THE EVALUATION
OF A
MODERN ARITIMETIC PROGRAM

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CHAPTER I

INTRODUCTION

Since the founding of the public elementary schools in America very few mathematical principles of contemporary interest have been introduced into the arithmetic program.[1] However, content and method in the arithmetic are now gradually being revised by our schools.

Many mathematicians who are familiar with the subject matter of the elementary curriculum have long believed that many of the elementary schools are not teaching arithmetic, but rather they are teaching computation. It is believed by many that the teaching of computation has made students so answer-minded that many things that come before the answer are forgotten. This emphasis on the answer is apparently due to a mistaken idea, on the part of many directing education, of what arithmetic is all about and failure to realize that ability to solve problems involves more than facility in computation. A change of emphasis in the teaching of arithmetic is required if the program is to bring about the frequent use of quantitative thinking in everyday experiences.

The writer believes that the elementary arithmetic program should be revised both as to content and approach and that approach is of greater importance. It is because of the belief that the approach to the fundamentals and to problem solving is of great importance that led to a desire to investigate the advantages and disadvantages of the modern

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[1] VanEngen, Henry and Hartung, Maurice L., The Theory of Arithmetic and Its Teaching, Scott, Foresman and Co., Chicago, 1960, p. 7

method of the teaching of arithmetic as compared with the traditional method by determining if there is a difference in student achievement.

Statement of the Problem

This investigation attempted to determine whether the mathematical achievement of students taught by the traditional approach to arithmetic was less than that of students taught by a modern approach to arithmetic.

Procedure

The procedure for this investigation was to first review the literature. Questions were listed to use in an interview of individuals serving as curriculum directors, principals, superintendents, and teachers representing different school systems throughout the United States. The interview furnished information as to experiences in evaluating various arithmetic programs. A school was selected for the study as were grades three through six. A committee consisting of teachers and principals was selected and they in turn selected a program to be used in the teaching of arithmetic. A control group and a experimental group of students and participating teachers were also selected. Tests were selected and administered to all students to determine the placement of each student at the beginning of the study. The teachers taught the selected groups. Tests were again given to determine the placement of each student at the end of the study. All test scores, teacher and student evaluations were recorded, evaluated and discussed.

Limitations

Certain limitations are evident in this study. The study is limited to a period of two years. Furthermore, only one program was observed with limited opinions of other programs. The evaluation is dependent on opinions and the validity and reliability of the chosen tests administered on a small selected sample of students. The teachers of the experimental group, because of their willingness to participate in the pilot program, tended to be more enthusiastic than the teachers of the control group.

Definition of Terms

Because the term "traditional arithmetic" has many variations in meaning, for the purpose of this paper it has been defined as the method of teaching arithmetic that stresses mechanical computation for the primary purpose of obtaining the answer to an example or problem.

The"modern arithmetic" as used in this paper is a method of teaching that stresses reasoning by the individual student and directed analysis of the structure of the problem that lead to the understanding of the computation.

"Action research" is defined as research based directly on programs carried on in public school classrooms.

CHAPTER II

REVIEW OF LITERATURE

The purpose of the review of literature is to bring together some of the opinions and experiences of others that are pertinent to a modern arithmetic curriculum. Available research was also reviewed. This chapter gives a brief history of modern developments in arithmetic, a brief discussion of some of the well known new programs and some results of other modern programs. A brief history of modern arithmetic follows.

History of Modern Arithmetic

One of the first educators to point in the direction of modern arithmetic was Warren Colburn, whose textbook, Intellectual Arithmetic, was published in 1821. He believed that teachers and students should develop their own rules by proceeding inductively. For fifty years after the publication of Colburn's textbook it greatly influenced the teaching of arithmetic.

Toward the end of the nineteenth century arithmetic in our schools became very rigid and formal. Then, in an attempt to reduce the amount of subject matter, the pendulum swung the other way to where social utility was stressed and over stressed until the late 1920's.[1] The National Society for the Study of Education added some stability to the field of arithmetic with the publication of the 1930 yearbook. Again arithmetic started back to being taught for meaning.

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[1]Buswell, G. T., "Arithmetic", Encyclopedia of Educational Research, p. 64, 1960.

The move toward modern arithmetic was slowed, somewhat, by World War II but in turn was greatly accelerated by Government and other foundation grants to various study groups in the period following the end of the war.

Modern Programs in Elementary Mathematics

Although until recently few textbook companies did little more than play with the modern approach to arithmetic a great number of individuals and groups with government and private foundations behind them did much in the planning and development of the "new" curriculum. A brief discussion of some of the more well known programs follows.

Greater Cleveland Mathematics Program (GCMP). The program had its start in 1959 in Cleveland and has been used with over 100,000 pupils in twenty-four systems in Greater Cleveland. More recently it has been published as a textbook series by the Science Research Associates, Inc. The GCMP was developed by experts in mathematics working jointly with classroom teachers. It was written to be a self-motivating program by emphasizing the "discovery method". Much stress is placed on in-service training for teachers. The design of the program makes it useable at every level. Geometric shapes are introduced in kindergarten and sets are used to unify the arithmetic.[2]

Operations of numbers are developed as an outgrowth of operations on sets. Emphasis is placed on the discovery of patterns. A major portion of mathematics is concerned with making proof. The readiness for making proof is established in the early grades. They investigate

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[2]G.C.M.P. Elementary Mathematics Series, 1960.

whether certain patterns are particular, or whether they hold in general. "Counting Men" are used to show how base ten numeration system works. Heavy emphasis is placed upon the "language" of mathematics. Besides the "Counting Men" the GCMP makes use of such manipulative materials as bead frame, ten frame, and Math Discovery Board. "Discovery, challenge, and ease of creative imagination are the true hallmarks of the GCMP approach."[3]

School Mathematics Study Group. The School Mathematics Study Group (SMSG), one of the most widely known and used, was created in March, 1958. The group was directed by Dr. E. G. Begle of Yale, now of Stanford University. Although the group did not begin work on elementary grades until 1960 they have experimental units at all levels now. The group from the start has been made up of classroom teachers below the college level and college professors.[4]

Approximately 32,000 children in grades four through six are using the 1961-1962 SMSG materials.[5]

The SMSG mathematics is written so as to preserve and strengthen the pupil's native interest and enthusiasm in mathematics. In the texts the pupil is directed to perform experiments which are related to his present experience but at the same time provide for his future needs. The mathematics included in these texts is still usable and sound at the college level.[6] Careful attention was given to precise definitions,

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[3]The Arithmetic Teacher, B. H. Gundlach, "The Greater Cleveland Mathematics Program", 8: 192-195, April 1961.

[4]Moise, Edwin, "The New Mathematics Program", The School Review, 70-87, Spring 1962.

[5]Weaver, J. Fred, "The School Mathematics Study Group Project on Elementary School Mathematics: A Progress Report", The Arithmetic Teacher, 8:436-439, December 1961.

[6]Hess, Adrian, "Trends in The Teaching of Arithmetic", p 49.

vocabulary and to structure. The language of sets is introduced at the kindergarten. The pupil learns to talk and think in the language of sets. Geometric concepts are included in each of the grades K-3. The K-3 series however, is not intended to represent a complete program of instruction.[7]

Madison Project. The Madison Project was directed by Robert B. Davis of Syracuse University and is a two year program in algebra suitable for presentation to children as early as grade three. The course has been taught to gifted fourth, fifth graders and low I. Q. seventh graders. The program is characterized by the absence of exposition, concepts are introduced early, class conversations and no lectures, motivation by successful experiences, and the stressing of the "light" touch.[8]

Maryland Project. The Maryland Project is designed for use in the junior high program.

The precise use of language, where mathematical concepts are involved, is considered important. The differentiation between mathematical concepts and the physical world is built gradually. The two kinds of reasoning -- induction and deduction, are both encouraged. No special emphasis is put on learning vocabulary.[9] The SMSG used the results of the project in the earlier writing groups.

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[7]SMSG Center Personnel "The School Mathematics Study Group Project on Elementary School Mathematics, Grades K-3", The Arithmetic Teacher 9:459-461, December 1962.

[8]Smith, Krouse and Atkinson, "Mathematics Program" The Educators Encyclopedia, p. 310.

[9]Ibid., p. 311.

University of Illinois Committee. The University of Illinois Committee started as a high school committee but later formed an Illinois Arithmetic Project. Soon after the experiment started it became clear that two primary considerations must be kept in mind in the development of the course. These were (1) keep the language of the textbook and teacher as unambiguous as possible (2) encourage discovery of generalization by the students.[10]

Students are expected to figure out for themselves methods of solving many exercises without being given a rule and numerous examples by the text or by the teacher.

Geometry for Primary Grades. Geometry for Primary Grades was written by Newton Hawley and Patrick Suppes and published by L. W. Singer and Company and began in 1958.

The pupils made the common geometric constructions using a straightedge and compass. Instruction was given in the correct techniques for using the tools. No definitions were given in the book. The teacher explained the meaning of words the children did not understand. Individual help was given to those who needed it. There was a geometry session each school day for ten to twenty minutes. No formal tests were given to check on achievement. The work was intended for all students and not just for the "gifted". Students were encouraged to attempt solutions and to give reasons for rejecting incorrect solutions.[11]

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[10]UICSM Project Staff, "The University of Illinois School Mathematics Program", The School Review, 65:457, Winter 1957.

[11]Hess, Op. cit., p.7

Seeing Through Arithmetic. The first elementary arithmetic text to be published and widely used and without a grant from one of the numerous foundations was Seeing Through Arithmetic, written by Henry VanEngen, Maurice L. Hartung and others, and published by the Scott, Foresman and Company in 1954. This series stresses that the student look for the structure of the problem situation and to understand the process. It has the following characteristics:

1. It uses the equation as an algorithm from the beginning.
2. It introduces division with a form other than the traditional long division method. This is done to avoid the difficulty of estimating the number of times a divisor is contained in a given dividend and is designed to give better understanding of what actually takes place in division.
3. It introduces ratio and proportion in the fifth and sixth grade.
4. It teaches the solution of percentage problems by the use of proportions. This is introduced at the sixth grade level.

Although Seeing Through Arithmetic is considered by many, as much more conservative than the SMSG and other study groups, it was in use in the classroom before most of these study groups were in operation. It appears that other text book companies, now just beginning to publish modern forms of arithmetic text, are following the pattern set down by VanEngen and his co-authors.

"Strands" Report. The State of California issued a report on mathematics through their advisory committee on mathematics, entitled the "Strands" Report. This is being used as a guide for the adoption of text

books in the State of California. It solidifies the opinions of many about the modern arithmetic programs and in all probability will serve as a guide for future text book publications.

One publisher, Holt, Reinhart and Winston has already published a series in conformance with the California report.

In explaining the philosophy, that along with basic topics in the K-8 curriculum other material is necessary, the California Curriculum Committee stated:

> But these are not enough. There have been remarkable advances in this century in the clarity and precision of mathematical discourse. We have come to see that many apparently diverse notions are special cases of a few underlying concepts. We believe it essential that mathematical instruction bring some of this economy of thought to the student. Consequently, we recommend a planned and conscious development of certain strands which contribute to this clarity, precision and economy. These are the strands labeled Sets, Functions, Logic and Mathematical Sentences.[12]

Results of Some Modern Arithmetic Programs

There are very few published results based on action research of the modern arithmetic programs. Two of the results that were based on a comparison of traditional arithmetic and modern arithmetic were used in this study.

In recent evaluations of the SMSG text, conducted in Minneapolis and St. Paul experimental and control groups, were used and were tested with traditional tests.

There was no significant difference in the progress of the two

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[12]Curriculum Committee for The State of California "The "Strands" Report", 1962.

groups in enacting traditional work. However, the experimental group was exposed to a number of basic mathematical concepts which the control group did not have. It was believed by all who participated that the experiment was a success and gave evidence of materials such as SMSG over traditional texts.[13]

In an evaluation of topics in modern mathematics conducted by Hugh I. Peck,[14] control groups and experimental groups were used. The control group used traditional arithmetic while the experimental group used the Scott Foresman, Seeing Through Mathematics.

An evaluation of student achievement in arithmetic was made by matching students studying the new mathematics units on the basis of scores made on Otis Tests with randomly selected pupils throughout Kanawha County in the same grade.

A study of the data shows that there was very little difference found between the control and the experimental group.

The conclusion drawn points out that both groups performed on the same level in all areas tested.

Mr. Peck states:

> These data indicate that students in the new mathematics perform as well on tests designed to measure the traditional arithmetic as did students in the traditional arithmetic program. This would seem to refute the argument that the modern approach to mathematics does not provide competency in number combinations and in applications to problem solving.[15]

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[13]School Mathematics Study Group, Newsletter No. 15, April 1963.

[14]Peck, Hugh I., Director of Guidance Counseling and Testing for The Kanawha County Schools, Charleston, West Virginia.

[15]Peck, Hugh I., "An Evaluation of Topics in Modern Mathematics" The Arithmetic Teacher, 10:277-279, May 1963.

Both of the above evaluators agreed that more controlled evaluations need to be performed and carried out in more detail over a longer period of time.

The field of elementary arithmetic is in a state of flux but not enough large scale evaluations of the experimental programs are being conducted thus causing a delay in the shift from the traditional. The procedure for the evaluation of the arithmetic program for this study is discussed next.

CHAPTER III

PROCEDURE IN EVALUATING THE ARITHMETIC PROGRAM

The procedure in evaluating the arithmetic program was to review the program, administer tests and discuss the comments of the teachers and students.

The Program In Arithmetic

During the school year 1961-1962, a pilot program was conducted involving eight classes, eight teachers and two hundred twenty-three students, in two separate schools of the Helena, Montana School System. The classes chosen for the pilot program were determined by the willingness of the teacher to try a new program.

Because of the apparent success of the pilot program the new arithmetic series was placed in all classes and all schools for the school year 1962-1963.

To keep teacher problems at a minimum and improve their knowledge of arithmetic, several grade level meetings were held throughout the year to discuss problems and new concepts. Workshops were conducted by professional consultants, a college extension course was offered and individual conferences were held with teachers by principals and others capable of giving professional help. The schools' professional library was also improved in the field of mathematics. To help proper student attitude several public explanations of the program were made at Parent-Teacher Association meetings and through newspaper releases.

The teachers for the experimental group were selected on the

basis of their willingness to teach a pilot program and tended to be more enthusiastic than the teachers of the control group, otherwise both groups of teachers would be classified as at the same level of proficiency. The classroom procedure in teaching both control and experimental groups were the same in all details but the material presented.

Selecting Students To Participate In Evaluating The Arithmetic Program

Students participating in the pilot program were selected at random and it was this group, from one school, that was used as the experimental group.

To select students for the control group however, I. Q. tests had to be administered to all students in the school. The teacher evaluation of scholastic ability for each student was also compiled. The control group was selected by picking students outside of the pilot program with I. Q. scores and teacher evaluation of students scholastic ability as equal as possible to those in the experimental group. The differences between the experimental group and the control group were then compared.

Testing The New Arithmetic Program

Each year at the end of the first semester a standardized test was given to every student participating in the study. The test, Iowa Test of Basic Skills, has two sections for testing arithmetic. Section A-1 tests arithmetic concepts and Section A-2 tests problem solving. It

is designed for testing the traditional type of arithmetic.

Since the students in the elementary schools are not grouped by academic ability the students that were in the pilot classes the previous year did not stay together in classes the following year but were distributed throughout other classes in the school. However, for test score tabulation the students who had been in the pilot program were treated as one group by scoring their tests separately and recording their scores as a group.

Test scores used in this study were taken from the test given at the end of the first semester of 1961-1962 and the end of the first semester of 1962-1963.

The arithmetic achievement, for the period covering one year, of these two groups were then compared. The frequency of error in the various areas of arithmetic concepts and problem solving were also compared.

Student-Teacher Interviews On Reactions To The Arithmetic

At the end of the evaluation period each student of the experimental group was interviewed individually to determine student attitude and to tabulate student evaluations of the program.

The teachers were also interviewed for the purpose of determining attitudes and teachers' evaluations. Any problems that were revealed during the interviews were tabulated and classified.

Complete planning and a continuous evaluation along with adequate record keeping was an important part of this procedure. The data obtained by observing the program, the test results and the student-teacher interviews are discussed next.

CHAPTER IV

RESULTS OF EVALUATING THE ARITHMETIC PROGRAM

The results of evaluating the new arithmetic for this study were reviewed in this order: the reaction of the students and teachers to the new program, and the results of the tests that were administered.

Student-Teacher Reaction To The New Arithmetic

The students interviewed were asked two questions. 1. Do you like the new arithmetic more than you did the arithmetic you had before? 2. What is your favorite subject?

The response to the questions indicated that the students definately liked the new arithmetic better than the traditional.[1] The level of differences was determined by calculation of Chi squared[2] and converting Chi squared to the level by the use of appropriate table.[3] The fifth and seventh grade response showed significant differences at the .01 level, while in the sixth grade it was significant at the .001 level.[4]

For comparison of the response to the question of the favorite subject, a survey made in January, 1958, of 152 seventh grade students was used. 71 of the 152, or 47%, believed that arithmetic was their favorite subject. This compares with 37 of the 54 seventh grade students, or 69%, who believed arithmetic was their favorite subject in May, 1963.

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[1] Table 1, page 23

[2] $\chi^2 = \sum \frac{(O-E)^2}{E}$ where O = observed frequencies and E = expected frequencies.

[3] Downie & Heath, Basic Elementary Statistics.

[4] See footnote 2, page 16.

The comparison of these percentages proved to be significant at the .0015 level.[5]

The interview with the teachers showed the teachers who had been teaching the program for the second year were more enthusiastic than those who had taught it for the first time. However, no teacher listed any problems or dislikes other than a few minor administrative procedures.

The Differences Between the Experimental Group and the Control Group in I.Q. and Teacher Scholastic Evaluation of the Student

The differences between the experimental group and the control group in I.Q. and teacher scholastic evaluation of the student was considered not to be significant. A statistical analysis, by Chi squared, of the difference in teacher scholastic evaluation showed no significant difference between the control group and the experimental group.

The level of differences for the I.Q. scores was determined by calculating $\tilde{z}$ and reference to the appropriate tables.[6] Values of $\tilde{z}$ ran from .36 to .66 which determined that there was no significant difference between the two groups.

After observing that there was no significant difference between the two groups at the beginning of the study the achievement test results were analyzed next.

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[5]Downie & Heath, Basic Elementary Statistics, p. 76.

[6]$\tilde{z}$ scale used from Downie & Heath, Basic Elementary Statistics, p. 137.

Results of Testing the Two Arithmetic Programs

A statistical analysis of the results of testing the two arithmetic programs was discussed next. The Iowa Test of Basic Skills is constructed in such a way that by hand scoring and tabulating the errors it is possible to classify the errors as to the basic concepts with which they are connected.

With the exception of one concept in each of the fourth, sixth, and seventh grades, all areas of the test showed a significant difference in favor of the experimental group.[7] The concepts showing no significant differences as a result of a statistical analysis were: in the seventh grade, per cent; in the sixth grade, geometric figures; in the fourth grade, fractions. All other areas showed a difference in favor of the experimental group at least at the .10 level.[8]

The concept of whole numbers showed the greatest difference in all grades, being significant at the .001 level in the fourth, fifth, and sixth grades and at the .01 level in the seventh grade.[9] All grades showed a significant difference in favor of the experimental group in problem solving.

The new arithmetic used in this study definitely showed an advantage over the traditional in the opinion of the students and teachers and in the test results in the areas of problem solving and basic concepts. The conclusions that were drawn as the result of this study are discussed in the following section.

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[7] Table 2, page 24

[8] Chi square formula was used. See table 3, page 25

[9] Ibid

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The summary, conclusions and recommendations of this study, will emphasize the apparent need for a change in elementary arithmetic, the advantage of the modern arithmetic over the traditional and the need for more research.

Summary

It has been the purpose of this study to determine whether the mathematical achievement of students taught by the traditional approach to arithmetic was less than that of students taught by a modern approach to arithmetic.

Many have advocated a change in the traditional approaches and concepts in the teaching of arithmetic. Because of this, many experimental programs have been developed. The review of literature in this study discussed some of the established experimental programs and studies now being made. Although there seems to be agreement on most of the materials used in these programs, there is a lack of action research proving their worth. The evaluations that were reviewed indicated a preference for modern programs.

The procedure in evaluating the new program points out that the program should be planned in advance in every detail, both in execution and evaluation. After the arithmetic program was selected the students in the experimental and control groups were chosen by a careful comparison. After the teachers who had been assigned to the groups taught the prescribed

courses, tests were administered to both groups and the differences in scores analyzed.

The results of this study were significantly in favor of the new arithmetic over the traditional in all areas both in the opinions of those involved and the test results.

Conclusions

The four conclusions drawn from this study involve changes in arithmetic, a research proven program and the specific arithmetic program reviewed in this study.

The recent developments which have taken place in the mathematics curriculum are bringing about definite changes in elementary arithmetic. There are both additions to the basic concepts and changes in methods, in reasoning and algorisms.

What program will be accepted as the "one" has not, as yet, been determined. With the many different experimental programs now in existance and the many study groups that have reports on what constitutes a good mathematics program, it is believed that with some well developed action research to point the way, a program that will be uniformly accepted will gradually be developed.

It is also the conclusion of the writer than VanEngen's Seeing Through Arithmetic, the arithmetic program in this study, is proceeding in the right direction. The results of this study show that, at least in some situations, students in modern arithmetic have an advantage over students in traditional arithmetic for learning the basic concepts. It is conceded, however, that this advantage may be due in part to the simple

fact that the program is new and therefore met with more enthusiasm on the part of both teachers and students.

The writer also concludes that over-all procedure in initiating a new program is important if the desired results are to be attained and that the program used in this study is mathematically sound as well as acceptable to students and teachers.

Recommendations

It is recommended that further detailed and exhaustive evaluations over a longer period of time be made and that these evaluations be in the form of action research.

In the light of the results of this study it is further recommended that educators interested in the improvement of the curriculum realize that changes in mathematics and its teaching are inevitable and that they carefully review the new programs, materials, and services available and adopt a modern program suitable to their particular local needs.

APPENDIX

TABLE 1. STUDENT RESPONSES TO THE QUESTION: DO YOU LIKE THE NEW ARITHMETIC MORE THAN YOU DID THE ARITHMETIC YOU HAD BEFORE?

Group	Liked it better	Didn't like it as well	Didn't know	Were not asked
Grade 7	26	6	20	2
Grade 6	41	3	15	2
Grade 5	29	9	21	1

TABLE 2. RESULTS OF I. Q. TESTS, ACHIEVEMENT TESTS AND TEACHER EVALUATIONS OF STUDENTS IN THE ARITHMETIC PROGRAM

Group*	Number in group	Results of I. Q. Group Tests				Results of Group Achievement in Iowa Test of Basic Skills									Teacher Evaluation of scholastic achievement				
		High	Low	Mean	Std. deviation	Errors in problem solving	Number of errors in arithmetic concepts**												
							N	W	F	D	P	M	G	R	A	B	C	D	F
A_7	25	132	91	112	7.1	239	35	4	59	73	45	28	76	6	8	8	7	2	0
B_7	29	127	97	111	7.3	381	60	17	92	109	53	57	135	20	4	13	10	1	1
A_6	31	129	86	104	5.6	291	26	11	44	49		29	11	5	6	13	9	2	1
B_6	30	127	87	103.5	5.4	402	67	51	81	69		61	14	22	5	12	11		2
A_5	29	121	85	101	6.0	328	39	26	31			49		16	2	7	15	3	2
B_5	31	122	85	102	6.1	420	69	77	76			101		29	2	6	17	4	2
A_4	26	125	94	110	6.3	200	40	10	10			41			4	7	12	2	1
B_4	25	128	95	109	6.7	250	99	40	18			97			4	7	12	1	1

*Group A with $1\frac{1}{2}$ years of the new program
Group B $\frac{1}{2}$ year

**Areas in arithmetic concepts:
N - number system
W - whole numbers
F - fractions
D - decimals
P - per cents
M - standard measures
G - geometric figures

TABLE 3. CHI SQUARE VALUES USED IN DETERMINING THE SIGNIFICANCE OF THE DIFFERENCES IN ARITHMETIC TEST RESULTS TABULATED IN TABLE 2

Groups	Problem Solving	Arithmetic Concepts							
		N	W	F	D	P	M	G	R
	χ^2	χ^2							
A_7 - B_7	9.25	3.74	7.12	3.47	3.02	.01	3.33	9.45	6.26
A_6 - B_6	21.11	19.17	26.48	11.92	4.02		12.08	.47	11.04
A_5 - B_5	5.68	7.09	23.32	17.00			15.70		3.13
A_4 - B_4	5.11	26.73	18.63	2.59			24.78		

The degrees of freedom associated with each of the χ^2 computed is equal to one.

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