A theoretical design comparison of a structural frame in prestressed and conventional reinforced concrete

by Mete Teoman

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University

Abstract:
The purpose of this thesis is to compare a structure designed in prestressed, and in conventional reinforced concrete. Both designs employ the same theoretical frame, and to obtain an accurate comparison the same live load is used. Allowable stresses in materials are as near the Same as codes and differing theories of the two systems will permit.

The thesis is divided into three parts. The first part covers the design of a theoretical frame in prestressed concrete. The design follows theories and practices used today.

The second part covers the design of the same theoretical frame in conventional reinforced concrete. The methods used follow those employed in this country.

The third part offers a comparison of the resultant structures of the two different designs.

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M. Teoman
Mete Teoman

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COMPARISON

ABSTRACT

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The thesis is divided into three parts. The first part covers the design of a theoretical frame in prestressed concrete. The design follows theories and practices used today.

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INTRODUCTION

History

Continual improvement in concrete has made the present-day product much superior to the concrete used only thirty years ago. A high degree of uniformity in quality has been attained, and important advances in methods for analysis of indeterminate structures have resulted in more accurate determination of stresses than ever before.

The history of conventional reinforced concrete is well known to people associated with the field, but the development of prestressed concrete is relatively new.

The basic principle of prestressing has been used in wooden barrel construction for many centuries. Metal bands placed around wooden staves to form a barrel are tightened, subjecting them to tensile prestress which creates compressive prestress between the staves, enabling them to resist hoop tension caused by internal liquid pressure.

This principle was not used for concrete until about 1886, when P. H. Jackson[1] of San Francisco was granted patents for tightening steel tie rods in concrete arches to serve as floor slabs.

Soon after, C. E. W. Doehring[2] of Germany secured a patent for concrete reinforced with metal that had tensile stress applied to it before the slab was loaded.

Both of these early methods were unsucessful because the low prestress in the steel was lost as a result of the shrinkage and creep of concrete.

1. "Prestressed Concrete Structures T. Y. Lin John Wiley & Sons N.Y.1955 pl
2. Ibid p. 1

In 1908, C. R. Steiner[3] of the United States suggested retightening the reinforcing rods after some shrinkage had taken place in the concrete.

Successful development of prestressed concrete is attributed to E. Freyssinet[4] of France, who began using high-strength steel wire for prestressing in 1928. These wires have an ultimate strength as high as 250,000 psi and a yield point strength of around 180,000 psi and are pre-stressed to about 150,000 psi before any shrinkage is taken place in the concrete.

E. Hoyer[5] of Germany first successfully used the method of pre-tensioning in which the steel is bonded to the concrete without end anchorage. His system consists of stretching wires between two buttresses several hundred feet apart, erecting form work between the units, placing the concrete, and cutting the wires after the concrete has hardened.

In 1939, Freyssinet[6] developed conical wedges for end anchorages and designed double-acting jacks which tensioned the wires and then thrust the male cones into the female cones for anchoring them.

Professor G. Magnel[7] of Belgium developed the Magnel system in 1940, in which two wires were stretched at a time and anchored with a simple metal wedge at each end.

Prestressed concrete first became important about 1940, perhaps partly as a result of a steel shortage in Europe. Much less steel is needed for prestressed than for reinforced concrete, since the steel used in

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3. Ibid p. 2
4. Ibid p. 3
5. Ibid p. 3
6. Ibid p. 4
7. Ibid p. 4

prestressed concrete is of higher strength. France and Belgium[8] led all other countries in the use of prestressed concrete.

The development of prestress concrete in the United States[9] followed a different course. Instead of prestressed concrete beams and slabs, or linear prestressing, circular prestressing was developed, especially in storage tanks.

Basic References Used

Gustave Magnel's book "Prestressed Concrete" was used as the main reference for the first part of this thesis, because the book embodies a good combination of theory and practical design problems in prestressed concrete. M. Guyon's "Béton Précontraint" and T. Y. Lin's "Prestressed Concrete Structures" were also followed.

In the second part of this thesis, "Reinforced Concrete Design" by Sutherland and Reese was the main reference together with the current A.C.I. Building Codes and Design Handbooks.

In designing girders and beams, uniformity was maintained by using I shapes and same size of wires in prestressed design, and rectangular shapes and same size of main reinforcements in conventional concrete design.

This thesis does not contain basic theories of the two designs, since such information is already available, but does employ them.

8. Ibid p. 4
9. Ibid p. 5

The Frame

The frame to be used in both designs is a single-story concrete rigid frame structure as shown in fig. 1.

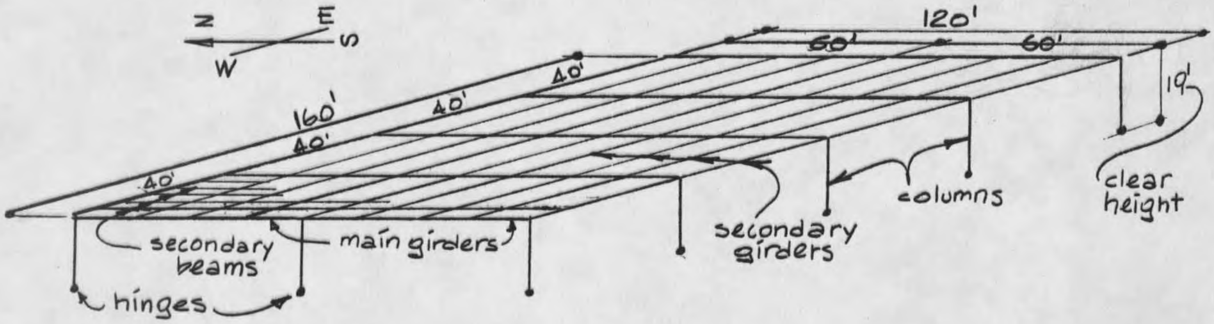


fig. 1
The Theoretical Frame

Center to center spacing between the columns in the E-W direction is 40', and center to center spacing between the columns in the N-S direction is 60'.

The slabs, secondary beams and secondary girders are the simply-supported type, and the main girder frames consist of two main girders and three columns of rigid-frame design. The spacing of secondary beams and secondary girders is given in fig. 2.

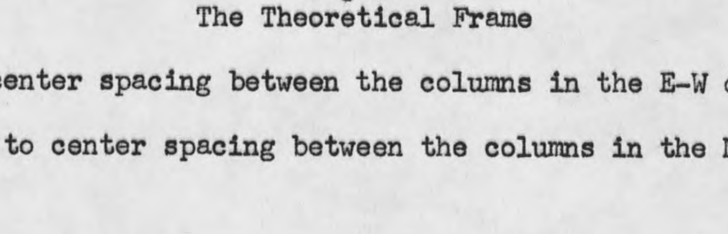


fig. 2
Spacing of Individual Members

Horizontal loads, such as earthquake loads and wind pressures, are omitted in the designs because they have the same effect on both designs of the structure. Reinforced concrete footings are assumed to be used in both designs and are therefore omitted.

The wide spacing of columns makes possible a good comparison of the two designs of beams and girders. Main girders and columns designed as a rigid frame result in more economical sections.

Loads on the Frame

Roof Live Loads:

Snow load	= 25# per sq. ft. (flat roof)
Superimposed load	= 30# per sq. ft.

Roof Dead Loads:

Lean concrete	= 20# per sq. ft.
Built-up roofing	= 5# per sq. ft.

Roof live loads and roof dead loads are the same in both designs.

Allowable Stresses

In Prestressed Design:

If the concrete is made carefully and with good aggregate a crushing resistance of 8,800 psi can be obtained without difficulty and consequently a working stress of 2,200 psi could then be adopted. For the tensile stresses due to shearing forces, the same stresses as in conventional reinforced concrete are acceptable. Tensile stresses due to bending moments do not, in theory, exist in fully-prestressed concrete, but there is no reason why some tensile stress in the top fibre during prestressing, and some tensile stress in the lower fibre under the greatest probable load, should not be permitted.

The working stress in wires is assumed to be 155,000 psi. Proportion of the initial stretching force that remains permanently is generally assumed to be 0.85, and this value is accepted in the design.

$f'_c = 8,800$ psi

$f_c = 2,200$ psi

$f_s = 155,000$ psi

Tensile stress due to shearing forces (no web reinforcement).

$$0.03\, f'_c = 0.03\,(8,800) = 264 \text{ psi}$$

Tensile stress due to shearing forces (with properly designed web reinforcement)

$$0.12\, f'_c = 0.12\,(8,800) = 1,056 \text{ psi}$$

Tensile stress due to bending moments = 230 psi

$\eta = 0.85$

7mm round wires used in the design have an equivalent diameter in inches of 0.276, and an area of 0.0596 sq. in.

Because concrete can carry compressive load better without being precompressed by steel, and horizontal forces do not exist in the design, conventional methods are used in column designs.

In Conventional Design:

Although the working compressive stress in concrete in prestressed design is assumed to be 0.25 f'_c, in conventional design, in order to follow A.C.I. building code requirements of 0.45 f'_c, we have to use $f'_c = 4,890$ psi to get $f_c = 2,200$ psi.

$f'_c = 4,890$ psi

$f_c = 2,200$ psi

$f_s = 20,000$ psi

$n = \frac{30,000}{f'_c} = 6.13$

With the value of $\frac{f_s}{nf_c} = 1.48$, a very close estimate can be made for j and k, where j is the ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth and k is the ratio of distance (kd) between extreme fiber and neutral axis to effective depth.

$j = 0.866$

$k = 0.403$

Allowable unit shearing stress:

beams with no web reinforcement $= 0.03\, f'_c = 0.03\,(4890) = 146.7$ psi

beams with web reinforcement $= 0.12\, f'_c = 0.12\,(4890) = 586.8$ psi

bond unit stress for deformed bars $= 350$ psi

Symbols and Notations

In Prestressed Concrete Design:

A, cross-sectional area of beam and girder.

A_g, gross area of concrete section.

A_s, cross-sectional area of the reinforcement or stretched wires.

a, length of the end-blocks of beam and girder.

add, additional.

b, width of rectangular beam or width of web in I-beams or girders; also the effective width (the width of the beam or girder at the end minus the width of the openings for the cables) in end-blocks of beam and girder; also the least lateral dimension of column.

C, constant relating to the ratio of stresses; also ratio of allowable concrete stress, f_a, in axially loaded column to allowable fibre stress for concrete in flexure.

C.O., carry-over factor.

c_n, compressive stress at the neutral axis.

c_x, compressive stress in the end-blocks of beam and girder.

c_z, bending stress in the end-blocks of beam and girder.

c_{ab}, c_{at}, calculated stresses in the bottom and top fibres respectively due to w_a.

c_{db}, c_{dt}, calculated stresses in the bottom and top fibres, due to loads acting at the time of prestressing.

comp., compressive, compression.

D, total depth of slab; also $D = \frac{t^2}{2R^2} =$ a factor, usually varying from 3 to 9 (the term R as used here is the radius of gyration of the entire column section.)

D.F., distribution factor

D.L., dead load

e, eccentricity of the stretched steel from the centroid; also eccentricity of the resultant load on a column, measured from the gravity axis.

e_A, eccentricity applicable to point A.

e_{A_1}, e_{B_1}, etc., actual eccentricity of the cable at sections A_1, B_1, etc.

F.E.C., far-end condition

F.E.M., fixed-end moment.

f_a, average allowable stress in the concrete of an axially loaded reinforced concrete column.

f_c, permissible compressive stress in the concrete.

f'_c, ultimate compressive strength of concrete.

f_s, permissible tensile stress in the steel.

f_t, permissible tensile stress in the concrete.

I, moment of inertia about the horizontal centroidal axis.

I_b, moment of inertia of main girder.

I_c, moment of inertia of column.

K, a non-dimensional coefficient used in calculating c_z, $K = 5\left(-1 + \frac{12z^2}{a^2} + \frac{16z^3}{a^3}\right)$, in end-block designs.

K_1 a non-dimensional coefficient used in calculating v in end-block designs. $K_1 = 5\left(\frac{1}{4} + \frac{z}{a} - \frac{4z^3}{a^3} - \frac{4z^4}{a^4}\right)$.

L, length of span; also length of column.

L.L., live load.

M, bending moment.

M_a, bending moment due to w_a.

M_d, bending moment due to w_d.

M_{d+a}, bending moment due to w_d and w_a acting together.

max., maximum.

mom., moment

$\sum M_q$, summation of moments around q.

N, axial load applied to conventional reinforced column.

n, ratio of modulus of elasticity of steel to that of concrete: assumed as equal to $\frac{30,000}{f'_c}$

N.A., neutral axis.

no., number, used in bar designation (no. 2 bars, no. 3 bars, etc.).

P, total allowable axial load on a column

P_i, initial stretching force.

p_g, ratio of the effective cross-sectional area of vertical reinforcement to the gross area A_g.

p_t, principal tensile stress.

Q_1, factor for moment of resistance; for a rectangular beam

$$Q_1 = \frac{0.775 f_c + f_t}{6}$$

Q_m, moment about the neutral axis of the area of a section on one side of the neutral axis.

R, vertical component of force in an inclined cable.

r, radius of gyration of the concrete section.

req'd, required.

sec., section; also secondary.

t, overall dimension of column.

u, unit str., unit stress.

V, shearing force (V_{AE}, shearing force at AE.).

v, shearing stress

ΣV, summation of shearing forces.

w, uniformly-distributed load (dead or live).

w_a additional load per unit length applied after the prestress has been established.

w_d, load per unit length acting when the prestress is being established.

w_{d+a}, addition of loads w_d and w_a.

x, distance from one support of any point in a beam or girder; also used in coordinate system of the end-block designs; also the distance from extreme fibre in compression to axis of zero stress in cracked-section design of column.

x_L, $\frac{x}{L}$ used in the equation for the shape of cables of continuous girder.

y_1, y_2, distances from the centroid to the top and bottom fibres respectively.

y, distance from the centroid to any fibre.

z, used in coordinate system of the end-block designs.

η, proportion of P_i that remains permanently; generally $= 0.85$.

ϕ, diameter of wires.

In Conventional Reinforced Concrete Design:

In addition to some of the symbols and notations used in the prestress concrete design:

A_t, area of temperature steel.

A_v, total area of web reinforcement in tension within a distance of s (measured in a direction parallel to that of the main reinforcement).

d, effective depth of flexural members.

f_c, actual stress in concrete.

f_s, actual stress in steel.

f_v, tensile unit stress in web reinforcement.

I_T, moment of inertia of transformed beam or girder areas.

j, ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth.

k, ratio of distance (kd) between extreme fibre and neutral axis to effective depth.

neg., negative.

$\sum_o$, sum of perimeters of bars.

poz., positive.

R, $\frac{M}{bd^2}$ coefficient of resistance.

s, spacing of stirrups in a direction parallel to that of the main reinforcement.

t, total depth of slab.

temp., temperature.

u, bond stress.

V_c, shear carried by concrete.

V_s, excess of the total shear over that permitted on the concrete.

w_a, all loads per unit length on the beam or girder, except their dead loads per unit length.

w_d, dead load per unit length of beam or girder.

x, distance (kd) between extreme fibre and neutral axis.

Z_c, $\frac{I_T}{x}$ used in moment of inertia method of finding actual extreme compressive fibre stress of concrete.

Z_s, $\frac{I_T}{d-x}$ used in moment of inertia method of finding actual tensile stress in steel.

PART I

DESIGN OF THE THEORETICAL FRAME IN PRESTRESSED CONCRETE

A. Design of the Roof Slab

Precast slabs are designed as simply-supported beams of 12" width, supported by the top flanges of secondary beams.

$L = 6.66'$

$w_a = 20 + 5 + 55 = 80\ \#/\square' = 80\ \#/'$ for each ft. of width.

Total depth (D)

$$M_a = \frac{80 \times 6.66^2 \times 12}{8} = 5{,}330 \text{ IN-LB}$$

$$Q_1 = \frac{0.775 f_c + f_t}{6} = \frac{0.775(2200) + 230}{6} = \frac{1935}{6} = 323$$

$$\text{total depth } D = \sqrt{\frac{M_a}{Q_1 b}} = \sqrt{\frac{5330}{323 \times 12}} = 1.170''$$

Use $D = 2''$ (with min. fireproofing cover)

Initial stretching force & wires

$$w_d = 150 \times \frac{2}{12} = 25\ \#/\square' \ , \qquad M_d = \frac{25 \times 6.66^2 \times 12}{8} = 1665 \text{ in-lb}$$

$$c_{dt} = c_{db} = \frac{1665 \times 6}{12 \times 2^2} = 208 \text{ psi}$$

$$c_{at} = c_{ab} = \frac{5330 \times 6}{12 \times 2^2} = 666 \text{ psi} \qquad \overset{f_c}{2200} > \overset{C_{dt} + C_{at}}{208 + 666}$$

$$C = \eta \frac{f_c - c_{dt} - c_{at}}{c_{db} + c_{ab} - f_t} = \frac{0.85(2200 - 208 - 666)}{208 + 666 - 230} = 1.75$$

$$r^2 = \frac{I}{A} = \frac{D^2}{12} = \frac{4}{12} = 0.333$$

$$e_A = \frac{(1 - C)\, r^2}{y_1 + C y_2} = \frac{(1 - 1.75)\, 0.333}{1 + 1.75 \times 1} = -0.091''$$

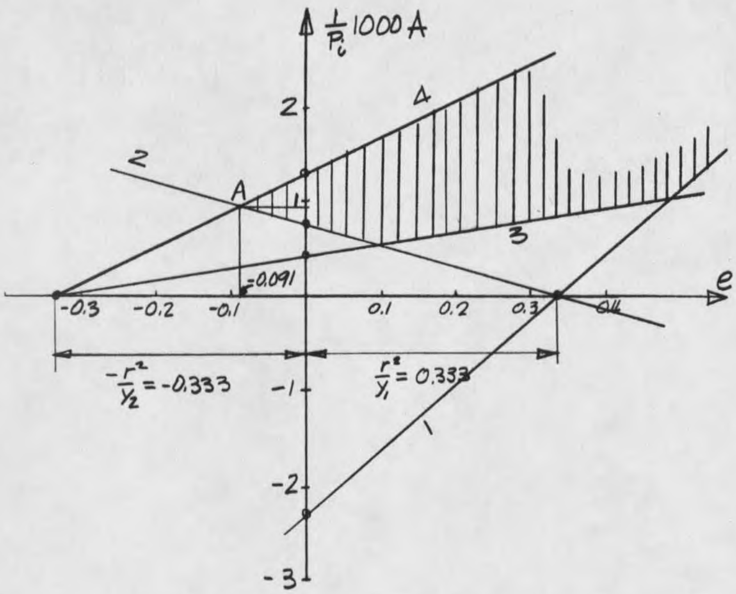
Ordinates for $e = 0$ (fig 3)

Line 1. $-\dfrac{1}{(c_{dt}+f_t)A} = -\dfrac{1}{(208+230)A} = -\dfrac{228}{1000\,A}$

Line 2. $+\dfrac{1}{(f_c-c_{dt}-c_{at})A} = +\dfrac{1}{(2200-208-666)A} = +\dfrac{0.755}{1000\,A}$

Line 3. $+\dfrac{1}{(f_c+c_{db})A} = +\dfrac{1}{(2200+208)A} = +\dfrac{0.416}{1000\,A}$

Line 4. $+\dfrac{n}{(c_{db}+c_{ab}-f_t)A} = +\dfrac{0.85}{(208+666-230)A} = +\dfrac{1.320}{1000\,A}$



$\dfrac{r^2}{y_1} = \dfrac{r^2}{y_2} = 0.333$

fig. 3.
The acceptable values
for P_i & e.

According to fig. 3

$\dfrac{1000\,A}{P_i} = 0.96 \qquad \underline{P_i} = \dfrac{1000\times 2\times 12}{0.96} = \underline{25{,}000^{\#}} \qquad \dfrac{P_i}{f_s} = \dfrac{25{,}000}{155{,}000} = 0.161^{\square''}$

<u>Use 3 $\phi = 0.276''$ (7mm) wires</u>

<u>Shearing stresses.</u>

ordinates of fig. 4 (due to w_a) $OA = \dfrac{80\times 6.66}{2} = 267.0^{\#}$

" $DB = \dfrac{267}{4} = 66.6^{\#}$

(due to w_d) $OC = \dfrac{25\times 6.66}{2} = 83.4^{\#}$

(due to bending up wires) $CE = 0$

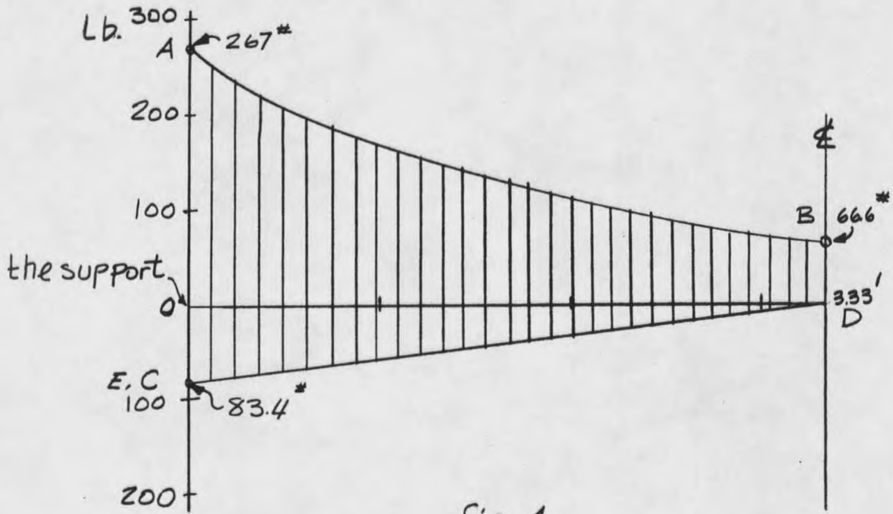


fig. 4.
Shearing stresses

Net shear at the support (fig. 4.) $= 267.0 + 83.4 = 350.4^{\#}$

Max. intensity of shearing stresses at the support

$$v = \frac{V_{AE}\, Q_m}{b\, I} = \frac{350.4 \times 6}{12 \times 8} = 21.9 \text{ psi}$$

$$V_{AE} = 350.4^{\#}$$

$$Q_m = \frac{2}{2} \times 12 \times \frac{2}{4} = 6 \text{ in}^3$$

$$I = \frac{12 \times 2^3}{12} = 8 \text{ in}^4$$

Horizontal compressive stress $= C_n = \frac{P_i}{A} = \frac{25{,}000}{2 \times 12} = 1040$ psi

Principal tensile stress $= P_t = \sqrt{v^2 + \frac{C_n^2}{4}} - \frac{C_n}{2}$

$$= \sqrt{21.9^2 + \frac{1040^2}{4}} - \frac{1040}{2} = \underline{2 < 230 \text{ O.K.}}$$

No further investigation is required since the greatest shearing stress is only <u>21.9 psi</u>.

B. Design of Secondary Beams

Secondary beams are simply-supported by the secondary girders.

Dimension calculations

$L = 15'$ $\qquad w_a = (25+30+50)\,6.66 = 700\ \#/_1$

$$M_a = \frac{700 \times 15^2 \times 12}{8} = 236{,}000 \text{ in-lb}$$

$$\frac{M_a}{0.775 f_c + f_t} = \frac{236{,}000}{(0.775 \times 2200) + 230} = 122 \text{ in}^3$$

$$\frac{M_a}{0.925 f_c + f_t} = \frac{236{,}000}{(0.925 \times 2200) + 230} = 104 \text{ in}^3$$

The trial section should have

$$\frac{I}{y_1} \geqslant 122 \text{ in}^3$$

$$\frac{I}{y_2} \geqslant 104 \text{ in}^3$$

First trial section

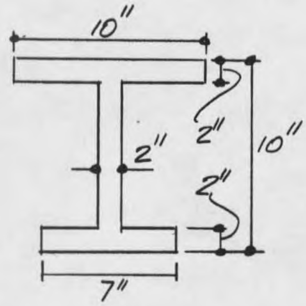


fig. 5.
first trial section.

properties of the cross section in fig.5

$A = 46\ \square''$

$y_2 = 5.52''$

$y_1 = 4.48''$

$I = 578 \text{ in}^4$

$r^2 = 12.58 \text{ in}^2$

$$\frac{I}{y_1} = \frac{578}{4.48} = 129 > 122 \text{ O.K.} \quad , \quad \frac{I}{y_2} = \frac{578}{5.52} = 105 > 104 \text{ O.K.}$$

Initial stretching force & wires

$$w_d = \frac{150}{144} \times 46 = 48\ \#/_1 \qquad M_d = \frac{48 \times 15^2 \times 12}{8} = 16{,}200 \text{ in-lb}$$

$$c_{dt} = \frac{16{,}200 \times 4.48}{578} = 126\ \#/\square''$$

$$c_{db} = \frac{16{,}200 \times 5.52}{578} = 155\ \#/\square''$$

$$c_{at} = \frac{236{,}000 \times 4.48}{578} = 1830\ \#/\square''$$

$$c_{ab} = \frac{236{,}000 \times 5.52}{578} = 2255\ \#/\square''$$

$f_c \qquad c_{dt} + c_{at}$

$2200 > 126 + 1830$

$$C = \eta \frac{f_c - c_{dt} - c_{at}}{c_{db} + c_{ab} - f_t} = 0.85 \frac{2200 - 126 - 1830}{155 + 2255 - 230} = 0.0952$$

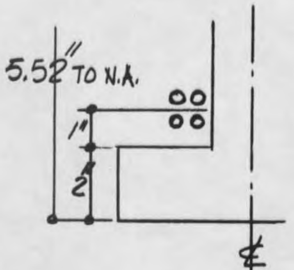


fig. 7.
Arrangement of wires.

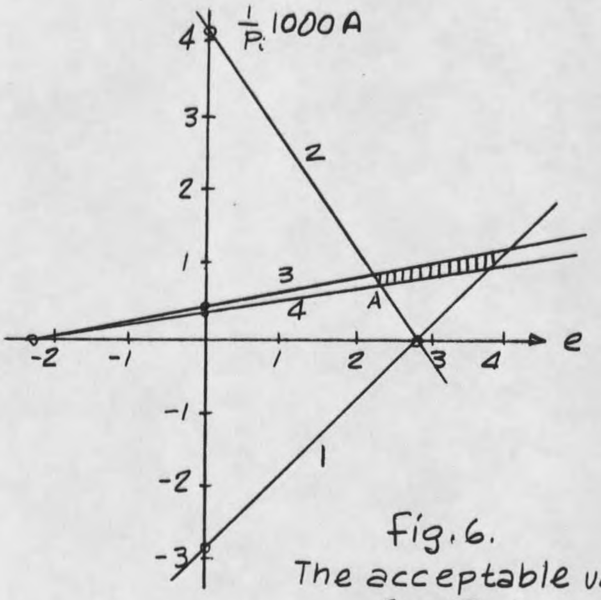


Fig. 6.
The acceptable values for P_i & e

$$e_A = \frac{(1-C)\,r^2}{y_1 + C y_2} = \frac{(1-0.0952)\,12.58}{4.48 + (0.0952 \times 5.52)} = 2.27''$$

two cables each having two layers of wires are used. as shown in fig 7 max. eccentricity obtainable is:

$$5.52 - (2+1) = 2.52'' > 2.27'' \text{ O.K.}$$

Ordinates for $e=0$ (fig. 6)

$$\text{Line 1.} -\frac{1}{(c_{dt} + f_t)A} = -\frac{1}{(126 + 230)A} = -\frac{2.81}{1000A}$$

$$\text{Line 2.} +\frac{1}{(f_c - c_{dt} - c_{at})A} = +\frac{1}{(2200 - 126 - 1830)A} = \frac{4.10}{1000A}$$

$$\text{Line 3} +\frac{1}{(f_c + c_{db})A} = +\frac{1}{(2200 + 155)A} = \frac{0.43}{1000A}$$

$$\text{Line 4} +\frac{\eta}{(c_{db} + c_{ab} - f_t)A} = +\frac{0.85}{(155 + 2255 - 230)A} = \frac{0.39}{1000A}$$

Because line 3 is above line 4 one of the fundamental formulae cannot be satisfied.

$$-\eta \frac{P_i}{A}\left(1 + \frac{e y_2}{r^2}\right) + c_{db} + c_{ab} \leq f_t$$

$-0.85\,(990)\,(2.1) + 155 + 2255 > 230$ <u>the section is not acceptable</u>

Second trial Section

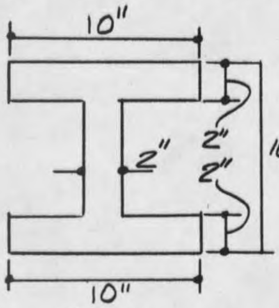


fig.8.
Second trial section

Properties of the cross section in fig.8

$A = 52^{\square''}$

$y_1 = y_2 = 5''$

$I = 689 \text{ in}^4$

$r^2 = 13.2 \text{ in}^2$

$$\frac{r^2}{y_1} = \frac{r^2}{y_2} = \frac{13.2}{5} = 2.64''$$

$$\frac{I}{y_1} = \frac{I}{y_2} = \frac{689}{5} = 138 > 122 \quad \text{O.K.}$$

Initial stretching force & wires

$$\omega_d = \frac{150}{144}\,52 = 54.1\ \#/_1 \quad , \quad M_d = \frac{54.1 \times 15^2 \times 12}{8} = 18{,}300 \text{ in-lb.}$$

$$c_{dt} = c_{db} = \frac{18{,}300 \times 5}{689} = 133 \text{ psi}$$

$$c_{at} = c_{ab} = \frac{236{,}000 \times 5}{689} = 1710 \text{ psi}$$

$$f_c \quad c_{dt} + c_{at}$$

$$2200 > 133 + 1710$$

$$C = 0.85\,\frac{2200 - 133 - 1710}{133 + 1710 - 230} = 0.188$$

$$e_A = \frac{(1 - 0.188)\ 13.2}{5 + (0.188 \times 5)} = 1.81''$$

max e obtainable $= 5 - (2+1) = 2.00'' > 1.81''$ O.K.

Ordinates for $e=0$ (fig.9)

$$\text{Line 1.} \ -\frac{1}{(133+230)A} = -\frac{2.76}{1000A}$$

$$\text{Line 2.} \ +\frac{1}{(2200-133-1710)A} = +\frac{2.80}{1000A}$$

$$\text{Line 3.} \ +\frac{1}{(2200+133)A} = +\frac{0.43}{1000A}$$

$$\text{Line 4.} \ +\frac{0.85}{(133+1710-230)A} = +\frac{0.53}{1000A}$$

from fig. 9 $\quad \dfrac{1000A}{P_i} = 0.94 \qquad \underline{P_i} = \dfrac{1000 \times 52}{0.94} = \underline{56{,}000^{\#}}$ ✓

or using formula:

$$\underline{P_i} = \frac{(c_{db} + c_{ab} - f_t)\,A}{\eta\left(1 + \frac{e y_2}{r^2}\right)} = \frac{(133 + 1710 - 230)\,52}{0.85\left(1 + \frac{2}{2.64}\right)} = \underline{56{,}000^{\#}} ✓$$

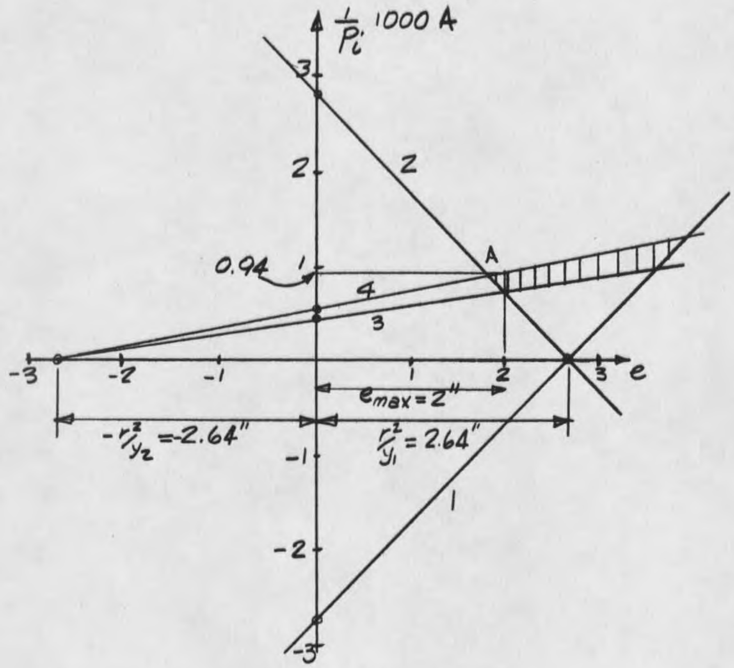


fig. 9.
The acceptable values
for P_i & e

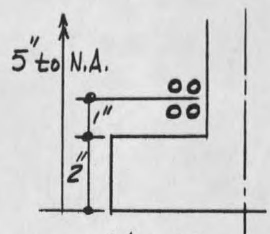


fig. 10.
Arrangement of wires

$$A_s = \frac{56{,}000}{155{,}000} = 0.361^{\square''}$$

Use 8 - $\phi = 0.276$ (7mm) wires 4 in each cable (fig. 10)

Investigation of the stresses in the beam.

top fibre $\left[\frac{P_i}{A}\left(1-\frac{ey_1}{r^2}\right)+C_{dt}+C_{at} = \frac{56{,}000}{52}\left(1-\frac{2}{2.64}\right)+133+1710 = 2101 < 2200 \right.$ O.K.

bottom fibre $\left[\frac{P_i}{A}\left(1+\frac{ey_2}{r^2}\right)-C_{db} = \frac{56{,}000}{52}\left(1+\frac{2}{2.64}\right)-133 = 1762 < 2200 \right.$ O.K.

$-\eta\frac{P_i}{A}\left(1+\frac{ey_2}{r^2}\right)+C_{db}+C_{ab} = -\frac{0.85\times 56{,}000}{52}\left(1+\frac{2}{2.64}\right)+133+1710 = 230 = 230$ O.K.

<u>Bending up cables</u>

We designed the section where the bending moment was the greatest. But since there is no moment at the ends we can bend up the cables.

Ordinates of the curves d & $d+a$ (fig. 12, fig. 13)

@ ℄ top fibre, compressive stress $= \frac{M_d y_1}{I} = \frac{18{,}300 \times 5}{689} = \underline{133 \text{ psi}}$ (d)

top fibre, compressive stress $= \frac{M_{d+a} y_1}{I} = \frac{254{,}300 \times 5}{689} = \underline{1843 \text{ psi}}$ (d+a)

Because the section is symmetrical the tensile stresses in the bottom fibres are equal to the compressive stresses in the top fibres

@ quarter point of span

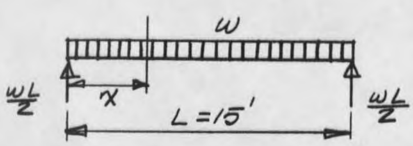


fig. 11.
Load & reactions

from fig. 11

$$M_x = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{wx}{2}(L-x)$$

$w_d = 54.1$ #/, $w_{d+a} = 754.1$ #/

when $x = 3.75'$

$$M_d = \frac{54.1(3.75)\times 12}{2}(15-3.75) = 13{,}700^{\text{in-lb}}$$

$$M_{d+a} = \frac{754.1(3.75)12}{2}(15-3.75) = 191{,}000^{\text{in-lb}}$$

top fibre, comp. stress $= \frac{13{,}700 \times 5}{689} = \underline{99.5 \text{ psi} \quad (d \text{ curve})}$

top fibre, comp stress $= \frac{191{,}000 \times 5}{689} = \underline{1386.0 \text{ psi} \quad (d+a \text{ curve})}$

Ordinates of the stress curves due to P_i & e (fig. 12 and fig. 13)

$$A_i B_i \sim \frac{P_i}{A} = \frac{56{,}000}{52} = 1077 \text{ psi}$$

$$A_u B_u \sim \frac{\eta P_i}{A} = 0.85 \times 1077 = 917 \text{ psi}$$

$$B_i C_i \sim \frac{P_i e y_2}{Ar^2} = \frac{56{,}000 \times 2 \times 5}{52 \times 13.2} = 816 \text{ psi}$$

$$B_u C_u \sim \eta \frac{P_i e y_2}{Ar^2} = 0.85 \times 816 = 695 \text{ psi}$$

$C_i \sim 1077 + 816 = 1893$ psi, $C_u \sim 917 + 695 = 1613$ psi

$1843 - 1613 = 230$ psi which is permissible tensile stress. O.K.

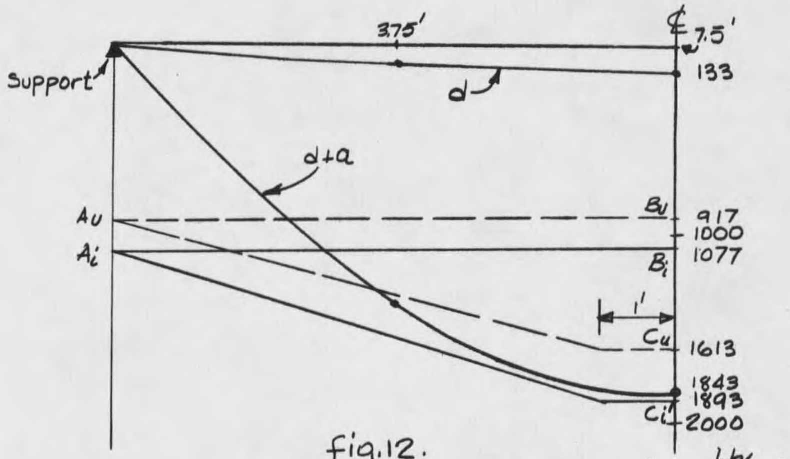


fig.12.
Stresses in the bottom fibre

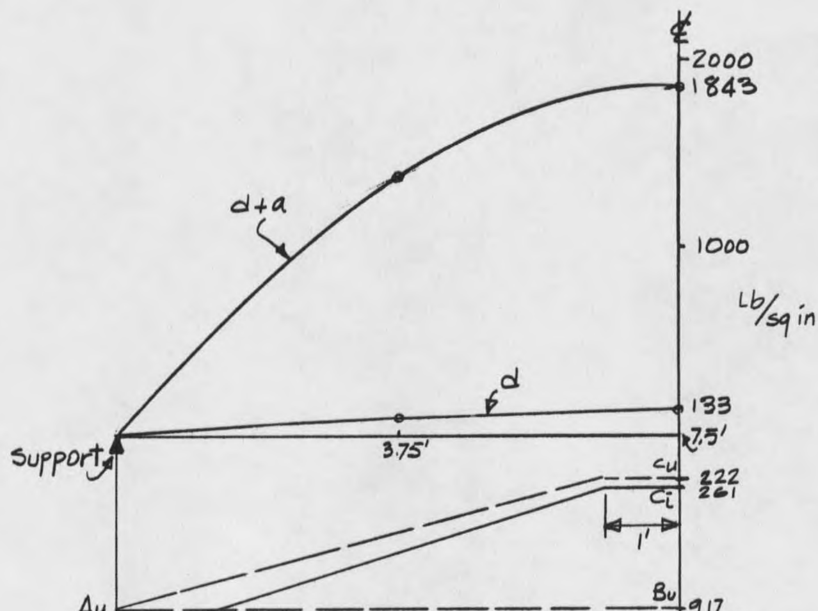


fig.13.
Stresses in the top fibre

Curve A_uC_u is above the curve (d+a) (fig 12). Max. tensile stress which will occur will be 230 psi which is permissible. There will be no tensile stress in top fibers. It is necessary to deviate the cable at a point 1' from mid span. In this case also two special blocks, the centers of which are 1' from mid span, are provided and each has two steel bars projecting from the web as is shown in fig.14

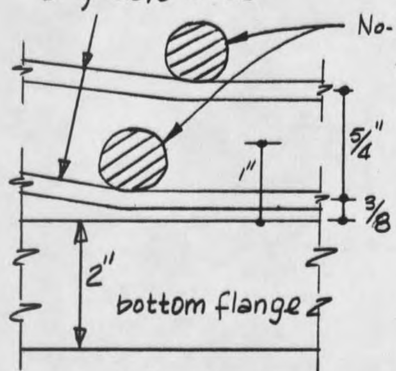
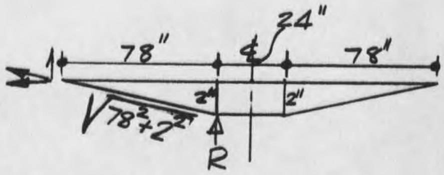
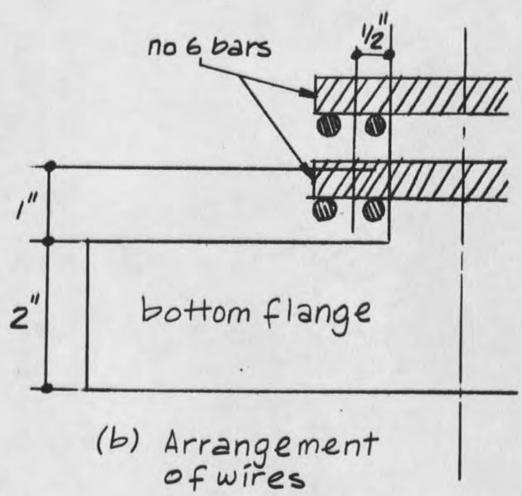


fig.14.
Deviation of cables.



(a) The force R



(b) Arrangement of wires

fig.15.

As shown in fig 15 (a) and (b) each pair of bars must resist as cantilevers the force R, which for each of the two cables is:

$$R = \frac{56{,}000 \times 2}{2\sqrt{78^2+2^2}} = 717^{\#}$$

Assume two no 6 bars are used, the stresses due to bending are:

$$= \frac{717 \times 0.5 \times 32}{2 \times \pi \times 0.75^3} = 4350 \text{ psi}$$

The two bars which bear the reaction from the cables on both sides of the web, exert an average pressure on the concrete of

$$\frac{2 \times 717}{2 \times 2 \times 0.75} = 478 \text{ psi}$$

Shearing stresses

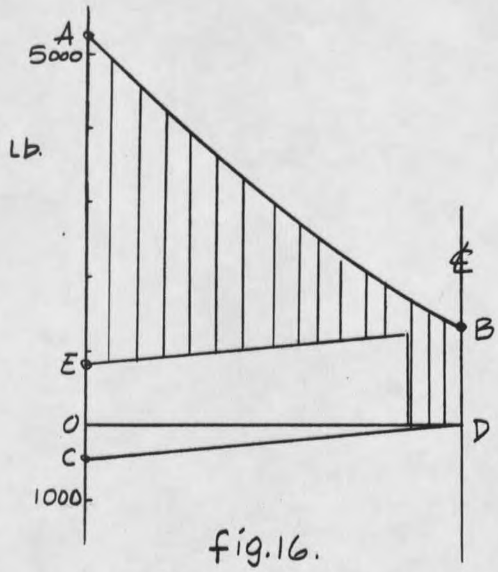


fig. 16.
shearing stresses

Ordinates of fig. 16

$$OA = \frac{700 \times 15}{2} = 5250^{\#}$$

$$DB = \frac{5250}{4} = 1313^{\#}$$

$$OC = \frac{54.1 \times 15}{2} = 406^{\#}$$

$$CE = 2 \times 717 \times 0.85 = 1219^{\#}$$

$$V_{AE} = 5250 + 406 - 1219 = 4437^{\#}$$

$$Q_m = 10 \times 2 \times 4 + 3 \times 2 \times 1.5 = 89 \text{ in}^3$$

$$v = \frac{V_{AE}\, Q_m}{b\, I} = \frac{4437 \times 89}{2 \times 689} = 286 \text{ psi}$$

$$c_n = \frac{P_i}{A} = \frac{56{,}000}{52} = 1078 \text{ psi}$$

$$P_t = \sqrt{v^2 + \frac{c_n^2}{4}} - \frac{c_n}{2} = \sqrt{(286)^2 + \frac{(1078)^2}{4}} - \frac{1078}{2} = \underline{71 \text{ psi} < 230 \text{ psi}}$$

<u>O.K.</u>

C. Design of Secondary Girders

Secondary girders will be designed as simply-supported girders length of 40'. End secondary girders are carrying only half of the load and should be designed seperately. But for simplicity of design they will be assumed to carry the same load as the rest of the secondary girders.

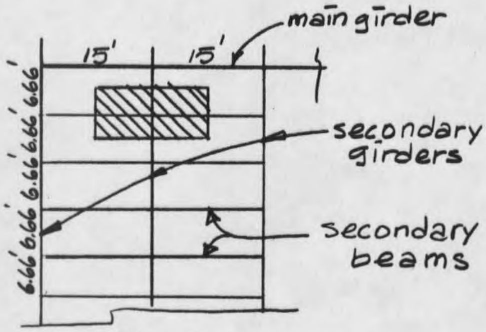


fig. 17.
Load distribution

one secondary beam carries the following wts. (refer to fig. 17)

wt of the beam: $54.1 \times 15 = 812^{\#}$

total load on beam: $105 \times 15 \times 6.66 = 10500^{\#}$

$11{,}312^{\#}$

This load can be assumed to be uniformly distributed along the secondary girder.

$$w_a = \frac{11{,}312}{6.66} = 1700\ \#/'$$

$$M_a = \frac{1700 \times 40^2 \times 12}{8} = 4{,}080{,}000 \text{ in-lb}$$

assume symmetrical section is used

$$\frac{M_a}{0.775 f_c + f_t} = \frac{4{,}080{,}000}{0.775 \times 2200 + 230} = 2110 \text{ in}^3 \qquad \frac{I}{y_2} = \frac{I}{y_1} \geqslant 2110 \text{ in}^3$$

First trial section

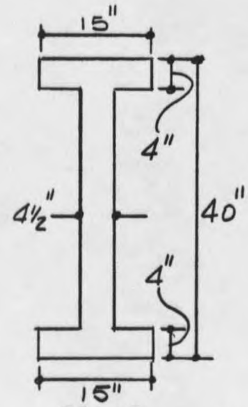


fig. 18.
first trial section

properties of the cross section in fig 18

$A = 264\ \square''$

$y_1 = y_2 = 20''$

$I = 51{,}400 \text{ in}^4$

$r^2 = 193.5 \text{ in}^2$

$$\frac{I}{y_1} = \frac{I}{y_2} = \frac{51{,}400}{20} = 2570 > 2110 \text{ O.K.}$$

$$\frac{r^2}{y_1} = \frac{r^2}{y_2} = \frac{193.5}{20} = 9.67 \text{ in}$$

Initial stretching force & wires

$w_d = \frac{150}{144} 264 = 275 \#/_1$, $M_d = \frac{275 \times 40^2 \times 12}{8} = 660{,}000$ in-lb

$c_{dt} = c_{db} = \frac{660{,}000 \times 20}{51{,}400} = 257$ psi

$c_{at} = c_{ab} = \frac{4{,}080{,}000 \times 20}{51{,}400} = 1585$ psi

$f_c \quad c_{dt} + c_{at}$

$2200 > 257 + 1585$

max e obtainable (assumed) $= 20.00 - (4 + 3.08) = 12.92''$

ordinates for e=0 (fig.19)

Line 1, $- \frac{1}{(257+230)A} = - \frac{2.05}{1000\,A}$

Line 2, $+ \frac{1}{(2200-257-1585)A} = + \frac{2.79}{1000\,A}$

Line 3, $+ \frac{1}{(2200+257)A} = + \frac{0.407}{1000\,A}$

Line 4, $+ \frac{0.85}{(257+1585-230)A} = + \frac{0.527}{1000\,A}$

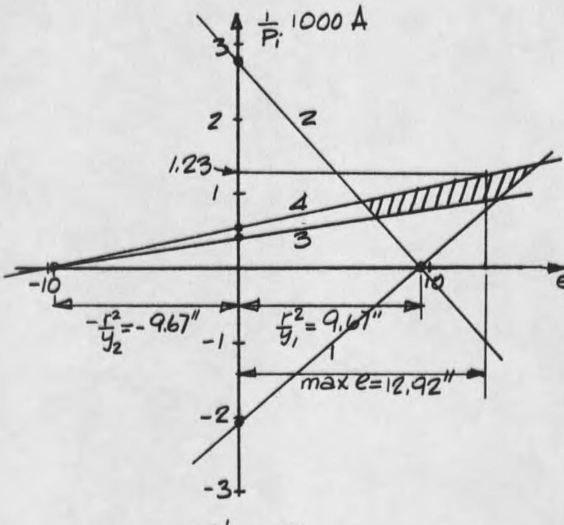


fig.19.
The acceptable values for P_i & e

from fig. 19

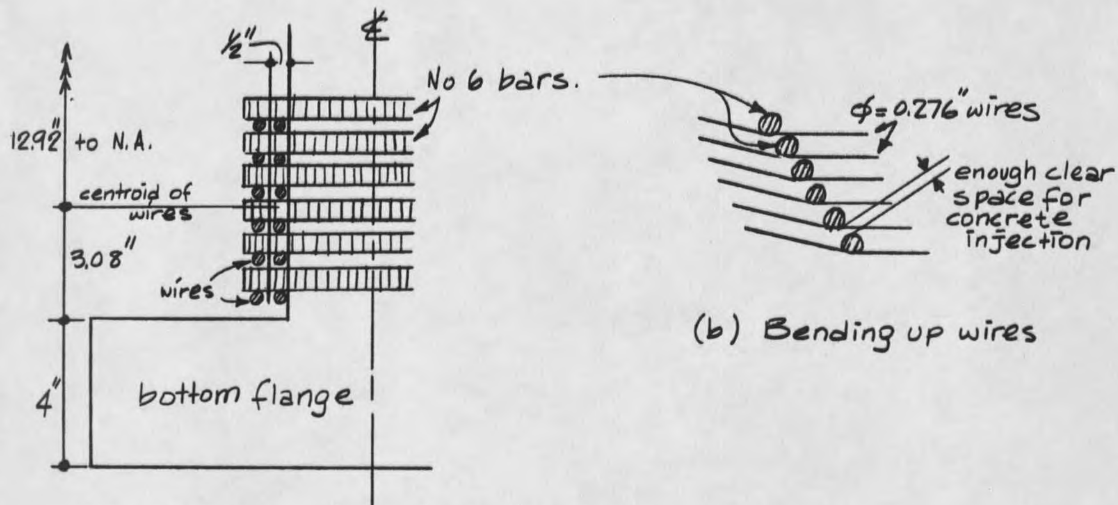
$\frac{1}{P_i} 1000\,A = 1.23$

$P_i = \frac{1000\,(264)}{1.23}$

$= \underline{214{,}300^{\#}}$

$A_s = \frac{214{,}300}{155{,}000} = 1.385 \,\square''$

Use 24 $\phi = 0.276''$ wires, 12 in each cable (fig 20a)



(a) Arrangement of wires & bars

(b) Bending up wires

fig. 20.

The stresses in the section

top fibre

$$\frac{P_i}{A}\left(\frac{ey_1}{r^2}-1\right)-c_{dt} = \frac{214{,}300}{264}\left(\frac{12.92}{9.67}-1\right)-257 = \underline{15 < 230 \text{ O.K.}}$$

$$-\eta\frac{P_i}{A}\left(\frac{ey_1}{r^2}-1\right)+c_{dt}+c_{at} = -0.85\,\frac{214{,}300}{264}\left(\frac{12.92}{9.67}-1\right)+257+1585 = \underline{1610 < 2200 \text{ O.K.}}$$

bottom fibre

$$\frac{P_i}{A}\left(1+\frac{ey_2}{r^2}\right)-c_{db} = \frac{214{,}300}{264}\left(1+\frac{12.92}{9.67}\right)-257 = \underline{1641 < 2200 \text{ OK}}$$

$$-\eta\frac{P_i}{A}\left(1+\frac{ey_2}{r^2}\right)+c_{db}+c_{ab} = -0.85\,\frac{214{,}300}{264}\left(1+\frac{12.92}{9.67}\right)+257+1585 = \underline{229 < 230 \text{ O.K.}}$$

Bending up cables

Ordinates of the curves d & d+a (fig.22, fig.23)

@ ℄

Tensile stresses in the bottom fibres equal compressive stresses in the top fibres, due to M_d & M_{d+a}

$$\frac{M_d\, y_1}{I} = \frac{660{,}000 \times 20}{51{,}400} = 257 \text{ psi} \quad \text{(d curve)}$$

$$\frac{M_{d+a}\, y_1}{I} = \frac{4{,}740{,}000 \times 20}{51{,}400} = 1842 \text{ psi} \quad \text{(d+a curve)}$$

@ quarter point of span

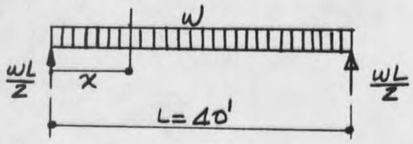


fig. 21.
Load & reactions

From fig. 21

$$M_x = \frac{wx}{2}(L-x)$$

$w_d = 275$ #/, $w_a = 1700$ #/

when $x = 10'$

$$M_d = \frac{275 \times 10 \times 12}{2}(40-10)$$
$$= 495{,}000 \text{ in-lb}$$

$$M_{d+a} = \frac{1975 \times 10 \times 12}{2}(40-10) = 3{,}558{,}000 \text{ in-lb}$$

Stresses in bottom and top fibres

$$\frac{495{,}000 \times 20}{51{,}400} = 192.5 \text{ psi} \quad \text{(d curve)}$$

$$\frac{3{,}558000 \times 20}{51{,}400} = 1382 \text{ psi} \quad \text{(d+a curve)}$$

Ordinates of the stress curves in fig. 22 and in fig. 23

$A_i B_i$ — $\frac{214{,}300}{264} = 813$ psi

$A_u B_u$ — $0.85 \times 813 = 691$ psi

$B_i C_i$ — $\frac{214{,}300 \times 12.92}{264 \times 9.67} = 1085$ psi

$B_u C_u$ — $0.85 \times 1085 = 922$ psi

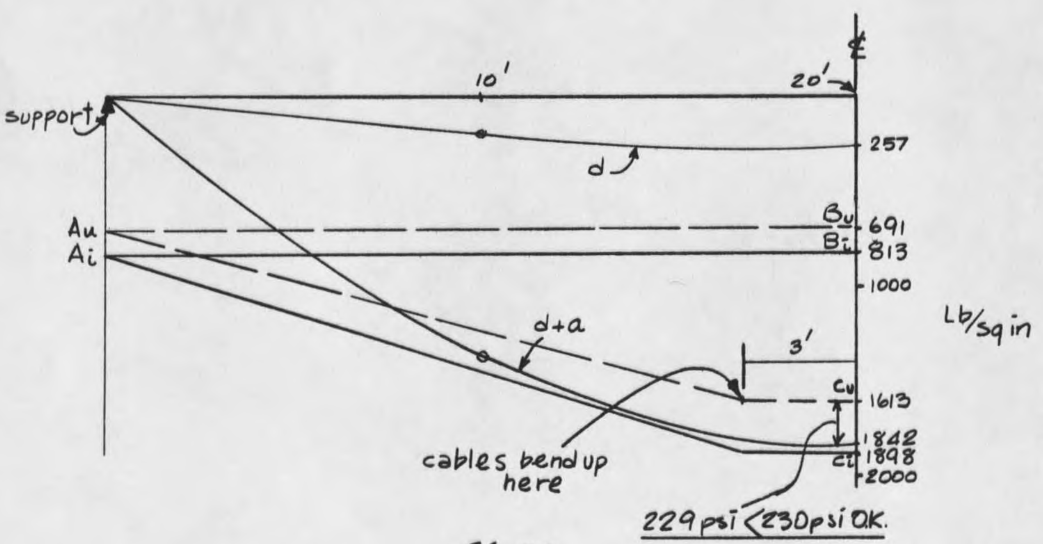


fig. 22.
stresses in the bottom fibre

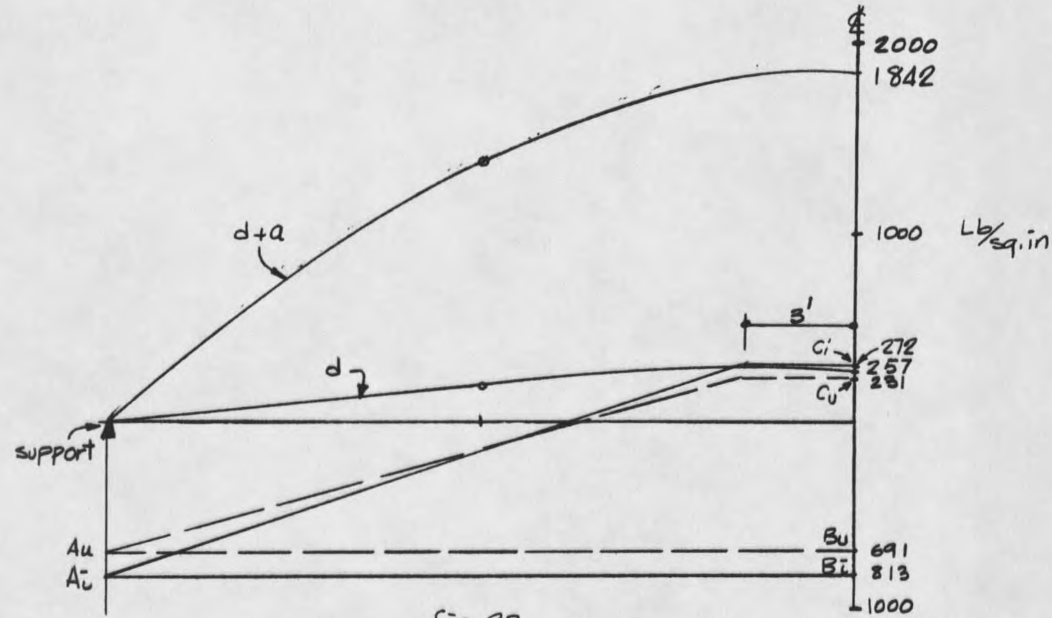


fig. 23.
stresses in the top fibre

Curve AuCu is above the curve d+a (fig.22). Max. tensile stress which will occur will be 229 psi which is permissible. There will be very small tensile stress in top fibers (fig.23). Deviation point of cables is 3' from mid span.

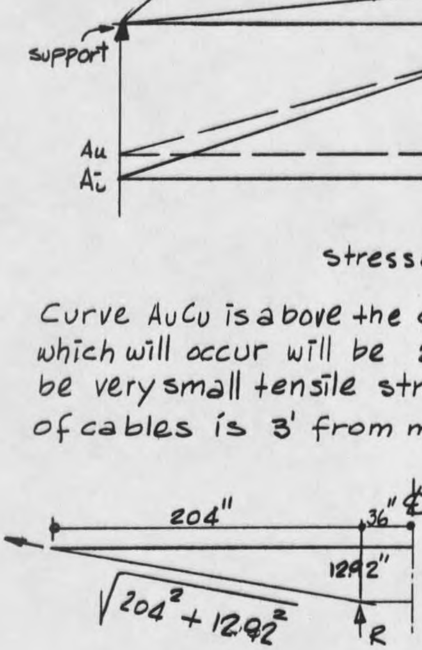


fig. 24.
the force R

from fig.24
The vertical component R of the initial prestress in one of the cables is:

$$R = \frac{214{,}300 \times 12.92}{2\sqrt{204^2+12.92^2}} = 6780^{\#}$$

$\frac{6780}{6} = 1129^{\#}$ for each layer of 2 wires (fig.20 (a))

$M = 1129 \times 0.5 = 564.5$ in.-lb in each No 6 bars

$$\text{stress} = \frac{564.5 \times 32}{\pi \times 0.75^3} = \underline{13{,}690 \text{ psi} < 20{,}000 \text{ psi O.K.}}$$

the average pressure on the concrete is:

$$\frac{2 \times 1129}{4.5 \times 0.75} = \underline{669 \text{ psi}}$$ this is smaller than permissible O.K.

Shearing stresses

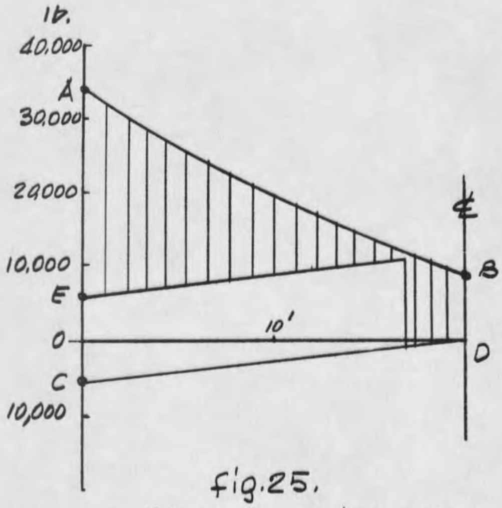


fig.25.
Shearing stresses

Ordinates of fig.25

$$OA = \frac{1700 \times 40}{2} = 34{,}000^{\#}$$

$$DB = \frac{34{,}000}{4} = 8{,}500^{\#}$$

$$OC = \frac{275 \times 40}{2} = 5{,}500^{\#}$$

$$CE = 2 \times 6780 \times 0.85 = 11{,}527^{\#}$$

$$V_{AE} = 34{,}000 + 5500 - 11{,}527 = 27{,}973^{\#}$$

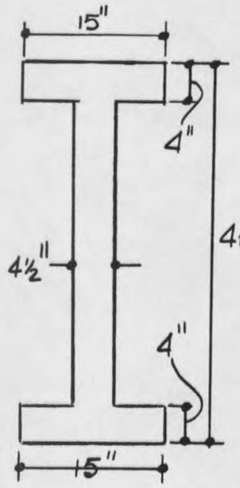
$$Q_m = 15 \times 4 \times 18 + 4.5 \times 16 \times 8 = 1656\ \text{in}^3$$

$$v = \frac{27{,}973 \times 1656}{4.5 \times 51{,}400} = 200\ \text{psi}$$

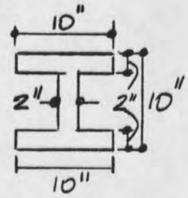
$$c_n = \frac{P_i}{A} = \frac{214{,}300}{264} = 813\ \text{psi}$$

$$P_t = \sqrt{v^2 + \frac{c_n^2}{4}} - \frac{c_n}{2} = \sqrt{200^2 + \frac{813^2}{4}} - \frac{813}{2} = \underline{46\ \text{psi} < 230\text{psi OK.}}$$

D. Design of the Ends of Secondary Beams

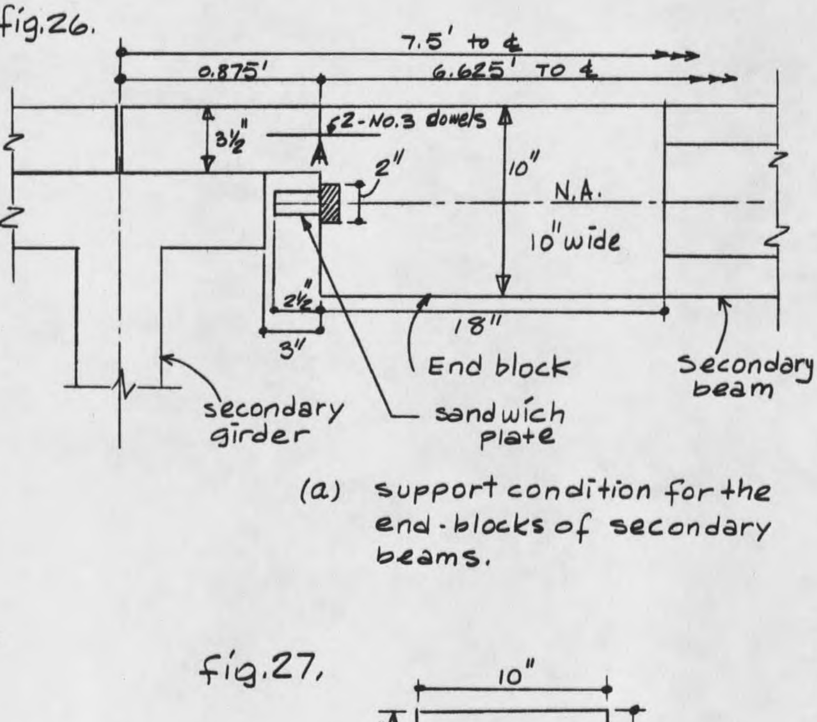


(b) secondary girder section

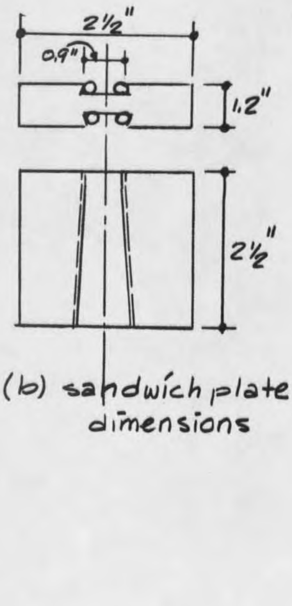


(a) Secondary beam section

fig.26.



(a) support condition for the end-blocks of secondary beams.



(b) sandwich plate dimensions

fig.27.

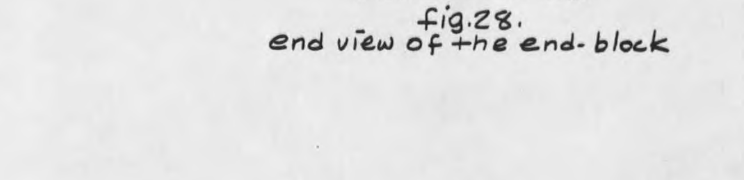


fig.28.
end view of the end-block

depth required @ section A

$w_{d+a} = 754.1$ #/'   direct shear @ section A $= 754.1 \times 6.625 = 5000$#

allowable unit shearing stress with no web reinforcement $= 0.03 f'_c$
$= 0.03 \times 8800 = 264$ psi

$$\text{depth required} = \frac{5000}{264 \times 10} = 1.9''$$

Use depth = $3\frac{1}{2}''$, provide 2-No 3 dowels

bearing plates

unit bearing stress $= 1.75 f_c = 1.75 \times 2200 = 3850$ psi

$$\text{gross area req'd} = \frac{56{,}000}{3{,}850} + (2 \times 1.2 \times 0.9) = 16.71 \square''$$

with a horizontal dimension of 10" the vertical dimension of the plate will be 1.671". Use 2"x 10"

stresses in the end-block

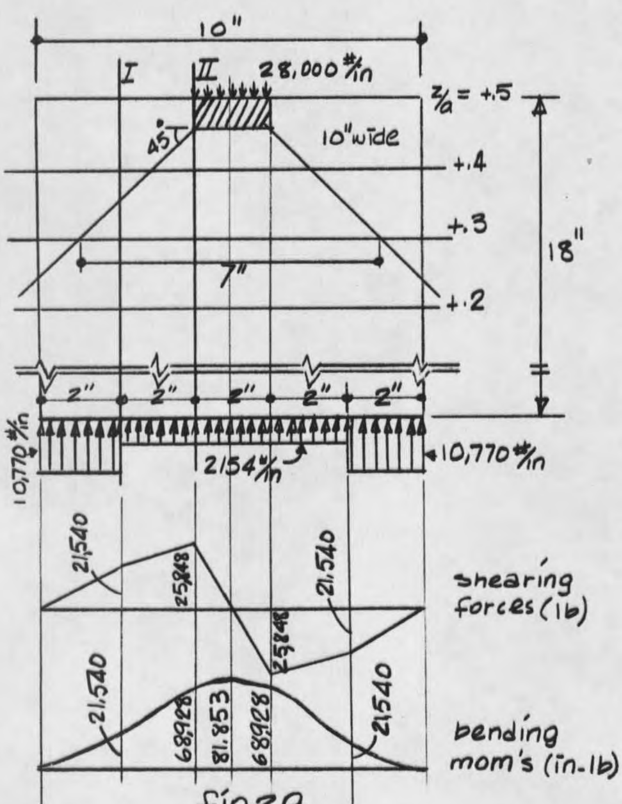


fig.29.
shears and moments at the end-block.

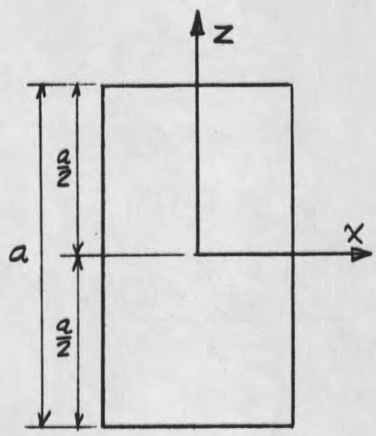


fig 30.
coordinate system

$$\frac{56{,}000}{2} = 28{,}000 \text{ \#/in}$$

$$\frac{56{,}000}{52} = 1{,}077 \text{ \#/}\square''$$

$$1077 \times 10 = 10{,}770 \text{ \#/in}$$

$$1077 \times 2 = 2{,}154 \text{ \#/in}$$

Table I

calculated stresses in end-block of the secondary beams (psi)

section	z/a	compressive stress C_x	bending stress C_z	Shearing stress V	P_t
I	+0.3	800	+21	306 +67	-129
	+0.2	560	-16	308 +67	-201
	+0.1	560	-36	259 +67	-180
	0.0	560	-42	187 +67	-135
	-0.1	560	-37	115 +67	-88
	-0.2	560	-27	57 +67	-52
II	+0.3	800	+68	368 +67	-135
	+0.2	560	-52	369 +67	-279
	+0.1	560	-115	310 +67	-283
	0.0	560	-133	224 +67	-239
	-0.1	560	-119	138 +67	-177
	-0.2	560	-86	68 +67	-113

calculations of the table I

effective width $= 10 - 2 = 8''$

from fig. 29.

when $z/a = +0.3$ $\quad C_x = \dfrac{56{,}000}{7 \times 10} = 800$ psi

for others $\quad C_x = \dfrac{56{,}000}{10 \times 10} = 560$ psi

$$C_z = K \frac{M}{ba^2} \qquad \text{where } K = 5\left(-1 + \frac{12z^2}{a^2} + \frac{16z^3}{a^3}\right) \qquad b = 8'', \; a = 18''$$

@ sec. I $\quad M = 21{,}540 \quad \dfrac{M}{ba^2} = 8.31$

@ sec II $\quad M = 68{,}928 \quad \dfrac{M}{ba^2} = 26.6$

z/a	K	M/ba^2 (I)	M/ba^2 (II)
+0.3	+2.560	× 8.31 = +21 psi	× 26.6 = +68
+0.2	-1.960	× " = -16	× " = -52
+0.1	-4.320	× " = -36	× " = -115
0.0	-5.000	× " = -42	× " = -133
-0.1	-4.480	× " = -37	× " = -119
-0.2	-3.240	× " = -27	× " = -86

$$V = K_1 \frac{V}{ba} \qquad \text{where } K_1 = 5\left(\frac{1}{4} + \frac{z}{a} - \frac{4z^3}{a^3} - \frac{4z^4}{a^4}\right)$$

@ sec. I $\quad V = 21{,}540 \quad \dfrac{V}{ba} = 149.5$

@ sec II $\quad V = 25{,}848 \quad \dfrac{V}{ba} = 179.5$

z/a	K_1	V/ba (I)	V/ba II
+0.3	2.048	× 149.5 = 306 psi	× 179.5 = 368 psi
+0.2	2.058	" = 308	= 369
+0.1	1.728	" = 259	= 310
0.0	1.250	" = 187	= 224
−0.1	0.768	" = 115	= 138
−0.2	0.378	" = 57	= 68

there are three shearing forces to be considered in addition to those taken into account above. these are due to:

DL −406#
LL −5250#

and to the slope of the cables: initially 1435# ultimately 1219#

this means that we have to consider either, 1435 − 406 = 1029#

or, (1219 − 406) − 5250 = 4437#

the greater of these forces develops a maximum shearing stress of:

$$\frac{1.5 \times 4437}{10 \times 10} = 67 \text{ psi}$$

$$P_t = \frac{C_x + C_z}{2} - \sqrt{v^2 + \left(\frac{C_x - C_z}{2}\right)^2}$$. ten. principal stresses are found by this formula.

It is seen that maximum tensile principal stress 283 psi exceeds the permissible value of 264 psi, reinforcement should be provided. If it be assumed that the stress 283 psi exists on every horizontal plane of half the length of the end-block, the corresponding total tensile force is: 0.5 × 283 × 18 × 8 = 20,400#

The total area of 18" by 8" is however separated into three areas by the holes for the cables, central area = 2", each side areas = 3"

The force corresponding to each of these areas is:

$20{,}400 \times \frac{2}{8} = 5100$# $20{,}400 \times 3/8 = 7650$#

The reinforcement required: <u>for central area</u> $\frac{5100}{20{,}000} = 0.255$ □"

<u>Use 3-No 3 Bars</u>

<u>for each side area</u> $\frac{7650}{20{,}000} = 0.383$ □"

<u>Use 4-No 3 Bars</u>

Some no 3 horizontal bars will also be provided at the same spacing as the vertical bars near cable holes.

E. Design of the Main Girder Frame.

Two main girders & three columns will be designed as rigid frames.

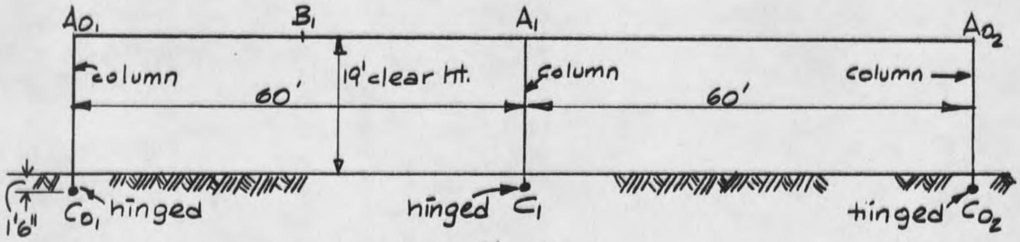


fig. 31.
The main girder frame

load on main girder

$1700 + 275 = 1975 \ \#/_1$ $\qquad \underline{w_a} = \dfrac{1975 \times 40}{15} = \underline{5270 \ \#/_1}$

First trial section

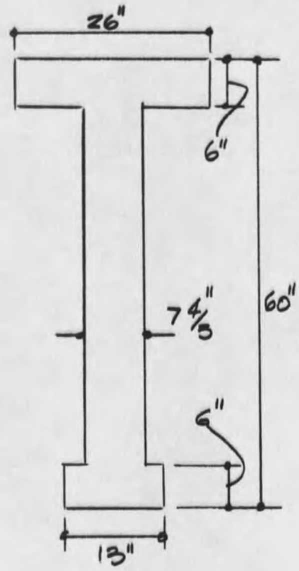


fig. 32.
first trial section

properties of the cross section in fig. 32

$A = 609 \ \square''$

$y_2 = 33.4''$

$y_1 = 26.6''$

$I_b = 241{,}300 \ in^4$

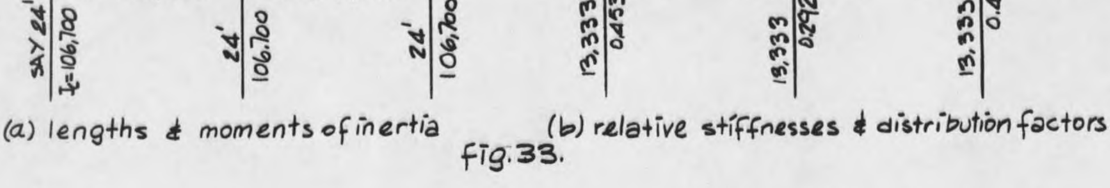
$r^2 = 397 \ in^2$

$\dfrac{r^2}{y_1} = 14.92''$, $\dfrac{r^2}{y_2} = 11.89''$

$w_d = \dfrac{150}{144}\ 609 = 635 \ \#/_1$

Assume $40'' \times 20''$ columns

$I_c = \frac{1}{12} bt^3 = \frac{1}{12} \times 20 \times 40^3 = 106{,}700 \ in^4$



(a) lengths & moments of inertia $\qquad$ (b) relative stiffnesses & distribution factors

fig. 33.

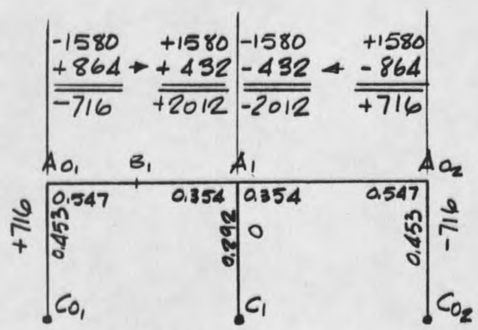
relative stiffnesses (from fig 33(a))

for columns : $3\,\frac{I_c}{L} = 3\,\frac{106,700}{24} = 13,333$

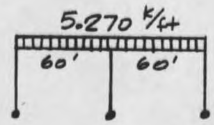
for girders : $4\,\frac{I_b}{L} = 4\,\frac{241,300}{60} = 16,080$

relative stiffnesses & distribution factor are shown in fig 33(b).

Moments due to additional load & DL.

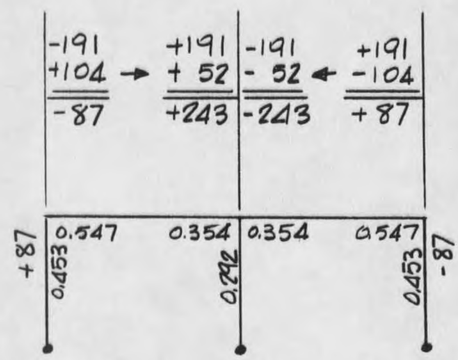


(b) moment distribution for additional load on the girder

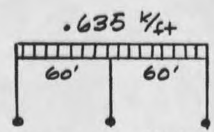


(a) loading for (b)

$$F.E.M. = \frac{wL^2}{12} = \frac{5.270 \times 60^2}{12} = 1580^{1\text{-}k}$$



(d) moment distribution for DL of the girder



(c) loading for (d)

$$F.E.M. = \frac{.635 \times 60^2}{12} = 191^{1\text{-}k}$$

fig. 34.

reactions due to moment & uniform load for additional load & D.L. on the girder.

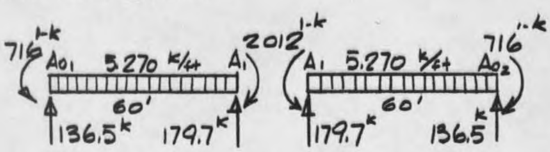


fig. 35.
reactions for additional load

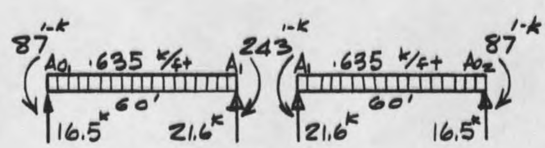


fig. 36.
reactions for D.L.

Now we have toconsider the condition of which only one of the spans loaded with L.L. and both spans loaded with D.L (D.L. of add. load)

L.L. on structure = 55 #/□'

L.L. on main girder = 55 × 40 = 2200 #/'

D.L on main girder = 5270 − 2200 = 3070 #/'

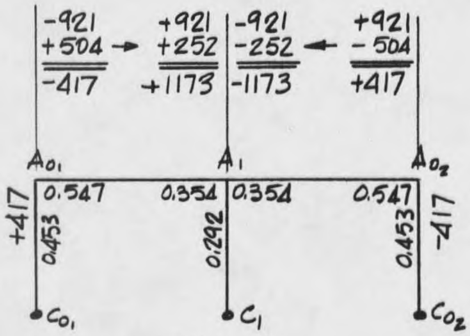


fig.37.
moment distribution for D.L. on the girder.

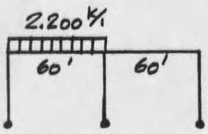
$$F.E.M = \frac{3.070 \times 60^2}{12} = 921^{\,k\text{-}k}$$

Table II
distribution & carry-over factors

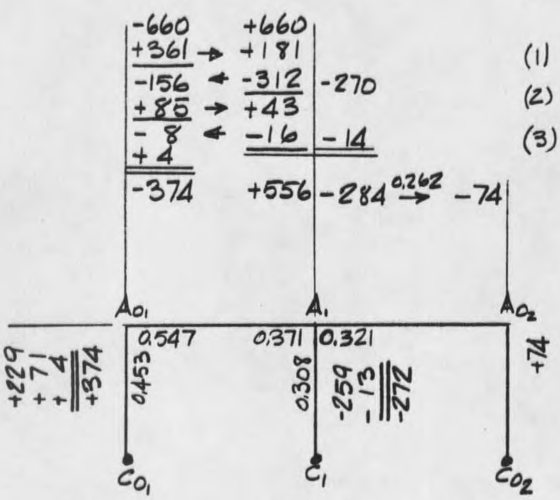
member	F.E.C.[1]	I/L	F.E.C. × I/L	D.F[2]	C.O.[3]
$A_{01}C_{01}$	4-1	4.450	13.333	0.453	0
$A_{01}A_1$	4	4.025	16.090	0.547	0.500
			Σ=29.413		
A_1A_{01}	4	4.025	16.100	0.371	0.500
A_1C_1	4-1	4.450	13.340	0.308	0
A_1A_{02}	4-0.547	4.025	13.900	0.321	0.262

Σ=43.340

(1) F.E.C. = far end condition
(2) D.F. = distribution factor
(3) C.O. = carry-over factor



(a) loading for(b)



(b) moment distribution for L.L. on the girder.

fig. 38.

$$F.E.M. = \frac{2.200 \times 60^2}{12} = 660^{\,1\text{-}k}$$

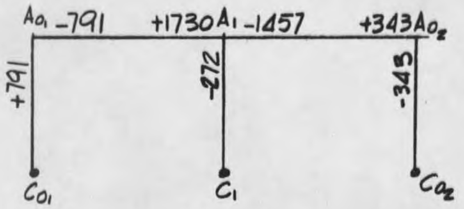


fig.39.
final moments for L.L. on one span & D.L. on both spans.

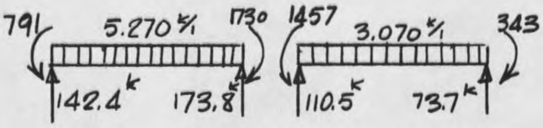


fig.40.
reactions on the girder for LL on one span & D.L. on both spans.

before continuing the girder design further, the columns must be checked.

Design of column A_1C_1

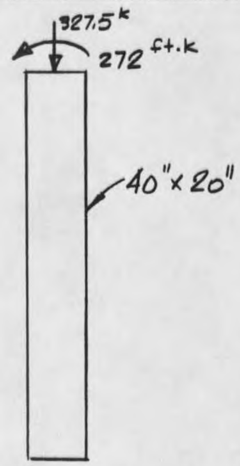


fig.41.
Axial load & mom. on column A_1C_1

given:

$f_c = 2200$ psi

$f_s = 20,000$ psi

$n = \frac{30,000}{f'_c} = \frac{30,000}{8800} = 3.41$

$N = 327.5^k$

$M = 272^{ft\cdot k}$

equivalent axial load $P = N\left[1 + \frac{CDe}{t}\right]$

from fig.41 $e = \frac{M}{N} = \frac{272 \times 12}{327.5} = 9.97''$

$\frac{e}{t} = \frac{9.97}{40} = 0.249 < 1.00$ O.K.

$f_a = 0.8\left[\frac{0.225\,(8800) + 20000\,(.01)}{1 + (3.41 - 1)(.01)}\right] = 1703$

$P_g = .01$ (assumed)

$C = \frac{f_a}{0.45 f'_c} = \frac{1703}{0.45 \times 8800} = 0.431$ ass. $D = 5.5$

$$P = 327.5\left[1 + \frac{0.431 \times 5.5 \times 9.97}{40}\right] = 521^k$$

max. allowable load $= P = 0.8\ A_g\left[0.225 f'_c + f_s P_g\right]$

$= 0.8\,(40 \times 20)\left[0.225 \times 8800 + 20,000 \times 0.01\right]$

$= \underline{1395^k > 521^k \text{ O.K.}}$

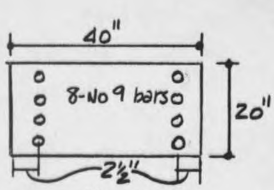


fig.42.
column cross sect.

Design of column A_oC_o

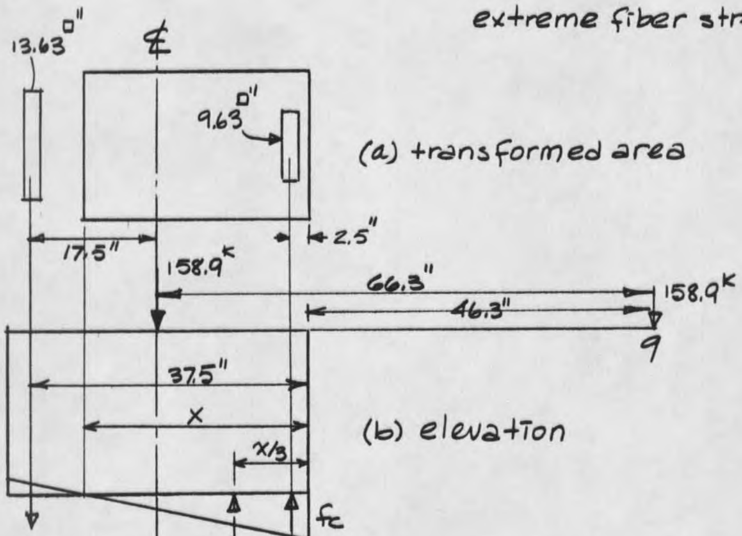
for uniformity the same section as used in col. A_1C_1 will be eccepted and checked.

$N = 158.9^k$
$M = 878$ ft.-k

$$e = \frac{M}{N} = \frac{878 \times 12}{158.9} = 66.3'' > 40''$$

design shall be made for a cracked section

extreme fiber stress in comp. = $0.45 f'_c$
= 3960 psi



(a) transformed area

(b) elevation

fig. 43.

from fig. 43.(b)

$$\Sigma M_q = 0 \qquad \frac{f_c}{2}(20x)\left(46.3+\frac{x}{3}\right)+f_c\left(\frac{x-2.5}{x}\right)9.63\,(46.3+2.5)$$

$$-f_c\left(\frac{37.5-x}{x}\right)13.63\,(46.3+37.5)-158.9\times 66.3 = 0$$

which gives $3.33\, f_c x^3 + 463\, f_c x^2 + 1613\, f_c x - 10{,}550\, x - 43{,}975\, f_c = 0$

$$\Sigma V = 0 \qquad \frac{f_c}{2}(20x)+f_c\left(\frac{x-2.5}{x}\right)9.63 - f_c\left(\frac{37.5-x}{x}\right)13.63 - 158.9 = 0$$

which gives $f_c = \dfrac{158.9\, x}{(10x^2 + 23.26x - 536.1)}$

if we put this value of f_c in equation above, we get

$$x^3 - 61x^2 + 20x - 2510 = 0$$

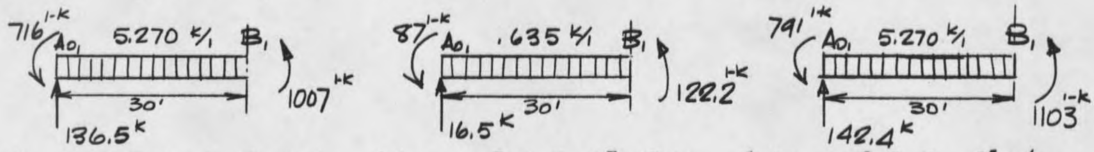
$x \cong 61.7''$ which means there is no tension in concrete.

$f_c = 258$ psi < 3960 psi O.K.

steel does not carry any load.

first trial section for girder (continued)

Moments @ midspan B_1



(a) mom. @ B_1 for fig. 35 (b) mom. @ B_1 for fig. 36 (c) mom @ B_1 for fig. 40

fig. 44.

max. moments @ B_1 for D.L. 122.2^{1-k}, for additional load 1103^{1-k}

The critical bending moments in main girder are:

@	B_1	at the time of prestressing	122,200 ft-lb	[fig. 44 (b)]
		Later is added	1,103,000 ft-lb	[fig. 44 (c)]
@	A_1	at the time of prestressing	243,000 ft-lb	[fig. 36]
		later is added	2,012,000 ft-lb	[fig. 35]

fibre stresses.

@ B_1

$$c_{dt} = \frac{122{,}200 \times 26.6 \times 12}{241{,}300} = 162$$

$$c_{db} = \frac{122{,}200 \times 33.4 \times 12}{241{,}300} = 203$$

$$c_{at} = \frac{1{,}103{,}000 \times 26.6 \times 12}{241{,}300} = 1460$$

$$c_{ab} = \frac{1{,}103{,}000 \times 33.4 \times 12}{241{,}300} = 1831$$

$$\underset{c}{2200} > \underset{c_{dt} + c_{at}}{162 + 1460}$$

@ A_1

$$c_{dt} = \frac{243{,}000 \times 26.6 \times 12}{241{,}300} = 321$$

$$c_{db} = \frac{243{,}000 \times 33.4 \times 12}{241{,}300} = 403$$

$$c_{at} = \frac{2{,}012{,}000 \times 26.6 \times 12}{241{,}300} = 2660$$

$$c_{ab} = \frac{2{,}012{,}000 \times 33.4 \times 12}{241{,}300} = 3340$$

$$\underset{c}{2200} < \underset{c_{dt} + c_{at}}{321 + 2660}$$

ordinates for $e=0$

@ B_1

Line 1. $$-\frac{1}{(c_{dt}+f_t)A} = -\frac{1}{(162+230)A} = -\frac{2.55}{1000A}$$

line 2. $$+\frac{1}{(f_c-c_{dt}-c_{at})A} = +\frac{1}{(2200-162-1460)A} = +\frac{1.73}{1000A}$$

Line.3. $+\dfrac{1}{(f_c + C_{db})A} = +\dfrac{1}{(2200 + 203)A} = +\dfrac{0.416}{1000A}$

Line 4. $+\dfrac{\eta}{(C_{db}+C_{ab}-f_t)A} = +\dfrac{0.85}{(203+1831-230)A} = +\dfrac{0.471}{1000A}$

@ A_1 Line 1. $-\dfrac{1}{(C_{dt}+f_t)A} = -\dfrac{1}{(321+230)A} = -\dfrac{1.815}{1000A}$

Line 2. $-\dfrac{\eta}{(C_{dt}+C_{at}-f_c)A} = -\dfrac{0.85}{(321+2660-2200)A} = -\dfrac{1.087}{1000A}$

Line 3. $+\dfrac{1}{(f_c + C_{db})A} = +\dfrac{1}{(2200+403)A} = +\dfrac{0.384}{1000A}$

Line 4. $+\dfrac{\eta}{(C_{db}+C_{ab}-f_t)A} = +\dfrac{0.85}{(403+3340-230)A} = +\dfrac{0.242}{1000A}$

we can stop the design at this point, because line 3 is above line 4 no solution is possible for e & P_i together. @ A_1

Second trial section for girder

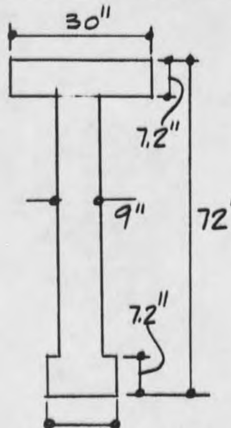


fig.45.
second trial section

properties of the cross section in fig.45

$A = 842\ \square''$
$y_2 = 40.1''$
$y_1 = 31.9''$
$I_b = 482{,}000\ in^4$
$r^2 = 573\ in^2$

$\dfrac{r^2}{y_1} = \dfrac{573}{31.9} = 17.95\ in$, $\dfrac{r^2}{y_2} = \dfrac{573}{40.1} = 14.29\ in$

$w_d = \dfrac{150}{144} \times 842 = 877\ \#/'$

increase the column dimension 20" to 26" in order to throw some more moment to column

$I_c = \dfrac{1}{12} \times 26 \times 40^3 = 138{,}700\ in^4$

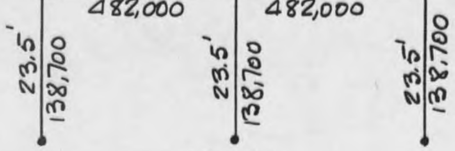


fig.46.
Lengths & moments of inertia

from fig.46

$3\ \dfrac{I_c}{L} = 3\ \dfrac{138{,}700}{23.5} = 17{,}700$

$4\ \dfrac{I_b}{L} = 4\ \dfrac{482{,}000}{60'} = 32{,}100$

$+$ 49,800

$+$ 32,100

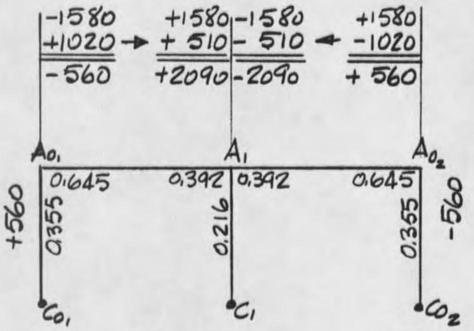
81,900

distribution factors

A_0, C_{0_1} $\frac{17,700}{49,800} = 0.355$ $A_1 C_1$ $\frac{17,700}{81,900} = 0.216$

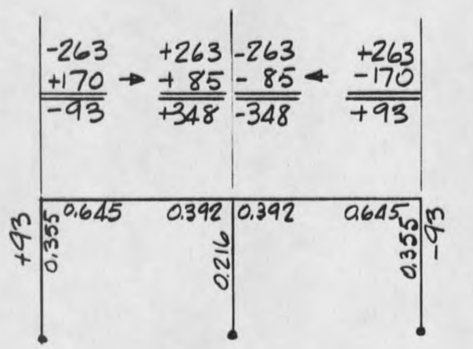
A_0, A_1 $\frac{32,100}{49,800} = 0.645$ $A_1 A_{0_1}, A_1 A_{0_2}$ $\frac{32,100}{81,900} = 0.392$

Moments due to additional load ¢ DL.



F.E.M. = 1580 ft-k

(a) mom. distribution for additional load on the girder.

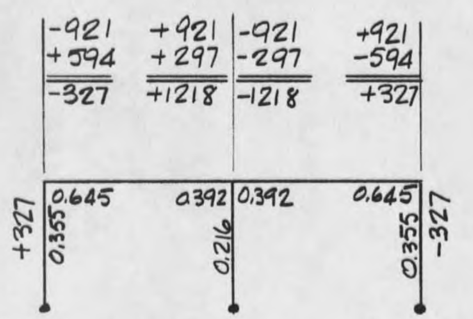


$$F.E.M = \frac{877 \times 60^2}{12} = 263$$

(b) mom. distribution for D.L. of the girder.

fig. 47.

the condition of which only one of the spans loaded with L.L. and both spans loaded with D.L. on the girder:



F.E.M. = 921 ft-k

fig. 48.

mom. distribution for D.L. on the girder.

Table III
Distribution & carry-over factors

member	F.E.C.[1]	I/L	FEC × I/L	D.F.[2]	C.O.[3]
A_1Ao_1	4	8030	32,120	0.419	0.500
A_1C_1	4-1	5900	17,700	0.231	0
A_1Ao_2	4-0.645	8030	26,900	0.350	0.212

Σ=76,720

(1) F.E.C. = far end condition
(2) D.F. = distribution factor
(3) C.O. = carry-over factor

F.E.M. = 660^{1-k}

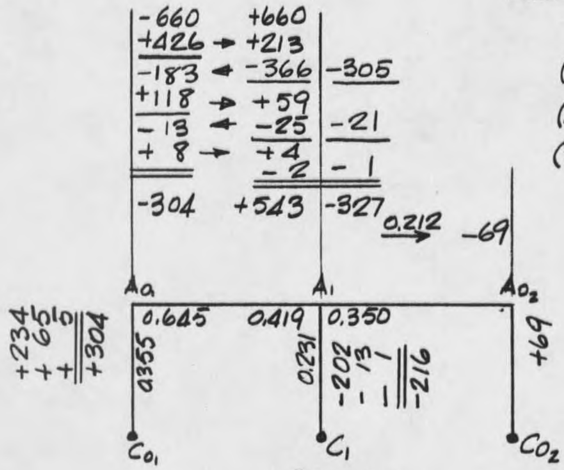


fig. 49.
mom. distribution for L.L. on the girder.

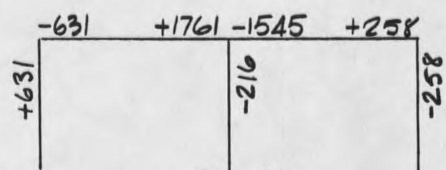


fig. 50.
final mom. for LL on one span & D.L. on both spans

reactions.

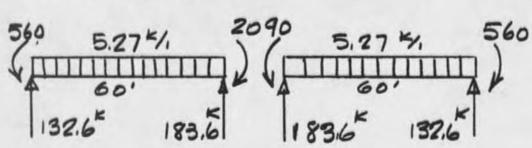


fig. 51.
reactions for additional load

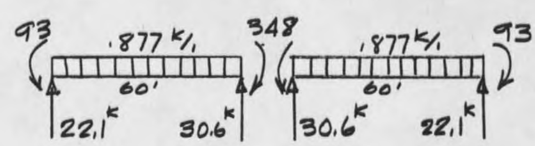


fig. 52.
reactions for D.L.

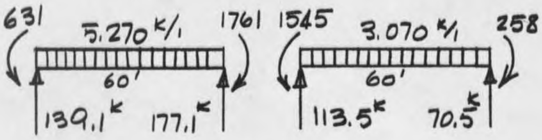
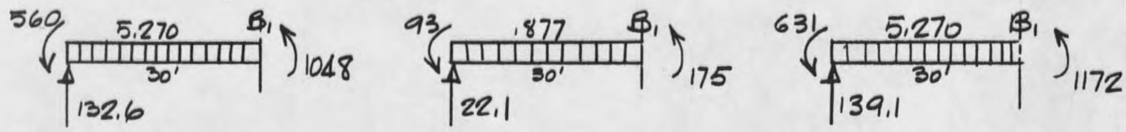


fig. 53.
reactions for LL on one span & DL on both spans.

Moments @ midspan B_1



(a) mom. @ B_1 for fig. 51 (b) mom. @ B_1 for fig. 52 (c) mom. @ B_1 for fig. 53

fig. 54.

The critical bending moments in main girder are:

@	B_1	at the time of prestressing	$175{,}000^{\text{ft-lb}}$	[fig 54(b)]
		later is added	$1{,}172{,}000^{\text{ft-lb}}$	[fig 54(c)]
@	A_1	at the time of prestressing	$348{,}000^{\text{ft-lb}}$	[fig. 52]
		later is added	$2{,}090{,}000^{\text{ft-lb}}$	[fig. 51]

fibre stresses.

@ B_1

$$C_{dt} = \frac{175{,}000 \times 12 \times 31.9}{482{,}000} = 139$$

$$C_{db} = \frac{175{,}000 \times 12 \times 40.1}{482{,}000} = 175$$

$$C_{at} = \frac{1{,}172{,}000 \times 12 \times 31.9}{482{,}000} = 931$$

$$C_{ab} = \frac{1{,}172{,}000 \times 12 \times 40.1}{482{,}000} = 1170$$

@ A_1

$$C_{dt} = \frac{348{,}000 \times 12 \times 31.9}{482{,}000} = 276$$

$$C_{db} = \frac{348{,}000 \times 12 \times 40.1}{482{,}000} = 347$$

$$C_{at} = \frac{2{,}090{,}000 \times 12 \times 31.9}{482{,}000} = 1660$$

$$C_{ab} = \frac{2{,}090{,}000 \times 12 \times 40.1}{482{,}000} = 2085$$

With the fibre stresses obtained above, line 3 is still above line 4. No solution is possible, for e & P_i together, @ A_1.

Third trial section

increase the width of bottom flange of the second trial section to 21".

properties of the new section:

$A = 885 \,\square''$

$y_2 = 38.4''$

$y_1 = 33.6''$

$I = 524{,}000 \text{ in}^4$

$r^2 = 592 \text{ in}^2$

$$\frac{r^2}{y_1} = \frac{592}{33.6} = 17.61 \text{ in}$$

$$\frac{r^2}{y_2} = \frac{592}{38.4} = 15.41 \text{ in}$$

if we increase column width to 28" distribution factors in mom. distributions will not change. We don't have to consider the small increase in D.L. of girder. $w_d = \frac{150}{144} \times 885 = 921 \,\#/_1$

fibre stresses

@ B_1

$$c_{dt} = \frac{175{,}000 \times 12 \times 33.6}{524{,}000} = 135$$

$$c_{db} = \frac{175{,}000 \times 12 \times 38.4}{524{,}000} = 154$$

$$c_{at} = \frac{1{,}172{,}000 \times 12 \times 33.6}{524{,}000} = 872$$

$$c_{ab} = \frac{1{,}172{,}000 \times 12 \times 38.4}{524{,}000} = 996$$

$$2200 > 135 + 872$$

@ A_1

$$c_{dt} = \frac{348{,}000 \times 12 \times 33.6}{524{,}000} = 268$$

$$c_{db} = \frac{348{,}000 \times 12 \times 38.4}{524{,}000} = 306$$

$$c_{at} = \frac{2{,}090{,}000 \times 12 \times 33.6}{524{,}000} = 1607$$

$$c_{ab} = \frac{2{,}090{,}000 \times 12 \times 38.4}{524{,}000} = 1836$$

$$2200 > 268 + 1607$$

ordinates of fig. 55 for $e = 0$

@ B_1 (full lines)

Line 1. $-\frac{1}{(135+230)A} = -\frac{2.740}{1000A}$

Line 2. $+\frac{1}{(2200-135-872)A} = +\frac{0.838}{1000A}$

Line 3. $+\frac{1}{(2200+154)A} = +\frac{0.425}{1000A}$

Line 4. $+\frac{0.85}{(154+996-230)A} = +\frac{0.924}{1000A}$

@ A_1 (dash lines)

$-\frac{1}{(268+230)A} = -\frac{2.010}{1000A}$

$+\frac{1}{(2200-268-1607)A} = +\frac{3.080}{1000A}$

$+\frac{1}{(2200+306)A} = +\frac{0.399}{1000A}$

$+\frac{0.85}{(306+1836-230)A} = +\frac{0.445}{1000A}$

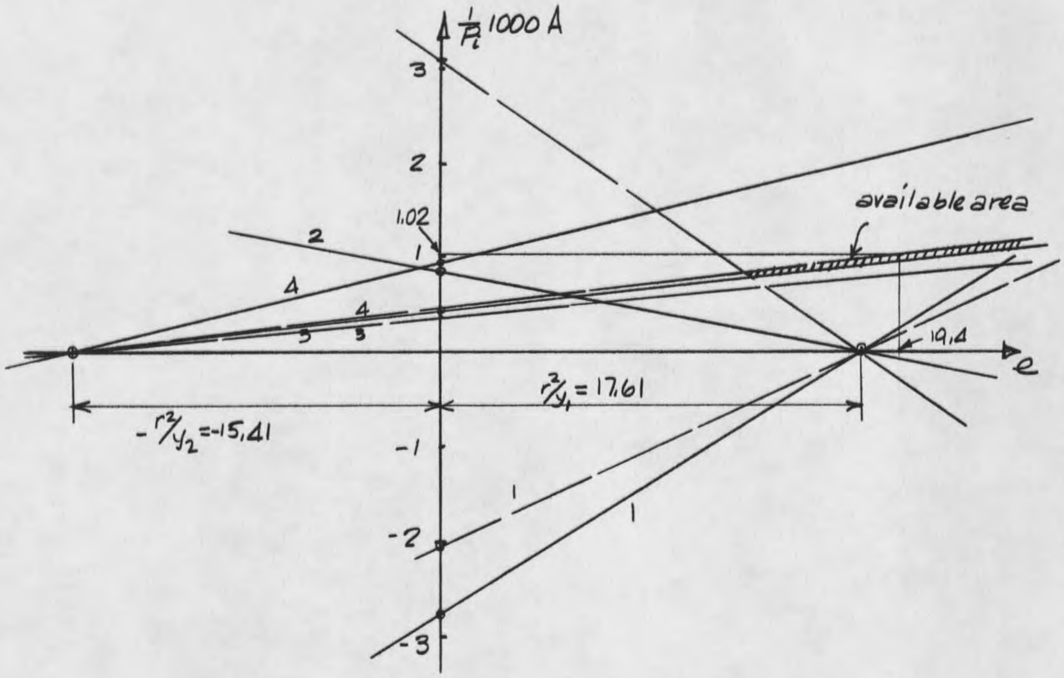


fig.55.
the acceptable values for P_i & e

Assume that max. eccentricities obtainable are :

for the bottom section $\quad 38.4 - (7.2 + 7) = 24.2''$
for the top section $\quad 33.6 - (7.2 + 7) = 19.4''$

case I : it is sufficient for the purpose of avoiding secondary bending moments, due to prestressing, @ A_1 by satisfying the formula $\quad 0.600\, e_{A_1} + 0.8375\, e_{B_1} = 0$

at the same time eccentricities should not exceed the max. values given above

$$e_{A_1} < 19.4''$$
$$e_{B_1} < 24.2''$$

if $e_{A_1} = 19.4''$ $\quad e_{B_1} = -\frac{0.600}{0.8375} e_{A_1} = -13.9'' < -24.2''$ O.K.

case II : The case in which there is no slope at the ends of the girder (@ A_0 & A_1) can also be adopted.

for this $\quad 8 e_{B_1} + 7 e_{A_0} = 0 \quad$ is sufficient.

The aim of this supposition is to avoid secondary bending moments. Indeed, it is intended to prestress the girder and the head of the columns at the same time and so long as the girder has no slope at its ends the column will not be subjected to bending due to the prestressing

with $e_{A_1} = +19.4''$, $e_{B_1} = -7/8\, e_{A_1} = -7/8\; 19.4 = -16.97'' < -24.2$ O.K.

in this case however the eccentricity of $+19.4''$ must also be provided @ A_{O_1} or A_{O_2}, as will be shown later this is possible

from fig. 55

with $e = 19.4''$ $\quad \frac{1000A}{P_i} = 1.02 \quad \underline{P_i} = \frac{1000 \times 885}{1.02} = \underline{867{,}000^{\#}}$

<u>steel</u>

$A_s = \frac{867{,}000}{155{,}000} = 5.6^{\square''}$

<u>Use 96- $\phi = 0.276''$ wires 48 each side 12 layers of 4 wires</u>

<u>Stresses @ A_O (A_{O_1} or A_{O_2}) due to eccentricity of wires</u>

top fibres $\frac{867{,}000}{885}\left(1 + \frac{19.4}{17.61}\right) = \underline{2050 \text{ psi (comp.)} < 2200 \text{ psi OK}}$

bottom fibres $\frac{867{,}000}{885}\left(1 + \frac{19.4}{15.41}\right) = \underline{-250 \text{ psi (tens.)}}$

this small tensile stress may be permissible.

eccentricities in case II are accepted

$e_{A_O} = +19.40''$
$e_{B_1} = -16.97''$
$e_{A_1} = +19.40''$

<u>the shape of the cable</u>

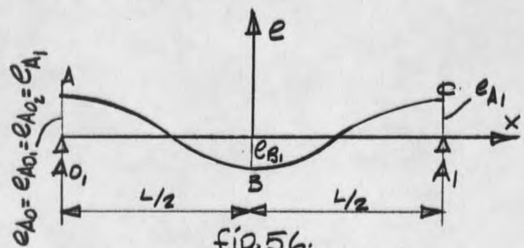
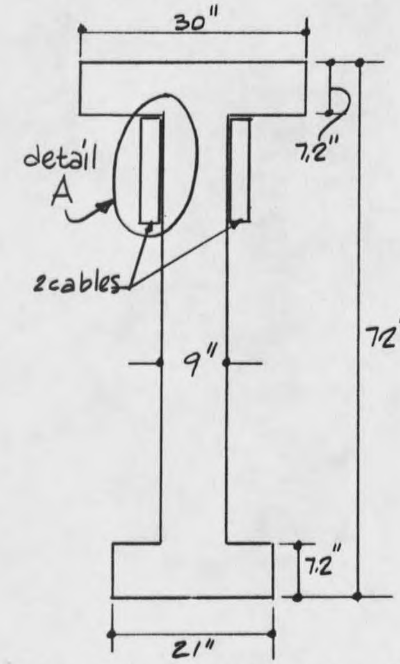


fig. 56.
the shape of the cable

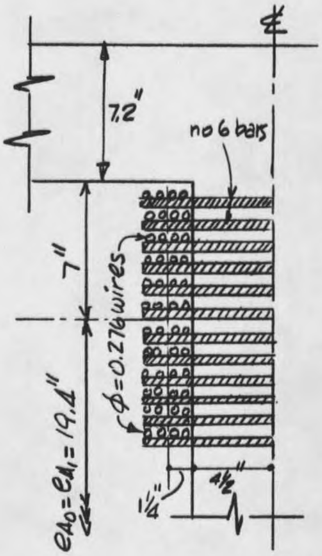
as is shown in fig. 56 the shape of cable ABC is a parabola of the fourth degree, the equation of which is:

$$e_x = 8(e_{B_1} - e_{A_1})(2x_L^2 - 1)x_L^2 + e_{B_1}$$

where $x_L = \frac{x}{L}$, e_{B_1} & e_{A_1} are algebraical values the signs of which depend on whether the cable is above or below the axis of the girder.



(a) section of main girder @ A_1 or @ A_0



(b) detail A

fig. 57.

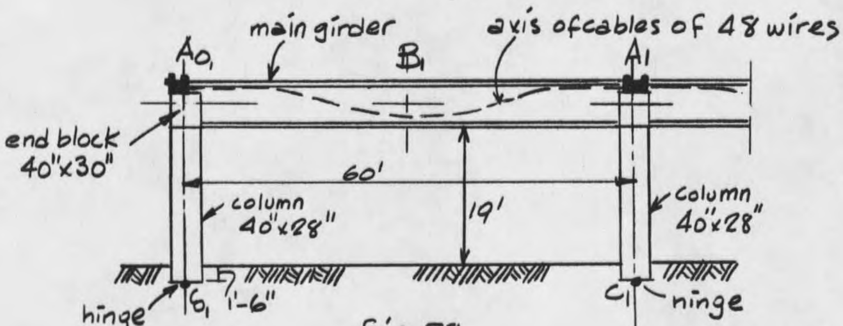


fig. 58.
final main girder frame

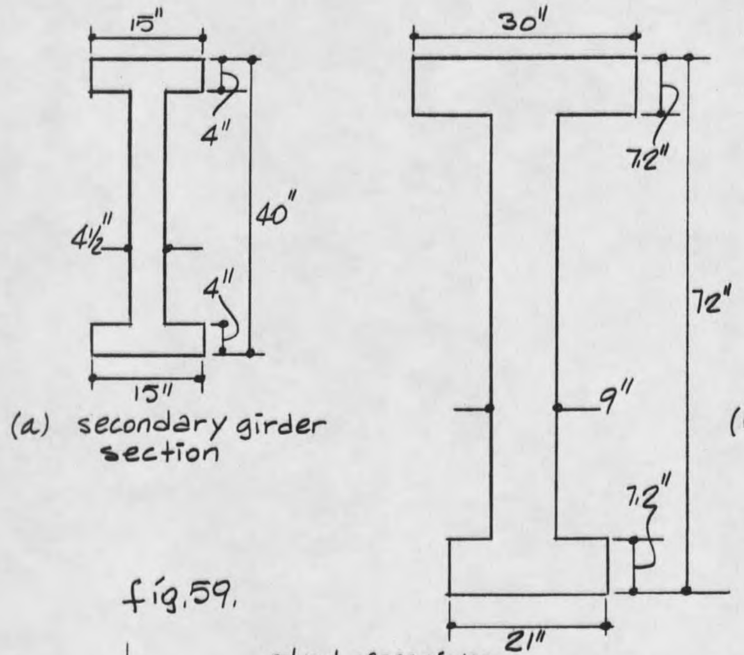
Columns are understressed, but these factors should be dwelt upon:

1. Large columns provides good dimensions for the end-block of girder.
2. Wind & eartquake loads are omitted in the design.
3. Large columns provides smaller girder section
4. Minimise the steel area req'd.

<u>steel area in columns</u>

$0.01 \times 1120 = 11.20^{\square''}$ <u>use 12-No 9 bars</u>

F. Design of the Ends of Secondary Girders



(a) secondary girder section

(b) main girder section

fig. 59.

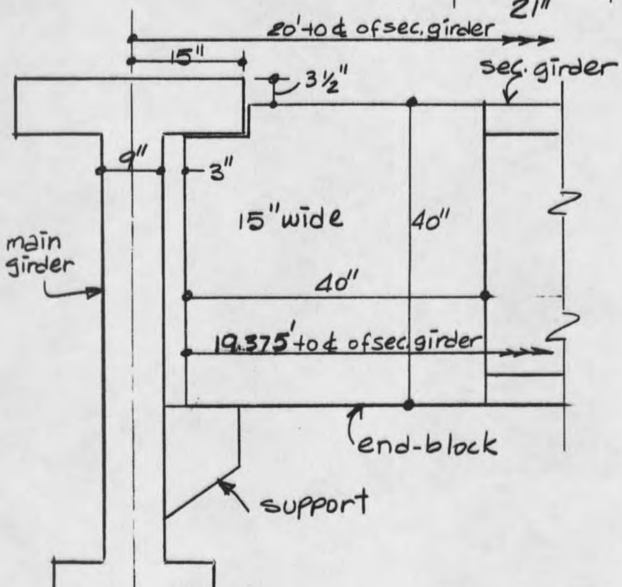


fig. 60.
support condition for the end-block of secondary girders.

<u>support</u>

from fig. 60

shear = 1975 × 19.375
= 38,200#

Area req'd = $\frac{38{,}200}{264}$ = 145 □"

with a horizontal dimension of 15" vertical dimension of 9.7" is req'd.

<u>use 15" × 12"</u>

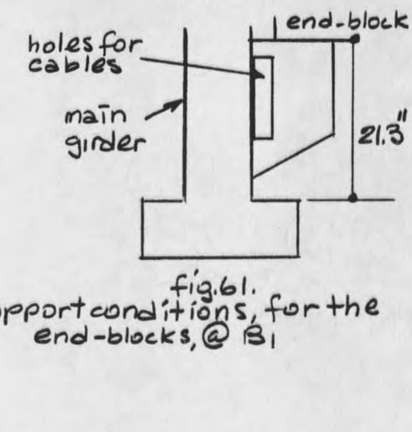


fig. 61.
support conditions, for the end-blocks, @ B1

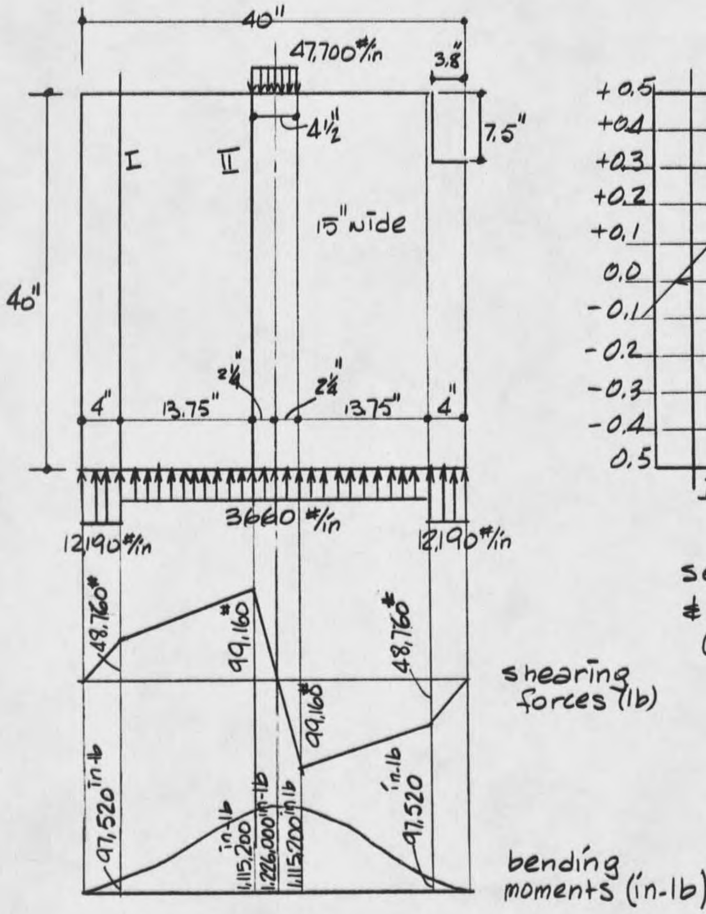


fig 62
shear & mom. at the end-block

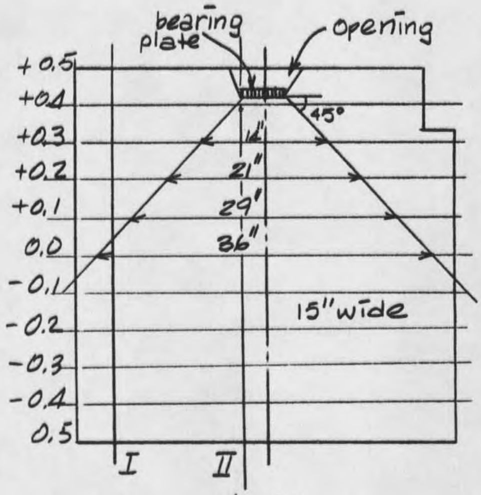


fig. 63.
sections used in calculations & compressive stress distribution @ the end-block.

$\frac{214,300}{4.5} = \underline{47,700}$ #/in

$\frac{214,300}{264} = 813$ #/in

$813 \times 15 = \underline{12,190}$ #/in

$813 \times 4.5 = \underline{3,660}$ #/in

<u>bearing plates</u>

unit bearing stress = $1.75 f_c = 1.75 \times 2200 = 3850$ psi

gross area req'd = $\frac{214,300}{3850} + (2 \times 0.9 \times 3.6) = 62.2^{\square''}$

with a horizontal dimension of 15" the vertical dimension will be 4.15" <u>use 15" x 4½"</u>

Table IV
calculated stresses in the end-block of the sec. girders (psi)

section	z/a	comp. stress c_x	bending stress c_z	shearing stress v	P_t
II	+0.3	1021	+ 137	389 +70	- 57
	+0.2	680	- 105	391 +70	- 318
	+0.1	493	- 232	328 +70	- 407
	0.0	397	- 268	238 +70	- 389
	-0.1	357	- 240	146 +70	- 309
	-0.2	357	- 173	72 +70	- 208

calculations for Table IV (for more critical section, section II)

effective width $=15-2=13''$

$C_z = K\frac{M}{ba^2}$ $b=13'$, $a=40''$

from fig 62

@ sec II $M = 1,115,200^{\text{in Lb}}$, $\frac{M}{ba^2} = 53.6$

z/a	K	M/ba^2	C_z
+0.3	+2.560	× 53.6 =	± 137 psi
+0.2	−1.960	× " =	− 105 psi
+0.1	−4.320	× " =	− 232 psi
0.0	−5.000	× " =	− 268 psi
−0.1	−4.480	× " =	− 240 psi
−0.2	−3.240	× " =	− 175 psi

$v = K_1 \frac{V}{ba}$

from fig 62.

@ sec II $V = 99,160^{\#}$, $\frac{V}{ba} = 190$

z/a	K_1	V/ba	v
+0.3	2.048 ×	190 =	389 psi
+0.2	2.058 ×	" =	391 psi
+0.1	1.728 ×	" =	328 psi
0.0	1.250 ×	" =	238 psi
−0.1	0.768 ×	" =	146 psi
−0.2	0.378 ×	" =	72 psi

from fig.63

$z/a = +.3$ $C_x = \frac{214,300}{14 \times 15} = \underline{1021}$

$z/a = +.2$ $C_x = \frac{214,300}{21 \times 15} = \underline{680}$

$z/a = +.1$ $C_x = \frac{214,300}{29 \times 15} = \underline{493}$

$z/a = 0.0$ $C_x = \frac{214,300}{36 \times 15} = \underline{397}$

for others $C_x = \frac{214,300}{40 \times 15} = \underline{357}$

extra shearing forces to be considered:

D.L. $-5500^{\#}$

L.L. $-34,000^{\#}$

to the slope of the cables

initially $13,560^{\#}$

ultimately $11,527^{\#}$

$13,560 - 5500 = \underline{8060^{\#}}$

$(11,527 - 5500) - 34,000 = \underline{27980^{\#}}$

maximum of these forces will produce a shearing stress of:

$$\frac{1.5\,(27,980)}{15 \times 40} = \underline{70\,psi}$$

It is seen that maximum tensile principal stress 407 psi exceeds the permissible 264 psi, reinforcement should be provided. It is again assumed that the stress 407 psi exists on every horizontal plane of half the length of the end-block.

total tensile force $= 0.5 \times 407 \times 40 \times 13 = 106,000^{\#}$

central area $= 4\frac{1}{2}''$

each side areas $= 4\frac{1}{4}''$

the forces corresponding to these areas are: $106,000\,\frac{4.5}{13} = 36,800^{\#}$

$106,000\,\frac{4.25}{13} = 34,600^{\#}$

reinforcement req'd.

for central area $\frac{36,800}{20,000} = 1.84\,\square''$ <u>Use 10- No 4 bars</u>

for each side area $\frac{34,600}{20,000} = 1.73\,\square''$ <u>Use 9 - No 4 bars</u>

Some no 4 horizontal bars will also be provided at the same spacing as the vertical bars near cable holes.

G. Design of the Ends of Main Girders

Actually end-blocks of main girders & columns act together, but to make the design possible we will separate them and check the end-blocks.

Bearing plates

unit bearing stress = 3850 psi

gross plate area req'd = $\frac{867{,}000}{3850} + (2 \times 2.5 \times 7.2) = 261 \square''$

USE 30" × 9"

If P_i was applied @ N.A of the end-block the unit compressive stress, at the other end where the I section meets the end-block, would be : $\frac{867{,}000}{885} = 980$ psi

Actually it is applied with an eccentricity of 19.40". This will increase the unit stress on the top & will decrease it on the bottom as is shown in fig. 64.

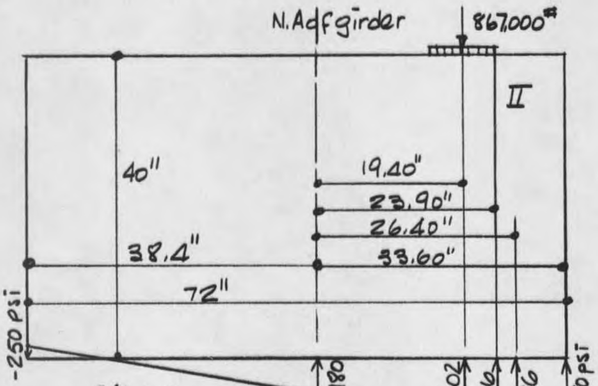


fig. 64
unit stress distribution

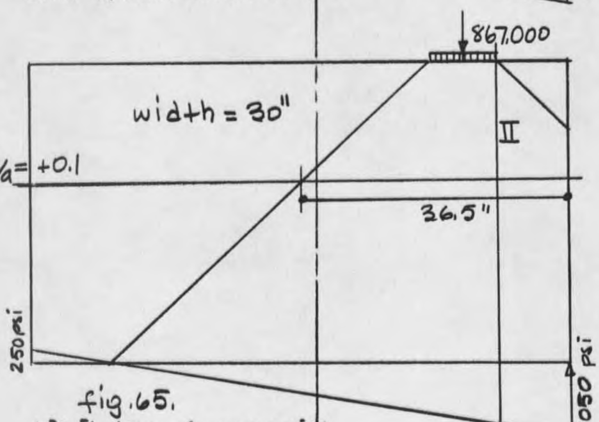


fig. 65.
distribution of compressive stress.

unit stress = $\frac{P_i}{A} \pm \frac{My}{I}$

$\frac{M}{I} = \frac{867{,}000 \times 19.40}{524{,}000} = 32.07 \,\#/in^3$

$\frac{P_i}{A} = 980$ psi

y = 33.60"
unit str. = 980 + 32.07 × 33.6 = 2050 psi

y = 26.40"
unit str. = 980 + 32.07 × 26.40 = 1826 psi

y = 23.90"
unit str. = 980 + 32.07 × 23.90 = 1746 psi

y = 19.40"
unit str. = 980 + 32.07 × 19.40 = 1602 psi

y = 0
unit str. = 980 psi

y = -38.4"
unit str. = 980 - 32.07 × 38.4
= -250 psi

From the experience of previous designs only section II & $z/a = +0.1$ will be inspected.

shear @ section II

$$\frac{(2050+1826)\,30}{2} \times 7.2 = 418{,}000$$
$$\frac{(1826+1746)\,9}{2} \times 2.5 = \underline{+\,40{,}200}$$
$$\underline{458{,}200\,\#} \text{ shear @ sec. II}$$

mom. @ section II

$$418{,}000 \times 6.10 = 2{,}550{,}000$$
$$40{,}200 \times 1.25 = \underline{50{,}000}$$
$$\underline{2{,}600{,}000} \text{ in-lb} \quad \text{mom @ sec. II}$$

for end-block $I = \frac{1}{12} \times 30 \times 72^3 = 933{,}000 \text{ in}^4$

$$C_x = \frac{867{,}000}{36.5 \times 30} + \frac{(867{,}000 \times 1946)\,18.25}{933{,}000} = 1120 \text{ psi}$$

$$C_z = -4.320\, \frac{2{,}600{,}000}{25 \times 40^2} = -281 \text{ psi}$$

$$V = 1.728\, \frac{458{,}200}{25 \times 40} = 791 \text{ psi}$$

$$P_t = \frac{C_x + C_z}{2} - \sqrt{V^2 + \left(\frac{C_x - C_z}{2}\right)^2} = \frac{1120 - 281}{2} - \sqrt{791^2 + \left(\frac{1120+281}{2}\right)^2} = -635$$

$635 > 264$ reinforcement should be provided

It is again assumed that the stress 635 psi exists on every horizontal plane of half the length of the end-block.

total tensile force $= 0.5 \times 635 \times 40 \times 25 = 318{,}500\#$

central area $= 9''$

two side areas $= 8''$

the forces corresponding to these areas are:

$$318{,}500 \times 9/25 = 114{,}500\#$$
$$318{,}500 \times 8/25 = 102{,}000\#$$

<u>reinforcement req'd</u>

for central area $\frac{114{,}500}{20{,}000} = 5.73\,\square''$ <u>Use 10- no 7 bars</u>

for each side areas $\frac{102{,}000}{20{,}000} = 5.10\,\square''$ <u>Use 9- no 7 bars</u>

Some horizontal bars at the same spacing will also be provided as the vertical bars near cable holes. (no 7 bars)

Web reinforcement in main girders

@ A_1

$$\text{shear} = 139.1 + 22.1 = 161.2^k$$

no 4 ∅ stirrups are to be used ($A_s = 0.40^{\square''}$)

$$\text{spacing} = \frac{72 \times 0.40 \times 40{,}000}{161{,}200} = 7''$$

yield point of stirrups 40,000 psi

@ A_0 shear = 214.2^k

$$\text{spacing} = \frac{72 \times 0.40 \times 40{,}000}{214{,}200} = 5''$$

the placing of stirrups should be continued for a length of 10' from the columns.

PART II

DESIGN OF THE SAME THEORETICAL FRAME IN CONVENTIONAL REINFORCED CONCRETE

A. Design of the Roof Slab

The one-way slab will be designed as simply-supported beams of 12" width, supported on two secondary beams.

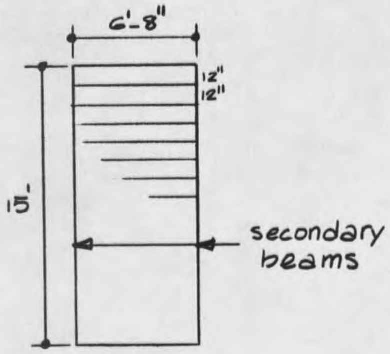


fig. 66. precast slab-block dimensions

Slab thickness

$L = 6.66'$

$b = 12''$

$w_a = 80\ \#/\square'$

assume 4" of slab thickness

$w_d = 150 \times \frac{4}{12} = 50\ \#/\square'$

$w_a + w_d = 130\ \#/\square' = 130\ \#/'$

$$M = \frac{w_{a+d} L^2}{8} = \frac{130 \times 6.66^2}{8} = 721 \text{ ft-lb}$$

$$d^2 = \frac{2M}{f_c b k j} = \frac{2\,(721)\,12}{2200 \times 12 \times .403 \times .866} = 1.881$$

$d = 1.37''$
$\quad .75''$ (3/4" concrete protection)
$\quad .25''$ (one half of bar diameter)
$\quad \overline{2.37''}$

use $t = 4''$ $d = 4 - 1 = 3''$

Shear

$$V = \frac{130 \times 6.66}{2} = 433\#, \quad v = \frac{V}{bjd} = \frac{433}{12 \times 0.866 \times 3} = 13.9 \text{ psi} < 146.7 \text{ psi}$$

O.K.

no slab reinforcement due to shear is req'd.

coef. of resistance $= R = \frac{M}{bd^2} = \frac{721}{1 \times 3^2} = 80.1$ psi < 384 psi for balanced reinforcement

Beam is underreinforced. Steel is the determining factor & concrete is not stressed to capacity which is a more economical case

for a balanced design $R = \frac{1}{2} f_c k j = \frac{1}{2}\, 2200 \times 0.403 \times 0.866$
$= \underline{384 \text{ psi}}$

Steel

$$A_s = \frac{M}{f_s\, j\, d} = \frac{721 \times 12}{20{,}000 \times 0.866 \times 3} = 0.167^{\square''}$$

Use no 4 bars @ 12" c/c

Bond

max. allowable bond unit stress = 350 psi

$$u = \frac{V}{\Sigma_0\, j\, d} = \frac{433}{1.57 \times 0.866 \times 3} = \underline{106 \text{ psi} < 350 \text{ psi O.K.}}$$

where $\Sigma_0 = \pi D = \pi \times 0.5 = 1.57''$

Temperature steel

$$A_t = 0.0025\, bd = 0.0025\,(12)\,3 = 0.09^{\square''/'}$$

Use no 3 bars 12" c/c

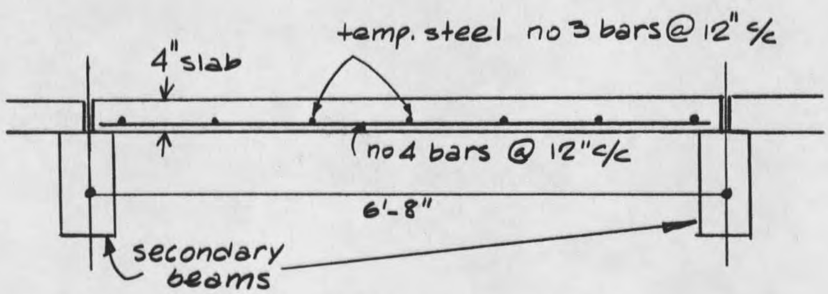


fig 67
slab reinforcement

B. Design of Secondary Beams

Secondary beams will be designed as simple beams of 15' length, supported by two secondary girders.

Load

$$w_a = 25+30+20+5+50 = 130\ \#/\square'$$

$$\times 6.66 = 866\ \#/'$$

$$\text{assume } w_d = \underline{150\ \#/'}$$

$$1016\ \#/'$$

Section

$$M = \frac{1016\,(15)^2}{8} = 28{,}580^{\text{ft-lb}} \qquad bd^2 = \frac{M}{R} = \frac{28{,}580\times 12}{384} = 892\ in^3$$

with $b = 8'$ $d = 10.50$
1.50 (cover)
.71 (½ bar diam.)
12.71 (total depth)

$\underline{w_d = 106\ \#/' < 150\ \#/'}$

assume $w_d = 90\ \#/'$, $w_{d+a} = 956\ \#/'$

$$M = \frac{956\,(15)^2}{8} = 26{,}900^{\text{ft-lb}} \quad , \quad bd^2 = \frac{26{,}900\times 12}{384} = 840\ in^3$$

$b = 6"$ $d = 11.82$
1.50
.71
14.03"

$\underline{\text{use } b = 6", d = 12", \text{depth} = 14\tfrac{1}{4}"}$

$w_d = 89\ \#/' < 90\ \#/'$ O.K.

Shear

$$V = \frac{956\times 15}{2} = 7170^{\#} \quad , \quad v = \frac{V}{bjd} = \frac{7170}{6\times .866\times 12} = \underline{115\,psi < 146.7\,psi}$$

no stirrups req'd $\underline{\text{O.K.}}$

Steel

$$A_s = \frac{M}{f_s jd} = \frac{26{,}900\times 12}{20{,}000\times .866\times 12} = 1.555\ \square"$$

use 1- no 11 bar

Bond

$$u = \frac{V}{\Sigma_o jd} = \frac{7170}{4.43\times .866\times 12} = \underline{155.5\,psi < 350\,psi\ \text{O.K.}}$$

Final section

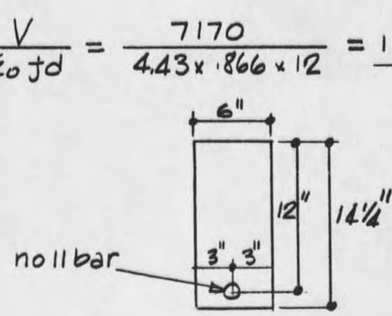


fig. 68.
secondary beam section

C. Design of Secondary Girders

Secondary girders will be designed as simply-supported girders of 40' length, supported by main girders.

load

$w_a = 956^{\#/\prime}$
$\times 15 = 14{,}330^{\#}$ $\quad \dfrac{14330}{6.66} = 2150^{\#/\prime}$
$600^{\#/\prime}$ (assumed w_d)
$2750^{\#/\prime}$ $(w_d + w_a)$

Section

$$M = \frac{2750 \times 40^2}{8} = 550{,}000^{ft\cdot lb}, \quad bd^2 = \frac{550{,}000 \times 12}{384} = 17{,}190\ in^3$$

assume 2 layers of no 11 bars $\quad b = 15''$ $\quad d = 33.83''$
1.50'' (cover)
1.41'' (bar diam)
.71 (½ diam)
37.45'' depth

use $b = 15''$, $d = 34''$, depth $= 38''$

$w_d = 594^{\#/\prime} < 600^{\#/\prime}$ O.K.

Shear

assume clear dist. between supports 38.0'

$$V = \frac{2750 \times 38}{2} = 52{,}250^{\#}, \quad v = \frac{52{,}250}{15 \times .866 \times 34} = 118.2\,psi < 146.7\,psi \text{ O.K.}$$

no web reinf. req'd

Steel

$$A_s = \frac{550{,}000 \times 12}{20{,}000 \times .866 \times 34} = 11.2^{\square''}$$

use 8-no11 bars two layers of 4

Bond

$$u = \frac{52{,}250}{35.44 \times .866 \times 34} = 50\,psi < 350\,psi \text{ O.K.}$$

Final Section

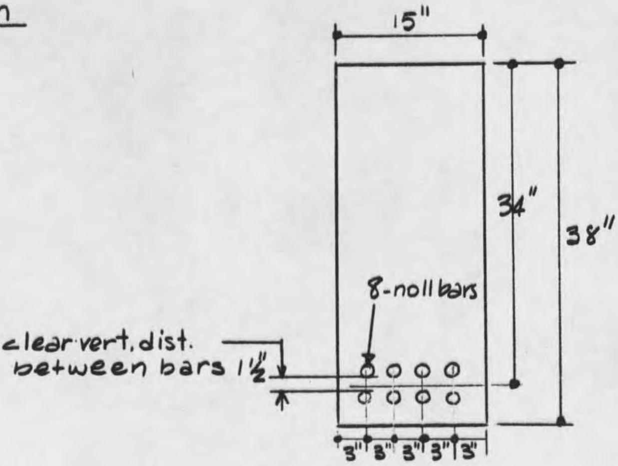


fig 69
Secondary girder section

D. Design of the Main Girder Frame

Two main girders and three columns will be designed as rigid-frames

Girder

load

$$w_a = \frac{2750 \times 40}{15} = 7340 \,\#/_1$$
$$1600 \,\#/_1 \quad (\text{assumed } w_d)$$
$$8940 \,\#/_1 \quad (w_a + w_d)$$

Section

Assume : $3\frac{I_c}{L} = 1$, $4\frac{I_b}{L} = 3$

Moments & shear

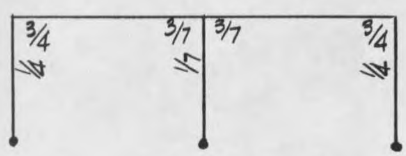
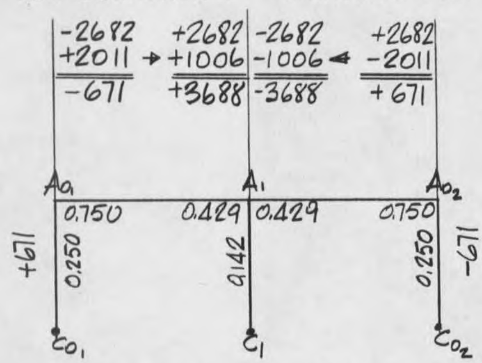
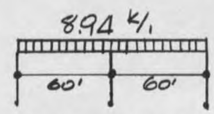


fig.70.
distribution factors

1st. condition : moments due to DL & LL on main girder



(b) moment distribution for DL & LL on main girder



(a) loading for (b)

$$F.E.M. = \frac{8.94 \times 60^2}{12} = 2682^{1-k}$$

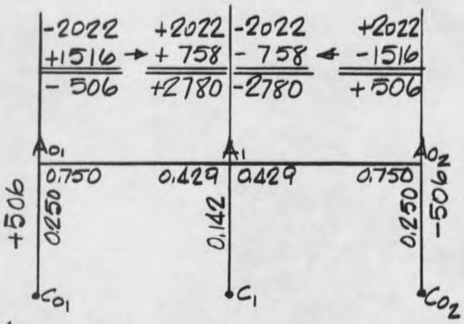
fig.71.

2nd. condition: consider the condition of which only one of the spans loaded with snow & superimposed loads and both spans loaded with DL's.

TABLE V
distribution & carry-over factors

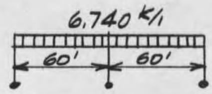
member	F.E.C.[1] × I/L	D.F[2]	C.O.[3]
A_1Ao_1	3	0.466	0.500
A_1C_1	1	0.155	0
A_1Ao_2	2.44	0.379	0.154

(1) F.E.C. = Far End Condition
(2) D.F. = Distribution Factor
(3) C.O. = Carry-over factor



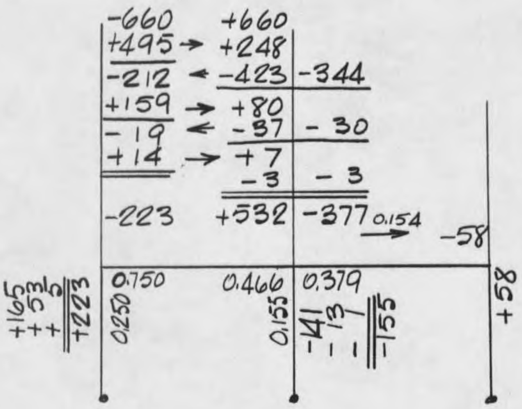
(b) moment distribution for D.L on main girder.

total load on maingirder = 8940 #/
L.L. " " " = 2200 #/
D.L " " " 6740 #/

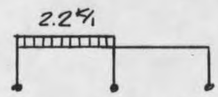


(a) loading for (b)

$$F.E.M = \frac{6.740 \times 60^2}{12} = 2022^{ft\,k}$$



(d) moment distribution for snow & superimposed loads on the girder.



(c) loading for (d)

$$F.EM = \frac{2.2\ (60)^2}{12} = 660^{\,'-k}$$

fig. 72.

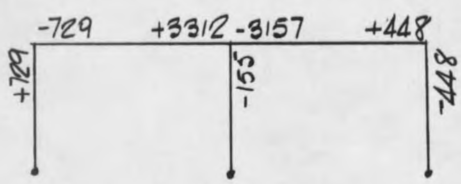


fig. 73.
final moments from fig. 72 (b) and (d)

Max moment = 3,688,000 $^{ft\,\#}$ @ A_1 fig 71(b)

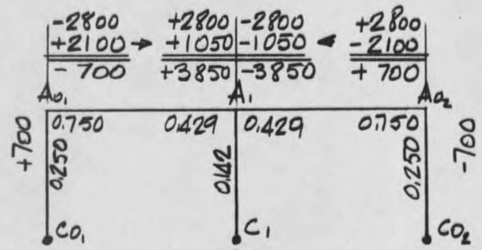
$$bd^2 = \frac{M}{R} = \frac{3.688.000 \times 12}{384} = 115,000 \text{ in}^3 \qquad b = 24'' \quad d = 69.4''$$

the wt. of this section will exceed 1600 #/ that was assumed.

Assume $w_d = 2000^{\#}/_1$ $\qquad w_a + w_d = 9340^{\#}/_1$

moments & shear

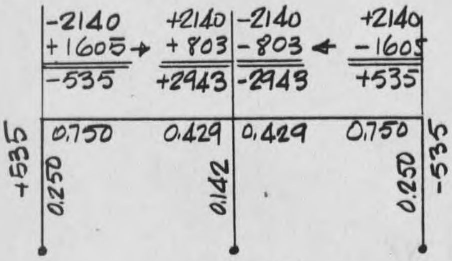
1st. condition.



$$F.E.M. = \frac{9.340 \times 60^2}{12} = 2800^{1-k}$$

fig.74.
moment distribution for DL & LL on main girder

2nd condition



D.L. on main girder = $7140^{\#}/_1$

$$F.E.M. = \frac{7.140 \times 60^2}{12} = 2140^{1-k}$$

fig.75.
mom. distribution for DL on main girder

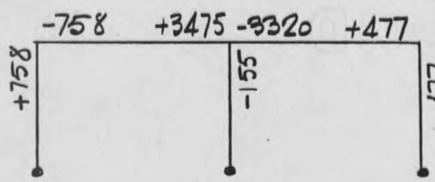


fig.76.
final moments from fig.72(d) & fig.75

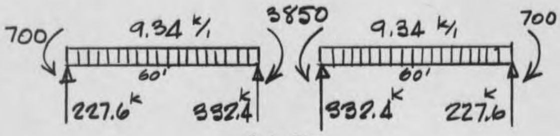


fig.77.
reactions due to loading of the 1st. condition

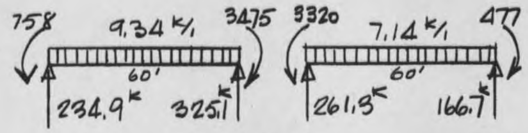


fig.78.
reactions due to loading of the 2nd condition

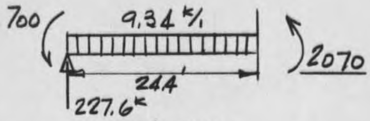


fig.79.
max. poz. moment for the 1st condition

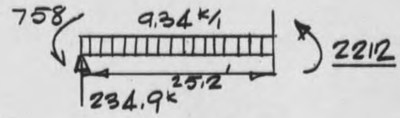


fig.80.
max. poz. moment for the 2nd condition

max. mom $= 3{,}850{,}000^{ft\text{-}lb}$ (neg. mom @ A_1, fig 74)

$$bd^2 = \frac{M}{R} = \frac{3{,}850{,}000 \times 12}{384} = 120{,}200 \text{ in}^3$$

use $b = 24\tfrac{1}{2}''$ $d = 72''$ total depth $= 77\tfrac{1}{4}''$

$w_d = 1972^{\#}/_1 < 2000^{\#}/_1$ assumed O.K.

Shear max $V = 332{,}400^{\#}$ @ A_1

$$v = \frac{V}{bjd} = \frac{332{,}400}{24.5 \times 0.866 \times 72} = 217.5 \text{ psi} > 146.7 \text{ psi}$$ stirrups are req'd

Compression

$$R = \frac{M}{bd^2} = \frac{3{,}850{,}000 \times 12}{24.5 \times 72^2} = 365 < 384$$ concrete is not stressed to capacity. design is economical.

Steel

@ A_1 neg. steel.

$$A_s = \frac{M}{f_s j d} = \frac{3{,}850{,}000 \times 12}{20{,}000 \times .866 \times 72} = 37.1 \,\square''$$

use 24 - no11 bars 3 layers of 8 bars

poz. steel.

$$A_s = \frac{2{,}212{,}000 \times 12}{20{,}000 \times .866 \times 72} = 21.3 \,\square''$$

use 14 - no11 bars 2 layers $\begin{cases} \text{top layer } 6 \\ \text{bot. layer } 8 \end{cases}$

@ A_0 neg. steel.

$$A_s = \frac{758{,}000 \times 12}{20{,}000 \times .866 \times 72} = 7.3 \,\square''$$

use 6 - no 11 bars 1 layer

Bond max. allowable bond unit str. (top bars) = 245 psi

@ A_0

$$u = \frac{V}{\Sigma_0 jd} = \frac{234{,}900}{26.58 \times .866 \times 72} = 141.5 \text{ psi} < 245 \text{ psi O.K.}$$

@ A_1

$$u = \frac{332{,}400}{106.32 \times .866 \times 72} = 50 \text{ psi} < 245 \text{ psi O.K.}$$

moment of inertia & actual stresses.

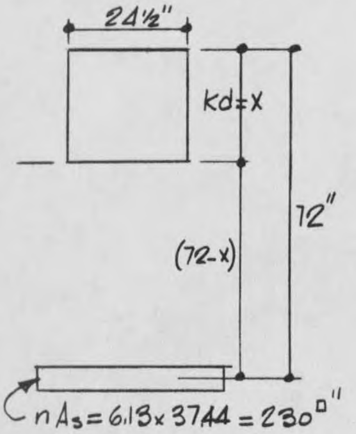


fig. 81
transformed area.

from fig. 81

$$24.5\,(x)\,\frac{x}{2} = 230\,(72-x)$$

$$x^2 + 18.8x - 1352 = 0$$

$$x = 28.5''$$

$$I_T = \frac{24.5\,(28.5)^3}{3} + 230\,(43.5)^2 = 623{,}500 \text{ in}^4$$

$$Z_c = \frac{623{,}500}{28.5} = 21{,}850 \text{ in}^3$$

$$Z_s = \frac{623{,}500}{6.13 \times 43.5} = 2{,}339 \text{ in}^3$$

$$M = 3{,}850{,}000 \text{ ft-lb}$$

$$f_c = \frac{3{,}850{,}000 \times 12}{21{,}850} = 2110 \text{ psi}$$

$$f_s = \frac{3{,}850{,}000 \times 12}{2{,}339} = 19750 \text{ psi}$$

Before continuing the girder design further, the columns must be checked.

Column designs

previously it was assumed that: $3\frac{I_c}{L} = 1$, $4\frac{I_b}{L} = 3$

$I_b = I_T = 623{,}500 \text{ in}^4$ $\qquad 4\,\frac{623{,}500}{60} = 41{,}600 \quad \div 3 = 13{,}830$

With a clear distance from floor to the bottom of the main girder of 19', the column length will be 23.7'

$$I_c = \frac{13{,}830 \times 23.7}{3} = 109{,}200 \text{ in}^4$$

$$I_c = \frac{1}{12}bt^3 = 109{,}200$$

use $b = 24\frac{1}{2}''$, $t = 35''$

Column A_1C_1

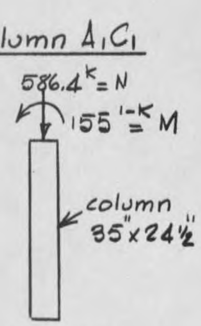


fig. 82.
axial load & mom on colum A_1C_1

from fig. 82

$$e = \frac{M}{N} = \frac{155 \times 12}{586.4} = 3.17'' < 35''$$

$$f_a = 0.8\left[\frac{0.225 f'_c + f_s p_g}{1 + (n-1)\,p_g}\right] = 0.8\left[\frac{0.225\,(4890) + 20{,}000 \times 0.01}{1 + (6.13-1)\,0.01}\right]$$

$$= 989$$

where p_g is assumed to be 0.01

$$C = \frac{f_a}{0.45 f'_c} = \frac{989}{0.45 \times 4890} = 0.449$$

assume $D = 5.5$

equivalent axial load $= P = N\left[1+\frac{cDe}{t}\right] = 586.4\left[1+\frac{0.449\times5.5\times3.17}{35}\right] = 718^k$

max allowable load $= P = 0.8\,A_g\,(0.225 f'_c + f_s\,p_g)$

$= 0.8(35\times24.5)(0.225\times4890+20{,}000\times0.01)$

$= \underline{893^k > 718^k \text{ O.K.}}$

<u>Steel</u>

$p_g = \frac{A_s}{A_g} = 0.01 \qquad A_s = 0.01\times35\times24.5 = 8.57\,\square''$

<u>use 6-no11 vert. bars</u>

<u>use no 3 ties @ 18" c/c</u>

$I_c = \frac{24.5\times35^3}{12} + 5.13\times9.36\times14.75^2 = 98{,}500 \text{ in}^4$

The difference between assumed $I_c = 109{,}200$ and actual $I_c = 98{,}500$ can not make much effect on stiffness factors. <u>The section will be accepted</u>

<u>Column A_oC_o</u>

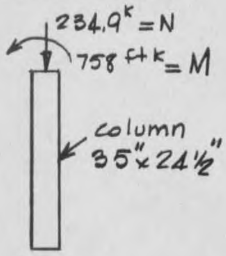


fig. 83.
axial load & mom
on column A_oC_o

for uniformity the same section used in column A_1C_1 will be accepted and checked.

from fig. 83

$e = \frac{758\times12}{234.9} = 38.7 > 35$

even if the design should be made for a cracked section it is very probable that the section is all under compression and would carry the equivalent axial load. so the uncracked section design will be checked first.

$f_a = 989 \quad , \; C = 0.449 \quad , \; D = 5.5$

equivalent axial load $= P = 234.9\left[1+\frac{0.449\times5.5\times38.7}{35}\right] = \underline{876^k < 893^k \text{ O.K.}}$

<u>the section will be accepted</u>

Columns again are overdesigned, with future effects of horizontal wind & earthquake loads on the frame in mind. These horizontal loads might be brought to bear from both sides, N-S or E-W. To resist the horizontal loads of E-W direction, columns should be designed as cantilevers and this will considerably increase the section.

Girder (continued)

Stirrups

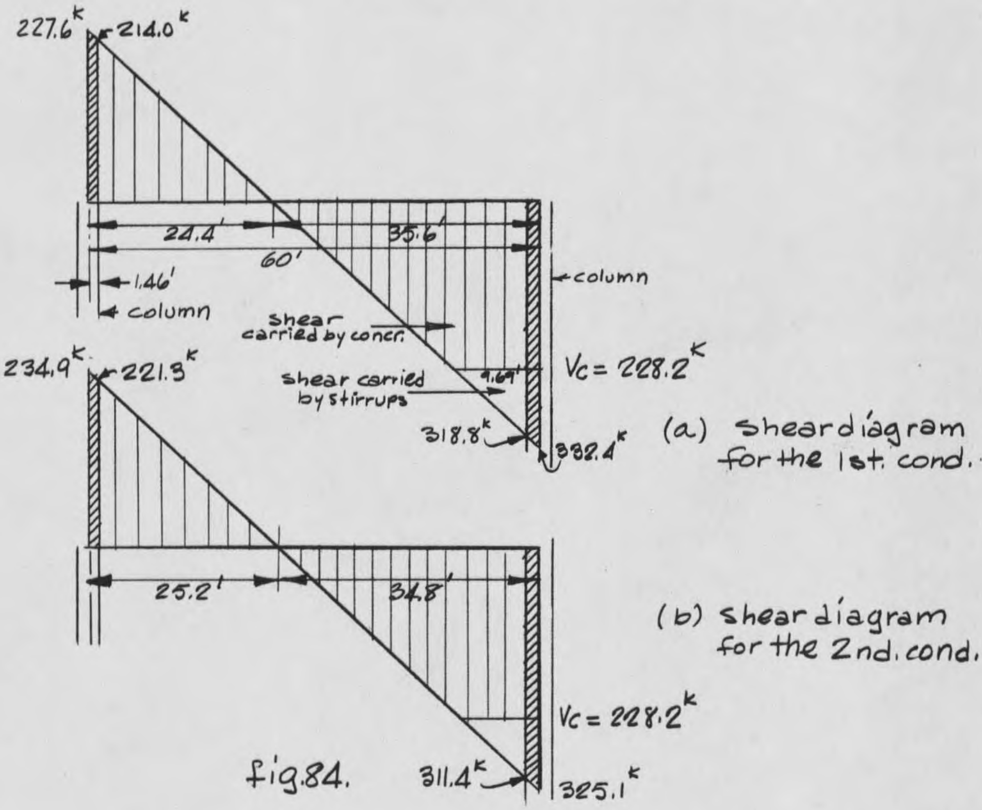


fig.84.

Increase girder width to 25.0" for stirrup cover. new columns 35"x25"

shear carried by concrete $V_c = v b j d = 146.7 \times 25 \times 0.866 \times 72 = 228.2^k$

As is seen in fig.84 only right side needs web reinforcement.

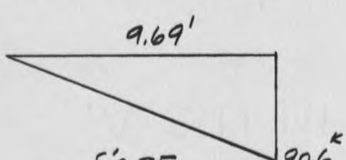


fig.85.
shearcarried by stirrups

try no 3 stirrups $A_v = 0.22\ \square''$

$sV_s = A_v f_v jd = 0.22(20)(0.866)72 = 274.1 \ \frac{k\,in}{stirrup}$

$\Sigma sV_s = 90.6 \ \frac{116.1}{2} = 5260^{k\,in}$

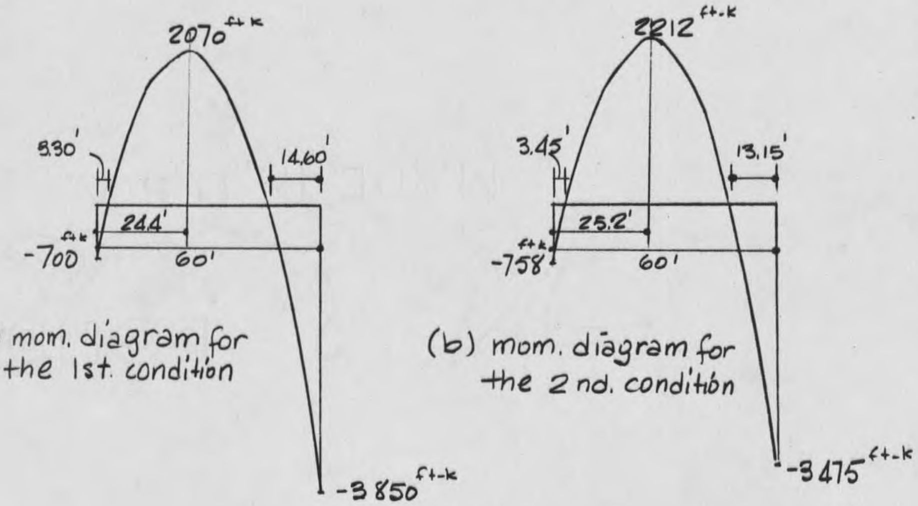
number of stirrups $= \frac{5260}{274.1} = 19.2$

use 20-no 3 stirrups

c to c spacing of stirrups starting from the face of col A_1C_1
2", 3", 3", 3", 3", 3", 4", 4", 4", 4", 4", 5", 5", 5", 5", 6", 7", 9", 12".

Bending up bars

Moment diagrams are drawn for both conditions as shown in fig. 86.



(a) mom. diagram for the 1st. condition

(b) mom. diagram for the 2nd. condition

fig. 86.

Mom. that is carried by the 8 bars of the bottom layer of poz. steel

$$M = A_s f_s j d = \frac{12.48 \times 20{,}000 \times 0.866 \times 72}{12} = 1{,}295{,}000^{ft\text{-}lb}$$

As will be seen from fig. 87 a 30' length of 6 poz. top layer bars will be sufficient to carry the max. poz. moment. These bars, then, can be bent to a 45° to afford negative reinforcement.

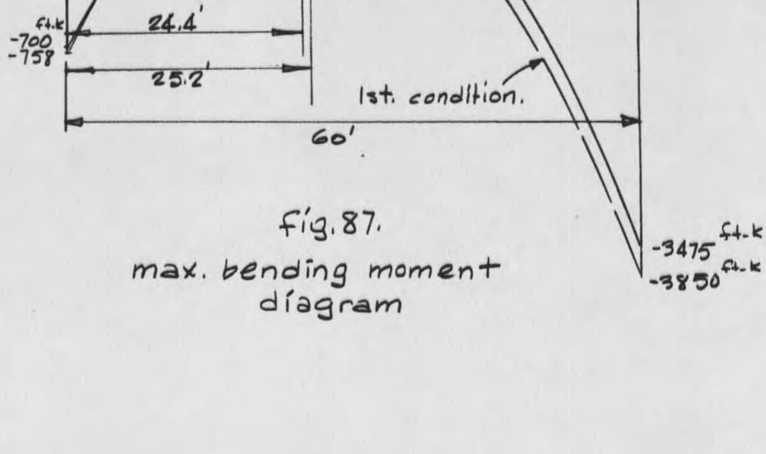


fig. 87.
max. bending moment diagram

Final sections and arrangement of reinforcement.

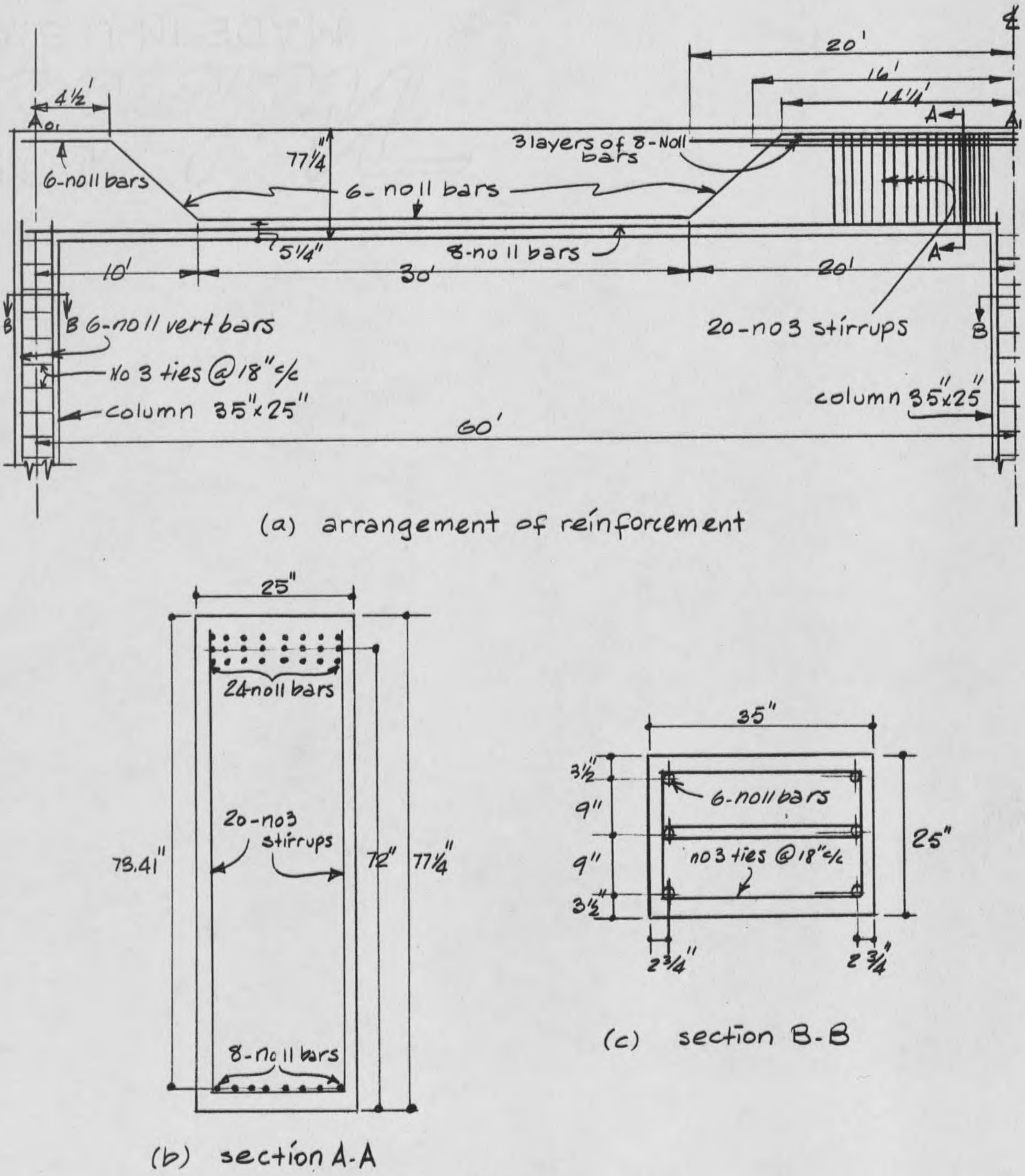


fig. 88.

PART III

COMPARISON

Of Resultant Structures of the Two Different Designs.

Results of the two different designs are tabulated in table VI. This table compares the cross sectional areas of the members and main reinforcements only.

table VI.
Results of the Two Different Designs

Member	Prestressed design		Conventional design		Savings in material in prestressed design	
	Concrete area in^2	Main steel area in^2	Concrete area in^2	Main steel area in^2	Concrete %	steel %
Roof slab	24/'	0.179	48/'	0.200	50.0	10.5
Sec. beam	52	0.477	85.5	1.560	39.2	69.4
Sec. girder	264	1.430	570.0	12.480	53.7	88.5
Main girder	885	5.722	1931.3	21.840	54.2	73.8
Column	1120	12.000	875.0	9.360	-28.0	-28.1

As seen from table VI, with the exception of columns, all members in prestressed design show comparatively large savings in concrete and steel. In prestressed design column sections of rigid frames could be decreased; however, a decrease in column sections would necessitate an increase in girder sections, possibly resulting in a less economical structure. Beams and girders of prestressed concrete design result in much smaller dead loads on columns which simply support them, thereby enabling the use of columns with smaller cross sections. Theoretically, due to light loads on the columns and large column sections used, no reinforcement is required for the columns of rigid frames of prestressed design; but, as required in the A.C.I. Building Code, a minimum steel area is provided in the design. In general, it can be said that columns of continuous frames in prestressed design result in heavier sections than the columns of continuous frames in conventional design. The differences in column sizes of both designs can be decreased to a minimum, but in few cases can the column sizes of rigid frames in prestressed design be smaller than the columns of rigid frames in conventional design. Large sized columns in prestressed rigid frames have some of the following advantages: they provide good dimensions for the end-blocks of beams or girders which is very important for the end anchorage of wires; they provide smaller beam or girder sections; they minimize the steel required which is very important in some countries where steel shortages exist.

Table VI shows that as the design of members progresses, from roof slab to main girder, an increasing saving of steel in prestressed design is obtained. The steel saving of 73.8 per cent in the main girder shows a decrease because an average of maximum and minimum main steel areas is used in conventional design.

The steel area required in prestressed design is much less than the steel area required in conventional design, but the cost will offset this large saving of steel in prestressed design because high-tensile steel used in prestressed concrete is more expensive than the low tensile steel used in conventional concrete. One of the reasons large steel areas are required in conventional design is that high-strength concrete is used. High strength concrete is desirable, even necessary with prestressed design, but in conventional design, using concrete of high-strength results in a smaller section, calling for more reinforcement and ends with a more costly design.

Using the same high-strength concrete in both designs enabled us to make a good comparison between them which resulted in savings up to 54.2 per cent concrete in the prestressed design.

I sections proved to be very economical in prestressed design, whereas, no advantage was observed in using I sections in conventional design. The rectangular shapes proved to be satisfactory in the latter design.

The reader will observe that more time and attention were required on prestressed design than on conventional design.

The designs indicate advantages in using prestressed concrete for long spans; also slenderness of members in prestressed concrete offers opportunity for artistic development.

As can be noted from the designs of different members in prestressed design, both steel and concrete are subjected during prestressing to high stresses. This means that materials which can stand prestressing are likely to be strong enough to carry service loads.

Another point observed from the two different designs is that shear in prestressed concrete is reduced because of the inclination of wires, diagonal tension is further reduced because of the existance of prestress.

Other points to be noted in the use of prestressed concrete are that, more auxiliary materials, such as end-anchorages, hydraulic jacks, etc., are required for prestressing; more formwork is needed since I shapes are used throughout the design of prestressed concrete; also because of the complication of the design, more labor and supervision will be required in the construction process. However, with the repetition of the units in the design the importance of this last consideration can be greatly reduced.

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