PRESERVICE TEACHERS’ CONSTRUCTION OF COMPUTATIONAL THINKING PRACTICES THROUGH MATHEMATICAL MODELING ACTIVITIES

by

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DEDICATION

This dissertation is dedicated to my late uncle, Adetunji Adebusuyi (1971 – 2008). I miss you so much! I wish you were here to see the manifestations of the beautiful future you wanted for me. Even in your eternal sleep, I believe you are proud of the Adewale I have become.

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ABSTRACT

The importance of learning computational thinking practices in K-12 settings is gaining momentum in the United States and worldwide. As a result, studies have been conducted on integrating these practices in mathematics teaching and learning. However, there is little study that focuses on how to prepare pre-service teachers who will teach the practices in K-12 settings. I investigated how pre-service teachers collaborated to develop computational thinking practices when working on modeling activities with computational tools. To carry out this research, I studied nine pre-service teachers working on modeling tasks for a semester. Five participants recorded their screens and were invited to participate in a stimulated recall interview.

Using the interactional analysis procedures, findings showed that the presence of computational tools influenced the *positioning (leadership and distributed authority) and collaborative processes (dividing and offloading labor, giving and receiving feedback, accommodation, and refining ideas)* pre-service teachers used during modeling. This study found that pre-service teachers used ten computational thinking practices, which are sub-grouped into four broader practices – *data practices, mimicking and mathematizing, model exploration and extension, and model communication.* This dissertation also found that pre-service teachers' mathematical knowledge and their ability to code were interdependent.

From a research point of view, this study extends our knowledge of the social constructivist theory of doing research in the context of pre-service teachers engaging in modeling activities with computational tools. From the teacher education perspective, this study emphasizes the need to consider the impact of computational tools on the interactions of pre-service teachers during modeling. The study also reveals the need to structure the mathematical modeling curriculum to lead to a better learning experience for pre-service teachers.

CHAPTER ONE

THE PROBLEM

Introduction

For more than two decades, educators have discussed the relevance of practices and skillsets from the computer science field in school curricula. Although this development is not new to mathematics educators, it has received a significant reawakening in recent times as more researchers are exploring the integration of computer science practices into the teaching and learning of mathematics (e.g., Cetin, 2016; DeJarneatte, 2019; Lockwood, 2019; Wiedemann et al., 2020). Lockwood and Mørken (2021) classified reasons for incorporating these computer science practices in mathematics into labor market rationale, computational thinking rationale, computational literacy rationale, and the equity of participation rationale. The focus of this study is the computational thinking rationale.

Following early work that involved computational practices (e.g., Perlis, 1962; Papert, 1980, 1996), Wing (2006) reintroduced the idea of computational thinking into K-12 education after little attention to the topic by researchers and educators caused by children's limited computer access, inconclusive findings among researchers about what computing should achieve in mathematics learning, and varied perspectives of the use and impact of early computational tools among educators (Benton et al., 2017). Wing (2006) called for educators to revisit the potential impact of computational practices in learning and stated that everyone (children of all grades) should learn computational thinking. The quest to teach computational practices in the early years of the 21st century propelled mathematics educators into a space of research that would allow investigation into their teaching and learning.

Wing (2014) described computational thinking as “the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer - human or machine – can effectively carry out.” Weintrop and colleagues (2016) argued that Wing’s (2014) definition is central to computer science and stated the need to break computational thinking down into a set of well-defined and measurable skills, concepts, and practices central to the field of science and mathematics. In a sense, their ideas are closely related to defining computational thinking as a form of procedural fluency that learners need to acquire when solving mathematical problems. Their taxonomy of practices focuses on applying computational thinking to mathematics and science and consists of four main categories: data practices, modeling and simulation practices, computational problem practices, and systems thinking practices (Weintrop et al., 2016). Many of these practices associated with computational thinking have been identified to relate to the mathematical modeling practices and mathematical thinking practices (CCSSM, 2010, more on this in Chapter two). In this study, computational thinking practices refer to actions pre-service teachers take when using computational tools to carry out the modeling processes.

Background

The inclusion of computational thinking practices in science and mathematics has widely been received among different policymakers (e.g., Common Core guidelines, 2010; National Research Council, 2010; and NGSS, 2013), researchers (e.g., Wing, 2006; Weintrop et al., 2016; Lockwood, 2019, Yadav et al., 2017), technological organizations (e.g., Google), and individuals (e.g., Wolfram - TED Talk, 2010) among others. For instance, the authors of Next Generation Science Standards (2013) emphasized the growing importance of computation and digital technologies across the science disciplines and thereby suggested that science's teaching and learning require authentic investigation, which is grouped into eight distinct practices. Some of these practices emphasized computational thinking. Similar educational outcomes were suggested by the Common Core guidelines (CCSS, 2010), whereby students are required to use technological tools to explore and deepen their understanding of concepts.

The advent of new and more sophisticated technologies expands the scope by which instructors can illustrate, represent, demonstrate, and facilitate mathematics instructions. While in-service teachers seem to possess appreciable knowledge of a good number of the practices identified in the science and mathematics standards, many practicing teachers still struggle with the skills required to teach and learn computational thinking practices (Weintrop et al., 2016). As the problem-solving process is not in any way new to teachers or educators in general, one can argue that a problem-solving process characterized by problem formulation, logical organizing and analyzing data, data representation through abstraction, among others, is a common process for teachers in mathematics classrooms. However, formulating problems that enable learners to use a computer and other tools to solve them or represent data through abstraction (e.g., models and simulations) is not easy for many teachers (Cetin, 2016; Wiedemannn et al., 2020). While there are professional development programs that help in-service teachers acquire these practices (Wiedemannn et al., 2020), research has suggested that instructing pre-service teachers in the use of computational thinking practices in their teacher education course can help them develop a better understanding of how it can be applied to the classroom (Yadav et al., 2017; US Department of Education, 2016).

Despite the intense pursuit of research to help students acquire computational thinking skills, many of these studies focus mainly on K-12 education, thereby leaving out similar rigorous studies at post-secondary education (Cetin, 2016; Lockwood and Mørken, 2021). That is, researchers engage in studies that investigate how computational thinking skills can become part of K-12 education (e.g., Weintrop, 2016; Wing, 2006, Cetin, 2016; Benton et al., 2017; Benton et al., 2018; Misfeldt and Ejsing-Duun, 2018) through integration into the existing curriculum without having done much at the post-K-12 level (Cetin, 2016; Lockwood and Mørken, 2021). Even with a handful of studies that involved college students, Cetin (2016) reported scarcity in studies that involve pre-service teachers. Almost five years after Cetin had called for studies regarding pre-service teachers, Lockwood and Mørken (2021) still identified scarcity and called for studies into pre-service teachers' learning of computing practices in their mathematics learning. In this dissertation, I share the view of Lockwood and Mørken (2021) that computational thinking practices could happen via a variety of technological tools. On top that, computational thinking practices can also be explored in an unplugged situation. Because they are already in use in the course in which this study is set, spreadsheets will be the computational tool studied in this research.

DeJarnette (2019) stated that the current popularity of integrating computer programming into mathematics could be attributed to the relative ease and easy access of modern technology as compared to the early technologies (e.g., the Logo programming). She further stated that computer technologies for learning mathematics differ in how students interact with the objects of those environments and, thereby, with the mathematical ideas students encounter through those environments. Since this study's focus is, in part, to investigate how pre-service teachers interact with the computational tool, I consider a spreadsheets Program – a graphical programming environment that is easy to use and provides a visual environment that allows students to put in codes and get output through the user interface.

Educators recommended a teaching and learning atmosphere that moves away from a teacher-centered approach to adopting student-centered and constructivist methods (Lee, 2007). As described by Solvie and Kloerk (2007):

> "Constructivism refers to bodies of knowledge as human constructs, as Phillips (2000) described, built up over time and influenced by politics, ideologies, values, and power structures that work to preserve this knowledge. In this way, constructivism refers to the social construction of knowledge and knowledge about the external world. Preservice teachers consider the material presented to them (external bodies of knowledge) and construct meanings and understandings as they reflect on and make sense of what they have experienced, thereby creating knowledge, not simply acquiring it (Phillips, 2000)."

In this study, I assume that knowledge constructs being influenced by politics, ideologies, values, and power structures among learners would be typical of knowledge construction when pre-service teachers work in groups on computational activities via a spreadsheet program environment – one of the most common programs available with individual computers and can capture, present, and manipulate data arranged in rows and columns. The assumption of social knowledge construction situates well with mathematical modeling activities since modelers are expected to share ideas in an egalitarian way when formulating problems, making choices and assumptions, building models, revising models, and reporting models. Thus, in this study, I investigated the forms of individual and group perspectives used by pre-service teachers to construct computational thinking practices and interpret such knowledge construction. In this study, knowledge construction is a process by which pre-service teachers generate ideas and understandings. In this regard, pre-service teachers are constructing knowledge when they go beyond knowledge reproduction by generating ideas and understandings that are new to them. The focus of classroom instruction should be on helping students to learn and experience this process.

Research Problem

Computational approaches to solving human problems have proven to be successful over the years and are becoming more relevant daily. Considering this, employers have recently emphasized the need for job seekers to possess computational thinking practices to solve the job's problems (Weintrop et al., 2016; Wiedemannn et al., 2020). Thus, it becomes imperative to build human capacity for such computational adventure in learners. Also, this type of skill acquisition is important for students to be abreast of skills needed for quality learning and be equipped as citizens of the world in this age. For this reason, the K-12 education in the United States and around the world should prepare learners who are ready for work or college by equipping with skillsets in solving numerous problems faced by humanity. Unarguably, if the computational skillsets are so important, we need teachers who possess the experience needed to teach students to accomplish these goals.

The ability to teach mathematical concepts does not translate to the ability to teach computational thinking practices. For this reason, efforts have been made to create professional development for practicing teachers to learn computational thinking practices (Weintrop et al., 2016). However, Weintrop et al. (2016) among other scholars, stated that a major challenge confronting computational thinking practices' teaching and learning is having adequate numbers of teachers who can teach the computational skillsets. Likewise, Cetin (2016) identified a lack of readiness of graduates who possess the skillsets to teach computational thinking practices in K-12 settings and consequently, research suggested that instructing pre-service teachers in computational thinking can help them develop a better understanding of how it can be applied to the classroom (Yadav et al., 2017).

Therefore, this dissertation seeks to investigate how a mathematics teacher education program provides a platform for pre-service teachers to learn to use computational tools such as spreadsheet program to engage in computational practices as mathematicians would do and thereby become a human resource to teach young learners. This study, in part, responds to the call by Lockwood and Mørken (2021) that the Research on Undergraduate Mathematics Education (RUME) community should strive to answer the questions of ways courses for pre-service teachers can be used to support integrating computing (computational thinking practices) into K-12 classrooms. In addition to that, they called for investigations into the kinds of issues pre-service teachers face when learning and teaching computing (computational thinking practices), and how they can be best prepared.

Research Questions

The purpose of this study is to investigate how pre-service teachers construct computational thinking skillsets when they work on modeling activities with computational tools. I, therefore, investigate three major questions.

1. What is the nature of pre-service teachers' interactions with peers and facilitators during modeling tasks in the presence of computational tools?

2. How does the development of computational thinking practices support pre-service teachers' mathematical and statistical thinking practices?
3. What computational thinking practices do pre-service teachers evidence during modeling?

Importance of the Study

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. The authors of CCSSM (2010) stated that the standards were created to ensure that all students graduate from high school with the skills and knowledge necessary to succeed in college, career, and life, regardless of where they live. All these educational goals seek a K-12 education that prepares students to be ready to take on the challenges they may encounter as citizens of the world. Central to achieving these goals is the place of teachers who will guide learners to develop computational thinking practices. However, it is concerning that not many teachers are equipped with the requisite knowledge to facilitate computer science skillsets needed in classrooms (Weintrop, 2016; Yadav et al., 2017). Moreover, having computer science as a standalone subject is complex as such an idea is faced with political, administrative, and educational challenges. Even in states where computer science has become an integral part of learning, studies (e.g., Cetin, 2016) established that college graduates who are equipped with the skillsets end up picking up careers in the industries. As a result of such adventure for graduates, classrooms do not have an adequate number of teachers who can teach students to learn the necessary skills (Lockwood and Mørken, 2021).

Educators have now realized the above stated challenges and thus strive to integrate the needed skillsets in the existing mathematics and curricula, thereby allowing students to be exposed to computational thinking skills through mathematics (Cetin, 2016; Lockwood and Mørken, 2021). Therefore, the integration of computational skills and mathematics requires a mathematics education for pre-service teachers to provide an avenue for their conceptualization of computational thinking practices. This study's objectives transcend what computational thinking skillsets pre-service teachers acquired; in addition, it also investigates how they socially construct the skillsets, how it influences their mathematical practices, and how they can infuse them in a mathematics lesson plan. In addition to the ongoing studies on the integration of mathematics and computational skillsets, this study explores pre-service teachers using and developing concepts of computational thinking practices in their mathematics learning context. Pre-service teachers will develop knowledge to integrate computational thinking practices into teaching mathematics. Preservice teachers will become aware and potentially ready to teach the skills to their future students. On top of that, this study will inform educators and researchers about the influence of computational thinking eliciting activities on pre-service teachers developing computational thinking skillsets through mathematics.

Findings from this study will inform educators and other stakeholders on strategies to employ when planning to teach computational thinking skillsets in professional development or in the classroom. This study will provide a lens through which to understand how pre-service teachers develop computational thinking skillsets and thereby give teacher educators directions to embed computational thinking in pre-service education mathematics courses and provide answers to the kinds of issues pre-service teachers face when learning computational thinking practices. Also, findings from this study will direct future studies in the context of integrating computational thinking in pre-service teachers' mathematics learning. Finally, this study will contribute to efforts in place to bring computational thinking practices into school curricula.

CHAPTER TWO

REVIEW OF THE LITERATURE

Introduction

The primary goal of this chapter is to review literature that discusses the main ideas of this dissertation, namely computational thinking practices, mathematical modeling practices, and mathematical thinking practices, and to discuss the conceptual framework guiding the study. To begin with, the first section documents definitions of computational thinking practices, mathematical thinking practices, and mathematical modeling practices. The first section also discusses how the three practices relate. The second section discusses the historical development of computational thinking from 1962, when the concept of computational thinking was referred to as algorithmic thinking to the present time, when its integration into K-12 curriculum is on the rise. The following section narrates why it is important to bring computational thinking practices into mathematics with a subsection that discusses mathematical modeling activities and computational tools. In section four, I discuss research on teaching and learning computational thinking practices in mathematics. This section also explains the relevance of the integration to learners and to teaching mathematics. It also documents the roles of teachers in the integration of computational thinking practices and mathematics. The following discussion focuses on teacher preparation to teach computational thinking practices in mathematics through professional development and effectively adapting it in pre-service teacher mathematics education. A summary of the first part is discussed before transitioning to the conceptual framework.

Computational Thinking Practices, Mathematical Thinking Practices, and Mathematical Modeling Practices

Computational thinking, which uses the power of logic, algorithm, abstraction, and precision, promotes a unique way of thinking about problems (Angeli et al., 2016; Yadav et al., 2017; Calao et al., 2015; Augustine, 2005). Wing (2014) described computational thinking as "the thought processes involved in formulating a problem and expressing its solution(s) in such a way that a computer - human or machine - can effectively carry out." Mathematical thinking is a way of looking at problems, breaking them down to their essentials, whether it is numerical, structural, or logical, and then analyzing the underlying patterns (Sylvester, 2016). Also, mathematical modeling is a process used by mathematicians to represent, understand, and solve real-world problems (Lesh and Harel, 2003; Bliss et al., 2018; Ang, 2019).

Educators and researchers have repeatedly identified the processes involved when users engage in computational thinking to include abstraction, decomposition, pattern generalization, visualization, problem-solving, parallel thinking, and algorithmic thinking (National Research Council, 2010; Weintrop et al., 2016; Furber, 2012; CSTA and ISTE, 2011, Shelby, 2015). Scholars have identified mathematical thinking and mathematical modeling processes engaged in by users as closely related to that of computational thinking. Table 1 shows some processes/practices of mathematical modeling, mathematical thinking, and computational thinking.

Table 1: Some processes/practices of mathematical modeling, mathematical thinking, and computational thinking

Mathematical Modeling	Mathematical Thinking	Computational Thinking
Formulating problems Defining problems Making assumptions Constructing models Solving models Interpreting solutions Validating solutions Refining solutions (CCSSM, 2010; Ang, 2019; Bliss et al., 2018)	Decomposition Identifying assumptions Revising solutions Specializing Generalizing Conjecturing Formula representation Abstract reasoning (Stephens, 2018; Stacey, 2006; Sylvester, 2016; Knuth, 1985)	Decomposition Pattern recognition Generalization Algorithmic Thinking Parallel Thinking Abstraction Mathematical Thinking Programming Computational Problem Solving (Shelby, 2015; Weintrop, 2015; Stephens, 2018)

A careful look at Table 1 shows that what mathematicians and scientists do when they engage in the three activities are closely related. In fact, computational thinking has been mentioned as one particular form of mathematical thinking (Stephens, 2018; Stacey, 2006; Sylvester, 2016). Also, the term *computational thinking* has been defined in the form of a taxonomy of practices - data practices, modeling and simulation practices, computational problem-solving practices, and systems thinking practices (Weintrop et al., 2016). These practices are closely related to mathematics. (Weintrop et al., 2016). Furthermore, members of the mathematical community value computing (computational practice) as a distinct mathematical practice that may foster computational fluency among students (Lockwood, Thomas, and DeJarnette, 2019). Finally, computational thinking practices are becoming an integral part of mathematics and the broader community (Buteau et al., 2017).

Thus, computational thinking practices can be described as toolsets for mathematical thinking processes; practices that provide chances to explore mathematical thinking processes;

and present opportunities to redefine how mathematical processes can be accomplished. As conceptualized in this study, Figure 1 shows how mathematics is used as a means to engage in modeling while computational tools create affordances to engage in a thought process that enables users to develop computational thinking practices. In the process of using computational tools to carry out aspect of mathematical modeling, modelers use and develop computational thinking practices. In addition, they use and develop mathematical thinking practices.

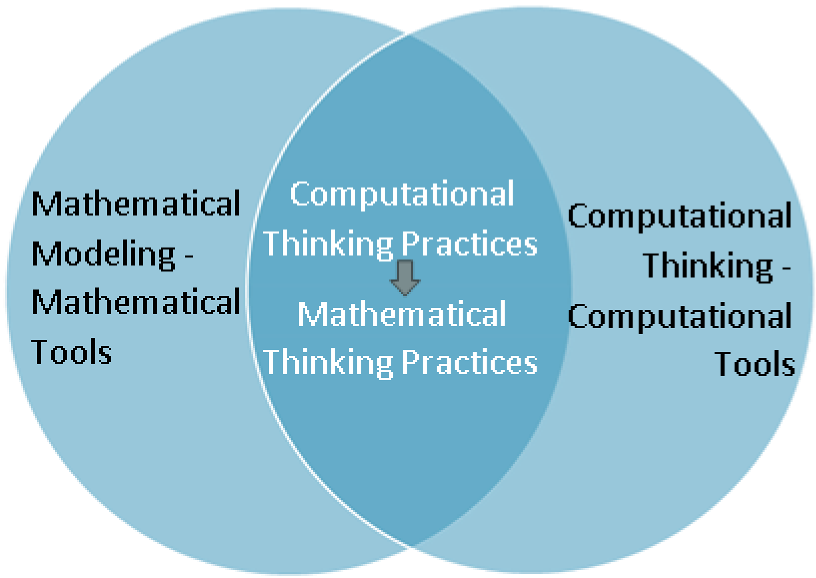

Figure 1: Representation of how practices of mathematical modeling, mathematical thinking, and computational thinking are related

Using mathematical modeling as mathematical tools and engaging in computational thinking through computational tools when developing a solution to a modeling problem allows modelers to engage in computational thinking practices that enhance mathematical thinking practices. The terms *practices* and *processes* have been used interchangeably by researchers. Henceforth, I will be using *practices* in this dissertation.

Following Wing's (2006) advocacy for computational thinking's relevance in K-12 education, many definitions have been developed to explain the term in the school curriculum. Efforts to coin the definition of computational thinking include the definitions put forward by the National Research Council (2010), Furber (2012), Computer Science Teachers Association (CSTA), and the International Society for Technology in Education (ISTE). Furber (2012) defined computational thinking as “the process of recognizing aspects of computation in the world that surrounds us and applying tools and techniques from computer science to understand and reason about natural and artificial systems and processes." According to CSTA and ISTE, computational thinking is a problem-solving process that involves formulating problems to use a computer and other tools to help solve them. In the following section, I will explore some history behind the evolution of computational thinking practices.

Historical Development of Computational Thinking Practices

While envisioning how the world would be impacted by machine automation, Alan Perlis (1962), who received the first ACM A.M Turing Award, concluded that programming should be integrated into liberal higher education. The term computational thinking (formally referred to as algorithmic or logical thinking) was first used by Seymour Papert in 1980 and later in 1996. This perspective, furthered by MIT scientists led by Seymour Papert in the 1980s, prescribed that all children should learn computational thinking rather than just featuring in higher education as proposed by Alan Perlis two decades earlier. The group later developed the Logo programming language to support Piagetian learning (Logo, 2015) and to help students think mathematically and logically (Cansu and Cansu, 2019). The Logo program developers considered the constructivist's view in developing the Logo programming environment for learners because they perceived learning as a fundamentally individual activity, which was explained in Piagetian terms – learning-by-making. Logo program developers and educators around that time had envisioned great potentials in its ability to revolutionize the teaching and learning of mathematics. However, these potentials did not come to life due to various challenges, ranging from replacing computing practices with information communication technology, to a computer science branch that focuses more on using technology than creating technology (Brown et al., 2014). Other challenges include teachers' experience in facilitating computing activity (Benton et al., 2017; 2018). Benton et al. (2018) also highlighted concentration on mastering programming syntax rather than the code's semantic meaning as a challenge that faced Logo users. These challenges, among others, limited the success of Logo programming in K-12 education.

Wing (2006) brought back the concept of computational thinking into the education system, particularly computer science. Educators did not waste time to see the potential of integrating computer science skillsets in students' learning as educators and policymakers at the time were defining educational goals for the new century and how to accomplish those goals (Ang, 2004; Cetin, 2016; Stacey, 2006; CCSSM, 2010; NGSS, 2013). While educators (e.g., Lockwood and Mørken, 2021) call for more studies in this field, new technologies are being developed to alleviate previous challenges of integrating computational thinking practices and mathematics.

Why Bring Computational Thinking Practices and Mathematics Together?

Educators, policymakers, and the government were concerned about the United States' performance in science and mathematics at the international level (Augustine, 2005). Augustine (2005) reported that US students perform more poorly overall on problem-solving skills than students in most participating countries on international assessments. He implied that most students lack the knowledge and skills required to secure desirable jobs and participate fully in a technologically sophisticated society. Various researchers and educators (e.g., Papert, 1980; Resnick et al., 2009; Weintrop, 2016; Lockwood, 2019; Lockwood and Mørken, 2021; DeJarnette, 2019) had extensively discussed the role of mathematics in developing knowledge and skills in learners to enable them to function as citizens of the world. I discuss how computational thinking practices, mathematical modeling, and mathematics practices are complementary in the following subsections.

Mathematical Modeling and Computational Thinking Practices

Mathematical modeling is a process by which users represent or describe real-world problems, find a solution(s) to the problems, or better understand the problems through mathematical practices. Put differently, mathematical modeling, an emerging branch of K-12 mathematics learning, is a process by which users develop and use mathematical tools and computational tools to represent, understand, and solve real-world problems (Lesh and Harel, 2003; Ang, 2004, 2019; Bliss et al., 2018). Many scholars have described mathematical modeling as a process (e.g., English, 2016; Carlson et al., 2016) which involves a series of actions or steps taken to achieve a particular end – solution to mathematical problems or understanding of the authentic scenarios (Ang, 2019; Bliss et al., 2018). The processes of mathematical modeling have been identified by various scholars and represented severally using the modeling cycle (Figure 2).

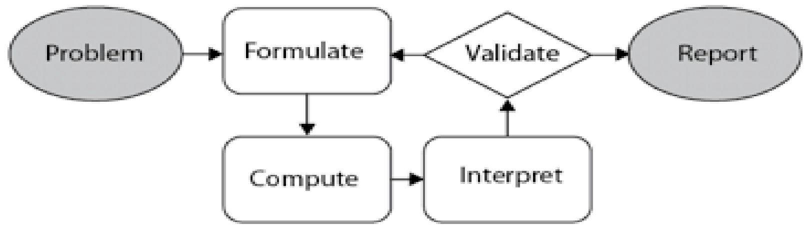

Figure 2: The mathematical modeling cycle from CCSSM (2010)

The series of actions or steps to be taken (Figure 2) creates space for computational tools (Bliss et al., 2018) through which users can develop computational thinking practices. For example, suppose I have a dataset consisting of two variables which I intend to model a relationship; I may have to engage in a modeling process that involves brainstorming, building the model, getting a solution, analyzing the results, and reporting the results (Bliss et al., 2018). In some situations, modelers' quest to build solutions starts with finding data or simulating in situations where data are not readily available. Modelers may also need to build a function that will connect to other parts of the larger model, which will warrant modelers to write code in the computational tools adopted. While getting a solution, modelers may need to write code to calculate some quantity repeatedly or may need to solve a complicated equation. Sometimes, modelers may need to vary a model parameter over a range of values to understand how sensitive the model is to the value. Finally, modelers may need to evaluate the model by visualizing it. By engaging in the above-mentioned endeavors, modelers will have to deploy some computational thinking from choosing the right computational tools, thinking with the tools in a computational language that yields results, and interpreting the results back to the real-world.

As mathematics educators and other stakeholders in the field of mathematics education believe that mathematical modeling creates space that could link mathematics and statistics to real-world, its inclusion in mathematics curriculum is not new. Many European countries' mathematics curricula featuring mathematical modeling dated back to the mid-1990s (Kaiser and Sriraman, 2006; Ang, 2019). As a result, the teacher education curriculum has integrated teacher learning of mathematical modeling in many parts of the world, including the United States. Some teacher preparation colleges now teach mathematical modeling to prospective teachers as part of their course requirements for graduation (Yadav et al., 2017). This development is a great improvement in teacher learning of mathematical modeling, unlike how it was some years back when prospective teachers did not have the opportunity to learn mathematical modeling as students themselves. There are also government and private-funded professional development for practicing mathematics teachers to enhance the inclusion of mathematical modeling in mathematics instruction. That said, I explore how pre-service teachers develop computational thinking practices through a mathematical modeling course designed for teachers being prepared to teach high schoolers. The nature and implementation of the activities used in the class provided spaces for pre-service teachers to explore math practices. That is, participants were able to make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning.

When constructing a model or solving mathematical modeling tasks, modelers can use mathematical tools and/or computational tools to accomplish a solution. Computational tools can be used in minor instances where calculations involve much addition, subtraction, and multiplication to a major scenario where modelers might simulate real-life events. Popular among these tools are spreadsheet, MATLAB, Python, and R because they allow users to show various approaches, are used by high school students, and are also relevant in industry (Bliss et al., 2018). While choosing a suitable tool is an essential decision modeler must make (Bliss et al., 2018), the instructor in this study used spreadsheet because it is free and readily available to classroom teachers, familiar to K-12 teachers, and students, and does not require users to learn a new programming language.

Research on Teaching and Learning of Computational Thinking Practices in Mathematics

Several studies have been carried out in K-12 settings on the importance, challenges, and relevance of integrating computational thinking in mathematics (Weintrop et al.,2016; Lockwood, 2019). Likewise, research is becoming popular among educators on the inclusion of computational thinking at the college level (Yadav et al., 2017). There is an enhanced interest in introducing programming in school in many countries as after-school coding clubs and formally as school curriculum (Benton et al., 2017).

Programming became a compulsory part of England's school curriculum in September 2014 (Benton et al., 2017) and some other European countries at that time. While computational thinking is placed in direct curricular connection to design and engineering in England and Sweden, computational thinking practices are integrated into Estonia and France's mathematics curricula (Misfeldt and Ejsing-Duun, 2015). Despite this development around developed nations globally, combined mathematical-computer programming is currently atypical in middle and secondary grades in the United States (DeJarnette, 2019). However, DeJarnette stated that efforts persist in their integration. One explanation for this situation in the United States is that early advocates for computer science practices considered these practices to be standalone. However, Wiedemannn et al. (2020) stated that while the number of schools offering computer science in the US is on the rise, it is estimated that only a little over a third of public high schools in the US currently offer computer science classes. On top of that, a standalone computer science course is considered elective and, as expected, will fail to entice a varied population of students, thereby aggravating the low participation rates of women and other underrepresented groups in computing fields (Wilensky et al., 2014). Just about sixteen states have policies in place that guarantee computer science courses for all high school students (Wiedemann et al.,2020). I note here that programming in this study is viewed as a sub-practice of computational problem-solving practices (as described by Weintrop et al., 2016).

New technologies developed in recent times have made Papert's ideas about computing practices in mathematics more realistic. Students can easily use some visual programming environments as interactive platforms to understand real-world contexts using mathematics. Based on the program model output, students may revisit their initial assumptions, add or remove parameters, modify both the algorithm and the code (when applicable) and run new tests. Students can repeat such actions until they arrive at a satisfying model. Popular among computational tools used to investigate computing practices in mathematics include the Scratch program (e.g., Benton, 2017, 2018; DeJarnette, 2019, Cetin, 2016), Python (e.g., Lockwood, 2019), and R program (e.g., Wiedemann et al., 2020). In what follows, I present the results of studies that focus on computational practices in mathematics teaching and learning.

Integrating Computational Thinking Practices and

Math learning in K-12 and post K-12

Studies have shown some positive results when researchers integrate computational thinking practices and mathematics. The idea of integrating computational practices and mathematics has helped researchers to explore the relevance of computational thinking in mathematics to learners, its relevance to teaching, as well as the role of teachers in its teaching and learning.

Integrating computational practices and mathematics enables learners to engage and implement a challenging programming concept through which they can explore mathematical concepts such as the coordinate system, units, rotation, permutation, and combination, among others (Benton et al., 2018; Misfeldt and Ejsing-Duun, 2015; Lockwood, 2019; Wiedemann et al., 2020). The integration extends computational thinking's reach to student populations that otherwise will not have access to if schools depend on teaching computer science as a standalone course (Weidemann, 2020). It also improves students' engagement and promotes deep mathematical reasoning (Benton et al., 2018; Misfeldt and Ejsing-Duun, 2015).

Teachers have important roles in connecting students’ activities while using computational tools in mathematics (Misfeldt and Ejsing-Duun, 2015). For example, teachers use their questioning skills to support students' discussion and thinking about mathematics when using computational tools to solve problems (Misfeldt and Ejsing-Duun, 2015). Lockwood (2019) showed that computational thinking practice can be exploited as an intervention in teaching mathematical concepts. Her study focused on how computing in the form of elementary programming tasks used as potential interventions might be leveraged to help college students reason about mathematical concepts of permutation and combination. She had students engage in programming directly by writing and running code or evaluating excerpts or code output. Lockwood suggested that students in the study were engaged in computational thinking by considering what the computer would output through the use of the “! =” and “>” symbols in the Python program. Lockwood found that by thinking carefully about what the program was doing in terms of those symbols and thinking about both what those commands told the computer and how the computer implemented and carried them out, the students understood how the outputs were being generated. A very important aspect of such intervention is the role played by the teacher. Thus, teachers have to be well equipped to give the needed support for learners when learning mathematics is integrated with computational thinking practices (Weintrop et al., 2016).

Teacher preparation to teach Computational Thinking Practices.

Teachers make a significant difference in student academic achievement (Berry et al., 2010). Put differently, teachers’ classroom effectiveness has been identified as one important factor affecting youngster’s academic growth (Sanders and Horn, 1998). Exposing students to computational thinking ideas improves students' problem-solving skills (Akcaoglu and Koehler, 2014; Calao et al., 2015) and their understanding of mathematical practices (Calao et al., 2015). Stacey (2006) noted that providing opportunities for students to learn about mathematical thinking requires considerable mathematical thinking on the part of the teachers. Thus, if computational thinking is so important in K-12 settings, there is the need to prepare teachers accordingly.

Many approaches have been considered to introduce teachers to core concepts of computational thinking; however, Yadav et al., (2017) stated that most current efforts to embed computational thinking focus on professional development for in-service teachers. In what follows, I present in-service teacher learning, pre-service teacher learning, and lessons about how to teach computational thinking practices to teachers effectively.

Professional Development: Teacher Learning of Computational Thinking Practices

Professional development for in-service teachers is part of efforts made by educators to expose teachers to computational thinking ideas. For example, Benton et al. (2017) designed a curriculum that embedded computational thinking practices for teachers as part of professional development to teach in-service teachers pedagogical knowledge of computational thinking practices. Teachers in this professional development engaged in a 2-day per year training to learn core computational thinking skills. While they reported that a particular teacher who had developed the confidence to use strategies and tools provided during the professional development session was able to act them out with pupils explicitly, they found that several of the teachers found it challenging themselves to envisage the outcomes of scripts for the two algorithms used and also to explain them to pupils. One can argue that a 2-day/year professional development is too limited in the timeframe for teachers to learn deep computational thinking practices. My argument here is not that in-service teachers cannot construct meaningful computational thinking practices during professional development, but such construction might not provide adequate opportunities for teachers to learn computational thinking practices sufficiently to feel comfortable incorporating them into their own teaching of mathematics.

In another study, Yadav et al. (2014) conducted a week-long module that incorporated computational thinking for pre-service teachers. The study showed that most of the pre-service teachers could define computational thinking as a problem-solving skillset that does not necessarily involve using computers and that can be implemented in classrooms. Although Yadav et al. (2014) suggested that a week-long module enabled pre-service teachers to define computational thinking, it is unlikely that it will enable them to acquire enough knowledge to embed it in their classrooms. A possible reason for this conclusion was confirmed in a study by Yang et al., (2018), in which preliminary results showed that pre-service teachers gradually improved their understanding throughout their participation in the course they were exposed to in the study. These immediate findings by Yang and colleagues support the idea that appreciable knowledge and skills needed to infuse computational thinking skills in classrooms may not be best provided to teachers through professional development with a limited time frame.

Teacher Education: Preservice Teacher Learning of Computational Thinking Practices

Research suggested scarcity of studies focusing on pre-service teachers' learning of computing practices in mathematics (e.g., Boulay, 1980; ISTE, 2016; Yadav et al., 2017; Lockwood and Mørken, 2021). For example, Yadav et al., (2017) stated that most current efforts to embed computational thinking focus on in-service professional development, while Lockwood and Mørken (2021) called for more studies in pre-service teachers learning of computational thinking practices. Available studies had looked into different ways pre-service teachers could learn computational thinking ideas. While some of these methods have been successful, the same cannot be said about others. In the study conducted by Boulay (1980), pre-service teachers' reactions to learning programming were mixed. Most students found the experience pleasant and stated that it increased their understanding of mathematical topics tackled. However, some pre-service teachers pointed that they were more confused just as the mathematics it was supposed to help. One identifiable reason for the mixed reactions was that most studies in the early years of computing in mathematics centered on programming, a situation that, according to Benton et al. (2017), caused inconclusive findings. Consequently, too much focus on programming concepts may create an imbalance between what pre-service teachers should focus on or what the teacher educator should introduce to student teachers to learn. Boulay viewed that such programming issues were diverting attention from mathematical issues.

Researchers also tried to situate computational thinking ideas in a general course or be integrated into a standalone technological course taken by student teachers (e.g., Guzdial, 2008; Ali and Smith, 2014; Yang et al., 2018). These approaches encountered various challenges and, as such, caused a dip from achieving the intended purpose of equipping pre-service teachers with computational thinking practices to use in classrooms. For example, students in a general class come from different backgrounds, thereby creating a situation for computer science instructors to teach computational thinking instructions at different depth levels (Ali and Smith, 2014). In some cases, such general computer science courses are a pre-requisite for advanced computer science courses, thereby mandating instructors to focus more on what to cover, not what students should learn as applicable to their subject domain. Ali and Smith (2014) stated that such general computer science class approach creates an imbalance in what students of varied backgrounds should learn from the program. Succinctly, Yadav et al., (2017) noted a limited understanding of how to engage pre-service teachers from other content areas in computational thinking skills relevant to their subjects. A direct impact of this approach on pre-service teachers is that they encounter difficulties in either selecting appropriate computing tools or infusing computational thinking concepts into the context of disciplinary content and pedagogy (Yang et al., 2018).

Thus, I argue that infusing computational thinking practices in a standalone technological course or any other general course provides an imaginary learning situation that may not represent pre-service teachers' authentic experience in developing computational thinking skillsets for mathematics teaching. What then is the ideal approach to introduce pre-service teachers to computational thinking practices?

How to Effectively Teach Computational Thinking Practices to Mathematics Teachers

For computational thinking to become an integral part of K-12 education, Lye and Koh (2014) posited a critical need to prepare teachers who are well trained to infuse computational thinking practices in their everyday pedagogical activities. While it is essential to introduce K-12 learners to computational thinking ideas, it is imperative to prepare teachers to take up classroom responsibility. Researchers had suggested incorporating computational thinking practices into pre-service teachers' mathematics learning mainly because they will have the opportunity to develop computational thinking ideas in the context of their subject matter. Such an opportunity will also provide the needful pedagogy required to teach in K-12 settings (US Department of Education, Office of Technology, 2016; Yadav et al., 2017; Yang et al., 2018). One of the four guiding principles of The National Education Technology Plan, Office of Educational Technology (US Department of Education, 2016) is to ensure pre-service teachers' experiences with educational technology are program-deep and program-wide rather than one-off courses separate from their mathematics courses. The department emphasized the active use of technology to enable learning and teaching through creation, production, and problem-solving. It thereby recommended a teacher preparation program that prepares teacher candidates to integrate technology with instruction in meaningful ways to support PK-12 students' learning. Yadav et al., (2017) posited that pre-service teacher education is an opportune time to provide future teachers with the knowledge and understanding they require to integrate computational thinking into their curricula and practice successfully.

Yadav et al., (2017) emphasized the importance of embedding computational thinking curricula in teacher education and provided recommendations for how teacher educators might be able to do it. They found that pre-service teachers' views about computational thinking encompass a broad spectrum of concepts, from merely using computers to using computational tools to solve problems. They identified that pre-service teachers' view of computational thinking ideas connects to other forms of thinking, such as logical thinking. Another valuable result came from Calao et al. (2015), who found that exposing students to computational thinking ideas improves students' problem-solving skills and their understanding of mathematical practices. When trying to infuse computational thinking in a technology course, Yang, Mouza, and Pan (2018) found a dip in the representation of computational thinking in practice. This result is not surprising as a standalone technological course does not represent the authentic experience a student teacher might encounter, like when computational thinking is infused in their mathematics learning.

In conclusion, to successfully infuse computational thinking knowledge and skills in K-12 education, we must equip pre-service teachers with appropriate skills that can be applied in their curricular context. Thus, this study seized the opportunity of embedding computational thinking ideas in pre-service teachers’ mathematics learning and not in any standalone computer science class or related educational technological class because mathematics courses enable pre-service teachers to acquire new ways to think about teaching and learning in mathematics and provide opportunities for pedagogical ways of doing, acting, and being as a teacher. I presume that a computational thinking and mathematics integrated class for pre-service teachers has the potential to bring together knowledge about mathematics and computational thinking practices suitable for children of a particular age group, knowledge of relevant pedagogy for the children, classroom nature of the children, and how the children will respond to teaching and learning of mathematics in a particular situation. These opportunities may not show forth in any other platform that integrates computational thinking ideas in pre-service teacher training. The pre-service teachers' mathematics class also provides an avenue for them to learn about learners' frustrations when learning computational thinking ideas through mathematics since they also experience similar encounters as learners. In particular, this dissertation considers mathematical modeling class for pre-service teachers.

Theoretical Framework

This study is guided by the social constructivist theory (SCT) propounded by Vygotsky (1978). Social constructivist theory is one of the theories that share a family resemblance with constructivist theory. Constructivism is the theory that says learners construct knowledge rather than just passively taking in information (Sobels et al., 2012; Adams, 2006). As people experience the world and reflect upon those experiences, they build their representations and incorporate new information into their pre-existing knowledge. While the social constructivist theory of learning shares this view, Vygotsky (1978) situated this world experience in social interaction with a more knowledgeable individual in the learner's world rather than Piaget's idea that individual learners should construct knowledge internally. While social constructivist theory is now widely used in different fields, Jerome Brunner (1966) first applied the theory in education.

Vygotsky (1978) emphasized the importance of interaction with people and tools such as language and computers to mediate knowledge construction. To Vygotsky, social interaction plays a crucial role in the process of cognitive development. Learning, to a social constructivist, is a process whereby social influences and interactions with others play significant roles in assuring that learners enculturate into a community of practice. During this process, learners generate the ability to advance and cultivate a shared meaning, thereby transferring knowledge to the group members. Vygotsky identified that a child's development evolves in two phases: first, on a social level and later, on the individual level. He had coined the first phase inter-psychological level (knowledge constructed between people), while the other phase, intra-psychological represents the level at which learners had internalized the knowledge constructed and could add their personal values.

Vygotsky referred to a more knowledgeable other as anyone who has a better understanding or a higher ability level than the learner concerning the learning assignment at hand. The more knowledgeable other is generally referred to as a teacher, coach, other learner, or older adult (Dawood, 2012; Sobels et al., 2012; Abtahi, 2017). Although researchers have documented the role of the more knowledgeable other and its alternation among participants during social learning (e.g., Graven and Lerman, 2014), Abtahi (2017) identified two issues that are relevant to this study. On the one hand, she questioned the possibility of identifying or pointing to an “other” that is more knowledgeable in some interactions. Secondly, she queried what might be the knowledge of the more knowledgeable other. In mathematical modeling activities, we want modelers to bring various perspectives to address the problem at hand; hence, this study does not emphasize any “more knowledgeable individual”; rather, the study focuses on how pre-service teachers co-construct knowledge. By co-construction of knowledge, I mean an active process that happens through social engagement. The process of constructing knowledge happens by building on people’s prior knowledge and experience.

Facilitators of modeling activities think about the multiple perspectives modelers might bring into modeling space, thereby preparing them to help modelers during the modeling process either by attending to questions that might move the learners forward or through scaffolding that might help learners (Adeolu, 2020). In this sense, the instructor's role in this study is to provide guidance or act as a coach, when necessary, and to co-construct computational thinking practices with pre-service teachers. Put differently, learners and instructor in this study co-construct knowledge; that is, no one is ascribed the role of knowing more than the other; instead, they interact to construct knowledge. Next, I consider computational tools in this study as agencies (Wertsch, 1998) or mediating tools (Vygotsky, 1978; Johnson, 2016) through which learners and instructor socially interact to construct computational thinking practices. That is, the computational tools used in this study are agencies (material environments) through which the agents (learners and instructor) act (Wertsch, 1998) to construct computational thinking practices.

Vygotsky (1978) identified the relationship between the subject (learner), mediating tools, and object (outcome) using the mediation triangle (Figure 3).

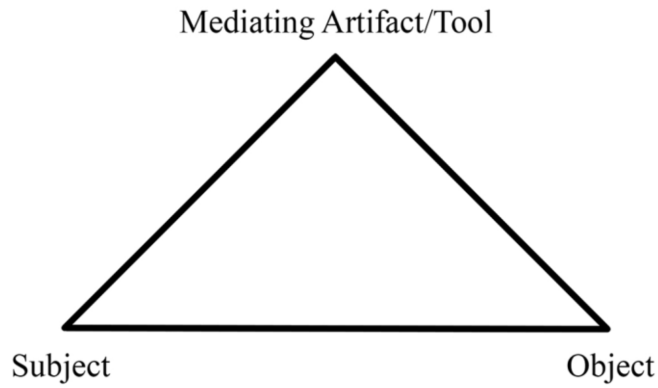

Figure 3: Vygotsky's basic mediation tool adapted from Johnson (2016).

In Vygotsky's view, the subject refers to an individual using the mediating artifact/tool to achieve the learning goals. Vygotsky's student, Leontiev (1978) expanded Vygotsky's idea of the subject to articulate the ways in which individuals come together to work on activities and are guided by goals or motives. Within group activity, there is a division of labor and rules or norms that the community follows (Leontiev, 1978; Johnson, 2016). Like Leontiev's view, I conceptualize pre-service teachers as individuals constructing conceptual understandings of computational thinking practices through their participation in mathematical modeling activities in a social context. One of the major goals of this study is to determine the nature of interaction that exists when pre-service teachers learn collectively.

Vygotsky also identified the zone of proximal development (ZPD) as the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers. However, this study would complete the last sentence with "other peers" rather than "more capable peers" since the goal of working together is to constitute shared meanings/understandings of the modeling tasks. That said, the zone of proximal development recognizes that individuals often exhibit higher skill levels through the assistance, support, encouragement, and coaching of other people and tools such as language and computers (Dawood, 2012).

Vygotsky emphasized that children and adults are both active agents in the process of a child's development. When applying to teaching, both the teacher and students are regarded as active agents in the learning process. The teacher's intervention in children's learning is necessary, but learners’ interaction is seen as crucial in the learning process. Since multiple perspectives form significant part of knowledge construction (Lesh and Doerr, 2003), this study provides pre-service teachers with the opportunity to construct knowledge through their social interaction with the instructor and peers.

While Vygotsky used the term more knowledgeable others, Bruner (1966) first used the term scaffolding to represent more knowledgeable others in education settings. Scaffolding is a strategy employed by teachers during facilitation to give a group of learners assistance when the attention is needed and then withdraw. The scaffolding strategy helps to move learning to the zone of proximal development. Since we wanted learners in the modeling class to engage in meaningful problem-solving, the facilitator used the scaffolding strategy to drive the lesson throughout the semester. In practice, a social constructivist teacher gives the necessary support to learners without taking away the opportunity to learn quality mathematics and also creates a suitable learning environment for learners to enhance their knowledge development through collaboration – which entails sharing responses, ideas, drafts, and finished work with others.

Figure 4 captures how the social constructivist theory is applicable in the context of pre-service teachers' (PSTs) construction of computational thinking practices (CTPs) in this study.

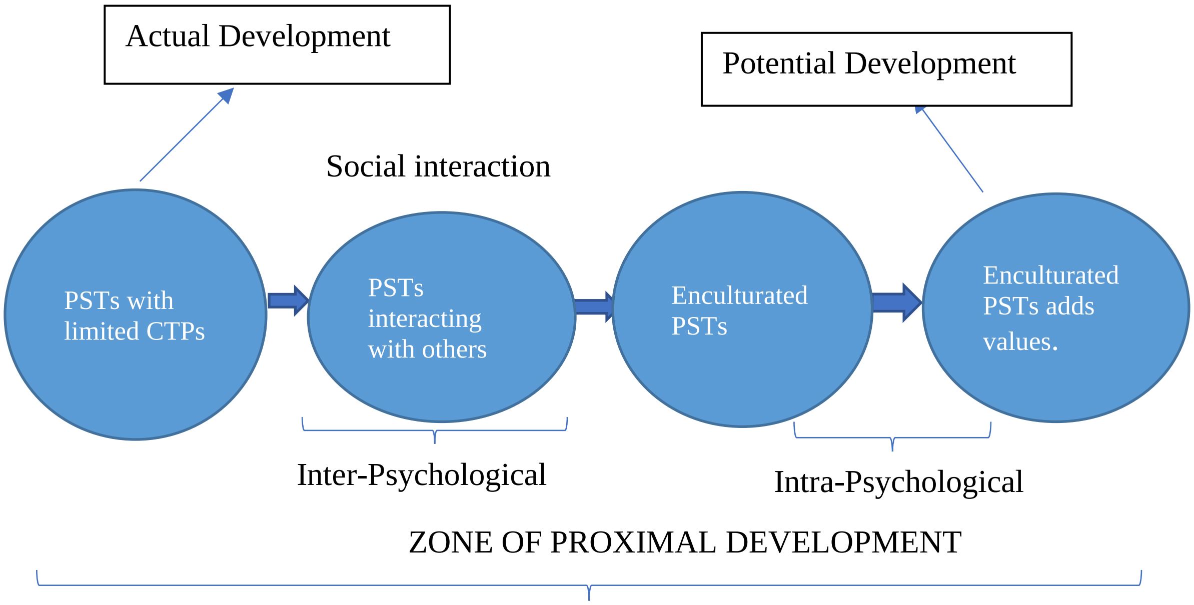

Figure 4: Social Constructivist Theory in Preservice Teachers' Construction of computational thinking practices

<u>Actual Development Level</u>

This level represents the knowledge pre-service teachers possess about computational thinking practices before interacting with others (including tools). It represents what pre-service teachers can learn on their own without the assistance of any significant other. The knowledge at this level is always limited. At this stage, learners make sense of the situation and form their independent processes (initial ideas) which become the basis of discussions with the group (Adeolu, 2022). Vygotsky (1978) noted that learners should be given the opportunity to move from that limited cognition to their zone of proximal development through interaction with teachers, coaches, parents, or tools such as language and computers. A major goal of this study is to investigate how such opportunity to learn cooperatively afford pre-service teachers to construct computational thinking practices.

Inter-psychological

This level represents where pre-service teachers interact with instructors and peers to construct computational thinking practices. Also, computational tools serve as agencies through which the participants act or interact to construct computational thinking practices. Although spreadsheet was the primary tool adopted for the class, participants used other computational tools such as R and Java programs. The instructor in the study used the reciprocal teaching method during facilitation. Reciprocal teaching involves interactive dialogues between the teacher and a small group of students (Palincsar and Brown, 1984). The instructor, adopting a social constructivist perspective in this study, creates a context for learning in which students can become engaged in exciting activities that encourage and facilitate learning. The idea of knowledge co-construction is not confined to teacher-pupil interaction alone like the case with the behaviorist. Instead, this study acknowledges the need for pre-service teachers-pre-service teachers' interaction. In the modeling activities, instructor and pre-service teachers interact with themselves and with computational tools to help pre-service teachers make sense of the tasks and to develop ideas to solve problems. In the same vein, pre-service teachers interact among themselves and with the tool to make assumptions, develop models, test models, and revise models. According to Adams (2006), social constructivism does not remove the teacher's need; instead, it redirects teacher activity towards providing a safe environment in which student knowledge construction and social mediation (Wertsch, 1998) is paramount. Omrod (1995) noted that the process of scaffolding the learning journey is the key teacher requisite. Figure 5 depicts the kind of social interaction between pre-service teachers, instructor, and computational tool.

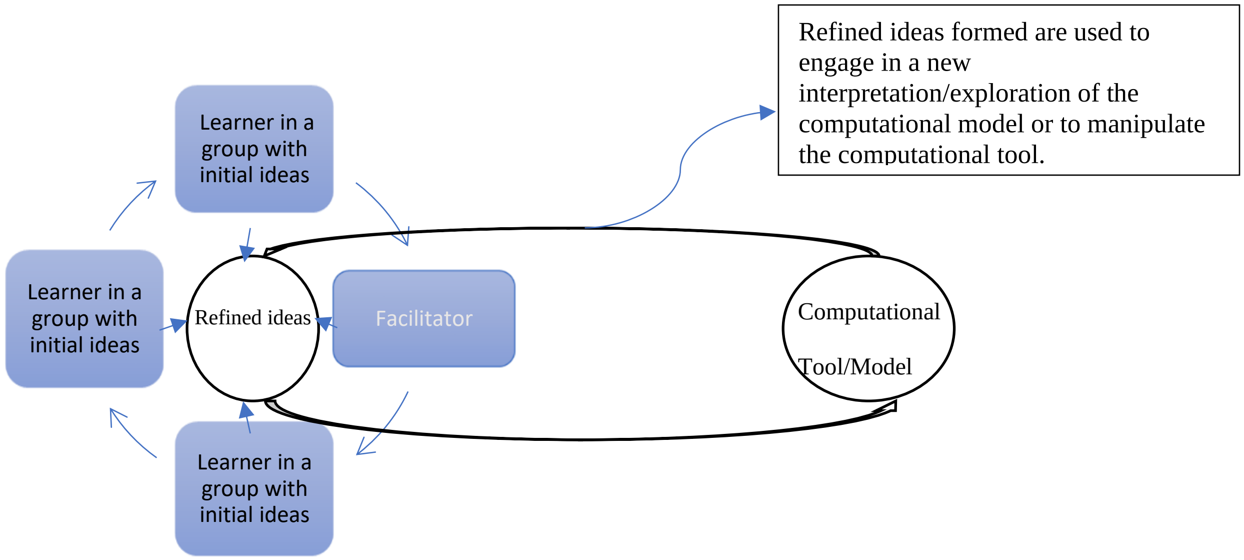

Figure 5: Social Interaction between learners, instructor, and computational tool to construct computational thinking practices adapted from Adeolu (2022)

In this kind of social interaction, pre-service teachers use available resources such as their existing knowledge, instructor, or any form of resource to generate shared (refined) ideas used to engage in further interaction with the computational tool. This model for social interaction does not place much emphasis on the instructor, thus, the reason for a faded lettering in the figure. Furthermore, the interaction with the tool is seen in form of a mediated action through which pre-service teachers develop refined ideas that is equivalent to the knowledge constructed in this study. There is a close link between this model and Vygotsky's (1978) mediation triangle displayed in Figure 3. The relationship between these models are further unpacked in chapter 4.

Among others, this study considers a real-world activity that is very concerning to every citizen of the world at the time of the study (simulating disease spread). Adams (2006) describes such learning adventure as a learning situation that makes school learning authentic. Bereiter

(2001) stated that school learning that connects to learners' wider, personal agenda is more likely to transfer between home and school. The instructor's role is to facilitate a teaching perspective well shared among social constructivist teachers (Copley, 1992), that is, act as a coach (Adeolu, 2020). The instructor works to provide opportunities to construct knowledge and understanding with a different teaching identity (Adams, 2006) whereby the focus is on activities that provide learner-world, case-based learning to enable authentic, context-oriented, and reflective practice within a collaborative and social environment (Rice and Wilson, 1999). In adopting the activities for the study, the instructor considered the question "how is this meaningful for pre-service teachers given their life-world" as a future teacher and citizen of the world. A thorough description of the activities can be found in chapter three of this dissertation.

What does the term *mediated action* connote in this study? Vygotsky (1978) emphasized the importance of tools such as language and computers to mediate knowledge construction. Wertsch (1998) referred to this knowledge construction as mediated actions performed through mediational means (or cultural tools). Mediated action is a way of researching how people (agents) use all kinds of tools (agencies), both physical and psychological, to structure their interactions, communicate with each other, and think (Wertsch, 1998). A powerful feature of such action is that the effect of one (agent or agency) cannot be separated from the other. This type of action involves focusing on agents and their cultural tools – the mediators of actions. I have described pre-service teachers and instructor as agents in this study, while computational tools are regarded as mediational means or cultural tools (Wertsch, 1998). As a sociocultural study, the essence of examining agents and mediational means (computational tools) in mediated action is to examine them as they interact. An appreciation of how mediational means or cultural tools are involved in action forces one to live in the middle. In particular, it forces researchers to go beyond the individual agent when trying to understand the forces that shape human action. Wertsch (1998) concluded that any attempt to reduce the analyses of mediated action to one or the other of these elements (agents and cultural tools) runs the risk of destroying the phenomenon under observation. Thus, the study focuses on the interactions between pre-service teachers and the computational tools in constructing computational thinking practices through modeling activities integrated into their mathematics learning.

Land and Hannafin (2000) noted that adopting a constructivist approach in a technology-rich environment promotes technology’s full potential. To do this, the instructor will facilitate research-oriented activities for pre-service teachers that would allow them to process information, ask questions, solve problems, and make decisions. This process allows pre-service teachers not to be imparted with knowledge as a behaviorist would suggest. Instead to acquire knowledge through an inquiry process and interaction with the instructor, colleagues, and cultural tools.

Another significant role of the instructor is using their assistance during teaching episodes as assessment opportunities. The instructor not only teaches but also gains insights into pre-service teachers' computational thinking practices construction and what relevant modifications are needed in their learning. Besides, the zone of proximal development opens up possibilities for peer assessment (Adams, 2006). The instructor allows pre-service teachers to share their distributed expertise as learners who had formed a community of practice. The term *assessment* of what pre-service teachers learned in this study is not an objective test of truth. Instead, we seek information about what pre-service teachers know about the learning process.

Enculturated pre-service teachers

At this level, pre-service teachers have acquired knowledge and understanding from others. Each pre-service teacher comes out enculturated into the practices learned, thereby becoming a sophisticated individual ready to attack other problems or future technical problems.

Intra-psychological

Preservice teachers have developed self-confidence and acceptance from others, now can add personal values to reach the zone of proximal development. Vygotsky (1978) termed this level the individual level whereby learners add personal values to the knowledge co-constructed at the inter-psychological level to pursue future endeavors. In the context of this study, I do not presume that pre-service teachers will directly use the activities or the computational tool with their students; rather, I envisage that pre-service teachers would become enculturated learners who could add personal values to the knowledge constructed for use in their classrooms.

In sum, the second part of the outgoing chapter discusses the conceptual framework guiding the study. This study extends our knowledge of the social constructivist views of learning and also extends the Vygotskyan mediation triangle. In the next chapter, I will discuss the research methodology that enables the study to be situated in a social constructivist worldview.

CHAPTER THREE

METHODOLOGY

Introduction

I investigated pre-service teachers' construction of computational thinking practices in a mathematical modeling course by applying the social constructivist learning theory. In this study, I investigated three research questions:

- What is the nature of pre-service teachers’ interactions with peers and facilitators during modeling tasks in the presence of computational tools?
- How does the development of computational thinking practices support pre-service teachers' mathematical and Statistical thinking practices?
- What computational thinking practices do they develop during modeling?

Research (e.g., Augustine, 2005) has shown the need to prepare students in secondary school who are sophisticated in problem-solving skills to compete favorably on the global stage. Central to preparing high school graduates with the essential problem-solving skills is the role of computational thinking practices and the need to prepare future teachers ready to help children develop those skills to function as citizens of the world. Among other potential relevance of the study, it documented how pre-service teachers constructed computational thinking practices in a mathematical modeling course and how such knowledge construction helped their mathematical and statistical thinking practices and vice versa. The study hopes to inform educators and other stakeholders on strategies to employ when planning to facilitate computational practices in professional development or the classroom. It can also give directions on future studies in integrating computational practices in pre-service teachers' mathematics learning.

I used the social constructivist theory to investigate how pre-service teachers constructed computational thinking practices through interaction with others (instructors, peers, and computational tools) to become enculturated individuals who could add their values to use computational thinking practices for problem-solving. This chapter is organized into seven sections. In the first part, I discussed my positionality statement. Next, I justified the suitability of the qualitative method used in the study. In the following section, I discussed the research design, which focused on the participants and settings, the study design, and the implementation of the study's activities. In the fourth section, I discussed the data collection procedures, including the interview protocol. Next, I discussed the data management and data analysis procedures, including NVivo as the computer program in this study. In the sixth section, I discussed the ethical considerations and trustworthiness of the study. Lastly, I summarized the chapter.

Positionality Statement

As the researcher in this study, it is important to provide my position related to pre-service teachers' construction of computational thinking practices in mathematics learning. Throughout this study, I have guarded my opinions and thoughts regarding the phenomenon under investigation, which could have significantly influenced data collection and data analysis.

My experiences teaching secondary mathematics in Nigeria informed my understanding of how students learn mathematics. Although, as a mathematics learner, I prefer to work independently, I saw the value of group work in mathematics. I was exposed to programming in

R in graduate school, and further reading expanded my research interests in computational thinking in mathematics.

The interpretive lens used in this study is social constructionism. Social constructionism is a way to understand the world in which humans function through the viewpoints and possibilities of others (teachers, peers, tools, etc.) operating within it (Creswell and Poth, 2018; Wertsch, 1998; Vygotsky, 1981). This study's ontological assumption is that the participants will construct computational thinking practices in mathematics differently, creating different realities and viewpoints on my investigations (Lesh and Doerr, 2003). The epistemological assumption in this study is that the participants will honestly share their experiences with the researcher, as their interviews and other data sources will be taken as truth and analyzed as such (Creswell and Poth, 2018). The axiological assumption for this study is that my own experiences as the researcher will impact my interpretation of how the participants interact, solve problems, and share their experiences.

Justification of the Qualitative Method Used

Teachers are responsible for teaching K-12 learners the needed problem-solving skills to compete favorably as world citizens. In chapter two, I reviewed studies that support the infusion of computational thinking practices in pre-service teachers' mathematics classes during their teacher training. I investigated how pre-service teachers interacted with one another to construct computational thinking practices via integrating computational activities in their mathematics learning. Following Merriam's (2009) perspectives, this study assumed that reality is socially constructed; that is, there is no single reality. I treated reality as manifestations of multiple realities (Lesh and Doerr, 2003). Thus, I was interested in documenting pre-service teachers' construction of knowledge through multiple realities.

The qualitative research method presents a systematic process through which researchers can understand how people interpret their experiences of how they construct their worlds, and what meaning they attribute to their experiences (Merriam, 2009). I found the qualitative research method a valuable tool to gain insight into pre-service teachers' construction of computational thinking practices, understand their discoveries, and interpret their interpretations to explain their worlds. The qualitative research approach was very relevant to this study since the goal was not to test hypotheses; rather, to understand the meaning pre-service teachers constructed and how they made sense of their world. Furthermore, how they made sense of their experiences as they engaged with the world they were interpreting.

I applied the qualitative case study approach to give an in-depth description and analysis of pre-service teachers' construction of computational thinking practices. Case study research is a qualitative approach in which the investigator explores a bounded system (a case) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving multiple sources of information (e.g., observations, interviews, audiovisual material, and documents and reports). It reports a case description and case-based themes (Creswell, 2007). I considered a case study for this study because it allowed me to:

- bound the study by pre-service teachers constructing computational thinking practices in a mathematical modeling class.
- focus on each group of pre-service teachers working together as the embedded unit of analysis.
- collect data from multiple sources.

- obtain detailed (rich qualitative) information about how pre-service teachers interact with peers and tools, how they construct computational thinking practices, the challenges they face, and how they overcome.
- focus on mediated actions between pre-service teachers, instructor, and computational tools (Vygotsky, 1978).

For this study's purpose, I investigated a single case (with an embedded unit of analysis) of secondary school pre-service teachers in a real-life setting over one semester, interacting with others (instructor, peers, tools) as they constructed computational thinking practices. Figure 6 shows how the study was bounded as a class with embedded groups of pre-service teachers (Yin, 2009).

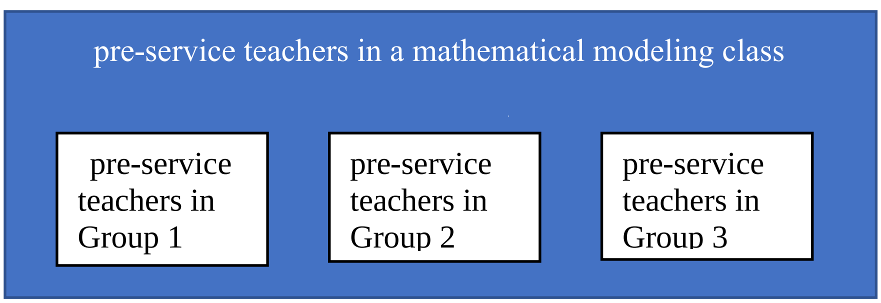

Figure 6: Case Study with Embedded Units of Analysis

A case study requires various data sources. Multiple sources such as surveys, observation, interviews, audiovisual recordings, and student work were used to collect data as representations of multiple realities of pre-service teachers' construction of computational thinking practices through mathematical modeling activities. In chapter four, I discussed my descriptions of these multiple realities as related to the study. Since this study was purely an observational case study (Merriam, 2009) where the primary data collection was via observation, supplemented with other forms, the study's goal centered on describing pre-service teachers' computational thinking practices construction rather than predicting their future behavior. Although I did not focus on generalization, the end-product is a detailed, thick description of pre-service teachers' construction of computational thinking practices; therefore, knowledge learned from this study can be transferred to another situation. That is, findings in this type of study can apply or be useful in similar contexts (Bloomberg and Volpe, 2012).

Finally, this single case study presents a basic data analysis - inductive and deductive - by drawing on the constant comparative method of data analysis (Merriam, 2009; Creswell, 2018; Miles et al., 2014). This chapter covers more about this under the section “data management and data analysis.”

Research Design

A research design is a plan to guide the research process by laying out how a study will move from the research purpose and questions to the findings. It is a thorough preparation process used to collect and analyze data to expand the understanding of the topic under investigation (Abutabenjeh and Jaradat, 2018).

Participants and Settings

Nine secondary pre-service teachers enrolled in a course for future secondary teachers to learn mathematical modeling in the Fall of 2021. The class was considered the case for the study, whereby participants were divided into three to four groups throughout the modeling activities. In this study, I focused on two of the modeling activities designed for the class (see next section for more); however, references were sometimes made to other activities used in the class. I focused on these two tasks because they took place over several weeks and required pre-service teachers to start the modeling activities from data creation. All participants (with pseudonyms - Amelia, Mia, Chloe, Mateo, Zoey, Owen, Aurora, Ivy, and Asher) were enrolled in a four-year mathematics education program at a public university in the Rocky Mountain West of the United States and were all seeking to be certified as secondary mathematics teachers. That is, they were expected to teach mathematics at middle and high school levels after graduation.

About the facilitator and the class

The table below shows basic features of the facilitator and the nature of the class. More

Table 2: Feature of the facilitator and the nature of the class

Facilitator	Nature of the Class
• The facilitator is a faculty member at the university where the study was conducted. • She possesses at least six years of instructing pre-service teachers in mathematics. • Specifically, her expertise is in teaching and learning mathematical modeling, and she also conducts research with pre-service teachers.	• The activities used in the class were real-world tasks that allowed participants to explore aspects of the tasks from different approaches. That is, there is no single way of tackling the problems. • Participants used computational tools to navigate the modeling processes. • In general, the nature of the class was flexible and allowed students to move from group to group, helping them to learn collectively. • In addition, the end goal of the course was to enable pre-service teachers to engage in problem-solving tasks through modeling.

Design and Implementation of Modeling (Computational) Activities

The facilitator considered the social constructivist approach in designing and implementing this study's tasks. However, I met with her to discuss tasks throughout the course. Following studies that suggest a positive connection between teachers' preparation in their subject matter and their performance and impact in the classroom and the recommendation for the infusion of computational thinking practices in pre-service teachers' learning, I considered *mathematical modeling for teachers* as a suitable mathematics class for the integration of computational activities for pre-service teachers (e.g., Cetin, 2016; Yadav et al., 2017; Yang et al., 2018; Edutopia, 2001; Guzdial, 2008; Ali and Smith, 2014; Bliss et al., 2018). During this class – a course for future secondary teachers to learn mathematical modeling, the instructor facilitated various modeling tasks with the students; however, this study focused on two of the modeling activities (Table 3). In some instances, I referenced one other task (the Ilesha task) in chapter four. In the Ilesha task, participants collected information about water usage and were tasked to predict the volume of water that would be needed in thirty years' time.

The instructor enacted all the activities to engage pre-service teachers in mathematical modeling with spreadsheets. However, other computational tools such as R and Java were used as convenient for some participants. In chapter two, I cited the following reasons for choosing the spreadsheet program with future secondary mathematics teachers:

- Free and readily available to classroom teachers
- Familiar to teachers and students
- Do not require users to learn a new programming language

Each activity was designed to run for at least 2.5 hours in a week. For more on the activities, see Appendix A. The facilitator guided students using the reciprocal strategy and scaffolding as they worked through the activities. Table 3 shows the description of the modeling activities.

Table 3: Description of modeling activities

Task	Description	Evidence of Computational Thinking Practices
Bobcat Population: How does Florida's Bobcat population change under different conditions?	This activity allowed pre-service teachers to explore the behavior of a bobcat population using growth rate data from the state of Florida (from Cox et al., *Closing the Gaps in Florida's Wildlife Habitat Conservation System).* They used computational tools to carry out statistical investigations on mean, standard deviation, and normal distribution, thus allowing them to engage in mathematical practices through a statistical lens.	Pre-service teachers used a spreadsheet program to: • simulate data for the bobcat population under three different environmental conditions • visualize dataset • create curve fitting • write a spreadsheet code to calculate some quantity over a given period • make prediction
Disease Spread: How do population size, infectiousness, and duration of illness influence the length and severity of a disease outbreak?	This activity allowed pre-service teachers to simulate disease spread in (un)plugged situations (without/with computer) and used mathematical concepts such as probability and rate of infection to model disease outbreaks.	Pre-service teachers used a spreadsheet program to: • simulate data • build sub-model that will connect to other parts of the larger model • perform curve-fitting • vary a model parameter over a range of values for sensitivity analysis • visualize model • make prediction

Data Collection

Data collection is the process through which researchers gather information to answer the questions under investigation. Qualitative researchers (e.g., Creswell and Poth, 2018) consider the process of data collection to ensure the construction of realities through multiple sources of knowledge. This idea resonates deeply with the lens through which I investigated the questions in this study. I considered reality as a product of knowledge from different perspectives among a community of learners. Thus, concerning the problem under investigation, the theoretical perspective, and the study's purpose, I employed four data sources – survey, classroom observation, student work, and stimulated recall interviews. In what follows, I describe each of these data sources and how they are used to obtain rich data.

Survey

I used a written survey format to collect some background data about the participants in this study. See Appendix B for the survey used. The background data I collected include participants' prior experience using spreadsheets or any computational tool(s) to solve mathematical problems, teaching experience, and mathematics background.

Classroom Observation

I attended the semester-long class in Fall 2021, observing pre-service teachers' activities before implementing the modeling activities through the implementation and post-implementation. Observing pre-service teachers before implementing this study's activities enabled me to become familiar with the students, thereby not making the setting strange to students as participants and me as the researcher. Classroom observation is essential in this study as it provides a natural setting where the phenomenon under investigation occurs. Merriam (2009) writes that observation, when possible, gives researchers a firsthand encounter with the phenomenon of interest. As the researcher, I assumed the position of observer as a participant. In this way, pre-service teachers knew my activities in the class as my participation in the group activities was secondary to my role of gathering information (Gold, 1958). In general, I observed participants at the group and individuals level. I did not directly get involved in their modeling processes since my major assignment was to collect data. However, there were instances where participants asked for my help setting up the screen recordings and using some functions in spreadsheets. When I helped participants with spreadsheets functions, I assumed the role of an assisting instructor.

I observed and interacted closely enough with all groups to establish an insider's identity without participating extensively in the activities constituting the core of group membership (Adler and Adler, 1998). I took notes as much as possible during observation about the participants, the setting, the participants' activities or behaviors, and what they were doing. Merriam's (2009) idea of writing observations as soon as possible was used, with a reflective process that served as the initial form of my interpretation of the data. In addition to observation, Merriam advises that researchers record participants since it is most difficult to capture every event. Merriam's idea of recording is relevant in this study because it was practically impossible to observe three groups sitting apart simultaneously; hence, swivl technology was used to capture students' participation in its entirety. In this way, I could move around to take notes of my observations. I generated field notes and video data for analysis with this technological aid.

Student Work

Student work is a form of artifact in this study, that is, material object produced by participants (Merriam, 2009). The purpose of using this method of data collection was to retrieve students’ solutions (e.g., spreadsheets, model report) to assigned tasks to learn more about pre-service teachers' learning, their computational thinking practices construction, and their mathematical reasoning. Student work was important in this study as it helped to retrieve an organized write-up of solutions I collected through video recordings. The instructor guided pre-service teachers to work in groups on the tasks, and they submitted solutions that included their thought processes, spreadsheet manipulations, etc.

Stimulated Recall Interviews

Qualitative researchers (e.g., Merriam, 2009; Creswell, 2013) opined that it is most difficult to observe how respondents have organized the world and the meanings they attach to what goes on in the world constructed. Patton (2002) states that the purpose of interviewing is to allow researchers to enter into participants' perspectives. Although I observed pre-service teachers in their natural settings as they worked on computational activities integrated into mathematical modeling course, it was necessary to gain entrance into their perspectives to understand how they organized the world they constructed and their meanings (Busse and Ferri, 2003).

Dexter (1970) noted that interview is necessary when the researcher wants a better understanding of perspective or more data. For the purpose of this study and as suggested by Merriam (2009) and Creswell (2018), I used an interview procedure that has elements of a semi-structured and task-based formats. While the semi-structured, open-ended nature was used to ask pre-service teachers questions about their interaction with peers and tools, including their challenges and how they overcame them, a task-based portion was used to gain access to pre-service teachers' previous mathematical reasoning and usage in constructing computational thinking practices. The term *previous* refers to participants activities in the class during the semester.

I went through the screen recordings, class recordings (using swivl), and fieldnotes to identify instances relevant to the questions of the study. I sliced videos of each participant based on time stamps where those important instances took place and made a slideshow presentation where each slide had underlying questions that probed further into what participants were doing at such instances. The term *instances* refer to places that were significant to collaboration, facing and resolving challenges, using mathematical or statistical ideas, etc. Five participants - Mia, Chloe, Zoey, Mateo, and Amelia participated in a single interview that lasted for about 90 minutes on average for each participant. During the face-to-face interview, I forwarded the slides to the respective participant while they played each slice and responded to the question at the top of the slide. Sometimes, participants had to watch two slides (clips of their actions during class) to respond to a question. On the one hand, interviewing pre-service teachers allowed me to collect more data, and on the other hand, it enabled me to triangulate the survey data, field notes, video data, and student work.

I modeled the interview guide following Burke (1972), Merriam (2009), Patton (2002), and Creswell's (2018) suggestions for asking quality questions that reflect the purpose and the theoretical perspective guiding the study. They identified qualities such as open-ended-ness, questions that connect to theoretical perspective and purpose of study, and questions that are easy to understand and digest with no need for clarification. Burke (1972) suggests that a sociocultural researcher is interested in the "what, where, who, how, and why" of the participant's worldview or interpretation of a phenomenon. The interview guide (see Appendix C) designed for this study was tried out with a scholar who knew much about the purpose and the study's theory. This idea was crucial for trying out the questions to know their suitability for gathering the intended information from pre-service teachers (Merriam, 2009). Furthermore, some committee members looked into the interview guide for validity.

Data Management and Analysis

Data management begins the process of data analysis (Creswell and Poth, 2018). Managing data in any qualitative research starts at an early stage. The researcher organizes the data into digital files and creates a file naming system to ensure materials can be easily located in large databases of text or recordings. Besides, researchers convert data into an analyzable format and make plans for long-term secure file storage (Creswell and Poth, 2018). As Miles et al., (2014) advise, I kept each form and unit of data as a separate computer file in a categorized folder. For example, a folder labeled “observation” contains field notes of events in the class, while a folder dubbed “transcripts” contains corrected transcripts from swivl video recordings of class events, stimulated recall interviews, and participants' screen recordings. Furthermore, I placed all the data, including the reduced (“transcripted” and “corrected”) data, in one “working” file on my personal computer (with password) uploaded on the university office 365 cloud storage for backup.

I used the NVivo program to manage, shape, and analyze the data. An NVivo program does not automatically analyze qualitative data for the user. However, it enables selective monitoring of data and the assigned codes in multiple configurations for the researcher's review of analytic thinking about the assemblages and meanings (Miles et al., 2014). To put it succinctly, despite using the NVivo program for analysis, I still have the responsibility to engage in great thinking (hard analytic thinking) through my internal hard drive (Stake, 1995). In sum, I used the NVivo program to store and organize the data; locate and sort text or recordings associated with a code or theme; retrieve and review familiar passages or segments that relate to two or more code labels. I also used it to compare and relate code labels, conceptualize different levels of abstraction, represent and visualize codes and themes, and finally, document and manage memos into codes.

Data analysis in qualitative research consists of preparing and organizing (as above) the data for analysis; then reducing the data into themes through a process of coding and condensing the codes (categorizing); and finally, representing the data in a discussion (Creswell and Poth, 2018; Miles et al., 2014). Put differently, data analysis is the systematic process of using the information gathered during fieldwork to answer the investigation questions. In the following sections, I described how the various data collected were analyzed in relation to each research question (see Table 5 for a summary).

Methods of Analysis

I adopted interactional analysis procedures. These analytical procedures acknowledged that knowledge and actions are fundamentally social in origin, organization, and use and are situated in social and material ecosystems (Henderson and Jordan, 1995). This analysis ascertains that expert knowledge and practice are not particularly located in the heads of individuals but as situated in the interactions among community members engaged with the material world (Vygotsky, 1978, Henderson and Jordan, 1995, Wertsch, 1998). Unlike behaviorist ideas of knowledge as being individualistic, interactionists look for evidence of learning by tracing the coordination of the mental activities of at least two individuals. Mental activities could be through their conversations (Cobb and Bauersfeld, 1995).

Social interactionists opined that the basic data for theorizing knowledge and practice are rooted in the details of social interactions naturally occurring in everyday interactions among members of a community of practice (Henderson and Jordan, 1995). I transcribed all the recorded data following Schütte, Friesen, and Jung (2019). I readied them for a sequential analysis by following a step-by-step procedure to investigate the internal structure of the interaction. The interactional analysis can include the following steps:

1. Structure of the interactional unit,
2. Displaying transcript of selected sequence
3. General description of each sequence,
4. Turn-by-turn analysis and
5. Summary of the interpretation.

It should be noted that these steps are not a linear sequence of how the data should be interpreted but are principles for the process of interpretation in which several of the steps can be repeated. An interactional unit in this study refers to an entire modeling activity, for example, the disease spread activity. The structure of the interactional unit is the beginning and the end of a chunk that meaningfully constitute a complete analytical premise. In this study, the interactional unit is structured according to themes that emerge within the interaction of the participants as they worked on the modeling activities. The themes emerged using the inductive method of analysis. I presented several sequences (in quotes) of the interactional unit in chapter four to give the readers summaries of the analyzed interactions. Next, I detailed general descriptions of the sequences to give the intrinsic meaning of the text (quotes).

Furthermore, I gave a turn-by-turn analysis of the sequences. The goal is to arrive at conclusive interpretations of consecutive actions: “The question of the turn-by-turn analysis, therefore, is: How other participants of the interaction react to an utterance, how do they seem to interpret the utterance, how is it developed further collectively, what is made of the situation collectively?" (Schütte et al., in ICME13, 2016, pp 117). Lastly, I presented the summary of the interpretation. This summary serves as a basis for generating theory as it represents the transition between the detailed interpretations of the Interactional Analysis and the first theorizations based on these interpretations (Schütte et al., 2019).

To generate themes, I adopted the inductive method of analysis. The inductive method of analysis allows codes to emerge progressively in data. This method helps understand how participants worked together and captured their perspectives on the modeling tasks and the tools. The inductive method was used to answer research questions one and two, while the deductive approach was used to answer research question three. The deductive analysis method allows me to compare data units to mathematical modeling, mathematical thinking, and computational thinking processes. To do this, I used practices (see Table 1) associated with mathematical practices and computational thinking practices as indicators to identify segments of data where participants exhibit related practices. Table 4 shows examples of some deductive codes used, sample codes, and examples in data.

Table 4: Deductive Codes and Examples in Data

Category	Sample code	Example in data
Mathematical practices	Constructing model	Pre-service teachers generate models that represent a solution to the problem.
	Identifying assumptions	Pre-service teachers decide on attributes that are important to answer the question.
	Generalizing	Pre-service teachers show evidence of moving from specific to general.

Table 4 Continued

Computational Thinking Practices	Creating data	Pre-service teachers created data that allowed them to answer the problem.
	Exploring data	They generated geometrical representations that support the data.

Figure 7 (Trochim and Donnelly, 2006) shows the flow of inductive and deductive analysis methods in this study. In the inductive method, I read through interactive sequences, identified patterns, developed tentative hypotheses, and then made conclusions. In the deductive method, I considered the elements of the standards for mathematical and statistical practices (CCSSM, 2010; SET, 2015), developed hypotheses of finding these elements in the data, read through the interactive sequences, and then confirmed the presence of those elements.

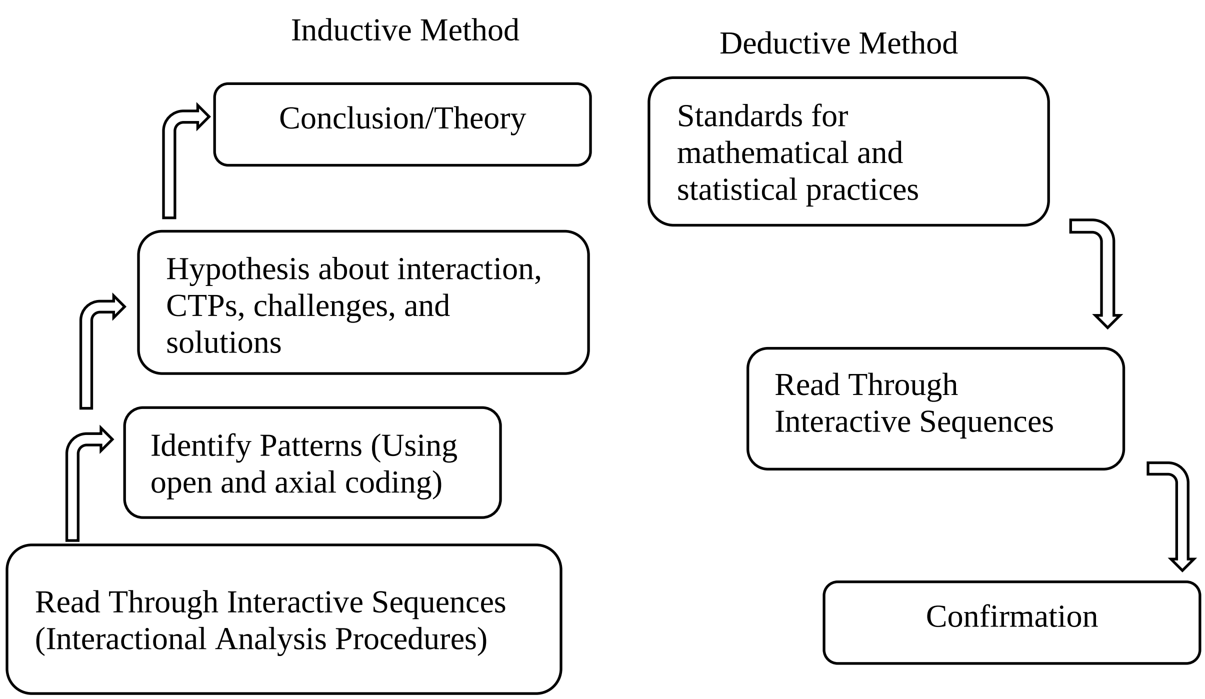

Figure 7: Flow of Inductive and Deductive Methods of Analysis (Trochim and Donnelly, 2006)

Throughout the analysis, I used the process of open and axial coding. The open coding process allowed me to identify and code data units through words/phrases repeatedly used, showing participants' perspectives about constructing computational thinking practices and researcher-generated codes. Also, the axial coding process enabled me to group codes identified during open coding into themes through the interpretation and reflection on meaning. To begin with, I used open and axial coding to analyze the survey data, which is already in written form. Analyzing the survey data went alongside data collection to know more about participants' backgrounds and previous experiences, which influenced the questions I asked participants during the stimulated recall interview. In what follows, I discussed the analysis of each data type with respect to the primary research question(s).

Observations

Analysis of the data collected from close observation of pre-service teachers in their natural setting provided insights to answer research questions one through three. That is: the data provided access to see participants' interactions with peers, instructor, and tools, computational thinking practices participants developed, the challenges they encountered and how they went about the challenges, and the mathematical thinking and practices they used in the modeling process. The challenges participants encountered were mathematical and computational; however, this study focused on the computational challenges and how participants resolved them. Analyzing observational data commenced as soon as the data were gathered to avoid having too much data to deal with after the data collection process (Merriam, 2009). I updated the field notes after every class during data collection to obtain a thorough description of participants' collaborations, how they resolved challenges, and other aspects of the study that I intended during the observation. The idea of updating my write-up was important because it is not always possible to write detailed notes at the moment (while observing). Simultaneously, I gave a reflection (observer's comment) on what was observed. The idea of writing reflections (memoing) enabled me to give interpretation to the data collected. Writing up the fieldnotes and memoing commenced from the first data collected, and this helped me to obtain information about aspects to ask participants during interviews.

For the inductive analysis, I identified any "useful but smallest pieces of information" called segments or units of data that reveal information relevant to the questions and stimulate the reader to think beyond the particular segments (Lincoln and Guba, 1985). Creswell (2013) called this identification "open-coding." By "useful but smallest pieces of information," I refer to portions of the data that can stand alone (structure of interactional unit) to answer what challenges participants encountered and how they dealt with those challenges. At this phase, I was open to anything possible at the beginning of the data analysis to develop codes (Creswell and Poth, 2018; Miles et al., 2014).

I used in-vivo coding - words or phrases repeatedly used by participants, researcher-generated codes, and data units that provided insights into participants' perspectives and worldviews to conduct my first cycle of coding (open-coding) for the data (Miles et al., 2014). The initial codes were grouped into what qualitative researchers called categories or themes. Corbin and Strauss (2007) described the process of grouping codes that seem to go together as axial coding or analytical coding. Richards (2005) noted that this form of coding comes from the interpretation and reflection on meaning. Thus, I engaged in axial coding to group the codes generated from open coding to generate a list referred to by Merriam (2009) as a primitive outline or classification system reflecting the study's recurring regularities or patterns. This list of categories or themes that spanned units of data previously identified became items into which subsequent data were sorted (deductive analysis).

The process described above is termed an “inductive” way of categorizing data units by constantly comparing codes generated to other units of the data corpus (Creswell and Poth, 2018; Miles et al., 2014). Merriam (2009) stated that the researcher would most likely be thinking in a more deductive rather than inductive mode when no new information is coming forth from the data. The researcher is at the level of testing the established categories against the data. In this study, the volume of available data allowed me to reach such saturation. I must note here that my description above was the same whenever I used data from observations to answer any research question or mention interactional analysis procedures or inductive methods of analysis.

I used the swivl video recorder to record pre-service teachers as they worked in groups in the classroom, thus generating video data for analysis. Recordings from the video became another level of data to answer how pre-service teachers in this class interacted with each other and with other mediating means available to generate computational thinking practices. It showed evidence of challenges faced by students and how they overcame the challenges. The video data also enabled me to see how pre-service teachers used mathematics in their work. I watched the video data several times to develop content logs of participants’ activities in the class. Considering the data's purpose, I used the online paid audio/video transcription services (Otter) to extend portions that responded to the research questions by transcribing these portions. I listened to the video data to correct the text data, thereby getting the data ready for analysis. After the initial preparation and organizing of the video data (i.e., conversion to text data and storing it), I read the transcripts in their entirety several times. This reading enabled me to immerse myself in the details, construct an analytic memo alongside, and then employ interactional analysis procedures (Schütte et al., 2019) and inductive analysis method to reduce the text data into themes through categorical aggregation (Creswell and Poth, 2018). I also looked through the video to identify important instances where pre-service teachers used signs or took actions that could not be retrieved as text to communicate and interact with other mediating means. For example, Chloe and Owen left their respective groups to form a new group. This situation was significant as Chloe mentioned during interviews that she was not learning in her former group. Other instances include Asher leaving Owen to collaborate with Ivy and Aurora.

Stimulated Recall Interviews

Interview data were converted into analyzable text (transcripts) through an online paid transcription platform (Otter). I made necessary corrections to the transcripts, read the transcripts several times to immerse myself in the data, and made some jottings (memoing) before venturing into interactional analysis procedures. I then applied the inductive and deductive approaches to analyze the transcripts. For all these methods of analyzing the interview data, I followed the processes discussed under observations since the interview data were converted into a written format. Furthermore, I used the practices identified in Table 1 as indicators (for the deductive method) to answer *what computational thinking practices pre-service teachers constructed as they worked on mathematical modeling activities and the mathematical thinking and practices they used in the process.* I have described the operational definitions of these practices (e.g., Table 4, more in chapter 4) as related to this study.

Student Work and Artifacts

The data comprised solutions to students' group tasks and written reflections on their work. I used these data primarily to respond to *pre-service teachers' computational thinking practices while working on the tasks.* I read through the data and wrote reflections alongside (memoing), open coded the data using inductive and deductive codes, and conducted axial coding to understand how codes related to one another and bring about themes.

Table 5: Summary of Data Analysis Strategies and the Primary Research Questions Addressed

Data	Analysis Strategies	Primary Research Question Addressed
Fieldnotes	Reflected and developed fieldnotes as soon as possible. Open coded transcript using inductive and deductive codes Conducted axial coding across data sources to understand how codes related to one another and to bring about themes Analytic memoing throughout the analysis Interaction analysis procedures	1. How does the presence of computational tools influence pre-service teachers’ interactions with peers and the facilitator during modeling tasks? What challenges do they encounter, and how do they overcome such challenges? 3. What computational thinking practices do pre-service teachers evidence during modeling?
Video	Prepared data in written form (transcription) Watched the video and annotated the transcript Open coded transcript using inductive and deductive codes Conducted axial coding across data sources to understand how codes related to one another and to bring about themes Analytic memoing throughout the analysis Interaction analysis procedures	1. How does the presence of computational tools influence pre-service teachers’ interactions with peers and the facilitator during modeling tasks? What challenges do they encounter, and how do they overcome such challenges? 2. How does the development of computational thinking practices support pre-service teachers' mathematical and statistical thinking practices?

Table 5 Continued

Interview	Prepared data in written form (transcription) Read through to clean data and annotated transcript (memoing) Open coded transcript using inductive and deductive codes Conducted axial coding across data sources to understand how codes related to one another and to bring about themes Interaction analysis procedures	3. What computational thinking practices do pre-service teachers evidence during modeling? 2. How does the development of computational thinking practices support pre-service teachers' mathematical and statistical thinking practices?
Student Work	Report on Solutions and Lesson Plan – The data is in written form. Read through the data and annotated (memoing) Open coded transcript using inductive and deductive codes Conducted axial coding across data sources to understand how codes related to one another and to bring about themes	3 What computational thinking practices do they evidence during modeling?
Survey	Survey data was in written form. Read and annotated Open coded transcript using inductive and deductive codes Conducted axial coding across data sources to understand how codes related to one another and to bring about themes	Helped to gain access to participants' backgrounds and in data triangulation.

<u>Timeline</u>

In sum, Table 6 shows the timeline for data collection, data analysis, and discussion.

Data Collection	Survey	1st – 3rd week of Fall, 2021
	Classroom Observation	Fieldnotes: 1st - Last week of the semester (Fall, 2021) Video: During each modeling activity (Fall, 2021)
	Student Work	Start – End of the semester (Fall, 2021)
	Interview	End of all the modeling activities (Fall, 2021).

Table 6 Continued

Data Analysis	Data Management	1st week of data collection-End of analysis (Fall, 2021 – Summer, 2022).
	Coding	From the first data collected to the end (Fall, 2021 – Summer, 2022)

Ethical Considerations, Trustworthiness and Reliability, and Limitation of the Study

Ethical issues related to participants' protection from harm and disclosure of comprehensive findings are some of the challenges researchers encounter during data analysis and representation processes (Creswell and Poth, 2018). For this study's purpose, I assigned aliases (pseudo names) to avoid disclosing the participants' real identities (Creswell and Poth, 2018). I conducted member-checking of my interpretation and analysis with participants during the stimulated recall interviews and provided opportunities for sharing procedures of analysis and results with participants. These strategies reduced ethical issues related to participants' limited access to analysis procedures and how findings are represented. The strategies also allowed participants to validate or invalidate my interpretation of their participation, thereby enhancing confidence in the data interpretations (Creswell and Poth, 2018). Furthermore, as sociocultural research, I presented multiple perspectives to discuss the complex nature of pre-service teachers' participation in the activities to avoid siding with participants and disclosing only positive results.

Qualitative researchers endeavor to answer two important questions to validate their findings and interpretations. These are questions of whether the account is valid and by what standards, and how to evaluate the quality of qualitative research (Merriam, 2009; Creswell and

Poth, 2018). For this study, I shared the terms used by Eisner (1991) – *structural corroboration, consensual validation, and referential adequacy* and by Creswell and Poth (2018) – *validation to emphasize a process* to establish the credibility or trustworthiness of my findings and interpretations. In structural corroboration, Eisner (1991) recommends that researchers seek a confluence of evidence through multiple types of data to ascertain confidence about researchers' observations, interpretations, and conclusions. Eisner (1991) noted that by consensual validation, researchers seek the opinion of competent others that the description, interpretation, and evaluation of the data are right. By referential adequacy, Eisner (1991) noted the importance of criticism to illuminate the subject matter and to bring about a more complex and sensitive human perception and understanding of the study. Creswell and Poth (2018) used the term "validation to emphasize a process" to assess the accuracy of the findings, as best described by the researcher, the participants, and the readers. Thus, I present strategies to ensure the validity and trustworthiness of this study's results from the researcher's perspectives, the participants, and the readers.

From the researcher's perspective, I engaged in corroborating evidence by triangulating multiple data sources and clarifying the researcher's bias or engaging in reflexivity (Creswell and Poth, 2018). To do this, I sought corroborating evidence from different sources and theories to shed light on a theme or perspective derived from the data. As described in the section under data collection, it is important to note that the initial data generated (e.g., observation and survey) formed an integral part of the subsequent data collection (e.g., interviews). I also used the *in-the-moment questioning* method to provide a truthful interpretation of the data ultimately. For instance, I constantly moved from group to group, observed, and asked questions from participants in situ to be sure I had the correct interpretation of the actions.

From the participants' perspective, I solicited participants' views of the credibility of the initial findings and interpretations by taking analyses, interpretations, and conclusions back to the participants so that they could judge the accuracy and credibility of the account (Merriam and Tisdell, 2015; Creswell Poth, 2018). Furthermore, as reported earlier, a semester-long engagement and observation during the study allowed me to get familiarized with the participants before data collection (Creswell and Poth, 2018), thereby enhancing my observation quality.

From the reader's or reviewer's perspectives, having a peer review or debriefing of the data and research process (Creswell and Poth, 2018), an external check was carried out by members of my committee. They checked to keep me honest by asking questions about methods, meanings, and interpretations. A major limitation to this study was that the study did not investigate the effect of the nature of interaction on the quality of models developed. For instance, the study did not consider how *vertical and horizontal* leaderships found in this study would influence the quality of models developed from within and across groups. Another limitation was my inability to conduct member checking with my participants at the end of the entire analysis. This was because the study was completed two semesters after data collection.

Despite the limitations mentioned above, findings in this study significantly support future teachers' knowledge of content and pedagogy in integrating computational thinking practices in mathematics teaching and learning. More specifically, this study provides strategies to educators when planning to teach computational thinking practices, provides a lens on how pre-service teachers develop computational thinking practices, expands Vygotsky's mediating triangle, and provides directions for future studies.

Summary

In the outgoing chapter, I discussed the methodology adopted for this study. I described the justification for a qualitative research method, procedures for data collection, and analysis. In the last section, I discussed the ethical considerations, issues of trustworthiness and reliability, and limitations of the study. In the next chapter, I present the findings from this study.

CHAPTER FOUR

FINDINGS

Introduction

The first three chapters have laid the foundations for the study, examined supporting literature and the theoretical underpinnings, and the methodology considered to investigate pre-service teachers' construction of computational thinking practices through mathematical modeling activities. Findings from this study show that meaningful forms of interaction lead to quality communication that engages pre-service teachers in computational, mathematical, and statistical thinking practices. Simply put, interaction contributes a great deal to the construction of computational thinking practices and exhibiting mathematical and statistical thinking practices.

This chapter presents the findings suggested by analyzing the data collected. I organized the chapter by discussing the findings of each research question one through three and concluded by summarizing the findings.

Research Question 1

What is the nature of pre-service teachers' interactions with peers and facilitator during modeling tasks in the presence of computational tools?

In research question one, I investigated the types and nature of interactions that pre-service teachers engaged in when they worked collaboratively on mathematical modeling tasks using computational tools to assist them in generating solutions (models). In what follows, I make two claims about the nature of interaction when pre-service teachers worked collaboratively on modeling tasks in the presence of computational tools.

- The presence of computational tools influenced the ways participants positioned themselves with peers and the facilitator during modeling.
- The presence of computational tools influenced the collaborative processes pre-service teachers used during modeling tasks.

Figure 8 shows the two claims and the interactions that manifested during the modeling activities.

Figure 8: Interactions in the presence of computational tools

Claim 1

The presence of computational tools influenced the ways participants positioned themselves with peers and the facilitator during modeling.

The term *positioning* in this study refers to how pre-service teachers identified themselves or others in a social context during modeling activities. *Positioning* can also refer to the roles pre-service teachers assumed during an interaction. I present this claim under two subheadings – *Leadership and Distributed authority.*

Leadership

Data show that the presence of computational tools during modeling created a space for group members who had expertise in using a particular tool to take over the leadership of the modeling process consciously or unconsciously within the groups. In this study, *leadership* describes when a participant identified themselves or another group member as the person who drove the modeling task, computation, coding, and/or interactions. The presence of computational tools naturally introduced how pre-service teachers position themselves with the ability to use the tools. While some participants were less confident in using the tool, others who were skillful in using the tools automatically took charge of the modeling process. Thus, computational use competence puts the less-skillful participants in a position to agree to ideas from the more skillful individuals.

I found that pre-service teachers appreciated activities that enabled them to construct knowledge independently rather than being told what to do by the facilitator. Moreover, they described having the opportunity to drive their learning by themselves as beneficial. For example, Mia stated:

Excerpt 1

> “I think having that collaborative environment is really nice, especially in this class, because the students really get to lead the class, which is cool. So, I thought that was cool that all of us felt like we could lead our project in different directions instead of like, here's what we want you to do for every single thing. So, I thought that was cool that each person took the project in a different direction.” [12/14/21, Interview data].

While this study supports having the opportunity to lead as beneficial, analysis shows that *leadership* could be overwhelming and impact groupmates negatively if some members are dominating during discussion. This impact may be because groupmates who were dominating consciously or unconsciously shut down others from leading or expressing their thought processes; all they did was listen and agree to whatever ideas the dominating groupmates presented to the group. Consider the following excerpt:

Excerpt 2

> Mia: So, I thought we had to develop a model, scenarios one and two, then create a third situation and develop a model for that. I think we need three equations.
>
> Facilitator: Potentially!
>
> Mia: Potentially?
>
> Facilitator: Yeah
>
> Chloe: We are definitely going to explore Scenarios One and Two based on these two questions. And then we are developing a third one,
>
> Mia: Right, yeah. But I'm still going to create an equation for Scenario One and Two to predict.
>
> Chloe: Yeah. Yeah,
>
> Mia: whatever it is.
>
> Chloe: Yeah. Yep. Right. [10/6/21: Video data].

It is important to note that Mia has better expertise in using the R programing language in her group. Excerpt 2 is one of the examples of the interactions where Chloe agreed to Mia's ideas on how to proceed on the disease modeling task. When some of the videos were played back during the stimulated recall interview, Chloe recounted how frustrating it was to learn with someone whose opinion must be respected. Chloe described Mia as a type-A student whom everything had to go her way and did not help explain things to her and Amelia. In her words, Chloe said: "We were kind of just like, I guess, take the lead if you are not going to explain what you are doing." [12/09/21, Interview]. Recalling how the interaction went in her group suggested that she was not allowed to develop her analysis and understanding of the task or express her thought process but was in a situation where she had to agree to Mia's idea to figure out equations that represented the model.

Data from this study further suggest that the presence of a dominating group member lowers groupmates' self-confidence in approaching a task or giving adequate contributions. During one of the lessons, Asher, for example, stated that "I do not know much of Java. I stopped learning it in high school because I sucked at it." [10/4/21, Video data]. Before this comment, Asher stated some ideas on how to go about the disease spread task but seemed to struggle to find ways to implement it in Java. In another example, when I asked Chloe about an interaction that happened when she moved away to join Owen, she had this to say:

Excerpt 3

> *"*...That was a good interaction. I felt it was an interaction where I felt confident in my abilities, and I could share them with someone else. And unlike the disease spread, Mia did not share them with me, so I could not learn. Whereas I had the opportunity to share my findings with Owen so he could learn, and in that process, I learned too*."* [12/09/21, Interview, Chloe]

Pre-service teachers in this study acknowledged being leaders but not dominating ones. For example, Mia and Mateo were very conscious that they drove the modeling processes in their groups; however, they believed that everyone was learning in the groups. Analysis of interactions in their groups and the stimulated recall interview with participants reveal that groupmates basically listened throughout most interactions. Although both Mia and Mateo believed that their actions created opportunities for groupmates to contribute or lead, data analysis shows that groupmates in both groups were only following Mia's and Mateo's lead. For example, Mateo said:

Excerpt 4

> "So, it was kind of doing most of the work. But even then, I didn't really feel that hindered the group because they were still helping, like, figuring out what we wanted to look at and like taking our data and analyzing it. I was mainly just saying like, you know, I was just getting us our data, while they were like using it and then figuring out what they want me to focus the code on." [12/07/21, Interview data].

In the group interactions, Owen and Chloe, respectively in Mateo and Mia's groups, were almost silent throughout the disease spread modeling task. At the same time, Asher and Amelia, who were respectively with Mateo and Mia, forced themselves to contribute to their group's discussions. Asher was seen trying to check Google for an idea that would work for Mateo's usage of the Java program, but in the end, he stated that it was unproductive because he did not know much about programming in Java. An occurrence in the class was that Chloe left her group and joined Owen to form a group of two, while Mateo practically worked on most of the project by himself. One explanation for this move could be that both Owen and Chloe were not satisfied with the discussions in their previous groups. In addition, creating a new group and the form of interactions therein showed that they were not satisfied with the discussions in their previous groups and wanted to work more collaboratively.

I found two forms of leaders that pre-service teachers exhibited. During peer discussions among pre-service teachers, *leadership* could be vertical or horizontal. A *leader* having the vertical trait is that in which ideas flow from top to bottom – mainly from one person to the group. In contrast, *horizontal leadership* occurs when group members construct ideas simultaneously by allowing every member to give input when developing the ideas. A characteristic of *vertical leadership* is that a single group member owns the ideas and preaches them to others in ways that compel groupmates to take the ideas. However, in *horizontal leadership*, the explainer mostly thinks of an idea and gives room for teammates to build the process together. For example, Zoey recounted how they went about extending the disease spread data. Zoey said, “I remember, when we first started looking at it, Aurora was in my group, and she said, I want more data.” From this suggested idea, Zoey and Ivy started to raise ideas on how they needed to recreate features embedded in the available data. For example, they came up with the idea of using the random number generator to introduce probability into the data. Also, they worked together to identify online dice to introduce randomness into the data.

Another important finding is that pre-service teachers who led vertically were vocal and always asked questions if another person was raising suggestions or writing codes. However, that was not the same with pre-service teachers who led horizontally. These set of pre-service teachers reported that knowledge should be constructed together rather than one person authoring and owing to the ideas. Thus, they became frustrated when working in a group with groupmates who led from top to bottom (e.g., Excerpt 3).

In the outgoing section, I established how the presence of computational tools in a modeling space created opportunities for group members to lord their computational ideas onto others. I also presented *vertical and horizontal leadership* as forms that participants explored during an interaction. In the following section, I present findings related to *positioning* in terms of *distributed authority*.

Distributed Authority

I explored the positioning of participants when they faced challenges and shared them with peers or facilitator who helped resolve them. In this context, distributed authority refers to entities and solution-seeking network participants considered to help them resolve challenges. Students often turn to peers and their instructor when faced with challenges. However, the presence of computational tools during modeling expanded solution-seeking networks from peers and facilitator to other resources whose ideas counted. For example, Mateo considered using online resources to resolve his challenges rather than reaching out to peers or the facilitator. One explanation for such a solution-seeking pathway is that Mateo positioned online resources over reaching out to peers or the facilitator. Nevertheless, this study reveals that the facilitator, in most cases, was the last stop during a solution-seeking process. However, there were a few times when the challenges could not be resolved even with the presence of the facilitator. In such situations, participants persevered to resolve the challenges themselves. Figure 9 shows the forms of solution-seeking networks found in this study.

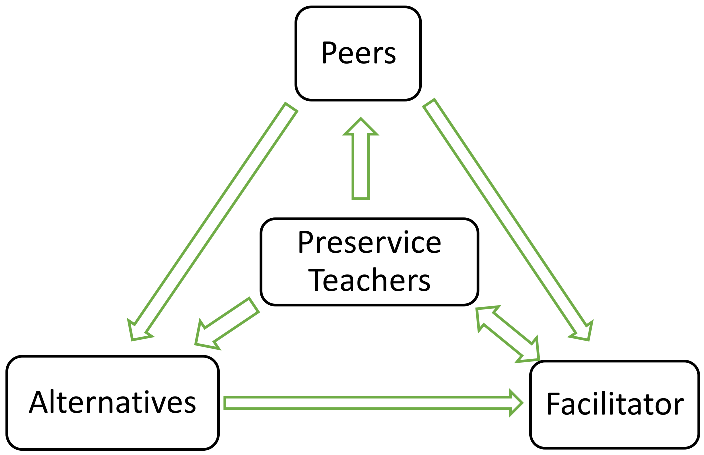

Figure 9: Solution-seeking networks when pre-service teachers resolved challenges

When a problem was not resolved at each node (peers, alternatives, and facilitator), they moved from peers to the facilitator or from peers to the alternatives. In another approach, the pre-service teachers having challenges turned to peers, and if the issue was not resolved, they used an alternative approach. If the solution was not found, they moved on to the facilitator. The following paragraphs show how participants used the solution-seeking network during modeling.

Pre-service teachers interacted with peers to share their challenges within the group and later across groups to further explore the possibilities of resolving the problem with peers. Excerpt 33 (Appendix D) shows the interaction within and across groups between Mia, Amelia, Chloe, and Owen to figure out why Mia and Amelia's graphic representation of the bobcat population differed from what Owen and Chloe had. The interactional unit also shows how the facilitator supported the two groups by guiding their thought processes. Although the facilitator intervened, pre-service teachers kept investigating how to fix the problem, which Owen eventually did. That is, the facilitator's intervention in the interaction did not in any way lead to how the problem was resolved; therefore, the data in Excerpt 33 (Appendix D) is an example of

Preservice teachers – Peers' interaction to resolving challenges.

Data also reveal that pre-service teachers interacted directly with the facilitator to resolve their challenges. Consider the interaction below:

Excerpt 5

> Mia: The 1/3 is saying like I met someone who was sick, and I have a 1/3 possibility of getting sick. We're trying to find the probability for how likely it is that I'll come across someone who's sick, plus their certain probability of them actually giving it to me.
>
> Facilitator: So, to me, it looks like you're on the right track. Um, because you've got the set-up, the first part is a compound probability. So, you've got your chance of getting the disease at a one out of four and 20 chances of encountering a person who's already sick, right?
>
> Mia: um uhm
>
> Facilitator: And why are you dissatisfied with that? That seems reasonable!
>
> Mia: because I want to be able to vary, like in our new one that we're creating, we're changing it from 1/3 to one half. So, I want to be able to change like type in ¼.
>
> Facilitator: … So, one way you might think about it is like creating some reference cells. So, I made one sheet with my parameters that I was going to look at, like what was important to me. So, my probability of infection, this is a regular rock, paper, scissors scenario, my population size, the number of people I have initially infected, and how long they're sick. And then my two models, so these are actually scenario two, and then some people scenario three, so recovering and immune, recover and not immune in the two models. [10/6/21: Video data]

In the example above, Mia wanted to vary the probability of infection but could not set it up in the spreadsheet. After discussing the issue with the facilitator, she learned the idea of creating reference cells. This is an example of pre-service teachers – facilitator interaction when resolving challenges faced by pre-service teachers. Interestingly, creating reference cells became another challenge for Mia, which she struggled with in the following class. In the following class, she discussed this newer challenge with Amelia, who also did not know how to succeed with the spreadsheet program. During their interaction, the facilitator came to their rescue to give them the ideas needed to create reference cells in spreadsheet. Excerpt 6 gives the interaction, which also serves as an example of Preservice teachers – Peers – Facilitator interaction in solving a problem.

Excerpt 6:

> Mia: In spreadsheet, do you know how to reference a different sheet? That's what I left not doing last time.
>
> Amelia: No. So, I went through and typed them all in separately because it kept wanting to refer to the wrong cells.
>
> Facilitator: Did you want it to refer to the same cell every time?
>
> Amelia: Yeah,
>
> Facilitator: Use the $ sign Infront if you want it to keep going back to b2; like =b2
> Amelia: yeah, because it kept referring to the one below it and I didn't want it to
>
> Facilitator: Right, I think it's called an absolute reference versus like referencing the cell above me
>
> Amelia and Mia: Yep. Got it. [10/6/21, video data]

Another type of interaction is Preservice teachers – Alternatives. In this interaction, pre-service teachers directly consulted Google and other forms of online resources to figure out solutions to the challenges they were experiencing. Although this study shows that pre-service teachers used this method, it remains unpopular among most participants in the study. One of the participants, Mateo, responded to why he used this method, unlike his colleagues who asked peers before asking the facilitator or using the alternative routes.

Excerpt 7:

> Researcher: Did you use any human resources to resolve your challenges?

> Mateo: I might guess, I guess, if you count, like, the people on forums as a resource like, you know, eight-year-old questions on like a Stack Overflow form of like, how do I get this as a human resource? But if not, then I count that it's just like the general level.
>
> Researcher: What about Classroom?
>
> Mateo: Classroom? I don't think so. No.
>
> Researcher: Instructor or your colleagues
>
> Mateo: No, all my resources and help were from searches.
>
> Researcher: why?
>
> Mateo: Yeah, so I mean, I'm not like opposed to asking other people for help. I'll definitely ask if I need help. It's just that like, I'm kind of much more independent, and especially on online stuff like this, like, I've spent a lot of time, you know, in the computer science classes, just figuring out how to get the best resources online, and how to do online, stuff like that. So, it's like, I didn't really feel the need to go up to someone, and hope that they had an answer to my question when I knew, like, if I just do this, then I can find the exact solution I need. And also, just because like, once again, they weren't that big, they were just like, how does this work? You know, if I were to go up to someone in class, and they're like, oh, I also don't know, then, you know, it's like, that didn't really help. And then I was stuck again, and once like, I just look it up. So, the problems weren't really requiring, or my point, at least not really requiring that much of a big collaborative exploration to them. It was more just like, you know, I can really quickly find this exact solution I need online, so I'm just gonna do that to save my time and other people as well. [12/07/21, Interview]

It is evident from the data above that Mateo's background in computer science influenced his decision to use a human resource in the classroom or not. He disclosed during interview that he was an expert in using Java compared to other persons in the class. This personal conviction made him prefer the alternative routes to others.

Lastly, the type of interaction pre-service teachers showed when resolving challenges is the Preservice teachers – Peers – Alternatives – Facilitator network. In this approach, pre-service teachers with challenges asked peers for solutions and then used the alternative method if peers could not solve the problem. In the end, they considered the facilitator when they could not get a solution using the alternative method. When Zoey was asked how she solved the problem of referencing a single cell, she said: “ I think I first asked my group if they knew. And if they didn't know that I typically googled it. And then if I couldn't find it, then I would ask the facilitator.”

In the outgoing section, I discussed findings regarding how pre-service teachers interacted with peers, facilitator, and other forms of resources to address their challenges. I identified five forms of interactions when pre-service teachers attempted to resolve challenges. Figure 9 gives the diagrammatic representation of pre-service teachers with modeling challenges turning to peers, facilitators, or alternative approaches.

Claim 2

The presence of computational tools influenced pre-service teachers' collaborative processes during modeling tasks

I identified four processes through which pre-service teachers engaged in interactions during modeling. These processes are: dividing and offloading labor, accommodation, giving and receiving feedback, and refining ideas. The presence of computational tools influenced how participants engaged in these processes during interactions. The following sections detail how pre-service teachers engaged in the processes during interactions on modeling activities.

Dividing and offloading labor

The presence of computational tools during modeling created situations for some group members to be more skillful in using the tools, thereby making participants divide labor amongst themselves according to expertise. *Dividing and offloading labor* refer to situations where pre-service teachers were tasked to carry out a part of the modeling processes on behalf of other group members. It involves situations where a group member checked peers' work to authenticate the ideas. This type of interaction also involves participants getting the tool to do computations or produce visuals while telling the computer what to do (coding).

Participants in this study had varied strengths, especially with computational tools. Using computational tools during modeling activities created the need for pre-service teachers to engage in division of labor so that group members could work on aspects of the task they had better abilities. Participants in this study reveal that group members divided tasks among members per individual specializations. Participants sometimes referred to these specializations as individual strengths that helped them move the task forward. When the researcher asked Mia and Zoey how learning evolved for them and other group members when working collaboratively on the modeling tasks, Mia said:

Except 8.

> "Um, I think we all relied a lot on each other's different strengths, which was cool. So, if I rely on your strengths in this part of the project, you rely on my strengths in other parts of the project. At the same time, we are getting better at each other's strengths, because we see what the other person is doing" [12/14/21, Interview data].

Except 9

Zoey said:

> "I think any group I worked with, we had different strengths. So, like one person might figure out a formula to use, and then they would share that with us, and we could help them with the data. It's just that everybody had a different take about how to go about the tasks" [12/08/21, Interview data]

In the excerpts above, it is evident that group members possessed individual specialized capabilities they could rely on to achieve a common goal. The specialties participants possessed include a better ability in mathematics and using the computational tool for coding. During the individual demonstration of such specialties, other group members could learn from their peers to get better at others' capabilities, thereby contributing to the positive effects collaborative learning had on participants. For example, Mia further explained how allowing those who were better at using the spreadsheet program helped her learn to use the spreadsheet program in the class. She said: “In spreadsheets, I definitely did not have the skills. I do not think I had the skills that everybody else did. And so, someone else would do the spreadsheets, and then I would try to do it with them.” [12/14/21, Interview]. Mia’s statement shows that dividing labor resulted in knowledge interdependence – sharing knowledge during learning. In addition, it extended beyond pre-service teachers' specialties to use computational tools to get things done faster and more accurately by offloading labor to the computational tools. For example, one participant, Chloe, said, “Making computer do what is in my head but much faster and more accurate.” For example, instead of counting the S, R, and I’s in the disease spread data by hand, participants used the COUNTIF function in excel.

Participants in this study revealed that they could concentrate on their areas of specialization, thereby reducing the workload that could have led to enormous stress as they worked on the task. In Chloe’s words, she said:

Except 10.

> “I think being in groups is also another way that eases my mind of having to do so much work. It's kind of like a brain trick I play on myself like, if I'm sharing the work with another person, I don't have to stress myself out enough to like, do the whole thing. But if I'm like, you're assigned to do this alone, I stress myself out and I want to perfect it [12/09/21, Interview data].”

In sum, participants engaged in labor division to ensure that individuals' expertise and specialties are explored to build a useful model. Such interaction allowed pre-service teachers to share the workload, thus reducing stress among group members.

<u>Giving and receiving feedback</u>

Giving and receiving feedback is a form of joint venture referring to a give-and-take interaction between participants and the computational tool via the user interface. The interactions involved testing ideas and getting feedback that amplified participants' cognitive strengths. Amplifying cognitive strengths means that participants' ideas produced a desirable outcome. They moved on with the other parts of the model development or had a situation where they had to modify their ideas and test again. In any case, participants learned something through the feedback on the interface.

Pre-service teachers interacted with the computational tools by inputting code ideas they had on the interface. This interaction can be referred to as testing coding ideas and getting feedback. This study found that pre-service teachers interacted with the computational tool at an individual, within the group, and across group levels to test their initial ideas. They either got a desirable output, an error, or an undesirable output as feedback. When the code ran and pre-service teachers got a desirable output, they moved on to other parts of the modeling process.

Suppose pre-service teachers got an error or an undesirable output. In that case, they would find ways to resolve the problem and then test the refined ideas with the computational tool. The output from the tool's interface would become the medium through which pre-service teachers received feedback on their input. Sometimes, the tool suggested where the error came from or what the error was about. Through this feedback, pre-service teachers learned that the ideas they used for the coding worked or needed to revisit the codes. I found that pre-service teachers could identify a situation whereby their code would run but could obtain an undesirable output. Such a situation occurred because pre-service teachers knew how the outcome of the code should be on the user interface. This situation means that pre-service teachers usually knew what the computational tool should return as feedback. When they got a different result, they had to revisit their computation.

My understanding that pre-service teachers knew how the output should be suggests the "giving and getting feedback" nature of interaction in this section. Put differently, pre-service teachers knew what to do and how to achieve that but required the speed, efficiency, and the ability of the tools to process the large dataset to obtain the desired output. Thus, while pre-service teachers needed the tool's speed, efficiency, and capabilities, they needed to provide codes that the tool could execute. Chloe described this "*giving and getting feedback*" process between her and the tool as making the tool do what was in her head but with speed and versatility. Summing these ideas gives an operational definition of computational thinking as the process whereby users understand what the computer is doing versus what they are doing – that is, knowing what the user inputs into the computer and what the computer is telling them, and how the computer carries out the process.

The interaction in this situation can be likened to a human-computer exchange requiring users to input some instructions that the computer will understand. These user's instructions are the cognitive ideas (initial and refined) pre-service teachers possessed to control the computational tool. When pre-service teachers interacted with the computational tool, they interacted cognitively by typing codes that the computer would understand. In addition, they cognitively decoded the feedback from the computer interface. In this study, data show that participants first developed a mathematical idea, coded the mathematical idea, and finally got output as the feedback that determined the next step in the modeling process. Consider the excerpt below:

Excerpt 11

> "I wanted to figure out something that will count for me. So, I think that's my thought of using the countif function - to figure out how many susceptible people there were; how many infected people were there for each student in each round" [12/09/21, Interview – Chloe].

Excerpt 11 refers to Chloe's interaction with spreadsheets to use the COUNTIF function. The analysis of Chloe's interaction with the spreadsheets program when using the COUNTIF function reveals back-and-forth attempts of inputting and receiving feedback through the user interface. In her initial attempt, she typed in s+Q2A17B2D2:V2 and got #NAME? – an error that signified that something needed to be corrected in the syntax. She tried again by typing =COUNTIF(B2:V2,s) and got zero as feedback. Although "0" is not an error, Chloe knew that the code was not doing what she wanted or was not outputting a desirable result. Next, she went to Google to look it up. By typing the COUNTIF function in the search portion, she got the following example, =COUNTIF(A2:A5, "London"). From this example, Chloe realized what was missing in her code and then modified the code to =COUNTIF(B2:V2, "S") and arrived at a desirable output of 20.

In the above computer - human interaction, Chloe was in cognitive control of the instructions inputted into spreadsheet. She possessed the mathematical idea of counting the number of susceptible and infected. However, she wanted spreadsheet to apply its speed and versatility in achieving the desired output. Thus, she needed to develop a mathematical code that spreadsheet could understand and interpret to give the expected output. During the interactions, Chloe was supplying the codes while spreadsheet was automating the counting process with speed and accuracy. At the same time, Chloe could think for the program on how to write the code to yield a desirable output. In addition, Chloe's ability to know that zero was not a desirable output solidifies that users should know what the computational tool should be outputting or when they have not arrived at a desirable output. On the part of the tool, spreadsheet communicated to Chloe by giving #NAME? to alert her that there was an error with the syntax. However, it required a cognitive ability of Chloe to fix this error to obtain the desired output. The human-computer interaction described above is an example of the giving and getting feedback between pre-service teachers and computational tools during modeling.

Giving or supplying codes to the tools and getting feedback enabled participants to offload labor to the tools and persist in solving problems by trying and adjusting codes to obtain the desirable output. In addition, giving and receiving feedback interaction also featured within and across groups where participants discussed what they had done in expectation of hearing the perspectives of the others to move their model development forward.

Accommodation

Accommodation refers to situations where participants respectfully welcomed others' ideas and found ways to move their solution forward through the ideas. For accommodation to hold, peers listened to the group's presenter(s) and then moved on with the suggested ideas. In a sense, some participants were positioned as leaders while others were positioned as listeners. This situation is a direct consequence of the presence of computational tools that introduced *leadership* during collaborative work. While we may think group members understand the processes discussed or the ideas presented to remove further actions such as questioning the explainers' ideas, data show that sometimes, participants agreed to the ideas with an underlying frustration. The agreement might not be because they learned anything from the ideas or understood the process. In general, this study found that accommodation can be of two types – one that enhances the collaborative process, such as refining ideas within the group, and one that leads to frustration and termination of the collaborative process in the ongoing discussion. Next, I discuss these two types under Peer – Peer accommodation and Students – Facilitator accommodation.

Peer–Peer Accommodation

During a discussion on how to go about the modeling process, pre-service teachers accommodated others' ideas, allowing them to further use the collaborative process of ideas refinement in the ongoing discussion. In the interaction, they agreed to carry out the ideas suggested by group members by refining the ideas to achieve a common goal. More on ideas refinement is discussed below. During Ivy, Aurora, and Zoey's work on the disease spread task, Aurora suggested that they should extend the data beyond 20 rounds. Each round represented a physical simulation of an infected person moving around in the system to meet persons who could contract the infection. Group members were excited about the suggestion and decided to explore the idea. Zoey recounted that it was a group decision to increase the number of rounds. She said: "I remember, when we first started looking at it, Aurora was in my group, and she said, I want more data." She further stated that the facilitator told them they could create more data and eventually explored Aurora's idea. The acceptance of the idea launched group members to find different ways to create data that possessed the characteristics of the data provided by the facilitator. They used an online aid to roll dice and generate random numbers. When she was asked how they extended the data, Zoey said:

Excerpt 12

> "We used the random number generator to mimic the chicken dance we did where it was randomly going into someone. And then Aurora came up with the idea of rolling the dice. And then when I copied over, like two beats one, those three things would determine if the person they're seeing becomes infected. And then I would go and fill in that row and then carry on to the next." [10/6/21, Video data]

Zoey's account of how they increased the data points shows she was excited about the exploration. She also stated how extending the dataset allowed her to learn about different ways to introduce randomness in the data. Alternatively, a similar interaction in another group shows that a group member agreed to an idea with an underlying frustration. Recall Excerpt 2, which detailed how Chloe agreed to Mia's ideas but with an underlying frustration that she disclosed during the stimulated recall interview.

Students – Facilitator Accommodation

The roles of the facilitator during modeling processes included guiding pre-service teachers' thought processes, initializing interactions, questioning pre-service teachers' thought processes, and being resourceful. These roles necessitated that the facilitator and participants engage in some interactions that resulted in accommodation from students. Similar to the discussion above, this study found that interactions between the facilitator and pre-service teachers could lead to accommodation that promotes collaborative process of ideas refinement or accommodation that frustrates students. Data show that interactions between pre-service teachers and the facilitator resulting in accommodation mostly promoted collaborative processes of ideas refinement.

However, cases where accommodation led to frustrations were also found in the data. For example, while working on the disease spread task, Mia wanted two independent variables (number of rounds and probability of getting infected) to predict the number of infected people. Before this time, she suggested that she wanted to find equations representing the model she was building, and the facilitator responded that she could do that. Mia believed she could comfortably achieve that with the R program but not with spreadsheet. Although Mia moved forward with R, she later said it would have been better if the instructor showed her how to do it in spreadsheet to add the knowledge to her knowledge of spreadsheet. In this sense, the presence of R programming created situations that led to a participant agreeing to the facilitator's ideas but with frustration. This situation could result from the facilitator trying to allow Mia to use the tool she was comfortable using.

In the outgoing section, I discussed accommodation as a form of interaction in a modeling class with a computational tool. Although one may anticipate a similar interaction in any learning space using group tasks, the presence of computational tools that influenced participants' *positioning* during learning made accommodation pronounced in this study. That said, accommodation of ideas could be from pre-service teachers to peers or from the facilitator to pre-service teachers. In either form, accommodation of ideas could enhance the collaborative process that warranted participants to refine ideas or lead to frustration and termination of the collaborative process in the ongoing discussion. The following section thoroughly discusses findings related to the collaborative process of refining ideas during modeling in the presence of computational tools.

Refining Ideas

The presence of computational tools introduced challenges for pre-service teachers when going through the modeling processes. The challenges they faced included coding in the computational tool to obtain a desirable output. Thus, they developed initial ideas that necessitated refinement through negotiations with peers and the facilitator. Earlier, I discussed findings related to interactions between participants and the computational tools, how users supplied codes to the tools, got feedback, and then retested if there was a need. This ongoing section focuses on interactions between pre-service teachers within and across groups interactions whereby they took some actions to refine their ideas. In this case, participants might test the refined ideas with the tools.

During interactions that involved refining ideas, participants engaged in a back-and-forth probing using mathematics and statistics, personal experience, and questions to ensure that they have admissibly shared meaning of the situation. During this interaction, mathematical and statistical concepts were used to negotiate the feasibility of the ideas or approaches raised by a group member. In the process of probing, participants developed analytical and communication skills. Consider a within-the-group interaction between Owen, Asher, and Mateo to identify the initial number of infected persons during the disease spread task:

Excerpt 13

Asher: one person initially infected?

> Mateo: So are we saying like when we have one person at each round, they just only see one other person and so on. So, what is the chance that we have (Asher interrupts)
>
> Asher: Uhm one in three?
>
> Owen: The odds change each time you come into contact with him reversing or keeping positive
>
> Mateo: uhm, I think it's.... (Owen interrupts)
>
> Owen: It's like when you flip a coin, you get one or two chances to get it right. Then you go flip it again, you got another one two chances to get a certain situation.
>
> Mateo: I think that's if you're trying to decipher picking out like a heads and tails or heads and heads, but these are isolated events. It's for that one. Although we could apply that to see if like what are the chances that they continue infecting people over time. But in each case, it would just be similar. And scenario one constant for two, the same thing, but just after three rounds.
>
> Owen: Oh, yeah [10/4/21: Video data].

During modeling processes, pre-service teachers relied on their mathematical knowledge to probe the ideas of others. In the discussion, Mateo and Owen exchanged ideas from probability to inform the choices they were making for the model. While Owen thought they should look at it as a 50:50 chance, Mateo suggested they look at it differently as the events were isolated and not as Owen described them.

This study shows that pre-service teachers debated among themselves when deciding on choices, assessing models, trying to figure out a problem/error, or experimenting. Pre-service teachers also mostly debated with the facilitator to discuss possibilities with the computational tools and how to proceed with the ongoing modeling task. While probing is mainly initiated by both the facilitator and pre-service teachers, pre-service teachers mostly initiate a debate within and across groups. However, the facilitator might come around to contribute to the ongoing argument. In the excerpt below, pre-service teachers were confused about how to go about the simulation for the bobcat population as structured in the task (see appendix). Consider the following excerpt:

Excerpt 14

> Mia: 50 rounds for 20 years.
>
> Facilitator: Okay. So, it runs a simulation across 20 years. And so, each time you need to go into that distribution, and pull out a different thread.
>
> Mia: So, we need to simulate 20 years 50 times?
>
> Facilitator: Yes,
>
> Owen: yes. So, you can.... (Mia interrupts)
>
> Mia: Okay, so we have, like, the end population after 20 years, right? So, like, do you want us to simulate that number 50 times or the N population for each year, but three times, so like, 50 simulations of your 150 simulations?
>
> Facilitator: So, the way that I set out is like if you put your years like zero to 20. Every year, this column is a year, that's one simulation, and it's going to use one birth rate, one death rate. All the years, like the way that I interpreted it; all deviation get that rate: birth rate, death rate. Does that make sense? Like I see what you're saying. I would say like year one has a different birth and death rate from year two from year three from year four in a single system simulation. Was that what you're saying?
>
> Amelia and Mia: Yes, yes.
>
> Facilitator: Like, yeah, the way I'm thinking about this is, you go into your distribution to get your survival rate and you use those to simulate............ You go again… [10/18/21, Video data]

Here, pre-service teachers debated with the facilitator to move the modeling process forward, especially when they were confused about what next to do. This type of debate helped participants clarify any misconceptions or confusion regarding the task, and in the process, they refined ideas that needed refinement.

Summary

So far, I have attempted to answer research question one – how the presence of computational tools influenced pre-service teachers' interactions with peers and facilitator during modeling activities. I presented findings to the question using two claims: the presence of computational tools influenced participants' *positioning* and interactional processes. I discussed findings related to *positioning* under *leadership and distributed authority* and the findings related to *processes* under *dividing and offloading labor, accommodation, giving and receiving feedback, and refining ideas.* The following sections present the computational thinking practices developed through the above processes.

Research Question 2

How does the development of computational thinking practices support pre-service teachers' mathematical and Statistical thinking practices?

The nature of the tasks used in this study shows the dimensions of processes and proficiencies that pre-service teachers are expected to develop in the class. Since the activities were primarily data-driven, findings in this section correspond to the processes and proficiencies used by pre-service teachers that complement their statistical and mathematical content knowledge. Thus, I present the findings to research question two by exploring participants' mathematical practices through a statistical lens, focusing on common core practices of modeling with mathematics and using tools strategically (CCSSM, 2010; SET, 2015).

Mathematical Practices Through a Statistical Lens

In the following sections, I will discuss definitions of each mathematical and statistical practice and then identify elements that characterize such practices in this study. By "element", I am referring to attributes of each mathematical or statistical practice. For example, applying mathematics to answer statistical questions, using equations to describe how one quantity depends on another, or using geometrical representations to describe structure, identifying important quantities, interpreting mathematical results in the context of the situation, and possibly improving the model will count as elements of "model with mathematics" (CCSSM, 2010; SET, 2015). Also, considering the available tools, detecting possible errors, using technology to visualize the results of varying assumptions, exploring consequences and comparing predictions with data, and identifying relevant external mathematical resources will count as elements of "*use appropriate tools strategically.*" In general, the description of each practice will identify the elements that are supported by data in this study. Furthermore, I will discuss key findings that extend our knowledge of the elements of each practice in the context of pre-service teachers in this study.

MSP4 Model with mathematics

When building a model, pre-service teachers acknowledged the importance of knowing the mathematical ideas that back up the modeling processes. Zoey stated during the interview that "knowing the mathematics reinforced what she was trying to do with the data". She added that she had to "know the math she wanted to use before she could create a meaningful model." In addition, using computational tools aided pre-service teachers in thinking about how they used mathematical concepts and why they used them. For example, Chloe narrated that "working with a spreadsheet helped her translate her mathematics brain to code format". Thus, knowing the math and the ability to code are interdependent in that the former helped her understand what the tool was doing computationally. She suggested that "a con of using a computational tool for modeling is not knowing what the tool is doing mathematically, which will eventually impede the interpretation of the results in the context of the problem." In what follows, I will discuss empirical support from this study focusing on how pre-service teachers applied mathematical equations and graphical representations to solve real-world problems. I will briefly discuss how they identified important quantities before moving to how they interpreted mathematical results in the context of the problem and possible suggestions to improve the models. Lastly, I will discuss an extension to the standard for mathematical practice four to include - students proficient in identifying existing models and comparing them to identify what works best for the situation.

Working on activities in this study created opportunities for pre-service teachers to apply mathematical knowledge in solving real-world problems. Such an opportunity helped them see the actual applications of mathematics. Consider the excerpt below:

> "I really liked that we worked on those tasks. Modeling is like, taking a real-world problem and applying our math ideas to it so that we can fix the real-world problem or begin to think about it. So, I thought that was super cool. And it made me want to do the math more. And so, like, if I didn't get something, I was more inclined to work harder at it. So, I think in that way, it strengthened my math skills because I had more perseverance in it because these problems meant something, especially like our Ilesha project was great because it was even in this state, and the rest of them are super useful too, like, obviously, the disease project has to do with our world today like all these problems are things that we actually have to worry about and deal with. So, I think working on activities that matter strengthened my mathematical perseverance" [12/14/21, Interview data - Mateo].

The excerpt above shows that activities that have real or immediate impact on modelers' society can lead them to persevere mathematically. Mateo described this adventure as seeing the practical usage of mathematics because it allowed them to see beyond learning the mathematical concepts. Now, they had the opportunity to learn mathematics in the context of known real-world problems, which made the math work fun to deal with.

In another example, when Zoey discussed the bobcat population model during the interview, she said, “I took the initial population, multiplied that by the birth rate (or growth rate). Then, I added that to the previous bobcat population. So, I used the cell above to multiply and then added what the new population would be” [12/08/21, Interview data – Zoey]. Here, Zoey explained how she created the mathematical model that later translated into what she inputted on the computational tool.

Participants in this study demonstrated *making sense of quantities and their relationships* in when working on modeling activities with computational tools. They analyzed and interpreted the mathematical equations of graphical representations (models) they derived when creating models. For example, during the stimulated recall interview, Zoey discussed how her group used the bar chart to make sense of the susceptible, recovering, and infectious populations and how they were changing perspectives toward each other over different rounds. Pre-service teachers also made sense of quantities in a computational model by analyzing and interpreting how the dependent variable related to the independent variable. For example, participants interpreted the mathematical equation - Number of Infected = -7.127e-16 + 2.228e-16*Round + 63*Probability of Susceptible Person Being Infected. They documented how the equation responded to different values in their simulations. Thus, modelers make sense of quantities, how they relate, and how the independent quantities contribute to the dependent quantity.

This study found that pre-service teachers transferred statistical knowledge into interpreting and assessing mathematical models – equations and graphs. For example, participants used their understanding of the R-squared values to identify a useful mathematical model. Mia annulled the usefulness of the equation $y = 0.0914x^2 - 0.488x + 1.3297$ as a good predictor for the probability of infected persons in the disease spread task as she regarded and the y and x values as translations of each other. Thus, Mia and groupmates reconsidered other polynomials for what they considered fit. Similarly, Zoey used the idea of R-squared value to identify what equation(s) fit the models they had in her group (Excerpt 29). The awareness of statistical principles (e.g., r-squared value, variability in data) is important for modelers to draw conclusions from data and assess models since it is expedient to communicate how they arrive at such conclusions.

In addition to the elements identified from CCSSM (2010) definition of the mathematical and statistical practice 4, data in this study suggest an extension. Statistically and mathematically proficient students, in addition to building models, can identify existing models and compare them using some statistical or mathematical principles to select the best model for the situation under investigation. Participants identified existing models that could fit the situation they were exploring. For example, Mateo described using the logarithmic growth function to explore the Bobcat population problem. To be successful in using an existing model, modelers must be aware of the behavior of the model, its relevance to the situation, and its limitations. They must also develop proficiency in interpreting how the model explains the situation they are modeling and suggest some improvements to the assumptions that would make the model they found better explain the problem. Furthermore, modelers who have developed this proficiency will also be able to compare different models, including the models they built, and use statistical and mathematical known ideas to select the best model for the situation. Consider the following excerpt:

Excerpt 15

> "I used this Logarithmic growth function for a population starting at 100 and having a carrying capacity of 200. And then to get our Bobcats to fit that group, the line for their group is going to be like a little bit above it all the time, based on the data trucks that we have. So, we take that number in that specific year minus whatever year we get whatever value we get from that function. And that results in the number of bobcats we can hunt. So, randomly, let's say if we had like 150 bobcats this year, we subtract that number from this number. And then we just hunt that, and I'll force them to fit on this curve. Compared to the other part of it, this one is a little bit kind of realistic. Because we reach about 200, like 250 to 300 years later, but it sounds a bit slow. So, this one is not realistic" [10/20/21, Video data].

In Excerpt 15, Mateo and his groupmates realized that the model they had found was not realistic and termed it a *slow model*. They manipulated this model to fit the situation whereby they could get the Bobcat population to stabilize at 200 in about 50 years. Their third adjustment to the logarithmic model got the Bobcat population to stabilize in about 100 years. In another group, Chloe and Owen compared two models (linear and exponential) that they built themselves. From the three models considered in Mateo's group, they were able to identify the best model that stabilized the population at 200 in the least number of years. Similarly, Chloe and Owen concluded that the exponential model presented a better model when compared with the linear model. Thus, students who have developed the art of modeling with mathematics will also understand how to find existing models, tweak or manipulate them to suit the situation, and then select the best that models the problem. Consider the following excerpt:

Excerpt 16

> "So, for part B, where we were asked to find where like stabilizers occur at 200, or where the population would reach 200, we figured we would look at the graphs more than the equations. So, looking at the graphs, we thought it would be linear. However, looking at it further, it's more definitely exponential growth. So, taking that into account, we changed those linear equations to exponential equations. And so, when we graphed those, we could see better how that would stabilize eventually. But in thinking about that, when we used our table of values and stretched it out to get to 200, we got around 42 years." [12/07/22, Interview, Mateo]

Chloe and Owen compared linear and exponential graphs to identify models that best represent the situation in the excerpt above. Comparing models across the two groups, Chloe and Owen's model yielded stability at the least number of years. While Mateo and his groupmates arrived at a model that stabilized the Bobcat population around 50 years, Chloe and Owen developed a model that stabilized the Bobcat population in about 42 years. I consider this, an important students' move that should add to the extension of being proficient in modeling with mathematics. Thus, modelers should also not just find existing models but also build theirs and vice versa.

<u>MSP5 Use appropriate tools strategically</u>

From reporting findings in this study from research question 1 to this point and continuing in research question 3, I document how participants used various tools to build their models. An extension (or contribution) to this mathematical and statistical practice five from this study is that mathematically and statistically proficient students will be able to use known tools, not just available tools, and can adapt to using new ones. For example, Mia and Mateo, respectively, had mastery of the R and Java programs but could adapt to using the spreadsheets program, which was the class's primary (available) tool.

In addition to considering and using available tools, participants in this study recognized the opportunity to learn mathematical concepts through the computational tool. For example, Chloe and Zoey stated that “writing codes in spreadsheet strengthened their algebraic knowledge of mathematics”. Mateo recognized that coding in Java transcends just knowing the math. That it extends to understanding how it applies to other aspects of model building, especially its setup in the coding space. Thus, mathematically and statistically proficient students will understand that their knowledge of mathematics should inform the usage of the tools and vice versa. Finally, students who are proficient in MSP5 will understand that the computational tool used in modeling is primarily available to speed up their mathematical thinking as the bulk of the work stuck in their ability to understand the situation mathematically and then invite the tool to expedite the process. This idea is similar to what Chloe said about spreadsheets - doing my mathematical brain for me.

In the previous sections and excerpts (e.g., 12, 17), I cited instances where Zoey, Aurora and Ivy used relevant external math resources such as the random number generator and online dice rolling to extend data for the disease spread task. Thus, showing participants' proficiency in identifying relevant external math resources. Mia also described using an Online coin tosses simulator to help her simulate the disease spread data with characterizations such as randomness and probabilities. Again, several discussions in the next sections described how pre-service teachers visualized results of varying assumptions, explored their consequences, and compared predictions with data.

Research Question 3

What computational thinking practices do they develop during modeling activities?

In Chapter 2, I described mathematical modeling as a process by which users represent or describe real-world problems, find a solution, or better understand the problems through mathematical practices. On top of that, I described that modeling processes require a series of actions or steps to be taken, which create space to use computational tools. Computational tools can be used to formulate a problem, compute, interpret, validate, and report. This study found that these modeling processes develop computational thinking practices.

Earlier, I presented an operational definition of computational thinking as the process whereby users understand what the computer is doing versus what they are doing – that is, knowing what the user inputs into the computer and what information the computer returns, and how the computer carries out the process. These processes can call for various strategies (including abstraction and modeling), practice, skill acquisition, and improvement. Thus, computational thinking practices (CTP) describe instances where participants show actions/practices that significantly contribute to the model development using computational tools. Table 7 shows actions I identified based on practices in the literature, and other practices suggested from data in this study.

Table 7: Computational thinking practices grouped into four practices.

Data Practices	**Mimicking and Mathematizing**	**Model Exploration and Extension**	**Model Communication**
Creating data (Weintrop et al., 2016)	Mimicking codes and models	Conjecturing and generalizing	Communicating, interpreting, and connecting model to the real world
Data exploration	Mathematizing codes and models	Manipulating parameters and making discoveries	Assessing models
Data manipulation (Weintrop et al., 2016)		Prompting and exploring	

This study also shows a close relationship between mathematical modeling processes and the computational thinking practices found in this study.

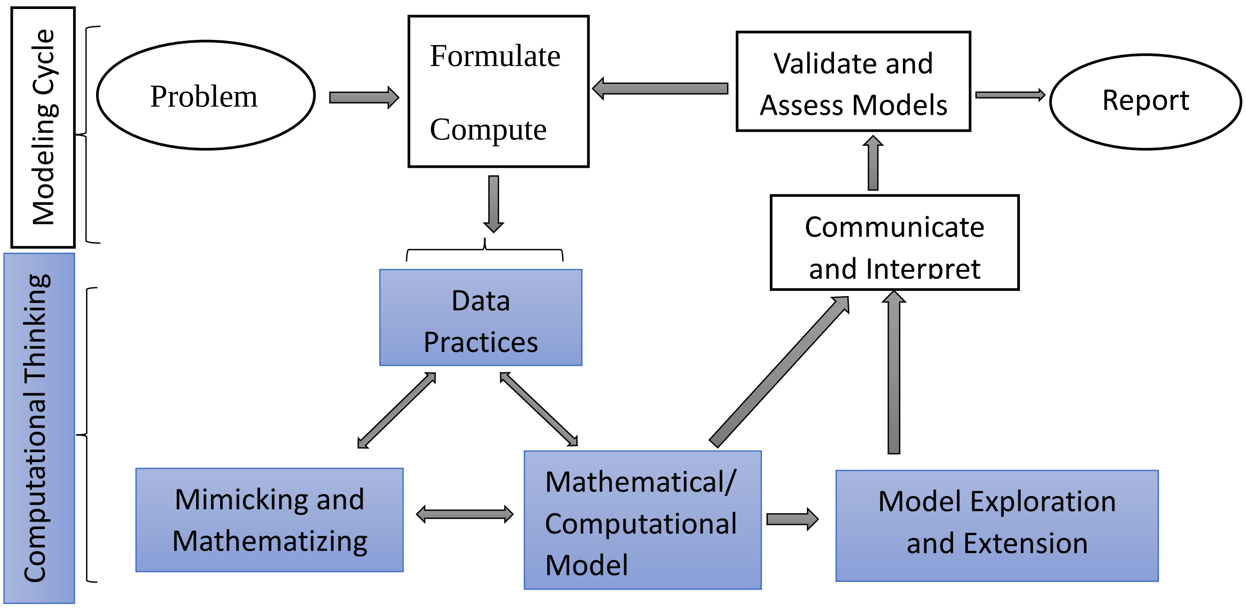

Figure 10: Connection between Mathematical modeling cycle (CCSSM, 2010) and computational thinking practices

In what follows, I discuss each computational thinking practice and thereafter discuss the connection between the mathematical modeling cycle and computational thinking practices.

<u>Creating Data</u>

Creating data describes instances where pre-service teachers generated new data or extended available data through simulations to develop a model. It involves situations where participants used computers or other online resources (e.g., spreadsheet, Java, random number generator) to simulate data or engage in some activities (e.g., playing rock-paper-scissors) that enabled them to create data. To extend existing data, pre-service teachers first made sense of the existing metadata – a summary of basic information about data, making finding and working with particular instances of data more accessible. During the disease spread modeling activity, the facilitator verbally explained the metadata – how the data was generated and what each data point represented. To extend the available data in Zoey's group, they mimicked the physical simulation they had used when simulating disease spread. In the physical simulation, *chicken dancing around* introduced randomness in the system. The rock-paper-scissors game signified the probability that an infectious person meeting a susceptible person would be able to pass on the infection. When asked about their approach, Zoey said:

Excerpt 17

> "We wanted to keep the probability of the 33% that you're infected. So, we wanted to find something that would represent that. So, we decided on using a three-sided die on the internet. And then, we also wanted to keep the randomness with the person they were seeing. So, we use the random number generator there" [12/08/21, Interview data].

It is evident from excerpt 17 that group members (Zoey, Ivy, and Aurora) knew the underlying mathematical story in the facilitator's disease spread data and mimicked the idea. They decoded what the rock-paper-scissors were introducing into the data and found ways to replicate that into the data extension. When Ivy was describing what they did, she said:

Excerpt 18

> Ivy: We wanted to have more data; we went through the class data. So it was like simulating rock, paper scissors. And then we did a random number generator to see who can see someone around. So, we picked our dice that goes from one to three, and then, um, I can't remember how we did it. So, we created like, a hierarchy. So, I think it was like, three is greater than two; two is greater than one; and one is greater than three...
>
> Facilitator: Okay!
>
> Ivy: So then, when we rolled the dice, like, the person that was infected was on, have like the left dice, and then the other person had the correct one. And then, we used the random number generator to get the other student that would be infected. Then, we just kept rolling the dice.

the facilitator: Uh huh.

Ivy: And then if it was an infected person, that person didn't play again. [10/6/21, video data].

While excerpt 17 describes participants' understanding of how the physical simulation introduced randomness and probability into the data, excerpt 18 describes the processes group members employed to actualize the attributes in the extended datapoints. Participants described the use of Spreadsheets to simulate data as a mathematical process. One of the participants – Zoey said, “we used different formulas to get data, and then got more data from using the formula” [12/08/21, Interview data]. When describing how they used the spreadsheets program, Amelia said, “we were able to use it to create more data based on the previously set parameters like the bobcat population rates” [10/6/21, video data].

Exploring data

Data exploration refers to the first steps of the study of data. Analysts use data visualization and statistical techniques to describe dataset characterizations, such as size, quantity, and accuracy, to uncover insights from the data. *Data exploration* in this study describes situations where pre-service teachers used graphics to represent their data. It reveals instances where participants explored and visualized data to uncover insights from data or identified areas or patterns for analysis. Put differently, one can simply put it as what pre-service teachers in this study did with a dataset.

This study found that pre-service teachers explored data just by manually scanning through the dataset to get to know the data they were working with. An example of this was when Mia noticed that the susceptible datapoints contained capital “S” and lower “s”. She was not sure if they meant different things and that prompted her to ask the instructor of the significance of upper and lower case “s”. In a similar circumstance, Amelia and Chloe realized the need to clean the dataset provided by the facilitator. In the data, the number of infected persons jumped from one to three, and they were confused since the initial rounds only had one infected person who could only play the rock-paper-scissors game with one person. They should have at most two but not three infected persons in round seven if she won. The facilitator responded that it must have been an error when the data was recorded on spreadsheets. The screen recording retrieved from Chloe shows how she kept scrolling across the dataset before she responded to Mia’s query about why the equation she found was not working for their model. Thus, it is evident that pre-service teachers scanned through the data to understand the data points and identify any errors in the data.

Next, this study found that pre-service teachers explored data by using visuals to understand the effect of a parameter, understand the relationship between two or more variables, make predictions, understand the effect of data size, and understand the effect of data size long-term behavior of data. For example, Mia stated in her report, “I found both stochastic effects to be extremely pronounced because the boxplots of the stochastic effect models changed drastically each time the data regenerated in excel.” [12/15/21, Artifacts data].

From the quote, Mia pointed out how the visuals helped her understand that stochasticity influences the model. She could understand which stochastic effect was more pronounced through the boxplot. Participants worked with variables such as susceptible population, recovery population, and the number of infected persons on the disease spread task. When Zoey was asked about the relevance of bar chart in her group's work she said, “so, we used the bar chart to represent the susceptible, recovering, and infected population, just so we could see how the three were changing perspectives to each other over the different rounds" [12/08/21, Interview].

This exploration of data allowed Zoey and groupmates to understand the relationship between the three variables which launched them wanting to know what would happen if they had more data. In her analysis of the bar chart (see the screenshot below, Figure 11), Zoey explained that "she could tell how, at the beginning, the blue was starting to drop off, and then it leveled off a little bit." Whereas the red was more of a curve. And then the yellow kind of went up. This analysis of the data prompted her group to wanting to know more about the data. Consider the question they raised in the excerpt below:

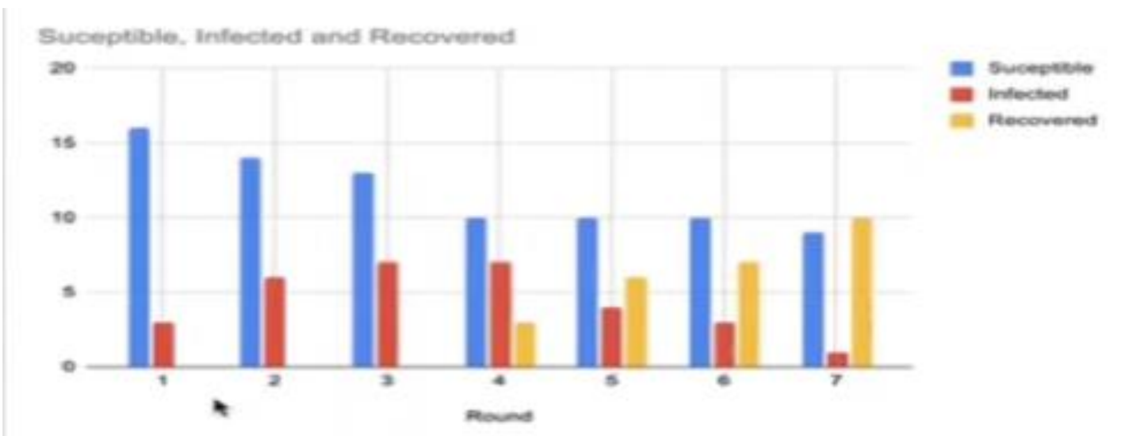

Figure 11: Bar chart representing the relationship between susceptible, infected, and recovered persons in Zoey group's disease spread model [10/6/21: Video data].

Excerpt 19

> "What if the simulation had continued, what would have happened? So, we wanted to truly see if the simulation continued what would happen. We wanted to just add more data to be able to do that because we didn't feel like we had enough to be able to make a definitive - this is what's going to happen so far." [10/6/21, video data]

The excerpt above shows how the chart revealed to participants that they needed more data to see the long-term behavior of the data and the relationship that would exist between the variables as time goes on. In a similar scenario, Chloe narrated how working with Owen allowed them to see a clearer end behavior of the bobcat dataset. She maintained that their initial thought was that the graph was linear but by stretching out the data, they could get a clearer picture of how it was not a linear relationship but exponential as the data grew.

Manipulating data

Data manipulation is the process of changing or altering data to make it more readable and organized. For example, you can arrange data alphabetically to speed up finding useful information. In this study, data manipulation describes instances where pre-service teachers adjusted data to make it organized and easier to read. Data show that pre-service teachers altered the dataset by quantifying data points to get helpful information from the dataset or gain insights into the dataset. In addition, they altered the dataset to suit the question they wanted to answer. For example, when Chloe was asked about using the COUNTIF function, she said,

Excerpt 20

> "My whole thought process was like, there's got to be something that counts it for me, so I don't have to sit here and count at all. And I heard someone else say count if function, I was like, I'm gonna go look that up and figure it out." [12/09/21, Interview data, Chloe].

In this excerpt, Chloe was changing the lettered data into numbers so that she could use some tools such as graphics to see the behavior of the dataset. When describing working with data, Amelia narrated how her group manipulated data during the class: "We realized that we could not get enough data from Ilesha to make our results meaningful. Instead, we took general statements and altered them to be able to mirror Ilesha as close as possible" [12/14/21, Interview, Amelia]. This idea explains how Amelia and her groupmates altered the limited information of Ilesha at their disposal to create enough data that would enable them to answer their modeling questions.

Data manipulation is an integral part of modeling. One of the participants, Chloe, recounted how data organization helped her work. She said, "So being able to organize even if you are given data or have to build the data, organizing it allows you to see it in different ways than you would by just looking at a spreadsheet of numbers" [12/09/21, Interview data, Chloe]. She cited that the inability of her and Owen to figure out how to organize data during the Bobcat population simulations probably made them lose quickly in figuring out the task. She stated that they could not understand what the results were telling them because they could not have a proper organization of the data. Zoey mentioned that their ability to organize their data made it easy for them to report the results.

Mimicking codes and models

Mimicking describes situations whereby pre-service teachers found quick and clever ways to overcome difficulties and copy or imitate existing codes or models found to go through the remainder of the modeling processes. Pre-service teachers looked up for ways to get something done and then emulate the ideas. This study found that pre-service teachers mimicked existing models, codes, computational tool structures, facilitators, peers, and mathematical models.

Data show that pre-service teachers were resourceful when replicating existing data characterization to create or extend data. In the following excerpt, Zoey described how her group mimicked the physical simulation through a random number generator and rolling dice to extend the disease spread data.

Excerpt 21

> "So, to see a person, I would do the random number generator to mimic the chicken dance we did where it was randomly going into someone. And then Aurora came up with this idea of rolling the dice. So, the person on the left would be like, who was infected, and then the person on the right was the person they were seeing" [12/08/21, Interview data - Zoey].

To achieve the goal of extending the data, Zoey's group members searched the internet to find strategies to use that would introduce randomness and probability into the data. After this, they mimicked the metadata to recreate and extend the data (see data exploration above). Pre-service teachers imitated the structure of computational tools in ways that suit their ongoing tasks. When responding to how Mia used spreadsheets, she said:

Excerpt 22:

> "Uhm, well, I kind of tried to make it look like how my data looks in R, and I would just have different columns with different information like okay, this is the number of bobcats for this year. And this is a number of bobcats after a catastrophe happened. So, like each column would have different information in it, and it would be separated like that." [12/14/21, Interview data]

In excerpt 22, Mia explained how she was using her R-programming knowledge to inform the structure of her data in spreadsheets. In this sense, she mimicked the R structure to set up her modeling in spreadsheets. In a similar version, Mateo responded to the researcher's question of why he was switching between spreadsheets and Java programs. He explained that the provided disease spread data was in spreadsheet format, having rows and columns and that he was trying to emulate rows and columns in spreadsheets in a readable way in Java. When describing how he achieved his goal, he stated that:

Excerpt 23

> “And I figured if I move person five to number two, person number four spot, I'm not changing person five, I'm just changing where they are in the output. So, I'm not affecting the data in any way. I'm just affecting readability. So that's why I kept looking back and forth to know what I was shooting for, but then like, how could I improve on it. So, it's taking out the formatting differently.” [12/07/21, Interview, Mateo]

In another example, Mateo explained that he was looking at the code in the disease spread model to see how he set up certain array. He said, “it's like mimicking that in this one.” He further explained that it was like making sure he was using the right functions and commands. Both Mia and Mateo showed how they mimicked ideas from the computational tool they were familiar with to navigate their work in spreadsheets.

Pre-service teachers in this study mimicked an existing model to inform their model building, make choices, or choose variables. These existing models comprise those developed by the facilitator, models from students' previous classes, and models from the internet. During my observation, participants struggled to know the first argument to use in the NORM.INV () function in spreadsheet. However, I was surprised to see that Mia effortlessly put in the RAND () in the NORM.INV (). This move was surprising since getting the NORM.INV () function to work at that time was a major challenge for all the participants. When responding to how she figured it out, Mia said:

Excerpt 24

> “The only reason I knew that was because I took a picture of how the facilitator did it on her example. She was like, look at the norm. Like, that was the example that she showed

us in spreadsheet. And I was like, Oh, cool. I took a picture of it, so that I would remember that later. Otherwise, I would have no idea.” [12/14/21, Interview data]

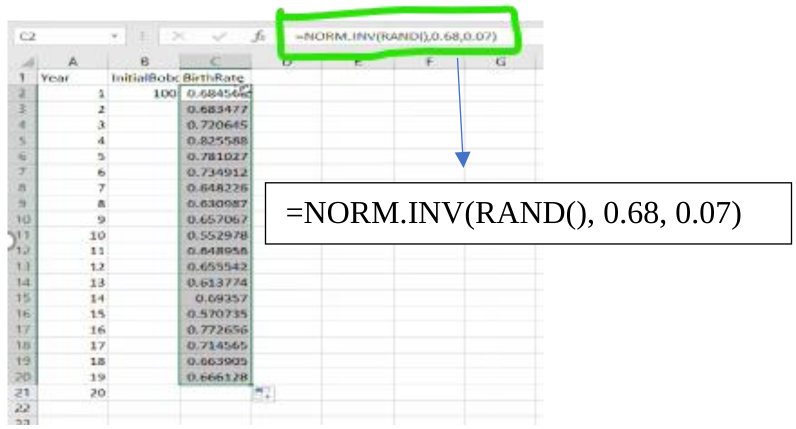

Figure 12: Using Norm.INV function [10/27/21, Video data]

Mateo also attributed their smooth running of scenario three of the disease spread task to their ability to build the model on their ideas in scenarios one and two. In this case, Mateo and his groupmates saw what he called interesting relationships between susceptible, infected, and recovered persons. Their observation propelled them to ask questions about what would happen to the relationships they observed if they introduced people who could never get sick, vaccination, natural immunity, or people that are flash vaccinated at the beginning. Thus, mimicking the ideas they had used in scenarios one and two formed the basis of how they developed scenario three, their choices, and how they chose variables. In another example, participants identified existing models that could fit the situation they were exploring. Mateo described using the logarithmic growth function to explore the Bobcat population problem.

Participants in this study also mimicked and adjusted existing codes to inform their codes. To do this, they would look at what other people had done to see if that could help; if not, they looked elsewhere. For example, Mateo said:

Excerpt 25

> "So, I was looking into Java packages just because I'm dealing with lists. Now, I'm trying to combine them, I'm trying to edit them. I'm editing combined versions of them, so it's just like a bunch of, is there something in here that could lighten the load for me a bit? Is the package here? Is there a function? Is there the utility of a list or array lists that I could use to make it easier?" [12/07/21, Interview data]

In some instances, they looked at their previous coding class to see if they could draw on some ideas. For example, Amelia explained how she wanted to use her coding of the SIR model in MATLAB from the previous class when working on the disease spread model. She said she could find the coding, but not helpful because MATLAB was not supported on her MacBook.

Mathematizing Codes and Models

Mathematizing codes and models refers to instances pre-service teachers treated or regarded coding mathematically. It involved instances where modelers generated mathematical expressions as models. In some cases, participants used graphs to represent models. To do this, participants might write out their ideas mathematically before coding in the computational tool. For example, Zoey acknowledged that she had to remotely do some mathematical thinking before writing a formula that spreadsheet could execute and represent the model she wanted. Mateo, who worked with Java for some time in the class, shared a similar experience with me during the stimulated recall interview. He said:

Excerpt 26

> "For something like coding, it is definitely much more of inequality, random, and maybe matrices kind of thing where you just have to know about them. And like, how you can get what you want out of them, as well as like, set them up." [12/07/21, Interview, Mateo].

In terms of setting modeling/coding up mathematically, for example, the facilitator tasked participants to think about the situation mathematically to develop a diagrammatic model that determines the Bobcat population. Figure 13 shows an example of mathematical model participants developed when working on the Bobcat population task.

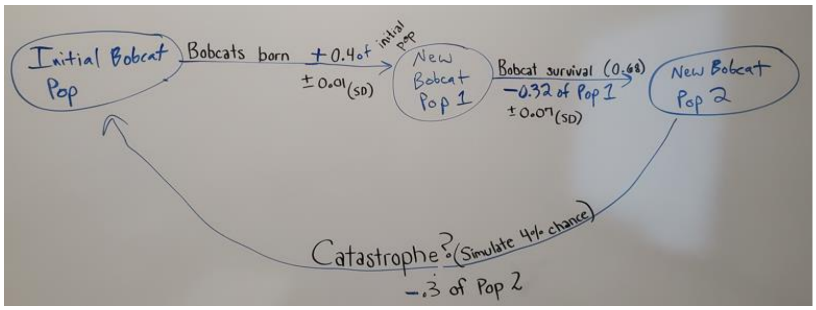

Figure 13: Example of mathematical model [10/25/21, Class Observation – Picture]

When responding to the mathematical model above, Chloe said that it created the right mindset for her about how to code things up in spreadsheets. She stated further that the idea was developed during the Bobcat population task, and it became a viable tool for her for the remainder of the class. Mia reported how her group transformed the mathematical idea into codes in spreadsheet.

Excerpt 27

> "For this model, we start with an initial bobcat population of 100. Then a catastrophe is implemented at a rate of 4% that decreases both the birth rate and survival rate by 30% by using the formula IF(RAND ()<=0.04,0.3,0). Next, the birth rate is implemented using the formula NORM.INV(RAND (),0.4,0.1)-C2, where C2 is the formula for the catastrophe. Then we have a new bobcat population using the formula B2+B2*D2, where B2 is the initial population for that year and D2 is the birth rate. Now, I implemented the survival rate on this new population using the formula NORM.INV(RAND(),0.68,0.07)-C2. Again, we have a new population using the formula E2*F2, where E2 is new population 1 and F2 is the survival rate. Finally, this gives us our final bobcat population for

> that year, that then goes to be the initial bobcat population for the next year and the cycle starts over.” [12/14/21, Artifacts].

In the above excerpt, Mia and her groupmates transformed the mathematical model they developed into codes in spreadsheet. Writing out the solution mathematically helped participants better understand what the computational tool was doing. They realized that they were the designer while the tool was just executing their ideas. Chloe put this as “I wanted the computer to do my brain 1000 times my speed” The above illustration suggests an example of how pre-service teachers mathematized model and translated the ideas into coding.

Pre-service teachers also developed the practice of mathematizing model. Participants in this study always looked for a way to mathematically present their model in the form of an equation that could predict the dependent variables. For example, Chloe said she was trying to figure out a way where she could incorporate the number of the year (independent variable) into the function to get the population of the Bobcat. This idea was like what Mia called a “prediction model." Consider the excerpt below:

Excerpt 28

> “Based on the data obtained from these trials, we plotted points in graphs and used excel to find the line of best fit for each. This data was then inputted into R to create a linear regression. The data yielded a prediction model for the number of infected people based on what round it is and the probability of a susceptible person being infected. For simulation one, the model we came up with is Number of Infected = -7.127e-16 + 2.228e-16*Round + 63*Probability of Susceptible Person Being Infected.” [12/15/21: Artifacts data].

In addition to using mathematical expressions to represent models, pre-service teachers also used graphs to represent models. The following pictures show some graphs participants used to represent models during the class.

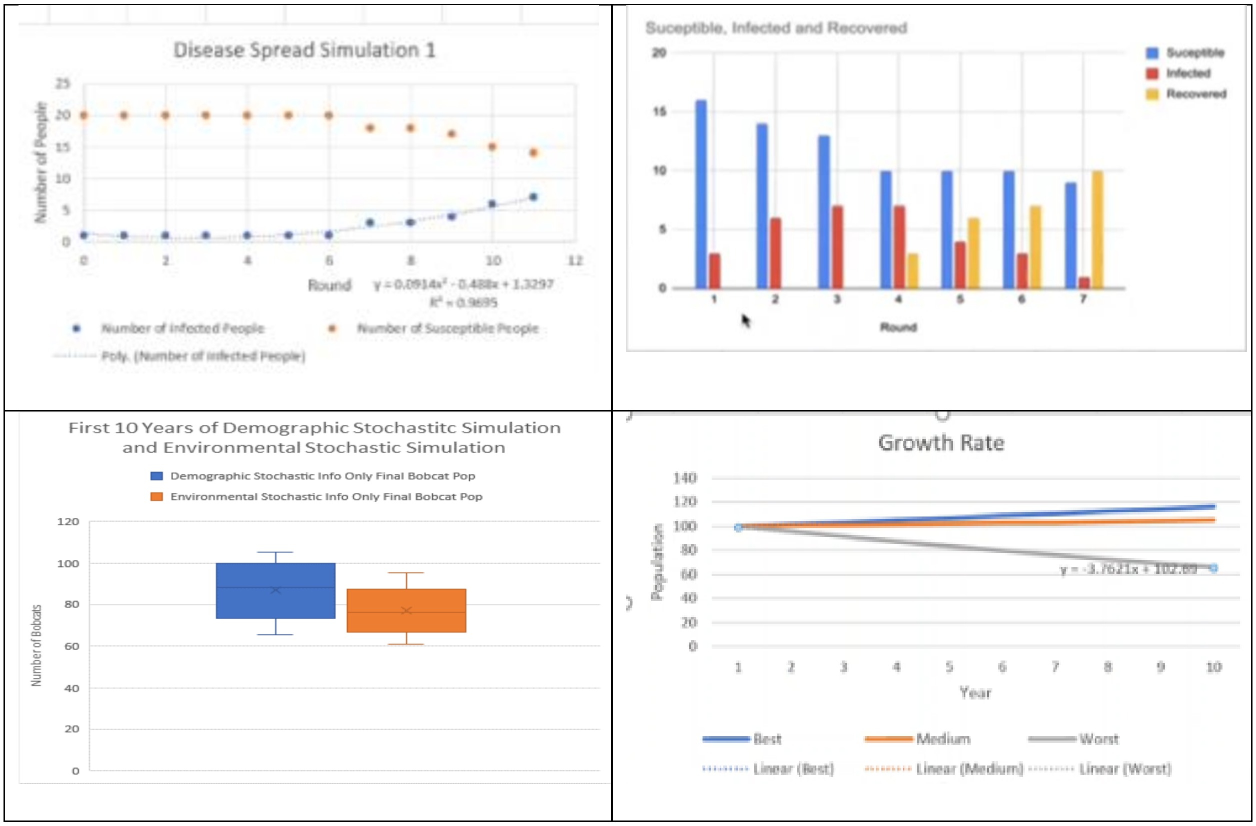

Figure 14: Examples of graphs representing models (10/27/21, Video data)

Pre-service teachers analyzed the models, gave interpretations, and then revised them if necessary. For example, analysis of the bar chart informed Zoey and her groupmates of the need to extend their data (See Exploring Data).

Communicating, Interpreting, and

Connecting Model to Real World

Interpreting model and connecting to real-world describes instances where pre-service teachers interpreted the model (solution) they developed. It also involves connecting the model (solution) to the real world or giving an interpretation of the model (solution) in the context of the real problem they were investigating. It describes situations when learners explain what they observed from the computational model in the context of the task and make connections to the real world. Participants used this practice to ensure fairness, accountability, and transparency, giving them enough confidence to use the models they built in real-world context.

This study also found that interpreting a model could be as basic as interpreting how a parameter affects a model. For example, when interpreting the disease spread model, Amelia interpreted the $1/3$ in her formula to mean the possibility of meeting a sick person and getting sick. That is, the likelihood of a susceptible person coming across another person who is sick and the probability of them giving the infection to the susceptible. Interpreting a model is important because we cannot just build a model and expect anyone who encounters the model to understand what the model is about or what they mean contextually. This study reveals that pre-service teachers used this practice by communicating what the model represented through reports, peers, or facilitator.

Participants in this study represented their models through mathematical equations and graphs (e.g., Figure 15). In the boxplots used by Mia to represent the stochastic effect model (Figure 14), she discussed the relevance of variability in the model. In her discussion, she took into account the variability we experience in our world with the demographic and environmental stochastic effect and thus making it a realistic predictor of Bobcats populations. During the stimulated recall interview with her, she interpreted the Bobcat population model below (Excerpt 29).

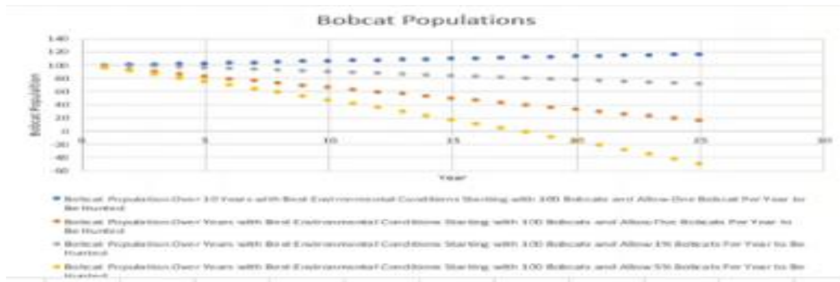

Figure 15: Screenshot of Bobcat Population Model (Graph of Bobcat population against Year)

Excerpt 29

> "In this model, the facilitator wanted us to do different situations. Here I did four and I wanted to see how the Bobcat populations changed overtime. Then I put them all on one graph so that I could see how each different impact changed the bobcat population relative to each other. Obviously, whatever this bottom one is, decreases the population most drastically. And then this top one increases the population, like it keeps it the highest, but it's a pretty flat change, there's no drastic changes" [12/14/21, Interview data].

In Excerpt 29, Mia interpreted the various conditions she was investigating to identify what condition was yielding increase and what conditions was yielding a decrease in the Bobcat population. Zoey reported that the bar chart (Figure 14) was representing the susceptible, recovering, and infected population in a way that shows how the three were changing perspectives to each other over different rounds. She further interpreted that "the blue started to drop off, and then leveled off a little bit whereas the red was more of a curve, and the yellow seemed to shoot up". From this interpretation, she could describe the behavior of the variables. She further stated that after they extended the data, there was no one available to spread the disease after round 15 and that the simulation ended, so it did not matter if it got to round 16 or 20. Using a mathematical equation, Mia, Amelia, and Chloe interpreted the mathematical model they came up with in scenario two of the disease spread model as follows:

Table 8: Interpretation of a Mathematical Model [12/15/21, Artifact]

Mathematical Model	**Interpretation**
Number of Infected = 0.1237 - 0.4042*Round + 42.3525*Probability of Susceptible Person Being Infected.	Our model addresses the question we asked because it tells us how many people will be infected based on what day it is and what the probability of a susceptible person being infected is. Hypothetically, if the day/round were 0 and the probability of a susceptible person being infected were 0, 0.1237 people would be infected. For every additional round, 0.4042 additional people become infected. For every one unit increase in probability of a susceptible person being infected, 42.3525 additional people become infected.

When Zoey was asked about the equation she had for the Bobcat population, she said the "y" would be the number of Bobcats and "x" would be the year and that the simulation started with a population of 100, which was why they had y intercept. The interpretation of the algebraic variables in the context of the Bobcat population makes much sense to an outsider who encounters the model.

Participants also communicated the limitations of the model they developed by either talking about possible improvements or how the model might not be a good predictor of the dependent variable. Mia discussed the need to incorporate conditions of different regions in the Bobcat model because there are Bobcats all over the place in the US, and not every region is the same in regard to the Bobcat population. Similarly, when giving an interpretation to the potential mathematical model to predict the probability of getting infected during the disease spread task, Mia pointed out that the model $y = 0.0914x^2 - 0.488x + 1.3297$ is not a good predictor of the probability of getting infected. She concluded this because the R-squared value yielded one, a situation she interpreted as just a mere translation of one column that would always yield a straight line (Figure 16).

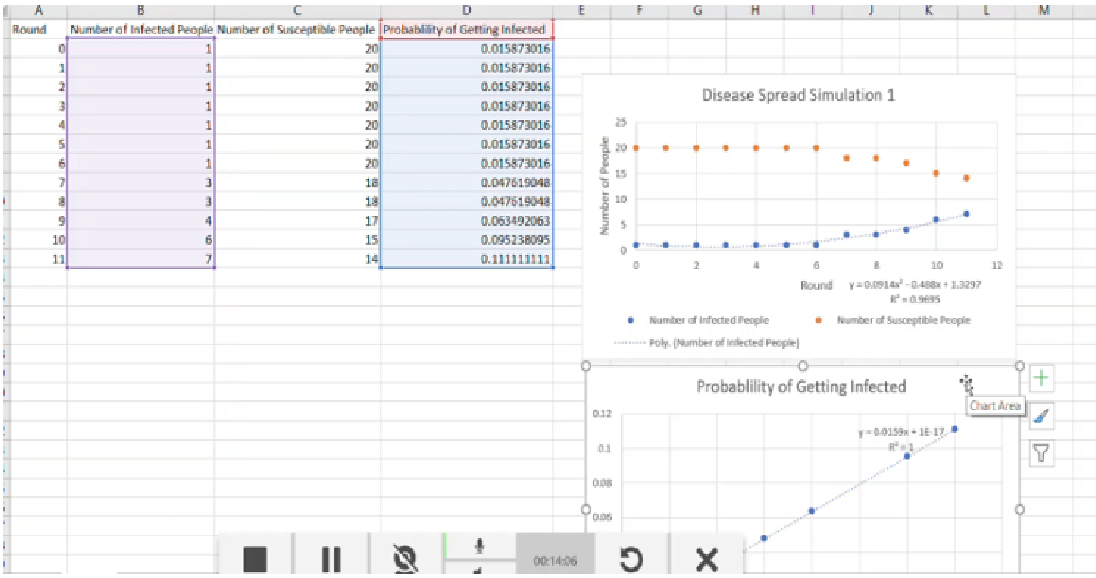

Figure 16: Screenshot of Mia's Work

Zoey also mentioned that she would provide her clients with information regarding under what conditions are the mathematical models effective and that they are also a rough estimate of the situation. The data discussed above show that participants interpreted the model in the context of the real world or the problem at hand and identified areas of improvement. Interpreting a model in the context of the problem is important as the end-user of the model would be aware of the limitations.

The ability to interpret a model shows participants' ability about the contribution of variables and parameters in the models. This ability enabled them to know what variables contributed and to what extent when compared to other variables. It helped participants identify which variables to consider and which ones to drop when revising the model. During the disease spread model for scenario three, Mateo, Asher, and Owen narrated that their analysis of the variables they considered in scenarios one and two helped determine what variables they should consider for scenario three.

Assessing Models

Assessing models describes instances participants shared their models with peers within and across groups to verify their thought processes and how the models they had, compared to what others had, and if they were doing the right thing. When assessing models, participants compared models to a real-world system to analyze the model's overall accuracy. This study found that pre-service teachers assessed models through mathematical or statistical knowledge and peers.

Participants in this study relied on the mathematical and statistical knowledge to assess models. Thus, previous mathematical or statistical knowledge may influence how pre-service teachers think about the model they develop and how to proceed from the assessment. For example, Mia’s concern about the R-squared value of the mathematical model for disease spread $y = 0.0914x^2 - 0.488x + 1.3297$ showed that her statistical knowledge of R-squared values helped assess the model's usability. Mia regarded this model as dumb because it is just the same number translated into other numbers, for example, plotting the set of 1, 2, 3, and 4 (x-values) against 2, 4, 6, and 8 (y-values). She said: “I was trying to predict the probability of getting infected based on the number of infected people, but then our R squared value is one because it's the same number.” [12/14/21, Interview, Figure 16, Mia]

In a similar scenario, Zoey recounted how she used the R-squared values to identify a better model. In the excerpt below, she discussed how R-squared values helped her to identify a useful model.

Excerpt 30

> Zoey: I think I found the R squared. I was just playing around with things at first to kind of see, and then I realized the R squared value when I was checking that I had

> the right type of equation. Like if I had linear and exponential, the R-squared helped me to see what equation fit the model better.
>
> Researcher: Ah I see, so the R square is telling you the information about if the model you've considered is a useful model.
>
> Zoey: Yeah, it's finding which type of equation would work best for what I had.
>
> Researcher: Interesting. So, what would you see on your R square value that would make you say this is not quite okay.
>
> Zoey: I think I just found the one that would be closest to one. And then definitely anything that was below point nine, I think I really looked to see if something was wrong or if there was a different one that fit better. But I felt comfortable in the high 90s. [10/8/21, video data]

In the above excerpt, Zoey revealed the usefulness of statistical knowledge of the R-squared values. This idea also helped her and Mia to revisit their models.

Next, pre-service teachers assessed models through peers across groups. Assessing a computational model across groups is common when modelers work on building a computational model. In this study, pre-service teachers assessed their models across groups when dealing with a complete model or writing codes. This approach was helpful in that it helped participants access different perspectives on writing codes or building a model. Mia discussed the need to assess models across groups to be able to ask the group what they did differently to arrive at their model. She further stated that such an assessment propelled her to revisit her work to see where she went wrong. In terms of group work, participants would go back to their groups and check through their work to figure out the problem. Sometimes, the two groups would join forces together to identify the problem. Chloe described this type of model assessment as helpful in that the approach helped them get better at each other's strengths, especially in mathematical skills and coding in spreadsheets.

Prompting and Exploring

Prompting describes instances where participants asked/got questions about variables, how to code, ideas to consider, etc. Prompting is different from probing in that it is usually followed by action/interaction with the tool rather than justifying an idea. Exploring describes instances where participants dived deep into the model. In this case, they tried out ideas as they occurred to them and adjusted along the way. That is, they experimented and watched for what happened. If they did not obtain the desired results, they revised the ideas by seeking help or persevering.

The actions that followed prompting mostly led to exploration with the computational tools. Pre-service teachers could get prompt from the facilitator and peers and then proceeded with exploration with the computational tool. The prompts could be about parameters, coding, variables, data simulation, or the model they had developed. In *excerpt 19*, group members prompted themselves on the data provided and their initial analysis of the data. They wanted to know what would happen if the simulation had continued. So, they decided to add data.

In the Bobcat population task, the facilitator prompted participants to consider different situations of the growth rate, birth rate, death rate, and survival rate. In Mia's discussion about her exploration of the prompt (Excerpt 29), she narrated how she put all the four situations on one graph to see how each parameter impact changed the bobcat population relative to each other. Similarly, Mateo and his groupmates deliberated on what population they should consider for the disease model. This prompt made Mateo explore Java in a way to set up his codes in Java so that they could input any value for the initial population and watch how it would affect the model.

The facilitator prompted participants on coding to check out how to use the COUNTIF function in spreadsheet. Earlier, I documented Chloe's exploration of how to get the COUNTIF function to count the disease spread data. Pre-service teachers also prompted themselves to explore ideas with the computational tool. For example, Zoey and her groupmates asked what would happen if the disease spread simulation had continued. In their exploration, they investigated how to use the random number generator and online rolling of dice to generate more data (Excerpt 19). In a similar version, pre-service teachers could be prompted about the model itself. For example, Amelia discussed how her group got the Bobcat population to stabilize at 200 (Excerpt 31).

Excerpt 31

> "And then to get it to grow to 200, I basically just took my years and my best growth and I just drag them out until it reached 200. And I ended up taking about 42 years. To get it to stabilize, basically we're just hunting off as many as many bobcats as we're adding. So, then I kind of stayed that the same way to about 200. And then you go to the next one and then to get it to stabilize I kind of did it in the same fashion, I just left it decreasing as it was for 15 years. And then in order to stabilize it, I also just guessed random numbers until it stayed the same, which ended up being adding about 1.5% back. That number relates to the number of previous years is 4.5% of whatever was there that year. And then to get it to stabilize at 200, I did the same thing. I let it go on, or I actually increased the population by 8.96% for 16 years. And then that's how I got it to reach 200. And then from there, I decrease how much we're adding and added only 4.5% in order to get it to stabilize at around 200" [10/18/21, Video data].

The excerpt above shows that pre-service teachers tried to adjust the ideas they used when they explored with a computational tool.

Parameter manipulation and discovering

Parameter manipulation and discovery describe an action involving tweaking the parameter to investigate its sensitivity to the model and making discoveries. The discoveries pre-service teachers made during this process were the outcome they were trying to get at from the beginning of the task or a particular way to write code to get the job done. For example, Amelia set up her Bobcat model in a spreadsheet with parameters (Figure 17, red portion) that she manipulated to arrive at stability.

	Amount Hunted Per Year		Proportion Hunted Per Year	
End of Year	1	5	0.01	0.05
1	100.676	96.676	100.0592	94.5622
2	101.3666	93.29429976	101.3395	93.34547527

Figure 17: Screenshot of Bobcat Population Model [10/20/21, video data]

When discussing how she manipulated the parameter, she said the following:

Excerpt 32

> "So, I started off by just setting up everything in spreadsheet so all my orange headings can be changed. So, you can change any of my proportions, you can change the input amount of bobcats and you can change how many you're adding or subtracting. So, then all of my cells basically have an equation that relates back to the cell above it, and then whatever proportion or ratio that we're changing by. And then, to reach stabilization, I basically just plugged in a bunch of different proportions. And then I kept adding it until it stayed the same number" [10/20/21, Video data, Amelia].

In this example, Amelia set up her spreadsheets to enable her to input different values to explore the sensitivity of the model and how changing the parameters (values in orange in Figure 17) would result in reaching stability, that is, a bobcat population of 200. The data also reveal that Amelia needed to set up the model so that the variables in the model would respond to any changes made to the parameter. Mateo described this process as ensuring that the code was adjusting to the variability. When modelers build a computational model, they need to set up the codes so that changing one parameter will cause the entire model to change by itself without any further adjustment. It is also important that the formula changes accordingly to avoid miscalculations that could render the model irrelevant to parameter manipulation. Participants in this study reported that they had the likely behavior of the model in their mind to know if the model was responding accurately to the manipulations and if not, they would have to figure out where the problem was.

Parameter manipulation helped modelers automate the modeling process in the computational tool to avoid running many simulations. Amelia recounted that setting up parameters helped her avoid redoing all the calculations again since spreadsheet would go off of the code she had typed to simulate a completely new situation. In terms of discovery, parameter manipulation helped modelers to discover a solution to task just by exploring the parameter sensitivity to the model. They can always adjust this sensitivity to arrive at a desired solution. In this case, modelers avoid running expensive simulations in terms of space and time. For example, Zoey stated that “In my parameters, I had 'number hunted' and then I could change that between like one to five and then a percentage…. So, I could continue to change that value in the parameters, and I can keep track of changes” [12/08/21, Interview data].

<u>Conjecturing and Generalizing</u>

Conjecturing involves making predictions about the model and what would happen in the long run. Consider the following example: “For every additional round, 0.4042 additional people become infected. For every one unit increase in probability of a susceptible person being infected, 42.3525 additional people become infected [12/15/21, Artifacts, Table 8].” The quote herein shows the statistical inference Mia, Amelia, and Chloe reported for the disease spread model they developed. This inference shows how they generalized the mathematical model for any future predictions. Thus, conjecturing and generalizing refers to scenarios where pre-service teachers predicted what would happen based on their interpretation, manipulation, discovery, and exploration of the codes or the computational model. Often, conjecturing led to generalizing. That is, in generalizing, participants extended their conjectures to the future behavior of the situation.

In most cases, modelers go through a sequence of other computational thinking practices before they can make predictions and generalizations. For example, Chloe narrated how she worked with Owen to generate the Bobcat population dataset, then moved to data exploration through graphical representations, interpreted the visuals, and forecasted what would happen by looking at the data past ten years. She said, “I think by looking at the data that we have created beyond year 10, we can see that even if you hunt 1% a year, the Bobcats can still grow” [12/09/21, Interview data]. A similar situation occurred when Zoey and her groupmates extended the disease spread data to predict the overall behavior of the model.

Mathematical Modeling and Computational Thinking Practices Cycle

Many mathematical modeling cycles feature the use of computational tools. This study found that the mathematical modeling processes involved using computational tools through which participants developed computational thinking practices. Figure 2 shows one of the modeling cycles used in mathematics education worldwide. In the cycle, modelers move from formulating problems to computing. This phase opens up the use of computational tools in this study which then creates space to discuss the connection between the mathematical modeling cycle and computational thinking practices. Figure 10 shows the connection between the modeling cycle in Figure 2 and the computational thinking practices found in this study.

Figure 10 is an extension of the common core state standards for the mathematical modeling cycle. This extension adds computational thinking practices found with pre-service teachers to the CCSSM (2010) modeling cycle. The portions “formulate and compute” in the CCSSM (2010) cycle open up opportunities for pre-service teachers to use computational tools in the modeling process. When pre-service teachers formulated a model, they created data, explored it, and engaged in data manipulation. They mimicked and mathematized codes and models to generate a mathematical/computational model in this process.

The three parts – data practices, mimicking and mathematizing, and mathematical/computational model - are iterative as pre-service teachers revisited each portion until they were satisfied with the model to move on (refined mathematical/computational model). In these three portions, pre-service teachers created geometrical, graphical, algebraic, or statistical representations of variables identified as important to the model. After generating the mathematical/computational model, participants could move to communicate and interpret or engage in model exploration and extension depending on the nature of the model developed. At this phase, pre-service teachers would interpret the model in terms of the original situation, make predictions, and generalize to a broader context.

Next, they moved on to validate the conclusions by comparing them with the situation and checking in with peers’ work to assess the model informally. If there was a need for improvement, they moved back to formulating and computing. Otherwise, they would report on the conclusions and the reasoning behind them. Throughout this cycle, choices, assumptions, and approximations are present.

Conclusion

I started the chapter by giving a brief introduction and then answering the research questions one after the other. In the chapter, I used data from multiple participants' perspectives to answer how pre-service secondary mathematics teachers interacted with computational tools, peers, and instructor to construct computational thinking practices while working on mathematical modeling tasks. Next, I responded to the types of computational thinking practices pre-service teachers constructed, the challenges they encountered, and how they overcame the challenges. Lastly, I presented answers on how computational thinking practices eliciting tasks supported pre-service teachers' mathematical and statistical thinking practices.

One can rephrase the findings in this chapter under the influence of computational tools on pre-service teachers' forms of interactions during modeling activities that enhance the development of computational thinking practices and promote mathematical and statistical practices. In the findings reported throughout the chapter, data show that pre-service teachers engaged in meaningful conversations with peers and the facilitator, which influenced how they created models, assessed models, interpreted models, made conjectures, and generalized, among others. Thus, in the end, the practices supported the standards for mathematical and statistical practices on how pre-service teachers made sense of problems, persevered on problems, reasoned abstractly and quantitatively, etc. For example, when pre-service teachers partnered with peers, they persevered, made better sense of problems, constructed viable arguments, critiqued the reasoning of others, better used computational tools, and collectively looked for and made use of structure. Furthermore, the collaboration helped create better models with mathematics, assess models, communicate and interpret models, etc.

In sum, meaningful forms of interaction lead to quality communication that engages pre-service teachers in computational, mathematical, and statistical thinking practices. Simply put, interaction is at the heart of constructing computational thinking practices and exhibiting mathematical and statistical thinking practices. Finally, Table 9 shows the summary of the interactions, computational thinking practices, and the standards for mathematics and statistical practices supported by data from the study.

Table 9: A Summary of the interactions, computational thinking practices, and the standard for mathematics and statistical practices supported by data from the study.

Types of Interactions found with pre-service teachers in a computational modeling space	Computational Thinking Practices developed by pre-service teachers	Standards of Mathematical and Statistical Thinking Practices used by pre-service teachers when building computational model
Leadership • Vertical leadership • Horizontal leadership Distributed Authority Division of Labor Giving and receiving feedback Accommodation • That enhances collaborative processes of refining ideas	Data Practices • Creating data • Data exploration • Data manipulation Mimicking and Mathematizing • Mimicking codes and models • Mathematizing codes and models Model exploration and extension • Conjecturing and Generalization • Parameter manipulation and Discovering • Prompting and Exploring	Model with mathematics. Use appropriate tools strategically.

Table 9 Continued

• That leads to frustration and termination of collaborative process Refining ideas • Debating Probing	Model Communication • Assessing models Communicating and interpreting models	

In the next chapter, I present my discussion of the findings, limitations of the findings, and my recommendations.

CHAPTER FIVE

SUMMARY, IMPLICATIONS, RECOMMENDATIONS, AND CONCLUSIONS

Introduction

The need to build human capacity to use computational approaches to solve problems faced in their daily lives is increasing every day. These human capacities are now considered an important part of K-12 education here in the US and worldwide. In Chapter 2, I presented an argument based on existing research literature that we need teachers who possess the skills or those who have experienced the construction of computational thinking practices as learners themselves to be able to teach the younger generation. Although professional development helps practicing teachers develop these skills, Darling-Hammond et al. (2017) noted that many professional development initiatives appear ineffective in supporting changes in teachers' practices and student learning. They identified the lack of time for implementing new instructional approaches during the school day or year as one of the reasons for the ineffectiveness of professional development for teachers.

Another challenge of integrating teachers' learning or construction of computational thinking practices in a professional development program is that it is expensive, short-lived, and may not be accessible to all teachers in every school in the US. Research suggests that instructing pre-service teachers in computational practices can help them better understand its applications in classrooms and, thus, preferred to engaging them in short-term training lacking sustainable knowledge acquisition (Yadav et al., 2017). The primary aim of this study was to investigate how mathematics teacher education programs provide a platform for pre-service teachers to learn computational practices to become a human resource to teach young learners.

This study aims to partly respond to an important question raised by Lockwood and Mørken (2021) on investigating how pre-service teachers' courses can be used to support integrating computational thinking practices into K-12 classrooms. To respond to such an initiative, I investigated how pre-service teachers constructed computational thinking practices collaboratively and how such learning supported mathematical and statistical practices. In my investigation, I explored three questions:

- What is the nature of pre-service teachers' interactions with peers and facilitators during modeling tasks in the presence of computational tools?
- How does the development of computational thinking practices support pre-service teachers' mathematical and Statistical thinking practices?
- What computational thinking practices do they develop during modeling?

I employed the social constructivist theory to investigate how pre-service teachers collaboratively developed computational thinking practices with peers, tools, and facilitator. I studied nine pre-service teachers and one facilitator in a mathematical modeling class for secondary teachers for a semester. Although the primary computational tool for the class was the spreadsheet program, participants used other tools such as Java and R programs. I collected data ranging from classroom observations to video recordings, screen recordings, stimulated recall interviews, student work (artifacts), and surveys. I analyzed the data collected using the interactional analysis procedures, inductive, and deductive analysis methods. This chapter will summarize my findings – how I answered the research questions, implications for practice, what the findings mean, why they are essential, and their real-world applications; recommendations for research – academic, practical, and real-world suggestions. Lastly, I will wrap up this research story.

Summary of Findings

In what follows, I will discuss the summary of the findings under the following headings:

- Nature of pre-service teachers' interaction with computational tools, peers, and instructor
- Mathematical and statistical practices supported during the construction of computational thinking practices.
- Computational thinking practices developed by pre-service teachers during interaction with peers, tools, and facilitator.

Nature of pre-service teachers' interaction with computational tools, peers, and instructor

Studies show that collaboration in mathematics learning positively impacts students' learning (e.g., Chan and Clarke, 2017). Many positive outcomes of collaborative learning encourage practitioners to use strategies such as group work, gallery walks, and think-pair-share, among others, in their classrooms. Seidouvy and Schindler (2019) divided research on students' collaborative learning into two aspects, one that focuses predominantly on the investigation and evaluation of the outcomes of collaborative adventures (product-focused) and the other that focuses on the collaborative process itself. This study reveals the dynamics of collaborative activity as a process in a mathematical modeling classroom. In chapter four, I documented the influence of computational tools on the positioning and processes pre-service teachers employed when working collaboratively on modeling tasks. I described the influence of computational tools on the positioning of pre-service teachers during modeling under *leadership and distributed authority.* Similarly, I discussed the influence of computational tools on interaction processes under the *division of labor, giving and receiving feedback, accommodation, and refining ideas*.

This study identifies *leadership* as a form of *positioning* during interaction. The presence of computational tools influences the *leadership* roles participants exhibited when collaborating on modeling activities. Learners are expected to either lead discussion or listen in a group work involving two or more learners. Chi and Wylie (2014) stated that students benefit from peer discussions if they act as explainers and listeners during an activity. Everyone in the group will have the opportunity to engage constructively and interactively. Studies identify issues such as lack of contribution from participants, poor communication, dominating group members as common issues that students may encounter when working collaboratively (Burke, 2011; Baines et al., 2015). Likewise, this study found that the presence of computational tool during modeling contributes to the emergence of some group members as dominating. James et al. (2008) suggested that peer discussions' pros can be limited by the presence of these dominating students.

While it is vital for groupmates to be explainers and listeners (Chi and Wylie, 2008), having those that dominate could take away the opportunity for others to learn constructively during peer interactions. It is important to have explainers in the group who can drive discussions and work with others to get the job done. However, this study finds that some group members consciously or unconsciously become dominating in leading the modeling processes as a result of their expertise in using computational tools better than other group member. This study finds that self-identified leader in a group often lord their ideas on other group members, including making others to use a computational tool that they are comfortable using.

This study finds two kinds of leadership forms of interaction. Leadership can be vertical or horizontal. In vertical leadership, ideas flow from top to bottom. In this case, the explainer leads the modeling process and, in some cases, prescribes the computational tool to be used. Groupmates are made to follow the ideas set by a dominating leader. In horizontal leadership, ideas are collectively developed among group members. Each member's ideas are considered, justified, probed, and debated to form a collective knowledge in the group. In horizontal leadership, power belongs to everyone.

Participants in this study were found to experience challenges that warranted them to assume different positionings in resolving them. This study finds that pre-service teachers attribute their challenges mostly to using computational tools. Many studies on how students get group challenges resolved have been focusing on the roles of teachers in making groups function effectively and also peer support (e.g., Burke, 2011; Vollet et al., 2017). This study finds that the presence of computational tools extend channels through which pre-service teachers can explore to navigate challenges faced during modeling.

Figure 9 shows the interaction pathways in form of distributed authority when resolving challenges. Each layer in the pathway is positioned as a resource that could help participants figure out the challenges. This study supports previous studies that identify teachers and peers as essential when resolving group problems as all pathways found in this study but one feature as resources in resolving group problems.

One way that participants in this study pursued the promise of collaborative learning was by dividing and offloading tasks. In this case, participants realized that group members should work on the aspects of the task they were comfortable with and then report to the group to develop collective ideas. The more activity such as modeling requires knowledge interdependence, collaborative problem-solving opportunities, and striving for a common goal, the better chance it will have at achieving the promise that collaborative learning offers (Shinder, 2009). This study shows that dividing labor results in knowledge interdependence among participants working collaboratively within the group because they rely on individual specialized capabilities to achieve a common goal. This effort to learn from peers attests to the positive effects of collaborative learning in any learning space. This finding supports Vygotsky's (1978) view that learners generate the ability to advance and cultivate a shared meaning, thereby transferring knowledge to group members.

When pre-service teachers engage in labor division, they share duties, thereby reducing stress on the part of group members. Connecting with other members of the group reduces stress because pre-service teachers can share problems with others who are experts in some areas, thereby reducing the mental stress that pre-service teachers engage in when developing computational models. Thus, this study is adding to the handful of studies on active-learning techniques (e.g., Cooper et al., 2018; Hsu et al., 2021) that reduce stress and anxiety in students during learning. Bliss et al., (2018) described the use of computational tools in going through the modeling processes such as in creating data, interpreting data, and assessing models. This study found that pre-service teachers offloaded tasks to the computational tools to help them with speed and accuracy. In this sense, the computational tools carry out the modeling processes that participants possess with speed and versatility.

A direct consequence of offloading tasks to the computational tools is a process of interaction found in this study as *giving and receiving feedback* - a give-and-take interaction between participants and the computational tool via the user interface. In this form of interaction, the tool provides feedback on its interface that enables user to either move on with other part or adjust and try the computation again. Here, the computational tool has the speed while the user has the computational technique to write codes that the tool understands.

While describing mediated action, Wertsch (1998) mentioned that a powerful feature of such action is that the effect of one (agent or agency) cannot be separated from the other. This assumption holds in this study as both humans and the tools depend on each other to derive results during an interaction. This study identifies outputs on the user interface as desirable, error, and undesirable output. When the tool outputs a desirable result, pre-service teachers move on with the modeling process. Otherwise, they adjust the codes and try again. This study finds that the first process in this *give and take* interaction is that user develops a mathematical idea of the situation, encodes the mathematical idea, and finally gets output as the feedback that determines the next step in the modeling process.

Next, this study finds accommodation as another form of the collaborative process in the presence of computational tools during modeling. Accommodation involves situations where participants respectfully welcome others' ideas and find ways to move their solution forward through the ideas either by proceeding to the collaborative process of refinement or termination of any collaborative process. Several studies (e.g., Cooper et al., 2018; Hsu et al., 2021);

Sofroniou and Poutus, 2016) suggest the positive outcomes of having students collaborate on tasks and the importance of having explainers and listeners during group discussions. Although this study finds that interaction in the form of accommodation can enhance the collaborative process of refinement, it shows that sometimes, interactions in a group lead to frustrations and termination of the collaborative process in the ongoing discussion when there is a dominating student.

This finding parallels studies by Eddy et al. (2015) and James et al. (2008), which suggest that a dominating student in a group stands as a barrier to effective participation in a group. Eddy et al. (2015) reported similar findings among college students in a biology classroom. Data from their study showed evidence of a barrier toward group participation due to students' exclusion from the discussion by the actions of their groupmates. Similarly, James et al. (2008) highlighted external factors such as the presence of a dominating student as a limiting factor to the ability to participate effectively in a group.

This study shows that accommodation could be from pre-service teachers to peers or facilitators to pre-service teachers. Facilitators in a modeling process should be aware of a situation like this in their classrooms and find ways to help students whose ideas are suppressed or whose thought processes are not allowed to manifest. This study finds that the disadvantages of students' ideas being suppressed in the group include losing their self-confidence in going forward on the task. Furthermore, this study shows that some students being silent in a group does not necessarily mean that they cannot develop ideas to navigate the task; they only experience a shut down because of the presence of a dominating peer.

Finally, the study finds that pre-service teachers refine ideas through interaction by probing and debating others' ideas. Sofroniou (2016) found that group work permits students to develop analytical and communication skills in the mathematics classroom. Furthermore, NCTM (2014) reports that collaborative work in mathematics education plays an essential role in students' question acquisition, while Koçak et al., (2009) suggest that such an opportunity helps learners criticize constructively in a mathematics classroom. When engaging in such analytical discussions, pre-service teachers use various questions to carry out their probing and debating, enabling them to criticize others' work constructively. This form of collaborative process whereby pre-service teachers ask questions empowers them to learn constructively (inquiry process) rather than knowledge flowing from one person to others. Thus, the facilitator acts as a coach (Smallhorn et al., 2015; Adeolu, 2020) in such a learning space to ensure that the lessons are student-centered and that discoveries in form of refined ideas are made in the process.

This finding supports the idea that the inquiry form of learning improves engagement with content and assists in developing analysis and critical thinking skills (Smallhorn et al., 2015). Adeolu (2022) found that learners and facilitators use different questions to enhance interactions during modeling. He suggests that using questions that allow learners to engage in deep thinking helps promote interactions that could lead to refined ideas. He reports that modelers and facilitator use questions to probe initiatives flowing within and across groups. When the listener is satisfied with the justifications for the probes, both parties conclude and move on. Otherwise, they enter into a debate.

Modeling interaction when pre-service teachers construct computational thinking practices within and across groups.

This section presents how pre-service teachers interact within and across groups. Figure 18 presents an interactional model developed through data from this study. The interactional model shows the nature of interaction when pre-service teachers collaboratively work on modeling tasks in the presence of computational tools as mediating artifacts. The model has three phases, namely Phase 1, Phase 2, and Phase 3, with each phase representing Vygotsky's mediating triangle discussed in chapter 2. All these phases are inscribed in a giant triangle that generally represents Vygotsky's mediating triangle. The phases in the giant triangle represent pre-service teachers' moves within and across groups, how the presence of computational tools affects their positioning in the groups, and the processes of interactions.

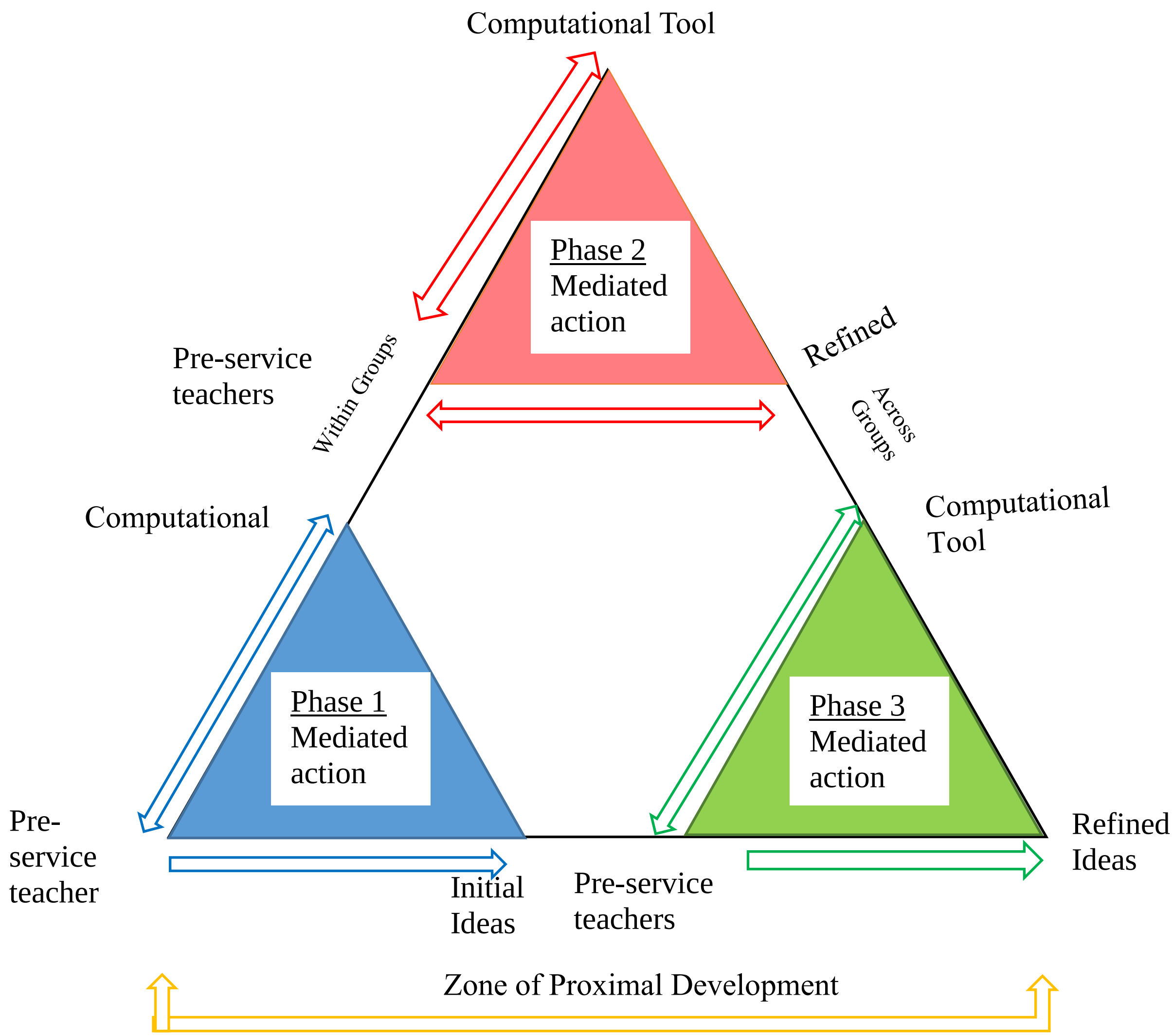

Figure 18: 3-Phase Nature of Interaction with Pre-service Teachers During Modeling in the Presence of Computational Tools.

Phase 1 (Within the group)

Here, pre-service teachers work through individual processes to develop limited cognition of the situation, which becomes the topic of discussion in phase 2. That is, they made sense of the situation without interaction or less interaction with others. In phase 1, a pre-service teacher (subject) interacts with the computational tool to generate initial ideas, also known as individual interpretations (Schütte et al., 2019). At this level, Schütte et al., (2019) opined that the participants develop individual preliminary interpretations of the situation based on their individual experiences and knowledge. These initial interpretations of situations develop in anticipation of possible attempts of other participants to negotiate them and adjust if the need arises (Krummheuer, 1992).

The limited cognition at this phase can be about the task in general or the ideas to write codes and build the models in the computational tool. More often than not, pre-service teachers experience confusion about the task in general or become less/more confident because of the computational tools available. As a result of such confusion or subjects' identity with the tool, the formation of positioning occurs for them.

The only interactional process subjects use in Phase 1 is giving and receiving feedback, whereby pre-service teachers and computational tools engage in a give-and-take interaction. The pre-service teacher gets feedback from the user interface that may become the initial ideas for further discussions in Phase 2. Put differently, the limited cognition (initial ideas) becomes an object/outcome that the pre-service teacher discusses with peers working together in the same group (within the group) in Phase 2.

Phase 2 (Within the group)

In phase 2, pre-service teachers work within the group through collective processes. That is, by interacting with others. The facilitator may be part or not. If a division of labor occurs, pre-service teachers go back to phase 1. They go back to phase 1 because they need to use their expertise to work on the portions of the tasks assigned to them. If there's no division of labor, subjects collectively refine ideas by probing and debating and then move to phase 3 if the need arises.

In Phase Two, the individual interpretation of the situation formed in phase 1 become the basis for peer discussion and can involve the facilitator. These discussions helped pre-service teachers refine the initial ideas they had constructed independently. In phase one, the formation of positioning takes place, determining how they act during discussions in phase two. Thus, pre-service teachers use vertical and horizontal leadership in this phase. They are also positioned as resources that peers can use to resolve challenges and other solution networks as resources.

Leontiev (1978) expanded Vygotsky's idea of the subject to articulate how individuals come together to work on activities and are guided by goals or motives. In the same sense, this model extends Leontiev's perspectives. When subjects divide labor, they move back to phase one to form initial ideas that become the topic of discussions in phase two. Otherwise, they discuss the initial ideas they had formed from phase one. Voigt (1995) discussed that participants involved in an interaction attempt to attune the individual interpretations of the situation to each other, ideally leading to the production of taken-as-shared meaning. This taken-as-shared meaning or a working consensus is what this study referred to as refined ideas that participants generated after interactions with peers, facilitator, and tools. In this phase, collective processes involve using the solution-seeking networks to navigate challenges. They engage in collective processes through which pre-service teachers give and receive feedback from computational tools and accommodate others' ideas.

Phase 3 (Across the group)

The ideas developed in phase 2 become the initial ideas pre-service teachers discuss in phase 3 through collective processes. Again, the facilitator may take part or not. If both groups are satisfied with the refined ideas developed in this phase, they move on to the other parts of the task. Otherwise, they go back to either phase 1 or phase 2 to go through the triangular cycle again. In this phase, groups meeting together may use the solution-seeking networks to navigate the challenges they face. Like in phase 2, pre-service teachers engage in collective processes through which they give and receive feedback from computational tools and accommodate others' ideas. In addition, one group can refine ideas with other group members by probing, debating, and justifying ideas from each group. A group can probe other groups' ideas through mathematics and statistical knowledge, personal experiences, or questions mostly resulting to both groups better understanding the ideas being deliberated during the exchange of ideas.

The refined ideas formed in phase three strengthen groups' understanding of the situation; otherwise, subjects involved may go back to Phase 2 or Phase 1 to revise the process. Just as in Phase 2, the roles of the facilitator in Phase 3 include guiding participants' thought processes, initializing interaction, probing their thought processes, and being resourceful. In conclusion, pre-service teachers develop individual ideas that represent the actual development they can construct when they work alone. However, they move their ideas forward and become enculturated into the community of practice when they interact with others in phases two and three. With such interaction and development of refined ideas, pre-service teachers get to their zone of proximal development and can add personal values to their learning in their subsequent tasks. Sometimes, time may not permit pre-service teachers to get to phase 3. I recommend that facilitators ensure that pre-service teachers get to phase 3 to extend their zone of proximal development.

Mathematical and statistical practices supported during the construction of computational thinking practices.

This study finds that pre-service teachers use mathematical and statistical thinking in developing a computational model. This study finds that many elements identified in the standards for mathematics practices (CCSSM, 2010) and mathematical practices through a statistical lens (SET, 2015) are used by pre-service teachers when building models. The National Council of Teachers of Mathematics (NCTM) describes five process standards that highlight ways of developing and using content knowledge: problem-solving, reasoning and proof, communication, connections, and representations (Principles and Standards for School Mathematics, 2000). The National Research Council (NRC) breaks down mathematical proficiency into five interconnected strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Adding It Up, 2001). The Common Core State Standards for Mathematics (CCSSM) (2010) builds on the processes and proficiencies outlined by NCTM and NRC in its eight Standards for Mathematical Practice. In their document, CCSSM describes the connection of practice standards to mathematical content as follows:

> "The Standards for Mathematical Practice describe how developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary-, middle-, and high-school years. Designers of curricula, assessments, and professional development should all attend to the need to connect mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination

> of procedure and conceptual understanding. Expectations that begin with the word "understand" are often especially good opportunities for connecting the practices to the content." (Source: Statistical Education of Teachers, 2015)"

Complete documentation can be found in the Statistical Education of Teachers (2015) and in the Common Core State Standards for Mathematics (CCSSM, 2010). However, I focused on the practices of modeling with mathematics and using tools strategically.

The elements used by pre-service teachers in this study include applying mathematics to answer statistical questions, using equations to describe how one quantity depends on another, or using geometrical representations to describe structure, identifying important quantities, interpreting mathematical results in the context of the situation, and possibly improving the model as elements of *"model with mathematics."* Also, considering the available tools, detecting possible errors, using technology to visualize the results of varying assumptions, exploring consequences and comparing predictions with data, and identifying relevant external mathematical resources as elements of "*use appropriate tools strategically.*

This study finds, as an extension of the practice of model with mathematics that students who have developed the art of modeling with mathematics can also understand how to find existing models, tweak or manipulate them to suit the situation, and then select the best that models the problem. Lastly, this study finds, as an extension of the mathematical practice of using tools strategically, that mathematically and statistically proficient students can understand that their knowledge of mathematics should inform the usage of the tools and vice versa.

Computational thinking practices developed by pre-service teachers during interaction with peers, tools, and facilitator.

This study identifies ten major computational thinking practices as shown in Table 7. Modelers sometimes encounter a situation where they need to create data or extend a dataset to obtain a clearer understanding of the model. Weintrop et al. (2016) suggested that the practice of data creation is important because sometimes, it is not possible to observe and measure some phenomena in situ because some events happened billions of years ago, and we want to understand them. Like in this study, we cannot inject people with diseases to carry out how the disease spread. Thus, we need a scientific approach in form of physical simulation to represent the situations we want to study.

This study finds it essential for modelers to understand the underlying assumptions for the data they want to create or extend. They also need to obtain the background information about the data. When modelers identify underlying assumptions and data stories, the next thing to do is to find ways to represent them. This study finds that working in groups creates a space whereby modelers can derive refined ideas through multiple perspectives on introducing the assumptions in the data they are creating or extending. In general, a meaningful form of interaction is needed for all groupmates to understand information embedded in the data and find resources to mimic the information before coding them up in the computational tool. My description of what modelers should understand and do when creating or extending data extends Weintrop et al. (2016) study that identifies data creation as a data practice.

Data exploration describes situations where pre-service teachers used visuals to represent data to uncover insights. Visualization is an essential component of any knowledge-building endeavor that mathematicians use to analyze and share data (Weintrop et al., 2016). This study finds that modelers can use visuals to understand the effect of a parameter, understand the relationship between two or more variables, make predictions, and understand the effect of data size and its long-term effect on the model. This study finds that modelers explore data by manually scanning the dataset to understand the data points and identify any errors. Next is data manipulation. Data manipulation is altering or adjusting a dataset for a better data organization that enables easy reading, getting helpful information, or ensuring that the data suits the questions under investigation.

Pre-service teachers are resourceful by looking for existing codes or models and then mimicking them to facilitate their model development. During the modeling process, modelers can look for quick and clever ways to overcome difficulties and then try to sync what they found with the problem. Mimicking is a valuable practice when pre-service teachers build models. However, this study finds that an essential procedure when modelers lookup for existing codes or models is that they should be able to compare different codes/models they found to ensure they have a model that best represents the situation. In addition, this study finds it essential for modelers to develop their models since some existing models may not adequately suit the problems they are modeling. In addition, pre-service teachers must be skillful to ensure that what they found is helpful to the problem and that group members understand why the model suits their investigation. Modelers can use models developed by the facilitator, models from previous classes, or those found on the internet to inform their work.

While creating models, modelers mathematize codes and models. To mathematize codes and models, pre-service teachers generate mathematical expressions or graphs to represent models. To mathematize codes and models, modelers have to think about the situation mathematically and develop a diagrammatic representation for the problem (Figure 13). The ability to develop a mathematical representation of the phenomenon helps pre-service teachers to develop a better understanding of how to code in a computational tool. In addition to that, it helps modelers to have a better understanding of what the tool is doing since they are the designer while the tool is executing the ideas.

The most important aspect of model building is to ensure it is useful in the context of the problem. This study finds that communicating, interpreting, and connecting models to the real world is an essential computational thinking practice modelers should develop. It is an idea whereby modelers explain the solution (model) they found or built in the task context to ascertain fairness, accountability, transparency, and confidence in using the models (Sakar, 2018; Adeolu, 2020). This important practice in model building ensures that modelers do not just throw models out there but ensure that whoever encounters the model understands it to the last detail. Furthermore, interpreting the model enables modelers to see the usefulness of the computational model beyond mathematical expression/representation (Table 8). This study also finds it essential to communicate limitations to models and places for improvements to give the confidence to use the models.

Another computational thinking practice found to be important is model assessment. When assessing models, modelers share models with peers within and across groups to compare what they have developed. The CCSSM (2010) categorized this process of modeling as validation whereby modelers assess or compare the models they built to reality. This study finds that modelers depend on their mathematical and statistical abilities to assess models. Thus, pre-service teachers must develop robust mathematical and statistical knowledge that will help them assess the usability or compatibility of the model they come up with. Assessing models across groups allows modelers to gain perspectives from others which could help them see their work differently or help other groups gain better insights into their model. Modelers also use prompting and exploring to navigate the modeling process. When pre-service teachers receive prompts (questions) about variables or how to code, they explore with the computational tool by trying and adjusting ideas based on the feedback received from the tool's user interface.

Pre-service teachers use a computational thinking practice called parameter manipulation and discovery. This practice involves tweaking parameters to investigate their sensitivity to the model and making discoveries. This practice helps pre-service teachers automate the modeling process to avoid running many simulations. The last computational thinking practice identified in this study is conjecturing and generalizing. Pre-service teachers are always in the business of making predictions about the model and also generalizing the predictions to other situations. To make predictions about a model, modelers must be equipped to create and interpret the model in the context of the problem.

In conclusion, the use of the practices identified above is not in any particular order. The practices can feature in the modeling processes as the need arises. However, data from this study reveal that the practice of mimicking and mathematizing cut across every other practice found in this study. In this sense, for example, modelers can mimic codes to create data, mathematize to explore or manipulate data. When modelers make predictions, it is apparent that they use mathematical ideas to make such predictions and generalize. To this end, I visualize the other eight practices revolving around mimicking and mathematizing codes and models (Figure 19).

However, it is possible the assumption that mimicking and mathematizing codes and models are at the center is applicable when pre-service teachers are using new computational tools. It is therefore essential to look into the idea more broadly.

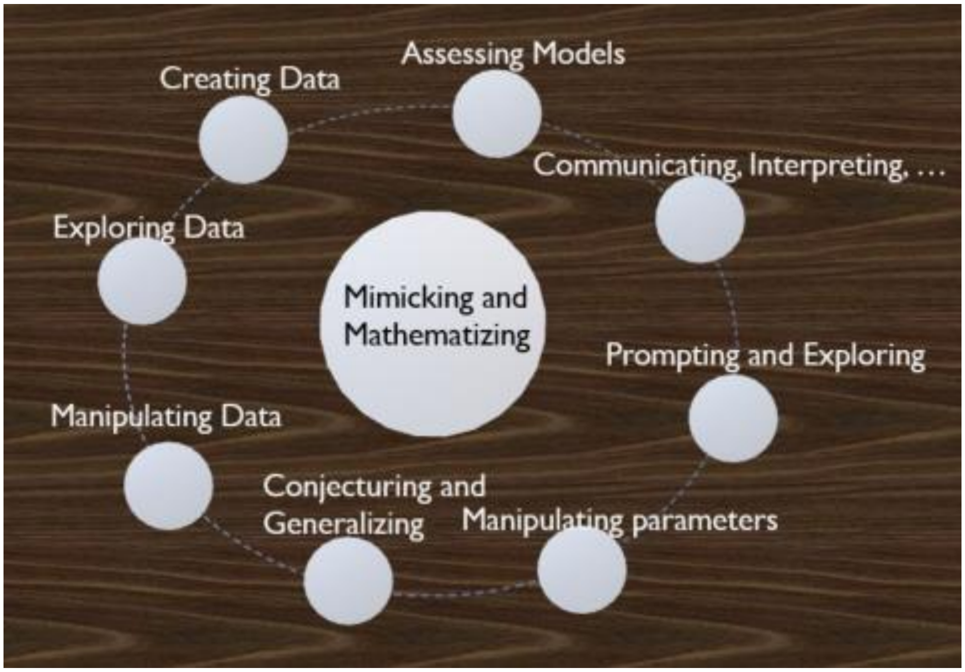

Figure 19: Computational thinking practices revolving around mimicking and mathematizing codes and models

Implications for practice

The findings discussed in chapter four and summarized above are significantly relevant to integrating computational thinking practices in pre-service teachers' mathematics education. Studies show that computational thinking practices are essential in K-12 education, which necessitate training teachers (Yadav et al. 2017). Researchers (e.g., Yadav et al., 2017) suggest that professional development for teachers may not sufficiently engage teachers in developing the requisite knowledge needed to facilitate the practices in their classrooms. Other studies (e.g., Weintrop et al., 2016; Lockwood and Mørken, 2021) suggest the need for pre-service teachers to construct the knowledge while others (e.g., Cetin, 2016) argued that teacher preparation is a better platform to introduce potential teachers to the construction of computational thinking practices through their integration in mathematics classes. Lockwood and Mørken (2021) push for studies investigating how mathematics courses for teacher preparation could help to teach computational thinking practices to pre-service teachers.

This study investigated how pre-service teachers can collaborate in a mathematical modeling class to construct computational thinking practices. This study finds that the presence of computational tools influences the positioning of pre-service teachers and the processes of modeling they engage in during modeling activities. Furthermore, this study identifies ten computational thinking practices relating to data-driven tasks and finds a link between computational thinking practices, mathematical thinking practices, and statistical thinking practices. In what follows, I will discuss what these findings mean, why they are important, and the real-world applications of the findings.

This study shows that pre-service teachers engage in *leadership and distributed authority* as *positioning* during modeling activities. They engage in *dividing and offloading labor, giving and receiving feedback, accommodation, and refining ideas* as *processes* of interaction during modeling. As a positioning, pre-service teachers consciously or unconsciously engage in *leadership* in their groups. The two forms of *leadership* identified with pre-service teachers are vertical and horizontal leadership. This study shows that computational tools naturally position some group members who are more skillful in using a particular computational tool as leaders in their groups. This positioning mostly makes ideas flow from top to bottom, thereby making groupmates accommodate ideas with underlying frustrations and termination of collaborative processes. This occurrence is predominant in a group where only one member is skillful in using a computational tool. They unconsciously make groupmates accept their ideas and structure the task in their ways.

In chapter two, I discussed how Abtahi's (2017) idea of not identifying any more knowledgeable others (MKOs) during interaction in opposition to Vygotsky's (1978) and Wertsch's (1998) ideas about the relevance of a more knowledgeable other. While group members must contribute meaningfully during modeling, this study finds that the presence of a computational tool always introduces group member(s) that is positioned higher in using the tool and that can influence the processes of interactions and modeling. It is therefore vital to know that modeling involving computational tools will always introduce a more knowledgeable other into modeling processes. Although this finding is not new and has already been identified in Vygotsky's (1978) and Wertsch's (1998) works, it reminds us of the influence of the presence of the mediating tool during an interaction. However, it is essential to worry about the presence of more knowledgeable pre-service teachers in using computational tools on the quality of developed models. The knowledge of assessing the models created across groups is beyond the scope of this study; however, it is a thing to consider when discussing the presence of a more knowledgeable other introduced during modeling as a result of the presence of a computational tool. Therefore, facilitators should look for ways to minimize the influence of group members leading vertically during modeling as this study does not show how their influence may affect the model development.

Next, I will explain the collaborative processes of interaction during modeling in the presence of computational tools. As a process, pre-service teachers interact with peers, facilitator, and tools through *dividing and offloading labor, giving and receiving feedback, accommodation, and refining ideas*.

In this study, labor division means that pre-service teachers are aware of individual strengths and the need to invest these capabilities in the aspect of the tasks they are working on. Pre-service teachers acknowledge that individuals working in a group possess different abilities that others can rely on to complete the modeling task. As a result, facilitators should be aware that pre-service teachers naturally identify their strengths in a modeling space and tend to share labor. On the positive side, such initiatives help pre-service teachers share the task according to their ability, thereby reducing workload that may translate to academic stress. One downside to this is that pre-service teachers may not care to understand what other group members achieve on other parts of the task, thereby completing the task in the name of the group but not having the deep understanding of other parts worked upon by groupmates. They are comfortable with getting the task behind them and moving on. In this sense, facilitators must encourage group members to always engage in ideas refinement through probing and debating.

Pre-service teachers engage in ideas refinement to bring improvement or clarification to the ideas for group members. During refinement, changes could be made to the ideas when pre-service teachers engage in probing or debate. While working in groups has always been a strategy recommended by educators (Burke, 2011; Baines et al., 2015), this study reveals that pre-service teachers can appear smiling with groupmates but experience unsaid frustrations. Thus, the role of the facilitator should also include assessing students working on modeling activities in groups by asking each member to share their understanding of the ideas they are using in the group.

This study identifies five ways by which pre-service teachers can go about resolving the problems they encounter. Pre-service teachers can reach out to peers and facilitators or use alternative means (e.g., Google search) to resolve their challenges. In this sense, pre-service teachers utilize immediate help by considering peers and facilitators as firsthand resources to resolve problems. On a rare occasion, they consider the alternative route as firsthand resource in solving their problems. Sometimes, pre-service teachers do not get their problems resolved using these firsthand resources; thus, they need to turn to other resource(s). In this sense, they reach out to another resource if they cannot solve the problem at the initial trial. Pre-service teachers should be aware of the resources available to them in the modeling space.

This study reveals that pre-service teachers can move from peers to facilitators or from peers to alternatives, and if not resolved, they move to the facilitators (Figure 9). This movement means that the facilitator is significant in getting problems resolved during modeling activities. As a result, the facilitator should be ready for any questions that pre-service teachers could raise during modeling or at least know how to use the alternative resources to help students solve problems. No single teacher can or is expected to know all the directions students might pursue or any computational problems that could arise in modeling space, but they should be ready to conduct mini research to resolve problems with students (Adeolu, 2022).

The above-described ways pre-service teachers resolve challenges are also applicable when encountering computational tools problems. During a *giving and receiving feedback* form of interaction, pre-service teachers engage in a give and take exchange of ideas with tools or peers. This means that exchanges are made during interaction in which pre-service teachers can swap ideas with others. This concept is useful when pre-service teachers assess models or check-in with peers to move the modeling process forward. This study reveals that pre-service teachers develop mathematical codes and structures that they input into the computational tool and get feedback. Writing codes is important for pre-service teachers to model with computers. This study reveals that pre-service teachers know what they want to model or what they want to achieve with the computational tool. They need the tool for versatility, speed, and the ability to process big data at a time or to run multiple simulations by just changing a parameter or through any model exploration. This study finds that pre-service teachers better understand results from the tools when they understand them mathematically before modeling with the computer or coding their ideas. Thus, pre-service teachers must understand the situation they are modeling mathematically before coding it up in the computational tool.

This study finds ten computational thinking practices associated with pre-service teachers working on data-driven mathematical modeling activities. This activity type means that computational thinking practices may differ in another context where the tasks are not data driven. For example, data practices identified in this study may not feature in agent-based modeling, whereby students explore existing agents in the system. This study finds that pre-service teachers use mathematical and statistical practices to develop computational thinking practices. In addition, this study supports existing studies (e.g., Bliss et al., 2018) on the relevance of using a computational tool to navigate modeling processes. This study reveals the intersection of the mathematical modeling processes and computational thinking practices pre-service teachers use during modeling (Figure 10). In a sense, the processes of modeling and developing computational thinking practices are interwoven. Chapter four showed how these practices are interrelated to the modeling processes and a modeling cycle containing modeling processes and computational thinking practices (Figure 10). This development means that students develop, or use known computational thinking practices during modeling.

This study finds that pre-service teachers use mathematical and statistical practices when developing or using computational thinking practices during modeling. In a sense, pre-service teachers must come into the modeling space with mathematical and statistical knowledge that helps them navigate the modeling process. For example, when engaging in data practices, they need mathematical and statistical ideas to understand the problem set involving terms such as probability, standard deviation, functions, and randomness. On top of that, they need mathematical ideas to mathematize codes and models and to communicate, interpret, assess, and connect models to the real world.

This study finds that the process of interaction enables pre-service teachers to negotiate ideas thoroughly in developing computational thinking practices and using mathematics and statistics practices in the process. This concept means that a class setting that allows pre-service teachers to interact will help them develop computational thinking practices during modeling. This study finds that interaction within and across groups is essential for thoroughly refining ideas. Thus, a class should embrace pre-service teachers moving across groups to go through modeling processes.

The importance of this study is that we now understand the nature of interaction pre-service teachers undergo when they work collaboratively on modeling tasks in the presence of computational tools. We now understand an extension to Vygotsky's (1978), Leontiev’s (1978), and Wertsch’s (1998) ideas about mediating triangle when pre-service teachers interact with the computational tool during modeling activities. In this sense, this study extends our understanding of collaborative work. It adds to our understanding of mediated actions with tools and the expansion of Vygotsky's (1978) and Leontiev's (1978) work on social constructivism. This study identifies the intersections between the modeling process and computational thinking practices and ascertains that the standards for mathematical and statistical practices are engrained in developing computational thinking practices.

We now understand the con with the presence of computational tools during modeling. First, this study reveals that the presence of computational tools in modeling activity concentrates the challenges pre-service teachers face in dealing with how to navigate the tool and subdue challenges with the use of mathematics. Thus, we do not know if pre-service teachers would experience greater or different challenges with the use of mathematics in this space if the tools were removed from the modeling process. Secondly, the presence of computational tools positions computationally more skillful students as leaders who lead vertically (from top to bottom) because they think the tools they have mastered are the best to navigate the problems, thereby sometimes leaving groupmates at a disadvantage. With the presence of this kind of vertical leadership, we now understand that pre-service teachers might accommodate peers' ideas. This accommodation is not because they understand but because they view those groupmates' ideas as more powerful, limiting what they learn and reducing their confidence during modeling that involves interactions with peers and tools. Lastly, this study adds to the very few studies on pre-service teachers using computational tools during modeling activities.

The knowledge gained from this study will enable the mathematics education for pre-service teachers to be more improved with the curriculum development that aims at teaching computational thinking practices. In terms of the real-world applications, I will discuss the implications of the findings from this study to pre-service teacher education and the integration of computational thinking practices in mathematics learning. To do this, I organize the applications under the headings: *teacher preparation and research in mathematics education.*

Teacher Preparation

Previous studies suggested that students' problem-solving skills and understanding of mathematics practices improve when they are exposed to computational thinking ideas (Akcaoglu and Koehler, 2014; Calao et al., 2015). Therefore, it becomes imperative that we prepare teachers equipped to train future students with these skills. I discussed in chapter two that teacher training programs for future teachers provide ample opportunities to prepare teachers to embrace and develop computational thinking practices. On the one hand, teachers' effectiveness has been identified as one important factor affecting students' academic growth (Sanders and Horn, 1998). Thus, future teachers need to develop ways to be effective in teaching computational thinking practices to their students. On the other hand, I argued in chapter two that future teachers need to experience computational thinking practices as learners themselves. Such an experience will give them firsthand integration of computational thinking practices in mathematics learning. This study provides opportunities to structure mathematics education that support pre-service teachers to develop the experiences needed for future assignments as teachers. I discuss these opportunities under two sub-headings: *subject matter and learning experiences.*

Subject Matter

When designing curriculum, the next element to consider after the objectives of the curriculum is the subject matter. It is an element or a medium through which the objectives are accomplished. Subject matter refers to the body of knowledge that the student will take away when the course is done. The assumed primary objective of pre-service teachers' mathematics education in this study is that graduates from the program have developed computational skill sets and experiences that position them to teach future learners. Findings from this study suggest various ways by which the subject matter can be structured for a better experience for pre-service teachers' mathematics education. This study reveals that using computational tools creates the biggest challenge for pre-service teachers and positions group members as leaders where groupmates accommodate their ideas with frustration. Therefore, this study opens doors for what could be done to improve the integration of computational thinking practices in pre-service teachers' mathematics education. I discuss various suggestions and recommendations later in this chapter.

Learning Experience

The next most important thing educators consider is the method or how to situate learners experience in the class. This study provides us with the nature of interaction pre-service teachers engage in, the challenges they face in this process, how they move to resolve the challenges, and the effect of using computational tools. Therefore, facilitators are equipped with pedagogical awareness to consider when setting up the class. For example, this study shows that a facilitator should be concerned about students' backgrounds with computational tools other than those used in the class. They should be concerned about groupings of students or at least find a common denominator with the use of tools in the class. Facilitators should be aware of the processes pre-service teachers engage in to refine their ideas. I discuss various suggestions and recommendations later in this chapter.

Research in Mathematics Education

Studies that focus on pre-service teachers developing computational thinking practices are few in the field. This study not only understands the types of computational thinking practices pre-service teachers develop during modeling but also reveals the types and nature of interactions in the collaborative process. This study extends our understanding of the social constructivist theory of doing research in the context of pre-service teachers engaging in modeling activities with tools. Therefore, this study adds to what we know in the field and introduces newer pathways we can explore to study pre-service teachers developing computational thinking practices during mathematical modeling activities. I discuss various suggestions and recommendations below.

Recommendations

The following suggestions are essential to ensure that pre-service teachers have a great learning environment when modeling with computational tools.

Teaching and Learning - Social Interaction

- Labor division should be encouraged to allow pre-service teachers to work on aspects of their strengths. However, facilitators should ensure that ideas are properly refined so that all group members understand the contributions of groupmates to the model development.
- Facilitators should be aware of the positioning and processes of interaction that happen during modeling with computational tools.

- Facilitators should encourage students to probe and debate ideas to minimize accommodation that leads to frustration and terminates collaborative processes. In addition, facilitators can ask group members about the ideas of others in their group to ensure that they are not just accommodating ideas to get pass the task but accommodating because they understand.
- Facilitators should mostly use tasks that directly affect pre-service teachers as it helps them persevere in using mathematics.
- Facilitators should alert pre-service teachers on the use of alternatives resource as a useful strategy for resolving challenges.
- Facilitators should encourage *across the group* ideas' refinement to help pre-service teachers get to phase 3 Figure 18.
- Peer discussion norms should be established at the beginning of the class to minimize vertical leadership during an interaction.

Teaching and Learning - Computational Tools

- A single computational tool should be recommended for the class to ensure that students have a similar experience during modeling. Otherwise, pre-service teachers should be grouped based on their computational backgrounds if possible.
- To ensure all students have equal opportunities modeling with a computational tool, a two-week use of the tool should be entrenched at the beginning of the class. Facilitators can lead this teaching by giving examples of how the tool is used in solving problems.
- Some statistical and mathematical ideas should be revised alongside the two-week learning of the computational tool. For example, concepts such as probability,

randomness, mean, median, standard deviation, normal distributions, charts and data representations, functions, etc., should be revised at least with definitions and practical examples.

Recommendation for Research

- Studies should be conducted to investigate the practice of mimicking in mathematics learning.
- Future studies should consider a study design that focuses on single computational use.
- Future studies should consider more participants to extend the scope of findings.
- Future studies should consider tasks that are not solely data-driven to capture the types of computational thinking practices developed in such situations.
- Future studies should make an effort to conduct stimulated recall interviews with all participants.
- Future studies should look into the effect of vertical and horizontal leadership on the model development. For example, how does vertical and horizontal leadership affect the kind of model developed?
- There is a need to study pre-service teachers' perspectives when using computational tools during modeling.

Conclusion

This study has been successful in investigating how the presence of computational tools influence the nature and types of interaction among pre-service teachers during modeling. The study also investigated the computational tools pre-service teachers developed, the challenges they encountered, how they resolved them, and explored the relevance of mathematical and statistical practices during modeling with data-driven tasks.

And so distinguished reader, this work comes down to the fact that the presence of computational tool influences the positioning and processes of interactions pre-service teachers engage in when working collaboratively on modeling activities. They encounter challenges mostly attributed to computational tools and use five pathways to resolve the challenges (Figure 9). They develop ten computational thinking practices relevant to participants and the nature of tasks used in this study (Table 7, Chapter 4). These practices are related to pre-service teachers' mathematical and statistical practices in their mathematics learning. In general, findings from this study extend our understanding of the knowledge development when pre-service teachers interact with tools, peers, and facilitators.

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APPENDICES

APPENDIX A

MODELING ACTIVITIES: BOBCAT POPULATION & DISEASE SPREAD

Bobcat Population

Most species of wild cats are endangered including the bobcat. In this activity, you will explore the behavior of a bobcat population using growth rate data from the state of Florida (from Cox et al., *Closing the Gaps in Florida's Wildlife Habitat Conservation System).*

Consider the three annual growth rates for bobcats under various environmental conditions.

Best ($r = 0.01676$)

Medium ($r = 0.00549$)

Worst ($r = -0.04500$)

Assume these growth rates are constant from year to year and consider each growth rate as representing a different region in Florida. Construct a spreadsheet tracking three bobcat populations, each initially consisting of 100 individuals, over a period of 10 years under the three types of environmental conditions. Plot all three simulations on a single graph and label all aspects of your graph.

Repeat part 1 over a period of 25 years.

Under the best conditions the population is growing. Several management plans have been discussed. The first is to allow one bobcat per year to be hunted. The second is to allow five bobcats per year to be hunted. The third is to allow one percent of the animals to be hunted. The last is to let five percent of the animals be hunted. Construct a simulation which compares these strategies over 10 years and 25 years. Which of these strategies result in a stable population?

Continuing with the theme of part 3, experiment to find strategies which cause the population under the best conditions to stabilize. Find a strategy which causes the population to rise to approximately 200 animals and then stabilize.

Under the worst conditions the population is declining. Proposed management plans include adding three animals per year, adding 10 animals per year, adding one percent of the population each year, and adding five percent of the population each year. Compare these strategies for 10 and 25 years. Which strategies cause the population to stabilize?

Experiment to find strategies which will cause the population under the worst conditions to stabilize at 50 and at 200 animals.

Disease Spread

https://drive.google.com/file/d/1F_NMhNdKWz8EiIoryBSJccAJtAAc61U_/view

APPENDIX B

SURVEY

<u>What this survey is about -</u>

This survey will help me understand more about your experiences using computers and learning mathematics. I may invite you to participate in an interview to talk more about your answers. Refusing to participate will not affect your grade in M 428, your standing in your program, or your relationship with Montana State University in any way. You will get a $5 gift card for filling out the survey. Remember that your responses to this questionnaire are voluntary and that you can choose not to answer certain questions or stop at any time. Furthermore, you will not be identified by name in any research or publications resulting from this study.

First Name: ________________________ Last Name: ________________________

Signature: ___________________________________ Date: ___________

Contact Information

Email: ______________________________________ Phone: ___________________

What mathematics courses did you take in high school?

What mathematics courses did you take in college?

When and how have you used technology in your mathematics courses?

How often do you work with others when you do mathematics? Describe your experience when learning mathematics with peers.

What experiences have you had with computer science or computer programming? These experiences can be in or out of school.

On a scale of 1 to 5, with 5 being the highest and 1 being the lowest, how confident are you using computers when

Creating and giving presentations _____

Using spreadsheets   _____

Writing computer programs or using tools such as NetLogo, CODAP, etc. _____

Describe any situations (K-12 through college) where you felt you were engaging in computer science “stuff” in or outside of school. What activities did you do? What tools or programs did you use, if any?

What challenges did you encounter and how did you overcome the challenges in 7?

Do you have any teaching experience? _________ If yes, please describe. Please, include anytime you use any computer tools in your teaching and how the tools were used.

What is your gender? _____________

How many years have you been in college? __________

Thank you for your time

APPENDIX C

INTERVIEW

Interview Protocol

Introduction

Thank you for volunteering to participate in this interview. Your involvement is very helpful, important, and central to knowing more about your participation in using a spreadsheet program in a mathematical modeling class. The main goal of this study is to investigate how pre-service teachers construct computational thinking practices in a mathematical modeling class and how such learning experiences help them understand how computational thinking can be part of secondary mathematics teaching and learning. This interview is in no way to evaluate your modeling ability, rather, it is a means to document what I can learn about pre-service teachers constructing computational and mathematical practices through you.

I will be recording our conversation. Remember, you can choose to stop the interview at any time. Do you have any questions before we get started? Are you ready to begin?

Part A

1. which of the activities stood out to you throughout the class? Why?

2. Tell me about working with your group members on this activity?

3. Can you describe how your group worked on the activities in the spreadsheet program to actualize the outcomes presented in your group?

4. What challenges did you encounter when building models using spreadsheet program?

5. How did you overcome the challenges?

6. What math ideas were useful in your group when working on the activities?

7. How did working on these activities strengthen your knowledge of those mathematics ideas?

8. How do you feel about the process concerning mathematics learning?

<u>Part B</u>

This part will be emerging from participants' solution to project. That is, I will be asking questions based on how participants report their solution, how they used mathematics in their solutions, etc.

Example: In the disease spread model presented in your group (show the participant a copy), you used line graph to show how the number of infected of people compared to the number that recovered. Can you tell me why you took this step?

APPENDIX D

EXCERPT 33

Mia: What part are you guys working on?

Owen: Part D

Mia: Okay, can I check my plots with you to make sure they look the same?

Chloe: For the best?

Mia: Yeah. Uhm, that's not how mine looks like

Owen: How's your equation, how does your equation look like?

Mia: So, what is this for?

Chloe: This is the best?

Mia: I know, but like (Chloe interrupts)

Chloe: This is over 10 years.

Mia: Yeah, but then, so are you subtracting one bobcat a year or 1%?

Chloe: So, different colors represent... So, the blue is the standard, just like not change at all. That is, one is hunted. Orange - five is hunted, then 1%, then 5%.

Amelia: What are you adding to it? Like adding 10, adding one percent? Like if you're introducing more.

Chloe: We haven't done that because we're confused.

Mia: What are those?

Chloe: Oh, that's only for the worst. Owen did that one but it's for the worst condition or worst growth rate because we were trying to get it. I don't know where we're trying to get it to 200 or what was the point?

Owen: For the worst?

Chloe: Yeah

Owen: I just did all the conditions required for bad. So basically, for the best and medium. I guess the medium is from when everything is extra. But I did these four scenarios. And then for the worst. I get these four.

Chloe: Oh, I didn't see it. I definitely missed this and taken away from the work.

Amelia: that makes sense.

Mia: So, I'm confused.

Owen: I'm sorry.

Mia: I'm confused by your graphs. Because these are mine. This is over 10 years

Owen: Can you show me; click on one of your cells and show me your equation?

Mia: Yeah, but what are you guys doing, what is this? Like, why is it doing that?

Chloe: That's a great question.

Mia: Like, what is this data?

Chloe: I'm confused on why it doesn't start at 100

Mia: Well, it looks like you've chunked it out like you took like, from year 11 to year 25 instead of one through 25 on this one.

Chloe: There's potential that I didn't do that.

Mia: I was just wondering like why

Chloe: I probably miss clicked. Let's see what happens if I tried to highlight the entire thing as I intended. Oh no, it's the same thing.

Owen: That's weird.

Chloe: That is very weird.

Owen: Oh, I know. I know why. Okay. Look at the x axis.

Chloe: Yep.

Owen: You're starting basically at 12 Almost.

Chloe: Yeah. but why isn't it starting? Here? Just assume that we're starting at like (Amelia interrupts)

Amelia: maybe because it's like so close.

Mia: You didn't put the days in there? Did you?

Chloe: Oh, yea, we did.

Mia: Because like sometimes I forget to put the year in there and it just starts on whatever cell it is.

Owen: maybe the first graph is just the first 10 years and then the 25 years can be the next kind of deal.

Chloe: Yeah, so we can't just match those graphs

Facilitator: Can you look at your range on your graph itself

Chloe: which one?

Facilitator: Oh, I don't know. Try clicking just in the middle for now. Is there a place in there where you can look at the range for your X values? Like just kind of see.

Chloe: max and min?

Amelia: Yeah

Facilitator: I don't think it's starting at 12, though, like, yeah, no, that's what I'm thinking. It's just that the first grid line is showing as 12. So, do you want to change what's shown on your graph, like in terms of your data? Or do you just want to change your axes?

Owen: I think, see if we can get like, show all 25 years.

Mia: Let me know which one you think stabilizes the population.

Chloe: Okay, now I'm really confused.

Amelia: Well, that's fun. Okay.

Facilitator: So maybe that's what I'll say.

Chloe: I like the fact that we graphed this....

Amelia: we're just looking at like the best one. Right?

Mia: we go and just setting up the numbers so that we can see. I highlighted one through 25.

Amelia: I did year one. And then I just took the one before it. And I multiplied that by the growth rate.

Chloe: So, I knew I thought the same thing.

Facilitator: There we go.

Chloe: We just had number 11 through 25 Rather than here, 11 years.

Facilitator: Okay. Okay. Yep. Okay, cool. I'm glad I could offer my presence.

Chloe: Okay. Mia, we fixed it.

Owen: Mia, there we go. Okay, now you can check.

Chloe: Okay, yes, I felt a little bit okay.