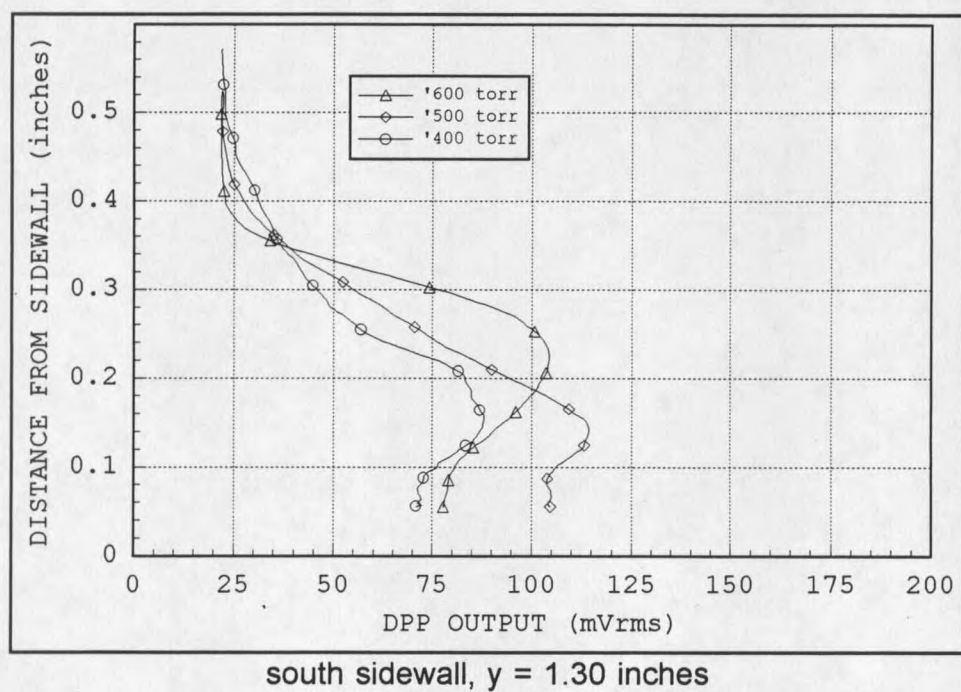
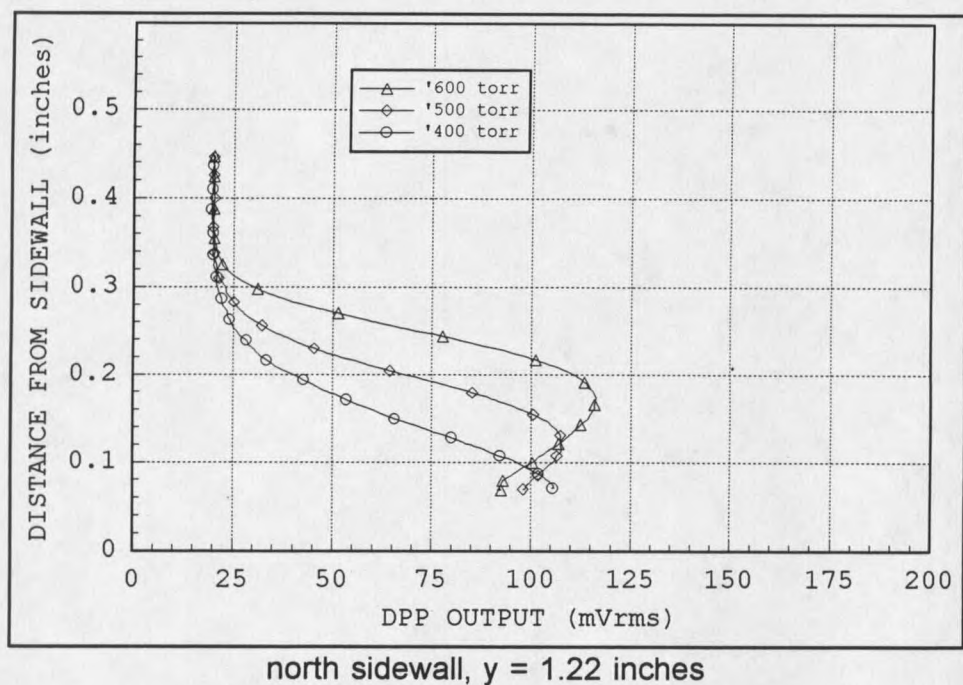


Figure 52 shows the profiles at $x = 11"$. In the recording of data for figure 52 there was a problem with signal noise on the north sidewall at $y = 1.30"$ which produced the anomalous readings evident at 400 and 500 torr.

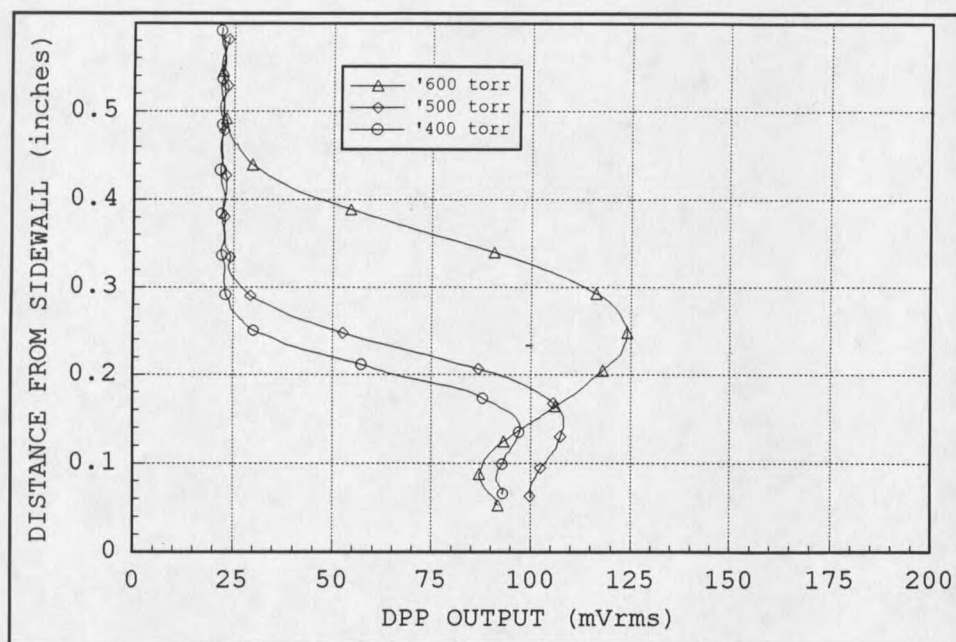
The profiles of the north and south sidewalls generally indicated similar patterns especially at $y = 0.9"$ and 1.3 . At these two positions it was possible to make a direct comparison of the north and south sidewalls. At $y = 0.9"$ on both sidewalls the profiles indicated turbulence at 600 and 500 torr. At 400 torr the status of the boundary layer was not clear although there appeared to be evidence of a more laminar profile.

At $y = 1.3"$ the 600 torr profile appeared turbulent on both sidewalls. At 500 torr profiles on both sidewalls indicated a transitional boundary layer. At 400 torr the profiles showed some indication of a laminar boundary layer.

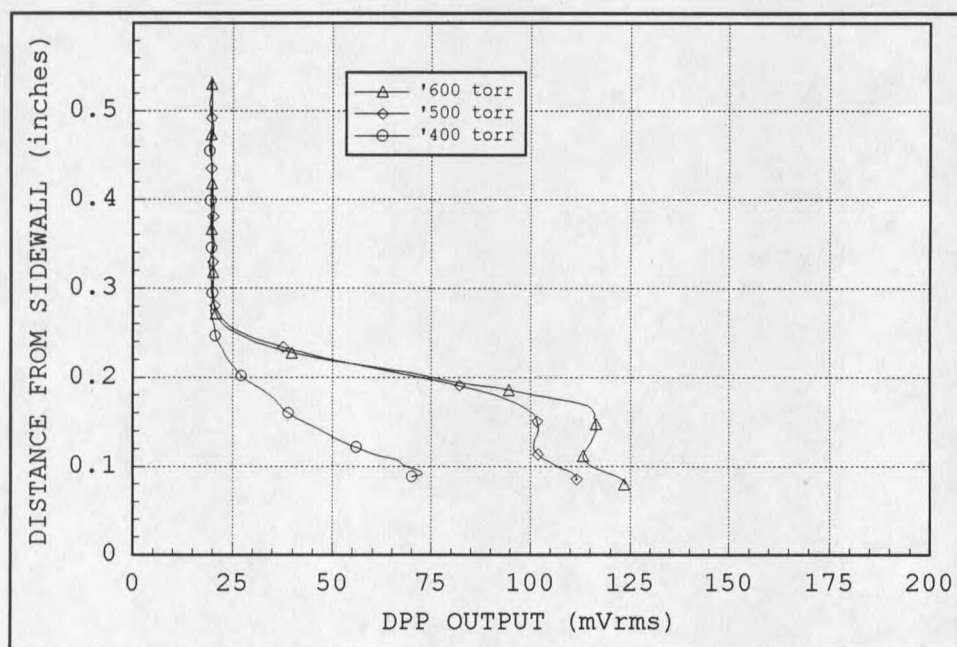
At the y positions nearest the floor it was not possible to compare the results of the two sidewalls. However, the profiles at all pressures on both sidewalls did appear to indicate a turbulent boundary layer. The large offset of the 600 torr trace on the north sidewall at $0.7"$ was believed to be due to an error in the recording of the data.

Figure 52.

Boundary layer profiles on north and south sidewalls at $x = 11$ inches.



north sidewall, $y = 0.70$ inches



south sidewall, $y = 0.50$ inches

Figure 52.--Continued

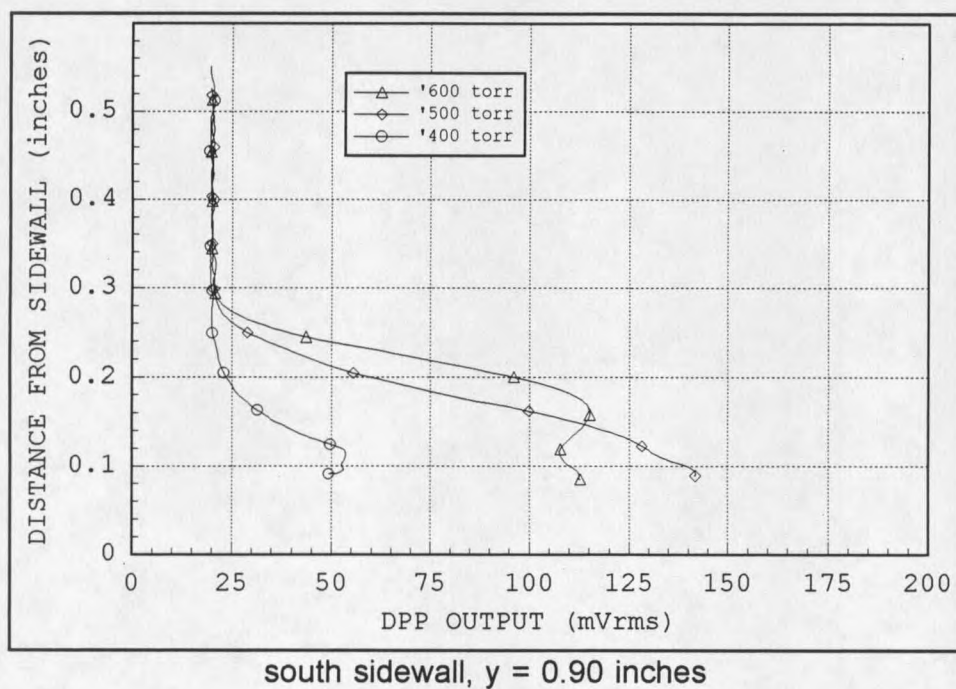
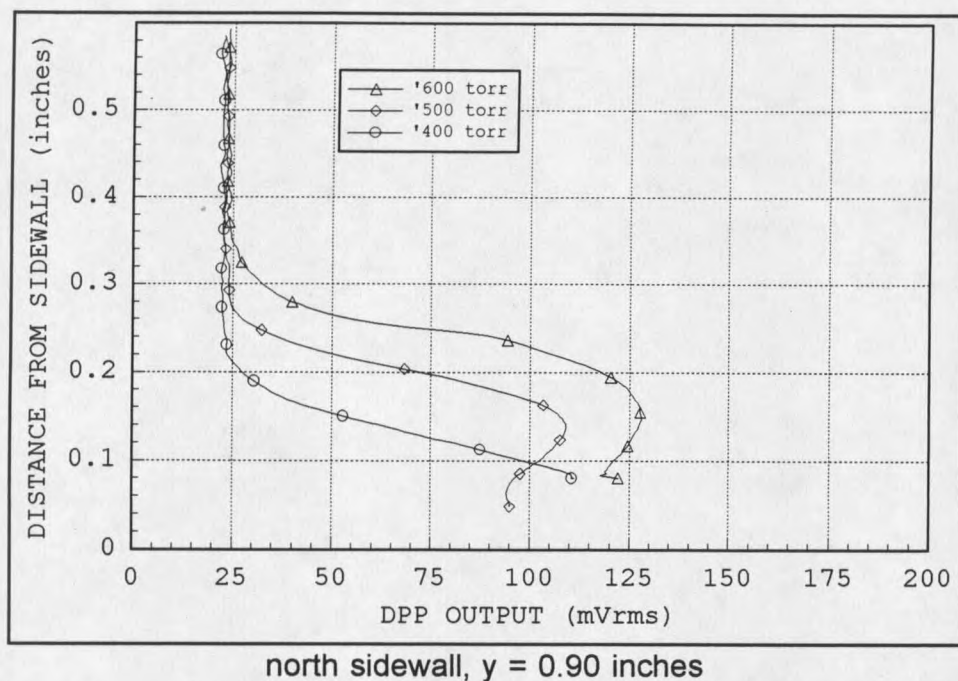
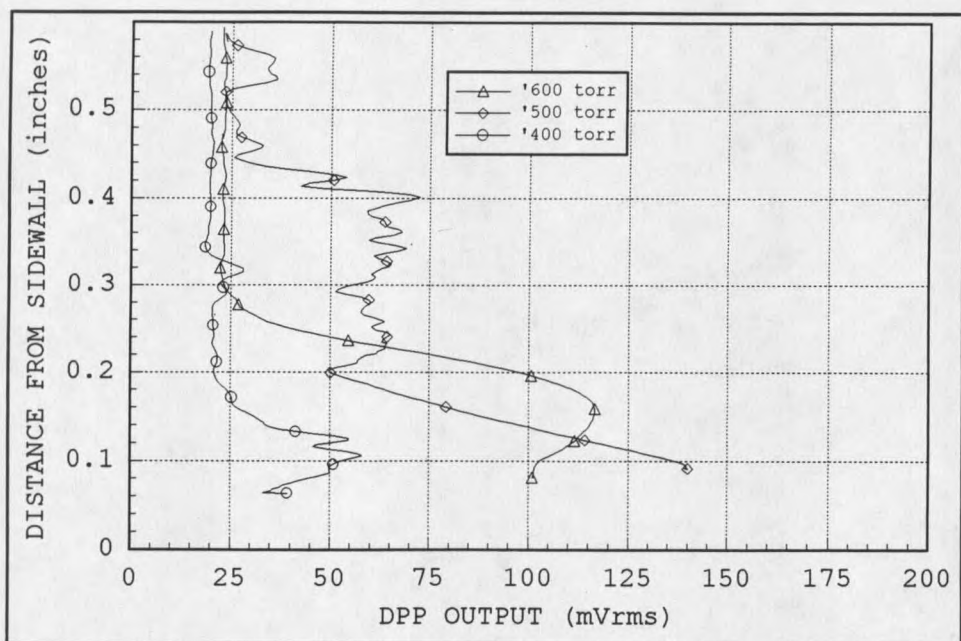
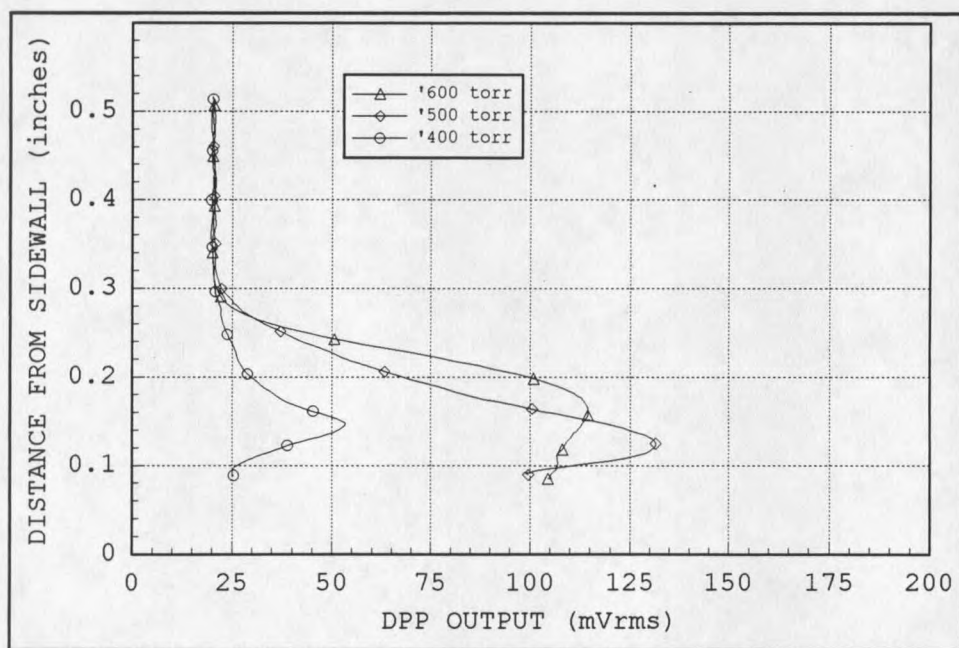


Figure 52.--Continued

north sidewall, $y = 1.30$ inchessouth sidewall, $y = 1.30$ inches

CONCLUSIONS

One of the key points observed throughout the investigation was the fact that a turbulent boundary layer produced a lower signal level from the DPP than a transitional boundary layer. This was due to the fact that the transitional boundary layer combined high frequency fluctuations with relatively large amplitude low frequency fluctuations. For this reason the maximum output signal from the DPP tended to occur prior to the boundary layer becoming fully turbulent. With the boundary layer fully turbulent the low frequency fluctuations were generally negligible leaving only the high frequency fluctuations thus producing a lower overall output signal from the DPP. Dramatic increases in the DPP output were also observed with the first departure of the boundary layer from fully laminar conditions. This was primarily due to the rapid increase of the low frequency fluctuations.

Transition Process

Based on the results of the first portion of the investigation, the DPP vs. P_o survey, it was found that transition of the boundary layer growing on the nozzle of the MSU-SWT exhibited four distinct regions. These regions were defined as completely laminar, laminar unstable, unstable with bursts and fully turbulent. These four regions corresponded to the four regions typically found in the general boundary layer transition process.

One of the most significant findings of this investigation was the evidence

of low frequency pressure fluctuations in the boundary layer preceding the occurrence of turbulent bursts. This finding introduced the possibility that transition of the nozzle boundary layer was driven by instability and not dominated by surface roughness. Two other findings also supported this possibility.

First, it was found that the low frequency fluctuations had a definite tendency to amplify with increasing Re'_{∞} . Sometime during this period of growth the first appearance of turbulent bursts was observed. Once the boundary layer became fully turbulent the low frequency activity diminished to a negligible level. This progression of low frequency amplification leading to turbulence is a classic example of boundary layer instability.

Second, it was found during the x-wise transition survey that the transition process required a substantial portion of the nozzle length for completion. At 600 torr the nozzle boundary layer remained transitional over nearly the entire nozzle length. In transition dominated by surface roughness the boundary layer is expected to proceed from laminar to turbulent in an abrupt manner. During the entire investigation there was never any evidence of transition occurring abruptly on the nozzle of the SWT.

Effects of Screen

Overall the screen seemed to have little effect in altering the boundary layer transition process. There were some definite differences between the results with screens and without, but these differences were mainly limited to the lower

pressures.

One of the most notable effects of the screen was observed in the transition maps and the frequency spectra. This was the apparent ability of the screen to delay the development of low frequency fluctuations in the boundary layer. However, this result was limited to the lower pressures and had no effect in delaying the onset of actual turbulence (the appearance of bursts). Another curious effect was that at low pressure the frequency of turbulent bursts tended to be lower with screens than without screens.

At higher pressure the screen had almost no effect on the transition process. The only noticeable difference was a slight increase in high frequency activity with the screen in place. This result indicated the screen had a slightly adverse effect on delaying transition at higher pressure.

Spatial Distribution of Transition

It was difficult to arrive at any definite results concerning transition on the tunnel sidewalls due to the highly non-uniform flow encountered there. The most stable characteristic of the sidewall boundary layer was the peak signal level. At constant pressure the peak signal level remained fairly constant at all y and x locations. The boundary layers on both sidewalls did appear to be similar in nature indicating the possibility of symmetrical boundary layers on the north and south sidewalls. Considering all transition results from the sidewalls provided some indication that transition occurred sooner on the sidewalls than on the floor.

One of the key conclusions drawn from the profiles on the floor was that the location of transition did vary across the width of the tunnel. At $x = 14.5$ " and 11" the profiles indicate rather clearly that the onset of turbulence on the nozzle surface occurs near the sidewalls prior to the onset of turbulence near the vertical tunnel centerline. This result also clarified the abnormal distribution of the peak signal level across the width of the tunnel.

At fixed conditions the major change in the floor boundary layer was the peak signal level at the various z positions. At $x = 16.5$ " and 14.5" the signal tended to be high near the center of the tunnel and lower near the sidewalls. At $x = 11$ " the reverse was true, with high signal levels near the sidewalls and lower signal levels near the centerline. These two results were readily explained by the onset of turbulence near the sidewalls prior to the onset near the vertical centerline.

At $x = 16.5$ " the low signal level near the sidewalls indicated turbulence while the high signal level near the centerline indicated a transitional boundary layer. At $x = 14.5$ " the effect was more pronounced. The boundary layer signal near the sidewalls remained relatively unchanged, while the central positions exhibited rapid decreases in signal level with decreasing pressure indicating the boundary layer approaching fully laminar conditions. At $x = 11$ " the three central profiles showed a definite trend toward fully laminar conditions with decreasing pressure, while the positions nearest the sidewall clearly indicated a transitional boundary layer.

Although the boundary layer appeared non-uniform across the width of the tunnel floor, there was strong evidence suggesting the boundary layer was symmetric about the vertical centerline of the tunnel. Consideration of the sidewall results tend to indicate the vertical centerline to be a complete plane of symmetry for the SWT. Although it could not be verified with the existing experimental setup, it is possible that the horizontal centerline of the tunnel could also be a plane of symmetry with the boundary layer on the upper half of the wind tunnel resembling the boundary layer on the lower half. To this extent the only evidence obtained was provided by the transition maps. The maps from the ceiling being nearly identical to the maps from the floor.

Concluding Remarks

The main problem addressed by this investigation was the development of "quiet" wind tunnels. Radiated noise from turbulent boundary layers on wind tunnel surfaces is currently the main source of freestream disturbances in conventional wind tunnels. By attempting to develop an understanding of the boundary layer transition process on the wind tunnel surfaces themselves the results of this investigation have direct implications in the development of future "quiet" wind tunnels.

Although the investigation fell short of the ultimate goal, a full stability analysis of a boundary layer developing on the nozzle of a $M_\infty = 3.0$ wind tunnel, sufficient evidence was encountered to indicate the transition process was

instigated by boundary layer instability not associated with surface roughness. It is felt that a thorough investigation of this instability concerning its origins and behavior would be an additional step towards the understanding of boundary layer transition on wind tunnel surfaces. With understanding of this process comes the eventual possibility of its prediction and control. If it were possible to control or modify the boundary layer transition process on wind tunnel surfaces, this would provide a key element in the development of "quiet" wind tunnels suitable for transition research on models.

It is important to understand that the ultimate goal of understanding boundary layer transition represents an immensely complex task. For decades countless scientists and researchers from several different fields have been working towards a better understanding of the boundary layer transition process. The research conducted during this investigation addresses only a small and highly specific portion of the overall problem. It is hoped that the findings of this investigation will provide some insight and guidance for future investigations of this nature.

Suggested Future Research

It is felt that a complete boundary layer stability analysis would be the next logical step based on the findings of this investigation. Several suggestions for this type of investigation are as follows.

The DPP would be the instrument of choice for this type of investigation.

This probe has the desired frequency response characteristics and provides an acceptable resolution in the boundary layer. The DPP has the additional advantage of responding only to pitot pressure fluctuations as opposed to hot film anemometers which respond to pressure, temperature and velocity fluctuations. This fact greatly simplifies the data reduction process.

Prior to any actual measurements, several points must be addressed. First, a suitable method of determining probe placement in the boundary layer must be developed. It is necessary that the probe be placed in the boundary layer in such a manner that all fluctuations, both low and high frequency, be visible to the probe. Second, a suitable amplifier must be found in order to reduce the signal noise generated in the recording of data to acceptable levels. Third, if quantitative stability results are sought, the DPP must be properly calibrated to determine the output characteristics.

The boundary layer stability should be considered as a function of x at constant pressure. It would be necessary for the measurements to be made with the smallest increment in x position possible. An x increment of 0.25" or less should be suitable. Ideally the measurements should be evenly spaced over the entire portion of the boundary layer being considered.

Only certain portions of the transition process need to be considered. The portion of the boundary layer most critical to this type of investigation is the laminar unstable zone. Measurements made in the boundary layer should encompass the completely laminar region and continue up to the first appearance of turbulent

bursts.

The main requirements of the stability analysis will be the frequency spectrum of the DPP output signal as a function of x . From the frequency spectra it is possible to determine the amplification rates of individual spectrum components. It should then be possible to determine which spectrum components and amplification rates lead to the first appearance of turbulent bursts. The final step would be to compare the results with compressible boundary layer stability theory.

The nozzle should be the primary surface of interest. Initially one z position and two pressures should be sufficient to determine the type of instability leading to transition. The next step would be to consider z positions across the width of the nozzle and also positions on the sidewalls. A final step would be to consider the effects of turbulence screens.

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REFERENCES CITED

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APPENDIX

Program for Calculation of Boundary Layer Properties

Figure 53.

Program for calculation of boundary layer properties.

```

* this code is used to compute various boundary layer properties
* for a compressible flow with variable free stream properties
*****
*   written by: ROB MUELLER
*   date:      2/25/93
*   for:       MSU-SWT
*****
      PROGRAM BL
*****
* this is the main line portion of the program its basic function
* is the organization of the various functions and subroutines
* necessary for the calculations
*****
* including the global declarations
      INCLUDE 'COMMON.INF'

* setting the GLOBAL tolerance for the root function
      TOL = 1D-6

* setting values of GLOBAL parameters
*   s-refers to standard conditions taken to be 20 Celcius, 1 atm
*   u-dynamic viscosity at STP
*   v-kinematic viscosity at STP
*   C-proportionality constant relating absolute viscosity to
*   temperature
      Ts = 273D0
      Ps = 99000D0
      us = 16.8D-6
      vs = 13.0D-6
      C = 1D0

* variables
      DOUBLE PRECISION Z0

* begin program BL

* calling the subroutine to input the stagnation conditions
      CALL USR_INPUT(Z0)

* reading in the nozzle coords
      CALL GET_NOZ()

* calling the subroutine to calculate Me(x), Ue(x) and dUe(x)
      CALL CALC_A()

      do 200 i = 1, pts, 1
        write(*,*) 'mach number', Me(i)
200    continue

* obtaining lambda as a function of x
      CALL DIF_SOLN(Z0)

* calculating the boundary layer growth properties
      CALL BL_PROPS()

      END

```

Figure 53.--Continued

```

SUBROUTINE USR_INPUT(P3)
*****
* subroutine to input the stagnation conditions and the initial value
* required for the solution of the differential equation
*****
* including the GLOBAL parameters
  INCLUDE 'COMMON.INF'

* functions called
  DOUBLE PRECISION SQRT

* parameters
*   P3-initial condition for the differential equation
  DOUBLE PRECISION P3

* begin subroutine
  WRITE(*,*) 'ENTER THE STAGNATION PRESSURE (mm Hg)      :>'
  READ(*,*) Po
  WRITE(*,*) 'ENTER THE STAGNATION TEMPERATURE (DEGREES F):>'
  READ(*,*) To
  WRITE(*,*) 'ENTER THE INITIAL VALUE OF Z AT THE SONIC THROAT:>'
  READ(*,*) P3

* converting temperature to degrees kelvin and pressure to pascals
  Po = (Po/760D0)*101300D0
  To = (To + 460D0)*(5D0/9D0)
  END

SUBROUTINE CALC_A()
*****
* calculates and stores Me(x), Ue(x), dUe(x)
*****
* including the global parameters
  INCLUDE 'COMMON.INF'

* functions called
  DOUBLE PRECISION VEL, DIFF_VEL, ROOT

* constant parameters
*   W-----tunnel width (3", 0.0762m)
*   AST---area at the sonic throat (0.001370706m^2)
*   dUx0--velocity derivative with respect to position
*           at the throat (assumed a value of 4000 (m/s)/m)
*   dUxEND-velocity derivative at final position (assumed to be zero)
*   M1----initial guess of the mach number (M1 = 1 at sonic throat)
  DOUBLE PRECISION dUx0, M1
  W      = 0.0762D0
  AST    = 0.001370706D0
  dUx0   = 4000D0
  dUxEND = 0D0
  M1     = 1D0

* variables
  DOUBLE PRECISION A_RATIO, ALC, Y1, X1, X2
  INTEGER i

```

Figure 53.--Continued

```

* begin subroutine CALC_A
  Me(1) = M1
  Ue(1) = VEL(Me(1), To)
  dUe(1) = dUx0
  DO 100 i=2, PTS, 1
    Y1 = Y(i)*(2.54D0/100D0)
    ALC = 2D0*Y1*W
    A_RATIO = ALC/AST
    Me(i) = ROOT((Me(i-1) + 1D0), A_RATIO, TOL, 1)
    Ue(i) = VEL(Me(i), To)
100  CONTINUE

  DO 200 i=2, PTS - 1, 1
    X1 = X(i-1)*(2.54D0/100D0)
    X2 = X(i+1)*(2.54D0/100D0)
    dUe(i) = DIFF_VEL(Ue(i-1), X1, Ue(i+1), X2)
200  CONTINUE

  dUe(PTS) = dUxEND

  END

*****
* these are the final routines used to determine the actual boundary
* layer properties
*****
  SUBROUTINE BL_PROPS()
*****
* main organizational subroutine to direct the computation of the
* various quantities of interest
*****
* including the global parameters
  INCLUDE 'COMMON.INF'

* functions called
    DOUBLE PRECISION DELTA, DEL ST, THETA, LOCAL T, LOCAL_P
    DOUBLE PRECISION PROP1, PROP2, PROP3, TAU, SQRT
*   PROP1-bl thickness
*   PROP2-displacement
*   PROP3-momentum thickness
*   TAU-wall shear

* variables
    DOUBLE PRECISION BL(MAX), DISP(MAX), MOM(MAX), SHEAR(MAX)
    DOUBLE PRECISION P1, P2, P3, Te, Pe
*   P1-capital delta
*   P2-capital delta star
*   P3-capital theta
    INTEGER i

* begin subroutine PROP

* calculating the three thicknesses and the wall shear
* all properties will be returned in inches and psi
* i.e. BL, MOM, DISP, SHEAR

```

Figure 53.--Continued

```

DO 100 i=1, PTS, 1

  Te = LOCAL_T(Me(i), To)
  Pe = LOCAL_P(Me(i), Po)

  C = ((Ts + 110.3D0)/(Te + 110.3D0))*SQRT(Te/Ts)

  P1      = DELTA(LAM(i), dUe(i), Me(i), vs)
  P2      = DEL_ST(P1, LAM(i))
  P3      = THETA(P1, LAM(i))
  BL(i)   = PROP1(P1, C, Pe, Te, Ps, Ts, Me(i), LAM(i))
  MOM(i)  = PROP3(P3, C, Pe, Te, Ps, Ts)
  DISP(i) = PROP2(P1, P2, C, Pe, Te, Ps, Ts, Me(i), LAM(i))
  SHEAR(i) = TAU(P1, C, us, Pe, Ps, Ue(i), LAM(i))

100  CONTINUE

* call the subroutine to save the results to disk
  CALL SAVE_PROP(BL, MOM, DISP, SHEAR)

END

      DOUBLE PRECISION FUNCTION DELTA(Ye, dU, M1, v)
*****
* returns the value of capital delta
*****
* functions called
      DOUBLE PRECISION CLMN6, SQRT
* parameters
      DOUBLE PRECISION Ye, dU, M1, v
* variables
      DOUBLE PRECISION A, B

      IF(Ye .EQ. 0) THEN
        DELTA = 0
      ELSE
        A      = CLMN6(M1)
        B      = (Ye*v)/(dU*A)
        DELTA = SQRT(B)
      END IF
END

      DOUBLE PRECISION FUNCTION DEL_ST(D, Ye)
*****
* returns the value of capital delta star
*****
* parameters
      DOUBLE PRECISION D, Ye

      DEL_ST = D*(36D0 - Ye)/120D0
END

      DOUBLE PRECISION FUNCTION THETA(D, Ye)
*****
* returns the value of capital theta
*****

```

Figure 53.--Continued

```

* parameters
  DOUBLE PRECISION D, Ye

  THETA = (D/315D0)*(37D0 - Ye/3D0 - (5*(Ye**2))/144D0)
  END

  DOUBLE PRECISION FUNCTION PROP1(D,C,Pe,Te,Ps,Ts,Ml,Ye)
  *****
  * returns the boundary layer thickness
  *****
  * functions called
    DOUBLE PRECISION SQRT
  * parameters
    DOUBLE PRECISION D, C, Pe, Te, Ps, Ts, Ml, Ye
  * variables
    DOUBLE PRECISION g, A1, A2, A3
    g = 1.4D0

    A1 = ((g - 1D0)/2D0)*(Ml**2)
    A2 = 1 + A1*(-1D-4*(Ye**2) - 9.4D-3*Ye + 0.4175D0)
    A3 = D*(Te/Ts)*SQRT(Ps*C/Pe)
    PROP1 = (A3*A2)*(100D0/2.54D0)
  END

  DOUBLE PRECISION FUNCTION PROP2(D,DS,C,Pe,Te,Ps,Ts,Ml,Ye)
  *****
  * returns the displacement thickness
  *****
  * functions called
    DOUBLE PRECISION SQRT
  * parameters
    DOUBLE PRECISION D, DS, C, Pe, Te, Ps, Ts, Ml, Ye
  * variables
    DOUBLE PRECISION g, A1, A2, A3
    g = 1.4D0

    A1 = ((g - 1D0)/2D0)*(Ml**2)
    A2 = DS + D*A1*(-1D-4*(Ye**2) - 9.4D-3*Ye + .4175D0)
    A3 = (Te/Ts)*SQRT(Ps*C/Pe)
    PROP2 = (A3*A2)*(100D0/2.54D0)
  END

  DOUBLE PRECISION FUNCTION PROP3(O, C, Pe, Te, Ps, Ts)
  *****
  * returns the value of the momentum thickness
  *****
  * functions called
    DOUBLE PRECISION SQRT
  * parameters
    DOUBLE PRECISION O, C, Pe, Te, Ps, Ts

    PROP3 = ((Te/Ts)*O*SQRT(Ps*C/Pe))*(100D0/2.54D0)
  END

```

Figure 53.--Continued

```

      DOUBLE PRECISION FUNCTION TAU(D, C, u, Pe, Ps, Ul, Ye)
      *****
      * returns the value of the wall shear
      *****
      * functions called
      DOUBLE PRECISION SQRT
      * parameters
      DOUBLE PRECISION D, C, u, Pe, Ps, Ul, Ye

      IF(D .EQ. 0) THEN
        TAU = 1D0
      ELSE
        TAU = ((Ul*u/D)*(2D0 + Ye/6D0)*SQRT(Pe*C/Ps))*(14.7D0/101300D0)
      END IF
      END

      *****
      * this is the subroutine to determine lambda as a function of x
      * based on the four step Runge-Kutta method
      *****
      SUBROUTINE DIF_SOLN(Z0)
      *****
      * subroutine to organize and process the Runge_Kutta differential
      * equation solution used to obtain lambda as a function of x
      *****
      * including the global parameters
      INCLUDE 'COMMON.INF'
      * functions called
      DOUBLE PRECISION ROOT, VEL, DIFF_VEL
      * parameters
      * initial value of Z corresponding to the first x location considered
      DOUBLE PRECISION Z0
      * variables
      DOUBLE PRECISION Y2, M2, Ue2, dUe2, K1, K2, K3, K4, DZ
      DOUBLE PRECISION Z, L
      DOUBLE PRECISION H, ALC, A_RATIO, Xa, Xb
      INTEGER i
      * initial guess of lambda
      L = 0.1D0

      * begin subroutine DIF_SOLN

      DO 100 i=1, PTS-1, 1

        H = (X(i+1) - X(i))*(2.54D0/100D0)
        Y2 = ((Y(i+1) + Y(i))/2D0)*(2.54D0/100D0)
        ALC = 2D0*Y2*W
        A_RATIO = ALC/AST
        M2 = ROOT((Me(i) + 1D0), A_RATIO, TOL, 1)
        Ue2 = VEL(M2, To)
        Xa = X(i)*(2.54D0/100D0)
        Xb = X(i+1)*(2.54D0/100D0)
        dUe2 = DIFF_VEL(Ue(i), Xa, Ue(i+1), Xb)

        Z = Z0

```

Figure 53.--Continued

```

CALL RUNGE(Me(i), Ue(i), dUe(i), Z, L, K1, TOL, H, i)
LAM(i) = L
Z      = Z0 + (1D0/2D0)*K1

CALL RUNGE(M2, Ue2, dUe2, Z, L, K2, TOL, H, i)
Z      = Z0 + (1D0/2D0)*K2

CALL RUNGE(M2, Ue2, dUe2, Z, L, K3, TOL, H, i)
Z      = Z0 + K3

CALL RUNGE(Me(i+1), Ue(i+1), dUe(i+1), Z, L, K4, TOL, H, i)
DZ     = (1D0/6D0)*(K1 + 2D0*(K2 + K3) + K4)
Z0     = Z0 + DZ

100  CONTINUE

* setting the final value of lambda to zero
  LAM(PTS) = 0

  END

  SUBROUTINE RUNGE(C4, C2, C3, C7, C9, C11, TOL, H, i)
*****
* actual runge-kutta calculations, parameters with Cx identify the
* columns of the runge-kutta work sheet
*****
* functions called
  DOUBLE PRECISION FM, Fn2, CLMN6, ROOT
* parameters
  DOUBLE PRECISION C2, C3, C4, C7, C9, C11, TOL, H
* variables
  DOUBLE PRECISION C5, C6, C8, C10, EST
  integer i

* begin subroutine RUNGE
  C5 = FM(C4)
  C6 = CLMN6(C4)
  C8 = C7*C3*C6

  IF(C9 .GT. 12) THEN
    EST = 1D0
  ELSE
    EST = C9 - 1D0
  END IF

  IF(C8 .EQ. 0) THEN
    C9 = 0
  ELSE
    C9 = ROOT(EST, C8, TOL, 2)
  END IF

  C10 = Fn2(C9)
  C11 = (2D0/C2)*(C8*C5 + C10)*H

  END

```

Figure 53.--Continued

```

*****
* function definitions for the boundary layer code; these functions
* correspond to routines necessary for the Runge-Kutta solution
* of the differential equation to obtain lambda as a function of x
*****

      DOUBLE PRECISION FUNCTION ROOT(x0,VAL,TOL, EQ)
*****
* this function returns the root of the specified function
* either the function Fn (f1) or the mach number from the area ratio
*****
* parameters
*   x0--the initial guess
*   VAL--the value of the function used to find the corresponding root
*   TOL--tolerance level used in determining the root
*   EQ--1 for mach number      2 for f1 of lambda
      DOUBLE PRECISION x0, VAL, TOL
      INTEGER EQ

* variables
*   x1,x2  estimates of the root
*   V1,V2  residuals of the function at x1 and x2
*   a,b    slope and intercept of the line between (V1,x1) and (V2,x2)
*   n,max  loop index and max # of iterations for convergence
*   h      step size
      DOUBLE PRECISION x1, x2, V1, V2, a, b, h
      INTEGER n, MAXX

* functions called
      DOUBLE PRECISION ABS, Fn1, SLOPE, INTCPT, M_NUM

***** begin function ROOT

      MAXX = 10000
      n = 0

      IF(EQ .EQ. 2) THEN
        IF(ABS(Fn1(x0, VAL)) .LT. TOL) THEN
          ROOT = x0
          RETURN
        ELSE
          x1 = x0
        END IF
      ELSE
        IF(ABS(M_NUM(x0, VAL)) .LT. TOL) THEN
          ROOT = x0
          RETURN
        ELSE
          x1 = x0
        END IF
      END IF

* loop while n < MAXX
100  CONTINUE
      IF(n .LT. MAXX) THEN

```

Figure 53.--Continued

```

      IF(x1 .EQ. 0) THEN
        h = TOL
      ELSE
        h = TOL*x1
        x2 = x1 + h
      END IF

      IF(EQ .EQ. 2) THEN
        V1 = Fn1(x1, VAL)
        V2 = Fn1(x2, VAL)
      ELSE
        V1 = M_NUM(x1, VAL)
        V2 = M_NUM(x2, VAL)
      END IF

      a = SLOPE(V1, x1, V2, x2)
      b = INTCPT(a, V2, x2)
      x1 = b

      IF(EQ .EQ. 2) THEN
        IF(ABS(Fn1(x1, VAL)) .LT. TOL) THEN
          ROOT = x1
          RETURN
        ELSE
          n = n + 1
        END IF
      ELSE
        IF(ABS(M_NUM(x1, VAL)) .LT. TOL) THEN
          ROOT = x1
          RETURN
        ELSE
          n = n + 1
        END IF
      END IF
      GOTO 100
    END IF
  * end while loop
  * if the root is not found in MAXX iterations, then return 0
  ROOT = 0D0
  END

      DOUBLE PRECISION FUNCTION Fn1(y,f)
      *****
      * this function computes the residual of the specified function
      * evaluated at y with the constant f as a parameter
      *****
      * parameters
        DOUBLE PRECISION y, f, A

      * this is the function whose root is sought
        A = y*((1D0/315D0)*(37D0 - y/3D0 - (5D0*(y**2D0))/144D0))**2D0)
        Fn1=A - f
      END

```

Figure 53.--Continued

```

      DOUBLE PRECISION FUNCTION M_NUM(y, f)
      *****
      * this function returns the residual of the mach number dependence on
      * area ratio for the estimate y for a ratio of f
      *****
      * parameters
      DOUBLE PRECISION y, f
      * variables
      DOUBLE PRECISION A, B, C, g, K
      g = 1.4D0
      K = (g + 1D0)/(2*(g - 1D0))
      A = 2/(g + 1D0)
      B = (g - 1D0)/(g + 1D0)
      C = y*f

      M_NUM = (A + B*(y**2D0))**K - C
      END

```

```

      DOUBLE PRECISION FUNCTION SLOPE(x1,y1,x2,y2)
      *****
      * computes the slope of the line between (x1,y1) and (x2,y2)
      *****
      * parameters
      DOUBLE PRECISION x1, y1, x2, y2

      SLOPE = (y2 - y1)/(x2 - x1)
      END

```

```

      DOUBLE PRECISION FUNCTION INTCPT(a,x,y)
      *****
      * determine the intercept of the line with slope a through point (x,y)
      *****
      * parameters
      DOUBLE PRECISION a, x, y

      INTCPT = y - a*x
      END

```

```

      DOUBLE PRECISION FUNCTION Fn2(y)
      *****
      * determine the value of the function f2 at the supplied value of lambda
      * this corresponds to column 10 of the Runge-Kutta Algorithm
      *****
      * parameters
      DOUBLE PRECISION y
      * variables
      DOUBLE PRECISION A, B

      A = (1D0/315D0)*(37D0 - y/3D0 - 5D0*(y**2D0)/144D0)
      B = (2D0 - 2D0*y/15D0 + (y**2D0)/120D0)
      Fn2 = A*B
      END

```

Figure 53.--Continued

```

      DOUBLE PRECISION FUNCTION FM(M)
      *****
      * mach number function (column 5 in the Runge-Kutta algorithm)
      *****
      * parameters
      *   M-local mach number
      *     DOUBLE PRECISION M
      * variables
      *     DOUBLE PRECISION g
      *     g = 1.4D0

      FM = (M**2D0 - 4D0)/(2D0 + (g - 1D0)*M**2D0)
      END

```

```

      DOUBLE PRECISION FUNCTION CLMN6(M)
      *****
      * returns the value of column 6 in the Runge-Kutta algorithm
      *****
      * parameters
      *   M-local mach number
      *     DOUBLE PRECISION M
      * variables
      *     DOUBLE PRECISION g
      *     g = 1.4D0

      CLMN6 = 1D0 + ((g - 1D0)/2D0)*(M**2D0)
      END

```

```

      *****
      * these are function definitions for the various relations to
      * determine local flow properties from the stagnation conditions
      *****

```

```

      DOUBLE PRECISION FUNCTION LOCAL_T(P1, P2)
      *****
      * calculates the boundary layer edge temperature from the mach number
      * and stagnation temperature
      *****
      * parameters
      *   P1-local mach number
      *   P2-stagnation temperature
      *     DOUBLE PRECISION P1, P2
      * variables
      *     DOUBLE PRECISION g
      *     g = 1.4D0

      LOCAL_T = P2/(1D0 + ((g - 1D0)/2D0)*P1**2)
      END

```

```

      DOUBLE PRECISION FUNCTION LOCAL_P(P1, P2)
      *****
      * calculates the local pressure
      *****
      * parameters
      *   P1-local mach number
      *   P2-stagnation pressure
      *     DOUBLE PRECISION P1, P2

```

Figure 53.--Continued

```

* variables
  DOUBLE PRECISION g, K
  g = 1.4D0
  K = g/(g - 1D0)

  LOCAL_P = P2/(1D0 + ((g - 1D0)/2)*P1**2)**K
  END

  DOUBLE PRECISION FUNCTION SOUND(P1)
  *****
  * calculates the local speed of sound
  *****
  * functions called
    DOUBLE PRECISION SQRT
  * parameters
  *   P1-local temperature
    DOUBLE PRECISION P1
  * variables
    DOUBLE PRECISION g, R
    g = 1.4D0
    R = 289.69D0

    SOUND = SQRT(g*P1*R)
  END

  DOUBLE PRECISION FUNCTION VEL(P1, P2)
  *****
  * calculates the boundary layer edge velocity
  *****
  * functions called
    DOUBLE PRECISION LOCAL_T, SOUND
  * parameters
  *   P1-local mach number
  *   P2-stag temp
    DOUBLE PRECISION P1, P2
  * variables
    DOUBLE PRECISION Te, a

    Te = LOCAL_T(P1, P2)
    a = SOUND(Te)
    VEL = a*P1
  END

  DOUBLE PRECISION FUNCTION DIFF_VEL(U1,X1,U2,X2)
  *****
  * returns the first derivative of the boundary layer edge velocity
  * based on the two point numerical differentiation technique
  *****
  * parameters
  *   U1,X1 - Ue at X1
  *   U2,X2 - Ue at X2
    DOUBLE PRECISION U1, X1, U2, X2

    DIFF_VEL = (U2 - U1)/(X2 - X1)
  END

```

