



Fully-developed turbulent flow in smooth and rough-walled pipe
by John Leonard Gow

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
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Abstract:

Fully-developed turbulent flow in both smooth and rough-walled pipes was investigated in a Reynolds number range 30,000 - 500,000. Experimental mean velocity profiles, root-mean-square fluctuating velocity profiles, cross-correlations of the fluctuating velocities, and friction factor data are presented in the thesis for flow in a smooth pipe and two sand-roughened pipes: $R/\epsilon = 208$ and $R/\epsilon = 26.4$. The quantity R/ϵ is the ratio of the actual pipe radius to the average sand particle size. All velocity data that were obtained were with hot-wire anemometers employing standard measuring techniques.

The smooth pipe data agree very well with the data of previous investigators and satisfy the Navier-Stokes equations for fully-developed axisymmetric turbulent flow. For Reynolds numbers less than 50,000, the data obtained using the $R/\epsilon = 208$ rough pipe exhibit; essentially the same characteristics as the smooth tube data. However, for Reynolds numbers greater than 50,000 in the $R/\epsilon = 208$ rough pipe and for all flow rates investigated in the $R/\epsilon = 26.4$ rough pipe, the data differ from smooth pipe data; the difference being a function of roughness size as well as Reynolds number.

The rough pipe experimental data do not agree with empirical data obtained, employing the Navier-Stokes equations for fully-developed axisymmetric turbulent flow; although the experimental data do agree with limited rough-pipe data of other investigators. Apparently, the axisymmetric, fully-developed flow model prescribed for smooth pipes does not represent the turbulent flow in rough-walled pipes.

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TABLE OF CONTENTS

CHAPTER	PAGE
VITA	ii
ACKNOWLEDGMENTS	iii
LIST OF TABLES	v
LIST OF FIGURES	vi
NOMENCLATURE	ix
ABSTRACT	xii
I. INTRODUCTION	1
II. LITERATURE REVIEW	3
III. EXPERIMENTAL SYSTEM	16
Apparatus	16
Procedures	28
IV. ANALYTICAL CONSIDERATIONS	37
V. EXPERIMENTAL RESULTS	41
Data Reduction	41
Mean Velocity Distribution	42
Fluctuating Velocity Correlations	54
RMS Fluctuating Velocities	61
Friction Factors	61
Discussion of Results	71
VI. CONCLUSIONS AND RECOMMENDATIONS	82
APPENDIX	84
LITERATURE CITED	88

LIST OF TABLES

		Page
I	U ⁺ vs. Log Y ⁺ Equations from Previous Investigations	15
II	U ⁺ vs. Log Y ⁺ Equations from Present Investigations	76

LIST OF FIGURES

Figure		Page
1	Schematic of Experimental System	17
2	Entrance Section	18
3	Test Section	19
4	Mean Velocity Boundary Layer Probe	22
5	X-Array $\overline{uv}$ Correlation Probe	23
6	X-Array $\overline{uw}$ Correlation Probe	24
7	Actual Sand Size and Shape - $R/\epsilon = 208$	26
8	Actual Sand Size and Shape - $R/\epsilon = 26.4$	26
9	Representative Spacing in $R/\epsilon = 208$ Rough Tube	27
10	Representative Spacing in $R/\epsilon = 26.4$ Rough Tube	27
11	Wire Orientation for $\overline{uv}$ Correlation Measurements	32
12	Wire Orientation for $\overline{uw}$ Correlation Measurements	32
13	U/U_{max} vs Y/R - Profile for Smooth Pipe at $Re = 175,000$	44
14	U/U_{max} vs Y/R - Profile for $R/\epsilon = 208$ Pipe at $Re = 52,000$	45
15	U/U_{max} vs Y/R - Profile for $R/\epsilon = 208$ Pipe at $Re = 170,000$	46

Figure		Page
16	$U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 208$ Pipe at $Re = 480,000$	47
17	$U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 26.4$ Pipe at $Re = 34,500$	48
18	$U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 26.4$ Pipe at $Re = 75,000$	49
19	$U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 26/4$ Pipe at $Re = 170,000$	50
20	U^+ vs $\log Y^+$ - Profile for Smooth Pipe .	51
21	U^+ vs $\log Y^+$ - Profile for $R/\epsilon = 208$ Pipe	52
22	U^+ vs $\log Y^+$ - Profile for $R/\epsilon = 26.4$ Pipe	53
23	$\overline{uv}/U_*^2$ vs Y/R for Smooth Pipe	55
24	$\overline{uv}/U_*^2$ vs Y/R for $R/\epsilon = 208$ Pipe	56
25	$\overline{uv}/U_*^2$ vs Y/R for $R/\epsilon = 26.4$ Pipe	57
26	$\overline{uw}/U_*^2$ vs Y/R for Smooth Pipe	58
27	$\overline{uw}/U_*^2$ vs Y/R for $R/\epsilon = 208$ Pipe	59
28	$\overline{uw}/U_*^2$ vs Y/R for $R/\epsilon = 26.4$ Pipe	60
29	RMS Fluctuating Velocities Measures in Smooth Pipe	62
30	RMS Fluctuating Velocities Measured in $R/\epsilon = 208$ Pipe at $Re = 52,000$	63
31	RMS Fluctuating Velocities Measured in $R/\epsilon = 208$ Pipe at $Re = 170,000$	64

Figure		Page
32	RMS Fluctuating Velocities Measured in $R/\epsilon = 208$ Pipe at $Re = 480,000$. . .	65
33	RMS Fluctuating Velocities Measured in $R/\epsilon = 26.4$ Pipe at $Re = 34,500$. . .	66
34	RMS Fluctuating Velocities Measured in $R/\epsilon = 26.4$ Pipe at $Re = 75,000$. . .	67
35	RMS Fluctuating Velocities Measured in $R/\epsilon = 26.4$ Pipe at $Re = 170,000$. .	68
36	Turbulence Intensities for Three Degrees of Pipe Roughness	69
37	Friction Factor vs Reynolds Number for Three Degrees of Pipe Roughness . .	70
38	Sketch of Mean Velocity Profile with Fluctuations Superimposed	73

NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
A, A', B, B'	Emperical constants
D	Pipe diameter
e_1, e_2	Linearized voltage signals
$\sqrt{e_1^2}, \sqrt{e_2^2}$	Root-mean-square linearized voltage signals
f	Friction factors
h	Distance between parallel plates
I	Hot wire current
K	Calibration constant
K_s	Equivalent sand diameter
$\ln$	Logarithm to base e
$\log$	Logarithm to base 10
P	Mean pressure at any point in the pipe
r	Radial coordinate
R	Pipe radius
R'	Hot wire resistance
Q	Flow rate
T	Temperature
Re	Reynolds number = $\frac{U_{avg} D}{\nu}$
U	Mean velocity in axial direction

<u>Symbol</u>	<u>Description</u>
U_{avg}	Bulk mean velocity
U_{max}	Centerline mean velocity
U_*	Shear velocity $(\tau_o / \rho)^{1/2}$
U^+	Dimensionless mean velocity (U/U_*)
u	Instantaneous fluctuation component in axial direction
$\sqrt{\overline{u^2}}$	Root-mean-square of fluctuating velocity in axial direction
V	Mean velocity component in radial coordinate direction
v	Instantaneous value of fluctuating velocity component in radial direction
$\sqrt{\overline{v^2}}$	Root-mean-square of fluctuating velocity in radial coordinate direction
W	Mean velocity component in angular coordinate direction
w	Instantaneous value of fluctuating velocity in angular direction
$\sqrt{\overline{w^2}}$	Root-mean-square of fluctuating velocity in angular coordinate direction
y	radial distance, $y = 0$ is the wall'
Y^+	Dimensionless distance parameter (yU_*/ν)
z	Axial coordinate
ϵ	Average sand grain size
$\epsilon U_*/\nu$	Roughness parameter

<u>Symbol</u>	<u>Description</u>
ρ	Air density
ν	Kinematic viscosity
θ	Angular coordinate
τ	Turbulent shear stress
τ_0	Wall shear stress
∇^2	$\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

ABSTRACT

Fully-developed turbulent flow in both smooth and rough-walled pipes was investigated in a Reynolds number range 30,000 - 500,000. Experimental mean velocity profiles, root-mean-square fluctuating velocity profiles, cross-correlations of the fluctuating velocities, and friction factor data are presented in the thesis for flow in a smooth pipe and two sand-roughened pipes: $R/\epsilon = 208$ and $R/\epsilon = 26.4$. The quantity R/ϵ is the ratio of the actual pipe radius to the average sand particle size. All velocity data that were obtained were with hot-wire anemometers employing standard measuring techniques.

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The rough pipe experimental data do not agree with empirical data obtained employing the Navier-Stokes equations for fully-developed axisymmetric turbulent flow; although the experimental data do agree with limited rough-pipe data of other investigators. Apparently, the axisymmetric, fully-developed flow model prescribed for smooth pipes does not represent the turbulent flow in rough-walled pipes.

I. INTRODUCTION

Fully developed turbulent flow over a smooth boundary has been extensively examined both theoretically and experimentally. Although strictly empirical, the accepted theories for flow in a smooth-walled tube agree well with experimental data and have been successfully applied in areas of heat transfer and hydraulic design. However, smooth tube theory has not been adequately modified to predict velocity profiles, shear stress distributions, or convective heat transfer characteristics of the flow in tubes with rough boundaries.

The rough surface has been shown by Dippery¹ to be a more effective heat transfer surface when compared with the smooth surface in a restricted range of Prandtl and Reynolds numbers. The larger surface area is, of course, a factor; but the marked difference in velocity profiles is also thought to be important. Since neither acceptable theory, nor adequate experimental velocity data has been available, an evaluation of the effect of the velocity profile has not been possible.

Consequently, the purpose of this investigation was to collect information on the properties of flow in sand-roughened pipes that might be eventually employed to more fully explain the heat transfer characteristics of rough-walled pipes.

An examination of the momentum and continuity equations governing turbulent pipe flow shows that the principle quantities of interest are the temporal mean velocity profiles and fluctuating velocity correlations. Root-mean-square values of fluctuating velocity components and friction factors are also presented for completeness.

II. LITERATURE REVIEW

A review of the literature indicates the considerable amount of work undertaken in the investigation of turbulent flow. However, close scrutiny of the textbooks which treat turbulent theory and present experimental data, reveals that only a few individuals have actually been responsible for the advancement and understanding of this complex phenomenon. Presented here is a review of the pertinent references dealing with the theoretical and experimental accomplishments for fully-developed turbulent flow in both smooth and rough surface conduits.

The important theories of turbulent flow can be classed into two categories, empirical and statistical. Unlike laminar flow, turbulent flow theory has never been fully manageable from momentum or energy considerations alone. The empirical theory, historically first, attempted to describe the phenomenon on the basis of mean temporal velocities. The statistical theory is based on a knowledge of the fluctuating velocity components and their correlations.

In the late 1800's, Reynolds modified the Navier-Stokes equations, generally applicable only to laminar flow, to include the fluctuating velocity components and the apparent turbulent shear stresses. This was done in an attempt to

illustrate the role played by the fluctuating velocities in the transfer of momentum. The Reynolds transformation is presented in a thorough manner by Corcoran, Opfell and Sage.² Employing the concept of turbulent stresses, Prandtl proposed his mixing length theory³ to explain the high friction factors encountered in turbulent flow. Prandtl reasoned that the high friction factors were due to an exchange of momentum between fluid layers as particles moved transversely to the direction of mean flow. The term "mixing length" was introduced to describe the distance a particle would have to travel in order to attain the same momentum as that of a different layer. Prandtl developed an expression for the turbulent shear stress and introduced his universal velocity distribution equation which has remained one of the most useful for predicting point mean velocities. The equation proposed by Prandtl is

$$U = \frac{U_*}{K} \ln y + C \quad (2.1)$$

where U_* is the shear velocity, K is a universal empirical constant obtained from experimental data, and C is a constant of integration that must be evaluated from boundary conditions. Evaluation of this integration constant at the pipe centerline yields the velocity deficiency equation

$$\frac{U_{\max} - U}{U_*} = \frac{1}{K} \ln \frac{R}{y} \quad (2.2)$$

Perhaps the best known adaption of this equation is the universal logarithmic velocity equation for large Reynolds number flow in smooth tubes. This equation has the form

$$U^+ = A + B \ln Y^+ \quad (2.3)$$

where A and B are again empirical constants, $U^+ = U/U_*$ and $Y^+ = yU_*/\nu$.

Subsequent investigations by Prandtl revealed that a thin laminar region existed at boundary for which it can be shown from shear stress considerations that

$$U^+ = Y^+ \quad (2.4)$$

Employing this postulate and Blasius' friction factor equation, Prandtl also introduced the power law for turbulent velocity distribution as

$$\frac{U}{U_{\max}} = (y/R)^{1/n} \quad (2.5)$$

where the exponent n varies, between six and ten, as a function of Reynolds number.

Later von Karman introduced his similarity hypothesis² in an attempt to determine the dependence of mixing length on space coordinates. He assumed the turbulent fluctuations differ from point to point only by time and length scale factors. von Karman assumed the mixing length satisfied the

equation

$$l = K \left| \frac{dU/dy}{d^2U/dy^2} \right| \quad (2.6)$$

where K is an empirical constant to be determined from experimental data. The mixing length thus can be considered independent of velocity magnitude and dependent only on the first and second derivation of U with respect to y.

Applying the similarity hypothesis, combined with the expression for turbulent shear stress as proposed by Prandtl, the universal velocity distribution law of von Karman is established as

$$\frac{U_{\max} - U}{U_*} = \frac{1}{K} [n(1 - \sqrt{1 - y/R}) + \sqrt{1 - y/R}] \quad (2.7)$$

A graphical comparison of von Karman's and Prandtl's universal velocity distribution equations is presented in Schlichting.⁴ It must be noted here, as pointed out by Schlichting, that neither equation is valid at the wall or at the pipe centerline.

Taylor presented a series of papers treating the statistical considerations of isotropic and homogeneous flow in 1935. These papers and others by Dryden, Kolmogoroff, von Karman and Lin are presented by Friedlander and Topper.⁵ However, only the works of Taylor have been applied in

boundary layer flow to produce a useful equation for predicting velocity profiles. As a result of his theoretical investigations, Taylor proposed the vorticity transport hypothesis.² For a fully developed flow he considered the axial-direction equation of motion and introduced the vorticity vector into the equation. After time averaging the equation and employing a modification of the Reynold's shear stress equation as proposed by Prandtl, Taylor developed the same equation for the turbulent shear stress as did von Karman with the similarity hypothesis. However, Taylor evaluated the velocity distribution employing all the boundary conditions for the centerline of the pipe, and thus the resulting equation

$$\frac{U_{\max} - U}{U_*} = \frac{\sqrt{2}}{K} \sin^{-1} [\sqrt{y/R} - \sqrt{y/R} \sqrt{1 - (y/R)}] \quad (2.8)$$

agrees well with experimental data at the centerline, but yields a negative infinite mean velocity at the wall which is, of course, not valid.

Wang⁶ has developed yet another empirical velocity distribution equation which agrees with experimental data better than do the equations of Prandtl and von Karman. Wang produced an expression for the mixing length from experimental velocity profiles and, employing Prandtl's mixing length theory, derived the following equation:

$$\begin{aligned} \frac{U_{\max} - U}{U_*} = & 2.5 \left[\ln \frac{1 + \sqrt{1 - y/R}}{1 - \sqrt{1 - y/R}} - 2 \tan^{-1} \sqrt{1 - y/R} \right. \\ & - 0.572 \ln \frac{1 - y/R + 1.75 \sqrt{1 - y/R} + 1.53}{1 - y/R - 1.75 \sqrt{1 - y/R} + 1.53} \\ & \left. + 1.14 \tan^{-1} \frac{1.75 \sqrt{1 - y/R}}{1.53 - (1 - y/R)} \right]. \end{aligned} \quad (2.9)$$

However, because of its complexity, the equation has never been employed as a useful tool.

In 1953, Pai presented papers^{7,8} treating turbulent flow strictly from a consideration of the Navier-Stokes equations by employing the Reynold's transformation. Pai derived the same equation for the $\overline{uv}$ correlation as did Laufer,¹²

$$\overline{uv} = v \frac{dU}{dr} + \frac{r}{R} U_T^2; \quad (2.10)$$

however, he integrated this equation to yield an expression for the mean velocity valid for both laminar and turbulent flow. Employing experimental mean velocity data, Pai developed empirical equations for the $\overline{uv}$ correlation and turbulent mean velocity profile. Excellent agreement with experimental smooth tube $\overline{uv}$ correlation data is indicated by Knudsen and Katz.³

General acceptance of Pai's theory is not indicated, because references to it are not found in newer textbooks

and recent investigations continue to employ the earlier empirical theories derived from mixing length considerations.

Nikuradse has presented a complete set of mean velocity and friction factor data for both smooth and rough tubes.⁹ He correlated his data to Prandtl's mixing length theory and evaluated the empirical constants in the logarithmic equation to yield the equation, for smooth tube flow,

$$U^+ = 5.5 + 2.5 \ln Y^+ \quad (2.11)$$

for data in the turbulent core. However, Knudsen and Katz³ point out that Nikuradse shifted his original data to correlate with the laminar sublayer hypothesis, and the validity of the constants is questioned.

Nikuradse divided the U^+ versus $\log Y^+$ distribution curve into three distinct parts: the laminar sublayer, for which $U^+ = Y^+$ according to Prandtl; a buffer region in which the transition from laminar to turbulent flow results; and the completely turbulent region. von Karman analyzed the existing data and found the extent of the laminar sublayer to be between $Y^+ = 0$ and $Y^+ = 5$. Between $Y^+ = 5$ and $Y^+ = 70$ is the buffer or transition layer and for $Y^+ > 70$ is the turbulent core.

Nikuradse's data, when fitted in these regions, yielded the following:

$$\begin{array}{ll} \text{Laminar} & U^+ = Y^+ \\ \text{transition} & U^+ = -3.05 + 3.5 \ln Y^+ \\ \text{turbulent} & U^+ = 5.5 + 2.5 \ln Y^+ \end{array} \quad (2.12)$$

Nikuradse's friction factor data is compared with Prandtl's universal law of friction for smooth pipes in Schlichting.⁴ Excellent agreement is seen.

Deissler¹⁰ has presented data for fully developed flow in smooth tubes and the equation

$$U^+ = 3.8 + 2.78 \ln Y^+ \quad (2.13)$$

best fitted his data for $Y^+ > 26$ in the turbulent core region. For $Y^+ < 26$, he derived the empirical equation

$$Y^+ = \frac{1}{n} \frac{\int_0^{nU^+} e^{-[(nU^+)^2/2]} d(nU^+)}{(1/\sqrt{2\pi}) e^{-[(nU^+)^2/2]}} \quad (2.14)$$

where the value of n was found experimentally.

Laufer conducted investigations in both circular pipes¹¹ and rectangular tubes¹² with smooth boundaries. He presented experimental data correlated to theoretical equations derived from the Navier-Stokes equations. The work consisted of a determination of mean velocity profiles, fluctuating velocity profiles and $\overline{uv}$ correlation measurements. The treatment of

the Navier-Stokes equations yielded two important results: namely an equation for predicting the $\overline{uv}$ correlation from mean velocity profile data and the fact that the $\overline{vw}$ correlation must be zero for all points in the pipe. The values of $\overline{uv}$ calculated by using the mean velocity profile are subject to error only in calculating the slope of the velocity profile from experimental data. Such calculated values have been shown to agree very well with measured $\overline{uv}$ values.

Many other experimental investigations have been conducted in smooth tubes and a list of the mean velocity logarithmic equations for the turbulent core is presented in Table I as taken from Corcoran, Page, Schlinger and Sage.¹³ (See page 15)

A recent investigation by Richardson and McQuivery¹⁴ contains fluctuating velocity data for both smooth and rough boundary open channel flow. RMS measurements of the axial fluctuating components for smooth boundary flow are compared to Laufer's data and good agreement is indicated. A comparison of their data to that of the present investigation for flow over a rough boundary is presented in Chapter V.

Although theory of turbulent flow in rough tubes is lacking, experimental data has been presented. The most notable is the investigation of Nikuradse.⁹ Nikuradse experimented with artificially roughened pipes and determined

mean velocity profiles for a number of pipe radius to sand grain height ratios. His roughness was obtained by cementing sand grains of a given average size to the inside of pipes of various diameters.

Nikuradse's data was correlated to smooth tube flow by altering Prandtl's universal logarithmic equation to contain the roughness height ϵ . The equation is

$$U^+ = 2.5 \ln \frac{Y}{\epsilon} + C \quad (2.15)$$

where C was determined experimentally as a function of the roughness parameter $\epsilon U^+/\nu$. Nikuradse also fitted straight lines to the U^+ versus $\log Y^+$ data in the turbulent core and found the slopes of these lines constant and equal to the slope obtained in smooth tubes. However, the ordinant intercepts of these straight lines decreased with increasing values of the roughness parameter. For values of the roughness parameter less than five, the U^+ versus $\log Y^+$ distribution coincided with the smooth tube curve. Friction factors were also predicted with excellent accuracy from Prandtl's universal law of friction for smooth pipes.⁴ For values of the roughness parameter greater than seventy, the friction factors were found to be independent of Reynold's number and dependent only on the actual roughness size ϵ . Nikuradse thus divided the flow into three regimes:

$$\frac{\epsilon U_*}{v} \leq 5 \quad - \quad \text{Hydraulically smooth}$$

$$5 \leq \frac{\epsilon U_*}{v} \leq 70 \quad - \quad \text{Transition from smooth to rough flow}$$

$$\frac{\epsilon U_*}{v} \geq 70 \quad - \quad \text{Completely rough.}$$

A detailed comparison is made of Nikuradse's data and the data of the present investigation in Chapter VI.

In 1944, Moody¹⁵ presented extensive friction factor data for commercially rough pipes. The Moody Diagram remains today the most useful method for predicting friction head losses for turbulent flow in rough tubes.

Schlichting,⁴ attempting to correlate friction factors for various roughness shapes and distributions to those of Nikuradse's sand roughness, introduced the concept of equivalent roughness. Friction factor data can be substituted into von Karman's friction factor law⁴ for completely rough flow,

$$f = \frac{1}{(2 \log(R/K_s) + 1.74)^2} \quad (2.16)$$

to calculate the equivalent sand roughness K_s . Similar measurements have been made by Streeter¹⁶ in pipes roughened with machined grooves of different forms and distribution.

Townes¹⁷ studied open channel flow over rough boundaries and noted that the mean velocity distribution was in close agreement with the logarithmic universal equation of Prandtl in the fully turbulent section of the boundary layer. Data close to the boundary is inconsistent with Prandtl's laminar sublayer hypothesis. However, it is noted that the flow in the so-called laminar sublayer is quite complicated, more often than not involving velocity components in all three coordinate directions.

The only data available until now for fluctuating velocity correlations in rough pipes has been presented by Logan and Jones.¹⁸ They measured the $\overline{uv}$ correlation and turbulence intensities following an abrupt increase in surface roughness. A comparison of their data is also presented in Chapter V.

TABLE I

Investigator	Equation
Nikuradse	$U^+ = 5.5 + 2.5 \ln Y^+$
Deissler	$U^+ = 3.8 + 2.78 \ln Y^+$
Laufer	$U^+ = 5.5 + 2.91 \ln Y^+$
Wattendorf	$U^+ = 4.0 + 2.68 \ln Y^+$
Skinner	$U^+ = 6.0 + 2.71 \ln Y^+$

III. EXPERIMENTAL SYSTEM

A. Apparatus

The experimental system illustrated schematically on Figure (1) was employed throughout the investigation. Hot-wire anemometry was used for all velocity measurements.

Air was discharged from a centrifugal fan into the entrance section through eighty feet of flexible duct. Flow rates were controlled with a manual damper on the inlet side of the fan. The entrance section, pictorially illustrated in Figure (2), contained a baffle plate and a series of screens to filter the air and dampen turbulence from the fan and return line. The cross-section of the entrance section is reduced in the transition section to match the test pipe diameter. Seventy feet of one-foot diameter aluminum irrigation pipe was used to ensure fully developed flow at the test section. The pipe was suspended with adjustable brackets so near perfect alignment was possible. The test section (Figure (3)) is mounted on the downstream end of the test pipe and consists of a machined section of one-foot diameter and a traversing mechanism for radial positioning of the hot-wire. This mechanism consists of two parallel cylindrical rods which are spanned by the probe support holder. Motion of the holder on the rods is provided by ball-bearing guides. A micrometer screw feed is mounted on the holder to allow hot-wire movements

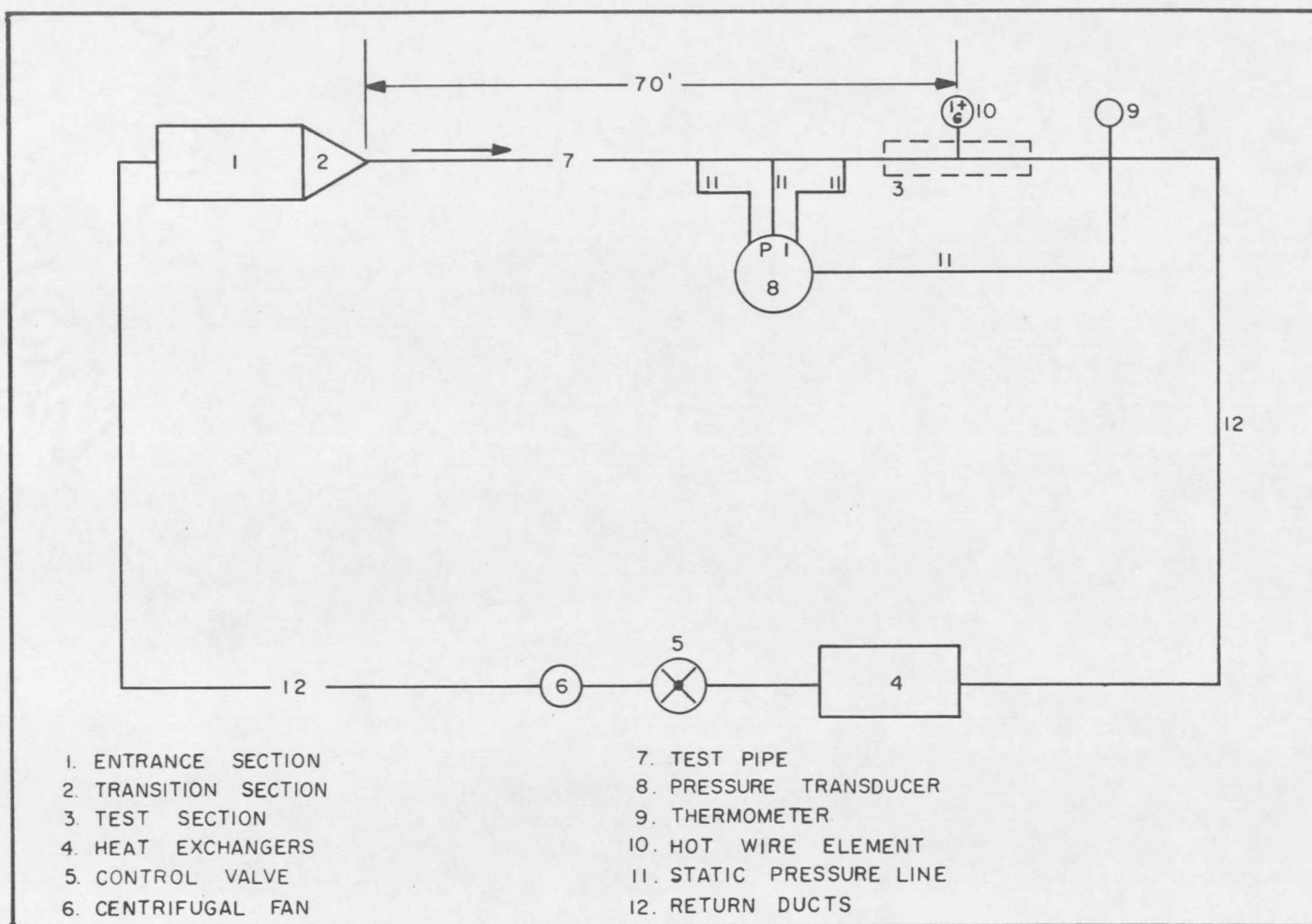


Figure 1. Schematic of Experimental System

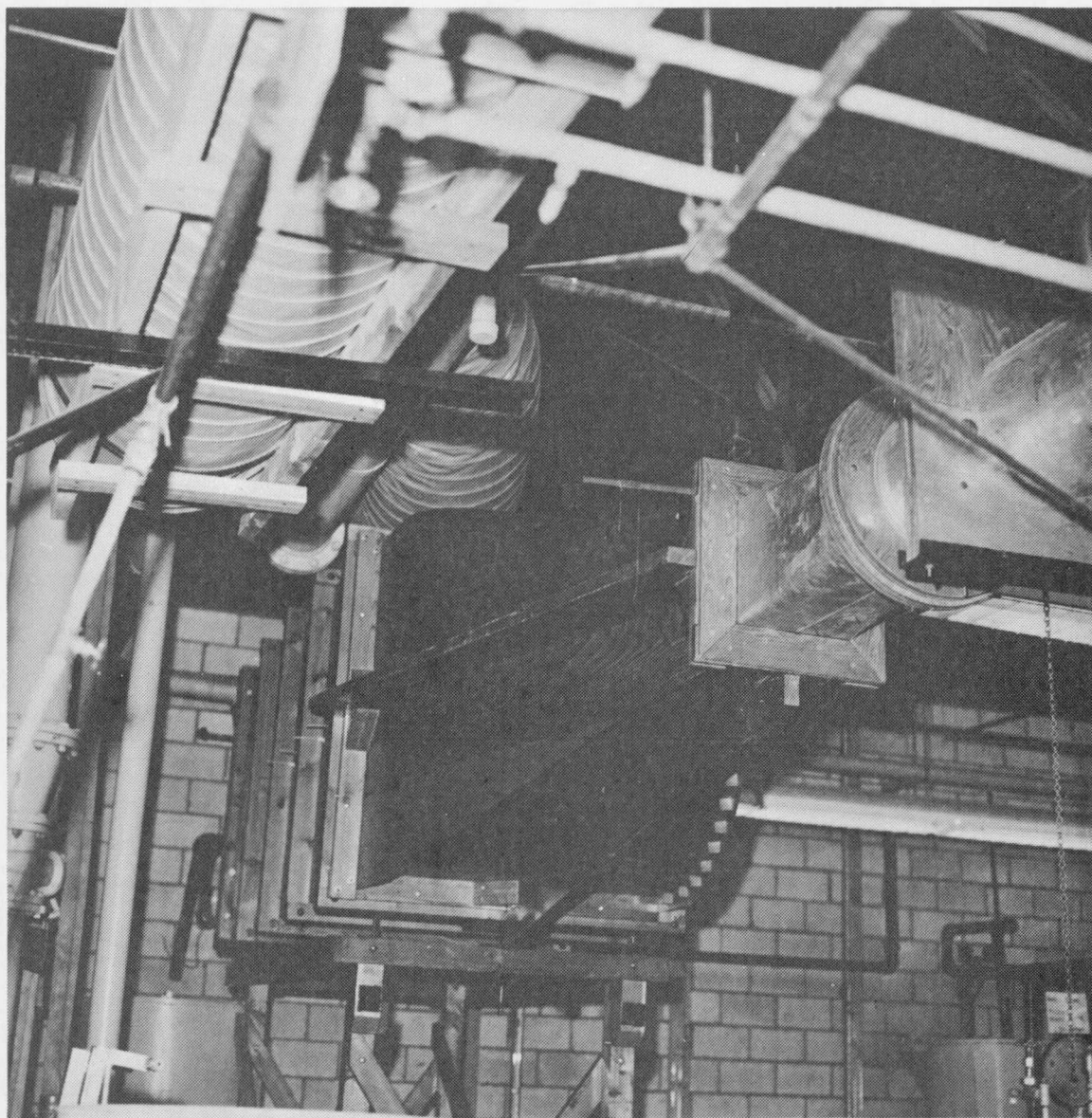


Figure 2. Entrance Section

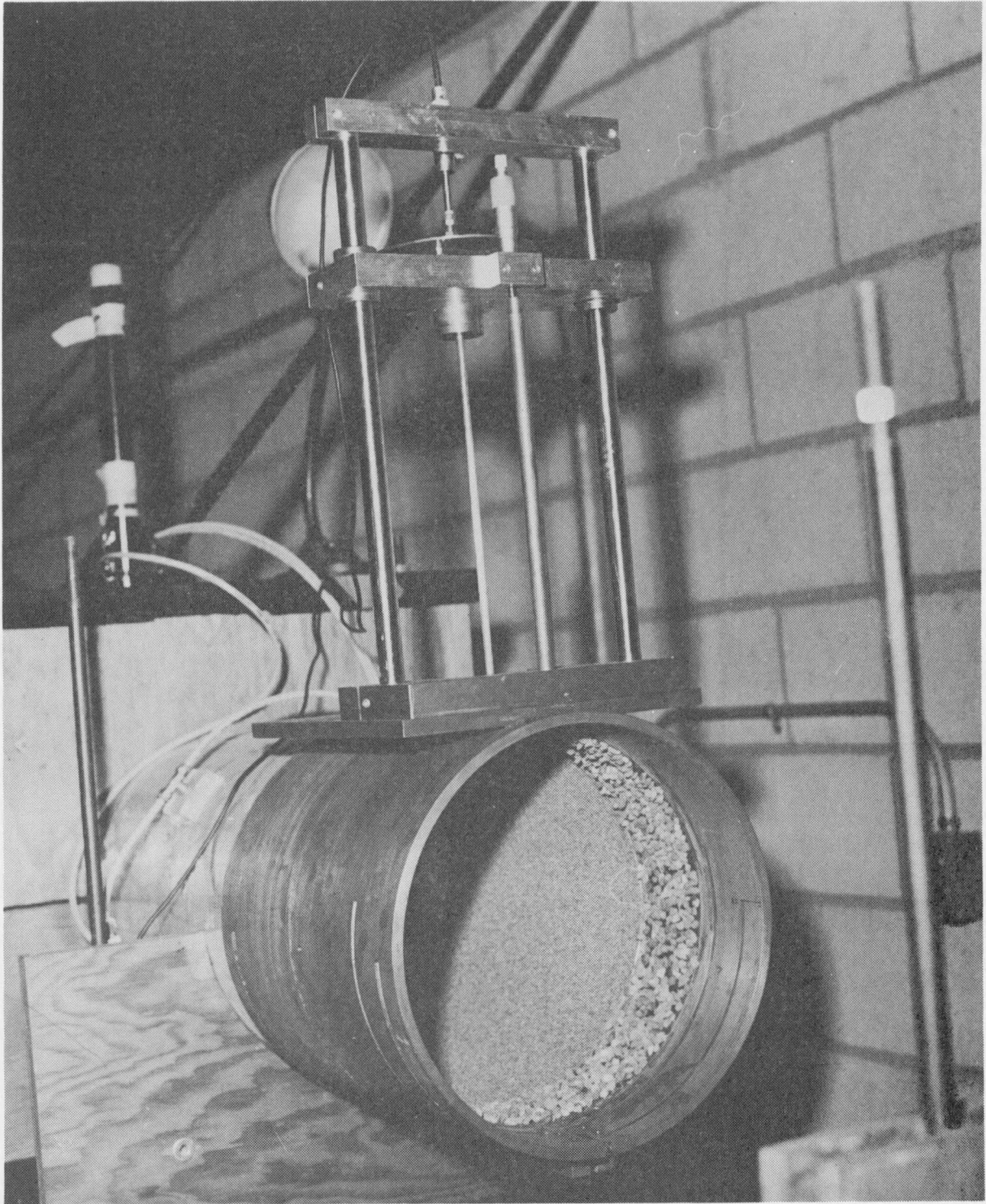


Figure 3. Test Section

as small as 0.001 inch. The probe support is free to rotate in the plane perpendicular to the pipe axis in order that the hot-wire could be positioned in any orientation necessary.

To accurately determine the distance from the wire to the wall during a run, a zero distance was measured by lowering the wire towards a polished spot on the machined surface until the wire and its image coincided.

Static pressure taps were installed in the pipe wall and connected to a taut diaphragm pressure transducer to measure the pressure drop in the pipe. Four taps 90° apart, were installed at each axial location and connected in parallel to one line that led to the transducer. Thus at each location, the static pressure recorded was the average pressure at the four static taps. The dynamic pressure head was determined for each run with a stagnation pitot tube permanently mounted one-foot downstream from the test section. The tube was positioned in the center of the pipe, thus allowing determination of the centerline mean velocity.

All pressure lines were connected to a manifold. An MKS Instruments, Inc. differential pressure transducer, with a range of ± 1.0 mm Hg and an accuracy of $.00005$ mm Hg, was employed to measure pressure differentials between any two lines connected to the manifold. For large Reynolds number runs, the dynamic head and absolute pressure measurements

were made with a Meriam water manometer with a full range of ± 20 inches of water and an accuracy of 0.001 inches water.

Two automobile radiators, through which water was circulated, were used as heat exchangers to maintain relatively constant air temperature during a test. The temperature of the air in the pipe was monitored using a conventional mercury thermometer.

The experimental system was made a closed system by connecting the various equipment with flexible ducts. A closed system was desirable because of adverse affects of dust on hot-wire response and calibration.

All velocity profile measurements were made with Thermo-Systems, Inc. hot-wire equipment. Calibrated constant temperature anemometers and linearizing circuits were coupled to yield voltage signals proportional to velocity. A Hewlett-Packard true RMS voltmeter was employed to measure the root-mean-square of the fluctuating velocity components; while, a Hickock DC digital voltmeter was used for measurement of both rms and mean voltage signals with an accuracy of 0.1%. This accuracy is limited since signal fluctuations hampered visual reading. A TSI correlator with built-in sum and difference circuits was employed to measure cross-correlation components. All probes, Figures (4), (5) and (6), and 0.00015 inch diameter copper-coated tungsten hot wires were supplied by TSI.

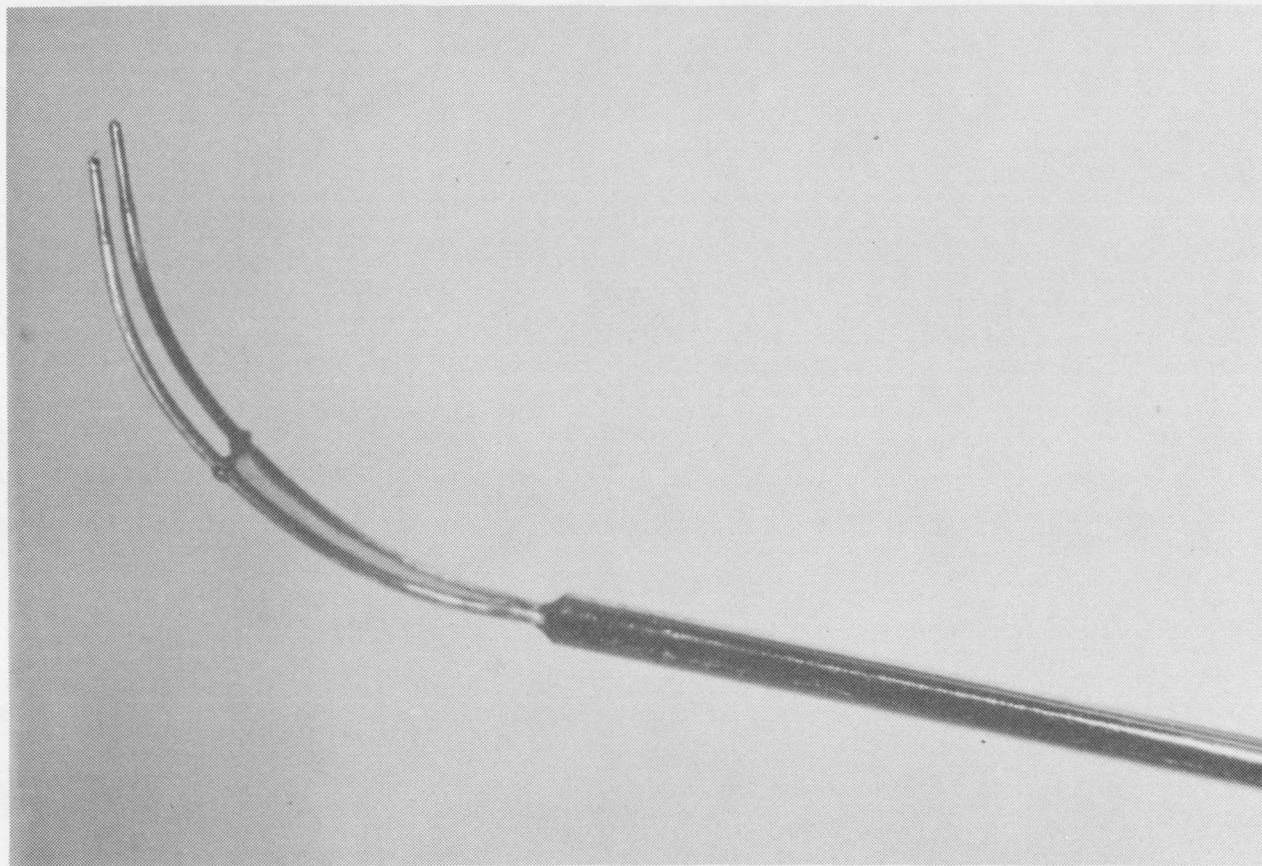


Figure 4. Mean Velocity Boundary Layer Probe

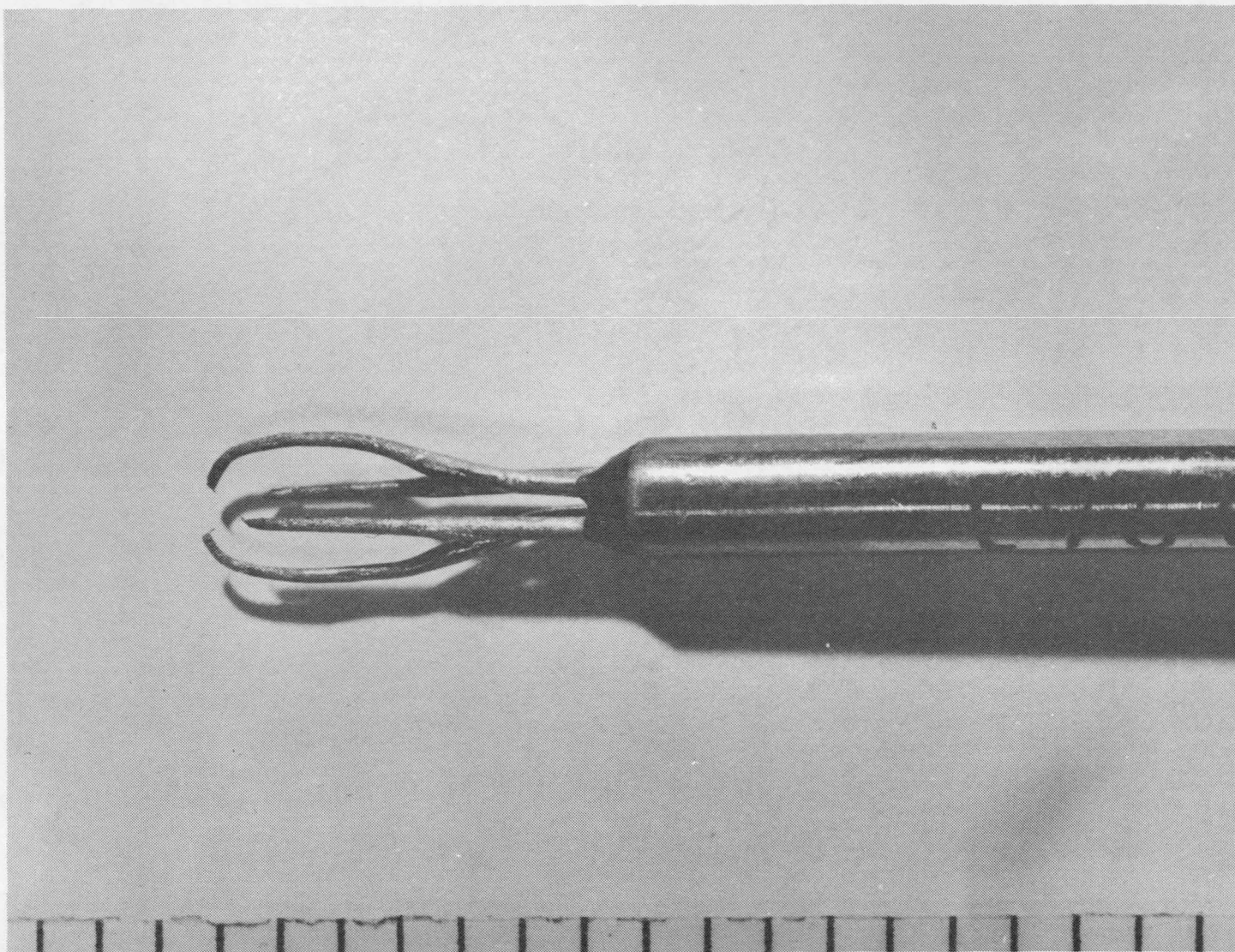


Figure 5. X-Array $\overline{uv}$ Correlation Probe

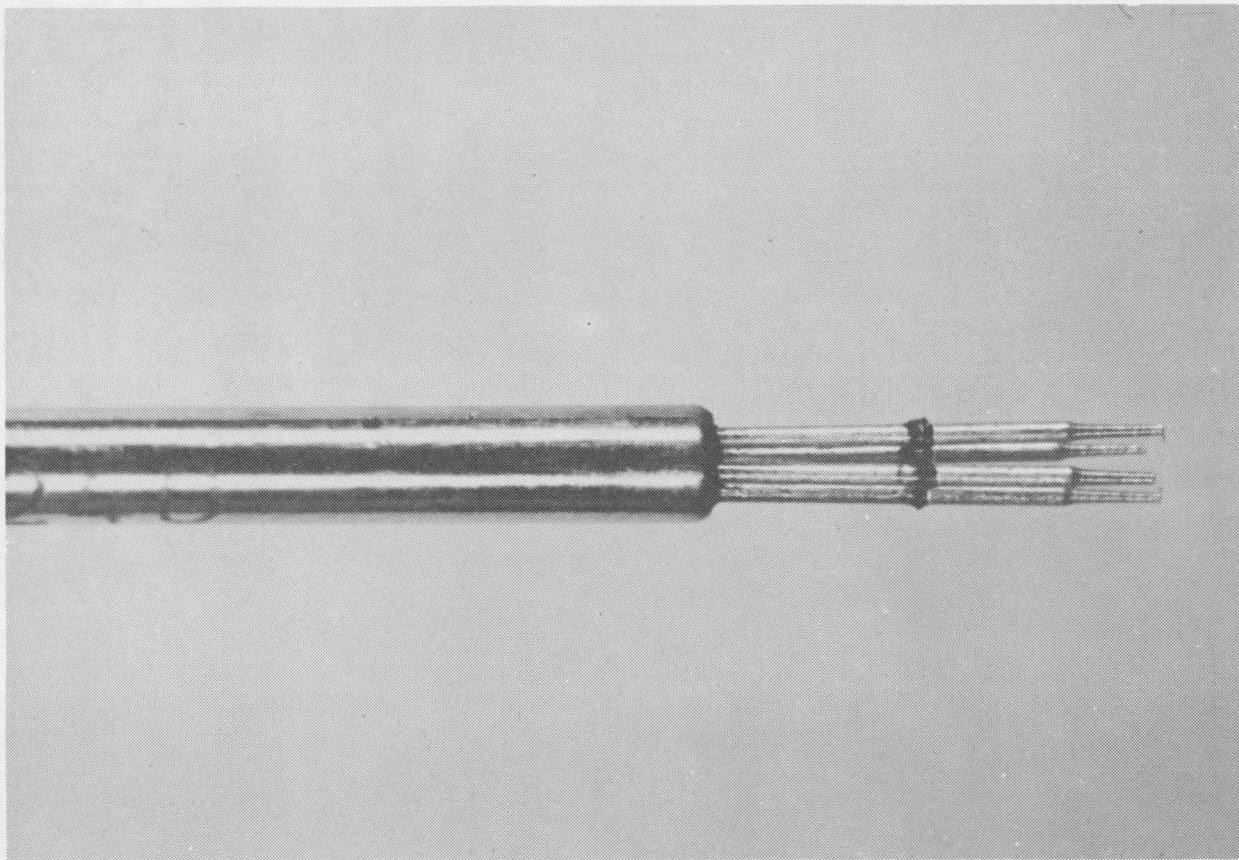


Figure 6. X-Array $\overline{uw}$ Correlation Probe

A complete list of equipment specifications and dimensions is given in the Appendix.

The two roughnesses were obtained by gluing sand on the pipe wall with water-soluble glue. The sand was sifted and scaled with U.S. Standard sieves and a conventional shaker; and washed before gluing to remove dirt and dust. This procedure made possible attainment of a uniform roughness. The sand grain sizes were taken to be the average between two sieve sizes. The smaller roughness, which averaged 0.0286 inch, was attached to the pipe wall by swabbing the entire interior of the pipe with a thin layer of glue, placing the sand in one end, and rotating the pipe until the complete surface area was covered. The larger roughness size was determined from sieve sizes to be 0.225 inch average. Considerably more difficulty was encountered in coating the pipe surface due to the large particle size. The weight of the individual pieces forced gluing to be confined to successive sectors of the surface approximately one-fifth the pipe circumference.

Figures (7) and (8) illustrate the actual individual sand sizes and shapes; while Figures (9) and (10) show the spacing attained with both roughness. Measurements of friction factors indicate both cementing procedures produced a uniform sand surface.

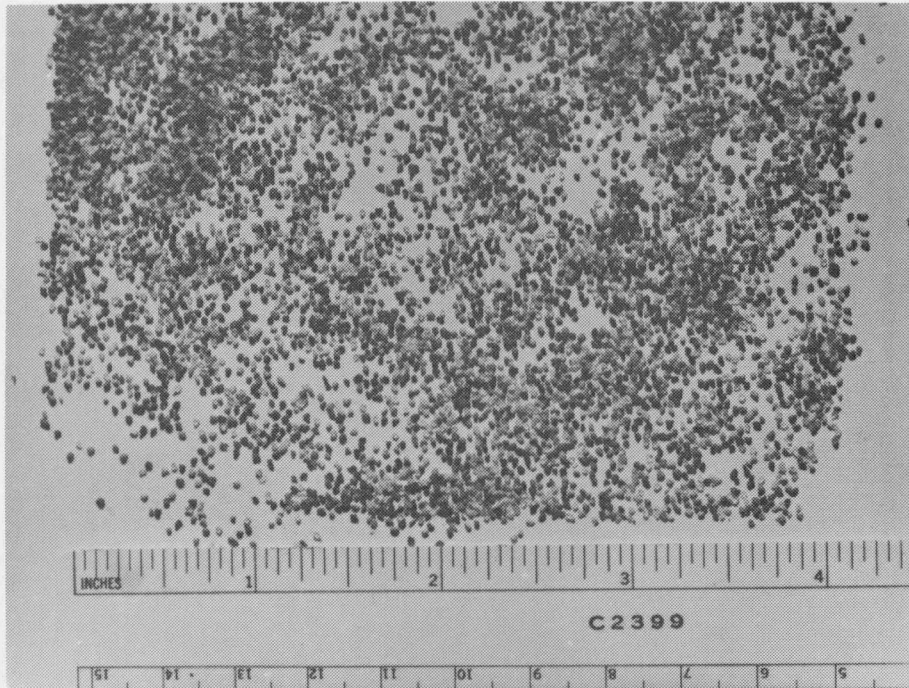


Figure 7. Actual Sand Size and Shape - $R/\epsilon = 208$



Figure 8. Actual Sand Size and Shape - $R/\epsilon = 26.4$

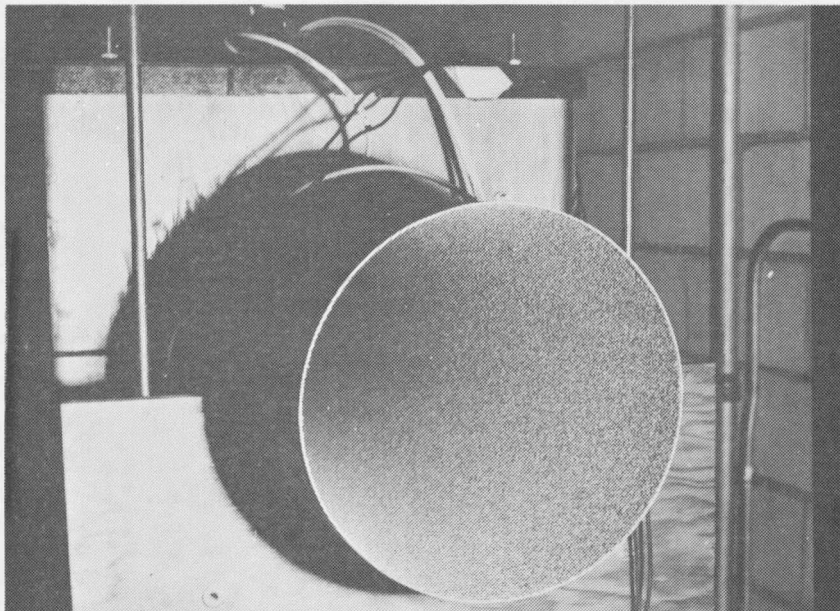


Figure 9. Representative Spacing in $R/\epsilon = 208$
Rough Tube

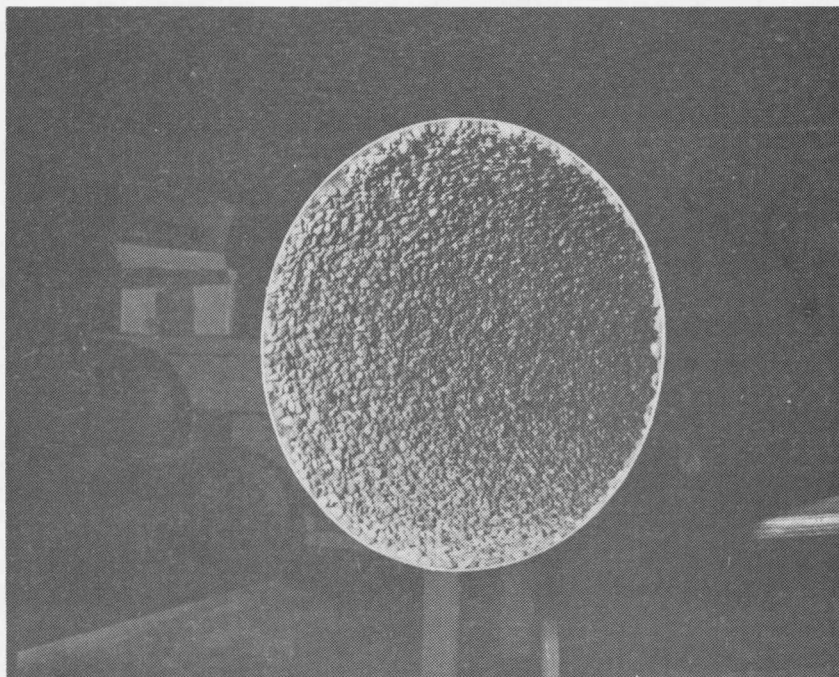


Figure 10. Representative Spacing in $R/\epsilon = 26.4$
Rough Tube

B. Procedures

The application of hot-wire anemometry to turbulent flow measurements has been wide spread and the advantages and limitations are well known. Laufer^{11,12} made all his fluctuating velocity measurements with hot-wires. The theory concerning measurements with hot-wires is not presented here, but reference is made to the series of manuals produced by Thermo Systems, Inc. for the operation of their equipment and to a series of papers presented at the 1967 International Symposium on Hot Wire Anemometry which have been collected and edited by Melnik and Weske.¹⁹ With few exceptions, the operational procedures outlined by TSI were employed during this investigation.

The basic principle of hot-wire anemometry is the determination of the convective heat transfer from a heated wire as the fluid moves past the wire. The amount of heat loss is controlled by the temperature difference between the wire and the surrounding fluid, the fluid density, specific heat, and velocity. The wire is connected to the anemometer such that it comprises one leg of a Wheatstone bridge. As the fluid velocity past the sensor changes, assuming other quantities are constant, the amount of heat loss also changes, thus altering the current needed through the wire to maintain it at a constant temperature. The output

voltage of the anemometer bridge is proportional to the current through the hot-wire.

A linearizing circuit is employed to first square the anemometer output, suppress the voltage signal, and then square the signal again to obtain a voltage signal which is a linear function of the fluid velocity over the calibrated range of the linearizer. At zero fluid velocity, the output of the linearizer must be suppressed to zero as natural convection from the hot-wire, conduction to the hot-wire needle supports, and other minor losses are present and produce a voltage reading even at zero fluid velocity. The power dissipated in a hot-wire can be expressed by King's equation,²¹

$$I^2 R' = A + B \sqrt{U'}$$

where U' is the instantaneous velocity past the hot-wire and A and B are functions of the overheat ratio, the fluid Nusselt number, and the length of the hot-wire. The quantity A is the intersection of the $I^2 R$ versus U' calibration curve at zero velocity. Suppression of the linearized voltage signal subtracts the natural convection and conduction losses actually present at zero velocity. For most hot-wire applications, especially in air, the quantity A is very nearly equal to the actual losses present at zero velocity. Thus after two squaring operations in the linearizer, the effective cooling velocity past the sensor is a linear function of the output

voltage.

The average dc component of the total voltage was measured by time averaging the total linearized signal and reading with a digital voltmeter. The total signal was also fed into a true RMS voltmeter which first squares the fluctuating AC component, time averages, and then takes the square root. The root-mean-square is necessary since the time average of the random fluctuating signal is zero.

For mean velocity profile runs, the probe wire, Figure (4), was orientated perpendicular to the mean flow and the mean and RMS fluctuating voltage readings were recorded at a sufficient number of points on the radius to clearly illustrate the profiles. The centerline mean voltage was divided by the centerline velocity obtained from the dynamic head measurements to determine the calibration constant. The complete velocity profile was thereby determined by multiplying the point mean voltages by this constant. Recalibration of the constant was made at several intervals during a run because of temperature changes of the system air, and electronic drift in the hot-wire equipment. The RMS velocities in the axial direction were obtained by dividing the calibration constant by the point RMS voltages.

To measure the correlation of fluctuating signals from two wires, x-array probes, Figures (5) and (6), are employed.

For the $\overline{uv}$ correlation, the wires are orientated in the Z-R plane as shown in Figure (11), and the TSI correlator was used to find the sum and difference of the signals from both wires. For two such signals e_1 and e_2 , the correlation is the time average of the product of the two. For sensors of equal sensitivity with a configuration in the Z-R plane it can be shown that

$$e_1 = K_1(u+v) \quad (3.1)$$

$$e_2 = K_2(u-v) \quad (3.2)$$

where K is a constant of proportionality. The basic assumptions in deriving equations (3.1) and (3.2) are that the fluctuating components are much smaller than the point mean velocities, and, that the effective cooling velocity past the sensor follows the cosine law.

For wires of equal sensitivity it can be shown that

$$\overline{u^2} = K (\overline{e_1 + e_2})^2 \quad (3.3)$$

$$\overline{v^2} = K (\overline{e_1 - e_2})^2 \quad (3.4)$$

$$\overline{uv} = K^2 (\overline{e_1^2} - \overline{e_2^2}) \quad (3.5)$$

and the normalized correlation coefficient can thus be written

$$\frac{\overline{uv}}{\sqrt{\overline{u^2}} \sqrt{\overline{v^2}}} = \frac{\overline{e_1^2} - \overline{e_2^2}}{\sqrt{(\overline{e_1 + e_2})^2} \sqrt{(\overline{e_1 - e_2})^2}} \quad (3.6)$$

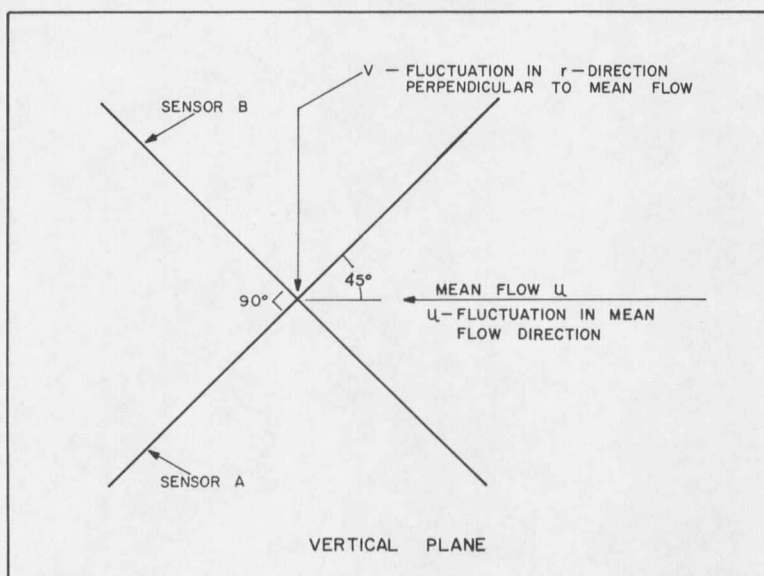


Figure 11. Wire Orientation for $\overline{uv}$ Correlation Measurements

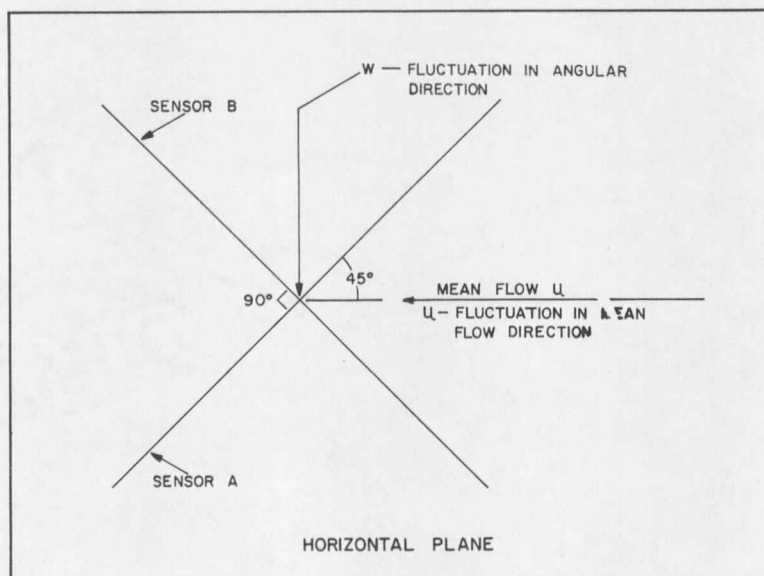


Figure 12. Wire Orientation for $\overline{uw}$ Correlation Measurements

where $\overline{e_1^2}$ and $\overline{e_2^2}$ are the respective RMS values of the fluctuating signals squared. A similar argument can be considered for the normalized $\overline{uw}$ correlation if the wires are orientated as in Figure (12).

To obtain the constant in equations (3.3), (3.4) and (3.5), the mean and RMS fluctuating voltage data were obtained from one of the wires when rotated perpendicular to the mean flow. The wires were returned to the orientation in Figure (11) or (12), as the case may be, and the RMS values of e_1 , e_2 , $e_1 + e_2$ and $e_1 - e_2$, obtained for each point on the mean velocity profile. Evaluation of the axial RMS velocity component, as outlined before, allows the calculation of K from equation (3.3) and $\overline{v^2}$ or $\overline{w^2}$ and the $\overline{uv}$ or $\overline{uw}$ correlations, depending on the probe in use, can be found from equations (3.4) and (3.5).

Since it is very difficult to obtain two wires of equal sensitivity, they were forced to be essentially equal experimentally by subjecting each to identical flow conditions and adjusting the electronic gain on the correlator to obtain equal RMS values from each wire.

Other procedures for evaluating the correlation components are compiled in the TSI correlator manual. However, in most cases additional equipment is required. It will become evident in Chapter V that reliable data were obtained

employing the least amount of equipment.

The average velocity for each run was calculated from a numerical integration of the experimentally determined velocity profile, and friction factors were calculated from the experimental velocity data using the following definition of the friction coefficient, f ,

$$f = \frac{8\tau_0}{\rho U_{avg}^2} = \frac{8U_*^2}{U_{avg}^2} \quad (3.7)$$

However, it was inconceivable to make the number of runs necessary to fully establish the dependency of the friction factor on Reynolds number. Consequently, a method for approximating friction factor values from pressure drop, dynamic head, and temperature data was established.

By definition the average velocity, U_{avg} , is

$$U_{avg} = \frac{Q}{Area} = \frac{\int U d(area)}{Area} \quad (3.8)$$

For smooth tubes the logarithmic equation

$$U^+ = A + B \ln Y^+ \quad (3.9)$$

predicts point mean velocities quite well except at the pipe centerline and near the wall. In rough pipes the equation

$$U^+ = A' + B' \ln y/\epsilon \quad (3.10)$$

is also adequate for flow in the fully-rough regime,

$\epsilon U_* / \nu > 70$, except at the centerline and near the wall. Substitution of equations (3.9) and (3.10) into equation (3.8) yields expressions for the average velocity for smooth and fully-rough flow conditions.

$$\text{Smooth} - U_{avg} = \int_0^R \frac{(A + B \ln Y^+)}{\pi R^2} \quad (3.11)$$

$$\text{Rough} - U_{avg} = \int_0^R \frac{(A' + B' \ln y/\epsilon)}{\pi R^2} \quad (3.12)$$

After integration the equations

$$\text{Smooth} - \frac{U_{avg}}{U_*} = (A - \frac{3B}{2} + B \ln \frac{U_* R}{\nu}) \quad (3.13)$$

$$\text{Rough} - \frac{U_{avg}}{U_*} = (A' - \frac{3B'}{2}) + B' \ln \frac{R}{\epsilon} \quad (3.14)$$

were subtracted from their original counterparts (3.9) and (3.10), to yield identical results.

$$\frac{U}{U_*} = \frac{U_{avg}}{U_*} + (3B/2) + B \ln \frac{Y}{R} \quad (3.15)$$

where B actually represents both B and B'. Employing the definition of the friction factor the equation

$$\frac{U}{U_{avg}} = 1 + \sqrt{f/8} (3B/2 + B \ln y/R) \quad (3.16)$$

was evaluated at the centerline, $y = R$, $U = U_{max}$, and the average velocity is found to be

$$U_{avg} = U_{max} - (3B/2 \cdot U_*) \quad (3.17)$$

The centerline velocity, U_{max} , was calculated using dynamic pressure head measurements, and the density of the air was calculated using the ideal gas law.

Thus the average velocity was obtained from dynamic head, pressure drop, and temperature data. The friction factor was calculated using the results of equations (3.17) and (3.7).

IV. ANALYTICAL CONSIDERATIONS

The statistical theories of Taylor and others were greatly simplified by assuming isotropic and homogeneous turbulence, which implies a zero mean velocity gradient throughout the pipe. This assumption is, of course, not valid for boundary layer flow. However, Laufer's fluctuating velocity data¹² illustrates isotropy at the centerline of the pipe. From the following treatment of the Reynolds equations, it is apparent that centerline isotropy is not a necessary condition, as was shown by Pai.⁸

The Reynolds equations for incompressible flow in the three coordinate directions

$$\begin{aligned} \frac{U \partial U}{\partial z} + \frac{V \partial U}{\partial r} + \frac{W}{r} \frac{\partial U}{\partial \theta} = -\frac{1}{\rho} \frac{\partial P}{\partial z} - \left(\frac{\partial \overline{u^2}}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{uv}) + \frac{1}{r} \frac{\partial}{\partial \theta} \overline{uw} \right) \\ + \nu \nabla^2 U \end{aligned} \quad (4.1)$$

$$\begin{aligned} \frac{U \partial V}{\partial z} + \frac{V \partial V}{\partial r} + \frac{W}{r} \frac{\partial V}{\partial \theta} - \frac{W^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - \left(\frac{\partial \overline{uv}}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{v^2}) \right. \\ \left. + \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{vw}) - \frac{\overline{w^2}}{r} \right) + \nu \left(\nabla^2 V - \frac{V}{r^2} - \frac{2}{r^2} \frac{\partial W}{\partial \theta} \right) \end{aligned} \quad (4.2)$$

and

$$\begin{aligned} \frac{U \partial W}{\partial z} + \frac{V \partial W}{\partial r} + \frac{W}{r} \frac{\partial W}{\partial \theta} + \frac{VW}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial \theta} - \left(\frac{\partial}{\partial z} \overline{uw} + \frac{\partial}{\partial r} (\overline{vw}) \right. \\ \left. + \frac{1}{r} \frac{\partial}{\partial \theta} (\overline{w^2}) - \frac{2 \overline{vw}}{r} \right) + \nu \left(\nabla^2 W + \frac{2}{r^2} \frac{\partial W}{\partial \theta} - \frac{W}{r^2} \right) \end{aligned} \quad (4.3)$$

are obtained from the Navier-Stokes equations by introducing the fluctuating velocity terms and time averaging each equation. For axially symmetric fully-developed pipe flow, the mean velocity components V and W are zero, all partial derivatives with respect to θ are zero because of symmetry, and all velocity terms are independent of the axial coordinate z .

The axial mean velocity component is then a function of radial position only, and the equations reduce to

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{1}{r} \frac{d}{dr} (r \overline{uv}) + \nu \left(\frac{d^2 \overline{U}}{dr^2} + \frac{1}{r} \frac{d\overline{U}}{dr} \right) \quad (4.4)$$

$$\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{1}{r} \frac{d}{dr} (r \overline{v^2}) + \frac{\overline{w^2}}{r} \quad (4.5)$$

$$\frac{d}{dr} (\overline{vw}) + \frac{\overline{vw}}{r} = 0 \quad (4.6)$$

At the centerline of the pipe, $r = 0$, the pressure gradient in the r direction must be zero; therefore, upon differentiation of equation (4.5) it is immediately seen that

$$\overline{v^2} + r \frac{d\overline{v^2}}{dr} = \overline{w^2} \quad (4.7)$$

or

$$\overline{v^2} = \overline{w^2} \quad (4.8)$$

It cannot be shown that the axial component, $\overline{u^2}$, need equal $\overline{v^2}$ and $\overline{w^2}$ at the centerline.

Integrating equation (4.6) and evaluating at the pipe wall where $\overline{vw}$ is zero, it follows that $\overline{vw}$ is zero for all values of r .

Rearranging equation (4.4),

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{1}{r} \frac{d}{dr}(r\overline{uv}) + \frac{v}{r} \frac{d}{dr}\left(r \frac{dU}{dr}\right) \quad (4.9)$$

or

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = -\frac{1}{r} \frac{d}{dr} r(\overline{uv} - v \frac{dU}{dr}) \quad , \quad (4.10)$$

and noting that the pressure drop per unit length is constant in fully developed flow, equation (4.10) can be integrated to yield

$$\frac{r^2}{2\rho} \frac{\partial P}{\partial z} = - (r\overline{uv}) + r v \frac{dU}{dr} + C_1 \quad . \quad (4.11)$$

At the pipe centerline, $r = 0$, therefore, $C_1 = 0$ and

$$\frac{r}{2\rho} \frac{\partial P}{\partial z} = - \overline{uv} + v \frac{dU}{dr} \quad . \quad (4.12)$$

At the pipe wall, $r = R$ and $\overline{uv} = 0$, so

$$\left. \frac{v dU}{dr} \right|_{r=R} = \frac{R}{2\rho} \frac{dP}{dz}$$

and

$$\frac{R}{2\rho} \frac{dP}{dz} = \frac{\tau_o}{\rho} = U_*^2 \quad . \quad (4.13)$$

Equation (4.11) is finally becomes

$$\overline{uv} = v \frac{dU}{dr} + \frac{r}{R} U_*^2, \quad (4.14)$$

which is the equation originally derived by Laufer.¹² The $\overline{uv}$ correlation can be obtained directly from mean velocity and pressure drop data.

However, starting with equation (4.12)

$$\frac{dU}{dr} = \frac{r}{2v\rho} \frac{dP}{dz} + \frac{\overline{uv}}{v} \quad (4.12)$$

and integrating

$$U = \frac{1}{4v\rho} \frac{dP}{dz} r^2 \Big|_R^r + \int_R^r \frac{\overline{uv}}{v} dr \quad (4.15)$$

or

$$U = \frac{1}{4v\rho} \frac{dP}{dz} (r^1 - R^2) + \int_R^r \frac{\overline{uv}}{v} dr \quad (4.16)$$

Equation (4.16), originally derived by Pai,⁸ is an expression for the mean velocity in either turbulent or laminar flow; because, if $\overline{uv}$ were zero, as would be the case in laminar flow, (4.16) reduces to the well known Poiseuille equation for laminar flow.

Equations (4.14) and (4.16) have never been extensively evaluated for turbulent flow in rough tubes because of the limited amount of experimental data available. The data of this investigation, Chapter V, indicates equation (4.14) is not valid for flow in rough tubes.

V. EXPERIMENTAL RESULTS

The range of Reynolds numbers examined during this investigation was 30,000 to 500,000. The upper limit was established by the physical capacity of the experimental system; whereas, at Reynolds numbers less than approximately 30,000, fluctuations about the reduced magnitudes of the linearized voltage and static pressure drop signals made manual data recording near impossible and erroneous. In the smooth tube and the $R/\epsilon = 208$ rough tube the Reynolds number range examined was between 50,000 and 480,000. Reynolds numbers as low as 30,000 were investigated in the rough tube with $R/\epsilon = 26.4$, since the large wall roughness produced measurable static pressure drops. However the upper Reynolds number limit was approximately 250,000. At flow rates larger than $Re = 250,000$, roughness particles were dislodged from the wall and carried downstream by the air flow. Should one of these sand grains strike a hot-wire probe or the statnation pilot tube, permanent and costly damage would result.

A. Data Reduction

The reduction of experimental data was accomplished by a digital computer program which was written to convert linearized voltage, pressure, and temperature data to velocity and shear stress data, and to calculate the properties of the

system air during each run. To ascertain the consistency of the reduced data, subroutines for least-squares curve-fitting and error analysis were included in the computer program. A correction for temperatures of the mercury manometers and compensations for radiation losses from the hot-wire, from Wills,²¹ were incorporated in the reduction program to reduce these sources of error.

The computer output was punched into card decks to facilitate plotting, which was accomplished with an X-Y analog plotter coupled to a Hewlett Packard 2116-A digital computer. The velocity and shear stress data were plotted in several ways, to obtain what appears to be the most meaningful representation of the data in so far as physical significance and comparison with other investigators is concerned. Many of these plots are presented in this chapter, and their physical significance is discussed in a later section.

B. Mean Velocity Distribution

Figures 13 through 19 illustrate the mean velocity distributions for the three degrees of relative roughness -- smooth, $R/\epsilon = 208$, and $R/\epsilon = 26.4$. Figure 13 represents smooth tube flow at a Reynolds number of 175,000. Figures 14, 15 and 16 illustrate the mean velocity distribution at Reynolds numbers of 52,000, 170,000, and 480,000 respectively

for the $R/\epsilon = 208$ boundary roughness. Figures 17, 18 and 19 represent the distribution at 34,500, 75,000, and 154,000 Reynolds numbers respectively for the $R/\epsilon = 26.4$ roughness.

Figure 20 is a plot of U^+ versus $\log Y^+$ in the smooth tube representative of the Reynolds numbers investigated. All smooth pipe data for $Y^+ > 70$, lie within $\pm 10\%$ of the straight line equation

$$U^+ = 4.55 + 2.75 \ln Y^+ \quad (5.1)$$

The introduction of surface roughness produced different mean velocity profiles. Figures 21 and 22 illustrate the velocity profile, U^+ versus $\log Y^+$, for flow in the roughened tubes at the Reynolds numbers indicated on each plot. The profiles shown in Figure 21 represent the transition region as defined by Nikurade's rough tube flow regimes, although the roughness parameter for the 52,000 Reynolds number is 6.7 which approaches the hydraulically smooth region. The curves of Figure 22 all represent fully-rough flow conditions.

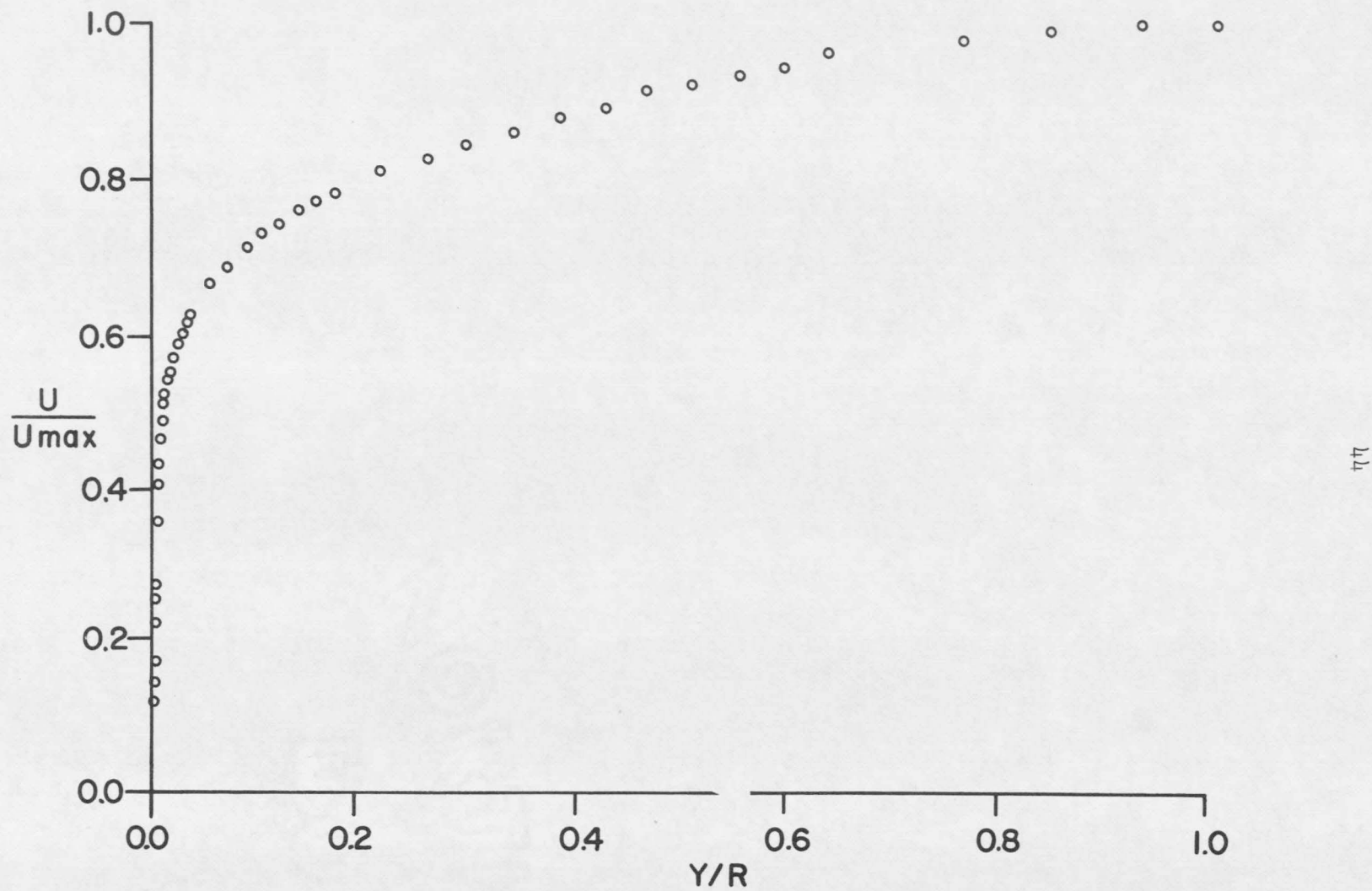


Figure 13. U/U_{max} vs Y/R - Profile for Smooth Pipe at $Re = 175,000$

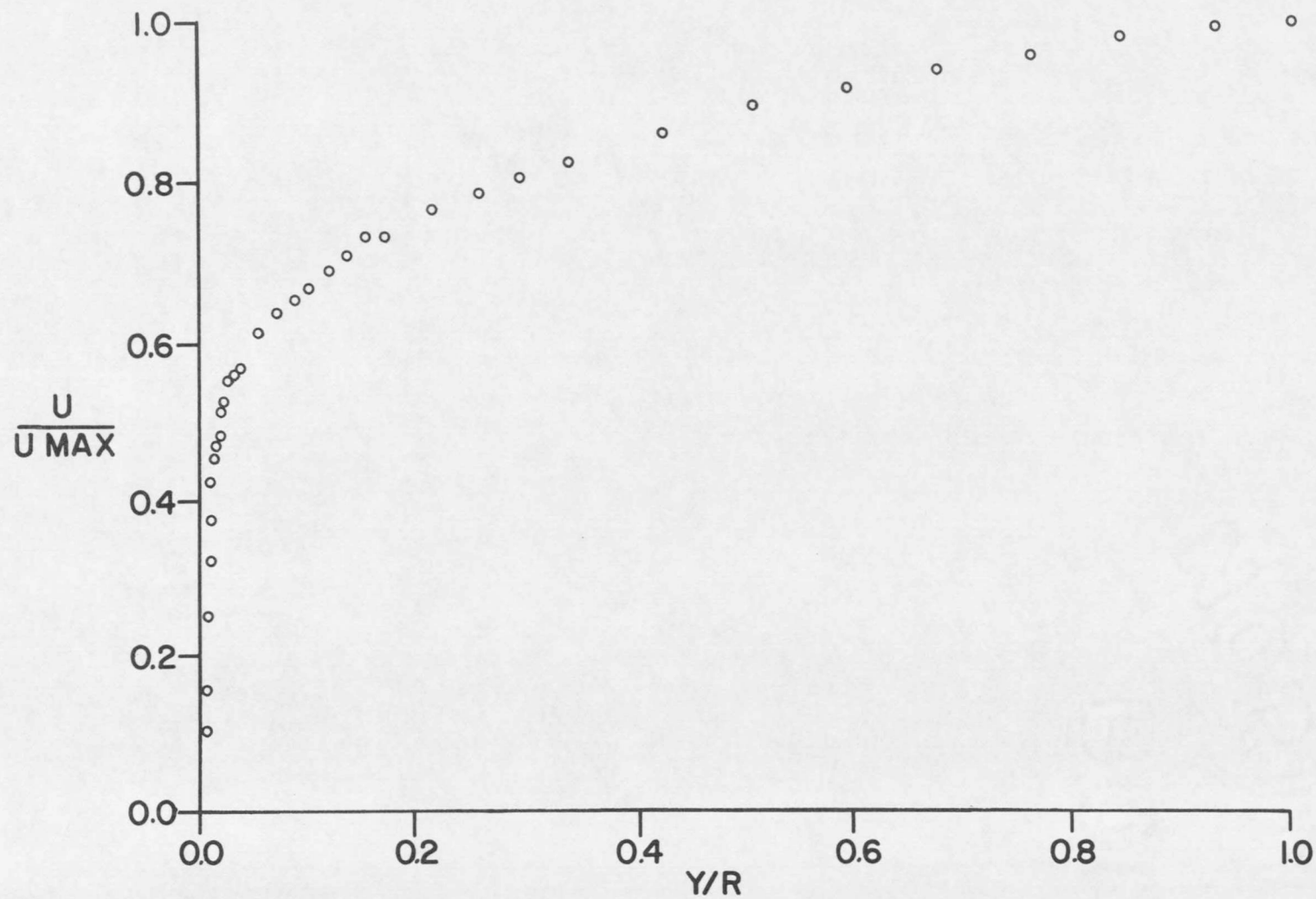


Figure 14. U/U_{max} vs Y/R - Profile for $R/\epsilon = 208$
Pipe at $Re = 52,000$

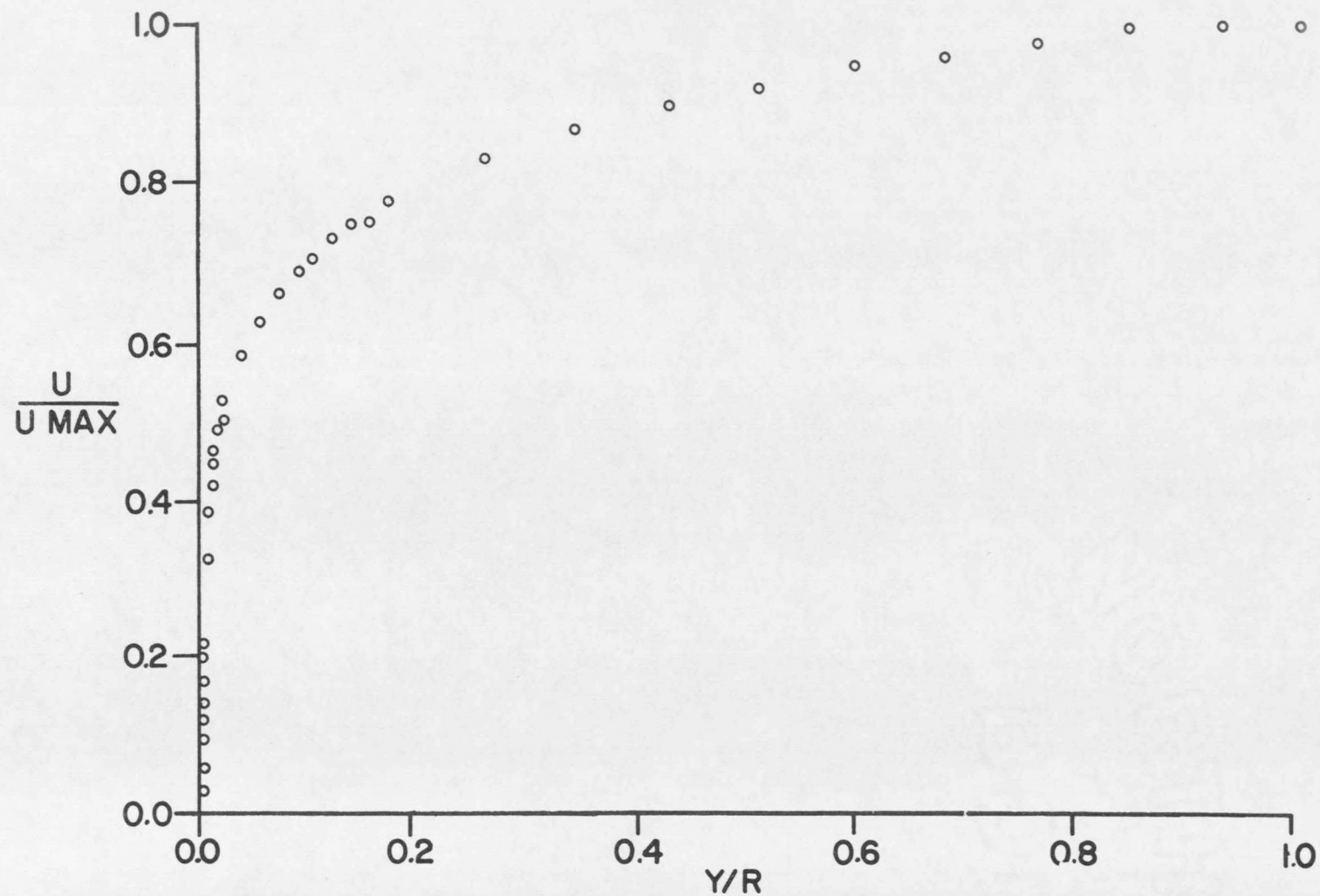


Figure 15. U/U_{max} vs Y/R - Profile for $R/\epsilon = 208$
Pipe at $Re = 170,000$

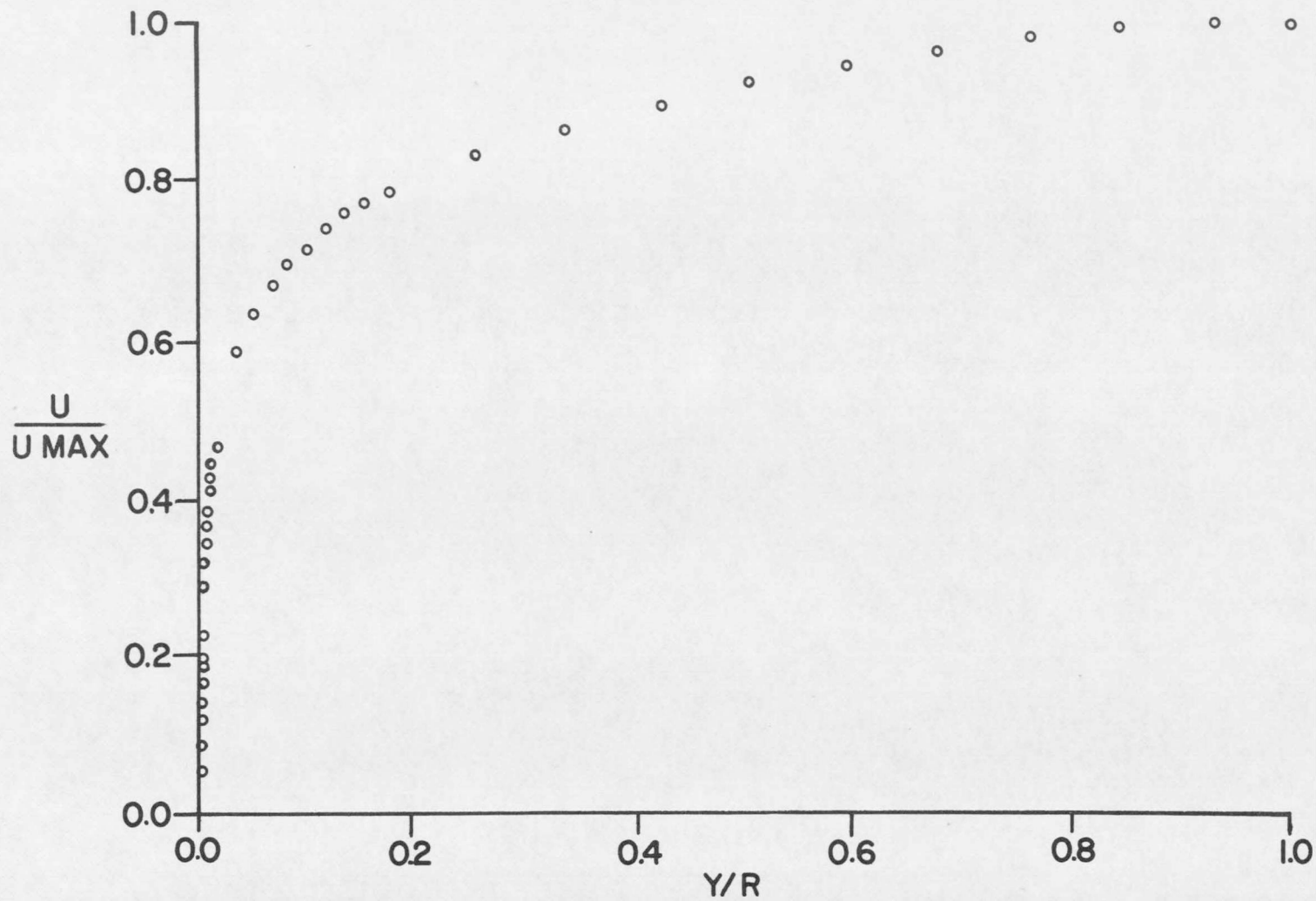


Figure 16. U/U_{max} vs Y/R - Profile for $R/\epsilon = 208$
Pipe at $Re = 480,000$

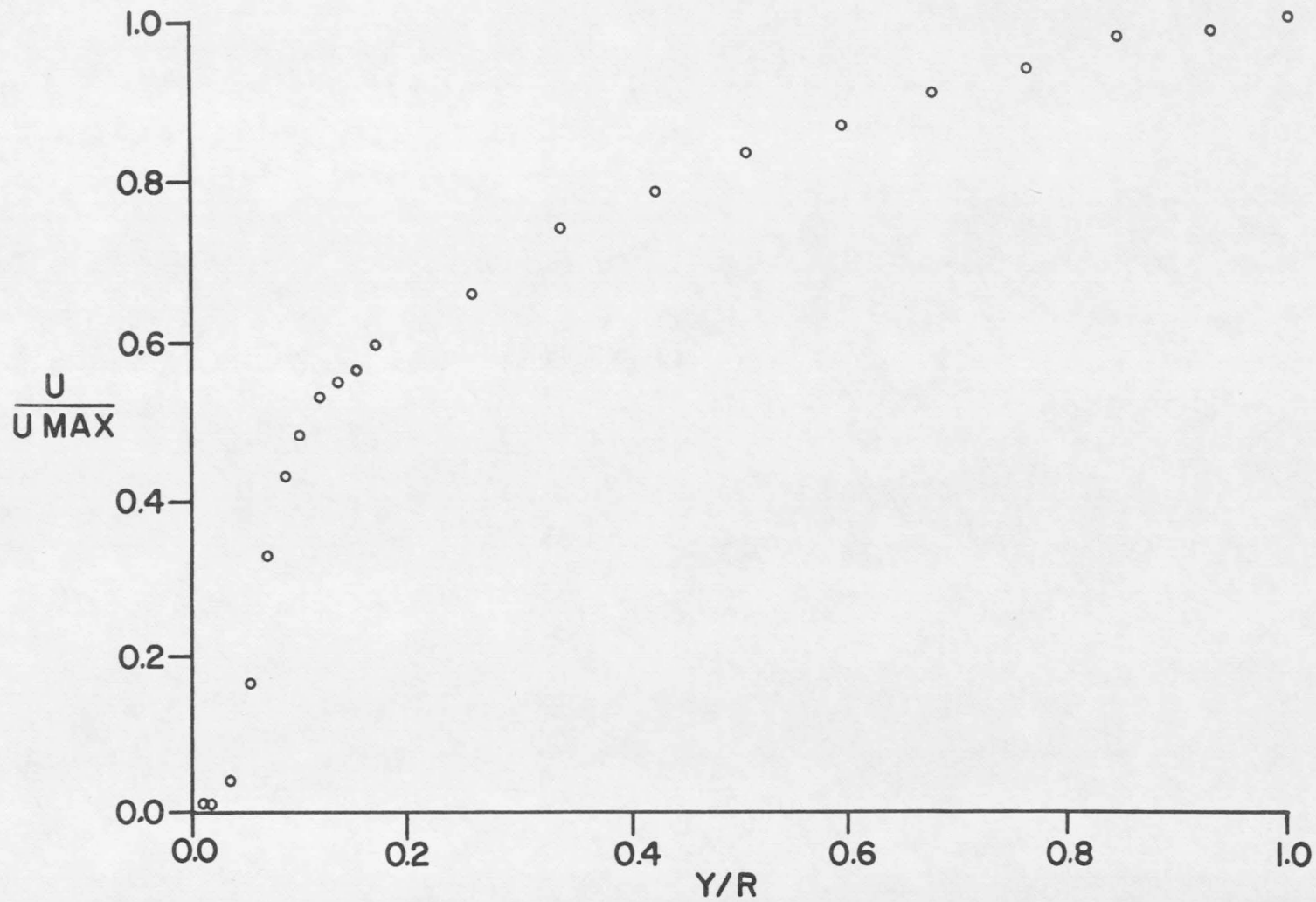


Figure 17. $U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 26.4$
Pipe at $Re = 34,500$

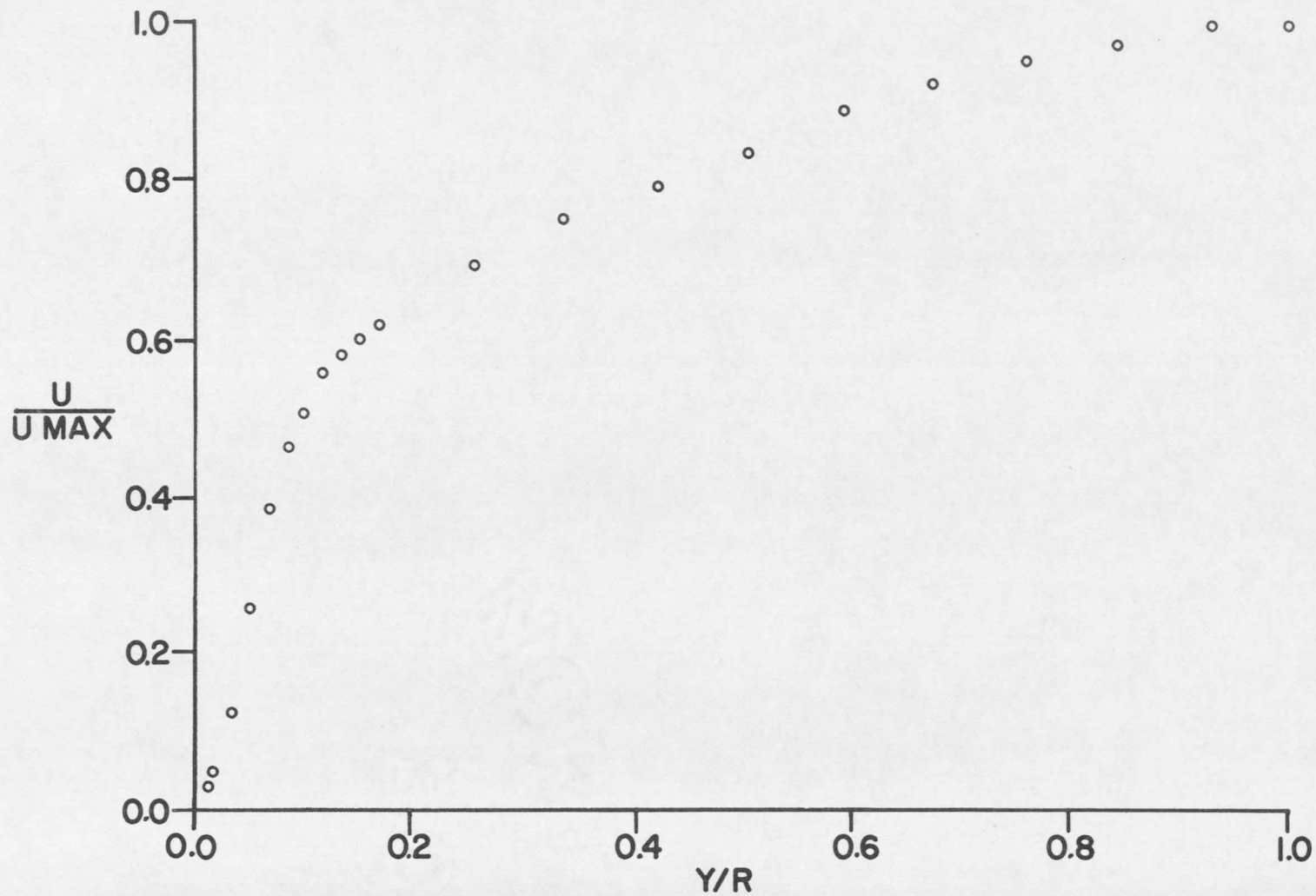


Figure 18. $U/U_{\max}$ vs Y/R - Profile for $R/\epsilon = 26.4$
Pipe at $Re = 75,000$

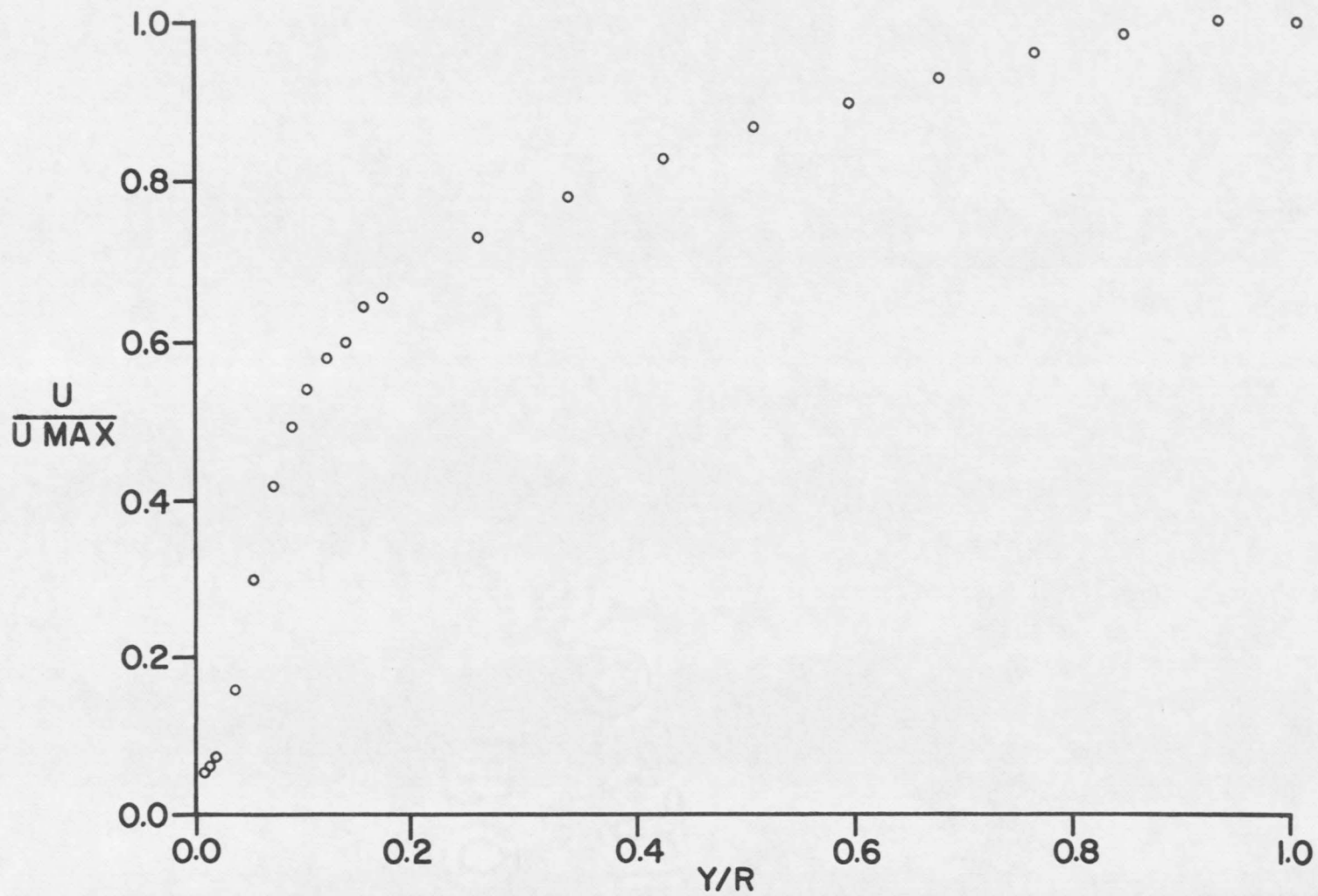


Figure 19. U/U_{max} vs Y/R - Profile for $R/\epsilon = 26.4$
Pipe at $Re = 170,000$

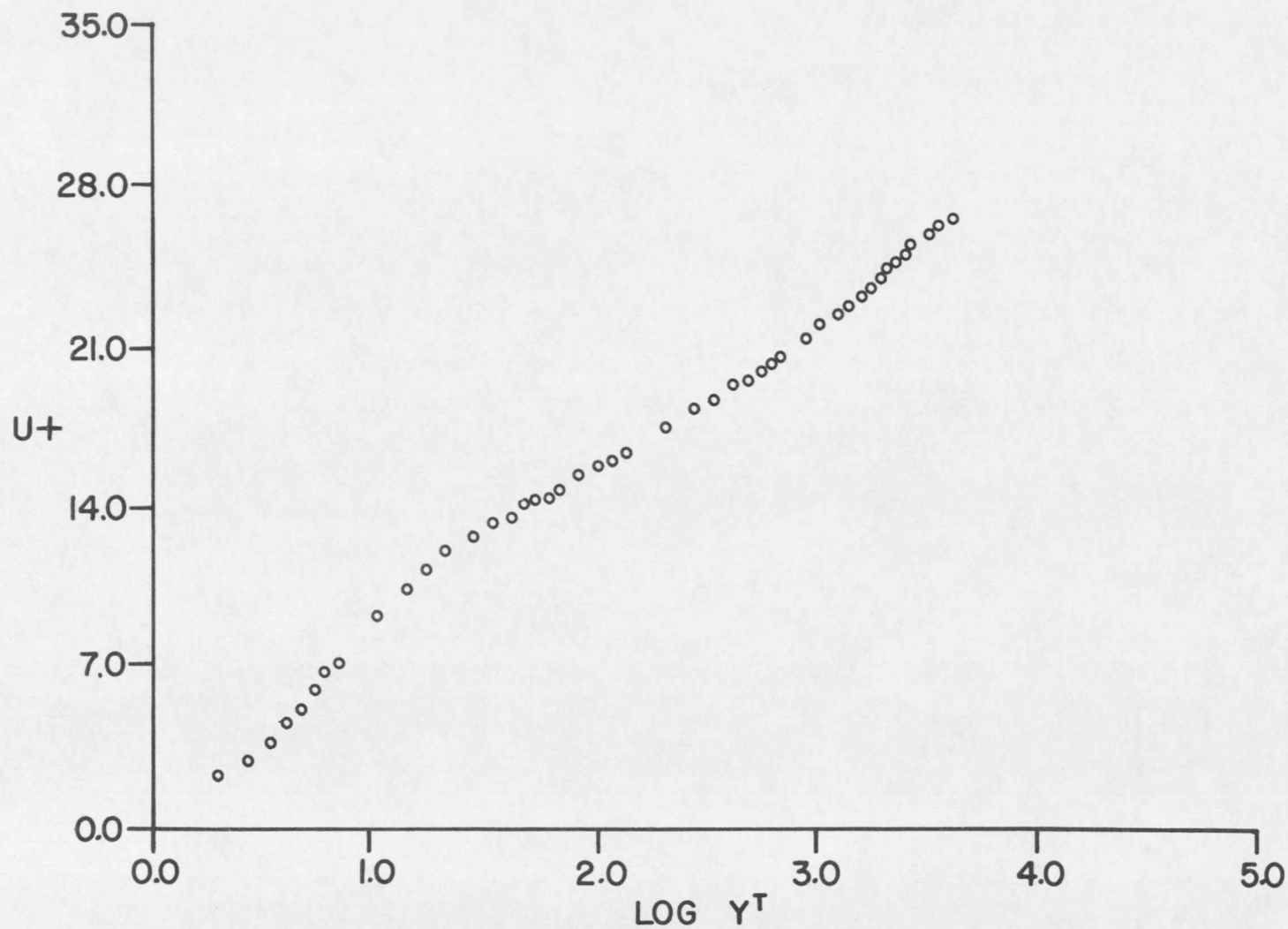


Figure 20. U^+ vs $\text{Log } Y^+$ - Profile for Smooth Pipe

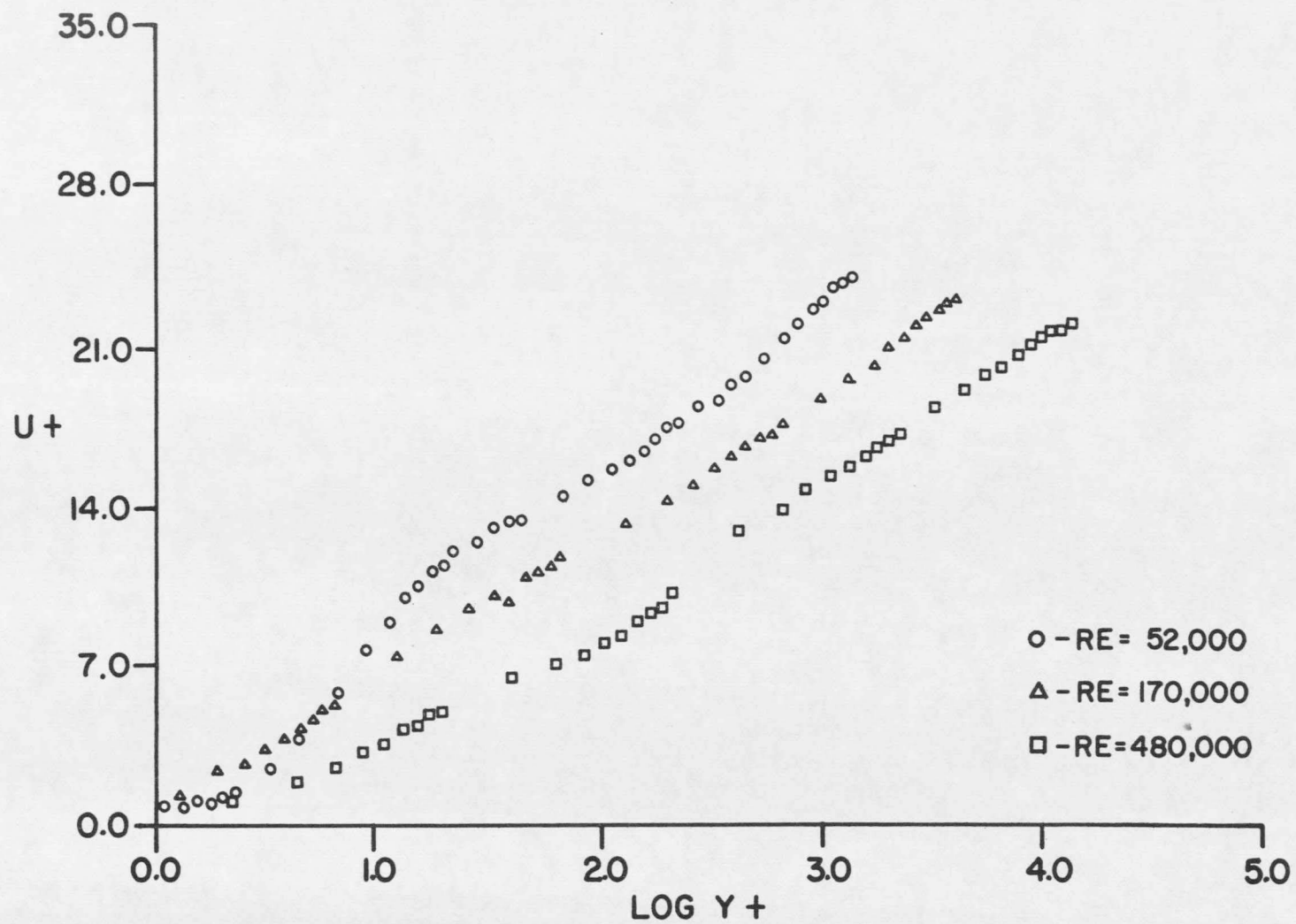


Figure 21. U^+ vs $\log Y^+$ - Profile for $R/\epsilon = 208$
 Pipe

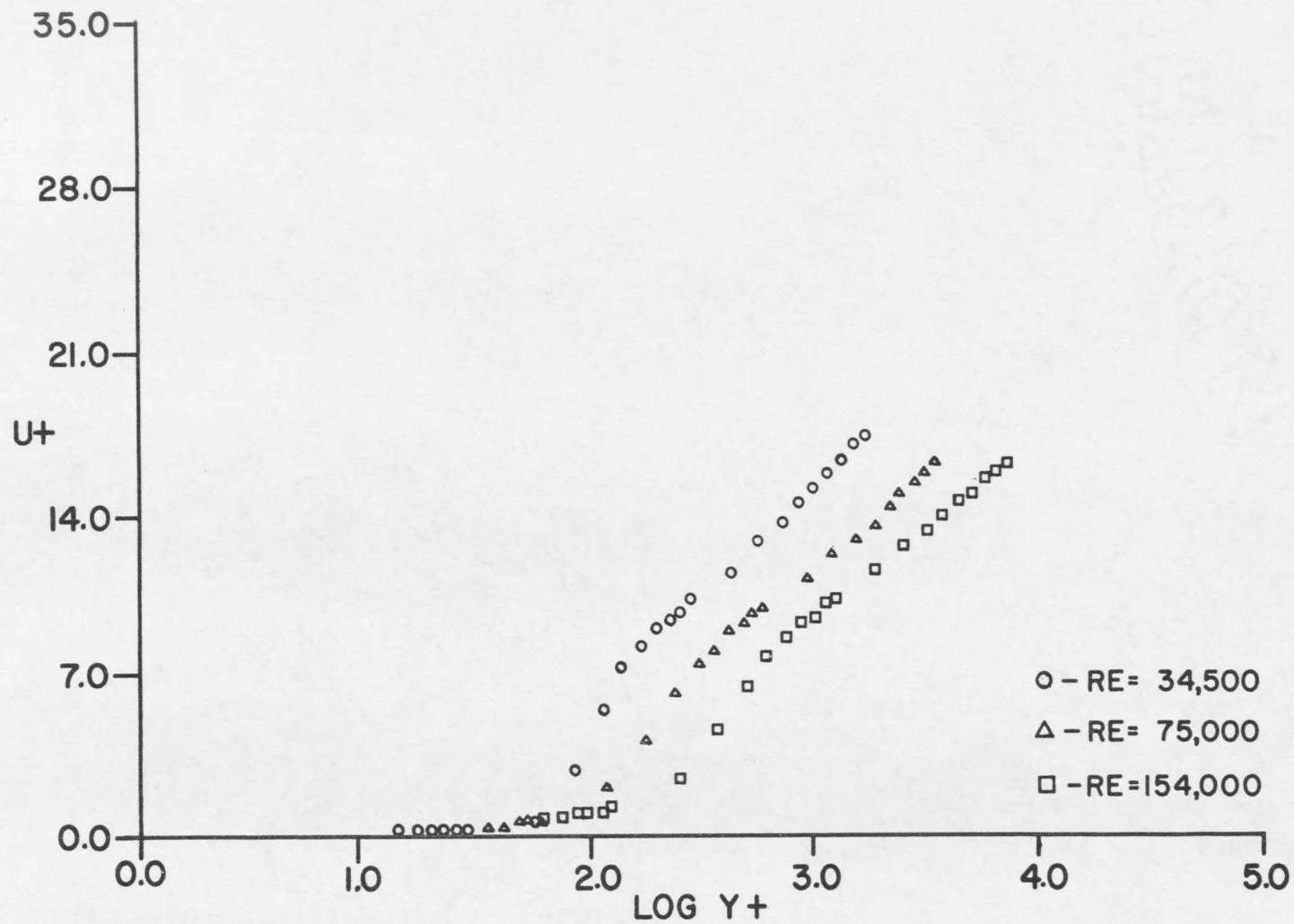


Figure 22. U^+ vs $\log Y^+$ - Profile for $R/\epsilon = 26.4$
 Pipe

C. Fluctuating Velocity Correlations

As shown in equations (4.14) and (4.16), the $\overline{uv}$ correlation is of prime importance. Employing the procedure outlined in Chapter III, the $\overline{uv}$ correlation was measured in both smooth and rough pipes. Figure 23 illustrates the experimental values of $\overline{uv}/U_*^2$ for the smooth tube, compared with the predicted values from equation (4.14). Similar measurements in rough tubes exhibit a different nature as indicated by Figures 24 and 25. The $\overline{uv}/U_*^2$ correlation decreases with increasing Reynolds numbers for both degrees of roughness examined. However, the predicted values from equation (4.14) are essentially independent of Reynolds number and wall roughness and are nearly identical to the predicted values for the smooth tube. The solid curves in Figures 23, 24 and 25 represent the predicted values for $\overline{uv}/U_*^2$ for all the Reynolds numbers indicated on each plot.

As a matter of completeness, the $\overline{uw}$ correlation was also examined. The $\overline{uw}/U_*^2$ parameter was experimentally found to be nearly zero for the smooth and $R/\epsilon = 208$ rough tube as indicated by Figures 26 and 27. The apparent increase in $\overline{uw}/U_*^2$ in Figure 26 at y/R values less than 0.1 is misleading since absolute values were plotted and the points actually represent both positive and negative measurements. Figure 28 illustrates a non-zero $\overline{uw}$ correlation for the $R/\epsilon = 26.4$ rough

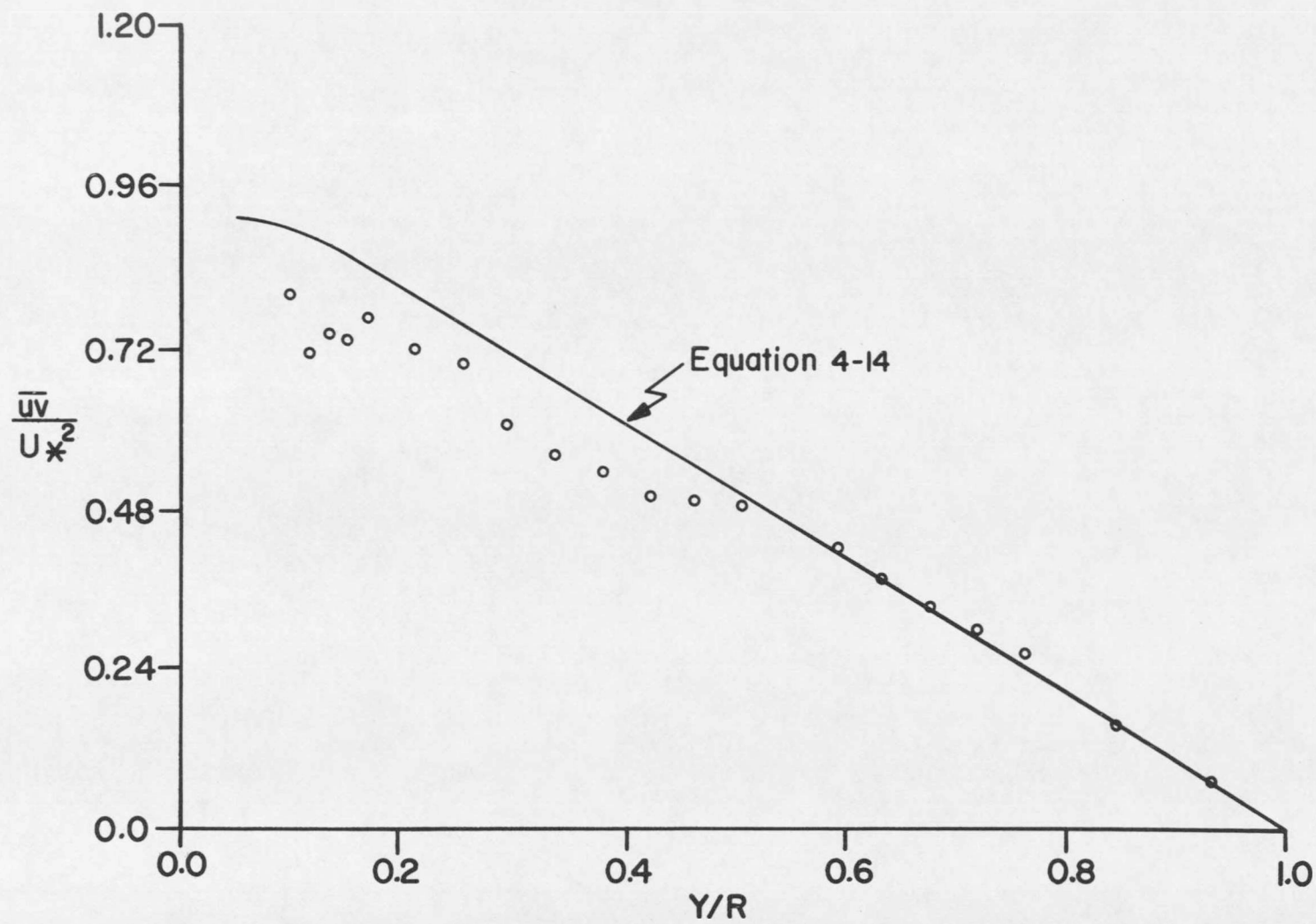


Figure 23. $\overline{uv}/U_*^2$ vs Y/R for Smooth Pine

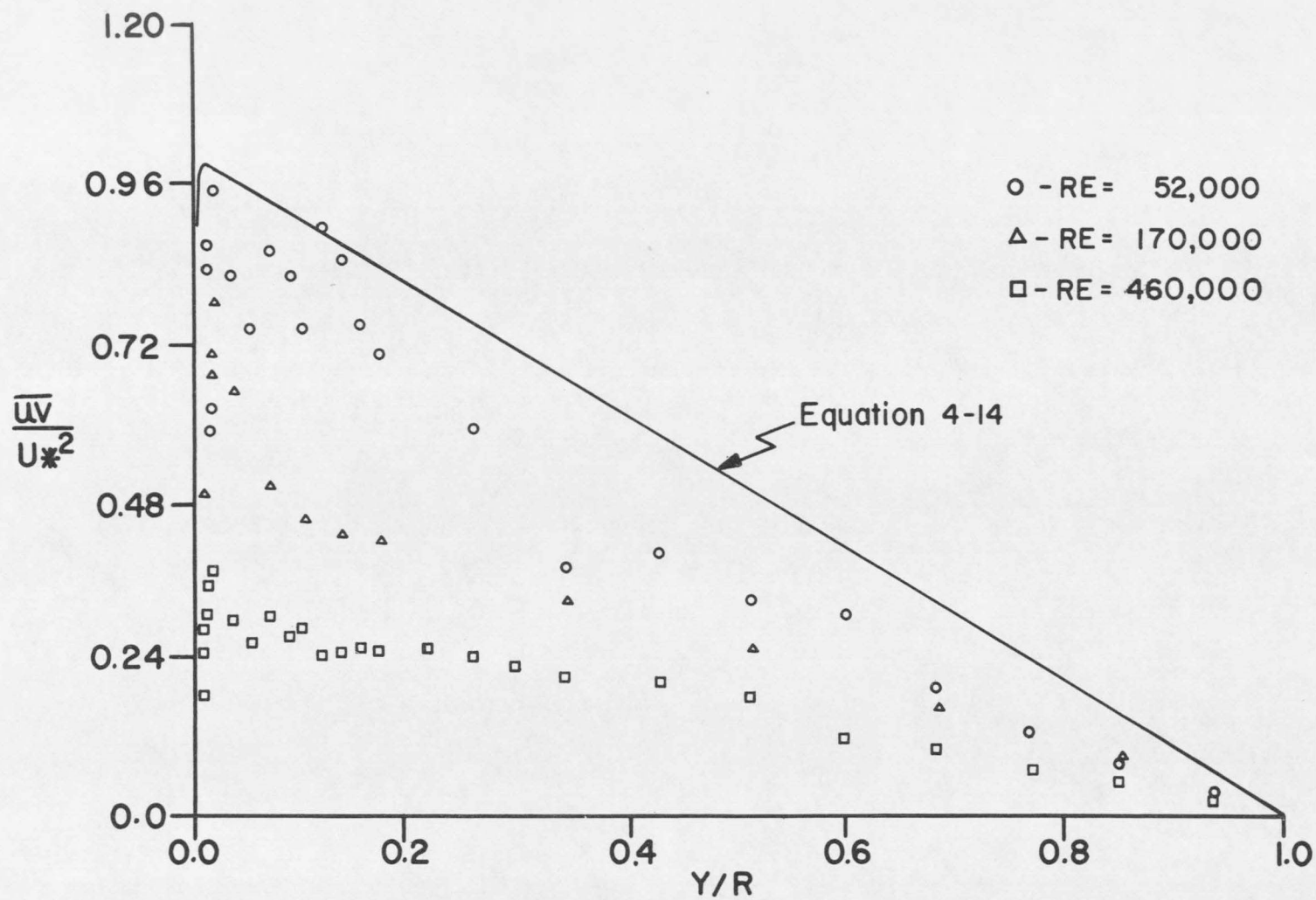


Figure 24. $\overline{uv}/U_*^2$ vs Y/R for $R/\epsilon = 208$ Pipe

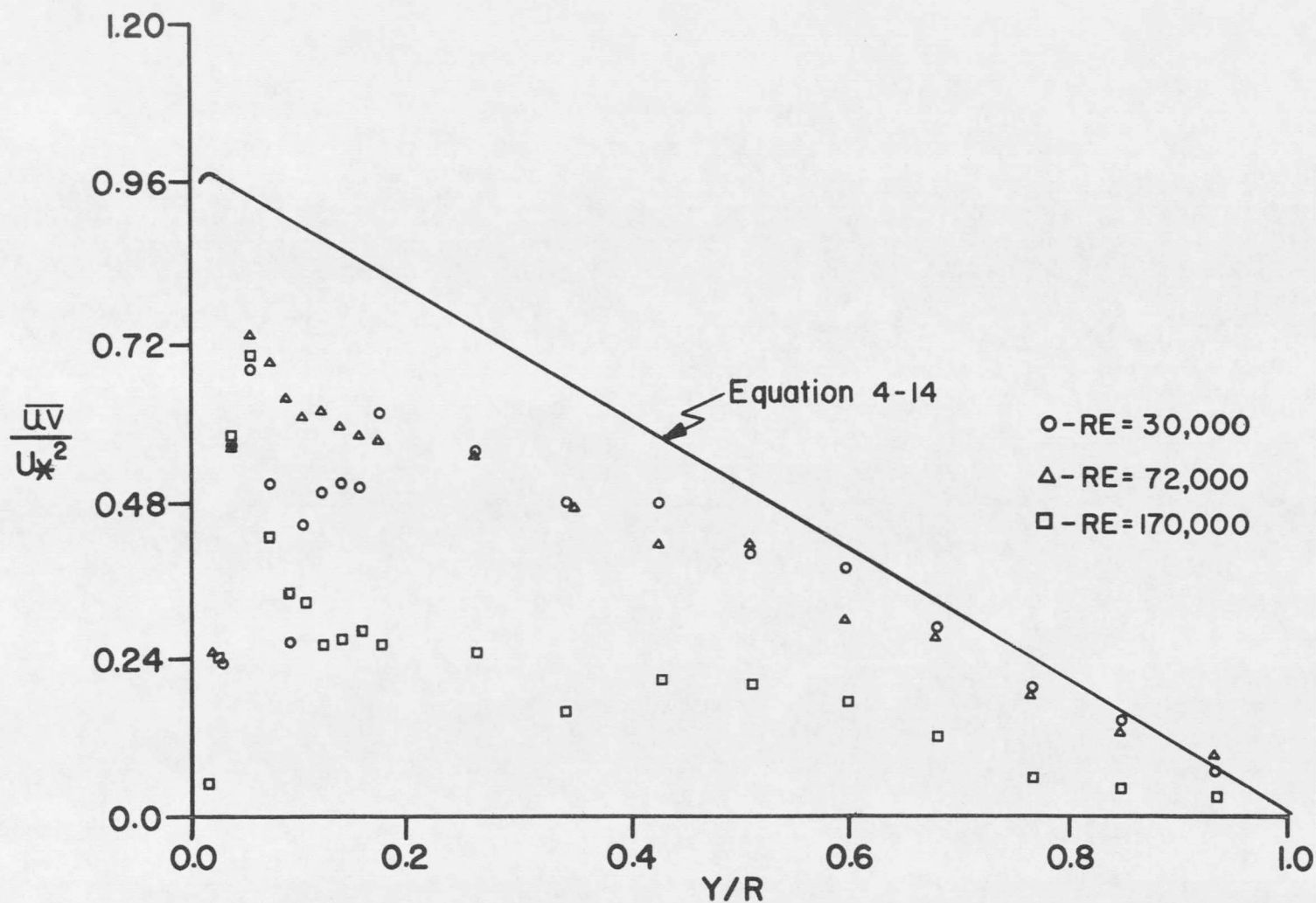


Figure 25. $\overline{uv}/U_*^2$ vs Y/R for $R/\epsilon = 26.4$ Pipe

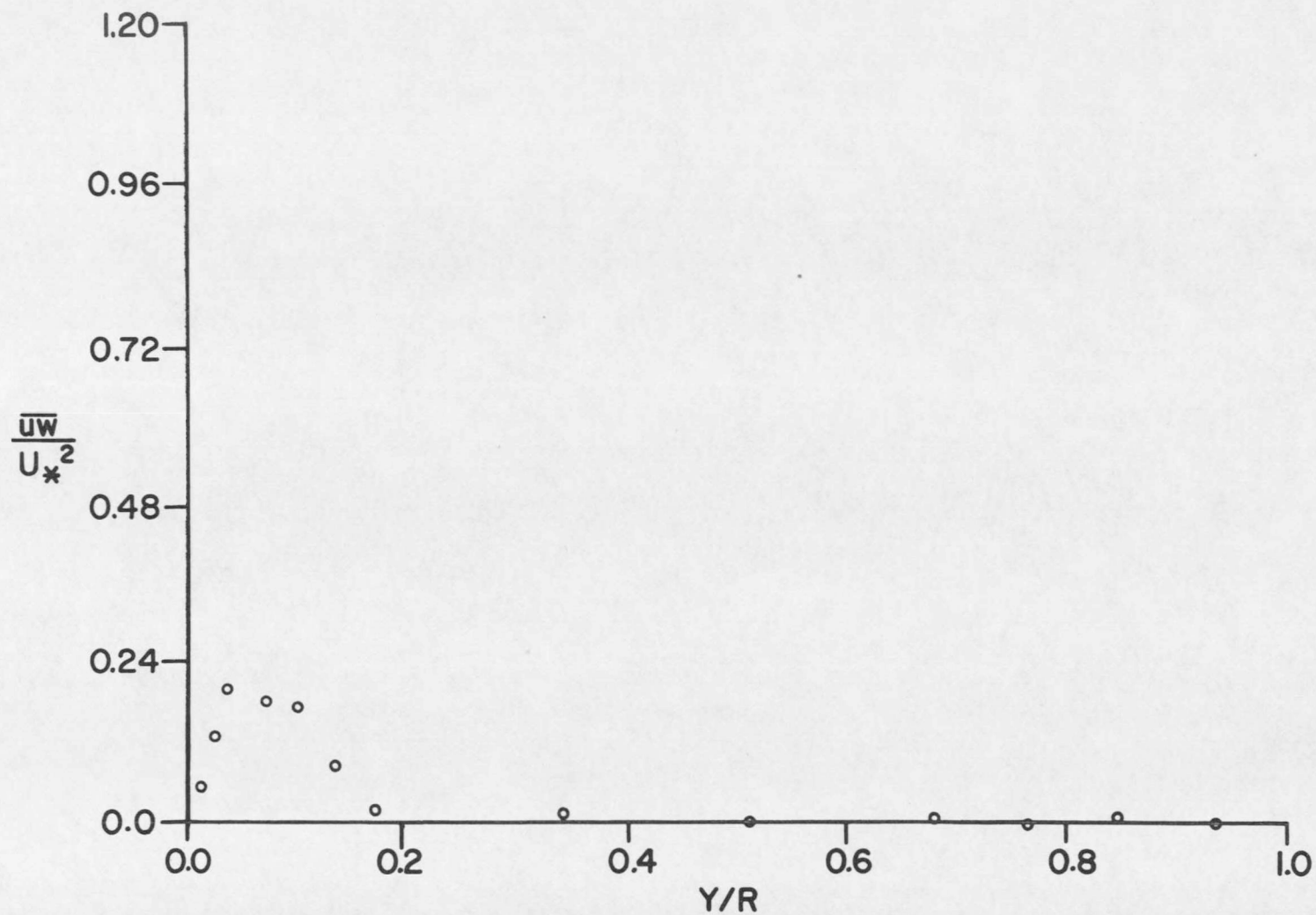


Figure 26. $\overline{uw}/U_*^2$ vs Y/R for Smooth Pipe

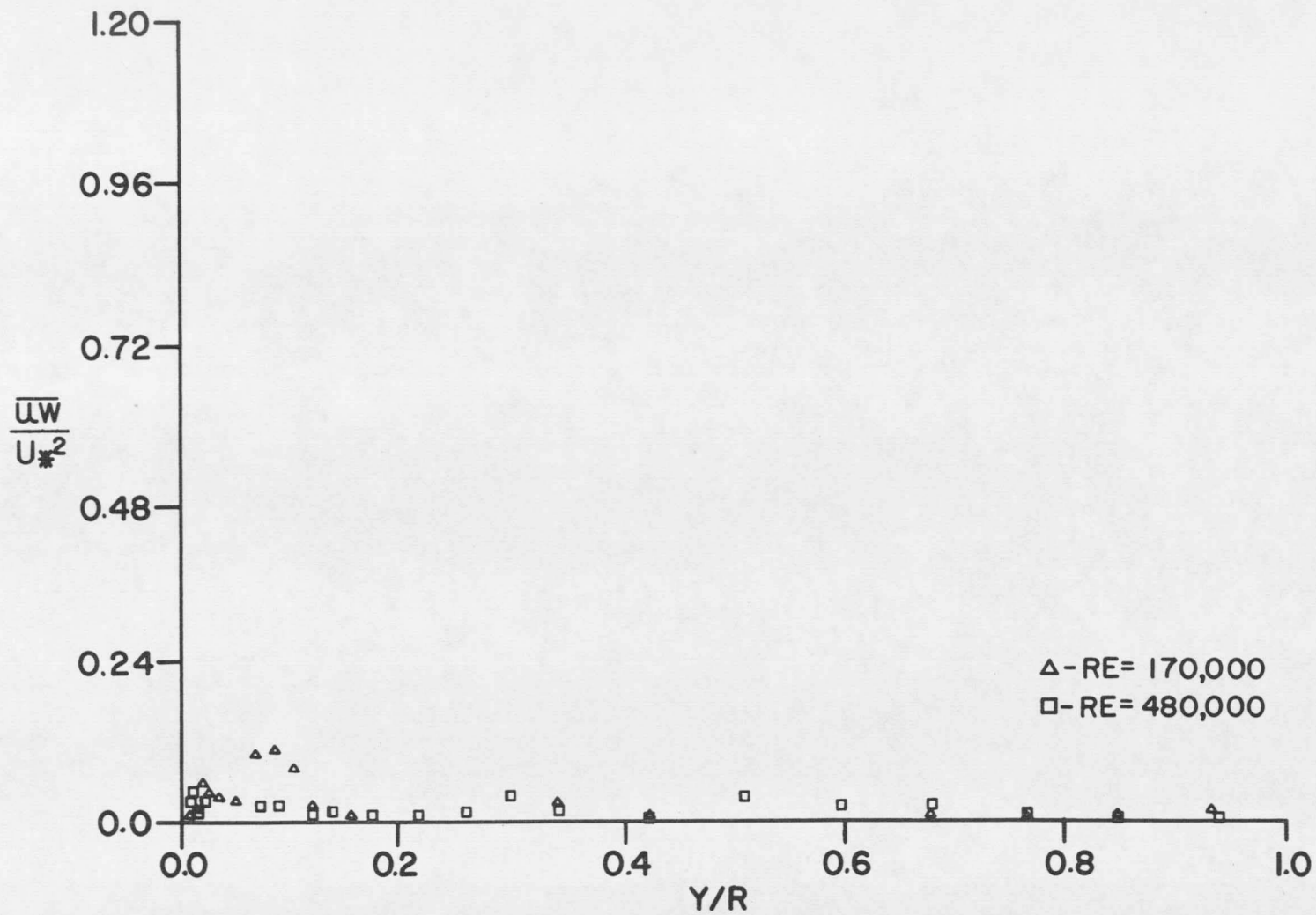


Figure 27. $\overline{uw}/U_*^2$ vs Y/R for $R/\epsilon = 208$ Pipe

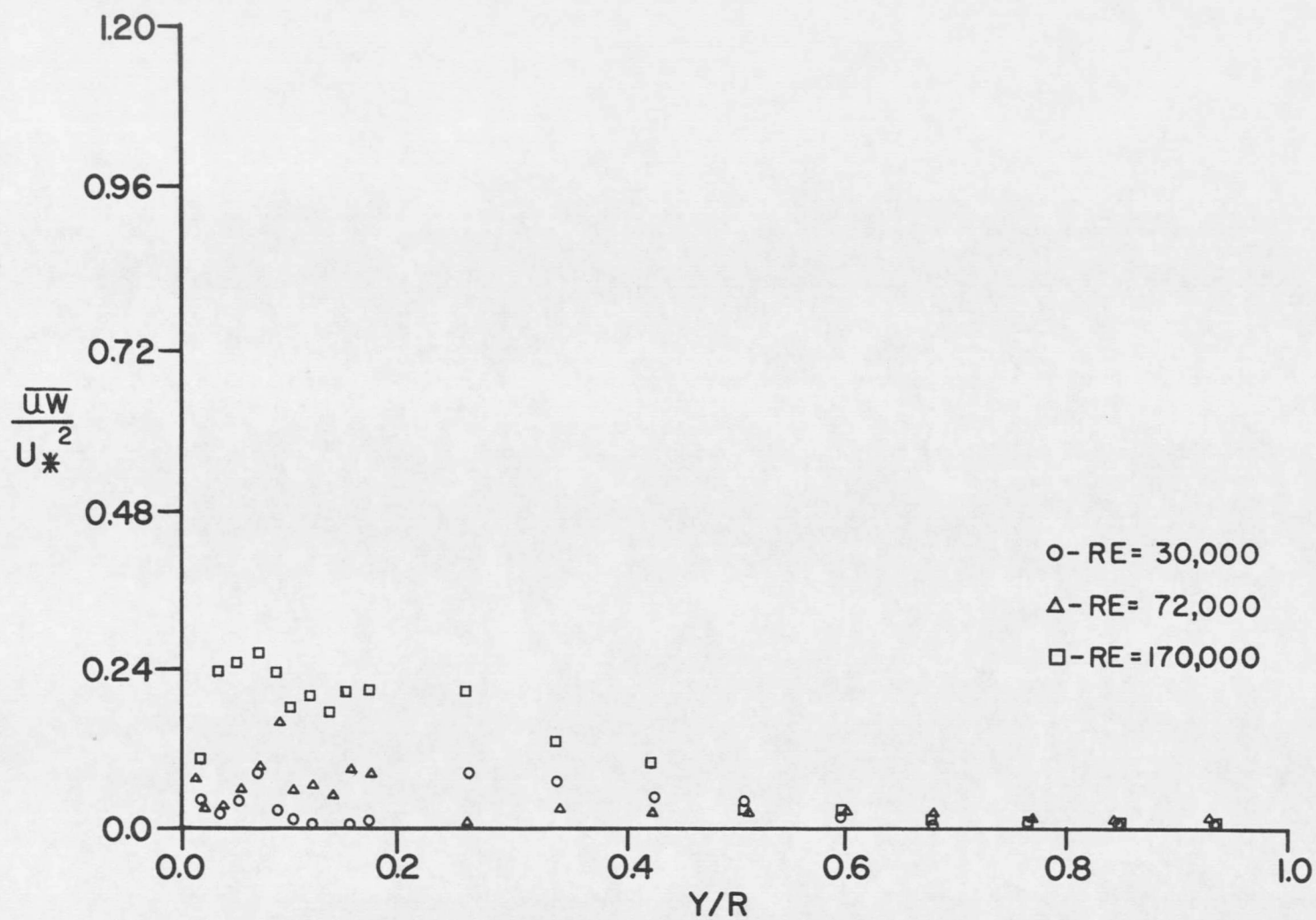


Figure 28. $\overline{uw}/U_*^2$ vs Y/R for $R/\epsilon = 26.4$ Pipe

tube for the two larger Reynolds numbers indicated on the plot.

D. RMS Fluctuating Velocities

Figures 29 through 36 illustrate the RMS value of the fluctuating velocity components measured in the three coordinate directions for each flow situation investigated. It is immediately obvious that centerline isotropy is not present. However the $\sqrt{\bar{v}^2}$ and $\sqrt{\bar{w}^2}$ values are nearly equal on all the plots. A $\sqrt{\bar{w}^2}$ curve is not presented in Figure 30 as equipment failures prevented its measurement.

Turbulence intensity plots, $\sqrt{\bar{u}^2}/U$ versus y/R , representative of all three pipe roughness conditions are illustrated in Figure 36.

E. Friction Factors

Figure 37 is the logarithmic plot of friction factor versus Reynolds number obtained for the three pipe roughness conditions. The solid curve represents Prandtl's universal law of friction factors for smooth tubes,³

$$\frac{1}{\sqrt{f}} = 2.0 \log (UD/\nu \sqrt{f}) - 0.8 \quad (5.2)$$

Friction factor values in the rough pipes were obtained primarily with the procedure outlined in Chapter III.

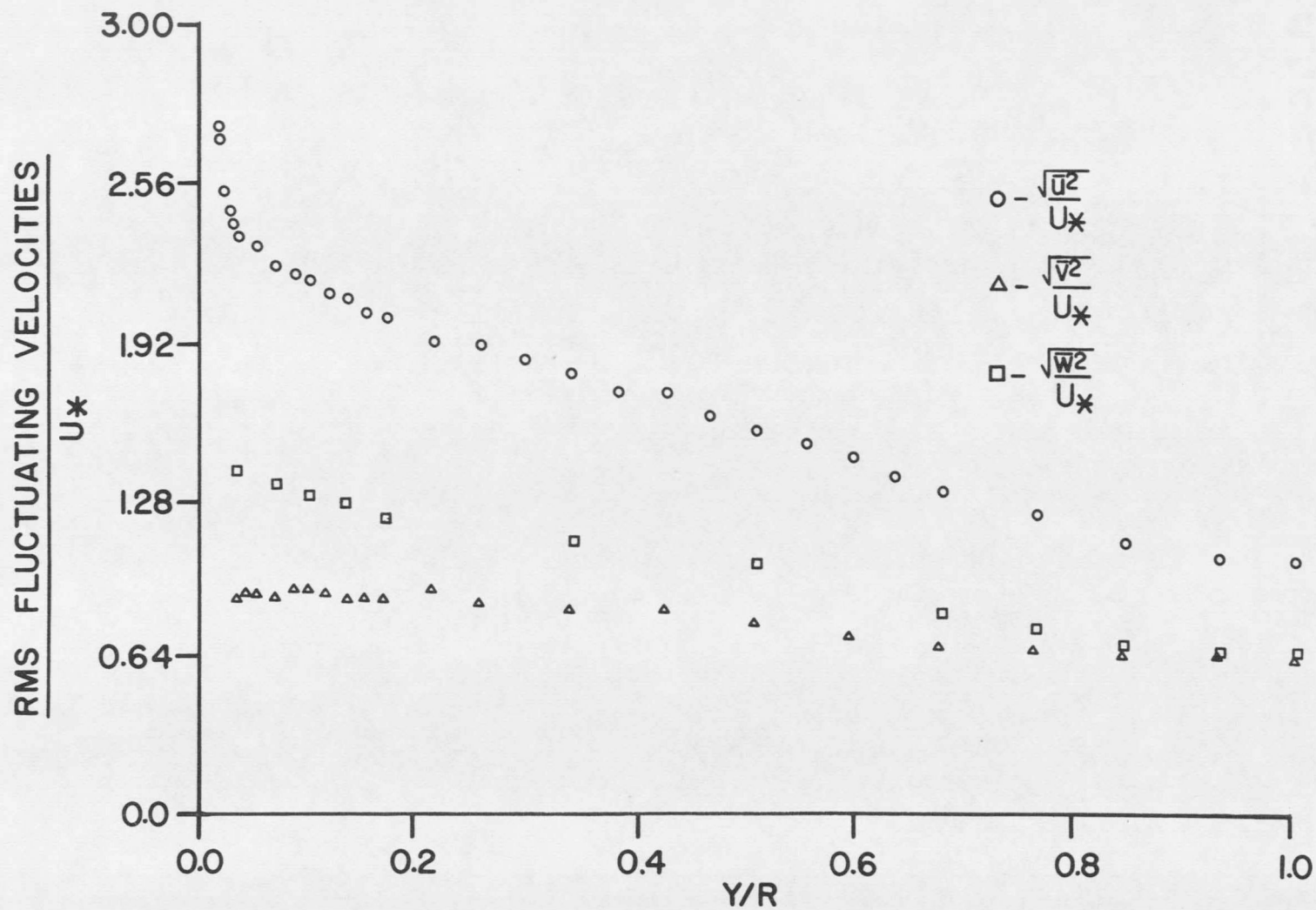


Figure 29. RMS Fluctuating Velocities Measured in Smooth Pipe

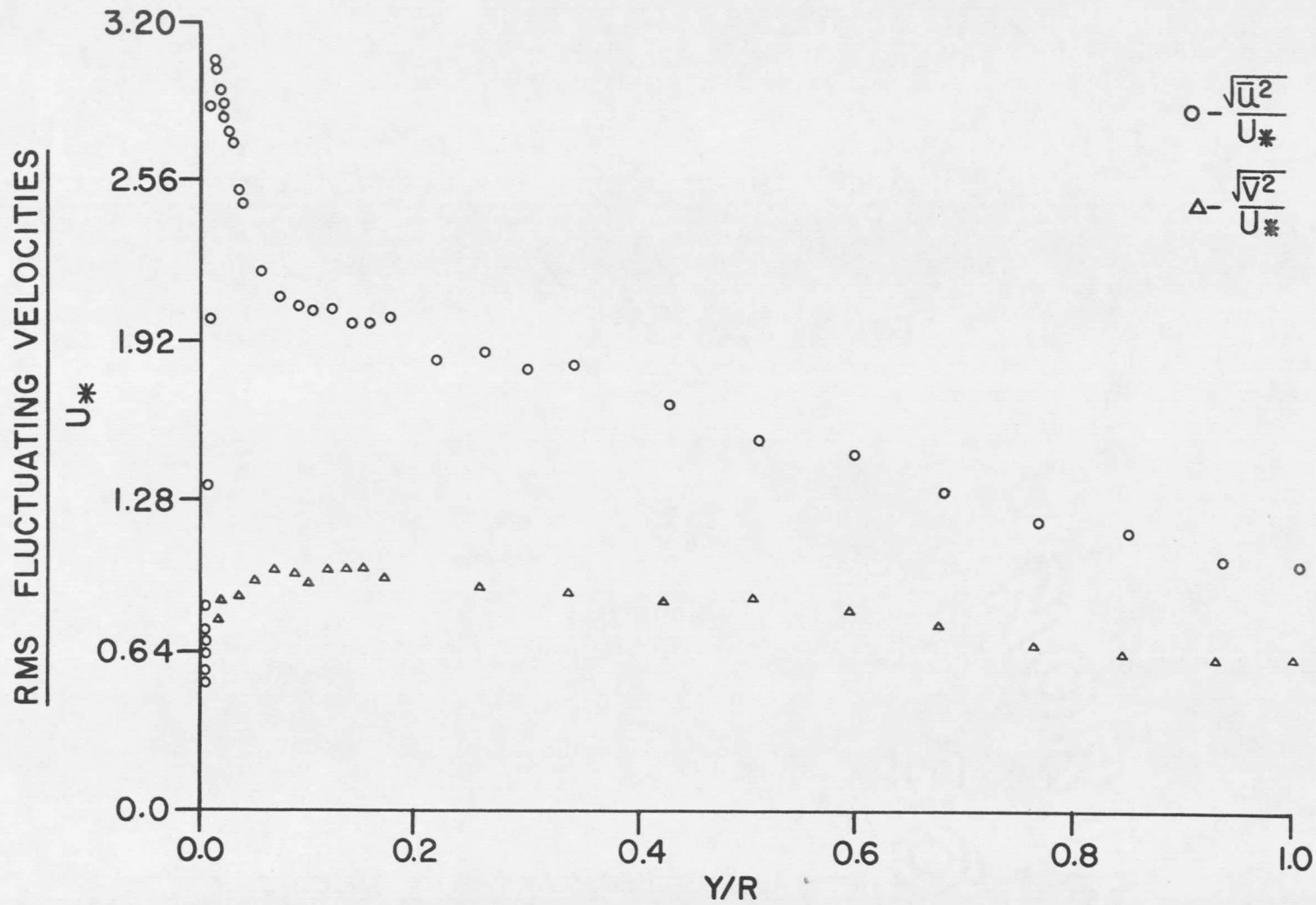


Figure 30. RMS Fluctuating Velocities Measured
in $R/\epsilon = 208$ Pipe at $Re = 52,000$

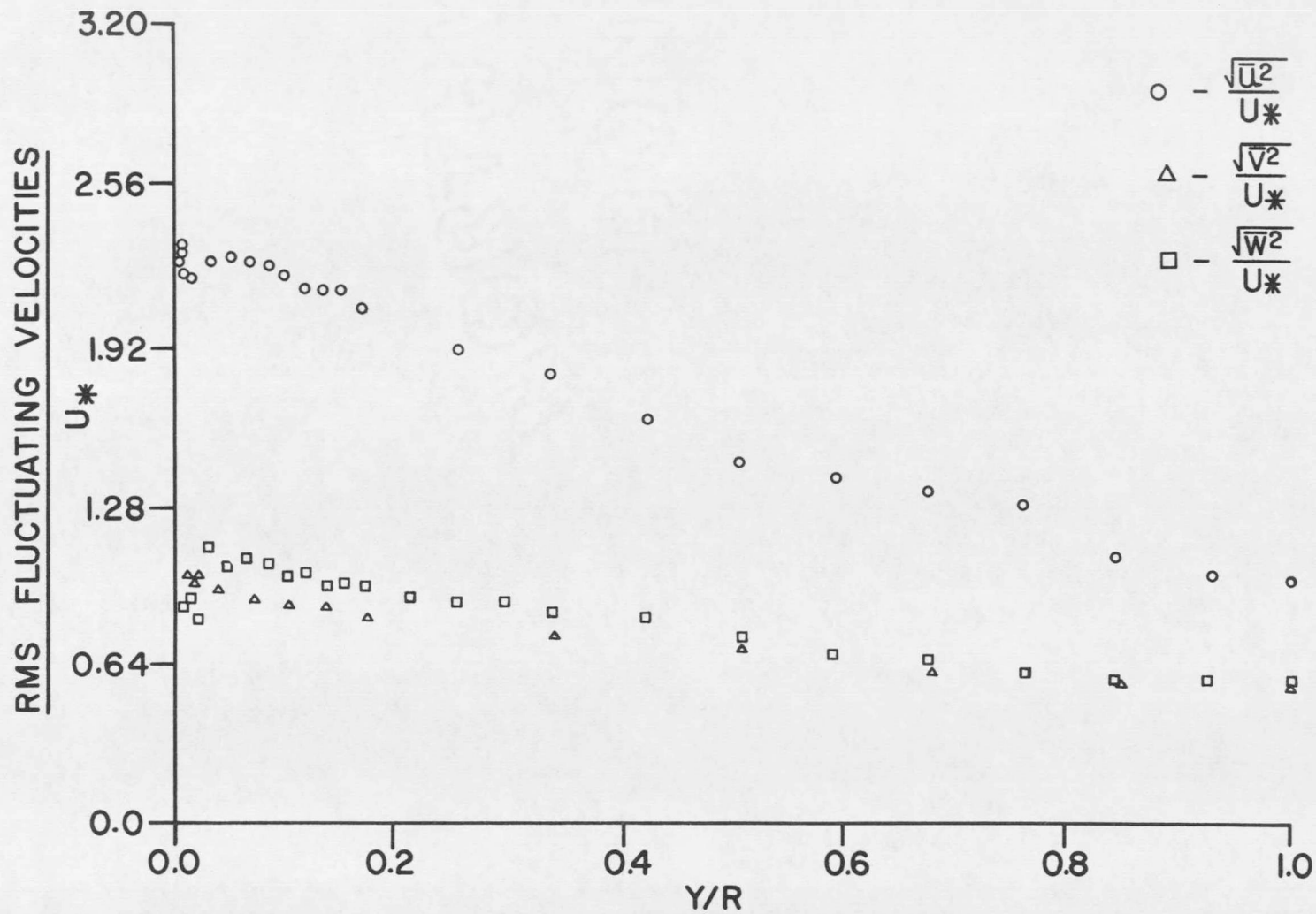


Figure 31. RMS Fluctuating Velocities Measured
in $R/\epsilon = 208$ Pipe at $Re = 170,000$

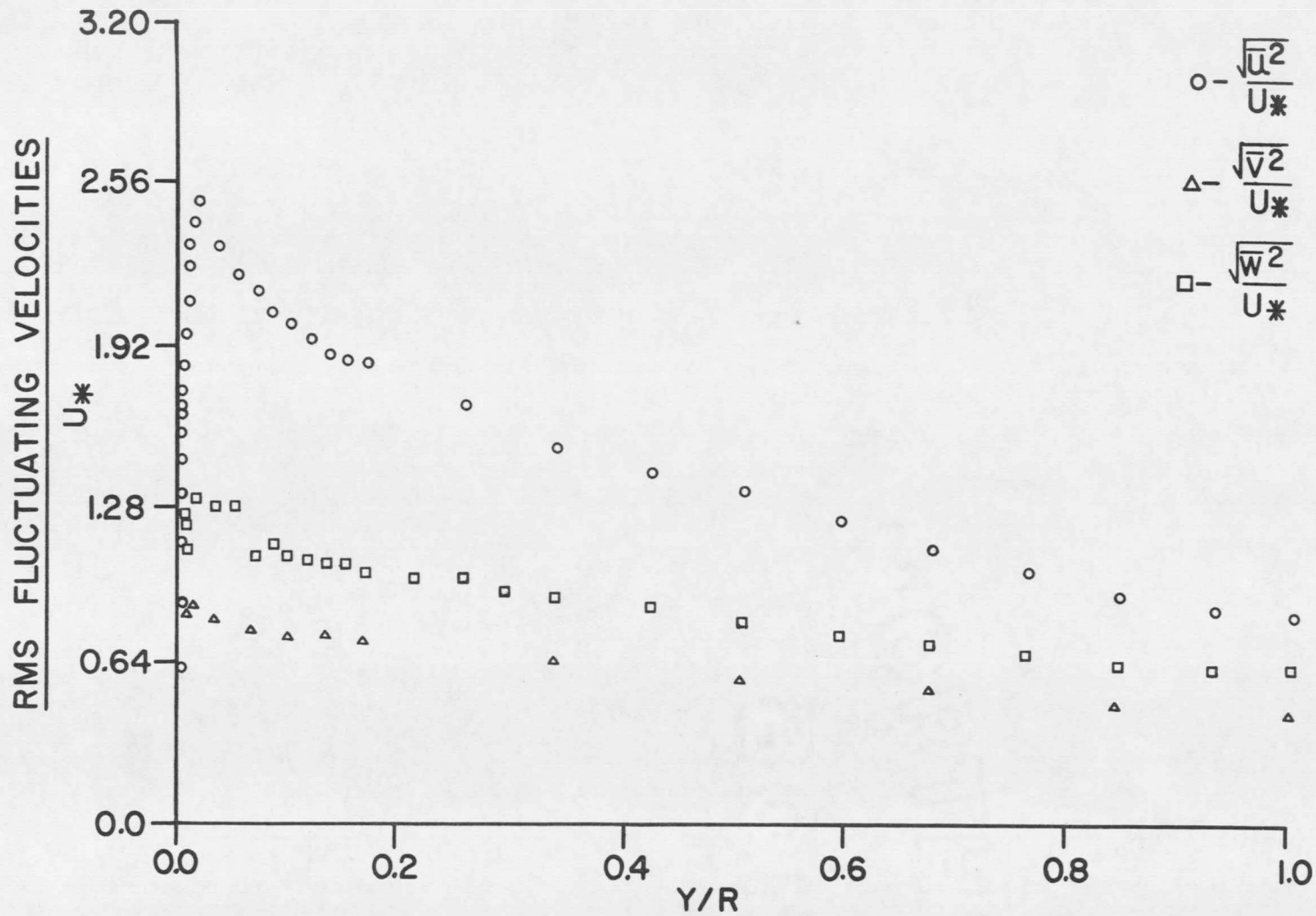


Figure 32. RMS Fluctuating Velocities Measured
in $R/\epsilon = 208$ Pipe at $Re = 480,000$

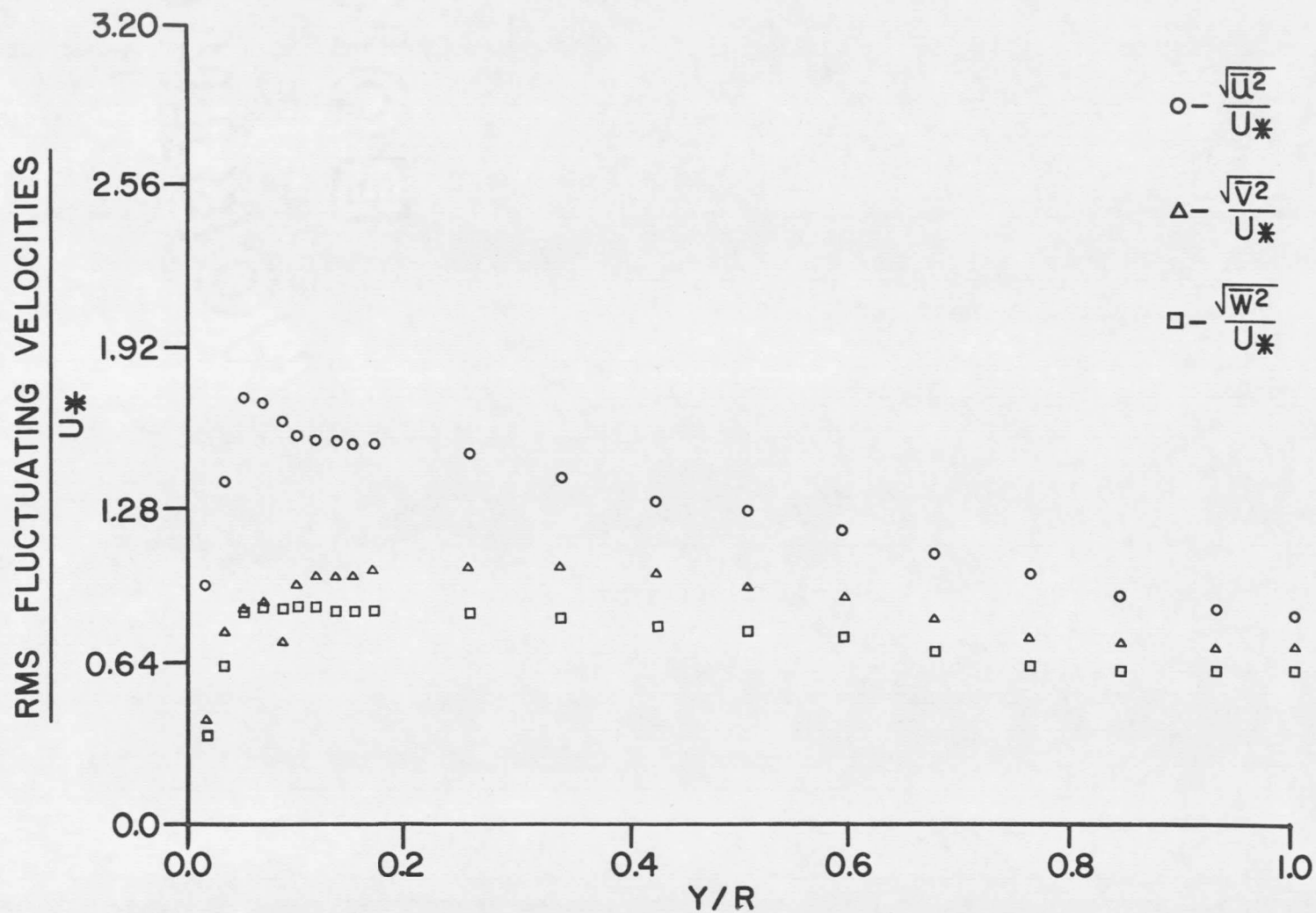


Figure 33. RMS Fluctuating Velocities Measured in $R/\epsilon = 26.4$ Pipe at $Re = 34,500$

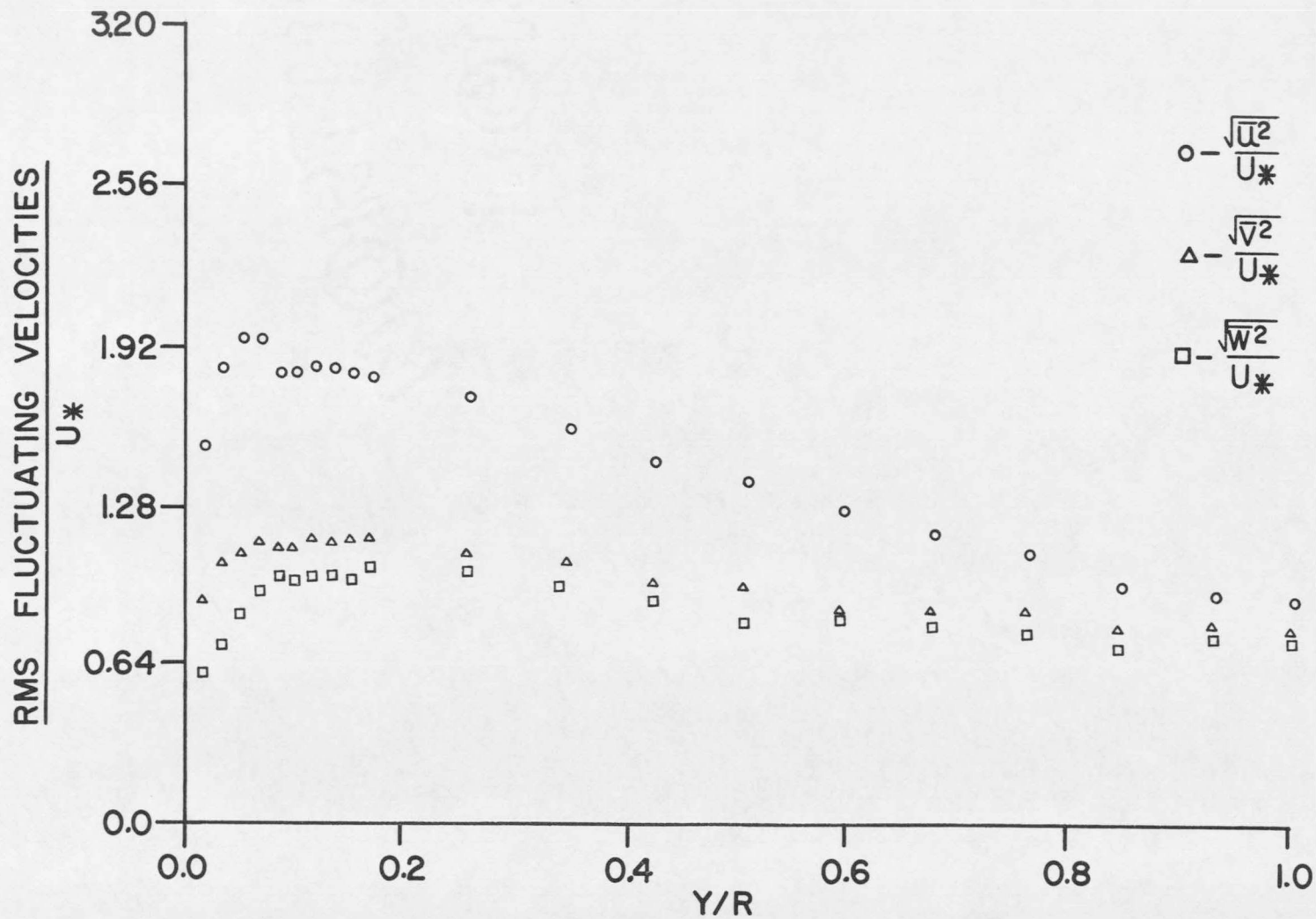


Figure 34. RMS Fluctuating Velocities Measured
in $R/\epsilon = 26.4$ Pipe at $Re = 75,000$

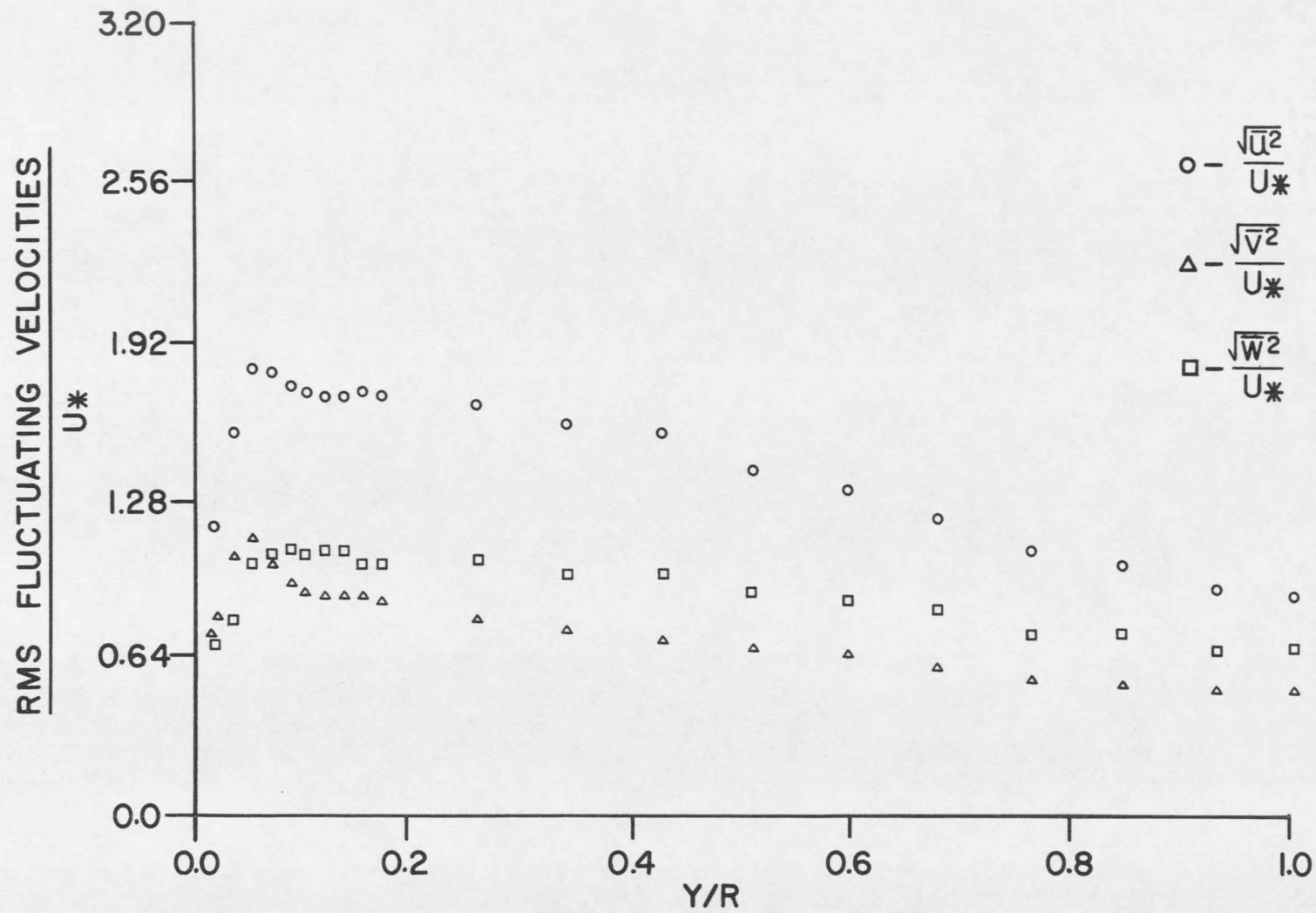


Figure 35. RMS Fluctuating Velocities Measured
in $R/\epsilon = 26.4$ Pipe at $Re = 170,000$

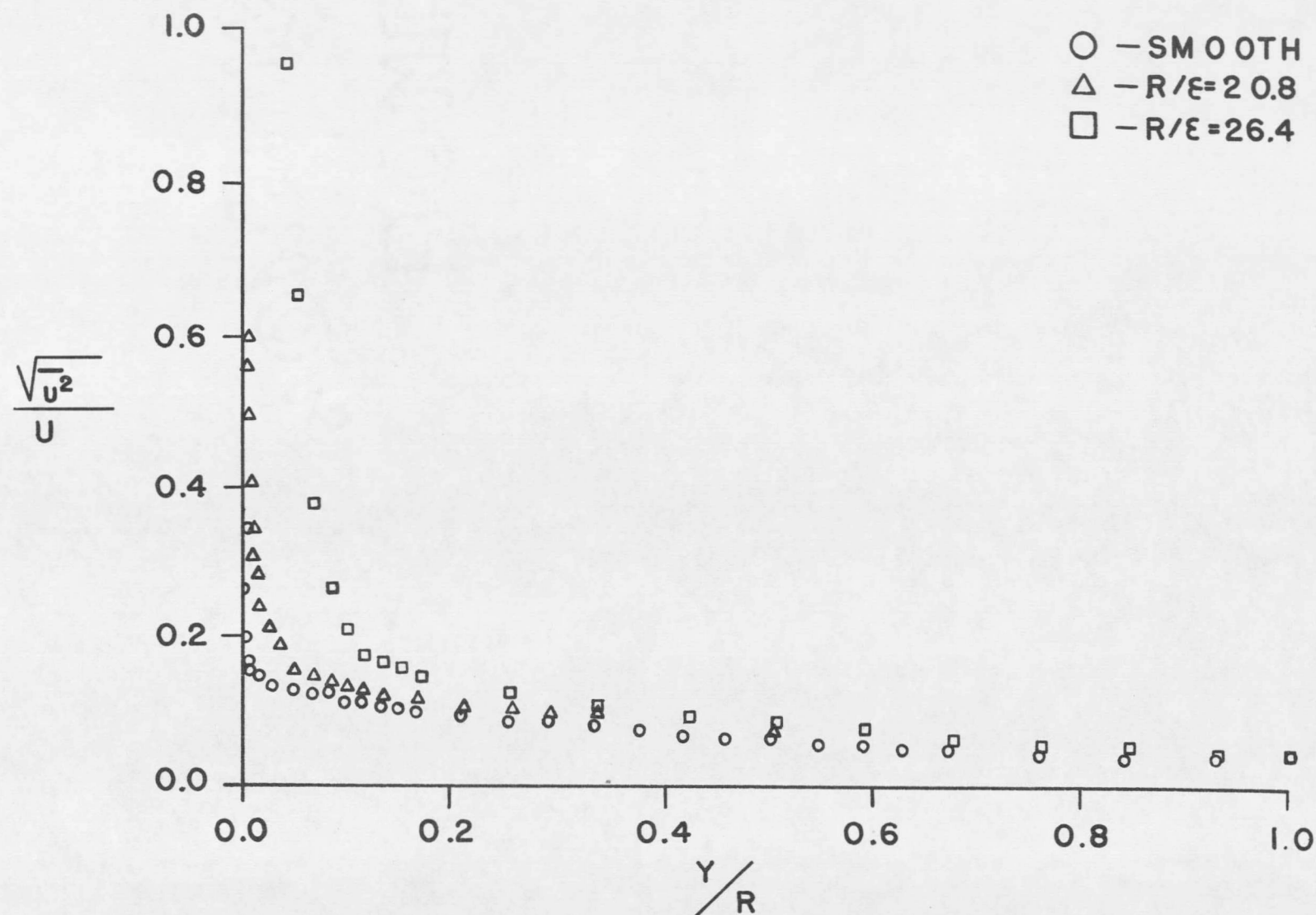


Figure 36. Turbulence Intensities for Three Degrees of Pipe Roughness

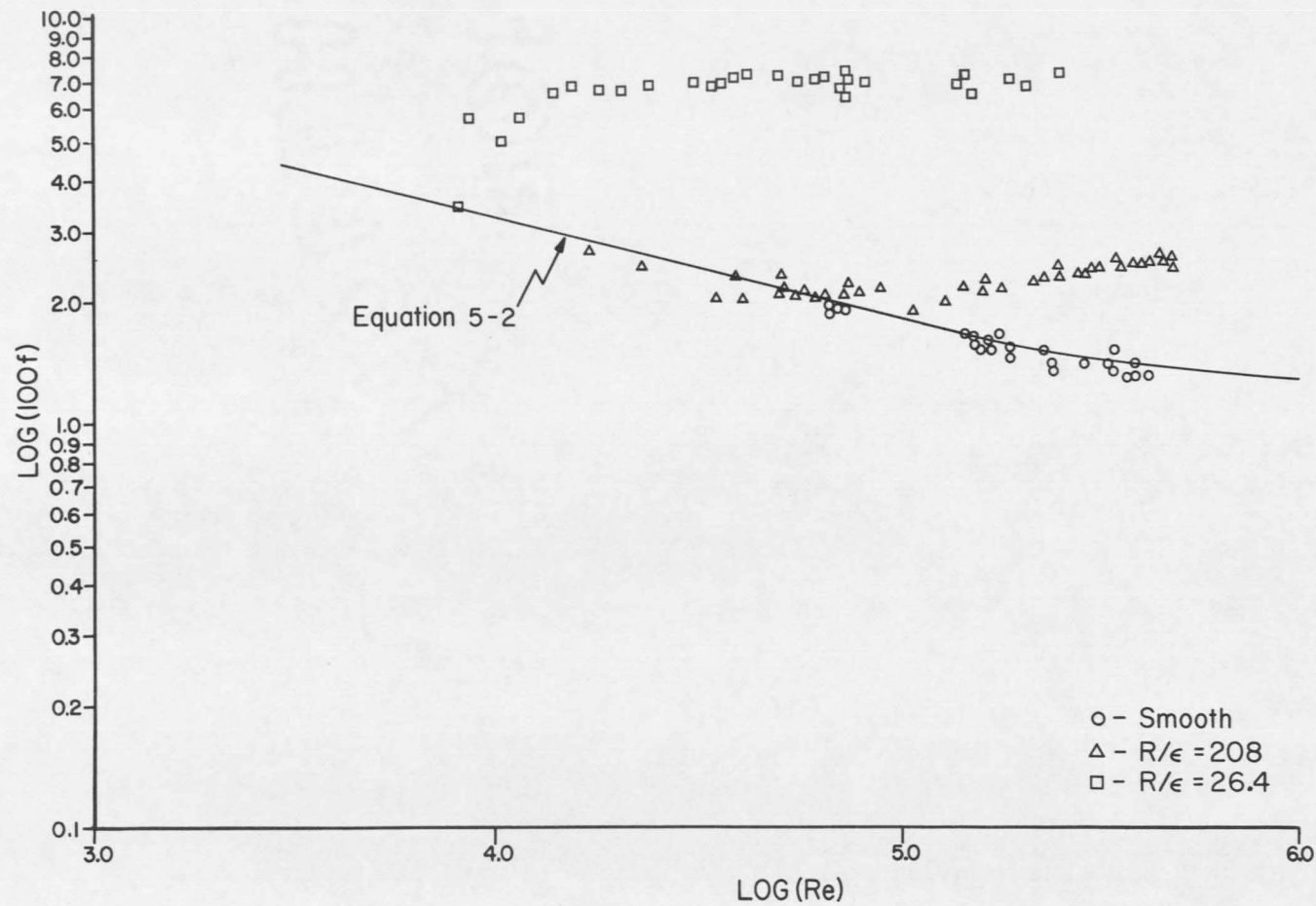


Figure 37. Friction Factor vs Reynolds Number
for Three Degrees of Pipe Roughness

F. Discussion of Results

Mean velocity profile, fluctuating velocity correlations, RMS values of the fluctuating velocity components, and friction factor data were presented in the preceding sections, for fully-developed turbulent flow in smooth and rough tubes. To establish measuring procedures, smooth tube data were obtained first. The validity of the smooth tube data was ascertained by comparing the results with those of other investigators and by the use of relationships derived from the Navier-Stokes equations.

For thirty individual mean velocity profiles determined in the smooth tube in the Reynolds number range 50,000 through 480,000, the straight-line equation

$$U^+ = 4.55 + 2.75 \ln Y^+$$

best fitted the data, for $Y^+ > 70$. The maximum deviation was $\pm 10\%$. This equation agrees very well with the results of previous investigations as is shown in Table I of Chapter II. In fact, it nearly represents the average of all equations listed in Table I.

The smooth pipe data of Laufer¹² was compared with the RMS values of the fluctuating velocity components and cross-correlation values obtained in the smooth tube of this investigation. Laufer's RMS values of the fluctuating velocity components showed centerline isotropy which is not

shown in the data of this investigation. However, the $\sqrt{v^2}$ and $\sqrt{w^2}$ values of this investigation are nearly equal at the centerline, which is the only isotropy requirement of the Navier-Stokes equations. The centerline value of $\sqrt{u^2}/U_*$ is observed from Figure 29 to be approximately 0.8, which agrees very closely with Laufer's data. In fact, all three RMS values of the fluctuating velocity components closely resemble the profile shapes presented by Laufer.¹²

Equation (4.14), originally derived by Laufer,¹² yielded empirical values of the $\overline{uv}/U_*^2$ correlation from experimental mean velocity data from the smooth tube, as is illustrated in Figure 23. Close agreement with experimental values was obtained.

Experimental values for the $\overline{uw}$ correlation are not available in the literature; however, assuming axisymmetric and fully-developed pipe flow, the $\overline{uw}$ correlation can be shown to be zero for all points on the radius. Figure 38 approximates the mean velocity profile for turbulent flow in a pipe. Fluctuations about the mean velocity profile in the angular direction give rise to no net momentum transfer in the axial direction and, therefore, no fluctuation in the axial direction. It is reasonable to infer the time average of the product of the two, u and w , which is the $\overline{uw}$ correlation, is zero.

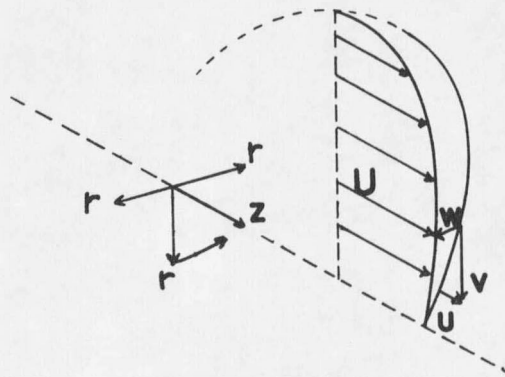


Figure 38. Sketch of Mean Velocity Profile with Fluctuations Superimposed

This argument was substantiated with $\overline{uw}$ measurements in the smooth tube as illustrated in Figure 26.

The quality of the experimental smooth pipe data verified the procedures employed. Consequently, the same procedures were followed for measurements in the rough tubes.

Neither the hydraulically smooth nor the fully-rough flow regions were attained with the $R/\epsilon = 208$ rough pipe, if the regimes set forth by Nikuradse are employed as the standard. The largest roughness parameter attained was 65 and the lowest 6.7. The transition region, $5 < \epsilon U_* / \nu < 70$, was extensively examined with the $R/\epsilon = 208$ rough tube. The fully-rough region was thoroughly investigated with the $R/\epsilon = 26.4$ rough tube, as, even at Reynolds numbers of 20,000, the friction factors calculated were independent of Reynolds number.

The mean velocity distribution plots clearly illustrate the effect of boundary roughness as the $U/U_{\max}$ curves are changed from the very blunt form of the smooth tube, to a more pointed profile in the rough tubes. It is evident, however, that for all three pipe conditions, the profile becomes more blunt with increasing Reynolds numbers. This tendency has also been cited by Nikuradse.⁹

The logarithmic mean velocity profiles also illustrate the pronounced effect of boundary roughness. The profile at a

Reynolds number of 52,000 and $R/\epsilon = 208$, in Figure 19 exhibits essentially the same characteristics as the smooth tube plot of Figure 18, indicating that the flow is nearly hydraulically smooth, and the influence from the boundary roughness is negligible. The roughness parameter value, $\epsilon U_* / \nu$, for this run was 6.7 and the friction factor agreed to within 4% of the smooth tube line of Figure 37. These results agree well with Nikuradse's rough tube flow regimes. However, for increasing roughness parameters, the curves shift to the right and downward.

The slopes of the straight line sections of these curves, for $Y^+ > 70$ and for the $R/\epsilon = 208$ rough tube, remain essentially constant, but the ordinate intercept decreases with increasing roughness parameter values. This shift is primarily due to increases in the wall shear stress, as evident from the friction factor data. Table II shows the best-fit equations obtained for each profile illustrated in Figures 18, 19 and 20. The slope of these lines is seen to be a slight function of Reynolds number and roughness size. The consistency of the data is also indicated.

In evaluating the friction factors in the rough tubes the procedure outlined in Chapter IV yielded values comparable with friction factors determined from the velocity profile runs. The constants A , A' , B , and B' of equations (3.9) and

TABLE II

Pipe Condition	Re. No.	Equation	Average Deviation
Smooth	50,000 - 480,000	$U^+ = 4.55 + 2.75 \ln Y^+$	4.25%
$R/\epsilon = 208$	52,000	$U^+ = -.43 + 3.32 \ln Y^+$	1.24%
$R/\epsilon = 208$	170,000	$U^+ = -1.5 + 2.97 \ln Y^+$	0.55%
$R/\epsilon = 208$	480,000	$U^+ = -5.62 + 2.93 \ln Y^+$	1.42%
$R/\epsilon = 26.4$	34,500	$U^+ = -12.6 + 4.03 \ln Y^+$	1.24%
$R/\epsilon = 26.4$	75,000	$U^+ = -12.5 + 3.50 \ln Y^+$	1.39%
$R/\epsilon = 26.4$	154,000	$U^+ = -12.8 + 3.27 \ln Y^+$	0.70%

(3.10) were obtained from equation (5.1) and from

$$U^+ = 4.06 + 3.65 \ln y/\epsilon \quad (5.3)$$

which was the best fit curve for the mean velocity profile data in the fully-rough region using the $R/\epsilon = 26.4$ rough pipe. The friction factor values shown in Figure 37 for the $R/\epsilon = 208$ rough tube were predicted employing the smooth tube equation (5.2), since velocity profiles in the fully-rough region were not obtained. For values of the roughness parameter less than five, the friction factor values agree to within $\pm 8\%$ with equation (5.1). For roughness parameters

greater than seventy, the friction factors are independent of Reynolds number. The equivalent sand diameter, as proposed by Schlichting,³ was calculated to be 0.0280 for the $R/\epsilon=208$ rough tube. The actual sand size used was 0.0286 inches diameter.

The two methods of obtaining friction factors in the $R/\epsilon = 26.4$ rough tube produced values which agree to within $\pm 5\%$. However, the equivalent sand diameter was calculated to be 0.524 inches which is more than twice the actual sand size employed.

The RMS values of the fluctuating velocity components as measured in the rough tubes are essentially the same as those measured in the smooth tube. The $\sqrt{\bar{u}^2}/U_*$, $\sqrt{\bar{v}^2}/U_*^2$, and $\sqrt{\bar{w}^2}/U_*$ values appear to be independent of Reynolds number and relative roughness. The $\sqrt{\bar{v}^2}$ and $\sqrt{\bar{w}^2}$ values are very nearly equal at the pipe centerline. The peak value of the $\sqrt{\bar{u}^2}/U_*$ plot was observed to be a function of Reynolds number. For low Reynolds numbers, the maximum value of $\sqrt{\bar{u}^2}/U_*$ was reached at y/R values greater than the individual sand grain heights; whereas, for increased Reynolds numbers, the maximum value was observed at the top of the roughness particles.

The turbulence intensity measurements at the pipe centerline were also found to be independent of Reynolds number and roughness. However, near the boundary the turbulence

intensity is definitely a function of relative roughness as indicated in Figure 36.

Logan and Jones¹⁸ have presented data for turbulent flow following an increase in pipe wall roughness which substantiates the turbulence intensity measurements of this investigation. For a relative roughness of $R/\epsilon = 55$ they reveal turbulence intensities near the rough wall of twice the magnitude of the smooth tube intensities at the same y/R position.

Richardson and McQuivery¹⁴ have presented RMS values of the fluctuating velocity components for open channel flow with rough boundaries. Their dimensionless parameter, $\sqrt{\bar{u}^2}/U_{avg}$, exhibited a definite Reynolds number dependency at all points in the boundary layer. The data of this investigation do not exhibit a noticeable dependency on Reynolds number at the pipe centerline, although there is a noticeable increase in $\sqrt{\bar{u}^2}/U_{avg}$ with increasing Reynolds numbers away from the centerline. Comparative $\sqrt{\bar{v}^2}$ and $\sqrt{\bar{w}^2}$ data for rough tubes were not found in the literature.

The $\overline{uv}/U_*^2$ parameter represents the ratio of the turbulent shear stress in the R-Z plane to the wall shear stress τ_0 , and the data of Figures 23, 24 and 25 indicate that this ratio is a linear function of radial position except in close proximity to the boundary. Equation (4.14) was employed to

predict the $\overline{uv}$ correlation in rough tubes as well as in the smooth tube. However, the experimentally measured values of $\overline{uv}/U_*^2$ in rough tubes decreases with increasing Reynolds numbers, while the curves predicted from equation (4.14) remain essentially the same as those measured in the smooth tube. This drastic discrepancy indicates either: 1) the experimental data is erroneous, or 2) the reduction of the Navier-Stokes equations to obtain equation (4.14) is not valid for flow in rough tubes, except possibly, in the hydraulically smooth regime.

Logan and Jones¹⁸ also presented values of the $\overline{uv}$ cross-correlation, for fully-developed flow in a $R/\epsilon = 55$ rough tube, which was a linear function of radial position throughout a limited range extending from the pipe centerline. However, a numerical comparison cannot be made since their values of the $\overline{uv}$ cross-correlation were non-dimensionalized with the centerline mean velocity in the smooth tube at the entrance of the rough tube.

The values of the $\overline{uw}$ cross-correlations measured in the rough tubes are shown in Figures 27 and 28. The data for the $R/\epsilon = 208$ rough tube were found to be nearly zero at all points on the radius. This substantiates the earlier arguments concerning the nonexistence of this cross-correlation in fully-developed, axisymmetric flow. However, Figure 28 shows

a definite $\overline{uw}$ correlation existing in the $R/\epsilon = 26.4$ rough tube at 170,000 Reynolds number.

The accuracy of the experimental data was questioned, when it was observed that Equation (4.14) did not predict the experimental values of the $\overline{uv}$ cross-correlation. However, the equipment was checked, procedures re-analyzed, and additional runs made to determine the repeatability of the data. Three runs were made at 483,000; 460,000 and 450,000 Reynolds numbers, and the maximum deviation for the values of the $\overline{uv}$ cross-correlation when at the same Y/R positions was 35% which occurred at the wall. The average deviation of values in the center of the pipe was calculated to be $\pm 8\%$. The frequency spectrum of the fluctuating components was also measured. It was found that 70% of the energy of the fluctuating velocity components was below 1000 cps, and none was observed above 16,000 cps. This is well within the frequency response limitations of the hot-wire equipment.

The accuracy of the rough pipe data close to the boundary is questionable as turbulence intensity values as high as 80% were observed. The assumptions regarding hot-wire response at these intensities are no longer valid. For turbulence intensities of 50%, errors in the order of 13% will result, while for a 20% intensity the error will be less than 2%.²¹ However, in all flow cases investigated, turbulence

intensities smaller than 10% were observed in the inner 75% of the cross-sectional area of the pipe. Thus the discrepancies between predicted $\overline{uv}$ and measured $\overline{uv}$ values could not be accounted for by experimental errors.

VI. CONCLUSIONS AND RECOMMENDATIONS

Fully-developed turbulent flow in both smooth and rough pipes has been extensively examined and the results presented in this thesis. The investigation was conducted to provide velocity profile data in rough pipes which can be employed to further analyze the heat transfer characteristics of rough walled pipes.

The mean velocity profiles, RMS values of the fluctuating velocity components and friction factor values agree well with data from earlier investigations and substantiate the results obtained from the Navier-Stokes equations. However, the experimental values of the $\overline{uv}$ cross-correlation in rough pipes do not agree with the empirically predicted values. The logical conclusion as drawn from the experimental data and their apparent validity, is that the Navier-Stokes equations as reduced in Chapter IV, which so excellently predicted values of the smooth pipe $\overline{uv}$ cross-correlation, are not valid for fully-developed flow in rough walled tubes. No attempt was made to reach any general conclusion regarding the obvious discrepancies. However, it is recommended that a detailed re-examination of the Navier-Stokes equations be made, to perhaps conceive a different flow model that might explain these discrepancies.

It is also recommended that additional investigations be conducted with pipes with different relative roughness values so as to include the effect of particle roughness shape as well as size.

APPENDIX

APPENDIX

Equipment Specifications

Thermo-Systems Heat Flux System

Hot Wires

Material -- copper plated tungsten
Diameter -- 0.00015 inch
Length -- 0.060 inches

Anemometers - Model 1010A

Controls -- Stability, bridge balance, zero
 suppression, square wave generator,
 overheat resistance, output filters
Meter Accuracy -- 0.5%
Frequency Response -- DC - 50 K cps
Output Noise Level -- 0.4 mv RMS
Dimensions -- 19 in. w. x 8-3/4 in. h. x 15-1/2 in. d.

Linearizers - Model 1005B

Controls -- Gain adjust and zero adjust for each
 squaring circuit, variable gain for
 each squaring circuit, zero suppression.

Input -- 0 - 15 volts
Output -- 0 - 15 volts
Frequency Response -- 100 K cps
Accuracy -- 1% full scale
Dimensions -- 19 in. w. x 8-3/4 in. h. x 15-1/2 in. d.

RMS Voltmeter - Model 1060

Frequency range -- 0.1 cps - 500 K cps
Voltage range -- 1 mv - 300 volts
Time constant -- 0 - 100 sec.
Dimensions -- 6 in. w. x 15 in. d. x 8 in. h.

Correlator - Model 1015B

Two inputs and outputs

Controls -- Function switch to select output:

e_1 , e_2 , e_1+e_2 , e_1-e_2

Variable gain on each input and output.

Noise level -- <1 mv RMS

Input range -- 0 - 50 volts

Frequency Response -- 0 - 200 K cps

Dimensions -- 5.7 in. w. x 7 in. h. x 9.5 in. d.

Hewlett Packard RMS Voltmeter - Model 3400A

Voltage range - 1 mv - 300 volts

Frequency range -- 10 cps - 10 M cps

Accuracy -- $\pm 1\%$

Dimensions -- 5-1/8 in. w. x 6-1/3 in. h. x 11 in. d.

Hickock D. C. Digital Voltmeter - Model DMS-3200

Voltage Range -- 0 - ± 1000 volts

Counter response -- Pulse rates to 10^6 PPS

Dimensions -- 9-1/4 in. w. x 6-3/8 in. l. x 12-7/8 in. d.

Wind Tunnel Equipment

Test Pipe -- aluminum irrigation pipe

Average radius -- 5.94 inches

Standard Deviation -- 0.04 inches

Lengths -- 15 - 20 feet

Entrance Section (Figure 2)

Filters, screens, and baffle plate

Cross-section reduced from 4 feet rectangular to circular one-foot diameter in 7 feet as half sine wave.

Test Section

Consists of one-foot diameter, 4 inches long, machined section; moveable probe support holder; Micrometer screw feed.

MKS Baratron Type 77 Pressure Meter

Calibrated accuracy -- 0.0005 mm Hg
Type 77-H Pressure Head -- diaphragm with
capacitance sensor
Differential range -- ± 1.1 mm Hg

Type 77-M x R indicator
Null balancing - decade readout

Pipe Roughness - River Sand

R/t = 208 Average sand grain size determined from
U.S. Standard Sieves, #20 - #28 = 0.0286 in.

R/ = 26.4 Average sand grain size determined from
Taylor Standard Sieves, #3 - #4 = 0.225 in.

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