An economic evaluation of nitrogen fertilization of Montana winter wheat
by Bradley Eugene Garnick

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Applied Economics
Montana State University

Abstract:
Relationships between yield and protein response of winter wheat to nitrogen fertilizer and various soil and climatic variables were determined using data from 43 fertility experiments conducted in Montana during the years 1967, 1968, 1970, and 1971. Relationships were estimated using a generalized nonlinear least squares algorithm.

Additive and multiplicative error models were examined. Explanatory variables were applied nitrogen, applied phosphorus, April through July precipitation, NO3-N to 4 feet in early spring, and soil water to 4 feet in early spring.

Variables important in explaining yield response were first 1 determined using an additive error model. Precipitation, NO3-N, and applied nitrogen were important in explaining yield response. A multiplicative error model was then estimated and additive and multiplicative models were compared. Multiplicative models were chosen for estimating yield and protein response based on the properties associated with the error term. Important variables for explaining protein response were precipitation, NO3-N, applied nitrogen, and applied phosphorus.

Estimated yield and protein response equations were used to determine optimal rates of nitrogen application under varying nitrogen-wheat price conditions. Optimal rates were first determined without a protein premium structure. Protein response and a protein premium structure were then included to illustrate the magnitude of the effect protein premiums may have on optimal nitrogen fertilization practices. High levels of soil nitrate did not prevent economic application of nitrogen fertilizer. As expected, protein premiums increased optimal rates of nitrogen application under most conditions.

In the final section, a 25-percent marginal rate of return was specified and used to derive optimal nitrogen rates without a protein premium. Specification of this marginal rate of return reduced optimal nitrogen rates.

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Signature Bradley Eugene Harnick

Date July 18, 1979

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BRADLEY EUGENE GARNICK

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of

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Deserving of special note are my parents for their continued encouragement and support during my academic career.

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ABSTRACT

Relationships between yield and protein response of winter wheat to nitrogen fertilizer and various soil and climatic variables were determined using data from 43 fertility experiments conducted in Montana during the years 1967, 1968, 1970, and 1971. Relationships were estimated using a generalized nonlinear least squares algorithm. Additive and multiplicative error models were examined. Explanatory variables were applied nitrogen, applied phosphorus, April through July precipitation, NO_3-N to 4 feet in early spring, and soil water to 4 feet in early spring.

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CHAPTER I

INTRODUCTION

Statement of the Problem

Nitrogen fertilizer usage in Montana has increased from 10,932 tons in 1965 to over 52,784 tons in 1976. Nitrogen fertilizers have increased yields and profitability of winter wheat production on many soils. Montana winter wheat producers need precise information concerning the rates of nitrogen fertilizer application best suited for their situation. Rates of nitrogen fertilizer application that produce the highest yields seldom result in the highest profits.

The costs of the inputs necessary to produce a winter wheat crop have risen considerably in the 1970's. During this same time period, extremely volatile price levels have prevailed in winter wheat markets. Winter 1977 prices received by Montana wheat producers are approximately 50 percent of 1973-74 price levels. Nitrogen fertilizer prices have also fluctuated widely from a level of 10 cents per pound of actual N in 1970 to over 30 cents per pound in 1974. Winter 1977 nitrogen price is 20 cents per pound of N. With rapidly changing input and output prices, it is imperative that producers strive to determine profit-maximizing levels of nitrogen fertilizer use. An uneconomic fertilizer application could result in a considerable loss of profits to wheat producers.

The criteria used by farmers for making fertilizer decisions vary considerably. Fertilizer applications are often made with little regard for changing price structures. In addition, differences in response to nitrogen arising from varying soil and/or moisture conditions are not well defined. If wheat farmers are to maximize profits with respect to fertilizer costs, it is necessary for them to have a decision criterion for nitrogen fertilizer application which is applicable under variable moisture and soil fertility conditions.

In order to make economic decisions regarding the level of nitrogen to apply to winter wheat, two types of information are needed. First, the decision maker needs information concerning the physical response of wheat to nitrogen. Two types of response must be quantified: (a) incremental yields forthcoming from different levels of nitrogen application, and (b) incremental changes in protein percentage associated with different levels of nitrogen. Second, the decision maker must have input cost information for nitrogen and product price information for wheat, including protein premium structures. If this information is available, basic economic logic can be used to determine optimum levels of nitrogen application (6).

Purpose of the Study

The general purpose of the study is to develop and present a profit maximizing decision criterion based on the best information available which will be of use to Montana winter wheat producers. In order to accomplish this end, the specific objectives of the study are to:

1) Estimate yield and protein response of winter wheat to applied nitrogen fertilizer and important soil and climatic variables.
2) Determine the optimal levels of nitrogen application given specified protein premiums, wheat prices, and nitrogen fertilizer costs.
3) Assemble and present the derived data and information in a form which can be used by Montana wheat producers to make economically rational decisions concerning nitrogen fertilizer use.

The chapters which follow discuss the means used to complete these steps. More precisely, Chapter II presents a review of previous work on wheat response to nitrogen fertilization. Chapter III discusses the specification and statistical estimation of yield and protein response to nitrogen fertilizer. In Chapter IV, input and output prices are introduced and economic logic is used to derive optimal application rates. Chapter V offers concluding remarks and suggestions for future work.

CHAPTER II

LITERATURE REVIEW

The ability to predict wheat yield and yield response is necessary if decision makers are to determine optimal levels of fertilizer application. A considerable amount of research has been completed which has studied the effects of measurable soil and climatic variables on the response of wheat to applied fertilizer. Significant increases in yields and protein content of the grain have been attributed to nitrogen fertilizers (9, 14, 18, 19). Results from this and other research suggest that a number of factors influence yield-protein relations. Variation in crop management practices can have a substantial effect on grain response. Seeding rates, row spacing, varieties, tillage practices, and land-use systems are all thought to influence wheat yields (12, 21, 26).

Studies which have attempted to explain wheat yield relations have been quite varied in terms of the explanatory variables included and the geographic areas studied. Early work by Fisher (10) utilized linear regression techniques to examine the effects of rainfall on wheat yields. Response curves were estimated giving the expected change in yield for an additional inch of precipitation falling above the average at any time of the year. Similar techniques were used by researchers in India (11). Results from their study indicated that rainfall distribution accounted for 75 percent of the total variation in yields on unfertilized lands.

Robertson (23) used a factorial yield-weather model to analyze 50 years of spring wheat yield and weather data from southwestern Saskatchewan. Precipitation for the summer-fallow period and for May, June and August; global radiation for May; and maximum temperature for June, and July proved to be the most important variables in explaining wheat yield variation. Later research in southwestern Saskatchewan by Read and Warder (22) found growing season precipitation to be more important than stored soil moisture in explaining yield and protein variability on unfertilized plots. On fertilized plots, stored soil moisture exerted a greater influence on yield and protein content of the grain than did growing season precipitation. They concluded variables which could be measured before seeding had the greatest influence on spring wheat response to nitrogen fertilizer.

Bauer et al. (3) correlated rainfall and stored availabe moisture with barley and spring wheat response. Total moisture accounted for 40.3 percent of the yield response to nitrogen fertilizer. Bair and Robertson (2) used maximum and minimum temperature, rainfall, and stored soil moisture to predict wheat yields. In western Oklahoma, positive and significant correlations between wheat yields and soil moisture at seeding, growing season precipitation, temperature during the ripening period, and soil moisture in the spring were reported by Eck and Tucker (8). The relationships, however, did not yield satisfactory prediction equations.

Recently Black (4) published results indicating that, under Montana conditions, successful annual cropping of winter grains requires 8 to 10 inches of water from stored soil moisture and growing season precipitation. In Texas (1), October through June rainfall affected wheat yields under summer fallow conditions. Approximately 5 inches of rainfall were required during this time period before any measurable amount of grain was produced. Each additional inch received during this period increased production by about 2 bushels per acre.

Other studies have related grain protein to moisture components in the environment. Under irrigated conditions in Mexico (9), grain protein increased as the available moisture percentage at time of irrigation decreased. Protein content of the grain was decreased by small applications of nitrogen but was increased by large application rates. Brengle (5) correlated available soil moisture with grain protein in Colorado. Significant negative correlations existed between the two variables. In the Great Plains (26), increases in rainfall and soil moisture decreased grain protein.

Levels of soil fertility have also been shown to have a marked effect on yields and protein content. Significant positive relationships between these response components and soil nitrate nitrogen have been documented in the literature (20, 28). In Nebraska, Terman et al. (29) reported grain yield-protein relationships to be tempered by soil nitrate levels. Under adequate moisture conditions and low available soil nitrate levels, response to applied nitrogen was high and protein levels were low. At higher soil nitrate levels, yield response to applied nitrogen was low and the chief effect of applied nitrogen was to increase grain protein. Young et al. (32) included soil nitrate at seeding, total nitrogen content, and organic matter in an analysis of spring wheat response. Soil nitrate at seeding was significant in the model at the .01 level.

Taylor et al. (28) indicated the effect of soil nitrate nitrogen on yields was most beneficial when growing season precipitation was high. Results published by Smika et al. (27) suggest soil moisture at seeding has an influence on soil nitrate-yield relationships. When soil moisture at seeding was included in the analysis, the largest grain yields were obtained where soil nitrate levels were lowest. With the exclusion of soil moisture at seeding, no relationships existed between soil nitrate and grain yields either with or without nitrogen fertilizer. Researchers in Montana (7) attributed the lack of response to nitrogen fertilizer to pre-existing high soil nitrate levels.

Recent efforts to quantify yield and protein responses under Montana conditions were made by Jackson (15). A stepwise multiple regression program was used to analyze winter wheat data from 47 summer fallow locations. Prediction equations were generated for potential grain yields. Predicted potential yields were then used along with soil and climatic variables to formulate models to estimate the nitrogen fertilizer rates required to achieve the predicted potential yields. Finally, grain protein models were developed using potential yields, nitrogen fertilizer rates, and soil and climatic factors. Only one observation from each available set of data was picked for use in formulating these models. Growing season precipitation, soil organic matter, and evaporation rates during the first half of the growing season were important in explaining potential yields. Soil moisture measurements proved to be nonsignificant in these models. When nitrogen fertilizer requirements were estimated, soil nitrate nitrogen, potential yield, available soil water, and evaporation rates during the first half of the growing season were the most important variables. Important variables for estimating protein content were potential yield, soil nitrate nitrogen, nitrogen fertilizer requirements, organic matter, and growing season rainfall. Positive relationships were reported between protein content and soil nitrate nitrogen, soil organic matter, and nitrogen fertilizer requirements. Multiple correlation coefficients for potential yield, nitrogen fertilizer requirement, and grain protein equations were 41 percent, 58 percent, and 41 percent, respectively. Models were also generated to predict yield and percent protein on unfertilized plots and residual soil nitrate after harvest.

Results from these studies suggest that a number of soil and climatic variables and variable interactions influence grain response.

In these experiments, soil nitrate and applied nitrogen were positively correlated with grain yield and protein content. Soil water and precipitation were also positively correlated with grain yield but negatively correlated with protein content. Variability in the success obtained using these factors along with other soil and climatic variables to predict yield and/or protein response may be explained by variation in individual soil characteristics, the distribution of soil water and soil nitrate within the top 4 feet of the soil profile, management practices, and many other factors. It is hoped that the following research will contribute to a better understanding of these relationships.

CHAPTER III

ESTIMATION OF YIELD AND PROTEIN RESPONSE

Source of Data

The data used for the analysis were collected from a series of nitrogen top-dressing experiments conducted by the Montana Cooperative Extension Service and the Montana Agricultural Experiment Station. Forty-three sets of data representing 38 locations were included in the final statistical analysis. Site locations and investigators are listed in the Appendix.

Data were selected according to their ability to meet a predetermined set of criteria. Initial selection was limited to experimental plots located on summer fallow land with good stands of a recommended variety of hard red winter wheat. Experimental sites characterized by hail damage were excluded from the analysis.

Observations on a predetermined set of explanatory variables to be used in the estimation of yield and protein response were also required. These explanatory variables included available soil nitrate measured to a depth of 4 feet in early spring, April through July precipitation, and available soil water measured to 4 feet in early spring. In many cases moisture content in the soil was not expressed as inches of available soil water. As a result, it was necessary to convert these measurements to inches of available water using the best information available. In addition, dates for the installation of rain gauges varied between experiments and were not available for a limited number of locations. Hence, precipitation records from nearby weather stations were used to replace missing rainfall data. A standard soil analysis with measurements on organic matter and phosphorus and potassium content present in the top 6 inches of soil was desired, but was not available for all locations.

Individual experiments were conducted using a randomized complete block design with three replications. Rates of broadcast nitrogen common to all locations were 0, 20, 40, 60, and 80 pounds per acre of spring-applied nitrogen in the form of ammonium nitrate. Potassium and phosphorus were also applied in the spring at rates of 25 and 20 pounds per acre respectively for 1967 and 1968, and at rates of 25 and 40 pounds per acre for 1970 and 1971. Rates represented actual pounds of elemental phosphorus and potassium. In cases where starter fertilizer was applied in the fall, total amounts of applied phosphorus and nitrogen were somewhat larger.

Varieties of winter wheat represented in the data were Winalta, Cheyenne, and Warrior.

Algebraic Specification of the Response Function

The selection of an algebraic equation to use when fitting crop response functions warrants careful consideration. In the selection procedure, it is important to take into account the nature of the phenomenon under investigation and any limitations or assumptions which may be imposed by a particular model. The functional form of the response function is important; of equal importance are the assumptions made concerning the specifications of the error term. The specification of the regression disturbance or error term will be discussed in detail later in this section.

For the purpose of this study, polynomial equations were selected for estimating yield and protein response. The properties associated with this type of equation are well suited to the data under consideration. This can be readily observed if the behavior of a second degree polynomial with one explanatory variable is examined. The equation is expressed in the following form:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_1^2 \tag{3.1}$$

where: Y = total output;

X_1 = units of the variable resource;

α = level of output when X_1 equals zero; and

β_1 and β_2 = parameters of the equation.

If we expect total product to reach a maximum and then decline, we would hypothesize β_2 to be negative. Thus the equation lends itself to situations where both positive and negative marginal products exist. The marginal product curve is linear for second degree polynomials, although this restriction is overcome when cubic terms or higher order interactions are included. Extensions and transformations of equation

(3.1) are easily obtained and allow considerable flexibility in the specification of crop response functions.

Attempts to use polynomial equations to fit crop-yield data have generally been quite successful (13). Research workers investigating yield relationships have demonstrated that the influence of explanatory variables is often dependent on the levels of other variables present. Consequently, effects are not additive and interactions occur between variables. The models for this analysis were formulated under the hypothesis that interactions of this nature do occur. More precisely, two models were specified and used to arrive at parameter estimates for relevant explanatory variables. Models differed in the specification of the regression disturbance and in the assumptions made regarding nonlinear characteristics of the equations.

Additive Error Model

The first model employed in the estimation of yield and protein response is represented by (3.2).

$$\begin{aligned} Y = {} & b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 \\ & + b_7X_7 + b_8X_8 + b_9X_9 + b_{10}X_{10} + b_{11}X_{11} + b_{12}X_{12} \\ & + b_{13}X_{13} + b_{14}X_{14} + b_{15}X_{15} + b_{16}X_{16} + b_{17}X_{17} \\ & + b_{18}X_{18} + \varepsilon_{iL} + U_L \end{aligned} \tag{3.2}$$

where: Y = estimated yield;

b_0 = intercept;

X_i's = independent variables designated in Table 1 on page 24;

b_i's = estimated regression coefficients;

ε_{iL} = random variation associated with the i^{th} treatment at location L; and

U_L = random variation due to location L.

By definition (3.2) is an intrinsically linear third degree polynomial. That is, the equation is nonlinear with respect to the variables but linear with respect to the parameters to be estimated. The model can be converted to a form which is more obviously linear in the parameters by redefining the variables. In this case, redefinition of second and third degree polynomial terms as additional variables in a linear form will convert the equation to an obviously linear relationship in the parameters.

The regression disturbance in (3.2) is additive and composed of the terms ε_{iL} and U_L. Although the separation of these components is not possible in the actual estimation, the logic and method of incorporation of these error terms are extremely important to the study and are thus outlined.

The first source of variation in the model is represented by ε_{iL}. Specifically, ε_{iL} is defined as a random disturbance which is independently and identically distributed across treatments, i, and locations, L. The effects of random variation in soil and drainage characteristics within experimental plots are examples of factors inherent in this term. The usual properties (16) are associated with ε_{iL} and are outlined below.

ε_{iL} is normally distributed

$$E(\varepsilon_{iL}) = 0$$

$$E(\varepsilon_{iL}^2) = \sigma_\varepsilon^2$$

$$E(\varepsilon_{iL}\ \varepsilon_{jL}) = 0 \text{ for } i \neq j$$

$$E(\varepsilon_{iL}\ \varepsilon_{iL'}) = 0 \text{ for } L \neq L'$$

A second disturbance is introduced into the additive model when observations from heterogeneous locations are used to estimate yield response. Obviously, an effect due to location exists between experiments and therefore introduces a major source of variation into the model. This location effect is represented by U_L in (3.2) and is defined as a random disturbance common to all treatments at a particular location. More specifically, U_L is added to all ε_{iL} for $i = 1, \ldots, n$, where U_L is independent and identically distributed across locations with $E(U_L) = 0$. If we let $V_{iL} = \varepsilon_{iL} + U_L$ and further assume that ε_{iL} and U_L are independent across locations and treatments, then

$$\begin{aligned} \mathrm{Cov}\ (V_{iL}, V_{jL}) &= E(V_{iL}\ V_{jL}) = E(\varepsilon_{iL} + U_L)(\varepsilon_{jL} + U_L) \\ &= E(U_L^2) = \sigma_u^2, \text{ for } i \neq j. \end{aligned}$$

When ρ_{ij} denotes the correlation between treatments i and j at a given location then

$$\rho_{ij} = \frac{\text{Cov }(V_{iL}, V_{jL})}{\sigma^2} = \frac{\sigma_u^2}{\sigma^2} = \rho$$

where: $\sigma_u^2 = \text{Var }(U_L)$ and $\sigma^2 = \text{Var }(V_{jL})$.

Since we assumed ε_{iL} was identically distributed across treatments and locations and that ε_{iL} and U_L were independently distributed, it follows that Var (V_{iL}) is constant across locations and treatments as shown below,

$$\text{Var }(V_{iL}) = E(V_{iL}^2) = E(\varepsilon_{iL} + U_L)^2 = E(\varepsilon_{iL}^2) + E(U_L^2)$$

$$= \sigma_\varepsilon^2 + \sigma_u^2 = \sigma^2$$

Inherent in this property is the assumption that the effect of U_L is identical across all treatment levels of nitrogen at a specific location. However, this assumption may not be valid in all cases, as we might expect that the effect of U_L would differ as nitrogen increases across treatments. As a result, an alternative definition of V_{iL} is

$$V_{iL} = \alpha_i(\varepsilon_{iL} + U_L) \tag{3.3}$$

where α_i is a proportional factor which reflects a treatment-specific effect on both components of the regression disturbance. Intuitively, differential variability exists over treatments in the error term. Therefore:

$$\text{Cov }(V_{iL},V_{jL}) = E(V_{iL}\ V_{jL}) = E[\alpha_i(\varepsilon_{iL} + U_L)\alpha_j(\varepsilon_{jL} + U_L)]$$

$$= E[\alpha_i\alpha_j(\varepsilon_{iL} + U_L)(\varepsilon_{jL} + U_L)] = \alpha_i\alpha_j E(U_L^2)$$

$$= \alpha_i\alpha_j\sigma_u^2$$

and

$$\text{Var }(V_{jL}) = E(V_{jL}^2) = E[\alpha_j(\varepsilon_{jL} + U_L)\alpha_j(\varepsilon_{jL} + U_L)]$$

$$= \alpha_j^2(\sigma_\varepsilon^2 + \sigma_u^2) = \alpha_j^2\sigma^2$$

This latter derivation shows the nature of the heteroskedastic variances between treatment residuals. If the correlations between treatment residuals are examined, then

$$\rho_{ij} = \frac{\text{Cov }(V_{iL},V_{jL})}{\alpha_i\alpha_j\sigma^2} = \frac{\alpha_i\alpha_j\sigma_u^2}{\alpha_i\alpha_j\sigma^2} = \frac{\sigma_u^2}{\sigma^2} = \rho$$

Correlations between treatment residuals are therefore constant and equal to ρ but variances are unequal across treatment levels. The correlation ρ is equal to the ratio of the variance of the location specific effect to the total variance. This latter specification was used in the estimation of yield and protein response.

Multiplicative Error Model

A second possible specification of this response function is a multiplicative error model. This model can be expressed in natural numbers as

$$\begin{aligned} Y = (\alpha &+ b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 \\ &+ b_7X_7 + b_8X_8 + b_9X_9 + b_{10}X_{10} + b_{11}X_{11} + b_{12}X_{12} \\ &+ b_{13}X_{13} + b_{14}X_{14} + b_{15}X_{15} + b_{16}X_{16} + b_{17}X_{17} \\ &+ b_{18}X_{18})\varepsilon_{iL}U_L \end{aligned} \tag{3.4}$$

where once again the explanatory variables maintain the same definitions as those in (3.2). In this equation, the error components are multiplied by the mean response function as opposed to the additive disturbance previously described.

The parameters of (3.4) are estimated by taking the natural logarithm of both sides of the equation. Performing this transformation gives

$$\begin{aligned} \log_e Y = \log_e (\alpha &+ b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 \\ &+ b_6X_6 + b_7X_7 + b_8X_8 + b_9X_9 + b_{10}X_{10} + b_{11}X_{11} \\ &+ b_{12}X_{12} + b_{13}X_{13} + b_{14}X_{14} + b_{15}X_{15} + b_{16}X_{16} \\ &+ b_{17}X_{17} + b_{18}X_{18}) + \log_e \varepsilon_{iL} + \log_e U_L \end{aligned} \tag{3.5}$$

which is an intrinsically nonlinear model. That is, (3.5) is nonlinear with respect to the parameters as well as being nonlinear with respect to the variables.

The two components of the multiplicative regression disturbances in (3.4) are defined in the same manner as ε_{iL} and U_L in the additive model. However, two additional assumptions regarding the expected values of ε_{iL} and U_L are necessary in the multiplicative framework. The assumptions are outlined below.

$$E(\varepsilon_{iL}) = 1$$

$$E(U_L) = 1$$

Taking the expected value of (3.4) with these assumptions results in (3.6),

$$\begin{aligned} E(Y) = \alpha &+ b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_6X_6 \\ &+ b_7X_7 + b_8X_8 + b_9X_9 + b_{10}X_{10} + b_{11}X_{11} + b_{12}X_{12} \\ &+ b_{13}X_{13} + b_{14}X_{14} + b_{15}X_{15} + b_{16}X_{16} + b_{17}X_{17} \\ &+ b_{18}X_{18} \end{aligned} \tag{3.6}$$

Again, it is important to recognize that the separate components of V_{iL}, defined as ε_{iL} and U_L, are not actually in the calculations. However, an argument analogous to that presented for the additive model can be applied to (3.5) to show the existence of nonzero covariances between treatments at specific locations. Similarly, heteroskedastic variances and constant correlations also exist when differential variability occurs across treatments in the error term.

Estimation Procedures

Parameters for these models were estimated using a generalized nonlinear program written by O. Burt, Montana State University. The interested reader is referred to Malinvaud, Chapter 9 (17) for a discussion of the statistical theory underlying the development of the program. The assumptions previously outlined concerning the regression disturbances were specified in the program.

The technique of maximum likelihood estimation is used to arrive at parameter estimates for nonlinear models. Although the concept of maximum likelihood estimation is not new, the technique is one which may be unfamiliar to the reader. Therefore, a brief discussion of the general principles associated with this form of estimation is presented.

Intuitively, the method of maximum likelihood estimation is based on the idea that certain populations are more likely to generate a given sample than other populations. When the nature of a population is estimated from a random sample of data, maximum likelihood estimates can be defined in the following manner:

> If a random variable X has a probability distribution f(x) characterized by parameters $\theta_1, \theta_2, \ldots, \theta_k$ and if we observe a sample $X_1, X_2, \ldots, X_n$, then the maximum likelihood estimators of $\theta_1, \theta_2, \ldots, \theta_k$ are those values of these parameters that would generate the observed sample most often (16).

The problem thus becomes one of finding the values of $\theta_1, \theta_2, \ldots, \theta_k$ which maximize the probability of the observed sample values. The objective of finding these values can be accomplished through the formulation of a likelihood function. If the sample observations are independent, the likelihood function takes on the formula of the joint probability distribution of the sample represented in equation (3.7).

$$L = g(X_1, X_2, \ldots, X_n) = f(X_1)\, f(X_2) \ldots f(X_n) \tag{3.7}$$

We can now maximize (3.7) with respect to $\theta_1, \theta_2, \ldots, \theta_k$ and obtain estimators $(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k)$ of the unknown parameters. More specifically, $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k$ are estimators obtained by the method of maximum likelihood estimation if the following condition is satisfied:

$$L(X_1, X_2, \ldots, X_n; \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k) > L(X_1, X_2, \ldots, X_n; \tilde{\tilde{\theta}}_1, \ldots, \tilde{\tilde{\theta}}_k)$$

where $\tilde{\tilde{\theta}}_1, \ldots, \tilde{\tilde{\theta}}_k$ are any other estimators of $\theta_1, \theta_2, \ldots, \theta_n$. Since the logarithm of L is a monotonic transformation of L, the maximization procedure can be accomplished by maximizing the function

$$L = \log_e L$$

Solutions are then obtained by setting the partial derivatives of L with respect to each parameter equal to zero and solving simultaneously.

Statistical Tests

Ratios of the likelihood functions generated in the estimation procedure described can be used to evaluate the significance of explanatory variables in the models. The test is based on the property that the negative of twice the logarithm of a likelihood function ratio is asymptotically distributed as χ^2 (30). The chi-square test statistic is given by (3.8)

$$T \log_e \left[\frac{D_r}{D_g}\right] \sim \chi_q^2 \tag{3.8}$$

where: T = number of experiments (43);

D = determinant of the residual covariance matrix;

g = general model with fewer restrictions;

r = restricted model; and

q = difference in the number of parameters between the two models.

This statistical test is particularly useful in evaluating the relevance of an explanatory variable which is a component of more than one term in the response equation. Such is the case with soil moisture, soil nitrate, precipitation, and applied nitrogen. The test is also appropriate for models with linear constraints on parameter values.

T-ratios for individual coefficients were also used to evaluate and compare models. Standard errors for linear combinations of variables were examined to determine the forecasting ability of the model.

Estimation Procedures

Initial estimates of yield response relationships were made using additive regression disturbances. Table 1 contains a listing of variable descriptions and associated means, standard deviations, and coefficients of variation. Parameters for each of these variables were estimated and the nature of the response function was evaluated. Equations were also generated to test the significance of soil nitrate, precipitation, soil moisture, and applied phosphorus in explaining yield response to nitrogen fertilizer. This set of equations is presented in Table 2. Chi-square tests were used to determine variables important in explaining yield response in this set of equations.

Computational costs involved in the estimation of multiplicative models did not allow duplication of the procedures outlined above. Therefore, significant variables determined in the previous section were used when multiplicative models were generated. A comparison was then made between multiplicative and additive models to determine the response function best suited for use in developing fertilizer recommendations. T-ratios for identical terms in the two models were initially used as a criterion for comparing models in this section of the analysis.

TABLE 1. VARIABLES USED IN DEVELOPING PREDICTIVE EQUATIONS FOR YIELD AND PERCENT PROTEIN OF WINTER WHEAT.

Variable Designation	Variable Description	Units	Mean	Standard Deviation	Adjusted Coefficient of Variation
Y	Grain yield	bu/A	38.00	8.74	0.54
X_1	Total applied phosphorus	lb/A	44.49	15.99	0.63
X_2	Binary variable (Warrior)		0.07	0.26	3.66
X_3	Binary variable (Cheyenne)		0.12	0.32	2.76
X_4	Total applied nitrogen (N)	lb/A	43.47	28.62	0.66
X_5	N^2	lb/A	2704.30	2857.40	0.96
X_6	Soil NO_3-N in 4' of soil (SN)	lb/A	105.41	105.34	1.12
X_7	$(SN)^2$	lb/A	22157.00	71255.00	3.24
X_8	Avail. soil water in 4' of soil (SW)	in.	4.84	1.72	0.64
X_9	$(SW)^2$	in.	26.33	17.40	0.80
X_{10}	Total April-July Precipitation (PREC)	in.	6.76	2.01	0.63
X_{11}	$(PREC)^2$	in.	49.73	29.83	0.81
X_{12}	N x PREC		297.23	224.67	0.76
X_{13}	SN x N		4471.70	5943.70	1.33
X_{14}	SN x SW		625.81	569.37	1.12
X_{15}	N x SW		282.33	247.71	0.88
X_{16}	SN x PREC		697.01	654.99	1.03
X_{17}	SW x PREC		43.52	26.28	0.81
X_{18}	N^2 x PREC		18581.00	19684.00	1.06

TABLE 2. EQUATIONS USED TO TEST THE SIGNIFICANCE OF SOIL WATER, APPLIED PHOSPHORUS, PRECIPITATION, AND SOIL NITRATE IN EXPLAINING YIELD RESPONSE.

Eq. No.	Equation
3.9	$Y = 20.27 + 0.0947X_1 + 9.292X_2 + 3.009X_3 - 0.04X_4 + 0.000338X_5 - 0.127X_6 + 0.000056X_7 - 6.154X_8 + 0.577X_9 + 7.67X_{10} - 0.468X_{11} + 0.0297X_{12} - 0.000162X_{13} + 0.0117X_{14} - 0.00303X_{15} + 0.0103X_{16} - 0.167X_{17} - 0.000163X_{18}$
3.10	$Y = 1.421 + 0.0648X_1 + 10.97X_2 + 3.609X_3 - 0.066X_4 + 0.000373X_5 + 0.00339X_6 - 0.0000316X_7 + 7.787X_{10} - 0.57X_{11} + 0.0306X_{12} - 0.000147X_{13} + 0.00727X_{16} - 0.000171X_{18}$
3.11	$Y = 2.877 + 11.229X_2 + 2.657X_3 - 0.066X_4 + 0.000383X_5 + 0.012X_6 - 0.00003X_7 + 8.088X_{10} - 0.568X_{11} + 0.0308X_{12} - 0.000148X_{13} + 0.00508X_{16} - 0.000174X_{18}$
3.12	$Y = 29.40 + 12.875X_2 + 4.609X_3 + 0.135X_4 - 0.000691X_5 + 0.0464X_6 - 0.0000389X_7 - 0.000167X_{13}$
3.13	$Y = 4.634 + 12.515X_2 + 1.666X_3 - 0.0939X_4 + 0.000486X_5 + 8.422X_{10} - 0.561X_{11} + 0.0312X_{12} - 0.000173X_{18}$

Determining Relevant Variables in the Additive Error Model

Behavior of estimated response functions varied considerably with the inclusion and exclusion of soil water variables. Results consistent with conventional theory were obtained only when soil water variables were excluded from the statistical analysis. A comparison of the behavior of equations (3.9) and (3.10) in Table 2 illustrates this point. Estimated parameters for (3.9) showed that increasing rather than decreasing marginal response existed for soil nitrate and soil moisture over all ranges of the data, with incremental response being negative at lower levels of these variables. In this model diminishing marginal response existed only for applied nitrogen and precipitation. However, when soil water variables were eliminated in (3.10), the estimated response function exhibited diminishing marginal response for each of the resources under consideration.

This result is difficult to understand. The significant change in the nature of the response function raises the question of the validity of including the soil water variables as measured in the data in the model. Soil water measurements at 15 locations were reported as inches of available soil water in the data source but actually represented some other measurement. As a result, it is possible that difficulties encountered in converting these soil water measurements to inches of available soil water together with other sources of measurement error severely limited the accuracy of soil water variables. In order to determine the importance of soil water variables in the model, a chi-square test statistic was calculated using the residual covariance matrices of (3.9) and (3.10). The null hypothesis tested was expressed as

$$\text{Ho:}\quad b_8 = b_9 = b_{14} = b_{15} = b_{17} = 0$$

where the b_i's refer to estimated parameters in (3.9). Each of these parameters is associated with a variable which includes soil water. Results from this test indicated that the null hypothesis could not be rejected at the .10 level of significance. Conclusions from previous studies cited in the literature review suggest that soil water is very important in explaining yield response. However, in this analysis soil water appears to only confound the behavior of estimated response functions. Based on these results, soil water was eliminated as an explanatory variable in the analysis which means that the net effects of soil water variation are moved to the error term of the model.

The importance of applied phosphorus was also examined by applying similar techniques to equations (3.10) and (3.11) in Table 2. The chi-square test statistic calculated from these regression equations showed that applied phosphorus was of little consequence in explaining yield variability. The coefficient for linear applied phosphorus was not significantly different from zero at the .10 level. Therefore, it too was excluded from the analysis.

With the exclusion of applied phosphorus and soil water, the model predicting yield can be expressed in the form of equation (3.11). Investigation of the statistical results for this model indicates that seasonal precipitation and soil nitrate are very important in explaining yield response to nitrogen fertilizer. The null hypothesis tested for precipitation can be expressed as

$$\text{Ho:} \quad b_{10} = b_{11} = b_{12} = b_{16} = b_{18} = 0$$

where the b_i's refer to estimated coefficients for terms in (3.11). Similarly, the null hypothesis used to test the significance of soil nitrate was expressed as

$$\text{Ho:} \quad b_6 = b_7 = b_{13} = b_{16} = 0$$

where, once again, the b_i's symbolize coefficients in equation (3.11) in Table 2. Parameters for terms which included precipitation as a variable were significantly different from zero at the .001 level. Thus, we can be 99.9 percent confident that precipitation contributes to the explanation of yield variation in this set of experiments. Similarly, coefficients for soil nitrate terms were significantly different from zero at the .01 level. Comparison of these confidence levels suggests that precipitation is the more important of the two variables.

Comparison of Additive and Multiplicative Error Models

Important variables from the additive analysis were next used to generate a model with a multiplicative disturbance term. Regression results for comparable multiplicative and additive models are presented in Tables 3 and 4 respectively. Models differ only in the specification of the disturbance term and in associated properties discussed earlier.

In both instances, estimated response functions are characterized by maximum yields and declining marginal productivity to the resources under consideration. Similarly, signs of estimated coefficients do not vary between models. A comparison of yields predicted by the two models for specified levels of precipitation and soil nitrate is presented in Tables 5 and 6. Predicted yields are similar over most ranges of the data.

Examination of the statistical significance of estimated parameters reveals that t-ratios for the multiplicative model are higher for all but four terms. The significance of linear soil nitrate, applied nitrogen x soil nitrate, and the two binary variables were reduced with the use of the multiplicative disturbance. However, differences between corresponding significance levels in the two

TABLE 3. REGRESSION COEFFICIENTS FOR MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.14).

Variable Description	Regression Coefficient	t-value
Intercept	3.925300	0.380
Variety 2 (Warrior)	11.248000 ***	2.687
Variety 3 (Cheyenne)	1.108200	0.410
N	-0.096910 *	-1.532
N^2	0.000654	0.894
SN	0.001654	0.037
SN^2	-0.000035	-0.925
PREC	7.834700 ***	2.856
$PREC^2$	-0.580110 ***	-3.443
N x PREC	0.035205 ***	3.768
N x SN	-0.000119 **	-1.816
PREC x SN	0.008295 *	1.511
N^2 x PREC	-0.000223 *	-2.106

Multiple R^2 = 36.8%

* p = .10 ** p = .05 *** p = .005

N = Total applied nitrogen (lb/A).
SN = Total NO_3-N to 4' in early spring (lb/A).
PREC = Total April-July precipitation (in.).

TABLE 4. REGRESSION COEFFICIENTS FOR THE ADDITIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.11).

Variable Description	Regression Coefficient	t-value
Intercept	2.876600	0.249
Variety 2 Binary (Warrior)	11.229000 ***	3.431
Variety 3 Binary (Cheyenne)	2.656600	0.991
N	-0.066648	-0.924
N^2	0.000383	0.453
SN	0.011962	0.250
SN^2	-0.000030	-0.803
PREC	8.088100 ***	2.682
$PREC^2$	-0.568450 ***	-2.959
N x PREC	0.030841 ***	2.985
N x SN	-0.000148 ***	-2.191
PREC x SN	0.005084	0.926
N^2 x PREC	-0.000174 *	-1.453

Multiple R^2 = 39.1%

* p = .10 ** p = .05 *** p = .005

N = Total applied nitrogen (lb/A).
SN = Total NO_3-N to 4' in early spring (lb/A).
PREC = Total April-July precipitation (in.)

TABLE 5. WINTER WHEAT YIELDS PREDICTED BY THE MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN (NO_3-N to 4' = 90 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
lb N/A	Yield (bu/A)									
0	24.31	28.83	32.19	34.39	35.43	35.31	34.03	31.59	27.99	23.23
20	24.26	29.40	33.38	36.19	37.05	38.34	37.67	35.85	32.86	27.72
40	24.21	29.78	34.19	37.44	39.54	40.47	40.24	38.85	36.30	32.59
60	24.14	29.97	34.64	38.15	40.50	41.69	41.72	40.59	38.30	34.85
80	24.06	29.97	34.72	38.31	40.74	42.01	42.12	41.07	38.86	35.49
100	23.97	29.78	34.43	37.93	40.26	41.43	41.45	40.30	37.99	34.52

TABLE 6. WINTER WHEAT YIELDS PREDICTED BY THE ADDITIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN (NO_3-N to 4' = 90 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
lb N/A	Yield (bu/A)									
0	24.23	28.80	32.23	34.52	35.68	35.69	34.58	32.32	28.93	24.40
20	24.43	29.54	33.52	36.36	38.06	38.63	38.06	36.35	33.50	29.52
40	24.51	30.03	34.42	37.67	39.78	40.75	40.59	39.29	36.86	33.28
60	24.49	30.28	34.93	38.45	40.83	42.08	42.18	41.14	38.99	35.68
80	24.35	30.27	35.06	38.71	41.22	42.59	42.83	41.93	39.89	36.72
100	24.10	30.02	34.79	38.43	40.94	42.30	42.53	41.62	39.58	36.40

models are small. As a result, the selection of a model for use in developing fertilizer recommendations was based on properties associated with the two error terms. In the multiplicative framework, standard errors of the estimate are directly proportional to the mean. Hence, for agronomic data this form of specification would appear to make sense intuitively. Based on this intuitive appeal, the multiplicative model was chosen to estimate yield.

Careful evaluation of the multiplicative model reveals that yields are depressed at high levels of precipitation. This phenomenon is rather difficult to explain, particularly when one considers that the experiments were conducted under dryland conditions. Under semi-arid conditions, moisture has often been thought to be the factor most limiting grain yields. Hence, this result does not seem plausible in lieu of <u>a priori</u> information.

One possible explanation for this occurrence may be found in the distribution of these high levels of precipitation. If large amounts of rainfall were received during a short time period, adverse effects on yield could result from increased weed infestation and disease. Methods analyzed to control weeds were not recorded for this group of experiments. As a result, competition with weeds for available nutrients and moisture may have confounded response results. It is the opinion of this researcher that the yields predicted by the model at these high levels of precipitation do not reflect normal yield relationships. After consultation with other researchers, a decision was reached to place a constraint on the net effects of precipitation in the model. The constraint involved the marginal product of precipitation. Values were specified for precipitation, applied nitrogen, and soil nitrate in order to force the first derivative of yield with respect to precipitation (marginal product of precipitation) equal to zero at a specified level. In this case, the first derivative was forced to equal zero at a level of 100 pounds of applied nitrogen, 13 inches of seasonal precipitation, and 10 pounds of available soil nitrate. Specifically, the constraint can be expressed as

$$b_{10} + 26b_{11} + 100b_{12} + 10b_{16} + 10{,}000b_{18} = 0$$

where b_i's refer to regression parameters in the multiplicative model. Parameters were then estimated with this constraint on the model.

Constrained Multiplicative Error Model

Table 7 contains regression results for the constrained model. T-ratios for terms which included precipitation as a variable were generally reduced by the imposition of the constraint. The greatest decline in significance occurred in the linear and quadratic precipitation terms which is not surprising since the constraint forces less curvature on the quadratic relationship. Both the linear and quad-

TABLE 7. REGRESSION COEFFICIENTS FOR THE CONSTRAINED MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT YIELD (EQ. 3.15).

Variable Description	Regression Coefficient	t-value
Intercept	25.902000 ***	3.091
Variety 2 Binary (Warrior)	12.806000 ***	2.864
Variety 3 Binary (Cheyenne)	3.266100	1.166
N	-0.084284	-1.240
N^2	0.000275	0.354
SN	-0.006196	-0.122
SN^2	-0.000038	-0.914
PREC	0.959140	0.554
$PREC^2$	-0.105220 *	-1.414
N x PREC	0.033287 ***	3.287
N x SN	-0.000114 **	-1.751
PREC x SN	0.010248 *	1.623
N^2 x PREC	-0.000165 *	-1.452

Multiple R^2 = 32.5%

* p = .10 ** p = .05 *** p = .005

N = Total applied nitrogen (lb/A).
SN = Total NO_3-N to 4' in early spring (lb/A).
PREC = Total April-July precipitation (in.).

ratic terms dropped below the .05 level of significance. Comparison of Tables 5 and 8 shows that the constrained function is much flatter over the range of precipitation values than the unconstrained equation.

As with the other models, care must be exercised in evaluating the signs of estimated parameters. Ordinarily, a positive linear term and a negative quadratic term indicate diminishing returns to the variable input under consideration. However, in an analysis of this type, the effects of interaction terms must be taken into account before the nature of the response function can be determined. Second degree interaction terms are of the hypothesized sign. For instance, a positive coefficient for b_{12} indicates that the effect of applied nitrogen is most beneficial when precipitation levels are high. Similarly, the coefficient for the interaction between soil nitrate and precipitation is also positive. A negative coefficient for b_{13} implies that soil nitrate and applied nitrogen are substitutes by nature. This result is in agreement with previous research (20). In the case of the third degree interaction between applied nitrogen squared and precipitation, the expected sign is not clear, as _a priori_ information was not available for this term.

Interactions appear to be very important in explaining yield variation in the model. The t-ratio for the interaction between applied nitrogen and precipitation suggests that this term exerts

TABLE 8. WINTER WHEAT YIELDS PREDICTED BY THE CONSTRAINED MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN (NO_3-N to 4' = 90 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
lb N/A	Yield (bu/A)									
0	29.73	30.88	31.81	32.54	33.05	33.35	33.45	33.33	33.00	32.46
20	29.75	31.50	33.03	34.35	35.47	36.37	37.06	37.54	37.82	37.88
40	29.59	31.80	33.80	35.60	37.18	38.55	39.70	40.65	41.39	41.92
60	29.25	31.80	34.14	36.36	38.18	39.88	41.38	42.66	43.73	44.60
80	28.74	31.49	34.03	36.35	38.47	40.38	42.07	43.56	44.84	45.90
100	28.05	30.87	33.47	35.87	38.06	40.04	41.80	43.36	44.70	45.84

the greatest degree of influence in this model. The coefficient for this term was significantly different from zero at the .001 level. The next most important term seems to be the interaction between soil nitrate and applied nitrogen. These are the only terms significant at least at the .05 level. However, caution is advised in using t-ratios as sole indicators of terms to be included. In this analysis, linear and quadratic terms are an integral part of the response function and are therefore essential to the model. Even though these terms appear to be of little consequence when taken one at a time, they are nevertheless necessary components of the equation.

Regression results also indicated that the correlation ρ between the treatment residuals was .89. Since ρ was shown to be the ratio of the variance associated with location specific effects to the total variance, approximately 90 percent of the unexplained variance is due to random effects associated with different locations.

Estimated Conditional Mean Yields

Estimated conditional mean yields and their associated standard errors were calculated for varying levels of soil nitrate and applied nitrogen. Yields are designated as conditional mean yields to account for the use of expected values for precipitation and precipitation squared in the equation. Rates of applied nitrogen varied from 20 to 100 pounds in 20-pound increments while soil nitrate values ranged from 30 to 210 pounds in 60-pound increments. Only precipitation was held constant. In this case, expected values for seasonal precipitation and seasonal precipitation squared were calculated using a rainfall probability distribution from Great Falls, Montana. Expected values for seasonal precipitation and seasonal precipitation squared were 7.47 and 62.38 respectively.

Standard errors calculated by the program for the estimated conditional mean yields associated with each combination of variables were used to construct 95 percent confidence intervals. The formula used in constructing the intervals was:

$$\hat{Y}_o - S\, t_{n-2,\ \alpha/2} \leq Y_o \leq \hat{Y}_o + S\, t_{n-2,\ \alpha/2}$$

where: Y_o = conditional mean yield;

$\hat{Y}$ = estimated conditional mean yield;

S = standard error for the estimated value of the mean, $\hat{Y}_o$; and

$t_{n-2,\ \alpha/2}$ = t value for n-2 degrees of freedom and $\alpha/2$ level of significance.

The resulting confidence intervals are presented in Table 9. As expected, intervals were narrowest for values of soil nitrate very close to its mean. In theory, the variance associated with the difference between Y_o and $\hat{Y}_o$ becomes smaller as the distance between the explanatory variable and its mean becomes smaller. If we look at these results, we see that the estimated conditional mean yield

TABLE 9. 95-PERCENT CONFIDENCE INTERVALS FOR YIELD WHEN EXPECTED PRECIPITATION EQUALS 7.47 INCHES AND EXPECTED PRECIPITATION SQUARED EQUALS 62.38 INCHES.

Total Applied Nitrogen	NO_3-N in 4' of Soil	Estimated Conditional Mean Yield	Std. Error*	95% Confidence Level Lower Limit	95% Confidence Level Upper Limit
lb N/A	lb N/A				
20	30	32.11	1.48	29.51	35.31
40	30	34.17	1.56	31.11	37.23
60	30	35.47	1.65	32.24	38.70
80	30	35.99	1.76	32.04	38.94
100	30	35.75	1.99	31.85	39.65
20	90	35.92	1.08	33.80	38.04
40	90	37.85	1.18	35.54	40.16
60	90	39.00	1.26	36.53	41.47
80	90	39.39	1.37	36.70	42.08
100	90	39.01	1.61	35.85	42.17
20	150	39.46	1.69	36.15	42.77
40	150	41.25	1.76	37.80	44.70
60	150	42.27	1.83	38.68	45.86
80	150	42.52	1.91	38.78	46.26
100	150	42.00	2.10	37.88	46.12
20	210	42.72	2.47	37.88	47.56
40	210	44.38	2.54	39.40	49.36
60	210	45.26	2.61	40.14	50.38
80	210	45.38	2.70	40.09	50.67
100	210	44.72	2.87	29.09	50.35

* = Standard error of the estimated conditional mean yield.

generated by this model is within 2.5 bushels of the conditional mean yield 95 percent of the time when soil nitrate equals 90 pounds, applied nitrogen equals 60 pounds, and yields are averaged across precipitation levels for the Great Falls location. However, the width of the interval nearly doubles for soil nitrate values over 200 pounds.

Linear combinations of parameters were also specified to produce the following relationship in applied nitrogen

$$Y = a_o + a_1N + a_2N^2$$

where: Y = conditional expected yield given soil nitrate;

a_o = intercept or $(b_6SN + b_7SN^2 + b_{10}PREC + b_{11}PREC^2 + b_{16}SN \times PREC)$;

$a_1 = b_4 + b_{12}PREC + b_{13}SN$; and

$a_2 = b_5 + b_{18}PREC^2$.

Once again, soil nitrate values were varied from 30 to 210 pounds in 60 pound increments while expected values for precipitation and precipitation squared were used.

Equations generated using this procedure are contained in Table 10. The parameter for nitrogen squared did not vary between equations, as precipitation squared assumed only one value in the equation for a_2. However, parameters for the intercept and linear nitrogen varied as different values for soil nitrate were introduced into the model. In all cases, parameters for this quadratic in nitrogen were significantly

TABLE 10. YIELD EQUATIONS WHEN PRECIPITATION EQUALS 7.47 INCHES, PRECIPITATION SQUARED EQUALS 62.38 INCHES, AND SOIL NITRATE IS SET AT THE VALUES GIVEN.

	Equation
(A)*	$Y = \underset{(1.456)}{28.584^{***}} + \underset{(0.0234)}{0.161N^{***}} - \underset{(0.000255)}{.00096N^{2\,***}}$
(B)	$Y = \underset{(1.456)}{32.535^{***}} + \underset{(0.0234)}{0.154N^{***}} - \underset{(0.000255)}{.00096N^{2\,***}}$
(C)	$Y = \underset{(1.655)}{36.214^{***}} + \underset{(0.0231)}{0.147N^{***}} - \underset{(0.000255)}{0.00096N^{2\,***}}$
(D)	$Y = \underset{(2.421)}{39.619^{***}} + \underset{(0.0238)}{0.141N^{***}} - \underset{(0.000255)}{0.00096N^{2\,***}}$

*

(A) = NO_3-N to 4' in soil equals 30 lb/A.

(B) = NO_3-N to 4' in soil equals 90 lb/A.

(C) = NO_3-N to 4' in soil equals 150 lb/A.

(D) = NO_3-N to 4' in soil equals 210 lb/A.

*** p = .005

different from zero at least at the .005 level. The relatively small change in the magnitude of the parameters for these response functions suggests that high levels of soil nitrate do not prevent yield response to nitrogen fertilizer. Previous research has suggested that lack of response to nitrogen fertilizer can often be attributed to pre-existing high levels of soil nitrate in the soil.

Assumption Concerning Soil Water

In Chapter IV nitrogen fertilizer recommendations are presented for varying levels of soil nitrate and total expected water, where total expected water is defined as the sum of inches of soil water in the top 4 feet of soil and the expected level of April through July precipitation. Recommendations are formulated using estimated parameters from the constrained model. Since the net effect of soil water variation was actually moved to the error term when soil water was excluded from the analysis, these recommendations are best suited to soil water conditions very near the mean level of soil water in the sample which was 5 inches. In order to extrapolate to other ranges of soil water values, it is important to make the assumption that an inch of soil water above or below the mean level is equivalent to an inch of precipitation above or below the expected level. Five inches was added to each precipitation figure to arrive at column headings for total expected water. As a result, the fertilizer recommendations presented for 10 inches of total water actually represent a precipitation level of 5 inches plus 5 inches of available soil water. Shortcomings associated with this procedure are obvious. The model is not capable of predicting yields for simultaneously high levels of soil water and precipitation. Nevertheless, with the use of expected precipitation in the decision criteria, there is still considerable leeway for soil water variation in the recommendations presented.

Protein Estimation

The equation used to predict percent protein is presented in Table 11. An additional interaction between precipitation squared and applied nitrogen was included in this model. A multiplicative disturbance term was specified and there were no linear constraints on the model.

Wheat protein percentages predicted by this equation for varying levels of applied nitrogen and precipitation are presented in Table 12. Values for soil nitrate and applied phosphorus were constant in this table. The relationship between applied nitrogen and percent protein was positive over most ranges of the data. Applied nitrogen did have a negative influence on protein content when low levels of precipitation and high levels of applied nitrogen occurred simultaneously. No apparent reason is suggested for this phenomenon, but the most likely explanation is statistical imprecision in the fitted

TABLE 11. REGRESSION COEFFICIENTS FOR THE MULTIPLICATIVE MODEL PREDICTING WINTER WHEAT PROTEIN CONTENT (EQ. 3.16).

Variable Description	Regression Coefficient	t-value
Intercept	15.987000	5.144
Applied phosphorus	0.041327 ***	2.843
Variety 2 (Warrior)	-2.062300 ***	-2.994
Variety 3 (Cheyenne)	0.521000	0.750
N	-0.002686	-0.111
N^2	-0.000321 *	-1.452
SN	0.018979 *	1.408
SN^2	-0.000017 *	-1.647
PREC	-2.156700 ***	-2.661
$PREC^2$	0.120540 **	2.319
N x PREC	0.016061 ***	2.991
N x SN	-0.000077 ***	-4.057
PREC x SN	0.001153	0.734
N^2 x PREC	0.000042 *	1.370
N x $PREC^2$	-0.001241 ***	-3.685

Multiple R^2 = 57.6%

* p = .10   ** p = .05   *** p = .005

N = Total applied nitrogen (lb/A).
SN = Total NO_3-N to 4' in early spring (lb/A).
PREC = Total April-July precipitation (in.).

TABLE 12. PERCENT PROTEIN OF WINTER WHEAT PREDICTED BY THE MULTIPLICATIVE MODEL BASED ON APRIL-JULY PRECIPITATION AND TOTAL APPLIED NITROGEN (NO_3-N to 4' = 90 lb/A and Applied Phosphorus = 25 lb/A).

Total Applied Nitrogen	Total April-July Precipitation (in.)									
	3	4	5	6	7	8	9	10	11	12
lb N/A	Percent Protein									
0	13.51	12.30	11.33	10.61	10.12	9.88	9.87	10.11	10.59	11.31
20	13.98	12.94	12.08	11.42	10.95	10.67	10.58	10.69	10.98	11.47
40	14.29	13.45	12.74	12.18	11.76	11.48	11.34	11.35	11.49	11.78
60	14.45	13.84	13.32	12.89	12.55	12.30	12.15	12.09	12.12	12.24
80	14.46	14.11	13.80	13.54	13.32	13.14	13.00	12.91	12.86	12.85
100	14.31	14.26	14.20	14.14	14.07	13.99	13.91	13.81	13.72	13.61

polynomial response. The marginal product for nitrogen declined in magnitude as applied nitrogen increased for fixed soil nitrate and moisture conditions, thus showing diminishing marginal returns to this variable input. However, this relationship occurred only when April through July precipitation was less than 7 inches. When precipitation was 7 inches or more, increasing returns characterized the effect of applied nitrogen on protein content. Ordinarily, we would expect diminishing marginal returns to exist to applied nitrogen over all ranges of the data. The fact that increasing marginal returns occurred at higher levels of precipitation could be explained by the actual relationship existing between precipitation and wheat yields in the experiments examined. If higher levels of precipitation actually had a detrimental effect on yields in the experiments examined, then the effect of applied nitrogen on grain protein could conceivably be positive and increasing. This proposition follows from the idea that protein content of the grain is generally increased by nitrogen uptake only after the nitrogen requirements for vegetative growth are met. Thus, the increased levels of nitrogen would be distributed among fewer bushels of wheat, with a large portion of this nitrogen being available for protein increases.

Precipitation appears to have a negative effect on protein content in most instances. However, increases in percent protein were associated with increased precipitation when simultaneously high levels of soil nitrate and applied nitrogen occurred. Evidently, adequate supplies of nitrogen were available to meet all plant requirements. High levels of precipitation also had a positive effect on grain protein when low levels of applied nitrogen were present, but as can be seen in Table 12 (at 90 pounds of NO_3-N), the positive effect is weak and probably explained by statistical imprecision.

The effect of soil nitrate was as hypothesized. Positive and diminishing marginal returns with respect to soil nitrate characterized this resource over all ranges of the data.

T-ratios in Table 11 indicate that all but three parameter estimates were significantly different from zero at least at the .10 level of significance. Parameters for interactions between nitrogen and precipitation, nitrogen and soil nitrate, and nitrogen and precipitation squared were all significantly different from zero at least at the .005 level. Linear precipitation, applied phosphorus, and the binary variable for Warrior were also significant at this level. The parameter for this binary variable indicates that grain protein is decreased by approximately 2 percent when the variety used is Warrior rather than Winalta. However, the lack of observations for this variety warrants caution in the acceptance of this result. Warrior represented 3 of the 43 locations.

The significance of applied phosphorus was also examined through the use of a chi-square test. Results from this test indicated the coefficient for applied phosphorus was significantly different from zero at the .001 level. This is consistent with the result obtained in Table 11. However, it is surprising that applied phosphorus would be this important in the estimation of protein response, for in the yield equations, applied phosphorus was of little consequence in explaining yield variability.

CHAPTER IV

OPTIMAL FERTILIZER POLICIES

Unlimited Capital

With the introduction of input and output prices, equation (4.1) on the following page can be used to determine profit maximizing levels of nitrogen application. In this analysis we are concerned with the specification of the optimal application of a single variable input, nitrogen. Moisture and soil nitrate levels are held fixed at their means in the experimental data for purposes of the illustrative economic analysis. The assumption is made that input and output markets are perfectly competitive, which implies that prices are fixed constants for a given decision.

First-order conditions for profit maximization require that the value of the marginal product be equal to the marginal cost of the input. This can be shown by formulating a profit equation of the following nature:

$$\pi = P_w Y - P_n N - K \tag{4.0}$$

where: P_w = price of wheat per bushel;

Y = wheat yield in bushels per acre;

P_n = price per pound of actual nitrogen;

N = pounds of nitrogen applied per acre; and

K = fixed costs per acre.

In this instance, profit is maximized when the marginal profit from the use of applied nitrogen is equal to zero. Taking the derivative of (4.0) with respect to applied nitrogen gives the following equation for marginal profit when (3.15) is substituted for Y in (4.0):

$$\frac{\partial \pi}{\partial X_4} = P_w(b_4 + 2b_5X_4 + b_{12}X_{10} + b_{13}X_6 + 2b_{18}X_4X_{10}) - P_n \tag{4.1}$$

Setting (4.1) equal to zero and adding P_n to both sides of the equation results in (4.2):

$$P_w(b_4 + 2b_5X_4 + b_{12}X_{10} + b_{13}X_6 + 2b_{18}X_4X_{10}) = P_n \tag{4.2}$$

The expression on the left is the value of the marginal product of nitrogen while P_n once again refers to the price per pound of actual nitrogen. This is identical to equating the marginal product of nitrogen equal to the ratio of the price of nitrogen to the price of wheat. Dividing (4.2) by P_w illustrates this result.

$$(b_4 + 2b_5X_4 + b_{12}X_{10} + b_{13}X_6 + 2b_{18}X_4X_{10}) = \frac{P_n}{P_w} \tag{4.3}$$

By specifying constant values for precipitation (X_{10}) and soil nitrate (X_6) in (4.3), we can solve for the quantity of nitrogen (X_4) which maximizes profits under these given conditions. Equation (4.4) expresses the optimal rates of nitrogen application in this study.

$$X_4 = \frac{\frac{P_n}{P_w} - b_4 - b_{12}X_{10} - b_{13}X_6}{2b_5 + 2b_{18}X_{10}} \tag{4.4}$$

Second-order conditions for profit maximization require that solutions to (4.4) be limited to strictly concave regions of the production function. Therefore, as the price of nitrogen increases relative to the price of wheat, the profit-maximizing amount of nitrogen decreases and vice versa.

Limited Capital

The decision rule outlined in the preceding section assumed that unlimited capital was available to wheat producers. This assumption may be unrealistic for typical farming situations. When capital is limited, profits are maximized if inputs are allocated among alternative uses in a way such that the marginal return per dollar invested is equal among all input categories. In other words, the last dollar invested in each input category returns a dollar amount which is greater than one and is equal for all possible uses of inputs. This decision rule encompasses the total farming operation and assumes that input-output relationships are known for alternative uses of inputs.

The marginal rate of return associated with using different levels of nitrogen application in this analysis is given by the following equation:

$$r = \frac{P_w(b_4 + 2b_5X_4 + b_{12}X_{10} + b_{13}X_6 + 2b_{18}X_4X_{10}) - P_n}{P_n} \qquad (4.5)$$

where r equals the marginal rate of return expressed as a fraction. In the derivation of optimal rates of application in the preceding section, capital was unlimited and nitrogen was applied up to the point where the last dollar invested returned exactly one dollar. Therefore, the marginal rate of return or r was equal to zero.

Fertilizer recommendations were derived under unlimited capital conditions by equating the marginal product of nitrogen with the input-output price ratio. Price ratios ranged from .01 to .15 in increments of .01. Prices of wheat used in this section did not take into account protein response.

Nitrogen Fertilizer Recommendations Without Protein Response

In Table 13 nitrogen fertilizer recommendations are presented for a price ratio equal to .09. This closely approximates the Winter 1977 nitrogen price of 20 cents per pound of N while the base price for Hard Red Winter Wheat was close to $2.20 per bushel. Rates of nitrogen application are based on total available soil nitrate in the top 4 feet of soil in early spring and the sum of expected April through July precipitation and total plant available water measured to 4 feet in early spring. Use of the latter sum is contingent on the assumption outlined earlier concerning the relative effects of an inch of soil water and an inch of precipitation.

TABLE 13. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	16	32	43	51	57	61	65	68
40	0	0	0	15	31	42	50	56	61	64	68
60	0	0	0	13	29	41	49	55	60	64	67
80	0	0	0	11	28	39	48	54	59	63	66
100	0	0	0	10	27	38	47	53	58	62	66
120	0	0	0	8	25	37	46	52	58	62	65
140	0	0	0	7	24	36	45	52	57	61	65
160	0	0	0	5	23	35	44	51	56	60	64
180	0	0	0	3	22	34	43	50	55	60	63
200	0	0	0	2	20	33	42	49	55	59	63
220	0	0	0	0	19	32	41	48	54	58	62

Investigation of the recommendations presented in Table 13 reveals that rates of nitrogen application are greater than zero only after total expected water is greater than 10 inches. We would expect that as input-output prices become more favorable, fertilization would begin at less favorable water conditions. This is in fact the case. In Table 14 recommendations are presented for varying price ratios and levels of total expected water when soil nitrate equals 90 pounds per acre. For a price ratio of .15, nitrogen application begins when total water equals 13 inches. However, for a price ratio equal to .03 or less, nitrogen is applied when total expected water equals 9 inches.

In Table 13 and each of the tables presented in the Appendix, optimal nitrogen rates decline as soil nitrate levels increase. The linear relationship between nitrogen rates and soil nitrate at a given level of water is due to the specification of the response function. As equation (4.4) is solved for optimal rates of nitrogen at a given level of precipitation, soil nitrate is the only variable in the equation. Since the soil nitrate variable in the interaction between applied nitrogen and soil nitrate was linear, each additional unit of soil nitrate will decrease the optimal rate of nitrogen application by a constant amount. This constant will of course vary as different levels of precipitation are substituted into (4.4). Precipitation had the opposite effect on application rates. Optimal

TABLE 14. OPTIMAL RATE OF NITROGEN BASED ON THE INPUT-OUTPUT PRICE RATIO AND TOTAL EXPECTED WATER WHEN NO_3-N TO 4' EQUALS 90 LB/A (No Protein Premium).

Input-Output Price Ratio	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
	Nitrogen (lb/A)										
.01	0	37	56	66	73	77	80	83	85	86	87
.02	0	24	47	58	67	72	76	79	81	83	85
.03	0	11	38	52	61	68	72	75	78	80	82
.04	0	0	29	45	56	63	68	72	75	77	79
.05	0	0	20	38	50	58	64	68	72	74	77
.06	0	0	11	31	44	53	60	65	68	72	74
.07	0	0	2	24	39	48	56	61	65	69	71
.08	0	0	0	18	33	44	51	57	62	66	69
.09	0	0	0	11	27	39	47	54	59	63	66
.10	0	0	0	4	22	34	43	50	56	60	63
.11	0	0	0	0	16	29	39	46	52	57	61
.12	0	0	0	0	10	25	35	43	49	54	58
.13	0	0	0	0	5	20	31	39	46	51	55
.14	0	0	0	0	0	15	27	36	43	48	53
.15	0	0	0	0	0	10	23	32	39	45	50

nitrogen rates increased as total expected water increased holding soil nitrate constant.

In Figure 1 recommended rates of nitrogen application are plotted against corresponding price ratios for selected levels of soil nitrate and total expected water. Rates of application ranged from 74 pounds per acre for a price ratio of .01 to 0 pounds for a price ratio slightly less than .14. Under Winter 1977 price conditions, 27 pounds of nitrogen would be recommended.

Introduction of Protein Premium

Historically, Montana winter wheat producers have received a premium for high protein wheat. The structure of this protein premium has varied from year to year. During 1973-75 there were times when the premium paid for high protein wheat was negligible. During other periods, premiums in the 50- to 75-cent range have not been uncommon. Certainly the ability to accurately predict protein premium structures would be desirable for decision-making purposes. Results from the estimation of protein response relationships in Chapter III indicate that protein content increases with the application of nitrogen fertilizer as water variables are held constant.

Researchers in Montana (25) concluded that three factors strongly influenced the protein premium received by Montana wheat producers. These factors were the level of wheat exports, the total supply of

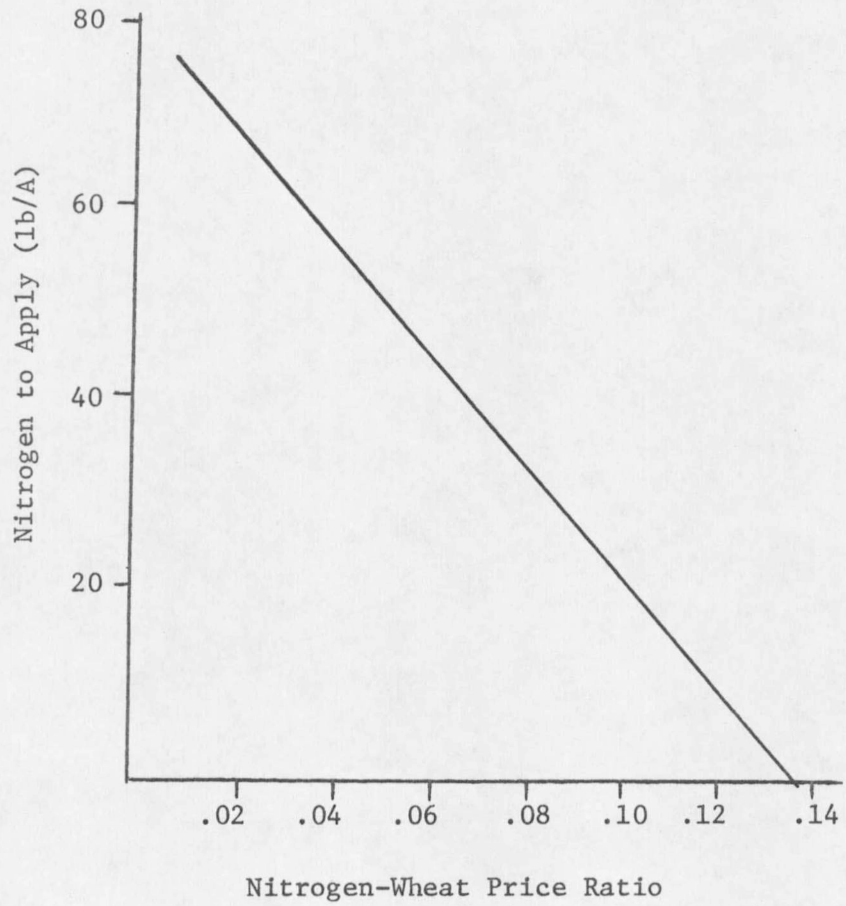

Figure 1. Recommended Rates of Nitrogen Application Plotted Against Varying Nitrogen-Wheat Price Ratios When No Protein Premium Exists (Total NO_3-N to 4' = 90 lb/A and Total Expected Water = 12 inches).

high protein wheat in the United States, and the average protein content of the high protein wheat. This research provided a method of forecasting protein premiums once causal factors were quantified. However, the information necessary to predict protein premiums is not available to the individual wheat producer at a time when the producer can react. Therefore, a profit-maximizing decision criterion is developed which uses historical protein premium structures in determining optimal rates of nitrogen application. The primary purpose of this section of the analysis is to illustrate the magnitude of the effect protein premiums may have on optimal nitrogen fertilization practices.

Yield and protein equations generated in Chapter III were used in developing fertilizer recommendations which included protein response in the decision criteria. Profit equations similar to (4.6) were formed under unlimited capital situations. Once again, input and output prices were designated as given parameters determined in perfectly competitive input and output markets.

Profit equations for this section of the analysis can be expressed as:

$$\pi = P_{Bw}Y + P_{m}Y - P_{n}N - K \tag{4.6}$$

where: P_{Bw} = base price of wheat per bushel (less than 11 percent protein content);

Y = wheat yield in bushels per acre;

P_m = protein premium associated with predicted protein content;

P_n = price per pound of nitrogen;

N = pounds of nitrogen applied per acre; and

K = fixed costs per acre.

Values for P_m were determined using observations from 10 years of protein premium data. Protein premium structures associated with the third Friday of September for each of the years in the 1967-76 period were used to calculate the following average protein premium structure:

less than 11 percent	=	\$.00
11 percent to 12 percent	=	.01
12 percent to 13 percent	=	.09
13 percent to 14 percent	=	.19
14 percent to 15 percent	=	.28
15 percent to 16 percent	=	.37
16 percent to 17 percent	=	.44
greater than 17 percent	=	.48

The selection of data to reflect protein premiums near harvest was completely arbitrary. Due to the discrete nature of the protein premium structure, it was necessary to maximize (4.6) with respect to applied nitrogen by examining total profit figures when constant values for precipitation and soil nitrate were specified. A computer program calculated profit levels associated with nitrogen application rates ranging from 0 to 100 pounds in 1-pound increments. The rate of nitrogen application which produced the greatest profits was then selected. A price of 20 cents per pound of actual nitrogen was used in the calculations. Optimal rates of application were determined for a price ratio equal to .09.

Nitrogen Fertilizer Recommendations With Protein Response

Fertilizer recommendations developed using this discrete protein premium structure are presented in Table 15. Obviously, the optimal level of nitrogen with a protein premium is going to be at least as great as the optimal level without a protein premium when identical soil and climatic conditions exist. However, we might expect that in some situations optimal rates of application would actually increase due to protein response and associated price increases. This does in fact occur. Rates are increased in a number of cases, although considerable discontinuity exists in the optimal levels. This discontinuity is due to the discrete nature of the protein premium structure. Optimal rates of nitrogen application often occurred at nitrogen levels which were just sufficient to boost protein percentages into the next premium category, thus boosting the per bushel price of wheat. Problems associated with using these recommendations are evident. Unless protein content is actually increased to the point where the additional protein premium is realized, the application of these higher rates of nitrogen will actually diminish profits.

TABLE 15. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (With Protein Premium Structure and Applied Phosphorus = 25 lb/A).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	6	31	47	81	86	90	94	97	97	77
40	0	22	47	62	71	77	81	85	87	90	93
60	3	10	35	52	62	68	72	75	76	77	76
80	0	0	24	41	52	82	85	87	89	91	93
100	13	16	13	59	67	73	76	78	78	78	75
120	0	4	31	48	57	63	66	92	93	94	95
140	0	0	19	37	47	79	81	82	82	81	77
160	0	13	8	58	66	70	72	72	70	65	67
180	0	0	30	47	56	60	62	61	88	87	82
200	0	0	18	36	46	50	51	80	77	71	67
220	0	15	6	24	35	71	71	69	63	63	66

A more desirable method of deriving optimal nitrogen rates with a discrete protein premium structure would use expected values for profit to derive optimal nitrogen rates. In order to use this approach, it would be necessary to calculate the joint probabilities for yield and percent protein associated with every combination of applied nitrogen, soil nitrate, and total expected water. Using those probabilities, the expected profit associated with every rate of nitrogen application at a given level of soil nitrate and total water could be calculated. The rate of nitrogen application which produced the highest expected profit figure would then be chosen. Although this method would certainly be superior to the procedure used to generate Table 15, the computational costs involved in deriving these optimal rates prevented its use.

An alternative procedure was to fit a continuous function to the optimal nitrogen values presented in Table 15. This approach has a certain amount of appeal, since a distribution of optimal nitrogen rates will result from variations in the actual level of precipitation received. Recommendations in Table 16 resulted from fitting this function. A comparison of Tables 16 and 13 shows that recommended rates were increased from 0 to 41 pounds by the inclusion of protein response and the associated protein premium. When total expected water equals 12 inches and available soil nitrate equals 100 pounds per acre, a nitrogen rate of 27 pounds per acre would be recommended

TABLE 16. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 (With Protein Premium Structure and Applied Phosphorus = 25 lb/A).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	21	41	57	71	81	88	92	93	91	85
40	0	18	37	53	67	77	84	89	90	89	84
60	0	17	35	50	63	74	81	86	88	87	84
80	0	15	33	48	61	71	79	84	86	86	84
100	0	15	32	47	59	69	77	82	85	85	84
120	0	14	31	45	57	67	75	80	83	84	83
140	0	14	30	44	56	65	73	78	81	83	82
160	0	13	29	42	54	63	71	76	79	80	80
180	0	12	27	40	51	61	68	73	76	77	77
200	0	10	27	38	48	57	64	69	72	73	72
220	0	7	21	34	45	53	60	65	67	68	67

without the protein premium. With the protein premium, the recommended rate in Table 16 for identical soil and climatic conditions increases to 59 pounds.

Further investigation of Tables 13 and 16 reveals that nitrogen fertilization begins at less favorable water conditions when the protein premium is included. Without the protein premium, nitrogen fertilization begins at 11 inches of total expected water. In Table 16 nitrogen fertilization begins at 9 inches of total expected water. This difference results from the positive relationship existing between applied nitrogen and protein content of the grain when soil nitrate and total water are held constant. In general, inclusion of a protein premium appears to increase the optimal rate of nitrogen application in all but the most unfavorable moisture conditions.

Risk and Uncertainty

An element of risk or uncertainty is associated with using any fertilizer level. The willingness and ability of farmers to assume this risk depends on their capital and equity positions as well as their attitudes toward risk aversion. Under favorable conditions, a conservative individual with limited capital and equity may be reluctant to apply more than a few pounds of nitrogen. On the other hand, individuals with little risk aversion may apply rates very close to those determined in the preceding sections. Due to these diverse attitudes toward risk, analysts are faced with the dilemma of making fertilizer recommendations which are consistent with each individual's preferences. Recommendations presented in the preceding sections are applicable only if we assume that wheat producers are indifferent to risk.

Different methods have been used to address the problem of risk. In this project, a method used by Ryan et al. (24) and Tollini et al. (31) will be used. This approach assumes that the farmer is willing to apply fertilizer only up to the point where the marginal rate of return on fertilizer investment is equal to some specified amount. This method is a rather crude approach to the problem, since the level of risk is not specifically considered. However, detailed analysis of more sophisticated techniques is beyond the scope of this study.

Without a protein premium, this decision criteria can be expressed as

$$\frac{P_w(b_4 + 2b_5X_4 + b_{12}X_{10} + b_{13}X_6 + 2b_{18}X_4X_{10}) - P_n}{P_n} = r$$

where P_w is the price per bushel of wheat, P_n is the price per pound of nitrogen, r is the desired marginal rate of return expressed as a fraction, and the expression in parentheses is the marginal product of nitrogen from equation (3.15). When the individual is indifferent to risk and when capital is unlimited, r would be equal to zero.

A marginal rate of return equal to 25 percent was specified and used to derive nitrogen recommendations. Specification of a marginal rate of return equal to 25 percent simply means that the wheat producer is willing to apply nitrogen fertilizer only up to the point where the last dollar invested in nitrogen returns exactly $1.25. The producer applies fertilizer as long as r is greater than or equal to 25 percent. Recommendations derived using this criterion are presented in Table 17 for a price ratio equal to .09. Protein response was not included in the derivation of these recommendations. The effect of specifying this marginal rate of return can be observed by comparing Tables 17 and 13. Second-order conditions for profit maximization require that profit-maximizing rates of nitrogen application be limited to strictly concave regions of the production function. As a result, optimal rates of application will decrease as the marginal rate of return increases in magnitude. If total expected water equals 12 inches, rates of application are decreased by approximately 12 pounds when a marginal rate of return of 25 percent is specified. However, as total expected water increases, the difference between optimal rates as presented in Tables 17 and 13 becomes smaller. This relationship is due to the nature of the estimated response function for yield. The rate of change or slope of the marginal product

TABLE 17. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .09 AND THE MARGINAL RATE OF RETURN EQUALS .25 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	0	19	32	41	48	54	59	62
40	0	0	0	0	18	31	40	48	53	58	62
60	0	0	0	0	17	30	39	47	53	57	61
80	0	0	0	0	15	29	39	46	52	57	60
100	0	0	0	0	14	28	38	45	51	56	60
120	0	0	0	0	13	27	37	44	50	55	59
140	0	0	0	0	11	26	36	44	50	55	59
160	0	0	0	0	10	24	35	43	49	54	58
180	0	0	0	0	9	23	34	42	48	53	57
200	0	0	0	0	8	22	33	41	47	53	57
220	0	0	0	0	6	21	32	40	47	52	56

curve for nitrogen from yield equation (3.15) is constant at fixed soil moisture and soil nitrate conditions. However, as moisture conditions become more favorable, the slope of this curve increases. As a result, the difference between optimal rates in Tables 17 and 13 declines as total expected water increases.

CHAPTER V

SUMMARY AND CONCLUSIONS

Data from 43 winter wheat fertility experiments were used to estimate yield and protein response equations for winter wheat grown in Montana under summerfallow conditions. Experiments were conducted by the Montana Cooperative Extension Service and the Montana Agricultural Experiment Station during the years 1967, 1968, 1970, and 1971. Observations on April through July precipitation, total applied phosphorus, total applied nitrogen, and available soil nitrate and available soil water measured in the top 4 feet of the soil in early spring were used as explanatory variables in estimating winter wheat response relationships. Response relationships were estimated using third-degree polynomial equations. Equations with additive and multiplicative disturbance terms were specified and parameters were estimated using a generalized nonlinear program with maximum likelihood estimators.

Important variables for estimating yield response were first determined using an additive regression disturbance. Important variables determined in this portion of the analysis were next used to estimate multiplicative models. Multiplicative and additive models were then compared to determine the yield equation best suited for use in developing nitrogen fertilizer recommendations.

In the additive regression work, the inclusion of soil water variables appeared to confound the behavior of estimated yield equations. A chi-square test statistic indicated terms which included soil water as a variable were not significantly different from zero at the .10 level. As a result, soil water was eliminated from the analysis. Difficulties encountered in converting soil water measurements to inches of available soil water may explain the poor results obtained for this explanatory variable. Applied phosphorus was also excluded in this portion of the analysis. Only applied nitrogen, precipitation, and soil nitrate were significant in explaining yield response. Terms which included precipitation were significantly different from zero at the .001 level. Similarly, terms with soil nitrate as a variable were significantly different from zero at the .01 level.

Relevant variables in the estimation of additive models were then used to estimate a multiplicative model. Comparison of corresponding multiplicative and additive models revealed that differences between t-ratios for terms in the two models were small. However, for agronomic data, the multiplicative error term would appear to make more sense intuitively, since standard errors of the estimate are directly proportional to the mean. Therefore, the model proposed for estimating yield includes a multiplicative disturbance term and can be expressed in natural logarithms as:

$$\log_e Y = \log_e(25.902 + 12.806X_2 + 3.266X_3 - .0843X_4 + .000275X_5 - .00620X_6 - .000038X_7 + .959X_{10} - .105X_{11} + .0333X_{12} - .000114X_{13} + .0102X_{16} - .000165X_{18}) + \log_e \varepsilon_{iL} + \log_e U_L$$

where: Y = estimated yield;

X_2 = binary variable for variety 2 (Warrior);

X_3 = binary variable for variety 3 (Cheyenne);

X_4 = total applied nitrogen (N);

X_5 = N^2;

X_6 = available soil nitrate in 4' of soil (SN);

X_7 = $(SN)^2$;

X_{10} = total April-July precipitation (PREC);

X_{11} = $(PREC)^2$;

X_{12} = N x PREC;

X_{13} = SN x N;

X_{16} = SN x PREC;

X_{18} = N^2 x PREC;

ε_{iL} = random variation associated with the i^{th} treatment at location L; and

U_L = random variation due to location L.

The behavior of this response function was characterized by maximum yields and diminishing marginal returns to each of the variable resources under consideration. Variables X_2, X_{12}, and X_{13} were significant at least at the .05 level. Parameters for this model were estimated with a constraint on the equation which prevented yields from declining at higher precipitation values.

The model proposed for estimating grain protein also included a multiplicative disturbance term. This model is expressed in natural logarithms as:

$$\begin{aligned}\log_e Y = \log_e(&15.987 + .0413X_1 - 2.0623X_2 + 0.521X_3 \\ &- 0.00269X_4 - .000321X_5 + 0.019X_6 - 0.000018X_7 \\ &- 2.157X_{10} + 0.121X_{11} + 0.0161X_{12} - 0.000077X_{13} \\ &+ 0.00115X_{16} + 0.000042X_{18} - 0.00124X_{19}) \\ &+ \log_e \varepsilon_{iL} + \log_e U_L\end{aligned}$$

where: Y = estimated grain protein (%);

X_1 = total applied phosphorus;

X_2 = binary variable for variety 2 (Warrior);

X_3 = binary variable for variety 3 (Cheyenne);

X_4 = total applied nitrogen (N);

$X_5 = N^2$;

X_6 = available soil nitrate in 4' of soil (SN);

$X_7 = (SN)^2$;

X_{10} = total April-July precipitation (PREC);

$X_{11} = (PREC)^2$;

X_{12} = N x PREC;

X_{13} = SN x N;

X_{16} = SN x PREC;

X_{18} = N^2 x PREC;

X_{19} = N x $(PREC)^2$;

ε_{iL} = random variation associated with the i^{th} treatment at location L; and

U_L = random variation due to location L.

In this equation, precipitation had a negative influence on grain protein over most ranges of the data. Increases in percent protein were associated with increased precipitation when simultaneously high levels of soil nitrate and applied nitrogen occurred. High levels of precipitation also had a positive effect on grain protein when very low levels of applied nitrogen were present. The effect of soil nitrate on protein content was positive in all cases. Applied nitrogen also had a positive effect on grain protein in most instances. Variables X_1, X_2, X_{10}, X_{11}, X_{12}, X_{13}, and X_{19} were significant at least at the .05 level. It was somewhat surprising that applied phosphorus would be significant in explaining protein response but not yield response. This result may warrant further investigation of the influence of applied phosphorus on protein content.

Fertilizer recommendations were next developed using estimated yield and protein response functions. In order to include soil water as a decision variable when making fertilizer recommendations, the assumption was made that an inch of soil water above or below the mean level of 5 inches of soil water was equivalent to an inch of precipitation above or below the expected precipitation level. The first set of recommendations developed did not include protein response. Optimal rates of nitrogen application were determined under unlimited capital conditions for nitrogen-wheat price ratios ranging from .01 to .15 in increments of .01. Optimal rates were presented for varying levels of soil nitrate and total expected water, where total expected water was defined as the sum of available soil water in the top 4 feet of soil in early spring and expected April through July precipitation.

As expected, optimal rates declined as available soil nitrate increased. At a given level of expected water, optimal rates decreased by a constant amount as soil nitrate increased. This relationship was due to the specification of the yield response function. On the other hand, optimal rates of nitrogen application increased as water conditions became more favorable at a given level of soil nitrate. As input-output price ratios became more favorable, nitrogen fertilization began at less favorable water conditions.

For individual wheat producers to use these recommendations, they must have information on the available soil nitrate present in the top 4 feet of soil in early spring, the available soil water present in the top 4 feet of soil in early spring, and the level of expected

April through July precipitation for each particular area. Given this information and nitrogen and wheat prices, optimal rates of nitrogen application can be determined directly from the tables. For instance, under Winter 1977 nitrogen and wheat price conditions, the nitrogen-wheat price ratio is approximately .09; thus, recommendations presented in Table 13 are applicable. If soil tests indicate that 100 pounds of available soil nitrate and 5 inches of available soil water are present in the top 4 feet of soil in early spring and the level of expected April through July precipitation is equal to 7 inches, then 27 pounds of nitrogen fertilizer would be recommended.

Recommendations were also derived using estimated protein reponse relationships. Protein premium quotations associated with the third Friday of September for each of the years in the 1967-76 period were used to calculate an average protein premium structure. The selection of this date was arbitrary but represents quotations following harvest. Profit equations which included the protein premium were then formulated. Due to the discrete nature of this protein premium structure, it was necessary to determine optimal rates by examining total profit figures. A computer program performed the necessary calculations. Optimal rates were calculated for a price ratio which closely approximated Winter 1977 price conditions (price ratio = .09). Results from this procedure showed that a considerable degree of discontinuity existed between the optimal rates presented in Table 15.

As a result, a continuous function was fitted to the values in Table 15 to better determine the influence of protein premiums on optimal nitrogen recommendations. Results from fitting this function showed that the protein premium increased optimal rates of application from 0 to 41 pounds. Nitrogen fertilization began at less favorable water conditions when a protein premium was included.

In the final section of this study, a 25-percent marginal rate of return was specified and used to derive optimal nitrogen rates without a protein premium. Specification of this marginal rate of return simply means that the wheat producer is willing to apply nitrogen fertilizer only up to the point where the last dollar invested in nitrogen returns exactly $1.25. This procedure represents a crude approach to the problem of addressing risk and uncertainty when ferilizer recommendations are made. Results showed that optimal nitrogen rates were reduced from 0 to 16 pounds when a 25-percent marginal rate of return was specified and the price ratio was equal to .09. As total expected water increased, the difference between optimal rates derived with and without the 25-percent marginal rate of return declined. These comparisons were made using Tables 13 and 17 in Chapter IV. Investigation of more sophisticated techniques of examining risk and uncertainty is suggested as an area for future research. Additional regression work examining protein response relationships and the influence of soil nitrate on yield response is also recommended as well as use of an expected value criterion for including protein premiums. Finally, more precise techniques are needed for measuring and recording site specific information to be used in estimating winter wheat response relationships. Use of neutron probes may aid in the acquisition of more accurate soil water measurements. Similarly, a uniform system for recording information would be helpful. Records of seeding rates, row spacing, varieties, planting and harvesting dates, dates when rain gauges are installed and soil tests are taken, and any other management practices would be helpful to the researcher.

APPENDIX

SITE LOCATIONS AND INVESTIGATORS

	Site Location	Year	Investigator
1.	Luther Auer Farm, Broadview, MT	1967	Chas. M. Smith[1/]
2.	Ben Becker Farm, Billings, MT	1967	" " "
3.	Ivan Dahlman Farm, Forsyth, MT	1967	" " "
4.	Don Kronebush Farm, Conrad, MT	1967	" " "
5.	J. P. Pelley Farm, Perma, MT	1967	" " "
6.	David Ross Farm, Terry, MT	1967	" " "
7.	Myron Schultz Farm, Bloomfield, MT	1967	" " "
8.	Wilbert Schweigert Farm, Baker, MT	1967	" " "
9.	Howard Wegner Farm, E. Helena, MT	1967	" " "
10.	Russell Anderson Farm, Harrison, MT	1968	" " "
11.	George Baudel Farm, Floweree, MT	1968	" " "
12.	Ivan Dahlman Farm, Forsyth, MT	1968	" " "
13.	Loyal Deardorff Farm, Brady, MT	1968	" " "
14.	David Enos Farm, Baker, MT	1968	" " "
15.	John Fadhl Farm, Angela, MT	1968	" " "
16.	Verle Jones Farm, Glendive, MT	1968	" " "
17.	Ivor Lund Farm, Circle, MT	1968	" " "
18.	Clark MacDonald Farm, Great Falls, MT	1968	" " "
19.	David Ross Farm, Terry, MT	1968	" " "
20.	Wilbert Schweigert Farm, Baker, MT	1968	" " "
21.	Dave Albin Farm, Sidney, MT	1970	Roger L. Wilson[2/]
22.	Fred Bergstrom Farm, Brady, MT	1970	" " "
23.	Ken Coulter Farm, Jordan, MT	1970	" " "
24.	Kermit Erickson Farm, Vida, MT	1970	" " "
25.	Leo Kahm Farm, Circle, MT	1970	" " "
26.	Jerry Obergfell Farm, Sidney, MT	1970	" " "
27.	Tom Stanton Farm, Jordan, MT	1970	" " "
28.	Harold Theilman Farm, Bloomfield, MT	1970	" " "
29.	Fred Berkrum Farm, Cut Bank, MT	1970	Harold A. Houlton[3/]
30.	Art Christofferson Farm, Malta, MT	1970	" " "
31.	Sherman Doucette Farm, Wagner, MT	1970	" " "
32.	John Jergenson Farm, Chinook, MT	1970	" " "
33.	Wilbur Rolston Farm, Kremlin, MT	1970	" " "
34.	Fred Elling Farm, Rudyard, MT	1971	" " "
35.	Fransen Bros. Farm., Dunkirk, MT	1971	" " "
36.	Giles Gregoire Farm, Havre, MT	1971	" " "
37.	John Jergenson Farm, Chinook, MT	1971	" " "
38.	Lloyd Kaercher Farm, Havre, MT	1971	" " "
39.	Leo Kraff Farm, Hingham, MT	1971	" " "
40.	Curt Lakey Farm, Chester, MT	1971	" " "
41.	James Reinowski Farm, Kremlin, MT	1971	" " "
42.	Wilbur Rolston Farm, Kremlin, MT	1971	" " "
43.	Leonard Wavrick Farm, Havre, MT	1971	" " "

1/ Formerly Extension Soils Scientist, Montana State University. Now serving as Department Head at North Dakota State University.

2/ Extension Soil Scientist, Montana State University.

3/ Assistant Soil Scientist, Central and Northern Branch Experiment Stations.

APPENDIX TABLE 1. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .01 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	7	47	63	72	77	81	84	86	87	88	90
40	2	44	61	70	76	80	83	85	86	88	89
60	0	41	59	69	75	79	82	84	86	87	88
80	0	38	57	67	73	78	81	83	85	86	88
100	0	35	55	65	72	77	80	82	84	86	87
120	0	32	53	64	71	75	79	81	84	85	87
140	0	30	51	62	69	74	78	81	83	84	86
160	0	27	49	61	68	73	77	80	82	84	85
180	0	24	47	59	67	72	76	79	81	83	85
200	0	21	45	57	66	71	75	78	81	82	84
220	0	18	43	56	64	70	74	77	80	82	83

APPENDIX TABLE 2. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .02 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	34	54	65	72	76	79	82	84	86	87
40	0	31	52	63	70	75	79	81	83	85	86
60	0	28	50	62	69	74	78	80	82	84	86
80	0	25	48	60	68	73	77	79	82	84	85
100	0	23	46	58	66	72	76	79	81	83	84
120	0	20	44	57	65	71	75	78	80	82	84
140	0	17	42	55	64	70	74	77	80	82	83
160	0	14	40	54	62	68	73	76	79	81	83
180	0	11	38	52	61	67	72	75	78	80	82
200	0	8	36	51	60	66	71	75	77	80	81
220	0	5	33	49	59	65	70	74	77	79	81

APPENDIX TABLE 3. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .03 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	21	45	58	66	71	75	78	81	83	84
40	0	18	43	56	65	70	74	78	80	82	84
60	0	16	41	55	63	69	73	77	79	81	83
80	0	13	39	53	62	68	73	76	79	81	82
100	0	10	37	52	61	67	72	75	78	80	82
120	0	7	35	50	59	66	71	74	77	79	81
140	0	4	33	48	58	65	70	73	76	79	81
160	0	1	31	47	57	64	69	73	76	78	80
180	0	0	29	45	56	63	68	72	75	77	79
200	0	0	27	44	54	62	67	71	74	77	79
220	0	0	24	42	53	60	66	70	73	76	78

APPENDIX TABLE 4. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .04 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	8	36	51	60	67	71	75	77	80	82
40	0	6	34	49	59	65	70	74	77	79	81
60	0	3	32	48	58	64	69	73	76	78	80
80	0	0	30	46	56	63	68	72	75	78	80
100	0	0	28	45	55	62	67	71	75	77	79
120	0	0	26	43	54	61	67	71	74	76	79
140	0	0	24	41	52	60	66	70	73	76	78
160	0	0	22	40	51	59	65	69	72	75	77
180	0	0	20	38	50	58	64	68	72	74	77
200	0	0	17	37	49	57	63	67	71	74	76
220	0	0	15	35	47	56	62	66	70	73	75

APPENDIX TABLE 5. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .05 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
<u>lb N/A</u>	Nitrogen (lb/A)										
20	0	0	27	44	55	62	67	71	74	77	79
40	0	0	25	42	53	61	66	70	74	76	78
60	0	0	23	41	52	60	65	69	73	75	78
80	0	0	21	39	51	59	64	69	72	75	77
100	0	0	19	38	49	57	63	68	71	74	76
120	0	0	17	36	48	56	62	67	71	73	76
140	0	0	15	34	47	55	61	66	70	73	75
160	0	0	13	33	46	54	61	65	69	72	75
180	0	0	10	31	44	53	60	64	68	71	74
200	0	0	8	30	43	52	59	64	68	71	73
220	0	0	6	28	42	51	58	63	67	70	73

APPENDIX TABLE 6. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .06 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	18	37	49	57	63	67	71	74	76
40	0	0	16	35	48	56	62	67	70	73	76
60	0	0	14	34	46	55	61	66	70	73	75
80	0	0	12	32	45	54	60	65	69	72	74
100	0	0	10	31	44	53	59	64	68	71	74
120	0	0	8	29	42	52	58	63	67	71	73
140	0	0	6	27	41	51	57	63	67	70	73
160	0	0	4	26	40	49	56	62	66	69	72
180	0	0	1	24	39	48	55	61	65	69	71
200	0	0	0	23	37	47	55	60	64	68	71
220	0	0	0	21	36	46	54	59	64	67	70

APPENDIX TABLE 7. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .07 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	9	30	43	52	59	64	68	71	74
40	0	0	7	28	42	51	58	63	67	70	73
60	0	0	5	27	41	50	57	62	66	70	71
80	0	0	3	25	39	49	56	61	66	69	72
100	0	0	1	24	38	48	55	61	65	68	71
120	0	0	0	22	37	47	54	60	64	68	71
140	0	0	0	20	35	46	53	59	63	67	70
160	0	0	0	19	34	45	52	58	63	66	69
180	0	0	0	17	33	44	51	57	62	66	69
200	0	0	0	16	32	42	50	56	61	65	68
220	0	0	0	14	30	41	49	56	60	64	67

APPENDIX TABLE 8. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .08 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	23	38	48	55	60	65	68	71
40	0	0	0	21	36	46	54	59	64	67	70
60	0	0	0	20	35	45	53	59	63	67	70
80	0	0	0	18	34	44	52	58	62	66	69
100	0	0	0	17	32	43	51	57	62	65	68
120	0	0	0	15	31	42	50	56	61	65	68
140	0	0	0	14	30	41	49	55	60	66	67
160	0	0	0	12	29	40	48	54	59	63	67
180	0	0	0	10	27	39	47	54	59	63	66
200	0	0	0	9	26	38	46	53	58	62	65
220	0	0	0	7	25	37	45	52	57	61	65

APPENDIX TABLE 9. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .10 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	9	26	38	47	53	58	62	66
40	0	0	0	8	25	37	46	52	57	62	65
60	0	0	0	6	24	36	45	51	57	61	64
80	0	0	0	4	22	35	44	51	56	60	64
100	0	0	0	3	21	34	43	50	55	60	63
120	0	0	0	1	20	33	42	49	54	59	63
140	0	0	0	0	19	31	41	48	54	58	62
160	0	0	0	0	17	30	40	47	53	58	61
180	0	0	0	0	16	29	39	46	52	57	61
200	0	0	0	0	15	28	38	46	51	56	60
220	0	0	0	0	13	27	37	45	51	56	59

APPENDIX TABLE 10. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .11 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	2	21	33	42	49	55	59	63
40	0	0	0	1	19	32	41	48	54	59	62
60	0	0	0	0	18	31	40	48	53	58	62
80	0	0	0	0	17	30	40	47	53	57	61
100	0	0	0	0	15	29	39	46	52	57	60
120	0	0	0	0	14	28	38	45	51	56	60
140	0	0	0	0	13	27	37	44	50	55	59
160	0	0	0	0	12	26	36	44	50	55	59
180	0	0	0	0	10	24	35	43	49	54	58
200	0	0	0	0	9	23	34	42	48	53	57
220	0	0	0	0	8	22	33	41	47	53	57

APPENDIX TABLE 11. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .12 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	0	15	28	38	46	52	56	60
40	0	0	0	0	14	27	37	45	51	56	60
60	0	0	0	0	12	26	36	44	50	55	59
80	0	0	0	0	11	25	35	43	49	54	58
100	0	0	0	0	10	24	34	42	49	54	58
120	0	0	0	0	8	23	34	42	48	53	57
140	0	0	0	0	7	22	33	41	47	52	57
160	0	0	0	0	6	21	32	40	46	52	56
180	0	0	0	0	5	20	31	39	46	51	55
200	0	0	0	0	3	19	30	38	45	50	55
220	0	0	0	0	2	18	29	37	44	50	54

APPENDIX TABLE 12. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .13 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	0	9	24	34	42	48	53	58
40	0	0	0	0	8	23	33	41	48	53	57
60	0	0	0	0	7	21	32	40	47	52	56
80	0	0	0	0	5	20	31	40	46	51	56
100	0	0	0	0	4	19	30	39	45	51	55
120	0	0	0	0	3	18	29	38	45	50	54
140	0	0	0	0	2	17	28	37	44	49	54
160	0	0	0	0	0	16	28	36	43	49	53
180	0	0	0	0	0	15	27	35	42	48	53
200	0	0	0	0	0	14	26	35	42	47	52
220	0	0	0	0	0	13	25	34	41	47	51

APPENDIX TABLE 13. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .14 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	0	4	19	30	38	45	50	55
40	0	0	0	0	2	18	29	38	44	50	54
60	0	0	0	0	1	17	28	37	44	49	54
80	0	0	0	0	0	16	27	36	43	48	53
100	0	0	0	0	0	15	26	35	42	48	52
120	0	0	0	0	0	13	25	34	41	47	52
140	0	0	0	0	0	12	24	33	41	46	51
160	0	0	0	0	0	11	23	33	40	46	51
180	0	0	0	0	0	10	22	32	39	45	50
200	0	0	0	0	0	9	22	31	38	44	49
220	0	0	0	0	0	8	21	30	38	44	49

APPENDIX TABLE 14. OPTIMAL RATES OF N APPLICATION BASED ON NO_3-N TO 4' AND TOTAL EXPECTED WATER WHEN INPUT-OUTPUT PRICE RATIO = .15 (No Protein Premium Structure).

NO_3-N to 4' in Early Spring	Expected April-July Precipitation (in.) Plus Plant Available Water (in.) to 4' in Early Spring										
	8	9	10	11	12	13	14	15	16	17	18
lb N/A	Nitrogen (lb/A)										
20	0	0	0	0	0	14	26	35	42	48	52
40	0	0	0	0	0	13	25	34	41	47	52
60	0	0	0	0	0	12	24	33	40	46	51
80	0	0	0	0	0	11	23	32	40	46	50
100	0	0	0	0	0	10	22	32	39	45	50
120	0	0	0	0	0	9	21	31	38	44	49
140	0	0	0	0	0	8	20	30	37	44	49
160	0	0	0	0	0	7	19	29	37	43	48
180	0	0	0	0	0	5	18	28	36	42	47
200	0	0	0	0	0	4	17	27	35	42	47
220	0	0	0	0	0	3	17	27	35	41	46

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