MONTANA STATE UNIVERSITY

The maximum deflection of a blast loaded cantilever beam by the modified Galerkin method
by Daniel Michael Koszuta

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University
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Abstract:
This paper presents an analytical method for calculating the maximum deflection of a rectangular cantilever beam subjected to blast loading. The method is based on the modified Galerkin method and uses a nonlinear stress-strain relation together with the nonlinear geometry changes of large deflection motion.

The paper also presents a method whereby the approximate solution can be improved. By variation of the shape functions used the corresponding equation residuals are studied. A preferred or "better residual" is defined and then used to select the shape function which best satisfies the governing equation of motion.

Results show that a one term Galerkin solution gives satisfactory accuracy. Both the linearized and nonlinear beam equations are solved. Residual properties of both the linear and nonlinear problems are found to be identical. This suggests first solving the linearized version of a physical system by this method and using the shape functions which give good results in the linear case to solve the nonlinear version.

The investigation revealed that combining shape functions resulted in an averaging of their residuals which can be used to good advantage when attempting to reduce equation residuals.

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December, 1969

ACKNOWLEDGMENT

The author is indebted to the United States Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland for providing the funds necessary for the completion of this work.

The valuable assistance of Professor D. O. Blackketter is also gratefully acknowledged.

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ABSTRACT

This paper presents an analytical method for calculating the maximum deflection of a rectangular cantilever beam subjected to blast loading. The method is based on the modified Galerkin method and uses a nonlinear stress-strain relation together with the nonlinear geometry changes of large deflection motion.

The paper also presents a method whereby the approximate solution can be improved. By variation of the shape functions used the corresponding equation residuals are studied. A preferred or "better residual" is defined and then used to select the shape function which best satisfies the governing equation of motion.

Results show that a one term Galerkin solution gives satisfactory accuracy. Both the linearized and nonlinear beam equations are solved. Residual properties of both the linear and nonlinear problems are found to be identical. This suggests first solving the linearized version of a physical system by this method and using the shape functions which give good results in the linear case to solve the nonlinear version. The investigation revealed that combining shape functions resulted in an averaging of their residuals which can be used to good advantage when attempting to reduce equation residuals.

CHAPTER I: INTRODUCTION

The classical linearized theory of beam deflections is known well and has been used extensively with good results in some engineering problems encountered in the past. This theory is based on the restrictive assumptions of small deflections and a linear stress-strain relation. Both assumptions eliminate highly nonlinear terms from the governing equations of motion. Previous works [1,2][1] have included some nonlinear aspects of beam motion but still only approach the general form of the equation of motion as developed by Eringen [3]. Another popular approach used in the past [4-8] has been the Rigid-Plastic theory, but in most cases it is again restricted to small or moderately large deflections. The literature presently available to the author suggests a need to investigate more of the nonlinear terms involved in the general governing equations.

The purpose of this paper is to present an approximate method of solution for the large deflection motion of a blast loaded rectangular cantilever beam. Fewer restrictive assumptions have been made in the derivation of an equation of motion which is a better approximation to the general form. An inverse tangent function was used to approximate a true stress-strain relation. An approximate solution for the maximum deflection is obtained by the modified Galerkin method. This method involves choosing deflection shape functions which reduce the continuous system to one of a finite number of degrees of freedom. The difficulty

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[1] Numbers in brackets refer to literature consulted.

of solving nonlinear equations led to a one term solution of the motion. A procedure to improve the approximate solution by mode shape variation is then described and evaluated. To aid this study the method is applied first to the classical beam equation using the corresponding exact eigenfunctions with variation. The results of mode shape variation in the linear case are studied, then the same technique is used on the nonlinear equation of motion. In this case the deflection shape of the nonlinear motion [9] is assumed to be approximately that of the linearized beam.

CHAPTER II: SOLUTION METHOD

2.1 The Galerkin Method as a Weighted Residual Method

Many physical problems encountered in engineering, such as the large deflections of a beam, lead to nonlinear mathematical models. The present methods available do not provide exact solutions in such cases. The engineer, quite practically, turns to methods which yield approximate solutions. The method of weighted residuals [10], of which the Galerkin method is one type, is such a method. The method involves assuming a particular form of the solution which is a sum of products of space and time functions. The space or shape functions are known from assumptions made through experience and intuition in most cases. Nonlinear problems make the choice of shape functions much more difficult. The time functions are unknown. The assumed solution, being inexact, will not satisfy the system model but will yield an error termed the equation residual and/or boundary residuals. These residuals are weighted in different fashions resulting in differential equations to be solved for the time functions.

The accuracy of the resulting approximate solution depends on the accuracy of the chosen shape functions. Finlayson and Scriven [10] point out the difficulties involved and the scant knowledge available on convergence, especially in a nonlinear problem. Crandl [11] tacitly assumed that convergence occurs with an increasing number of terms in the assumed solution.

Consider the mathematical model in two independent variables for $y(x,t)$:

$$N\left[y(x,t)\right] - \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad \text{for } x \text{ in } V,\ t > 0 \qquad \text{(Eq. 1)}$$

where $N\left[y\right]$ denotes a general differential operator involving spacial derivatives of y in the three dimensional domain V with boundary S, and t represents time.

The inital and boundary conditions are:

$$y(x,0) = y_0(x) \qquad \text{for } x \text{ in } V$$

$$y(x,t) = f_s(x,t) \qquad \text{for } x \text{ on } S \qquad \text{(Eq. 2)}$$

A trial solution is assumed of the form:

$$y^*(x,t) = y_s(x,t) + \sum_{i=1}^{n} q_i(t)\ \theta_i(x) \qquad \text{(Eq. 3)}$$

where $y_s(x,t)$ satisfies all the nonhomogeneous boundary conditions and $\theta_i(x)$ satisfies all homogeneous boundary conditions; i.e.,

$$y_s = f_s \qquad \text{for } x \text{ on } S$$

$$\theta_i = 0 \qquad \text{(Eq. 4)}$$

Substitution of the assumed solution Equation (3) into the governing Equation (1) yields the equation residual:

$$R\left[y^*(x,t)\right] = N(y^*) - \frac{\partial^2 y^*}{\partial t^2} \qquad \text{(Eq. 5)}$$

The residual is a measure of the exactness of the assumed solution. When the assumed solution is the exact solution, the residual will be identically zero throughout the domain V. It is a premise of this paper that the magnitude and distribution of the residual can be used to judge the merit of the assumed solution.

The next step in the procedure is an approximation to the ideal case of a zero residual. The weighted integrals of the residual are set equal to zero over the domain of interest:

$$\int_V W_j \, R(y^*) \, dV = 0 \qquad j = 1,2,3,\ldots,n \qquad \text{(Eq. 6)}$$

where the W_j are prescribed weighting functions and can be chosen in several different ways [10]. Each criterion for choosing the weighting functions corresponds to a particular weighted residual method. The Galerkin method uses the chosen space modes as weighting functions;

$$\int_V \theta_j \, R(y^*) \, dV = 0 \qquad j = 1,2,3,\ldots,n \qquad \text{(Eq. 7)}$$

Equations (7) represent a system of n ordinary second order differential equations in time for the time modes $q_i(t)$. They may be linear or nonlinear, coupled or uncoupled, depending on the spacial operator $N(y)$ and the orthogonality of the space modes with respect to the residual. The original initial conditions are imposed on the $q_i(t)$, term by term, thus completing the approximate solution, Equation 3.

2.2 The Modified Galerkin Method

The modified Galerkin method is nearly identical to the Galerkin method. As discussed by Anderson [12], it is used for problems which present great difficulty in choosing shape functions which satisfy all boundary conditions. The modified Galerkin method permits the use of functions which satisfy only the displacement boundary conditions. As an example, let $y(x,t)$ represent the nonlinear large deflection motion of a cantilever beam fixed at $x = 0$ and free at $x = L$. The displacement boundary conditions at the wall are:

$$y(0,t) = 0 \qquad \text{(zero displacement)}$$

$$\frac{\partial y(0,t)}{\partial x} = 0 \qquad \text{(zero slope)} \qquad \text{(Eq. 8)}$$

which can be easily satisfied. The force boundary conditions of zero moment and shear at the free end are very difficult to satisfy since these expressions are highly nonlinear. Using space modes $\Theta_i(x)$ which satisfy only the displacement boundary conditions will result in errors in force at the free end termed boundary residuals. The modified Galerkin method includes the boundary errors in the weighted residual condition, Equation (7):

$$\int_V \Theta_j \, R(y^*) \, dV + E_M \, \Phi_j + E_V \, \Theta_j = 0 \qquad j = 1,2,3,\ldots,n \qquad \text{(Eq. 9)}$$

where E_M and E_V are the moment and shear force errors at the free end.

Φ_j and Θ_j are weighting terms usually chosen as a rotation and displacement of the free end in the j_{th} space mode. Equations (9) similarly result in n differential equations in the time modes q_i.

CHAPTER III: FORMULATION OF SYSTEM MODELS

3.1 The Blast Loaded Linear Beam

The derivation of the linearized equation of motion of a cantilever beam is well known and is presented here only for easy comparison with that of the nonlinear large deflection equation of motion.

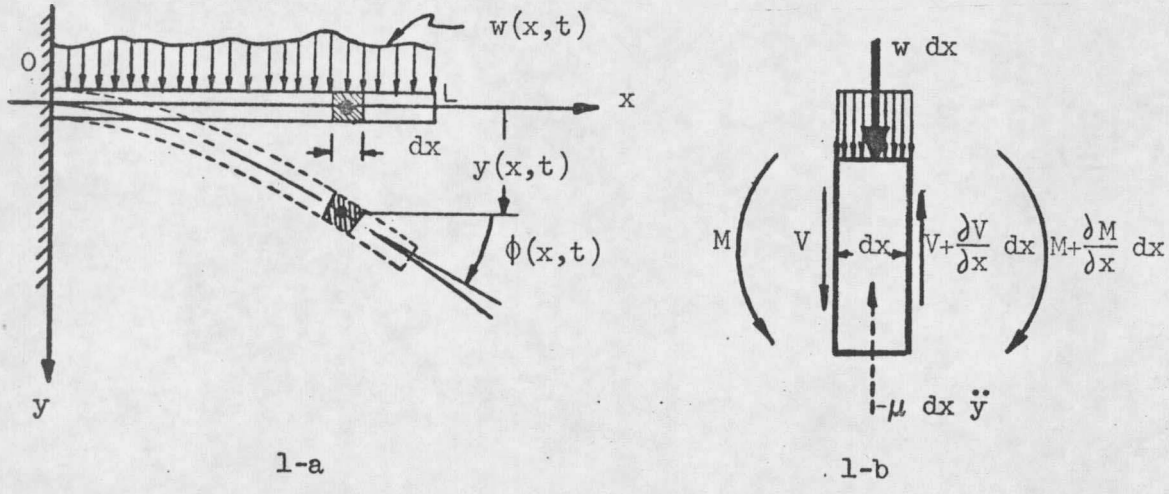


Figure 1. Linear Beam Deflection and Elemental Forces

Referring to Figure 1, it is assumed that there exists a neutral plane at the beam center, initially on the x-axis, which does not deflect longitudinally. Neglecting shear deformation and assuming that cross-sectional planes remain plane, let y(x,t) represent the motion of the neutral plane. The beam is fixed at x = 0 and free at x = L. The ex-ternal loading is represented by w(lbs/in). Referring to the free body diagram (Figure 1-b) of a beam element of length dx, we can write the equation of motion from a Newtonian force balance in the y direction:

$$\downarrow + \sum \text{External Forces} - \text{Inertia Force} = 0$$

$$w\,dx + V - \left(V + \frac{\partial V}{\partial x}\,dx\right) - \mu\,dx\,\frac{\partial^2 y}{\partial t^2} = 0$$

or: $$w - \frac{\partial V}{\partial x} - \mu\,\frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 10)}$$

where V = Shear Force (psi)

μ = Linear Beam Density (lbsm/in)

t = Time (sec)

Summing moments and neglecting rotational inertia yields:

$$\frac{\partial M}{\partial x} = V \qquad \text{(Eq. 11)}$$

where M = Internal Moment (in-lbs).

Substituting Equation (11) into Equation (10):

$$\downarrow + \qquad w - \frac{\partial^2 M}{\partial x^2} - \mu\,\frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 12)}$$

Note that it was assumed that geometry changes do not effect the direction of forces. Assuming a linear stress-strain relation and small deflection, from elementary beam theory [13] we have:

$$\frac{\partial y}{\partial x} = \tan\phi \approx \phi$$

and $$M = EI\,\frac{\partial^2 y}{\partial x^2} \qquad \text{(Eq. 13)}$$

where ϕ = Deflection Angle at x (Radians)

I = Cross-Sectional Area Moment of Inertia

E = Constant Modulus of Elasticity (psi)

Substituting Equation (13) into Equation (12) results in the final equation of motion

$$+ \quad w(x,t) - EI \frac{\partial^4 y}{\partial x^4} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 14)}$$

An exponential function is used to describe the pressure loading caused by the blast shock wave as presented in [14].

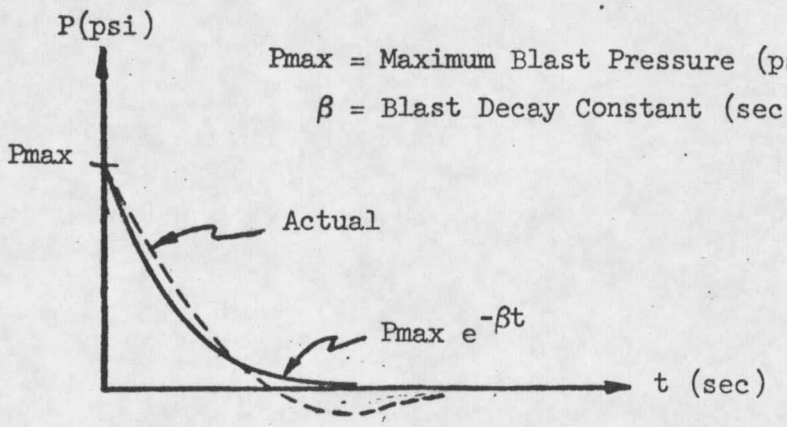


Figure 2. Blast Load Approximation

Note that we have assumed that the blast pressure is independent of x. The external loading function is now:

$$w(t) = D\ P_{max}\ e^{-\beta t} \qquad \text{(lbsf/in)}$$

where D = Beam Width (in.). The governing equation of motion becomes:

$$\downarrow + \quad \alpha\, e^{-\beta t} - EI \frac{\partial^4 y}{\partial x^4} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 15)}$$

where $\alpha = D\ P_{max}$.

3.1.2 Exact Solution of the Linear Beam Motion

The linear mathematical model describing the beam motion due to blast loading is a nonhomogeneous, inital-boundary value problem:

governing equation: $$\mu \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = \alpha\, e^{-\beta t}$$

Boundary Conditions:

$y(0,t) = 0$ (zero displacement at wall) B.C. (1)

$$\frac{\partial y(0,t)}{\partial x} = 0 \quad \text{(zero slope at wall)} \qquad \text{B.C. (2)}$$

$$\frac{\partial^2 y(L,t)}{\partial x^2} = 0 \quad \text{(zero moment at free end)} \qquad \text{B.C. (3)}$$

$$\frac{\partial^3 y(L,t)}{\partial x^3} = 0 \quad \text{(zero shear at free end)} \qquad \text{B.C. (4)}$$

The solution is assumed to be a sum of a homogeneous part and a particular part:

$$y(x,t) = y_H(x,t) + y_P(x,t) \qquad \text{(Eq. 16)}$$

The particular part has the form

$$y_P(x,t) = \Phi(x)\, e^{-\beta t} \qquad \text{(Eq. 17)}$$

where $\Phi(x)$ is an unknown function of x.

We next impose the conditions that y_H satisfies the homogeneous equation of motion and y_P satisfies the nonhomogeneous equation of motion. Substituting the particular solution Equation (17) into Equation (15), applying the appropriate transformed boundary conditions and De Moivre's theorem [15] for the extraction of roots, results in:

$$y_P(x,t) = \left\{ e^{\epsilon x}(D_1 \operatorname{Sin} \epsilon x + D_2 \operatorname{Cos} \epsilon x) + e^{-\epsilon x}(D_3 \operatorname{Sin} \epsilon x + D_4 \operatorname{Cos} \epsilon x) + \frac{\alpha}{\mu\beta^2} \right\} \cdot e^{-\beta t}$$

where $$\epsilon = \frac{\sqrt{2}}{2} \left(\frac{\mu\beta^2}{EI}\right)^{1/4}$$

and the constants D_1, D_2, D_3, and D_4 are found from the boundary conditions on $\phi(x)$:

$$\Phi(0) = \Phi'(0) = \Phi''(L) + \Phi'''(L) = 0$$

The technique of separation of variables is used to obtain the homogeneous solution presented by Wylie [16],

$$y_H(x,t) = \sum_{n=1}^{\infty} X_n(x)\, T_n(t)$$

and

$$X_n(x) = (\text{Cos } z_n + \text{Cosh } z_n)(\text{Sin } \gamma_n x - \text{Sinh } \gamma_n x) - (\text{Sin } z_n + \text{Sinh } z_n)(\text{Cos } \gamma_n x - \text{Cosh } \gamma_n x) \qquad \text{(Eq. 17a)}$$

$$T_n(t) = E_n \text{ Sin } \omega_n t + F_n \text{ Cos } \omega_n t$$

where the z_n are the roots to the frequency equation:

$$\text{Cos } z \text{ Cosh } z = -1$$

and

$$\omega_n = \sqrt{\frac{EI}{\mu}} \left(\frac{z_n}{L}\right)^2 \qquad \text{(natural frequencies)}$$

$$\gamma_n = \sqrt[4]{\frac{\mu\, \omega_n^2}{EI}}$$

Application of initial conditions gives the coefficients E_n and F_n:

$$F_n = - \frac{\int_0^L \Phi(x)\, X_n(x)\, dx}{\int_0^L X_n^2(x)\, dx} \qquad \text{(Eq. 17b)}$$

$$E_n = - \frac{\beta F_n}{\omega_n}$$

thus resulting in the complete solution

$$y(x,t) = \sum_{n=1}^{\infty} X_n(x)\, T_n(x) + \Phi(x)\, e^{-\beta t}$$

3.2 Equation of Motion for Large Deflections of a Blast Loaded Cantilever Beam

Referring to Figure 3, the effect of geometry changes due to large deflections will now be included. Unlike the general form of the equations of motion developed by Eringen [3], normal elemental forces are neglected and a nonlinear stress-strain relation is used. Cross-sectional planes are assumed to remain plane. Again let $y(x,t)$ represent the motion of the neutral plane of a beam fixed at $x = 0$ and free at $x = L$, neglecting shear deformation and longitudinal deflection. Since the duration of the blast load is very short it is assumed that geometry changes do not affect the direction of the pressure force. Results justified this. End deflections of only $(.01)L$ occurred after a 99% decay of the pressure function.

Consider the forces on the beam element of length Δx in the deformed position. Summing forces in the positive y direction we get:

$$\downarrow + \quad V \cos \phi + w \Delta x - \left(V + \frac{\partial V}{\partial x} \Delta x\right) \cos \left(\phi + \frac{\partial \phi}{\partial x} \Delta x\right) - \mu \Delta x \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 18)}$$

where: V = Total Shear Force (psi)

ϕ = Deflection Angle at x (radians)

w = Loading Function, $w = \alpha e^{-\beta t}$ (lbsf/in)

μ = Linear Beam Density (lbsm/in)

t = Time (sec)

M = Internal Moment (in-lbs)

Summing moments about the right face, using counterclockwise as positive, and neglecting the effect of rotational inertia, we get:

$$M + V\Delta x \cos\phi + V\Delta y \sin\phi + (w\Delta x)\frac{\Delta x}{2} - \left(M + \frac{\partial M}{\partial x}\Delta x\right) = 0$$

rearranging,

$$V = \frac{\partial M}{\partial x}\cos\phi \qquad \text{(Eq. 19)}$$

therefore $$\frac{\partial V}{\partial x} = -\frac{\partial M}{\partial x}\frac{\partial \phi}{\partial x}\sin\phi + \frac{\partial^2 M}{\partial x^2}\cos\phi \qquad \text{(Eq. 20)}$$

Substitution of Equations (19) and (20) into (18) gives the governing equation of motion in terms of the displacement y and the internal moment M:

$$w(t) - \left\{\frac{\partial^2 M}{\partial x^2}\cos^2\phi - \frac{\partial M}{\partial x}\frac{\partial \phi}{\partial x}\sin 2\phi\right\} - \mu\frac{\partial^2 y}{\partial t^2} = 0$$

(Eq. 21)

Note that neglecting the rotational inertia force enabled elimination of the shear force from Equation (18).

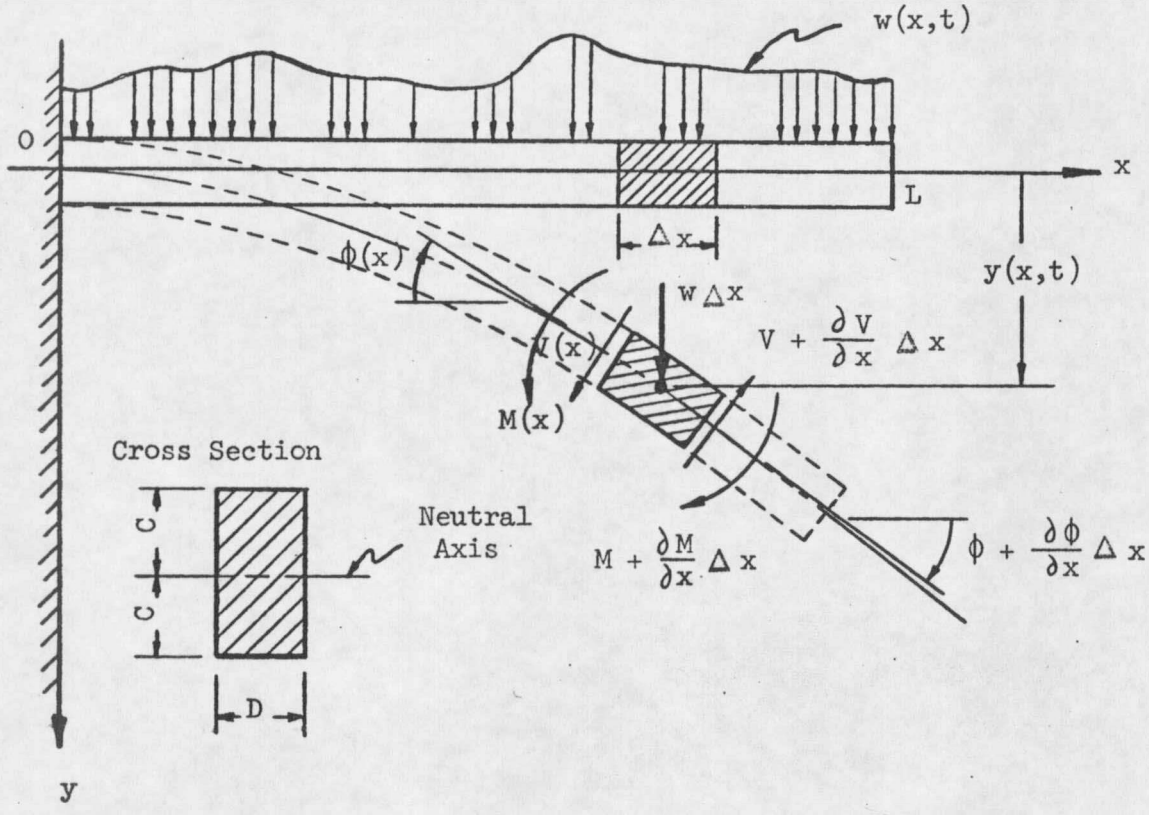


Figure 3. Large Beam Deflection and Elemental Forces

It is now desirable to eliminate the internal moment M from Equation (21). To accomplish this, a relation between the moment and displacement is needed. Referring to Figure 4, we begin by defining the strain (ϵ) of a longitudinal fiber at a distance z from the neutral plane. From the definition of the neutral plane and assuming symmetrical deformation about it, it can be seen that

$$\epsilon(x,z) = \frac{z d\phi}{dx} \quad \text{(in/in)} \qquad \text{(Eq. 22)}$$

which conforms to the popular sign convention of positive for tension and negative for compression.

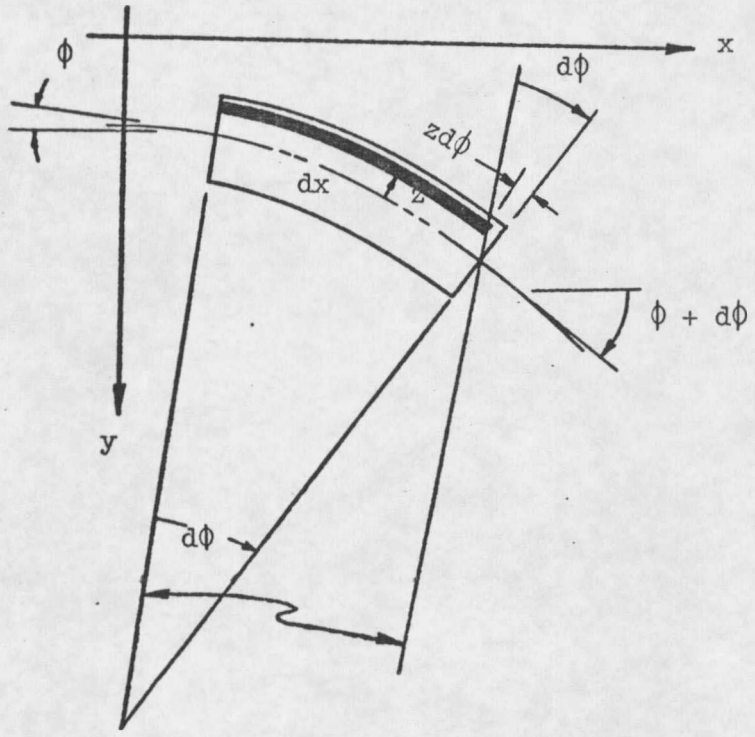


Figure 4. Fiber Deformation Defining Strain.

The true stress-strain relation of a material (shown in Figure 8) can be approximated by the relation

$$\sigma = \frac{1}{b} \tan^{-1} (a \quad) \qquad \text{(psi)} \qquad \text{(Eq. 23)}$$

where σ = stress (psi),

and a and b are constants which are selected for best fit. The internal moment can now be defined

$$M = D \int_{-c}^{c} \sigma \, z dz = \frac{D}{b} \int_{-c}^{c} \tan^{-1} (a \epsilon) \, z dz \qquad \text{(Eq. 24)}$$

Integrating Equation (24) results in

$$M = \frac{D}{b} \left\{ c^2 \tan^{-1} a\phi' c + (\tan^{-1} a\phi' c - a^2 \phi' c)/a^2 \phi'^2 \right\} \qquad \text{(Eq. 25)}$$

where ϕ = Deflection Angle

$\phi' = \partial \phi / \partial x$

Differentiation of Equation (25) gives the necessary relations to eliminate the internal moment from Equation (21):

$$\frac{\partial M}{\partial x} = \frac{2D\phi''}{a^2 b} \left\{ \frac{a\phi' c - \tan^{-1} a\phi' c}{\phi'^3} \right\} \qquad \text{(Eq. 26)}$$

and

$$\frac{\partial^2 M}{\partial x^2} = \frac{2D}{a^2 b} \left\{ \phi''^2 \left[3(1 + a^2 \phi'^2 c^2) \tan^{-1} (a\phi' c) - 3a\phi' c - 2(a\phi' c)^3 \right] / \phi'^4 (1 + a^2 \phi'^2 c^2) + \phi''' \left[a\phi' c - \tan^{-1} a\phi' c \right] / \phi'^3 \right\} \qquad \text{(Eq. 27)}$$

The final elimination to obtain an equation of motion only in terms of the displacement is to define the deflection angle ϕ as:

$$\phi = \tan^{-1} \frac{\partial y}{\partial x} \qquad \text{(Eq. 28)}$$

Substitution of Equations (26), (27), and (28) into Equation (21) gives the final governing equation of motion in terms of the displacement y only, which for brevity is not written here. Needless to say, the complete equation of motion is highly nonlinear, thus an approximate solution is the most reasonable approach.

To complete the nonlinear mathematical model, the initial and boundary conditions are defined as:

Initial Conditions:

$y(x,0) = 0$ (zero initial displacement) I.C. (1)

$\frac{\partial y(x,0)}{\partial t} = 0$ (zero initial velocity) I.C. (2)

Boundary Conditions:

$y(0,t) = 0$ (zero displacement at wall) B.C. (1)

$\frac{\partial y(0,t)}{\partial x} = 0$ (zero slope at wall) B.C. (2)

$$M(L,t) = 0 = \frac{D}{b}\left[c^2 \tan^{-1}(a\phi' c) + (\tan^{-1}(a\phi' c) - a\phi' c)/ a^2\phi'^2\right]_{x=L}$$

(zero moment at free end) B.C. (3)

$$V(L,t) = 0 = \left[\frac{\partial M}{\partial x} \text{ Cos } \phi \right]_{x=L}$$

(zero shear at free end) B.C. (4)

Note the complexity of the moment and shear relations compared with that of the linear case. Shape functions which satisfy these conditions are difficult to find, hence the modified Galerkin method was used. It was found that the static deflection shape of a linearized cantilever beam with a constant loading function did satisfy all boundary conditions. Further investigation showed that the condition for satisfying B.C. (3) and B.C. (4) was:

$$(\phi')_{x=L} = \left(\frac{\partial \phi}{\partial x}\right)_{x=L} = 0$$

CHAPTER IV: APPLICATION OF THE MODIFIED GALERKIN METHOD AND COMPUTATIONAL PROCEDURE.

4.1 The Modified Galerkin Method Applied to the Linearized Problem.

A one term Galerkin solution of the linearized beam problem was studied so that comparison of the results to the exact solution could be made. The availability of an exact solution gave a foundation for evaluation of the method itself and the mode shape variation technique to be presented. We begin by restating the initial-boundary value problem which models the motion of the beam.

Equation of motion:

$$\alpha e^{-\beta t} - EI \frac{\partial^4 y}{\partial x^4} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \qquad \text{(Eq. 15)}$$

Initial conditions:

$$y(x,0) = 0 \qquad \text{I.C. (1)}$$

$$\frac{\partial y(x,0)}{\partial t} = 0 \qquad \text{I.C. (2)}$$

Boundary conditions:

$$y(0,t) = 0 \qquad \text{B.C. (1)}$$

$$\frac{\partial y(0,t)}{\partial x} = 0 \qquad \text{B.C. (2)}$$

$$EI \frac{\partial^2 y(L,t)}{\partial x^2} = 0 \qquad \text{B.C. (3)}$$

$$EI \frac{\partial^3 y(L,t)}{\partial x^3} = 0 \qquad \text{B.C. (4)}$$

First, a form of the solution is assumed corresponding to Equation (3) of Chapter II. Since the boundary conditions are all homogeneous, y_s is zero. As was shown in Chapter II, the final step in the process, Equation (9), is the solution of a system of differential equations in $q_i(t)$. A two term solution in the nonlinear case would result in two coupled nonlinear equations in q_1 and q_2. The difficulty of solving such a system of equations led to using a one term approximation which was used in both the linear and nonlinear cases.

$$y^*(x,t) = q(t)\, \theta\, (x) \qquad \text{(Eq. 29)}$$

As can be seen from the linear boundary conditions, selection of shape functions which satisfy all of them can be made with relative ease. Again, to aid the linear-nonlinear comparison, mode shapes satisfying only the displacement boundary conditions were used.

Substitution of Equation (29) into Equation (15) results in the equation residual:

$$R(x,t) = \alpha\, e^{-\beta t} - EI \frac{\partial^4 y^*}{\partial x^4} - \mu \frac{\partial^2 y^*}{\partial t^2}$$

or $$R(x,t) = \alpha e^{-\beta t} - EI\, q\, \theta'''' - \mu\, \ddot{q}\, \theta \qquad \text{(Eq. 30)}$$

where $$\ddot{q} = \frac{d^2 q}{dt^2} \quad \text{and} \quad \theta'''' = \frac{d^4 \theta}{dx^4}$$

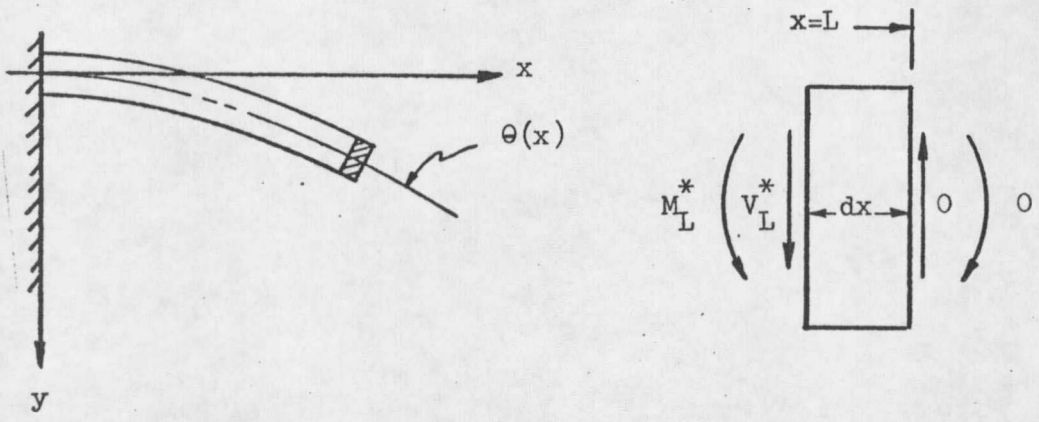


Figure 5. Boundary Residuals

Referring to Figure 5, consider a beam element at x = L. Since the shape function $\theta(x)$ does not satisfy the force boundary conditions of zero moment and shear at x = L, there will be boundary residuals. Summing forces on the end element, using the same sign convention as used in deriving the equation of motion, we have

$$\circlearrowleft_{(+)} \quad E_M = M(y^*)_{x=L} - 0 = EI \left.\frac{\partial^2 y^*}{\partial x^2}\right|_{x=L} = EI\, q\, \theta_L'' \qquad \text{(Eq. 31)}$$

$$\downarrow + \quad E_V = V(y^*)_{x=L} - 0 = EI \left. \frac{\partial^3 y^*}{\partial x^3} \right|_{x=L} = EI\, q\, \theta_L''' \qquad \text{(Eq. 32)}$$

Note that each term in the equation of motion and the boundary conditions are in units of force. The residuals can then be thought of as errors in force. The first term in the equation residual, Equation (30), is an inertial force error,

$$E_{IN} = -\mu\, \ddot{q}\, \theta \qquad \text{(Eq. 33)}$$

The second term is an error force related to internal beam forces,

$$E_{IF} = -EI\, q\, \theta''' \qquad \text{(Eq. 34)}$$

The weighted residual condition, Equation (9), can be interpreted as the total virtual work (ΔW_T) of the error forces resulting from a virtual displacement of the coordinate q; i.e.,

$$\delta y = \delta q \cdot \theta$$

and $$\Delta W_T = \int_0^L R(x,t)\, \delta y\, dx + E_M(-\delta \phi) + E_V\, \delta y = 0 \qquad \text{(Eq. 35)}$$

corresponding to the principle of virtual work. Another interpretation of Equation (35) is that it is a condition which distributes the error over the domain of interest ($0 \le x \le L$). Note the weighting terms for the boundary residuals. The moment error is weighted by the

resulting virtual rotation at x = L. Considering positive virtual work:

$$-\delta\phi = -\delta\left(\left.\frac{\partial y^*}{\partial x}\right|_{x=L}\right) = -\delta q\ \theta_L' \qquad \text{(Eq. 35a)}$$

and the shear error is weighted by the virtual displacement at x = L; both terms having units of work.

The weighted residual condition, Equation (35), then leads to a second order differential equation in the unknown q(t),

$$\left\{\mu \int_0^L \theta^2\, dx\right\} \ddot{q} + \left\{EI \int_0^L \theta''''\, \theta\, dx + EI\, \theta_L''\, \theta_L' - EI\, \theta_L'''\, \theta_L\right\} q = \left\{\int_0^L \theta\, dx\right\} \alpha\, e^{-\beta t} \qquad \text{(Eq. 36)}$$

where

$$\bar{M} = \mu \int_0^L \theta^2\, dx \qquad \text{(Eq. 37)}$$

is the generalized mass term in the q coordinate system,

and

$$\bar{K} = EI\left\{\int_0^L \theta''''\, \theta\, dx + \theta_L''\, \theta_L' - \theta_L'''\, \theta_L\right\} \qquad \text{(Eq. 38)}$$

is the generalized stiffness.

The differential equation in q has the form:

$$\bar{M}\,\ddot{q} + \bar{K}\,q = A\,e^{-\beta t} \qquad \text{(Eq. 39)}$$

where

$$A = \alpha \int_0^L \Theta \, dx$$

Solution of Equation (39) with the initial conditions

$$q(0) = 0$$

$$\frac{dq(0)}{dt} = 0$$

completes the approximate solution of the motion

$$y^*(x,t) = q(t)\,\Theta\,(x)$$

4.2 Linear Beam Computer Program

Fortran IV-H level computer programs for the linear and non-linear problems were written to aid the analysis. An incremental plotter was used to plot equation residuals for evaluation of mode shape variations. Listings of the plotting routines are not included. The plotting routines were based on an XDS Sigma VII machine language character generator which undoubtedly would not be compatible with other machines. Plot data is array stored before plotting, allowing easy insertion of compatible plot statements.

The necessary input is read in by statements 200 and 206 (see Appendix A). The program was designed for ease of mode shape changes. Statement functions were used to define the mode shape and its first four derivatives. Changing the mode shape requires changing only statements 51 through 59. Shape functions normalized to 1 at x = L were used, thus q(t) and $\dot{q}(t)$ represents the displacement and velocity of the free end. Integrations were performed by Simpson's 1/3 rule [17]. Refer to Appendix A for more detail on the linear program.

Referring to Equation (36), the program first performs the necessary integrations for $\bar{M}$, $\bar{K}$, and A. Note that the first term in $(\bar{K}q)$ represents the virtual work of internal error forces (WI):

$$WI(q) = \int_0^L (EI\ \theta'''\ q\ \theta)dx \qquad \text{(Eq. 40)}$$

The integrand of WI is a weighted internal force residual

$$FYY(q,x) = (EI\ \theta'''\ \theta)q = K_1 q \qquad \text{(Eq. 41)}$$

The remaining two terms are the virtual work of boundary force errors:

$$WM(q) = (EI\ \theta_L''\ \theta_L')q = K_2 q \qquad \text{(virtual work of boundary moment error)} \qquad \text{(Eq. 42)}$$

$$WV(q) = -(EI\ \theta_L'''\ \theta_L)q = K_3 q \qquad \text{(virtual work of boundary shear error)} \qquad \text{(Eq. 43)}$$

The total virtual work of internal forces (DW) is then linear in q:

$$DW(q) = \bar{K}q = (K_1 + K_2 - K_3)q \qquad \text{(Eq. 44)}$$

The solution of $q(t)$ and $\dot{q}(t)$ from Equation (39) is then generated:

$$q(t) = \frac{A\alpha}{\bar{M}\beta^2 + \bar{K}} \left\{ \beta\sqrt{\frac{\bar{M}}{\bar{K}}}\,\text{Sin}\,\sqrt{\frac{\bar{K}}{\bar{M}}}\,t + e^{-\beta t} - \text{Cos}\,\sqrt{\frac{\bar{K}}{\bar{M}}}\,t \right\} \qquad \text{(Eq. 45)}$$

$$\dot{q}(t) = \frac{A\alpha}{\bar{M}\beta^2 + \bar{K}} \left\{ \beta\left[\text{Cos}\,\sqrt{\frac{\bar{K}}{\bar{M}}}\,t - e^{-\beta t}\right] + \sqrt{\frac{\bar{K}}{\bar{M}}}\,\text{Sin}\,\sqrt{\frac{\bar{K}}{\bar{M}}}\,t \right\} \qquad \text{(Eq. 46)}$$

The maximum deflection of the free end is found by noting the end displacement (q) corresponding to the velocity ($\dot{q}$) being zero. The approximate solution being complete, the equation residual R(x,t), Equation (30), can be generated:

$$R(x,t) = R1 + R2 - R3$$

where

$R1 = \mu\,\ddot{q}\,\Theta$ (inertial residual)

$R2 = EI\,q\,\Theta''''$ (internal force residual)

$R3 = \alpha\,e^{-\beta t}$ (forcing term)

4.3 The Modified Galerkin Method Applied to the Nonlinear Problem.

Solution of the large deflection motion of a cantilever beam by the modified Galerkin method proceeds in like manner to that of the linearized problem. The integrations performed and the computer programming is somewhat more complex.

We begin with the equation of motion (21); after substitution of Equations (26), (27), and (28), it has the form:

$$-\mu \frac{\partial^2 y}{\partial t^2} - F_x(y) + \alpha e^{-\beta t} = 0 \qquad \text{(Eq. 47)}$$

where $F_x(y)$ is a nonlinear differential operator operating on $y(x,t)$. As mentioned, a one term solution was assumed:

$$y^*(x,t) = q(t)\,\theta(x) \qquad \text{(Eq. 48)}$$

Substitution of Equation (48) into (47) results in the equation residual:

$$R(x,t) = -\mu \ddot{q}\,\theta - f(q,x) + \alpha e^{-\beta t} \qquad \text{(Eq. 49)}$$

where $f(q,x)$ is a result of $F_x(y^*)$.

The boundary force residuals are similar to Equations (31) and (32) and are obtained from Equations (19) and (25).

$$E_M = M(y^*) \qquad \text{(Eq. 50)}$$

$$E_V = V(y^*) \qquad \text{(Eq. 51)}$$

Similarly, the inertia force error is

$$E_{IN} = -\mu \ddot{q} \theta \qquad \text{(Eq. 52)}$$

and the internal force error is

$$E_{IF} = -f(q,x) \qquad \text{(Eq. 53)}$$

The weighted residual condition is:

$$\int_0^L R(x,t)\, \delta y\, dx + E_M(-\delta \phi) + E_V\, \delta y = 0$$

where

$$\delta \phi = \delta(\tan^{-1} \frac{\partial y}{\partial x})_L = \left. \frac{\delta q}{1 + q^2 \theta'^2} \right|_{x=L} \qquad \text{(Eq. 53b)}$$

$$\delta y = (\delta q)\theta$$

which results in a nonlinear differential equation in q

$$\left\{ \int_0^L \mu \theta^2\, dx \right\} \ddot{q} + \left\{ \int_0^L f(q,x)\, \theta\, dx + E_M/(1 + q^2 \theta'^2)_L - E_V \theta_L \right\} = \left\{ \int_0^L \theta\, dx \right\} \alpha e^{-\beta t} \qquad \text{(Eq. 54)}$$

where similarly

$$\bar{M} = \int_0^L \mu \theta^2\, dx$$

$$A = \alpha \int_0^L \theta\, dx$$

But note that the stiffness term is now a nonlinear function of q, Equation (54) has the form

$$\bar{M}\,\ddot{q} + \delta\, W(q) = A\, e^{-\beta t} \qquad \text{(Eq. 55)}$$

where

$$\delta\, W(q) = \int_0^L f(q,x)\,\theta\, dx + E_M/(1 + q^2\,\theta'^2)_L - E_V\,\theta_L \qquad \text{(Eq. 56)}$$

with the initial conditions

$$q(0) = \dot{q}(0) = 0 \qquad \text{(Eq. 57)}$$

Solution of Equation (55) completes the approximate solution.

$$y^*(x,t) = q(t)\,\theta(x)$$

The corresponding weighted internal force function is

$$FYY(q,x) = f(q,x)\,\theta \qquad \text{(Eq. 58)}$$

The virtual work of the internal force error is:

$$WI = \int_0^L f(q,x)\,\theta\, dx \qquad \text{(Eq. 58a)}$$

The virtual work of the boundary moment error is:

$$WM = M(y^*)/(1 + q^2\,\theta^2)_{x=L} \qquad \text{(Eq. 58b)}$$

and the virtual work of the boundary shear error is:

$$WV = V(y^*)\,\theta \Big|_{x=L} \qquad \text{(Eq. 58c)}$$

4.4 Computer Program for the Nonlinear Problem

4.4.1 Nonlinear Program Input and Output

Input and output of the nonlinear program follows that of the linear program except for an exact solution. Mode shape changes are made in an identical manner. Input of stress constants is required for the inverse tangent fit of the true stress-strain relation.

4.4.2 Computational Procedure of Nonlinear Beam Program

The main difference of the nonlinear program is in the computational procedure for q(t). The solution of Equation (55) required a numerical approach as a result of the nonlinearity of the virtual work term $\delta W(q)$. Referring to Figure 6, a small interval Δq was chosen and δW was approximated by a straight line within the interval.

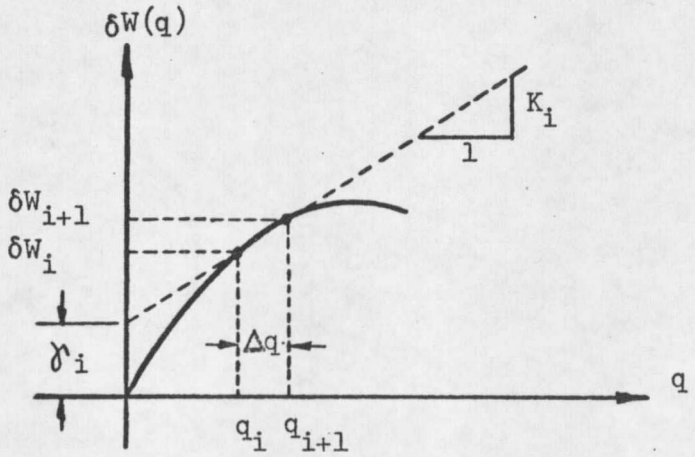


Figure 6. Approximation of Virtual Work Function

$$\delta W(q) = K_i q + \gamma_i \quad \text{for} \quad q_i \leq q(t) \leq q_{i+1} \qquad \text{(Eq. 57)}$$

where

$$K_i = \frac{\delta W_{i+1} - \delta W_i}{\Delta q}$$

Within the interval q(t) approximately satisfies,

$$\bar{M}\ddot{q} + K_i q = A\, e^{-\beta t} - \gamma_i \quad \text{for} \quad q_i \leq q(t) \leq q_{i+1} \qquad \text{(Eq. 58)}$$

with the initial conditions at q_i

$$q(t_i) = q_i$$

$$\dot{q}(t_i) = \dot{q}_i$$

The program steps through the solution, Equation (59), of q(t) in this manner stopping the calculation at the maximum deflection.

$$q(t) = E_1 \operatorname{Sin}\sqrt{\frac{K_i}{\bar{M}}}\, t + E_2 \operatorname{Cos}\sqrt{\frac{K_i}{\bar{M}}}\, t + c_1 e^{-\beta t} + c_2 \qquad \text{(Eq. 59)}$$

where

$$c_1 = \frac{A}{\bar{M}\beta^2 + K_i} \quad , \quad c_2 = -\frac{\gamma_i}{K_i} \quad \text{for } q_i \leq q(t) \leq q_{i+1}$$

and E_1 and E_2 are determined from initial conditions at q_i. Refer to Appendix B for more detail on the nonlinear program.

CHAPTER V: RESULTS AND CONCLUSIONS

5.1 The Mode Variation Technique

As mentioned in Chapter II, if the assumed solution chosen for the modified Galerkin method is the true solution then the residual, Equation (5), will be identically zero. It seems to follow, therefore, that the smallest residual magnitude, distributed uniformly over the interval $0 \leq x \leq L$, would denote the best approximation. The "best" refers to that which best satisfies the governing equation of motion. Four mode shape functions were chosen, three of which could be varied. All shape functions were normalized to 1 at $x = L$. The functions used were:

1) $$\theta_1(x) = (x^4 - 4Lx^3 + 6L^2x^2)/3L^4$$

which is the static deflection shape of a linearized beam;

2) $$\theta_2(x) = \frac{X_1(x) + a_2 X_2(x)}{X_1(L) + a_2 X_2(L)}$$

where $X_1(x)$ and $X_2(x)$ are the first two eigenfunctions, Equation (17a), of the linearized beam motion, and a_2 is the variation parameter;

3) $$\theta_3(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)}$$

and the final shape function used was:

4) $$\theta_4(x) = \frac{z_1(x) + a_4 z_2(x)}{x_1(L) + a_4 z_2(L)}$$

where

$$z_1(x) = \theta_1(x) \qquad \text{(the static deflection shape)}$$

and $$z_2(x) = 1 - \cos \frac{\pi x}{2L}$$

Shape functions θ_2, θ_3, and θ_4 could be varied by variation of the constants a_2, a_3, and a_4. Evaluation of the particular shape function was made by the size of the residual and the stiffness of the reduced system reflected by the magnitude of the maximum deflection.

5.2 Problem Data

The same problem data was used for all runs for uniformity of results. Experimental results were not available; therefore an arbitrary problem was chosen. Figure 7 shows the dimensions of the beam used.

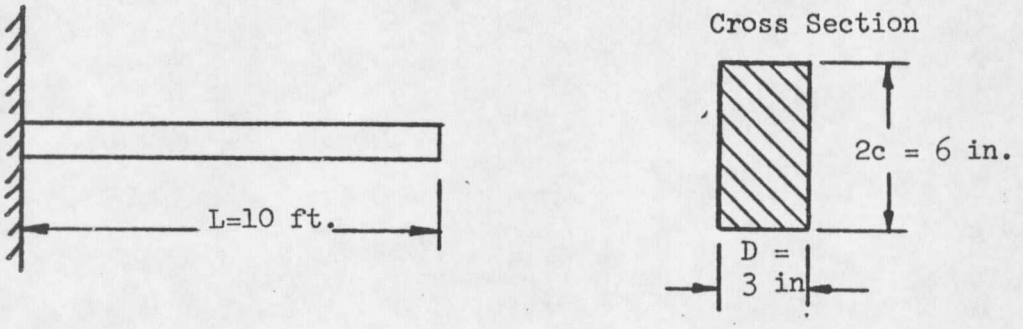


Figure 7. Beam Dimensions

Figure 8 shows the stress-strain relation used. The modulus of elasticity of aluminum was used for the linear problem ($E = 10^7$ psi). The nonlinear relation used had the same initial slope and an ultimate stress of 65,500 psi.

The impulse (I) of the blast pressure was chosen to insure deflections into the nonlinear range:

$$I = \frac{P_{max}}{\beta} = \frac{6000}{3000} = 2 \text{ psi-sec}$$

5.3 Results of Linear Beam Problem

Table I is a summary of maximum end deflection results for both the linear and nonlinear problems. The mode shape variation investigation proceeded as indicated by the order in the table.

First, the accuracy of the method was tested. The normalized exact first eigenfunction, X_1, of the linear beam problem was used as a shape function in the Galerkin approximation. As expected, this yielded the true first mode motion:

$$y(x,t) = X_1(x)\, T_1(t) + y_p(x,t)$$

Figure 9 compares the Galerkin results of end deflection with the true motion, illustrating the results that can be expected from a correct choice of the shape function in a one term approximation.

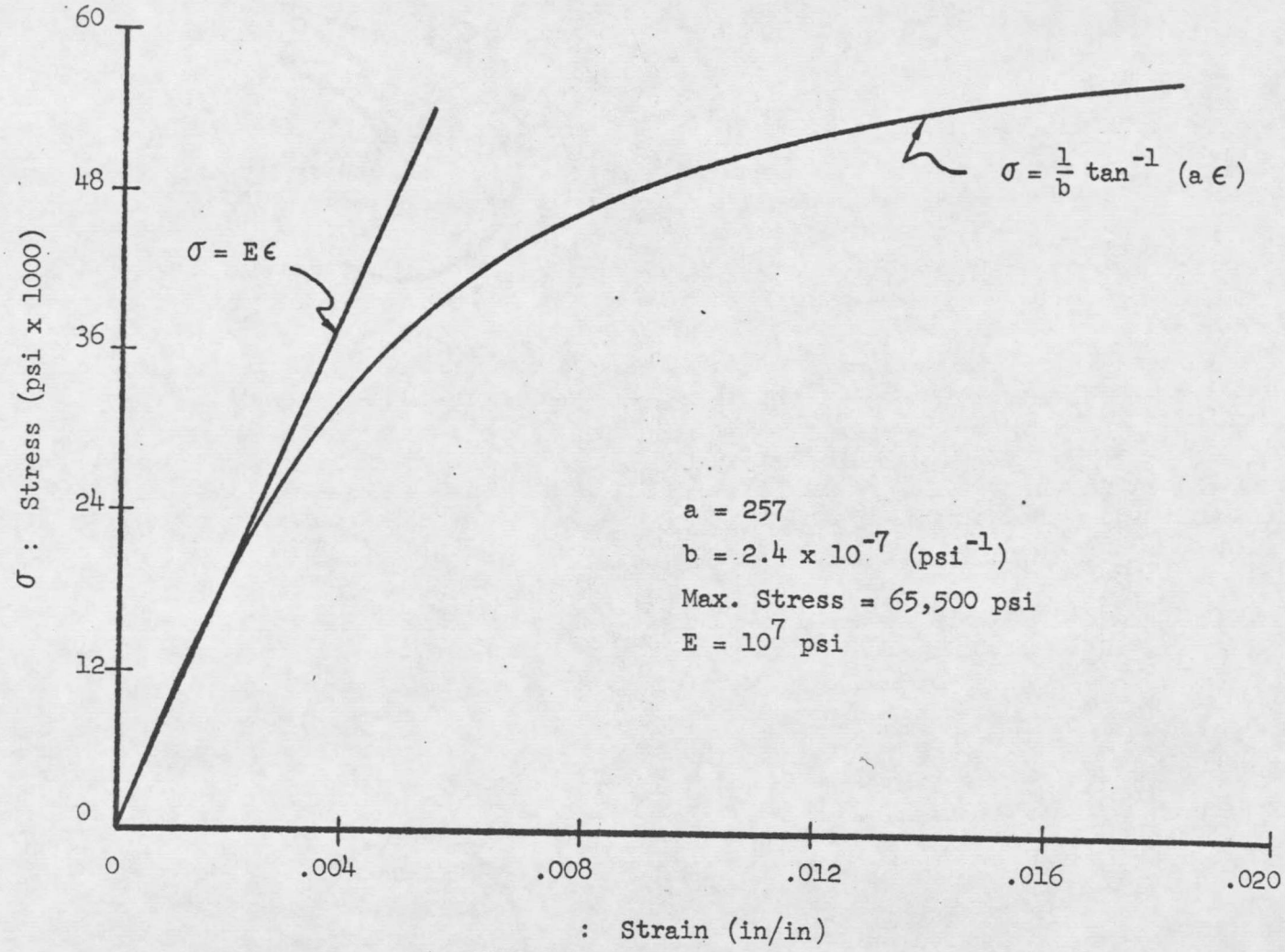


Figure 8. Stress-Strain Approximation

Table I. Linear-Nonlinear Comparison of Deflection Results

Shape Function $\theta(x)$	Max. End Deflection	
	Linear	Nonlinear
$\theta(x) = X_1(x)/X_1(L)$	24.24	31.92
$\theta(x) = \dfrac{X_1(x) + 0.3\,X_2(x)}{X_1(L) + 0.3\,X_2(L)}$	15.36	14.48
$\theta(x) = \dfrac{X_1(x) + X_2(x)}{X_1(L) + X_2(L)}$	3.62	10.85
$\theta(x) = \dfrac{X_1(x) + 0.1\,X_1^2(x)}{X_1(L) + 0.1\,X_1^2(L)}$	24.36*	31.38*
$\theta(x) = \dfrac{X_1(x) + 0.25\,X_1^2(x)}{X_1(L) + 0.25\,X_1^2(L)}$	24.12	30.12
$\theta(x) = \dfrac{X_1(x) + 0.5\,X_1^2(x)}{X_1(L) + 0.5\,X_1^2(L)}$	23.15	28.05
$\theta(x) = \dfrac{X_1(x) - 0.2\,X_1^2(x)}{X_1(L) - 0.2\,X_1^2(L)}$	22.91	31.12
$\theta(x) = \dfrac{x^4 - 4Lx^3 + 6L^2x^2}{3L^4}$	24.02	32.44*
$\theta(x) = 1 - \cos(\pi x/2L)$	23.81	28.31
$\theta(x) = \dfrac{z_1(x) + 5z_2(x)}{z_1(L) + 5z_2(L)}$	24.09	29.12
$\theta(x) = \dfrac{z_1(x) + z_2(x)}{z_1(L) + z_2(L)}$	24.36*	30.64

* Denotes "best" shape function.

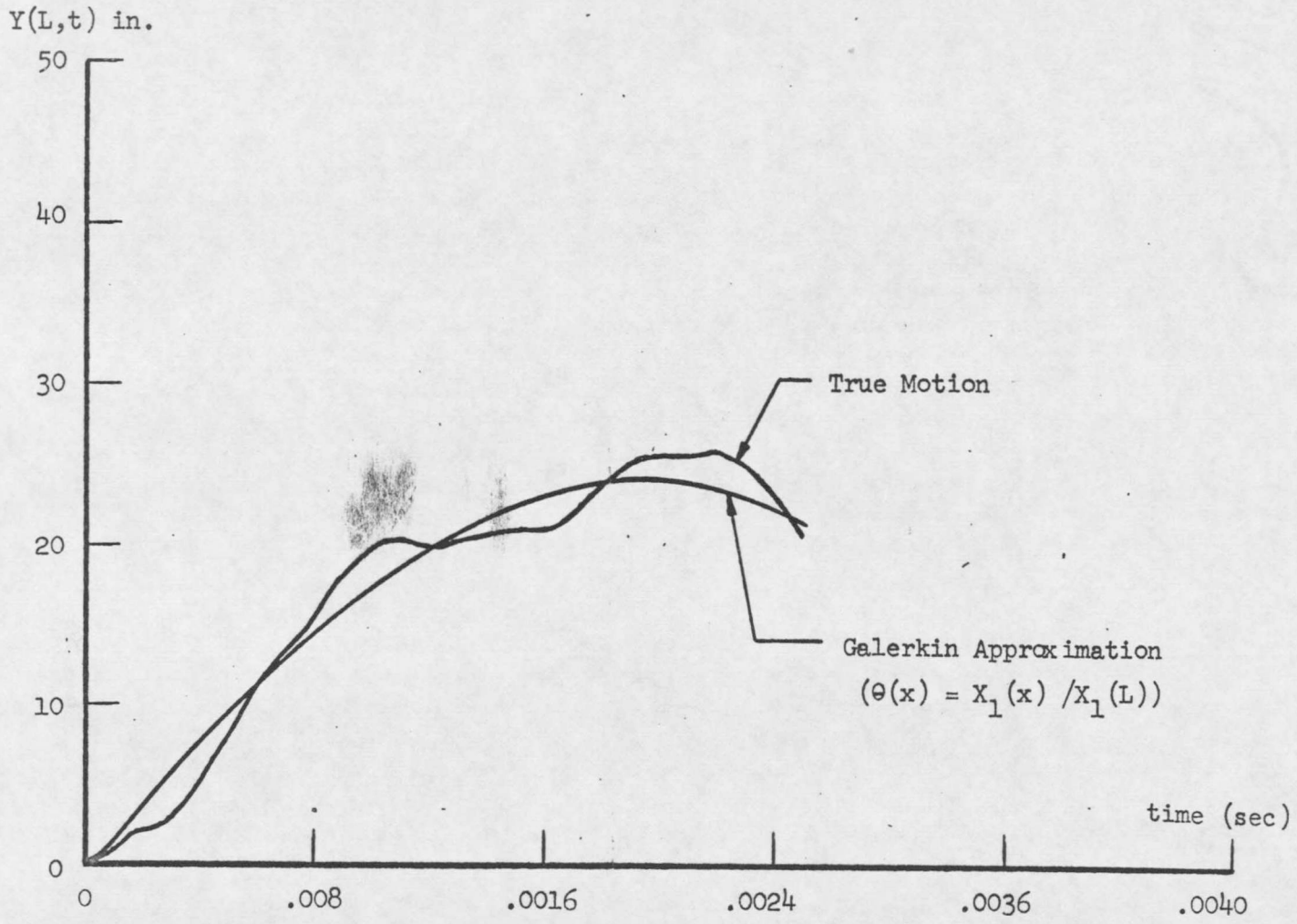


Figure 9. Approximation of True Linear Beam Motion by the Modified Galerkin Method.

The beam, though, is not being forced in a manner which results in only the first mode motion. It was then theorized that a shape function, other than $X_1(x)$, could be chosen which would give "better" results. As suggested by Anderson [12], a "better" shape function will result in a reduced system stiffness, thus resulting in larger deflections. The criterion used here to select the best shape function was to choose that which resulted in a smaller and "better" shaped equation residual. The "better residual" will be defined in this Section.

The first attempt at an improvement over X_1 was the selection of a linear combination of the first and second eigen functions $X_1(x)$ and $X_2(x)$, respectively.

$$\Theta(x) = \frac{X_1(x) + a_2 X_2(x)}{X_1(L) + a_2 X_2(L)}$$

As the shape variation parameter a_2 was increased, including more of the second mode, the equation residual increased tremendously and oscillated wildly over the interval of interest: $0 \leq x \leq L$. Maximum magnitudes were of the order of 10^5 lbs/in. The system became stiffer, resulting in smaller deflections. From the magnitude of the residual and the decreased deflections it was deduced that this combination was a poor choice and that shape functions similar to $X_1(x)$ would be more desirable.

Final deformation shapes shown by Bodner and Symonds [5] suggested that a shape having a larger curvature at the wall ($x = 0$) would be a better choice. A linear combination of $X_1(x)$ and $X_1^2(x)$ was chosen, namely:

$$\theta(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)}$$

Figure 10 illustrates the residual changes resulting from variation of a_3. Note that the maximum magnitudes of the residuals have been reduced greatly over that of the previous combination. Referring back to Table I, it can be seen that variation of the parameter a_3 resulted, in one case ($a_3 = 0.1$), in a larger deflection than that obtained when using only $X_1(x)$, ($a_3 = 0$). All other variations of a_3 resulted in smaller deflections.

With the aid of Figure 10 the "better residual" can now be defined. As an approximation to the ideal case of an identically zero residual, the preferred residual is of smaller magnitude over the interval $0 \leq x \leq L$. Thus the approximate solution exhibits less error over the interval and better satisfies the governing equation of motion. When judging the relative merits of two residuals, the best is that which conforms to the definition over a greater portion of the interval.

Referring to Figure 10, consider the residuals resulting from the variations $a_3 = 0.1$ and $a_3 = 0.5$. The residual resulting from $a_3 = 0.1$ has a smaller magnitude than that of $a_3 = 0.5$ over the entire length of the beam. In keeping with our definition, the shape function with $a_3 = 0.1$ is then judged as a better approximation. Deflection results showed $a_3 = 0.1$ giving the largest deflection, which was also slightly greater than that obtained from using $X_1(x)$ or $a_3 = 0$. It can be seen from Figure 10 that the $a_3 = 0.1$ residual is of smaller magnitude over a greater portion of the interval than that resulting from $a_3 = 0$, thus again $a_3 = 0.1$ is judged best. Similar arguments can be made for each variation on a_3 resulting in $a_3 = 0.1$ as best.

Two shapes, which through physical considerations were chosen as being reasonable approximations, were then used giving favorable results. They were the static deflection shape

$$z_1(x) = \frac{x^4 - 4Lx^3 + 6L^2x^2}{3L^4}$$

and

$$z_2(x) = 1 - \text{Cos}\ \frac{\pi x}{2L}$$

Figure 11 shows their corresponding residuals.

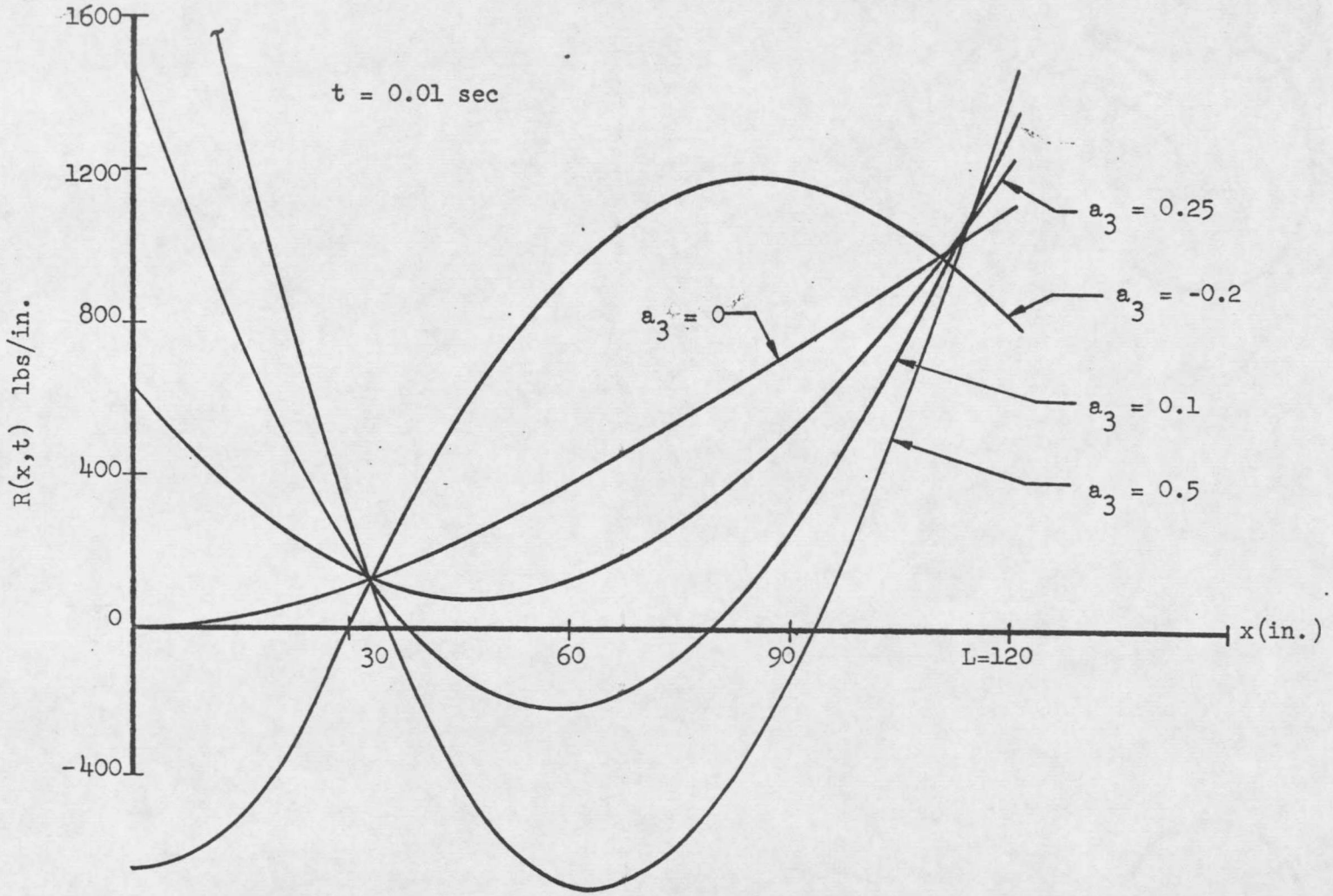


Figure 10. Comparison of Equation Residuals of the Shape Variation $\theta = X_1 + a_3 X_1^2$ (Linear Beam).

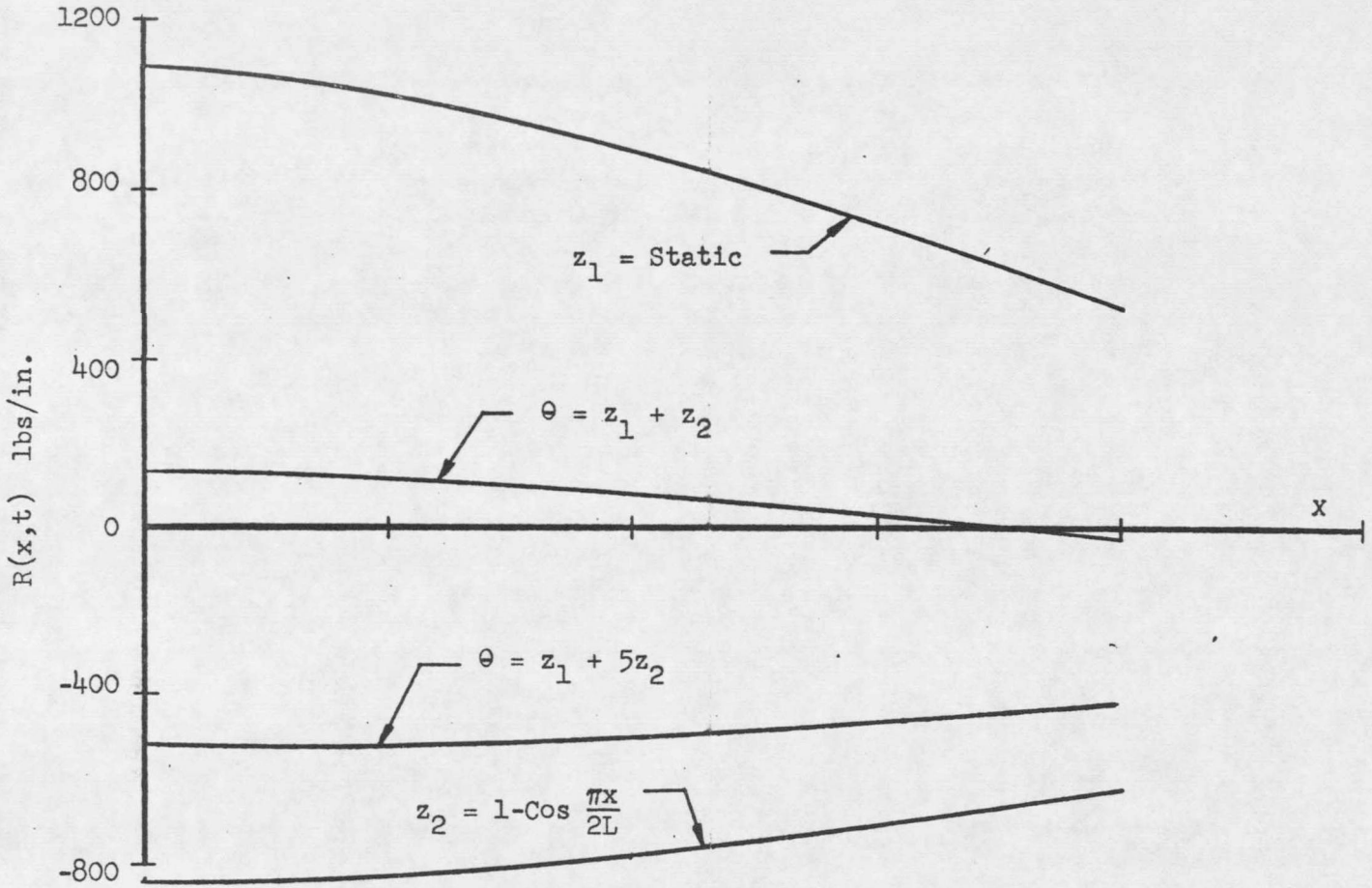


Figure 11. Residual Averaging -- Linear Beam

A final linear combination was then used, namely:

$$\theta(x) = \frac{z_1(x) + a_4\, z_2(x)}{z_1(L) + a_4\, z_2(L)}$$

Referring to Figure 11, by our definition the above combination with $a_4 = 5$ is a better approximation than either z_1 or z_2 alone. Deflection results show that this combination gives a larger deflection.

Note that the residual resulting from $a_4 = 5$ is approximately 0.6 of that resulting from $z_2(x)$ alone. This suggested decreasing the amount of the cosine function in the combination $\theta(x)$. The parameter a_4 was decreased to 1.0, resulting in a greatly decreased residual. Note that the combination of the shape functions resulted in an averaging of residuals in some sense. This property, if repeatable, could be used to good advantage when attempting to minimize equation residuals. Further variation of a_4 could result in further decrease of the residual. Again large deflection results coincided with the selection of $a_4 = 1$ being best.

It is interesting to note that a maximum end deflection of 24.36 inches resulted from two different mode shapes having differently shaped residuals. Figure 12 illustrates this point. From our definition the $z_1 + z_2$ residual is better, yet $X_1 + .1\, X_1^2$ gives the same maximum deflection results. It is speculated that the latter may give poorer results for deflections at other points on the beam. Higher

order deflections such as slope or curvature may not be approximated as well, either.

5.4 Results of Nonlinear Beam Problem

In general, results of the nonlinear beam problem followed the same patterns as the linear problem, using exactly the same shape function variations. Maximum deflections occurred for different shapes as a result of the expected different deflection shape. The overall maximum deflection achieved by mode variation was that of the static deflection shape whose residual was determined as best according to the definition.

Figure 13 illustrates residuals resulting from the variation on the combination

$$\theta(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)}$$

for the nonlinear problem. Similarly, the variation ($a_3 = 0.5$) giving the smallest deflection resulted in the "worst" residual, while the variation ($a_1 = 0$) giving the largest deflection of this combination resulted in the "best" residual. All nonlinear deflections were larger than the linear case. This is a result of the system stiffness decreasing as deflection increased approaching the fully plastic state for the nonlinear problem. This is reflected in the decreasing slope of the nonlinear stress-strain relation used.

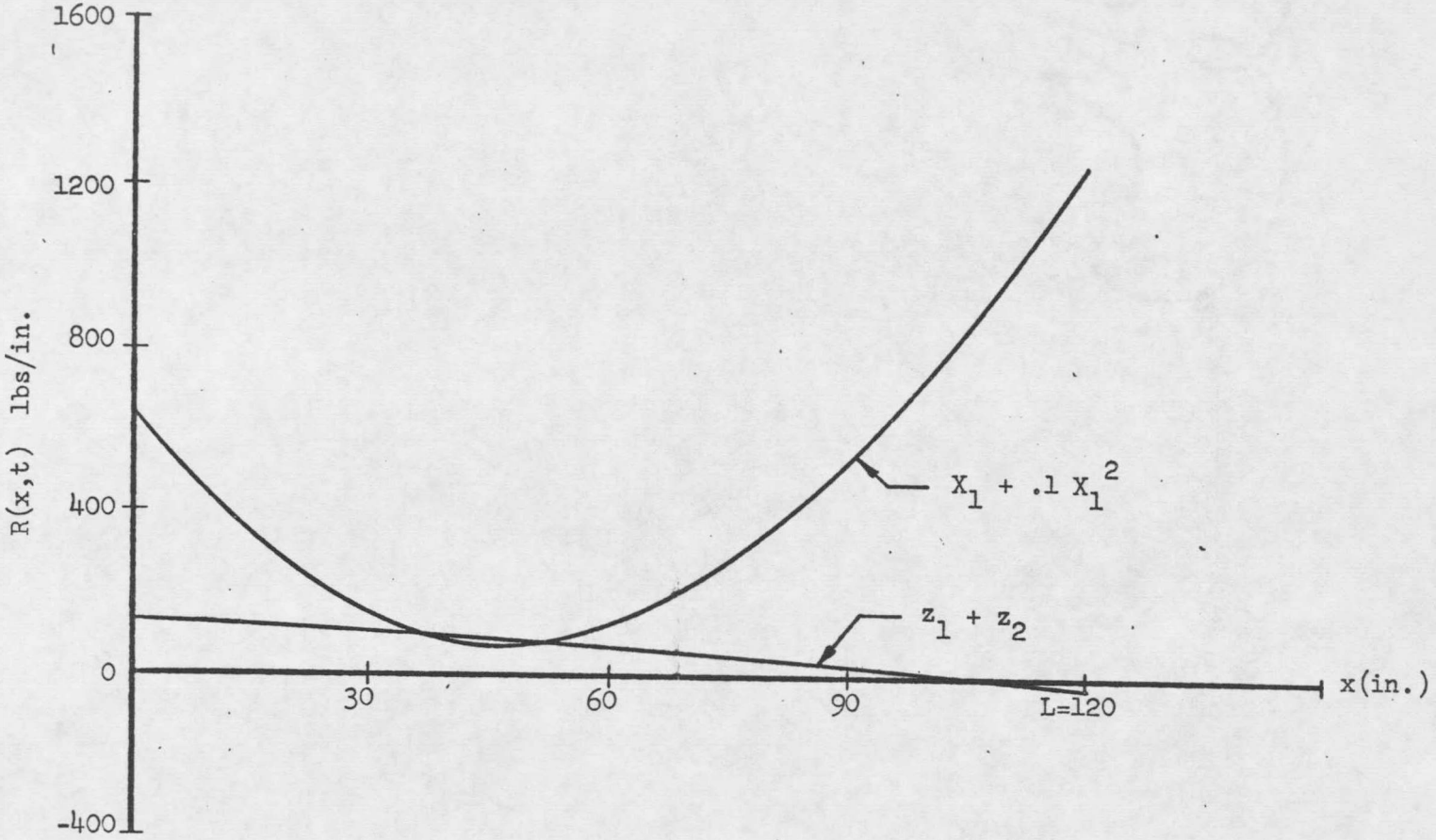


Figure 12. The "Better" Residual -- Linear Beam

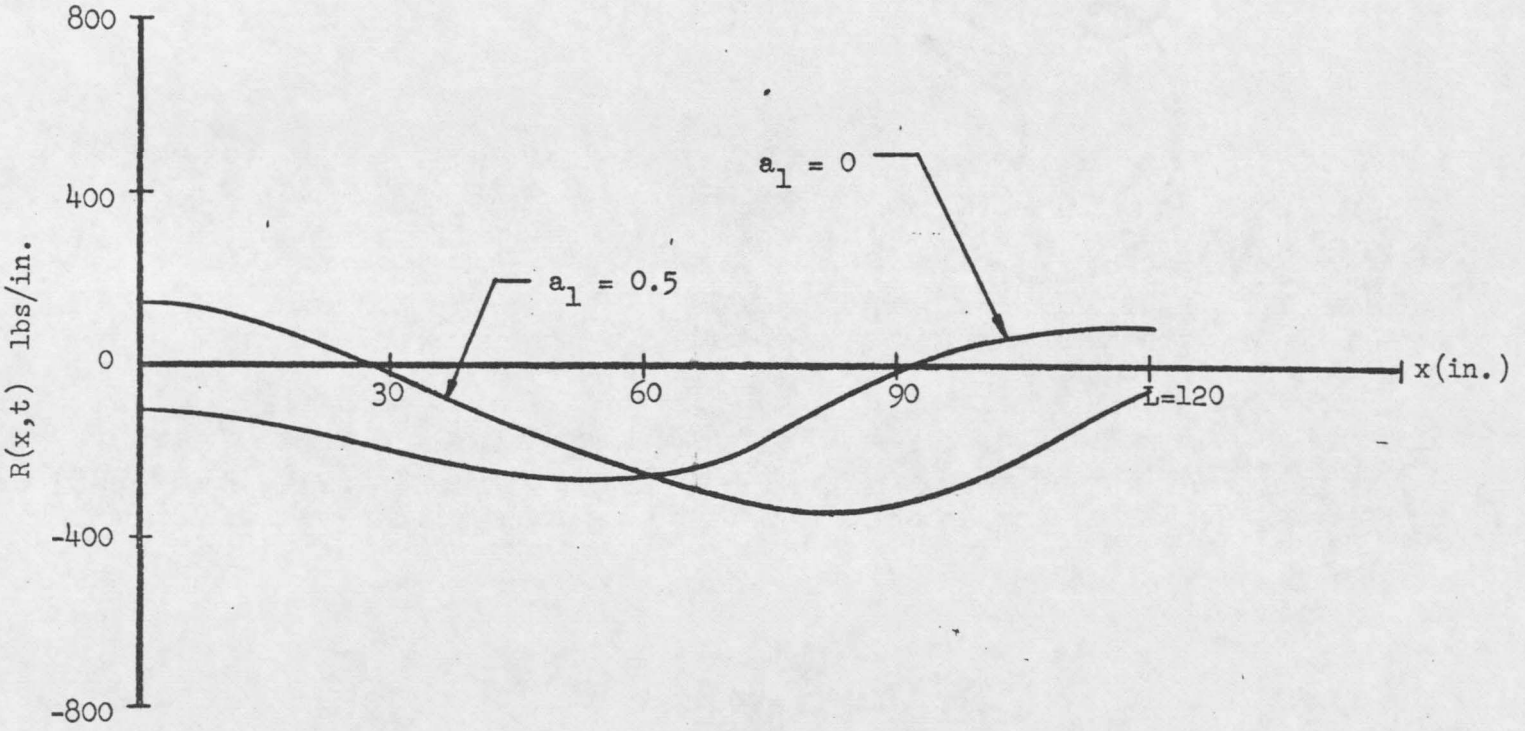


Figure 13. Residuals of Variation on $X_1^2 + a_1 X_1^2$ -- Nonlinear Beam

The nonlinear problem exhibited the same averaging of residuals from the combination of $z_1(x)$ and $z_2(x)$ as the linear problem. This is illustrated in Figure 14. Note the small magnitude of the static shape residual which gave the largest deflection.

Introduction of the second eigenfunction $X_2(x)$ similarly gave small deflections and residuals of large magnitude.

5.5 Conclusions and Further Studies

From comparison of the approximate solution of the linearized beam deflection to the true motion (Figure 9), it appears that the one term modified Galerkin solution has merit. Selection of a shape function similar to the first mode shape is preferred. Inclusion of a shape similar to the second mode shape is not. This corresponds to forcing a deflection in the second mode at a frequency near the first natural frequency requiring a large amount of energy, thus increasing the stiffness of the reduced system.

In both the linear and nonlinear cases, separately, differences in deflection results were small between shape functions judged best. Refer to Table I. This suggests that the method is a stable one where reasonable shapes will give reasonable answers.

Comparison of the deflection results presented here should be made to experimental results. It would be a simple task to modify the computer programs to the experimental beam dimensions and cross-

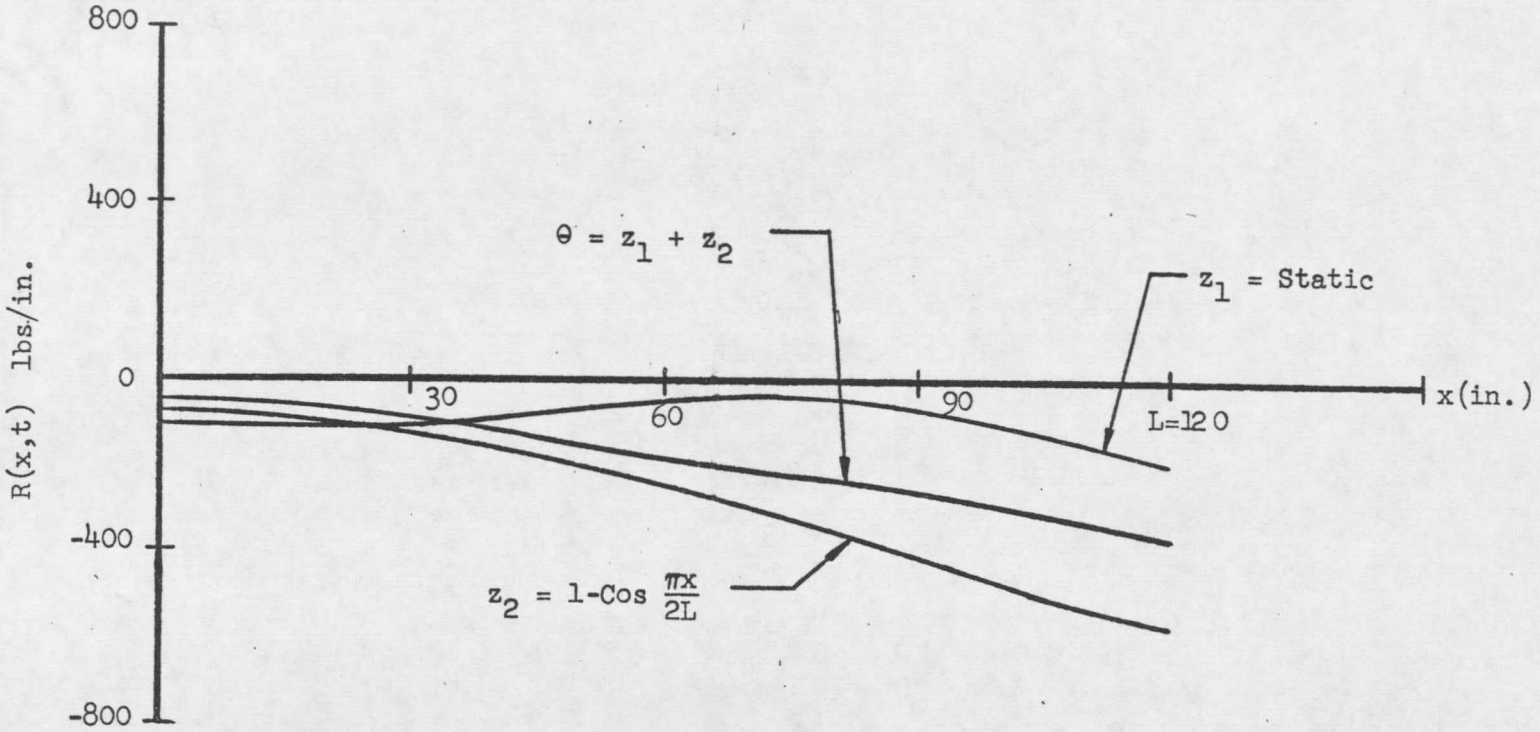


Figure 14. Residual Averaging -- Nonlinear Beam

sectional geometry. The "better residual" criterion presented may give more dependable analytic results.

The method presented for selection of a best shape function by the preferred residual definition also appears to have merit. The averaging property of the system residual found in this investigation could prove to be valuable if this is a general property. This suggests a technique to reduce equation residuals. Two shape functions which resulted in reasonably flat residuals, one of positive value and the other negative, must first be found. These shapes could then be combined in some manner to reduce the residual, as illustrated in Figure 11. The large deflections resulting from shape functions judged best corresponds to the previously implied shape function criterion. That is, the best shape function gives the largest deflection. This correspondence strengthens the merit of both approaches.

The similarity of residual properties and deflection results between the linear and nonlinear cases suggests an approach to solving nonlinear system models. First investigate a linearized version which in general will have an exact solution available. Solve the linear problem by the modified Galerkin method noting the shape functions which give the best results. Use these same functions for a modified Galerkin solution of the nonlinear system. If both solutions exhibit similar properties, as did the beam problems, it does not seem unreas-

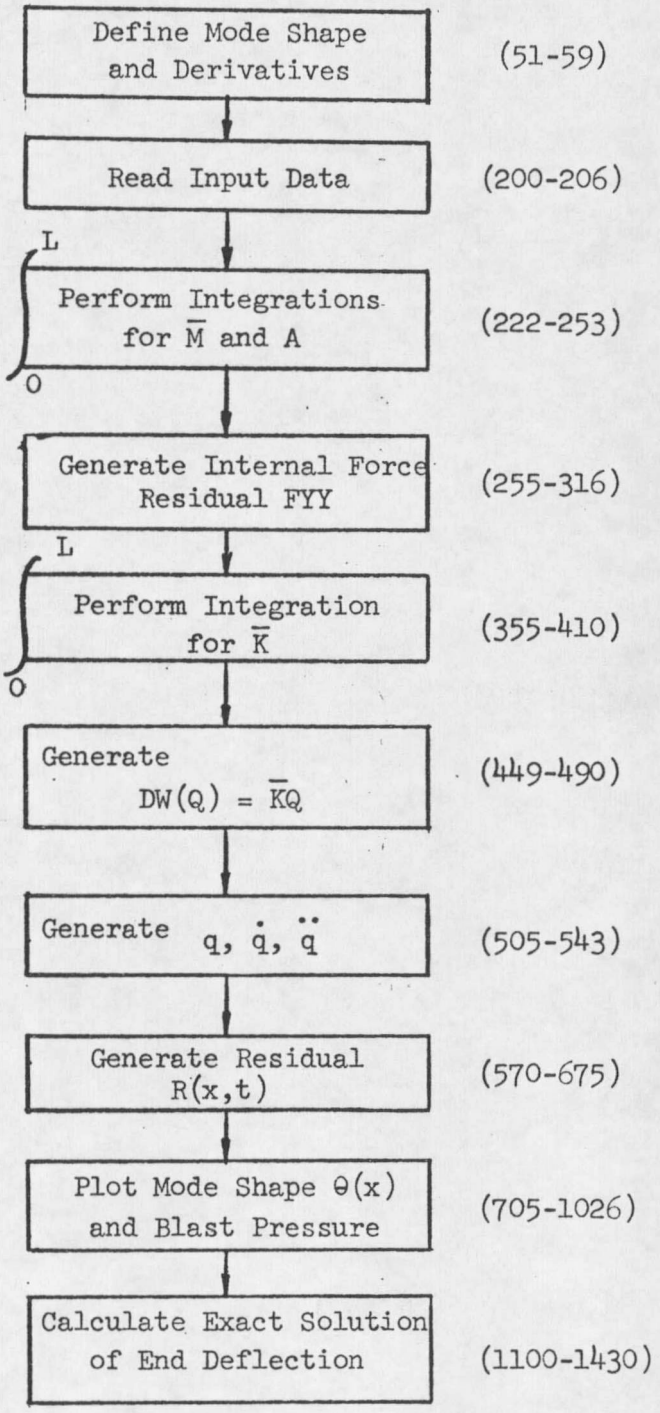
onable to assume that the solution of the nonlinear system will possess the same merit.

The shape function selection technique presented should be investigated further to determine its merits in different problems. Other preferred residual definitions could prove to be more desirable. Possibly the minimization of area under the absolute value of the residual is more significant. Another possibility is minimizing the area under the square of the residual. This corresponds to the method of Least Squares (see [10]) which is difficult to apply in a nonlinear case.

APPENDIX

APPENDIX A

A-1. Flow Chart of Linear Beam Program.



A-2. Linear Program Nomenclature

Variable Name	Description
AA	Augmented Coefficient Matrix, Exact Solution
ALPHA	$\alpha = P_{max}\, D$
BETA	β = Blast Decay Constant
BL	L = Beam Length (inches)
BM	$\bar{M}$ = Generalized Mass
C	2C = Height of Beam Cross-section (inches)
CAPX	Exact Eigenfunctions $X_n(x)$
D	D = Thickness of Beam (inches)
DD	Coefficients of Particular Solution
DT	Time Increment (sec)
DQ	Deflection Increment (inches)
DEF	Exact End Deflection
E	Modulus of Elasticity
EP	$\epsilon = \frac{\sqrt{2}}{2} \sqrt[4]{\mu\beta^2/EI}$
EPL	$\epsilon(L)$
P	$P = A = \alpha \int_0^L \Theta \, dx$
PT	Blast Pressure Function
PMAX	Maximum Blast Pressure
QT	End Displacement (inches)
QDT	End Velocity (in/sec)
QDDT	End Acceleration (in/sec^2)

A-2. Linear Program Nomenclature (continued)

Variable Name	Description
RHO	Material Density (lbs/in^3)
SK	$\bar{K}$ = Generalized Stiffness
THETA, Y	$\theta(x)$, $y(x) = q(t)\,\theta(x)$
THETA1, YX1	θ', y'
THETA2, YX2	θ'', y''
THETA3, YX3	θ''', y'''
THETA4, YX4	θ'''', y''''
U	μ = Linear Beam Density (lbsm/in)
X	Beam Length Coordinate (inches)
YP	Exact Particular Solution

A-3. Linear Program Listing

```
C     MAXIMUM DEFLECTION OF A CANTILEVER BEAM UNDER BLAST LOAD
C     MODIFIED GALERKIN METHOD USED WITH LINEAR STRESS-STRAIN RELATION
C     AND SMALL DEFLECTION GEOMETRY.
   45 DIMENSION XA(500),YA(500),YYA(500),YYYA(500),YB(500),AA(4,5),DD(4)
   46 DIMENSION W(50),Z(50),G(50),FF(50)
      CZ1(BL,X)=(X**4-4*BL*X**3+6*BL**2*X**2)/3/BL**4
      CZ11(BL,X)=(X**3-3*BL*X**2+3*BL**2*X)*4/3/BL**4
      CZ12(BL,X)=4*(X-BL)**2/BL**4
      CZ13(BL,X)=8*(X-BL)/BL**4
      CZ14(BL,X)=(8/BL**4)*(X+1-X)
      CZ2(BL,X)=1-COS(3.1415923*X/2/BL)
      CZ21(BL,X)=(3.1415923/2/BL)*SIN(3.1415923*X/2/BL)
      CZ22(BL,X)=(3.1415923/2/BL)**2*COS(3.1415923*X/2/BL)
      CZ23(BL,X)=(3.1415923/2/BL)**3*SIN(3.1415923*X/2/BL)*(-1)
      CZ24(BL,X)=(3.1415923/2/BL)**4*COS(3.1415923*X/2/BL)*(-1)
      THETA(BL,X)=(CZ1(BL,X)+ 0.50*CZ2(BL,X))/(1+ 0.50)
      THETA1(BL,X)=(CZ11(BL,X)+ 0.50*CZ21(BL,X))/(1+ 0.50)
      THETA2(BL,X)=(CZ12(BL,X)+ 0.50*CZ22(BL,X))/(1+ 0.50)
      THETA3(BL,X)=(CZ13(BL,X)+ 0.50*CZ23(BL,X))/(1+ 0.50)
      THETA4(BL,X)=(CZ14(BL,X)+ 0.50*CZ24(BL,X))/(1+ 0.50)
   60 PT(PMAX,BETA,T)=PMAX*EXP(-BETA*T)
   70 CAPX(Z,G,X)=(COS(Z)+COSH(Z))*(SIN(G*X)-SINH(G*X))
     1 -(SIN(Z)+SINH(Z))*(COS(G*X)-COSH(G*X))
  100 Y(Q,BL,X)=Q*THETA(BL,X)
  105 YX1(Q,BL,X)=Q*THETA1(BL,X)
  110 YX2(Q,BL,X)=Q*THETA2(BL,X)
  115 YX3(Q,BL,X)=Q*THETA3(BL,X)
  120 YX4(Q,BL,X)=Q*THETA4(BL,X)
  125 F(E,C,D,BL,X)=2*E*D*C**3*THETA4(BL,X)*THETA(BL,X)/3
  188 C1(P,ALPHA,BM,BETA,SK)=P*ALPHA/(BM*BETA**2+SK)
  190 QT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*(BETA*SQRT(BM/SK)*
     1SIN(SQRT(SK/BM)*T)+EXP(-BETA*T)-COS(SQRT(SK/BM)*T))
  192 QDT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*(BETA*(COS(SQRT(S
     1K/BM)*T)-EXP(-BETA*T))+SQRT(SK/BM)*SIN(SQRT(SK/BM)*T))
  195 QDDT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*(-BETA*SQRT(SK/B
     1M)*SIN(SQRT(SK/BM)*T)+BETA**2*EXP(-BETA*T)+SK/BM*COS(SQRT(SK/BM)*T
     2))
  198 PHI(EP,D1,D2,D3,D4,C0,X)=EXP(EP*X)*(D1*SIN(EP*X)+D2*COS(EP*X))
     1+EXP(-EP*X)*(D3*SIN(EP*X)+D4*COS(EP*X))+C0
  199 YP(EP,D1,D2,D3,D4,C0,BETA,X,T)=PHI(EP,D1,D2,D3,D4,C0,X)*EXP(-BETA
     1*T)
  200 READ(105,201) E,C,D,BL,RHO,PMAX,BETA
  201 FORMAT(7F10.0)
C     N=EVEN NUMBER OF INTEGRATION STEPS -- J=# DATA POINTS IN Q
  205 IF(E) 2000,2000,206
  206 READ(105,207)   N,J
  207 FORMAT(2I5)
  208 PI=3.1415927
  210 CALL PINIT
C     PLOT INTERNAL FORCE FUNCTION   FYY(Q,X)
  211 CALL AXIS(1,0.0,150.,-200.,800.,6.5,4.5)
  212 WRITE(108,213)
  213 FORMAT(1H1,////////////////28X,'MAXIMUM DEFLECTION OF A CANTILEVER
     1BEAM'//28X,'BY:   DANIEL M. KOSZUTA'/34X,'AEROSPACE & MECHANICAL E
     2NGINEERING DEPT.'/34X,'MONTANA STATE UNIVERSITY')
  215 WRITE(108,216) BL,C,D,E,RHO,PMAX,BETA
  216 FORMAT(1H1///23X,'INPUT DATA:        BEAM LENGTH ..........  BL=',F10
     1.4/40X,'HALF BEAM HEIGHT .....  C=',F10.4/40X,'BEAM WIDTH .......
     2..  D=',F10.4/40X,'MODULUS OF ELASTICITY.  E=',E12.4/40X,'MATERIA
```

```
   3L DENSITY .....RHB=!,F7.4/40X,!MAX BLAST PRESSURE....PMAX=!,F8.2/40
   4X,!BLAST DECAY CONST....BETA=!,F8.2/////)
222 DT=0.0001
227 NP1=N+1
230 H=BL/N
235 X1=0.
240 XN=BL
241 QMAX=BL
242 BM=H*(THETA(BL,X1)**2+THETA(BL,XN)**2)/3
243 P=H*(THETA(BL,X1)+THETA(BL,XN))/3
244 DO 249 K=1,2
245 NK=N-K
246 DO 249 L=K,NK,2
247 X=L*H
248 P=P+4*H*THETA(BL,X)/3/K
249 BM=BM+4*H*THETA(BL,X)**2/3/K
250 DQ=QMAX/J
252 J1=J+1
253 BM=C*D*RHB*BM/16.085/12.
254 ALPHA=PMAX*D
255 DO 316 L=1,3
260 IF(L-1) 200,270,265
265 IF(L-2) 200,280,290
270 Q=DQ
275 GO TO 295
280 Q=(J/6)*DQ
285 GO TO 295
290 Q=(J/4)*DQ
295 DO 310 I=1,NP1
300 XA(I)=(I-1)*H
305 YA(I)=Q*F(E,C,D,BL,XA(I))
310 WRITE(108,315) XA(I),YA(I),Q
312 CALL PLOTS(1,1,NP1,0.0,150.,-200.,800.,6.5,4.5,XA,YA)
313 IF(L-1) 200,316,314
314 LKK=L+1
    CALL PLOTS(LKK,1,NP1,0.0,150.,-200.,800.,6.5,4.5,XA,YA)
316 CONTINUE
315 FORMAT(10X,!X=!,E12.5,10X,!FYY=!,E12.5,10X,!Q=!,E12.5)
350 WRITE(108,351)
351 FORMAT(1H1)
355 SK1=H*(F(E,C,D,BL,X1)+F(E,C,D,BL,XN))/3
360 DO 380 K=1,2
365 NK=N-K
370 DO 380 L=K,NK,2
375 X=L*H
380 SK1=SK1+4*H*F(E,C,D,BL,X)/3/K
400 SK2=2*E*D*C**3*THETA2(BL,BL)*THETA1(BL,BL)/3
405 SK3=2*E*D*C**3*THETA3(BL,BL)*THETA(BL,BL)/(-3)
410 SK=SK1+SK2+SK3
415 WRITE(108,416) SK1,SK2,SK3,SK
416 FORMAT(10X,!K1=!,E12.5,10X,!K2=!,E12.5,10X,!K3=!,E12.5,10X,!K=!,E1
   12.5)
418 WRITE(108,419) SK1,SK2,SK3,SK
419 FORMAT(5X,!WI=!,E12.5,' * Q!,10X,!WM=!,E12.5,' * Q!, 7X,!WV=!,E12.
   15,' * Q!, 7X,!DW=!,E12.5,' * Q!/////)
449 JJ2=J/2+1
450 DO 480 I=1,JJ2
455 XA(I)=(I-1)*DQ
```

```
  470 WV=S<3*XA(I)
  475 YA(I)=SK*XA(I)
  480 WRITE(108,481) XA(I),WI,WV,WM,YA(I)
  481 FORMAT(5X,'Q=',E12.5,5X,'WI=',E12.5,5X,'WV=',E12.5,5X,'WM=',E12.5,
     15X,'DW=',E12.5)
C     PLOT VIRTUAL WORK FUNCTION  DW=SK*Q
  485 CALL AXIS(1,0.0, 75.,0.0, 75000.,6.5,4.5)
  490 CALL PLOTS(1,1,JJ2,0.0, 75.,0.0, 75000.,6.5,4.5,XA,YA)
  500 WRITE(108,351)
  505 DO 530 I=1,250
  510 XA(I)=(I-1)*DT
  515 YA(I)=QT(P,ALPHA,BM,BETA,SK,XA(I))
  520 YYA(I)=QDT(P,ALPHA,BM,BETA,SK,XA(I))
  525 YYYA(I)=QDDT(P,ALPHA,BM,BETA,SK,XA(I))
  530 WRITE(108,531) XA(I),YA(I),YYA(I),YYYA(I)
  531 FORMAT(5X,'T=',E12.5,5X,'QT=',E12.5,5X,'QDT=',E12.5,5X,'QDDT=',E12
     1.5)
C     PLOT DEFLECTION QT   AND VELOCITY QDT
  535 CALL AXIS(1,0.0,0.04,0.0,50.,6.5,4.5)
  540 CALL PLOTS(1,1,250,0.0,0.04,0.0,50.,6.5,4.5,XA,YA)
  542 CALL AXIS(1,0.0,0.04,0.0,2500.,6.5,4.5)
  543 CALL PLOTS(1,1,250,0.0,0.04,0.0,2500.,6.5,4.5,XA,YYA)
  550 WRITE(108,351)
  570 DO 675 L=1,3
  575 IF(L-1) 200,590,580
  580 IF(L-2) 200,600,610
  590 T=5*DT
  591 YMIN=-4000.
  592 YMAX=16000.
  595 GO TO 615
  600 T=40*DT
  601 YMIN=-400.
  602 YMAX=1600.
  605 GO TO 615
  610 T=100*DT
  611 YMIN=-400.
  612 YMAX=1600.
  615 DO 645 I=1,NP1
  620 XA(I)=(I-1)*H
  625 YA(I)=RHO*C*D*THETA(BL,XA(I))*QDDT(P,ALPHA,BM,BETA,SK,T)/193.02
  630 YYA(I)=2*E*D*C**3*THETA4(BL,XA(I))*QT(P,ALPHA,BM,BETA,SK,T)
  635 YYYA(I)=D*PT(PMAX,BETA,T)
  640 YB(I)=YA(I)+YYA(I)-YYYA(I)
  645 WRITE(108,646) T,XA(I),YA(I),YYA(I),YYYA(I),YB(I)
  646 FORMAT(1X,'T=',E12.4,3X,'X=',E12.4,3X,'R1=',E12.4,3X,'R2=',E12.4,
     13X,'R3=',E12.4,3X,'RES=',E12.5)
C     PLOT RESIDUAL AND COMPONENTS
  650 CALL AXIS(1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5)
  655 CALL PLOTS(1,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YA)
  656 CALL PLOTS(3,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YA)
  660 CALL PLOTS(1,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYA)
  661 CALL PLOTS(4,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYA)
  665 CALL PLOTS(1,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYYA)
  666 CALL PLOTS(5,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYYA)
  670 CALL PLOTS(1,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YB)
  671 CALL PLOTS(9,1,NP1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YB)
  675 CONTINUE
  700 WRITE(108,351)
  705 DO 725 I=1,NP1
  710 XA(I)=(I-1)*H
```

```
  720 YA(I)=THETA(BL,XA(I))
  725 WRITE(108,726) XA(I),YA(I)
  726 FORMAT(10X,'X=',E12.5,10X,'THETA=',E12.5)
C     PLOT MODE SHAPE  THETA(BL,X)
  730 CALL AXIS(1,0.0,150.,0.0,1.25,6.5,4.5)
  731 CALL PLOTS(1,1,NP1,0.0,150.,0.0,1.25,6.5,4.5,XA,YA)
 1000 DXA=0.002/50.
 1005 DO 1020 I=1,51
 1010 XA(I)=(I-1)*DXA
 1015 YA(I)=PT(PMAX,BETA,XA(I))
 1020 WRITE(108,1021) I,XA(I),I,YA(I)
 1021 FORMAT(35X,'T(',I2,')=',E12.5,10X,'PRES(',I2,')=',E12.5)
C     PLOT PRESSURE FUNCTION  P(T)=PMAX*EXP(-BETA*T)
 1025 CALL AXIS(1,0.0,0.002,0.0,10000.,6.5,4.5)
 1026 CALL PLOTS(1,1,51,0.0,0.002,0.0,10000.,6.5,4.5,XA,YA)
C     CALCULATE EXACT SOLUTION
C     FIRST FIND COEFFICIENTS OF PARTICULAR SOLUTION
 1100 AA(1,1)=1.
 1101 AA(1,2)=1.
 1102 AA(1,3)=1.
 1103 AA(1,4)=-1.
 1104 AA(1,5)=0.
 1105 AA(2,1)=0.
 1106 AA(2,2)=1.
 1107 AA(2,3)=0.
 1108 AA(2,4)=1.
 1109 U=C*D*RHO/193.02
 1110 AA(2,5)=-ALPHA/U/BETA**2
 1111 EP=0.5*2.**0.5*(3*U*BETA**2/2/E/D/C**3)**0.25
 1112 EPL=EP*BL
 1113 EX=EXP(EPL)
 1114 EXM=EXP(-EPL)
 1115 CS=COS(EPL)
 1116 SN=SIN(EPL)
 1117 AA(3,1)=EX*CS
 1118 AA(3,2)=-EX*SN
 1119 AA(3,3)=-EXM*CS
 1120 AA(3,4)=EXM*SN
 1121 AA(3,5)=0.
 1122 AA(4,1)=EX*(CS-SN)
 1123 AA(4,2)=-EX*(SN+CS)
 1124 AA(4,3)=EXM*(SN+CS)
 1125 AA(4,4)=EXM*(CS-SN)
 1126 AA(4,5)=0.
 1127 CO=-AA(2,5)
 1130 CALL SOLTN(AA,DD,4,5)
C     PRINT COEFFICIENTS OF PARTICULAR SOLUTION & THE PARTICULAR
C     SOLUTION EVALUATED AT THE BEAM END.
 1135 WRITE(108,1136)
 1136 FORMAT(1H1////20X,'PARTICULAR SOLUTION'////)
 1140 DO 1141 I=1,4
 1141 WRITE(108,1142) I,DD(I)
 1142 FORMAT(20X,'DD(',I1,')=',E12.5)
 1150 WRITE(108,1136)
 1160 DO 1175 I=1,250
 1165 T=(I-1)*DT
 1170 YPL=YP(EP,DD(1),DD(2),DD(3),DD(4),CO,BETA,BL,T)
 1175 WRITE(108,1176) T,YPL
 1176 FORMAT(15X,'T=',E12.5,10X,'YP(BL,T)=',E12.5)
```

```
 1196 FORMAT(1H1////5X,'W(NN)=NATURAL FREQUENCIES'///)
C     CALCULATE NATURAL FREQUENCIES OF HOMOGENEOUS SOLUTION
 1200 DO 1285 NN=1,50
 1205 IF(NN-1) 1500,1210,1215
 1210 ZO=1.9
 1211 GO TO 1230
 1215 IF(NN-2) 1500,1220,1225
 1220 ZO=4.67
 1221 GO TO 1230
 1225 ZO=Z(NN-1)+PI
 1230 IC=0
 1235 ZN=ZO+(COS(ZO)+1/COSH(ZO))/(SIN(ZO)+SINH(ZO)/COSH(ZO)**2)
 1240 IC=IC+1
 1245 DZ=ABS(ZN-ZO)
 1250 IF(DZ-1.E-5) 1265,1265,1255
 1255 IF(IC-20) 1260,1265,1265
 1260 ZO=ZN
 1261 GO TO 1235
 1265 Z(NN)=ZN
 1270 DZN=Z(NN)-Z(NN-1)
 1275 W(NN)=SQRT(2*D*C**3*E/3/U)*(Z(NN)/BL)**2
 1280 G(NN)=(U*W(NN)**2*3/2/E/D/C**3)**0.25
 1285 WRITE(108,1286) NN,W(NN),NN,G(NN),NN,Z(NN),DZ,DZN
 1286 FORMAT( 2X,'W(',I2,')=',E12.5,5X,'G(',I2,')=',E12.5,5X,'Z(',I2,')=
     1',E12.5,5X,'DZ=',E12.5,5X,'DZN=',E12.5)
C     CALCULATE HOMOGENEOUS SOLUTION COEFFICIENTS  F(NN)
 1300 WRITE(108,1301)
 1301 FORMAT(1H1///20X,'HOMOGENEOUS COEFFICIENTS'///)
 1305 DO 1355 NN=1,50
 1310 F1=H*(PHI(EP,DD(1),DD(2),DD(3),DD(4),CO,0.0)*CAPX(Z(NN),G(NN),0.0)
     1+PHI(EP,DD(1),DD(2),DD(3),DD(4),CO,BL)*CAPX(Z(NN),G(NN),BL))/3
 1315 F2=H*(CAPX(Z(NN),G(NN),0.0)**2+CAPX(Z(NN),G(NN),BL)**2)/3
 1320 DO 1345 K=1,2
 1325 NK=N-K
 1330 DO 1345 L=K,NK,2
 1335 X=L*H
 1340 F1=F1+4*H*PHI(EP,DD(1),DD(2),DD(3),DD(4),CO,X)*CAPX(Z(NN),G(NN),X)
     1/3/K
 1345 F2=F2+4*H*CAPX(Z(NN),G(NN),X)**2/3/K
 1350 FF(NN)=-F1/F2
 1355 WRITE(108,1356) NN,FF(NN)
 1356 FORMAT(20X,'F(',I2,')=',E12.5)
C     CALCULATE DEFLECTION (DEF) OF BEAM END USING 10,20,30,40,&50 TERMS
 1380 DO 1430 I=1,5
 1385 NT=I
 1390 WRITE(108,1391) NT
 1391 FORMAT(1H1///10X,'DEFLECTION OF BEAM END',10X,'USING',I3,' TERMS'/
     1///)
 1395 DO 1425 L=1,250
 1400 T=(L-1)*DT
 1401 XA(L)=T
 1405 DEF=0.0
 1410 DO 1415 J=1,NT
 1415 DEF=DEF+FF(J)*CAPX(Z(J),G(J),BL)*(COS(W(J)*T)-BETA*SIN(W(J)*T)/W(J
     1))
 1420 DEF=DEF+YP(EP,DD(1),DD(2),DD(3),DD(4),CO,BETA,BL,T)
 1421 YA(L)=DEF
 1425 WRITE(108,1426) T,DEF
 1426 FORMAT(20X,'T=',E12.5,10X,'DEF=',E12.5)
```

```
C     SOLUTION OF SIMULTANEOUS EQUATIONS BY GAUSSIAN ELIMINATION
C     N  NUMBER OF SIMULTANEOUS EQUATIONS
C     M  NUMBER OF COLUMNS IN THE AUGMENTED MATRIX
C     L = N-1
C     A(I,J)  ELEMENTS OF THE AUGMENTED MATRICIES
C     I  MATRIX ROW NUMBER
C     J  MATRIX COLUMN NUMBER
C     JJ TAKES ON VALUES OF THE ROW NUMBERS WHICH ARE POSSIBLE PIVOT
C     ROWS, EVENTUALLY TAKING ON THE VALUE IDENTIFYING THE ROW HAVING
C     THE LARGEST PIVOT ELEMENT
C     BIG  TAKES ON VALUES OF THE ELEMENTS IN THE COLUMN CONTAINING
C     POSSIBLE PIVOT ELEMENTS, EVENTUALLY TAKING ON THE VALUE OF THE
C     ELEMENT USED
C     TEMP  TEMPORARY NAME USED FOR THE ELEMENTS OF THE ROW SELECTED TO
C     BECOME THE PIVOT ROW, BEFORE THE INTERCHANGE IS MADE
C     K  INDEX OF A DO LOOP TAKING ON VALUES FROM 1 TO N-1, IT IDENTIFIES
C     THE COLUMN CONTAINING POSSIBLE PIVOT ELEMENTS
C     KP1  K+1
C     AB  ABSOLUTE VALUE OF A(I,K)
C     QUOT  QUOTIENT A(I,K)/A(K,K)
C     X(I)  UNKNOWNS OF THE SET OF EQUATIONS BEING SOLVED
C     SUM  SUMMATION OF A(I,J)*X(J) FROM J=I+1 TO N
C     NN  INDEX OF A DO LOOP TAKING ON VALUES FROM 1 TO N-1
C     IP1  I+1
      SUBROUTINE SOLTN(A,X,N,M)
      DIMENSION A(N,M),X(N)
      L=N-1
      DO 12 K=1,L
      JJ=K
      BIG=ABS(A(K,K))
      KP1=K+1
      DO 7 I=KP1,N
      AB=ABS(A(I,K))
      IF (BIG-AB) 6,7,7
    6 BIG=AB
      JJ=I
    7 CONTINUE
      IF (JJ-K) 8,10,8
    8 DO 9 J=K,M
      TEMP = A(JJ,J)
      A(JJ,J)= A(K,J)
    9 A(K,J)= TEMP
   10 DO 11 I=KP1,N
      QUOT = A(I,K)/A(K,K)
      DO 11 J=KP1,M
   11 A(I,J)=A(I,J)-QUOT*A(K,J)
      DO 12 I=KP1,N
   12 A(I,K)=0.
      X(N)=A(N,M)/A(N,N)
      DO 14 NN=1,L
      SUM=0.
      I=N-NN
      IP1=I+1
      DO 13 J=IP1,N
   13 SUM=SUM+A(I,J)*X(J)
   14 X(I)=(A(I,M)-SUM)/A(I,I)
      RETURN
      END
```

```
1429 CALL AXIS(1,0.0,0.04,0.0,50.,6.5,4.5)
1430 CALL PLOTS(1,1,250,0.0,0.04,0.0,50.,6.5,4.5,XA,YA)
1500 GO TO 200
2000 END
```

SUBPROGRAMS

BF!PIN	BF!SL	BF!FN	BF!ITF	COS	SIN	EXP	C
SINH	SQRT	BF!S3	BF!FI	BF!SF	BF!II	PINIT	A
BF!S6	PLOTS	BF!FR	SOLTN	ABS	BF!SX		

PROGRAM ALLOCATION

E0C.0	E	E0D.0	C	E0E.0	D	E0F.0	BL	E10.
E11.0	PMAX	E12.0	BETA	E13.0	N	E14.0	J	E15.
E16.0	DT	E17.0	NP1	E18.0	H	E19.0	X_1	E1A.
E18.0	QMAX	E1C.0	BM	E1D.0	P	E1E.0	K	E1F.
E20.0	L	E21.0	X	E22.0	DQ	E23.0	J_1	E24.
E25.0	Q	E26.0	I	E27.0	LKK	E28.0	SK1	E29.
E2A.0	SK3	E2B.0	SK	E2C.0	JJ2	E2D.0	W1	E2E.
E2F.0	WV	E30.0	T	E31.0	YMIN	E32.0	YMAX	E33.
E34.0	U	E35.0	EP	E36.0	EPL	E37.0	EX	E38.
E39.0	CS	E3A.0	SN	E3B.0	CO	E3C.0	YPL	E3D.
E3E.0	ZO	E3F.0	IC	E40.0	ZN	E41.0	DZ	E42.
E43.0	F1	E44.0	F2	E45.0	NT	E46.0	DEF	
E47.0	XA	103B.0	YA	122F.0	YYA	1423.0	YYYA	1617
1808.0	AA	181F.0	DD	1823.0	W	1855.0	Z	1887
1889.0	FF							

PROGRAM SIZE 18EB

PROGRAM END

Appendix B

B-1. Nonlinear Program Nomenclature

Variable Name	Description
A	Stress Constant a, Eq. (23)
APC	Argument of $\tan^{-1}$, a ϕ'c. Eq. (25)
ALPHA	$\alpha = P_{max}D$, Eq. (15)
B	Stress Constant (psi^{-1}), Eq. (23)
BMX1	$\partial M/\partial x$, Eq. (26)
BMS1	$\partial M/\partial x$ with series for $\tan^{-1}$
BMX2	$\partial^2 M/\partial x^2$, Eq. (27)
BMX2S	$\partial^2 M/\partial x^2$, with series for $\tan^{-1}$
BML	Moment at x = L
BMS	Moment with series for $\tan^{-1}$
BETA	Blast Decay Constant (sec^{-1})
BL	Beam Length (in.)
C	2C = Beam Height (in.)
C1, C2	Constants of Eq. (59)
D	Beam Thickness (in.)
DT	Time Increment
DW	Virtual Work, Eq. (56)
DX	Beam Length Coordinate Increment, (in.)
DQ	Displacement Increment, (in.)
DWN & DWN1	δW_i & δW_{i+1}, Fig. 6
E1, E2	Constants of Eq. (59)

B-1 Nonlinear Program Nomenclature (continued)

Variable Name	Description
FYY	Weighted Internal Force, Eq. (58)
FIN	Internal Force Error, Eq. (53)
GN	Straight Line Intercept $_i$, Fig. 6
P	$P = \int_0^L \Theta \, dx$
PHI	Deflection Angle ϕ, Fig. 3
PHI1	ϕ / x
PHI2	$\partial^2\phi / \partial x^2$
PHI3	$\partial^3\phi / \partial x^3$
PHIL	$\delta\phi$, Eq. (53b)
PT	Blast Pressure Function (psi) Fig. 2
QNT	Free End Deflection (in.)
QDNT	Free End Velocity (in/sec)
QDDT	Free End Acceleration (in/sec^2)
QDOTO	Initial Velocity
QO	Initial Displacement
RHO	Material Density (lbsm/in^3)
RES	Equation Residual, Eq. (49)
T	Time (sec)
THETA, Y	$\Theta(x)$, $y^*(x,t)$

B-1 Nonlinear Program Nomenclature (continued)

Variable Name	Description
THETA1, YX1	θ', $y^{*'}$
THETA2, YX2	θ', $y^{*''}$
THETA3, YX3	θ''', $y^{*'''}$
THETA4, YX4	θ'''', $y^{*''''}$
U	Linear Density (lbsm/in)
WI	Internal Work, Eq. (58a)
WM	Boundary Moment Work, Eq. (58b)
WV	Boundary Shear Work, Eq. (58c)

B-2. Nonlinear Program Listing

```
C       MAXIMUM DEFLECTION OF A CANTILEVER BEAM ASSUMING LARGE
C       DEFLECTIONS AND A NONLINEAR STRESS-STRAIN RELATION.
   45 DIMENSION XA(500),YA(500),XXA(500),YYA(500),XB(500),YB(500)
      CZ1(BL,X)=(X**4-4*BL*X**3+6*BL**2*X**2)/3/BL**4
      CZ11(BL,X)=(X**3-3*BL*X**2+3*BL**2*X)*4/3/BL**4
      CZ12(BL,X)=4*(X-BL)**2/BL**4
      CZ13(BL,X)=8*(X-BL)/BL**4
      CZ14(BL,X)=(8/BL**4)*(X+1-X)
      CZ2(BL,X)=1-COS(3.1415923*X/2/BL)
      CZ21(BL,X)=(3.1415923/2/BL)*SIN(3.1415923*X/2/BL)
      CZ22(BL,X)=(3.1415923/2/BL)**2*COS(3.1415923*X/2/BL)
      CZ23(BL,X)=(3.1415923/2/BL)**3*SIN(3.1415923*X/2/BL)*(-1)
      CZ24(BL,X)=(3.1415923/2/BL)**4*COS(3.1415923*X/2/BL)*(-1)
      THETA(BL,X)=(CZ1(BL,X)+ 0.50*CZ2(BL,X))/(1+ 0.50)
      THETA1(BL,X)=(CZ11(BL,X)+ 0.50*CZ21(BL,X))/(1+ 0.50)
      THETA2(BL,X)=(CZ12(BL,X)+ 0.50*CZ22(BL,X))/(1+ 0.50)
      THETA3(BL,X)=(CZ13(BL,X)+ 0.50*CZ23(BL,X))/(1+ 0.50)
      THETA4(BL,X)=(CZ14(BL,X)+ 0.50*CZ24(BL,X))/(1+ 0.50)
   60 PT(PMAX,BETA,T)=PMAX*EXP(-BETA*T)
  100 Y(Q,BL,X)=Q*THETA(BL,X)
  105 YX1(Q,BL,X)=Q*THETA1(BL,X)
  110 YX2(Q,BL,X)=Q*THETA2(BL,X)
  115 YX3(Q,BL,X)=Q*THETA3(BL,X)
  120 YX4(Q,BL,X)=Q*THETA4(BL,X)
  125 PHI(Q,BL,X)=ATAN(YX1(Q,BL,X))
  130 PHI1(Q,BL,X)=YX2(Q,BL,X)/(1+YX1(Q,BL,X)**2)
  135 PHI2(Q,BL,X)=((1+YX1(Q,BL,X)**2)*YX3(Q,BL,X)-2*YX1(Q,BL,X)*YX2(Q,B
     1L,X)**2)/(1+YX1(Q,BL,X)**2)**2
  140 PHI3(Q,BL,X)=((1+YX1(Q,BL,X)**2)**2*YX4(Q,BL,X)-2*(1+YX1(Q,BL,X)**
     12)*(3*YX1(Q,BL,X)*YX2(Q,BL,X)*YX3(Q,BL,X)+YX2(Q,BL,X)**3)+8*YX1(Q,
     2BL,X)**2*YX2(Q,BL,X)**3)/(1+YX1(Q,BL,X)**2)**3
  145 BMX1(Q,BL,X,A,B,C,D)=2*D*PHI2(Q,BL,X)/A**2/B*(A*PHI1(Q,BL,X)*C-ATA
     1N(A*PHI1(Q,BL,X)*C))/PHI1(Q,BL,X)**3
  150 BMX2(Q,BL,X,A,B,C,D)=2*D/A**2/B*(PHI2(Q,BL,X)**2*(3*(1+A*A*C*C*PHI
     11(Q,BL,X)**2)*ATAN(A*PHI1(Q,BL,X)*C)-3*A*PHI1(Q,BL,X)*C-2*(A*PHI1(
     2Q,BL,X)*C)**3)/PHI1(Q,BL,X)**4/(1+A*A*PHI1(Q,BL,X)**2*C*C)+PHI3(Q,
     3BL,X)*(A*PHI1(Q,BL,X)*C-ATAN(A*PHI1(Q,BL,X)*C))/PHI1(Q,BL,X)**3)
  151 R(APC)=(1/(1+APC**2))+6*(-APC/15+APC**3/35-APC**5/63+APC**7/99)
  152 S(APC)=1/3-APC**2/5+APC**4/7-APC**6/9+APC**8/11
  153 BMX2S(Q,BL,X,A,B,C,D,APC)=2*D*A*A*C**4*PHI2(Q,BL,X)**2*R(APC)/B
     1+2*D*A*C**3*PHI3(Q,BL,X)*S(APC)/B
  154 FYS(Q,BL,X,A,B,C,D,APC)=THETA(BL,X)*(BMX2S(Q,BL,X,A,B,C,D,APC)*COS
     1(PHI(Q,BL,X))**2-BMX1(Q,BL,X,A,B,C,D)*PHI1(Q,BL,X)*SIN(2*PHI(Q,BL,
     2X)))
  155 FY(Q,BL,X,A,B,C,D)=THETA(BL,X)*(BMX2(Q,BL,X,A,B,C,D)*COS(PHI(Q,BL,
     1X))**2-BMX1(Q,BL,X,A,B,C,D)*PHI1(Q,BL,X)*SIN(2*PHI(Q,BL,X)))
  160 VL(Q,BL,A,B,C,D)=BMX1(Q,BL,BL,A,B,C,D)*COS(PHI(Q,BL,BL))
  165 BML(Q,BL,A,B,C,D)=D/B*(C*C*ATAN(A*PHI1(Q,BL,BL)*C)+(ATAN(A*C*PHI1(
     1Q,BL,BL))-A*C*PHI1(Q,BL,BL))/A**2/PHI1(Q,BL,BL)**2)
  170 THETAL(BL)=THETA(BL,BL)
  175 PHIL(Q,BL)=THETA1(BL,BL)/(1+Q*Q*THETA1(BL,BL)**2)
  180 WV(Q,BL,A,B,C,D)=VL(Q,BL,A,B,C,D)*THETAL(BL)
  185 WM(Q,BL,A,B,C,D)=BML(Q,BL,A,B,C,D)*PHIL(Q,BL)
  186 SKN(DWN1,DWN,DQ)=(DWN1-DWN)/DQ
  187 GN(DWN1,DWN,DQ,Q0)=DWN-Q0*SKN(DWN1,DWN,DQ)
  188 C1(P,ALPHA,BM,BETA,DWN1,DWN,DQ)=P*ALPHA/(BM*BETA**2*SKN(DWN1,DWN,D
     1Q))
  189 C2(DWN1,DWN,DQ,Q0)=-GN(DWN1,DWN,DQ,Q0)/SKN(DWN1,DWN,DQ)
  190 E1(DWN1,DWN,DQ,Q0,BM,TN,DDOTO,P,ALPHA,BETA)=SIN(SQRT(SKN(DWN1,DWN,
```

```
     1DQ)/BM)*TN)*(QO-C1(P,ALPHA,BM,BETA,DWN1,DWN,DQ)*EXP(-BETA*TN)-C2(D
     2WN1,DWN,DQ,QO))+SQRT(BM/SKN(DWN1,DWN,DQ))*COS(SQRT(SKN(DWN1,DWN,DQ
     3)/BM)*TN)*(QDOTO+BETA*C1(P,ALPHA,BM,BETA,DWN1,DWN,DQ)*EXP(-BETA*TN
     4))
  191 E2(DWN1,DWN,DQ,QO,BM,TN,QDOTO,P,ALPHA,BETA)=COS(SQRT(SKN(DWN1,DWN,
     1DQ)/BM)*TN)*(QO-C1(P,ALPHA,BM,BETA,DWN1,DWN,DQ)*EXP(-BETA*TN)-C2(D
     2WN1,DWN,DQ,QO))-SQRT(BM/SKN(DWN1,DWN,DQ))*SIN(SQRT(SKN(DWN1,DWN,DQ
     3)/BM)*TN)*(QDOTO+BETA*C1(P,ALPHA,BM,BETA,DWN1,DWN,DQ)*EXP(-BETA*TN
     4))
  192 FIN(Q,BL,X,A,B,C,D)=          (BMX2(Q,BL,X,A,B,C,D)*COS(PHI(Q,BL,
     1X))**2-BMX1(Q,BL,X,A,B,C,D)*PHI1(Q,BL,X)*SIN(2*PHI(Q,BL,X)))
  193 BMS(B,C,D,APC)=2*D*C**2/B*(APC/3-APC**3/15+APC**5/35-APC**7/63
     1+APC**9/99)
  194 BMS1(Q,BL,X,A,B,C,D,APC)=2*A*C**3*D*PHI2(Q,BL,X)/B*(1./3.-APC**2/5
     1+APC**4/7-APC**6/9+APC**8/11)
  195 FINS(Q,BL,X,A,B,C,D,APC)=BMX2S(Q,BL,X,A,B,C,D,ARC)*COS(PHI(Q,BL,X)
     1)**2-BMS1(Q,BL,X,A,B,C,D,APC)*PHI1(Q,BL,X)*SIN(2*PHI(Q,BL,X))
  196 WVS(Q,BL,A,B,C,D,APC)=BMS1(Q,BL,BL,A,B,C,D,APC)*COS(PHI(Q,BL,BL))
     1*THETA(BL,BL)
  197 WMS(Q,BL,B,C,D,APC)=BMS(B,C,D,APC)*PHIL(Q,BL)
  200 READ(105,201) A,B,C,D,BL,RHO,PMAX,BETA
  201 FORMAT(8F10.0)
      N=EVEN NUMBER OF INTEGRATION STEPS --- J= # DATA POINTS FOR STIFFNESS K
  205 IF(A) 2000,2000,206
  206 READ(105,207)   N,J
  207 FORMAT(2I5)
  210 CALL PINIT
  212 WRITE(108,213)
  213 FORMAT(1H1,///////////////28X,'MAXIMUM DEFLECTION OF A CANTILEVER
     1BEAM'/28X,'BY:   DANIEL M. KOSZUTA'/34X,'AEROSPACE & MECHANICAL E
     2NGINEERING DEPT.'/34X,'MONTANA STATE UNIVERSITY')
  215 WRITE(108,216) BL,C,D,A,B,RHO,PMAX,BETA
  216 FORMAT(1H1///23X,'INPUT DATA:      BEAM LENGTH ..........  BL=',F10
     1.4/40X,'HALF BEAM HEIGHT .....  C=',F10.4/40X,'BEAM WIDTH ........
     2...  D=',F10.4/40X,'STRESS CONSTANTS .....  A=',F10.4/57X,'.....
     3  B=',F10.8/40X,'MATERIAL DENSITY .....RHO=',F7.4/40X,'MAX BLAST P
     4RESSURE...PMAX=',F8.2/40X,'BLAST DECAY CONST...BETA=',F8.2////)
  220 WRITE(108,221)
  221 FORMAT(10X,'Q',14X,'WI',14X,'WV',14X,'WM',14X,'DW',15X,'K'//)
  222 DT=0.0001
  223 T=0.
  224 TN=0.
  225 QDOTO=0.
  226 QO=0.
  227 NP1=N+1
  230 H=BL/N
  235 X1=0.
  240 XN=BL
  241 QMAX=BL
  242 BM=H*(THETA(BL,X1)**2+THETA(BL,XN)**2)/3
  243 P=H*(THETA(BL,X1)+THETA(BL,XN))/3
  244 DO 249 K=1,2
  245 NK=N-K
  246 DO 249 L=K,NK,2
  247 X=L*H
  248 P=P+4*H*THETA(BL,X)/3/K
  249 BM=BM+4*H*THETA(BL,X)**2/3/K
  250 DQ=QMAX/J
  251 Q=0.
  252 J1=J+1
```

```
253 BM=C*D*RHO*BM/16.085/12.
254 ALPHA=PMAX*D
255 DO 900 M=1,J1
260 APC=A*C*PHI1(Q,BL,X1)
261 IF(APC-0.1) 262,262,263
262 FYY=FYS(Q,BL,X1,A,B,C,D,APC)
    GO TO 264
263 FYY=FY(Q,BL,X1,A,B,C,D)
264 WI=H*FYY/3
    XA(1)=X1
    YA(1)=FYY
265 IF(Q-4*DQ) 267,266,267
266 WRITE(108,296) Q,X1,FYY,APC
267 APC=A*C*PHI1(Q,BL,XN)
268 IF(APC-0.1) 269,269,270
269 FYY=FYS(Q,BL,XN,A,B,C,D,APC)
    GO TO 271
270 FYY=FY(Q,BL,XN,A,B,C,D)
271 WI=WI+H*FYY/3
    XA(NP1)=XN
    YA(NP1)=FYY
272 IF(Q-4*DQ) 275,273,275
273 WRITE(108,296) Q,XN,FYY,APC
275 DO 299 K=1,2
280 NK=N-K
285 DO 299 L=K,NK,2
286 X=L*H
287 APC=A*C*PHI1(Q,BL,X)
288 IF(APC-0.1) 289,289,292
289 FYY=FYS(Q,BL,X,A,B,C,D,APC)
290 GO TO 293
292 FYY=FY(Q,BL,X,A,B,C,D)
293 WI=WI+4*H*FYY/3/K
    LP1=L+1
    XA(LP1)=X
    YA(LP1)=FYY
294 IF(Q-4*DQ) 299,295,299
295 WRITE(108,296) Q,X,FYY,APC
296 FORMAT(13X,'Q=',E12.5,5X,'X=',E12.5,5X,'FY=',E12.5,5X,'APC=',E12.5
   1)
299 CONTINUE
300 IF(M-2) 315,310,315
310 CALL AXIS(1.0,0.0,150.,-200.,800.,6.5,4.5)
311 MS=1
312 GO TO 324
315 IF(M-J/6-1) 320,316,320
316 MS=3
317 GO TO 324
320 IF(M-J/4-1) 350,321,350
321 MS=4
324 WRITE(108,998)
325 DO 330 JL=1,NP1
330 WRITE(108,331) Q,JL,XA(JL),JL,YA(JL)
331 FORMAT(20X,'Q=',E12.5,10X,'X(',I2,')=',E12.5,10X,'FYY(',I2,')=',E1
   12.5)
C     PLOT INTERNAL FORCE FUNCTION.
335 CALL PLOTS(1,1,NP1,0.0,150.,-200.,800.,6.5,4.5,XA,YA)
336 IF(MS-1) 200,350,337
337 CALL PLOTS(MS,1,NP1,0.0,150.,-200.,800.,6.5,4.5,XA,YA)
350 APCL=A*PHI1(Q,BL,BL)*C
```

```
    IF(APCL=0.1) 351,351,352
351 WVV=WVS(Q,BL,A,B,C,D,APCL)
    WMM=WMS(Q,BL,B,C,D,APCL)
    GO TO 353
352 WVV=WV(Q,BL,A,B,C,D)
    WMM=WM(Q,BL,A,B,C,D)
353 DW=WI=WVV+WMM
354 STIF=DW/Q
355 WRITE(108,356) Q,WI,WVV,WMM,DW,STIF
356 FORMAT(4X,5(E12.5,4X),E12.5)
360 XXA(M)=Q
361 YYA(M)=DW
400 DWN1=DW
410 IF(Q) 510,510,420
420 SKB=SKN(DWN1,DWN,DQ)
430 IF(SKB) 560,445,445
445 CI=CI(P,ALPHA,BM,BETA,DWN1,DWN,DQ)
450 CL=C2(DWN1,DWN,DQ,QO)
455 EI=E1(DWN1,DWN,DQ,QO,BM,TN,QDOTO,P,ALPHA,BETA)
460 EL=E2(DWN1,DWN,DQ,QO,BM,TN,QDOTO,P,ALPHA,BETA)
465 WRITE(108,466) BM,P,ALPHA,BETA,DWN1,DWN,SKB,QDOTO,QO,TN,CI,CL,EI,
   1EL
466 FORMAT(//10X,'BM=',E12.5,10X,'P=',E12.5,10X,'ALPHA=',E12.5,10X,
   1'BETA=',E12.5/10X,'DWN1=',E12.5,10X,'DWN=',E12.5,10X,'SKB=',E12.5/
   210X,'QDOTO=',E12.5,10X,'QO=',E12.5,10X,'TN=',E12.5/10X,'C1=',E12.5
   3,10X,'C2=',E12.5,10X,'E1=',E12.5,10X,'E2=',E12.5)
470 QNT=EI*SIN(SQRT(SKB/BM)*T)+EL*COS(SQRT(SKB/BM)*T)+CI*EXP(-BETA*T)
   1+CL
    QDOT=CI*BETA**2*EXP(-BETA*T)-SKB/BM*(EI*SIN(SQRT(SKB/BM)*T)+EL*COS
   1(SQRT(SKB/BM)*T))
471 QDNT=SQRT(SKB/BM)*(EI*COS(SQRT(SKB/BM)*T)-EL*SIN(SQRT(SKB/BM)*T))
   1-BETA*CI*EXP(-BETA*T)
472 WRITE(108,473) QNT,QDNT,T
473 FORMAT(10X,'QNT=',E12.5,10X,'QDNT=',E12.5,10X,'T=',E12.5)
474 IF(T) 475,475,489
475 QNT=0.
476 QDNT=0.
477 IB=1
478 IA=1
479 IF(IB-1) 487,480,487
480 XB(IA)=T
481 YB(IA)=QNT
482 YB(IA+250)=QDNT
    XB(IA+250)=QDOT
    IF(IA-250) 483,483,545
483 IA=IA+1
484 IB=2
485 T=T+DT
486 GO TO 470
487 IB=1
488 GO TO 485
489 IF(QNT-Q) 490,490,495
490 IF(QDNT) 550,550,479
495 QO=QNT
500 QDOTO=QDNT
505 TN=T
510 DWN=DWN1
515 GO TO 900
545 WRITE(108,546)
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  550 WRITE(108,551) QNT,QDNT,T
  551 FORMAT(1H1,/////10X,'QNT=',E12.5,10X,'QDNT=',E12.5,10X,'T=',E12.5)
  555 GO TO 904
  560 WRITE(108,561) SKB
  561 FORMAT(1H1/////10X,'NEGATIVE STIFFNESS  SKB=',E12.5)
  565 GO TO 950
  900 Q=M*DQ
  904 WRITE(108,998)
  905 DO 906 ML=1,M
  906 WRITE(108,907) ML,XXA(ML),ML,YYA(ML)
  907 FORMAT(20X,'Q(',I2,')=',E12.5,10X,'DW(',I2,')=',E12.5)
C     PLOT VIRTUAL WORK FUNCTION DW.
      CALL AXIS(1,0.0,.75,.0.0,75000.,6.5,4.5)
  908 CALL PLOTS(1,1,M,0.0,.75,.0.0,75000.,6.5,4.5,XXA,YYA)
  909 WRITE(108,998)
  910 IP=IA-1
  911 DO 913 MN=1,IP
  912 IS=MN+250
  913 WRITE(108,914) MN,XB(MN),MN,YB(MN),IS,YB(IS),IS,XB(IS)
  914 FORMAT( 5X,'T(',I3,')=',E12.5, 5X,'Q(',I3,')=',E12.5, 5X,'QDOT(',I
     13,')=',E12.5,5X,'QDDT(',I3,')=',E12.5)
C     PLOT DEFLECTION OF BEAM END Q(T).
      CALL AXIS(1,0.0,0.04,0.0,50.,6.5,4.5)
      CALL PLOTS(1,1,IP,0.0,0.04,0.0,50.,6.5,4.5,XB,YB)
      DO 915 I=1,IP
  915 YB(I)=YB(I+250)
C     PLOT VELOCITY OF BEAM END.
      CALL AXIS(1,0.0,0.04,0.0,2500.,6.5,4.5)
      CALL PLOTS(1,1,IP,0.0,0.04,0.0,2500.,6.5,4.5,XB,YB)
      U=RHO*C*D/193.02
  916 DX=DQ
  920 DO 949 K=1,3
  921 IF(K-1) 200,922,925
  922 T=2*DT
      YMIN=-4000.
      YMAX=16000.
  923 IA=2
  924 GO TO 933
  925 IF(K-2) 200,926,930
  926 IH=IP/2
  927 T=2*IH*DT
      YMIN=-400.
      YMAX=1600.
  928 IA=IH+1
  929 GO TO 933
  930 T=2*(IP-1)*DT
  931 IA=IP
  932 YMIN=-400.
      YMAX=1600.
  933 Q=YB(IA)
  934 QDDT=XB(IA+250)
      PRS=PT(PMAX,BETA,T)*D
  935 DO 943 I=1,J1
  936 X=(I-1)*DX
  937 APC=A*C*PHI1(Q,BL,X)
  938 IF(APC-.1) 939,939,940
  939 FIIN=FINS(Q,BL,X,A,B,C,D,APC)
      GO TO 941
  940 FIIN=FIN(Q,BL,X,A,B,C,D)
  941 XXA(I)=U*THETA(BL,X)*QDDT
```

```
      XXA(I+250)=FIIN
      YYA(I)=PRS
      RES=XXA(I)+XXA(I+250)-PRS
  942 XA(I)=X
  943 YA(I)=RES
C     PLOT RESIDUAL AND COMPONENTS.
      CALL AXIS(1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5)
      WRITE(108,998)
      DO 944 I=1,J1
      ISS=I+250
  944 WRITE(108,999) T,I,XA(I),I,XXA(I),ISS,XXA(ISS),I,YYA(I)
      WRITE(108,998)
      CALL PLOTS(1,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,XXA)
      CALL PLOTS(3,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,XXA)
      DO 945 I=1,J1
  945 XXA(I)=XXA(I+250)
      CALL PLOTS(1,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,XXA)
      CALL PLOTS(4,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,XXA)
      CALL PLOTS(1,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYA)
      CALL PLOTS(5,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YYA)
      DO 946 I=1,J1
  946 WRITE(108,947) I,XA(I),I,YA(I),T,IA,XB(IA)
  947 FORMAT(5X,'X(',I2,')=',E12.5,5X,'RES(',I2,')=',E12.5,5X,'T=',E12.5
     1,5X,'XB(',I3,')=',E12.5)
  948 CALL PLOTS(1,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YA)
  949 CALL PLOTS(9,1,J1,0.0,150.,YMIN  ,YMAX  ,6.5,4.5,XA,YA)
  950 DXA=BL/J
  954 WRITE(108,998)
  955 DO 970 I=1,J1
  960 XA(I)=(I-1)*DXA
  965 YA(I)=THETA(BL,XA(I))
  970 WRITE(108,971) I,XA(I),I,YA(I)
  971 FORMAT(35X,'X(',I2,')=',E12.5,10X,'THETA(',I2,')=',E12.5)
C     PLOT MODE SHAPE.
  975 CALL AXIS(1,0.0,150.,0.0,1.25,6.5,4.5)
  976 CALL PLOTS(1,1,J1,0.0,150.,0.0,1.25,6.5,4.5,XA,YA)
  998 FORMAT(1H1)
  999 FORMAT(1X,'T=',E12.4,4X,'X(',I3,')=',E12.5,4X,'R1(',I3,')=',E12.5,
     14X,'R2(',I3,')=',E12.5,4X,'R3(',I3,')=',E12.5)
 1000 DXA=0.002/50.
 1004 WRITE(108,998)
 1005 DO 1020 I=1,51
 1010 XA(I)=(I-1)*DXA
 1015 YA(I)=PT(PMAX,BETA,XA(I))
 1020 WRITE(108,1021) I,XA(I),I,YA(I)
 1021 FORMAT(35X,'T(',I2,')=',E12.5,10X,'PRES(',I2,')=',E12.5)
C     PLOT PRESSURE FUNCTION OF BLAST.
 1025 CALL AXIS(1,0.0,0.002,0.0,10000.,6.5,4.5)
 1026 CALL PLOTS(1,1,51,0.0,0.002,0.0,10000.,6.5,4.5,XA,YA)
 1500 GO TO 200
 2000 END
```

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