The determination of reactor parameters of the Montana State College sub-critical assembly
by Tushar Kumar Chowdhury

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Physics
Montana State University


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This thesis is a report of the research, in which the infinite multiplication factor of the Montana State College natural uranium, water moderated subcritical assembly has been obtained both theoretically and experimentally. Theoretically the infinite multiplication factor has been calculated from the four-factor formula to which the simple diffusion theory has been applied. Experimentally the thermal utilization has been determined by the foil-activation method; two-mil-thick Indium foil has been used. The theoretically calculated value of the thermal utilization is (0.774) whereas the experimentally determined value is 0.780. The material and the geometrical bucklings have been determined experimentally with the scintillation counter and from these the infinite multiplication factor has been calculated to be (0.911). The simple diffusion theory yields (0.960) as the value of the infinite multiplication factor. Finally the effective multiplication factor has been found to be (0.814) and the subcritical multiplication is (5.38).

# THE DETERMINATION OF REACTOR PARAMETERS OF THE MONTANA STATE COLLEGE SUB-CRITICAL ASSEMBLY

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## INTRODUCTION

Plutonium emits alpha-particles which interact with a light element such as beryllium to produce neutrons of 5 Mev or higher energy. The sub-critical reactor at Montana State College employs this type of poly-energetic Pu-Be source of neutrons. Such fast neutrons lose most of their kinetic energies as a result of elastic scattering with the nuclei of the material, called moderator, in which the source is embedded. After a number of scattering collisions, the energy of a neutron is reduced to $\sim$0.025 ev, which corresponds to the average kinetic energy of the atoms and molecules of the moderator. This energy is called thermal energy and the neutrons possessing it are called thermal neutrons. Neutrons with energies above thermal value (0.025 ev) are epithermal neutrons.

Uranium-nuclei (specifically $U^{235}$) have high interaction probability (i.e. high fission cross-section) for the thermal neutrons. The $U^{235}$ nucleus absorbs a neutron and the resulting compound nucleus is so unstable that it either returns to the ground state by emission of a gamma-ray or breaks up into two parts emitting on the average $2.5 \pm 0.1$ neutrons of high energy (approximately 2 Mev). These fast neutrons are then slowed down in the moderator to thermal neutrons which then produce fission of some other U-nuclei. Thus the chain reaction continues.

In the Montana State College reactor, natural uranium (0.71 per cent $U^{235}$) and ordinary distilled water are the respective fuel and moderator.

The gamma-rays produced in interaction of neutron with nuclei induce some photofission and thereby help to maintain the chain reaction. However, their contribution to the chain-reacting system is less than one per cent (1).

Two essentially distinct physical phenomena are associated with the uranium-fission chain reaction. One covers the individual nuclear processes such as fission, neutron capture and neutron scattering; these are quantum-mechanical in character, and their theory is non-classical. The second one, which is the process of diffusion and leakage of neutrons, is of fundamental importance in a nuclear chain reaction. This process is classical. Insofar as the theory of neutron diffusion is concerned, the mathematical study of chain reactions is an application of classical, not quantum-mechanical technique.

In a heterogeneous thermal reactor, fission in the uranium lump produces fast neutrons. These neutrons then diffuse in accordance with the laws of diffusion theory until they are finally captured in the moderator, or in the uranium, or until they escape from the boundary.

If a reactor has exactly the critical size, each neutron produced by fission will, on the average, give rise to exactly one new fission, and the neutron density will remain constant. The ratio of the number of neutrons in one generation to that in the preceding generation is called the reproduction factor or the criticality factor "k" and for an exactly critical assembly, $k=1$.

Critical reactors are designed in such a way that "k" can be made slightly larger than one. The reactor can then be made super-critical ($k>1$), just critical ($k=1$), or subcritical ($k<1$) by means of control rods. Since the Montana State College reactor can never become critical, it is called a subcritical reactor.

In this paper all calculations for the determination of the reactor parameters are based on classical theories. Here "one-group theory" is applied, i.e. it is assumed that all neutrons are generated, diffused, and absorbed at a single energy which corresponds to energy in the thermal region.

All theoretical calculations are made for the steady state.

## DESCRIPTION OF THE SUBCRITICAL ASSEMBLY

Form and Composition: The assembly contains about 2300 kg of natural uranium in the form of ring-shaped cylindrical slugs clad with Al-Si alloy (density 2.79 gm/cm$^3$) immersed in about 500 gallons of distilled water. The fuel elements are distributed heterogeneously in the central part of the reactor, i.e., the reactor core. The horizontal cross-section of such distribution of the fuel tubes is shown in Figure 1 below.

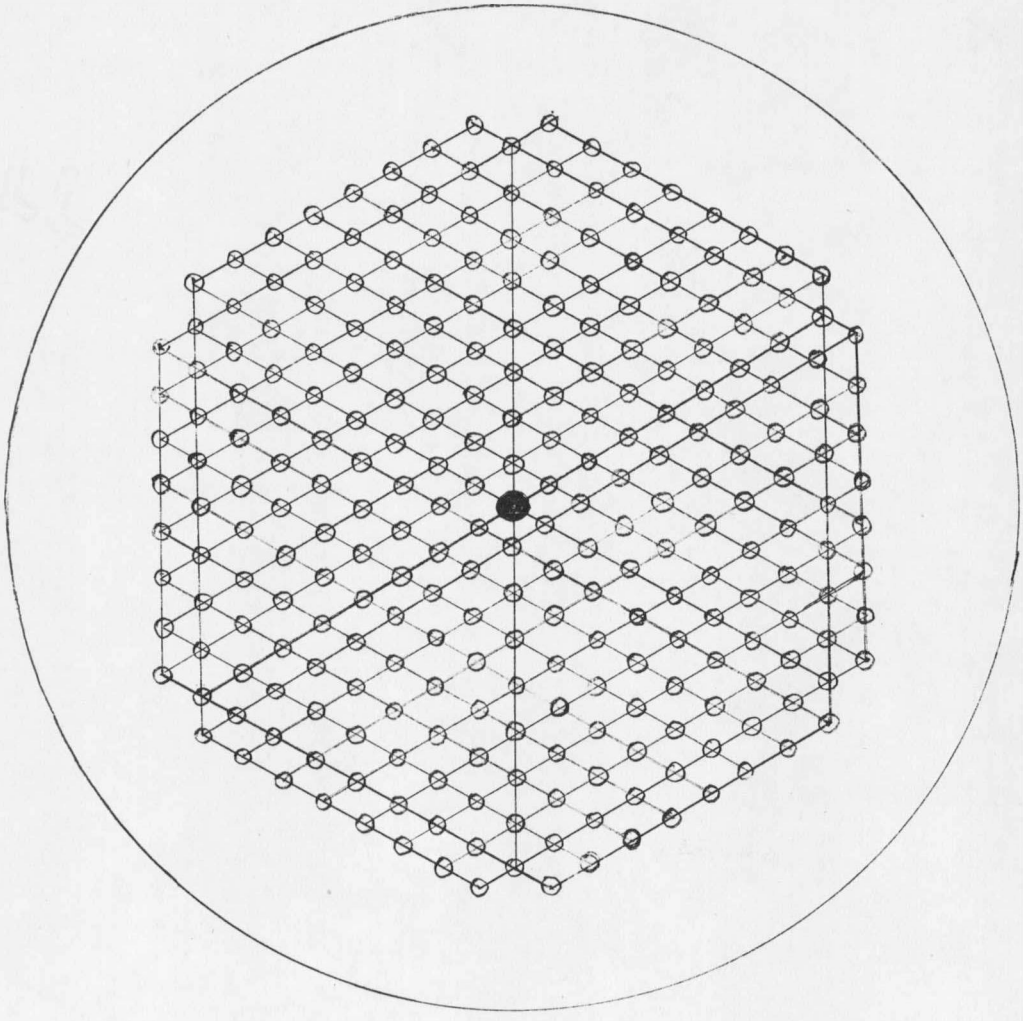


Figure 1. Horizontal Cross Section of the Assembly

The vertical section of the assembly is shown in Figure 2. This assembly contains 264 aluminum tubes each holding 5 uranium slugs. The average dimensions of a slug are given below:

Outside diameter of the aluminum-clad slug, $D_1$ = 2.90 cm.

Inside diameter of the aluminum-clad slug, $D_2$ = 0.98 cm.

Length = 20.56 cm

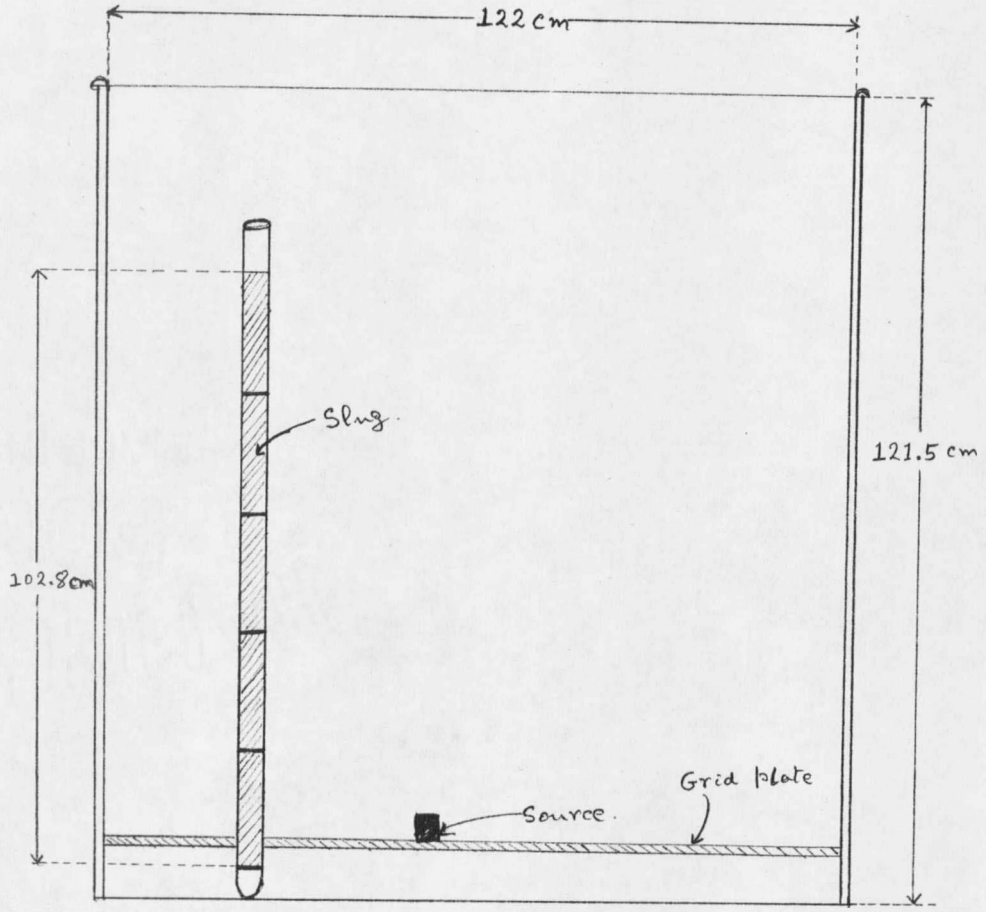



Figure 2. Vertical Section of the Assembly

The reactor core isconsidered to be composed of 264 cells of parallelogram cross-section; each cell includes one tube of fuel element at the center of the cell and the moderator surrounds the fuel element (Figure 3). To simplify the calculation of the different parameters, each of these cells has been replaced by a circular cell of the same area as that of the parallelogram; this is the so-called unit cell (Figure 4).

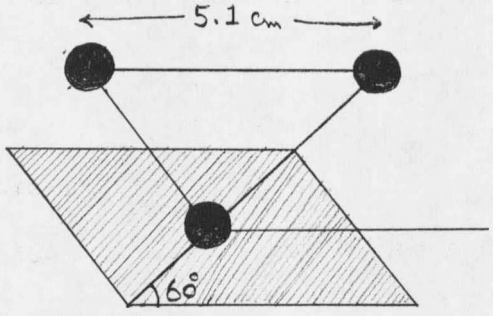
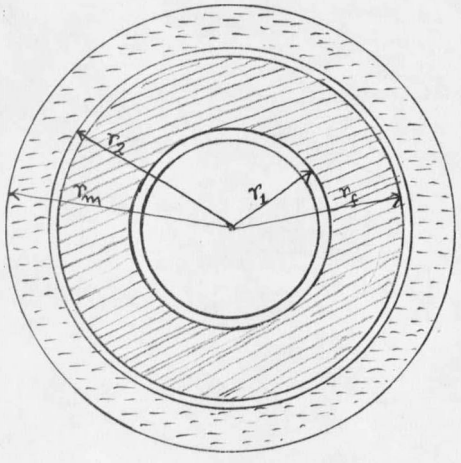



Figure 3. Parallelogram cell within the reactor, with the fuel rod at the center.




$r_1$ = inside radius of the fuel portion = 0.635 cm.
$r_f$ = outside radius of the fuel portion = 1.350 cm.
$r_2$ = inside radius of the moderator cylinder = 1.715 cm.
$r_m$ = outside radius of the moderator cylinder = 2.650 cm.

Figure 4.
Cross section of the unit cell.

## THEORY

Introduction: In order to determine the multiplication factor of the finite reactor, it is necessary to calculate the multiplication factor of the infimite reactor. If the reactor medium is assumed to be infinite, all the neutrons will remain inside the reactor, i.e., no leakage takes place. The infinite multiplication factor is obtained from the four-factor formula:

$$k_\infty = \eta f p \epsilon \qquad (1)$$

These four factors are defined in the following pages.

1) $\eta$ -- the average number of fast fission neutrons emitted as a result of the capture of one thermal neutron by fuel material, i.e., by either $U^{235}$ or $U^{238}$. On the average (2.5±0.1) fast neutrons are produced by each thermal neutron fission. It should be noted that the thermal neutron induced fission probability of $U^{235}$ is much greater than that of $U^{238}$, and also that the neutrons captured in fuel do not all necessarily lead to fission. The preponderance of $U^{238}$ (99.28%) in natural uranium causes a marked decrease in the value of $\eta$ from the average number (2.5 ± 0.1) of fast neutrons released per slow neutron fission:

$$\eta = \nu \frac{\Sigma_f}{\Sigma_{fuel}} \qquad (2)$$

where $\Sigma_f$ is the macroscopic cross-section for slow neutron fission, and $\Sigma_{fuel}$ is the total macroscopic cross-section for absorption of thermal neutrons, including both fission and non-fission capture processes, in

the fuel material. The macroscopic cross-section is defined as $\Sigma_i = N_i \sigma_i$ where $N_i$ is the number of nuclei of the i-th kind per $cm^3$ and $\sigma_i$ is the corresponding microscopic cross-section. If $N^{235}$ and $N^{238}$ represent the respective numbers of atoms of the two isotopes $U^{235}$ and $U^{238}$ per $cm^3$ of natural uranium, equation (2) may be written as

$$\eta = \nu \frac{N^{235} \sigma_f^{235}}{N^{235} \sigma_f^{235} + N^{235} \sigma_c^{235} + N^{238} \sigma_c^{238}}$$

$$= \nu \frac{\sigma_f^{235}}{\sigma_f^{235} + \sigma_c^{235} + R\, \sigma_c^{235}} \qquad (3)$$

where $\nu$ is 2.5 neutrons per thermal fission, R is the isotopic ratio of $U^{238}$ to $U^{235}$ in natural uranium, $\sigma_f$ and $\sigma_c$ are the respective microscopic fission and non-fission capture cross-sections.

2) $\underline{f}$ -- the thermal utilization. The fast neutrons originating from the Pu-Be source are slowed down to thermal energies by the moderator nuclei. The thermal neutrons will diffuse for some time, the energy distribution remaining essentially constant, until they are ultimately absorbed by fuel, by moderator, or by some other materials present in the assembly--aluminum in the present case. The absorption cross-section of aluminum for thermal neutron is very small. Hence the main absorbing materials for thermal neutrons, present in the assembly, will be the fuel and the moderator. Of the thermal neutrons, therefore, a

fraction "f" will be absorbed in fuel material. The value of "f" is represented by:

$$f = \frac{\text{Thermal neutrons absorbed in fuel}}{\text{Total thermal neutrons absorbed in fuel and moderator}} \tag{4}$$

If the fuel and the moderator are uniformly distributed throughout a system, it is clear that the nuclei of both materials will be exposed to the same neutron flux at all energies. If, on the other hand, the two materials are physically separated, the neutron density at a given energy will differ in the two media. In the case of thermal flux, there is, in fact, a depression in the neutron population throughout the fuel region, since most of the neutrons in the fuel region are fast. The unit-cell model (Figure 4) is used to calculate the thermal utilization. If $V_f$ and $V_m$ denote the volumes of the fuel and moderator regions respectively in the unit cell, and $\phi_f(\vec{r})$ and $\phi_m(\vec{r})$ denote the thermal fluxes in the respective regions, the thermal utilization can be defined as

$$f \equiv \frac{\int_{V_f} \Sigma_{af}\, \phi_f(\vec{r})\, d\vec{r}}{\int_{V_f} \Sigma_{af}\, \phi_f(\vec{r})\, d\vec{r} + \int_{V_m} \Sigma_{am}\, \phi_m(\vec{r})\, d\vec{r}} \tag{5}$$

where the average fluxes $\overline{\phi_i(\vec{r})}$ can be expressed as:

$$\overline{\phi_i(\vec{r})} \equiv \frac{1}{V_i} \int \phi_i(\vec{r})\, d\vec{r}.$$

The equation (4) may be written as

$$f = \frac{\Sigma_{af} \nu_f \bar{\phi}_f}{\Sigma_{af} \nu_f \bar{\phi}_f + \Sigma_{am} \nu_m \bar{\phi}_m}$$

$$\text{or,} \quad f = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am} \frac{\nu_m}{\nu_f} \frac{\bar{\phi}_m}{\bar{\phi}_f}} \qquad (6)$$

3) p--the resonance escape probability. During the slowing down process from fast neutron energy (~2 Mev) to thermal neutron energy ( ~ 0.025 ev), the neutrons pass through an energy region where the probability of non-fission interaction with the absorbers e.g., $U^{238}$ etc., is exceptionally large. This particular energy region is termed the resonance region. Due to this non-fission resonance absorption, not all the fast neutrons reach thermal energies. The fraction of fast (fission) neutrons which escape capture while being slowed down to a particular energy E is called the resonance escape probability, and is represented by p(E) or simply by "p". For a thermal-neutron reactor, E represents thermal energy. The principal resonance absorber is $U^{238}$, which is present in the greatest abundance in natural uranium, and the term "resonance absorption" has become almost synonymous with the resonance absorption by $U^{238}$; most of the prominent resonance peaks for the $U^{238}$ absorber appear in the energy range of 6.7 ev to 66 ev.

For a heterogeneous system, the resonance escape probability is approximately (within an error of one per cent) given by:

$$p(E) \approx \exp\left[ - \frac{N_f V_f \bar{\Phi}_f}{\xi_m \Sigma_{sm} V_m \bar{\Phi}_m} \int (\sigma_{af})_{eff} \frac{dE'}{E'} \right] \qquad (7)$$

where $V_f$ and $V_m$ are the volumes of the fuel and moderator respectively in the unit-cell of unit length; $\bar{\Phi}_f$ and $\bar{\Phi}_m$ are the average values of the resonance flux in the interior of the fuel slug and in the moderator respectively. The ratio $\bar{\Phi}_m / \bar{\Phi}_f$ is called the disadvantage factor for resonance neutrons. $\Sigma_{sm}$ is the macroscopic scattering cross-section of the moderator for resonance neutrons. $\xi_m$ is the average logarithmic energy decrement per collision with the moderator nuclei, i.e. if $E_1$ is the energy of the neutron before and $E_2$ that after a collision, $\xi_m$ is given by

$$\xi_m = \overline{\ln (E_1 / E_2)}$$

The integral in equation (7) is called the effective resonance integral, i.e.,

$$\text{Effective resonance integral} \equiv \int (\sigma_{af})_{eff} \frac{dE'}{E'} \qquad (8)$$

the integration being carried over the resonance region.

$(\sigma_{af})_{eff}$ is defined by:

$$(\sigma_{af})_{eff} \equiv \frac{\sigma_{af}}{1 + \frac{N_f \sigma_{af}}{\Sigma_s}} \qquad (9)$$

The quantity $\Sigma_s / N_f$, the total macroscopic scattering cross-section divided by the number of atoms of uranium per $cm^3$, may be regarded as the scattering cross-section associated with each atom of the absorber. The calculation of the resonance escape probability in any particular case requires a knowledge of the resonance disadvantage factor $\bar{\phi}_m / \bar{\phi}_f$ which is determined from the physical properties and the geometry of the heterogeneous unit cell. For this purpose a quantity $f_r$, called the resonance utilization, because of its analogy to the familiar thermal utilization (Equation 6) is introduced. It is defined as the ratio of the neutrons absorbed by the fuel in the resonance region to the total number of resonance neutrons produced. As a result, equation (7) can be transformed into:

$$p(E) \approx \exp\left[ - \frac{N_f}{\xi_m \Sigma_{sm}} \left( \frac{V_f \Sigma_{af} \bar{\phi}_f}{V_m \Sigma_{am} \bar{\phi}_m} \right) \frac{\Sigma_{am}}{\Sigma_{af}} \int (\sigma_{af})_{eff} \frac{dE'}{E'} \right]$$

$$= \exp\left[ - \frac{N_f \Sigma_{am}}{\xi_m \Sigma_{sm} \Sigma_{af}} \cdot \frac{f_r}{1 - f_r} \int (\sigma_{af})_{eff} \frac{dE'}{E'} \right] \qquad (10)$$

where the evaluation of $f_r$ is based on the relationship given by equation (6) as

$$f_r = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am} \frac{V_m}{V_f} \frac{\bar{\phi}_m}{\bar{\phi}_f}} \qquad (11)$$

where all the quantities involved have the values corresponding to the resonance neutrons.

The value of the effective resonance integral (Ref. 6) is given by

$$\int (\sigma_{af})_{eff} \frac{\alpha E'}{E'} = \frac{\xi_m \Sigma_{sm} \Sigma_{af}}{N_f \Sigma_{am}} \qquad (12)$$

Substitution of equation (12) in Equation (10) yields:

$$p(E) \approx \exp\left(-\frac{f_r}{1-f_r}\right) \qquad (13)$$

4) $\epsilon$ -- the fast fission effect factor. So far, it has been considered that the fast (fission) neutrons are produced only from the fissions by thermal neutrons. Actually other fission neutrons are produced by fast neutrons. Before the fast neutrons have slowed down appreciably, some will be captured by $U^{235}$ and $U^{238}$ nuclei, and will cause fission. At neutron energies greater than about one Mev, fast-neutron fission in natural uranium will come from the $U^{238}$ nuclei. Allowance for this effect may be made by introducing the fast-fission factor, denoted by $\epsilon$ and defined as the ratio of the total number of fast neutrons produced by fission due to neutrons of all energies (fast and thermal) to the number resulting from thermal-neutron fission.

Calculations of the above four factors are shown in the following pages.

Once $k_\infty$ is determined from the four-factor formula, the

multiplication constant for the finite medium which is customarily called the effective multiplication constant, $k_{eff}$, can be calculated from the relationship

$$k_{eff} = k_{\infty} e^{-B_g^2 M^2} \tag{14}$$

where $B_g^2$ is termed as the geometric buckling given by the equation $B_g^2 = (2.405/R_o)^2 + (\pi/H_o)^2$. $R_o$ and $H_o$ are the respective extrapolated radius and height of the reactor and 2.405 is the first zero of Bessel Function $J_o$; $M^2$ is the migration area for the natural uranium, water moderated reactor.

The subcritical multiplication is given in terms of $k_{eff}$ by

$$\mu = \frac{1}{1 - k_{eff}} \tag{15}$$

CALCULATION of $\eta$ - The average number of fast neutrons produced by the capture of a thermal neutron in the fuel is calculated from the equation (3) i.e.,

$$\eta = \nu \frac{\sigma_f^{235}}{\sigma_f^{235} + \sigma_c^{235} + R\,\sigma_c^{238}} \qquad (3)$$

where $R = \frac{N^{238}}{N^{235}}$

$\simeq \frac{99.28}{0.71}$

$= 139.8$

and $\sigma_f^{235}$ = fission cross section of $U^{235}$ = 582* barns

$\sigma_c^{235}$ = non-fission cross section of $U^{235}$ = 106* barns

$\sigma_c^{238}$ = non-fission cross section of $U^{238}$ = 2.8* barns

$\nu$ = average number of neutrons given out by a fissionable uranium nucleus

$= 2.5$

Substituting the above values in equation (3),

$$\eta = 1.35$$

* These values of the cross sections are taken from the Nuclear Engineering Handbook, Etherington (McGraw-Hill Book Co., Inc., 1958).

## CALCULATION OF THE THERMAL UTILIZATION, f

Theory: In the case of the heterogeneous reactor, the following three assumptions have been made for the determination of "f":

(1) The slowing down density is constant in the moderator and zero in the uranium. In other words, it is assumed that thermal neutrons are produced uniformly throughout the moderator but not at all in the uranium.

(2) The lattice is divided up into 264 identical unit cells, each cell having the fuel rod at the center. It is supposed that a parallelogram cross-section can be replaced by a circular cross section of the same area.

(3) The simple diffusion theory is applicable within both the moderator and the fuel regions.

With the above assumptions, the thermal utilization "f" is calculated. According to the first assumption, the neutrons may be treated as monoenergetic and therefore one-group theory can be applied. The thermal diffusion equations are as follows:

In the fuel, $$D_f \nabla^2 \phi_f(\vec{r}) - \Sigma_{af} \phi_f(\vec{r}) = 0 \qquad \text{.....(16)}$$

In the moderator, $$D_m \nabla^2 \phi_m(\vec{r}) - \Sigma_{am} \phi_m(\vec{r}) + q = 0 \qquad \text{.....(17)}$$

where "q" is the source term, equivalent to the number of neutrons

becoming thermal in the moderator per $cm^3$, and $\phi_m(\vec{r})$ and $\phi_f(\vec{r})$ denote the thermal neutron fluxes in the moderator and fuel respectively. The D's are the diffusion coefficients and the $\sum_a$'s the macroscopic absorption cross-sections for the thermal neutrons in the respective media.

The boundary conditions are:

(i) $\phi_f(\vec{r})$ is finite at the center of the cell, i.e. at $r = 0$.

(ii) $\frac{d\phi_m(\vec{r})}{dr} = 0$ at $r = r_m$

where $r_m$ is the outside radius of the moderator in the unit cell (Figure 4).

(iii) $\alpha\, 2\pi r_f j_+^{(f)}(r_f) + 2\pi r_2 \beta j_-^{(m)}(r_2) = 2\pi r_2 j_+^{(m)}(r_2)$

(iv) $2\pi r_f j_-^{(f)}(r_f) = \alpha(1-\beta)\, 2\pi r_2 j_-^{(m)}(r_2)$

} taking the cylindrical cell of unit length.

where, $\beta$ is the fraction of neutrons, leaving the moderator at the surface $r = r_2$ heading inward, which miss the fuel lump, $r_f$ and $r_2$ are respectively the outside radius of the fuel rod and the inside radius of the moderator ring of the unit cell, and

$j_+$ and $j_-$ are the partial neutron currents, the plus sign denoting the neutron current flowing out from the center of the cell and the minus sign denoting the neutron current flowing towards the center of the cell.

$\alpha$ is the transmission coefficient of the cladding material and the tube material (both aluminum) and is defined as the fraction of the neutrons incident upon the surface of a lump of the material which pass through the lump without being absorbed. For aluminum, $\alpha$ has been experimentally found to be unity. (Appendix I).

Equation (16) can be written as

$$\nabla^2 \phi_f(\vec{r}) - k_f^2 \phi_f(\vec{r}) = 0 \qquad \text{.....(16-a)}$$

where $k_f^2 \equiv \Sigma_{af} / D_f$

In cylindrical co-ordinates,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

In the present case, the flux distribution is axially symmetric (i.e. dependent only on the radial distance r), hence the diffusion equation (16a) can be written in cylindrical co-ordinates as:

$$\frac{d^2\phi_f(r)}{dr^2} + \frac{1}{r}\frac{d\phi_f(r)}{dr} - k_f^2 \phi_f(r) = 0 \qquad (16\text{-b})$$

The general solution of the differential equation (16b) is:

$$\phi_f(r) = A I_o(k_f r) + A' K_o(k_f r) \qquad (18)$$

where $I_o$ and $K_o$ are the Bessel's functions of zero order and second kind; A and A' are constants.

With the boundary condition (i),

at $r = 0$, $\phi_f(r) \neq \infty$

hence $A' = 0$

Therefore, $$\phi_f(r) = A I_o(\kappa_f r) \tag{18a}$$

Likewise, equation (17) becomes

$$\nabla^2 \phi_m(\vec{r}) - \kappa_m^2 \phi_m(\vec{r}) + \frac{q}{D_m} = 0 \tag{17a}$$

where, $\kappa_m^2 \equiv \Sigma_{am} / D_m$,

and the general solution is

$$\phi_m(r) = B' I_o(\kappa_m r) + B'' K_o(\kappa_m r) + \frac{q}{\Sigma_{am}} \tag{19}$$

where $B'$ and $B''$ are constants.

With the boundary condition (ii),

$$0 = \kappa_m B' I_1^*(\kappa_m r_m) - \kappa_m B'' K_1(\kappa_m r_m)$$

or, $$B'' = B' \frac{I_1(\kappa_m r_m)}{K_1(\kappa_m r_m)}$$

Therefore, $$\phi_m(r) = B \left[ I_o(\kappa_m r) K_1(\kappa_m r_m) + I_1(\kappa_m r_m) K_o(\kappa_m r) \right] + \frac{q}{\Sigma_{am}} \tag{19a}$$

* $\left[ \frac{d}{dr} \{ I_o(\kappa r) \} = \kappa I_1(\kappa r) \text{ and } \frac{d}{dr} \{ K_o(\kappa r) \} = -\kappa K_1(\kappa r) \right]$

where, $$B = B' / K_1(k_m r_m) \tag{19'}$$

or $$\phi_m(r) = B\,G(r) + q/\Sigma_{am} \tag{19b}$$

where, $$G(r) \equiv I_o(k_m r) K_1(k_m r_m) + I_1(k_m r_m) K_o(k_m r) \tag{19''}$$

Now, the boundary conditions (iii) and (iv) can be written in slightly different forms as follows:

$$r_f\, j_+^{(f)}(r_f) + r_2 \beta\, j_-^{(m)}(r_2) = r_2\, j_+^{(m)}(r_2) \tag{iii}$$

and $$r_f\, j_-^{(f)}(r_f) = (1-\beta)\, r_2\, j_-^{(m)}(r_2)\,. \tag{iv}$$

The partial currents are given in terms of the diffusion coefficient D (Ref. 11) by the following equations (for an arbitrary space point $\vec{r}$ in an infinite medium):

$$j_+(\vec{r}) = \frac{1}{4}\Phi(\vec{r}) - \frac{D}{2}\left|\nabla\Phi(\vec{r})\right|\cos\theta_r \tag{20a}$$

and $$j_-(\vec{r}) = \frac{1}{4}\Phi(\vec{r}) + \frac{D}{2}\left|\nabla\Phi(\vec{r})\right|\cos\theta_r \tag{20b}$$

where $\theta_r$ is the co-latitude of the gradient of the flux, $\Phi(\vec{r})$.

If $\Phi(\vec{r}) \equiv \phi(r)$ where r is the single co-ordinate of the system, the relations 20a and 20b become $j_+(r) = \frac{1}{4}\phi(r) - \frac{D}{2}\frac{d\phi(r)}{dr}$ (20c)

and $j_-(r) = \frac{1}{4}\phi(r) + \frac{D}{2}\frac{d\phi(r)}{dr}$ (20d)

in which case the gradient of neutron flux is in the direction of r.

Therefore,

$$j_\pm^{(f)}(r_f) = \frac{\phi_f(r_f)}{4} \mp \frac{D_f}{2}\frac{d\phi_f(r)}{dr}\Big|_{r=r_f} \quad (21a)$$

and $$j_\pm^{(m)}(r_2) = \frac{\phi_m(r_2)}{4} \mp \frac{D_m}{2}\frac{d\phi_m(r)}{dr}\Big|_{r=r_2} \quad (21b)$$

Substituting the above values of the partial currents given by the relations (21a) and(21b), the boundary condition (iii) becomes

$$r_f\frac{\phi_f(r_f)}{4} - \frac{D_f r_f}{2}\frac{d\phi_f(r)}{dr}\Big|_{r=r_f} + \beta r_2\frac{\phi_m(r_2)}{4} + \beta\frac{r_2 D_m}{2}\frac{d\phi_m(r)}{dr}\Big|_{r=r_2} = \frac{r_2\phi_m(r_2)}{4} - \frac{r_2 D_m}{2}\frac{d\phi_m(r)}{dr}\Big|_{r=r_2} \quad (iii)$$

and the boundary condition (iv) becomes

$$r_f\frac{\phi_f(r_f)}{4} + \frac{D_f r_f}{2}\frac{d\phi_f(r)}{dr}\Big|_{r=r_f} = (1-\beta)r_2\frac{\phi_m(r_2)}{4} + (1-\beta)\frac{r_2 D_m}{2}\frac{d\phi_m(r)}{dr}\Big|_{r=r_2} \quad (iv)$$

Adding the two relations for the boundary conditions (iii) and (iv), and multiplying through by 2,

$$r_f \phi_f(r_f) = (1-\beta) r_2 \phi_m(r_2) - 2\beta r_2 D_m \left.\frac{d\phi_m(r)}{dr}\right|_{r=r_2} \qquad \text{(iii)}'$$

Subtracting (iii) from (iv)

$$r_f D_f \left.\frac{d\phi_f(r)}{dr}\right|_{r=r_f} = r_2 D_m \left.\frac{d\phi_m(r)}{dr}\right|_{r=r_2} \qquad \text{(iv)}'$$

Substituting the values of $\phi_f(r)$ and $\phi_m(r)$ from the equations (18a) and (19b) respectively in (iv)',

$$r_f D_f \kappa_f A I_1(\kappa_f r_f) = r_2 D_m B G'(r_2) \qquad (22)$$

Where, $G'(r_2)$ denotes the derivative of $G(r)$ with respect to r at the point $r = r_2$.

Equation (22) may be written in a slightly different form as

$$B = AM \qquad (22a)$$

where $$M = \frac{r_f D_f \kappa_f}{r_2 D_m} \frac{I_1(\kappa_f r_f)'}{G'(r_2)} \qquad (22)'$$

Substituting the values of $\phi_f$ and $\phi_m$ in (iii)'

$$r_f A I_0(\kappa_f r_f) = B\left[(1-\beta) r_2 G(r_2) - 2\beta r_2 D_m G'(r_2)\right] + (1-\beta) r_2 \frac{q}{\Sigma_{am}} \qquad (23)$$

where, $$= AMP + (1-\beta) r_2 q/\Sigma_{am} \qquad (23)''$$

$$P \equiv (1-\beta) r_2 G(r_2) - 2\beta r_2 D_m G'(r_2) \qquad (23)'$$

Equation (23) may be written, after transposition, as

$$A\left[\gamma_f I_o(k_f\gamma_f) - MP\right] = (1-\beta)\gamma_2 \frac{q}{\Sigma_{am}}$$

or,
$$A = \frac{(1-\beta)\gamma_2 q}{\Sigma_{am}\left[\gamma_f I_o(k_f\gamma_f) - MP\right]} \tag{24}$$

Thus the values of A and B can now be calculated. The computations are shown on the succeeding pages. Once these constants are calculated, the fluxes $\phi_f(r)$ and $\phi_m(r)$ can be determined easily from the equations (18a) and (19b). Then in the infinite medium, the thermal utilization "f" is calculated from equation (6), i.e.,

$$f = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am}\frac{V_m}{V_f}\frac{\bar{\phi}_m}{\bar{\phi}_f}} \tag{6}$$

where the average thermal fluxes $\bar{\phi}_f$ and $\bar{\phi}_m$ in the fuel and the moderator respectively are given by

$$\bar{\phi}_f \equiv \frac{1}{V_f}\int_{V_f} \phi_f(r)\, dV \tag{25a}$$

and
$$\bar{\phi}_m \equiv \frac{1}{V_m}\int_{V_m} \phi_m(r)\, dV \tag{25b}$$

$V_f$ and $V_m$ are the respective volumes of the fuel and the moderator within the cylindrical unit cell of unit length.

## NUMERICAL CALCULATION OF THE THERMAL UTILIZATION "f"

$k_m = 0.37\ cm^{-1}$*  $k_f = 0.632\ cm^{-1}$*

$D_m = 0.16$ cm *  $D_f = 0.785$ cm**

$\sum am = 0.022\ cm^{-1}$ *  $\sum af = 0.3135\ cm^{-1}$ **

$r_m = 2.650$ cm  $r_1 = 0.635$ cm  $r_f = 1.350$ cm  $r_2 = 1.715$ cm

$k_f r_f = 0.853$  $k_m r_2 = 0.6345$  $k_m r_m = 0.980$

$I_o(k_f r_f) = 1.190$  $I_o(k_m r_2) = 1.103$  $I_o(k_m r_m) = 1.255$

$I_1(k_f r_f) = 0.467$  $I_1(k_m r_2) = 0.333$  $I_1(k_m r_m) = 0.551$

$K_o(k_f r_f) = 0.522$  $K_o(k_m r_2) = 0.734$  $K_o(k_m r_m) = 0.433$

$K_1(k_f r_f) = 0.780$  $K_1(k_m r_2) = 1.207$  $K_1(k_m r_m) = 0.623$

$k_f r_1 = 0.401$

$I_1(k_f r_1) = 0.204$ .

All the above values have been taken from the Table of Bessel Functions by Gray, Mathews, and Macroberts, and compared with the values in the Tables of the Ref. 17.

The quantities, $\beta$ G(r), G'($r_2$), P, M, A, and B are computed in the following pages.

* These data are obtained from page 165 or Reference 11.

** These data are obtained from pages 6-96 and 6-97 of Reference 4.

The missing-probability $\beta$ has been computed by Newmarch (11) from the relationship

$$\beta \equiv \frac{(1-r) + \gamma_F f(r)}{1 + \gamma_F f(r) + r\gamma_F}$$

where

$$r \equiv \frac{r_f}{r_2} = \frac{1.350}{1.715} = 0.787$$

$$f(r) \equiv 1 - \frac{2}{\pi} \sin^{-1} r - \frac{2r}{\pi} (1-r^2)^{\frac{1}{2}}$$

$$= 0.114$$

and

$$\gamma_F = \frac{2 D_f \phi_f' (r_f)}{\phi_f (r_f)}$$

(prime denoting the derivative with respect to r)

$$= \frac{2 D_f K_f AI_1 (k_f r_f)}{AI_o (k_f r_f)}$$

$$= 0.389$$

Therefore,

$$\beta = \frac{(1-0.787) + (0.389)(0.114)}{1 + (0.389)(0.114) + (0.787)(0.389)}$$

$$= 0.1904$$

From equation (19)",

$$G(r_2) = I_o (k_m r_2) K_1 (k_m r_m) + I_1 (k_m r_m) K_o (k_m r_2)$$

$$= 1.095$$

$$G'(r_2) = k_m \left[ I_1(k_m r_2)\, K_1(k_m r_m) - I_1(k_m r_m)\, K_1(k_m r_2) \right]$$

$$= -0.169$$

From equation (23)',

$$P = (1-\beta) r_2 G(r_2) - 2\beta r_2 D_m G'(r_2)$$

$$= 1.537$$

and from (22)',

$$M = \frac{r_f D_f k_f}{r_2 D_m} \frac{I_1(k_f r_f)}{G'(r_2)}$$

$$= -6.752$$

From equation (24),

$$A = \frac{(1-\beta) r_2 \, q}{\Sigma_{am} \left[ r_f I_0(k_f r_f) - MP \right]} = 5.266\, q$$

And from equation (22a),

$$B = AM$$

$$= -35.56\, q$$

The fluxes $\phi_f(r)$ in the fuel and $\phi_m(r)$ in the moderator are then given by equations (18a) and (19b) ( in neutrons - $cm^{-2}$ - $sec^{-1}$) as:

$$\phi_f(r) = 5.266\, q\, I_0(k_f r) \qquad (26a)$$

and $$\phi_m(r) = 45.459 - 35.56\, G(r)\, q \qquad (26b)$$

The average of $\phi_f(r)$ is:

$$\bar{\phi}_f = \frac{1}{V_f}\int_{V_f}\phi_f(r)\,dV = \frac{5.266}{\pi(r_f^2 - r_1^2)}\,q\int_{r_1}^{r_f} 2\pi r\, I_o(k_f r)\,dr.$$

where $r_f$ and $r_1$ are respectively the outside and inside radii of the fuel cylinder in the unit cell (Figure 4).

Therefore,

$$\bar{\phi}_f = \frac{2(5.266)}{(r_f^2 - r_1^2)}\,q\left[\frac{r_f I_1(k_f r_f) - r_1 I_1(k_f r_1)}{k_f}\right] \text{ neutrons-cm}^{-2}\text{sec}^{-1} \qquad (27a)$$

since $$\int_{r_1}^{r_f} r I_o(k_f r)\,dr = \left.\frac{r I_1(k_f r)}{k_f}\right|_{r_1}^{r_f}$$

Finally, after substituting the numerical values of the quantities involved in (27a),

$$\bar{\phi}_f = 5.889\, q \text{ neutrons-cm}^{-2}\text{sec}^{-1} \qquad (27b)$$

Simiarly, $$\bar{\phi}_m = 45.459\, q + \frac{B}{\pi(r_m^2 - r_2^2)}\int_{r_2}^{r_m} G(r)\, 2\pi r\, dr$$

$$= 45.459\, q - \frac{2(35.56)}{(r_m^2 - r_2^2)}\,q\int_{r_2}^{r_m} r\, G(r)\,dr \qquad (28b)$$

where $r_m$ and $r_2$ are respectively the outside and inside radii of the moderator cylinder in the unit cell. (Figure 4).

Now,

$$\int_{r_2}^{r_m} rG(r)dr = K_1(k_m r_m)\int_{r_2}^{r_m} r I_0(k_m r)\,dr + I_1(k_m r_m)\int_{r_2}^{r_m} r K_0(k_m r)\,dr \qquad (28)'$$

The first part of the right-hand-side of the above equation (28)' equals

$$K_1(k_m r_m)\left[\frac{r_m I_1(k_m r_m) - r_2 I_1(k_m r_2)}{k_m}\right]$$

$$= 1.503$$

and the second part of the same, i.e.,

$$I_1(k_m r_m)\int_{r_2}^{r_m} r K_0(k_m r)\,dr = I_1(k_m r_m)\left[\frac{-r K_1(k_m r)}{k_m}\right]_{r_2}^{r_m}$$

$$= 0.617$$

Therefore, after substituting the above values in equation (28)',

$$\int_{r_2}^{r_m} rG(r)\,dr = 2.120$$

Thus,

$$\bar{\phi}_m(r) = 45.45q - \frac{2(35.56)}{(r_m^2 - r_2^2)}q\,(2.120)$$

or,

$$\bar{\phi}_m = 8.51q \text{ neutrons-cm}^{-2}\text{-sec}^{-1} \qquad (28c)$$

The values of $\phi_f(r)$ and $\phi_m(r)$ as given by equations (26a) and 26(b) at different distances, r, from the center of a unit cell are tabulated below:

TABLE I - THERMAL NEUTRON FLUX IN THE FUEL REGION WITHIN A UNIT CELL

| r (cm) | $k_f r$ | $I_0(k_f r)$* | $\phi_f(r)$ (q neutrons per cm$^2$ per sec) |
|---|---|---|---|
| 0.90 | 0.569 | 1.080 | 5.687 |
| 0.95 | 0.600 | 1.092 | 5.750 |
| 1.00 | 0.632 | 1.102 | 5.803 |
| 1.05 | 0.664 | 1.113 | 5.861 |
| 1.10 | 0.695 | 1.124 | 5.919 |
| 1.15 | 0.727 | 1.136 | 5.982 |
| 1.20 | 0.758 | 1.148 | 6.045 |

TABLE II - THERMAL NEUTRON FLUX IN THE MODERATOR REGION WITHIN A UNIT CELL.

| r | $k_m r$ | $I_0(k_m r)$* | $K_0(k_m r)$** | G(r) | $\phi m(r)$ (q neutrons per cm$^2$ per sec). |
|---|---|---|---|---|---|
| 1.75 | 0.647 | 1.107 | 0.719 | 1.086 | 6.832 |
| 1.80 | 0.666 | 1.114 | 0.697 | 1.078 | 7.116 |
| 2.00 | 0.740 | 1.42 | 0.620 | 1.053 | 8.005 |
| 2.20 | 0.814 | 1.172 | 0.553 | 1.035 | 8.645 |
| 2.30 | 0.851 | 1.189 | 0.523 | 1.029 | 8.859 |
| 2.40 | 0.888 | 1.207 | 0.494 | 1.024 | 9.037 |
| 2.50 | 0.925 | 1.225 | 0.469 | 1.021 | 9.143 |

The above values of the Bessel Functions are taken from the Table of Bessel Functions: * (Reference 7)

** (Reference 17)

Using the values given in Tables I and II, the thermal neutron flux distribution at the vertical medial plane of the unit cell is shown in Figure 5, which depicts the depression of thermal neutrons within the fuel region.

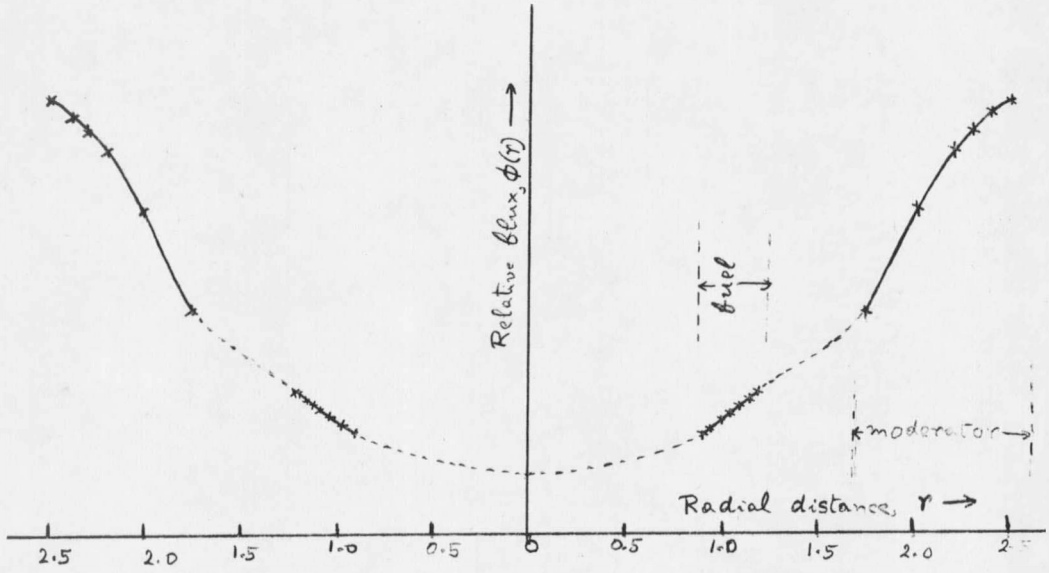



Figure 5 - Thermal neutron flux distribution in a vertical plane through the origin of a unit cell.

The flux across the vertical section of the heterogeneous reactor is shown below in Figure 6.

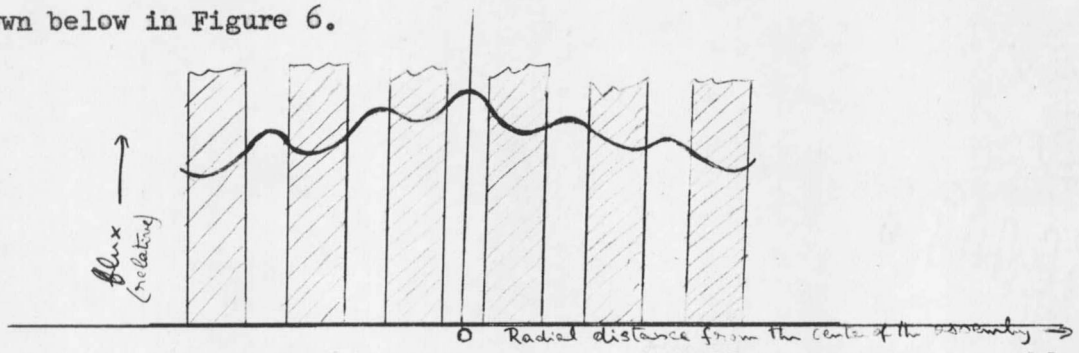



Figure 6 - Flux across the vertical section of the heterogeneous assembly.

Using the average values of $\bar{\Phi}_f(r)$ and $\bar{\Phi}_m(r)$, as given in equations (27b) and 28b), the thermal utilization "f" is calculated from the equation (6), i.e.

$$f = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am} \frac{V_m}{V_f} \frac{\bar{\phi}_m}{\bar{\phi}_f}}$$

$$= \frac{0.3135}{0.3135 + .022 \frac{r_m^2 - r_2^2}{r_f^2 - r_1^2} \frac{8.519}{5.889}}$$

or, $$f = 0.774 \qquad (29)$$

## CALCULATION OF THE RESONANCE ESCAPE PROBABILITY - "p" :

The value of the resonance escape probability can be calculated from equation (13), $p(E) = \exp(-f_r/1-f_r)$ (13)

where $f_r$ is given by equation (11) as

$$f(r) = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am}\frac{v_m}{v_f}\cdot\frac{\bar{\phi}_m}{\bar{\phi}_f}} \qquad (11)$$

The disadvantage factor $\bar{\phi}_m / \bar{\phi}_f$ (Ref 11) is now to be determined in a manner similar to that of the determination of the thermal utilization. The resonance neutron group constants are entirely different from those for thermal neutrons. (For the thermal neutrons, $k_m = 0.37\ cm^{-1}$, $D_m = 0.16$ cm, $k_f = 0.632\ cm^{-1}$, and $D_f = 0.785$ cm, while for the resonance neutrons, $k_m^* = 0.885\ cm^{-1}$, $D_m = 0.585^*$ cm, $k_f = 0.44^*\ cm^{-1}$, and $D_f = 0.65^{**}$ cm.)

Since very little slowing down occurs in the fuel, a uniform source of resonance neutrons, $q_1$ ($cm^{-3}sec^{-1}$), in the moderator may be assumed. Thus, the diffusion equations for the resonance neutrons are:

$$\nabla^2\phi_f(r) - k_f^2\,\phi_f(r) = 0 \qquad (30a)$$

and

$$\nabla^2\phi_m(r) - k_m^2\,\phi_m(r) + q_1 = 0 \qquad (30b)$$

where $q_1$ is the source term corresponding to the resonance neutrons.

* Page 669, reference (18).

** Page 666 reference (18).

The boundary conditions are the same as in the case of thermal neutron diffusion for the calculation of the thermal utilization.

Solving the differential equations (30a) and (30b), in the same way as is done for the calculation of "f", the following solution can be obtained:

$$\phi_f(r) = A I_0(k_f r) \tag{31a}$$

and

$$\phi_m(r) = B G(r) + q_1 / \Sigma_{am} \tag{31b}$$

A, B, and G(r) satisfy the same relationships as those in the case of the thermal utilization, the only difference being the numerical values due to the changed values of the group constants k, $\Sigma_a$, and D and the source term $q_1$.

## NUMERICAL CALCULATION OF "p"

For natural uranium, $k_m = 0.885\ cm^{-1}$ $\quad k_f = 0.44\ cm^{-1}$

$D_m = 0.585$ cm $\quad D_f = 0.65$ cm

$\Sigma_{am} = -.46^*\ cm^{-1}$ $\quad \Sigma_{af} = 0.1258^*\ cm^{-1}$

| | | |
|---|---|---|
| $k_f r_f = 0.594$ | $k_m r_2 = 1.518$ | $k_m r_m = 2.345$ |
| $I_0(k_f r_f) = 1.090$ | $I_0(k_m r_2) = 1.666$ | $I_0(k_m r_m) = 2.920$ |
| $I_1(k_f r_f) = 0.310$ | $I_1(k_m r_2) = 1.000$ | $I_1(k_m r_m) = 2.180$ |
| $K_0(k_f r_f) = 0.785$ | $K_0(k_m r_2) = 0.208$ | $K_0(k_m r_m) = 0.075$ |
| $K_1(k_f r_f) = 1.321$ | $K_1(k_m r_2) = 0.270$ | $K_1(k_m r_m) = 0.090$ |

$k_f r_1 = 0.279$ $\quad I_1(k_f r_1) = 0.141$

These values are calculated from $k_i^2 \equiv \Sigma_{ai} / D_i$

To find A and B, first $G(r_2)$, $\beta$, P, and M are calculated as is done with the thermal utilization.

(i) $G(r_2) = I_0(k_m r_2) K_1(k_m r_m) + I_1(k_m r_m) K_0(k_m r_2)$

$= 0.6034$

(ii) $G'(r_2) = k_m [I_1(k_m r_2) K_1(k_m r_m) - I_1(k_m r_m) K_1(k_m r_2$

$= -0.4413$

(iii) $\beta \equiv \frac{(1-r) + \gamma_F f(r)}{1 + \gamma_F f(r) + r \gamma_F}$,

where r = 0.787 and f(r) = 0.114. (Page 29),

$$\gamma_F = \frac{2 D_f \phi_f'(r_f)}{\phi_f(r_f)},$$

where $\phi_f(r) = A I_0(k_f r)$

and $\phi_f'(r) = A k_f I_1(k_f r)$

or $$\gamma_F = \frac{2 D_f k_f I_1(k_f r_f)}{I_0(k_f r_f)}$$

$= 0.163$

Therefore,

$$\beta = \frac{(0.213) + (0.163)(0.114)}{1 + (0.163)(0.114) + (0.787)(0.163)}$$

$= 0.2019$

(iv) $$P \equiv (1-\beta) r_2 G(r_2) - 2\beta r_2 D_m G'(r_2)$$

$$= 1.005$$

(v) $$M \equiv \frac{r_f D_f k_f}{r_2 D_m} \cdot \frac{I_1(k_f r_f)}{G'(r_2)}$$

$$= -0.2704$$

Therefore,

$$A = \frac{(1-\beta) r_2}{\Sigma_{am}\left[r_f I_0(k_f r_f) - MP\right]} q_1 \qquad (32a)$$

$$= 1.707\, q_1$$

and $$B = AM \qquad (32b)$$

$$= -0.4616$$

Finally, from (31a) and (31b), the resonance neutron fluxes (neutrons - $cm^{-2}$ - $sec^{-1}$) are: $$\phi_f(r) = 1.707\, q_1 I_0(k_f r) \qquad (33a)$$

and $$\phi_m(r) = -0.4616 G(r) q_1 + 2.1739 q_1 \qquad (33b)$$

Therefore, $$\bar{\phi}_f(r) = \frac{2(1.707)}{(r_f^2 - r_1^2)} q_1 \int_{r_1}^{r_f} r I_0(k_f r)\, dr \qquad (34a)$$

$$= 1.798\, q_1 \text{ neutrons-cm}^{-2}\text{-sec}^{-1}$$

and $$\bar{\phi}_m(r) = 2.1739\, q_1 + \frac{2(-0.4616)}{(r_m^2 - r_2^2)}\, q_1 \int_{r_2}^{r_m} r\, G(r)\, dr.$$

$$= 2.1739\, q_1 - \frac{0.9232}{4.081}\,(0.9662)$$

or, $$\bar{\phi}_m(r) = 1.9559\, q_1 \quad \text{neutrons-cm}^{-2}\text{-sec}^{-1} \tag{34b}$$

Therefore from equation (11),

$$f_r = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am}\,\frac{V_m}{V_f}\,\frac{\bar{\phi}_m}{\bar{\phi}_f}}$$

$$= 0.0805$$

and from equation (12),

$$p = \exp\left(-\frac{f_r}{1-f_r}\right)$$

or, $$p = 0.916 \tag{35}$$

CALCULATION OF THE FAST FISSION EFFECT FACTOR, $\epsilon$ and $k_\infty$

Theory: Fission neutrons may undergo various reactions with $U^{238}$ such as non-fission capture, fission, and elastic scattering. Considering all these, an expression for $\epsilon$ is given in terms of P which is the probability that a fission neutron, formed inside the fuel element, will make a single collision with a uranium nucleus before escaping into the moderator (Ref 1), as follows:

$$\epsilon = 1 + \frac{(\nu - 1 - \sigma_c/\sigma_e)(\sigma_f/\sigma)P}{1 - \frac{1}{\sigma}(\nu\sigma_f + \sigma_c)P} \qquad (17)$$

where subscripts c, f, and e refer to the non-fission capture, fission, and elastic scattering respectively, and $\sigma$ is the total cross section including that for inelastic scattering. For natural uranium,

$\sigma_c$ = 0.04 barns

$\sigma_f$ = 0.29 "

$\sigma_e$ = 1.50 "

$\sigma_c$ = 2.47 "

$\sigma$ = 4.30 "

Substituting the above values in equation (17), $\epsilon = 1 + 0.095P/(1 - 0.521P)$

From the talbe (Ref. 2), P is 0.28; therefore,

$$\epsilon = 1 + (0.095)(0.28)/[1 - (0.521)(0.28)] = 1.003 \qquad (36)$$

Therefore, for the infinite reactor,

$$K_\infty = (1.35)(0.774)(0.916)(1.003) = 0.960 \qquad (37)$$

## EXPERIMENTAL DETERMINATION OF THE THERMAL UTILIZATION "f"

In the heterogeneous assembly, the thermal utilization is defined by equation (6),

$$f = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{am} \frac{V_m}{V_f} \frac{\bar{\phi}_m}{\bar{\phi}_f}} \qquad (6)$$

The average thermal neutron fluxes $\bar{\phi}_m$ and $\bar{\phi}_f$ have been determined by the foil activation method. Thin indium foils have been used both in the fuel and in the moderator. The foils have been activated for about 40 hours and by that time the activity attains almost its saturation point. A neutron reacts with the indium isotope In-115 to produce the radioactive In-116. The $\beta$ —activity of these In-116 nuclei has been measured with the Geiger counter. The foils are activated not only by the thermal neutrons but also by the epithermal and the fast neutrons. In the determination of the thermal utilization it is essential to know the ratio of the density of neutrons with energies above thermal to that of the thermal neutrons. In this connection, a new quantity called the "cadmium ratio" is defined as:

$$\text{Cadmium ratio, } R_{cd} = \frac{\text{Saturated activity of the bare foil}}{\text{Saturated activity of the cadmium-covered foil.}}$$

Metallic cadmium absorbs thermal neutrons of energies up to 0.4 ev. The epithermal and fast neutrons pass through the cadmium cover and react with the foil inside the cover.

The values of cadmium ratios measured at several locations in the reactor are given in appendix III. The average of these is the $R_{cd}$ used below.

If $A_{th}$ and $A_{ep}$ denote respectively the saturated activity of the thermal neutrons and that of the neutrons of energies higher than thermal, then the cadmium ratio may be written as

$$(A_{th} + A_{ep}) / A_{ep} = R_{cd}$$

$$\text{or,} \quad A_{th}/A_{ep} = R_{cd} - 1 \tag{38}$$

Therefore, from equation (36)

$$A_{th}/(A_{th} + A_{ep}) = \frac{R_{cd} - 1}{R_{cd}}$$

$(R_{cd} - 1)/R_{cd}$ is the ratio of the thermal neutron saturated activity to the total neutron saturated activity. This factor is inserted in the 3rd and 5th columns of Table III as the correction factor for the thermal neutrons.

TABLE III - Average Thermal Neutron Fluxes by the Foil Activition Method

| Foil No.: | Count Rate | | | | $\bar{\Phi}_m/\bar{\Phi}_f = C_1/C_2$ |
|---|---|---|---|---|---|
| | In the moderator * | | In the fuel * | | |
| | Net (—Back-ground) | Corrected $C_1$ (x.7251) | Net (—Back-ground) | Corrected ($C_2$), (x 0.6337) | |
| 1 | 286 | 207 | 241 | 153 | 1.353 |
| 2 | 283 | 205 | 232 | 147 | 1.394 |
| 3 | 174 | 126 | 141 | 89 | 1.415 |
| 4 | 190 | 138 | 150 | 95 | 1.452 |
| 5 | 202 | 146 | 171 | 108 | 1.352 |
| 6 | 270 | 196 | 230 | 146 | 1.342 |
| | | | | Total | - 8.308 |

The average ratio, $\bar{\phi}_m / \bar{\phi}_f = 1.384$

The thermal utilization can now be calculated from equation (6), after substitution of the values of the $\sum_a$'s and the V's from page 28, as

$$f = \frac{.3135}{.3135 + .022 \dfrac{\pi(r_m^2 - r_2^2)}{\pi(r_f^2 - r_1^2)}\ 1.384}$$

or, $f = 0.780$ (39)

* All the foils have been activated for equal intervals of time and the beta-counts have been taken after the elapse of the same time after removal of the sample from the reactor.

## EXPERIMENTAL DETERMINATION OF THE EXTRAPOLATED BOUNDARIES AND THE BUCKLING

To determine the extrapolated radius of the subcritical assembly, the scintillation counter* was used. The vertical position of the scintillator inside the reactor was kept fixed (the source being at the bottom of the reactor). The radial distances of the scintillator were changed and the corresponding count rates were taken (the scintillator was always situated very close to the fuel rod but on the side away from the source, to maintain identical locations in the different unit cells). These count rates are plotted against the radial distances of the counter from the center of the Pu-Be source. Two sets of readings were taken for two different vertical positions of the scintillation counter. It was found that the extrapolated lines of both the plots (Graph I) meet the abscissa at the same point. This point is considered to lie on the extrapolated radial boundary of the reactor, and consequently the distance of this point on the abscissa corresponds to the extrapolated radius of the reactor. It will be noticed that the radial flux plots (Graph I) fit very nearly the Bessel function $J_0(r)$, and there is a particular value of r where the plots meet the abscissa i.e., where $J_0(r) = 0$. This is a consequence of the boundary condition that the flux vanishes at the extrapolated boundary. The particular value of r for which $J_0(r) = 0$ is the extrapolated radius $R_0$. The data and the plots are given on the following pages.

*In a different experiment, it was found that the scintillation counter is least efficient for fast neutrons. Hence the readings directly obtained from the counter correspond to the thermal neutrons.

Trial # 1:

Vertical distance of the scintillation counter from the center of the source = 41.5 cm.

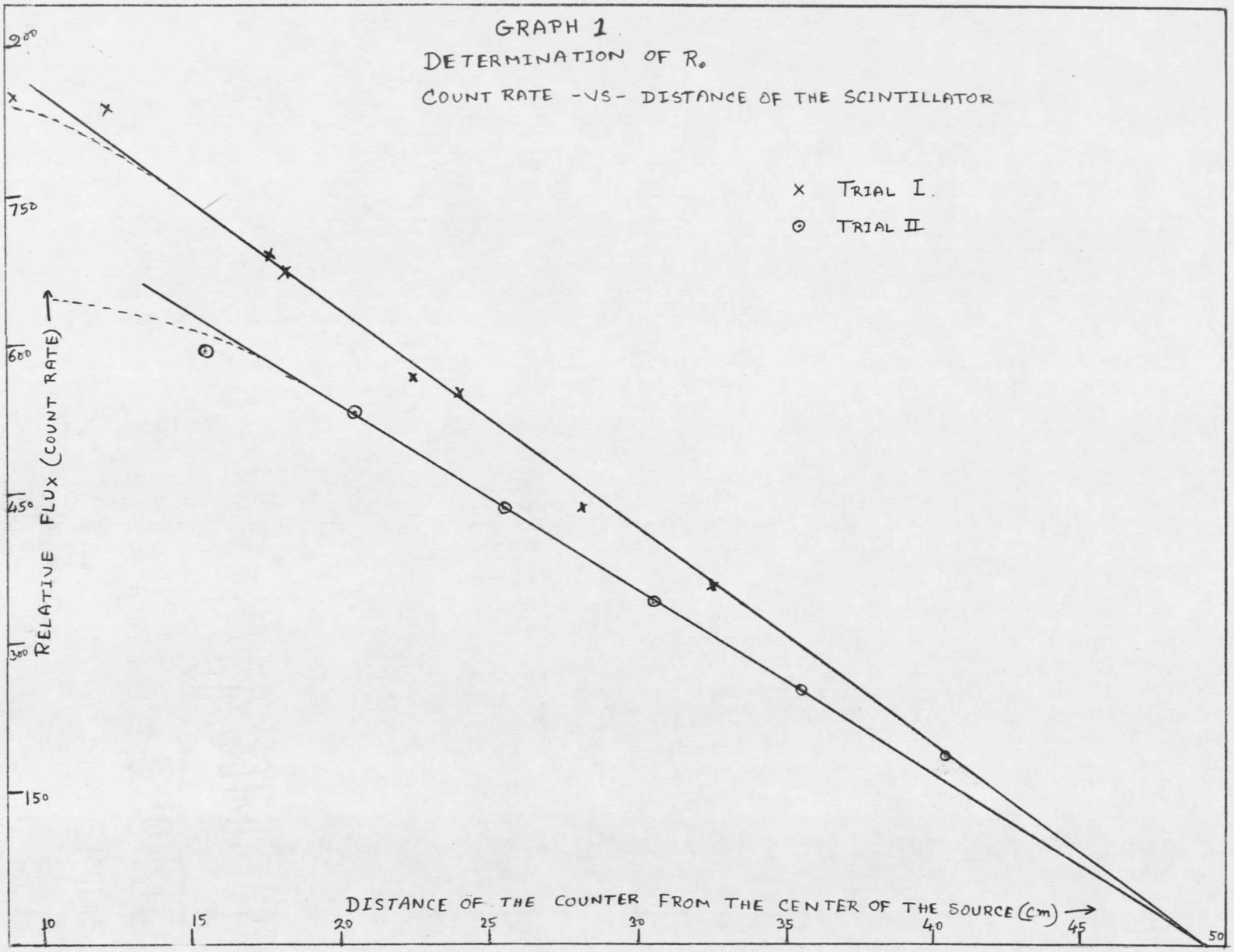
| Radial distance of the scintillator | Counts per minute |
|---|---|
| 7.1 cm | 918 |
| 8.2 " | 871 |
| 12.2 " | 841 |
| 17.5 " | 689 |
| 18.1 " | 674 |
| 22.3 " | 571 |
| 23.9 " | 558 |
| 28.2 " | 440 |
| 32.5 " | 360 |

Trial # 2:

Vertical distance = 46.5 cm

| Radial distance | Counts per minute |
|---|---|
| 15.4 cm | 595 |
| 20.4 " | 532 |
| 25.5 " | 435 |
| 30.5 " | 340 |
| 35.5 " | 252 |
| 40.4 " | 192 |

From the graph on the next page, $R_o = 49.3$ cm.
The dotted lines on the graph correspond to the $J_o$'s.


GRAPH 1
DETERMINATION OF $R_o$
COUNT RATE -VS- DISTANCE OF THE SCINTILLATOR
× TRIAL I
⊙ TRIAL II
RELATIVE FLUX (COUNT RATE)
900
750
600
450
300
150
DISTANCE OF THE COUNTER FROM THE CENTER OF THE SOURCE (cm)
10
15
20
25
30
35
40
45
50

The material buckling $B_m^2$ is given by the relationship

$$B_m^2 = (2.405^*/R_o)^2 - (\gamma)^2 \qquad (40)$$

where $R_o$ is the extrapolated radius, and $\gamma$ is the slope of the plot of ln A against the vertical distance of the scintillator from the center of the Pu-Be source, A being the relative activity ( i.e., the observed count rate). The data for the determination of $\gamma$ are given below:

The radial distance of the scintillator from the center of the source = 16.4 cm.

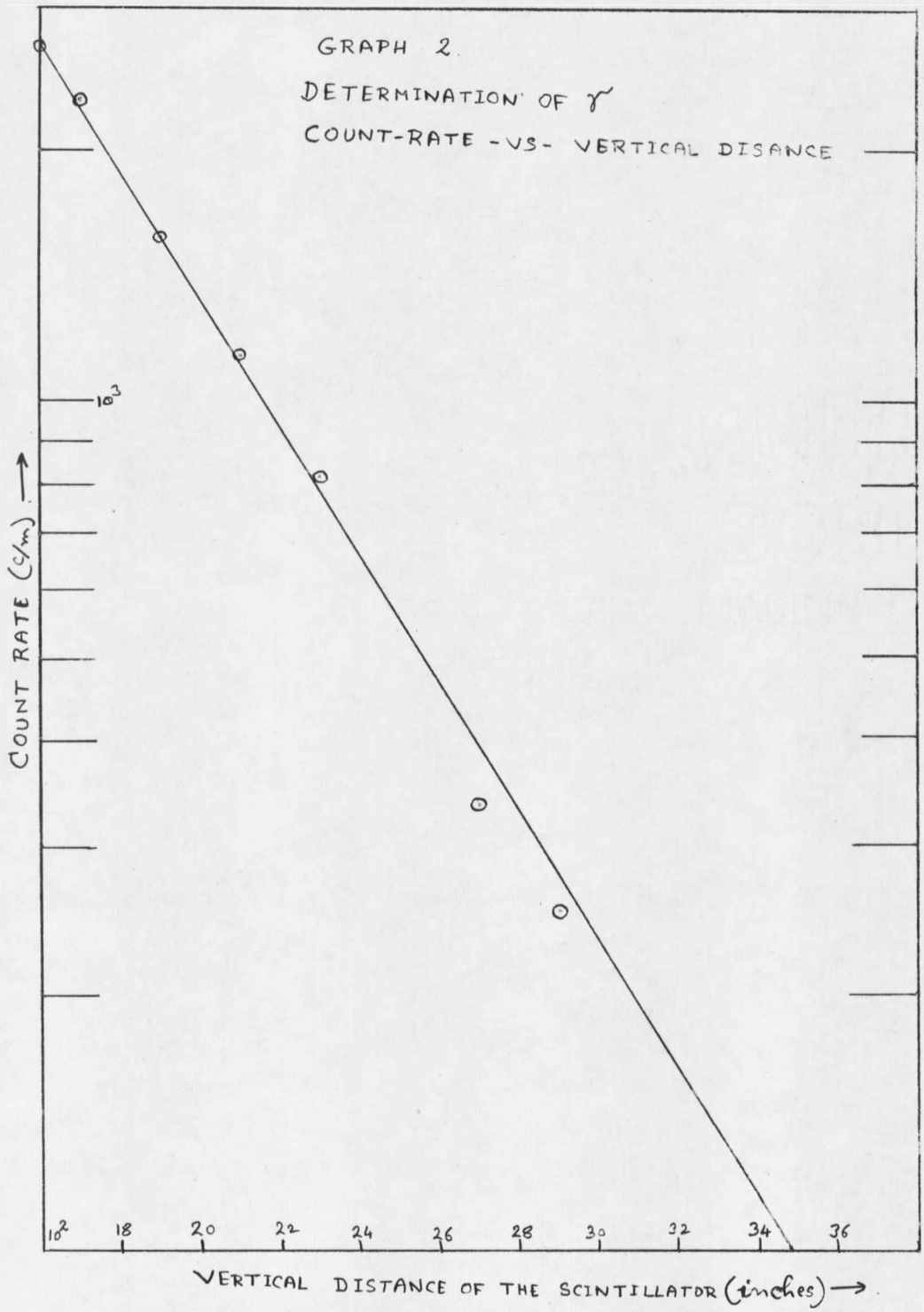
| Vertical distance of the scintillator | Counts per minute |
|---|---|
| 16 inches | 2690 |
| 17 " | 2315 |
| 19 " | 1580 |
| 21 " | 1140 |
| 23 " | 813 |
| 27 " | 335 |
| 29 " | 252 |

The relative activities are plotted against the vertical distances as shown in Graph # 2 and from the plot, $\gamma = 0.071 \ \text{cm}^{-1}$.

Therefore, $B_m^2 = (2.405/49.3)^2 - (0.071)^2$

$= -2.6615 \times 10^{-3} \ \text{cm}^{-2}$. (41)

* 2.405 is the first zero of Bessel Function $J_o$.


GRAPH 2
DETERMINATION OF γ
COUNT-RATE -VS- VERTICAL DISANCE
COUNT RATE (c/m) →
$10^3$
$10^2$
18
20
22
24
26
28
30
32
34
36
VERTICAL DISTANCE OF THE SCINTILLATOR (inches) →

The geometrical buckling can be obtained (Ref. 3) from the relationship

$$B_g^2 = (2.405/R_o)^2 + (\pi/H_o)^2, \qquad (42)$$

where $H_o$ is the extrapolated height of the reactor.

The extrapolated height is calculated from the relationship (Ref. 11)

$$H_0 = H + 0.7104 \lambda tr, \qquad (43)$$

where, H is the geometrical or actual height of the reactor and $\lambda_{tr}$ is the transport mean free path of a thermal neutron in water.

The height of a number of fuel rods was measured, the average value being 102.8 cm. The value of $\lambda_{tr}$ is 0.48 cm*.

Substituting these values in the equation (43),

$$H_o = 103.14 \text{ cm.}$$

Then, from equation (42),

$$B_g^2 = 3.307 \times 10^{-3} \text{ cm}^{-2}. \qquad (44)$$

* Page 6-86, Ref. 4.

## EVALUATION OF $k_\infty$, $k_{eff}$, and $\mu$

The infinite multiplication factor, $k_\infty$ is obtained from

$$k_\infty = \exp (B_m{}^2 M^2), \tag{45}$$

where $M^2$ is the migration area, given by the relationship

$$M^2 = L^2 + \tau; \tag{46}$$

$\tau$ is the Fermi age, and $L^2$ is the diffusion area of the heterogeneous reactor.

The diffusion area of the heterogeneous reactor is as follows: (Ref. 14)

$$L^2 = L_m^2 (1-f) + f L_f^2 \tag{47}$$

Therefore,

$$L^2 = (2.85^*)^2 \times (1-0.780) + 0.780 \times (1.55^*)^2$$

or, $L^2 = 3.66 \text{ cm}^2$ (48)

With $\tau = 30.4^* \text{ cm}^2$,

$$M^2 = 34.06 \text{ cm}^2 \tag{48}$$

Hence,

$$k_\infty = \exp (-2.66 \times 10^{-3}) \times (34.06)$$

$$= 0.911 \tag{49}$$

From equations (14), (44) and (48),

$$k_{eff} = 0.814 \tag{50}$$

Therefore, $\mu$ can be obtained from (15) as

$$\mu = \frac{1}{1-k_{eff}} = 5.38$$

* Pages 6-86 and 6-88, Ref. 4

## DISCUSSIONS

(1) In the present work, I used the simple first-order diffusion theory. In a heterogeneous reactor, better results would be obtained, if the transport theory were used. The Boltzman transport equation is more fundamental in expressing the principle of conservation of neutrons. In this treatment, the dependent variable is the angular distribution of neutron velocity vector, $n(\vec{r}, v, \vec{\Omega})$ which is defined as the number of neutrons at $\vec{r}$ per unit volume that are traveling with speed $v$ per velocity interval in the direction $\vec{\Omega}$ per unit solid angle. The quantity $n(\vec{r}, v, \vec{\Omega})\, v$ is called the vector flux and is merely the number of neutrons traveling in the direction $\vec{\Omega}$ which cross a unit area normal to this direction per unit time. In the diffusion theory the dependent variable has been reduced to the scalar neutron flux with the assumption that the angular distribution of the neutron velocity vector is isotropic, and independent of energy.

If it is assumed that neutrons deflected in scattering collisions with nuclei may proceed in any direction with equal probability (spherically symmetric or isotropic scattering) and that all neutrons have a single constant velocity, the transport theory reduces to diffusion theory. If it is further assumed that the neutron density does not change rapidly with distance over a few scattering mean free paths, usually a few cm, then diffusion theory reduces to elementary or first-order diffusion

theory.

The latter assumptions limit the elementary theory to systems in which the ratio of $\sigma_a$ to $\sigma_s$ is relatively small, i.e., to weakly absorbing media. The assumption of isotropic scattering leads to correct estimates of the flux at distances of a few mean free paths from sources and from boundaries between materials having significantly different absorbing and scattering properties. Estimates of flux close to such sources or boundaries may be greatly in error if made on the basis of diffusion theory.

(2) Assumption (1) on page 20 is accurate if the cell size, i.e., its separation between lumps, is not large compared to the slowing down distance of fission neutrons. In the present case, the slowing down distance $\sqrt{\tau}$ is about 5.5 cm and the cell diameter is 5.3 cm. Hence assumption (1) is not quite correct.

Assumption (2), in which a parallelogram is replaced by a circle of the same area has been introduced to simplify the mathematics, since a problem with axial symmetry is easier to handle than one with angular dependence. In this approximation only zeroth order cylindrical harmonics are involved. This implies an error in estimating the neutron flux. Assumption (3) is accurate if: 1) the fuel dimensions are not large compared to a mean free path; 2) if the system does not absorb neutrons very heavily; and 3) if the system contains no sources. None of these assumptions is strictly correct in our uranium-lattice. Errors due to the use of elementary diffusion theory in our system are obviously present.

(3) The one-group method assumes that all neutrons are thermal. No allowance is made for the fact that fast neutrons escaping from the reactor core have a longer average life in the reflector (water-medium surrounding the actual reactor core) than do thermal neutrons. These fast neutrons, therefore, have a higher probability of being scattered back into the core. Furthermore, neutrons, entering the reflector with energies above the resonance level are moderated in the reflector; then they return to the core as thermal neutrons, having completely avoided the resonance capture to which they would have been subjected if they had reached thermal energies in the core. Both these factors tend to increase the number of neutrons available at the end of any particular neutron generation which would consequently increase the value of the effective multiplication constant, $k_{eff}$.

(4) In the calculation of 'f', the denominator of the right hand side of equation (6) should contain an extra term, $\Sigma_{ac} \bar{\phi}_c V_c$ , which corresponds to the absorption of thermal neutrons in the constituent materials of the unit cell (other than the fuel and water), such as aluminum. The contribution of this term to the denominator is negligible because the macroscopic absorption cross section of aluminum $\Sigma_{ac}$ is very small.

(5) For the determination of the average thermal neutron flux, in the fuel, the indium foil was placed between two fuel rods, as shown in the figure on the next page.

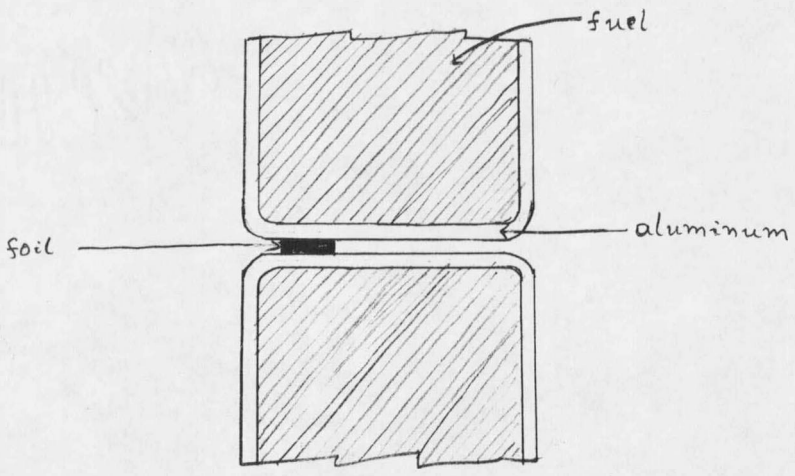



Figure 7

Location of the foil between the fuel rods

It is evident from the figure that the foil was not actually exposed to the thermal neutron flux, $\phi_f(r)$ but compositely to the thermal neutron flux, $\phi_m(r)$. This fact will decrease the experimental value of the disadvantage factor, $\bar{\phi}_m / \bar{\phi}_f$ and consequently increase the value of f.

(6) The experimental value of f is 0.78 whereas the theoretical value of the same is 0.77. The agreement between these values shows that one-group theory is adequate for the determination of the thermal utilization.

(7) The count rate from the activated indium foils was not large enough. Thus statistical fluctuation might cause considerable error in the relative activities of the foils. Larger count rates could be

obtained by using dysprosium foil which has very high thermal neutron absorption cross section (for Dy, $\sigma_a \sim 2700$ barns, and for In, $\sigma_a \sim 150$ barns). However, experimental difficulties prevented the use of dysprosium.

(8) The one-group treatment is an approximate one, but it is simple and leads to conclusions which are qualitatively correct.

(9) The experimental values of the thermal utilization, material and geometrical bucklings and cadmium ratio which have been obtained here can be used in further research work.

## Appendix I

The determination of the transmission coefficient of aluminum for thermal neutrons.

Bare indium foils and aluminum-covered indium foils were activated within the reactor for about 40 hours. In both the cases, a single foil was used and in the same location within the reactor. Gamma-rays with a few high energy beta-rays were counted in a well counter. The data are recorded below:

Background count-rate, at the start 8 c/m
at the end 10 c/m

Average = 9 c/m.

| Observation | Time after the removal from the reactor, t | Net count rate (-background), A | Saturated relative activity, $A_{sat} = A e^{\lambda t}$ | Transmission coefficient, $\alpha$ |
|---|---|---|---|---|
| Bare #1 | 12 min | 342 c/m | 392 c/m | 1.015 |
| Covered | 16 min | 330 c/m | 404 c/m | |
| Bare #2 | 20 min | 383 c/m | 494 c/m | 1.041 |
| Covered | 24 min | 381 c/m | 517 c/m | |

The decay constant $\lambda$ of In-116 $= \frac{.693}{\text{half-life of In-116}}$

$= \frac{.693}{54 \text{ min}} = 1.283 \times 10^{-2} \text{ min}^{-1}$

But, $\alpha$ cannot exceed unity, hence it is taken to be equal to unity. The higher values of $\alpha$ can be attributed to the statistics of the counter; the count rate was not high enough to minimize this statistic.

## Appendix II

## SYMBOLS USED

$B^2$ = buckling, $cm^{-2}$

D = diffusion coefficient, cm

f = thermal utilization

$f_r$ = resonance utilization

$K_\infty$ = infinite multiplication factor

$k_{eff}$ = effective multiplication factor for finite reactor

L = diffusion length, cm

$M^2$ = migration area, $cm^2$

p = resonance escape probability

q = slowing-down density, neutrons/$(cm^3)$(sec)

$\vec{r}$ = position vector

$V_f$ = volume of fuel cylinder of unit length, $cm^3$

$V_m$ = volume of moderator in the unit cell of unit length, $cm^3$

$\alpha$ = transmission coefficient of aluminum for the thermal neutrons

$\beta$ = missing-probability

$\epsilon$ = fast fission effect factor

$\eta$ = fission neutrons produced per neutron absorbed in fuel

$k$ = inverse diffusion length, $cm^{-1}$

$\nu$ = neutrons produced per fission

$\sigma$ = microscopic cross section, $cm^2$/nucleus or barns

$\sigma_a$ = microscopic cross section for absorption, barns

$\sigma_c$ = microscopic cross section for radiative capture, barns

$\sigma_f$ = microscopic cross section for fission, barns

$\sigma_s$ = microscopic cross section for scattering, barns

$\Sigma$ = macroscopic cross section ($N\sigma$), $cm^{-1}$

$\tau$ = Fermi age, $cm^2$

$\phi$ = neutron flux, for thermal neutrons unless otherwise specified, neutrons/$(cm^2)$(sec)

$\lambda_{tr}$ = transport mean free path, cm.

## Appendix III

The determination of the cadmium ratio in the moderator and the fuel.

The cadmium ratio has been defined on page 42 as the saturated activity of a bare foil, for example, indium, divided by the saturated activity of the indium foil completely covered with cadmium. Bare and cadmium covered indium foils were exposed to the neutron fluxes both in the fuel and in the moderator. The foils were activated for about 40 hours in all cases and the beta-activities were counted with a Geiger counter. The saturated activities were then obtained from the relationship $A_{\infty} = A \exp(\lambda \times t)$, where A is the corrected relative activity (i.e., observed count rate minus the background), $\lambda$ is the decay constant of In-116, and t is the time elapsed after removal of the foil from the reactor. The cadmium ratio in the fuel was determined by positioning the bare and the cadmium covered foils between two fuel rods on a line perpendicular to the axis of the fuel element. The cadmium ratio was determined at several locations in both the fuel and the moderator. The data are given below:

Data for the cadmium ratio in the moderator:

| Foil No.* | Bare foil | | Cadmium covered foil | | Cadmium -ratio, $R_{cd} = A_b/A_c$ |
|---|---|---|---|---|---|
| | time, t | corrected relative activity, $A_b$ | time, t | corrected relative activity, $A_c$ | |
| 1 | 4 min | 202 c/m | 4 min | 50 c/m | 4.04 |
| 2 | 4 " | 190 " | 4 " | 55 " | 3.45 |
| 3 | 4 " | 174 " | 4 " | 53 " | 3.29 |
| 4 | 14 " | 202 " | 14 " | 62 " | 3.26 |
| 5 | 22 " | 586 " | 22 " | 150 " | 3.90 |
| 6 | 25 " | 966 " | 25 " | 250 " | 3.86 |
| 7 | 28 " | 1489 " | 28 " | 412 " | 3.61 |
| 8 | 4 " | 283 " | 4 " | 76 " | 3.72 |
| 9 | 8 " | 560 " | 8 " | 150 " | 3.73 |
| 10 | 16 " | 380 " | 8 " | 108 " | 3.53 |

*These foils were positioned at different locations well within the reactor, (i.e., neither too close to the source nor to the boundary of the reactor)

The average $R_{cd}$ in the moderator is 3.638.

Therefore in the moderator the fraction of the thermal neutron density of the total neutron density, $(R_{cd} - 1)/R_{cd}$ is .7251.

Data for the cadmium ratio in the fuel:

| Foil No. | Bare foil | | Cadmium covered foil | | Cd-ratio $= A_b/A_c$ |
|---|---|---|---|---|---|
| | Time, t | corrected relative activity, $A_b$ | Time, t | corrected relative activity, $A_c$ | |
| 1 | 4 min | 232 c/m | 4 min | 81 c/m | 2.86 |
| 2 | 4 " | 131 " | 4 " | 49 " | 2.67 |
| 3 | 4 " | 171 " | 4 " | 65 " | 2.63 |
| 4 | 4 " | 265 " | 4 " | 95 " | 2.79 |
| 5 | 4 " | 410 " | 4 " | 150 " | 2.73 |
| 6 | 4 " | 384 " | 4 " | 143 " | 2.68 |

The average value of $R_{cd}$ in the fuel is then 2.73 and the fraction of thermal neutron density of the total neutron density is 0.6337.

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