An economic study on the optimum input mix for irrigated pastures

by Aldon Allen Turner

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Agricultural Economics
Montana State University

Abstract:
The purpose of this study was to maximize the returns to labor and fixed assets from feeder steers utilizing forage produced on irrigated orchardgrass pasture. A linear programming model was designed to analyze the economic alternatives of various application rates of irrigation water and nitrogen on the irrigated pasture. Irrigation water and nitrogen were not limited and were constrained only by their respective costs. Later, the quantities of irrigation water, nitrogen, and capital were sequentially varied and the effect on the returns noted. A model was also developed which analyzed the effect on returns to labor and fixed assets when marginal cost of water was varied from $4 per acre foot to $0 per acre foot of water.

The results of this study indicate that as the marginal cost of irrigation water goes to zero, the optimal application rate of irrigation water approaches a rate which keeps the soil at field moisture capacity at all time. As the marginal cost of irrigation water increased above about $1.50 per acre foot, the optimal application rate of irrigation water approached that application rate which just keeps the soil moisture above the wilting point for orchardgrass; approximately one-third the amount of water used when the marginal cost approached zero.

The optimal application of nitrogen was a direct function of the amount of water used, as the amount of water varied, the amount of nitrogen varied in about a 1:1 relationship. In general, the returns to labor and fixed assets were adversely affected more by rates of nitrogen which were in excess to the optimal amount than by application rates which were less than the optimal amount.

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AN ECONOMIC STUDY ON THE OPTIMUM INPUT MIX FOR IRRIGATED PASTURES

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Agricultural Economics

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Appreciation for the patience of one's wife is implicitly understood.

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ABSTRACT

The purpose of this study was to maximize the returns to labor and fixed assets from feeder steers utilizing forage produced on irrigated orchardgrass pasture. A linear programming model was designed to analyze the economic alternatives of various application rates of irrigation water and nitrogen on the irrigated pasture. Irrigation water and nitrogen were not limited and were constrained only by their respective costs. Later, the quantities of irrigation water, nitrogen, and capital were sequentially varied and the effect on the returns noted. A model was also developed which analyzed the effect on returns to labor and fixed assets when marginal cost of water was varied from $4 per acre foot to $0 per acre foot of water.

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The optimal application of nitrogen was a direct function of the amount of water used, as the amount of water varied, the amount of nitrogen varied in about a 1:1 relationship. In general, the returns to labor and fixed assets were adversely affected more by rates of nitrogen which were in excess to the optimal amount than by application rates which were less than the optimal amount.

CHAPTER I

INTRODUCTION

Justification

> "On irrigated lands, less grass has been grown in rotation systems than its value for soil fertility and immediate income justifies because farmers have not been educated to irrigate pastures and have been using unirrigated range lands and low, wet areas for grazing." 1/

Historically, the best irrigated land was planted to cash crops and only that land not suitable for cultivation was planted to pastures. 2/ Recently, Mr. D. E. Baldridge, Associate Agronomist at Huntley Branch Station, reported that much of the irrigated pasture in the Huntley Irrigation Project was in the odd corners and on the marginal land. However, he reported that in the past few years an increased interest in irrigated pastures has been expressed.

In 1965, 1,490,005 acres of irrigated crop land in Montana was harvested. 3/ All of this could conceivably be converted into irrigated pastures if the profitability of irrigated pastures was greater than any other crop. Wagner and Shepherd 4/ report that in California irrigated pastures produced forage that, although more costly than forage produced on range land, was a much cheaper source of nutrients

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1/ Grass, the Yearbook of Agriculture, 1948, U. S. Department of Agriculture, Washington, D. C., 1948, p. 141.

2/ Clee S. Cooper, D. E. Baldridge, and C. W. Roath, Selection and Management of Irrigated Pasture Mixtures, ARS, USDA, and Mont. Agr. Expt. Sta., 1966, p. 2.

3/ Montana Agricultural Statistics, SRS, Helena, August 1967, p. 10.

4/ Grass, op. cit., p. 129.

than most other livestock feeds. They reported that hay would have to be obtainable at from $3.80 to $7.50 per ton to be as cheap as feed obtained from irrigated pasture.

In general, the profitability of irrigated pastures has not been established, particularly not for Montana conditions. This thesis is dedicated to supplying a small part of the answer to the question of when, how, and why irrigated pastures are profitable. That is, what species, what level of fertilization, what level of irrigation, and what amount of capital should be applied to give maximum revenue net of variable costs. If this question can be answered, then it is a relatively simple matter to compute and compare the profitability of irrigated pasture with the profitability of other crops.

The beneficial aspects of irrigated pasture and hay to the land with respect to fertility, soil tilth, and weed control has been fairly well determined. 5/6/7/ However, it is extremely hard to put a monetary value on "improved soil tilth", "12 percent less weeds", etc. These intangibles will not be specifically covered, however, the reader should keep them in mind for they influence future profits.

5/ G. O. Bateman and W. Keller, Grass-Legume Mixtures for Irrigated Pastures for Dairy Cows, Utah Agr. Expt. Sta. Bul. 382, 1956, p. 230.

6/ H. D. Hughes, Maurice E. Heath, and Darrel S. Metcalf, Forages, the Science of Grassland Agriculture, Iowa State College Press, 1953, pp. 22, 44.

7/ Frank P. Thrasher, Clee S. Cooper, and Jesse M. Hodgson, "Competition of Forage Species with Canada Thistle as Affected by Irrigation and Nitrogen Levels," Weeds, Vol. II, No. 2, April 1963, p. 137.

Purpose and Objectives

The purpose of this thesis is to determine the optimal combination of grass species, nitrogen, water, and capital on irrigated pasture, and the optimal utilization scheme that will generate the most revenue to cover fixed costs and labor over a three-year time period.

An optimal economic solution will be investigated assuming unlimited supplies of water and nitrogen. While it is common for irrigated pastures to be productive for five to seven years, the study which developed the forage data for this thesis was terminated after only three years. This limitation allowed this study to consider no more than a three-year period. First, an optimal economic solution will be investigated assuming unlimited supplies of water and nitrogen. Then several alterations will be made to answer the following questions:

1) If water is scarce, or is not available, how much nitrogen should be applied?

2) If nitrogen is limited, how much water should one apply?

3) If capital is scarce, how should it be allocated?

The forage utilization activities will be two different weights of feeder steers: (1) buy at 400 pounds and (2) buy at 600 pounds; and two different feeding systems: (1) maximum feed and (2) limited feed; giving four alternatives. The one most profitable or the set of most profitable alternatives will be chosen to utilize the forage from the irrigated pasture.

The primary objective will be to determine the optimal combination of inputs which will maximize revenue to cover fixed costs and labor over a three-year period. This is assumed to be the goal of an operator of irrigated pastures. The marginal return on investment for an additional unit of nitrogen and/or water will also be calculated.

Hypothesis

The hypothesis can be stated as follows: There is some optimal combination of grass species, capital, fertilizer, water, and livestock on a given tract of land which will generate a larger profit than any other expressed combination of inputs, assuming irrigated pasture will be grown and utilized by feeder steers.

Limitations

Management of irrigated pasture is as critical as the input of any other resource. 8/9/10/ However, "good management" is a hard input to define and quantify. "Good management" was assumed in this study and was not considered a variable. This is definitely a limitation of the study. If accepted management techniques such as clipping and pasture rotation are not followed, results will be different than reported in this study and less meaningful from an application viewpoint.

8/ Cooper, *op. cit.*, p. 2.

9/ D. E. Baldridge, *Increased Production from Irrigated Pasture Through Fertilization and Management*, Mont. Agr. Expt. Sta., Huntley Branch Sta. Cir. 103, February 1965, p. 4.

10/ Frank B. Morrison, *Feeds and Feeding*, 22nd Ed., Morrison Publishing Co., Clinton, Iowa, 1958, p. 230.

This thesis is based on data obtained from well-drained Bozeman brown silt soil and the climate found at Bozeman, Montana. However, it is hoped that these restrictions are not too limiting, and if not the findings, the procedure will be applicable throughout Montana and surrounding areas.

CHAPTER II

THE DATA

Pasture Data

The irrigated pasture data is the result of an experiment conducted by the Montana Agricultural Experiment Station and the Agricultural Research Service, U. S. Department of Agriculture on Bozeman silt loam soil (brown phase) at Bozeman, Montana.

The experiment was conducted in a split-split-plot randomized block design of three replications. Irrigation treatments were assigned to main plots, nitrogen treatments to subplots, and species to sub-subplots which were 5 feet by 20 feet in size. Irrigation treatments were as follows:

1) No irrigation, natural precipitation only;
2) Irrigation when 90 percent of the available soil moisture had been depleted;
3) Irrigation when 60 percent of the available soil moisture had been depleted; and
4) Irrigation to keep the available soil moisture near field capacity.

Nitrogen was applied annually at treatment levels of 0, 50, 100, 200, and 400 pounds per acre respectively. 1/

Six species of grasses were grown consisting of two common sod-forming species, Troy bluegrass (*Poa pratenis L.*) and Lincoln smooth

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1/ Clee S. Cooper, Murray G. Kalges, and Jergen Schultz-Schaeffer, "Performances of Six Grass Species Under Different Irrigation and Nitrogen Treatments," *Agronomy Journal*, Vol. 54, 1962, p. 283.

bromegrass (Bromus inermis Leyss.); two common bunchgrass species, Potomac orchardgrass (Dactylis glomerata L.) and Alta tall fescue (Festuca arundinacea Schreb.); a dryland species, Russian wildrye (Elymus funceus Fisch.); and a wetland species, Reed canary grass (Phalaris arundinacea L.). Only one species was evaluated in this thesis. It was a bunchgrass species, Potomac orchardgrass. The reason for selecting this species is outlined below in the section entitled "Species."

Plots were seeded in the spring of 1956. At time of seeding, 175 pounds of available phosphorus (400 pounds of P_2O_5) per acre was applied. All plots were irrigated uniformly during the seeding year. Plots were clipped at the end of the growing season and the herbage removed. 2/ In the spring of the three following years before growth started, nitrogen was broadcast at the stated rates.

Plots were flood irrigated when available soil moisture had declined to the appropriate moisture percentage. The time of irrigation was determined by periodically sampling the soil moisture content at the 0 to 6 inch level in 1957 and the 0 to 9 inch level in 1958 and 1959. Since sampling all plots was not feasible due to the size of the experiment, samples were obtained under orchardgrass and bromegrass grown with 100 pounds of N per acre only. Each time water was applied, enough water was applied to wet the soil to at least 3 feet. Adequacy of irrigation was determined by sampling the soil with a probe following each irrigation. 3/

2/ Ibid.

3/ Ibid.

For those plots treated to maintain the available soil moisture at field capacity, the plots were irrigated often with only enough water to cover the plots. They were irrigated approximately twice a week in 1957 and three times a week in 1958 and 1959. At the time of each harvest, irrigation was suspended for approximately a week on all treatment levels. While part of the irrigation recrods for 1957 were lost, the number of irrigations in 1958 were 0, 3, 5, and 32 for Zero, Low, Medium, and High irrigation treatments respectively. For 1959, there were 0, 2, 5, and 27 irrigations for the same irrigation treatments. The number of irrigations in 1957 and the amount of water used for each treatment was assumed to be similar to the data for the years 1958 and 1959.

Four harvests were taken in 1957 and five in 1958 and 1959. In 1958 and 1959, the harvests were taken at 28-day intervals after the first harvest. Yield samples were taken by harvesting a 3 x 16 foot strip lengthwise through each plot. Samples were weighed green and subsampled for moisture content. Yields were computed and reported in terms of herbage containing 12 percent moisture. 4/

Previous Use

This data was originally generated for a study attempting to determine by statistical means the relative responses of the grass

4/ Ibid.

species and the legume species to the various treatment levels of nitrogen and water. Later, interest was attracted to the fact that varying stands of Canadian thistle (Cirsium arvense L. Scop.) were present in the experimental units and a study was conducted using this data plus data on the thistle stands to compare the competitive ability of the eight species under the various treatment levels. These results have been reported and published in several articles. 5/6/7/8/

Problems and Limitations

A limitation with respect to this data is that the experiment was a split-split-plot randomized block design and this study does not utilize the special properties of this design. However, the very well-done work already published using statistical techniques on this data has already utilized the advantages of this design. This thesis uses the data for a linear programming economic model with no intention of redeveloping the research already done.

A problem was present in the pasture data since in 1957 only four harvests were taken, while in 1958 and 1959 five harvests were taken.

5/ Clee S. Cooper, "A Comparison of Birdsfoot Trefoil and Ladino Clover Under Varying Irrigation and Fertility Levels," Agronomy Journal, Vol. 53, 1961, pp. 180183.

6/ Clee S. Cooper and R. F. Eslick, "Influence of Pasture Type and Management Practice Upon Barley Yields," Agronomy Journal, Vol. 55, 1963.

7/ Cooper, Klages, Schultz-Schaeffer, op. cit.

8/ Frank P. Thrasher, Clee S. Cooper, and Jesse M. Hodgson, "Competition of Forage Species with Canada Thistle as Affected by Irrigation and Nitrogen Levels," Weeds, Vol. II, No. 2, April 1963.

In order to enter the data into a relatively simple linear program, the 1957 data had to be transformed to five harvests or the 1958 and 1959 data had to be transformed to four harvests. At the expense of a considerably larger tableau, nonuniform number of cuttings could have been incorporated into the linear program. However, since the smaller program used pushed the upper limit of the computer capacity as it was, this was a critical factor.

TABLE I. HARVEST DATES FOR 1957, 1958, AND 1959 RESPECTIVELY.

Cutting	1957	1958	1959
Cutting 1	June 11	May 25-29	June 1-4
Cutting 2	July 17-24	June 23-26	June 28-July 2
Cutting 3	Aug. 19-20	July 21-25	July 27-30
Cutting 4	Sept. 25	Aug. 18-22	July 24-28
Cutting 5	---	Sept. 15-19	Sept. 21-25

While the cuttings for 1958 and 1959 do not exactly match, they were assumed to have been cut at the same time. The 1957 data was transformed to correspond with the 1958 and 1959 data by using a straight line interpolation method. Growth was assumed to start May 1 and grow at a constant rate. The five cuttings were estimated as follows:

June 1 Cutting: (Cutting 1) = 31/42 of June 11, 1957 cutting;

June 27 Cutting: (Cutting 2) = 11/42 of June 11 cutting plus 16/39 of July 17 cutting;

July 26 Cutting: (Cutting 3) = 23/39 of July 17 cutting plus 6/30 of August 19 cutting;

August 21 Cutting: (Cutting 4) = 24/30 of August 19 cutting; and

September 20 Cutting: (Cutting 5) = all of September 25 cutting.

The fractions are computed by:

$$\frac{\text{Number of possible growing days to computed cutting date}}{\text{Number of possible growing days to actual cutting date}}$$

For instance, if the forage started growing on May 1 and grew until the computed cutting date of June 1, 31 days of growth would have occurred. This is the numerator in the fraction for the June 1 cutting. If the forage started growing on May 1 and grew until it was actually cut on June 11, 42 days of growth would have occurred; this is the denominator of the fraction for the June 1 cutting. The same procedure was used to determine the fractions for the subsequent cuttings, complicated only by having to find a fraction for two different cuttings instead of one.

The straight line growth line assumed is only an approximation of the true growth curve. Morrison 9/ reports that in Pennsylvania a test pasture cut every month yielded only 66 percent as much dry matter over a five-year period as a pasture cut only twice a season. Therefore, one would expect to find the pasture cut four times a year yielding more than pasture cut five times a year. How much more is unknown.

The linear program was structured such that total digestible nutrient was the measure of forage production. Therefore, the forage data, given in terms of dry matter, was transformed to pounds of TDN.

9/ Morrison, *op. cit.*, p. 230.

After studying the various figures for TDN published in Morrison's Feeds and Feeding for various stages of maturity of pasture and hay, a figure of 62.2 percent TDN was chosen. This figure is relatively arbitrary and may be in error up to 5 percent either way. However, it is the best estimate possible and was used with this in mind. The raw data was given as percentages of a ton so the data was multiplied by 1,244 to get pounds of TDN. The data in terms of pounds of TDN was then used in the program.

Species

One species of grass, Potomac orchardgrass was included in this analysis for the following reasons.

Orchardgrass is a very palatable grass which is highly recommended for inclusion in an alfalfa-grass mixture for hay. It compared very favorably with the other grasses in this experiment in terms of forage produced. Because of its general popularity and excellent production, it was selected.

Bromegrass was originally chosen for this study also. Because of computer limitations it was eliminated. It is a sod-forming grass which is palatable and resistant to stress caused by overgrazing. Also, it is winter hardy, more so than orchardgrass. In the statistical studies already completed on this data, bromegrass proved to be one of the highest producers.

There were four other grasses in the original experiment; Reed canary grass, Russian wildrye, Troy bluegrass, and Alta tall fescue. These were eliminated from the study for the following reasons.

Reed canary grass is a species of grass adapted to poorly drained soils and there it is probably unsurpassed as a pasture grass. On well-drained, irrigated land it did not produce as well as other species. 10/

Russian wildrye is a dryland grass. It did not do as well as orchardgrass or bromegrass in an irrigated situation . 11/

Alta tall fescue was considered for this study because of its excellent growth in irrigated pastures. However, it lacks palatability and cattle do not make maximum gains on it.

Troy bluegrass is a variety of Kentucky bluegrass which was included in the forage experiment because of its characteristic of producing forage early, before most other species start producing. This specie is not popular in Montana, mainly because of a shortage of seed.

The salinity of the soil on which these grasses were grown was kept very close to neutral, Ph of about 7. Thus, there was no advantage of growing species with high Ph tolerances such as Alta tall fescue. Likewise, with drainage no problem, the species adapted to poorly drained soils; Reed canary grass, for instance, did not have any advantage.

Livestock Data

Four different livestock alternatives were considered. The program could select any one or all of the four alternatives. The four were:

10/ Cooper, Klages, Schultz-Schaeffer, op. cit.

11/ Ibid.

(1) Buy 400-pound steers in the spring and feed them at a maximum rate until it was more profitable to sell them than to keep them or until October 15, (2) Buy 600-pound feeder steers and feed them at a maximum rate until it was more profitable to sell them than to keep them or until October 15, (3) Buy 400-pound steers in the spring and feed them at a limited rate until they could be sold on the same criteria as above, and (4) Buy 600-pound feeder steers in the spring and feed them at a limited rate until it was more profitable to sell them than to keep them or until October 15.

These four were chosen because they are faily uncomplicated. They are not like a cow-calf operation, for instance, where one must worry about winter feed requirements, cow culling, and breeding problems. The purpose of this paper was to study the affects of inputs on irrigated pasture production, and the feeding operation was used as a means of measuring the value of pasture produced. Because of the emphasis of the study, the feeder operation was simplified by several assumptions. Death loss was assumed to be zero. Transportation costs were estimated and shrink effects were estimated. The livestock were assumed to be of choice quality and required very little management.

In general, it is difficult to feed steers on pasture at any other rate than all they will eat except by chopping the forage and hauling it to them. Two other possibilities exist, however. One is to feed concentrates or grain to enable the steer to consume more TDNs than he would normally get from pasture. The other alternative is to limit the time the steer is allowed to graze per day.

The only alternative for limiting feed to steers feasible for this program is to limit the time the steers are allowed on pasture per day. This requires more labor and is more costly. However, the results will show that in general the limited feed alternatives were not profitable anyway so the problem of how to limit feed is of little importance to this thesis.

Feed Data

The feed requirements of the livestock alternatives were based upon Morrison's Feeds and Feeding. 13/ The animals receiving maximum feed were assumed to gain about 2 pounds per day when they were on feed and gaining. The feed requirements for "growing beef cattle, fed for rapid growth" were used. 14/ The animals receiving limited feed were assumed to gain about 1 pound per day and the feed requirements were taken from "wintering beef calves," until the animals reached 600 pounds and then "wintering yearlings" was used. 15/ As the animals gained during the summer, the feed requirements were straight line interpolated from the data given to give an average requirement per time period. For instance, a 400-pound feeder steer fed for rapid growth needs 6.7 pounds of TDN per day, and a 500-pound animal needs 7.8 pounds of TDN per day. Therefore by straight-line interpolation, a

13/ Morrison, op. cit.

14/ Ibid.

15/ Ibid.

450-pound animal needs an average of 6.7 and 7.8 or 7.25 pounds of TDN per day.

Protein, dry matter, and TDN were the only feed requirements considered. The amount of TDN, protein, and dry matter produced per pound of dry orchardgrass and bromegrass was based upon Morrison's Feeds and Feeding. For both orchardgrass and bromegrass, it was assumed that 66 percent of the total feed available would be grazed. This figure will be affected by different grazing systems, but given the assumption that all four alternatives utilize the same percentage an increase or decrease of this figure will simply add or subtract a given percent to the final revenue and will not affect the optimal solution. If young orchardgrass or bromegrass pasture is grazed by steers, when the TDN requirement for the steers is satisfied, the protein requirement is very liberally satisfied and the dry matter requirement is adequately satisfied. Thus, the only requirement considered was TDN since when that requirement was satisfied the other two were also included.

Price Data

The price data is the weighted average of choice feeder steers in the Kansas City market. 16/ Price data for the last 10 years

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16/ Livestock and Meat Statistics, Sta. Bul. 333, U. S. Department of Agriculture, 1963 and supplements for 1963 to 1968.

(58-67) was obtained for choice feeder steers by weight and by months. The months considered were May, June, July, August, September, and October. The feeder weight classifications were: 300-500 pounds, 500-700 pounds, and 700-900 pounds.

After obtaining the price data a weighted average was determined by using a weighted sum of the digits method. The latter years were weighted heavier than the early years. The actual formula used is as follows:

$$\frac{(10 \times 67 \text{ data})+(9 \times 66 \text{ data})+(8 \times 65 \text{ data})+ \ldots +(2 \times 59 \text{ data})+(1 \times 58 \text{ data})}{55}$$

This resulted in a set of data which recognized the fluctuations of the market over the past 10 years, but which weights the fluctuations and prices received for the past few years heavier than the price level 10 years ago.

It is acknowledged that Billings, Montana, data probably would have been more appropriate than Kansas City data. However, continuous data for Billings was not available. For the particular weights and grades involved, Kansas City price data was regressed against Billings data for the purpose of determining if Billings data was significantly different from Kansas City data. The results are printed in Appendix A. In general, it was found that there was a significant difference, however, the difference was simply a matter of a constant. In other words, given Y = BX + A, B was not significantly different from 1, but A was significantly different from 0. Thus, any difference in the price data used and the "real" market in Montana will simply be an addition or subtraction of a fixed amount, and will not affect the optimal solution. For instance, if a 400-pound steer is bought at $24 per hundred pounds, and sold at $22.80 (a 5 percent decrease) when he is 600-pounds a $40.80 revenue will be realized. If the 400-pound steer was bought instead at $25 and sold at 600 pounds at $23.75 per hundred (a 5 percent decrease) a $42.50 revenue will be realized. Hence, as long as the trend over the summer is the true trend, whether one buys at $24 per hundred at the start of the summer and sells later at a 5 percent decrease in price per hundred or buys at $25 per hundred and sells at a 5 percent decrease makes little difference.

The real risk involved in the feeding operation is the price risk. If a farmer buys 400-pound steers at $25 per hundred, feeds them all summer and then sells his 600-pound steers for $18 per hundred, he has gotten an income of $8 for his effort. Out of this has to come a charge for selling, transportation expenses, and other fixed costs. The price data used is only an estimate at best of what is reasonable to expect the prices to be over a typical summer. The prices fluctuate enough from year to year and month to month that these variabilities discount any bias introduced by using Kansas City data.

As the steers grew during the summer, they were progressively placed into a higher weight category when figuring the selling price. When an animal fell on or close to the dividing line between two weight classes, an average was taken of the two prices respectively and considered the price per hundred. This eliminated the sudden drops in price which would have resulted if the averages were not taken. For example, an animal grown from 400 pounds to 500 pounds would be in the 300-500 pound weight category and priced accordingly. As soon as it gained another 10 pounds, it went into the 500-700 pound category and the price dropped substantially. Hence, if an average was not taken, one would find that the 10 pounds put on from 500 to 510 were extremely unprofitable because of the arbitrary price drop when the 500 pound dividing line was crossed. A price drop was still there, but by averaging the two price data to determine the price at the dividing line, the impact of the dividing line on the price was cushioned.

Miscellaneous Data

The fertilizer used in the grass experiment was ammonia nitrate. The price quoted by a local elevator 17/ September 17 was $94 per ton. This figures out to 14.8 cents per pound of actual nitrogen. Cheaper nitrogen can be purchased, however, since no application charge was assessed, this high price also partly paid for application.

Hay was grown and valued as it stood in the field. Assuming the hay to be of top quality, and allowing for custom hay making, a figure of $5.66 per ton as it stood in the field was arrived at. This figure was arrived at by valuing stacked, good quality grass-hay at $18.16 per ton and subtracting the average custom cost of mowing, raking, baling, and stacking of $12.50. 18/

17/ Montana Elevator Co., 712 East Mendenhall, Bozeman, Montana.

18/ LeRoy D. Luft, Rates for Custom Work in Montana, Cooperative Ext. Service, Cir. 242, Montana State University, May 1968, p. 4.

The variable cost of water in the original solution was valued at zero. The reasoning is that it is generally the practice for ditch companies to charge a flat rate for the water and the farmer uses up to a certain amount for a fixed rate. Therefore, up to the ceiling amount, and it is usually very generous for old water rights, water must be considered a fixed cost. Alternative solutions were also calculated with marginal water costs varied, so if one is interested in a solution with water having a variable cost other than zero, these solutions will also be available in Chapter V.

CHAPTER III

LINEAR PROGRAM

Definition

"The general linear programming problem can be described as follows: Given a set of m linear inequalities or equations in r variables, we wish to find nonnegative values of these variables which will satisfy the constraints and maximize or minimize some linear function of the variables." 1/

In general, the linear program is concerned with solving a special type of programming problem. It is concerned with problems in which all relations among the variables are linear, both in the constraints and the function to be optimized.

Mathematically, this can be stated as follows: Given m inequalities in r variables of the form $A_{ij}X_{ij}[\leq = \geq]B_i$ i=1 ... m, j=1 ... r, we seek values of the variables X_j satisfying the above equation and the equation $X_j \geq 0$ which will maximize or minimize a linear objective function $Z = \sum C_jX_j$ j=1 ... r. 2/

Each vector of X_0 unknown is called an activity. For instance, the vector consisting of all X_1 is an activity and the level the activity is entered into the program represents the value X_1 takes on throughout the program. Each vector of $A_{0j}X_{0j}[\leq = \geq]B_0$ is called a constraint and B_0 is said to constrain the program.

An important characteristic of linear programming is its ability to pick an optimal set of activities. This can be done by other methods,

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1/ G. Hadley, Linear Programming, Addison-Wesley Publishing Co., Inc., 1963, p. 4.

2/ Ibid.

budget studies for instance, however linear programming is a method which assures one that he has a set of activities chosen from the set of considered activities which will maximize the objective function. Budget studies cannot guarantee this unless every possible combination of activities are tried. This is inconvenient with a problem with more than a few score activities.

The traditional way to maximize a function which is constrained by the availability of resources is to use classical marginal analysis and Lagrangian multipliers. This approach assumes a continuous production function. In this problem a production function was difficult to isolate. While a linear program implicitly implies a production function, the function does not have to be computed and it does not have to be continuous.

"It is one of the unfortunate facts of life that the numbers that come out of any analysis are related to the numbers that went into the analysis, and not even linear programming is going to change that." 3/ A disadvantage of linear programming is its tendency to give the weight of computer technology and general mathematics to an analysis conducted with bad data. As with any analysis, one must not forget to consider the accuracy of the data and how apropos the model is. Thus, the linear programming model used in this thesis must be looked at as a tool to analyze the problem and not an end in itself.

3/ Walter W. Garvin, Introduction to Linear Programming, McGraw-Hill Book Co., Inc., New York, 1960, p. 49.

Assumptions

There are several assumptions included in a linear program structure. They are, according to Dantzig 4/ and Handley 5/:

Proportionality

This assumption makes the resources necessary to use an activity at the level of one exactly half of the amount of resources necessary to use the activity at the level of two, and so on. In other words, the input of resources into an activity is a linear function of the output of the activity.

Nonnegativity

While it is mathematically possible to have negative quantities of resources, activities, and final output, it is meaningless in an economic sense. Therefore, for almost all applied problems using linear programming, the assumption of nonnegativity is entered into the program. This assumption does not limit the use and meaning of the program, it simply forces the mathematics of the program to conform to the real world situation.

Linear Objective Function

This assumption requires that each activity add or subtract a set amount to the objective function, and if the activity is used at some

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4/ George B. Dantizg, Linear Programming and Extensions, Princeton University Press, 1963, p. 32.

5/ Hadley, *op. cit.*, p. 5.

level, the amount of contribution to the objective function is simply the amount contributed by one unit of the activity times the amount the activity is actually used. This denies decreasing or increasing returns to scale for a particular activity. In a perfectly competitive market this assumption is not restrictive.

Divisibility

This assumption deals with the divisibility of the activities and constraints. In general, for an optimal solution, the activities will be entered in units and parts of units. For instance, an activity may be entered at 4.897 units. Likewise, a constraint consisting of four units may be divided evenly between three activities.

This assumption is, in general, not limiting since most problems deal with resources which can be divided (acres of land, for instance) and activities which may be used in noninteger levels. Even those activities which cannot be used at levels other than integer levels are not limited by this assumption if they are activated at fairly high levels. For instance, if the program activates a hog raising activity at 150.987 units, it can reasonably be assumed that 151 units of hogs can be raised without greatly affecting the optimality of the solution. However, if the hog raising activity was activated at the level of 1.545, then a very marked change in the optimal solution may be experienced if 1.545 is rounded to either 1 or 2. In cases such as this, a refinement of linear programming, integer programming is used.

Additivity

Additivity is the assumption which forces each equation to entirely account for the resource available. The program may use all of the resource, or it may "save" some of the resource, but in either case, it must account for all of the resource. A slack is an an activity commonly used to "save" a resource. If the resource is not constraining some of the resource may best be "wasted" or "saved" by a slack activity.

These assumptions limit the use of the linear program very little. However, the program is usually only a model of the real situation and must be considered as such. Most analyses of empirical data first set up a model to approximate the actual situation and then analyze the model. Linear programming shares this disadvantage with almost all other optimizing tools. Fortunately in this case, the amount the model deviates from the real situation is small enough that a very good approximation of the problem can be made, and the results of the analysis are apropos to the original problem.

CHAPTER IV

OPTIMIZING MODELS

Several models were developed and analyzed in this thesis. The purpose of the first and most general was to determine the optimal allocation of fertilizer, water, capital, and feeder steers onto 100 acres of land. This model and following models made use of only one species of grass, orchardgrass. The models that were developed after the first were concerned with limiting an input and its affect on the solution. Each of the inputs; water, fertilizer, and capital were individually parametrically varied.

The General Model

The general model consisted of five general subparts. They are: (1) forage producing activities, (2) forage utilizing activities, (3) transfer activities and material balance equations, (4) objective function and capital equations, and (5) the right-hand side. These five parts will be analyzed separately and in the order given. A model of the tableau is given in Figure 1. A listing of the complete model will be included in Appendix B.

Forage Producing Activities

The forage producing activities consisted of a dense, 24 x 20 matrix. There were five possible rates that nitrogen could have been applied and four different rates of irrigation.

The raw grass data was transformed as explained in Chapter II to have the number of cuttings and the time of cuttings correspond in each of the three years. A model of the forage producing activities is shown in Figure 2.

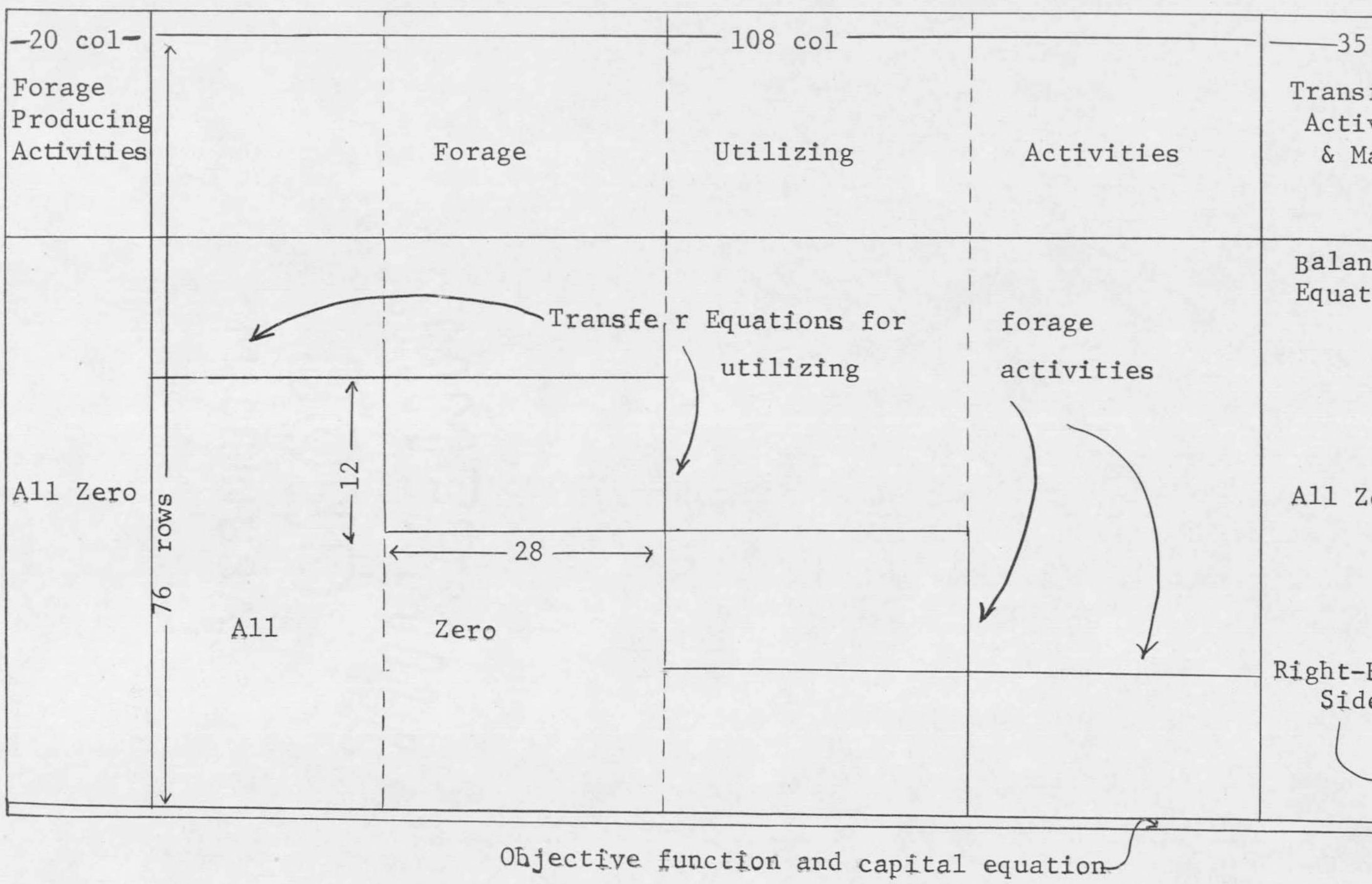

Figure 1. Schematic of Tableau. (One inch equals approximately 20 rows or columns)

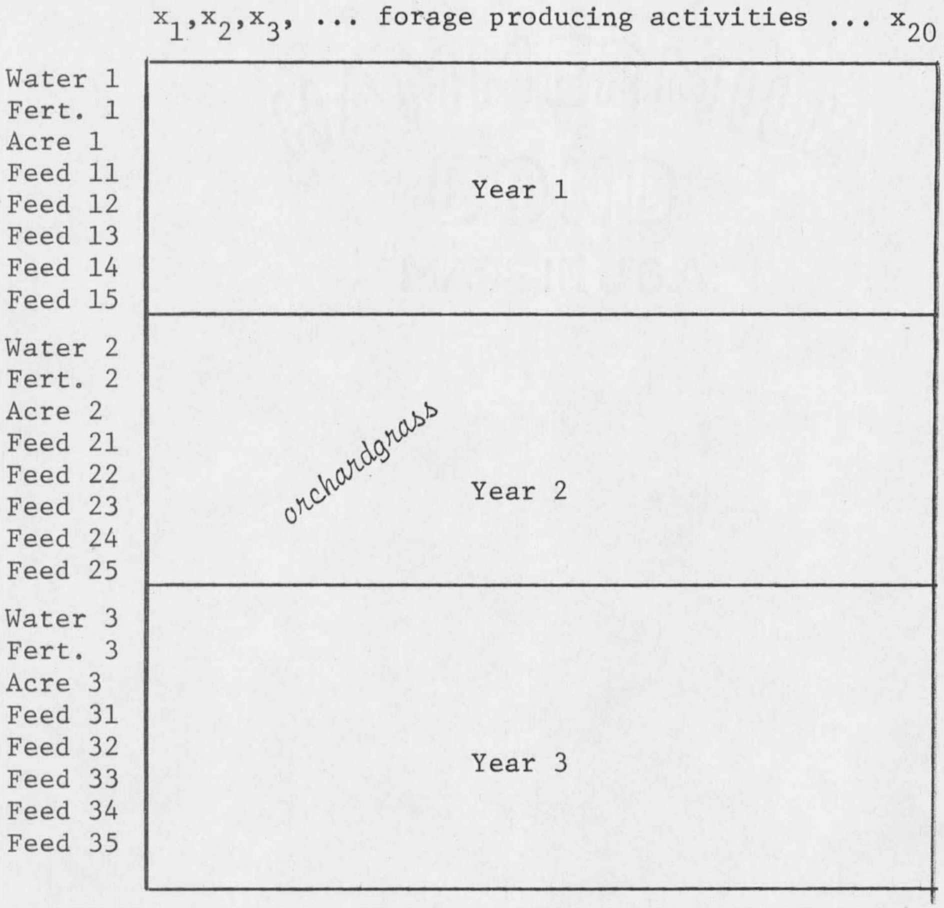

Figure 2. Schematic of Forage Producing Activities.

The coefficients in the fertilizer rows are the stated rate which corresponded to the respective treatments. The coefficients in the water rows are estimates of how much water was actually applied. While the plots were irrigated when percentage of soil moisture reached a predetermined percentage, the actual amount of water applied was unknown. An estimate of water applied was obtained by finding the number of applications of irrigation water during the season and multiplying by the estimated quantity of water per application. This came out to 0, 8, 17, and 25 acre inches of water per acre for the Zero, Low, Medium, and High irrigation treatments. Each unit of activity X_i was considered one acre, so in the acre row, a 1 was entered for each activity.

Feed 11, 12 ... 15; 21 ... 25; 31 ... 35 represented the feed producing rows. The data from the pasture experiment was entered in the appropriate columns and rows. This data had been transformed into pounds of total digestible nutrients produced, and the data was entered into the program in this form.

Forage Utilizing Activities

The forage utilizing activities were activities which allowed feeder steers of various weights and under various feeding systems to consume the forage. This set of activities formed a relatively sparse matrix 76 x 104. There were four possible alternative types of feeder steers available. They were as follows: (1) buy 400-pound feeders and feed them all they could eat, (2) buy 400-pound feeders and limit in some way the available TDN to approximately two-thirds as much TDN as the same weight steer on maximum feed was getting, (3) buy 600-pound steers and feed them all they could eat, and (4) buy 600-pound steers and feed them at the limited rate.

Because of the emphasis of the thesis, only a simple model was set up to utilize the forage. No grain or hay was fed to the steers at any time. A rotation type grazing system, four pastures with the steers on each pasture seven days, was assumed because this type of grazing system corresponded more closely to the way the forage was measured in the grass data experiment.

Several assumptions were made in the livestock operation. They were: (1) the steers had to be transported to and from a market which was 60 miles away, (2) the cattle were on the pasture for two weeks before the first gain was put on and then they gained at 2 pounds per day for those on maximum feed and 1 pound per day for those on limited feed, (3) there was a 2 percent shrink when the animals were sold, and (4) there was a $3.50 per head commission charge at the time of sale.

The first assumption, that of transportation, was necessitated by the observed fact that very few ranchers have their irrigated pasture close enough to a market to eliminate loading the animals into trucks for transporting to and from the market. It was assumed that the pasture was 60 miles from market. A shorter or longer haul from ranch to market will not affect the optimality of the solution, it is a directly linear addition or subtraction to the revenues of the system.

The figure chosen as the cost of transporting the steers was from Harston and Richards. 1/ For the average haul of under 100 miles, $.064 per ton per mile was used. This figure was converted to a cost of transporting 100 pounds of steer 60 miles. The resulting figure, $.192 per hundredweight was added to the price of feeder steers when they were purchased, and subtracted when they were sold.

Winn 2/ indicates that an animal that is transported and marketed under normal Montana conditions requires from 20 to 35 days of feeding to regain the weight lost due to shrinkage. When a rancher buys an animal, most of the shrink has already taken place, and the rancher expects only slight shrinkage from time of buying until the animal is put on pasture. However, it does take an animal a few days to become acquainted with its surroundings and to start eating and gaining at a maximum rate. Because of these factors, it was assumed the animal would weigh the same 14 days after purchase as when purchased and from then on gain at the indicated rate. A shortening of this period may have some effects on the optimal solution, while a lengthening will again reduce revenue by a set percentage. A drastic shortening of this adjustment period make it profitable to buy feeders in May and sell them one month later, in June. Because of transportation

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1/ Clive R. Harston and Jack Richards, Montana Livestock Transportation, Mont. Agr. Expt. Sta. Bul. 592, Bozeman, 1965, p. 31.

2/ Hugh J. Winn, Preconditioning of Feeder Calves and Its Effect on Shrinkage, Agr. Expt. Sta. Research Report 13, Bozeman, May 1960, p. 6.

costs previously mentioned, and other costs that will be discussed in the following paragraphs, it is very unlikely that this one month feeding would be profitable, even if the adjustment period was shortened to zero days. This possibility was not fully explored, however.

A 2 percent shrink was assumed when the animals were sold. This is certainly not excessive. Harston 3/ reports that a 2 to 7 percent shrink can be expected under these conditions, that is, cattle taken from green pastures and hauled 60 miles to market. Once more, this figure is not too important to the optimal solution. A raising of this figure will reduce the revenue by a given percentage but will change the optimal solution very little since all types of livestock activities are affected in almost the same way.

Another charge which was taken into account was direct marketing costs, that is, stock yard expenses. There is a brand inspection fee in Montana of 20 cents per head; a Montana Beef Breeders Council fee of 5 cents per head; and a selling fee of approximately $2.50 per head. This charge of $3.50 per head is paid by the seller. Therefore, this was subtracted from the price expected when the animals were sold.

The feeders were assumed to be bought on or about May 15. They were put onto pasture and from thereafter a decision as to sell them or keep them was made at the fifteenth of each month. It was possible

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3/ Clive R. Harston, Shrinkage Depends on How You Market, Mont. Agr. Expt. Sta. Cir. 22, Bozeman, April 1959, p. 4.

to buy a large number of steers on May 15 and sell them on June 15, July 15, August 15, September 15, or October 15. Those that had not been sold during the summer were sold on October 15 since it was assumed that no cattle were wintered. While technically possible to buy cattle on May 15 and sell them on June 15, this possibility was almost rule out by the added charges associated with buying and selling the livestock and the adjustment period. In reality, one could not get as much as he paid for an animal on May 15 until he had fed the animal at least one full month. That is, if he bought an animal on May 15 and sold it on June 15 he took a loss on the transaction even before fixed costs such as charges for land, fences, etc., were considered.

If an animal was kept the entire summer, it was on feed 153 days. During that time the animals fed at a maximum rate gained 235 pounds or an average of 1.54 pounds per day. The cattle on limited feed gained 115 pounds or .75 pound per day. These figures were arrived at by figuring the animals on maximum and limited feed would gain 2 pounds and 1 pound per day respectively when they were gaining. From this was subtracted the 14 days of adjustment in which they made no gain and a 2 percent shrink at the time of marketing. The figures actually used in the tableau are summarized in Appendix B.

The square boxes in the lower half of the matrix of forage utilizing activities in the schematic shown in Figure 1 symbolizes the transfer equations used to allow the possibility of selling the feeder cattle at the end of each month. Each of the four squares are identical.

Transfer Activities and Material Balance Equation

This is a sparse 30 x 30 matrix which handles the numerous details of the program and forces the tableau to closely approximate the real world situation. This also includes several activities which automatically record and cause the computer to output the levels at which each resource is used.

Because of the importance of this section to the construction of the program, this section is presented here in detail. Only the portion that has nonzero elements is included.

First, there were slacks for water. A slack is an accounting activity which allows a resource to be entered into a tableau without limit or allows the program to "waste" some of the available resource. For example, if 10 acre feet of water is available and the program can use only 8 acre feet, then a slack is entered into the program which either implicitly or explicitly uses the other 2 acre feet. There is no profit associated with the slack; it is an accounting activity.

The water resource in the program was assumed to be unlimited, that is, the program could use all of the water it economically could put to use. Therefore, the water constraint was entered as zero and an artificial slack was entered with a negative one in the water equation. Thus, as water was used in the program, the artificial slack was activated, using negative quantities of water, and in effect producing water. Since the equation had to equal its constraint, zero in this case, the water "produced" had to equal the water used. Thus, by

	Slack water 1	Slack water 2	Slack water 3	Hay trans. 1	Hay 1	Hay trans. 2	Hay 2	Hay trans. 3	Hay 3	Slack land 1	Slack land 2	Slack land 3	Transfer 1	Transfer 2	Slack forage 1	Transfer 3	Transfer 4	Slack forage 2	Transfer 5	Transfer 6	Slack forage 3	Slack fert. 1	Slack fert. 2	Slack fert. 3	Buy capital 1	Buy capital 2	Buy capital 3	Sell capital 1	Sell capital 2	Sell capital 3
Water 1	-1																													
Fert. 1																						-1								
Acre 1										-1																				
Feed 11				-1																										
Feed 12				+1	-1																									
Feed 13													-1																	
Feed 14													+1	-1																
Feed 15														+1	-1															
Water 2		-1																												
Fert. 2																							-1							
Acre 2											-1																			
Feed 21						-1																								
Feed 22						+1	-1																							
Feed 23																-1														
Feed 24																+1	-1													
Feed 25																	+1	-1												
Water 3			-1																											
Fert. 3																								-1						
Acre 3												-1																		
Feed 31								-1																						
Feed 32								+1	-1																					
Feed 33																			-1											
Feed 34																			+1	-1										
Feed 35																				+1	-1									
BeCap																									1			-1		
Cap 1																									-1.08	1		1.04	-1	
Cap 2																										-1.08	1		1.04	-1
Cap 3																											-1.08			-1.04

Figure 3. Schematic of Transfer Activities and Material Balance Equations.

observing the level at which the artificial slack for water was entered, it was possible to determine at a glance the amount of water used in the program. Since there were three years considered, there were three water constraints and therefore, three artificial slacks concerned with water.

With the species used and in this geographic area irrigated pasture, or any pasture for that matter, normally produces most of the forage in the late spring and early summer and the forage production is limited in early spring, the late summer, and fall. It is difficult for any livestock activity to entirely utilize all of the forage throughout the growing season. The two ways of achieving full utilization of the pasture is by adding or subtracting animal units as the need arises, or to utilize some of the excess production as hay or silage. The latter alternative was used in this thesis. The first grazing of the grass forage was around June 1 and this was too early to cut hay. Therefore, pasture not utilized in the first harvest was transferred to the second harvest. At the second harvest, the excess production was utilized as hay. Morrison 4/ indicates that forage allowed to grow two months would produce more dry matter than the same forage cut every month. No calculation was made to correct for this. The units of production were TDN and as grass matures it decreases in percentage of TDN. While more dry matter is produced

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4/ Morrison, op. cit., p. 230.

un a hay crop, the forage produced is lower in TDN and therefore more closely corresponds the TDN's stated in this program. Because there were three years considered, there were six activities concerned with producing hay, three to transfer the forage from one harvest to the next, and three to account for the hay cut. The possibility of not producing any pasture at all had to be left open. Thus, there had to be three slacks for land so the program could leave land idle.

Three transfer equations were entered to enable the program to borrow capital, one for each year. There was no limit on the amount of capital the program could borrow. However, an 8 percent interest charge was assessed. Three more transfer equations were entered to allow the program to save its excess capital. These equations gave the program a 4 percent credit for saving capital.

The total amount of fertilizer used was not constrained. Three transfer equations were entered which allowed the program to buy all of the fertilizer it could use at a price of 14.8 cents per pound.

Starting with the third grazing, the program is designed to allow for possible conservation of pasture for use in later months. That is, the unused forage at harvest three was transferred to harvest four, and the unused forage of harvest four was transferred to harvest five. Again, according to Morrison, the grass probably produced more forage when it was not grazed and allowed to grow two months before harvesting than that grass cut every harvest. However, as the grass gets more mature, the percentage TDN goes down, and while the amount of forage is more than the simple addition of the two harvests, the pounds of TDN is about the same.

In addition to the transfer equations for transferring forage from harvest three to harvest five, there were three slacks which allowed the program to leave some forage unused. The actual tableau, as far as the transfer equations are concerned is summarized in Appendix B.

The Objective Function

The objective function was a function which maximized the capital generated within the system. Working capital was introduced into the system by borrowing at the beginning of each year. The working capital was invested in cattle, water, and fertilizer to raise forage and feed it at a profit to steers. The profit from raising steers in year 1 was used as working capital in year 2 and likewise the profit in year 2 was transferred to year 3. The final amount of working capital which the program had generated at the end of the third year was maximized and was called the objective function.

An example of what took place will be illustrated on the following page. Note this is an illustration and not the actual activities or matrix elements.

	Hay 1	Hay 2	Hay 3	Fert. 1	Fert. 2	Fert. 3	Buy steers 1	Sell steers 1	Buy steers 2	Sell steers 2	Buy steers 3	Sell steers 3	Buy cap 1	Buy cap 2	Buy cap 3	Sell cap 1	Sell cap 2	Sell cap 3
BeCap				-.14			-100						1			-1		
Cap 1	+.00566				-.14			150	-100				-1.08	1		1.04	-1	
Cap 2		+.00566				-.14				150	-100			1.08	1		1.04	-1
Cap 3			+.00566								+1.				1.08			-1.04

Figure 4. Schematic of Capital Equations.

For instance, the "buy steers 1" represents activities, there were four, which bought steers in year 1. The $100 entry in the Becap row represents the cost of buying a steer. In the actual matrix this costs was either $110.52 or $150.72, depending on the weight of the steer purchased. Likewise, the "sell steers 1" represents the activities which sell steers in year 1. Even if the steer was only fed for one month, the proceeds of its sale went into Cap 1 and was not available to provide working capital for year 1.

Note that in general a minus sign indicates a using of capital while a plus sign indicates a contribution of capital. This order was reverse in "Cap 3," the objective function. The computer program used could only minimize the objective function. Since this was a maximization problem, the objective function was multiplied by a negative one. This effectively changed the problem to a minimization problem which the computer could handle from a maximization problem which the computer could not handle.

There were four functions, one of which was the objective function which allowed the program to generate revenue. They were: Becap; Cap 1; Cap 2; and Cap 3. Becap was the equation which starting capital was subtracted from. In order to buy a steer in year 1 it was necessary to subtract capital from "Becap." The net revenue from the first year's operation went into Cap 1. The working capital for year 2 was subtracted from Cap 1 and the net revenue from year 2 was put into Cap 2. Year 3 did the same, it subtracted capital from

Cap 2 and put the net revenue into Cap 3, the objective function. Thus, the objective function, by maximizing the net revenue of year 3, maximized the net revenue of all of the years.

Right-Hand Side

The right-hand side had only four nonzero elements. For "Becap" a figure of 2,000 was entered, effectively giving the program $2,000 of free capital. The "Acre" equations for each year had a limit of 100 which was entered in the right-hand side. These equations allowed the program to use 100 acres or less for pasture in each of the three years.

The decision to use 100 acres was an arbitrary one. Because of the proportionality and divisibility properties of linear programming, any number of acres would have given an identical mix of inputs. The output would be a direct linear function of the number of acres allowed.

Because of the action of the material balance and transfer equations, the rest of the right-hand side were zeroes.

Alternative Models

After the initial general model was constructed and optimized, several revisions were made on it. These revisions, mainly parametric revisions of a matrix element or constraint, were designed to obtain more information from the model.

Fertilizer Varied

In this model only a limited amount of fertilizer was made available. The program was designed to force the use of all the available fertilizer. After the model was constructed, the amount of nitrogen available per acre was varied from 0 to 400 pounds of actual nitrogen in 50-pound increments. The return on investment in terms of revenue or costs was calculated and the input mix was noted. These results are summarized in the next chapter.

Water Varied

In this model the amount of available water was parametrically varied from 0 to 25 acre inches of water per acre per year in 5-acre inch intervals. Nitrogen was unconstrained at 14.8 cents per pound and capital was unconstrained at 8 percent interest.

Marginal Water Costs Varied

In the general model the variable cost of water was assumed to be zero. In this model, fertilizer was again unlimited at a cost of 14.8 cents per pound and the variable cost of water was varied from $0 per acre foot to $4 per acre foot in 50-cent increments. Capital was available at .08 present interest in unlimited amounts.

Working Capital Varied

In this model, water was again set at zero variable cost, fertilizer was unlimited at 14.8 cents per pound, and the amount of capital that could be used was limited. In this model it was impossible to borrow in year 2 or year 3, so any capital requirements for that year had to go through year 1. A charge of 8 percent per year was made for working capital. The amount of working capital available in year 1 was varied from $3,000 to $24,000 in $3,000 increments. Also, another run was made with working capital limited to $1,000. The marginal return on working capital was calculated and the results of this, like the other results, are summarized in the following chapter.

CHAPTER V

RESULTS AND CONCLUSIONS

The solution data for the different models are summarized below, both verbally and in table form. Feeding 400-pound steers at the fastest possible rate was the optimal way to utilize the forage. Every model bought enough 400-pound steers in May to properly utilize the forage grown and all of the steers were fed until October.

Although all of the borrowed working capital had a cost of 8 percent, the capital generated in each of the first two years was available in the subsequent years at zero interest. Likewise, $2,000 of free capital was given every model at the start of year one and this $2,000 also was available in each of the years at zero interest. Corrections for the $2,000 were made by hand in the following tables so the figures printed here may not exactly match the raw output.

Because the borrowed working capital had an 8 percent interest charge, all of the models had, at optimum, a marginal value product for capital of 8 percent per year. However, the return to capital did not equal 8 percent. Because of the free capital previously mentioned, the return to total capital used decreased to something less than 8 percent.

General Model

The general irrigated pasture-feeder steer system represented by the general model described in the previous chapter, was optimized. The value of the objective function at optimum after correcting for the $2,000 of free capital was $9,438.70.

This figure represents the maximum possible return to cover fixed costs and labor over a three-year period. It is gross revenue minus the cost of purchasing the steers, the cost of the nitrogen, the variable cost of water, and a 8 percent interest charge on working capital. Since the only variable costs were considered in the model, it would be necessary to subtract fixed costs from this figure to determine the addition to net worth or to determine profit.

The average return to cover fixed costs and labor each year varies from year to year. The breakdown of the optimal solution is given in Tables II through V.

The general model indicated that 25,624.8 pounds of actual nitrogen per year was optimal. Since 100 acres of pasture were grown this averages 256 pounds of actual nitrogen per acre per year.

Water was applied at the rate of 2,500 acre inches per year. This represented the amount which will keep 100 acres of pasture at field moisture capacity at all times.

The 100 acres of pasture were fertilized in the following manner: 28.12 acres with 400 pounds of actual N per acre, and 71.88 acres with 200 pounds of actual N per acre. The 71.88 acres of pasture were cut for hay on or about June 25. The growth pattern for 28 acres of pasture fertilized with 400 pounds of N very closely approximated the forage demands of 158 head of 400-pound feeder steers. This indicated that approximately 72 percent of the pasture must be deferred from grazing in the spring when the pasture is lush and

TABLE II. ANALYSIS OF GENERAL OPTIMAL SOLUTION, FIRST YEAR.

Activity	H_2O Treat	N Treat	Amount	Cost	Revenue	Revenue to Cover Fixed Costs and Repairs
	In./Acre	Lbs./Acre		---------Dollars--------		
Pasture--orchardgrass	25	400	28.12/A	3,715.60	--	
Pasture--orchardgrass	25	200	71.88/A			
Hay--orchardgrass	25	200	71.88/A	--	1,447.83	
			255.80 T			
Transfer--July-Aug			---			
Transfer--Aug-Sept			27.10 T	--	--	
Transfer--Sept slack			--	--	--	
Cattle--buy 400# rapid growth			158.00Hd	17,462.16	23,546.74	
Free Capital			$2,000.00	--	--	
Borrow Year 1			$19,217.12	1,537.37	--	
				22,715.13	24,994.57	2,279.44

TABLE III. ANALYSIS OF GENERAL OPTIMAL SOLUTION, SECOND YEAR.

Activity	H_2O Treat	N Treat	Amount	Cost	Revenue	Revenue to Cover Fixed Costs and Repairs
	In/Acre	Lbs./Acre		--------Dollars--------		
Pasture--orchardgrass	25	400	28.12/A	3,715.60		
Pasture--orchardgrass	25	200	71.88/A			
Hay--orchardgrass	25	200	67.00/A		1,480.66	
			261.60 T			
Transfer--July-Aug			21.30 T	--	--	
Transfer--Aug-Sept			39.70 T			
Waste--Sept Waste			0			
Cattle--buy 400# rapid growth			189.00Hd	20,888.28	28,166.67	
Free capital			$2,000.00			
Capital from Year 1			$2,279.44			
Borrow Capital Year 2			$20,268.40	1,621.47		
				26,225.35	29,657.33	3,421.98

TABLE IV. ANALYSIS OF GENERAL OPTIMAL SOLUTION, THIRD YEAR.

Activity	H_2O Treat	N Treat	Amount	Cost	Revenue	Revenue to Cover Fixed Costs and Repairs
	In./Acre	Lbs./Acre		---------	Dollars	--------
Pasture--orchardgrass	25	400	28.12/A	3,715.60		
Pasture--orchardgrass	25	200	71.88/A			
Hay--orchardgrass	25	400	28.12/A			
Hay--orchardgrass	25	200	29.40/A		865.41	
			152.90 T			
Transfer--July-Aug			52.20 T	--	--	
Transfer--Aug-Sept			56.30 T	--	--	
Waste--Sept Waste			0	--	--	
Cattle--buy 400# rapid growth			211.00Hd	23,319.72	31,455.33	
Free Capital			$2,000.00			
Capital from Year 1 + Capital from Year 2			$5,701.42			
Borrow Capital			$19,373.48	1,549.88		
				28,585.20	332,310.74	3,725.54

TABLE V. ANALYSIS OF GENERAL OPTIMAL SOLUTION, ALL THREE YEARS.

Item		Amount		
Revenue to Cover fixed Costs and Labor	Year 1	$ 2,279.44	revenue to fixed costs and labor	$9,426.96*
	Year 2	3,421.98		
	Year 3	3,725.54		
		$11,426.96		

*The objective function computed this figure to be $9,438.70. The slight difference is due to rounding error.

producing the most growth. Later, after the first cutting of hay, all of the pasture is then entered into the rotation system.

As previously mentioned, the hay was valued at $5.66 per ton as it stood in the field. Hay represented a significant contribution to revenue, $1,477.83 the first year.

The first year forage was transferred from the August 22 grazing to the September 20 grasing in order to pasture all of the livestock until October 15. The forage deferred to later time periods was grown on about 43 acres of pasture fertilized with 200 pounds of N or on about 20 acres of pasture fertilized with 400 pounds of N.

The first year, 158 head of 400-pound feeder steers were bought May 15 and sold October 15. They were all fed at the rapid rate, that is, all they would eat. The net revenue from this transaction was the major revenue for the first year and was $5,084.58.

The capital requirements for the first year were $21,217.12. Two thousand dollars of this was free capital which was initially introduced into the program and $19,217.12 was borrowed at 8 percent, resulting in an interest of $1,537.37.

The second year the same combination of fertilization was carried out on the 100 acres of pasture. Fewer acres were committed to hay, but because of better forage production per acre, more tons of hay were cut.

The second year, the forage transferred from the July 25 grazing to the August 22 grazing represented 21.3 tons and 39.7 tons were transferred to the September grazing. These represented 19 and 25 acres respectively if the grass from the pasture fertilized with 400 pounds of N was considered. If the pasture fertilized with 200 pounds of N was considered these represented 28.5 and 68.0 acres, respectively.

The second year, 189 steers were bought in May and sold in October. Again, the steers were bought at 400 pounds and fed all they would eat. They generated $7,278.39 of net revenue.

The capital requirement of year 2 was $24,547.84. It is larger than in year 1 because more steers were bought. The $2,000 of free capital was used at zero interest, the $2,279.44 of revenue generated in year 1 was used at zero interest, and $20,268.40 was borrowed at 8 percent interest, resulting in a $1,621.47 charge. This is about 6.64 percent interest on total capital used.

In year 3, all of the pasture fertilized with 400 pounds of N and 29.5 acres of land fertilized with 200 pounds of N were cut for hay. Less hay was cut than in either of the preceding two years.

The forage transferred from July to August was 52.2 tons while 56.3 tons were transferred from August to September. This represents 46 and 78 acres respectively if the land was fertilized with 200 pounds of N per acre.

The third year, 211 head of steers were purchased in May and sold in October. Again they were bought at 400 pounds and fed at the maximum rate. They generated $8,125.61 of revenue.

The capital requirement for the third year was $27,074.90. The original $2,000 and $5,701.42 of revenue generated the preceding two years was used at zero percent interest. The remaining capital requirement of $19,373.48 was borrowed at a cost of $1,547.88. This is a return of 5.73 percent for all capital used. The total revenue and its components is summarized in Table V.

The implication of these results are that if water is free of variable costs or available at a very low variable cost, then the best policy is to irrigate very heavily. At the same time, it is best to fertilize at a heavy rate, over 200 pounds of actual N per acre. This results in a few practical problems. In year 2, for instance, only 33 acres were in the rotation system for the first two months. If these acres were divided up into four pastures of approximately 8 acres each, and the livestock grazed in rotation, 189 head of steers will be grazing 8 acres. Trampling will be a problem and will probably reduce production.

Disregarding the decrease, if any, in production due to trampling and the increase, if any, due to the added fertilizer in the droppings, the outstanding thing about this solution is its heavy rates of irrigation and fertilization.

Fertilizer Constrained

Many operators of irrigated pastures for several reasons do not or cannot apply fertilizer in the quantities demanded by the optimal solution. The operator may have cattle on hand but a severely limited amount of capital, thereby allowing him to pasture steers but forcing him to limit his investment in fertilizer. A table summarizing the affects of limiting the amount of nitrogen available is presented on the following page.

A few operators of irrigated pastures feel that 256 pounds of nitrogen per acre is too much nitrogen to apply to irrigated pasture under any circumstance and they may very well be right. Only about .1 percent decrease in profit is experienced if nitrogen is constrained to 200 pounds per acre. However, if the operator applies 44 pounds of nitrogen more than optimum, the operator experiences about a 7 percent drop in profits. Therefore, it is much better to apply slightly less than the optimal amount of nitrogen than to apply slightly more.

Note that as the available nitrogen is constrained the use of water also decreases. However, there seems to be two different rates

TABLE VI. EFFECT OF VARYING QUANTITY OF NITROGEN (OTHER INPUTS FREE TO VARY).

Available Nitrogen Lbs/Acre	Average Annual Return Per Acre 1/ Dollars	Level of Input Use: Land Mix Percent	Water In./Acre	Nitrogen Lbs./Acre	400-Pound Feeder Steers Hd/A/Yr.	Working Capital Invested Dol/Acre/Year
					.482	40.76
0	28.32	100.0	8	0	.892	64.60
					.411	00.00
		27.4	8	0	.598	53.65
50	29.53	63.5	25	50	.996	93.41
		9.1	25	200	.859	23.75
					.806	84.03
100	30.22	50.0	8	0	1.219	105.49
		50.0	25	200	1.112	57.90
					.967	109.42
150	30.82	25.0	8	0	1.383	133.73
		75.0	25	200	1.462	106.52
					1.129	134.81
200	34.41	100.0	25	200	1.546	161.98
					1.813	155.13
		75.0	25	200	1.533	186.94
250	31.45	25.0	25	400	1.848	200.53
					2.080	190.46
		71.9	25	200	1.584	192.17
256	31.46	28.1	25	400	1.886	202.68
					2.113	193.78

(table continued)

TABLE VI. (continued).

Available Nitrogen Lbs/Acre	Average Annual Return Per Acre 1/ Dollars	Level of Input Use: Land Mix Percent	Water In/Acre	Nitrogen Lbs/Acre	400-Pound Feeder Steers Hd/A/Yr.	Working Capital Invested Dol/Acre/Year
		50.0	25	200	1.820	226.20
300	30.23	50.0	25	400	2.150	242.14
					2.347	229.01
		25.0	25	200	2.096	264.10
350	28.63	75.0	25	400	2.436	282.14
					2.592	265.80
					2.404	305.65
400	25.10	100.0	25	400	2.592	306.31
					2.663	285.05

1/ Average annual revenue to cover fixed costs and labor per acre.

at which water should be applied, either just enough to keep the plants from wilting or enough to keep the soil at field moisture capacity at all times.

All of the figures in Table VI are given in terms of per acre per year. For instance, addition to capital is in terms of addition per acre per year.

Amount of Water Varied

Occasionally an operator cannot or will not apply the optimal amount of water to irrigated pasture. This may be because labor is a constraining factor and the operator cannot find the time to irrigate often enough to keep the ground at field moisture capacity during the hot weeks of the summer. It may also be because the water is simply not available in the quantities necessary to apply the optimal amount.

With this problem in mind, an analysis was conducted assuming the marginal cost of water to be zero and the quantities of water limited. The results of this analysis are summarized in Table VII.

Revenue was constricted very noticeably when no water was available; cut by more than 50 percent, in fact. However, after 8 acre inches per acre per year was available, revenue per acre was not affected too much. When water was constrained from 25 acre inches per acre per year to 10 acre inches per acre, a decrease of 60 percent, the revenue per acre per year was decreased only about 8 percent.

TABLE VII. EFFECT OF VARYING QUANTITY OF WATER (ALL OTHER INPUTS FREE TO VARY).

Water Available Acre In/Acre/Yr.	Average Annual Return Per Acre 1/ Dollars	Level of Resource Use: Land Mix Percent	Water In./Acre	Nitrogen Lbs/Acre	400-Pound Feeder Steers Head/Acre	Working Cap Invested/Ac Dollars
					.385	22.56
0	15.40	100.0	0	0	.254	0
					.196	0
					.466	29.25
5	23.67	37.5	0	0	.653	25.76
		62.5	8	0	.330	0
					.748	69.50
10	29.23	88.2	8	0	1.111	79.95
		11.8	25	400	.701	0
		58.8	8	0	1.247	138.66
15	30.05	10.4	25	200	1.534	141.58
		30.8	25	400	1.318	83.66
		29.4	8	0	1.415	165.41
30	30.76	41.1	25	200	1.710	172.13
		29.5	25	400	1.716	138.70
25	31.46	71.9	25	200	1.584	192.17
		28.1	25	400	1.886	202.68
					2.113	193.78

1/ Average annual revenue to cover fixed costs and labor per year.

This same phenomenon is shown in the section on "Cost of Water Varied." The use of water decreased to about 9 acre inches per acre per year and then changed very little as the cost of water decreased.

Cost of Water Varied

It is not reasonable to assume that all operators of irrigated pasture will have water available in unlimited amounts and at zero variable cost. This analysis is for those situations where an operator has water available in unlimited amounts but at a variable cost. This is the case when an operator is pumping from a well or stream. Then the water is available only at an added cost and cost is a direct function of the amount of water used.

The variable cost of water was incremently varied from $0 to $4 per acre foot. The results of this analysis are summarized in Table VIII. No difference is detected in the quantity of water used when it has a variable cost of zero and when it has a variable cost of 50 cents per acre foot. After this point, as the cost is systematically raised, change takes place.

Between $1.50 variable cost per acre foot of water and $4.00 variable cost per acre foot, the change in water use is small and the change is regular. Notice that each time the variable cost raises 50 cents, the return drops 47 cents per acre per year. The number of livestock per acre per year and the capital requirements vary regularly as the variable cost of water is raised.

TABLE VIII. EFFECT OF VARYING VARIABLE COST OF WATER.

Variable Cost of Water Per Acre Foot	Average Annual Return Per Acre 1/ Dollars	Level of Input Use: Land Mix Percent	Water In./Acre	Nitrogen Lbs/Acre	400-Pound Feeder Steers Head/Acre	Working Cap. Invested/Acre Dollars
					1.584	192.17
0	31.46	71.9	25	200	1.886	202.68
		28.1	25	400	2.113	193.78
					1.584	193.21
.50	30.25	71.9	25	200	1.886	204.85
		28.1	25	400	2.113	197.11
					1.190	131.33
1.00	28.28	68.7	8	0	1.470	134.23
		31.3	25	400	1.180	69.42
					.720	67.10
1.50	27.87	89.4	8	0	1.080	79.39
		10.6	25	400	.670	0
					.710	66.31
2.00	27.40	89.7	8	0	1.080	79.21
		10.2	25	400	.660	0
					.700	65.52
2.50	26.93	90.2	8	0	1.080	79.02
		9.8	25	400	.650	0
					.696	64.75
3.00	26.47	90.6	8	0	1.070	78.84
		9.4	25	400	.640	0

(table continued)

TABLE VIII. (continued).

Variable Cost of Water Per Acre Foot	Average Annual Return Per Acre 1/ Dollars	Level of Input Use				
		Land Mix Percent	Water In./Acre	Nitrogen Lbs/Acre	400-Pound Feeder Steers Head/Acre	Working Cap. Invested/Acre Dollars
					.687	63.99
3.50	26.01	90.9	8	0	1.060	78.66
		9.1	25	400	.635	0
					.679	63.24
4.00	25.57	91.3	8	0	1.054	78.48
		8.7	25	400	.626	0

1/ Revenue to cover fixed costs and to labor.

It is in the region between 50 cents and $1.50 variable cost per acre foot of water where the most change takes place. Revenue per acre dropped $2.38 per acre per year and the actual water used dropped from 25 inches per acre to approximately 9.8 inches per acre per year. The amount of nitrogen used dropped from 256 pounds per acre to 42 pounds per acre per year on the average.

The general recommendations can be summarized as follows: If the variable cost of water is less than 75 cents per acre foot of water, use the same solution as was generated for water with a variable cost of zero. If water costs more than $1.50 variable cost per acre foot, then some combination of heavy-fertilization, heavy-irrigation and light-irrigation, no-fertilization is optimal. Devote most of the land to the light-irrigation and no-fertilization combination.

Capital Constrained

Many operators of irrigated pastures find themselves short of operating capital. The purpose of this analysis is to determine the effects, if any, a restriction of operating capital will have on the optimal solution and profit.

Capital which was generated in year 1 was available at zero interest in year 2 and 3. Likewise, the capital generated in year 2 was available in year 3 at zero interest. The capital constraint was the amount of capital available to be borrowed at 8 percent interest at the start of year 1.

Year 2 was the year the capital requirement was the largest because more forage per acre was produced that year. More cattle had to be bought that year than in year 1, resulting in a demand for more capital. The capital generated from year 1 was available, but was not enough to satisfy the added demand. Year 3 had more demand for capital than year 1, but the capital generated in years 1 and 2 offset this added demand and until the capital constraint got below $30 per acre available capital, the available capital in year 3 was not all being used.

The results of this analysis are summarized in Table XI. The data is all listed in terms of per acre per year to make it easier for an operator with an odd number of acres to calculate the optimal solution for his particular situation.

The unconstrained solution borrowed $192.17, $202.68, and $193.78 per acre respectively in years 1, 2, and 3. The return to capital was equal to the cost of capital and was 8 percent in this case. The return to capital did not increase too much until the capital was limited to $60 per acre per year. At that figure the return to capital jumped from 10 percent compound interest to 15.4 percent compound interest. If only $10 of capital per acre per year is available, 31.5 percent compound return is obtained, a very respectable return indeed.

If an operator has no money of his own to invest but can borrow money at 8 percent, he can profitably borrow up to the amount listed under the title "Capital Requirement" for his particular situation.

TABLE IX. EFFECT OF VARYING THE QUANTITY OF WORKING CAPITAL.

Available Capital Per Acre/Year	Average Annual Return Per Acre 1/	Return to Working Capital	Level of Input Use			
			Land	Water	Nitrogen	400-Pound Feeder Steers
Dollars	Dollars	Comp. Pct.	Pct.	In/Acre	Lbs/Acre	Head/Acre
			71.9	25	200	1.584
Unconstrained	31.462	8.0	28.1	25	400	1.886
						2.113
						1.266
180	29.701	8.1	92.0	25	200	1.649
			8.0	25	400	1.904
						1.015
150	29.276	10.0	18.0	8	0	1.431
			82.0	25	200	1.566
						.826
120	28.577	10.0	47.0	8	0	1.240
			53.0	25	200	1.157
						.637
90	27.880	10.0	76.0	8	0	1.049
			24.0	25	200	.744
						.482
60	26.808	15.4	100.0	8	0	.839
						.411
						.271
30	21.418	23.3	100.0	8	0	.496
						.411
						.091
10	17.304	31.5	100.0	8	0	.258
						.410

1/ Revenue to cover fixed costs and labor.

If he can only borrow money at a high rate of interest, say 15 percent, then he can still profitably borrow $60 per acre. See Table XI, column "Return."

Implications for Further Research

This thesis set up a model which, if other data were used, can be useful in determining the value of other crops and types of grazing schemes. For instance, if a set of activities were introduced which would have explored the possibility of harvesting the forage with a cow-calf operation instead of feeder steers, it would have been possible to compare a cow-calf operation with the present study and feeder operation. Hay was harvested in the present study and then sold. It is more reasonable to assume the hay would be fed on the farm, probably to a herd of cows. With a cow-calf operation, it would have been possible to have fed the hay during the winter, and hay may have become an effective constraint.

This identical model would work if data from dryland pastures were substituted for the forage producing activities. It would be impossible to vary irrigation water in that case, but information could be gathered on the effect capital and nitrogen, or any other fertilizer for that matter, had on the model.

This thesis used only one grass species out of a possible six species. If computer facilities were not limiting, it would be relatively easy to enter all six grass species into the model. This would give a more general solution.

This thesis used data from pure stands of each grass species. More production can be obtained if grass species are mixed or if a legume-grass mix is grown. 1/ Data from a mixture of grass species or a legume-grass mix could very easily be substituted for the "forage producing activities."

Using the data obtained from the optimum solution of this thesis it should be fairly easy to compare the profitability of irrigated pasture with other crops. The cost of establishing a stand, labor costs, and fixed costs could be subtracted from addition to capital figure and profit determined. It would then be fairly easy to compare this profit figure with the profit of other crops.

A model similar to the model used in this thesis could give much useful information if data were available in a slightly different form. Instead of having a plot of ground irrigated and fertilized at a predetermined rate all of the years of an experiment, it would be helpful if the plots were irrigated and fertilized at one rate the first year and different rates in the subsequent years. This would give information on how efficient fertilizer and irrigation water is in each year of the plant's life. If it was possible to irrigate heavily the first year and not at all the second year, it would, with the right model, be possible to determine the

1/ Morrison, op. cit.

marginal value product for water each year of a plant's life. Perhaps it is best to allocate a scarce water supply to first-year pasture and no water at all to two-year old pasture. Questions such as this could be easily answered.

The experiment necessary to obtain this type of data would have to be a large one. To get one replication of an experiment on one grass species with four levels of irrigation and five levels of fertilization in each of three years, it would require 8,000 experimental units. Some compromising would reduce this considerably, but the shear size of the experiment has discouraged any experiments of this sort to date.

APPENDICES

APPENDIX A

Monthly Kansas City data was regressed against monthly Billings data to determine if any trends existed and particularly to determine if β in the equation $Y = X\beta + \alpha$ was significantly different from one.

Three different regressions were computed. They were: Good 500-800 pound stocker feeder steers, Choice 500-800 pound stocker feeder steers, and Choice 250-500 pound stocker feeder calves.

A different source of data was used in this analysis than in the actual thesis. The source used here is Stocker-Feeder Cattle Prices by Thomas M. Stubblefield. The data is monthly average prices received at selected markets for the appropriate class of livestock for the years 1957-1961. 1/

1/ Thomas M. Stubblefield, An Analysis of Prices for Stocker-Feeder Cattle at Selected Markets in the United States, 1957-1961, Tech. Bul. 172, Agr. Expt. Sta., University of Arizona, Tucson, 1966, p. 47.

TABLE A-I. REGRESSION DATA FOR CHOICE 500-800 POUND STOCKER FEEDER STEERS.

Location	May	June	July	August	September	October
Kansas City	28.02	27.06	26.98	26.84	26.44	26.09
Billings	26.58	26.25	25.75	25.25	25.16	24.96

	Mean	Standard Deviation	Variance
Kansas City	26.905	.6569	.4315
Billings	25.658	.6497	.4221

(Kansas City) = β(Billings) + α

$Y = .9256X + 3.1556$

t test of β from 0 = 4.55

t test of β from 1 = .2034

R square = .8381

St. error of estimate .295

Source	df	ss	ms	F
due to regression	1	1.808	1.808	20.71
about regression	4	.349	.087	
total	5	2.157		

TABLE A-II. REGRESSION DATA FOR GOOD 500-800 POUND STOCKER FEEDER STEERS.

Location	May	June	July	August	September	October
Kansas City	25.50	24.77	24.67	24.31	24.06	23.76
Billings	24.97	24.79	24.37	23.96	23.77	23.20

	Mean	Standard Deviation	Variance
Kansas City	24.512	.6129	.3757
Billings	24.193	.6359	.4044

(Kansas City) = β (Billings) + α

$Y = .9196X + 2.2646$

t test of β from 0 = 6.37

t test of β from 1 = .56

R square = .9103

St. error of estimate .2052

Source	df	ss	ms	F
due to regression	1	1.710	1.710	40.61
about regression	4	.168	.042	
total	5	1.878		

TABLE A-III. REGRESSION DATA FOR CHOICE 250-500 STOCKER FEEDER CALVES.

Location	May	June	July	August	September	October
Kansas City	29.84	28.99	28.69	28.47	28.67	28.58
Billings	29.33	29.09	28.51	28.51	28.43	28.09

	Mean	Standard Deviation	Variance
Kansas City	27.463	.5639	.3180
Billings	28.660	.4567	.2113

(Kansas City) = β (Billings) + α

$Y = 1.1032X - 4.1533$

t test of β from 0 = 4.11

t test of β from 1 = .38

R square = .8086

St. error of estimate = .276

Source	df	ss	ms	F
due to regression	1	1.286	1.286	16.90
about regression	4	.304	.076	
total	5	1.590		

TABLE A-IV. CATTLE PRICES USED BY MONTHS AND WEIGHTS.* 1/

Month	800-1,000	500-800	300-500	Mean
May	23.18	24.93	27.44	25.18
June	23.33	24.13	27.40	24.95
July	23.06	23.70	26.75	24.50
August	23.41	24.05	26.74	24.73
September	23.13	24.11	26.85	24.69
October	22.66	24.06	26.62	
	23.13	24.16	26.97	

*Source: Livestock and Meat Statistics, Stat. Bul. 333, USDA, 1963 and supplements for 1963 to 1968.

1/ The data was originally from Livestock and Meat Statistics. It was weighted as explained on page 17, resulting in the above table.

APPENDIX B

The following is a listing of the actual program by columns. The model is described in the chapter "Optimal Models." This listing will not work as it is. To change this listing to a form acceptable by a computer, remove the "new column" statement.

Explanation of Columns

<u>Forage Producing Activities--G311Y1 - G345Y1</u>

Col. 1--G--grass

Col. 2--3--orchardgrass

Col. 3--1-4--type of irrigation

Col. 4--1-5--type of fertilization

5--Y--year
6--1--year 1
} meaningless in context actually used

<u>Forage Utilizing Activities</u>

40Rg11-40Rg25--buy 400 steers, feed at maximum rate

60Rg11-60Rg25--buy 600 steers, feed at maximum rate

40Lg11-40Lg25--buy 400 steers, feed at limited rate

60Lg11-60Lg25--buy 600 steers, feed at limited rate

<u>Transfer Activities</u>

hay T1
hay T2
hay T3
} transfer forage from cutting 1 to cutting 2. 1-3 stands for respective year

hay 1
hay 2
hay 3
} hay actually cut in each year

Scap 1 to Scap 3--save capital each year

Bcap 1 to Bcap 3--borrow capital each year

fert Y1 - fert Y2--fertilizer for each year

Sacre 1 - Sacre 3--slacks for land each year

Swat 1 - Swat 3--slacks for water each year

Trag 31 - Trag 53--transfer grass from cutting 3 to cuttings 4 and 5 for each year

```
BEGIN
ROW        CAP3
RHS
                    ACRE1     100.
                    ACRE2     100.
                    ACRE3     100.
END
MATR
NEW COLUMN        (Start forage producing activities)
          G311Y1    WAT1        0.
          G311Y1    ACRE1       1.
          G311Y1    FEED11   1153.19
          G311Y1    FEED21    493.87
          G311Y1    FEED31    202.77
          G311Y1    FEED41    171.30
          G311Y1    FEED51     41.05
          G311Y1    FERT1       0.
          G311Y1    CAP1        0.
          G311Y1    WAT2        0.
          G311Y1    FEED12    568.51
          G311Y1    FEED22    335.88
          G311Y1    FEED32    161.72
          G311Y1    FEED42     49.76
          G311Y1    FEED52     62.20
          G311Y1    ACRE2       1.
          G311Y1    FERT2       0.
          G311Y1    CAP2        0.
          G311Y1    FEED13     95.79
          G311Y1    FEED23    323.44
          G311Y1    FEED33    174.16
          G311Y1    FEED43     24.88
          G311Y1    FEED53     12.44
          G311Y1    ACRE3       1.
          G311Y1    FERT3       0.
          G311Y1    WAT3        0.
          G311Y1    CAP3        0.
NEW COLUMN
          G312Y1    WAT1        0.
          G312Y1    ACRE1       1.
          G312Y1    FEED11   1889.64
          G312Y1    FEED21    823.66
          G312Y1    FEED31    252.66
          G312Y1    FEED41     97.74
          G312Y1    FEED51     28.61
          G312Y1    FERT1      50.
          G312Y1    CAP1        0.
          G312Y1    WAT2        0.
          G312Y1    FEED12    991.47
          G312Y1    FEED22    572.24
```

G312Y1	FEED32	157.99
G312Y1	FEED42	49.76
G312Y1	FEED52	53.49
G312Y1	ACRE2	1.
G312Y1	FERT	50.
G312Y1	CAP2	0.
G312Y1	FEED13	460.28
G312Y1	FEED23	605.83
G312Y1	FEED33	227.65
G312Y1	FEED43	28.61
G312Y1	FEED53	12.44
G312Y1	ACRE3	1.
G312Y1	FERT3	50.
G312Y1	WAT3	0.
G312Y1	CAP3	0.
NEW COLUMN		
G313Y1	WAT1	0.
G313Y1	ACRE1	1.
G313Y1	FEED11	1472.90
G313Y1	FEED21	701.12
G313Y1	FEED31	493.49
G313Y1	FEED41	97.74
G313Y1	FEED51	28.61
G313Y1	FERT1	100.
G313Y1	CAP1	0.
G313Y1	WAT2	0.
G313Y1	FEED12	1277.59
G313Y1	FEED22	998.93
G313Y1	FEED32	314.72
G313Y1	FEED42	199.04
G313Y1	FEED52	153.01
G313Y1	ACRE2	1.
G313Y1	FERT2	100.
G313Y1	CAP2	0.
G313Y1	FEED13	659.32
G313Y1	FEED23	837.21
G313Y1	FEED33	419.23
G313Y1	FEED43	99.52
G313Y1	FEED53	16.17
G313Y1	ACRE3	1.
G313Y1	FERT3	100.
G313Y1	WAT3	0.
G313Y1	CAP3	0.
NEW COLUMN		
G314Y1	WAT1	0.
G314Y1	ACRE1	1.
G314Y1	FEED11	2147.14
G314Y1	FEED21	1120.28
G314Y1	FEED31	594.79

G314Y1	FEED41	292.22
G314Y1	FEED51	195.30
G314Y1	FERT1	200.
G314Y1	CAP1	0.
G314Y1	WAT2	0.
G314Y1	FEED12	1318.64
G314Y1	FEED22	870.80
G314Y1	FEED32	422.96
G314Y1	FEED42	344.59
G314Y1	FEED52	161.72
G314Y1	ACRE2	1.
G314Y1	FERT2	200.
G314Y1	CAP2	0.
G314Y1	FEED13	833.48
G314Y1	FEED23	961.61
G314Y1	FEED33	543.63
G314Y1	FEED43	145.55
G314Y1	FEED53	37.32
G314Y1	ACRE3	1.
G314Y1	FERT3	200.
G314Y1	WAT3	0.
G314Y1	CAP3	0.
NEW COLUMN		
G315Y1	WAT1	0.
G315Y1	ACRE1	1.
G315Y1	FEED11	2491.96
G315Y1	FEED21	1276.95
G315Y1	FEED31	654.92
G315Y1	FEED41	329.50
G315Y1	FEED51	236.36
G315Y1	FERT1	400.
G315Y1	CAP1	0.
G315Y1	WAT2	0.
G315Y1	FEED12	1538.83
G315Y1	FEED22	767.55
G315Y1	FEED32	422.96
G315Y1	FEED42	526.21
G315Y1	FEED51	344.59
G315Y1	ACRE2	1.
G315Y1	FERT2	400.
G315Y1	CAP2	0.
G315Y1	FEED13	588.41
G315Y1	FEED23	721.52
G315Y1	FEED33	472.72
G315Y1	FEED43	170.43
G315Y1	FEED53	87.08
G315Y1	ACRE3	1.
G315Y1	FERT3	400.
G315Y1	WAT3	0.

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          G315Y1    CAP3        0.
NEW COLUMN
          G321Y1    WAT1        8.
          G321Y1    ACRE1       1.
          G321Y1    FEED11   1912.00
          G321Y1    FEED21    794.19
          G321Y1    FEED31    221.77
          G321Y1    FEED41    194.47
          G321Y1    FEED51    103.25
          G321Y1    FERT1       0.
          G321Y1    CAP1        0.
          G321Y1    WAT2        8.
          G321Y1    FEED12    730.23
          G321Y1    FEED22    638.17
          G321Y1    FEED32    339.61
          G321Y1    FEED42    547.36
          G321Y1    FEED52     74.64
          G321Y1    ACRE2       1.
          G321Y1    FERT2       0.
          G321Y1    CAP2        0.
          G321Y1    FEED13    277.41
          G321Y1    FEED23    575.97
          G321Y1    FEED33    327.17
          G321Y1    FEED43    103.25
          G321Y1    FEED53     12.44
          G321Y1    ACRE3       1.
          G321Y1    FERT3       0.
          G321Y1    WAT3        8.
          G321Y1    CAP3        0.
NEW COLUMN
          G322Y1    WAT1        8.
          G322Y1    ACRE1       1.
          G322Y1    FEED11   1635.84
          G322Y1    FEED21    743.07
          G322Y1    FEED31    294.06
          G322Y1    FEED41    221.68
          G322Y1    FEED51     74.64
          G322Y1    FERT1      50.
          G322Y1    CAP1        0.
          G322Y1    WAT1        8.
          G322Y1    FEED12   1377.11
          G322Y1    FEED22    476.45
          G322Y1    FEED32    215.21
          G322Y1    FEED42    133.11
          G322Y1    FEED52     90.81
          G322Y1    ACRE2       1.
          G322Y1    FERT2      50.
          G322Y1    CAP2        0.
          G322Y1    FEED13    522.48
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G322Y1	FEED23	646.88
G322Y1	FEED33	323.44
G322Y1	FEED43	120.67
G322Y1	FEED53	28.61
G322Y1	ACRE3	1.
G322Y1	FERT3	50.
G322Y1	WAT3	8.
G322Y1	CAP3	0.

NEW COLUMN

G323Y1	WAT1	8.
G323Y1	ACRE1	1.
G323Y1	FEED11	1668.98
G323Y1	FEED21	780.21
G323Y1	FEED31	339.65
G323Y1	FEED41	258.96
G323Y1	FEED51	74.64
G323Y1	FERT1	100.
G323Y1	CAP1	0.
G323Y1	WAT2	8.
G323Y1	FEED12	1800.07
G323Y1	FEED22	659.32
G323Y1	FEED32	269.95
G323Y1	FEED42	220.19
G323Y1	FEED52	78.37
G323Y1	ACRE2	1.
G323Y1	FERT2	100.
G323Y1	CAP2	0.
G323Y1	FEED13	696.64
G323Y1	FEED23	998.93
G323Y1	FEED33	522.48
G323Y1	FEED43	245.07
G323Y1	FEED53	28.61
G323Y1	ACRE3	1.
G323Y1	FERT3	100.
G323Y1	WAT3	8.
G323Y1	CAP3	0.

NEW COLUMN

G324Y1	WAT1	8.
G324Y1	ACRE1	1.
G324Y1	FEED11	2294.96
G324Y1	FEED21	1035.85
G324Y1	FEED31	466.66
G324Y1	FEED41	581.41
G324Y1	FEED51	273.68
G324Y1	FERT1	200.
G324Y1	CAP1	0.
G324Y1	WAT2	8.

G324Y1	FEED12	2086.19
G324Y1	FEED22	808.60
G324Y1	FEED32	638.17
G324Y1	FEED42	447.84
G324Y1	FEED52	149.28
G324Y1	ACRE2	1.
G324Y1	FERT2	200.
G324Y1	CAP2	0.
G324Y1	FEED13	746.40
G324Y1	FEED23	1165.63
G324Y1	FEED33	837.21
G324Y1	FEED43	398.08
G324Y1	FEED53	53.49
G324Y1	ACRE3	1.
G324Y1	FERT3	200.
G324Y1	WAT3	8.
G324Y1	CAP3	0.
NEW COLUMN		
G325Y1	WAT1	8.
G325Y1	ACRE1	1.
G325Y1	FEED11	2740.51
G325Y1	FEED21	1289.31
G325Y1	FEED31	684.11
G325Y1	FEED41	913.93
G325Y1	FEED51	767.55
G325Y1	FERT1	400.
G325Y1	CAP1	0.
G325Y1	WAT2	8.
G325Y1	FEED12	1903.32
G325Y1	FEED22	916.83
G325Y1	FEED32	970.32
G325Y1	FEED42	787.45
G325Y1	FEED52	273.68
G325Y1	ACRE2	1.
G325Y1	FERT2	400.
G325Y1	CAP2	0.
G325Y1	FEED13	675.49
G325Y1	FEED23	974.05
G325Y1	FEED33	954.15
G325Y1	FEED43	829.75
G325Y1	FEED53	174.16
G325Y1	ACRE3	1.
G325Y1	FERT3	400.
G325Y1	WAT3	8.
G325Y1	CAP3	0.
NEW COLUMN		
G331Y1	WAT1	17.
G331Y1	ACRE1	1.

G331Y1	FEED11	960.14
G331Y1	FEED21	468.43
G331Y1	FEED31	233.54
G331Y1	FEED41	191.45
G331Y1	FEED51	49.76
G331Y1	FERT1	0.
G331Y1	CAP1	0.
G331Y1	WAT2	17.
G331Y1	FEED12	733.96
G331Y1	FEED22	513.77
G331Y1	FEED32	298.56
G331Y1	FEED42	348.32
G331Y1	FEED52	149.28
G331Y1	ACRE2	1.
G331Y1	FERT2	0.
G331Y1	CAP2	0.
G331Y1	FEED13	547.36
G331Y1	FEED23	924.29
G331Y1	FEED33	419.23
G331Y1	FEED43	170.43
G331Y1	FEED53	12.44
G331Y1	ACRE3	1.
G331Y1	FERT3	0.
G331Y1	WAT3	17.
G331Y1	CAP3	0.

NEW COLUMN

G332Y1	WAT1	17.
G332Y1	ACRE1	1.
G332Y1	FEED11	2372.28
G332Y1	FEED21	1003.35
G332Y1	FEED31	275.85
G332Y1	FEED41	134.02
G332Y1	FEED51	37.32
G332Y1	FERT1	50.
G332Y1	CAP1	0.
G332Y1	WAT2	17.
G332Y1	FEED12	1235.29
G332Y1	FEED22	481.43
G332Y1	FEED32	220.19
G332Y1	FEED42	730.23
G332Y1	FEED52	99.52
G332Y1	ACRE2	1.
G332Y1	FERT2	50.
G332Y1	CAP2	0.
G332Y1	FEED13	650.61
G332Y1	FEED23	792.43
G332Y1	FEED33	419.23

G332YL	FEED43	177.89
G332Y1	FEED53	16.17
G332Y1	ACRE3	1.
G332Y1	FERT3	50.
G332Y1	WAT3	17.
G332Y1	CAP3	0.
NEW COLUMN		
G333Y1	WAT1	17.
G333Y1	ACRE1	1.
G333Y1	FEED11	2221.31
G333Y1	FEED21	979.38
G333Y1	FEED31	338.24
G333Y1	FEED41	221.68
G333Y1	FEED51	90.81
G333Y1	FERT1	100.
G333Y1	CAP1	0.
G333Y1	WAT2	17.
G333Y1	FEED12	1882.17
G333Y1	FEED22	1069.84
G333Y1	FEED32	385.64
G333Y1	FEED42	444.11
G333Y1	FEED52	161.72
G333Y1	ACRE2	1.
G333Y1	FERT2	100.
G333Y1	CAP2	0.
G333Y1	FEED13	933.00
G333Y1	FEED23	1011.37
G333Y1	FEED33	538.65
G333Y1	FEED43	245.07
G333Y1	FEED53	33.59
G333Y1	ACRE3	1.
G333Y1	FERT3	100.
G333Y1	WAT3	17.
G333Y1	CAP3	0.
NEW COLUMN		
G334Y1	WAT1	17.
G334Y1	ACRE1	1.
G334Y1	FEED11	2298.64
G334Y1	FEED21	1108.55
G334Y1	FEED31	541.76
G334Y1	FEED41	463.51
G334Y1	FEED51	170.43
G334Y1	FERT1	200.
G334Y1	CAP1	0.
G334Y1	WAT2	17.
G334Y1	FEED12	2002.84
G334Y1	FEED22	1222.85
G334Y1	FEED32	543.63

G334Y1	FEED42	605.83
G334Y1	FEED52	190.33
G334Y1	ACRE2	1.
G334Y1	FERT2	200.
G334Y1	CAP2	0.
G334Y1	FEED13	779.99
G334Y1	FEED23	1397.01
G334Y1	FEED33	970.32
G334Y1	FEED43	447.84
G334Y1	FEED53	111.96
G334Y1	ACRE3	1.
G334Y1	FERT3	200.
G334Y1	WAT3	17.
G334Y1	CAP3	0.
NEW COLUMN		
G335Y1	WAT1	17.
G335Y1	ACRE1	1.
G335Y1	FEED11	2706.46
G335Y1	FEED21	1318.14
G335Y1	FEED31	698.63
G335Y1	FEED41	725.50
G335Y1	FEED51	497.60
G335Y1	FERT1	400.
G335Y1	CAP1	0.
G335Y1	WAT2	17.
G335Y1	FEED12	2189.44
G335Y1	FEED22	1144.48
G335Y1	FEED32	737.69
G335Y1	FEED42	1153.19
G335Y1	FEED52	394.35
G335Y1	ACRE2	1.
G335Y1	FERT2	400.
G335Y1	CAP2	0.
G335Y1	FEED13	493.87
G335Y1	FEED23	1219.12
G335Y1	FEED33	1032.52
G335Y1	FEED43	858.36
G335Y1	FEED53	286.12
G335Y1	ACRE3	1.
G335Y1	FERT3	400.
G335Y1	WAT3	17.
G335Y1	CAP3	0.
NEW COLUMN		
G341Y1	WAT1	25.
G341Y1	ACRE1	1.
G341Y1	FEED11	877.29
G341Y1	FEED21	408.72
G341Y1	FEED31	191.15
G341Y1	FEED41	198.51
G341Y1	FEED51	83.35

G341Y1	FERT1	0.
G341Y1	CAP1	0.
G341Y1	WAT2	25.
G341Y1	FEED12	650.61
G341Y1	FEED22	335.88
G341Y1	FEED32	174.16
G341Y1	FEED42	307.27
G341Y1	FEED52	177.89
G341Y1	ACRE2	1.
G341Y1	FERT2	0.
G341Y1	CAP2	0.
G341Y1	FEED13	164.21
G341Y1	FEED23	538.65
G341Y1	FEED33	485.16
G341Y1	FEED43	215.21
G341Y1	FEED53	46.03
G341Y1	ACRE3	1.
G341Y1	FERT3	0.
G341Y1	WAT3	25.
G341Y1	CAP3	0.

NEW COLUMN

G342Y1	WAT1	25.
G342Y1	ACRE1	1.
G342Y1	FEED11	1724.21
G342Y1	FEED21	772.59
G342Y1	FEED31	289.64
G342Y1	FEED41	211.60
G342Y1	FEED51	115.69
G342Y1	FERT1	50.
G342Y1	CAP1	0.
G342Y1	WAT2	25.
G342Y1	FEED12	1601.03
G342Y1	FEED22	643.15
G342Y1	FEED32	319.71
G342Y1	FEED42	493.87
G342Y1	FEED52	223.92
G342Y1	ACRE2	1.
G342Y1	FERT2	50.
G342Y1	CAP2	0.
G342Y1	FEED13	725.25
G342Y1	FEED23	961.61
G342Y1	FEED33	600.85
G342Y1	FEED43	294.83
G324Y1	FEED53	90.81
G342Y1	ACRE3	1.
G342Y1	FERT3	50.
G342Y1	WAT3	25.
G342Y1	CAP3	0.

NEW COLUMN

G343Y1	WAT1	25.
G343Y1	ACRE1	1.
G343Y1	FEED11	2092.43
G343Y1	FEED21	879.52
G343Y1	FEED31	283.35
G343Y1	FEED41	322.44
G343Y1	Feed51	90.81
G343Y1	FERT1	100.
G343Y1	CAP1	0.
G343Y1	WAT2	25.
G343Y1	FEED12	2006.57
G343Y1	FEED22	750.13
G343Y1	FEED32	352.05
G343Y1	FEED42	410.52
G343Y1	FEED52	186.60
G343Y1	ACRE2	1.
G343Y1	FERT2	100.
G343Y1	CAP2	0.
G343Y1	FEED13	1165.63
G343Y1	FEED23	1235.29
G343Y1	FEED33	572.24
G343Y1	FEED43	261.24
G343Y1	FEED53	83.35
G343Y1	ACRE3	1.
G343Y1	FERT3	100.
G343Y1	WAT3	25.
G343Y1	CAP3	0.

NEW COLUMN

G344Y1	WAT1	25.
G344Y1	ACRE1	1.
G344Y1	FEED11	2405.42
G344Y1	FEED21	1137.40
G344Y1	FEED31	432.29
G344Y1	FEED41	635.82
G344Y1	FEED51	149.28
G344Y1	FERT1	200.
G344Y1	CAP1	0.
G344Y1	WAT1	25.
G344Y1	FEED12	2774.12
G344Y1	FEED22	1215.39
G344Y1	FEED32	750.13
G344Y1	FEED42	580.95
G344Y1	FEED52	355.88
G344Y1	ACRE2	1.
G344Y1	FERT2	200.
G344Y1	CAP2	0.
G344Y1	WAT2	25.

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          G344Y1     FEED13    1165.63
          G344Y1     FEED23    1824.95
          G344Y1     FEED33    1140.75
          G344Y1     FEED43     630.71
          G344Y1     FEED53     182.87
          G344Y1     ACRE3        1.
          G344Y1     FERT3      200.
          G344Y1     WAT3        25.
          G344Y1     CAP3         0.
NEW COLUMN
          G345Y1     WAT1        25.
          G345Y1     ACRE1        1.
          G345Y1     FEED11    2282.99
          G345Y1     FEED21    1135.19
          G345Y1     FEED31     798.36
          G345Y1     FEED41    1360.31
          G345Y1     FEED51     799.89
          G345Y1     FERT1      400.
          G345Y1     CAP1         0.
          G345Y1     WAT2        25.
          G345Y1     FEED12    2205.61
          G345Y1     FEED22     924.29
          G345Y1     FEED32    1107.16
          G345Y1     FEED42    1576.15
          G345Y1     FEED52     286.12
          G345Y1     ACRE2        1.
          G345Y1     FERT2      400.
          G345Y1     CAP2         0.
          G345Y1     FEED13     588.41
          G345Y1     FEED23    1712.99
          G345Y1     FEED33    1480.36
          G345Y1     FEED43    1231.56
          G345Y1     FEED53     394.35
          G345Y1     ACRE3        1.
          G345Y1     FERT3      400.
          G345Y1     WAT3        25.
          G345Y1     CAP3         0.
NEW COLUMN
          40RG11     BECAP-     110.52
          40RG11     FEED11-    290.
          40RG11     TRAN1        1.
NEW COLUMN
          40RG21     FEED21-    310.
          40RG21     TRAN1-       1.
          40RG21     TRAN2        1.
NEW COLUMN
          40RG31     FEED31-    338.
          40RG31     TRAN2-       1.
          40RG31     TRAN3        1.
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NEW COLUMN
        40RG41    FEED41-   359.
        40RG41    TRAN3-      1.
        40RG41    TRAN4       1.
NEW COLUMN
        40RG51    FEED51-   381.
        40RG51    CAP1      149.03
        40RG51    TRAN4-      1.
NEW COLUMN
        40RG12    CAP1-     110.52
        40RG12    FEED12-   290.
        40RG12    TRAN5       1.
NEW COLUMN
        40RG22    FEED22-   310.
        40RG22    TRAN5-      1.
        40RG22    TRAN6       1.
NEW COLUMN
        40RG32    FEED32-   338.
        40RG32    TRAN6-      1.
        40RG32    TRAN7       1.
NEW COLUMN
        40RG42    FEED42-   359.
        40RG42    TRAN7-      1.
        40RG42    TRAN8       1.
NEW COLUMN
        40RG52    FEED52-   381.
        40RG52    CAP2      149.03
        40RG52    TRAN8-      1.
NEW COLUMN
        40RG13    CAP2-     110.52
        40RG13    FEED13-   290.
        40RG13    TRAN9       1.
NEW COLUMN
        40RG23    FEED23-   310.
        40RG23    TRAN9-      1.
        40RG23    TRAN10      1.
NEW COLUMN
        40RG33    FEED33-   338.
        30RG33    TRAN10-     1.
        40RG33    TRAN11      1.
NEW COLUMN
        40RG43    FEED43-   359.
        40RG43    TRAN11-     1.
        40RG43    TRAN12      1.
NEW COLUMN
        40RG53    FEED53-   381.
        40RG53    CAP2-     149.03
        40RG53    TRAN12-     1.
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NEW COLUMN
        40RG14    CAP1       110.51
        40RG14    TRAN1-       1.
NEW COLUMN
        40RG24    CAP1       115.14
        40RG24    TRAN2-       1.
NEW COLUMN
        40RG34    CAP1       122.72
        40RG34    TRAN3-       1.
NEW COLUMN
        40RG44    CAP1       136.19
        40RG44    TRAN4-       1.
NEW COLUMN
        40RG54    CAP2       110.51
        40RG54    TRAN5-       1.
NEW COLUMN
        40RG64    CAP2       115.14
        40RG64    TRAN6-       1.
NEW COLUMN
        40RG74    CAP2       122.72
        40RG74    TRAN7-       1.
NEW COLUMN
        40RG84    CAP2       136.19
        40RG84    TRAN8-       1.
NEW COLUMN
        40RG94    TRAN9-       1.
        40RG94    CAP3 -     110.51
NEW COLUMN
        40RG04    CAP3-      115.14
        40RG04    TRAN10-      1.
NEW COLUMN
        40RG15    CAP3-      122.72
        40RG15    TRAN11-      1.
NEW COLUMN
        40RG25    CAP3-      136.19
        40RG25    TRAN12-      1.
NEW COLUMN
        60RG11    BECAP-     150.72
        60RG11    FEED11-    381.
        60RG11    TRAN13       1.
NEW COLUMN
        60RG21    FEED21-    394.
        60RG21    TRAN13-      1.
        60RG21    TRAN14       1.
NEW COLUMN
        60RG31    FEED31-    406.
        60RG31    TRAN14-      1.
        60RG31    TRAN15       1.
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NEW COLUMN
        60RG41    FEED41-    422.
        60RG41    TRAN15-      1.
        60RG41    TRAN16       1.
NEW COLUMN
        60RG51    FEED51-    437.
        60RG51    CAP1       184.12
        60RG51    TRAN16-      1.
NEW COLUMN
        60RG12    CAP1-      150.72
        60RG12    FEED12-    381.
        60RG12    TRAN17       1.
NEW COLUMN
        60RG22    FEED22-    394.
        60RG22    TRAN17-      1.
        60RG22    TRAN18       1.
NEW COLUMN
        60RG32    FEED32-    406.
        60RG32    TRAN18-      1.
        60RG32    TRAN19       1.
NEW COLUMN
        60RG42    FEED42-    422.
        60RG42    TRAN19-      1.
        60RG42    TRAN20       1.
NEW COLUMN
        60RG52    FEED52-    437.
        60RG52    CAP2       184.12
        60RG52    TRAN20-      1.
NEW COLUMN
        60RG13    CAP2-      150.72
        60RG13    FEED13-    381.
        60RG13    TRAN21       1.
NEW COLUMN
        60 RG23   FEED23-    394.
        60RG23    TRAN21-      1.
        60RG23    TRAN22       1.
NEW COLUMN
        60RG33    FEED33-    406.
        60RG33    TRAN22-      1.
        60RG33    TRAN23       1.
NEW COLUMN
        60RG43    FEED43-    422.
        60RG43    TRAN23-      1.
        60RG43    TRAN24       1.
NEW COLUMN
        60RG53    FEED53-    437.
        60RG53    CAP3-      184.12
        60RG53    TRAN24-      1.
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NEW COLUMN
        60RG14    CAP1      143.73
        60RG14    TRAN13-     1.
NEW COLUMN
        60RG24    CAP1      157.22
        60RG24    TRAN14-     1.
NEW COLUMN
        60RG34    CAP1      164.85
        60RG34    TRAN15-     1.
NEW COLUMN
        60RG44    CAP1      176.76
        60RG44    TRAN16-     1.
NEW COLUMN
        60RG54    CAP2      143.73
        60RG54    TRAN17-     1.
NEW COLUMN
        60RG64    CAP2      157.22
        60RG64    TRAN18-     1.
NEW COLUMN
        60RG74    CAP2      164.85
        60RG74    TRAN19-     1.
NEW COLUMN
        60RG84    CAP2      176.76
        60RG84    TRAN20-     1.
NEW COLUMN
        60RG94    CAP3-     143.73
        60RG94    TRAN21-     1.
NEW COLUMN
        60RG04    CAP3-     157.22
        60RG04    TRAN22-     1.
NEW COLUMN
        60RG15    CAP3-     164.85
        60RG15    TRAN23-     1.
NEW COLUMN
        60RG25    CAP3-     176.76
        60RG25    TRAN24-     1.
NEW COLUMN
        40LG11    BECAP-    110.52
        40LG11    FEED11-   289.
        40LG11    TRAN25      1.
NEW COLUMN
        40LG21    FEED21-   300.
        40LG21    TRAN25-     1.
        40LG21    TRAN26      1.
NEW COLUMN
        40LG31    FEED31-   315.
        40LG31    TRAN26-     1.
        40LG31    TRAN27      1.
```

```
NEW COLUMN
         40LG41    FEED41-    329.
         40LG41    TRAN27-      1.
         40LG41    TRAN28       1.
NEW COLUMN
         40LG51    FEED51-    343.
         40LG51    CAP1       119.43
         40LG51    TRAN28-      1.
NEW COLUMN
         40LG12    CAP1-      110.52
         40LG12    FEED12-    289.
         40LG12    TRAN29       1.
NEW COLUMN
         40LG22    FEED22-    300.
         40LG22    TRAN29-      1.
         40LG22    TRAN30       1.
NEW COLUMN
         40LG32    FEED32-    315.
         40LG32    TRAN30-      1.
         40LG32    TRAN31       1.
NEW COLUMN
         40LG42    FEED42-    329.
         40LG42    TRAN31-      1.
         40LG42    TRAN32       1.
NEW COLUMN
         40LG52    FEED52-    343.
         40LG52    CAP2       119.43
         40LG52    TRAN32-      1.
NEW COLUMN
         40LG13    CAP2-      110.52
         40LG13    FEED13-    289.
         40LG13    TRAN33       1.
NEW COLUMN
         40LG23    FEED23-    300.
         40LG23    TRAN33-      1.
         40LG23    TRAN34       1.
NEW COLUMN
         40LG33    FEED33-    315.
         40LG33    TRAN34-      1.
         40LG33    TRAN35       1.
NEW COLUMN
         40LG43    FEED43-    329.
         40LG43    TRAN35-      1.
         40LG43    TRAN36       1.
NEW COLUMN
         40LG53    FEED53-    343.
         40LG53    CAP3-      119.43
         40LG53    TRAN36-      1.
```

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NEW COLUMN
          40LG14     CAP1       106.97
          40LG14     TRAN25-      1.
NEW COLUMN
          40LG24     CAP1       111.50
          40LG24     TRAN26-      1.
NEW COLUMN
          40LG34     CAP1       114.75
          40LG34     TRAN27-      1.
NEW COLUMN
          40LG44     CAP1       119.92
          40LG44     TRAN28-      1.
NEW COLUMN
          40LG54     CAP2       106.97
          40LG54     TRAN29-      1.
NEW COLUMN
          40LG64     CAP2       111.50
          40LG64     TRAN30-      1.
NEW COLUMN
          40LG74     CAP2       114.75
          40LG74     TRAN31-      1.
NEW COLUMN
          40LG84     CAP2       119.92
          40LG84     TRAN32-      1.
NEW COLUMN
          40LG94     CAP3-      106.97
          40LG94     TRAN33-      1.
NEW COLUMN
          40LG04     CAP3-      111.50
          40LG04     TRAN34-      1.
NEW COLUMN
          40LG15     CAP3-      114.75
          40LG15     TRAN35-      1.
NEW COLUMN
          40LG25     CAP3-      119.92
          40LG25     TRAN36-      1.
NEW COLUMN
          60LG11     BECAP-     150.72
          60LG11     FEED11-    381.
          60LG11     TRAN37       1.
NEW COLUMN
          60LG21     FEED21-    392.
          60LG21     TRAN37-      1.
          60LG21     TRAN38       1.
NEW COLUMN
          60LG31     FEED31-    399.
          60LG31     TRAN38-      1.
          60LG31     TRAN39       1.
```

```
NEW COLUMN
          60LG41    FEED41-   404.
          60LG41    TRAN39-     1.
          60LG41    TRAN40      1.
NEW COLUMN
          60LG51    FEED51-   414.
          60LG51    CAP1      156.26
          60LG51    TRAN40-     1.
NEW COLUMN
          60LG12    CAP1-     150.72
          60LG12    FEED12-   381.
          60LG12    TRAN41      1.
NEW COLUMN
          60LG22    FEED22-   392.
          60LG22    TRAN41-     1.
          60LG22    TRAN42      1.
NEW COLUMN
          60LG32    FEED32-   399.
          60LG32    TRAN42-     1.
          60LG32    TRAN43      1.
NEW COLUMN
          60LG42    FEED42-   404.
          60LG42    TRAN43-     1.
          60LG42    TRAN44      1.
NEW COLUMN
          60LG52    FEED52-   414.
          60LG52    CAP2      156.26
          60LG52    TRAN44-     1.
NEW COLUMN
          60LG13    CAP2-     150.72
          60LG13    FEED13-   381.
          60LG13    TRAN45      1.
NEW COLUMN
          60LG23    FEED23-   392.
          60LG23    TRAN45-     1.
          60LG23    TRAN46      1.
NEW COLUMN
          60LG33    FEED33-   399.
          60LG33    TRAN46-     1.
          60LG33    TRAN47      1.
NEW COLUMN
          60LG43    FEED43-   404.
          60LG43    TRAN47-     1.
          60LG43    TRAN48      1.
NEW COLUMN
          60LG53    FEED53-   414.
          60LG53    CAP3-     156.26
          60LG53    TRAN48-     1.
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NEW COLUMN
          60LG14    CAP1      140.72
          60LG14    TRAN37-     1.
NEW COLUMN
          60LG24    CAP1      144.38
          60LG24    TRAN38-     1.
NEW COLUMN
          60LG34    CAP1      151.16
          60LG34    TRAN39-     1.
NEW COLUMN
          60LG44    CAP1      152.95
          60LG44    TRAN40-     1.
NEW COLUMN
          60LG54    CAP2      140.62
          60LG54    TRAN41-     1.
NEW COLUMN
          60LG64    CAP2      144.38
          60LG64    TRAN42-     1.
NEW COLUMN
          60LG74    CAP2      151.16
          60LG74    TRAN43-     1.
NEW COLUMN
          60LG84    CAP2      152.95
          60LG84    TRAN44-     1.
NEW COLUMN
          60LG94    CAP3-     140.62
          60LG94    TRAN45-     1.
NEW COLUMN
          60LG04    CAP3-     144.38
          60LG04    TRAN46-     1.
NEW COLUMN
          60LG15    CAP3-     151.16
          60LG15    TRAN47-     1.
NEW COLUMN
          60LG25    CAP3-     152.95
          60LG25    TRAN48-     1.
NEW COLUMN
          HAYT1     FEED11-     1.
          HAYT1     FEED21      1.
NEW COLUMN
          HAY1      FEED21-     1.
          HAY1      CAP1         .00566
NEW COLUMN
          HAYT2     FEED12-     1.
          HAYT2     FEED22      1.
NEW COLUMN
          HAY2      FEED22-     1.
          HAY2      CAP2         .00566
```

```
NEW COLUMN
        HAYT3     FEED22-     1.
        HAYT3     FEED23      1.
NEW COLUMN
        HAY3      FEED23-     1.
        HAY3      CAP3-        .00566
NEW COLUMN
        SCAP1     BECAP-      1.
        SCAP1     CAP1        1.04
NEW COLUMN
        SCAP2     CAP1-       1.
        SCAP2     CAP2        1.04
NEW COLUMN
        SCAP3     CAP2-       1.
        SCAP3     CAP3-       1.04
NEW COLUMN
        BCAP1     BECAP       1.
        BCAP1     CAP1-       1.08
NEW COLUMN
        BCAP2     CAP1        1.
        BCAP2     CAP2-       1.08
NEW COLUMN
        BCAP3     CAP2        1.
        BCAP3     CAP3        1.08
NEW COLUMN
        SACRE1    ACRE1       1.
NEW COLUMN
        SACRE2    ACRE2       1.
NEW COLUMN
        SACRE3    ACRE3       1.
NEW COLUMN
        FERTY1    FERT1-      1.
        FERTY1    BECAP-       .145
NEW COLUMN
        FERTY2    FERT2-      1.
        FERTY2    CAP1-        .145
NEW COLUMN
        FERTY3    FERT3-      1.
        FERTY3    CAP2-        .145
NEW COLUMN
        SWAT1     WAT1-       1.
NEW COLUMN
        SWAT2     WAT2-       1.
NEW COLUMN
        SWAT3     WAT3-       1.
NEW COLUMN
        TRAG31    FEED33-     1.
        TRAG31    FEED43      1.
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NEW COLUMN
         TRAG32    FEED43-     1.
         TRAG32    FEED53      1.
NEW COLUMN
         TRAG33    FEED53-     1.
NEW COLUMN
         TRAG21    FEED32-     1.
         TRAG21    FEED42      1.
NEW COLUMN
         TRAG22    FEED42-     1.
         TRAG22    FEED52      1.
NEW COLUMN
         TRAG23    FEED52-     1.
NEW COLUMN
         TRAG11    FEED31-     1.
         TRAG11    FEED41      1.
NEW COLUMN
         TRAG12    FEED41-     1.
         TRAG12    FEED51      1.
NEW COLUMN
         TRAG13    FEED51-     1.
NEW COLUMN
END
SOLVE
```

LITERATURE CITED

Baldridge, D. E. Increased Production from Irrigated Pasture Through Fertilization and Management. Mont. Agr. Expt. Sta., Huntley Branch Station Circular 103, February 1965.

Bateman, G. O. and W. Keller. Grass-Legume Mixtures for Irrigated Pastures for Dairy Cows. Utah Agr. Expt. Sta. Bulletin 328, 1956.

Cooper, Clee S. "A Comparison of Birdsfoot Trefoil and Ladino Clover Under Varying Irrigation and Fertility Levels," Agronomy Journal, Vol. 53, 1961.

______________. The Establishment and Production of Birdsfoot Trefoil-Grass Compared to Alfalfa-Grass Mixtures Under Several Cultural Practices. Mont. Agr. Expt. Sta. Bulletin 603, Bozeman, June 1966.

Cooper, Clee S., D. E. Baldridge, and C. W. Roath. Selection and Management of Irrigated Pasture Mixtures. ARS, USDA and Mont. Agr. Expt. Sta., 1966.

Cooper, Clee S. and R. F. Eslick. "Influence of Pasture Type and Management Practice Upon Barley Yields," Agronomy Journal, Vol. 55, 1963.

Cooper Clee S., Murray G. Klages, and Jergen Schultz-Schaeffer. "Performances of Six Grass Species under Different Irrigation and Nitrogen Treatments," Agronomy Journal, Vol. 54, 1962.

Dantzig, George B. Linear Programming and Extensions. Princeton University Press, 1963.

Garvin, Walter W. Introduction to Linear Programming. McGraw-Hill Book Co., Inc., New York, 1960.

Hadley, G. Linear Programming. Addison-Wesley Publishing Co., Inc., 1963.

Harston, Clive R. Shrinkage Depends on How You Market. Mont. Agr. Expt. Sta. Circular 22, Bozeman, April 1959.

Harston, Clive R. and Jack Richards. Montana Livestock Transportation. Mont. Agr. Expt. Sta. Bulltin 592, Bozeman, 1965.

Hughes, H. D., Maurice E. Heath, and Darrel S. Metcalf. Forages, the Science of Grassland Agriculture. Iowa State College Press, 1953.

Luft, LeRoy D. Rates for Custom Work in Montana. Cooperative Extension Service, Montana State University, Circular 242, Bozeman, May 1968.

Morrison, Frank B. Feeds and Feeding. 22nd Ed., Morrison Publishing Co., Clinton, Iowa, 1958.

Thrasher, Frank., Clee S. Cooper, and Jesse M. Hodgson. "Competition of Forage Species with Canada Thistle as Affected by Irrigation and Nitrogen Levels," Weeds, Vol. II, No. 2, April 1963.

United States Department of Agriculture. Grass, The Yearbook of Agriculture. Government Printing Office, Washington, D. C., 1948.

_______________. Livestock and Meat Statistics. Stat. Bul. 333, and supplements for 1963 to 1968, Government Printing Office, Washington, D. C.

_______________. Montana Agricultural Statistics. SRS, Helena, Montana, August 1967.

Winn, Hugh J. Preconditioning of Feeder Calves and Its Effects on Shrinkage. Agr. Expt. Sta. Research Report 13, Bozeman, May 1960.