Parametric effects on slope stability analysis
by Robert Lorin Sanderson

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

Abstract:
The purpose of this investigation was to determine the relative effect of soil parameter and failure surface variance on the factor of safety against sliding in two selected heterogeneous finite slopes. Two actual finite slope cross-sections taken from Montana Highway Commission files were used in the study. One section was a typical highway fill placed on a multilayered subbase. The other was a typical cut section through gently dipping bedded material overlain by a considerable depth of alluvium.

The method of analysis used was a series of digital computer solutions based on the "method of slices." The method of slices was adapted for use on a combination circular and planar failure surface for analysis of the cut section stability. For both trial problems, soil parameters, soil boundaries, and failure surfaces were systematically changed in order that the effect of these variables on slope stability could be observed.

The results of the investigation revealed that certain patterns of failure arc locations can be expected for specific soil conditions.

Changes in any one soil property, i.e., unit cohesion, unit weight, or internal friction angle, will result in a change in the failure arc location and thus a change in the relative stability. It was determined that the use of a fixed or "best guess" failure arc can be used with a high degree of accuracy if the arc position is within the characteristic failure pattern of the cross-section. If the fixed arc is not representative of the typical failure condition, an equally high degree of inaccuracy results.

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Signature Robert Lorin Anderson Jr.

Date November 24, 1969

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by

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A thesis submitted to the Graduate Faculty in partial
fulfillment of the requirements for the degree

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ABSTRACT

The purpose of this investigation was to determine the relative effect of soil parameter and failure surface variance on the factor of safety against sliding in two selected heterogeneous finite slopes. Two actual finite slope cross-sections taken from Montana Highway Commission files were used in the study. One section was a typical highway fill placed on a multilayered subbase. The other was a typical cut section through gently dipping bedded material overlain by a considerable depth of alluvium.

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The results of the investigation revealed that certain patterns of failure arc locations can be expected for specific soil conditions. Changes in any one soil property, i.e., unit cohesion, unit weight, or internal friction angle, will result in a change in the failure arc location and thus a change in the relative stability. It was determined that the use of a fixed or "best guess" failure arc can be used with a high degree of accuracy if the arc position is within the characteristic failure pattern of the cross-section. If the fixed arc is not representative of the typical failure condition, an equally high degree of inaccuracy results.

CHAPTER I

INTRODUCTION

Approach to Stability Analysis

Men of scientific interest and background have been studying the problems of the stability of earth slopes for many years. At first, theory behind movement of soil and rock was essentially undeveloped and men relied on experience and sound judgment in analyzing stability problems. Gradually as the theory was advanced, more and more of a gap formed between quantitative and qualitative methods of slope stability analysis. Geologists continued to rely on actual field conditions and the geologic history of an area as a basis for final conclusions. Engineers have followed closely the development of quantitative slope stability theory based on the fundamentals of mechanics and the use of various theoretical methods has become widespread.

Many different quantitative approaches to the problem of determining the stability of slopes are presently in use. Definite limitations to the application of the quantitative approach exist and must be recognized by the engineer. Basically, all quantitative methods assume failure occurs on either a circular or planar surface. While these two basic approaches can be applied with varying degrees of accuracy to many actual slope stability problems, it needs to be recognized that a great many situations exist that do not lend themselves to straight-forward quantitative analysis. In many cases, the particular geologic structure in the unstable area is such that a definite failure arc or failure plane cannot rationally be predicted. Such problems must then be analyzed using basic geologic considerations.

Quantitative Analysis Limitations

The fact that the numerous quantitative methods have limitations, as far as general application to slope stability problems is concerned, is well-known and well-documented in the literature. Terzaghi and Peck (1948) made a brief statement concerning those limitations as follows:

> Because of the extraordinary variety of factors and processes that may lead to slides, the conditions for the stability of slopes usually defy theoretical analysis. Stability computations based on test results can be relied on only when conditions specified (in each method of analysis) are strictly satisfied. Moreover, it should always be remembered that various undetected discontinuities of sliding, or thin seams of water-bearing sand, may completely invalidate the results of computations.

Eckel (1958) covers the subject of mathematical stability analyses limitations as well as any of the literature searched. Eckel states in part that " . . . the analyses cannot be made for every type of landslide and for any type a number of assumptions based on idealized conditions and materials will be required. It is impossible to treat mathematically all of the variables imposed by nature."

If mathematical approaches to slope stability analysis have limitations, and these limitations are known, of what use are the results of such analyses? It is easy to see how quantitative estimates of the magnitude of forces involved in a sliding mass would aid in designing adequate corrective measures. Also, the knowledge of these forces and their magnitudes determined from an existing slide, may be applied to the prediction of sliding in an adjacent area of similar geologic structure. The economic feasibility of maintaining or correcting unstable slopes is usually based on analytical stability analysis.

Factor of Safety Against Sliding

Most quantitative methods of stability analysis give a factor of safety against sliding as the end result. Although factors of safety may be calculated in many different ways, the quotient of resisting and activating forces along the potential failure surface is most commonly used. The value of the factor of safety is only as good as the assumptions made in applying a particular analysis. The factor of safety is a good indicator of relative stability, however, and is often used in this manner.

Data Required for Analysis

Obtaining the precise information required to apply a quantitative slope stability analysis is not ordinarily a straight-forward accumulation of data. First, and possibly most important, the general geology of the existing or potential slide area must be known. The engineer must know the properties of the materials contained in the slope. These properties include the strength parameters, unit cohesion and angle of internal friction, and the soil unit weight. An adequate drilling, sampling, and laboratory testing program must be formulated at this stage of the investigation in order to obtain the above information.

Before continuing with the above mentioned procedures, however, the engineer must decide how thorough an investigation is warranted or necessary. With unlimited finances, time, and equipment, it is possible to accurately determine conditions of slope stability where quantitative methods of analysis are applicable. Precise determination of the soil parameters mentioned above, for instance, is time consuming and costly. Most often, however, representative samples are taken from drill holes, soil parameters are averaged, and factors of safety against sliding are calculated along assumed representative failure arcs. It is known that in any given embankment a certain combination of possible values of soil parameters, slope, and slope height, along with a particular failure arc location, will give a minimum factor of safety against sliding. Because of the usual case of limited time and funds, however, only representative cases based on experience are evaluated, and the true minimum condition is not necessarily determined.

Need for Investigation

There is a definite need for information regarding the seriousness of not knowing the conditions of the most critical sliding situation. One way of developing this data is to determine the sensitivity of the factor of safety against sliding to minor change in each of the soil parameters and the position of the failure arc. With a knowledge of the effect of each of these changes on the accuracy of quantitative slope stability analyses, the engineer should do a better job of determining how extensive a field and laboratory investigation need be in a specific case.

Previous work in isolating the effect of individual parameters has been varied and incomplete. Taylor (1948) described one method of illustrating the effect by defining what he called a stability number. The stability number is:

$$\frac{cD}{\gamma H}$$

where: cD = the mobilized cohesion in pounds per square foot

γ = soil unit weight in pounds per cubic foot, and

H = slope height in feet.

Taylor's results show the relationship between stability number and slope angle for various angles of internal friction. The relationships are shown for simple, homogeneous, finite slopes with fixed failure arc positions. Relative stability of more complex slopes could only be approximated from the use of these relationships.

In the usual slope stability problem confronting the engineer, the slope geometry is a function of the soil characteristics. In a highway design situation, a cut or fill slope angle is proposed or desired, and the height of the existing or proposed slope has been determined from requirements of grade. Therefore, the most difficult task is to determine the soil parameters and then determine, in turn, the position of the critical failure surface.

Purpose of Investigation

The primary objective of this study was to investigate the effect of soil parameter and failure surface variance on the relative stability of two selected heterogeneous finite slopes. Two typical slope stability problems, a highway cut section and a highway fill section, were used in this investigation. Stability analyses were conducted on these slopes to determine minimum factors of safety against sliding for selected combinations of soil parameters and failure arc positions. From the results of these computations, an analysis was made of the effect of using average or approximate values of the soil parameters and approximate positions of the critical failure surface.

CHAPTER II

DEFINITIONS OF TERMINOLOGY AND REVIEW OF SLOPE FAILURE CONDITIONS

Finite Slope Description

The basic finite slope configuration is shown in Figure 1. As shown in the figure, more than one soil type may be represented in the typical cross-section. The slope itself may be at any angle, and the ground surface above and below the slope may or may not be horizontal. For the purposes of discussion in this study, all cross-sections will be represented by the basic (x,y) coordinate system as shown in Figure 1.

Soil Characteristics

Three basic properties describe each soil included in a slope stability problem. They are the soil unit weight, soil unit cohesion, and the angle of internal friction. The soil unit weight, γ, is defined as the total weight of the soil per unit volume.

The two remaining properties, unit cohesion and internal friction, are directly responsible for the shearing strength of a particular soil and thus are very important in all slope stability calculations. Shearing strength has been defined by Spangler (1963) as "the property which enables soil to maintain equilibrium on a sloping surface, such as a natural hillside, the backslope of a highway or railway cut, or the sloping sides of an embankment, levee, or earth dam."

Mass-Movement Classification

In a typical finite slope situation, such as that shown in Figure 1, several different types of failure can take place. Classification of various landslide and mass-movement types is well-covered in the literature. The most thorough geological classification is that of Sharpe (1938).

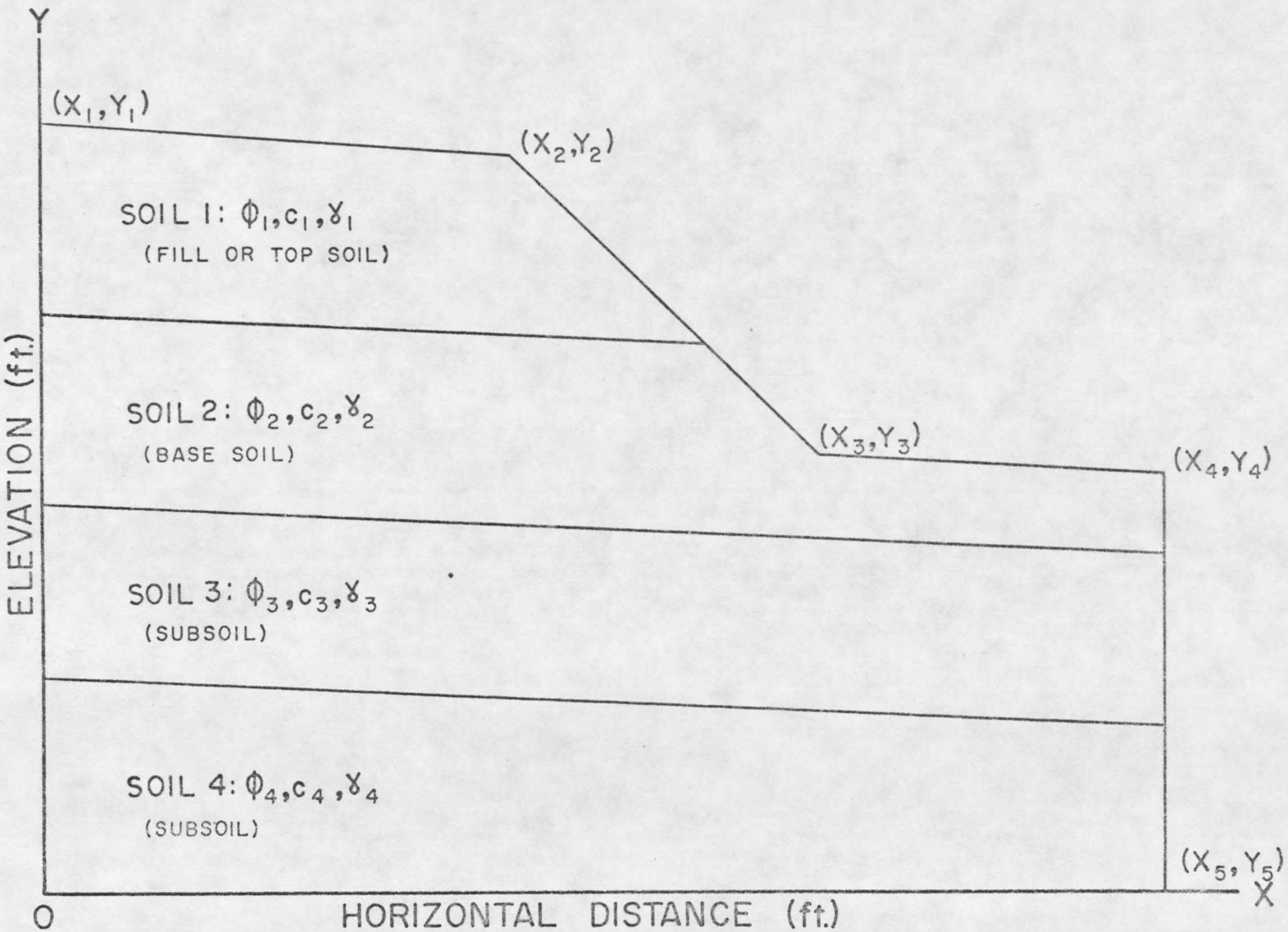

Figure 1. Basic configuration of heterogeneous finite slope.

Sharpe classified mass-movement into two distinct categories; that of slides and flows. He noted one basic distinction between the two types of movement. If a slope or shear plane separates the moving mass from the stable ground, the movement is termed a slide. If no distinct slip plane exists, the movement is plastic or viscous and has occurred due to continuous deformation of the material involved. The latter type of movement is classed as a flow. These distinctions must be qualified, however, because cases seldom exist where only "slippage" or "flowage" occurs. Some cases of flowage may actually start as slippage and vice-versa.

Basically, then, Sharpe (1938) divided all forms of mass-movement into four groups, and each group into subgroups:

1. Slow flowage
    a. Rock-creep
    b. Talus-creep
    c. Soil-creep
    d. Rock-glacier creep
    e. Solifluction
2. Rapid flowage
    a. Earthflow
    b. Mudflow
    c. Debris-avalanches
3. Sliding
    a. Slump
    b. Debris-slide
    c. Debris-fall
    d. Rockslide
    e. Rockfall
4. Subsidence

Of these four categories, only the third contains types of failures in which an actual shear surface could be defined. Of these types of failure, only that of slump, debris-slide, and rockslide occur on a shear surface. Those of debris-fall and rockfall are of a free-falling nature, and a sliding surface, as such, cannot be defined.

Quantitative Analysis Application

From the above discussion, it can be seen that the mathematical principles are limited in application to but a few cases in the overall mass-movement classification. That the other categories do exist and are a common threat to man-made structures is well-known. It is thus important that a complete perspective of the sliding or mass-movement possibilities is recognized and the analytical analysis, therefore, has limitations.

In summary, the types of slope failure most often analyzed analytically are slump, debris-slide, and rockslide. As defined by Sharpe (1938), the slump is " . . . the downward slipping of a mass of rock or unconsolidated material of any size, moving as a unit or as several subsidary units, usually with backward rotation on a more or less horizontal axis parallel to the cliff or slope from which it descends." The slump type of slide has a definite shear surface which is usually partially, and quite often wholly, arc-shaped.

The debris-slide is defined by Sharpe as a " . . . rapid downward movement of predominantly unconsolidated and incoherent earth and debris in which the mass does not show backward rotation but slides or rolls forward, forming an irregular hummocky deposit which may resemble morainal topography. Debris-slides are very common and are usually quite small. They occur on most talus slopes or any other slope which is steep enough that a minor disturbance may cause sliding of unconsolidated material. A definite sliding plane is often difficult to determine as the material moves in a rolling or tumbling manner.

The third shear surface type of slide according to Sharpe's system is a rockslide. He defines a rockslide as " . . . the downward and usually rapid movement of newly detached segments of the bedrock sliding on bedding, joint, or fault surfaces or any other plane of separation." Rockslides are not as common as slumps or debris-slides; however, several of the more destructive slope failures in history have been rockslides.

Planar block glide is a type of slide which is similar to the rockslide except that the planar block acts more as a single unit. The planar block glide category was introduced in a classification system developed by the Committee on Landslide Investigations of the Highway Research Board as reported by Eckel (1958). The classification system parallels Sharpe's rather closely, although several cause and effect relationships introduced are engineering oriented. One important difference in the methods of classification is the addition of the planar block glide category.

Eckel indicates that the planar block glide type of slide is actually quite common, but it has received very little coverage in the literature. Eckel describes the block glide as a mass progressing " . . . out, or down and out, as a unit along a more or less planar surface, without the rotary movement and backward tilting characteristic of slump." Eckel points out that the important difference between slump and block glide is revealed in the type of control that is necessary to prevent sliding from occurring or continuing to occur. As a slumping mass rotates along its failure arc, energy is gradually dissipated and the slide comes to a rest. A block glide, in contrast, will continue to slide indefinitely as long as the sliding surface remains inclined and a low value of shearing resistance exists. Control, then, may be a much greater problem with the block glide situation.

Quantitative Slope Stability Analysis

Many analytical approaches to stability analysis have been developed with regard to both planar and curved surfaces. Planar surface analyses simply involve the basic theory of a soil mass sliding on an inclined plane. The situation can be likened to Sharpe's rockslide definition, as a definite sliding surface is apparent. Also, block glide or block slide as discussed previously fits the planar sliding surface type of analysis.

Probably the most popular and most widely used method of analyzing failures on a curved surface is the method of slices. Eckel (1958) reports that this method was developed by W. Fellenius in 1926. Many variations of the basic method of slices have been developed and are in use. Also, many other mathematical solutions for slope failure along arc-shaped surfaces have been successfully applied. All use the same soil mechanics principles, however, and a brief explanation of the simplified method of slices will serve to illustrate the nature of these solutions.

Figure 2 shows the typical geometric cross-section of an arc-shaped failure surface on a finite slope.

The method of slices can be solved analytically or graphically and has been successfully adapted to digital computer programs. As

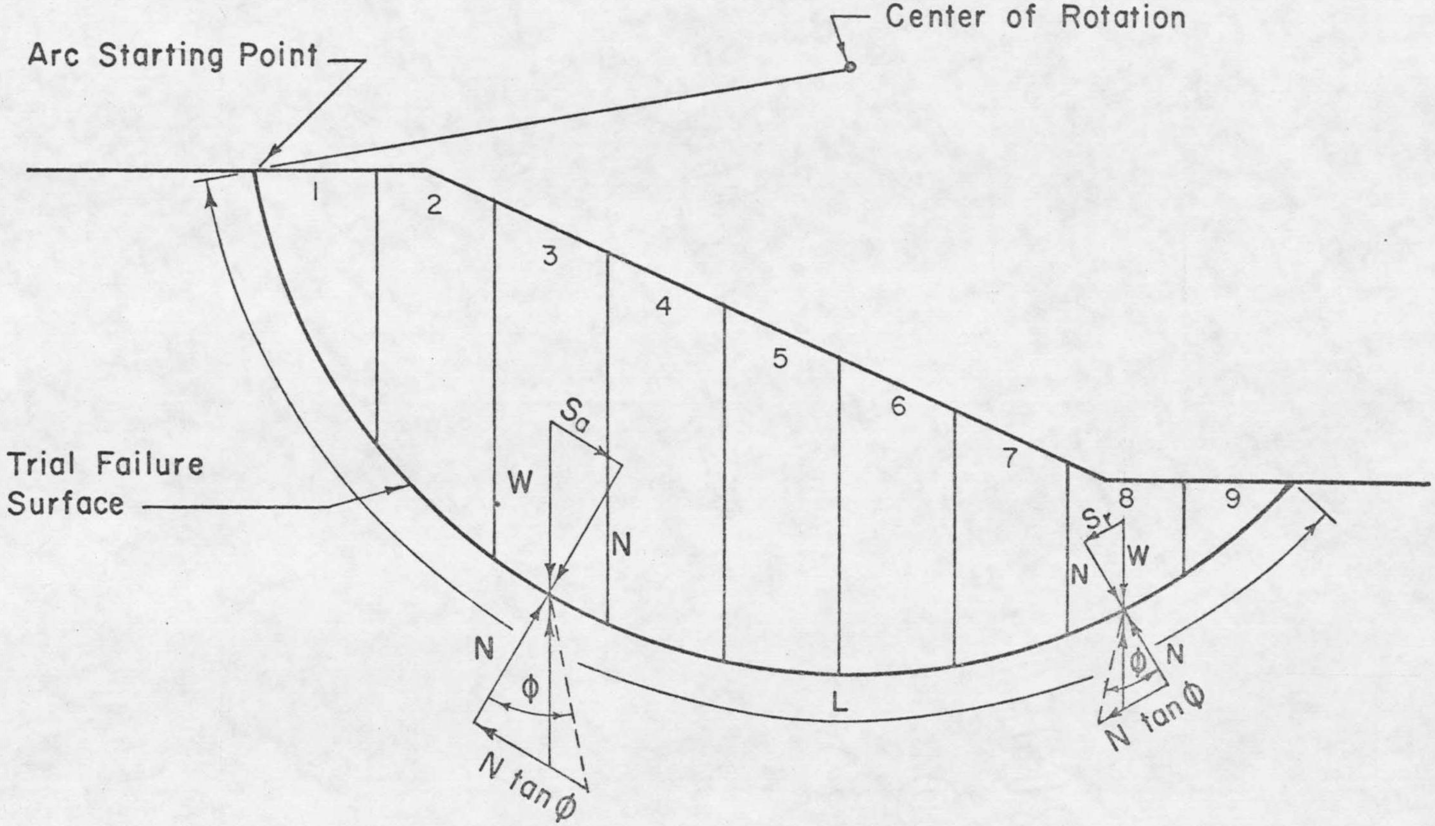

Figure 2. Finite slope with potential circular failure surface.

indicated previously, a factor of safety against sliding is the final result of this method of analysis. The solution is based on a failure arc which is a section of a circle. The total tangential weight component, which is the activating force, is the sum of all of the tangential components of the individual slices. The total resisting force is the sum of the resistance along the arc due to cohesion and friction of the individual slices, and the tangential weight component of those slices which fall to the right of the center of rotation. The final expression for the factor of safety against sliding is as follows:

$$F.S. = \frac{cbL + \Sigma N\tan\phi + \Sigma Sr}{\Sigma Sa}$$

where: c = unit cohesion in pounds per square foot

b = the slice depth normal to paper in feet

L = the arc length in feet

N = the normal component of slice weight in pounds

$\tan\phi$ = the coefficient of friction

Sr = the tangential weight component resisting in pounds, and

Sa = the tangential weight component activating in pounds.

Summary of Quantitative Approach

In all methods of quantitative slope stability analysis, one fundamental prerequisite stands out. That is a shear failure must occur or be expected to occur in the case of a potential slide situation, and it must be possible to define where that failure surface is or is most likely to be. Also, accurate knowledge of the soil properties, angle of internal friction, unit cohesion, and unit weight, must be available.

In all slope stability calculations, the effect of water conditions is important. Contained water above the water table tends to add weight to the soil and thus increase normal and tangential weight components. Static water below the water table has a bouyant effect on the soil and thus reduces the above mentioned components. Moving water or seepage conditions on the other hand, adds to the slide activating force. Seismic or other external forces also add to the slide activating forces. These and other theoretical factors, such as the consideration of shear forces along the sides of slices in the method of slices, affect all methods of analysis in a similar and consistent manner. In other words, a series of analyses made without including the effects of seismic, seepage, or slice side forces, could be expected to differ systematically from that which did include these factors. Also, these relative differences would be apparent no matter what method of analysis was used.

In this investigation, the simple method of slices was used as a means of analysis to determine the effect of the arc position and the systematic variation of the individual soil parameters. As indicated above, however, the relative results are applicable to all methods of analysis.

CHAPTER III

EXPERIMENTAL PROCEDURES

Slope Sections Studied

Two actual cross-sections taken from Montana Highway Commission files were used as a basis for this investigation. The first is a typical fill section located at Station 720+00 on Interstate Highway 15 near Emerson Junction in north-central Montana. The second is a typical cut section located at Station 1593+00 on Interstate Highway 15 near Hardy Creek. The selected cross-sections are shown in Figures 3 and 4.

For the purposes of this study, the cut and fill slope configurations originally proposed by the Highway Commission were used. In actuality, both sections have since been altered to control slide activity, but these changes were not incorporated in this study. These sections were chosen as realistic finite slope stability problems which confront an agency such as a highway department. Soil property values shown in Figures 3 and 4 were furnished by the Montana Highway Commission.

Use of Digital Computer

As stated previously, the method of slices was used as a means of determining stability of the various slope conditions in this study. A digital computer program for the method of slices made possible the calculation of vast amounts of needed data.

Initially it was desired to obtain a factor of safety for the configurations (see Figures 3 and 4) with the proposed slopes and the given soil conditions. This approach served two purposes: (1) to verify that the computer solution was comparable with the Highway Commission calculations, and (2) to provide a basis with which to compare later solutions

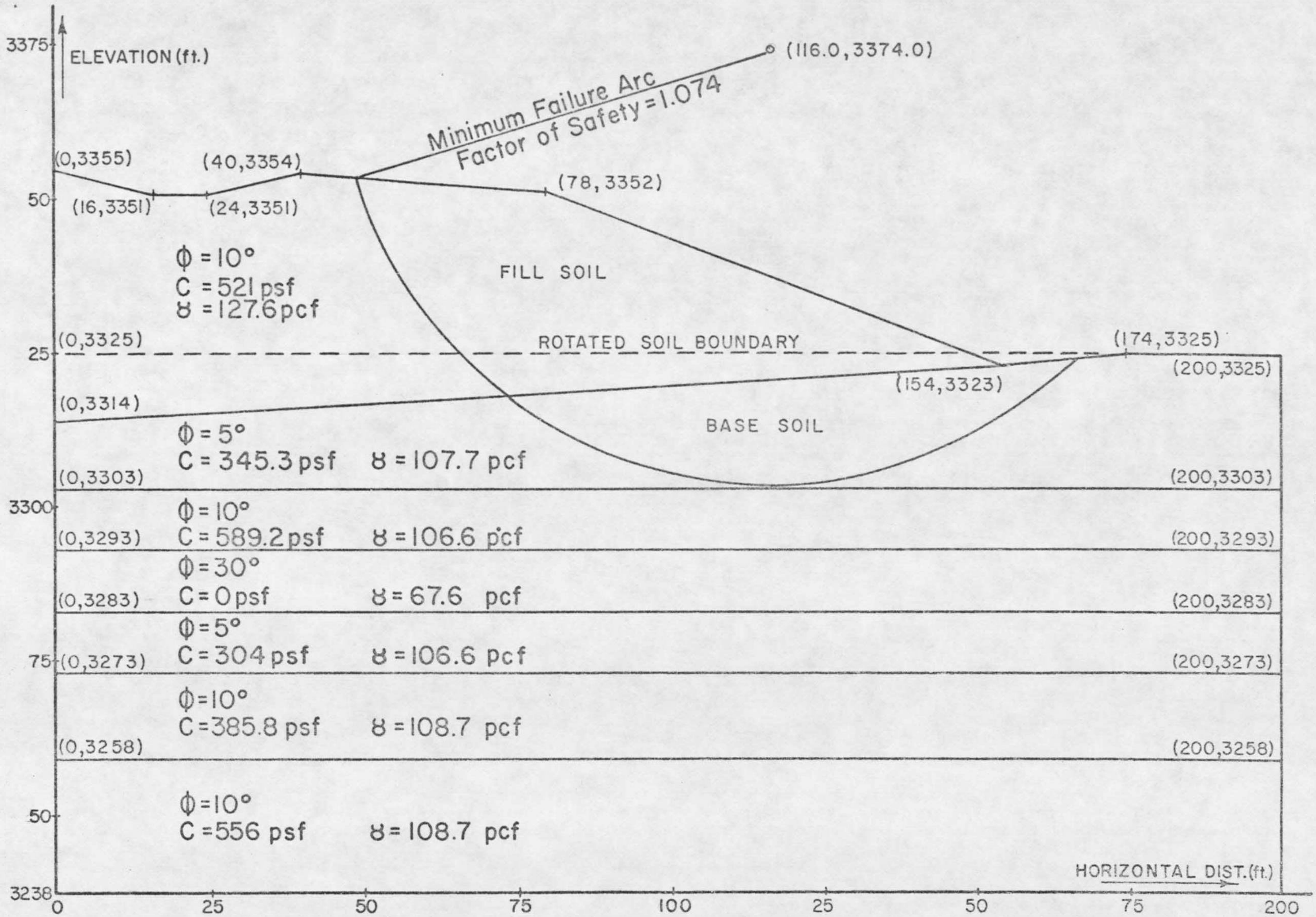

Figure 3. Geometry and soil conditions of highway fill section.

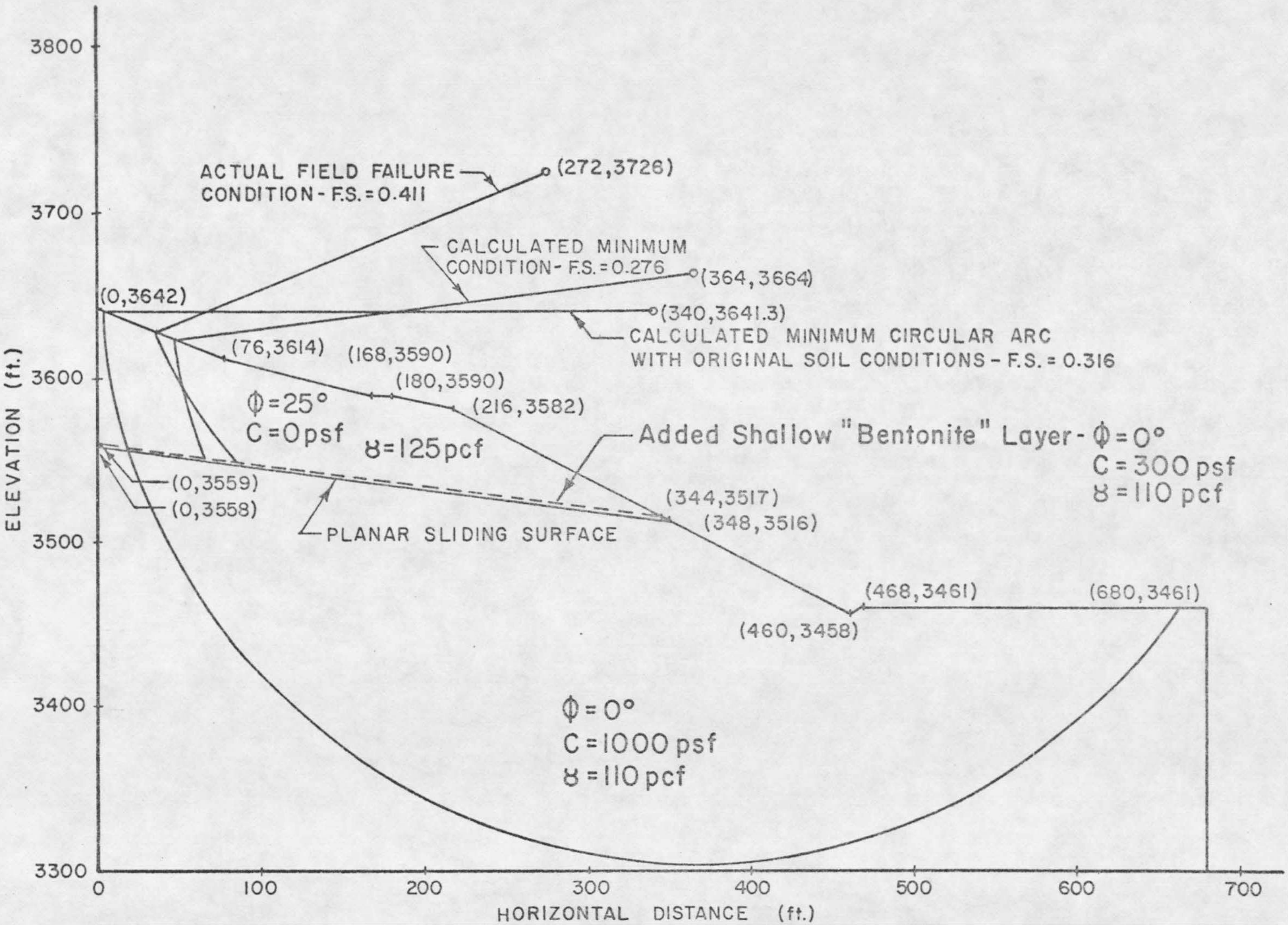

Figure 4. Geometry and soil conditions of highway cut section.

for different imposed soil conditions.

The computer program required modification to solve the type of stability problem shown in Figure 4. In this case, failure was known to have occurred along the planar surface indicated in the sketch. Also, a tension crack had been located along the upper surface and was determined to be the upper limit of sliding. The overall failure surface, then, could be approximated by a portion of a circular arc and a planar surface.

<u>Variation of Parameters</u>

The properties coefficient of friction, unit cohesion, and unit weight were systematically varied between specified limits. This was done by holding two of the variables constant while allowing the third to vary and continuing this process until all combinations were investigated. Unit weight was varied from 70 pcf to 130 pcf and unit cohesion from 0 psf to 1000 psf. The value of $\tan\phi$ was varied between the limits of 0.0 and 1.0 (0° to 45°) in increments of 0.1 or 0.25.

CHAPTER IV

PRESENTATION AND DISCUSSION OF RESULTS

Problem Description

The example problems (shown in Figures 3 and 4) were analyzed in several different ways to produce data that would illustrate the effect of independently changing soil parameters and failure surface positions. The fill problem, shown in Figure 3, is a typical homogeneous fill section placed on a stratified base. The slope condition, in this case, is most susceptible to a slump type of failure, and thus the failure arcs calculated in the analyses were circular.

The cut problem, shown in Figure 4, is a typical highway cut section through stratified material. As a result of the subsurface material being a hard shale, sliding could be expected to occur on or above this planar surface. In the analyses for this problem, the shale layer was assumed to be an impervious base, and the resulting failure surface was a combination of a circular arc and a planar surface.

The minimum factor of safety for the original soil conditions as shown in the fill section in Figure 3 is 1.074, and the failure arc passes through two soils. To determine how the factor of safety would be affected by a change in the original soil conditions: (1) the fill parameters were varied while the subsurface parameters were held constant; (2) the soil parameters directly below the fill were varied while the fill parameters were held constant; (3) the base and fill parameters were alternately varied as before, but the position of the failure arc was fixed; (4) the fill parameters were varied with a 5° change in the internal angle of friction of the base soil; (5) the value of the fill unit weight was fixed while the other fill parameters and the angle of internal friction of the base were allowed to vary; (6) all soil parameters for the entire cross-section were reduced ten percent and a new minimum solution was obtained; (7) the soil boundary between the fill and the base was moved and a solution for the original or "fixed" arc was obtained for the new cross-section; and (8) "fixed" arc solutions were obtained for alternately varying the fill and base parameters above and below the new soil boundary.

Minimum Solution--Variation of Fill Parameters

The first series of calculations for the fill section were for minimum factors of safety while the fill parameters were ranged over prescribed limits. The results are shown in Table I.

Figure 5 shows the failure arc positions represented by the minimum factors of safety in Table I. The arc positions shown are representative of all data accumulated during this study and illustrate several basic points of slump type slope failures. The longest and deepest failure arc shown is for fill soil properties of unit cohesion equal to 1000 psf and internal angle of friction equal to zero. The shortest and shallowest arcs are for lower unit cohesion and higher internal friction values.

Recalling the basic geometry of a circle, it is easy to visualize the reasons for this characteristic failure arc pattern. The circumference of a circle, a fraction of which is the failure arc in this case, is represented by the expression $2\pi r$ where r equals the radius of rotation. The resisting force due to the cohesive properties of the soil is dependent upon this arc length.

The resisting force due to the frictional properties of the soil is

TABLE I

FACTORS OF SAFETY FOR HIGHWAY FILL PROBLEM WITH VARIATION OF FILL SOIL PROPERTIES

γ (pcf)	Tangent ϕ 0.0	0.25	0.50	0.75	1.00	c (psf)
70	2.244	2.402	2.502	2.618	2.728	1000
100	1.535	1.712	1.798	1.875	2.034	
130	1.197	1.327	1.454	1.555	1.642	
70	1.958	2.103	2.263	2.346	2.524	750
100	1.371	1.528	1.630	1.724	1.775	
130	1.132	1.214	1.314	1.403	1.506	
70	1.693	1.897	2.061	2.141	2.191	500
100	1.203	1.375	1.460	1.545	1.638	
130	0.947	1.090	1.205	1.267	1.355	
70	1.122	1.583	1.720	1.844	1.949	250
100	0.773	1.179	1.291	1.375	1.483	
130	0.643	0.958	1.099	1.186	1.220	
70	0.000	0.666	1.333	1.570	1.657	0
100	0.000	0.665	1.113	1.227	1.334	
130	0.000	0.666	0.938	1.075	1.148	

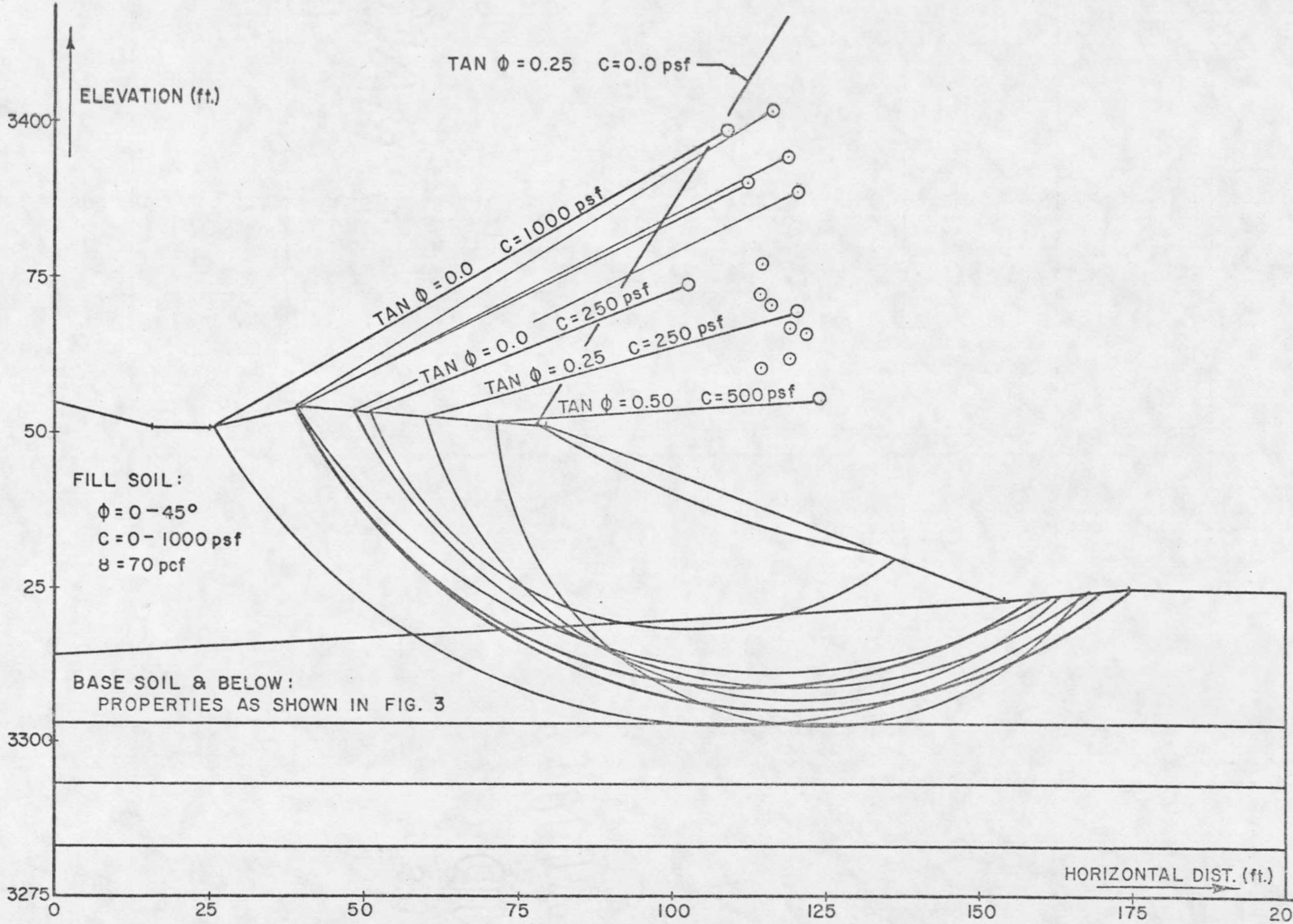

Figure 5. Typical failure surface pattern for highway fill section with variable fill soil properties.

dependent upon the soil weight acting perpendicular to the circular failure surface. The activating force is dependent upon the soil weight acting tangential to that surface. The weight of the soil included within a specific circular failure arc is in direct proportion to the total volume of that soil. The volume is represented by the depth of the slice perpendicular to the cross-section multiplied by the soil area within the arc. The area is thus some fraction of the area of a circle, which is expressed by πr^2.

Thus it can be seen that for low or non-existent values of internal friction, the search for a minimum failure condition will tend to move trial failure arcs deeper into the section. The resisting cohesive forces increase by the first power of the arc radius, whereas the activating tangential weight forces increase by the second power of the radius. This was found to continue deeper and deeper into the cross-section until a new soil condition was reached or until the search for a minimum condition resulted in a factor of safety of zero. In the case shown in Figure 5, the search for a minimum was successful because the second and third subsoils provided additional stability with angles of internal friction of 5° and 10° respectively.

The arc representing a unit cohesion of 250 psf and an angle of internal friction of zero is a further illustration of the above point. The arc is much shallower and shorter because less weight is required to equalize the smaller value of unit cohesion. Also, it can be noted that the additional stability developed as the arc encountered the second soil helped in concluding the search for the minimum condition.

Another interesting case is shown by the arc for a soil unit cohesion of zero and a tangent of the internal angle of friction equal to 0.25. This arc is shown following closely along the face of the slope. When no cohesive properties exist in a soil, the nature of the soil is similar to that of a granular material such as sand. From Taylor (1948) it is known that the angle of internal friction of a non-cohesive soil can be approximated by the determination of its angle of repose. Thus it could be expected that a fill material placed at a slope exceeding the value of its angle of repose, would be most unstable along that slope. This is the case here, since the friction angle is 14° and the slope angle is 21°.

As shown in Figure 5, all other combinations of soil properties produce failure arcs beginning along the upper reaches of the top surface and exiting at or just beyond the toe of the slope. In the majority of cases the arcs encompass both the fill and the second soil. Only in those cases, as mentioned above, where stable conditions exist in the upper two soils, does the search for a minimum arc go below the second soil boundary.

Figure 6 shows the relationship between unit cohesion and the minimum factor of safety for selected values of the tangent of the angle of internal friction. Figure 6 serves to demonstrate that the factor of safety is sensitive to a small change in cohesion for low values of the angle of internal friction. The region of greatest sensitivity is for coefficient of friction values from 0.0 to 0.50 and for unit cohesion values of 0.0 to 500 psf. Beyond unit cohesion values of approximately 500 psf, the rate of change of the factor of safety is relatively uniform for all values of the coefficient

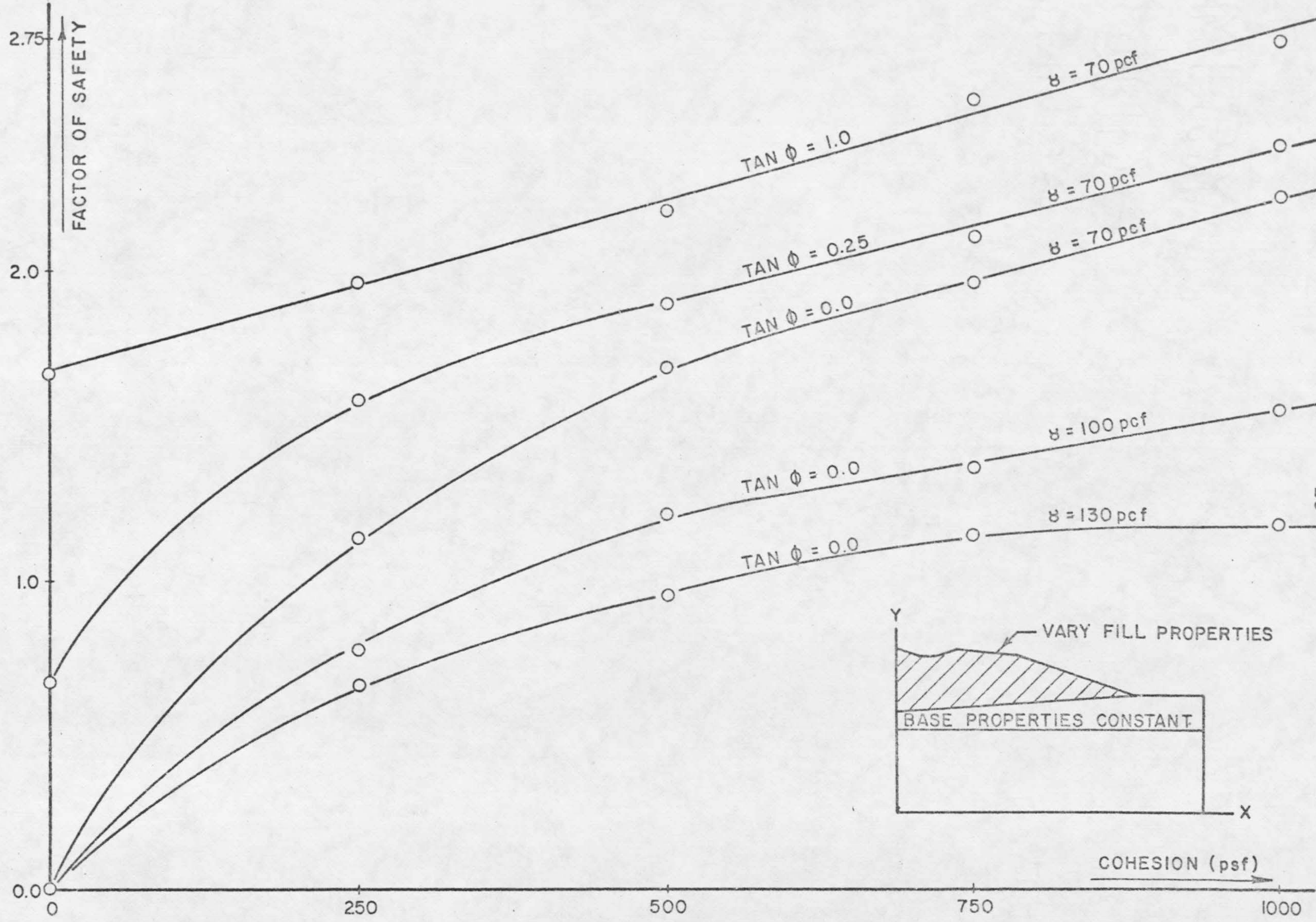

Figure 6. Factor of safety versus cohesion for highway fill section with variable fill soil properties.

of friction.

These observations are further demonstrated in Figure 7. The graph shows specific curves for unit cohesion when the factor of safety is plotted against the tangent of the angle of internal friction. Again, the most sensitive portions of the curves are in that area where tanϕ varies from 0.0 to 0.50 and unit cohesion varies from 0.0 to 500 psf.

As would be expected, the lower fill unit weights result in higher factors of safety as shown in Figure 8. Figure 8 shows a typical family of curves for values of unit cohesion and coefficient of friction. The curve for tanϕ equal to 0.25 when the unit cohesion equals zero shows that change in unit weight for this material has virtually no affect on the factor of safety. This is again illustrative of the fact that the slope angle in this particular problem exceeds the angle of internal friction, and the instability imposed by this situation overrides that due to increased unit weight.

Figure 8 indicates the obvious fact that as the unit weight approaches zero the curves appear to become asymptotic to the vertical axis or in other words, the factor of safety against sliding would approach infinity at zero unit weight. At the other extreme, as the unit weight increases it appears that the curves become asymptotic to individual values of the factor of safety. This would indicate that change in values of unit weight above approximately 130 pcf would not significantly alter values of factor of safety against sliding.

Figure 8 illustrates that an increase in stability results from increasing values of unit cohesion and internal friction angle. The

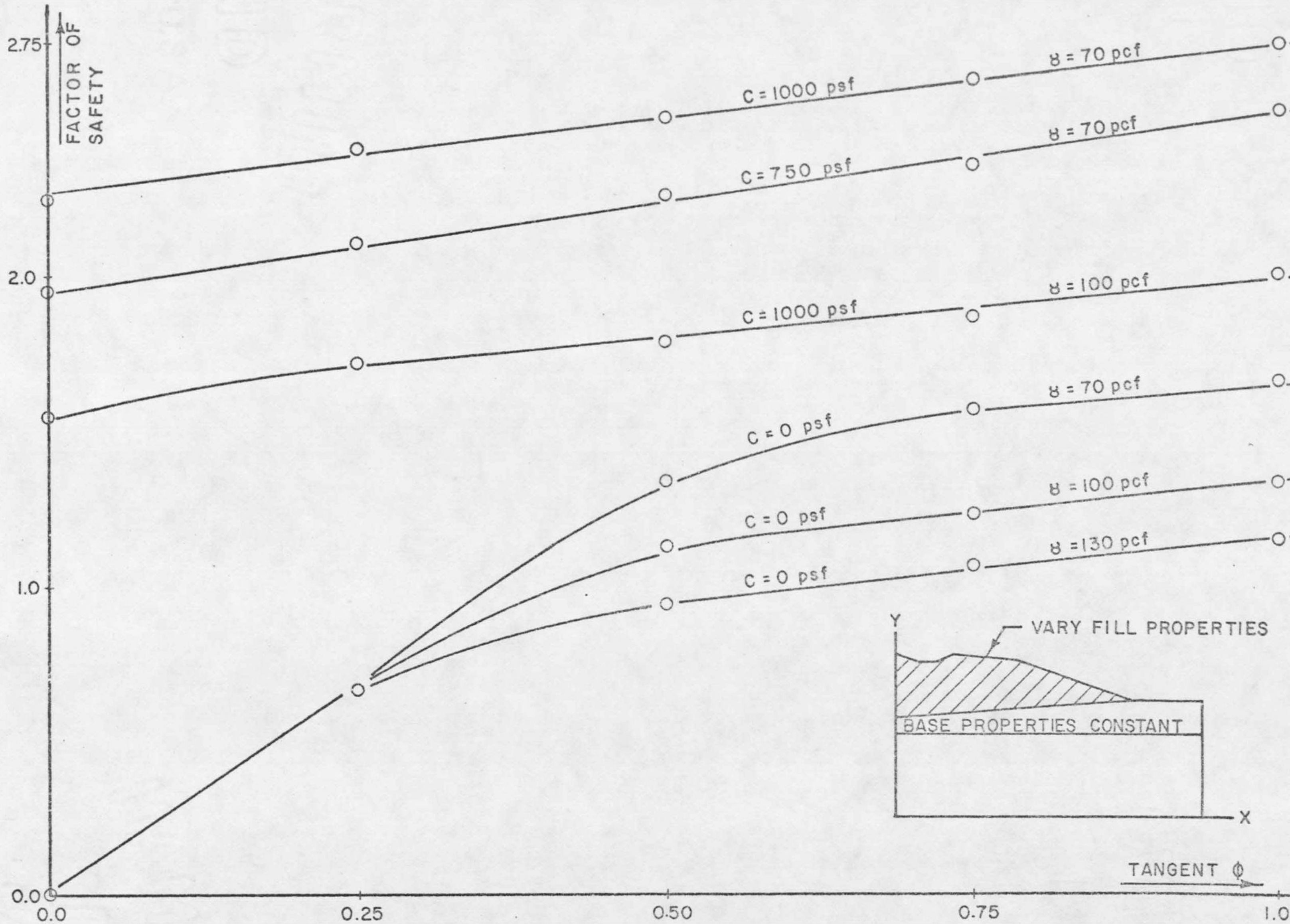

Figure 7. Factor of safety versus tangent ϕ for highway fill section with variable fill soil properties.

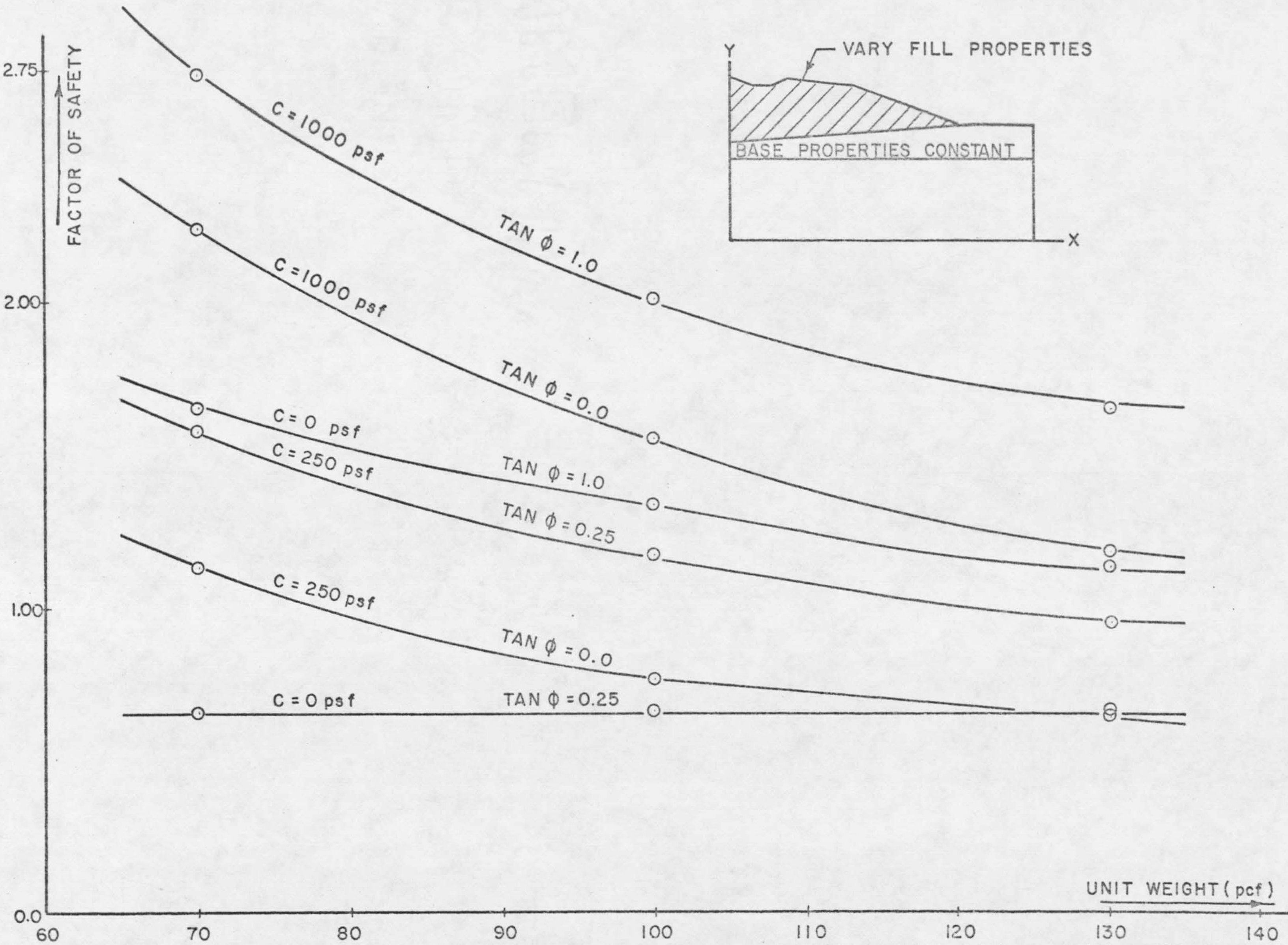

Figure 8. Factor of safety versus unit weight for highway fill section with variable fill soil properties.

highest factor of safety results from a lower value of unit weight (in this case 70 pcf), and the higher strength values for unit cohesion (1000 psf), and for the angle of internal friction (45°). Again, it is noted that combinations of higher values of unit weight and lower values of unit cohesion and angle of internal friction produce the only factors of safety below unity. It is interesting to note the similarity in factors of safety produced by low values of unit cohesion and angle of internal friction for a unit weight of 130 pcf.

Mathematical Reduction of Initial Data

The sensitivity of the factor of safety to change in the soil parameters can be expressed mathematically. A tabulation of the slopes between points figured from the data in Table I is shown in Table IIa, Table IIb, and Table IIc. The slope values represent the change in factor of safety per unit change in the variable plotted along the horizontal axis. The last columns in Table IIa and Table IIb list the average slopes for the parallel straight line portions of the curve families as was shown to occur in Figure 6 and Figure 7. As indicated by these values, the slope of the parallel curves, and thus the relative increase of the factor of safety, decreases with each increase in unit weight. It can be further stated that the greatest affect on the slope for unit weight increase is shown by the curve families with unit cohesion as the variable. This is also illustrated in Table IIc where slope values from data such as that plotted in Figure 8 are given. As shown, soils with high cohesive strength properties and low internal friction properties are more severely affected by an increase in unit weight than are soils with high

TABLE IIa

STRAIGHT LINE SLOPE VALUES FOR DATA POINTS GIVEN IN TABLE I
(Typical Plotted Curves Shown in Figure 6)

γ (pcf)	Tan φ	c (psf)				average	group average
		0-250	250-500	500-750	750-1000		
70	0.00	0.00450	0.00290	0.00106	0.00114	0.00110*	0.00108
	0.25	0.00367	0.00126	0.00082	0.00120	0.00101*	
	0.50	0.00155	0.00136	0.00081	0.00095	0.00117	
	0.75	0.00109	0.00119	0.00082	0.00109	0.00105	
	1.00	0.00117	0.00097	0.00133	0.00082	0.00107	
100	0.00	0.00310	0.00170	0.00062	0.00066	0.00067*	0.00067
	0.25	0.00206	0.00078	0.00061	0.00073	0.00067*	
	0.50	0.00071	0.00068	0.00068	0.00067	0.00068	
	0.75	0.00059	0.00068	0.00072	0.00060	0.00065	
	1.00	0.00059	0.00062	0.00055	0.00104	0.00070	
130	0.00	0.00258	0.00122	0.00074	0.00026	0.00050*	0.00049
	0.25	0.00117	0.00053	0.00050	0.00045	0.00048*	
	0.50	0.00064	0.00042	0.00044	0.00056	0.00052	
	0.75	0.00044	0.00032	0.00054	0.00061	0.00048	
	1.00	0.00029	0.00054	0.00060	0.00054	0.00049	

* average slope of linear portion of curve only

TABLE IIb

STRAIGHT LINE SLOPE VALUES FOR DATA POINTS GIVEN IN TABLE I
(Typical Plotted Curves Shown in Figure 7)

γ (pcf)	c (psf)	Tan ϕ				average	group average
		0.00-0.25	0.25-0.50	0.50-0.75	0.75-1.00		
70	0	2.66	2.66	0.95	0.35	0.650*	0.531
	250	1.85	0.55	0.496	0.420	0.458*	
	500	0.815	0.655	0.320	0.200	0.498	
	750	0.580	0.640	0.332	0.710	0.565	
	1000	0.631	0.400	0.463	0.440	0.484	
100	0	2.66	1.79	0.456	0.427	0.441*	0.433
	250	1.63	0.448	0.336	0.432	0.384*	
	500	0.688	0.340	0.340	0.372	0.435	
	750	0.629	0.407	0.376	0.204	0.404	
	1000	0.708	0.344	0.308	0.636	0.499	
130	0	2.67	0.688	0.548	0.292	0.420*	0.358
	250	1.26	0.564	0.348	0.136	0.242*	
	500	0.172	0.460	0.248	0.352	0.308	
	750	0.328	0.400	0.356	0.412	0.374	
	1000	0.520	0.508	0.403	0.348	0.445	

TABLE IIc

STRAIGHT LINE SLOPE VALUES FOR DATA POINTS GIVEN IN TABLE I
(Typical Plotted Curves Shown in Figure 8)

Tan ϕ	c (psf)	γ (pcf)	
		70-100	100-130
0.00	0	-0.00	-0.00
0.25	250	-0.0135	-0.00737
0.50	500	-0.0200	-0.0085
0.75	750	-0.0208	-0.0107
1.00	1000	-0.0231	-0.0131

* average slope of linear portion of curve only

frictional properties and low cohesive strength.

A method of making the previous results more useable is to simply combine the variables using the average slope values given in Table II. To determine the effect of changing a given variable, an equation of the following form was devised:

$$\text{New F.S.} = \text{Old F.S.} + \frac{\Delta\text{F.S.}}{\text{unit } c}(\Delta c) + \frac{\Delta\text{F.S.}}{\text{unit } \tan\phi}(\Delta\tan\phi) + \frac{\Delta\text{F.S.}}{\text{unit } \gamma}(\Delta\gamma) \tag{4.1}$$

where: F.S. = the factor of safety against sliding, and

Δ = the change in variable value.

Equation 4.1 can be used to advantage when determining the change in the factor of safety due to slight changes in some originally assumed soil condition. An example of the procedure used in solving the above equation is as follows: Assume that a "best guess" of soil conditions gave the fill material in the cross-section a unit weight of 100 pcf, a unit cohesion of 250 psf, and a coefficient of friction of 0.25. From Table I the value of the factor of safety is read directly at 1.179. Now suppose it is realized that the actual soil conditions are more nearly approximated by a unit weight of 120 pcf, a unit cohesion of 200 psf, and a coefficient of friction of 0.20. The change in factor of safety (ΔF.S.) for each variable can now be determined by averaging slope values. These are shown in Tables IIa, IIb, and IIc and were calculated directly from the data shown in Table I.

From Table IIa the slope is 0.00206 for a unit cohesion from zero to 250 psf, a unit weight of 100 pcf, and a coefficient of friction equal to 0.25. From Table IIb for a coefficient of friction from zero to 0.25, and a unit cohesion equal to 250 psf, the slope value is 1.63. A slope value of -0.00737 is read from Table IIc for a unit weight ranging from 100 pcf to 130 pcf, a unit cohesion equal to 250 psf, and a coefficient of friction equal to 0.25. These values can be placed in the equation as follows:

$$\text{New F.S.} = \text{Old F.S.} + (0.00206)\Delta c + (1.630)(\Delta\tan\phi) + (-0.00737)(\Delta\gamma) \qquad (4.2)$$

In the above example problem:

$$\Delta c = -50 \text{ psf},$$

$$\Delta\tan\phi = -0.05, \text{ and}$$

$$\Delta\gamma = +20 \text{ pcf}.$$

Then:

$$\text{New F.S.} = 1.179 + (0.00206)(-50) + (1.630)(-0.05) + (-0.00737)(20)$$

$$= 1.179 - 0.103 - 0.0815 - 0.1474$$

$$\text{New F.S.} = 0.85.$$

A plot of the data shown in Table I indicates an actual value of the new factor of safety equals 0.95. Thus, an error in excess of 10% results from assuming straight lines between data points, and using the resulting slope values for approximate interpolation. More accurate results could be obtained by exact interpolation using graphs of all data in Table I. Considerable time can be saved, however, by not having to plot all of the data. Less error could be expected in the portions of the data represented by nearly linear relationships.

The calculations for the new factor of safety reveal the soil parameter which has the most influence in any particular situation. In the example problem, the increase of 20 pcf in unit weight had more effect on the factor of safety than did the corresponding decrease in both unit cohesion and coefficient of friction. Emphasis should be placed on accurately determining the value of the unit weight in the field investigation for this example situation.

Minimum Solution--Variation of Base Parameters

Figures 3 and 5 show that in the solutions for the soil conditions shown in Figure 3 and for variation of fill soil properties, the soil layer directly beneath the fill has an important influence on the final results. Nearly all of the critical failure arcs pass through this "base" soil. As a second part of this investigation, a complete solution similar to that described for the fill soil was performed using the base soil parameters as variables, while the fill soil properties were assumed to be constant and equal to the respective values shown in Figure 3. The resulting minimum factors of safety for all soil property combinations are tabulated in Table III.

Figure 9 shows the failure arcs for the 70 pcf factors of safety tabulated in Table III. As before, these arcs are representative of the data taken for other unit weight conditions. It is interesting to note that two distinct bands of failure arcs can be distinquished in Figure 9. All arcs shown passing through or just below the base soil are for conditions of low internal friction. Even the arc for a coefficient of friction equal to zero and unit cohesion of 1000 psf is included in the upper

TABLE III

FACTORS OF SAFETY FOR HIGHWAY FILL PROBLEM WITH VARIATION OF BASE SOIL PROPERTIES

γ	Tangent ϕ					c
(pcf)	0.0	0.25	0.50	0.75	1.00	(psf)
70	1.492	1.608	1.687	1.765	1.840	
100	1.530	1.787	1.826	1.961		1000
130	1.577	1.818	1.847	1.837	2.064	
70	1.206	1.537	1.591	1.621	1.683	
100	1.253	1.845	1.763	1.883	1.889	750
130	1.267	1.831	1.851	1.852	1.865	
70	0.969	1.456	1.502	1.552	1.688	
100	0.963	1.730	1.849	1.797	1.837	500
130	0.980	1.777	1.836	1.893	1.877	
70	0.659	1.321	1.444	1.466	1.555	
100	0.662	1.442	1.631	1.741	1.843	250
130	0.674	1.511	1.844	1.940	1.852	
70	0.370	1.000	1.371	1.435	1.501	
100	0.378	1.129	1.558	1.600	1.685	0
130	0.371	1.212	1.677	1.348	1.881	

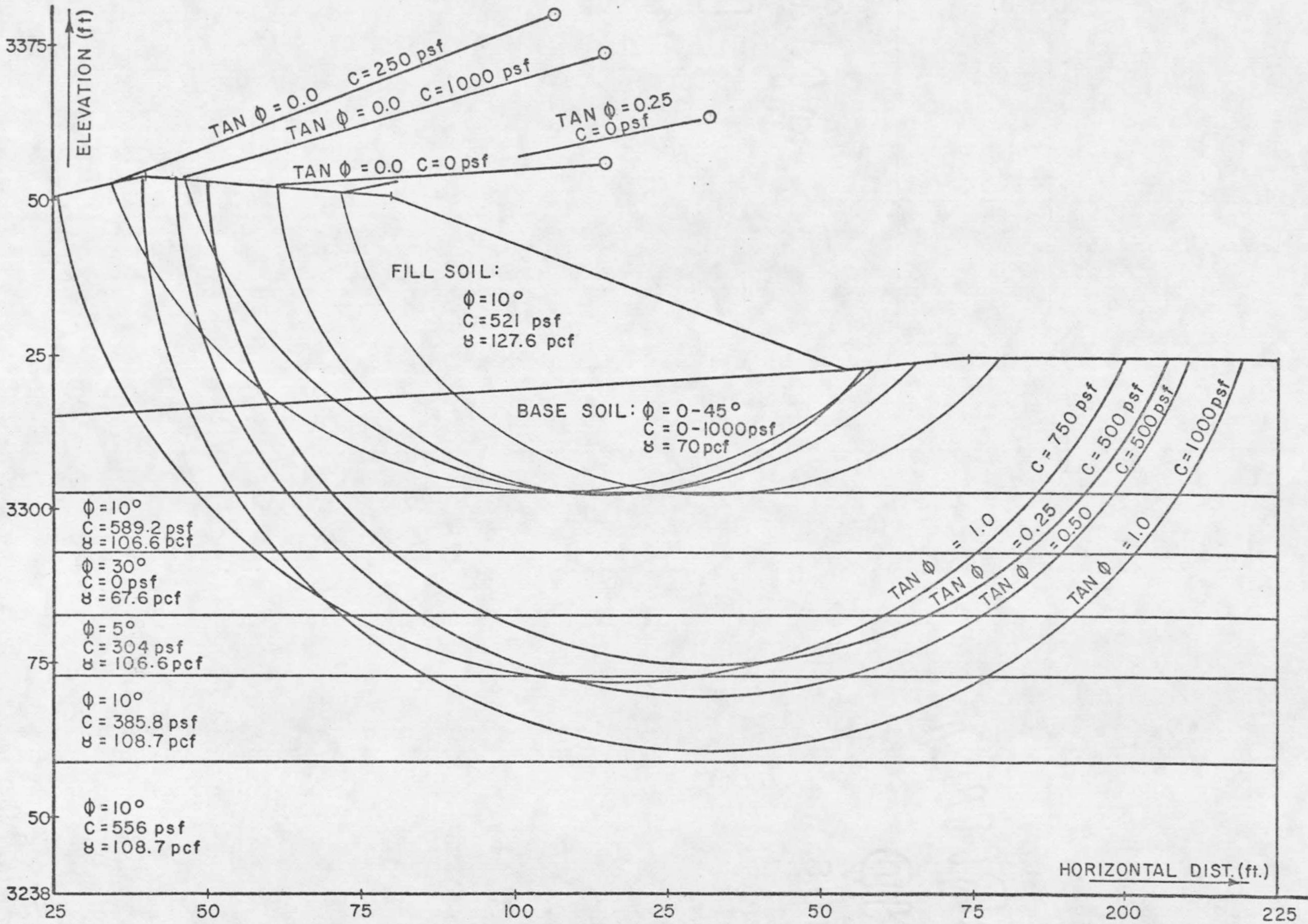

Figure 9. Typical failure surface pattern for highway fill section with variable base soil properties.

band of arcs. Correspondingly, all arcs in the lower band are for coefficients of friction ranging from 0.25 to 1.00. The lowest and longest arc is for the highest strength parameters used; that is, a coefficient of friction equal to 1.0 and a unit cohesion equal to 1000 psf.

The reason for the two bands of arcs is easily explained. Both the fill soil and the soil below the base soil (soil number 3) have angles of internal friction equal to 10°. When the base soil is given low angles of internal friction and low values of unit cohesion, a relatively short arc passing through the unstable base soil will produce a minimum factor of safety. If the entire arc passed through the fill material, the relatively stable soil would produce a high factor of safety. The same would be true if a major portion of the arc passed through the similar soil properties of soil number 3. As soon as the strength parameters in the base soil are assumed to increase, however, a new "low strength zone" must be found to produce minimum factors of safety. In the high strength base problem, the fill material, the base soil, and soils number 3 and 4 were avoided due to high strength conditions. The search for a minimum condition ended in soils 5 and 6 because at this point the arcs were finally long enough to develop enough resistance due to cohesion and to enclose an adequate amount of soil to produce significant frictional resistance. It can be seen that had soils 3 through 6 been given extremely low or zero values of the friction angle, the search for a minimum factor of safety could not have converged.

Figure 10 shows curves for the coefficient of friction and unit weight of the base soil when the factor of safety is plotted against

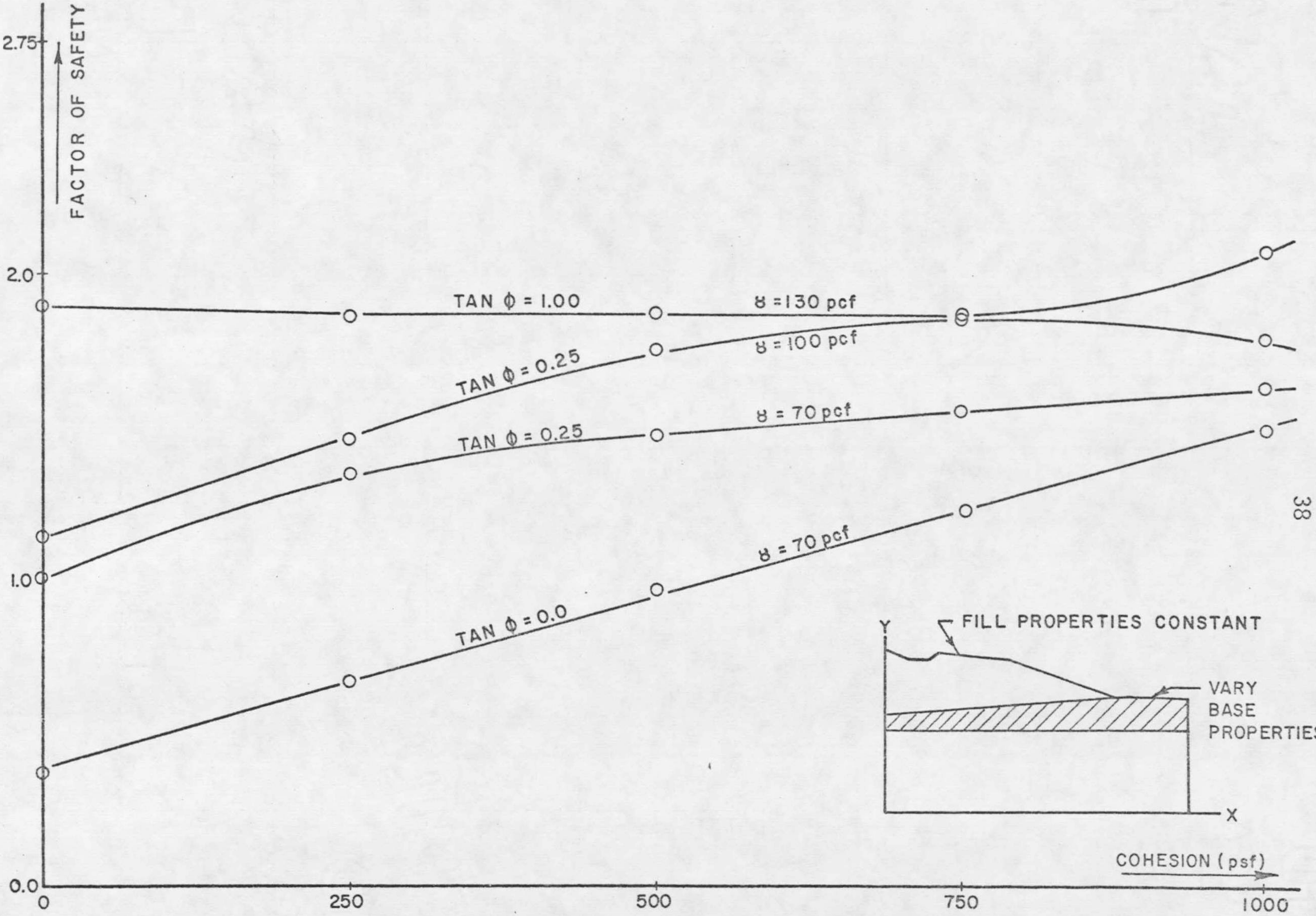

Figure 10. Factor of safety versus cohesion for highway fill section with variable base soil properties.

unit cohesion of the base soil. The curves in this figure show only basic resemblance to those of Figure 6 which were plotted for variable fill soil properties. Immediately apparent is the linear relationship for a coefficient of friction of zero. This is because the failure arcs for these low friction soils are in a uniform band as shown in Figure 9, and the factor of safety is increasing solely as a function of the increase in cohesion. When the base soil is given a slight amount of internal friction (except for the case of zero unit cohesion) the arcs move to a new region and the resulting factors of safety are again dependent upon combinations of strength parameters. Instead of the relatively uniform pattern of data characteristic of Figure 6, however, a rather irregular pattern results when the base soil parameters are varied, especially in the case of higher unit weights. The reason for the irregularity stems from the movement of the failure arc from a common band as shown in Figure 9 to a random location. This is due to the fact that the increase in unit weight causes the resistance due to friction at the base soil level to immediately increase far beyond what it was when no internal friction was available. The search for a minimum factor of safety then changes from arcs far below this new zone of strength to arcs passing through the fill material only. An arc entirely in the fill material is the minimum condition in Figure 10 for the combination of coefficient of friction equal to 1.0, unit cohesion equal to 1000 psf, and base soil unit weight equal to 130.0 pcf.

The immediate increase in strength due to the change from a zero value of internal friction is shown in all curves of Figure 11. Beyond

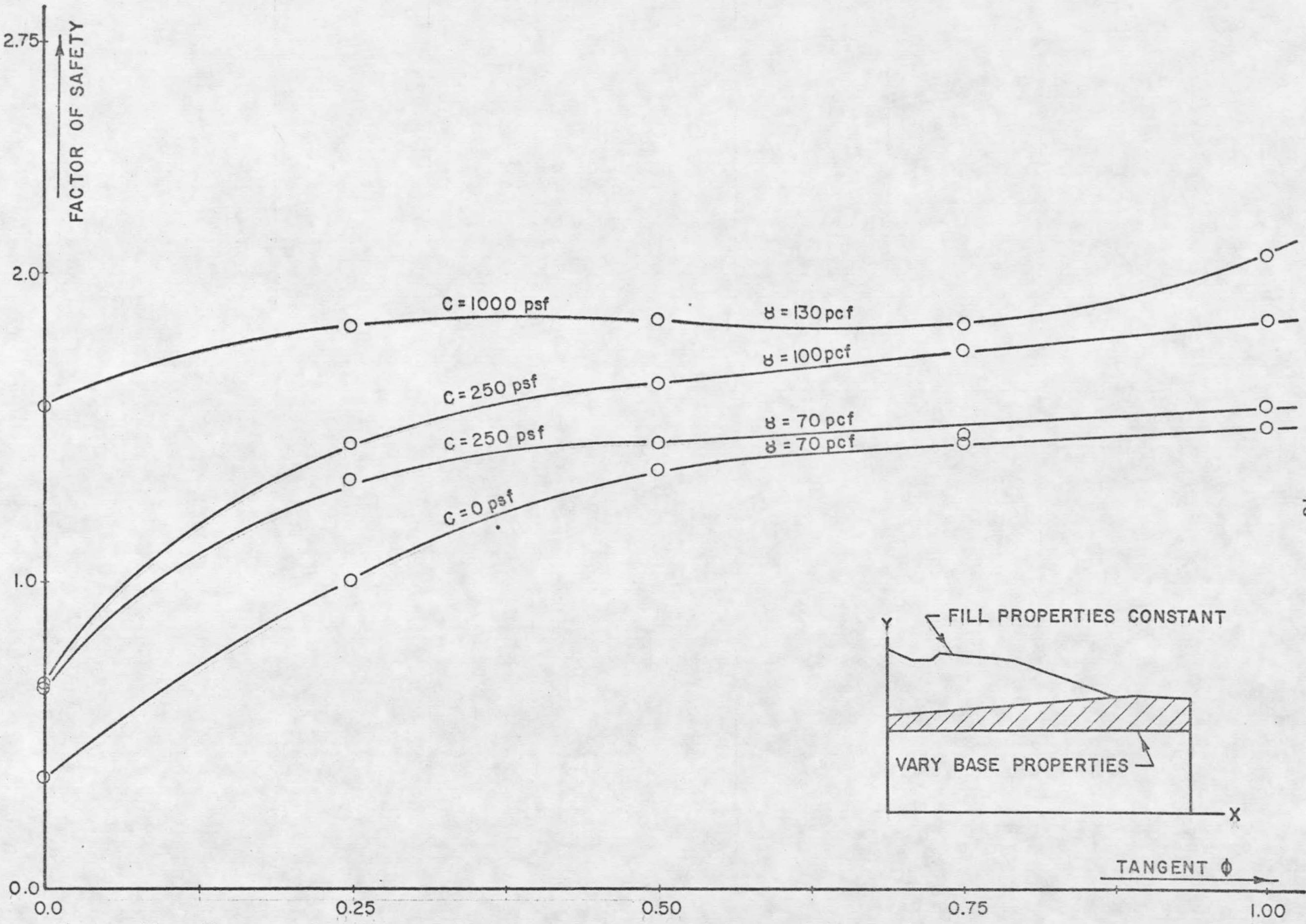

Figure 11. Factor of safety versus tangent ϕ for highway fill section with variable base soil properties.

the initial increase in the coefficient of friction (approximately from 0.0 to 0.50), additional increase does not appreciably alter the relative increase of strength. This is shown in the upper curves of both Figures 10 and 11. For high unit weight and strength parameter values in the base soil, the minimum failure arc moves up and out of this zone. The further increase in strength or weight of this soil has no bearing on the minimum solution for this particular problem.

Figure 12 shows curves for various combinations of internal friction and unit cohesion when the factor of safety is plotted against unit weight. The curves illustrate how the factor of safety is only slightly affected by increases in the base soil unit weight when the coefficient of friction equals zero. This is probably because cohesive resistance along the failure arc just compensates for the increased tangential weight component. In the case of zero unit cohesion, positive and negative tangential weight components must be nearly balanced in the base soil.

The curves of Figure 12 show substantial increases in strength result from increasing unit weight values when a noncohesive base soil is given some value of internal friction. As the unit weight is increased, the normal weight component of each individual slice is increased, and therefore when internal friction is available, the frictional resistance to sliding along a particular arc is increased.

The upper two curves shown in Figure 12 compare rates of increase for minimum factors of safety when combinations of relatively high values of internal friction and unit cohesion in the base soil are plotted. As

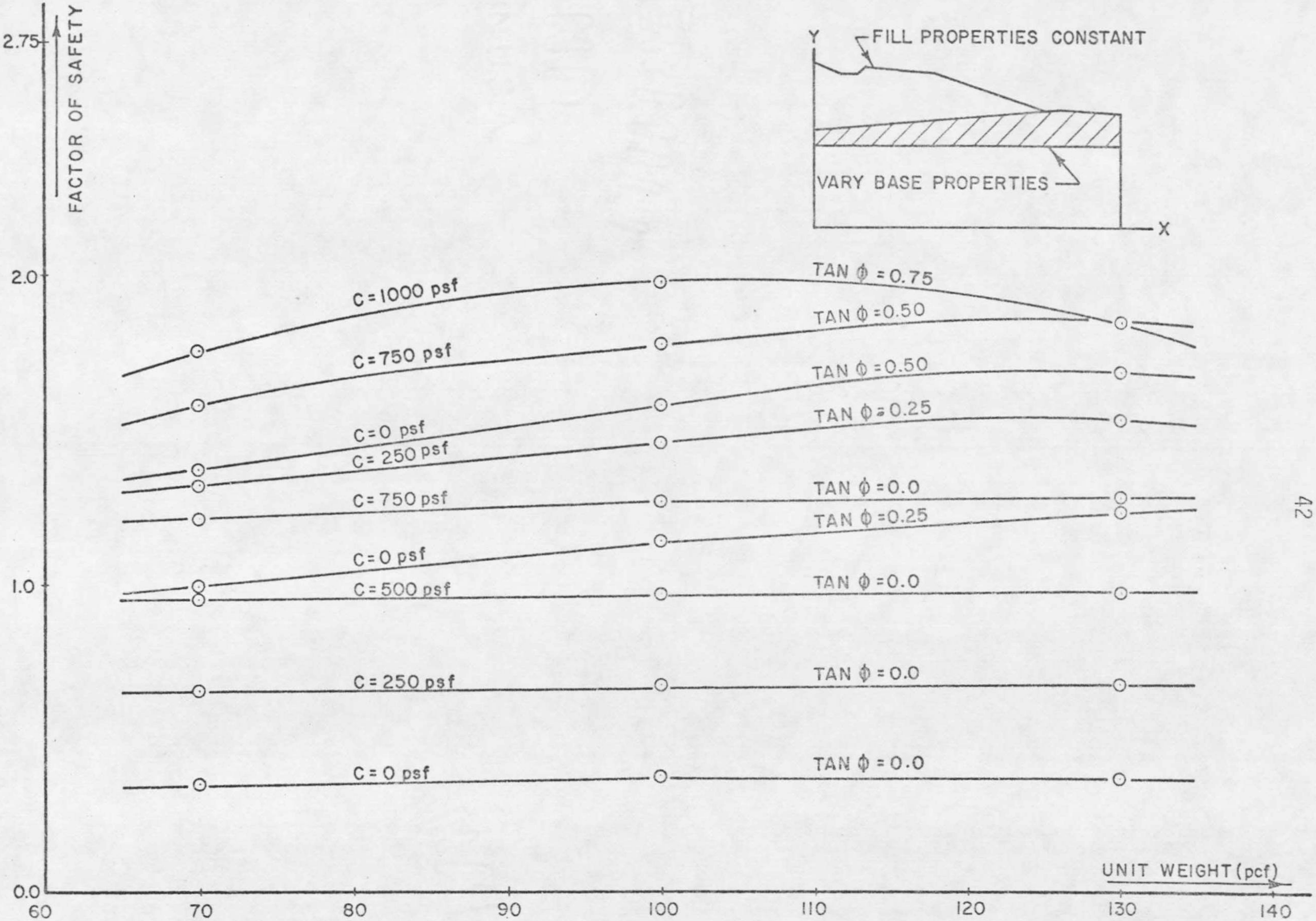

Figure 12. Factor of safety versus unit weight for highway fill section with variable base soil properties.

noted previously, low values of the strength parameters (coefficient of friction equal to 0.25 and unit cohesion equal to 250 psf) in the base soil allows the minimum failure arc to pass through it and thus an increase in the unit weight causes a uniform increase in the factor of safety against sliding. As shown, high values of the base soil strength parameters in combination with high unit weight causes the critical failure arc to not pass through the base soil, and thus the value of the factor of safety for the upper curves at 130 pcf is independent of the base soil properties.

Fixed-Arc Solution--Variation of Fill Parameters

To determine the importance of failure arc location in this non-homogeneous slope stability analysis, a series of solutions were performed for a fixed arc. In other words, an arc was chosen that was thought to be representative of the pattern of locations shown in Figures 5 and 9, and minimum factors of safety for the various assumed soil conditions were calculated for this arc. This would be similar to an actual slope stability problem where time and facilities limited calculation to only those conditions which experience and judgment determined to be critical. The arc chosen was that which produced the minimum factor of safety for the original soil conditions as shown in Figure 3.

Factors of safety calculated are shown in Table IV for various combinations of unit cohesion, coefficient of friction, and unit weight of the fill soil. For comparison with the results of the minimum-arc solutions shown in Table I, similar relationships were plotted for the fixed-arc solutions. These curves are shown in Figures 13, 14, and 15,

TABLE IV

FACTORS OF SAFETY FOR HIGHWAY FILL PROBLEM WITH FIXED ARC AND VARIATION OF FILL SOIL PROPERTIES

γ	Tangent ϕ					c
(pcf)	0.0	0.25	0.50	0.75	1.00	(psf)
70	2.222	2.352	2.482	2.612	2.743	1000
100	1.550	1.674	1.799	1.924	2.049	
130	1.210	1.332	1.454	1.528	1.698	
70	1.976	2.106	2.236	2.367	2.497	750
100	1.384	1.509	1.634	1.759	1.884	
130	1.086	1.207	1.330	1.452	1.574	
70	1.730	1.860	1.990	2.121	2.251	500
100	1.219	1.344	1.469	1.594	1.719	
130	0.961	1.084	1.205	1.328	1.450	
70	1.484	1.614	1.744	1.875	2.005	250
100	1.054	1.179	1.304	1.429	1.554	
130	0.837	0.959	1.081	1.203	1.326	
70	1.238	1.368	1.499	1.629	1.759	0
100	0.889	1.014	1.139	1.264	1.388	
130	0.712	0.835	0.957	1.079	1.201	

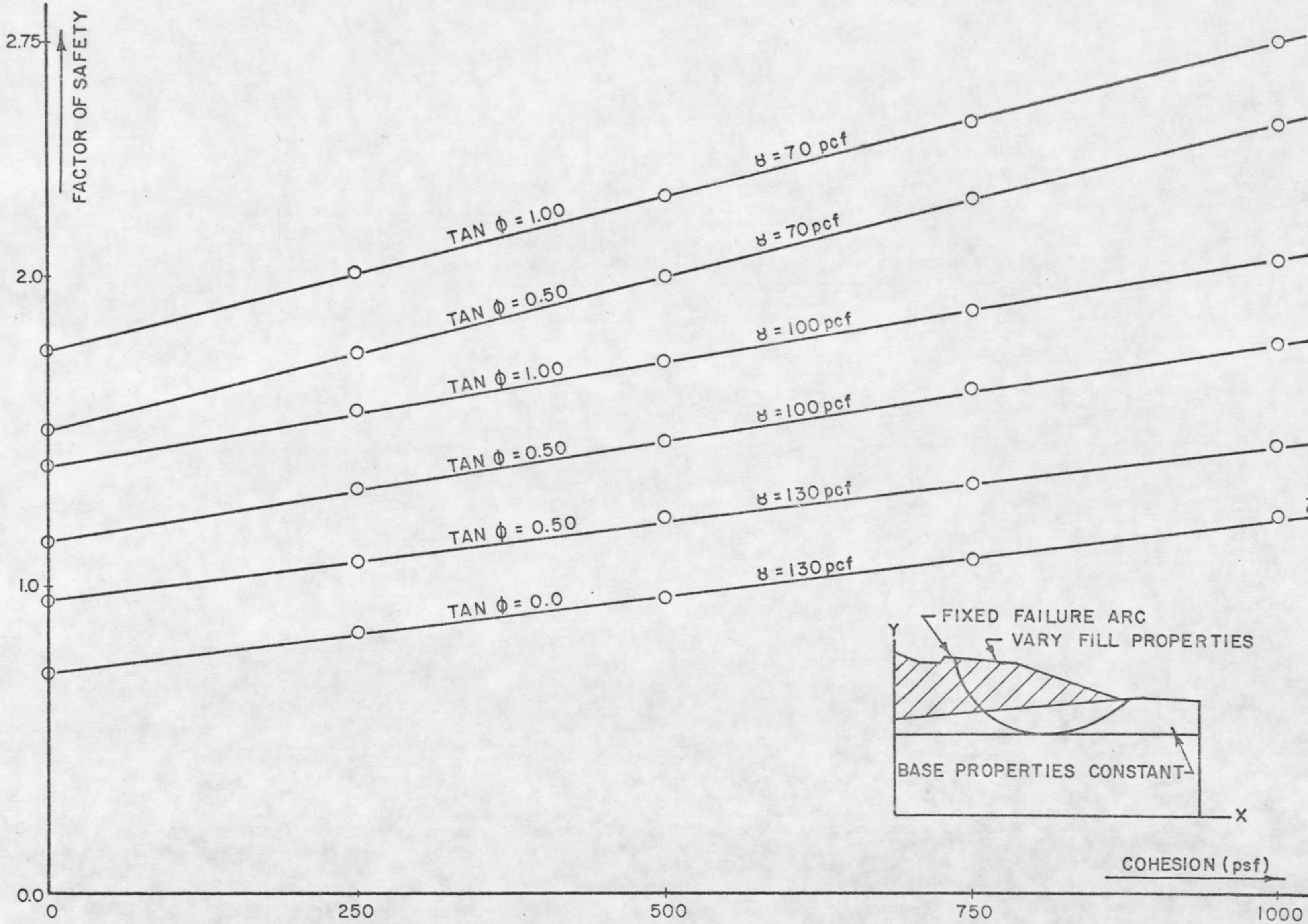

Figure 13. Factor of safety versus cohesion for highway fill section with fixed failure arc and variable fill soil properties.

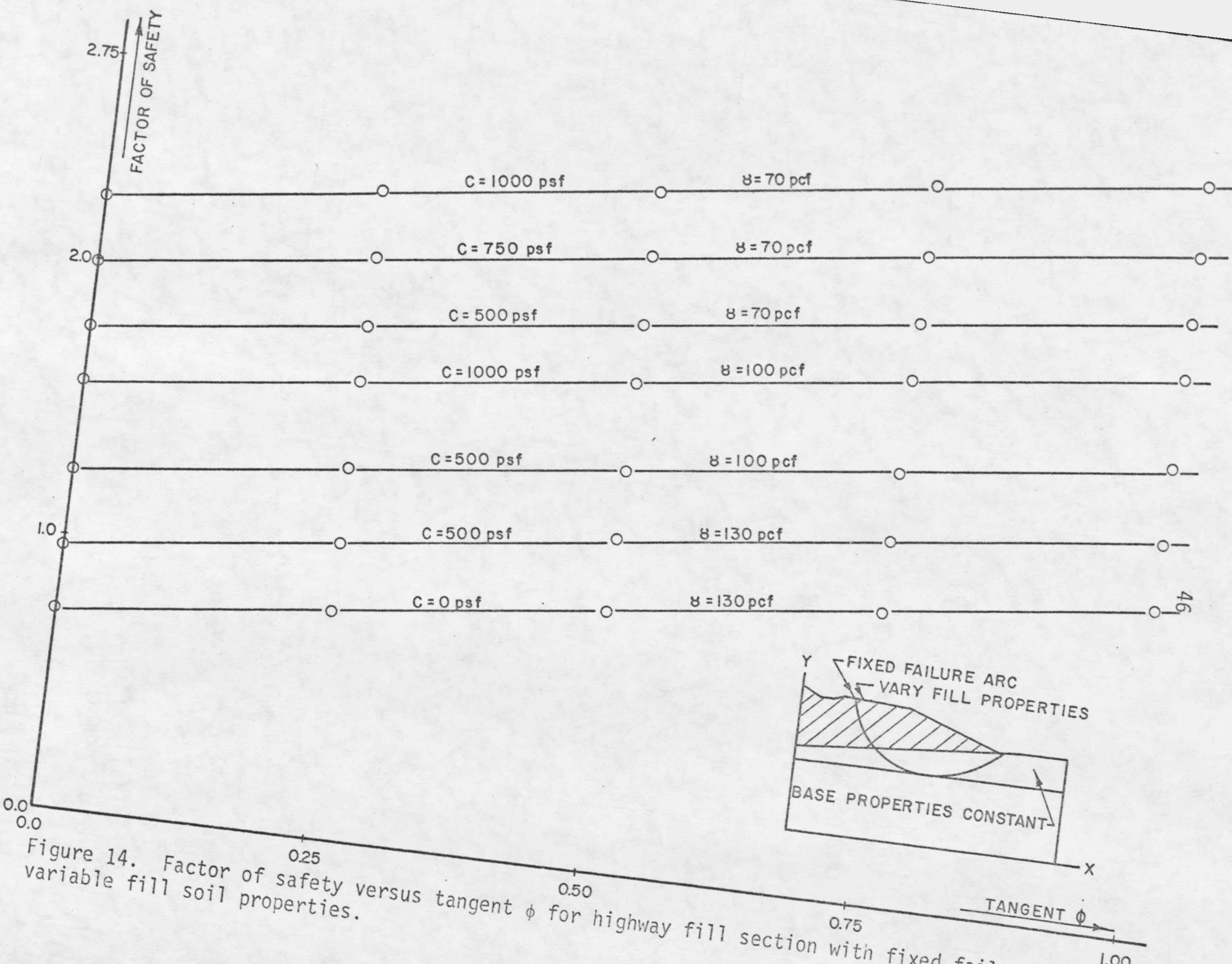

Figure 14. Factor of safety versus tangent ϕ for highway fill section with fixed failure arc and variable fill soil properties.

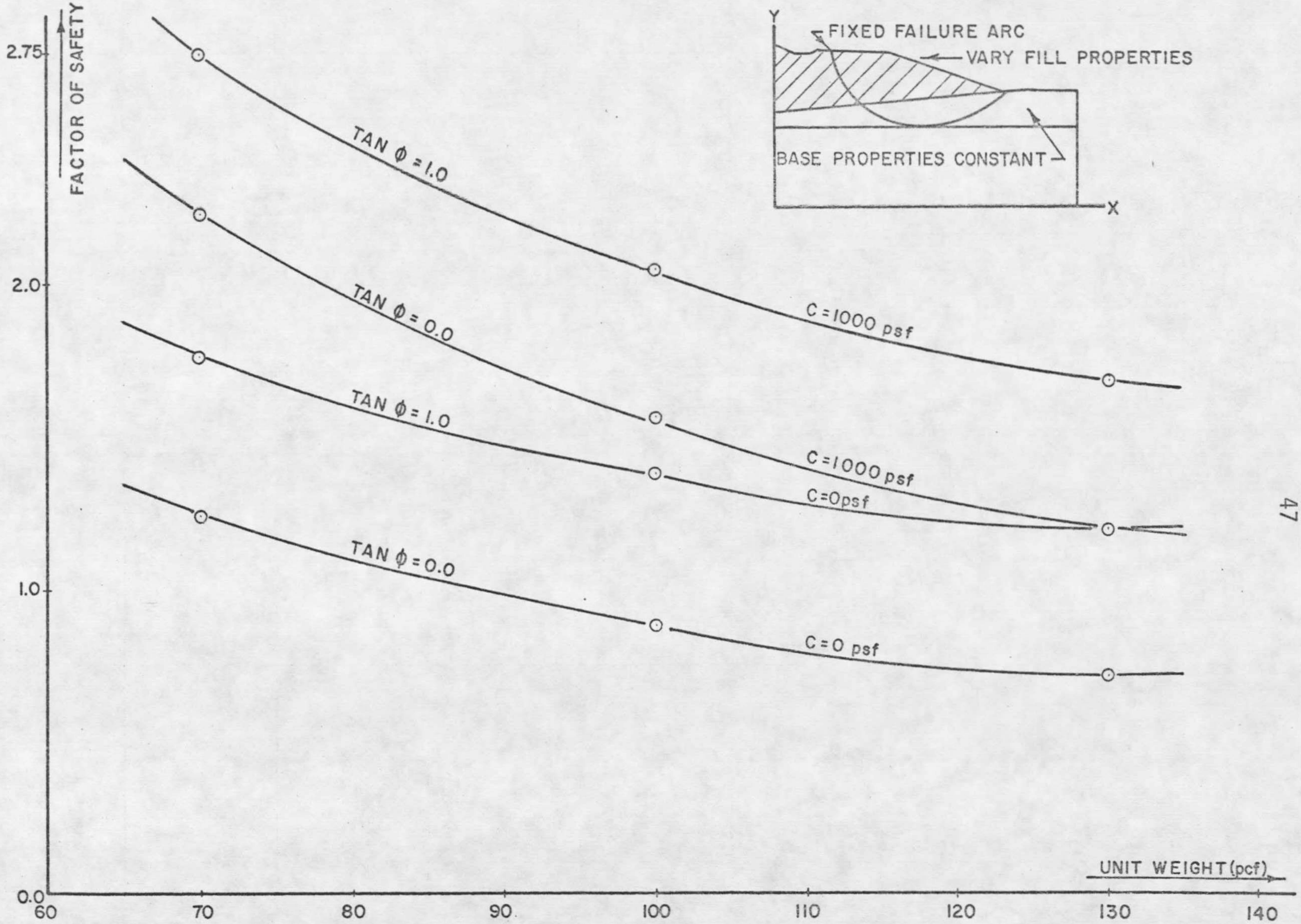

Figure 15. Factor of safety versus unit weight for highway fill section with fixed failure arc and variable fill soil properties.

and correspond to those shown previously in Figures 10 through 12.

The relationships shown in Figures 13 and 14 indicate that for a single arc, a nearly linear change in the factor of safety is caused by increasing the strength parameters. When the curves representing the fixed-arc solution are superimposed on those representing the minimum-arc solution, a distinct similarity in slopes and values of factor of safety are observed for the linear portions of the minimum arc curves. This is to be expected because the assumed fixed-arc represents an "average" failure arc in the band shown in Figure 5. It is also obvious that the portion of the curves representing combinations of low unit cohesion and coefficient of friction do not correspond. This fact is easily explained by again referring to Figure 5. When the fill soil is assumed to have low strength properties, it becomes the unstable unit of the over-all cross-section and the minimum factor of safety against sliding occurs along an arc passing through the fill only. The fixed arc is not representative of this condition, and will produce a higher factor of safety due to the strength of the base soil which it passes through.

A more complete comparison of minimum-arc and fixed-arc values is shown in Figure 16. In this figure, the differences in the calculated factors of safety are plotted against increasing values of the coefficient of friction. The major differences are for soils with a coefficient of friction between zero and 0.50 and unit cohesion between zero and 250 psf. For all other combinations of strength parameters, the differences range from zero to a maximum of plus or minus approximately 0.1. The differences are obviously not significant and the practical use of this knowledge is

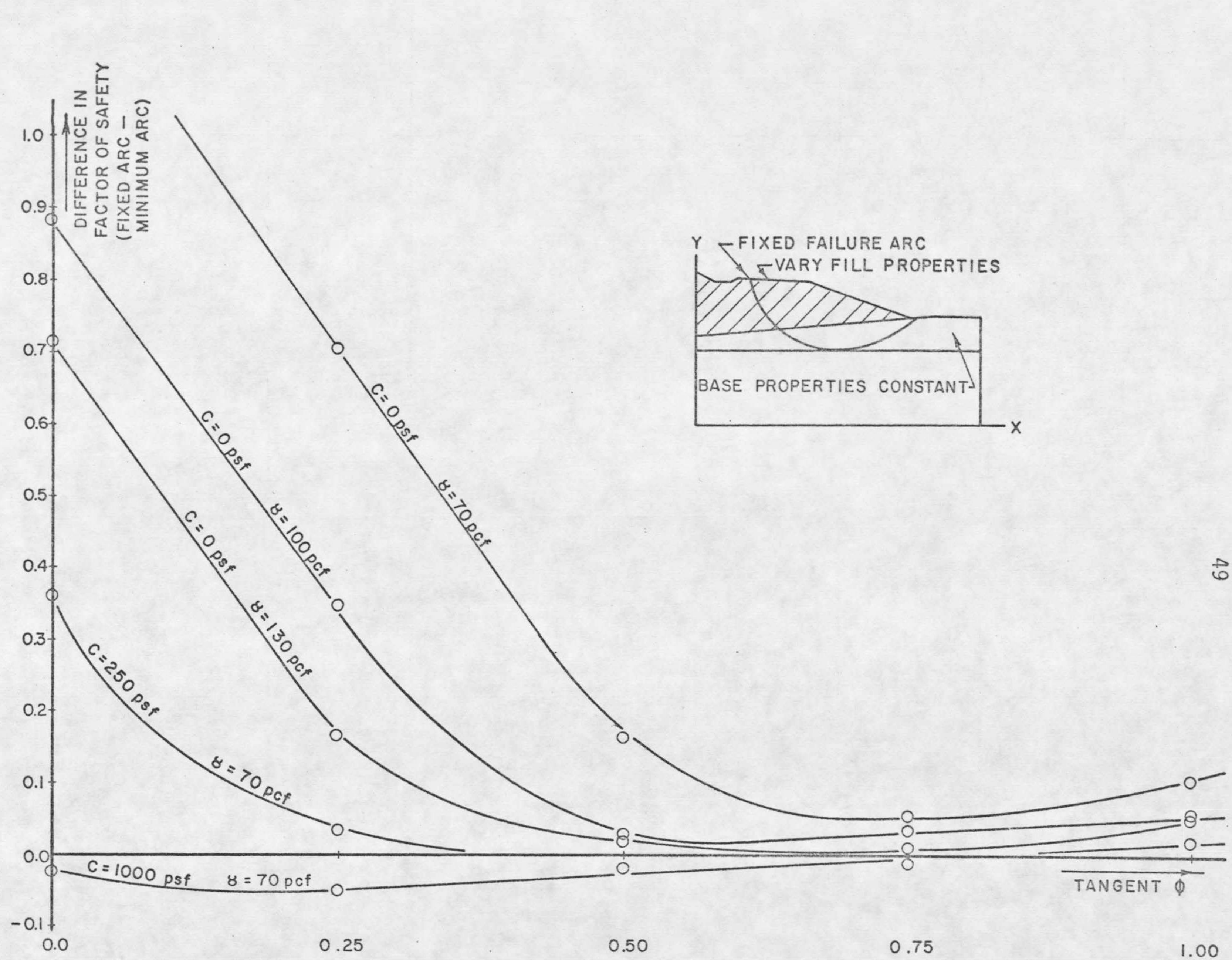

Figure 16. Fixed arc minus minimum arc factor of safety versus tangent ϕ for highway fill section with fixed failure arc and variable fill soil properties.

obvious.

Figure 15 shows factors of safety versus increasing unit weight. The curves are similar to those given in Figure 8. The comparable differences between the two figures due to low strength combinations are apparent, especially for the fill soil conditions of zero unit cohesion and zero coefficient of friction. For a soil with such properties there is obviously no strength and for any unit weight the factor of safety against sliding is zero. For a fixed-arc solution, however, some resistance is built up along the portion of the arc passing beneath the fill and a finite value for the factor of safety results. The lower curve shown in Figure 16 is therefore entirely erroneous. These values produce the points of greatest difference between the fixed-arc and minimum-arc solution as plotted in Figure 16.

<u>Fixed-Arc Solution--Variation of Base Parameters</u>

As a further check on the feasibility of using a fixed-arc analysis, the properties of the fill material were held constant at the original values shown in Figure 3 and the base properties were varied. Factors of safety for the same arc used before are tabulated in Table V, and plotted in the same manner as before in Figures 17, 18, and 19. It will be noted that the same linear relationships hold as when the fill properties were varied. Several basic differences are apparent, however. For a coefficient of friction equal to zero, all factors of safety calculated for all combinations of base unit weight and unit cohesion are nearly identical with those for the minimum-arc solutions. This is shown in the plots of the differences of these factors of safety shown in Figure 20. When

TABLE V

FACTORS OF SAFETY FOR HIGHWAY FILL PROBLEM WITH FIXED FAILURE ARC AND VARIATION OF BASE SOIL PROPERTIES

γ (pcf)	Tangent ϕ 0.0	0.25	0.50	0.75	1.00	c (psf)
70 100 130	1.516 1.546 1.575	2.255 2.395 2.542	2.994 3.245 3.505	3.734 4.094 4.469	4.473 4.943 5.432	1000
70 100 130	1.222 1.247 1.273	1.962 2.096 2.236	2.701 2.945 3.200	3.441 3.795 4.163	4.180 4.644 5.127	750
70 100 130	0.929 0.943 0.967	1.668 1.797 1.931	2.408 2.646 2.894	3.147 3.495 3.858	3.887 4.344 4.821	500
70 100 130	0.635 0.648 0.662	1.375 1.497 1.625	2.114 2.347 2.589	2.854 3.196 3.552	3.593 4.045 4.516	250
70 100 130	0.342 0.349 0.356	1.082 1.198 1.320	1.821 2.047 2.283	2.560 2.897 3.247	3.300 3.746 4.210	0

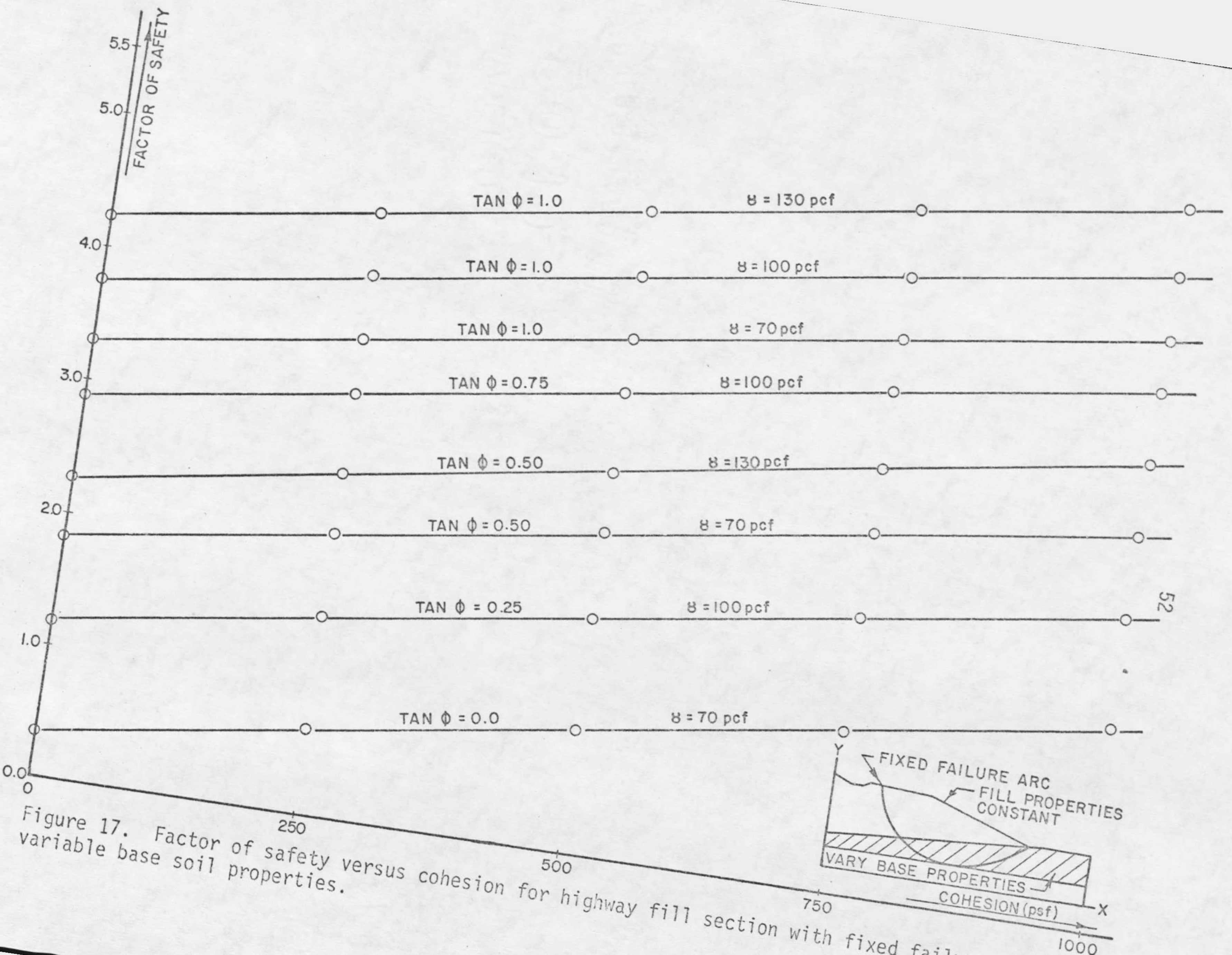

Figure 17. Factor of safety versus cohesion for highway fill section with fixed failure arc and variable base soil properties.

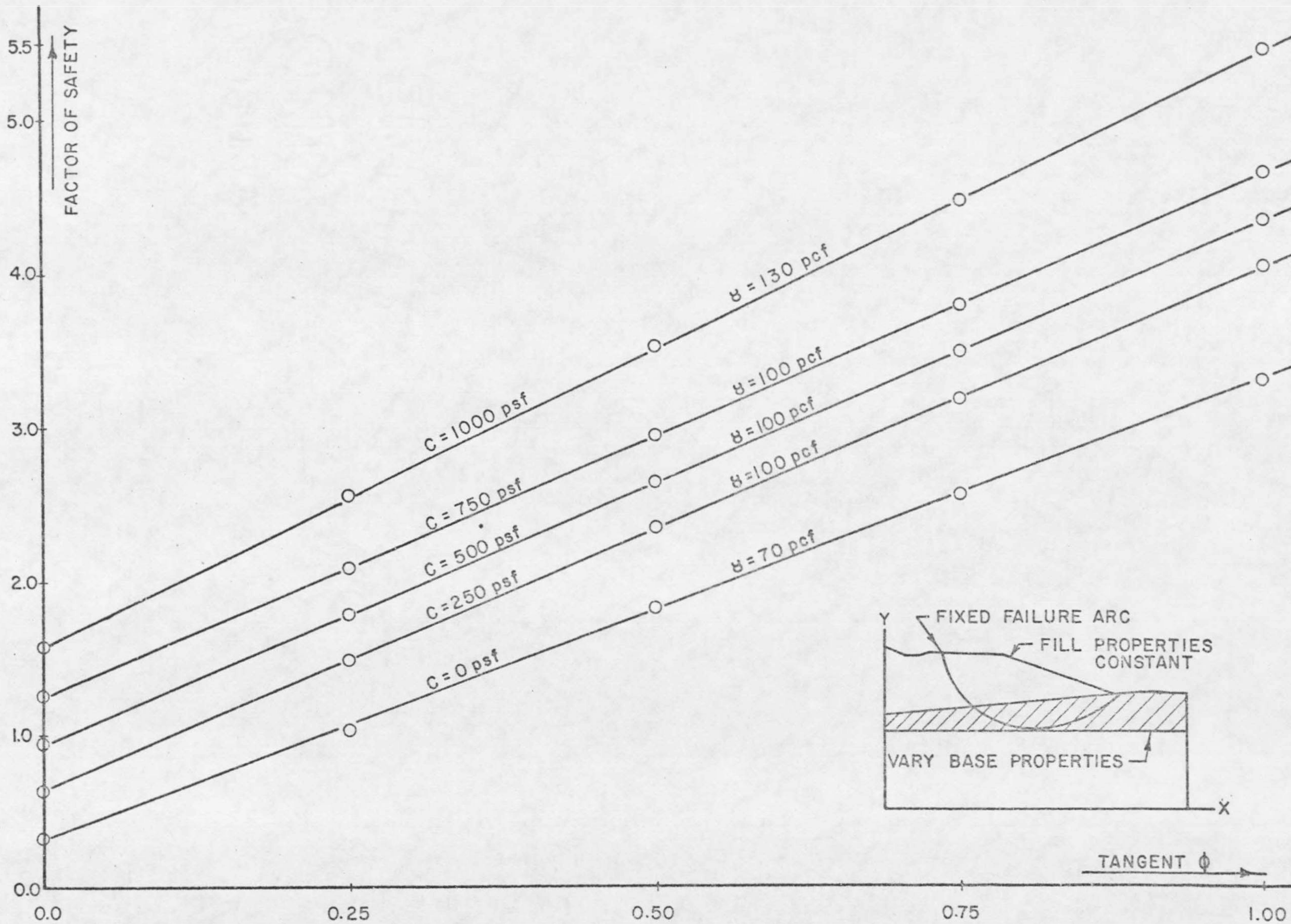

Figure 18. Factor of safety versus tangent ϕ for highway fill section with fixed failure arc and variable base soil properties.

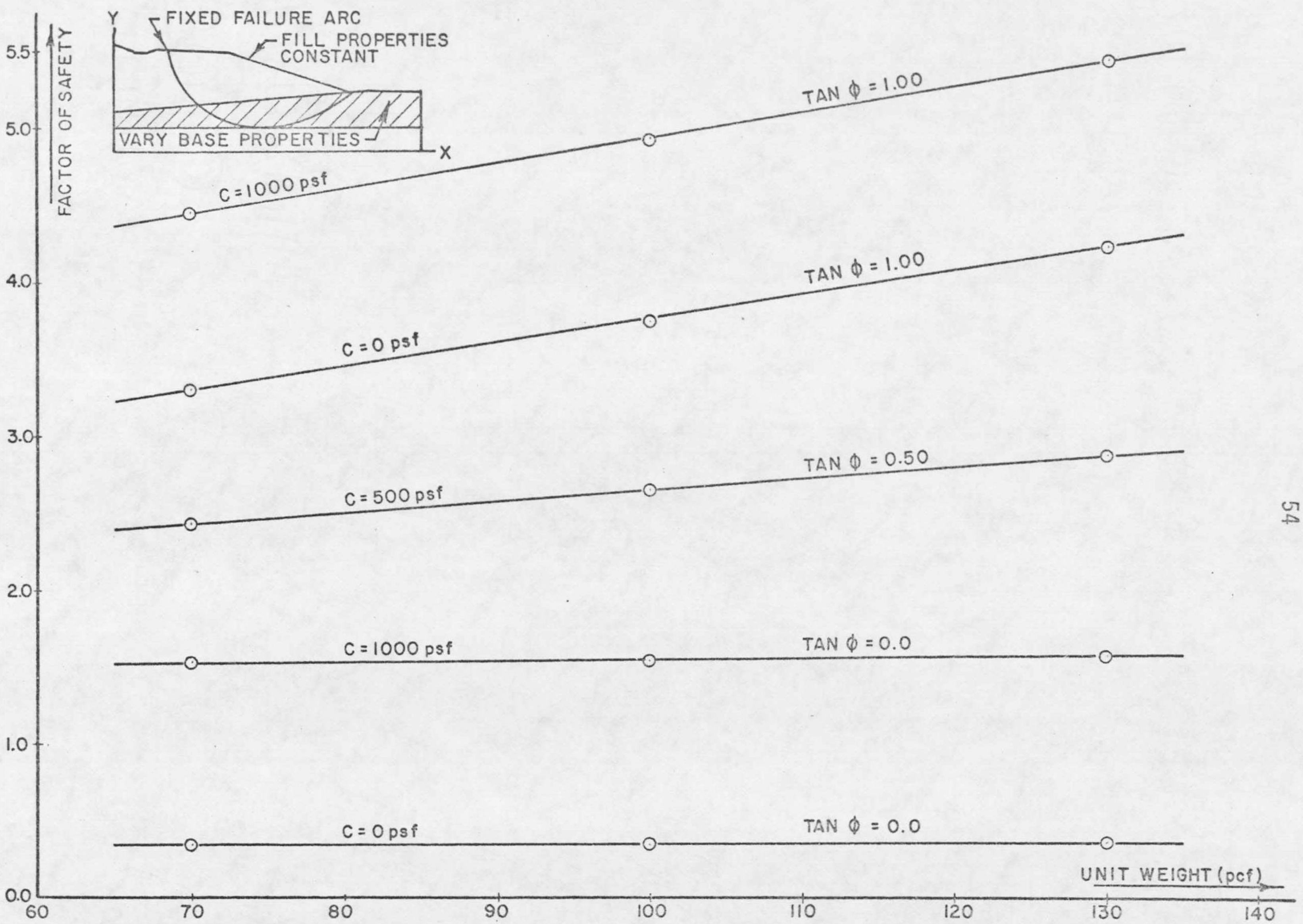

Figure 19. Factor of safety versus unit weight for highway fill section with fixed failure arc and variable base soil properties.

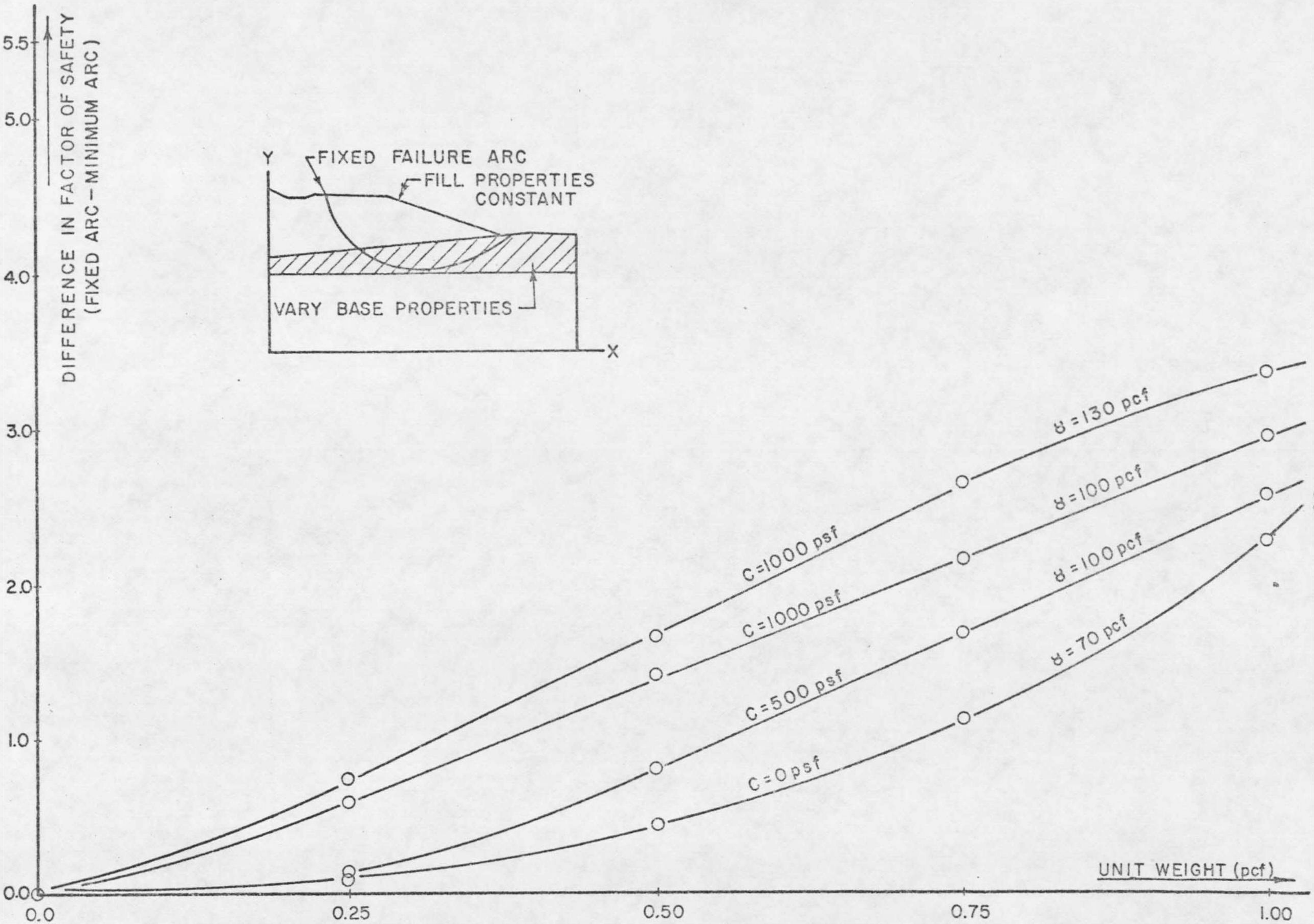

Figure 20. Fixed arc minus minimum arc factor of safety versus tangent ϕ for highway fill section with fixed failure arc and variable base soil properties.

no internal friction exists in the base soil, an increase in the unit weight of that soil has no effect on the slide resisting forces and a balancing effect occurs in the slide activating forces due to the position of the fixed arc. This can be seen in Figure 19 where all factors of safety plotted for zero coefficient of friction are identical for each increment of unit cohesion. As soon as the coefficient of friction is given a value, the resistance to sliding along the fixed arc is increased. Unit cohesion along the arc remains the same regardless of a change in unit weight, therefore; the rapid strength increases can be attributed solely to the property of internal friction. A comparison of upper and lower curves in Figure 19 supports this observation.

A look at Figure 20 reveals the fallacy of using a fixed-arc solution when varying the properties of the base soil. As the strength of the base soil increases, the resistance to sliding along the fixed arc through that soil increases proportionately. This increased resistance to sliding would cause a minimum-arc solution to search above or below the zone of strength for a more critical condition. In the fixed-arc case, however, the arc cannot move to a more critical position and the factors of safety calculated are exceedingly high. The resulting differences in the fixed-arc and minimum-arc factors of safety increase rapidly beyond the conditions of zero coefficient of friction, as shown in Figure 20.

Minimum Solution--Increased Base Strength

To further investigate the effect of the internal friction property in the base soil, a second complete set of solutions was performed for variable fill properties and fixed base properties. The base unit cohesion and unit weight were as shown in Figure 3, but the angle of internal friction was increased to 10°. Plotted data patterns and trends were similar to those shown previously in Figures 5 through 8. Values of the minimum factors of safety were consistently higher, however, for the new base soil conditions. The actual differences in factors of safety obtained in the two sets of calculations are shown plotted in Figure 21.

The over-all trend revealed in these plots is towards lower differences in factors of safety for higher fill unit weights. A check of failure locations indicates that in both cases where the fill unit weight is 70 pcf the minimum arcs pass through the base soil. The increased internal friction in the second case causes the large differences in the results as shown in Figure 21. Only a slight increase in the differences is caused by increase in fill soil strength. As the fill unit weight increases the failure arcs in both cases pass below the base soil, with the result that only short portions of the arcs pass through the base soil layer. Thus, lesser differences in the factors of safety occur for high fill unit weight. Figure 21 also indicates that combinations of high unit weight, coefficient of friction, and unit cohesion values cause a reversal of relative differences in factors of safety compared with combinations of high unit weight and low strength values.

The curves shown in Figure 22 represent a fill unit cohesion value of 500 psf and a coefficient of friction value of 0.50. Both curves demonstrate the trend towards less stability with increased fill unit weight. The consistent differences in over-all stability for the two values of base internal friction angle is graphically displayed by the two

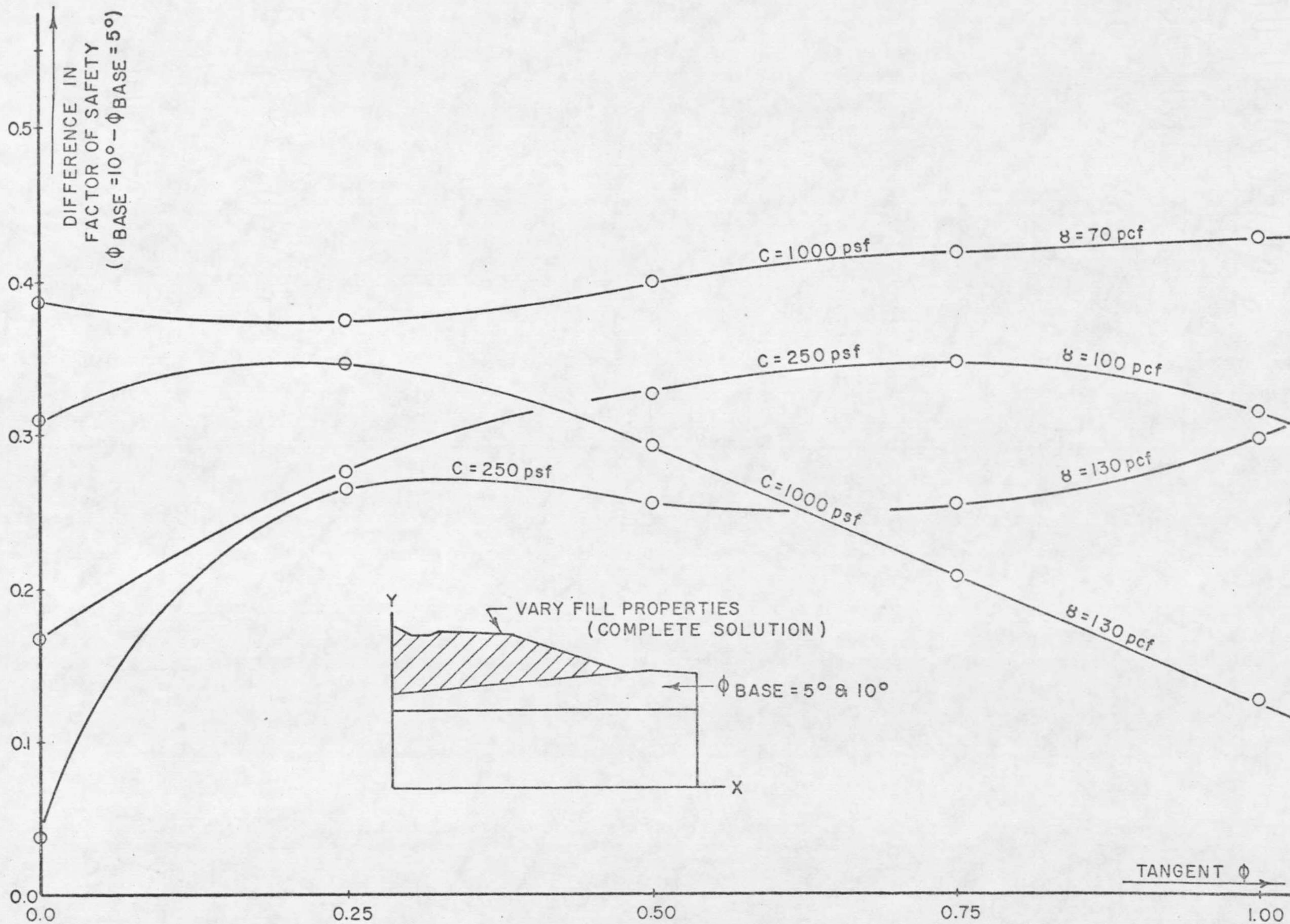

Figure 21. Difference in factor of safety versus tangent ϕ for highway fill section with variable fill soil properties and base soil friction angle of 5° and 10°.

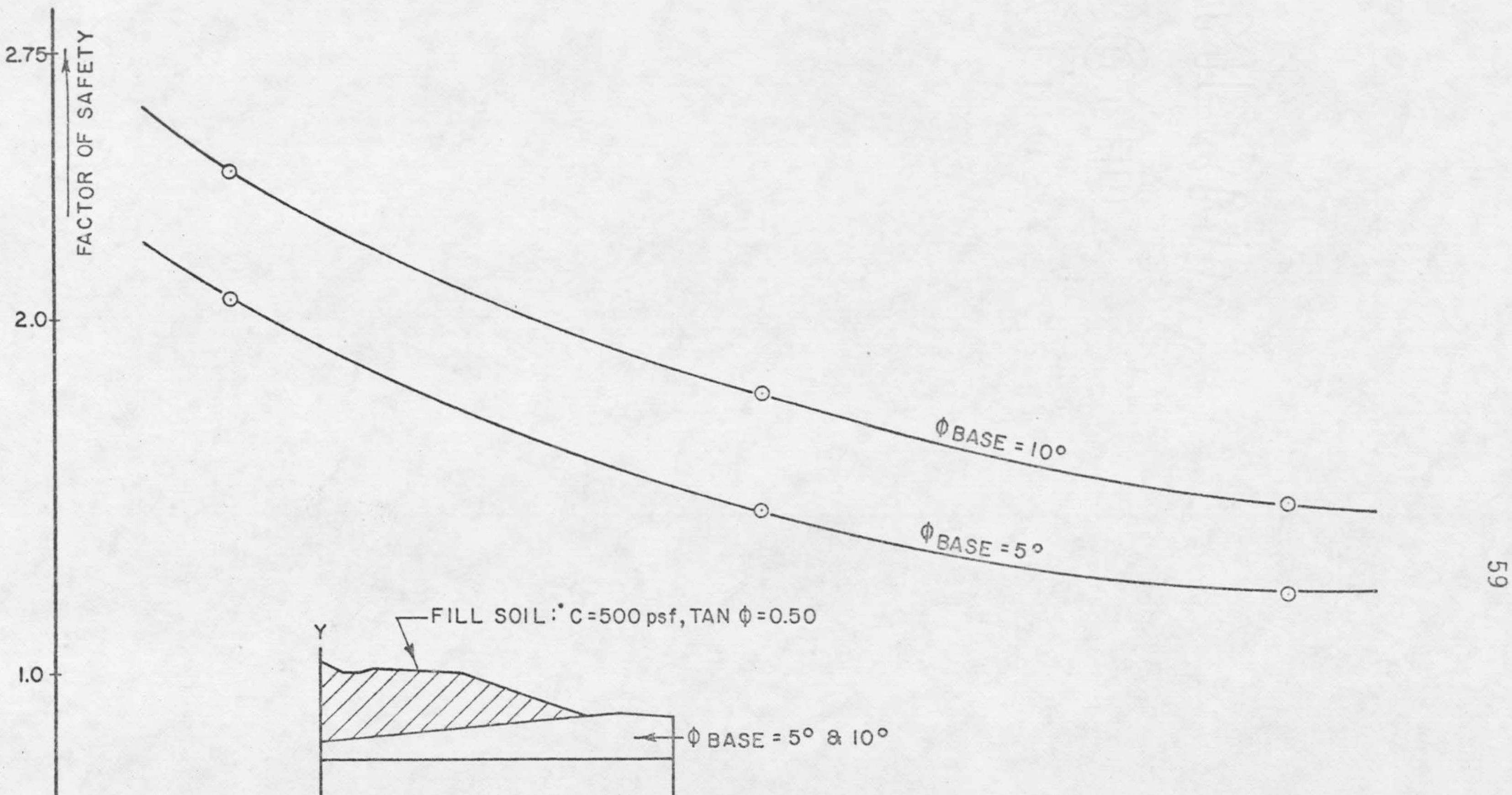

Figure 22. Factor of safety versus unit weight for highway fill section with base soil friction angles of 5° and 10°.

representative curves.

Minimum Solution--High Fill Unit Weight

The previous results indicate that high fill unit weights produce the most critical stability conditions regardless of strength property conditions. Also, it was shown that fixed-arc analyses become inaccurate when included base soil properties are allowed to vary. With these thoughts in mind, a series of results were accumulated for a constant fill unit weight of 130 pcf while the fill unit cohesion and the fill and base internal friction were allowed to vary. The arc starting position was fixed, but the center of rotation was allowed to move to a position of minimum stability. Results were obtained for base angles of internal friction equal to 0°, 5°, and 10°. A check of final arc positions reveals much the same pattern as shown in Figure 9. For a base angle of internal friction equal to 0°, the failure arcs range from entirely within the fill to just on top of the third soil for low and high fill strength combinations respectively. Only very low strength combinations are required in the fill to move the arc down into the weaker base soil. As the base friction angle is increased to 5°, more strength is required in the fill to move the failure arc into the base soil. The arcs still do not go below the base soil, however, because the third soil has a 10° angle of internal friction. When the base angle of internal friction is increased to 10°, however, high strength is required in the fill soil to cause the failure arc to pass into the base soil. The highest combinations of strength in the fill soil cause the failure arc to go below the base soil as was shown in Figure 9.

Figure 23 shows differences in the factors of safety calculated for base soil friction angles of 0° and 5°, and 5° and 10° respectively. The differences appear to be consistent for all values of the fill strength parameters except as expected for low unit cohesion and zero internal friction. The greatest differences occur between the solutions for base friction angles of 0° and 5°, and the differences tend to increase with increasing internal friction of the fill soil. The reason for the larger differences is the fact that any value of internal friction above zero immediately adds frictional resistance along the failure arc coupled with the previously existing resistance due to cohesion. Additional strength in the fill soil, as mentioned above, causes more of the arc to pass into the base soil and thus increases the difference in the factor of safety over the zero friction condition. As the base soil friction angle is increased from 5° to 10°, the arcs are forced into positions below the base level, and the resulting differences in factors of safety against sliding are less.

Minimum Solution--Reduction of All Parameters

All previous means of evaluating the sensitivity of the factor of safety in this study has involved changing soil parameters in one or two soils and varying or fixing failure arc positions. It was further desired, however, to determine the effect of changing all of the soil parameters in the entire cross-section. A complete search for a minimum solution was made for the cross-section shown in Figure 3, but with all properties reduced ten percent. The resulting minimum arc was identical to that of the original condition, and the minimum factor of safety was 1.035 as

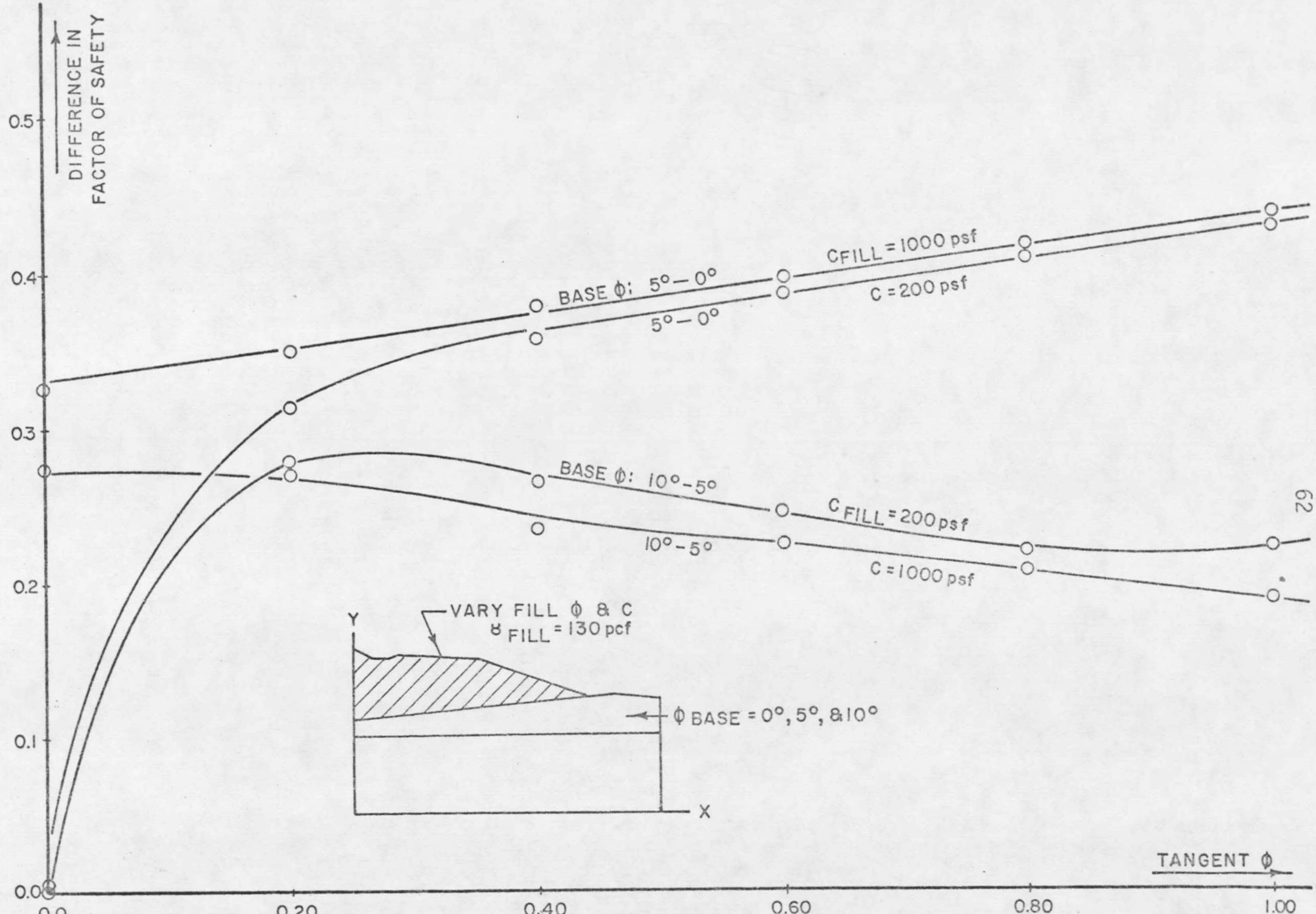

Figure 23. Difference in factor of safety versus tangent ϕ for highway fill section with fixed soil unit weight and base soil friction angles of 0°, 5°, and 10°.

compared to 1.074 for the original condition. The common arc postion is probably due to the compensating nature of reducing all soil property values. From the information related previously in this investigation, it is reasonable to assume that the reduced factor of safety is mainly a result of the reduced internal friction angle. The actual reduction is not of a magnitude that would be important in a slope stability analysis, however.

Fixed-Arc Solution--Movement of Soil Boundary

The factor of safety against sliding is sensitive to change of soil property values and change of failure arc location as shown above. Another variable within a stratified cross-section is the location of each soil boundary, and thus the relative quantity of each soil type. To determine the effect of moving a soil boundary, the cross-section in Figure 3 was altered as shown between the fill and base soils. A series of fixed-arc solutions was then performed for variation of both the fill and base soil properties. The previous fixed-arc location was used in these calculations. The results were then compared with those previously obtained for both the minimum-arc and fixed-arc solutions of the original cross-section. For the original soil properties as shown in Figure 3, the new calculated factor of safety along the fixed arc was 1.052. This compares with a value of 1.074 for the original minimum along this arc. The slight reduction is probably due to the elimination of a portion of the fill soil with a friction angle of 10°, and its replacement with base soil with a friction angle of 5°.

The results obtained for varying the base soil properties were very similar to those obtained for the fixed-arc solutions of the original cross-section. As the base soil strength was increased, the factors of safety against sliding calculated along the fixed arc became exceedingly high. As the base soil volume is greater in the new cross-section, the differences in the factors of safety were even greater for high base soil strength. The plot of the new and original factor of safety differences were otherwise similar to those shown in Figure 20.

Figure 24 shows the range of differences in factors of safety between the original minimum solution and the fixed-arc solution with the new soil boundary. Comparing these differences with those shown in Figure 15, it can be seen that the general trend is similar. The magnitude of the differences are distinctly greater, however, for the new soil boundary case. This again illustrates the increased influence of the base soil on the fixed-arc analysis. By using the same fixed arc as was chosen for the original soil conditions and subsequently moving the soil boundary, the average difference in the resulting solutions is increased more than four times. These results emphasize the importance of choosing a truly representative "best guess" or "fixed" failure arc, and also the importance of knowing precisely where soil boundaries are located.

Cut Section Investigation

To verify the results of the previous investigation involving a highway fill section, similar analyses were conducted on a naturally stratified highway cut section. The section is shown in Figure 4. The first step was to establish the minimum failure condition for the cross-

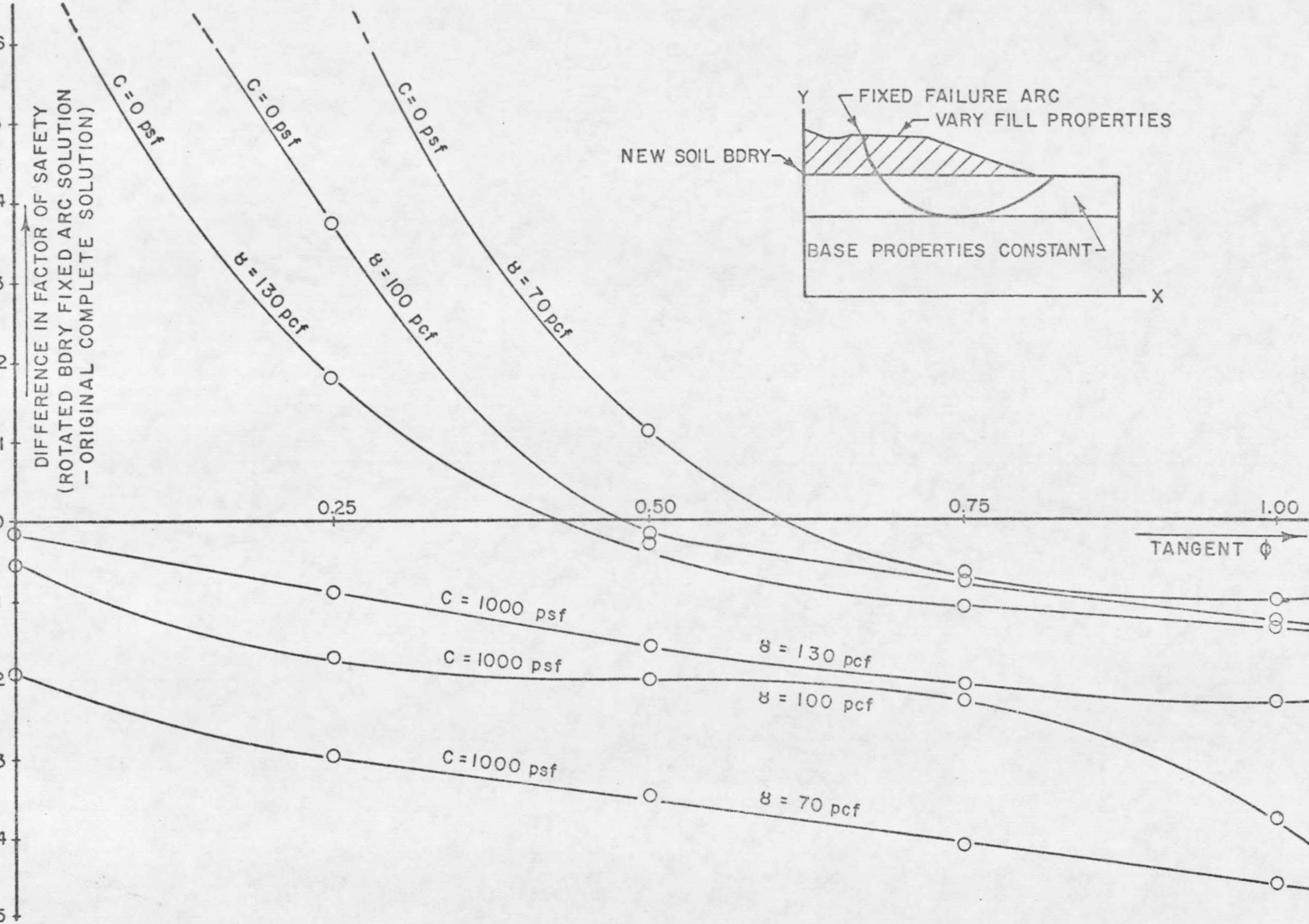

Figure 24. Rotated soil boundary-fixed arc minus original minimum factors of safety for highway fill section with variable fill soil properties.

section with the soil properties as shown, and with the shallow bentonite layer omitted. The result was a deep arc which encompassed the entire cross-section with a factor of safety against sliding equal to 0.316. From what has been developed in the previous investigation, the deep arc and low factor of safety is easily explained. It is the result of the zero angle of internal friction in the base soil. The search for a minimum solution moves the arc down into the frictionless soil, and, in this case, reaches a minimum when the counter rotation weight components and the cohesion begin to adequately resist the driving force due to weight.

It was determined in the field during the Highway Commission investigation, however, that the actual failure plane was at the location shown in Figure 4. With no more information than this, a solution was determined with this surface entered as an impervious base. The result was an arc following along the face of the cut section above this base, with a factor of safety equal to 0.944. This is a logical result, as the angle of internal friction of the soil above the base is 25° and the cut slope angle exceeds this by approximately 3°.

Further field investigation results shown in Highway Commission files indicated that sliding along the base occurred in a shallow zone of clay material similar to bentonite. This material was then entered into the cross-section as shown in Figure 4, and was given a zero angle of internal friction. The upper failure zone or tension crack shown in Figure 4 was also located from field information. With the actual field failure surface thus determined, a representative arc was assumed and a solution obtained. The result was a factor of safety against sliding along this surface of 0.411 which compared favorably with a Highway Department hand calculation of 0.427. The actual minimum found by the search routine was 0.276 along an almost vertical initial failure surface as shown in Figure 4.

Complete and Truncated Failure Arc--Variation of Top Soil Parameters

The investigation of the effect of changing soil parameters was continued using the arc closely resembling the actual field failure condition. First, a series of calculations were made for alternately varying the soil parameters above and below the actual failure surface, and the arc was allowed to pass into and through the base soil. Then, the arc was truncated at the actual failure surface and calculations were made for the planar sliding condition. Again, results were obtained for alternately varying the soil parameters in both the top soil and the shallow "bentonite" layer just above the sliding surface. The unit cohesion in the base soil was set at 300 psf to provide correlation between the two sets of results.

In all cases, the factors of safety calculated for the truncated arcs were greater than those for the complete circular arcs. This is because much of the driving force due to the individual slice weight is eliminated when the arcs are truncated. The planar failure surface is nearly horizontal, and the slide activating force due to unit weight above this surface is much less than that along the nearly vertical portions of the truncated circular arc.

This fact is illustrated in the plots of differences in factors of safety shown in Figure 25. The differences plotted represent results obtained when the top soil parameters were varied between the specified limits. Figure 25 indicates that for low top soil unit weight there is a relatively rapid and nearly linear increase in the difference in factors of safety for increasing values of the coefficient of friction. A check of tabulated factors of safety for the circular arc indicates that because the majority of the arc passes through the low strength base soil, there is only token decrease in stability due to increase in top soil weight. A wide spread difference in factors of safety occurs for the truncated arc with increasing top soil unit weight. This trend in results, and thus the pattern shown in Figure 25, is because the low unit weight along the planar sliding surface provides a very small slide activating force component compared with that of the higher unit weight value. Because the shallow "bentonite" layer on top of the planar surface has no internal friction the additional weight has no effect on the frictional resistance. The frictional resistance is increased along the circular portion of the arc, however. The low unit weight condition of Figure 25 is most affected by the increase in friction along the circular portion of the truncated arc. As the unit weight increases, the increased driving force along the arc begins to overcome the frictional and cohesional resisting forces. Cohesional resistance, of course, is not affected by unit weight, and because of the fixed failure surface length it is constant for all comparative situations. The combination of high top soil unit weight and strength causes the truncated-arc solutions to differ uniformly with those of the complete arc. This relationship

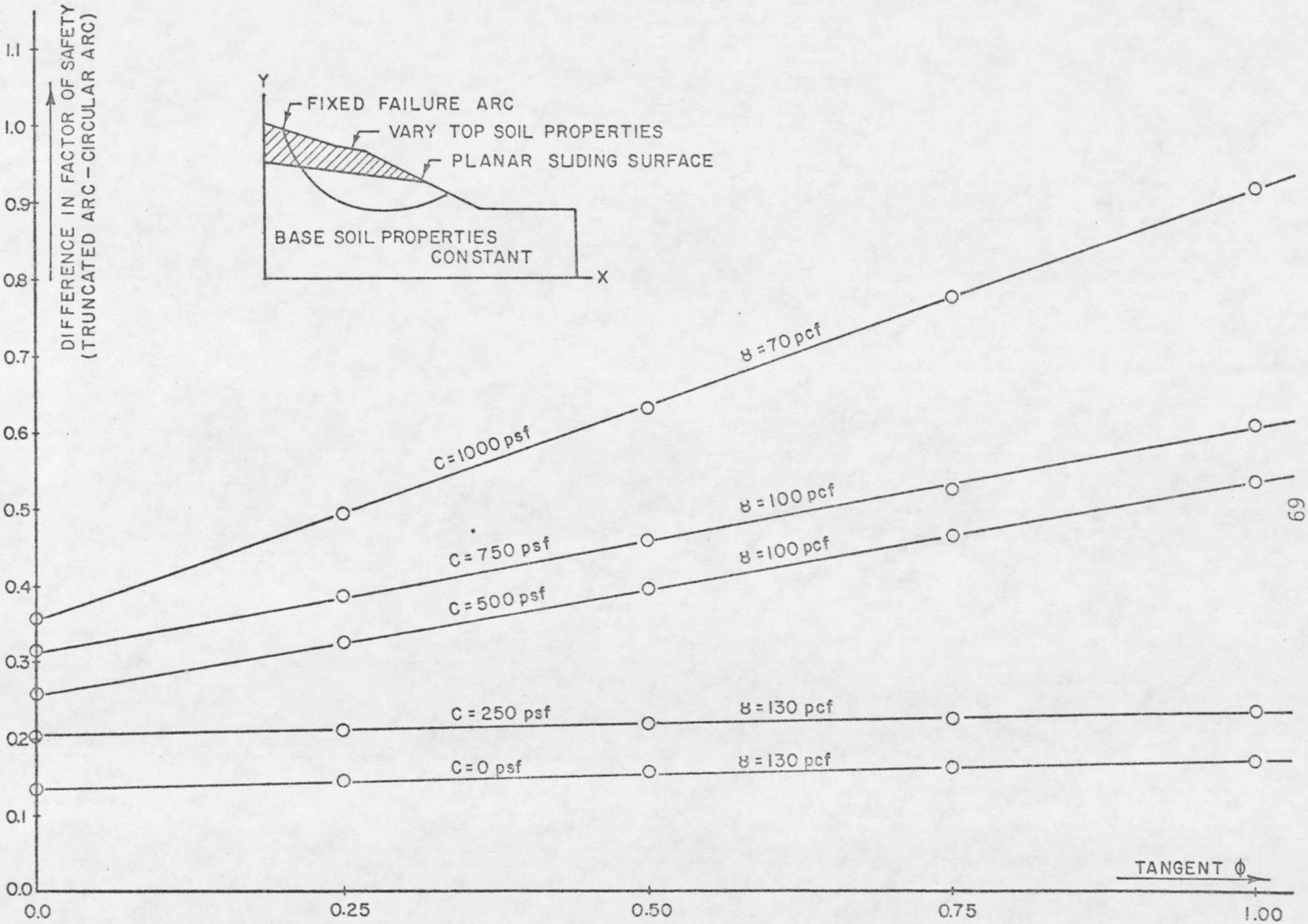

Figure 25. Truncated arc minus circular arc factors of safety versus tangent ϕ for highway cut section with variable top soil properties.

is shown in the lower two curves of Figure 25.

Complete and Truncated Failure Arc--Variation of Base Parameters

Variation of base soil parameters in the cut section produced results which were similar to those presented previously for fill section solutions. For increasing strength in the base soil, which affected a major portion of the failure surfaces in both the complete and truncated-arc solutions, exceedingly high values of the factor of safety were recorded. Again, only a slight increase in the internal friction value causes a rapid increase in stability. This proved to be even more of a trend in the planar surface type of failure. This is shown in the plotted differences in the factors of safety shown in Figure 26. For all values of base soil unit weight, the truncated-arc solutions show a consistent increase in the differences for corresponding increases in the coefficient of friction. This is true because the normal weight component along the planar surface is consistent and not subject to change in slope direction as in the case of the circular arc situation. The increase in frictional resistance is also consistent with increasing values of the coefficient of friction, and rapidly overcomes the much smaller weight-produced slide activating force.

The trend of the results is toward a narrower range of differences with increasing base soil unit weight. The base soil strength increases with an increase in unit weight, as the frictional resistance begins to take effect. The results of the planar surface solutions, however, are nearly the same for all conditions of unit weight since only a very thin base soil exists. The net result is that the differences in factors of safety for the two failure conditions become less with increased values

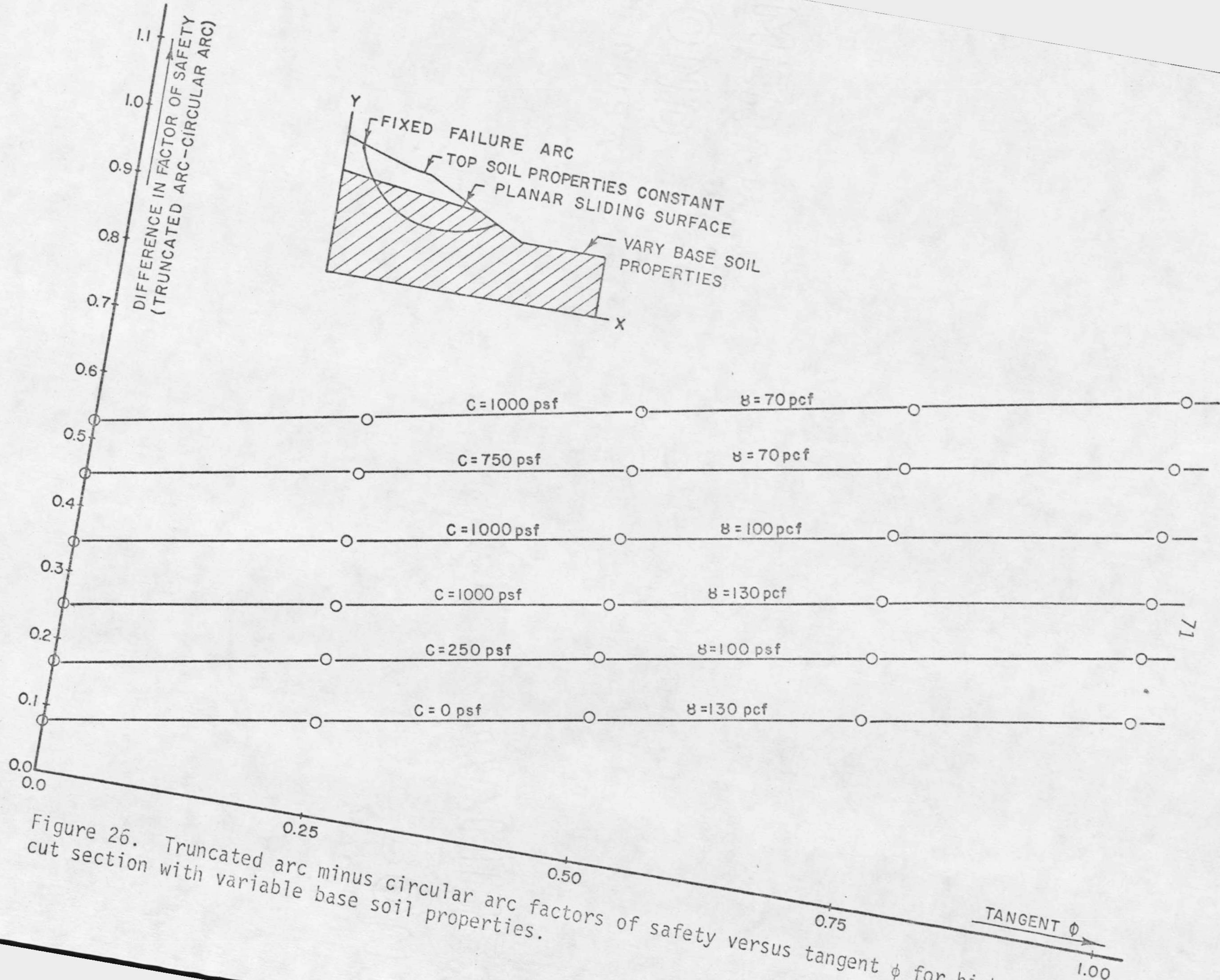

Figure 26. Truncated arc minus circular arc factors of safety versus tangent ϕ for highway cut section with variable base soil properties.

of base soil unit weight.

Trend of Complete and Truncated Failure Arc Solutions

The over-all trend in the results of this comparison reveal that for like conditions of material properties, failure is more likely to occur on a circular failure arc than on a similar but truncated arc which includes a planar sliding surface. This would indicate that planar sliding is indeed initiated by peculiarities or irregularities in the soil, since the path of least resistance in a uniform (but not necessarily homogeneous) soil is along a circular failure arc.

CHAPTER V

SUMMARY OF RESULTS

Conclusions

The investigation produced a wealth of information that has immediate practical application. The computer program used was altered in such a manner that in addition to its original ease of use, the output can be chosen to fit immediate needs. Through the use of the computer solution it was found that variation of soil parameters in both highway cut and fill sections produced results which contained basic patterns fundamental to the behavior of the individual situations. In this form, the results were more easily placed in perspective with the following observations being foremost:

1. Average soil strength combinations result in a common band of failure arcs; extremes cause a wider distribution of arcs. It is important to know soil properties most accurately when either unit cohesion or internal friction is low and especially when both are low.

2. The majority of effect on the factor of safety against sliding is in the unit cohesion range from zero to 500 psf and the coefficient of friction range from zero to 0.50. Beyond these values relative change becomes uniform.

3. The internal friction property appears to be the most influential soil characteristic. Zero values of internal friction in base soils cause unrealistic minimum solutions. The slightest value of internal friction (1° to 5°) will provide more consistent and representative results.

4. High strength in a base soil will cause minimum failure arcs to

avoid this zone. High strength can be considered to be a unit weight of 130 pcf, a unit cohesion of 500 psf and a coefficient of friction of 0.5. Unless all soils in a cross-section are of uniform strength, minimum failure arcs will tend to pass above or below zones of high strength. For this reason, it is poor practice to estimate a trial failure arc within a high strength soil zone.

5. For average soil conditions, it is possible to choose a trial failure arc that corresponds well with a complete minimum solution. Greatest differences in resulting factors of safety occur for low strength soils.

6. Fixed-arc solutions are less accurate for arcs passing through combinations of low and high strength soil zones, and highly inaccurate if the high strength zones have unit cohesion values above 500 psf and coefficients of friction above 0.25. In the latter case more care must be taken in determining individual soil properties and choosing representative failure arcs.

7. Minor shift of soil boundaries can greatly influence the validity of a fixed or trial arc solution. Increase or decrease of soil strength should be noted and a new trial arc chosen.

8. Partial circular and planar failure surfaces produce consistently higher factors of safety against sliding than corresponding complete circular arcs. Determination of which type of sliding will occur depends on accurate knowledge of the planar surface geometry and base soil properties.

<u>Application of Results</u>

The results of this investigation can be used directly in the analysis of slope stability problems. The limitations of quantitative solutions and the importance of geologic considerations must still be emphasized, however. In the typical approach to a stability problem, the geologic investigation provides the basic information upon which all further work depends. Understanding of the general geology of the potential slide area will help insure knowledge of what type of sliding, or classification of mass-movement, is to be expected. It will also reveal a general picture of the materials involved and their relationships to each other. Possible failure surfaces may also be detected.

If it appears that a quantitative method of analysis is feasible, the approach used and the information developed in this thesis may be helpful. As shown in the results of this study, the factor of safety against sliding is most sensitive in low strength and low unit weight situations. If reasonably accurate estimates of the soil properties in a cross-section can be made, a preliminary analysis resulting in probable zones of weakness can be made with the digital computer. If the soil boundaries and properties cannot be estimated with reasonable accuracy, a minimum of drilling and laboratory testing may be necessary. The search for a minimum solution will point out where the characteristic failure zones are, and, if desired, more accurate field and laboratory investigations may be made on these materials.

In further analyses, properties of specific soils may be varied between prescribed limits and the true sensitivity of the factor of safety to change in soil parameters can be visualized. This could be especially helpful in determining the properties required in a fill material to maintain stability in a specific cross-section. When the analysis has reached a stage of certain refinement, a representative fixed arc may be chosen for use in further investigations. Considerable computer time can be saved by using a fixed arc, and except for those precautions discussed previously, accuracy sacrificed is minimal.

As described previously, the stability of a slope, and thus the factor of safety against sliding along any particular failure surface, is sensitive to change in soil properties and soil quantities. Thus it was determined that soil boundaries in the heterogeneous cross-section have an important influence on stability. These boundaries exert their influence on the other stability variable, the failure surface position. Recognizing these facts, more meaning may be obtained from drill hole data. Soil zones which are obviously of low strength should be analyzed more accurately than those which are of obvious high strength. If as in Figure 3, the cross-section is made up of many soils, the deeper zones may be expected to have little influence on the over-all stability, especially if they have angles of internal friction in excess of 10°. Shallow zones of high strength soil will have minimum influence on the over-all section; however, extensive high strength zones will cause the failure arc to pass above. Base soils of extremely low internal friction will cause deep failure zones to develop. Soil testing in the high strength zones, then, need only be minimal. Sampling and testing in very low strength zones must be more thorough, even if the zone occurs at depth.

For complete solutions, such as those done in this study, simple mathematical relationships can be formulated to determine the effect each change in specific soil parameters has on the stability of the slope in question. Much data can be produced by the computer solution, and desired results may be quickly obtained without painstakingly plotting all of the data. This type of thorough solution could be done with only the basic cross-sectional geometry known. Once more exact soil data was established, a final value of the factor of safety against sliding could be taken directly from the previously developed data.

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