Xenharmonic Praxes: Compositional Studies in 31-EDO for Guitar

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Abstract

This thesis examines the creative, theoretical, and practical implications of composing for the guitar in thirty-one-tone equal temperament (*31-EDO*). Although 31-EDO has a deep connection to musical tradition, its application to the contemporary guitar remains largely unexplored within modern research circles. By combining historical contextualization with theoretical analysis, instrument modification, and compositional inquiry, this study investigates how microtonality can reshape the timbral identity, expressive range, and performative possibilities of the guitar in modern contexts.

The methodology of this research encompasses the modification of two traditional guitars. A classical guitar was configured to a *fretless* design, with adhesive *microfrets* subsequently applied to facilitate precise microtonal inflections. Complementing this, an adjustable microtonal fretboard was installed on a traditional electric guitar, allowing exploration of 31-EDO across differing instrumental contexts. These modifications resulted in a collection of compositional sketches that directly engage the 31-EDO system, illustrating both the practical challenges inherent in performing microtonal music and the expanded expressive possibilities made available through instrument modification.

In this context, *praxes* denotes distinct modes of *praxis*: historical, theoretical, physical, and compositional, through which musical knowledge is generated, collectively giving rise to a broader Xenharmonic *praxes.* This framework demonstrates the viability of 31-EDO as a coherent compositional environment and highlights practical pathways for composers, performers, and instrument builders seeking to expand the expressive potential of the guitar.

Chapter 1

Introduction

In recent years, microtonality has become an increasingly active area of academic interest, with composers, performers, and luthiers working to further musical praxes beyond the limitations of twelve-tone equal temperament (*12-TET*). Among the tuning systems receiving renewed attention, 31-EDO stands out for its connection to established historical lineage, structural clarity, and rich theoretical potential. Rooted in Just Intonation and Extended Meantone Theory, 31-EDO was later codified through the work of Vicentino, Huygens, and Fokker. It occupies a distinct position within the historical development of tuning systems. 31-EDO offers composers near-just thirds and a dense intervallic palette well-suited for use in contemporary compositional praxis. Despite its strengths and academic interest, the system remains underdeveloped with respect to the guitar, a situation shaped as much by material constraints as by historical ones.

The guitar's fixed-fret geometry permanently encodes 12-TET into the instrument's physical structure. Altering that geometry is not merely a theoretical gesture but a material transformation: fret spacing, scale length, and fretboard radius all become direct expressions of the tuning system. This project begins from that premise. Through collaboration with Gibson Guitars and the Bozeman Makerspace, including the installation of a custom 31-EDO fretboard designed by Tolgün Çoğulu, this research examines how instrument modification can lead to practical solutions for microtonal practice on the guitar.

Rather than simply transferring a tuning system onto a familiar instrument, this project seeks to understand how instruments reshape theoretical and compositional possibilities when situated within a Xenharmonic environment.

1.1 Background

Although the theoretical properties of 31-EDO are being increasingly well documented, its practical integration into guitar performance and composition has received comparatively little scholarly attention. Existing microtonal work for the guitar has largely focused on refretting strategies for quarter-tone systems or Just Intonation-derived tunings, many of which emphasize fixed tonal centers and localized pitch hierarchies. While these approaches have expanded the instrument's repertoire, they often remain constrained by the guitar's static frets and its inherited relationship to 12-TET.

In contrast, 31-EDO offers a uniform and highly flexible tuning environment that extends the philosophy of Xenharmony by decoupling pitch organization from fixed tonal centers. Like most microtonal systems, its application to the guitar demands a reconsideration of the instrument's nature.

1.2 Research Focus

This study investigates how adapting the guitar to 31-EDO reshapes compositional praxis through the material realization of an alternative tuning system. The research is guided by the following central questions: How does the physical implementation of 31-EDO on the guitar influence compositional decision-making? In turn, how does required instrument modification lend to the establishment of theoretical principles?

Rather than proposing a single prescriptive method, this project develops a set of interrelated experimental approaches that contribute to broader Xenharmonic praxes.

1.3 Methodology

Historical and theoretical study establish the structural and conceptual foundations of 31-EDO, while instrument modification provides the physical grounding through which tuning structures become musically actionable. Compositional work serves both as inquiry and demonstration, using a collection of sketches to articulate emerging praxis and refine theoretical understanding. This integrated approach positions the guitar as both a research instrument and a creative tool.

Chapter 2

Historical Lineage of 31-EDO

The exploration of alternative tuning systems emerges from a sustained historical effort to reconcile pitch organization with harmonic clarity working within the confines of the instrument's limitations. While 31-EDO itself occupies a relatively marginal position in Western historical literature, the problems it addresses recur throughout the history of tuning theory. For fretted instruments in particular, fixed-pitch layouts have repeatedly exposed the tension between tuning ideals and material realization.

2.1 Early Microtonal Practice

In the Renaissance, lutes were equipped with tied gut frets that were modular, which allowed performers to respond directly to the demands of different musical contexts. This design enabled performers to calibrate their instruments to various temperaments, subtly adjusting fret positions to accommodate Justly Intonated intervals and tempered systems. In the modern day, guitar design has largely forgotten this historical lineage.

Early evidence of enharmonic practice also appears in the form of *tastini* (or *tastinos*), small additional frets composed of gut, which would be inserted between standard tied frets on certain Renaissance and Baroque instruments, reflecting a musical environment in which sharps and flats were not assumed to be enharmonically equivalent. Together, these practices produced a hybrid tuning environment: neither strictly just nor tempered, but negotiated continuously.

2.2 31-EDO in Scholarship

Contemporary scholarship on 31-EDO is deeply informed by the work of Christiaan Huygens and Adriaan Fokker, whose investigations demonstrated that dividing the octave into a greater number of equal parts than twelve provides close, often perceptually indistinguishable, approximations of historically proposed tuning systems and select intervallic structures found in some non-Western musical traditions. 31-EDO represents one of the most refined and musically coherent extensions of these frameworks.

Despite its prominence within tuning theory scholarship, the application of 31-EDO to fretted instruments remains underdeveloped. Most modern explorations of 31-EDO take place in electroacoustic contexts, where temperament adjustments require no physical modification.

2.3 Positioning the Present Study

Contemporary scholarship on 31-EDO similarly highlights both the theoretical refinement of the system and the practical limitations of its application. Pioneering investigations formally established 31-EDO as both a mathematically and musically coherent system. This juxtaposition between theoretical possibility and instrumental constraint illuminates the critical need for research that integrates both domains.

Building upon these historical and scholarly foundations, the present study seeks to bridge the gap between theoretical knowledge and practical realization. By examining 31-EDO through historical praxis, theoretical modeling, and targeted instrument modifications, this research positions the guitar as both a site of inquiry and a creative medium.

This positioning firmly establishes the study within the existing body of scholarship while extending it into new performative and compositional terrain, making a direct contribution to both historical understanding and contemporary Xenharmonic praxis.

Chapter 3

Theoretical Foundations of 31-EDO

In 31-EDO, an octave is divided into thirty-one equal logarithmic steps. Let Δ denote the size of a single step, expressed in cents. The value of Δ is given by

$$\Delta = \frac{1200}{31} \approx 38.709677 \text{ cents.}$$

An interval of n steps in 31-EDO therefore corresponds to the frequency ratio

$$r(31)_n = 2^{(n/31)}$$

Because the division of an octave is uniform, all intervallic relationships in 31-EDO may be expressed either as ratios or in decimal form.

3.1 Intervallic Comparisons

To evaluate the accuracy with which 31-EDO approximates Justly Intonated intervals, it is useful to define a deviation function that measures the tuning error between a Just ratio and its nearest equal-tempered approximation. Let $p : q$ denote a Justly Intonated frequency ratio. The deviation of an n-step interval in 31-EDO from this ratio, expressed in cents, is given by

$$\varepsilon_{\mathrm{JI}}(n; p : q) = 1200 \log_2 \left(\frac{p/q}{2^{(n/31)}} \right)$$

This expression quantifies the extent to which a given equal-tempered interval deviates from the target (Justly Intonated) interval. Small absolute values of $\varepsilon_{\mathrm{JI}}$ correspond to close approximations, greater values indicate perceptually significant deviations.

It is also instructive to compare intervals in 31-EDO directly with those of 12-TET. A 12-TET interval of m semitones has the frequency ratio

$$r(12)_m = 2^{(m/12)}$$

The deviation, in cents, between a 12-TET interval of m semitones and an interval in 31-EDO of n steps is therefore

$$\varepsilon_{12\to 31}(m,n) = 1200 \log_2 \left(\frac{2^{(m/12)}}{2^{(n/31)}} \right)$$

This expression may be simplified to

$$\varepsilon_{12\to 31}(m,n) = 1200 \left(\frac{m}{12} - \frac{n}{31} \right)$$

Because this deviation function is linear in n, its rate of change with respect to step count is constant. Differentiating with respect to n yields

$$\frac{d\varepsilon}{dn} = -\frac{1200}{31}$$

This constant slope indicates that tuning error varies uniformly as one moves through successive steps of the 31-EDO lattice.

3.2 Fretboard Geometry

It is instructive to examine how these intervallic relationships manifest physically on the guitar. The geometric and ergonomic implications of the fretboard provide essential context for understanding practical approaches to instrument modification.

In the following expressions, L denotes the effective vibrating string length associated with a given fret position, measured relative to the instrument's full scale length, L_0. With thirty-one equal divisions per octave, fret spacing between adjacent notes decreases rapidly in the upper register. The difference in effective string length between adjacent frets may be approximated as

$$\Delta L \approx L_0 \left(2^{(n/31)} - 2^{(-(n+1)/31)} \right)$$

This discrete expression may be understood as the finite difference of the underlying continuous fret-position function. Treating n as a continuous variable, the local rate of change of fret position with respect to fret index is given by the derivative

$$\frac{dL}{dn} = -\frac{\ln 2}{31} L_0 \, 2^{(-n/31)}$$

This derivative formalizes the exponential compression of fret spacing along the neck: each successive fret reduces available physical space by a constant multiplicative factor. The negative sign indicates monotonic convergence toward the bridge, while the exponential term quantifies the increasingly fine spatial resolution required of the performer in higher positions.

This compression illustrates why certain high-register intervals may be impractical for practical performance applications, in practice:

- Harmonic playing is most comfortable in the middle register of the neck
- Upper-register clusters function effectively as timbral or textural resources that affect color, but are less suitable for rapid scalar motion

While 31-EDO theoretically supports extensive modulation and Xenharmonic reinterpretation, physical constraints impose practical limitations.

Chapter 4

Applied Praxis

This chapter documents the practical work undertaken to realize 31-EDO on the guitar. The focus is to examine how material and instrumental constraints shape the implementation of microtonal pitch systems on fretted instruments.

- The modification of an existing standard guitar through the addition of microfrets. These microfrets, fabricated from steel and conceptually analogous to historical tastinos, were specifically constructed by Dr. John Schneider for this project. They allow selected 31-EDO pitch sets to be realized temporarily on a standard or fretless guitar fingerboard.
- The installation of a purpose-built, custom adjustable microtonal fretboard fabricated by Dr. Tolgün Çoğulu designed for use on an electric guitar.

These approaches illustrate the practical limitations of retrofitting standard 12-TET guitars and highlight the advantages of dedicated microtonal instrument modifications. All modifications and luthiery were conducted by the author of this paper for a more direct engagement with the research objective.

4.1 Approach One: Fretless Congifuration

One pragmatic response to this limitation is to prefigure the instrument to a specific functional subset of 31-EDO rather than attempting to realize the entire system simultaneously. This approach is achieved through the use of microfrets, which function as localized microtonal operators. Conceptually, this practice parallels historical gut-fret techniques employed on Renaissance lutes as discussed earlier.

When conceptualizing the physics of strings with respect to the fingerboard of the guitar the principle of scale length explains the impracticality of a full

31-EDO system with a complete full fret layout. For a scale length L_0, the distance from the nut to the nth fret

$$d_n^{(12)} = L_0\left(1 - 2^{(-n/12)}\right), \quad d_n^{(31)} = L_0\left(1 - 2^{(-n/31)}\right)$$

where n denotes the fret index.

A direct comparison of these expressions demonstrates the impracticality of retrofitting a standard guitar with a complete set of additional full-width frets to accommodate all thirty-one divisions of the octave. The resulting fret density, particularly in the upper register, renders clean chordal articulation effectively impossible, as adjacent fret spacing becomes too narrow to support reliable left-hand positioning.

For emulating cultures whose music exists outside of the Western tradition, converting the guitar to a fretless configuration becomes a necessary strategy rather than an optional one. Removing fixed frets restores continuous pitch control and further optimizations that allow performers to negotiate alternative tuning systems through embodied intonation.

In an interview conducted as part of this research, Dr. John Schneider described his early use of interchangeable and magnetically attached fretboards, which allowed him to reconfigure separate fingerboards for precise tuning systems, reflecting the need for a modular system for the guitar when situated within individual Xenharmonic praxis. It was in this work that he began making microfrets. Schneider further noted that he eventually abandoned this specific practice; the magnets used to swap fingerboards interfered with the instrument's natural resonance.

Though an exceptional tool, in practice the use of microfrets introduces several interrelated constraints. The added height of the frets can subtly alter the effective speaking length of the string under finger pressure, introducing small but perceptible intonation deviations. As fret spacing decreases, manufacturing tolerances narrow considerably. Sub-millimeter placement errors may correspond to several cents of pitch deviation. The adhesive on the frets is another issue that must be consistently monitored. A performer should exercise care with finger pressure, as excessive force may inadvertently displace the microfrets and affect pitch accuracy.

While microfrets provide valuable insight into the spatial organization of 31-EDO on the guitar and serve as an effective exploratory tool, they ultimately prove impractical for sustained performance praxis without the use of multiple instruments or interchangeable fingerboards.

4.2 Approach Two: Adjustable Microtonal Fingerboard

To address the limitations of a non-modular system, the second approach employed in this project uses a fully custom, adjustable microtonal fretboard designed and fabricated by Dr. Tolgün Çoğulu. This modular system was initially designed with the intention of accurately replicating scale types beyond the scope of Western music theory.

This approach replaces the standard fingerboard entirely, resulting in a stable, fully fretted modular system that eliminates issues of impracticality, adhesion, fret height inconsistency, and localized intonational drift. While the underlying logic of adjustable pitch organization remains historically grounded in Renaissance practice, the execution offers a contemporary, mechanically reliable realization.

The process began by carefully heating the fretboard with an iron while using a razor blade to gently separate it from the neck. Remaining adhesive was cleaned with alcohol, after which the neck surface was meticulously sanded and planed to achieve a completely flat and level platform. Once prepared, the new fretboard was bonded and clamped until fully cured. During installation, careful attention was given to the neck joint to ensure a precise fit and to maintain the original scale length and neck geometry; additional considerations were made at the *Floyd-Rose* locking nut to preserve proper string alignment and tension. Following curing, finish touch-ups were applied as necessary to match the original appearance.

All measurements taken for the custom fingerboard were captured using a digital caliper to ensure exact specifications. No additional fretwork was required as all fret material was temporarily removed from their grooves during clamping. It is important to ensure that the microfrets designed for this system are fully seated in their grooves, as they tend to lift when moved. During initial testing, it was observed that *glissando*, when executed with excessive pressure, can displace microfrets; therefore, performers should exercise care when using this technique. When using bending techniques, the performer ought to rely on longer barred microfrets that span two to three strings to avoid snagging on notches.

Fretboard installation was made possible through collaboration with Gibson Guitars and the Bozeman Makerspace, whose facilities supported all luthiery tasks required for this project.

Chapter 5

Compositional Sketches in 31-EDO

This chapter presents the ideas contained in a collection of compositional sketches that emerged from this project. Rather than presenting finished works as discrete case studies, the discussion focuses on the praxis of composition relative to the constraints inherent in the project. These examples are intended to illustrate how individual praxes lead to compositional insight.

Within this framework, the instrument may be organized into functional regions based on pitch subsets drawn from the full 31-EDO spectrum. Instead of engaging the entire temperament, the composer can configure strings or localized areas of the fretboard to emphasize particular subsets. Modulation and harmonic expansion are achieved through spatial reorientation. The guitarist approaches the fretboard as a set of spatial fields in which intervallic relationships are distributed relatively across strings and sections.

5.0.1 Mothra [6] Sketch

The Mothra [6] subset is a hexatonic *moment-of-symmetry* (MOS) scale derived from the iterative stacking of a single intervallic *generator*. Rather than engaging the 31 -EDO system in its entirety, this sketch restricts itself to the subset in order to investigate the compositional and instrumental implications of a specific pitch organization.

The generator employed is the six-step interval of 31-EDO, measuring

$$g = 6 \cdot \frac{1200}{31} \approx 232.26 \text{ cents.}$$

In the context of 31-EDO, this interval is commonly described as a *supermajor* 2nd.

The subset can be synthesized through repeated application of this generator beginning from a reference pitch class 0. This process may be formalized by the generator function

$$f(k) = k \cdot 6 \pmod{31},$$

with $k = 0, 1, \ldots, 5$. Evaluating this function yields the pitch-class set

$$\text{Mothra}\,[6] = \{0, 6, 12, 18, 24, 30\}$$

The primodal expression of this subset is therefore

$$28 : 32 : 37 : 42 : 48 : 55$$

Because $\gcd(6, 31) = 1$, iterating the generator would eventually traverse all thirty-one pitch classes before returning to the octave.

The Mothra [6] subset therefore produces the following intervals above the fundamental pitch (root): *supermajor 2nd, ultramajor 3rd, perfect 5th* (approximating Just Intonation), *supermajor 6th*, and *ultramajor 7th*, culminating with the return to the octave. The restriction to six elements is not a consequence of cyclic closure but marks the point at which the generator-derived collection first exhibits the defining MOS property. In this case, the MOS corresponds to a 51Ls *machinoid*.

Theoretically, Mothra [6] is well-regarded for its ability to generate smooth, melodic lines. The *major* 7th sonority is intrinsic to this tuning system. The hexatonic nature of this scale and its inherent symmetry produce a smooth, dream-like quality within the subset. Each note in the subset forms a perfect 5th relationship with another respective pitch-class in the subset. Mothra is most commonly employed as a 5-, 6-, or 11-note subset derived from its generator.

On the guitar, Mothra [6] was realized by mapping its pitch classes across adjacent strings and localized fretboard regions. Accurate performance and compositional inquiry required careful measurement of string lengths and precise placement of fret material. Composers and performers ought to consider the scale-length relationships described in acoustic theory.

5.0.2 Orwell [9] Sketch

The *Orwell* collection takes its name as a literary reference to George Orwell, specifically invoking the number 84 from his world-renowned dystopian novel *1984*. In 84-TET, a 19-step interval is equivalent to a *subminor* 3rd (*sin3*) interval. Just as Orwell's novel explores themes of constrained structures within society, this tuning system emphasizes harmonic contrasts and the interplay of closely spaced intervals. In 31-EDO, a 7-step interval approximates the same *sin3* of 84-TET.

One of the first compositional uses of Orwell, and more specifically Orwell [9], was in a piece by Juhani Nuorvala for cello and (31-EDO) organ, commissioned by the Adriaan Fokker.

It is best practice to explain the synthesis of this scale by first describing the construction of Orwell [5] as this demonstrates how many times the sin3 interval can be stacked before reaching the octave. Orwell [9] is created from Orwell [5] through subset expansion.

For Orwell [5], the generator is as follows:

$$g = 5 \cdot \frac{1200}{31} \approx 193.55 \text{ cents.}$$

Through repeated application of the generator:

$$f(k) = k \cdot 5 \pmod{31}, \quad k = 0, 1, \ldots, 4$$

Evaluating this function yields the pitch-class set for Orwell [5].

A seven-step diesis leap can then be broken up in six distinct ways. For creating Orwell [9] in this case, we split the 7-step diesis into a neutral 2nd and a minor 2nd interval corresponding to 4-step and 3-step diesis respectively. When this process is iteratively stacked, we synthesize the Orwell [9] subset. Choosing to start with the neutral or minor 2nd is not arbitrary; following the generator in either direction produces mirrored subsets. This can be observed in the MOS patterns of the two structures:

LsLsLsLss (neutral 2nd) | sLsLsLssL (minor 2nd)

Nonatonic Orwell favors linear projections and emphasizes harmonic exploration over melodic development. Composers and performers can exploit these features to generate novel harmonic textures, leveraging the subset's intervallic structure for functional experimentation. Further exploration of different segmentations of the 7-step diesis may reveal additional harmonic implications yet to be discovered.

5.1 Electroacoustic Sketch

In this work, a solo electric guitar fitted with an adjustable microtonal fingerboard operates within a fixed subset of 31-EDO while electronic processes articulate microtonal modulation and spectral expansion.

Time-based effects play a central role in this interaction. A chorus pedal with a delay time set near 0ms is employed to generate closely spaced pitch replicas whose deviation corresponds to specific steps within the diesis of 31-EDO, producing *shimmer* chords. Modulation and precise configuration of the delay time can be used to reinforce different steps in the diesis. In performance, this effect can be realized using a Boss CE-2 chorus pedal (or similar) with the pedals activation precisely notated in the score. The Boss CE-2 does not natively support modulation of its knobs (though it can be structurally re-modified to do so). For live performance a Max/MSP patch was created as an option to forego the use of standard pedals and allow for finely-tuned modulation. Notation practices used for shimmer chords in this piece are being concurrently researched and developed by Levi McClain through community-driven inquiry.

The use of a *Sustaniac*-Pickup System operated in harmonic mode is necessary for live performance of this piece. This system employs an electromagnetic driver built into the pickup to induce continuous vibration in the string, creating a feedback loop that sustains oscillation without the need to re-pluck said string. By selectively reinforcing upper partials $f_n = n \cdot f_0, \quad n = 2, 3, 4, \ldots$ rather than the fundamental f_0, the system maintains a stable harmonic spectrum.

The process can be described as a feedback loop applied to the dry signal of the guitar:

$$\text{String} \to \text{Pickup} \to \text{Driver} \to \text{Sustained Oscillation} \to \text{String}$$

Electronic processing in this sketch functions both as accompaniment and as an extension of the compositional praxis. Microtonal pitch content introduced through signal processing is treated as extension of the pitch material that would be produced by the respective dry signal.

5.2 Future Research into Compositional Praxis

In addition to the sketches realized in this project, several further approaches emerged for applying 31-EDO to the guitar. These techniques were not developed into complete works within the project's scope, but function as prospective directions for future research. In particular, it was discovered that methods which configure open-string pitch relationships may be treated as precise microtonal *operators*, enacting control over harmonic reference points without requiring further modification of the fretboard.

5.2.1 *Scordatura* as a Microtonal Operator

Within a 31-EDO context, *scordatura* functions as a microtonal operator acting on the open string pitch-class set of the instrument. Scordatura allows the composer to selectively tune open strings to specific subsets or pitch classes within the 31-EDO diesis. A composer can emphasize particular harmonic relationships, reinforce generators, or create linear/melodic trajectories that would be less accessible within standard tuning systems. For example, one might tune a set of open strings to form a nonatonic subset similar to Orwell [9], or select complementary pitch classes to highlight a hexatonic generator like in Mothra [6]. This flexibility enables targeted exploration of Xenharmony and facilitates the articulation of both the harmonic and melodic goals of the composer.

This approach should be considered a complementary tool alongside fretted pitch material, allowing the composer to shape both the harmonic and melodic possibilities of the instrument within 31-EDO.

5.2.2 Capos as Microtonal Operators

In a compositional praxis, *capos* acquire a theoretical significance beyond their conventional use. In 31-EDO, the octave is divided into thirty-one equal steps. A capo placed at physical position p on the neck may therefore be understood to transpose the instrument by an integer number n of 31-EDO steps. This corresponds to a pitch displacement of $n\Delta$ cents, where $\Delta = \frac{1200}{31}$.

Under this interpretation, the capo functions as a microtonal operator acting on the fundamental frequency f of the open strings, such that

$$\mathrm{capo}(p) : f \mapsto f \cdot 2^{(n/31)}$$

where n denotes the effective 31-EDO step displacement induced by the capo placement at position p. This distinction allows for nonstandard capo angles and other mechanically induced microtonal deviations that affect each string independently. When controlled carefully, open string pitch-class sets can be configured with a high degree of precision, within the mechanical tolerances of the capo itself.

This effect may be described as a first-order perturbation of the effective vibrating string length. If a capo introduces a small displacement δL_i relative to the nominal length L_i of a given string i, the resulting frequency shift may be approximated by

$$\frac{\delta f}{f} \approx -\frac{\delta L_i}{L_i}$$

Expressed in cents, the corresponding pitch deviation is

$$\delta c \approx -1200 \log_2 \left(1 + \frac{\delta L_i}{L_i}\right)$$

For sufficiently small deviations, the logarithmic term admits a first-order Taylor expansion,

$$\log_2(1+x) \approx \frac{x}{\ln 2}, \quad |x| \ll 1\,,$$

yielding the linear approximation

$$\delta c \approx -\frac{1200}{\ln 2} \frac{\delta L_i}{L_i}$$

This approximation formalizes the observation that small, string-specific geometric deviations introduced by an angled capo produce proportionally small and predictable microtonal pitch shifts in the open strings.

Coupled with scordatura, this technique enables composers to specify microtonal pitch sets for the open strings. When the capo is installed at an angle, the resulting displacement δL_i varies across strings, producing distinct, string-specific pitch deviations rather than a uniform transposition.

Chapter 6

Conclusion: *Looking Forward*

This thesis has examined 31-EDO from historical, theoretical, applied, and compositional perspectives, establishing a broader framework of Xenharmonic praxes. By situating tuning theory within the material constraints of the guitar, this research demonstrates that microtonal systems are not merely abstract constructs, but embodied environments that shape individual praxis.

The compositional sketches generated through this project serve as new additions to the guitar repertoire. These pieces illustrate how musical structure in 31-EDO emerges from the interplay between historical understanding, theoretical modeling, and the physicality of the instrument.

Applied praxis was primarily explored through the use of microfrets and the adjustable microtonal fretboard. Further experiments generated the concept of microtonal operators. Scordatura and angled capos reveal the guitar itself as a medium through which theoretical principles become tangible in individual ways with respect to the nature of the guitar.

Looking beyond the immediate scope of 31-EDO, the implications of this research point toward a broader vision of Xenharmonic praxes. John Schneider remarked to me that the future of microtonality lies in its ubiquity: when these systems are widely understood and accepted, their use need not be feared or considered exotic. By providing concrete methodologies for instrument modification, compositional exploration, and integration of electronic processes, this thesis contributes to the normalization and accessibility of broader Xenharmonic praxes.

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