



Preliminary analysis and hanger adjustment of tied arch bridges
by William Edward Beyer

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Engineering Mechanics
Montana State University
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Abstract:

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By using matrix structural analysis, the effects of certain parameters upon tied arch behavior are investigated. The parameters include rise to span ratio, hanger spacing, ratio of areas of rib and tie, and ratio of moments of inertia of rib and tie. The geometry of an existing span was used for analysis.

The results of the parametric study are portrayed graphically for a range of the parameters. Methods for analysis of dead load moment and hanger tension adjustment are developed. Finally a preliminary design example is considered.

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of

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in

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Bozeman, Montana

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William Edward Beyer

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ABSTRACT

Preliminary design of a tied arch bridge is complex due to the many possible parameters of the problem. After obtaining a design the minimization of dead load moment is an important consideration. Similarly, obtaining proper tensions in the hangers of a tied arch bridge is very important, to prevent overstressing of the arch.

By using matrix structural analysis, the effects of certain parameters upon tied arch behavior are investigated. The parameters include rise to span ratio, hanger spacing, ratio of areas of rib and tie, and ratio of moments of inertia of rib and tie. The geometry of an existing span was used for analysis.

The results of the parametric study are portrayed graphically for a range of the parameters. Methods for analysis of dead load moment and hanger tension adjustment are developed. Finally a preliminary design example is considered.

CHAPTER I

INTRODUCTION

Description

Tied arch bridges are distinguished from other forms of arch bridges by the presence of a tie chord. Figure 1 shows a typical profile of a tied arch bridge and gives the nomenclature of the parts of the structure. Tied arch behavior is similar to that of a self-anchored suspension bridge. In both cases the effects of live load structural distortion are practically eliminated. The tie girder carries the thrust of the arch rib. Hence, tied arches are ideally suited for sites where foundation conditions will not permit an economical substructure, which could carry the thrust of a conventional arch. Tied arches are also used where moderate span lengths are required with a maximum clearance.

History

The history of the tied or bowstring arch in America can be traced back to Trees' and King's truss, [3]. Trees' and King's truss consisted of tubular iron arches, with a tie beam attached to each end of the arch which supported the roadway. The patent on Trees' and King's truss expired Oct. 1, 1878. Some of the trusses designed by Squire Whipple in the mid-nineteenth century also possessed some

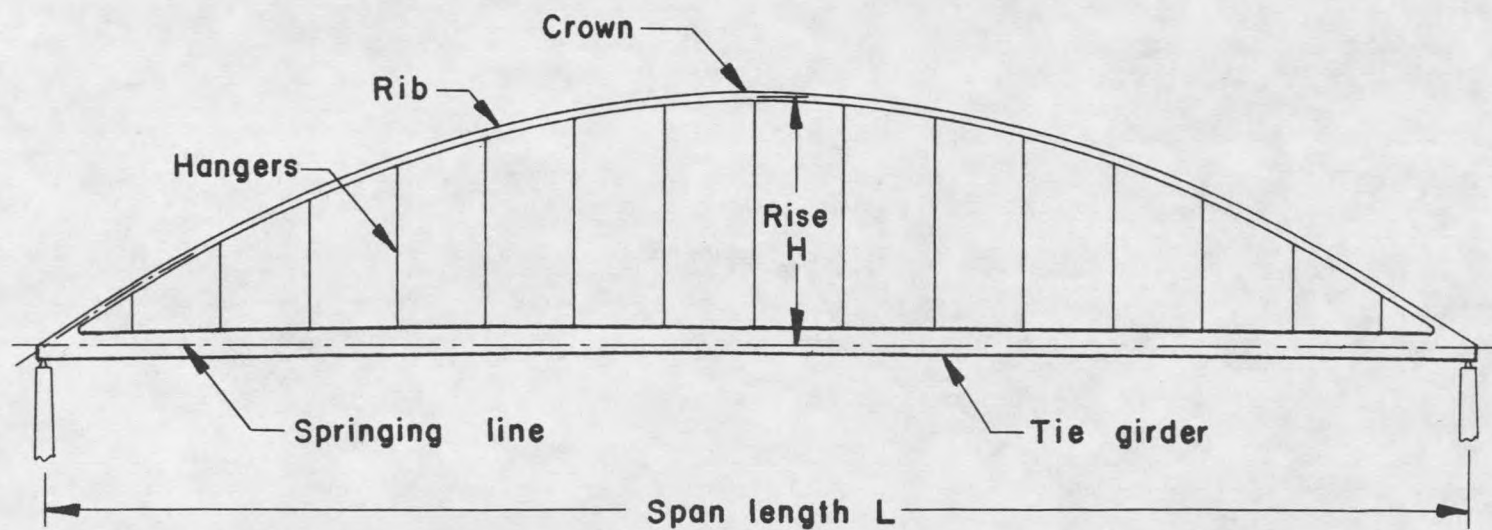


Figure 1. Tied arch bridge nomenclature.

of the features of tied arch construction. The tied arch was also being widely used in the mid-nineteenth century in Europe. The first bowstring arch was at Lugao in Hungary, designed by Hoffman and Medersbach in 1833, [48]. Austrian engineer Joseph Langer designed a truss-tied arch in 1859, [61].

Originally tied arch bridges such as the Tacony-Palmyra [55] or the West End [57] were designed with deep ribs, to carry the majority of the live load moment. In 1941, J. M. Garrelts revolutionized the design of tied arch bridges in America, with the design of the St. Georges Bridge. In the St. Georges Bridge the tie was made about 13 times as stiff as the rib, and therefore carried the majority of the live load moment, [7, 36].

Virtually all tied arch spans constructed today utilize the stiff tie and slender rib concept. In addition to superior aesthetics Garrelts gave the following advantages for this type of tied arch, [7]:

1. The erection will generally require less falsework because the tie girders will support erection equipment over longer spans.
2. The demands on erection equipment are reduced by reason of the fact that the heaviest members are lifted only to the level of the deck rather than to the top of the span. This may in some designs eliminate the necessity of special creeper travelers moving along the rib and permit the use of more common equipment operating from the deck.
3. Although no direct comparison in weight has been made as yet, it can be pointed out that some economy may result from the shorter length of the heavy tie as compared to the longer length of a heavy rib. This saving is partly offset by heavier splices.
4. Since the tie girder will always have little, if any, compression, the ends of the members at splice points need

not be milled. All milling of the splices in the curved rib is therefore confined to lighter, smaller members.

5. The connections at the ends of the span appear to be simplified when the deep girders are horizontal or nearly so, and the transfer of the vertical shear into the shoes is readily accomplished by detail parts that are easier to fabricate. The erection in this panel is much less difficult because a milled splice for the first rib member can generally be located above the tie girder so that the first section of the rib can be erected after the girder has been placed on the shoe and the first hanger erected. No temporary support for a heavy, inclined section of rib is required.

Figure 2 shows a sketch of a typical modern steel tied arch bridge, utilizing a stiff tie and slender rib.

In the 1950's design of tied arches underwent further refinements and improvements. The advent of high strength steels made tied arches competitive economically for much longer spans and heavier loads. Additionally the development of the digital computer led to a much better understanding of tied arch behavior. The Fort Pitt Bridge and Fort Duquesne Bridge in Pittsburgh are two notable double deck tied arch structures constructed in 1957, and 1960 respectively, [63, 65].

By using an orthotropic deck, the dead load of a bridge can be greatly reduced. Two notable tied arch bridges utilizing orthotropic decks are the Port Mann Bridge in 1963, [11, 15, 41], and the award winning Fremont Bridge in 1973, [14]. The Fremont Bridge is the longest tied arch bridge in history with a 1200 foot main span.

Today the science of tied arch bridges is still expanding. In other countries inclined hangers (Nielsen System) are being used [10, 24, 34, 35]. In some cases the entire deck is being used to act as

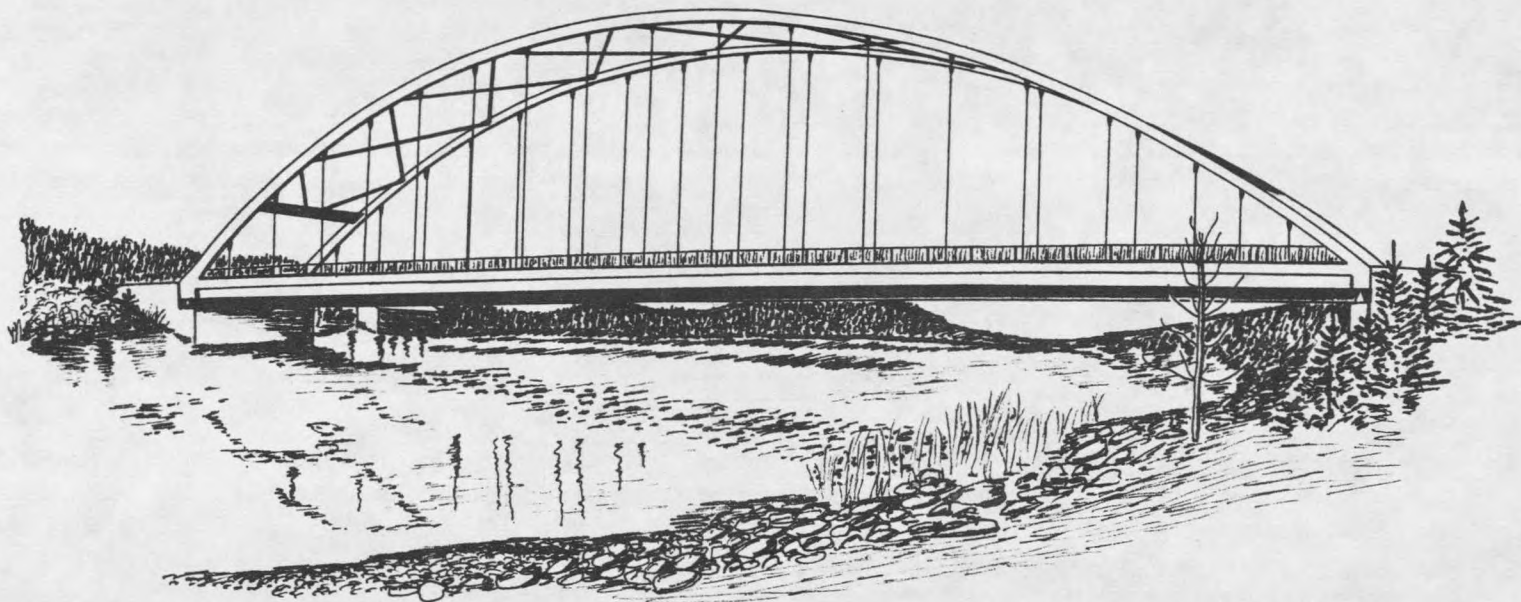


Figure 2. Typical modern steel tied arch bridge.

the tie [48]. For increased stability inclined tied arches are being tried [20, 22, 29]. The use of prestressed concrete for the ribs and for the deck is being investigated [29, 54]. These are all topics which would be of interest for further research and are beyond the scope of this thesis.

CHAPTER II

MINIMIZATION OF DEAD LOAD MOMENTS IN A TIED ARCH

The principle of moment minimization

One of the advantages in the use of an arch is having the ability to eliminate practically all of the dead load moments. To accomplish this the arch axis should be shaped to match the dead load funicular polygon or moment diagram. The moment diagram should be that of an equivalent simply supported beam carrying the dead load of the rib, tie, and the loads of the floor system and upper lateral bracing. If the arch axis is made up of straight members rather than curved members, it is only possible to minimize dead load moments but not totally eliminate them. This is due to the local bending effects between hangers caused by the self weight of the members. Additionally the lengths of the arch members must be fabricated to eliminate dead load deformations.

Application of the principle to a frame example

Figure 3 shows an example of a simple beam loaded with a third point loading and the corresponding moment diagram.

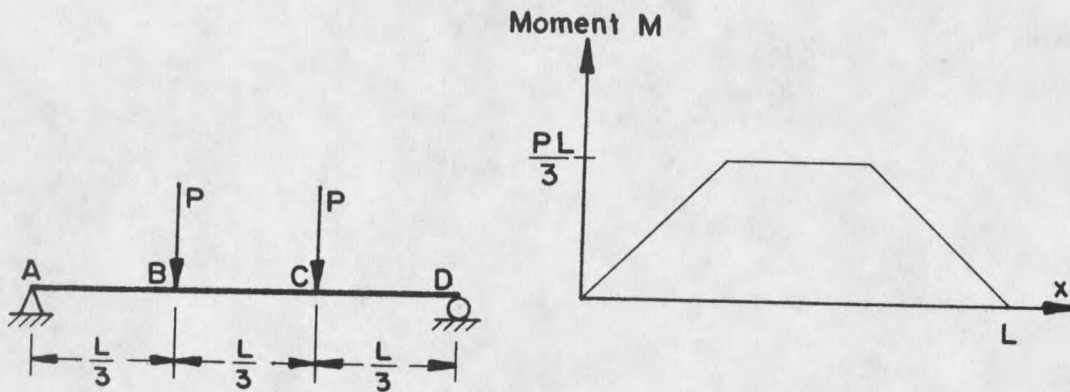


Figure 3. Simply supported beam and corresponding moment diagram.

By reshaping the beam of Figure 3 into a frame corresponding to the moment diagram it can be seen from Figure 4 that the loads P are now being carried primarily axially. There will be secondary moments induced into the frame due to the deformation of the frame.

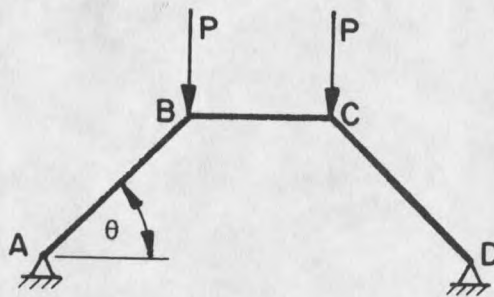


Figure 4. Shaping beam to match funicular polygon.

Also from Figure 4 it can be seen that as the angle θ decreases the axial load in each member increases. Therefore, for an arch, as the rise to span ratio decreases the axial forces in the rib and tie

will increase. Basic tied arch action is accomplished by replacing the rocker at point D of Figure 4 with a roller and adding a tie member AD as shown in Figure 5.

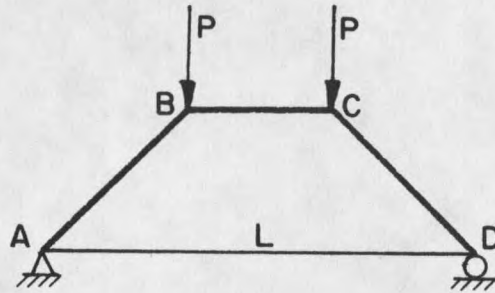


Figure 5. Basic tied arch action.

Under dead load action the arch axis will shorten due to the axial thrust. This effect is called rib shortening. Similarly tie lengthening and hanger stretching will occur due to axial tension. These effects cause secondary moments in the rib and tie which can become quite large. The moments due to rib shortening and tie lengthening can be minimized by fabricating the arch rib longer by a certain amount, and by fabricating the tie shorter by a certain amount. Hangers which are not adjustable also would need to be fabricated shorter by certain amounts. In effect what this does is camber the arch equal and opposite to the deflections induced by the dead load [14], as shown in Figure 6.

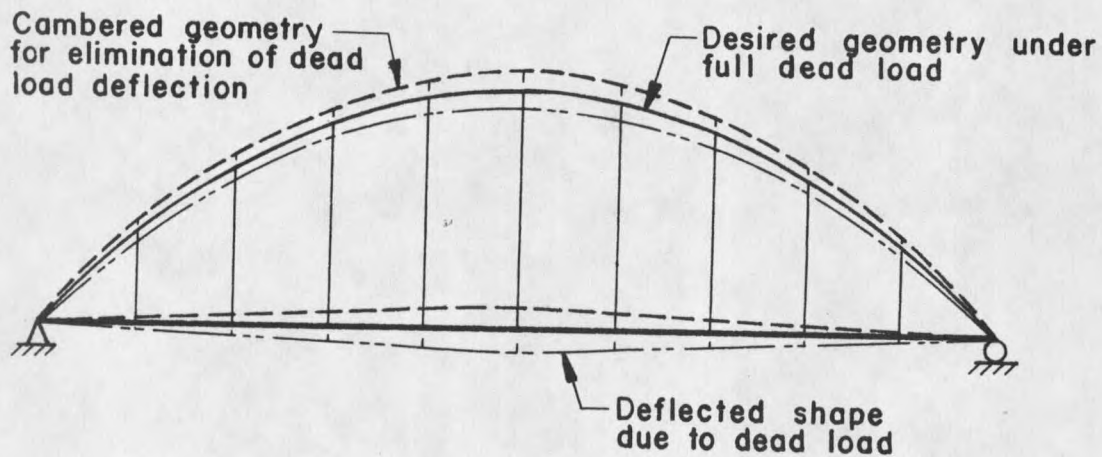


Figure 6. Cambering arch to eliminate dead load deflection.

Referring again to Figure 4 the forces in members AB and CD are found from statics to be $P(\csc\theta)$, and for member BC, $P(\cos\theta)$. The horizontal thrust at points A and D is also equal to $P(\cos\theta)$. The equation for axial deformation in a prismatic member is given by:

$$\delta = \frac{FL}{AE} \quad (1)$$

where:

δ = Axial deformation
 F = Axial force
 L = Length of member
 A = Area of member
 E = Modulus of elasticity

Noting that the lengths of members AB and CD are equal to $(\sec\theta)L/3$, the required additional fabricated length to eliminate axial shortening is found from Equation (1) to be:

$$\delta = \frac{P(\csc\theta)(\sec\theta)L}{3AE}$$

Similarly for member BC:

$$\delta = \frac{P(\cos\theta)L}{3AE}$$

The tie member AD in Figure 5 is required to carry the horizontal thrust, $P(\cos\theta)$, of members AB and CD. Therefore member AD should be fabricated short by an amount:

$$\delta = \frac{-P(\cos\theta)L}{AE}$$

In addition to fabricating the members to compensate for changes in length, the geometry of the connections between members must be fabricated to the ideal undeformed shape of the structure. This insures that moments are not induced by angular rotations at the joints due to the changes in lengths of the members. For the frame of Figure 5 this means that angles BAD and CDA would equal θ , and angles ABC and DCB would equal $180^\circ - \theta$.

Application of the principle to a tied arch

The basic ideas presented in the simple frame example can readily be applied to a highly statically indeterminate structure, such as a tied arch. To accomplish this it is convenient to first consider the tie separately, and then consider the entire arch.

For most tied arch bridges the dead load of the floor system including the concrete slab will be nearly uniform across the span. Possible variances in the loading may occur due to different sizes of the lateral floor bracing. Also the cross section of the tie could vary across the span. For a simply supported beam carrying a uniform load the moment diagram is parabolic. Referring to Figure 7, if the

axis of the beam is shaped to a parabola, the moment sum about point B at the centerline yields:

$$Fh + \left(\frac{WL}{2}\right) \frac{L}{4} - \left(\frac{WL}{2}\right) \frac{L}{2} = 0$$

where:

h = Height at centerline
 L = Length of span
 W = Uniform load per unit length
 F = Horizontal force

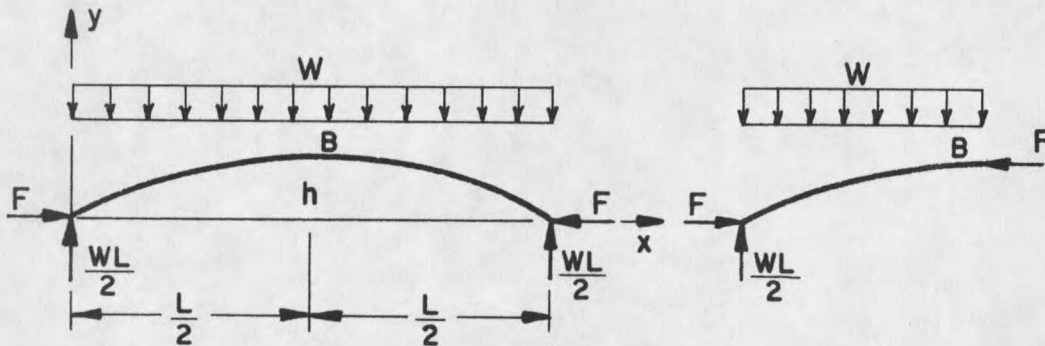


Figure 7. Solving for horizontal force for parabolically shaped beam.

Solving for the horizontal force:

$$F = \frac{WL^2}{8h} \quad (2)$$

If the floor loads are truly uniform across the span the hanger forces will turn out to all be equal as shown in Figure 8.

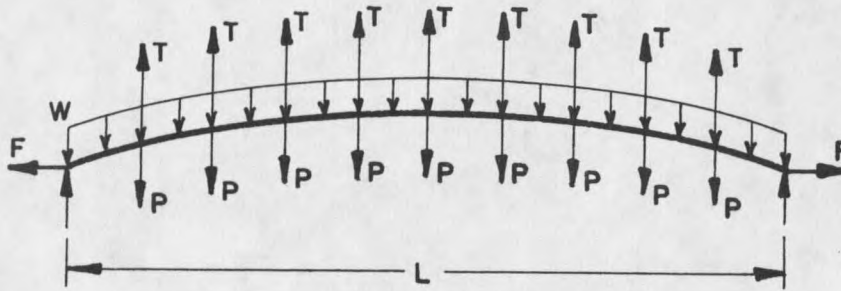


Figure 8. Tie carrying uniform loading.

Noting that the force F in Figure 8 is tensile and opposite in sense to the force F in Figure 7, the net resultant of the the hanger forces T acting upon the tie is an upward uniform loading. Obviously the closer that the hangers are spaced, the more closely a uniform loading is approximated. This means local dead load moments will be smaller.

Referring again to Figure 8, although there are two unknowns F and T , T can be written in terms of F by substitution into Equation (2):

$$F = \left(\frac{T - P}{S} - W \right) \frac{L^2}{8h} \quad (3)$$

where:

- L = Length of tie
- T = Hanger tension
- F = Horizontal force in tie
- W = Weight of tie per unit length
- P = Floor system load
- S = Distance between hangers
- h = Height of tie at centerspan

Equation (3) demonstrates that the tie in itself can be treated as a parabolic arch. For the case of a non-uniform floor loading the net loading can be considered as the sum of a uniform loading P plus a non-uniform loading ΔP_i , as shown in Figure 9.

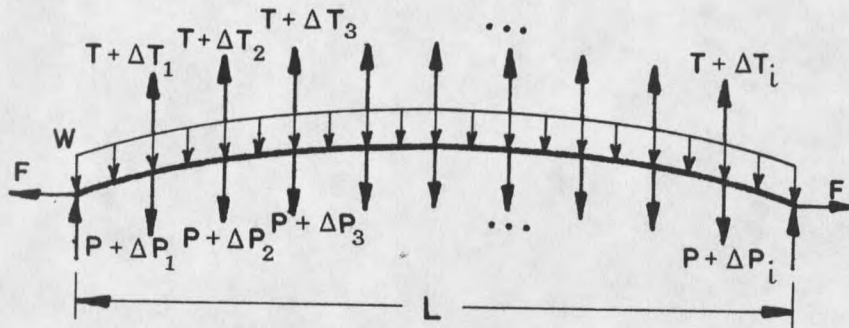


Figure 9. Tie carrying non-uniform loading.

For this case the hanger forces will no longer be equal. From Figure 9 it is apparent however that the non-uniformity of the loading can be directly transmitted into the hangers and carried to the rib. If the weight of the tie varies across the span, the non-uniformity can also be carried into the hangers, although some dead load moment will exist. Therefore the tie can still be modeled as a uniformly loaded parabolic arch.

If the tie is not cambered, the hanger forces will be equal to:

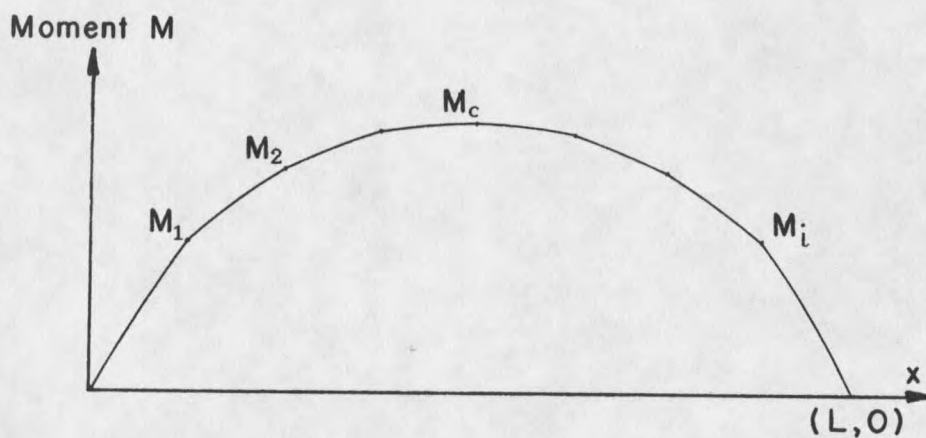
$$T = P + \Delta P_i + WS \quad (4)$$

where:

- T = Hanger tension
- P = Floor system load
- W = Weight of tie per unit length
- S = Distance between hangers

This will not produce the minimum dead load moment, since the geometry of the tie is not conforming to the moment diagram of a similarly loaded simply supported beam. For minimizing the dead load moments in the tie it is probably easiest to shape the tie as a parabola for uniform loading. Any adjustments for non-uniform loading can be made in the geometry of the rib.

The initially assumed geometry of the rib should be checked. If there are no extremely abrupt changes in load across the span, good results can usually be achieved by initially assuming the rib profile to be parabolic. The dead load moment diagram should be constructed for the entire arch. Loads due to the upper lateral bracing and the floor system are treated as concentrated loads applied at the hanger locations. The weights of the rib and tie are treated as distributed loads. A typical shape of a dead load moment diagram for a tied arch is shown in Figure 10.



- M_i = Internal arch dead load moment
 at hanger location i
 M_c = Internal arch dead load moment
 at center of bridge

Figure 10. Dead load moment diagram for a tied arch.

The horizontal force F can be determined by equating the internal couple caused by F to the internal dead load moment M_c at the center of the bridge. Referring to Figure 11, and assuming the height H of the rib at the center of the bridge to remain unchanged:

$$F(H-h) = M_c$$

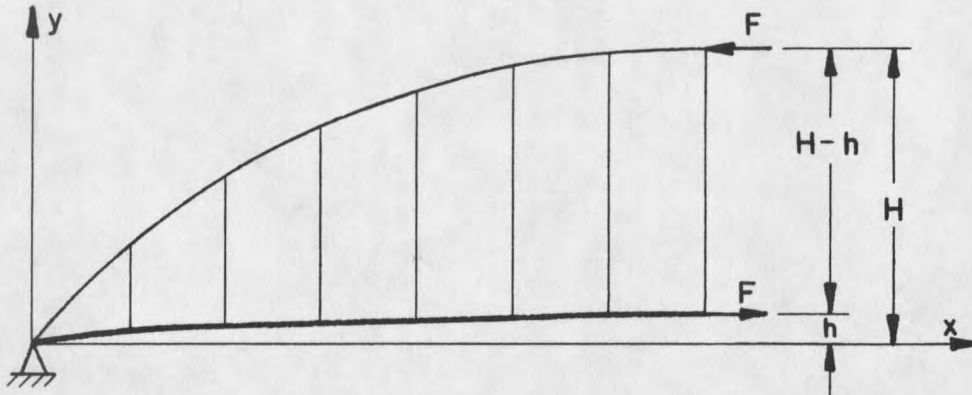


Figure 11. Solving for horizontal force for parabolically shaped arch.

Solving for the horizontal force:

$$F = \frac{M_c}{H - h} \quad (5)$$

Now that the tensile force F in the tie is known, the remainder of the rib geometry can be determined from:

$$H_i = \frac{M_i}{F} + h_i \quad (6)$$

where:

- H_i = Ordinates of rib profile
- h_i = Ordinates of tie profile (parabolic)
- M_i = Internal arch dead load moment at hanger location i

The ordinates of the rib profile should be set equal to the values of H_i . If the values of H_i calculated from Equation (6) differ greatly from the initially assumed values, it may be necessary to recalculate the weights of the rib members based upon the new geometry. The dead load moment diagram would have to be modified accordingly.

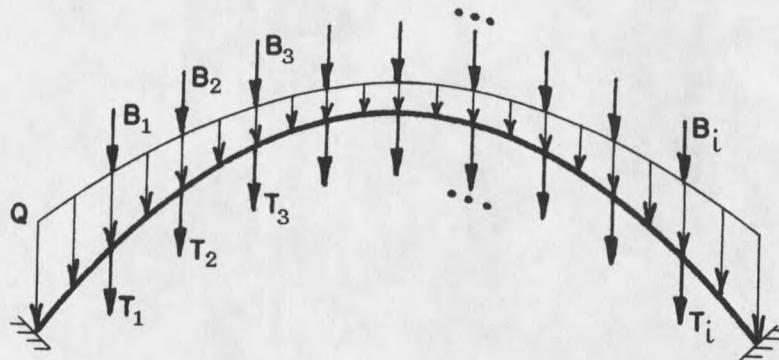
The hanger forces can now be determined by eliminating F from Equations (3) and (5), and adding the non-uniform loading ΔP_i :

$$T_i = \frac{8ShM_c}{L^2(H-h)} + P + \Delta P_i \quad (7)$$

Referring to Figure 9, the value of $P + \Delta P_i$ in Equation (7) represents the total dead load floorbeam reaction at hanger location i .

Computer analysis

The axial loads induced in the rib members can now be determined most readily by computer, considering the ends of the rib as fixed. The forces could also be calculated by dividing the horizontal force F at centerspan by the cosines of the angles of inclination of the rib segments. The loaded rib is shown in Figure 12. The distributed loads would be entered as equivalent member end actions.



- B_i = Weight of upper bracing
 at hanger location i
 T_i = Hanger tension at location i
 Q = Weight of rib per unit length

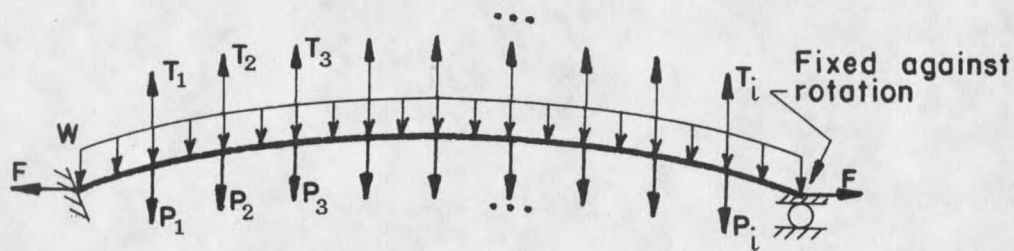
Figure 12. Mathematical model of rib for computer analysis.

The axial forces computed in the analysis are now entered into the data as axial member end actions. As mentioned previously the connections must be fabricated to the undeformed geometry. Additionally if the members are curved the radius of curvature must be fabricated to the undeformed shape.

A second computer analysis should show practically zero deflection at all joints and small moments due to local bending between joints. If the axial loads computed in this second analysis are larger than the axial loads input as member end actions from the first analysis, it will be necessary to repeat the process. The axial forces from the second analysis are entered as member end actions and another analysis is made. Convergence to the solution is very rapid,

usually within one or two tries, and occurs when the calculated axial forces equal the input axial member end actions. Substitution of the axial forces into Equation (1) determines the required lengthening adjustments for the rib members.

Although the tension in the tie is already known, an analysis should be made to determine the deflections of the joints. The tie can be modeled as fixed at one end and free to translate in the horizontal direction at the other, as shown in Figure 13.



- T_i = Hanger tension at location i
 P_i = Floor load at hanger location i
 W = Weight of tie per unit length

Figure 13. Mathematical model of tie for computer analysis.

An analysis is made of the tie similar to the rib analysis including axial member end actions equal to the tensile force F . When calculating the member end actions it may be necessary to adjust the force F to account for the camber in the tie. This is accomplished by:

$$AML = \frac{F}{\cos \theta} \quad (8)$$

where:

AML = Axial member end action
 F = Horizontal force
 θ = Angle of inclination of tie segment
 from horizontal

The final joint deflections are determined and should be very small. Substitution of the axial force in the tie into Equation (1) yields the required shortening adjustments for the tie members.

The required fabrication lengths of the hangers are determined by first computing the relative vertical displacements between the rib and the tie at each hanger location:

$$\delta y_i = \delta y_R - \delta y_T \quad (9)$$

where:

δy_i = Relative vertical displacement
 between rib and tie of hanger i
 δy_R = Vertical displacement of rib joint of hanger i
 δy_T = Vertical displacement of tie joint of hanger i

The values of δy_i should be very small if rib shortening effects are properly eliminated. The required shortening adjustments for the hangers are determined from:

$$\delta l_i = - \frac{T_i l_i}{E A} + \delta y_i \quad (10)$$

where:

T_i = Hanger tension at location i
 l_i = Length of hanger i
 E = Modulus of elasticity
 A = Area of hanger
 δl_i = Length that hanger i is
 to be fabricated short

The calculation of δy_i and δl_i would not be necessary for adjustable length hangers. In any case it is good to make a final dead load analysis of the entire arch by inputting the computed member end actions for the rib and the tie, and converting the δl_i of Equation (10) to equivalent axial member end actions for the hangers. The final analysis should indeed show that the dead load moments are much smaller than those of a comparable analysis without the geometry changes or the axial deformations taken into account.

CHAPTER III

HANGER TENSION ADJUSTMENT

The structural shapes used for hangers in tied arch bridges generally are box, I or W shapes, or wire rope. Designs using wire rope hangers can be incorporated with provisions for adjusting the tension in the hangers. Typically a threaded strand socket is used at the connection of the hanger and the tie. Designs using solid shaped hangers usually are not provided with means of tension adjustment. Non-adjustable arches require very good quality control during fabrication, to insure that secondary stresses due to misfit are not induced into the arch. Vibration problems have occurred with the use of H shaped hangers, such as the breakage which occurred in the Tacony-Palmyra Bridge, [56].

Arches that are provided with adjustment must be "tuned", sometimes at several phases of the construction. The most convenient times during construction for hanger adjustment are after all steelwork has been erected, and after the deck slab has been completed. Failure to achieve the correct design tensions in the hangers could result in serious overstressing of the rib and tie.

In order to carry out an adjustment analysis the following must be known:

- 1.) The actual tension in each hanger cable.
- 2.) The desired tension in each hanger cable.
- 3.) The relationship between the force induced in cable u due to a unit axial deformation applied to cable v.

The biggest problem in the analysis is determining the actual hanger tensions. Several methods are available for accomplishing this:

- 1.) Force - deflection method.
- 2.) Frequency of vibration method.
- 3.) Strain gages.

Methods of determining hanger tensions

1.) Force-deflection method

By applying a known horizontal force P at a known distance e between two points on a tightly stretched cable, the tension T can be found from static equilibrium. Referring to Figure 14 and equating the sum of horizontal forces to zero yields:

$$-P + T(\sin\alpha) + T(\sin\beta) = 0$$

where:

P = Known applied horizontal force
 T = Tension in cable

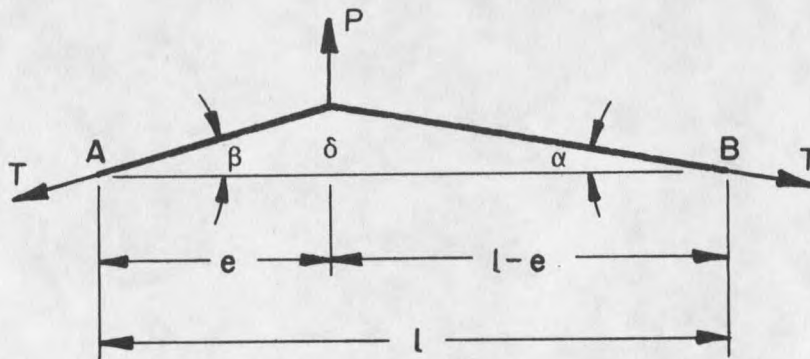


Figure 14. Application of a horizontal force to a tightly stretched cable.

Solving for the tension:

$$T = \frac{P}{\sin\alpha + \sin\beta} \quad (11)$$

The angles α and β can be determined from the relationships:

$$\alpha = \tan^{-1}\left(\frac{\delta}{h-e}\right) \quad \text{and:} \quad \beta = \tan^{-1}\left(\frac{\delta}{e}\right)$$

This method is commonly used because of the simplicity and accuracy.

2.) Frequency of vibration method

The tensions in a tightly stretched cable can be determined from measurements of the frequency of vibration and mode shape. Referring to Figure 15 it can be shown that the frequencies of vibration of a tightly stretched cable satisfy the one-dimensional wave equation:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (12)$$

where:

T = Tension in cable
 g = Acceleration due to gravity
 w = Weight per unit length of cable
 l = Length of cable

and: $a^2 = \frac{Tg}{w}$

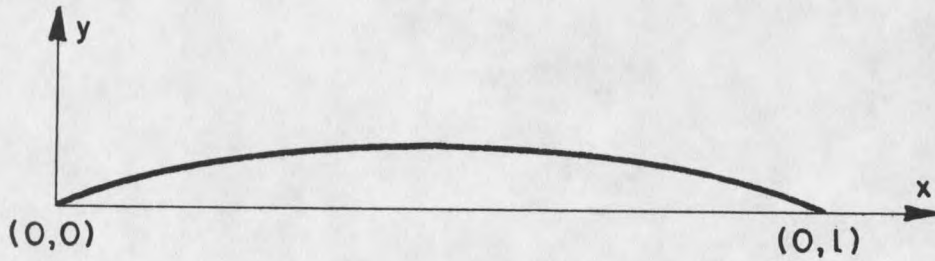


Figure 15. Fundamental mode of vibration of a tightly stretched cable.

After trying a solution of the form $y(x,t) = \chi(x)\tau(t)$ which employs separation of variables, substitution into Equation (12) yields:

$$\frac{\tau''}{\tau} = a^2 \frac{\chi''}{\chi} = \mu \quad (\mu \text{ constant}) \quad (13)$$

The general solution of Equation (13) is found to be:

$$y(x,t) = (A \cos \lambda t + B \sin \lambda t)(C \cos \frac{\lambda x}{a} + D \sin \frac{\lambda x}{a}) \quad (14)$$

for $\mu = \lambda^2$ and $\lambda < 0$

This is a periodic function repeating itself each time t increases by $2\pi/\lambda$. Therefore the frequency of vibration is given by:

$$f = \frac{\lambda}{2\pi} \quad (15)$$

Noting in Equation (14) that the solution $A = B = 0$ produces no motion, this solution is rejected as a trivial solution. The boundary conditions are seen to be $y(0,t) = 0$ and $y(l,t) = 0$. Substitution of the boundary conditions into Equation (14) yields $C = 0$. Eliminating

the possibility $D = 0$ as another trivial solution the following is obtained:

$$\sin\left(\frac{\lambda_1 l}{a}\right) = 0 \quad \text{or:} \quad \frac{\lambda_k l}{a} = k\pi, \quad k = 1, 2, 3, \dots$$

Solving for λ_k and substituting into Equation (15) yields:

$$f_k = \frac{ak}{2l}$$

Therefore the expression for the natural frequencies of vibration of a tightly stretched cable is given by:

$$f_k = \frac{k}{2l} \sqrt{\frac{Tg}{w}} \quad (16)$$

where:

f_k = Frequency for the k th mode
 k = Mode of vibration

To determine the tension, the hanger is vibrated and the number of cycles per unit time is measured either visually or electronically. The mode shape must be visually determined.

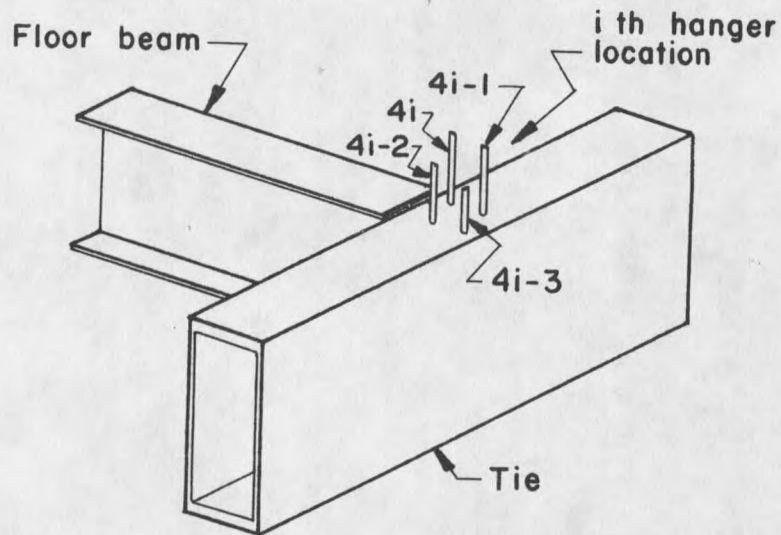
3.) Strain gages

Strain gages could also be utilized for determining hanger tensions. The cost of this method would probably be greater than that of other methods. It would be important to attach a strain gage to a hanger before any stress is induced into the hanger. After placing the hanger in the arch the tension can be determined directly, by reading the value from a strain indicator. By using a multi-channel strain indicator it would be possible to monitor the strains in all

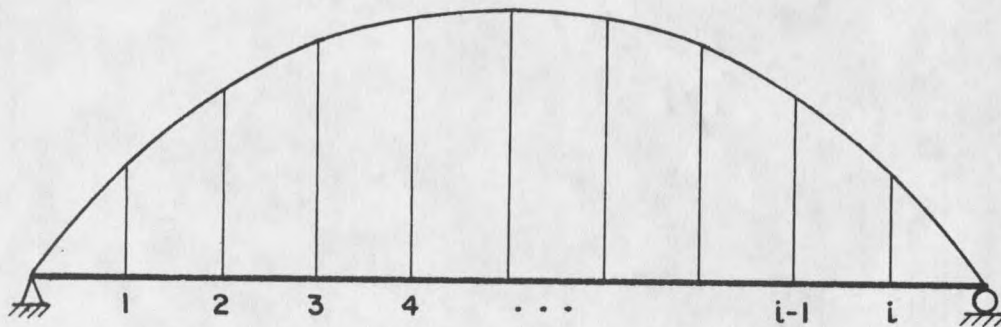
hangers simultaneously. This would be advantageous during the adjustment process.

Analysis

The preliminary step for the adjustment analysis is determining the actual tension in each hanger. For hangers with multiple cables the tension in each individual cable must be determined as they probably will not be equal. The analysis can be carried out most readily using matrix structural analysis on a computer. Most tied arch bridges can be modeled as if each arch behaves as a planar frame. Therefore no interaction occurs between arches due to the floor beams or the upper lateral bracing. If there is interaction between arches a space frame analysis can be used. Figure 16 shows the assumed hanger and cable numbering conventions. The analysis is written assuming four cables per hanger, giving a total of $4i$ cables per arch. The procedure applies for any number of cables however.



Cable Numbers



Hanger Numbers

Figure 16. Cable and hanger numbering conventions for adjustment analysis.

The required tension adjustments $\{\Delta T\}$ for all cables are obtained from the relationship:

$$\{\Delta T_u\} = \{T_u\} - \{I_u\} \quad \text{for } u = 1, 4i \quad (17)$$

where:

$\{\Delta T_u\}$ = Required tension adjustments

$\{T_u\}$ = Final desired tensions

$\{I_u\}$ = Actual cable tensions

Starting with cable 1 at hanger location 1 it is necessary to develop an influence coefficient matrix $[K_{uv}]$. The matrix relates the force induced in cable u due to a unit axial deformation applied to cable v . The matrix $[K_{uv}]$ therefore has the dimension $4i$ by $4i$. Using this influence coefficient matrix the required tension adjustments $\{\Delta T\}$ can be related to the required length adjustments $\{D\}$ by the equation:

$$[K_{uv}]\{D_u\} = \{\Delta T_u\} \quad \text{for } u = 1, 4i ; v = 1, 4i \quad (18)$$

After partitioning the matrix $[K_{uv}]$ in terms of hanger locations, Equation (18) can be rewritten:

$$[k_{mn}]\{d_m\} = \{\Delta t_m\} \quad \text{for } m = 1, i ; n = 1, i \quad (19)$$

In expanded notation:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & \dots & k_{1n} & \dots & k_{1i} \\ k_{21} & k_{22} & k_{23} & \dots & k_{2n} & \dots & k_{2i} \\ k_{31} & k_{32} & k_{33} & \dots & k_{3n} & \dots & k_{3i} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ k_{m1} & k_{m2} & k_{m3} & \dots & k_{mn} & \dots & k_{mi} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ k_{i1} & k_{i2} & k_{i3} & \dots & k_{in} & \dots & k_{ii} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_m \\ \vdots \\ d_i \end{Bmatrix} = \begin{Bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \vdots \\ \Delta t_m \\ \vdots \\ \Delta t_i \end{Bmatrix}$$

where:

$$[k_{mn}] = \begin{bmatrix} K_{4m-3, 4n-3} & K_{4m-3, 4n-2} & K_{4m-3, 4n-1} & K_{4m-3, 4n} \\ K_{4m-2, 4n-3} & K_{4m-2, 4n-2} & K_{4m-2, 4n-1} & K_{4m-2, 4n} \\ K_{4m-1, 4n-3} & K_{4m-1, 4n-2} & K_{4m-1, 4n-1} & K_{4m-1, 4n} \\ K_{4m, 4n-3} & K_{4m, 4n-2} & K_{4m, 4n-1} & K_{4m, 4n} \end{bmatrix}$$

$$\{d_m\} = \begin{Bmatrix} D_{4m-3} \\ D_{4m-2} \\ D_{4m-1} \\ D_{4m} \end{Bmatrix} \quad \text{and:} \quad \{\Delta t_m\} = \begin{Bmatrix} \Delta T_{4m-3} \\ \Delta T_{4m-2} \\ \Delta T_{4m-1} \\ \Delta T_{4m} \end{Bmatrix}$$

Assuming that the connections between the hangers and the rib and tie are very stiff the deflection due to bending of the connections can be neglected. Therefore all cables of a hanger deflect equally. With this assumption the influence coefficients relating the cables $4m$, $4m-1$, $4m-2$, and $4m-3$, at a particular hanger m can be written:

$$K_{4m-3, 4m-3} = K_{4m-2, 4m-2} = K_{4m-1, 4m-1} = K_{4m, 4m} = AML - S_{mm} \quad (20i)$$

and:

$$K_{4m-3, 4m-2} = K_{4m-3, 4m-1} = K_{4m-3, 4m} = -S_{mm} \quad (20ii)$$

$$K_{4m-2, 4m-3} = K_{4m-2, 4m-1} = K_{4m-2, 4m} = -S_{mm} \quad (20iii)$$

$$K_{4m-1, 4m-3} = K_{4m-1, 4m-2} = K_{4m-1, 4m} = -S_{mm} \quad (20iv)$$

$$K_{4m, 4m-3} = K_{4m, 4m-2} = K_{4m, 4m-1} = -S_{mm} \quad (20v)$$

where:

S_{mm} = Force induced in three cables at hanger location m due to application of a unit deformation to the forth cable at hanger location m .

AML = Force required to produce unit deformation in the forth cable.

In Equations (20) and (21) S_{mm} is assumed to be positive to produce positive tensile forces. Substituting (20i) - (20v) into the expression for $[k_{mn}]$ in Equation (19), and noting that $m = n$, the following is obtained:

$$[k_{mn}] = S_{mm} \begin{bmatrix} \frac{AML - S_{mm}}{S_{mm}} & -1 & -1 & -1 \\ -1 & \frac{AML - S_{mm}}{S_{mm}} & -1 & -1 \\ -1 & -1 & \frac{AML - S_{mm}}{S_{mm}} & -1 \\ -1 & -1 & -1 & \frac{AML - S_{mm}}{S_{mm}} \end{bmatrix} \quad \text{for } m = n \quad (21)$$

Similarly, the forces induced in the four cables at hanger location m due to the unit deformation of a cable at location n will be equal. This can be expressed as:

$$[k_{mn}] = S_{mn} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{for } m \neq n \quad (22)$$

where:

S_{mn} = Force induced in a cable at hanger location m
due to application of a unit deformation to a
cable at hanger location n .

The coefficient S_{mn} in Equation (22) is assumed positive to produce positive tensile forces. When applying the unit deformation to a cable for calculating S_{mm} and S_{mn} , it is usually more convenient to convert to an equivalent member end action:

$$AML = \frac{AE}{l} \quad (23)$$

where:

AML = Equivalent member end action
A = Area of cable
E = Modulus of elasticity
l = Length of cable

The remaining three cables at the hanger being loaded can be combined into a single equivalent member. Similarly the four cables at all other hangers can be combined into single equivalent members.

Since the arch geometry is symmetric, S_{mm} and S_{mn} only have to be calculated for half of the hangers.

With the formation of the $[K_{uv}]$ matrix, the required length adjustments to be made to the hangers are solved for from Equation (18):

$$\{D_u\} = [K_{uv}]^{-1} \{\Delta T_u\} \quad (24)$$

The length adjustments $\{D_u\}$ can also be written in terms of the required turns of the adjusting nuts:

$$\{N_u\} = \{D_u\}(p) \quad (25)$$

where:

$$\begin{aligned} \{N_u\} &= \text{Required turns of the adjusting nuts} \\ \{D_u\} &= \text{Required length adjustments} \\ p &= \text{Pitch of the thread (turns/length)} \end{aligned}$$

When performing the adjustment to the bridge, hangers should be alternately tightened on each side of the arch. This should minimize the possibility of overstressing the rib or tie during adjustment. If any hanger tensions vary excessively from the design values, it may be necessary to compute the stresses in the arch. Since this is a closed form solution the tightening sequence will not affect the final results. There is the distinct possibility however of overstressing the rib or the tie during adjustment. This may have to be checked.

CHAPTER IV

EFFECTS OF VARIOUS PARAMETERS UPON TIED ARCH BEHAVIOR

Discussion of parameters

Preliminary analysis of a tied arch is complex due to the many possible geometric and member parameters for a structure. The effects of some of the various parameters on the behavior of a tied arch were investigated. The parameters considered in this discussion are:

- 1.) Rise to span ratio.
- 2.) Ratio of moments of inertia of rib and tie.
- 3.) Ratio of areas of rib and tie.
- 4.) Hanger spacing.

Other important design parameters which must be considered include:

- 1.) Type of rib, tie, and deck.
- 2.) Type of joint at connection of rib and tie.
- 3.) Tie depth to span ratio.
- 4.) Rib depth to span ratio.
- 5.) Curved rib versus segmental rib.

Tied arches can be designed within a wide range of the various parameters. The question of economics must be considered in conjunction with design efficiency. For example, structurally it is desirable to space the hangers as close as possible. Economically this is not feasible, even though the dead load of the structure will decrease. Similarly if the hanger spacing becomes too large the savings in materials and labor will be more than offset by the cost of the increased dead load. Again this makes an uneconomic alternative.

Therefore when choosing the hanger spacing, the cost of installing a hanger must be considered along with the cost per foot of floor system and rib.

Aesthetics also form a very important part of the selection of parameters. An arch which uses straight segments between panel points may not have a pleasing appearance if the hangers are spaced too far apart. This would produce sharp angular changes at the panel points and break the continuity of the rib. Aesthetics can also enter into the consideration of rib and tie depths. Many times a certain tie depth is determined which will correspond with the girder depths of the approach spans.

Table 1 gives some values of the typical parameters used for various bridges with span lengths ranging from 116 feet to 1200 feet. Richardson [37] compiled photos and overall dimensions of various arch bridges, and is an excellent source of information for preliminary designs. The parameter ranges found from Table 1 primarily of interest for this study are:

1. Arch rise to span ratio typically lies within $1/5$ and $1/6.5$.
2. For arches with deep ties and shallow ribs, tie depth to span ratio typically lies within $1/50$ and $1/70$.
3. Ratio of moments of inertia of rib to tie for solid rib and tie bridges typically lies within $1/20$ and $25/1$.
4. Ratio of span to hanger spacing typically lies within 10 and 20.
5. Ratio of areas of rib to tie for solid rib and tie bridges typically lies within 0.6 and 1.5.

Table 1¹. Parameters of various tied arch bridges.

Name of Bridge Route No. / River Spanned Location / Date Constructed	No. of Lanes	Type of Tie	Type of Rib	Length	Rise	$\frac{L}{H}$	No. of Panels	Rib Crown Depth	Tie Center Depth	$\frac{\text{Length}}{\text{Tie Depth}}$	$\frac{A_R}{A_T}$	$\frac{I_R}{I_T}$	Hangers	Notes
Old Mill Road Old Mill Road, Chagrin Cleveland, OH, 1959	2	I	Channel	116.0'	25.0'	4.64	7	1.5'	3.0'	38.67	0.64^2	0.13^2	12WF31	
North Fork Stillaguamish U.S. 410, N.Fork Stillaguamish Cicero & Snohomish Co., WA, 1966	2	Box	Box	278.7'	51.0'	5.50	11	2.0'	4.0'	69.67	0.81^2	0.22^2	8x8x3/8 Tubing	
Bonar A9 Highway, Dornoch Firth Scotland, 1975	2	I	Box	341.5'	61.4'	5.56	17	2.3'	5.6'	61.20	1.08^3	0.17^3	64mm Coil Cable	
Leavenworth Centennial Missouri River Leavenworth, KA, 1955	2	Box	Box	420.0'	80.0'	5.25	13	2.8'	6.2'	67.74	0.74^2	0.15^2	HP12x53	Two Arches
Fort Duquesne Allegheny River Pittsburgh, PA, 1960	8	Truss	Box	423.0'	64.6^6	6.55	9	4.0'	23.5'	18.00	0.95^7	0.02^7	2-3/4" Wire Rope	Double Deck
St. Georges, Dupont Hwy. Chesapeake & Delaware Canal St. Georges, DE, 1940	4	Box	Channel	539.4'	104.1'	5.18	15	3.4'	9.0'	59.93	0.73^3	0.08^3	Box	
Fort Henry Ohio River Wheeling, WVa, 1955	4	Plates	Box	577.5'	110.9'	5.20	15	9.2'	2.8'	204.1	1.46^2	26.7^2	W14x103	
LaSalle - Peru Route 412, Illinois River LaSalle, IL, 1984	4	I	Box	620.5'	123.6'	5.00	17	4.0'	11.0'	56.41	0.82^2	0.12^2	1-5/8" Wire Rope	

Table 1¹ Continued. Parameters of various tied arch bridges.

Name of Bridge Route No. / River Spanned Location / Date Constructed	No. of Lanes	Type of Tie	Type of Rib	Length	Rise	$\frac{L}{H}$	No. of Panels	Rib Crown Depth	Tie Center Depth	$\frac{\text{Length}}{\text{Tie Depth}}$	$\frac{A_R}{A_T}$	$\frac{I_R}{I_T}$	Hangers	Notes
Fort Pitt Monongahela River Pittsburgh, PA, 1957	8	Truss	Box	750.0'	122.2' ⁶	6.14	15	5.4'	24.9'	30.10	0.94 ⁵	0.04 ⁵	3-1/4" Wire Rope	Double Deck
Glenfield I - 79, Ohio River Neville Island, PA, 1974	6	Box	Box	750.0'	124.4'	6.03	15	4.0'	12.5'	60.00	0.64 ²	0.06 ²	2-7/8" Wire Rope	
West End Ohio River Pittsburgh, PA, 1932	4	Plates	Truss	778.0'	151.0' ⁴	5.15	28	25.0'	3.3'	233.6	1.21 ²	183 ²	4" Wire Rope	Truss Arch Rib
Sherman Minton I - 64, Ohio River Louisville, KY, 1961	6	Box	Truss	797.5'	140.0' ⁴	5.70	22	30.0'	3.2'	251.6	1.86 ²	328 ²	2-1/2" Wire Rope	Truss Rib Double Deck
Mobile Arch I - 65, Mobile River Delta Mobile, AL, 1977	2	Box	Box	800.0'	136.0'	5.88	16	4.9'	14.0'	57.14	1.09 ²	0.15 ²	1-5/8" Wire Rope	
Port Mann, Trans-Canada Hwy. Lower Fraser River New Westminster, BC, 1962	4	Box	Box	1200.0'	230.0'	5.22	24	4.4'	12.0'	Half through tied arch with orthotropic deck and 360' sidespans.				
Fremont I - 405, Willamette Portland, OR, 1973	8	Box	Box	1255.3'	448.3'	3.00	28	4.0'	18.0'	Half through tied arch with orthotropic deck and 448' sidespans.				

1. Data collected from Richardson [37]
and drawings from various consulting firms.
2. Values taken at center of span.
3. Values taken at 0.20 point of span.
4. From springing line to lower chord of rib truss.
5. Values taken at 0.25 point of span.
6. From top chord of tie truss to rib.
7. Values taken at 0.44 point of span.

To study the effects that the parameters have upon tied arch behavior, the Mobile Arch Bridge, designed by the consulting firm of Howard Needles Tammen & Bergendoff was selected for analysis. The geometry for the Mobile Arch Bridge is given in Figure 17. The cases considered for study are given in Table 2.

Table 2. Cases considered for parametric study of tied arch behavior.

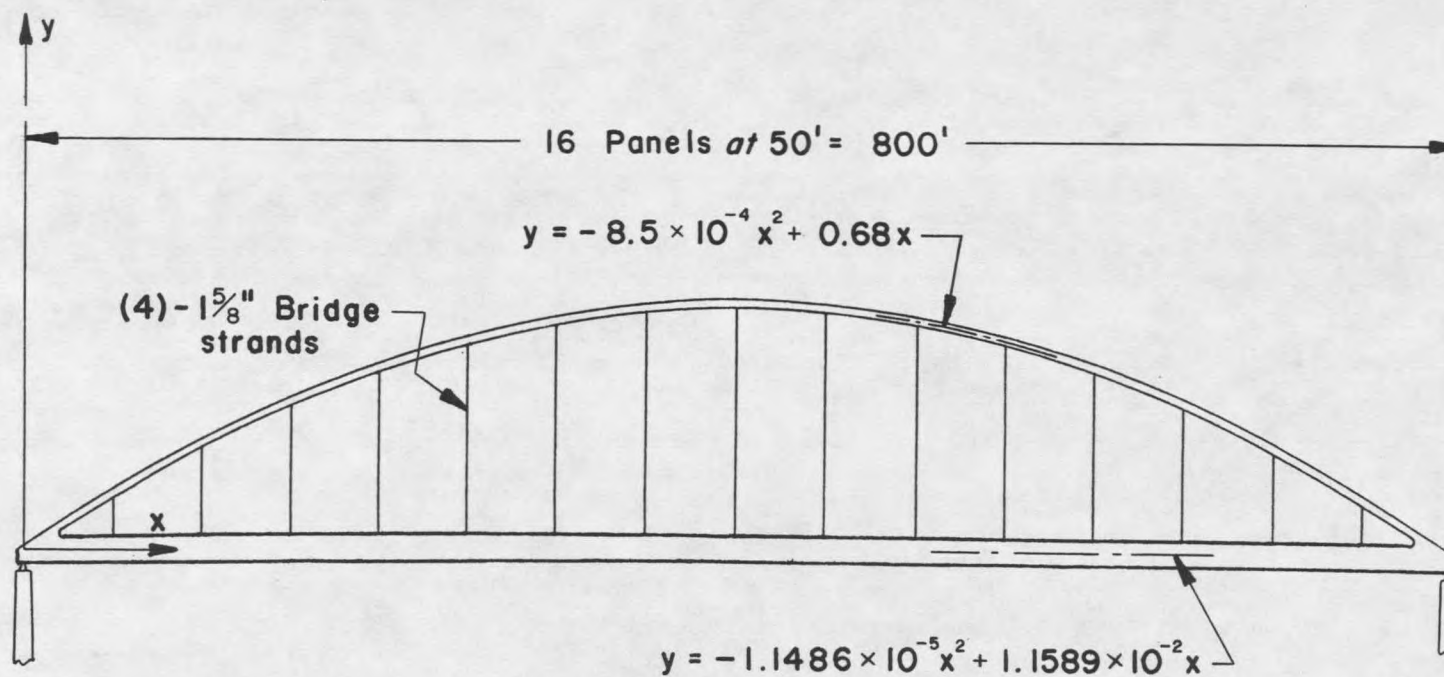
Case No.	$\frac{I_R}{I_T}$	$\frac{A_R}{A_T}$	s
1/20-12	1/20	0.6	12
1/20-16	1/20	0.6	16
1/20-20	1/20	0.6	20
1/10-10	1/10	1.0	10
1/10-12	1/10	1.0	12
1/10-16	1/10	1.0	16
1/10-20	1/10	1.0	20
1/10-24	1/10	1.0	24
1-12	1.0	1.0	12
1-16	1.0	1.0	16
1-20	1.0	1.0	20
20-12	20.0	1.5	12
20-16	20.0	1.5	16
20-20	20.0	1.5	20

where:

$\frac{I_R}{I_T}$ = Ratio of moments of inertia of rib to tie at centerspan

$\frac{A_R}{A_T}$ = Ratio of areas of rib to tie at centerspan

s = Number of panels



Arch rise at centerspan, $H = 136'$
 Tie rise at centerspan, $h = 2.3'$

Figure 17. Mobile Arch Bridge geometry.

It was decided to keep the cross sectional area of the arch at centerspan constant at 600 in^2 . Similarly the total moment of inertia of the rib plus tie was held constant at $1.5 \times 10^6 \text{ in}^4$. The rib and tie areas and moments of inertia were held constant across the span. The hanger area remained constant at 8.3 in^2 . All of the cases were run for a rise to span ratio (H/L) of 1/5.9. Some of the cases were repeated for an H/L ratio of 1/5.0.

The analysis was carried out by computer using matrix structural analysis. The member element used in the analysis allowed axial and flexural deformations, and gave a linear first order solution. Therefore axial-flexural interaction was not accounted for. The size effects of the connection at the rib and tie were also neglected.

The results were analyzed in terms of:

1. Rib and tie moment influence lines.
2. Hanger forces.
3. Rib and tie deflection.
4. Proportion of live load moment carried by the rib and by the tie.

Rib and tie moment influence lines

Figures 18 through 23 show the graphs of the rib and tie moment influence lines for three of the 16 panel cases. The H/L ratio for all cases is 1/5.9. The curves look similar to those predicted by Garrelts [7].

In all cases the rib moment influence lines tend to be more rounded near the peaks. This indicates distribution of moment to adjacent panels, as the rib deforms. The tie moment influence lines show much sharper peaks, especially when I_R/I_T becomes large. This

indicates more localized bending at the point of load application. All curves indicate the same shape of moment envelopes, although the amplitudes vary.

Several cases were repeated for an H/L ratio of 1/5.0. The results indicated insignificant changes in the amplitude and shape of the moment envelopes.

Figures 24 through 31 show the graphs of the rib and tie moment influence lines for four of the $I_R/I_T = 1/10$ cases, for different hanger spacings. The H/L ratio for all cases is 1/5.9. The results indicate that the live load moment envelope does not change significantly for different hanger spacings.

By using Figures 18 through 31 the loading cases for maximum rib and tie live load moment can be determined. The results are shown in Table 3.

Table 3. Locations of maximum rib and tie live load moment.

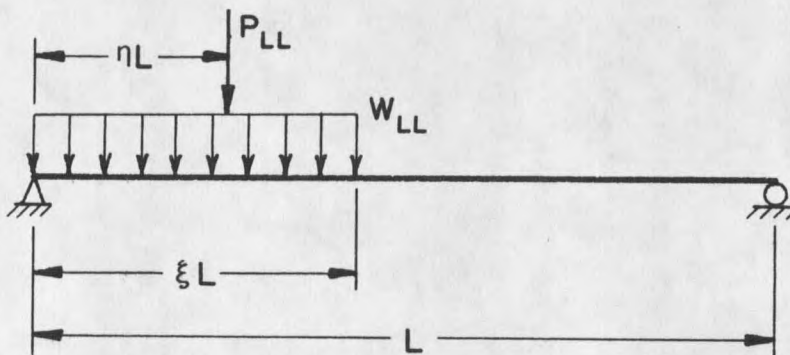
Case No.	Rib or Tie	Distributed Load				Concentrated Load			
		Positive ξ	L_P	Negative ξ	L_N	Positive η	L_P	Negative η	L_N
1/20-16	Rib	0.38	0.25	0.56	0.75	0.19	0.19	0.31	0.75
	Tie	0.38	0.25	0.56	0.75	0.19	0.19	0.31	0.75
1/10-10	Rib	0.40	0.30	0.50	0.80	0.20	0.20	0.30	0.70
	Tie	0.40	0.20	0.50	0.80	0.20	0.20	0.30	0.70
1/10-12	Rib	0.42	0.25	0.50	0.75	0.25	0.25	0.33	0.75
	Tie	0.42	0.25	0.50	0.75	0.17	0.17	0.33	0.75
1/10-20	Rib	0.40	0.25	0.55	0.75	0.20	0.20	0.30	0.75
	Tie	0.40	0.25	0.55	0.75	0.20	0.20	0.30	0.75
1/10-24	Rib	0.42	0.25	0.54	0.75	0.21	0.21	0.29	0.71
	Tie	0.42	0.25	0.54	0.75	0.21	0.21	0.29	0.75
1-16	Rib	0.38	0.25	0.56	0.75	0.19	0.19	0.31	0.75
	Tie	0.38	0.25	0.56	0.75	0.19	0.19	0.31	0.75
20-16	Rib	0.38	0.25	0.56	0.75	0.19	0.19	0.31	0.75
	Tie	0.38	0.19	0.56	0.81	0.19	0.25	0.31	0.75

where:

L_P = Location of maximum positive live load moment, as a fraction of the span length L

L_N = Location of maximum negative live load moment, as a fraction of the span length L

η and ξ are fractions of the span length L as shown:



The absolute maximum rib and tie moments may not occur at a hanger location, but between them. This is due to the dead load of the members. Table 3 indicates that maximum positive live load moment occurs between the 0.20 and 0.25 points of a span. Maximum negative live load moment occurs near the 0.75 point. Therefore the flanges of the rib and tie might require larger areas in these portions of a span.

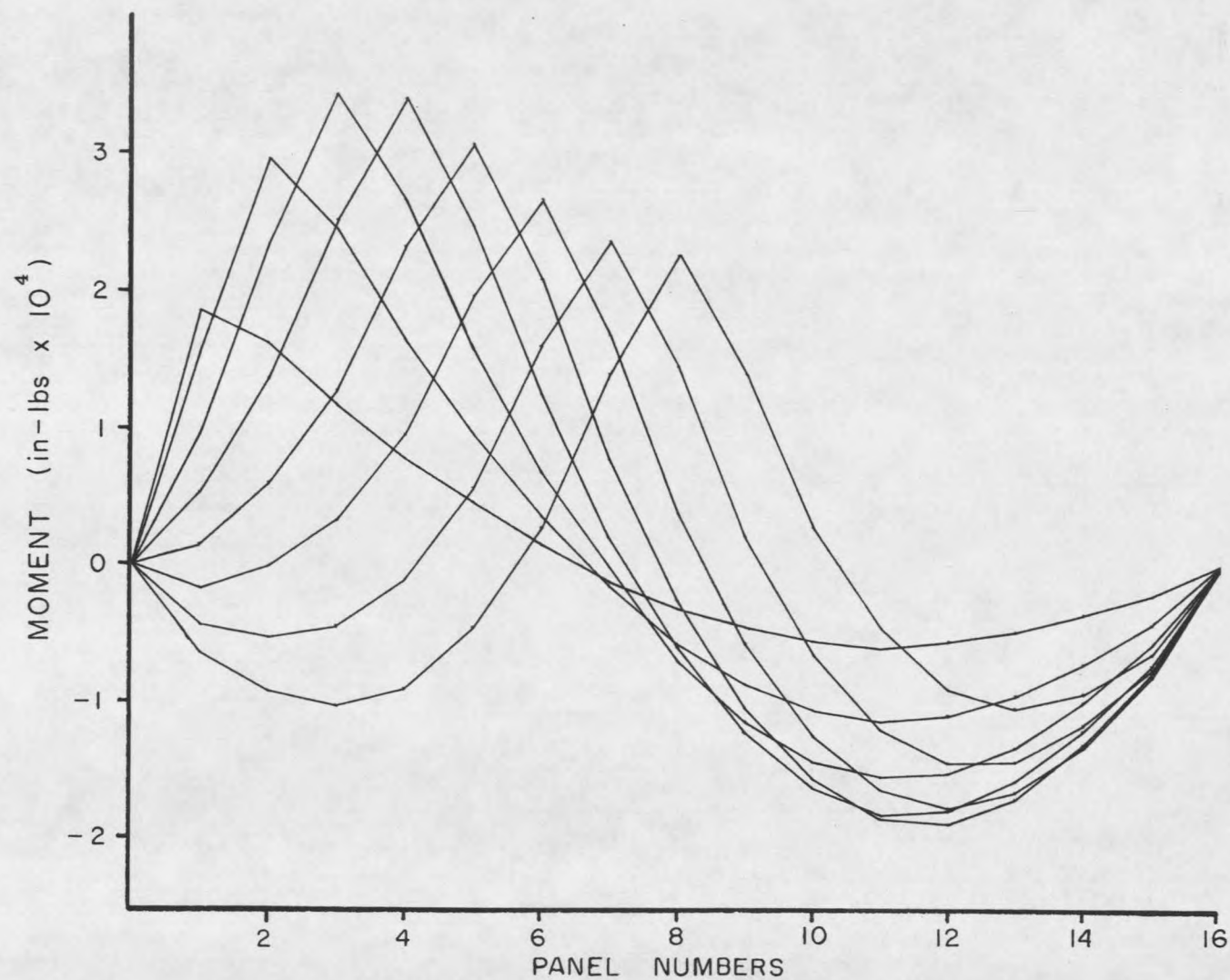


Figure 18. Graph of rib moment influence lines.
 $A_R/A_T = 0.6$; $I_R/I_T = 1/20$; 16 Panels

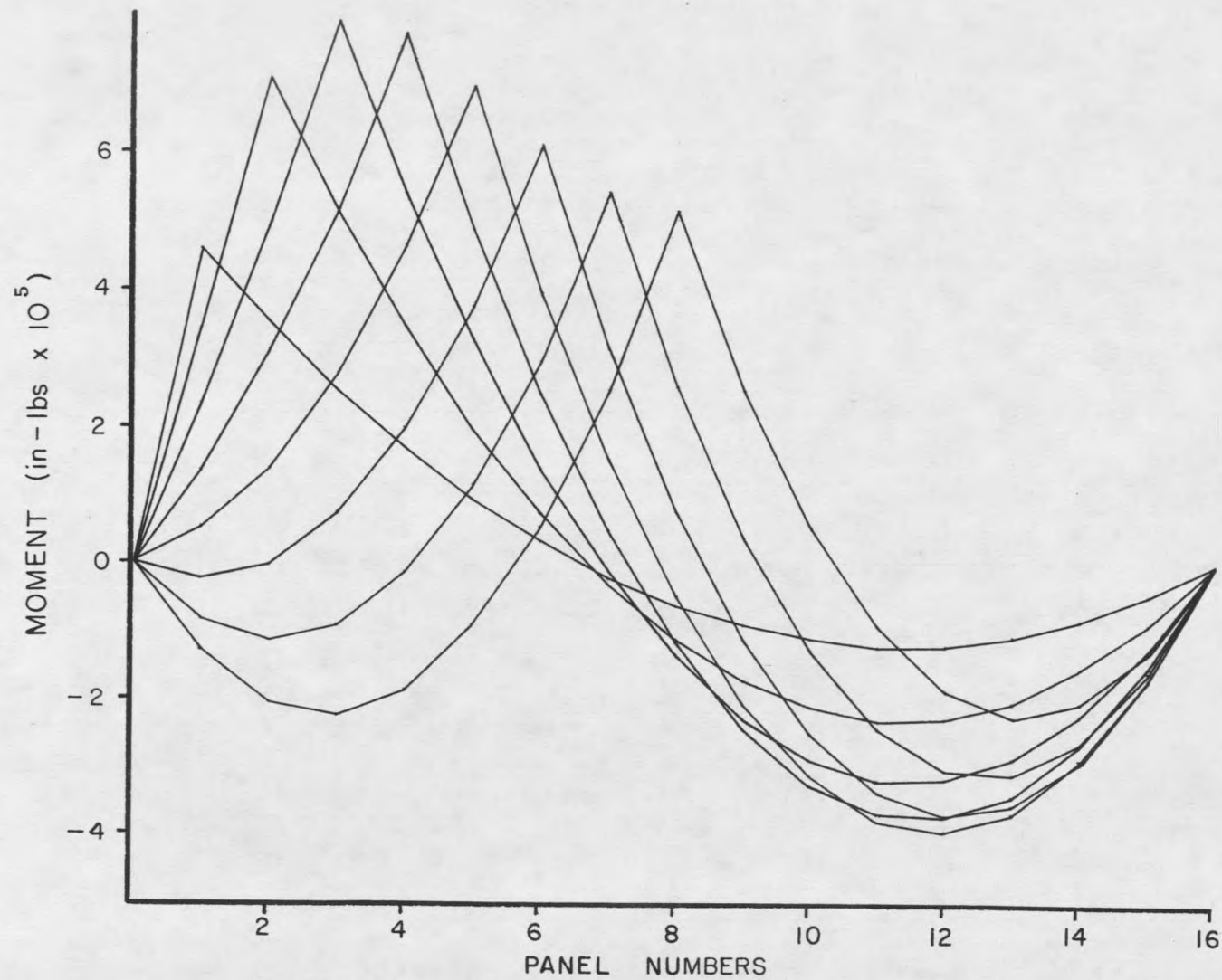


Figure 19. Graph of tie moment influence lines.
 $A_R/A_T = 0.6$; $I_R/I_T = 1/20$; 16 Panels

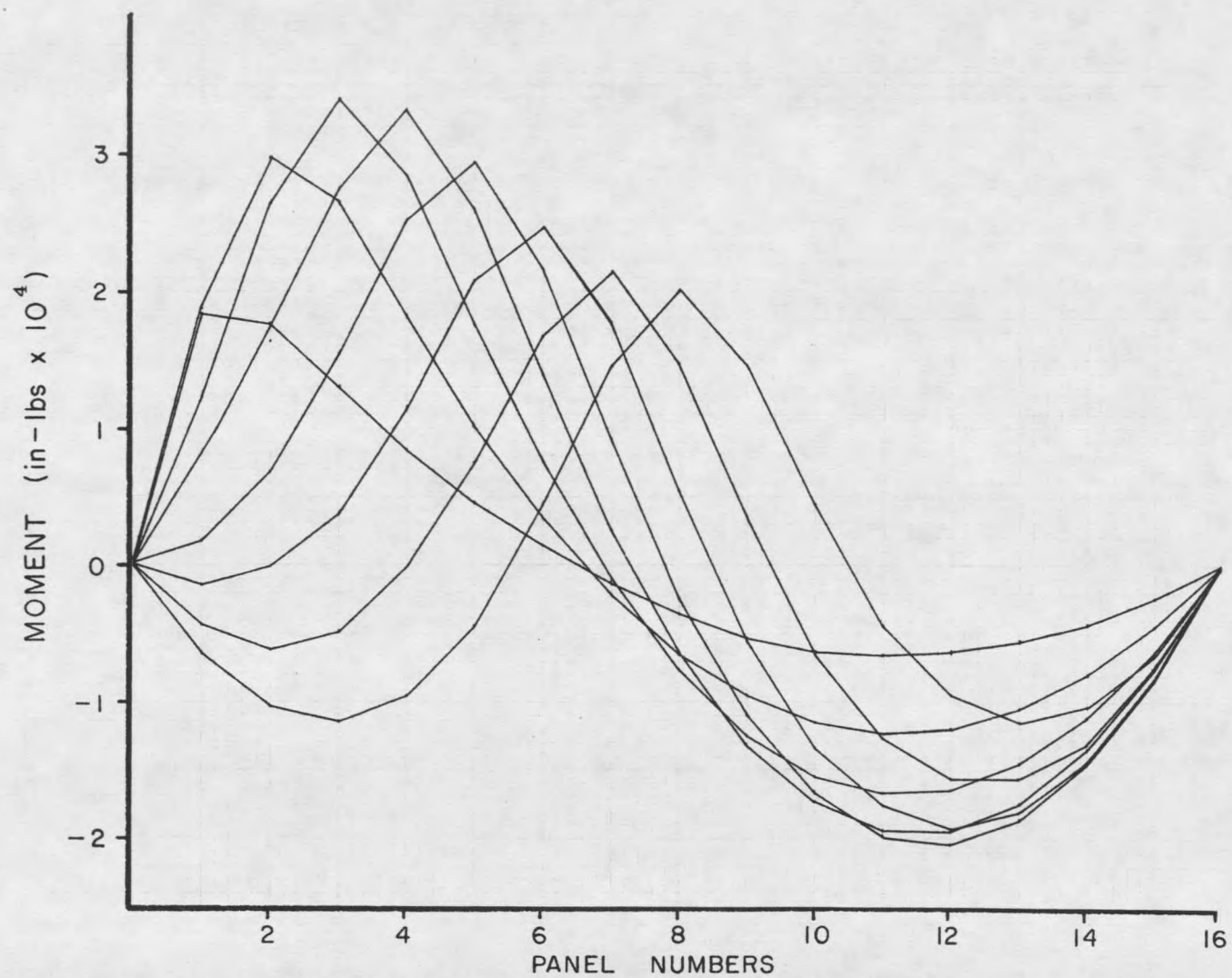


Figure 20. Graph of rib moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1.0$; 16 Panels

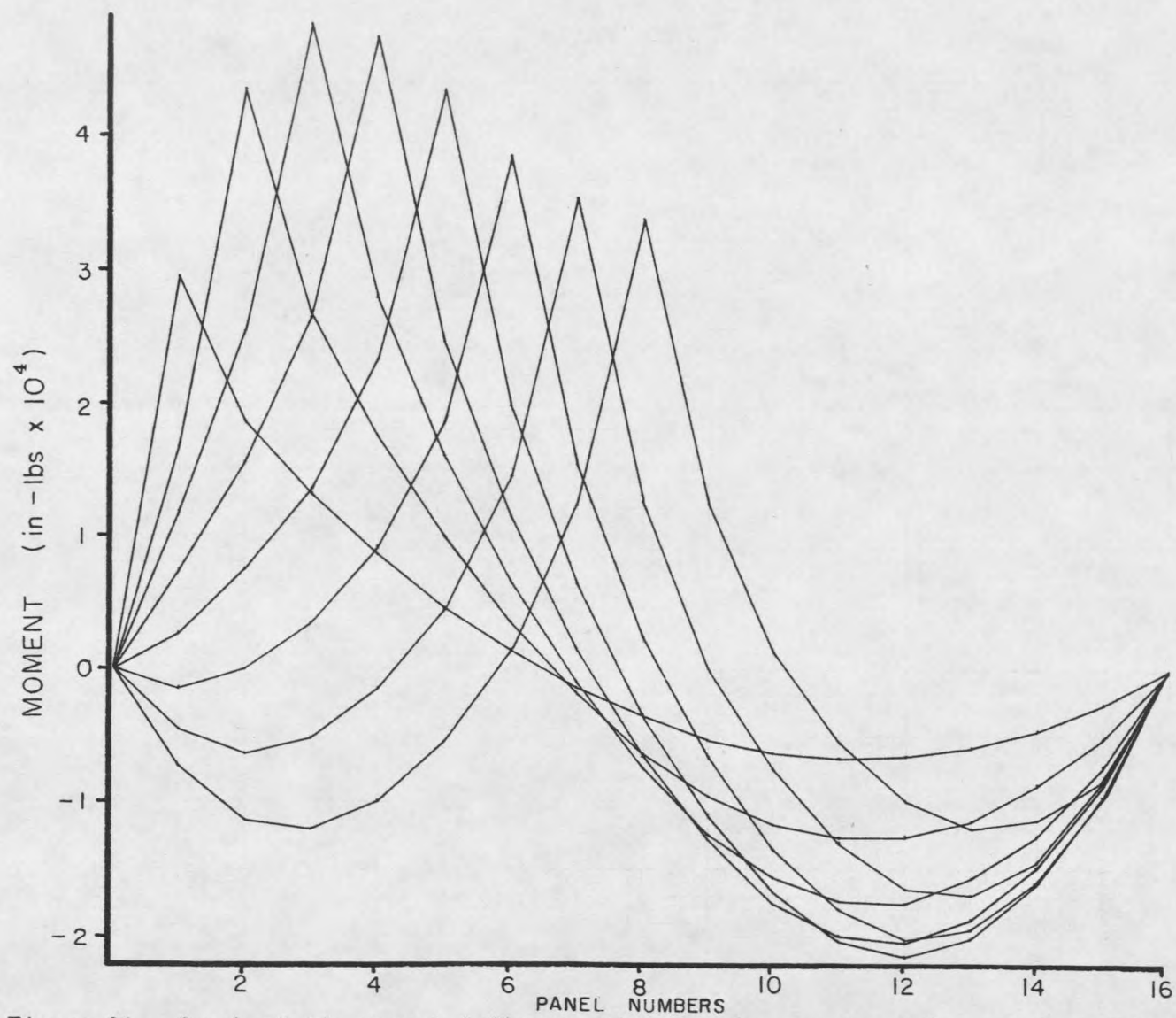


Figure 21. Graph of tie moment influence lines. $A_R/A_T = 1.0$; $I_R/I_T = 1.0$; 16 Panels

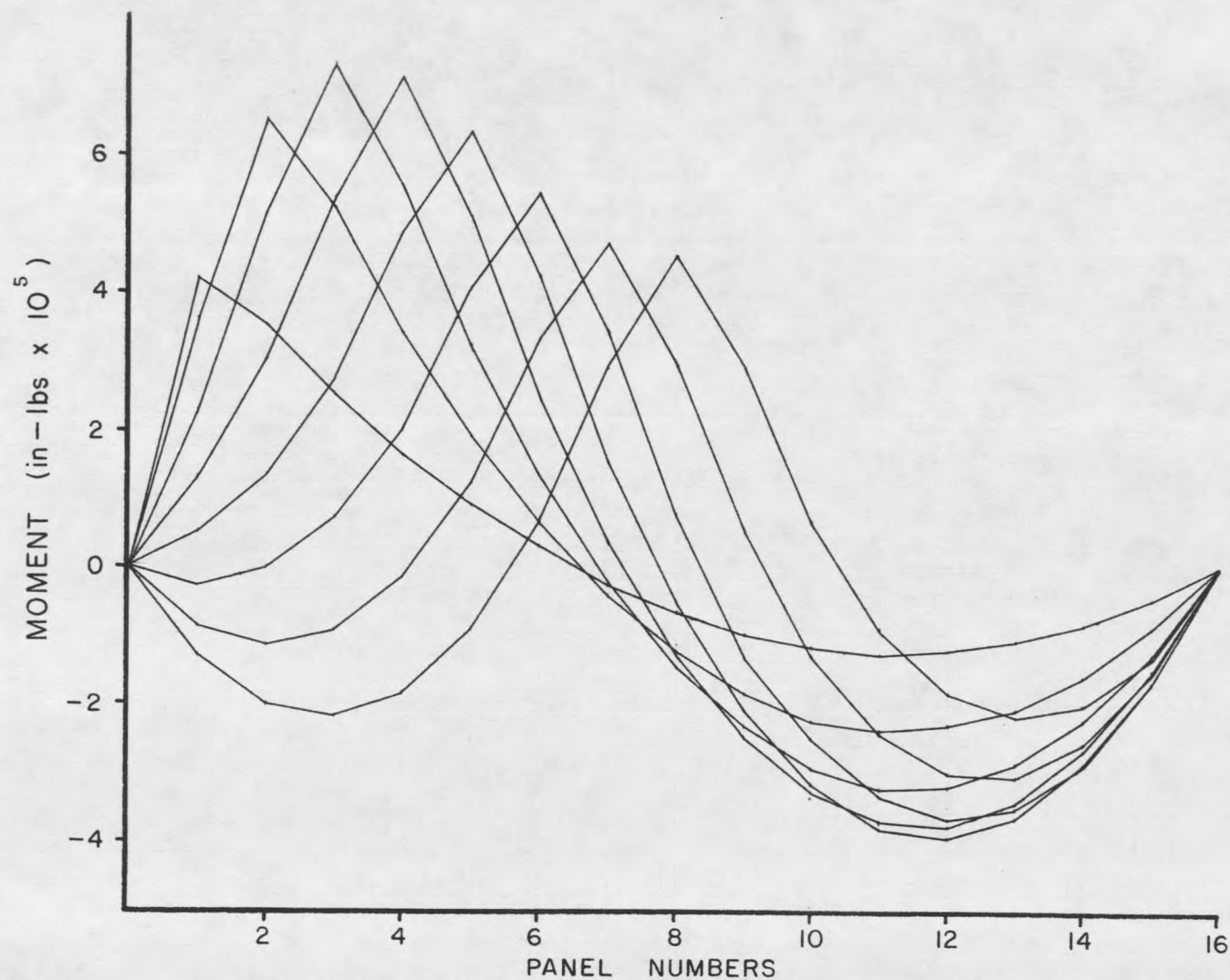


Figure 22. Graph of rib moment influence lines.
 $A_R/A_T = 1.5$; $I_R/I_T = 20$; 16 Panels

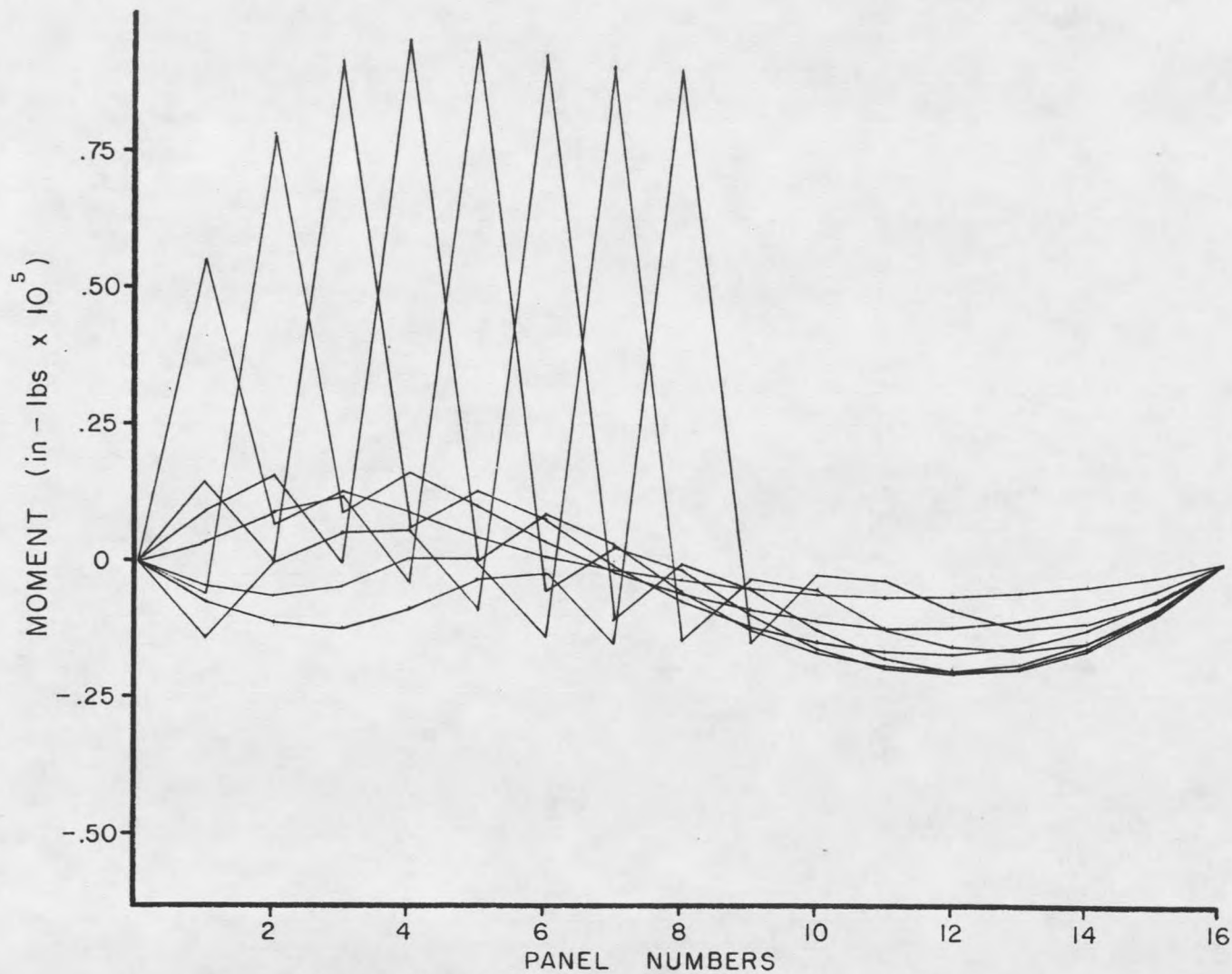


Figure 23. Graph of tie moment influence lines.
 $A_R/A_T = 1.5$; $I_R/I_T = 20$; 16 Panels

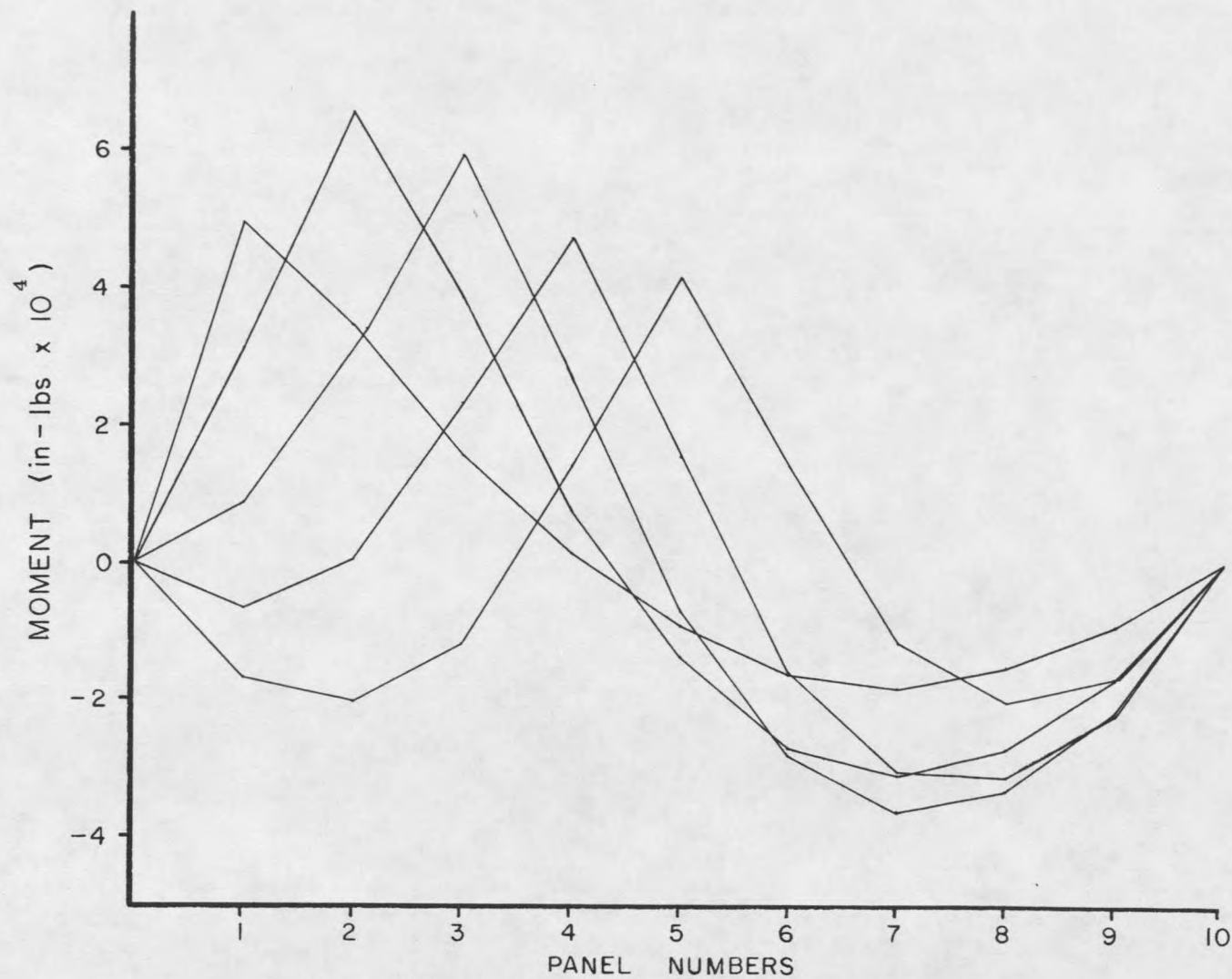


Figure 24. Graph of rib moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 10 Panels

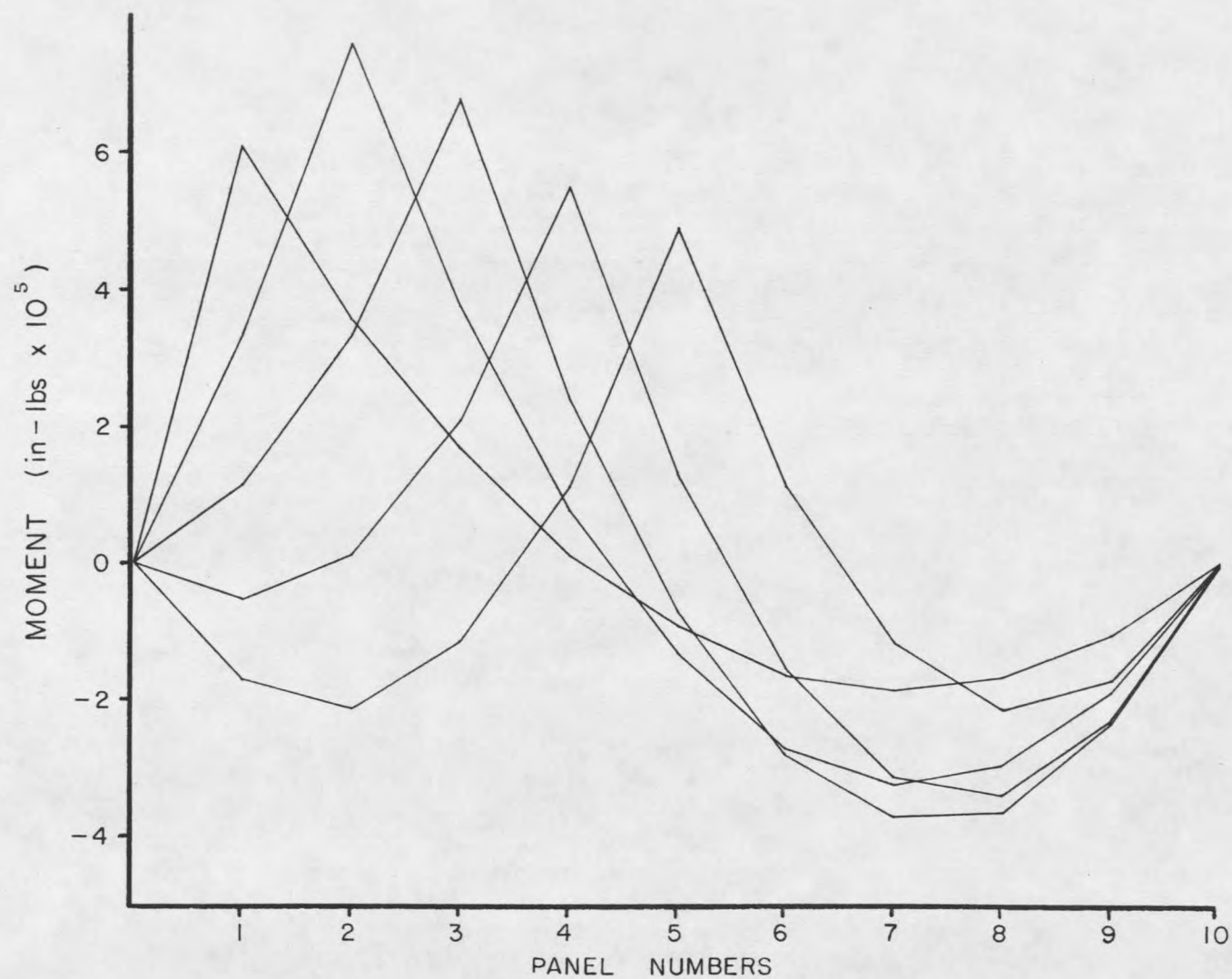


Figure 25. Graph of tie moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 10 Panels

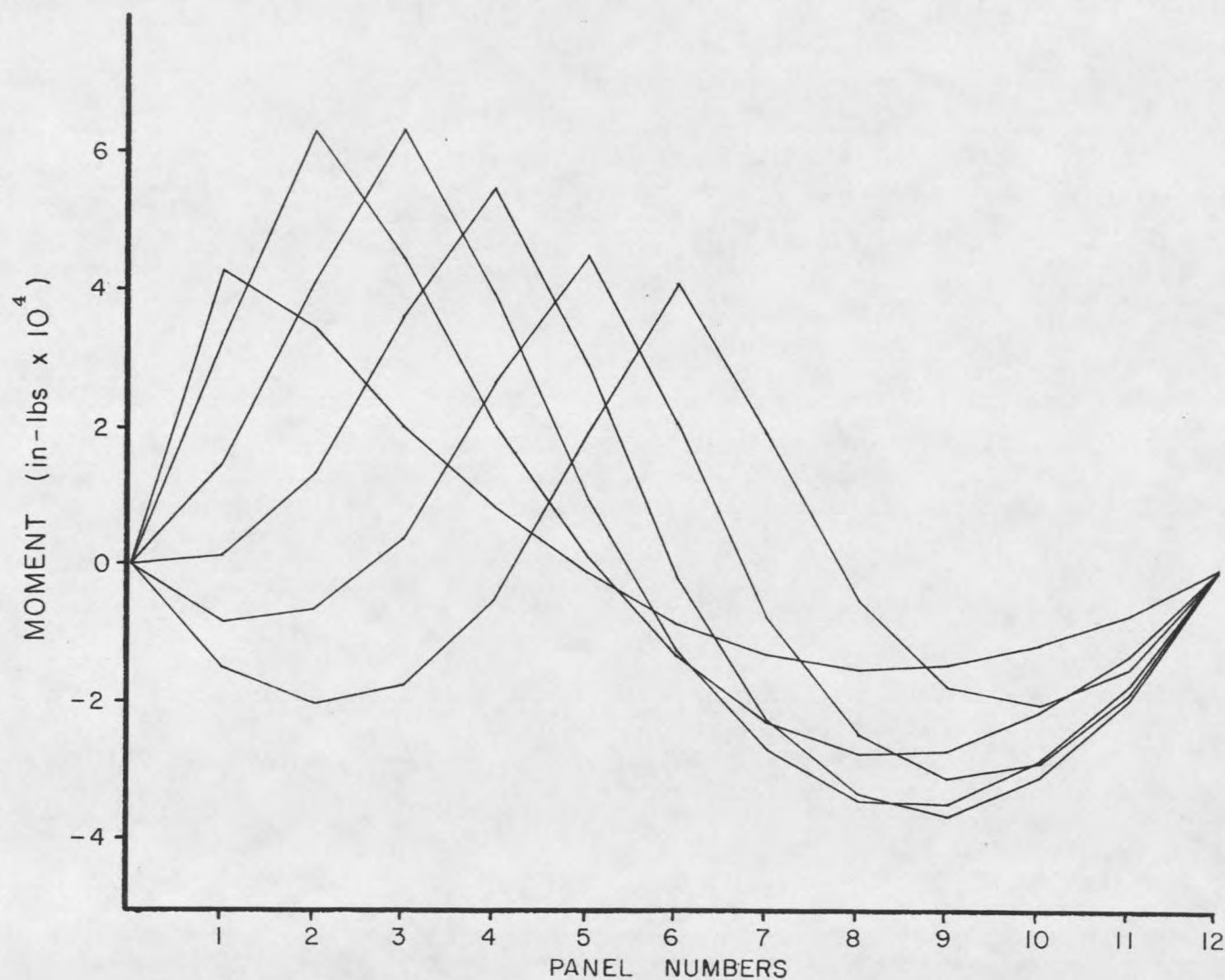


Figure 26. Graph of rib moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 12 Panels

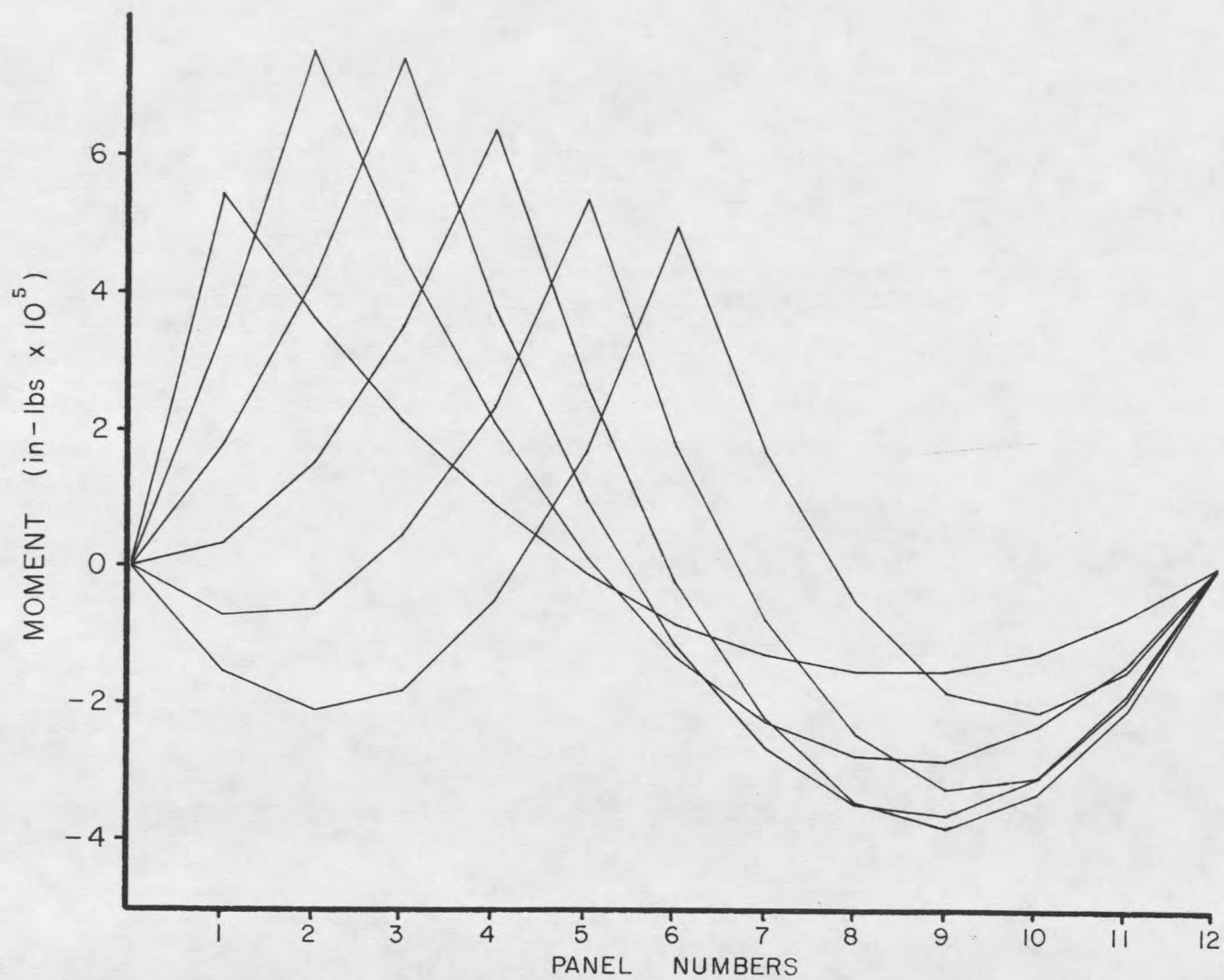


Figure 27. Graph of tie moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 12 Panels

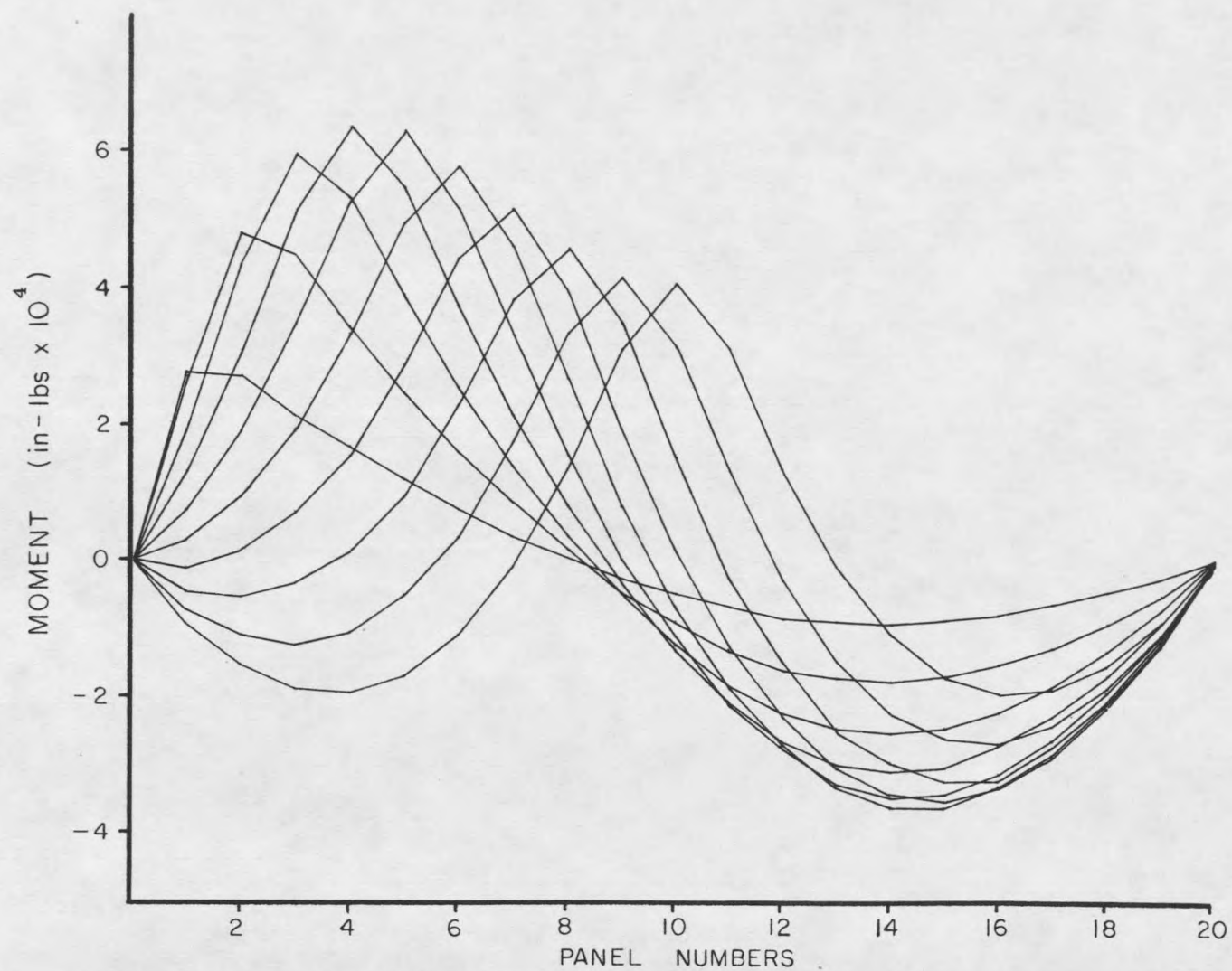


Figure 28. Graph of rib moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 20 Panels

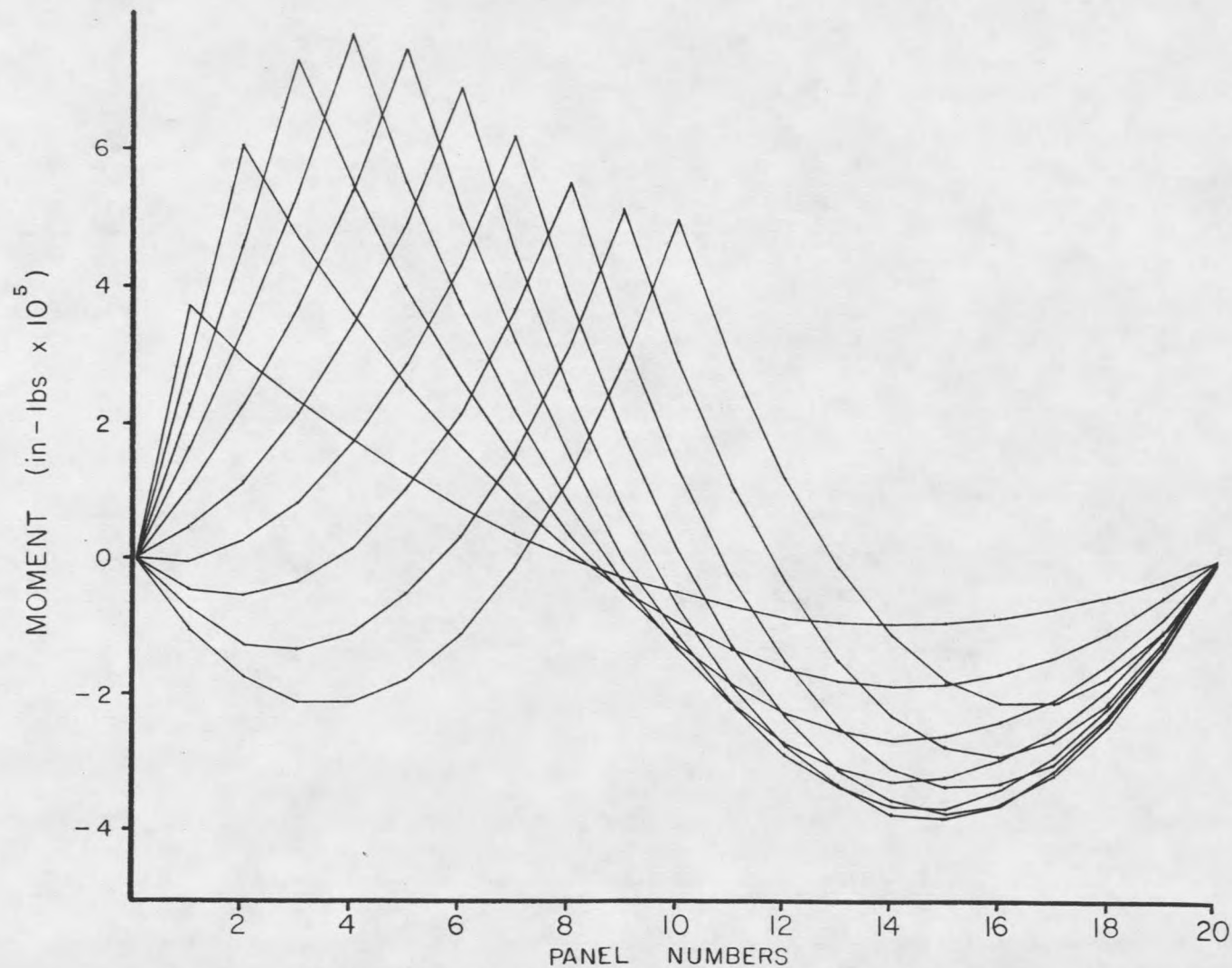


Figure 29. Graph of tie moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 20 Panels

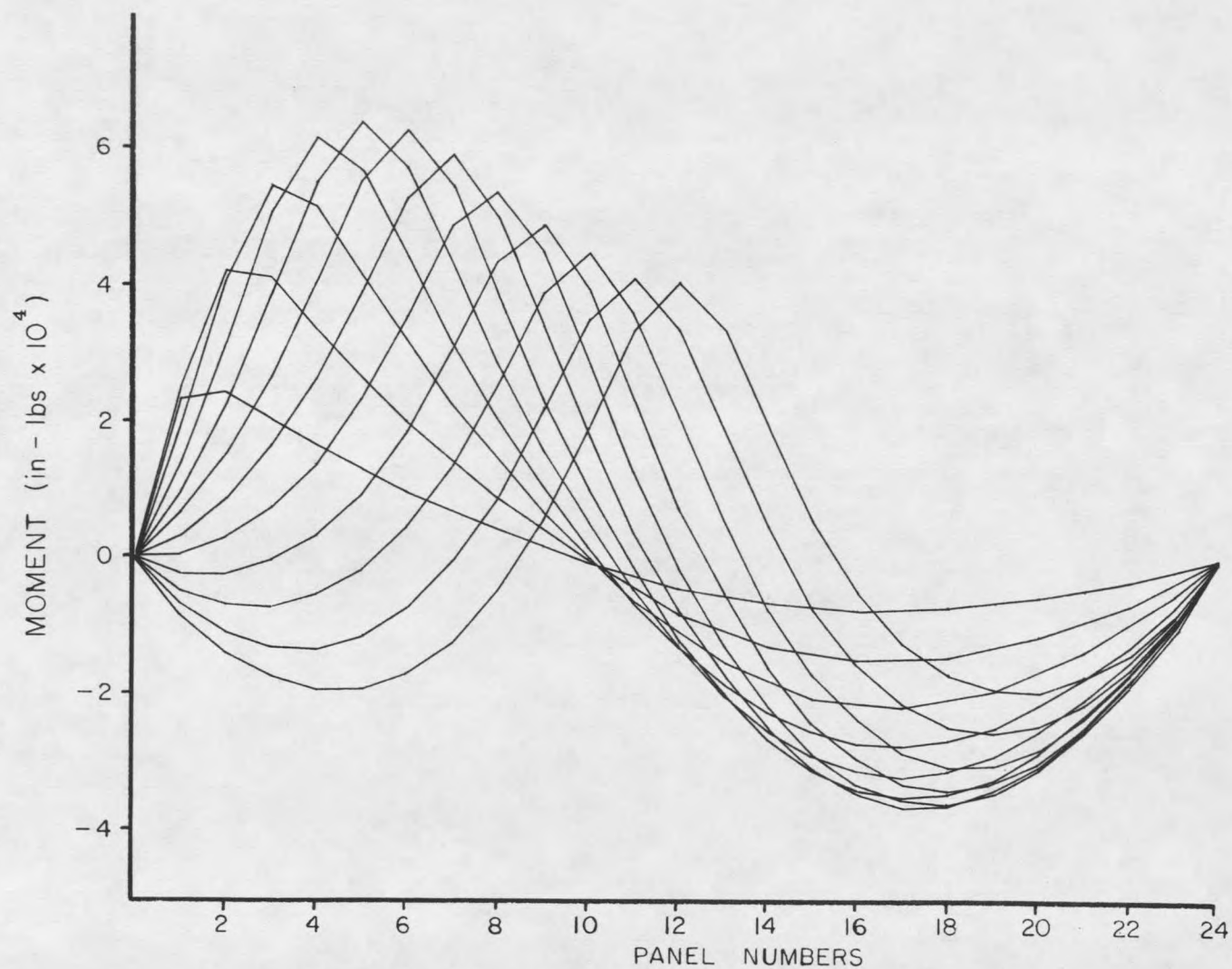


Figure 30. Graph of rib moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 24 Panels

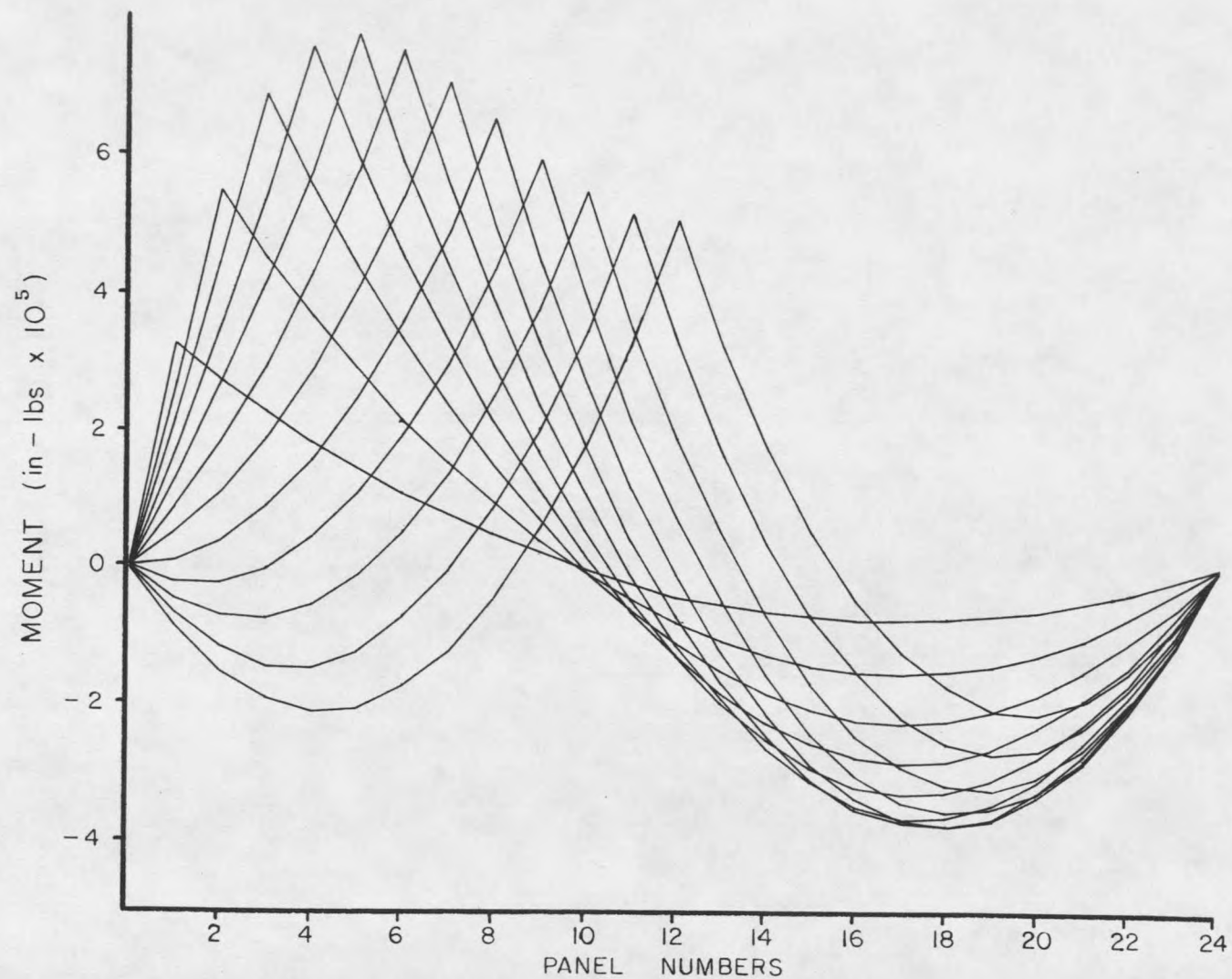


Figure 31. Graph of tie moment influence lines.
 $A_R/A_T = 1.0$; $I_R/I_T = 1/10$; 24 Panels

Hanger Forces

The live load forces carried by the hangers in a tied arch are highly dependent upon the I_R/I_T ratio and the hanger spacing. The percentage of load carried into the hangers due to application of loads directly under the hangers were computed for several of the cases. The results for the hangers carrying the largest percentage of load are listed in Table 4, and graphed in Figures 32 and 33.

Table 4. Hangers carrying maximum force due to live load.

Case No.	H/L	ψ (Pct.)	Hanger Location
1/20-16	1/5.0	12.50	Center
1/20-16	1/5.9	12.60	Center
1/10-10	1/5.9	21.60	Center
1/10-12	1/5.9	18.50	Center
1/10-16	1/5.9	14.40	Center
1/10-20	1/5.9	11.80	Center
1/10-24	1/5.9	10.10	Center
1-16	1/5.0	27.00	Center
1-16	1/5.9	28.00	Center
20-16	1/5.0	74.60	Second from end
20-16	1/5.9	77.00	Second from end

where:

ψ = Percentage of a unit load carried by
a hanger due to application of a unit
load at the hanger

Figure 32 shows that for a bridge with a stiff rib and slender tie, most of the live load is transmitted into the hangers. The hanger forces decrease towards the center of the bridge. This occurs because the relative stiffness of the rib to tie decreases towards the center of the bridge. The case of the smaller H/L ratio only increased the hanger forces by about 3-1/2%.

Similarly, Figure 32 shows that for a bridge with a stiff tie and slender rib most of the live load is carried by bending moment in the tie. The hanger forces increase towards the center of the bridge. This is due to being away from the ends of the bridge where the relative stiffness of the rib to tie increases. The hanger forces were virtually unaffected by changes in the H/L ratio for the 1/20-16 cases.

When the size effects of the large joint between the rib and the tie are taken into account, the tensions in the end hangers decrease. This is due to the increased stiffness of the arch due to the joint.

Figure 33 demonstrates the effect of hanger spacing upon hanger forces, for the $I_R/I_T = 1/10$ cases. Increasing the hanger spacing for a given I_R/I_T ratio increases the hanger forces.

From this analysis it becomes apparent that for bridges with deep ties and shallow ribs, the critical hanger carrying maximum live load force is the center hanger. The loading to produce the maximum force would be a uniform load across the bridge, with a concentrated load at the center, or the maximum floor beam reaction could be used. An initial estimate of the maximum hanger force can be obtained from the relationship:

$$T_{\max} = \frac{8ShM_L}{L^2(H-h)} + qS + \psi P + T_{DL} \quad (26)$$

where:

- S = Distance between hangers
- q = Uniform live load across the span
- P = Concentrated live load applied at center hanger
- h = Height of tie at centerspan
- L = Length of span
- ψ = Coefficient estimated from Table 4
- H = Height of rib at centerspan
- T_{DL} = Hanger tension due to dead load
(Found from Equation (7).)
- M_L = Internal arch live load moment
at the center of the bridge

The critical hanger carrying maximum live load force for bridges with deep ribs and shallow ties, was the second hanger from either end. For bridges with wide hanger spacings the end hanger could be the critical hanger. The stiffness and size of the joint connecting the rib and tie would affect this.

The loading necessary to provide the maximum hanger force for this case would be a uniform load across the span, and a concentrated load applied at the second hanger from the end. An estimate of the hanger force can be obtained from Equation (26).

Values of ψ can be roughly interpolated from Table 4 for various values of I_R/I_T , and different hanger spacings. For example, if $I_R/I_T = 1/8$ and $s = 12$, the value of ψ could be estimated at 0.21. For $I_R/I_T = 1/10$ and $s = 12$, ψ would be 0.19. For bridges with deep ties, a conservative value of ψ would be 0.3.

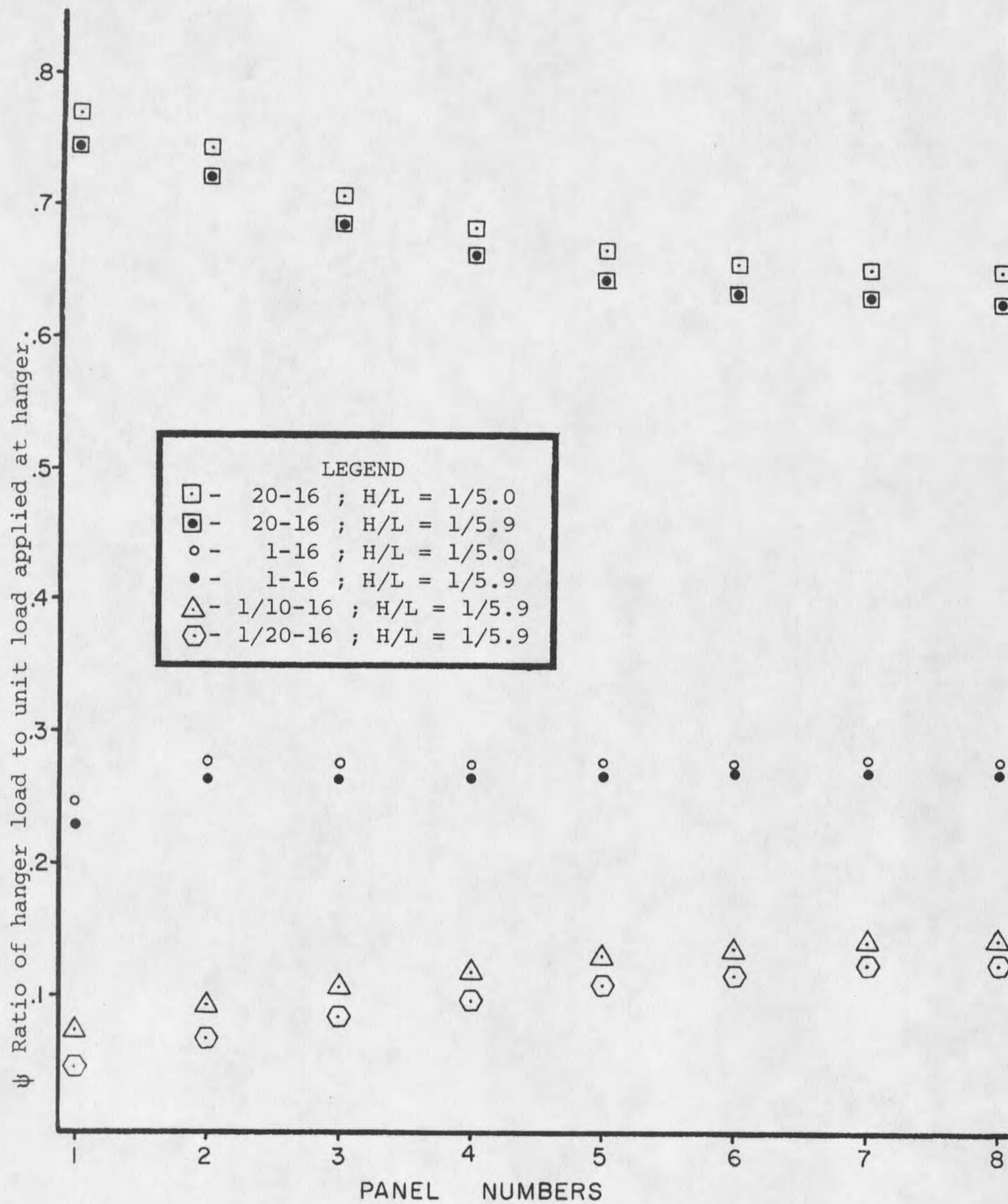


Figure 32. Graph showing effect of I_R/I_T ratio upon hanger forces.

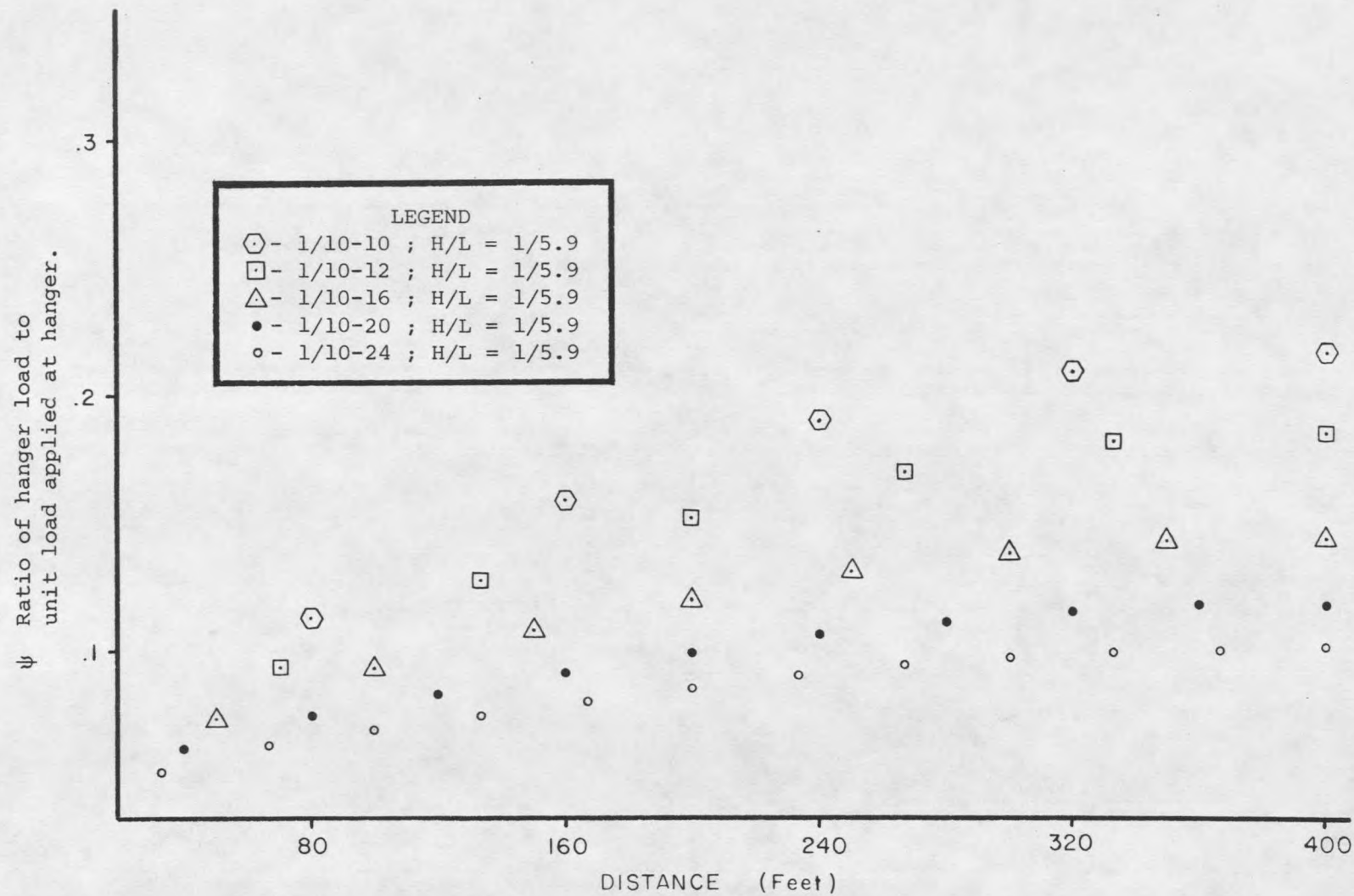
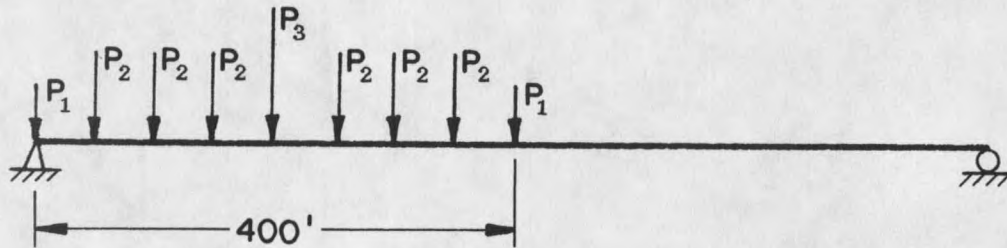


Figure 33. Graph showing effect of hanger spacing upon hanger forces.

Rib and tie deflection

For the $s = 16$ spacing case the following half span loading was used:



where:

$$\begin{aligned} P_1 &= 22.5 \text{ kips} \\ P_2 &= 45.0 \text{ kips} \\ P_3 &= 70.2 \text{ kips} \end{aligned}$$

It was found that the largest live load deflection occurred under half span loading, with a concentrated load near the 0.25 point. Graphs of the rib and tie deflections are shown in Figures 34 and 35. The graphs show similar deflections for the rib and tie for a wide range of the various parameters. This verifies the classical theory of tied arches in assuming hanger elongation negligible compared with the rib and tie deflection, [7, 30]. Therefore for a given span length, loading, and cross-sectional area, the live load deflections are rather insensitive to changes in the I_R/I_T , A_R/A_T , or H/L ratios. The point of inflection also remained nearly unchanged, near the center of the arch.

Two additional cases were tried for determining deflections. The 1/20-16 and 20-16 cases were analyzed with the total cross-sectional

area of the arch reduced by one-half, to 300 in². the results indicate that the deflections still did not change drastically. The deflection increased for the loaded segment, and decreased for the unloaded segment. In addition, the point of inflection shifted to the right.

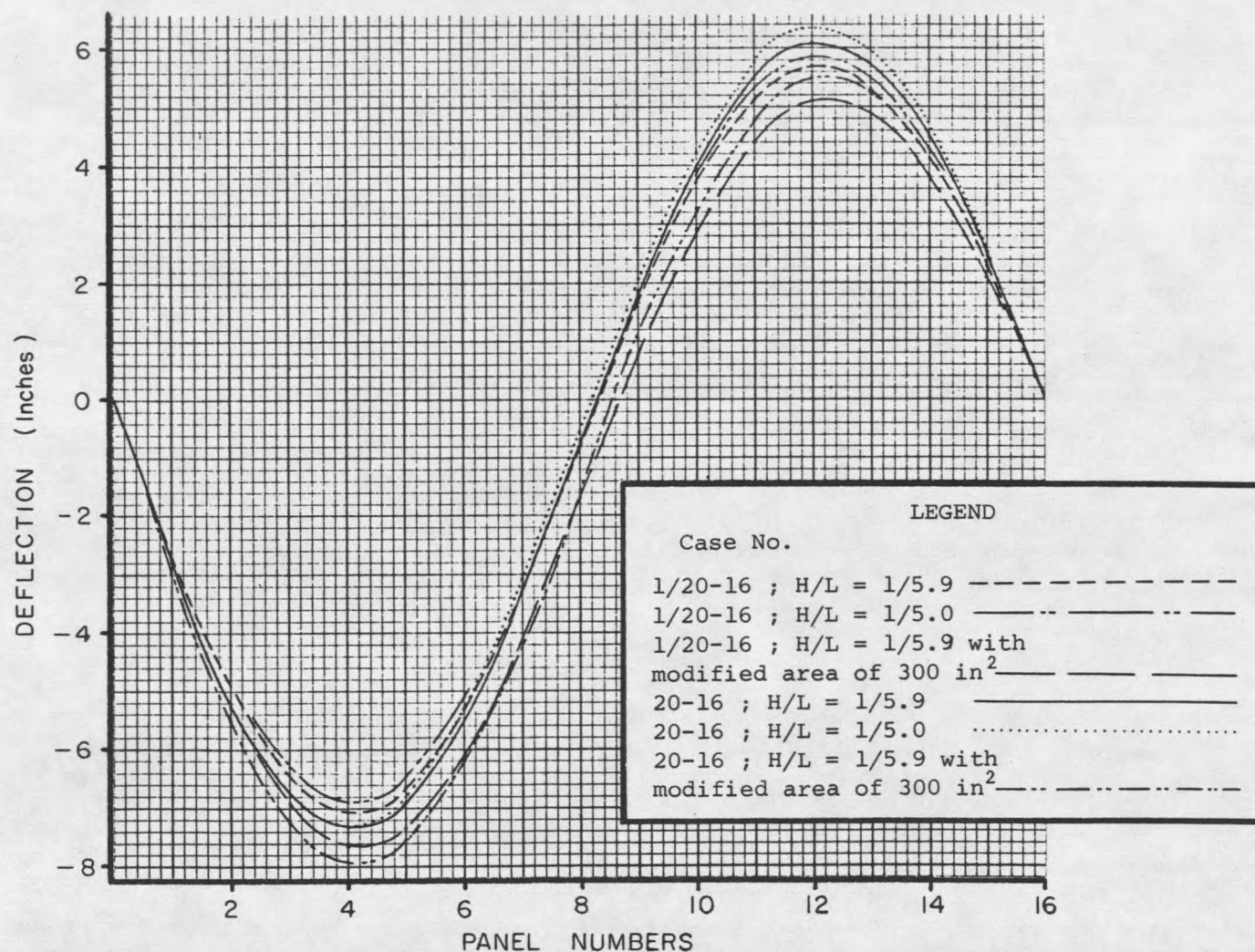


Figure 34. Graph of rib deflection for various cases.

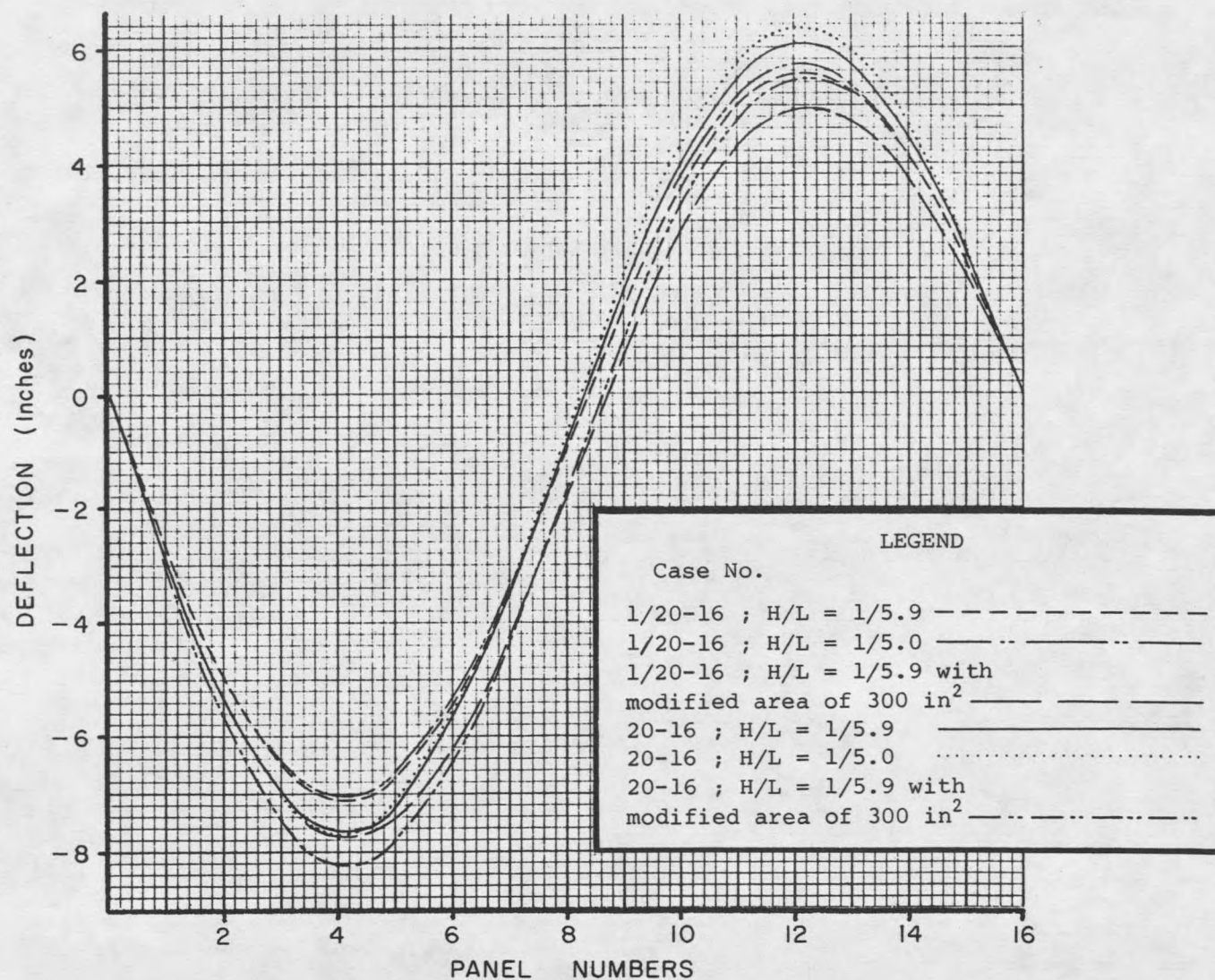


Figure 35. Graph of tie deflection for various cases.

Proportion of live load moment carried by the rib and by the tie

When designing the rib and tie it is necessary to know what proportion of the live load moment is being carried by the rib and by the tie. This will obviously be strongly influenced by the I_R/I_T ratio. In recent times the trend has been to use a stiff tie which carries the majority of the live load moment, and a slender rib. Garrelts [7] developed the following relationships for proportioning the live load moment:

$$M_T = M_L \left(\frac{I_T}{I_T + I_R \cos \alpha} \right) \quad (27)$$

and:

$$M_R = M_L \left(\frac{I_R \cos \alpha}{I_T + I_R \cos \alpha} \right) \quad (28)$$

where:

- M_T = Live load moment in tie
- M_R = Live load moment in rib
- M_L = Moment resisted by rib and tie ($M_T + M_R$)
- I_T = Moment of inertia of tie
- I_R = Moment of inertia of rib
- α = Angle of inclination of the rib with respect to a horizontal line

If the curvature of the tie cannot be neglected, I_T should be replaced by $I_T(\cos \phi)$ in Equations (27) and (28). ϕ would be the angle of inclination of the tie with respect to a horizontal line.

The biggest problem with using Equations (27) and (28) for preliminary analysis is determining M_L . Figure 36 indicates how live load moment is carried by a tied arch. Requiring equilibrium at a section of the arch, the following is obtained:

$$M = M_L + Fl \quad (29)$$

where:

M = Simple beam live load moment
 F = Horizontal force in rib and tie
 l = Distance between rib and tie at any section
 M_L = Moment resisted by rib and tie ($M_R + M_T$)

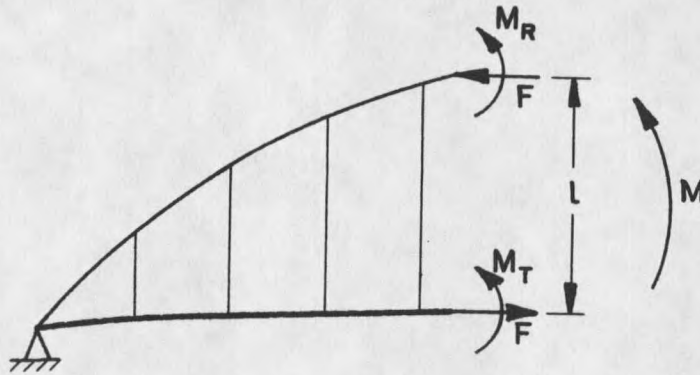


Figure 36. Live load moment in a tied arch.

From Figure 36 it is apparent that due to equilibrium the force F will remain constant across the span. Therefore the moment M_L in Equations (27) and (28) could be found from Equation (29) if the force F is known. Garrelts [7] uses a virtual work solution to solve for the force F :

$$F = \frac{\frac{\sum M(h - H) \Delta L}{I_{tr}}}{\frac{\sum (h - H)^2 \Delta L}{I_{tr}} + \frac{\sum \Delta L}{A_T} + \frac{\sum \sec^2 \alpha (\Delta L)}{A_T}} \quad (30)$$

where:

M = Simple beam live load moment
 H = Ordinates of rib profile
 h = Ordinates of tie profile
 $I_{tr} = I_T + I_R (\cos \alpha)$
 A_T = Area of tie

From computer analysis the locations of zero rib and tie moment for several cases were determined. The results are tabulated in Table 5. Since M_L is nearly zero at these locations the force F can be found from Equation (29):

$$F = \frac{M}{l} \quad (31)$$

where:

F = Horizontal force in rib and tie
 M = Simple beam live load moment at location determined from Table 5
 l = Distance between rib and tie

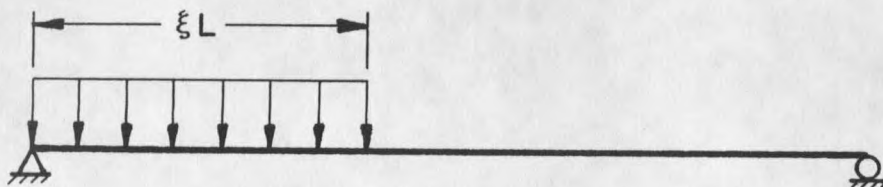
Table 5. Computer results for locations of zero rib and tie moment.

Case No.	ξ	Φ Rib	Y Tie	Case No.	ξ	Φ Rib	Y Tie	Case No.	ξ	Φ Rib	Y Tie
20 - 12	.58	.553	.593	20 - 16	.56	.542	.572	20 - 20	.55	.534	.559
	.50	.522	.532		.50	.517	.525		.50	.515	.521
	.42	.486	.473		.44	.491	.479		.45	.494	.483
	.33	.458	.412		.38	.468	.432		.40	.475	.445
1 - 12	.58	.550	.562	1 - 16	.56	.539	.548	1 - 20	.55	.532	.540
	.50	.521	.526		.50	.516	.520		.50	.514	.517
	.42	.488	.483		.44	.492	.488		.45	.495	.491
	.33	.461	.454		.38	.471	.464		.40	.477	.471
1/10 - 12	.58	.552	.557	1/10 - 16	.56	.541	.544	1/10 - 20	.55	.534	.537
	.50	.522	.524		.50	.517	.519		.50	.514	.516
	.42	.487	.486		.44	.492	.491		.45	.494	.493
	.33	.460	.459		.38	.470	.469		.40	.476	.475
1/20 - 12	.58	.553	.557	1/20 - 16	.56	.542	.545	1/20 - 20	.55	.534	.537
	.50	.522	.525		.50	.518	.520		.50	.515	.516
	.42	.487	.487		.44	.492	.491		.45	.494	.494
	.33	.459	.459		.38	.469	.469		.40	.476	.475

where:

Φ = Location of zero rib live load moment,
as a fraction of the span length L
 Y = Location of zero tie live load moment,
as a fraction of the span length L

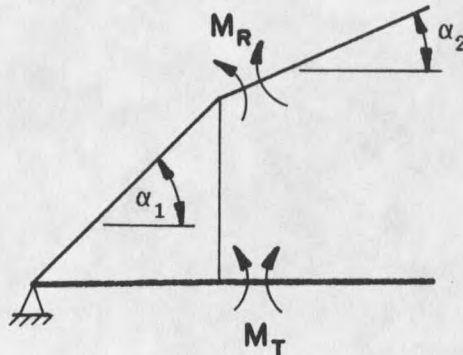
ξ is a fraction of the span length L as shown:



After approximating the force F , the moment M_L is determined from Equation (29). For preliminary analysis it is sufficient to calculate M_L near the critical locations of largest moment. The approximate moment carried by the rib and the tie is then found by substitution into Equations (27) and (28). The critical locations for positive and negative live load moment were given in Table 3.

Tables 6 and 7 compare the computer results with the results of Equations (27) and (28) for proportioning the live load moment between the rib and the tie, for various cases. The data indicates that Equations (27) and (28) give good results for preliminary analysis. The values are usually accurate to within 10%. The solution is not very good at the end panels, at centerspan, or near any concentrated loads however. This is due to localized bending effects.

For segmental ribs it was found that better results are obtained from Equations (27) and (28) by using $\cos\alpha$ from the previous segment, rather than from the segment being calculated. For example, to calculate the values at hanger location 1:



$$\frac{M_R}{M_R + M_T} = \frac{I_R \cos\alpha_1}{I_T + I_R \cos\alpha_1} \quad \text{and:} \quad \frac{M_T}{M_R + M_T} = \frac{I_T}{I_T + I_R \cos\alpha_1}$$

Table 6. Proportion of live load moment carried by the rib and by the tie, for $H/L = 1/5.9$ cases. Comparison of computer results with results using Equations (27) and (28).

Hanger No.	Case No. 1/20-16				Case No. 1/10-16			
	Computer		Equation		Computer		Equation	
	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$
1	.0365	.9635	.0404	.9596	.0727	.9273	.0777	.9223
2	.0423	.9577	.0419	.9581	.0808	.9192	.0805	.9195
3	.0439	.9561	.0433	.9567	.0841	.9159	.0831	.9169
4	.0444	.9556	.0446	.9554	.0841	.9159	.0854	.9146
5	.0462	.9538	.0457	.9543	.0881	.9119	.0874	.9126
6	.0465	.9535	.0466	.9534	.0885	.9115	.0891	.9109
7	.0458	.9542	.0473	.9527	.0862	.9138	.0902	.9098
8	.0176	.9823	.0476	.9524	.0174	.9826	.0908	.9092
9	.0468	.9532	.0476	.9524	.0890	.9110	.0908	.9092
10	.0478	.9522	.0473	.9527	.0912	.9088	.0902	.9098
11	.0468	.9532	.0466	.9534	.0896	.9104	.0891	.9109
12	.0455	.9545	.0457	.9543	.0869	.9131	.0874	.9126
13	.0442	.9558	.0446	.9554	.0847	.9153	.0854	.9146
14	.0425	.9575	.0433	.9567	.0822	.9178	.0831	.9169
15	.0453	.9547	.0419	.9581	.0845	.9155	.0805	.9195

Hanger No.	Case No. 1.0-16				Case No. 20-16			
	Computer		Equation		Computer		Equation	
	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$
1	.4519	.5481	.4574	.5426	.9288	.0712	.9440	.0560
2	.4681	.5319	.4667	.5333	.9407	.0593	.9459	.0541
3	.4780	.5220	.4753	.5247	.9492	.0508	.9477	.0523
4	.4705	.5295	.4829	.5171	.9318	.0682	.9492	.0508
5	.4890	.5110	.4893	.5107	.9492	.0508	.9504	.0496
6	.4848	.5152	.4944	.5056	.9359	.0641	.9514	.0486
7	.4623	.5377	.4980	.5020	.9075	.0925	.9520	.0480
8	.0483	.9517	.4997	.5003	3.397	-2.40	.9523	.0477
9	.4780	.5220	.4997	.5003	.9480	.0520	.9523	.0477
10	.4988	.5012	.4980	.5020	.9706	.0294	.9520	.0480
11	.4981	.5019	.4944	.5056	.9626	.0374	.9514	.0486
12	.4896	.5104	.4893	.5107	.9565	.0435	.9504	.0496
13	.4824	.5176	.4829	.5171	.9537	.0463	.9492	.0508
14	.4724	.5276	.4753	.5247	.9437	.0563	.9477	.0523
15	.4692	.5308	.4667	.5333	.9503	.0497	.9459	.0541

Table 7. Proportion of live load moment carried by the rib and by the tie, for $H/L = 1/5.0$ cases. Comparison of computer results with results using Equations (27) and (28).

Hanger No.	Case No. 1/20-16				Case No. 1/10-16			
	Computer		Equation		Computer		Equation	
	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$
1	.0344	.9656	.0385	.9615	.0688	.9312	.0741	.9259
2	.0404	.9596	.0402	.9598	.0775	.9225	.0773	.9227
3	.0425	.9575	.0420	.9580	.0822	.9178	.0805	.9195
4	.0433	.9567	.0436	.9564	.0832	.9168	.0836	.9164
5	.0458	.9542	.0451	.9549	.0872	.9128	.0863	.9137
6	.0463	.9537	.0463	.9537	.0881	.9119	.0884	.9116
7	.0456	.9544	.0471	.9529	.0860	.9140	.0900	.9100
8	-.047	1.047	.0476	.9524	-.551	1.551	.0908	.9092
9	.0467	.9533	.0476	.9524	.0884	.9116	.0908	.9092
10	.0474	.9526	.0471	.9529	.0905	.9095	.0900	.9100
11	.0462	.9538	.0463	.9537	.0885	.9115	.0884	.9116
12	.0450	.9550	.0451	.9549	.0853	.9147	.0863	.9137
13	.0432	.9568	.0436	.9564	.0823	.9177	.0836	.9164
14	.0414	.9586	.0420	.9580	.0799	.9201	.0805	.9195
15	.0437	.9563	.0402	.9598	.0815	.9185	.0773	.9227

Hanger No.	Case No. 1.0-16				Case No. 20-16			
	Computer		Equation		Computer		Equation	
	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$	$\frac{M_R}{M_L}$	$\frac{M_T}{M_L}$
1	.4385	.5615	.4444	.5556	.9392	.0608	.9412	.0588
2	.4560	.5440	.4559	.5441	.9456	.0544	.9437	.0563
3	.4718	.5282	.4670	.5330	.9487	.0513	.9460	.0540
4	.4656	.5344	.4770	.5230	.9385	.0615	.9480	.0520
5	.4905	.5095	.4856	.5144	.9569	.0431	.9497	.0503
6	.4878	.5122	.4924	.5076	.9467	.0533	.9510	.0490
7	.4686	.5314	.4972	.5028	.9235	.0765	.9519	.0481
8	.9644	.0355	.4997	.5003	1.002	-.002	.9523	.0477
9	.4654	.5346	.4997	.5003	.9269	.0731	.9523	.0477
10	.4897	.5103	.4972	.5028	.9575	.0425	.9519	.0481
11	.4890	.5110	.4924	.5076	.9541	.0459	.9510	.0490
12	.4836	.5164	.4856	.5144	.9483	.0517	.9497	.0503
13	.4743	.5257	.4770	.5230	.9450	.0550	.9480	.0520
14	.4667	.5333	.4670	.5330	.9471	.0529	.9460	.0540
15	.4592	.5408	.4559	.5441	.9355	.0645	.9437	.0563

CHAPTER V

PRELIMINARY ANALYSIS

There are many methods of approaching a preliminary design of a statically indeterminate structure, such as a tied arch. One of the best methods is to examine other similar designs. An experienced designer can probably make an initial layout of a tied arch with nearly correct proportions and member sizes.

Preliminary tied arch analysis is simplified over conventional arch analysis in that the live load structural deformation does not cause appreciable moment magnification. From the results of this study it was found that two simplifying assumptions can be made, to aid in preliminary analysis:

- 1.) Hanger deformation is small compared with rib and tie deformations.
- 2.) Live load moment in the rib and tie near the center of the bridge is carried primarily by the couple caused by the forces in the rib and tie.

Using assumption 1 the approximation can be made that the moment arms of the force components of the rib and tie do not change appreciably. Assumption 2 enables the axial forces in the rib and tie to be easily approximated. The simple beam live load moment divided by the distance between the rib and tie near centerspan yields the force in the tie. As a general guideline for tied arch analysis the following steps can be considered:

A.) Initial geometric assumptions

Initial geometric assumptions must be made regarding geometric parameters such as:

- 1.) Ratio of rib to tie stiffness.
- 2.) Rise to span ratio.
- 3.) Number of panels.
- 4.) Tie and rib depths.
- 5.) Camber in the tie.
- 6.) Segmental or curved rib.

Table 1, or Richardson [37], can be used as aids in making these geometric assumptions.

B.) Idealized geometry

The coordinates of the joints of the rib and tie should be calculated. Good results are obtained by assuming parabolic rib and tie profiles initially. A line drawing of the arch at this point will tell much about the aesthetics of the design.

C.) Design of the floor system

The slab and stringer system (if used) is designed as in a conventional girder bridge. The assumption regarding the number of panels will affect this. The stringers can be designed as continuous, but joints should be placed periodically to prevent interaction of the stringers and deck slab with the ties. It is possible to design for this interaction but this will not be considered in this thesis.

Lateral floor bracing must be designed to resist lateral wind shear forces. It is customary to design the bracing for the end panels and use the same member sizes for the other panels.

For long spans however it may be advantageous to reduce dead load by changing member sizes.

The floor beams are designed for dead and live loads. Usually the floorbeams are hybrid plate girders or trusses. The floorbeams should be cambered to eliminate dead load deflection. When designing the tie, the rotations at the connections to the floorbeams will have to be compensated for.

D.) Upper lateral bracing

Upper lateral bracing must be designed to resist lateral wind shear forces and to give stability to the arch. The usual bracing types are X, K, or vierendeel.

E.) Rib and tie analysis

One method for preliminary rib and tie analysis is to first design the rib sections for maximum axial force. The tie is then designed for the case of reduced axial force plus bending moment.

To calculate the maximum axial forces, the forces caused by the dead load of the floor system and upper lateral bracing can be easily approximated. The force caused by the weight of the rib and tie is unknown however.

One method proposed by Richardson [37] for estimating the weight of the rib and tie, for tied arches of low alloy high strength steel, is to use the empirical relationships:

$$R = 0.088 + 0.000321(L) \quad (32)$$

and:

$$W = \frac{R(D + q)}{1 - R} \quad (33)$$

where:

- R = Ratio of weight of rib and tie to total load
- L = Length of span (feet)
- W = Weight of rib plus tie (lbs/ft)
- D = Dead load on arch (lbs/ft), excluding weight of rib plus tie
- q = Equivalent live load plus impact (lbs/ft), on arch when the arch is fully loaded

Another method proposed in this thesis is to express the force caused by the weight of the tie in terms of allowable stresses. The force caused by the weight of the rib is expressed as a function of the rib area at centerspan. The total horizontal force in the rib and tie can be expressed by the relationship:

$$F = F_{DL} + F_{LL} + F_{WT} + F_{WR} \quad (34)$$

where:

- F_{DL} = Force due to dead load of floor system and upper lateral bracing
- F_{LL} = Force due to live load
- F_{WT} = Force due to weight of tie
- F_{WR} = Force due to weight of rib

If an average cross-sectional area of tie is considered, the force F_{WT} can be expressed as:

$$F_{WT} = \frac{\rho A_T L^2}{8(H - h)} \quad (35)$$

where:

- ρ = Density of material (for steel, $\rho = 0.283$ ksi)
- L = Length of span
- H = Height of rib at centerspan
- h = Height of tie at centerspan
- A_T = Average area of tie

If the tie was a pure tension member under all loading conditions the area A_T in Equation (35) could be expressed as:

$$A_T = \frac{F}{\sigma} \quad (36)$$

where:

σ = Allowable tensile stress of tie material

Equation (36) underestimates the required area of tie though because of the required increased section to resist bending moment. Equation (36) could be used if a reduced allowable stress in the tie, $C_1\sigma$, is assumed. The value of $C_1\sigma$ must be estimated. In the design example which follows, C_1 was assumed at 0.5 and 0.6. Substitution into Equation (35) yields:

$$F_{WT} = \frac{\rho FL^2}{8C_1\sigma(H-h)} \quad (37)$$

Analytically determining an expression for the force due to the weight of the rib is very difficult. The problem is further complicated by increasing the cross-sectional area near the ends of the bridge. An approximate method for estimating F_{WR} was developed using computer analysis. A solution for F_{WR} as a function of the area of the rib at centerspan is assumed:

$$F_{WR} = \frac{\zeta \rho A_R L^2}{H - h} \quad (38)$$

where:

- ζ = Coefficient determined from computer analysis
- ρ = Density of the material (for steel $\rho = 0.283$ ksi)
- A_R = Area of rib at crown
- L^R = Length of span
- H = Height of rib at centerspan
- h = Height of tie at centerspan

The coefficients ζ were determined for two cases:

- 1.) Area of rib near springing line equals 1.16 times the area at the crown.
- 2.) Area of rib near springing line equals 1.20 times the area at the crown.

The area was assumed to change at five approximately equal distances across the span length, using the Mobile Arch Bridge geometry. The results for the two cases are given in Table 8, for various H/L ratios.

Table 8. Values of coefficient ζ .

H/L	(Case 1)	(Case 2)
5.00	0.13916	0.14135
5.25	0.13858	0.14065
5.50	0.13782	0.14004
5.88	0.13716	0.13923
6.00	0.13704	0.13907
6.30	0.13654	0.13869

Equation (38) was tried for several cases and gave accurate results.

Substitution of Equations (37) and (38) into Equation (34) gives the result:

$$F \left(1 - \frac{\rho L^2}{8C_1 \sigma (H - h)} \right) = F_{DL} + F_{LL} + \zeta \left(\frac{\rho A_R L^2}{H - h} \right) \quad (39)$$

Equation (39) expresses F in terms of A_R . The rib sections are estimated based upon the maximum axial force loading condition, assuming no moment. For this loading case live load extends across the span and computer analysis verifies that the moments are small. Another relationship between F and A_R is obtained by using the AASHTO equation for allowable stress in a concentrically loaded column. To do this an initial estimate of the web size is necessary.

The tie sections are estimated based upon combined tension and bending. Live load bending moments in the tie are approximated from Equations (27) and (29). The axial force for maximum moment loading cases will be considerably less than that of the maximum axial force case.

F.) Hanger analysis

The hangers are selected based upon the maximum force predicted from Equation (26). Although hanger areas could be varied across the span usually the same size hangers are used.

G.) Computer analysis for live load

After determining preliminary member sizes the maximum live load moment cases are analyzed by computer. It is best to check the live load analysis first because the accuracy of the dead load analysis will be greater. If any members are seriously oversized or undersized they can be modified accordingly and the computer analysis repeated.

H.) Minimization of dead load moment

If the member sizes are satisfactory from the live load analysis the dead load moment is minimized according to the procedure discussed in Chapter II. The rib geometry must be adjusted. Also the arch members must be shortened or lengthened appropriately.

I.) Final dead load analysis

The final dead load computer analysis incorporates the modified geometry and adjusted member lengths. It is important to remember that the weights of the members are treated as equivalent member end actions for computer purposes. When determining member stresses due to dead load the distributed loadings must be considered.

J.) Final stress analysis and deflection check

With all member sizes known the arch should be analyzed as a three-dimensional space frame. Different loading combinations must be considered, including wind and earthquake forces. AASHTO permits increased allowable stresses for various loading combinations. Using the space frame analysis the adequacy of the lateral bracing can be checked.

The overall stability of the arch must be considered. FHWA [46] discusses in-plane buckling. The arch rib is treated as a column, with a KL factor equal to approximately one half of the length of the arch axis. Lateral buckling effects are not as pronounced in laterally braced arches. The possibility does exist however that both arches could buckle laterally, and may

have to be investigated. Godden [8] did a study on the lateral buckling of tied arches. His results show that the hangers exert a considerable stabilizing effect on the arch.

Finally a deflection check should be made on the arch. AASHTO limits the deflection to $1/800$ of the span. Maximum deflection will occur under half span loading with a concentrated load at the quarter point.

CHAPTER VI

DESIGN EXAMPLE

Problem

Develop a preliminary design for the rib, tie, and hangers, for a 600 foot steel tied arch bridge. The bridge is for an interstate highway, and is to carry four lanes of HS20-44 loading.

The design will be based upon AASHTO, "Standard Specifications for Highway Bridges", 1977.

SolutionA.) Initial geometric assumptions

For preliminary analysis assume:

- 1.) Tie will be stiffer than rib.
- 2.) H/L ratio of 1/5.5.
- 3.) 15 panels.
- 4.) Tie to have constant depth of approximately ten feet. (1/60 of span).
- 5.) Rib to be segmental, and constant depth of approximately four feet.
- 6.) Tie to camber about two feet from springing line to center-span.
- 7.) Joint size effects of the large rigid connection between the rib and the tie will not be considered. This could be accounted for in the computer analysis.

The tie will be designed as parabolic for uniform floor load with non-uniform floor loads being carried by the hangers. Assume the rib profile initially to be parabolic.

B.) Idealized geometry

Figure (37) shows the geometry of the arch, and the coordinate system to be used in the computer analysis. From assumption (2) of part (A) the equation of the rib profile is found to be:

$$y = -1.01010 \times 10^{-4} x^2 + 0.72727x \quad (x, y \text{ in inches})$$

From assumption (6) of part (A) the equation of the tie profile is found to be:

$$y = -1.85190 \times 10^{-6} x^2 + 0.01333x \quad (x, y \text{ in inches})$$

Table 9 gives the rib and tie profile coordinates.

Table 9. Rib and tie profile coordinates.

<u>Rib coordinates</u>			<u>Tie coordinates</u>		
Joint No.	x (in)	y (in)	Joint No.	x (in)	y (in)
1	0	0.00	1	0	0.00
2	480	325.82	17	480	5.97
3	960	605.09	18	960	11.09
4	1440	837.82	19	1440	15.36
5	1920	1024.00	20	1920	18.77
6	2400	1163.64	21	2400	21.33
7	2880	1256.73	22	2880	23.04
8	3360	1303.27	23	3360	23.89
9	3840	1303.27	24	3840	23.89
10	4320	1256.73	25	4320	23.04
11	4800	1163.64	26	4800	21.33
12	5280	1024.00	27	5280	18.77
13	5760	837.82	28	5760	15.36
14	6240	605.09	29	6240	11.09
15	6720	325.82	30	6720	5.97
16	7200	0.00	16	7200	0.00

Table 10 gives the rib member lengths, and cosines of the angles of inclination for the members.

Table 10. Rib member geometry.

Member	Length (in)	$\text{Cos}\alpha$
1	580.131	0.8274
2	555.298	0.8644
3	533.452	0.8998
4	514.856	0.9323
5	499.896	0.9602
6	488.948	0.9817
7	482.267	0.9953
8	480.000	1.0000

where:

α = Angle of inclination of rib member from horizontal line

7 = Joint numbers

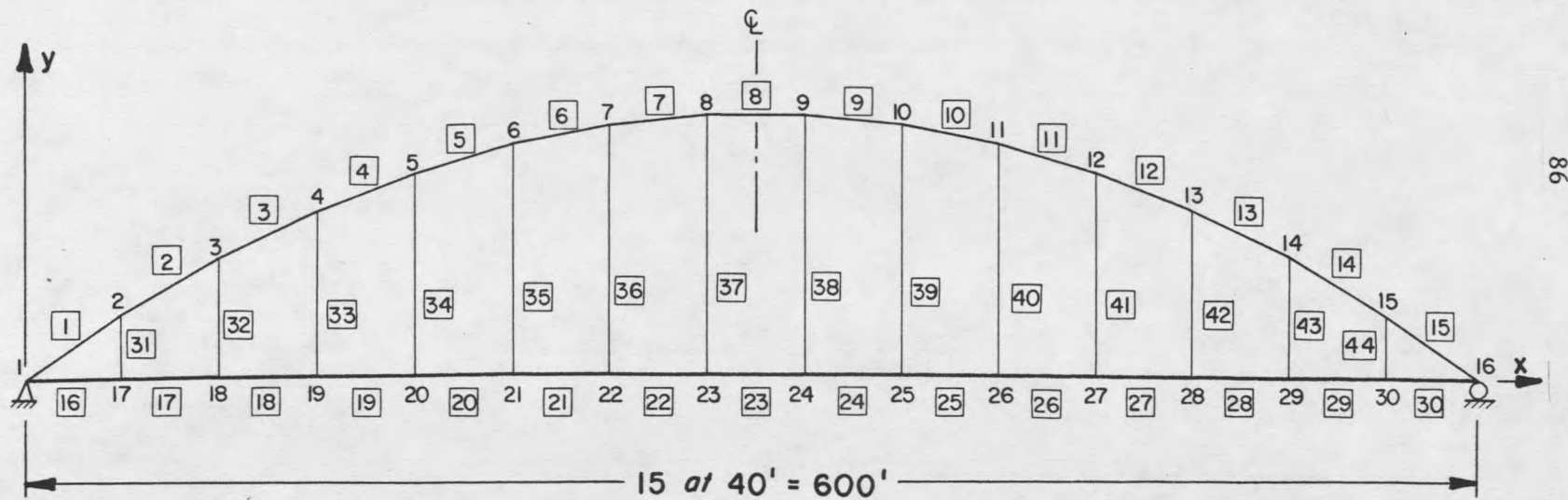


Figure 37. Assumed geometry of arch.

C.) Design of floor system

Figure 38 shows the assumed cross section of the floor.

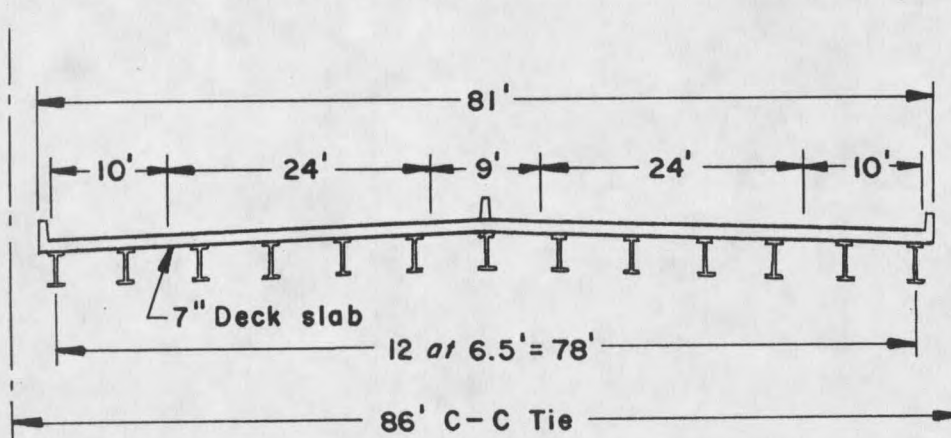


Figure 38. Cross section of floor.

Stringer design:

Assume stringers to be three-span, continuous, non-composite beams.

Initial dead load per stringer (kips/ft):

Assumed stringer weight	0.100
7-in concrete slab	0.569
Concrete haunch	0.030
Total	0.699

Superimposed dead load on 13 stringers (kips/ft):

Bridge median	0.030
Railings	0.060
Parapets	1.004
Future wearing surface	2.310
Total	3.404

Total uniformly distributed load:

$$0.699 + \frac{3.404}{13} = 0.961 \text{ kips/ft}$$

Concentrated load due to diaphragm: 0.32 kips

Impact factors:

For moments, and reactions at external supports:

$$I = \frac{50}{40 + 125} = 0.300$$

For reactions at internal supports:

$$I = \frac{50}{80 + 125} = 0.244$$

Maximum positive moment at 0.4 point (ft-kips):

Dead load	125.0
Live load	207.9
Impact	62.4
Total	395.3

Maximum negative moment at interior support (ft-kips):

Dead load	155.9
Live load	150.0
Impact	45.0
Total	350.9

Solving for the beam size, assuming an allowable bending stress of 20 ksi:

$$\text{Required } S = \frac{395.3(12)}{20} = 237.2 \text{ in}^3$$

Use W 27 x 94, $S = 243 \text{ in}^3$

Shear was satisfactory. Revise dead load to 0.955 kips/ft.

Maximum reactions at end support (kips):

Dead load	15.4
Live load	35.1
Impact	10.5
Total	61.0

Maximum reactions at internal support (kips):

Dead load	42.4
Live load	43.7
Impact	<u>10.7</u>
Total	96.8

Design of lateral floor bracing

For preliminary analysis assume that the lateral bracing of the floor acts independently of the lateral bracing of the rib. After trial members are chosen this can be checked using a space frame analysis. The allowable stresses were increased by 25%, for dead load plus wind load. At this point in the analysis it is best to over-design the bracing somewhat, in case if the tie depth needs to be deepened.

Assumed wind load against tie:	75 psf
Tie depth:	10 ft
Depth of parapet, railing, and exposed stringer:	4.5 ft

Total uniform wind load (kips/ft):

$$0.075(10 + 4.5) = 1.09 \text{ kips/ft}$$

Figure 39 shows the assumed layout of the lateral floor bracing.

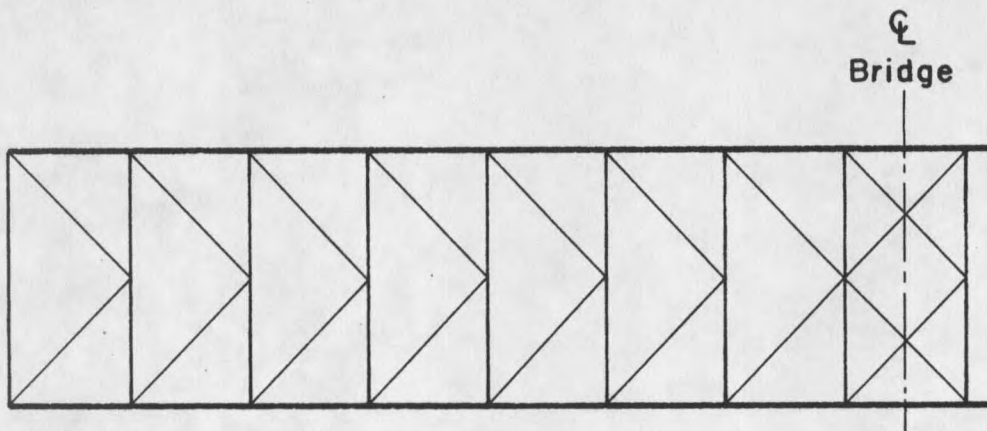
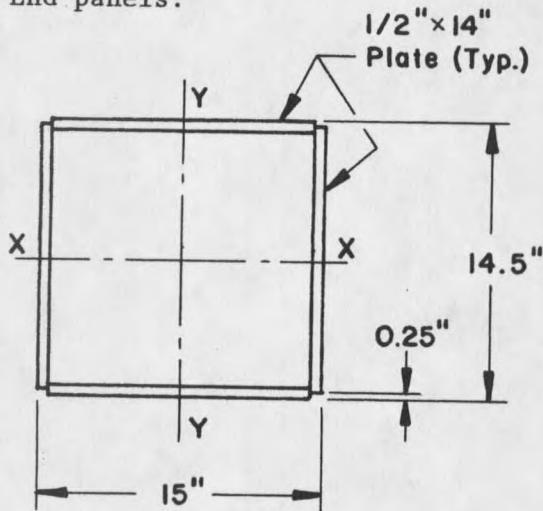


Figure 39. Plan of lateral floor bracing.

Basing the axial loads in the members upon the shear force carried in each panel the following member sizes were chosen:

End panels:



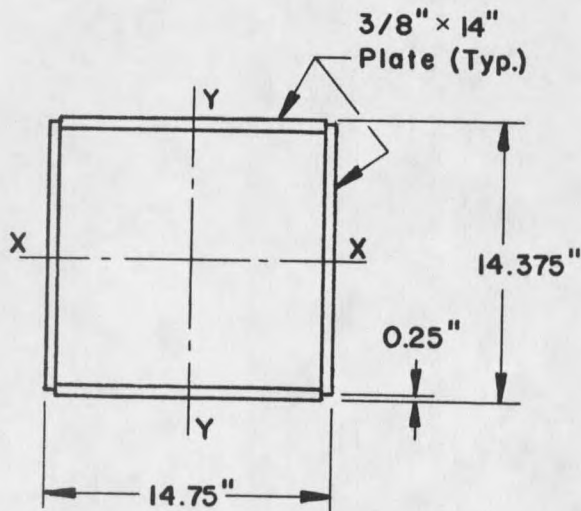
$$\text{Area} = 28 \text{ in}^2$$

$$I_{xx} = 914.7 \text{ in}^4$$

$$I_{yy} = 964.5 \text{ in}^4$$

$$\text{Weight} = 0.095 \text{ kips/ft}$$

Interior panels:



$$\text{Area} = 21 \text{ in}^2$$

$$I_{xx} = 686.0 \text{ in}^4$$

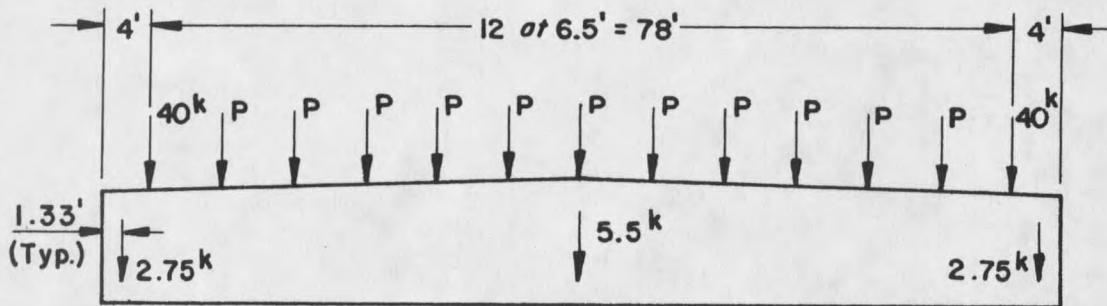
$$I_{yy} = 714.3 \text{ in}^4$$

$$\text{Weight} = 0.071 \text{ kips/ft}$$

Design of floorbeam

Floorbeams were designed as hybrid plate girders using A588 steel for the web, and A514 steel for the flanges near the center of the beam. Camber was assumed at about 1.5% for drainage.

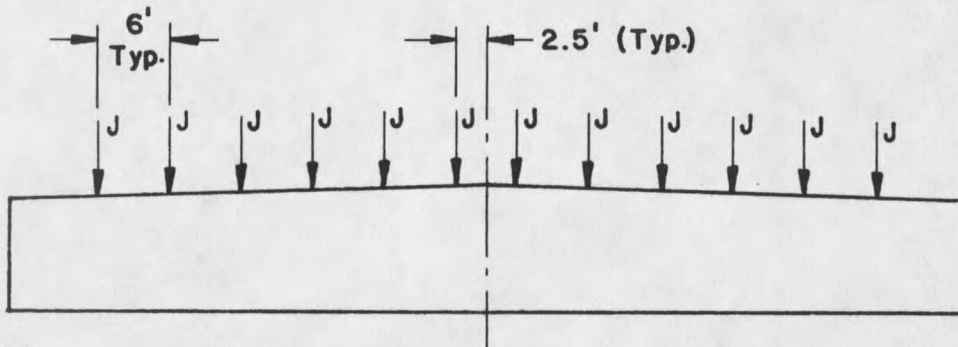
Dead loading:



where:

$$P = 40 \text{ kips}$$

Live loading:



where:

$$J = 43.7 \text{ kips}$$

Assume dead load of beam to be 0.40 kips/ft.

Maximum moment at center of beam (ft-kips):

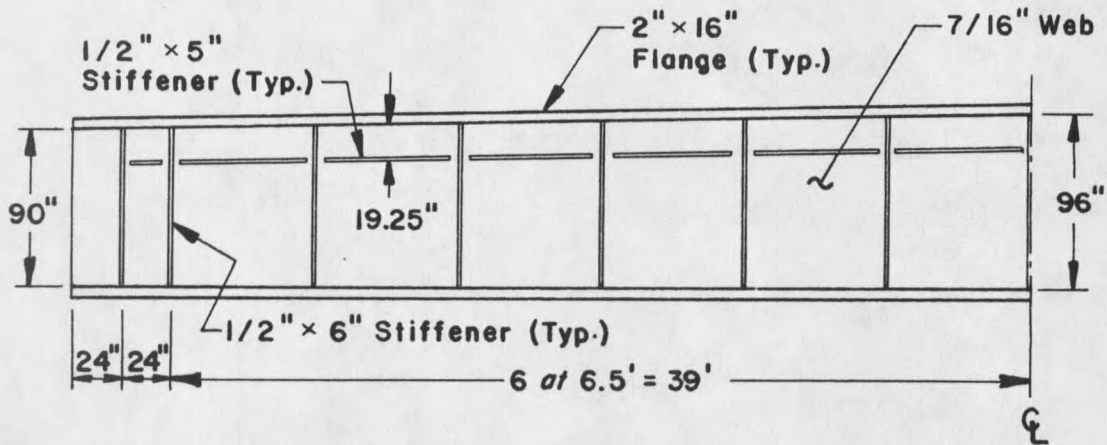
Dead load	6545
Live load	6686
Impact	1631

Reduce live load plus impact moment due to more than four lanes
being loaded:

$$0.75(6686 + 1631) = 6238 \text{ ft-kips}$$

Design moment : $6238 + 6545 = 12783 \text{ ft-kips}$

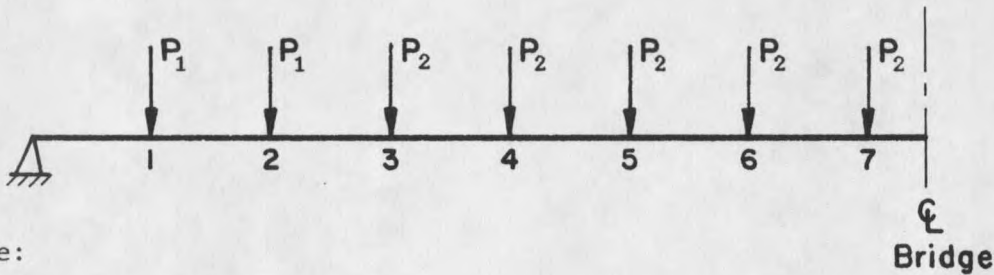
Final floorbeam design:



Floorbeam weight: 0.36 kips/ft

Dead load reaction to tie, excluding weight of bracing: 265.1 kips

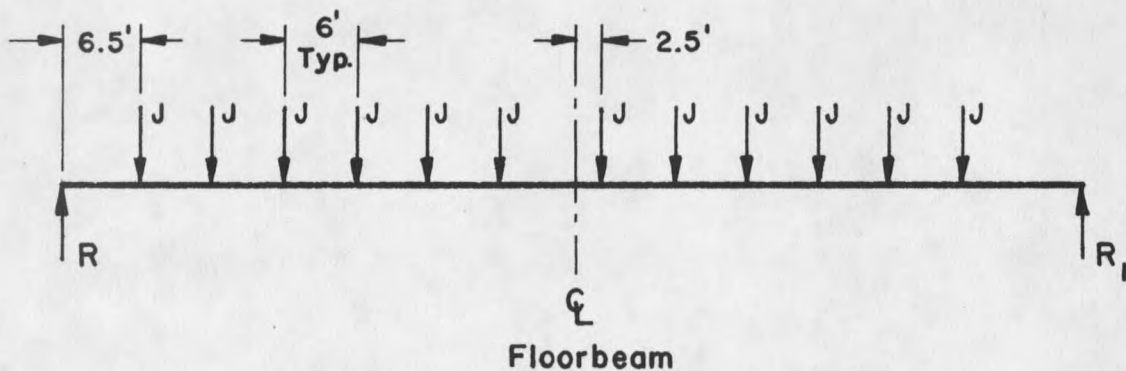
Total floorbeam dead load reactions to tie:



where:

$$\begin{aligned} P_1 &= 273.4 \text{ kips} \\ P_2 &= 271.3 \text{ kips} \end{aligned}$$

Live load to produce maximum reaction to tie:



where:

$$J = 43.7 \text{ kips}$$

Maximum floorbeam live load reaction to tie:

$$R = 6.28J = 274.4 \text{ kips}$$

Including impact and multiple lane reduction factors: $R = 256 \text{ kips}$

Uniform lane loading:

$$\frac{6.28J}{2J/\text{lane}} = 3.14 \text{ lanes}$$

D.) Upper lateral bracing

Assumed wind load against rib: 75 psf
Assumed rib depth: 4 ft

Total uniform wind load:

$$0.75(4) = 0.30 \text{ kips/ft}$$

Figure 40 shows the assumed layout of the upper lateral bracing.

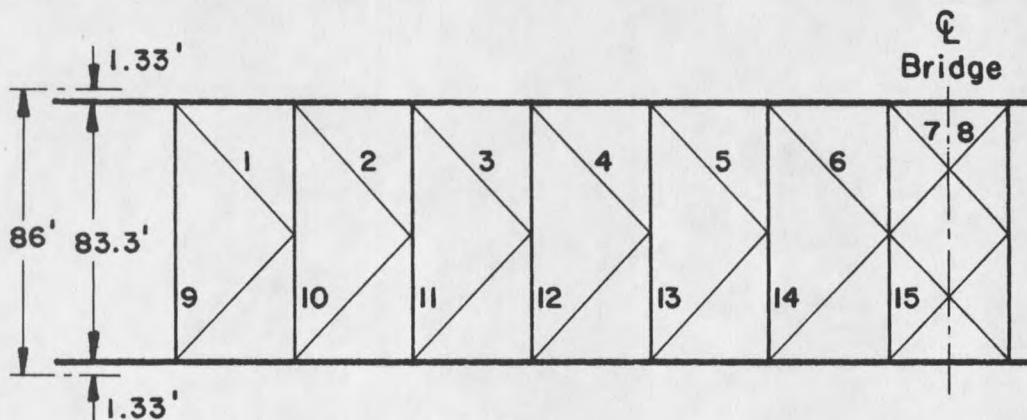
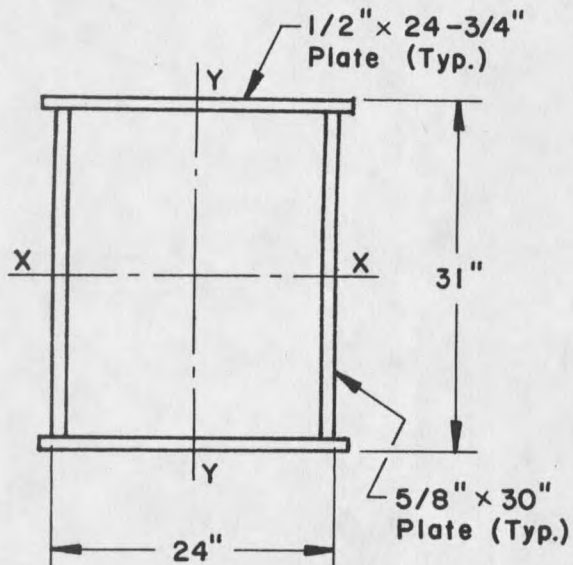


Figure 40. Plan of upper lateral bracing.

The following member sizes were selected:

Member 9:



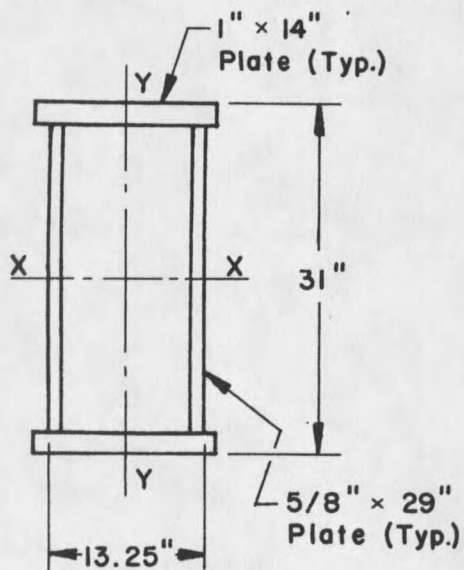
$$\text{Area} = 62.25 \text{ in}^2$$

$$I_{xx} = 8568 \text{ in}^4$$

$$I_{yy} = 6386 \text{ in}^4$$

$$\text{Weight} = 0.21 \text{ kips/ft}$$

Members 1-8, 10-15:



$$\text{Area} = 64.25 \text{ in}^2$$

$$I_{xx} = 8841 \text{ in}^4$$

$$I_{yy} = 1902 \text{ in}^4$$

$$\text{Weight} = 0.22 \text{ kips/ft}$$

Table 11 gives the lengths and weights of the upper lateral bracing members.

Table 11. Lengths and weights of upper lateral bracing members.

Member	Length (ft)	Weight (kips)
1	62.272	13.6
2	60.928	13.3
3	59.807	13.1
4	58.920	12.9
5	58.278	12.7
6	57.889	12.6
7	57.759	12.6
8	57.759	12.6
9	83.334	17.6
10-15	83.334	18.2

E.) Rib and tie analysis

Apply AASHTO uniform lane loadings to the rib and tie:

$$\begin{aligned}
 W &= 3.14(0.64) = 2.0 \text{ kips/ft} \\
 P_M &= 3.14(18.0) = 56.5 \text{ kips (for moment)} \\
 P_S &= 3.14(26.0) = 81.6 \text{ kips (for shear)}
 \end{aligned}$$

Impact for rib and tie, assuming loaded member length of $0.4L$:

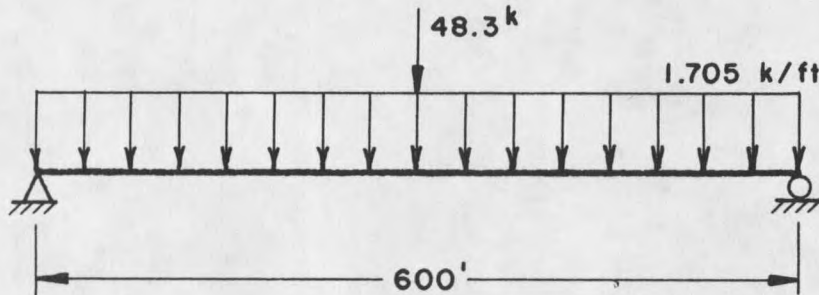
$$I = \frac{50}{240 + 125} = 0.14$$

Applying impact and multiple lane reduction factors to the lane loadings:

$$\begin{aligned}
 W &= 1.705 \text{ kips/ft} \\
 P_M &= 48.3 \text{ kips} \\
 P_S &= 69.8 \text{ kips}
 \end{aligned}$$

Begin the analysis by considering the rib and tie sections at the center of the bridge, under maximum axial force due to live load.

Live loading for maximum axial force:



Maximum moment at centerspan: $M_C = 83970$ ft-kips

Horizontal force in rib and tie due to live load:

$$F_{LL} = \frac{83970(12)}{1303.27 - 23.89} = 787.6 \text{ kips}$$

Assume dead load of floor system and upper lateral bracing to approximate the uniform load:

$$W = \frac{271 + 22}{40} = 7.33 \text{ kips/ft}$$

Maximum moment at centerspan: $M_C = 329630$ ft-kips

The corresponding horizontal force at centerspan would be:

$$F_{DL} = \frac{329630(12)}{1303.27 - 23.89} = 3091.8 \text{ kips}$$

Assume initially that the tie is a pure tension member for this loading case. The dead and live load moments will be accounted for by initially assuming a reduced allowable stress for the tie in Equation (37). The tie is to be made of A588 steel, with an allowable tensile stress of 27 ksi. Assume initially that the allowable stress is reduced to 13.5 ksi.

Horizontal force due to weight of tie:

$$F_{WT} = \frac{F(.283 \times 10^{-3})7200^2}{8(13.5)(1303.27 - 23.89)}$$

or: $F_{WT} = 0.10618(F)$

where:

F = Horizontal force in rib and tie due to total dead and live loads acting on the arch

The horizontal force due to the weight of the rib is estimated from Equation (38). For this example the area of the rib at the springing line will be estimated at 1.16 times the area at centerspan. The factor ζ is found from Table 8 to be 0.1378.

Horizontal force due to the weight of the rib:

$$F_{WR} = \frac{A_R(.283 \times 10^{-3})(.1378)7200^2}{1303.27 - 23.89}$$

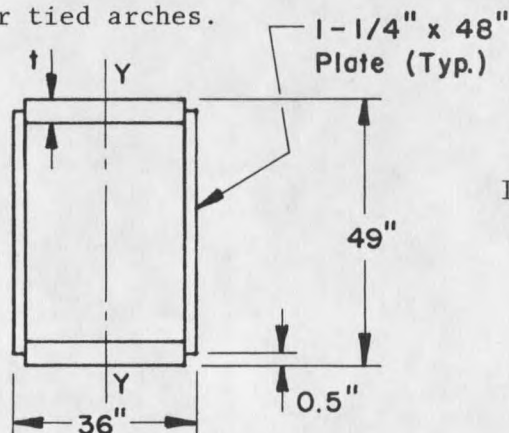
or: $F_{WR} = 1.5804(A_R)$

Substituting into Equation (34):

$$F = 787.6 + 3091.8 + 0.10618F + 1.5804A_R$$

or: $F = 4340.2 + 1.7681A_R$ (40)

A trial rib section will be assumed, based upon an unknown flange thickness. The overall dimensions were chosen from dimensions of other similar tied arches.



$$A_R = 120 + 67t$$

$$I_{yy} = 93.52A_R + 25004$$

The rib section properties will be kept the same for members 7,8, and 9. Therefore the allowable stress will be based upon member 7 or 9 where the axial force, and kl/r will be slightly larger.

Axial force in rib: $F/\cos\alpha \approx 1.005F$

The rib will be made of A588 steel, and the AASHTO formula for a concentrically loaded column will be used to determine an approximate rib size. To account for the small dead and live load moments a reduced allowable stress of 20 ksi will be used in the AASHTO formula:

$$\frac{1.005F}{A_R} = 20 - .00103 \left(\frac{L}{r} \right)^2$$

$$\text{or: } F = 19.9A_R - \frac{238.51A_R^2}{I_{yy}} \quad (41)$$

Substituting the expression for I_{yy} and equating Equations (40) and (41) the following is obtained:

$$A_R^2 + 32.58A_R - 74474 = 0$$

Solving for A_R and the flange thickness:

$$A_R = 257.1 \text{ in}^2$$

$$t = 2.04 \text{ in} \quad \text{Use 2 in. with } A_R = 254 \text{ in}^2$$

Solving for the force F from Equation (40):

$$F = 4340.2 + 1.7681(254) = 4789.3 \text{ kips}$$

Check the assumed weight of tie:

$$F_{WT} = 0.10618(4789.3) = 508.5 \text{ kips}$$

Solving for the area of the tie from Equation (35):

$$508.5 = \frac{(.283 \times 10^{-3})A_T 7200^2}{8(1303.27 - 23.89)}$$

$$A_T = 354.77 \text{ in}^2$$

This is not a reasonable value for A_T , being too large. Therefore the initial estimate of allowable stress in the tie will be increased to 16 ksi.

After repeating the above procedure with $\sigma = 16$ ksi the following values are obtained:

$$\begin{aligned} A_R &= 251.7 \text{ in}^2 \\ t &= 1.97 \text{ in} && \text{Use 2 in. with } A_R = 254 \text{ in}^2 \\ F &= 4702 \text{ kips} \\ F_{WT} &= 421.3 \text{ kips} \\ A_T &= 293.9 \text{ kips} \end{aligned}$$

The values look reasonable, A_T may be a little large. Using a force F of 4702 kips, the remainder of the rib member sizes can be estimated. The flange thickness will be assumed to change at panels 1,2,4, and 7. The procedure will be illustrated for member 4.

$$\begin{aligned} \text{Force in member:} & \quad 4702/0.9323 = 5044 \text{ kips} \\ \text{Length of member:} & \quad 514.84 \text{ in} \end{aligned}$$

Allowable stress:

$$\frac{5044}{A_R} = 20 - .00103 \frac{(514.84)^2 A_R}{I_{yy}}$$

$$A_R^2 + 18.12 A_R - 80458 = 0$$

Solving for A_R and the flange thickness:

$$\begin{aligned} A_R &= 274.7 \text{ in}^2 \\ t &= 2.31 \text{ in} && \text{Use 2-3/8 in. with } A_R = 279.13 \text{ in}^2 \end{aligned}$$

The other rib member sections were solved for in a similar manner. A summary of the preliminary rib member sizes is given in Table 12.

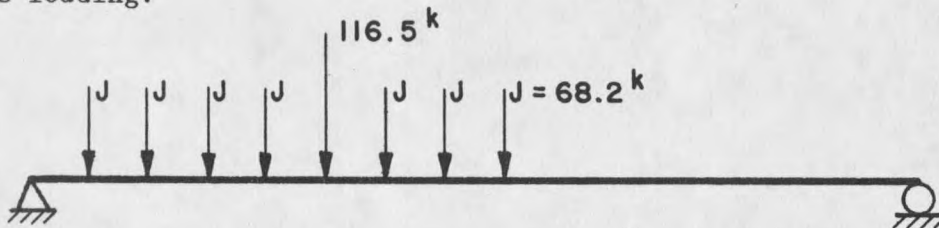
Table 12. Summary of preliminary rib member sizes.

Member	t (in)	A_R (in ²)	I_R (in ⁴)
1	3.00	321.00	129369
2-3	2.75	304.25	121571
4-6	2.38	279.13	109520
7-8	2.00	254.00	97042

Maximum negative moment

To obtain maximum negative moment, load approximately 0.56 of the span with uniform load. In addition place a concentrated load at the 0.3 point. From Table 5, assume $M_L = 0$ at hanger 8.

Live loading:



Simple beam live load moment at 0.53 point: $M = 50338$ ft-kips

Horizontal force induced in the rib and tie:

$$\frac{50338(12)}{1303.27 - 23.89} = 472.1 \text{ kips}$$

Table 13 shows the calculated distribution of maximum negative live load moment to be carried by the rib and the tie.

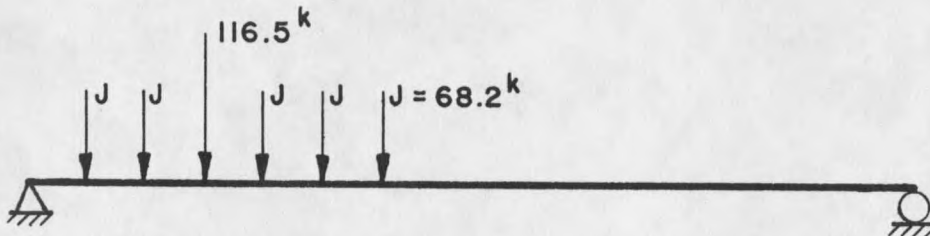
Table 13. Rib and tie maximum negative live load moment distribution.

Hanger	Simple Beam Moment (in-kips)	Couple (in-kips)	M_L (in-kips)
1	198780	151033	47747
2	364824	280487	84337
3	498120	388366	109754
4	598694	474670	124024
5	666528	539399	127129
6	678444	582548	95896
7	657624	604123	53501
8	604056	604123	-67
9	517764	582548	-64784
10	431472	539399	-107927
11	345180	474670	-129490
12	258888	388366	-129478
13	172584	280487	-107903
14	86292	151033	-64741

Maximum positive moment

To obtain maximum positive moment, load approximately 0.40 of the span with uniform load. In addition place a concentrated load at the 0.2 point. From Table 5, assume $M_L = 0$ at hanger 7.

Live loading:



Simple beam live load moment at 0.47 point: $M_C = 33645$ ft-kips

Horizontal force induced in rib and tie:

$$\frac{33645(12)}{1303.27 - 23.89} = 316 \text{ kips}$$

Table 14 shows the calculated distribution of maximum positive live load moment to be carried by the rib and tie.

Table 14. Rib and tie maximum positive live load moment distribution.

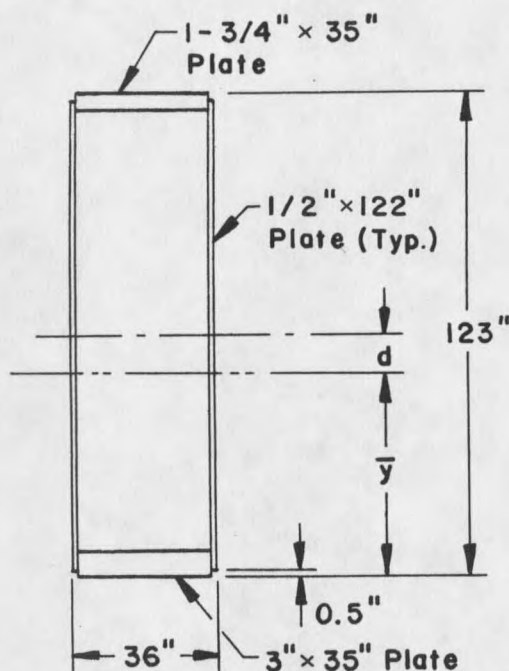
Hanger	Simple Beam Moment (in-kips)	Couple (in-kips)	M_L (in-kips)
1	169130	101073	68057
2	305530	187704	117826
3	409190	259897	149293
4	456940	317653	139287
5	471940	360970	110970
6	454200	389846	64354
7	403740	404284	-544
8	353270	404284	-51014
9	302810	389846	-87036
10	252340	360970	-108630
11	201860	317653	-115793
12	151400	259897	-108497
13	100930	187704	-86774
14	50467	101073	-50606

Begin the preliminary tie design in the region of maximum positive moment.

Horizontal force due to dead load (including weight of rib plus tie):

$$F_{TDL} = 4702 - 787.6 = 3914.4 \text{ kips}$$

An assumed section is selected. This section was obtained by trial and error by varying the flange areas. The web plate sizes were obtained from dimensions of other similar bridges as found in Richardson [37], and held constant.



$$A = 288.25 \text{ in}^2$$

$$\bar{y} = 52.53 \text{ in}$$

$$d = 8.974 \text{ in}$$

$$I = 747535 \text{ in}^4$$

Approximate positive moment carried by the tie:

$$M_T = \left(\frac{747535}{747535 + (.8998)121571} \right) 149293$$

$$M_T = 130235 \text{ in-kips}$$

Check stress:

$$\frac{316 + 3914.4}{288.25} + \frac{130235(52.526)}{747535} = 23.8 < 27$$

The section must also be checked in the region of maximum negative moment.

Approximate negative moment carried by the tie:

$$M_T = \left(\frac{747535}{747535 + (.9602)109520} \right) 129490$$

$$M_T = 113520 \text{ in-kips}$$

Check stress:

$$\frac{472 + 3914.4}{288.25} + \frac{113520(61.5 + 8.974)}{747535} = 25.9 < 27$$

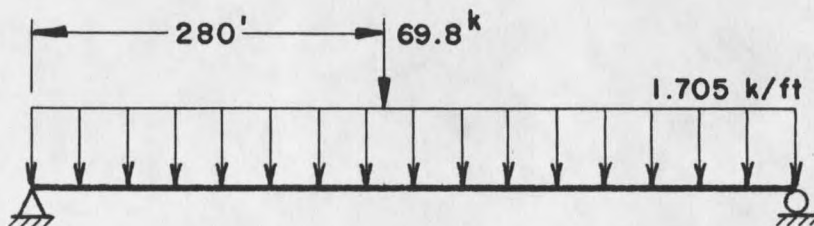
This is satisfactory for preliminary analysis. The upper flange thickness of the tie was held fixed at 1.75 inches. The lower flange was changed at several locations, and the summary of the assumed tie sections is given in Table 15.

Table 15. Summary of preliminary tie member sizes.

Member	t (in)	A _T (in ²)	$\bar{y}$ (in)	I _T (in ⁴)
16	1.75	244.50	61.50	601556
17	2.50	270.75	55.74	697154
18-19	3.00	288.25	52.53	747535
20	2.50	270.75	55.74	697154
21	2.00	253.25	59.44	639832
22-23	1.75	244.50	61.50	601556

F.) Hanger analysis

The hangers subjected to the largest forces will be the center hangers. The loading will be assumed to uniform across the span with the AASHTO concentrated load for shear placed under one of the two center hangers.



Moment at center of bridge:

$$M_C = 86497 \text{ ft-kips}$$

In order to estimate how much of the 69.8 kips will be carried into the hanger, the ratio of the rib to tie moments of inertia at centerspan must be calculated.

$$\frac{I_R}{I_T} = \frac{97042}{601556} = 0.16$$

From Figure 32, assume ψ equals 0.23.

Weight of tie at centerspan: $W = 244.5(.0034) = 0.83 \text{ kips/ft}$

The hanger force due to the force in the tie is found to be:

$$T_F = \left(\frac{86497(12)}{1303.27 - 23.89} + 3914 \right) \frac{8(23.89)(480)}{7200^2} = 8.4 \text{ kips}$$

Maximum hanger force (kips):

69.8 kip concentrated load	16.1
Weight of tie	33.2
Uniform live load	68.2
Concentrated floor load	271.3
Force in the tie	8.4
Total	397.2

Use (4) 1-5/8 in. dia. bridge strands, class A coating, breaking strength = 162 tons, $A = 6.36 \text{ in}^2$.

G.) Computer analysis for live load

The computer analysis is carried out using a plane frame program. The member element considers flexural and axial deformations.

The computer results for maximum positive live load moment are given in Table 20 (Appendix). The following comparisons can be made:

	Preliminary Analysis	Computer	% Error
Force in tie	316 kips	311 kips	+1.6%
Max tie moment	130235 in-kips	135000 in-kips	-3.5%

Similarly the computer results for maximum negative moment are given in Table 21 (Appendix). The following comparisons can be made:

	Preliminary Analysis	Computer	% Error
Force in tie	472 kips	475 kips	-0.6%
Max tie moment	113520 in-kips	116000 in-kips	-2.1%

The computer results for the maximum axial force case are given in Table 22 (Appendix). The following comparisons can be made:

	Preliminary Analysis	Computer	% Error
Force in tie	787.6 kips	763 kips	+3.2%
Max tie moment	-----	19100 in-kips	-----
Max rib moment	-----	2590 in-kips	-----

The comparisons indicate a good correlation between the hand calculations and the computer results. The results are sufficiently close that the preliminary member sizes should be reasonable. The results for the maximum axial force case verify that the live load moments are small. Thus neglecting these moments in preliminary analysis is justified. The maximum moment in the tie appeared in the vicinity of the concentrated load, at centerspan.

The stresses will be rechecked for the tie section under maximum moment at the quarter point. It is important to note that at this stage in the analysis the dead load forces are still estimated. The accuracy of the estimate of the dead load forces should be fairly good however.

Rechecking the stresses in the tie:

$$\frac{311 + 3914.4}{288.25} + \frac{135000(52.526)}{747535} = 24.1 < 27$$

and:

$$\frac{475 + 3914.4}{288.25} + \frac{116000(61.5 + 8.974)}{747535} = 26.2 < 27$$

In a similar fashion the stresses in the other sections of the tie should be checked. The rib stresses must also be checked, considering the rib segments as eccentrically loaded columns.

For the purposes of this example it will be assumed that all member sizes are satisfactory. If any member sizes needed to be adjusted, they can be adjusted before performing the dead load analysis.

H.) Minimization of dead load moment.

A preliminary dead load analysis was performed to determine the weights of all of the members. The results are given in Table 23 (Appendix). The computer analysis calculated a force in the tie of 3870 kips. This compares very well with the estimated value of 3914 kips.

The initially assumed rib geometry should be checked and modified as required. Figure 41 shows the dead loads applied to the arch.

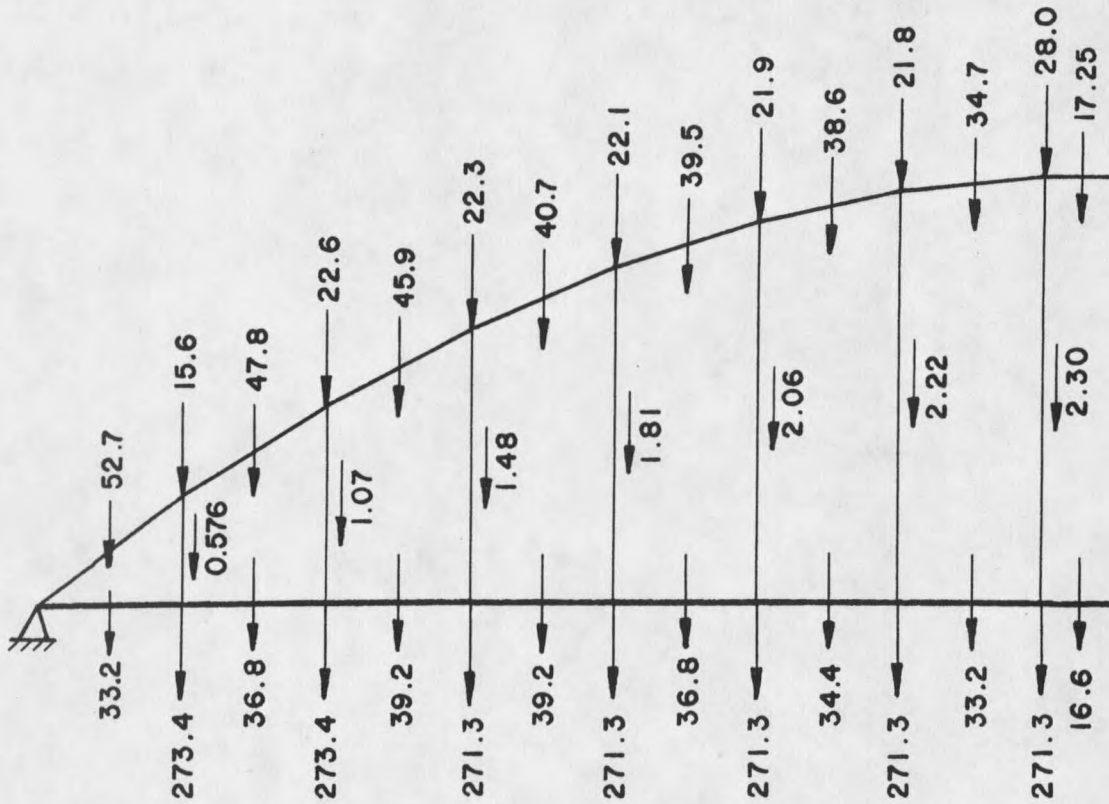
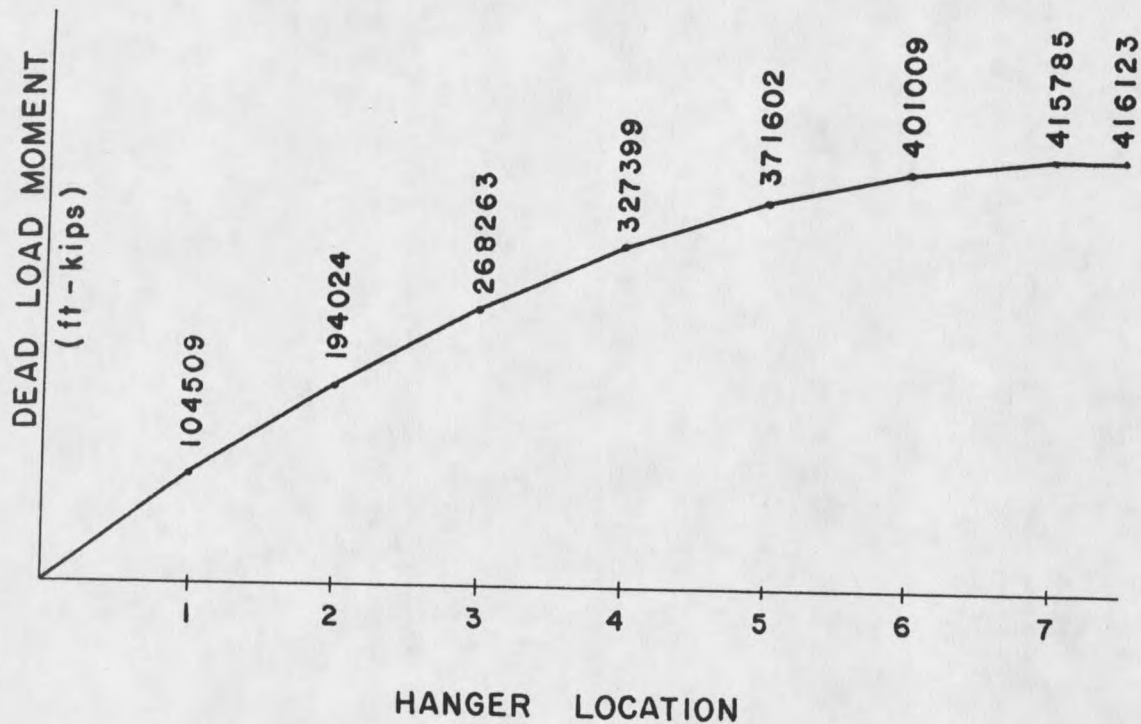


Figure 41. Dead load of arch.

The corresponding moment diagram is found to be:



The horizontal force F in the rib and tie is found from the moment at the center of the bridge:

$$F = \frac{416123(12)}{1303.27 - 23.89} = 3903 \text{ kips}$$

The ordinates of the rib should now be determined from Equation (6). This will equate the internal dead load moment of the arch to the couple caused by F . The modified ordinates are given in Table 16.

Table 16. y Coordinates of rib profile.

Joint No.	Original (in)	Modified (in)
1	0.00	0.00
2	325.82	327.28
3	605.09	607.62
4	837.82	840.14
5	1024.00	1025.37
6	1163.64	1163.82
7	1256.73	1255.95
8	1303.27	1302.23
center	1303.27	1303.27

The results show that the initial assumption of a parabolic rib is indeed good. The maximum difference in values is 2.5 inches or about 0.4%. It should be noted in Table 16 that since the center height of the bridge was held constant at 1303.27 inches, the center member should be cambered about an inch to account for the dead load of the center rib and tie members. Similarly the other members could be cambered to minimize bending between panel points.

Hanger forces

In the tie the force F will cause an equivalent upward uniform load given by:

$$W = \frac{8(3903)(23.89)}{7200^2} = 0.0144 \text{ kips/in}$$

Converting to upward concentrated loads:

$$T = 0.0144(480) = 6.9 \text{ kips}$$

Calculating the tension in hanger 1, including the weight of the hanger:

$$\text{At rib: } T_1 = 6.9 + 273.4 + \frac{33.2 + 36.8}{2} + \frac{.576}{2} = 315.6 \text{ kips}$$

$$\text{At tie: } T_1 = 315.6 - 0.576 = 315.0 \text{ kips}$$

The tensions for the other hangers are similarly found and tabulated in Table 17.

Table 17. Hanger tensions under dead load.

Hanger	Tension at rib	Tension at tie
1	315.6	315.0
2	318.8	317.8
3	318.1	316.7
4	317.1	315.3
5	314.8	312.8
6	313.1	310.9
7	312.6	310.3

Rib analysis

With the hanger forces being known the rib can be analyzed as shown in Figure 12. The results for the rib analysis without modifying the geometry are given in Table 24 (Appendix). The results using the modified geometry are given in Table 25 (Appendix). When modifying the geometry an extra joint, no.17', was added at the crown of the arch to account for the camber of the center member.

Comparison of the results of Tables 24 and 25 indicate that the geometry modification has helped reduce the moments, near the ends of the arch. The axial member end actions to eliminate axial deflections can be determined from the results of Table 25. They can also be computed by dividing the force F at centerspan by the cosines

of the angles of inclination of the rib segments. The results which include the elimination of the axial deflections are given in Table 26 (Appendix), and indicate a further reduction of moment. Referring to Table 26 it can also be seen that the deflections have become very small.

Tie analysis

The tie can be analyzed in a similar manner as the rib, and loaded as shown in Figure 13. The tie would be fabricated short to compensate for the 3903 kip tensile force. The results for the tie are given in Table 27 (Appendix), and again show very small deflections of the joints.

I.) Final dead load analysis

The member end actions for eliminating the stretch of the hangers must be calculated. The procedure will be illustrated for hanger no. 1.

From Tables 26 and 27 the relative deflection between joints 2 and 17 is found to be:

$$\delta y = -0.0371 - (-0.00201) = -0.03509 \text{ in}$$

Thus from Equation (10) the required length that hanger 1 must be fabricated short is:

$$\delta l = 0.03509 + \frac{315.3(327.28 - 5.97)}{6.36(30000)} = 0.566 \text{ in}$$

Rewriting δl in terms of an equivalent member end action for computer analysis:

$$AML = \frac{0.566(6.36)(30000)}{327.28 - 5.97} = 336.1 \text{ kips}$$

The values of the member end actions for the remainder of the hangers are tabulated in Table 18.

Table 18. Member end actions for eliminating hanger stretch.

Hanger	δy (in)	$\frac{1}{AE}$ (in/kips)	AML (kips)
1	-.035	1.684×10^{-3}	336.1
2	-.082	3.127×10^{-3}	344.4
3	-.092	4.323×10^{-3}	338.7
4	-.058	5.276×10^{-3}	327.1
5	.012	5.988×10^{-3}	311.8
6	.104	6.462×10^{-3}	295.9
7	.197	6.700×10^{-3}	282.0

Now that the required axial lengthening or shortening effects are known for all of the members, a final dead load analysis is made of the entire arch. The computer results are given in Table 28 (Appendix). Comparison of the moments of Tables 23 and 28 show that the dead load moments have become smaller, especially in the tie. Comparison of the hanger tensions (members 31-44) from Table 28 with the computed values in Table 17 indicates a good correlation.

J.) Final stress analysis

The stresses due to dead load must be included with the stresses due to live load to determine if a member is satisfactory. The estimated axial loads were within 2% of the computer solution. In general the dead load moments are small enough that they will not cause much increase in stress, but they should be considered. For example, at hanger location 5, the stress on the lower tie flange due to dead load moment would be:

$$\sigma = \frac{1840(59.44)}{639832} = 0.17 \text{ ksi}$$

A final computer analysis for half span uniform loading with a concentrated load at hanger 4 was made. The results are given in Table 29 (Appendix). A check of the maximum deflection yields:

$$\frac{8.26}{7200} < \frac{1}{800}$$

Hanger tension adjustment

A hypothetical hanger tension adjustment example will be considered. The procedure will follow the method described in Chapter III. Suppose for simplicity that the bridge of the previous example contains hangers consisting of (2) 2-5/16 inch dia. wire ropes, with an area of 3.21 in². Assume that the tensions in all cables were measured after the deck slab was completed, and for one arch were found to be:

Cable No.	Tension (kips)	Cable No.	Tension (kips)
1	156	15	152
2	161	16	157
3	148	17	159
4	158	18	165
5	160	19	151
6	151	20	158
7	159	21	150
8	164	22	154
9	153	23	161
10	156	24	156
11	162	25	149
12	160	26	160
13	151	27	151
14	143	28	154

Assuming the desired tensions to be the values shown in Table 28, the following adjustments are required:

Cable No.	Desired Tension (kips)	Adjustment (kips)	Cable No.	Desired Tension (kips)	Adjustment (kips)
1	158.25	2.25	15	155.50	3.50
2	158.25	-2.75	16	155.50	-1.50
3	159.75	11.75	17	157.00	-2.00
4	159.75	1.75	18	157.00	-8.00
5	159.50	0.50	19	158.00	7.00
6	159.50	8.50	20	158.00	0.00
7	159.00	0.00	21	159.00	9.00
8	159.00	-5.00	22	159.00	5.00
9	158.00	5.00	23	159.50	-1.50
10	158.00	2.00	24	159.50	3.50
11	157.00	-5.00	25	159.75	10.75
12	157.00	-3.00	26	159.75	-0.25
13	155.50	4.50	27	158.25	7.25
14	155.50	12.50	28	158.25	4.25

In order to develop the influence coefficient matrix the member end actions corresponding to a unit deformation must be computed, for each cable. For cable 1 the member end action is found to be:

$$\text{AML} = \frac{(3.21)(30000)(1)}{327.28 - 5.97} = 299.7 \text{ kips}$$

The values of AML for the other cables are tabulated in Table 19.

Table 19. Member end actions used for developing the influence coefficient matrix.

Cable No.	AML (kips)	Cable No.	AML (kips)
1	299.71	8	95.67
2	299.71	9	84.29
3	161.43	10	84.29
4	161.43	11	78.11
5	116.76	12	78.11
6	116.76	13	75.33
7	95.67	14	75.33

One half of the influence coefficient matrix is shown in Table 30 (Appendix). The other half of the matrix is the same as that shown in Table 30, with the rows inverted, and the columns reversed. Row 1 would become row 28, row 2 becomes row 27, etc.

The final length adjustments are tabulated in Table 31 (Appendix). Subsequently the solution can be checked by converting the adjustments of Table 31 to equivalent member end actions and performing an analysis of the entire arch. The results for this are given in Table 32. Comparison of the actual cable tensions (members 31 - 58) with the desired adjustments shows that a satisfactory solution is obtained.

SUMMARY

A tied arch behaves similarly to a self-anchored suspension bridge. In the self-anchored suspension bridge the cables carry tensile force and no moment. In a tied arch by designing a stiff tie to carry the live load, the rib carries compressive force with little moment. By using high strength steels the size of the rib can be reduced to a minimum size based upon slenderness effects.

One of the major advantages in using any arch is being able to carry the dead load of the structure primarily by axial forces. This produces a more efficient use of the materials. By shaping the axis of the arch correctly, and eliminating axial deformations, the dead load moments in a tied arch can be substantially reduced. Since the predominant dead load of a tied arch bridge is the floor system, and is nearly uniform, the rib should be nearly parabolic. It is easiest to design the tie as a parabola, and make any adjustments for non-uniform loading to the rib geometry.

The axial forces carried by the rib and the tie are approximately proportional to the rise to span ratio. A flatter arch will have larger axial forces. A study could be made to investigate if there is an optimum rise to span ratio, for various grades of steel. Although the axial forces decrease with larger rise to span ratios the slenderness ratios increase. Thus there is a tradeoff in economy. The I_R/I_T ratio primarily determines how much live load moment is being

carried by the rib and by the tie. The hanger spacing does not appreciably affect the live load moment envelope. This is because the live load moment envelope is primarily a function of the span length and the rise to span ratio. The primary effects of hanger spacing on tied arch design is in the dead load and aesthetics. Hangers spaced further apart will cause increased dead load due to the longer panel lengths. The A_R/A_T ratio does not affect the overall behavior of a tied arch substantially. The A_R/A_T ratio is governed by allowable stresses, and therefore depends greatly upon what types of steel are being used. The A_R/A_T ratio does affect the maximum live load deflection more than the I_R/I_T ratio, but not greatly.

After a tied arch bridge is constructed it is very important to obtain the minimum dead load moment which was designed for. This can only be accomplished by obtaining the correct design hanger tensions. By measuring the actual hanger tensions an analysis can be made using influence coefficients to obtain the required tension adjustments. Failure to do this can cause substantial dead load moment and subsequent overstressing of the arch. Although no tightening sequence is absolutely necessary, each arch should be investigated and a tightening sequence developed. The process is similar to truing up a bicycle wheel.

The horizontal component of force in the rib and tie due to dead load can be determined by dividing the dead load moment at centerspan by the distance between the rib and the tie. Computer analyses showed that a similar approximation can be made for live load by locating the approximate location of zero rib and tie moment. This enables a

designer to make a quick initial estimate of member sizes for preliminary analysis. The computer analyses also verified that the rib and tie deflections are nearly the same. Therefore the moment arms of the horizontal components of force do not change appreciably.

The graphs of the rib and tie moment influence lines indicate that the moment envelopes are rather insensitive to changes in the various parameters. This strongly suggests that a tied arch with a slender rib carrying compressive force may have an optimum design, in terms of minimum dead load. The rib and tie sizes of the design example were determined on the basis of allowable stresses, but undoubtedly this is not the most efficient design configuration. The rather large I_R/I_T ratio suggests this. Questions may be raised as to the initially assumed rib and tie depths at the beginning of the example. By developing the design approach based upon allowable stress it may be possible to develop a set of design curves for tied arches. The method used in the example could easily be programmed into a computer. The solution could be iterative, with the rib and tie depths being varied. There should be a point at which the dead load of the rib and tie is a minimum. Thus adding or subtracting material from the rib or tie would increase the overall dead load of the arch. This would be an interesting topic for further research.

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APPENDIX

Table 20. Computer results for maximum positive live load moment.

MAXIMUM POSITIVE LIVE LOAD MOMENT

JOINT DISPLACEMENTS AND SUPPORT REACTIONS						
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	-.719E-02	-.236E-01	.353E+03	.000E+00
2	.220E+01	-.328E+01	-.627E-02	.000E+00	.000E+00	.000E+00
3	.369E+01	-.589E+01	-.438E-02	.000E+00	.000E+00	.000E+00
4	.441E+01	-.742E+01	-.190E-02	.000E+00	.000E+00	.000E+00
5	.448E+01	-.765E+01	.966E-03	.000E+00	.000E+00	.000E+00
6	.414E+01	-.656E+01	.354E-02	.000E+00	.000E+00	.000E+00
7	.370E+01	-.438E+01	.540E-02	.000E+00	.000E+00	.000E+00
8	.340E+01	-.153E+01	.628E-02	.000E+00	.000E+00	.000E+00
9	.338E+01	.144E+01	.586E-02	.000E+00	.000E+00	.000E+00
10	.361E+01	.393E+01	.438E-02	.000E+00	.000E+00	.000E+00
11	.391E+01	.559E+01	.246E-02	.000E+00	.000E+00	.000E+00
12	.408E+01	.626E+01	.300E-03	.000E+00	.000E+00	.000E+00
13	.392E+01	.589E+01	-.183E-02	.000E+00	.000E+00	.000E+00
14	.326E+01	.459E+01	-.359E-02	.000E+00	.000E+00	.000E+00
15	.204E+01	.252E+01	-.496E-02	.000E+00	.000E+00	.000E+00
16	.299E+00	.000E+00	-.538E-02	.000E+00	.105E+03	.000E+00
17	.617E-01	-.333E+01	-.642E-02	.000E+00	.000E+00	.000E+00
18	.109E+00	-.600E+01	-.452E-02	.000E+00	.000E+00	.000E+00
19	.140E+00	-.758E+01	-.195E-02	.000E+00	.000E+00	.000E+00
20	.159E+00	-.784E+01	.842E-03	.000E+00	.000E+00	.000E+00
21	.172E+00	-.679E+01	.345E-02	.000E+00	.000E+00	.000E+00
22	.184E+00	-.461E+01	.547E-02	.000E+00	.000E+00	.000E+00
23	.199E+00	-.171E+01	.634E-02	.000E+00	.000E+00	.000E+00
24	.219E+00	.126E+01	.588E-02	.000E+00	.000E+00	.000E+00
25	.244E+00	.377E+01	.444E-02	.000E+00	.000E+00	.000E+00
26	.270E+00	.544E+01	.245E-02	.000E+00	.000E+00	.000E+00
27	.292E+00	.611E+01	.308E-03	.000E+00	.000E+00	.000E+00
28	.307E+00	.577E+01	-.171E-02	.000E+00	.000E+00	.000E+00
29	.313E+00	.451E+01	-.348E-02	.000E+00	.000E+00	.000E+00
30	.310E+00	.249E+01	-.480E-02	.000E+00	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.384E+03	.111E+02	-.289E+04	-.384E+03	-.111E+02	.935E+04
2	.367E+03	.110E+02	-.935E+04	-.367E+03	-.110E+02	.155E+05
3	.349E+03	.553E+01	-.155E+05	-.349E+03	-.553E+01	.184E+05
4	.334E+03	-.388E+00	-.184E+05	-.334E+03	.388E+00	.182E+05
5	.323E+03	-.522E+01	-.182E+05	-.323E+03	.522E+01	.156E+05
6	.315E+03	-.126E+02	-.156E+05	-.315E+03	.126E+02	.946E+04
7	.311E+03	-.173E+02	-.946E+04	-.311E+03	.173E+02	.112E+04
8	.311E+03	-.154E+02	-.112E+04	-.311E+03	.154E+02	-.627E+04
9	.314E+03	-.110E+02	.627E+04	-.314E+03	.110E+02	-.116E+05
10	.318E+03	-.535E+01	.116E+05	-.318E+03	.535E+01	-.142E+05
11	.324E+03	.715E-01	.142E+05	-.324E+03	-.715E-01	-.142E+05
12	.333E+03	.223E+01	.142E+05	-.333E+03	-.223E+01	-.130E+05
13	.344E+03	.382E+01	.130E+05	-.344E+03	-.382E+01	-.110E+05
14	.356E+03	.699E+01	.110E+05	-.356E+03	-.699E+01	-.711E+04
15	.366E+03	.148E+02	.711E+04	-.366E+03	-.148E+02	.150E+04
16	-.310E+03	.132E+03	.289E+04	.310E+03	-.132E+03	.603E+05
17	-.310E+03	.938E+02	-.603E+05	.310E+03	-.938E+02	.105E+06
18	-.311E+03	.618E+02	-.105E+06	.311E+03	-.618E+02	.135E+06
19	-.311E+03	-.186E+02	-.135E+06	.311E+03	.186E+02	.126E+06
20	-.312E+03	-.523E+02	-.126E+06	.312E+03	.523E+02	.101E+06
21	-.312E+03	-.835E+02	-.101E+06	.312E+03	.835E+02	.608E+05
22	-.312E+03	-.118E+03	-.608E+05	.312E+03	.118E+03	.439E+04
23	-.311E+03	-.899E+02	-.439E+04	.311E+03	.899E+02	-.388E+05
24	-.311E+03	-.646E+02	.388E+05	.311E+03	.646E+02	-.698E+05
25	-.311E+03	-.406E+02	.698E+05	.311E+03	.406E+02	-.893E+05
26	-.311E+03	-.164E+02	.893E+05	.311E+03	.164E+02	-.971E+05
27	-.311E+03	.110E+02	.971E+05	.311E+03	-.110E+02	-.918E+05
28	-.312E+03	.389E+02	.918E+05	.312E+03	-.389E+02	-.732E+05
29	-.312E+03	.647E+02	.732E+05	.312E+03	-.647E+02	-.421E+05
30	-.312E+03	.846E+02	.421E+05	.312E+03	-.846E+02	-.150E+04
31	-.309E+02	.000E+00	.000E+00	.309E+02	.000E+00	.000E+00
32	-.368E+02	.000E+00	.000E+00	.368E+02	.000E+00	.000E+00
33	-.368E+02	.000E+00	.000E+00	.368E+02	.000E+00	.000E+00
34	-.352E+02	.000E+00	.000E+00	.352E+02	.000E+00	.000E+00
35	-.376E+02	.000E+00	.000E+00	.376E+02	.000E+00	.000E+00
36	-.347E+02	.000E+00	.000E+00	.347E+02	.000E+00	.000E+00
37	-.282E+02	.000E+00	.000E+00	.282E+02	.000E+00	.000E+00
38	-.259E+02	.000E+00	.000E+00	.259E+02	.000E+00	.000E+00
39	-.246E+02	.000E+00	.000E+00	.246E+02	.000E+00	.000E+00
40	-.247E+02	.000E+00	.000E+00	.247E+02	.000E+00	.000E+00
41	-.279E+02	.000E+00	.000E+00	.279E+02	.000E+00	.000E+00
42	-.283E+02	.000E+00	.000E+00	.283E+02	.000E+00	.000E+00
43	-.264E+02	.000E+00	.000E+00	.264E+02	.000E+00	.000E+00
44	-.204E+02	.000E+00	.000E+00	.204E+02	.000E+00	.000E+00

Table 21. Computer results for maximum negative live load moment.

MAXIMUM NEGATIVE LIVE LOAD MOMENT

JOINT DISPLACEMENTS AND SUPPORT REACTIONS						
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	-.619E-02	-.208E-01	.414E+03	.000E+00
2	.188E+01	-.283E+01	-.546E-02	.000E+00	.000E+00	.000E+00
3	.322E+01	-.519E+01	-.422E-02	.000E+00	.000E+00	.000E+00
4	.398E+01	-.684E+01	-.251E-02	.000E+00	.000E+00	.000E+00
5	.422E+01	-.754E+01	-.281E-03	.000E+00	.000E+00	.000E+00
6	.406E+01	-.711E+01	.212E-02	.000E+00	.000E+00	.000E+00
7	.373E+01	-.556E+01	.429E-02	.000E+00	.000E+00	.000E+00
8	.346E+01	-.307E+01	.592E-02	.000E+00	.000E+00	.000E+00
9	.343E+01	-.648E-01	.638E-02	.000E+00	.000E+00	.000E+00
10	.368E+01	.284E+01	.550E-02	.000E+00	.000E+00	.000E+00
11	.409E+01	.508E+01	.366E-02	.000E+00	.000E+00	.000E+00
12	.440E+01	.627E+01	.122E-02	.000E+00	.000E+00	.000E+00
13	.435E+01	.624E+01	-.140E-02	.000E+00	.000E+00	.000E+00
14	.373E+01	.502E+01	-.366E-02	.000E+00	.000E+00	.000E+00
15	.241E+01	.281E+01	-.549E-02	.000E+00	.000E+00	.000E+00
16	.457E+00	.000E+00	-.602E-02	.000E+00	.180E+03	.000E+00
17	.671E-01	-.290E+01	-.569E-02	.000E+00	.000E+00	.000E+00
18	.121E+00	-.535E+01	-.439E-02	.000E+00	.000E+00	.000E+00
19	.163E+00	-.705E+01	-.261E-02	.000E+00	.000E+00	.000E+00
20	.194E+00	-.780E+01	-.471E-03	.000E+00	.000E+00	.000E+00
21	.221E+00	-.744E+01	.198E-02	.000E+00	.000E+00	.000E+00
22	.245E+00	-.590E+01	.431E-02	.000E+00	.000E+00	.000E+00
23	.272E+00	-.340E+01	.593E-02	.000E+00	.000E+00	.000E+00
24	.303E+00	-.380E+00	.646E-02	.000E+00	.000E+00	.000E+00
25	.339E+00	.258E+01	.563E-02	.000E+00	.000E+00	.000E+00
26	.378E+00	.485E+01	.368E-02	.000E+00	.000E+00	.000E+00
27	.412E+00	.605E+01	.124E-02	.000E+00	.000E+00	.000E+00
28	.438E+00	.605E+01	-.124E-02	.000E+00	.000E+00	.000E+00
29	.455E+00	.490E+01	-.350E-02	.000E+00	.000E+00	.000E+00
30	.460E+00	.276E+01	-.524E-02	.000E+00	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.577E+03	.385E+01	-.374E+04	-.577E+03	-.385E+01	.598E+04
2	.555E+03	.770E+01	-.598E+04	-.555E+03	-.770E+01	.103E+05
3	.531E+03	.541E+01	-.103E+05	-.531E+03	-.541E+01	.131E+05
4	.512E+03	.430E+01	-.131E+05	-.512E+03	-.430E+01	.154E+05
5	.496E+03	.165E+01	-.154E+05	-.496E+03	-.165E+01	.162E+05
6	.483E+03	-.650E+01	-.162E+05	-.483E+03	.650E+01	.130E+05
7	.476E+03	-.130E+02	-.130E+05	-.476E+03	.130E+02	.674E+04
8	.475E+03	-.165E+02	-.674E+04	-.475E+03	.165E+02	-.117E+04
9	.479E+03	-.173E+02	.117E+04	-.479E+03	.173E+02	-.950E+04
10	.487E+03	-.116E+02	.950E+04	-.487E+03	.116E+02	-.152E+05
11	.496E+03	-.354E+01	.152E+05	-.496E+03	.354E+01	-.169E+05
12	.510E+03	.832E+00	.169E+05	-.510E+03	-.832E+00	-.165E+05
13	.526E+03	.392E+01	.165E+05	-.526E+03	-.392E+01	-.144E+05
14	.545E+03	.874E+01	.144E+05	-.545E+03	-.874E+01	-.956E+04
15	.561E+03	.205E+02	.956E+04	-.561E+03	-.205E+02	.236E+04
16	-.474E+03	.929E+02	.375E+04	.474E+03	-.929E+02	.408E+05
17	-.475E+03	.657E+02	-.408E+05	.475E+03	-.657E+02	.724E+05
18	-.475E+03	.456E+02	-.724E+05	.475E+03	-.456E+02	.942E+05
19	-.475E+03	.239E+02	-.942E+05	.475E+03	-.239E+02	.106E+06
20	-.475E+03	.380E+01	-.106E+06	.475E+03	-.380E+01	.108E+06
21	-.476E+03	-.592E+02	-.108E+06	.476E+03	.592E+02	.791E+05
22	-.476E+03	-.758E+02	-.791E+05	.476E+03	.758E+02	.427E+05
23	-.475E+03	-.953E+02	-.427E+05	.475E+03	.953E+02	-.299E+04
24	-.475E+03	-.117E+03	.299E+04	.475E+03	.117E+03	-.593E+05
25	-.475E+03	-.776E+02	.593E+05	.475E+03	.776E+02	-.966E+05
26	-.475E+03	-.404E+02	.966E+05	.475E+03	.404E+02	-.116E+06
27	-.475E+03	.351E+00	.116E+06	.475E+03	-.351E+00	-.116E+06
28	-.476E+03	.422E+02	.116E+06	.476E+03	-.422E+02	-.956E+05
29	-.476E+03	.818E+02	.956E+05	.476E+03	-.818E+02	-.563E+05
30	-.477E+03	.112E+03	.563E+05	.477E+03	-.112E+03	-.236E+04
31	-.419E+02	.000E+00	.000E+00	.419E+02	.000E+00	.000E+00
32	-.490E+02	.000E+00	.000E+00	.490E+02	.000E+00	.000E+00
33	-.475E+02	.000E+00	.000E+00	.475E+02	.000E+00	.000E+00
34	-.490E+02	.000E+00	.000E+00	.490E+02	.000E+00	.000E+00
35	-.544E+02	.000E+00	.000E+00	.544E+02	.000E+00	.000E+00
36	-.525E+02	.000E+00	.000E+00	.525E+02	.000E+00	.000E+00
37	-.495E+02	.000E+00	.000E+00	.495E+02	.000E+00	.000E+00
38	-.470E+02	.000E+00	.000E+00	.470E+02	.000E+00	.000E+00
39	-.406E+02	.000E+00	.000E+00	.406E+02	.000E+00	.000E+00
40	-.380E+02	.000E+00	.000E+00	.380E+02	.000E+00	.000E+00
41	-.415E+02	.000E+00	.000E+00	.415E+02	.000E+00	.000E+00
42	-.427E+02	.000E+00	.000E+00	.427E+02	.000E+00	.000E+00
43	-.404E+02	.000E+00	.000E+00	.404E+02	.000E+00	.000E+00
44	-.314E+02	.000E+00	.000E+00	.314E+02	.000E+00	.000E+00

Table 22. Computer results for maximum axial force.

MAXIMUM LIVE LOAD AXIAL FORCE

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	-.642E-03	-.158E-01	.502E+03	.000E+00
2	.909E-01	-.232E+00	-.353E-03	.000E+00	.000E+00	.000E+00
3	.168E+00	-.472E+00	-.529E-03	.000E+00	.000E+00	.000E+00
4	.263E+00	-.780E+00	-.659E-03	.000E+00	.000E+00	.000E+00
5	.347E+00	-.114E+01	-.735E-03	.000E+00	.000E+00	.000E+00
6	.404E+00	-.150E+01	-.718E-03	.000E+00	.000E+00	.000E+00
7	.420E+00	-.182E+01	-.561E-03	.000E+00	.000E+00	.000E+00
8	.390E+00	-.202E+01	-.213E-03	.000E+00	.000E+00	.000E+00
9	.342E+00	-.202E+01	.213E-03	.000E+00	.000E+00	.000E+00
10	.313E+00	-.182E+01	.561E-03	.000E+00	.000E+00	.000E+00
11	.329E+00	-.150E+01	.718E-03	.000E+00	.000E+00	.000E+00
12	.386E+00	-.114E+01	.735E-03	.000E+00	.000E+00	.000E+00
13	.470E+00	-.781E+00	.659E-03	.000E+00	.000E+00	.000E+00
14	.565E+00	-.472E+00	.529E-03	.000E+00	.000E+00	.000E+00
15	.642E+00	-.232E+00	.353E-03	.000E+00	.000E+00	.000E+00
16	.733E+00	.000E+00	.642E-03	.000E+00	.502E+03	.000E+00
17	.541E-01	-.334E+00	-.737E-03	.000E+00	.000E+00	.000E+00
18	.103E+00	-.700E+00	-.798E-03	.000E+00	.000E+00	.000E+00
19	.149E+00	-.110E+01	-.862E-03	.000E+00	.000E+00	.000E+00
20	.194E+00	-.152E+01	-.891E-03	.000E+00	.000E+00	.000E+00
21	.242E+00	-.194E+01	-.840E-03	.000E+00	.000E+00	.000E+00
22	.291E+00	-.230E+01	-.651E-03	.000E+00	.000E+00	.000E+00
23	.341E+00	-.253E+01	-.255E-03	.000E+00	.000E+00	.000E+00
24	.391E+00	-.253E+01	.254E-03	.000E+00	.000E+00	.000E+00
25	.442E+00	-.231E+01	.651E-03	.000E+00	.000E+00	.000E+00
26	.491E+00	-.194E+01	.840E-03	.000E+00	.000E+00	.000E+00
27	.539E+00	-.152E+01	.891E-03	.000E+00	.000E+00	.000E+00
28	.584E+00	-.110E+01	.862E-03	.000E+00	.000E+00	.000E+00
29	.630E+00	-.700E+00	.798E-03	.000E+00	.000E+00	.000E+00
30	.679E+00	-.334E+00	.737E-03	.000E+00	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.915E+03	-.109E+02	-.509E+04	-.915E+03	.109E+02	-.122E+04
2	.883E+03	.255E+00	.122E+04	-.883E+03	-.255E+00	-.108E+04
3	.848E+03	.716E+00	.108E+04	-.848E+03	-.716E+00	-.699E+03
4	.819E+03	.828E+00	.699E+03	-.819E+03	-.828E+00	-.273E+03
5	.795E+03	.154E+01	.274E+03	-.795E+03	-.154E+01	.496E+03
6	.778E+03	.228E+01	-.496E+03	-.778E+03	-.228E+01	.161E+04
7	.767E+03	.202E+01	-.161E+04	-.767E+03	-.202E+01	.259E+04
8	.763E+03	.676E-03	-.259E+04	-.763E+03	-.676E-03	.259E+04
9	.767E+03	-.202E+01	-.259E+04	-.767E+03	.202E+01	.161E+04
10	.778E+03	-.228E+01	-.161E+04	-.778E+03	.228E+01	.497E+03
11	.795E+03	-.154E+01	-.496E+03	-.795E+03	.154E+01	-.273E+03
12	.819E+03	-.827E+00	.273E+03	-.819E+03	.827E+00	-.699E+03
13	.848E+03	-.715E+00	.699E+03	-.848E+03	.715E+00	-.108E+04
14	.883E+03	-.256E+00	.108E+04	-.883E+03	.256E+00	-.122E+04
15	.915E+03	.109E+02	.122E+04	-.915E+03	-.109E+02	.509E+04
16	-.763E+03	.636E+01	.509E+04	.763E+03	-.636E+01	-.203E+04
17	-.763E+03	-.264E+01	.203E+04	.763E+03	.264E+01	-.330E+04
18	-.763E+03	.128E+01	.330E+04	.763E+03	-.128E+01	-.269E+04
19	-.763E+03	.559E+01	.269E+04	.763E+03	-.559E+01	-.739E+01
20	-.763E+03	.928E+01	.842E+01	.763E+03	-.928E+01	.444E+04
21	-.763E+03	.130E+02	-.444E+04	.763E+03	-.130E+02	.107E+05
22	-.763E+03	.177E+02	-.107E+05	.763E+03	-.177E+02	.191E+05
23	-.763E+03	-.623E-03	-.191E+05	.763E+03	.623E-03	.191E+05
24	-.763E+03	-.177E+02	-.191E+05	.763E+03	.177E+02	.107E+05
25	-.763E+03	-.130E+02	-.107E+05	.763E+03	.130E+02	.445E+04
26	-.763E+03	-.927E+01	-.445E+04	.763E+03	.927E+01	-.337E+01
27	-.763E+03	-.559E+01	.393E+01	.763E+03	.559E+01	-.269E+04
28	-.763E+03	-.127E+01	.269E+04	.763E+03	.127E+01	-.330E+04
29	-.763E+03	.264E+01	.330E+04	.763E+03	-.264E+01	-.203E+04
30	-.763E+03	-.637E+01	.203E+04	.763E+03	.637E+01	-.509E+04
31	-.605E+02	.000E+00	.000E+00	.605E+02	.000E+00	.000E+00
32	-.735E+02	.000E+00	.000E+00	.735E+02	.000E+00	.000E+00
33	-.739E+02	.000E+00	.000E+00	.739E+02	.000E+00	.000E+00
34	-.733E+02	.000E+00	.000E+00	.733E+02	.000E+00	.000E+00
35	-.733E+02	.000E+00	.000E+00	.733E+02	.000E+00	.000E+00
36	-.743E+02	.000E+00	.000E+00	.743E+02	.000E+00	.000E+00
37	-.760E+02	.000E+00	.000E+00	.760E+02	.000E+00	.000E+00
38	-.760E+02	.000E+00	.000E+00	.760E+02	.000E+00	.000E+00
39	-.743E+02	.000E+00	.000E+00	.743E+02	.000E+00	.000E+00
40	-.733E+02	.000E+00	.000E+00	.733E+02	.000E+00	.000E+00
41	-.733E+02	.000E+00	.000E+00	.733E+02	.000E+00	.000E+00
42	-.739E+02	.000E+00	.000E+00	.739E+02	.000E+00	.000E+00
43	-.735E+02	.000E+00	.000E+00	.735E+02	.000E+00	.000E+00
44	-.605E+02	.000E+00	.000E+00	.605E+02	.000E+00	.000E+00

Table 23. Computer results for preliminary dead load analysis.

PRELIMINARY DEAD LOAD ANALYSIS

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	-.481E-02	-.729E-01	.266E+04	.000E+00
2	.967E+00	-.192E+01	-.315E-02	.000E+00	.000E+00	.000E+00
3	.161E+01	-.357E+01	-.308E-02	.000E+00	.000E+00	.000E+00
4	.206E+01	-.508E+01	-.269E-02	.000E+00	.000E+00	.000E+00
5	.227E+01	-.633E+01	-.214E-02	.000E+00	.000E+00	.000E+00
6	.230E+01	-.729E+01	-.156E-02	.000E+00	.000E+00	.000E+00
7	.219E+01	-.794E+01	-.991E-03	.000E+00	.000E+00	.000E+00
8	.198E+01	-.829E+01	-.343E-03	.000E+00	.000E+00	.000E+00
9	.173E+01	-.829E+01	.342E-03	.000E+00	.000E+00	.000E+00
10	.152E+01	-.795E+01	.990E-03	.000E+00	.000E+00	.000E+00
11	.141E+01	-.729E+01	.156E-02	.000E+00	.000E+00	.000E+00
12	.144E+01	-.633E+01	.214E-02	.000E+00	.000E+00	.000E+00
13	.165E+01	-.508E+01	.269E-02	.000E+00	.000E+00	.000E+00
14	.210E+01	-.357E+01	.308E-02	.000E+00	.000E+00	.000E+00
15	.275E+01	-.192E+01	.315E-02	.000E+00	.000E+00	.000E+00
16	.371E+01	.000E+00	.481E-02	.000E+00	.266E+04	.000E+00
17	.283E+00	-.235E+01	-.482E-02	.000E+00	.000E+00	.000E+00
18	.535E+00	-.453E+01	-.426E-02	.000E+00	.000E+00	.000E+00
19	.767E+00	-.642E+01	-.360E-02	.000E+00	.000E+00	.000E+00
20	.993E+00	-.798E+01	-.288E-02	.000E+00	.000E+00	.000E+00
21	.123E+01	-.917E+01	-.210E-02	.000E+00	.000E+00	.000E+00
22	.148E+01	-.998E+01	-.128E-02	.000E+00	.000E+00	.000E+00
23	.173E+01	-.104E+02	-.427E-03	.000E+00	.000E+00	.000E+00
24	.198E+01	-.104E+02	.426E-03	.000E+00	.000E+00	.000E+00
25	.224E+01	-.998E+01	.128E-02	.000E+00	.000E+00	.000E+00
26	.249E+01	-.917E+01	.210E-02	.000E+00	.000E+00	.000E+00
27	.272E+01	-.798E+01	.288E-02	.000E+00	.000E+00	.000E+00
28	.295E+01	-.642E+01	.360E-02	.000E+00	.000E+00	.000E+00
29	.318E+01	-.453E+01	.426E-02	.000E+00	.000E+00	.000E+00
30	.343E+01	-.235E+01	.482E-02	.000E+00	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Weights - Kips

DEAD LOAD ANALYSIS

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.467E+04	-.195E+02	-.210E+05	-.464E+04	.631E+02	-.296E+04
2	.450E+04	.260E+02	.296E+04	-.447E+04	.153E+02	.194E+02
3	.432E+04	.238E+02	-.178E+02	-.430E+04	.175E+02	.169E+04
4	.416E+04	.197E+02	-.169E+04	-.415E+04	.183E+02	.204E+04
5	.404E+04	.196E+02	-.204E+04	-.403E+04	.184E+02	.235E+04
6	.395E+04	.188E+02	-.234E+04	-.394E+04	.191E+02	.227E+04
7	.389E+04	.183E+02	-.227E+04	-.389E+04	.162E+02	.278E+04
8	.387E+04	.173E+02	-.278E+04	-.387E+04	.172E+02	.278E+04
9	.389E+04	.162E+02	-.278E+04	-.389E+04	.183E+02	.227E+04
10	.394E+04	.191E+02	-.227E+04	-.395E+04	.188E+02	.235E+04
11	.403E+04	.184E+02	-.235E+04	-.404E+04	.196E+02	.205E+04
12	.415E+04	.183E+02	-.205E+04	-.416E+04	.197E+02	.169E+04
13	.430E+04	.175E+02	-.169E+04	-.432E+04	.238E+02	.212E+02
14	.447E+04	.153E+02	-.218E+02	-.450E+04	.260E+02	-.296E+04
15	.464E+04	.631E+02	.296E+04	-.467E+04	-.195E+02	.210E+05
16	-.387E+04	.979E+02	.210E+05	.387E+04	-.647E+02	.180E+05
17	-.387E+04	.376E+02	-.180E+05	.387E+04	-.861E+00	.273E+05
18	-.387E+04	.284E+02	-.273E+05	.387E+04	.108E+02	.315E+05
19	-.387E+04	.223E+02	-.315E+05	.387E+04	.169E+02	.328E+05
20	-.387E+04	.167E+02	-.328E+05	.387E+04	.201E+02	.319E+05
21	-.387E+04	.151E+02	-.319E+05	.387E+04	.193E+02	.309E+05
22	-.387E+04	.162E+02	-.309E+05	.387E+04	.171E+02	.307E+05
23	-.387E+04	.166E+02	-.307E+05	.387E+04	.166E+02	.307E+05
24	-.387E+04	.170E+02	-.307E+05	.387E+04	.162E+02	.309E+05
25	-.387E+04	.193E+02	-.309E+05	.387E+04	.151E+02	.320E+05
26	-.387E+04	.202E+02	-.320E+05	.387E+04	.166E+02	.328E+05
27	-.387E+04	.169E+02	-.328E+05	.387E+04	.223E+02	.315E+05
28	-.387E+04	.108E+02	-.315E+05	.387E+04	.284E+02	.273E+05
29	-.387E+04	-.894E+00	-.273E+05	.387E+04	.377E+02	.180E+05
30	-.387E+04	-.648E+02	-.180E+05	.387E+04	.980E+02	-.210E+05
31	-.254E+03	.000E+00	.000E+00	.253E+03	.000E+00	.000E+00
32	-.309E+03	.000E+00	.000E+00	.308E+03	.000E+00	.000E+00
33	-.313E+03	.000E+00	.000E+00	.311E+03	.000E+00	.000E+00
34	-.314E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
35	-.316E+03	.000E+00	.000E+00	.313E+03	.000E+00	.000E+00
36	-.316E+03	.000E+00	.000E+00	.314E+03	.000E+00	.000E+00
37	-.314E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
38	-.314E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
39	-.316E+03	.000E+00	.000E+00	.314E+03	.000E+00	.000E+00
40	-.316E+03	.000E+00	.000E+00	.313E+03	.000E+00	.000E+00
41	-.314E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
42	-.313E+03	.000E+00	.000E+00	.311E+03	.000E+00	.000E+00
43	-.309E+03	.000E+00	.000E+00	.308E+03	.000E+00	.000E+00
44	-.254E+03	.000E+00	.000E+00	.253E+03	.000E+00	.000E+00

MEMBER	WEIGHT
1	.527E+02
2	.478E+02
3	.459E+02
4	.407E+02
5	.395E+02
6	.386E+02
7	.347E+02
8	.345E+02
9	.347E+02
10	.386E+02
11	.395E+02
12	.407E+02
13	.459E+02
14	.478E+02
15	.527E+02
16	.332E+02
17	.368E+02
18	.392E+02
19	.392E+02
20	.368E+02
21	.344E+02
22	.332E+02
23	.332E+02
24	.332E+02
25	.344E+02
26	.368E+02
27	.392E+02
28	.392E+02
29	.368E+02
30	.332E+02
31	.576E+00
32	.107E+01
33	.148E+01
34	.181E+01
35	.206E+01
36	.222E+01
37	.230E+01
38	.230E+01
39	.222E+01
40	.206E+01
41	.181E+01
42	.148E+01
43	.107E+01
44	.576E+00

Table 24. Computer results for dead load
analysis of rib - unmodified geometry.

RIB WITHOUT MODIFIED GEOMETRY

JOINT DISPLACEMENTS AND SUPPORT REACTIONS						
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	.000E+00	.386E+04	.268E+04	.215E+05
2	.137E+00	-.704E+00	-.195E-02	.000E+00	.000E+00	.000E+00
3	.526E+00	-.191E+01	-.228E-02	.000E+00	.000E+00	.000E+00
4	.766E+00	-.299E+01	-.165E-02	.000E+00	.000E+00	.000E+00
5	.753E+00	-.366E+01	-.790E-03	.000E+00	.000E+00	.000E+00
6	.587E+00	-.395E+01	-.196E-03	.000E+00	.000E+00	.000E+00
7	.367E+00	-.402E+01	.239E-04	.000E+00	.000E+00	.000E+00
8	.121E+00	-.403E+01	.166E-04	.000E+00	.000E+00	.000E+00
9	-.122E+00	-.403E+01	-.187E-04	.000E+00	.000E+00	.000E+00
10	-.368E+00	-.402E+01	-.255E-04	.000E+00	.000E+00	.000E+00
11	-.588E+00	-.395E+01	.195E-03	.000E+00	.000E+00	.000E+00
12	-.754E+00	-.366E+01	.790E-03	.000E+00	.000E+00	.000E+00
13	-.767E+00	-.299E+01	.165E-02	.000E+00	.000E+00	.000E+00
14	-.526E+00	-.192E+01	.228E-02	.000E+00	.000E+00	.000E+00
15	-.137E+00	-.704E+00	.196E-02	.000E+00	.000E+00	.000E+00
16	.000E+00	.000E+00	.000E+00	-.386E+04	.268E+04	-.216E+05

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Weights - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.470E+04	.437E+02	.215E+05	-.467E+04	-.101E+00	-.883E+04
2	.449E+04	.380E+02	.883E+04	-.447E+04	.332E+01	.798E+03
3	.431E+04	.269E+02	-.798E+03	-.429E+04	.144E+02	.414E+04
4	.415E+04	.178E+02	-.414E+04	-.414E+04	.202E+02	.353E+04
5	.403E+04	.142E+02	-.352E+04	-.402E+04	.237E+02	.113E+04
6	.394E+04	.141E+02	-.113E+04	-.393E+04	.239E+02	-.127E+04
7	.388E+04	.166E+02	.126E+04	-.388E+04	.179E+02	-.160E+04
8	.386E+04	.172E+02	.159E+04	-.386E+04	.173E+02	-.159E+04
9	.388E+04	.179E+02	.159E+04	-.388E+04	.166E+02	-.126E+04
10	.393E+04	.239E+02	.126E+04	-.394E+04	.141E+02	.114E+04
11	.402E+04	.238E+02	-.113E+04	-.403E+04	.142E+02	.353E+04
12	.414E+04	.202E+02	-.353E+04	-.415E+04	.178E+02	.415E+04
13	.429E+04	.144E+02	-.415E+04	-.431E+04	.269E+02	.798E+03
14	.447E+04	.330E+01	-.802E+03	-.449E+04	.380E+02	-.884E+04
15	.467E+04	-.118E+00	.884E+04	-.470E+04	.437E+02	-.216E+05

DEAD LOAD ANALYSIS

MEMBER	WEIGHT
1	.527E+02
2	.478E+02
3	.459E+02
4	.407E+02
5	.395E+02
6	.386E+02
7	.347E+02
8	.345E+02
9	.347E+02
10	.386E+02
11	.395E+02
12	.407E+02
13	.459E+02
14	.478E+02
15	.527E+02

Table 25. Computer results for dead load
analysis of rib - modified geometry.

RIB WITH MODIFIED GEOMETRY

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	.000E+00	.387E+04	.268E+04	.125E+05
2	-.198E-01	-.473E+00	-.119E-02	.000E+00	.000E+00	.000E+00
3	.172E+00	-.134E+01	-.178E-02	.000E+00	.000E+00	.000E+00
4	.381E+00	-.235E+01	-.190E-02	.000E+00	.000E+00	.000E+00
5	.485E+00	-.333E+01	-.173E-02	.000E+00	.000E+00	.000E+00
6	.469E+00	-.414E+01	-.134E-02	.000E+00	.000E+00	.000E+00
7	.343E+00	-.471E+01	-.817E-03	.000E+00	.000E+00	.000E+00
8	.121E+00	-.497E+01	-.171E-03	.000E+00	.000E+00	.000E+00
9	-.122E+00	-.497E+01	.169E-03	.000E+00	.000E+00	.000E+00
10	-.344E+00	-.471E+01	.815E-03	.000E+00	.000E+00	.000E+00
11	-.470E+00	-.414E+01	.134E-02	.000E+00	.000E+00	.000E+00
12	-.486E+00	-.333E+01	.173E-02	.000E+00	.000E+00	.000E+00
13	-.383E+00	-.235E+01	.191E-02	.000E+00	.000E+00	.000E+00
14	-.173E+00	-.135E+01	.178E-02	.000E+00	.000E+00	.000E+00
15	.195E-01	-.474E+00	.119E-02	.000E+00	.000E+00	.000E+00
16	.000E+00	.000E+00	.000E+00	-.387E+04	.268E+04	-.125E+05
17	-.545E-03	-.499E+01	-.132E-05	.000E+00	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Weights - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.471E+04	.302E+02	.125E+05	-.468E+04	.134E+02	-.759E+04
2	.450E+04	.271E+02	.759E+04	-.447E+04	.142E+02	-.399E+04
3	.431E+04	.255E+02	.399E+04	-.429E+04	.159E+02	-.143E+04
4	.416E+04	.224E+02	.143E+04	-.414E+04	.155E+02	.366E+03
5	.403E+04	.215E+02	-.366E+03	-.402E+04	.164E+02	.163E+04
6	.395E+04	.204E+02	-.163E+04	-.394E+04	.175E+02	.233E+04
7	.389E+04	.180E+02	-.233E+04	-.389E+04	.165E+02	.269E+04
8	.387E+04	.472E+00	-.270E+04	-.387E+04	.168E+02	.740E+03
9	.387E+04	.167E+02	-.749E+03	-.387E+04	.535E+00	.269E+04
10	.389E+04	.165E+02	-.270E+04	-.389E+04	.180E+02	.233E+04
11	.394E+04	.175E+02	-.233E+04	-.395E+04	.204E+02	.164E+04
12	.402E+04	.164E+02	-.164E+04	-.403E+04	.215E+02	.374E+03
13	.414E+04	.155E+02	-.374E+03	-.416E+04	.224E+02	-.142E+04
14	.429E+04	.158E+02	.142E+04	-.431E+04	.255E+02	-.399E+04
15	.447E+04	.142E+02	.399E+04	-.450E+04	.272E+02	-.760E+04
16	.468E+04	.134E+02	.760E+04	-.471E+04	.302E+02	-.125E+05

DEAD LOAD ANALYSIS

MEMBER	WEIGHT
1	.528E+02
2	.479E+02
3	.459E+02
4	.406E+02
5	.395E+02
6	.386E+02
7	.347E+02
8	.173E+02
9	.173E+02
10	.347E+02
11	.386E+02
12	.395E+02
13	.406E+02
14	.459E+02
15	.479E+02
16	.528E+02

Table 26. Computer results for dead load analysis of rib - modified geometry. Axial shortening effects eliminated.

RIB WITH MODIFIED GEOMETRY AND MEMBER END ACTIONS

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.469E+04	.000E+00	.000E+00	-.469E+04	.000E+00	.000E+00
2	.448E+04	.000E+00	.000E+00	-.448E+04	.000E+00	.000E+00
3	.430E+04	.000E+00	.000E+00	-.430E+04	.000E+00	.000E+00
4	.415E+04	.000E+00	.000E+00	-.415E+04	.000E+00	.000E+00
5	.402E+04	.000E+00	.000E+00	-.402E+04	.000E+00	.000E+00
6	.394E+04	.000E+00	.000E+00	-.394E+04	.000E+00	.000E+00
7	.389E+04	.000E+00	.000E+00	-.389E+04	.000E+00	.000E+00
8	.387E+04	.000E+00	.000E+00	-.387E+04	.000E+00	.000E+00
9	.387E+04	.000E+00	.000E+00	-.387E+04	.000E+00	.000E+00
10	.389E+04	.000E+00	.000E+00	-.389E+04	.000E+00	.000E+00
11	.394E+04	.000E+00	.000E+00	-.394E+04	.000E+00	.000E+00
12	.402E+04	.000E+00	.000E+00	-.402E+04	.000E+00	.000E+00
13	.415E+04	.000E+00	.000E+00	-.415E+04	.000E+00	.000E+00
14	.430E+04	.000E+00	.000E+00	-.430E+04	.000E+00	.000E+00
15	.448E+04	.000E+00	.000E+00	-.448E+04	.000E+00	.000E+00
16	.469E+04	.000E+00	.000E+00	-.469E+04	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Weights - Kips

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	.000E+00	.388E+04	.268E+04	.368E+04
2	.250E-01	-.371E-01	-.114E-03	.000E+00	.000E+00	.000E+00
3	.541E-01	-.882E-01	-.775E-04	.000E+00	.000E+00	.000E+00
4	.607E-01	-.103E+00	.215E-04	.000E+00	.000E+00	.000E+00
5	.480E-01	-.722E-01	.108E-03	.000E+00	.000E+00	.000E+00
6	.277E-01	-.477E-02	.171E-03	.000E+00	.000E+00	.000E+00
7	.990E-02	.857E-01	.201E-03	.000E+00	.000E+00	.000E+00
8	.450E-03	.178E+00	.181E-03	.000E+00	.000E+00	.000E+00
9	-.456E-03	.178E+00	-.181E-03	.000E+00	.000E+00	.000E+00
10	-.991E-02	.857E-01	-.201E-03	.000E+00	.000E+00	.000E+00
11	-.277E-01	-.478E-02	-.171E-03	.000E+00	.000E+00	.000E+00
12	-.480E-01	-.722E-01	-.108E-03	.000E+00	.000E+00	.000E+00
13	-.607E-01	-.103E+00	-.214E-04	.000E+00	.000E+00	.000E+00
14	-.541E-01	-.882E-01	.775E-04	.000E+00	.000E+00	.000E+00
15	-.250E-01	-.371E-01	.114E-03	.000E+00	.000E+00	.000E+00
16	.000E+00	.000E+00	.000E+00	-.388E+04	.268E+04	-.368E+04
17	-.279E-05	.203E+00	-.633E-08	.000E+00	.000E+00	.000E+00

DEAD LOAD ANALYSIS

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.471E+04	.246E+02	.368E+04	-.468E+04	.190E+02	-.207E+04
2	.451E+04	.221E+02	.207E+04	-.448E+04	.192E+02	-.128E+04
3	.432E+04	.211E+02	.128E+04	-.430E+04	.202E+02	-.104E+04
4	.417E+04	.188E+02	.104E+04	-.415E+04	.191E+02	-.110E+04
5	.404E+04	.187E+02	.110E+04	-.403E+04	.192E+02	-.123E+04
6	.396E+04	.185E+02	.123E+04	-.395E+04	.194E+02	-.146E+04
7	.390E+04	.170E+02	.146E+04	-.390E+04	.175E+02	-.156E+04
8	.388E+04	.435E+00	.156E+04	-.388E+04	.168E+02	-.352E+04
9	.388E+04	.168E+02	.352E+04	-.388E+04	.435E+00	-.156E+04
10	.390E+04	.175E+02	.156E+04	-.390E+04	.170E+02	-.146E+04
11	.395E+04	.194E+02	.146E+04	-.396E+04	.185E+02	-.123E+04
12	.403E+04	.192E+02	.123E+04	-.404E+04	.187E+02	-.110E+04
13	.415E+04	.191E+02	.110E+04	-.417E+04	.188E+02	-.104E+04
14	.430E+04	.202E+02	.104E+04	-.432E+04	.211E+02	-.128E+04
15	.448E+04	.192E+02	.128E+04	-.451E+04	.221E+02	-.207E+04
16	.468E+04	.190E+02	.207E+04	-.471E+04	.246E+02	-.368E+04

MEMBER WEIGHT

MEMBER	WEIGHT
1	.528E+02
2	.479E+02
3	.459E+02
4	.406E+02
5	.395E+02
6	.386E+02
7	.347E+02
8	.173E+02
9	.173E+02
10	.347E+02
11	.386E+02
12	.395E+02
13	.406E+02
14	.459E+02
15	.479E+02
16	.528E+02

Table 27. Computer results for dead load analysis of tie.
Axial lengthening effects eliminated.

TIE

JOINT DISPLACEMENTS AND SUPPORT REACTIONS

JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT
1	.000E+00	.000E+00	.000E+00	-.624E-03	-.318E+02	.167E+04
2	.447E-04	-.201E-02	-.808E-05	.000E+00	.000E+00	.000E+00
3	.105E-03	-.647E-02	-.102E-04	.000E+00	.000E+00	.000E+00
4	.155E-03	-.111E-01	-.868E-05	.000E+00	.000E+00	.000E+00
5	.185E-03	-.146E-01	-.578E-05	.000E+00	.000E+00	.000E+00
6	.201E-03	-.169E-01	-.358E-05	.000E+00	.000E+00	.000E+00
7	.207E-03	-.183E-01	-.205E-05	.000E+00	.000E+00	.000E+00
8	.209E-03	-.190E-01	-.736E-06	.000E+00	.000E+00	.000E+00
9	.209E-03	-.190E-01	.736E-06	.000E+00	.000E+00	.000E+00
10	.211E-03	-.183E-01	.205E-05	.000E+00	.000E+00	.000E+00
11	.217E-03	-.169E-01	.358E-05	.000E+00	.000E+00	.000E+00
12	.233E-03	-.146E-01	.578E-05	.000E+00	.000E+00	.000E+00
13	.263E-03	-.111E-01	.868E-05	.000E+00	.000E+00	.000E+00
14	.313E-03	-.647E-02	.102E-04	.000E+00	.000E+00	.000E+00
15	.373E-03	-.201E-02	.808E-05	.000E+00	.000E+00	.000E+00
16	.418E-03	.000E+00	.000E+00	.000E+00	-.318E+02	-.167E+04

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Weights - Kips

MEMBER END ACTIONS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	-.390E+04	.168E+02	.167E+04	.390E+04	.165E+02	-.160E+04
2	-.390E+04	.185E+02	.160E+04	.390E+04	.183E+02	-.153E+04
3	-.390E+04	.197E+02	.153E+04	.390E+04	.194E+02	-.146E+04
4	-.390E+04	.197E+02	.146E+04	.390E+04	.195E+02	-.141E+04
5	-.390E+04	.185E+02	.141E+04	.390E+04	.183E+02	-.134E+04
6	-.390E+04	.173E+02	.134E+04	.390E+04	.171E+02	-.128E+04
7	-.390E+04	.166E+02	.128E+04	.390E+04	.166E+02	-.127E+04
8	-.390E+04	.166E+02	.127E+04	.390E+04	.166E+02	-.127E+04
9	-.390E+04	.166E+02	.127E+04	.390E+04	.166E+02	-.128E+04
10	-.390E+04	.171E+02	.128E+04	.390E+04	.173E+02	-.134E+04
11	-.390E+04	.183E+02	.134E+04	.390E+04	.185E+02	-.141E+04
12	-.390E+04	.195E+02	.141E+04	.390E+04	.197E+02	-.146E+04
13	-.390E+04	.194E+02	.146E+04	.390E+04	.197E+02	-.153E+04
14	-.390E+04	.183E+02	.153E+04	.390E+04	.185E+02	-.160E+04
15	-.390E+04	.165E+02	.160E+04	.390E+04	.168E+02	-.167E+04

DEAD LOAD ANALYSIS

MEMBER	WEIGHT
1	.332E+02
2	.368E+02
3	.392E+02
4	.392E+02
5	.368E+02
6	.344E+02
7	.332E+02
8	.332E+02
9	.332E+02
10	.344E+02
11	.368E+02
12	.392E+02
13	.392E+02
14	.368E+02
15	.332E+02

Table 28. Computer results for final dead load analysis
of entire arch - modified geometry.
Axial deformation effects eliminated.

FINAL DEAD LOAD ANALYSIS

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K
1	.469E+04	.000E+00	.000E+00	-.469E+04	.000E+00	.000E+00
2	.448E+04	.000E+00	.000E+00	-.448E+04	.000E+00	.000E+00
3	.430E+04	.000E+00	.000E+00	-.430E+04	.000E+00	.000E+00
4	.415E+04	.000E+00	.000E+00	-.415E+04	.000E+00	.000E+00
5	.402E+04	.000E+00	.000E+00	-.402E+04	.000E+00	.000E+00
6	.394E+04	.000E+00	.000E+00	-.394E+04	.000E+00	.000E+00
7	.389E+04	.000E+00	.000E+00	-.389E+04	.000E+00	.000E+00
8	.387E+04	.000E+00	.000E+00	-.387E+04	.000E+00	.000E+00
9	.389E+04	.000E+00	.000E+00	-.389E+04	.000E+00	.000E+00
10	.394E+04	.000E+00	.000E+00	-.394E+04	.000E+00	.000E+00
11	.402E+04	.000E+00	.000E+00	-.402E+04	.000E+00	.000E+00
12	.415E+04	.000E+00	.000E+00	-.415E+04	.000E+00	.000E+00
13	.430E+04	.000E+00	.000E+00	-.430E+04	.000E+00	.000E+00
14	.448E+04	.000E+00	.000E+00	-.448E+04	.000E+00	.000E+00
15	.469E+04	.000E+00	.000E+00	-.469E+04	.000E+00	.000E+00
16	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
17	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
18	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
19	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
20	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
21	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
22	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
23	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
24	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
25	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
26	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
27	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
28	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
29	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
30	-.390E+04	.000E+00	.000E+00	.390E+04	.000E+00	.000E+00
31	-.336E+03	.000E+00	.000E+00	.336E+03	.000E+00	.000E+00
32	-.344E+03	.000E+00	.000E+00	.344E+03	.000E+00	.000E+00
33	-.339E+03	.000E+00	.000E+00	.339E+03	.000E+00	.000E+00
34	-.327E+03	.000E+00	.000E+00	.327E+03	.000E+00	.000E+00
35	-.312E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
36	-.296E+03	.000E+00	.000E+00	.296E+03	.000E+00	.000E+00
37	-.282E+03	.000E+00	.000E+00	.282E+03	.000E+00	.000E+00
38	-.282E+03	.000E+00	.000E+00	.282E+03	.000E+00	.000E+00
39	-.296E+03	.000E+00	.000E+00	.296E+03	.000E+00	.000E+00
40	-.312E+03	.000E+00	.000E+00	.312E+03	.000E+00	.000E+00
41	-.327E+03	.000E+00	.000E+00	.327E+03	.000E+00	.000E+00
42	-.339E+03	.000E+00	.000E+00	.339E+03	.000E+00	.000E+00
43	-.344E+03	.000E+00	.000E+00	.344E+03	.000E+00	.000E+00
44	-.336E+03	.000E+00	.000E+00	.336E+03	.000E+00	.000E+00

UNITS:

Displacements - Inches

Shears - Kips

Moments - Inch-kips

Axial - Kips

Table 28.-- Continued. Computer results for final dead load analysis of entire arch - modified geometry. Axial deformation effects eliminated.

JOINT DISPLACEMENTS AND SUPPORT REACTIONS							UNITS:
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT	
1	.000E+00	.000E+00	-.127E-03	.815E-04	.266E+04	.000E+00	Displacements - Inches
2	.467E-01	-.720E-01	-.142E-03	.000E+00	.000E+00	.000E+00	Shears - Kips
3	.759E-01	-.127E+00	-.667E-04	.000E+00	.000E+00	.000E+00	Moments - Inch-kips
4	.769E-01	-.134E+00	.431E-04	.000E+00	.000E+00	.000E+00	
5	.586E-01	-.928E-01	.130E-03	.000E+00	.000E+00	.000E+00	Axial - Kips
6	.339E-01	-.159E-01	.190E-03	.000E+00	.000E+00	.000E+00	Weights - Kips
7	.136E-01	.803E-01	.202E-03	.000E+00	.000E+00	.000E+00	
8	.423E-02	.155E+00	.888E-04	.000E+00	.000E+00	.000E+00	
9	.200E-02	.155E+00	-.888E-04	.000E+00	.000E+00	.000E+00	
10	-.733E-02	.803E-01	-.202E-03	.000E+00	.000E+00	.000E+00	
11	-.277E-01	-.159E-01	-.190E-03	.000E+00	.000E+00	.000E+00	
12	-.523E-01	-.928E-01	-.130E-03	.000E+00	.000E+00	.000E+00	
13	-.706E-01	-.134E+00	-.431E-04	.000E+00	.000E+00	.000E+00	
14	-.696E-01	-.127E+00	.667E-04	.000E+00	.000E+00	.000E+00	
15	-.404E-01	-.720E-01	.142E-03	.000E+00	.000E+00	.000E+00	
16	.623E-02	.000E+00	.127E-03	.000E+00	.266E+04	.000E+00	
17	.874E-03	-.393E-01	-.444E-04	.000E+00	.000E+00	.000E+00	
18	.133E-02	-.501E-01	-.585E-05	.000E+00	.000E+00	.000E+00	
19	.164E-02	-.485E-01	.972E-05	.000E+00	.000E+00	.000E+00	
20	.191E-02	-.429E-01	.122E-04	.000E+00	.000E+00	.000E+00	
21	.222E-02	-.384E-01	.573E-05	.000E+00	.000E+00	.000E+00	
22	.257E-02	-.381E-01	-.379E-05	.000E+00	.000E+00	.000E+00	
23	.294E-02	-.407E-01	-.418E-05	.000E+00	.000E+00	.000E+00	
24	.330E-02	-.407E-01	.417E-05	.000E+00	.000E+00	.000E+00	
25	.367E-02	-.381E-01	.378E-05	.000E+00	.000E+00	.000E+00	
26	.402E-02	-.384E-01	-.573E-05	.000E+00	.000E+00	.000E+00	
27	.433E-02	-.429E-01	-.123E-04	.000E+00	.000E+00	.000E+00	
28	.460E-02	-.485E-01	-.972E-05	.000E+00	.000E+00	.000E+00	
29	.490E-02	-.501E-01	.585E-05	.000E+00	.000E+00	.000E+00	
30	.536E-02	-.393E-01	.444E-04	.000E+00	.000E+00	.000E+00	

MEMBER END ACTIONS

DEAD LOAD ANALYSIS

MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K	MEMBER	WEIGHT
1	.474E+04	.236E+02	.272E+04	-.471E+04	.200E+02	-.170E+04	1	-.528E+02
2	.454E+04	.217E+02	.170E+04	-.451E+04	.197E+02	-.114E+04	2	-.479E+02
3	.435E+04	.209E+02	.114E+04	-.433E+04	.204E+02	-.103E+04	3	.459E+02
4	.419E+04	.188E+02	.103E+04	-.418E+04	.191E+02	-.112E+04	4	.406E+02
5	.407E+04	.187E+02	.112E+04	-.406E+04	.192E+02	-.125E+04	5	.395E+02
6	.398E+04	.181E+02	.125E+04	-.397E+04	.198E+02	-.168E+04	6	.386E+02
7	.393E+04	.156E+02	.168E+04	-.392E+04	.189E+02	-.246E+04	7	.347E+02
8	.391E+04	.173E+02	.246E+04	-.391E+04	.173E+02	-.246E+04	8	.345E+02
9	.392E+04	.189E+02	.246E+04	-.393E+04	.156E+02	-.168E+04	9	.347E+02
10	.397E+04	.198E+02	.168E+04	-.398E+04	.181E+02	-.125E+04	10	.386E+02
11	.406E+04	.192E+02	.125E+04	-.407E+04	.187E+02	-.112E+04	11	.395E+02
12	.418E+04	.191E+02	.112E+04	-.419E+04	.188E+02	-.103E+04	12	.406E+02
13	.433E+04	.204E+02	.103E+04	-.435E+04	.209E+02	-.114E+04	13	.459E+02
14	.451E+04	.197E+02	.114E+04	-.454E+04	.217E+02	-.170E+04	14	.479E+02
15	.471E+04	.200E+02	.170E+04	-.474E+04	.236E+02	-.272E+04	15	.528E+02
16	-.391E+04	.127E+02	-.272E+04	.391E+04	.205E+02	.868E+03	16	.332E+02
17	-.391E+04	.156E+02	-.868E+03	.391E+04	.211E+02	-.452E+03	17	.368E+02
18	-.391E+04	.180E+02	.452E+03	.391E+04	.212E+02	-.123E+04	18	.392E+02
19	-.391E+04	.187E+02	.123E+04	.391E+04	.205E+02	-.167E+04	19	.392E+02
20	-.391E+04	.180E+02	.167E+04	.391E+04	.187E+02	-.184E+04	20	.368E+02
21	-.391E+04	.176E+02	.184E+04	.391E+04	.169E+02	-.167E+04	21	.344E+02
22	-.391E+04	.180E+02	.167E+04	.391E+04	.152E+02	-.101E+04	22	.332E+02
23	-.391E+04	.166E+02	.101E+04	.391E+04	.166E+02	-.101E+04	23	.332E+02
24	-.391E+04	.152E+02	.101E+04	.391E+04	.180E+02	-.167E+04	24	.332E+02
25	-.391E+04	.169E+02	.167E+04	.391E+04	.176E+02	-.184E+04	25	.344E+02
26	-.391E+04	.187E+02	.184E+04	.391E+04	.180E+02	-.167E+04	26	.368E+02
27	-.391E+04	.205E+02	.167E+04	.391E+04	.187E+02	-.123E+04	27	.392E+02
28	-.391E+04	.212E+02	.123E+04	.391E+04	.180E+02	-.453E+03	28	.392E+02
29	-.391E+04	.211E+02	.451E+03	.391E+04	.156E+02	.867E+03	29	.368E+02
30	-.391E+04	.205E+02	-.869E+03	.391E+04	.127E+02	.272E+04	30	.332E+02
31	-.317E+03	.000E+00	.000E+00	.316E+03	.000E+00	.000E+00	31	.578E+00
32	-.320E+03	.000E+00	.000E+00	.319E+03	.000E+00	.000E+00	32	.107E+01
33	-.320E+03	.000E+00	.000E+00	.318E+03	.000E+00	.000E+00	33	.148E+01
34	-.319E+03	.000E+00	.000E+00	.317E+03	.000E+00	.000E+00	34	.181E+01
35	-.317E+03	.000E+00	.000E+00	.315E+03	.000E+00	.000E+00	35	.206E+01
36	-.315E+03	.000E+00	.000E+00	.313E+03	.000E+00	.000E+00	36	.222E+01
37	-.312E+03	.000E+00	.000E+00	.310E+03	.000E+00	.000E+00	37	.230E+01
38	-.312E+03	.000E+00	.000E+00	.310E+03	.000E+00	.000E+00	38	.230E+01
39	-.315E+03	.000E+00	.000E+00	.313E+03	.000E+00	.000E+00	39	.222E+01
40	-.317E+03	.000E+00	.000E+00	.315E+03	.000E+00	.000E+00	40	.206E+01
41	-.319E+03	.000E+00	.000E+00	.317E+03	.000E+00	.000E+00	41	.181E+01
42	-.320E+03	.000E+00	.000E+00	.318E+03	.000E+00	.000E+00	42	.148E+01
43	-.320E+03	.000E+00	.000E+00	.319E+03	.000E+00	.000E+00	43	.107E+01
44	-.317E+03	.000E+00	.000E+00	.316E+03	.000E+00	.000E+00	44	.578E+00

Table 29. Computer results for maximum arch deflection.
Half-span loading with concentrated load at
quarter point.

JOINT DISPLACEMENTS AND SUPPORT REACTIONS							UNITS:
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT	
1	.000E+00	.000E+00	-.705E-02	.950E-02	.304E+04	.000E+00	Displacements - Inches
2	.217E+01	-.324E+01	-.624E-02	.000E+00	.000E+00	.000E+00	Shears - Kips
3	.368E+01	-.589E+01	-.460E-02	.000E+00	.000E+00	.000E+00	Moments - Inch-kips
4	.448E+01	-.759E+01	-.235E-02	.000E+00	.000E+00	.000E+00	Axial - Kips
5	.463E+01	-.807E+01	.428E-03	.000E+00	.000E+00	.000E+00	
6	.436E+01	-.721E+01	.315E-02	.000E+00	.000E+00	.000E+00	
7	.394E+01	-.515E+01	.533E-02	.000E+00	.000E+00	.000E+00	
8	.363E+01	-.224E+01	.659E-02	.000E+00	.000E+00	.000E+00	
9	.360E+01	.943E+00	.642E-02	.000E+00	.000E+00	.000E+00	
10	.384E+01	.372E+01	.498E-02	.000E+00	.000E+00	.000E+00	
11	.419E+01	.566E+01	.297E-02	.000E+00	.000E+00	.000E+00	
12	.441E+01	.653E+01	.609E-03	.000E+00	.000E+00	.000E+00	
13	.427E+01	.625E+01	-.178E-02	.000E+00	.000E+00	.000E+00	
14	.360E+01	.492E+01	-.376E-02	.000E+00	.000E+00	.000E+00	
15	.228E+01	.273E+01	-.534E-02	.000E+00	.000E+00	.000E+00	
16	.385E+00	.000E+00	-.585E-02	.000E+00	.280E+04	.000E+00	
17	.667E-01	-.327E+01	-.634E-02	.000E+00	.000E+00	.000E+00	
18	.119E+00	-.595E+01	-.469E-02	.000E+00	.000E+00	.000E+00	
19	.157E+00	-.769E+01	-.247E-02	.000E+00	.000E+00	.000E+00	
20	.183E+00	-.826E+01	.155E-03	.000E+00	.000E+00	.000E+00	
21	.202E+00	-.751E+01	.287E-02	.000E+00	.000E+00	.000E+00	
22	.221E+00	-.555E+01	.515E-02	.000E+00	.000E+00	.000E+00	
23	.242E+00	-.270E+01	.655E-02	.000E+00	.000E+00	.000E+00	
24	.268E+00	.514E+00	.658E-02	.000E+00	.000E+00	.000E+00	
25	.299E+00	.340E+01	.527E-02	.000E+00	.000E+00	.000E+00	
26	.332E+00	.545E+01	.315E-02	.000E+00	.000E+00	.000E+00	
27	.360E+00	.640E+01	.734E-03	.000E+00	.000E+00	.000E+00	
28	.381E+00	.618E+01	-.161E-02	.000E+00	.000E+00	.000E+00	
29	.392E+00	.489E+01	-.369E-02	.000E+00	.000E+00	.000E+00	
30	.392E+00	.272E+01	-.524E-02	.000E+00	.000E+00	.000E+00	
MEMBER END ACTIONS							
MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K	
1	.523E+04	.310E+02	-.634E+03	-.520E+04	.126E+02	.599E+04	
2	.500E+04	.310E+02	-.599E+04	-.497E+04	.103E+02	.118E+05	
3	.479E+04	.273E+02	-.118E+05	-.477E+04	.140E+02	.153E+05	
4	.462E+04	.221E+02	-.153E+05	-.460E+04	.158E+02	.169E+05	
5	.448E+04	.165E+02	-.169E+05	-.447E+04	.214E+02	.157E+05	
6	.438E+04	.827E+01	-.157E+05	-.437E+04	.296E+02	.105E+05	
7	.432E+04	-.520E+00	-.105E+05	-.432E+04	.350E+02	.193E+04	
8	.430E+04	-.980E+00	-.193E+04	-.430E+04	.355E+02	-.682E+04	
9	.432E+04	.393E+01	.682E+04	-.432E+04	.306E+02	-.132E+05	
10	.438E+04	.115E+02	.132E+05	-.438E+04	.264E+02	-.169E+05	
11	.447E+04	.179E+02	.169E+05	-.448E+04	.200E+02	-.174E+05	
12	.460E+04	.209E+02	.174E+05	-.462E+04	.170E+02	-.164E+05	
13	.477E+04	.245E+02	.164E+05	-.479E+04	.168E+02	-.143E+05	
14	.496E+04	.280E+02	.143E+05	-.499E+04	.133E+02	-.103E+05	
15	.518E+04	.381E+02	.103E+05	-.521E+04	.549E+01	-.781E+03	
16	-.430E+04	.126E+03	.634E+03	.430E+04	-.925E+02	.517E+05	
17	-.430E+04	.964E+02	-.517E+05	.430E+04	-.596E+02	.892E+05	
18	-.430E+04	.727E+02	-.892E+05	.430E+04	-.335E+02	.115E+06	
19	-.430E+04	.468E+02	-.115E+06	.430E+04	-.762E+01	.128E+06	
20	-.430E+04	-.269E+02	-.128E+06	.430E+04	.637E+02	.106E+06	
21	-.430E+04	-.506E+02	-.106E+06	.430E+04	.850E+02	.734E+05	
22	-.430E+04	-.753E+02	-.734E+05	.430E+04	.108E+03	.293E+05	
23	-.430E+04	-.106E+03	-.293E+05	.430E+04	.139E+03	-.293E+05	
24	-.430E+04	-.729E+02	.293E+05	.430E+04	.106E+03	-.723E+05	
25	-.430E+04	-.408E+02	.723E+05	.430E+04	.752E+02	-.100E+06	
26	-.430E+04	-.858E+01	.100E+06	.430E+04	.454E+02	-.113E+06	
27	-.430E+04	.277E+02	.113E+06	.430E+04	.115E+02	-.109E+06	
28	-.430E+04	.639E+02	.109E+06	.430E+04	-.248E+02	-.879E+05	
29	-.430E+04	.974E+02	.879E+05	.430E+04	-.606E+02	-.500E+05	
30	-.430E+04	.122E+03	.500E+05	.430E+04	-.891E+02	.779E+03	
31	-.354E+03	.000E+00	.000E+00	.353E+03	.000E+00	.000E+00	
32	-.363E+03	.000E+00	.000E+00	.362E+03	.000E+00	.000E+00	
33	-.362E+03	.000E+00	.000E+00	.361E+03	.000E+00	.000E+00	
34	-.363E+03	.000E+00	.000E+00	.361E+03	.000E+00	.000E+00	
35	-.362E+03	.000E+00	.000E+00	.360E+03	.000E+00	.000E+00	
36	-.359E+03	.000E+00	.000E+00	.357E+03	.000E+00	.000E+00	
37	-.352E+03	.000E+00	.000E+00	.350E+03	.000E+00	.000E+00	
38	-.347E+03	.000E+00	.000E+00	.345E+03	.000E+00	.000E+00	
39	-.346E+03	.000E+00	.000E+00	.344E+03	.000E+00	.000E+00	
40	-.348E+03	.000E+00	.000E+00	.345E+03	.000E+00	.000E+00	
41	-.354E+03	.000E+00	.000E+00	.352E+03	.000E+00	.000E+00	
42	-.356E+03	.000E+00	.000E+00	.354E+03	.000E+00	.000E+00	
43	-.355E+03	.000E+00	.000E+00	.354E+03	.000E+00	.000E+00	
44	-.343E+03	.000E+00	.000E+00	.343E+03	.000E+00	.000E+00	

Table 30. One half of the influence coefficient matrix
for an adjustment analysis.

INFLUENCE COEFFICIENT MATRIX (Kips/inch)

.204E+03	-.959E+02	-.220E+02	-.220E+02	.102E+01	.102E+01	.147E+01	.147E+01	.250E+00	.250E+00	-.633E-01	-.633E-01	-.380E-01	-.380E-01
-.959E+02	.204E+03	-.220E+02	-.220E+02	.102E+01	.102E+01	.147E+01	.147E+01	.250E+00	.250E+00	-.633E-01	-.633E-01	-.380E-01	-.380E-01
-.220E+02	-.220E+02	.110E+03	-.511E+02	-.152E+02	-.152E+02	.313E+00	.313E+00	.119E+01	.119E+01	.279E+00	.279E+00	-.351E-01	-.351E-01
-.220E+02	-.220E+02	-.511E+02	.110E+03	-.152E+02	-.152E+02	.313E+00	.313E+00	.119E+01	.119E+01	.279E+00	.279E+00	-.351E-01	-.351E-01
.102E+01	.102E+01	-.152E+02	-.152E+02	.820E+02	-.348E+02	-.123E+02	-.123E+02	-.125E+00	-.125E+00	.106E+01	.106E+01	.310E+00	.310E+00
.102E+01	.102E+01	-.152E+02	-.152E+02	-.348E+02	.820E+02	-.123E+02	-.123E+02	-.125E+00	-.125E+00	.106E+01	.106E+01	.310E+00	.310E+00
.147E+01	.147E+01	.313E+00	.313E+00	-.123E+02	-.123E+02	.683E+02	-.274E+02	-.107E+02	-.107E+02	-.293E+00	-.293E+00	.979E+00	.979E+00
.147E+01	.147E+01	.313E+00	.313E+00	-.123E+02	-.123E+02	-.274E+02	.683E+02	-.107E+02	-.107E+02	-.293E+00	-.293E+00	.979E+00	.979E+00
.250E+00	.250E+00	.119E+01	.119E+01	-.125E+00	-.125E+00	-.107E+02	-.107E+02	.608E+02	-.235E+02	-.978E+01	-.978E+01	-.372E+00	-.372E+00
.250E+00	.250E+00	.119E+01	.119E+01	-.125E+00	-.125E+00	-.107E+02	-.107E+02	-.235E+02	.608E+02	-.978E+01	-.978E+01	-.372E+00	-.372E+00
-.633E-01	-.633E-01	.279E+00	.279E+00	.106E+01	.106E+01	-.293E+00	-.293E+00	-.978E+01	-.978E+01	.565E+02	-.216E+02	-.930E+01	-.930E+01
-.633E-01	-.633E-01	.279E+00	.279E+00	.106E+01	.106E+01	-.293E+00	-.293E+00	-.978E+01	-.978E+01	-.216E+02	.565E+02	-.930E+01	-.930E+01
-.380E-01	-.380E-01	-.351E-01	-.351E-01	.310E+00	.310E+00	.979E+00	.979E+00	-.372E+00	-.372E+00	-.930E+01	-.930E+01	.545E+02	-.208E+02
-.380E-01	-.380E-01	-.351E-01	-.351E-01	.310E+00	.310E+00	.979E+00	.979E+00	-.372E+00	-.372E+00	-.930E+01	-.930E+01	-.208E+02	.545E+02
-.295E-03	-.295E-03	-.322E-01	-.322E-01	-.151E-01	-.151E-01	.301E+00	.301E+00	.877E+00	.877E+00	-.365E+00	-.365E+00	-.906E+01	-.906E+01
-.295E-03	-.295E-03	-.322E-01	-.322E-01	-.151E-01	-.151E-01	.301E+00	.301E+00	.877E+00	.877E+00	-.365E+00	-.365E+00	-.906E+01	-.906E+01
.677E-02	.677E-02	-.141E-02	-.141E-02	-.320E-01	-.320E-01	-.128E-01	-.128E-01	.290E+00	.290E+00	.874E+00	.874E+00	-.365E+00	-.365E+00
.677E-02	.677E-02	-.141E-02	-.141E-02	-.320E-01	-.320E-01	-.128E-01	-.128E-01	.290E+00	.290E+00	.874E+00	.874E+00	-.365E+00	-.365E+00
.517E-02	.517E-02	.757E-02	.757E-02	-.198E-02	-.198E-02	-.307E-01	-.307E-01	-.910E-02	-.910E-02	.290E+00	.290E+00	.877E+00	.877E+00
.517E-02	.517E-02	.757E-02	.757E-02	-.198E-02	-.198E-02	-.307E-01	-.307E-01	-.910E-02	-.910E-02	.290E+00	.290E+00	.877E+00	.877E+00
.551E-02	.551E-02	.900E-02	.900E-02	.112E-01	.112E-01	.764E-03	.764E-03	-.306E-01	-.306E-01	-.128E-01	-.128E-01	.301E+00	.301E+00
.551E-02	.551E-02	.900E-02	.900E-02	.112E-01	.112E-01	.764E-03	.764E-03	-.306E-01	-.306E-01	-.128E-01	-.128E-01	.301E+00	.301E+00
.531E-02	.531E-02	.769E-02	.769E-02	.106E-01	.106E-01	.112E-01	.112E-01	-.198E-02	-.198E-02	-.319E-01	-.319E-01	-.151E-01	-.151E-01
.531E-02	.531E-02	.769E-02	.769E-02	.106E-01	.106E-01	.112E-01	.112E-01	-.198E-02	-.198E-02	-.319E-01	-.319E-01	-.151E-01	-.151E-01
.447E-02	.447E-02	.615E-02	.615E-02	.769E-02	.769E-02	.902E-02	.902E-02	.756E-02	.756E-02	-.138E-02	-.138E-02	-.322E-01	-.322E-01
.447E-02	.447E-02	.615E-02	.615E-02	.769E-02	.769E-02	.902E-02	.902E-02	.756E-02	.756E-02	-.138E-02	-.138E-02	-.322E-01	-.322E-01
.328E-02	.328E-02	.448E-02	.448E-02	.533E-02	.533E-02	.553E-02	.553E-02	.519E-02	.519E-02	.680E-02	.680E-02	-.272E-03	-.272E-03
.328E-02	.328E-02	.448E-02	.448E-02	.533E-02	.533E-02	.553E-02	.553E-02	.519E-02	.519E-02	.680E-02	.680E-02	-.272E-03	-.272E-03

Table 31. Required cable length adjustments.

CABLE	LENGTH ADJUSTMENT (Inches)
1	.62575E+00
2	.60906E+00
3	.21232E+01
4	.20612E+01
5	.36988E+01
6	.37673E+01
7	.53008E+01
8	.52485E+01
9	.68874E+01
10	.68518E+01
11	.80464E+01
12	.80720E+01
13	.90604E+01
14	.91666E+01
15	.92643E+01
16	.91979E+01
17	.88400E+01
18	.87632E+01
19	.81747E+01
20	.80916E+01
21	.68532E+01
22	.68114E+01
23	.48494E+01
24	.48922E+01
25	.28080E+01
26	.27399E+01
27	.88775E+00
28	.87774E+00

Table 32. Computer results for checking adjustment analysis.

JOINT DISPLACEMENTS AND SUPPORT REACTIONS							UNITS:
JOINT	X DISPL.	Y DISPL.	ROTATION	X REAC.	Y REAC.	MOMENT	
1	.000E+00	.000E+00	.307E-02	.273E+00	-.204E+00	.000E+00	Displacements - Inches
2	-.639E+00	.933E+00	.980E-03	.000E+00	.000E+00	.000E+00	
3	-.737E+00	.110E+01	-.105E-03	.000E+00	.000E+00	.000E+00	
4	-.667E+00	.950E+00	-.452E-03	.000E+00	.000E+00	.000E+00	Shears - Kips
5	-.559E+00	.667E+00	-.758E-03	.000E+00	.000E+00	.000E+00	
6	-.437E+00	.236E+00	-.976E-03	.000E+00	.000E+00	.000E+00	
7	-.346E+00	-.244E+00	-.103E-02	.000E+00	.000E+00	.000E+00	Moments - Inch-kips
8	-.305E+00	-.690E+00	-.695E-03	.000E+00	.000E+00	.000E+00	
9	-.306E+00	-.879E+00	-.159E-03	.000E+00	.000E+00	.000E+00	
10	-.310E+00	-.902E+00	.430E-06	.000E+00	.000E+00	.000E+00	Axial - Kips
11	-.304E+00	-.865E+00	.244E-03	.000E+00	.000E+00	.000E+00	
12	-.231E+00	-.607E+00	.862E-03	.000E+00	.000E+00	.000E+00	
13	-.260E-01	-.712E-01	.125E-02	.000E+00	.000E+00	.000E+00	
14	.244E+00	.491E+00	.981E-03	.000E+00	.000E+00	.000E+00	
15	.373E+00	.716E+00	-.258E-03	.000E+00	.000E+00	.000E+00	
16	-.118E+00	.000E+00	-.305E-02	.000E+00	-.322E+00	.000E+00	
17	-.178E-01	.155E+01	.334E-02	.000E+00	.000E+00	.000E+00	
18	-.335E-01	.315E+01	.326E-02	.000E+00	.000E+00	.000E+00	
19	-.455E-01	.464E+01	.296E-02	.000E+00	.000E+00	.000E+00	
20	-.537E-01	.597E+01	.254E-02	.000E+00	.000E+00	.000E+00	
21	-.582E-01	.706E+01	.201E-02	.000E+00	.000E+00	.000E+00	
22	-.596E-01	.787E+01	.133E-02	.000E+00	.000E+00	.000E+00	
23	-.590E-01	.831E+01	.500E-03	.000E+00	.000E+00	.000E+00	
24	-.575E-01	.834E+01	-.374E-03	.000E+00	.000E+00	.000E+00	
25	-.567E-01	.796E+01	-.117E-02	.000E+00	.000E+00	.000E+00	
26	-.579E-01	.723E+01	-.190E-02	.000E+00	.000E+00	.000E+00	
27	-.623E-01	.615E+01	-.257E-02	.000E+00	.000E+00	.000E+00	
28	-.707E-01	.479E+01	-.307E-02	.000E+00	.000E+00	.000E+00	
29	-.833E-01	.323E+01	-.339E-02	.000E+00	.000E+00	.000E+00	
30	-.996E-01	.158E+01	-.344E-02	.000E+00	.000E+00	.000E+00	
MEMBER END ACTIONS							
MEMBER	AXIAL J	SHEAR J	MOMENT J	AXIAL K	SHEAR K	MOMENT K	
1	.348E+02	.106E+02	.170E+05	-.348E+02	-.106E+02	-.109E+05	
2	.339E+02	.134E+02	.108E+05	-.339E+02	-.134E+02	-.340E+04	
3	.268E+02	.384E+01	.340E+04	-.268E+02	-.384E+01	-.135E+04	
4	.231E+02	.235E+01	.135E+04	-.231E+02	.235E+01	-.256E+04	
5	.249E+02	.449E+01	.255E+04	-.249E+02	-.449E+01	-.312E+03	
6	.228E+02	.156E+00	.311E+03	-.228E+02	.156E+00	-.387E+03	
7	.235E+02	.994E+01	.385E+03	-.235E+02	-.994E+01	.441E+04	
8	.224E+02	.485E+01	-.441E+04	-.224E+02	.485E+01	.208E+04	
9	.230E+02	.465E+01	-.208E+04	-.230E+02	.465E+01	-.158E+03	
10	.216E+02	.733E+01	.156E+03	-.216E+02	-.733E+01	.343E+04	
11	.226E+02	.254E+01	-.343E+04	-.226E+02	-.254E+01	.470E+04	
12	.274E+02	.855E+01	-.470E+04	-.274E+02	.855E+01	.295E+03	
13	.289E+02	.806E+01	-.289E+03	-.289E+02	.806E+01	-.400E+04	
14	.347E+02	.149E+02	.400E+04	-.347E+02	.149E+02	-.123E+05	
15	.422E+02	.219E+02	.123E+05	-.422E+02	.219E+02	-.250E+05	
16	-.228E+02	.283E+02	-.170E+05	.228E+02	.283E+02	.344E+04	
17	-.228E+02	.288E+02	-.344E+04	.228E+02	.288E+02	-.104E+05	
18	-.226E+02	.153E+02	.104E+05	.226E+02	.153E+02	-.177E+05	
19	-.225E+02	.628E+01	.177E+05	.225E+02	.628E+01	-.208E+05	
20	-.225E+02	.113E+02	.208E+05	.225E+02	.113E+02	-.262E+05	
21	-.225E+02	.421E+01	.262E+05	.225E+02	.421E+01	-.282E+05	
22	-.225E+02	.122E+02	.282E+05	.225E+02	.122E+02	-.340E+05	
23	-.225E+02	.481E+01	.340E+05	.225E+02	.481E+01	-.317E+05	
24	-.225E+02	.680E+01	.317E+05	.225E+02	.680E+01	-.284E+05	
25	-.225E+02	.314E+01	.284E+05	.225E+02	.314E+01	-.299E+05	
26	-.225E+02	.394E+01	.299E+05	.225E+02	.394E+01	-.280E+05	
27	-.226E+02	.179E+02	.280E+05	.226E+02	.179E+02	-.194E+05	
28	-.227E+02	.200E+02	.194E+05	.227E+02	.200E+02	-.980E+04	
29	-.228E+02	.305E+02	.979E+04	.228E+02	.305E+02	.484E+04	
30	-.230E+02	.419E+02	.486E+04	.230E+02	.419E+02	.250E+05	
31	-.225E+01	.000E+00	.000E+00	.225E+01	.000E+00	.000E+00	
32	-.275E+01	.000E+00	.000E+00	.275E+01	.000E+00	.000E+00	
33	-.118E+02	.000E+00	.000E+00	.118E+02	.000E+00	.000E+00	
34	-.175E+01	.000E+00	.000E+00	.175E+01	.000E+00	.000E+00	
35	-.499E+00	.000E+00	.000E+00	.499E+00	.000E+00	.000E+00	
36	-.850E+01	.000E+00	.000E+00	.850E+01	.000E+00	.000E+00	
37	-.438E-02	.000E+00	.000E+00	.438E-02	.000E+00	.000E+00	
38	.500E+01	.000E+00	.000E+00	-.500E+01	.000E+00	.000E+00	
39	-.500E+01	.000E+00	.000E+00	.500E+01	.000E+00	.000E+00	
40	-.200E+01	.000E+00	.000E+00	.200E+01	.000E+00	.000E+00	
41	.500E+01	.000E+00	.000E+00	-.500E+01	.000E+00	.000E+00	
42	.300E+01	.000E+00	.000E+00	-.300E+01	.000E+00	.000E+00	
43	-.450E+01	.000E+00	.000E+00	.450E+01	.000E+00	.000E+00	
44	-.125E+02	.000E+00	.000E+00	.125E+02	.000E+00	.000E+00	
45	-.350E+01	.000E+00	.000E+00	.350E+01	.000E+00	.000E+00	
46	.150E+01	.000E+00	.000E+00	-.150E+01	.000E+00	.000E+00	
47	.200E+01	.000E+00	.000E+00	-.200E+01	.000E+00	.000E+00	
48	.800E+01	.000E+00	.000E+00	-.800E+01	.000E+00	.000E+00	
49	-.700E+01	.000E+00	.000E+00	.700E+01	.000E+00	.000E+00	
50	.206E-03	.000E+00	.000E+00	-.206E-03	.000E+00	.000E+00	
51	-.900E+01	.000E+00	.000E+00	.900E+01	.000E+00	.000E+00	
52	-.500E+01	.000E+00	.000E+00	.500E+01	.000E+00	.000E+00	
53	.150E+01	.000E+00	.000E+00	-.150E+01	.000E+00	.000E+00	
54	-.350E+01	.000E+00	.000E+00	.350E+01	.000E+00	.000E+00	
55	-.107E+02	.000E+00	.000E+00	.107E+02	.000E+00	.000E+00	
56	.244E+00	.000E+00	.000E+00	-.244E+00	.000E+00	.000E+00	
57	-.725E+01	.000E+00	.000E+00	.725E+01	.000E+00	.000E+00	
58	-.425E+01	.000E+00	.000E+00	.425E+01	.000E+00	.000E+00	

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