

Basic analysis and design procedures for optimizing transmission via meteor trails
by Richard Raymond Stone

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Electrical Engineering
Montana State University
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Abstract:
Herein, analytical tools in the form of graphs and approximating formulae are developed for the Meteor Burst Communication System. Further, design (synthesis) procedures are developed which allow the MBCS designer to determine the parameter (e. g., bit rate, message duration, power levels) that give rise to a maximum average information rate within the constraints imposed on these parameters and/or the probability of error. Various modes of operation are defined and evaluated.

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BASIC ANALYSIS AND DESIGN PROCEDURES FOR OPTIMIZING TRANSMISSION VIA METEOR TRAILS

by

RICHARD RAYMOND STONE

A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Electrical Engineering

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Bozeman, Montana

February, 1977

ACKKNOWLEDGMENTS

The author wishes to express his sincere gratitude to his adviser, Dr. D. N. March, who suggested that he pursue the area of Meteor Burst Communication System analysis and design, and for securing sponsorship from Boeing Company for the initial research leading to this thesis.

The author also would like to thank his wife for her typing and patience throughout the development of this thesis.

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ABSTRACT

Herein, analytical tools in the form of graphs and approximating formulae are developed for the Meteor Burst Communication System. Further, design (synthesis) procedures are developed which allow the MBCS designer to determine the parameter (e. g., bit rate, message duration, power levels) that give rise to a maximum average information rate within the constraints imposed on these parameters and/or the probability of error. Various modes of operation are defined and evaluated.

CHAPTER I

INTRODUCTION

1.1 Historical Review

In the 1950's, it was experimentally determined that with only moderate power levels, VHF forward scatter off ionized meteor trails at high altitudes produced high signal-to-noise ratios at considerable distances.[5] And, it was determined that reciprocity existed between the communication terminals. Although the meteor trails occurred only intermittently and were short-lived, it was evident that they provided an ideal channel for low-rate remote data sampling. Such communication systems, assumed to be digital throughout, were called Meteor Burst Communication Systems (MBCS).

1.2 MBCS Operation

The MBCS is assumed to be a digital communication system that operates as follows in the forward scatter mode (see Figure 1.1). A base station transmits a continuous wave, VHF, interrogate signal that is received by a remote station when a properly oriented meteor trail forms between these stations. If the signal received at the remote station is above a given threshold, indicating a meteor trail that will provide a reasonably good forward scatter medium, messages are transmitted back to the base station via that meteor trail. The meteor trail disperses with

time and gives rise to a signal of decaying strength at the base station. Hence, after some time, the remote station must stop transmitting. This constitutes a full cycle of MBCS operation.[4]

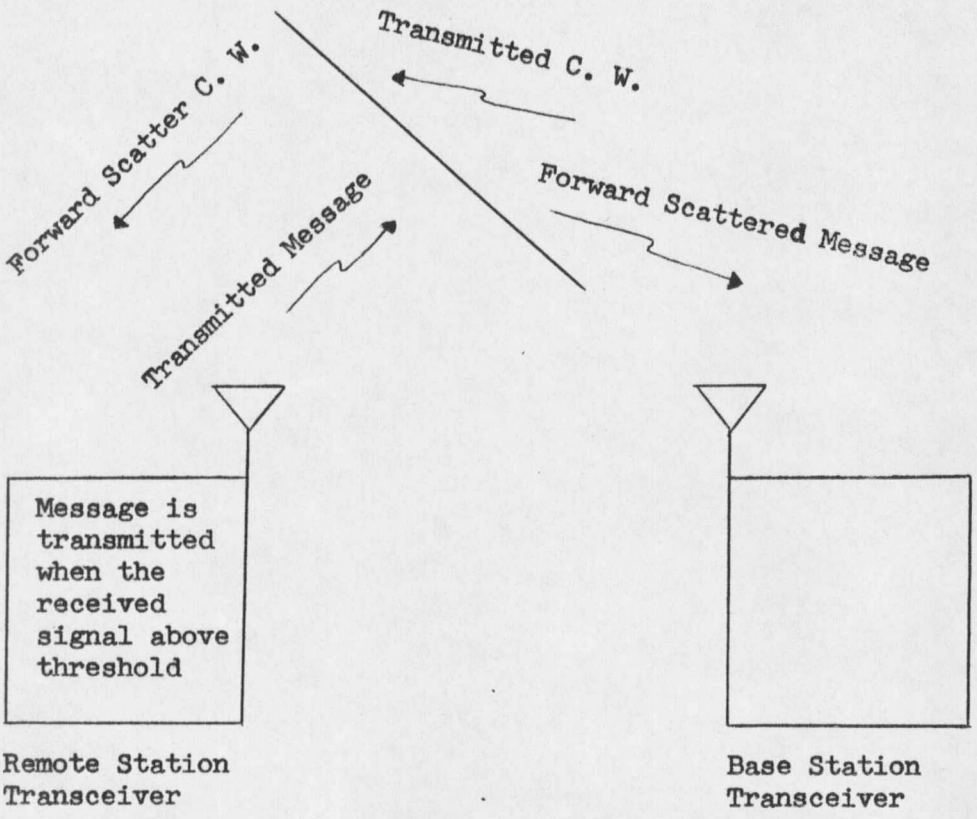


Figure 1.1
The Meteor Burst Communication System

1.3 Types of Meteor Trails

There are two types of meteor trails. They are the overdense and underdense meteor trails. Overdense meteor trails are

infrequent compared to underdense meteor trails, and the signals reflected from such meteor trails are erratic in nature, and generally produce high error rates. As a result, simple error detection methods are adequate to reject the data received from transmissions during which the forward scatter medium is an overdense meteor trail. Hence, it is assumed that the forward scatter medium consists of only underdense meteor trails--simply called meteor trails in much of what follows.

1.4 Initializing MBCS Base Station Receiver

Underdense meteor trails are characterized by the fact that they cause reflected signals' strength to decay nearly exponentially, after a finite rise time, as shown in Figure 1.2. Prior to receiving the binary information sequence from the remote station via an underdense meteor trail, an initializing binary sequence is received at the base station. This initializing binary sequence serves several purposes. First, it acts as a delay that is of sufficient duration to allow most signals reflected from the underdense meteor trails to reach their peak values before the binary information sequence is received. Secondly, bit phase coherence of the receive clock is established at this time. And thirdly, any coding or other logic devices are zeroed at this time.

This process of initializing the Base Station receiver is of no analytic interest herein. Hence, for simplicity, it is assumed that the rise time of each reflected signal is zero. Further, it is assumed that bit phase coherence of the receive clock and zeroed code and other logic devices exist prior to receiving the reflected signal. Under these assumptions, the entire transmission consists solely of the binary information sequence, and so, we have a convenient method of restricting the discussion to the interval of analytical interest, $[0, t_d]$ of Figure 1.2.

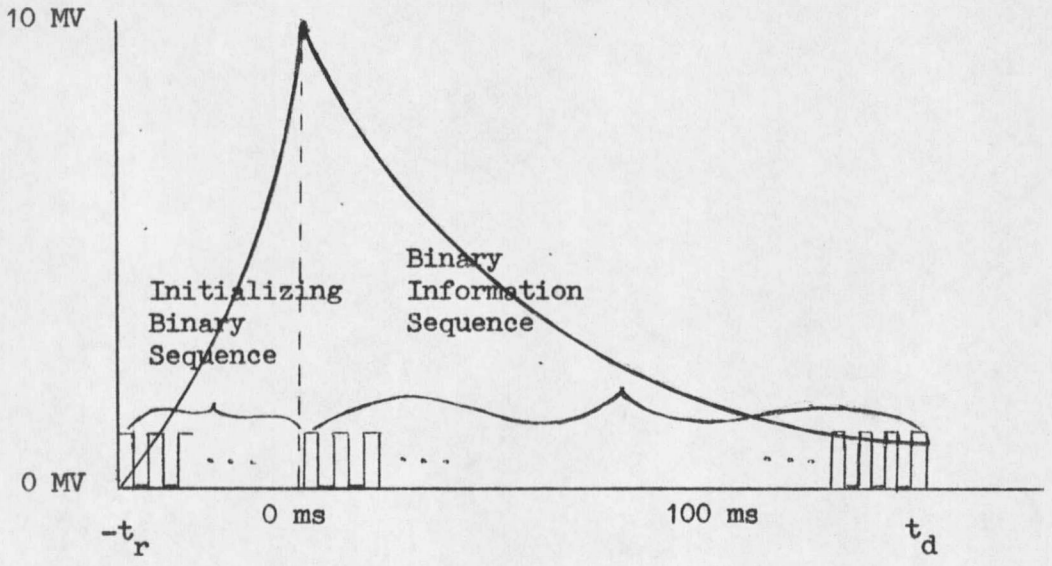


Figure 1.2
Typifying Envelope (rms value) of Signal
Recovered at Base Station Via Underdense Meteor Trail

1.5 Model of Meteor Trails

The meteor trails of the MBCS are assumed to be underdense and to produce reflected signals that decay nearly exponentially from an initial peak value. This is equivalent to assuming an antennuation factor of the form Ae^{-t/t_D} for the forward scatter medium, where t_d is the time in milliseconds for the reflected signal to decay to 36.8 per cent of its initial value.

Further, Gaussian white noise of zero mean is assumed to be added to the signal during transmission. It is denoted by n(t).

Finally, a linear time-varying phase error u(t) is assumed to be introduced by meteor trail movement due to high altitude winds.[5]

As a result of these considerations, the meteor trail model is assumed to be that shown in Figure 1.3.

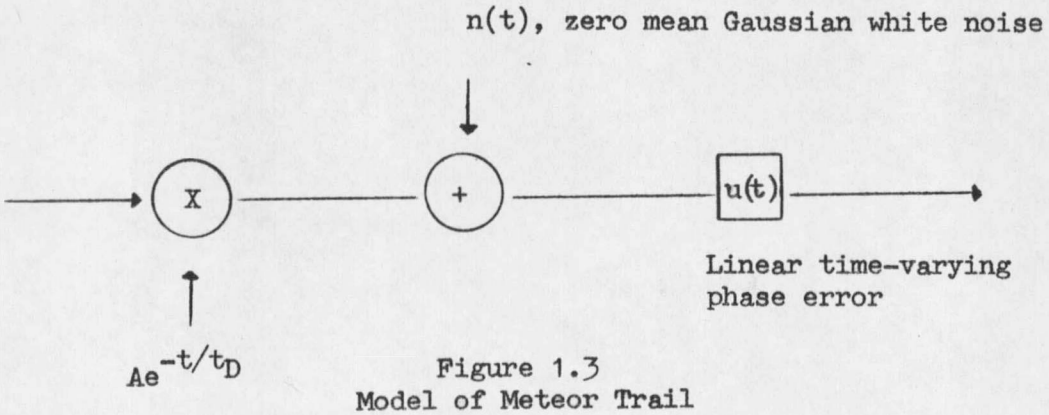


Figure 1.3
Model of Meteor Trail

Further, it is assumed that a given meteor trail provides the same forward scatter medium to the remote station as to the base station. That is, meteor trail model reciprocity is assumed.[4]

The MBCS, together with a forward scatter medium consisting of a single fixed meteor trail, is denoted by MBST. Further, for convenience, the MBCS, together with a forward scatter medium consisting of an ensemble of meteor trails, is denoted by MBG.

1.6 The Problem Considered

Three problems were considered. First, an optimum receiver was conceptually developed for the received signal plus noise identified in Figure 1.4.

Secondly, analytical tools in the form of graphs and approximating formulae were developed. Using these, the outcomes expected for a given set of MBCS operants and parameters may be determined. Thirdly, synthesis (design) procedures were developed for choosing the parameters which maximize (approximately) the outcomes within a given probability of error, bandwidth, and power constraint.

MBCS OPERANTS AND PARAMETERS

1. Mode of Operation
2. Code type
3. C. W. signal power
4. Information signal power
5. Remote station threshold
6. Bit rate
7. Message duration

MBCS RECEIVED SIGNAL PLUS NOISE

$$n(t) + Ve^{-t/t_D}\cos\left(W_c t + u(t)\right)$$

- $n(t)$: Gaussian white noise of zero mean
- Ve^{-t/t_D}: Independent random variables
- $u(t)$: Linear phase error

MBCS OUTCOMES

1. Average amount of Information/Transmission
2. Average information rate
3. Probability of bit, word, message errors
4. Signal-to-noise ratio

Figure 1.4
Illustrating Problems Considered

CHAPTER II

DETECTION

2.1 Base Station Detection Scheme Implementation

The MBCS is assumed to be a digital communication system with the specific base station receiver implementation shown in Figure 1.4. Phase-shift keying yields maximal error performance among the various PCM systems in the presence of additive Gaussian noise, provided that phase coherence can be maintained at the detector.[17] The phase errors introduced by meteor trail drift due to high altitude winds rarely exceed the limits within which phase coherence at the detector can be maintained.[5] Hence, excluding the antennuation factor, the PSK system with the correlator of Figure 1.4 yields maximal error performance among the various PCM systems. That this is also the case when the antennuation factor is included is shown in the following two sections.

An error-detecting and/or -correction scheme is quite useful in meeting probability of error constraints for transmission via underdense meteor trails and excluding data from overdense meteor trails. For simplicity of implementation, evaluation of error performance, and effectiveness in error correction and detection, an algebraic cyclic code of block length k that corrects r errors per block length is assumed to be used.[12, 13]

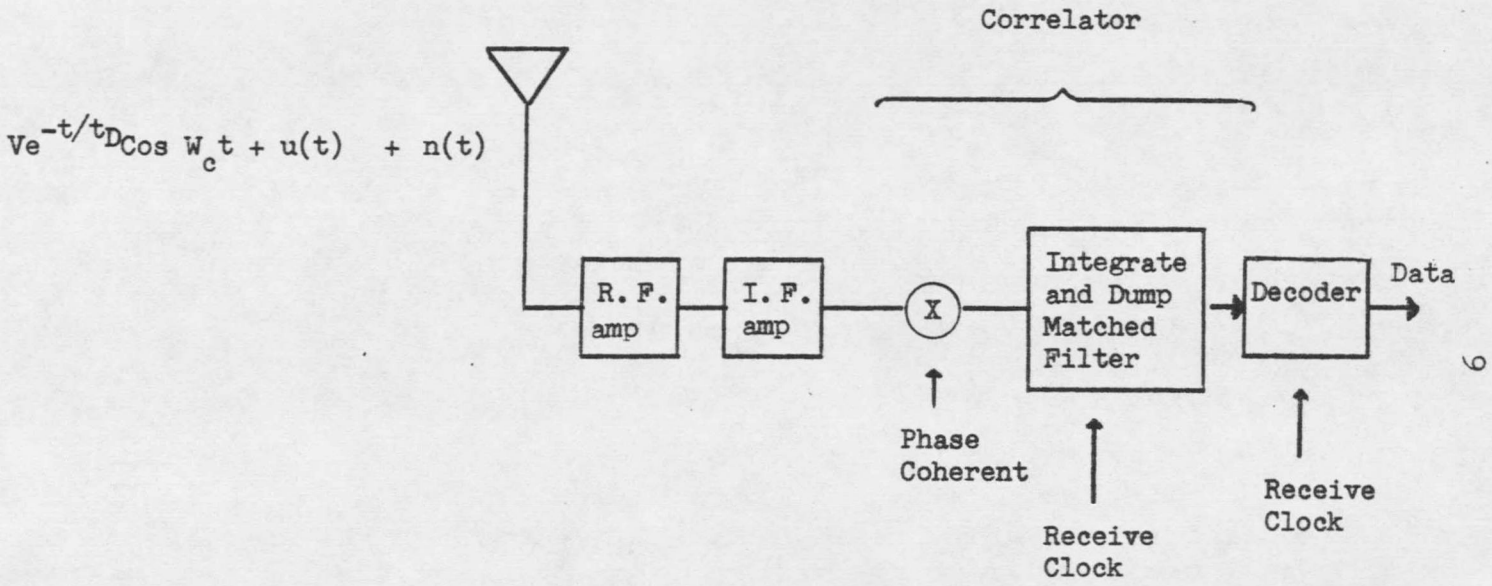


Figure 2.1
Optimum MBCS Detection Scheme

2.2 Detector Input Noise

The predetection linear filtering of the base station receiver, together with the additive Gaussian white noise of zero mean, n(t), gives rise to zero mean bandlimited Gaussian noise with spectral density

$$G(f) = \begin{cases} N & \text{if } f_0 - 2w < |f| < f_0 + 2w, \\ 0 & \text{otherwise} \end{cases}$$

where f_0 is the center frequency of the I. F. amplifier and 2w its bandwidth. However, the only portion of the power spectrum of any input that contributes to the detector output is that portion in the frequency range $f_0 - 2w < |f| < f_0 + 2w$. Hence, the effect of the bandlimited noise on the detector output is exactly the same as if the noise input was additive Gaussian white noise of zero mean and power spectral density N. As a result of this, it will be assumed that the noise input to the detector is this type and it will be denoted by $n_w(t)$.

2.3 Detector Output Signal-to-Noise Ratio

The optimum filter for the signal plus noise $n_w(t) + \overline{S}(t)$ must have an impulse response of the form $h(t) = k\overline{S}(T-t)$, where k is an arbitrarily chosen positive constant and T is the duration of the binary signal $\overline{S}(t)$.[2] The correlator shown in Figure 2.2 yields the same output as an optimum filter with impulse response

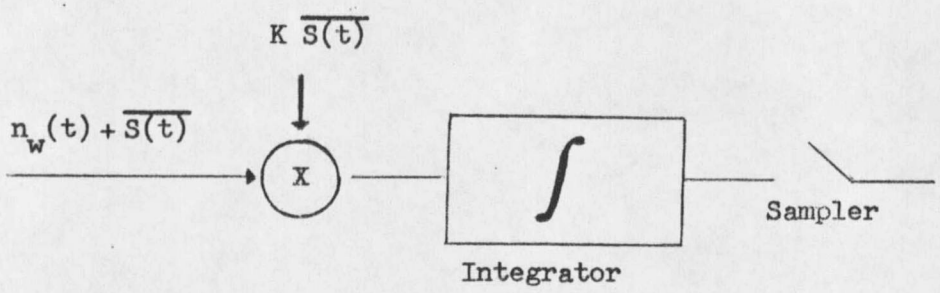


Figure 2.2
General Correlator

$k\bar{S}(T-t)$. That is, the output of an optimum filter with $h(t) = k\bar{S}(T-t)$ is

$$Y(t) = \int_{-\infty}^{\infty} \left[n_w(x)+\bar{S}(x)\right] h(t-x)dx = \int_{0}^{t} \left[n_w(x)+\bar{S}(x)\right] k\bar{S}(T-t+x)dx$$

at time t. Hence, at sample t, t = T, the output would be

$$y(T) = \int_{0}^{T} k\left[n_w(x) + \bar{S}(x)\right]\bar{S}(x)dx,$$

the output of the correlator of Figure 2.2.

If we let $\bar{S}(t) = \pm A\text{Cos}w_0t$ and $k = 1/A$, then the correlator becomes that shown in Figure 2.3. This correlator yields the same output as the optimum filter would. As a result, the ratio of the output normalized signal power-to-mean noise power (s) is the same as that of an optimum filter, namely,

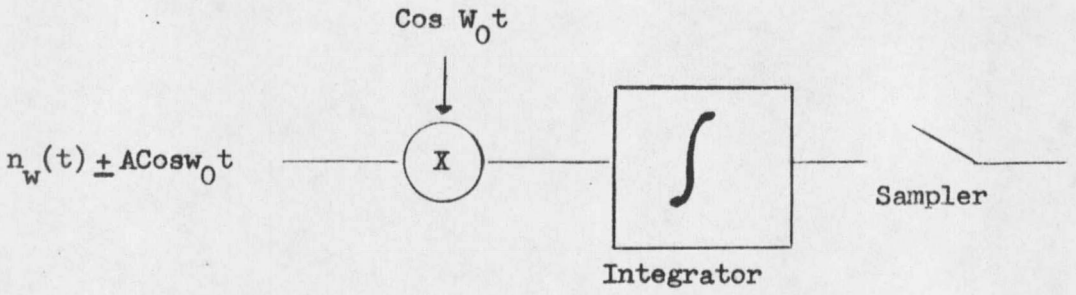


Figure 2.3
Specific Correlator

$$(2\text{-}1)\quad \frac{\int_{-\infty}^{\infty} \bar{S}(t)^2 dt}{N} = \frac{A^2 t}{2N} = s$$

As concluded in the previous section, it may be assumed that the input noise to the detector is $n_w(t)$. Hence, the MBCS detector input signal plus noise, assuming phase coherence can be maintained in spite of the linear time varying phase error, is equivalently $n_w(t) + \bar{S}(t) = n_w(t) \pm Ve^{-t/t_D}\text{Cos}w_0t$, where e^{-t/t_D} is the fade factor corresponding to the fade in signal strength during transmission. Typically, $t_D > 100T$, where T is the duration of a binary digit. Thus, Ve^{-t/t_D} is approximately constant over the duration of a binary digit. As such, Ve^{-t/t_D} may be identified with the coefficient A of the correlator input of Figure 2.3. As such, it is evident that the correlator of Figure 2.3 (equivalently the synchronous detector of Figure 2.1) is an optimum detector for the

signals whose forward scatter medium consisted of an underdense meteor trail. Also, from 2-1, the signal-to-noise ratio at the detector output is

$$(2\text{-}2) \quad s(t) \cong \frac{(Ve^{-t/t_D})^2 T}{2N}$$

at sample time t. Equivalently, assuming equality holds, we have

$$(2\text{-}3) \quad s(t) = \frac{(Ve^{-t/t_D})^2}{2NB} \text{, where } B = 1/T = \text{binary digit rate.}$$

And, defining $S(t) = 10\log s(t)$ db, 2-3 may be expressed as

$$(2\text{-}4) \quad S(t) = 10\log_{10} \frac{(Ve^{-t/t_D})^2}{2NB} \text{, or,}$$

$$(2\text{-}5) \quad S(t) = \frac{(-8.68)}{t_D} t + 10\log_{10} \frac{V^2}{2NB} \text{, or,}$$

$$(2\text{-}6) \quad S(t) = \frac{(-8.68)}{t_D} t + S(0)$$

Equation 2-6 says that initially (i.e., the first binary signal sampled over a given meteor trail) we have S(0) at the detector output. As time goes on, we drop from this initial value at the rate of $\frac{8.68}{t_D} t$ db in t ms. In Figure 2.4, Equation 2-6 is sketched for $t_D = 40$ ms and $S(0) = 32$ db.

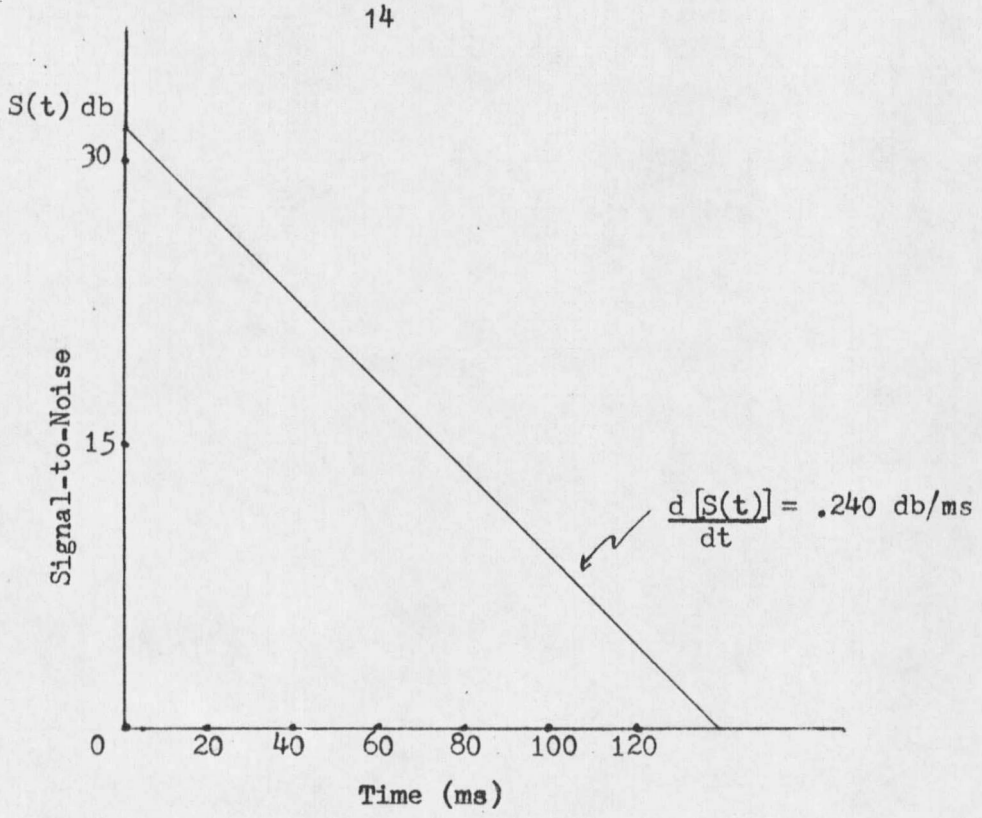


Figure 2.4
Exemplifying S(t) from Underdense Meteor Trail Reflection

CHAPTER III

MBST ERROR ANALYSIS

3.1 MBST Bit Errors, $P_{eb}(t)$

Once the detector output is sampled, a decision is made as to whether a zero or a one was received. Since for each bit the sampled output is the same as it would be for an optimum filter, the probability of a bit error is the same. This may be expressed as a function of the signal-to-noise ratio at the detector output. The graphical equivalence of this function is sketched in Figure 3.1.[9]

From the curve of S(t) in Figure 2.4 and $P_{eb}(S)$ in Figure 3.1, we may easily construct the curve $P_{eb}(t)$ which expresses the probability of a bit error for the MBST for the bit detected (sampled) at time t. This construction is shown in Figure 2.1. Here, Graph 3 is obtained by reflecting the points of the curve in Graph 3 through the curve in Graph 2 as indicated by the dashed lines.

We summarize these results in the following Theorem.

Theorem 1

The bit detected (sampled) at time t at the base station of a MBST system has a signal-to-noise ratio given by (see Figure 2.4).

(a) $S(t) = \frac{-8.68t}{t_D} + S(0)$ db

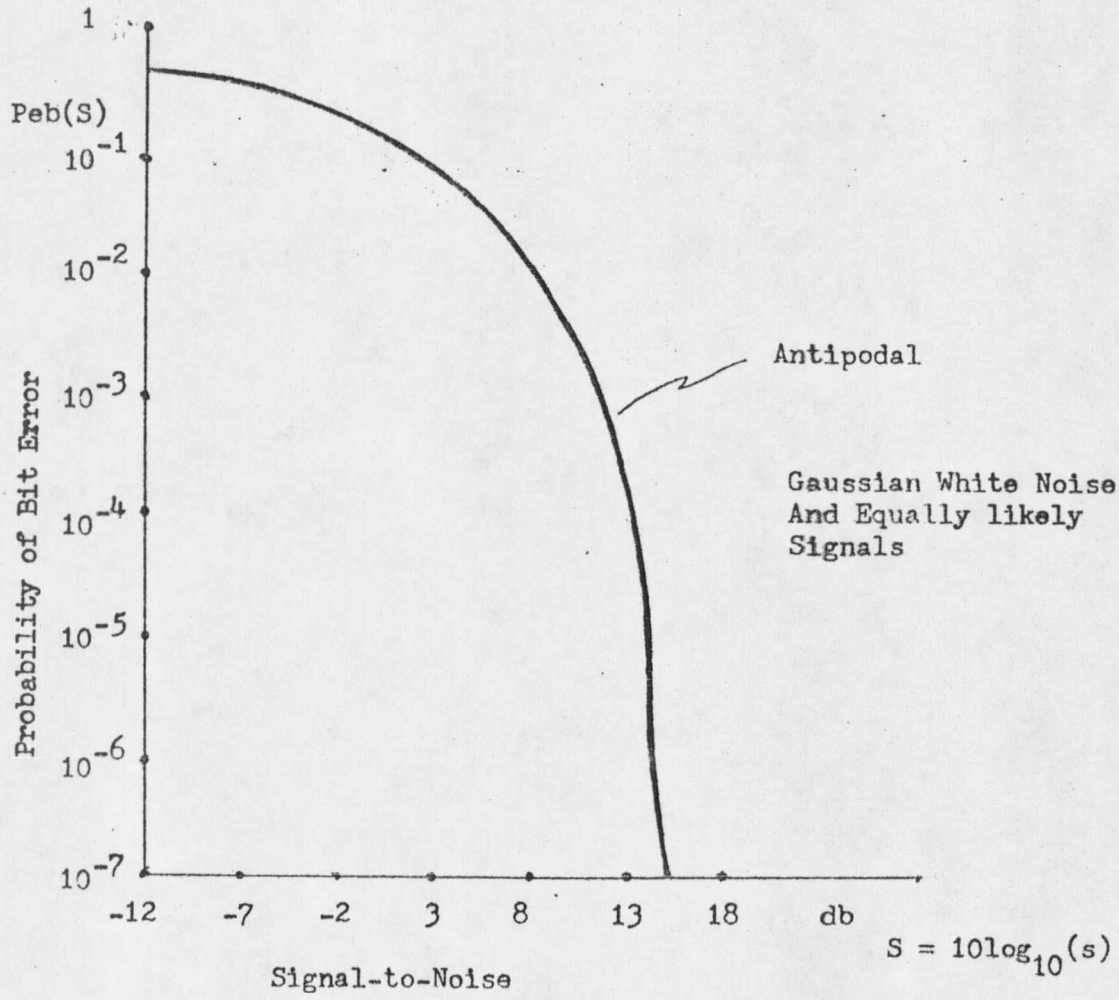


Figure 3.1
Probability of Bit Error for MBCS

where the signal at the detector input is equivalently $Ve^{-t/t_D}\text{Cos}W_0t$.

In this case,

(b) $S(0) = 10\log_{10}(\frac{V^2}{2NB})$ db

Further, the probability that this bit is in error is given by $P_{eb}(t)$ or equivalently $P_{eb}[S(t)]$, which is constructable in graphical form by the technique shown in Figure 3.2. And, finally, the synchronous detector (see Figure 2.1) is optimal in the sense of minimizing the probability of error for signals whose forward scatter medium consisted of an underdense meteor trail.

3.2 Bandwidth Equivalent

Since a typical binary signal of binary digit rate B has a bandwidth of approximately[3] (i.e., first zero crossing)

(3-1) $W = 2B$

any expression involving B, as for example Theorem 1 (b), may immediately be translated to an equivalent expression in terms of bandwidth. The equations will be expressed throughout only in terms of B and the bandwidth equivalent understood but not explicitly expressed except when convenient.

Example 1

To exemplify the use of Theorem 1, suppose that the MBST has t_D = 40 ms, S(0) = 29 db and B = 2000 bit/second. (This

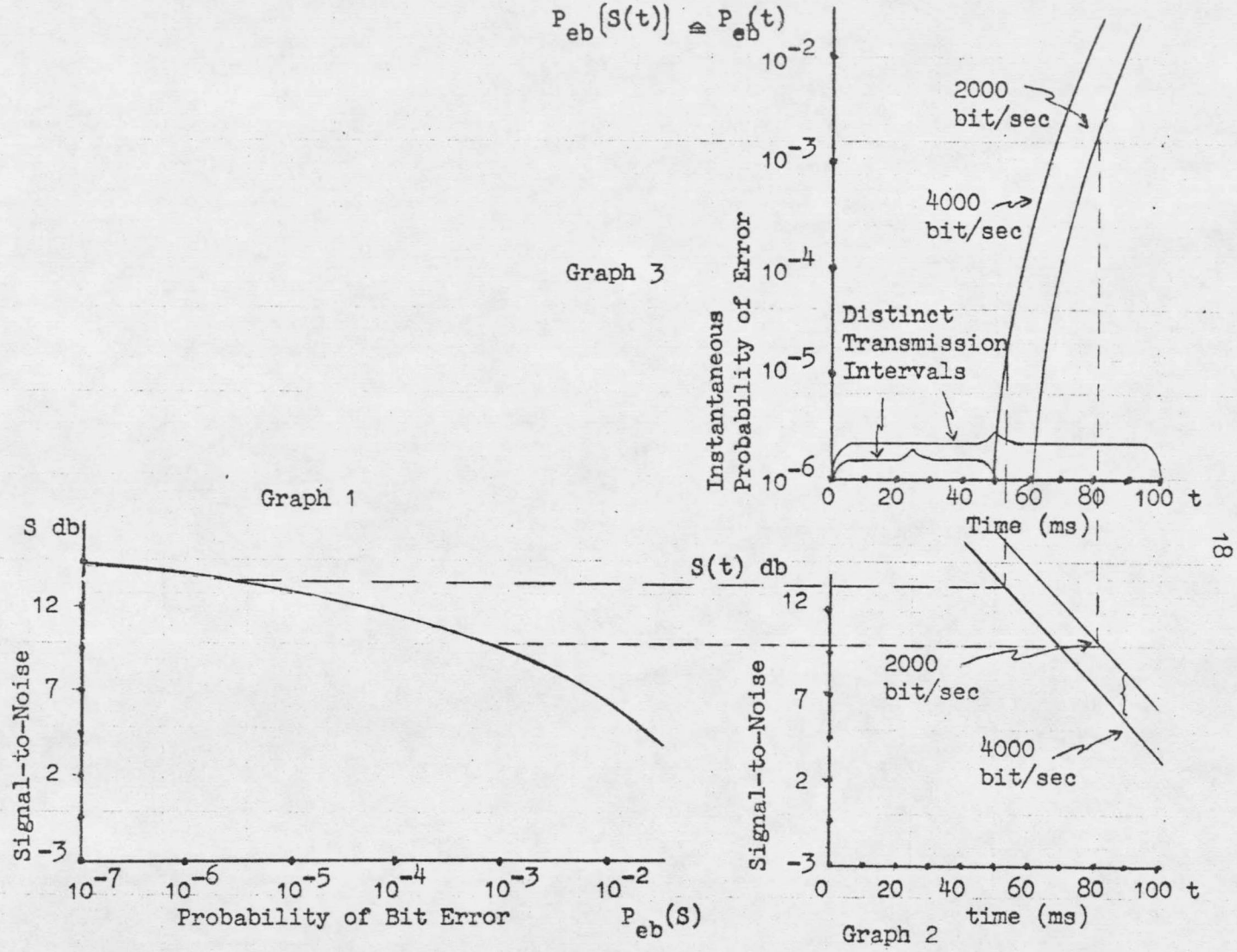


Figure 3.2
Construction of $P_{eb}(t)$ for MBST

S(t) curve, together with the corresponding $P_{eb}(t)$ curve, is shown in Figure 3.2.) Further, suppose that to perform a certain task, it is required that 200 bits be transmitted per transmission. This would require 100 ms when B = 2000 bit/second = 2 bits/ms and is indicated on the time axis of Graph 3 of Figure 2.1. Examination of this graph indicates that during the last 20 ms of the transmission the MBST is operating at a high probability of bit error. This problem can be remedied if the bit rate is increased to 4000 bit/second. (This requires W = 2 x 4000 hz as opposed to W = 2 x 2000 hz in the former case.) That is, with B doubled, S(∞) is reduced by 3 db, by Theorem 1, part (b). This is manifested as a translation of the S(t) curve by 3 db and a corresponding translation of the $P_{eb}(t)$ curve as shown in Figure 2.1. But, with B = 4000 bit/second = 4 bit/ms, we require only 50 ms to transmit 200 bits. As indicated in Graph 3, this allows operation at a low probability of bit error throughout the transmission.

3.3 Normal Operating Conditions of MBST

A word is a sequence of k binary digits. The duration of a word is assumed to be less than or equal to $t_D/10$. (Since the signal-to-noise ratio drops at the rate of $(\frac{8.68}{t_D})t$ db at the detector output, according to Theorem 1, the restriction on the duration of the word implies that there will be less than 1 db

variation in signal-to-noise over a word.)

A message is the sequence of M words transmitted during a given transmission. The duration of a message is assumed to be between $t_D/2$ and t_D. (This implies a variation in signal-to-noise ratio between 4 and 9 db over a message.)

The instantaneous probabilities of a bit and word error are assumed to be less than 1/100 and 1/2.5, respectively. Experimental data [11] indicate that these conditions are usually satisfied except when the forward scatter medium is an overdense meteor trail (easily identified by the high bit error content of the received data) and these data are excluded.

3.4 $P_{eb}(t_0,t_1)$

In Figure 3.3, a typical word of a message of typical duration is shown. Also, the signal-to-noise drop at the detector output and probability of a bit error, $P_{eb}(t)$ and $P_{eb}(S)$, are considered over the word interval $[t_0,t_1]$ of the message. In what follows, it is assumed that $P_{eb}(t)$ and $P_{eb}(S)$ are constant for the duration of a word. Since the greatest variation of S for a word is less than 1 db, it is evident from Figure 3.3 that $P_{eb}(t)$ and $P_{eb}(S)$ are well approximated by constant values in a word interval. The constant value of probability of a bit error assigned to the bit of the word received via MBST in the interval $[t_0,t_1]$ will be

denoted by $P_{eb}(t_0,t_1)$. $P_{eb}(t_0,t_1)$ is taken to be the mean value of $P_{eb}(t)$ in the interval $[t_0,t_1]$,

$$\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt,$$

or, equivalently, the mean value of $P_{eb}(S)$ over the interval $[S(t_1),S(t_0)]$,

$$\frac{1}{S(t_1)-S(t_0)}\int_{S(t_0)}^{S(t_1)} P_{eb}(S)ds.$$

(The equivalence is evident by observing that since $P_{eb}(t) = P_{eb}[S(t)]$, where $S(t) = \frac{-8.68}{t_D}t + S(0)$, a change of variables yields

$$\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt = \frac{1}{S(t_1)-S(t_0)}\int_{S(t)_0}^{S(t_1)} P_{eb}(S)ds.)$$

3.5 $P_{k,r}(t_0,t_1)$

Let $P_{k,r}(t_0,t_1)$ be the probability of exactly r bit errors in the word received via MBST in the interval $[t_0,t_1]$, as typified in Figure 3.3.

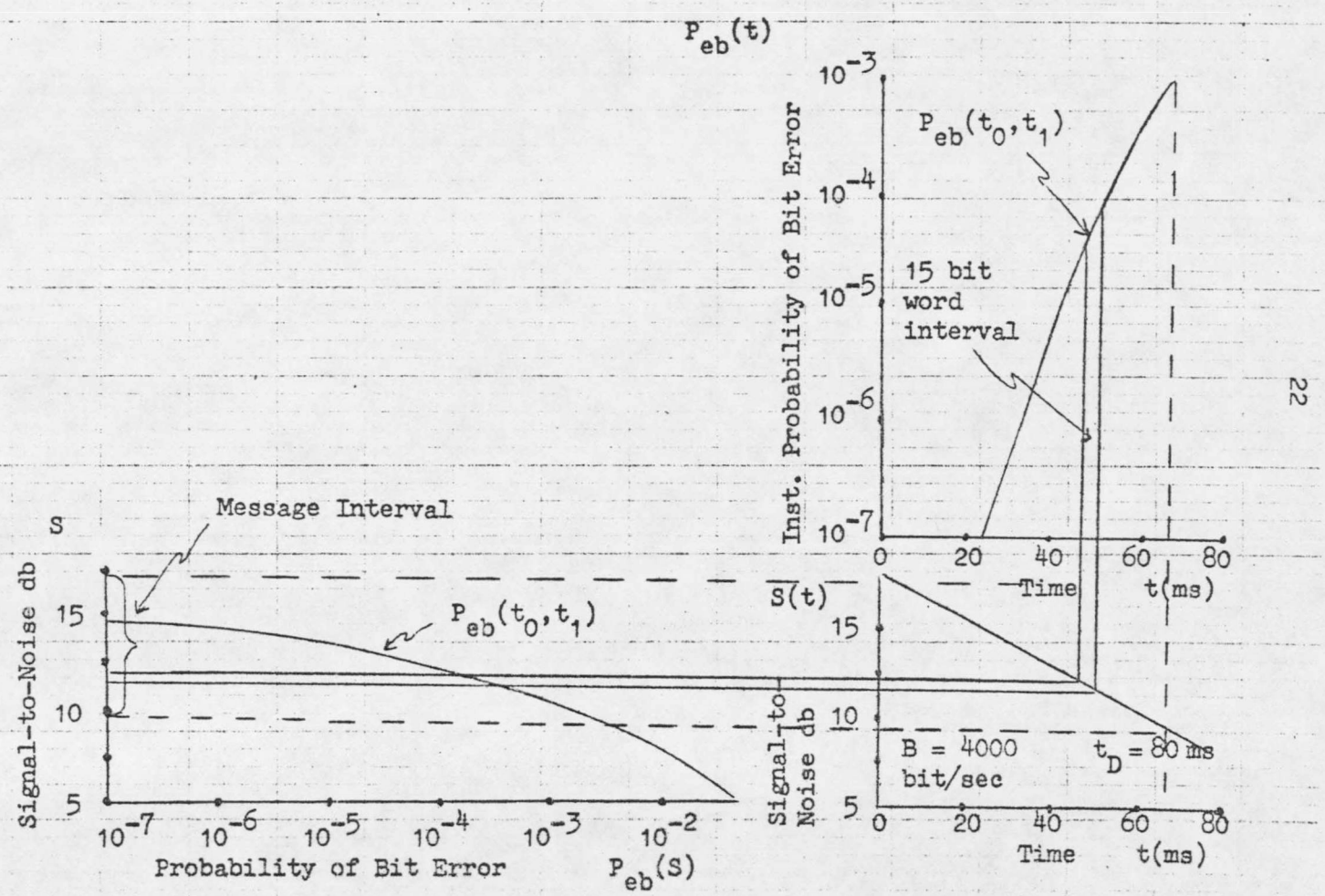


Figure 3.3
Typifying Message and Word Intervals

Theorem 2

For MBST, with $r > 0$,

$$P_{k,r}(t_0,t_1) \cong \frac{1}{t_1-t_0}\int_{t_0}^{t_1}\binom{k}{r} P_{eb}(t)^r dt$$

$$\cong \frac{1}{S(t_1)-S(t_0)}\int_{S(t_0)}^{S(t_1)}\binom{k}{r} P_{eb}(S)^r ds$$

That is, $P_{k,r}(t_0,t_1)$ is the mean of $\binom{k}{r} P_{eb}(t)^r$ over the interval $\left[t_0,t_1\right]$, or equivalently by a change of variables, the mean of $\binom{k}{r} P_{eb}(S)^r$ over the interval $\left[S(t_1),S(t_0)\right]$. Furthermore, $P_{k,r}(t_0,t_1)$ is less than the value of these means.

Proof.

Since the noise at the detector input was (w.l.o.g.) assumed to be white and Gaussian, the probability of error in distinct positions of a word are independent of each other. [10] Hence, the probability of exactly r errors for the message received in the interval $\left[t_0,t_1\right]$ is

$$(3\text{-}2)\quad P_{k,r}(t_0,t_1) = \binom{k}{r}\left[P_{eb}(t_0,t_1)\right]^r\left[1-P_{eb}(t_0,t_1)\right]^{k-r}$$

as the conditions of the Bernoulli's Trial problem are met. [6]

Since

$$P_{eb}(t_0,t_1) = \frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt,$$

3-2 may be expressed as

$$(3\text{-}3)\quad P_{k,r}(t_0,t_1) = \binom{k}{r}\left(\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt\right)^r\left(1-\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt\right)^{k-r}$$

The right most factor of this product is approximately 1. That is, it has its least value

$\left(1-\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt\right)^k$, for $r = 0$. This is also the value of the entire product in 3-3 when $r = 0$ and so represents the probability of receiving a correct word in the interval $[t_0,t_1]$ which is very close to 1. Hence, $P_{k,r}(t_0,t_1)$ is well approximated by but less than

$$\binom{k}{r}\left(\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt\right)^r$$

By Holder's Inequality [13]

$$(3\text{-}4)\quad \binom{k}{r}\left(\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)dt\right)^r \le \binom{k}{r}\frac{1}{t_1-t_0}\int_{t_0}^{t_1} P_{eb}(t)^r dt$$

Since equality is approached in 3-4 as $P_{eb}(t)$ approaches a constant value in the interval $[t_0, t_1]$, and in fact $P_{eb}(t)$ is approximately constant in this word interval, as previously discussed, $P_{k,r}(t_0, t_1)$ is well approximated by but is less than

$$\binom{k}{r} \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} P_{eb}(t)^r dt$$

Q. E. D.

Example 2

(A) Figure 3.4 shows $\binom{15}{r} P_{eb}(S)^r$ for r = 1, 2, 3. It may be used in the same way that $P_{eb}(S)$ is used. That is, if S is the assumed constant value of signal-to-noise ratio at the time the bits of word W is detected, then the probability of exactly r errors occurring in W is approximately

$\binom{15}{r} P_{eb}(S)^r$, but slightly less,

(B) The construction technique of Figure 3.2 produced $P_{eb}(t)$ from $P_{eb}(S)$ and S(t). Similarly, we may produce

$\binom{k}{r} P_{eb}(t)^r$ from $\binom{k}{r} P_{eb}(S)^r$ and S(t). This is done for k = 15 and r = 1, 2, 3 in Figure 3.5. Here, the exact meaning of Theorem 2 becomes evident.

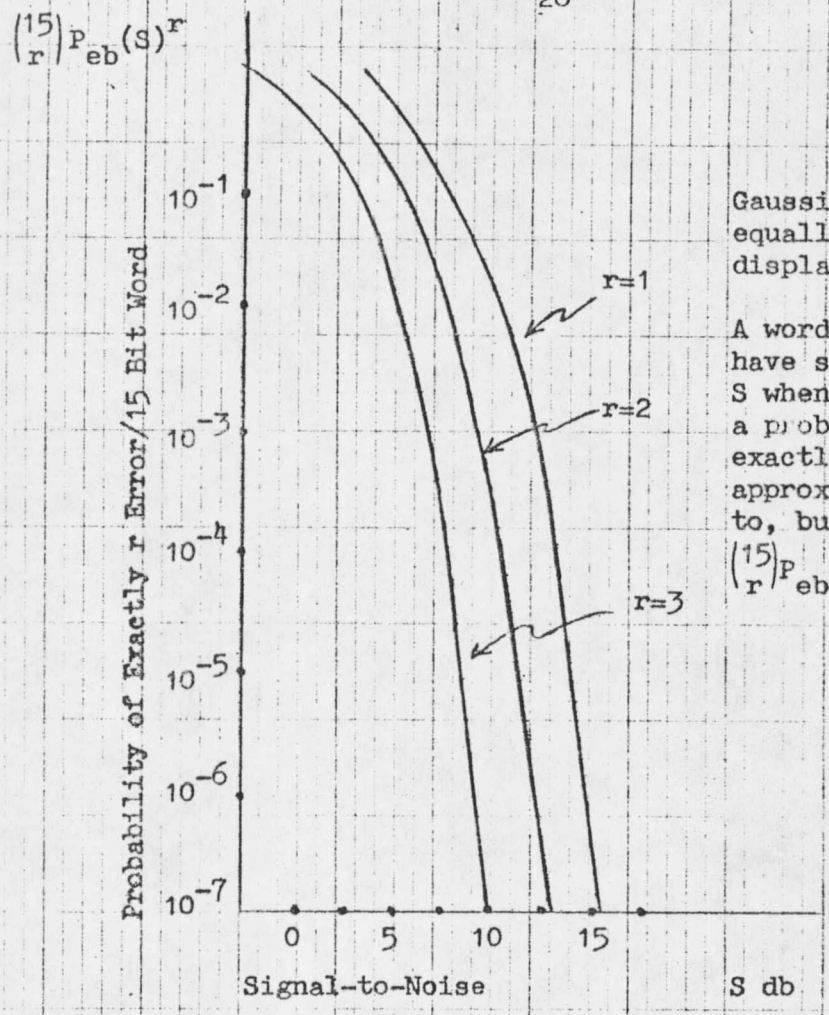


Figure 3.4
Exemplifying $\binom{k}{r} P_{eb}(S)^r$ vs. S

3.6 $P_{k,r}(0,t_d)$

Let $P_{k,r}(0,t_d)$ be the probability that the received word has exactly r errors, given that the message interval is $[0,t_d]$.

Theorem 3

For MBST, with $r > 0$,

$$P_{k,r}(0,t_d) \cong \frac{1}{t_d}\int_0^{t_d}\binom{k}{r}P_{eb}(t)^r dt$$

$$\cong \frac{1}{S(t_d)-S(0)}\int_{S(0)}^{S(t_d)}\binom{k}{r}P_{eb}(S)^r ds$$

That is, $P_{k,r}(0,t_d)$ is the mean of $\binom{k}{r}P_{eb}(t)^r$ over the interval $[0,t_d]$, or equivalently by a change of variables, the mean of $\binom{k}{r}P_{eb}(S)$ over the interval $[S(t_d),S(0)]$. Furthermore, $P_{k,r}(0,t_d)$ is less than these mean values.

Proof.

Let M be the number of words received in the interval $[0,t_d]$ and let the corresponding word intervals be $[t_{i-1},t_i]$, where $[t_i - t_{i-1}] = kT$, and T is the duration of a bit and k the number of bits per word. Since a word is equally likely to be received in any one of these M intervals,

$$\sum_{i=1}^{M} \left(\frac{1}{M}\right) P_{k,r}(t_{i-1},t_i), \text{ for } r > 0$$

is the expected probability of exactly r errors for the words received in the message interval $[0,t_d]$. From Theorem 2, it is apparent that this sum is well approximated by, but less than,

$$\sum_{i=1}^{M} \left(\frac{1}{M}\right)\left(\frac{1}{t_i-t_{i-1}} \int_{t_{i-1}}^{t_i} \binom{k}{r} P_{eb}(t)^r dt\right)$$

But $M(t_i-t_{i-1}) = M(kt) = td$, the duration of the message. Hence, this sum may be expressed as

$$\frac{1}{t_d} \sum_{i=1}^{M} \int_{t_{i-1}}^{t_i} \binom{k}{r} P_{eb}(t)^r dt = \frac{1}{t_d} \int_{0}^{t_d} \binom{k}{r} P_{eb}(t)^r dt.$$

As such, the Theorem holds. Q. E. D.

$P_{k,r}(0,t_d)$ may be found for k = 15 and r = 1, 2, 3, by obtaining the mean of $\binom{k}{r} P_{eb}(t)^r$, over the message interval $[0,t_d]$ directly from Figure 3.5. (Equivalently, the mean of $\binom{k}{r} P_{eb}(S)^r$ over $[S(t_d),S(0)]$ could be used.) However, this is inconvenient and time-consuming to do accurately. Hence, we approximate the mean of $\binom{k}{r} P_{eb}(t)^r$, equivalently $\binom{k}{r} P_{eb}(S)^r$,

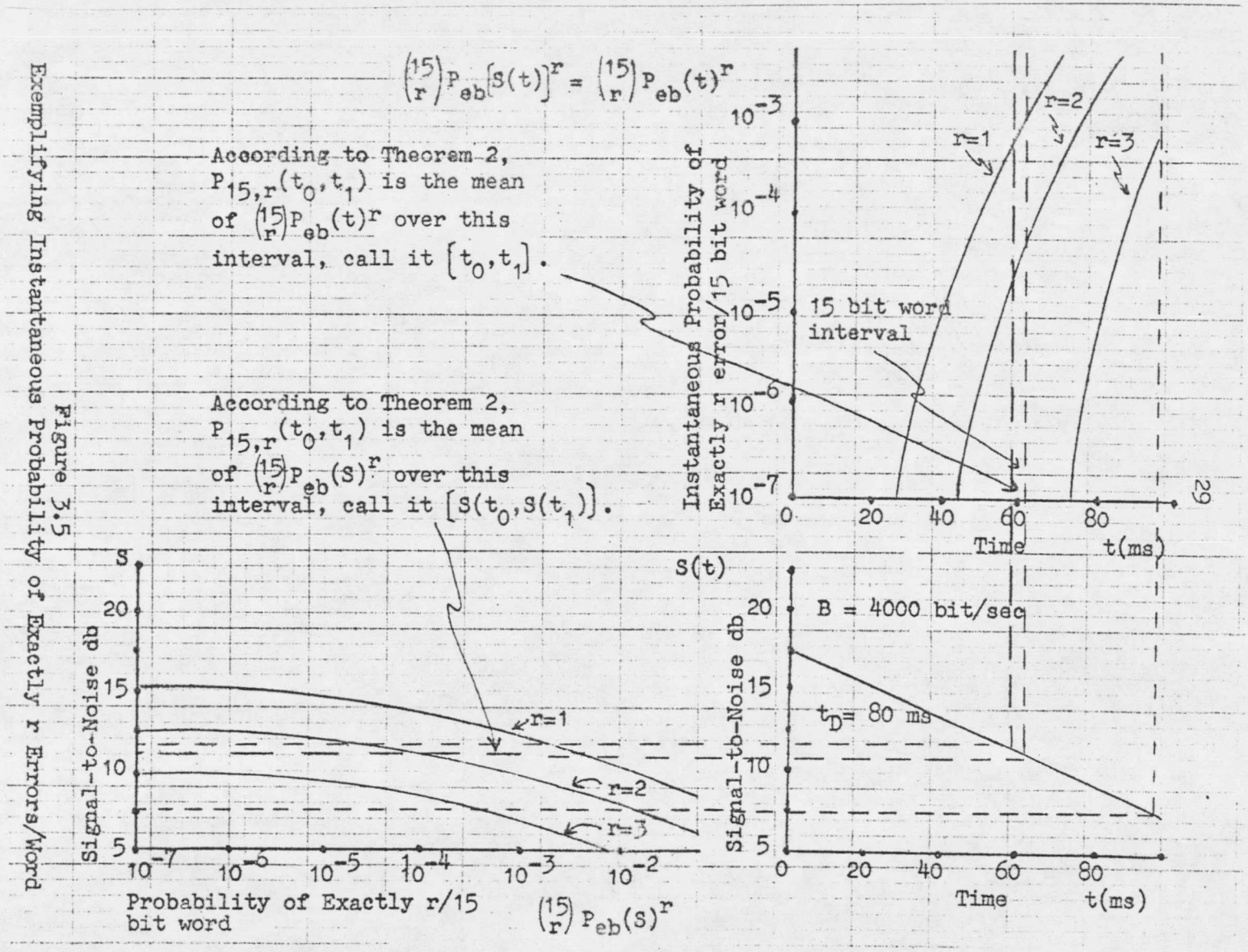


Figure 3.5 Exemplifying Instantaneous Probability of Exactly r Errors/Word

over the message interval $[0,t_d]$ by a function that is more convenient to use.

Theorem 4

For MBST, with $r > 0$,

$$P_{k,r}(0,t_d) \cong \binom{k}{r} \frac{P_{eb}[S(t_d)]^r}{10 \cdot r}$$

and $P_{k,r}(0,t_d)$ is less than this approximate value.

Proof.

From Theorem 3,

$$P_{k,r}(0,t_d) \cong \frac{1}{S(t_d)-S(0)} \int_{S(0)}^{S(t_d)} \binom{k}{r} P_{eb}(S)^r ds,$$ but less than this approximate value.

Let $Y(S)$ be the linear approximation to $\log_{10} P_{eb}(S)$ shown in Figure 3.6. Since $Y(S) \cong \log_{10} P_{eb}(S)$ and is greater than this approximate value, $10^{Y(S)} \cong P_{eb}(S)$ and is greater than this approximate value. Hence,

$$P_{k,r}(0,t_d) = \frac{1}{S(t_d)-S(0)} \int_{S(0)}^{S(t_d)} \binom{k}{r} 10^{Y(S) \cdot r} ds,$$

but less than this approximate value. Since $Y(S) = \frac{-5}{7} S + Y(0)$, a change of variables yields

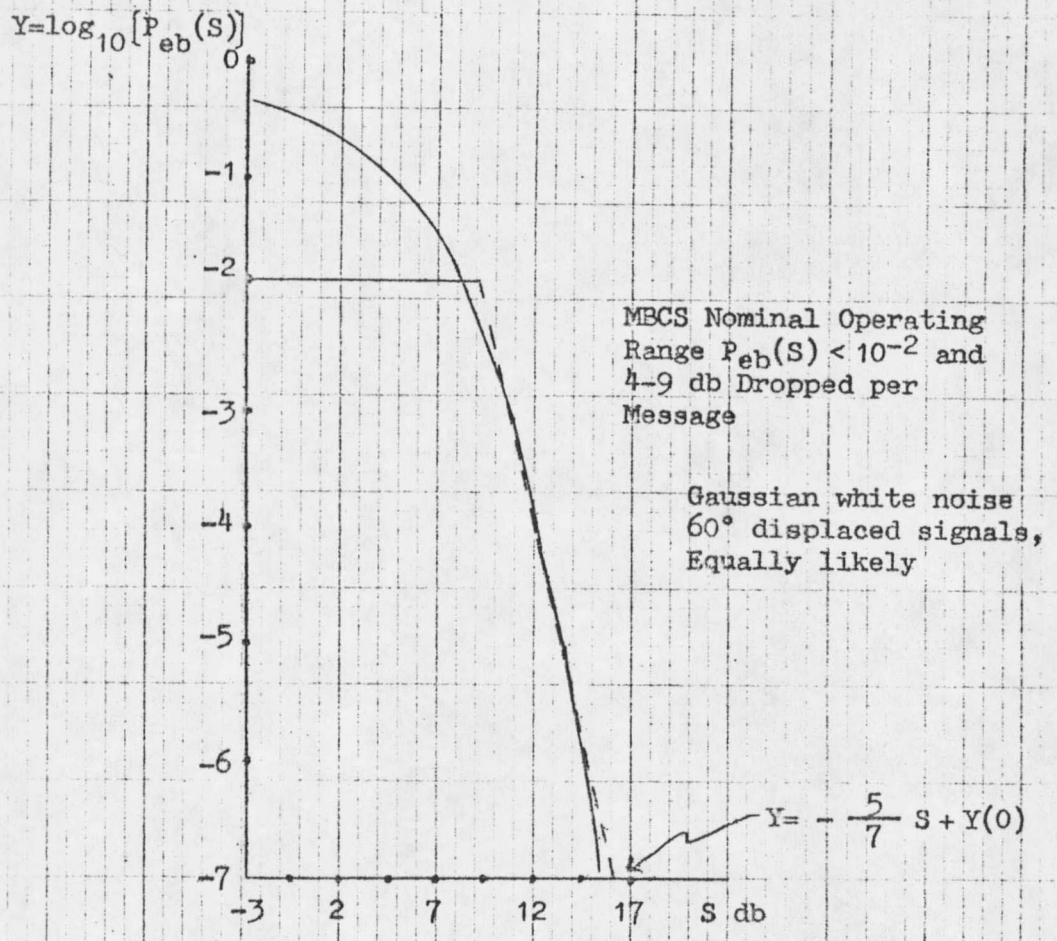


Figure 3.6

Exemplifying $\log_{10}\left[P_{eb}(S)\right]$ Linearized

$$\frac{1}{S(t_d)-S(0)}\int_{S(0)}^{S(t_d)}\binom{k}{r}10^{Y(S)\cdot r}ds = \frac{1}{Y_1-Y_0}\int_{Y_0}^{Y_1}\binom{k}{r}10^{Y\cdot r}dY,$$

where $Y_1 = Y\left[S(t_d)\right]$ and $Y_0 = Y\left[S(0)\right]$. Upon integration, we obtain

$$\frac{1}{Y_1-Y_0}\int_{Y_0}^{Y_1}\binom{k}{r}10^{Y\cdot r}dY = \binom{k}{r}\frac{10^{Y_1\cdot r} - 10^{Y_0\cdot r}}{r(Y_1-Y_0)\ln(10)}$$

But $10^{Y(S)} = P_{eb}(S)$, hence the quotient, denote it by Q, may be expressed as

$$Q = \binom{k}{r}\frac{P_{eb}\left[S(t_d)\right]^r - P_{eb}\left[S(0)\right]^r}{2.3r\left\{\log_{10}P_{eb}\left[S(t_d)\right] - \log_{10}P_{eb}\left[S(0)\right]\right\}}$$

From Figure 3.6, it is evident that when $S(t_d) > S(0) - 4$ db (as is required by the definition of message) $P_{eb}\left[S(0)\right]$ is negligible in comparison to $P_{eb}\left[S(t_d)\right]$. It is also evident from Figure 3.5 that $\left\{\log_{10}P_{eb}\left[S(t_d)\right] - \log_{10}P_{eb}\left[S(0)\right]\right\}$ varies in value between 3 and 6 over the normal operating range (Figure 3.6). Hence, assuming $P_{eb}\left[S(0)\right]$ is negligible,

$$\binom{k}{r}\frac{P_{eb}\left[S(t_d)\right]^r}{6.9r} < Q < \binom{k}{r}\frac{P_{eb}\left[S(t_d)\right]^r}{13.8r}$$

Since the bounds on Q differ from $\binom{k}{r}\frac{P_{eb}[S(t_d)]^r}{10r}$ by an amount that is well within the error introduced by the graphical method of obtaining $\binom{k}{r}P_{eb}(S)^r$, Q is assumed to have this value. Then, the Theorem holds.

Q. E. D.

Example 3

The values of $\binom{k}{r}\frac{P_{eb}(S)^r}{10\cdot r}$ and $\binom{k}{r}\frac{P_{eb}(t)^r}{10\cdot r}$ for k = 15 and r = 1, 2, 3 may be obtained graphically from the curve of $\binom{k}{r}P_{eb}(S)^r$ and $\binom{k}{r}P_{eb}(t)^r$ in Figure 3.5 by dividing by 10, 20, and 30, respectively. Thus, as indicated by the dotted line of Figure 3.5, if the MBST designer chooses to use a single error-correcting code and have $P_{15,2}(0,t_d) < \frac{1}{2000}$, he must terminate the message interval at t_d = 95 ms for the assumed MBST.

3.7 $P^*_{k,r}(0,t_d)$

Another probability that is very useful to the MBST designer is the probability that at least r errors occur in a k bit word received in a message interval $[0,t_d]$, $P^*_{k,r}(0,t_d)$.

Theorem 5

For MBST, with $r > 0$,

$$P^*_{k,r}(0,t_d) \cong P_{k,r}(0,t_d).$$

Proof.

From a comparison of (ratio) $P_{k,r+1}(0,t_d)$ to $P_{k,r}(0,t_d)$ we obtain $P_{eb}\left[S(t_d)\right]\frac{(k-r)^r}{(r+1)^2}$. Since this is a decreasing function of r, it has a maximum value of $P_{eb}\left[S(t_d)\right]\frac{k-1}{4}$ and it occurs at r = 1. Then, $\frac{P_{k,r+1}(0,t_d)}{P_{k,r}(0,t_d)} < P_{eb}\left[S(t_d)\right]\frac{k-1}{4} < P_{eb}\left[S(t_d)\right]\frac{k}{4}$. Further, $\frac{P_{eb}\left[S(t_d)\right]^k}{4} < \frac{1}{10}$ when $P_{eb}\left[S(t_d)\right] < \frac{1}{2.5k}$. Hence, $\frac{P_{k,r+1}(0,t_d)}{P_{k,r}(0,t_d)} < \frac{1}{10}$ when $P_{eb}\left[S(t_d)\right] < \frac{1}{2.5k}$. $P_{eb}\left[S(t_d)\right]$ is well approximated by the mean of $P_{eb}(t)$ over the terminal word. That is,

$$P_{eb}\left[S(t_d)\right] \cong \frac{1}{\Delta t}\int_{t_d-\Delta t}^{t_d} P_{eb}(t)dt,$$ where Δt is duration of the terminal word. Hence,

$$P_{eb}\left[S(t_d)\right] < \frac{1}{2.5k} \text{ if } \frac{1}{\Delta t}\int_{t_d-\Delta t}^{t_d}\binom{k}{1}P_{eb}(t)^1dt < \frac{1}{2.5}.$$

By Theorem 2, this condition is satisfied when the probability of a single error in the terminal word is less than $\frac{1}{2.5}$. This

is less stringent than assuming that the instantaneous probability of a word error is less than $\frac{1}{2.5}$ which was assumed to be a normal operating condition of MBST.

Hence,

$$\frac{P_{k,r+1}(0,t_d)}{P_{k,r}(0,t_d)} < \frac{1}{10}.$$

Observe that

$$P^*_{k,r_0}(0,t_d) = \sum_{r=r_0}^{r_0+n} P_{k,r}(0,t_d), \text{ where } k = r_0+n.$$

Since $P_{k,r+1}(0,t_d) < \frac{1}{10} P_{k,r}(0,t_d)$,

$$P_{k,r_0}(0,t_d) < P^*_{k,r_0}(0,t_d) =$$

$$\sum_{r=r_0}^{r_0+n} P_{k,r}(0,t_d) < P_{k,r_0}(0,t_d)\left[1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 \cdots \left(\frac{1}{10}\right)^{11}\right]$$

Inasmuch as the right most inequality is approximately equal to one, the Theorem holds.

Q. E. D.

CHAPTER IV

MBST OPTIMIZATION

4.1 Channel Capacity of MBST

The remote station accepts signals from external devices. These signals are sampled, encoded and transmitted bit by bit. It will be assumed that encoding is accomplished, using the Huffman encoding technique. Huffman encoding is generally so efficient that the information content of each sample is essentially equal to the number of binary digits required to encode the sample. Stated differently, the information content of a sample that occurs with probability p is $\log_2(\frac{1}{p})$ bits. This is a lower bound on the number of binary digits required to encode such a sample, and through huffman encoding this lower bound is on the average nearly achieved. As such, with Huffman encoding, the remote station acts as an information source from which we realize nearly 1 bit/binary digit, the maximum amount of information per binary digit. Further, Huffman encoding implies that equally likely binary signals are transmitted. The amount of information per binary digit is reduced from 1 bit/binary digit due to information losses in the channel. The extent of these losses is easily computed for MBST over a given word interval $[t_0,t_1]$ where MBST behaves as a Binary Symmetric Channel (BSC). That is, the BSC model is applicable whenever

two conditions are satisfied (see Appendix A). First, each binary digit must have the same probability of error, p. This condition is satisfied over $[t_0,t_1]$ by the assignment of $p = P_{eb}(t_0,t_1)$. Secondly, each binary digit must behave independently. This is the case for the binary digits of MBCS as the noise is Gaussian and white of zero mean. [9] This model is shown in Figure 4.1.

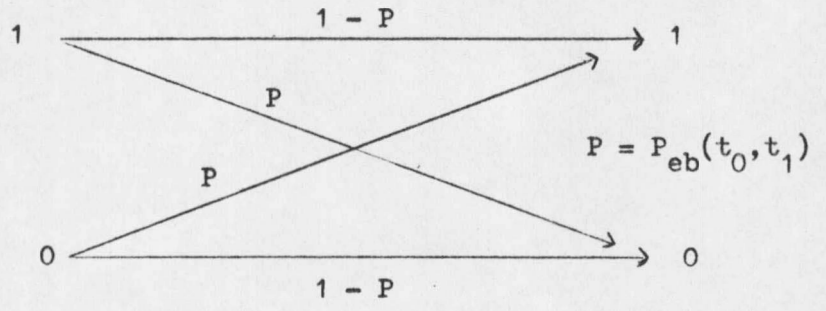


Figure 4.1
MBST as BSC Over $[t_0,t_1]$

The maximum amount of information received per binary digit during a given transmission of MBST is called the channel capacity of MBST. We denote the channel capacity of MBST restricted to $[t_0,t_1]$ by $C(t_0,t_1)$. It is well known that for the BSC the channel capacity is achieved if and only if equally likely binary signals are transmitted. [16] This condition is necessarily met when Huffman encoding is used. Hence, $C(t_0,t_1)$ is achieved and

$$C(t_0,t_1) = 1 + p\log_2 p + (1-p)\log_2(1-p)$$

where $p = P_{eb}(t_0,t_1)$.

For MBST, even in the worst case situation, without the Algebraic code, $P_{eb}(t_0,t_1) < \frac{1}{100}$, and so $C(t_0,t_1) > .93$. Hence, it may be assumed that the channel capacity of MBST exceeds .95. Let the channel capacity of MBST be denoted by $C[B,t_d]$, as it necessarily depends upon B and t_d.

Theorem 6 (A)

The amount of information received per transmission of the MBST is $I(B,t_d) = C[B,t_d] \cdot B \cdot t_d$ and $C[B,t_d] > .95$.

Proof.

Through Huffman encoding, the remote station behaves as an information source, achieving nearly 1 bit/binary digit. Hence, the total amount of information transmitted is nearly equal numerically to the total number of binary digits transmitted. Further, equally likely binary signals result from Huffman encoding; as such, the BSC achieves its capacity, $C[B,t_d]$, which exceeds .95.

Q. E. D.

$C[B,t_d]$ is a complicated function of B and t_d and not important to determining the choice of B and t_d that approximately

optimize $I(B,t_d)$ because variations of $C[B,t_d]$ are confined to the range $.95 \leq C[B,t_d] < 1$, whereas $I(B,t_d)$ may vary by a factor of 2 or more over the range of t_d and B. Hence, we state the following Theorem as a simple gross measure of I.

Theorem 6 (B)

$I(B,t_d) \cong B \cdot t_d$, where the approximation is within 5 % of the true value.

Proof.

This is immediate from Theorem 6 (A).

Q. E. D.

4.2 Coded Equivalent

In an algebraic code of block length k, if n bits are information (k-n parity check bits), the code rate is n/k. The total amount of information received per transmission when the code is used is $I = (n/k)\ C'[B,t_d]B \cdot t_d$ where $C'[B,t_d]$ is the channel capacity when the code is being used. If $C[B,t_d]$ is the channel capacity with no code (previously considered) $1 > C'[B,t_d] > C[B,t_d] > .95$. Hence, $I \cong n/k\ B \cdot t_d$, by Theorem 6.

As such, any expression involving I, for example, Theorem 6, may immediately be translated to an equivalent expression, assuming a code of rate n/k had been used, by replacing I by n/kI.

This simply states that n of each k bits are used for information and k-n for redundancy. Such equivalent forms are understood to exist in most cases, but are stated explicitly in some instances in what follows.

4.3 Minimum Signal-to-Noise, S_0, s_0

For intuitive purposes, consider the Graphs of $\left[P_{eb}(S),\ S(t)\right.$ and $\left.P_{eb}(t)\right]$ depicted in Figure 4.2. Here, the dashed line indicates the minimum terminal signal-to-noise allowed at the

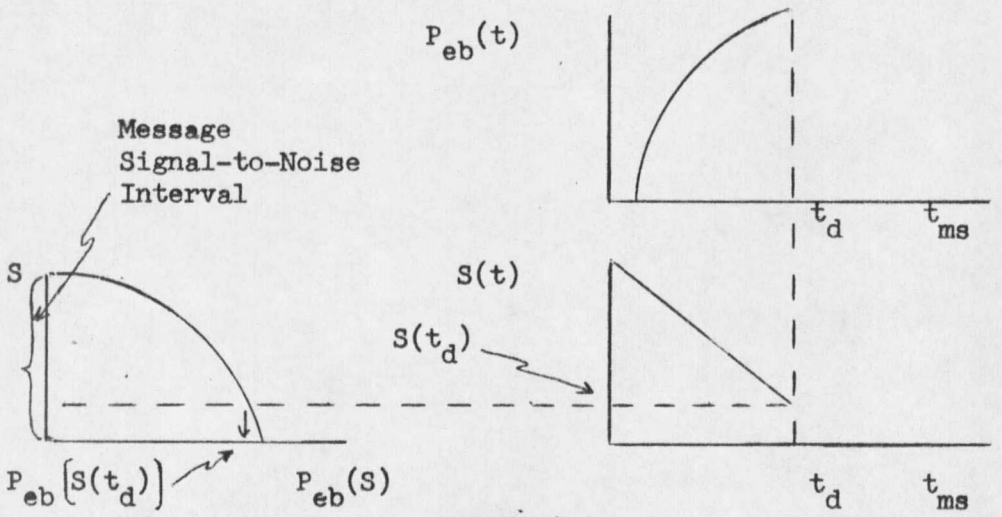


Figure 4.2
Representing $P_{eb}(S)$, $S(t)$, $P_{eb}(t)$

detector output. This is determined by the probability of error constraint. For instance, if we require that the probability of receiving a word containing at least r errors must be less than P, then by Theorem 6,

$$P^{*}_{k,r}(0,t_d) \cong \binom{k}{r} \frac{P_{eb}[S(t_d)]^r}{10 \cdot r} = P, \text{ and so,}$$

$$P_{eb}S(t_d) \cong \sqrt[r]{\frac{10 \cdot rP}{\binom{k}{r}}} .$$

Having computed this value, the corresponding $S(t_d)$ may be determined graphically as shown in Figure 4.2. Other error constraints may be specified and the $\left[\binom{k}{r}P_{eb}(S)^r, S(t), \binom{k}{r}P_{eb}(t)^r\right]$ curves as well as the $\left[P_{eb}(S), S(t), P_{eb}(t)\right]$ curves may be used to determine the minimum signal-to-noise, $S(t_d)$, for which the constraint is satisfied. Henceforth, the minimum signal-to-noise required to satisfy the probability of error constraint will be denoted by S_0 db, and too, we let $S_0 = 10\log_{10}(s_0)$.

By Equation 2-3, the signal-to-noise at the detector output at sample time t is $s(t) = \frac{(Ve^{-t/t_D})^2}{2BN}$. The function and its relationship to the minimum signal-to-noise s_0 is shown in Figure 4.3.

4.4 Optimum Message Duration, t_d, and Bit Rate, B for MBST

As shown in Figure 4.3, the probability of error constraint requires that $\frac{(Ve^{-t_d/t_D})^2}{2NB} = s_0$. If the MBST is fixed except for t_d and B, where $t_D/2 < t_d < t_D$ and $B < W/2$, there are a variety of choices of B and t_d for which the relationship $\frac{(Ve^{-t_d/t_D})^2}{2NB} = s_0$ is satisfied, and so give rise to operation within the probability

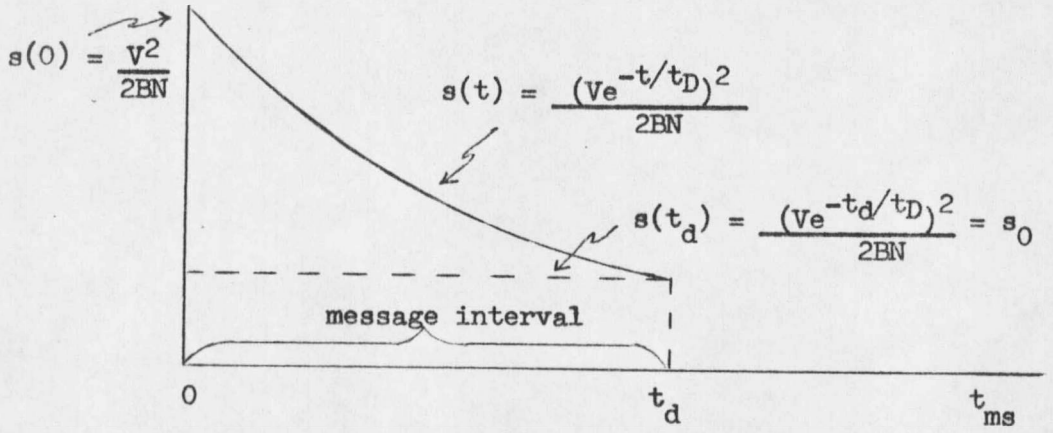


Figure 4.3
Signal-to-Noise Variation Over Message

of error constraint. By Theorem 6, for a given B and t_d, the total amount of information received per transmission is within 5 % of $B \cdot t_d$. That is, $I \cong B \cdot t_d$ bits. Hence, there exists a tradeoff between message duration and bit rate that critically affects I. The following Theorem indicates how to choose the best t_d and B pair that satisfy the probability of error constraint s_0.

Theorem 7

To optimize I, choose the message duration that maximizes the bit rate within the probability of error constraint. That is, probability of error constraint requires that

$$s_0 = \frac{(Ve^{-t_d/t_D})^2}{2BN}, \text{ thus } B = \frac{(Ve^{-t_d/t_D})^2}{2Ns_0}.$$ When t_d is chosen to be the least value constrained by $t_D/2 < t_d < t_D$ for

which B is constrained by $B < W/2$, I is optimized.

Proof.

$I \cong B \cdot t_d$ and as stated in the hypothesis, in order to satisfy the probability of error constraint, it is required that

$$s_0 = \frac{(Ve^{-t_d/t_D})^2}{2BN} \text{ or equivalently, } B = \frac{(Ve^{-t_d/t_D})^2}{2s_0N} .$$

Hence, $I(t_d) \cong \frac{(Ve^{-t_d/t_D})^2}{2s_0N} t_d$. Upon differentiating with respect to t_d, we have

$$\frac{dI(t_d)}{dt_d} = \frac{(Ve^{-t_d/t_D})^2}{2s_0N} \left(1 - \left(\frac{2t_d}{t_D}\right)\right) = B\left(1 - \frac{2t_d}{t_D}\right)$$

Since $B > 0$, the sign of the derivative is completely determined by the factor $1 - 2t_d/t_D$. It is positive, indicating $I(t_d)$ is increasing, for $t_d < t_D/2$, and negative, indicating $I(t_d)$ is decreasing for $t_d > t_D/2$. Hence, $I(t_d)$ has a single maximum value and it occurs at $t_d = t_D/2$.

Since $B = \frac{(Ve^{-t_d/t_D})^2}{2s_0N}$, it decreases as t_d increases. It may be necessary to choose $t_D/2 < t_d$ in order to reduce B such that $B \leq W/2$, the bandwidth constraint. Since $I(t_d)$ increases as t_d decreases for $t_D/2 < t_d$, the least t_d for which $B = \frac{(Ve^{-t_d/t_D})^2}{2Ns_0} \leq W/2$ yields the largest $I(t_d)$ within the constraints on the probability of error, bandwidth and message duration.

Observe that if there is no t_d with its constraint that is large enough so that B remains within the bandwidth constraint, then the probability of error constraint would have to be relaxed in order that the MBST could be designed to meet it.

Q. E. D.

4.5 An Intuitive View of Optimally Choosing t_d and B

Intuitively, the problem may be viewed as follows. The detector output signal-to-noise ratio is

$$\frac{(Ve^{-t/t_D})^2}{2NB} = s(t) = \left[\frac{(Ve^{-t/t_D})^2}{2} \cdot T\right] 1/N$$

Since the input to the detector is $n_w(t) \pm Ve^{-t/t_D}\text{Cos}W_0t$, the input signal energy is $\frac{(Ve^{-t/t_D})^2}{2} \cdot T$. The probability of error constraint requires $\frac{(Ve^{-t/t_D})^2}{2} \cdot T \geq s_0N$.

That is, only those bits whose signal energy at the detector input that are above s_0N satisfy the probability of error constraint. If the bit duration T is chosen to be large (i.e., low bit rate) as in Figure 4.4 (b), the bit energy $\frac{(Ve^{-t/t_D})^2}{2} \cdot T$ (the area under the curve corresponding to that bit) remains larger than s_0N (two squares in Figure 4.4) for a longer period of time than when T is chosen as in Figure 4.4 (a) (higher bit rate). However,

more bits (areas) satisfy the requirement of remaining larger than s_0N at the higher bit rate. Hence, in this latter case, the energy is simply more efficiently distributed among the bits.

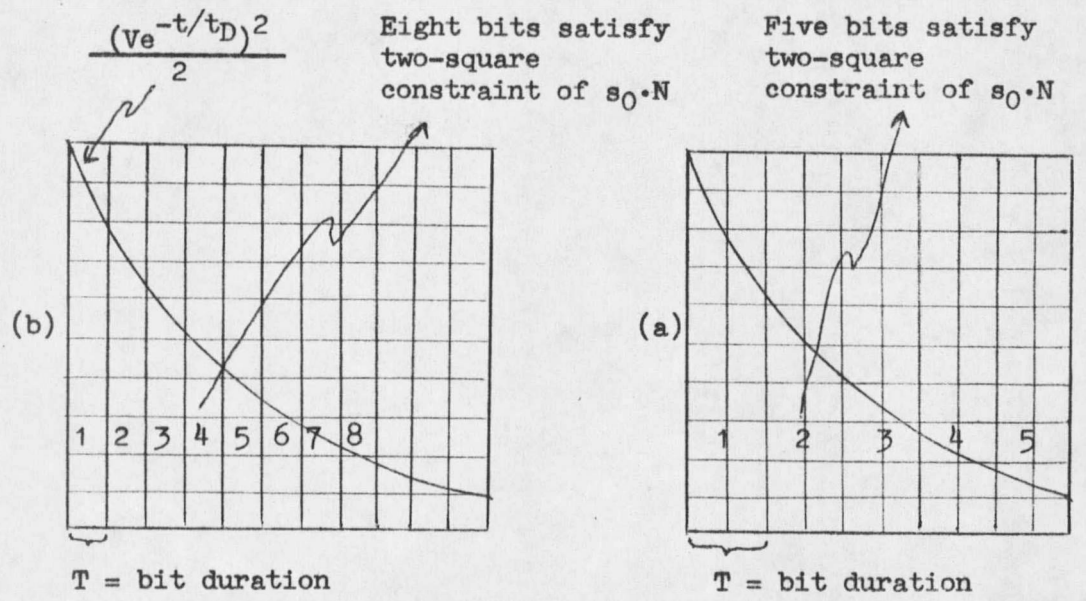


Figure 4.4

4.6 MBST Design Procedure and Equations

To begin with, t_D and N are assumed to be known, and too, the probability of error constraint, P, and amount of information received per transmission required to accomplish the task are assumed to be known.

Then, k, the number of bit/word, is chosen. Since the duration of a word is required to be less than 1/10 the time decay constant, we must have $k \cdot T < t_D/10$.

Next, s_0, the minimum detector output signal-to-noise ratio required to satisfy the probability of error constraint, is determined. If P is a constraint on the instantaneous bit or word error, the $\left[P_{eb}(S),\ S(t),\ P_{eb}(t)\right]$ or $\left[\binom{k}{r}P_{eb}(S)^r,\ S(t),\ \binom{k}{r}P_{eb}(t)^r\right]$ curves may be used directly. If P is a constraint on the probability of receiving a bit or word in error,

$$P^*_{k,r}(0,t_d) \cong P_{k,r}(0,t_d) \cong \binom{k}{r}\frac{P_{eb}\left[S(t_d)\right]^r}{10\cdot r} \quad \text{is used}$$

(Theorems 4 and 5). If an algebraic correction and/or detection is being used, the curves and formula are still applicable. For example, if a double error-correcting code is used and it is required that the probability of receiving a word with at least 3 errors is less than 10^{-4}, we require

$$P^*_{15,3}(0,t_d) = \binom{15}{3}\frac{\left(P_{eb}\left[S(t_d)\right]\right)^3}{30} = 10^{-4},$$

when the block length is $k = 15$. Then $P_{eb}\left[S(t_d)\right]$ may be determined. And correspondingly, $S(t_d) = S_0 = 10\log_{10}(s_0)$, so that s_0 may be determined.

Finally, the message duration that optimizes I within the probability of error constraint (equivalent) s_0 and bandwidth constraint is chosen. Let V be the initial peak signal voltage at the detector input. V must be large enough so that

(4-1) $B = \frac{(Ve^{-t_d/t_D})^2}{2Ns_0} \leq W/2$ for some t_d in the range $t_D/2 < t_d < t_D$.

According to Theorem 7, choose the least t_d such that 4-1 holds and I will be optimum for that message's duration. If we denote this t_d by $t_d = t_0$, we have

$I \cong B \cdot t_0 = \frac{(Ve^{-t_0/t_D})^2}{2Ns_0} t_0$. Observe that the information rate under these conditions is $I/t_0 = B$. Since $B = \frac{(Ve^{-t_0/t_D})^2}{2Ns_0}$, or in db we have

$$10\log_{10}\frac{V^2}{2} = 10\log_{10}B + 10\log_{10}N + \left(\frac{-8.68}{t_D} t + S_0\right)$$

Initial normalized Signal Power in db

Information Rate in db

Power Density in db

S(0), the Required Initial Signal-to-Noise

Signal-to-Noise Equivalent of Probability of Error Constraint

This equation indicates the tradeoffs between the various choices involved.

4.7 The Effect of Order on Meeting Constraints on I

The probability of error constraint must be satisfied in

order for a solution to exist to the design problem. After that, the constraints may be encountered in the following ordereds, (t_0,V,B), (t_0,B,V), (B,V,t_0), (B,t_0,V), (V,B,t_0), (V,t_0,B). It is interesting to consider how I is affected in some of the different cases.

Case I

For $(t_0 = t_D/2,\ B = W/2,\ V < V_{max})$.

I has its maximum value of $I \cong B \cdot t_0 = W/2 \cdot t_D/2$, within the probability of error constraint. If V is increased beyond where

$$\frac{V^2e^{-1}}{2Ns_0} = W/2,$$

a new more stringent probability of error constraint is being satisfied and so energy is being wasted.

Case II

For $(t_0 = t_D/2, B < W/2, V < V_{max})$,

$$I = \left(\frac{V^2e^{-1}}{2Ns_0}\right) \cdot \left(\frac{t_D}{2}\right)$$

($\frac{V^2e^{-1}}{2Ns_0}$ → B; $\frac{t_D}{2}$ → t_0)

Observe that as V increases, I decreases as the square of V, as B does. This is a very rapid increase in I at the expense of bandwidth. On the other hand, for $(B = W/2,\ t_D/2 < t_0 < t_D$,

$V < V_{max}$), $I = W/2 \cdot t_0$ where $W/2 = B = \frac{(Ve^{-t_0/t_D})^2}{2Ns_0}$, or

equivalently,

$$t_0 = t_D/2 \log_e\left(\frac{V^2}{WNs_0}\right), \text{ so } I = \underset{\downarrow}{(W/2)}\left(\frac{t_D \log_e V}{WNs_0}\right)_{\swarrow t_0}.$$

t_0 may be increased if V is increased and B will remain equal to W/2. But this results in a much slower increase of I. Specifically, I increases only logarithmically ($\log_e V$) with V. Hence, once the bandwidth constraint is met, further increase in I by increasing the message duration at the expense of V is hardly worthwhile.

Example 4

MBST Design Problem.

It is assumed that the channel consists of a meteor trail with a time decay constant t_D = 100 ms and that the power spectral density $N = 1 \times 10^{-16}$ watts/cycle. It is determined that the MBST must transfer 400 bits of information per transmission. Further, the words are taken to be 30-bit sequences and not more than one in one hundred words can be in error. What is the minimum power required to accomplish this task?

Design Solution.

1. $I = 400$ bit/transmission $\cong B \cdot t_d$.

2. Minimum power corresponds to optimal operation, so choose $t_d = t_D/2$. That is, $t_D/2 = 100$ ms$/2 = 50$ ms. Then, the required bit rate is $B = I/t_d = 400$ bits$/50$ ms $= 8000$ bits/second. And, this requires a bandwidth (low pass) of $2 \times 8000 = 16{,}000$ hz.

3. The probability of error constraint requires that

$$P^*_{30,1}(0,t_d) < 1/100, \text{ but } P^*_{30,1}(0,t_d) \cong P_{30,1}(0,t_d)$$

$$\cong \binom{30}{1} \frac{P_{eb}[S(t_D/2)]}{10 \cdot 1} = \frac{1}{100},$$

or equivalently, $P_{eb}\left(S\left[\frac{t_D}{2}\right]\right) = \frac{10}{(30)(100)} = \frac{1}{300}$. From the Graph of $P_{eb}(S)$ (Figure 3.1) this requires that $S(t_D/2) = 10$ db.

4. The drop of signal-to-noise ratio during the transmission is $\left(\frac{8.68}{t_D}\right) t_0$ evaluated at $t_0 = t_D/2$. That is, 4.34 db.

5. Hence, the initial signal-to-noise ratio, $S(0)$, must be

$$10\log_{10} \frac{V^2}{2NB} = S(0) = 4.34 + 10 \text{ db}$$

4.34: dropped during transmission

10 db: must have S above to meet probability of error constraint

6. Thus,

$$10\log(V^2/2) = 10\log_{10}(8000) + 10\log_{10}(1\text{x}10^{-16}) + 14.34 \text{ db}$$

$$= 53.3 \text{ db} - 160 \text{ db} = 106.7 \text{ db}$$

Hence, $(V^2/2) = 25$ u u watts, or $V/\sqrt{2} = 5$ u volts.

Thus, it requires a 5 u v rms detector input signal to transmit 400 bits with 1/100 words of 30 bits each in error over the given meteor trail.

It is worthwhile to compare the amount of information we would realize if we assume the same conditions but transmitted instead for $t_D = 100$ ms rather than $t_D/2 = 50$ ms. $S(t_D) =$ 10 db as the probability of error constraint has not changed. But the signal-to-noise dropped during the transmission is now $\left(\frac{8.68}{t_d}\right) t_0$ evaluated at $t_0 = t_D$. That is, 8.68 db. Hence, we need

$$10\log_{10} \frac{V^2}{2NB} = S(0) = 18.68 \text{ db or}$$

$$10\log_{10}(B) = 10\log_{10} \frac{(5\times10^{-6})^2}{1\times10^{-16}} = 18.68 \text{ db}$$

or B = 3,140 bit/second.

Since we are transmitting for 100 ms,

I = 3.14 bit/ms x 100 ms = 314 bits.

As a result, doubling the time duration causes a reduction of about 3/4 in the amount of information received. Of course, the bandwidth is 2 x 3,140 in the second case, as opposed to 2 x 8000 in the first case.

4.8 MBST Utility

There are three basic reasons for considering MBST operation. First, MBG (the MBCS with a forward scatter medium consisting of an ensemble of meteor trails) may be viewed as an ensemble of MBST systems with a common implementation. Secondly, it is useful to know how the MBCS responds on a per-trail basis. And, thirdly, it is possible to implement an adaptive system in which all received signals appear to have been transmitted over a single meteor trail.

4.9 MBG Adaptive System (Approximate MBST Operation)

The simplest (conceptually) adaptive system is one in which the amplification, and hence signal power, at the remote station is adjusted, based on the strength of the interrogate signal, to a level which insures that the signal received at the base station has essentially a fixed initial detector input value, V_0. If this is done and t_D has a narrow variance over the various meteor trails, the signal input to the detector would, except for small variance, be $V_0 e^{-t/t_D} \mathrm{Cos} W_0 t$ for each transmission. And near optimal operation within the various constraints would be achieved for each transmission.

Alternately, if Ve^{-t/t_D} is the envelope of the interrogate signal, t_0 seconds after reception its value is Ve^{-t_0/t_D}.

(Recall that $t_D/2 < t_0 < t_D$.) If $t_0 = at_D$, then $Ve^{-t_0/t_D} = Ve^{-a}$. Hence, if the transmission continues only until the interrogate signal decays to Ve^{-a} (e^{-a} is fixed value), the duration of the message transmitted is t_0. This would yield near optimal operation within the various constraints for each transmission.

In either case, the MBG adaptive system would behave as an MBST system operating nearly optimally for each transmission.

CHAPTER V

MBG AS ENSEMBLE OF MBST

5.1 MBG (Nonadaptive) Normal Operating Conditions

The MBG is considered to be an ensemble of MBST with a common implementation. The normal operating conditions of MBG are assumed to be those of an MBST with $t_D = \overline{t_D}$ (see Section 3.3), where $\overline{t_D}$ is the mean time decay constant over the ensemble. That is, each MBST in the ensemble is assumed to have the same word duration, $kT \leq \overline{t_D}/10$, message duration, $\overline{t_D}/2 < t_d < \overline{t_D}$, bit rate, $B < W/2$, and probability of error constraint (equivalent) s_0, where s_0 represents a constraint more stringent than the requirement that the instantaneous probability of a bit and word error is less than 1/100 and 1/2.5, respectively.

5.2 Applicability of Theorems 1-7 to MBG

The normal operating conditions of MBG are slightly weaker than assuming that the normal MBST operating conditions hold for each MBST in the ensemble. However, Theorems 1-7 apply, being affected only in the closeness of the approximations. The degree of approximation variation due to the slightly weaker conditions is negligible, as will be argued. And, the form of the MBG normal operating conditions stated is useful and simple to work with.

With the exception of the message duration, $\overline{t_D}/2 < t_d < \overline{t_D}$, and word duration, $kT < \overline{t_D}/2$, the normal operating conditions of

MBST are satisfied for each MBST in the ensemble. Experimental data indicate that the time decay constants of the ensemble are typically constrained by $\overline{t_D}/2 < t_D < 2\overline{t_D}$ (discussed in Time Decay Constraints Section.) Assuming this to be the case, these inequalities imply that the message duration is constrained by $t_D/4 < \overline{t_D}/2 < t_d < \overline{t_D} < 2t_D$ and the word duration by $kT < t_d/5$. This is a relaxation of the normal operating conditions of MBST which require $t_D/2 < t_d < t_D$ and $kT < t_d/10$. But the arguments leading to Theorems 1-7, with these numerical values for the constraints would lead to the same conclusion with negligible variation in the closeness of the approximations resulting. (The assumption of $\overline{t_D}/2 < t_D < 2\overline{t_D}$ is not critical to the results obtained but simplifies and unifies the discussion involved. Further, t_D is apparently distance dependent, but appears to be constrained about the mean, $\overline{t_D}$, as indicated, for a fixed distance between transmitter and receiver.)

5.3 Apparent Threshold

The signal-to-noise ratio at the MBG detector output over the course of several transmissions is represented in Figure 5.1. Here, it is assumed that the message duration is t_d^*.

Recall that if the interrogate signal received at the remote station is above a certain level, the threshold, a transmission

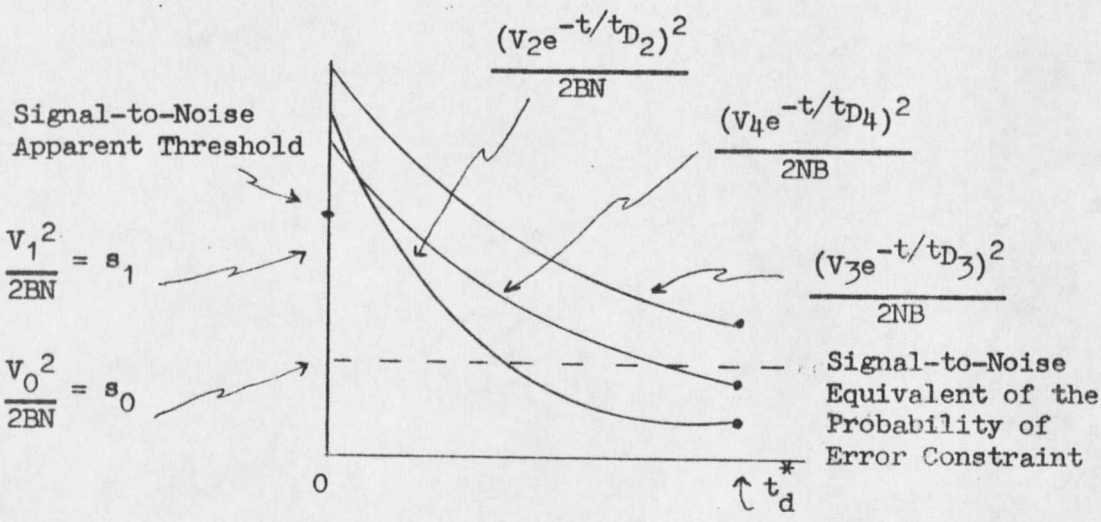


Figure 5.1
Apparent Threshold

is returned, using that same meteor trail as a forward scatter medium. By reciprocity of the meteor trail model, the signal received at the base station must be above a corresponding level. This level, measured at the detector input, is called the apparent threshold and is denoted by V_1. s_1 is the equivalent signal-to-noise apparent threshold as indicated in Figure 5.1.

5.4 Useable Meteor Trails

Figure 5.1 illustrates that only one of the signals satisfies the probability of error constraint, s_0, namely, the one that remains above s_0 for the duration of the message, t_d milliseconds. Any signal whose initial signal-to-noise is above s_0 has some

probability of satisfying the probability of error constraint. Such a signal is said to have had a useable meteor trail acting as the forward scatter medium. (Equivalently, any signal whose initial detector input signal envelope is above V_0 is said to have a useable meteor trail acting as the forward scatter medium.)

Let A and B denote distinct modes of operation of a MBCS system operating within the normal MBG operating conditions. Suppose that for both modes of operation, P_{trans}, B, s_0, and the meteor trail ensemble are the same. These systems necessarily have the same set of useable meteor trails. If there are a large number of useable meteor trails involved, say M, the average amount of information received per useable meteor trail may be computed and an absolute comparison of the two modes of operation on this basis can be made. This is the most important reason for introducing the notion of useable meteor trails.

Throughout, it is assumed that $s_1 > s_0$, equivalently $V_1 > V_0$. This implies that the apparent threshold is always chosen so that each signal received at the base station is useable, that is, has some probability of satisfying the probability of error constraint. However, even when s_1 is strictly greater than s_0, some useable meteor trails are not used by the MBCS. This is the case in certain modes of operation (discussed later) in which

it is desirable to sacrifice some useable meteor trails so that a simpler implementation is realizable.

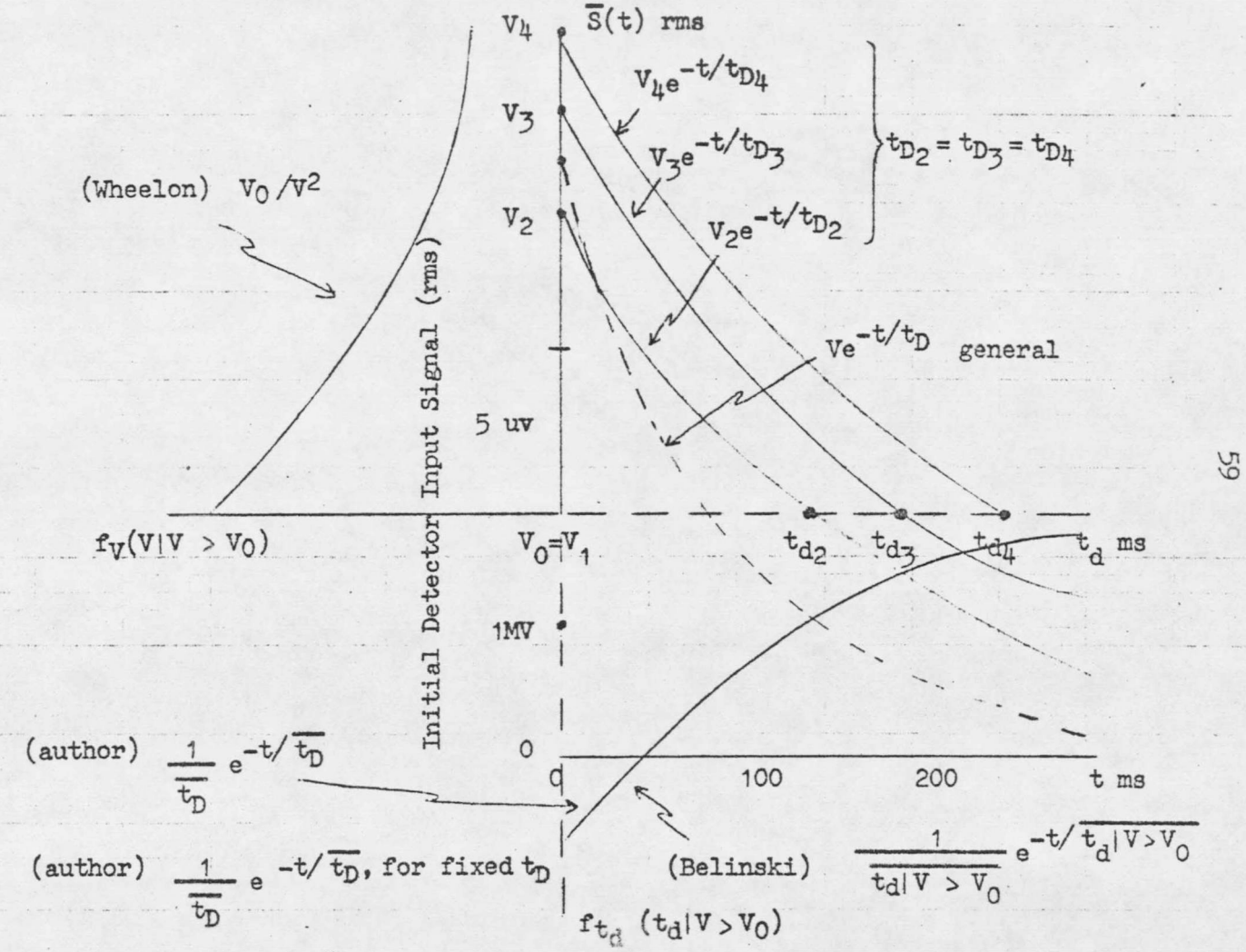


Figure 5.2
Depicting Probability Density Functions for MBG

CHAPTER VI

MBG STATISTICAL CONSIDERATIONS

6.1 Statistical Notation

In what follows, we denote the probability density function (p.d.f.) of a random variable x by $f_x(x)$, with $f_x(x|M)$ being the p.d.f. of x restricted to M. Further, $P_x(x < x_0)$ and $P_x(x < x_0|M)$ are the corresponding probabilities that $x < x_0$. Similarly, the joint p.d.f. of the random variables x and y is denoted $f_{x,y}(x,y)$ with $f_{x,y}(x,y|M)$ being the joint p.d.f. restricted to M.

In addition, mean values of x, and x restricted to M, are denoted by $\bar{x}$ and $\overline{x|M}$.

6.2 MBG Statistics

In Figure 5.2, the statistics of fundamental importance to MBG are depicted. Here, the apparent threshold, V_1, is assumed to coincide with the equivalence of the probability of error constraint, V_0. This yields a set of statistics over the entire set of useable meteor trails.

The first statistic considered is the p.d.f. of the duration, t_d, above V_0 over the set of useable meteor trails.

Theorem 8

The random variable, t_d, the duration above V_0 for useable

meteor trail envelopes, Ve^{-t/t_D}, is independent of V_0. Furthermore for t_D fixed value,

$$f_{t_d}(t_d|t_D) = 1/t_D\ e^{-1/t_D}$$

and over the set of all t_D,

$$f_{t_d}(t_d) = \int_{t_D} f_{t_d}(t_D)\ 1/t_D\ e^{-t_d/t_D}\ dt_D$$

Proof.

By A. D. Wheelon, [8] the p.d.f. of V for meteor trails yielding envelopes Ve^{-t/t_D} is Q/V^2, where Q is a positive constant. Since it is necessary that $\int_{V_0}^{\infty} Q/V^2\ dV = Q/V_0 = 1$ when the p.d.f. of V is restricted to be for $V > V_0$, $Q = V_0$. And

$$(6\text{-}1) \quad f_V(V|V > V_0) = V_0/V^2$$

Each envelope resulting from the assumed underdense meteor trail forward scatter medium approximately must satisfy the relationship $Ve^{-t_d/t_D} = V_0$. That is, it decays to V_0 after time t_d. Hence,

$$(6\text{-}2) \quad V = V_0 e^{t_d/t_D}$$

and so we have

$$(6\text{-}3) \quad f_V(V|V > V_0) \frac{dV}{dt_d} = f_{t_d}(t_d|V > V_0|t_D)$$

for $0 < t_d < \infty$, by the standard technique of transforming random variables. That is, we have the p.d.f. of t_d, the duration above V_0, for useable meteor trails yielding envelopes Ve^{-t/t_D} with fixed time decay constant t_D. Now, substituting 6-1 and 6-2 in 6-3, we obtain

$$(6\text{-}4) \quad f_V(V|V > V_0) \frac{dV}{dt_d} = \frac{V_0}{V^2} \frac{dV}{dt_d} = \frac{V_0}{(V_0 e^{t_d/t_D})^2} \left(V_0 \frac{de^{t_d/t_D}}{dt_d}\right).$$

Hence,

$$(6\text{-}5) \quad f_{t_d}(t_d|V > V_0|t_D) = 1/t_D \; e^{-t_d/t_D}$$

Observe that V_0 was canceled in 6-4 and as a result $f_{t_d}(t_d|V > V_0)$ is independent of it, and so we may express 6-5 as

$$(6\text{-}6) \quad f_{t_d}(t_d|t_D) = 1/t_D \; e^{-t_d/t_D}.$$

t_d is a random variable that is a function of two independent random variables V and t_D. [15] Hence [7]

$$(6\text{-}7) \quad f_{t_d}(t_d|V > V_0) = \int_0 f_{t_d}(t_d|V > V_0|t_D) f_{t_d}(t_D) dt_D$$

But, $f_{t_d}(t_d|V > V_0|t_D) = f_{t_d}(t_d|t_D)$ as mentioned in obtaining 6-6, hence

$$f_{t_d}(t_d|V > V_0) = \int_0^\infty f_{t_d}(t_d|t_D) f_{t_d}(t_D) dt_D = f_{t_d}(t_d).$$

and so t_d is independent of V_0 and the Theorem holds as a result of this and 6-6.

Q. E. D.

At first glance, this theorem seems rather surprising. That is, it implies that the probability of a useable meteor trail being above a given level V_0 for a duration, say t_d^*, is independent of the level chosen. Figure 6.1 illustrates why this is the case. Here, t_D was chosen fixed for clarity and the comments on the mean will be discussed later.

It is evident that since the p.d.f. of V and the decaying envelopes remain in the same relationship to each other regardless of the level involved, the p.d.f. of t_d doesn't change as the level does.

Using the p.d.f.'s introduced in Theorem 8, we may compute $\overline{t_d}$ and $\overline{t_d|t_D}$.

Theorem 9

The mean duration above V_0 for useable meteor trails yielding

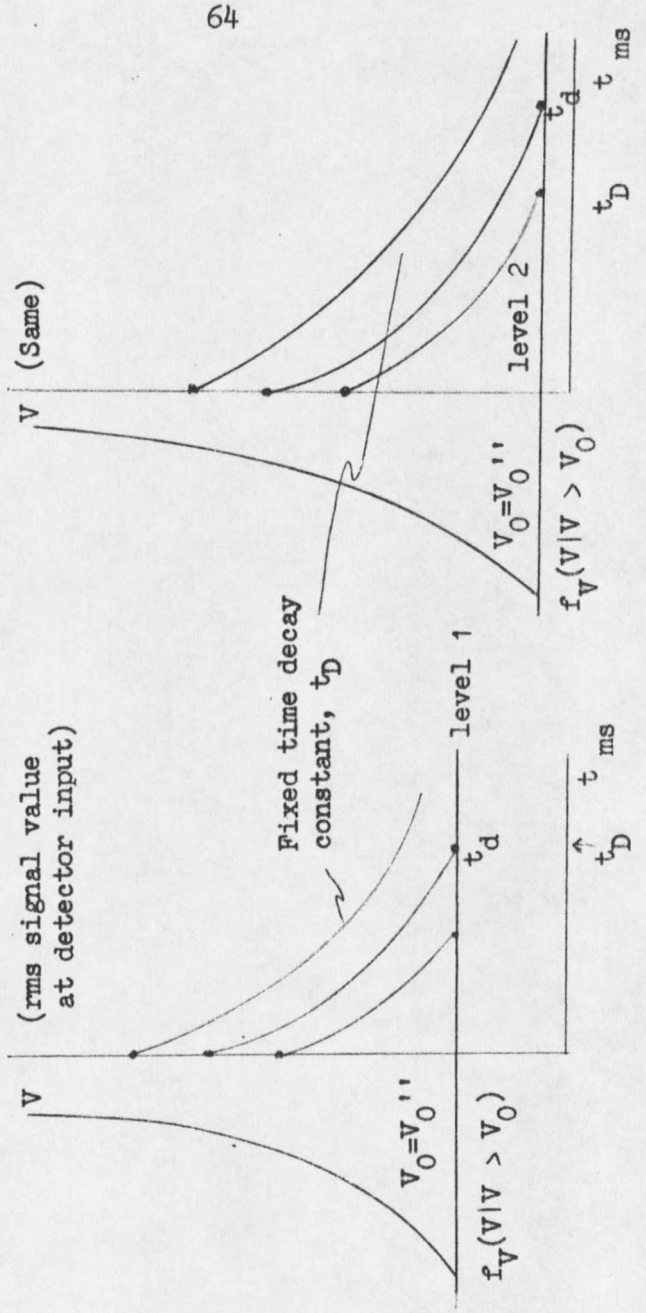


Figure 6.1
Illustrating Why t_d is Independent of V_0

envelopes Ve^{-t/t_D} with fixed t_D, is t_D. Notationally,

$$\overline{t_d|t_D} = t_D$$

And, when t_D is not fixed, it is $\overline{t_D}$. Notationally,

$$\overline{t_d} = \overline{t_D}$$

Proof.

$$\overline{t_d|t_D} = \int_{t_d} t_d f_{t_d}(t_d|t_D)dt_d, \text{ and by Theorem 8,}$$

$$(6\text{-}8) \quad \overline{t_d|t_D} = \int_0 t_d/t_D\, e^{-t_d/t_D}\, dt_d = t_D$$

Similarly,

$$\overline{t_d} = \int_{t_d} t_d f_{t_d}(t_d)dt_d, \text{ and by Theorem 8}$$

$$(6\text{-}9) \quad \overline{t_d} = \int_{t_d} t_d \left(\int_{t_D} f_{t_d}(t_D)\ 1/t_D\ e^{-t_d/t_D} dt_D \right) dt_d. \text{ Reversing the}$$

order of integration yields

$$\overline{t_d} = f_{t_d}(t_D) \left(\int_{t_d} t_d/t_D\, e^{-t_d/t_D} dt_d \right) dt_D$$

and from 6-8 this becomes

$$(6\text{-}10) \quad \overline{t_d} = \int_{t_D} t_D f_{t_d}(t_D)\,dt_D = \overline{t_D}$$

By 6-8 and 6-10, the Theorem holds.

Q. E. D.

Again, it is convenient that the means are independent of V_0. The importance of these statistical results will be discussed in the context of various modes of MBG operation in later sections.

6.3 Experimental Determination of MBG Statistics

The curve $a \cdot e^{-at}$ may easily be constructed on natural logarithm paper as shown in Figure 6.2. Here, the two points chosen to construct $a \cdot e^{-at}$ were $(0,a)$ and $(\ln(a)/a, 1)$.

$\overline{t_D}$:

The experimental data of Figure 6.3, excluding the bounding curves, was obtained by Belinski.[1] It is the number of meteor trails (normalized to this convenient graphical range) that remained above a certain threshold, say V_0, for a duration of t_d seconds. He assumed p.d.f. for t_d, that is, $1/\overline{t_d|V > V_0}\; e^{-t_d/\overline{t_d|V > V_0}}$ and constructed the curve $4e^{-4t}$ as the best fit to the data. From this, $1/\overline{t_d|V > V_0} = 4$ or $\overline{t_d|V > V_0} =$

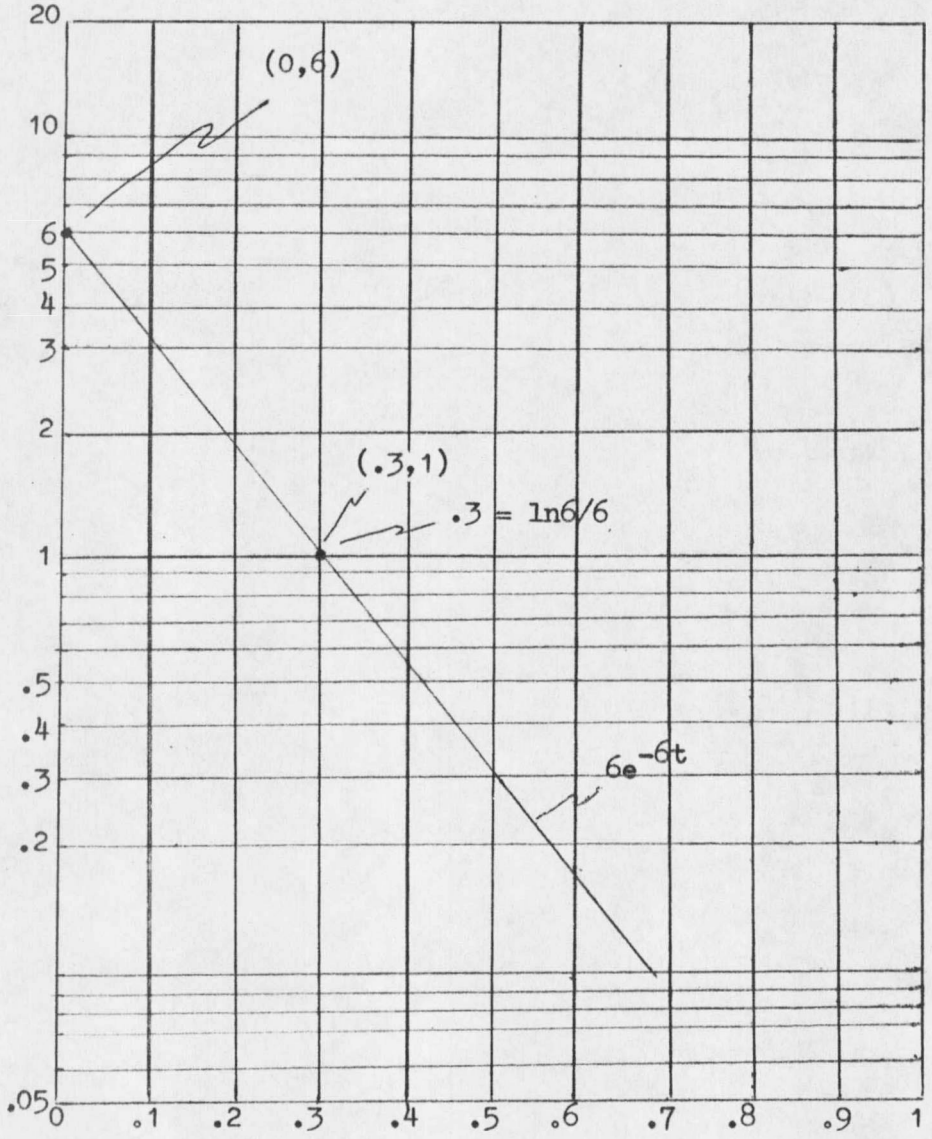


Figure 6.2
Constructing ae^{-at} Using (0,a) and (lna/a, 1)

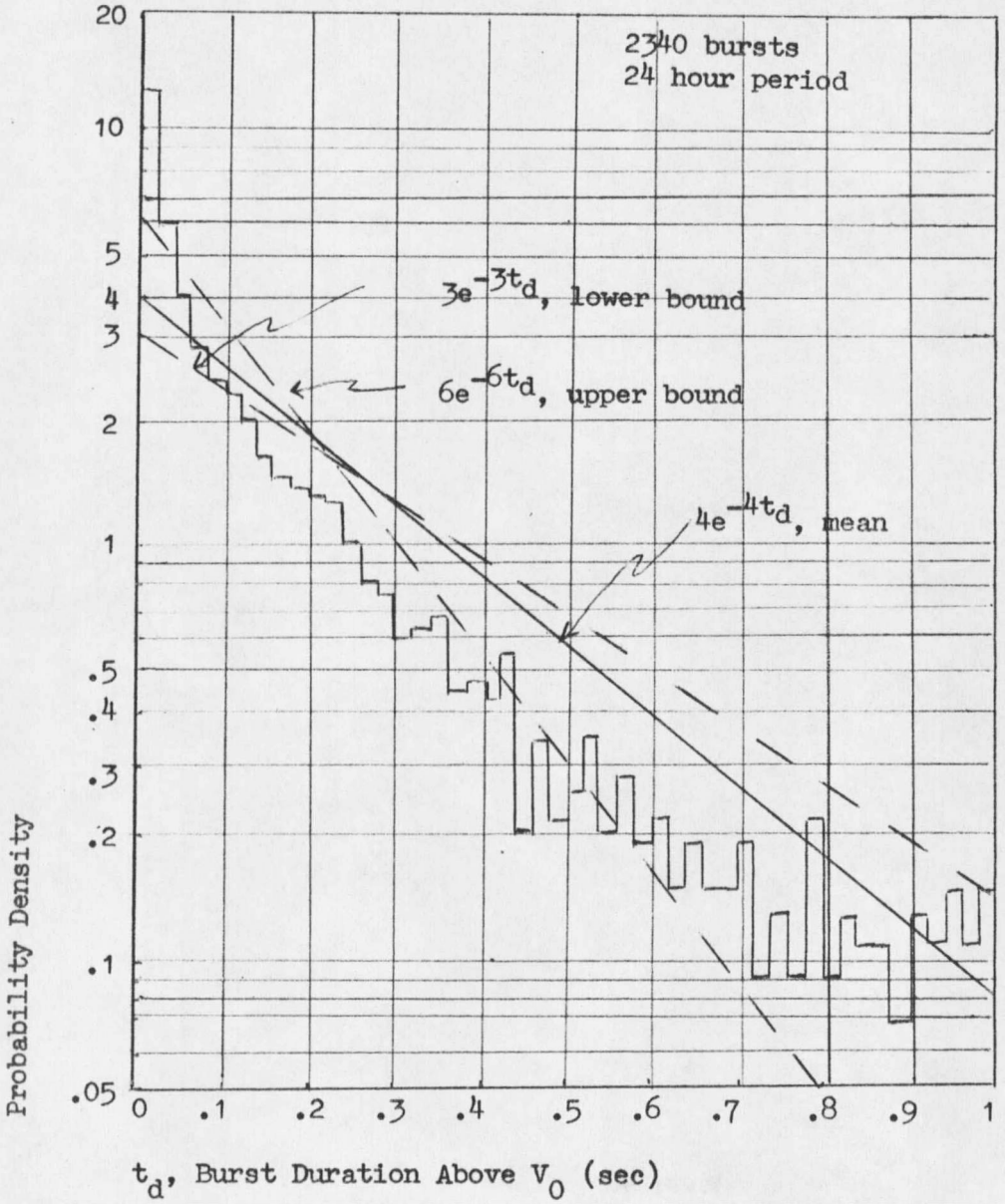


Figure 6.3
MBG Experimental Data

.25 sec. But from Theorem 9, $\overline{t_d | V > V_0} = \overline{t_d} = \overline{t_D}$. Hence, $\overline{t_D}$ = .25. Thus, Theorem 9 gives rise to a simple method of determining $\overline{t_D}$, using the duration of t_d above a fixed but arbitrarily chosen level, V_0.

$f_{t_d}(t_d)$:

Experimental data for Higgens [1] and others also indicate that the p.d.f. of t_d was exponential. It was implicitly assumed by them that the p.d.f. of t_d was independent of V_0. Based on these results and Theorems 8 and 9, the p.d.f. of the time duration, t_d, for useable meteor trails yielding envelopes, Ve^{-t/t_D}, is taken to be

$$f_{t_d}(t_d) = 1/\overline{t_D}\ e^{-t_d/\overline{t_d}} = 1/\overline{t_D}\ e^{-t_d/\overline{t_D}}$$

Observe that if the variance of t_D is very small about $\overline{t_D}$, then $f_{t_d}(t_D)$ approaches a unit impulse and consequently the p.d.f. determined using Wheelon's result and a change of variables,

$$f_{t_d}(t_d) = \int_{t_D} 1/t_D\ e^{-t/t_D} f(t_D) dt_D, \text{ approaches } 1/t_D\ e^{-t/\overline{t_D}},$$

the experimentally determined p.d.f. of t_d.

$\overline{t_D}/2 < t_D < 2\overline{t_D}$:

Using Theorems 8 and 9 and the experimental technique

discussed above, the constraints on t_D may be easily determined by using bounding curves on the experimental data of Figure 6.3. (There, two dashed lines were constructed by the author, namely, $3e^{-3t_d}$ and $6e^{-6t_d}$. They were called the lower and upper bound, respectively, as they act as such over the typical operating range of MBCS, $[0, 200\ ms]$).

If t_D^* was the time decay constant for even a small number of envelopes, Ve^{-t/t_D^*}, that comprised Belinski's experimental data, there would exist data points in the Graph of Figure 6.3 to indicate that. That is, there would be points at or about the curve $1/t_D^*\ e^{-t_d/t_D^*}$, because by Theorem 8, this is the p.d.f. of the time duration, t_d, above V_0 for envelopes of the form Ve^{-t/t_D^*}. Hence, no points about this curve would imply that the probability of such an event is zero. This in turn would imply that envelopes of the form Ve^{-t/t_D^*} were extremely sparse, or nonexistent, with the same implication about t_D^*. As a result, the bounds on t_D may be found by constructing bounding curves on the data of the form $1/t_D\ e^{-t/t_D}$ and observing the values of t_D for these bounding curves.

Example 5

$3e^{-3t_d} = 1/.333\ e^{-t_d/.333}$ and $6e^{-6t_d} = 1/.166\ e^{-t_d/.166}$ are the bounding curves for the data of Figure 6.3 and so $t_D = .333$

and $t_D = .166$ are the bounding values of t_D. Hence, $.166 < t_D < .333$ for that set of meteor trails and $\overline{t_D} = .25$ as previously determined. Then, $\overline{t_D}/2 < t_D < 2\overline{t_D}$ for Belinski's experimental data. The experimental data of Higgins also shows a similar variance of t_D about the mean, $\overline{t_D}$.

Total Experimental Data Required:

If it is assumed that $\overline{t_D}/2 < t_D < 2\overline{t_D}$, the only experimental data needed at the fixed remote site is $\overline{t_D}$ (equivalent $\overline{t_d}$) and N, the two-sided power spectral density. If the constraints on t_D are desired on a per-remote-site basis, the bounding curve technique of example is required.

It is important to note that the value of $\overline{t_D}$ is very much affected by the baseline distance (i.e., distance between remote and base station) and geographical location.[4] Hence, it is necessary to determine N and $\overline{t_d}$ (equivalently t_d) for each remote site.

CHAPTER VII

MBG OPTIMIZATION

7.1 MBG in the Terminal Threshold (TT) Mode

In the TT Mode, the base station is assumed to have a terminal threshold detector after the last I. F. amplifier stage (i.e., at the detector input) that samples the signal at t_d' ms after it is received. If it is above a certain level, V_0 (rms) volts, the message is accepted; otherwise, it is discarded. t_d' is assumed to be the duration of the message, and V_0 is the voltage equivalent of the probability of error constraint.

Theorem 10

In the TT Mode, the average amount of information received via MBG is $\overline{I}_{TT}(t_d') \cong Bt_d' e^{-t_d'/\overline{t_D}}$ bits/useable meteor trails, where $\overline{t_D}/2 < t_d' \leq \overline{t_D}$. And, it has a maximum value at $t_d' = \overline{t_D} = \overline{t_d}$ of

$$\text{Max}\overline{I}_{TT} \cong B\overline{t_d}\, e^{-1} = B\overline{t_D}\, e^{-1} \text{ bits/useable meteor trail.}$$

Furthermore, it is assumed that in this mode the apparent threshold, V_1, coincides with the equivalence of the probability of error constraint, V_0. As such, the maximum bit rate which allows the probability of error constraint s_0 to be satisfied may be expressed as

$$B = V_1^2 / 2Ns_0 = W/2$$

and when operating optimally, Theorems 1-7 apply to each MBST in the ensemble which constitute the MBG if there are at least

10 words per message.

Proof.

$\overline{I_{TT}}$:

Not all useable meteor trails yield signals that satisfy the probability of error constraint, s_0. That is, some signal envelopes are initially above V_0, but not for the full message duration, t_d'. The MBST corresponding to such a transmission is not in the ensemble of MBST constituting MBG as it does not satisfy the probability of error constraint s_0 as required under the MBG normal operating condition. (This would be detected by the TT detector. The prohibition of a meteor trail from being used as a forward scatter medium for MBG messages at this point is no different in principle from having the prohibition invoked by the threshold at the remote station. In either case, except for the energy loss in transmission, the system behaves as though no meteor trail had appeared.)

Since each MBST in the ensemble constituting MBG satisfies the probability of error constraint, s_0, and Theorems 1-7 are applicable for each MBST under normal MBG operating conditions, the amount of information received is $I = B \cdot t_d'$ for each such MBST. After the occurrence of M (a large number) useable meteor trails, the Law of Large Numbers implies that the amount of information

received is approximately

$$I_{TT} \cong M \cdot P_{t_d}(t_d > t_d') \cdot I \cong M \cdot P_{t_d}(t_d > t_d')(B \cdot t_d') \text{ bits, where}$$

$P_{t_d}(t_d > t_d')$ is the probability that the detector input signal envelopes, Ve^{-t/t_D}, remain above V_0 for the duration of the message t_d' (see Figure 5.1 and 5.2). Since

$$f_{t_d}(t_d) = 1/\overline{t_D}\ e^{-t_d/\overline{t_D}},$$

$$(7\text{-}1) \qquad P_{t_d}(t_d > t_d') = \int_{t_d'} 1/t_D\ e^{-t_d/\overline{t_D}}\ dt_d = e^{-t_d'/\overline{t_D}}.$$

Hence,

$$I_{TT} \cong MBt_d'e^{-t_d'/\overline{t_D}}$$

and so the average amount of information per useable meteor trail is

$$(7\text{-}2) \qquad I_{TT}/M \triangleq \overline{I_{TT}} = Bt_d'e^{-t_d'/\overline{t_D}}$$

in the TT mode.

Max $\overline{I_{TT}}$:

Further,

$\frac{d\overline{I}_{TT}(t_d')}{dt_d'} = Be^{-t_d'/\overline{t_D}}\left(1 - t_d'/\overline{t_D}\right)$. Since $Be^{-t_d'/\overline{t_D}}$

is positive, the factor $(1 - t_d'/\overline{t_D})$ completely determines the sign of the derivative. Hence, for $t_d' < \overline{t_D}$, the derivative is positive, indicating $\overline{I}_{TT}(t_d')$ is increased, and for $t_d' > \overline{t_D}$ the derivative is negative, indicating $\overline{I}_{TT}(t_d')$ is decreasing. As such, $\overline{I}_{TT}(t_d')$ has a single maximum value and it occurs at $t_d' = \overline{t_D}$. That is,

(7-3) $\quad \text{Max}\overline{I_{TT}}(t_d') = \overline{I}_{TT}(\overline{t_D})$

Max B :

Let MBST* be an MBST in the ensemble constituting MBG that just meets the probability of error constraint. That is, (see Figure 5.1) its signal-to-noise envelope, $(Ve^{-t/t_D})^2/2BN = s^*(t)$, decays to s_0 exactly at the end of the message. Then $(Ve^{-t_d'/t_D})^2/2BN = s^*(t_d') = s_0 = V_0^2/2BN$, and $B = V_0^2/2Ns_0$. Any further increase in B would cause $s^*(t_d')$ to be less than s_0, and consequently a violation of the probability of error constraint. Hence, Max $B = V_0^2/2Ns_0$. It has been assumed that in the TT mode the apparent threshold, V_1, coincides with V_0. Hence,

(7-4) $\quad \text{Max } B = V_1^2/2Ns_0$

Min k :

Finally, under optimum operating conditions, $t_d' = \overline{t_D}$. As

such, the normal MBG operating condition of $kT < \overline{t_D}/10$ for the word duration may be expressed as $kT < t_d'/10$. But, if the message, Bt_d' binary digits, is subdivided into at least 10 words, the number of bits per word, k, necessarily satisfies the condition $k < Bt_d'/10$ or equivalently $kT < t_d'/10$. Hence, the MBG normal operating condition of $kT < \overline{t_D}/10$ is also met. Theorems 1-7 are applicable for each MBST in the ensemble constituting the MBG, as all the normal operating conditions have been met. As a result of this and 7-2, 7-3, 7-4, the Theorem holds as stated.

Q. E. D.

7.2 Design Procedure for MBG, TT Mode

To maximize $\overline{I}_{TT}$, choose the MBG parameters as follows, in accordance with Theorem 10.

1. Message duration: $t_d' = \overline{t_D} = \overline{t_d}$
2. Bit rate: $B = W/2$
3. Apparent threshold: $V_1 = \sqrt{s_0 NW}$, where s_0 is chosen in 5.
4. Word duration: at least 10 words/message
5. Probability of error constraints, s_0:

Let P be the probability of receiving a word with at least (or exactly) r errors (Theorem 5). Each MBST in the ensemble of MBG satisfies P when the minimum signal-to-noise, s_0, throughout the message duration satisfies the relation

$P = \binom{k}{r} \frac{P_{eb}[s_0]^r}{r \cdot 10}$ (assuming that P is at least as stringent as an instantaneous probability of bit and word error of 1/100 and 1/2.5 respectively). Hence, the MBG meets its probability of error constraint for this s_0. Further, an instantaneous probability of error constraint may be satisfied for MBG by having each MBST in the ensemble satisfy it as in the above case.

6. Adjust the terminal threshold detector (at the I. F. output) so that t_d' after the signal is detected it will be sampled again and discarded if and only if the sample voltage is below V_1.

7. Use a cyclic code for correcting single errors and/or detecting parity error at least, so that data from overdense meteor trails may be detected and discarded.

Example 6

MBG TT Mode Design Problem.

Assume that for a given MBG in the TT mode, $N = 1 \times 10^{-16}$ watt/Hertz, $\overline{t_D} = 100$ ms, W = 8000 Hertz, and an even parity code of block length 15 is used for error detection. Further, suppose it is required that on the average there are less than 1 word error/ 4 messages, and the maximum information rate is desired.

Design Solution.

1. Message duration = 100 ms
2. Bit rate = 8000/2 bit/sec. = 4000 bit/sec.

3. We have 15 bit/block (i.e., 15 bits/word) and 100 ms • 4 bit/ms = 400 bit/message, or 400/15 ≅ 25 words per message. Thus, 1 word/4 message is equivalent to 1/100 words in error. Let

$$1/100 = \binom{15}{1} \frac{P_{eb}[s_0]^1}{1 \cdot 10} \cong \text{Probability of receiving a word}$$

in error (Theorem 5). Then, $P_{eb}[s_0] = 1/150$, and from Figure 3.1, this requires that $S_0 \cong 5$ db $= 10\log_{10} s_0$ or $s_0 = 10^{.5} \cong 3.1$.

4. Hence, $V_1 = \sqrt{(3.1)\ (1\text{x}10^{-16}\ \text{watt/cycle})\ (8000\ \text{cycle/sec})}$

$$V_1 \cong \sqrt{25 \text{ x } 10^{-13}\ \text{watt}}$$

$$V_1 \cong 1.58\ \text{u V} = \text{Terminal Threshold Voltage}$$

The parity code only recognizes multiple errors. These are typical from the overdense meteor trails and hence act to reject such data at very little cost to the information rate over the underdense trails. Also, an assumption that the maximum information rate occurs when Max $\bar{I}$ occurs was made. This is correct and will be justified later.

7.3 MBG Apparent Threshold Mode

In the Apparent Threshold (AT) Mode, the only threshold at the base station is V_1. V_1 must be chosen greater than V_0 to minimize the number of useable meteor trails that do not remain above the probability of error constraint (equivalent), V_0, for

the full message duration, t_d'. In Figure 7.1, it is evident how V_1 should be chosen. As $\overline{t_D}/2$ is the assumed decay time of the most rapidly decaying envelope, when V_1 is chosen just large enough so that $V_1 e^{-t_d'/\overline{t_D}/2} \geq V_0$, all of the envelopes remain above V_0 for the full message duration, t'_d. Hence, we need $V_1 \geq V_0 e^{\frac{t_d'}{(\overline{t_d}/2)}}$

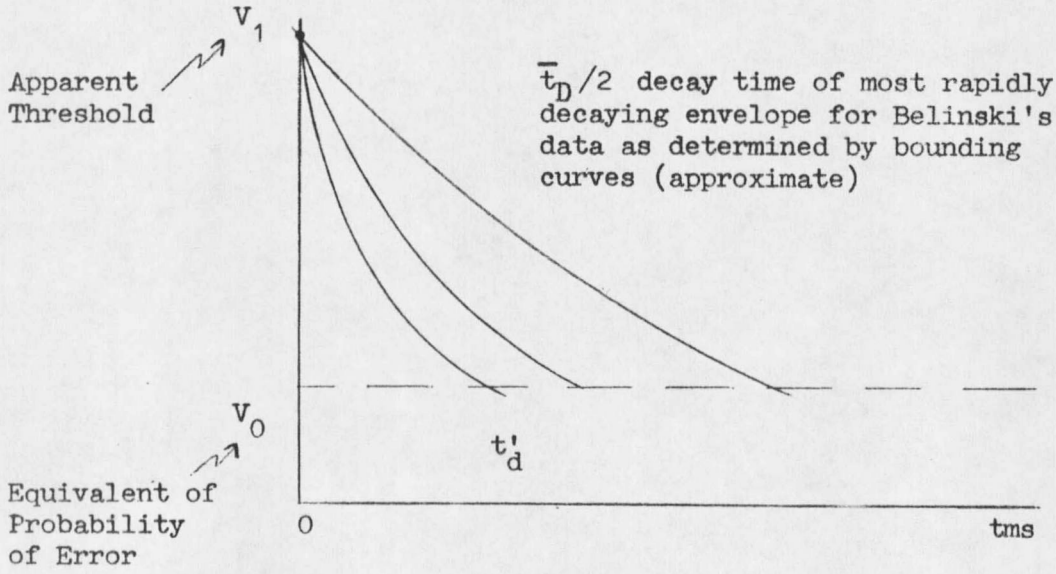


Figure 7.1
Choosing Apparent Threshold

The fastest decay time can be found by the method of constructing bounding curves exemplified in Exemple 5. $\overline{t_D}/2$ was the approximate lower constraint on the time decay constants

for Belinski's data and probably in most cases the variance about the mean is constraint above $\overline{t_D}/2$. (Higgins' data indicates a similar variance for t_D.)

Satisfying the probability of error constraint on a per-trail basis with the apparent threshold alone is clearly not as well defined a task as for the TT mode. But if the minimum time decay constant is about $\overline{t_D}/2$, as it will be assumed to be herein, and only on rare occasions is this violated, the probability of error found by assuming $\overline{t_D}/2$ is absolutely the minimum decay time should well approximate the actual probability of error.

Theorem 11

In the AT mode, assuming $\overline{t_D}/2 \leq t_D$, the average amount of information received via MBG is $\overline{I}_{AT}(t_d') \cong Bt_d' e^{\frac{-t_d'}{(\overline{t_D}/2)}}$ bit/useable meteor trail, where $\overline{t_D}/2 \leq t_d' < \overline{t_D}$. The maximum value of $\overline{I}_{AT}(t_d')$ occurs at $t_d' = \overline{t_D}/2$ and is

$$\text{Max}\overline{I}_{AT} = \overline{I}_{AT}(\overline{t_D}/2) \cong B\,\overline{t_D}/2\,e^{-1} \text{ bits/useable meteor trail.}$$

Further, it is assumed that the apparent threshold has the value $V_1 = V_0 e^{\frac{t_d'}{\overline{t_D}/2}}$ in general and $V_1 = V_0 e$ when $\overline{I}_{AT}$ is maximized.

Finally, the bit rate is assumed to be equal to

$B = \dfrac{\left[V_1 e^{-t_d'/(\overline{t_D}/2)}\right]^2}{2Ns_0}$ in general, and to equal $B = \dfrac{(V_1 e^{-1})^2}{2Ns_0}$

and $\overline{I}_{AT}$ is maximized. And when operating maximally, Theorems 1-7 apply to each MBST in the ensemble which constitute the MBG if there are at least 5 words per message.

Proof.

$\overline{I}_{AT}$:

Each MBST in the ensemble constituting MBG satisfies the probability of error constraint s_0 and other normal MBG operating conditions. Hence, Theorems 1-7 are applicable to each. As such, for each MBST in the ensemble, the amount of information received is $I \cong B \cdot t_d'$. After the recurrence of M (a large number) of useable meteor trails, the Law of Large Numbers implies that the amount of information received is approximately

$$I_{AT} = M \cdot P_V (V > V_1 | V > V_0) B \cdot t_d',$$

where $P_V(V > V_1 | V > V_0)$ is the probability that a useable meteor trail $(V > V_0)$ is in the ensemble of MBG meteor trails $(V > V_1)$ As assumed in the Theorem, $V_1 = V_0 e^{t_d'/(\overline{t}/2)}$. This choice was discussed and illustrated in the introduction to the AT mode. As such,

$$I_{AT} = M \cdot P_V(V > V_0 e^{t_d'/(\overline{t_D}/2)} | V > V_0) B \cdot t_d'$$

The p.d.f. of V for $V > V_0$ is found to be V_0/V^2, using Wheelon's

general form of Q/V^2. Hence,

$$P_V\left(V > V_0 e^{t_d'/(\overline{t_D}/2)} \mid V > V_0\right) \int_{V_{00}} V_0/V^2 \, dV = e^{-t_d'/(\overline{t_D}/2)}$$

where $V_{00} = V_0 e^{t_d'/(\overline{t_D}/2)}$. As such, I_{AT} becomes

$$I_{AT} = M \cdot B t_d' e^{-t_d'/(\overline{t_D}/2)}$$

and so, the average amount of information received in the AT mode is approximately

$$(7\text{-}5) \quad I_{AT}/M \triangleq \overline{I}_{AT} = B \cdot t_d' e^{-t_d'/(\overline{t_D}/2)}$$

$\text{Max}\overline{I}_{AT}$:

Further,

$$d \frac{\overline{I}_{AT}(t_d')}{dt_d'} = Be^{-t_d'/(\overline{t_D}/2)}\left[1 - t_d'/(\overline{t_D}/2)\right]$$

Since the first factor is positive, the sign of the derivative is completely determined by the factor $\left[1 - t_d'/(\overline{t_D}/2)\right]$. It is positive for $t_d' < \overline{t_D}/2$, indicating $\overline{I}_{AT}$ is increasing, and negative for $t_d' > \overline{t_D}/2$ indicating $\overline{I}$ is decreasing. Hence, $\overline{I}_{AT}$ has a single maximum at $t_d' = \overline{t_D}/2$ of

$$(7\text{-}6) \quad \text{Max}\overline{I}_{AT} = B \cdot \overline{t_D}/2 \, e^{-1}$$

Max B:

Let MBST* be the MBST in the ensemble constituting MBG that just meets the probability of error constraint. That is, (see Figure 7.1) its signal-to-noise envelope,

$$\frac{(Ve^{-t/t_D})^2}{2BN} = s(t),$$

decays to s_0 exactly at the end of the message. MBST* would have $V_1e^{-t/(\overline{t_D}/2)}$ as its envelope. Hence, $\frac{V_1e^{-t_d'/(\overline{t_D}/2)})^2}{2NB} = s_0$, and so,

$B = \frac{(V_1e^{-t_d'/(t_D/2)})^2}{2Ns_0} < W/2$ is the maximum allowable value of B. That is, any further increase in B would decrease the signal-to-noise to the point where the probability of error constant s_0 is violated. When operating maximally, at $t_d' = \overline{t_D}/2$, this constant becomes

$$(7\text{-}7) \quad B = \frac{(V_1e^{-1})^2}{2Ns_0}.$$

Min k:

Under optimal operating conditions, $t_d' = \overline{t_D}/2$. As such, the normal operating conditions of $kT < \overline{t_D}/10$ on the word duration may be expressed as $kT < t'_d/5$. But, if the message, Bt_d' binary digits, is subdivided into at least 5 words, the number

of bits per word, k, necessarily satisfies the condition $k < Bt_d'/5$ or equivalently $kT < t_d'/5$. Hence, the MBG normal operating condition of $kT < \overline{t_D}/10$ is also met.

Theorems 1-7 are applicable for each MBST in the ensemble constituting MBG, as all the normal operating conditions have been met. As a result of this, and 7-5, 7-6, and 7-7, the Theorem holds as stated.

Q. E. D.

7.4 Design Procedure for MBG, AT Mode

To maximize $\overline{I}_{AT}$, choose the parameters as follows, in accordance with Theorem 11.

1. Message duration: $t_d' = \overline{t_D}/2$
2. Bit rate: $B = W/2$
3. Apparent threshold: $V_1 = \sqrt{s_0 NW}$ where s_0 is chosen in 5.
4. Word duration: at least 5 words/message
5. Probability of error constraint, s_0:

Let P be the probability of receiving a word with at least (or exactly) r errors (Theorem 5). Each MBST in the ensemble of MBG satisfies P when the minimum signal-to-noise, s_0, throughout the message duration is such that

$$P = \binom{k}{r} \frac{P_{eb}[s_0]^r}{r \cdot 10}$$ (assuming that P is at least as stringent as an instantaneous probability of bit and word error, or 1/100

or 1/2.5, respectively.) Hence, the MBG meets its probability of error constraint for this s_0. Further, an instantaneous probability of error constraint may be satisfied for MBG by having each MBST in the ensemble satisfy it as in the above case.

6. Use a cyclic code for correcting single error and/or detecting parity errors at least, so that data from overdense meteor trails may be detected and discarded.

Example 7

MBG AT Mode Design Problem.

Assume that for a given MBG in the AT mode, $N = 1 \times 10^{-16}$ watts/hz and $\overline{t_D} = 100$ ms, W = 8000 hertz, and an even parity code of block length 15 is used for error detection. Further, it is required that on the average there are less than 1 word error/4 messages, and maximum information rate is desired.

Design Solution.

1. Message duration = 200 ms/2 = 50 ms

2. Bit rate = 8000 hz/2 = 4000 bit/sec

3. We have 15 bits/block and 50 ms • 4 bits/ms = 200 bit/message, or 200/15 $\cong$ 13 words per message. Thus, 1 word/4 messages is equivalent to about 1/50 words in error. Let

$$\frac{1}{50} = \binom{15}{1} \frac{P_{eb}[s_0]^1}{1 \cdot 10} \cong \text{Probability of receiving a word in}$$

error (Theorem 5). Then, $P_{eb}(s_0) \cong 1/75$ (close enough to 1/100 so that Theorem 5 is applicable). From Figure 2.1, this requires that $s_0 \cong 3$ db $= 10\log s_0$, or $s_0 = 10^{.3} \cong 2$.

4. Hence,

$$V_1 = e\sqrt{(2)\ (1 \times 10^{-16}\ \text{watt/cycle})\ (8000\ \text{cycle/sec})}$$

$$V_1 \cong e\sqrt{1.6 \times 1 \times 10^{-12}\ \text{watt}}$$

$$V_1 \cong e\ (1.26)\ \text{uV}$$

$$V_1 = 3.43\ \text{uV} = \text{Apparent Threshold}$$

Again, the parity code only recognizes multiple errors. These are typical from the overdense meteor trail, and hence, act to reject such data at very little cost to the information rate over the underdense trails. Also, as mentioned previously, an implicit assumption is made that the maximum information rate occurs when max $\overline{I}_{AT}$ occurs. This is correct and will be justified later.

CHAPTER VIII

MBG OVERVIEW

8.1 Comparison of the Three Modes of MBG Operation

Theorem 12

While operating optimally, the information received via MBG is

(Adaptive Mode) Max $\overline{I}_A = (W\overline{t_D})/4$ (1) bit/useable meteor trail

(TT Mode) Max $\overline{I}_{TT} = (W\overline{t_D})/4$ (.73) bit/useable meteor trail

(AT Mode) Max $\overline{I}_{AT} = (Wt_D)/4$ (.37) bit/useable meteor trail

Proof.

Max $\overline{I}_{TT} = W/2 \cdot \overline{t_D} \cdot e^{-1} = (W\overline{t_D})/4$ .73 by Theorem 10.

Max $\overline{I}_{AT} = W/2 \cdot \overline{t_D}/2 \cdot e^{-1} = (W\overline{t_D})/4$ .37 by Theorem 11.

In the adaptive mode, one transmits optimally over each meteor trail, and, hence, realizes Max $I_A = W/2 \cdot t_D/2$ bit/useable meteor trail when the decay time constant is t_D, as this is optimal MBST operation within the bandwidth constraint $B < W/2$, by Theorem 7. Hence, on the average, the maximum information received is

$$\text{Max } \overline{I}_A = \int_{t_D} W/2 \;\; t_D/2 \, f_{t_d}(t_D) dt_D \text{ bit/useable meteor trail.}$$

Hence, Max $\overline{I}_A = W/2 \; t_D/2$ bit/useable meteor trail.

Q. E. D.

8.2 Effect of Varying Power on MBG (Nonadaptive)

The following Theorem is evident to the point that no proof is required, but is stated for referencing convenience.

Theorem 12

With respect to the nonadaptive MBG system:

(A) If the continuous wave interrogate signal is increased in power by a factor of $k \geq 1$, the rms value of this signal is increased by $\sqrt{k}$. This has the same effect on the MBST ensemble insofar as the base station is concerned) as decreasing the remote station threshold by a factor of $\sqrt{k}$.

(B) Alternately, by meteor trail model reciprocity, ensembles of MBST having the same statistics (insofar as the base station is concerned) would result from a decrease in remote station transmitter power by a factor of k, as would result from an increase in interrogate signal power by a factor of k.

Theorem 13

When the C. W. interrogate signal is increased in power by a factor of $k \geq 1$, the number of transmissions is increased by a factor of k.

Proof.

From the results of A. D Wheelon, the p.d.f. of the interrogate signal (rms) at the remote station is Q/V^2, where Q is a positive constant. Let V_1^* be the voltage (rms) threshold at the remote station. The p.d.f. for the interrogate signals with rms voltages above V_1^* is V_1^*/V^2. (That is,

$\int_{V_1^*}^{\infty} Q/V^2 \, dV = Q/V_1^* = 1$. Hence, $Q = V_1^*$, and so $f_V(V|V > V_1^*) = V_1^*/V^2)$.

Suppose that the power in the continuous wave interrogate signal is increased by a factor of k. Then, if the normalized signal power of a given interrogate signal had been V^2, it is kV^2 after the power increase, and correspondingly, the rms value of that interrogate signal would change from V to $(\sqrt{k})V$. However, the remote station threshold is unaffected by this power increase. It is still V_1^*. Consequently, the probability of receiving an interrogate signal above V_1^* (the remote station threshold) after the power increase, that would also have been above V_1^* before the power increase, is

$$P_V(V > (\sqrt{k})\, V_1^* | V > V_1^* = \int_{(\sqrt{k})V_1} V_1^*/V^2 \, dV = V_1^*/\sqrt{k}\, V_1^* = 1/\sqrt{k}$$

That is, of the total number of transmissions occurring with increased power, only $1/\sqrt{k}$ of these transmissions would have occurred at the reduced power level. Hence, an increase in the number of transmissions by a factor of $\sqrt{k}$ resulted when the power was increased by a factor of k.

Q. E. D.

To counteract the effect of increasing the power in the

continuous wave interrogate signal (i.e., increasing the probability of error resulting from the equivalent remote station power reduction discussed in Theorem 12), the remote station signal power must be increased. When the signal power of the signal being transmitted and received via the remote station are increased by a factor, k, we say the MBG system power has been increased by the factor k.

Theorem 14

When the MBG nonadaptive system power is increased by a factor of $k \geq 1$, the average rate at which information is received is increased by $\sqrt{k}$, given all other parameters are held constant. However, the ensemble of MBST is invariant to a system power change (as far as the base station is concerned). As a result, the average amount of information received per useable meteor trail and the probability of error is invariant to a change in system power.

Proof.

As shown in Theorem 13, the increase in transmission rate by a factor of $\sqrt{k}$ is due to the fact that meteor trails giving rise to interrogate signals of strength $V_1^*/\sqrt{k} < V < V_1^*$ before the power increase give rise to interrogate signals of strength $V_1^* < V^* < V_1^*\sqrt{k}$ after the power increase. And, these meteor trails give rise to transmissions after the power increase that would not have occurred before the power increase.

In addition, the range of interrogate signals values $V_1^*/\sqrt{k} < V < \infty$ becomes $V_1^* < V < \infty$ after the power increase. Hence, the p.d.f. of V over this range is $V_1^*/(V)^2$ for both power levels because it has the same form Q/V^2 and is restricted to the same range of values $V_1^* < V < \infty$. By the reciprocity of the meteor trail model, and a symmetric argument, the p.d.f. of the initial values of the envelopes at the detector input, Ve^{-t/t_D}, does not vary with the system power increase either. As a result, the ensemble of MBST constituting the MBG nonadaptive system does not vary (as far as the base station is concerned). Thus, the average amount of information and probability of error computed for each mode of operation is invariant to changes in the system power.

Q. E. D.

8.3 Summary, Conclusions, and Recommendations

Receiver Optimization

An optimum receiver was conceptually developed for the MBCS, assuming the signals received at the base station were predominately of the form

$$n(t) + V e^{-t/t_D} \cos(W_c t + u(t))$$

Gaussian white noise	Independent random variables	Carrier frequency	Linear phase error

After the base station has been initialized (Figure 1.4), and when correctable messages are received.

Probability of Error

The time response of the probability of (bit)(word) errors and the signal-to-noise ratio at the time of sampling by the detector were represented graphically, together with the effects of varying bit rate and power, for a fixed base station received signal. Further, a simple expression relating the minimum signal-to-noise over the message received to the probability of receiving a k bit word with exactly and/or at least r errors was derived.

Message Duration (Not including the initializing time) and Bit Rate that Optimize Information Rate

The probability of error was assumed to be constrained by accepting only those signals at the base station whose minimum initial or terminal signal-to-noise ratio at the time of sampling by the detector was above a fixed level. The specific type of probability of error constraint implemented determines distinct modes of operation. Herein, the Adaptive mode (A), Terminal Threshold mode (TT), and Apparent Threshold mode (AT) were defined and evaluated with respect to average information, $\overline{I}$, and average information rate, $\overline{R}$. Further, the bit rate, B, and time duration,

t_d, that optimize $\overline{I}$ and $\overline{R}$ were determined. It was shown that within the usual system constraints of normal operating condition (Figures 5.1, 3.3), $\overline{I}$ and the value of B and t_d which optimize $\overline{I}$ and $\overline{R}$ are independent of the probability of error constraint and system power. These optimizing values of B and t_d are $(W/2, t_D/2)$, $(W/2, \overline{t_D})$ and $(W/2, t_D/2)$ in the A, TT and AT modes, respectively. The respective optimum values of $\overline{R}$ and $K\overline{Wt_D}$, $.73\ K\overline{Wt_D}$, and $.37\ K\overline{Wt_D}$, expressed in bits/time, where k is a constant determined by the code rate of the block code being used and the average transmission rate.

The TT mode of operation is the best overall for these reasons. It is easily implemented, allows quite a rigid probability of error constraint to be adhered to, and yields a comparatively high information rate.

Statistical Results

It was shown that $\overline{t_D} = \overline{t_d} \triangleq$ mean time duration above a fixed voltage level exceeding the noise level. Hence, all of the results on optimization involving $\overline{t_D}$ may equivalently be expressed in terms of $\overline{t_d}$, a statistic that is easily determined experimentally at each remote site.

In addition to this statistical result, a simple method of determining the bounds on t_D from the probability density function

of t_d (easily determined experimentally) was given. These bounds on t_D are an important consideration for operation in the very common AT mode.

Effect of Power Variation on Average Information Rate

It was shown that the transmission rate, and correspondingly, the average information rate, of the nonadaptive MBCS systems increases by a factor of $\sqrt{k}$ when the system signal power (i.e., both interrogate and information signal power) is increased by $k \geq 1$. Further, it was shown that the average amount of information per transmission is not affected by a system power increase.

Design Procedures

In addition, design procedures that optimize the information rate within the constraints on bandwidth, power, and error were given and exemplified for each of the modes of operation.

Recommendations

The results do agree with the limited experimental results available, but I recommend that a complete experimental test procedure be written and carried out.

APPENDICES

APPENDIX A

On a given channel, if the words are sequences of k equally likely binary digits with the property that the probability of error in any position of a word is independent of the probability of error in any other position of that word and is the same, say P, for each position of that word, then the channel capacity is given by

$$C = 1 - \left[P\log(P) + (1-P)\log_2(1-P)\right] \text{ bits/binary digit}$$

As such, the channel may without loss of generality be assumed to be the BSC with probability of bit error P.

Proof.

For lack of a specific reference, the argument involved in satisfying this will be explicitly given.

The channel capacity is equal to the maximum information transmitted per symbol (k bits/word) minus the information per symbol loss due to the noise in the channel (k' bits/word). For each word transmitted (e.g., for k = 7, $\lfloor 1111000 \rfloor$) an error vector is produced by the noise source (e.g., $\lfloor 0000001 \rfloor$) and added (i.e., componentwise exclusive or) to the transmitted message and produces a corresponding received message (e.g., $\lfloor 1111001 \rfloor$).

Since the probability of a given error vector occurring

is the same for each equally likely transmitted message, a symmetric channel model results. As such, the channel is unifor uniform [16] and

$k' = -\sum_{i=0}^{2^k-1} P_1 \log_2 P_i$, where P_i is the probability of occurrence of the error vector corresponding to the binary representation of i. Since error vectors of equal hamming weight have equal P_i, we have

$$k' = -\sum_{r=0}^{k} \binom{k}{r} \cdot p^r \cdot (1-P)^{k-r} \log_2 \left[p^r \cdot (1-P)^{k-r}\right].$$

Using the identities $\sum_{r=0}^{k} \binom{k}{r} X^r \cdot r = kX(1 + X)^{k-1}$ and

$\sum_{r=0}^{k} \binom{k}{r} X^r = (1 + X)^k$, k reduces to

$-kP\left[\log_2(1/P) + (1 - P)\log_2 1/1-P\right]$ bits/word, and so, the channel capacity is given by k-k' bit/word.

$k\left[1 - (P\log_2(1/P) + (1 - P)\log_2 1/1-P\right]$, or

$C = 1 - \left[P\log_2 1/P + (1 - P)\log_2 1/1-P\right]$ bits/binary digit.

Q. E. D.

APPENDIX B (SYMBOL LIST)

Originating Section	Symbol		Meaning
5.3	AT	$\triangleq$	Apparent Threshold
2.2	B	$\triangleq$	Binary digit rate
4.1	$C(t_0,t_1)$	$\triangleq$	Maximum amount of information received per binary digit
4.1	$C(B,t_d)$	$\triangleq$	Maximum amount of information received per binary digit during a given transmission of MBST
4.1	$C'(B,t_d)$	$\triangleq$	$C(B,t_d)$ in the case when an algebraic code is being used
4.1	$I(B,t_d)$	$\triangleq$	Amount of information received during a given transmission of MBST
7.1, 5.3	I_{TT}, I_{AT}	$\triangleq$	Amount of information received via M useable meteor trail in the Terminal and Apparent Threshold modes, respectively
7.1, 5.3	$\overline{I}_{TT}, \overline{I}_{AT}$	$\triangleq$	Average amount of information received per transmission in the Terminal and Apparent Threshold modes, respectively
1.5	MBST	$\triangleq$	Meteor Burst Communication System, assuming a fixed meteor trail as the forward scatter medium
1.5	MBG	$\triangleq$	Meteor Burst Communication System, assuming an ensemble of meteor trails as the forward scatter medium
3.5	$\binom{n}{r}$	$\triangleq$	$\frac{n!}{(n-r)!\ r!}$

1.5	$n(t)$	$\triangleq$	Zero mean Gaussian white noise added to the signal in the channel
2.2	$n_w(t)$	$\triangleq$	Zero mean Gaussian white noise of power spectral density N added to the signal prior to detection
2.2	N	$\triangleq$	Power spectral density of $n_w(t)$
3.1	$P_{eb}(S)$	$\triangleq$	Probability of a bit error as a function of S
3.1	$P_{eb}(t)$	$\triangleq$	Probability of a bit error as a function of S(t)
3.1	$P_{eb}S[S(t)]$	$\triangleq$	Probability of a bit error as a function of S(t)
3.4	$P_{eb}(t_0,t_1)$	$\triangleq$	Probability of a bit error assigned to the bits detected (sampled) at time t where $t_0 \leq t \leq t_1$
3.5	$P_{k,r}(t_0,t_1)$	$\triangleq$	Probability of receiving a k bit word with exactly r error in the interval of time $t_0 \leq t \leq t_1$
3.7	$P^*_{k,r}(t_0,t_1)$	$\triangleq$	Probability of receiving a k bit word with at least r errors in the time interval $t_0 \leq t \leq t_1$
2.2	S(t) db	$\triangleq$	Signal-to-noise ratio, S db, at the time t of detector sampling
2.2	S(t)	$\triangleq$	$10\log_{10}S(t)$
2.2	$\overline{S(t)}$	$\triangleq$	Signal at time t of detector sampling

5.3	S_1	$\triangleq$	$10\log_{10}S_1$, where $S_1 = V_1^2/2BN$
4.3	S_0	$\triangleq$	$10\log_{10}S_0$, where $S_0 = V_0^2/2BN$
7.1	TT	$\triangleq$	Terminal Threshold
1.4	$(0,t_d)$	$\triangleq$	Transmission interval of MBCS
1.5	t_D	$\triangleq$	Time decay constant of a given reflected signal envelope
7.1	t_d^*, t_d'	$\triangleq$	Fixed values of t_d
2.2	T	$\triangleq$	Duration of a binary digit
5.3	V_1	$\triangleq$	Apparent Threshold
5.3	V_0	$\triangleq$	Least initial rms detector input signal value of a signal reflected from a useable meteor trail
5.3	V	$\triangleq$	Detector input signal (rms) value
1.5	u(t)	$\triangleq$	Linear phase error
3.2	W	$\triangleq$	First zero crossing bandwidth of a random binary signal with equally likely binary values
3.5	w.l.o.g.	$\triangleq$	Without loss of generality

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