



X-ray observations and theoretical modeling of Active Galactic Nuclei
by Michael James Kellen

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in
Physics

Montana State University

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Abstract:

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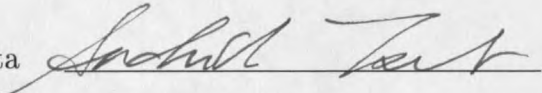
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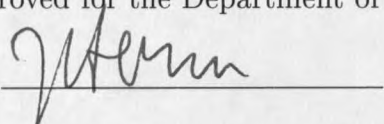
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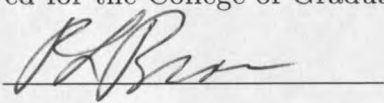
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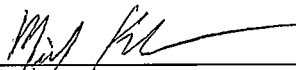
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ABSTRACT

The archival ASCA observation of the Seyfert 1 galaxy NGC 4593 is thoroughly analyzed across the entire calibrated X-ray band. In addition to the typical power law spectrum, several emission and absorption features are discovered. Temporal analysis of several of these features indicates significant variability in correlation to the variations in the continuum luminosity and each other. Several possible models are put forth to describe the spectrum.

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Chapter 1

INTRODUCTION

We begin with a review of the current observational and theoretical state-of-the-art in the study of the central engines of Active Galactic Nuclei (AGNs). First, we will present the definition and observational history of AGN. From there we will narrow our focus to the high energy central engines of these objects and the associated X-ray emissions. Next we will present currently viable theoretical models of the physical processes in the central engine and discuss their strengths and shortcomings. We shall then present an additional model which attempts to address these shortcomings.

1.1 The Active Galaxy

We review the discovery of the various varieties of active galaxies and how they are differentiated. Some of the more interesting problems in understanding them are pointed out. There are several features which are commonly accepted as distinguishing 'active' galaxies from 'normal' ones:

- A compact, bright galactic nucleus
- Rapid variations in luminosity
- A strong nonstellar continuum

The critereon of compactness is not always applied. Radio-loud active galaxies (discussed in section 1.1.2) are dominated by regions of radio emission which extend far outside the host galaxy.

1.1.1 History

Many radio sources were discovered in the late 1940s, but with the identification of a radio source with the extragalactic optical source Cygnus A (Baade & Minkowski, 1954), multiwavelength observations of active galaxies became a vital tool. Active galaxies have been observed in all wavelengths now, up to and including TeV (yes, erg!) gamma rays (T.C. Weekes, 1996).

The variability of some AGNs was used by Matthews & Sandage (1963) to place the first limits on the size of the optical emission region. The variability on time scales of weeks requires scales of light-weeks or less (by causality arguments). The energy density calculated from this scale and the observed luminosity made it impossible for this energy to be stellar in origin. Later discovery of intraday variations (Oke, 1967) pushed these energy densities still higher.

An early discovery in the field of X-ray astronomy was that much of the luminosity of active galaxies is emitted in the X-ray band (Elvis, *et al.*, 1978). This is in fact such a common feature of AGN that it can almost be considered a criterion for being "active".

1.1.2 The Active Galaxy Zoo

Active galaxies are often divided into two categories based upon their relative radio luminosity. These classes are radio loud and radio quiet. Radio loud active galaxies appear to have multiple sources of radio energy, generally a radio quiet "core" with an additional luminous, extended region of emission (Antonucci, 1993).

Radio loud objects can also be classified by the physical location of the

radio source relative to the galaxy. Some radio loud galaxies display kilo- to megaparsec scale lobes of radio emission, some symmetric, some distorted (apparently by the local intergalactic medium) (Jennison & Das Gupta, 1953). These galaxies are referred to as lobe-dominated. Other radio loud galaxies emit most of their radio luminosity from a small region close to the core of the galaxy. These are referred to as core-dominated.

Core-dominated radio loud active galaxies show a highly variable and polarized optical-near IR component. If this component dominates other optical spectral features, the object is referred to as a BL Lac object. If the spectrum sometimes displays broad emission lines, the object is classified as an Optically Violently Variable quasar (OVV). Both categories are collectively referred to as *blazars* (attributed to Spiegel, see Angel & Stockman, 1980). This optical/IR component is understood to be the high energy tail of the core radio process (Landau, *et al.*, 1986).

Lobe-dominated radio loud active galaxies, as well as radio quiet active galaxies, can be further subdivided into two types. Type 2 galaxies have spectra which show narrow permitted and forbidden emission lines. Line widths are usually of < 1000 km/s. Type 1 galaxies show additional broad emission lines with widths on the order of $10,000$ km/s. The relative line strengths in

type 1 galaxies indicate that optical depth effects are often important. Type 1 galaxies also have strong, variable nonthermal continua.

The lower luminosity (*ie*, closer) active galaxies can be seen to be spirals. These are classified as Seyfert galaxies, discussed in the next section. Higher luminosity AGN (usually at greater distances) can sometimes be identified as ellipticals, and are referred to as quasars (QSOs). It has also been argued that ultraluminous infrared galaxies are in fact highly luminous Type 2's, which are otherwise underrepresented.

Galaxies at the lowest "active" limit of core luminosity are Low Ionization Nuclear Emission Regions (LINER)s (Heckman, 1980). These are spirals which show some variable nuclear emission features. Photoionization models developed for Seyfert galaxies have been successfully applied (with lower ionization parameters) to these objects (Ferland & Netzer, 1983), suggesting that they are the weakest objects in the extended family of active galaxies.

1.1.3 Seyferts

Seyfert galaxies are defined in two general classes (Seyfert, 1943). Type 1 Seyferts are spiral galaxies with bright, point-like nuclei in the optical and X-ray bands. They have broad permitted emission line features in their

spectra in the optical and UV ($\delta v \geq 3000$ km/sec). Their optical continuum is dominated by nonthermal emission. Type 2 Seyferts show strong, narrow ($200 < \delta v < 2000$ km/sec) forbidden emission lines instead of the broad lines of the Type 1's. They also have little or no nonthermal component to their optical spectrum.

These two classes are not, in practice, quite so distinct. In fact, as we shall discuss later, most (but not all) of the differences can be reconciled with a single physical model. Several "transitional" classes are sometimes assigned (1.5, 1.8 and 1.9) based upon the relative strengths of the broad and narrow line components (Osterbrock & Matthews, 1986).

1.2 The Heart of the Matter

AGN are visible at many scales and in many parts of the electromagnetic spectrum. By studying the similarities and differences shown by different classes of active galaxies, we can try to formulate a model of the physical processes which drive them.

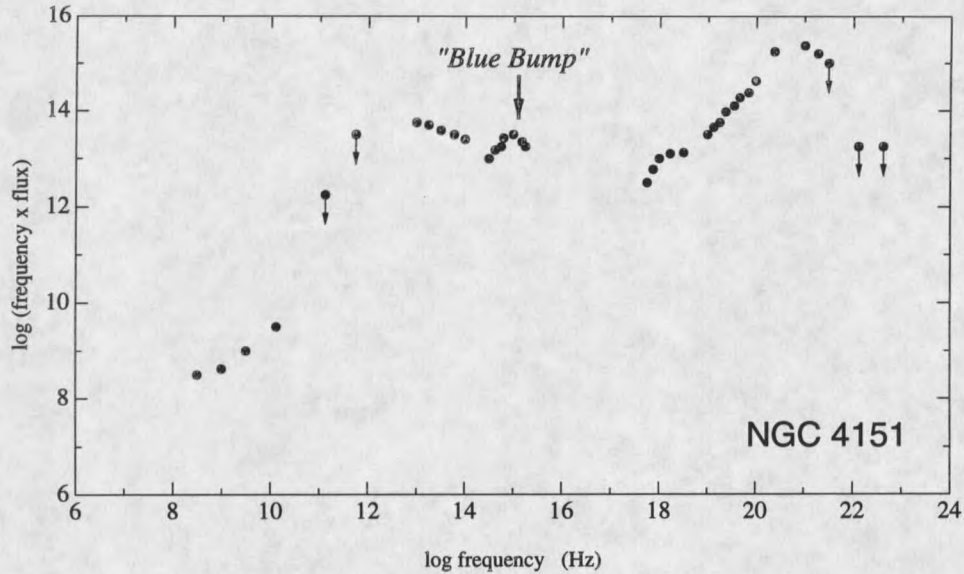


Figure 1.1: Multiwavelength Spectrum of a radio quiet active galaxy (after Schlickeiser, 1980)

1.2.1 Size and Energy

The intimate relationship between wavelength and energy allow us to use different wavelengths of light to study different scales in an AGN. The highest energies are emitted very close to the central engine, and so can be used to probe it. The rapid variations seen in the high energy emissions imply large scale changes on the order of hours or days, with correspondingly small characteristic distances determined by the light travel time.

Figure 1.1 shows the spectrum of a radio-quiet AGN from the radio to the hard X-ray. The x-axis indicates the log of the frequency, while the y-axis

indicates the log of the frequency times the flux at that frequency. These axes are chosen because the resulting plot has the property that equal areas under a curve represent equal luminosities. Equivalently, a horizontal line would indicate that equal amounts of energy are being emitted at each wavelength. One can also rescale the graph into units of log photon energy and log of photon energy times the flux at that energy without changing the shape of the plotted data.

In the optical and ultraviolet, we often see rapid variability (Optically Violently Variables being the most obvious example) as well as strong spectral features. In addition to Hydrogen emission lines, the visible-UV region of radio-quiet AGNs contains an excess of photons relative to the generally power law nature seen in other wavelengths. This so-called "blue bump" (indicated on the figure) can be fit by a thermal profile of a few tens of thousands of Kelvins (Pringle, 1981). Also visible in the optical are narrow linear structures referred to as jets (Curtis, 1918). These structures extend on the parsec scale in the visible.

Radio observations, on the other hand, are often used to study the kilo- to megaparsec scale radio jets and lobes of the radio-loud active galaxies (Miley, 1980). It should be stressed that the radio luminosity can be of roughly the

same order as that in higher energy bands, but is likely generated *in situ* by relativistic particles in a strong magnetic field (Wardel, et al., 1986; Jones, 1988; Begelman, et al., 1984).

1.2.2 Gross Properties of the X-ray Emission

The general character of the X-ray emission from active galaxies seems to fall into two categories, radio-loud and radio-quiet. Those AGN which are classified as radio-loud have stronger X-ray emission as well (Worall, *et al.*, 1987). Their spectra tend to lack the detailed features at higher energies (> 6 keV). In the unified model presented in section 1.3.2, we shall discuss the possible relation of relativistic jets of material to the distinction between these two types.

The X-ray emissions of radio-quiet AGN are characterized by rapid, high amplitude variability (Snyder, *et al.*, 1980). Changes happen on the order of days, hours (Tennant & Mushotzky, 1983) or even minutes (Kunieda, *et al.*, 1990) and be of very large amplitudes ($\delta I/I \sim 1$). They are also highly luminous in the X-ray, with 5-40% of their bolometric luminosity being emitted in this region of the spectrum (Ward, *et al.*, 1987).

The spectral shape of these radio-quiet AGN in the X-ray from 2-20 keV

has been known to be roughly a power law since the late 70's (Mushotzky, 1978a,b; Ives, *et al.*, 1976; Stark, *et al.*, 1977). These observations also showed the first detections of iron K-line (6.4 keV) emission. Samples with larger numbers of objects have since found that both of these features are common to radio-quiet AGN and that the photon spectral index is narrowly clustered around $\Gamma = 1.7$ (Mushotzky, *et al.*, 1980). The fact that less energy is observed at the lower end of the power law is attributed to absorption by a column of neutral Hydrogen:

$$N(E) = \text{const.} \times E^{-\Gamma} \times e^{-\sigma(E)N_H} \quad \text{photons/s/cm}^2/\text{keV} \quad (1.1)$$

where N_H is the column density and $\sigma(E)$ is the effective cross-section for neutral Hydrogen atoms.

These findings have continued to be seen as the observations have been extended and improved. Observations using the *Ginga* Large Area Counter (LAC) have shown that a significant fraction of all Seyferts have Fe K line features far in excess of the level to be expected from material in the line of sight (Matsuoka, *et al.*, 1990; Pounds, *et al.*, 1989; Nandra, 1991).

The *Ginga* spectra also showed a flattening in the power law spectrum above 8 keV in about 40% of observed Seyferts (Nandra, 1991). This has been

attributed to reprocessing of the intrinsic emitted spectrum by additional cold gas which either partially covers or reflects the incident spectrum (Guilbert & Rees, 1988; Lightman & White, 1988).

With the advent of *ASCA*, additional features have begun to be seen in the X-ray spectra. In particular, absorption and emission features in the soft X-ray (.5-1 keV) range have been seen in some objects. In a few objects, these features are even seen to be varying relative to the continuum. (George, *et al.*, 1997)

1.2.3 High Energy Spectral Features

Recent (OSSE) observations of the higher energy spectra of Seyfert galaxies (Madjeski, *et al.*, 1995; Zdziarski, *et al.*, 1995) have been used to place limits upon the shape of the X-ray spectrum of Seyferts above about 100 keV. The best fits found to these observations require an exponential cut-off in the power law at about 100-150 keV.

The same studies have placed upper limits upon the strength of any electron-positron annihilation line in the spectra of Seyferts. Nondetection requires that the generated line have an equivalent width of less than 300 keV.

1.3 The Central Engine

In attempting to understand the physical processes taking place in the central engine of AGNs, we are not faced with a completely blank slate. There are constraints upon the possible models placed by observational evidence as well as purely physical reasons to prefer one model to another.

1.3.1 Popular Conclusions

The natural curiosity associated with such a strong energy source with such a small characteristic size has made theorizing about the central engine of active Galaxies a major passtime for extragalactic astronomers. For some time, theoretical arguments from several sources (see, for example, Salpeter, 1964) have suggested that mass accretion onto supermassive black holes could be the explanation.

In addition to scale arguments, which push the energy densities beyond simple stellar generation, the observations of highly relativistic, even “superluminal” jets of material suggest relativistically compact generators (Bridle & Pearley, 1984). Accretion efficiencies and AGN luminosities predict black hole masses from between 10^6 and $10^{9.5} M_{\odot}$. Unfortunately, AGN are much less common now (at $z \leq 1$) than they have been in the past ($z \geq 2$). This

implies many dead or dormant AGN engines in nearby galaxies.

Rees (1996) presents several compelling observational arguments for the existence of supermassive black holes in the nuclei of other galaxies. The strongest is the precise mapping of gas motions in the core of NGC 4258 by observing maser emission from H_2O . The 1.3 cm emission can be resolved to within 1 km/sec and angular resolution better than 0.5 milliarc seconds. The observed rotational speeds describe Keplerian motion around a central mass of $3 \times 10^7 M_\odot$ (Watson & Wallin, 1994; Miyoshi, *et al.*, 1995).

Ford, *et al.* (1994) detected an orbiting disk of hot gas in M87 with rotational speeds of about 550 km/s, which requires a central mass on the order of $3 \times 10^9 M_\odot$. This is a much higher mass than can be accounted for by the observed stars in the inner region. Recent Hubble Space Telescope Imaging Spectrograph (STIS) observations have fairly conclusively detected the doppler signature of a supermassive black hole in the core of M84 (Bower, *et al.*, 1997). With rotational velocities of over 400 km/s within 26 light years of the central mass and clearly Keplerian motion, the calculated central mass is $3 \times 10^8 M_\odot$. This supplements an earlier survey of 27 nearby galaxies which detected massive black holes in three nearby normal galaxies (NGC 3379 (also known as M105), NGC 3377 and NGC 4486B, a companion galaxy to

M87) and concluded that massive (million or more $M_{\odot}$) black holes may exist in every galaxy, and that they may be an inevitable product of formation (Gebhardt, *et al.*, 1997). Radio observations of our own Galactic Center indicate a high mass central object ($\sim 10^6 M_{\odot}$ in the inner $\sim .1$ A.U.), with an orbiting disk or torus of gas and a thick molecular torus at a distance of about 200 pc (Morris & Serabyn, 1996). The interested reader is directed to the *Annual Review* article by Kormendy and Richstone (1995) for a more detailed discussion.

The existence of the iron fluorescent emission line at 6.4 keV implies a relatively large amount of nearly neutral material. The equivalent width (EW) of this line is sometimes quite broad ($50 < \text{EW} < 350$ eV) (Nandra, 1991). A thermal emission line at 6.7 keV can usually be excluded. The strength of the line requires that much of the solid angle seen by the source must be occupied by high column density gas ($> 10^{22}$ atoms/cm²) (George & Fabian, 1991).

By combining 12 *Ginga* spectra to increase the signal-to-noise, Pounds, *et al.* (1990) found a flattening of the X-ray spectral slope above about 10 keV. They also found evidence in some Seyferts for an absorption feature from ionized iron above 7 keV. Two major theories have been put forth to

explain these features, with the distinction being primarily one of viewing angle.

One possibility is that the intrinsic spectrum is reflected by an optically thick medium which covers a large solid angle for the source (Lightman & White, 1988; Guilbert & Rees, 1988; George & Fabian, 1991). The reflection is strongest at higher energies, because of the ionization energies of the medium. Above the energy of the iron absorption edges (around 7-9 keV), the reflection increases to peak at about 20-30 keV before being curtailed by Compton down-scattering and reduced opacity at higher energies. This model is attractive on several levels. We have a ready-made reflector, that being the accretion disk of the central black hole. The reflection process provides a method of exciting fluorescence of the iron line, as well as the depletion seen at 7-8 keV, due to the reduced reflectivity near the ionization edges of iron. Fits based upon this model find a steeper intrinsic X-ray power law ($\Gamma \sim 1.9$). The lower value which has been found observationally is attributed to our inability to resolve the high energy flattening until recently (Pounds, *et al*, 1990; Nandra, 1991).

The other possibility is that of partial covering of the source. If there is material which irregularly and incompletely obscures the line of sight, much

of the flux below 6 keV will be absorbed by the high column densities. Iron line fluorescence as well as iron absorption features in excess of those to be expected from the softer energy results are a by-product of this model. In this case, the intrinsic power law is reduced in flux below 6 keV, creating a “flattening” which is in fact due to the diminishing opacity at higher energies. A physical model which provides a source for this absorber involving spherical accretion was put forth by Sivron & Tsuruta (1993) and discussed in detail in Sivron (1995).

1.3.2 The “Standard Model”

An idealized model of active galaxies which unifies many of the different types has arisen from many sources, as summarized in Antonucci (1993). The differences in spectral features and variability are put forward to be due to differences in the orientation of a nonspherical system.

The so-called Standard Model is illustrated in Figure 1.2. It consists of a central engine, implicitly assumed to be a massive black hole with an accretion disk in the inner milliparsec or so of the galaxy. This engine is the source of our nonthermal continuum and our visible-UV excess (“blue bump”). This strong energy source also drives the broad emission lines,

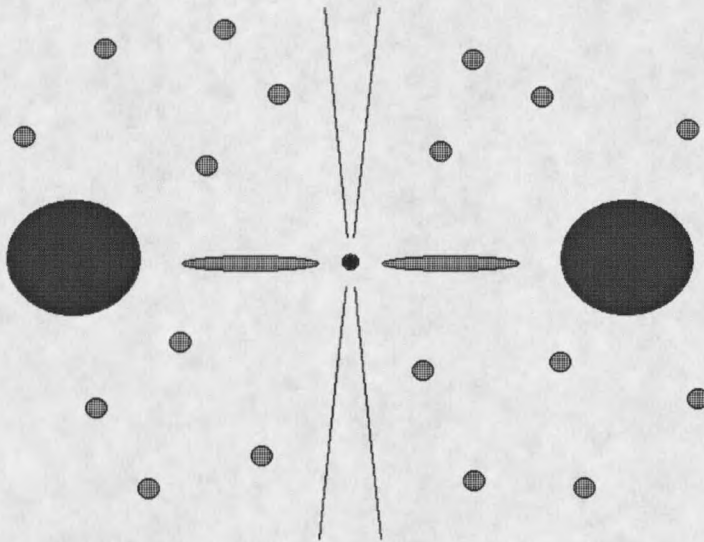


Figure 1.2: Schematic of the “Standard Model” for active galaxies

which are generated in the surrounding gas at about .1 to 1 pc from the engine. In addition to this, an optically thick torus is located at a greater distance (hundreds of parsecs) and roughly coplanar with the accretion disk, which obscures the central engine from certain lines of sight. Other, smaller, molecular clouds are posited to be scattered throughout the 10 to 100 parsec scale region to explain narrow emission line features. If the object exhibits a polar jet, that feature can be included in this model, and in fact is invoked in some cases as being nearly along the line of sight as an explanation of a highly variable and featureless spectrum, such as that seen in blazars.

These features are not simply pulled out of thin air. The reasonability of the black hole/accretion disk system was discussed earlier. The other major feature is the obscuring torus. This element has been derived historically from the attempt to unify the different types of Seyfert galaxies, but as we have seen, the distinction between Seyferts and other types of active galaxies has become blurred as observations have improved.

The lack of broad emission lines and weak to nonexistent featureless optical continua of Seyfert 2's can be attributed to one of two possibilities: either they are not present, or they are obscured from view. The latter possibility is the one which has been better supported by other evidence. Excesses of dust (IR) emission (Rowan-Robinson, 1977) support the existence of optically thick dust in some objects. More recently, Hubble Space Telescope observations show aligned dusty tori in some LINERs. Figure 1.3 shows the central few parsecs of NGC 4261 as seen by the HST. The detail image clearly shows a disc structure. Other observers have noted that the extrapolated nonthermal continuum is insufficient to generate the observed Ly α flux, implying that the narrow line emitting gas must be exposed to ionizing flux not visible to us (Neugebauer, *et al.*, 1980).

One final argument for this unifying model is based on NGC 1068. While

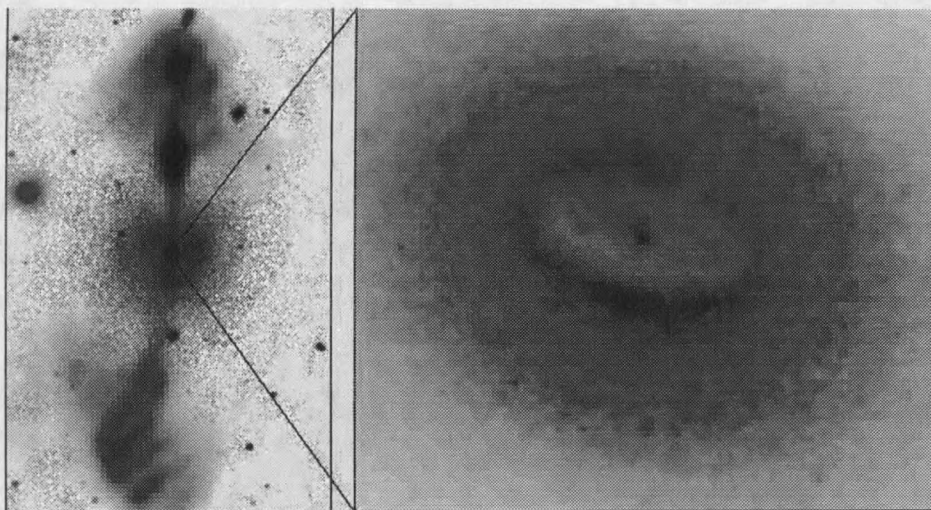


Figure 1.3: Dust Torus in NGC 4261.(Jaffe, *et al.*, 1993)

usually defined as a Seyfert 2 galaxy (but see Osterbrock & Matthews, 1986), if observed purely in polarized light, it has a solidly Seyfert 1 spectrum. This means that the spectrum of an obscured Seyfert 1 nucleus is being scattered into our line of sight by an anisotropically distributed medium (Antonucci & Miller, 1985). Wisconsin Ultraviolet Photo-Polarimeter Experiment (WUPPE) observations show the same level of wavelength-independent polarization in the UV spectrum (Code, *et al.*, 1993). This argues that the scattering is from free electrons rather than dust grains. This is consistent with the position angle of the polarization (perpendicular to the radio axis, Antonucci, 1983, Brindle, *et al.*, 1990 a & b), which places the scatterer along

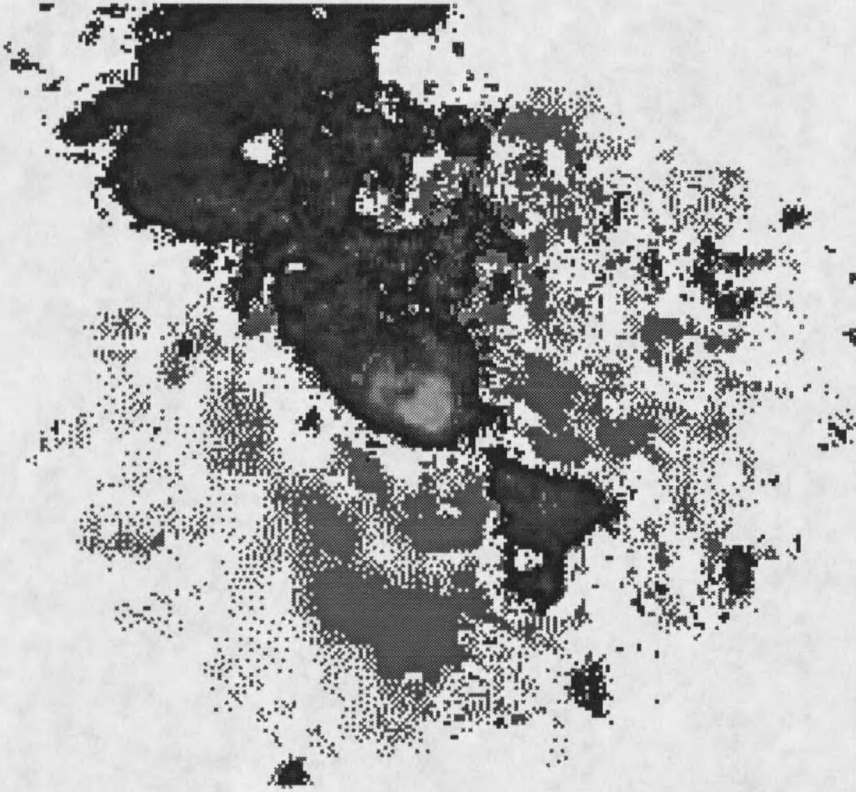


Figure 1.4: Ionization cones in NGC 5728

the polar axis of the central engine. Figure 1.4 is a Hubble Space Telescope image of the Seyfert 2 galaxy NCG 5728. The lighter grey areas indicate the plane of the host galaxy. The more detailed areas clearly show two conical regions of ionized emission in the UV, aligned with the jet axis. (see also, Evans, *et al.*, 1991)

1.3.3 Nonstandard Problems

The Standard Model is, of course, an idealization of what we actually expect is going on. We can expect reality to diverge from this model on a case by case basis. Some of the likely problems are worth mentioning for completeness.

It has been recognized for some time that the axis of the nuclear engine need not be (and most likely is not) aligned with the axis of the host galaxy. Recent work by Begelman (1996) has shown that precession instabilities exist even within the black hole/accretion disk system. So our naive coaxial model is in fact unlikely.

Observations of transitional and variable Seyferts ($>1.5 - 2$) sometimes show less polarization in the nonthermal continuum than in the (reflected) broad lines (Tran, *et al.*, 1992). This may imply extension of the nonthermal continuum into the line of sight (ie, above the obscuring torus). This could imply either an extended coronal emitter or a thinner obscuring torus.

Other authors have suggested that there may be two different classes of Seyfert 2 (Neff & Hutchings, 1992; Moran, *et al.*, 1992). The former authors put forth that Seyfert 2s occupy disjoint regions on a far IR color-magnitude diagram. The latter group notes that in all cases for which a hidden Seyfert 1 has been claimed, the object has a high radio luminosity and so may not

represent less radio-loud objects.

In the region of the spectrum of greatest interest here, namely the X-ray band, Mulchaey, *et al.* (1992) found a general support for this type of model, but found that there were fewer high luminosity Seyfert 2's than should be expected based upon the Seyfert 1 population. If one allows the obscuring torus to be dense enough to obscure even the X-ray, this can be somewhat ameliorated. They also conclude that the opening angle of the torus is probably greater than the canonical 30° in higher luminosity Seyferts. The value they find based upon polarization data, however, conflicts with the value found by statistical argument based upon the relative densities of Type 1 and 2 Seyferts. (Lawrence, 1991)

1.4 Non-homogeneous Modeling

The theoretical focus of this paper is on modeling the central engine of AGN. Many of the difficulties encountered by theorists can in fact be eliminated by taking into account an element which is often neglected computationally, despite its obvious presence in the "Standard Model". That feature is geometry. In particular, most models to date have been isotropic and ho-

mogeneous. As we have just seen, neither condition is realistic. Also, if we wish to distinguish between the cases of reflection and partial covering, each intrinsically nonisotropic and/or nonspherical, we will need to include these effects.

1.4.1 Models To-date

Compton scattering of soft UV photons by energetic electrons and positrons is a promising model of the X through γ -ray emission mechanism in radio-quiet AGNs. The energetic particles can either be assumed to have been produced via a thermal process (Sunyaev & Titarchuk, 1986; Lightman, 1982) or non-thermally (Guilbert, Fabian & Rees, 1983; Fabian, et al., 1986; Lightman & Zdziarski, 1987; Done & Fabian, 1989; Zdziarski, et al., 1990; Coppi & Blandford, 1990; Zdziarski & Coppi, 1991). Thermal models without pair production have also been explored (Bjornsson & Svensson, 1992) with some success.

The nonthermal production process is energetically favored when it is possible, but the models presented in the above references have all failed to satisfy observational constraints placed by the observed γ -ray background and the unobserved e^-e^+ annihilation line. This has led people to consider

thermal pair production models to be more viable. (Haardt & Maraschi, 1991)

1.4.2 Model to be presented

Tritz (1990) and Tsuruta & Kellen (1995) applied a simple one-half dimensional model in which the problem of the nonthermal pair cascade was re-examined. The consideration of the effects of geometry upon the resultant spectrum produced a markedly different result from what had previously been seen.

The scheme presents a radiative transport model in which the energy injection all occurs at the base of the transport atmosphere. A physical example would be injection of energy from an accretion disk into the surrounding medium via shocks. The model only calculated the effects of linear, outward streaming radiation (hence, half-dimensional). The density of electron positron pairs above the injection source was determined entirely by detailed balance calculations at each altitude. The resultant optical depth calculated was not high enough to conflict with the assumption of outward streaming, but was high enough to suggest that reflection effects should be added to the model. This would also allow comparison with the hypothesis that the iron line features and break in the power law at about 7 keV (the hard X-ray

“bump”, see section 1.3.1) are a result of reflection of the power law spectrum off of a cold disk.

The extension of this model to full self-consistency is the thrust of the theoretical research presented here. Chapter 3 will detail the physical concepts to be included in the calculations, while chapter 4 will outline the computational algorithm and the results it produces.

1.5 Back in the Real World

All of this is very nice, but we need ultimately to be able to connect it back up with observation. We shall begin with a detailed analysis of the *ASCA* observation of the Seyfert 1 galaxy NGC 4593 in chapter 2. After this, we will return to the problem of high energy radiative transport in the central engine of an AGN and develop a model which is inherently better suited to our assumptions about the physical makeup of this region. We will use this model to construct a set of algorithms to calculate the emergent spectrum. Finally, the model will be compared to observational results.

Chapter 2

ANALYSIS OF NGC 4593

We start by defining the tools and methods to be applied in the analysis. Some of the characteristics of the instrument are discussed, and the results of previous analyses summarized. Next the time-dependant behaviour of the object is analyzed and used to suggest areas of interest in the analysis of the X-ray spectrum. Finally, a detailed analysis of the X-ray spectrum is presented, with specific attention to those components in the soft X-ray (< 1 keV) band.

The XSPEC data analysis package which was used in the modeling of this source underwent three major revisions during the course of the analysis. Two of those were translated by the author to local hardware which was

unsupported by NASA, in collaboration with Keith Arnaud, primary author of the package.

2.1 Notes on Statistics

The low number of photons received in the X-ray from an average Seyfert, the inability to clearly resolve most of the spectral features and the short integration times available, force us to obtain the details of an X-ray spectrum by constructing a model which is then fit to the observation. We then vary the free parameters of the model in an attempt to find a model which most accurately reflects the observation. Under our assumption that the only variations of the observation from the "correct" model are due to noise, we make a statistical comparison between the two and attempt to minimize this statistic. Exactly which comparison to use is determined by our knowledge (or estimate) of the nature of the random deviations.

2.1.1 Chi-Squared: Gaussian Noise

The χ^2 statistic is widely used in modeling because it is well understood and provides an estimate of the quality of the fit in addition to setting the free

parameter(s). It derives from the assumption of a Gaussian distribution of values around an expectation value, with a width σ_i for each observational bin:

$$\chi^2 = \sum_{i=1}^N \frac{[(y_i) - y(x_i)]^2}{\sigma_i^2} \quad (2.1)$$

where N is the number of bins, y_i is the observed count rate in the bin, $y(x_i)$ is the number of counts predicted by the model for that bin and σ_i is the estimated uncertainty of the measurement.

The χ^2 statistic compares the variance of two distributions under the assumption that their individual fluctuations are Gaussian. If the two agree, $\chi^2 \sim N$ and the model is said to fit the data. One simply attempts to find the best fit (ie, lowest value of χ^2), bearing in mind that values of χ^2 which are much below N may not be trustworthy, as they imply that our errors are smaller than σ_i .

The problem with this approach is that we must have a sufficient number of photons available to justify our assumption of Gaussian statistics (about 15 to 20 per bin). This can be difficult to achieve in the higher energy bins, even over a long observation. This is often done by combining energy bins until enough counts are in each, but naturally this reduces our spectral

resolution.

2.1.2 The C Statistic: Poisson Noise

If the count rate is low, Poissonian statistics are a better estimator. This is the basis of the "C statistic" (Cash, 1979):

$$C = 2 \sum_{i=1}^N (y(x_i) - y_i \log(y(x_i)) + \log(y_i)!) \quad (2.2)$$

with the same meanings as in the previous section.

The minimum of the C statistic indicates the maximum likelihood that the data were produced by the selected model. This means that one can only determine the best possible fit with the given model. There is no way to determine if the derived model even vaguely resembles the data. We are merely promised that this is the best possible set of parameters for this model.

Another problem with the C statistic is that we cannot simply subtract the background from the observation without invalidating our Poissonian statistics. We must model the background as well as the data, adding many more parameters to the fitting process.

Confidence	Change in Fit Statistic			
	1 param	2 params	3 params	4 params
.20	0.06	0.45	1.00	1.65
.68	1.00	2.30	3.50	4.70
.90	2.71	4.61	6.25	7.78
.99	6.63	9.21	11.3	13.3

Table 2.1: Relation of Confidence to the Change of Statistic

(Lampton, Margon and Bowyer, 1976)

2.1.3 Confidence Ranges

Several different models may each give a good fit to the observation, or, in the case of the C statistic, we may not be able to determine exactly how good our fit is. In these cases, our primary interest is directed to the range of parameter space over which our fit is acceptable (*ie*, the best available for the model). This is sometimes expressed in terms of *confidence intervals*.

The confidence level is the probability that the "correct" value falls within the error range defined by a change in the fit statistic. Some commonly used confidence levels are listed in table 2.1 ($\Delta\chi^2$ has the same probability distribution as ΔC , so this table applies to either). One moves away from

the “best” value until the fit statistic changes by the indicated amount for the number of variable parameters. For a single parameter, 3σ is equivalent to a change in fit statistic of 9.00 or a confidence of about 97%. A one σ error range is a 90% confidence level.

2.1.4 The Right Tool

From these two choices we must determine which fit statistic we wish to use for our analysis of NGC 4593. Each has its own strengths and weaknesses. The requirement of rebinning to use χ^2 means we may lose some high energy spectral sensitivity. The requirement of modeling the background for the use of the C statistic adds complications. For several reasons, χ^2 proves to be the correct choice.

We shall shortly see that the X-ray background has a significant effect upon the observed spectrum in the higher energy bins. The requirement that we model this background would greatly complicate our efforts.

Our observation contains several channels which received no counts. Trying to fit to a model to a zero value will cause the C statistic to become singular ($\log 0$). In practice, this slows the fitting procedure drastically.

Our primary interest lies in determining if the models presented are, in

fact, a good representation of the data. Only χ^2 can allow us to compare the validity of different models.

Finally, comparison of the binned to unbinned data shows that no significant features are lost in rebinning. The heaviest rebinning takes place in the 5-10 keV range, in which the primary spectral feature is the Fe K line. This line is clearly apparent after rebinning. A minor *caveat* here is that by definition, absorption features will be areas of lower counts. To avoid rebinning these features away, we group by a regular number of channels (increasing at higher energies to account for the fewer number of events) rather than simply grouping *ad hoc* to 20 counts.

2.2 Previous Analyses

Nandra, *et al.* (1997) included data from this same observation in their survey of relativistic effects in Seyfert 1 galaxies. Their study focused more strongly upon the average spectral features of eighteen Seyfert galaxies and only the high energy (> 3 keV) fitting was reported. Where possible, the values found in their fitting have been used for comparison.

Santos-LLeó, *et al.* (1994) performed multifrequency monitoring of NGC

4593 using EXOSAT observations in the X-ray. Their focus was most heavily upon the broadband spectral energy distribution, and little more was done with the X-ray data than to constrain the X-ray power law to $\Gamma \sim 1.75$.

2.3 The Observation

The Japanese X-ray observing satellite ASCA was used to gather the data to be presented here. ASCA has four nested foil X-ray telescopes with an effective area of 1300 cm^2 at 1 keV and an angular resolution of 2.9 arc min. Two of the telescopes are used by X-ray Charge-Coupled Devices (CCDs) referred to as the Solid State Imaging Spectrometers (SIS). The other two are used by Gas Scintillation Imaging Spectrometers (GIS). The SIS detectors are designed and calibrated to detect X-rays in the energy range between .4 and 10 keV with a resolution of 50 eV at 5.9 keV. The GIS detectors are effective from 1 to 12 keV and have an energy resolution of 12 eV at 5.9 keV.

NGC 4593 was observed by *ASCA* on January 9 and 10, 1994 for 98 ksec using all four of the available instruments. Data was taken in both the MEDIUM and HIGH telemetry rates. The object is classified as a type 1 Seyfert galaxy and is located at $12^h37^m05^s - 5.344^\circ$. The principle investi-

gator for the observation was Dr. Hideyo Kunieda.

The standard criteria for the rejection of bad data (angle from the earth, cosmic ray coincidence, CCD overflow, passage through the South Atlantic Anomaly, known damaged pixels) were applied to the raw data, which were then extracted for analysis using the FTOOLS (v 3.5)/XSELECT (v 1.3) package. Data from both the SIS and GIS detectors were used in gain-corrected (pulse invariant, or PI) mode. The timing analysis was performed using the abovementioned FTOOLS package, while the spectral fitting package was XSPEC (v 9.0). None of these tools is native to the hardware used, but the results of the software port have been confirmed using the Guest Observer Facility at NASA's Goddard Space Flight Center.

The HEASARC calibration database provided the response matrices for the instruments used. For the SIS detectors, the 1994 November 9 matrices were used. For the GIS detectors, the 1995 March 6 release was used. Each of these are the most recent available. The relative normalizations of each detector are influenced by the exact position of the source in the focal plane. This requires fits performed with multiple detectors to include a multiplicative constant which was determined by fitting.

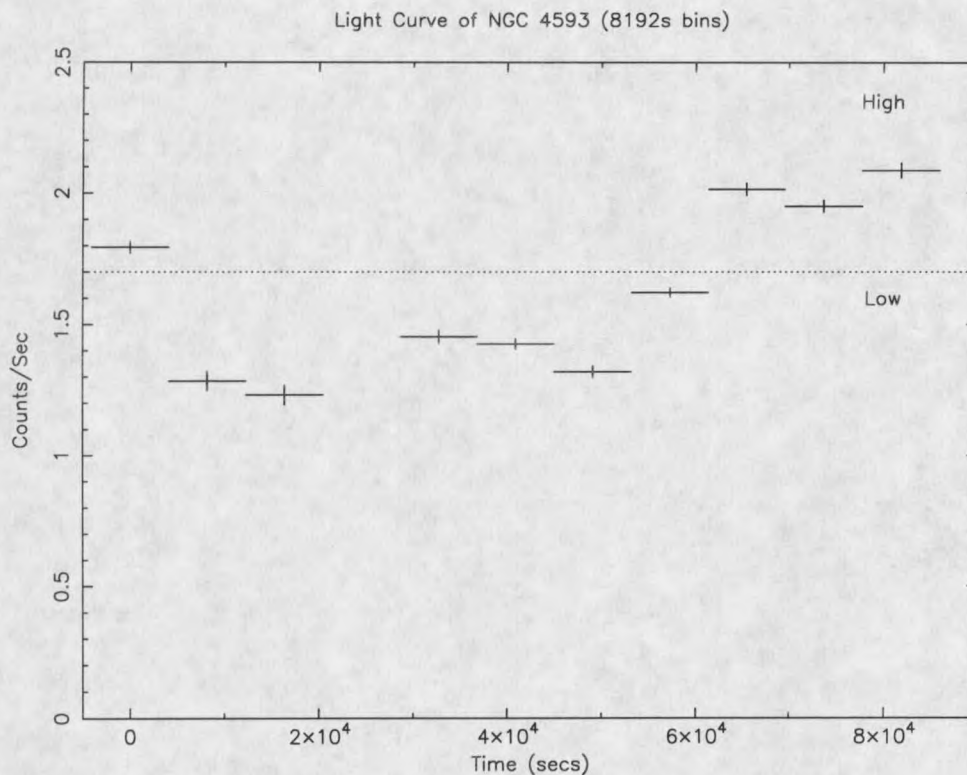


Figure 2.1: The light curve of NGC 4593, binned in 8192 second intervals.

2.4 Timing Analysis

The light curve of NGC 4593 is shown in Figure 2.1. The data have been combined from the original integration time of a few seconds into 4 ksec bins. The x-axis of the figure is the time in seconds from the beginning of the observation. The y-axis indicates the average number of photon counts per second in the bin. The figure shows that the luminosity varied strongly over time scales of just a few hours during the course of the observation,

from a low of about 1.25 counts/sec to almost twice that value. The dotted line indicates a artificial division of the data set into two luminosity states. Separate analyses of these states will provide a useful insight into the changes in the spectrum of the object with the continuum luminosity.

It is interesting to examine whether the variability seen is present in all energy bands or is a function of the X-ray energy. Figure 2.2 shows the light curve of NGC 4593 in several different energy bands. The axes are the same as in the previous figure. The top box in the figure shows the light curve of the hardest X-rays (7-10 keV) and is effectively constant throughout the observation. The next box shows the variations with time of the medium (3-5 keV) X-rays. This curve shows some variations of the same general shape as the overall light curve (shown in the bottom box for comparison). The third box shows the light curve of the soft X-rays (less than 1 keV). This band accounts for the majority of the counts and displays the continuum variations most strongly. This will guide our attention in the next section, where we look more closely at the components of the X-ray spectrum. There is no significant spectral variability shown in the comparison of the medium and high energy bands, indicating a relatively fixed power law.

Analysis of the power spectrum of the curve showed no significant regular

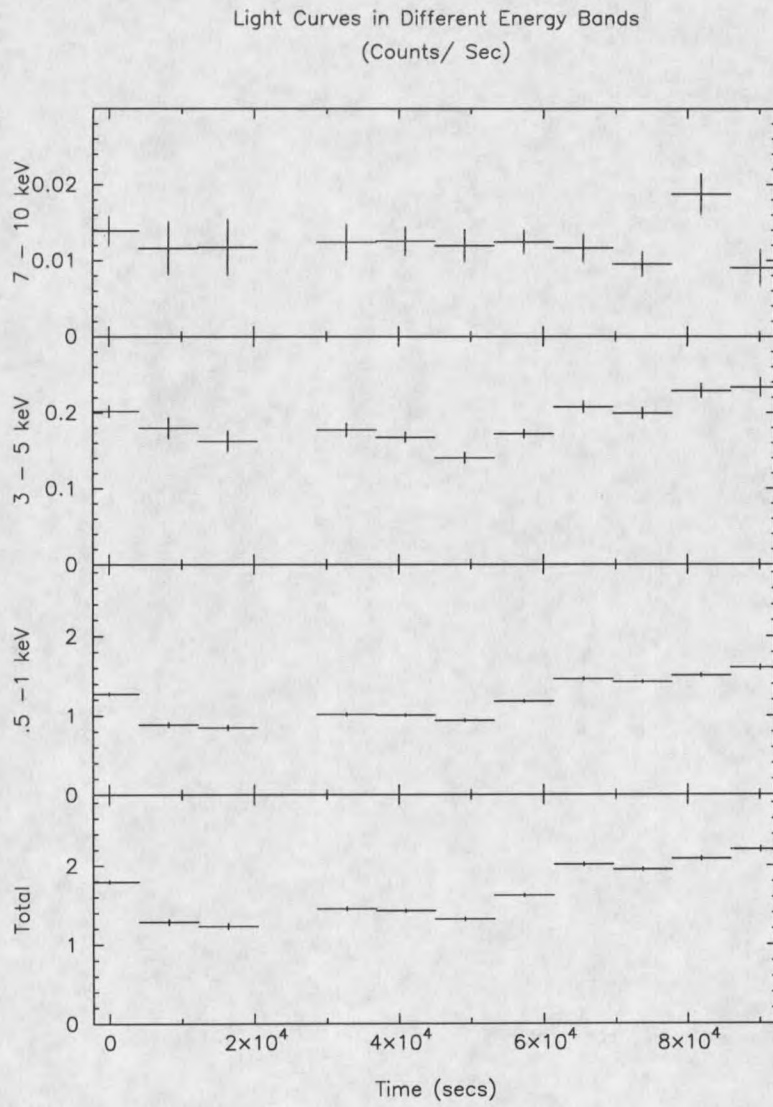


Figure 2.2: Light curve in various X-ray bands

variations (other than the satellite's orbit). This is not unexpected, as no regular variability has yet been seen in an AGN.

In addition to the simple morphological division into a high and low state, it can be useful to consider phenomenological divisions of the data. By dividing the light curve up on the basis of increasing, decreasing or (relatively) constant flux, we can study dynamic rather than averaged quantities. This analysis method was suggested by Kunieda, and will prove to be of great importance in section 2.5.3.3 when we study time-dependant spectral features.

2.5 Spectral Analysis

The analysis of the X-ray spectrum will be broken into several parts. First, we must examine the effects of non-source X-rays upon the observation to insure that our analysis is not contaminated by unrelated phenomena. Then we shall model the hard and medium X-ray regime (>2 keV). After that we will analyze the soft X-ray (< 2 keV) observations using two complementary methods. Finally, we shall discuss the conclusions which can be drawn from the models found.

2.5.1 Background

Although NGC 4593 is a relatively bright AGN, it still gives us only a couple of photon events each second in the X-ray band. This makes the careful elimination of background (non-source) photons a vital step in the analysis of the spectrum. There are two distinct methods for estimating the background X-ray flux.

One method is to use the "average" blank field image. The majority of the counts in these fields are measured by pointing the instrument at the Earth, which is opaque to any sources and so the only events gathered will be non-source events. This method is commonly used, but is not really valid if the X-ray background in the direction of the observed object is large. The same physical section of the detector is used for the background measurements as was selected for the observation to avoid differences in telescope sensitivity.

The other method is to use a "near sky" observation. Either the instrument is repointed at an empty part of the sky near the observed object, or data from an empty region in the same observation is taken. The sensitivity of the instrument varies across the field, therefore care must be taken to select a region with similar response and sensitivity to that of the region in which the image of the object lies. An example near sky background region

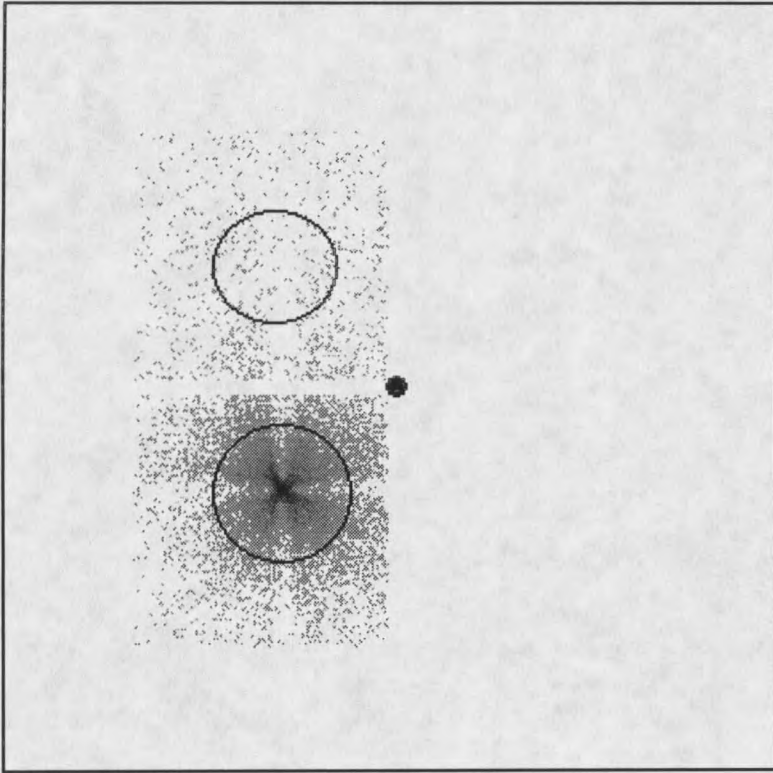


Figure 2.3: SAOimage plot indicating object and near sky background regions selection is shown in Figure 2.3. The figure shows an unclipped SIS 0 image. The center of the focal plane is indicated by the large black dot in the center of the frame. The lower circle is an example of selected region of the source, while the upper circle is an example near sky background.

As was noted earlier, the “blank sky” method requires that your object be in a relatively X-ray quiet part of the sky. Figure 2.4, from SkyView¹,

¹<http://skyview.gsfc.nasa.gov/>

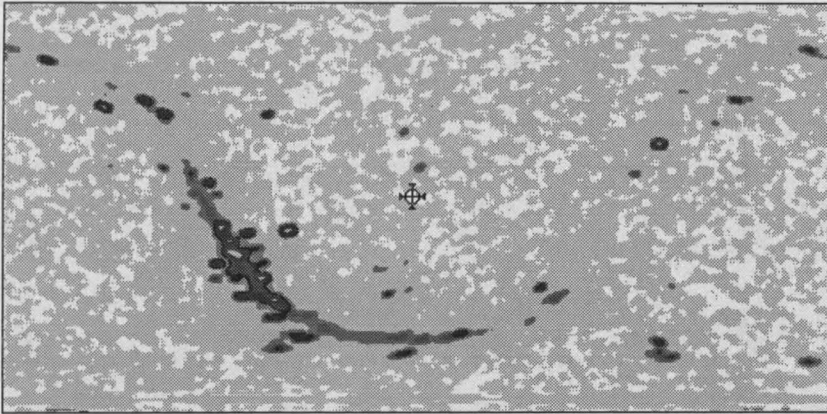


Figure 2.4: The Diffuse X-ray background. NGC 4593 at center.

shows a survey of the X-ray sky. NGC 4593 is indicated by the cross hairs at the center. The darker regions in the figure indicate regions of the sky with a higher X-ray background. As we can see, the X-ray background is relatively low in this area of the sky.

The background may be low, but its effects are certainly not negligible. Figures 2.5 a and b show comparisons of the spectra seen by the SIS 0 (on the left) and GIS 2 (on the right) instruments before (upper line) and after (lower line) background subtraction. The x-axis is the channel energy, while the y-axis is the flux in that channel times the energy of the channel. The SIS data is fairly unchanged by this subtraction, reflecting the lower sensitivity of these instruments to the higher energy X-rays. The greater sensitivity of the

GIS detectors at high energies makes for a radical change in spectral shape seen when the background is subtracted.

As a final check, the two backgrounds obtained were compared. As each should be random noise, no general differences in the spectrum should be seen if both methods are valid for this observation. As we can see in Figure 2.6, this is not the case. Each panel displays the background spectrum obtained for a single detector. In each, the solid markers indicate the near sky background, while the dotted ones indicate the blank sky background. The x-axis shows the channel energy, while the y-axis indicates the number of counts per second in that channel. The two methods are similar at high energies, but in the soft X-ray we see a great deal more flux in the near sky background in most of the detectors. In the following analysis, we shall in general use the near sky background. This is felt to be a more conservative approach, as it is based strictly upon observation. Occasional comparisons with the blank sky background will be made where appropriate.

2.5.2 The Hard X-ray Spectrum

It is customary if not compulsory to begin the analysis of the spectrum of a Seyfert galaxy by examining the hard X-ray (> 2 keV) portion. The

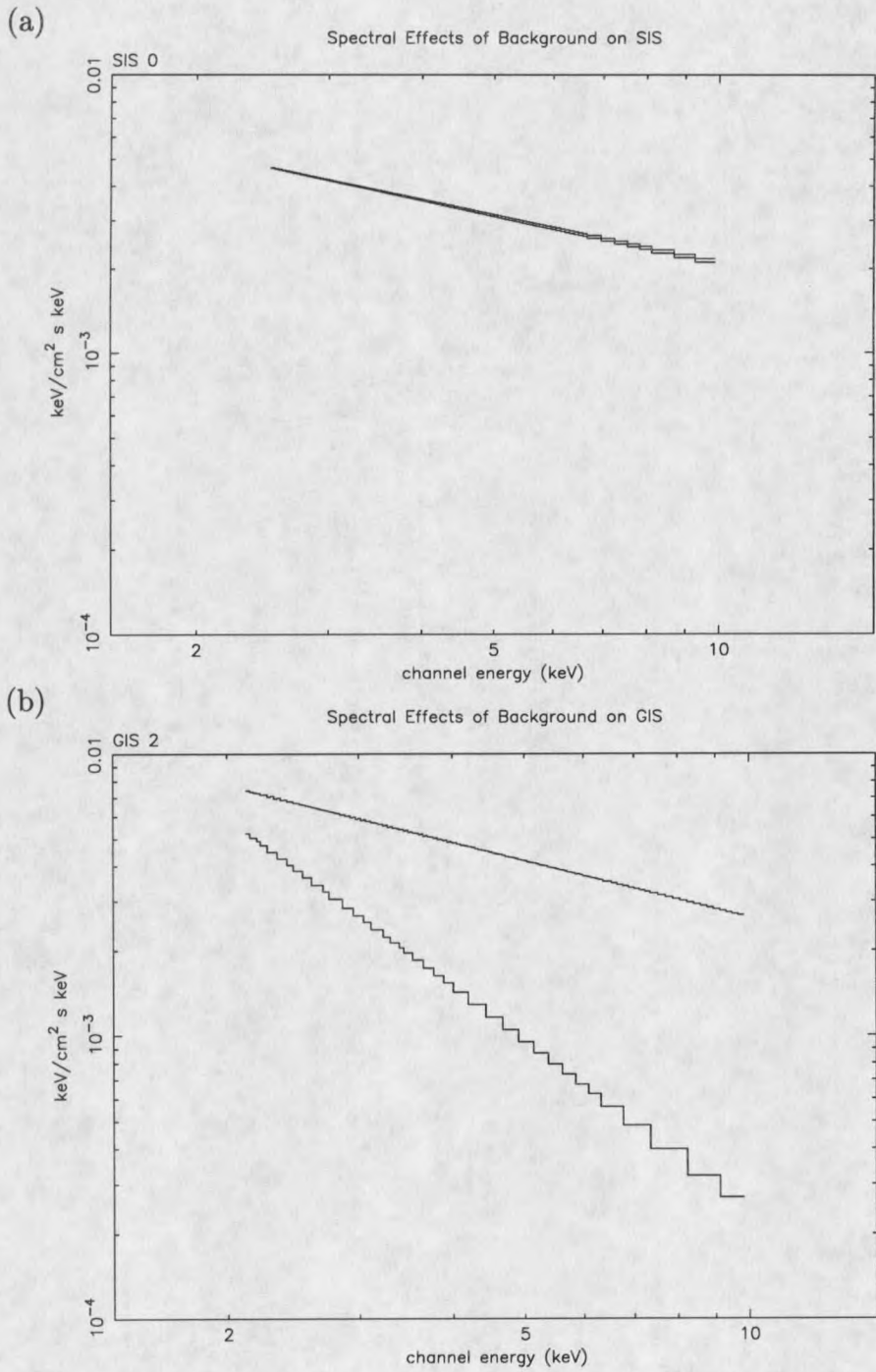


Figure 2.5: Comparison of detectors, with and without backgrounds

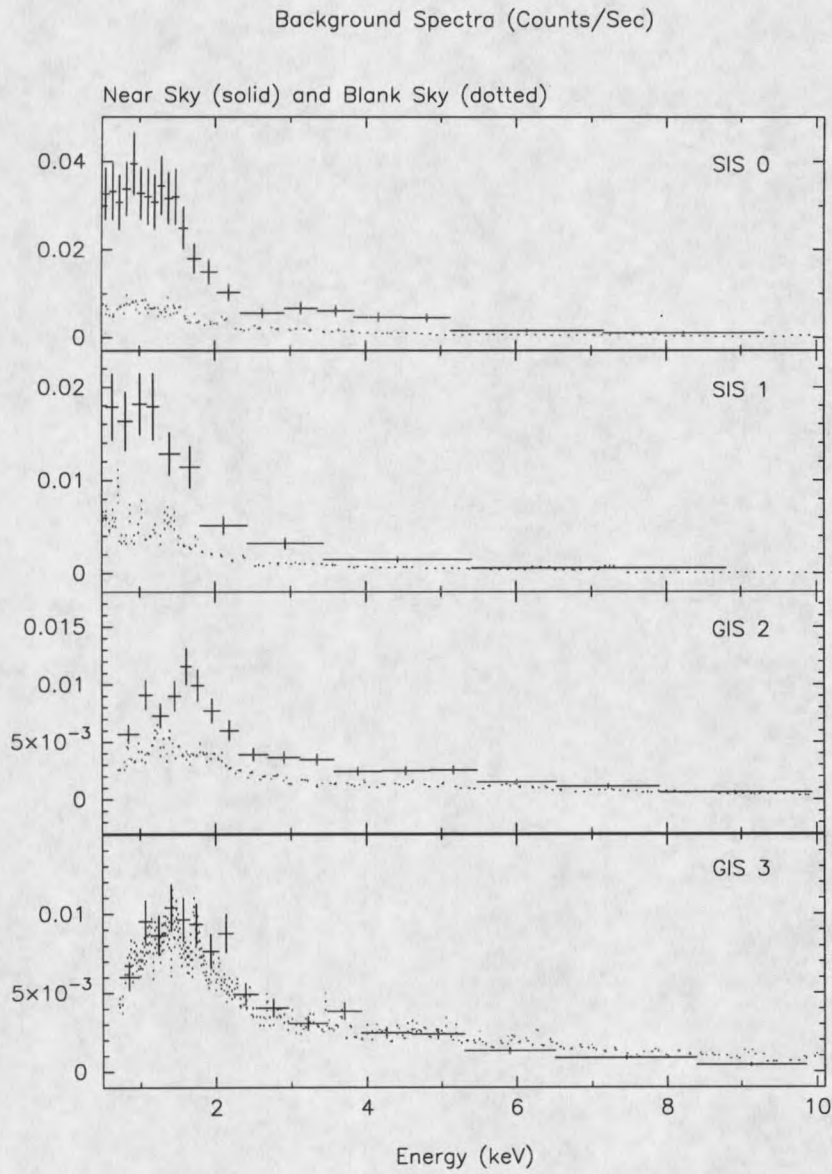


Figure 2.6: Background Spectra for the different methods

lower degree of variability mentioned in section 2.4 allows us to more easily determine the gross properties of the object before turning our attention to specific spectral features. We shall begin with the most prominent feature of all, the power law, then attempt to model the hard excess and finally examine the iron line.

2.5.2.1 Power Law

The X-ray spectra of Seyferts are characterized by a power law (section 1.1.3). A quick glimpse at the spectrum of the object after deconvolution with the instrument response shows that this object is no exception. Figure 2.7 shows the observed spectrum. The x-axis is the channel energy and the y-axis is the flux times the energy. The solid line shows the best fitting power law to the data. We modeled the power law by simultaneously fitting data from all four detectors over the higher energy bands (2-10 keV). To avoid any possible effects due to the iron K line and edge (at 6.4 and 7.1 keV, respectively), the region from 5.7 keV to 7.5 keV was also excluded from the fit. The soft X-rays were excluded to avoid the effects of several other absorption features.

The fitted parameters of the power law are shown in Table 2.2. In addition to listing the best fit values for the slope of the power law, the table lists

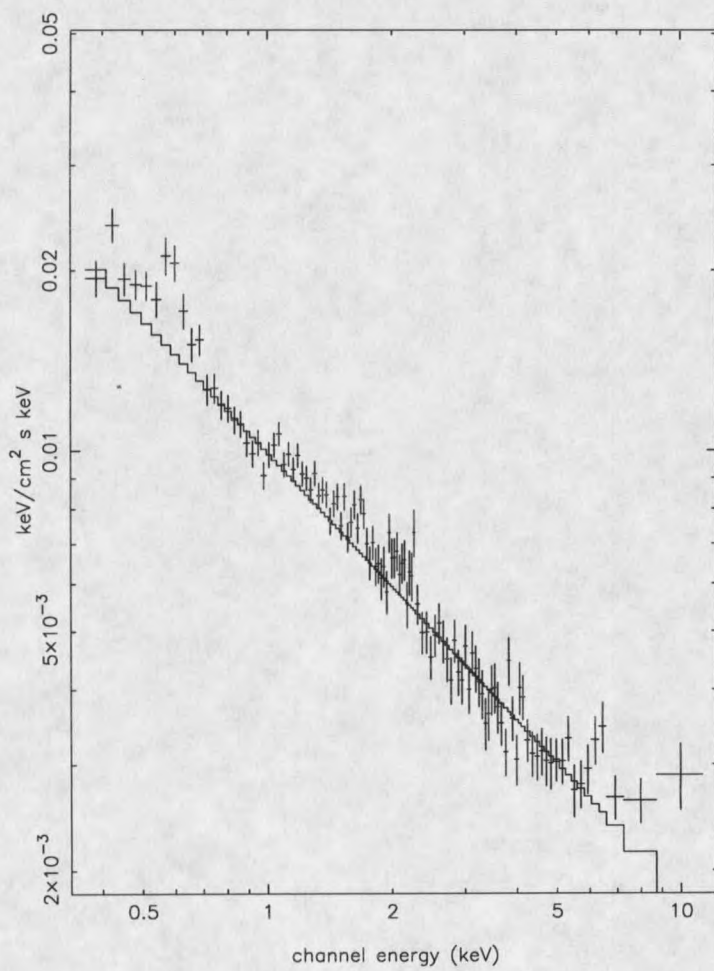


Figure 2.7: A power law fit to the data.

the luminosity of the model over the energy range under consideration, the χ^2 for the model for the best fitting parameters and the number of degrees of freedom in the model. Errors listed are 3σ confidence for variations in the parameter of interest. The fits provide tolerable but incomplete models of the spectrum, as indicated by the comparison of χ^2 to the number of degrees of freedom in the fit. The two are of roughly the same magnitude, but sufficiently different to suggest that further features remain to be determined. For illustrative purposes, fits using the blank sky background as well as those with no background subtraction at all are also shown in the table. The results do not differ by much due to the similarity noted earlier in the high energy range of the backgrounds. The low flux in the background in this regime causes even those values found with no background subtraction to be substantially the same. The slightly flatter fit when no background is subtracted is due to the presence of extra high energy counts, while the slightly steeper power law in the blank sky fit is due to fewer low energy counts being subtracted than in the near sky fit.

As was also mentioned in the timing analysis, the observation can be divided into a high and a low luminosity state. Fits to the observation show no significant variation in the power law slope over the course of the

observation. This is very much in accord with our timing analysis of the hard X-rays.

The high energy X-ray continuum can be taken to be a simple unbroken power law ($E^{-\Gamma}$) with a slope of $\Gamma = 1.74 \pm .09$. This value agrees well with that found in Santos-LLeó, *et al.* (1994) ($\Gamma = 1.75 \pm .05$) using EXOSAT data. Nandra, *et al.* (1997) determined a spectral slope of $\Gamma = 1.78 \pm .05$ using the same data set.

2.5.2.2 High Energy Tail

The above fit was obtained using a simple power law. As we discussed in section 1.3.1 however, many Seyferts exhibit a flattening of the power law above 7 keV. This feature is present in NGC 4593, as can be seen in figure 2.7. The last few high energy bins are well above the value suggested by our simple power law fit. For the fits to this feature, the energy range considered is extended out as far as we have sufficient counts for gaussian statistics.

Three different models were tested against this feature. The simplest (though hardest to justify physically) was a broken power law, in which the slope of the power law is allowed to change sharply at some arbitrary energy. The second model tested was an ionized reflection model presented in Done,

Data Set	Luminosity ($10^9 L_\odot$)	Slope (Γ)	χ^2	d.o.f.
No Background				
Entire Observation	$2.89 \pm .18$	$1.70 \pm .05$	1151.	795.
High State Only	$3.16 \pm .23$	$1.73 \pm .06$	521.4	451.
Low State Only	$2.56 \pm .26$	$1.66 \pm .09$	455.3	451.
Blank Sky Background				
Entire Observation	$2.73 \pm .34$	$1.83 \pm .08$	1331	795.
High State Only	$3.00 \pm .32$	$1.86 \pm .09$	579.9	451.
Low State Only	$2.41 \pm .38$	$1.78 \pm .12$	682.4	451.
Near Sky Background				
Entire Observation	$2.81 \pm .31$	$1.74 \pm .09$	1008.	726.
High State Only	$3.05 \pm .36$	$1.79 \pm .10$	541.9	477
Low State Only	$2.48 \pm .42$	$1.70 \pm .13$	521.7	477

Table 2.2: Fitting Parameters for the hard X-ray power law

et al. (1992) and distributed as part of the XSPEC data analysis package. In this model the power law continuum is allowed to self-consistently ionize the surrounding material (see section 3.1.5 for a discussion of the method). The final model was one in which partial covering by an optically thick (neutral) absorber is employed.

The *ad hoc* broken power law model simply tries to fit the data without offering a physical reason for it. In this case the power law changes slope rather sharply just above the iron line energy at 6.6 keV (in the region of excluded data). The fit parameters (table 2.3) still show no significant differences between the two luminosity states. For each data set the table lists the low energy power law slope, the energy at which the power law slope changes and the slope above that energy as well as the measure of the goodness of fit. The errors listed for each of the model parameters overlap in each luminosity state. Note that no upper limit could be placed upon the break energy for the low luminosity state, meaning that an unbroken power law is not ruled out. The near equality of χ^2 and the number of degrees of freedom indicates that this is an acceptable model of the high energy spectrum.

The ionized reflector model (Done, *et al.*, 1992) calculates the spectrum from a material in an arbitrary ionization state illuminated by a power law.

Data Set	Model Parameter	Value	χ^2	d.o.f
Entire Observation	Low Energy Slope	$1.78 \pm .07$	448.7	436.
	Break Energy	$6.62 \pm .66$ keV		
	High Energy Slope	$0.26 \pm .60$		
High State Only	Low Energy Slope	$1.81 \pm .09$	234.2	220.
	Break Energy	$6.80 \pm .98$ keV		
	High Energy Slope	$0.13 \pm .86$		
Low State Only	Low Energy Slope	$1.72 \pm .13$	209.2	220.
	Break Energy	$5.97_{-.92}$ keV		
	High Energy Slope	$0.60 \pm .74$		

Table 2.3: Fit parameters for the Broken Power Law model

The opacities used in the reflection calculation are obtained by solving the Saha equation (see section 3.1.5) for each ionized species. The ionization state of the medium is parameterized by $\xi = L/nR^2$ where L is the 5 eV to 300 keV power law luminosity, n is the density of the material and R is the distance of the reflector from the source. Fitting placed no useful constraints upon the inclination angle of the reflector, nor upon the solid angle subtended by the reflector as seen from the source, so these parameters are not listed in the table of model parameters (table 2.4). The fraction of the escaping photons is a measure of the isotropy of the power law emission. The much lower than unity values indicate that this is not an isotropic source. The ionization parameters cannot be confidently distinguished from zero, indicating that the reflector is primarily neutral in this object. This model also provides an acceptable description of the high energy spectrum.

The partial covering model modifies the existing power law by obscuring it with a dense column of neutral material. The existing model is multiplied by

$$f \exp -N_H \sigma(E) + (1 - f), \quad (2.3)$$

where f is the fraction of the source covered by the material, N_H is the column

Data Set	Model Parameter	Value	χ^2	d.o.f.
Entire Observation	Power Law Slope	$1.94 \pm .09$	378.1	394
	Fraction Escaping	$.179^{+.075}_{-.044}$		
	Ionization Parameter	$0^{+21.7}$		
High State Only	Power Law Slope	$1.95 \pm .11$	196.1	198.
	Fraction Escaping	$.183^{+.108}_{-.053}$		
	Ionization Parameter	$0^{+22.5}$		
Low State Only	Power Law Slope	$1.94 \pm .16$	179.4	198.
	Fraction Escaping	$.168^{+.143}_{-.058}$		
	Ionization Parameter	$.026^{+324}$		

Table 2.4: Ionized Reflection Model Fit Parameters

density, and $\sigma(E)$ is the photoelectric cross-section. The parameters for fits with this model are shown in table 2.5. The table lists the intrinsic power law slope, model luminosity, covering fraction and covering column depth for each data set. The higher derived luminosities than in the other models are accounted for by the fact that many of these photons are absorbed by the covering material. There is a significant change in the column depth of the covering material between the high and low states as well as a strong change in the intrinsic power law slope. This model also produces an acceptable description of the observation, but unlike the other two strongly suggests that the power law slope varies with luminosity.

All three models each provided approximately equivalent fits to the data set. For any of the models, the fit is substantially better than that for the simple power law of the previous section, indicating that the high energy tail is statistically present. Unfortunately, we are unable to rule out or select one of the models based upon the fit. This is not as large an obstacle as it might sound. All three of these models are effectively identical below about 5 keV, and furthermore, they are all simple power laws in that regime. For our soft X-ray modeling efforts, we will consider any of these hard spectrum models to be equivalent to a simple power law with a slope of $\Gamma = 1.86 \pm .07$, which

Data Set	Model Parameter	Value	χ^2	d.o.f.
Entire Observation	Power Law Slope	$1.87 \pm .07$	436.1	435.
	Luminosity ($10^9 L_\odot$)	$3.48 \pm .09$		
	Covering Fraction	$.713 \pm .088$		
	N_H ($\times 10^{22} \text{cm}^{-2}$)	83.7 ± 18.7		
High State Only	Power Law Slope	$1.93 \pm .09$	223.4	219.
	Luminosity ($10^9 L_\odot$)	$3.42 \pm .09$		
	Covering Fraction	$.664 \pm .110$		
	N_H ($\times 10^{22} \text{cm}^{-2}$)	68.0 ± 19.4		
Low State Only	Power Law Slope	$1.74 \pm .12$	205.8	219.
	Luminosity ($10^9 L_\odot$)	$2.84 \pm .11$		
	Covering Fraction	$.862 \pm .159$		
	N_H ($\times 10^{22} \text{cm}^{-2}$)	$144.3^{+65.4}_{-28.9}$		

Table 2.5: Partial Covering Fit Parameters

is the average slope of all three models, and within the 3σ errors of each fit.

2.5.2.3 Iron Line

The excess of X-rays above the power law at about 6.4 keV is assumed to be associated with iron K-line emission (see section 1.3.1). We must first include this region of the spectrum for consideration. Figure 2.8 shows the contribution of each energy bin to the total χ^2 of the model we have developed when this region is examined. The x-axis indicates the channel energy. The y-axis indicates the contribution to χ^2 due to that channel times the sign of the data minus the model. There is clearly a large excess above our current model in the area between 6 and 7 keV in every detector.

The simplest model of this excess is a narrow gaussian line. The best parameters to fit this model to the data are shown in table 2.6. The table indicates the best fit energy of the line, the width of the gaussian, the equivalent width of the line and the measures of the model fit. The low number of counts in this part of the spectrum made it impossible to determine errors on the line energy or width to the 3σ confidence level. Less strict error estimates are indicated. The best fit was to a single narrow gaussian line, with an equivalent width of about 86^{+47}_{-82} eV, but only in the lower luminosity state

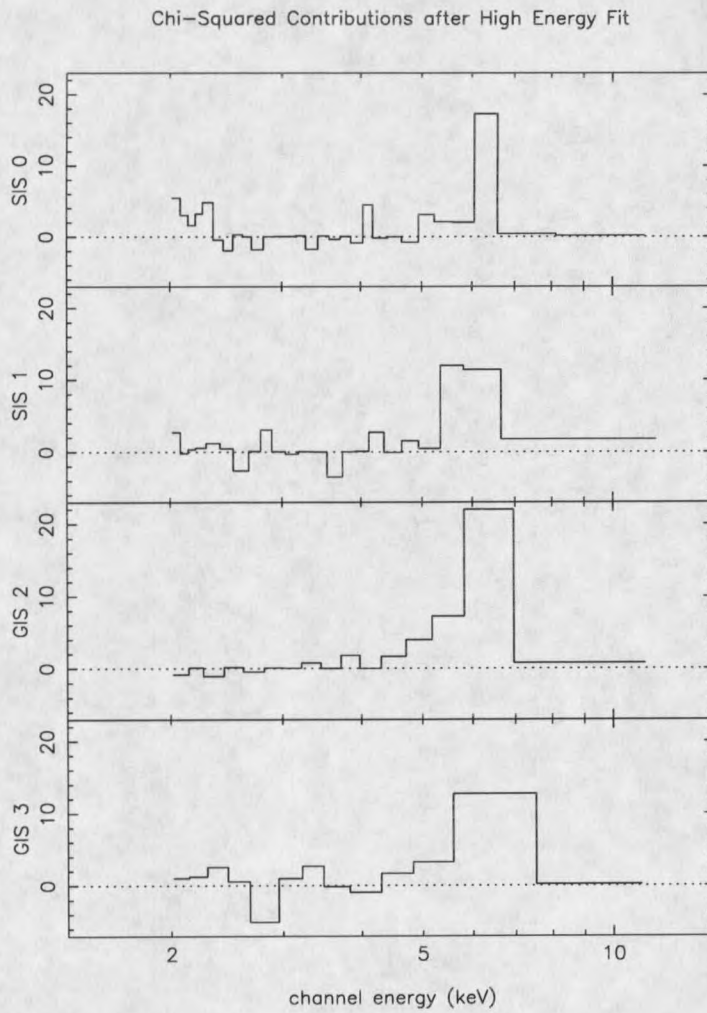


Figure 2.8: χ^2 residuals after high energy fit (rebinned for clarity)

can we even claim to see a line at the desired level of confidence. The line energy determination was hampered by the relatively low number of counts, but was found to be about 6.3 keV. No real limits could be placed upon the width of the line, but the best fit was obtained with a line width less than the bin size. The value found here is an only slightly narrower line profile than the 90^{+40}_{-30} eV equivalent width seen by Nandra, *et al.*, but well within their given 1σ errors.

A more complex model of the iron line emission was discussed in section 1.3.1. It has been suggested that the iron line feature is generated by the hot material in the assumed accretion disk. A model of the line profile resulting from the variations in temperature and rotational velocity in the disk is included in XSPEC (see Fabian, *et al.*, 1988) and was also tested against this observation. The best fits of this model to the data are shown in the lower part of table 2.6. Few error estimates could be obtained from this observation and there was no significant improvement in the fit of the model. The low luminosity state could not be fit at all with this model.

Iwasawa, *et al.* (1996) and Yaqoob, *et al.* (1997) have each reported rapid variability in the iron line profile of separate Seyfert galaxies. To examine this possibility, we hold the other model parameters fixed and vary only the

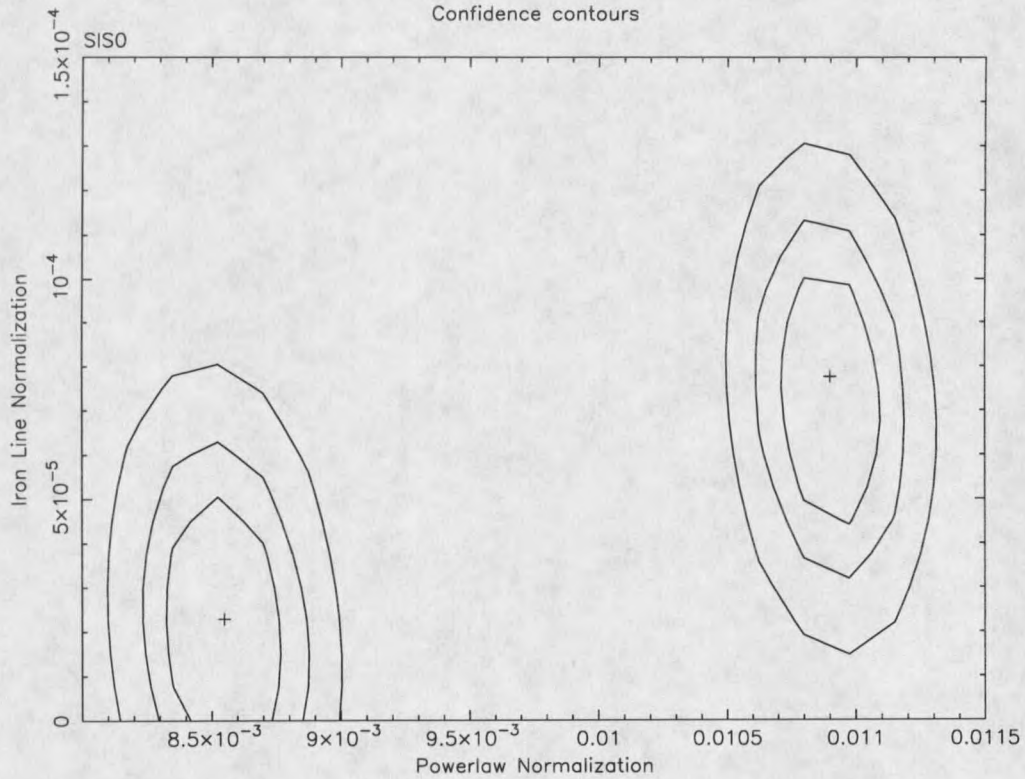


Figure 2.9: High and Low State iron Line Normalization Contours

normalization of the power law and the narrow gaussian. By calculating the value of χ^2 for our model at each pair of values, we generate a contour plot of the region of parameter space which best fits the observation. This allows us to easily compare model parameters in each state to see if they are changing significantly. Figure 2.9 shows the results of this procedure for both the high and low luminosity states. The x-axis indicates the continuum normalization and is proportional to the luminosity. The high luminosity state is therefore

Simple Gaussian Line					
Data Set	Line Energy	Width	Equ.Width	χ^2	d.o.f.
Entire Observation	$6.33 \pm .74^\dagger$ keV	$0.0^{+.15}$	54^{+56} eV	485.5	490
High State Only	$6.34 \pm .10^\dagger$ keV	0.0	97.5^{+29} eV	239.1	247.
Low State Only	$6.28_{-.38}^\dagger$ keV	$0.01^{+.16}$	75.0 ± 66 eV	239.9	247.
Schwarzschild Accretion Disk Line Profile					
Data Set	Line Energy	Equivalent Width		χ^2	d.o.f.
Entire Observation	6.34 ± 1.2 keV	54^{+23}_{-54} eV		482.1	490
High State Only	6.21keV	95^{+33}_{-95}		238.0	247.
Low State Only	No valid fit obtained				247.
† Only a 2σ error could be obtained					

‡ Only a 1σ error could be obtained

Table 2.6: Fitting Parameters for iron K line.

indicated by the set of contours to the right side of the plot. The y-axis shows the normalization of the gaussian line and is related to the equivalent width of the line.

2.5.3 The Soft X-ray Spectrum

Figure 2.10 shows the data from the entire observed spectrum along with the model we have developed in our analysis of the hard X-rays. The x-axis is again the average energy of the data bin, while the y-axis is the flux times the energy of the bin. The continuum of our model overestimates the X-ray flux from about .7 to around 1 keV and perhaps underestimates the flux below .6 keV. We must therefore continue to develop our model to include these spectral features.

2.5.3.1 Galactic Hydrogen Absorption

The first and most obvious additional effect is the absorption due to neutral Hydrogen in our Galaxy. Measurements of the 21 cm radiation due to the spin flip transition in neutral hydrogen set the column depth of Galactic H to be 2.1×10^{20} atoms per cm^2 in the direction of NGC 4593 (Stark, *et al.*, 1992). We cannot allow our model to fit to less than this amount of neutral hydrogen. In fact, our data fitting software attempts to do so if the hard X-ray model is not held fixed (LeRoux, *et al.*, 1997). With the power law fixed to the value determined from the hard X-ray data, we find a neutral column depth of $(3.03 \pm .68) \times 10^{20}$ atoms per cm^2 . The χ^2 of this fit is

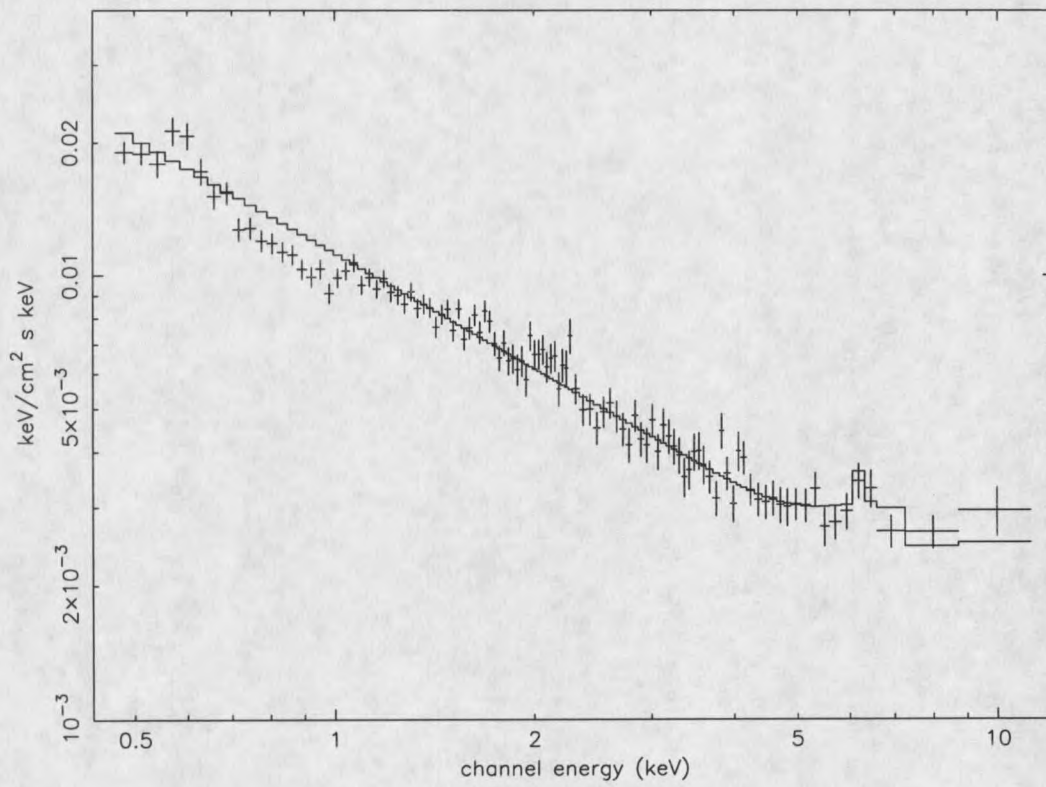


Figure 2.10: The X-ray spectrum shown with the hard X-ray model.

957.7, with 646 degrees of freedom. This is an improvement of $\Delta\chi^2 \sim 143$. The value was the the same for each of the luminosity states being analyzed. This value was then fixed for the remainder of the analysis.

2.5.3.2 A Possible Soft Emission Line

Figure 2.11 shows the ratio of the observation to the model we have developed to this point. The top two panels show the high luminosity state data from the SIS detectors, while the bottom pair show the low luminosity state data from the same instruments. The x-axis indicates the channel energy, while the y-axis is the value of the ratio. If the model were a good description of the data, these plots should show only scatter around the value of 1 (shown by the dotted lines in each panel). There are still strong features at about .6, .7 and perhaps .8 keV which remain to be modeled.

The common excess in all detectors over our model at about .6 keV is suggestive of an emission line, and coincides with the energy of an emission line of O VI. We attempt to model this feature by a narrow gaussian with an energy of .58 keV. Table 2.7 shows the best fit model parameters for this line. Unfortunately, one must be extremely careful with the low energy channel calibration in ASCA observations. Even a minor correction to the

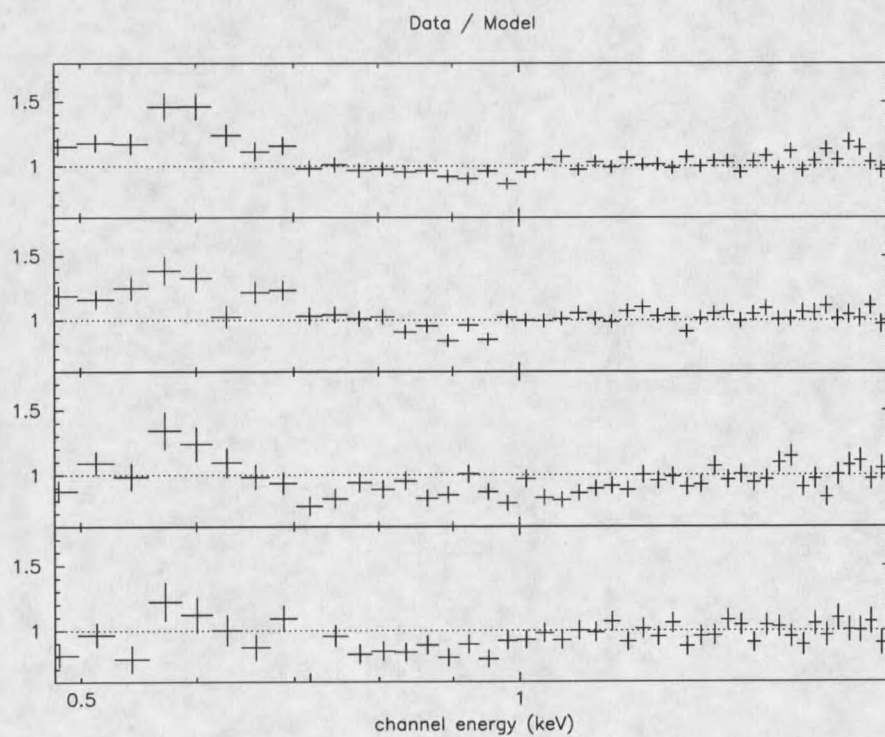


Figure 2.11: Soft X-Ray Residuals showing several features

Data Set	Line Energy	Equivalent Width	χ^2	d.o.f.
Entire Observation	$.583 \pm .018$ keV	45.7 ± 14.8 eV	871.3	646.
High State Only	$.577 \pm .023$ keV	61.3 ± 18.7 eV	390.5	324.
Low State Only	$.594 \pm .037$ keV	26.3 ± 20.8 eV	395.1	324.
MCG-6-30-15	$.587 \pm .021$ keV	$17.2^{+34.3}_{-17.2}$ eV	N/A	
Crab Nebula	<i>No valid fit found</i>			

Table 2.7: Oxygen VI Emission Line Fit

instrument's response can cause artifacts in the spectrum. To see if this was the case, the spectra of other objects observed by ASCA were examined in the same region of the spectrum. The resulting parameters of these fits can also be seen in Table 2.7. For each data set examined, the table lists the best fit for the line energy, the equivalent width of the line and the model's χ^2 for the available number of degrees of freedom. The line could not be fit with a wide (resolved) gaussian.

Perhaps of greatest interest is the strong variation seen in this line between the two luminosity states. Figure 2.12 shows another set of confidence contours. Again, we hold all of the model parameters but the continuum normalization and the normalization of the emission line gaussian fixed and

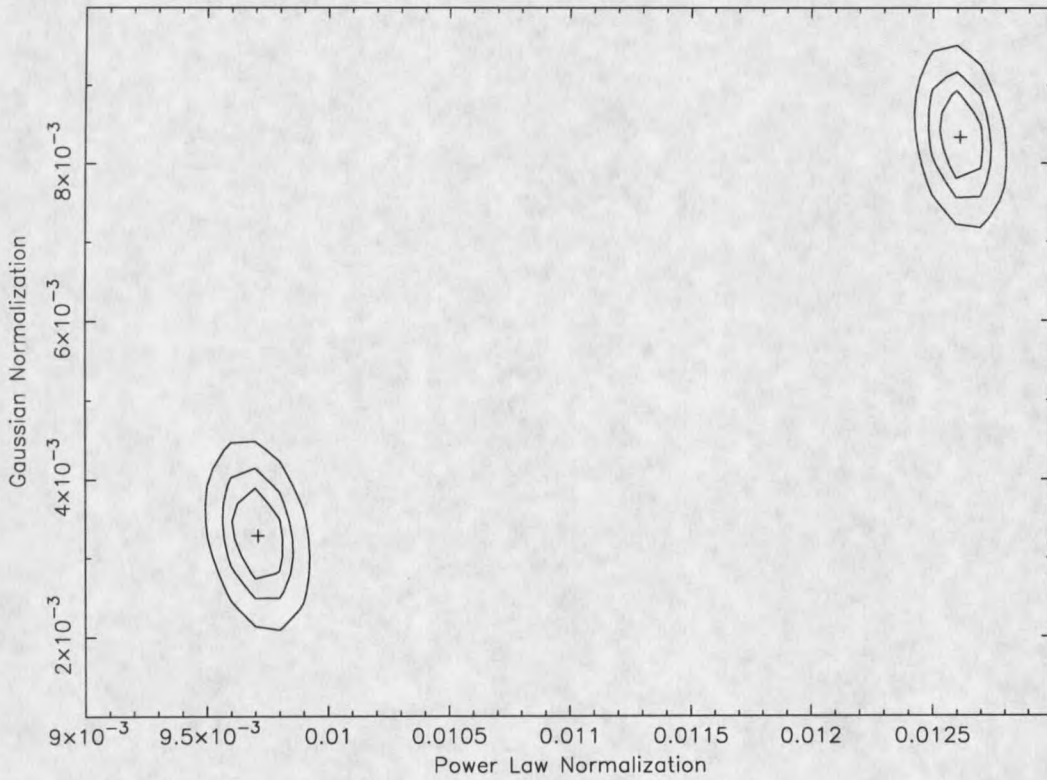


Figure 2.12: Confidence contours of Powerlaw v. OVI Gaussian

plot the values of χ^2 found for the model in each of the two luminosity states at several values of the two free parameters. The x-axis shows the continuum normalization, while the y-axis indicates the normalization of the gaussian. Each contour line shows an additional one sigma variation of the model at that point in parameter space from the best fit.

2.5.3.3 Soft X-ray Absorption Features

Having now accounted for Galactic effects upon the soft X-rays and a likely O VI line, we examine the ratio of the data to the model and see that there are still significant features remaining to be fit. Figure 2.13 shows the ratio of the data to the model we have developed in the two luminosity states. The upper panel shows the high state while the lower panel shows the low state. The most obvious deviations from our model are the points which fall below the dotted line around .9 keV in the upper panel and between .7 and .9 keV in the lower panel.

We begin to model these features with a pair of absorption edges. Table 2.8 lists the model parameters for the best fit to the data for this model. The table shows the best fit edge energy in keV for each edge on the indicated data set as well as the optical depth of that absorption edge.

Instead of an *ad hoc* modeling in which edges are added and fit as they seem to be needed, we also use a model which properly calculates the relative ionization species as a function of ionization parameter (Magdziarz & Zdziarski, 1995). Table 2.9 lists the column depth of the absorber and the ionization parameter for the best fit to each data set using this model.

In Figure 2.14 we again see a plot of confidence contours. In this plot

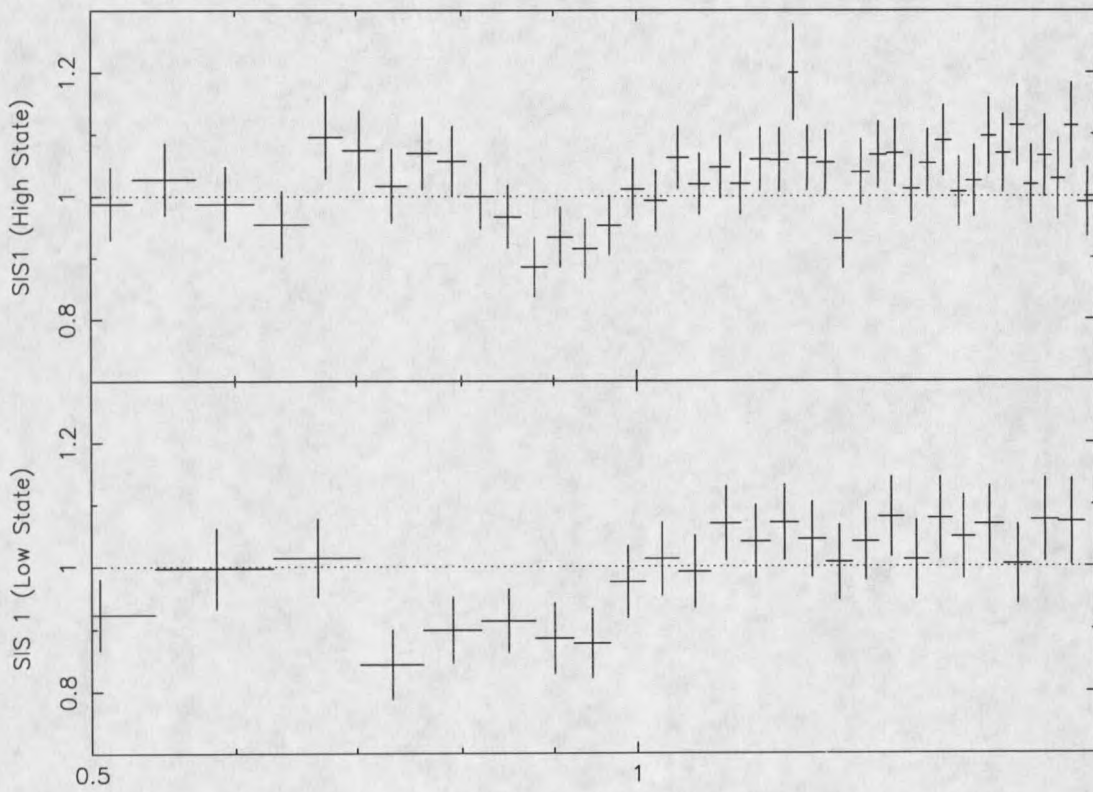


Figure 2.13: Ratio of the observed soft x-rays to the model.

Data Set	Model Parameter	Value	χ^2	d.o.f
Entire Observation	Edge Energy	$.712 \pm .059$	781.5	648
	Optical Depth	$.119 \pm .062$		
	Edge Energy	$.837 \pm .059$		
	Optical Depth	$.085 \pm .055$		
High State Only	Edge Energy	.749	372.3	326
	Optical Depth	$.022^{+.072}$		
	Edge Energy	$.816 \pm .068$		
	Optical Depth	$.132 \pm .068$		
Low State Only	Edge Energy	$.688 \pm .050$	321.5	326
	Optical Depth	$.288 \pm .119$		
	Edge Energy	$.885^{+.203}$		
	Optical Depth	$.056^{+.098}$		

Table 2.8: Parameters of fit using two absorption edges

Data Set	Model Parameter	Value	χ^2	d.o.f
Entire Observation	$N_H (\times 10^{22} \text{cm}^{-2})$	$.078 \pm .063$	663.8	645
	Ionization Parameter	$9.36^{+94}_{-8.71}$		
High State Only	$N_H (\times 10^{22} \text{cm}^{-2})$	$.059 \pm .052$	387.6	324
	Ionization Parameter	$33.0^{+226}_{-24.4}$		
Low State Only	$N_H (\times 10^{22} \text{cm}^{-2})$	$.091 \pm .031$	318.8	324
	Ionization Parameter	$1.10^{+3.6}_{-.93}$		

Table 2.9: Ionized absorber model fit parameters

our two free parameters are the power law normalization along the x-axis and the ionization parameter along the y-axis. Each contour indicates the limit of an additional one sigma difference in the fit of the model with those parameters from the best fit value. The contour set to the right corresponds to the high luminosity state data, while the set to the left is from the low luminosity state.

The two luminosity states clearly occupy separate parts of ionization parameter space. As we would expect, the more luminous continuum corresponds to a higher level of ionization in the fitted spectrum. This relationship is discussed in greater detail in section 2.6.

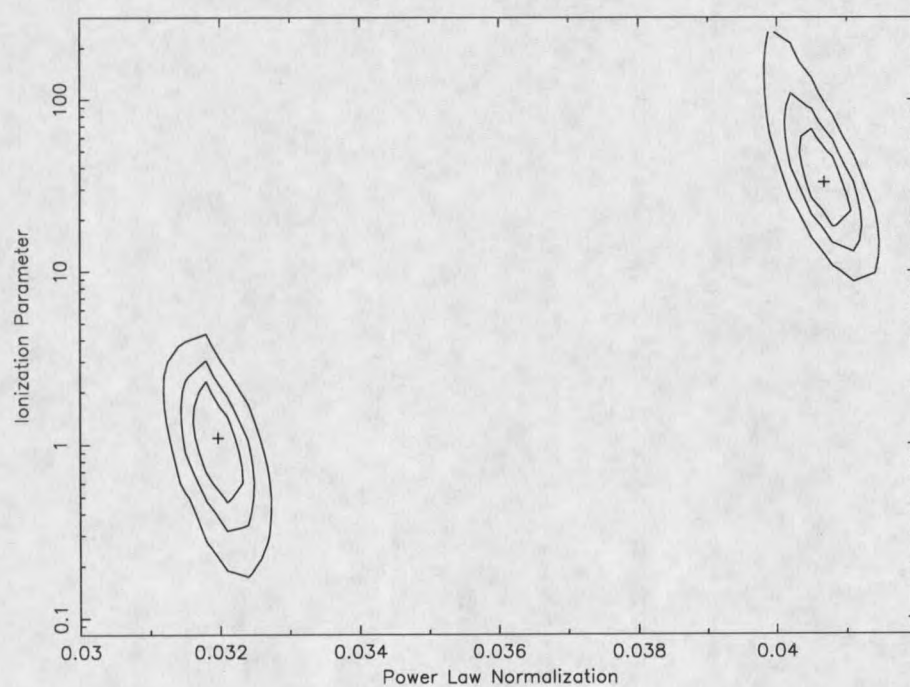


Figure 2.14: Ionization parameter confidence contours

2.5.3.4 An Alternate View

Kunieda has suggested an alternative to the usual practice of breaking up the observation according to the absolute luminosity of the object. His suggestion was to bin the data phenomenologically instead. Rather than dividing the data up into a "high" and a "low" state, we now combine parts of the light curve in which the continuum is changing in the same manner. All of the bins which contain a general upward trend are added together into an "increasing" group. All those which display a downward trend form a "decreasing" group.

The continuum level of each bin in a group is different from each of the others, so it would be difficult to try and compose them into a single spectrum. We *can* compare the flux within a bin in a given channel or set of channels to the flux in a different channel in the same bin. This allows us to measure the *relative* flux in channels which contain spectral features to those which have been found to be featureless in the analysis of the full spectrum. This method allows us to study transitions between states rather than the states themselves.

Figure 2.15 shows the result of applying this approach to the soft X-ray absorption features. The plot on the top left uses data only from the "increasing" group, while the plot on the top right uses only "decreasing"

bins. In each of the two top plots, the x-axis is the change in the flux at and just above .739 keV (the edge energy of O VII) over the course of a 4 ksec bin as a fraction of the flux in the continuum. The y-axes are fraction of the O VIII absorption edge flux at .871 keV relative to the same continuum. The bottom plot is the complete light curve of the object, as in Figure 2.1. The increasing group shows a correlation between the change in O VII and the change in O VIII, whereas the decreasing group shows little or no variation in O VIII despite strong changes in the O VII flux.

2.6 Summary & Discussion of Our Results

The parameters of our final models contain a more detailed analysis of the available ASCA data for this object than has previously been performed. Previous work has only constrained the power law and quickly analyzed the iron line feature of NGC 4593. The values found elsewhere for these features are consistent with our more detailed analysis.

Although we were unable to distinguish between the three models tested in section 2.5.2.2 for the high energy tail of the X-ray spectrum, the fit was greatly improved by the inclusion of this feature in our model. We found some

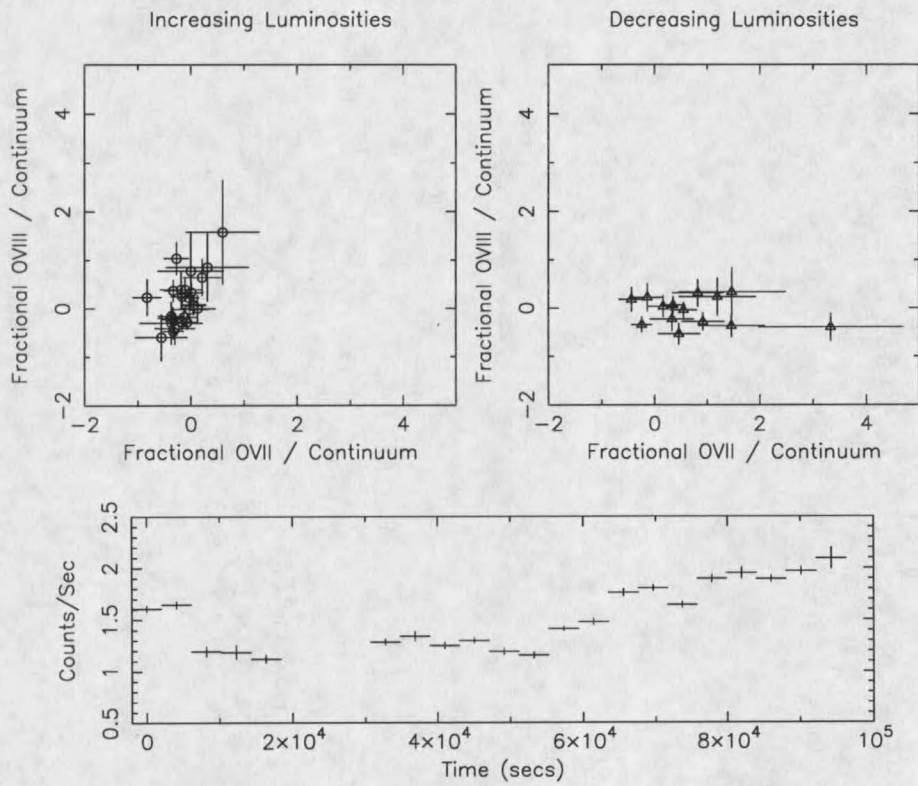


Figure 2.15: Fractional Changes in Absorption Edge Depth

form of hard X-ray flattening to be present at the $> 99\%$ level of confidence.

An iron K line was found to be present and contribute a significant improvement to the fit of the model. Exact fitting of the feature was hampered by the low number of total counts in the observation. The line appears to be narrow. We saw no significant variability in the equivalent width of the line with continuum luminosity. There are simply not enough photons available to distinguish between a gaussian line profile and the characteristic "double-horned" profile of an accretion disk. In the absence of a compelling reason to use the more complicated model, we adopt the narrow gaussian as the final element in our model of the hard X-ray spectrum.

The extension of our hard X-ray model to lower energies is not sufficient to explain all of the features in the spectrum. We found several soft X-ray features which deviated substantially from the simple power law model. None of these features has previously been detected in NGC 4593.

Neutral hydrogen has a definite effect upon the observed spectrum. The fitted value exceeds the value found for Galactic neutral hydrogen above the 3σ level. This tells us that there is other neutral material in the line of sight from us to NGC 4593 or with the Seyfert galaxy itself.

The solid detection of a narrow line component at about .58 keV we iden-

tify with an emission line of O VI. This feature is definitely present over the entire observation. The non-detection in the Crab, and lack of a significant detection in MCG -6-30-15 (otherwise rich in soft features) supports the conclusion that this feature is real and not an artifact of instrument calibration. The confidence contours of the relationship between the line normalization and that of the power law show a correlation between the source luminosity and the line strength. The line becomes stronger at higher luminosities. This is also seen in the fitted values of the line's equivalent width in Table 2.7. At greater than the 3σ level of confidence, the normalization of the line varies strongly enough that the two states do not overlap. This is the first detection of a variable O VI emission line in a Seyfert 1 galaxy.

The fitted energy ranges of the absorption edges correspond to the absorption edges of OVII (.739 keV) and OVIII (.871 keV). The values for the absorption edge energies in each luminosity state are consistent with each other and the values found for the entire observation. The values of the optical depths for these edges cannot confidently claim detection of the higher ionization state edge when the source luminosity is low, nor can we claim to see the lower ionization state during the high luminosity period of the observation. The same can be said of the ionized absorber model, in which

we see a significantly higher ionization parameter with increased luminosity. As was discussed in Section 1.1.3, Seyfert galaxies often show evidence of absorption features in the soft X-ray regime. This clear variation in the soft features has only been reported in one other source (MCG-6-30-15, Otani, 1996), and in that source, the opposite trend was seen from that discovered here (deeper low ionization edges at higher flux).

If we accept that the feature is indeed indicative of partially ionized gas in the line of sight, it is obvious that the relative amounts of O VII and O VIII are changing. Our analysis of the absorption features using the Kunieda method shows strongly different behaviour in the increasing and decreasing phenomenological groups. In the decreasing group, the OVIII depth is not changing significantly, but the OVII edge is. If the ionization is due to increased photon flux from the source, the ionization process will be relatively quick, having a linear dependence upon the density of the gas

$$\Gamma \propto n_{Ovii} \int_{\nu_{ionize}}^{\infty} F(\nu) \sigma(\nu) d\nu \quad (2.4)$$

where $F(\nu)$ is the flux at a given energy and $\sigma(\nu)$ is the photoionization cross-section. On the other hand, the recombination time scale is much slower, as the recombinations are collisionally driven, and so depend upon

the gas density squared:

$$\Gamma \propto n_{Oviii} n_e \sigma. \quad (2.5)$$

This is in fact exactly what we are seeing. When the luminosity increases, both the OVII and OVIII change. Some of the OVII population is ionized to OVIII, and some of the OVIII population is completely ionized. The fact that both species react in the same way at the same time bolsters our assumption that the two exist in the same region of space near the object. When the luminosity decreases, the O VIII does not respond nearly as strongly if indeed, at all. This implies that the O VIII density is lower than the density of O VII.

Two different model spectra of NGC 4593 are shown in figure 2.16. The upper figure is a theoretically motivated one in which features are included based upon our unified model of Seyfert galaxies. It shows the best fit spectrum using the reflection model for the hard X-rays and the ionized absorber in the soft X-ray. The bottom figure is an observationally motivated one in which features are included because they fit the data. It uses the partial covering model for the high energy regime, while the lower energy features are specifically assigned absorption edges. Each model is statistically

equivalent for this data.

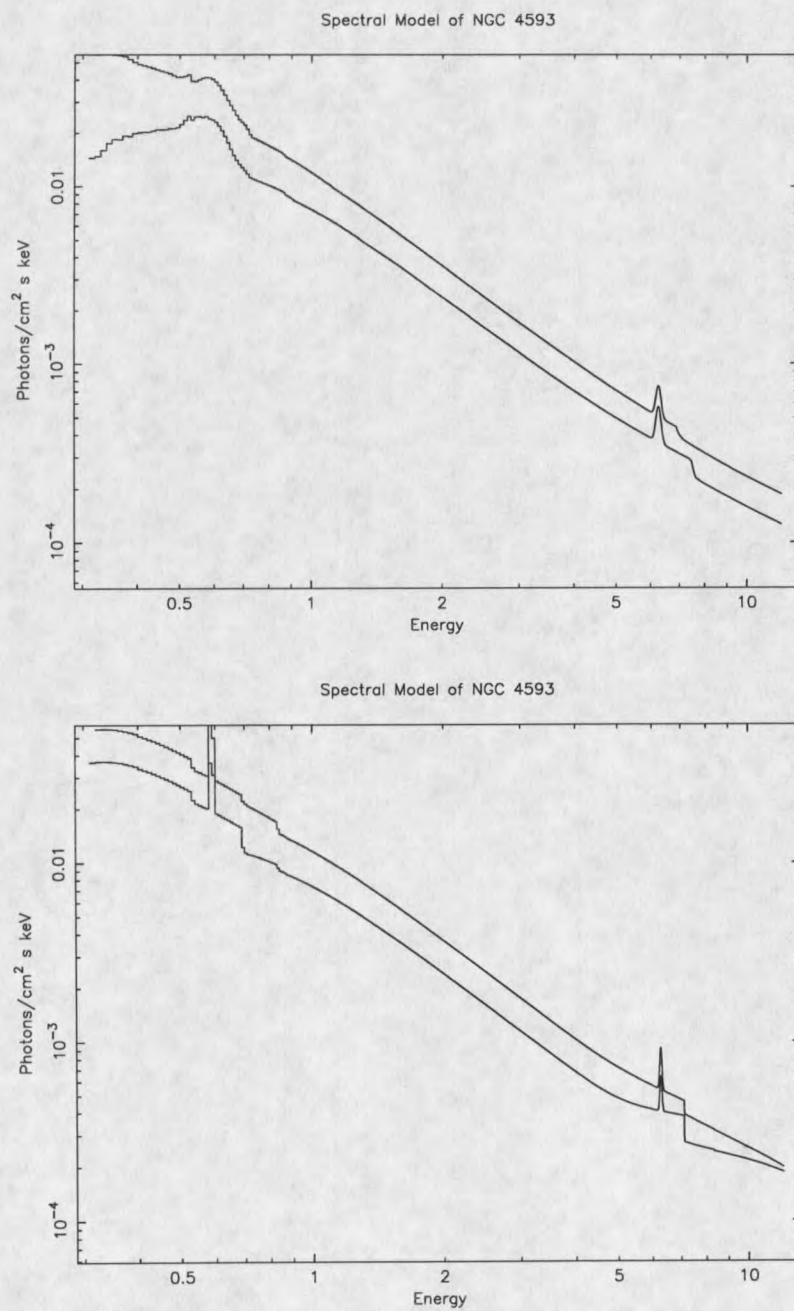


Figure 2.16: Model Spectra for Ionized Absorber and *ad hoc* Lines and Edges

Chapter 3

ELECTRON- POSITRON/PHOTON PHYSICS

Observational evidence (see section 1.3.2) as well as theoretical predictions give us strong reasons to believe that relativistic electron-positron pairs play an important part in the generation and processing of many parts of the spectra of AGN. From the generally power law character of the spectrum, to the wavelength independent polarization, to the unavoidable pair production effects from the highest observed photons ($1 \text{ TeV} \gg m_e c^2$) we can see that

to understand these objects, we must first understand pair plasmas.

I apologize in advance for the use of the CGS system of units in the following discussion. The use of these archaic units is pervasive in electromagnetism, and personal preference was sacrificed in favor of making the comparison with other sources easier for the interested reader.

3.1 Radiation Processes

Many different physical processes will need to be considered to accurately model the spectra of AGN. There are many ways in which radiation can interact with matter, even neglecting the pair creation and annihilation processes, which will be discussed in the next section.

3.1.1 Compton Scattering

Compton scattering is the interaction of a high energy photon with an electron (or positron) of lower energy. When the electron has a higher energy than the photon, the process is called inverse-Compton scattering, but the physics is the same. In the regime where ($h\nu \ll mc^2$), the problem reduces to simpler Thomson scattering.

Recall that in Thomson scattering, the incoming photons can be approximated as a continuous electromagnetic wave scattering off of the dipole moment of the charge (Rybicki & Lightman, 1979). The charge does not recoil from the collision, meaning that we have elastic scattering, and so our only really interesting quantity is the differential cross-section:

$$\frac{d\sigma_T}{d\Omega} = \frac{1}{2}r_0^2(1 + \cos^2 \theta) \quad (3.1)$$

where $r_0 = e^2/m_e c^2$.

When we extend ourselves to allow for the quantum mechanical effects due to photon momentum and a full treatment of the cross-section, we have Compton scattering. Simple conservation relationships allow us to calculate the change in the energy ϵ of the outgoing photon and electron as

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{m_e c^2}(1 - \cos \theta)} \quad (3.2)$$

Or in terms of the wavelength,

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos \theta). \quad (3.3)$$

The small magnitude of this shift relative to long wavelengths shows us why Thomson (*ie*, elastic) scattering is recovered in the low energy limit.

The details of the changes in the differential cross-section are beyond the scope of this review, but the final expression is relatively straightforward. Known as the Klein-Nishina formula, the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \frac{\epsilon_1^2}{\epsilon^2} \left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2 \theta \right). \quad (3.4)$$

We note in passing that this reduces to equation 3.1 for $\epsilon_1 \sim \epsilon$, which is again elastic scattering. This modification to the elastic formula reduces the cross-section at high photon energies, as we might expect. The total Klein-Nishina cross-section is

$$\sigma_{KN} = \frac{3}{4} \sigma_T \left[\frac{1+x}{x^3} \left\{ \frac{2x(1+x)}{1+2x} - \ln(1+2x) \right\} + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right], \quad (3.5)$$

where $x \equiv h\nu/m_e c^2$.

Coppi and Blandford (1990) provide a series of useful approximations for the angle averaged scattering rates and resultant energy distribution. They take great pains to point out that several commonly used approximations for these quantities deviate strongly from the exact value in the highest energy regimes. This area is most critical for the nonthermal pair cascade process which will be discussed in section 3.3. The scattering rate approximation used in our calculations is

$$R(\omega, \gamma) \approx \frac{3c\sigma_T}{8\gamma\omega} \left\{ \left[1 - \frac{2}{\gamma\omega} - \frac{2}{(\gamma\omega)^2} \right] \ln(1+2\gamma\omega) + \frac{1}{2} + \frac{4}{\gamma\omega} - \frac{1}{2(1+2\gamma\omega)^2} \right\}, \quad (3.6)$$

where γ is the electron's relativistic energy factor [$\doteq (1 - \beta^2)^{-1/2}$] and ω is the photon energy in terms of $m_e c^2$.

In addition to knowing the scattering rate, we must also be able to determine the resulting distribution of energies of the scattering particles. Conservation of energy allows us to worry about only one of the two. A common approximation to this function is a delta function about the mean scattering energy, which is itself often approximated only roughly (see, for example, Tritz, 1990). Although a useful approximation is suggested by Coppi and Blandford for the lower energies, more accurate results are found by using even a coarse interpolation table near the high energy limits. These tables were calculated by the author from the exact distribution given by Jones (1968), *modulo* corrections for misprints (see Appendix A).

3.1.2 Bremsstrahlung Radiation

Also known as free-free emission, bremsstrahlung ("braking radiation") is the radiation emitted by electrons (and positrons) undergoing acceleration due to the electrostatic fields of other charged particles. Again, relativistic effects complicate an otherwise simple picture. At lower energies, the dominant contribution comes from the electrostatic field of any surrounding ions, as the dipole moment of two like particles in collision is a constant of the motion and the higher charge on the ions also generates a greater deflection force.

In the rest frame of an electron approaching a nucleus with charge Ze to within a distance b , the electric field seen is

$$\vec{\mathcal{E}} \sim \frac{E}{mc^2} \frac{Ze}{b^2}, \quad (3.7)$$

with E being the energy of the electron. The power radiated by an accelerated charged particle is found from the Larmor formula (Jackson, 1964)

$$P = \frac{2}{3} \frac{e^2}{c^3} \left(\frac{F}{m} \right)^2, \quad (3.8)$$

allowing us to estimate the (Lorentz invariant) instantaneous power radiated by the electron as

$$P(E, Z) \sim \frac{2Z^2 e^6}{3m^2 c^3 b^4} \left(\frac{E}{mc^2} \right)^2. \quad (3.9)$$

If the electron travels through a medium with N ions per cm^3 , then the number of collisions per second it experiences (as a function of the impact parameter) is $n(b) db = 2\pi c N b db$. The duration of a collision is approximately $\tau \sim b/c$. From all of this, we can calculate the power emitted over the entire collision as

$$P(E, Z) \sim \frac{4\pi N Z^2 e^6}{3m^2 c^3} \left(\frac{E}{mc^2} \right)^2 \int_{b_{\min}}^{\infty} \frac{db}{b^2}. \quad (3.10)$$

A fully quantum mechanical treatment (Heitler, 1944) shows that the minimum impact parameter is of the order of the electron wavelength $b_{\min} \sim$

$\hbar E/m^2 c^3$. Performing the integration and substituting this minimum value in, we find

$$P(E, Z) = \frac{4\pi N Z^2 e^6}{3\hbar m c^2} \frac{E}{m c^2}, \quad (3.11)$$

which is a fair approximation to the fully quantum mechanical result of

$$P(E, Z) = \frac{4N Z^2 e^6}{\hbar m c^2} \frac{E}{m c^2} \ln \left(\frac{183}{Z^{1/3}} \right). \quad (3.12)$$

We must not forget that in the rest frame of the electron there is also a magnetic field of comparable strength ($\vec{\mathcal{H}} \sim \beta \vec{\mathcal{E}}$), thus we also have an electromagnetic pulse of duration $\gamma\tau$. We treat this as a distribution of (virtual) photons with a gaussian frequency distribution. We are then faced with a set of photons to scatter off of the electron, *ie* Thomson or Compton scattering. We shall first address ourselves to Thomson scattering to simplify the following discussion.

The width of this gaussian distribution in frequency space is E/bmc , which is large for the high energy electrons. We therefore approximate the spectrum of the virtual photons $j(\nu')$ as being independent of frequency for $\nu' < mc^2/h$ (Thomson regime). The number spectrum of the incoming photons is then

$$n(\nu') d\nu' = \frac{j(\nu') d\nu'}{h\nu'} \sim \frac{Ze d\nu'}{\hbar bc \nu'}. \quad (3.13)$$

However, Thomson scattering is frequency independent, so the outgoing number spectrum will have the same $1/\nu$ shape.

If the scattered photon makes an angle φ with the direction of motion of the electron, then in the observer's frame, the frequency seen is

$$\nu = \nu' \frac{1 + \beta \cos \varphi}{\sqrt{1 - \beta^2}}. \quad (3.14)$$

This still leaves the spectral shape of the distribution unchanged, with

$$n(\nu) d\nu \propto \frac{d\nu}{\nu}. \quad (3.15)$$

The scattered photons cannot exceed the initial energy of the electron, so we have

$$j(E, \nu) = n(E, \nu) h\nu = \begin{cases} j_0, & \nu \leq E/h \\ 0, & \nu > E/h \end{cases}. \quad (3.16)$$

Combining Eq. 3.12 and Eq. 3.16, we have

$$j_0 \frac{E}{h} = P(E, Z). \quad (3.17)$$

This allows us to determine j_0 as

$$j_0 = \frac{8\pi N Z^2 e^6}{m^2 c^4} \ln \left(\frac{183}{Z^{1/3}} \right). \quad (3.18)$$

If instead of a single electron, we have a spectrum of particles with some distribution $n(E)$, we must integrate the above over this distribution to find the emitted spectrum,

$$J(\nu)d\nu = \int_{E=0}^{\infty} n(E)j(E, \nu) dE d\nu. \quad (3.19)$$

After substituting Eq. 3.16 into this, we note that our lower bound is altered by the condition on the frequency, leaving a final spectrum of

$$J(\nu) d\nu = \int_{E=h\nu}^{\infty} n(E) j_0 dE d\nu. \quad (3.20)$$

The ion-electron interaction above will only be important in our calculations at the surface of the accretion disk. In the region above the disk, there are only electrons and positrons. The calculation of electron-electron and electron-positron bremsstrahlung cross sections and spectra are "a straightforward application of quantum electrodynamics" (Haug, 1984) and also "extremely lengthy." The full expression for the differential cross-section (see Appendix B) takes the place of the Thomson cross-section in the preceding discussion.

Thermal bremsstrahlung assumes that the emitting particles are in a thermal distribution of speeds. The cooling rates and emission spectra of thermal electron-positron pair plasmas have been fairly well fit by other researchers (Haug, 1985; Stepney & Guilbert, 1983; Dermer, 1984, 1986). These rates were used by the author to generate interpolation tables to speed calculation.

Nonthermal bremsstrahlung rates are conceptually handled in the same way. The proper cross-section is somewhat different (Alexanian, 1968) and

so a separate set of tables must be generated, but the computation itself is straightforward.

3.1.3 Synchrotron Radiation

When highly energetic charged particles move in a magnetic field, they describe a circular orbit about the field lines. The radiation emitted by this continuous acceleration is called synchrotron radiation. Although the model we develop here neglects magnetic fields (and hence, synchrotron radiation), the mechanisms are quickly sketched here for completeness. Synchrotron radiation is seen in jets and can provide a power law spectrum under certain conditions.

Synchrotron (also known as magnetobremßstrahlung) radiation derives from the Lorentz force

$$\vec{F} = \frac{e}{c}(\vec{v} \times \vec{\mathcal{H}}). \quad (3.21)$$

The cross-product shows that the force upon a charged particle moving along the magnetic field line will always result in a helical motion with a frequency that depends upon the field strength and the energy of the particle. When we make suitable corrections for relativistic effects, we find the Larmor frequency

to be

$$\nu_L = \frac{e\mathcal{H}_\perp}{2\pi mc} \frac{mc^2}{E} \quad (3.22)$$

where $\mathcal{H}_\perp$ is the magnetic field strength perpendicular to the velocity vector of the particle. The emitted radiation is beamed by relativistic effects into a cone of size $1/\gamma$. A distant observer thus sees pulses of radiation at the rate of the Larmor frequency with a duration of

$$\Delta T \sim \frac{e\mathcal{H}}{mc} \left(\frac{E}{mc^2} \right)^2, \quad (3.23)$$

the Fourier transform of which gives us the spectrum of the radiation. This consists of many harmonics of the Larmor frequency up to a maximum

$$\nu_m \sim \frac{1}{2\pi\Delta T} \sim \frac{e\mathcal{H}_\perp}{2\pi mc} \left(\frac{E}{mc^2} \right)^2. \quad (3.24)$$

The maximum emission of the spectrum occurs at about $.44\nu_m$ (Hillier, 1984).

The power radiated must be Lorentz invariant and must reduce to the proper classical limit. Bless (1968) has shown that the total power radiated is given by

$$P(E, \mathcal{H}) = \frac{2e^4\mathcal{H}_\perp^2}{3m^2c^3} \left(\frac{E}{mc^2} \right)^2. \quad (3.25)$$

If we wish to determine the spectrum emitted by a distribution of electrons $n(E)dE$, we must evaluate the integral

$$J(\nu) d\nu = \int_E n(E) P(E, \mathcal{H}) dE d\nu. \quad (3.26)$$

This problem is often simplified by taking the emission to be monochromatic at the peak frequency of the emission

$$J(\nu) \, d\nu = \int_E n(E) P(E, \mathcal{H}) \, \delta(\nu - \nu_p) \, d\nu. \quad (3.27)$$

Note that the value of the peak emission frequency ν_p is proportional to E^2 (Eq. 3.24).

A possible distribution of electrons is a simple power law

$$n(E) dE = n_0 E^{-m} dE. \quad (3.28)$$

This distribution is easily generated in shock fronts by the first order Fermi process. When used with Eq. 3.27, we find a spectrum of the form

$$J(\nu) \propto \nu^{-(m-1/2)}, \quad (3.29)$$

which is also a power law (such as we see in Seyfert galaxies).

3.1.4 Absorption & Optical Depth

The observed high column depths of neutral Hydrogen, the assumed accretion disk and especially the proposed partial covering model for the high energy “tail” of the X-ray spectrum can all involve the absorption of photons by

un- or partially ionized atoms. The ability of an absorber species to interact with a photon is usually parameterized by σ_ν , the absorption cross section at a given frequency. The *optical depth* is defined as:

$$\tau_\nu = \int_{s_0}^s \sigma_\nu n(s') \, ds' \quad (3.30)$$

where n is the number density of the absorber. The probability that a photon of a given frequency ν will pass through the medium without being absorbed is $e^{-\tau_\nu}$. An object is called optically “thin” when τ_ν is less than 1, and optically “thick” if τ_ν is greater than 1. (Note that although the term *optical depth* is used, this applies to every wavelength. Do not be confused by the tendency of astronomers to use optical as a synonym for visible.)

3.1.5 Photoionization and Fluorescence

If an absorbed photon has sufficient energy, the absorber can be ionized. This adds free electrons (but not positrons) to the surrounding medium. The ions can also later recombine, emitting fluorescence photons isotropically. The structure and strength of the resulting emission lines depend upon the relative densities of the various ions (the *ionization parameter*), the temperature and the physical distribution of the material. Iron has a particularly high fluorescent yield, and so tends to dominate the line emission in the X-ray.

If the system of ions and neutral atoms is in thermal equilibrium, we use the Saha equation to calculate the relative amounts of each ionization state in the absorber. In the simplest case of a neutral atom and its first excited state, we proceed from the assumption of thermal equilibrium (ie, Boltzmann statistics) and write

$$\frac{dN_0^+(v)}{N_0} = \frac{g}{g_0} \exp \left[-\frac{\chi_I + \frac{1}{2}m_e v^2}{kT} \right], \quad (3.31)$$

where χ_I is the ionization potential, N_0 is the number of ground state atoms and $dN_0^+(v)$ is the differential number of ground state ions with their free electron velocity between v and $v+dv$. The factors g_0 and g are the statistical weights for the atom to be in the ground state and

$$g_0^+ \frac{2dx_1dx_2dx_3dp_1dp_2dp_3}{h^3} \quad (3.32)$$

respectively. g_0^+ is the statistical weight for the ion to be in its ground state, and the phase space differentials are constrained by Boltzmann statistics such that

$$dx_1dx_2dx_3 = 1/n_e$$

$$dp_1dp_2dp_3 = 4\pi m_e^3 v^2 dv$$

(n_e is the number density of electrons) and the factor of 2 is from the two possible spin states. Integration over velocity yields

$$\frac{N_0^+ n_e}{N_0} = 2 \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} \frac{g_0^+}{g_0} e^{-\chi_I/kT}. \quad (3.33)$$

Moving now to a general case, we apply the Boltzmann law

$$N_i = \frac{N}{\Xi} g_i e^{-E_i/kT} \quad (3.34)$$

where N is constrained to be the total number of atoms (including all ionization states) and Ξ is the partition function

$$\Xi = \sum_i g_i e^{-E_i/kT}. \quad (3.35)$$

This gives us

$$\frac{N_0}{N} = \frac{g_0}{\Xi(T)}, \quad \frac{N_0^+}{N^+} = \frac{g_0^+}{\Xi^+(T)} \quad (3.36)$$

from which we obtain Saha's equation

$$\frac{N^+ n_e}{N} = 2 \frac{\Xi^+(T)}{\Xi(T)} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_I/kT}. \quad (3.37)$$

Ξ and Ξ^+ are the partition functions which corresponds to the total number of neutral atoms N and the total number of ionized ones N^+ . Similar equations can be derived to relate any two adjacent ionization states.

3.2 The Pair Plasma

In addition to all of the above processes, we have the nonclassical effects of particle creation and annihilation to contend with at high energies. The detailed physics of pair plasmas is an entire field unto itself, and so we shall present only the relevant highlights.

3.2.1 Pair Annihilation

When an electron and a positron annihilate, two or more gamma rays are generated. Hillier (1984) performs a quick calculation based on the two and three photon creation cross sections to estimate their relative importance.

$$\sigma_{2\gamma} = \frac{\pi r_0^2 c}{v} \quad \sigma_{3\gamma} = \frac{4}{3}(\pi^2 - 9)\alpha \frac{r_0^2 c}{v} \quad (3.38)$$

$\alpha (= e^2/\hbar c)$ is the fine structure constant, $r_0 (= e^2/m_e c^2)$ is the classical electron radius and v is the relative velocity of the antiparticles. A quick resort to a calculator shows that the two photon process is over 370 times more likely. We therefore neglect all but the two photon process in this work.

The pair annihilation cross-section for the case of isotropic and unpolarized incident particles is (Hillier, 1984)

$$\sigma(\beta') = \frac{1}{2} \int d\sigma = \frac{3\sigma_T(1 - \beta'^2)}{32\beta'} \left[\frac{3 - \beta'^4}{\beta'} \ln \left(\frac{1 + \beta'}{1 - \beta'} \right) - 2(2 - \beta'^2) \right], \quad (3.39)$$

where σ_T is the Thomson scattering cross-section and β' is the centre of mass speed. The factor of one-half in front of the integral is necessary to avoid overcounting identical particles. From the cross-section, we calculate the annihilation rate as

$$R(\gamma_-, \gamma_+) = c \int_{-1}^{+1} \frac{d\mu}{2} f_{kin}(\beta_-, \beta_+, \mu) \sigma(\beta_-, \beta_+, \mu), \quad (3.40)$$

where $\sigma(\beta_-, \beta_+, \mu)$ is the cross-section (from Eq.3.39) with

$$\beta'^2 = \frac{\gamma_+ \gamma_- (1 - \mu \beta_+ \beta_-) - 1}{\gamma_+ \gamma_- (1 - \mu \beta_+ \beta_-) + 1} \quad (3.41)$$

and

$$f_{kin}(\beta_-, \beta_+, \mu) = \sqrt{\beta_-^2 + \beta_+^2 - \beta_-^2 \beta_+^2 (1 - \mu^2) - 2\beta_- \beta_+ \mu}. \quad (3.42)$$

While this integral can be done analytically, sufficient accuracy is obtained using the approximation presented in Coppi & Blandford (1990). In our calculations, we use their rate approximation

$$R(\gamma_-, \gamma_+) \approx R(x) = \frac{3\sigma_T c}{8x} [\sqrt{x} + \ln(x)], \quad (3.43)$$

where $x = \gamma_- \gamma_+$. Note that the previous work (Tritz, 1990) incorrectly used the nonrelativistic limit of this rate

$$R(\gamma_-, \gamma_+) \rightarrow \frac{3}{8} c \sigma_T.$$

The average energy of the generated photons ($\langle \omega \rangle = h\nu/m_e c^2$) can be easily seen by energy conservation and the fact that photons are identical particles to be $(\gamma_- + \gamma_+)/2$. The dispersion about this value, $\langle \omega^2 \rangle$ is found by integrating

$$\langle \omega^2 \rangle = \frac{c}{2R(\gamma_-, \gamma_+)} \int_{-1}^{+1} \frac{d\mu}{2} f_{kin}(\mu) \int_{-1}^{+1} d\alpha \left[\frac{d\sigma(\alpha, \mu)}{d\alpha} \right] \bar{\omega}^2(\alpha, \mu), \quad (3.44)$$

where α is the cosine of the centre of mass scattering angle. The other quantities are

$$\frac{d\sigma}{d\alpha} = \frac{3\sigma_T}{16\beta'\gamma'^2} \left[\frac{1 + \beta'^2(2 - \alpha^2)}{(1 - \beta'^2\alpha^2)} - \frac{2\beta'^4(1 - \alpha^2)^2}{(1 - \beta'^2\alpha^2)^2} \right], \quad (3.45)$$

$$\bar{\omega}^2(\alpha, \mu) = \frac{\gamma'^2}{(1 - \beta_{CM}^2)} \left\{ 1 + 2\alpha\beta_{CM} \cos \xi' + \frac{\beta_{CM}^2}{2} [1 - \alpha^2 + \cos^2 \xi' (3\alpha^2 - 1)] \right\}, \quad (3.46)$$

$$\beta_{CM} = \frac{\sqrt{\gamma_-^2 \beta_-^2 + \gamma_+^2 \beta_+^2 + 2\gamma_+ \gamma_- \beta_+ \beta_- \mu}}{\gamma_+ + \gamma_-}, \quad (3.47)$$

and

$$\cos \xi' = \frac{1}{\beta_{CM} \beta'} \left(\frac{\gamma_+ - \gamma_-}{\gamma_+ + \gamma_-} \right). \quad (3.48)$$

In the computational models to be presented, this expression was numerically integrated and tabulated for interpolation at run-time.

3.2.2 Pair Creation

Aside from a factor of two (electrons and positrons not being identical), the cross-section for pair creation is nearly the same as that for annihilation (Eq. 3.39). The only other difference is a phase space factor, so we have

$$d\sigma_{\gamma\gamma\rightarrow ee} = \beta'^2 d\sigma_{ee\rightarrow\gamma\gamma} \quad (3.49)$$

with

$$\sigma_{\gamma\gamma\rightarrow ee} = 2\beta'^2 \sigma_{ee\rightarrow\gamma\gamma}. \quad (3.50)$$

The pair creation rate is given by

$$R(\omega_1, \omega_2) = c \int_{-1}^{\mu_{max}} \frac{d\mu}{2} (1 - \mu) \sigma(\omega_1, \omega_2; \mu), \quad (3.51)$$

where ω is once again the photon energy in terms of $m_e c^2$ and we take $\beta' = \sqrt{1 - 2/\omega_1 \omega_2 (1 - \mu)}$ in $\sigma(\omega_1, \omega_2; \mu)$, the cross section obtained from equations 3.39 and 3.50. The upper limit on the integration is fixed by the need for β' to remain real. Since the energies only appear as a product, we define $x = \omega_1 \omega_2$. Again, Coppi & Blandford (1990) provide several useful approximations, of which we use

$$R(x) \approx 0.652 c \sigma_T \frac{x^2 - 1}{x^3} \ln(x) \Theta(x - 1), \quad (3.52)$$

where $\Theta(y)$ is the step function (cannot create particles with less than the rest mass energy).

Once again, energy conservation and symmetry fix $\langle \gamma \rangle = (\omega_1 + \omega_2)/2$ for us, and we find the dispersion by numerically integrating and tabulating

$$\langle \gamma^2 \rangle = \frac{c}{R(\omega_1, \omega_2)} \int_{-1}^{+1} \frac{d\mu}{2} (1 - \mu) \int_{-1}^{+1} d\alpha \left[\frac{d\sigma(\alpha, \mu)}{d\alpha} \right] \bar{\gamma}^2(\alpha, \mu). \quad (3.53)$$

The additional terms here are

$$\frac{d\sigma}{d\alpha} = \frac{3\sigma_T \sqrt{\omega'^2 - 1}}{16\omega'^3} \left\{ \frac{\omega'^2 + (\omega'^2 - 1)(2 - \alpha^2)}{\omega'^2 - (\omega'^2 - 1)\alpha^2} - \frac{2(\omega'^2 - 1)^2(1 - \alpha^2)^2}{[\omega'^2 - (\omega'^2 - 1)\alpha^2]^2} \right\}, \quad (3.54)$$

$$\bar{\gamma}^2(\alpha, \mu) = \frac{\omega'^2}{(1 - \beta_{CM}^2)} \left\{ 1 + 2\alpha\beta'\beta_{CM} \cos \xi' + \frac{\beta_{CM}^2 \beta'^2}{2} [1 - \alpha^2 + \cos^2 \xi' (3\alpha^2 - 1)] \right\}, \quad (3.55)$$

$$\beta_{CM} = \frac{\sqrt{\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\mu}}{\omega_1 + \omega_2}, \quad (3.56)$$

$$\cos \xi' = (\omega_1 - \omega_1)/\sqrt{2\omega_1\omega_2\mu}, \quad (3.57)$$

and $\omega' = \gamma' = 1/\sqrt{1 - \beta'^2}$.

These terms are often (Svensson, 1987; Lightman & Zdziarski, 1987; Tritz, 1990) used to construct a "pair creation optical depth" in radiative transfer models. For an arbitrary distribution of photons $n(\omega)d\omega$, the absorption probability per unit path-length is

$$\frac{d\tau_{\gamma\gamma}}{ds(\omega)} = \frac{1}{c} \int_0^\infty dx R(\omega, x) n(x). \quad (3.58)$$

For the specific case of a simple power law distribution, Svensson (1987) has shown that this function is approximated by

$$\frac{d\tau_{\gamma\gamma}}{ds(\omega)} \sim 0.2\sigma_T n \left(\frac{1}{\omega}\right) \left(\frac{1}{\omega}\right). \quad (3.59)$$

This approximation was used by Tritz(1990) in his SHOWER algorithm. Unfortunately, Coppi & Blandford (1990) show that it overestimates in general the optical depth (sometimes by more than a factor of two) and drastically overestimates the depth at $\omega \sim 1$, which may lead to incorrect estimations of the annihilation line feature.

3.3 Nonthermal Cascades

Combinations of processes in the preceding two sections can strongly reprocess an input photon distribution in the presence of electrons and positrons. Even in their absence, if the photon distribution contains photons with energies exceeding the rest mass of an electron it will create its own “atmosphere” of electron-positron pairs, which will then have the same effects.

If the initial photon distribution contains many photons with energies far in excess of the electron rest mass, pair creation can generate a *nonthermal pair cascade*. In this process, the high energy photon creates an electron

and positron which are themselves highly energetic. These particles then Compton upscatter lower energy photons to gamma ray energies. These upscattered photons can then generate more pairs, continuing the process. This rapidly transfers energy from the hard gamma rays to the softer energies.

3.4 Previous Work

Other researchers have traditionally presented homogeneous, isotropic models of the central engine of AGN. The rapid depletion of the high energy photons through the nonthermal pair cascade process caused those models to be considered first. That same process produced too many electron positron pairs, which in turn produced models with a strong annihilation line feature. This feature cannot be reconciled with observation (Done & Fabian, 1989). Theoretical work now concentrates on models which use a thermal distribution of high energy particles rather than a nonthermal one.

Tritz (1990) presented a simple radiative transfer calculation which addressed the inclusion of geometry in a nonthermal model. The primary model was a half-dimensional algorithm called SHOWER. The SHOWER code begins with a single emission surface (either a sphere or an infinite plane)

which emits a fixed photon spectrum. The model explicitly disregards back-scattering. All radiation continues to move away from the source surface. This allows the result to be simply evaluated by integration outwards from the surface. The code itself determines that optical depths for realistic input parameters are usually of order unity, so the assumption of zero backwards scattering is neither entirely correct nor strongly inconsistent. By limiting the injection to the bottom of the atmosphere (unlike other models), high energy photons are unable to escape without some measure of processing. This allows the annihilation line photons to be reduced along with the gamma ray input, producing a model which can actually satisfy both observational constraints.

At each step in altitude from the surface, detailed balance of the electron-positron pair reaction rates at equilibrium is used to fix the photon spectrum at that height:

$$0 = \dot{n} = \dot{n}_{cre} + \dot{n}_{ann} + \dot{n}_{in} + \dot{n}_{out} + \dot{n}_{inscatter} + \dot{n}_{outscatter}, \quad (3.60)$$

where the various subscripted terms are the rates of change at each energy due to photon creation, photon annihilation, incoming photons from above, below or model injection, outgoing photons to other parts of the atmosphere,

scatterings into that energy and scatterings out of that energy. The rates must be balanced *at all energies* in the calculation. The simplest term here is the annihilation rate

$$\dot{n}_{ann}(\gamma) = \int_0^\infty d\gamma' \int_0^\infty d\gamma'' R(\gamma', \gamma'') n(\gamma') n(\gamma''). \quad (3.61)$$

Similar expressions apply for the other reaction rates.

The dominant effects in the evolution of the atmosphere are Compton scattering and the pair creation and annihilation process. When there are photons with energies above the pair creation threshold, pairs will be created. Some of these particles will Compton scatter lower energy photons (especially if there is an additional thermal component) and so effectively transfer energy from the higher energy photons to the lower. As this process continues, fewer and fewer high energy photons remain. If the photon spectrum is not locally replenished, as one moves away from the source there will be fewer photons capable of generating pairs. The density of pairs drops, depleting the only remaining supply of high energy, pair annihilation.

Preliminary work had also been done to extend this code to include back-reaction. The reaction rate packages CRESOR and ANNSOR were developed and tested with a Monte Carlo algorithm which used the scattering angle information as well. This package was inefficient and the approximation

errors became so fully encoded that it was necessary to start over with a new approach.

3.5 Current Advancements

The exact rate equations used in the SHOWER code were not always the most accurate, as was mentioned in sections 3.1 and 3.2. The differences in the results when these rates were improved (Tsuruta & Kellen, 1995) can be seen in figure 3.1. The two larger panels on the left show output spectra from the SHOWER code. The x-axis indicates the log of the photon energy, the y-axis indicates the log of the photon energy times the flux at that energy. The upper panel shows the output spectrum using the cruder approximations of Tritz (1990), while the lower panel shows the benefits of the improved microphysics in Tsuruta & Kellen (1995). The three smaller panels on the right compare, from top to bottom, the optical depth, pair density and temperature between the two implementations of SHOWER. The x-axis in each case is the log of the altitude in source radii. The y-axes show the log of the relevant physical parameter for each panel. The dotted line in the top panel indicates an optical depth of unity. In each panel, a

solid line indicates the results found by Tritz, while the dashed line indicates the improved results from Tsuruta & Kellen. Qualitatively, the proper cross-section increases the downscattering effect, making the object more compact and depleting the high energy photons more quickly. It is encouraging when greater detail *improves* the fit of the model to reality.

Also of interest with the improved rate calculations is the break in the power law spectrum which occurs at about 100 keV (see figure 3.1). As was mentioned in section 1.2.3, recent OSSE observations are best fit with an (unexplained) power law cut off at about this energy. This feature was not discussed in Tsuruta & Kellen (1995) but is carefully examined in section 4.4.6.

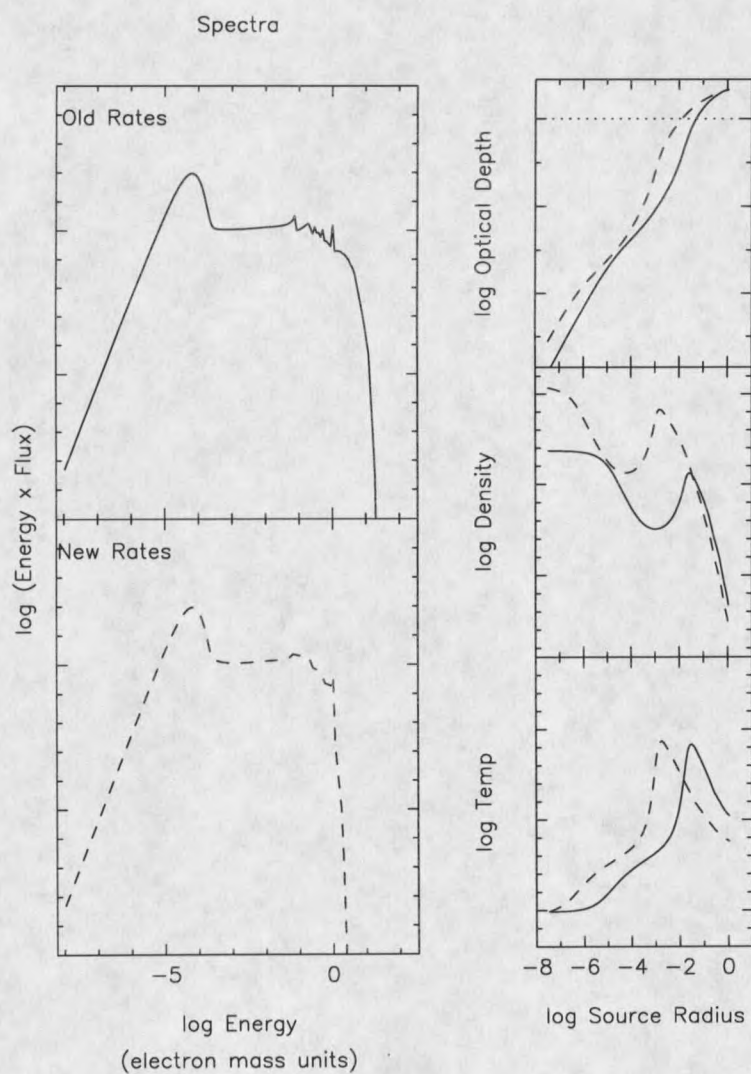


Figure 3.1: Results from Tritz, 1990 (solid) and Tsuruta & Kellen, 1995 (dashed)

Chapter 4

BATH

The primary shortcoming of the SHOWER algorithm upon which this research is based is the outward-only streaming of radiation it employs. This is also the source of its speed, as direct calculations can be made instead of iteratively approaching the solution. The code presented here allows for radiation to propagate in both directions, and must therefore approach a self-consistent solution via relaxation methods. Something which is more thorough, bidirectional, relaxed and *much, much* slower than a SHOWER could only be a BATH. (The less paranomastically inclined may prefer to think of this as referring to the radiation bath in which the pair plasma is immersed.)

4.1 A Conceptual Overview

The inherent difficulty in solving any scattering problem is the possibility of the scattered particles interacting with their original source environment. This nonlinear behaviour is often solved in either a Monte Carlo method or an iterative method. In a Monte Carlo solution, individual particles are followed as they propagate from the source through the surrounding medium. A suitably large number of particles are separately simulated to produce a statistical model of the resulting behaviour of the entire system. In an iterative method, an estimated solution is input into a simpler linear model. The output of that first calculation is taken as the input for the next iteration. This process continues until the input and output are the same, at which point the model is said to have converged on the solution.

In developing the BATH algorithm, we are guided by our previous results. Simulations using the SHOWER algorithm (Tsuruta & Kellen, 1995) show relatively low optical depths, which is to say a low number of scattering events. Analysis of the differential cross-section indicates that forward scatterings are much more likely than backward scatterings, especially at higher energies. These effects combine to make backward scattering events relatively rare. This means that the degree of nonlinear behaviour in the system is low

and suggests that the iterative approach will converge fairly quickly.

Our conceptual model of the system begins with the injection of both a hard and a soft photon distribution. The soft photon spectrum is generally any distribution in which the photon energy is less than the rest mass of the electron. This component is necessary in the model to provide a population of lower energy photons for the pair cascade process by which we intend to process our highest energy injected photons. We justify the inclusion of this component theoretically by noting the difficulty in naturally creating a distribution with no soft photons and observationally by noting that photons below the rest energy of an electron are in fact observed. The hard photon spectrum is also generally defined and is allowed to be any distribution in which there are photons with energies higher than the rest mass of an electron. The hard spectrum provides a source of photons capable of producing electron-positron pairs in the model, which are needed to generate the observed power law. The algorithm can also accept an arbitrary injection of electrons and positrons, although this was not done in this study.

The injected photons are then allowed to propagate. In our iterative approach, we use our injected spectrum as the input in our first round calculation of a linear model. Our linear model is a modified form of that used

by the SHOWER algorithm. We shall consider only a single dimension, height above an opaque "bottom" of the system, tacitly identified with the accretion disk. The "top" of the system is the height above which no significant number of scattering events will occur. As such it depends upon the input to the model and so may vary from iteration to iteration. The injected photons are then allowed to propagate under the condition of photon continuity from their injection point to the ends of the system. As they propagate, pair creation events will cause a plasma of electrons and positrons to form above the base of the system. We refer to this plasma as a corona or an atmosphere interchangeably hereafter.

We begin at the base of the corona and calculate the incident spectrum at that height. The incident spectrum in a layer consists of the upward moving photon spectrum from the layer below it, the downward moving photon spectrum from the layer above it and any photons which are injected at that height. The energy distribution of electrons and positrons must also be determined. This consists of the distribution output by the previous iteration (if any) plus any injected distribution. The distributions of photons and pairs are then scattered off of each other. Photon-photon and particle-particle processes are also computed. The resulting distribution of pairs is

then stored for the next iteration. The photons are divided according to the relative flux of initial upward and downward photons (injected photons are treated as being isotropic) on conservation of momentum arguments. The upward and downward emitted spectra are also stored for later use.

We then proceed to the next height to be considered and repeat the process. The upward emission from the previous layer's calculation is used as input from below. The downward emission from above is determined from the previous iteration through the atmosphere (if any). The process is repeated as we progress upwards through the atmosphere. When the top is reached we reverse the direction of integration and proceed downwards. Processing continues until an equilibrium is reached.

4.2 Details

Our first complication is the *raison d'être* of the model, geometry. Two underlying one dimensional geometries can be used in the model. The base of the atmosphere is either spherical (spherical symmetry) or an infinite sheet (linear symmetry). The spherical case has application if the region considered is far away compared to the size of the emitting region ($d \gg R$), where the

source appears as a point, and very close to the source, where the curvature effects are small and the surface approximates a plane. The infinite plane model provides a better insight into the intermediate ranges ($d \sim R$) as well as instructive contrasts for future work on the finite disk.

Having selected one of the available geometries, the photon continuity equation is then

$$\frac{d\tilde{n}(z)}{dz} = \frac{\dot{\tilde{n}}_{source}}{c} + \frac{\dot{\tilde{n}}_{reaction}}{c} \quad (\text{planar}) \quad (4.1)$$

or

$$\frac{d\tilde{n}(r)}{dr} = \frac{\dot{\tilde{n}}_{source}}{c} + \frac{\dot{\tilde{n}}_{reaction}}{c} - \frac{2\tilde{n}}{r} \quad (\text{spherical}), \quad (4.2)$$

where $\tilde{n}$ is the photon number density spectrum. The source term refers to any additional local input of energy in the model (*ie*, injection). All of the real physics is contained in the reaction term, which is calculated using the various physical processes detailed in chapter 3. Each of these terms is discussed in detail below. The final term in the spherical symmetry case is due to geometric dilution ($\tilde{n} \propto 1/r^2$).

This equation is integrated outwards by means of an adaptive Cash-Karp integrator for an arbitrary vector of differential equations, loosely based upon that presented by Press, *et al.* (1986). The integrator is an embedded fifth order Runge-Kutta formula. The routine performs six calculational steps

instead of the four in a standard Runge-Kutta integration method. The additional steps allow us to obtain slightly higher integrational accuracy (fifth order in step size) as well as calculate the truncation error. This is then used to adjust the size of the next vertical step for maximum efficiency and accuracy. Each step was required to be within the desired error tolerance.

The adaptive step sizes are not in general the same in any two iterations. This means that the exact altitude of the layer calculations varies from iteration to iteration. To obtain the particle and photon spectra at the point of calculation on a given iteration, we interpolate between the nearest layer results from the previous iteration. Since all of these calculations occur at the same height/radius, we suppress this coordinate for clarity in the remaining discussion.

4.2.1 Photon Source Function

In determining the source distribution of photons (both in energy and location), we are left with a wide range of choices. Our only constraints are that we must have a soft photon population off of which we can Compton scatter the pair population, and we must have a hard photon population to generate the pairs. The computer code is designed to allow us to input any spectral

distribution we can devise, for maximum generality.

A synchrotron-style hard photon power law and a nearly blackbody accretion disk soft photon spectrum were the primary input spectra used in most of the calculational runs, but to avoid the danger of initial assumptions coloring the results, we also considered some simple but highly unphysical input spectra. Results will be presented for input distributions which are monoenergetic in either or both of the hard and soft populations. There is also no requirement that we use a nonthermal hard photon spectrum. The output from a purely thermal hard photon input was also tested.

The other question is that of the physical location of the source(s) in the atmosphere. The SHOWER code injected both the hard and soft spectrum at the bottom of the atmosphere and *only* there. If we can associate the soft photons with thermal emission from the disk, this is not unreasonable, but for maximum generality we cannot make this assumption. The hard photons could be from any location in the atmosphere and could be generated by shocks in the disk or nearby clouds, magnetic flare events or some unknown process. The generalized input of the BATH algorithm enables the effects of injection altitude upon the model to be examined. Several possibilities are assessed, including the simplest, "everything at the bottom" approach

employed by Tritz.

The quasi-blackbody spectrum of an accretion disk can be taken to be the integrated flux from a set of blackbodies at temperatures

$$T(r) = \left\{ \frac{3GM\dot{M}}{8\pi r^3 \sigma_{SB}} \left[1 - \sqrt{\frac{R_G}{r}} \right] \right\}^{1/4}, \quad (4.3)$$

where R_G is the gravitational radius of the accreting mass (Pringle, 1981).

Both this form and a single temperature blackbody were tried as input soft photon sources.

In general, the photon flux from a given source is defined by the compactness parameter

$$l_s = \frac{L_s \sigma_T}{R m_e c^2} = \frac{4\pi R \sigma_T}{m_e c^2} \int F(x) dx, \quad (4.4)$$

with $F(x)$ being the flux at energy $x (= h\nu/m_e c^2)$ for the given injection source and σ_T being the Thomson cross-section. The soft and hard photon compactness parameters are separate quantities, as the physical processes which generate them and perhaps their physical location (and hence the value of R) may be quite different.

4.2.2 Reaction Rate Function

The reaction rate function contains all of the physical processes of scattering, creation and annihilation. It can be broken down into several pieces

$$\dot{\tilde{n}}_{reaction} = \dot{\tilde{n}}_{\gamma\gamma} + \dot{\tilde{n}}_{\gamma e} + \dot{\tilde{n}}_{ee}. \quad (4.5)$$

The $\gamma\gamma$ subscript refers to the photon-photon interactions (ie, pair creation), the γe to photon-particle scattering (including Compton scattering and the $\gamma + e^\pm \rightarrow e^\pm + e^+ + e^-$ creation process) and the ee to particle-particle interactions (annihilation, bremsstrahlung).

The SHOWER algorithm enforced detailed balance of each energy bin at each altitude by treating the photons and electron-positron pairs as each consisting of two separate populations, thermal and nonthermal. This method was rejected in the current model on the grounds that a given photon or particle reaction rate is strictly energy dependant, and is insensitive to the source of that energy. Also, single-pass calculation of the equilibrium state was only possible under a large set of simplifications of the underlying physics. Instead we applied the general physics of the scattering processes in an iterative manner until equilibrium values were obtained.

The condition for detailed balance of pair creation and annihilation is

$$0 = \int_0^\infty dx \int_0^\infty dx' [R_{ann}(x, x')N(x)N(x') - R_{cre}(x, x')n(x)n(x')], \quad (4.6)$$

where x is the energy in units of $m_e c^2$, $n(x)$ and $N(x)$ refer to the photon and pair number density spectra, respectively, and R_{ann} and R_{cre} are given in equations 3.40 and 3.51. The possible reaction matrices are repeatedly

applied over the course of the simulation until pair balance is achieved, indicating that the equilibrium output for the given input has been reached.

The probability that a photon of energy ω and another photon of energy ω' will interact to form an electron-positron pair, one of which has energy γ is

$$\frac{2}{\sqrt{2\pi}\langle\gamma^2\rangle} R(\omega, \omega') \exp\left(-\frac{\gamma - (\omega + \omega')/2}{\langle\gamma^2\rangle}\right), \quad (4.7)$$

where $R(\omega, \omega')$ is the pair creation rate (Eq 3.51) and $\langle\gamma^2\rangle$ is the dispersion (Eq 3.53) about the average energy. The factor of two accounts for the possibility of either particle having the desired energy. The total number of particles of energy γ produced is thus

$$\dot{N}_+(\gamma) = 2 \int_0^\infty d\omega \int_0^\infty d\omega' \frac{n(\omega)n(\omega')}{\sqrt{2\pi}\langle\gamma^2\rangle} R(\omega, \omega') \exp\left(-\frac{\gamma - (\omega + \omega')/2}{\langle\gamma^2\rangle}\right). \quad (4.8)$$

Similar rate equations are needed for every physical interaction considered. The reaction rate and dispersion function are different for each process, of course.

4.2.3 The Code

The code itself is written in an Object-Oriented style using C++. Object-Oriented programming places the emphasis upon the things being manip-

ulated (in this case, spectra, particles, distributions and layers in the atmosphere) rather than on the actions performed. Although in general less compact, C++ code is highly modular and so the individual parts can be developed, tested and improved without major reworking of the entire program. The header files for the classes defined in BATH are included in Appendix C.

Storage of intermediate results was performed using on-the-fly file compression to accommodate the large data sets. For N spectral bins in M atmosphere layers, each iteration generates $3N \times M$ double precision values. For cases in which we wish to observe the evolution of the model with each iteration, all of these values must be stored. The resulting files grow rapidly large without some sort of compression routine.

The simplest object defined was a RANGE. A RANGE is a logarithmic interval between a high and a low value. Various functions for setting, changing and comparing ranges are also defined as part of the RANGE object.

A continuous collection of RANGES constitutes a SPECTRUM. The associated functions allow comparison, creation and modification of spectra. The object-oriented approach makes this collective object more intuitive to use in programs than an array of RANGES, even though that is essentially what it is. The collective creation and change functions also insure that there are

no discontinuities in the SPECTRUM. Functions were also provided to alter the default equal binning of the ranges. The programmer can increase the density of bins and narrow their width and/or reduce the density by widening or conjoining bins at will. This allows computational power to be applied more strongly to more interesting parts of the SPECTRUM.

A DISTRIBUTION is a SPECTRUM plus a set of values, one for each RANGE in the SPECTRUM. In addition to the usual creation and modification functions, it is possible to completely change the SPECTRUM object. The old DISTRIBUTION is rebinned to conform to the new set of spectral bins. A programmer can create a DISTRIBUTION of any type of object. In this code, photons and electron-positron pairs were placed in DISTRIBUTIONS.

A collection of the upward going photon, downward going photon and electron-positron pair DISTRIBUTIONS along with a RANGE of altitudes make up a LAYER. It is at the LAYER level that the particle reactions enter the code, as we now have multiple DISTRIBUTIONS which can interact. Each LAYER must be stored for use in the next iteration of the model. Six of these are generated in each upward step of the Cash-Karp integrator.

A set of LAYERS which are continuous in altitude from the bottom of the corona to the top form an ATMOSPHERE. This object implements the

integration of photon continuity as well as the interpolation of the input to each LAYER calculation.

The input for a given simulation was an ATMOSPHERE object. The ATMOSPHERE object was created by a utility for composing an input DISTRIBUTION object into an ATMOSPHERE at a requested height and of the requested thickness. The interpolative procedure used to determine the input distributions during each iteration smoothed the effects of discrete source LAYERS. The DISTRIBUTION objects were generated by several utilities. The exact program used depended upon the desired photon or particle spectrum.

4.3 Reality Checks

Before plunging headlong into running the simulations, we must determine if the code is functioning as claimed. Of course, if there existed a code to which it could be compared, this would not make a useful thesis project. We can, however, take several limiting cases and compare the results with those obtained by algorithms which implement those cases.

4.3.1 The SHOWER Limit

If we take the limit in which $\sigma(\theta > \pi/2) \rightarrow 0$ (ie. no back scattering), we should recover the modified SHOWER solutions seen in Tsuruta & Kellen (1995). Figure 4.1 shows that this is the case. In this plot of the output spectra, the x-axis is the log of the photon energy in units of the electron rest mass and the y-axis is the log of the energy of the photons times the flux at that energy. The dotted line indicates the results from Tsuruta and Kellen, while the solid line shows the results of the BATH calculation under the constraint of no backward scattering. Only that portion of the total spectrum considered which shows any discernible differences is shown. The minor differences seen are due to the inclusion of effects ignored previously, such as nonthermal pair annihilation. This accounts for the slightly stronger annihilation line radiation and lower photon flux above the electron rest mass energy in the BATH results. In the case where only the identical physical processes modelled by Tritz are used, the differences are imperceptible.

4.3.2 The Thermal Limit

The left side of figure 4.2 shows the input spectrum (top left) for a purely thermal case as well as the resulting output (bottom left). Each spectrum

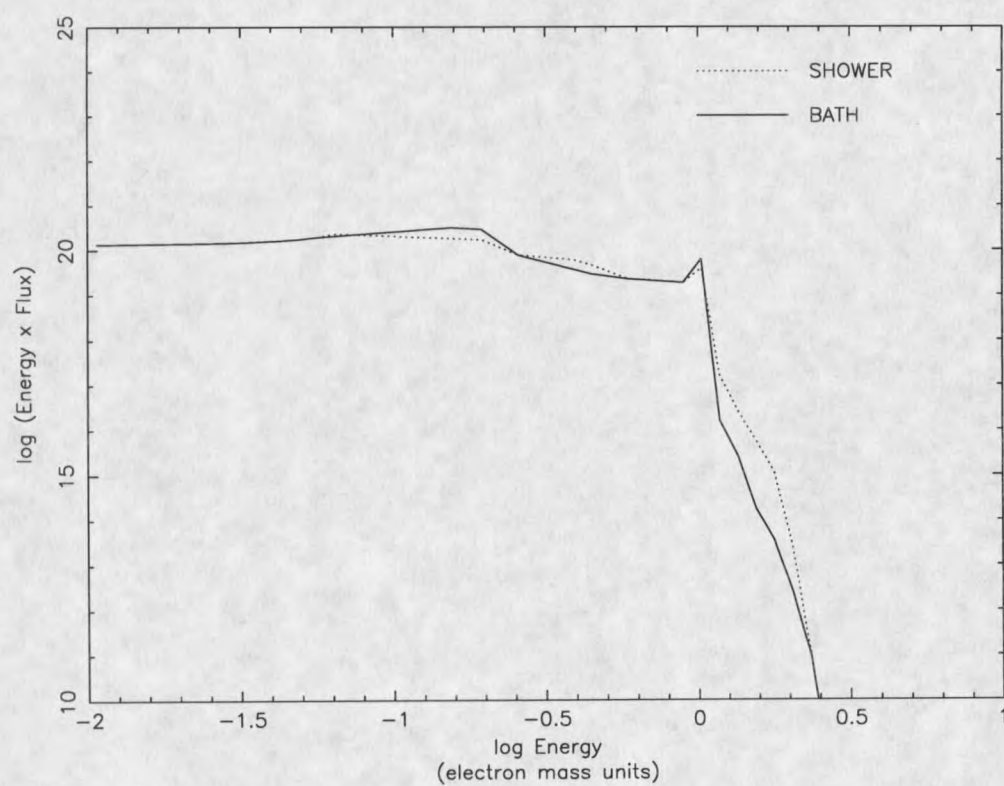


Figure 4.1: Comparison between SHOWER and BATH without backward scattering

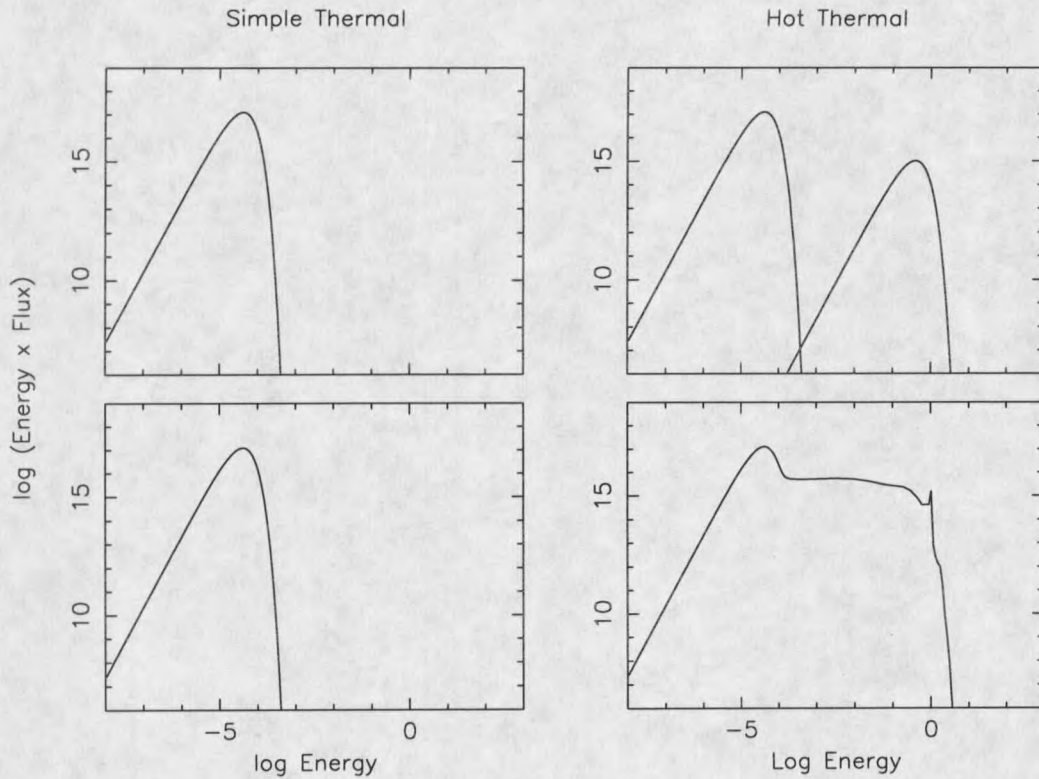


Figure 4.2: BATH results in the limit of no hard photon source

is plotted with the log of the photon energy along the x-axis and the log of the photon energy times the flux at that energy along the y-axis. The input spectrum is a single blackbody. In the case where no hard photon source is injected, and thus no pairs are produced, we should see no great processing of the input spectrum. The complete lack of photons capable of generating electron positron pairs leads to no spectral processing to speak of.

In the case of a highly energetic thermal input spectrum ($kT \sim m_e c^2$),

we should expect some processing by pair plasma physics. The right side of figure 4.2 shows these results under the same conditions. The top right panel shows the two input blackbodies, while the bottom right indicates the model output.

4.3.3 The Monte Carlo Limit

Although the underlying physics in the ANNSOR and CRESOR packages written by Tritz are oversimplified, they can provide a rough guide to the correctness of the results presented here. Tritz only modelled monoenergetic soft and hard energy inputs in low resolution bins (4 altitudes, 2 directions and 22 energy bins), so the graphical comparison in Figure 4.3 is not entirely fair. The figure shows the resulting spectra from two simulations under the Bath algorithm and one by the Monte Carlo method employed by Tritz. The dashed line indicated the results from Tritz' model with the altitude, direction and energy binning listed above. The solid line indicates the results of the BATH simulation with the same input parameters. The dotted line shows the result produced by the BATH algorithm when constrained to 22 energy bins and four altitude steps. Tritz noted depletion occurred at both the soft and hard ends of the spectrum in his Monte Carlo codes, but was unsure if

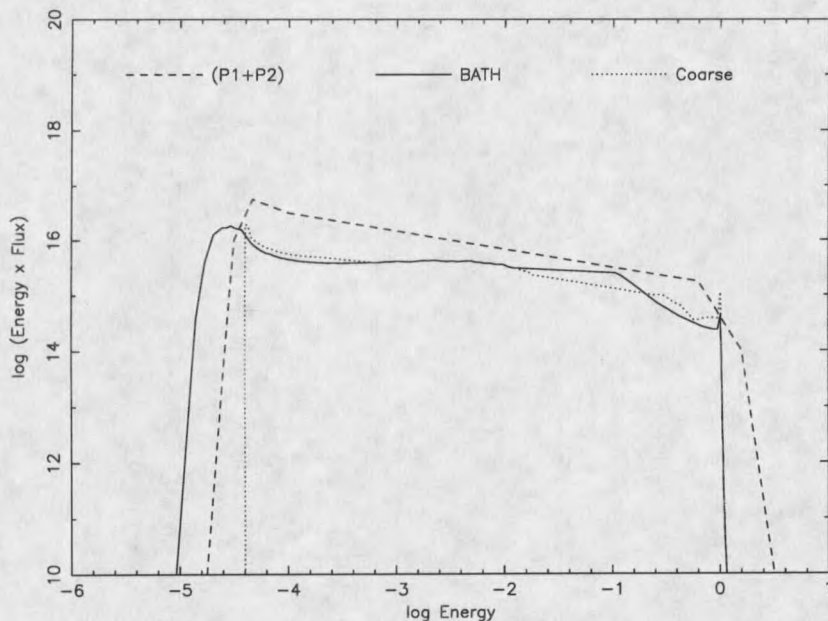


Figure 4.3: Comparison of BATH to Tritz's (P1+P2) Method

this was an artificial effect due to the coarseness of his altitude binning. The apparent trend of the BATH algorithm's soft photon cutoff under reduced resolution supports this conjecture.

4.4 Results and Constraints

Having performed the above checks to convince ourselves of the accuracy of the simulation, we turn now to the results and predictions made by the simulation, as well as the robustness of these answers. We shall also use available observational data to place some constraints upon the input parameters to

the model.

4.4.1 Input Dependencies

The BATH algorithm continues to display the same near total indifference to the shape of the input spectrum shown by SHOWER. Figure 4.4 shows a set of input spectra (on the left) and the resulting output from BATH (on the right). Each panel displays the log of the photon energy in units of electron mass on the x-axis and the log of the flux times the energy in arbitrary flux units on the y-axis. The top pair shows an input consisting of a thermal soft photon population and a monoenergetic hard photon source. The output panel shows the soft population as well as an X-ray power law which breaks at about 100-200 keV and displays a small annihilation line. The next pair down shows the input as simply two monoenergetic populations. The output still contains many of the spectral features of interest: the power law, the break and the annihilation line. The small variations in the power law are effects from the discreteness of the spectral bins in combination with the delta function injection. The third pair shows in input of a monoenergetic soft component plus a hard power law up to a maximum input energy of just under 1 GeV. The output spectrum still displays the same general shape.

The bottom pair of panels shows an input spectrum consisting of two thermal distributions. The output is qualitatively the same in the X-ray regime as in the other panels.

While the shape of the soft photon spectrum at low energies is sensitive to the exact input, the X- and γ -ray spectrum is not. This frees us from the concern that a major bias was introduced in the choice of injected spectrum. It also means that this method will not be useful as a probe of the seed spectrum in AGN.

This insensitivity is a direct result of the inhomogeneous atmosphere paradigm used in designing the model. In the homogeneous models constructed by others, the input spectrum is injected at all heights, even the uppermost layers of the atmosphere. This reduces the amount of material through which some of the input photons must travel to escape. The result in their models is therefore more sensitive to the input choice. In our BATH model, the injected spectrum is input only at specified altitudes, usually far below the top of the atmosphere. The higher levels in the atmosphere process all of the photons emitted from below. The variations in the spectrum at lower layers of the atmosphere due to the input spectrum are washed out in the upper layers by scattering and pair processes. This can be seen in

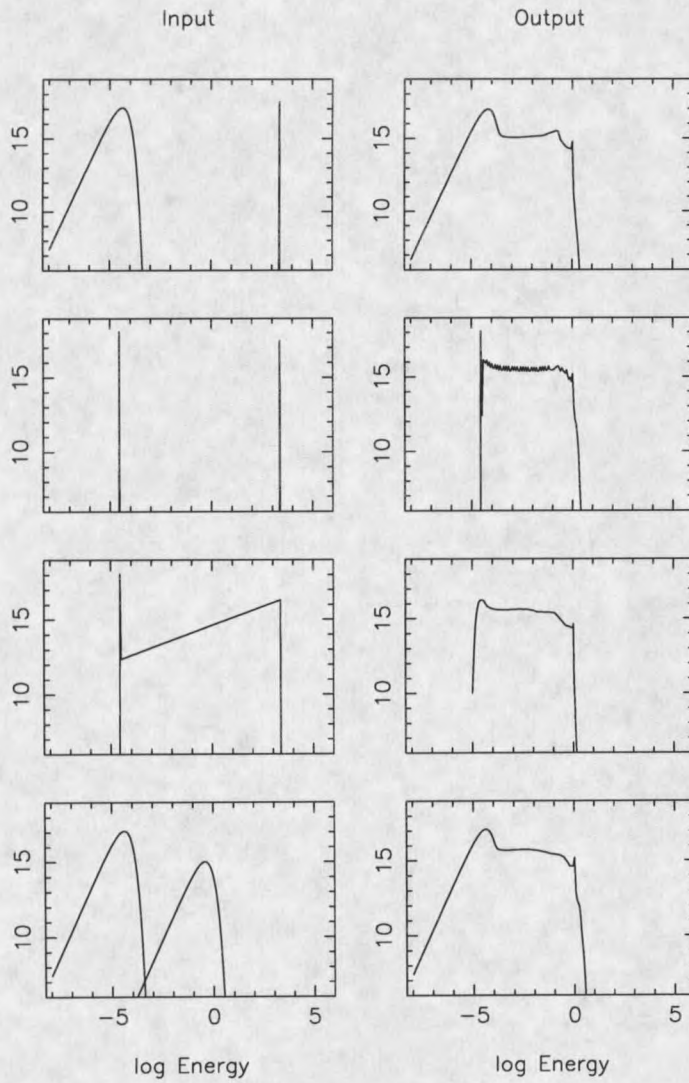


Figure 4.4: Insensitivity to the input spectral shape

Figure 4.5. Recall that injection altitude can also be varied in our model. The five panels show the output of the BATH model under variations of only this one parameter. The x-axis in each specifies the log of the photon energy in electron rest mass units. The y-axis is the log of the flux times the energy of the photons in arbitrary units. In the upper left corner of each panel the altitude at which the input spectrum was injected is listed in terms of the source region size. The uppermost panel shows the output spectrum if the injection height is at the surface of the atmosphere. It is identical to the input spectrum. Each of the other panels shows the effects upon the spectral shape caused by placing the source lower in the atmosphere. The bottom panel shows the results for injection at the base of the atmosphere *a la* the SHOWER model.

The variation of the resulting spectrum from BATH with hard photon compactness can be seen Figure 4.6. Recall that the compactness is proportional to the luminosity of the source divided by the radius (altitude) at which that source is emitted (Eq. 4.4). A more compact source is thus either more luminous, smaller or both. The output spectra for various values of this model parameter are shown in the figure. The compactness of the hard photon source injected in each case is shown in the upper right. In each case

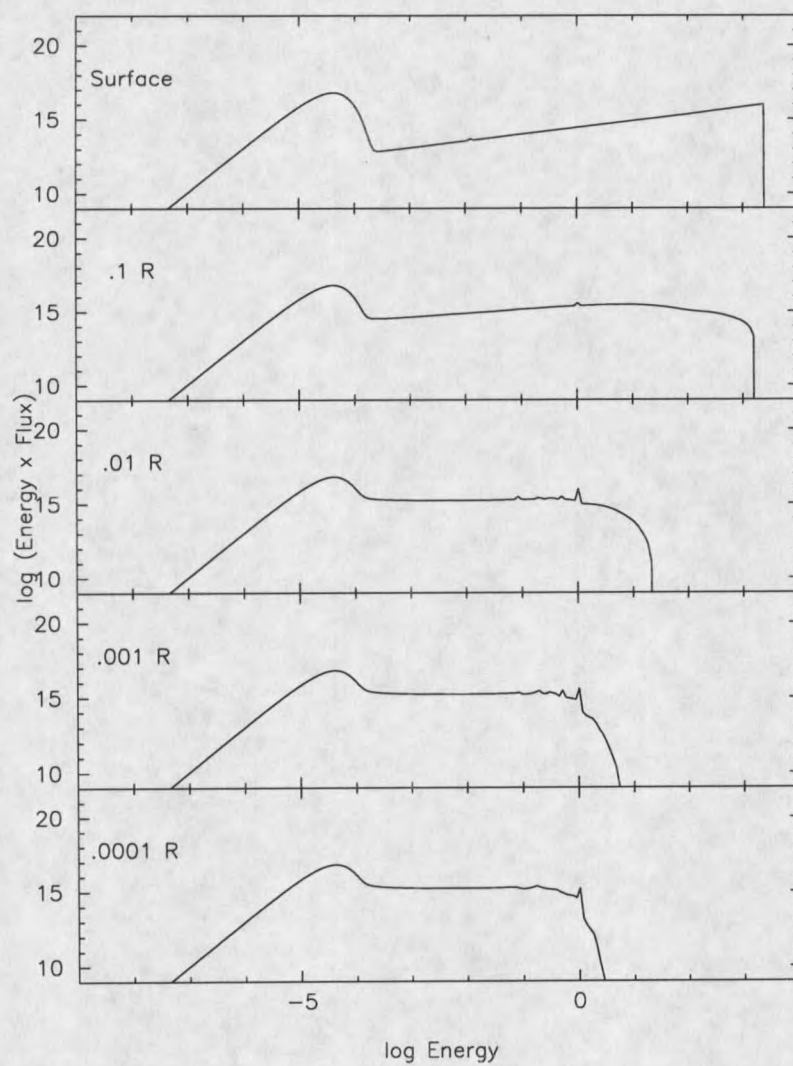


Figure 4.5: Variations in source layer altitude

the injection altitude is fixed at the base of the atmosphere. The soft photon injection was also held fixed.

These model results show that the output spectrum from the BATH model is nearly independent of the input spectrum. Only variations in the compactness of the source have an appreciable effect upon the result. More compact sources are less sensitive to the input spectrum than less compact sources. The altitude dependant simulations tell us that the hard photon injection cannot occur near the top of the atmosphere. The greatest range of parameter space will be available when the injection occurs at the bottom of the atmosphere.

4.4.2 Annihilation Radiation

The major failing of some other nonthermal pair models has been the overproduction of a .511 MeV line due to electron-positron annihilation radiation (see section 1.4.1). The inability of these models to fit observational constraints can be laid entirely at the feet of their homogeneous energy injection assumption. The BATH algorithm produces a line as strong or even stronger than that seen in other models, but that line is produced low enough in the atmosphere that additional processing has time to reduce the intensity.

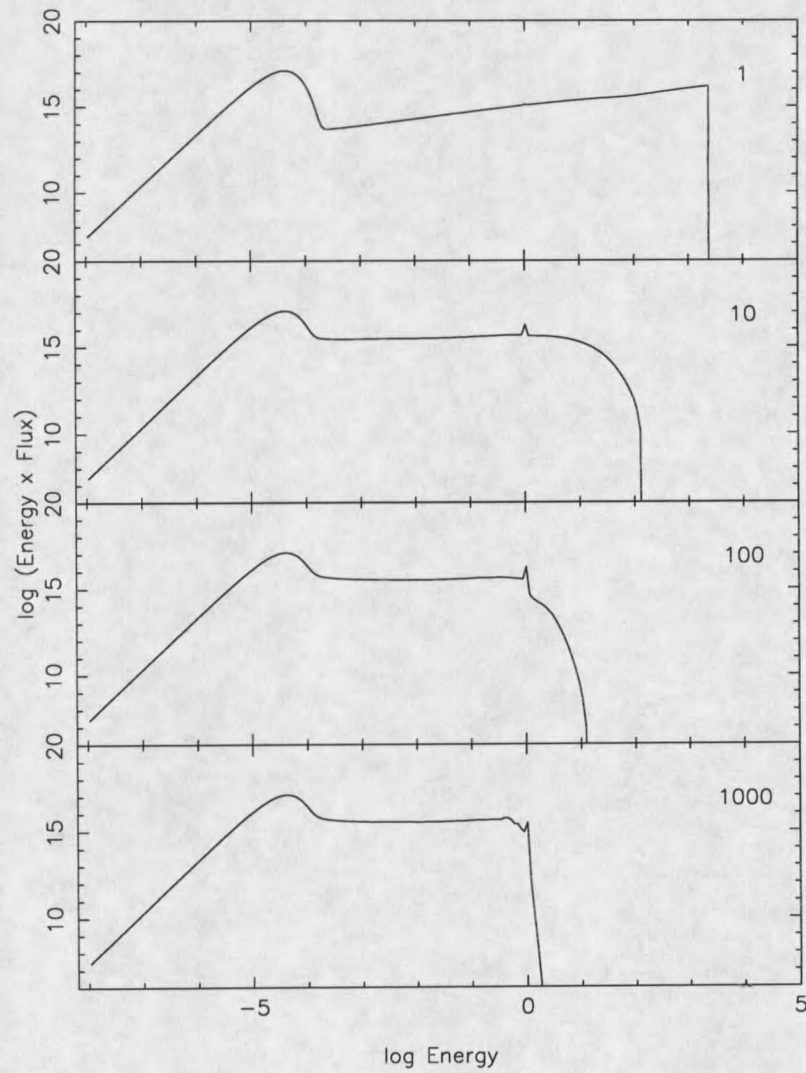


Figure 4.6: Results for varying compactness values

The primary input parameter determining the strength of the annihilation line is the hard compactness. A more compact source produces annihilation radiation deeper in the atmosphere, and therefore shows a weaker emitted line feature. Luminosity, of course, is also a factor. The curves in Figure 4.7 show the equivalent width of the annihilation line (based upon a power law fit to the continuum just above it) as a function of hard compactness (see section 4.2.1). The ratio of soft to hard compactness l_s/l_h is given to the right of each curve. The dashed curve indicates a very high energy powerlaw (cut off energy $\sim 10^4 m_e c^2$) which will produce a strong pair cascade, while the solid line indicates a lower energy ($\sim 10 m_e c^2$) model in which pair cascades are unlikely. High energy observations of Seyferts have failed to detect the annihilation radiation line feature, which places an upper limit upon the equivalent width of the line (Done & Fabian, 1989). The shaded area indicates regions prohibited by observational constraints. As you can see, a wide range of parameter space is still available.

4.4.3 The Gamma-Ray Background

The γ -ray sky background places limits upon the allowable high energy photon flux from Seyferts (Done & Fabian, 1989; Lightman & Zdziarski, 1987).

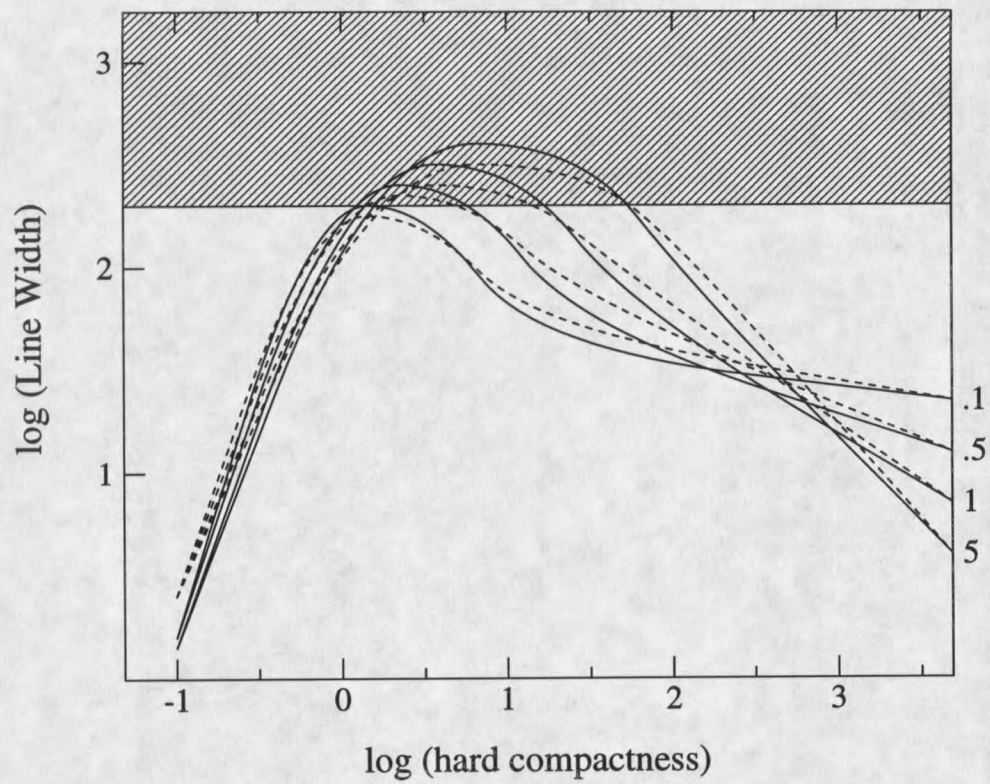


Figure 4.7: Equivalent width of the Annihilation line in several models

If the model presented here is to describe the output spectra of Seyferts, it should not predict an output in excess of observation. The γ -ray luminosity of the output model times the surface density of Seyferts on the sky should not exceed the observed background (Done & Fabian, 1989; Tsuruta & Kellen, 1995).

Figure 4.8 shows the ratio of γ -rays produced by the BATH model to the maximum which could be reconciled with the observed background. This parameter is most closely tied to the hard compactness, which is the horizontal axis. The y-axis indicates the value of the ratio, with a value of unity indicating that the model would require that only Seyferts contribute to the observed background. Models which produce more γ -ray luminosity than this are excluded. Models which produce less γ -ray luminosity allow for the possibility of other objects contributing to the background. The dashed lines indicate a higher energy input hard spectrum, as before. The numbers near the bottom end of the curves indicate the ratio of soft to hard compactness. The more compact sources are more heavily processed, and so lose more high energy photons, making it easier for them to comply with observational constraint. Many models can be found which lie within the allowed region.

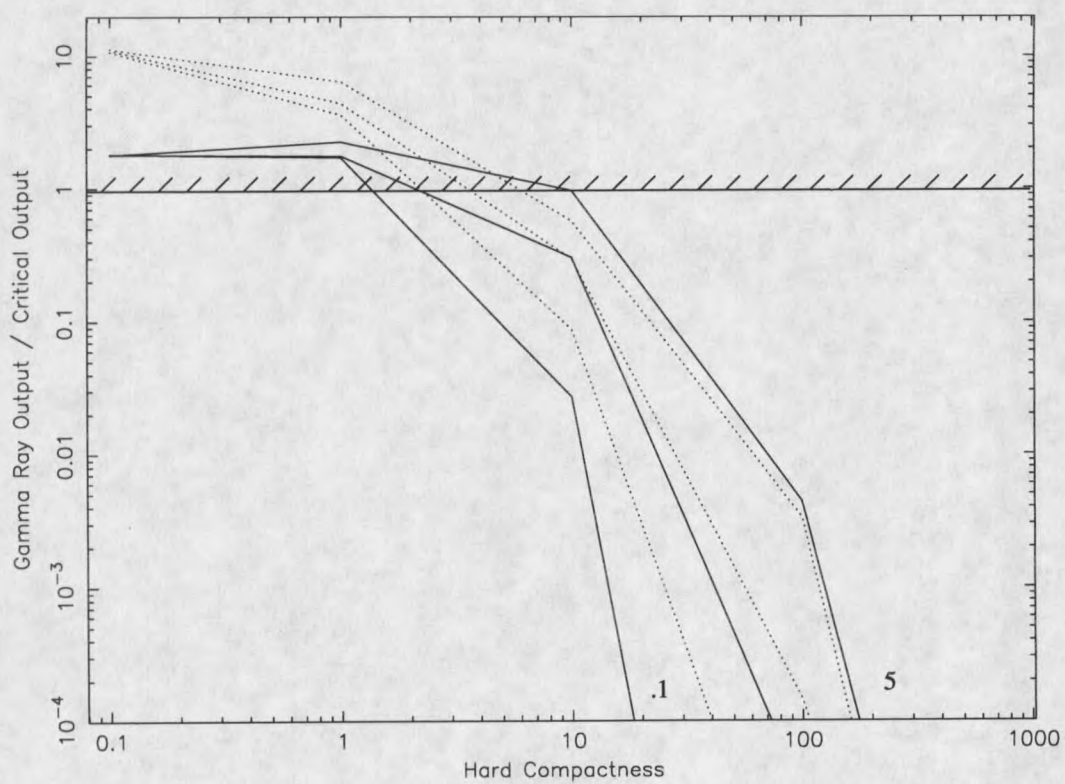


Figure 4.8: Ratio of Gamma-Ray production to observed background

4.4.4 X-ray Power Law

The output X-ray (energy) power law approaches 2.0 asymptotically as the pair cascade becomes saturated (Tritz, 1990). We define the X-ray compactness (l_x) as in equation 4.4, with the X-ray flux being defined as the model output between 2 and 10 keV. We also fit this region of the spectrum with a power law for comparison with observation. The ratio of the soft compactness to that in the X-ray, l_s/l_x is observationally constrained to be within the range from 1 to 300 (Done & Fabian, 1989; but modified as Tsuruta & Kellen, 1995), while the power law (energy) spectral index should be between 1.7 and 2.0 to be in accord with observation (see section 1.2.2).

The fitted power law slope is plotted against hard compactness in Figure 4.9. The ratio of input soft to hard compactness is indicated for each line at the right of the plot. The top panel indicates a set of models with a lower maximum energy cut off in the hard photon spectrum. The lower panel shows the results for models in which the high energy cut off is high enough for several generations of pair cascades to occur.

The trend towards a slope at 2.0 at high values of hard compactness is a general trait of the pair cascade (Tritz, 1990). In the cases where the soft source luminosity (compactness) is much greater than the hard, the cascade

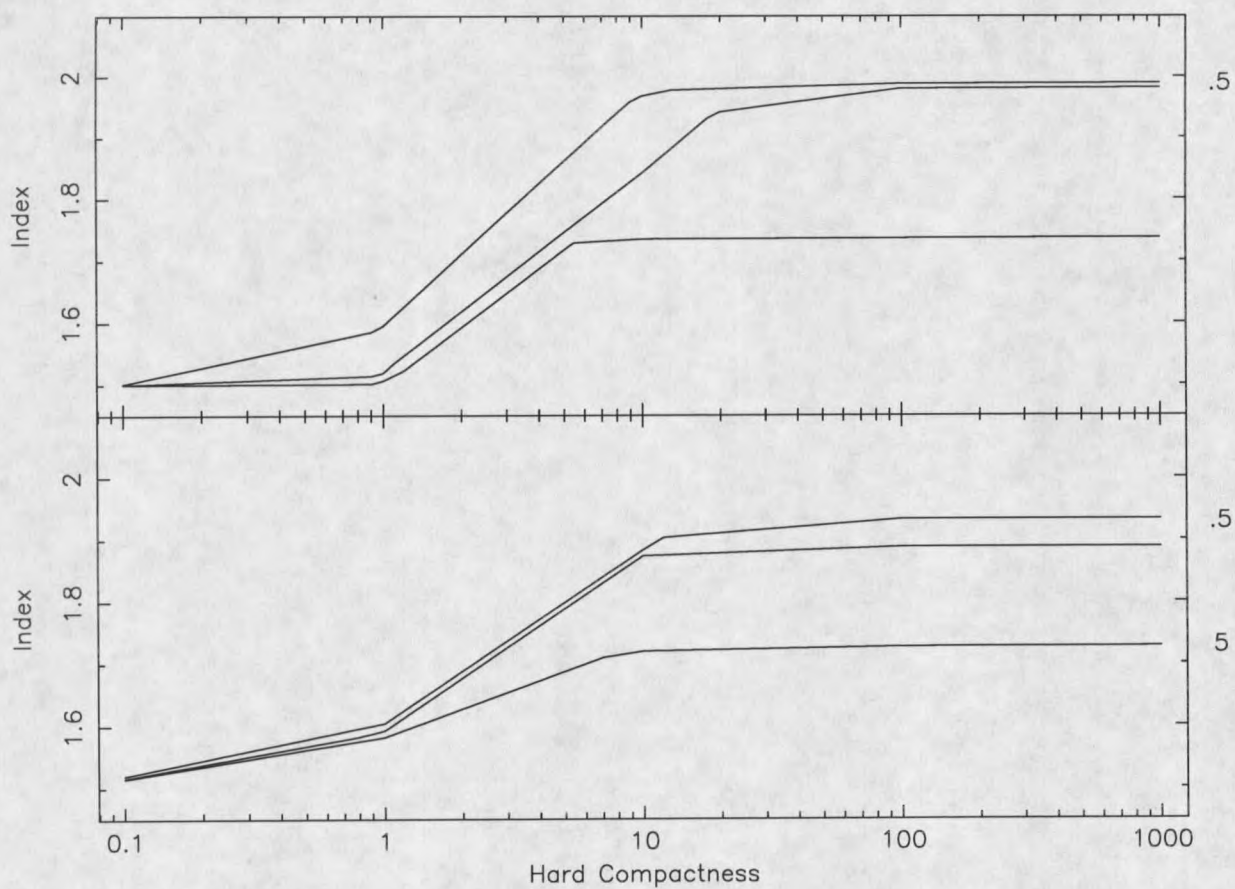


Figure 4.9: Power Law Fits to Model Output

is unable to reach saturation and approaches a lesser value. Once the model enters the allowed region, it does not leave, so this observational limit places only a lower bound upon the hard compactness.

Figure 4.10 plots the ratio of the X-ray to soft compactness versus the injected hard compactness. The ratio of input soft to hard compactness is 5 in the uppermost model runs and .1 in the lowest pair. The solid lines show the results from an input spectrum with a maximum cut off energy which allows for few pair cascades. The dotted lines indicate model runs with a higher maximum energy cut off in the injected hard photon power law.

This observational constraint allows us to limit the ratio of input soft to hard compactness to be roughly unity ($.1 < l_s/l_h < 5$). The rapid fall off in X-ray compactness with increasing hard compactness seen in the runs with a large pair cascade population (dotted lines) illustrates the efficiency with which this process transfers energy from a few high energy photons down into the X-ray at a low altitude.

4.4.5 Upon Reflection

The general insensitivity of the output spectra to changes near the bottom of the atmosphere explains the lack of effect in including reflection from the

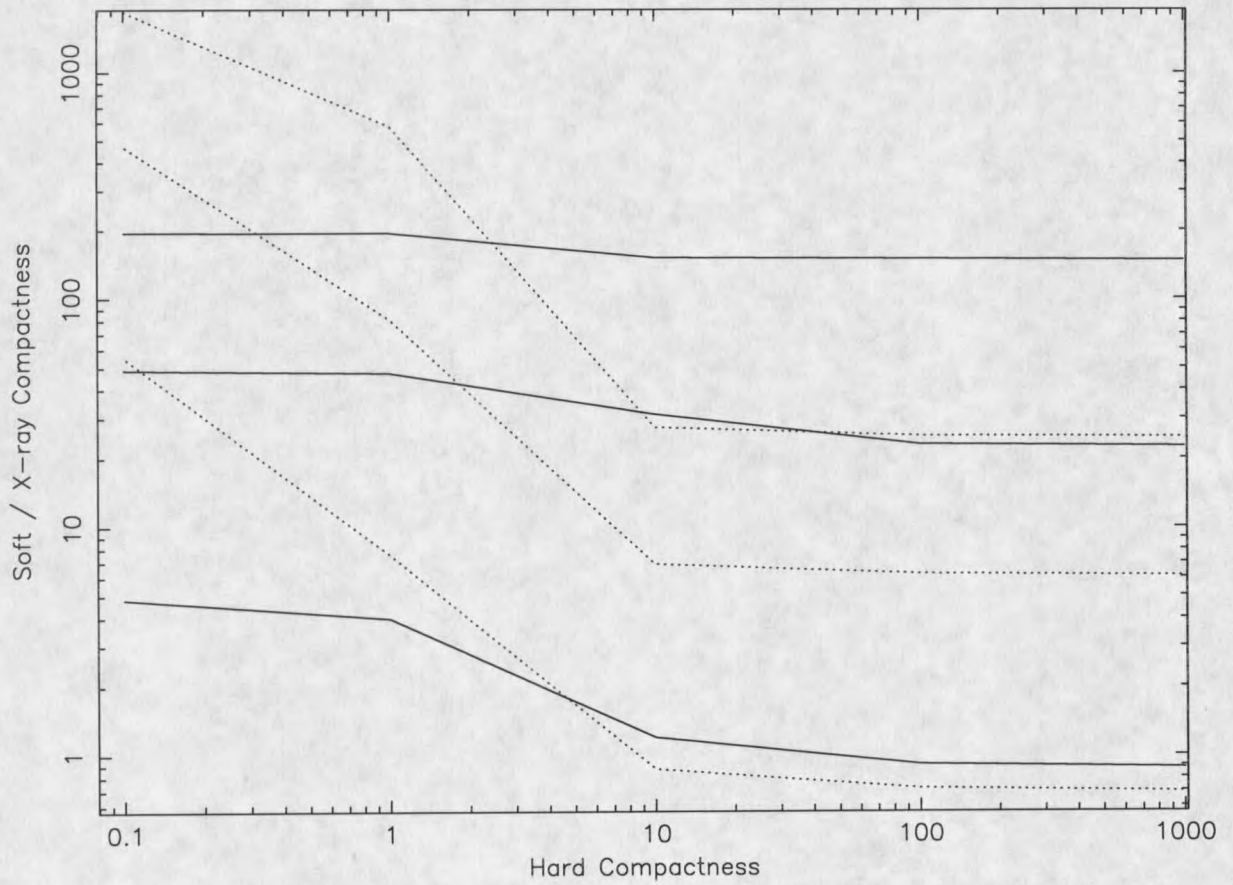


Figure 4.10: X-ray Compactness Ratios from Model

bottom of the atmosphere. Although the spectral effects are included at low altitudes they are processed out further up in the atmosphere. Any minor spectral features near the bottom of the atmosphere are quickly dispersed by the Compton scattering in the atmosphere. A possible way for reflection effects to be seen would be in far off-axis lines of sight in a higher dimensional model. This would require that the accretion disk and hypothetical optically thick torus (see section 1.3.2) not be coplanar if we were to see this feature.

Another possibility is that the reflection seen is not in fact due to the accretion disk, and that we are seeing exactly what we should see. Weaver, *et al.* (1996) were able to rule out the possibility of accretion disk reprocessing in the Seyfert galaxy NGC 2992 by virtue of a ten year delay in the response of reflection features to changes in continuum luminosity.

4.4.6 The 100 keV Break

The exponential cutoff seen in these data at about 100 keV shows that the same feature in the modified SHOWER results (mentioned in section 3.5) is an actual prediction and not an artifact of a single model. This is exciting, in that it predicts (albeit *post facto*) observational evidence, but unexpected in that no physics specific to this energy range is apparent in the construction

of the model.

This feature only begins to develop near the top of the atmosphere. At this point, the supply of higher energy photons is almost completely exhausted. The total energy necessary to create a pair is (of course) $2m_e c^2$. When there are few photons above this threshold, and none far above it, they must interact with photons of nearly the same energy to create a pair. Thus the range of photons which can “partner” with hard photons is originally quite broad, but shrinks rapidly towards $h\nu \sim m_e c^2$ as the system evolves. More photons are created at the annihilation line energy, but the average energy loss in a single Compton scattering event is roughly (Tritz, 1990)

$$\frac{\Delta x}{x} = 4 \frac{k_B T}{m_e c^2} - \frac{x}{1+x}, \quad (4.9)$$

where $x = h\nu/m_e c^2$ and T is the effective temperature of the pairs, about $1/3 m_e c^2$ in this part of the atmosphere. That makes for a $\Delta x/x \sim .8$, or about $.4 \text{ MeV}/c^2$ leaving an energy “no man’s land” below the electron energy from which photons are depleted by creation events, but rarely returned by scattering. The nonzero dispersion of the scattering events contributes to the smoother fall off than this naive calculation would predict.

4.5 Discussion

The BATH algorithm presented here is a highly generalized one dimensional model of radiative transport including pair processes. It can be used to study thermal, nonthermal and combination input spectra. The model properly reduces to special case results found in other studies under the same limiting conditions. It is intended to be easily improved and extended without requiring a great deal of programming skill to use.

The BATH models do not allow us to place useful constraints upon the input spectrum. The dependence on depth of the importance of the input spectrum suggests that this limitation will be relaxed in a higher dimensional model. If the soft and hard photon injection sources have different spatial distributions, then their relative depths in the atmosphere will vary with position. This would affect the observed spectrum at some angles.

Observational constraints do allow us to place limits upon the values of the input soft and hard compactness in the models. Our results show relatively small dependence of these limits upon the shape or maximum energy range of the input spectrum. The observed range in ratios of soft to X-ray compactness limits $5 > l_s/l_h > .1$. Requiring that the X-ray power law fall between 1.7 and 2.0 forces up the lower limit of this constraint to

$l_s/l_h > .3$ and also constrains $l_h > 2-6$, depending upon the input. Limiting the models to produce a valid amount of γ -ray flux restrict us to values of $l_h > 5-20$. Our final observational constraint is placed by the nondetection of the electron-positron annihilation line, requiring $l_h > 10-80$.

A natural result of the conceptual model is the prediction of the observed 100 -150 keV break in the power law. This feature is unavoidable in any model in which the input photon supply becomes depleted near the surface. It begins to appear in models with $l_h > 100-200$. No observational constraint could be placed by this feature.

Chapter 5

CONCLUSIONS

The primary conclusion to be reached is that faster computers are better. Both the observational modeling and the theoretical simulations are highly computationally intensive and would not have been possible to complete without total monopolization of computing resources. The other obvious conclusion is that it is easier and faster to use precompiled analysis software than to try and port it to the local hardware.

5.1 Introduction

We began with an overview of the topic of active galaxies, and Seyfert galaxies in particular. The common features of the X-ray spectrum of Seyfert galaxies

were described. Several common conclusions based upon these observations were presented. Current models of the central engine of AGN and their counter-arguments were presented. The state of the art in high energy models was also reviewed. Finally, the intended thrust of this thesis was outlined.

5.2 Observation

Chapter 2 presented a detailed account of spectral fitting to an observation of the Seyfert 2 galaxy NGC 4593 with several realistic models. Possibly the most useful instruction the author received in this process was to be as conservative as possible in fitting, retaining only model components which significantly improve the fit, and carefully limiting the number of free parameters.

The detailed analysis of NGC 4593 has netted many interesting results which have previously not been seen in this object . The most striking measurements are the detection of variability in the O VI emission line and the correlated changes in the ionization states of oxygen VII and VIII with the continuum luminosity. The highly featured soft continuum of this source deserves more detailed observation.

The hard spectrum results are unremarkable, but certainly consistent with the cursory examinations by other groups of researchers. The inability to resolve the iron line is again an argument for longer observations of this object.

The phenomenological binning approach suggested by Kunieda bore striking results in the timing analysis of the soft X-ray band. Once again, with better statistics, the error bars on the relative OVII-OVIII plots could be constrained well enough to make some direct calculation of the density of the ionized species.

5.3 Particle Theory

In chapter 3 we reviewed the many physical processes which were to be included in our model. First we detailed the physics of various photon/particle interactions, with special attention to the most important of these, Compton scattering. Several common errors in the approximations commonly applied were indicated.

Our discussion of particle creation and annihilation detailed the reaction rates and energy dispersions of these processes. We also reviewed the phe-

nomenon of the pair cascade and discussed how it rapidly processes high energy photons into low energy pairs.

We reviewed the models presented by Tritz (1990) and discussed some of their results and assumptions. The errors in previous works were corrected and the effects of these corrections discussed, along with some of the results.

5.4 Computational Model

The results presented in Chapter 4 show us that the nonthermal pair disk-corona model for the high energy spectra of Seyfert 1's is very much alive and well. Many of the results presented here are intensifications of findings from the simple half-dimensional SHOWER code, supporting the original claim that the simplifying assumptions were not a great deviation from a more realistic approach.

We developed an object-oriented C++ class library for the general computation of one dimensional pair plasma atmospheres and verified that it reduced to all of the testable limits. The opacity inherent in an object oriented approach will allow future researchers to improve upon the class implementation without the complete rewriting that was required to improve the physics

employed in SHOWER.

Many corners of parameter space were explored using the BATH relaxation algorithm based upon the developed class library. The nature of several model dependencies was determined, in one case extending a possible reason for an unexplained observational constraint. Some of the features included in the model were shown to have little impact upon the results in the one dimensional case. The ability of the model to conform with observation over a broad range of input parameters was confirmed and the boundaries of that space delineated.

5.5 Concluding Remarks

The iterative relaxational approach used here to generate the spectral models is not well suited for use in fitting to observational data. The need to rapidly recalculate the model spectrum based after changes in the input parameters is in strong conflict with the accuracy over speed design strategy employed in BATH. Bridging this gap would allow qualitative comparison of the nonthermal model to observational data, greatly speeding the evolution of nonthermal models.

The art of data analysis is practiced in many ways by many people. No two of my instructors held the same or even fully compatible opinions on the subject. The author's advice is to abide by the cardinal rules of data analysis: add nothing, delete only what you can justify and do not allow preconceptions to taint your method.

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APPENDICES

Appendix A

The exact distribution of photons after a Compton scattering event is calculated in Jones (1968). Coppi and Blandford (1990) note apparent misprints in the published article, and give the corrected distribution as

$$\begin{aligned} \frac{dP}{dz}(\omega; \omega', \gamma) = & \frac{3\sigma_T c \omega}{16\gamma^4 \omega'^2 R(\omega', \gamma)(1-\beta z) \sqrt{\beta^2 + \epsilon^2 + 2\beta\epsilon z}} \\ & \times \left[2y_0 k - ak^2 + \frac{(1+y_0^2) - 2ay_0 k + a^2 k^2}{\sqrt{a^2 - b^2}} \right. \\ & \left. + \frac{1}{k^2(1-\beta z)} \left\{ k^2 + \frac{k^2 a(2b-a)}{(a-b)\sqrt{a^2-b^2}} + \frac{a(1-y_0)^2 + 2kb^2(1-y_0) - ba^2 k^2}{(a^2-b^2)^{3/2}} \right\} \right], \end{aligned} \quad (A.1)$$

where $\epsilon = \omega'/\gamma$, $k = \gamma/\omega$, $a = 1 - \beta z - (1 - y_0)/k$, $b = \delta/k$ and in the following, $\rho = \omega/\omega'$:

$$y_0 = \frac{(\epsilon + \beta z)(\rho + \epsilon\rho - 1 + \beta z)}{\rho(\beta^2 + \epsilon^2 + 2\beta\epsilon z)}$$

and

$$\delta = \beta \sqrt{1 - z^2} \frac{\sqrt{\rho^2 \beta^2 + 2\rho\epsilon(1-\rho)(1-\beta z) - (\rho - 1 + \beta z)^2}}{\rho(\beta^2 + \epsilon^2 + 2\beta\epsilon z)}.$$

The Compton scattering energy distribution is then found by integrating

$$P(\omega; \omega', \gamma) = \int_{z_-}^{z_+} dz \frac{dP}{dz}, \quad (A.2)$$

where

$$z_- = \max \left[-1, \beta^{-1} \{ 1 - \rho(d + \sqrt{d^2 - 1/\gamma^2}) \} \right],$$

and

$$z_- = \min \left[+1, \beta^{-1} \{ 1 - \rho(d - \sqrt{d^2 - 1/\gamma^2}) \} \right],$$

with $d = 1 + \epsilon - \epsilon\rho$. Equation A.1 is easily integrated numerically and the results tabulated for use.

Appendix B

The expression in Haug (1985) is derived from the electron-electron bremsstrahlung cross-section by means of a symmetry argument and substitution rule. In the c.m. system for the scattering of an electron and positron with momenta $\vec{p}$ and $\vec{q}$ into $\vec{p}'$ and $\vec{q}'$ plus a photon $\vec{k}$, we have

$$\vec{p} + \vec{q} = \vec{p}' + \vec{q}' + \vec{k} \quad (B.1)$$

and

$$\epsilon_+ = \epsilon_- \equiv \epsilon, \quad (B.2)$$

where $\epsilon_{\pm}$ is the energy of the particle under consideration.

Two invariants are defined,

$$\tau = 2\epsilon^2 - 1, \quad \rho^2 = 4\epsilon(\epsilon - k) \quad (B.3)$$

and two symmetric products are constructed,

$$\kappa_1 = k(\epsilon - p \cos \theta), \quad \kappa_2 = k(\epsilon + p \cos \theta). \quad (B.4)$$

In these terms, the following formula for the differential cross-section is given:

$$\begin{aligned} \frac{1}{\pi}(\rho^2 - 4)^{1/2} \int A d\Omega_{q'} = & (\rho^2 - 4)^{1/2} \left\{ \frac{9\tau+4}{4\kappa_1\kappa_2} - \frac{3}{4\kappa_1^2}(\rho^2 + 3) - \frac{\kappa_2\rho^2}{4\kappa_1^3} + \right. \\ & + \frac{\rho^2}{\kappa_1(\tau^2-1)} \left[\frac{\kappa_2^2}{\kappa_1^2} - \frac{2\kappa_2\tau}{\kappa_1^2} + \frac{\kappa_2\tau}{2\kappa_1} - \frac{1}{2} \right] + \frac{1}{2(\tau+1)} \left[\frac{\kappa_2}{\kappa_1} - \frac{1}{\tau+1} \right] \\ & - \frac{1}{\kappa_1+\kappa_2} + \frac{1}{(\kappa_1+\kappa_2)^2} \left[\frac{\kappa_1\kappa_2-2.5\rho^2}{\tau+1} - \frac{\rho^2}{2(\tau+1)^2} - 4\rho^2 \right] \\ & + \frac{\kappa_1\kappa_2\rho^2}{(\kappa_1+\kappa_2)^4} \left[4 - \frac{1}{\tau+1} \right] + \frac{1}{\rho^2} \left[\frac{4\tau}{\kappa_1\kappa_2} - \frac{4}{\kappa_1^2} + \frac{4\kappa_2}{\kappa_1} + 3 \right] \\ & + \frac{\rho^2-4}{12\kappa_1\rho^4} \left[(\rho^2 - 4) \left(\frac{\tau}{\kappa_2} - \frac{1}{\kappa_1} \right) - 4\kappa_2 \right] \Big\} \\ & + X_1 \left[\frac{2\tau+3}{\kappa_2\rho^2} - \frac{2}{\kappa_1\kappa_2\rho^2} - \frac{1}{\kappa_1\rho^2} + \frac{3\tau-1}{2\kappa_1\kappa_2} + \frac{\kappa_2}{2\kappa_1(\tau+1)} - \frac{1}{\kappa_1} \right] + X_2 \left[\tau + 1 - \frac{1}{2(\tau+1)} \right] \\ & - \frac{X_3}{2\kappa_1} - \frac{\tau+1-\kappa_2}{2\kappa_1\kappa_2\rho^2} X_4 - \tau^2 X_5 + \rho \frac{L_1}{W_1} \left\{ \frac{1}{2(\tau+1)} \left[\frac{1}{\kappa_1} (3\kappa_2^2 + 2\kappa_2 - 4) + 3\kappa_2 + 6 \right] \right. \\ & + \frac{5\tau-11\kappa_1+1}{2\kappa_1} - \frac{13\tau+7}{2\kappa_2} + \frac{2\rho^2}{\kappa_1^2} - \frac{2\tau\rho^2}{\kappa_1\kappa_2} \\ & + \frac{1}{2\rho^2} \left[7\kappa_1 + 4\kappa_2 - 10\tau + 2 + \frac{1}{\kappa_1} (\kappa_2^2 - \kappa_2\tau + \kappa_2 + 4) \right. \\ & \left. \left. - \frac{1}{\kappa_2} (11\kappa_1\tau + 4\tau + 9\kappa_1 - 8) - \frac{8}{\kappa_1^2} + \frac{8\tau}{\kappa_1\kappa_2} \right] \right\} \\ & + 4\rho \frac{L_2}{W_2} \left\{ \kappa_1 - \tau + 2 - \frac{2\tau}{\kappa_2} + \frac{1}{\kappa_1} \left[\left(\tau - \frac{5}{4}\kappa_2 \right) \rho^2 + 4\tau^2 - 3\kappa_2\tau + 2\kappa_2^2 \right] \right. \\ & + \frac{\kappa_2}{2(\tau+1)} (\tau - \kappa_1 - 2\kappa_2) - \frac{\kappa_2^3-3\kappa_2+\tau}{\kappa_1(\tau+1)} + \frac{1}{2(\kappa_1+\kappa_2)} (3\kappa_2\tau + \kappa_1\kappa_2 - 4\tau^2 - 4\tau - \kappa_2 + 2) \\ & \left. - \frac{2\tau^3}{\kappa_1(\kappa_1+\kappa_2)} + \frac{\kappa_1-1}{(\kappa_1+\kappa_2)(\tau+1)} \right\} \\ & + \rho L_3 \left\{ \frac{1}{\tau+1} \left[1 - \frac{3\kappa_2}{\kappa_1} + \frac{1}{\kappa_1\kappa_2} - \frac{8}{\kappa_1+\kappa_2} + 2 \frac{2\tau-\kappa_1\kappa_2}{(\kappa_1+\kappa_2)^2} + \frac{4\kappa_1\kappa_2}{(\kappa_1+\kappa_2)^3} + \frac{4\kappa_1\kappa_2}{(\kappa_1+\kappa_2)^4} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{(\tau+1)^2} \left[1 - \frac{2}{\kappa_1 + \kappa_2} - \frac{2}{(\kappa_1 + \kappa_2)^2} \right] - \frac{6}{\kappa_1} - \frac{8}{\kappa_1^2} - \frac{4\rho^2}{\kappa_1 \kappa_2} \\
& + \frac{5\rho^2}{(\kappa_1 + \kappa_2)^2} - (\rho^2 + 6) \frac{2\kappa_1 \kappa_2}{(\kappa_1 + \kappa_2)^4} + \frac{2}{\kappa_1 \rho^2} \left[\frac{1}{\kappa_2} - \kappa_2 - 2(\tau + 1) \right] \Big\} \\
& + \frac{\rho L_4}{\kappa_1 \kappa_2 \sqrt{\tau^2 - 1}} \left\{ 2\kappa_1^2 + \frac{5}{2}\kappa_1 \kappa_2 - \tau(\rho^2 - 2) + \frac{\kappa_1}{2}\rho^2 + 2 + 2\frac{\kappa_2}{\kappa_1}(2\tau^2 - \kappa_2^2 - 2\kappa_2\tau - \rho^2 \right. \\
& \quad \left. - 1) + \frac{1}{\kappa_1 + \kappa_2}(4\tau^3 - 3\kappa_1 \kappa_2 \tau + \kappa_1 \kappa_2) + \frac{1}{\tau + 1} \left[\kappa_1^2 - 2\kappa_1 - 1 + \frac{2\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} + 2\kappa_2 \frac{\kappa_2 + 1}{\kappa_1} \right] \right. \\
& \quad \left. + \frac{1}{\rho^2} \left[3\tau(\kappa_1 + \kappa_2)^2 - 2\kappa_1 \kappa_2(\tau + 1) + 4\tau^2 - 2\tau = 2 = 4\tau \frac{\kappa_2}{\kappa_1} \right] \right. \\
& \quad \left. + \kappa_2 \frac{\kappa_1 + \kappa_2}{\tau^2 - 1} \left[\tau - \frac{\kappa_2}{\kappa_1} \right] \left[4\tau \frac{\tau - \kappa_2}{\kappa_1^2} - 1 \right] \right\} + \{ \kappa_1 \leftrightarrow \kappa_2 \} \quad (B.5)
\end{aligned}$$

with the following terms:

$$\begin{aligned}
L_1 &= \ln \frac{\rho(\tau - \kappa_2 - 1) + \sqrt{\rho^2 - 4} W_1}{2\kappa_1}, & L_2 &= \ln \left(1 + \frac{\kappa_2 \rho^2 \sqrt{\rho^2 - 4} W_2}{4\kappa_1(\kappa_1 + \kappa_2)} \right) \\
L_3 &= \ln \frac{\rho + \sqrt{\rho^2 - 4}}{2}, & L_4 &= \ln \left(\frac{\tau^2 - 1 + (\rho/2)\sqrt{(\rho^2 - 4)(\tau^2 - 1)}}{\kappa_1 + \kappa_2} - \tau \right) \\
W_1 &= \sqrt{(\tau - \kappa_2)^2 + 2\kappa_1 - 1}, & W_2 &= \sqrt{\kappa_2 [\kappa_2 \rho^2 (\rho^2 - 4) + 8\kappa_1(\kappa_1 + \kappa_2)]} \\
R &= (\tau + 1)(\kappa_2 - \kappa_1) - \kappa_2(\kappa_1 + \kappa_2), & S &= 2\kappa_1 \kappa_2 \tau - \kappa_1^2 - \kappa_2^2 \\
X_1 &= \frac{R}{W_1^2} \left[\sqrt{\rho^2 - 4} + (\kappa_2 - \tau + 1) \frac{\rho L_1}{W_1} \right] \\
X_2 &= \frac{2\rho}{\kappa_1^2(\tau^2 - 1)} \left[\rho \sqrt{\rho^2 - 4} \frac{\kappa_1 \tau - \kappa_2}{\kappa_1 + \kappa_2} + (\tau^2 - \kappa_2 \tau + \kappa_1 - 1) \frac{2L_4}{\sqrt{\tau^2 - 1}} \right] \\
X_3 &= \frac{\rho R}{W_1^2} \left[2 \frac{L_1}{W_1} + \frac{\kappa_2 - \tau + 1}{2\kappa_1^2} \rho \sqrt{\rho^2 - 4} \right] \\
X_4 &= \frac{1}{W_1^4} \left[(\kappa_2 - \tau + 1) \sqrt{\rho^2 - 4} \left(R^2 - \frac{\rho^2}{2} S \right) + [(\kappa_2 - \tau + 1)^2 R^2 - 2\kappa_1^2 S] \frac{\rho L_1}{W_1} \right] \\
X_5 &= \frac{\rho}{\kappa_1^2(\tau^2 - 1)} \left\{ \rho \sqrt{\rho^2 - 4} \left[1 - \frac{3S}{\kappa_1^2(\tau^2 - 1)} \right] \right. \\
& \quad \left. + \frac{2L_4}{\sqrt{\tau^2 - 1}} \left[2(\tau + 1) - \frac{\kappa_2}{\kappa_1} \rho^2 - \frac{3S}{\kappa_1^2(\tau^2 - 1)} \left(\kappa_1 + \kappa_1 - \frac{\tau - 1}{2} \rho^2 \right) \right] \right\}
\end{aligned}$$

Appendix C

```

#ifndef HAVE_RANGE_H
#define HAVE_RANGE_H

#define W_INVALID_RANGE 1

#define SUCCESS 0

#define E_INVALID_RANGE -1

class Range
{
    friend ostream &operator<<(ostream &, const Range &);

    friend istream &operator>>(istream &, Range &);

public:
    Range(double = 0, double = 0);    // constructor

    ~Range();                        // destructor

    Range(Range &source);            // copy constructor

    int operator<(Range &) const;    // entirely lower range than

    int operator>(Range &) const;    // entirely higher range than

    int operator==(Range &) const;   // exactly equal range

    int operator!=(Range &) const;   // not exactly equal range

    int operator&(Range &) const;    // ranges overlap

```

```

int operator&(double) const; // value is in range

int setLow(double);          // Set low boundary

int setHigh(double);         // Set high boundary

int setRange(double, double); // Set both

Range &operator=(Range &);   // assignment operator

double getLow() const;       // Return low boundary

double getCenter() const;    // Return center of range

double getHigh() const;      // Return high boundary

private:

    double low_val;

    double high_val;

};

#endif /* HAVE_RANGE_H */


---


#ifndef HAVE_SPECTRUM_H

#define HAVE_SPECTRUM_H

#include "Range.H"

extern int SpectralResolution;

extern double LowestEnergy;

extern double HighestEnergy;

```

```

#define E_INVALID_BIN      -1

#define E_INVALID_INDEX    -2

#define E_INVALID_SPECTRUM -3

class Spectrum

{

    friend ostream &operator<<(ostream &, const Spectrum &);

    friend istream &operator>>(istream &, Spectrum &);

public:

    Spectrum(int = SpectralResolution,

             double = LowestEnergy,

             double = HighestEnergy);

    ~Spectrum();

    Spectrum(Spectrum &);

    Spectrum &operator=(Spectrum &);

    int  operator==(Spectrum &) const;

    int  operator!=(Spectrum &) const;

    int  getSize()      const;

    Range getERange()    const;

    Range getBin(int)    const;

```

```

    int    getBinNo(double) const;

    int    setSize(int);

    int    setERange(Range);

    int    setBin(int, Range);

private:

    int    num_bins;

    Range   eRange;

    Range   *bin;

};

#endif /* HAVE_SPECTRUM_H */


---


#ifndef HAVE_DIST_H

#define HAVE_DIST_H

#include "Interpolator.H"

#include "Spectrum.H"

#define E_INVALID_DENSITY -1

class Dist
{

    friend ostream &operator<<(ostream &, const Dist &);

    friend istream &operator>>(istream &, Dist &);

```

public:

```
    Dist(Spectrum *);  
    ~Dist();  
    Dist(Dist &);  
    Spectrum  getSpectrum()      const;  
    Spectrum  *getSpecPtr()      const;  
    double    getDensity(int)     const;  
    int        getDensityNo(double) const;  
    int        setSpectrum(Spectrum *);  
    int        setDensity(int, double);  
    int        addDensity(int, double);  
    Dist       &operator=(Dist &);  
    Dist       operator+(Dist &);  
    Dist       &operator+=(Dist &);  
    Dist       operator-(Dist &);  
    Dist       &operator-=(Dist &);
```

private:

```
    Spectrum  *eSpec;  
    double    *density;
```

```

    Interpolator interp;

};

#endif /* HAVE_DIST_H */


---


#ifndef HAVE_LAYER_H

#define HAVE_LAYER_H

extern int verbose;

#define NUM_DISTS    3

#define DUpPhoton    0

#define DPairs       1

#define DDnPhoton    2

class Layer

{

    friend ostream &operator<<(ostream &, const Layer &);

    friend istream &operator>>(istream &, Layer &);

public:

    Layer(Spectrum *, double = 0, double = 0);

    // ~Layer();

    Layer(Layer &);

    Range  getHRange() const;

```

```

    Spectrum getSpectrum() const;

    Dist    getDist(int) const;

    int     setHRange(Range);

    int     setSpectrum(Spectrum *);

    int     setDist(int, Dist);

    Layer   &operator=(Layer &);

    double  operator-(Layer &);

    double  OneStep(double);

private:

    Range    hRange;

    Spectrum *eSpec;

    Dist     *dist;

};

#endif /* HAVE_LAYER_H */


---


#ifndef HAVE_INTERP_H

#define HAVE_INTERP_H

#define SUCCESS      0

#define E_INTERP_FAILURE -1

class Interpolator

```

```

{
    friend ostream &operator<<(ostream &, const Interpolator &);
    friend istream &operator>>(istream &, Interpolator &);

public:
    Interpolator(int = 0);
    ~Interpolator();
    Interpolator(Interpolator &);
    void clear();
    int addPoint(double, double);
    double getEstimate(double) const;

private:
    int num_points;
    double *x, *y;
};

#endif /* HAVE_INTERP_H */


---


#ifndef HAVE_3DARRAY_H
#define HAVE_3DARRAY_H
#define SUCCESS 0

class ThreeDArray

```

```

{

    friend ostream &operator<<(ostream &, const ThreeDArray &);

    friend istream &operator>>(istream &, ThreeDArray &);

public:

    ThreeDArray(int = 1, int = 1, int = 1);                // constructor

    ~ThreeDArray();                                        // destructor

    ThreeDArray(ThreeDArray &);                            // copy constructor

    ThreeDArray &operator=(ThreeDArray &);                // assignment operator

    double operator()(int, int, int);

    int     setVal(int, int, int, double);

    int     subCube(ThreeDArray &src,int, int, int);

private:

    double *idx2ptr(int, int, int) const;

    int     x_size, y_size, z_size;

    double *values;

};

#endif /* HAVE_3DARRAY_H */


---


#ifndef HAVE_LOOKUP_H
#define HAVE_LOOKUP_H

```

```

#include "Spectrum.H"

#define SUCCESS      0

#define E_LOOKUP_FAILURE -1

class LookupTable

{

    friend ostream &operator<<(ostream &, const LookupTable &);

    friend istream &operator>>(istream &, LookupTable &);

public:

    LookupTable(Spectrum *,Spectrum *,Spectrum *);

    // ~LookupTable();

    LookupTable(LookupTable &);

    LookupTable &operator=(LookupTable &);

    int    setXSpectrum(Spectrum *);

    int    setYSpectrum(Spectrum *);

    int    setZSpectrum(Spectrum *);

    int    setValue(int, int, int, double);

    int    setValue(double, double, double, double);

    double getValue(int, int, int);

    double getValue(double, double, double);

```

```
double getEstimate(double, double, double);
```

```
private:
```

```
Spectrum *xSpec;
```

```
Spectrum *ySpec;
```

```
Spectrum *zSpec;
```

```
ThreeDArray values;
```

```
};
```

```
#endif /* HAVE_LOOKUP_H */
```

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