

POLARIZATION IMAGING OF THE MOON
IN SUPPORT OF NIGHTTIME REMOTE SENSING

by

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DEDICATION

To my parents, who have supported me as a scientist and in every aspect of my life. From teaching me multiplication tricks to driving me to the train station every day for a summer for my first research experience, I would not have gotten here without your support.

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ABSTRACT

The Moon is a valuable calibration target for on-orbit satellite instruments because it provides a stable source outside of the atmosphere, but effective lunar calibrations require an accurate characterization of the moonlight in the observation geometry. Similarly, the Moon can be used as the source of illumination in nighttime remote sensing applications, but knowing the moonlight properties improves understanding of the remote sensing data. Radiometric models of the Moon that predict the spectral radiance at a given geometry have been developed from large datasets of lunar radiance measurements. However, for accurate lunar calibrations of polarization-sensitive instruments and for polarimetric nighttime remote sensing, it is critical to account for the polarization state of the moonlight.

Moonlight has a polarization state that depends on wavelength and the lunar geometry, but its polarization state has not been characterized as thoroughly as its radiance. This dissertation addressed two knowledge gaps: there were no previously published measurements of disk-averaged moonlight polarization as a function of phase angle for wavelengths longer than 520 nm, and there was no published Moon polarization model that predicted the polarization from the lunar geometry. To address these gaps, a method was developed to record calibrated polarization images of the Moon, and the disk-averaged degree of linear polarization was calculated from the images. The method was initially used to measure the polarization in a broad spectral band (responsive from 400 nm to 1000 nm). Spectral filters were added to the imaging system to measure the moonlight polarization in five spectral bands. The data in 450-nm and 520-nm bands agreed well with previously published measurements, and the data in 650-nm, 800-nm, and 900-nm bands provided the first characterization of polarization as a function of phase angle at these wavelengths. Finally, empirical models were developed using these data and previously published data to predict polarization from the lunar geometry in spectral bands from 336 nm to 900 nm with estimated uncertainties of ± 0.01 or less. These measurements and the resulting model could enable more accurate lunar calibration for instruments currently on orbit and improve a variety of nighttime remote sensing applications.

INTRODUCTION

Satellites have been orbiting the Earth for decades. The number of satellites continues to grow rapidly, and with missions focused on everything from predicting the spread of malaria to characterizing ice in the Arctic, Earth-observing satellites make a critical impact on solving some of humanity’s biggest problems [1]. According to one estimate, at least 845 instruments on 327 Earth observing satellite missions are currently operating or planned for launch by 2035 [2]. One limitation to the effectiveness of Earth-observing satellite missions is how well the instruments on the satellite can be calibrated while in orbit. The level of uncertainty in the top-of-atmosphere signal (measured directly by a satellite instrument) impacts the quality of the environmental variables derived from the retrieval algorithms [3–5]. Instruments typically undergo rigorous characterization and calibration before leaving the lab [6], but the launch and time in space cause calibration constants to change. Some methods of on-orbit calibration for reflective solar bands (ultraviolet, UV, through shortwave infrared, SWIR, spectral bands) include recording measurements of on-board calibration lamps, the Sun through a solar diffuser, pseudo-invariant targets on Earth, and the Moon. Lunar calibration is advantageous in that it does not require on-board calibrators that degrade nor atmospheric corrections, but it does require accurate characterizations of the Moon.

Multiple research projects have been directed at characterizing the irradiance of the Moon as a function of lunar phase angle, other Moon geometry variables, and wavelength to enable and improve lunar calibration [7–10]. Radiometric models of the Moon have enabled successful lunar calibrations of on-orbit satellite instruments [3, 11–15]. However, a corresponding polarimetric model of the Moon does not yet exist, and it has been recognized that more polarization measurements are needed to develop a model [16]. The development of

a polarization model would enable more accurate lunar calibrations of polarization-sensitive satellite instruments. The ability to predict the polarization state of moonlight as a function of phase angle and wavelength would also be useful for nighttime passive polarimetric remote sensing, where the Moon is a primary source of illumination and its polarization state could affect the polarization state of light reflected and scattered by objects.

This dissertation describes work towards enabling more accurate lunar calibration of polarization-sensitive satellite instruments and more accurate nighttime passive polarimetric remote sensing. To advance toward this goal, a method to measure the polarization state of the Moon was developed and used to record polarization measurements in a broad spectral band. Then, spectral filters were added to the system to characterize moonlight polarization in five visible and near infrared bands, providing the first disk-averaged polarization measurements of the Moon as a function of wavelength in bands longer than 600 nm. Finally, these measurements were combined with measurements from previous studies to develop a polarization model of the Moon at wavelengths from 336 nm to 900 nm that depended on phase angle and other lunar geometry variables. The rest of this introduction discusses on-orbit calibration methods, radiometric models of the Moon, and previous studies of the polarization state of the Moon. It also discusses some previous nighttime passive remote sensing work and polarimetric remote sensing of cloud thermodynamic phase. Finally, it lists the objectives and main contributions of this work.

On-orbit calibrations

Common calibration targets for on-orbit satellite instrument calibration or inter-comparison include the Sun, on-board lamps, pseudo-invariant calibration sites on Earth, deep convective clouds, and the Moon. Each target has its own drawbacks, and typically multiple methods are combined to provide satellite data with the most accurate calibration. This section focuses on calibration methods for reflective solar bands (i.e., not thermal bands).

Two example satellite sensor systems that will be mentioned throughout this discussion are the Moderate Resolution Imaging Spectroradiometer (MODIS) and the Plankton, Aerosol, Cloud, ocean Ecosystem (PACE) mission. MODIS is a widely used instrument aboard two Earth-observing satellites [2]. The MODIS instruments are used for ocean color products such as chlorophyll concentration [17], cloud products such as cloud thermodynamic phase and cloud-top temperature [18], and land products such as evapotranspiration [19]. The PACE mission is a newer important satellite with instruments with and without polarization-sensitivity. A primary goal of the PACE mission is to continue the record of satellite-based ocean color measurements (including measurements from MODIS) using a spectrometer, but the satellite also includes two polarimeters [20]. The combination of the three instruments on PACE enables retrieval of many ocean, cloud, and atmosphere products. The many applications supported by just these two satellites highlights the importance of accurate on-orbit calibrations.

On-orbit calibration targets

The Sun provides a stable, on-orbit source of illumination that is viewed by satellites without any atmospheric effects. However, the Sun is significantly brighter than the scenes that Earth-observing satellite instruments are designed to measure, so a solar diffuser must be used to reduce the amount of light incident on the instruments. If the solar diffuser's performance is characterized before launch and its degradation while on orbit due to UV exposure can be monitored, viewing the Sun through a solar diffuser can provide absolute radiometric calibration. MODIS and the Visible Infrared Imaging Radiometer Suite (VIIRS) both used ratioing radiometers as solar diffuser stability monitors (SDSM) [21, 22]. The MODIS SDSM consisted of an integrating sphere and nine filtered detectors to match different MODIS bands. Each calibration event consisted of a dark view, a direct solar view, and a solar diffuser view. The solar diffuser degradation could nominally be tracked

with the ratio between the direct solar view and the solar diffuser view, but additional considerations needed to be taken into account such as the use of a solar diffuser screen to decrease the brightness of the solar diffuser when viewed by high-gain bands and a design error that caused an angle-dependent variation in the response of the SDSM direct solar view [21]. A different approach was used by the Ocean Color Instrument on PACE: two solar diffusers were used at different frequencies, one used on a daily basis to perform the solar calibration and one used on a monthly basis to track changes in the daily diffuser [23]. The Sea-viewing Wide Field-of-view Sensor (SeaWiFS) employed a solar diffuser without any means of monitoring its degradation, so the solar diffuser calibration data was only used to monitor short-term effects, and other calibration methods were used for long-term trending [3]. The Orbiting Carbon Observatory-2 (OCO-2) used a solar diffuser to view the Sun on most orbits, and used less frequent lunar calibration observations to track the solar diffuser degradation [24]. Thus, the Sun has been used successfully as a calibration target for absolute radiometric calibration, but at the expense of added components and complexity to the satellite instrument.

Some satellite instruments includes lamps that provide an on-orbit calibration reference [24–26]. The challenge in this approach is isolating whether changes in the signal from the lamp during calibration measurements are due to changes in the lamp brightness or the detector response. Some satellites include photodiodes as lamp monitors, which can be placed strategically [25] or used in conjunction with data about the lamp current and voltage [26] to isolate lamp changes from detector changes. Another strategy is to include multiple lamps used at different frequencies, using the less-frequently-used lamp to monitor the stability of the more-frequently-used lamp. Similar to solar diffusers, lamps can provide effective monitoring of on-orbit performance but also increase complexity.

Pseudo-invariant calibration sites (PICS) are temporally stable sites on Earth that can be used for vicarious calibration. The most useful sites tend to be in the Sahara desert

because of their limited rainfall and sparse human population, but Arctic and Antarctic sites are also useful, especially for polar orbiting satellites [27]. There is a long record of using PICS for long-term monitoring [28] and inter-calibration [29], and more recent work has explored absolute radiometric calibration [27, 30]. The Radiometric Calibration Network (RadCalNet) is an effort to develop instrumented sites for radiometric calibration of satellites. There are sites in four countries (Railroad Valley Playa, USA, La Crau, France, Baotou, China, Gobabeb, Namibia), and the sites contain instruments to measure the surface properties and the atmosphere to provide an SI-traceable estimate of the top-of-atmosphere radiance [30]. Deep convective clouds (DCCs) can be used similarly: they are near Lambertian and have both predictable and constant monthly mean albedos [31]. This has enabled inter-comparison through coincident measurements by two satellites [31], calibration by simulating the top-of-atmosphere reflectance of the DCCs [32], and calibration by using the radiance measured by a reference satellite [33].

The Moon is an extremely stable reflector of sunlight that has a reflectance similar to Earth scenes and is viewed by an on-orbit satellite instrument from outside of the atmosphere [16]. Thus, lunar calibration does not require a diffuser and corresponding diffuser monitor or an atmospheric correction. On the other hand, the Moon has a brightness that varies significantly with phase angle and a spatially variable surface, so accurate calibrations require a model that can account for the dependence on the phase angle and what part of the lunar surface is viewed by the instrument [7]. Commonly used models currently only achieve radiometric uncertainties of about 5% [16]. The small angular size of the Moon also means that it could not easily fill all pixels of an imaging instrument designed to view a wide swath on Earth. Therefore, lunar calibration cannot fill every on-orbit calibration need but provides an invaluable supplement to other calibration methods. Examples of successful lunar calibrations include MODIS bands from 412 nm to 940 nm with spatial resolutions from 0.25 to 1 km [11, 34], Planet Dove bands from 420 nm to 860 nm with approximately

3-km resolution [13], and SeaWiFS bands from 412 nm to 865 nm with 1.1-km resolution [3]. A typical lunar calibration image from one of these instruments had approximately 8 pixels in the cross-track direction [3, 34] and had to account for oversampling (i.e. two pixels measuring the same part of the lunar surface) in the along-track direction. After averaging all pixels in the image and accounting for oversampling, the average radiance of the Moon measured by the satellite was compared to the lunar radiance predicted by a radiometric model.

Radiometric models of the Moon

To develop the Moon as an on-orbit radiometric reference, several studies measured or have been planned to measure the spectral irradiance of the Moon. The RObotic Lunar Observatory (ROLO) and resulting ROLO lunar model is used as the basis of many lunar calibrations [7], and multiple recent projects seek to improve upon the ROLO model [8–10].

The ROLO model is based on a six-year record of measurements from a dedicated observatory at the USGS field center in Flagstaff, Arizona. The observatory consisted of two telescopes that recorded images in bands from 350 nm to 2200 nm. The images were corrected for atmospheric extinction, calibration to radiance, and summed across all pixels to calculate the lunar irradiance. The lunar irradiance was converted to disk-equivalent reflectance for each band using the solid angle of the Moon and the solar spectral irradiance in the band, and the disk-equivalent reflectance was fit to an empirical function of lunar geometry variables. The form of the ROLO model is

$$\begin{aligned} \ln A_k = & \sum_{i=0}^3 a_{ik} g^i + \sum_{j=1}^3 b_{jk} \Phi^{2j-1} + c_1 \theta + c_2 \phi + c_3 \Phi \theta + c_4 \Phi \phi \\ & + d_{1k} e^{-g/p_1} + d_{2k} e^{-g/p_2} + d_{3k} \cos [(g - p_3)/p_4], \end{aligned} \quad (1)$$

where A_k is the disk-equivalent reflectance in band k , g is the absolute phase angle, θ and

ϕ are the observer selenographic latitude and longitude, and Φ is the Sun selenographic longitude. The absolute phase angle is the angle between the Sun, the Moon, and the observer. The observer selenographic latitude and longitude give the location on the Moon that is closest to the observer, and the Sun selenographic latitude and longitude give the location on the Moon that is closest to the Sun (though only Sun selenographic longitude is in the model). The observer selenographic latitude and longitude are referred to as the libration variables, and they describe which part of the lunar surface is seen by the observer. The libration variables vary with phase angle on an 18-year cycle. The Sun selenographic latitude and longitude describe what part of the lunar surface is illuminated. The coefficients for absolute phase angle and Sun selenographic longitude are different for each band, but a shared set of coefficients are used for the terms that depend on libration. The exponential terms have different coefficients for each wavelength and describe the opposition effect, and the cosine term addressed a correlation present in the residuals.

One recent project to provide more high-accuracy measurements of lunar irradiance is the Airborne LUnar Spectral Irradiance (air-LUSI) instrument. High-altitude flights occurred in 2018, 2019, and 2022 and are scheduled for 2025 on NASA’s ER-2 aircraft, providing higher accuracy measurements of lunar irradiance than ROLO from 250 nm to 1050 nm because of the reduction in atmospheric effects [8, 35, 36]. Another project called the Lunar Irradiance Model of ESA (LIME) conducted measurements with a lunar radiometer operating at spectral bands centered at 340 nm, 380 nm, 440 nm, 500 nm, 675 nm, 870 nm, 937 nm, 1020 nm, and 1640 nm [10]. The Langley plot method was used to derive the top-of-atmosphere lunar irradiance, and measurements were conducted at the Izaña Atmospheric Observatory, a high-altitude location optimal for obtaining many high-quality Langley plots [10, 37]. Two space-borne instruments, ARCSTONE [9] and Climate Absolute Radiance and Refractivity Observatory (CLARREO) Pathfinder [38], will provide high-accuracy measurements of the Moon from outside of the atmosphere.

Current state of lunar calibration of polarized instruments

Some on-orbit instruments with polarization-sensitivity that performed lunar calibration include MODIS and VIIRS, OCO-2, and the Hyper-Angular Rainbow Polarimeter (HARP2) on PACE. MODIS has a polarization sensitivity due to its design, but it is not a polarimeter. This polarization sensitivity was characterized as less than 4% prelaunch [39], but it became as high as a 30% sensitivity for at least one band while on orbit [34]. Residual oscillations remaining after applying an improved lunar irradiance model to MODIS lunar observations were speculated to be significantly caused by the polarization of moonlight, and mitigating these errors by accounting for the polarization of moonlight was proposed as future work [34]. The VIIRS instruments also have polarization sensitivity [40] and perform lunar calibrations [12], but there appears to be no discussion of potential errors due to the polarization of moonlight in the literature (and the moonlight was even assumed to be unpolarized in one analysis [40]). The moonlight polarization may have a very small effect because the polarization sensitivity of VIIRS did not change like that of MODIS [41] and because of the tight constraints on the lunar-view geometry [12].

OCO-2 is a three-channel spectrometer with linear polarizers installed in front. It performs lunar calibration, and a polarization correction is performed on the lunar observations. This correction is based on many observations of OCO-2 with a similar phase angle but different librations as well as a preliminary polarization model component of LIME [42].

The HARP2 instrument on the PACE satellite is a polarimeter and conducts lunar observations for calibration at phase angles of $\pm 7^\circ$. The polarization of moonlight at $\pm 7^\circ$ is only approximately 1-2%, so the maximum errors in lunar calibration would only be 1-2% if the polarization was not taken into account, and more accurate lunar calibration will likely require some method of accounting for moonlight polarization.

Previous lunar polarization studies

While measurements of the polarization state of moonlight date back as far as observations by Arago in 1811 [43], there are still some knowledge gaps, particularly how the polarization varies with phase angle for wavelengths longer than 600 nm and how much the polarization depends on libration. Previous studies of the polarization of moonlight fall into two primary categories: disk-averaged polarization measurements characterize the average polarization state of the illuminated fraction of the Moon, and disk-resolved measurements characterize the polarization state of particular regions of the Moon. It should be noted that "moonlight" is used in this dissertation to refer to the sunlight scattered and reflected by the lunar surface and measured by a sensor pointed directly at the Moon (rather than the light from the Moon that has been scattered by the sky).

The disk-averaged polarization of moonlight was measured in a few important studies. Lyot established the shape of the curve of disk-averaged polarization as a function of phase angle with 61 observations of the Moon in a broad visible band [43]. The first study of the spectral variation of disk-averaged polarization was conducted in 1970 by Coyne and Pellicori [44]. They measured the disk-averaged degree of linear polarization (DoLP) as a function of phase angle in spectral bands ranging from 336 nm to 520 nm. The following conclusions were made by Coyne and Pellicori [44]. Polarization was found to decrease monotonically with wavelength, and the phase angle corresponding to maximum polarization depending slightly on wavelength. The polarization was higher for positive phase angles than negative. Also, the authors corroborated Lyot's suggestion that this asymmetry could be attributed to the ratio of low-albedo areas to high-albedo areas during positive and negative phases. Finally, the observed polarization varied between lunar cycles, which the authors concluded was caused by libration differences.

A few additional studies measured the disk-averaged polarization state at other

wavelengths. Forbes and Welch measured the DoLP at 1.25 and 2.24 μm but only at a phase angle of -80.9° [45]. Snow et al. measured the disk-averaged DoLP from 220 nm to 300 nm at a wide range of phase angles [46]. These measurements were made from the Solar Stellar Irradiance Comparison Experiment (SOLSTICE), a scanning-grating monochromator instrument designed to observe the full disk of the Sun. While not a polarimeter, the instrument has a strong polarization sensitivity, so the researchers derived polarization information from measurements of the Moon obtained at different orientations (resulting from the roll angle of the satellite) [46]. These studies established a general understanding of how the disk-averaged polarization varies with lunar phase angle and provided a thorough characterization of moonlight polarization for UV, green, and blue bands across lunar phase angles. However, there are still gaps in the literature, including: 1) an understanding of how the disk-averaged polarization varies with phase angle beyond 600 nm, and 2) a model for predicting the disk-averaged polarization, similar to the existing radiometric models of the Moon.

Many experiments have been performed to conduct disk-resolved polarization measurements of the Moon. While they do not directly address these two knowledge gaps, they add to the existing body of knowledge about the polarization of moonlight. Disk-resolved studies are useful for studying the lunar surface itself and could one day enable pixel-wise lunar calibration. Disk-resolved polarization measurements have been conducted with narrow-FOV instruments [47, 48] that can be scanned to different regions of the lunar surface and with images where each pixel contains information about a different lunar region [49–52]. These measurements have been used to study many aspects of moonlight polarization vary with lunar terrain. Some specific properties that have been studied in different lunar regions include the maximum DoLP, corresponding phase angle, and its relationship with albedo [47–49, 53]; the minimum DoLP, corresponding phase angle, and its relationship with albedo [54]; and the inversion angle (where the signed DoLP crosses through 0 around phase angles

of $\pm 23^\circ$) and its relationship with albedo [48, 54]. Many of these studies characterized these properties in multiple spectral bands. The spectral disk-resolved measurements have been used to infer properties of the lunar surface in different regions, such as the median grain size [51, 55], the thicknesses of the surfaces of lunar regolith particles [50], and potentially the index of refraction [56].

One recent paper recorded polarization images of the Moon using a division-of-focal-plane polarization method [57]. This paper was published at approximately the same time as the paper in Ch. 3. There are a few significant differences between their method and the method in this dissertation. To calibrate the imaging system, they used a polarized standard star and a zero-polarization standard star.

One other recent investigation of the polarization of moonlight was conducted by the same team that created LIME [10]. The instrument that recorded irradiance measurements also had polarized channels, so it was able to measure the disk-averaged DoLP. A fourth-power polynomial of phase angle was fit to the data recorded over an approximately one-year period, and the fits were made available online [58] and described in a preprint [59]. However, the method was not able to determine the sign of the DoLP, provided noisy raw data, and the resulting fits did not agree closely with measurements in similar bands recorded by Coyne and Pellicori. Once issues with the noisiness of the raw data are resolved, this dataset could provide an important characterization of moonlight polarization.

Nighttime passive remote sensing with the Moon

Nighttime passive remote sensing could also benefit from improved characterization of the polarization state of moonlight as a function of phase angle. The Moon is the primary source of ultraviolet through shortwave infrared illumination at nighttime, so passive remote sensing measurements in these parts of the spectrum are measurements of scattered or reflected moonlight. Thus, the amount and polarization state of that scattered or reflected

moonlight depends on the polarization state of the moonlight itself. Some studies have directly measured moonlight to infer properties of the atmosphere, such as aerosol optical depth [60, 61] and the presence of trace gases [62, 63] and smoke layers [64]. However, some of these studies used instruments that are likely to have polarization sensitivity [65], which would create errors if the polarization state of the moonlight was ignored (and the lunar phase angle was not close to 0°). Other studies measured moonlight scattered by aerosols [66, 67], but they also did not account for the polarization of moonlight. Wang et al. acknowledged that more accurate data are needed before they could update their work to account for the polarization state instead of assuming randomly polarized moonlight [67].

In the Optical Remote Sensor Laboratory, cloud thermodynamic phase (CTP) has been measured previously with passive polarimetric methods. We initially began recording polarization images of the Moon to explore how much the polarization of moonlight would impact CTP measurements conducted at night. While the measurements of moonlight polarization became the focus of this dissertation, one chapter is included on improving passive polarimetric measurements of CTP. Specifically, the use of low-cost commercial polarization imagers and an improved classification algorithm that depends on scattering angle are demonstrated. This section provides some background on why it is important to measure CTP and how it can be measured.

Cloud thermodynamic phase

Cloud thermodynamic phase (CTP) refers to whether a cloud contains liquid water droplets or ice crystals (or a combination of the two). CTP is itself an important parameter in climate models, and it must be known to enable retrieval of many other cloud remote sensing parameters, such as particle shape and size [18, 68, 69]. Cloud properties, such as height, size, shape, and thermodynamic phase, can affect optical propagation and radiative transfer through a cloud [68, 70] and control whether a cloud has a heating or cooling effect,

thereby impacting Earth’s radiation balance [71–73].

Because CTP has such an important effect on the overall behavior of clouds, increasing the amount of time and areas for which CTP data are available will improve our climate models. This research directly supports the goal of increasing CTP data availability, by progressing toward the first low-cost, widely deployable method for nighttime CTP measurements that are vital for climate models. The ability to measure CTP at night is particularly important during the Arctic winter, when the polar night lasts for weeks or months. Monitoring and understanding climate change in the Arctic, where change is occurring most rapidly, is critical to understanding both global climate change and how Arctic change influences midlatitude weather patterns [74, 75]. More specifically, the effect of clouds and atmospheric constituents on the surface radiation balance is considered an under-measured component in the Arctic system [74], and CTP is a key parameter in knowing how clouds affect radiation [71–73].

In addition, cloud-aerosol interactions are known to play a significant role in anthropogenic radiative forcing, but with high uncertainty [76]. This makes furthering our understanding of aerosol-cloud interactions crucial for reducing climate uncertainty, and nighttime CTP data advance this understanding. Also, this method could enable real-time detection of dangerous aircraft icing conditions, particularly the presence of supercooled liquid clouds (liquid clouds below freezing temperature) [77]. The combination of CTP detection with a low-cost polarization imager and cloud temperature measurements with a low-cost thermal imager calibrated to obtain science-grade measurements [78] could form the basis of a new system for identifying supercooled liquid clouds.

Various methods have been employed to measure CTP, including *in situ* measurements, active remote sensing, and passive remote sensing. Some *in situ* methods use scattering-based [79] or polarization-based [80] measurements to distinguish between liquid water droplets and ice crystals, but they all require intake of cloud particles into the instrument. *In situ*

measurements tend to be the most accurate and have been used to validate remote sensing measurements, but remote sensing tends to be cheaper and can achieve more consistent long-term monitoring over a greater spatial extent [81]. Active remote sensing methods for measuring CTP include polarization-sensitive lidar [82–86] and radar systems [85]. These systems rely on the cross-polarization ratio of the light backscattered by clouds to identify ice and liquid. Active remote sensing methods can provide additional information like cloud height and do not rely on an external source of illumination, but they also tend to require more power and cost more than passive remote sensing methods.

One passive method uses shortwave radiance ratios to distinguish between liquid water droplets and ice crystals [18, 87–89]. Liquid and ice water have different absorption spectra in the shortwave infrared part of the spectrum, so liquid and ice can be distinguished by comparing the ratios of radiances in at least two different spectral bands (if those spectral bands are chosen at wavelengths where the liquid and ice water absorption curves have different shapes). These methods relying on SWIR radiance ratios work well for optically thick clouds, and methods adding polarization information to the SWIR bands have attempted to improve classification for optically thin clouds [90]. This method has been used by ground-based [87, 89] and satellite-based [18, 88] instruments.

Polarimetric passive remote sensing has also been explored with ground-based instruments. This method is based on simulations performed by Knobelspiesse et al. showing that the algebraic sign of the Stokes S_1 parameter in a scattering plane reference frame depends on the thermodynamic phase of the cloud [91]. Specifically, positive values of S_1 correspond to liquid clouds, and negative values of S_1 correspond to ice clouds. The scattering plane contains the observer, the sun, and the scatterer. S_1 is the difference in the parallel and perpendicular polarization states, where parallel and perpendicular are referenced to the scattering plane. These simulations predicted that classifying liquid and ice based on S_1 should work best for optically thin clouds. The Optical Remote Sensor Laboratory was

the first to apply this idea to all-sky polarization imaging for passive remote sensing of CTP under sunlight and performed successful retrievals using a relatively expensive, science-grade polarization imager [92].

Objectives

The research in this dissertation involved using a division-of-focal-plane (DoFP) polarimeter to address the two areas (moonlight polarization and passive polarimetric CTP retrievals) discussed in the introduction. The first three objectives listed below related to better characterizing the polarization state of moonlight, which could improve on-orbit lunar calibration of polarization-sensitive satellite instruments. These satellite instruments contribute to a wide range of Earth science applications, and calibration of polarization-sensitive instruments is particularly useful with the recent launch of the Plankton, Aerosol, Cloud, ocean Ecosystem (PACE) satellite (which carries two polarimeters) and the planned Atmosphere Observing System - Sky satellite (which will carry one polarimeter). The most important contribution of this dissertation was using the DoFP polarimeter to measure the disk-averaged DoLP of moonlight as a function of phase angle for spectral bands longer than 600 nm, which had not been published previously. Additional contributions included developing a relatively simple method that could be used to measure the disk-averaged lunar polarization in an arbitrary spectral band and developing a model of disk-averaged polarization as a function of phase angle for spectral bands between 350 nm and 900 nm. The fourth objective related to improving passive polarimetric remote sensing of CTP. Widespread measurements of CTP enabled by low-cost, passive instrumentation could contribute to improving understanding of how clouds impact Earth's radiation balance, and this research explored the capabilities of a low-cost, passive polarimeter.

1. Develop a method to record calibrated polarization images of the Moon and use them

to calculate disk-averaged DoLP as a function of lunar phase angle.

2. Extend use of Moon imaging system to characterize the polarization of moonlight in visible and near infrared spectral bands.
3. Model the lunar disk-averaged polarization as a function of lunar geometry variables for bands in the ultraviolet, visible, and near infrared.
4. Improve passive polarimetric capabilities for sensing CTP by using lower-cost instrumentation and developing a classification that accounts for scattering angle.

Chapter 2 provides an overview of quantifying and measuring the polarization state of light with a DoFP polarimeter and of displaying, calibrating, and processing polarimetric data. Chapters 3 - 6 describe Objectives 1 - 4 respectively.

POLARIMETRIC IMAGING

This dissertation discusses multiple applications of polarimetric imaging, and there was significant overlap between the instruments and methods used in those applications. This chapter provides an overview of polarized light and how polarimetric data can be displayed, measured, calibrated, and processed. These tasks are explained in the context of a division-of-focal-plane (DOFP) polarization camera, the type of polarimeter used in this work.

Stokes Vector

The polarization state of light can be defined in terms of observables using the Stokes vector. The Jones vector also defines the polarization state of light, but it uses parameters based on the electric fields, which cannot be measured directly at optical frequencies. The Stokes vector is a four-element vector, consisting of four parameters:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(I_{0^\circ} + I_{45^\circ} + I_{90^\circ} + I_{135^\circ}) \\ I_{0^\circ} - I_{90^\circ} \\ I_{45^\circ} - I_{135^\circ} \\ I_{RHC} - I_{LHC} \end{bmatrix}, \quad (2)$$

where I_{0° , I_{45° , I_{90° , and I_{135° are measurements through a polarizer oriented at 0° , 45° , 90° , or 135° , respectively, relative to a common reference. Depending on the polarimeter, these measurements (and therefore the Stokes parameters) could be reported in units of power, radiance, or irradiance or as dimensionless peak-normalized quantities.

The first Stokes parameter S_0 describes the total power, radiance, or irradiance of the measured light, regardless of its polarization state. The second and third Stokes parameters relate to the linear polarization of the light, and the fourth relates to the circular polarization [93, 94]. The amount of circularly polarized light is expected to be small for light scattered

by the Moon [95]. Circularly polarized light can arise in the atmosphere from certain aerosol types or multiple scattering in clouds, but the amount from multiple scattering is expected to be orders of magnitude smaller than the linear polarization [96, 97], so only the linear Stokes vector was used to describe the polarization state of light in this work.

Two useful quantities can be calculated from the linear Stokes vector. The degree of linear polarization (DoLP), calculated as

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}, \quad (3)$$

quantifies the fraction of the light that is linearly polarized. The angle of polarization (AoP), given by

$$AoP = \frac{1}{2} \arctan \left(\frac{S_2}{S_1} \right), \quad (4)$$

describes the dominant direction of oscillation of the electric-field vector of the electromagnetic wave. The linear Stokes vector can be written in terms of S_0 , DoLP, and AoP as follows:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_0 * DoLP * \cos(2 * AoP) \\ S_0 * DoLP * \sin(2 * AoP) \end{bmatrix}. \quad (5)$$

Linearly polarized light has three degrees of freedom, so it is possible to completely describe linearly polarized light in terms of S_0 , DoLP, and AoP.

It should be noted that because the linear Stokes vector consists of three parameters, it is possible to retrieve the Stokes vector, DoLP, and AoP with three measurements instead of four. If three measurements are used, the ideal set of linearly polarized measurements are I_{0° , I_{60° , and I_{120° [98, 99]. In this work, the four measurements in Eq. (2) were used because of the design of the commercial polarization camera used.

Displaying polarimetric data

When an imaging polarimeter measures the linear Stokes vector, every pixel measures a value for each Stokes parameter, so the DoLP and AoP can be calculated at each pixel. To display and interpret these data, an image using a colormap to show each Stokes vector, DoLP, and AoP must be created. The ideal colormap to use for each variable depends on both the properties of the variable, such as whether it is monotonic, periodic, or diverging, and the interpretation goal [100]. Figure 1 shows an example of the colormaps used throughout this work for S_0 , S_1 , S_2 , DoLP, and AoP. These colormaps were created based on the principles described in [100] using code for the CAM02 perceptually uniform colorspace [101] (except for S_0 , shown by MATLAB’s default “parula” [102] or grayscale). The scene shown in Fig. 1 has a laptop screen on the bottom left, an external monitor on the right, and a ball and unpolarized sunglasses on the bottom right. The laptop screen had an AoP around 90° and $S_1 < 0$, the external monitor had an AoP around 0° and $S_1 > 0$, and the AoP of the sunglasses varied significantly because of stress birefringence in the plastic lenses.

Both S_0 and DoLP are monotonic variables, and the goal in interpreting images of these variables is to determine the value of the variable in a particular part of the image and how that compares to the rest of the image. Because of this goal and the high spatial frequencies likely to be contained in an image of S_0 or DoLP, a suitable colormap is achromatic and monotonically increasing in lightness. The colormaps shown in Figs. 1(a) and 1(d) have these properties, and different colormaps are used for easy distinction between the two variables. S_1 and S_2 are diverging variables centered around 0, and therefore should be shown with diverging colormaps. The colormaps shown in Figs. 1(b) and 1(c) are diverging colormaps, increasing in colorfulness with absolute value but using different hues to display positive and negative values. Finally, since AoP is periodic data and AoP images often have high spatial frequency, the colormap used is periodic but has luminance variation.

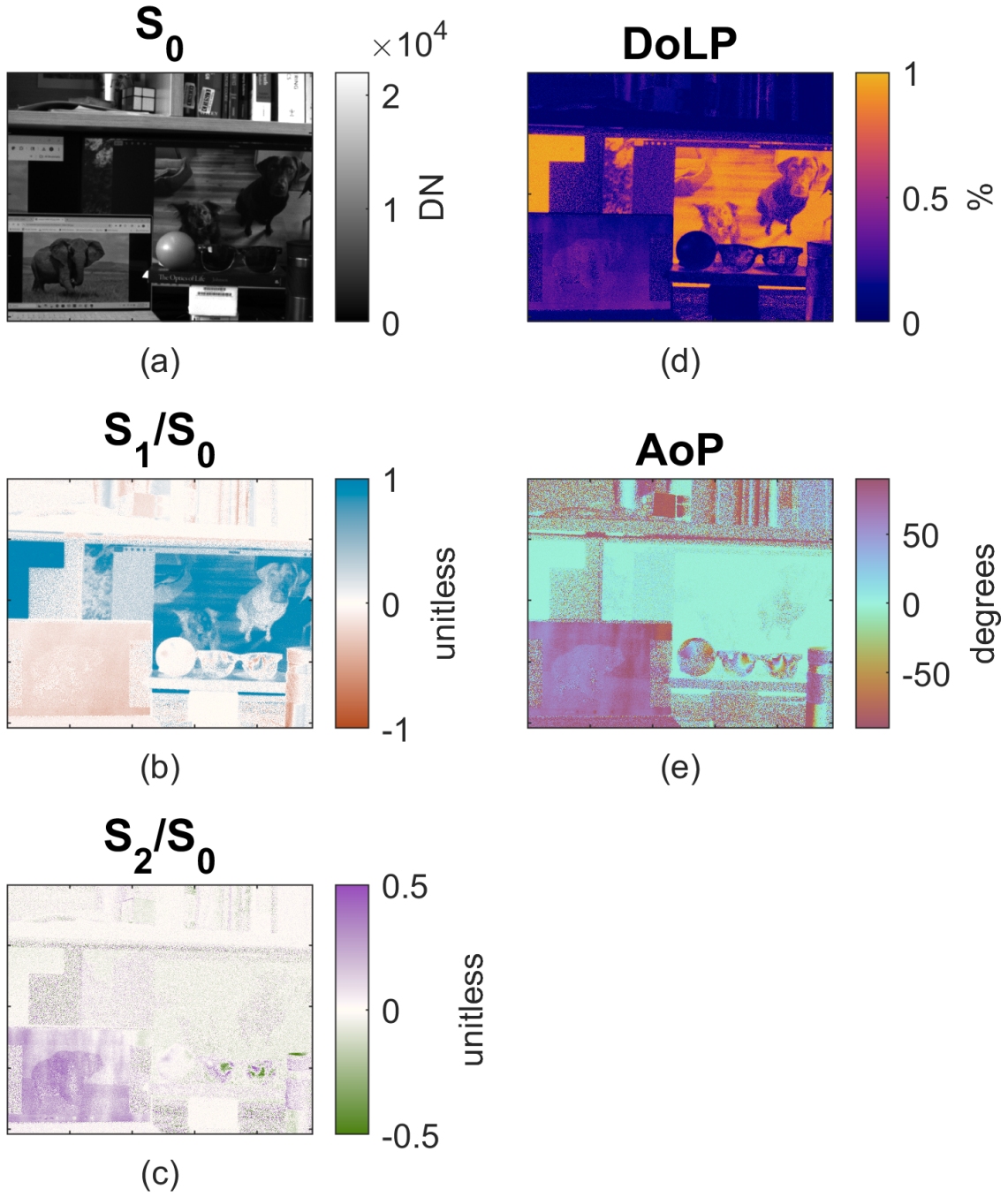


Figure 1: Example image of computer monitors, unpolarized sunglasses, and a ball showing the colormaps used for (a) S_0 , (b) S_1 , (c) S_2 , (d) DoLP, and (e) AoP.

Division-of-focal-plane (DoFP) polarization cameras

A division-of-focal-plane (DoFP) polarization camera is a type of polarimeter that records images in multiple polarization states simultaneously. It uses an array of wire-grid polarizers on the detector array, so each pixel measures light through a polarizer of a particular orientation. The panchromatic DoFP camera used in this work was a FLIR Blackfly BFS-U3-51S5P-C, and the RGB DoFP camera used was a FLIR Blackfly BFS-U3-51S5PC-C. The panchromatic version used 2×2 arrays of subpixels to form a superpixel, where the four subpixels comprising a superpixel had polarizers oriented at 0° , 45° , 90° , and 135° , relative to the bottom edge of the detector. This layout is depicted in Fig. 2. The RGB version used the same polarizer layout but had an additional array of color filters to form a repeating 4×4 group of subpixels, where the top left polarization superpixel measured all four polarization states in the red spectral band, the top right and bottom left polarization superpixels measured the four polarization states in the green band, and the bottom right polarization superpixel measured the four polarization states in the blue band.

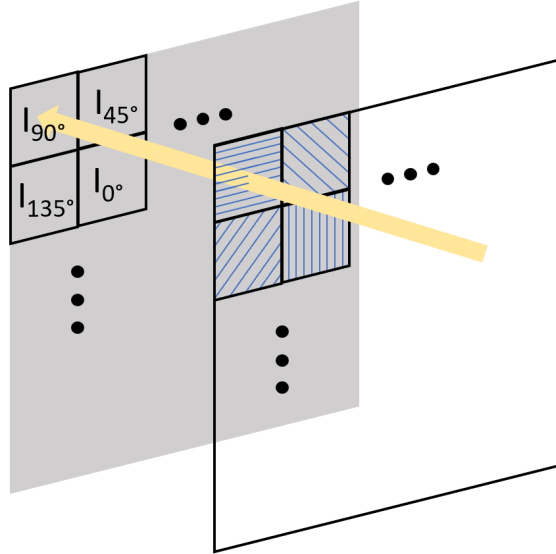


Figure 2: Layout of the wire-grid polarizer array on the panchromatic DoFP polarization camera. Reprinted with permission from [103] © Optica Publishing Group.

There are several common architectures for imaging polarimeters [94]. DoFP polarization cameras were suitable for this work because of their useful advantages (simultaneous acquisition of all four polarization images, mechanical stability, simplicity) and acceptable disadvantages [instantaneous field of view (IFOV) misregistration]. Division-of-time polarimeters record images in different polarization states sequentially by rotating a polarizer or retarder, changing the retardance of a liquid crystal variable retarder, etc. This polarimeter architecture can result in motion artifacts: any movement of the polarimeter or scene between sequential measurements is falsely perceived as resulting from the polarization of the scene. Both the Moon and clouds can move quickly [relative to the fields of view (FOVs) of the imaging systems], so simultaneous acquisition of the polarization images can eliminate this type of error and simplify interpretation of the measurements. Division-of-amplitude and co-boresighted systems acquire multiple polarization states simultaneously but require multiple detectors, which increases cost and requires higher mechanical stability so that the relative alignment between the detectors is fixed, which can result in a relatively large system [94]. For the Moon imaging application, which required steering the imager on a tripod, having a small system with no concerns about changing the relative alignment between detectors was advantageous.

The most significant drawback to the DoFP architecture is IFOV misregistration, which results from the subpixels within a superpixel measuring slightly different parts of the scene. This can cause errors in the polarization state retrieved, especially when the scene has spatial frequency content high enough that there is significant intensity variation across the superpixel [104]. The interpolation method described in the “Demosaicking method” section on page 36 was used to mitigate these errors.

The DoFP polarization camera was deployed as a part of multiple imaging systems throughout this work. The imaging systems are described in more detail in the relevant chapters, but a brief overview is given here to provide context for how the calibrations were

performed. First, for the Moon polarization imaging application described in Ch. 3, two imaging systems were used with the DoFP polarization camera: a 2-m telescope and a 300-mm telephoto lens. The Celestron Schmidt-Cassegrain telescope had a focal length of 2.032 m and an aperture diameter of 20.32 cm. The 300-mm telephoto was set to f/11, so it had an aperture of approximately 27 mm in diameter, and it had a physical front lens diameter of 72 mm. Both had a full-angle FOV of less than 3° . In Ch. 4, the 300-mm lens was used with a step-down adapter to hold 25-mm spectral filters, so the illuminated part of the front lens was decreased to 25 mm. These two systems will be referred to as “the telescope imaging system” and the “300-mm lens imaging system” through the rest of this chapter. Next, for the CTP application described in Ch. 6, the DoFP polarization camera was used with a fisheye lens, which was 56.5 mm in (physical) diameter and had a full-angle FOV of approximately 180° . This will be referred to as the “fisheye imaging system.” Finally, for the polarized water contrast measurements in the Appendix, the polarization camera was used with a lens so the FOV along the long dimension of the detector was approximately 18° , and this will be referred to as the “ 18° -FOV imaging system.”

Calibration methods

Four primary calibrations and characterizations were performed on the polarimetric imaging systems used throughout this project. This section describes the calibration methods, but many of the calibration constants are reported in subsequent chapters. To qualitatively assess the accuracy of these calibrations, particularly the polarimetric calibration, we aimed to measure a partially polarized scene with a predictable DoLP and AoP. We conducted some measurements of an air-water interface and ultimately realized that the measurements were useful beyond a cursory validation of the polarimetric calibration. Those measurements were reported in a journal paper [105], which is included in the Appendix on page 153.

Spatial characterization

The purpose of spatially characterizing these imaging systems was to determine the FOV, IFOV, and center pixel of a system, which enabled creating a pixel-angle map. The pixel-angle map related a pixel number (a location in image space) to the angle its corresponding ray made with the optical axis in object space. Assuming a linear relationship between pixel distance and angle, the following equations described this relationship for an all-sky image recorded with the on-axis ray pointed at the zenith and the top of the camera pointing north:

$$\theta = P * \sqrt{(X - X_{center})^2 + (Y - Y_{center})^2} \quad (6)$$

$$\phi = \arctan \left(\frac{X_{center} - X}{Y_{center} - Y} \right) \quad (7)$$

where X and Y were the coordinates of the pixel of interest, X_{center} and Y_{center} were the coordinates of the pixel corresponding to the on-axis ray, P was the constant IFOV in degrees/pixel determined in the characterization, and θ and ϕ were the zenith and azimuth angle the part of the sky imaged by the pixel of interest, respectively. Note that the column numbers increased to the right, the row numbers increased downward, and north ($\phi = 0^\circ$) was at the top and east ($\phi = 90^\circ$) to the left in an all-sky image, so the azimuth angle increased counterclockwise in the image. While not all imaging systems were used to record all-sky images, it was still important to know the FOV or IFOV of these systems. The assumption of a linear relation between pixel distance and angle was confirmed for the 18° -FOV imaging system and the fisheye imaging system. Since the aberration distortion generally decreases with field angle, it was fair to assume the imaging systems with a smaller FOV (the 300-mm lens imaging system and the telescope imaging system) were also described by the linear relationship.

Because of the unique challenges presented by each imaging system, three methods were used to measure the IFOV of the imaging systems. The first method, used for the 18° -FOV

imaging system, involved recording an image of a checkerboard-patterned target with black and white squares of known size. The distance from the imaging system to the target was recorded, so the angular size of each square could be calculated with the geometry shown in Fig. 3 and the following equations:

$$\theta_1 = \arctan(d/z) \quad (8)$$

$$\theta_2 = \arctan(2d/z) - \arctan(d/z) \quad (9)$$

$$\theta_n = \arctan(nd/z) - \arctan((n-1)d/z) \quad (10)$$

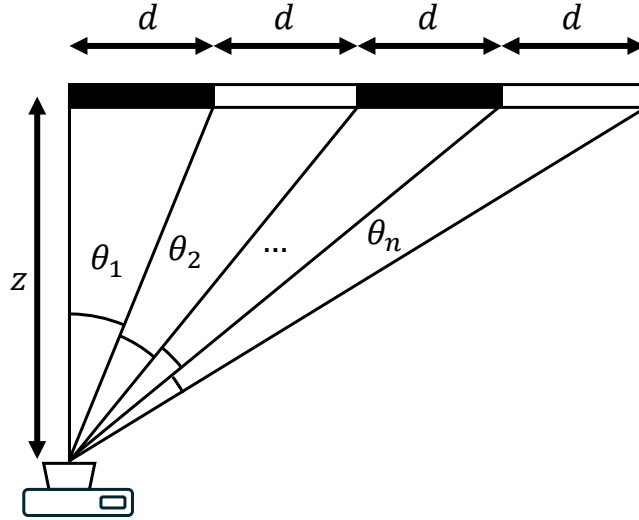


Figure 3: Geometry relating the size of a checkerboard square to the angle seen by the camera.

In general, the angle corresponding to each square decreased with increasing distance from the on-axis ray. However, in the case of an imaging system with a narrow FOV, the small-angle approximation was applied, so $\theta_1 = \theta_2 = \dots = \theta_n = d/z$, and the angle

corresponding to each square was constant. This method was used with the imaging system for the polarized water measurements. Images were recorded at three distances, so the checkerboard target filled different amounts of the FOV (the closest distance was used to fill the FOV to look for uniformity across the whole FOV in both the vertical and horizontal directions, while the longest distance was used to look for variation near the edge of the FOV). MATLAB's edge detection algorithm (*edge* [106]) was used to find the indices of the boundaries between the white and black squares in the image, and the indices were subtracted to find the number of pixels across each square. Each square filled many rows and columns of the image, so dozens of values of the number of pixels per square were calculated for each square. The number of pixels filling each square varied by a few pixels across all the squares without an apparent systematic spatial pattern in the variation, so the angle per square calculated with the small-angle approximation was divided by the mean number of pixels per square to calculate the angle per pixel. Finally, the three values of angle per pixel determined at the three distances were averaged to obtain the final measurement of IFOV of the imaging system. This imaging system had a full-angle FOV of approximately 18° in the longest dimension, so it was possible to simultaneously fill the image with the checkerboard target and view many of the squares on the target while maintaining a reasonable distance between the camera and the target. However, this was not possible for the imaging systems used for both the Moon polarization and all-sky applications, so different methods were used.

The imaging systems used to record Moon polarization images had narrow FOVs and long focal lengths, so recording an in-focus image of the checkerboard with multiple squares showing would have required a long distance and accurate measurement of that distance. Instead, the second spatial calibration target was used: the Moon itself. The position of the Moon relative to the Earth can be calculated to a high degree of accuracy using ephemeris calculation tools. We used the JPL Horizons tool [107] to calculate the angular diameter of the Moon as seen from Earth (specifically, the latitude and longitude of Bozeman) at the

time an image was recorded. Then, a circle was drawn around the image of the Moon using the *drawcircle* tool in MATLAB [108], and the diameter of the circle in pixels was related to the angular diameter to find the linear relationship between pixel distance and angle in object space. Using *drawcircle* was important to identify the diameter of the Moon in pixels as if it were full.

Finally, a third method was used to perform a spatial calibration of the fisheye imaging system. Because the FOV was so large, the checkerboard target (or the Moon) could not fill a significant part of the FOV. Instead of recording multiple images while moving the target, a fixed line source was imaged as the imaging system was rotated. This setup is depicted in Fig. 4. The line source was created by placing a narrow slit in front of an integrating sphere, and the slit width was chosen so that the image of the line was approximately one pixel wide. The imaging system was placed on a rotation stage, so the location of the image of the slit could be related to an angle in object space (the angle between the slit and the on-axis ray in object space, labeled “rotation angle” on Fig. 4). It was important to align the imaging system with the slit such that the slit was imaged onto the center pixel of imaging system (which corresponded to the on-axis ray) when the rotation stage was set to 0° , and it was important that the imaging system was rotated about its nodal point.

The center column of pixels was determined by imaging a vertical line object and rotating the imaging system until the image of the line object had no curvature. The row where the line object was imaged with no curvature was the center row. The same process was repeated with the camera mounted at a 90° angle to find the center row. The nodal point was found by imaging two vertical line objects, approximately one meter and three meters from the camera, with the farther away object (object A) obscured by the closer one (object B). When the imaging system was rotated about the nodal point, object A remained obscured by object B regardless of the rotation of the system. When the imaging system was rotated about any other point, object A could sometimes be seen to the side of object B.

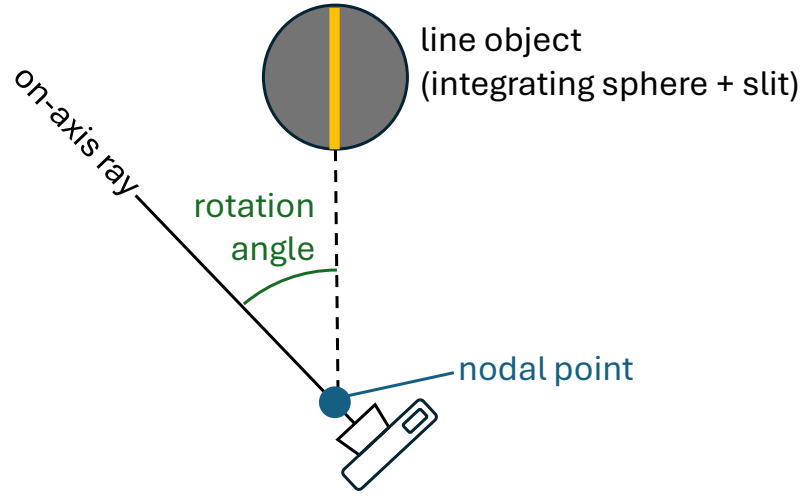


Figure 4: Setup for spatial characterization of a system with a wide FOV. The imaging system was rotated about its nodal point, and the pixel number of the slit in the image was related to the angle of the slit relative to the on-axis ray.

The imaging system was placed on a rotation mount such that it was possible to adjust the point of rotation, and the nodal point was found by experimentally determining the point of rotation at which object A was always obscured by object B.

Once the center pixel and nodal point were determined, the slit was imaged as the imaging system was rotated from -92° to $+92^\circ$ in 4° steps. The pixel distance of the image of the slit from the center pixel was fitted as a function of imaging system rotation angle. The residuals of a linear fit were found to have no systematic pattern suggesting a higher-order fit was not needed. Thus, the slope of this linear fit provided a constant value describing pixels per degree across the FOV, and the inverse of the slope gave the IFOV in degrees. As a simple verification that the spatial characteristics of the fisheye imaging system were circularly symmetric, an image of a brick wall was recorded. The brick height and the distance from the wall to the nodal point were measured, the vertical angular extent of each brick was calculated, and the number of columns filling each brick (along the center row) was divided by

the vertical angular extent. The IFOV measured was similar enough to the IFOV measured by the rotating-slit experiment to suggest that there was no difference between the horizontal and vertical characterizations, but the IFOV obtained in the horizontal characterization was used because it came from a more controlled experiment.

Spectral characterization

The purpose of the spectral characterization was to understand the part of the spectrum to which the imager was sensitive. This was necessary for both correct interpretation of the data produced by the imager and for performing a radiometric calibration on the imager (to be discussed further in the next section). The end result of the spectral characterization was a relative spectral response (RSR) curve, which showed the normalized response of the camera as a function of wavelength. The RSR was measured using a grating monochromator, a device with a broadband lamp, a diffraction grating, and a slit that can tune a narrowband output across a wide range of wavelengths. The procedure for measuring the RSR of a DoFP polarization camera with a grating monochromator is described completely in my MS professional paper [103], but a brief summary is included here.

The general procedure for measuring the RSR of an imaging uses the following steps:

1. Place an optical power meter at the monochromator output and measure the optical power of the monochromator output as it is swept through the wavelength range to be characterized.
2. Place the imaging system to be characterized at the monochromator output and record measurements as the output is swept across the wavelength range. Average the pixels measuring the monochromator output to increase signal-to-noise ratio.
3. Divide the measurement recorded by the detector by the optical power at each wavelength to account for fluctuations in the optical power of the monochromator

output across the wavelength range.

4. Normalize the curve to have a maximum of one.

However, there were some additional considerations necessary for characterizing a DoFP polarization camera with a grating monochromator. Because of the diffraction grating and reflective optics, the monochromator output had a polarization state that varied with wavelength. The polarization of the monochromator output is shown as a function of wavelength in Fig. 5. If not accounted for, using the above steps to calibrate each subpixel, which measured a polarized quantity, would have resulted in an incorrect RSR. This incorrect RSR would be the product of two effects: the true RSR and the polarization state of the monochromator output relative to the angle of the polarizer for that subpixel. To address this, we measured an unpolarized quantity (S_0) at each superpixel instead of the polarized quantity at each subpixel. This avoided the issue of the polarization state of the monochromator output changing with wavelength; though it did combine the RSRs of four subpixels and measure the effective RSR of the superpixel. In summary, in Step 2 above, we recorded S_0 at each superpixel instead of the polarized signal at each subpixel.

To measure S_0 accurately required correcting differences in relative response of the subpixels. Both nonuniformities in the sensitivity of the pixels themselves and in the polarized transmission of the polarizers on top of the pixels contributed to errors in the value of S_0 . To account for these errors, we obtained a flat-field correction matrix by calculating the multiplicative inverse of the mean of several images of an unpolarized source. The flat-field correction matrix was multiplied by each image before calculating S_0 .

For the work described in [103], we used a bare detector without a lens or other optics. The methods here generally can extend to measuring the RSR of an imaging system, but with some limitations. Other projects in the Optical Remote Sensor Laboratory have successfully measured the RSR of an imaging system [109, 110], but we found that we could not directly

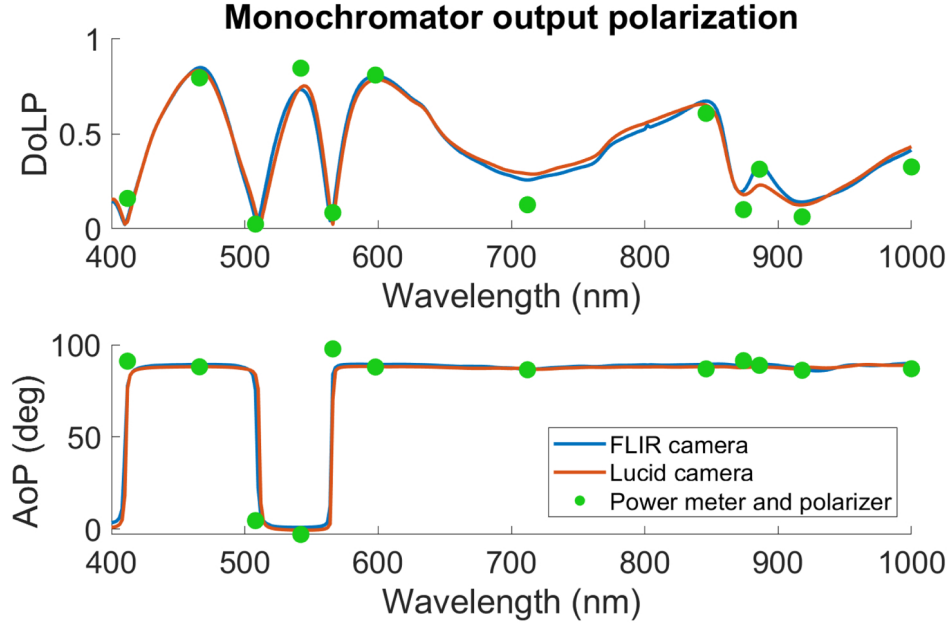


Figure 5: Polarization of the monochromator output measured with two DoFP polarization cameras (labeled “FLIR” and “Lucid”) and by rotating a polarizer in front of a power meter. The polarization state was described in terms of (top) DoLP and (bottom) AoP. Reprinted with permission from [103] © Optica Publishing Group.

characterize the 300-mm lens imaging system accurately because of its long focal length. To obtain an in-focus image of the monochromator output, the imaging system had to be placed approximately 2 m from the monochromator. However, the power meter measured the monochromator output from a distance of approximately 2 cm. The monochromator output diverged at different angles depending on the wavelength, so placing the power meter and the imaging systems at different distances caused errors in the RSR. It was not possible to place the imaging system at the same distance as the power meter and perform the calculations with out-of-focus images because the images were so severely out-of-focus that not all of the light reached the detector and the amount of signal per pixel was extremely low. The solution we adopted was to characterize the elements individually. The detector RSR was measured with the procedure above, but the 300-mm lens RSR was measured with a spectrometer and an integrating sphere. The spectrum of the integrating sphere was measured directly and through the lens, and the spectrum through the lens was divided by the spectrum of

the integrating sphere to obtain the transmission spectrum. The same process was used to characterize the transmission spectrum of the filters that were used with the 300-mm lens imaging system. The detector RSR and transmission spectra were multiplied together to obtain the RSR of the full 300-mm lens imaging system. The RSR of the 300-mm lens imaging system with and without spectral filters is shown in Chapter 4.

Radiometric calibration

The purpose of a radiometric calibration was to relate the DN measured by each pixel to the radiance of part of the scene imaged by the corresponding pixel. This calibration was performed using an integrating sphere with a NIST-traceable spectral radiance calibration as the source of known radiance incident on the system. The basic procedure for performing the radiometric calibration was to record measurements with the imaging system at multiple (at least two) integrating sphere radiance levels and perform a linear fit at each pixel relating DN to radiance. While relatively straightforward to perform the linear fit, it was important to calculate the radiance across the bandwidth of the imaging system correctly.

The imaging systems used in this work had relatively broad spectral bands, so each measurement was an integration of the incident light across a range of wavelengths. The integrating sphere manufacturer provided a calibration describing the spectral radiance ($W/(m^2 \text{ sr nm})$) of the output, but the radiance ($W/(m^2 \text{ sr})$) of the output needed to be calculated by integrating the spectral radiance as a function of wavelength. The band-integrated radiance was calculated as the integral of the spectral radiance of the output weighted by relative sensitivity of the imaging system to light at each wavelength (i.e., the RSR), according to the following equation:

$$L_{band} = \int L_{\lambda,source}(\lambda) * RSR(\lambda) d\lambda, \quad (11)$$

where L_{band} is the measured band-integrated radiance, $L_{\lambda,source}(\lambda)$ is the spectral radiance

of the integrating sphere output, and $RSR(\lambda)$ is the RSR of the imaging system being calibrated. Theoretically, the bounds of the integral were the wavelengths where the RSR had a value of zero; in practice, the bounds of the integral were controlled by the range over which we performed the spectral characterization. Figure 6 shows the RSR for the 300-mm lens system with no spectral filters, the spectral radiance of the integrating sphere across the same spectral range, and the product of the two (the weighted spectral radiance). The band-integrated radiance is the area under the weighted spectral radiance curve.

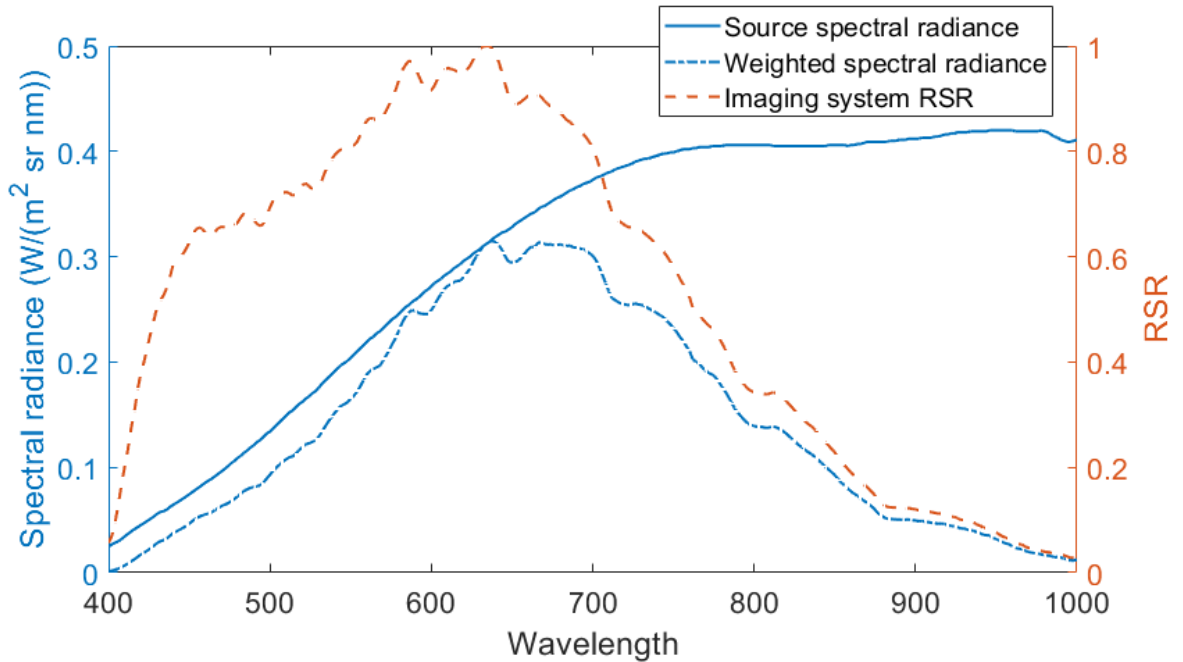


Figure 6: The “weighted spectral radiance” was calculated as the product of the spectral radiance of the integrating sphere (“source spectral radiance”) and the “imaging system RSR”. The area under the weighted spectral radiance curve was the band-integrated radiance that was related to DN measured by the imaging system.

Polarimetric calibration

The purpose of the polarimetric calibration was to correct polarimetric errors in the imaging system so that the measured Stokes vector was as close as possible to the true Stokes vector incident on the system. There were two types of errors that the polarimetric

calibration corrects for this type of polarimeter: polarimetric aberrations in the lenses and non-uniformity in the polarizers on the detector. In the polarimetric calibration, we measured a 3x3 system matrix (mathematically equivalent to a Mueller matrix) that related the incident linear Stokes vector to the measured linear Stokes vector. This was described mathematically by the following equations:

$$\mathbf{S}_{meas} = \mathbf{M}_{sys} \mathbf{S}_{inc}, \quad (12)$$

where $\mathbf{S}_{meas}$ was the measured or output Stokes vector, $\mathbf{M}_{sys}$ is the system matrix, and $\mathbf{S}_{inc}$ was the input Stokes vector, and they were defined as

$$\mathbf{S}_{meas} = \begin{bmatrix} S_{0,meas} \\ S_{1,meas} \\ S_{2,meas} \end{bmatrix}, \mathbf{M}_{sys} = \begin{bmatrix} m_{00} & m_{10} & m_{20} \\ m_{01} & m_{11} & m_{21} \\ m_{02} & m_{12} & m_{22} \end{bmatrix}, \text{ and } \mathbf{S}_{inc} = \begin{bmatrix} S_{0,inc} \\ S_{1,inc} \\ S_{2,inc} \end{bmatrix}. \quad (13)$$

Then, the inverse of this system matrix could be multiplied by any measured Stokes vector to infer the incident Stokes vector. Since each superpixel measured the linear Stokes vector, each superpixel had its own system matrix that was found during calibration.

The mathematical basis for calculating the system matrix from measurements is described briefly here, and the setup used in the lab is described in Ch. 3 Sec. 3 and shown in Fig. 11. The calibration setup used a polarizer on a rotation stage in front of an integrating sphere to create an incident polarization state known to a high degree of accuracy. Measurements were recorded at three calibration polarizer angles, creating three pairs of known incident and measured Stokes vectors. The three incident Stokes vectors were combined to form a 3x3 matrix, $[\mathbf{S}_{inc}]$, and the three measured Stokes vectors were combined to form $[\mathbf{S}_{meas}]$. These matrices were used to calculate the system matrix as

$$\mathbf{M}_{sys} = [\mathbf{S}_{meas}][\mathbf{S}_{inc}]^{-1}. \quad (14)$$

Combining at least three pairs of incident and measured Stokes vectors was necessary to constrain all nine elements of the system matrix. This equation was solved at each superpixel. However, this procedure could not be followed directly for the telescope imaging system because of its large aperture.

The telescope imaging system had an aperture larger than the aperture of our integrating sphere or high-extinction-ratio wire-grid polarizer, so we were not able to fill the telescope aperture with a known Stokes vector. We instead used a flat-field correction to account for the nonuniformity between the subpixel polarizers, and we assumed the polarimetric aberrations of the telescope were low. The uniform source of light used for the flat-field correction was the integrating sphere at a close distance, so it was extremely out-of-focus. The process for creating the flat-field correction, the validity of the low-polarimetric-aberration assumption, and the results of comparing the 300-mm lens imaging system calibration with the telescope imaging system calibration are discussed in Ch. 3 “Calibrations” section on page 50.

Processing data from DoFP polarization cameras

To convert a raw image from the DoFP polarization camera into meaningful images of Stokes parameters, DoLP, and AoP, two steps beyond calibrations are necessary. First, a reference frame must be chosen for the AoP images. Second, a demosaicking algorithm mitigating IFOV-misregistration effects must be applied.

Choice of AoP reference frame

Polarization is the direction of oscillation of the electric field in an electromagnetic wave, so to fully describe the polarization state it is necessary to specify the direction relative to

a meaningful and unambiguous reference frame. This is referred to as the AoP reference frame, and an AoP of 0° corresponds to a polarization state parallel to the reference frame. However, it is important to note that S_1 and S_2 also depend on the reference frame [Eq. (5)]; only S_0 and DoLP are independent of reference frame.

A few different reference frames were used throughout this work. In the measurements of the polarized contrast of underwater scenes presented in the Appendix, the instrument plane reference frame was used, where an AoP of 0° corresponded to a polarization state parallel to the long edge of the detector. The camera was always oriented so that the long edge of the detector was parallel to the ground, and an AoP of 0° corresponded to horizontal. For the all-sky polarization measurements presented in Ch. 6, the scattering plane reference frame was used. The scattering plane is the plane containing the source, the scatterer, and the observer, and it is useful for comparing the measured polarization state to that of Rayleigh scattered light [111]. To convert images from the instrument plane to the scattering plane reference frame, code originally written by Nathan Pust was slightly modified and used. Figure 7 shows an all-sky image of AoP on a cloud-free day referenced to the instrument plane and the scattering plane. For most of the sky, the AoP referenced to the scattering plane is 90° , corresponding to Rayleigh scattering. For images of the AoP of the Moon shown in Chs. 3 and 4, the instrument plane was also used as the reference frame. However, the plane of vision (the plane containing the Sun, the Moon, and the observer) was used to determine the sign of disk-averaged DoLP. This is described in greater detail in the image processing and analysis section of Ch. 3.

Demosaicking method

A raw image from the detector of a DoFP polarization camera contains interlaced images of the different polarization states. “Demosaicking” is the processing of separating the images of different polarization states so that images of the Stokes parameters, DoLP,

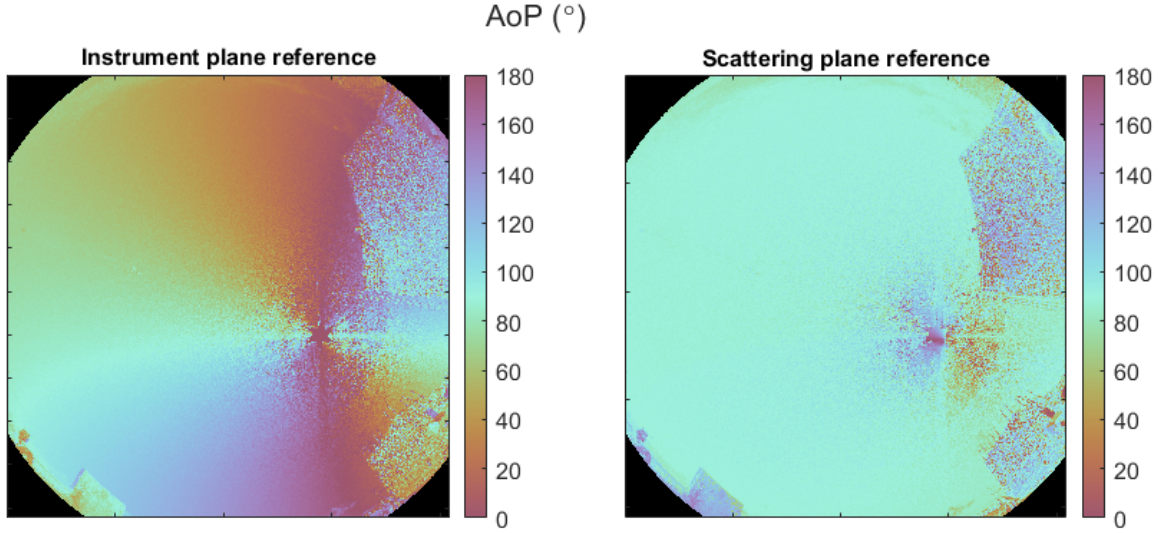


Figure 7: Example all-sky image of AoP referenced to the instrument plane (left) and the scattering plane (right).

and AoP can be calculated. The simplest demosaicking method consists of assuming a reconstruction point at the center of the each superpixel, and assigning the value of the four subpixels within the superpixel to that reconstruction point. However, this method is subject to IFOV misregistration errors, because each subpixel actually measures a slightly different part of the scene. To address these errors, interpolation algorithms are used to calculate a better estimate of the value of each polarization state at the reconstruction point, using multiple of the nearest neighboring pixels of the corresponding polarization state.

The interpolation algorithm used in this work was the bilinear interpolation algorithm presented by Ratliff, LaCasse, and Tyo as Method 3 in [104]. This method used reconstruction points at every vertex between subpixels (except for those nearest to the edges of the image). The estimated value of each polarized image at the reconstruction point was calculated as a weighted average of the nearest three subpixels with the corresponding polarization state, with the weight determined by the distance from the reconstruction point to the subpixel. The vertices closest to the edge of the image had fewer neighboring pixels

within the same distance so were not used as reconstruction points. Thus, a raw image with dimensions of 2048 rows and 2448 columns produced interpolated images of the Stokes parameters, DoLP, and AoP with dimensions of 2045 rows and 2445 columns.

Images of the Moon recorded with the 300-mm telephoto lens are shown in Fig. 8 using the simplest demosaicking method (labeled “No interpolation”) and the bilinear interpolation algorithm (labeled “Interpolation”). Without interpolation, areas with the strongest intensity gradient show a falsely high DoLP and an incorrect AoP which is related to the direction of the gradient. Areas with the highest gradient include the edge of the Moon as well as the edges of some maria. After interpolation, this effect is reduced significantly, though the edge of the Moon still shows somewhat inflated values of DoLP.

Polarimetric imaging applications in this dissertation

The instruments, calibration methods, and image processing methods described here were used in three applications in this dissertation. First, the polarization camera was used with a telescope and a telephoto lens for recording images of the Moon in Chs. 3-5. Second, the polarization camera was used with a fisheye lens for recording all-sky images of clouds in Ch. 6. Third, the polarization camera was used with a lens with an 18°FOV for recording images of underwater objects in the Appendix.

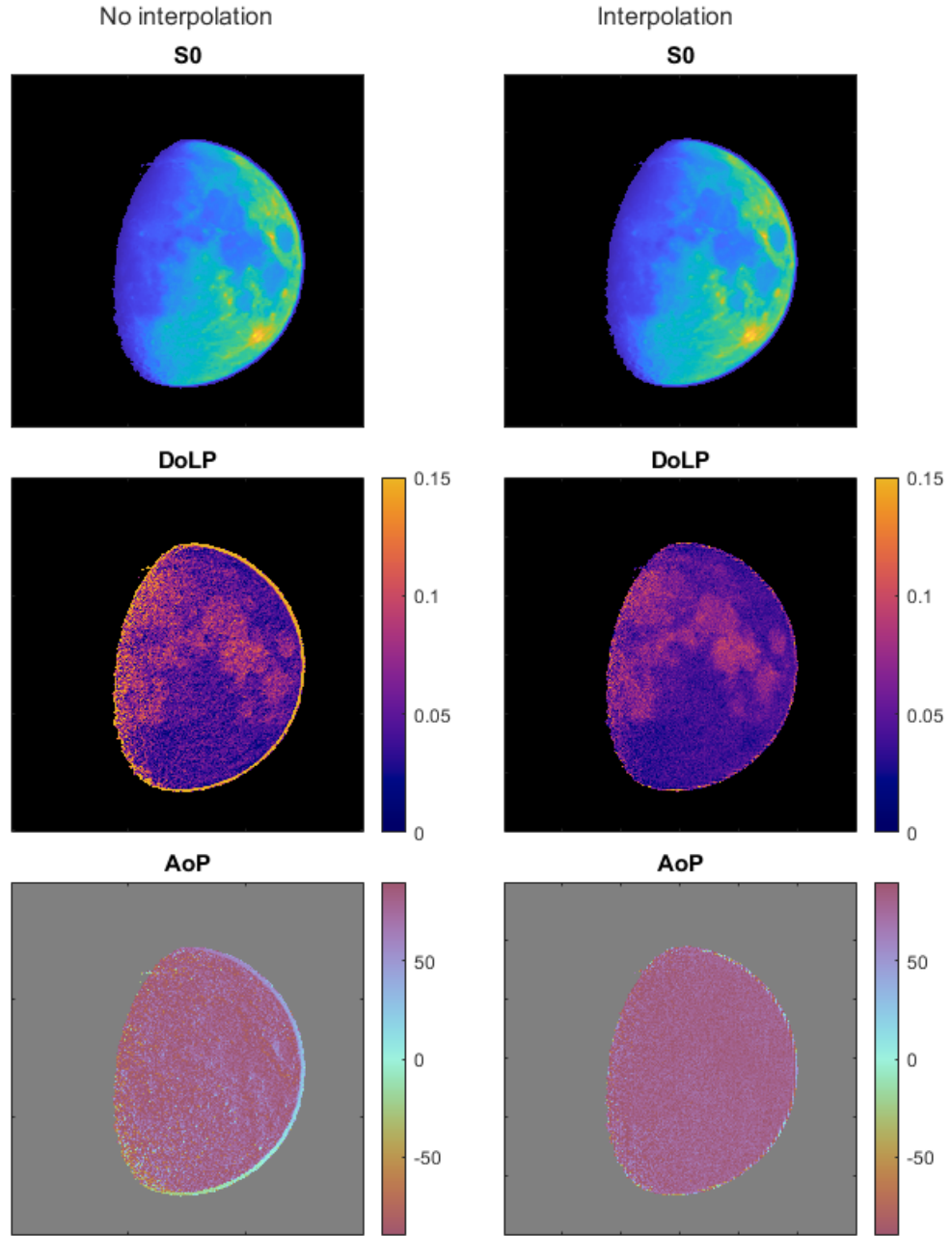


Figure 8: Comparison of S_0 , DoLP, and AoP before and after interpolation

POLARIZATION IMAGES OF THE MOON AS A FUNCTION OF THE LUNAR
PHASE

Contribution of Authors and Co-Authors

Manuscript in Chapter 3

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Contributions: Investigation, formal analysis, data curation, methodology, software, validation, visualization, writing - original draft

Co-Author: Joseph A. Shaw

Contributions: Conceptualization, funding acquisition, methodology, resources, project administration, supervision, writing - reviewing & editing

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Abstract

Moonlight has a partial polarization state that varies with the lunar phase angle. This needs to be characterized for the Moon to be used as a calibration source for polarization-sensitive instruments or as the illumination source in nighttime passive remote sensing. To contribute to this characterization, a pixelated polarization imager was calibrated with a telescope and with a telephoto lens and used to record images of the Moon at 40 phase angles from -138° to 125° . High-resolution images of relative intensity, degree of linear polarization (DoLP), and angle of polarization (AoP) were recorded in a spectral band from 400-1000 nm and were used to calculate disk-averaged DoLP values. These images are made available to readers. Results showed higher DoLP in regions of lower reflectivity (the Umov effect), higher DoLP for waning phases (maximum DoLP of 8.3%) than for waxing (maximum DoLP of 6.8%), and consistent DoLP to disk-averaged values published previously for wavelengths contained within our broader band.

Introduction

The Moon has a partial polarization state that varies with the lunar phase. The randomly polarized light from the Sun becomes partially polarized upon scattering from the Moon's surface. The scattered light is partially polarized with a degree and angle of polarization depending on regolith particle size and composition [53, 55] and scattering angle. Thus, the polarization state of the Moon must be characterized as function of the lunar phase angle. Moonlight is the primary source of natural illumination at night for humans, animals [112], and insects [113], but it is also an illumination source for nighttime remote sensing and a calibration source for satellite-borne instruments. Knowing how the polarization state of the Moon depends on phase angle can extend the utility of moonlight in these two applications.

The ability to perform nighttime remote sensing using moonlight can increase the time in which passive remote sensing is possible by a significant fraction since the Moon is in the night sky 50% of the time on average. Nighttime passive remote sensing could be particularly valuable in the Arctic, where nighttime lasts for not just a fraction of each day but for several months. Monitoring and understanding climate change in the Arctic, where change is occurring most rapidly, is critical to understanding both global climate change and how Arctic change influences midlatitude weather patterns [74, 75]. Several studies have used moonlight to perform passive remote sensing, including ones that directly measured moonlight [60–64, 114] and ones that measured moonlight scattered from aerosols [66, 67]. Only one of these studies addressed the phase-dependent polarization state of moonlight, where the authors assumed randomly polarized moonlight but acknowledged that this is not completely true and that more accurate data are needed to account for this effect [67]. Further, some of these studies used instruments that are likely to have polarization sensitivity, which would create errors if ignored [65]. Our interest in moonlight polarization is that we have developed ground-based polarimetric methods to perform remote sensing of cloud thermodynamic phase [90, 92] and we seek to extend those methods to operate using moonlight illumination, so we need to better understand the polarization state of moonlight to assess its potential impact on the polarization of scattered light observed by the imager.

Additionally, much work has been devoted to characterizing the Moon as a calibration source for instruments operating in the visible through shortwave infrared [7, 8, 10, 16, 115] and even microwave [116] wavelengths. Satellites have used lunar calibrations for correcting instrument drift [3, 11, 13, 14] as well as for instrument cross-calibrations [41, 117]. However, without accounting for the polarization state of moonlight, the utility of lunar calibrations is limited for instruments that have polarization sensitivity, either by design or inadvertently. Both instruments designed to measure polarization and instruments with unintentional polarization sensitivity will suffer from errors if the polarization state of the source is not

taken into account [39, 65, 118–120]. Researchers working on a radiometric model of the Moon acknowledged that further measurements must be recorded before a complementary polarization model can be developed [16].

Previous studies have measured the polarization state of the Moon[44–51, 53, 54], and a short review of relevant polarization studies is included in Section 3. The two primary types of polarization studies relevant to this work are disk-averaged and disk-resolved measurements, which characterize the total polarization state of moonlight and the spatially resolved polarization state, respectively. Other studies have measured the polarimetric properties of soils from Apollo samples [121], have been planned to conduct polarization measurements from a lunar orbit [122], and have recorded polarimetric measurements of Earthshine [123].

In this work, we used a division-of-focal-plane (DoFP) polarization camera to record images of moonlight polarization at a variety of lunar phase angles in a broad panchromatic band ($\sim 400\text{--}1000\text{ nm}$). Specifically, we used a camera with a polarized sensor from Sony[124] that has higher polarization contrast and lower cost relative to previous DoFP sensors. While a variety of studies on moonlight polarization have already been published, there is evidence in the literature that the existing data are not yet sufficient to develop a quality model of moonlight polarization that can be used by the lunar calibration and nighttime remote sensing communities [16, 67]. Our measurements add to the existing body of data, and we make those data available in Dataset 1 (Ref. [125]). Specifically, Dataset 1 contains one set of S_0 , degree of linear polarization (DoLP), and angle of polarization (AoP) images as comma-separated values (CSV) files at each of the 40 phase angles at which we recorded data, and all raw data (including multiple polarimetric images at most phase angles) and processing code are available in Ref. [126]. The availability of these data will enable researchers to predict with higher accuracy than before how the polarization state of the Moon will affect their measurements, including accounting for spatial variation of polarization and field-dependent

instrument polarization response.

Many applications will require moonlight polarization measurements in narrow spectral bands. Additional narrow-band data are also necessary to refine a wavelength-dependent model of the polarization of moonlight, which is important for lunar calibration of polarization-sensitive instruments [16]. Therefore, the next step in our study is to record similar data in some common narrower visible and near infrared (VNIR) spectral bands by adding bandpass filters to the front of our telephoto lens. The measurement technique we describe also can be adapted easily to enable other researchers to obtain Moon polarization measurements with low-cost equipment in their chosen spectral band, whether they require the disk-averaged polarization state in a spectral band not yet published or spatially resolved polarization images in that spectral band.

The remainder of this paper, which expands on an earlier conference paper [127] by adding a more complete analysis, briefly reviews previous moonlight polarization studies, describes the instruments we used to record these measurements, presents the disk-averaged DoLP as a function of lunar phase angle as well as polarization images at various phases, and discusses the similarity of our results to previously published work.

Background

Disk-averaged measurements describe the total polarization state of the moonlight and can be obtained with a single-pixel polarimeter with a field of view (FOV) capturing the full lunar disk[44], with scans of a narrow-FOV single-pixel polarimeter[45], or with spatial averages of images of the lunar disk (this work). Disk-averaged measurements are useful for on-orbit calibration of polarization-sensitive instruments that capture the whole Moon in the FOV (or the instantaneous FOV of a pixel). Disk-resolved measurements can consist of measurements of specific locations on the lunar surface recorded with an instrument with a narrow-FOV[47, 48] or images in which each pixel provides information about a different

location on the lunar surface[49]. Disk-resolved measurements could be useful for on-orbit calibration of instruments with a small FOV, understanding disk-average measurements at different phases, or for drawing scientific conclusions about the lunar surface in various regions [50]. Table 1 lists several studies of lunar polarization.

Table 1: Summary of Moon polarization measurements (θ_p = lunar phase angle).

Paper	Spectral bands	Measurement description
1964 - Gehrels, Coffeen, & Owings [47]	360, 540, 940 nm	DoLP vs. θ_p for eight lunar regions. Typical θ_p range of -75° to $+100^\circ$ (varied by lunar region)
1967 - Forbes & Welch [45]	1.25, 2.24 μm	Disk-averaged DoLP at $\theta_p = -80.9^\circ$
1970 - Coyne & Pellicori [44]	336, 367*, 383, 449*, 519* nm	Disk-averaged DoLP vs. θ_p . Typical θ_p range of -130° to $+140^\circ$ (varied by spectral band)
1971 - Dollfus & Bowell [48]	Eight bands in 0.327 to 1.05 μm	DoLP vs. θ_p for 14 lunar regions. Typical θ_p range of 0° to $+100^\circ$ (varied by lunar region and spectral band)
1992 - Shkuratov, Opanasenko, and Kreslavsky [54]	420, 650 nm	Polarization properties (e.g., min. & max. DoLP, inversion angle) of 31 regions
2005 - Korokhin & Velikodsky [49]	461, 669 nm	Polarization images for phase angles of maximum DoLP
2007 - Shkuratov et al. [50]	420, 650 nm	Images of DoLP at $\theta_p = 88^\circ$
Continued on next page		

Table 1 – continued from previous page

Paper	Spectral bands	Measurement description
2012 - Snow, Holsclaw & McClintock [46]	220 to 300 nm	Disk-averaged DoLPs for $-80^\circ < \theta_p < 80^\circ$
2015 - Jeong et al. [51]	373.8, 443.5, 558.6, 676.3, 818.9 nm	Maps of maximum DoLP from polarization images in $22^\circ < \theta_p < 121^\circ$
Venkatesulu and Shaw (this study)	Panchromatic (~ 400 -1000 nm)	Polarization images and disk-averaged DoLP vs. θ_p for $-138^\circ < \theta_p < 125^\circ$
* Filters replaced and shifted to slightly different wavelengths partway through study.		

Methodology and Instrumentation

In this work, a commercial division-of-focal-plane polarization camera (FLIR Blackfly BFS-U3-51S5P-C) was deployed at the focus of two imaging systems (a telescope and a telephoto lens) to obtain polarization images of the Moon. Only one DoFP camera was used, and we swapped it between the imaging systems. Images were recorded at a variety of phase angles from -138° to $+125^\circ$ across multiple lunar cycles. This section describes the camera used to record polarization images, the two imaging systems and the procedure used with each to record images, the calibration of each system, and the image processing and analysis.

The polarization camera captured panchromatic images simultaneously in four polarization states (0° , 45° , 90° , and 135° , relative to the long axis of the detector, “horizontal”). An array of wire-grid polarizers on top of the detector controlled the polarization state measured by each pixel. The polarizers were arranged in superpixels made up of 2×2 groups

of pixels, and the polarizers on the four subpixels were oriented such that each superpixel measured intensity in the four polarization states (the top left subpixel measured 90° , the top right 45° , the bottom right 0° , and the bottom left 135°), where intensity was measured in flat-field-corrected digital numbers (DN) that were proportional to W/m^2 of optical power incident on each pixel. This layout was repeated across the detector array so that an image in each polarization state was formed. Images of the linear Stokes parameters, DoLP, and AoP were calculated from the images of the four polarization states using Eqs. 15-19.

$$S_0 = \frac{1}{2}(I_{0^\circ} + I_{45^\circ} + I_{90^\circ} + I_{135^\circ}) \quad (15)$$

$$S_1 = I_{0^\circ} - I_{90^\circ} \quad (16)$$

$$S_2 = I_{45^\circ} - I_{135^\circ} \quad (17)$$

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0} \quad (18)$$

$$AoP = \frac{1}{2} \arctan \frac{S_2}{S_1}. \quad (19)$$

The AoP was referenced to the 0° subpixels, so that an AoP of 0° corresponded to a polarization state parallel to the bottom edge of the image, since no information about the pointing of the lenses or telescope was recorded. The AoP images were interpreted relative to the plane of vision to obtain signed values of DoLP, as described in Sec. 3.

These data were recorded with the polarization camera at the focus of a 300-mm telephoto lens or a 2032-mm telescope (Fig. 9). Two 300-mm lenses were used to record images: a Nikon lens with a maximum aperture of f/4 (AF-S 300mm, f/4) and a Nikon lens with a maximum aperture of f/4.5 (Nikkor*ED 300mm 1:4.5 IF AIS). They were both set to f/11 and calibrated separately. The instantaneous FOV (IFOV) of each subpixel was 0.000669° ($11.7 \mu\text{rad}$ or approximately 4.5 km on the part of the Moon's surface normal

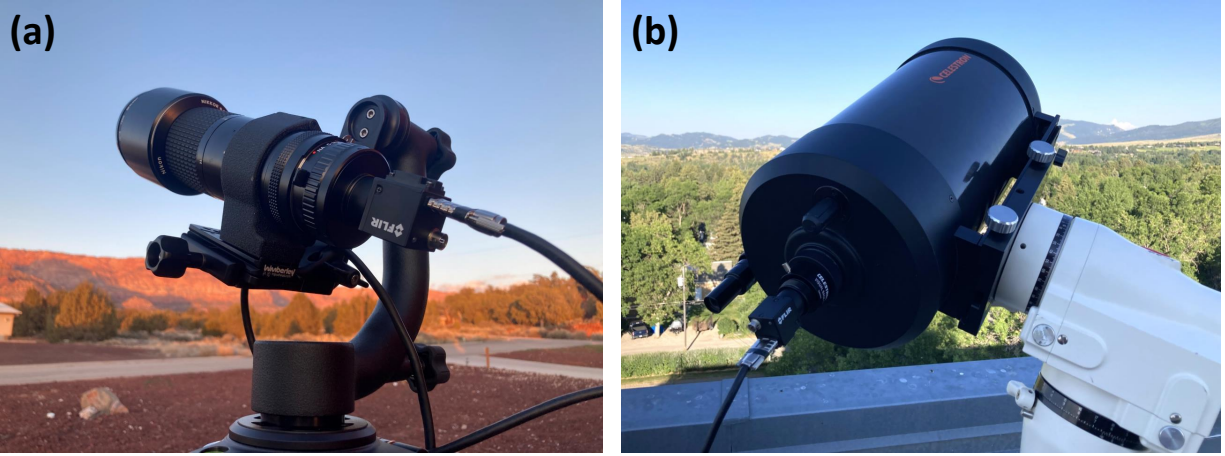


Figure 9: Photos of the DoFP polarization camera deployed at the focus of (a) a Nikon telephoto lens with a 300-mm focal length and (b) a Schmidt-Cassegrain telescope with a 2-m focal length. Reprinted with permission from [128] © Optica Publishing Group.

to the observer-Moon direction) when the camera was used with the 300-mm lenses. The IFOV was calculated using the number of pixels across the Moon’s diameter and the Moon’s diameter reported by the JPL Horizons tool [107] at the time the image was recorded. The Moon was aligned within the frame so that the Moon and a dim ghost image did not overlap. Each night when we used the 300-mm lens system, multiple images were recorded, and the disk-averaged DoLP was calculated from each individual image. The number of images varied based on the time duration of favorable atmospheric conditions and variations in pointing accuracy.

The telescope was a Celestron Schmidt-Cassegrain telescope with an f-number of $f/10$ and a focal length of 2.032 m. The IFOV of each subpixel was 0.000102° ($1.78 \mu\text{rad}$ or approximately 0.7 km on the part of the Moon’s surface normal to the observer-Moon direction) when the camera was used with the telescope (calculated in the same way as the 300-mm lens IFOV). Since the FOV of the telescope was narrow, only a fraction of the Moon could be recorded in one frame; thus, multiple images were recorded and stitched together to form one image of the Moon. The telescope was steered west of the Moon, and several frames were recorded as the Moon passed through the field of view of the telescope.

This process was repeated five to nine times with the telescope aimed to different altitudes to fully capture the dimension perpendicular to the Moon’s motion. This process took an average of 29 minutes, and the longest amount of time over which frames were recorded for one full image was 38 minutes, which corresponded to a phase angle change of 0.2° . The images were stitched together, and overlapping parts of images were averaged to produce the final image. The imaging stitching process is described in further detail in Section 3. An example of a single frame is shown in Fig. 10, and examples of the full, stitched images are shown in Section 3.

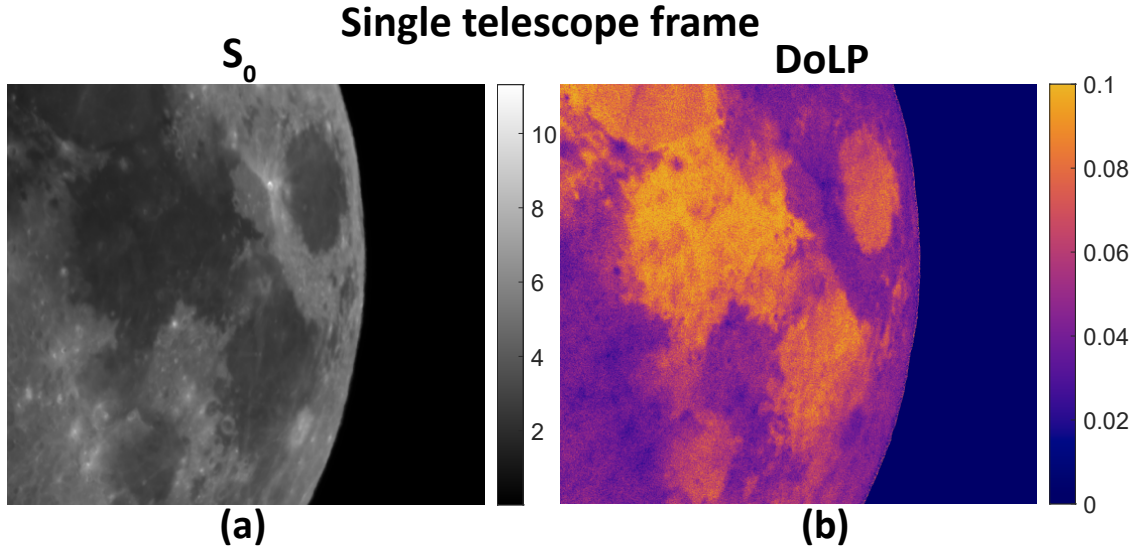


Figure 10: These images show an example of a single frame recorded with the telescope imaging system, processed as (a) S_0 (in $\text{DN}/\mu\text{s}$) and (b) DoLP (on a scale of 0 to 1). See Visualization 1 for high-resolution version. Reprinted with permission from [128] © Optica Publishing Group.

Calibrations

The two imaging systems were calibrated with different methods because of optical and practical differences between the systems. The 300-mm lenses were polarimetrically calibrated with the setup shown in Fig. 11. The setup used an integrating sphere (Labsphere USLR-V12F-NDNN-P) as the source of illumination, and a high-extinction-ratio wire-grid

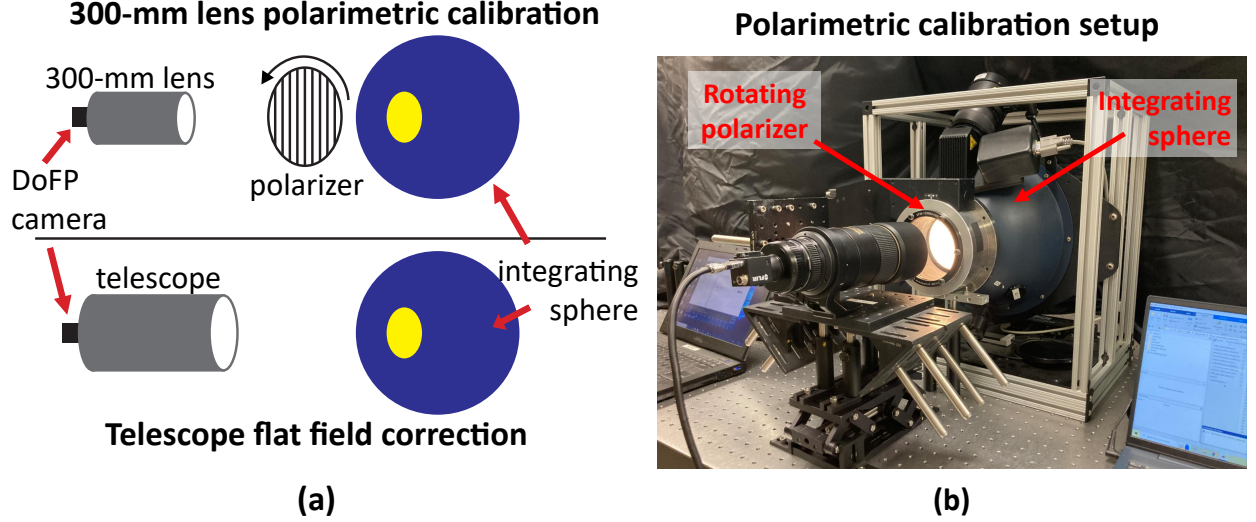


Figure 11: (a) Top shows the schematic of setup used to calibrate the camera with the 300-mm lens, which includes an integrating sphere as the source of illumination and a rotating polarizer to control the polarization state incident on the imaging system. Bottom shows the schematic for the camera with a telescope, which includes only the integrating sphere as a source of randomly polarized illumination. (b) Photograph of the 300-mm lens calibration setup in the lab. Reprinted with permission from [128] © Optica Publishing Group.

polarizer (Meadowlark Optics Versalight, aluminum wire grid on UV-grade fused silica substrate) in a precision rotation stage (Newport RV160CC) to control the polarization state of the incident light.

We performed the polarimetric calibration to find the 3x3 system matrix $[M_{sys}]$ relating the incident linear Stokes vector $[S_{inc}]$ to the measured linear Stokes vector $[S_{meas}]$ (i.e., the linear Mueller matrix relating the incident and transmitted/reflected Stokes vectors). This calibration was performed based on a method used in our group previously to calibrate polarization imagers[129]. The linear Stokes vectors had three elements since no circularly polarized measurements were recorded. We rotated the polarizer to angles of 0° , 60° , and 120° and recorded measurements of these three calibration scenes. We found the system matrix that satisfied

$$[S_{meas}] = [M_{sys}][S_{inc}] \quad (20)$$

at each pixel for these three images, where $[S_{meas}]$ was calculated from all four subpixels in the recorded polarization images using Eqs. 15-17 and $[S_{inc}]$ was calculated from the angle of the calibration polarizer. Since the system matrix was normalized so that the mean of m_{00} was 1, we did not need to include the radiance of the scene in the incident Stokes vector and instead set $S_0 = 1$. Images of each element of the system matrix are shown in Fig. 12. The system matrix for an ideal polarimeter would have diagonal elements equal to 1 and off-diagonal elements equal to 0. Finally, the inverse system matrix $[M_{sys}]^{-1}$ was calculated at each pixel so that the incident (polarimetrically corrected) Stokes vector could be calculated from the measured Stokes vector as

$$[S_{inc}] = [M_{sys}]^{-1}[S_{meas}]. \quad (21)$$

It was sufficient to characterize the response of the system only to linearly polarized light because we did not expect to measure any circularly polarized moonlight. That is, even if m_{03} , m_{13} , m_{23} were non-zero, S_3 was expected to be zero or extremely small.

Calibration accounted for two types of polarimetric errors that can arise in each system. First, the polarimetric aberrations in the imaging system can create a difference between the polarization state entering the system and the polarization state incident on the detector (after passing through all the lens elements). Second, random pixel-to-pixel variation and non-uniformity in the transmission of the polarizers on the detector pixels can create a difference between the polarization state incident on each superpixel and the polarization state measured by that superpixel. The first type of error will be referred to as imaging system error and the second type will be referred to as detector error. In previous work, we found that this DoFP camera with no lens measured a mean DoLP greater than 5% for a randomly polarized source when the detector error was not corrected [103]. This magnitude of detector error depended on the angle of the incident light, so when the randomly polarized

Normalized polarimetric system matrix

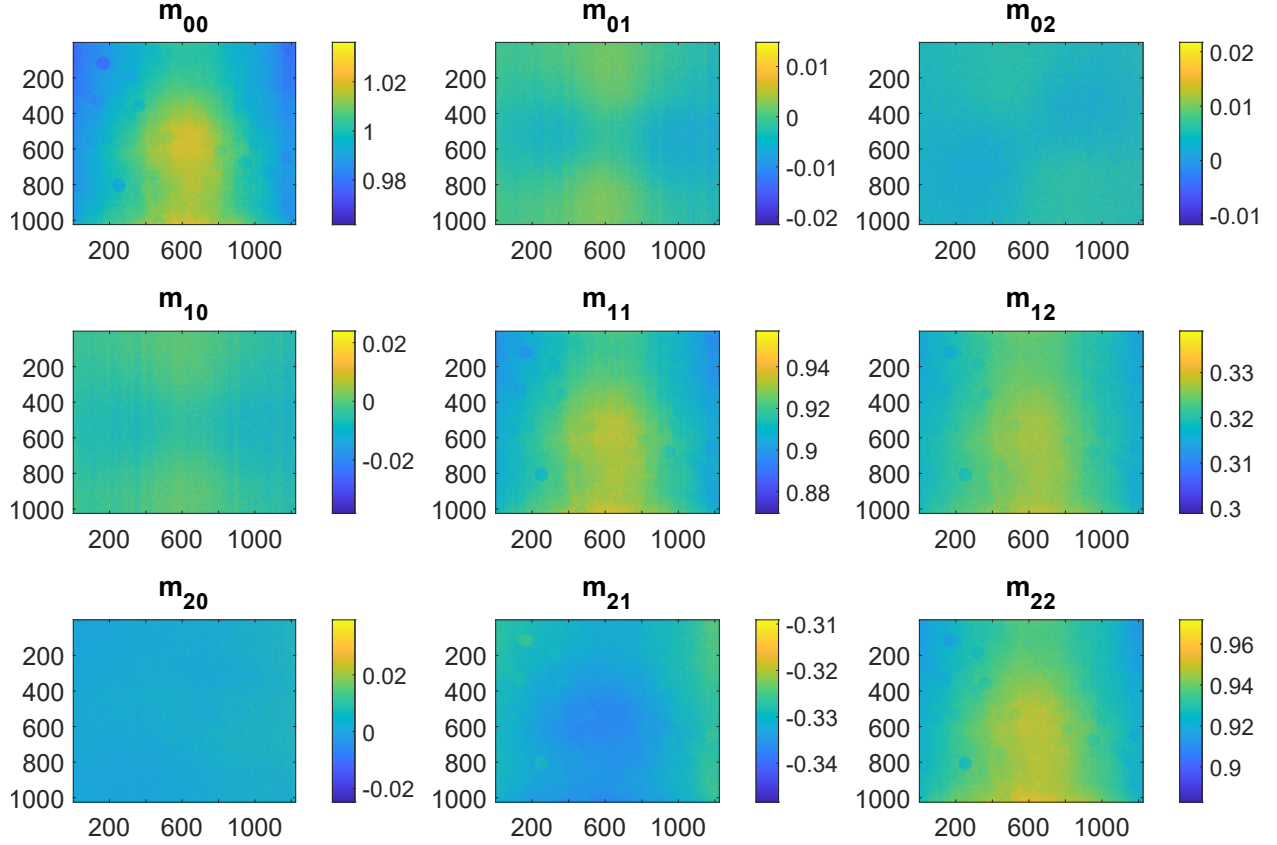


Figure 12: System matrix of the polarization camera used with the 300-mm lens. Each pixel has its own system matrix, so each element is shown as an image. Reprinted with permission from [128] © Optica Publishing Group.

source was imaged by the camera with a narrow-field telescope, the detector error was smaller than 5% but still present (see Table 2).

The polarimetric calibration performed on the 300-mm lenses yielded system matrices that described both imaging system and detector error, so the inverse system matrices corrected for both. However, to perform a polarimetric calibration of the telescope, we would have needed integrating sphere and polarizer apertures that were larger than what we had available. Instead, we performed a flat-field correction that corrects the majority of the polarimetric error in the system. The same telescope model was measured previously to

have a depolarization smaller than 0.1% [130]. This was expected since the f/10 Schmidt-Cassegrain telescope design is symmetric about the axis with weakly curved mirrors, a design which lends itself to small polarimetric aberrations. Thus, the imaging system error was small, and we only needed to correct for the detector error.

The flat-field correction was developed with the following measurements and calculations. *Measurement:* We recorded 100 images of an out-of-focus integrating sphere, such that the detector was uniformly illuminated with randomly polarized light. Also, the light incident on the detector had an angular distribution similar to that of the light incident on the detector when the Moon was imaged, which is important since the transmission of the polarizers on the detector depends strongly on incidence angle. *Calculation:* The 100 images were averaged, and the multiplicative inverse of the average was calculated on a pixel-by-pixel basis. The resulting array was normalized to have a mean of one, and the normalized array was the flat-field correction matrix. Thus, the flat-field correction was performed on a raw image through pixel-by-pixel multiplication of the correction matrix and raw image, and the corrected image had units of DN.

Any measured polarization in the raw images of the randomly polarized integrating sphere scene was caused primarily by random variations in pixel responsivity within the four subpixels in a superpixel or by differences in transmission of the four polarizers within a superpixel (and not caused by telescope polarimetric aberrations or scene polarization), so the flat-field correction canceled out these sources of false polarization. Figure 13 shows the flat-field correction matrix for the camera used with the telescope. The effective S_0 correction shown in Fig. 13(a) accounts for variation in measured total intensity resulting from superpixel-to-superpixel nonuniformity across the detector. The 10 x 10 group of subpixels shown in the raw correction in Fig. 13(b) illustrates how the flat-field correction modified the polarized intensity measured at each individual subpixel so that the overall polarization state measured by each superpixel was modified to correct the detector error. This is further

illustrated in Table 2, where the mean and standard deviation of each uncorrected and corrected polarized intensity are shown for an individual image of the randomly polarized scene used to create the flat field. In histograms of each quantity, the pixel values appeared normally distributed so the mean and standard deviation were a good description of the distribution. The mean values of the four subpixels were closer together after the flat-field correction, and the DoLP calculated from the mean values of the four subpixels decreased after the correction. To assess how well the flat field corrected the telescope images, the same part of the sky was imaged with the telescope and the 300-mm lens system about 45 seconds apart. We ensured that we were imaging the same part of the sky by steering a distant tree into the corner of the frame. The average DoLP measured by the telescope in a 0.9×1.1 mrad area was within 0.6% of the average DoLP measured by the 300-mm lens in an equivalently sized area.

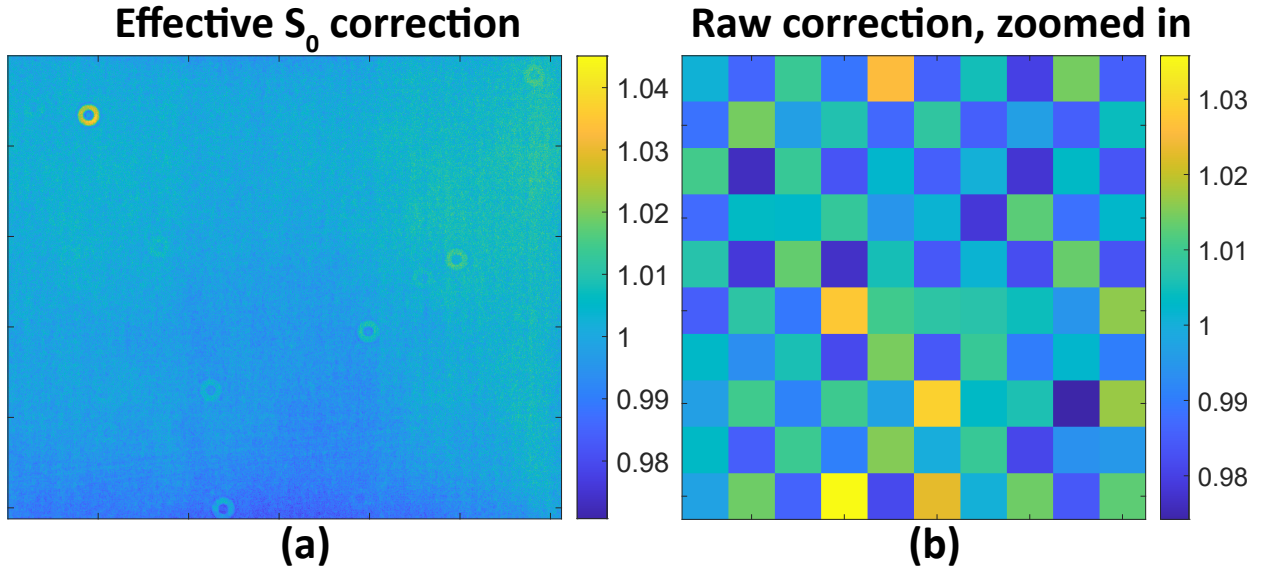


Figure 13: Flat-field correction for the DoFP polarization camera used with the telescope. These two visualizations show (a) the effective flat-field correction for each S_0 superpixel and (b) the raw correction zoomed in to the top left square of 10×10 subpixels (with no demosaicing algorithm, so adjacent pixels show different polarization states). The small circles in (a) are ghost images. Reprinted with permission from [128] © Optica Publishing Group.

Table 2: Summary of polarized quantities measured with a randomly polarized input scene, before and after flat-field correction. Units of polarized intensities are DN/ μ s.

Quantity	I_{0°	I_{45°	I_{90°	I_{135°	DoLP
Uncorrected	3.984 ± 0.080	4.089 ± 0.077	4.009 ± 0.077	4.070 ± 0.080	0.40%
Corrected	4.036 ± 0.074	4.033 ± 0.073	4.039 ± 0.074	4.037 ± 0.073	0.040%

We also characterized the extinction ratio of our polarization camera and explored the spectral weighting of the data within our broad spectral band. We measured the extinction ratio through six bandpass filters and with no filter to quantify the maximum polarimetric uncertainty in camera measurements. The extinction ratio was calculated as the signal measured by a subpixel when the incident polarization state was parallel to the subpixel transmission axis divided by the signal measured by a subpixel when the incident polarization state was perpendicular. It was measured using the high-extinction-ratio polarizer and integrating sphere described above. Table 3 shows the average extinction ratio of a group of subpixels with nearly on-axis rays as a function of wavelength. The extinction ratio decreased with wavelength, with the lowest broadband extinction ratio measured as 68 (average of 82.5), giving a maximum polarimetric uncertainty of $1/68$ or 1.5% (average of 1.2%). To investigate the effect of this decreasing contrast ratio on the measurements of Moon polarization, we multiplied our camera’s relative spectral response (RSR) [103] with a typical lunar spectral radiance curve (assuming flat lens and telescope transmittance). The lunar radiance curve was generated with MODTRAN5 using our 1.6 km ground altitude, the 1976 US Standard Atmosphere, 420 ppm CO_2 , rural aerosols with 23 km visibility, and a direct viewing path to the Moon at 45° zenith angle. The resulting weighted lunar radiance shown in Fig. 14 has a maximum at 561 nm with half-maximum points at 439 nm and 717 nm, for a full-width, half-maximum (FWHM) bandwidth of 278 nm. Only approximately 27% of the received signal lies beyond 671 nm where the minimum extinction ratio is 74, so the low extinction ratios at the longer wavelengths have a limited effect on the overall uncertainty (70% of the signal is at wavelengths for which the polarization contrast is greater than 100). Further,

Table 3: Measured polarization extinction ratio in different spectral bands for nearly on-axis rays. Wavelengths are denoted as center wavelength (FWHM bandwidth) in nm.

Wavelength	440(10)	450(50)	550(10)	650(50)	671(10)	800(50)	broadband
I_{0°	296	356	203	103	89	54	81
I_{45°	517	419	223	128	116	74	102
I_{90°	317	340	205	107	95	55	79
I_{135°	624	359	173	93	74	45	68

the spectral radiance of the Moon is shifted towards blue relative to the spectral radiance of our integrating sphere, so the broadband polarimetric contrast was higher when measuring the Moon than when measuring the integrating sphere.

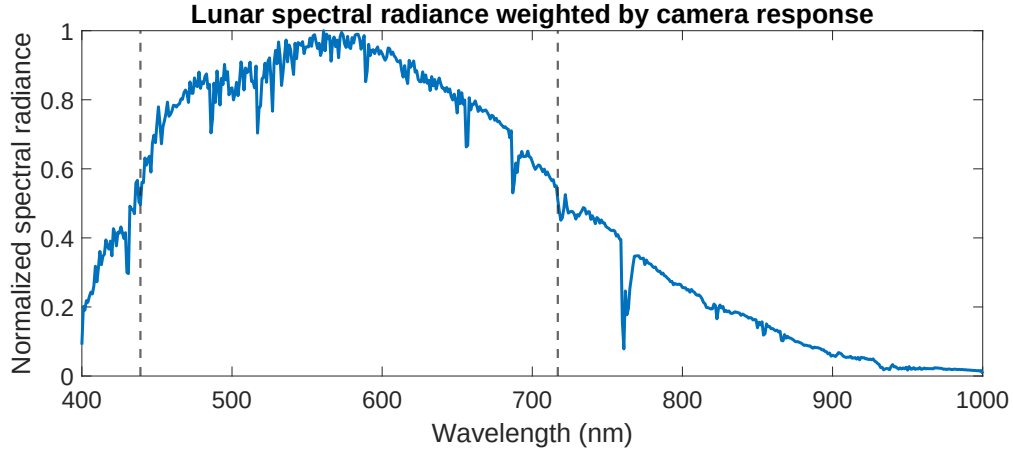


Figure 14: Spectral radiance of the Moon weighted by the spectral response function of the camera and normalized to a maximum of 1 at 561 nm. Dashed lines show the half-maximum points. Reprinted with permission from [128] © Optica Publishing Group.

Image processing and analysis

This section describes the processing performed on each image. The phase angle of the Moon for each image was calculated using the JPL Horizons tool [107]. For images recorded by the 300-mm lenses, the following steps were performed to process the images and calculate a disk-averaged DoLP value:

1. **Subtract dark frame.** A dark image was not available for each night, so the average of ten dark images recorded on eight different nights was subtracted from each raw

image. The ten dark image means varied by less than 1% and the mean dark signal was less than 1% of the mean Moon signal in 99% of images recorded with the 300-mm lenses and less than 0.2% of the mean Moon signal in 66% of images.

2. **Convert raw data to a count rate.** Each image was divided by the exposure time used to record it, so images were in units of $\text{DN}/\mu\text{s}$.
3. **Apply polarimetric calibration.** At each pixel, the polarimetric system matrix (for the corresponding 300-mm lens) was multiplied by the Stokes vector to obtain the polarimetrically corrected Stokes vector.
4. **Subtract background sky pixels.** Following the process described by Schmude [131], the average I_{0° , I_{45° , I_{90° , and I_{135° signal was calculated in a background region chosen manually. In order of priority, the background region was chosen within each frame so that it was away from the image of the Moon, away from the ghost image, and in a part of the sky with a similar scattering angle to that of the sky in front of the Moon.
5. **Apply demosaicing interpolation algorithm.** A bilinear interpolation algorithm was used for demosaicing of the polarization subpixel pattern on the detector. We implemented Method 3 described by Ratliff et al.[104], in which we calculated I_{0° , I_{45° , I_{90° , and I_{135° at each reconstruction point from the nearest three pixels with that polarizer orientation. Edge artifacts were reduced but still present after this algorithm was applied, and the disk-averaged DoLP was not changed by the algorithm.
6. **Create images and a mask of Moon pixels.** Images of the Stokes parameters, DoLP, and AoP were calculated with the interpolated subpixel images. The remaining steps are only for determining disk-averaged values. A mask of Moon pixels was created so that the polarization state of the Moon (and not the surrounding sky) could be

calculated. The mask consisted of all adjacent pixels in the S_0 image that were greater than a manually selected brightness threshold between 3% and 5% of the mean value of the S_0 image to obtain the mask that appeared to best include Moon pixels and exclude sky pixels.

7. **Calculate average polarimetric quantities of masked pixels.** For each image, the mask was used to calculate the spatially averaged S_0 , S_1 , and S_2 . The disk-averaged DoLP was calculated from the spatially averaged Stokes parameters. For each phase angle, the DoLP plotted in Fig. 24 was calculated as the mean of the multiple disk-averaged DoLPs calculated from the multiple images recorded at each phase angle. The standard deviation of the DoLP was calculated by propagating the standard deviation of the quantities I_{0°/S_0 , I_{45°/S_0 , I_{90°/S_0 , and I_{135°/S_0 through the equation $DoLP = \sqrt{(\frac{I_{0^\circ}}{S_0} - \frac{I_{90^\circ}}{S_0})^2 + (\frac{I_{45^\circ}}{S_0} - \frac{I_{135^\circ}}{S_0})^2}$ [132]. The standard deviation can be propagated this way only if the components are uncorrelated [132], but random atmospheric fluctuations were correlated across the four parameters when we measured the four polarization states simultaneously, so we divided each of the four polarized measurements by S_0 to remove these correlated fluctuations (arising from turbulence assumed to have no polarization dependence [133]).

The DoLP images were calculated using Eq. 18, so they only took on positive values. The disk-averaged DoLP values were also calculated with Eq. 18, but the sign of the disk-averaged DoLP was determined by inspecting AoP images and comparing the AoP to the plane of vision (containing the Sun, Moon, and observer) so that positive and negative disk-averaged DoLP values were possible. A positive DoLP corresponded to light polarized perpendicular to the plane of vision, and a negative DoLP indicated light polarized parallel to the plane, consistent with the definition used elsewhere in the literature[48]. The direction of the plane of vision was determined by finding the line

of cusps in the images, which was the line joining the two points on the circumference of the lunar disk between the illuminated and unilluminated regions, and the plane of vision is perpendicular to the line of cusps. When the DoLP was near 0, the AoP images appeared noisy, so the AoP was determined by identifying the maximum on a histogram of AoP values for all pixels.

Images recorded with the telescope system were processed in a similar manner. The steps below were performed to process the images and calculate a disk-averaged DoLP value (here we only specify details that are distinct from the process described above). A MATLAB script was developed to perform background correction for the 300-mm lens images because some of these images were recorded when the Sun was above or barely below the horizon, but all telescope images were recorded when the Sun was at least 10° below the horizon. Therefore, the sky signal was insignificant in the telescope measurements, and no background correction was performed (the sky signal was also insignificant for the majority of the 300-mm lens measurements, but since the script needed to be developed for a few measurements, the background correction was performed for all measurements).

1. **Subtract dark frame.** The same dark frame as above was subtracted from each image. The dark signal was small compared to the signal from the Moon: the mean dark signal was less than 1% of the mean signal of the Moon in 100% of images recorded with the telescope and less than 0.2% of the mean signal of the Moon in 80% of images.
2. **Convert raw data to a rate.** Process same as above.
3. **Apply flat-field correction.** Each image was multiplied by the flat-field correction for the exposure time of that image.
4. **Apply demosaicing interpolation algorithm.** Process same as above.

5. **Combine many frames to form one image of the Moon.** Several frames were stitched together to create a single Moon image. We combined the images by manually finding locations on adjacent frames with the same features on the lunar surface and translating and rotating frames to co-locate those features [134]. We intentionally chose small features on the lunar surface, so the typical translation uncertainty was estimated as 5 pixels, and the maximum translation uncertainty was estimated as 10 pixels, or approximately 7 km on the part of the Moon’s surface normal to the observer-Moon direction. A rotation was only performed when the telescope was steered to a new direction. When two features were selected to rotate two images to the same angle, the features selected were at least 500 pixels apart, so a 10-pixel uncertainty would correspond to a maximum rotation uncertainty of approximately 1.2° . For the seven phase angles where we recorded data with the telescope, the average number of frames used to create one full image of the Moon was 46, with a minimum of 29 and a maximum of 58.
6. **Create images and a mask of Moon pixels.** Process same as above. S_0 , DoLP, and AoP images were calculated with the stitched image, and the next step is for calculating disk-averaged values.
7. **Calculate average polarimetric quantities of masked pixels.** The disk-averaged DoLP was calculated from the spatially averaged Stokes parameters as described for the 300-mm lens images. One value of DoLP was calculated for each phase angle. The sign of the DoLP was determined using the same process described above.

Results

Here we present both disk-averaged and disk-resolved results. Our measurements of disk-averaged DoLP are shown in black and red in Fig. 24, and measurements from the Coyne

and Pellicori G band are shown in green[44]. Error bars show twice the standard deviation of multiple measurements of DoLP. However, the error bars do not capture all of the uncertainty in the data. The maximum polarimetric uncertainty was estimated as 1.5% in Section 3. In addition, the data were collected over multiple lunar cycles, and lunar librations could affect the observed polarization state since the lunar terrain affects the polarization state [16], potentially increasing variability around the fit lines. Four measurements where clouds were visible in the vicinity of the Moon were labeled as having potential cloud contamination, though no clouds were visible by eye directly in front of the Moon for those measurements. There are two sets of fit lines on this plot. The mean DoLPs were used to fit a modified Rayleigh function separately for negative phase angles less than -35° and for positive phase angles greater than 35° , where the resulting fits were

$$DoLP(\theta_p) = \frac{\sin^2(\theta_p - \theta_0)^W}{1 + \cos^2(\theta_p - \theta_0) + D}, \quad (22)$$

where $\theta_0 = -7.90$, $W = 0.87$, and $D = 13.84$ for negative phase angles and $\theta_0 = 7.72$, $W = 0.91$, and $D = 11.43$ for positive phase angles. The modified Rayleigh function was fit only over the phase angle range used previously in the literature[49, 51] for positive DoLP values. The mean and standard deviation of the disk-averaged DoLPs recorded at each phase angle are shown in Table 4. We also fit a fourth-order polynomial (separately for the negative and positive phase angles) to the mean DoLP, which provided an empirical fit to the shape of the measured DoLP vs. θ_p curve for all measured phase angles. At least a cubic term was required to produce the two inflection points in the curve, but a fourth-order polynomial represented the data at the small phase angles more closely. No constant term was included, so the fit had $DoLP = 0$ at $\theta_p = 0$. The resulting polynomial fits were

$$DoLP(\theta_p) = a\theta_p^4 + b\theta_p^3 + c\theta_p^2 + d\theta_p, \quad (23)$$

where $a = 1.95 * 10^{-9}$, $b = 6.87 * 10^{-7}$, $c = 7.03 * 10^{-5}$, and $d = 0.00143$ for negative phase angles, and $a = 2.72 * 10^{-9}$, $b = -8.88 * 10^{-7}$, $c = 8.59 * 10^{-5}$, and $d = -0.00165$ for positive phase angles.

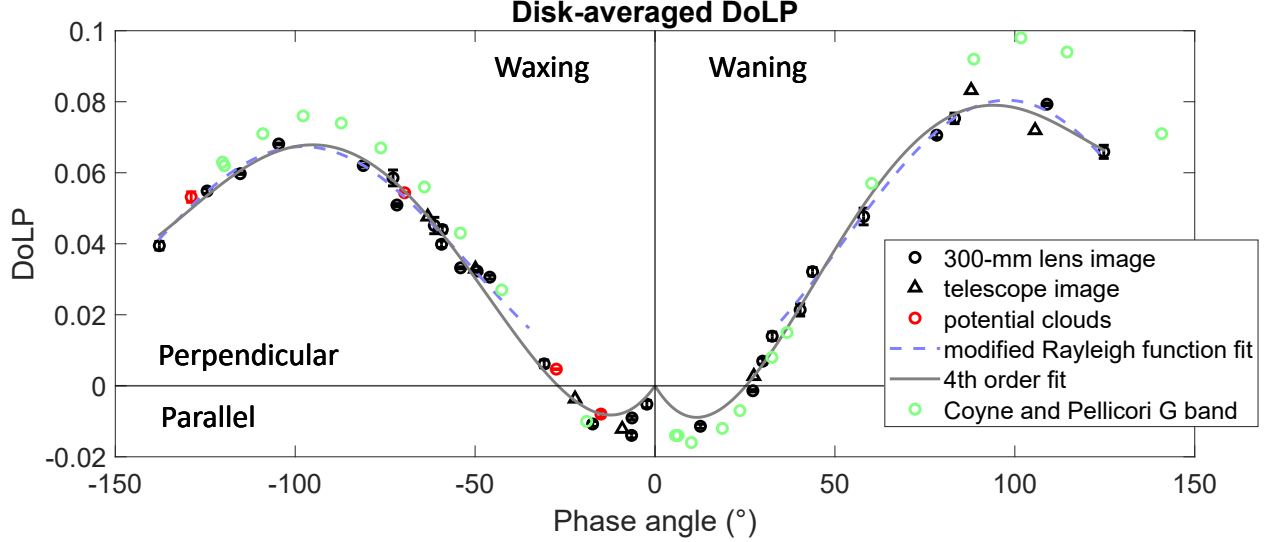


Figure 15: Disk-averaged DoLP as a function of lunar phase angle. Error bars show twice the standard deviation of multiple measurements recorded on the same night. A modified Rayleigh function fit is shown for phase angles with an absolute value greater than 35° and a fourth order polynomial fit is shown for the full phase angle range. Reprinted with permission from [128] © Optica Publishing Group.

Table 4: List of DoLP measurements recorded. θ_p is the lunar phase angle in degrees, DoLP (%) is reported with the mean (μ) and standard deviation (σ) when available, θ_M is the lunar elevation angle, θ_S is the solar elevation angle, and n shows the number of 300-mm lens images used to calculate DoLP ('T' is listed when DoLP was calculated from a stitched-together telescope image).

θ_{P} [°]	DoLP		$\theta_{\text{M}}, \theta_{\text{S}}$ [°]	n	θ_{P} [°]	DoLP		θ_{S} [°]	n
	$(\mu \pm \sigma)$ [%]					$(\mu \pm \sigma)$ [%]			
-137.8	3.95 ± 0.06	6.2, -15.1	10		-17.3	-1.08 ± 0.03	15.9, -20.4	7	
-128.9	5.32 ± 0.07	23.2, -23.3	3		-15.0	-0.80 ± 0.04	46.1, -31.5	5	
-124.5	5.49 ± 0.03	16.2, -12.7	6		-9.1	-1.22	13.9, -17.3	T	
-115.3	5.97 ± 0.01	10.6, -45.7	3		-6.5	-1.40 ± 0.04	63.8, -59.2	4	

-104.6	6.81 ± 0.01	54.0, -13.9	3	-6.4	-0.91 ± 0.02	13.0, -17.8	4
-81.1	6.20 ± 0.02	64.5, -24.0	2	-2.3	-0.52 ± 0.05	37.8, -36.2	11
-72.8	5.85 ± 0.11	63.7, -28.0	7	12.6	-1.14 ± 0.03	15.2, -25.6	5
-71.7	5.09 ± 0.02	35.8, -15.6	5	27.2	-0.14 ± 0.02	20.9, -16.0	2
-69.6	5.44	64.0, -34.0	1	27.5	0.27	17.2, -14.0	T
-63.2	4.76	20.0, -16.3	T	29.9	0.69 ± 0.04	10.8, -21.7	8
-61.3	4.51 ± 0.12	41.3, -11.1	13	32.6	1.40 ± 0.06	49.3, -60.3	8
-59.3	3.99 ± 0.05	37.1, -15.4	7	40.3	2.14 ± 0.09	17.2, -21.4	9
-59.1	4.40 ± 0.05	67.6, -15.6	13	43.8	3.21 ± 0.06	42.1, -60.9	6
-54.0	3.32 ± 0.015	23.2, -10.7	2	58.0	4.77 ± 0.12	29.1, -9.7	4
-49.9	3.29	20.7, -15.4	T	78.3	7.06 ± 0.04	13.9, -11.1	4
-49.4	3.23 ± 0.01	36.2, -13.8	4	83.3	7.53 ± 0.07	25.1, 0.8	4
-45.9	3.06 ± 0.03	51.9, -51.4	4	87.9	8.32	31.0, -17.3	T
-30.8	0.62 ± 0.05	18.7, -15.6	7	105.6	7.19	19.7, -11.4	T
-27.4	0.47 ± 0.005	68.5, -58.3	3	108.9	7.93 ± 0.02	16.0, -11.1	4
-22.2	-0.37	16.8, -18.1	T	124.7	6.59 ± 0.09	40.2, -6.5	13

Next, the disk-resolved results are shown for images recorded by both the telescope and the 300-mm lens systems. We present two examples of S_0 , DoLP, and AoP images recorded by the telescope system. Images are shown for the two example lunar phases, phase angles of -63.2° and 27.5° , in Figs. 16 and 17. We also present two examples of S_0 and DoLP images recorded by the 300-mm lens system, at lunar phase angles of -81.1° and 78.3° , in Fig. 18. The color maps used for DoLP and AoP images in Figs. 16 - 18 were created with code for the CAM02 perceptually uniform colorspace[101].

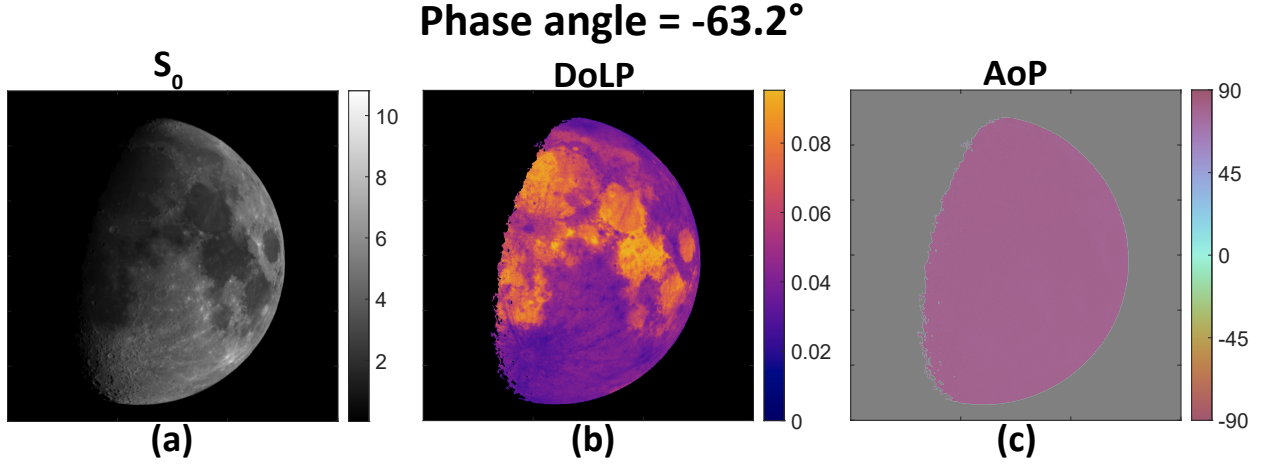


Figure 16: Polarization images of the Moon at a phase angle of -63.2° recorded with the telescope. (a) S_0 image in units of $DN/\mu s$, (b) DoLP image, where DoLP is given as a fraction out of 1, and (c) AoP image, in units of degrees. See Visualization 2 for high-resolution version. Reprinted with permission from [128] © Optica Publishing Group.

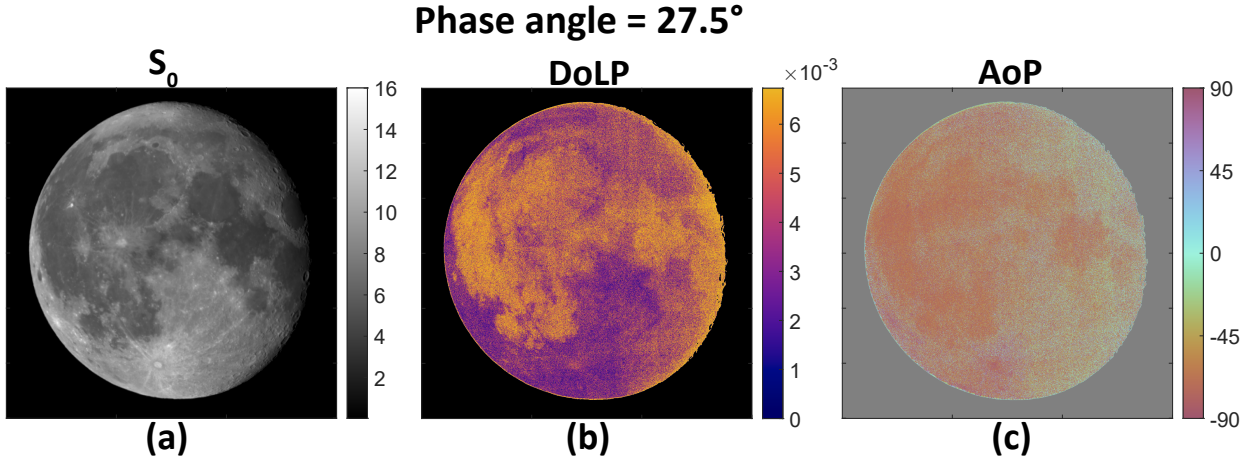


Figure 17: Polarization images of the Moon at a phase angle of 27.5° recorded with the telescope. (a) S_0 image in units of $DN/\mu s$, (b) DoLP image, where DoLP is given as a fraction out of 1, and (c) AoP image, in units of degrees. See Visualization 3 for high-resolution version. Reprinted with permission from [128] © Optica Publishing Group.

Discussion

In this section, we describe trends that can be observed in the DoLP plot and images. First, we observed that the shape of our disk-averaged DoLP curve in Fig. 24 matched well with what has been published previously for narrower bands within our broad spectral

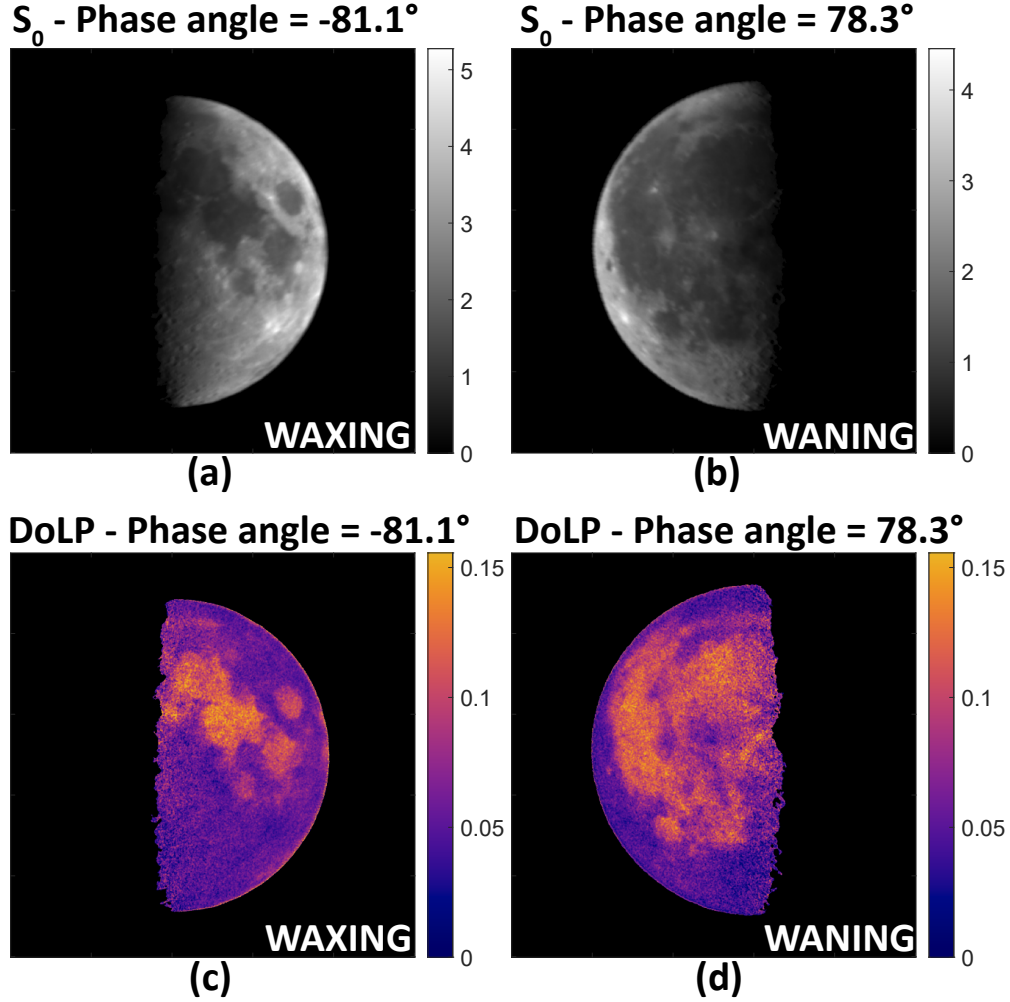


Figure 18: Corresponding S_0 and DoLP images of the Moon at a waxing phase and a waning phase, recorded with the 300-mm lens. In the (a) waxing S_0 image, a small fraction of the illuminated lunar surface has a low albedo, and in the (c) waxing DoLP image, those corresponding areas have a higher DoLP. In the (b) waning S_0 image and (d) waning DoLP image, a larger fraction of the illuminated region has low albedo and high DoLP. See Visualization 4 for high-resolution version. Reprinted with permission from [128] © Optica Publishing Group.

band [44]. The DoLP was close to 0% for a nearly full Moon (at the smallest phase angle we measured, -2.3° , the DoLP was 0.5%). As the absolute value of phase angle increased, the Moon decreased and then increased in polarization until the inversion angle (measured between -27.4° and -22.2° for waxing phases and near 27.3° for waning phases) where the DoLP was again 0%. As described in Section 3, negative DoLP corresponded to a polarization

state parallel to the plane containing the Moon, the Sun, and the observer. Past the inversion angle, the DoLP was positive. The DoLP increased with the absolute value of phase angle until it reached its maximum near approximately 100° , where it began to decrease again. The maximum DoLP measured in this work was 6.8% for the waxing (negative) phase angles and 8.3% for the waning (positive) phase angles. The disk-averaged DoLP values we measured were lower than those measured by Coyne and Pellicori in a spectral band centered at 519 nm [44] (shown in green on Fig. 24), which was consistent with the trend that DoLP decreases with wavelength (our spectral band was centered at 581 nm). Also, Fig. 24 shows that the measurements from the 300-mm lens and telescope agreed well. The telescope measurements at 87.9° and 105.6° visually fell furthest away from the grouping of 300-mm points and the modified Rayleigh function fit, but they were still well within the 1.5% maximum polarimetric uncertainty from the contrast ratio of the detector.

Some specific aspects of these trends can be observed in the images we recorded. In the AoP image in part (c) of Fig. 16, the line of cusps was approximately vertical, so the plane of vision was approximately horizontal. The AoP was uniformly near 90° (relative to the bottom edge of the image), so the polarization state of the Moon was approximately perpendicular to the plane of vision. The corresponding disk-averaged DoLP at -63.2° in Fig. 24 was a fairly large and positive value. In part (c) of Fig. 17, the plane of vision is also horizontal. However, the marial regions showed an AoP near 90° whereas the continental regions showed an AoP closer to 0° . This is consistent with previous observations[48], which showed that the continental regions have a slightly higher inversion angle. In this image, the continental regions were polarized parallel to the plane of vision, so the inversion angle of the continental region is above 27.5° , the phase angle at which the image was recorded. The marial regions were polarized perpendicular to the plane of vision, so the inversion angle in the marial region is below 27.5° . We also observed in both Figs. 16 and 17 that the parts of the Moon that appear darker in the S_0 image had higher DoLPs, so that the DoLP images

resembled (inversely) albedo maps of the Moon. This is called the Umov effect, which is the phenomenon that darker surfaces tend to be more highly polarized, and it has been measured in previous work [44–48]. We also noted that the maximum DoLP was higher for positive phase angles than for negative phase angles; this was also measured in previous work [44, 46]. This resulted from the surface of the Moon itself. There were more dark maria illuminated by the Sun and visible from Earth when the Moon was waning than when it was waxing. This effect can be observed when comparing the images in Fig. 18: the image on the left shows a waxing phase that had less area of low albedo and high polarization than the image on the right, which shows a waning phase.

In addition to the ways in which polarization images can help us understand and visualize the features observed on the DoLP curve, the polarization images can have additional value for lunar scientists. Polarization images provide detailed data about the polarization state in each pixel, which could once only be determined by scans, enabling extensive studies of the relationship between the maximum polarization, maximum polarization angle, minimum polarization, minimum polarization angle, inversion angle, and albedo [131]. These properties can in turn provide information about the lunar surface roughness and mean grain size of the regolith [50, 51]. However, it has been noted that one of the major areas in which progress can be made is through recording additional digital polarization images [131]. We provide such a data set that includes publicly available high-resolution images of DoLP at a wide range of phase angles [125, 126].

Conclusion

We presented panchromatic images of moonlight polarization recorded with a pixelated polarization camera. Images were recorded across 40 lunar phase angles spread over the range from -138° to 125° , corresponding to phases from approximately a 13% waxing crescent to a 22% waning crescent. Results showed higher DoLP in regions of lower reflectivity (the

Umov effect), higher DoLP for waning phases (maximum DoLP of 8.3%) than for waxing (maximum DoLP of 6.8%), and consistent DoLP to disk-averaged values published previously for wavelengths contained within our broader band. These images provide insight into the spatial distribution of polarized moonlight, which is key to interpreting disk-averaged values. Moreover, these images are necessary to predict how the spatial distribution of polarized moonlight would affect measurements by an instrument with a field-dependent polarization response, which could enable higher accuracy in lunar calibration of such instruments. We also used our images to calculate disk-averaged DoLP, and our images and DoLP values agreed well with the existing moonlight polarization measurements in the literature. The method presented here uses readily available, low-cost instrumentation and could be adapted to measure disk-averaged DoLP in narrow spectral bands by adding spectral filters, which will be explored in future work. The combination of this methodology and the publicly available high-resolution polarization images will contribute to decreasing the uncertainty when lunar calibration methods are used with polarization-sensitive instruments and improving passive VNIR polarimetric remote sensing at nighttime.

Parts of Manuscript

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Disclosures

The authors declare no conflicts of interest.

Data Availability

Some data underlying the results presented in this paper are available in Dataset 1, Ref. [125]. Specifically, a set of processed images of S_0 , DoLP, and AoP, for each phase angle, like those images presented in Figs. 16 - 18, are contained in Dataset 1. The remainder of the data underlying the results are available in Ref. [126], including all sets of polarization images, processing code, and calibration files.

VISIBLE AND NEAR INFRARED MOON POLARIZATION ACROSS THE LUNAR
CYCLE

Contribution of Authors and Co-Authors

Manuscript in Chapter 4

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Contributions: Investigation, formal analysis, conceptualization, data curation, methodology, software, validation, visualization, writing - original draft

Co-Author: Elyse C. Duley

Contributions: Investigation, software

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Abstract

Moonlight has a polarization state that depends on the lunar phase angle and wavelength. Specifically, properties that vary with wavelength include the maximum degree of linear polarization (DoLP) and phase angles of maximum and zero DoLP. Polarization measurements of the Moon are important for calibrating on-orbit satellite instruments with polarization sensitivity and for reducing systematic errors in nighttime passive remote sensing data. However, there are limited disk-averaged measurements and images of Moon polarization beyond 600 nm, which limits on-orbit calibration accuracy in this spectral range. To address this, polarization images of the Moon were recorded using a calibrated pixelated polarization camera with 450-nm, 520-nm, 650-nm, 800-nm, and 900-nm filters. Disk-averaged measurements in the 450-nm and 520-nm bands agreed well with previous measurements. To our knowledge, this paper provides the first disk-averaged DoLP values in 650-nm, 800-nm, and 900-nm bands as a function of phase angle.

Introduction

In the ultraviolet, visible, and shortwave infrared parts of the electromagnetic spectrum, moonlight is reflected sunlight. Therefore, the properties of moonlight, including its brightness, color, and polarization state, depend on the reflection and scattering processes that occur before the light originally from the Sun reaches the Earth. Multiple large research projects have been directed at characterizing the spectral irradiance from the Moon [7–10]. The reflectance spectrum of the Moon increases with wavelength, so moonlight is shifted towards the red relative to sunlight; specifically, the albedo at 700 nm is more than 40% greater than 485 nm. Moonlight irradiance at Earth increases with decreasing phase angle, the angle between the Sun, the Moon, and the observer (i.e., there is higher moonlight irradiance during the full Moon with a phase angle close to 0° than during the new Moon

with a phase angle close to 180°). There is a rapid increase in irradiance that occurs for phase angles smaller than approximately 5° known as the opposition effect, which is a departure from Lambertian behavior and has been studied [135]. This characterization work and the resulting models of lunar irradiance as a function of wavelength, phase angle, and libration have enabled on-orbit calibrations [3, 39, 136] and inter-comparisons [41] of many satellite instruments.

Studies of the polarization state of moonlight have also been conducted, but an equivalent model of lunar polarization does not yet exist. Developing a model of the polarization state of moonlight as a function of phase angle and wavelength could enable more accurate on-orbit lunar calibrations for polarization-sensitive satellite instruments [137]. Studies of Moon polarization can generally be divided into two categories: disk-averaged measurements characterize the total polarization state of the visible disk, while disk-resolved measurements provide information about the spatially resolved polarization state. Disk-averaged measurements are more commonly used in lunar calibration, with satellite instruments averaging the pixels filled by the Moon [136] or even designed so the Moon fits in the instantaneous field of view of a pixel [138]. Disk-averaged measurements could also be useful for understanding the polarization state of the illumination for passive nighttime remote sensing measurements. Disk-resolved measurements are most useful in studying the Moon itself, with the local polarization state depending on properties such as the mean grain size and composition of the regolith [53, 55], but they could enable pixelwise calibration of satellite instruments in the future.

A few studies have conducted disk-averaged measurements of the polarization state of moonlight. The most thorough characterization as a function of wavelength was conducted by Coyne and Pellicori, who measured the disk-averaged polarization in five spectral bands from 336 nm to 534 nm over a phase angle range of approximately -130° to $+140^\circ$ [44]. Another study measured the disk-averaged polarization from 220 nm to 300 nm in a phase

angle range of -80° to $+80^\circ$ [46]. The only disk-averaged measurements beyond the visible wavelengths were recorded by Forbes and Welch at 1.25 and 2.24 μm but only at a single phase angle of -80.9° [45]. Several studies have also measured the disk-resolved Moon polarization, including point measurements of specific regions on the Moon [47, 48, 54] and images of the full visible disk [49–51, 57, 128]. A more comprehensive description of the existing literature was included in our previous publication presenting broadband Moon polarization measurements [128].

In this work, we measured the disk-averaged and disk-resolved polarization state of moonlight in five spectral bands centered at 450 nm, 520 nm, 650 nm, 800 nm, and 900 nm using ground-based polarization images of the Moon. Images of the polarization state were recorded with a calibrated division-of-focal-plane polarization camera and averaged to obtain the disk-averaged degree of linear polarization (DoLP). The raw images, calibration data, and code underlying these results are included in a public repository [139]. An abbreviated version of this study was presented in the proceedings of the Polarization Science and Remote Sensing XII conference at SPIE Optics & Photonics 2025, but this paper provides more details on the calibration and image processing procedures as well as details on the modified Rayleigh function fits to the data and estimated uncertainties. A table including all processed disk-averaged values in Fig. 21 is available in the repository as well. The remainder of this paper describes the instrumentation and image processing, presents example images and the measurements of DoLP as a function of phase angle, and discusses the measurements including agreement with previous results and trends in the data.

Instrumentation and Image Processing

The polarization images of the Moon presented in this work were recorded with a low-cost, commercial division-of-focal-plane (DoFP) polarization camera, a 300-mm lens, and five spectral filters. On each night when data were recorded, images of the Moon were

recorded with no filter and with each spectral filter. This section describes the imaging system, calibration process, procedure for recording images, and image processing.

Imaging system

The imaging system was a DoFP polarization camera (FLIR Blackfly BFS-U3-51S5P-C), which simultaneously recorded images through 0° , 45° , 90° , and 135° linear polarizers (relative to the bottom edge of the image). The camera had a micro-polarizer array on the CMOS sensor, with the polarizers arranged in 2x2 groups of pixels (“superpixels”) so each superpixel measured all four polarization states. These four radiance measurements (referred to as L_{0° , L_{45° , L_{90° , L_{135°) were used to calculate the three linear Stokes parameters (S_0 , S_1 , S_2), degree of linear polarization (DoLP), and angle of polarization (AoP) at each superpixel with Eqs. 24-28, creating an image of each quantity. The camera was placed at the focal plane of a 300-mm lens (Nikkor*ED 300mm 1:4.5 IF AIS), and spectral filters were attached to the front of the lens with a 72-to-25-mm step-down adapter. Four spectral filters with a 50-nm full-width half-maximum centered at 450 nm, 650 nm, 800 nm, and 900 nm were used, as well as a Kron-Cousins V filter centered at 520 nm (90-nm bandwidth).

$$S_0 = 0.5 (L_{0^\circ} + L_{45^\circ} + L_{90^\circ} + L_{135^\circ}) \quad (24)$$

$$S_1 = L_{0^\circ} - L_{90^\circ} \quad (25)$$

$$S_2 = L_{45^\circ} - L_{135^\circ} \quad (26)$$

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0} \quad (27)$$

$$AoP = \frac{1}{2} \arctan \frac{S_2}{S_1} . \quad (28)$$

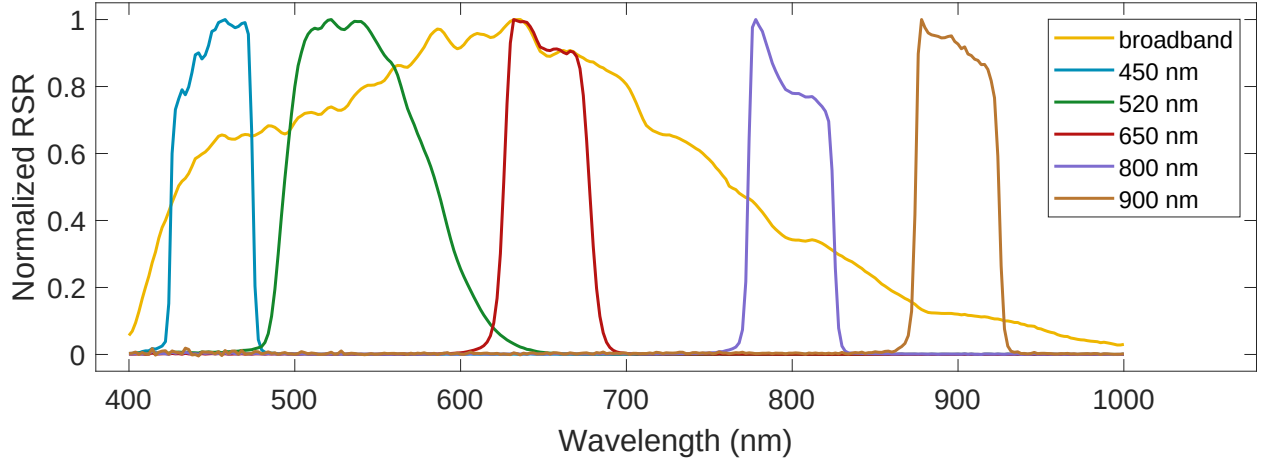


Figure 19: Relative spectral response (RSR) of the imaging system without a filter (labeled ‘broadband’) and of the imaging system through each filter (labeled by their center wavelengths). Each curve is normalized to a maximum of one.

Calibrations

To understand the system’s sensitivity to different wavelengths of incident light, the relative spectral response (RSR) was measured for the system without a filter and through each filter. First, the RSR of the bare detector (no lens or filter) was measured using a monochromator (Princeton Acton SP-150), according to the process described in [103]. Second, the transmission of the lens as a function of wavelength was measured using a spectrometer (Ocean Optics USB4000) and an integrating sphere (Labsphere USLR-V12F-NDNN-P). The spectral transmission curve was calculated as the spectrum of the integrating sphere measured through the lens normalized by the spectrum without the lens. The bare detector RSR was multiplied by the spectral transmission of the lens and normalized to have a maximum of one to obtain the broadband RSR of the system with no filters (Fig. 19). Finally, the spectral transmission curve of each filter was measured in the same way as the lens. The spectral transmission of each filter was multiplied by the broadband RSR and normalized to have a maximum of one to obtain the RSR through each filter, which are also shown in Fig. 19, labeled with the filter center wavelength.

In addition to characterizing the spectral response of the system, the polarimetric response of the system had to be measured. The polarimetric response of the system was described with a 3x3 system matrix that related the linear Stokes vector measured to the linear Stokes vector of the incident light (i.e., a non-normalized linear Mueller matrix). This was measured using the integrating sphere and a high-extinction-ratio wire-grid polarizer (Meadowlark Optics Versalight) on a precision rotation stage (Newport RV160CC). This setup created a known linear Stokes vector, and the 3x3 system matrix was calculated with the measured Stokes vector and the known incident Stokes vector for calibration polarizer angles of 0°, 60°, and 120°. A unique system matrix was recorded at each superpixel for every combination of filter, exposure time, and gain setting. Applying the inverse system matrix to the measured linear Stokes vector corrected both polarimetric effects of the lens and polarimetric errors resulting from imperfections in the polarizers on the detector. This calibration method was the same as the one described in greater detail by Venkatesulu and Shaw [128], with the exception that in this work the incident Stokes vector was calculated in units of radiance using the calibration of the integrating sphere and the normalized RSRs measured for each band, so the system matrix converted the raw images from digital numbers to units of radiance $[W/(m^2sr)]$.

Image acquisition and processing

On each night when data were recorded, we recorded at least one, but typically three, images of the Moon through each filter. Images were recorded with no filter and through each filter every night, unless clouds or other unfavorable imaging conditions prevented us from measuring through all filters. The imaging system was pointed so that the Moon was towards the edge of the frame, minimizing overlap between the primary Moon image and a dim ghost image (a source of unwanted signal). The exposure time and gain settings were chosen so that the maximum digital number in the image (excluding hot pixels) was less than

90% of the saturation value, and a dark image was recorded with the same exposure time and gain settings. The phase angle of the Moon at the time when the image was recorded was found using the JPL Horizons tool [107].

Images were processed with the following steps to obtain calibrated polarization images:

1. Subtract the dark image.
2. Apply the polarimetric calibration for the corresponding filter, exposure time, and gain.
3. Find the background signal in each polarization state from a region of the image away from the Moon and ghost image. Subtract the background signals from the corresponding images.
4. Use the bilinear interpolation algorithm described by Ratliff et al. [104] to decrease edge effects from the DoFP architecture.
5. Calculate S_0 , S_1 , S_2 , DoLP, and AoP images with Eqs. 24 - 28.
6. Create a mask of Moon pixels using a brightness threshold on the S_0 image.

The background signal subtraction was performed to remove any signal from sunlight scattered by the atmosphere in front of the Moon and was discussed further in [128]. Also, the polarization image of the Moon was formed by light that was forward scattered by the atmosphere, and forward scattering does not change the polarization state. Thus, we did not expect propagation through the atmosphere to affect the polarization state of the moonlight. This assumption was confirmed when we recorded images at a variety of elevation angles and aerosol conditions and found consistent polarization states.

Then, to calculate disk-averaged DoLP values:

7. Calculate the mean S_0 , S_1 , and S_2 of Moon pixels

8. Calculate average DoLP from the mean S_0 , S_1 , and S_2
9. Determine the algebraic sign of DoLP.

The most common convention for the algebraic sign of DoLP is positive values correspond to light polarized perpendicular to the plane of vision (the plane containing the Sun, the Moon, and the observer) and negative values correspond to light polarized parallel to the plane. While Eq. 27 only yields positive values, we used AoP images to determine whether the light was polarized parallel or perpendicular to the plane of vision and assigned a positive or negative value to the DoLP accordingly. More details on all image processing steps are detailed by Venkatesulu and Shaw [128].

Disk-averaged and disk-resolved polarization measurements

The method presented in Section 4 was used to obtain polarization images of the Moon on 35 nights over the phase angle range -124° to $+122^\circ$, and these polarization images were used to calculate the disk-averaged DoLP. Figure 20 shows example images recorded at a phase angle of -92° (a waxing Moon approximately 48% full), with each image displaying the DoLP in each band. The colorbars were set to span the same total range (0 - 0.28) for all six images, but the range from white to the maximum hue for each image was set from zero to three times the disk-averaged DoLP in that spectral band. This allowed for visual comparison of the disk-averaged DoLP across spectral bands without decreasing the color contrast for bands with lower polarization. The threshold between maximum hue and black was chosen based on the disk-averaged DoLP rather than the maximum to avoid the possibility of setting the threshold based on the value of a noisy superpixel.

The disk-averaged DoLPs for all measurements are shown in Fig. 21 as a function of phase angle for each spectral band. Modified Rayleigh functions were fit to data with phase angles greater than 35° in magnitude, with the equations for the positive and negative phase

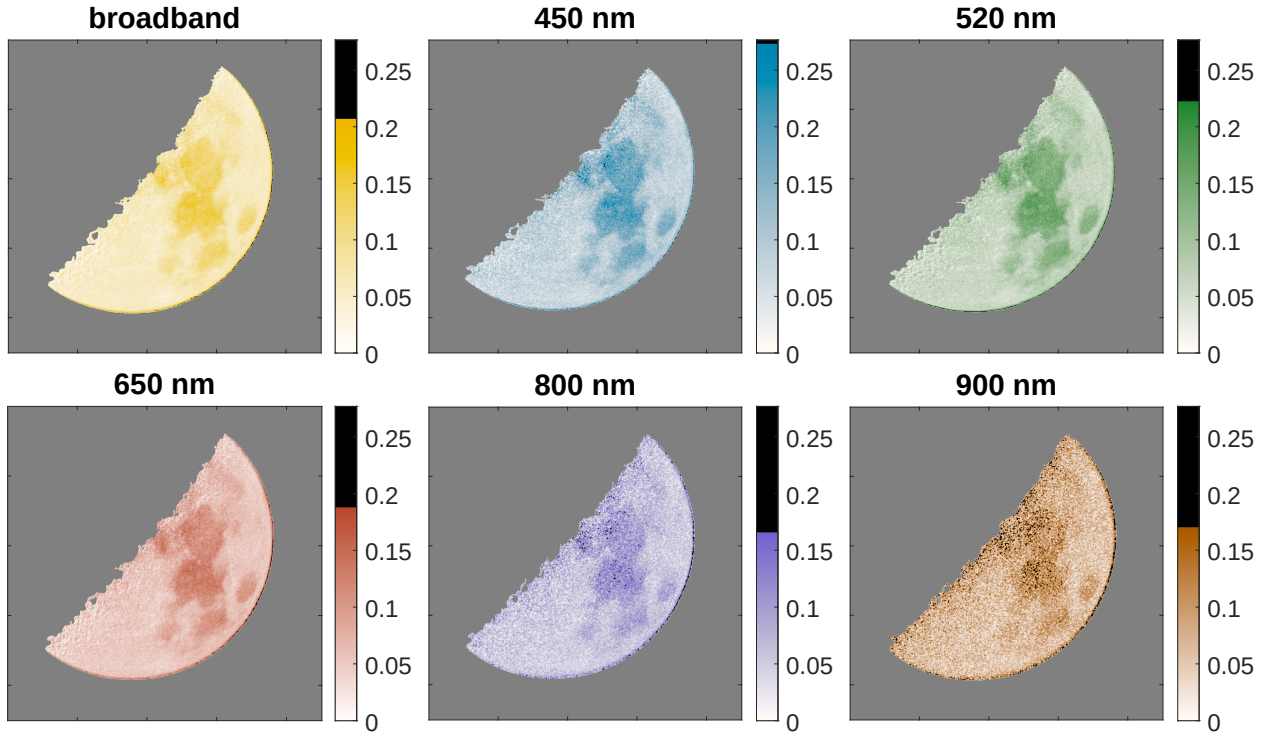


Figure 20: Disk-resolved DoLP of the Moon at a phase angle of -92° , measured without a filter (broadband) and with five spectral filters. The threshold where the colorbar becomes black was set to three times the disk-averaged DoLP in that band.

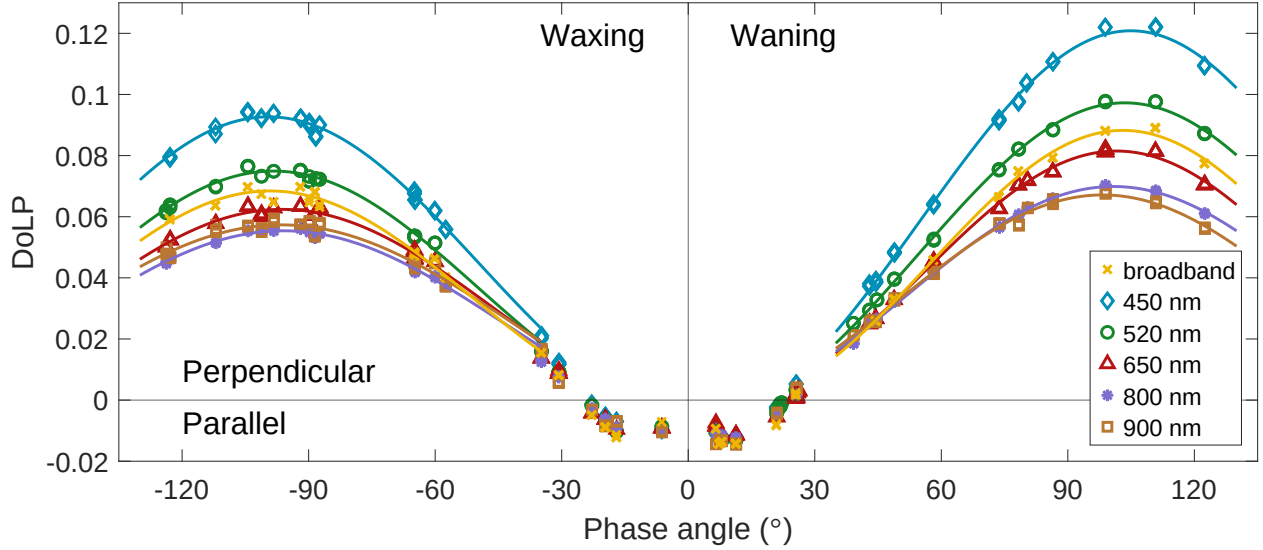


Figure 21: Disk-averaged degree of linear polarization of the Moon as a function of lunar phase angle, measured without a filter (broadband) and with five spectral filters.

angles fit separately. The range of phase angles over which the modified Rayleigh function was fit was based on previous literature [49, 51]. The equation for the modified Rayleigh function was

$$DoLP(\theta_p) = \frac{\sin^2(\theta_p - \theta_0)^W}{1 + \cos^2(\theta_p - \theta_0) + D}, \quad (29)$$

where θ_p was the phase angle, and W , θ_0 , and D are fitted terms. The resulting fits are shown in Table 5, as well as the phase angle and value of maximum DoLP according to the fits.

The noise on the measurements was quantified in two ways. First, the standard deviation was calculated for each set of observations, where a set of observations consisted of at least three consecutive DoLP measurements in one spectral band. There were at least 25 sets of observations for each spectral band, resulting in at least 25 calculations of the standard deviation of the DoLP for each band. For the 450-nm channel, the mean standard deviation was 2.1×10^{-4} and the maximum standard deviation was 5.7×10^{-4} , which would correspond to a 0.42% and 1.1% variation on a DoLP of 0.05. The mean and maximum

Table 5: Coefficients for the modified Rayleigh function, maximum DoLP, and phase angle of maximum DoLP for each spectral bands.

Wavelength	broadband	450 nm	520 nm	650 nm	800 nm	900 nm
Waxing						
W	0.8396	0.7669	0.8142	0.775	0.766	0.7071
θ_0	9.196	9.52	8.323	6.362	6.017	6.537
D	13.61	9.79	12.35	15.03	17.05	16.44
$DoLP_{max}$	0.0684	0.0927	0.0749	0.0624	0.0554	0.0573
$\theta_{p,max}$	-99.2	-99.5	-98.3	-96.4	-96.0	-96.5
Waning						
W	0.8844	0.7424	0.784	0.8726	0.7975	0.8307
θ_0	13.07	14.92	13.48	11.56	10.83	8.256
D	10.33	7.276	9.282	11.27	13.31	13.9
$DoLP_{max}$	0.0882	0.1208	0.0973	0.0815	0.0699	0.0671
$\theta_{p,max}$	103.1	104.9	103.5	101.6	100.8	98.3

standard deviations were $1.6 * 10^{-4}$ and $9.0 * 10^{-4}$ for the 520-nm channel, respectively; $2.0 * 10^{-4}$ and $9.2 * 10^{-4}$ for the 650-nm channel; $1.7 * 10^{-4}$ and $5.9 * 10^{-4}$ for the 800-nm channel; $3.2 * 10^{-4}$ and $1.3 * 10^{-4}$ for the 900-nm channel; and $1.3 * 10^{-4}$ and $5.3 * 10^{-4}$ for the broadband channel.

Second, we used 100 repeated measurements of a stable integrating sphere source to characterize the signal-to-noise ratio (SNR) of each pixel for the broadband channel and for each filter. The gain was set to the same setting used for all images of the Moon, and the exposure time was set to a typical setting used when recording images (0.05 s for the broadband channel and 0.1 s for the rest). The integrating sphere brightness was then set in the middle of the dynamic range of the sensor. This did not provide an exact match to the brightness of the Moon, which changed with phase angle anyway, but it did allow us to characterize the camera in a representative situation. The mean SNR of the individual subpixels was 45.0 for the 450-nm channel, 79.3 for the 520-nm channel, 65.8 for the 650-nm channel, 29.7 for the 800-nm channel, 23.0 for the 900-nm channel, and 90.3 for the broadband channel. Then, we calculated the condition number describing how much noise in the individual subpixel measurements propagated to polarimetric error in the Stokes

vector calculated at a superpixel and thus how much lower the polarimetric SNR was than the SNRs reported above [99]. We found that mean condition number across all superpixels was 1.4395 for the 450-nm channel, 1.4357 for the 520-nm channel, 1.4432 for the 650-nm channel, 1.4617 for the 800-nm channel, 1.4769 for the 900-nm channel, and 1.4494 for the broadband channel. While the polarimetric SNR of one superpixel was fairly low, many superpixels were averaged to calculate the disk-averaged DoLP (over 10^4 superpixels for a thin crescent Moon), yielding a final SNR greater than 10^3 for all spectral channels.

These two analyses provided different characterizations of the noise contained in a measurement. The first approach using the standard deviation of multiple measurements captured additional effects such as potential atmospheric effects, so it made sense that the first approach gave a larger estimate of the noise. Thus, we believe it gave a more realistic characterization of the noise contained in the measurements.

Discussion

The data presented here generally followed trends established in the literature. First, for all six bands, the shapes of the DoLP vs. phase angle curves were consistent: the DoLP was zero at the full Moon (phase angle of 0°), took on negative values (i.e., parallel to the plane of vision) until the inversion angle near $\pm 20^\circ$, increased to a maximum near $\pm 100^\circ$, and then decreased again. The maximum DoLP was higher for waning phases (positive phase angles) than for waxing phases (negative phase angles). Second, the DoLP was generally found to decrease with increasing wavelength; this trend was observed for disk-averaged DoLP values for bands less than 534 nm by Coyne and Pellicori [44] and for disk-resolved values of specific locations on the Moon from the UV to NIR by Dollfus and Bowell [48]. Finally, the phase angle of maximum DoLP was found to depend slightly on wavelength. Coyne and Pellicori observed this dependence and found that the exact phase angle of maximum DoLP depended on libration [44], so we would not necessarily expect to observe the same phase

angles corresponding to the maximum. However, we did observe the same patterns observed by Coyne and Pellicori continuing to longer wavelengths [44]. For waning phases the phase angle of maximum DoLP decreased monotonically with increasing wavelength; however, for waxing phases, while the phase angle of maximum DoLP did still decrease in magnitude with wavelength, it was a smaller change and not a monotonic trend.

An interesting exception to the trend that DoLP decreases with wavelength was that the 900-nm band exhibited slightly higher polarization than the 800-nm band for the waxing phases. The difference between the 800-nm and 900-nm channels was greater than the maximum noise observed (as estimated by the standard deviation of repeated measurements). We investigated this further by averaging the DoLP of maria and highland regions for each spectral channel for a pair of Moon images recorded at approximately -92° and $+86^\circ$. The maria and highland regions were chosen manually; it should be noted that different locations were chosen for negative and positive phases based on the areas of the Moon that were illuminated. We found that the 900-nm channel was more polarized than the 800-nm channel only for the highland regions that were illuminated during negative phases. Also, we used the ROLO model to investigate the disk-averaged albedo of the Moon in the NIR [7]. For both positive and negative phase angles with a magnitude between 40° and 100° , the albedo was lower for bands around 935 nm than bands around 870 nm, but the magnitude of the decrease was larger for negative phase angles than for positive phase angles. It is well known that DoLP increases as albedo decreases (Umov's effect), and our images showed this effect. Thus, this relative change in albedo predicted the model partially supported our observation that the 900 nm channel had a higher polarization state for only the negative phase angles.

Beyond these general trends, the measurements recorded through the 450-nm and Kron-Cousins V (520-nm) filter also show close agreement with the data presented by Coyne and Pellicori in the same spectral bands[44]. To investigate their agreement quantitatively, we performed the following two-step process. First, using the modified Rayleigh function

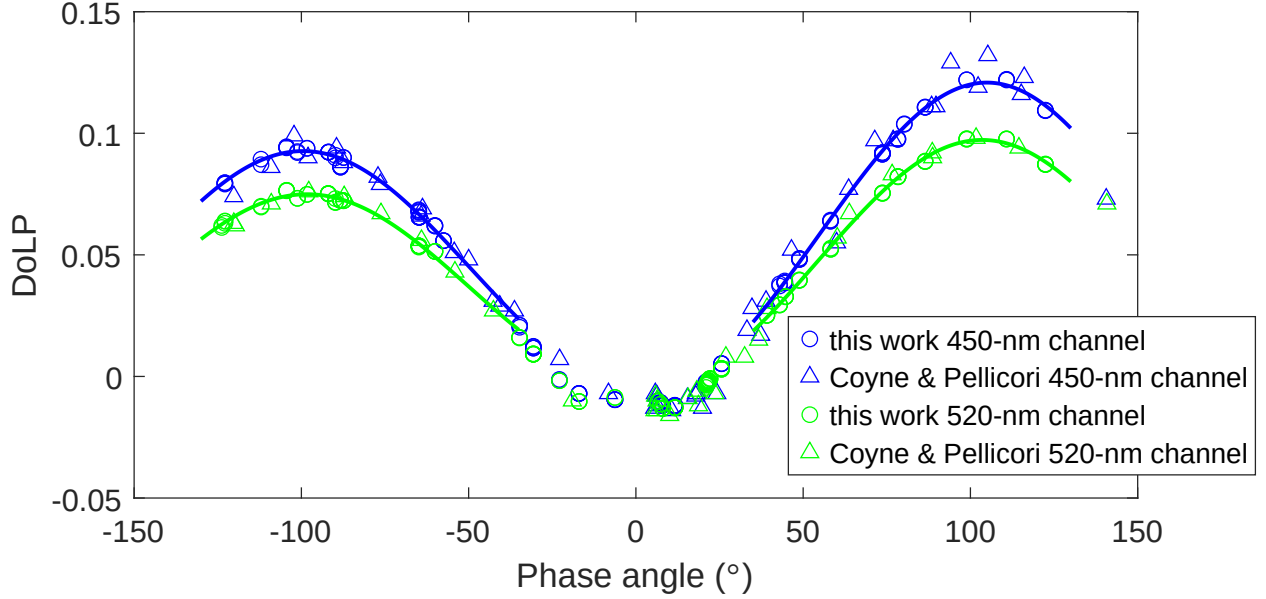


Figure 22: Disk-resolved DoLP of the Moon in 450-nm and 520-nm channels measured in this study and by Coyne and Pellicori [44]. The solid lines show the modified Rayleigh function fit to the data from this study.

determined by our measurements for the 450-nm and 520-nm bands, we predicted the DoLP at the phase angles measured by Coyne and Pellicori[44] and calculated the residuals (i.e., the difference between our prediction and their measurement). Second, we performed a t-test on the residuals and found no evidence that the mean of the residuals was different from zero (p-value of 0.95 for the 450-nm channel and 0.46 for the 520-nm channel). The two sets of measurements and the modified Rayleigh function fit to our data were plotted together as a visual comparison in Fig. 22. This close agreement supported the accuracy of our calibration and processing methods, thus supporting the accuracy of the new DoLP measurements in the 650-nm, 800-nm, and 900-nm spectral bands.

Conclusions

In this work, we recorded polarization images of the Moon through five spectral filters across phase angles from -124° to $+122^\circ$. Two spectral bands measured, centered at 450

nm and 520 nm, agreed closely with previous measurements. To our knowledge, our measurements with the other three spectral-bands, centered at 650 nm, 800 nm, and 900 nm, provided the first published disk-averaged DoLP values at those wavelengths. These measurements displayed expected trends, including the negative polarization close to the full Moon and the decrease in polarization with increasing wavelength. The disk-resolved images of DoLP we recorded could contribute to studies of lunar regolith. Finally, the disk-averaged values of DoLP presented in this paper could contribute to more accurate lunar calibration of on-orbit polarization-sensitive instruments and more accurate passive polarimetric remote sensing for instruments operating near 650 nm, 800 nm, or 900 nm.

Parts of Manuscript

Disclosures

The authors declare that there are no financial interests, commercial affiliations, or other potential conflicts of interest that could have influenced the objectivity of this research or the writing of this paper.

Code, Data, and Materials Availability

The datasets and software for this research are archived and freely available in the repository authored by Venkatesulu et al.[139], accessible at <https://doi.org/10.6084/m9.figshare.c.7835132.v1>.

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MODEL OF DISK-AVERAGED MOON POLARIZATION

Contribution of Authors and Co-Authors

Manuscript in Chapter 5

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Abstract

The Moon can be used as a stable calibration target for on-orbit satellite instruments without the need for solar diffusers or atmospheric correction. Radiometric models of the Moon have been developed to enable lunar calibration; however, to the authors' knowledge, a polarization model of the Moon to improve lunar calibration of polarization-sensitive instruments has not been published. In this work, a model of the disk-averaged degree of linear polarization in terms of lunar geometry variables was developed for nine spectral bands (336 nm, 365 nm, 383 nm, 450 nm, 519 nm, 650 nm, 800 nm, 900 nm, and a broadband channel with a response from 400 nm to 1000 nm) using measurements from three previously published datasets. Specifically, a model dependent on phase angle and Sun selenographic longitude was used for all nine channels, and a second model incorporating libration variables (observer selenographic latitude and longitude) was used for the three channels with the most observations. The uncertainty in degree of linear polarization predicted by the simpler model was estimated as ± 0.01 , and the uncertainty for the model with libration dependence was estimated as ± 0.005 .

Introduction

Earth-observing satellites contribute critical measurements to applications ranging from monitoring Earth's radiation budget [140, 141] to estimating evapotranspiration for better understanding of the water cycle [19] to predicting the spread of malaria [142]. The quality of data from these satellites depends directly on the calibration of the onboard instruments [4, 5]. It is also important to be able to compare measurements from different satellite instruments for maintaining long term records [141] or obtaining more frequent measurements of a particular location on Earth.

There are a variety of methods to perform calibration and inter-comparison of satellite

instruments. Solar calibration can be performed using a solar diffuser to track changes in the response of the reflective solar bands of the instrument [21–23, 143, 144]; however, due to degradation of solar diffusers, it is common to use solar diffuser stability monitors [21, 22] or multiple solar diffusers at different temporal intervals [23]. Some satellites include calibration sources such as lamps [145] or blackbodies [146] to monitor the performance of detectors operating in different spectral ranges, but the stability of these sources over time must also be monitored. Other calibration targets include pseudo-invariant calibration sites (PICS) on Earth’s surface and deep convective clouds [33], which can be used for long-term trending, inter-calibration, and even absolute radiometric calibration [27]. However, these methods require some atmospheric correction to infer the signal from the calibration target, inducing additional uncertainties. The Moon has increasingly been used as a target for on-orbit calibration, avoiding some of the challenges associated with other calibration methods. The surface of the Moon is extremely stable and a similar reflectance to Earth scenes, so lunar calibration does not require a solar diffuser for use as a calibration target and is not limited in accuracy by the performance of a stability monitor. The Moon is also outside the atmosphere, so lunar calibration is also not limited in accuracy by the performance of an atmospheric correction.

Multiple studies measured or plan to measure the spectral irradiance of the Moon to enable and improve lunar calibration [7–10, 38]. In particular, the model based on measurements from the RObotic Lunar Observatory (ROLO) has been widely used [7]. The Moon has since been used as a reference for on-orbit calibrations [3, 12, 24, 39, 147, 148] and inter-comparisons [41] of many satellite instruments. However, for satellite-borne polarimeters or other instruments with polarization sensitivity, the polarization of the Moon would create errors in lunar calibration if not accounted for. The existing radiometric measurements and models focused on the disk-averaged signal from the Moon, so a complementary model of disk-averaged degree of linear polarization (DoLP) of the Moon

would contribute to reducing polarimetric errors. The lack of an existing model and its potential benefit has been acknowledged in the literature [16].

This paper introduces a polarization model of the Moon that predicts the disk-averaged DoLP as a function of lunar geometry variables based on disk-averaged DoLP measurements across nine spectral channels ranging from 336 nm to 900 nm. Modeling the disk-averaged DoLP is useful for supplementing the models of disk-averaged lunar radiance that are currently used in lunar calibrations (where all the pixels imaging the Moon are averaged). This remainder of this paper describes how the datasets were combined, explains the model and when libration was used, presents the resulting fits with example plots, and shows example predictions of DoLP.

Dataset

We combined data from Coyne and Pellicori [44] with data that we previously recorded (broadband data published [128] and publicly available [126] and data in visible and near infrared spectral bands submitted for publication and publicly available [139]). We recorded high-spatial-resolution images in those studies, which could theoretically contribute to a disk-resolved model of the Moon, but our objective for this model was to predict the disk-averaged DoLP. This section describes our methods for manipulating those data to form the combined dataset that is the basis of the lunar polarization model.

We recorded measurements of the disk-averaged DoLP in six channels: “broadband” [128] and five narrower bands centered at 450 nm (full-width half-maximum bandwidth of 50 nm), 520 nm (90 nm), 650 nm (50 nm), 800 nm (50 nm), and 900 nm (50 nm) [139]. The broadband channel had a spectral response across the visible and near infrared (VNIR), from approximately 400 nm to 1000 nm. Coyne and Pellicori recorded measurements of the disk-averaged DoLP in spectral bands centered at 336.0 nm (effective bandwidth of 33 nm), 362.5 nm (36 nm), 367.0 nm (44 nm), 383.0 nm (25 nm), 449.0 nm (110 nm), 450.2 nm (49

Model without libration	336 nm	365 nm	383 nm	450 nm	519 nm	650 nm	800 nm	900 nm	broad-band
Model with libration				450 nm	519 nm				broad-band
Data source (paper #)	1			1 & 3		3			2

Figure 23: Summary of the the source of data for each of the nine channels: Paper 1 refers to data from Coyne and Pellicori [44], Paper 2 refers to data from Venkatesulu and Shaw [128], and Paper 3 refers to data presented in a submitted work by Venkatesulu et al. and made publicly available [139]. The figure also describes which channels were fit to models with and without libration variables.

nm), 518.7 nm (92 nm), and 534.0 nm (56 nm) [44]. Over the whole measurement period, the filters centered at 336.0 nm and 383.0 nm were always used; the filters centered at 367.0 nm, 449.0 nm, and 518.7 nm were used for their first measurement period and replaced by filters centered at 362.5 nm, 450.2 nm, and 534.0 nm, respectively. We chose to combine the data recorded with the 367.0-nm and 362.5-nm filters into one set of measurements, referred to by a wavelength of 365 nm. Similarly, we combined data recorded with the 449.0-nm and 450.2-nm filters as well as the measurements we recorded in a 450-nm band into one set of measurements, referred to by a wavelength of 450 nm. Finally, we combined Coyne and Pellicori’s 518.7-nm data with the measurements we recorded in a 520-nm channel, referred to by a wavelength of 519 nm. We chose to exclude measurements in the 534.0-nm channel because they did not sufficiently sample phase angles as well as measurements from Coyne and Pellicori collected during a lunar eclipse. In summary, we fit a model for nine spectral channels: 336 nm, 365 nm, 383 nm, 450 nm, 519 nm, 650 nm, 800 nm, 900 nm, and broadband. Figure 23 summarizes how we combined the three datasets as well as which channels were fitted to models with and without libration variables (see “Libration usage” on page 98).

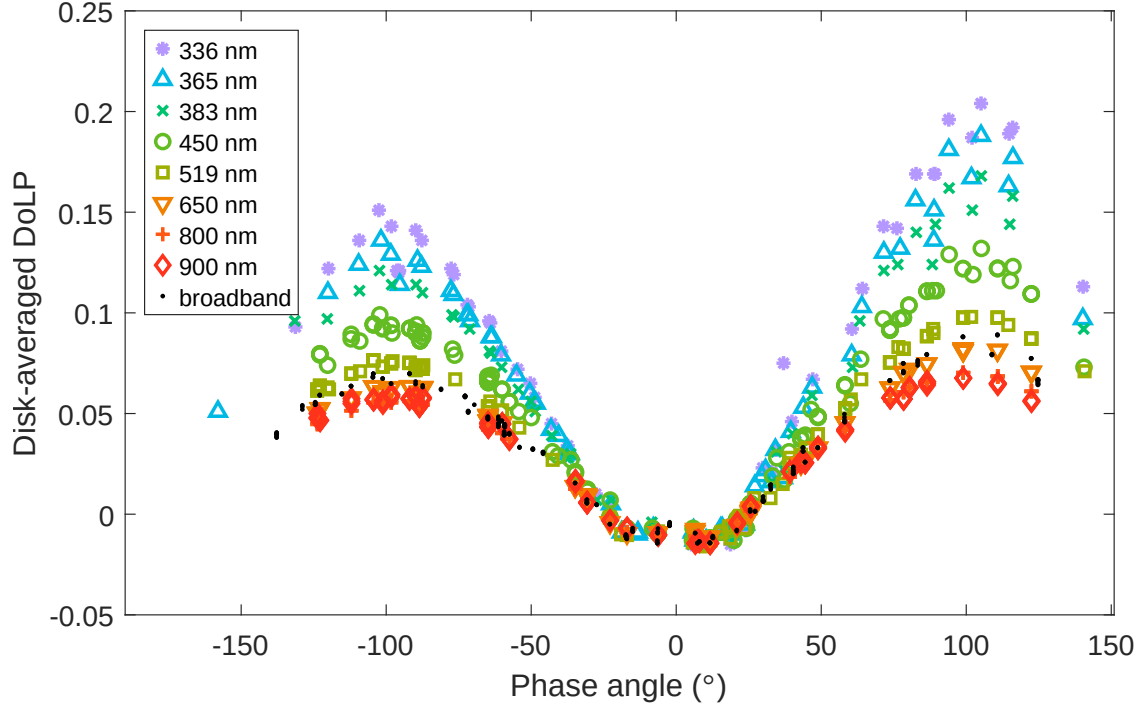


Figure 24: Disk-averaged DoLP measurements for the nine spectral channels that were modeled.

Finally, we used the JPL Horizons tool [107] to calculate the absolute phase angle, Sun selenographic longitude, and observer selenographic latitude and longitude corresponding to the date of each observation. We found that the difference in measurement location for Coyne and Pellicori’s dataset and our datasets made a negligible change in the calculated lunar geometry variables, so we performed all calculations with one measurement location (Bozeman, MT USA). The DoLP measurements in the nine spectral channels are shown as a function of signed phase angle in Fig. 24.

Lunar polarization model

Our objective for the lunar polarization model was to predict the disk-averaged DoLP from the lunar geometry for the spectral bands where data were recorded. Thus, we used lunar geometry terms as independent variables in the model. Ultimately, this could enable

predictions of the DoLP observed at the geometry of an on-orbit polarization-sensitive satellite and increase the accuracy of its lunar calibration. Following a similar approach to Kieffer and Stone’s lunar irradiance model (the ROLO model) [7], we considered including the absolute phase angle, Sun selenographic longitude, and the observer selenographic latitude and longitude as independent variables in the model. The absolute phase angle was directly related to the angle at which sunlight was scattered and reflected by the lunar surface toward the observer, so the polarization state of observed moonlight depended strongly on the absolute phase angle. The Sun selenographic longitude controlled which part of the lunar surface was illuminated. Different regions of the lunar surface had different properties (such as lunar regolith particle size), so the polarization of moonlight varied with lunar region. Thus, we accounted for the effect on the disk-averaged polarization by using the Sun selenographic longitude as an independent variable. The observer selenographic latitude and longitude (referred to as “libration”) controlled which part of the Moon was seen by the observer, similarly affecting the disk-averaged polarization. However, the libration/lunar phase cycle was 18 years, so we only used the libration variables when we had sufficient sampling of the possible libration values (see “Libration usage” section on page 98).

The ROLO model had absolute phase angle terms up to the third power and odd Sun selenographic longitude terms up to the fifth power, with different coefficients for each spectral band. It also had first-power interaction terms between the libration variables and Sun selenographic longitude, with one set of coefficients applied to all spectral bands. There were also terms to describe the opposition effect, a sharp increase in brightness at small phase angles, which was not relevant for a polarimetric model.

We allowed absolute phase angle terms up to the fourth power because this was above the minimum needed to capture the two inflection points present in the polarization curve (Fig. 24). We initially allowed all Sun selenographic longitude terms up to the fifth power, found that the second and fourth power terms were generally negligible, and followed

the ROLO model in excluding them. Also, there was the following physical reasoning to exclude the even terms. The signed phase angle and the Sun selenographic longitude were geometrically related and thus highly correlated. However, we used the absolute phase angle, so it had a similar magnitude to the Sun selenographic longitude but not always the same sign. Approximately, the absolute phase angle terms created the shape of the curve that was symmetric for waxing and waning phases (i.e., the positive and negative signed phase angles/Sun selenographic longitudes), and the Sun selenographic latitude terms accounted for the asymmetry between the waxing and waning phases. The second- and fourth-power terms had the same effect for positive and negative values of Sun selenographic latitude, so they did not create any differences between the waxing and waning phases and essentially were already accounted for by the second- and fourth-power absolute phase angle terms. Thus, we decided to use only the odd terms like the ROLO model. Finally, we included libration terms and interactions with Sun selenographic libration for some spectral channels as described in the next section.

The form of the model when libration terms were not used was

$$\begin{aligned} DoLP = a_1g^4 + a_2g^3 + a_3g^2 + a_4g \\ + b_1\Phi^5 + b_2\Phi^3 + b_3\Phi + c, \end{aligned} \tag{30}$$

where g was the absolute phase angle and Φ was the Sun selenographic longitude. The form of the model including libration terms was

$$\begin{aligned} DoLP = a_1g^4 + a_2g^3 + a_3g^2 + a_4g + b_1\Phi^5 + b_2\Phi^3 + b_3\Phi \\ + c_1\theta + c_2\phi + d_1\theta\Phi + d_2\phi\Phi + e, \end{aligned} \tag{31}$$

where g was the absolute phase angle, Φ was the Sun selenographic longitude, θ was the observer selenographic latitude, and ϕ was the observer selenographic longitude.

Libration usage

To determine whether and how to include libration terms in the model for each spectral channel, we first investigated the sampling of the libration space. We plotted the observer selenographic latitude and longitude of all observations, shown in Fig. 25. The plot showed that the 450-nm, 519-nm, and broadband channels had moderate sampling across the libration space, but the other six spectral channels did not. Thus, the libration terms could not be used for these six spectral channels individually.

However, we considered whether it was possible to use one set of coefficients for the libration terms for all spectral channels as in the ROLO model [7] with the following steps. First, we fit all channels individually with libration terms (Eq. 31). Second, the nine sets of coefficients c_1 , c_2 , d_1 , and d_2 were averaged. Third, the models were refit again using the form of Eq. 31 but using the average values for c_1 , c_2 , d_1 , and d_2 to find the optimal values of a_1 - a_4 , b_1 - b_3 and e separately for each channel. The mean absolute residuals was the same or higher using this approach compared to not accounting for libration at all (using Eq. 30 separately for each channel). This suggested that one set of libration coefficients did not adequately describe the behavior across all spectral channels. To further explore this behavior, we examined the residuals of the model without libration (Eq. 30). Figure 26 shows the residuals as a function of observer selenographic latitude and longitude for the all nine channels combined as well as for the 450-nm, 519-nm, and broadband channels individually. The residuals were plotted with a best-fit plane to enable easier identification of trends. The residuals for all channels considered together (top left in Fig. 26) had a best-fit plane close to horizontal, indicating that the DoLP did not depend on observer selenographic latitude and longitude across all channels. This explained why using the libration coefficients averaged across all channels showed no improvement. This conclusion was further supported by the residuals depending on the libration variables in different ways in the 450-nm, 519-nm, and broadband channels (e.g., the residuals tended to be positive for negative observer

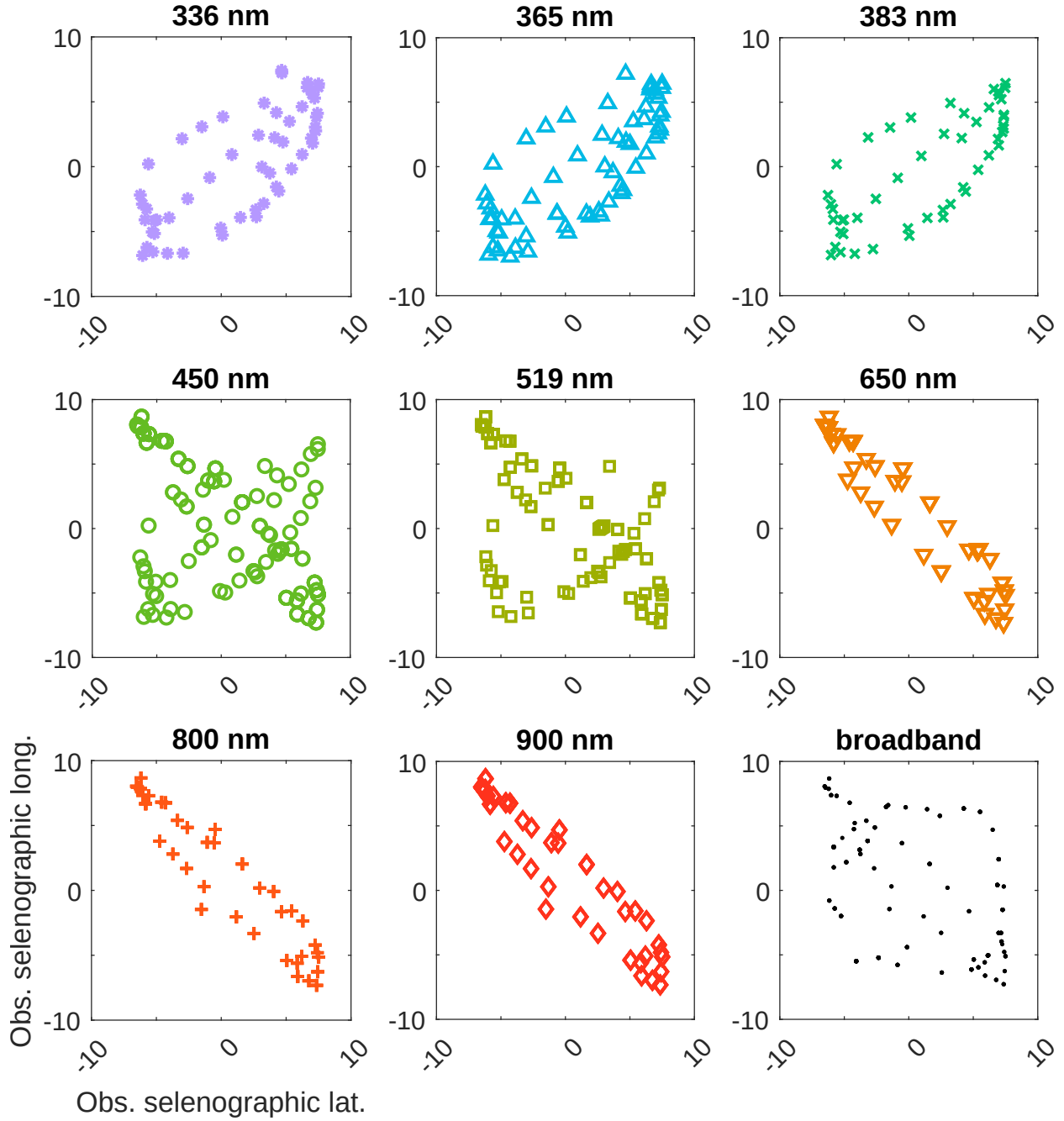


Figure 25: Sampling of libration space for the nine spectral channels that were modeled. Each point shows the observer selenographic latitude and longitude where a measurement was recorded.

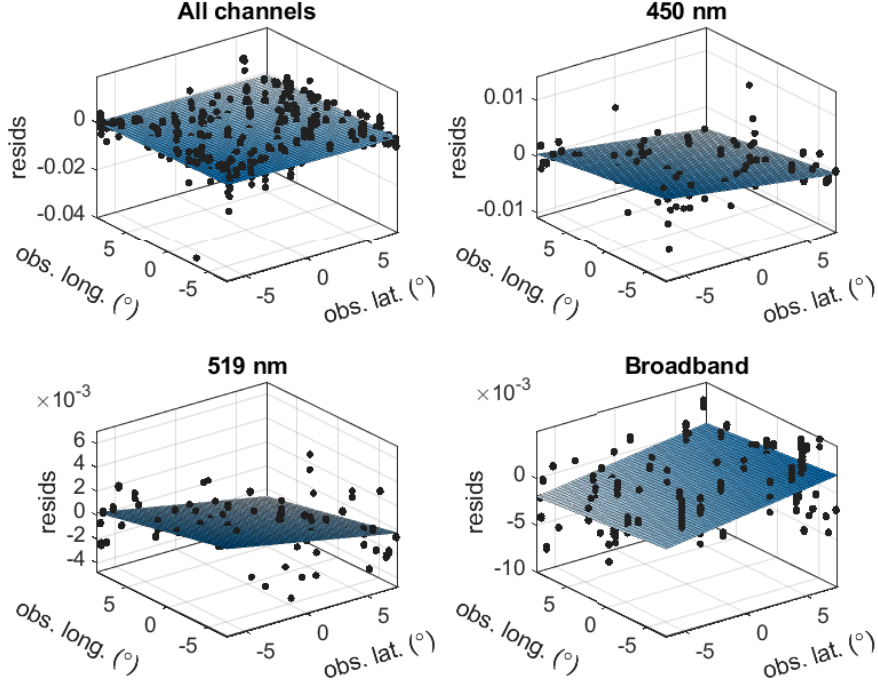


Figure 26: Residuals after fitting to Eq. 30, shown with a best-fit plane. The residuals had little dependence on observer selenographic latitude and longitude when all channel were considered together, but a stronger dependence when the 450-nm, 519-nm, and broadband channels considered individually.

selenographic latitudes for the 450-nm channel, but the broadband channels had negative residuals for negative observer selenographic latitudes).

Based on these findings, we made the following conclusion about how to include libration variables in the model: (1) one set of libration coefficients could not be used across all nine channels, and (2) only some channels had enough sampling of the libration space to use Eq. 31 fitted individually for the channel. Thus, we created fits to Eq. 30 without libration for all nine channels, but we only created additional fits to Eq. 31 with libration for the 450-nm, 519-nm, and broadband channels. A graphical summary outlining which models were created for the nine spectral channels and their data source is shown in Fig. 23.

Models without libration

This section presents the models without libration variables. The fits for all nine channels are shown in Table 6. To more easily show the coefficients in the table, the equation was rewritten as

$$\begin{aligned} DoLP = & (a'_1 \cdot 10^{-10}) g^4 + (a'_2 \cdot 10^{-7}) g^3 + (a'_3 \cdot 10^{-5}) g^2 + (a'_4 \cdot 10^{-4}) g \\ & + (b'_1 \cdot 10^{-12}) \Phi^5 + (b'_2 \cdot 10^{-8}) \Phi^3 + (b'_3 \cdot 10^{-5}) \Phi + (c' \cdot 10^{-2}), \end{aligned} \quad (32)$$

and the values of the modified coefficients are shown.

Table 6: Modified coefficients corresponding to Eq. 32 for the models without libration variables for all nine channels.

Coefficient	336 nm	365 nm	383 nm	450 nm	519 nm
a'_1	1.1989	8.4664	10.952	9.6088	13.239
a'_2	-3.6683	-5.2831	-5.6712	-4.6601	-5.0789
a'_3	5.6242	6.8038	6.8692	5.3827	5.2345
a'_4	-2.8837	-7.8868	-8.6177	-5.0548	-5.0611
b'_1	1.8610	2.6377	2.3512	1.4425	0.70004
b'_2	-4.5324	-5.6974	-5.2002	-3.2085	-1.6708
b'_3	-1.0859	7.4533	7.3101	2.3292	-1.7735
c'	-1.4209	-0.88819	-0.83303	-1.04260	-1.1686
Coefficient	650 nm	800 nm	900 nm	broadband	
a'_1	9.3045	11.060	9.2387	10.380	
a'_2	-3.9033	-3.9604	-3.4070	-4.4411	
a'_3	4.1384	3.7637	3.1347	4.8588	
a'_4	-3.4020	-1.5472	1.3255	-6.0330	
b'_1	1.2704	0.63014	0.71832	0.68355	
b'_2	-2.6193	-1.3188	-1.2907	-1.2548	
b'_3	3.1473	-0.80328	0.38393	-2.7667	
c'	-1.0286	-1.3008	-1.6289	-0.87868	

As examples, the measured and fitted data are shown together along with plots of the residuals for the 383-nm channel in Fig. 27 and for the 650-nm channel in Fig. 28. For both channels shown and for the other channels not shown, the residuals were approximately normally distributed and did not show a strong trend with fitted DoLP, which were two

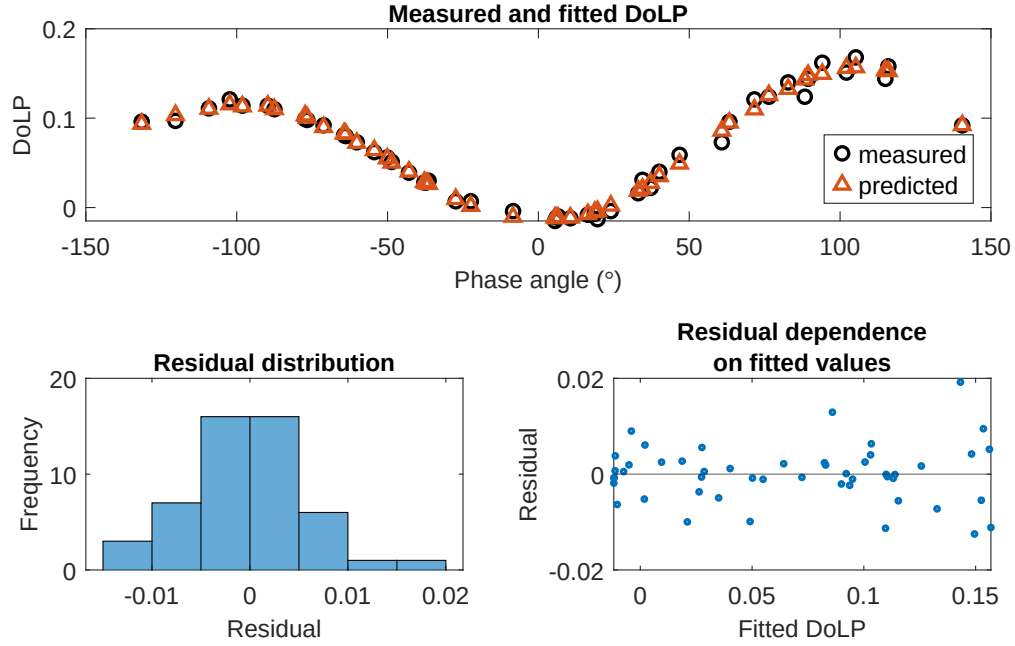


Figure 27: Model without libration variables (Eq. 30) for the 383-nm channel, with the measured and predicted DoLP values plotted together (top) and the distribution of the residuals (bottom left) and the dependence of the residuals on fitted DoLP (bottom right) shown.

important assumptions to satisfy.

For the 383-nm band, the mean absolute residual was 0.0043, and all residuals were between -0.013 and 0.020. It appeared that a major contribution to the residuals was not accounting for libration. For the predicted values in the phase angle range of 60° to 116° (which appeared to fall farthest from the measured values), almost all negative residuals corresponded to a positive observer selenographic latitude, and almost all positive residuals corresponded to a negative observer selenographic latitude. This was explained by the Moon's asymmetry across the equator (when more lunar surface areas with low albedo and high polarization were illuminated and visible to the observer, the disk-averaged DoLP was higher).

For the 650-nm channel, the mean absolute residual was 0.0010, and all residuals were between -0.0036 and 0.0040. The agreement between the measured and predicted DoLP in

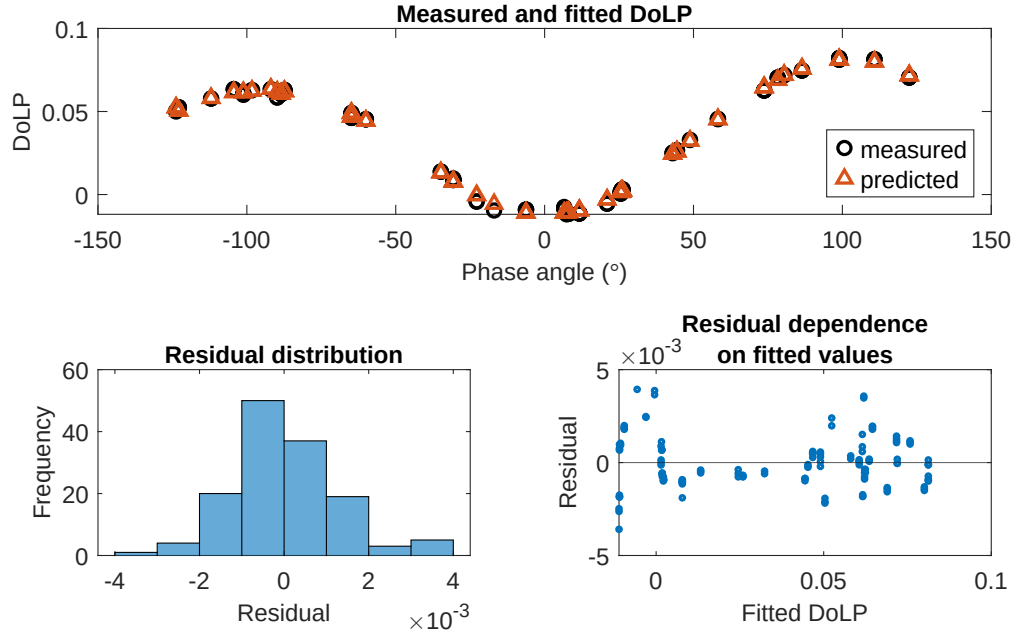


Figure 28: Model without libration variables (Eq. 30) for the 650-nm channel, with the measured and predicted DoLP values plotted together (top) and the distribution of the residuals (bottom left) and the dependence of the residuals on fitted DoLP (bottom right) shown.

Fig. 28 was closer than that for 383 nm, but the most likely cause was again libration. Because the 650-nm dataset was recorded over approximately one year and fewer lunations than the 383-nm data set, all observations with a phase angle greater than 50° had a negative value of observer selenographic latitude. Thus, even though libration variables were not included in the model, the fit for 650 nm was optimized for negative values of observer selenographic latitude. If the 650-nm dataset spanned more lunations, the residuals would likely have been larger. Another reason that the residuals were smaller for 650 nm than for 383 nm be that the DoLP had a lower magnitude for 650 nm than for 383 nm (e.g., the observed maximum for waning phases was 0.168 for the 383-nm channel and 0.0822 for the 650-nm channel).

The trends observed for the 383-nm and 650-nm channels were consistent through the rest of the channels not shown. The channels with datasets spanning more lunations tended

Table 7: Mean and maximum residuals for the models without libration variables.

Channel	$mean resid $	$max resid $
336 nm	0.0056	0.038
365 nm	0.0046	0.022
383 nm	0.0043	0.019
450 nm	0.0021	0.014
519 nm	0.0014	0.0070
650 nm	0.0010	0.0039
800 nm	0.00087	0.0026
900 nm	0.0015	0.0043
broadband	0.0024	0.0073

to have higher residuals and an apparent relationship between the sign of the residuals and the sign of the libration variables. On the other hand, the channels with datasets spanning fewer lunations tended to have lower residuals, but it is likely that the residuals would increase if more data were recorded at different librations. In addition, residuals generally decreased with wavelength, but the channels with the longest wavelengths had datasets that spanned fewer lunations, so these effects were not independent. The mean and maximum absolute residuals are included in Table 7.

Models with libration

For the channels with moderate sampling of the libration space (450 nm, 519 nm, and broadband), a second model was fit to Eq. 31 with libration variables. To simplify the numbers shown in the table, the coefficients were modified according to Eq. 33, and the modified coefficients are included in Table 8.

Table 8: Modified coefficients corresponding to Eq. 33 for the models with libration variables for the 450-nm, 519-nm, and broadband channels.

Coefficient	450 nm	519 nm	broadband
a'_1	1.0165	1.3362	1.0207
a'_2	-4.8169	-5.1745	-4.3082
a'_3	5.5038	5.3708	4.6710
a'_4	-5.2069	-5.5156	-5.3352
b'_1	18.503	8.4055	6.6049
b'_2	-3.7386	-1.8795	-1.3161
b'_3	3.5234	-1.8541	-1.5805
c'_1	3.0301	2.5576	-1.4996
c'_2	1.6673	1.2406	1.4851
d'_1	-6.7661	-1.9864	-0.24788
d'_2	-5.5464	-0.92855	-2.5665
e'	-1.1156	-1.1779	-0.91466

$$\begin{aligned}
DoLP = & (a'_1 \cdot 10^{-9}) g^4 + (a'_2 \cdot 10^{-7}) g^3 + (a'_3 \cdot 10^{-5}) g^2 + (a'_4 \cdot 10^{-4}) g \\
& + (b'_1 \cdot 10^{-13}) \Phi^5 + (b'_2 \cdot 10^{-8}) \Phi^3 + (b'_3 \cdot 10^{-5}) \Phi \\
& + (c'_1 \cdot 10^{-4}) \theta + (c'_2 \cdot 10^{-4}) \phi \quad (33) \\
& + (d'_1 \cdot 10^{-6}) \theta \Phi + (d'_2 \cdot 10^{-6}) \phi \Phi \\
& + (e' \cdot 10^{-2}).
\end{aligned}$$

The measured and modeled data for the 450-nm channel are shown in Fig. 29. The measured and predicted DoLP values qualitatively appeared to match well, and the predictions matched the rapidly varying shape of the measured data near a phase angle of 100° better than the model without libration. The mean absolute residual was 0.0017, and all residuals were between -0.0071 and 0.011. For the 519-nm fit, the mean absolute residual was 0.0013, and all residuals were between -0.0053 and 0.0041. For the broadband fit, the mean absolute residual was 0.0021, and all residuals were between -0.0060 and 0.0048. All three fits with libration had smaller residuals than the corresponding fits without libration.

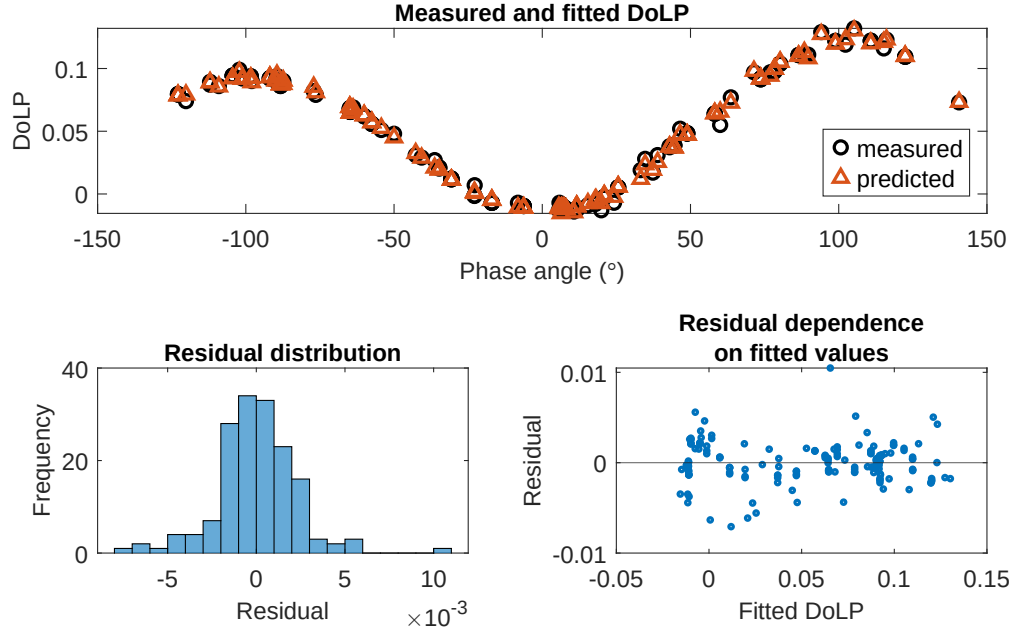


Figure 29: Model with libration variables (Eq. 31) for the 450-nm channel, with the measured and predicted DoLP values plotted together (top) and the distribution of the residuals (bottom left) and the dependence of the residuals on fitted DoLP (bottom right) shown.

As a secondary confirmation that the model was fitting to a libration effect and not noise, we fit the same model but with a subset of the data that only included measurements outside the rapidly varying region of 85° to 116° . We fit the new model and used it to predict the DoLP for all observations (including 85° to 116°). The mean absolute residual was 0.0017 and all residuals were between -0.0072 and 0.011. Figure 30 shows the measured and predicted DoLP values together as well as the residuals. Even though some data were excluded in fitting the model, the residuals were extremely similar to the model using the whole dataset, and the qualitative agreement between the measured and predicted values in the range of 85° to 116° did not appear to decrease. In addition to this confirmation, we noticed the same pattern that was described for the 383-nm channel: the measurements and predictions that had a higher DoLP in the range from 85° to 116° tended to correspond to observations with positive observer selenographic latitudes.

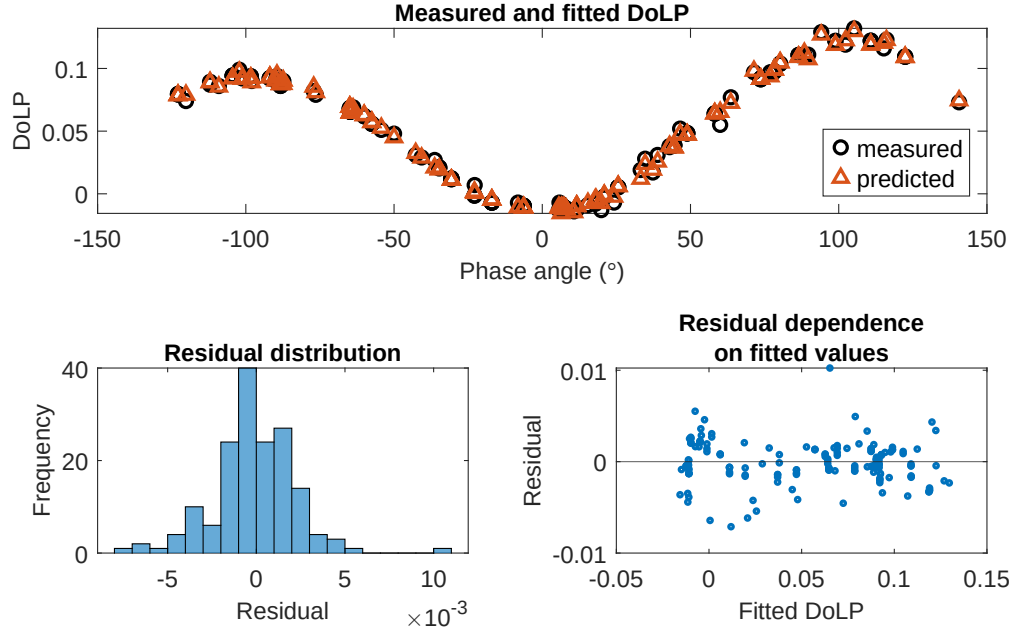


Figure 30: Model with libration variables (Eq. 31) for the 450-nm channel fitted with a subset of the data excluding 85° to 116° , with the measured and predicted DoLP values plotted together (top) and the distribution of the residuals (bottom left) and the dependence of the residuals on fitted DoLP (bottom right) shown.

Example predictions

As an example, we predicted the disk-averaged DoLP at six phase angles and zero libration for all channels except broadband and plotted as a function of wavelength in Fig. 31. For 450 nm and 519 nm, we used the fits that included libration variables. Even though the libration variables were set to zero, the other coefficients were slightly different for the fits with and without libration, so this was not the same as using the fits without libration variables.

As expected, the predictions showed the same trends that were identified in the previously published datasets [44, 139]. The DoLP decreased with wavelength, with the exception of the 900-nm channel which had a higher DoLP than the 800-nm channel for waxing phases. The DoLP was negative for a phase angle of 15° and positive for phase angles 30° and greater. The phase angle of maximum DoLP also appeared to decrease as

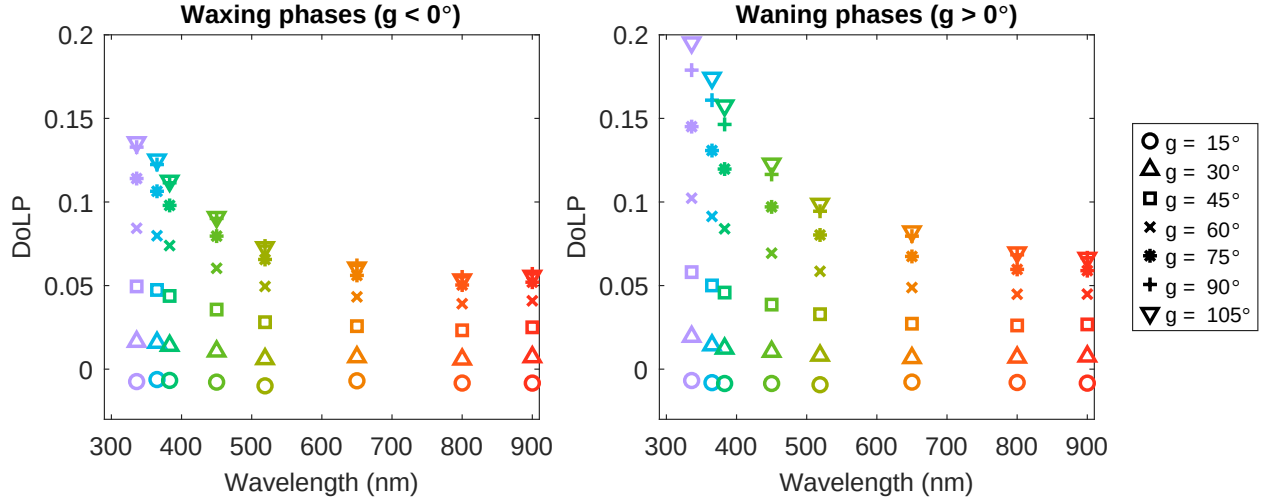


Figure 31: Predicted disk-averaged DoLP for waxing (left) and waning (right) phases. DoLP was predicted for phase angles (g) ranging from 15° to 105° in 15° -steps and zero libration.

wavelength increased. This was seen most clearly for waning phases, as the predicted DoLP was higher for 105° than 90° for the UV channels but predicted values grew closer together as wavelength increased.

Using predictions like the ones in Fig. 31, we could use the model results to predict the disk-averaged DoLP that would be observed by an arbitrary instrument operating between 336 nm and 900 nm. This could be achieved by predicting the DoLP as a function of wavelength for the relevant lunar geometry and then interpolating to estimate the DoLP in the spectral band of the instrument.

Estimating the inversion angle

We also used the model to estimate the inversion angle at which the polarization flips from negative to positive, which could be useful for identifying when an on-orbit instrument could use the disk average of the Moon as an unpolarized calibration target. We predicted DoLP values for phase angles from 19° to 25° in 0.1° steps and zero libration, and the phase angle at which the DoLP had a minimum absolute value was reported as the inversion angle. The results are summarized in Fig. 32.

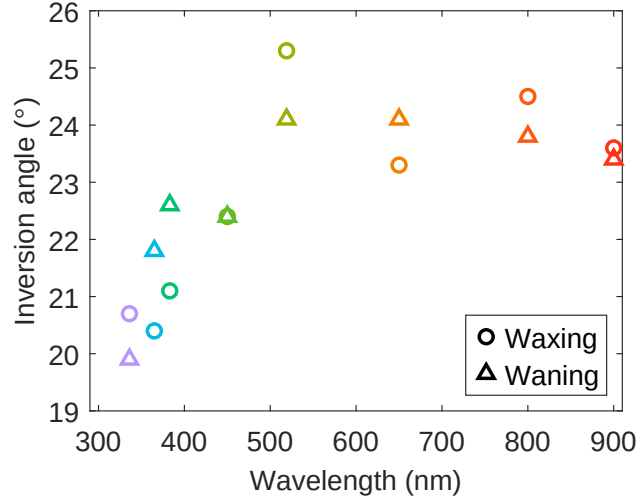


Figure 32: Inversion angle estimated with the model, assuming zero libration.

The inversion angle generally increased with wavelength, which has been reported previously [48]. However, the specific value of the inversion angle appeared to change considerably with the values of the libration variable. For example, at an absolute phase angle of 20.9° , Sun selenographic longitude of -20.6° , observer selenographic latitude of 3.0° , and observer selenographic longitude of 0.2° , the predicted DoLP in the 450-nm channel was -0.0012. However, at a higher absolute phase angle of 24.2° , Sun selenographic longitude of -26.8° , observer selenographic latitude of -5.8° , and observer selenographic longitude of -4.1° , the predicted DoLP in the 450-nm channel was -0.0024. Assuming similar rates of DoLP increase for both cases, this suggested the inversion angle could change by approximately 3° depending on the libration values. The values in Fig. 32 were within 3° of estimates of the inversion angle previously made by Dollfus and Bowell [48] (but there is ambiguity in the lunar region(s) described by their measurements).

Prediction accuracy

The accuracy with which this model can be used to predict the disk-averaged DoLP depends on several factors. First, the uncertainty of the measured data, especially systematic errors, would affect the resulting model. The close agreement between the two sources of data

combined in the 450-nm and 519-nm channels suggested there were not major systematic errors. Also, the extent to which the empirical model captured the true variation of moonlight polarization also affects the overall accuracy. For instance, there was likely a very small dependence on Sun selenographic latitude that was ignored. Similarly, the limited sampling of the libration/lunar phase cycle decreased the accuracy of the model. Particularly for the 650-nm, 800-nm, and 900-nm channels, the fits represented the closest match to the data for only a small range of the possible values of the libration variables and therefore would not fit as well to data recorded at other values of the libration variables. Finally, when using the model to predict the DoLP observed by a certain instrument at a certain geometry, unless the instrument has the same spectral response as one of the nine channels used in this work, there would be some uncertainty resulting from interpolating to a different wavelength and/or bandwidth. While we did not attempt to combine the effects of the known uncertainties in this list (and could not include the unknown uncertainties), the residuals were generally larger than the reported measurement uncertainties [44, 128, 139]. Based on the residuals for the channels without small libration coverage, we believe that a realistic estimate for the models without libration variables was ± 0.01 and for the models with libration variables ± 0.005 .

Conclusion

The Moon polarization models presented in this work can be used to predict the disk-averaged DoLP with the lunar geometry in eight spectral channels ranging from 336 nm to 900 nm as well as in a VNIR broadband channel. The models followed the approach used by Kieffer and Stone’s lunar irradiance model [7]. For the 450-nm, 519-nm, and broadband channel, the predicted DoLP depended on the value of the libration variables (observer selenographic latitude and longitude) but the rest of the models did not. We used the models to predict the disk-averaged DoLP at a set of illustrative phase angles as well as phase angles

between 19° and 25° to estimate the inversion angle. We estimated the uncertainty for the models with libration was ± 0.005 and for the models without libration was ± 0.01 . A longer set of observations spanning more of the libration/lunar phase cycle would be necessary to reduce this uncertainty.

While the DoLP can only be predicted directly at eight spectral channels, the predicted DoLP values varied relatively smoothly as a function of wavelength, so interpolation could enable predicting the DoLP in an arbitrary spectral band between 336 nm and 900 nm. This could be useful for increasing the accuracy of lunar calibration of polarization-sensitive satellite instruments. It could also be possible to use the model to estimate when a satellite instrument would have a lunar-view opportunity close to the inversion angle and perform lunar calibration with a very low DoLP. With the recent and planned launches of polarimeters [149, 150], the improved lunar calibration accuracy enabled by these Moon polarization models could ultimately impact an increasing amount of polarimetric environmental remote sensing data.

PASSIVE POLARIZATION IMAGING OF CLOUD THERMODYNAMIC PHASE

This dissertation originally began with a focus on cloud thermodynamic phase (CTP) retrievals, with a specific interest on how well a passive polarimetric CTP retrieval method could work under moonlight illumination at night. Polarization measurements of moonlight were initially recorded to understand how the disk-averaged polarization varied across the lunar cycle and across the spectral range of the planned CTP measurements. As the potential utility of additional measurements and a model of Moon polarization became apparent, it became the primary focus of this dissertation. However, some CTP analysis was performed, and the methodology, measurements, and results are described in this chapter.

Two potential improvements to a passive polarimetric method of ground-based CTP retrievals were explored in this work. First, this work is the first demonstration of CTP retrievals using an commercial, off-the-shelf, ultra-low-cost polarization imager. This will contribute to enabling widespread ground-based CTP monitoring; previous demonstrations of this method used more expensive instrumentation that is less conducive to widespread deployment. Second, this work demonstrated an improvement in classification accuracy when the scattering angle was included as a predictor variable. Previous work showed that a dependence on scattering angle existed, but this study was the first to demonstrate that the relationship could be used for more accurate classifications. Nighttime retrievals were not explored enough to draw any conclusions, but a brief description is included.

Methodology

The basis for ground-based passive polarimetric remote sensing of CTP was first described by Knobelspiesse et al., who used a vector radiative transfer simulation and measurements from a polarized AERONET Cimel instrument [91]. They showed that the P_{12} phase matrix element (which described how much scattered light was parallel or

perpendicular to the scattering plane) was positive or slightly negative for liquid clouds and more negative for ice clouds. This was true for the scattering angles seen by a ground-based system (in an example simulation, this was true from approximately 20° to 110°). This resulted in simulations generally showing $S_1/S_0 > 0$ for liquid clouds and $S_1/S_0 < 0$ for ice clouds when S_1 was referenced to the scattering plane (with exceptions at small scattering angles). The difference between S_1/S_0 tended to decrease for larger optical depths and to change with wavelength. The measurements with the Cimel instrument showed some promise, but most Cimel algorithms use DoLP instead of S_1 , there were challenges [91]. This technique was demonstrated more successfully with an all-sky imaging polarimeter by the Optical Remote Sensor Laboratory (ORSL) [92]. The all-sky polarimeter used science-grade components and cost approximately \$100k, and the data were used to explore many of the same relationships as Knobelspiesse et al., including the relationships between S_1/S_0 and both scattering angle and wavelength.

In this work, a low-cost polarization camera with a fisheye lens recorded all-sky polarization images. The images were processed as images of S_0 , S_1 , S_2 , DoLP, and AoP. The images of S_0 and DoLP were used to identify cloudy pixels in the all-sky image, and the S_1 image was used for CTP classification. A dual-polarization lidar was deployed simultaneously to obtain a verification measurement of CTP at the zenith. The S_1 values of cloudy pixels in a cone around zenith were averaged, and the resulting dataset of S_1 values and CTP classification from the lidar was used to explore the best way to predict CTP from the polarization camera data.

Polarization camera

The all-sky polarization images in this experiment were recorded with the division-of-focal-plane polarization camera and fisheye lens described in Ch. 2. The all-sky images were used to calculate the normalized quantity S_1/S_0 for cloudy pixels, and S_1/S_0 was

ultimately used to classify the clouds as comprised of liquid water droplets or ice crystals. The polarization camera cost approximately \$2500, and the fisheye lens was less than \$1000. The system was held by a light-weight tripod, and the image acquisition code was written in MATLAB and ran on a laptop. There were no spectral filters in the system, so the spectral response ranged from approximately 400 nm to 1000 nm (the response of the detector is shown by the “Imaging system RSR” curve in Fig. 6). The procedures described in Ch. 2 were used to determine the center pixel, instantaneous field of view (IFOV), and the polarimetric response of the center of the field of view (FOV). The center pixel (i.e., the pixel corresponding to the zenith when the camera is leveled with the optical axis perpendicular to the ground) had a row and column number of 1058 and 1234, respectively. The IFOV was 0.0736° per pixel, or there were 13.6 pixels per degree. The polarimetric response of the cone with a half angle of 5° in the center of the FOV was described by a 3x3 system matrix for each superpixel, and enabled comparing calibrated values of S_1/S_0 with a verification measurement of CTP at the zenith.

Each time measurements were recorded, the polarization camera was set up on the roof near the dual-polarization lidar. The top of the camera was aligned approximately to the North, and a level was used to ensure the optical axis of the camera was pointed to the zenith. First, one dark image was recorded at exposure times of 400 μs , 800 μs , 1200 μs , 2000 μs , and 4000 μs , and all images of the sky were recorded at one of these exposure times. Second, an alignment image was recorded at an exposure time of 40 μs . The purpose of the alignment image was to verify the position of the Sun, and in most conditions, this low exposure time resulted in an image where the Sun pixels were saturating but the nearby sky pixels were not, so the position of the Sun could be identified accurately. This image was later used to correct for differences between the top-of-camera direction and true North. Finally, the exposure time(s) for recording suitable images for the cloud conditions at the time were determined. One or more exposure times out of the possible list of five above were

chosen with the goal of obtaining images with digital numbers in the middle of the dynamic range of the camera sensor for the pixels near the zenith but without too large of a region saturating near the Sun. If weather permitted after all images were recorded, another set of dark images or another alignment image was sometimes recorded.

The all-sky images were processed to obtain the value of S_1/S_0 referenced to the scattering plane as follows.

1. The Sun position was identified in the alignment image as the centroid of the saturating pixels, and it was compared to the expected Sun position based on the time when the image was recorded. The difference in azimuth angle in the alignment image and the expected azimuth based on the time (or the mean difference if multiple alignment images were recorded) was used to determine the actual top-of-camera direction relative to North. If there was no useful alignment image because thick clouds obscured the Sun position, it was assumed that the top-of-camera direction was North.
2. The dark (or the mean of multiple darks) recorded at the same exposure time as the image was subtracted.
3. A region of cloudy pixels within the 5° -half-angle cone around the zenith was manually identified by looking for areas with higher S_0 and lower DoLP.
4. The system matrix was multiplied by the Stokes vector for all pixels in the cone with a 5° -half-angle around the zenith.
5. The bilinear interpolation algorithm for demosaicking was applied, as described in the “Demosaicking method” section on page 36.
6. The AoP reference frame was changed from the instrument plane to the scattering plane using the IFOV of the system and the top-of-camera direction, as described in the “Choice of AoP reference frame” section on page 35. S_1 and S_2 were calculated in

the scattering plane reference frame using the definition of the linear Stokes vector in Eq. 5 as a function of AoP in the scattering plane reference frame as well as S_0 and DoLP, which were reference frame invariant.

7. The mean S_1/S_0 was calculated of all pixels in the mask of cloudy pixels.

This process resulted in a measurement of S_1/S_0 that could be compared to the lidar measurements. At this time, the images are only calibrated and used to predict CTP at the zenith, but code is written so that a system matrix for the full-field of the fisheye lens instead of only the 5° -cone could easily be substituted in the future. Steps 1, 2, 5, and 6 were used to produce uncalibrated all-sky images for visualization.

Dual-polarization lidar

The verification measurement of CTP was provided by a dual-polarization lidar that was operated simultaneously [130]. Dual-polarization lidar is a widely used method to measure CTP. The lidar transmitted a linearly polarized beam, and the receiver measured two linearly polarized states: one parallel to the transmitted beam and one perpendicular. The ratio of the signal in the perpendicular state to the signal in the parallel state, the cross-polarization ratio, is related to the cloud composition. Nominally, when the transmitted beam interacts with a spherical liquid water droplet, the only light that returns to the receiver (i.e., the backscattered light) has either (1) reflected directly from the front and back of the droplet, so the angle of incidence on the dielectric boundaries is 0 and the polarization of the backscattered light is unchanged relative to the transmitted beam, or (2) propagated as a surface wave 180° around the droplet, which also does not change the polarization state of the backscattered light. On the other hand, light that is backscattered by a hexagonal ice crystal undergoes multiple internal reflections with the crystal that result in a different polarization state at the receiver. This enables using the cross-polarization ratio to distinguish between liquid and ice clouds: the clouds comprised of liquid water droplets

have a lower cross-polarization ratio, and the clouds comprised of ice crystals have a higher one. However, multiple scattering can also increase the cross-polarization ratio measured for liquid clouds and complicate interpretation of the lidar signals.

Example lidar signals for liquid and ice clouds are shown in Figs. 33 and 34, with the intensity of the co-polarized signal, cross-polarized signal and cross-polarization ratio plotted as a function of height. It should be noted when there were no clouds, the cross-polarization ratio was a division of two signals at the noise floor, so the ratio could take on non-physical values and appear very noisy. An example liquid cloud at an altitude of approximately 2.2 km above the ground is shown in Fig. 33. This lidar measurements was recorded on 23 September 2024 at 1:49 PM MDT, when the ground temperature recorded by the ORSL Weather Station was 21.4°C. The co-polarized signal was stronger than the cross-polarized signal, and the cross-polarization ratio was approximately 0 at the bottom of the cloud and slowly started increasing. This increase was likely due to multiple scattering: a single scattering event with a liquid water droplet does not change the polarization state of backscattered light, but multiple scattering events can (and the number of scattering events increases with depth into the cloud).

Figure 34 shows the same set of plots but for an ice cloud approximately 6.9 km above the ground on 7 July 2023, when the ground temperature was 22.4°C. The co-polarized signal was still stronger than the cross-polarized signal, but they were much closer in magnitude than the liquid cloud, with a cross-polarization ratio around 0.35. This higher cross-polarization ratio suggested that this was an ice cloud. The relatively low amount of backscattered light and the cross-polarization ratio around 0.35 even at the bottom of the cloud both distinguished this from cases where multiple scattering increased the cross-polarization ratio. Also, a 22°-halo was visible around the Sun during the afternoon when this lidar signal was recorded, which was an indicator that the clouds were comprised of ice crystals (the 22°-halo results from a specific ray path through ice crystals).

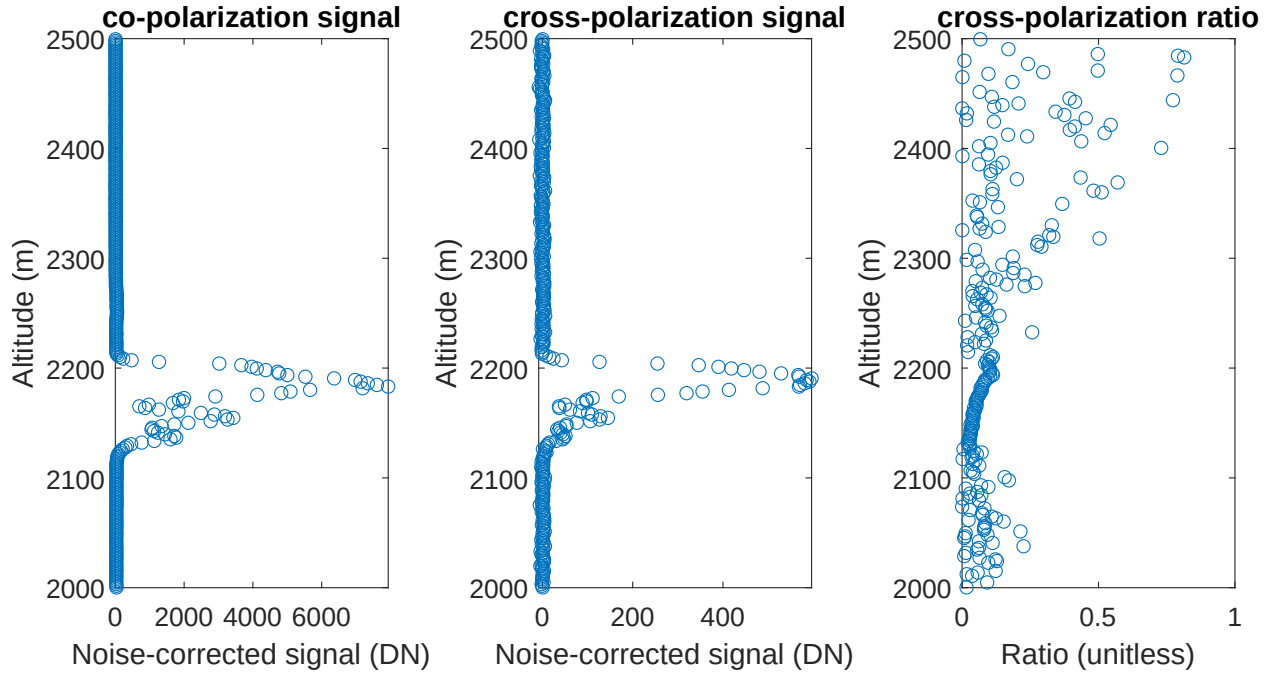


Figure 33: Co-polarized signal, cross-polarized signal, and cross-polarization ratio as a function of altitude above ground for a liquid cloud on 23 September 2024 at 1:49 PM MDT.

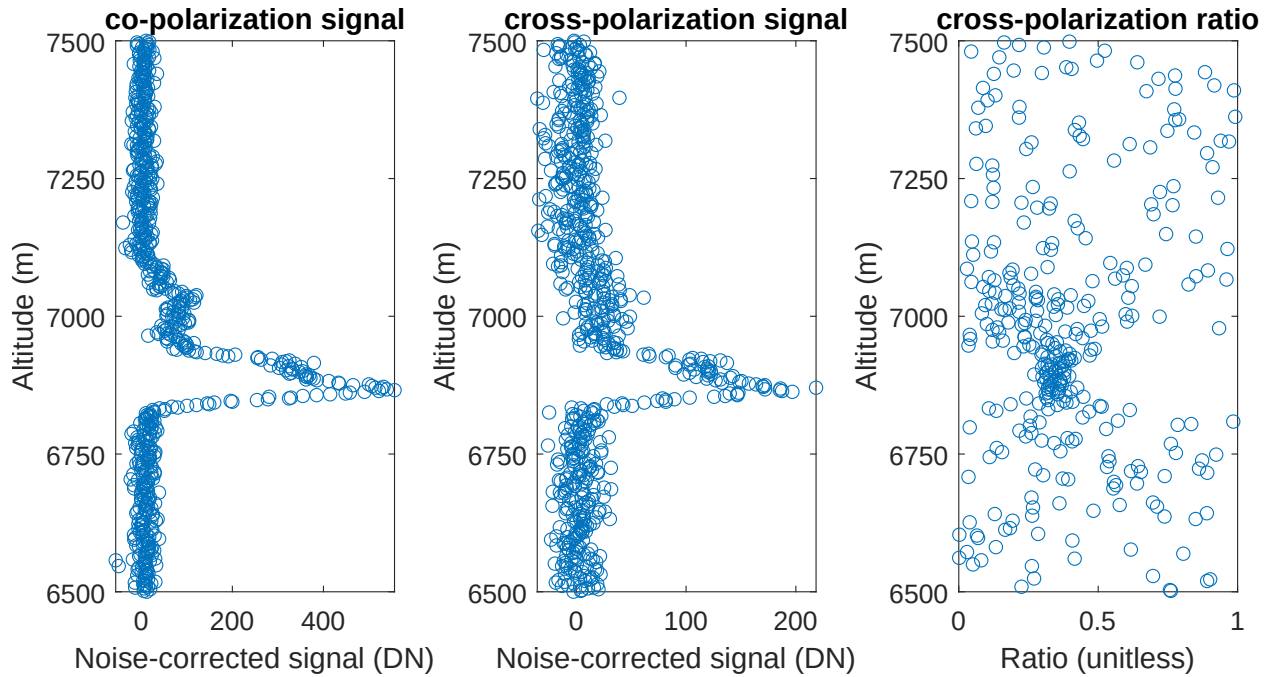


Figure 34: Co-polarized signal, cross-polarized signal, and cross-polarization ratio as a function of altitude above ground for a ice cloud on 7 July 2023 at 2:20 PM MDT.

Results over 16 days

Daytime all-sky images were recorded on 28 days, but only 16 days yielded measurements that were used to assess how well the low-cost polarization camera images could distinguish CTP. The other days of measurements sometimes had no clouds at the zenith, a signal from the dual-polarization lidar that could not easily be identified as liquid or ice, or no lidar signal at all (we had intermittent problems with the lidar from October 2023 until the computer in the lidar system was replaced by Matthew McClelland in April 2024). First, a summary of all observations of the mean S_1/S_0 of cloudy pixels near the zenith is shown in Fig. 35; the observations are shown as a function of the scattering angle of the zenith pixel.

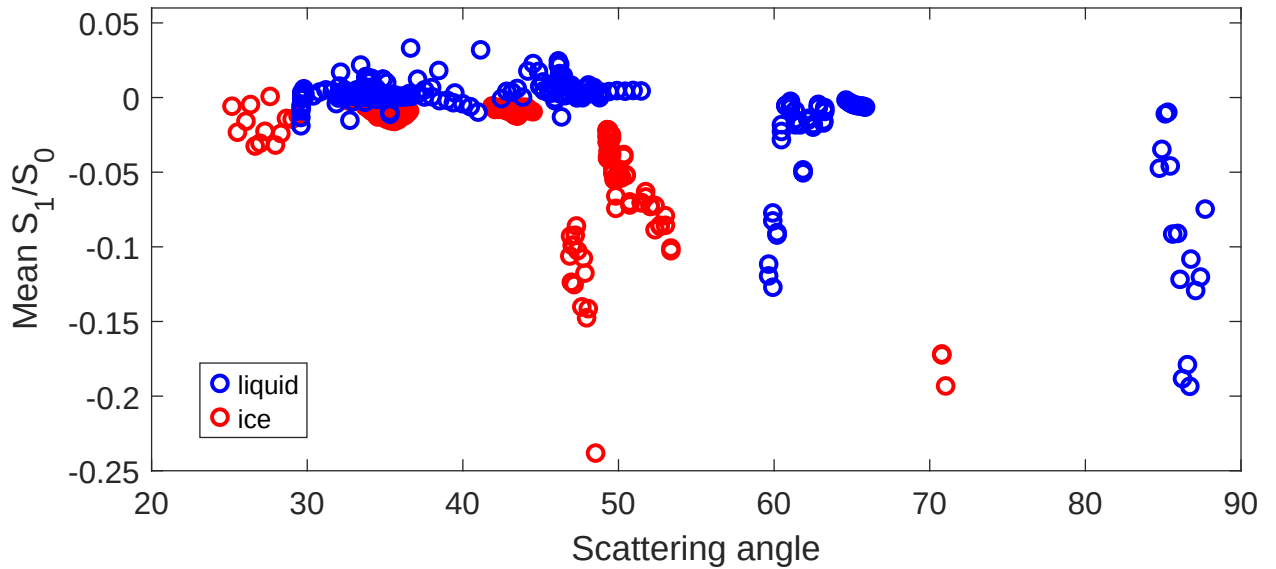


Figure 35: Observations of S_1/S_0 of cloudy pixels near the zenith on 16 days, with the observations colored according to the classification as liquid or ice from the dual-polarization lidar.

Second, two example images of different clouds conditions are shown. Figure 36 shows an all-sky image with ice clouds, and Fig. 37 shows an all-sky image with liquid clouds. The S_0 and DoLP images looked relatively similar for both cloud types: the pixels with clouds were brighter and less polarized than the background sky. The S_0 and DoLP images were

examined simultaneously to create a mask of cloudy pixels near the zenith. The difference between liquid and ice clouds was more pronounced in the S_1/S_0 and AoP images. Ice clouds tended to have a more negative value of S_1/S_0 , corresponding to an AoP near 90° , resulting in an AoP image that looked similar to an AoP image for a clear sky. However, liquid clouds tended to have positive or only slightly negative values of S_1/S_0 , so many cloudy pixels in Fig. 37 had an AoP deviating from 90° . Because the S_1/S_0 threshold between liquid and ice clouds was not expected to be exactly 0 and was expected to depend on phase angle [91, 92], an image of S_1/S_0 alone is not sufficient for CTP identification. However, the general patterns can be visualized with these images. In future work, it could be possible to perform a pixel-wise classification with the method described in the next section and generate an image where each pixel is colored according to the CTP classification.

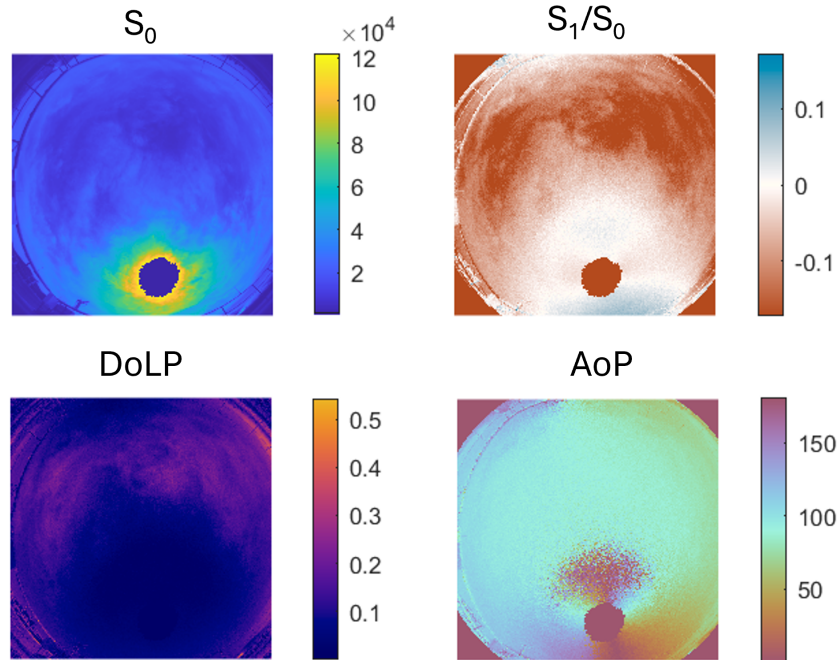


Figure 36: Example all-sky polarization images of ice clouds. S_0 has units of digital number, S_1/S_0 and DoLP are unitless, and AoP has units of degrees.

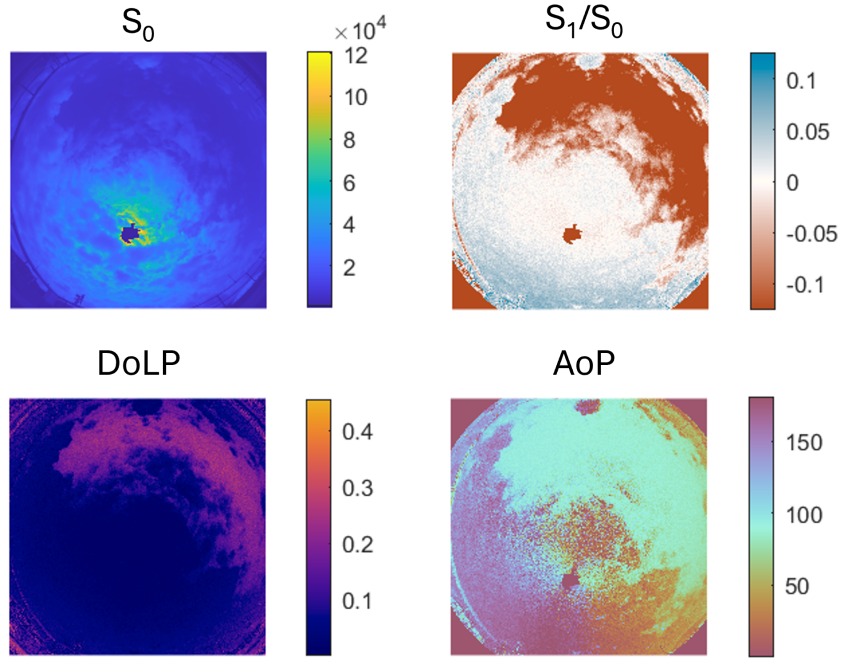


Figure 37: Example all-sky polarization images of liquid clouds. S_0 has units of digital number, S_1/S_0 and DoLP are unitless, and AoP has units of degrees.

Nighttime measurement example

Nighttime measurements are more challenging than daytime measurements for several reasons: the Moon is only in the sky nominally half of the nighttime and has a variable brightness and polarization, the amount of light from the source is orders of magnitude lower, and light pollution is a source of strong background signal. An example of a cloudy nighttime all-sky image is shown in Fig. 38. Clouds are visible in the S_0 , and the corresponding part of the DoLP image has a low DoLP. However, the band of maximum polarization 90° from the Moon is difficult to see. Also, the clear sky parts of the image do not have an AoP around 90° and a negative S_1/S_0 . The cloudy pixels appear to be less noisy than the sky, which makes sense because they are brighter. Too few measurements were recorded at night to perform any statistical analysis, but this example image illustrates some of the challenges associated with nighttime CTP measurements.

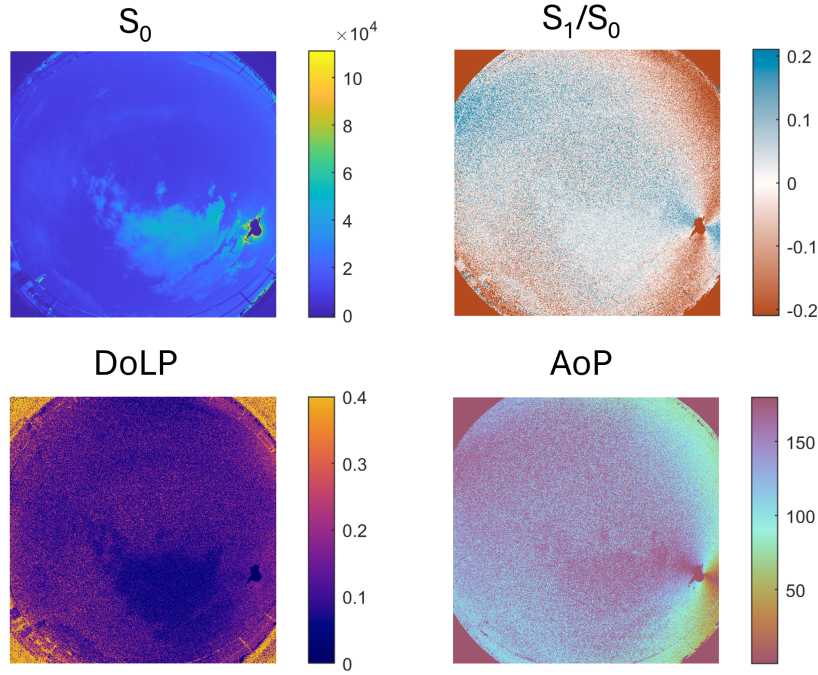


Figure 38: Example all-sky polarization images of clouds at night. S_0 has units of digital number, S_1/S_0 and DoLP are unitless, and AoP has units of degrees.

Classification results

Two logistic regressions were used to classify CTP as liquid or ice in the daytime measurements. The first used only the mean S_1/S_0 as the predictor variable, and the second used the zenith angle and mean S_1/S_0 as predictor variables. First, using only a dependence on S_1/S_0 , the best fit of logit of the probability of a liquid cloud (or the “log odds”) was found:

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = 0.739 + 21.9(S_1/S_0), \quad (34)$$

where p is the probability that a cloud was comprised of liquid water droplets. Thus, the resulting S_1/S_0 threshold between more likely to be a liquid cloud ($p > 0.5$) and more likely to be an ice cloud ($p < 0.5$), was $S_{1,\text{threshold}} = -0.034$, similar to the threshold of -0.04 used in [92]. This is shown by the horizontal yellow line in Fig. 39, and the number of correct

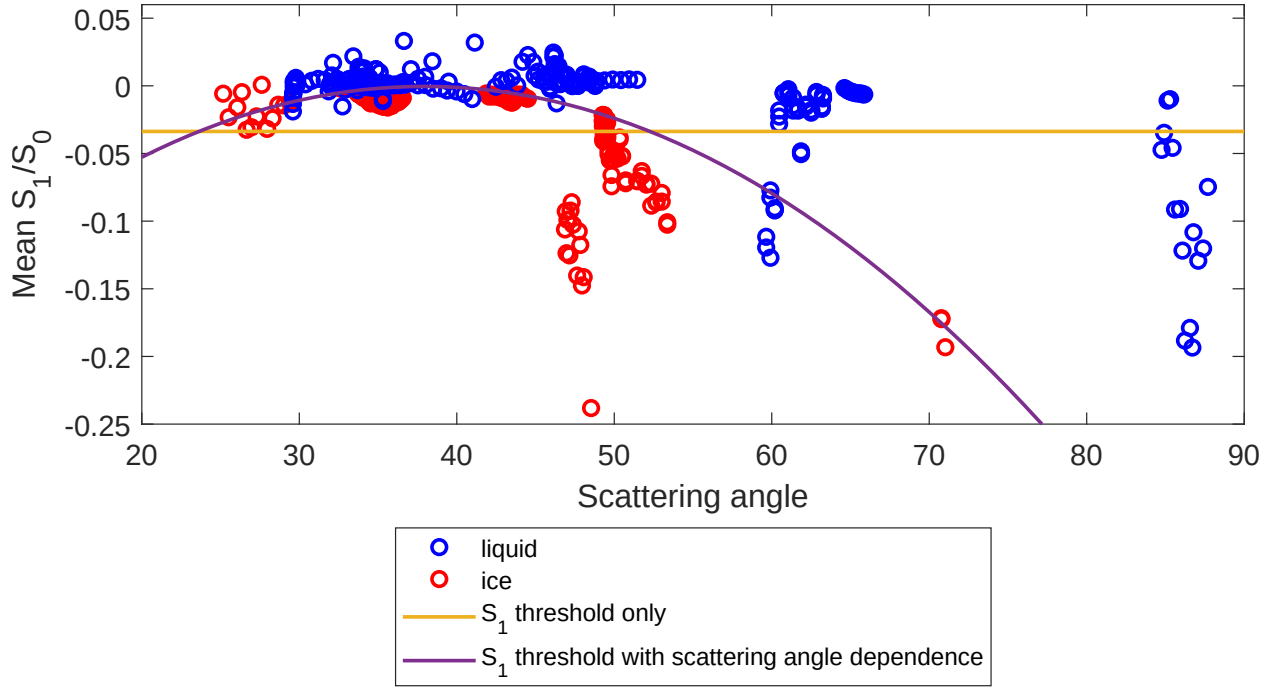


Figure 39: Observations of S_1/S_0 of cloudy pixels near the zenith colored according to their classification from the dual-polarization lidar, shown with a threshold based only on the value of S_1/S_0 (the horizontal yellow line) and a threshold with a dependence on scattering angle (the curved purple line).

and incorrect predictions are shown in Table 9.

Second, the best fit of the logit of the probability of a liquid cloud adding a scattering angle dependence was found:

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right) = 38.9 - 2.05 \theta_s + 0.027 \theta_s + 166.2(S_1/S_0), \quad (35)$$

where p is the probability that a cloud was comprised of liquid water droplets and θ_s is the scattering angle of the zenith pixel. In this case, the threshold at $p = 0.5$ is a function of scattering angle and is shown by the curved purple line in Fig. 39. The number of correct and incorrect predictions are shown in Table 10. Thus, adding a scattering angle dependence to the S_1/S_0 threshold increased the overall accuracy from 66.7% to 89.8%.

Table 9: Summary of prediction accuracy for the classification described by Eq. 34.

	Lidar class: ice	Lidar class: liquid
Predicted ice	60	24
Predicted liquid	116	220

Table 10: Summary of prediction accuracy for the classification described by Eq. 35.

	Lidar class: ice	Lidar class: liquid
Predicted ice	153	20
Predicted liquid	23	224

Example: 23 June 2023

The conditions on 23 June 2023 provided an interesting case study because there was a persistent high layer of ice clouds and an intermittent low layer of liquid clouds. This is shown in Fig. 40. The extremely low DoLP across the whole image suggested that there was a cloud layer across the whole sky. However, some areas of the image had a negative S_1/S_0 and AoP of 90° , consistent with ice clouds, and other areas had a positive S_1/S_0 and AoP rotated from 90° , consistent with liquid clouds. A low liquid cloud beneath a higher ice cloud was most obvious in the top part of the S_1/S_0 and AoP images.

The presence of two cloud layers was also supported by the lidar data. The top plot of Fig. 41 shows a time-height plot of the cross-polarization ratio. The lidar data only reflects the lower liquid clouds when they are at the zenith, so even though a low layer can only be seen around 21:29 UTC and 21:58 UTC in the lidar data, there could have been low liquid clouds elsewhere in the image at other times.

The classification algorithm is demonstrated in the bottom plot of Fig. 41. Using the time of day, the scattering angle of the zenith pixel was determined and then used to calculate the values of S_1/S_0 for $p = 0.5$ (solid black line) and for $p = 0.05$ and $p = 0.95$ (dashed black lines). There was a visible correlation between when mean S_1/S_0 (black circles) fell above the threshold and when there was a low layer of liquid clouds in the time-height plot: some

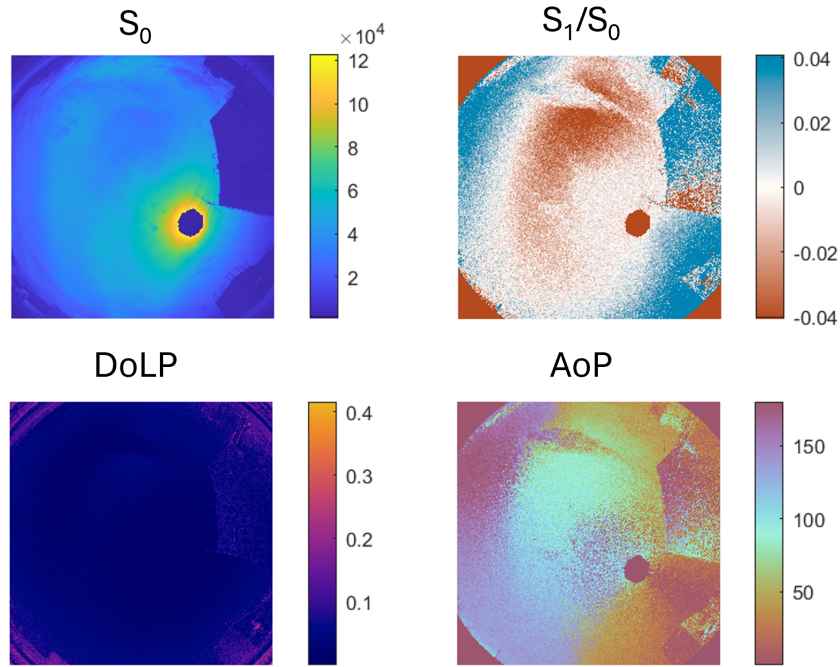


Figure 40: Example all-sky polarization images on a day with a high layer of ice clouds and low layer of liquid clouds. S_0 has units of digital number, S_1/S_0 and DoLP are unitless, and AoP has units of degrees.

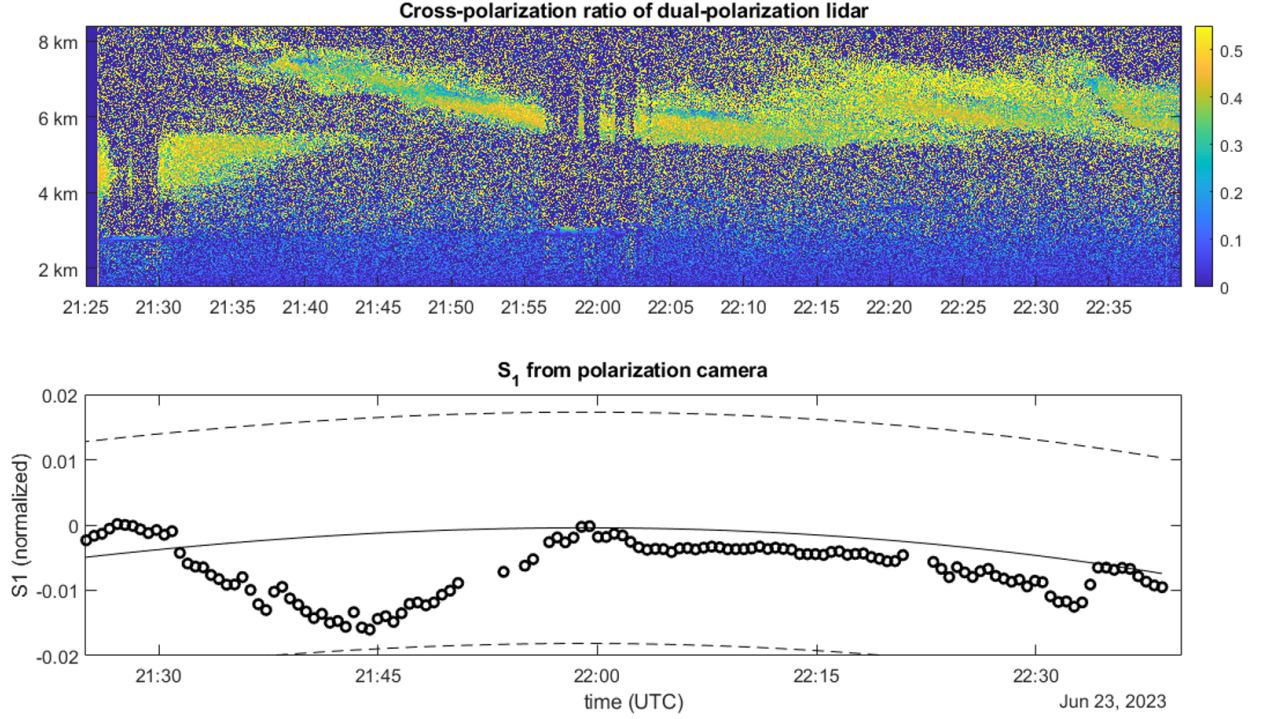


Figure 41: Cross-polarization ratio of lidar (top) and mean S_1/S_0 (bottom) for a day with a persistent high layer of ice clouds and an intermittent layer of low clouds. The data from both the lidar and polarization camera suggest the presence of liquid clouds around 21:29 UTC and 21:58 UTC.

liquid clouds were also identified by the polarization camera around 21:29 UTC and 21:58 UTC. However, the liquid clouds appeared for a longer time in the lidar signal than the polarization camera signal near 21:58 UTC, so the polarization camera appeared to falsely identify some liquid clouds as ice.

Discussion and future work

The classification algorithm using an S_1/S_0 threshold that depended on scattering angle performed fairly well (approximately 90% accuracy). The logistic regression determining the threshold was fit to over 400 individual observations, but these observations were made over sixteen days and thus do not represent completely independent observations. They also

do not evenly and completely sample the range of possible scattering angles. To obtain more reliable estimates of the accuracy of this method, more measurements spread out over the range of possible scattering angles are needed. This will require both recording more observations and a polarimetric calibration of the full field of the fisheye lens. The full-field polarimetric calibration will enable observations of S_1/S_0 for a larger range of scattering angles (so far, the scattering angles observed were limited by the possible Sun zenith angles because only zenith pixels were processed).

Besides low-cost instrumentation and more accurate classification algorithms, other potential ways to improve passive polarimetric CTP sensing include using the spectral dependence of S_1/S_0 and exploring nighttime measurements. It was shown previously that different spectral bands observed different values of S_1/S_0 [90–92], and that using a red and a blue band in particular could provide better separation between liquid and ice clouds [92]. A low-cost polarization camera with spectral filters like the all-sky polarization imager developed by the ORSL that was presented in [110] could provide the measurements needed to take advantage of this dependence and improve CTP retrievals, and the ORSL has already begun recording all-sky images on cloudy days with the dual-polarization lidar providing a verification method.

Also, demonstrating passive nighttime CTP retrievals using moonlight illumination could provide an important step towards enabling widespread CTP monitoring, especially in the Arctic, where polar night can last for weeks or months. A series of nighttime observations under different lunar phases was not recorded in this work because moonlight polarization images became the focus of the dissertation. As a first step, the moonlight polarization measurements recorded in this work could be used as an input to a simulation exploring how a weakly polarized source changes passive polarimetric CTP retrievals relative to an unpolarized source. To reach an experimental answer to how well polarimetric CTP retrievals work under moonlight illumination, a significantly expanded dataset is needed to be able to

perform a similar analysis to what was done for the daytime measurements and to be able to examine the effect of other variables such as lunar phase and light pollution.

CONCLUSION

The polarization of moonlight depends on wavelength, lunar phase angle, and other lunar geometry variables. Knowledge gaps remain with these dependencies, most notably a lack of disk-averaged measurements at wavelengths longer than 520 nm and model describing polarization. Addressing these knowledge gaps contributes to improving the accuracy of lunar calibration of satellite instruments and nighttime remote sensing. This dissertation addressed these two knowledge gaps by: 1) providing measurements of the disk-averaged polarization of moonlight as a function of phase angle at wavelengths of 650 nm, 800 nm, and 900 nm, and 2) developing a model of moonlight polarization across UV to NIR spectral bands as a function of lunar geometry variables. This dissertation also contributed improvements to a passive polarimetric method for sensing cloud thermodynamic phase (CTP) by demonstrating the feasibility of using low-cost instrumentation and a more accurate classification algorithm.

Lunar calibration is advantageous over other on-orbit calibration methods such as solar calibration, on-board lamps, or pseudo-invariant calibration sites because it provides a stable target outside of the atmosphere, thus not requiring atmospheric correction or additional on-board hardware to monitor calibration target stability. However, lunar calibrations of polarization-sensitive satellite instruments would result in errors if the polarization state of moonlight is not accounted for. The polarization measurements and model of the Moon recorded in this work enable other researchers to account for those errors more accurately. Nighttime polarimetric remote sensing could experience similar errors if using the Moon as a source of illumination without accounting for its polarization. Remote sensing of aerosol properties, trace gases, and smoke layers has been demonstrated using the moonlight as the source. Accounting for moonlight polarization in similar applications and other future work could reduce polarimetric errors. The passive polarimetric remote sensing application

that inspired the measurements of moonlight polarization conducted in this work was nighttime remote sensing of CTP, which was previously demonstrated by the Optical Remote Sensor Laboratory using sunlight illumination. While this dissertation did not thoroughly explore nighttime remote sensing of CTP since the focus shifted to moonlight polarization measurements, this work presented here addressed some of the initial steps towards that goal. Four objectives were pursued in this dissertation to address the knowledge gaps listed above and provide preliminary progress towards improved passive polarimetric remote sensing of CTP.

First, a method to record calibrated polarization images of the Moon and calculate disk-averaged DoLP was developed. This method as well as broadband moonlight polarization measurements were published [128]. This was useful because it used low-cost instrumentation and could easily be extended to other spectral bands. It yielded high-resolution images of the Moon which could enable studying different regions of the lunar surface and their properties, and it yielded the disk-averaged DoLP measurements that are more useful for using the Moon as a calibration target or a source for nighttime passive remote sensing.

Second, the Moon polarization imaging method was used to characterize the polarization of the Moon in five visible and near infrared spectral bands. This addressed the first knowledge gap mentioned above: previously, published measurements of Moon polarization across the lunar cycle only extended from the UV to 520 nm. These measurements were submitted for publication and the disk-averaged DoLP values were made available [139]. We plan to share these data with the ARCSTONE team for use in their lunar polarization model supporting operation of the ARCSTONE satellite [9]. The new measurements in this dissertation improve understanding of how the polarization state of the Moon changes for longer wavelengths. This is similarly important for lunar calibration and nighttime remote sensing for instruments operating in these bands. These measurements confirmed the general trend that the DoLP decreased as wavelength increased but also uncovered that the 900-nm

band had a higher DoLP than the 800-nm band for waxing phases.

Third, the measurements from the first two objectives were combined with measurements from the literature to develop a model of disk-averaged polarization as a function of lunar geometry variables. Eight spectral bands from 336 nm to 900 nm and a broad spectral band were modeled. All channels were modeled with dependence on lunar phase angle and sun selenographic longitude only (resulting in an estimated uncertainty of ± 0.01), and some channels were additionally modeled with dependence on the libration variables as well (resulting in an estimated uncertainty of ± 0.005). This work was prepared for submission to a journal. The model can be used to predict the polarization directly in any of the nine channels modeled, and interpolation could enable predicting the polarization in any other band between 336 nm and 900 nm. We plan to share this model with the calibration team for OCO-2 and OCO-3 to augment the model of lunar polarization that they currently used to correct their lunar calibration observations [24, 42] as well as with the calibration team for HARP2 [137].

These achievements provide an important first step towards more accurate lunar calibration, especially for spectral bands from 520 nm to 900 nm. However there are some additional needs to keep refining the accuracy. Recording more measurements in these spectral bands from 336 nm to 900 nm at different parts of the libration/lunar phase cycle will enable a model that accounts for libration more accurate for all bands. Also, validation measurements should be recorded to test the accuracy of spectral interpolation between bands in the Moon polarization model. Finally, Moon polarization measurements at wavelengths from approximately 900 nm to $2.2\ \mu\text{m}$ would be useful; many satellites have instruments with reflected solar bands in this spectral range and there are currently very few Moon polarization measurements published in this range.

In the fourth objective, improvements to a passive polarimetric method of cloud thermodynamic phase sensing were explored. This included using lower-cost instrumentation,

which could enable widespread monitoring, and developing a classification method that accounts for scattering angle, which could improve accuracy. Using the low-cost imager and classification method using scattering angle, an accuracy of approximately 90% was achieved for measurements recorded over sixteen days. The next steps in this study include obtaining a larger sample size and a polarimetric calibration over the full-field of the fisheye lens. Other important things to study in the future include improving CTP classifications by using spectral filters and exploiting the dependence of S_1 on wavelength with a system like the one presented in [110] and exploring nighttime CTP measurements under moonlight illumination. This dissertation could contribute to better understanding nighttime CTP measurements by allowing prediction of the polarization state of the Moon during observations. Also, the demonstration of CTP measurements with a broadband polarization camera could be important because using a broad bandwidth could improve signal-to-noise ratio in nighttime measurements over a method that requires narrow spectral bands.

The most important contribution in this dissertation was measurements and a model of the polarization state of moonlight as a function of lunar geometry, particularly for wavelengths of 650 nm, 800 nm, and 900 nm. These measurements and the resulting model address the two knowledge gaps detailed above and will contribute to more accurate lunar calibration of on-orbit polarization-sensitive instruments and nighttime remote sensing.

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APPENDIX

POLARIZATION ENHANCEMENT OF CONTRAST THROUGH A WATER SURFACE

Contribution of Authors and Co-Authors

Manuscript in Appendix A

Author: Erica Venkatesulu

Contributions: Investigation, formal analysis, methodology, software, validation, writing - original draft, visualization

Author: Kyndra L. Buglione

Contributions: Investigation

Author: Nathaniel J. Field

Contributions: Investigation

Author: Kirstin D. Doney

Contributions: Supervision, project administration, funding acquisition, writing - review & editing

Author: Joseph A. Shaw

Contributions: Conceptualization, supervision, project administration, funding acquisition, writing - review & editing, resources

Manuscript Information

Erica Venkatesulu, Kyndra L. Buglione, Nathaniel J. Field, Kirstin D. Doney, Joseph A. Shaw

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Abstract

When imaging underwater scenes from above the water surface, the reflection from the air-water interface creates an obscuring background that varies with illumination and viewing angles. It is well known that the reflected light is horizontally polarized, and using a vertically transmitting polarizing filter is a common technique to improve the contrast of underwater scenes. However, to our knowledge, no quantitative measurements of polarization-enabled contrast enhancement have been reported in the literature. In this work, panchromatic and RGB division-of-focal-plane polarization cameras were used to record images of black and white tiles submerged in water for determining contrast as a function of viewing angle, both without a polarizer and with a vertical polarizer. Experiments were conducted in two outdoor locations and in a black tub indoors with controlled color and brightness of the reflected background. The maximum contrast through a vertical polarizer occurred near the Brewster angle, but the amount of contrast enhancement (the ratio of contrast through a polarizer to contrast without a polarizer) was found to increase until much larger angles. Also, the observed changes in contrast resulting from changing properties of the reflected background were consistent with the Fresnel reflection coefficients.

Introduction

It is well known that using a vertical polarizer improves how well an underwater scene can be viewed through the water surface. It is also well known that this improvement arises from the vertical polarizer blocking the horizontally polarized light reflected from the horizontal water surface. However, we have been unable to find reports in the literature of experiments performed to quantify the increase in contrast of an underwater scene viewed through the water surface with a polarizer. Quantitative understanding of this effect could be valuable in many remote sensing contexts. For example, our research group

uses hyperspectral and multispectral imagers to measure benthic algae under the surface of the water in rivers [109, 151]. The surface reflections can reduce the signal-background ratio or cause errors due to the polarization response of the imagers [152]. A quantitative understanding of the polarization-enabled contrast enhancement will help inform how well the addition of a polarizer to these imagers could mitigate the above effects, and the controlled experiments in this work will produce such a quantitative understanding, along with curves that will aid qualitative understanding.

Much work has been devoted to exploring the role of polarization in aquatic environments, through both measurements and simulations. One area of interest is in using polarization to improve imaging within water, when both the scene and the imager are underwater. Many of these techniques fundamentally rely on a difference in the polarization states of light scattered by the scene and by the water. Improvements in underwater imaging have been explored by researchers using active systems to control the illumination and detection polarization states [153–156]. The benefits of polarization in passive systems relying on sunlight illumination have also been explored, including studies that assumed a target with a low degree of polarization [157, 158], studies that considered the polarization state of targets [159–161], and a study that used hyperspectral polarization imaging [162]. Additionally, biological research has shown that contrast enhancement is an important function of polarization-sensitive vision in aquatic invertebrates [163–165].

Another area of interest is in understanding the polarization of the water surface, including the polarization of reflected skylight, water-leaving radiance, and glints. Early studies made observations of the polarization of light observed at sea [166] and measured the polarization of light reflected from water as a function of incidence angle [167]. The polarization pattern of skylight reflected by the water surface has been measured and simulated for various solar angles and wind speeds [168–171]. Others have explored how to remove the effect of the polarized surface reflection to understand the polarization of the

water-leaving radiance [172–174]. One study even used polarized measurements of the water surface to detect marine oil spills [175]. Similar work in the infrared part of the spectrum has dealt with the polarization of the sea surface emission[65, 176–180] and studied how using a polarizer can improve the contrast between a man-made object on the surface of the water and sea surface emission[181–183]. Finally, researchers have explored the polarization properties of glints and how much using a polarizer improves the contrast between water with glints and targets on top of the water, in the visible[184–187] and infrared [176, 177, 188] parts of the spectrum. These studies address the polarization of skylight and sunlight reflection from a water surface, which is a dominant effect in how well objects under the water surface can be seen.

Some studies address more directly the question of improving imaging of underwater objects from above. Fluid lensing is a non-polarized method of imaging underwater objects which relies on surface waves[189]. Two studies explore how much using a polarizer improves the contrast of underwater objects [190, 191]. Dolin and Turlaev modeled the polarized and unpolarized contrasts of an object relative to the bottom of the water body as a function of angle of incidence, taking into account parameters including solar zenith and azimuth angles, azimuth angle of the imager, sky radiance, and the optical characteristics of the water [190]. They also measured the contrast at one incidence angle and set of conditions, and they compared it to the model using measured and estimated model parameters describing measurement conditions. Wang et al. used a polarization camera to explore the suppression of glints and surface reflections for improving imaging of underwater objects, but they focused on characterizing an image improvement algorithm with a limited set of incidence angles [191].

In this work, we present a simple, informative set of results that provide measurements of geometries similar to those modeled by Dolin and Turlaev [190], and we emphasize the physical interpretation of those results. These results were presented in an earlier conference

paper [192], and some text and figures are reused with permission, but this paper presents additional analysis and shows the results from modifying the Dolin and Turlaev model[190] to match one of our experiments. The remainder of this paper outlines the relevant background theory, describes the instruments and experimental setup, presents measurements of degree of linear polarization (DoLP) and contrast enhancement, discusses the effects present in those measurements, and compares the measured contrast to modeled results.

Background

The polarization of light reflected from a dielectric boundary is determined by the difference in reflectance of the s - and p -polarization states, which are governed by the Fresnel equations. For a horizontal boundary like a still water surface, the plane of incidence (containing the directions of incident and reflected light) is vertical, so the s -polarization state is horizontal (perpendicular to the plane of incidence), and the p -polarization state is vertical (parallel to the plane of incidence). Assuming a horizontal boundary, the horizontal and vertical reflectances are given as

$$R_H = R_s = \left(\frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right)^2 \quad (36)$$

$$R_V = R_p = \left(\frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} \right)^2, \quad (37)$$

where n_1 is the index of refraction in air, n_2 is the index of refraction in water, θ_i is the angle of incidence, and θ_t is the angle of transmission, calculated with Snell's law. Using these equations, the DoLP of initially unpolarized light after reflection from the horizontal surface is

$$DoLP = \frac{R_H - R_V}{R_H + R_V}. \quad (38)$$

The Fresnel reflectances and the DoLP are plotted as a function of incidence angle in Fig. 42. At the Brewster angle, $\theta_B = \arctan(n_2/n_1)$, the p -polarized reflectance goes to zero and the reflected light is completely horizontally polarized with $DoLP = 1$. This leads to the frequently discussed use of a vertical polarizer to suppress the reflected background light, thereby increasing the contrast of underwater objects seen from above the water. However, this simple treatment does not tell us the contrast that can be achieved or the degree to which the contrast is improved by the polarizer. Furthermore, influence of scattering in the water and from the bottom, as well as partially polarized skylight, are neglected. This provides motivation for our experiments and the resulting quantitative plots of contrast and the polarization-enabled contrast enhancement factor.

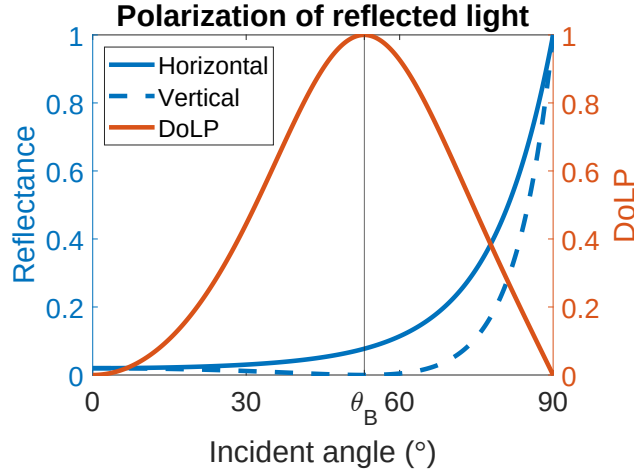


Figure 42: Fresnel reflectances for horizontal and vertical polarization states incident on an air-water interface ($n_{air} = 1$ and $n_{water} = 1.33$) and DoLP. The Brewster angle is $\theta_B = 53.1^\circ$. Reprinted with permission from [105] © Optica Publishing Group.

Instruments and Methodology

We conducted experiments to measure the contrast of an underwater scene through the water surface as a function of viewing angle. We used polarization cameras to record vertically polarized and unpolarized images, and we calculated the contrast through a vertical

polarizer and without a polarizer from those images. The remainder of this section describes the cameras and their calibrations, the black and white tiles comprising the underwater scene, the experimental setup, and the contrast calculations.

The polarization imagers used throughout this project were commercial division-of-focal-plane (DoFP) polarization cameras based on polarization-sensitive sensors from Sony: one was a broadband panchromatic camera (FLIR Blackfly BFS-U3-51S5P-C) sensitive to visible and near infrared (NIR) wavelengths and the other RGB (FLIR Blackfly BFS-U3-51S5PC-C), both capturing images simultaneously in four polarization states (0° , 45° , 90° , and 135° relative to horizontal). The DoFP polarization cameras used an array of nano-wire polarizers placed over the detector to control the polarization state measured by each pixel. The panchromatic camera used superpixels of four independently polarized pixels, while the RGB camera used a Bayer color filter over four 2×2 polarization superpixels to create a polarized RGB superpixel. The top left 2×2 group of pixels was sensitive to red light, the top right and bottom left were sensitive to green, and the bottom right was sensitive to blue. This layout was repeated across the detector array so that an image in each polarization state (and color) was formed. The images in the four polarization states were used to calculate images of the first Stokes parameter S_0 , which quantifies the total irradiance:

$$S_0 = \frac{1}{2}(I_{0^\circ} + I_{45^\circ} + I_{90^\circ} + I_{135^\circ}). \quad (39)$$

The DoLP of the light was calculated from the linear Stokes parameters as

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}, \quad (40)$$

where $S_1 = I_{0^\circ} - I_{90^\circ}$ and $S_2 = I_{45^\circ} - I_{135^\circ}$ (Eqs. 38 and 40 are equivalent for a horizontal surface).

We used these polarization cameras with 25-mm lenses that had a full-angle field of view

(FOV) of approximately 18° . We performed polarimetric, spectral, and spatial calibrations for each system. The polarimetric calibration used an integrating sphere and high-contrast wire-grid polarizer to measure the polarimetric response system matrix, which we inverted and multiplied by raw images to obtain polarimetrically corrected images. Next, we used a grating monochromator to measure the relative spectral response (RSR) of the cameras, shown in Fig. 43, according to the process described previously [103]. The panchromatic camera was used with a NIR filter for one experiment (the filter’s transmission was measured with an integrating sphere and a Ocean Optics USB4000 spectrometer to be less than 1% from 400 nm to 693 nm and increased to above 95% from 752 nm to 1000 nm). The data recorded with the panchromatic camera without the filter are sometimes referred to in this paper as “broadband data” and the data recorded with the filter as “NIR data.” Finally, we used a black-and-white checkerboard target to perform a spatial calibration and found that the instantaneous pixel FOV (IFOV) did not vary significantly across the FOV. This was expected since changes in IFOV across the field result from distortion, a wave aberration that varies as the cube of the field angle, and these lenses had a fairly small field. The full-angle IFOV for a RGB superpixel was 0.0307° and the full-angle IFOV for a panchromatic superpixel was 0.0154° .

In all experiments, a white tile and a black tile were placed underwater, so the reflection from the water surface decreased the observed contrast between the tiles. Images were recorded at a variety of viewing angles to characterize the difference between the contrast through a vertical polarizer and the contrast without a polarizer as a function of viewing angle. Since the water surface was completely still indoors and calm outdoors, the surface reflection was specular, so the viewing angle of the camera was equal to the incidence angle of skylight (or the diffuse background reflection) on the surface. Thus, for the remainder of this paper, we refer to the view angle of the camera to specify the incidence angle of skylight. The ceramic tiles used in these experiments were originally brown and painted

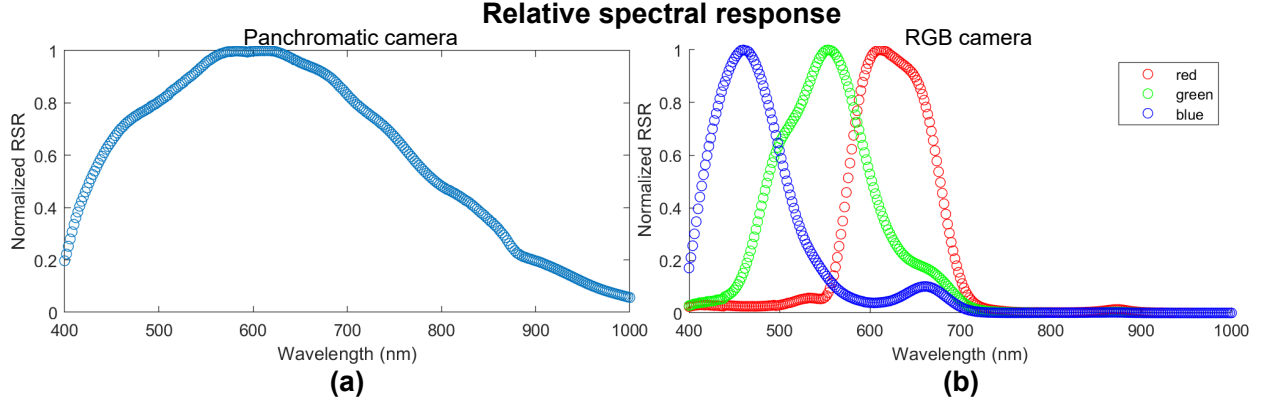


Figure 43: Relative spectral response measured for (a) the panchromatic DoFP polarization camera and (b) the RGB DoFP polarization camera with an 18° full-angle FOV lens. Figure reused with permission [192] and reprinted with permission from [105] © Optica Publishing Group.

white and black using Krylon flat paint. We measured the reflectance spectrum of both tiles illuminated by a halogen lamp, using a spectrometer and a 99%-reflective Spectralon reflectance panel. After a rapid increase starting around 400 nm, the white tile reflectance decreased gradually from approximately 94% at 450 nm to 84% at 900 nm while wet and 84% to 77% while dry. The black tile reflectance was fairly uniform over that spectral range, with a reflectance of approximately 1.5% while wet and 6% while dry.

The contrast was calculated from the images according to

$$contrast = \frac{white - black}{white + black}, \quad (41)$$

where *white* was the signal measured by pixels imaging the white tile and *black* was the signal measured by pixels imaging the black tile. Each signal was calculated as the mean digital number of at least 100 pixels near the center of the tile, where the center of the tile was chosen manually and the exact number of pixels averaged depended on the distance from the tiles to the camera. The mean S_0 values of the black and white tiles were used to calculate the contrast without looking through a polarizer, and the I_{90° values were used

to calculate the contrast looking through a vertically oriented polarizer. The polarization camera imaged the tiles from various positions above the water surface. At each position, the camera was aligned so the tiles were close to the center of the FOV of the camera, a digital level was used to measure the angle of the on-axis ray from vertical, and an adjustment was calculated with the IFOV value to determine the view angle for the part of the FOV where the tiles were located.

Measurements were recorded at Bozeman Pond in Bozeman, Montana USA, with the tiles in a water depth of approximately 45 cm, and later at Bozeman Beach with tiles in a water depth of approximately 5 cm. For the measurements at Bozeman Beach, the camera was pointed approximately 230° southwest while the Sun position varied from 154° to 170° azimuth and from 59° to 63° zenith. For the measurements at Bozeman Pond, the camera was pointed approximately due south while the Sun position varied from 172° to 187° azimuth at 53° zenith. This geometry placed the specular reflection near the tiles, but we were careful to avoid overlap of the specular reflection and the tiles in the image.

In the indoor laboratory measurements, the tiles were placed in a black tub and covered by approximately 2 cm of water. Sheets were attached to the wall behind the tub to provide relatively uniform and diffuse illumination of different colors (white, gray, and blue were used). The tiles were approximately 2 m from the camera, and the sheet was approximately 0.5 m behind the tiles. The scene was illuminated with a halogen lamp in a soft box diffuser, which was approximately 5 m away from the tiles and placed to avoid the shadow of the camera on the scene. The polarization of the gray sheet was measured as 3.5% when viewed with the polarization camera from the approximate position of the tiles. The gray sheet had the lowest reflectance and therefore the highest DoLP, so the white and blue sheets had a DoLP smaller than 3.5% (according to the Umov effect[193]).

Figure 44 shows a photograph of the setup at Bozeman Pond as well as a photograph and diagram of our indoor setup, including the tiles in a black tub and a white sheet attached

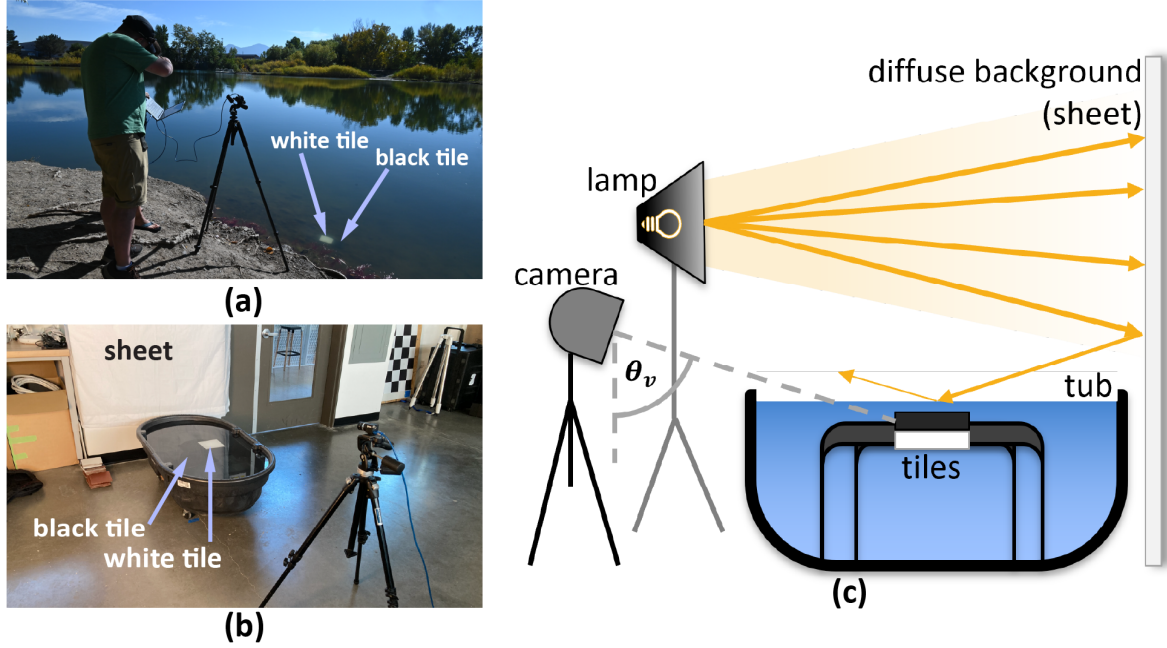


Figure 44: Photographs of setup for measurements recorded at (a) Bozeman Pond, where the black tile is to the right of the white tile and (b) indoors, where the black tile is to the left of the white tiles. (c) A diagram showing the geometry of indoor measurements. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

to the wall behind the tub. Figure 45 shows an example of uncalibrated I_{90° , I_{0° , and DoLP images recorded at Bozeman Beach. This image was recorded with the polarization camera pointed approximately 230° southwest with the Sun at 161° azimuth and 61° zenith, with a fairly uniform layer of stratus clouds in the sky.

For experiments using the RGB camera, we observed some evidence that the camera settings changed between images. However, we believe this had at most a minor effect on the contrast curves presented here since contrast and DoLP both use ratios in which multiplicative changes across whole images canceled out when we divided the white tile signal by the black tile signal or one polarization state by another.

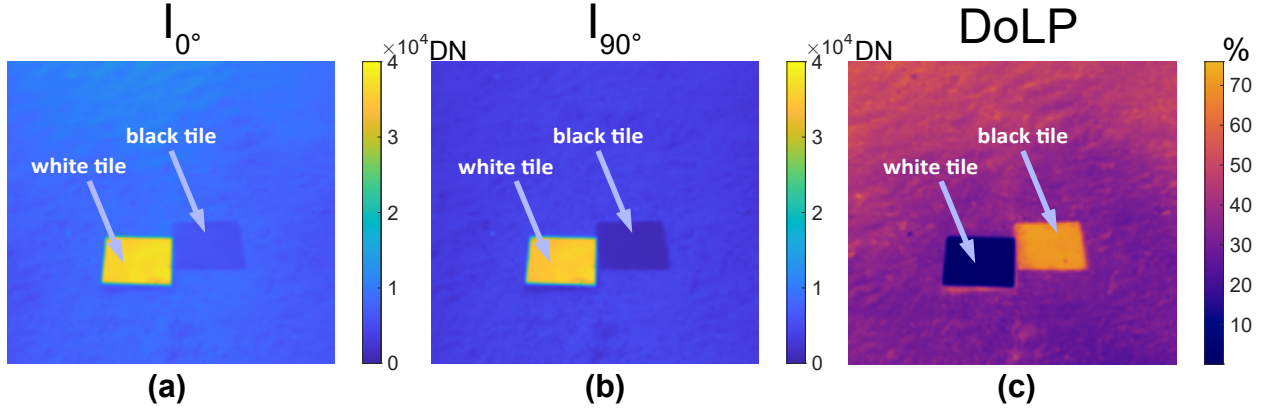


Figure 45: Example image recorded by the DoFP polarization camera, processed to show an image of (a) I_{0° , (b) I_{90° , and (c) DoLP. Water surface ripples are visible in the I_{0° image but nearly impossible to see in the I_{90° image, and the contrast between the tiles is better in the I_{90° image. Reprinted with permission from [105] © Optica Publishing Group.

Results

Here we show the primary results in the form of graphs of contrast as a function of view angle. Specifically, the following three parameters are plotted vs. view angle: 1) S_0 contrast from Eq. (41), representing what would be measured by an unpolarized camera; 2) I_{90° contrast from applying Eq. (41) to only the vertically polarized (90°) subpixels to remove the horizontally polarized reflection; and 3) the ratio I_{90°/S_0 to describe the polarization-enabled contrast enhancement. First, we present the results from the two outdoor experiments. The panchromatic camera results are shown in Fig. 46, and the RGB camera results are in Fig. 47. Figure 46 shows that the contrast with a vertical polarizer (the I_{90° contrast) was higher than the contrast without a polarizer (the S_0 contrast) and that the ratio between the two increased with the incidence angle. Figure 47 shows the same two trends and also shows that the contrast ratio was significantly higher in the blue channel than the red and green. We hypothesized that this difference was because of the relative brightness of the sky in those three channels (i.e., the color of the sky).

We next conducted a series of indoor experiments to explore the effect of background

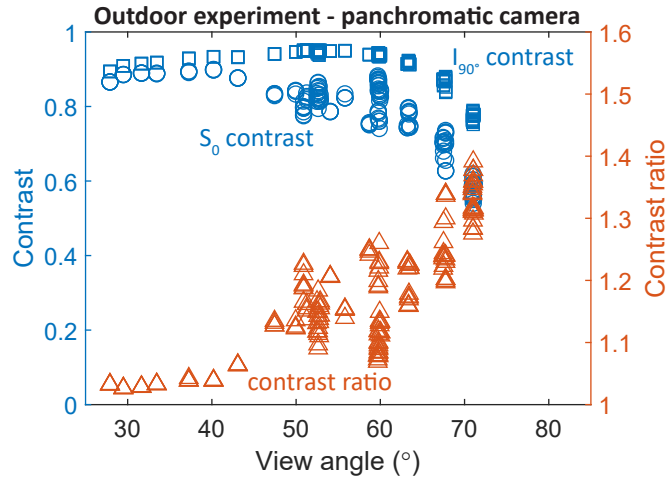


Figure 46: Results of outdoor experiment with panchromatic camera. Polarization-enabled contrast enhancement measured by a panchromatic camera with tiles under approximately 5 cm of water at Bozeman Beach. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

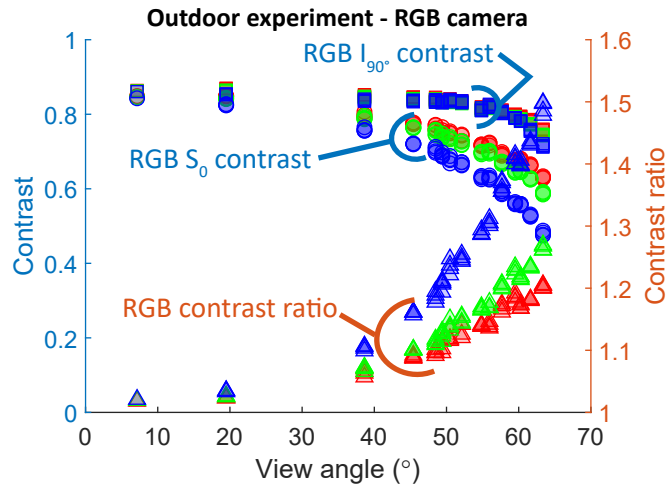


Figure 47: Results of outdoor experiment with RGB camera. Polarization-enabled contrast enhancement measured by RGB camera with tiles under approximately 45 cm of water at Bozeman Pond. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

brightness and color, investigate what happens at higher angles of incidence than we could feasibly measure outside, and avoid the effects of a nonuniform sky or small water surface waves. The first indoor measurements are shown in Fig. 48, where a white sheet was used as the relatively uniform background that reflected off the surface of the water. The data in Fig. 48 were used as a baseline to which other measurements were compared. Before changing variables such as the brightness of the background, we took measurements of the tiles without any water to provide insight to the angle dependence of the tiles themselves. The data with and without water covering the tiles are shown in Fig. 49 and were used when interpreting other measurements to separate surface effects from tile effects. Figure 50 presents the same data in Fig. 49 (the results of the experiments with and without water) but processed as DoLP as a function of view angle to provide insight into the polarization of the tiles.

The next experiments allowed us to explore the influence of background brightness and color. Figure 51 presents data from the panchromatic camera with both white and gray sheet backgrounds, showing that the darker background produced less reflected radiance and lower contrast enhancement. The background color was changed for the next experiment. Indoor measurements using the RGB camera are shown in Fig. 52 for a white sheet background (left) and blue sheet background (right). The white background produced essentially equal red, green, and blue curves, while the blue background produced a higher polarization-enabled contrast enhancement in the blue channel. Finally, the last measurements were recorded with the panchromatic camera and a NIR filter. These are presented in Fig. 53, which shows measurements with and without a NIR filter that blocked wavelengths shorter than 750 nm.

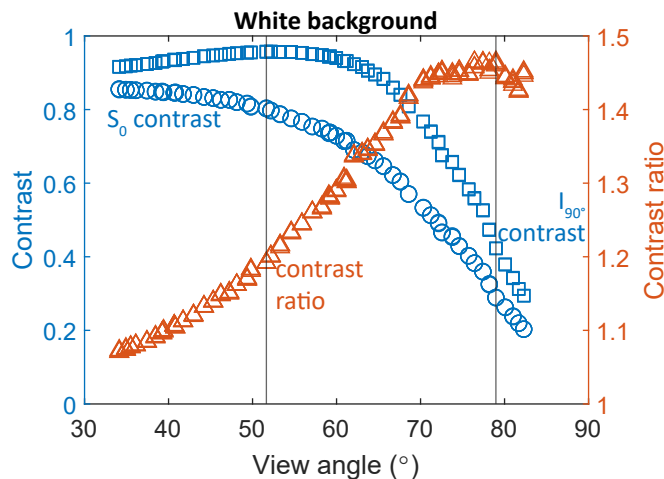


Figure 48: Results of indoor experiment with white sheet background.

Polarization-enabled contrast enhancement measured with black and white tiles under approximately 2 cm of water. Measurements collected with panchromatic camera, white sheet background (reflectance of the white sheet was 92%), and halogen lamp illumination. Vertical lines show the location of the maximum I_{90° contrast and maximum contrast ratio. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

Discussion

The experiments conducted outdoors demonstrate a few interesting and important trends. After a discussion of Figs. 46 and 47, this section consists of explanations of the indoor data in Figs. 48 to 53 to explore the effect of different variables and better understand the outdoor data. It should be noted in all of these measurements that the maximum possible contrast between the tiles depended on the reflectance of the tiles and would only approach a value of 1 if the reflectance of the black tile was 0.

Outdoor measurements

Figure 46 shows data recorded outside on an overcast day with the panchromatic camera. The maximum I_{90° contrast occurred at 51.3° and the maximum S_0 contrast occurred at 40.2° . The contrast ratio appeared to increase steadily up to the maximum angle measured (71°), but the noise in the S_0 contrast data strongly affected the contrast

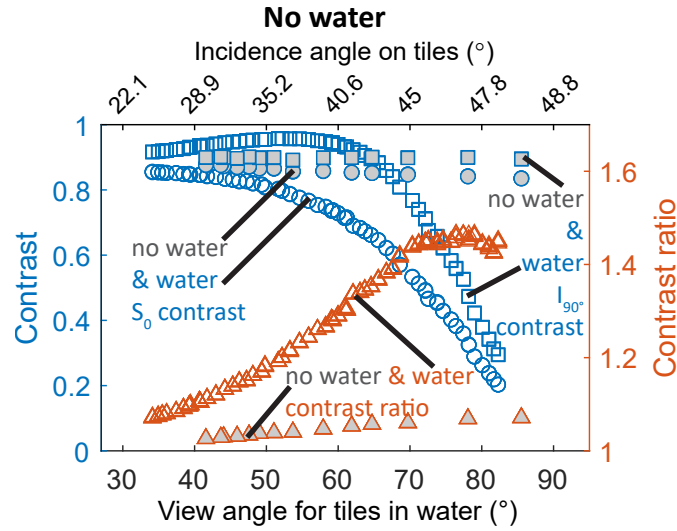


Figure 49: Results of indoor experiment with no water covering the tiles. Polarization-enabled contrast enhancement measured by panchromatic camera with white sheet background compared to that measured without water covering the tiles. Reprinted with permission from [105] © Optica Publishing Group.

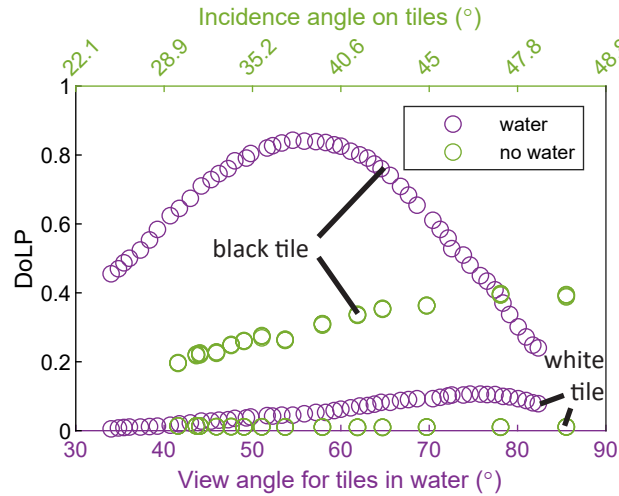


Figure 50: Results of indoor experiment with no water covering the tiles. DoLP of tiles measured by panchromatic camera with and without water, with the white sheet background. The data with water describe the DoLP of both the light reflected from the surface and from the tiles, while the data without water only describe light reflected from the tiles. Reprinted with permission from [105] © Optica Publishing Group.

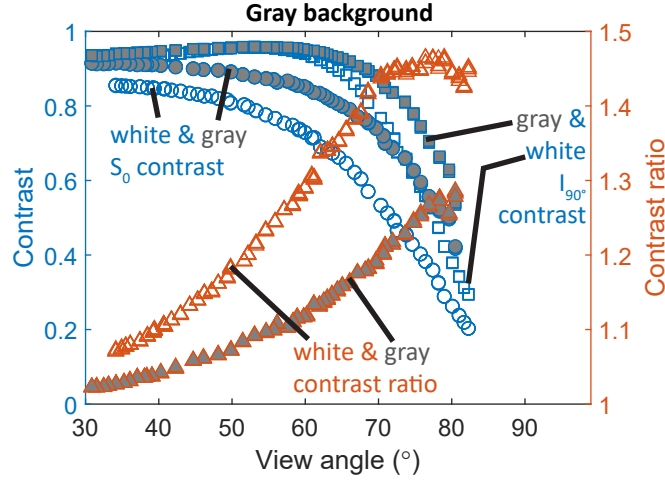


Figure 51: Results of indoor experiment with gray sheet background. Polarization-enabled contrast enhancement measured by panchromatic camera with white sheet background compared to that measured with gray sheet background (reflectance of the gray sheet was 36%). Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

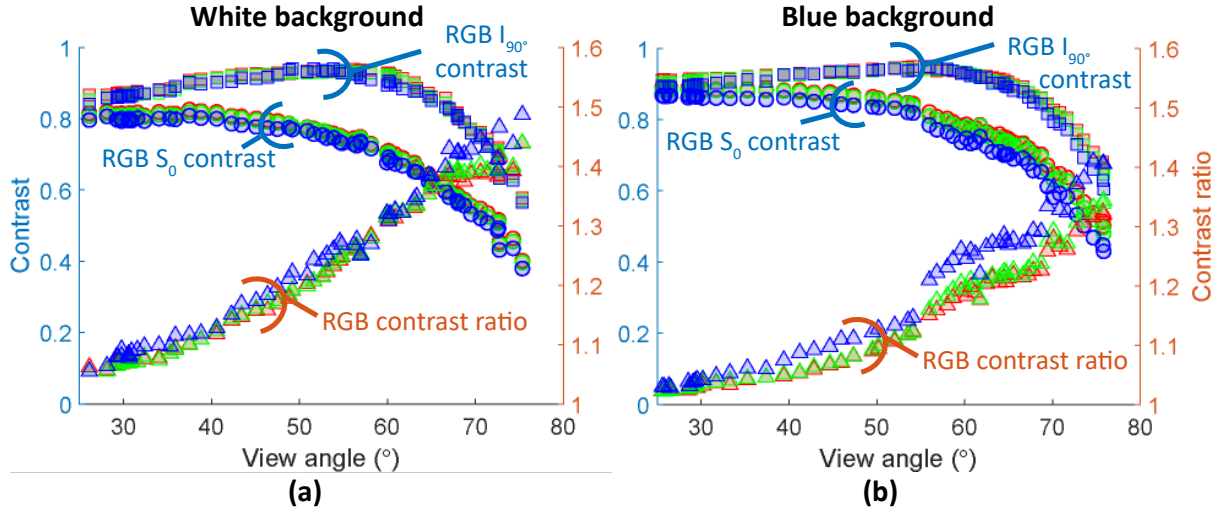


Figure 52: Results of indoor experiment with blue sheet background. Polarization-enabled contrast enhancement measured by RGB camera with white sheet background compared to that measured with blue sheet background. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

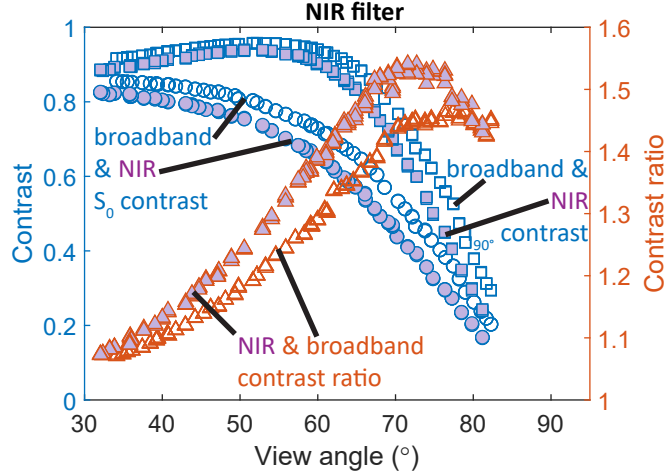


Figure 53: Results of indoor experiment with NIR filter. Polarization-enabled contrast enhancement measured by panchromatic camera with white sheet background compared to that measured with a NIR filter on the camera. Figure adapted from [192] and reprinted with permission from [105] © Optica Publishing Group.

ratio since it was in the denominator. Because the S_0 contrast data are noisier than the I_{90° contrast data, we concluded this noise was due to small waves on the surface and non-uniformities in the sky background from the clouds. Also, because of the overcast sky, the skylight reflected from the water surface likely had a low DoLP, so we did not consider the sky polarization to affect the amount of reflected light.

Similar patterns were present in the RGB data shown in Fig. 47. The maximum I_{90° contrast occurred at an angle of 19.5° , the maximum S_0 contrast occurred at the smallest view angle measured, 7.1° , and the contrast ratio increased monotonically in all three channels. These data were recorded on a clear sky day with a completely calm surface, which explained why they were less noisy than the data recorded with the panchromatic camera. Both the S_0 and I_{90° contrast curves were lower in the RGB experiment, which was likely because of the increased water depth and turbidity. Finally, these data showed that the I_{90° contrast was similar for all three channels, but the S_0 contrast was lowest in the blue and highest in the red. Therefore, the highest contrast enhancement was observed in the blue and the lowest in the red. We interpreted this as a result of the blue sky (i.e., the reflected background was

brightest for the blue channel).

Indoor measurements

To interpret the differences between experiments, we begin by making observations about the white sheet background measurements in Fig. 48 and comparing them to the outdoor panchromatic measurements. In the white sheet background measurements, the maximum contrast between the white and the black tiles as viewed through a vertical polarizer occurred at 51.6° , very close to the Brewster angle ($\theta_B = 53.1^\circ$ for $n_{water} = 1.33$). This made sense because at the Brewster angle the reflectance for vertically polarized light was 0, so the reflected light was 100% horizontally polarized. Thus, the vertical polarizer nearly completely suppressed the surface reflection. This angle was similar to the angle of maximum I_{90° contrast observed in the outdoor data with the panchromatic camera (Fig. 46). However, in the measurements with the white sheet background, the S_0 contrast decreased continuously with angle, while in the outdoor measurements, the S_0 contrast reached a maximum at 40.2° . A monotonically decreasing S_0 contrast was consistent with a monotonic increase of the total reflectance (horizontal plus vertical) with angle. A potential explanation for the increase in S_0 contrast in the outdoor measurements for small view angles was changing sky conditions throughout the measurement period.

A final observation about the white sheet background measurements was that the maximum value of the polarization-enabled contrast enhancement occurred at a view angle of 79° , well beyond the Brewster angle. This angle depended on the relative increase of the vertically and horizontally polarized reflection components. While the S_0 contrast decreased monotonically with angle because of both the vertically polarized and horizontally polarized components, the I_{90° contrast decreased only at large angles where the vertically polarized reflection began to increase. At even larger near-grazing angles, the vertically polarized reflectance approached the horizontally polarized reflectance (see Fig. 42), so it is only

at these grazing view angles that the benefit of the vertical polarizer started to decrease. However, since the vertical polarizer always blocked the horizontally reflected component, it always provided some contrast enhancement (i.e., the ratio maintained a value above 1). Thus, these observed contrasts strongly depended on suppression of the surface reflection but could also be affected by the angular dependence of the contrast of the tiles, so we measured the contrast of the tiles without submerging them in water.

Measurements of the tiles without a water surface reducing the contrast are compared to those with water reflecting a white sheet background in Fig. 49. To account for refraction, the bottom axis in Fig. 49 shows the view angle of the camera with the tiles underwater and the top axis shows the corresponding angle of incidence on the tile after refraction. Thus, measurements of the tiles above water are shown only for incidence angles less than approximately 48° , the angle corresponding to a camera view angle near 90° in air. We did not consider the impact of refraction on the angle of illumination because the illumination was diffuse. The contrast between the tiles without water decreased slowly as a function of angle. Specifically, the above water S_0 contrast changed by 5% over the range of incidence angles on the tile and the above water I_{90° contrast changed by less than 1%. If the contrast between the white and black tiles were falling off more quickly, this could have shifted the angle at which we observed the maximum contrast between the underwater tiles; that is, even if the reflection were maximally suppressed at the Brewster angle, the contrast could be no higher than the contrast of the scene beneath the water. Thus, the slowly changing contrast between the tiles gave further evidence that the main factor in observing maximum contrast close to the Brewster angle was maximum suppression of the surface reflection, and the decreasing contrast likely explained why the maximum polarized contrast was observed at an angle slightly smaller than the Brewster angle.

However, it was unexpected that the contrast through a polarizer was higher for the tiles measured underwater than for the tiles measured without water to a viewing angle of

approximately 65° (empty squares are higher than filled squares in Fig. 49). This was likely because the tiles were allowed to dry before measurements were made without water, and the black tile looked darker when it was wet, which in turn increased the contrast between the tiles. The decrease in reflectance of wet surfaces has been explained for multiple-scattering media[194] and rough absorbing surfaces [195]. This same increase in contrast was not observed without a polarizer (filled circles are higher than empty circles in Fig. 49) because of the horizontally polarized surface reflection. Although there was an increase in unpolarized contrast when the tiles were wet, the decrease in contrast due to the surface reflection was larger.

These measurements are further compared in Fig. 50, where the DoLP was plotted as a function of angle for the black and white tiles, with and without water covering the tiles. For both the underwater and above-water measurements, we observed that the black tile was more polarizing than the white tile. This is explained by the Umov effect [193]. Also, we observed that the DoLP was higher for both the white and black tiles when they were underwater, primarily because of the highly polarized surface reflection. However, the underwater white tile still had a relatively low DoLP (which can also be observed in the image in Fig. 45). The measured light can be thought of as a superposition of polarized light from the surface and relatively unpolarized light from the bright, white tile and thus had a low DoLP. This decrease in DoLP in shallow waters due to the reflectance of the bottom surface has been studied previously[196]. The key takeaway from these observations is that the scene itself does not have a strongly angle-dependent contrast and therefore does not affect measurements of contrast enhancement of underwater tiles significantly.

The comparison between the white and the gray background in Fig. 51 also has features that can be explained by considering the surface reflection. First, the gray reflection obscured the tiles less than the white reflection, so the contrast in S_0 was lower for the white background than for the gray. Second, the contrast through a polarizer was very similar for

the white and the gray backgrounds at small view angles. Even though the white background resulted in a brighter reflection, the vertically polarized reflectance was small at these angles, so neither reflection had a significant effect on the observed contrast. Instead, the contrast of the tiles themselves primarily controlled the contrast observed at these small angles. Third, after the Brewster angle, the vertically polarized reflectance started to increase, so more reflected light passed through the polarizer. This resulted in more contrast-reducing background light from a brighter background (white background) than for a darker one (gray background), and therefore a more rapidly decreasing I_{90° contrast for the white background than the gray one.

The effect of background color was also investigated in a set of measurements recorded with the RGB DoFP polarization camera. Measurements were recorded with white and blue sheet backgrounds (Fig. 52). As expected, the blue channel had the lowest contrast in S_0 when the background was blue, but the blue reflection was largely suppressed when the vertical polarizer was used. Thus, the contrast in I_{90° was nearly the same for all three channels until higher view angles, when the vertically polarized reflection coefficient started to increase substantially. In comparison, the red, green, and blue channels were found to have more similar curves when the background was white. In other words, the polarization-enabled contrast enhancement was largest with the blue background because the background was brighter in that band. This supports our interpretation of the higher blue contrast ratio arising from the blue sky in the outdoor experiment (Fig. 47).

The last sets of indoor measurements were recorded with the panchromatic polarization camera with a NIR filter. In the comparison of the white sheet measurements made with and without the NIR filter (Fig. 53), the S_0 and I_{90° contrasts with the NIR filter decreased relative to the broadband contrasts. We believe the observed decrease was caused by both lower transmission through the water in the NIR part of the spectrum and by a lower tile contrast in the NIR. First, the transmission through the water at a depth of 2 cm and a view

angle of 60° outside of the water was 98.5% at the center of the broad spectral response of the panchromatic camera and 87.5% at the center of the response of the panchromatic camera with the NIR filter. These transmission values were calculated from the water absorption values given by Hale and Querry[197]. This decreased the amount of signal from the tiles relative to the signal from the background. Second, the measured reflectance spectrum (described in Sec. A) of the white tile decreased with wavelength, and the measured spectrum of the black tile was constant, so the contrast between the tiles themselves was lower in the NIR. Another small difference between the broadband and NIR measurements is the average index of refraction of the water, but the index of water changes from $n = 1.339$ at 400 nm to $n = 1.327$ at 1000 nm [197]. Thus, the Brewster angle changes from 53.25° to 53.00° , and this probably was not a cause of any effects we observed.

The exact angle of maximum contrast enhancement was different for each of these experiments. As mentioned in the discussion of the measurements with the white sheet background, the relative increase in the vertically and horizontally polarized reflected components determined the angle of maximum contrast enhancement. However, this relative increase in the amount of reflected light depended on both the Fresnel reflectances and the properties of the background itself. A brighter background created a larger reflection, and the polarization state of the background (particularly in the outdoor measurements) could have affected the amount of reflected light as well. This explained the different angles of maximum contrast enhancement. On the other hand, the angle of maximum vertically polarized contrast depended primarily on the tiles and the Brewster angle, which explained why a similar angle of maximum I_{90° contrast was observed for most experiments.

Modeled results

Finally, we compared some of our measurements to the model published by Dolin and Turlaev by implementing a simplified version of their model. The purpose of this comparison

was to evaluate how well the sources of polarization considered in the model could explain the observed contrast and therefore add support for our interpretations, not to generally validate the model. The comparison was performed for the indoor experiment with the tiles underwater and the white background (shown in Fig. 48) as well as the indoor experiment with the gray background (shown in Fig. 51). The simplified model used the following parameters and modifications:

- Reflectance of the white tile $R_{white} = 0.89$ and reflectance of the black tile $R_{black} = 0.015$ from the spectrometer measurements described in Sec. A.
- Reflectance of the white sheet background $R_{sheet,white} = 0.92$ and reflectance of the gray sheet $R_{sheet,gray} = 0.36$, determined with a 99% Spectralon panel.
- No added radiance or extinction from the water column because the water depth was only ~ 2 cm (i.e., $L_{water} = 0$ and $T = T_t = 1$ instead of using Eqs. (4) and (7) from Ref. [190], respectively).
- The background radiance was calculated assuming the sheet was a Lambertian reflector of incident irradiance. The irradiance incident on the sheet and the tiles were calculated using the approximate geometry of the scene and an arbitrary lamp brightness which canceled out (instead of using Eqs. (8)-(14) from Ref. [190] to calculate L_{sky}). Specifically, the lamp was considered a point source at a distance much further than the distance between the tiles and the sheet background at a zenith angle of 55° .
- We used the Michelson contrast instead of the Weber contrast [198].

The modeled contrast is plotted as a function of angle with the measured contrast in Fig. 54 for the indoor experiment with the white sheet background in Fig. 54(a) and with the gray sheet background in Fig. 54(b). The strong qualitative agreement between the measurements and the model suggested that the sources of polarization considered in the

model (transmission of illumination into the water, transmission of light from scene out of the water, and reflection of background from the water surface) were the primary factors affecting the contrast observed from above the water. This supports our interpretation that the measured contrast curves depended primarily on the background reflection from the water surface. Potential sources of disagreement between the model and the measurement include the assumptions that the tiles were Lambertian reflectors, reflecting both polarization states equally, and that the tiles were only illuminated by the lamp (in reality, the tiles were also illuminated by light reflected from the background).

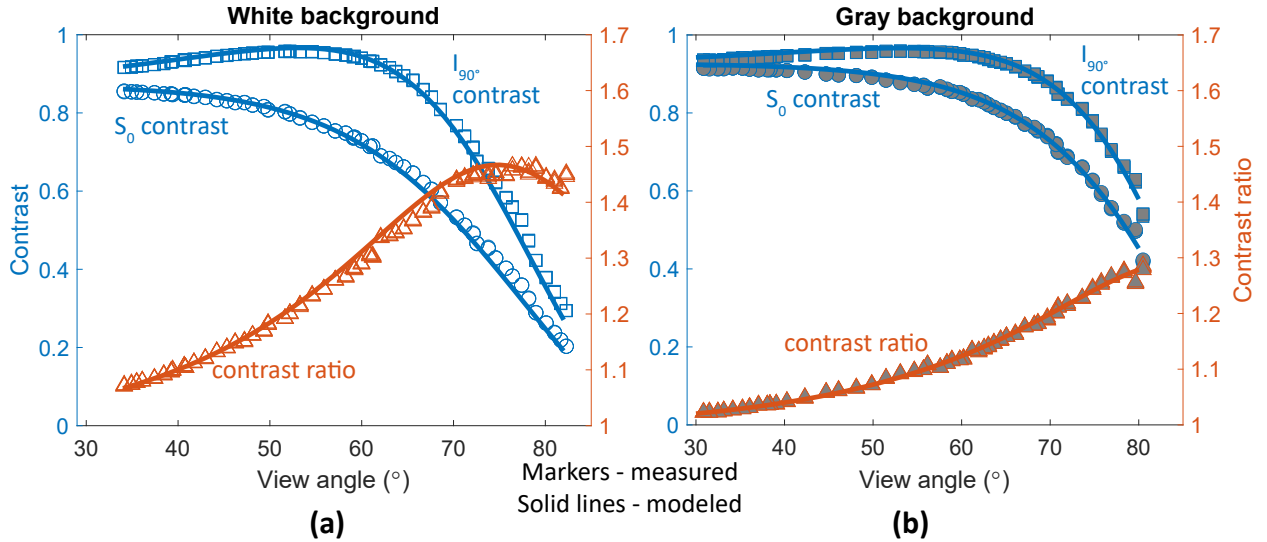


Figure 54: Comparison of the measured and modeled vertically polarized contrast, unpolarized contrast, and contrast enhancement for the indoor experiments using the white and gray sheet backgrounds. Reprinted with permission from [105] © Optica Publishing Group.

The model was also used to briefly explore the effect of angle of illumination. We ran the model with the parameters corresponding to the white background experiment and varied the zenith angle of the lamp. As the zenith angle increased, the tiles (horizontal surfaces) were illuminated from a more glancing angle, and the sheet background (a vertical surface) was illuminated from an angle closer to normal. Thus, the brightness of the background increased relative to the brightness of the tiles. The model results were consistent with this

change in relative brightness; the model results for a small zenith angle were more similar to the results from the gray sheet experiment, while the results for a larger zenith angle were more similar to the white sheet experiment.

Conclusion

We measured the polarization-enabled contrast enhancement of an underwater scene consisting of a black tile and a white tile. The contrast of the underwater scene observed above the water depends on the degree to which the underwater scene contrast changes with incidence angle. Our measurements showed that the contrast of the ceramic tiles we used did not change strongly with angle, so we would expect these results to be representative of other materials that behave similarly (i.e., materials that produce a fairly diffuse reflection). While the phenomenology explaining the horizontal polarization of light reflected from an air-water interface is well known and vertical polarizers are commonly used to improve imaging of underwater scenes, we are not aware of any published quantitative measurements of the contrast enhancement of an underwater scene resulting from use of a polarizer. We made the following observations from the quantitative measurements presented here:

- The maximum contrast between an underwater black and white object occurred when viewing the scene through a vertical polarizer at view angles close to the Brewster angle.
- The maximum polarization-enabled contrast enhancement occurred at an angle much larger than the Brewster angle.
- The effect of background brightness and color on the observed contrast was primarily explained by the Fresnel reflectance.

In other words, the underwater scene was most visible at an angle near the Brewster angle when using a vertical polarizer, but the amount of benefit from using a vertical polarizer was

maximized at a much larger angle. This leads to the following recommendations. In the case of imaging without a polarizer, the view angle should be set as close to 0° as possible for best imaging. With a vertical polarizer, the view angle should be set close to the Brewster angle for best imaging. In cases where the view angle is constrained by other factors, this work showed that the exact amount of contrast enhancement will depend on factors such as the brightness and color of the reflected background, but the curves presented here could help in predicting the amount of improvement. In future work the effect of the polarization of the underwater scene and of the background should be investigated. The degree of linear polarization of the background could be significant in outdoor applications because of sky polarization and could be experimentally investigated by imaging the scene with various parts of the sky reflecting from the water surface.

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Disclosures

The authors declare no conflicts of interest.

Data Availability Statement

The data underlying the results presented in this paper are available in Ref. [199]. The processing codes were corrupted after the data were interpreted and figures were created, so they are not available.