

THE ALCHIAN AND ALLEN THEOREM: THEORY AND EVIDENCE

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Applied Economics

MONTANA STATE UNIVERSITY-BOZEMAN
Bozeman, Montana

February 1998

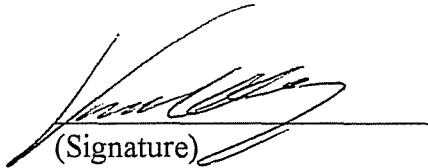
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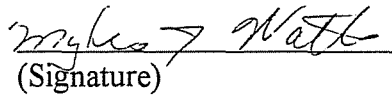
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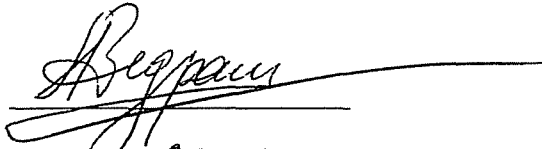
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ACKNOWLEDGEMENTS

I would like to thank my graduate advisor, Dr. Vincent Smith, for his guidance and support throughout my work on this thesis. I also wish to express my sincere appreciation to my advisory committee: Dr. Ronald Johnson, for providing expertise at the most difficult points of my thesis work, and Dr. Randal Rucker, for his helpful suggestions that raised the overall quality of this thesis.

I want to thank my friends and my mother for their continual encouragement and moral support.

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ABSTRACT

The Alchian and Allen theorem states that when a common per unit fee is added to the prices of high and low quality goods, the relative price of the high quality good falls, and its relative consumption increases. The theorem has been analyzed in the literature under the assumption that the prices of the goods are exogenous. This thesis presents a spatial equilibrium model that drops this restriction. The comparative statics analysis developed in this thesis does not support the theorem proposition in the general case. The theorem is then tested empirically on U.S. hard wheat exports. Transport costs, represented by ocean grain freight rates, are not found to have a significant influence on the average quality of U.S. wheat exports for the group of importing countries with a high per capita incomes that have not received U.S. export subsidies. However, transport costs are shown to be positively related with the average quality of U.S. wheat exports for the group of low-income importing countries. Additionally, it is shown that there exists an inverse relationship between the relative price of high quality wheat and the average quality of U.S. wheat exports for high-income countries.

CHAPTER 1

INTRODUCTION

The Alchian and Allen theorem states that when identical per unit fees are added to the prices of high and low quality goods, the relative price of the high quality good falls, and, as a consequence, its relative consumption increases. This proposition first appeared in the literature in Alchian and Allen's textbook, *University Economics*, in 1964. The theorem generated a substantial debate in the literature that has been devoted mostly to specifying the conditions necessary for the theorem to hold. Numerous examples of possible applications of the theorem have been developed. Many of them have been rejected because they did not comply with the theorem's assumptions. The two major contributions that created the theoretical foundation for the Alchian and Allen proposition were the articles by Borcharding and Silberberg (1978) and Umbeck (1980). In these studies, the authors used comparative statics to derive the theorem and identified several important assumptions regarding the conditions necessary for the theorem to hold. The articles indicated that the theorem has a number of real world applications, but comparatively little empirical work has been carried out to test the implications of the theorem. Empirical studies have often been limited because one or more of the theorem assumptions have been violated by the situation in which the theorem's predictions are

being tested. Therefore, to test the theorem empirically, its assumptions need to be clearly stated and shown to be satisfied in each case.

The theorem has been tested exclusively on goods that can be consumed directly without any transformation. It has not been examined in applications where the goods under consideration are inputs in a production process. In this thesis, the Alchian and Allen theorem is applied to a commodity, wheat, that is consumed almost solely as an input to various production processes. This fact makes the analysis of wheat quality different from the examples found in the literature on the theorem.

In addition, the theorem has never before been applied to international transactions, which are usually more economically complex than domestic ones. Examples from the literature on the theorem have exclusively focused on domestic transactions in the United States. When a high-quality good is shipped out over country borders rather than over state borders, *ceteris paribus*, the theorem should hold by analogy. This thesis examines changes in the quality of U.S. wheat exports as the transport costs represented by ocean grain freight rates change.

An important feature of the comparative statics analysis that has been done by Borcharding and Silberberg (1978) is that prices are assumed to be exogenous. Only Umbeck (1980) paid attention to a possible price endogeneity, but did not propose a mathematical model that would incorporate such endogeneity. If the prices of the two goods under consideration are exogenous, an increase in the transport costs is equivalent to increasing the prices by the same amount. This is not true if the prices are endogenous. A change in the transport costs would affect the prices of the high and low quality goods

in both exporting and importing local markets. The corresponding demand shifts influence the relative quantities of the high and the low quality goods. Under such conditions, the predictions of the theorem may not be the same as when the commodities' prices are exogenous. Therefore, a new model reflecting the flexibility of prices is needed. Such a model better approximates real world conditions in international markets. The comparative statics results of the new model can be different from those of the model with exogenous prices. This thesis presents comparative statics analysis for a case in which the goods are inputs to production and prices are endogenous.

In addition, the Alchian and Allen proposition in its original formulation does not include the case when a per-unit subsidy is applied to the two goods under consideration. The effect of a per-unit subsidy on prices is directly opposite to the effect of a per-unit fee. Subsidies are common in international wheat trade, and their effects cannot be neglected. In fact, a large portion of the U.S. wheat exports received some form of a subsidy during the last decade.

This study considers the quality composition of wheat exported from the United States. The goal is to examine how transportation cost changes alter the relative quantities of high and low quality exported wheat. If this relationship is found, the theorem must be useful for individuals and organizations involved in international commodity trade, because the theorem helps to predict relative quality changes in exports when transportation or other per-unit transaction costs change.

Chapter 2 presents a literature review. The theoretical model is formulated in Chapter 3. Chapter 4 discusses quality-determining criteria for wheat and the assumptions

of the theorem. Chapter 5 describes the data used in the estimation procedures, which are presented in Chapter 6. A summary and conclusions are provided in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

Introduction

The topic of this thesis comprises several research areas: the Alchian and Allen theorem, the relationship between wheat quality and wheat prices, the roles of wheat quality and the transport costs in international trade. The purpose of the literature review chapter is to summarize the literature on each of these areas.

The Alchian-Allen Theorem

In 1964, Alchian and Allen proposed a theory to explain why the better grapes were shipped out of California to the eastern United States. The proposition is that when transport costs are added to the prices of the high and low quality good, the relative consumption of the high quality good increases.

The two seminal works on the Alchian and Allen theorem were articles by Borcharding and Silberberg (1978) and by Umbeck (1980). Later Kaempfer and Brastow (1985) and Bertonazzi, Maloney, and McCormick (1993) presented empirical findings

supporting the Alchian and Allen proposition, and Cowen and Tabarrok (1995) made a theoretical contribution to the theorem.

In their economics textbook, Alchian and Allen (1964) noted that empirical verification is often required for the implications of the law of demand. They listed a set of real world events that illustrated these implications. Some of these events were: second hand goods are more heavily consumed near the place of manufacture than farther away; Asian countries import disproportionately more expensive American cars rather than cheaper models; and "luxuries" are disproportionately represented in international trade. They also used a detailed numerical example to explain why better California grapes are exported to New York. They assumed that the transportation costs added to the prices are the same for the two classes of items. The key element in their explanation was that "the price of the better item, relative to the poorer item, is lower after shipment than it is at the place of manufacture before shipment." Alchian and Allen used this argument as indirect evidence of the validity of the law of demand.

Borcherding and Silberberg (1978) generalized the Alchian and Allen proposition by introducing the notion of a per item fee that can be, for instance, the transport costs or a per-unit tax. They also pointed out that the theorem's conclusions do not depend on whether the goods are shipped to the consumers or the consumers are shipped to the goods. Borcherding and Silberberg argued that the Alchian and Allen proposition does not follow mathematically from the law of demand. They derived comparative statics showing the conditions under which the proposition holds. The conditions were that, first, real income must be constant and equal across the buyers in the two places, and, second, the

high and low quality goods must be close substitutes. Finally, Borcharding and Silberberg derived several implications to illustrate the generality of the Alchian and Allen proposition.

The article by Umbeck (1980) was a crucial advance towards stating the conditions necessary for the theorem to hold. The most important of these conditions was that the per unit fixed charge should not bring any utility per se to the consumer. This immediately refuted many of the implications proposed by Borcharding and Silberberg. Then, Umbeck reformulated the settings in which the Alchian and Allen proposition holds and showed that the proposition's prediction follows directly from the law of demand. In his argument, when a consumer buys apples, he or she actually seeks apple juice, which can be called the quality attribute. The more juice apples contain, the higher quality they are. To buy juice, the container (the apple peel) must be purchased. When transport costs are added to the price of the container, consumers buy apples with more juice in them to minimize the expenditures on containers having no economic value. The demand for the quality attribute (apple juice) shifts outward for those consumers who continue to purchase apples. However, whether a consumer is willing to purchase apples after the transport costs increase depends on the consumer's willingness to pay, as noted by Umbeck. To measure the consumer surplus, one needs to use compensated demand functions. This assumption is implicit and went unnoticed by Umbeck in his proof that the Alchian and Allen theorem follows from the law of demand.

Kaempfer and Brastow (1985) considered a case when only one unit of either high or low quality good is consumed, for example, a university education or parking permit.

They argued that a fixed fee does not change the relative prices of different quality goods in such a case. The reason for this is simultaneous income compensation that is equal to the fixed fee. As a result, the relative price of different quality goods remains the same, but the relative price of "any as opposed to no quality" changes. If the consumers of low quality have a smaller consumer surplus than the consumers of high quality, the former consumers would be more likely to leave the market than the latter, and the average quality consumed would rise as a result. The authors used data on University of Washington parking fees. They showed empirically that, although the theorem does not hold at the level of individual choice, it holds at the aggregate level.

Bertonazzi, Maloney, and McCormick (1993) examined an application of the theorem to the case when the goods are not shipped to consumers, but when consumers travel to buy the goods. This application was first suggested by Borchering and Silberberg. Using a unique data set on individual ticket purchases of football games, Bertonazzi et al. showed that the farther the visitors traveled, the more they were willing to pay for the football tickets; that is they bought higher quality seats. No correlation between visitors' income and distance which they traveled was assumed to exist in the study.

Cowen and Tabarrok (1995) addressed the conditions under which the Alchian-Allen theorem holds and found that some conditions had been misspecified. For example, the authors clarified questions about the types of fees that would cause the changes in quality described by Alchian and Allen. Any additional fees not incurred on a per-unit basis, such as import or export permits, do not change relative prices and do not create a

substitution effect between high and low quality goods. Based on these considerations, several theorem implications were shown to be invalid and were re-formulated in a way that satisfied the theorem assumptions.

Hedonic Price Analysis and Its Application to Wheat

The application of hedonic price analysis to a commodity leads to a thorough consideration of the relative importance of the commodity's quality-determining criteria. The results of such studies can be used in many other areas beyond the hedonic price analysis models. Lancaster (1966) and Rosen (1974) laid the foundations of hedonic price analysis. Wilson (1989), Larue (1991), Espinosa and Goodwin (1991), and Ahmadi-Esfahani and Stanmore (1994) evaluated the importance of various wheat quality characteristics. Their findings help to identify the main wheat quality-determining criteria for wheat in this paper.

Lancaster (1966) suggested a new approach to consumer theory. The traditional theory considered goods as direct objects of a consumer's utility function. Lancaster argued that it is the characteristics of the good and not the good itself from which utility is derived. This approach allowed the improved consumption theory to be applied when new goods are introduced or when the quality of the goods changes.

Rosen (1974) presented the idea that the observed price of a good is a function of the good's characteristics or attributes. Each attribute has an implicit or hedonic price which can be obtained by regressing the price of the good on the quantities of attributes.

Unlike Lancaster's model, in which the goods serve as inputs to a consumer's "self-production function" of the final consumption characteristics, Rosen's model separates producers and consumers. Producers create the product's characteristics that, along with consumers' tastes, determine market clearing product prices and, consequently, hedonic prices.

Wilson (1989) used a hedonic price function to estimate the implicit prices of wheat characteristics. Wheat protein content was used as an approximation for protein quality. The function was estimated for two United States and two international locations. However, this did not allow him to distinguish the effects of several quality characteristics on the wheat price because many wheat types were traded only in certain markets and not in others. The price-quantity information on these wheat types was, consequently, insufficient.

Larue (1991) differentiated wheat by end-use and by country of origin and applied the hedonic price model. Different end-uses led to classification of wheat into four types: high-protein wheats, medium-protein wheats, low-protein wheats, and durum wheats. None of the wheat classes produced in the United States were included in the second category. Among the findings were a statistically significant (at a 10% or higher level) difference between the end-use categories and also between wheat characteristics and wheat price, a variation in the protein premium over time and between the end-use categories, and the existence of correlation between wheat and flour characteristics. The article emphasized that "wheat purchased for different end-uses should be considered as related, but different products".

Espinosa and Goodwin (1991) applied a hedonic price model to wheat. They classified wheat characteristics into (1) traditional characteristics which include protein, test weight, moisture content, and total defects, and (2) alternative characteristics, which directly measure the milling and baking properties of wheat and include milling rating, falling number, theoretical flour yield, wet-gluten content (milling properties), dough water absorption, mixing time, dough stability, and the valorimeter measure (baking properties).

The authors ran regressions for each of the two types of characteristics and then for the combination of them. They pointed out a high degree of correlation among the alternative quality characteristics and also between test weight and moisture content.

The conventional quality characteristics found to be significant were percent moisture, protein, and test weight. The alternative characteristics were also found statistically significant in their influence on wheat price. The article concluded that both types of characteristics influence wheat end-use qualities and wheat prices independently of each other.

Ahmadi-Esfahani and Stanmore (1994) implemented a model which was similar to that of Espinosa and Goodwin, but had a different classification of quality attributes. They applied it to Australian wheat and flour exports without differentiating wheat by its end-use. Wheat characteristics included protein, test weight, hardness, thousand-kernel weight, foreign material, ash content, flour extraction, nonmillable material, and amylase activity. Flour characteristics included flour protein, color grade, flour ash, development time, water absorption, and stability. As one would expect with this set of variables,

multicollinearity problems were encountered because these wheat characteristics are interrelated. The results indicated that protein, thousand-kernel weight, ash content, and nonmillable material had statistically significant impacts on wheat price. However, the coefficient for nonmillable material was positive and significant. This implies that consumers are willing to pay a premium for a higher content of nonmillable material in wheat, while such wheat is discounted. The authors explained the positive coefficient by asymmetric information.

Wheat Quality and International Wheat Trade

An important aspect of U.S. wheat trade is how foreign buyers value the imported wheat in terms of quality. Several papers address this topic. Papers by Mercier (1993), Leath (1995), Haley (1990), Pick, Webb, Dusch, and Gudmunds (1994), and Webb, Haley, and Leetmaa (1995) studied individual importers' quality demands. Darr and Gribbons (1985), Larue and Lapan (1990), Devadoss and Meyers (1990), Wilson and Gallagher (1990), Wilson and Preszler (1992), and Larue and Lapan (1992) investigated the relationships between price, quality characteristics, and quantity of wheat traded at the international markets.

Mercier (1993) addressed the issue of U.S. wheat purity. Major countries importing U.S. wheat were divided on the basis of income and type of marketing system. For countries with low income and state trading agencies, price and credit were shown to be of higher priority than wheat quality. The study reported that differences in the price of

wheat between exporters, which could be caused by export subsidies, export credits, and food aid programs, have more impact on import decisions than quality factors. Mercier argued that, under certain conditions, mandatory cleaning of U.S. wheat before exporting may lead to a net benefit for U.S. producers and help the United States maintain its market share in some countries that have special demands to wheat cleanliness.

Leath (1995) examined the role of test weight in determining the value of U.S. wheat. Special attention was paid to Soft Red Winter wheat, and it was found unnecessary to lower the minimum test weight levels in the grading scale, in contrast to a U.S. wheat growers proposition. The New Crop Quality Reporting Program data for 1986-89 crop years and the Grain Inspection Monitoring System inspection data for 1986-94 fiscal years were used. Leath found that test weight affects the productivity, efficiency, and operating costs of flour milling. He also found that the tested effect on profitability of using wheat with a lower test weight is similar to the discounts actually applied in the marketplace. In his survey, flour millers suggested that test weight is a good indicator of overall quality and soundness, and most millers informed him that, for a given class, they rejected shipments with test weights below a certain minimal level. The role of quality in an importer's purchase decisions was found to be in a direct relationship with the income level of an importing country.

Haley (1990) proposed a three-stage theory of wheat import demand. A partial-equilibrium Static World Policy Simulation (SWOPSIM) framework was used for analysis focusing on the importers rather than on the exporters. An Armington-type trade model was used instead of a spatial equilibrium model, and the results were shown to be different

between the two models. Between-class and between-supplier elasticities were calculated for major wheat importing countries. The results were used to analyze the benefits and costs of the U.S. Export Enhancement Program (EEP). The EEP was shown to have significantly smaller export revenue expansion, if wheat was assumed to be a heterogeneous rather than a homogeneous good.

Webb, Haley, and Leetmaa (1995) argued that mandatory cleaning U.S. wheat before exporting it could bring net gains to the U.S. wheat producers. Their conclusion relied upon interviews with the buyers of U.S. wheat. The model used was SWOPSIM, as in Haley (1990). Webb et al. also analyzed four scenarios based upon whether the United States cleans all export wheat or only wheat going to dockage-sensitive markets, and whether U.S. export demand increases. The results indicate that the U.S. wheat industry would gain if wheat exported to dockage-sensitive markets was cleaned.

Pick, Webb, Dusch, and Gudmunds (1994) evaluated the relative importance of U.S. wheat quality characteristics without using hedonic price estimation procedures. Instead, they surveyed buyers of U.S. wheat about their quality needs. They divided the factors leading to a purchase decision into quality criteria, which included various quality characteristics and purchase criteria (price, credit availability, overall quality, and company relationships). The results indicated that price and quality are the only purchase criteria important for both high and low income importing countries. Among the quality criteria, protein content and gluten were highly valued. Moisture level, nonmillable material and falling number ranked lower and were followed by shrunkens and broken, test weight, and total defects. The authors also used supply and attainment indices that showed the

buyers' dissatisfaction with the levels of protein and nonmillable material in U.S. wheat exports.

Darr and Gibbons (1985) examined the effects of wheat prices and the U.S. dollar exchange rate on U.S. wheat exports. Wheat was differentiated by class and country of origin. As the U.S. dollar appreciated against the currencies of most importers during 1981-84, U.S. wheat became less price competitive. In terms of importers' currencies, the price of U.S. wheat went up, and the United States lost a part of its market share because of this in the early 1980s.

Larue and Lapan (1990) used a general equilibrium model to predict how changes in imported wheat quality affect import volumes. The approach was based on the paper by Lancaster (1966): a domestic good has one unit and an imported good has q units of the quality characteristic. This was also very similar to the "juice idea" suggested by Umbeck (1980). One result of the model was that if the quality of the imported good goes up, then imports increase in the case of an elastic import demand or decrease in the case of an inelastic import demand. A central feature of this model was that wheat prices were assumed to be endogenous.

Another prediction was for the case when substitutability between the foreign and domestic wheat depends on the volume of the foreign wheat. An improvement of milling technology makes less foreign wheat necessary to replace a unit of domestic wheat. Domestic production increases, but the direction of the change in the volume of imports depends on the relative sizes of the income and substitution effects.

Devadoss and Meyers (1990) investigated the possibility that the U.S. wheat export demand elasticity is not constant. A nonspatial, partial equilibrium world wheat trade model was used to estimate the domestic wheat demand elasticity. The model did not identify trade flows between specific regions or consider wheat as a heterogeneous commodity. However, government programs and trade restrictions were included in the model. It was shown that the export demand elasticity for U.S. wheat would more than double if trade restrictions, such as quotas and tariffs, were lifted.

Wilson and Gallagher (1990) applied a model developed by Case to estimate the responsiveness of wheat class market shares to price and preference changes. Wheat prices were assumed to be exogenous. In some markets, wheat quality was found to be more important, while in other markets prices were more important in determining wheat class shares.

Wilson and Preszler (1992) used an input characteristics model to analyze the influence of price, quality, and quality characteristics uncertainty on import demand in the United Kingdom. Canadian wheat was found to have lower variability in quality characteristics than U.S. wheat. The authors argued that this caused the United Kingdom buyers to purchase relatively more Canadian wheat for use in the blending of domestically produced wheat.

Larue and Lapan (1992) investigated the fact that both the United States and Canada exceed minimum contractual requirements for quality in wheat exports, but the United States exceeded its minimum contractual obligations by less than Canada did. This was explained by the differences in the exporting countries' reputation and wheat

inspection standards. The United States has many wheat exporting companies, so that a single firm does not internalize all the benefits of higher quality margins because of the free rider problem. This is the case when the importing country does not possess a complete ability to differentiate the wheat of one exporting firm from the other. Therefore, the authors argued that the quality margin of U.S. wheat exports is lower.

Transport Costs

The influence of transport costs on international wheat trade has been examined by Binkley (1983), Goodwin and Schroeder (1991), and Hsu and Goodwin (1995). Roehner (1996) examined the role of the transport costs in a spatial equilibrium framework.

Binkley (1983) examined the impact of marketing costs, such as grain freight rates, on wheat price stability. He reported a statistically significant influence of transport rates on grain price differences between international locations.

Goodwin and Schroeder (1991) investigated dynamic relationships among wheat prices in different international locations. Changes in freight rates were found to cause a slow adjustment (starting after two or more months and persisting for eight months after the shock) in international wheat prices. On the other hand, exchange rate shocks caused an immediate adjustment in wheat prices.

Hsu and Goodwin (1995) examined the ocean grain transport market. One of their findings was that ocean grain freight rates are determined within a competitive market for shipping services. Grain freight rates were found to be responsive to new ship deliveries,

fuel prices, and lagged freight rate shocks. A grain shipment response to a shock in freight rates was not found to be statistically significant. However, significant responses were found for a shock in lagged fuel prices and lagged (four months) grain shipments. The authors did not include grain prices in the model to explain grain shipment variation.

Roehner (1996) considered spatial price differentials of commodities under uncertainty. He examined effects of changing transport costs on volume of trade, price volatility, and spatial price differentials. He developed and implemented a stochastic version of the Enke-Samuelson model and found, among other things, that decreases in transport costs reduce price volatility. An important observation in the article was that fluctuations in tariff levels mask the effect of transport costs on the prices in the international markets.

The articles discussed in this chapter contain results that will be used further in this study. Among the most important findings are the analysis of the Alchian and Allen theorem in the articles by Borcharding and Silberberg (1978) and Umbeck (1980), the discussion of the relative importance of wheat quality characteristics, the principle of differentiating wheat by end-use in the article by Larue (1991), and the classification of wheat importing countries by Mercier (1993).

The theoretical results by Borcharding and Silberberg (1978) form the starting point for the next chapter, while the findings related to wheat quality are extensively used in Chapters 4 and 6.

CHAPTER 3

THEORY

Introduction

Borcherding and Silberberg (1978) showed that the Alchian and Allen theorem prediction holds for the case of two regions and three goods. They assumed that the buyers of each good are final consumers. The theorem has yet to be applied to the cases where the goods under consideration are inputs in production processes. In addition, the literature on the Alchian and Allen theorem has never addressed the effects of a per-unit subsidy in lieu of a per-unit fee because subsidies are not observed in domestic transactions. Since this thesis is about international trade, the role of subsidies needs to be discussed. Another feature of Borcherding and Silberberg's comparative statics analysis was that prices were exogenous. This significantly weakens the relevance of the results derived from their model for real world applications. This chapter presents a spatial equilibrium model in which prices are considered endogenous and are allowed to adjust as the transport costs change. These three issues—goods being inputs to production, a per-unit subsidy, and prices being endogenous—will be examined in this chapter.

The Theorem Applied To Production

Borcherding and Silberberg (1978) and Umbeck (1980) concluded that the Alchian and Allen theorem holds for the consumption of two goods that are close substitutes if certain assumptions are satisfied. There is nothing in their analysis that would preclude the application of the theorem to the same goods when they are used as inputs to production provided that the additional assumptions are satisfied. One can assume that in Umbeck's example a person purchasing the apples does not eat them, but decides to make apple cider and sell it. In either case, the buyer is interested in the quantity of juice the apples contain. The conclusions of Umbeck's paper did not require any specific use of apples purchased. The relevant requirement was that the buyer is looking for more of the quality attribute per unit ("container")—apple juice per apple—and the higher the amount of apple juice in each apple, the more utility the buyer derives from using the same number of apples. A consumer receives higher satisfaction or utility from eating higher quality apples. Analogously, a producer earns higher profits from using higher quality apples for making cider. This idea leads to the following model.

Assume that there are three inputs, x_1 , x_2 , and x_3 . Let x_1 and x_2 be the quantities of the high quality and low quality inputs, respectively, and x_3 be a composite of all other inputs, such as labor and capital. Then w_1 , w_2 , and w_3 are the factor costs ($w_1 > w_2$), P is the output price, and $f(x_1, x_2, x_3)$ is the production function. Assuming a perfectly competitive market, all producers are price-takers and maximize their profits:

$$\max \pi = P * f(x_1, x_2, x_3) - w_1 * x_1 - w_2 * x_2 - w_3 * x_3$$

The solution to this problem gives the following unconditional factor demand functions:

$$x_1 = x_1(w_1, w_2, w_3, P); x_2 = x_2(w_1, w_2, w_3, P); x_3 = x_3(w_1, w_2, w_3, P).$$

The factor-demand functions are homogeneous of degree zero in the factor costs and the output price (Silberberg 1990).

The extension of the Alchian and Allen theorem to the production processes is that as a transportation cost t is added to the prices of the first two inputs, the relative use of the high quality input to the low quality input increases. This can be written as:

$$\frac{\partial(x_1 / x_2)}{\partial t} > 0 \quad (1)$$

Following similar manipulations as in Borcharding and Silberberg (1978), it can be shown that:

$$\frac{\partial(x_1 / x_2)}{\partial t} = (x_2 \frac{\partial x_1}{\partial t} - x_1 \frac{\partial x_2}{\partial t})(1 / x_2^2) \quad (2)$$

When the transport costs increase, the prices of the high and low quality inputs at the importing market increase, and, consequently, the output price increases at that market. If this industry producers are price-takers in the market for labor and other inputs that constitute the composite input, then the price of the third input does not change as the transport costs change. Therefore, the effect of a change in the transport costs on exports is equal to the sum of the effects of changes in the first two factor costs and the output price. The relationship between factor costs and the transport costs is linear, but the

relationship between the output price and the transport costs may not be linear. This results in the following equation:

$$\frac{\partial x_i}{\partial t} = \frac{\partial x_i}{\partial w_1} + \frac{\partial x_i}{\partial w_2} + \frac{\partial x_i}{\partial P} \frac{\partial P}{\partial t}, \quad i = 1, 2.$$

Based upon this relationship and (2), we obtain:

$$\frac{\partial(x_1/x_2)}{\partial t} = \left(\frac{x_1}{x_2}\right) \left(\frac{e_{11}}{w_1} + \frac{e_{12}}{w_2} + \frac{e_{1p}}{P} \frac{\partial P}{\partial t} - \frac{e_{21}}{w_1} - \frac{e_{22}}{w_2} - \frac{e_{2p}}{P} \frac{\partial P}{\partial t} \right) \quad (3)$$

where the e_{ij} 's are unconditional factor demand elasticities, and the e_{ip} 's are the cross-elasticities of factor demands with respect to the output price. Since unconditional factor-demand functions are homogeneous of degree zero in factor costs and the output price, applying Euler's theorem to the first two inputs yields:

$$\sum_{j=1}^3 e_{ij} + e_{ip} = 0, \quad i=1, 2 \quad (4)$$

After substituting e_{12} and e_{22} into equation (3), it follows that:

$$\frac{\partial(x_1/x_2)}{\partial t} = \left(\frac{x_1}{x_2}\right) \left[\frac{e_{11}}{w_1} - \frac{e_{11}}{w_2} - \frac{e_{13}}{w_2} - \frac{e_{1p}}{w_2} - \frac{e_{21}}{w_1} + \frac{e_{21}}{w_2} + \frac{e_{23}}{w_2} + \frac{e_{2p}}{w_2} + \frac{1}{P}(e_{1p} - e_{2p}) \frac{\partial P}{\partial t} \right] \quad (5)$$

$$\frac{\partial(x_1/x_2)}{\partial t} = \left(\frac{x_1}{x_2}\right) \left[(e_{11} - e_{21}) \left(\frac{1}{w_1} - \frac{1}{w_2} \right) + (e_{23} - e_{13} + e_{2p} - e_{1p}) \frac{1}{w_2} + \frac{1}{P}(e_{1p} - e_{2p}) \frac{\partial P}{\partial t} \right] \quad (6)$$

Analogously to Borchering and Silberberg's logic, as inputs x_1 and x_2 become closer substitutes, and neither of the first two inputs are close substitutes to the composite input, e_{23} tends to be equal to e_{13} . Because the effect of the output increase or decrease (resulting from a change in the output price) is expected to increase or decrease the use of very close substitute inputs in a very similar way, e_{1p} and e_{2p} must be almost equal, and

$(e_{2p} - e_{1p})$ must be close to zero. This conclusion is also similar to that in Borcharding and Silberberg's paper. If $\frac{\partial P}{\partial t}$ does not approach infinity, then $(e_{1p} - e_{2p}) \frac{\partial P}{\partial t}$ is also close to zero. As the first two inputs become closer substitutes, $(e_{1l} - e_{2l})$ will approach negative infinity ($e_{2l} > 0$, $e_{1l} < 0$), and the whole first term will approach positive infinity. So the condition for the sign of equation (6) to be positive is that the high and low quality inputs must be close substitutes. If this is the case then the Alchian and Allen theorem holds when the goods under consideration are used as inputs to the production of other goods, assuming that producers are price-takers, and prices are exogenous.

Interestingly, the comparative statics derivation could be identical to that of Borcharding and Silberberg, if the cost-minimization model was used instead of the profit maximization model. In production theory, the cost-minimization model, which assumes that output is fixed, gives a rise to conditional factor demand elasticities that are equivalent to compensated demand elasticities in consumption theory. Borcharding and Silberberg applied the expenditure-minimization model that holds consumers' real income constant. In both cases, demand functions are homogeneous of degree zero in commodity prices, and the equations following from Euler's theorem are identical.

Per-Unit Subsidy

In this section, the effect of a per-unit export subsidy on the quality of exports is discussed in the Alchian and Allen framework. Analogous to a per-unit fee, a per-unit

subsidy is assumed to be the same for the high and low quality goods. The export subsidy makes the high quality good more expensive relative to the low quality good in the importing country. The effect of a per-unit subsidy on the relative price in the importing country is opposite to the effect of a per-unit fee. According to the Alchian and Allen theorem, the effects of a new per-unit subsidy should be an increase in the consumption of the low quality good relative to that of the high quality good in the importing country. In other words, the effect of a new per-unit subsidy on the relative quantities of exports is opposite to the effect of a per-unit fee. The comparative statics in the previous section also provide this result. If transport costs fall as a result of a new subsidy lowering the prices, the sign of the derivative (6) would be negative; that is more of the low quality good is exported relative to the high quality good.

In a situation when both a per-unit subsidy and a per-unit fee are simultaneously applied to the prices of the high and low quality goods, the total effect on the quality of exports cannot be predicted a priori. It will depend on the relative sizes of the subsidy and the fee.

Spatial Equilibrium Framework

In this section, the assumption that prices are exogenous is relaxed. When Borcharding and Silberberg (1978) examined the comparative statics of a change in the transportation cost, they used a partial equilibrium analysis ignoring secondary effects in the domestic market. Such effects—both price and consumption changes—affect

consumption patterns at the foreign market in turn. Therefore, when all the prices and quantities are interdependent, a spatial equilibrium model should be used instead.

The first step, which is covered in this section, is the introduction of the spatial equilibrium framework and showing how the price endogeneity and the relationship between the high and low quality goods influences the analysis.

We start with a model that considers two countries. Each country produces two goods that are of different qualities. We assume that buyers are identical in the two countries in terms of their preferences and income.

In our analysis, we use an approach applied by Umbeck (1980). He suggested that both high and low quality apples have a common quality attribute (juice). A high quality apple has more units of juice than a low quality apple, and this fact determines the quality of the apples. This can be written for Country 1 as:

$$p_1^h = w_1^h * q_1^h, p_1^l = w_1^l * q_1^l,$$

where p_1^h and p_1^l are the prices per container of the high and low quality goods, w_1^h and w_1^l are the prices per unit of the quality attribute for the high and low quality goods, q_1^h and q_1^l are the quantities of the quality attribute, correspondingly. Similarly, for Country 2:

$$p_2^h = w_2^h * q_2^h, p_2^l = w_2^l * q_2^l,$$

where p_2^h and p_2^l are the prices per container of the high and low quality goods, w_2^h and w_2^l are the prices per unit of the quality attribute for the high and low quality goods, q_2^h and q_2^l are the quantities of the quality attribute, correspondingly.

We proceed by making the assumption that the two goods are perfect substitutes. This means that the dollar price per unit of the quality attribute is the same for both goods. However, the goods contain a different number of units of the quality attribute. In our notation, this can be written as:

$$w_1^h = w_1^l \text{ and } q_1^h = \alpha * q_1^l, \quad \alpha > 1.$$

$$\text{So that } \frac{p_1^h}{p_1^l} = \alpha.$$

The same is true for Country 2; that is,

$$w_2^h = w_2^l \text{ and } q_2^h = \alpha * q_2^l, \quad \alpha > 1.$$

Therefore,

$$\frac{p_2^h}{p_2^l} = \alpha.$$

To summarize, when there is no trade between Country 1 and Country 2, and the domestic markets are in competitive equilibrium, the following is true:

$$\frac{p_1^h}{p_1^l} = \alpha \text{ and } \frac{p_2^h}{p_2^l} = \alpha \tag{7}$$

If (7) is satisfied, a consumer in either country is indifferent between a unit of the high quality good and α units of the low quality good.

Now, we allow for the goods to be traded internationally, and Country 1 is assumed to be the exporter of both goods. There is a fixed per-unit transport cost t , which is the same for both goods. A competitive equilibrium at the export-import markets for each of the two goods implies that:

$$p_2^h \leq p_1^h + t \text{ and } p_2^l \leq p_1^l + t. \quad (8)$$

If both goods are actually traded internationally, then the inequalities (8) hold as equalities; that is,

$$p_2^h = p_1^h + t \text{ and } p_2^l = p_1^l + t. \quad (9)$$

Thus,

$$\frac{p_2^h}{p_2^l} = \frac{p_1^h + t}{p_1^l + t} < \frac{p_1^h}{p_1^l}.$$

This inequality says that the high quality good is relatively cheaper than the low quality good in Country 2 (the importing country), and the low quality good is relatively cheaper than the high quality good in Country 1 (the exporting country). The inequality also says that conditions (7) and condition (9) cannot hold at the same time. They are mutually exclusive. In other words, a situation in which all goods are consumed in both countries and are also exported from Country 1 to Country 2 is impossible.

Now we drop the assumption that the two goods are perfect substitutes. If the two goods are substitutes, but not perfect substitutes, then condition (7) does not have to hold because the following inequalities are possible: $w_1^h \neq w_1^l$, $w_2^h \neq w_2^l$, and $w_1^h \neq w_2^h$ (the prices of a unit of the quality attribute may not be equal for the two goods). Consequently, the local markets in autarky can be in equilibrium when

$$\frac{p_1^h}{p_1^l} \neq \alpha \text{ and/or } \frac{p_2^h}{p_2^l} \neq \alpha.$$

There is no contradiction between the equilibrium conditions of the international and the local markets: the international trade in, and domestic consumption of, both goods

can coexist if the two goods are not perfect substitutes. However, the two goods are required to be close substitutes to emphasize that they share the same quality attribute(s), and the elasticity of substitution between them is much higher than that between an unrelated (or composite) third good and one of the two goods.

The Spatial Equilibrium Model and Comparative Statics

The previous section showed that the Alchian and Allen theorem can be applied under the spatial equilibrium framework if the two goods are close, but not perfect, substitutes. In this section, a third composite good is introduced and assumed to have a different transportation cost from the first two goods under consideration. Both countries are assumed to have linear demand equations and fixed supply of each good.

The quantity demanded of each good depends on the price of the good and the prices of the other two goods. Thus, the inverse system of demand equations is:

$$\begin{aligned}
 a_1^h - b_1^{hl} y_1^l - b_1^{hh} y_1^h - b_1^{h3} y_1^3 &= p_1^h, \\
 a_1^l - b_1^{ll} y_1^l - b_1^{lh} y_1^h - b_1^{l3} y_1^3 &= p_1^l, \\
 a_1^3 - b_1^{3l} y_1^l - b_1^{3h} y_1^h - b_1^{33} y_1^3 &= p_1^3, \\
 a_2^h - b_2^{hl} y_2^l - b_2^{hh} y_2^h - b_2^{h3} y_2^3 &= p_2^h, \\
 a_2^l - b_2^{ll} y_2^l - b_2^{lh} y_2^h - b_2^{l3} y_2^3 &= p_2^l, \\
 a_2^3 - b_2^{3l} y_2^l - b_2^{3h} y_2^h - b_2^{33} y_2^3 &= p_2^3,
 \end{aligned} \tag{10}$$

where y_i^h , y_i^l , y_i^3 are the quantities demanded and p_i^h , p_i^l , p_i^3 are the prices of the high quality good, the low quality good, and the composite good, correspondingly, in the i th

Country, $i = 1, 2$. All coefficients in these equations are assumed to be positive and constant over time; that is $\forall b_i \geq 0, i = 1, 2$.

This system of equations can be written in matrix form as:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad (11)$$

$$\text{where } p_1 = \begin{pmatrix} p_1^h \\ p_1^l \\ p_1^3 \end{pmatrix}, p_2 = \begin{pmatrix} p_2^h \\ p_2^l \\ p_2^3 \end{pmatrix}, a_1 = \begin{pmatrix} a_1^h \\ a_1^l \\ a_1^3 \end{pmatrix}, a_2 = \begin{pmatrix} a_2^h \\ a_2^l \\ a_2^3 \end{pmatrix}, y_1 = \begin{pmatrix} y_1^h \\ y_1^l \\ y_1^3 \end{pmatrix}, y_2 = \begin{pmatrix} y_2^h \\ y_2^l \\ y_2^3 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} b_1^{hh} & b_1^{hl} & b_1^{h3} \\ b_1^{lh} & b_1^{ll} & b_1^{l3} \\ b_1^{3h} & b_1^{3l} & b_1^{33} \end{pmatrix}, B_2 = \begin{pmatrix} b_2^{hh} & b_2^{hl} & b_2^{h3} \\ b_2^{lh} & b_2^{ll} & b_2^{l3} \\ b_2^{3h} & b_2^{3l} & b_2^{33} \end{pmatrix}.$$

Matrix equation (11) makes it easy to see how the inverse demand equations can be converted into direct demand equations $y_i = y_i(p'_i), i = 1, 2$. The fact that we started with the inverse, instead of direct, system of demand equations does not affect the analysis.

We can use two relationships. The first is that:

$$\begin{pmatrix} p_2^h \\ p_2^l \\ p_2^3 \end{pmatrix} = \begin{pmatrix} p_1^h \\ p_1^l \\ p_1^3 \end{pmatrix} + \begin{pmatrix} t_{12} \\ t_{12} \\ -t_{12}^3 \end{pmatrix}. \quad (12)$$

This relationship follows from the assumption that all goods are traded internationally, the first two goods have the same unit transport costs, and that international markets are in competitive equilibrium. If at least one good is not traded internationally, a relationship between the prices of this good in the two countries would be represented by inequalities

in (8). Consequently, equation (12) cannot be obtained, and further algebraic transformations presented below are not valid. The second relationship is that:

$$\begin{pmatrix} y_1^h \\ y_1^l \\ y_1^3 \end{pmatrix} = \begin{pmatrix} s_1^h \\ s_1^l \\ s_1^3 \end{pmatrix} - \begin{pmatrix} x_{12}^h \\ x_{12}^l \\ -x_{12}^3 \end{pmatrix}, \text{ and } \begin{pmatrix} y_2^h \\ y_2^l \\ y_2^3 \end{pmatrix} = \begin{pmatrix} s_2^h \\ s_2^l \\ s_2^3 \end{pmatrix} + \begin{pmatrix} x_{12}^h \\ x_{12}^l \\ -x_{12}^3 \end{pmatrix}, \quad (13)$$

where x_{12}^h , x_{12}^l , and x_{12}^3 are respectively the exports of the high quality good, the low quality good, and the third good from Country 1 to Country 2; s_i^h , s_i^l , and s_i^3 are the quantities supplied of the high quality good, the low quality good, and the composite good, correspondingly, in the i th Country, $i = 1, 2$; t_{12} and t_{12}^3 are the transport costs for the first two goods and the composite good, correspondingly.

These relationships imply that all three goods are purchased in both countries. Because Country 1 exports the first two goods, we assume that Country 2 exports the composite good. This assumption does not influence the analysis. It is preferred over such assumptions as "Country 1 exports the composite good" or "there is no trade of the composite good" because the balance of payments constraint can only be met if Country 2 exports the composite good to Country 1 in exchange for the two goods under consideration. It is interesting to note that Borcharding and Silberberg (1978) did not specify whether the composite good is exported or not, apparently because they were using a partial equilibrium framework.

Substituting equations (12) and (13) into matrix equation (11) gives:

$$\begin{pmatrix} p_1 \\ p_1 \end{pmatrix} + \begin{pmatrix} 0 \\ t \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} s_1 - x \\ s_2 + x \end{pmatrix},$$

$$\text{where } s_1 = \begin{pmatrix} s_1^h \\ s_1^l \\ s_1^3 \end{pmatrix}, s_2 = \begin{pmatrix} s_2^h \\ s_2^l \\ s_2^3 \end{pmatrix}, x = \begin{pmatrix} x_{12}^h \\ x_{12}^l \\ -x_{12}^3 \end{pmatrix}, t = \begin{pmatrix} t_{12} \\ t_{12} \\ -t_{12}^3 \end{pmatrix}.$$

Multiplying out gives:

$$\begin{pmatrix} p_1 \\ p_1 \end{pmatrix} + \begin{pmatrix} 0 \\ t \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} - \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \begin{pmatrix} -x \\ x \end{pmatrix}.$$

Now we can split this matrix equation into a system of equations:

$$\begin{cases} p_1 = a_1 - B_1 s_1 + B_1 x \\ p_1 = a_2 - t - B_2 s_2 - B_2 x \end{cases} \quad (14)$$

From this system of equations, we can obtain expressions for vectors x and p_1 ; that is,

$$x = (B_1 + B_2)^{-1}(a_2 - a_1) - (B_1 + B_2)^{-1}t - (B_1 + B_2)^{-1}B_2 s_2 + (B_1 + B_2)^{-1}B_1 s_1, \quad (15)$$

and

$$p_1 = (B_1^{-1} + B_2^{-1})^{-1}(B_1^{-1}a_1 + B_2^{-1}a_2) - (B_1^{-1} + B_2^{-1})^{-1}B_2^{-1}t - (B_1^{-1} + B_2^{-1})^{-1}(s_1 + s_2) \quad (16)$$

The goal is to obtain the signs of the following comparative statics results:

$$\text{a) } \frac{\partial (x_{12}^h / x_{12}^l)}{\partial t_{12}} \quad (17)$$

$$\text{b) } \frac{\partial (p_2^h / p_2^l)}{\partial t_{12}} \quad (18)$$

If all three goods are exported and the international markets are in competitive equilibrium, then:

$$\frac{\partial(p_2^h / p_2^l)}{\partial t_{12}} = \frac{\partial(\frac{p_1^h + t_{12}}{p_1^l + t_{12}})}{\partial t_{12}} = \frac{1}{(p_1^l + t_{12})^2} \left((p_1^l + t_{12}) \frac{\partial p_1^h}{\partial t_{12}} - (p_1^h + t_{12}) \frac{\partial p_1^l}{\partial t_{12}} + p_1^l - p_1^h \right) \quad (19)$$

that is we can use expression (16) to take this derivative.

The above partial derivatives are equal to the total derivatives with respect to t_{12} because supply is fixed in the model, and because equations (15) and (16) are essentially functions of one variable: t_{12} . In other words, we are looking for the signs of

$$\frac{d(x_{12}^h / x_{12}^l)}{d t_{12}} \quad \text{and} \quad \frac{d(p_2^h / p_2^l)}{d t_{12}}$$

which we obtain by differentiating (15) and (16) with respect to t_{12} . To obtain the comparative statics results (17) and (18), matrix expressions (15) and (16) are used. The complete comparative statics analysis is presented in Appendix A. The interpretation of the results and conclusions are presented in this section.

Neither the numerator nor the denominator in expressions (A1.3) and (A1.6)-(A1.7) in Appendix A can be signed unambiguously unless additional assumptions about the high and low quality goods are made. Borchering and Silberberg (1978) came to the same conclusion about $\frac{\partial(x_{12}^h / x_{12}^l)}{\partial t_{12}}$ for the case where prices are exogenous.

An additional assumption that the first two goods are very close substitutes can be made to further investigate the comparative statics. Under that assumption, Borchering and Silberberg showed $\frac{d(x_{12}^h / x_{12}^l)}{d t_{12}}$ to have a positive sign, which becomes more likely

as the two goods become closer substitutes. They also used the inverse relationship of $\frac{P_2^h}{P_2^l}$ with the transport costs. This inverse relationship is the moving force behind the Alchian and Allen effect on the quality of exports.

We have assumed in our model that all three goods are purchased in both countries and traded internationally. If these assumptions are violated, then either (12) or (13) is violated and further algebraic transformations and comparative statics analysis cannot be done in the model. The previous section showed that when two goods are perfect substitutes, the situation described by the assumptions of the model is impossible. It showed that when the two goods more closely approach being perfect substitutes, a corner solution becomes more likely, and exports or consumption of one of the goods will cease. Therefore, the additional assumption that the first two goods are very close substitutes cannot be applied in the case of the spatial equilibrium model.

Based on the results (A1.3) and (A1.6)-(A1.7) and the above considerations, we conclude that, in the general case, the Alchian and Allen proposition cannot be derived by comparative statics if prices are endogenous. Furthermore, it cannot be shown for the general case that the price of the high quality good decreases relative to the price of the low quality good in the importing country when the transport costs go up.

Nevertheless, the predictions of the Alchian and Allen theorem can hold in the real world. The elements of B_i in (A1.3) and (A1.6)-(A1.7) can be such that the sign of (A1.3) is positive and the signs of (A1.6)-(A1.7) are such that the relative price in (18) goes

down as the transport costs go up. On the other hand, the sign of $\frac{d(x_{12}^h / x_{12}^l)}{dt_{12}}$ can be negative when the coefficients B_i have other values. An infinite number of numerical examples can be used to illustrate these two possibilities. Furthermore, the coefficients can reflect additional assumptions about the goods and the countries. One can find assumptions that would limit possible sets of coefficients in B_i to such sets that the Alchian and Allen theorem would always hold under those assumptions, but not under different assumptions.

The exogenous price assumption impairs an economic model that is supposed to explain the Alchian and Allen effects in the real world. A model that incorporates price endogeneity has been developed in this chapter to examine the generality of the Alchian and Allen theorem. In the more general case examined here, however, changes in the mix of exported goods cannot be unambiguously predicted when the transport costs or other per-unit fees change.

CHAPTER 4

WHEAT QUALITY AND THE THEOREM ASSUMPTIONS

This study tests the implication of the Alchian and Allen theorem that when a fixed transportation cost is added to the price of wheat, the proportion of total wheat exports consisting of high quality wheat increases. To apply the theorem to wheat, however, it needs to be determined whether wheat as a commodity satisfies the assumptions of the theorem.

Wheat Quality

An important step of the subsequent empirical procedures is to divide wheat into high and low quality categories. This requires identifying the criteria that distinguish high quality wheat from low quality wheat. In their analysis, Alchian and Allen were not specific about the exact meaning of quality. Nevertheless, articles by Lancaster (1966), Rosen (1974), and Umbeck (1980) provide useful insights into the definition of quality. To summarize the ideas given in these articles, a good is of higher quality when it contains larger amounts of the quality attributes (apple juice, in Umbeck's paper) per unit than another lower quality good.

To apply this quality definition to wheat, it must first be noted that wheat is a multidimensional, heterogeneous good used as an input to many production processes in the baking industry and for making pasta. Wheat quality can be described as having three dimensions: (1) intrinsic characteristics; (2) physical condition; and (3) uniformity. The last refers to a variation in wheat quality, either physical or intrinsic, within a shipment or between shipments (Hyberg, et al. 1994). Each of these characteristics affects the wheat's performance in terms of its processing and end-use properties.

Intrinsic characteristics include the biochemical and structural properties inherent in wheat. Most of these characteristics are associated with specific varieties of wheat. Wheat can be divided into six major classes: Hard Red Spring (HRS), Hard Red Winter (HRW), Soft Red Winter (SRW), Durum, Soft White, and Hard White wheat. Each class of wheat has different properties and, consequently, different end-uses. While HRW and HRS are used for yeast breads, SRW and Soft White are used for cakes, pastries, and crackers, and white wheats are used for oriental noodles. Durum wheat is used almost exclusively for pasta. The suitability of wheat for these various purposes depends mainly on its protein content and also gluten¹ properties.

The physical condition dimension of wheat quality is reflected in the classification of wheat by grade. Within each class, wheat is assigned one of five U.S. numerical grades, and wheat that does not meet the requirements for U.S. grades Nos. 1, 2, 3, 4, or 5, is assigned U.S. Sample grade. The grade assignment is based on many factors, such as test

¹ Gluten is a tenacious, elastic substance (found especially in wheat flour) that gives cohesiveness to dough (Mercier 1993).

weight, moisture, damaged kernels, and purity. Of these, test weight is the most important grade-determining factor (Hyberg, et al. 1994).

Several studies have investigated the relative importance of different wheat quality characteristics (Larue 1991, Espinosa and Goodwin 1991, and Mercier 1993). Two general approaches have been utilized: (1) calculating the hedonic prices of various quality characteristics; and (2) surveying wheat buyers about their own valuations of wheat quality. Both types of studies indicate that protein content and protein (gluten) quality are the main wheat quality characteristics in terms of wheat's end-use quality.

Measuring gluten quality requires laboratory testing and is impractical for export terminals, where large quantities of wheat arrive and leave daily. Information about milling and baking properties is not usually specified in wheat purchase contracts. Protein content specification, however, is a necessary feature of a wheat purchase contract, and protein content is a good estimate of the value of wheat shipped. Other quality dimensions do not have such an impact on the properties and quality of the final product (bread, pasta, cookies, etc.) as wheat intrinsic characteristics. Given the importance of protein content, in this study protein content is used as the sole criteria to differentiate wheat by quality for the purposes of testing the Alchian and Allen theorem application to wheat trade.

It has already been mentioned that protein content, along with wheat class and physical quality characteristics, determines the products for which wheat can be used. However, the relationship between protein content and wheat quality varies among wheat classes. Wheat with a higher protein content is not necessarily considered to be a higher quality wheat. A good example of this is Soft Red Winter wheat, for which a very high

protein content lowers the quality of the final products, such as cookies and biscuits. In the case of durum wheat, the main criterium of quality is not protein content, but the quality of semolina². Only hard wheats (Hard Red Spring and Hard Red Winter) are consistent in terms of the direct relationship between protein content and wheat quality. For those wheats, protein content exhibits a direct positive correlation with the wheat sample's quality, and, consequently, the sample will have a higher market value. Because HRW and HRS are the only wheats for which protein content is an unambiguous proxy for quality, they are the only wheats used in the empirical analysis presented in this study.

Assumptions

Several important assumptions needed for application of the Alchian and Allen theorem have been clarified in the literature on the theorem. However, virtually all of these assumptions have been made for the case when the goods in question enter a consumer's utility function directly. This paper considers wheat, a commodity that enters a production function as an input. Therefore, several assumptions need to be restated for the case of production, and we need to determine whether wheat complies with the other assumptions. One important assumption of the original model, that real income is held constant, has been restated earlier in this paper in the second section of Chapter 3 describing the profit maximization model. Other assumptions are considered below.

² Semolina is a coarse separation of endosperm extracted from durum wheat to make pasta (Mercier 1993).

First, Borcharding and Silberberg imposed an important restriction on the kinds of goods to which the theorem can be applied. Specifically, they must be close substitutes and different grades of one class of commodities, and one of these two goods must be more expensive (p. 134). Hard wheats used for making yeast bread and other similar end-uses represent such a class of commodities. Larue (1991) provides evidence for this view. Protein content determines whether a hard wheat sample is a high or a low quality sample, and high protein wheat is more expensive than low protein wheat. Borcharding and Silberberg's constraint that the two kinds of wheat should be close substitutes is satisfied and becomes an important determinant of the structure of the empirical analysis.

To apply the Alchian and Allen theorem to production we must assume that the producers who are willing to pay the per-unit transportation charge have the same production function and input costs as those not paying the charge. This assumption is analogous to the assumption that consumers paying the transportation charge do not have systematically different preferences and incomes from those not paying the charge (Cowen and Tabarrok 1995). In the present analysis, this assumption implies that the technologies employed in the United States and in the wheat importing countries are not significantly different. Milling, flour and dough processing, and baking require many steps and involve the use of a large variety of equipment and technologies. The desired properties of the final product determine the choice of milling methods and baking technologies. Therefore, it is reasonable to compare only most general features of commercial milling and baking industries. The literature (Pylar 1988, Matz 1984) provides evidence that technologies in these industries have developed at a similar pace throughout the world; consequently, the

assumption that the United States and wheat importing countries are at similar technological levels is based in reality.

Another assumption in the original setting for the Alchian and Allen theorem was that the fixed charge must be on units of the goods exchanged and not dependent on the amount of the quality attribute. In the apple example, it was assumed that high quality apples did not use more space or require additional care during transportation, so the transport costs were the same for both kinds of apples. In this paper, the same assumption—equal costs for shipping high and low quality wheat—is made for wheat.

The empirical analysis of this thesis considers international wheat trade. International trade is subject to various tariffs, permits, and customs duties which do not exist if the trade is conducted domestically. The analysis does not change, however, for international transactions. Any additional fees not incurred on a per-unit basis, such as import or export permits, do not change relative prices and do not create a substitution effect (Cowen and Tabarrok 1995). The same is true about lump-sum subsidies. On the other hand, additional per-unit fees affect the price ratio of the high to low quality wheat in the same way as the transport costs. Transport costs are just one of several types of per-unit fees incurred in international transactions.

CHAPTER 5

DATA

Description of the Data Sets

Data on U.S. wheat export shipments were obtained from the Export Grain Inspection Service (EGIS) data set provided by the United States Department of Agriculture. The data consists of all export shipments for the time period from December 1982 to July 1997. The data set contains 128 variables providing information on the origin, destination, carrier, date, type, size, and quality of each wheat shipment (represented by a record in the dataset). More than 90 variables describe the exported wheat's quality, including grade, test weight, protein content, dockage, moisture, foreign material, damaged kernels, falling number, and wheat hardness. A significant weakness of the data set is that it does not contain wheat price data.

First, data on U.S. wheat export shipments were separated into categories of high and low quality using a protein content criteria. (The details are provided in the next section.) Then, aggregate monthly data were obtained by adding the quantities (measured in metric tons) of all shipments during each month.

Freight rate data were obtained from the International Grains Council's World Grain Statistics and cover the time period from July 1990 to June 1995 on a monthly

basis. Based on geographical distance from the United States, six routes were selected to represent the destinations of U.S. wheat exports: the European Union (from U.S. Atlantic ports); USSR (from the U.S. Gulf); Venezuela (from the U.S. Gulf); China (from the U.S. Gulf); Japan (from the North Pacific); and Egypt (from the U.S. Gulf). The U.S. ports were selected for these destinations based on the average quantity of wheat shipped from them annually. The only exception was the European Union destination for which the Great Lakes ports were the main origin of exports, but the data were usually missing from three to five months every year because of winter closure of the St. Lawrence Seaway, through which wheat is to be shipped. A simple average of the freight rates over these six routes was calculated and used in the regressions. Freight rates were deflated by the GDP deflator obtained from various issues of the Survey of Current Business.

Data on wheat prices were obtained from various issues of the International Grains Council's World Grain Statistics. Monthly averages of f.o.b. prices in U.S. ports were calculated by the publisher and covered the same time period as the freight rate data. The prices of No. 2 Dark Northern Spring (DNS) 14% and No. 2 Hard Red Winter (HRW) Ordinary Protein (12.5%) in the U.S. Gulf ports were used for estimation purposes.

Procedures with the Data

This section describes the necessary transformations performed on the data set to estimate the model that is specified in the next chapter. U.S. wheat export shipments are divided into two groups by the country of destination. The principle used here to classify

countries was developed by Mercier (1993) and Leath (1995) and is based mostly on per capita incomes, quality consciousness, and wheat marketing systems. The first group consists of countries believed to have wheat quality, i.e. wheat end-use performance characteristics, as their main purchase criteria. The second group consists of countries that have wheat price as their main purchase criteria, whereas quality is of secondary importance. According to Mercier, the prices of wheat also play an important role for the countries that belong to the first group, but only when the choice is between wheats with very similar levels of end-use quality characteristics.

Mercier found that importers from the first group typically are industrial countries and countries where private buyers make wheat quality the most important factor in import decision-making. In contrast, countries from the second group have lower per capita incomes and pay more attention to wheat prices and credit availability when choosing a source for their imported wheat. Most of these countries are beneficiaries of the U.S. Export Enhancement Program (EEP) and other forms of export subsidies. These countries are further separated from the high-income countries because the subsidies they receive affect the quality of wheat imported in the opposite way of the transport costs and other per-unit fees. Chapter 3 showed that export subsidies should lower the quality of wheat exported. Therefore, the relationship between wheat quality and transport costs can be confounded. The lists of countries that belong to each group are presented in Tables 1 and 2 in Appendix B. Some countries, for example, Switzerland, Austria, and Sweden, are not included in either list because they have not been reported to import any wheat from

the United States since 1983. The division of countries into low-income and high-income created two datasets, and the same econometric procedures were applied to each.

The next step is to divide wheat shipments into the high and low quality shipments. In the first section of Chapter 4, wheat protein content was identified as the main determinant of wheat quality. This choice has also been supported by the author's personal communications with International Grains Council associates. Only shipments of Hard Red Winter (HRW) wheat and Hard Red Spring (HRS) wheat, also known as Dark Northern Spring (DNS) wheat, were selected for estimation procedures because, as noted in Chapter 4, it is important to ensure that tests of the Alchian and Allen theorem are based on commodities that are close substitutes. According to International Grains Council associates, the level of protein content that divides wheat into high and low quality categories is between 13% and 14%. Thus, 13.5% protein content is chosen as the demarcation between high and low quality wheat in this study. HRS and HRW wheat exports with 13.5% and less protein content are considered to be the low quality shipments, and those with protein content above 13.5% are considered to be high quality exports. Quality differentiated price data in the importing countries are available only for Japan. Therefore, the prices at the U.S. wheat export terminals listed in the previous section, together with ocean grain freight rates, are used to represent the price of U.S. wheat in importing countries.

Because the level of protein content dividing wheat into high and low quality is 13.5%, the price of No. 2 HRW Ordinary Protein (12.5%) is used as the price of the low quality wheat, and the price of No. 2 HRS 14% is used as the price of the high quality

wheat. The relative price, which is an explanatory variable in the model specifications, is calculated as the ratio of the price of No. 2 HRS 14% to that of No. 2 HRW Ordinary Protein. This explanatory variable does not need to be deflated because it is a price ratio.

Shipping contracts are signed several months before the related shipments take place (Hsu and Goodwin 1995). There are a variety of contractual arrangements for wheat exports. Grain purchase and shipping contracts are signed separately. Wheat purchase contracts can specify several sources of price information: wheat prices at the futures markets, prices at the wheat export terminals on the day of the shipment or at an earlier date, or prices established privately between exporter and buyer. Similarly, shipping contracts can specify rates based on numerous intertemporal arrangements and signals from the markets for ocean grain shipping services. Wheat prices reported for Gulf ports are based on bids made for wheat to be delivered within thirty days, and ocean grain freight rate is quoted in the International Grains Council publications for cargo ships that become available for loading not earlier than three weeks after the date of the quotation. Therefore, both contemporaneous and lagged freight rate and relative price variables are included in the estimation models reported in the next chapter.

CHAPTER 6

PROCEDURES AND RESULTS

The theoretical model developed in Chapter 3 and the discussion of wheat quality in Chapter 4 identify the major factors that influence the quantity and quality of wheat exports. In this chapter, an empirical model is developed, and the implications of the Alchian and Allen theorem are tested for the case of international wheat trade. The chapter proceeds as follows: first, the variables used in regressions are described; next, the steps of processing the data are outlined; finally, the results of each econometric model are reported.

Selecting The Variables

The Alchian and Allen hypothesis, when applied to international wheat markets, suggests that the quantity of high quality wheat exported will increase relative to that of low quality wheat as high quality wheat becomes relatively cheaper in the importing country due to an increase in transport costs and/or other per-unit fees. This direct relationship between relative exports and transport costs is empirically tested in this chapter.

The estimation model has two explanatory variables, transport costs and relative prices. Transport costs are represented by the grain freight rate deflated by the GDP deflator (1992 = 100). The relative price variable is included in the regression based on the following considerations. Equation (12) in Chapter 3 includes transport costs and supplies of high and low quality wheat as the variables that influence the average quality of exports. The supply variable should not be included in the regression because only changes in the annual supplies of high and low quality wheat influence the average quality of exports; monthly supply changes are meaningless in that regard. Therefore, equation (15) in Chapter 3 must be converted into a form where the ratio of the relative exports is the dependent variable, and the transport costs, along with the prices of the high and low quality wheat, are the independent variables.

One potential dependent variable is the ratio of the quantity of high quality wheat to low quality wheat exported by the United States. However, when the dependent variable is specified in this way, it has a high degree of variation at the monthly aggregate level. Therefore, the dependent variable used in the regressions is the ratio of high quality exports to the sum of all exports, both high and low quality, during a particular month. The variable is bounded between 0 and 1 and measures the proportion of total wheat exports that consist of high quality wheat exported each month.

The estimated model has the following form:

$$\left(\frac{x_{12}^h}{x_{12}^h + x_{12}^l} \right)_t = INTERCEPT + RATE * \left(\frac{t_{12}}{GDPDF} \right)_t + RELPR * \left(\frac{p_1^h}{p_1^l} \right)_t + \varepsilon_t \quad (20)$$

where $GDPDF$ is the GDP deflator, $RATE$ and $RELPR$ are the coefficients for the transport costs and relative price of the high quality wheat, respectively.

Different combinations of the lags of the two explanatory variables are used in the regressions. Given common business practices, there is a reason to assume that the freight rates may have equal or longer lags than prices. It is also assumed that the majority of wheat shipping contracts are signed not more than two months before the shipment takes place, and hence the maximum lag in the estimation model is two periods. These considerations limit the number of regressions to fifteen for each group importers.

Estimation

The manipulations with the data described in the previous chapter resulted in the creation of two datasets, Group 1 (high-income importing countries), and Group 2 (low-income importing countries) importers of U.S. wheat. Table 3 provides a statistical summary of the key variables used in the regressions. An important question is whether the share of high quality wheat in total wheat exports to the high-income countries is statistically different from the share for the low-income countries.

It is assumed that the variables $PERCENT1 = x_{12}^h / (x_{12}^h + x_{12}^l)$, where the quantities are the exports to Group 1 importers, and $PERCENT2 = x_{12}^h / (x_{12}^h + x_{12}^l)$, where the quantities are the exports to Group 2 importers, have independent normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. First, the variances of the two

distributions are compared. If they are found to be equal, then the means of the two distributions are compared. These two steps constitute a test of whether the distributions of *PERCENT1* and *PERCENT2* are statistically different.

Table 3. Summary Statistics on Selected Variables for Group 1 and Group 2 U.S. Wheat Importers.

Variable Name	Number of Observations	Mean	Standard Deviation	Minimum	Maximum
Group 1 x_{12}^h (metric tons)	60	244772.5	63553.6	76384.1	389666.2
Group 1 x_{12}^l (metric tons)	60	229912.3	77480.5	96745.1	521160.8
Group 1 $x_{12}^h / (x_{12}^l + x_{12}^h)$	60	.5198	.0952	.2548	.6967
Group 2 x_{12}^h (metric tons)	60	227538.7	144932.2	72199.8	897057.3
Group 2 x_{12}^l (metric tons)	60	579929.5	209576.6	132654.8	973648.8
Group 2 $x_{12}^h / (x_{12}^l + x_{12}^h)$	60	.2849	.1101	.1179	.5991
t_{12} (U.S. \$)	60	23.42	2.17	17.41	26.08
p_1^h (U.S. \$)	59	169.2	38.05	115.0	267.0
p_1^l (U.S. \$)	60	153.97	35.64	113.0	258.0
$t_{12} / GDPDeflator$	60	.232	.0265	.1648	.2636
p_1^h / p_1^l	59	1.1034	.1154	.9805	1.4161

The hypothesis $H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$ is tested against a two sided alternative hypothesis

$$H_1: \frac{\sigma_1^2}{\sigma_2^2} \neq 1.$$

The test statistics is $F = \frac{s_1^2}{s_2^2}$, where s_1^2 and s_2^2 are the observed values of variances of PERCENT1 and PERCENT2, and has an F distribution with $r_1 = n_1 - 1$, $r_2 = n_2 - 1$ degrees of freedom (n_1, n_2 are the numbers of observations—the sizes of random samples—for Group 1 and Group 2, respectively).

From Table 3, it follows that $n_1 = n_2 = 60$, $s_1 = 0.0952$, $s_2 = 0.1101$. The critical region for an $\alpha = 0.05$ significance level test is given by :

$$\frac{s_1^2}{s_2^2} \geq F_{.025}(59, 59) \text{ and } \frac{s_1^2}{s_2^2} \leq F_{.975}(59, 59) = \frac{1}{F_{.025}(59, 59)}.$$

Since $\frac{s_1^2}{s_2^2} = .7477$, $F_{.025}(59, 59) = 1.67$, and $F_{.975}(59, 59) = .5988$, we fail to reject

the null hypothesis at the 5 percent level of significance. This implies that the variances of the two datasets can be considered equal.

Now the hypothesis $H_0: \mu_1 - \mu_2 = 0$ is tested against $H_1: \mu_1 - \mu_2 \neq 0$. The test relies on the t-statistic:

$$T = \frac{\bar{\mu}_1 - \bar{\mu}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} (1/n_1 + 1/n_2)}},$$

where T has a t distribution with $r = n_1 + n_2 - 2$ degrees of freedom. The critical region is $|t| \geq t_{0.025}(118) = 1.98$ at an $\alpha = 0.05$ significance level. As $\bar{\mu}_1 = .5198$ and $\bar{\mu}_2 = .2849$, it follows that $T = 12.501 > 1.98$, and therefore H_0 is rejected. Thus, the wheat shipment data for the two groups of U.S. wheat importers appears to have statistically different

distributions. One reason for the difference between the means of these distributions is that income and subsidy effects are different for the two groups. Although data on these two variables are not available, the important piece of information we have about the values of these variables is that they are quite different between the two groups. The subsidy variable is zero for Group 1 and positive for Group 2, and the average per capita income of countries in Group 1 is significantly higher than that of countries in Group 2. Therefore, we proceed by estimating separate models for each group importers.

The models are initially estimated using Ordinary Least Squares procedures. The Durbin-Watson statistic is calculated to check the presence of autocorrelation in the error terms, a common problem for estimations based on time series data. Autocorrelation lowers the efficiency of OLS parameter estimates and biases the standard error estimates. Therefore, the presence of autocorrelation must be tested, and the econometric model must be corrected if necessary.

The regressions for Group 1 importers do not exhibit serial correlation according to the Durbin-Watson test. In contrast, serial correlation is present in all of the regressions for Group 2 importers. This regularity implies that an important variable is omitted in the regressions for Group 2. The omitted variable could be the amount of export subsidies received by these importers since one criteria used to classify importers of U.S. wheat into the two groups was whether an importer received any export subsidies from the United States. The countries in Group 2 received substantial subsidies for purchasing U.S. wheat from several U.S. programs. The inclusion of a subsidy variable into the regressions for Group 2 could possibly improve the results, but data on subsidies was not available.

As noted above, models were initially estimated by Ordinary Least Squares methods. The estimated models for Group 1 importers have no serial correlation in the error term, and the results for Group 1 are obtained by performing an OLS estimation. The OLS results for Group 2 importers must be corrected for serial correlation in the error term according to the Durbin-Watson test. The description of the correction procedures follows.

After establishing that correction for autocorrelation is needed, the error term structure is determined to be one of three types: Autoregressive (AR), Moving Average (MA), and Autoregressive, Moving Average (ARMA) of different orders.

Selecting the order of the autoregressive error model for an AR type of the error term is an important step in obtaining correct regression results. Generalized Durbin-Watson tests should not be used to decide on the autoregressive order because the higher-order tests assume the absence of lower-order autocorrelation. A standard way to find the order of an AR model is to utilize the stepwise autoregressive model. The stepwise model initially fits a high-order model with many autoregressive lags and then sequentially removes autoregressive parameters until all remaining parameters have significant t-tests. Since seasonality produces autocorrelation at a seasonal lag, the initial value of the lag order should be higher than a potential seasonality. For monthly data used for estimation in this work, the initial lag is specified at 12 periods. This procedure results in an AR(1,9) model. Serial correlation in the error term is corrected by simultaneously estimating the regression coefficients and the autoregressive coefficients of the lagged error terms using the maximum likelihood method.

The best form of the error term is selected from AR(1,9), ARMA(1,1), ARMA(1,2), ARMA(2,1), ARMA(2,2), MA(1), MA(2), and MA(3) models based on the pattern of partial autocorrelation coefficients and the lowest mean squared error of the residuals. For each combination of the dependent variables, an AR(1,9) model has the lowest root mean square error. The first and the ninth lagged error terms have the highest of partial autocorrelation coefficients, which confirms that AR(1,9) should be preferred. Other models often failed to eliminate serial correlation completely or did not converge. Even when they were applied successfully, the intercept and relative price coefficient estimates were insignificant. This also supports the choice of an AR (1,9) model to correct for serial correlation.

The results of the regressions for the two groups of importers are presented in Table 4 and Table 5. Each table represents one group of countries (Group1 are "rich" and Group2 are "poor" countries) for the time period of 1990-96. *RELPR* is the estimated coefficient of the relative price ratio, and *RATE* is the estimated coefficient of the deflated freight rate. The number following each variable name indicates the number of periods by which the variable is lagged; for example, *RATE2* means that $\frac{t_{12}}{p_1}$ is lagged by two

periods; that is, $\left(\frac{t_{12}}{p_1}\right)_{t-2}$ is used in the regression.

The results for Group 1 importers show that the coefficients for the relative prices are negative and statistically significant. If relative prices for several time periods are included in a regression, no relative price variable coefficient is significant, and the

contemporaneous relative price coefficient has a much higher t-value. The coefficients for the freight rate are statistically significant in only one regression. In this regression, the freight rate lagged two periods is negative and statistically significant at the 10% level, and the freight rate lagged one period is positive and statistically significant at the 10% level. The coefficient for the contemporaneous freight rate is not found to be significant in any of the regressions.

The results for Group 2 importers differ from the Group 1 results in several ways. One difference is that the estimated coefficients for the relative prices have the highest t-statistic for the relative price variable lagged one period, although they reached a 10% significance level only in two regressions. In contrast, the contemporaneous relative price coefficient has the highest significance level in the regressions for Group 1. Several conclusions can be drawn from these results. First, high-income countries appear to concentrate on the relative price in their wheat purchase decisions more than the low-income countries, and the price information from the month preceding the shipment date has more influence on the quality of U.S. wheat for the low-income importers. Second, relative prices are correlated between the consequent periods. The relative importance of the latter fact is small because such correlation is not reflected in the regressions for Group 2 importers to the same extent as for Group 1. The t-values of all coefficients for the relative price variables are consistently smaller when several variables are combined in one regression. Thus, the correlation between the relative prices in different periods cannot be completely disregarded. This multicollinearity is partially responsible for the insignificant coefficients for some of the relative price variables.

Table 4. Estimation Results for Group 1 (High-Income) U.S. Wheat Importers (t-statistics in parentheses)

Regression Number	INTERCEPT	RATE	RATE1	RATE2	RelPr	RelPr1	RelPr2
1	.74 (3.53)*	.26 (.52)			-.25 (-2.18)**		
2	.74 (3.87)*		.28 (.59)		-.26 (-2.33)**		
3	.83 (4.5)*			.004 (.01)	-.28 (-2.58)**		
4	.73 (3.39)*	.17 (.12)	.13 (.09)		-.25 (-2.05)**		
5	.73 (3.83)*		2.29 (1.65)	-2.09 (-1.55)	-.23 (-2.09)**		
6	.73 (3.4)*	-.02(-.01)	2.31 (1.16)	-2.09 (-1.53)	-.23(-1.9)***		
7	.69 (3.22)*		.25 (.48)			-.21(-1.76)***	
8	.79 (3.98)*			-.03 (-.06)		-.24(-2.09)**	
9	.64 (2.94)*		2.32 (1.53)	-2.09 (-1.46)		-.16 (-1.28)	
10	.72 (3.4)*		.31 (.6)		-.28 (-1.13)	.03 (.12)	
11	.83 (4.24)*			-.01 (-0.2)	-.26 (-1.04)	-.02 (-.09)	
12	.66 (3.08)*		2.83(1.85)***	-2.51(-1.76)***	-.38 (-1.52)	.19 (.68)	
13	.82 (3.74)*			-.14 (-.26)			-.24(-2.01)**
14	.79 (3.57)*			-.04 (-.08)		-.23 (-.89)	-.01 (-.04)
15	.86 (3.72)*			.03 (.05)	-.26 (-1.03)	-.08 (-.23)	.06 (.23)

*, **, *** - significant at 1%, 5%, 10%, respectively.

Table 5. Estimation Results for Group 2 (Low-Income) U.S. Wheat Importers (t-statistics in parentheses)

Regression Number	INTERCEPT	RATE	RATE1	RATE2	RelPr	RelPr1	RelPr2
1	-.22(-.75)	2.36(3.75)*			-.03(-.2)		
2	-.22(-.87)		2.56(4.69)*		-.08(-.59)		
3	.002(.01)			1.91(3.26)*	-.15(-.97)		
4	-.29(-.99)	.55(.48)	2.15(2.08)**		-.04(-.28)		
5	-.11(-.42)		2.99(2.82)**	-.7(-.64)	-.13(-.89)		
6	-.19(-.65)	.85(.67)	2.64(2.21)**	-1.04(-.85)	-.08(-.55)		
7	.02(.06)		2.15(3.47)*			-.21(-1.34)	
8	.3(1.13)			1.51(2.61)**		-.33(-2.29)**	
9	.04(.14)		2.24(1.93)***	-.19(-.19)		-.21(-1.34)	
10	-.03(-.09)		2.13(3.39)*		.09(.51)	-.3(-1.48)	
11	.24(.8)			1.61(2.6)**	.16(.83)	-.46(-2.4)**	
12	.06(.2)		2.17(1.76)***	-.16(-.14)	.05(.26)	-.27(-1.26)	
13	-.19(-.58)			2.4(3.55)*			-.08(-.45)
14	-.01(-.03)			2.14(3.03)*		-.24(-1.29)	.06(.28)
15	-.02(-.04)			2.16(3.08)*	.04(.22)	-.32(-1.47)	.09(.39)

*, **, *** - significant at 1%, 5%, 10%, respectively.

Two other differences between Groups 1 and 2 must be noted. First, the intercept is not significant in the regressions for Group 2. Second, the coefficient for the freight rate is positive and statistically significant in each regression for Group 2. The freight rate variable that has the highest t-statistic tends to be the variable lagged one period, which is especially notable in the regressions where several freight rate variables are used.

For both groups of importers, all coefficients for the relative price variables that are significant are also negative. The negative sign of the coefficients implies that when the relative price of the high quality good increases, its share in the total exports falls. This is a common result for both groups of importers and supports a general expectation of the reverse relationship between the relative price of the high quality wheat in the United States and the relative quantity of the high quality wheat exported. The only difference between the groups is that contemporaneous prices matter the most for the group of the high income importers, and the prices from the previous month have the most influence on the quality of wheat bought by the low income importers.

Although the freight rate is positive and significant for Group 2 importers, the freight rate is found to have little or no influence on the purchase decisions of Group 1 importers. In the case when they are found statistically significant, their interpretation in the Alchian and Allen framework is ambiguous. They fail either to support or contradict the hypothetical relationship between the transport costs and the relative quality of exports suggested by the theorem. Nevertheless, the results for the low-income importers provide empirical support for the Alchian and Allen proposition.

CHAPTER 7

CONCLUSIONS

The general objective of this study has been to examine the Alchian and Allen proposition that when a per unit fee is added to the prices of high and low quality goods, the relative price of the high quality good falls, and, as a consequence, its relative consumption increases. An important aspect of the theoretical foundation of the proposition has been considered in this thesis, specifically, the assumption that the prices are exogenous in the Alchian and Allen framework. This assumption had been implicitly used by Borcharding and Silberberg (1976) in their influential study and did not receive proper attention in the subsequent literature on the theorem. The assumption of exogenous prices was relaxed in this study, and a spatial equilibrium model was proposed to incorporate the endogeneity of prices. The comparative statics results did not provide a theoretical support for the Alchian and Allen proposition in the general case.

An important intermediate step in the transition from the exogenous to endogenous price models was the finding that when the two goods under consideration are perfect substitutes, they cannot be traded simultaneously in both international and local markets, assuming that the markets are in competitive equilibrium. Based on this finding and the comparative statics results, the effect of the transport cost change on the relative price is ambiguous for the general case when the prices are endogenous. This fact also

underlines the impossibility to theoretically prove the Alchian and Allen proposition for the general case.

Despite the lack of theoretical support, the Alchian and Allen theorem reveals itself as an empirical regularity. A few easy-to-observe, real world examples of the theorem are reported in literature. This paper considered wheat as another possible illustration of the Alchian and Allen principle. No Alchian and Allen's transport costs-quality relationship has been tested and reported for wheat in the literature yet. Wheat as a commodity has a certain distinction from the usual examples of the Alchian and Allen theorem application. It is an input to production processes, rather than a commodity consumed directly. This distinction required some minor changes in the model, discussed in the first part of the theory section, but it was not an obstacle for the testing of the Alchian and Allen proposition on wheat. The main problem with wheat is the difficulty defining wheat quality for the purpose of testing the theorem and the lack of quality-differentiated price information in both domestic and international wheat markets.

This study relied on a comprehensive dataset on U.S. wheat exports. Protein content was chosen as the quality determining criteria for wheat. The importers of U.S. wheat were divided into two groups based on their income, wheat quality consciousness, and the receipts of U.S. export subsidies. Econometric results were obtained separately for these two groups of the importers, and substantial differences were found between the estimated regression coefficients. The main difference was that transport costs, represented by the ocean grain freight rate in the econometric model, were not a factor that influenced the quality of wheat bought by the high-income importing countries, but

were a significant factor in the low-income importers' wheat purchases. Transport costs were found to have a positive relationship with the portion of high quality wheat in U.S. wheat exports to the low-income countries. This supported the prediction of the Alchian and Allen theorem that an increase in the transport costs or other per-unit fees leads to an increase in the quality of the commodity exported.

The second difference between the results for the two groups of importers was that the relative price of high quality wheat was found to be statistically significant for the group of the high-income importers, but it was not significant for the relative price in all but one regression for the low-income countries. The only similarity between the estimated coefficients for the relative price variable for the two groups was that the coefficients were negative, which confirmed the common sense expectation that an increase in the relative price of the high quality good would lead to a decrease in its relative quantity exported.

To summarize the results of this study, the Alchian and Allen theorem received empirical support for the low-income importers of U.S. wheat, but not for the high-income importers. The proposition was shown not to be a universal economic law under the endogenous price condition. The combination of the theoretical findings and the empirical results of this study constitute a significant step towards a greater understanding of the quality change phenomenon stated in the Alchian and Allen proposition and created an opportunity for U.S. grain producers to use the discovered empirical regularities.

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APPENDICES

APPENDIX A

First, $\frac{\partial x_{12}^h}{\partial t_{12}}$ and $\frac{\partial x_{12}^l}{\partial t_{12}}$ are obtained in order to calculate $\frac{\partial(x_{12}^h/x_{12}^l)}{\partial t_{12}}$.

$$\frac{\partial x}{\partial t_{12}} = -(B_1 + B_2)^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Thus,

$$\frac{\partial x_{12}^h}{\partial t_{12}} = \begin{pmatrix} -1 & 0 & 0 \end{pmatrix} * (B_1 + B_2)^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad (\text{A1.1})$$

and

$$\frac{\partial x_{12}^l}{\partial t_{12}} = \begin{pmatrix} 0 & -1 & 0 \end{pmatrix} * (B_1 + B_2)^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (\text{A1.2})$$

We denote $U = B_1$, $V = B_2$, $C = (B_1 + B_2)^{-1} = (U + V)^{-1}$ to simplify the notation.

Then,

$$\frac{\partial(x_{12}^h/x_{12}^l)}{\partial t_{12}} = \frac{1}{(x_{12}^l)^2} \left(\frac{\partial x_{12}^h}{\partial t_{12}} x_{12}^l - \frac{\partial x_{12}^l}{\partial t_{12}} x_{12}^h \right) = \frac{1}{(x_{12}^l)^2} [(c_{21} + c_{22})x_{12}^h - (c_{11} + c_{12})x_{12}^l].$$

This expression is calculated below:

$$\frac{\partial(x_{12}^h - x_{12}^l)}{\partial t_{12}} = \frac{1}{(x_{12}^l)^2} * \frac{1}{\det(U + V)} * [(u_{21}u_{33} + u_{21}v_{33} + v_{21}u_{33} + v_{21}v_{33} - u_{23}u_{31} - u_{23}v_{31} - v_{23}u_{31} - v_{23}v_{31} - u_{11}u_{33} - u_{11}v_{33} - v_{11}u_{33} - v_{11}v_{33} + u_{13}u_{31} + u_{13}v_{31} + v_{13}u_{31} + v_{13}v_{31})x_{12}^h - (u_{23}u_{32} - u_{22}u_{33} - u_{22}v_{33} - v_{22}u_{33} - v_{22}v_{33} + u_{23}v_{32} + v_{23}u_{32} + v_{23}v_{32} + u_{12}u_{33} + u_{12}v_{33} + v_{12}u_{33} + v_{12}v_{33} - u_{13}u_{32} - u_{13}v_{32} - v_{13}u_{32} - v_{13}v_{32})x_{12}^l], \quad (\text{A1.3})$$

where

$$\det(U + V) = -v_{31}v_{13}u_{22} + u_{11}v_{22}u_{33} + u_{21}u_{13}v_{32} + u_{31}v_{12}u_{23} - u_{31}u_{13}v_{22} + u_{11}v_{22}v_{33} + v_{21}v_{13}u_{32} + u_{11}u_{22}u_{33} + u_{11}u_{22}v_{33} - u_{11}u_{23}u_{32} - u_{11}u_{23}v_{32} - u_{11}v_{23}u_{32} - u_{11}v_{23}v_{32} + v_{11}u_{22}u_{33} + v_{11}u_{22}v_{33} + v_{11}v_{22}u_{33} + v_{11}v_{22}v_{33} - v_{11}u_{23}u_{32} - v_{11}u_{23}v_{32} - v_{11}v_{23}u_{32} - v_{11}v_{23}v_{32} - u_{21}u_{12}u_{33} - u_{21}u_{12}v_{33} - u_{21}v_{12}u_{33} - u_{21}v_{12}v_{33} + u_{21}u_{13}u_{32} + u_{21}v_{13}u_{32} + u_{21}v_{13}v_{32} - v_{21}u_{12}u_{33} - v_{21}u_{12}v_{33} - v_{21}v_{12}u_{33} - v_{21}v_{12}v_{33} + v_{21}u_{13}u_{32} + v_{21}u_{13}v_{32} + v_{21}v_{13}v_{32} + u_{31}u_{12}u_{23} + u_{31}u_{12}v_{23} + u_{31}v_{12}v_{23} - u_{31}u_{13}u_{22} - u_{31}v_{13}u_{22} - u_{31}v_{13}v_{22} + v_{31}u_{12}u_{23} + v_{31}u_{12}v_{23} + v_{31}v_{12}u_{23} + v_{31}v_{12}v_{23} - v_{31}u_{13}u_{22} - v_{31}u_{13}v_{22} - v_{31}v_{13}v_{22}. \quad (\text{A1.4})$$

Similarly, we obtain the parts of the expression for $\frac{\partial(p_2^h / p_2^l)}{\partial t_{12}}$.

$$\begin{aligned} \frac{\partial p_2^h}{\partial t_{12}} &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} * (B_1^{-1} + B_2^{-1})^{-1} B_2^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \\ &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} * (B_2(B_1^{-1} + B_2^{-1}))^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \\ &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} * (B_2 B_1^{-1} + I)^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{A1.5}$$

where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the identity matrix.

The same computation procedures give

$$\frac{\partial p_1^l}{\partial t_{12}} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} * (B_2 B_1^{-1} + I)^{-1} * \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

To obtain expressions $\frac{\partial p_1^h}{\partial t_{12}}$ and $\frac{\partial p_2^l}{\partial t_{12}}$, procedures similar to those for the relative exports can be used.

We denote $U = B_1$, $V = B_2$, $C = B_2 B_1^{-1} + I = V * U^{-1} + I$, $D = C^{-1}$ for notation convenience.

Then $\frac{\partial p_1^h}{\partial t_{12}} = d_{11} + d_{12}$ and $\frac{\partial p_2^l}{\partial t_{12}} = d_{22} + d_{21}$.

The final expressions for $\frac{\partial p_1^h}{\partial t_{12}}$ and $\frac{\partial p_2^l}{\partial t_{12}}$ are:

$$\begin{aligned} \frac{\partial p_1^h}{\partial t_{12}} &= \frac{r}{\det(C)} \\ &* (r^2 - v_{21}u_{12}u_{33}r - v_{21}u_{12}u_{33}v_{32}u_{13}u_{21} - v_{21}u_{12}u_{33}v_{33}u_{11}u_{22} + v_{21}u_{12}^2u_{33}v_{33}u_{21} \end{aligned}$$

$$\begin{aligned}
& -v_{21}u_{13}u_{32}v_{32}u_{11}u_{23} + v_{21}u_{13}^2u_{32}v_{32}u_{21} - v_{21}u_{13}u_{32}v_{33}u_{12}u_{21} - v_{22}u_{11}u_{33}v_{31}u_{13}u_{22} \\
& + v_{22}u_{11}^2u_{33}v_{33}u_{22} - v_{22}u_{11}u_{33}v_{33}u_{12}u_{21} - v_{22}u_{13}u_{31}v_{31}u_{12}u_{23} + v_{22}u_{13}^2u_{31}v_{31}u_{22} \\
& - v_{22}u_{13}u_{31}v_{33}u_{11}u_{22} - v_{23}u_{11}u_{32}v_{31}u_{12}u_{23} + v_{23}u_{11}^2u_{32}v_{32}u_{23} - v_{23}u_{11}u_{32}v_{32}u_{13}u_{21} \\
& + v_{23}u_{12}^2u_{31}v_{31}u_{23} - v_{23}u_{12}u_{31}v_{31}u_{13}u_{22} - v_{23}u_{12}u_{31}v_{32}u_{11}u_{23} + v_{21}u_{13}u_{32}r + v_{22}u_{11}u_{33}r \\
& - v_{22}u_{13}u_{31}r - v_{23}u_{11}u_{32}r + v_{23}u_{12}u_{31}r + rv_{31}u_{12}u_{23} - rv_{31}u_{13}u_{22} - rv_{32}u_{11}u_{23} + rv_{32}u_{13}u_{21} \\
& + rv_{33}u_{11}u_{22} - rv_{33}u_{12}u_{21} + v_{21}u_{12}u_{23}v_{32}u_{13}u_{31} + v_{21}u_{12}u_{23}v_{33}u_{11}u_{32} - v_{21}u_{12}^2u_{23}v_{33}u_{31} \\
& + v_{21}u_{13}u_{22}v_{32}u_{11}u_{33} - v_{21}u_{13}^2u_{22}v_{32}u_{31} + v_{21}u_{13}u_{22}v_{33}u_{12}u_{31} + v_{22}u_{11}u_{23}v_{31}u_{13}u_{32} \\
& - v_{22}u_{11}^2u_{23}v_{33}u_{32} + v_{22}u_{11}u_{23}v_{33}u_{12}u_{31} + v_{22}u_{13}u_{21}v_{31}u_{12}u_{33} - v_{22}u_{13}^2u_{21}v_{31}u_{32} \\
& + v_{22}u_{13}u_{21}v_{33}u_{11}u_{32} + v_{23}u_{11}u_{22}v_{31}u_{12}u_{33} - v_{23}u_{11}^2u_{22}v_{32}u_{33} + v_{23}u_{11}u_{22}v_{32}u_{13}u_{31} \\
& - v_{23}u_{12}^2u_{21}v_{31}u_{33} + v_{23}u_{12}u_{21}v_{31}u_{13}u_{32} + v_{23}u_{12}u_{21}v_{32}u_{11}u_{33} + v_{11}u_{12}u_{33}v_{32}u_{13}u_{21} \\
& + v_{11}u_{12}u_{33}v_{33}u_{11}u_{22} - v_{11}u_{12}^2u_{33}v_{33}u_{21} + v_{11}u_{13}u_{32}v_{32}u_{11}u_{23} - v_{11}u_{13}^2u_{32}v_{32}u_{21} \\
& + v_{11}u_{13}u_{32}v_{33}u_{12}u_{21} + v_{12}u_{11}u_{33}v_{31}u_{13}u_{22} - v_{12}u_{11}^2u_{33}v_{33}u_{22} + v_{12}u_{11}u_{33}v_{33}u_{12}u_{21} \\
& + v_{12}u_{13}u_{31}v_{31}u_{12}u_{23} - v_{12}u_{13}^2u_{31}v_{31}u_{22} + v_{12}u_{13}u_{31}v_{33}u_{11}u_{22} + v_{13}u_{11}u_{32}v_{31}u_{12}u_{23} \\
& - v_{13}u_{11}^2u_{32}v_{32}u_{23} + v_{13}u_{11}u_{32}v_{32}u_{13}u_{21} - v_{13}u_{12}^2u_{31}v_{31}u_{23} + v_{13}u_{12}u_{31}v_{31}u_{13}u_{22} \\
& + v_{13}u_{12}u_{31}v_{32}u_{11}u_{23} + v_{11}u_{12}u_{33}r - v_{11}u_{13}u_{32}r - v_{12}u_{11}u_{33}r + v_{12}u_{13}u_{31}r + v_{13}u_{11}u_{32}r \\
& - v_{13}u_{12}u_{31}r - v_{11}u_{12}u_{23}v_{32}u_{13}u_{31} - v_{11}u_{12}u_{23}v_{33}u_{11}u_{32} + v_{11}u_{12}^2u_{23}v_{33}u_{31} \\
& - v_{11}u_{13}u_{22}v_{32}u_{11}u_{33} + v_{11}u_{13}^2u_{22}v_{32}u_{31} - v_{11}u_{13}u_{22}v_{33}u_{12}u_{31} - v_{12}u_{11}u_{23}v_{31}u_{13}u_{32} \\
& + v_{12}u_{11}^2u_{23}v_{33}u_{32} - v_{12}u_{11}u_{23}v_{33}u_{12}u_{31} - v_{12}u_{13}u_{21}v_{31}u_{12}u_{33} + v_{12}u_{13}^2u_{21}v_{31}u_{32} \\
& - v_{12}u_{13}u_{21}v_{33}u_{11}u_{32} - v_{13}u_{11}u_{22}v_{31}u_{12}u_{33} + v_{13}u_{11}^2u_{22}v_{32}u_{33} - v_{13}u_{11}u_{22}v_{32}u_{13}u_{31} \\
& + v_{13}u_{12}^2u_{21}v_{31}u_{33} - v_{13}u_{12}u_{21}v_{31}u_{13}u_{32} - v_{13}u_{12}u_{21}v_{32}u_{11}u_{33}), \tag{A1.6}
\end{aligned}$$

and

$$\frac{\partial p_1^i}{\partial t_{12}} = \frac{r}{\det(C)}$$

$$\begin{aligned}
& * (r^2 + rv_{31}u_{12}u_{23} - rv_{31}u_{13}u_{22} - rv_{32}u_{11}u_{23} + rv_{32}u_{13}u_{21} + rv_{33}u_{11}u_{22} - rv_{33}u_{12}u_{21} \\
& + v_{21}u_{22}u_{33}v_{32}u_{11}u_{23} - v_{21}u_{22}^2u_{33}v_{33}u_{11} + v_{21}u_{22}u_{33}v_{33}u_{12}u_{21} - v_{21}u_{23}^2u_{32}v_{32}u_{11} \\
& + v_{21}u_{23}u_{32}v_{32}u_{13}u_{21} + v_{21}u_{23}u_{32}v_{33}u_{11}u_{22} + v_{22}u_{21}u_{33}v_{31}u_{12}u_{23} + v_{22}u_{21}u_{33}v_{33}u_{11}u_{22} \\
& - v_{22}u_{21}^2u_{33}v_{33}u_{12} - v_{22}u_{23}^2u_{31}v_{31}u_{12} + v_{22}u_{23}u_{31}v_{31}u_{13}u_{22} + v_{22}u_{23}u_{31}v_{33}u_{12}u_{21} \\
& + v_{23}u_{21}u_{32}v_{31}u_{13}u_{22} + v_{23}u_{21}u_{32}v_{32}u_{11}u_{23} - v_{23}u_{21}^2u_{32}v_{32}u_{13} + v_{23}u_{22}u_{31}v_{31}u_{12}u_{23} \\
& - v_{23}u_{22}^2u_{31}v_{31}u_{13} + v_{23}u_{22}u_{31}v_{32}u_{13}u_{21} - v_{21}u_{22}u_{33}r + v_{21}u_{23}u_{32}r + v_{22}u_{21}u_{33}r \\
& - v_{22}u_{23}u_{31}r - v_{23}u_{21}u_{32}r + v_{23}u_{22}u_{31}r - v_{21}u_{12}u_{23}v_{32}u_{21}u_{33} + v_{21}u_{12}u_{23}^2v_{32}u_{31} \\
& - v_{21}u_{12}u_{23}v_{33}u_{22}u_{31} - v_{21}u_{13}u_{22}v_{32}u_{23}u_{31} - v_{21}u_{13}u_{22}v_{33}u_{21}u_{32} + v_{21}u_{13}u_{22}^2v_{33}u_{31} \\
& - v_{22}u_{11}u_{23}v_{31}u_{22}u_{33} + v_{22}u_{11}u_{23}^2v_{31}u_{32} - v_{22}u_{11}u_{23}v_{33}u_{21}u_{32} - v_{22}u_{13}u_{21}v_{31}u_{23}u_{32} \\
& + v_{22}u_{13}u_{21}^2v_{33}u_{32} - v_{22}u_{13}u_{21}v_{33}u_{22}u_{31} + v_{23}u_{11}u_{22}^2v_{31}u_{33} - v_{23}u_{11}u_{22}v_{31}u_{23}u_{32} \\
& - v_{23}u_{11}u_{22}v_{32}u_{21}u_{33} - v_{23}u_{12}u_{21}v_{31}u_{22}u_{33} + v_{23}u_{12}u_{21}^2v_{32}u_{33} - v_{23}u_{12}u_{21}v_{32}u_{23}u_{31} \\
& - v_{11}u_{22}u_{33}v_{32}u_{11}u_{23} + v_{11}u_{22}^2u_{33}v_{33}u_{11} - v_{11}u_{22}u_{33}v_{33}u_{12}u_{21} + v_{11}u_{23}^2u_{32}v_{32}u_{11} \\
& - v_{11}u_{23}u_{32}v_{32}u_{13}u_{21} - v_{11}u_{23}u_{32}v_{33}u_{11}u_{22} - v_{12}u_{21}u_{33}v_{31}u_{12}u_{23} - v_{12}u_{21}u_{33}v_{33}u_{11}u_{22} \\
& + v_{12}u_{21}^2u_{33}v_{33}u_{12} + v_{12}u_{23}^2u_{31}v_{31}u_{12} - v_{12}u_{23}u_{31}v_{31}u_{13}u_{22} - v_{12}u_{23}u_{31}v_{33}u_{12}u_{21} \\
& - v_{13}u_{21}u_{32}v_{31}u_{13}u_{22} - v_{13}u_{21}u_{32}v_{32}u_{11}u_{23} + v_{13}u_{21}^2u_{32}v_{32}u_{13} - v_{13}u_{22}u_{31}v_{31}u_{12}u_{23} \\
& + v_{13}u_{22}^2u_{31}v_{31}u_{13} - v_{13}u_{22}u_{31}v_{32}u_{13}u_{21} + v_{11}u_{22}u_{33}r - v_{11}u_{23}u_{32}r - v_{12}u_{21}u_{33}r \\
& + v_{12}u_{23}u_{31}r + v_{13}u_{21}u_{32}r - v_{13}u_{22}u_{31}r + v_{11}u_{12}u_{23}v_{32}u_{21}u_{33} - v_{11}u_{12}u_{23}^2v_{32}u_{31} \\
& + v_{11}u_{12}u_{23}v_{33}u_{22}u_{31} + v_{11}u_{13}u_{22}v_{32}u_{23}u_{31} + v_{11}u_{13}u_{22}v_{33}u_{21}u_{32} - v_{11}u_{13}u_{22}^2v_{33}u_{31} \\
& + v_{12}u_{11}u_{23}v_{31}u_{22}u_{33} - v_{12}u_{11}u_{23}^2v_{31}u_{32} + v_{12}u_{11}u_{23}v_{33}u_{21}u_{32} + v_{12}u_{13}u_{21}v_{31}u_{23}u_{32} \\
& - v_{12}u_{13}u_{21}^2v_{33}u_{32} + v_{12}u_{13}u_{21}v_{33}u_{22}u_{31} - v_{13}u_{11}u_{22}^2v_{31}u_{33} + v_{13}u_{11}u_{22}v_{31}u_{23}u_{32}
\end{aligned}$$

$$+v_{13}u_{11}u_{22}v_{32}u_{21}u_{33} + v_{13}u_{12}u_{21}v_{31}u_{22}u_{33} - v_{13}u_{12}u_{21}^2v_{32}u_{33} \\ + v_{13}u_{12}u_{21}v_{32}u_{23}u_{31})$$

(A1.7)

where

 $\det(C) =$

$$\begin{aligned} & r^3 + v_{11}u_{22}u_{33}r^2 + v_{11}u_{22}^2u_{33}^2v_{22}u_{11}^2v_{33} - 2v_{11}u_{22}u_{33}^2v_{22}u_{11}v_{33}u_{12}u_{21} \\ & - 2v_{11}u_{22}^2u_{33}^2v_{22}u_{13}u_{31}v_{33}u_{11} + 2v_{11}u_{22}u_{33}v_{23}u_{11}^2u_{32}v_{32}u_{23} - 2v_{11}u_{22}u_{33}^2v_{23}u_{11}u_{32}v_{32}u_{13}u_{21} \\ & - 2v_{11}u_{22}u_{33}^2v_{23}u_{12}u_{31}v_{32}u_{11}u_{23} + v_{11}u_{22}u_{33}^2v_{22}u_{11}r - v_{11}u_{22}u_{33}^2v_{22}u_{13}u_{31}r \\ & - v_{11}u_{22}u_{33}^2v_{23}u_{11}u_{32}r - v_{11}u_{22}u_{33}^2rv_{32}u_{11}u_{23} + v_{11}u_{22}^2u_{33}^2rv_{33}u_{11} - v_{11}u_{22}u_{33}^2rv_{33}u_{12}u_{21} \\ & - 2v_{11}u_{22}u_{33}^2v_{22}u_{11}^2u_{23}v_{33}u_{32} + 2v_{11}u_{22}u_{33}^2v_{22}u_{11}u_{23}v_{33}u_{12}u_{31} \\ & + 2v_{11}u_{22}u_{33}^2v_{22}u_{13}u_{21}v_{33}u_{11}u_{32} - v_{11}u_{22}^2u_{33}^2v_{23}u_{11}^2v_{32} + 2v_{11}u_{22}^2u_{33}^2v_{23}u_{11}v_{32}u_{13}u_{31} \\ & + 2v_{11}u_{22}u_{33}^2v_{23}u_{12}u_{21}v_{32}u_{11} - v_{11}u_{23}u_{32}r^2 + 2v_{11}u_{23}u_{32}v_{22}u_{11}u_{33}v_{33}u_{12}u_{21} \\ & + 2v_{11}u_{23}u_{32}v_{22}u_{13}u_{31}v_{33}u_{11}u_{22} - v_{11}u_{23}^2u_{32}^2v_{23}u_{11}^2v_{32} + 2v_{11}u_{23}u_{32}^2v_{23}u_{11}v_{32}u_{13}u_{21} \\ & + 2v_{11}u_{23}^2u_{32}^2v_{23}u_{12}u_{31}v_{32}u_{11} - v_{11}u_{23}u_{32}v_{22}u_{11}u_{33}r + v_{11}u_{23}u_{32}^2v_{23}u_{11}r - v_{11}u_{23}u_{32}v_{23}u_{12}u_{31}r \\ & + v_{11}u_{23}^2u_{32}^2rv_{32}u_{11} - v_{11}u_{23}u_{32}rv_{32}u_{13}u_{21} - v_{11}u_{23}u_{32}rv_{33}u_{11}u_{22} + v_{11}u_{23}^2u_{32}^2v_{22}u_{11}^2v_{33} \\ & - 2v_{11}u_{23}^2u_{32}^2v_{22}u_{11}v_{33}u_{12}u_{31} - 2v_{11}u_{23}u_{32}^2v_{22}u_{13}u_{21}v_{33}u_{11} - 2v_{11}u_{23}u_{32}v_{23}u_{11}u_{22}v_{32}u_{13}u_{31} \\ & - 2v_{11}u_{23}u_{32}v_{23}u_{12}u_{21}v_{32}u_{11}u_{33} - v_{12}u_{21}u_{33}r^2 + 2v_{12}u_{21}u_{33}^2v_{21}u_{12}v_{33}u_{11}u_{22} \\ & - v_{12}u_{21}^2u_{33}^2v_{21}u_{12}^2v_{33} + 2v_{12}u_{21}^2u_{33}^2v_{21}u_{13}u_{32}v_{33}u_{12} + 2v_{12}u_{21}u_{33}^2v_{23}u_{11}u_{32}v_{31}u_{12}u_{23} \\ & - 2v_{12}u_{21}u_{33}^2v_{23}u_{12}u_{31}v_{31}u_{23} + 2v_{12}u_{21}u_{33}^2v_{23}u_{12}u_{31}v_{31}u_{13}u_{22} + v_{12}u_{21}u_{33}^2v_{21}u_{12}r \\ & - v_{12}u_{21}u_{33}^2v_{21}u_{13}u_{32}r - v_{12}u_{21}u_{33}^2v_{23}u_{12}u_{31}r - v_{12}u_{21}u_{33}^2rv_{31}u_{12}u_{23} \\ & - v_{12}u_{21}u_{33}^2rv_{33}u_{11}u_{22} + v_{12}u_{21}^2u_{33}^2rv_{33}u_{12} - 2v_{12}u_{21}u_{33}^2v_{21}u_{12}u_{23}v_{33}u_{11}u_{32} \\ & + 2v_{12}u_{21}u_{33}^2v_{21}u_{12}^2v_{33}u_{31} - 2v_{12}u_{21}u_{33}^2v_{21}u_{13}u_{22}v_{33}u_{12}u_{31} - 2v_{12}u_{21}u_{33}^2v_{23}u_{11}u_{22}v_{31}u_{12} \\ & + v_{12}u_{21}^2u_{33}^2v_{23}u_{12}^2v_{31} - 2v_{12}u_{21}^2u_{33}^2v_{23}u_{12}v_{31}u_{13}u_{32} + v_{12}u_{23}u_{31}r^2 \\ & - 2v_{12}u_{23}u_{31}v_{21}u_{12}u_{33}v_{33}u_{11}u_{22} - 2v_{12}u_{23}u_{31}v_{21}u_{13}u_{32}v_{33}u_{12}u_{21} \\ & - 2v_{12}u_{23}u_{31}^2v_{23}u_{11}u_{32}v_{31} - 2v_{12}u_{23}u_{31}^2v_{23}u_{12}v_{31}u_{13}u_{22} \\ & - v_{12}u_{23}u_{31}^2v_{21}u_{12}u_{33}r - v_{12}u_{23}u_{31}^2v_{23}u_{11}u_{32}r + v_{12}u_{23}u_{31}^2v_{23}u_{12}r + v_{12}u_{23}^2u_{31}^2rv_{31}u_{12} \\ & - v_{12}u_{23}u_{31}^2rv_{31}u_{13}u_{22} - v_{12}u_{23}u_{31}^2rv_{33}u_{12}u_{21} + 2v_{12}u_{23}^2u_{31}^2v_{21}u_{12}v_{33}u_{11}u_{32} \\ & - v_{12}u_{23}^2u_{31}^2v_{21}u_{12}^2v_{33} + 2v_{12}u_{23}u_{31}^2v_{21}u_{13}u_{22}v_{33}u_{12} + 2v_{12}u_{23}u_{31}v_{23}u_{11}u_{22}v_{31}u_{12}u_{33} \\ & + 2v_{12}u_{23}u_{31}v_{23}u_{12}u_{21}v_{31}u_{13}u_{32} + v_{13}u_{21}u_{32}r^2 - 2v_{13}u_{21}^2u_{32}^2v_{21}u_{12}u_{33}v_{32}u_{13} \\ & - 2v_{13}u_{21}u_{32}^2v_{21}u_{13}v_{32}u_{11}u_{23} + v_{13}u_{21}^2u_{32}^2v_{21}u_{13}^2v_{32} - 2v_{13}u_{21}u_{32}^2v_{22}u_{11}u_{33}v_{31}u_{13}u_{22} \\ & - 2v_{13}u_{21}u_{32}^2v_{22}u_{13}u_{31}v_{31}u_{12}u_{23} + 2v_{13}u_{21}u_{32}^2v_{22}u_{13}u_{31}v_{31}u_{22} - v_{13}u_{21}u_{32}^2v_{21}u_{12}u_{33}r \\ & + v_{13}u_{21}u_{32}^2v_{21}u_{13}r - v_{13}u_{21}u_{32}^2v_{22}u_{13}u_{31}r - v_{13}u_{21}u_{32}^2rv_{31}u_{13}u_{22} - v_{13}u_{21}u_{32}^2rv_{32}u_{11}u_{23} \\ & + v_{13}u_{21}^2u_{32}^2rv_{32}u_{13} + 2v_{13}u_{21}u_{32}^2v_{21}u_{12}u_{23}v_{32}u_{13}u_{31} + 2v_{13}u_{21}u_{32}^2v_{21}u_{13}u_{22}v_{32}u_{11}u_{33} \\ & - 2v_{13}u_{21}u_{32}^2v_{21}u_{13}^2u_{22}v_{32}u_{31} + 2v_{13}u_{21}u_{32}^2v_{22}u_{11}u_{23}v_{31}u_{13} + 2v_{13}u_{21}^2u_{32}^2v_{22}u_{13}v_{31}u_{12}u_{33} \\ & - v_{13}u_{21}^2u_{32}^2v_{22}u_{13}^2v_{31} - v_{13}u_{22}u_{31}r^2 + 2v_{13}u_{22}u_{31}v_{21}u_{12}u_{33}v_{32}u_{13}u_{21} \\ & + 2v_{13}u_{22}u_{31}v_{21}u_{13}u_{32}v_{32}u_{11}u_{23} + 2v_{13}u_{22}^2u_{31}^2v_{22}u_{11}u_{33}v_{31}u_{13} + 2v_{13}u_{22}u_{31}^2v_{22}u_{13}v_{31}u_{12}u_{23} \\ & - v_{13}u_{22}^2u_{31}^2v_{22}u_{13}^2v_{31} - v_{13}u_{22}u_{31}^2v_{21}u_{13}u_{32}r - v_{13}u_{22}u_{31}^2v_{22}u_{11}u_{33}r \\ & + v_{13}u_{22}u_{31}^2v_{22}u_{13}r - v_{13}u_{22}u_{31}^2rv_{31}u_{12}u_{23} + v_{13}u_{22}^2u_{31}^2rv_{31}u_{13} - v_{13}u_{22}u_{31}^2rv_{32}u_{13}u_{21} \\ & - 2v_{13}u_{22}u_{31}^2v_{21}u_{12}u_{23}v_{32}u_{13} - 2v_{13}u_{22}^2u_{31}^2v_{21}u_{13}v_{32}u_{11}u_{33} + v_{13}u_{22}^2u_{31}^2v_{21}u_{13}^2v_{32} \\ & - 2v_{13}u_{22}u_{31}^2v_{22}u_{11}u_{23}v_{31}u_{13}u_{32} - 2v_{13}u_{22}u_{31}^2v_{22}u_{13}u_{21}v_{31}u_{12}u_{33} - v_{21}u_{12}u_{33}r^2 + v_{21}u_{13}u_{32}r^2 \\ & + v_{22}u_{11}u_{33}r^2 - v_{22}u_{13}u_{31}r^2 - v_{23}u_{11}u_{32}r^2 + v_{23}u_{12}u_{31}r^2 + r^2v_{31}u_{12}u_{23} - r^2v_{31}u_{13}u_{22} \\ & - r^2v_{32}u_{11}u_{23} + r^2v_{32}u_{13}u_{21} + r^2v_{33}u_{11}u_{22} - r^2v_{33}u_{12}u_{21} - rv_{21}u_{12}u_{33}v_{32}u_{13}u_{21} \\ & - rv_{21}u_{12}u_{33}v_{33}u_{11}u_{22} + rv_{21}u_{12}^2u_{33}v_{33}u_{21} - rv_{21}u_{13}u_{32}v_{32}u_{11}u_{23} + rv_{21}u_{13}^2u_{32}v_{32}u_{21} \\ & - rv_{21}u_{13}u_{32}v_{33}u_{12}u_{21} - rv_{22}u_{11}u_{33}v_{31}u_{13}u_{22} + rv_{22}u_{11}^2u_{33}v_{33}u_{22} - rv_{22}u_{11}u_{33}v_{33}u_{12}u_{21} \\ & - rv_{22}u_{13}u_{31}v_{31}u_{12}u_{23} + rv_{22}u_{13}^2u_{31}v_{31}u_{22} - rv_{22}u_{13}u_{31}v_{33}u_{11}u_{22} - rv_{23}u_{11}u_{32}v_{31}u_{12}u_{23} \end{aligned}$$

$$\begin{aligned}
& +rv_{23}u_{11}^2u_{32}v_{32}u_{23} - rv_{23}u_{11}u_{32}v_{32}u_{13}u_{21} + rv_{23}u_{12}^2u_{31}v_{31}u_{23} - rv_{23}u_{12}u_{31}v_{31}u_{13}u_{22} \\
& -rv_{23}u_{12}u_{31}v_{32}u_{11}u_{23} + rv_{21}u_{12}u_{23}v_{32}u_{13}u_{31} + rv_{21}u_{12}u_{23}v_{33}u_{11}u_{32} - rv_{21}u_{12}^2u_{23}v_{33}u_{31} \\
& +rv_{21}u_{13}u_{22}v_{32}u_{11}u_{33} - rv_{21}u_{13}^2u_{22}v_{32}u_{31} + rv_{21}u_{13}u_{22}v_{33}u_{12}u_{31} + rv_{22}u_{11}u_{23}v_{31}u_{13}u_{32} \\
& -rv_{22}u_{11}^2u_{23}v_{33}u_{32} + rv_{22}u_{11}u_{23}v_{33}u_{12}u_{31} + rv_{22}u_{13}u_{21}v_{31}u_{12}u_{33} - rv_{22}u_{13}^2u_{21}v_{31}u_{32} \\
& +rv_{22}u_{13}u_{21}v_{33}u_{11}u_{32} + rv_{23}u_{11}u_{22}v_{31}u_{12}u_{33} - rv_{23}u_{11}^2u_{22}v_{32}u_{33} + rv_{23}u_{11}u_{22}v_{32}u_{13}u_{31} \\
& -rv_{23}u_{12}u_{21}v_{31}u_{13}u_{32} + rv_{23}u_{12}u_{21}v_{32}u_{11}u_{33} - v_{21}u_{22}^2u_{33}^2v_{12}u_{11}^2v_{33} \\
& +2v_{21}u_{22}^2u_{33}^2v_{12}u_{13}u_{31}v_{33}u_{11} - 2v_{21}u_{22}u_{33}^2v_{13}u_{11}^2u_{32}v_{32}u_{23} + 2v_{21}u_{22}u_{33}^2v_{13}u_{12}u_{31}v_{32}u_{11}u_{23} \\
& -v_{21}u_{22}u_{33}^2v_{12}u_{11}^2r + v_{21}u_{22}u_{33}^2v_{12}u_{13}u_{31}r + v_{21}u_{22}u_{33}^2v_{13}u_{11}u_{32}r + 2v_{21}u_{22}u_{33}^2v_{12}u_{11}^2u_{23}v_{33}u_{32} \\
& -2v_{21}u_{22}u_{33}^2v_{12}u_{13}u_{21}v_{33}u_{11}u_{32} + v_{21}u_{22}^2u_{33}^2v_{13}u_{11}^2v_{32} - 2v_{21}u_{22}u_{33}^2v_{13}u_{12}u_{21}v_{32}u_{11} \\
& -2v_{21}u_{23}u_{32}v_{12}u_{13}u_{31}v_{33}u_{11}u_{22} + v_{21}u_{23}^2u_{32}^2v_{13}u_{11}^2v_{32} - 2v_{21}u_{23}^2u_{32}^2v_{13}u_{12}u_{31}v_{32}u_{11} \\
& +v_{21}u_{23}u_{32}^2v_{12}u_{11}u_{33}r - v_{21}u_{23}u_{32}^2v_{13}u_{11}^2r + v_{21}u_{23}u_{32}^2v_{13}u_{12}u_{31}r - v_{21}u_{23}^2u_{32}^2v_{12}u_{11}^2v_{33} \\
& +2v_{21}u_{23}u_{32}^2v_{12}u_{13}u_{21}v_{33}u_{11} + 2v_{21}u_{23}u_{32}^2v_{13}u_{12}u_{21}v_{32}u_{11}u_{33} + v_{22}u_{21}^2u_{33}^2v_{11}u_{12}^2v_{33} \\
& -2v_{22}u_{21}^2u_{33}^2v_{11}u_{13}u_{32}v_{33}u_{12} - 2v_{22}u_{21}u_{33}^2v_{13}u_{11}u_{32}v_{31}u_{12}u_{23} + 2v_{22}u_{21}u_{33}^2v_{13}u_{12}u_{31}v_{31}u_{23} \\
& -v_{22}u_{21}u_{33}^2v_{11}u_{12}^2r + v_{22}u_{21}u_{33}^2v_{11}u_{13}u_{32}r + v_{22}u_{21}u_{33}^2v_{13}u_{12}u_{31}r - 2v_{22}u_{21}u_{33}^2v_{11}u_{12}^2u_{23}v_{33}u_{31} \\
& +2v_{22}u_{21}u_{33}^2v_{11}u_{13}u_{22}v_{33}u_{12}u_{31} + 2v_{22}u_{21}u_{33}^2v_{13}u_{11}u_{22}v_{31}u_{12} - v_{22}u_{21}^2u_{33}^2v_{13}u_{12}^2v_{31} \\
& +2v_{22}u_{23}u_{31}v_{11}u_{13}u_{32}v_{33}u_{12}u_{21} + 2v_{22}u_{23}^2u_{31}^2v_{13}u_{11}u_{32}v_{31}u_{12} - v_{22}u_{23}^2u_{31}^2v_{13}u_{12}^2v_{31} \\
& +v_{22}u_{23}u_{31}^2v_{11}u_{12}u_{33}r + v_{22}u_{23}u_{31}^2v_{13}u_{11}u_{32}r - v_{22}u_{23}u_{31}^2v_{13}u_{12}^2r + v_{22}u_{23}^2u_{31}^2v_{11}u_{12}^2v_{33} \\
& -2v_{22}u_{23}u_{31}^2v_{11}u_{13}u_{22}v_{33}u_{12} - 2v_{22}u_{23}u_{31}^2v_{13}u_{11}u_{22}v_{31}u_{12}u_{33} + 2v_{23}u_{21}^2u_{32}^2v_{11}u_{12}u_{33}v_{32}u_{13} \\
& -v_{23}u_{21}^2u_{32}^2v_{11}u_{13}^2v_{32} + 2v_{23}u_{21}u_{32}^2v_{12}u_{11}u_{33}v_{31}u_{13}u_{22} - 2v_{23}u_{21}u_{32}^2v_{12}u_{13}u_{31}v_{31}u_{22} \\
& +v_{23}u_{21}u_{32}^2v_{11}u_{12}u_{33}r - v_{23}u_{21}u_{32}^2v_{11}u_{13}r + v_{23}u_{21}u_{32}^2v_{12}u_{13}u_{31}r \\
& -2v_{23}u_{21}u_{32}^2v_{11}u_{12}u_{23}v_{32}u_{13}u_{31} + 2v_{23}u_{21}u_{32}^2v_{11}u_{13}u_{22}v_{32}u_{31} - 2v_{23}u_{21}u_{32}^2v_{12}u_{11}u_{23}v_{31}u_{13} \\
& +v_{23}u_{21}^2u_{32}^2v_{12}u_{13}^2v_{31} - 2v_{23}u_{22}u_{31}v_{11}u_{12}u_{33}v_{32}u_{13}u_{21} - 2v_{23}u_{22}u_{31}v_{12}u_{11}u_{33}v_{31}u_{13} \\
& +v_{23}u_{22}^2u_{31}^2v_{12}u_{13}^2v_{31} + v_{23}u_{22}u_{31}v_{11}u_{13}u_{32}r + v_{23}u_{22}u_{31}v_{12}u_{11}u_{33}r - v_{23}u_{22}u_{31}^2v_{12}u_{13}r \\
& +2v_{23}u_{22}u_{31}^2v_{11}u_{12}u_{23}v_{32}u_{13} - v_{23}u_{22}^2u_{31}^2v_{11}u_{13}^2v_{32} + 2v_{23}u_{22}u_{31}v_{12}u_{11}u_{23}v_{31}u_{13}u_{32} \\
& +v_{31}u_{22}^2u_{33}^2v_{12}u_{11}^2v_{23} + 2v_{31}u_{22}u_{33}^2v_{13}u_{11}^2u_{32}v_{22}u_{23} - 2v_{31}u_{22}u_{33}^2v_{12}u_{11}^2u_{23}v_{23}u_{32} \\
& -v_{31}u_{22}^2u_{33}^2v_{13}u_{11}^2v_{22} + v_{31}u_{22}u_{33}^2v_{12}u_{11}u_{23}r - v_{31}u_{22}^2u_{33}^2v_{13}u_{11}^2r + v_{31}u_{22}u_{33}^2v_{13}u_{12}u_{21}r \\
& -v_{31}u_{23}^2u_{32}^2v_{13}u_{11}^2v_{22} + v_{31}u_{23}^2u_{32}^2v_{12}u_{11}^2v_{23} - v_{31}u_{23}^2u_{32}^2v_{12}u_{11}^2r + v_{31}u_{23}u_{32}^2v_{12}u_{13}u_{21}r \\
& +v_{31}u_{23}u_{32}^2v_{13}u_{11}u_{22}r - v_{32}u_{21}^2u_{33}^2v_{11}u_{12}^2v_{23} - 2v_{32}u_{21}u_{33}^2v_{13}u_{12}u_{31}v_{21}u_{23} \\
& +2v_{32}u_{21}u_{33}^2v_{11}u_{12}^2u_{23}v_{23}u_{31} + v_{32}u_{21}^2u_{33}^2v_{13}u_{12}^2v_{21} + v_{32}u_{21}u_{33}^2v_{11}u_{12}u_{23}r \\
& +v_{32}u_{21}u_{33}^2v_{13}u_{11}u_{22}r - v_{32}u_{21}^2u_{33}^2v_{13}u_{12}^2r + v_{32}u_{23}^2u_{31}^2v_{13}u_{12}^2v_{21} - v_{32}u_{23}^2u_{31}^2v_{11}u_{12}^2v_{23} \\
& -v_{32}u_{23}^2u_{31}^2v_{11}u_{12}^2r + v_{32}u_{23}u_{31}^2v_{11}u_{13}u_{22}r + v_{32}u_{23}u_{31}^2v_{13}u_{12}u_{21}r + v_{33}u_{21}^2u_{32}^2v_{11}u_{13}^2v_{22} \\
& +2v_{33}u_{21}u_{32}^2v_{12}u_{13}^2u_{31}v_{21}u_{22} - 2v_{33}u_{21}u_{32}^2v_{11}u_{13}^2u_{22}v_{22}u_{31} - v_{33}u_{21}^2u_{32}^2v_{12}u_{13}^2v_{21} \\
& +v_{33}u_{21}u_{32}^2v_{11}u_{13}u_{22}r + v_{33}u_{21}u_{32}^2v_{12}u_{11}u_{23}r - v_{33}u_{21}^2u_{32}^2v_{12}u_{13}^2r - v_{33}u_{22}^2u_{31}^2v_{12}u_{13}^2v_{21} \\
& +v_{33}u_{22}^2u_{31}^2v_{11}u_{13}^2v_{22} + v_{33}u_{22}u_{31}^2v_{11}u_{12}u_{23}r - v_{33}u_{22}^2u_{31}^2v_{11}u_{13}^2r \\
& +v_{33}u_{22}u_{31}^2v_{12}u_{13}u_{21}r.
\end{aligned} \tag{A1.8}$$

and

$$\begin{aligned}
r &= \det U = \det \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \\
&= u_{11}u_{22}u_{33} - u_{11}u_{23}u_{32} - u_{21}u_{12}u_{33} + u_{21}u_{13}u_{32} + u_{31}u_{12}u_{23} - u_{31}u_{13}u_{22}.
\end{aligned} \tag{A1.9}$$

APPENDIX B

Table 1. The Quantity of U.S. Hard Wheat Exported to Group 1 Importers,
July 1990 - June 1995

Number	Country	Quantity of U.S. Hard Wheat, (metric tons)
1	BELGIUM	500699
2	CANADA	2296869
3	FINLAND	29403
4	HONG KONG	318467
5	ICELAND	1488
6	ITALY	1296550
7	JAPAN	11981304
8	MEXICO	1528333
9	NETHERLANDS	57177
10	NEW ZEALAND	108802
11	NORWAY	357391
12	PORTUGAL	11811
13	REPUBLIC OF SOUTH AFRICA	1375655
14	SINGAPORE	103325
15	SOUTH KOREA	4201970
16	TAIWAN	3531189
17	SPAIN	26410
18	TURKEY	246508
19	UNITED KINGDOM	23698
20	VENEZUELA	1927231
	TOTAL	29924278

Table 2. The Quantity of U.S. Hard Wheat Exported to Group 2 Importers, July 1990 - June 1995

Number	Country	Quantity of U.S. Hard Wheat, (metric tons)
1	ALBANIA	152060
2	ALGERIA	2759852
3	ANGOLA	11551
4	ARMENIA	486592
5	BANGLADESH	1788852
6	BARBADOS	92301
7	BELIZE	74247
8	BENIN	184609
9	BOLIVIA	644032
10	BOSNIA	15519
11	BRAZIL	1019509
12	BULGARIA	10000
13	BURKINA	11241
14	BYELARUS	99999
15	CAMEROON	310836
16	CANARY ISLANDS	123020
17	CHILE	107622
18	CHINA	9695284
19	COLOMBIA	979697
20	COSTA RICA	537293
21	CYPRUS	170782
23	DJIBOUTI	10433
24	DOMINICAN REPUBLIC	649491
25	ECUADOR	938735
26	EGYPT	7000242
27	EL SALVADOR	489572
28	ESTONIA	32639
29	ETHIOPIA	420487
30	GABON	8800
31	GEORGIA	784428
32	GHANA	620450
34	GRENADA	47834
35	GUADELOUPE	18925
36	GUATEMALA	908265
37	GUYANA	180706
38	HAITI	68764
39	HONDURAS	575835

Table 2. The Quantity of U.S. Hard Wheat Exported to Group 2 Importers, July 1990 - June 1995 (Continued)

Number	Country	Quantity of U.S. Hard Wheat, (metric tons)
40	INDIA	1025478
41	INDONESIA	261989
42	ISRAEL	2805569
43	IVORY COAST	56223
44	JAMAICA	173323
45	JORDAN	2216201
46	KENYA	633429
47	KUWAIT	212005
48	KYRGRZSTAN	292841
49	LATVIA	57764
50	LEBANON	560958
51	LESOTHO	1757
52	LIBERIA	5000
53	MALAYSIA	336083
54	MALI	22142
55	MALTA	134075
56	MARTINIQUE	5297
57	MAURITANIA	53654
58	MOLDOVA	110394
59	MONGOLIA	30000
60	MOROCCO	1956631
61	MOZAMBIQUE	53989
62	NAMIBIA	21000
63	NETHERLANDS ANTILLES	16213
64	NICARAGUA	341137
65	NIGERIA	2508321
66	PAKISTAN	256088
67	PANAMA	366993
68	PERU	1185882
69	PHILIPPINES	5682603
70	POLAND	219493
71	ROMANIA	333763
72	RUSSIA	2307679
73	RWANDA	1548
74	SENEGAL	36070
75	SIERRA LEONE	92503
76	SLOVENIA	196460
77	SOMALIA	9480

Table 2. The Quantity of U.S. Hard Wheat Exported to Group 2 Importers, July 1990 - June 1995 (Continued)

Number	Country	Quantity of U.S. Hard Wheat, (metric tons)
78	SRI LANKA	2139351
79	SAINT VINCENT AND THE GRENADINES	133244
80	SUDAN	364685
81	SURINAME	126402
82	TAJKISTAN	93084
83	TANZANIA	67275
84	THAILAND	989428
85	TOGO	371709
86	TRINIDAD	388860
87	TUNISIA	1408877
88	TURKMENISTAN	119506
89	UGANDA	7475
90	UKRAINE	209826
91	USSR	8511685
92	UZBEKISTAN	1549272
93	YEMEN	746313
94	YUGOSLAVIA	598
95	ZAIRE	540208
96	ZIMBABWE	152729
	TOTAL	74531068