Convergence structures on homeomorphism groups
by Wayne Richard Park

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Mathematics
Montana State University

Abstract:
In this work the concept of a homeomorphism group H(X) is analyzed in the category of convergence structures as developed by H. Fischer. In Chapter II, some general results are shown concerning lattice pro-perties of convergence structures on function sets. In Chapter III, a convergence structure ς on H(X) is developed which is the coarsest of the admissible convergence group structures on H(X). This struc-ture is compared with the convergence structure of continuous conver-gence on H(X). These concepts are then generalized in the construc-tion of convergence transformation groups over a convergence space.

In Chapter IV, X is given a uniform convergence structure. Uniform convergence structures are then constructed on subgroups of H(X).

In each case, the question is asked whether these uniform convergence structures induce convergence group structures on the underlying ho-meomorphism subgroup.

CONVERGENCE STRUCTURES ON HOMEOMORPHISM GROUPS

by

WAYNE RICHARD PARK

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Mathematics

Approved:

Louis C. Barrett
Head, Major Department

Richard M. Gillette
Chairman, Examining Committee

K Goering
Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

June, 1971

The author wishes to express his utmost appreciation to Professor Richard M. Gillette for his encouragement and guidance in this work.

TABLE OF CONTENTS

ABSTRACT

In this work the concept of a homeomorphism group H(X) is analyzed in the category of convergence structures as developed by H. Fischer. In Chapter II, some general results are shown concerning lattice properties of convergence structures on function sets. In Chapter III, a convergence structure σ on H(X) is developed which is the coarsest of the admissible convergence group structures on H(X). This structure is compared with the convergence structure of continuous convergence on H(X). These concepts are then generalized in the construction of convergence transformation groups over a convergence space. In Chapter IV, X is given a uniform convergence structure. Uniform convergence structures are then constructed on subgroups of H(X). In each case, the question is asked whether these uniform convergence structures induce convergence group structures on the underlying homeomorphism subgroup.

INTRODUCTION

In 1959 H. Fischer formalized the concept of a convergence structure as a generalization of a topology. The generalization is a good one in that many of the fundamental ideas of topology carry over to this new category, and in addition, as the category is so much fuller, we can find convergence structures satisfying certain properties for which no such topologies exist.

The primary intent of this work is to show similar results with respect to homeomorphism groups. R. Arens has analyzed the properties of various topologies on the group of homeomorphisms, H(X), of a topological space X. Basically, meaningful results are obtained only after the assumption that the topological space X is at least locally compact and Hausdorff. In this work, by beginning with the assumption that X is given just an arbitrary convergence structure and considering homeomorphisms in this new category, we analyze convergence structures on the new H(X). In particular, we construct a convergence structure σ on H(X) which is the coarsest of the admissible convergence group structures on H(X). By an admissible convergence group structure we mean the convergence structure guarantees the continuity of the group operations and the evaluation mapping. Properties of this σ convergence structure are analyzed and compared with those of γ_c, the convergence structure of continuous convergence. A counterexample using the rational numbers shows that these two convergence structures are distinct.

The topological generalization of a homeomorphism group is a topological transformation group on a topological space X. We define the concept of a convergence transformation group on a convergence space X and obtain a characterization of these groups by a mapping property. Finally we define a convergence structure on a convergence transformation group in an analogous way to that of σ on H(X). It is shown that this convergence structure has properties similar to those of σ in this more general setting.

In the last chapter we let X be an arbitrary uniform convergence space. We construct uniform convergence structures on both the homeomorphism group H(X) and on U(X), the subgroup of uniformly continuous homeomorphisms of X. These uniform convergence structures are analyzed with respect to the question of whether they induce convergence group structures on the underlying homeomorphism group.

CHAPTER I

Let $(S,\leq)$ be a partially ordered set with the additional property that every subfamily $\{\mathcal{F}_\alpha\}$ of S has an infimum $\bigwedge_\alpha \mathcal{F}_\alpha$ in S. A $\wedge$-ideal is a subfamily T of S which satisfies:

1. $\mathcal{F}, \mathcal{F}' \in T \Rightarrow \mathcal{F} \wedge \mathcal{F}' \in T$, and

2. $\mathcal{F} \in T, \mathcal{F}' \geq \mathcal{F} \Rightarrow \mathcal{F}' \in T$.

A (proper) filter $\mathcal{F}$ on a non-empty set X is a $\wedge$-ideal in $(\mathcal{P}(X), \subseteq)$ with the additional requirement that $\emptyset \notin \mathcal{F}$. A filter base $\mathcal{B}$ is a family of subsets of X satisfying

1. $B, B' \in \mathcal{B} \Rightarrow$ there exists a $B'' \in \mathcal{B}$ such that $B'' \subseteq B \cap B'$,

and

2. $\emptyset \notin \mathcal{B}$.

We say that $\mathcal{B}$ generates a filter $\mathcal{F}$ if $\mathcal{B} \subseteq \mathcal{F}$ and for every $F \in \mathcal{F}$ there exists a $B \in \mathcal{B}$ such that $B \subseteq F$, and in this case we write $\mathcal{F} = [\mathcal{B}]$. The family of filters F(X) on a non-empty set X forms a partially ordered set with infima with $\leq$ defined by $\mathcal{F} \leq \mathcal{F}' \Leftrightarrow \mathcal{F} \subseteq \mathcal{F}'$. If $\{\mathcal{F}_\alpha\}$ is an arbitrary family of filters on X, then $\bigwedge_\alpha \mathcal{F}_\alpha$ is just $\bigcap_\alpha \mathcal{F}_\alpha$ and if for each α, $\mathcal{B}_\alpha$ is a filter base for $\mathcal{F}_\alpha$, then $\bigwedge_\alpha \mathcal{F}_\alpha$ is generated by the filter base $\{\cup B_\alpha \mid B_\alpha \in \mathcal{B}_\alpha\}$.

A convergence structure τ on a non-empty set X is defined if for each $x \in X$ there is assigned a $\wedge$-ideal, τx, of filters of subsets of X with the added requirement that the filter of supersets of the point x, denoted by $\overset{o}{x}$, also is a member of τx. If X has a convergence structure τ, then (X,τ) is called a convergence space. If the filter $\mathcal{F} \in \tau x$, we say $\mathcal{F}$ converges to x.

If τ, τ' are two convergence structures on X, we say τ is coarser than τ', ($\tau \leq \tau'$), or τ' is finer than τ if for each $x \in X$, $\tau'x \subseteq \tau x$. With this, the family of convergence structures on X forms a complete lattice. [7, p.275].

If τ is a convergence structure on X, let $\psi\tau$ be the convergence structure on X defined by $\psi\tau x = \{\mathcal{F}' \in F(X) \mid \mathcal{F}' \geq \bigwedge_{\mathcal{F}\in\tau x} \mathcal{F}\}$. These convergence structures are called principal and the corresponding spaces are called principal convergence spaces. Every topology T on X defines a convergence structure τ_T on X, namely, $\tau_T\, x = \{\mathcal{F} \in F(X) \mid \mathcal{F} \geq \mathcal{V}(x)\}$ where $\mathcal{V}(x)$ is the neighborhood filter of x defined by the topology T. It should be noted if $T \leq T'$ for two topologies T and T', then $\tau_T \leq \tau_{T'}$. For every convergence structure τ on X there is a finest topology $\bar{\omega}\tau$ which is coarser than τ. Namely, consider those sets A in X such that for each $x \in A$, $A \in \mathcal{F}$ for each $\mathcal{F} \in \tau x$. These sets are called the τ-open sets and for each $x \in X$, they generate a neighborhood filter $\mathcal{V}(x)$. So for each $x \in X$, $\bar{\omega}\tau x = \{\mathcal{F} \in F(X) \mid \mathcal{F} \geq \mathcal{V}(x)\}$. It is clear that for each convergence structure τ on X we have the relations $\bar{\omega}\tau \leq \psi\tau \leq \tau$.

If $f\colon X \to Y$ is an arbitrary mapping between the non-empty sets X and Y, then for any $\mathcal{F} \in F(X)$, $\{f(F) \mid F \in \mathcal{F}\}$ is a filter base on Y. Hence, let $f(\mathcal{F})$ denote the filter that this filter base generates. A function $f\colon (X,\tau) \to (Y,\tau')$ between convergence spaces is said to be continuous if for each $x \in X$, $f(\tau x) \subseteq \tau' f(x)$. A bi-continuous bijection between convergence spaces is called a homeomorphism.

If $\mathcal{F}$, $\mathcal{F}'$ are filters on X, X' respectively, $\mathcal{F} \times \mathcal{F}'$ is the filter on $X \times X'$ generated by the filter base $\{F \times F' \mid F \in \mathcal{F}, F' \in \mathcal{F}'\}$. If (X, τ) and (X', τ') are two convergence spaces and $\mathcal{G}$ is a filter on $X \times X'$, the product convergence structure $\tau \times \tau'$ on $X \times X'$ is given by:

$$\mathcal{G} \in (\tau \times \tau')(x,x') \iff \begin{cases} p_1(\mathcal{G}) \in \tau x, \text{ and} \\ p_2(\mathcal{G}) \in \tau' x'. \end{cases}$$

where p_1 and p_2 are the projection mappings. It is easy to see that the continuity of a mapping $f\colon (X \times X', \tau \times \tau') \to (X'', \tau'')$ is equivalent to the condition:

$$\left.\begin{array}{l} \mathcal{F} \in \tau x \\ \mathcal{F}' \in \tau' x' \end{array}\right\} \Rightarrow f(\mathcal{F} \times \mathcal{F}') \in \tau'' f(x,x') \text{ for all } (x,x') \in X \times X'.$$

The usual properties of products, valid for topologies, hold also in this more general setting. For a more detailed analysis see [7, p.290-291].

If (X,τ) and (Y,τ') are convergence spaces and Y^X denotes the family of all continuous functions from X to Y, then for any $H \subseteq Y^X$, the convergence structure of continuous convergence γ_c is defined on H by $\mathcal{F} \in \gamma_c f \iff$ for all $x \in X$ and for all $\Phi \in \tau x$, $\mathcal{F}(\Phi) = \omega(\mathcal{F} \times \Phi) \in \tau' f(x)$ where $\omega\colon H \times X \to X$ is the evaluation mapping defined by $\omega(f,x) = f(x)$. It should be noted that inherent in this definition is the minimum of conditions to guarantee the continuity of ω.

For an extensive analysis of the properties of γ_c see [4] and [2].

Let X be a non-empty set. If Δ represents the set $\{(x,x) \mid x \in X\}$ then $[\Delta]$ is the filter of all supersets of Δ in $X \times X$. If F and G are subsets of $X \times X$, let $F^{\Delta} = \{(y,x) \mid (x,y) \in F\}$ and also, $F \circ G = \{(x,y) \mid$ there exists a $z \in X$ such that $(x,z) \in F$ and $(z,y) \in G\}$ (note that composition reads left to right). Then for filters $\mathcal{F}$ and $\mathcal{G}$ on $X \times X$, let $\mathcal{F}^{\Delta} = [\{F^{\Delta} \mid F \in \mathcal{F}\}]$ and also $\mathcal{F} \circ \mathcal{G} = [\{F \circ G \mid F \in \mathcal{F}, G \in \mathcal{G}\}]$. For this latter filter to exist it is clear that $F \circ G$ must be non-empty for all $F \in \mathcal{F}$ and $G \in \mathcal{G}$.

A uniform convergence structure $\mathcal{J}$, is a $\wedge$-ideal of filters on $X \times X$ satisfying:

1. $[\Delta] \in \mathcal{J}$,

2. $\mathcal{F} \in \mathcal{J} \Rightarrow \mathcal{F}^{\Delta} \in \mathcal{J}$, and

3. $\mathcal{F}, \mathcal{G} \in \mathcal{J} \Rightarrow \mathcal{F} \circ \mathcal{G} \in \mathcal{J}$, whenever this latter filter exists. The uniform convergence structure $\mathcal{J}$ induces a convergence structure $\tau_{\mathcal{J}}$ on X by the definition:

$$\mathcal{F} \in \tau_{\mathcal{J}} x \Leftrightarrow \mathcal{F} \times \overset{\circ}{x} \in \mathcal{J}.$$

$(X, \tau_{\mathcal{J}})$ is then said to be a uniform convergence space. The family of uniform convergence structures on X forms a complete lattice where $\mathcal{J} \leq \mathcal{J}' \Leftrightarrow \mathcal{J} \subseteq \mathcal{J}'$. If $\{\mathcal{J}_\alpha\}$ is a family of uniform convergence structures on X, then $\tau_{\sup_\alpha \mathcal{J}_\alpha} = \sup_\alpha \tau_{\mathcal{J}_\alpha}$. Finally, a function $f: (X, \tau_{\mathcal{J}}) \to (Y, \tau_{\mathcal{J}'})$ is said to be uniformly continuous if $(f \times f)\, \mathcal{J} \subseteq \mathcal{J}'$.

If (X,τ) is a convergence space and $(Y,\tau_{\mathcal{J}})$ a uniform convergence space with $\tau_{\mathcal{J}}$ induced from the uniform convergence structure $\mathcal{J}$, then for any non-empty $F \subseteq Y^X \times Y^X$ and C some non-empty subset of X, let

$$[F,C] = \{(f(x),f'(x)) \mid (f,f') \in F,\ x \in C\} .$$

Moreover, for any filter $\mathcal{F} \in F(Y^X \times Y^X)$, let $[\mathcal{F},C]$ be the filter in $Y \times Y$ that is generated by $\{[F,C] \mid F \in \mathcal{F}\}$. Now let μ_C be the family of filters on $Y^X \times Y^X$ defined by

$$\mathcal{F} \in \mu_C \iff [\mathcal{F},C] \in \mathcal{J}.$$

μ_C is a uniform convergence structure on Y^X and is called the <u>uniform convergence structure of uniform convergence on C</u>. If $\mathcal{C}$ is a non-empty family of non-empty subsets of X, let $\mu_{\mathcal{C}} = \sup_{C\in\mathcal{C}} \mu_C$. $\mu_{\mathcal{C}}$ is then called the <u>uniform convergence structure of uniform convergence on the collection $\mathcal{C}$</u>. If $\mathcal{C} = \{\{x\} \mid x \in X\}$, $\mu_{\mathcal{C}}$ is called the <u>uniform convergence structure of simple convergence</u>. Here $\mathcal{F} \in \tau_{\mu_{\mathcal{C}}} f \iff$ $\mathcal{F}(x) = \omega(\mathcal{F} \times \{x\}) \in \tau_{\mathcal{J}} f(x)$ for all $x \in X$.

For more detail on the subject of uniform convergence structures, see [5].

CHAPTER II

We first observe some basic results concerning the lattice of topologies on function sets that are also valid in the lattice of convergence structures.

Let X and Y be convergence spaces and let Y^X denote the set of all continuous functions from X to Y. A convergence structure on Y^X is said to be conjoining if for every convergence space Z, the continuity of $\hat{\alpha}: Z \to Y^X$ implies the continuity of $\alpha: Z \times X \to Y$ where $\alpha(z,x) = \hat{\alpha}(z)(x)$. Secondly, a convergence structure on Y^X is said to be splitting if for every convergence space Z, the continuity of $\alpha: Z \times X \to Y$ implies the continuity of $\hat{\alpha}: Z \to Y^X$. A convergence structure on Y^X is said to be admissible if the evaluation mapping $\omega: Y^X \times X \to Y$ ($\omega(f,x) = f(x)$) is continuous.

For the following four theorems, convergence structures apply to the set Y^X.

Theorem 1: A convergence structure is conjoining if and only if that structure is admissible.

Theorem 2: A convergence structure which is finer than a conjoining structure is conjoining.

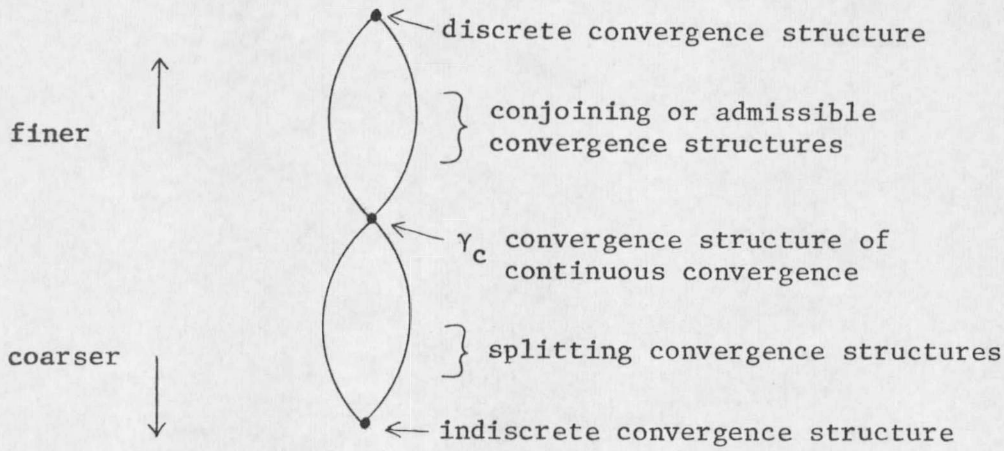
Theorem 3: A convergence structure which is coarser than a splitting structure is splitting.

Theorem 4: Any conjoining convergence structure is finer than any splitting structure.

The proofs of the above theorems are virtually identical to the proofs given in [6,p.274-275]. We demonstrate the technique for Theorem 1 only:

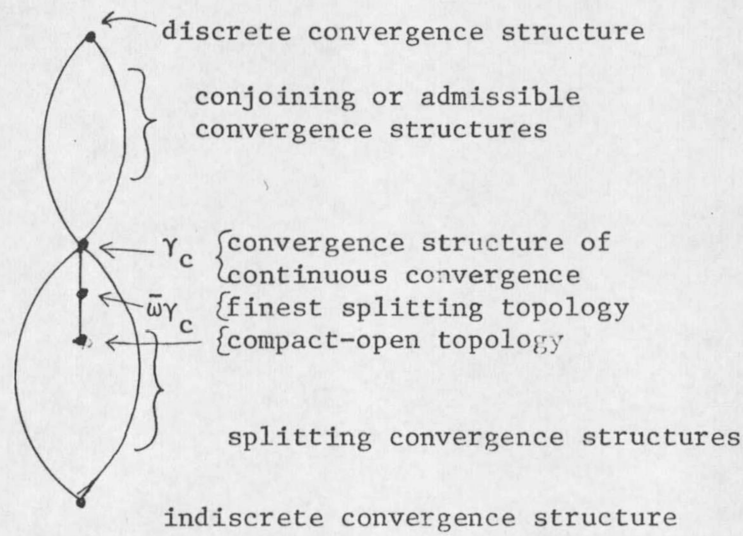
Proof: As $\omega\colon Y^X \times X \to Y$ is given by $\omega(f,x) = f(x)$, we have $\hat{\omega}\colon Y^X \to Y^X$ as the identity mapping since $\hat{\omega}(f)(x) = \omega(f,x) = f(x)$. Assuming then that τ is a conjoining convergence structure on Y^X, the continuity of $\hat{\omega}\colon(Y^X, \tau) \to (Y^X, \tau)$ as the identity mapping implies the continuity of $\omega\colon Y^X \times X \to Y$. Conversely, if ω is continuous and if $\hat{\alpha}\colon Z \to Y^X$ is continuous for some convergence space Z, then the composition of $Z \times X \xrightarrow{\hat{\alpha}\times id} Y^X \times X \xrightarrow{\omega} Y$ is continuous. But this composition is precisely $\alpha\colon Z \times X \to Y$ showing that τ is a conjoining structure.

The γ_c convergence structure of continuous convergence is both splitting and conjoining [2,Satz 2, p.7], so with the theorems above, γ_c is characterized as the unique finest splitting and coarsest conjoining or admissible convergence structure on Y^X. As the discrete convergence structure (topology) is conjoining and the indiscrete convergence structure (topology) is splitting, we can schematically represent these results as follows:



In the case X and Y are topological spaces we know first that the compact-open topology is always splitting and secondly, there is always a finest splitting topology for Y^X. This gives rise to the following diagram:

X,Y topological spaces



It is well known that when X is locally compact then $\gamma_c = \bar{\omega}\gamma_c$ = compact-open topology [6,p.275]. In general, γ_c is not a topology when X and Y are topological spaces.

CHAPTER III

Let H(X) represent the group of homeomorphisms of the convergence space (X,τ).

Definition: For each f in H(X) let σf consist of all filters $\mathcal{F}$ on H(X) such that:

1) for all $x \in X$ and for all $\Phi \in \tau x$, $\mathcal{F}(\Phi) \in \tau f(x)$, and
2) for all $x \in X$ and for all $\Phi \in \tau x$, $\mathcal{F}^{-1}(\Phi) \in \tau f^{-1}(x)$.

Here $\mathcal{F}(\Phi) = \omega(\mathcal{F} \times \Phi)$ where ω: $H(X) \times X \to X$ is the evaluation mapping, and $\mathcal{F}^{-1}$ is the filter with filter base $\{F^{-1} \mid F \in \mathcal{F}\}$ where $F^{-1} = \{f^{-1} \in H(X) \mid f \in F\}$.

Theorem 5: σ is a convergence structure on H(X) making $(H(X),\sigma)$ a convergence group.

Proof: A: $\overset{\circ}{f} \in \sigma f$

1) $\overset{\circ}{f}(\Phi) = f(\Phi) \in \tau f(x)$ as f is continuous.
2) $(\overset{\circ}{f})^{-1}(\Phi) = f^{-1}(\Phi) \in \tau f^{-1}(x)$ as f^{-1} is continuous.

B: $\mathcal{G} \geq \mathcal{F} \in \sigma f \Rightarrow \mathcal{G} \in \sigma f$

1) $\mathcal{G} \geq \mathcal{F} \in \sigma f \Rightarrow \mathcal{G}(\Phi) \geq \mathcal{F}(\Phi) \in \tau f(x) \Rightarrow \mathcal{G}(\Phi) \in \tau f(x)$.
2) $\mathcal{G} \geq \mathcal{F} \Rightarrow \mathcal{G}^{-1} \geq \mathcal{F}^{-1} \Rightarrow \mathcal{G}^{-1}(\Phi) \geq \mathcal{F}^{-1}(\Phi) \in \tau f^{-1}(x)$
$\Rightarrow \mathcal{G}^{-1}(\Phi) \in \tau f^{-1}(x)$.

C: $\mathcal{F},\mathcal{G} \in \sigma f \Rightarrow \mathcal{F} \wedge \mathcal{G} \in \sigma f$

1) $\mathcal{F}(\Phi)$, $\mathcal{G}(\Phi) \in \tau f(x) \Rightarrow \mathcal{F}(\Phi) \wedge \mathcal{G}(\Phi) \in \tau f(x)$ and as

$\mathcal{F}(\Phi) \wedge \mathcal{G}(\Phi) = (\mathcal{F} \wedge \mathcal{G})(\Phi)$ it follows $(\mathcal{F} \wedge \mathcal{G})(\Phi) \in \tau f(x)$.

2) $\mathcal{F}^{-1}(\Phi), \mathcal{G}^{-1}(\Phi) \in \tau f(x) \Rightarrow \mathcal{F}^{-1}(\Phi) \wedge \mathcal{G}^{-1}(\Phi) \in \tau f^{-1}(x)$ and similarly $\mathcal{F}^{-1}(\Phi) \wedge \mathcal{G}^{-1}(\Phi) = (\mathcal{F}^{-1} \wedge \mathcal{G}^{-1})(\Phi) = (\mathcal{F} \wedge \mathcal{G})^{-1}(\Phi) \Rightarrow (\mathcal{F} \wedge \mathcal{G})^{-1}(\Phi) \in \tau f^{-1}(x)$.

D: Continuity of composition: If $\mathcal{F} \in \sigma f$ and $\mathcal{G} \in \sigma g$, we want to show that $\mathcal{F} \circ \mathcal{G} \in \sigma(f \circ g)$.

1) $(\mathcal{F} \circ \mathcal{G})(\Phi) = \mathcal{F}(\mathcal{G}(\Phi)) \in \tau(f(g(x))) = \tau(f \circ g)(x)$.

2) $(\mathcal{F} \circ \mathcal{G})^{-1}(\Phi) = (\mathcal{G}^{-1} \circ \mathcal{F}^{-1})(\Phi) \in \tau(g^{-1}(f^{-1}(x))) = \tau(f \circ g)^{-1}(x)$.

E: Continuity of inversion: We need to show that $\mathcal{F} \in \sigma f \Rightarrow \mathcal{F}^{-1} \in \sigma f^{-1}$.

1) $\mathcal{F}^{-1}(\Phi) \in \tau f^{-1}(x)$ by part 2) of the definition.

2) $(\mathcal{F}^{-1})^{-1}(\Phi) = \mathcal{F}(\Phi) \in \tau f(x) = \tau(f^{-1})^{-1}(x)$ by part 1) of the definition.

<u>Theorem 6</u>: σ is the coarsest admissible convergence structure on H(X) such that H(X) is a convergence group.

Proof: By the definition of σ it is clear that σ is admissible. Let σ' be any convergence structure such that $(H(X), \sigma')$ is an admissible convergence group. We want to show $\sigma \leq \sigma'$, that is, for all $f \in H(X)$, $\sigma' f \subseteq \sigma f$. Let $\mathcal{F} \in \sigma' f$, and let $x \in X$, $\Phi \in \tau x$. As σ' is admissible $\omega(\mathcal{F} \times \Phi) = \mathcal{F}(\Phi) \in \tau f(x)$. Moreover, $\mathcal{F} \in \sigma' f$ implies $\mathcal{F}^{-1} \in \sigma' f^{-1}$, so again using admissibility, $\mathcal{F}^{-1}(\Phi) \in \tau f^{-1}(x)$ implies $\mathcal{F} \in \sigma f$.

<u>Theorem 7</u>: σ on H(X) has the following property:

$$\mathcal{F} \in \sigma f \quad \Rightarrow \quad \begin{cases} \text{for all } x \in X \text{ and for any filter } \Phi \text{ on } X \\ \mathcal{F}(\Phi) \in \tau f(x) \text{ if and only if } \Phi \in \tau x. \end{cases}$$

Proof: Assume $\mathcal{F} \in \sigma f$ and that Φ is some filter on X such that $\mathcal{F}(\Phi) \in \tau f(x)$. By 2) of the definition of σ, $\mathcal{F}^{-1}(\mathcal{F}(\Phi))$ belongs to $\tau f^{-1}(f(x)) = \tau x$. But $\Phi \geq \mathcal{F}^{-1}(\mathcal{F}(\Phi))$ which implies $\Phi \in \tau x$. The reverse implication is immediate from 1) of the definition of σ.

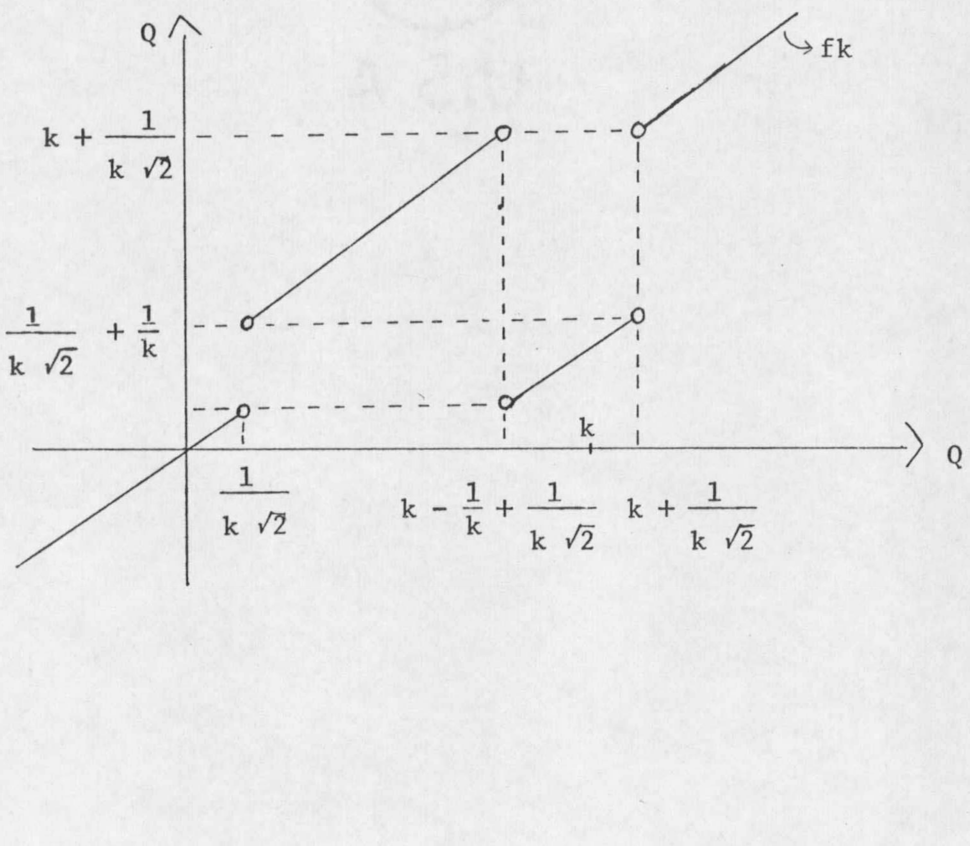
The convergence structure of continuous convergence, γ_c, applied to homeomorphism groups is given by:

$\mathcal{F} \in \gamma_c f$ in H(X) if and only if for all $x \in X$ and for all $\Phi \in \tau x$, $\mathcal{F}(\Phi) \in \tau f(x)$. As γ_c is the coarsest of all admissible convergence structures on H(X), and as σ is admissible, we have $\gamma_c \leq \sigma$. Arens' example [1,p.601], although presented in a different setting shows that $\sigma \neq \gamma_c$. To quickly see the distinctness of σ and γ_c though, consider the following example:

<u>Counterexample</u>: Let Q be the set of rational numbers with the usual topology. Let $\mathcal{F}$ be the filter on H(Q) generated by the filter base of homeomorphisms f_k: Q $\rightarrow$ Q defined by:

$$f_k(x) = \begin{cases} x & x < \dfrac{1}{k\sqrt{2}} \\ x + \dfrac{1}{k} & \dfrac{1}{k\sqrt{2}} < x < k - \dfrac{1}{k} + \dfrac{1}{k\sqrt{2}} \\ x - k + \dfrac{1}{k} & k - \dfrac{1}{k} + \dfrac{1}{k\sqrt{2}} < x < k + \dfrac{1}{k\sqrt{2}} \\ x & x > k + \dfrac{1}{k\sqrt{2}} \end{cases}$$

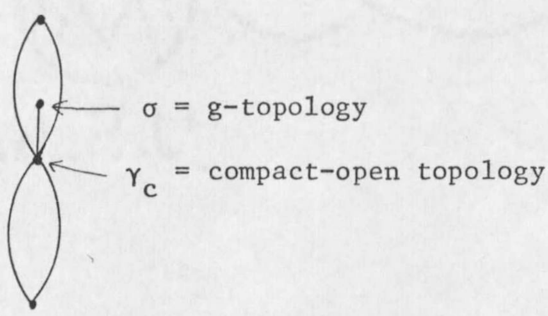
Graphically f_k has the following appearance:



It is clear that $\mathcal{F} \in \gamma_c$ id where id is the identity homeomorphism. However, if Φ is a filter on Q defined by a sequence $\{x_k\}$ of rationals such that for all k, $\frac{1}{k\sqrt{2}} < x_k < \frac{1}{k\sqrt{2}} + \frac{1}{k}$, then Φ converges to zero but $\mathcal{F}^{-1}(\Phi)$ does not converge. Hence $\mathcal{F} \notin \sigma$ id and therefore, in general, γ_c is strictly coarser than σ.

In case X is a locally compact Hausdorff space, σ and γ_c are topologies [1]. Namely, σ is the g-topology on H(X) defined by taking as subbasis the family of sets $\{(K, W)\}$ where K is closed in X, W is open in X, and either K is compact, or the complement of W in X is compact. The γ_c convergence structure is the familiar compact-open topology. We can picture this topological case as follows:

H(X) for locally compact Hausdorff topological X



If in addition to locally compact and Hausdorff, X is also locally connected, the σ and γ_c are identified [1,p.598].

In [1,p.596] Arens states the following theorem:

"Given any admissible topology for the group of homeomorphisms H of the rational number system, one can construct another admissible topology for H which is not weaker [finer in our sense] than the first."

This shows precisely that γ_c on H(Q) is not a topology.

The natural question arises now if similar results can be determined for σ on H(X) for arbitrary topological X. The problem appears to be much more involved than in the γ_c case and is left here as an open question.

<u>Theorem 8</u>: For any convergence group (G,τ), G isomorphically imbeds in $(H(G),\sigma)$. (We still interpret H(G) as the group of homeomorphisms of the convergence space (G,τ).)

Proof: Let $\theta: G \to H(G)$ be defined by $\theta(g) = \theta_g \in H(G)$ where $\theta_g(g') = gg'$. Clearly θ is a monomorphism. To show continuity, we need to show that if $\mathcal{F} \in \tau g$ for $g \in G$, then $\theta(\mathcal{F}) \in \sigma\theta(g) = \sigma\theta_g$. So, to satisfy the definition of σ, let $g' \in G$ and $\Omega \in \tau g'$. Then

1) $\theta(\mathcal{F})(\Omega)$ has as a filter-base $\{\theta(F)(P) \mid F \in \mathcal{F}, P \in \Omega\} =$

$\{\{\theta(x)(x') \mid x \in F \in \mathcal{F}, x' \in P \in \Omega\}\} =$

$\{\{xx' \mid x \in F \in \mathcal{F}, x' \in P \in \Omega\}\} =$

$\{F \circ P \mid F \in \mathcal{F}, P \in \Omega\}$.

Hence $\theta(\mathcal{F})(\Omega) = \mathcal{F} \circ \Omega \in \tau(gg') = \tau\theta_g(g')$ by the continuity of composition in G.

2) $(\theta(\mathcal{F}))^{-1}(\Omega)$ has as a filter-base $\{(\theta(F))^{-1}(P) \mid F \in \mathcal{F}, P \in \Omega\} =$

$\{\{(\theta(x))^{-1}(x') \mid x \in F \in \mathcal{F}, x' \in P \in \Omega\}\} =$

$\{\{(\theta_x)^{-1}(x') \mid x \in F \in \mathcal{F}, x' \in P \in \Omega\}\} =$

$\{\{x^{-1} \circ x' \mid x \in F \in \mathcal{F},\ x' \in P \in \Omega\}\} =$

$\{F^{-1} \circ P \mid F \in \mathcal{F},\ P \in \Omega\}$.

Hence $(\theta(\mathcal{F}))^{-1}(\Omega) = \mathcal{F}^{-1} \circ \Omega \in \tau(g^{-1} \circ g') = \tau\theta_{g^{-1}}(g') =$

$\tau\theta_g^{-1}(g')$ by the continuity of both of the group operations in G and the homomorphism property of θ.

Now let $\delta = \theta^{-1} \mid_{\theta(G)}$. Namely $\delta(\theta_g) = g$ where again $\theta_g(g') = gg'$. But note that $\delta(\theta_g) = \omega(\theta_g, e) = \theta_g(e) = ge = g$, where ω is again the evaluation mapping. This shows that the inverse mapping δ is none other than ω restricted to $\theta(G) \times \{e\}$. But ω is continuous since σ on H(G) is admissible. Hence δ is continuous and the proof is complete.

The natural generalization of a homeomorphism group is the concept of a topological transformation group. [8,p.40] Again by studying these groups in the category of convergence spaces we obtain slightly more general results than are obtained in the strictly topological case.

<u>Definition</u>: A convergence group G is a <u>convergence transformation group</u> on a convergence space X relative to a function $\hat{\theta}: G \times X \to X$ if

1) $\hat{\theta}$ is continuous,

2) $\hat{\theta}(e,x) = x$ for all $x \in X$, and

3) $\hat{\theta}(gg',x) = \hat{\theta}(g, \hat{\theta}(g',x))$ for all $g,g' \in G$ and $x \in X$.

If in addition

4) $\hat{\theta}(g,x) = x$ for all $x \in X$ implies that $g = e$,

then the convergence transformation group is said to be <u>effective</u>.

We first state a lemma, the proof of which is identical to the corresponding portion of the proof of Theorem 1.

Lemma 9: $(H(X),\sigma)$ has the conjoining property, that is for all convergence spaces Y, the continuity of $\alpha: Y \to H(X)$ implies the continuity of $\hat{\alpha}: Y \times X \to X$ where $\hat{\alpha}(y,x) = \alpha(y)(x)$.

Theorem 10: (G,τ) is a convergence transformation group on (X,τ') relative to $\hat{\theta}$ if and only if $\theta: G \to (H(X),\sigma)$ is a continuous homomorphism. $(\theta(g)(x) = \hat{\theta}(g,x))$

Proof: Again represent $\theta(g)$ by θ_g and $\theta_g(x) = \hat{\theta}(g,x)$. Then $\theta_{gg'}(x) = \theta(gg',x) = \hat{\theta}(g, \hat{\theta}(g',x)) = \hat{\theta}(g, \theta_{g'}(x)) = \theta_g(\theta_{g'}(x)) = (\theta_g \circ \theta_{g'})(x)$. Hence $\theta_{gg'} = \theta_g \circ \theta_{g'}$ and θ is a homomorphism.

To show the continuity of θ, let $g \in G$ and $\mathcal{G} \in \tau g$. We want to show that $\theta(\mathcal{G}) \in \sigma\theta_g$. So let $x \in X$ and let $\mathcal{F} \in \tau' x$.

1) $\theta(\mathcal{G})(\mathcal{F}) = \hat{\theta}(\mathcal{G},\mathcal{F}) \in \tau'\hat{\theta}(g,x) = \tau'\theta_g(x)$

2) As θ is a homomorphism $(\theta(\mathcal{G}))^{-1} = \theta(\mathcal{G}^{-1})$ and $\theta_g{}^{-1} = \theta_{g^{-1}}$. Now as G is a convergence group $\mathcal{G} \in \tau g$ implies $\mathcal{G}^{-1} \in \tau g^{-1}$ and repeating the procedure used in 1), we have $\theta(\mathcal{G}^{-1})(\mathcal{F}) \in \tau'\theta_{g^{-1}}(x)$ and hence $(\theta(\mathcal{G}))^{-1}(\mathcal{F}) \in \tau'(\theta_g)^{-1}(x)$.

Conversely if θ is a continuous homomorphism, then by Lemma 9, $\hat{\theta}: G \times X \to X$ is continuous. Moreover since θ is a homomorphism, $\hat{\theta}(e,x) = \theta_e(x) = id(x) = x$. Finally, $\hat{\theta}(gg',x) = \theta_{gg'}(x) = \theta_g(\theta_{g'}(x)) = \theta_g(\hat{\theta}(g',x)) = \hat{\theta}(g,\hat{\theta}(g',x))$ showing that (G,τ) is a convergence transformation group on (X,τ') relative to $\hat{\theta}$.

Corollary 11: G is effective if and only if θ is one-to-one.

As $(H(X),\sigma)$ is a convergence transformation group over X relative to the evaluation mapping ω, the natural question arises as to whether the σ construction on H(X) generalizes to arbitrary transformation groups on a convergence space X. The answer is affirmative as described by the following:

Let X,τ be a convergence space, G a group, and $\hat{\theta}$ a function, $\hat{\theta}: G \times X \to X$ satisfying

1) $\hat{\theta}(e,x) = x$ for all $x \in X$, and
2) $\hat{\theta}(gg',x) = \hat{\theta}(g,\hat{\theta}(g',x))$ for all $g,g' \in G$ and $x \in X$, and
3) $\hat{\theta} \mid \{g\} \times X$ is continuous for each $g \in G$.

Definition: For each f in G let $\kappa_{\hat{\theta}} f$ consist of all filters $\mathcal{F}$ on G such that:

1) for all $x \in X$ and for all $\Phi \in \tau x$, $\hat{\theta}(\mathcal{F} \times \Phi) \in \tau\hat{\theta}(f,x)$, and
2) for all $x \in X$ and for all $\Phi \in \tau x$, $\hat{\theta}(\mathcal{F}^{-1} \times \Phi) \in \tau\hat{\theta}(f^{-1},x)$

With this definition of $\kappa_{\hat{\theta}}$ on G the next five theorems follow immediately from the previous discussion of σ on H(X). The differences in proof are notational only.

Theorem 12: $\kappa_{\hat{\theta}}$ is a convergence structure on G making $(G,\kappa_{\hat{\theta}})$ a convergence transformation group on X relative to $\hat{\theta}$.

<u>Theorem 13</u>: $\kappa_{\hat{\theta}}$ is the coarsest of the convergence transformation group structures on G over X relative to $\hat{\theta}$.

<u>Theorem 14</u>: $\kappa_{\hat{\theta}}$ on G has the following property:

$$\mathcal{F} \in \kappa_{\hat{\theta}} f \quad \Rightarrow \quad \begin{cases} \text{for all } x \in X \text{ and for any filter } \Phi \text{ on } X \\ \hat{\theta}(\mathcal{F} \times \Phi) \in \tau\hat{\theta}(f,x) \text{ if and only if } \Phi \in \tau x. \end{cases}$$

CHAPTER IV

Throughout the following discussion we assume $(X,\tau_{\mathcal{J}})$ to be a uniform convergence space with $\tau_{\mathcal{J}}$ induced from the uniform convergence structure $\mathcal{J}$. [5,p.291] Again let H(X) be the family of homeomorphisms of the convergence space $(X,\tau_{\mathcal{J}})$, and now let U(X) represent automorphisms of X in this new setting, that is,

$$U(X) = \{f \in H(X) \mid f, f^{-1} \text{ uniformly continuous on } X\}$$

Theorem 15: U(X) is a subgroup of H(X).

Proof: Let $f,g \in U(X)$. If we can show $f\circ g$ is uniformly continuous on X, we are done since $(f\circ g)^{-1} = g^{-1}\circ f^{-1}$ will also be uniformly continuous on X. So let $\Phi \in \mathcal{J}$. Then $((f\circ g)\times(f\circ g))(\Phi) = (f\times f)((g\times g)(\Phi))$ which is in $\mathcal{J}$ by the uniform continuity of g and f respectively which demonstrates the uniform continuity of $f\circ g$.

Definition: Let $\mathcal{U}$ be the uniform convergence structure on U(X) defined by:

$$\mathcal{F} \in \mathcal{U} \text{ if and only if } [\mathcal{F},X] \in \mathcal{J}.$$

Note that this is the uniform convergence structure induced from the uniform convergence structure on X^X of uniform convergence on X. [5,p.301]

Theorem 16: $(U(X), \tau_{\mathcal{U}})$ is a convergence group.

Proof: Continuity of composition: Let $f,g \in U(X)$ and $\mathcal{F}, \mathcal{G} \in \tau_{\mathcal{U}}f, \tau_{\mathcal{U}}g$ respectively. We need to show $\mathcal{F}\circ\mathcal{G} \in \tau_{\mathcal{U}} f\circ g$. But $\mathcal{F} \in \tau_{\mathcal{U}}f \Rightarrow \mathcal{F}\times\overset{\circ}{f} \in \mathcal{U} \Rightarrow [\mathcal{F}\times\overset{\circ}{f}, X] \in \mathcal{J}$ and also, $\mathcal{G} \in \tau_{\mathcal{U}}g \Rightarrow \mathcal{G}\times\overset{\circ}{g} \in \mathcal{U} \Rightarrow [\mathcal{G}\times\overset{\circ}{g}, X] \in \mathcal{J}$. Similarly we want to show $[\mathcal{F}\circ\mathcal{G}\times\overset{\circ}{fg}, X] \in \mathcal{J}$.

Now $[\mathcal{G}\times\overset{\circ}{g}, X] = [\{\{(g'(x),g(x)) \mid g' \in G \in \mathcal{G}, x \in X\}\}]$

so that with f uniformly continuous on X we have

$\mathcal{H} = [\{\{(fg'(x),fg(x)) \mid g' \in G \in \mathcal{G}, x \in X\}\}$ also in $\mathcal{J}$. Now let

$\mathcal{H}' = [\{\{(f'g'(x),fg'(x)) \mid f' \in F \in \mathcal{F}, g' \in G \in \mathcal{G}, x \in X\}\}]$

$= [\{\{(f'(g'(x)),f(g'(x))) \mid f' \in F \in \mathcal{F}, g'(x) \in X\}\}]$

$= [\{\{(f'(x),f(x)) \mid f' \in F \in \mathcal{F}, x \in X\}\}]$ since g' is a homeomorphism,

$= [\mathcal{F}\times\overset{\circ}{f}, X]$ which belongs to $\mathcal{J}$ by assumption. But

$[\mathcal{F}\circ\mathcal{G}\times\overset{\circ}{fg}, X] = [\{\{((f'g')(x),(fg)(x)) \mid f' \in F \in \mathcal{F}, g' \in G \in \mathcal{G}, x \in X\}\}]$

$= \mathcal{H}'\circ\mathcal{H} \in \mathcal{J}$ which is what was needed.

Continuity of inversion: Let $\mathcal{F} \in \tau_{\mathcal{U}}f$, that is again, $[\mathcal{F}\times\overset{\circ}{f}, X] \in \mathcal{J}$. We need to show $[\mathcal{F}^{-1}\times \overset{\circ}{f}{}^{-1}, X] \in \mathcal{J}$. But

$[\mathcal{F}\times\overset{\circ}{f}, X] = [\{\{(f'(x),f(x)) \mid f' \in F \in \mathcal{F}, x \in X\}\}]$

$= [\{\{(f'(f'^{-1}(x)),f(f'^{-1}(x))) \mid f' \in F \in \mathcal{F}, x \in X\}\}]$

as each f'^{-1} is a homeomorphism of X,

$= [\{\{(x,f(f'^{-1}(x))) \mid f' \in F \in \mathcal{F}, x \in X\}\}] \in \mathcal{J}$.

So, using the uniform continuity of f^{-1}, we have

$[\{\{(f^{-1}(x), f^{-1}ff'^{-1}(x)) \mid f' \in F \in \mathcal{F}, x \in X\}\}]$

$= [\{\{(f^{-1}(x), f'^{-1}(x)) \mid f' \in F \in \mathcal{F}, x \in X\}\}]$

$= [f^{-1} \times F^{-1}, X] \in J$. But $[F^{-1} \times f^{-1}, X] = [f^{-1} \times F^{-1}, X]^{\Delta}$, hence $[F^{-1} \times f^{-1}, X] \in J$.

It should be noted that this last theorem will not hold in general for H(X) with the corresponding uniform convergence structure. To see this, simply let X represent the real numbers with the usual topology and let F be the filter generated by the sequence

$$f_n(x) = x^{1/3} - \frac{1}{n}$$

with the limit homeomorphism $f(x) = x^{1/3}$. Here F converges uniformly on X but clearly F^{-1} does not converge uniformly to f^{-1} on X.

To guarantee the continuity of inversion it would appear that we need to use new definitions in this setting, similar to those in the last chapter. We proceed in this manner.

Definition: Let C be any non-empty subset of X. Let ν_C be the family of filters on H(X) × H(X) defined by:

$$F \in \nu_C \iff \begin{cases} [F, C] \in J \\ [F^{-1}, C] \in J \end{cases}$$

Theorem 17: ν_C is a uniform convergence structure on H(X).

Proof: (This proof is primarily that of [5,p.300] with the appropriate modifications to apply to this new definition.)

First we note that ν_C actually does form a Λ-ideal:

1: If $F, F' \in \nu_C$ then $[F \wedge F', C] = [F, C] \wedge [F', C] \in J$, and $[(F \wedge F')^{-1}, C] = [F^{-1} \wedge F'^{-1}, C] = [F^{-1}, C] \wedge [F'^{-1}, C] \in J$.

2: If $\mathcal{F}' \geq \mathcal{F} \in \nu_C$, then $[\mathcal{F}', C] \geq [\mathcal{F}, C] \in \mathcal{J}$, and as $\mathcal{F}'^{-1} \geq \mathcal{F}^{-1}$, then $[\mathcal{F}'^{-1}, C] \geq [\mathcal{F}^{-1}, C] \in \mathcal{J}$.

Secondly we check the three requirements for a uniform convergence structure:

3: $[\Delta] \in \nu_C$

$[\Delta] = [\{\{(f,f) \mid f \in H(X)\}\}]$ so that $[[\Delta], C] =$
$[\{\{(f(x),f(x)) \mid f \in H(X),\ x \in C\}\}] \geq [\Delta_X] \in \mathcal{J}$. Moreover, as $[\Delta]^{-1} = [\{\{(f^{-1},f^{-1}) \mid f \in H(X)\}\}] = [\Delta]$, we have $[[\Delta]^{-1}, C] \in \mathcal{J}$.

4: $\mathcal{F} \in \nu_C$ implies $\mathcal{F}^\Delta \in \nu_C$

$[\mathcal{F}^\Delta, C] = [\mathcal{F}, C]^\Delta \in \mathcal{J}$, and $[(\mathcal{F}^\Delta)^{-1}, C] = [(\mathcal{F}^{-1})^\Delta, C] =$
$[\mathcal{F}^{-1}, C]^\Delta \in \mathcal{J}$.

5: $\mathcal{F}, \mathcal{G} \in \nu_C$ implies $\mathcal{F} \circ \mathcal{G} \in \nu_C$

We claim initially that $[\mathcal{F} \circ \mathcal{G}, C] \geq [\mathcal{F}, C] \circ [\mathcal{G}, C] \in \mathcal{J}$. To show this it suffices to show for basis elements $F \in \mathcal{F}$, $G \in \mathcal{G}$, that $[F, C] \circ [G, C] \supseteq [F \circ G, C]$. So let $(f(x),g(x)) \in [F \circ G, C]$ with $(f,g) \in F \circ G$. This implies there exists $h \in H(X)$ such that $(f,h) \in F$ and $(h, g) \in G$. Hence for $x \in C$, $(f(x),h(x)) \in [F, C]$ and $(h(x),g(x)) \in [G, C]$ which gives the desired result that $(f(x),g(x)) \in [F, C] \circ [G, C]$. Secondly, we need to show that $[(\mathcal{F} \circ \mathcal{G})^{-1}, C] \geq$ $[\mathcal{F}^{-1}, C] \circ [\mathcal{G}^{-1}, C]$ which is in $\mathcal{J}$ by assumption. As above, this will be true if for $F \in \mathcal{F}$ and $G \in \mathcal{G}$, that $[F^{-1}, C] \circ [G^{-1}, C] \supseteq [(F \circ G)^{-1}, C]$. So let

$(f^{-1}(x),g^{-1}(x)) \in [(F \circ G)^{-1}, C]$ with $x \in C$ and $(f,g) \in F \circ G$. This implies there exists $h \in H(X)$ such that $(f,h) \in F$ and $(h,g) \in G$, or equivalently, $(f^{-1},h^{-1}) \in F^{-1}$ and $(h^{-1},g^{-1}) \in G^{-1}$. Hence $(f^{-1}(x),h^{-1}(x)) \in [F^{-1}, C]$ and $(h^{-1}(x),g^{-1}(x)) \in [G^{-1}, C]$ for $x \in C$, and so $(f^{-1}(x),g^{-1}(x)) \in [F^{-1}, C] \circ [G^{-1}, C]$.

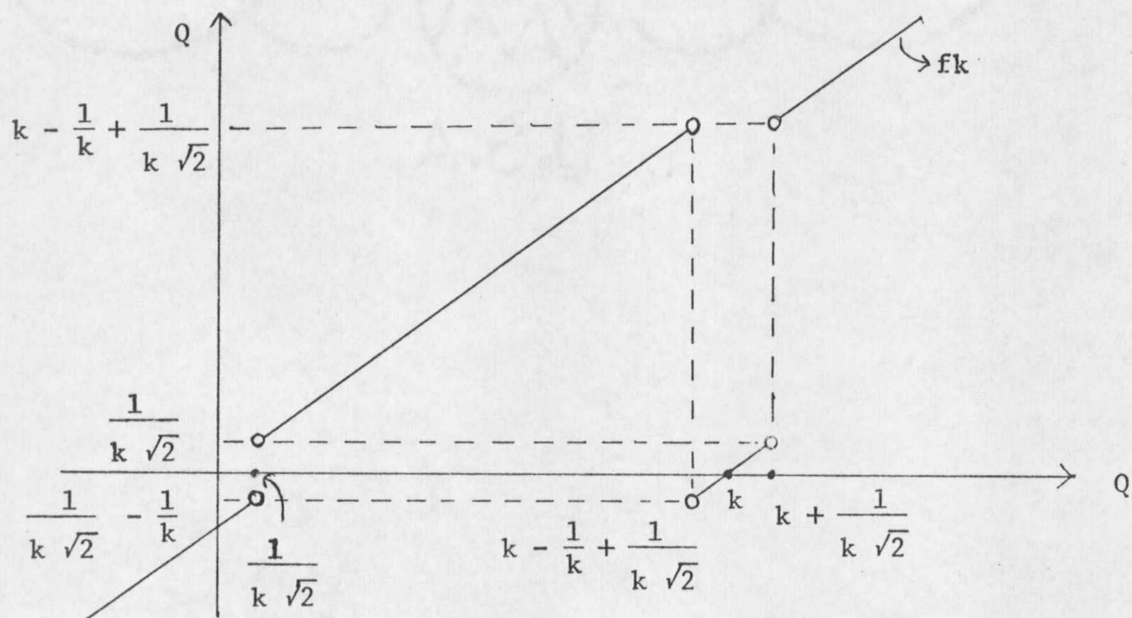
For convenience purposes we will say that if $\mathcal{F} \in \tau_{\nu_C}$ that $\mathcal{F}$ converges biuniformly to f on C, that is both $\mathcal{F}$ and $\mathcal{F}^{-1}$ respectively converge uniformly to f and f^{-1} on C.

Now if $\mathcal{C}$ is any non-empty family of non-empty subsets of X, form $\nu_{\mathcal{C}} = \sup_{C \in \mathcal{C}} \nu_C = \bigcap_{C \in \mathcal{C}} \nu_C$. $\nu_{\mathcal{C}}$ is then the uniform convergence structure of biuniform convergence on $\mathcal{C}$, that is $\mathcal{F} \in \tau_{\nu_{\mathcal{C}}}$ f if and only if $\mathcal{F}$ converges biuniformly to f on every $C \in \mathcal{C}$.

Initially, consider the case where $\mathcal{C} = \{\{x\} \mid x \in X\}$. Here $\mathcal{F} \in \tau_{\nu_{\mathcal{C}}}$ f if and only if $\mathcal{F}$ and $\mathcal{F}^{-1}$ converge simply to f and f^{-1} respectively. [5,p.301] An example similar to the example given in the previous chapter shows that this new uniform convergence structure is distinct from the uniform convergence structure of simple convergence, that is $\mathcal{F}$ converging simply to the homeomorphism f does not imply the simple convergence of $\mathcal{F}^{-1}$ to f^{-1}. Namely let Q be the set of rational numbers with the usual topology and let $\mathcal{F}$ be the filter generated by the sequence of homeomorphisms of Q given by:

$$f_k(x) = \begin{cases} x - \dfrac{1}{k} & x < \dfrac{1}{k\sqrt{2}} \\ x & \dfrac{1}{k\sqrt{2}} < x < k - \dfrac{1}{k} + \dfrac{1}{k\sqrt{2}} \\ x - k & k - \dfrac{1}{k} + \dfrac{1}{k\sqrt{2}} < x < k + \dfrac{1}{k\sqrt{2}} \\ x - \dfrac{1}{k} & x > k + \dfrac{1}{k\sqrt{2}} \end{cases}$$

Graphically, we have:



For each $x \in Q$ we have $\omega(F \times \{x\}) = F(x)$ converging to $id(x) = x$ but clearly $\omega(F^{-1} \times \{0\}) = F^{-1}(0)$ does not converge.

By our construction of ν_C, we know that inversion in $(H(X),\tau_{\nu_C})$ will be continuous, regardless of the nature of C. However when $C = \{\{x\} \mid x \in X\}$, the counterexample given in [3,p.74,(14)] is applicable here also and shows that composition in $(H(X),\tau_{\nu_C})$ is not continuous in general. Hence in general, $(H(X),\tau_{\nu_C})$ for $C = \{\{x\} \mid x \in X\}$ is not a convergence group.

The problem now appears to center on finding those families of subsets C of X for which composition in $(H(X),\tau_{\nu_C})$ is continuous. The question appears to be difficult to answer and in this paper is left as an open question. It seems unlikely that $C = \{\{X\}\}$ will suffice although this has not been determined. A more likely candidate for C is the family of all compact subsets of X. Investigation here has led to unanswered questions concerning relationships of compactness to uniform continuity. Hopefully, these questions can be analyzed successfully at a later date.

BIBLIOGRAPHY

[1] R. F. Arens, Topologies for Homeomorphism Groups, Amer. J. Math., 68 (1946), 593-610.

[2] E. Binz, H. H. Keller, Funktionenräume, Ann. Acad. Sci. Fenn. A.I. 383 (1966), 3-21.

[3] N. Bourbaki, Éléments de Mathématique, Livre III, Chapitre 10, Deuxième Édition, Hermann, Paris.

[4] C. H. Cook, H. R. Fischer, On Equicontinuity and Continuous Convergence, Math. Annalen 159 (1965), 94-105.

[5] C. H. Cook, H. R. Fischer, Uniform Convergence Structures, Math. Annalen 173 (1967), 290-306.

[6] J. Dugundji, Topology, Allyn and Bacon, Inc., Boston, 1966.

[7] H. R. Fischer, Limesräume, Math. Annalen 137 (1959), 269-303.

[8] D. Montgomery, L. Zippin, Topological Transformation Groups, Interscience Publishers, Inc., New York, 1955.