

Tentative shear design method for steel fiber reinforced concrete flat plates
by Charles Afamefuna Chukwuemeka Ofili

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University
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Abstract:
R. N. Swamy et al., S. J. Patel and M. E. Criswell have performed tests to determine the influence of steel fiber reinforcement on the shear strength of slab-column connections. The results of these tests are used to develop a tentative, empirical design method to proportion steel fiber reinforced concrete flat plate systems to resist punching shear-. The tensile strength of the steel fiber-concrete composite is predicted by a—combined crack-control composite materials theory. This theory takes into account bond deficiency and randomness of the steel fibers. The short-fall of this method is the fact that tests and test specimen configurations for evaluating material properties are yet to be standardized.

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May 23, 1984
Date

Fred F Udeen
Chairperson, Graduate Committee

Approved for the Major Department

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Date

Fred F Udeen
Head, Major Department

Approved for the College of Graduate Studies

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## ABSTRACT

R. N. Swamy et al., S. J. Patel and M. E. Criswell have performed tests to determine the influence of steel fiber reinforcement on the shear strength of slab-column connections. The results of these tests are used to develop a tentative, empirical design method to proportion steel fiber reinforced concrete flat plate systems to resist punching shear. The tensile strength of the steel fiber-concrete composite is predicted by a combined crack control-composite materials theory. This theory takes into account the bond deficiency and randomness of the steel fibers. The short-fall of this method is the fact that tests and test specimen configurations for evaluating material properties are yet to be standardized.

# INTRODUCTION

The connection between the slab and the column in a flat plate structure is generally the most critical part as far as its strength is concerned. This is because large bending moments as well as shearing forces are concentrated in this area.

The economy of the entire structure is, to a large extent, governed by the degree to which the ultimate strength of the connection between the slab and the column can be predicted and utilized. One of the main design problems of slab-column connections in flat plate structures is the punching shear failure at overloads. Punching shear failures are usually sudden and catastrophic and as such are undesirable.

To satisfy weight restrictions and other architectural or geometric constraints, slab-column connections are conventionally reinforced with shearheads, stirrups or bent up bars to enhance their punching shear strength. These methods of reinforcement have their faults. Shearheads cannot be used for exterior columns at the present state of art. The construction costs may be high for shearheads. Also, shearheads may cause excessive congestion of the reinforcement and perhaps difficulty in placing the concrete. Bent up bars and stirrups may also cause congestion of the reinforcement and their construction may be very labor-intensive. Bent up bars and stirrups have to be well anchored (ACI Code 12.14) to be effective. The limited depth of flat plates make the anchorage of these shear reinforcements difficult.

Several investigators (26, 27, 28, 29) have found that the addition of short discrete steel fibers to concrete, improves its tensile strength and ductility. Tests have been performed to determine the influence of steel fiber reinforcement on the shear strength of slab-column connections (26, 27, 28, 29). The test results show that steel fibers may be used singly or combined with conventional shear reinforcement to enhance the ductility and ultimate punching shear load in flat plate structures. Swamy et al. (26) observed that the addition of fibers in the vicinity of the column head was as effective in resisting punching shear as providing fibers over the entire plate.

The use of steel fibers for shear reinforcement also has its drawbacks. The fibers in the concrete are randomly oriented rather than being placed in the most advantageous direction. Also, failure in fiber reinforced concrete usually occurs by unstable crack propagation (before the strength of the fibers is fully utilized) when the fibers pull out and the interfacial shear stress reaches the ultimate bond strength. These problems make steel fibers less effective than conventional reinforcement in reinforcing concrete.

Whether steel fibers, shearheads, stirrups or bent up bars are used for shear reinforcement would depend on the conditions and functions of the flat plate structure. From the conditions listed above, steel fiber reinforcement would appear to be advantageous in thin concrete structures.

## Objective and Scope

The object of this thesis is to develop a tentative, empirical design method to proportion steel fiber reinforced concrete flat plate systems to resist punching shear. The design method may be used as an alternative to conventional methods to determine the volume fraction of steel fibers required to be added to a given flat plate structure to prevent punching shear failure of the plate. The size and thickness of the plate is chosen to satisfy geometric constraints, current ACI deflection requirements and other ACI serviceability requirements. The method proposed in this thesis might be a suitable alternative for the case where the flat plate is thin and the use of conventional shear reinforcement might cause congestion and anchorage problems of the shear and flexural reinforcement, thereby reducing their effectiveness. With the use of steel fibers as shear reinforcement this problem may be reduced or eliminated. It may also be used as an alternative to increasing the slab thickness of a flat plate that satisfies ACI serviceability requirements, but does not satisfy the punching shear requirements. Rather than increase the thickness of this slab, steel fibers may be added to enhance its punching shear strength. In this case, savings on concrete and satisfaction of any weight restrictions may be achieved.

This thesis examines the split cylinder tensile strength of fiber-concrete, and the shear strength and behavior of fiber-concrete flat plates. The results of 41 split cylinder tensile strength tests, together with possible factors governing these results were investigated

and tentative design recommendations based on these results were developed to be used to predict the split cylinder tensile strength of fiber-concrete. Next, tentative design recommendations based on the predicted split cylinder tensile strength of fiber-concrete, the behavior of fiber-concrete flat plates, and the result of tests on eight slab-column specimens, were developed to be used to predict the shear strength of fiber-concrete flat plates. Note that the above-mentioned design recommendations (in both cases) are tentative and are intended to be used as a guideline for further research. Therefore, caution should be exercised in their usage, until confirmed by further research.

# A REVIEW OF CURRENT ACI METHODS FOR THE DESIGN OF FLAT PLATES

Flat plates are two way slabs whose interiors are supported directly by columns, and whose exterior edges may or may not be supported by edge beams. The slab-column connection of a flat plate has neither column capitals nor drop panels, and a uniform slab thickness is used. Generally, flat plates are used for smaller panel size and lighter loading than slabs with interior beams.

Research on slabs shows that there is a complex dependence of shear strength on the flexural behavior of both the slab system and the intermediate area around the connection (2). In addition, it has been difficult to observe the internal inclined cracking of these slabs. Due to these problems, the mechanism of shear failure in slabs has not been completely described, and most current methods for the analysis of shear failure are empirical (1).

Several effects have been identified in the mechanism of shear failure in slabs to enable apparently safe design equations to be developed without a full understanding of the fundamental laws governing this phenomenon. These effects are discussed below.

## Shear Strength of Concrete Slabs Without Shear Reinforcement

The joint ASCE-ACI Task Committee 426 on Shear and Diagonal Tension has stated the following variables involved in the shear strength of slabs without shear reinforcement (1).

1. The concrete strength, $f'_c$.
2. The ratio of the side length, r, of the loaded area to the effective depth, d, of the slab.
3. The relationship (V/M) between shear and moment in the vicinity of the loaded area.
4. The column shape in terms of the ratio $\beta_c$ of the long side to the short side of a rectangular column.
5. Lateral restraints such as may be provided by stiff beams along the boundaries of a slab.
6. The rate of loading.

Flat plates may fail by either of two kinds of shear, namely, wide-beam shear or punching shear. Wide-beam shear has a tendency to occur in a slab whose length is substantial compared to its width. Two other factors that may prompt wide-beam action in a slab are the shape and size of the loaded area and the relative magnitude of the principal moments in the slab. For rectangular columns, as the size of the loaded area increases relative to the slab thickness, the behavior of the slab is more like that of a wide beam (3). Also for a loaded area of rectangular shape, if the maximum principal moment acts about the short side, a rapid transition from slab to beam behavior begins when the aspect ratio for the two sides exceeds two (3).

The critical section for wide-beam shear in a flat plate is taken a distance, d, from the column face and is parallel to the panel centerline in the transverse direction (Fig. 1). The failure occurs at an angle of approximately 45° with the column face (see Fig. 1). The total shear $V_u$ along the critical section equals the net pressure on the slab

times the area of the slab just to the right of the critical section (Fig. 1). The maximum value of $V_u$ if shear reinforcement is not used is $\phi V_c$ which equals $2\phi\sqrt{f'_c}$ bd according to the simplified method of ACI-11.3.1.1.(4). This gives the maximum required depth as:

$$d = \frac{V_u}{2\phi\sqrt{f'_c}\, b} \qquad (1)$$

Wide-beam shear may occur in flat plates, but it rarely controls the design.

A flat plate, heavily loaded with a concentrated load, may cause diagonal tension cracks to form around the load or column. These cracks extend into the compression area of the plate and encounter resistance near the load similar to the shear-compression condition in beams. Finally, failure occurs with the potential diagonal crack following the surface of a truncated cone or pyramid which is punched through the plate (Fig. 2). Circular loaded areas produce conical failure surfaces, and rectangular areas produce pyramid-shaped failure surfaces. For a square or rectangular column, the failure surface extends from the bottom of the plate, at the support, diagonally upward to the top surface. The angle of inclination of the pyramidal failure surface with the horizontal depends upon the nature and amount of reinforcement in the plate. It may range between about 20° and 45°. For a plate without shear reinforcement, this angle is approximately 45° (Fig. 2).

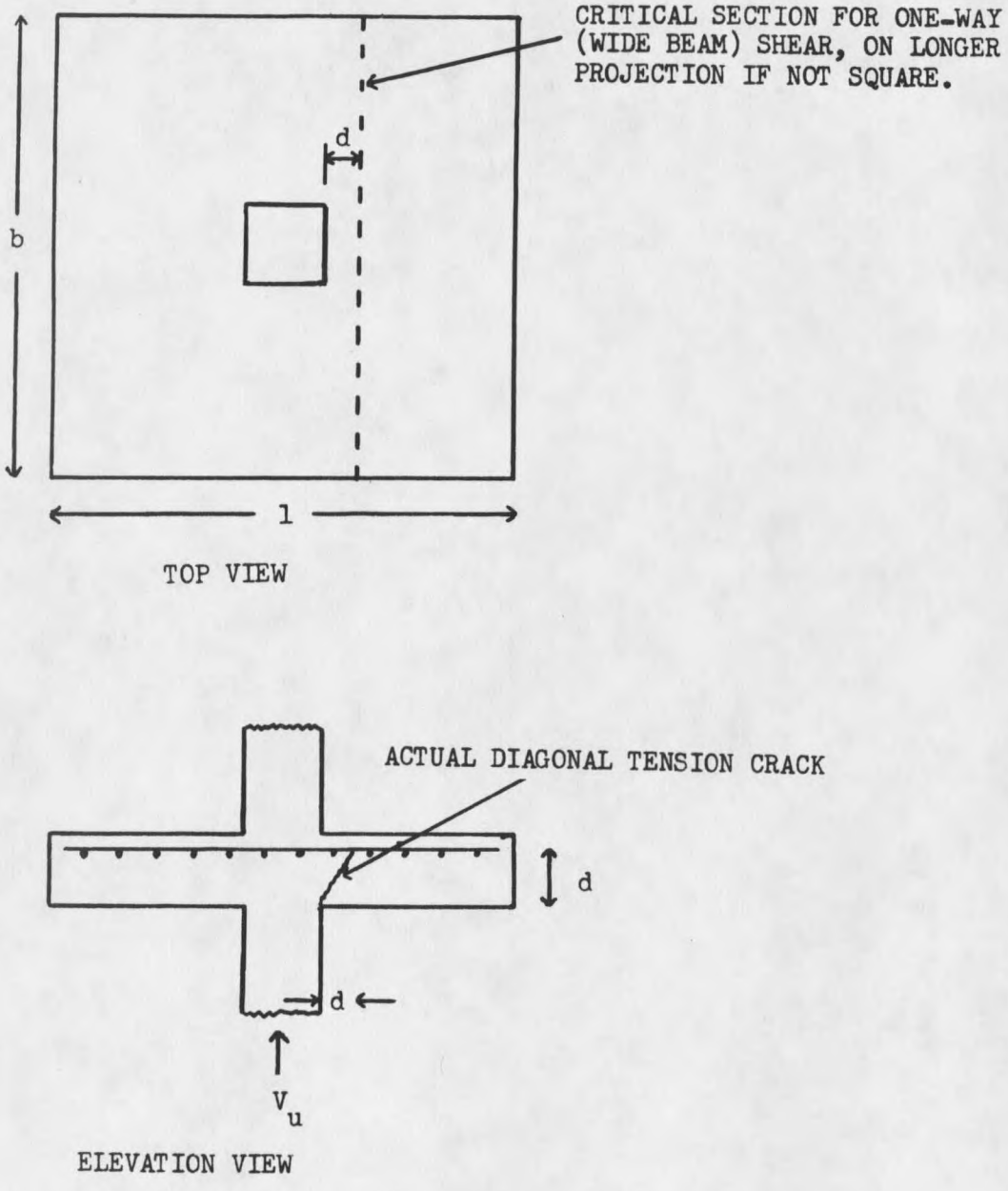
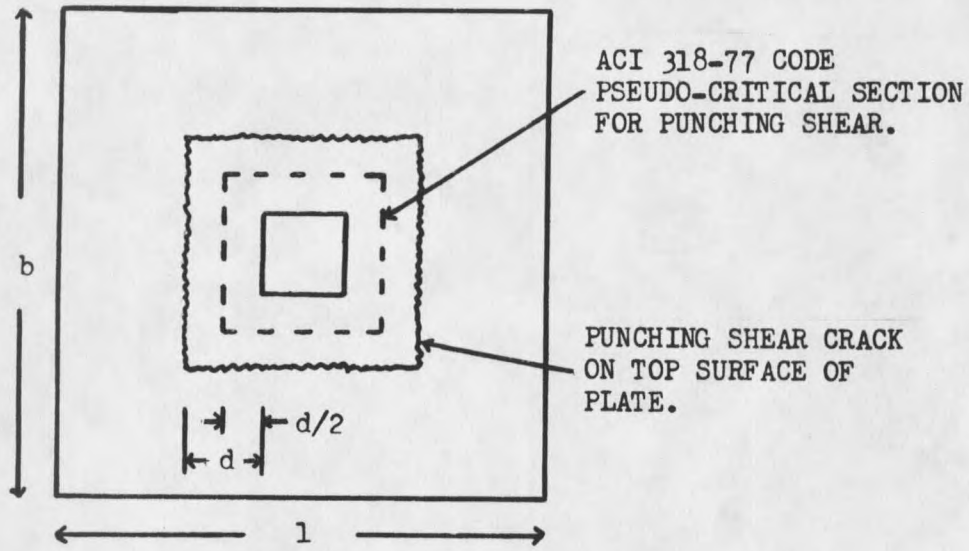
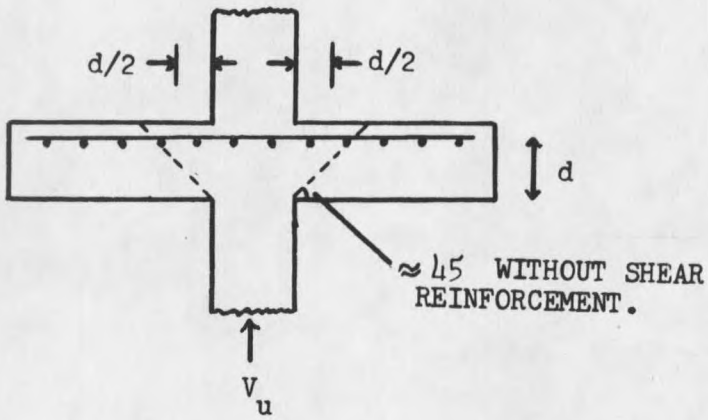



Figure 1. Failure Surface Defined by Wide-Beam Shear.




TOP VIEW




ELEVATION VIEW

Figure 2. Failure Surface Defined by Punching Shear.

The ASCE-ACI Joint Committee 426 recommended that the critical section for punching shear should be taken perpendicular to the plane of the plate and located at a distance of d/2 from the periphery of the column face or edge of the load as shown in Fig. 2. Calculation of the shear stress in test specimens at sections located at the column face, at a distance d from the column face, and at a distance d/2 from the column face indicate that the latter gives the most consistent results for all depths (5, 6). It also seems logical to choose this section since the inclined cracks usually formed first at a distance of approximately d/2 from the periphery of the face of the column (6).

The shear force, $V_u$, to be resisted is calculated as the total factored load on the area bounded by the panel centerlines and the pseudo-critical section (Fig. 2). In the failure region of the plate-column connection, the combination of the shearing stress, the horizontal compressive stresses due to negative bending moment, and the vertical or somewhat inclined compressive stress due to the reaction of the column provide a lateral confining effect which increases the shear strength of the concrete by a moderate amount. Also, a portion of the shear force may be resisted by the tension reinforcing steel (which passes through the "pyramid of rupture") of the plate by dowel action. The above-mentioned effects cause the nominal shear strength of flat plates to be generally greater than that for beams. The maximum nominal shear stress at the pseudo-critical section of a flat plate must not be greater than $4\sqrt{f'_c}\, b_o d$, if shear reinforcement is not used (4). $b_o$ is the perimeter along the critical section. For design purposes, $V_u \leq \phi V_c$ where $\phi$ is equal to 0.85 for shear.

Tests of punching shear around columns or loads indicate that the value of $V_c = 4\sqrt{f'_c}\, b_o d$ is unconservative when $\beta_c$ is larger than two (7). In such a case, the actual shear stress at punching failure varies from a maximum of about $4\sqrt{f'_c}$ around the ends of the column down to $2\sqrt{f'_c}$ or less along the long sides between the two end sections. Reflecting the test data, the ACI code provision for punching shear strength is:

$$V_c = (2 + \frac{4}{\beta_c})\sqrt{f'_c}\, b_o d \quad \leq 4\sqrt{f'_c}\, b_o d \qquad (2)$$

Unless $\beta_c$ is larger than 2, the expression involving $\beta_c$ does not control, and $V_c$ is limited to $4\sqrt{f'_c}\, b_o d$. For columns of nonrectangular cross section the ACI Code indicates that the perimeter $b_o$ must be of minimum length but need not approach closer than d/2 to the perimeter of the actual loaded area.

Shear Reinforcement in Flat Plates

In flat-plate floors where neither column capitals no drop panels are used, shear reinforcement is frequently necessary. Shear reinforcement may be needed if the span length of the plate is not to be reduced or the column size increased above that provided. It may also be used to satisfy thickness or weight restriction in flat-plate floors. The effect of the shear reinforcement at the supports of flat plates is to increase the punching shear strength of the plates. The type of shear reinforcements for slab-column connections reported in the literature can be divided into two basic types, shearhead reinforcement and bent bar and stirrup reinforcement.

Shearheads are steel I or channel shapes fabricated by welding into four identical arms that are at right angles and are continuous through the column section (Fig. 3). The cantilever like projections of the shearhead take up both moment and shear near the column. The main effect of the shearhead is to greatly increase the effective perimeter, $b_o$, of the critical section for shear and decrease the total shear force, $V_u$, for which the slabs must be designed. It also contributes to the negative bending resistance of the plate.

In the article by Corley et al. (8), they noted that provided the plastic moment capacity of the shearhead was adequate, the shearheads increased the shear capacity in proportion to the projection of the shearhead arm. The increase was at a rate slightly less than that expected for an increase in column size equal to the project of the shearhead arm. Therefore, when they proposed the design procedure incorporated in ACI 318-77, they specified that the critical section shall be at 0.75 of the shearhead projection beyond the face of the support and is defined so that the perimeter is a minimum. It need not approach closer than d/2 to the face of the support (Fig. 3). The maximum nominal shear stress, $V_n$, at this pseudo-critical section should be limited to $4\sqrt{f'_c}\, b_o d$. However, at a pseudo-critical section whose perimeter, $b_o$, is a minimum and located at a distance from the column face not less than d/2, $V_n$ should be limited to $7\sqrt{f'_c}\, b_o d$ when shearhead reinforcement is provided (4). These provisions apply only where shear is transferred at an interior column.

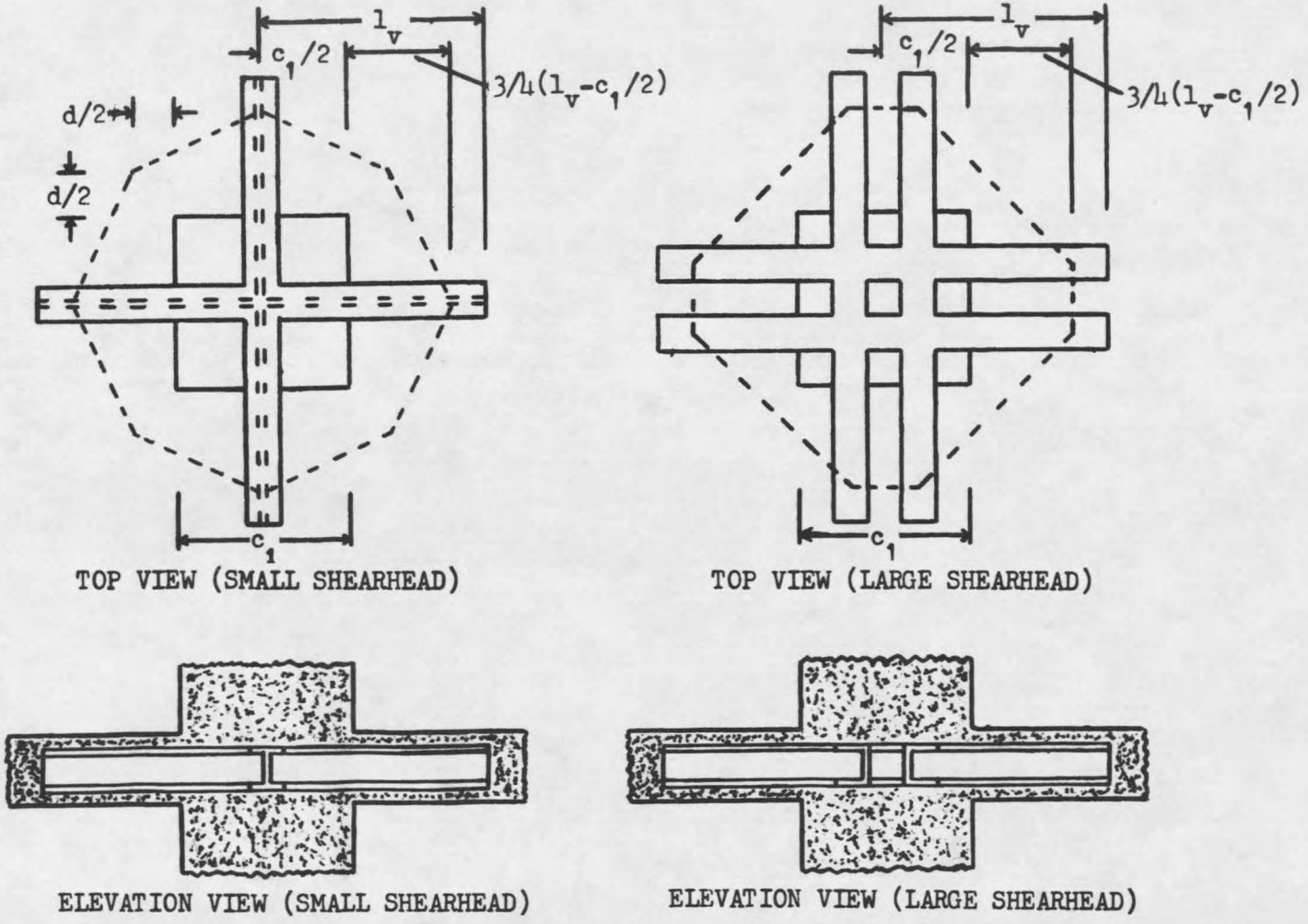



Figure 3. Shearhead Reinforcement in Flat Plates.

According to Corley et al. (8), the shear is constant throughout most of the length of a shearhead arm, and the part of the total shear carried by the shearhead arm is proportional to $\alpha_v$, where $\alpha_v$ is the relative stiffness of the shearhead arm to that of a composite cracked slab section with an effective width of $(c_2 + d)$, where $c_2$ is the width of the support measured perpendicular to the arm direction. The calculation of the composite cracked, transformed section should include the shearhead. According to the ACI Code, the value of $\alpha_v$ should not be less than 0.15. The steel shape used must not be deeper than 70 times its web thickness; and the compression flange must be located within 0.3d of the compression surface of the slab. In addition, the plastic moment capacity $M_p$ of the shearhead arm must be at least

$$\min M_p = \frac{V_u}{8\phi} [h_v + \alpha_v (L_v - \frac{c_1}{2})] \qquad (3)$$

where

$V_u$ = shear force around the periphery of the column face, due to factored loads

$h_v$ = depth of shearhead

$L_v$ = length of shearhead measured from column centerline

$\phi$ = 0.90, the capacity reduction factor for flexure

Equation (3) is to assure that the required shear capacity of the slab is reached before the flexural capacity of the shearhead is exceeded.

The following are some of the major considerations for detailing of bent bars and stirrups shear reinforcement (9). First, effective anchorage is difficult to achieve in a thin slab. Second, in order to

ensure ductility under overloads, the shear reinforcement should be extended at least twice the slab thickness beyond the distance at which it is no longer theoretically needed based on strength considerations. Third, unless ductility is mandatory, the increased labor costs resulting from the provision of shear reinforcement must be balanced against the cost of providing a similar increase in shear strength by increasing the thickness of the slab or the size of the column.

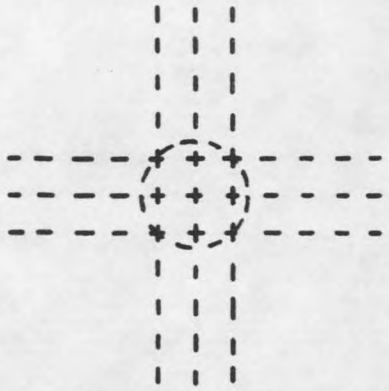
If bar shear reinforcement (i.e., bent bars or stirrups) is used, the limit value of the nominal shear strength $V_n$, calculated at the critical section d/2 from the support face, may be increased to $6\sqrt{f'_c}\, b_o d$ according to the ACI Code. In this case, the shear resistance of the concrete, $V_c$, is taken as $2\sqrt{f'_c}\, b_o d$, and the shear reinforcement must provide for the excess shear above $\phi V_c$. When bent bar shear reinforcement (Fig. 4) is used, the total bar area $A_v$ crossing the critical section, $b_o$, at a slope angle of $\alpha$ relative to the horizontal is easily obtained by equating the vertical component of the steel force to the excess shear force to be accommodated:

$$\phi A_v f_y \sin\alpha = V_u - \phi V_c \qquad (4)$$

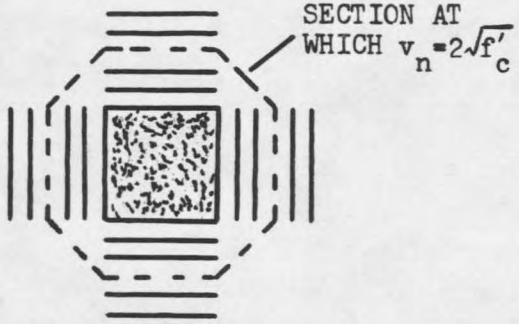
from which

$$A_v = \frac{V_u - \phi V_c}{\phi f_y \sin\alpha}$$

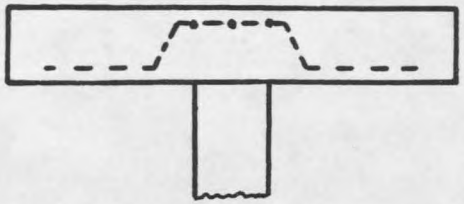
For vertical stirrup bars (Fig. 4), $\alpha = 90°$. Successive sections at increasing distances from the support must be investigated and reinforcement provided where $V_u$ exceeds the value given by equation (2). Only the center three-quarters of the inclined portion of the bent bars can be considered effective in resisting shear.



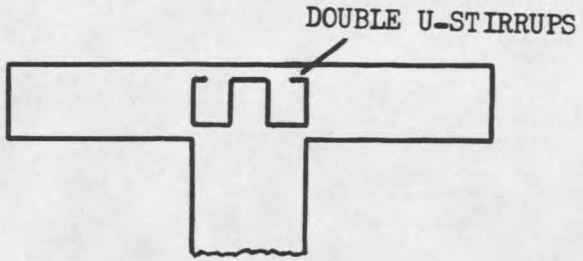
TOP VIEW (BENT BARS)




TOP VIEW (STIRRUPS)



ELEVATION VIEW (BENT BARS)




ELEVATION VIEW (STIRRUPS)

Figure 4. Bent Bar and Stirrups Shear Reinforcement.

## PROPERTIES OF FIBER REINFORCED CONCRETE

Fiber reinforced concrete is concrete made of hydraulic cements containing fine or fine and course aggregates and discontinuous discrete fibers (10). Fibers currently being used to reinforce concrete can broadly be classified into two types. Low modulus, high elongation fibers, such as polypropylene, are capable of large energy absorption characteristics, with little or no improvement in strength. They impart toughness and resistance to impact and explosive loading to the composite. High strength, high modulus fibers such as steel, glass, and asbestos, on the other hand, produce strong composites. And they impart strength and stiffness characteristics to the composite.

Steel fibers are available in various shapes (Fig. 5). These include: straight fibers (round, square or rectangular in cross-section), hooked fibers and crimped fibers. The crimped fibers may be crimped the full length of the fibers or at the ends only. Round steel fibers are produced by cutting or chopping wires with typical diameters of 0.010 to 0.030 in. Flat steel fibers having typical cross sections ranging from 0.006 to 0.016 in. in thickness by 0.010 to 0.035 in. in width are produced by shearing sheets or flattening wire. A common form of crimped fiber is the round fiber with an evenly spaced flat section (Duoform). The most common size of this is 0.016 in. round with 0.010 by 0.020 in. flat sections with fiber lengths of 0.4 to 1.5 in.

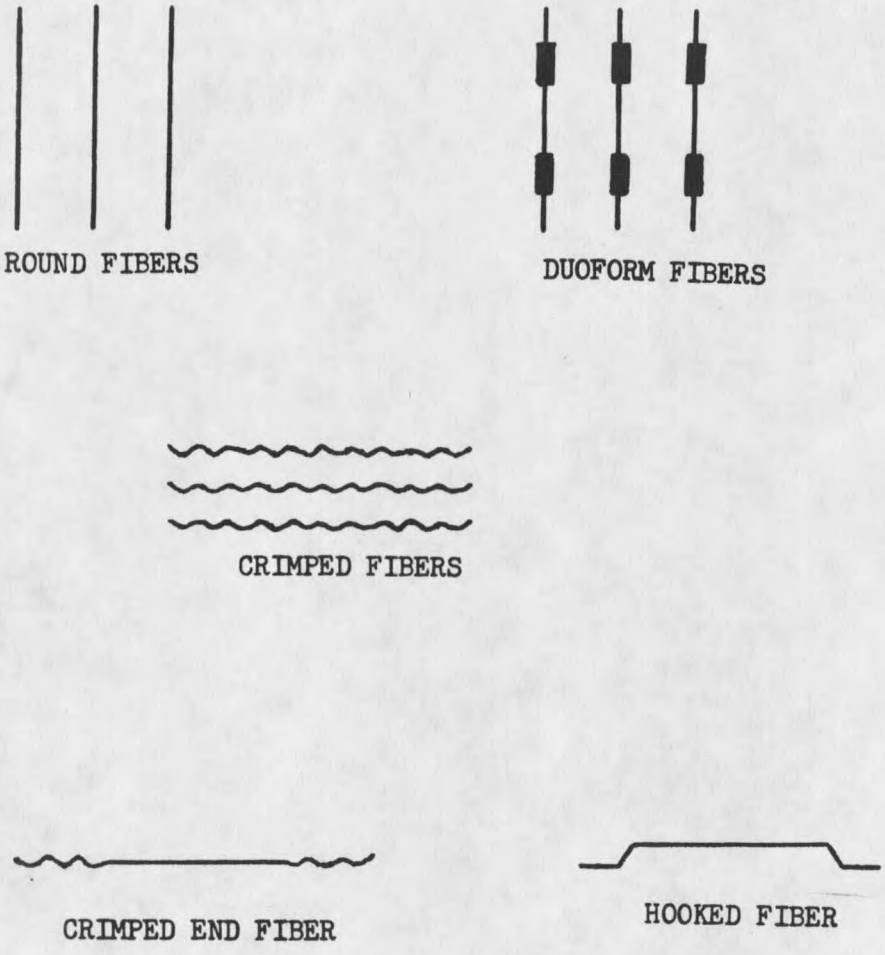



Figure 5. Different Types of Fibers.

A convenient numerical parameter that is used to describe a fiber is its aspect ratio. The aspect ratio is defined as the fiber length divided by an equivalent fiber diameter. Typical aspect ratios range from about 30 to 150 for length dimensions of 0.25 to 3 in. Aspect ratios greater than those stated above tend to cause balling of the fibers. These balls not only prevent the involved fibers from being uniformly distributed as reinforcement to other parts of the member, they also cause bleed holes and openings in freshly finished surfaces (11). In hardened concrete the "balls" act as stress risers due to the inhomogeneity their existence creates in the area. The critical length for bond development in most cases exceeds the maximum fiber length that can be mixed effectively with concrete. This causes the fiber-matrix bond to fail before the fiber has yielded, and, therefore, the strength of the fiber is not fully utilized. The typical tensile strength of steel fibers ranges from 40 to 400 Ksi, with an elastic modulus of approximately 29,000 Ksi.

Several ways have been investigated to improve the fiber-matrix bond in fiber concrete (12, 13). These include special surface coatings, surface indentations, e.g. crimped fibers, or the use of collated fibers. Collated fibers are a group of 10 to 30 fibers with hooked ends, glued together side by side with water-soluble adhesive. The bundling of the fibers creates a low aspect ratio when introduced to the mix. When the glue is dissolved by the water in the mix, the fibers separate as individual fibers with a higher aspect ratio, which is required for developing higher strength.

Since the concrete matrix has a lower failure strain than the steel fibers, it is the first to crack, leaving the fibers to span the crack and carry the load originally sustained by the matrix. For an effective reinforcement of the concrete matrix, the fibers must be able to carry the load sustained by the matrix before it cracked. The volume fraction of fibers necessary for this is called the critical volume fraction. For discontinuous steel fibers, the critical volume fraction is approximately 0.80% for 2-D random fibers and 1.55% for 3-D random fibers (14). In order to achieve a uniform distribution of the fibers and to avoid "balling" or segregation of the fibers in the fiber-concrete composite, the fiber content is generally limited to a maximum value of 4%, unless mixing techniques other than the conventional are used (11). Stated in reference 10 is what might be considered as the conventional mixing techniques for steel fiber concrete.

Other factors that contribute to an effective reinforcement of the concrete by the fibers are a high modular ratio and a greater degree of fiber alignment with the stress direction. The modular ratio of the fiber-concrete composite is defined as E (fiber)/E (matrix). To achieve maximum stress transfer to the fibers, the modular ratio of the composite should be as high as possible. Fibers which give a modular ratio that is less than one for a given composite cannot be expected to increase the tensile strength of the matrix (15).

In a concrete matrix reinforced with randomly oriented fibers, only those fibers which are parallel or nearly parallel to the tensile stress trajectories are effective in crack control. A correction must be applied to those fibers that are less effectively oriented in the volume

space. Romualdi et al. (16) proposed a correction in the form of an orientation efficiency factor. The orientation efficiency factor may be defined as the fraction of total volume of fibers which contributes to crack control in the direction of the tensile stress trajectories. For three-dimensional random orientation, in which the fibers can be oriented in any direction with equal probability, the orientation factor of 0.41 is exact (16). For two-dimensional random orientation of fibers, the orientation factor is 0.64.

The role of fibers is essentially to retard the propagation of cracks in a fiber-concrete composite. Since the concrete matrix has a lower failure strain than the steel fibers, it is the concrete matrix which cracks first. Additional concrete displacements in the neighborhood of the crack are restricted by the stiffer steel fibers which deform less, thereby exerting pinching forces at the crack tips. The pinching forces shall exist as long as the steel fibers do not fracture or debond. If the steel fibers do fracture or debond, the cracks will propagate and coalesce into a failure plane until the ultimate composite strength is reached. Final failure occurs when the interfacial shear stress reaches the ultimate bond strength with the fibers pulling out, or by the simultaneous yielding of the fibers and the matrix. The former is a more common occurrence.

In the early 1960's, Romualdi et al. (17, 18) showed analytically and experimentally that for a constant volume of reinforcement the stress required to extend a crack beyond the reinforcement confining it is inversely proportional to the square root of the reinforcement spacing and that the effect is not significant until the reinforcement

spacing is less than about 0.5 in. This assumed that the reinforcement was continuous and arranged parallel to one another and parallel to the direction of major principal stress. Romualdi et al. (16) also showed that similar results may be achieved with short lengths of wire in random orientation, but nearly uniform spacing throughout the concrete. Critical examination of the spacing concept proposed by Romualdi et al. and experimental results by other researchers tend to show that the spacing concept does not accurately predict the first crack strength of fibrous concrete (10). Swamy et al. have suggested a theory based on a dual criterion of crack control and composite mechanics to predict the flexural tensile strength of fiber concrete (19).

## THE TENSILE STRENGTH OF STEEL FIBER REINFORCED CONCRETE

Three major factors affect the transfer of stress from the matrix to fibers of concrete reinforced with short, discontinuous steel fibers that are randomly oriented and uniformly dispersed in the matrix. These are:

1. The critical fiber length or transfer length.
2. The fiber efficiency or orientation factor of random fibers.
3. The fiber-matrix interfacial bond.

The critical fiber length and interfacial bond are interdependent and related to the fiber aspect ratio.

The reinforcing action of fibers in a fiber-concrete composite occurs through the fiber-matrix interfacial bond stress. Consider a discontinuous aligned fiber of length, 1, and diameter, d, embedded in the surrounding matrix (Fig. 6). The fiber-concrete matrix is loaded to the point of failure by a certain uniaxial stress. The load is transferred from the matrix to the fiber by an average interfacial shear stress, $\tau$. For a long fiber whose length, 1, is greater than or equal to the critical length $1_c$, the longitudinal tensile stress in the fiber $\sigma_f$ varies from zero at the ends of the fibers to the fracture stress of $\sigma_{fu}$ (Fig. 7). For a short fiber with $1 < 1_c$, the longitudinal tensile stress in the fiber may not reach the value for fiber fracture (Fig. 7).

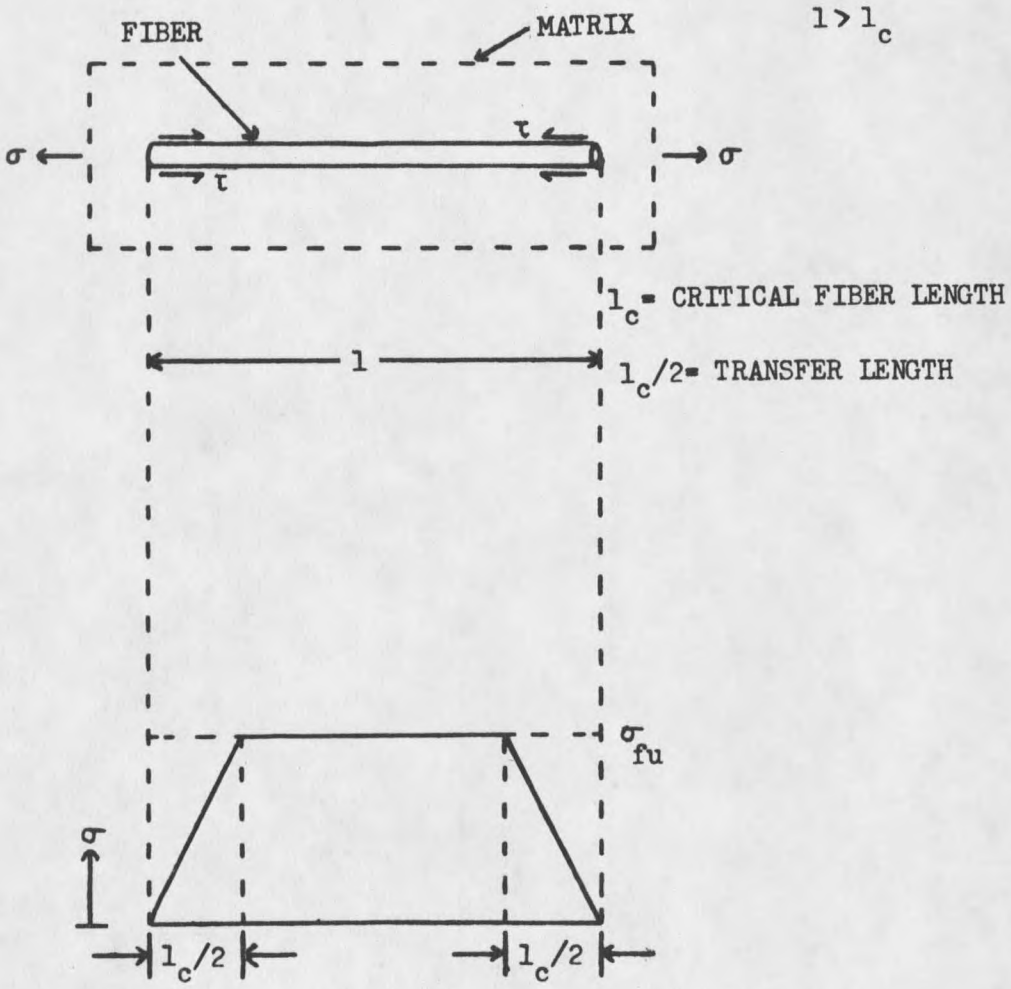



Figure 6. Usually Assumed Fiber Stress Distribution (Ref. 20).

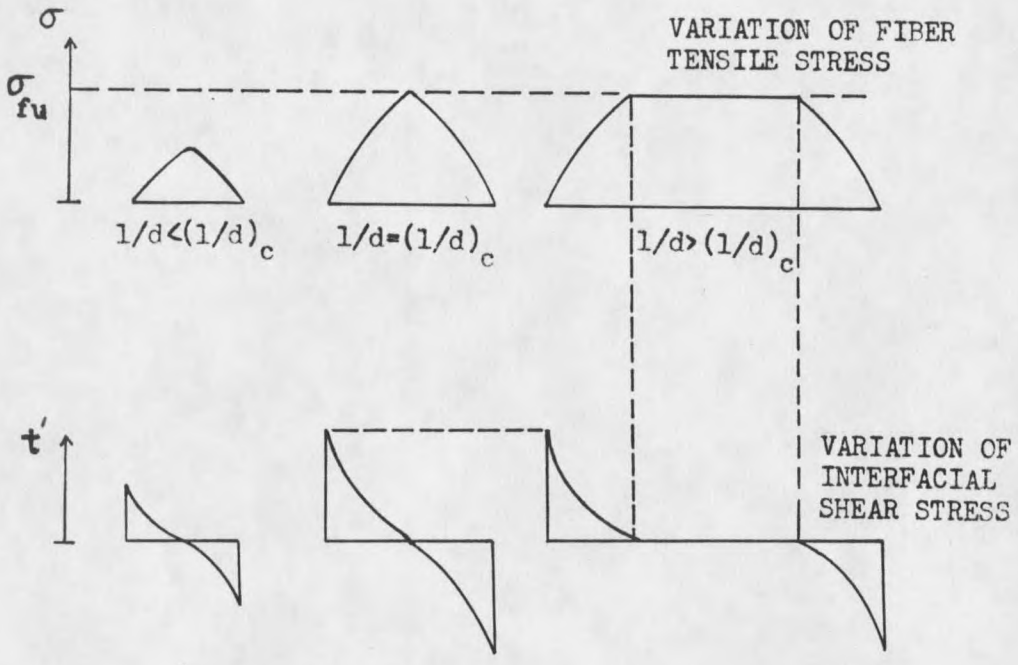



Figure 7. Schematic Variation of Interfacial Shear Stress and Fiber Tensile Stress (Ref. 20).

Therefore, pull out or sliding of the fiber at the fiber-concrete interface would be the form of failure.

The transfer length is defined as half the critical length, $l_c/2$. It is assumed that the fiber is of uniform diameter, d, and the stress is transferred uniformly between matrix and fiber. Now, the cross-sectional area of the fiber is:

$$\pi d^2/4$$

And the surface area of the fiber over the length, dl, is:

$$(\pi d)(dl)$$

If $d\sigma$ is the stress transferred in the distance dl, then from equilibrium conditions,

$$\frac{\pi d^2}{4} d\sigma = (\pi d\tau)dl$$

Which implies that:

$$\frac{d\sigma}{dl} = \frac{4\tau}{d} \tag{6}$$

From equation (6), the tensile stress, $\sigma_f$, in the fiber at the crack surface at pull-out is:

$$\sigma_f = \frac{4\tau Z}{d}$$

where Z is the length of the fiber being pulled out of the matrix. If Z is at least equal to half the critical length, $l_c/2$, then the stress in the fiber will become equal to $\sigma_{fu}$, the fiber fracture strength, and thus

$$\sigma_{fu} = \frac{4\tau}{d} \left(\frac{l_c}{2}\right) = \frac{2\tau l_c}{d} \tag{7}$$

From equation (7) it follows that the aspect ratio of the fiber must be

$$\frac{l_c}{d} \geq \frac{\sigma_{fu}}{2\tau} \tag{8}$$

for fiber fracture to occur.

If bond failure occurs, the frictional force per unit area between matrix and fiber may be adequate to impart a measure of ductility and crack resistance to the composite.

The interfacial bond stress between the steel fibers and matrix in a fiber reinforced composite may be considered to consist of two types (20):

1. The interfacial bond stress due to load transfer from matrix to fiber (Fig. 7).
2. The interfacial bond stress due to the presence of a crack (Fig. 8).

In case 1, the tensile stress is not constant along the length of the fiber (Fig. 7). It is zero at the ends and builds up towards the center of the fiber. Thus, in considering the strength of a discontinuous fiber composite, the average stress in the fiber must be used instead of the maximum stress. The average fiber stress over the length $l_c$ is $\sigma_{fu}/2$, if linear distribution is assumed (Fig. 7). However, if the fibers are much longer than $l_c$, the average stress will approach the maximum, $\sigma_{fu}$, and the fibers will effectively behave as continuous fibers. The shear stress is maximum at the fiber ends and decays to zero at a distance $l_c/2$ from the ends (Fig. 7). For long fibers with lengths greater than the critical length, the central zone of the fiber

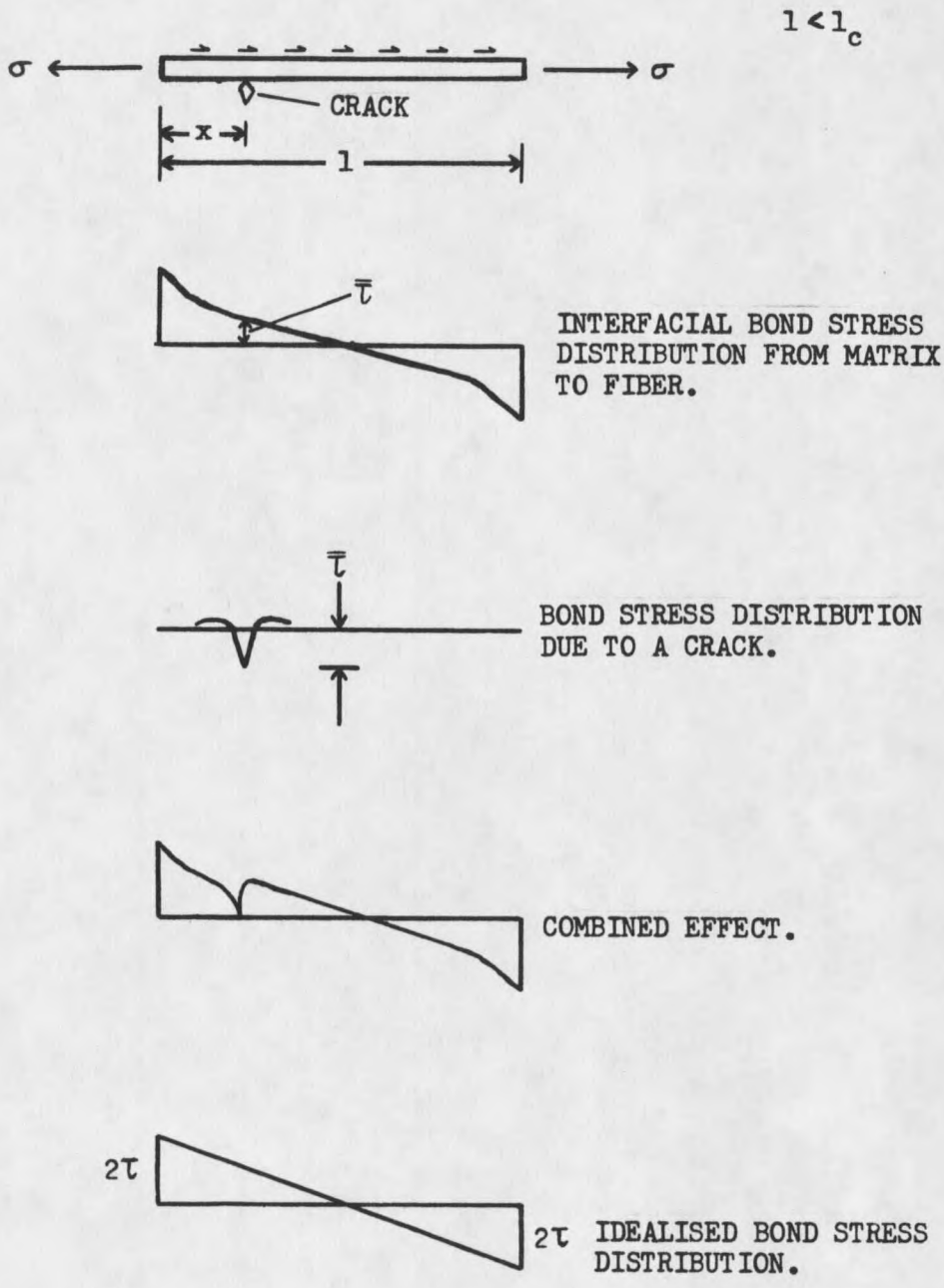



Figure 8. Schematic Bond Stress Distribution at Fiber Matrix Interface due to the Presence of a Crack (Ref. 20).

is free from shear stresses. For short fibers with $l < l_c$, the bond stress is present throughout the length of the fiber.

In case 2, the bond stress is zero at the plane of the crack and increases to a maximum a certain distance away from the crack. It then decreases with increasing distance from the crack edge (Fig. 8). A crack, therefore, disrupts the continuity of the bond stress distribution. This effect is more pronounced in short fibers. For $l > l_c$, the interfacial bond stress distribution due to the presence of a crack is the same as in Figure 7.

Two other main factors that affect the strength of fiber reinforced concrete are fiber length and fiber-fiber interaction. The strength of fiber-concrete composites with short discontinuous aligned fibers is lower than those with long continuous aligned fibers due to the difference in fiber stress distribution. For discontinuous fibers, the stress in the fiber is not constant along the entire length of fiber. Therefore, for strength computations the average stress in the fiber is used (20). The relation (14) between the average and maximum fiber stress can be written as

$$\sigma_{f,av} = K_L(\sigma_{f_{max}})$$

where $K_L$ is a constant and is equal to the length efficiency factor. Allen (14) has given the following values for $K_L$.

$$l \leq l_c, \qquad K_L = \frac{l}{2l_c} \tag{9}$$

$$l \geq l_c, \qquad K_L = 1 - \frac{l_c}{2l} \tag{10}$$

Equation 10 assumes a linear fiber stress distribution with an average stress (after debonding) of $\sigma_f/2$.

The ends of short discontinuous fibers in a fiber-concrete composite provide discontinuities which may create stress concentrations. The tensile stress normally assumed by the fiber without the discontinuity must be taken up by the surrounding fibers. This effect is called fiber-fiber interaction (21). Riley's theory predicts that discontinuous fibers can contribute a maximum of only 6/7 of their strength to the strength of the composite when fiber-fiber interaction is in effect (21).

Fiber reinforced concrete may fail in several different ways. Some of the main failure modes are: fiber pull-out, fiber failure in tension and matrix failure in tension. Fiber pull-out usually occurs when the fiber length is less than the critical fiber length, $l_c$. $l_c$ may be obtained from equation (8) as:

$$l_c = \frac{\sigma_f d}{2\tau} \tag{11}$$

In fiber failures of fiber-concrete composites, failure usually occurs in the composite at the instant of fiber fracture. Also, the failure strain of the matrix is greater than the failure strain of the fiber. In matrix failures, failure usually occurs in the composite when the failure strain of the matrix is reached.

## Description of Model

Concrete matrices, in general, have relatively high compressive strength, poor tensile strength, and brittleness at failure. The

addition of steel fibers to concrete may be used to enhance its tensile properties, delay cracking, and increase toughness. Specific applications are limited mainly by a lack of standardized procedures and equipment and insufficient information for assessing design properties. With standardized procedures and equipment, results can be effectively evaluated and compared. Methods for the prediction of fiber reinforced concrete strength have ranged from the application of linear-elastic fracture mechanics to a composite material approach which employs the law of mixtures of composite materials (10). The linear-elastic fracture mechanics approach may be used to evaluate the crack arrest action of the fiber reinforcement. This approach relates the strength of the composite to the spacing of the fiber reinforcement. The composite materials approach relates the strength of the composite to the volume, orientation and aspect ratio of the fibers. The latter approach is simpler and more popular than the former, and it is used in this thesis.

The following major assumptions are made in the application of the theory of the law of mixtures to fiber-concrete composites.

1. It predicts the causal effects of given dominant parameters, e.g. the volume fraction and aspect ratio of the fibers, and the bond and tensile strength of the matrix on the composite strength.
2. The fibers are short, discontinuous, uniformly distributed and randomly oriented in the matrix.
3. Failure of the fiber-concrete composite occurs by fiber pull-out.

4. Before failure, the fibers are fully bonded to the matrix, i.e., equal strains in fiber and matrix.

5. The Poisson's ratio in the fiber and matrix = 0.

The following is a derivation of the law of mixtures as applicable to fiber reinforced concrete (14).

Given a section of the fiber-concrete composite (Fig. 9), let the cross-sectional area of the composite,

$$A_c = 1 \tag{12}$$

Let the volume of the composite,

$$V_c = 1 \tag{13}$$

This assumes the section of the composite is of unit length. Now,

$$A_m = (A_c - A_f)$$

$$= (1 - A_f) \text{ since } A_c = 1 \tag{14}$$

Similarly, $$V_m = (1 - V_f) \tag{15}$$

If the section of the composite is subjected to a uniaxial tensile force, F, then from equilibrium considerations,

$$F = \sigma_c A_c = \sigma_f A_f + \sigma_m A_m \tag{16}$$

Substituting equations (12) and (14) into equation (16) gives:

$$\sigma_c = \sigma_f A_f + \sigma_m (1 - A_f) \tag{17}$$

In fiber technology, the amount of reinforcing fibers used in a composite is measured in volume fractions, $V_f$, not in area fractions, $A_f$. For a unit length of the composite and fibers,

$$V_c = A_c = 1, \text{ and } V_f = A_f$$

Therefore, $$\sigma_c = \sigma_f V_f + \sigma_m (1 - V_f) \tag{18}$$

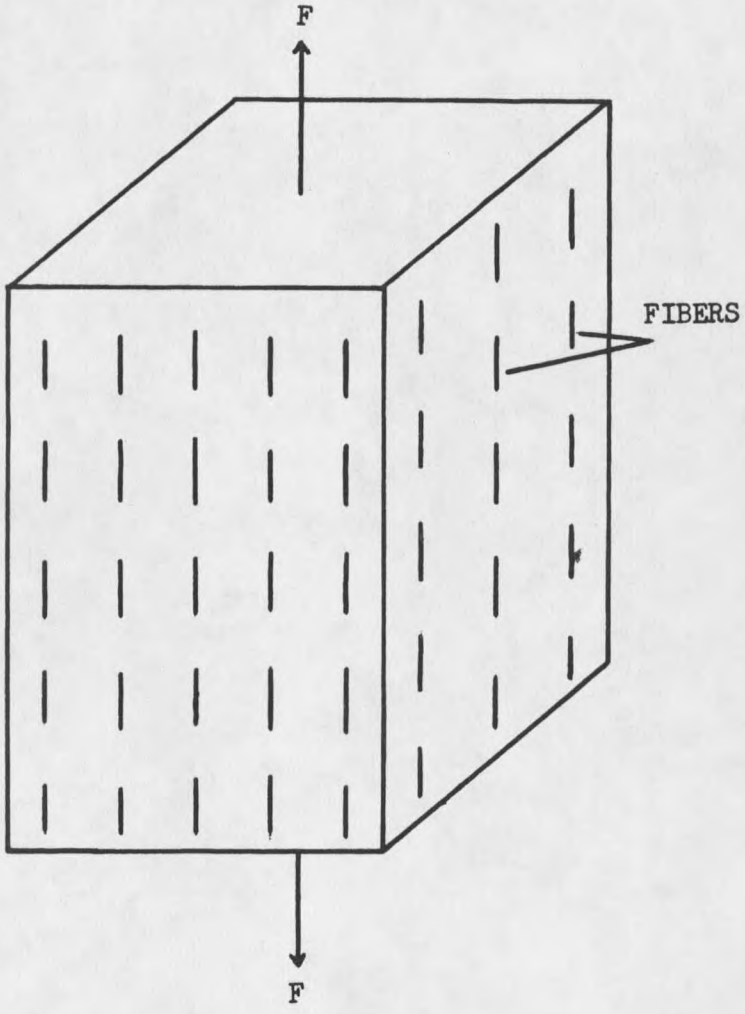



Figure 9. Section of Fiber-Concrete Composite Subjected to Uniaxial Tension.

Equation (18) assumes that the fibers are uniaxial and continuous, so it has to be modified for short, discontinuous, uniformly distributed and randomly oriented fibers. Also, at failure of such a composite, fiber pull-out invariably occurs since the effective length of each fiber in the direction of stress is less than the critical length, $l_c$. Therefore, the tensile strength of the fibers is not fully utilized and a correction has to be made for this effect. The above-mentioned effects could be accounted for if $V_f$ is replaced by an equivalent volume of fibers which are effective in the direction of stress and if $\sigma_f$ is considered to be the stress in the fiber caused by the fiber-matrix interfacial bond stress.

Figure 7, page 25, shows the assumed interfacial bond stress and fiber tensile stress distribution along the length of a fiber whose length is less than the critical length. It may be seen that the bond stress distribution is essentially linear except for the ends of the fiber. To simplify calculations, it is further assumed that the bond stress distribution is linear throughout the length of the fiber. Based on this assumption, it can be deduced that the average bond stress is half of the maximum bond stress, and the average tensile stress in the fiber may be obtained from equation (11) as

$$\sigma_f = 2\tau l/d \qquad (19)$$

The effective length and volume fraction of randomly oriented, uniformly distributed, short discontinuous fibers in the direction of the applied stress are equal to 0.41 l and 0.41 $V_f$ respectively (16). Hence, substituting the average tensile stress in the fiber (Equation 19), the

effective length of the fibers and the effective volume fraction of the fibers in equation (18) gives:

$$\sigma_c = \sigma_m(1 - V_f) + (2\tau 1/d)(V_f)(0.41)^2$$

$$= \sigma_m(1 - V_f) + 0.34\tau\ V_f 1/d \qquad (20)$$

The interfacial bond stress, $\tau$, may be mildly influenced by the composition and strength properties of the matrix, the curing regime of the specimen, chemical treatment (coating) of the fibers, and the geometric configuration of the fibers. However, the influence of these parameters on the bond stress are not very significant for mixes likely to be met in construction practice (19). Therefore, $\tau$ is regarded as a constant with a value in the range between 290.0 psi and 870.0 psi (22). This range may be different depending on the method used to obtain $\tau$.

Equation (20) is modified to the following equation below, to be used to predict the splitting tensile strength of fiber reinforced concrete cylindrical specimens.

$$\sigma_c = A\sigma_m(1 - V_f) + BV_f 1/d \qquad (21)$$

A and B in equation (21) are constants which shall be determined from a plot of the experimental results. A statistical analysis of the plot shall test the validity of equation (21). Constant A reflects the contribution of the matrix to the composite strength and the interfacial bond stress. Constant B depends on the bond strength between the fibers and the matrix and on the randomness of the fibers. The higher the bond strength and the better aligned the fibers are in the direction of the load, the higher the constant B. Constant B has units of stress.

Experimental Data and Discussion

Constants A and B (above-mentioned) and the validity of equation (21) were established by the test results of three independent investigators, all of whom were not concerned with the present problem but provided suitable data nonetheless. These investigators were: Shah and Naaman (23); Ghalib (24); and LaFraugh and Moustafa (25), and their tests shall be referred to as Test 1, Test 2 and Test 3 respectively. The above-mentioned investigators performed split cylinder tensile strength tests on fiber-concrete cylinder specimens, with different volume fractions and aspect ratios of fibers. Some of the characteristics of Tests 1, 2 and 3 are tabulated in Table 1.

Straight, smooth, rectangular steel fibers were used in Test 1. In Test 2, duoform fibers made of low carbon steel were used. The fineness modulus of the fine aggregate was 2.33 and type I cement was used. In Test 3, type III cement with 3/8 and 3/4 inch maximum size coarse aggregates were used for the cylinder specimens. The fineness modulus of the fine aggregates was 3.14 and the coarse aggregate content varied from 0 to 60% of the total aggregate weight. The two cement contents used were 470 Pcy. and 658 Pcy.

To obtain the values of the constants A and B in equation (21), equation (21) is modified to the form below:

$$\frac{\sigma_c}{V_f l/d} = A \cdot \frac{\sigma_m (1 - V_f)}{V_f l/d} + B \qquad (22)$$

The plot of $\frac{\sigma_c}{V_f l/d}$ versus $\frac{\sigma_m (1 - V_f)}{V_f l/d}$ should yield a straight line whose

Table 1. Some Characteristics of Tests 1, 2 and 3 (Ref. 23, 24, 25).

| Test | Mix Proportions | W/C Ratio | Matrix Comp. Strength, psi | Matrix Split Tensile Strength, psi | $V_f$ | l, in. | d, in. | Number of Tests |
|---|---|---|---|---|---|---|---|---|
| 1 | Mortar | Not given | 4020 | 422 | 1-3% | 1/2-1 | 0.0167[a] | 9 |
| 2 | 1:1.5:2[b] | 0.45 | 5206 | 474 | 1 & 1.5% | 0.71-2.3 | 0.0098-0.0256 | 8 |
| 3 | Not given | 0.45-0.63 | 4000 & 5230[c] | 485 & 525 | 0.5 -1.5% | 1 & 2.5 | 0.016 - 0.027 | 24[d] |

a Equivalent Diameter of Fiber.

b Ratio refers to cement:sand:water ratio.

c 22 Hours Compressive Strength (60% of 28 day $f'_c$).

d 4x8 in. cylinders used.

slope is equal to constant A and whose y-intercept is equal to constant B, if equation (21) is valid.

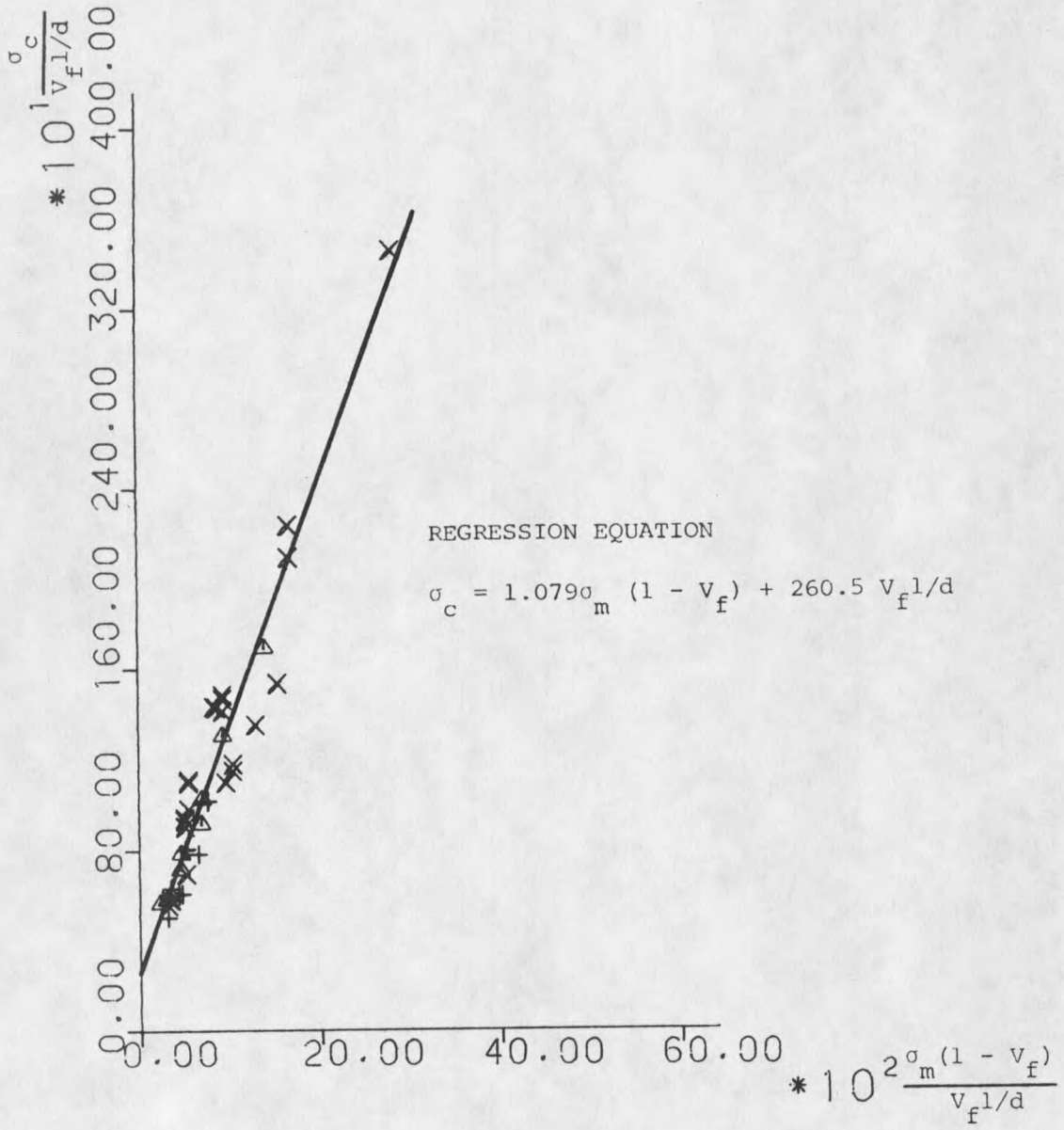
Figure 10 shows the plot of $\frac{\sigma_c}{V_f l/d}$ versus $\frac{\sigma_m(1 - V_f)}{V_f l/d}$, where the values of $\sigma_c$, the split tensile strength of the fiber reinforced cylinder specimens; $V_f$, the volume fraction of fibers in the specimen; $l/d$, the aspect ratio of the fibers; and $\sigma_m$, the split tensile strength of the unreinforced matrix, were obtained from the test results of the above-mentioned investigators. The distribution of the data points suggested that a straight line would be the best fit. The best fit straight line was obtained by a linear regression analysis and its equation is

$$\frac{\sigma_c}{V_f l/d} = 1.079 \frac{\sigma_m(1 - V_f)}{V_f l/d} + 260.5 \text{ (Units = psi)} \qquad (23)$$

Therefore, $\sigma_c = 1.079\ \sigma_m(1 - V_f) + 260.5\ V_f l/d$ (24)

where $\sigma_m$ is in psi, and constants A and B are 1.079 and 260.5 respectively. The coefficient of determination for the regression analysis was 0.922. This implies that 92.2% of the variation in the values of "Y" is accounted for by a linear relationship with "X". Equation (24) does not apply when $V_f = 0$. If constant A is greater than or equal to one, it implies that the matrix reached its ultimate strength. In theory, constant A is supposed to be 1, since it is assumed that the matrix reached its ultimate strength but the test results suggest otherwise. This is possibly due to the variability of concrete.

Tabulated in Table 2 is the ratio of $\sigma_c$ (predicted by equation 24) to $\sigma_c$ (obtained from test results). The average and standard




THE DATA ABOVE WERE OBTAINED FROM THE FOLLOWING THREE SOURCES:

| | |
|---|---|
| Δ | REF. 23 |
| + | REF. 24 |
| X | REF. 25 |

Figure 10. The plot of $\frac{\sigma_c}{V_f 1/d}$ versus $\frac{\sigma_m(1 - V_f)}{V_f 1/d}$.

Table 2. Comparison of Test Values with Values Predicted by Equation (24).

| $\sigma_c$ (Test), psi | $\sigma_c$ (Predicted), psi | $\frac{\sigma_c \text{ (Predicted)}}{\sigma_c \text{ (Test)}}$ |
|---|---|---|
| 506 | 529 | 1.05 |
| 549 | 602 | 1.10 |
| 643 | 676 | 1.05 |
| 586 | 568 | 0.97 |
| 705 | 680 | 0.96 |
| 787 | 793 | 1.01 |
| 614 | 607 | 0.99 |
| 722 | 759 | 1.05 |
| 1023 | 911 | 0.89 |
| 626 | 667 | 1.07 |
| 740 | 744 | 1.01 |
| 564 | 694 | 1.23 |
| 655 | 785 | 1.20 |
| 719 | 739 | 1.03 |
| 847 | 853 | 1.01 |
| 608 | 767 | 1.26 |
| 754 | 895 | 1.19 |
| 1040 | 802 | 0.77 |
| 590 | 694 | 1.18 |
| 830 | 703 | 0.85 |
| 640 | 612 | 0.96 |
| 920 | 802 | 0.87 |
| 800 | 703 | 0.88 |
| 1030 | 802 | 0.78 |
| 700 | 645 | 0.92 |
| 655 | 645 | 0.98 |
| 875 | 760 | 0.87 |
| 850 | 760 | 0.89 |
| 885 | 760 | 0.86 |
| 480 | 602 | 1.25 |
| 835 | 906 | 1.09 |
| 550 | 651 | 1.18 |
| 870 | 949 | 1.09 |
| 870 | 949 | 1.09 |
| 575 | 694 | 1.21 |
| 875 | 949 | 1.08 |
| 695 | 821 | 1.18 |

Table 2 (Continued)

| $\sigma_c$ (Test), psi | $\sigma_c$ (Predicted), psi | $\frac{\sigma_c \text{ (Predicted)}}{\sigma_c \text{ (Test)}}$ |
|---|---|---|
| 825 | 703 | 0.85 |
| 895 | 724 | 0.81 |
| 795 | 660 | 0.83 |
| 500 | 614 | 1.23 |

Average of $\frac{\sigma_c \text{ (Predicted)}}{\sigma_c \text{ (Test)}} = 1.019.$

Standard Deviation of $\frac{\sigma_c \text{ (Predicted)}}{\sigma_c \text{ (Test)}} = 0.142.$

deviation of these ratios are 1.019 and 0.142 respectively. Assuming that the ratio follows a normal distribution, then the percentage of the ratio that exceeds 1 is equal to 55.3%, which is large. If equation (24) is multiplied by a factor of 0.82, then the percentage of the ratio that exceeds 1 is equal to 7.9% which is a reasonable value. Therefore, a strength reduction factor of 0.82 should be applied to equation (24) unless the method of predicting the split tensile strength of fiber reinforced concrete is further refined by research. An alternative to the empirical equation for obtaining the split tensile strength of fiber reinforced concrete to be used in design is to perform tests and plot a design curve from the test results. The design curve may be either $\sigma_c$ versus $V_f l/d$ or $\frac{\sigma_c}{V_f l/d}$ versus $\frac{\sigma_m(1 - V_f)}{V_f l/d}$. From either of these curves, the value of $\sigma_c$ may be obtained for a given value of $V_f$, l and d.

Though this method might be safer, it is too lengthy to be practical for design application.

Other methods of obtaining the tensile strength of fiber reinforced concrete are: the split cube test, the modulus of rupture test and direct tensile test on prism specimens, which may be straight or shaped like a dog's bone. As of yet, there is no known standard specimen or test for the tensile strength of fiber reinforced concrete. In this thesis, the split cylinder tensile test method was used because there has been more research in the use of this method to predict the tensile strength of fiber reinforced concrete than any of the above-mentioned methods.

# DEVELOPMENT OF A TENTATIVE SHEAR DESIGN METHOD

In thin flat plates, the use of conventional shear and flexural reinforcements may cause excessive congestion of the reinforcement and difficulty in placing the concrete. Also, for very thin members, the conventional reinforcement may be impossible to place without violating minimum bar bend diameters, cover requirements and other geometric constraints. This problem has led several researchers to investigate a different alternative, which is the use of steel fibers as a possible means of shear reinforcement in slabs (26, 27, 28, 29). The above-mentioned investigators carried out a limited amount of tests on isolated simply supported specimens, assumed to represent a portion of a flat plate floor system near a column head. These tests were done to examine the resistance of fiber concrete flat plates to punching shear. Below is a summarized review of some of their findings.

## Review of Current Research

Swamy et al. (26, 27) tested 19 slab-column connections, 4 of which are of primary concern in this thesis. The size of the slab-column test specimens were such as to be located within the negative moment region around the interior column, and inside the line of contraflexure of a typical prototype flat plate. The concrete cube compressive strength was approximately 6.5 Ksi and the yield strength of the flexural reinforcement was approximately 67 Ksi. The test showed that the

presence of steel fibers resulted in substantial reductions in the deflection, flexural steel strain, concrete strain, and the rotation at the compressive face, compared to those of a plain concrete slab under equivalent loads. A fiber volume of about 1% showed an increase in ductility and energy absorption of about 100 and 300%, respectively. Ductility in this case is quantified in terms of the ratio of the deflection at 25% of the maximum load (after reaching maximum load) to the deflection at the first crack (as visually observed). The addition of fibers to the slabs showed a gradual and ductile punching failure, in contrast to the sudden punching shear failure shown by the plain concrete slab. The addition of fibers changed the failure surface from nearly square for the plain concrete connection to nearly circular and elliptical shapes as the fiber volume was increased to 1.2%. Conventional shear reinforcement (bent up bars in this case) was observed to be more effective than an equal volume of fiber reinforcement in resisting shear. It was noted that the number of cracks in the slabs with fibers was greater and much finer than the cracks in the plain concrete slab. As the fiber volume in the slab changed from 0.6% to 1.2%, the maximum punching shear load increased from 23 to 42% above that of a plain concrete slab. Providing fibers in the vicinity of the column head was observed to be as effective in resisting punching shear as providing fiber reinforcement over the entire slab.

In addition to some of the things mentioned above, Criswell's investigation (28) showed that addition of 1% volume of fibers to slabs with a flexural reinforcement ratio of 1.04% and 1.88% increases the punching strength by 27 and 21% respectively. Criswell tested four

slabs made of concrete with a matrix compressive strength of 6.31 Ksi and reinforced with steel of yield strength equal to 56.6 Ksi. The angle of the failure cone for the fiber-concrete slabs was not significantly different from 45°, and the location of the failure surface was at about the same distance from the column face as that of plain concrete slabs (33).

In addition to some of the observations mentioned above, Patel (29) observed that when the compressive flexural reinforcement ($\rho' = 0.32\%$) of a slab is replaced by an equal volume of fibers (volume = 0.575%), the maximum column load is the same for both cases. Results from four slabs with a tensile reinforcement ratio of 0.32% showed a 50% increase in strength when the fiber content was increased from 0.575 to 1.20 volume percent. Addition of fibers was noted to significantly increase the load needed for visible flexural cracking of the slab. Patel tested eight slabs whose concrete compressive strength was 5.01 Ksi.

## Description of Model

An analysis of the results of the above-mentioned investigators show that three major parameters influence the punching shear strength of fiber-concrete flat plates. These are:

1. The split cylinder tensile strength of the fiber-concrete composite.
2. The ratio of the effective depth, d, of the slab to the side length, r, of the loaded area.
3. The relative yield force of the flexural reinforcement, $\rho f_y$.

An increase in any of the above-mentioned parameters leads to an increase in the punching shear strength of a fiber-concrete flat plate. Note that while an increase in $\rho f_y$ increases the shear strength of slabs, the addition of extra flexural reinforcement for the sole purpose of increasing shear strength is an inefficient design. The addition of fibers to concrete enhances its tensile strength, but has little or no effect on its compressive strength. Also, the shear failure of a flat plate is primarily a tension failure. These two effects make the split tensile strength of fiber-concrete a more appropriate variable to predict the shear strength of fiber-concrete flat plates than the compressive strength (or any power of it) of the fiber-concrete.

After cracking of a fiber-concrete flat plate due to an applied load, the fiber reinforcement reduces the stress concentration at the tips of these shear cracks, thereby controlling their propagation. This action enhances the shear strength of concrete flat plates and maintains their structural integrity.

The method proposed in this thesis for the prediction of the shear strength of fiber-concrete flat plates shall be based on the following major assumptions:

1. Failure occurs when the principle tensile stresses produced by the shear force exceed the tensile strength of the fiber concrete.
2. The ratio of the sides of the column should not exceed two.
3. The shear carried by the compression zone of the slab is negligible.

4. Tensile flexural reinforcement affects the ultimate punching load of the slab.

5. The ratio of d/r is greater than or equal to 0.33.

As noted previously, Criswell (private communication) and Patel (29) observed that the addition of fibers to the concrete flat plates tested by them did not significantly change the location of the punching shear failure surface from the column face nor the angle the failure surface makes with the horizontal. For this reason, the pseudo-critical section for the nominal shear strength of a fiber-concrete flat plate is assumed to be located at a distance of d/2 from the periphery of the column. And the angle of inclination of the failure surface is assumed to be 45° with the horizontal (Fig. 11).

The following is the development of a tentative empirical method to predict the punching shear strength of fiber-concrete flat plates which follow the assumptions stated above.

Let $\sigma_s$ in Figure 11 be the principle tensile stress on the assumed failure surface of the plate. Then, the resultant of the diagonal tension forces acting on the surface of the failure cone at the moment of failure, $F_s$, is given by:

$$F_s = \sigma_s \int dA \tag{25}$$

Let $\sigma_s = K\sigma_c$, where K is a constant that relates the principle tensile stress on the failure surface of the plate to the split cylinder tensile strength of fiber-concrete. Therefore,

$$F_s = K\sigma_c \int dA \tag{26}$$

The vertical component of $F_s$ must be in equilibrium with the shear force $V_u$ (see Fig. 11) acting on the column. Therefore,

$$V_u = F_s \text{ Cos } \alpha \tag{27}$$

where $\alpha$ = the angle of the failure cone with respect to the horizontal. $\alpha$ is assumed to be 45°. Substituting equation (26) into equation (27) gives

$$V_u = K\sigma_c \text{ Cos } \alpha \int dA \tag{28}$$

where Cos $\alpha \int dA$ = area of the horizontal projection of the concrete failure surface (see Fig. 11).

$$\int dA = \frac{\pi}{4}\left[(r\sqrt{2} + 2d/\text{Tan } \alpha)^2 - (r\sqrt{2})^2\right] \tag{29}$$

Therefore, substituting into equation (28) and reducing terms gives

$$V_u = \sigma_c \text{ rd } [3.14K + 2.22K \text{ (d/r)}] \tag{30}$$

Now, b = 4r. Therefore, equation (30) becomes

$$V_u = \sigma_c \text{ bd } [0.785K + 0.555K \text{ (d/r)}] \tag{31}$$

So,

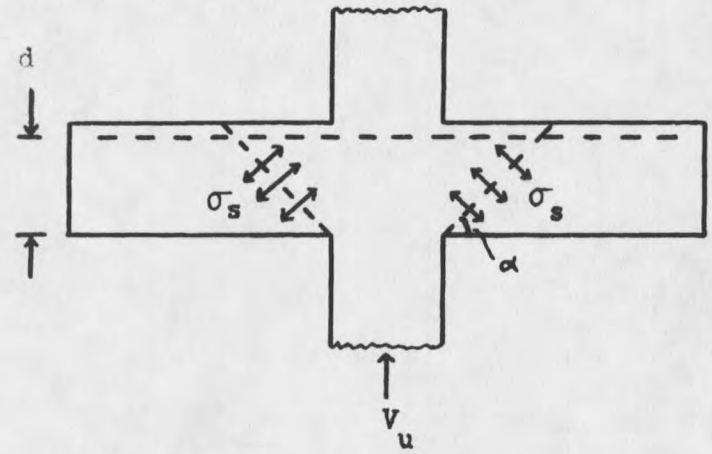
$$v_u = V_u/(bd) = [0.785K + 0.555K \text{ (d/r)}] \sigma_c \tag{32}$$

If the periphery of the critical section, $b_o$, for punching shear is assumed to be located at a distance d/2 from the periphery of the column, then equation (32) may be modified to the following

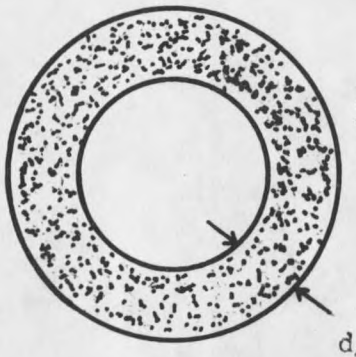
$$v_u = V_u/(b_o d) = [C' + D' \text{ (d/r)}] \sigma_c \tag{33}$$

where C' and D' are constants.

Several investigators (28, 30, 31, 32) have observed that the punching shear strength of flat plates is affected by the flexural




ELEVATION VIEW




TOP VIEW

Figure 11. Assumed Mode of Punching Failure of a Fiber-Concrete Flat Plate Connection.

reinforcement of the plate. Taking this into account, equation (33) is modified to the following

$$v_u = V_u/(b_o d) = [C\rho f_y + D\ (d/r)]\ \sigma_c \qquad (34)$$

where C and D are constants to be determined from a plot of the test results of Swamy et al. (26, 27) and Criswell (28). A statistical analysis of the plot shall test the validity of equation (34). Constant C reflects the contribution of the flexural reinforcement to the shear strength of the plate, and it has units of $(Ksi)^{-1}$ when $f_y$ is in Ksi. Constant D is dimensionless, and it reflects the effect of the d/r ratio on the shear strength of the plate.

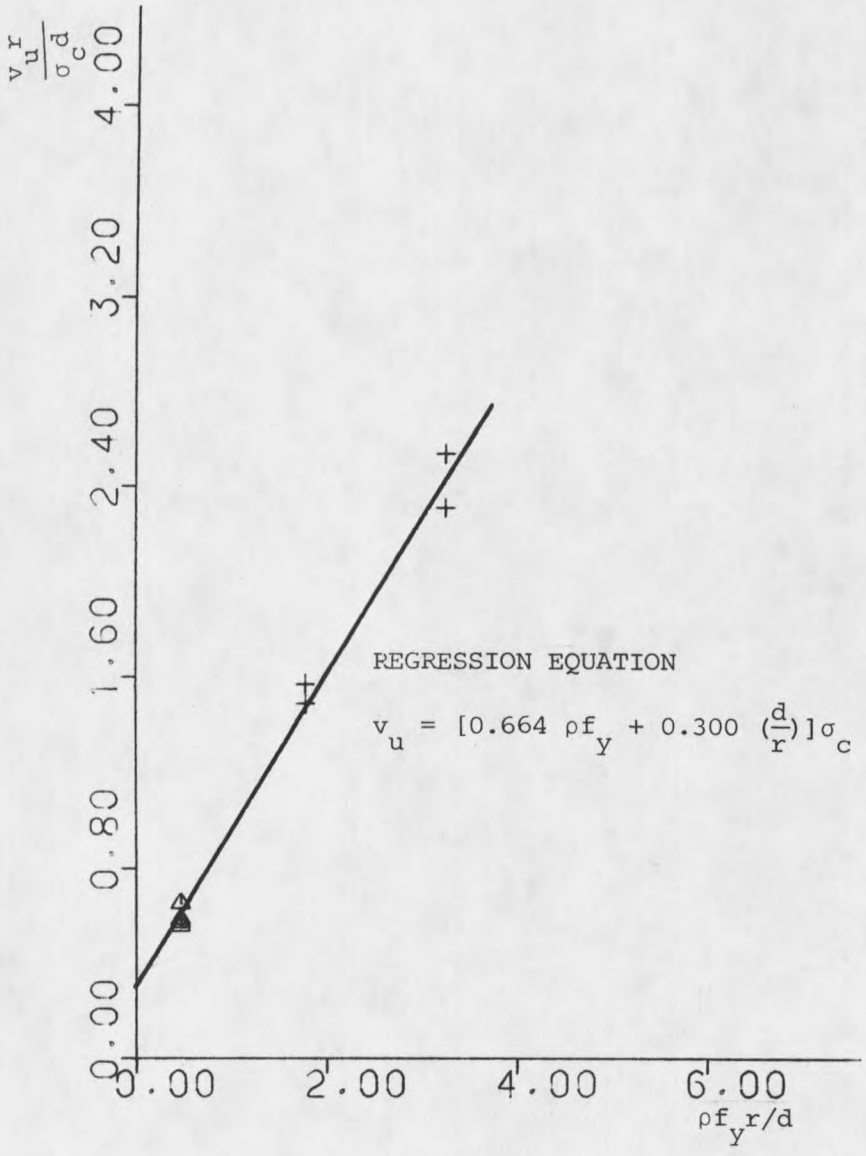
To obtain the values of the constants C and D in equation (34), equation (34) is modified to the form below so that a linear regression analysis may be performed on it.

$$v_u r/(\sigma_c d) = C\rho f_y r/d + D \qquad (35)$$

The plot of $v_u r/(\sigma_c d)$ versus $\rho f_y r/d$ should yield a straight line whose slope is equal to constant C and whose y-intercept is equal to constant D, if equation (35) is valid.

Figure 12 shows the plot of $v_u r/(\sigma_c d)$ versus $\rho f_y r/d$, where the variables in these terms were obtained from the test results of eight fiber-concrete flat plate specimens obtained by Swamy et al. (26, 27) and Criswell (28). The distribution of the data points suggested that a straight line would be the best fit. The best fit straight line was obtained by a linear regression analysis and its equation is

$$v_u r/(\sigma_c d) = 0.664\rho f_y r/d + 0.300$$




THE DATA ABOVE WERE OBTAINED FROM THE FOLLOWING TWO SOURCES:

Δ Ref. 26

\+ Ref. 28

Figure 12. The plot of $v_u r/(\sigma_c d)$ versus $\rho f_y r/d$.

Therefore,

$$v_u = [0.664\rho f_y + 0.300\ (d/r)]\sigma_c \qquad (36)$$

where $f_y$ is in Ksi, and constants C and D are 0.664 and 0.300 respectively. The coefficient of determination for the regression analysis was 0.991. This implies that 99.1% of the variation in the values of "Y" is accounted for by a linear relationship with "X". Equation (36) does not apply when d or r is equal to zero.

Tabulated in Table 3 is the ratio of $v_u$ [predicted by equation (36)] to $v_u$ (obtained from tests results). The average and standard deviation of these ratios are 1.008 and 0.061 respectively. And the coefficient of variance is 6.05%. Assuming that the ratio follows a normal distribution, then the percentage of the ratio that exceeds 1 is equal to 55.2%, which is large. If equation (36) is multiplied by a factor of 0.92, then the percentage of the ratio that exceeds 1 is equal to 10.3%, which is a reasonable value. Therefore, if equation (36) is to be used, a strength reduction factor of 0.92 or less should be applied to it. Other equations made up of various combinations of the terms in equation (36) were also analyzed statistically, but none of them yielded better results than equation (36) in predicting the punching shear strength of fiber-concrete flat plates. However, it must be noted that the statistical analysis was done on a very limited amount of data, therefore further research is needed to refine equation (36). Equation (36) should not be used for design, but as a guideline for further research.

Table 3. Comparison of Test Values with Values Predicted by Equation (36).

| $v_c$ (Test), psi | $v_c$ (Predicted), psi | $\frac{v_c \text{ (Predicted)}}{v_c \text{ (Test)}}$ |
|---|---|---|
| 250.6 | 232.9 | 0.93 |
| 308.8 | 319.4 | 1.03 |
| 333.2 | 353.4 | 1.06 |
| 356.2 | 387.4 | 1.09 |
| 311.1 | 292.6 | 0.94 |
| 395.8 | 390.9 | 0.99 |
| 504.2 | 486.2 | 0.96 |
| 611.1 | 649.6 | 1.06 |

Average of $\frac{v_u \text{ (Predicted)}}{v_u \text{ (Test)}} = 1.008.$

Standard Deviation of $\frac{v_u \text{ (Predicted)}}{v_u \text{ (Test)}} = 0.061.$

# CONCLUSIONS

Steel fiber reinforced concrete technology is currently in the material development and early application stage. And as such, design criteria and standards for the application of steel fiber reinforced concrete are rare. This thesis is directed to finding such criteria for application in the shear design of steel fiber reinforced concrete flat plates and to elicit further research in this seemingly promising area. The advantages of using steel fibers as shear reinforcement in concrete flat plates as an alternative to the conventional methods have been previously mentioned.

Swamy et al. (26) noted that bent up bars are more effective than fibers as shear reinforcement in concrete flat plates, but they are four times more labor intensive than the use of fibers. They also observed that providing fibers in the vicinity of the column head in a concrete flat plate, within the punching shear perimeter, was as effective in ultimate load capacity as providing fiber reinforcement over the entire plate.

Equation (36) was developed primarily from tests on simply-supported fiber reinforced plate-column specimens for which the ultimate shear at collapse was less than or approximately equal to that for yielding of the reinforcement across the full width of the plate, $V_{flex}$. An understanding is needed of fiber-concrete plate behavior where

ductility does develop prior to collapse. That understanding cannot be achieved from the test data.

The test results of Swamy et al. (26) and Criswell (28) were analyzed, and very simple criteria based on the principle tensile strength of the failure cone and the relative yield force of the flexural reinforcement, $\rho f_y$, were used to develop equation (36). Equation (36) showed good agreement with the test results and it may serve as a guideline for further research. The ratio of $(v_u)_{calc}/(V_u)_{test}$ had an average and standard deviation of 1.008 and 0.061 respectively. And the coefficient of variance was 6.05%. The largest $(v_u)_{calc}/(v_u)_{test}$ ratio among the test results was 1.09, and in this case equation (36) overestimated $(v_u)_{test}$ by 8.76%. The lowest $(v_u)_{calc}/(v_u)_{test}$ ratio among the test results was 0.93, and in this case equation (36) underestimated $(v_u)_{test}$ by 7.60%. If the ratio $(v_u)_{calc}/(v_u)_{test}$ is assumed to follow a normal distribution, and if equation (36) is multiplied by an under capacity factor of 0.92, then the percentage of this ratio that exceed 1 is approximately equal to 10.3%. Although these results are quite good for concrete, it should be kept in mind that only a limited number of tests were involved and, therefore, no attempt has been made to generalize the findings. Before doing so, a large number of tests must be carried out with a wider range of variables to give a comprehensive understanding of this problem.

# SUGGESTIONS FOR FURTHER STUDY

The following studies may be pursued for a better understanding of fiber-concrete behavior and to enhance its use as shear reinforcement in flat plates.

1. Study the effects of member size, creep and shrinkage on the shear strength of flat plates.
2. Study the economics and desirability of fiber-concrete.
3. Study the combined use of fiber reinforcement and conventional reinforcement as shear reinforcement in plates.
4. Study ways of improving the fiber-concrete interfacial stress and preventing failure by fiber pull-out.
5. Study ways to improve material property predictions and material test methods of fiber-concrete.

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# APPENDIX

## NOMENCLATURE

1. $K_L$ = length efficiency factor
2. $\sigma_{f,av}$ = average fiber stress
3. $\sigma_{f_{max}}$ = maximum fiber stress
4. $l_c$ = critical length of fiber
5. $l$ = length of fiber
6. $\sigma_f$ = fiber stress
7. $V_f$ = volume fraction of fibers
8. $d$ = diameter or equivalent diameter of fibers
9. $A_m$ = cross-sectional area of matrix
10. $A_c$ = cross-sectional area of composite
11. $A_f$ = area fraction of fibers
12. $V_m$ = volume of matrix
13. $V_c$ = volume of composite
14. $\sigma_c$ = split-tensile strength of composite
15. $\tau$ = interfacial bond stress of fiber-matrix composite
16. $V_{flex}$ = ultimate flexural capacity determined by yield line analysis
17. $\phi_o$ = ratio of ultimate shear capacity to ultimate flexural capacity
18. $V_u$ = ultimate shear capacity
19. $r$ = side length of equivalent square column

20. $\rho'$ = compression flexural reinforcement ratio

21. $\alpha$ = angle of failure cone with respect to the horizontal

22. $f_y$ = yield strength of flexural reinforcement

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