INSTRUCTIONAL PROBLEMS REVEALED THROUGH USE OF STANDARDIZED TESTS ADMINISTERED TO STUDENTS IN SUB-COLLEGE ALGEBRA AT MONTANA STATE COLLEGE

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CHAPTER I

INTRODUCTION

The increased use of the scientific method in our rapidly growing society has caused a much greater emphasis to be placed on the training of students in all fields of science. The vast role of science in the rapidly expanding field of industry has caused a heavy burden to be placed on institutions of higher education to give more students at least a general background in scientific methods and procedure. Mathematics is one area where this increased pressure is being felt. The demand for more scientists has influenced many students, who had not taken college preparatory work in high school, to attempt to gain a college education in spite of their lack of background. This has created a problem which has grown more serious with the ever-increasing number of students enrolled in college. This trend has been nationwide and at Montana State College has been even greater than normal. This has focused attention on the greatly increased enrollment in classes in sub-college mathematics and the need to investigate instructional problems and methods.

The Problem

The increased enrollment at Montana State College during the last five years has caused a serious problem in regard to the offering of courses in mathematics. The students entering

have had backgrounds in mathematics so varied that some students are ready to begin with an introductory calculus course. Others have had no training in mathematics beyond the basic arithmetic of the sixth, seventh, and eighth grades, and possibly general mathematics in the ninth grade. It is with this latter group that this investigation has been concerned. These are the students who have reached the college level totally unprepared to take the regular freshman mathematics courses and, therefore, must enroll in high school level elementary algebra and geometry. This sub-standard type of course has been designed to meet the need of a review for those persons entering or returning to college after a period of time away from study, as well as to meet the need of those who have never had the courses. It has been in the best interests of both the college and the student to find the best methods to accomplish this sub-college education in mathematics in the most efficient manner. The courses offered must cover the basic objectives of the material to be covered in as short a time as is practical to both the student and the institution. Without this type of course many potential scientists, engineers, educators, and technicians from other fields would be lost to society.

A course in elementary algebra has been offered at Montana State College in order to fulfill the needs of those students who have had insufficient mathematical background. The need was felt, as enrollment in this course grew, that

some way should be found to improve the course to better meet the needs of those students. This need to improve led to the selection of the topic of this investigation, "Instructional Problems Revealed Through Use of Standardized Tests Administered to Students in Sub-College Beginning Algebra at Montana State College."

The Procedure

The first procedure was to review the literature as presented in Chapter II. It was obvious from this review of the literature that little research had been conducted concerning the teaching of elementary algebra at the college level. It was, therefore, decided to test all freshman students enrolled in the elementary algebra course at Montana State College during the school year 1956-1957. An attempt was to be made, using the results of the test, to determine the weaknesses that still existed after completion of the course. It was planned that if suitable results were obtained the course material and methods of presentation could be altered to better meet the needs of the students. The same text book was to be used each quarter and the same teaching schedule would be followed with the minimum of adjustments made to meet the particular number of teaching days in each quarter. The text book selected for use was Elementary Algebra by Hawkes-Luby-Touton. A copy of the teaching schedule appears in Table I of the Appendix.

The test selected was the First Year Algebra test from the Bureau of Educational Measurements, Kansas State Teachers College.[1] The test was administered on the last day of fall quarter, the last day of winter quarter, and the last day of spring quarter to all students enrolled in the course. No tests were returned to the students in order that no student would have the opportunity to study the test beforehand. Forty minutes of time was allotted to each section being tested, as specified by the author of the test.

Limitations of the Investigation

There were three items that were concerned as possibilities which might bias the results obtained from such an investigation. The first of these was the problem involved with the use of more than one instructor. That individual differences in instructors can cause different points or concepts to be stressed has been clearly brought out by Clark and Fehr[2] in their statement "we learn that mathematics which we are taught." This limitation was minimized as much as possible by having all instructors supervised by an expert.[3]

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[1]Tucker, C. B., Professor of Education, Kansas State Teachers College, First Year Algebra, designed as a forty minute test for students having completed the first year of high school algebra.

[2]Clark, J. R., and Fehr, H. F., "Ways to Improve Instruction in Mathematics," in Twenty-first Yearbook of National Council of Teachers of Mathematics, p.348.

[3] Hess, Adrien L., Professor of Mathematics, Mathematics

A second limitation was the possibility of students repeating the course, and likewise the testing program. This was remedied by the exclusion of tests taken by those repeating. No data were used from these tests.

The third area of limitation existed in the possibility of having test questions passed on to students who were about to take the test by students who had taken the test the preceding quarter. Plans were made to have the tests carefully collected after each testing period and to not return the tests to the students in order to hold the "information leaks" to a minimum. As an additional precaution, it was not announced before the test what kind of a test would be given.

The preceding restrictions and corrective procedures are recommendations that may be found in any good textbook on testing. The procedures to be followed in observing the limitations, as discussed in the preceding material, were decided upon after a review of the current literature which is presented in Chapter II.

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CHAPTER II

REVIEW OF LITERATURE

A review of current literature was made to discover what was being done regarding research pertaining to elementary algebra courses being taught at the college level. Such periodicals as Review of Educational Research, Journal of Educational Research, The Mathematics Teacher, and others, both from the field of education and the field of mathematics, were reviewed. One fact remained outstanding after this review. Research relating to high school mathematics courses which are being taught at the college level has been either extremely rare or possibly has not been published. There was, however, a wealth of material covering the most desirable situations, techniques, and outcomes. There are certain specific aims, values, and purposes[1] which are commonly attributed to the subject of algebra that have been used for comparison and evaluation in order to improve methods and material. Such things as ability, understanding, and appreciation are representative of the goals sought by mathematicians and teachers of mathematics. Many excellent lists of aims, values and purposes have been made by many authorities . An excellent listing of these has been reported by the National Committee on Mathematical Requirements, as

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[1]The Mathematical Association of America, The Reorganization of Mathematics in Secondary Education, 658 pp.

follows:

A. Practical aims, i.e., of immediate or direct usefulness in life.
   1. Developing the ability, to apply the fundamental process of arithmetic.
   2. Understanding the language of algebra.
   3. Developing the ability to understand and use the algebraic technique.
   4. Developing the ability to understand and use graphs.
   5. Developing the ability to understand and to use quantitative ideas.

B. Disciplinary aims, i.e., related to mental thinking.
   1. Acquiring ideas and concepts used in quantitative terms.
   2. Developing the ability to think in quantitative terms.
   3. Acquiring the concept of dependence and of relationships.[2]

The basic aims involved in the teaching of the course in elementary algebra were determined by examining this National Committee list and other lists which were supported by authority and which could give a good basis for the evaluation of the test results. These aims will be briefly considered in order that the results of the tests used in this study may be viewed with the purpose of improvement of course methods and materials.

First, consideration must be given to the reasons that algebra is taught. The reasons that have been given for teaching this course are many and varied, and have been generalized by many authorities. One very good example of a reason for

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[2] Ibid., p.10.

teaching algebra is given in the following statement made by Kinney and Purdy:

> Algebra has been described as the most important labor-saving device invented by man ..... Although the answers to specific problems are sought, attention is focused on the examination and study of functional relationships.[3]

The material should be taught not only for the manipulation of the algebra itself but also for understanding of basic concepts. Such things as dependence, relationships, quantitative values, logical thinking, and expression have all been stressed as aims. Success in achieving the understanding of these basic aims rests not only with the material presented but also in the manner of presentation and the teaching techniques used.

One should consider first the area of problem solving since this area is directly associated with the ideas of logical thinking and relationship. This association must be clearly indicated to the student by the teacher. The teacher must help the student become oriented to the problem. The need of psychology in problem solving has been clearly brought out by Henderson and Pingry in a statement:

> In a sense a mathematics teacher is a psychology teacher teaching the psychology of problem solving. It is important that the students not only know what a problem is, and what some of the aspects of problem

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[3]Kinney, L. B., and Purdy, C. R., Teaching Mathematics in the Secondary School, p.59.

> solving are, but why it is desirable to solve problems in school.[4]

The teacher must present the student with the tools which enable him to solve the problem. These tools must then be used by the student, under the direction of the teacher, in order that the correct methods of attack upon the problem may be not only seen by the student but actually used. The value of this use of correct methods has been very well expressed by Stroud in the statement:

> Material high in associative value is for that reason comparatively easy to learn and for the same reason easily recalled, relearned or recognized afterward. Logical material, material capable of meaningful organization or reduction to some kind of system, comes within the operations of transfer of training, operations that facilitate recall as well as learning.[5]

All of these aims are of little value if the student can associate them with only the subject at hand. If the problem solving and logical thinking used in algebra are not put into use in the other phases of his daily life, then the objectives will not have been reached. Transfer of training does not occur automatically but comes only with the use of proper teach-techniques. The conditions governing the amount of transfer have been outlined by Orata:

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[4]Henderson, K. B., and Pingry, R. E., "Problem Solving in Mathematics," in Twenty-first Yearbook of National Council of Teachers of Mathematics, p. 249.

[5]Stroud, J. B., Psychology in Education, p.538.

> First, transfer is a fact, as revealed by nearly eighty percent of the studies; second, transfer is not an automatic process that can be taken for granted, but it is to be worked for .....; and third, the amount of the transfer is conditioned by many factors, among which are: age, mental ability; (Possibly) time interval between the learning and transfer; degree of stability attained by the learned pattern; "knowledge of directions, favorable attitude toward the learning situation, and efficient use of past experience"; accuracy of learning; "conscious acceptance by the learner of methods, procedures, principles, sentiments, and ideals"; meaningfulness of the learning situation; the personality of the subject- greater transfer in extroverts than in introverts; method of study; suitable organization of subject matter presentation; and provision for continuous reconstruction of experience.[6]

The students must also be given a good understanding of the actual language of mathematics. The definitions and the proper usage of the words and terms commonly found in the literature should be developed so that the work at hand may be properly done. These definitions and usages should be developed in such a way that the student first mentally visualizes the concept which is being discussed, or as Brune has stated:

> Pupils who get the idea and then name it tend to become less confused than pupils who attempt to learn the terms representing ideas which are as strange as the terms themselves.[7]

Probably the best way to gain understanding on the part of the

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[6]Orata, P. T., "Recent Research Studies on Transfer of Training with Implications for the Curriculum, Guidance and Personnel Work," Journal of Educational Research 35:93, Oct., 1941

[7]Brune, I. H., "Language and Mathematics," in Twenty-first Yearbook of National Council of Teachers of Mathematics, p.73.

students is to relate the classroom work to the experiences of the students. The understanding gained by relating the actual experiences to the symbols tie the symbols to life itself and show their importance. The need for relating the subject matter to real life experiences was noted in the report of the Harvard Committee which stated:

> Abstractions in themselves are meaningless unless connected with experience .... The teacher can do a great deal....; he can relate theoretical content to the student's life...., and he can deliberately simulate in the classroom situations from life....he can be persistent in directing the attention of the student from the symbols to the things they symbolize.[8]

The foregoing aims gave a base which could be used for the evaluation of the tests given. With these aims in mind, the program of testing was devised and put into operation. The explanation of this program is presented in Chapter III.

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[8]Harvard Committee Report, General Education in a Free Society, pp. 70-71.

CHAPTER III

THE TESTING PROGRAM

The First Year Algebra test was given at the end of each quarter to all students enrolled in Mathematics 001, Elementary Algebra. These tests were corrected and a tally was made of the number of errors[1] made on each particular question. Next a table was made which listed each individual question, the frequency of error, and relative frequency of error, both by quarter and also for the cumulative total of all three quarters. This compilation of data appears in the Appendix in Table 2. A frequency histogram was made to facilitate a comparison of individual questions and appears as Table 3 in the Appendix.

A careful study of each question as to content and degree of difficulaty was made after the series of tests had been graded and tabulated. This analysis was made for two purposes: First, it would show which questions had not been covered in the material presented to the students. Such questions then were lined in red on Table 2 of the Appendix and their frequencies were omitted to prevent weighting of some one particular category. These questions were omitted on the histogram, Table 3 of the Appendix. The second reason for the question analysis was to have a basis to group the questions into categories representing the fundamental

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[1] An error in this tally was a wrong answer.

operations of algebra. An arbitrary grouping, consisting of nine major groups, was designed to meet this need. Since some questions applied to more than one of these nine groups, the question was included in that group which it was felt was the most basic. This grouping appears as the Appendix Table 4, and shows the numbers of the questions considered in the evaluation. In addition are shown the questions, by group, which were deleted. The nine groups used represent problems dealing with fractions, ration and proportion, linear equations, exponents and radicals, factoring, signed numbers, algebraic expression, problem solving, and numerical evaluation. A final table was made to demonstrate which class of question was most often missed. The following equation was used to determine this factor:

$$R.F.(C) = \frac{f_1 + f_2 + f_3 + \cdots + f_r}{\sum_{i=1}^{r} n_i} \qquad (1)$$

$$= \frac{r_1 f_1 + r_2 f_2 + r_3 f_3 + \cdots + r_r f_r}{r} \qquad (2)$$

where $n_1 = n_2 = n_3 = \cdots n_r$, R.F.(C) is the relative frequency of any class "C", $r_i f_i$ is the relative frequency of the ith[2] question, and r_i is the number of questions in the ith class. This table gives an insight into which type or class of question gave the students the most difficulty. This data on

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[2]The ith question is any question in the range and is used here to generalize the notation.

classification of questions appears as Table 1.

TABLE 1. THE RELATIVE FREQUENCY OF CLASSES OF QUESTIONS AND THE RELATIVE FREQUENCY OF ERROR WITHIN CLASSES OF QUESTIONS

Class of question	Number of errors	Relative frequency of class	Relative frequency of error of class
Fractions	141	.0769	.3832
Ratio and proportion	160	.0769	.4348
Linear equations	169	.0769	.4592
Exponents and radicals	55	.0577	.1993
Factoring	100	.0962	.2174
Signed numbers	184	.1731	.2222
Algebraic expression	240	.1923	.2609
Problem solving	436	.1923	.4739
Evaluation	125	.0577	.4529

Examination of Table 1 will give an indication of the types of question most frequently missed. The first class of questions, for example, those dealing with fractions, had a relative frequency of error of approximately .38. This indicates that slightly over 38 per cent of the time when a question dealing with fractions appeared the students made an

error in the problem. The data contained in Table 1 indicates that the biggest failing on the part of the students arose with questions based on problem solving, linear equations, evaluation, and ratio, in that order. Following these classes was the class of questions covering fractions. The table shows quite a definite break here between the fraction class and the next lowest classes, those of algebraic expression, signed numbers, factoring, radicals, and exponents.

First one must consider the class which had the largest frequency of error, the class of questions relative to problem solving. These questions were missed over 47 per cent of the time. A good example of this type of question is problem 30 which appears in the test as follows:

> A rectangular yard is half again as long as it is wide. If the width is x feet, the amount of fencing needed to go around the yard is:
>
> (1) 5x (2) 8x (3) 6x (4) 4x

Since over half of the students missed this question, it is evident that a better method of presenting the relationship between length, width, and perimeter must be found. Since this aim is a very basic one to the course, the frequency of error figure is rather distressing. This idea was covered quite extensively in the course work preceding the testing program. Considering the actual class time spent covering this area, this question should have had a much lower frequency of error.

The second largest frequency of error was in the area of linear equations, where a percentage figure nearly equal to that of the problem solving class was noted. This type of question also involved such ideas as signed numbers, logical thinking, and vocabulary. Problem 10 from the test is an example of this type of problem and is stated as follows:

The value of x that satisfies the equation $3x + 7 = 16$ is called the:

1. root 2. unknown 3. radical 4. definite number

It is evident here that more time must be spent on the vocabulary and the proper use of mathematical terminology. Question number 10 was missed by nearly 60 per cent of the students. This material was also covered extensively in course work previous to testing and again the results were not the most gratifying.

The problems involving evaluation and those involving ratio and proportion also had large frequencies of error. A good example of the problems classified as evaluation was problem number 26 which reads as follows:

The value of $(a+b)^2$ for $a = 2$ and $b = 3$ is:

(1)13 (2) 25 (3) 36 (4) 625

The wrong answers which appeared most frequently were the first and third. These answers indicate a lack of care in reading and comprehending the problem. More care should be

taken to be sure students have an understanding of the problem before attempting to work out the answer. Carelessness in reading is also evident in the answers to question number 47 which is classified as a ratio and proportion problem. This problem appears in the test as follows:

> If Mary works 8 problems while John does 6, the ratio of Mary's work to John's is:
>
> (1) 6:8 (2) 3/4 (3) 4/3 (4) 3/2

The wrong answers that appeared most were the first and second. This seemed to indicate that both a poor understanding of the material and carelessness or lack of ability to read and comprehend contributed to the frequency of error.

Consideration of these results by class of question alone does not, however, give a clear idea as to the relative ease or difficulty of working any one problem. A single question which indicates that a basic concept is not fully understood by a large segment of the participating students may also give an indication that more work is required in some particular area. This is well illustrated by observing the results obtained on this test relative to question number five. This question was a multiple-choice question concerning signed numbers. The question reads as follows:

> $(-7) - (+3) =$:
>
> (1) -10 (2) -4 (3) 10 (4) 21

This particular question, an extremely basic signed number problem, was missed by 54 per cent of the students fall quarter, 57 per cent of the students winter quarter, and 50 per cent of the students spring quarter. This is an average of 55 per cent of the students for the entire year. Comparison of the percentage of this single question with the percentage of all questions covering the class of signed number problems of 22 per cent, indicates that the concept is not fully comprehended.

The continuity between the results quarter by quarter was quite good. A few individual questions varied a great deal, as was the case of question number eight. Approximately 61 per cent of the students missed this question winter quarter while in the spring quarter this percentage dropped to about 20 per cent. A trend was noted, in a few cases, where the percentage either rose or dropped with each successive quarter. Such trends occurred with questions 24 and 62. The relative frequency of error for question 62 rose each successive quarter while the opposite was true of question 24. These trends were not sufficient for any one class to base any prediction on future results. The conclusions that may be drawn from these results will be helpful in preparing teaching schedules and lesson plans covering this course in future quarters.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The test results showed some definite areas of understanding that must be given additional attention. The classes of questions concerning ratio and proportion, problem solving, evaluation, and linear equations appear to be the ones where more emphasis is needed. Specific questions, such as question number five, indicate that within classes of questions there is need for better instruction covering such things as notation and definition of terms. Further investigation is needed to establish more definite trends in some classifications of questions where the results were not consistent enough to permit any definite conclusions to be drawn.

The results of these tests showed that there were many areas where the desired outcomes were not being obtained. More emphasis is needed in areas shown to be troublesome. The course in elementary high school algebra taught in one quarter to college students was felt to be inferior to the same course taught in a full year to regular high school students. Since the time allowed at the college level is much shorter than at the high school level, more efficient use of the time must be made. Specific areas such as problem solving, evaluation, linear equations, ratio and proportion must receive increased attention. Additional

investigation of present teaching methods may reveal ways for better presentation and improved teaching aids. It is felt that while this investigation has revealed certain particular areas of deficiency, it has also shown that more research is needed to find ways of correcting these deficiencies.

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APPENDIX

Appendix Table 1

Teaching Schedule for Mathematics 001
Montana State College
Fall Quarter, 1956

TABLE 1. TEACHING SCHEDULE FOR MATHEMATICS 001, MONTANA STATE COLLEGE, FALL QUARTER, 1956

Week	Day		Pages	Comments
1	Oct.	1	1-22	Use arithmetic fractions and decimals for evaluation. Stress division by zero has no meaning. Include four fundamental operations with zero.
		2	26-47	
		3	48-54	
		4	55-60	
		5	60-63 (short quiz)	
2		8	64-70	Give axioms--omit transposition. Stress checking throughout quarter. Stress correct use of equal signs and respect for all signs.
		9	71-82	
		10	82-89	
		11	90-97	
		12	98-107 (short quiz)	
3		15	108-11	Stress method of setting up problem.
		16	112-115	
		17		M-Day
		18	115-117	
		19	118-123 (short quiz)	
4		22	123-127	
		23	128-130	
		24	130-131	
		25	132-138	
		26	138-140 (short quiz)	
5		29	141-144	
		30	145-146	
		31	157-164	
	Nov.	1	165-170	
		2	Hour Exam	
6		5	170-173	
		6	174-178	
		7	179-184	
		8	185-186	
		9	186-187	
7		12	188-190	
		13	191-196	
		14	196-198	
		15	198-203	
		16	Hour Exam	

TABLE 1 (Continued)

Week	Day	Pages	Comments
8	Nov. 19	204-211	Stress arithmetic fractions.
	20	211-215	
	21		Thanksgiving Vacation begins,
	22		Wednesday, 12 noon.
	23		
9	26	216-218	
	27	219-222	
	28	222-226	
	29	227-231	
	30	Hour Exam	
10	Dec. 3	231-237	
	4	238-243	
	5	243-246	
	6	247-249	
	7	250-253	
11	10	254-261	
	11	262-264	
	12	265-268	
	13	271-278	
	14		
12	17	Hour Test	
	18-21	Final Exam	There will be a 2-hour final exam in 001, but no makeup exam.

Appendix Table 2

Frequency of Error and Relative Frequency of Error of Questions Missed by Students in Elementary High School Algebra at Montana State College During the Testing Program 1956-1957

TABLE 2. FREQUENCY OF ERROR AND RELATIVE FREQUENCY OF ERROR OF QUESTIONS MISSED BY STUDENTS IN ELEMENTARY HIGH SCHOOL ALGEBRA AT MONTANA STATE COLLEGE DURING THE TESTING PROGRAM, 1956-1957.**

Ques. no.	Freq. fall*	Rel. freq. fall	Freq. winter*	Rel. freq. winter	Freq. spring±	Rel. freq. spring	Total freq.#	Total rel. freq.
1	14	.1522	6	.1176	1	.1000	21	.1373
2	12	.1304	8	.1568	0	0.0000	20	.1307
3	12	.1304	8	.1568	0	0.0000	20	.1307
4	3	.0326	2	.0392	0	0.0000	5	.0327
5	50	.5435	29	.5686	5	.5000	84	.5490
6	11	.1196	5	.0980	1	.1000	17	.1111
7	26	.2826	9	.1765	2	.2000	37	.2418
8	36	.3913	31	.6078	2	.2000	69	.4510
9	39	.4239	16	.3137	5	.5000	60	.3922
10	59	.6413	27	.5294	5	.5000	91	.5948
11	14	.1522	7	.1373	2	.2000	23	.1503
12	40	.4348	21	.4118	4	.4000	65	.4248
13	8	.0870	2	.0392	1	.1000	11	.0719
14	58	.6304	32	.6275	6	.6000	96	.6275
15	25	.2717	16	.3137	1	.1000	42	.2745
16	40	.4348	25	.4902	5	.5000	70	.4575
17	57	.6196	28	.5294	7	.7000	92	.6013
18	19	.2065	14	.2549	0	0.0000	33	.2157
19	23	.2500	21	.3725	5	.5000	49	.3203
20	13	.1413	6	.1176	0	0.0000	19	.1242

**Questions lined in red were omitted from this study as material not presented to the students.

*Fall Quarter, 92 students.

*Winter Quarter, 51 students.

±Spring Quarter, 10 students.

#Total, 153 students.

TABLE 2 (Continued)

Ques. no.	Freq. fall*	Rel. freq. fall	Freq. winter*	Rel. freq. winter	Freq. spring*	Rel. freq. spring	Total freq.#	Total rel. freq.
21	84	.9130	40	.7843	7	.7000	131	.8562
22	83	.9022	43	.8431	10	1.0000	136	.8889
23	6	.0652	3	.0569	0	0.0000	9	.0588
24	49	.5326	22	.4314	2	.2000	73	.4771
25	47	.5109	30	.5882	4	.4000	81	.5294
26	37	.4022	23	.4510	7	.7000	67	.4379
27	40	.4348	22	.4314	3	.3000	65	.4248
28	90	.9783	50	.9608	10	1.0000	150	.9804
29	13	.1413	9	.1745	1	.1000	23	.1503
30	46	.5000	28	.5490	4	.4000	78	.5098
31	5	.0543	6	.1157	0	0.0000	11	.0719
32	40	.4348	30	.5686	7	.7000	77	.5033
33	12	.1304	3	.0569	0	0.0000	15	.0983
34	26	.2826	22	.4314	1	.1000	49	.3203
35	66	.7174	43	.8235	6	.6000	115	.7516
36	12	.1304	13	.2400	3	.3000	28	.1830
37	36	.3913	23	.4510	3	.3000	62	.4052
38	28	.3043	22	.4314	3	.3000	53	.3464
39	6	.0652	2	.0392	1	.1000	9	.0588
40	10	.1087	4	.0784	3	.3000	17	.1111
41	56	.6087	38	.7451	4	.4000	98	.6405
42	27	.2935	21	.4118	1	.1000	49	.3203
43	72	.7826	39	.7647	9	.9000	120	.7843
44	78	.8478	47	.9019	10	1.0000	135	.8824
45	48	.5217	21	.4118	6	.6000	75	.4902

1 79807

TABLE 2 (Continued)

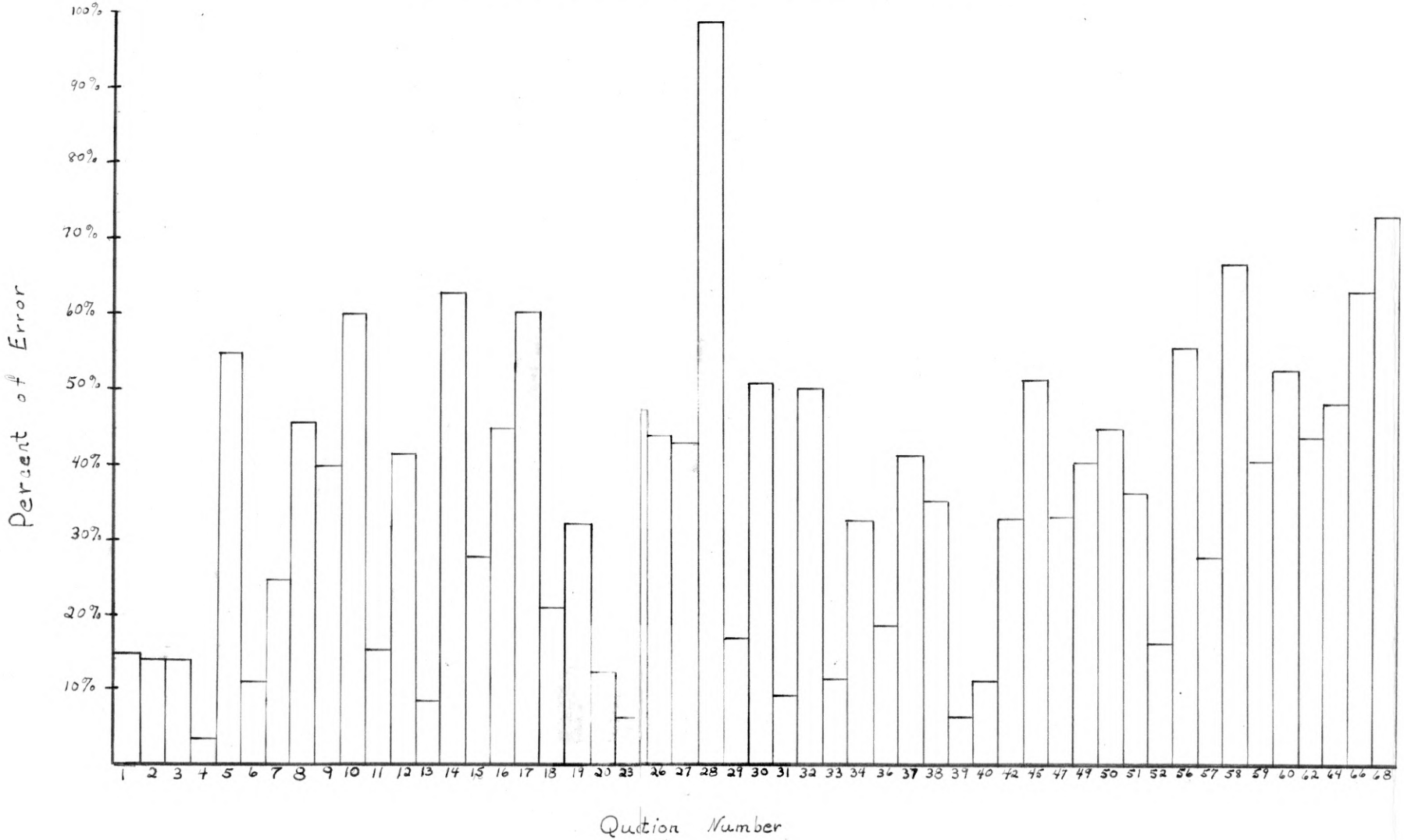
Ques. no.	Freq. fall*	Rel. freq. fall	Freq. winter*	Rel. freq. winter	Freq. spring*	Rel. freq. spring	Total freq.#	Total rel. freq.
46	72	.7826	38	.7451	8	.8000	118	.7712
47	27	.2935	17	.3333	5	.5000	49	.3203
48	72	.7826	37	.7255	7	.7000	116	.7582
49	35	.3804	21	.4811	2	.2000	58	.3791
50	37	.4022	28	.5490	5	.5000	70	.4575
51	34	.3696	17	.3333	4	.4000	55	.3595
52	11	.1196	12	.2353	3	.3000	26	.1699
53	73	.7935	40	.7843	10	1.0000	123	.8039
54	61	.6630	40	.7843	9	.9000	110	.7190
55	75	.8152	42	.8235	7	.7000	124	.8105
56	45	.4891	35	.6863	4	.4000	84	.5490
57	21	.2283	16	.3137	4	.4000	41	.2680
58	53	.5761	42	.8235	7	.7000	102	.6667
59	32	.3478	23	.4510	5	.5000	60	.3922
60-1	43	.4674	31	.6078	7	.7000	81	.5294
62-3	29	.3152	26	.5098	7	.7000	62	.4052
64-5	40	.4348	27	.5294	7	.7000	74	.4837
66-7	50	.5435	36	.7059	7	.7000	93	.6078
68-9	63	.6848	44	.8627	9	.9000	116	.7582

Appendix Table 3

Histogram of Percentage of Error of Questions Missed by Students in Elementary High School Algebra at Montana State College During the Testing Program 1956-1957

APPENDIX TABLE 3

HISTOGRAM OF PERCENTAGE OF ERROR OF QUESTIONS MISSED BY STUDENTS IN ELEMENTARY HIGH SCHOOL ALGEBRA AT MONTANA STATE COLLEGE DURING THE TESTING PROGRAM, 1956 — 1957



Appendix Table 4

Classification of Individual Questions into Type Form, Showing Both the Questions Used and those Deleted as Not Applicable

TABLE 4. CLASSIFICATION OF INDIVIDUAL QUESTIONS INTO TYPE FORM, SHOWING BOTH THE QUESTIONS USED IN EVALUATION AND THOSE DELETED AS NOT APPLICABLE

Class	Type form	Question number	Deleted questions
I	Fractions	16, 37, 38, 50	54
II	Ratio and proportion	28, 47, 52, 59	43
III	Linear equations	9, 10, 11, 17	21, 22, 25
IV	Exponents and radicals	4, 32, 33	35, 44, 46, 55
V	Factoring	36, 39, 40, 42, 56	
VI	Signed number	3, 5, 6, 8, 13, 15, 19, 20, 23	
VII	Algebraic expression	1, 2, 7, 12, 24, 29, 31, 34, 51, 57	41, 48, 53
VIII	Problem solving	14, 18, 27, 30, 45, 60-1, 62-3, 64-5, 66-7, 68-9	
IX	Evaluation	26, 49, 58	

Appendix

Every Pupil Scholarship Test, April 5, 1956, Bureau of Educational Measurements, Kansas State Teachers College, Emporia, First Year Algebra, by Charles B. Tucker, Kansas State Teachers College

TIONS: Read the ns for each part llow them. Answer ems first; return to later. You will have 40 minutes.

EVERY PUPIL SCHOLARSHIP TEST
April 5, 1956
Bureau of Educational Measurements
Kansas State Teachers College, Emporia

FIRST YEAR ALGEBRA

By Charles B. Tucker
Kansas State Teachers College
Emporia, Kansas

Possible Score 6 9

Number wrong and omitted .....

FINAL SCORE .....

.....Age ..... Grade .....

.....State ..... Date .....

PART I

TIONS: In the following problems there are four only one of which is correct. Select the correct answer e the number of the correct answer in the parenthesis ig the problem.

1. In the expression $\frac{2ax^2y}{3b}$, the numerical coefficient is:
   (1) 2 **(2) 2a**
   (3) $\frac{2a}{3b}$ **(4)** $\frac{2}{3}$

2. If the two equal sides of an isosceles triangle are each **a** inches long and the third side is **b** inches, then the perimeter, **P**, is given by the formula:
   (1) P=a+b **(2)** $P^2=a^2+b^2$
   (3) P=2a+b **(4) P=ab**

3. (−6) (−2) (4) = :
   (1) −24 (2) −48 (3) 48 (4) 12

4. $16x^2y^3z$ is a short method of expressing the quantity:
   (1) 16xxyyzz **(2) 16xxyyyz**
   (3) 16xyyyz **(4)** $16+x^2y^3z$

5. (−7) − (+3) = :
   (1) −10 (2) −4 (3) 10 (4) 21

6. If four negative numbers are multiplied together, their product:
   1. is negative 2. takes the sign of the largest number 3. is positive 4. is positive or negative depending upon whether it is even or odd

7. If **x** is an even integer, then 2x+½ is: **1. an even integer 2. the next consecutive integer 3. an odd integer 4. a mixed number**

8. 24 − (−3) = :
   (1) 27 (2) 21 (3) −8 (4) 72

( ) 9. If −2x=8, then: **(1) x is negative (2) x is positive (3) x may be either positive or negative (4) x=4**

( ) 10. The value of **x** that satisfies the equation 3x+7=16 is called the:
1. root 2. unknown 3. radical 4. definite number

( ) 11. If 3 is subtracted from each member of the equation x+6=y, the resulting equation is:
(1) x+6=y−3
(2) x+3=y
(3) x+3=y−3
(4) x+9=y+3

( ) 12. $7a^3$ and $7a^2$ are: **1. binomials 2. similar terms 3. equivalent terms 4. both powers of a**

( ) 13. −15x÷3= :
(1) −12x (2) −5x (3) −5 (4) 5x

( ) 14. If the square of a number **N** is two more than twice the number, then:
(1) $N^2-2N=2$
(2) $N^2-2N=-2$
(3) $N^2=2N-2$
(4) $(N-2)^2=2N$

( ) 15. (2x+3y) − (3x−2y) = :
(1) −x+y (2) −x+5y
(3) 5x+y (4) 5x−5y

( ) 16. $\frac{1}{a}+\frac{1}{b}$ may also be written correctly as:

(1) $\frac{a+b}{ab}$

(2) $\frac{b}{a}+\frac{a}{b}$

(3) $\frac{2}{ab}$

(4) $\frac{2}{a+b}$

( ) 17. To solve an equation such as $x+7=15$ is to find a value of **x** that makes each member of the equation:
1. equal to zero 2. indeterminate 3. identical 4. equal to eight

( ) 18. The sum of two numbers is $12x+8y$. One number is $6x-4y$. Then the other number is:
(1) $6x+12y$ (2) $6x+4y$
(3) $6x-8y$ (4) $18x+4y$

( ) 19. The quotient of two numbers is $-6x$. If the divisor is $2x$, then the dividend is:
(1) $-12x^2$ (2) $-4x$
(3) -4 (4) -3

( ) 20. $(3x+5)+(2x-5)=$:
(1) $5x$
(2) $5x+10$
(3) $6x^2-5x-25$
(4) x

( ) 21. The point (2,3) will lie on the graph at the equation:
(1) $x+y=-5$
(2) $3x-2y=5$
(3) $2x-y=1$
(4) $2x+3y=6$

( ) 22. The graph of which of the following equations is a straight line?
(1) $2x+3y=7$
(2) $xy=4$
(3) $x^2+y^2=9$
(4) none of these

( ) 23. In the expression $(x+3y)-(3x-2y)$:
1. it is correct to simply drop both sets of parentheses 2. the parentheses may be removed correctly if all signs are changed 3. the parentheses cannot be removed 4. if the parentheses are removed, the signs of (3x—2y) must be changed only

( ) 24. If $10t+u$ represents the number 46, where **t** is the ten's digit and **u** is the unit's digit, then **t**=:
(1) .4 (2) 4 (3) 40 (4) 6

( ) 25. A logical first step in the solution of the simultaneous equations
$\begin{cases}(1) & x-y=3 \\ (2) & 2x-3y=4\end{cases}$ would be to:
1. add the two equations
2. subtract the two equations
3. solve for x in equation (1)
4. solve for y in equation (2)

( ) 26. The value of $(a+b)^2$ for $a=2$ and $b=3$ is:
(1) 13 (2) 25 (3) 36 (4) 625

( ) 27. Five years ago John was just one-half as old as he will be five years from now. If **x** is his age now, then the equation which will give his age is:
(1) $x-5=\frac{1}{2}(x+5)$
(2) $x-5=\frac{1}{2}x+5$
(3) $2x-5=x+5$
(4) $2(x+5)=x-5$

( ) 28. The percent of antifreeze in a radiator that has a capacity of 20 quarts and contains **x** quarts of antifreeze is:
(1) $20-x$
(2) $\frac{x}{20}$
(3) $\frac{20-x}{100}$
(4) $5x$

( ) 29. The expression $\frac{1}{3}(a-b)$ means:
1. both a and —b are multiplied by $\frac{1}{3}$ 2. only the a is multiplied by $\frac{1}{3}$ 3. the $\frac{1}{3}$ is inverted and multiplied times (a—b) 4. the difference of a and b is divided by $\frac{1}{3}$

( ) 30. A rectangular yard is half again as long as it is wide. If the width is **x** feet, the amount of fencing needed to go around the yard is:
(1) 5x (2) 8x (3) 6x (4) 4x

( ) 31. To multiply $3x^2+2x+5$ by $4x$, one must: **(1) find the least common multiple (2) multiply each term of the trinomial by 4x (3) multiply the $3x^2$ by 4x and add 4x to $2x+5$ (4) multiply the $3x^2$, 2x and 5 by 4**

( ) 32. The expression $a^2b^2c^2$ contains:
1. no exponents
2. no numerical coefficient
3. equal coefficients
4. three factors each raised to the same power

( ) 33. $(a^2b)(b^2c)(c^2d) =$:
(1) $a^2b^2c^2d^2$
(2) $a^2b^2c^2d$
(3) $a^2b^3c^3d^3$
(4) $a^2b^3c^3d$

( ) 34. $ax^2+bx+c=0$ is:
1. a quadratic trinomial
2. a quadratic equation
3. a quadratic function
4. an identity

( ) 35. $\sqrt{3}$ is: **1. a rational number**
2. an irrational number
3. a binomial radical
4. the only square root of 3

( ) 36. $(7-ax)(5+ax) =$:
(1) 12
(2) $35-a^2x^2$
(3) $35-12ax+a^2x^2$
(4) $35+2ax-a^2x^2$

( ) 37. $\dfrac{x}{x^2+x} =$:
(1) $\dfrac{1}{x^2}$
(2) $\dfrac{1}{x}+1$
(3) $\dfrac{1}{x+1}$
(4) $\dfrac{1}{2x}$

( ) 38. If $\dfrac{3}{x-4} = \dfrac{1}{2}$, then $x=$:
(1) 10 (2) 2 (3) 8 (4) 6

( ) 39. $9x^2-16=$:
1. $(9x-1)(x+16)$
2. $(x+1)(9x-16)$
3. $(3x-4)^2$
4. $(3x-4)(3x+4)$

( ) 40. $2x^2+3x-35=$:
1. $(2x+7)(x-5)$
2. $(2x-7)(x+5)$
3. $(2x-5)(x+7)$
4. $(x-35)(2x+1)$

( ) 41. The formula for the solution of the general quadratic equation is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

For the equation $3x^2+13x-10=0$, the values to be used are:
(1) $a=3, b=13, c=10$
(2) $a=3, b=-13, c=10$
(3) $a=3, b=-13, c=-10$
(4) $a=3, b=13, c=-10$

( ) 42. If $(x-a)(x+b)=0$, then **x** must be:
1. equal to a 2. equal to −b
3. equal to either a or −b
4. equal to either a or b

( ) 43. When two variables have a constant ratio, they are said to: **1. be equal**
2. be independent variables
3. vary directly
4. be reciprocals of each other

( ) 44. $(\sqrt{2}+2)^2 =$:
(1) 8 (2) 6
(3) $4+\sqrt{2}$ (4) $6+4\sqrt{2}$

( ) 45. A car traveling at the rate of **r** miles an hour covers a distance **d** in **t** hours. If the rate of travel is changed to r/2 miles per hour, how many hours will it take the car to go **d** miles?
(1) d/r (2) 2d/r
(3) 2d (4) t/2

( ) 46. $\sqrt{2}\ \sqrt{10} =$:
(1) $4\sqrt{5}$
(2) $2\sqrt{3}$
(3) $2\sqrt{5}$
(4) cannot be further combined

( ) 47. If Mary works 8 problems while John does 6, the ratio of Mary's work to John's is:
(1) 6:8 (2) 3/4 (3) 4/3 (4) 3/2

( ) 48. The graph of a quadratic function is [**1. always 2. often 3. seldom 4. never**] a straight line.

( ) 49. If $V=\frac{1}{3}\pi r^2h$, then when $r=2$ and $h=3$, $V=$:
(1) 2π (2) 4π (3) 12π (4) 36π

( ) 50. $\frac{6}{a^2-b^2} \div \frac{12}{a-b} =$:

(1) $\frac{1}{2(a+b)}$

(2) $\frac{1}{2(a-b)}$

(3) $2(a+b)$

(4) $\frac{72}{(a-b)^3}$

( ) 51. The expression x^2-8x+..... will be a perfect square if the blank is replaced by:
(1) −64 (2) 64 (3) −16 (4) 16

( ) 52. If John can do $\frac{1}{x}$ of a piece of work in one hour, then in 5 hours he can do:

(1) $\frac{5}{x}$

(2) $5+\frac{1}{x}$

(3) $\frac{x}{5}$

(4) all of the work

( ) 53. In the formula $C=2\pi r$, r is the:
1. independent variable
2. dependent variable
3. constant of variation 4. ratio

( ) 54. To rationalize the denominator of the fraction $\frac{4}{2+\sqrt{2}}$ one should:

(1) multiply the fraction by $2-\sqrt{2}$
(2) divide the fraction by $2-\sqrt{2}$
(3) multiply both the numerator and denominator by $2-\sqrt{2}$
(4) subtract $\sqrt{2}$ from both the numerator and the denominator

( ) 55. $5\sqrt[3]{x^2}$ may also be written as:
(1) $5x^{\frac{2}{3}}$
(2) $(5x)^{\frac{2}{3}}$
(3) $\sqrt[3]{25x^2}$
(4) $5x$.

( ) 56. $x^2+y^2=$:
1. $(x+y)(x-y)$
2. $(x+y)^2$
3. $2(x+y)$
4. none of these

( ) 57. Three consecutive numbers are:
(1) $x+2, x+4, x+6$
(2) $x+1, x-1, x-2$
(3) $x+2, x+3, x+4$
(4) x, x^2, x^3

( ) 58. A common solution to the pair of equations $\begin{cases} 2x+y=4 \\ x+y=1 \end{cases}$ is:
1. (1,2) 2. (2,−1)
3. (3,−2) 4. (4,−4)

( ) 59. In constructing a bar graph showing his test grades in arithmetic, John used a scale of one-half inch=8 points. How many inches long did he make a bar to represent a grade of 88?
(1) 4 (2) 5½ (3) 11 (4) 16

PART II

DIRECTIONS: Solve the following problems and write the number of the answer in the parenthesis at the left. Each problem counts 2 points.

( ) 60-61. John is 2 years older than Henry. Henry is $\frac{5}{6}$ as old as John. John's age is:
(1) 30 (2) 24 (3) 18 (4) 12

( ) 62-63. An airplane traveled 480 miles in 3 hours, traveling with the wind. On the return trip, traveling against the wind, it took 4 hours. The normal speed of the plane in miles per hour was:
(1) 100 (2) 140 (3) 160 (4) 200

( ) 64-65. Frank has $1.60 in dimes and nickels. If he has 20 coins in all, how many nickels does he have?
(1) 6 (2) 8 (3) 10 (4) 12

( ) 66-67. The sum of the digits of a certain two-figured number is 5. If the digits are interchanged, the resulting number is 9 more than the original number. What is the original number?
(1) 14 (2) 41 (3) 23 (4) 32

( ) 68-69. An oval flower bed is made up of a square plot 7 feet on each side and a semi-circular plot on two opposite sides. How many feet of fencing will be required to enclose the flower bed? (Use $\pi=3\frac{1}{7}$.)
(1) 44 (2) 28 (3) 36 (4) 14π