



Determination of the temperature coefficient of reactivity of the Montana State College subcritical assembly

by Gary James Russell

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Physics at Montana State College

Montana State University

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Abstract:

This thesis is a report of the research in which the temperature coefficient of reactivity of the Montana State College subcritical assembly has been calculated. This assembly is a light water moderated, natural uranium system. In order to calculate the temperature coefficient of reactivity, the temperature effect on the diffusion length and buckling has been determined. The diffusion length has been measured at 10°, 27°, 50°, and 75° C. Experimentally, the variation of the diffusion length with temperature, $\Delta L/\Delta T$, is 0.00778 cm./°C; whereas the accepted variation is 0.006 cm./°C. The method used for measuring the diffusion length of thermal neutrons in light water utilizing a relatively high energy Pu-Be (α, n) source is outlined. The vertical as well as the radial buckling has been measured at 13°, 23°, 35°, and 45° C. The temperature coefficient of the material buckling, B^{am} , is found to be $-2.87 \times 10^{-3}/^{\circ}\text{C}$. The net temperature coefficient of reactivity (assuming k_{∞} is independent of temperature) is $-16.2 \times 10^{-5}/^{\circ}\text{C}$. The contribution to the temperature coefficient due to the change of k_{∞} with temperature [$1/\Delta T (\delta k_{\infty}/k_{\infty})$] is $-10.6 \times 10^{-5}/^{\circ}\text{C}$. Experimentally, the infinite multiplication factor at 23°C is 0.965. The calculated value of $-k_{\infty}$ is 0.960. Using the experimental value of k_{∞} , the effective multiplication factor is 0.869 and the subcritical multiplication is 7.64. The ratio of the volume of uranium to the volume of water is 0.097. The average coefficient of linear expansion, αT , for the natural uranium-water system is $8.4 \times 10^{-5}/^{\circ}\text{C}$. All measurements at both high and low temperatures have been made with a thermal neutron scintillation counter.

THE DETERMINATION OF THE TEMPERATURE COEFFICIENT
OF REACTIVITY OF THE MONTANA STATE COLLEGE
SUBCRITICAL ASSEMBLY

by

GARY J. RUSSELL

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ABSTRACT

This thesis is a report of the research in which the temperature coefficient of reactivity of the Montana State College subcritical assembly has been calculated. This assembly is a light water moderated, natural uranium system. In order to calculate the temperature coefficient of reactivity, the temperature effect on the diffusion length and buckling has been determined. The diffusion length has been measured at 10°, 27°, 50°, and 75° C. Experimentally, the variation of the diffusion length with temperature, $\Delta L/\Delta T$, is 0.00778 cm./°C; whereas the accepted variation is 0.006 cm./°C. The method used for measuring the diffusion length of thermal neutrons in light water utilizing a relatively high energy Pu-Be (α, n) source is outlined. The vertical as well as the radial buckling has been measured at 13°, 23°, 35°, and 45° C. The temperature coefficient of the material buckling, B_m^2 , is found to be $-2.87 \times 10^{-3}/^\circ\text{C}$. The net temperature coefficient of reactivity (assuming k_∞ is independent of temperature) is $-16.2 \times 10^{-5}/^\circ\text{C}$. The contribution to the temperature coefficient due to the change of k_∞ with temperature $[1/\Delta T (\delta k_\infty/k_\infty)]$ is $-10.6 \times 10^{-5}/^\circ\text{C}$. Experimentally, the infinite multiplication factor at 23°C is 0.965. The calculated value of k_∞ is 0.960. Using the experimental value of k_∞ , the effective multiplication factor is 0.869 and the subcritical multiplication is 7.64. The ratio of the volume of uranium to the volume of water is 0.097. The average coefficient of linear expansion, α , for the natural uranium-water system is $8.4 \times 10^{-5}/^\circ\text{C}$. All measurements at both high and low temperatures have been made with a thermal neutron scintillation counter.

INTRODUCTION

A problem of considerable importance to the control of nuclear reactors is the evaluation of the temperature coefficient of the reactor system. The temperature coefficient is defined as the change in the reactivity of the system due to a change in the operating temperature of the reactor. Therefore, the temperature coefficient is a measure of the inherent stability of the reactor. If the temperature of the reactor increases (this could be due to a change in the power demands of the reactor system) and if the temperature coefficient is positive, the multiplication constant and the reactivity will increase; this will in turn cause the rate of production of energy to increase and further raise the temperature of the system. A system of this nature would be unstable and would require a positive and continuous influence of a control system in order to maintain a steady state system. However, if the temperature coefficient is negative, the effect of a temperature rise on the neutron population (and therefore on the power production) would be to decrease the number of neutrons (and power), so that the temperature would then decrease. Thus, in this type of system, temperature "disturbances" would eventually disappear and the reactor would return to a steady state even in the absence of external control orders.

Ordinary reactors are designed to have negative temperature coefficients of reactivity to ensure adequate stability. These ordinarily lie in the range 10^{-3} to 10^{-2} % reactivity per degree centigrade.

In a subcritical assembly the reactivity, being negative and not

necessarily close to one, is not of too great interest. However, a subcritical system still has a temperature coefficient and this coefficient for the Montana State College subcritical assembly was calculated. This assembly is a light water moderated, heterogeneous, natural uranium system.

For the theoretical treatment, to be outlined later, it is convenient to separate the effect of temperature on reactivity into two main parts: the nuclear temperature coefficient, which is determined by the effect on the nuclear cross sections, and the density temperature coefficient, due to changes in density (and volume) of the system. The discussion which follows applies particularly to a homogeneous reactor system, but the general conclusions will hold, to a good approximation, for most heterogeneous reactors and were applied in the calculations of the temperature coefficients in this report.

DESCRIPTION OF THE HEATING AND COOLING MECHANISM

The aluminum tank that contains the moderator¹, reflector¹, and fuel¹ was covered with three inches of fiber glass insulation to reduce heat losses. A thin aluminum cover was then placed over the insulation in order to preserve the appearance of the subcritical assembly.

The complete heating and cooling system consisted of a pipe extending from the bottom of the tank, a series of valves, a pump (capacity ≈ 10 gal./min.), a heat exchanger and a pipe connecting the heat exchanger with the top of the subcritical assembly. In order to heat the assembly, steam (at about 40 psi) was introduced into the heat exchanger. Cooling was obtained by running cold ground water through the heat exchanger.

A diagram of the system is given below: (not to scale)

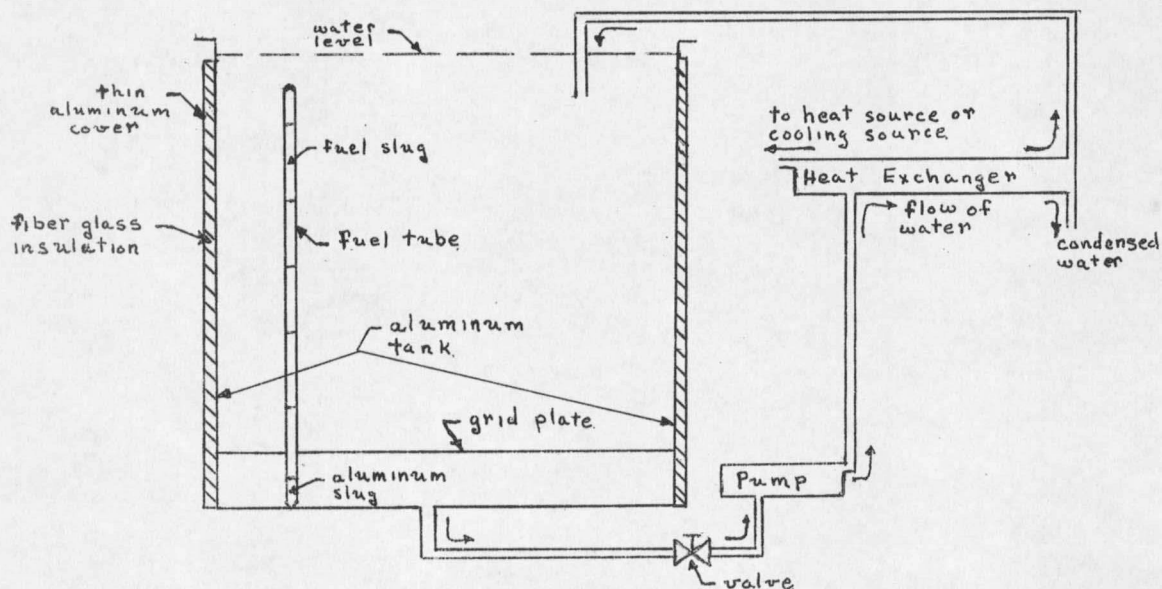


Figure 1. Heating and Cooling Mechanism.

1. There were $1.19 \times 10^6 \text{ cm}^3$ (approximately 315 gal.) of water and $1.15 \times 10^5 \text{ cm}^3$ of natural uranium.

THEORY

THE DIFFUSION LENGTH

Before a description of the diffusion length, L , is attempted, one has to understand what is meant by a thermal neutron. Imagine a fast neutron undergoing elastic scattering with molecules of hydrogen. After a number of scattering collisions, the velocity of the neutron is reduced to such an extent that it has approximately the same average kinetic energy as the molecules of the hydrogen medium in which it is undergoing scattering. The energy depends on the temperature of the medium and is called thermal energy. Neutrons whose energies have been reduced to values in this region are designated as thermal neutrons. At 25°C the energy of thermal neutrons is approximately 0.025 ev.²

In reactor physics, the diffusion length, L , of thermal neutrons in a medium is defined as $L = \sqrt{\frac{D}{\Sigma_a}}$, where D is the diffusion coefficient and Σ_a is the macroscopic absorption cross section of the particular scattering medium. The diffusion coefficient, in general, will be a function of the nuclear properties of the medium and of the neutron speed.

As seen from the defining equation of the diffusion length, it varies inversely as $\sqrt{\Sigma_a}$; hence, if the scattering medium has a relatively large absorption cross section (for light water and 0.025 ev neutrons, $\Sigma_a = 0.022$

-
2. In atomic studies, energies are frequently expressed in electron volt units. The electron volt, that is, 1 ev, is the energy acquired by any charged particle carrying a unit electric charge when it passes, without resistance, through a potential difference of 1 volt. It is equivalent to 1.6×10^{-12} ergs. The million electron volt (or Mev) unit is 1×10^6 ev or 1.6×10^{-6} erg.

cm.)³ the diffusion length will be small. It should be noted that L has the dimensions of length. For light water $L=2.85$ cm. at 25°C .⁴

In order to obtain a physical picture of just what the diffusion length is, consider a point source of thermal neutrons and let ϕ be the thermal neutron flux of neutrons per cm.² per sec. at a distance r . The rate at which neutrons are absorbed is then equal to $\Sigma_a \phi$ per cm.³ per sec., where Σ_a is the macroscopic absorption cross section of the medium. In a spherical shell element of radius r and thickness dr , that is, volume $4\pi r^2 dr$, surrounding the point source, the number of neutrons captured per second will be $4\pi r^2 dr \Sigma_a \phi$. This is a measure of the probability that a neutron will be absorbed within the element dr at a distance r from the source. It is preferable now to consider the second spatial moment of r ; therefore, the mean square distance, $\bar{r}^2$, which a neutron travels from its source to where it is absorbed is given by

$$\bar{r}^2 = \frac{\int_0^{\infty} r^2 (4\pi r^2 \Sigma_a \phi) dr}{\int_0^{\infty} 4\pi r^2 \Sigma_a \phi dr} \quad (1)$$

It can be shown⁵ that for a single point source in an infinite medium $\phi = \frac{e^{-Kr}}{4\pi Dr}$, where K is defined as $\sqrt{\frac{\Sigma_a}{D}}$.

Therefore, upon substituting ϕ into the equation for $\bar{r}^2$ one obtains

$$\bar{r}^2 = \frac{4\pi \Sigma_a \int_0^{\infty} \frac{r^4 e^{-Kr}}{4\pi Dr} dr}{4\pi \Sigma_a \int_0^{\infty} \frac{r^2 e^{-Kr}}{4\pi Dr} dr} = \frac{6/K^4}{1/K^2} = \frac{6}{K^2} \quad (2)$$

then,

$$K^2 = 6/\bar{r}^2 \quad (3)$$

3. Etherington, Nuclear Engineering Handbook (New York: McGraw-Hill Inc., 1958), p. 12-70.
4. Ibid., p. 1-20.
5. Meghreblian and Holmes, Reactor Analysis (McGraw-Hill Inc., 1960), p. 183.

However, by definition, $L = \sqrt{\frac{D}{\Sigma_a}}$ cm. or $L^2 = \frac{1}{\chi^2}$ and after substituting for χ^2 it is seen that

$$L^2 = \frac{r^2}{6} \quad (4)$$

so that the diffusion length squared is one-sixth of the mean square distance, as the crow flies, that a thermal neutron travels from the point at which it just becomes thermal to the point of capture.

For a right circular cylinder it can be shown⁶ that

$$\phi(z) = c e^{-\gamma z} \left\{ 1 - e^{-2\gamma(R-z)} \right\} \quad (5)$$

where z is measured from the plane where neutrons are thermalized and

γ is defined by

$$\gamma^2 = \left(\frac{1}{L} \right)^2 + \left(\frac{2.405}{\tilde{R}_0} \right)^2 \quad (6)$$

For distances not too close to the source and not too close to the top of the cylinder, the term in brackets is not very different from unity, then

$$\phi(z) = c e^{-\gamma z} \quad (7)$$

Therefore, the slope of the straight line portion of $\ln \phi(z)$ versus z yields γ . It should be noted that $\tilde{R}_0$ is the extrapolated radius, that is, the radius of the moderator configuration plus $0.71 \lambda_{tr}$, and 2.405 is the first zero of the Bessel function $J_0(x)$. Also, λ_{tr} is the transport mean free path.

In the thermal energy region the absorption cross sections may be assumed to follow the $1/v$ law. Since v is proportional to $\sqrt{T}$ and if σ_a is the microscopic absorption cross section at temperature T and σ_{a_0} is the value at T_0 , then

$$\sigma_a = \sigma_{a_0} \left(\frac{T_0}{T} \right)^{1/2} \quad (8)$$

6. Glasstone and Edlund, The Elements of Nuclear Reactor Theory (Van Nostrand Inc., 1952), p. 287.

Scattering cross sections do not change greatly with temperature and through the remainder of this report it will be assumed that they remain constant.

$$\text{Now, } L = \sqrt{\frac{D}{\Sigma_a}} \text{ and } D = \frac{4r}{3}, \Sigma_a = \frac{1}{\lambda_a} \quad (9)$$

$$\text{then, } L = \sqrt{\frac{1}{3} \lambda_a \lambda_s} \text{ or } L^2 \approx \frac{1}{3 \Sigma_a \Sigma_s} \quad (10)$$

This holds for substances of high atomic mass provided the medium does not absorb too strongly and scattering is isotropic in the laboratory systems.

$$\text{Then, } L^2 = L_0^2 \left(\frac{I}{I_0} \right)^{1/2} \quad (11)$$

if the effect of density changes on the number of nuclei per cm.³, which is involved in the macroscopic cross section, is ignored.

In contrast to charged particles which have a definable range, neutrons and γ -rays are stopped approximately exponentially. In this exponential approximation, use is made of the relaxation length, which is the thickness of material that causes a drop in intensity by a factor of e. For thermal neutrons the relaxation length is equal to the thermal diffusion length, which can be made quite small in absorbing materials.

In order to gain an additional physical significance of the diffusion length concept, consider a plane thermal neutron source in an infinite medium. It can be shown⁷ that the flux distribution in a nonmultiplying medium has an exponential behavior given by the factor $\exp(-x/L)$. Then,

7. Meghreblan and Holmes, Reactor Analysis (McGraw-Hill Inc., 1960) p. 182.

in this form one can identify the diffusion length L as an attenuation or relaxation length. Therefore, L is the distance from the source plane at which the neutron density is reduced to $1/e$ of its value at the source. One can think of the diffusion length, then, as a measure of the average depth of penetration into a medium by a source neutron. It should be noted that a proper measure of the penetration is the "crow-flight" distance from the source to the point of capture as was explained previously.

The thermal neutron diffusion length in a hydrogenous medium can be simply determined by measuring the variation of the thermal neutron density at relatively large distances from the center of the neutron source. The necessary requirement is that the slowing-down length of the source neutrons in the medium be less than or equal to the diffusion length, so that the diffusion length dominates at large distances from the source.

The square root of the Fermi age is called the slowing-down length. The Fermi age is related to the mean square distance traveled while slowing down.⁸ For thermal neutrons, of age τ_{th} , $\sqrt{\tau_{th}}$ is a measure of the net vector (crow-flight) distance traveled from their formation as fission neutrons to their attainment of thermal energy.

If the slowing-down length of the source neutrons is greater than the diffusion length of thermal neutrons in the medium, then, the thermal neutron flux or density will decay by means of something other than an exponential.

For an Ra-Be (γ, n) source the maximum neutron energy emitted is only

8. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 165.

0.7 Mev and the slowing-down length of 0.7 Mev neutrons is 2.45 cm.⁹ This distance of 2.45 cm. is less than the diffusion length of thermal neutrons in light water at 25°C which is 2.85 cm.. Thus, a direct measurement of the diffusion length in light water using a Ra-Be (α, n) source is possible. However, a Pu-Be (α, n) source emits neutrons of fairly high energies ranging from 5 to 12 Mev or more. The slowing-down length for fast neutrons of this energy in light water would be greater than 5.74 cm.¹⁰ Therefore, a direct measurement of the diffusion length in light water, utilizing a Pu-Be (α, n) source, is impossible owing to the fact that the slowing-down length of the source neutrons is greater than the diffusion length. A typical result for the diffusion length (obtained by making a direct measurement of the decay of Pu-Be neutrons in light water) is 5.08 cm.¹¹ The experimental procedure used and results found for measurements of L in light water, using a Pu-Be neutron source, are given later.

BUCKLING

One of the most important quantities measured in an exponential experiment is the material buckling, B_m^2 . The material buckling, B_m^2 , is the value of B^2 that satisfies the critical transcendental equation

$$\frac{k e^{-B^2 L}}{1 + L^2 B^2} = 1 \quad (12)$$

provided the continuous slowing-down (Fermi Age) model is applicable.

-
9. DeJuren and Rosenwasser, "Diffusion Length of Thermal Neutrons in Water," Journal of Research of National Bureau of Standards, October 1953, p. 203.
 10. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 166.
 11. Curtiss-Wright Research Division, "Laboratory Manual for the Subcritical Facility," p. 63.

Now, k_{∞} is the infinite multiplication factor of the medium, that is, k_{∞} (or k_{∞}) is the ratio of the average number of neutrons produced in each generation to the average number of corresponding neutrons in the preceding generation. In other words k_{∞} is essentially the value the effective multiplication factor would have if the system were infinitely large and there were no loss of neutrons by leakage. Again, λ is the Fermi age or the average (net vector) distance a fission neutron travels while slowing down to thermal energies.

B_m^2 is the parameter of a given fuel lattice which determines the critical size of the reactor. In the critical reactor, the spatial flux is given by

$$\nabla^2 Q(\vec{r}) + B_m^2 Q(\vec{r}) = 0 \quad (13)$$

The geometrical buckling, B_g^2 , is defined as the lowest eigenvalue that results from solving the wave equation

$$\nabla^2 Q(\vec{r}) + B_g^2 Q(\vec{r}) = 0 \quad (14)$$

with the boundary condition that $Q(\vec{r})$ shall be zero at the extrapolated boundary of the system.

The critical condition for a reactor system can be written as

$$B_m^2 = B_g^2 \quad (15)$$

In other words, the material buckling, B_m^2 , for the given multiplying medium is equal to the geometrical buckling, B_g^2 , of the critical system of a specified shape. Therefore, if B_g^2 is less than B_m^2 , the reactor will be larger than the critical size and the system will be supercritical, and if B_g^2 is greater than B_m^2 , the system will be subcritical.

In the case of the Montana State College subcritical assembly, which

is essentially cylindrical in form, B_m^2 is given by the expression

$$B_m^2 = \left(\frac{2.405}{\widetilde{R}_0} \right)^2 - \gamma^2 \quad (16)$$

where $\widetilde{R}_0$ is the extrapolated radius and γ is the reciprocal of the relaxation length. The first term on the right of equation (16) constitutes the radial buckling while γ^2 is referred to as the vertical buckling.

The geometric buckling for a finite, cylindrical reactor, in terms of its radius and height is

$$B_g^2 = \left(\frac{2.405}{\widetilde{R}_0} \right)^2 + \left(\frac{\pi}{\widetilde{H}_0} \right)^2 \quad (17)$$

where $\widetilde{H}_0$ is the extrapolated height of the system.

$$\widetilde{H}_0 = H_0 + 2(0.7104 \lambda_{tr}) \quad (18)$$

where H_0 is the actual height of the reactor and λ_{tr} is the transport mean free path of a thermal neutron in water. For light water the value of λ_{tr} is 0.48 centimeters.¹²

The flux distribution in a subcritical assembly does not satisfy the wave equation for a critical reactor, but for a relatively large assembly the thermal neutron flux variation at a distance from boundaries and the extraneous neutron source can be represented by

$$\nabla^2 \Phi(r, z) + B_m^2 \Phi(r, z) = 0 \quad (19)$$

where Φ is the thermal flux and B_m^2 , the material buckling, a constant for the particular assembly. The solution of equation (19) can be obtained for a cylindrical reactor by setting

$$\Phi(r, z) = \Theta(r) Z(z) \quad (20)$$

This is then substituted into equation (19) and the terms in $\Theta(r)$ and

12. Etherington, Nuclear Engineering Handbook (McGraw-Hill Inc., 1958) p. 6-86.

$Z(z)$ are set equal to the constants α^2 and r^2 so that

$$B_m^2 = \alpha^2 - r^2 \quad (21)$$

The general solution of equation (19) using the correct Laplacian operator is

$$\Phi(r, z) = \sum_{n=1}^{\infty} A_n J_0(\alpha_n r) \sinh r_n (\bar{H} - z) \quad (22)$$

Previous experimentation has shown that for flux measurements not too near the source the fundamental ($n=1$) term is sufficient for good accuracy.

Thus, the radial flux distribution becomes

$$\Phi(r) = Q(r) = A_1 J_0(\alpha r) \quad (23)$$

The constant α can be evaluated from the boundary condition that the flux falls to zero at the extrapolated radius $\bar{R}_0$ and is found to be $\frac{2.405}{\bar{R}_0}$.

Then, the radial flux distribution becomes

$$Q(r) = A_1 J_0\left(\frac{2.405}{\bar{R}_0} r\right) \quad (24)$$

It can be seen from equation (16) that α^2 is the radial buckling which is equal to $\left(\frac{2.405}{\bar{R}_0}\right)^2$.

Therefore, radial flux measurements at a constant height above the source should provide data from which a graph can be made which will yield the extrapolated radius. The extrapolated radius can be gotten from the graph of relative thermal neutron flux versus radial distance either by visual extrapolation or by fitting the graph to a Bessel curve. Using only the fundamental term ($n=1$) of equation (22), the flux variation along the z -axis may be represented as

$$Z(z) = Q(z) = A_2 \sinh r (\bar{H} - z) \quad (25)$$

or

$$\phi(z) = A_3 e^{-\gamma z} \left\{ 1 - e^{-2\gamma(\tilde{H}-z)} \right\} \quad (26)$$

For measurements taken not too near the top of the assembly

$$\phi(z) = A_3 e^{-\gamma z} \quad (27)$$

Therefore, if relative flux measurements are made in the core along lines parallel to the cylinder's axis γ can be determined from the semilogarithmic plot of activity versus vertical distance .

TEMPERATURE COEFFICIENTS

A temperature increase in a nuclear reactor will alter the reactivity¹³ for at least two reasons: first, the thermal neutron mean energy changes and, therefore, their absorption is affected since nuclear cross sections vary with energy; and, second, the mean free paths and the non-leakage probabilities are temperature dependent.

Nuclear Temperature Coefficients-- Previously, ignoring the effect of density changes on the number of nuclei per cm.³, it was shown that

$$L^2 = L_0^2 \left(\frac{T}{T_0} \right)^{1/2} \quad (28)$$

This means that the diffusion length is temperature dependent.

The Fermi age, τ , defined by¹⁴

$$\tau(E) = \int_E^{E_0} \frac{D}{\Sigma_s E} dE \quad (29)$$

will be temperature independent. This will be so because the temperature variation of τ will be determined by the effect on $\frac{D}{\Sigma_s}$ or $\frac{1}{3\Sigma_s^2}$ and Σ_s is regarded as being temperature independent.

The infinite multiplication, k_{∞} , is independent of temperature. The factor k_{∞} is defined through the four factor formula¹⁵

$$k_{\infty} = \eta \epsilon p f \quad (30)$$

13. The reactivity, ρ , is defined as

$\rho = \frac{k_{eff} - 1}{k_{eff}}$
where k_{eff} , the effective multiplication factor, is the ratio of the average number of neutrons produced by fission in each generation to the total number of corresponding neutrons absorbed or leaking out, on the average.

14. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 162.

15. Ibid., p. 298.

where η is the average number of neutrons liberated for each neutron absorbed, ϵ is the fast fission factor, p is the resonance escape probability and F is the thermal utilization. If all the absorbers present obey the $1/v$ law, then the thermal utilization will not be affected by temperature very much. The resonance escape probability may decrease somewhat as the temperature is raised while F increases slightly. However, as a reasonable approximation, since η and ϵ remain unchanged, it may be assumed that k_{∞} is independent of temperature.

For a large reactor, the effective multiplication factor is given by¹⁶

$$k_{eff} = \frac{k_{\infty}}{1 + M^2 B_g^2} \quad (31)$$

but,
$$\rho = \frac{k_{eff} - 1}{k_{eff}} \quad (32)$$

then,
$$\rho = \frac{k_{\infty} - 1 - M^2 B_g^2}{k_{\infty}} \quad (33)$$

but,
$$M^2 = L^2 + \gamma \quad (34)$$

then,
$$\rho = \frac{k_{\infty} - 1}{k_{\infty}} - \frac{B_g^2}{k_{\infty}} \left\{ L_0^2 \left(\frac{T}{T_0} \right)^{1/2} + \gamma \right\} \quad (35)$$

The nuclear temperature coefficient, at constant density, that is, the temperature coefficient of ρ due to changes in the cross sections is given by

$$\left(\frac{\partial \rho}{\partial T} \right)_d = - \frac{B_g^2 L_0^2}{2 k_{\infty}} \cdot \frac{1}{(T T_0)^{1/2}} \quad (36)$$

The subscript d indicates constant density and it is assumed that the $1/v$ law is applicable. The temperature coefficient at the temperature T is

16. Glasstone and Edlund, The Elements of Nuclear Reactor Theory (Van Nostrand Inc., 1952), p. 341.

obtained by setting $T = T_0$, so that

$$\left(\frac{\partial \rho}{\partial T}\right)_d = -\frac{B_g^2 L^2}{2 k_\infty T} \quad (37)$$

Density Temperature Coefficients-- An increase in temperature causes expansion of the reactor materials, and this affects the reactivity in two ways: first, by changing the mean free paths for absorption and scattering and, second, by an over-all change in the size of the system.

In a homogeneous system where density changes have the same effect on all components, it is possible to write

$$L^2 + \gamma = L_0^2 \left(\frac{d_0}{d}\right)^2 + \gamma_0 \left(\frac{d_0}{d}\right)^2 \quad (38)$$

or
$$M^2 = M_0^2 \left(\frac{d_0}{d}\right)^2 \quad (39)$$

then substituting into equation (33) it is seen that

$$\rho = \frac{k_\infty - 1}{k_\infty} - \frac{B_g^2 M_0^2}{k_\infty} \left(\frac{d_0}{d}\right)^2 \quad (40)$$

If the volume is held constant, B_g^2 is constant, and if it is assumed that the microscopic cross sections are constant, then

$$\left(\frac{\partial \rho}{\partial T}\right)_{B, G, A, S} = \frac{2 B_g^2 M_0^2}{k_\infty} \cdot \frac{d_0^2}{d^3} \cdot \frac{\partial d}{\partial T} \quad (41)$$

If α is the coefficient of linear expansion of the material, that is,

$$l = l_0 [1 + \alpha(T - T_0)], \text{ then } V = V_0 [1 + \alpha(T - T_0)]^3, \text{ so that}$$

$$d = \frac{d_0}{[1 + \alpha(T - T_0)]^3} \quad (42)$$

then
$$\frac{\partial d}{\partial T} = \frac{-3\alpha d_0}{[1 + \alpha(T - T_0)]^4} \quad (43)$$

substituting for $\frac{\partial d}{\partial T}$ from (43) into (41) gives

$$\left(\frac{\partial \rho}{\partial T}\right)_{B, G, A, S} = -\frac{6 B_g^2 M_0^2 \alpha}{k_\infty} \cdot \frac{d_0^3}{d^3} \cdot \frac{1}{[1 + \alpha(T - T_0)]^4} \quad (44)$$

At $T = T_0$ this becomes

$$\left(\frac{\partial \rho}{\partial T}\right)_{B, G, A, S} = -\frac{6 B_g^2 M_0^2 \alpha}{k_\infty} \quad (45)$$

The reactivity change due to an over-all change in the size of the reactor is small. The effect would be due to a change in the buckling; thus from (33)

$$\left(\frac{\partial \rho}{\partial B_g}\right)_{\Sigma_a, \Sigma_s} = -\frac{2 B_g M^2}{k_{\infty}} \quad (46)$$

For a finite cylinder¹⁷ $B_g^2 = \left(\frac{2.405}{\tilde{R}_0}\right)^2 + \left(\frac{\pi}{\tilde{H}_0}\right)^2$ (47)

where $\tilde{R}_0 = \tilde{R}_0 [1 + \alpha(T - T_0)]$ and $\tilde{H}_0 = \tilde{H}_0 [1 + \alpha(T - T_0)]$ (48)

now $\left(\frac{\partial \rho}{\partial T}\right)_{\Sigma_a, \Sigma_s} = \frac{\partial \rho}{\partial B_g} \frac{\partial B_g}{\partial T}$ (49)

where $\left(\frac{\partial B_g}{\partial T}\right) = \frac{\partial B_g}{\partial \tilde{R}_0} \frac{\partial \tilde{R}_0}{\partial T} + \frac{\partial B_g}{\partial \tilde{H}_0} \frac{\partial \tilde{H}_0}{\partial T}$ (50)

now $\left(\frac{\partial \tilde{R}_0}{\partial T}\right) = \alpha \tilde{R}_0$ and $\left(\frac{\partial \tilde{H}_0}{\partial T}\right) = \alpha \tilde{H}_0$ (51)

also $\frac{\partial B_g}{\partial \tilde{R}_0} = -\frac{2(2.405)^2}{\tilde{R}_0^3}$ and $\frac{\partial B_g}{\partial \tilde{H}_0} = -\frac{2\pi^2}{\tilde{H}_0^3}$ (52)

thus
$$\begin{aligned} \frac{\partial B_g}{\partial T} &= -\frac{2(2.405)^2 \alpha}{\tilde{R}_0^2} - \frac{2\pi^2 \alpha}{\tilde{H}_0^2} \\ &= -2\alpha \left\{ \left(\frac{2.405}{\tilde{R}_0}\right)^2 + \left(\frac{\pi}{\tilde{H}_0}\right)^2 \right\} = -2\alpha B_g \end{aligned} \quad (53)$$

Consequently, from (46), (49), and (53)

$$\left(\frac{\partial \rho}{\partial T}\right)_{\Sigma_a, \Sigma_s} = \frac{4\alpha B_g^2 M^2}{k_{\infty}} \quad (54)$$

The net temperature coefficient is obtained by taking the algebraic sum of equations (37) (45) and (54).

17. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 173.

DIFFUSION LENGTH EXPERIMENT

For the measurement of L all of the fuel tubes of the subcritical assembly were removed. A four curie Pu-Be (α, n) source (the source was a right circular cylinder of diameter 3.31 cm. and height of 5.85 cm.) was used. The source was positioned with its axis horizontal on a lucite table. The detector used throughout the diffusion length experiment was a thermal neutron scintillation crystal with the associated circuit. The thermal neutron crystal was a disc of plastic $1/4$ inch in diameter and $3/16$ of an inch thick. The plastic contained 95% enriched boron-10 and silver-activated zinc sulfide. The neutron probe consisted of an aluminum housing, $5/16$ inch outside diameter and 42 inches long, a light pipe, the thermal neutron crystal and a probe adapter with a screw cap. The rest of the circuit consisted of a preamplifier, amplifier, scalar, and timer. The lucite table only served the purpose of raising the source high enough so that measurements near to the source could be made. The table and source were then put into about 400 gallons of distilled light water (see figure 2).

Owing to the previously mentioned fact that the slowing down length of the neutrons given off by a Pu-Be (α, n) source is greater than the diffusion length of thermal neutrons in light water, a direct measurement of the diffusion length is not possible. Therefore, an aluminum cylinder of diameter 27.9 cm. and a height of 61.1 cm. was covered with a sheet of cadmium 20 mils thick. When this cadmium cylinder was placed into the water its surface area represented a physical boundary for neutrons whose

energy was less than or equal to 0.4 ev. This cylinder was centered on top of the lucite table directly over the source. Another sheet of cadmium 40 mils thick was used as a removable bottom of the cylinder. The cylinder was not covered on the top with cadmium for two reasons:

1. To facilitate raising, lowering, and positioning of the thermal neutron probe.
2. The cadmium covered cylinder was made long enough so that end effects due primarily to the open top of the cylinder would be smaller than the inherent source of error of the experiment.

When the cadmium covered cylinder with its cadmium bottom is in position over the source, there are effectively within the cylinder only neutrons whose energies prior to entering the cylinder are greater than 0.4 ev. Thus, if a vertical traverse at selected heights above the source is made with the thermal neutron probe, any counts recorded would be due to fast neutrons that were thermalized within the boundaries of the cylinder. Now, if the cadmium bottom is removed and a similar traverse is made at exactly the same heights, the increase in counts is due only to neutrons with energies less than 0.4 ev entering from the bottom of the cylinder. That is, the difference in the two trials, at corresponding heights, is equal to the number of thermal neutrons. Therefore, there is effectively present a plane source of thermal neutrons whose subsequent behavior can be analyzed to give the thermal neutron diffusion length.

This was the procedure followed throughout the experiment. A typical count of the number of thermal neutrons emanating from the plane

source was about 18,000 counts per minute. Fifteen minute counts were taken at every $1/2$ inch above the top of the lucite table. The same measurements were made with both the bottom cover on and off. Measurements were made at temperatures of 10° , 27° , 50° , and 75° C.

The particular crystal that was used in this experiment could not be heated over 65°C ; therefore, at the two high temperatures, 50°C and 75°C , an additional aluminum tube with a one inch outside diameter was used. One end of the tube was capped with an aluminum cover and sealed so that the tube would be watertight. Provisions were also made in the tube so that cold air could be blown into it and hot water could be added to it (see figure 3). This pipe was then centered inside the cadmium cylinder and placed directly over the source. Extreme care had to be taken in order to insure that the aluminum tube was exactly vertical. The neutron scintillation probe was then raised and lowered inside this watertight tube.

The procedure for making the measurements at 50° and 75°C differed from the procedure at 10 and 27°C in one respect. Before each measurement, other than the first, an amount of water at the correct temperature was added to the tube so that the water level in the tube was raised $1/2$ inch after the probe had been raised $1/2$ inch. This was accomplished by means of a hypodermic syringe. All the time that the neutron probe was inside the tube, cold air was blown into the tube to insure that the probe would not get too hot. Actually, there was an air gap of at least $1/4$ to $1/2$ inch between the thermal neutron crystal and the height of the water in

the tube (see figure 3).

It is essential that the flux within the cylinder be symmetric. The thermal neutron diffusion length is obtained from the relationship $\gamma^2 = (1/L^2) + \left(\frac{2.405}{R_0}\right)^2$, where γ is the slope of the straight line plot of $\ln$ [thermal neutron (counts/min)] versus vertical distance from the source (cm.). It should be noted that in this particular measurement the term $\left(\frac{2.405}{R_0}\right)^2$ was neglected. This factor would subtract a term from γ^2 of the order of magnitude of 0.025 cm., and the experimental error was much greater than this -- probably of the order of five times this figure.

As seen from figure 4, a plot of $\ln$ [counts/min. versus vertical distance], there is a position on the curve where the higher harmonics die out. It can be seen from the graph that the flux distribution in the range from 8.85 cm. to 17.74 cm. varies essentially as an exponential. In every trial it was the points in this region through which a least square straight line was fitted. The slope of this straight line yielded γ . It was found that the experimental variation of the diffusion length with temperature agrees fairly well with the accepted variation; however, the good agreement was obtained by fitting a least square line to the experimental points. The slope of this line was taken as the experimental value of $\Delta L / \Delta t$. Table V summarizes the findings of this experiment and compares these with the accepted values.

It should be noted that for this part of the experiment a long counting time is a necessity in order to obtain statistics that are good enough to give a relatively accurate value for the diffusion length. It

should also be pointed out that with this method of subtracting the two counts, one obtains just the thermal neutron count (within statistics).

This method eliminates the counts due to fast neutrons and gamma rays which to some extent activate the scintillation counter.

(not to scale)

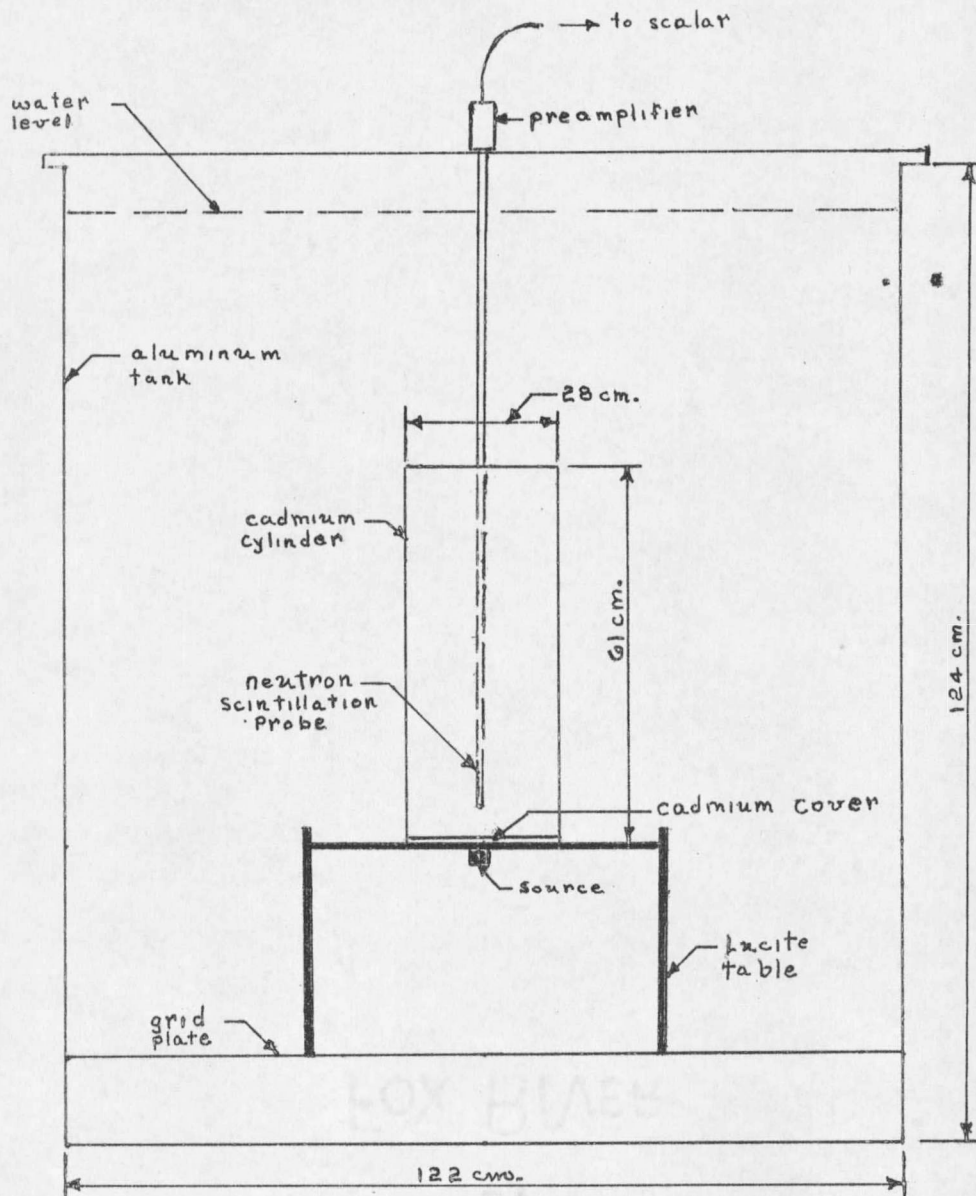


Figure 2. Illustration of the diffusion length experimental arrangement.

(not to scale)

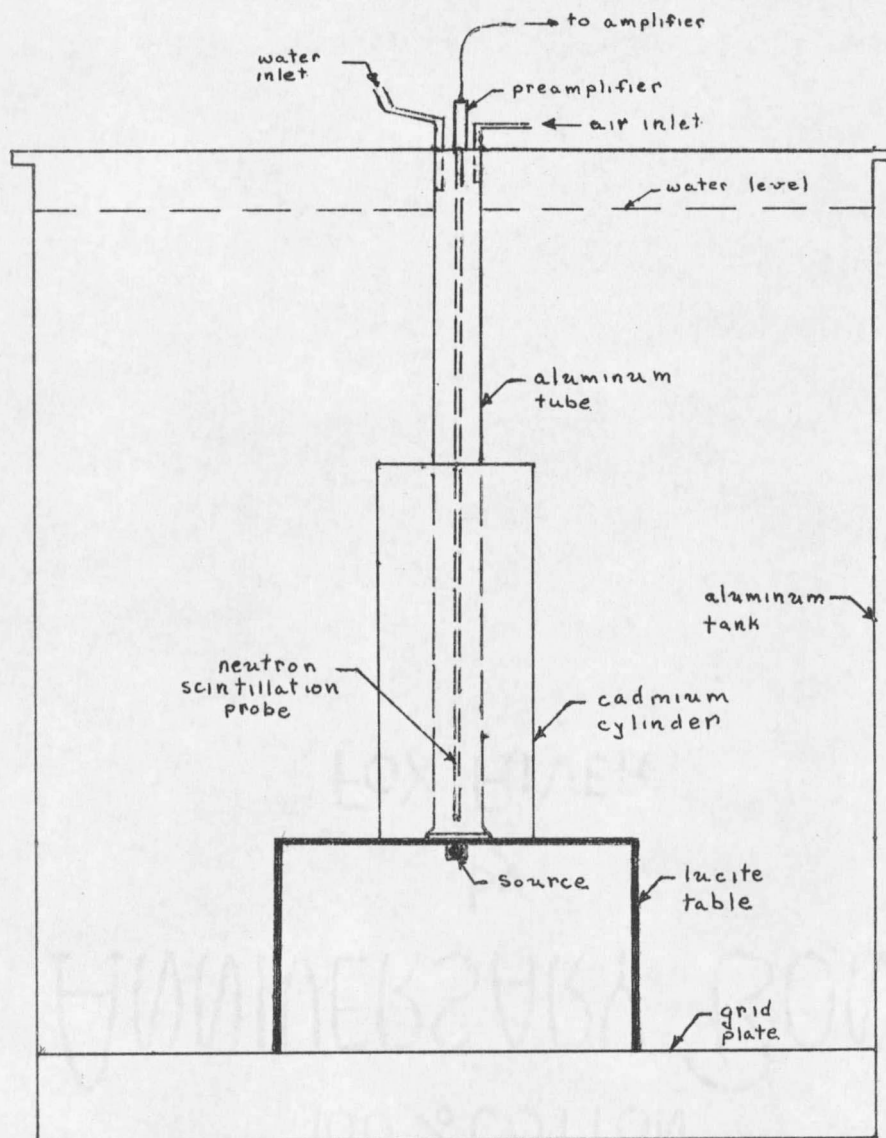


Figure 3. Illustration of the high temperature adapter tube.

TABLE I. Counts per minute $\pm$ error for the cadmium cylinder alone, cadmium cylinder plus cadmium cover, net number of thermal neutrons for the diffusion length experiment at $10^{\circ}\text{C} \pm 0.5^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	cadmium cylinder (counts/min. $\pm$ error)	cadmium cylinder plus cadmium cover (counts/min. $\pm$ error)	net number of thermal neutron (counts/min. $\pm$ error)
2.50	10	33,938.7 $\pm$ 58.1	16,066.9 $\pm$ 40.1	17,871.8 $\pm$ 70.6
3.77	10	31,578.4 $\pm$ 56.1	20,533.4 $\pm$ 45.3	11,045.0 $\pm$ 72.1
5.04	10	27,040.9 $\pm$ 52.0	20,640.2 $\pm$ 45.4	6,400.7 $\pm$ 69.0
6.31	10	22,160.4 $\pm$ 47.0	18,460.5 $\pm$ 43.0	3,699.9 $\pm$ 63.4
7.58	15	17,626.2 $\pm$ 34.2	15,221.6 $\pm$ 31.8	2,404.6 $\pm$ 46.6
8.85	15	13,732.0 $\pm$ 30.2	12,337.8 $\pm$ 28.7	1,394.2 $\pm$ 41.6
10.12	15	10,631.3 $\pm$ 26.6	9,772.8 $\pm$ 25.4	858.5 $\pm$ 36.8
11.39	15	8,145.1 $\pm$ 23.3	7,648.4 $\pm$ 22.6	496.7 $\pm$ 32.4
12.66	16	6,244.5 $\pm$ 19.9*	5,922.0 $\pm$ 19.8	322.5 $\pm$ 28.0
13.93	15	4,832.0 $\pm$ 17.9	4,605.6 $\pm$ 17.5	226.4 $\pm$ 25.0
15.20	15	3,699.2 $\pm$ 15.7	3,578.4 $\pm$ 15.4	120.8 $\pm$ 22.0
16.47	15	2,855.1 $\pm$ 13.8	2,791.5 $\pm$ 13.6	63.6 $\pm$ 19.4
17.74	15	2,196.5 $\pm$ 12.1	2,151.0 $\pm$ 12.0	45.5 $\pm$ 17.0

* 15 minute count

TABLE II. Counts per minute $\pm$ error for the cadmium cylinder alone, cadmium cylinder plus cadmium cover, net number of thermal neutrons for the diffusion length experiment at $27^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	cadmium cylinder (counts/min. $\pm$ error)	cadmium cylinder plus cadmium cover (counts/min. $\pm$ error)	net number of thermal neutron (counts/min. $\pm$ error)
2.50	10	35,459.7 $\pm$ 59.5	17,138.7 $\pm$ 41.4	18,321.0 $\pm$ 72.5
3.77	10	33,247.9 $\pm$ 57.6	21,105.6 $\pm$ 46.0	12,142.3 $\pm$ 73.6
5.04	10	28,404.0 $\pm$ 53.4	21,009.3 $\pm$ 45.8	7,394.7 $\pm$ 70.3
6.31	10	23,113.8 $\pm$ 48.1	18,867.7 $\pm$ 43.4	4,246.1 $\pm$ 64.6
7.58	10	18,423.3 $\pm$ 42.9	15,834.6 $\pm$ 39.8	2,588.7 $\pm$ 58.5
8.85	10	14,957.5 $\pm$ 38.6	12,860.6 $\pm$ 35.8	2,096.9 $\pm$ 52.6
10.12	10	11,570.4 $\pm$ 33.9	10,238.5 $\pm$ 32.0	1,331.9 $\pm$ 46.6
11.39	10	8,934.4 $\pm$ 29.8	8,083.0 $\pm$ 28.4	851.4 $\pm$ 41.2
12.66	10	6,873.5 $\pm$ 26.1	6,296.0 $\pm$ 22.9*	577.5 $\pm$ 36.4
13.93	11	5,259.5 $\pm$ 21.8	4,942.4 $\pm$ 22.2	317.1 $\pm$ 31.1
15.20	10	4,050.2 $\pm$ 20.2	3,802.3 $\pm$ 19.5	247.9 $\pm$ 28.0
16.47	10	3,089.9 $\pm$ 17.5	2,929.4 $\pm$ 17.1	160.5 $\pm$ 24.4
17.74	10	2,424.6 $\pm$ 15.5	2,278.5 $\pm$ 15.1	146.1 $\pm$ 21.6
19.01	10	1,871.1 $\pm$ 13.7	1,781.7 $\pm$ 13.3	89.4 $\pm$ 19.1

* 12 minute count

TABLE III. Counts per minute $\pm$ error for the cadmium cylinder alone, cadmium cylinder plus cadmium cover, net number of thermal neutrons for the diffusion length experiment at $50^\circ\text{C} \pm 1.0^\circ\text{C}$.

vertical distance from source (cm.)	time (min.)	cadmium cylinder (counts/min. $\pm$ error)	cadmium cylinder plus cadmium cover (counts/min. $\pm$ error)	net number of thermal neutron (counts/min. $\pm$ error)
2.50	10	31,337.5 $\pm$ 55.9	12,911.5 $\pm$ 36.0	18,426.0 $\pm$ 66.5
3.77	10	30,531.7 $\pm$ 55.2	16,742.7 $\pm$ 40.8	13,789.0 $\pm$ 68.6
5.04	11	27,817.6 $\pm$ 50.3	18,425.7 $\pm$ 42.9*	9,391.9 $\pm$ 68.0
6.31	10	24,204.9 $\pm$ 49.3	17,765.9 $\pm$ 42.1	6,439.0 $\pm$ 64.6
7.58	15	20,452.2 $\pm$ 36.9	15,960.8 $\pm$ 32.6	4,491.4 $\pm$ 49.3
8.85	15	16,765.0 $\pm$ 33.4	13,746.8 $\pm$ 30.2	3,018.2 $\pm$ 45.0
10.12	15	13,423.5 $\pm$ 30.0	11,445.8 $\pm$ 27.6	1,977.7 $\pm$ 40.8
11.39	15	10,712.0 $\pm$ 26.7	9,350.5 $\pm$ 25.0	1,361.5 $\pm$ 35.6
12.66	15	8,376.3 $\pm$ 23.6	7,564.2 $\pm$ 22.4	812.1 $\pm$ 32.6
13.93	15	6,557.0 $\pm$ 20.8	6,033.4 $\pm$ 19.2	523.6 $\pm$ 28.4
15.20	15	5,169.0 $\pm$ 18.5	4,785.4 $\pm$ 17.9	383.6 $\pm$ 25.8
16.47	15	4,031.7 $\pm$ 16.4	3,795.4 $\pm$ 15.7	236.3 $\pm$ 22.7
17.74	15	3,133.4 $\pm$ 14.5	2,982.5 $\pm$ 14.1	150.9 $\pm$ 20.2
19.01	15	2,445.1 $\pm$ 12.8	2,355.3 $\pm$ 12.5	89.8 $\pm$ 17.9
20.28	15	1,937.5 $\pm$ 11.4	1,877.8 $\pm$ 11.2	59.7 $\pm$ 16.0

* 10 minute count

TABLE IV. Counts per minute $\pm$ error for the cadmium cylinder alone, cadmium cylinder plus cadmium cover, net number of thermal neutrons for the diffusion length experiment at $75^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	cadmium cylinder (counts/min. $\pm$ error)	cadmium cylinder plus cadmium cover (counts/min. $\pm$ error)	net number of thermal neutron (counts/min. $\pm$ error)
2.50	10	27,985.3 $\pm$ 52.9	13,196.6 $\pm$ 36.2	14,788.7 $\pm$ 64.1
3.77	10	29,412.0 $\pm$ 54.2	16,655.8 $\pm$ 31.4*	12,756.2 $\pm$ 67.9
5.04	10	26,987.8 $\pm$ 51.9	17,567.3 $\pm$ 41.9	9,420.5 $\pm$ 66.8
6.31	10	23,391.2 $\pm$ 48.4	16,812.7 $\pm$ 41.0	6,578.5 $\pm$ 63.5
7.58	15	17,944.5 $\pm$ 34.6	14,857.8 $\pm$ 31.4	3,086.7 $\pm$ 46.8
8.85	15	15,555.5 $\pm$ 32.2	13,487.2 $\pm$ 30.0	2,068.3 $\pm$ 44.1
10.12	15	12,685.5 $\pm$ 29.1	11,252.1 $\pm$ 27.4	1,433.4 $\pm$ 40.0
11.39	15	10,205.1 $\pm$ 26.1	9,198.4 $\pm$ 24.6	1,006.7 $\pm$ 35.9
12.66	15	7,998.7 $\pm$ 23.1	7,432.3 $\pm$ 22.2	566.4 $\pm$ 32.0
13.93	15	6,378.0 $\pm$ 20.7	5,877.3 $\pm$ 19.8	500.7 $\pm$ 28.7
15.20	15	4,959.9 $\pm$ 18.2	4,696.1 $\pm$ 17.7	263.8 $\pm$ 25.4

* 13 minute count

TABLE V. Results of the diffusion length experiment.

experimental value of the diffusion length (L)	accepted value* of the diffusion length (L)	temperature
2.62 cm.	2.76 cm.	10°C±0.5°C
2.95 cm.	2.86 cm.	27°C±1°C
2.98 cm.	3.00 cm.	50°C±1°C
3.19 cm.	3.15 cm.	75°C±1°C
variation of experimental L with temperature	$\frac{\Delta L}{\Delta T} = 0.00778 \text{ cm./}^\circ\text{C}$	
variation of accepted L with temperature	$\frac{\Delta L}{\Delta T} = 0.006 \text{ cm./}^\circ\text{C}^*$	

* The accepted values are calculated using a base of 2.85 cm. at 25°C and a variation of 0.006 cm./°C as given in reference 4, page 2-9.

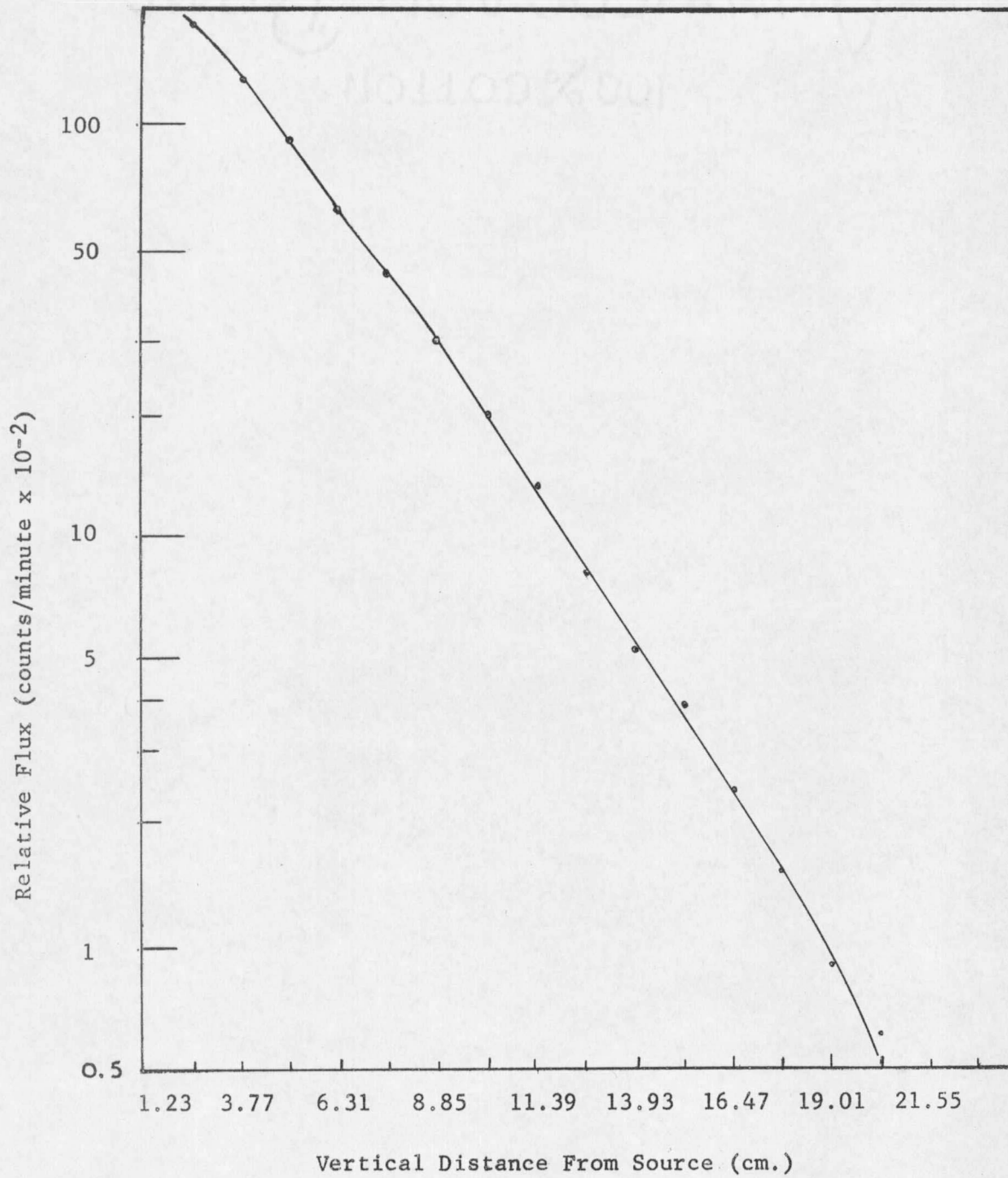


Figure 4. Illustration of the axial flux distribution.

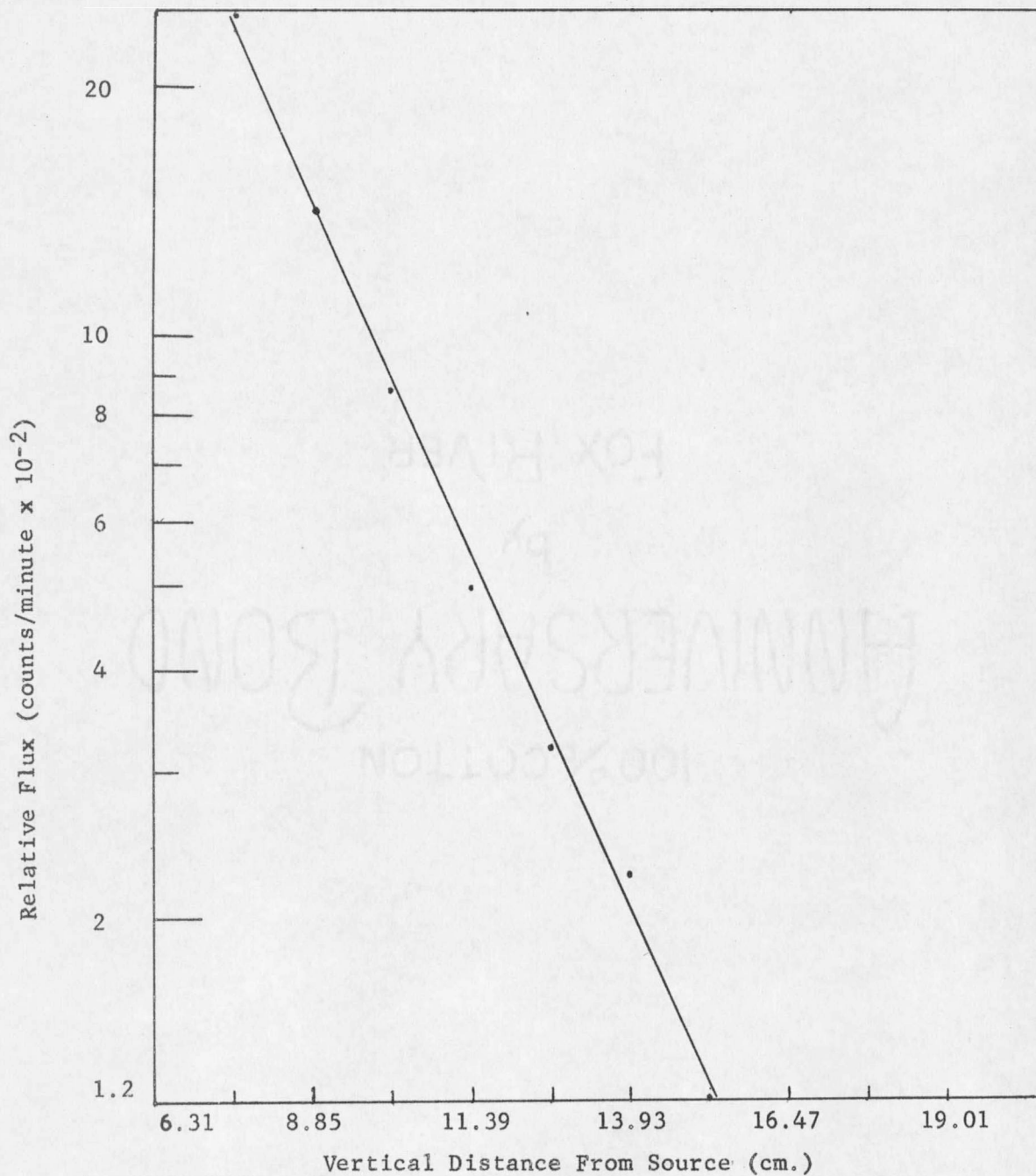


Figure 5. Illustration of the least square plot of relative flux versus distance from source for the diffusion length experiment.
Temperature = $10^{\circ}\text{C} \pm 0.5^{\circ}\text{C}$
Slope = 0.382 cm^{-1}
Diffusion length = 2.62 cm.

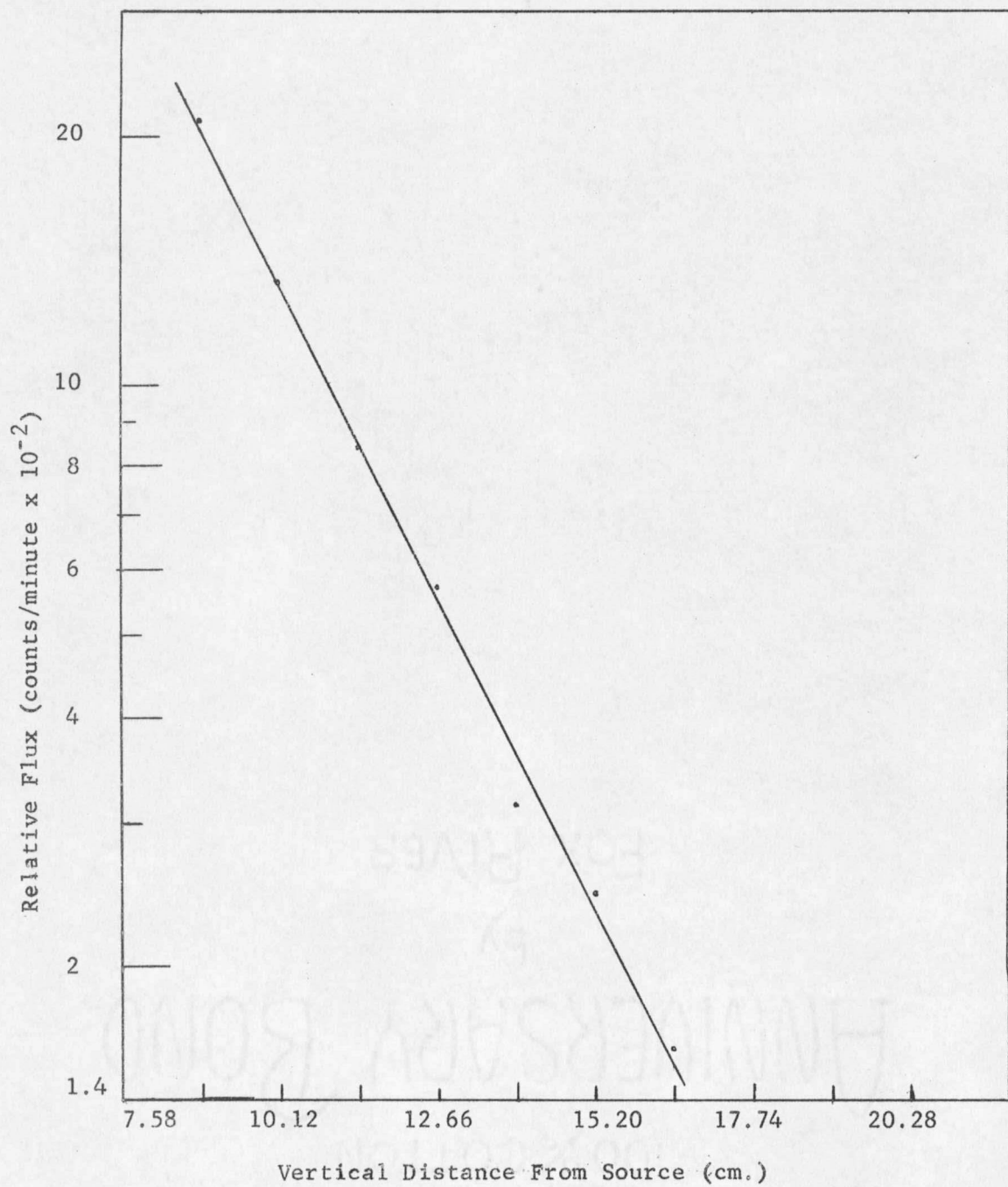


Figure 6. Illustration of the least square plot of relative flux versus distance from source for the diffusion length experiment.
Temperature = $27^{\circ}\text{C} \pm 1^{\circ}\text{C}$
Slope = 0.339 cm.^{-1}
Diffusion length = 2.95 cm.

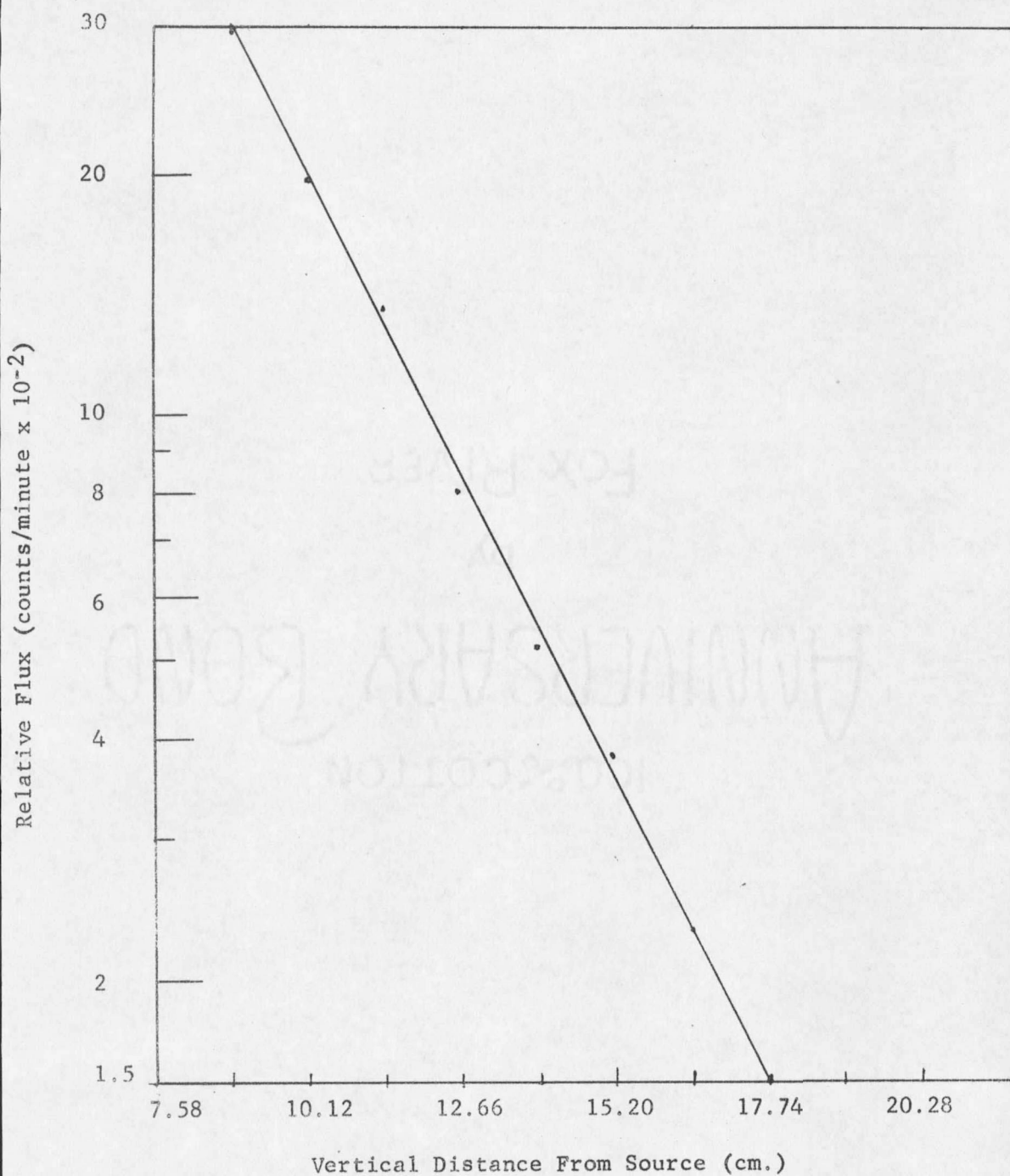


Figure 7. Illustration of the least square plot of relative flux versus distance from source for the diffusion length experiment.
Temperature = $50^{\circ}\text{C} \pm 1^{\circ}\text{C}$
Slope = 0.335 cm^{-1}
Diffusion length = 2.98 cm.

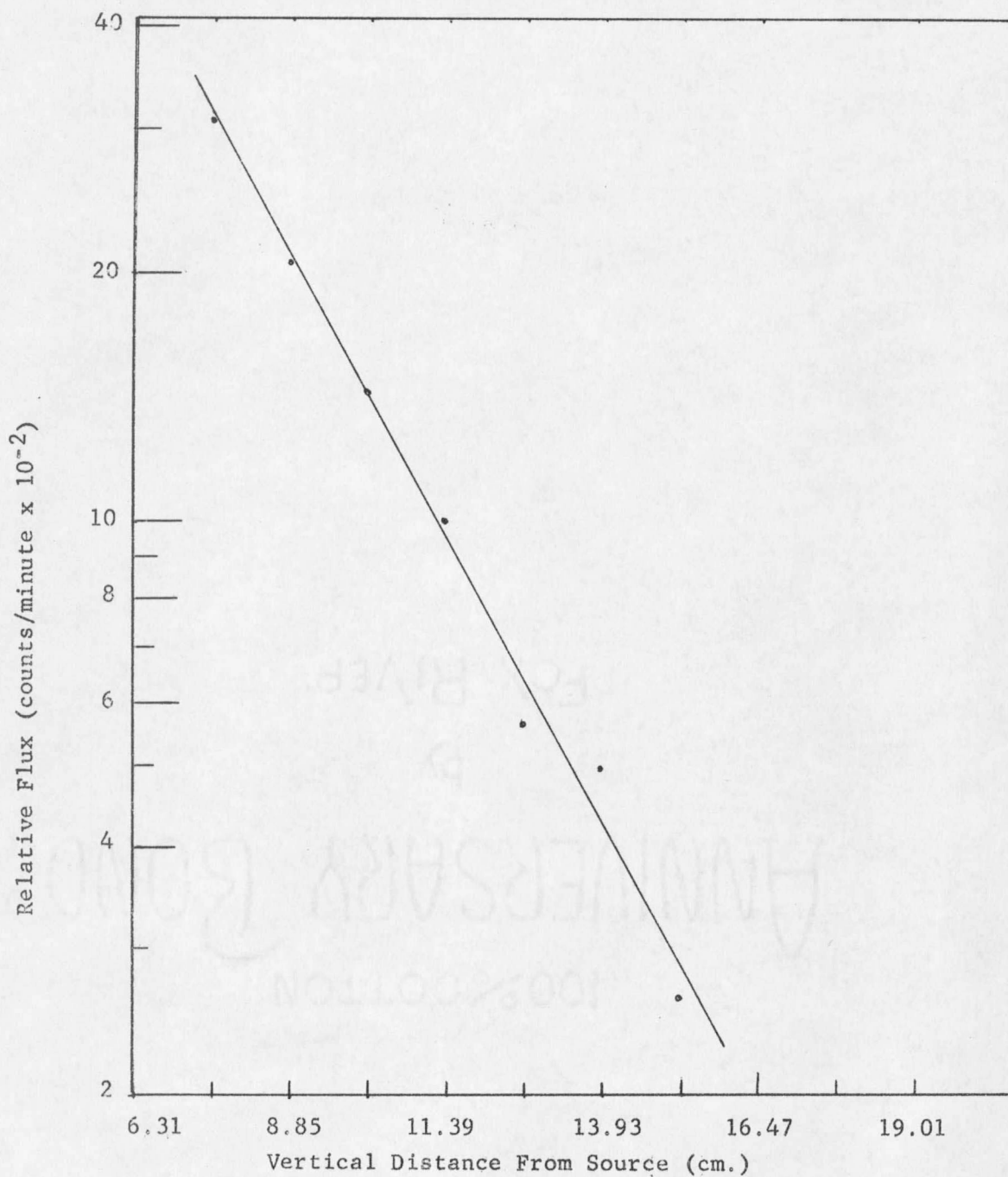


Figure 8. Illustration of the least square plot of relative flux versus distance from source for the diffusion length experiment.

Temperature = $75^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

Slope = 0.313 cm.^{-1}

Diffusion length = 3.19 cm.

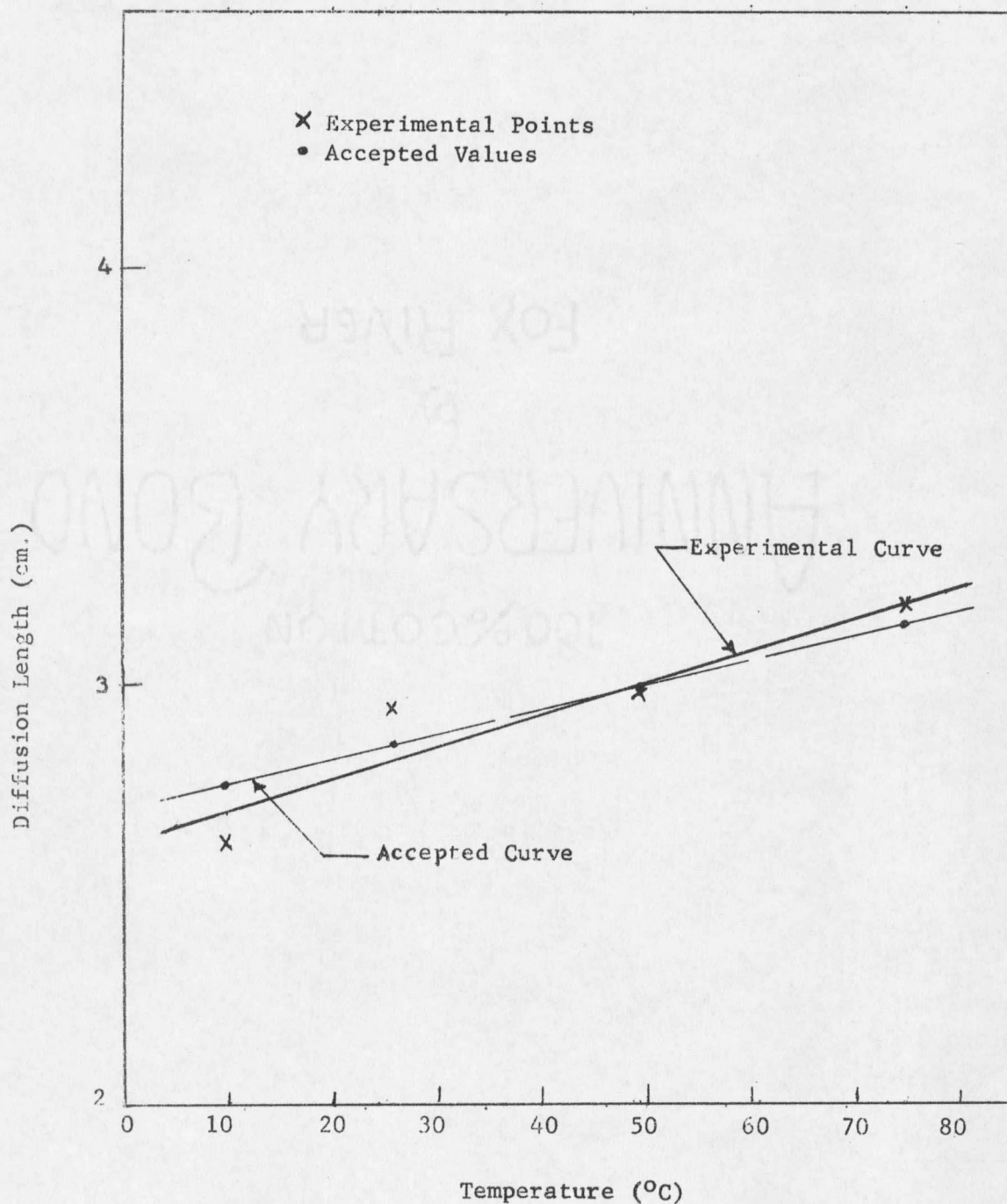


Figure 9. Illustration of the plot of diffusion length versus temperature for experimental points and accepted values.

$\Delta L / \Delta T = 0.00778 \text{ cm./}^\circ\text{C}$ - experimental

$\Delta L / \Delta T = 0.006 \text{ cm./}^\circ\text{C}$ - accepted

BUCKLING MEASUREMENTS

For the buckling measurements the core was loaded with 264 fuel tubes (2220 Kg. of natural uranium) in the form seen in Figure 10. The four curie plutonium-beryllium source (1.04×10^7 neutrons/second) was placed in the water-filled central tube (see Figure 11) which rested on the grid plate.

The same thermal neutron crystal and associated circuit that was used in the diffusion length experiment was also used in the determination of the buckling. The high temperature adapter tube that was used at temperatures of 50°C or greater was not used for the buckling measurements. It was anticipated that the buckling and diffusion length would be measured at the same temperatures, but this was not possible. The aluminum fuel tubes were capped with a rubber stopper to make them water tight. When the temperature of the system exceeded approximately 47°C , these stoppers became free of the tube and water would fill the tubes and surround the fuel slugs. Since this is a very undesirable situation, the temperatures used for both the vertical and radial buckling measurements were $13^{\circ}\text{C} \pm 1^{\circ}\text{C}$, $23^{\circ}\text{C} \pm 1^{\circ}\text{C}$, $35^{\circ}\text{C} \pm 1^{\circ}\text{C}$, and $45^{\circ}\text{C} \pm 1^{\circ}\text{C}$. A uniform temperature was maintained throughout the system by allowing the pump to circulate the water continuously.

Five minute counts were taken for both the vertical and radial buckling. At each position in the vertical buckling experiment a count was taken with the source in position and when the source had been removed. This was done to obtain the contribution due to spontaneous fission. For the radial buckling measurements background measurements were not taken.

Vertical Buckling - Vertical traverses were made at radial positions of 21.25, 17.44, and 19.35 cm. from the source at each temperature. The vertical distances at each radial position were 30.86, 38.48, 46.10, 53.72, 61.34, and 68.96 cm. from the source. The counts per minute were corrected for background and graphs of $\ln[\text{counts per minute}]$ versus vertical distance from the source were made for each temperature and radial position.

The values of γ , the slope of these graphs, were found from the data by the method of least squares; that is, by fitting the best straight line on the semi-logarithmic graph to the experimental points. There were at least two trials at each temperature; the average value of these then being used as the vertical buckling at that particular temperature. The exact same positions of the detecting tube were reproduced at each temperature, and the environment was kept the same. This was done so that the results of the measurements would have the same meaning. The results of the experiment are tabulated in table XIV.

Radial Buckling - Radial traverses were made at vertical positions of 48.64, 51.18, 53.72, and 56.26 cm. from the source at each temperature. The four trials were needed so that a fairly good average of the extrapolated radius could be found. At each of the above-mentioned heights, measurements were taken at every 2.54 cm. starting at 21.25 cm. from the source and going out to 39.03 centimeters. In some cases it was more difficult than others to visually extrapolate the curve, relative flux versus radial distance from the source, to zero. No attempt was made to fit any

of the curves to a Bessel curve. The same experimental conditions held for the vertical buckling experiment. The results are listed in table XIV.

The fuel tubes are distributed heterogeneously in the central part of the reactor, that is, the reactor core. The horizontal cross-section of this distribution is shown in the figure below.

(not to scale)

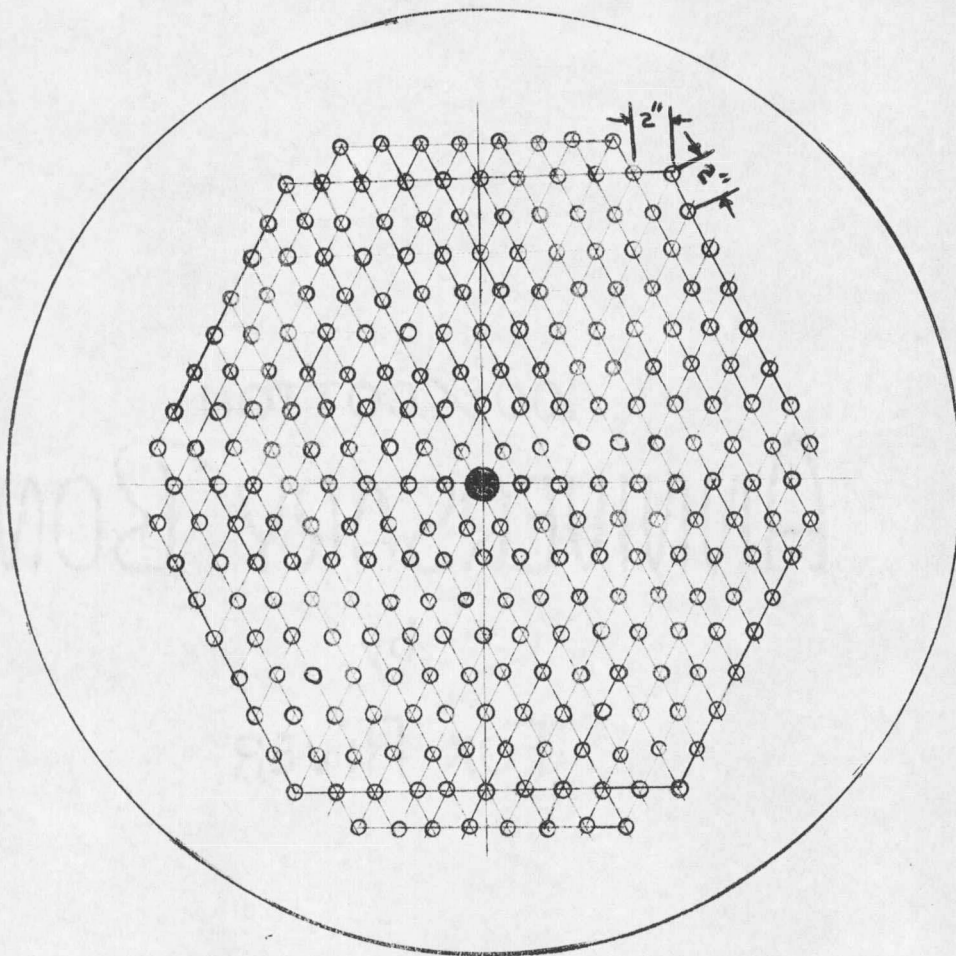


Figure 10. Illustration of the horizontal cross-section of the subcritical assembly.

(not to scale)

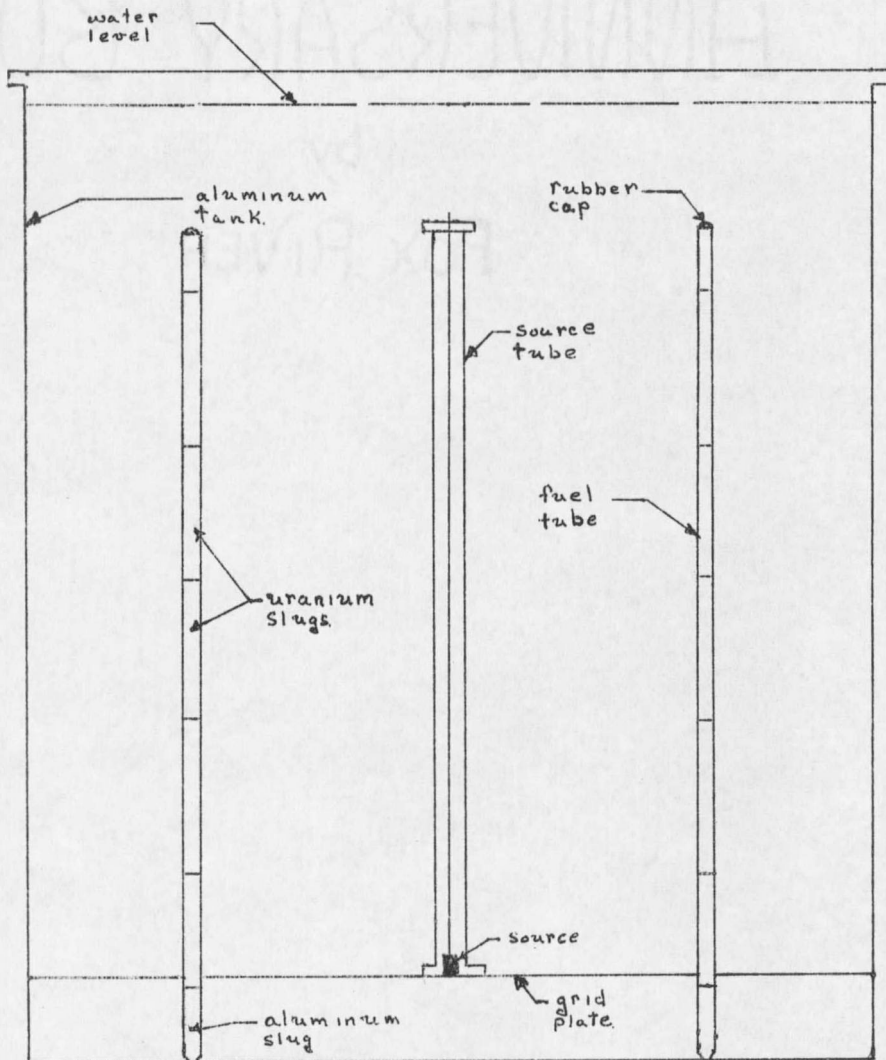


Figure 11. Illustration of the position of the source tube and fuel tubes.

TABLE VI. Distance, number of thermal neutrons, background, net number of thermal neutrons for vertical buckling at $13^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	counts per minute $\pm$ error	background counts per minute $\pm$ error	net counts per minute $\pm$ error
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radial distance from source = 21.25 cm.

30.86	5	1,132.2 $\pm$ 14.9	7.0 $\pm$ 1.3	1,125.2 $\pm$ 15.0
38.48	5	657.4 $\pm$ 11.5	8.0 $\pm$ 1.4	649.4 $\pm$ 11.6
46.10	5	428.8 $\pm$ 9.3	4.0 $\pm$ 1.0	424.8 $\pm$ 9.4
53.72	5	287.6 $\pm$ 7.6	5.0 $\pm$ 1.1	282.6 $\pm$ 7.7
61.34	5	161.8 $\pm$ 5.7	4.0 $\pm$ 1.0	147.8 $\pm$ 5.8
68.96	5	111.8 $\pm$ 4.7	7.0 $\pm$ 1.3	104.8 $\pm$ 4.8

radial distance from source = 19.35 cm.

30.86	5	1,331.6 $\pm$ 16.3	15.5 $\pm$ 2.0	1,316.1 $\pm$ 16.4
38.48	5	810.6 $\pm$ 12.7	12.5 $\pm$ 1.8	798.1 $\pm$ 12.8
46.10	5	511.6 $\pm$ 10.1	8.0 $\pm$ 1.4	503.6 $\pm$ 10.2
53.72	5	326.4 $\pm$ 8.1	13.5 $\pm$ 1.8	312.9 $\pm$ 8.2
61.34	5	202.0 $\pm$ 6.4	11.0 $\pm$ 1.2	191.0 $\pm$ 6.5
68.96	5	131.0 $\pm$ 5.1	16.0 $\pm$ 2.0	115.0 $\pm$ 5.2

TABLE VI. (Continued)

vertical distance from source (cm.)	time (min.)	counts per minute $\pm$ error	background counts per minute $\pm$ error	net counts per minute $\pm$ error
radial distance from source = 17.44 cm.				
30.86	5	1.470.8 $\pm$ 17.1	10.0 $\pm$ 1.6	1,460.8 $\pm$ 17.2
35.90	5	998.0 $\pm$ 14.1	11.0 $\pm$ 1.2	987.0 $\pm$ 14.2
41.02	5	740.6 $\pm$ 12.2	9.0 $\pm$ 1.5	731.6 $\pm$ 12.3
46.10	5	558.8 $\pm$ 10.6	5.5 $\pm$ 1.2	553.3 $\pm$ 10.7
51.18	5	407.8 $\pm$ 9.0	6.0 $\pm$ 1.2	401.8 $\pm$ 9.1
56.26	5	287.4 $\pm$ 7.6	5.0 $\pm$ 1.1	282.4 $\pm$ 7.7
61.43	5	209.6 $\pm$ 6.5	22.5 $\pm$ 2.4	187.1 $\pm$ 6.6
66.42	5	171.4 $\pm$ 5.9	20.0 $\pm$ 2.2	151.4 $\pm$ 6.0
68.96	5	141.0 $\pm$ 5.3	17.0 $\pm$ 2.1	124.0 $\pm$ 5.4

TABLE VII. Distance, number of thermal neutrons, background, net number of thermal neutrons for vertical buckling at $23^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	counts per minute $\pm$ error	background counts per minute $\pm$ error	net counts per minute $\pm$ error
radial distance from source = 17.44 cm.				
30.86	5	1,508.2 $\pm$ 17.4	8.0 $\pm$ 1.4	1,500.2 $\pm$ 17.5
35.90	5	1,059.8 $\pm$ 14.6	9.0 $\pm$ 1.5	1,050.8 $\pm$ 14.7
41.02	5	748.0 $\pm$ 12.2	8.5 $\pm$ 1.5	739.5 $\pm$ 12.3
46.10	6	571.8 $\pm$ 10.7	11.0 $\pm$ 1.7	560.8 $\pm$ 10.8
51.18	5	424.5 $\pm$ 8.4	7.0 $\pm$ 1.3	417.5 $\pm$ 8.5
56.26	5	296.8 $\pm$ 7.7	5.0 $\pm$ 1.1	291.8 $\pm$ 7.8
61.34	5	215.2 $\pm$ 6.6	5.0 $\pm$ 1.1	210.2 $\pm$ 6.7
66.42	5	170.2 $\pm$ 5.8	8.0 $\pm$ 1.4	162.2 $\pm$ 5.9
68.96	5	144.8 $\pm$ 5.4	7.0 $\pm$ 1.3	137.8 $\pm$ 5.5
radial distance from source = 21.25 cm.				
23.24	5	1,599.6 $\pm$ 17.9	6.4 $\pm$ 1.3	1,593.1 $\pm$ 18.0
30.86	5	1,194.8 $\pm$ 15.4	5.5 $\pm$ 1.2	1,189.3 $\pm$ 15.5
38.48	5	717.8 $\pm$ 12.0	5.5 $\pm$ 1.2	712.3 $\pm$ 12.1
46.10	5	449.4 $\pm$ 9.5	11.0 $\pm$ 1.7	438.4 $\pm$ 9.6
53.72	5	292.8 $\pm$ 7.7	5.0 $\pm$ 1.1	287.8 $\pm$ 7.8
61.34	5	183.4 $\pm$ 6.1	6.0 $\pm$ 1.2	177.4 $\pm$ 6.2
68.96	5	110.8 $\pm$ 4.7	4.5 $\pm$ 1.1	106.3 $\pm$ 4.8

TABLE VIII. Distance, number of thermal neutrons, background, net number of thermal neutrons for vertical buckling at $35^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	counts per minute $\pm$ error	background counts per minute $\pm$ error	net counts per minute $\pm$ error
radial distance from source = 17.44 cm.				
30.86	5	$1,568.8 \pm 17.6$	13.0 ± 1.8	$1,555.8 \pm 17.7$
38.48	5	952.0 ± 13.8	8.0 ± 1.4	944.0 ± 13.9
46.10	5	603.0 ± 11.0	8.5 ± 1.5	594.5 ± 11.1
53.72	5	373.6 ± 8.7	7.5 ± 1.4	366.1 ± 8.8
61.34	5	234.2 ± 6.8	7.0 ± 1.3	227.2 ± 6.9
68.96	5	156.6 ± 5.6	11.0 ± 1.7	145.6 ± 5.7
radial distance from source = 21.25 cm.				
30.68	5	$1,324.2 \pm 16.3$	8.0 ± 1.4	$1,316.2 \pm 16.4$
38.48	5	819.6 ± 12.8	9.5 ± 1.5	810.1 ± 12.9
46.10	5	517.4 ± 10.2	12.0 ± 1.7	505.4 ± 10.3
53.72	5	347.6 ± 8.4	9.5 ± 1.5	343.1 ± 8.5
61.34	5	217.0 ± 6.6	5.5 ± 1.2	211.5 ± 6.7
68.96	5	131.8 ± 5.1	5.0 ± 1.1	126.8 ± 5.2

TABLE IX. Distance, number of thermal neutrons, background, net number of thermal neutrons for vertical buckling at $45^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

vertical distance from source (cm.)	time (min.)	counts per minute $\pm$ error	background counts per minute $\pm$ error	net counts per minute $\pm$ error
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radial distance from source = 17.44 cm.

30.86	5	$1,661.6 \pm 18.3$	18.0 ± 2.1	$1,643.6 \pm 18.4$
38.48	5	945.2 ± 13.8	8.5 ± 1.5	936.7 ± 13.9
46.10	5	634.0 ± 11.3	6.5 ± 1.3	627.5 ± 11.4
53.72	5	410.6 ± 9.1	11.0 ± 1.7	399.6 ± 9.2
61.34	5	243.2 ± 7.0	9.0 ± 1.5	234.2 ± 7.1
68.96	5	170.6 ± 5.8	15.0 ± 1.9	155.6 ± 5.9

radial distance from source = 21.25 cm.

30.86	5	$1,412.2 \pm 16.8$	11.0 ± 1.7	$1,401.2 \pm 16.9$
38.48	5	855.4 ± 13.1	13.5 ± 1.8	841.9 ± 13.2
46.10	5	557.0 ± 10.6	13.5 ± 1.8	543.5 ± 10.7
53.72	5	350.0 ± 8.4	4.5 ± 1.1	345.5 ± 8.5
61.34	5	215.6 ± 6.6	5.0 ± 1.1	210.6 ± 6.7
68.96	5	142.6 ± 5.3	6.0 ± 1.2	136.6 ± 5.4

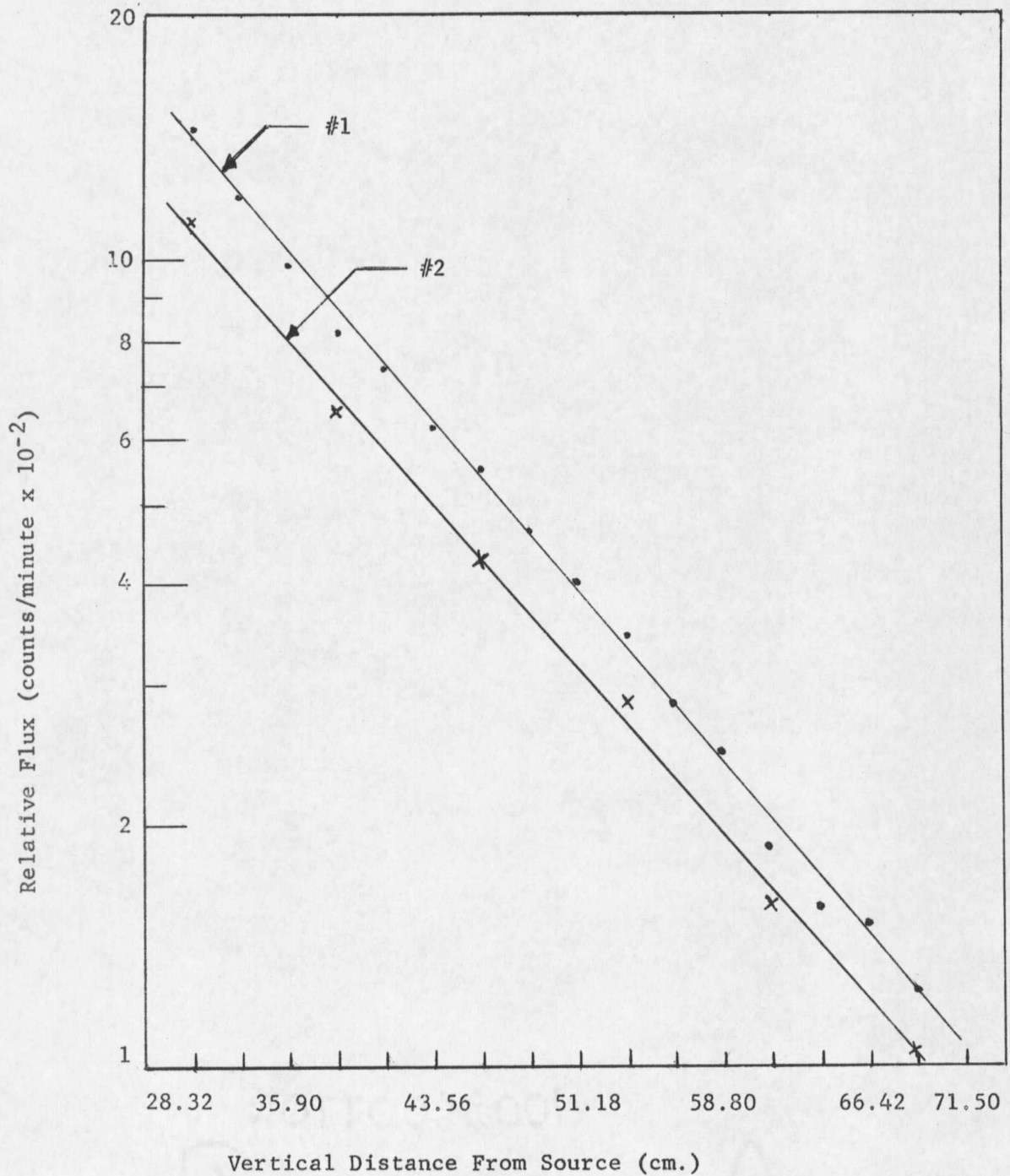


Figure 12. Illustration of the least square plot of relative flux versus vertical distance from source for the vertical buckling experiment.

Temperature = 13°C ± 1°C

Trial #1 at a radius of 17.44 cm.: $\gamma^2 = 41.10 \times 10^{-4} \text{ cm.}^{-2}$

Trial #2 at a radius of 21.25 cm.: $\gamma^2 = 38.36 \times 10^{-4} \text{ cm.}^{-2}$

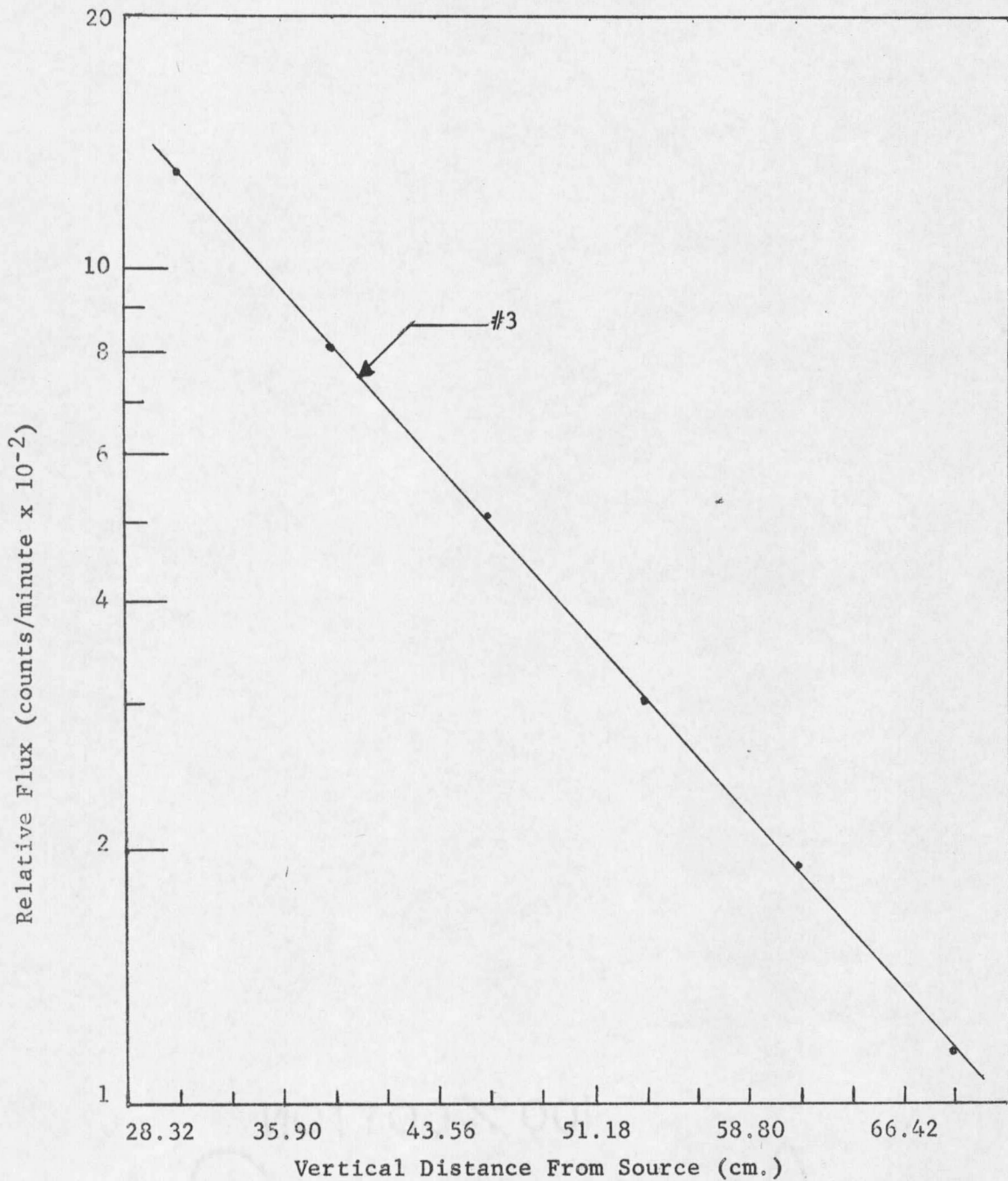


Figure 13. Illustration of the least square plot of relative flux versus vertical distance from source for the vertical buckling experiment.

Temperature = $13^{\circ}\text{C} \pm 1^{\circ}\text{C}$

Trial #3 at a radius of 19.35 cm.: $\gamma^2 = 40.40 \times 10^{-4} \text{ cm.}^{-2}$

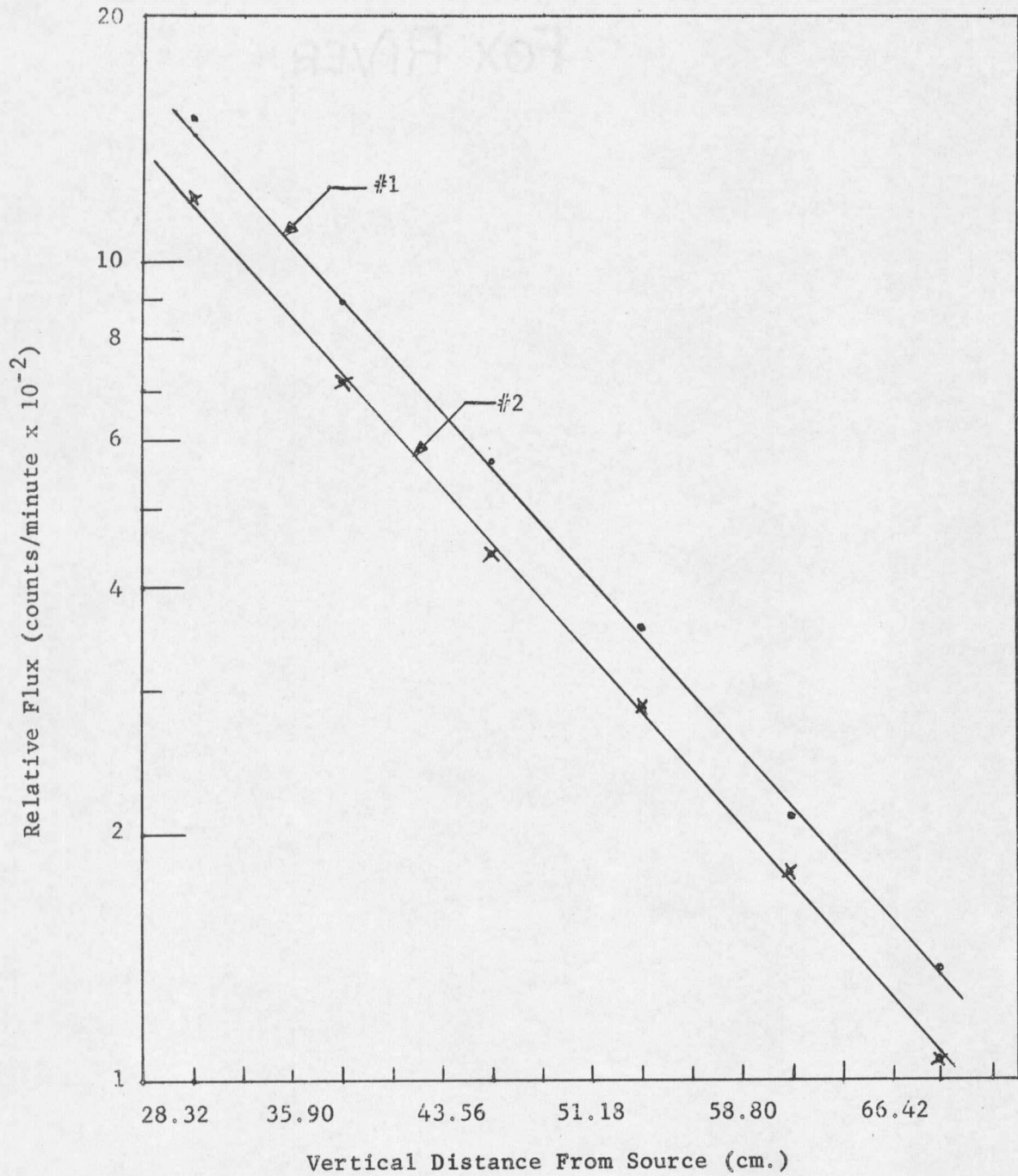


Figure 14. Illustration of the least square plot of relative flux versus vertical distance from source for the vertical buckling experiment.

Temperature = 23°C ± 1°C.

Trial #1 at a radius of 17.44 cm.: $\gamma^2 = 38.72 \times 10^{-4} \text{ cm.}^{-2}$

Trial #2 at a radius of 21.25 cm.: $\gamma^2 = 39.04 \times 10^{-4} \text{ cm.}^{-2}$

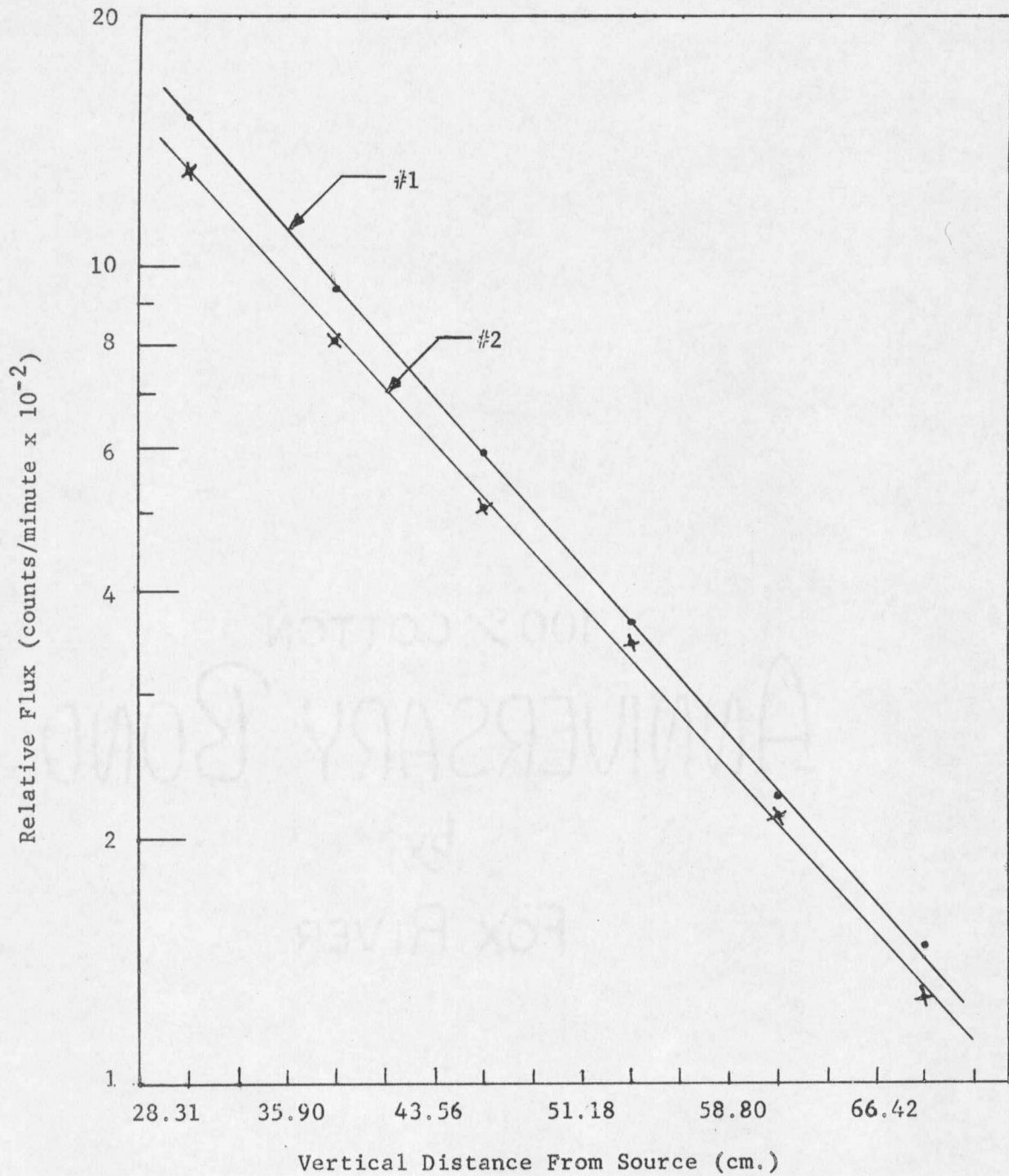


Figure 15. Illustration of the least square plot of relative flux versus vertical distance from source for the vertical buckling experiment.

Temperature = 35°C ± 1°C

Trial #1 at a radius of 17.44 cm.: $\chi^2 = 38.75 \times 10^{-4} \text{ cm.}^{-2}$

Trial #2 at a radius of 21.25 cm.: $\chi^2 = 36.51 \times 10^{-4} \text{ cm.}^{-2}$

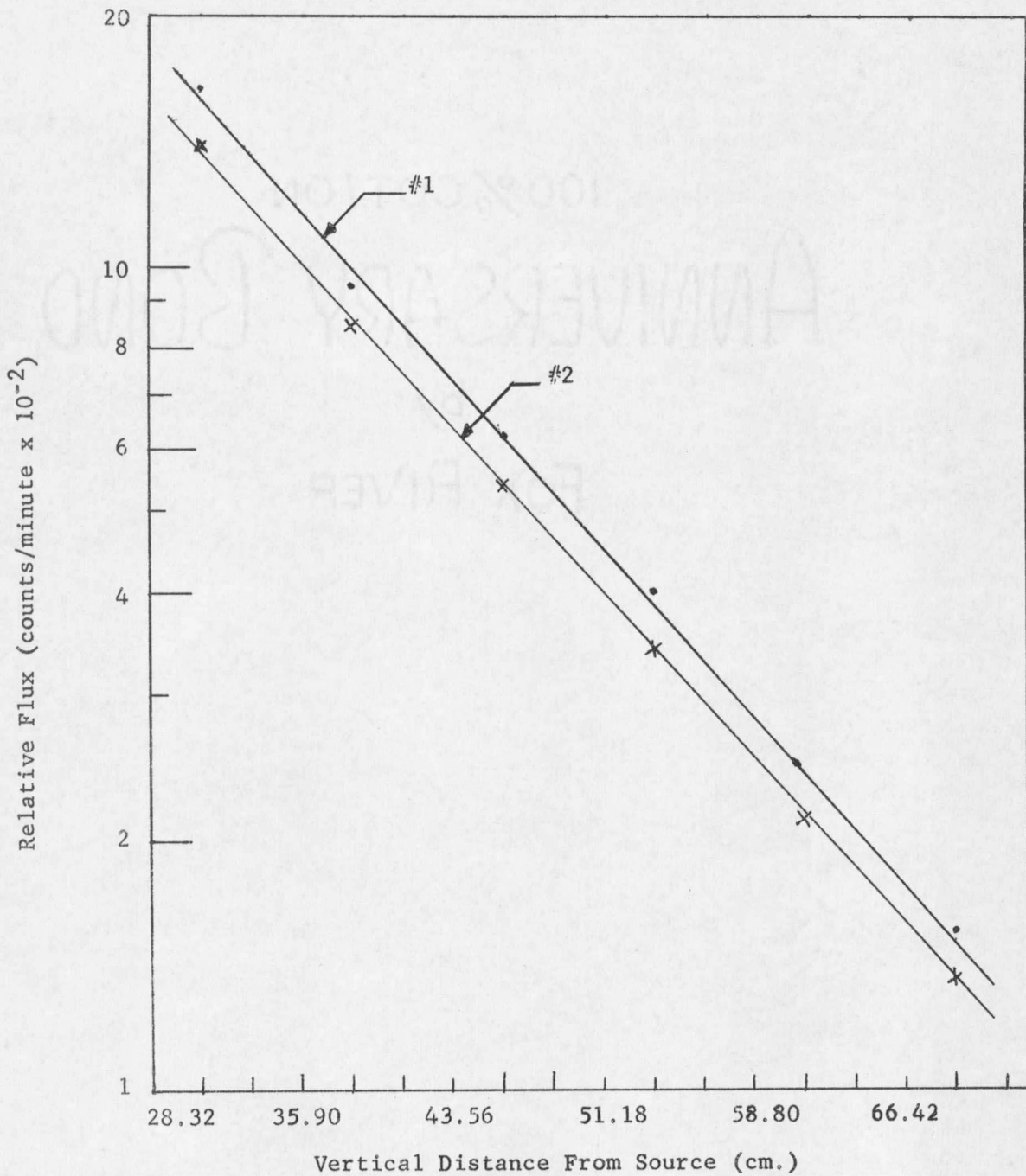


Figure 16. Illustration of the least square plot of relative flux versus vertical distance from source for the vertical buckling experiment.

Temperature = 45°C ± 1°C

Trial #1 at a radius of 17.44 cm.: $\gamma^2 = 37.80 \times 10^{-4} \text{ cm.}^{-2}$

Trial #2 at a radius of 21.25 cm.: $\gamma^2 = 37.13 \times 10^{-4} \text{ cm.}^{-2}$

TABLE X. Distance and number of thermal neutrons for radial buckling at $13^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

distance from source (cm.)	time (min.)	vertical distance from source = 48.64 cm.: counts/min. $\pm$ error	vertical distance from source = 51.18 cm.: counts/min. $\pm$ error	vertical distance from source = 53.72 cm.: counts/min. $\pm$ error	vertical distance from source = 56.26 cm.: counts/min. $\pm$ error
39.03	5	179.0 $\pm$ 6.0	147.2 $\pm$ 5.4	132.8 $\pm$ 5.2	110.2 $\pm$ 4.7
36.49	5	209.8 $\pm$ 6.5	171.4 $\pm$ 5.9	159.6 $\pm$ 5.7	135.2 $\pm$ 5.2
33.95	5	250.0 $\pm$ 7.1	202.4 $\pm$ 6.4	167.0 $\pm$ 5.8	152.8 $\pm$ 5.5
31.41	5	266.4 $\pm$ 7.3	232.0 $\pm$ 6.8	199.4 $\pm$ 6.3	177.2 $\pm$ 6.0
28.87	5	329.6 $\pm$ 8.1	284.2 $\pm$ 7.5	227.0 $\pm$ 6.7	186.2 $\pm$ 6.1
26.33	5	378.4 $\pm$ 8.7	330.8 $\pm$ 8.1	272.6 $\pm$ 7.4	230.6 $\pm$ 6.8
23.79	5	383.0 $\pm$ 8.7	356.8 $\pm$ 8.5	292.0 $\pm$ 7.7	244.6 $\pm$ 7.0
21.25	5	437.2 $\pm$ 9.4	377.7 $\pm$ 7.3	304.2 $\pm$ 7.8	244.8 $\pm$ 7.0

TABLE XI. Distance and number of thermal neutrons for radial buckling at $23^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

distance from source (cm.)	time (min.)	vertical distance from source = 48.64 cm.: counts/min. $\pm$ error	vertical distance from source = 51.18 cm.: counts/min. $\pm$ error	vertical distance from source = 53.72 cm.: counts/min. $\pm$ error	vertical distance from source = 56.26 cm.: counts/min. $\pm$ error
39.03	5	200.8 $\pm$ 6.3	157.6 $\pm$ 5.6	122.0 $\pm$ 4.9	118.4 $\pm$ 4.9
36.49	5	218.2 $\pm$ 6.6	187.0 $\pm$ 6.1	149.6 $\pm$ 5.5	127.8 $\pm$ 5.1
33.95	5	252.2 $\pm$ 7.1	212.0 $\pm$ 6.5	185.2 $\pm$ 6.1	148.8 $\pm$ 5.5
31.41	5	296.8 $\pm$ 7.7	255.0 $\pm$ 7.1	200.8 $\pm$ 6.5	184.4 $\pm$ 6.1
28.87	5	347.6 $\pm$ 8.3	284.4 $\pm$ 7.5	235.6 $\pm$ 6.9	197.4 $\pm$ 6.3
26.33	5	402.0 $\pm$ 9.0	319.8 $\pm$ 8.0	283.6 $\pm$ 7.5	244.4 $\pm$ 7.0
23.79	5	422.0 $\pm$ 9.0	366.6 $\pm$ 8.6	287.2 $\pm$ 7.6	254.4 $\pm$ 7.1
21.25	5	445.4 $\pm$ 9.4	391.8 $\pm$ 8.9	294.4 $\pm$ 7.7	270.2 $\pm$ 7.4

TABLE XII. Distance and number of thermal neutrons for radial buckling at $35^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

distance from source (cm.)	time (min.)	vertical distance from source = 48.64 cm.: counts/min. $\pm$ error	vertical distance from source = 51.18 cm.: counts/min. $\pm$ error	vertical distance from source = 53.72 cm.: counts/min. $\pm$ error	vertical distance from source = 56.26 cm.: counts/min. $\pm$ error
39.03	5	195.6 $\pm$ 6.3	175.2 $\pm$ 5.9	140.8 $\pm$ 5.3	119.2 $\pm$ 4.5
36.49	5	238.0 $\pm$ 6.9	200.0 $\pm$ 6.3	165.8 $\pm$ 5.8	146.6 $\pm$ 5.4
33.95	5	266.0 $\pm$ 7.3	226.0 $\pm$ 6.7	206.0 $\pm$ 6.3	170.4 $\pm$ 5.8
31.41	5	340.0 $\pm$ 8.2	275.6 $\pm$ 7.4	244.6 $\pm$ 7.0	192.8 $\pm$ 6.2
28.87	5	360.8 $\pm$ 8.5	303.0 $\pm$ 7.8	268.8 $\pm$ 7.3	217.2 $\pm$ 6.6
26.33	5	409.8 $\pm$ 9.1	369.8 $\pm$ 8.6	314.8 $\pm$ 7.8	259.6 $\pm$ 5.9
23.79	5	447.6 $\pm$ 9.5	401.2 $\pm$ 8.9	327.8 $\pm$ 8.1	270.8 $\pm$ 7.4
21.25	5	514.4 $\pm$ 10.1	432.4 $\pm$ 9.3	329.6 $\pm$ 8.1	297.8 $\pm$ 7.7

TABLE XIII. Distance and number of thermal neutrons for radial buckling at $45^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

distance from source (cm.)	time (min.)	vertical distance from source = 48.64 cm.: counts/min. $\pm$ error	vertical distance from source = 51.18 cm.: counts/min. $\pm$ error	vertical distance from source = 53.72 cm.: counts/min. $\pm$ error	vertical distance from source = 56.26 cm.: counts/min. $\pm$ error
39.03	5	197.8 $\pm$ 6.3	163.0 $\pm$ 5.7	127.2 $\pm$ 5.0	134.8 $\pm$ 5.2
36.49	5	208.8 $\pm$ 6.5	174.2 $\pm$ 5.9	152.4 $\pm$ 5.5	150.0 $\pm$ 5.5
33.95	5	252.0 $\pm$ 7.1	220.0 $\pm$ 6.6	196.2 $\pm$ 6.3	183.0 $\pm$ 6.1
31.41	5	296.4 $\pm$ 7.7	250.6 $\pm$ 7.1	210.4 $\pm$ 6.5	200.8 $\pm$ 6.4
28.87	5	352.4 $\pm$ 8.4	293.4 $\pm$ 7.7	241.8 $\pm$ 7.0	228.0 $\pm$ 6.8
26.33	5	377.8 $\pm$ 8.7	347.4 $\pm$ 8.3	278.4 $\pm$ 7.5	271.8 $\pm$ 7.4
23.79	5	437.4 $\pm$ 9.4	386.0 $\pm$ 8.8	308.8 $\pm$ 7.8	287.8 $\pm$ 7.6
21.25	5	441.0 $\pm$ 9.4	407.2 $\pm$ 9.0	320.8 $\pm$ 8.0	292.8 $\pm$ 7.7

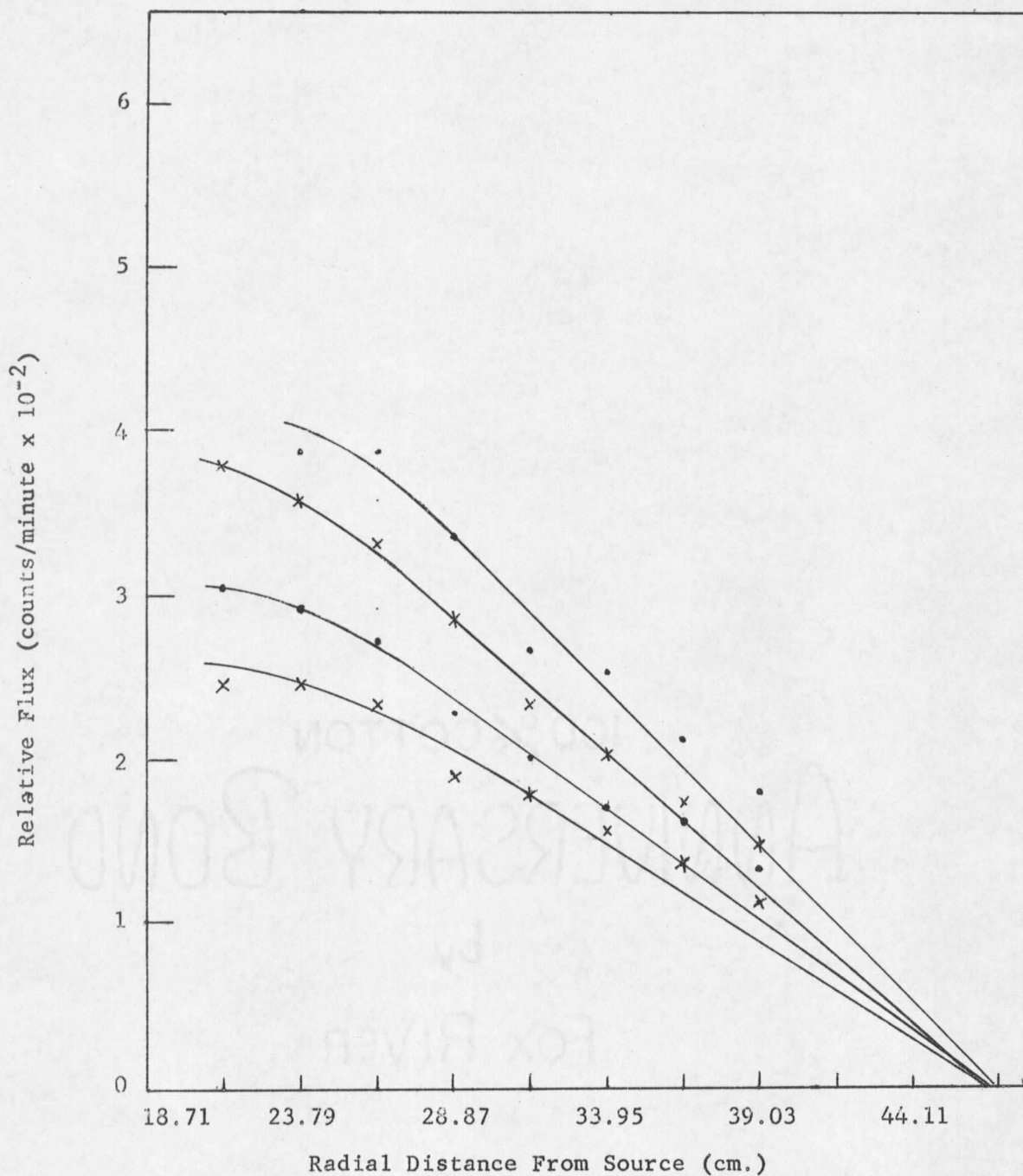


Figure 17. Illustration of the plot of relative flux versus radial distance from source for the radial buckling experiment.

Temperature = $13^{\circ}\text{C} \pm 1^{\circ}\text{C}$

Average $\bar{R}_0 = 46.17$ cm.

Radial buckling, $B_{r,}^2 = 27.12 \times 10^{-4}$ cm.⁻²

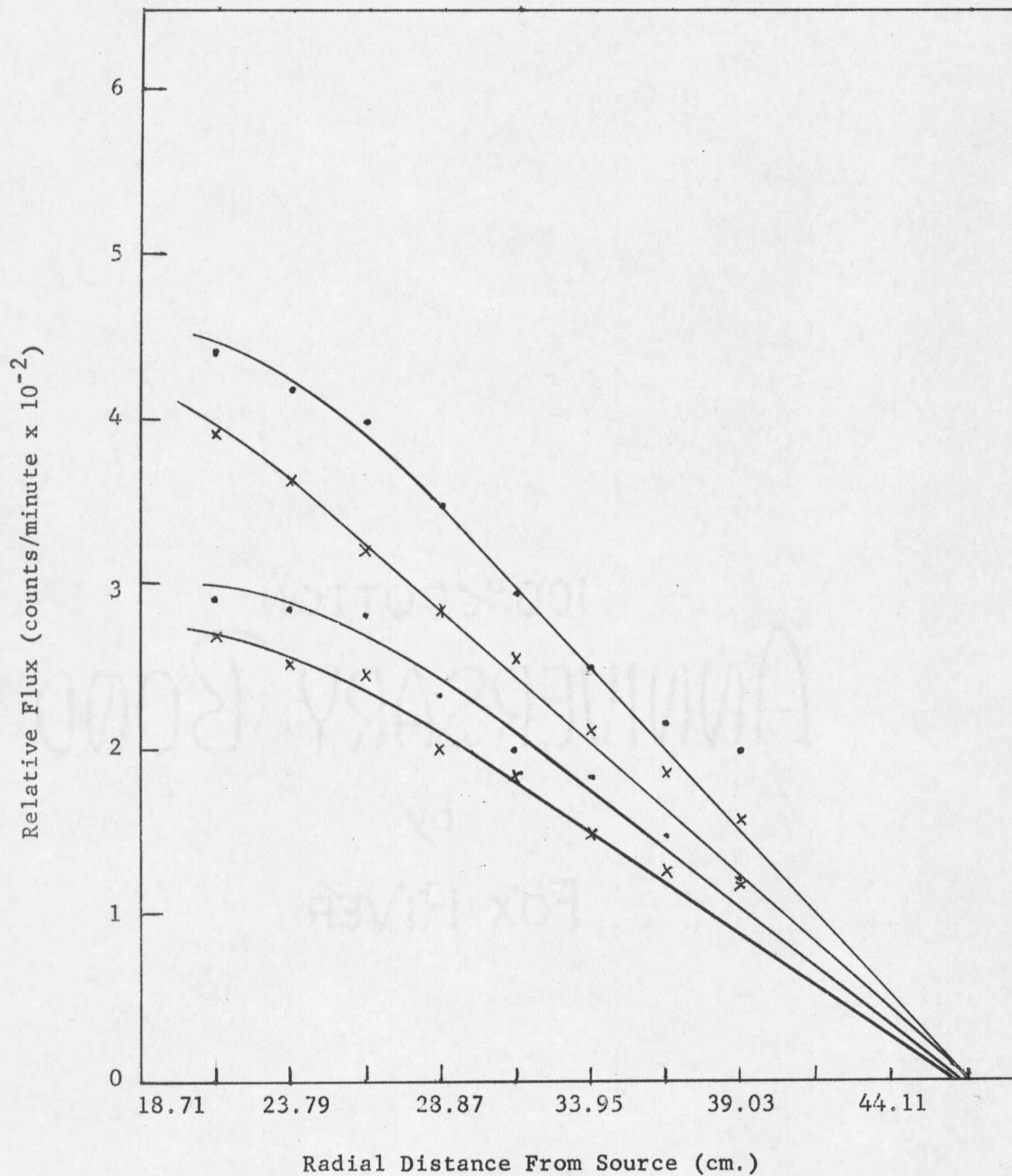


Figure 18. Illustration of the plot of relative flux versus radial distance from source for the radial buckling experiment.

Temperature = $23^{\circ}\text{C} \pm 1^{\circ}\text{C}$

Average $\bar{R}_0 = 46.77$ cm.

Radial buckling, $B_r^2 = 26.43 \times 10^{-4} \text{ cm.}^{-2}$

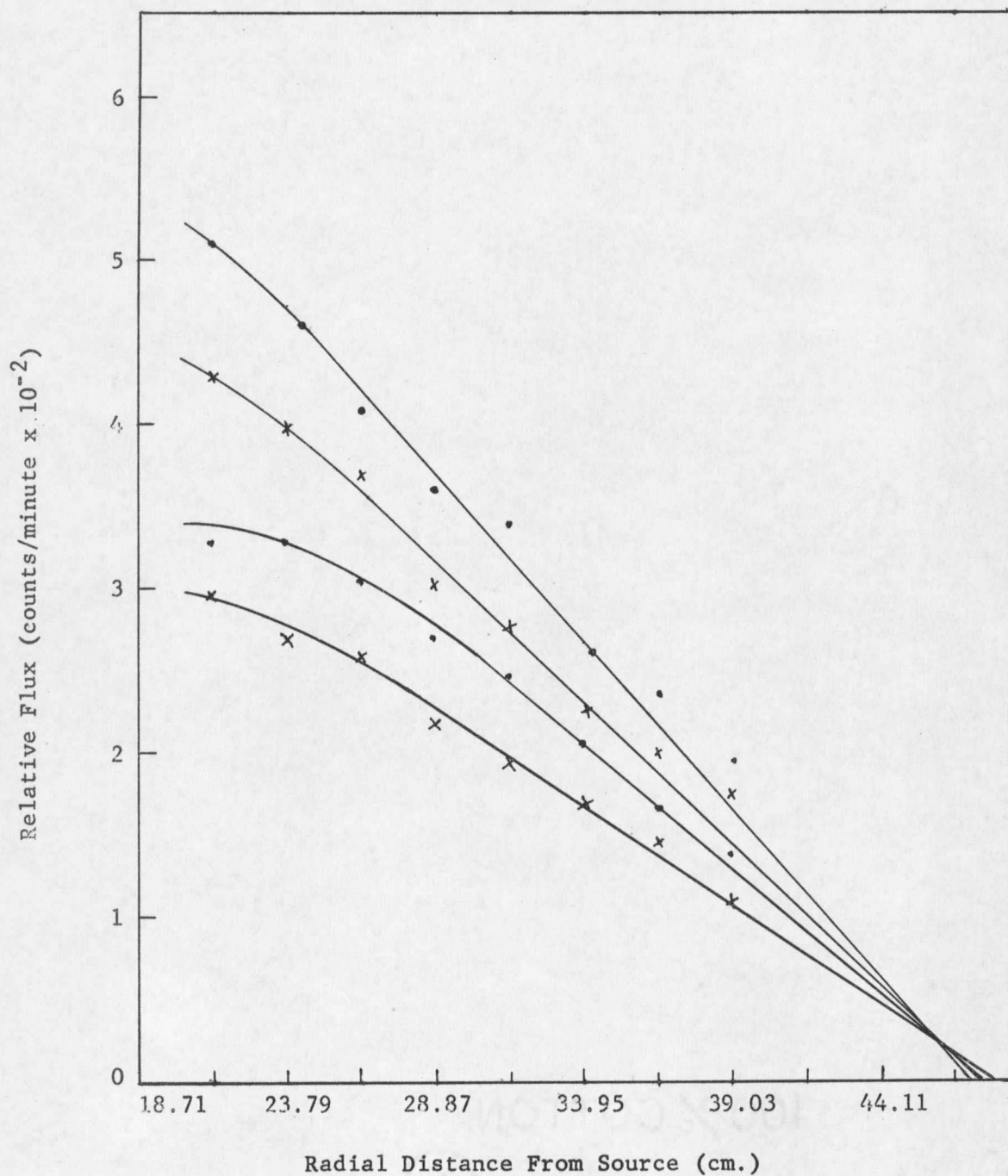


Figure 19. Illustration of the plot of relative flux versus radial distance from source for the radial buckling experiment.

Temperature = $35^{\circ}\text{C} \pm 1^{\circ}\text{C}$

Average $\bar{R}_0 = 47.52$ cm.

Radial buckling, B_r^2 , = 25.60×10^{-4} cm.⁻²

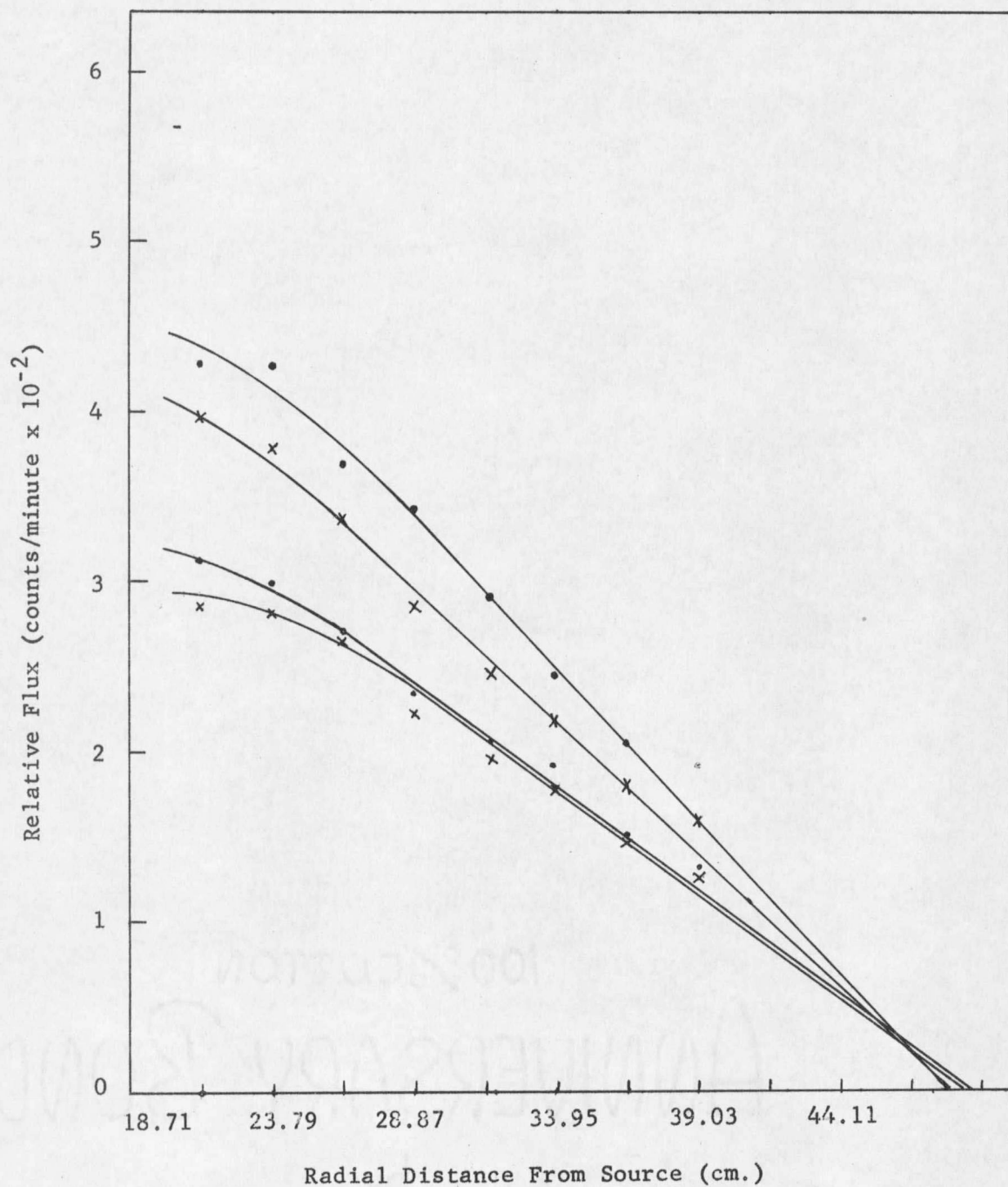


Figure 20. Illustration of the plot of relative flux versus radial distance from source for the radial buckling experiment.
 Temperature = $45^{\circ}\text{C} \pm 1^{\circ}\text{C}$
 Average $\bar{R}_0 = 48.16$ cm.
 Radial buckling, B_r^2 , = $24.93 \times 10^{-4} \text{ cm.}^{-2}$

TABLE XIV. Results of buckling experiment.

Temperature °C	Vertical Buckling B^2_v (cm. ⁻²)	Radial Buckling B^2_r (cm. ⁻²)	Buckling* B^2_m
13±1	39.96×10^{-4}	27.13×10^{-4}	-12.83×10^{-4}
23±1	38.88×10^{-4}	26.44×10^{-4}	-12.44×10^{-4}
35±1	37.63×10^{-4}	25.61×10^{-4}	-12.02×10^{-4}
45±1	37.47×10^{-4}	24.94×10^{-4}	-12.53×10^{-4}

* The values of B^2 are calculated from the equation $B^2_m = B^2_r - B^2_v$.

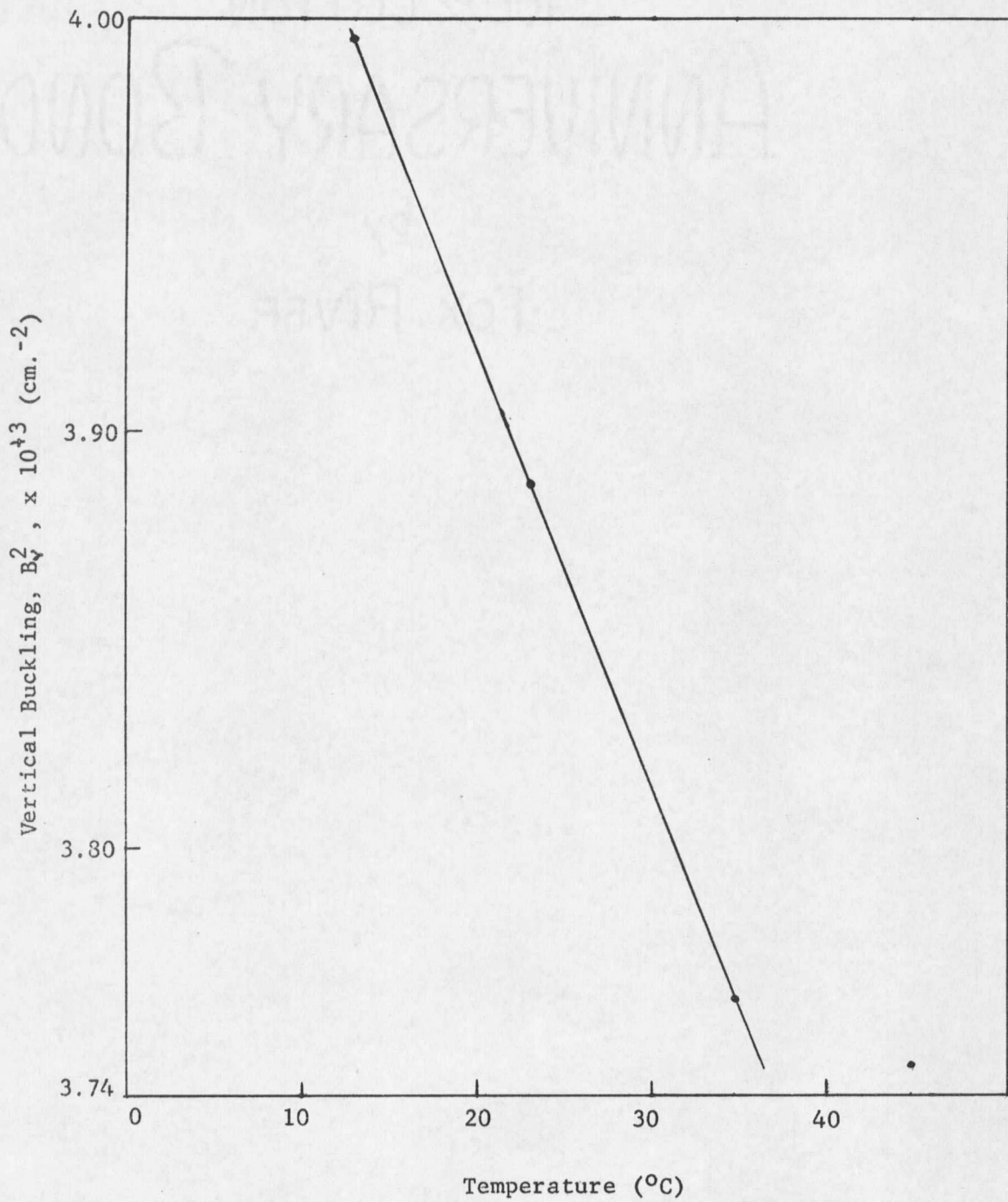


Figure 21. Illustration of the plot of vertical buckling versus temperature.

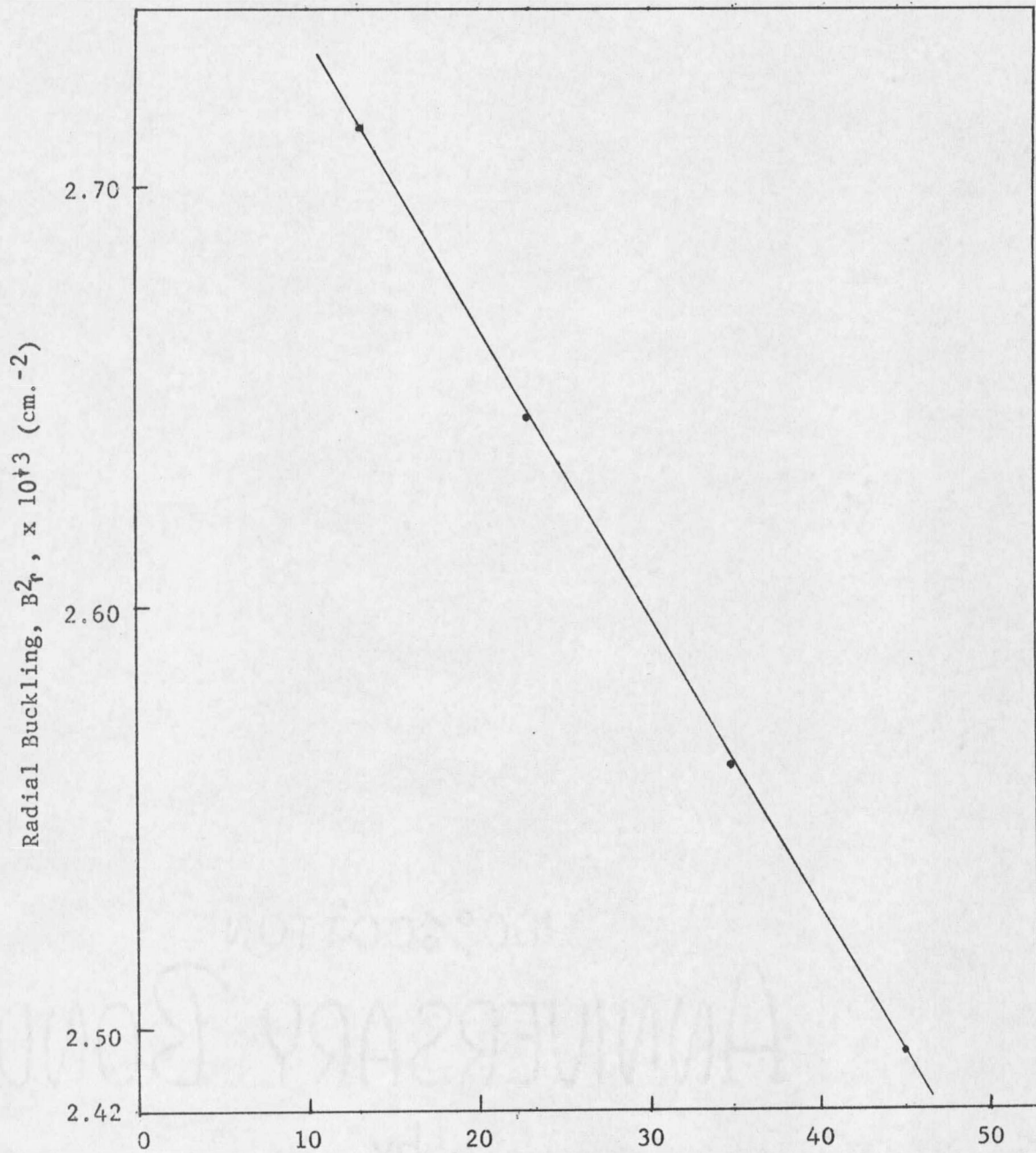


Figure 22. Illustration of the plot of radial buckling versus temperature.

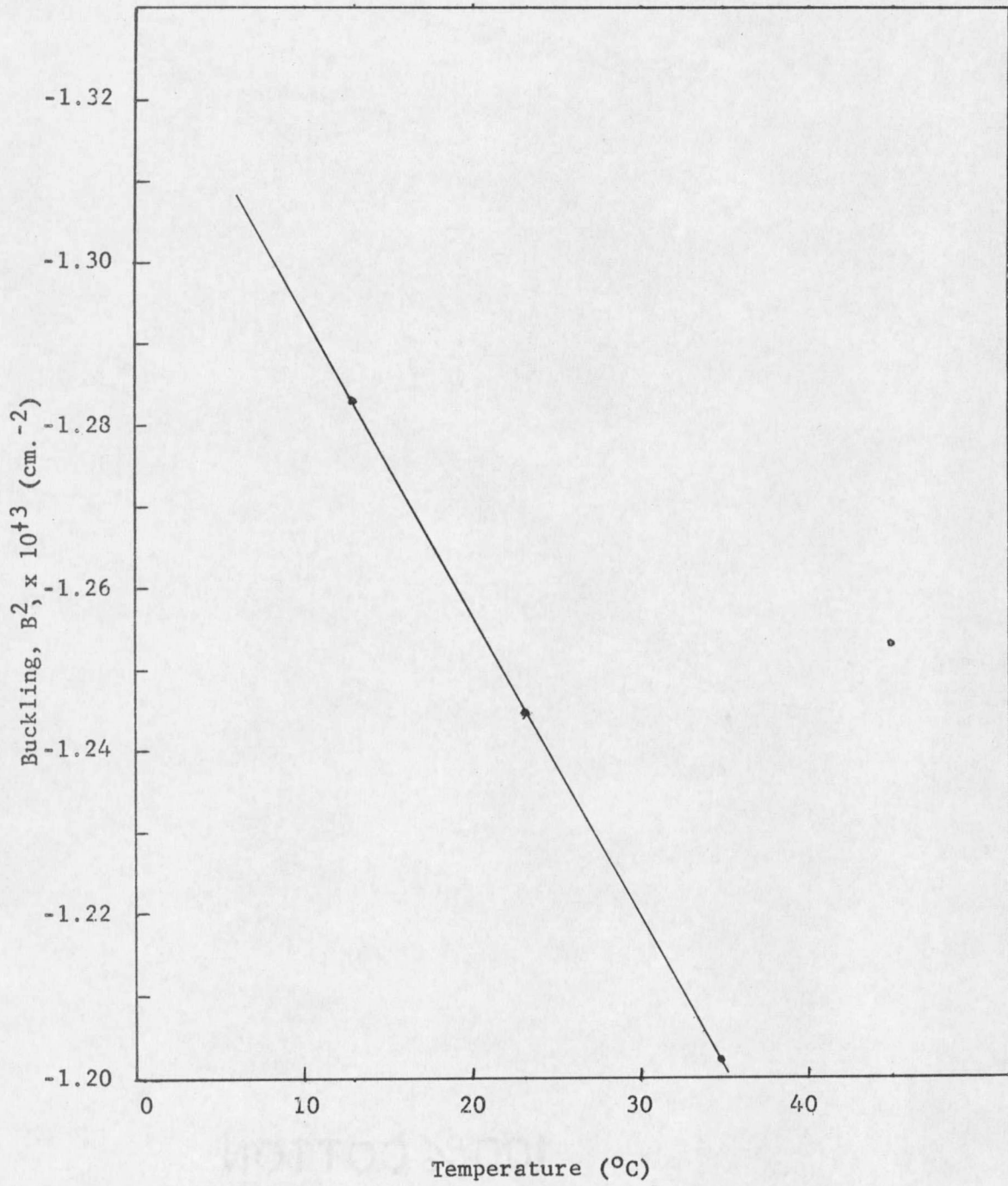


Figure 23. Illustration of the plot of variation of the buckling versus temperature.

CALCULATIONS

Temperature Coefficients of Reactivity:

All temperature coefficients are calculated at a temperature of 23° C.

$$\widetilde{R}_0 = 46.77 \text{ cm.}$$

$$\widetilde{H}_0 = 104.43 \text{ cm}$$

$$L_m = 2.80 \text{ cm.} = \text{experimental (moderator)}$$

$$T = 23^\circ \text{ C.}$$

$$\alpha_T = 8.4 \times 10^{-5}/^\circ\text{C.}$$

$$k_\infty = 0.960 = \text{calculated}^{18}$$

$$f = 0.774 = \text{calculated}^{18}$$

$$\gamma = 28 \text{ cm.}^2 \text{ }^{20}$$

$$B_m^2 = -12.44 \times 10^{-4} \text{ cm.}^{-2}$$

For a heterogeneous system the diffusion length is¹⁹

$$L_H^2 = L_m^2 (1-f) = (2.80)^2 (1-0.774) = 1.77 \text{ cm.}^2 \quad (55)$$

Now,

$$M_0^2 = L_H^2 + \gamma = 1.77 + 28 = 29.77 \text{ cm.}^2 \quad (56)$$

from (17),

$$B_g^2 = \left(\frac{2.405}{\widetilde{R}_0} \right)^2 + \left(\frac{\pi}{\widetilde{H}_0} \right)^2 = \left(\frac{2.405}{46.77} \right)^2 + \left(\frac{\pi}{104.43} \right)^2 = 35.60 \times 10^{-4} \text{ cm.}^{-2} \quad (57)$$

from (37),

$$\left(\frac{\partial \rho}{\partial T} \right)_d = \frac{B_g^2 L_H^2}{2 k_\infty T} = \frac{35.60 \times 10^{-4} \cdot 1.77}{2 \cdot 0.960 \cdot 2.3} = -14.3 \times 10^{-5} / ^\circ\text{C} \quad (58)$$

from (45),

$$\left(\frac{\partial \rho}{\partial T} \right)_{B_g^2, G_a, G_s} = - \frac{6 B_g^2 M_0^2 \alpha_T}{k_\infty} = - \frac{6 \cdot 35.60 \cdot 29.77 \cdot 8.4 \times 10^{-5}}{9.60 \times 10^{-1}} = -5.6 \times 10^{-5} / ^\circ\text{C} \quad (59)$$

from (54)

$$\left(\frac{\partial \rho}{\partial T} \right)_{B_g^2, G_s} = \frac{4 \alpha_T B_g^2 M_0^2}{k_\infty} = \frac{4 \cdot 8.4 \cdot 35.60 \cdot 29.77 \times 10^{-5}}{9.60 \times 10^{-1}} = +3.7 \times 10^{-5} / ^\circ\text{C} \quad (60)$$

18. Chowdhury, Thesis, Montana State College, May 1961, p.4.

19. Glasstone and Edlund, The Elements of Nuclear Reactor Theory (Van Nostrand Inc., 1952), p.280.

20. Argonne National Laboratory, Reactor Physics Constants (University of Chicago Press, 1958), p. 162.

The net temperature coefficient of reactivity is $-16.2 \times 10^{-5}/^{\circ}\text{C}$.

Temperature Coefficient of The Material Buckling:

The temperature coefficient of material buckling is defined as follows

$$\alpha_{B_m^2} \equiv \frac{\Delta B_m^2}{B_m^2(T) \Delta T} = \frac{B_m^2(T_2) - B_m^2(T_1)}{(T_2 - T_1) B_m^2(T_1)} \quad (61)$$

where

$$T_2 = 35^{\circ}\text{C} \text{ and } T_1 = 13^{\circ}\text{C}$$

then,

$$\alpha_{B_m^2} = \frac{(-12.02 \times 10^{-4}) - (-12.83 \times 10^{-4})}{(35 - 13)(-12.83 \times 10^{-4})} = -2.87 \times 10^{-3}/^{\circ}\text{C} \quad (62)$$

Infinite, Effective and Subcritical Multiplication Factors:

All multiplication factors were calculated at 23°C .

$$k_{\infty} = (1 + L_H^2 B_m^2) e^{+\gamma B_m^2} = (1 - 1.77 \cdot 1.244 \times 10^{-3}) e^{-2.8 \times 1.244 \times 10^{-2}} \quad (63)$$

$$k_{\infty} = 0.965$$

$$k_{\text{eff}} = \frac{k_{\infty} e^{-\gamma B_g^2}}{1 + L_H^2 B_g^2} = \frac{0.965 e^{-2.8 \cdot 3.560 \times 10^{-2}}}{1 + 1.77 \cdot 3.56 \times 10^{-3}} \quad (64)$$

$$k_{\text{eff}} = 0.869$$

$$\text{subcritical multiplication} \equiv k_s = \frac{1}{1 - k_{\text{eff}}} = \frac{1}{1 - 0.869}$$

$$k_s = 7.64 \quad (65)$$

Change in $k_{\infty}/k_{\text{eff}}$:

In obtaining equations (37), (45), and (54), it was assumed that k_{∞} and γ did not vary with temperature. In a reactor where the moderator is liquid this is not quite true. The variation of $\frac{\delta k_{\infty}}{k_{\infty}}$ can be obtained as follows:

Assuming $\frac{\delta \gamma}{\gamma} = \frac{\delta L_H^2}{L_H^2}$, it can be shown that

$$\frac{1}{\Delta T} \left(\frac{\delta k_{\infty}}{k_{\infty}} \right) = \frac{1}{\Delta T} \left\{ \frac{1}{1 + L_H^2 B_m^2} + \frac{\gamma}{L_H^2} \right\} (B_m^2 \delta L_H^2 + L_H^2 \delta B_m^2) \quad (66)$$

also,

$$\frac{\partial \gamma}{\partial T} = \frac{1}{k_{\text{eff}}} \frac{1}{\Delta T} \left(\frac{\delta k_{\text{eff}}}{k_{\text{eff}}} \right) = \frac{1}{\Delta T} \left\{ \frac{\delta k_{\infty}}{k_{\infty}} - \delta(\gamma B_g^2) - \delta(L_H^2 B_g^2) \right\} \frac{1}{k_{\text{eff}}} \quad (67)$$

now,

$$T_1 = 13^\circ \text{ C}$$

$$T_2 = 35^\circ \text{ C}$$

$$\Delta T = 22^\circ \text{ C}$$

$$\delta B_m^2 = + 0.81 \times 10^{-4} \text{ cm.}^{-2}$$

$$\delta L_H^2 = 0.22 \text{ cm.}^2$$

$$B_m^2 = -12.83 \times 10^{-4} \text{ cm.}^{-2}$$

$$L_H^2 = 1.67 \text{ cm.}^2$$

$$\gamma = 26.4 \text{ cm.}^2 \text{ (using the above assumption).}$$

then,

$$\frac{1}{\Delta T} \left(\frac{\delta k_\infty}{k_\infty} \right) = \frac{1}{22} \left\{ \frac{1}{1 - 1.67 \cdot 1.283 \times 10^{-3}} + \frac{26.4}{1.67} \right\} (-1.283 \times 10^{-3} \cdot 0.22 + 1.67 \cdot 0.81 \times 10^{-4})$$

thus,

$$\frac{1}{\Delta T} \left(\frac{\delta k_\infty}{k_\infty} \right) = -10.6 \times 10^{-5} / ^\circ \text{ C} \quad (68)$$

No attempt was made to substitute values into equation (67)

CONCLUSION

The results of the diffusion length experiment were fairly accurate. The more conventional ways of measuring the diffusion length employ either a thermal column of a reactor or a pulsed neutron source; however, this report shows that representative values of L (for light water at least) can be gotten using a Pu-Be (α, n) neutron source. This means that for laboratory demonstrations of the diffusion length or for relative values of L , the more elaborate, costly, and sometimes inaccessible units such as the thermal column and pulsed neutron source are not needed.

In order to obtain representative values of L using a Pu-Be source, there still has to be certain requirements met. Naturally, the more thermal neutrons there are, the better will be the statistics and the better will be the values of L . A four curie neutron source ($\approx 10^7$ neutrons/sec) was used in this experiment and a source larger than this would have been more desirable. Long counting times are needed for an experiment of this type so that the statistics will be better. The direct neutron counting devices should have a fairly level plateau and the power supply should be extremely stable. These are the basic necessities needed before a diffusion length experiment of this type can be run; however, the actual performance of the experiment has to be done very carefully.

It should be noted that no correction was made for any absorption of fast neutrons by cadmium. This was not done because the experiment itself was not accurate enough to warrant a correction of this nature.

The results obtained for the temperature coefficients were as expected for a subcritical reactor of this type. The net temperature coefficient of

reactivity (assuming k_{∞} not to change with temperature) was found to be $-16.2 \times 10^{-5}/^{\circ}\text{C}$. The actual contribution to the temperature coefficient due to a change in k_{∞} was found to be $-10.6 \times 10^{-5}/^{\circ}\text{C}$. Thus, the contribution to the temperature coefficient from $1/\Delta T (\delta k_{\infty} / k_{\infty})$ is very significant for this assembly. The importance of this contribution is partly due to the fact that this assembly is light water moderated. It should be noted that the temperature coefficient depends very much on the α_T that is used in the equations. The α_T is not too well defined for a heterogeneous system and in this paper it depends very much on the density of uranium that is used and also on the temperature interval over which $\bar{\beta}_{\text{H}_2\text{O}}$ is obtained.

The results from the buckling experiment were rather unique in that the material buckling was found to be negative. This can be interpreted as meaning that an infinite array (arranged exactly as in this assembly) of similar fuel elements and a light water moderator could never go critical. The measurements of the buckling must be performed with care in order to obtain conclusive results. Again, long counting times are needed to obtain good statistics.

The radial buckling is rather hard to determine because it is somewhat difficult to obtain the extrapolated radius. Four trials were run at each temperature so that a fairly reliable average of $\widehat{R}_0$ could be obtained. It is interesting to note that the extrapolated radius varied significantly with temperature.

The vertical buckling does not require as many trials as the radial buckling to obtain a good average value, but the more observations made the

better will be this average. A background measurement with the source removed should also be included in the vertical buckling experiment. This is necessary because when finding the slope of $\ln[\text{net number of thermal neutrons}]$ versus vertical distance from the source a least square straight line is fitted to the experimental points.

The experimental value of k_{∞} (0.965) is in good agreement with the calculated value (0.960).

Generally then, the results found in this experiment may be applied to further research and some of the procedures followed could be used either as a basis for more detailed investigation or as a basis for laboratory course work.

APPENDIX I

CALCULATION OF THE VOLUME OF URANIUM AND VOLUME OF WATER USED IN THE SUBCRITICAL ASSEMBLY

The subcritical assembly contained 264 fuel tubes. In each fuel tube there were five fuel slugs. There were two different lengths of slugs used and, on the average, there were three small and two large slugs per fuel tube. A description of the fuel slugs together with a tabulation of the computations used in obtaining the volume of uranium is given below.

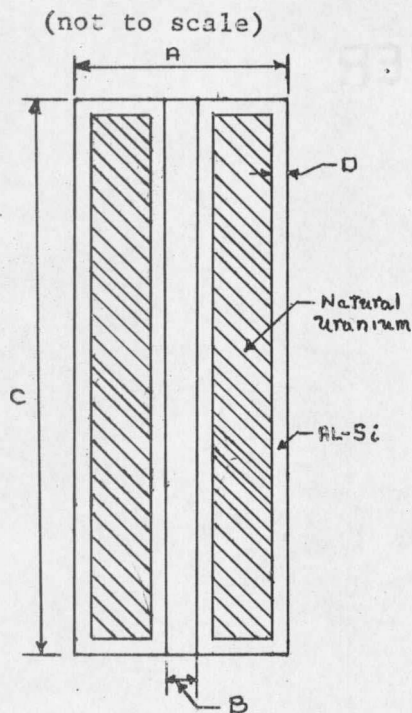


Figure 24. Diagram of fuel slug.

TABLE XV. Values and results used in obtaining the volume of uranium.

	LARGE SLUG	SMALL SLUG
A	2.90 cm.	2.90 cm.
B	0.96 cm.	0.96 cm.
C	21.35 cm.	20.35 cm.
D	0.129 cm.	0.129 cm.
Average Weight	1834 gm.	1747 gm.
No. of Slugs	528	792
Vol. of One Slug	89.8 cm. ³	85.5 cm. ³
Total Volume	4.74x10 ⁴ cm. ³	6.77x10 ⁴ cm. ³
Total Volume of URANIUM = 1.15x10 ⁵ cm. ³		
Total Weight of URANIUM = 2220 Kg.		
Density of AL-Si = 2.79 gm./cm. ³ ²⁰		
Density of URANIUM = 19.3 gm./cm. ³ ²¹		

20. Chowdhury, Thesis, Montana State College, May 1961, p. 8.

21. Etherington, Nuclear Engineering Handbook (McGraw Hill Inc., 1958), p. 10-18.

The following table contains the data used in obtaining the volume of water.

TABLE XVI. Values and results used in obtaining the volume of water.

Bare Tank	Voids*
radius 60.96 cm.	
depth of water 124.38 cm.	
volume of water $1.45 \times 10^6 \text{ cm.}^3$	Total Volume $0.26 \times 10^6 \text{ cm.}^3$
Net Volume of Water = $1.19 \times 10^6 \text{ cm.}^3$	

* Voids include any volume of material, other than water, that was present in the assembly tank; such as, fuel tubes, source holder, grid plate, experimental tube.

APPENDIX II

CALCULATION OF THE COEFFICIENT OF LINEAR EXPANSION,

The coefficient of linear expansion, α_T , used in this paper is an average value for water and natural uranium. It is defined as

$$\alpha_T = \frac{\bar{\beta}_{\text{total}}}{3} \quad (69)$$

where $\bar{\beta}_{\text{total}}$ is the average volume expansion coefficient. $\bar{\beta}_{\text{total}}$ is defined as

$$\bar{\beta}_{\text{total}} = \frac{\bar{\beta}_{\text{H}_2\text{O}} V_{\text{H}_2\text{O}} + \bar{\beta}_{\text{U}^{238}} V_{\text{U}^{238}}}{V_{\text{H}_2\text{O}} + V_{\text{U}^{238}}} \quad (70)$$

where

$$\begin{aligned} V_{\text{H}_2\text{O}} &= 1.19 \times 10^6 \text{ cm}^3 & \bar{\beta}_{\text{U}^{238}} &= 4.58 \times 10^{-5} / ^\circ\text{C} \\ V_{\text{U}^{238}} &= 1.15 \times 10^5 \text{ cm}^3 & \bar{\beta}_{\text{H}_2\text{O}} &= 27.2 \times 10^{-5} / ^\circ\text{C} \end{aligned}$$

$\bar{\beta}_{\text{H}_2\text{O}}$ is averaged over the temperature range of 10°C to 45°C and is calculated as follows:

$$\bar{\beta}_{\text{H}_2\text{O}} = \frac{V_{T_2} - V_{T_1}}{\bar{V}_{T_2, T_1}} \cdot \frac{1}{T_2 - T_1} \quad \text{where} \quad \begin{aligned} T_2 &= 45^\circ\text{C} \\ T_1 &= 10^\circ\text{C} \end{aligned} \quad (71)$$

at

$$\begin{aligned} 10^\circ\text{C} &\rightarrow V = 1.00027 \\ 45^\circ\text{C} &\rightarrow V = 1.00985 \end{aligned} \quad \text{then } \bar{V} = 1.00506$$

$$\bar{\beta}_{\text{H}_2\text{O}} = \frac{1.00985 - 1.00027}{1.00506} \cdot \frac{1}{35} = 27.2 \times 10^{-5} / ^\circ\text{C}$$

$$\bar{\beta}_{\text{total}} = \frac{27.2 \times 10^{-5} \cdot 11.9 \times 10^5 + 4.58 \times 10^{-5} \cdot 1.15 \times 10^5}{11.9 \times 10^5 + 1.15 \times 10^5}$$

$$\bar{\beta}_{\text{total}} = 25.2 \times 10^{-5} / ^\circ\text{C}$$

or

$$\alpha_T = 8.4 \times 10^{-5} / ^\circ\text{C}$$

APPENDIX III

TYPICAL CALCULATION

OF

LEAST SQUARE STRAIGHT LINE FIT FOR EXPERIMENTAL POINTS

Vertical Buckling - Temperature = 35°C ± 1°C - Radial Distance = 17.44 cm.

distance from source (cm.)	actual reading	$x_{obs.}$	$y_{obs.}$	$\ln y_{obs.}$	dev. = $\ln y_{obs.} - mx_{obs.} - b$
30.86	31.00	0.00	1,555.8	7.34974	7.34974-b.
38.48	28.00	3.00	944.0	6.85013	6.85013-3m-b.
46.10	25.00	6.00	594.5	6.38772	6.38772-6m-b.
53.72	22.00	9.00	366.1	5.90293	5.90293-9m-b.
61.34	19.00	12.00	227.2	5.42583	5.42583-12m-b.
68.96	16.00	15.00	145.6	4.98087	4.98087-15m-b.

$$f(m,b) = \sum dev.^2 = \text{constant} - 73.79444b + 6b^2 - 503.65218m + 495m^2 - 90mb.$$

$$\frac{\partial f(m,b)}{\partial b} = -73.79444 + 12b + 90m = 0$$

$$\frac{\partial f(m,b)}{\partial m} = -503.65218 + 990m + 90b = 0$$

$$m = -0.1581146666$$

$$b = 7.335396665$$

$$y = -0.1581146666x + 7.335396665$$

$$r^2 = 38.75 \times 10^{-4} \text{ cm.}^{-2}$$

APPENDIX IV

SYMBOLS USED

B^2	= buckling cm. ⁻²
B_{θ}^2	= radial buckling cm. ⁻²
B_v^2	= vertical buckling cm. ⁻²
D	= diffusion coefficient cm.
f	= thermal utilization
K_{∞}	= infinite multiplication
K_{eff}	= effective multiplication
L	= diffusion length of the moderator
L_H	= diffusion length of heterogeneous system
M^2	= migration area
p	= resonance escape probability
$\vec{r}$	= position vector
Σ_a	= macroscopic absorption cross section
σ_a	= microscopic absorption cross section
ϕ	= neutron flux n/cm. ² /sec.
$\bar{R}_0$	= extrapolated radius
R_0	= actual radius
$\bar{H}_0$	= extrapolated height
H_0	= actual height
λ_{tr}	= transport mean free path
α	= coefficient of linear expansion
τ	= Fermi age
d	= density

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