

TESTING ALTERNATIVE THEORIES OF GRAVITY
USING LOW FREQUENCY GRAVITATIONAL WAVES

by

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DEDICATION

I dedicate my PhD work to a few seminal people.

To Ron Smith, my high school astronomy teacher, who not only kindled my love of the night sky, but also encouraged abstract thought, and the logical deductions therein.

To Thomas Wade Sheets, my uncle, who died before he could see me get this degree, and with whom I had many conversations about the potential advancement of science and physics. He conferred values of high academic achievement to me.

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ABSTRACT

General Relativity aptly describes current gravitational observations. However, there is great theoretical interest in its validity in untested regimes. Alternative theories of gravity attempt to relax some of the assumptions made, leaving distinct signatures that are absent in Einstein's theory, namely the presence of alternative polarizations of gravitational waves that manifest from the emission of gravitational scalar and vector dipole radiation in black hole binaries. To study this lower order multipole of radiation, it is desirable to work in a regime where the quadrupolar tensor radiation of general relativity is as quiet as possible. This motivates working with supermassive black hole binaries in their slowly evolving inspiral phase, when they are well separated from merger, emitting low frequency gravitational waves.

Using a frequentist framework, we study the detectability of a stochastic background of each polarization using pulsar timing arrays, which is currently the most technically developed and viable method for studying low frequency gravitational waves, correlating the observed time delays of pulsars. We also find that astrometry, which measures transverse displacements of the apparent position of stars, turns out to have a very similar correlation structure as the time delays measured by pulsar timing arrays. We lastly study how effective using a pulsar timing array is at studying a loud, foreground binary with these alternative polarizations, using a Bayesian framework.

Low frequency gravitational wave astronomy proves advantageous for studying these exotic signatures.

CHAPTER ONE

INTRODUCTION

1.1 Opening Remarks

The scientific method could be argued to be the crown jewel of modern civilization. It has allowed humans to change the global landscape in a very short period of time relative to recorded history. The rate of change seems to be exponential with respect to the volume of knowledge accessible, technology being the byproduct of well understood scientific principles and human ingenuity. It acts as a feedback mechanism, reinforcing itself to further probe phenomena that exist outside of human senses and reveal the subtle inner workings of nature. Without it, humans would remain at the mercy of apophenia, the ability to see patterns where there are none, slaves to superstition.

Astronomy in particular has had a profound effect on human knowledge as well as culture. It is perhaps unsurprising that the mysterious and beautiful canvas of the night sky, attributed to other-worldly beings and being perpetually out of reach, would play a sacred role in how early humans understood their place in existence. As a result, they took keen note to record planetary paths and the unusual activities of comets, meteors, and supernovae, for a very long time without knowing what they were and believing them to be signs of doom. With the help of novel advancements in optics, Nicolaus Copernicus, Galileo Galilei, and Joannes Kepler demystified the nature of these objects with the introduction of the heliocentric model of the universe and laws of planetary motion. It would not be until Isaac Newton formulated his laws

of motion and theory of gravity in Principia Mathematica (along with the development of differential and integral calculus) that it was clear these celestial bodies followed many of the same rules as everyday objects.

Rene Descartes helped solidify the idea that the universe must be internally consistent and that it continues to follow the same rules even when no one is watching. In the first of his *Meditations on First Philosophy*, Descartes discussed the alternative to this, which would encompass having a “demon” tampering with things so as to mislead us of their true nature. This could be considered a trivial point in the colloquial sense since without consistency there would be no means to develop a logical basis for anything. This solidified a particular kind of epistemology, logical positivism, as modernity’s conventional school of thought. However, another incarnation of *Descartes’ demon* lies in the internal biases of human perception, against which, again, the scientific method is our only weapon.

Physics, and subsequently technology, saw unprecedented advancement over the last 200 years. The 19th century began much as the previous several hundred years transpired, with citizens riding horse carriages and lighting their homes with oil lanterns. By the end of that century, studies in electromagnetism allowed for the widespread use of electricity. Work in thermodynamics and statistical mechanics saw the development of combustion engines. By the 20th century, Albert Einstein presented his highly controversial theory of gravity, General Relativity, and improvements in widening the observation band of the electromagnetic spectrum helped pave the way to studying atomic physics, which not only opened the potential to harness the hidden power of the atom but saw the beginning of quantum theory. All of this culminated into developing state of the art devices and computers, manned space travel, nuclear reactors, the discovery of exotic, short-lived particles, among countless more. Every step forward has allowed the human race to use the inner

workings of nature to our advantage, answering the questions, “Where do we come from?” and “How far can we go?”

1.2 Einstein’s Theory of Gravity

General Relativity (GR) is surprisingly simple when considered in the context of first principles. Einstein expanded on his Special Theory of Relativity, which stated that the results of a physical experiment are independent of the position and velocity of the frame of reference. This was originally postulated by Galileo, but Einstein included the regime where the frame of reference was close to the speed of light c . Experiments in the 19th century had been done to measure the speed of light, all finding the same value. This was a perplexing result as the goal of some of these experiments were to find an absolute frame of reference. Incidentally, it was realized that when Maxwell’s equations were fully formulated into a wave equation, the speed of electromagnetic waves, cast in terms of fundamental constants, was incredibly close to that of c .

A sequitur interpretation put this property as the chief, hierarchical principal because it fundamentally puts a limit on the propagation of physical information from one region of spacetime to another. This meant that 2 events separated by a time Δt and distance $\Delta \mathbf{x}$ in one reference frame had the same spacetime interval defined by $\Delta s^2 = \Delta \mathbf{x}^2 - (c \Delta t)^2$ as that of any other reference frame. Individually, Δt and $\Delta \mathbf{x}$ change with the reference frame, but not Δs ; it is invariant. This results in some of the “weirder” time dilation and length contraction predictions of special relativity.

For GR, this statement must also be true in a curved spacetime, i.e. in gravitational fields. Consequently, Einstein also posited that gravitational mass and inertial mass were equivalent, and he built a geometric framework that could describe how the observed 3-acceleration due to gravity was merely an artifact of how the fabric

of spacetime was curved by the presence of matter. Mathematically this amounts to taking the simple action

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} R + S_M(\Psi, g_{\mu\nu}) \quad (1.1)$$

and applying variational calculus with respect to the metric $g_{\mu\nu}$, where R is the Ricci tensor, and S_M is the action of any matter fields Ψ . This yields Einstein's Field Equations (EFE) in terms of the Einstein tensor $G_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad (1.2)$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}. \quad (1.3)$$

Λ is the cosmological constant that drives accelerated cosmic expansion, and it can be considered a part of the matter stress energy tensor $T_{\mu\nu}$. This is where all of the equations of motion for various spacetime metrics come from.

His famous elevator thought experiment pointed out that if a person stood in a windowless elevator on Earth, the observer would not be able to tell if the elevator was on the surface of the Earth or accelerating at roughly 9.8m/s^2 in an empty vacuum. Likewise, if they were floating in empty space without acceleration, the observer could not distinguish that from free-falling towards a massive body. The only clear evidence would be to watch two light, floating objects in the elevator slowly drift together, suggesting that they are traveling on curved geodesics. While GR helped explain some shortcomings of Newton's theory of gravitation, it also had some surprising predictions. The existence of black holes followed from the fact that at some point a gravitational potential will be great enough such that a particle's escape velocity will

be greater than the speed of light, which is prohibited, and thus all information within the *event horizon* is censored. Since information cannot travel faster than the speed of light, including changes in a gravitational field, we inevitably get gravitational waves. And the expansion of the cosmos simply follows from applying the Friedmann-Robertson-Walker metric to Einstein's equations.

However, GR also has its own shortcomings, namely at very small and very large distance scales. In the former case, quantum mechanics (QM) and GR are fundamentally at odds with each other from a first principles viewpoint; GR maintains locality while QM does not. Furthermore, GR is not renormalizable, which is a requirement of quantum field theory. String theory and loop quantum gravity are the most studied *quantum gravity* alternatives that attempt to reconcile these differences.

At the opposite end, on galactic and intergalactic distance scales GR clashes severely with observations, at least when accounting for the only kind of matter that was accepted at the time, what is now called baryonic matter. The orbits of stars on the rims of galaxies are much too fast to be in conjunction with the theory unless *dark matter* is invoked, whose inclusion fixes this and other galactic evolution observations. However, not much else can be said about its nature besides that it provides the extra missing gravity we observe, and if we assume it is indeed matter, it must be weakly interacting. It is possible that the extra gravity may be the result of a modified gravitational interaction, but such a mechanism would have to fix things at least as well as the dark matter explanation, and the current convention is to accept the existence of heavy, weakly interacting matter.

At larger scales the dimming of standard candle supernova measurements suggests that the expansion of the cosmos is actually accelerating whereas GR predicts a deceleration for the quantities of matter, including dark matter, currently observed in an asymptotically flat spacetime. Einstein had initially proposed a term, the

cosmological constant Λ , which is referred to as *dark energy*, in his equations that would render the cosmos to be stationary, and while altering this term could render the acceleration in question, its inclusion had no justifiable basis at the time other than to make the cosmological evolution equations adhere to some preconceived bias. The observed cosmic expansion by Edwin Hubble about a decade after put to rest the conjecture that the universe was stationary, and Einstein would later call this the biggest blunder of his career. Indeed, we could call it “Einstein’s demon.”

There is also observational evidence from the early universe that conflicts with GR’s predictions. The *Flatness Problem* and the *Horizon Problem* respectively show that distant regions of the visible universe are too isotropic and too correlated to agree with Einstein. These motivated the introduction of an *inflation* epoch of the early universe where cosmic expansion was exponential, generally invoking a scalar inflaton field to serve this purpose. In this context, GR might not be wrong in that the EFEs are still correct, we simply did not account for all types of fields and energy, and as such the responsibility of reconciling the observations and theory may be deferred to a more correct quantum field theory to describe the scalar field. However, the mechanics of the inflation epoch are very poorly understood as it is necessary to understand quantum gravity at very high energy and density, much higher than can be replicated in a laboratory. Incidentally, it is possible that the observed accelerated expansion of the cosmos is related to this inflaton field.

GR does well when describing the orbits of celestial bodies, but it is far from perfect. We will not comment in detail the countless specific proposed solutions to these incongruities, but simply state that it is also theoretically viable to expect that Einstein’s theory, and thus its underlying principles, break down in some regime. In particle physics, many symmetries (such as Parity Invariance), which formerly may have been considered sacrosanct, are violated individually in certain cases, but a

higher overall symmetry is still satisfied. This may also be the case for what we can conjecture to be the “fully true” theory of gravity. Certain observational clues could lead the way in this crusade.

Ref. [28] provides an adequate examination of many extensions of GR; here we simply review some of the fundamental aspects therein that motivate our study.

1.2.1 Observation

Solar system observations have ardently defended the validity of GR in the weak-field approximation, but it is unclear how well the theory holds up in the strong-field regime, i.e. as the gravitational escape velocity of a particle becomes relativistic. White dwarfs, neutron stars, and black holes are *compact objects* whose gravitational vicinity can be considered strong-field. Interesting tests can be done to probe the surrounding spacetime of these bodies when they are isolated, for instance using the electromagnetic signatures of accretion disks and light bending from hot spots. However, we focus solely on the direct observation of spacetime perturbations caused by binary compact objects a la gravitational wave astronomy.

Gravitational wave astronomy can be divided into distinct frequency bands based on the detectors observing the band and the sources of GWs therein. The most prevalent method of gravitational wave astronomy involves measuring how GWs change the light travel time of a photon along some trajectory. The Laser Interferometer Gravitational-Wave Observatory (LIGO) [73] is an interferometer that studies high frequency GWs ($10 - 10^4 \text{Hz}$) caused by the mergers of stellar mass black holes and neutron stars by using laser interference patterns. Likewise, the upcoming Laser Interferometer Space Antenna (LISA) [17] will operate with much longer detector arms and thus probes intermediate GW frequencies ($10^{-5} - 1 \text{Hz}$) sourced by supermassive black hole (SMBH) binary mergers and other compact binaries of

intermediate and stellar mass. Also, binaries where a stellar to intermediate mass compact object is inspiraling into a SMBH emit GWs in LISA's band that will serve as a strong null test of GR. Because the interaction can be expanded in counting orders of the mass ratio, we will be able to see if GR's incongruities with observation stem simply from differences in mass scales.

Pulsar timing arrays (PTAs) [49, 78, 80], unlike interferometers, operate in the short wavelength approximation as the detector arms, pulsar lines-of-sight, are of the order of kiloparsecs. These galactic-scale gravitational wave detectors observe the low frequency band ($10^{-9} - 10^{-7}$ Hz), a region thought to be dominated by the collective hum of distant supermassive black hole binaries (SMBHBs), known as the stochastic gravitational wave background (SGWB), although other more obscure and exotic sources of GWs have been hypothesized in this band.

An up-and-coming method of low frequency GW astronomy is astrometry, the measuring of perturbations of the sky locations of distant electromagnetic sources. However, it is unclear how severely this method is crippled by the contamination of foreground astrophysical phenomena as a recent analysis of the second Gaia satellite data release, which contains an extensive catalog of such sources, has GW upper limits several orders of magnitude worse than PTAs [40].

Ultra-low frequency gravitational waves ($10^{-18} - 10^{-10}$ Hz) are thought to be sourced by primordial zero-point quantum-gravitational excitations that have been amplified by cosmic inflation. These GWs could potentially be studied using the polarization pattern of the cosmic microwave background (CMB). Specifically, these GWs leave an imprint on the background polarization pattern from the shift of the photon frequency; this is called the Sachs-Wolfe effect. The BICEP2 team initially claimed a detection of gravitational waves using this method [13] but neglected to properly account for the contribution from galactic dust [30]. We will not focus on

studying this band, but will later comment on the GR case in order to discuss some properties pertaining to superluminal GWs.

Our study focuses on the low frequency band using PTAs and astrometry.

1.2.2 Modified Gravity

We have already discussed some of the motivations for modifying GR. Here we briefly review the classes that these modified theories fall into. Lovelock's theorem is a generalization of the main principles and assumptions that build GR. It reads as follows:

In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term.

There are 4 unique ways around this theorem. The most common method to modify GR is to introduce additional fields, dynamical or nondynamical. The former involves introducing a dynamical scalar, vector, or tensor field, which can be added to the action. Therein, standard variation with respect to the metric yields one tensor differential equation, and another variation with respect to the additional field yields another, and we have a system of coupled equations that describe the evolution of all the fields. In the latter case, nondynamical fields are invoked by introducing nonlinear couplings with respect to the matter stress energy tensor, and GR is reproduced in the vacuum case for these theories, but not in the presence of matter.

Violating diffeomorphism invariance, is another way around Lovelock's theorem. Lorentz invariance, which is a subset of this, has been tested to great precision in the

matter sector, i.e. with atomic scale particles moving at relativistic speeds, but tests in the gravitational sector are lacking. Introducing extra dynamical fields generally encodes a level of Lorentz symmetry violations, so often the 2 categories are mutually inclusive.

Another artifact of Lorentz invariance is the prediction of a massless spin-2 graviton. There are strong constraints on a potential mass parameter for massive gravity theories, but these theories are still of interest, pertaining to cosmological problems.

The remaining 2 circumventions of Lovelock's theorem are violations of the weak-equivalence principle from nonminimal coupling to matter, which has been highly constrained, and the introduction of higher dimensions, whose scope is limited at the intermediate distance scale (i.e. with respect to compact object orbital dynamics). However, they tend to be motivated by quantum gravity and introduce extra fields when compactified into 4 dimensions.

As a result of relaxing the assumptions in GR, it is typical, but not necessary, to admit additional polarizations of GWs that are not allowed in Einstein's theory. Therefore, while a non-detection of these polarizations does not necessarily imply that Einstein's theory is correct, a detection would be unequivocal evidence that GR needs to be modified.

We focus our work on generic properties in the orbital dynamics of compact objects that emerge from certain modified theories, namely the emission of dipole scalar and vector radiation in alternative polarizations that are not allowed in GR. The power emitted from radiative processes in the slowly oscillating limit can be generically decomposed into a multipole expansion, with each successive radiative multipole being proportional to the square of an increased integer power in some characteristic speed or frequency. In the post-Newtonian (PN) context, this expansion

goes as square powers of the orbital velocity relative to the speed of light, $(v/c)^2$, and this in turn can be cast in terms of the orbital frequency with Kepler's 3rd Law. The dipole radiation is thus emitted at a lower radiative harmonic than the typical quadrupole signal of GR. However, GR GWs have already been detected from merging stellar-mass black holes, where $v \lesssim c$, and we a priori expect any modifications to be very small relative to GR in this regime. Considering our multipole expansion, this means that in order to study dipole radiation, we desire the binary to be well separated from merger such that $v/c \ll 1$ while still providing a strong GW signal, which will allow the dipole radiation to be the loudest permissible values relative to the quadrupole radiation. This naturally motivates studying SMBHBs in the low frequency band, and PTAs and astrometry are the best tools for this. The most likely GW source in this band is the SGWB.

In Chapter 2, we review many of the essential tools and theory required to proceed. We review some essential properties of pulsars that make them ideal for GW astronomy and the general model used in timing them. We then introduce frequentist statistical framework involving GW detection and the filter used to optimize the detection of a SGWB. We then analytically derive the response of a plane GW in a PTA as well as the specific the geometry of alternative GW polarizations. We end this chapter reviewing the essentials of Bayesian statistics, a tool which we return to frequently in later chapters. In Chapter 3, we study the detectability of a SGWB with each GW polarization individually with PTAs. We use the typical frequentist approach of filtering cross-correlated pulsar timing residuals against the optimal statistic introduced in Chapter 2 to build up signal-to-noise until a detection is made. Our work showed for the first time that in the strong-signal limit, the enhanced response from longitudinal GWs reduces the amount of useful information contained in the cross-correlation relative to the self-noise. We use analytical relationships to

show how the signal-to-noise behaves with each respective case. We subsequently used a Bayesian analysis method to put upper limits on the SGWB amplitudes of these various GW polarizations based on the NANOGrav PTA's 9 year data release. In Chapter 4, we calculate for the first time the set of correlation coefficients that correspond to detecting a SGWB with astrometry, and we show that the same properties pertaining to the signal-to-noise in Chapter 3 apply here as well. Chapter 5 lastly studies with a Bayesian framework how a loud, foreground SMBHB could be detected using PTAs. We find that the analysis is apt to extract information about alternative polarizations in the data. We present our conclusions and comment on future studies in Chapter 6.

CHAPTER TWO

THEORY

2.1 Pulsars

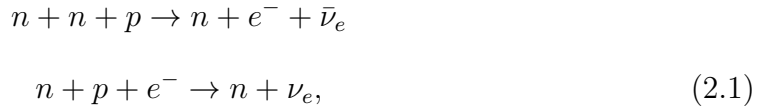
It is important to understand some qualitative features of pulsars that make them distinct from other celestial bodies, and thus render them advantageous for GW astronomy. Many undergraduate textbooks provide ample introductory information on compact objects and pulsars specifically. Here we follow Malcolm Longair's 3rd edition of *High Energy Astrophysics* [75].

When a star is above $8M_{\odot}$, it will most likely proceed in creating iron in its core. At some point afterward, hydrostatic equilibrium will become impossible to sustain, resulting in core collapse, which may cause an intense explosion of the star's outer layers called a supernova [109] although this is just one of many supernova mechanisms, whose specific classifications are outside the scope of this text. If the initial star's mass is somewhere in the range $M \in 15 - 20M_{\odot}$, a highly dense (~ 100 million tons/cm³) star composed of a degenerate neutron gas, called a neutron star (NS), will most likely remain at the center of the supernova remnant cloud. However, NSs can have progenitor masses greater or less than the range mentioned, and Longair acknowledges the complexity and breadth of scenarios that allow NSs to be born. A NS will tend to be highly magnetic and rapidly rotating due to angular momentum conservation and magnetic flux freezing via Faraday's Law. They have radii $R \simeq 10\text{km}$, temperatures $T \simeq 10^6 - 10^7\text{K}$, and magnetic fields $B \simeq 10^{12} - 10^{15}\text{G}$. They emit a wide range of EM signatures, radio waves to X-rays, which is how we observe them.

There is somewhat of a hierarchy involving compact objects. White dwarfs

(WDs) are composed of degenerate electron gas, and they are also leftover from supernovae. They also contain some proton and neutron matter. In a non-degenerate electron gas, the neutrons would β decay with a half-life of about 10.2 minutes, but degenerate electrons leave only high energy states at relativistic Lorentz factors, γ_L , to occupy for those products of β decay. Increasing the amount of subatomic matter in a WD causes the electrons to reach energies above the mass difference between protons and neutrons $E = \gamma_L m_e c^2 \geq (m_n - m_p) c^2 = 1.29 \text{ MeV}$, leading to an inverse β decay $p + e^- \rightarrow n + \nu_e$, turning protons and electrons into neutrons and electron neutrinos. This is called neutronization, and is the process by which WDs become NSs. Chandrasakhar limited WD masses to be $M_{\text{max,WD}} \simeq 1.4 M_\odot$; any greater and they would collapse into a NS. Similarly, for a NS, the Tolman-Oppenheimer-Volkov limit is $M_{\text{max,NS}} \simeq 2.17 M_\odot$ [79], the exact value depending on the NS equation of state; any greater and it would collapse into a black hole.

While the temperatures of NSs seem high, relative to the Fermi temperature these stars are actually quite cold. Furthermore, the primary cooling mechanism is not from their residual electromagnetic glow from the ambient temperature. In the core it is from neutrino emission via the modified URCA process [24, 114]



and there is comparable cooling in the NS crust from neutrino bremsstrahlung [50, 91, 113].

The matter inside neutron stars is highly exotic. The outer layers are mostly ($\sim 90\%$) composed of neutron degenerate matter, but the inner layers are not well understood as these densities cannot be achieved in laboratories on Earth. There

could be stable pions and/or kaons, a quark gluon plasma; many equations of state have been hypothesized and are still studied [32, 71, 88, 101].

A rapidly rotating neutron star's magnetic field tends not to be aligned with its axis of rotation. Therefore, it emits electromagnetic radiation due to its changing magnetic dipole moment, which is observed as evenly spaced radio pulses when the column of emission crosses the line-of-sight to Earth. After being observed by Anthony Hewish and Jocelyn Bell at Mullard Radio Astronomy Observatory at Cambridge in 1967 [62], they were dubbed *pulsars* although at the time the mechanism from which these pulses were originating was unclear. It was clear that the mechanism was unrelated to orbiting binaries because the emission of gravitational radiation decreases the semi-major axis a [90]

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3}{c^5} \frac{m_1 m_2 (m_1 + m_2)}{a^{7/2} (1 - e^2)^2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right), \quad (2.2)$$

where e here is the orbital eccentricity, and m_1 and m_2 are the masses of the respective orbiting bodies. Because the period of the binary P depends on Kepler's 3rd Law,

$$P = \frac{2\pi a^{3/2}}{(GM)^{1/2}}, \quad (2.3)$$

where $M = m_1 + m_2$, the change in the period decreases as

$$\left\langle \frac{dP}{dt} \right\rangle = \frac{\pi a^{1/2}}{(GM)^{1/2}} \left\langle \frac{da}{dt} \right\rangle, \quad (2.4)$$

whereas observations showed the period increased.

If the pulses were not due to orbital dynamics, perhaps they were from internal pulsations within whatever body it was. The speed of sound in a polytropic fluid is $v_s = \sqrt{(\partial p / \partial \rho)_s} = \sqrt{\gamma p / \rho}$, where γ is the adiabatic index, p is the pressure, and ρ is

the density; here we have expressed the equation of state as $p = K\rho^\gamma$. The bodies of interest here needed to be very dense, which made the only candidates WDs and NSs. The indices for a degenerate electron gas and degenerate neutron gas (corresponding to a crude model for white dwarf and neutron star) are respectively $\gamma = 4/3, 1$, along with standard density estimates $\rho = 10^{10}, 10^{16}\text{kg/m}^3$ [97], which meant that the period of pulsations, scaled as the time pulses take to traverse the body's diameter, can be roughly calculated to be $P_{WD} \approx 7\text{s}$, which was too slow to agree with observation, and $P_{NS} \approx 3\text{ms}$, which was too fast. The only remaining option was to compare estimations for the rotation from conservation of angular momentum, which yielded a good value for the NS. Gold [55] and Pacini [89] would later propose that these pulsars were rapidly rotating NSs.

Pulsars are born with periods on the order of milliseconds, but they slow due to braking mechanisms predominantly from electromagnetic radiation and neutron star wind, which is coupled to the magnetic field. Variable coupling between the crust and the liquid interior causes them to *glitch*, which briefly increases the period but also increases the spin-down torque. These are attributed with changes in the moment of inertia and especially magnetic field rearrangements.

However, if a pulsar is part of an accreting binary, it may spin back up and become a millisecond pulsar. Subsequently from the accretion, the magnetic field decreases, allowing the spin down to be slower than *non-recycled* pulsars, even though the frequencies are much higher. This means that the resulting pulses are very regular and highly stable. Timing these pulses against a template will reveal any deviations therefrom, revealing information about GWs.

2.2 Timing and Residuals

Having introduced pulsars, we now review some of the basic practices of pulsar timing mentioned in reference [77]. For the interested reader, the *Handbook of Pulsar Astronomy* [76] is a more in-depth resource in this regard.

The intrinsic time of arrival (TOA) of an n th pulse is a function of a pulsar's rotational period and its derivatives in the form of a Taylor Expansion

$$t_n = t_0 + nP_0 + \frac{1}{2}n^2\dot{P}_0P_0 + \frac{1}{6}n^3\ddot{P}_0P_0^2 + \mathcal{O}(\ddot{P}_0), \quad (2.5)$$

or alternatively in terms of its rotational frequency

$$\phi(t) = \phi(t_0) + f_0(t - t_0) + 1/2 \dot{f}_0(t - t_0)^2 + \mathcal{O}(\ddot{f}_0), \quad (2.6)$$

There is variability between individual pulses, so a pulse “folding” method against a template must be employed until an average pulse profile emerges $\sim 100 - 1000$ rotations per observing epoch. The observing cadence, which dictates the Nyquist frequency of GWs, is determined by the time separation between epochs as opposed to individual pulses.

After the TOA is measured at the telescope, it must be transformed into an inertial frame with respect to the pulsar, which assumed approximately to be the solar system barycenter (SSB). This involves a number of corrections

$$t = t_{\text{topo}} - t_{0,\text{topo}} + \Delta_{\text{clock}} - \Delta_{\text{DM}} + \Delta_{\text{R}\odot} + \Delta_{\text{E}\odot} + \Delta_{\text{S}\odot} + \Delta_{\text{R}} + \Delta_{\text{E}} + \Delta_{\text{S}}. \quad (2.7)$$

The relativistic terms are modified in alternative theories of gravity, but current solar system constraints show that these corrections would be of order pico-seconds [95].

The terms labeled “topo” are the topocentric (where it is on Earth) frame of the observatory, and Δ denotes a timing correction. “Clock” is the difference between the topocentric time and some terrestrial standard time, and “DM” corrects for a frequency-dependent dispersion measure since the pulses are traveling through the ionized interstellar medium (ISM), or colloquially electron clouds. Generally, the DM is measured in real-time by observing the pulses through 2 widely separated radio bands. However, the DM need not be stationary in practice, which is problematic. The remaining six terms denote corrections with respect to orbits of both Earth and the pulsar (if in a binary). R is the Roemer delay that accounts for light travel time across the orbit of the body, E is the Einstein delay due to time dilation of gravitational bodies, and S is the Shapiro delay, which accounts for the increased travel distance of a curved spacetime. All of this culminates into a timing model. The goal of PTAs is to subtract the observed TOAs from the timing model, rendering a set of *timing residuals*, which carry the signature of GWs.

Note that the orbital corrections encompass ephemerides within the solar system, namely Jupiter; knowing the correct position of Earth with respect to the SSB requires precise measurements of the positions of these other bodies. Current precision allows astronomers to know the position of Earth to about 10 km in an AU, which is actually not precise enough for pulsar timing, and a spurious GW detection with a dipolar pattern emerges. Luckily, the uncertainty can be marginalized in Bayesian analyses, and longer observation times still allow the prospect of a true GW detection despite this.

Of course there are other noise processes that make proper modeling more difficult. They come in two classes, distinguished by whether the noise power spectral density is flat over all frequencies or higher at lower frequencies, i.e. white and red. While white noise (WN) makes the GW signals of interest quieter, the effect can

be mitigated with longer data sets. Some examples are *radiometer noise*, due to the observing instruments, Earth’s atmosphere, and other radio contamination from space like the CMB and galactic synchrotron emission, and *interstellar scintillation*(ISS), scattering from ISM inhomogeneities. *Pulse jitter* refers to small scale variations in the pulse profile. This inevitably results from observing finite pulses, and can be mitigated by lengthening the observing epochs.

Red noise (RN) complicates the prospect of GW detection greatly for PTAs because GWs themselves appear as a kind of RN in PTA analyses. The ISM is turbulent and thus highly non-stationary, inducing chromatic (dependent on the observing radio wavelengths) RN. This results in kinks in observing frequencies that can be mistaken for a GW signal although the appropriate antenna patterns are noticeably absent from most other pulsars. Longer data sets, again, mitigate this effect.

Timing noise, which is considered intrinsic to the pulsar, is thought to be due to spin-down torque variations; this contaminates the measurement of the pulsar’s frequency derivatives. This is more common with non-recycled pulsars, but will become more important for MSPs as precision timing increases. However, the origin of timing noise is not well understood, and several models for timing noise can be found in Ref. [92].

The predominant candidate believed to be detected by PTAs is an isotropic stochastic gravitational wave background (SGWB), originating mostly from a large ensemble of SMBHBs originating from galactic mergers, and possibly other sources. They emit GWs that are too faint to be resolved individually; however, it is possible that a strong foreground signal is resolvable, and we discuss that later. In the absence of other noise, this will leave a very specific correlation between pulsar residuals

$R(t) = t_n - TOA$ [42, 61, 96] of the form

$$C_{ij}(\Theta) = \langle R_i^*(t) R_j(t) \rangle \equiv \int_{f_{\min}}^{f_{\max}} df S_h(f) \Gamma_{ij}(\Theta, f), \quad (2.8)$$

where the frequency bounds on the integral depend on the sampling limits, $S_h(f)$ describes the SGWB spectrum as a function of the frequency f , and $\Gamma_{ij}(\Theta)$ is an *overlap reduction coefficient* that depends on the separation angle in the sky Θ between the i th and j th pulsars and is independent of f in GR. The general C_{ij} function equals the variance of the timing between the pulsars due to GWs. The *Hellings-Downs curve*, $\Gamma(\Theta)$, is an *overlap reduction function* (ORF) that describes the coefficients at all angles for a GR background. We assume that $\langle R_i(t) \rangle = 0$.

2.3 Detection

GWs induce perturbations to pulsars that cannot be accounted for with the timing model alone. These perturbations will follow a distinct set of statistical properties, which we now introduce. Following Ref. [93], in order to detect a SGWB, we want some detection statistic S measured from the cross-correlations against some threshold value S_T . In the absence of a background, we expect only white, stationary, uncorrelated/stochastic, and a zero-mean Gaussian noise with standard deviation σ_0 , which gives a probability density function (PDF) of measuring a particular value of S as

$$p_0(S) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{S^2}{2\sigma_0^2}}. \quad (2.9)$$

We define a false-alarm probability (FAP) α_0 that determines how likely a given measurement above the threshold S_T is in the absence of a signal as

$$\alpha_0 = \int_{S_T}^{\infty} p_0(S) dS. \quad (2.10)$$

We can see here that for larger values of σ_0 , and thus wider PDFs, measuring some fixed value S_T becomes more probable.

On the other hand, a separate PDF can be defined if a signal is present (also assumed to be a stochastic Gaussian process), which carries a mean μ_1 and a different standard deviation σ_1 as

$$p_1(S) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(S-\mu_1)^2}{2\sigma_1^2}}, \quad (2.11)$$

which we can use to define a detection probability (DP) γ_D that we measure above S_T

$$\gamma_D = \int_{S_T}^{\infty} p_1(S) dS. \quad (2.12)$$

Solving the integrals of α_0 and γ_D in terms of S_T , we can recast γ_D in terms of α_0 with a complimentary error function, which yields

$$\gamma_D = \frac{1}{2} \operatorname{erfc} \left[\frac{\sqrt{2}\sigma_0 \operatorname{erfc}^{-1}(2\alpha_0) - \mu_1}{\sqrt{2}\sigma_1} \right]. \quad (2.13)$$

Given the shape of the complimentary error function, the highest DPs result from large μ_1/σ_1 values. σ_0 is usually known through experimental measurements, and most often $\sigma_1 \sim \sigma_0$ is taken in the weak-signal limit. In practice it often suffices to simply measure a value of S that gives an astoundingly small FAP, but strictly the

convention is to choose a FAP and DP, and find a corresponding S_T . We can see that in the limit $\sigma_1 \gg \sigma_0$, the DP becomes independent of the FAP as the complimentary error function remains of order unity even for minuscule arguments. In that regime, if μ_1 is $5\sigma_1$, which is regarded as “beyond a shred of doubt,” this is equivalent to a 0.9999997 DP. Compare this with a $5\sigma_0$ FAP of 0.0000003. We see that the 2 scenarios normalize to unity. However, we should be wary of overgeneralizing; a $5\sigma_0$ FAP yields only a 0.5 DP in the weak-signal limit, i.e. $\mu_1 = 5\sigma_1 \approx 5\sigma_0$. A $9\sigma_0$ measurement in S_T would render the DP quoted previously.

2.4 Optimal Detection Statistic

We now develop a filter to apply to S that maximizes μ_1/σ_1 , which will improve our detection prospects. While most references derive the optimal statistic in the weak-signal limit [15, 18], we use the same notation but take the liberty to include arbitrarily loud backgrounds in conjunction with reference [93], whose derivation is in matrix notation.

We define the detection statistic S as

$$S = \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' s_i(t) s_j(t') Q(t - t') \quad (2.14)$$

where $s_i(t) = R_i(t) + n_i(t)$ is the combined response of the i th pulsar from a GW residual R_i and any extra noise n_i , and $Q(t - t')$ is a filter function that accounts for the time lag. We assume that the noise is stochastic/uncorrelated

$$\begin{aligned} \langle n_i(t) \rangle &= 0, \quad \langle n_i(t) R_j(t) \rangle = 0 \\ \langle n_i(t) n_j(t) \rangle_{i \neq j} &= 0, \quad \langle n_i(t) n_i(t) \rangle = \langle n_i^*(f) n_i(f') \rangle = \frac{1}{2} \delta(f - f') P_{ii}(f). \end{aligned} \quad (2.15)$$

$P_{ii}(f)$ is the one-sided noise power spectral density of the i th pulsar.

We want the filter to maximize the signal-to-noise ratio (SNR) $\bar{\rho}$ defined as

$$\bar{\rho} = \frac{\mu_1}{\sigma_1} = \frac{\langle S_{ij} \rangle}{\sqrt{\langle S_{ij}^2 \rangle - \langle S_{ij} \rangle^2}}, \quad (2.16)$$

and we can see that a statistic threshold S_T corresponds to an SNR threshold ρ_T . It is possible to create a filter that maximizes DP, but this is substantially more difficult, and as Eq.(2.13) shows, the SNR is a sufficient proxy for detection. Explicitly we have

$$\langle S_{ij} \rangle = T \int_{f_{\min}}^{f_{\max}} df S_h(f) \Gamma_{ij}(f) \tilde{Q}(f), \quad \text{and} \quad (2.17)$$

$$\begin{aligned} \langle S_{ij}^2 \rangle = T \int_{f_{\min}}^{f_{\max}} df \Big([P_{ii}(f) + S_h(f) \Gamma_{ii}(f)] [P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] \\ + 2S_h^2(f) \Gamma_{ij}^2(f) \Big) \tilde{Q}(f)^2. \end{aligned} \quad (2.18)$$

It is implicit that there is a sum over all pulsars ij . With these, we define a positive-definite inner product from the total power spectra between two detectors as

$$\begin{aligned} (A, B) \equiv \int_{f_{\min}}^{f_{\max}} df A^*(f) B(f) \Big([P_{ii}(f) + S_h(f) \Gamma_{ii}(f)] [P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] \\ + 2S_h^2(f) \Gamma_{ij}^2(f) \Big), \end{aligned} \quad (2.19)$$

and it subsequently follows that

$$\mu_1 = T \left(\tilde{Q}, \frac{S_h \Gamma_{ij}}{[P_{ii}(f) + S_h(f) \Gamma_{ii}(f)] [P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] + S_h^2(f) \Gamma_{ij}^2(f)} \right), \quad (2.20)$$

and

$$\sigma_1^2 = T \left[(\tilde{Q}, \tilde{Q}) - \left(\tilde{Q}, \frac{S_h \Gamma_{ij}}{[P_{ii}(f) + S_h(f) \Gamma_{ii}(f)][P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] + S_h^2(f) \Gamma_{ij}^2(f)} \right)^2 \right]. \quad (2.21)$$

The optimal filter $\tilde{Q}(f)$ that maximizes $\bar{\rho}$ is then

$$\tilde{Q}(f) = \frac{S_h \Gamma_{ij}}{[P_{ii}(f) + S_h(f) \Gamma_{ii}(f)][P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] + S_h^2(f) \Gamma_{ij}^2(f)}, \quad (2.22)$$

which gives the optimal SNR

$$\hat{\rho} = T^{1/2} \int_{f_{\min}}^{f_{\max}} df \frac{(S_h(f) \Gamma_{ij}(f))^2}{[P_{ii}(f) + S_h(f) \Gamma_{ii}(f)][P_{jj}(f) + S_h(f) \Gamma_{jj}(f)] + S_h^2(f) \Gamma_{ij}^2(f)}. \quad (2.23)$$

An interesting aspect of this filter is that it applies for any detector seeking to detect a SGWB. Here, we have assumed PTA notation, but it is just as valid for astrometry or any other ORF describing a SGWB.

2.5 Shifts

In order to calculate a Γ_{ij} for PTAs (and thus the optimal statistic), we need to know the explicit form of the residual induced by GWs. The timing residual is simply the frequency shift of the pulse induced GWs, integrated in time, so we need to express the shift's dependence on a GW plane wave perturbation. While several sources have derived this quantity [18, 42, 48], we follow the intuitive approach in

Ref. [31]. We define the shift as

$$\frac{\Delta\omega}{\omega_0} \equiv \frac{\omega_0 - \omega_{\text{observed}}}{\omega_0}. \quad (2.24)$$

The response at a point $\mathbf{x}$ from a plane GW of frequency f propagating in direction $\mathbf{\Omega}$ is

$$h_{ij}^{TT}(t, \mathbf{\Omega}, \mathbf{x}) = h^P(f) \epsilon_{ij}^P(\mathbf{\Omega}) e^{2\pi i f(t - \mathbf{\Omega} \cdot \mathbf{x})}, \quad (2.25)$$

where $h(f)$ are the Fourier coefficients, and ϵ_{ij}^P is the polarization tensor of polarization P . In GR, gauge conditions constrain this to only plus and cross polarizations.

The metric we work with will be flat with a small spatial perturbation of

$$ds^2 = dt^2 + (\delta_{ij} + h_{ij}) dx^i dx^j. \quad (2.26)$$

We want to know the effect of the perturbation on a photon with unperturbed angular frequency ω_0 traveling from some source to an observer, with a worldline pointed in $\mathbf{n}$, $x_0^\mu = \omega_0(\lambda, -\lambda\mathbf{n})$, and the unperturbed 4-momentum follows as $k_0^\mu = \omega_0(1, -\mathbf{n})$. It is easily verified that the path given here is null. Using the geodesic equation, we can also verify that paths of stationary observers are geodesics, so we can set $x_{\text{observer}}^i(t) = 0$ and $x_{\text{source}}^i(t) = x_{\text{source}}^i = \text{constant}$. Subsequently, the affine parameter of the source is $\lambda_{\text{source}} = -|\mathbf{x}_{\text{source}}|/\omega_0$.

The observed photon frequency is simply the dot product of the photon's 4-momentum with the frame of the observer $\omega_{\text{observed}} = -g_{\mu\nu} k^\mu u^\nu = k^0(0)$, whose frame we have already taken to be stationary. We need to expand the photon path

and 4-momuntum perturbatively with

$$x^\mu(\lambda) = x_0^\mu(\lambda) + x_1^\mu(\lambda) \quad (2.27)$$

$$k^\mu(\lambda) = k_0^\mu(\lambda) + k_1^\mu(\lambda). \quad (2.28)$$

The shift originates from how GWs change the length of the photon path (and thus the travel time), so we can use the geodesic equation for a null path and the only non-vanishing Christoffel connection $\Gamma_{\nu\sigma}^\mu$ for the component of interest (0) to give

$$\frac{d^2 x^\mu}{d\lambda^2} = \frac{dk^\mu}{d\lambda} = -\Gamma_{\nu\sigma}^\mu \frac{dx^\nu}{d\lambda} \frac{dx^\sigma}{d\lambda} \quad (2.29)$$

$\xrightarrow{\Gamma_{ij}^0 = \frac{1}{2}h_{ij,0}, \quad dx^i/d\lambda = k^i}$

$$\frac{d^2 x_1^0}{d\lambda^2} = \frac{dk_1^0}{d\lambda} = -\frac{\omega_0^2}{2} n^i n^j h_{ij,0}, \quad (2.30)$$

where the perturbed 4-momentum remains first order with the contraction of the unperturbed 4-momentum and the first order metric perturbation. We assume a plane wave h_{ij} is traveling in direction $\mathbf{\Omega}$ evaluated at the retarded time of some position in space $\mathbf{x}$, $h_{ij}(t - \mathbf{\Omega} \cdot \mathbf{x}) = h_{ij}(\omega_0 \lambda (1 + \mathbf{\Omega} \cdot \mathbf{n}))$, which means

$$h_{ij,0} = \frac{\partial}{\partial t} h_{ij} = \frac{1}{\omega_0(1 + \mathbf{\Omega} \cdot \mathbf{n})} \frac{\partial}{\partial \lambda} h_{ij}, \quad (2.31)$$

which makes the shift

$$\begin{aligned} \frac{\omega_0 - \omega_{\text{observed}}}{\omega_0} &= -\frac{\omega_0}{2} n^i n^j \int_0^{\lambda_{\text{source}}} d\lambda' h_{ij,0}(\lambda') \\ &= -\frac{1}{2(1 + \mathbf{\Omega} \cdot \mathbf{n})} n^i n^j [h_{ij}(\lambda_{\text{source}}) - h_{ij}(0)]. \end{aligned} \quad (2.32)$$

Some definitions differ by a negative sign (which is an error), but in the context of

cross-correlations, this does nothing because of the quadrature in Eq.(2.8).

2.6 Alternative Gravitational Wave Polarizations

Now that we have the specific form of the response of a GW in a PTA, we need to find all allowed polarization tensors for the GWs. There are 2 unique polarizations in Einstein's theory of gravity. For a general metric theory of gravity we can have up to 6. These can be found by finding the independent excitation modes of the Riemann tensor using the Newman-Penrose formalism [87]. Here we follow reference [29] for its brevity. We assume a sinusoidal GW traveling in the $\mathbf{z}$ direction with a flat Minkowski background metric.

We introduce a quasi-orthonormal complex-null tetrad basis with 2 real null vectors $\mathbf{k}$ and $\mathbf{l}$ along with 2 complex vectors $\mathbf{m}$ and $\bar{\mathbf{m}}$, the bar denoting complex conjugation, which satisfy the relations

$$\mathbf{k} \cdot \mathbf{l} = 0, \quad \mathbf{m} \cdot \bar{\mathbf{m}} = -1, \quad \mathbf{k} \cdot \bar{\mathbf{m}} = \mathbf{k} \cdot \mathbf{m} = \mathbf{l} \cdot \bar{\mathbf{m}} = \mathbf{l} \cdot \mathbf{m} = 0, \quad (2.33)$$

and hence we can choose the Newman-Penrose tetrad

$$\mathbf{k} = -\frac{1}{\sqrt{2}}(1, 0, 0, 1), \quad \mathbf{l} = -\frac{1}{\sqrt{2}}(1, 0, 0, -1), \quad (2.34)$$

$$\mathbf{m} = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \bar{\mathbf{m}} = -\frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad (2.35)$$

where our coordinates are defined as (t, x, y, z) . With this basis, we can describe the Riemann tensor in terms of its irreducible parts, the Weyl tensor, the Ricci Tensor, and the Ricci scalar. Reference [43] showed that the number of independent components of all these tensors reduces into the following six components of the

Riemann tensor:

$$-\frac{1}{6}R_{lklk} \equiv A_{SL}, \quad (2.36)$$

$$-\frac{1}{2}\text{Re } R_{kl\bar{m}} \equiv A_{v,VL}, \quad (2.37)$$

$$-\frac{1}{2}\text{Im } R_{kl\bar{m}} \equiv A_{u,VL}, \quad (2.38)$$

$$-\text{Re } R_{l\bar{m}l\bar{m}} \equiv A_{+,TT}, \quad (2.39)$$

$$-\text{Im } R_{l\bar{m}l\bar{m}} \equiv A_{\times,TT}, \quad (2.40)$$

$$-R_{lml\bar{m}} \equiv A_{ST}, \quad (2.41)$$

where we have defined *Newman-Penrose amplitudes* on the right hand side of these equations. The subscripts will be explained in a moment.

It is convenient to introduce a driving-force matrix

$$S_{ij}(t) \equiv R_{i0j0}(u), \quad (2.42)$$

where u is the retarded time. The equation of geodesic deviation, which describes the perturbations introduced by GWs in an interferometer, is sourced by these Riemann tensor components. In terms of coordinates (t, x^i) this matrix takes the form

$$S = \begin{pmatrix} A_{+,TT} + A_{ST} & A_{\times,TT} & A_{v,VL} \\ A_{\times,TT} & -A_{+,TT} + A_{ST} & A_{u,VL} \\ A_{v,VL} & A_{u,VL} & A_{SL} \end{pmatrix} = \sum_P A_P(\mathbf{z}, t) \epsilon_P \quad (2.43)$$

Given this arrangement, we can see that the polarization tensors for each of our

Newman-Penrose amplitudes is defined by

$$\epsilon_{\text{SL}} = \mathbf{z} \otimes \mathbf{z}, \quad (2.44)$$

$$\epsilon_{\text{ST}} = \mathbf{x} \otimes \mathbf{x} + \mathbf{y} \otimes \mathbf{y}, \quad (2.45)$$

$$\epsilon_{\text{x,VL}} = \mathbf{x} \otimes \mathbf{z} + \mathbf{z} \otimes \mathbf{x}, \quad (2.46)$$

$$\epsilon_{\text{y,VL}} = \mathbf{y} \otimes \mathbf{z} + \mathbf{z} \otimes \mathbf{y}, \quad (2.47)$$

$$\epsilon_{+, \text{TT}} = \mathbf{x} \otimes \mathbf{x} - \mathbf{y} \otimes \mathbf{y}, \quad (2.48)$$

$$\epsilon_{\times, \text{TT}} = \mathbf{x} \otimes \mathbf{y} + \mathbf{y} \otimes \mathbf{x}. \quad (2.49)$$

These correspond to definite helicity states under rotations about $\mathbf{z}$. The first two are spin-0, a scalar longitudinal (SL) mode, whose perturbations are strictly along the propagation direction, and a scalar transverse (ST) “breathing” mode, which expands and contracts along all transverse directions. The next two are spin-1 shear vector longitudinal (VL) modes, whose distortion is partially along the propagation direction and along one of the transverse axes. The last two are the spin-2 tensor transverse (TT) modes of GR.

2.7 Bayesian Inference

Part of the scientific method involves collecting data d that could potentially reflect information about a given hypothesis $\mathcal{H}$. The question becomes how do we quantify the significance of the agreement/disagreement between them? Frequentist statistics tend to defer to FAPs like we introduced before, which mostly quantify if the data has no useful information, i.e. noise, or how far away from having no useful information the data is. However, it is far more useful in most contexts to see if the data agrees with a specific model of the truth that we assert, and this is what draws

the distinction between frequentist versus Bayesian statistics. Phil Gregory's text on Bayesian analysis is a great reference for this [57]. Our model depends on a number of parameters described by the vector $\vec{\theta}$, which are randomly distributed. The data updates our knowledge of certain values of $\vec{\theta}$ being more reflective of the truth via the posterior probability from Bayes theorem

$$p(\vec{\theta}|d, \mathcal{H}) = \frac{p(d|\vec{\theta}, \mathcal{H})p(\vec{\theta}|\mathcal{H})}{p(d|\mathcal{H})}. \quad (2.50)$$

Explicitly, given the data and some hypothetical assertion of the truth, the probability that the truth is reflected by specific parameter values $\vec{\theta}$ is equal to the likelihood that d could be drawn randomly from a distribution described by $\mathcal{H}$ times our prior knowledge of $\vec{\theta}$ being truthful, all normalized. The denominator in Eq.(2.50) is the *evidence* of $\mathcal{H}$, or the *marginalized likelihood*

$$p(d|\mathcal{H}) = \int d\vec{\theta} p(d|\vec{\theta}, \mathcal{H})p(\vec{\theta}|\mathcal{H}). \quad (2.51)$$

Eq.(2.51) shows that in this context it is merely a normalizing factor; however, when performing model selection, i.e. choosing one over another, it becomes crucial as we use the Bayesian odds ratio between models A and B

$$\mathcal{O} = \frac{p(d|\mathcal{H}_A) p(\mathcal{H}_A)}{p(d|\mathcal{H}_B) p(\mathcal{H}_B)}. \quad (2.52)$$

The first term is called the *Bayes Factor*, $\mathcal{B}$, and it reflects our confidence in one model over another based on the data. The second is the prior odds ratio, or prior belief in both models. In cases where we are maximally ignorant, it is customary to make all parameter priors uniform and any prior ratio unity, and we defer solely to $\mathcal{B}$ to provide information as to which model is preferable, making $\mathcal{B} \equiv \mathcal{O}$. We can use

old posterior distributions to serve as updated priors for newer data sets, and this can be shown to be equivalent to using all of the data with our originally ignorant prior.

In principle, we would be able to compute Eq.(2.50) directly. However, with current computational limitations, this is intractable. Imagine having a 10-dimensional parameter space and crudely choosing 10 unique values per parameter, at equal intervals, to evaluate. We would have to compute 10 billion points, which is very expensive, and we may not even be choosing values in a very intelligent way.

This motivates introducing a Markov Chain Monte Carlo (MCMC) integration that much more efficiently produces Eq.(2.50). It begins by choosing some initial value set $\vec{x}$ of the parameters, then proposing some jump to another set $\vec{y}$ using a proposal distribution $q(\vec{y}|\vec{x})$ that generates the choice randomly. The posterior probability is then evaluated and the jump is accepted with probability $\alpha(\vec{y}|\vec{x}) = \min(1, H(\vec{y}|\vec{x}))$, where the *Hastings ratio*, H , is

$$H_{\vec{x} \rightarrow \vec{y}} = \frac{p(\vec{y}|d) q(\vec{x}|\vec{y})}{p(\vec{x}|d) q(\vec{y}|\vec{x})}. \quad (2.53)$$

This is known as the Metropolis-Hastings algorithm. Often, proposal distributions are symmetric, i.e. $q(\vec{x}|\vec{y}) = q(\vec{y}|\vec{x})$, and the expression depends only on the posterior probabilities between $\vec{x}$ and $\vec{y}$. Furthermore, very biased priors can have a significant influence on the result, but for uniform priors, the first fraction in Eq.(2.53) simplifies to the likelihood ratio. We will discuss specific choices for the likelihood in future chapters. The purpose of the second fraction is to ensure reversibility in the jumps

in the Hastings ratio, known as *detailed balance*, which mathematically requires [56]

$$\begin{aligned} \int_A \int_B \pi(x) P(x, y) &= \int_B \int_A \pi(y) P(y, x) \\ &\equiv \int_A \pi(x) \int_B q(x|y) \alpha(x|y) = \int_B \pi(y) \int_A q(y|x) \alpha(y|x), \end{aligned} \quad (2.54)$$

where the transition kernel $P(x, y)$ has been simplified in terms of the proposal distribution and acceptance rate; we have omitted the algebra. This is especially important when dealing with transdimensional proposals, i.e. jumps between 2 models, and an additional Jacobian factor is required in Eq.(2.53). In order to be truly Markovian, the current location of the chain is independent of its previous history (although adaptive Metropolis algorithms use the chain's past history for jump proposals), and reversibility is required to achieve this.

Since our initial guess in the MCMC may not be a good one, there is usually a *burn-in period* to let the chain assume a region of high probability. It is difficult to gauge when the chain has converged, i.e. it reflects the true marginalized posterior distributions. Some convergence criteria exist, such as the Gelman-Rubin Statistic, which compares multiple chains. It is easier to demonstrate when the chain has failed to converge, generally with acceptance rates and autocorrelation lengths in the chain. An autocorrelation function (ACF) is defined in terms of the lag h of a future chain x_{t+h} with respect to some initial chain x_t as

$$\rho(h) = \frac{\sum_{\text{overlap}} [(x_t - \bar{x})(x_{t+h} - \bar{x})]}{\sqrt{\sum_{\text{overlap}} (x_t - \bar{x})^2} \times \sqrt{\sum_{\text{overlap}} (x_{t+h} - \bar{x})^2}}. \quad (2.55)$$

The qualitative shape of this function is $\rho(h) \propto \exp(h/L_{AC})$, where L_{AC} is the *autocorrelation length* (ACL), which quantifies approximately the distance between independent points in the chain, and we want to minimize this. The *acceptance rate*

is also merely a crude tool; it is said that a 50% rate is ideal for models with few parameters and 25% for higher dimensional models. Qualitatively, we can see how the 2 quantities work in tandem. If the proposals render the distances between the points very small, then they will be frequently accepted, and yield a high autocorrelation length. On the other hand, if the proposals keep asking to make jumps that are never accepted, the acceptance rate will be low, and the chain will seldom move, again yielding a high autocorrelation length. Hitting the sweet spot where the length is optimally low is the ideal scenario. This depends heavily on the proposals utilized and is the real, nontrivial art in constructing MCMCs.

While crude, often it suffices to look at the posteriors with the naked eye, which is the spirit of the Gelman-Rubin statistic. Firstly, we need to look at the chains to ensure that parameters are not stuck and are indeed getting good “mixing;” sufficiently mixed chains look like white noise when plotted. If the chain “drifts,” it is not achieving optimal mixing. Let’s entertain the situation where we have a simple, 1 dimensional MCMC. If we choose different initial points for our MCMC, let it run for 100,000 iterations, say, and our autocorrelation length is very low such that we can say that the distance between independent points is of order unity, we can compare the posteriors of these independent runs. If they seem visually indistinguishable, that tends to be good enough to assert convergence.

It is often the case that the chain will get stuck in a local maximum, but not the global maximum, which would be preferred. However, ideally the sampler will sample across all maxima. In a perfect world, the jump proposals would account for this, but often it is difficult to anticipate what the data prefers. Instead, we can run several chains in parallel, modifying the target distribution as

$$p(\vec{\theta}|d, \beta) = p(\vec{\theta})p(d|\vec{\theta})^\beta, \quad (2.56)$$

where $\beta \leq 1$ is an inverse temperature. “Hotter” chains flatten out the distribution such that the sampler can freely explore the prior space. Every so often, these chains communicate with the “cold” chain to see if it would prefer to explore other regions. The Hastings ratio of the proposal between i th and j th temperature chains is

$$H_{i \rightarrow j} = \frac{p(d|\vec{\theta}_i, \beta_j)p(d|\vec{\theta}_j, \beta_i)}{p(d|\vec{\theta}_i, \beta_i)p(d|\vec{\theta}_j, \beta_j)} \quad (2.57)$$

Incidentally, these higher temperature chains can be used to evaluate the evidence of the model from *thermodynamic integration* [74]. The evidence of a particular temperature chain is its tempered posterior probability integrated over the explored parameter values

$$Z(\beta) = \int d\vec{\theta} p(d|\vec{\theta}, \mathcal{H}, \beta) p(\vec{\theta}|\mathcal{H}). \quad (2.58)$$

So long as we have sufficiently incremented the temperature ladder, we can integrate over β

$$\ln p(d|\mathcal{H}) = \int_0^1 d\beta \langle \ln p(d|\vec{\theta}, \mathcal{H}, \beta) \rangle_\beta, \quad (2.59)$$

where we have exploited the prior’s independence of β , and the angle brackets denote the expectation value of the tempered chain’s likelihood. The rule of thumb is to space the temperature increments log-uniformly and to choose a maximum temperature of at least $T_{\max} = \text{SNR}^2$ since the temperature decreases the SNR qualitatively $\sim 1/\sqrt{T}$.

CHAPTER THREE

CONSTRAINING ALTERNATIVE THEORIES OF GRAVITY USING PULSAR
TIMING ARRAYS3.1 Contribution of Authors and Co-Authors

Co-Author: Neil J. Cornish

Contributions: Provided the correct variance calculation, the data analysis method and data, performed the analysis, and wrote the manuscript.

Author: Logan O’Beirne

Contributions: Provided approximate analytical expressions for the signal-to-noise ratio for all alternative polarizations, generated correlation plots and detection contours as consistency checks, cross-checked upper limits, and performed the data analysis.

Co-Author: Stephen R. Taylor

Contributions: Cross-checked correlation plots and detection contours, performed the data analysis, and provided figures for upper limits.

Co-Author: Nicolás Yunes

Contributions: Provided theoretical expressions for the spectrum.

3.2 Manuscript Information

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ABSTRACT

The opening of the gravitational wave window by ground-based laser interferometers has made possible many new tests of gravity, including the first constraints on polarization. It is hoped that within the next decade pulsar timing will extend the window by making the first detections in the nano-hertz frequency regime. Pulsar timing offers several advantages over ground-based interferometers for constraining the polarization of gravitational waves due to the many projections of the polarization pattern provided by the different lines of sight to the pulsars, and the enhanced response to longitudinal polarizations. Here we show that existing results from pulsar timing arrays can be used to place stringent limits on the energy density of longitudinal stochastic gravitational waves. However, unambiguously distinguishing these modes from noise will be very difficult due to the large variances in the pulsar-pulsar correlation patterns. Existing upper limits on the power spectrum of pulsar timing residuals imply that the amplitude of vector longitudinal and scalar longitudinal modes at frequencies of 1/year are constrained: $\mathcal{A}_{\text{VL}} < 4 \times 10^{-16}$ and $\mathcal{A}_{\text{SL}} < 4 \times 10^{-17}$, while the bounds on the energy density for a scale invariant cosmological background are: $\Omega_{\text{VL}} h^2 < 4 \times 10^{-11}$ and $\Omega_{\text{SL}} h^2 < 3 \times 10^{-13}$.

3.3 Introduction

The detection of gravitational waves from merging black hole binaries [2, 3, 7–9] and neutron stars [6, 10, 11] by the LIGO/Virgo collaboration has made possible many fundamental tests of gravity [1, 4, 7, 118], including the first studies of the polarization content of the waves [9, 12]. Alternatives to Einstein’s theory of gravity generically predict the presence of scalar and vector polarization states, in addition to the usual tensor modes [34, 108, 117]. Pulsars are also a tremendously valuable tool for probing these strong gravity effects. Pulsar timing observations of binary systems have been used to constrain the fraction of the emitted energy that goes into scalar and vector polarization states [70, 102, 103]. The LIGO/Virgo collaboration have produced upper limits on the emission of non-tensorial gravitational waves from isolated pulsars [5]. These results consider pulsars as *sources* of gravitational waves. Here we derive new limits on alternative theories of gravity by considering an array of milli-second pulsars as a *detector* of gravitational wave [52].

Pulsar timing is a complimentary approach to gravitational wave detection that uses milli-second pulsars as a natural galactic scale gravitational wave detector [65]. Possible sources in the nano-hertz frequency range probed by pulsar timing arrays include slowly inspiraling supermassive black hole binaries [27], cosmic string networks [39], and processes in the very early Universe, such as inflation or phase transitions [58]. Over fifty millisecond pulsars, widely distributed across the sky, are now monitored as part of the worldwide pulsar timing effort [105]. Each Earth-pulsar line of sight provides a different projection of the gravitational wave polarization pattern, offering a distinct advantage over existing ground-based interferometers which provide

very few independent projections. Moreover, pulsar timing arrays operate in the limit where the wavelengths are much shorter than the light path, while ground-based interferometers operate in the long-wavelength limit. The response to longitudinal polarizations is significantly enhanced relative to the transverse modes for pulsar timing [33, 38, 54, 72], but not for ground-based interferometers.

Here we show that existing results from pulsar timing arrays can be used to set stringent limits on the energy density in alternative polarization modes for both astrophysical and cosmological stochastic backgrounds. We derive expressions for the power spectra of gravitational waves from a population of supermassive black hole binaries. The power spectra for the scalar and vector modes include an additional dipole contribution, which impacts both the generation of the waves and the orbital decay. The measured power spectrum is further modified by the different response functions for scalar, vector and tensor modes. Published upper limits on the power spectrum of pulsar timing residuals can be converted into upper limits on the amplitude of each polarization mode. Note that in our analysis we consider all modes simultaneously. The bounds are particularly strong for the scalar longitudinal and vector longitudinal modes due to the enhanced response to these polarization states [72]. In principle our upper limits can be translated into bounds on the coupling constants for particular alternative theories, but this requires assumptions be made about the merger rate of black holes. For some theories of gravity our results provide no constraints: for example, black hole binaries in Brans-Dicke gravity are not thought to radiate any differently than in general relativity [60, 85, 115]. For a large class of theories, however, our results do provide constraints because black holes acquire either scalar or vector hair, and thus, emit dipole radiation when in a binary [110–112, 117].

The prospects for using pulsar timing to unambiguously *detect* the signature of alternative polarization states are much less promising. In contrast to upper

limits, which we derive by attributing the entire timing residual to gravitational waves, detection requires that we are able to separate signal from noise. Stochastic gravitational wave backgrounds imprint a tell-tale signature in the angular correlation pattern between pairs of pulsars [61,72]. However, we find that the correlation pattern for the longitudinal modes is obscured by their large intrinsic variance, making it extremely difficult to resolve the correlation pattern and separate signal from noise.

3.4 Computing the timing residuals

A stochastic gravitational wave background will produce correlated perturbations in the pulse arrival times measured for pulsars a and b with cross-spectral density

$$S_{ab}(f) = \sum_I \Gamma_{ab}^I(f) \frac{h_{c,I}^2(f)}{8\pi^2 f^3}, \quad (3.1)$$

where $h_{c,I}$ is the characteristic amplitude of the I^{th} polarization state, and Γ_{ab} is a geometrical factor that describes the correlation between the pulsars. Throughout we use geometric units $G = c = 1$. Astrophysical limits are traditionally reported in terms of the amplitude of the characteristic strain at a period of one year: $\mathcal{A} = h_c(f = \text{yr}^{-1})$, while cosmological limits are traditionally reported in terms of the energy density per logarithmic frequency interval, scaled by the closure density, $\Omega(f)h^2 = 2\pi^2 f^2 h_c^2(f)/(3H_{100}^2)$. Here $H_{100} = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and we write the Hubble constant as $H_0 = hH_{100}$.

The correlation pattern for the transverse tensor states of general relativity was first computed by Hellings and Downs [61]: $\Gamma_{ab}^{\text{TT}} = (1 + \delta_{ab})/3 + \gamma_{ab}(\ln \gamma_{ab} - 1/6)$, where δ_{ab} is the Kroneker delta, $\gamma_{ab} = (1 - \cos \theta_{ab})/2$, and θ_{ab} is the angle between the line of sight to pulsars a and b . Note that for this study we use un-normalized correlation functions since it is not possible to normalize the longitudinal modes

to unity when $a = b$. Instead we have $\Gamma_{aa}^{\text{TT}} = 2/3$. The correlation pattern for scalar transverse waves is $\Gamma_{ab}^{\text{ST}} = (1 + \delta_{ab})/3 - \gamma_{ab}/6$ [72]. Closed-form expressions for the scalar and vector longitudinal modes are not available, and have to be computed numerically. Approximate expressions for the autocorrelation terms have been found, and are given by $\Gamma_{aa}^{\text{VL}} \approx 2\ln(4\pi L_a f) - 14/3 + 2\gamma_E$ [72] and $\Gamma_{aa}^{\text{SL}} \approx \pi^2 f L_a/4 - \ln(4\pi L_a f) + 37/24 - \gamma_E$ [33], where γ_E is the Euler constant and L_a is the light travel time from pulsar a . For typical pulsar timing distances and observation frequencies, the quantity $f L_a$ is of order 10^2 to 10^4 , which implies that the response to longitudinal modes is much larger than to the transverse modes.

Binary systems of supermassive black holes are expected to be the dominant sources of gravitational waves in the pulsar timing band. Some alternative theories of gravity predict that these systems will generate scalar and vector dipole radiation (along with sub-dominant higher moments), in addition to the usual tensor quadrupole radiation. Rather than considering specific theories individually, we can derive a general form for the gravitational wave spectrum, which can then be constrained using existing bounds from pulsar timing observations. Turning these bounds into constraints on the coupling constants for specific theories would require more detailed calculations and assumptions about the number of supermassive black hole binaries. Our derivation is based on the analysis in Refs. [34, 94], and assumes that the binaries are in circular orbits, with the orbital decay dominated by gravitational wave emission. Neglecting higher moments, the gravitational wave signal from a slowly evolving binary has the generic form [34]

$$\mathbf{h}(t) = \mathbf{A}_D f(t)^{1/3} + \mathbf{A}_Q f(t)^{2/3} \quad (3.2)$$

where $\mathbf{A}_D$ and $\mathbf{A}_Q$ are the polarization tensors for the dipole and quadrupole modes,

scaled by masses, distances, and coupling constants. Note that pulsar timing is generally more sensitive to the dipole terms, since the binaries are well separated, while ground-based detectors are more sensitive to the quadrupole terms since the binaries are close to merger. We assume that any modifications to the conservative dynamics are sub-dominant compared to the modifications to the radiative sector, so that to leading order the frequency is related to the orbital separation by Kepler's law $f(t) \sim r(t)^{-3/2}$. The energy flux dE/dt in general relativity is computed from $\dot{h}^2 \sim f^2 h^2$ and an integration over a sphere surrounding the source. In alternative theories the energy flux will also include energy carried by any of the additional fields that must exist in the non-GR theory, but the frequency dependence will be the same since it follows from the multipole decomposition. Thus we have

$$\frac{dE}{dt} = B_D f^{8/3} + B_Q f^{10/3} \quad (3.3)$$

where B_D and B_Q are related to scalars formed from the squares of $\mathbf{A}_D$ and $\mathbf{A}_Q$ integrated over the sphere, along with additional factors that come from the scalar and vector degrees of freedom. Combining this with the Newtonian expression for the binding energy $E = -Gm\mu/r \sim f^{2/3}$, we have $dE/df \sim f^{-1/3}$, and

$$\frac{df}{dt} = \frac{dE/dt}{dE/df} = C_D f^{9/3} + C_Q f^{11/3} \quad (3.4)$$

Combining the expressions for $\mathbf{h}(t)$ and df/dt according to the formalism in Ref. [94]

yields

$$S_{ab}(f) = \left(\frac{1 + \kappa^2}{1 + \kappa^2 \left(\frac{y\mathbf{r}^{-1}}{f} \right)^{2/3}} \right) \left\{ \Gamma_{ab}^{\text{TT}} \mathcal{A}_{\text{TT}}^2 \left(\frac{y\mathbf{r}^{-1}}{f} \right)^{4/3} + [\Gamma_{ab}^{\text{ST}} \mathcal{A}_{\text{ST}}^2 + \Gamma_{ab}^{\text{VL}} \mathcal{A}_{\text{VL}}^2 + \Gamma_{ab}^{\text{SL}} \mathcal{A}_{\text{SL}}^2] \left(\frac{y\mathbf{r}^{-1}}{f} \right)^2 \right\} \frac{1}{8\pi^2 f^3}. \quad (3.5)$$

In deriving this expression we have assumed that background is dominated by binaries in a narrow mass range, so that ratio C_D/C_Q is approximately constant. Relaxing this assumption results in a broader knee in the spectrum around $f \sim \kappa^3 \text{yr}^{-1}$, but has little impact on the final results.

In addition to signals from binary black holes, the pulsar timing band may contain signals from a network of cosmic strings, or from processes in the early Universe, such as phase transitions or inflation. Computing the gravitational wave signature for each of these sources in a general way for alternative theories of gravity is outside the scope of our current work. One simple case that we can address is inflation, where general considerations imply that the scalar, vector and tensor modes that re-enter the horizon during the radiation dominated epoch will have a nearly scale-invariant spectrum, with the energy density per-logarithmic frequency interval Ωh^2 independent of frequency, and

$$S_{ab}(f) = \frac{3H_{100}^2}{16\pi^4 f^5} \sum_{I=\text{TT,ST,VL,SL}} \Gamma_{ab}^I(f) \Omega_I h^2. \quad (3.6)$$

3.5 Constraints on alternative polarizations

Existing results from pulsar timing studies can be used to place interesting constraints on the energy density of tensor and non-tensor polarization states. While

the Parkes Pulsar Timing array currently has the lowest published upper limit on the tensor amplitude [100], it is difficult to map those limits to constraints on other polarization states that have different spectra. Instead, we chose to use the Bayesian per-frequency upper limits on $h_c(f)$ derived by the NANOGrav collaboration [21], from which we can derive a likelihood function for $S_d(f) = S_{aa}(f)$. Since the NANOGrav bounds are for tensor modes, we have the mapping $S_d(f) = h_c(f)^2/(12\pi^2 f^3)$. Following Ref. [81], we model the per-frequency posterior distributions for h_c with Fermi functions:

$$p(h_c) = \frac{1}{\sigma \ln(e^{h_*/\sigma} + 1) (1 + e^{(h_c - h_*)/\sigma})} \quad (3.7)$$

with $\sigma \approx h_*/2$ and the turn-over point h_* related to the quoted 95% upper limits h_{95} by

$$h_{95} = h_* - \sigma \ln \left[(e^{h_*/\sigma} + 1)^{0.95} - 1 \right]. \quad (3.8)$$

The posterior distributions for $h_c(f)$ define a posterior distribution for $S_d(f)$, which we then use to define a likelihood for the model parameters $\mathcal{A}_I, \kappa$ or Ω_I from the product $\prod_f p(S_{aa}(f))$. Applying this procedure to a purely tensor theory yields the 95% upper limits $\mathcal{A} < 2 \times 10^{-15}$ and $\Omega h^2 < 7 \times 10^{-10}$, which are in reasonable agreement with the directly computed upper limits [21], $\mathcal{A} < 1.5 \times 10^{-15}$ and $\Omega h^2 < 4.2 \times 10^{-10}$. The discrepancies are likely due to imperfections in our fit to the $h_c(f)$ posterior distributions, and differences in the covariances between the noise model and the signal model in the per-frequency-bin versus full spectrum analyses. The bounds we derive will provide conservative upper limits on the alternative polarization states.

One additional caveat that pertains to using the previously derived bounds on $S_{aa}(f)$ to constrain alternative polarization states is that the original analysis combines the limits derived from multiple different pulsars, each at a different

distances from Earth. Ideally, we would use the per-pulsar bounds on $S_{aa}(f)$ and factor in the different distances to each pulsar, which enter into the response function for the longitudinal modes. This information, however, is not publicly available, and since the best timed pulsars are all at roughly the same distance from Earth, we simply assume that all the pulsars are at a distance of 1 ± 0.2 kpc from Earth [105], and marginalize over the uncertainty.

Results for the amplitudes of the astrophysical signal are shown in Fig. 3.1 assuming uniform priors on the amplitudes $0 \leq \mathcal{A} < 10^{-13}$ and decay parameter $0 \leq \kappa < 10$. Note that values of $\kappa > 10$ produce spectra in the PTA band that are identical to those with $\kappa = 10$, hence our choice of upper bound on the prior range. The 95% upper limits on the amplitudes are $\mathcal{A}_{\text{TT}} < 3 \times 10^{-15}$, $\mathcal{A}_{\text{ST}} < 2 \times 10^{-15}$, $\mathcal{A}_{\text{VL}} < 4 \times 10^{-16}$ and $\mathcal{A}_{\text{SL}} < 4 \times 10^{-17}$. The posterior distribution for the spectrum $S_d(f)$ is plotted against the NANOGrav 9-year upper limits [21] in Fig. 3.2. Repeating the analysis for the cosmological model in Eq. (3.6), we find 95% upper limits on the energy densities of $\Omega_{\text{TT+ST}}h^2 < 8 \times 10^{-10}$, $\Omega_{\text{VL}}h^2 < 4 \times 10^{-11}$ and $\Omega_{\text{SL}}h^2 < 3 \times 10^{-13}$. Note that we can only constrain the sum of energy density in the tensor and scalar transverse modes since they produce residuals with identical frequency dependence.

3.6 Detecting alternative polarization states

The unique angular correlation patterns imprinted by gravitational waves should allow us to distinguish between a stochastic gravitational wave background and the myriad sources of noise that impact pulsar timing. What we discovered when analyzing simulated signals came as a surprise: we found that the longitudinal modes made it very difficult to detect any correlation pattern, even in the zero noise limit. In effect, the longitudinal signals behave as noise. The signal we are looking for is the cross-spectrum of the timing residuals $S_{ab}(f) = \text{E}[r_a(f)r_b(f)]$, which have

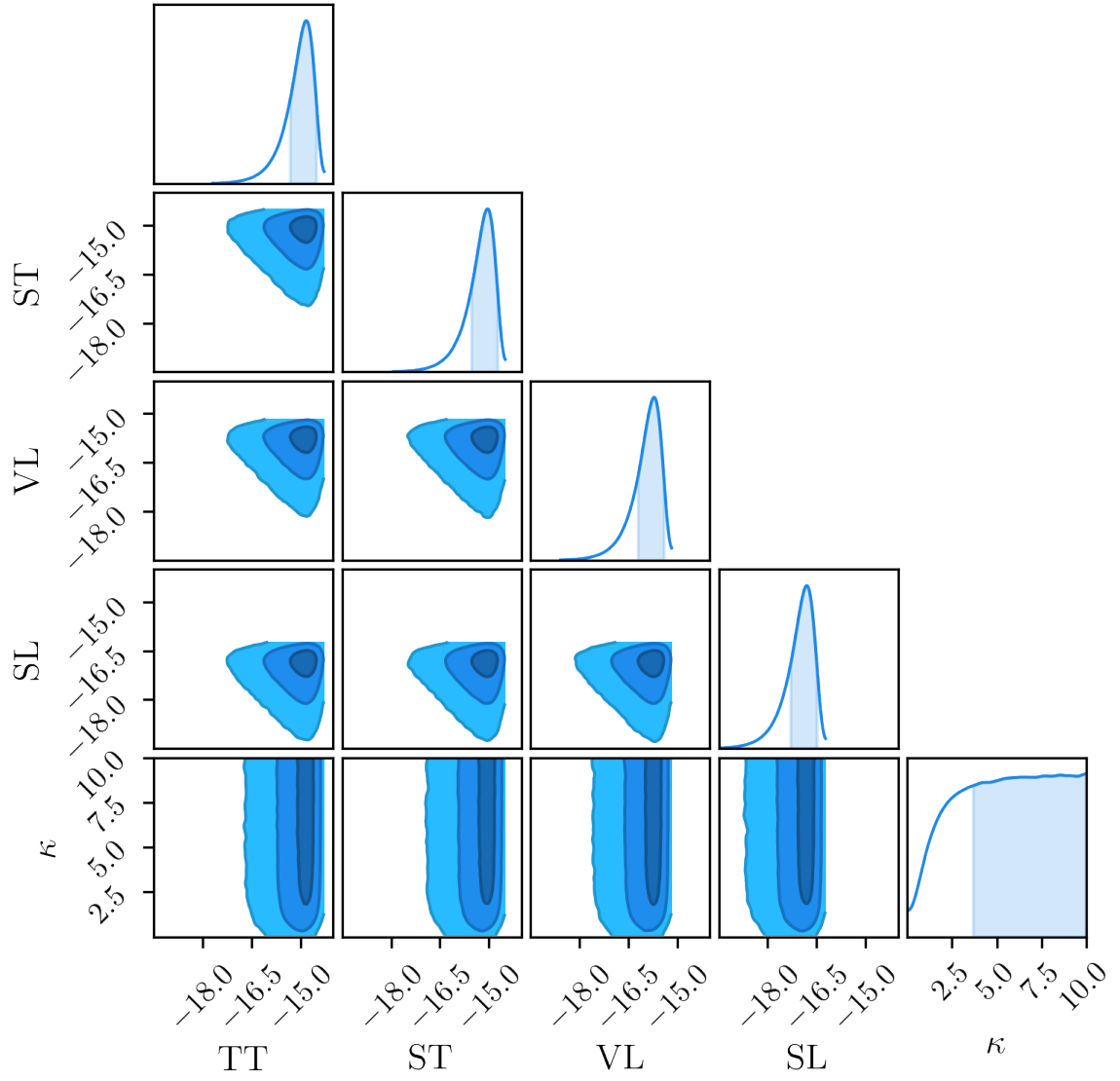


Figure 3.1: Slices through the posterior distribution for the astrophysical amplitudes $\mathcal{A}_{\text{TT}}$, $\mathcal{A}_{\text{ST}}$, $\mathcal{A}_{\text{VL}}$, $\mathcal{A}_{\text{SL}}$ and the decay parameter κ .

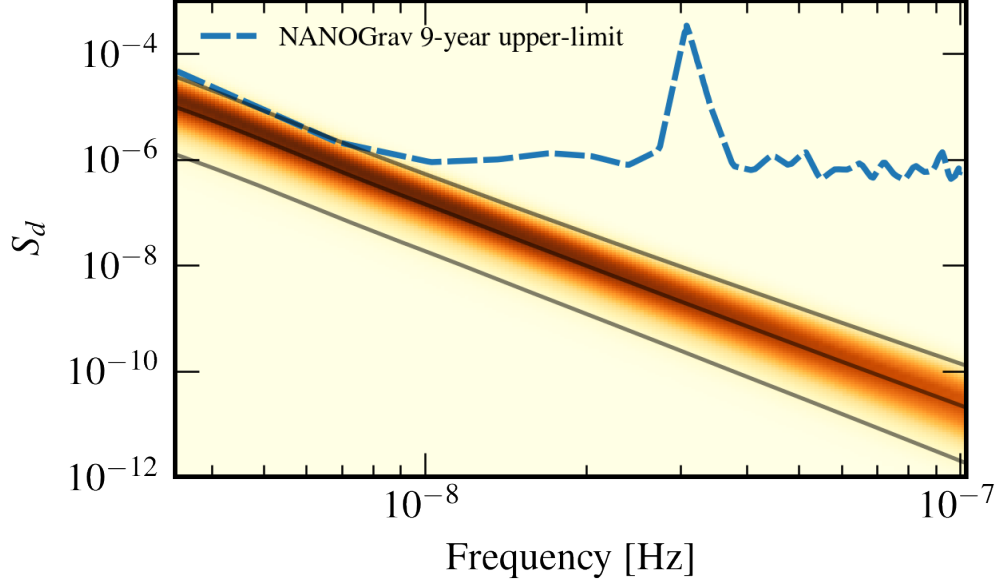


Figure 3.2: Posterior distribution for the combined spectrum $S_d(f)$ of the astrophysical model (shaded region) compared to the NANOGrav 9-year upper limits (dashed line).

variance $\sigma_{ab}^2 = \text{E}[r_a(f)r_b(f)r_a(f)r_b(f)] - \text{E}[r_a(f)r_b(f)]^2 = S_{aa}(f)S_{bb}(f) + S_{ab}^2(f)$. The variance of the longitudinal modes is very large due to the fL dependence in the auto-covariance terms. We can quantify this effect by computing the signal-to-noise ratio of the X_B correlation statistic [93]. We consider the observation-noise-free limit, for if the signal cannot be detected without noise, it will not be detectable with noise. In the zero observation-noise limit, the signal-to-noise ratio squared of the X_B statistic for the I^{th} polarization state is

$$\text{SNR}_B^2 = 2 \sum_f \sum_a^{N_p} \sum_{b>a}^{N_p} \frac{\Gamma_{ab}^{I^2}(f)}{\Gamma_{aa}^I(f)\Gamma_{bb}^I(f) + \Gamma_{ab}^{I^2}(f)}. \quad (3.9)$$

The Bayesian evidence for a correlation being present scales as SNR_B^2 . The angular dependence of the summand for each polarization state is shown in Fig. 3.3 for $f =$

10^{-8} Hz and $L = 1$ kpc. We see that the longitudinal modes accumulate most of their signal-to-noise ratio from pulsars with very small angular separations.

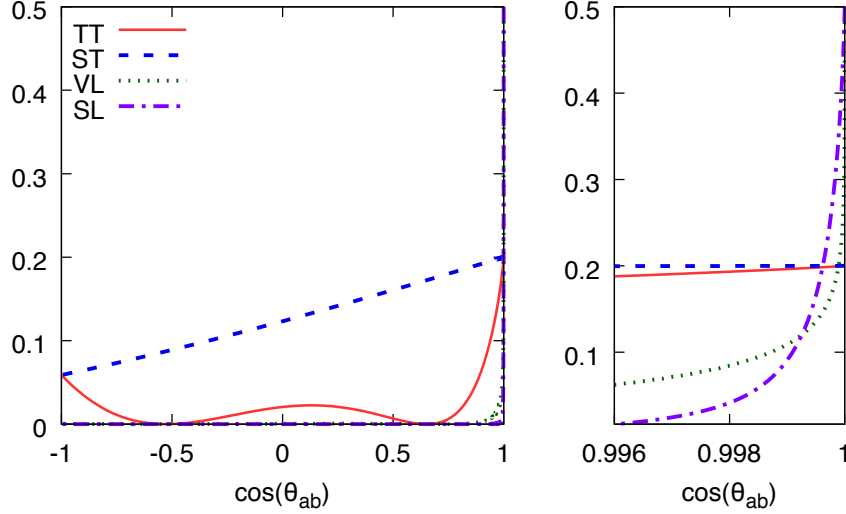


Figure 3.3: The contribution to the signal-to-noise ratio squared as a function of the cosine of the angle between the pulsars for the various polarization states. The panel on the right highlights the small region near zero angular separation.

The relative detectability of the various polarizations can be illustrated by considering the 46 pulsars from International Pulsar Timing Array [105]. Figure 3.4 shows the scaling of the signal-to-noise-ratio-squared as a function of the observation time T under the assumption that the signal dominates the noise for $3/T \leq f \leq \text{yr}^{-1}$. The vector longitudinal modes are a factor of ten harder to detect than the tensor modes, while the scalar longitudinal mode is a factor of a thousand harder to detect. With enough pulsars and a long enough observation time it will be possible to separate the scalar, vector and tensor modes, but the observational challenge is much greater than originally thought [72]. The difference in our conclusions can be traced to the original study using a detection statistic that neglects the auto-correlation terms.

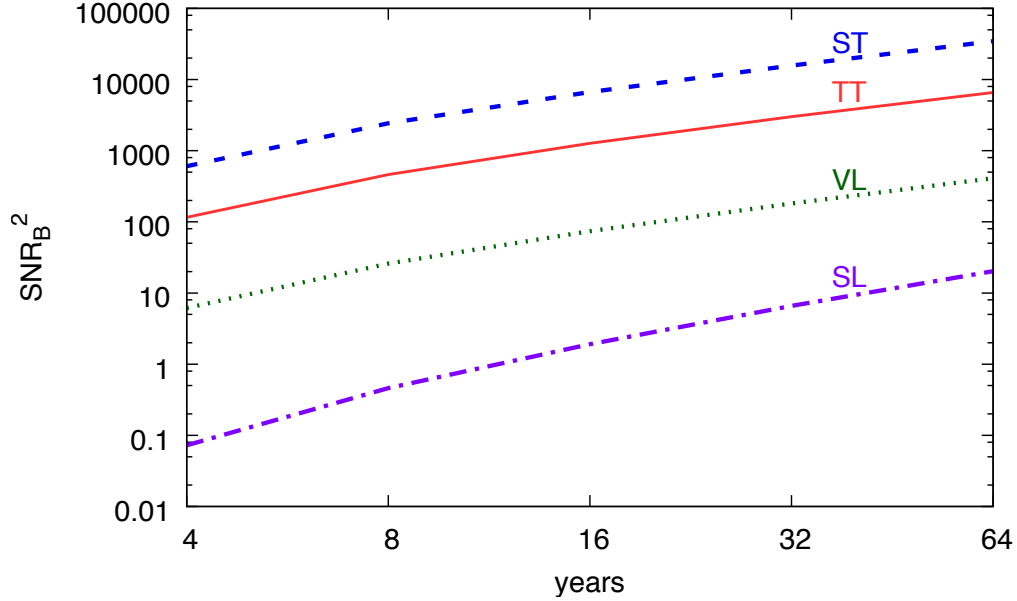


Figure 3.4: The growth in the signal-to-noise-ratio squared with time for each polarization mode, assuming that the 46 pulsars of the International Pulsar Timing Array are in the signal-dominated regime.

3.7 Summary

We have derived the first pulsar timing bounds on the amplitude of scalar and vector stochastic gravitational wave backgrounds for both astrophysical and cosmological sources. We have also pointed out that the “self-noise” produced by the strong response to longitudinal modes will make detecting alternative polarization states from a stochastic background very challenging. We hypothesize that observations of bright resolvable systems may provide the best opportunity to probe alternative polarization states using pulsar timing, since there the autocorrelation terms will contribute to the signal, not the noise.

Acknowledgments

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CHAPTER FOUR

CONSTRAINING THE POLARIZATION CONTENT OF GRAVITATIONAL
WAVES WITH ASTROMETRY4.1 Contribution of Authors and Co-Authors

Author: Logan O’Beirne

Contributions: Provided analytic expressions for the overlap reduction functions in the tensor basis and zero-angle expressions in the scalar basis, other miscellaneous tensor notation simplifications, and wrote the manuscript.

Co-Author: Neil J. Cornish

Contributions: Corrected and consulted in calculations, derived angular deflection, and helped write and edit the manuscript.

4.2 Manuscript Information

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*The notation has been modified from the original manuscript for uniformity with the dissertation.

ABSTRACT

Gravitational waves perturb the paths of photons, impacting both the time-of-flight and the arrival direction of light from stars. Pulsar timing arrays can detect gravitational waves by measuring the variations in the time of flight of radio pulses, while astrometry missions such as Gaia can detect gravitational waves from the time-varying changes in the apparent position of a field of stars. Just as gravitational waves impart a characteristic correlation pattern in the arrival times of pulses from pulsars at different sky locations, the deflection of starlight is similarly correlated across the sky. Here we compute the astrometric correlation patterns for the full range of polarization states found in alternative theories of gravity, and decompose the sky-averaged correlation patterns into vector spherical harmonics. We find that the tensor and vector polarization states produce equal power in the electric- and magnetic-type vector spherical harmonics, while the scalar modes produce only electric-type correlations. Any difference in the measured electric and magnetic-type correlations would represent a clear violation of Einstein gravity. The angular correlations functions for the vector and scalar longitudinal modes show the same enhanced response at small angular separations that is familiar from pulsar timing.

4.3 Introduction

Gravitational wave astronomy has made it possible to test gravity in the dynamical, strong field regime. The first binary black hole [3] and binary neutron star [10] detections have been used to carry out a wide range of tests, including placing stringent bounds on the difference in propagation speed of gravity and light [6, 36], constraining departures in the waveforms from the predictions of general relativity [4], and constraining the polarization content of the signals [5, 9, 12]. The existing array of ground based interferometers is not ideal for testing the polarization content of the signals as the two LIGO detectors and the single Virgo detector provide only a limited number of projections of the polarization. It takes at least 5 (mis-aligned) detectors to unambiguously resolve the polarization content [34]. The pulsar timing approach to detecting gravitational waves is better suited to measuring polarization [33, 37, 38, 72] as each pulsar line-of-sight provides a separate projection of the polarization state. The astrometric approach to detecting gravitational waves [31, 68, 86] is similarly well suited to constraining the polarization content, with astrometric missions such as Gaia [53] observing billions of stars across the sky.

Photon trajectories are perturbed by gravitational waves, resulting in time delays and changes in the apparent position of the source. Gravitational waves cause the relative position of two stars on the sky to oscillate in a tensor correlation pattern. The formal expression for this tensor two-point correlation pattern, valid for any gravitational wave polarization, was first derived by Book and Flanagan [31], however they only provided explicit expressions for the transverse-tensor modes of general relativity. Here we complete the derivation for the additional four polarization

states that are possible in other metric theories of gravity [108, 117]. Book and Flanagan [31] also derived the angular power spectrum, found by expanding the two-point correlation in terms of electric type (E) and magnetic type (B) vector spherical harmonics and integrating over the locations of the stars on the sky. This analysis is analogous to what is done when studying the polarization of the microwave background, where the polarization of the microwave photons are separated into curl-free E-modes and divergence-free B-modes. The tensor-transverse (TT) modes of general relativity produce identical angular correlation patterns for the EE and BB correlations at large angles, and differ slightly at small angular separations (the EB cross correlations vanish for all polarization states) [31]. Here we compute the angular power spectra for the additional non-Einsteinian polarization modes. The BB correlation vanishes for the scalar-transverse (ST) and the scalar-longitudinal (SL) polarizations while the EE correlation does not, indicating the the difference in the correlation functions $\Delta = (EE - BB)$ provides a powerful null test of general relativity. The vector-longitudinal (VL) polarizations evade this test since they produce identical EE and BB correlation patterns. Both the vector and scalar longitudinal modes produce an enhanced response at small angular separations that depends on the product of the gravitational wave frequency f and the distance to the stars L . The enhancement is logarithmic for VL and linear for SL. Precisely the same enhancements occur in the two-point correlation functions for these modes for pulsar timing [33, 38, 72]. This implies that astrometry missions like Gaia will be able to place much stronger constraints on the energy density in the longitudinal modes than for the transverse modes, as is the case for pulsar timing [37].

In the final stages of writing this paper a preprint was posted with an independent calculation of the astrometric two-point correlation functions for non-Einsteinian polarization modes [83]. After accounting for some differences in notation,

we verified that our expressions for the two-point functions agree. Their treatment stopped short of computing the E and B mode angular power spectra, and those results are reported here for the first time. We work in geometrical units with $G = c = 1$.

4.4 Summary of Results

Our calculation begins with the expression for the astrometric deflection $\delta\mathbf{n}(\mathbf{n}, \boldsymbol{\Omega}, t)$ for a star in the $\mathbf{n}$ direction perturbed by a gravitational wave source in the $\boldsymbol{\Omega}$ direction. Book and Flanagan [31] provided a formal expression for the deflection that is valid for any gravitational wave polarization. The next step is to compute the two-point correlation function $C^{ij}(\mathbf{n}, \mathbf{n}', f)$ which describe the tensor deflection pattern produced by an isotropic stochastic background of gravitational waves. Book and Flanagan [31] computed a formal expression for $C^{ij}(\mathbf{n}, \mathbf{n}', f)$ that is valid for all polarizations. In practice, to make a detection, we need to combine measurements of the angular deflections between many pairs of stars since the angular deflections between any two stars is expected to be small relative to the noise in the measurement. This is analogous to measurements of the cosmic microwave background, where correlations are computed over large areas of the sky and expressed in terms of angular power spectra. To compute the angular power spectra of the astrometric deflections we decompose the angular deflections into electric-type (E) and magnetic-type (B) vector spherical harmonics and compute the correlations EE, EB, BB averaged over the distribution of stars in the sky. Assuming a uniform distribution of stars, the correlations only depend on the angle $\Theta = \text{acos}(\mathbf{n} \cdot \mathbf{n}')$ between the stars, or equivalently, the multipole moment ℓ .

The astrometric deflection $\delta\mathbf{n}(\mathbf{n}, \boldsymbol{\Omega}, t)$ depends on the phase of the gravitational wave at the Earth and at the star. In the distant source limit, where the product of

the gravitational wave frequency f and the distance to the star L is large, the phase of the star term is uncorrelated between stars located at $L\mathbf{n}$ and $L'\mathbf{n}'$ unless $\Theta \approx 0$. For the transverse polarization states the star term can be safely ignored, but not so for the longitudinal modes. The general expression for the two-point correlation function for an isotropic stochastic background can be written as

$$C^{ij}(\mathbf{n}, \mathbf{n}', f) = \int d\Omega \langle \delta n^i(\mathbf{n}, \Omega, f) \delta n^j(\mathbf{n}', \Omega, f) \rangle = \frac{3H_0^2}{32\pi^2} \frac{\Omega_{\text{gw}}(f)}{f^3} H^{ij}(\mathbf{n}, \mathbf{n}'), \quad (4.1)$$

where H_0 is the Hubble constant, $\Omega_{\text{gw}}(f)$ is the energy density in gravitational waves per logarithmic frequency interval, scaled by the critical density $\rho_c = 3H_0^2/(8\pi)$, H^{ij} are the components of the deflection tensor

$$\mathbf{H}(\mathbf{n}, \mathbf{n}') = \alpha(\Theta)(\mathbf{a} \otimes \mathbf{a}) - \sigma(\Theta)(\mathbf{b} \otimes \mathbf{c}), \quad (4.2)$$

and the unit triad $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is defined by $\mathbf{a} = (\mathbf{n} \times \mathbf{n}')/\sin \Theta$, $\mathbf{b} = (\mathbf{n}(\mathbf{n} \cdot \mathbf{n}') - \mathbf{n}')/\sin \Theta$, $\mathbf{c} = (\mathbf{n}'(\mathbf{n} \cdot \mathbf{n}') - \mathbf{n})/\sin \Theta$. The angular correlations functions for the tensor transverse (TT), scalar transverse (ST) and vector longitudinal (VL) modes can be computed

in closed form in the distant star limit $fL \rightarrow \infty$, and are given by

$$\begin{aligned}
\alpha^{\text{TT}} &= \sigma^{\text{TT}} = (1/12)(7 \cos \Theta - 5) \\
&\quad - 8 \ln(\sin(\Theta/2))(\sin^6(\Theta/2)/\sin^2 \Theta) \\
\alpha^{\text{ST}} &= \frac{1}{12} \\
\sigma^{\text{ST}} &= \frac{1}{12} \cos(\Theta) \\
\alpha^{\text{VL}} &= \sigma^{\text{VL}} = \frac{\sin^2(\Theta/2)}{\sin^2(\Theta)} \left[8 \sin^2(\Theta/2) \ln(\sin(\Theta/2)) \right. \\
&\quad \left. + \frac{1}{3} (3 + 2 \cos(\Theta) - \cos(2\Theta)) \right]. \tag{4.3}
\end{aligned}$$

Plots of the TT, ST and VL correlation patterns are shown in Figure 4.1.

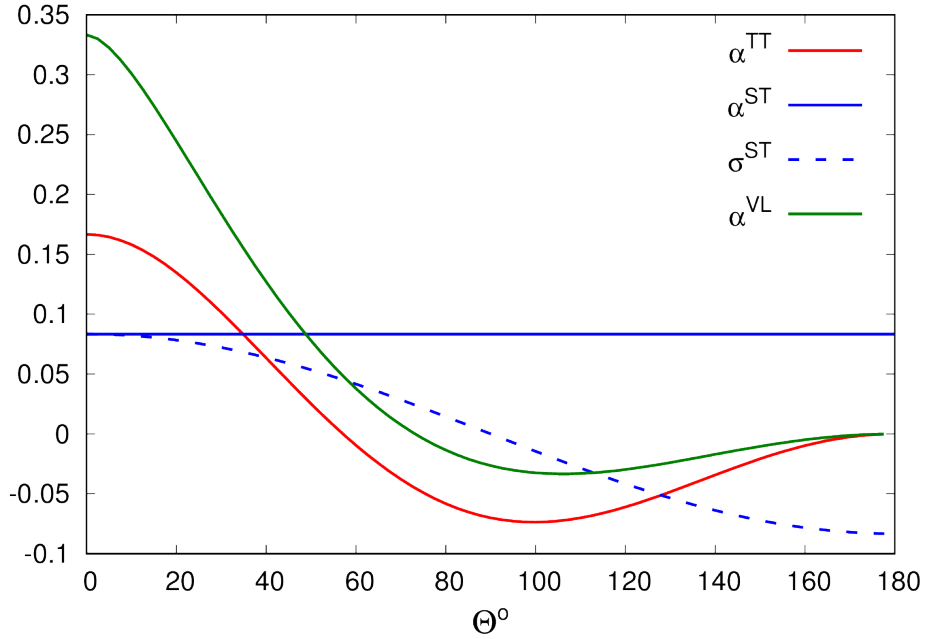


Figure 4.1: The analytic correlation functions $\alpha(\Theta)$ and $\sigma(\Theta)$ for the TT, ST and VL modes.

The scalar longitudinal (SL) case is more complicated as the star terms need

to be included to arrive at a finite expression at small angular separations. We were unable to derive a closed-form expression for the SL correlation functions that is valid for all angles, and instead quote results that are valid for $\Theta = 0$ and $\Theta \gg 1/(fL)$. For large angles we have

$$\begin{aligned}\alpha^{\text{SL}} &\approx -\frac{\sin^2(\Theta/2)}{3\sin^2(\Theta)} \left[1 + \cos(\Theta) + 3\ln(\sin(\Theta/2)) \right] \\ \sigma^{\text{SL}} &\approx -\frac{\sin^2(\Theta/2)}{6\sin^2(\Theta)} \left[4 + 5\cos(\Theta) + \cos(2\Theta) + 6\ln(\sin(\Theta/2)) \right].\end{aligned}\quad (4.4)$$

The divergence at $\Theta = 0$ in these expressions is regularized to a logarithmic dependence on the distance to the stars when the star terms are included. To leading order in $fL \gg 1$ we find

$$\begin{aligned}\alpha^{\text{SL}}(0) = \sigma^{\text{SL}}(0) = \\ \frac{1}{8} \left[\frac{\Phi_s^2 - \Phi_s'^2}{4\Phi_s\Phi_s'} \ln(\Phi_s^2 - \Phi_s'^2) + \ln(\Phi_s\Phi_s') - \frac{\Phi_s^2 + \Phi_s'^2}{4\Phi_s\Phi_s'} \ln(\Phi_s + \Phi_s') + \dots \right],\end{aligned}\quad (4.5)$$

where $\Phi_s = 2\pi fL$ and $\Phi_s' = 2\pi fL'$.

The angular power spectra can be expressed in terms of the multipole coefficients

$$C_{\ell m \ell' m'}^{QQ'} = \int d^2\Omega_{\mathbf{n}} d^2\Omega_{\mathbf{n}'} Y_{\ell m i}^{Q*}(\mathbf{n}) Y_{\ell' m' j}^{Q'}(\mathbf{n}') H_{ij}(\mathbf{n}, \mathbf{n}').\quad (4.6)$$

Here Q, Q' refer to the electric-type and magnetic-type vector spherical harmonics

$$\begin{aligned}\mathbf{Y}_{\ell m}^E(\mathbf{n}) &= (\ell(\ell+1))^{-1/2} \nabla Y_{\ell m}(\mathbf{n}) \\ \mathbf{Y}_{\ell m}^B(\mathbf{n}) &= (\ell(\ell+1))^{-1/2} (\mathbf{n} \times \nabla) Y_{\ell m}(\mathbf{n})\end{aligned}\quad (4.7)$$

The EB cross-correlations vanish due to the different parity of the E and B modes.

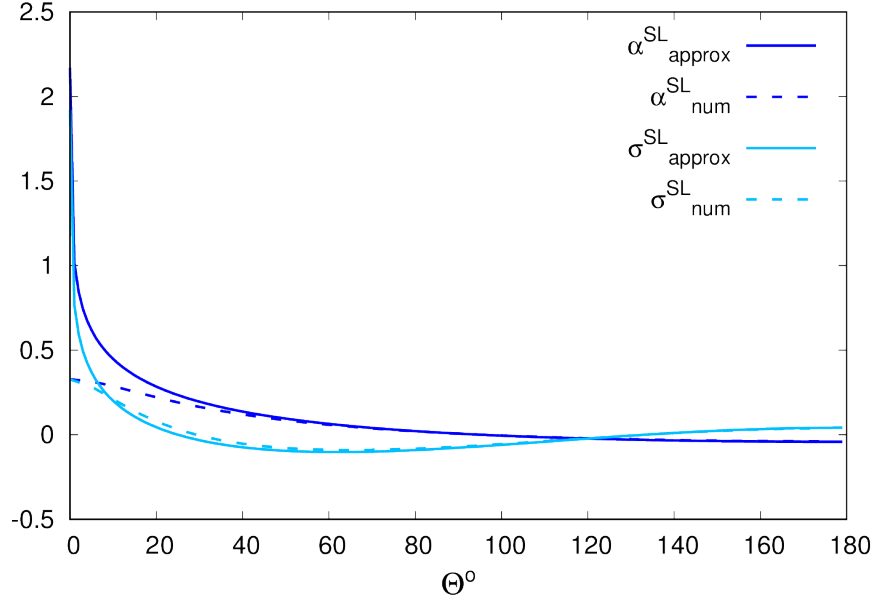


Figure 4.2: The correlation functions $\alpha(\Theta)$ and $\sigma(\Theta)$ for the SL mode. The solid lines show the analytic approximation, valid for $\Theta \gg 1/(fL)$. The dotted lines are found by numerical integration for the case $fL = fL' = 10$.

For an isotropic distribution of stars we find

$$C_{\ell m \ell' m'}^{QQ'} = \delta^{QQ'} \delta_{\ell \ell'} \delta_{mm'} \frac{1}{\ell(\ell+1)} \frac{4\pi}{2\ell+1} \mathbb{Q}\mathbb{Q}_\ell, \quad (4.8)$$

where

$$\mathbb{Q}\mathbb{Q}_\ell = \frac{2\ell+1}{4\pi} \int d\Theta P_\ell(\cos \Theta) \mathbb{Q}\mathbb{Q}(\Theta). \quad (4.9)$$

The formal expressions for the EE and BB correlation functions are:

$$\begin{aligned} \text{EE}(\Theta) &= \nabla_i \nabla'_j [H^{ij}(\mathbf{n}, \mathbf{n}')] \\ \text{BB}(\Theta) &= \nabla^l \nabla'^p [\epsilon_{ikl} \epsilon_{jmp} n^k n'^m H^{ij}(\mathbf{n}, \mathbf{n}')] . \end{aligned} \quad (4.10)$$

Note that $\text{EE}(\Theta)$ and $\text{BB}(\Theta)$, or equivalently EE_ℓ and BB_ℓ , are physical observables

that can be inferred from astrometric data, just as the EE and BB power spectra for the CMB polarization are physical observables. A method for extracting $C_{\ell m \ell' m'}^{QQ'}$, and hence EE_ℓ and BB_ℓ , from Gaia observations is described in Ref. [82].

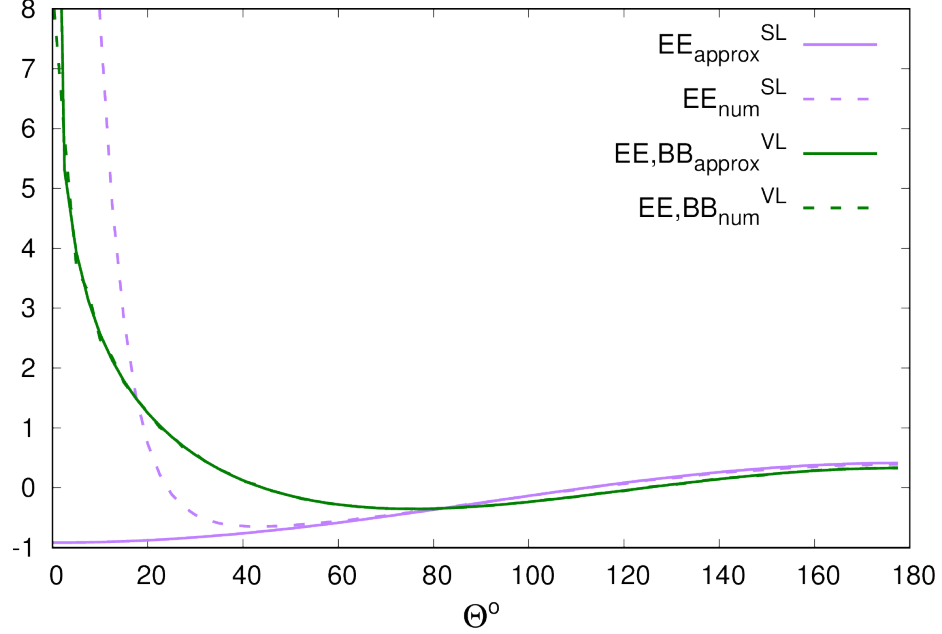


Figure 4.3: The upper plot shows EE and BB angular correlation functions for the longitudinal modes. The solid lines are the analytic approximations from Eq. (4.11) and the dotted lines are from a numerical evaluation with $fL = 10$.

At large angles we find that $EE(\Theta) = BB(\Theta)$ for the TT and VL modes, and for the scalar modes ST and SL we find $BB(\Theta) = 0$, as expected from the even parity of the scalar perturbations. Additional care must be taken when computing the angular power spectra as the derivatives acting on $H_{ij}(\mathbf{n}, \mathbf{n}')$ act on the phase of the star term,

and modify the behavior at small angles. In the distant star limit we find

$$\begin{aligned}
EE^{\text{TT}}(\Theta) &= \frac{2}{3} \left[2 - \left(\frac{1 - \cos\Theta}{2} \right) \right. \\
&\quad \left. + 6 \left(\frac{1 - \cos\Theta}{2} \right) \ln \left(\frac{1 - \cos\Theta}{2} \right) \right] \\
BB^{\text{TT}}(\Theta) &= EE^{\text{TT}}(\Theta) \\
EE^{\text{VL}}(\Theta) &\approx -\frac{1}{3} \left(3 + 4 \cos(\Theta) + 6 \ln(\sin(\Theta/2)) \right) \\
BB^{\text{VL}}(\Theta) &= EE^{\text{VL}}(\Theta) \\
BB^{\text{ST}}(\Theta) &= BB^{\text{SL}}(\Theta) = 0 \\
EE^{\text{ST}}(\Theta) &= \frac{1}{3} \cos\Theta \\
EE^{\text{SL}}(\Theta) &\approx -\frac{1}{12} (3 + 8 \cos \Theta), \tag{4.11}
\end{aligned}$$

Note that up to an overall scaling, the correlation pattern for the TT mode is *identical* to the Hellings-Downs curve from pulsar timing. Similarly, the EE correlation for the ST mode has the same form as the ST correlation function from pulsar timing, aside from an overall constant offset and scaling. The expressions for the scalar longitudinal and vector longitudinal modes are only valid for $\Theta \gg 1/(fL)$. Derivatives of the star terms become important at small angular separations. A complete numerical treatment would need to integrate over the radial distribution of stars, but to simplify the calculation we will assume that all the stars are at the same distance. Figure 4.3 compares the closed-form approximations to the correlation functions for the longitudinal modes to the numerical evaluation with $fL = 10$.

The derivatives of $H_{ij}(\mathbf{n}, \mathbf{n}')$ that appear in the definition of the EE and BB correlation functions enhance the importance of the star terms at small angular separations and produce an enhanced response. For the VL modes we find that the enhancement at $\Theta = 0$ scales as $\ln(fL)$, while for the SL mode we find that

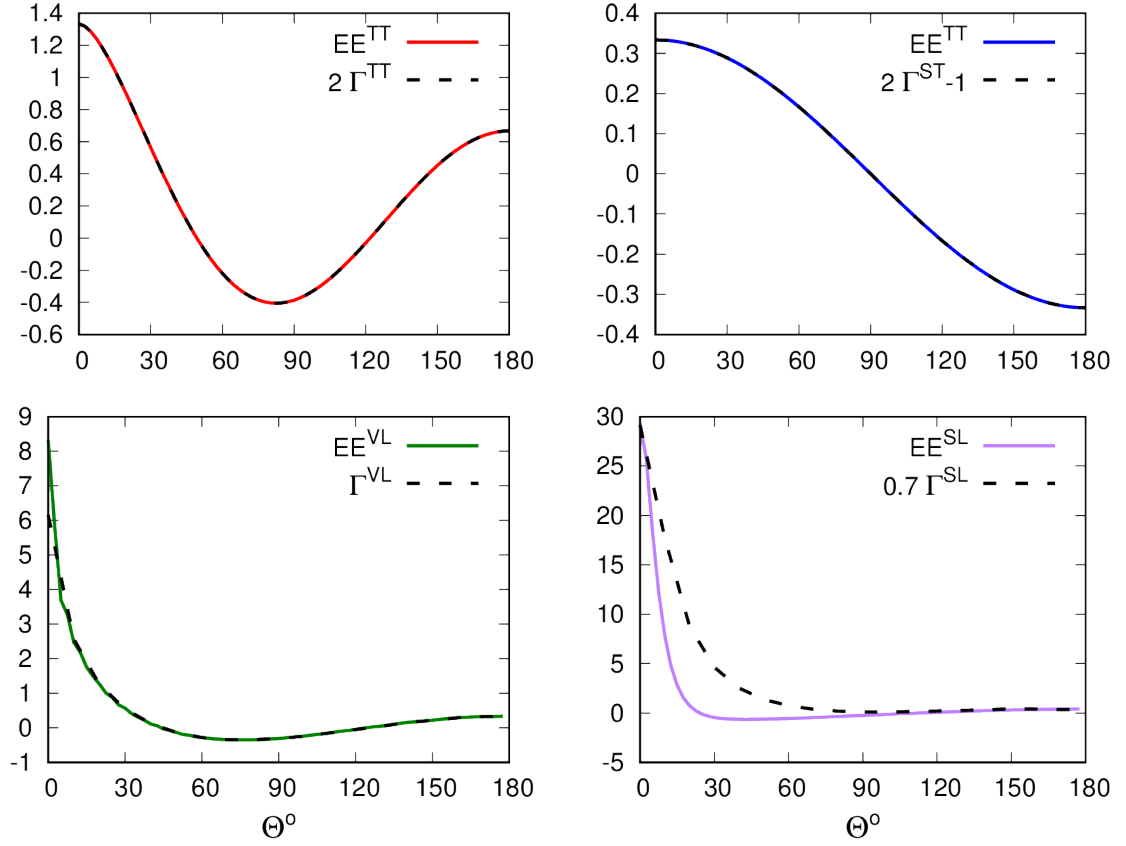


Figure 4.4: The EE angular correlation functions are compared to the corresponding correlation functions Γ for pulsar timing. Aside from overall scalings and offsets, the correlation functions for the transverse modes are identical. The vector longitudinal correlation functions are identical at large angles, but differ slightly at small angles, though both share a similar $\ln(fL)$ scaling at $\Theta = 0$. The correlation functions for the scalar longitudinal modes are qualitatively similar, but differ in their detailed form. They share a similar fL dependent scaling at $\Theta = 0$. The plots are shown for $fL = 10$.

the enhancement scales as fL . These enhancement factors are identical to those found for the VL and SL angular correlation functions for pulsar timing. This implies that astrometric observatories such as Gaia will be able to place much tighter constraints on the energy density of the longitudinal modes, as is the case for pulsar timing [37]. Figure 4.4 compares the numerically computed EE correlation functions to the corresponding two-point angular correlation functions Γ for pulsar timing [72]. Aside from an overall difference in scaling and a constant shift for the scalar mode, the correlation functions for the transverse modes are *identical* for astrometry and pulsar timing. The vector longitudinal correlation functions are identical at large angles, but differ slightly at small angles, though both scale as $\ln(fL)$ at small angles, albeit with different scaling coefficients. The correlation functions for the scalar longitudinal modes are qualitatively similar, but differ in their detailed form. They both scale as fL for small angles, but with different scaling coefficients.

4.5 Computing the two-point correlation functions

The angular deflection of a star at location $\mathbf{N} = \mathbf{n}L$ caused by a plane gravitational wave $h_{ij}(t, \mathbf{x}) = \Re[\mathcal{H}_{ij}e^{-2\pi if(t-\mathbf{\Omega}\cdot\mathbf{x})}]$ propagating in the $\mathbf{\Omega}$ direction is given by [31]

$$\delta n^i(\mathbf{n}, \mathbf{\Omega}, t) = \Re\left[\mathcal{R}^{ikl}(\mathbf{n}, \mathbf{\Omega})\mathcal{H}_{kl}e^{-2\pi ift}\right] \quad (4.12)$$

where

$$\mathcal{R}^{ikl}(\mathbf{n}, \mathbf{\Omega}) = \frac{1}{2}\left[\frac{(T_{\mathbf{n}_1}n^i + T_{\mathbf{\Omega}}\Omega^i)n^kn^l}{1 + \mathbf{\Omega} \cdot \mathbf{n}} - T_{\mathbf{n}_2}n^k\delta^{il}\right] \quad (4.13)$$

and

$$\begin{aligned}
T_{\mathbf{n}_1} &= 1 - \frac{i(2 + \boldsymbol{\Omega} \cdot \mathbf{n})}{\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})} e^{i\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})} \\
T_{\boldsymbol{\Omega}} &= 1 - \frac{i}{\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})} e^{i\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})} \\
T_{\mathbf{n}_2} &= \frac{1}{2} - \frac{i}{\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})} e^{i\Phi_s(1 + \boldsymbol{\Omega} \cdot \mathbf{n})}
\end{aligned} \tag{4.14}$$

with $\Phi_s = 2\pi fL$. To compute the two-point correlation function $C^{ij}(\mathbf{n}, \mathbf{n}', f)$ we define the orthonormal coordinate system $(\mathbf{m}, \mathbf{l}, \mathbf{n})$ and introduce the gravitational wave polarization tetrad $(\mathbf{u}, \mathbf{v}, \boldsymbol{\Omega})$:

$$\begin{aligned}
\boldsymbol{\Omega} &= \cos \theta \mathbf{n} + \sin \theta (\cos \phi \mathbf{m} + \sin \phi \mathbf{l}), \\
\mathbf{u} &= \frac{(-\sin \theta \cos \phi \mathbf{n} + \cos \theta \mathbf{m})}{\sqrt{\sin^2 \theta \cos^2 \phi + \cos^2 \theta}}, \\
\mathbf{v} &= -\frac{(\sin \theta \cos \theta \sin \phi \mathbf{n} + \sin^2 \theta \cos \phi \sin \phi \mathbf{m})}{\sqrt{\sin^2 \theta \cos^2 \phi + \cos^2 \theta}} \\
&\quad + \frac{(\sin^2 \theta \cos^2 \phi + \cos^2 \theta) \mathbf{l}}{\sqrt{\sin^2 \theta \cos^2 \phi + \cos^2 \theta}}.
\end{aligned} \tag{4.15}$$

The basis tensors for the various gravitational wave polarization states are then

$$\begin{aligned}
\boldsymbol{\epsilon}_{\text{TT}}^+ &= \mathbf{u} \otimes \mathbf{u} - \mathbf{v} \otimes \mathbf{v} \\
\boldsymbol{\epsilon}_{\text{TT}}^\times &= \mathbf{u} \otimes \mathbf{v} + \mathbf{v} \otimes \mathbf{u} \\
\boldsymbol{\epsilon}_{\text{ST}}^\odot &= \mathbf{u} \otimes \mathbf{u} + \mathbf{v} \otimes \mathbf{v} \\
\boldsymbol{\epsilon}_{\text{VL}}^u &= \mathbf{u} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{u} \\
\boldsymbol{\epsilon}_{\text{VL}}^v &= \mathbf{v} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{v} \\
\boldsymbol{\epsilon}_{\text{SL}}^\odot &= \boldsymbol{\Omega} \otimes \boldsymbol{\Omega}.
\end{aligned} \tag{4.16}$$

The $\mathbf{n}'$ vector can be written as

$$\mathbf{n}' = \cos \Theta \mathbf{n} + \sin \Theta (\cos \Phi \mathbf{m} + \sin \Phi \mathbf{l}). \quad (4.17)$$

The two-point correlation function $C^{ij}(\mathbf{n}, \mathbf{n}', f)$ can be written in terms of the tensor $H^{ij}(\mathbf{n}, \mathbf{n}') = \alpha(\Theta) a^i a^j - \sigma(\Theta) b^i c^j$. Written in the the $(\mathbf{m}, \mathbf{l}, \mathbf{n})$ coordinate system we have

$$\begin{aligned} \mathbf{a} &= (-\sin \Phi \mathbf{m} + \cos \Phi \mathbf{l}) \\ \mathbf{b} &= -(\cos \Phi \mathbf{m} + \sin \Phi \mathbf{l}) \\ \mathbf{c} &= -\sin \Theta \mathbf{n} + \cos \Theta (\sin \Phi \mathbf{m} + \cos \Phi \mathbf{l}). \end{aligned} \quad (4.18)$$

The angular correlation functions are given by

$$\begin{aligned} \alpha^P(\Theta) &= \frac{1}{4\pi \sin^2(\Theta)} \int d^2\Omega a^i \mathcal{R}_{ikl}(\mathbf{n}, \mathbf{\Omega}) \epsilon_{kl}^P(\mathbf{\Omega}) a^j \mathcal{R}_{jrs}(\mathbf{n}', \mathbf{\Omega})^* \epsilon_{rs}^P(\mathbf{\Omega}) \\ \sigma^P(\Theta) &= -\frac{1}{4\pi \sin^2(\Theta)} \int d^2\Omega b^i \mathcal{R}_{ikl}(\mathbf{n}, \mathbf{\Omega}) \epsilon_{kl}^P(\mathbf{\Omega}) c^j \mathcal{R}_{jrs}(\mathbf{n}', \mathbf{\Omega})^* \epsilon_{rs}^P(\mathbf{\Omega}) \end{aligned} \quad (4.19)$$

where $d^2\Omega = \sin \theta d\theta d\phi$. The correlation functions for the TT and VL modes are defined as being summed over the $+$, $\times$ or u, v states. The correlation functions for ST and SL only have a single contribution. The full expression for the correlation functions are lengthy. To render the expressions manageable we follow Ref. [31] and introduce the quantities $\kappa = \mathbf{n} \cdot \mathbf{\Omega}$, $\kappa' = \mathbf{n}' \cdot \mathbf{\Omega}$, $\lambda = \mathbf{n} \cdot \mathbf{n}'$, $\nu^2 = 1 - \kappa^2$, $\nu'^2 = 1 - \kappa'^2$, $\mu = \sin \Theta \mathbf{a} \cdot \mathbf{\Omega}$, which satisfy the identity $1 + 2\lambda\kappa\kappa' = \mu^2 + \lambda^2 + \kappa^2 + \kappa'^2$.

The correlation functions are then given by

$$\alpha^{\text{TT}}(\Theta) = \frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \left[\frac{\mu^2(\nu^2\nu'^2 - 2\mu^2)}{(1+\kappa)(1+\kappa')} T_{\Omega} T_{\Omega'}^* - \frac{\mu^2(\kappa'\kappa^2 - 2\lambda\kappa + \kappa')}{(1+\kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* \right. \\ \left. - \frac{\mu^2(\kappa\kappa'^2 - 2\lambda\kappa' + \kappa)}{(1+\kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* + (\lambda - \kappa\kappa')(1 - \lambda^2 - \mu^2) T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \right] \quad (4.20)$$

$$\sigma^{\text{TT}}(\Theta) = -\frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \times \\ \left[\frac{(\lambda\kappa - \kappa')(\lambda\kappa' - \kappa)(\nu^2\nu'^2 - 2\mu^2)}{(1+\kappa)(1+\kappa')} T_{\Omega} T_{\Omega'}^* \right. \\ - \frac{(\lambda\kappa - \kappa')[\lambda(\nu^2\nu'^2 - 2\mu^2) - \nu^2(\lambda - \kappa\kappa')]}{(1+\kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* \\ - \frac{(\lambda\kappa' - \kappa)[\lambda(\nu^2\nu'^2 - 2\mu^2) - \nu'^2(\lambda - \kappa\kappa')]}{(1+\kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* \\ \left. + [\lambda^2(\nu^2\nu'^2 - 2\mu^2 - \nu^2 - \nu'^2) \right. \\ \left. + \lambda\kappa\kappa'(\nu^2 + \nu'^2) + \nu^2\nu'^2] T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \right] \quad (4.21)$$

$$\alpha^{\text{ST}}(\Theta) = \frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \left[\mu^2(1 - \kappa)(1 - \kappa') T_{\Omega} T_{\Omega'}^* \right. \\ \left. + \mu^2\kappa'(1 - \kappa) T_{\Omega} T_{\mathbf{n}_2'}^* + \mu^2\kappa(1 - \kappa') T_{\mathbf{n}_2} T_{\Omega'}^* + (\mu^2\kappa\kappa') T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \right] \quad (4.22)$$

$$\sigma^{\text{ST}}(\Theta) = -\frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \left[(\lambda\kappa - \kappa')(\lambda\kappa' - \kappa)(1 - \kappa)(1 - \kappa') T_{\Omega} T_{\Omega'}^* \right. \\ + (\lambda\kappa - \kappa')(\lambda\kappa' - \kappa)(1 - \kappa)\kappa' T_{\Omega} T_{\mathbf{n}_2'}^* + (\lambda\kappa - \kappa')(\lambda\kappa' - \kappa)(1 - \kappa')\kappa T_{\mathbf{n}_2} T_{\Omega'}^* \\ \left. + [\lambda^2\nu^2\nu'^2 - \lambda(\lambda - \kappa\kappa')(\nu^2 + \nu'^2) + (\lambda - \kappa\kappa')^2] T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \right] \quad (4.23)$$

$$\begin{aligned}
\alpha^{\text{VL}}(\Theta) = & \frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \frac{4\mu^2 \kappa \kappa' (\lambda - \kappa \kappa')}{(1 + \kappa)(1 + \kappa')} T_{\Omega} T_{\Omega'}^* \\
& - \frac{2\mu^2 \kappa (\lambda - 2\kappa \kappa')}{(1 + \kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* - \frac{2\mu^2 \kappa' (\lambda - 2\kappa \kappa')}{(1 + \kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* \\
& + [\lambda \mu^2 - \kappa \kappa' (4\mu^2 + \lambda^2 - 1)] T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \Big] \quad (4.24)
\end{aligned}$$

$$\begin{aligned}
\sigma^{\text{VL}}(\Theta) = & -\frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \Big[\frac{4(\lambda \kappa - \kappa')(\lambda \kappa' - \kappa) \kappa \kappa' (\lambda - \kappa \kappa')}{(1 + \kappa)(1 + \kappa')} T_{\Omega} T_{\Omega'}^* \\
& - \frac{2(\lambda \kappa - \kappa') [2\lambda \kappa \kappa' (\lambda - \kappa \kappa') - \kappa (\kappa' - \kappa' \kappa^2 + \lambda \kappa)]}{(1 + \kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* \\
& - \frac{2(\lambda \kappa' - \kappa) [2\lambda \kappa \kappa' (\lambda - \kappa \kappa') - \kappa' (\kappa - \kappa \kappa'^2 + \lambda \kappa')]}{(1 + \kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* \\
& + [(\kappa^2 - \kappa'^2)^2 + \lambda^2 (\lambda^2 - 1) + \mu^2 (\mu^2 - 1)] T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \Big] \quad (4.25)
\end{aligned}$$

$$\begin{aligned}
\alpha^{\text{SL}}(\Theta) = & \frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \Big[\frac{\mu^2 \kappa^2 \kappa'^2}{(1 + \kappa)(1 + \kappa')} T_{\Omega} T_{\Omega'}^* \\
& - \frac{\mu^2 \kappa' \kappa^2}{(1 + \kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* - \frac{\mu^2 \kappa \kappa'^2}{(1 + \kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* + \mu^2 \kappa \kappa' T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \Big] \quad (4.26)
\end{aligned}$$

$$\begin{aligned}
\sigma^{\text{SL}}(\Theta) = & -\frac{1}{16\pi \sin^2(\Theta)} \int d^2\Omega \Big[\frac{(\lambda \kappa - \kappa')(\lambda \kappa' - \kappa) \kappa^2 \kappa'^2}{(1 + \kappa)(1 + \kappa')} T_{\Omega} T_{\Omega'}^* \\
& - \frac{(\lambda \kappa - \kappa')(\lambda \kappa' - \kappa) \kappa^2 \kappa'}{(1 + \kappa)} T_{\Omega} T_{\mathbf{n}_2'}^* - \frac{(\lambda \kappa' - \kappa)(\lambda \kappa - \kappa') \kappa'^2 \kappa}{(1 + \kappa')} T_{\mathbf{n}_2} T_{\Omega'}^* \\
& + [(\lambda^2 + 1) \kappa^2 \kappa'^2 - \lambda \kappa \kappa' (\kappa^2 + \kappa'^2)] T_{\mathbf{n}_2} T_{\mathbf{n}_2'}^* \Big] \quad (4.27)
\end{aligned}$$

with

$$\begin{aligned}
T_{\mathbf{n}_1} &= 1 - \frac{i(2 + \kappa)}{\Phi_s(1 + \kappa)} e^{i\Phi_s(1 + \kappa)} \\
T_{\mathbf{\Omega}} &= 1 - \frac{i}{\Phi_s(1 + \kappa)} e^{i\Phi_s(1 + \kappa)} \\
T_{\mathbf{n}_2} &= \frac{1}{2} - \frac{i}{\Phi_s(1 + \kappa)} e^{i\Phi_s(1 + \kappa)} \\
T_{\mathbf{n}'_1} &= 1 - \frac{i(2 + \kappa')}{\Phi'_s(1 + \kappa')} e^{i\Phi'_s(1 + \kappa')} \\
T_{\mathbf{\Omega}'} &= 1 - \frac{i}{\Phi'_s(1 + \kappa')} e^{i\Phi'_s(1 + \kappa')} \\
T_{\mathbf{n}'_2} &= \frac{1}{2} - \frac{i}{\Phi'_s(1 + \kappa')} e^{i\Phi'_s(1 + \kappa')} .
\end{aligned} \tag{4.28}$$

In the distant star limit, $\Phi_s, \Phi'_s \gg 1$, and away from $\theta = \pi$, the phase terms oscillate rapidly and can be discarded, allowing the integrals to be evaluated in closed form to give the expressions quoted in Eqs.(4.3,4.4). Without the phase terms, the α and σ integrands for SL mode diverge at $\theta = \pi$. Restoring the phase terms renders the integrands finite, but the integrals are then intractable, save at $\Theta = 0$, where the leading order terms scale logarithmically with fL , as shown in Eq.(4.5).

4.6 Computing the angular power spectra

In contrast with pulsar timing, where the small number of millisecond pulsars allow searches for gravitational waves to be performed directly in terms of the two-point correlation functions, the large number of stars available for an astrometric search make it more natural to integrate over the distribution of stars on the sky and compute angular correlation functions, as is done with the cosmic microwave background. To achieve this, the astrometric deflections $\delta \mathbf{n}$ are first decomposed in

terms of vector spherical harmonics:

$$\delta n^i(\mathbf{n}, \boldsymbol{\Omega}, f) = \sum_{\ell m} \delta n_{E\ell m}(f) \mathbf{Y}_{\ell m}^E(\mathbf{n}) + \delta n_{B\ell m}(f) \mathbf{Y}_{\ell m}^B(\mathbf{n}). \quad (4.29)$$

The angular power spectra are found by integrating the two-point correlation function $C^{ij}(\mathbf{n}, \mathbf{n}', f)$ over a uniform distribution of stars $\mathbf{n}, \mathbf{n}'$:

$$\begin{aligned} \Gamma_{\ell m \ell' m'}^{QQ'}(f) &= \int d^2\Omega_{\mathbf{n}} d^2\Omega_{\mathbf{n}'} Y_{\ell m i}^{Q*}(\mathbf{n}) Y_{\ell' m' j}^{Q'}(\mathbf{n}') C^{ij}(\mathbf{n}, \mathbf{n}', f) \\ &= \frac{3H_0^2 \Omega_{\text{gw}}(f)}{16\pi^3 f^3} \delta^{QQ'} \delta_{\ell\ell'} \delta_{mm'} C_{\ell}^Q(f), \end{aligned} \quad (4.30)$$

Here $Q = E, B$ label the electric and magnetic terms, and $C_{\ell}^Q(f)$ defines the angular power spectrum

$$\begin{aligned} C_{\ell}^Q(f) &= \int d^2\Omega_{\mathbf{n}} d^2\Omega_{\mathbf{n}'} Y_{\ell m i}^{Q*}(\mathbf{n}) Y_{\ell m j}^Q(\mathbf{n}') H^{ij}(\mathbf{n}, \mathbf{n}') \\ &= \frac{1}{\ell(\ell+1)} \int d(\cos \Theta) P_{\ell}(\cos \Theta) Q Q(\Theta). \end{aligned} \quad (4.31)$$

To arrive at this expression, the derivatives of the ordinary spherical harmonics that appear in the definitions of the vector spherical harmonics are converted to derivatives acting on the angular displacement tensor H_{ij} by integrating by parts, resulting in the formal expressions:

$$\begin{aligned} \text{EE}(\Theta) &= \nabla_i \nabla'_j [H^{ij}(\mathbf{n}, \mathbf{n}')] \\ \text{BB}(\Theta) &= \nabla^l \nabla'^p [\epsilon_{ikl} \epsilon_{jmp} n^k n'^m H^{ij}(\mathbf{n}, \mathbf{n}')] . \end{aligned} \quad (4.32)$$

Recalling the expression for H_{ij} from Eq. (4.33), we see that the derivatives will act on both the basis tensors and the α, σ correlation functions. Writing $\tilde{H}_{lp} =$

$\epsilon_{ikl}\epsilon_{jmp}n_k n'_m H^{ij}$ we find that

$$\tilde{\mathbf{H}}(\mathbf{n}, \mathbf{n}') = \sigma(\Theta)(\mathbf{a} \otimes \mathbf{a}) - \alpha(\Theta)(\mathbf{b} \otimes \mathbf{c}). \quad (4.33)$$

For the TT and VL modes $\alpha = \sigma$, so that $\tilde{H}_{ij} = H_{ij}$ and $\text{EE}(\Theta) = \text{BB}(\Theta)$. For the ST and SL modes $\alpha \neq \sigma$ and $\text{EE}(\Theta) \neq \text{BB}(\Theta)$. Indeed, a direct calculation shows that the magnetic-type correlation vanishes for the scalar modes, $\text{BB}^{\text{ST}}(\Theta) = \text{BB}^{\text{SL}}(\Theta) = 0$, which follows from the even parity of the scalar perturbations. Evaluating the derivatives that appear in Eq. (4.32) we find

$$\begin{aligned} \nabla_i a^i a^j &= \frac{(\mathbf{n} \cdot \mathbf{n}')n'^j - n^j}{\sin^2(\Theta)} \\ \nabla_i b^i c^j &= \frac{(\mathbf{n} \cdot \mathbf{n}')[(\mathbf{n} \cdot \mathbf{n}')n'^j - n^j]}{\sin^2(\Theta)} \\ \nabla_i \nabla'_j a^i a^j &= \frac{2(\mathbf{n} \cdot \mathbf{n}')}{\sin^2(\Theta)} \\ \nabla_i \nabla'_j b^i c^j &= -1 \\ \nabla_i \sigma(\Theta) &= -\sigma'(\Theta) \frac{n'_i - (\mathbf{n} \cdot \mathbf{n}')n_i}{\sin(\Theta)} \\ \nabla'_j \sigma(\Theta) &= -\sigma'(\Theta) \frac{n_j - (\mathbf{n} \cdot \mathbf{n}')n'_j}{\sin(\Theta)} \\ \nabla_i \nabla'_j \sigma(\Theta) &= \sigma'(\Theta) \left\{ \frac{\delta_{ij} - n_i n_j - n'_i n'_j + (\mathbf{n} \cdot \mathbf{n}')n_i n'_j}{-\sin(\Theta)} \right. \\ &\quad \left. + \frac{\cos(\Theta)[n'_i - (\mathbf{n} \cdot \mathbf{n}')n_i][n_j - (\mathbf{n} \cdot \mathbf{n}')n'_j]}{-\sin^3(\Theta)} \right\} \\ &\quad + \sigma''(\Theta) \frac{[n'_i - (\mathbf{n} \cdot \mathbf{n}')n_i][n_j - (\mathbf{n} \cdot \mathbf{n}')n'_j]}{\sin^2(\Theta)} \end{aligned} \quad (4.34)$$

and similarly for $\alpha(\Theta)$. Here the primes denote derivatives with respect to Θ . Combining these pieces together yields

$$\begin{aligned} \text{EE}(\Theta) &= -\sigma''(\Theta) + \frac{1}{\sin \Theta} \alpha'(\Theta) - 2 \frac{\cos \Theta}{\sin \Theta} \sigma'(\Theta) + \sigma(\Theta) \\ \text{BB}(\Theta) &= -\alpha''(\Theta) + \frac{1}{\sin \Theta} \sigma'(\Theta) - 2 \frac{\cos \Theta}{\sin \Theta} \alpha'(\Theta) + \alpha(\Theta) \end{aligned} \quad (4.35)$$

Care needs to be exercised in evaluating these expressions: each of the terms are separately infinite at $\Theta = 0$, and, moreover, the derivatives also act on the “star” terms, which changes the character of the integrand over the the source direction $\mathbf{\Omega}$. The longitudinal terms can no longer be evaluated in closed form for all Θ . Ignoring the star terms yield the expressions quoted in Eq.(4.11). These expressions are valid everywhere for the transverse modes, but are only valid for $\Theta \gg 1/(fL)$ for the longitudinal modes. Figure 4.5 compares analytic and numerical evaluations of $\alpha^{\text{VL}}(\Theta)$, $\alpha'^{\text{VL}}(\Theta)$, $\alpha''^{\text{VL}}(\Theta)$. The numerical evaluation includes the contribution from the star terms, while the analytic expressions do not. We see that star term plays an important role in determining the small angle behavior of $\alpha''^{\text{VL}}(\Theta)$. The same is true for $\alpha''(\Theta)$ and $\sigma''(\Theta)$ for all the polarization modes.

The correlation functions for the longitudinal modes are enhanced at small angles. To study the small angle behavior of the correlation functions in more detail we re-write Eq.(4.35) in terms of $\bar{\alpha}(\Theta, \mathbf{\Omega})$ and $\bar{\sigma}(\Theta, \mathbf{\Omega})$, where $\alpha(\Theta) = \int d\Omega \bar{\alpha}(\Theta, \mathbf{\Omega})$ and similarly for $\sigma(\Theta)$. The correlation functions are then given by $\text{EE}(\Theta) = \int d\Omega \text{ee}(\Theta, \mathbf{\Omega})$ and similarly for BB, where the integrands $\text{ee}(\Theta, \mathbf{\Omega})$ and $\text{bb}(\Theta, \mathbf{\Omega})$ are finite at $\Theta = 0$. Using these expressions we find several interesting results: First, the EE and BB correlation functions for the transverse traceless modes are *not* equal

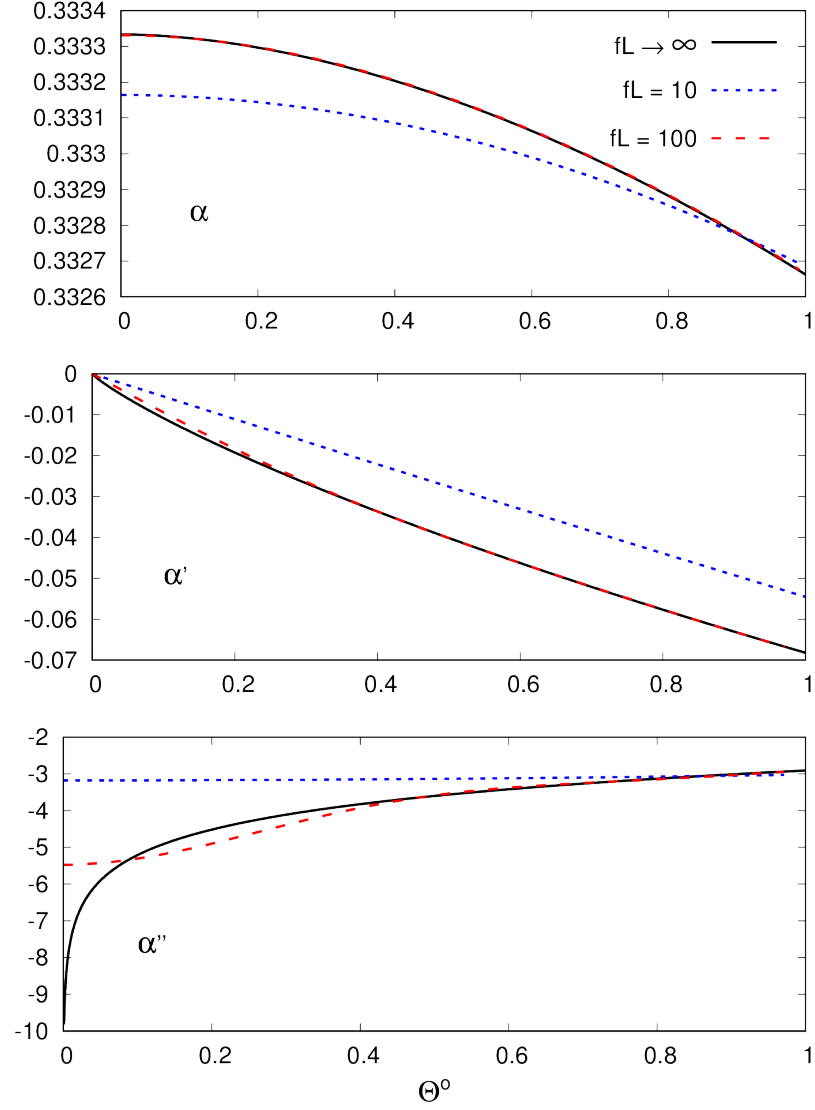


Figure 4.5: Comparing the analytic expressions without the star term for $\alpha^{\text{VL}}(\Theta)$, $\alpha'^{\text{VL}}(\Theta)$, $\alpha''^{\text{VL}}(\Theta)$ to a numerical evaluation with the star terms illustrates the differences that occur at small angles, which are especially significant for the $\alpha''^{\text{VL}}(\Theta)$ term. The star terms play an important role in determining the behavior of the $\text{EE}(\Theta)$ and $\text{BB}(\Theta)$ correlation functions for small angles. The α terms for $fL = 10$ and $fL = 100$ are compared to the expression without the star term ($fL \rightarrow \infty$). As fL becomes large, the star term is only important in a small region near $\Theta = 0$.

at small angles since

$$\begin{aligned} \text{EE}^{\text{TT}}(\Phi_s)|_{\Theta=0} &= \frac{22}{15} + \mathcal{O}\left(\frac{1}{\Phi_s^2}\right) \\ \text{BB}^{\text{TT}}(\Phi_s)|_{\Theta=0} &= \frac{8}{3} + \mathcal{O}\left(\frac{1}{\Phi_s^2}\right). \end{aligned} \quad (4.36)$$

The fact that $\text{EE}^{\text{TT}}(\Theta) \neq \text{BB}^{\text{TT}}(\Theta)$ at small angles was missed by Book and Flanagan [31] since they neglected the star terms. A similar small-angle departure from the Hellings-Downs curve occurs due to the pulsar terms [84]. Note that equality between EE and BB can not be used as a null test of general relativity at small angular separations. Second, we find that the VL correlations are enhanced by a leading factor

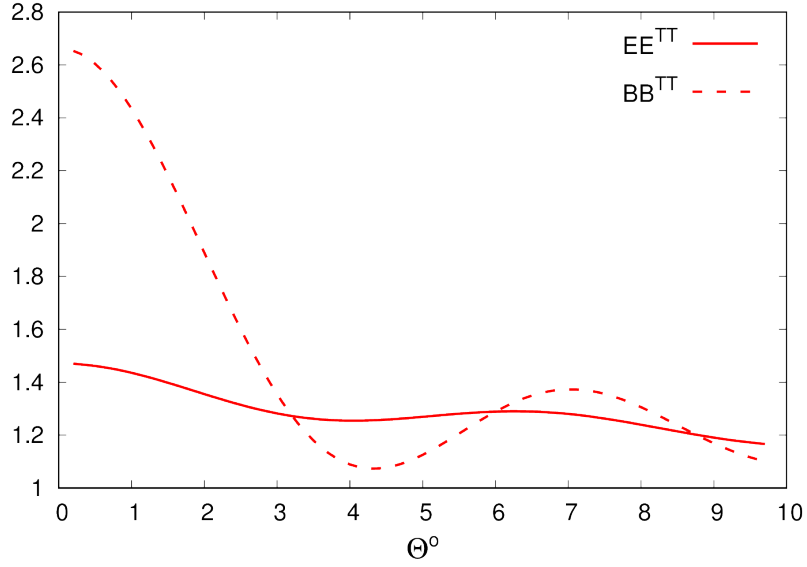


Figure 4.6: The EE and BB correlation functions for the TT mode at small angular separations. The star terms break the equality between EE and BB. The plots are shown for $fL = 10$.

that is logarithmic in fL :

$$\begin{aligned} \text{EE}^{\text{VL}}(\Phi_s)|_{\Theta=0} &= 2 \ln(2 \Phi_s) - \frac{38}{15} + 2\gamma_E + \mathcal{O}\left(\frac{1}{\Phi_s}\right) \\ \text{BB}^{\text{VL}}(\Phi_s)|_{\Theta=0} &= 2 \ln(2 \Phi_s) - \frac{14}{3} + 2\gamma_E + \mathcal{O}\left(\frac{1}{\Phi_s}\right). \end{aligned} \quad (4.37)$$

Third, the SL modes are enhanced by a leading factor that is linear in fL :

$$\begin{aligned} \text{EE}^{\text{SL}}(\Phi_s)|_{\Theta=0} &= \frac{1}{2} \left[-\frac{19}{10} + 3\gamma_E + \frac{\pi}{3} \Phi_s \right. \\ &\quad \left. - \log(2 \Phi_s) + \mathcal{O}\left(\frac{1}{\Phi_s}\right) \right], \end{aligned} \quad (4.38)$$

where γ_E is the Euler-Gamma constant. Note that as fL becomes large, the region in Θ where the star terms are important shrink as $1/(fL)$. Thus the region where the fL enhancement occurs gets smaller and smaller. For example, the scalar longitudinal EE correlation essentially becomes as step function starting at $\pi^2 fL/3$ at $\Theta = 0$ and rapidly dropping to $\sim -11/12$ for $\Theta > 1/(fL)$.

The enhanced response to longitudinal gravitational waves will allow us to place far more stringent bounds on the energy density in these modes. The majority of stars in the Gaia catalog are at distances of $L = 0.1 \rightarrow 10$ kpc [22] and the duration and cadence of the Gaia mission will allow us to probe gravitational frequencies between 3×10^{-9} Hz and 5×10^{-7} Hz [86]. Thus the typical values for fL will range between 30 and 5×10^5 , and the bounds for the SL mode in particular should be orders of magnitude tighter than for the usual TT modes of GR.

4.7 Discussion

Astrometry is a promising new approach for detecting gravitational waves [31, 68, 86]. Astrometry is particularly well suited to constraining the polarization content

of gravitational waves. The expressions for the two-point correlation functions and the E and B mode correlation patterns that we have computed here provide a starting point for the analysis of data from the Gaia mission [53]. Implementing such an analysis will be challenging: our expressions for the EE and BB correlations assumed a uniform distribution of stars on the sky, and we placed all the stars at the same distance from the Earth. In reality the Gaia's sky coverage will be non-uniform in location and distance, and techniques similar to those used in cosmic microwave background analyses will have to be used to extract the uniform correlation patterns from experiments with non-uniform sky coverage [63, 82]. The vanishing of the BB correlations for scalar polarizations and the near-equality of the EE and BB polarization modes for general relativity provides a potentially powerful null test. It will be interesting to assess how the power of this test is impacted by non-uniform sky coverage and observation noise. In addition to measuring the locations of billions of stars in our galaxy, Gaia will also measure the locations of thousands of quasars. These quasars are at vastly larger distances, making the fL dependent enhancements for the longitudinal modes much more significant. Pairs of nearby quasars, or individual strongly lensed quasars that produce multiple images, may well provide the strongest limits to the energy density of longitudinal polarization states.

Acknowledgments

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4.8 Appendix

In this appendix, we include additional work that was not included in the published manuscript.

4.8.1 Behavior of QQ mode coefficients

Following Book and Flanagan [31], we calculated the power law coefficients a_ℓ^{QQ} of the form:

$$a_\ell^Q \propto \frac{1}{\ell(\ell+1)} QQ_\ell \quad (4.39)$$

starting at $\ell = 2$, as it was said therein that modeling up to proper motion removes the information about the lower order harmonics. They normalized their coefficients equally between the EE and BB modes to unity because they neglected the star terms. We comment not on our normalization as it will depend on the relative amplitudes of the alternative modes, and since BB modes, where permitted, are not equal to EE modes when the star term is non-negligible, their respective contributions to the coefficients will not be equal. However, we comment on some features of the power laws for each polarization.

Figure 4.7 shows the EE and BB coefficients for the TT mode, both numerically and approximately without the star term including power law fit. We can see that the higher order harmonics deviate from the power law, the interpretation being that the small scale oscillations observed in Figure 4.6 are picked up in the integration of Eq.(4.31). The BB correlation oscillates more appreciably than EE at low angular separation, so the higher harmonics deviate more than EE for the TT mode. It would be expected that as we increase fL , the numerical expression relax down onto the power law.

Figure 4.8 similarly shows the EE coefficients for the VL mode; the BB

coefficients are not appreciably different for these harmonics, so they are omitted. Note the power law exponent is less steep than the TT case.

If we start at $\ell = 2$, then both scalar EE correlation coefficients vanish if we utilize their approximate forms. However, there is clearly large scale structure present in the SL EE at low angular separation. Figure 4.9 shows how the lowest harmonics pick out this structure, but the higher harmonics quickly drop to zero. If we increase fL in this case, the steep drop observed in this figure would manifest at lower harmonics.

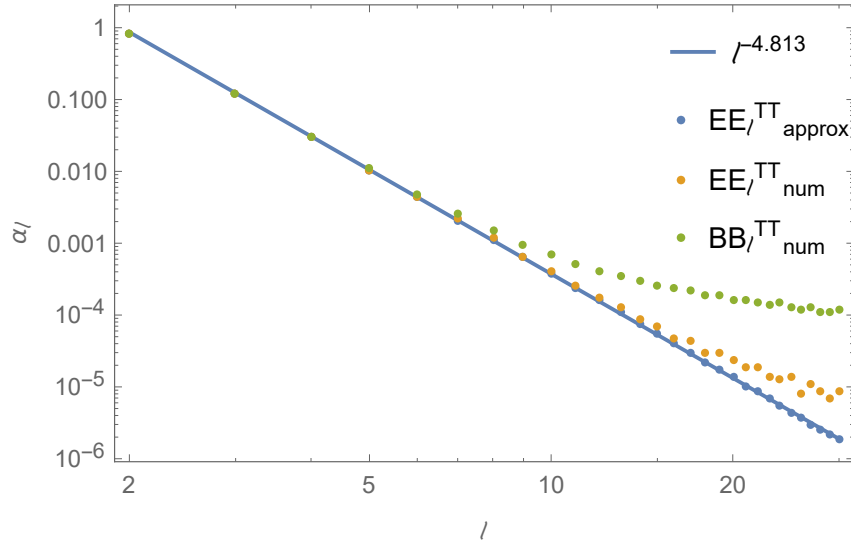


Figure 4.7: The EE and BB correlation coefficients a_ℓ^Q for the TT mode, evaluated at $fL = 10$, as a function of the harmonic index, ℓ . The power law of exponent -4.813 is the value of a best fit line.

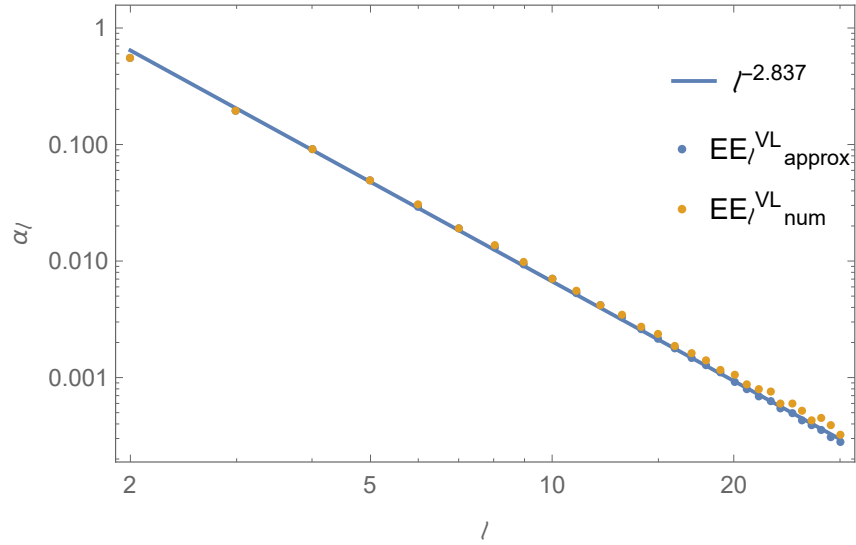


Figure 4.8: The EE correlation coefficients a_ℓ^Q for the VL mode, evaluated at $fL = 10$, as a function of the harmonic index, ℓ . The power law of exponent -2.837 is the value of a best fit line.

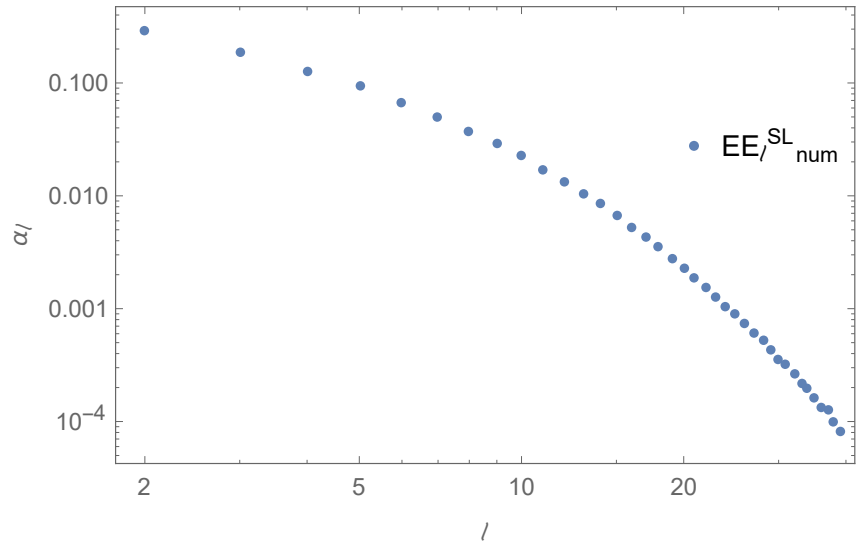


Figure 4.9: The EE correlation coefficients a_ℓ^Q for the SL mode, evaluated at $fL = 10$, as a function of the harmonic index, ℓ .

CHAPTER FIVE

CONSTRAINING ALTERNATIVE POLARIZATIONS OF CONTINUOUS
GRAVITATIONAL WAVES USING PULSAR TIMING ARRAYS5.1 Contribution of Authors and Co-Authors

Author: Logan O’Beirne

Contributions: Modified original continuous wave code, ran analyses, created plots, and wrote manuscript.

Co-Author: Neil J. Cornish

Contributions: Provided notes on the gravitational wave response and calculations for priors, helped develop the analysis formalism, and helped write and edit the manuscript.

Co-Author: Sarah J. Vigeland

Contributions: Provided original continuous wave code to modify, ran fixed-sky location upper limit runs and provided their plots.

Co-Author: Stephen R. Taylor

Contributions: Contributing author of enterprise software used to perform the analysis and consulted.

5.2 Manuscript Information

Logan O’Beirne, Neil J. Cornish, Sarah J. Vigeland, Stephen R. Taylor

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ABSTRACT

Pulsar timing arrays are sensitive to gravitational wave perturbations produced by individual supermassive black hole binaries during their early inspiral phase. Modified gravity theories allow for the emission of gravitational dipole radiation, which is enhanced relative to the quadrupole contribution for low orbital velocities, making the early inspiral an ideal regime to test for the presence of modified gravity effects. Using a theory-agnostic description of modified gravity theories based on the parametrized post-Einsteinian framework, we explore the possibility of detecting deviations from General Relativity using simulated pulsar timing array data, and provide forecasts for the constraints that can be achieved. We generalize the **enterprise** pulsar timing software to account for possible additional polarization states and modifications to the phase evolution, and study how accurately the parameters of simulated signals can be recovered. We find that while a pure dipole model can partially recover a pure quadrupole signal, there is little possibility for confusion when the full model with all polarization states is used. With no signal present, and using noise levels comparable to those seen in contemporary arrays, we produce forecasts for the upper limits that can be placed on the amplitudes of alternative polarization modes as a function of the sky location of the source.

5.3 Introduction

The dark energy and dark matter problems in cosmology and the unresolved reconciliation between General Relativity (GR) and quantum mechanics suggest that Einstein's theory of gravity is incomplete [28]. Gravitational wave (GW) astronomy provides a new arena to search for deviations from GR. One smoking gun signature would be the detection of additional polarization states. Many theories that violate the strong equivalence principle or Lorentz invariance allow for the emission of dipole radiation [34, 107, 118]. To search for this signature it is best to observe binary systems that are widely separated since deviations from pure quadrupole emission are enhanced for low velocity systems [34]. It is also advantageous to measure multiple independent projections of the polarization pattern [34].

The GW detections that have been made by the LIGO-Virgo instruments are of the high velocity final inspiral and merger phase, where there is only a small amplification of the dipole/tensor ratio. Moreover, there are presently a limited number of ground-based detectors, providing a limited number of projections of the radiation field, so it is difficult to decipher the polarization pattern [5, 9, 12, 34]. Pulsar timing arrays are well suited to constraining dipole emission since they observe in a frequency band where supermassive black hole binaries will be moving relatively slowly, and with dozens of pulsars in the array, they provide multiple projections of the radiation field.

Rapidly rotating neutron stars, known as pulsars, emit beams of electromagnetic radiation that are observed as radio pulses when the beam sweeps across the Earth. Millisecond pulsars that have been spun up due to accretion act as very stable clocks

whose pulse phases are known to high precision [66], allowing astronomers to search for slight perturbations in the times of arrival (TOA) of radio pulses caused by low frequency GWs [42, 48]. A collection of these comprise a pulsar timing array (PTA), a galactic scale GW detector. There are currently three distinct PTAs operating around the world [41, 64, 80] whose combined efforts comprise the International Pulsar Timing Array [105].

PTAs observe frequencies of approximately $10^{-9} - 10^{-7}$ Hz, and it is believed that the dominant source of GWs in this frequency band is produced by a population of supermassive black hole binaries (SMBHBs) in their slow, adiabatic inspiral phase [67, 98, 99]. Modeling suggests that the ensemble signal from multiple binary systems will be detect first, followed by the signals from the loudest individual systems [93]. PTAs probe a regime well before SMBHBs merge, where the systems have orbital velocities of order $v/c \sim 10^{-2} - 10^{-1}$, and where any dipole radiation will be enhanced by a factor of $10 - 100$ relative to the quadrupole.

Here we study how the signals from individual SMBHBs can be used to constrain alternative theories of gravity. We use the model independent parametrized post-Einsteinian formulation [34, 116] of modified gravitational theories to model simulated signals from individual SMBHBs that include all polarizations allowed by a general metric theory of gravity. We then use Bayesian inference to study the signals from simulated pulsar timing data sets. We explore how well the system parameters can be recovered, and the upper limits that can be placed when no signal is present in the data.

In Section 5.4, we describe the post-Einsteinian signal model and make comparisons with the GR model. Section 5.5 outlines the data generation and analysis methods. Section 5.6 explores how well the model parameters can be recovered from simulated data, and in the absence of a signal, how the upper limits on the amplitudes

of each polarization mode will depend on sky location. In Section 5.7 we present our conclusions. Throughout this paper, we use units where $G = c = 1$.

5.4 Signal model

Pulsar timing arrays encode GWs in the timing residuals, which are found by subtracting the timing model from the raw arrival times¹. A single pulsar's timing residual δt can be written:

$$\delta t = \mathbf{M} \cdot \delta \boldsymbol{\xi} + n + s, \quad (5.1)$$

where $\mathbf{M} \cdot \delta \boldsymbol{\xi}$ describes uncertainties in the timing model [45–47, 104], n is the white noise, and s is the GW signal. We omit red noise in our simulations as including it in the noise model significantly slows down the likelihood evaluations. Leaving out the red noise results in somewhat optimistic predictions for the signal extraction capabilities and the strength of the upper limits.

For a gravitational wave propagating in the $\boldsymbol{\Omega}$ direction we can introduce the orthonormal coordinate system

$$\begin{aligned} \boldsymbol{\Omega} &\rightarrow (-\sin \theta \cos \phi, -\sin \theta \sin \phi, -\cos \theta) \\ \mathbf{u} &\rightarrow (\sin \phi, -\cos \phi, 0) \\ \mathbf{v} &\rightarrow (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta), \end{aligned} \quad (5.2)$$

These are related to the principle axes $\mathbf{m}$, $\mathbf{n}$ of the source by a rotation angle ψ (the

¹The timing model includes relativistic effects such as the Shapiro time delay and Einstein delay, and these effects are modified in alternative theories of gravity. However, existing solar system constraints on these post-Newtonian effects, which scale with the PPN parameter γ , are of order pico-seconds [95], so we can safely use the standard GR timing model in our analysis

polarization angle) about the propagation direction:

$$\begin{aligned}\mathbf{m} &= \mathbf{u} \cos \psi + \mathbf{v} \sin \psi \\ \mathbf{n} &= -\mathbf{u} \sin \psi + \mathbf{v} \cos \psi,\end{aligned}\tag{5.3}$$

The basis tensors for the various gravitational wave polarization states are then:

$$\begin{aligned}\epsilon_{\text{TT}}^+ &= \mathbf{u} \otimes \mathbf{u} - \mathbf{v} \otimes \mathbf{v} = \cos(2\psi)\mathbf{e}^+ - \sin(2\psi)\mathbf{e}^\times \\ \epsilon_{\text{TT}}^\times &= \mathbf{u} \otimes \mathbf{v} + \mathbf{v} \otimes \mathbf{u} = \sin(2\psi)\mathbf{e}^+ + \cos(2\psi)\mathbf{e}^\times \\ \epsilon_{\text{ST}}^\odot &= \mathbf{u} \otimes \mathbf{u} + \mathbf{v} \otimes \mathbf{v} = \mathbf{m} \otimes \mathbf{m} + \mathbf{n} \otimes \mathbf{n} \\ \epsilon_{\text{VL}}^u &= \mathbf{u} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{u} = \cos(\psi)\mathbf{e}^u - \sin(\psi)\mathbf{e}^v \\ \epsilon_{\text{VL}}^v &= \mathbf{v} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{v} = \sin(\psi)\mathbf{e}^u + \cos(\psi)\mathbf{e}^v \\ \epsilon_{\text{SL}}^{\leftrightarrow} &= \boldsymbol{\Omega} \otimes \boldsymbol{\Omega},\end{aligned}\tag{5.4}$$

where

$$\begin{aligned}\mathbf{e}^+ &= \mathbf{m} \otimes \mathbf{m} - \mathbf{n} \otimes \mathbf{n} \\ \mathbf{e}^\times &= \mathbf{m} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{m} \\ \mathbf{e}^\odot &= \epsilon_{\text{ST}}^\odot \\ \mathbf{e}^u &= \mathbf{m} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{m} \\ \mathbf{e}^v &= \mathbf{n} \otimes \boldsymbol{\Omega} + \boldsymbol{\Omega} \otimes \mathbf{n} \\ \mathbf{e}^{\leftrightarrow} &= \epsilon_{\text{SL}}^{\leftrightarrow},\end{aligned}\tag{5.5}$$

and the subindices are labeled for the 2 tensor transverse (TT) modes of General Relativity (+ and $\times$), a scalar transverse (ST) “breathing” mode ($\odot$), 2 vector

longitudinal (VL) modes (u and v), and a scalar longitudinal (SL) mode ($\leftrightarrow$). We refer to the latter four as alternative polarizations (alt-pols). The timing residuals induced by polarization state A for a pulsar in the $\mathbf{p}$ direction is given by

$$r^A(t_e) = \frac{1}{2(1 + \boldsymbol{\Omega} \cdot \mathbf{p})} \mathbf{p} \otimes \mathbf{p} : (\mathbf{H}^A(t_e) - \mathbf{H}^A(t_p)), \quad (5.6)$$

where $t_p = t_e - L(1 + \boldsymbol{\Omega} \cdot \mathbf{p})$, t_e is the time at Earth, L is the distance to the pulsar and $\mathbf{H}^A = \int^t \mathbf{h}^A dt$ is the anti-derivative of the gravitational wave strain. Note that there are contributions from the presence of GWs at the pulsar and the Earth, indicated in Eq.(5.6). We assume all modes travel at the speed of light. However, Lorentz-violating and massive gravity allow for superluminal [25,26,44,51,59,110] and subluminal propagation [23,28,69,106,117] of non-Einsteinian modes, respectively. Superluminal modes decrease the effective luminosity distance to the binary and the pulsar frequency in that respective alt-pol's response. This would be an interesting extension to the current study, but we do not anticipate that the upper limits would be change significantly in this case as it only impacts the pulsar terms, which are less constraining than the Earth term due to uncertainties in the pulsar distances. In the case of massive gravity, the resulting dispersion relationship makes the analysis appreciably more complicated and is beyond the scope of this study.

We define the frequency dependent antenna patterns as

$$F^A = \left[\frac{|e^{-2\pi i f L(1 + \boldsymbol{\Omega} \cdot \mathbf{p})} - 1|}{2(1 + \boldsymbol{\Omega} \cdot \mathbf{p})} \right] (\mathbf{e}^A : \mathbf{p} \otimes \mathbf{p}). \quad (5.7)$$

The frequency dependent prefactor in square brackets oscillates rapidly for large fL , and Ref. [46] argued that the oscillating term should be dropped. Here we are required to include this factor to avoid singularities in the longitudinal antenna patterns when the pulsar is in the same direction as the source: $1 + \boldsymbol{\Omega} \cdot \mathbf{p} = 0$. Furthermore, as

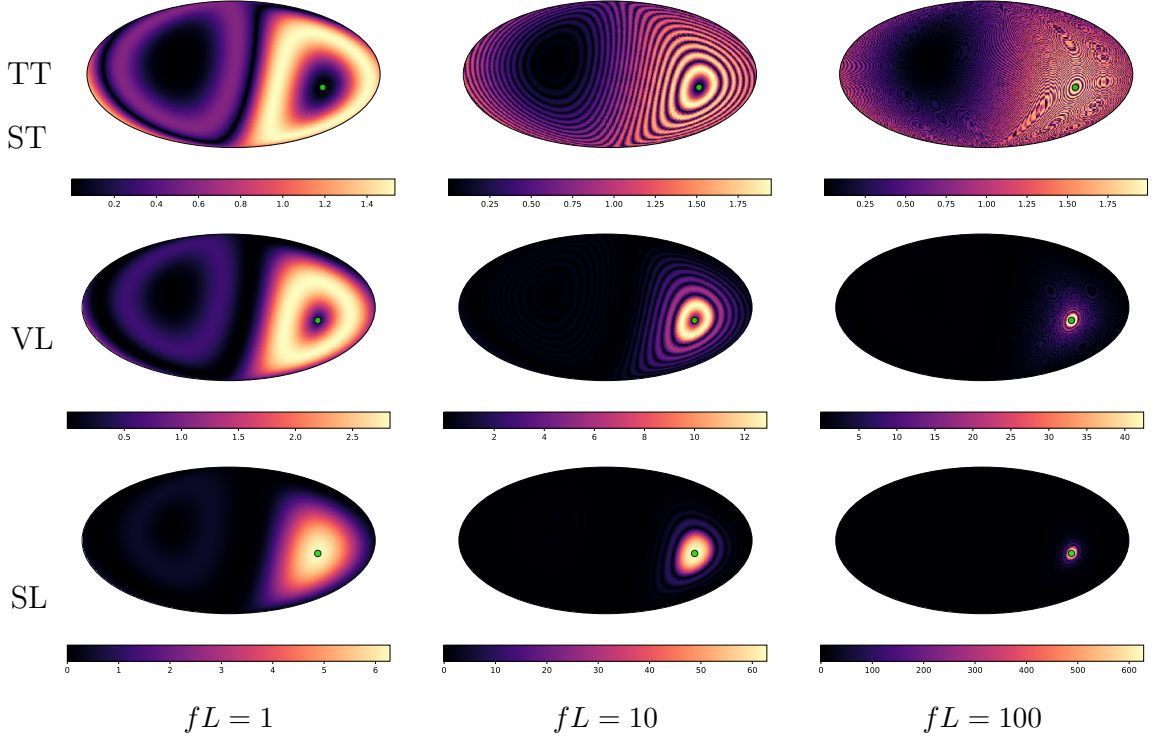


Figure 5.1: GW antenna pattern sky maps. The color gradient indicates how sensitive the response is to a GW based on the pulsar’s sky position. The GW is originating from green circle. From left to right, the columns correspond to values of $fL = 1, 10$, and 100 , respectively. From top to bottom, the polarizations are the rms of TT/ST, VL, and SL, respectively.

discussed in Ref. [35], its inclusion improves sky localization. Figure 5.1 shows the sky maps of various polarization antenna patterns at different values of fL – the number of gravitational wavelengths to the pulsar. For the longitudinal modes there is increased sensitivity in the direction of propagation [16, 33], proportional to δfL for the VL mode and fL for the SL mode where δ is the small angle subtended by the pulsar and GW propagation direction. The transverse modes are not significantly enhanced since the response scales as $\delta^2 fL$. The enhanced response for pulsars near the line of sight to the GW source increases with increasing fL , as can be seen going from left to right along the lower two rows in Figure 5.1. It is worth noting that

the transverse modes excite zero response in pulsars in exactly the same direction as the source, or in the antipodal direction, while the SL mode excites no response in pulsars that are oriented perpendicular to the line of sight to the source. Similarly, the VL modes excite no response in pulsars in towards or antipodal to the source, or in those perpendicular to the line of sight. All modes are less sensitive when the pulsar is the opposite direction to the GW source. To reiterate, the GW response to all polarization modes is largest when the pulsar is in almost the same sky direction as the source. The longitudinal modes have an enhanced response relative to the transverse modes in this respect, but the transverse modes have better sky coverage. Depending on where the GW source is located with respect to pulsars in the array, there is potential to have tighter constraints on the strains of longitudinal modes, or for the longitudinal modes to be completely undetectable to the array.

For the GW residuals in GR we have [35]

$$\begin{aligned}
\mathbf{H}^+ &= \mathbf{e}^+ \frac{\mathcal{M}^{5/3}}{d_L \omega^{1/3}} (1 + \cos^2 \iota) \cos \left(2 \int^t \omega dt + 2\Phi_0 \right) \\
&\quad \times (1 + \mathcal{O}(\dot{\omega}/\omega^2)), \\
\mathbf{H}^\times &= \mathbf{e}^\times \frac{2\mathcal{M}^{5/3}}{d_L \omega^{1/3}} \cos \iota \sin \left(2 \int^t \omega dt + 2\Phi_0 \right) \\
&\quad \times (1 + \mathcal{O}(\dot{\omega}/\omega^2)),
\end{aligned} \tag{5.8}$$

where d_L is the luminosity distance, $\mathcal{M}$ is the chirp mass, Φ_0 is the initial orbital phase of the binary, ω is the orbital angular frequency, and ι is the angle of inclination of the binary.

Note that the expression for the anti-derivative of the gravitational wave amplitude assumes that we are working in the slow-evolution limit. We can evaluate

the size of the errors that this introduces:

$$\mathbf{H}(t) = \int^t h(t) dt = \int^{\Phi(t)} \frac{A(\Phi)}{\omega(\Phi)} \sin \Phi d\Phi, \quad (5.9)$$

so that

$$\begin{aligned} \mathbf{H}(t) = \frac{A(\Phi)}{\omega(\Phi)} \cos \Phi + \left(\frac{A(\Phi)}{\omega(\Phi)} \right)' \sin \Phi \\ - \left(\frac{A(\Phi)}{\omega(\Phi)} \right)'' \cos \Phi + \dots, \end{aligned} \quad (5.10)$$

where the primes denote derivatives with respect to Φ . Note that $d/d\Phi = \omega^{-1}d/dt$.

The ratio of the second order term to the first order term is given by

$$\left[\ln \left(\frac{A_p}{\omega_p} \right) \right]' = -\frac{\dot{\omega}}{3\omega^2}. \quad (5.11)$$

In GR this is given to leading PN order by

$$\begin{aligned} \left[\ln \left(\frac{A_p}{\omega_p} \right) \right]' &= -\frac{32}{5} \mathcal{M}^{5/3} \omega^{5/3} \\ &= -3.8 \times 10^{-2} \left(\frac{\mathcal{M}}{10^{10} M_\odot} \right)^{5/3} \left(\frac{f_{\text{GW}}}{3 \times 10^{-7} \text{Hz}} \right)^{5/3}, \end{aligned} \quad (5.12)$$

which validates the dropping of higher order corrections in PTA analyses.

In the current NANOGrav analysis for continuous wave sources [14], upper limits are quoted on h_{TT} as a function of the TT-mode frequency $2\omega_0$, where ω_0 is the orbital frequency as measured at the Earth. In producing the upper limits the signal model is marginalized over the GW parameters

$$\vec{\lambda} \rightarrow (\theta, \phi, \Phi_0, \psi, \iota, \mathcal{M}, h_{\text{TT}}) \quad (5.13)$$

and the pulsar distances L_i . To allow for alternative theories of gravity, we need to enlarge the parameter set to

$$\vec{\lambda} \rightarrow (\theta, \phi, \Phi_0, \psi, \iota, \alpha_D, \alpha_Q, h_{\text{TT}}, h_{\text{ST}}, h_{\text{VL}}, h_{\text{SL}}) \quad (5.14)$$

where $h_{\text{ST}}, h_{\text{VL}}, h_{\text{SL}}$ are the amplitudes of the additional polarization modes, and the parameter α_D, α_Q scale the dipole and quadrupole contribution to the frequency evolution:

$$\frac{d\omega}{dt} = \alpha_D \omega^3 + \alpha_Q \omega^{11/3} \quad (5.15)$$

We have neglected higher order terms in the frequency evolution since they are negligible for slowly moving sources. The GR limit is recovered by setting $\alpha_D = 0$ and $\alpha_Q = \frac{96}{5} \mathcal{M}^{5/3}$. The wave tensors are given by [34, 59]

$$\begin{aligned} \mathbf{H}^+ &= \mathbf{e}^+ \frac{(1 + \cos^2 \iota)}{2} \frac{h_{\text{TT}}}{\omega} \cos(2\omega t + 2\Phi_0) \\ \mathbf{H}^\times &= \mathbf{e}^\times \cos \iota \frac{h_{\text{TT}}}{\omega} \sin(2\omega t + 2\Phi_0) \\ \mathbf{H}^\odot &= \mathbf{e}^\odot \sin \iota \frac{h_{\text{ST}}}{\omega} \cos(\omega t + \Phi_0) \\ \mathbf{H}^u &= \mathbf{e}^u \cos \iota \frac{h_{\text{VL}}}{\omega} \cos(\omega t + \Phi_0) \\ \mathbf{H}^v &= \mathbf{e}^v \frac{h_{\text{VL}}}{\omega} \sin(\omega t + \Phi_0) \\ \mathbf{H}^{\leftrightarrow} &= \mathbf{e}^{\leftrightarrow} \sin \iota \frac{h_{\text{SL}}}{\omega} \cos(\omega t + \Phi_0), \end{aligned} \quad (5.16)$$

where

$$\begin{aligned}
h_{\text{TT}} &= \frac{2\mathcal{M}}{d_L}(\mathcal{M}\omega)^{2/3} \\
h_{\text{ST}} &= \alpha_{\text{ST}} \frac{\mathcal{M}}{d_L}(\mathcal{M}\omega)^{1/3} \\
h_{\text{VL}} &= \alpha_{\text{VL}} \frac{\mathcal{M}}{d_L}(\mathcal{M}\omega)^{1/3} \\
h_{\text{SL}} &= \alpha_{\text{SL}} \frac{\mathcal{M}}{d_L}(\mathcal{M}\omega)^{1/3}
\end{aligned} \tag{5.17}$$

and $\alpha_{\text{ST},\text{VL},\text{SL}}$ are dimensionless couplings coefficients, which we treat as independent. Our h_{TT} is equivalent to h_0 in Eq.(20) of Ref. [14]. Note that we have neglected higher order post-Newtonian corrections to the amplitude. The gravitational coupling strength can be modified in alternative theories of gravity, but here we maintain the $G = 1$ scaling and absorb any changes via the coupling coefficients $\alpha_{\text{ST},\text{VL},\text{SL}}$.

We work on the assumption that any alternative polarization states are dominated by dipole emission, which is to be expected unless special symmetries eliminate the dipole contribution. We have also assumed that the tensor and vector waves are elliptically polarized, which should be the case for the leading order emission from a circular binary. Note that we can define h_{TT} to be positive, but we have to allow h_{ST} , h_{VL} , and h_{SL} to range over negative and positive values as the dipole charges can be negative or positive. We can use the data to derive bounds on the absolute values of the amplitudes.

In the GR case of Eq.(5.6), degeneracies exist in the timing residuals: $r^{GR}(\psi, \Phi_0, \Phi_{p,0}) = r^{GR}(\psi + \pi/2, \Phi_0 + \pi/2, \Phi_{p,0} + \pi/2)$, and trivially $r^{GR}(\psi) = r^{GR}(\psi + \pi)$ and $r^{GR}(\Phi_0, \Phi_{p,0}) = r^{GR}(\Phi_0 + \pi, \Phi_{p,0} + \pi)$. If the pulsar terms are considered unimportant noise, the transformation further simplifies. Similar degeneracies exist in our parameterization, namely $r(\psi, h_{VL}) = r(\psi + \pi, -h_{VL})$, and $r(\Phi_0, \Phi_{p,0}, h_{ST}, h_{SL}, h_{VL}) =$

$r^{GR}(\Phi_0 + \pi, \Phi_{p,0} + \pi, -h_{ST}, -h_{SL}, -h_{VL})$. The standard analysis in the GR case exploits these mappings to restrict the prior ranges on the polarization and phase parameters. Here we are permitted to do the same, so long as we include the sign freedom in the strain amplitudes.

Ideally we would choose priors on the source parameters that are similar to those used in the standard GR analysis, but this is difficult to do since the dipole radiation introduces additional terms into the frequency evolution. To cover a large range of possibilities we adopt scale-invariant priors that are log uniform in α_D and α_Q . In the GR case priors on the chirp mass $\mathcal{M}$ translate directly into priors on α_Q . The additional polarization modes will contribute to the quadrupole emission so the mapping is modified in a theory-dependent fashion. There is less guidance on what prior bounds to use for α_D . One way to set boundaries on the prior range for these parameters is to impose self-consistency conditions. Our model assumes that the signals do not evolve significantly during the duration of the observation, which implies that $\dot{\omega}T_{\text{obs}}^2 \ll 1$. The problem here is that even in the GR limit the self-consistency relation can be violated for the most massive systems at high frequencies. Setting $\dot{\omega}T_{\text{obs}}^2 = 1$ in the GR limit, we get

$$\mathcal{M}_{\text{max}} = 4 \times 10^7 M_{\odot} \left(\frac{3 \times 10^{-7} \text{Hz}}{f_{\text{max}}} \right)^{11/5} \left(\frac{10 \text{yr}}{T_{\text{obs}}} \right)^{6/5} \quad (5.18)$$

Turning this around and using a minimum mass chirp mass of $\mathcal{M} = 10^8 M_{\odot}$, the lowest value considered in the NANOGrav analyses, we see that the no-chirp condition is violated at $f = 1.9 \times 10^{-7} \text{Hz}$ for the lowest mass systems. For the highest mass systems with $\mathcal{M} = 10^{10} M_{\odot}$, the no-chirp condition is violated at $f = 2.4 \times 10^{-8} \text{Hz}$. Note that we are just requiring that the signal moves less than a frequency bin during the observations. The criteria really should be some small fraction of a bin.

Imposing the bound at highest frequencies effectively limits the allowed chirp masses at all frequencies. The alternative is to change the model and allow for frequency evolution, at least during the pulsar-to-Earth pulse travel time. Then we need to integrate $\omega(t)$ with respect to time, which can be done by recasting the integrand to the following form:

$$\begin{aligned}
\Phi(t) - \Phi_0 &= \int_{t_0}^t \omega(t) dt = \int_{\omega_0}^{\omega(t)} \frac{d\omega}{\alpha_D \omega^2 + \alpha_Q \omega^{8/3}} \\
&= 3 \frac{\alpha_Q^{3/2}}{\alpha_D^{5/2}} \left(\tan^{-1} \left(\sqrt{\frac{\alpha_Q \omega(t)^{2/3}}{\alpha_D}} \right) - \tan^{-1} \left(\sqrt{\frac{\alpha_Q \omega_0^{2/3}}{\alpha_D}} \right) \right) \\
&\quad + \frac{1}{\alpha_D} \left(\frac{1}{\omega_0} - \frac{1}{\omega(t)} \right) + \frac{3 \alpha_Q}{\alpha_D^2} \left(\frac{1}{\omega(t)^{1/3}} - \frac{1}{\omega_0^{1/3}} \right). \tag{5.19}
\end{aligned}$$

It is easier to understand the dipole correction in the PN framework when we take the limit $\alpha_D \rightarrow 0$ such that $\alpha_D \ll \alpha_Q \omega_0^{2/3}$, which to first order is:

$$\begin{aligned}
\Phi(t) - \Phi_0 &\approx \frac{3}{5 \alpha_Q} \left(\frac{1}{\omega_0^{5/3}} - \frac{1}{\omega(t)^{5/3}} \right) - \\
&\quad \frac{3 \alpha_D}{7 \alpha_Q^2} \left(\frac{1}{\omega_0^{7/3}} - \frac{1}{\omega(t)^{7/3}} \right) \tag{5.20}
\end{aligned}$$

We see how this corresponds to the GR case with a small correction. Unfortunately, the dipole term also makes $\omega(t)$ a transcendental function of time

$$\begin{aligned}
t - t_0 &= \int_{\omega_0}^{\omega(t)} \frac{d\omega}{\alpha_D \omega^3 + \alpha_Q \omega^{11/3}} \\
&= \frac{1}{4 \alpha_D^4} \left[2 \alpha_Q^3 \left(2 \ln \left(\frac{\omega_0}{\omega(t)} \right) + 3 \ln \left(\frac{\alpha_D + \alpha_Q \omega(t)^{2/3}}{\alpha_D + \alpha_Q \omega_0^{2/3}} \right) \right. \right. \\
&\quad \left. \left. 2 \alpha_D^3 \left(\frac{1}{\omega_0^2} - \frac{1}{\omega(t)^2} \right) + 3 \alpha_Q \alpha_D^2 \left(\frac{1}{\omega(t)^{4/3}} - \frac{1}{\omega_0^{4/3}} \right) \right. \right. \\
&\quad \left. \left. + 6 \alpha_Q^2 \alpha_D \left(\frac{1}{\omega_0^{2/3}} - \frac{1}{\omega(t)^{2/3}} \right) \right) \right] \tag{5.21}
\end{aligned}$$

With the full evolution included, it is less clear how we should choose maximum values for α_D , α_Q . One extreme might be to demand that the systems do not merge during the observation time, which we can define as when the Earth term frequency becomes infinite during the observation time:

$$T_{\text{merge}} = \frac{3\alpha_Q^3}{2\alpha_D^4} \left[\ln \left(\frac{\alpha_Q \omega_0^{2/3}}{\alpha_D + \alpha_Q \omega_0^{2/3}} \right) \right] + \frac{1}{2\alpha_D \omega_0^2} - \frac{3\alpha_Q}{4\alpha_D^2 \omega_0^{4/3}} + \frac{3\alpha_Q^2}{2\alpha_D^3 \omega_0^{2/3}} \quad (5.22)$$

In the context of GR, this corresponds to the condition $T_{\text{merge}} < T_{\text{obs}}$ where

$$\begin{aligned} T_{\text{merge}}^{GR} &= \frac{5}{256} \mathcal{M}^{-5/3} \omega^{-8/3} \\ &= 2 \text{ years} \left(\frac{10^{10} M_\odot}{\mathcal{M}} \right)^{-5/3} \left(\frac{10^{-7} \text{ Hz}}{f} \right)^{-8/3} \end{aligned} \quad (5.23)$$

More generally we can define $T_{\text{chirp}} = \omega/\dot{\omega}$. This quantity is similar to T_{merge} but is easier to compute for modified theories (in GR, $T_{\text{chirp}} = \frac{8}{3} T_{\text{merge}}$). Treating the dipole and quadrupole extremes separately we have the limits

$$\begin{aligned} \alpha_D &< \frac{1}{\omega_0^2 T_{\text{obs}}} \\ \alpha_Q &< \frac{1}{\omega_0^{8/3} T_{\text{obs}}} \end{aligned} \quad (5.24)$$

Here ω_0 is the initial orbital angular frequency at the Earth. The merger time is related to the chirp time by a factor less than unity, so we multiply Eq.(5.24) by a factor of one tenth to define the no-merger condition, which then defines the upper bounds on α_D and α_Q . For the lower limits we can choose values that we know apriori produce un-observable frequency changes: $\dot{\omega} L T_{\text{obs}} \ll 1$. Treating the dipole

and quadrupole extremes separately, we have the limits

$$\begin{aligned}\alpha_D &> \frac{1}{\omega_0^3 T_{\text{obs}} L} \\ \alpha_Q &> \frac{1}{\omega_0^{11/3} T_{\text{obs}} L}\end{aligned}\tag{5.25}$$

In other words, the GW becomes effectively monochromatic, with the pulsar frequency roughly equal to the Earth term frequency. We will need to use priors that depend on the Earth term frequency. Note that maximum values are a factor of $\omega_0 L$ larger than the minimum values. Consequently the prior range on the frequency evolution will be very different from the GR case, and this will impact the the upper limits. Note that depending on the choice of the radiative-loss coupling and the GW frequency, it is possible that just the pulsar term or just the Earth term falls in the observation band.

Since the distance to each pulsar L_i is not known to high precision, we marginalize over the distance to each pulsar. In principle the orbital phase seen at each pulsar, Φ_i , is determined by the time delay $L_i(1 + \boldsymbol{\Omega} \cdot \mathbf{p})$, but since the L_i are not well constrained we get phase wrapping in the pulsar signals that makes it very difficult to marginalize over the L_i . One way around this is to introduce independent phase terms Φ_i for each pulsar [35], and only keeping the L_i dependence in the pulsar frequencies. In effect this splits the pulsar distance into two parts, a large part on the order of 1 kpc, and a small correction of order $2\pi/\omega \sim 1$ pc.

5.5 Data Analysis Methods

To simulate the pulsar timing data, we used the `libstempo`² package `toasim.py` to generate the timing residuals for each pulsar. Using the 11 year data release pulsars [19] with ephemeris DE435 and the `fake_pulsar` function, we created a mock pulsar data set with an 11 year observation time at a 30 day cadence with random 1 day offsets to mimic the irregularity of pulsar observations. For all realizations, the TOA uncertainty assigned to all pulsars was $\sigma_{\text{TOA}} = 0.5\mu\text{s}$. We then added white noise of $\text{EFAC} = 1$ and $\sigma_{\text{EQUAD}} = 100\text{ns}$, where the total rms white noise is defined as $\sigma^2 = (\text{EFAC})^2\sigma_{\text{TOA}}^2 + \sigma_{\text{EQUAD}}^2$. To the pulsar distances we added a random Gaussian variate, proportional to its cataloged uncertainty. For pulsars whose distance is not well known, we used a distance of 1 kpc and assigned a 20% error in the distance. We then added a random uniform component proportional to the GW wavelength relative to the pulsar distance in order properly de-phase the pulsar and Earth terms.

For simulations with a GW signal, we used a modified version of the function `create_cw` to generate a continuous wave signal based on the model outlined in Section 5.4. We used a fixed quadrupolar GW frequency $f_{\text{GW}}^{\text{TT}} = 1 \times 10^{-8}\text{Hz}$ to ensure both the quadrupolar and dipolar signatures appear in the observation band at roughly the same sensitivity. This corresponds to an initial Earth term orbital frequency $\omega_0 = \pi f_{\text{GW}}^{\text{TT}}$. For all simulations we chose $\psi = \pi/4$, $\Phi_0 = \pi/4$, and $\cos \iota = 0.5$.

Ignoring the timing model for now and considering only white noise, the total

²<https://github.com/vallis/libstempo>

SNR of an alt-pol signal injection is equal to [93]:

$$\begin{aligned} \text{SNR}_{\text{inj}}^2 &= \frac{2}{S_n} \sum_i^{N_{\text{puls}}} \int_0^{T_{\text{obs}}} dt \left(\sum_A r_i^A(t_e) \right)^2 \\ &\equiv \frac{1}{\sigma^2} \sum_i^{N_{\text{puls}}} \sum_n^{N_{\text{TOA}}} \left(\sum_A r_i^A(t_n) \right)^2 \end{aligned} \quad (5.26)$$

where $r_i^A(t_e)$ is defined by Eq.(5.6), t_n is the n th TOA, and $S_n = 2\sigma^2\Delta t$, where Δt is the cadence. The sums are over the polarizations and pulsars. We normalize our injections using this definition, with each mode contributing roughly equal SNR^2 ; however, it is difficult to gauge what constitutes equal considering the complicated form of Eq.(5.26) when substituting in Eq.(5.6). We simply choose a target SNR and find the values of the amplitudes, individually, that achieve this target for a given sky location. We then add all the amplitudes together and rescale them to achieve the target SNR. Note that the normalization will be dependent on the configuration of the array, particularly with respect to longitudinal modes. If a SL longitudinal signal were directly behind a particular pulsar, virtually all SNR information is contained in that pulsar's residuals per our normalization procedure. If there were no enhancement but there were still many pulsars near the GW source, then the longitudinal amplitudes would be determined by the collective SNR of those pulsars, on the same order as transverse modes. If we had pulsars only in the sky region opposite to the GW source, the response would be reduced, requiring us to inject an appreciably louder signal.

We should emphasize that while we have normalized the injections according to Eq.(5.26), this definition is valid only for higher frequencies in the band because the fitting of the timing model in Eq.(5.1) reduces the sensitivity at lower frequencies, and this effects the SNR. We found the effective SNR by empirically computing the likelihood ratio by dividing the maximum likelihood value by the likelihood when

the GW amplitudes are set equal to zero. The log-likelihood ratio Λ scales with the measured SNR as $\Lambda = \text{SNR}_{\text{eff}}^2/2$, and we use this relation to define the effective SNR of the signal. See Table 5.1 for the corresponding effective and injected SNRs.

We used the same likelihood and Bayesian framework outlined in Ref. [20], which used NANOGrav’s software package **enterprise**³ to implement the search with **PTMCMCSampler**⁴. The common parameters in our search are indicated in Eq.(5.14). All non-amplitude parameter priors were uniform, except for the evolution couplings which were log uniform. For the alt-pol signals we added parameters that allowed the sign of the alt-pol strain amplitudes to be positive or negative. We assumed Gaussian priors on the pulsar distances L_i , centered on the observed value and with a standard deviation given by the measured uncertainty. A uniform prior was assumed for the pulsar initial phase terms.

We discuss seven main analyses, the first six of which are outlined in Table 5.1. The first analysis is on a simulated data set with all polarizations present, and uses the full polarization model to recover the signal. The purpose of this analysis is to test the analysis software and see how well the various model parameters can be recovered. The second analysis uses simulated data where a pure TT signal has been injected, and the recovery is done using each polarization individually. The third analysis is on pure noise realizations with the goal of comparing the upper limits that can be placed on the amplitude of each polarization mode. The fourth analysis uses the same noise-only data, but with the sky location of the source restricted to be near pulsar J1024-0719 at $(-0.13, 2.72)$; the choice was incidental as this pulsar was closest to one of our signal injections. The goal here is to investigate how the enhanced response in the longitudinal modes tends to push the inferred sky localization away from the

³<https://github.com/nanograv/enterprise>

⁴<https://github.com/jellis18/PTMCMCSampler>

pulsar locations. The fifth and sixth analyses use simulated data with a pure TT signal, and look at the upper limits that can be placed on the alt-pol modes when a TT signal is detected. The data sets differ in the sky location of the source, with the source for analysis six placed in the direction of the galactic center (GC), where the array has more pulsars. The seventh and final analysis uses noise-only data with noise levels similar to those found in contemporary timing arrays to produce upper limits as a function of sky location.

Data Simulation Parameters								
Analysis Model	Index	Strain Priors	Injected Strains	Injected SNR	Effective SNR	Sky Location ($\cos \theta, \phi$)	Injected Radiative-Loss	Figures
All modes	1	log-uniform	TT, ST, SL, VL	40	~ 20	(-.468, 4.647)	$\log_{10}(\alpha_D) = 3.5$ $\log_{10}(\alpha_Q) = 8.5$	5.2, 5.3, 5.4
Single mode only	2	log-uniform	TT	20	~ 10	(0, π)	$\log_{10}(\alpha_Q) = 8.5$	5.5
All modes, sky-averaged	3	uniform	-	0	0	-	-	-
All modes, restricted around J1024-0719	4	uniform	-	0	0	(-.13, 2.72)	-	5.6
All modes	5	uniform	TT	20	~ 10	(0, π)	$\log_{10}(\alpha_Q) = 8.5$	5.7
All modes	6	uniform	TT	20	~ 10	(-.468, 4.647)	$\log_{10}(\alpha_Q) = 8.5$	-

Table 5.1: The set-up for each data simulation and analysis

5.6 Results

The results of each analysis are described in the following subsections. The figure summary is as follows: Figures 5.2, 5.3, and 5.4 show the alt-pol parameter recovery for simulated signals with effective signal-to-noise ratio $\text{SNR}_{\text{eff}} \sim 20$. Figure 5.5 shows the results of a ST-only model search for simulated data with a TT-only signal. The ST model is able to recover the TT signal, but with biases in some parameters.

Figure 5.6 illustrates how the pulsar locations impact the upper limits analysis by creating “zones of avoidance” around the pulsar locations when no signal is present in the data. Figure 5.7 explores the limits that could be placed on the alt-pol amplitudes in likely event that the signal is a pure GR TT-mode. Figure 5.8 shows the amplitude upper limits as a function of sky location for each of the polarization modes when no signal is in the data.

Analysis 1: Full Alt-Pol Parameter Recovery

The full parameter recovery of alt-pol searches validates our ability to probe higher dimensional signals. An interesting and unexpected covariance is seen between the SL and ST modes as well as the SL and VL modes in Figure 5.2; apparently there exists a geometric degeneracy with respect to the array. This is an important aspect to account for in future studies. We have verified this disappears if we have a large enough amplitudes to discriminate the angle of inclination ι . Note that the dipole radiation is far less sensitive for these frequencies than we normalized in the injection. Sky localization is very reliable, and the mapped posterior for other parameters is reasonable although we can see that the posterior has local maxima away from the true injected values with some unexpected structure. Fitting for the timing model can have an effect on some of these other parameter posteriors, such as Φ_0 and $\cos \iota$ along with the amplitudes because the fitting procedure changes the shape of the waveform, as seen visually in Ref. [46]. Since the dipole signal exists at 5nHz, the lower order harmonic and any couplings to it are more effected by this than the quadrupole signal. Again, the effect becomes negligible for larger amplitudes. All evolution coupling posteriors appear like Figure 5.4, regardless of the nature of the injection. The reason for this is that while a mixed dipole/quadrupole injection yields the transcendental function of Eq.(5.21), the uncertainty in the pulsar distances allows

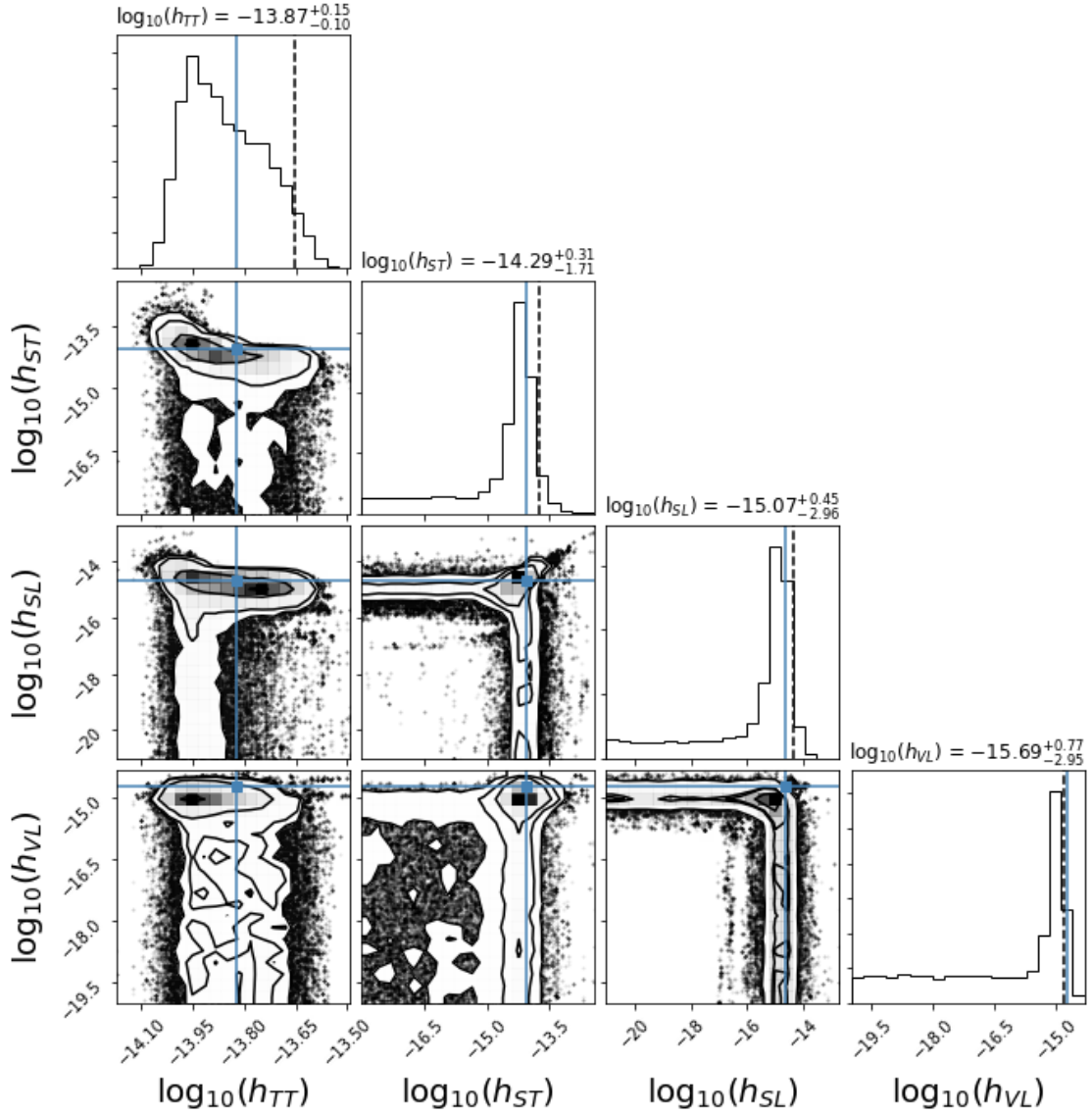


Figure 5.2: Joint posteriors for the strain parameters of an all modes search. The injected signal has all modes present and a total effective $\text{SNR} \sim 20$. The priors for the strains are log-uniform. The blue lines are the injected values of the strains, and the dotted lines are 95% quantile.

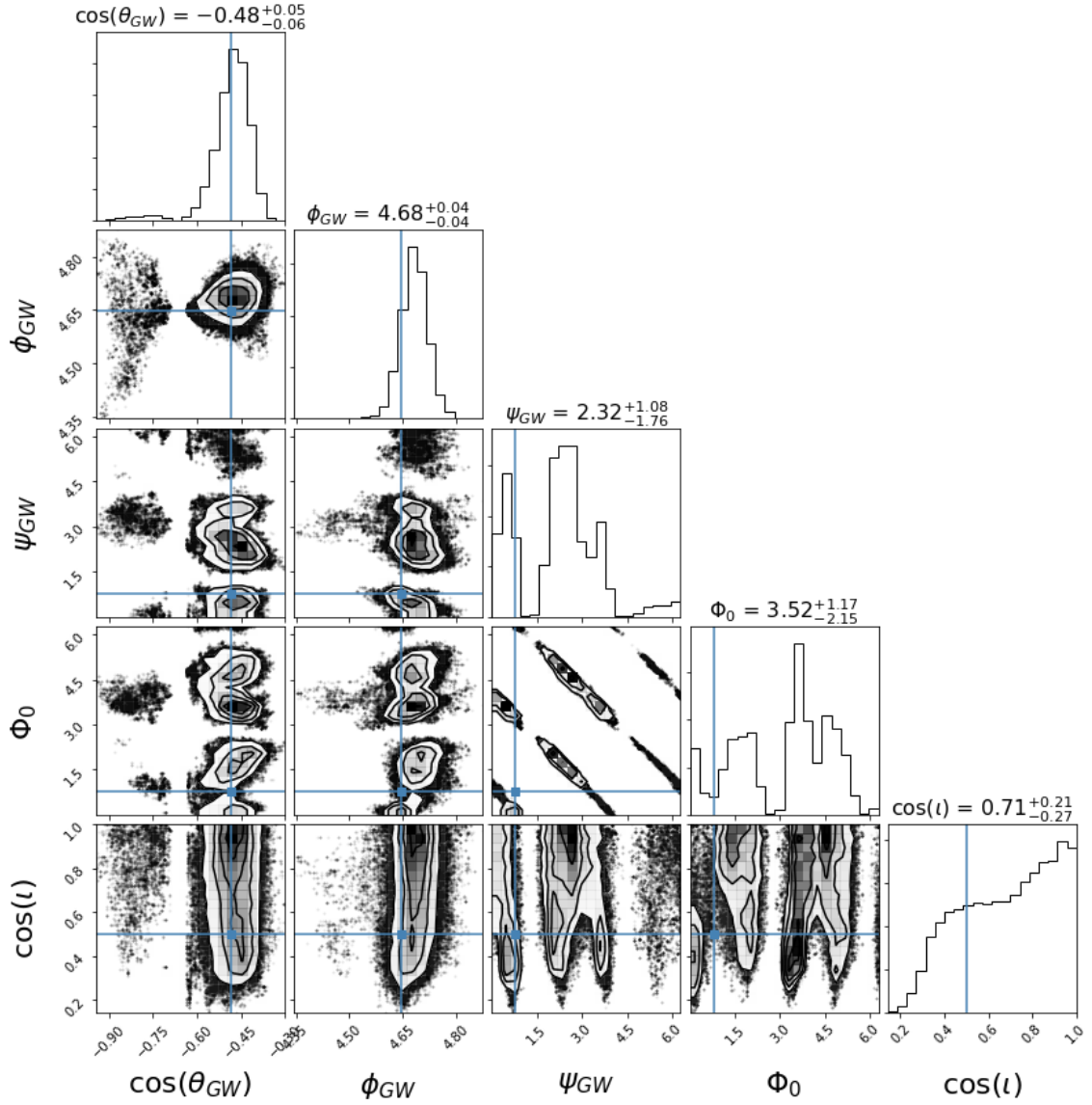


Figure 5.3: Joint posteriors for the sky parameters of an all modes search. The injected signal has all modes present and a total effective $\text{SNR} \sim 20$. The priors for the parameters shown here are all uniform. The blue lines are the injected values of the parameters. The GW is originating from behind the GC.

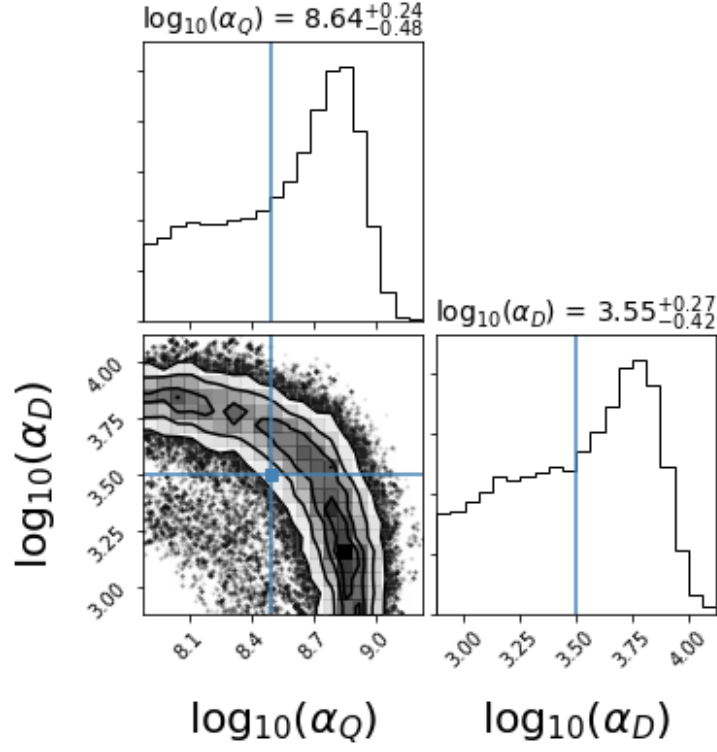


Figure 5.4: Joint posterior for the radiative-loss coupling parameters of an all modes search. The injected signal has all modes present and a total effective SNR ~ 20 . The priors for the parameters shown here are log-uniform. The blue lines are the injected values of the parameters.

a level of degeneracy with the non-mixed injections, and vice versa.

It should be noted that alt-pol injections render posterior modes in the pulsar phase parameters of pulsars near the GW source; all other pulsar phases sample uniformly. This is because the nearby pulsars dominate the SNR contribution of the longitudinal modes, so the pulsar phase becomes necessary to accurately describe the antenna pattern.

For very loud signals ($\text{SNR}_{\text{eff}} > 100$), the signs of the dipole charges lock on to the true values. However, for moderate to low SNR, the posterior is not very sensitive to the value of the sign, and frequently accepts jumps to opposite signed values because the difference in the likelihood is not very significant. As a check, we restricted the values of the signs to the true injected values and did not find an appreciable difference between the posterior distributions compared to the marginalized search. However, the periodic covariance between Φ_0 and ψ in Figure 5.3 is a result of the marginalization of the signs of the dipole strain and is much more selective in the restricted case.

Analysis 2: Single Polarization Recovery

To test if one signal could be mistaken for another, we searched a ST GW in phase with an injected TT mode of $\text{SNR}_{\text{eff}} \sim 10$ at $f_{\text{GW}} = 1 \times 10^{-8} \text{Hz}$. The physical motivation for this lies in the possibility of superluminal alt-pol GWs, which could arrive far earlier than the TT part of the signal, prompting an individual polarization search similar to tests performed on LIGO/Virgo data [5, 9, 12]. Figure 5.5 shows the results of one such analysis. We see that a pure ST model can recover a pure TT signal, albeit with a biased amplitude and α_D . We found that any single polarization search can yield a detection of a TT signal (with a biased sky localization for longitudinal modes), but that when all modes are included in the model the correct TT model

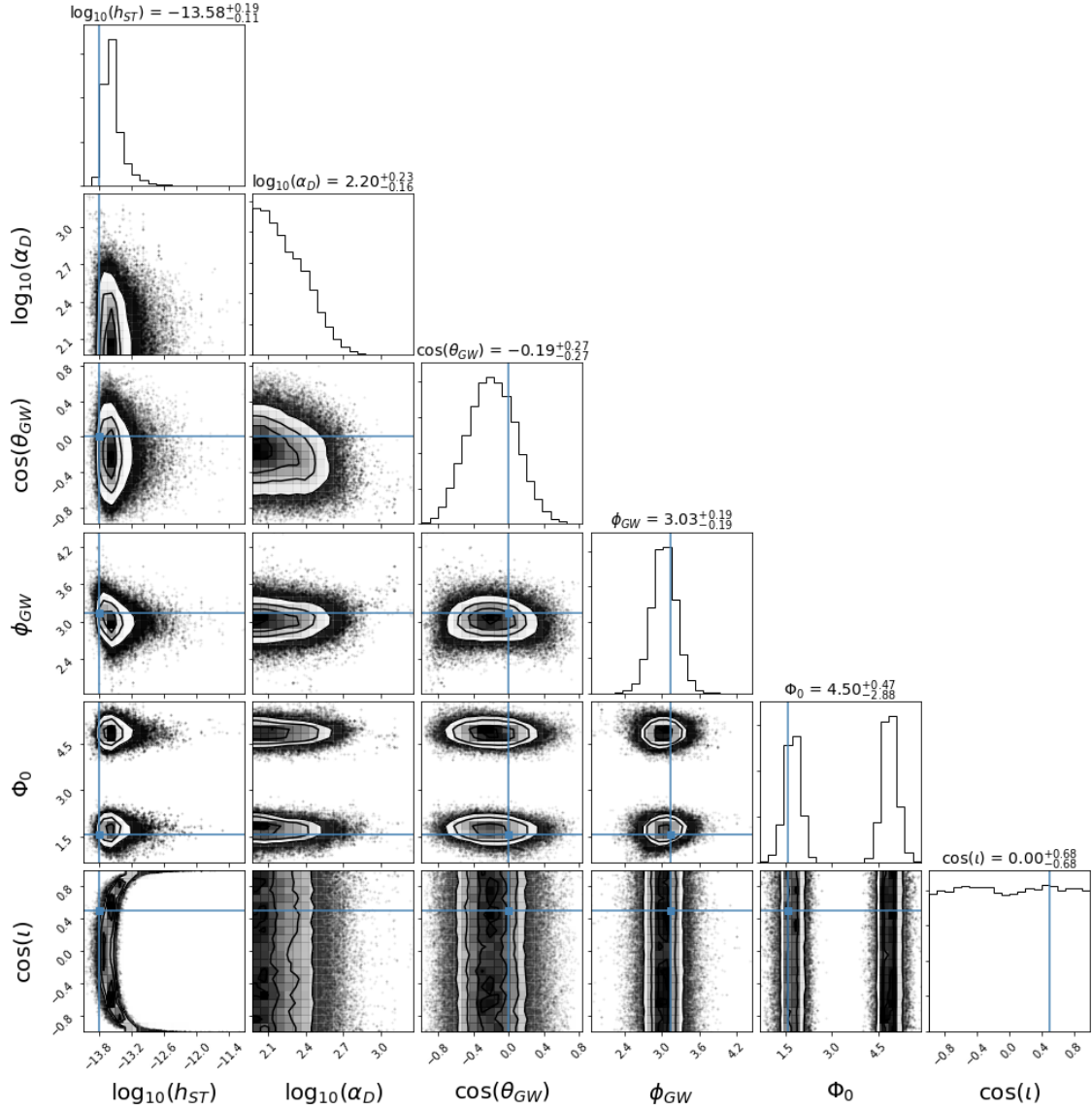


Figure 5.5: Joint posteriors for an analysis using a pure ST model on data with a pure TT signal with $\text{SNR}_{\text{eff}} \sim 10$ at $f_{\text{GW}} = 1 \times 10^{-8} \text{Hz}$. The ST model is able to recover the TT signal, albeit with a biased α_D and amplitude. Identical analyses with longitudinal modes render biases in the sky localization. Two truth lines have been modified. Eq.(5.16) shows that $\Phi_0 = \pi/4$ for a TT mode injection will be seen as $\Phi_0 = \pi/2$ for an ST mode that is in phase with that injection, and we have reflected that here. Also, since $\log_{10} \alpha_Q = 8.5$ for this injection, we recast the truth value in terms of the correct units, $\log_{10} \alpha_Q \omega^{2/3} = \log_{10} \alpha_D = 3.5$, which is outside the posterior.

is preferred (see Analysis 5 below). The longitudinal modes recover less of the TT signal and have poorer sky localization than either of the transverse modes because the response functions are very different. Incidentally, the ST search sky location and amplitudes are close to the injected TT mode values due to the array having similar geometric sensitivity to any transverse mode. We find the mapped posteriors agree well with the injected parameters of the TT signal when searching for the TT mode only.

Analysis 3: Sky-Averaged Upper Limits

For the following Analyses 3 and 4 we simulated noise-only data to perform an upper limit search. If we conjecture that a signal is present yet undetectable, we want to know the largest values the amplitudes can be. To this end we use uniform amplitude priors for upper limits rather than log-uniform. The other parameter priors are still uniform. We performed a marginalized search of the upper limits of all polarizations. While it is known that even for the TT mode a sky-averaged upper limit yields a sky location bias, the inclusion of longitudinal modes greatly exaggerates this effect. In the absence of any signal, the posterior is diminished in sky regions with many pulsars because the enhanced sensitivity to longitudinal modes forces the amplitudes to lower values. Since the posterior is the product of the likelihood and the prior, the likelihood will be no different at lower amplitudes, but the prior will penalize them, preferring the largest values possible, thus rendering the bias in sky location. For the simulated NANOGrav array with $0.5\mu\text{s}$ timing residuals we find sky-averaged upper limits of $h_{\text{TT}}^{95\%} < 2.0 \times 10^{-14}$, $h_{\text{ST}}^{95\%} < 1.3 \times 10^{-14}$, $h_{\text{VL}}^{95\%} < 8.6 \times 10^{-15}$, and $h_{\text{SL}}^{95\%} < 4.0 \times 10^{-14}$. Note that the projected limit on the TT mode is comparable to that found in the NANOGrav 11 year analysis [14], and we anticipate that the same data set will yield bounds on the other modes that are in line with these simulated

upper limits.

The sky location posteriors exhibit maxima at values where the response can remain geometrically hidden from the detector. The sky-averaged case provides a general proxy for the array’s sensitivity to localized GWs.

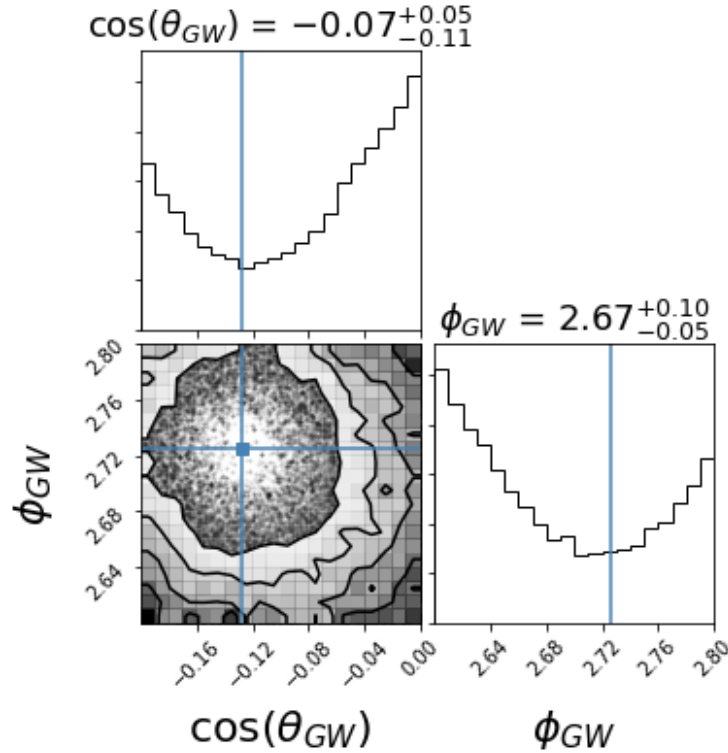


Figure 5.6: Joint posteriors of the sky location for an all modes upper limit search. No signal is present in this injection. All parameter priors are uniform except for α_D and α_Q , which are log-uniform. The sky location is restricted in a region around J1024-0719, whose position is indicated in blue lines. The enhanced response from longitudinal modes causes the posterior to peak away from the pulsar.

Analysis 4: Sky-Restricted Upper Limits

To further understand the nature of sky location bias, we performed a search nearly identical to Analysis 3, but restricted the sky location close to a pulsar, incidentally J1024-0719 in this case. Figure 5.6 shows the resulting sky location

posterior, which is peaked away from the pulsar, and we found the resulting upper limit on the SL mode dramatically reduced compared to the sky-averaged case. The VL mode is not as severely effected as the pulsar is close to the less sensitive region directly aligned with the GW source, seen in Figure 5.1. Again, the other parameter posteriors exhibit maxima where the response can remain hidden from the detector. This analysis confirmed that the enhanced response from pulsars was dictating the shape of the upper limit posteriors.

Analyses 5 and 6: Alt-Pol Upper Limits with TT Injection

We also performed alt-pol upper limit searches in the presence of a TT mode injection with $\text{SNR}_{\text{eff}} \sim 10$ for two separate sky locations, indicated in Table 5.1. The idea here is that the detection of the TT-mode would constrain the orbital frequency and sky location of the source, potentially resulting in stronger bounds on the alt-pol modes. Unfortunately this was not found to be the case. The joint posteriors for the amplitude parameters for Analysis 5 are shown in Figure 5.7. We found the resulting TT parameter posteriors recover the injected parameters as they did in Analysis 2, and that the alt-pol upper limits depend only on the sky location of the GW; the upper limits are more constrained if many pulsars are localized near the GW source. We find only for TT signals with SNR_{eff} of order unity do we get the aforementioned sky location bias mentioned in Analyses 3 and 4; the likelihood’s preference for the correct sky location is less significant in this case and starts to lose out to the alt-pol prior’s avoidance of nearby pulsars.

Analysis 7: Upper Limits as a function of Sky Location

We already established the sky location bias of Analyses 3 and 4, as well as the independence of the upper limits when a TT signal is present in Analyses 5 and 6. It is, therefore, permissive to search for upper limits as a function of fixed sky location.

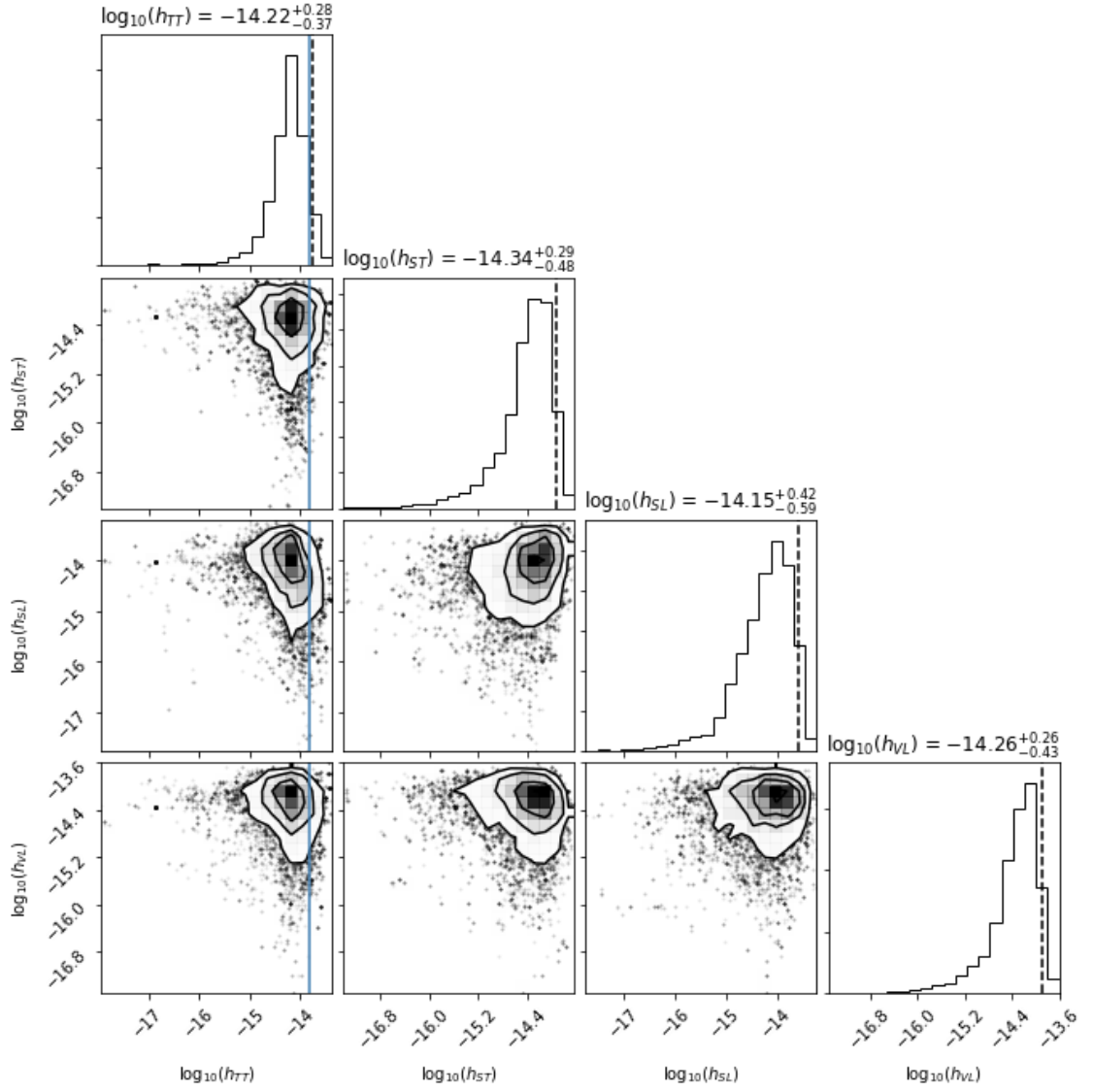


Figure 5.7: Joint posteriors of the strain upper limit strains in the presence of a pure GR signal with $\text{SNR}_{\text{eff}} \sim 10$ located at $(0, \pi)$.

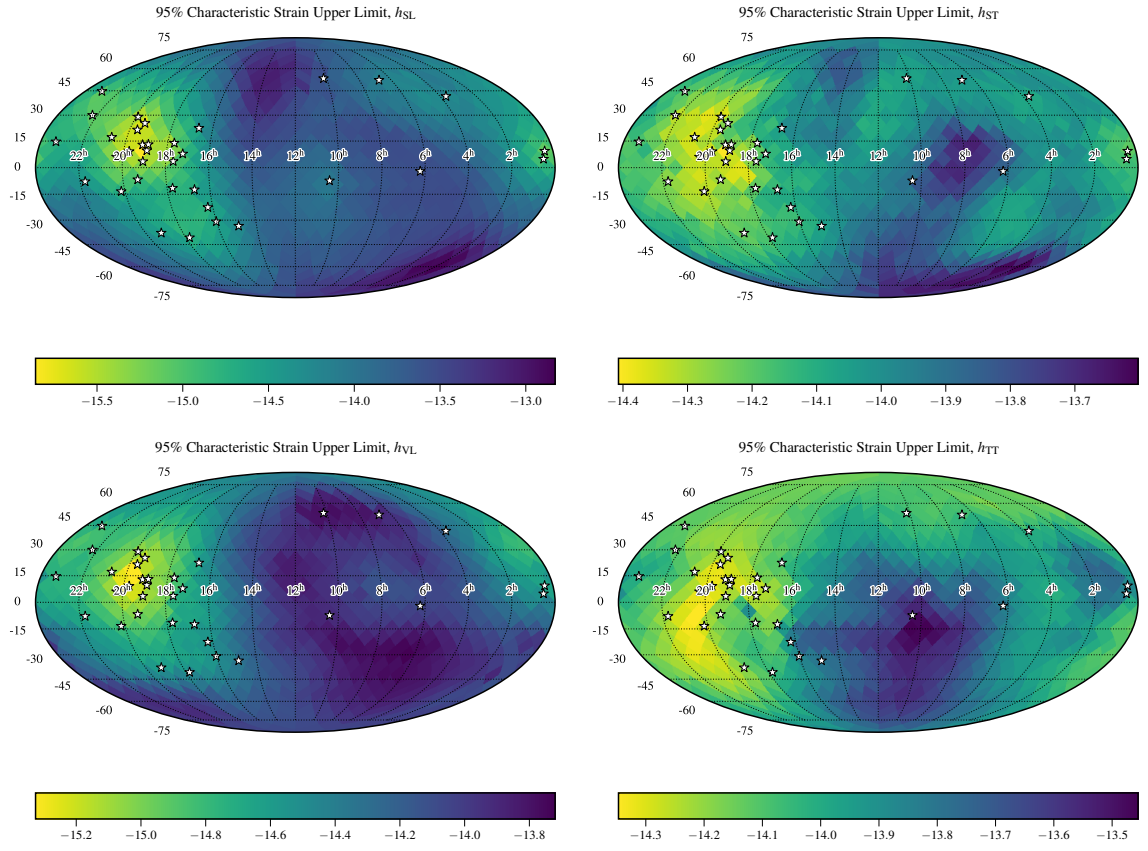


Figure 5.8: Sky maps of an all modes upper limit search at fixed sky locations for GW frequency $f_{\text{GW}}^{\text{TT}} = 1 \times 10^{-8} \text{Hz}$. From left to right, top to bottom, the plots correspond to SL, ST, VL, and TT strains.

This provides a more illuminating measure of the upper limit as a function of sky location when the upper limits span many orders of magnitude. In other words, we see in detail the array’s sensitivity to localized alt-pol GWs based on the source’s sky location. For a noise-only realization, the sky maps for marginalized strain upper limits at fixed sky locations are shown in Figure 5.8. We can see the enhanced sensitivity from longitudinal modes makes the upper limits more pronounced in sky regions with many more pulsars, namely the left side of the maps, as seen in the range of values indicated in the color bar. In the region of the sky that is well populated by pulsars, the upper limits on the amplitudes of the longitudinal modes are an order of

magnitude lower than for the transverse modes. This is consistent with the enhanced response to longitudinal modes for sources near to the sky location of a pulsar seen in Figure 5.1.

5.7 Conclusions

We showed that for a modified gravity GW of total $\text{SNR}_{\text{eff}} \sim 20$, we can detect the individual amplitudes of alt-pols even when their collective contribution to the SNR_{eff} is ~ 10 , and the analysis is not hindered by a higher dimensional model. This is encouraging as this contribution is only moderately loud, and our array is medium-sized with 34 pulsars. If significantly quieter alt-pols exist relative to GR a priori, we would need to rely on a very loud TT component being present to detect them, and the enhanced response due to longitudinal modes can prove advantageous in this respect.

We put upper limits on alt-pols in the presence of a TT mode, whose detection is unaffected by the search over additional strains. The values of the alt-pol upper limits depend on the location of the TT mode source, and thus on the sky location in general. We tested this for two separate cases, with one signal originating close to J1024-0719, which has few pulsar neighbors, and the second originating behind the GC, near many pulsars. The latter case rendered smaller alt-pol upper limits due the enhanced longitudinal response. We showed that over a sky-averaged upper limit search in the absence of any signal, the posterior probability is diminished in regions of the sky where longitudinal modes are enhanced. We subsequently showed the increased range of the strain values of longitudinal modes in our upper limit sky maps with fixed source sky locations.

We conclude that the upper limits set on alt-pol strains will be independent of the presence of a TT signal and depend only on sky location. The upper limits will be

meaningful if we detect a TT mode as it will allow us to place constraints on coupling constants of alternative theories of gravity relative to GR.

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CHAPTER SIX

CONCLUSION

We have calculated the cross-correlation signatures of all polarizations for both pulsar timing arrays and astrometry. We showed that the noise for any strong-signal cross-correlation is proportional to the zero-angle ORF of the method in question. While astrometric angular deflections provide less strong-signal self-noise from longitudinal modes compared to pulsar residuals, both of these methods are complementary. Not only do the two have the potential to confirm the same GW background, but other astrophysical studies, say with respect to the ISM, can be cross-referenced as there are similar effects on the observables of both methods. A future study could exploit the refractive effects of the ISM in order to decompose the chromatic and achromatic perturbations of angular deflections in various color filters in GAIA data and compare with the chromatic and achromatic components of pulsar residuals, and interesting studies are already being undergone with respect to the latter, comparing radio and X-ray perturbations.

It is advantageous that both of these methods operate in the low frequency GW band as it provides the most effective way to detect or constrain the -1PN corrections of modified gravity SMBHBs in their slowly evolving adiabatic inspiral phase. Additionally, the two methods incorporate distinct ORFs for each polarization, allowing the interpretation of any deviations from GR to be well understood.

We also showed how pulsar timing arrays are sensitive to individual SMBHBs in alternative gravity. With very little difficulty, an MCMC is capable of mapping the posterior probabilities of any extra parameters in alternative gravity present in some data set. The signals recorded in the data need not be very loud for the MCMC to

discriminate strongly among alternative polarizations.

The ultra-low frequency band of GWs, sourced by zero-point fluctuations in the early universe, would provide a medley of information pertaining to cosmic inflation if they could be detected. The value of these GW's amplitude depends on fundamental constants of nature and the time of horizon crossing (along with the specific nature of the inflation). As such, deviations from GR, namely the existence of superluminal alternative GWs, are suppressed relative to GR. The superadiabatic amplification of these zero-point fluctuations occur only while they exist inside the Hubble horizon, and any superluminal GWs would pierce the horizon at earlier times such that their amplitudes are no longer amplified. Therefore, even if GR GWs are detected in this band, the absence of detectable alt-pols in the band should be interpreted carefully. Indeed, we should be wary of the absence of a detection of alt-pols in any observation band; absence of evidence is not evidence of absence, and we must be cautious not to fall victim to our own epistemological “demons.” It is possible that foreground astrophysical contamination will render ultra-low frequency GWs undetectable indefinitely; therefore, the low frequency GW band may prove the most accessible for testing alternative gravity theories with these -1PN corrections, assuming that BHBs carry the relevant hair in these theories.

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