



Analysis for three-dimensional pipe structures by group relaxation  
by Nicholas Bassar

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering at Montana State College  
Montana State University  
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Abstract:

The group relaxation procedure for determining moments and reactions at the fixed ends of a three-dimensional pipe structure subjected to expansion resulting from temperature change utilizes principles of the neutral point method and the moment distribution process. The given pipe structure is subdivided into groups or system such that each system has one terminal that is common to all systems and the other terminal a fixed end. This subdivision has the advantage of requiring only the control of  $S$  degrees of freedom of the common terminal during the solution. Before permitting expansion the common terminal is restrained against rotation in each plane and against translation in each direction. When expansion occurs, moments throughout the structure, end reactions and restraints at the common terminal are induced.

The restraints at joint B may be released by either of two methods as follows: (1) By first allowing rotational equilibrium to occur and then releasing the restraints against translation. (2) By releasing the restraints against translation first and then allowing rotational equilibrium to occur\* but balancing shears each time the common terminal rotates.

In Appendix II the six neutral point equations are derived and are used to provide the following items needed to perform the moment distribution process and to eliminate the restraints against translation at the common terminal: (a) Distribution factors (b) Carry-over factors (c) Shear correction factors (d) Forces required to produce a unit translation of the common terminal For a pipe structure with from 1 to 4 degrees of freedom for translation it is probably more advantageous to use Method I. If there are more than 4 degrees of freedom for translation Method II appears to have merit\*

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PIPE STRUCTURES BY  
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*g. of the graduate committee*

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### ABSTRACT

The group relaxation procedure for determining moments and reactions at the fixed ends of a three-dimensional pipe structure subjected to expansion resulting from temperature change utilizes principles of the neutral point method and the moment distribution process. The given pipe structure is subdivided into groups or systems such that each system has one terminal that is common to all systems and the other terminal a fixed end. This subdivision has the advantage of requiring only the control of 3 degrees of freedom of the common terminal during the solution. Before permitting expansion, the common terminal is restrained against rotation in each plane and against translation in each direction. When expansion occurs, moments throughout the structure, end reactions and restraints at the common terminal are induced.

The restraints at joint B may be released by either of two methods as follows: (1) By first allowing rotational equilibrium to occur and then releasing the restraints against translation, (2) By releasing the restraints against translation first and then allowing rotational equilibrium to occur, but balancing shears each time the common terminal rotates.

In Appendix II the six neutral point equations are derived and are used to provide the following items needed to perform the moment distribution process and to eliminate the restraints against translation at the common terminal:

- (a) Distribution factors
- (b) Carry-over factors
- (c) Shear correction factors
- (d) Forces required to produce a unit translation of the common terminal

For a pipe structure with from 1 to 4 degrees of freedom for translation it is probably more advantageous to use Method I. If there are more than 4 degrees of freedom for translation Method II appears to have merit.

## INTRODUCTION

### Object

The primary objective of this thesis is to present a group relaxation procedure for determining moments and forces at the fixed ends of a three-dimensional pipe structure with 3 or more fixed ends subjected to expansion resulting from temperature change. A comparison of two methods by which the relaxation is executed is the secondary objective.

### Previous Investigation

Two available methods of evaluating the moments and forces at the fixed ends of a three-dimensional pipe structure with three fixed ends require the solution of a minimum of six<sup>1</sup> and a maximum of twelve<sup>2</sup> simultaneous equations.

The method involving six simultaneous equations utilizes the moment area principles and the principles of the neutral point method.<sup>3</sup> If the structure has more than 3 fixed ends, the number of simultaneous equations to be solved increases.

1. "Simplified Method of Analysis of Reactions Developed by Expansion in a Three-Anchor Piping System," by Boris Lochak, A.S.M.E. Trans., Vol. 66, 1944, pp. 311-318
2. "Design of Piping Systems," published by M. W. Kellogg Co., New York, N. Y., 1941.
3. "Theory of Modern Steel Structures," Vol. 2, by L. E. Grinter, published by the Macmillan Co., N. Y. pp. 206-210

The equations of the general method of indeterminate structures<sup>4</sup> along with the virtual work principles<sup>5</sup> are applied in the method involving twelve simultaneous equations. This method requires that the structure be cut back to a statically determinate one; then knowing that the freed ends cannot translate or rotate, the displacement due to expansion is eliminated with the aid of virtual work principles.

A third method<sup>6</sup> makes use of the moment distribution<sup>7</sup> process. Each member of the structure is allowed to expand while restrained against rotation. Then each joint is allowed to rotate with translation prohibited. After a joint rotates, the restraint against translation is released. This procedure of alternately allowing a joint to rotate and then the entire structure to translate will finally result in a condition whereby there will be no further tendency for translation or rotation when the joints are released, indicating that the structure has reached equilibrium.

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4. Ibid pp. 74-75

5. Ibid pp. 35-41

6. Discussion of "Moment-Distribution Analysis for Three Dimensional Pipe Structures," by R. C. DeHart, A.S.M.E. Trans., Vol. 66, 1944, pp. A240-A244

7. "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan published by John Wiley and Sons, Inc., N. Y. pp. 81-125

Importance

The group relaxation procedure requires the solution of only 3 simultaneous equations at any one time and thereby reduces the tedious task of solving six or more simultaneous equations as presented by two of the available methods. A physical picture of the effect of joint rotation and joint translation on the moments and on the forces at the fixed ends is presented by applying the group relaxation procedure which is of assistance to the engineer analyzing the structure.

While the solution given is for square corners, the method can be applied to pipe structures with bends at the corners. When quarter bends are a part of the pipe structure, their lengths must be modified when computing the centroid of an orthogonal projection and when computing the moments of inertia and products of inertia to account for the added flexibility due to flattening of the curved section when subjected to bending. A three-dimensional pipe structure with two fixed ends containing circular quarter bends has been solved by the neutral point method by S. W. Spielvogel.<sup>8</sup> The use of quarter bends reduces stress concentrations at the corners.

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8. "Stress Calculation for High Temperature Piping-II," by S. W. Spielvogel, Power, February 1941, pp. 67-69

### PROCEDURE

The pipe structure shown in Fig. 1<sup>9</sup> is subdivided into groups or systems with B being one terminal of each system and a fixed end the other terminal. These systems are AB, EDB, and GFCE. Joint B is restrained against rotation in each plane and against translation in the Y and Z directions before expansion is permitted. The effect of direct stress relative to elongation or contraction is to be neglected in this analysis; therefore AB must be allowed to expand. This permanently displaces joint B in the X direction by the amount of expansion of AB. After AB expands, it serves as a permanent restraint in the X direction.

For each orthogonal projection of the elastic areas,  $ds/EI$ , of each complex system as EDB and GFCE, the centroid or the neutral point is evaluated. For a member that appears as a point in an orthogonal projection, an equivalent length is used since in this plane the member acts in torsion rather than flexure. The equivalent length is equal to the product of the actual length of the member and a constant. This constant is equal to the ratio of  $EI$  to  $GJ$ . The coordinate axes pass through the neutral point. The moment

---

9. The pipe structure used is the same as the one given in closure to discussion of paper, "Moment Distribution Analysis for Three-Dimensional Pipe Structures," by R. C. DeHart, Journal of Applied Mechanics, A.S.M.E. Trans., Vol. 67, 1945, pp. A-188

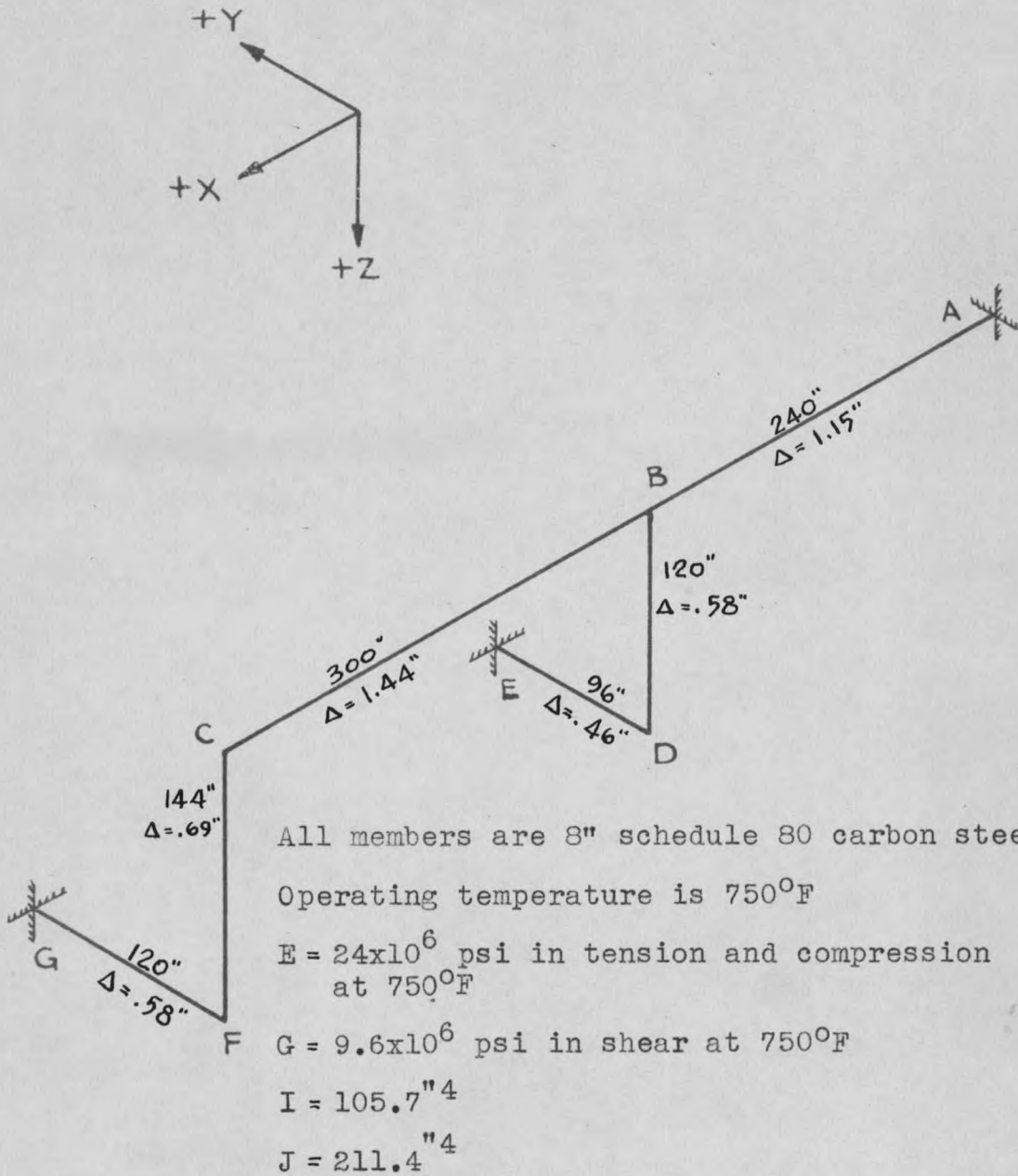


Fig. 1 Three-Dimensional Pipe Structure to be Analyzed

of inertia and the product of inertia of the elastic areas about the axes through the neutral point are computed and tabulated in Tables I and II. Since  $EI$  is constant in this problem and since it is in the denominator, multiplying equations 3b through 5b shown in Appendix II by  $EI$  removes this denominator. The terms in Table I and II are of this form.

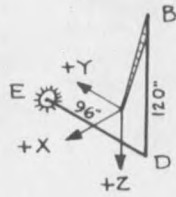
#### Sign Convention

The sign for translation is plus if the movement is in the positive direction of the axes as indicated in Tables I and II. The sign for a force is plus if it acts in the positive direction of the axes. Moment in a plane is positive if the action of the joint on the member is clockwise when the plane is viewed as indicated in Tables I and II.

#### Unit Translation Effect

The translation of the neutral point in one of the coordinate directions implies that joint B also translates in the same direction and the same amount, but it is restrained against translation in the other two directions and restrained against rotation in each plane. The neutral point forces  $F_x$ ,  $F_y$  and  $F_z$  corresponding to a positive unit translation in the X direction of the neutral point of system GFGB are computed from the neutral point equations 3b through 5b shown in Appendix II:

TABLE I: LOCATION OF NEUTRAL POINT AND ELASTIC PROPERTIES OF EDB SYSTEM



PROJECTION IN

X-Y PLANE	X-Z PLANE	Y-Z PLANE

CENTROID

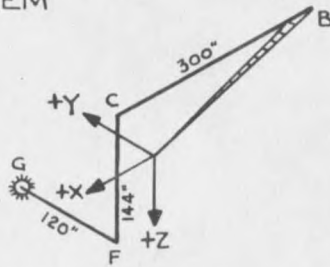
	L	Y	LY	X"	LX"		L	Z	LZ	X"	LX"		L	Z"	LZ"	Y"	LY"
ED	96	48	4608	0	0	ED	125x96	0	0	0	0	ED	96	0	0	48	4608
DB	$\frac{125 \times 120}{246}$	0	0	0	0	DB	$\frac{120}{240}$	-60	-7200	0	0	DB	$\frac{120}{216}$	-60	-7200	0	0
			4608						-7200						-7200		4608
			$\frac{4608}{246} = 18.7"$						$\frac{-7200}{240} = -30"$						$\frac{-7200}{216} = -33.3"$		$\frac{4608}{216} = 21.3"$

$I_{xy}$		$I_{xz}$		$I_{yz}$	
ED	$96 \times 29.3 \times 0 = 0$	ED	$125 \times 96 \times 30 \times 0 = 0$	ED	$96 \times 33.3 \times 26.7 = .854 \times 10^5$
DB	$125 \times 120 \times (-18.7) \times 0 = 0$	DB	$120 \times 30 \times 0 = 0$	DB	$120 \times (-26.7) \times (-21.3) = .682 \times 10^5$
					$1.536 \times 10^5$

$I'_{xx}$		$I_{xx}$		$I_{yy}$	
ED	$\frac{1}{12} \times 96^3 + 96 \times 29.3^2 = 1.561 \times 10^5$	ED	$125 \times 96 \times 30^2 = 1.080 \times 10^5$	ED	$96 \times 33.3^2 = 1.065 \times 10^5$
DB	$125 \times 120 \times 18.7^2 = .525 \times 10^5$	DB	$\frac{1}{12} \times 120^3 + 120 \times 30^2 = \frac{2.520 \times 10^5}{3.600 \times 10^5}$	DB	$\frac{1}{12} \times 120^3 + 120 \times 26.7^2 = \frac{2.295 \times 10^5}{3.360 \times 10^5}$
	$2.086 \times 10^5$				

$I'_{yy}$		$I_{zz}$		$I'_{zz}$	
ED	NEGLECT	ED	NEGLECT	ED	$\frac{1}{12} \times 96^3 + 96 \times 26.7^2 = 1.421 \times 10^5$
DB	NEGLECT	DB	NEGLECT	DB	$120 \times 21.3^2 = .544 \times 10^5$
					$1.965 \times 10^5$

TABLE II: LOCATION OF NEUTRAL POINT AND ELASTIC PROPERTIES OF GFCB SYSTEM



PROJECTION IN

X-Y PLANE	X-Z PLANE	Y-Z PLANE

CENTROID

	L	Y"	LY"	X"	LX"		L	Z"	LZ"	X"	LX"		L	Z"	LZ"	Y"	LY"	
GF	120	60	7200	0	0	GF	125+120	0	0	0	0	GF	120	0	0	60	7200	
FC	125+144	0	0	0	0	FC	144	72	10368	0	0	FC	144	72	10368	120	17280	
CB	300	0	0	150	45000	CB	300	144	43200	150	45000	CB	125+300	144	54000	120	45000	
	600		7200		45000		594		53568		45000		639		64368		69480	
	$\frac{7200}{600} = 12"$			$\frac{45000}{600} = 75"$			$\frac{53568}{594} = 90.2"$			$\frac{45000}{594} = 75.8"$			$\frac{64368}{639} = 100.7"$			$\frac{69480}{639} = 108.7"$		
$I_{XY}$					$I_{XZ}$					$I_{YZ}$								
GF	120	75	48	$= 4.320 \times 10^5$		GF	125	120	75.8	90.2	$= 10.26 \times 10^5$		GF	120	48.7	100.7	$= 5.885 \times 10^5$	
FC	125	144	75	$\times (-12) = -1.620 \times 10^5$		FC	144	75.8	18.2	$= 1.97 \times 10^5$		FC	144	72	11.3	$\times 28.7 = -4.67 \times 10^5$		
CB	300	0	75	$\times (-12) = 2.700 \times 10^5$		CB	300	0	74.2	$\times (-53.8) = -1.98 \times 10^5$		CB	125	300	0	$\times (-43.3) = -1.835 \times 10^5$		
				$5.400 \times 10^5$						$24.21 \times 10^5$						$7253 \times 10^5$		
$I'_{XX}$					$I_{XX}$					$I_{YY}$								
GF	$\frac{1}{12}$	120	3	$+ 120 \times 48^2 = 4.205 \times 10^5$		GF	125	120	90.2	$^2 = 12.204 \times 10^5$		GF	120	100.7	$^2 = 12.169 \times 10^5$			
FC	$\frac{1}{12}$	125	144	$\times 12^2 = .259 \times 10^5$		FC	$\frac{1}{12}$	144	144	$\times 18.2^2 = 2.965 \times 10^5$		FC	$\frac{1}{12}$	144	$\times 144 \times 28.7^2 = 3.674 \times 10^5$			
CB	$\frac{1}{12}$	300	12	$= .432 \times 10^5$		CB	$\frac{1}{12}$	300	53.8	$= 8.683 \times 10^5$		CB	125	300	$\times 43.3^2 = 7.031 \times 10^5$			
				$4.896 \times 10^5$						$23.852 \times 10^5$					$22.874 \times 10^5$			
$I'_{YY}$					$I_{ZZ}$					$I'_{ZZ}$								
GF	120	75	$^2 = 6.750 \times 10^5$		GF	125	120	75.8	$^2 = 8.618 \times 10^5$		GF	$\frac{1}{12}$	120	$\times 120 \times 48.7^2 = 4.286 \times 10^5$				
FC	125	144	$\times 75^2 = 10.125 \times 10^5$		FC	144	75.8	$^2 = 8.274 \times 10^5$		FC	144	11.3	$^2 = .184 \times 10^5$					
CB	$\frac{1}{12}$	300	$\times 300^3 + 300 \times 75^2 = 39.375 \times 10^5$		CB	$\frac{1}{12}$	300	$\times 300 \times 74.2^2 = 39.017 \times 10^5$		CB	125	300	$\times 11.3^2 = .479 \times 10^5$					
			$56.250 \times 10^5$					$55.909 \times 10^5$					$4.949 \times 10^5$					

$$5c) \quad -EI + 28.748 \times 10^5 F_x - 5.40 \times 10^5 F_y - 24.21 \times 10^5 F_z = 0$$

$$6c) \quad + 5.40 \times 10^5 F_x + 79.124 \times 10^5 F_y - 7.253 \times 10^5 F_z = 0$$

$$7c) \quad - 24.21 \times 10^5 F_x - 7.253 \times 10^5 F_y + 60.858 \times 10^5 F_z = 0$$

Similarly the neutral point forces corresponding to positive unit translation of the neutral point in the Y and Z directions are found. These are tabulated along with similar translations for system EDB in Tables V and VI. Determinants provide the best solution for this type of work since the denominator determinant is always the same for a given system. The denominator determinants are evaluated in Tables III and IV for systems EDB and GFGB respectively. System AB is a single member and is treated as such.

In Fig. 2 system GFGB is shown with the neutral point forces required to produce a positive unit translation of the neutral point in the X direction. The moment at B in the XY plane produced by this translation is:

$$M_{xy} = (.0058 \times 10^{-5} EI\#)(225") - (.0548 \times 10^{-5} EI\#)(12") = 16420\#\#$$

This moment is clockwise and it is the moment which joint B must exert on the system at B to prevent rotation. The absolute values of the lever arms used are obtained from Table II. The moment at G in the XY plane is:

$$M_{xy} = (.0548 \times 10^{-5} EI\#)(108") - (.0058 \times 10^{-5} EI\#)(75") = 139000\#\#$$

TABLE III : Value of Denominator Determinant for EDB System

$+(I_{xx} + I'_{xx})(I_{yy} + I'_{yy})(I_{zz} + I'_{zz})$	$5.686 \times 10^5 \times 3.360 \times 10^5 \times 1.965 \times 10^5$	$+37.532 \times 10^{15}$
$-(-I_{yz})^2 (I_{xx} + I'_{xx})$	$-(-1.536 \times 10^5)^2 \times 5.686 \times 10^5$	$-13.419 \times 10^{15}$
$-(-I_{xy})^2 (I_{zz} + I'_{zz})$	0	
$+2(-I_{xy})(-I_{xz})(-I_{yz})$	0	
$-(-I_{xz})^2 (I_{yy} + I'_{yy})$	0	
	$D_D =$	$+24.113 \times 10^{15}$

TABLE IV : Value of Denominator Determinant for GFCS System

$+(I_{xx} + I'_{xx})(I_{yy} + I'_{yy})(I_{zz} + I'_{zz})$	$28.748 \times 10^5 \times 79.124 \times 10^5 \times 60.858 \times 10^5$	$+138.431 \times 10^{18}$
$-(-I_{yz})^2 (I_{xx} + I'_{xx})$	$-(-7.253 \times 10^5)^2 \times 28.748 \times 10^5$	$- 1.512 \times 10^{18}$
$-(-I_{xy})^2 (I_{zz} + I'_{zz})$	$-(-5.40 \times 10^5)^2 \times 60.858 \times 10^5$	$- 1.775 \times 10^{18}$
$+2(-I_{xy})(-I_{xz})(-I_{yz})$	$2(-5.40 \times 10^5)(-24.21 \times 10^5)(-7.253 \times 10^5)$	$- 1.896 \times 10^{18}$
$-(-I_{xz})^2 (I_{yy} + I'_{yy})$	$-(-24.21 \times 10^5)^2 \times 79.124 \times 10^5$	$- 46.376 \times 10^{18}$
	$D_D =$	$+ 86.872 \times 10^{18}$

TABLE V: NEUTRAL POINT FORCES REQUIRED FOR A UNIT TRANSLATION OF NEUTRAL POINT OF EDB SYSTEM IN +X DIRECTION +Y DIRECTION +Z DIRECTION

SOLVING FOR NUMERATOR DETERMINANT (D<sub>x</sub>)

$\begin{aligned} & +(-\Delta X EI)(I_{YY}+I'_{YY})(I_{ZZ}+I'_{ZZ}) \\ & -(-\Delta X EI)(-I_{YZ})^2 \\ & -(-\Delta Y EI)(-I_{XY})(I_{ZZ}+I'_{ZZ}) \\ & +(-\Delta Y EI)(-I_{XZ})(-I_{YZ}) \\ & -(-\Delta Z EI)(-I_{XZ})(I_{YY}+I'_{YY}) \\ & +(-\Delta Z EI)(-I_{XY})(-I_{YZ}) \end{aligned}$	$\begin{aligned} & EI \times 3.360 \times 10^5 \times 1.965 \times 10^5 \\ & -EI \times (-1.536 \times 10^5)^2 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$	$\begin{aligned} & +6.602 \times 10^{10} EI \\ & -2.359 \times 10^{10} EI \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$
$D_x$	$+4.243 \times 10^{10} EI$	$D_x \quad 0$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>y</sub>)

$\begin{aligned} & -(-\Delta X EI)(-I_{XY})(I_{ZZ}+I'_{ZZ}) \\ & +(-\Delta X EI)(-I_{XZ})(-I_{YZ}) \\ & +(-\Delta Y EI)(I_{XX}+I'_{XX})(I_{ZZ}+I'_{ZZ}) \\ & -(-\Delta Y EI)(-I_{XZ})^2 \\ & -(-\Delta Z EI)(-I_{YZ})(I_{XX}+I'_{XX}) \\ & +(-\Delta Z EI)(-I_{XY})(-I_{XZ}) \end{aligned}$	$\begin{aligned} & 0 \\ & 0 \\ & EI \times 5.686 \times 10^5 \times 1.965 \times 10^5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$	$\begin{aligned} & 0 \\ & 0 \\ & +11.172 \times 10^{10} EI \\ & 0 \\ & 0 \\ & -EI \times (-1.536 \times 10^5) \times 5.686 \times 10^5 \\ & 0 \end{aligned}$
$D_y$	$0$	$D_y \quad +11.172 \times 10^{10} EI$
		$D_y \quad +8.733 \times 10^{10} EI$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>z</sub>)

$\begin{aligned} & -(-\Delta X EI)(-I_{XZ})(I_{YY}+I'_{YY}) \\ & +(-\Delta X EI)(-I_{XY})(-I_{YZ}) \\ & -(-\Delta Y EI)(-I_{YZ})(I_{XX}+I'_{XX}) \\ & +(-\Delta Y EI)(-I_{XY})(-I_{XZ}) \\ & +(-\Delta Z EI)(I_{XX}+I'_{XX})(I_{YY}+I'_{YY}) \\ & -(-\Delta Z EI)(-I_{XY})^2 \end{aligned}$	$\begin{aligned} & 0 \\ & 0 \\ & -EI \times (-1.536 \times 10^5) \times 5.686 \times 10^5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$	$\begin{aligned} & 0 \\ & 0 \\ & +8.733 \times 10^{10} EI \\ & 0 \\ & EI \times 5.686 \times 10^5 \times 3.360 \times 10^5 \\ & 0 \end{aligned}$
$D_z$	$0$	$D_z \quad +8.733 \times 10^{10} EI$
		$D_z \quad +19.105 \times 10^{10} EI$

NEUTRAL POINT FORCES

$F_x = D_x \div D_D$	$F_x = 4.243 \times 10^{10} EI \div 24.113 \times 10^{15} = .176 \times 10^{-5} EI$	$F_x = 0$
$F_y = D_y \div D_D$	$F_y = 0$	$F_y = 8.733 \times 10^{10} EI \div 24.113 \times 10^{15} = .362 \times 10^{-5} EI$
$F_z = D_z \div D_D$	$F_z = 0$	$F_z = 19.105 \times 10^{10} EI \div 24.113 \times 10^{15} = .792 \times 10^{-5} EI$

TABLE VI: NEUTRAL POINT FORCES REQUIRED FOR A UNIT TRANSLATION OF NEUTRAL POINT OF GFCB

SYSTEM IN +X DIRECTION +Y DIRECTION +Z DIRECTION

SOLVING FOR NUMERATOR DETERMINANT (D<sub>x</sub>)

$+(\Delta XEI)(I_{YY}+I'_{YY})(I_{ZZ}+I'_{ZZ})$	$EI \times 79.124 \times 10^5 \times 60.858 \times 10^5$	$+4.815 \times 10^{13} EI$	0	0	0
$-(-\Delta XEI)(-I_{YZ})^2$	$-EI \times (-7.253 \times 10^5)^2$	$-.053 \times 10^{13} EI$	0	0	0
$-(-\Delta YEI)(-I_{XY})(I_{ZZ}+I'_{ZZ})$	0		$-EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$+0.329 \times 10^{13} EI$	0
$+(\Delta YEI)(-I_{XZ})(-I_{YZ})$	0		$EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$+0.176 \times 10^{13} EI$	0
$-(-\Delta ZEI)(-I_{XZ})(I_{YY}+I'_{YY})$	0		0		$-EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$
$+(\Delta ZEI)(-I_{XY})(-I_{YZ})$	0		0		$EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$
		$D_x +4.762 \times 10^{13} EI$		$D_x +.505 \times 10^{13} EI$	$D_x \frac{+1.916 \times 10^{13} EI + .039 \times 10^{13} EI}{+1.955 \times 10^{13} EI}$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>y</sub>)

$-(-\Delta XEI)(-I_{XY})(I_{ZZ}+I'_{ZZ})$	$-EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$+0.329 \times 10^{13} EI$	0	0	0
$+(\Delta XEI)(-I_{XZ})(-I_{YZ})$	$EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$+0.176 \times 10^{13} EI$	0	0	0
$+(\Delta YEI)(I_{XX}+I'_{XX})(I_{ZZ}+I'_{ZZ})$	0		$EI \times 28.748 \times 10^5 \times 60.858 \times 10^5$	$+1.750 \times 10^{13} EI$	0
$-(-\Delta YEI)(-I_{XZ})^2$	0		$-EI \times (-24.21 \times 10^5)^2$	$-.586 \times 10^{13} EI$	0
$-(-\Delta ZEI)(-I_{YZ})(I_{XX}+I'_{XX})$	0		0		$-EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$
$+(\Delta ZEI)(-I_{XY})(-I_{XZ})$	0		0		$EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$
		$D_y +.505 \times 10^{13} EI$		$D_y +1.164 \times 10^{13} EI$	$D_y \frac{+2.08 \times 10^{13} EI + 1.31 \times 10^{13} EI}{+3.39 \times 10^{13} EI}$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>z</sub>)

$-(-\Delta XEI)(-I_{XZ})(I_{YY}+I'_{YY})$	$-EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$	$+1.916 \times 10^{13} EI$	0	0	0
$+(\Delta XEI)(-I_{XY})(-I_{YZ})$	$EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$	$+0.039 \times 10^{13} EI$	0	0	0
$-(-\Delta YEI)(-I_{YZ})(I_{XX}+I'_{XX})$	0		$-EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$	$+0.209 \times 10^{13} EI$	0
$+(\Delta YEI)(-I_{XY})(-I_{XZ})$	0		$EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$	$+0.131 \times 10^{13} EI$	0
$+(\Delta ZEI)(I_{XX}+I'_{XX})(I_{YY}+I'_{YY})$	0		0		$EI \times 28.748 \times 10^5 \times 79.124 \times 10^5$
$-(-\Delta ZEI)(-I_{XY})^2$	0		0		$-EI \times (-5.40 \times 10^5)^2$
		$D_z +1.955 \times 10^{13} EI$		$D_z +.340 \times 10^{13} EI$	$D_z \frac{+2.275 \times 10^{13} EI - .029 \times 10^{13} EI}{+2.246 \times 10^{13} EI}$

NEUTRAL POINT FORCES

$F_x = D_x \div D_D$	$F_x = 4.762 \times 10^{13} EI \div 86.872 \times 10^{18} = .0548 \times 10^{-5} EI$	$F_x = .505 \times 10^{13} EI \div 86.872 \times 10^{18} = .0058 \times 10^{-5} EI$	$F_x = 1.955 \times 10^{13} EI \div 86.872 \times 10^{18} = .0225 \times 10^{-5} EI$
$F_y = D_y \div D_D$	$F_y = .505 \times 10^{13} EI \div 86.872 \times 10^{18} = .0058 \times 10^{-5} EI$	$F_y = 1.164 \times 10^{13} EI \div 86.872 \times 10^{18} = .0134 \times 10^{-5} EI$	$F_y = .339 \times 10^{13} EI \div 86.872 \times 10^{18} = .0039 \times 10^{-5} EI$
$F_z = D_z \div D_D$	$F_z = 1.955 \times 10^{13} EI \div 86.872 \times 10^{18} = .0225 \times 10^{-5} EI$	$F_z = .340 \times 10^{13} EI \div 86.872 \times 10^{18} = .0039 \times 10^{-5} EI$	$F_z = 2.246 \times 10^{13} EI \div 86.872 \times 10^{18} = .0259 \times 10^{-5} EI$

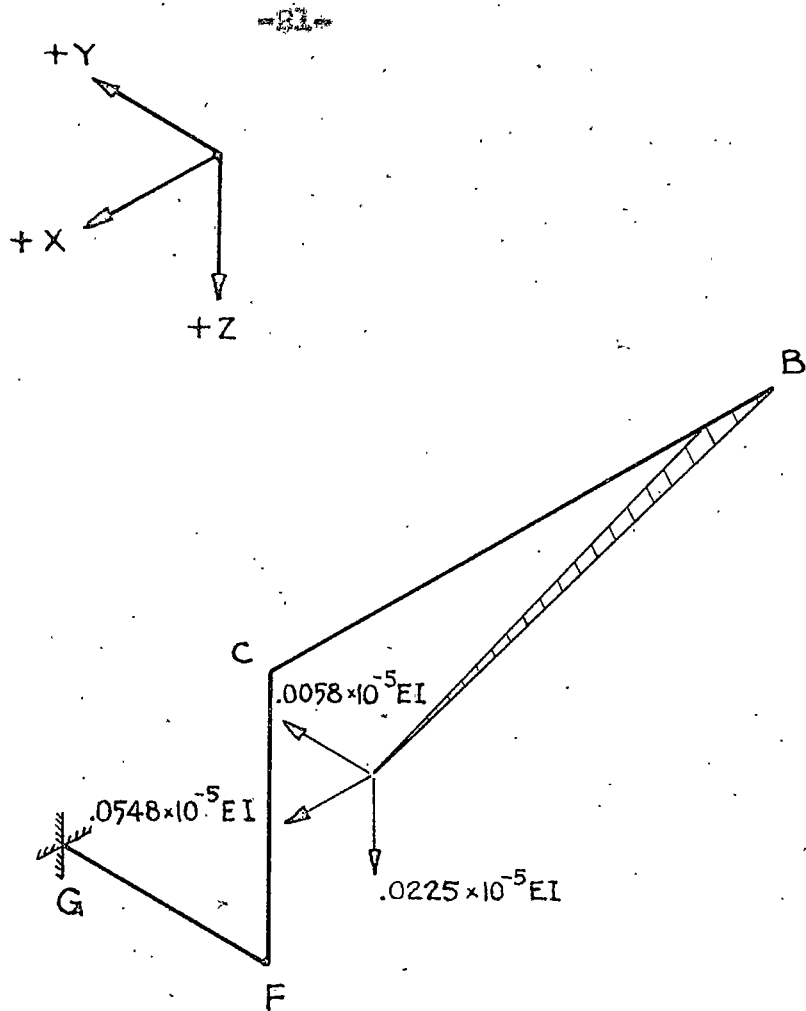


Fig. 2 Neutral Point Forces Required to Produce a Unit Translation of Neutral Point in Positive X Direction

This moment is also clockwise but it is the action of the member on the joint; therefore the action of the joint on the member is counterclockwise or a negative moment. In Table VIII the fixed end moments in each plane at B and at the fixed end are tabulated for each complex system. These moments are due to a positive unit translation of the neutral point. For system AB, the moments at B and A, due to a positive unit translation of B are evaluated from the formula

$$M = 6EI\Delta/L^2.$$

#### Expansion Forces and Moments

With the neutral point forces corresponding to unit translations of the neutral point, the reactions at G and E due to actual expansion are determined in Table VII. When member AB expands, the terminal B of each complex system must translate; however, when expansion takes place within a complex system, the terminal B can be restrained against translation. This occurs since the complex system can take up the change in length of the members by flexure while the single member system cannot. The restraints at B are equal but of opposite sign to the sum of the corresponding forces shown in Table VII. The only reaction at A is in the X direction and by the Laws of statics it is equal but of opposite sign to the sum of the forces in the X direction in Table VII.

With the moments for unit translation in Table VIII, it

TABLE VII : End Reactions in Pounds

	Reactions Due To Expansion			Corrections Due To					Final Reactions
				Rotation*			Translation*		
At A	$\Delta X = 0$	$\Delta Y = 0$	$\Delta Z = 0$	$M_{xy} = +100$	$M_{xz} = -83800$	$T_{yz} = -57500$	$\Delta Y F^{\Delta Y=1}$	$\Delta Z F^{\Delta Z=1}$	
$F_x$	+8730	+ 85	+ 394	- 1	- 690	neglect	- 39	- 88	+8391
$F_y$	0	0	0	neglect	0	0	+ 546	+ 33	+ 579
$F_z$	0	0	0	0	+ 524	0	+ 45	+1126	+1695
At E	$\Delta X = +1.15''$	$\Delta Y = +.46''$	$\Delta Z = +.58''$	$T_{xy} = +100$	$M_{xz} = -92600$	$M_{yz} = -507700$	$\Delta Y F^{\Delta Y=1}$	$\Delta Z F^{\Delta Z=1}$	
$F_x$	-5130	0	0	+ 1	+ 796	0	- 28	- 295	-4656
$F_y$	0	-5400	-5330	0	0	+5550	+1498	+3490	- 192
$F_z$	0	-4220	-11650	0	0	+2490	+2190	+11210	+ 20
At G	$\Delta X = +2.59''$	$\Delta Y = +.58''$	$\Delta Z = +.69''$	$M_{xy} = 0$	$M_{xz} = -54500$	$T_{yz} = -31000$	$\Delta Y F^{\Delta Y=1}$	$\Delta Z F^{\Delta Z=1}$	
$F_x$	-3600	- 85	- 394	0	- 106	neglect	+ 67	+ 383	-3735
$F_y$	- 381	- 197	- 68	0	- 28	+ 92	+ 164	+ 28	- 390
$F_z$	-1478	- 57	- 453	0	- 231	- 21	+ 48	+ 490	-1702

\* Rotation effect given in terms of moment in inch pounds.

**Table VIII: Moments in Inch Pounds at Fixed Ends of Each Complex System Due to a Positive Unit Translation of Neutral Point**

			$\Delta X = +1"$	$\Delta Y = +1"$	$\Delta Z = +1"$
System EDB	At B	$T_{xy}$	- 83500	0	0
		$M_{xz}$	+402000	0	0
		$M_{yz}$	0	+823000	+368000
	At E	$M_{xy}$	-345000	0	0
		$T_{xz}$	+134000	0	0
		$M_{yz}$	0	+295000	-1195000
System GFCB	At B	$M_{xy}$	+ 16420	+ 74700	+ 15420
		$M_{xz}$	- 53200	+ 14270	-116600
		$T_{yz}$	- 81	+ 13600	- 3140
	At G	$M_{xy}$	-139000	+ 9610	- 54200
		$T_{xz}$	+ 82200	+ 5770	+ 1675
		$M_{yz}$	- 47200	+ 23500	- 61400

**Table IX: Fixed End Moments in Inch Pounds Due to Expansion**

Plane	System GFCB		System EDB	
	At B	At G	At B	At E
X-Y	+ 96500	-392500	- 96000	-396900
X-Z	+226500	+217300	+462100	+154000
Y-Z	+ 5510	-151100	+592100	-826400

is possible to evaluate fixed end moments for any translation. For example, B of system GPCB moves in the positive X direction 1.15" due to expansion in AB. The member CB expands 1.44", making a total X translation of +2.59" in the system. The fixed end moment at B in the XY plane due to  $\Delta X = +2.59"$  is computed by multiplying  $(16420 \text{ \#/"})(+2.59") = 42500 \text{ \#}$ . To obtain the total fixed end moment at B in the XY plane, this value must be added to the moments produced at B in the XY plane due to the Y and Z expansions in the system GPCB. All fixed end moments due to expansion in the pipe structure are tabulated in Table IX.

#### Distribution Factors

The distribution factors at joint B are dependent upon the moment or torque at B required to produce a unit rotation at B in each plane of each system. For AB, the moment required for a unit rotation at B is obtained from  $M = 4EI\theta/L$  for flexure; and the torque required for a unit rotation is obtained from  $T = GJ\theta/L = EI/1.25L$  for torsion.

To find the moment required to produce a unit rotation at B in any plane of the complex system, B is given a negative unit rotation in that plane while it is fixed against translation. This rotation causes the neutral point to be displaced as shown in Fig. 6, Appendix III. The neutral point forces and moment act to restore the condition existing before rota-

tion and therefore produce a positive unit rotation at B. If this rotation is in the XY plane, equation 6b is used to obtain the moment at the neutral point corresponding to a unit rotation at B in the XY plane. Equation 7b and 8b are used when the rotation is in the XZ and YZ planes. By substituting the displacements of the neutral point, due to a unit rotation of B, into equation 3b, 4b and 5b, the neutral point forces required for a positive unit rotation of B are determined. These values are tabulated in Tables X and XI. The moments induced at the terminals of each system due to a unit rotation at B in each plane are tabulated in Table XII and are obtained by taking moments of the neutral point forces and moment about each terminal.

The distribution factor for a system at B in a given plane such as the XY plane is found by the ratio of  $M/\Sigma M$ , where M is the moment required to produce unit rotation at B for the system in question and  $\Sigma M$  is the sum of the moments required to produce unit rotation in the XY plane at B for all systems. Using Table XII in which the end moments for each system due to unit rotation of joint B are listed, the distribution factors at B in each plane are determined and tabulated in Table XIII.

#### Carry-over Factors

Rotation of the terminal B of a complex system in one

TABLE X : NEUTRAL POINT FORCES AND MOMENT REQUIRED FOR A UNIT POSITIVE ROTATION OF JOINT B OF EDB SYSTEM IN X-Y PLANE X-Z PLANE Y-Z PLANE

SOLVING FOR NUMERATOR DETERMINANT ( $D_x$ )

$\begin{aligned} &+(-\Delta XEI)(I_{YY}+I_{YY})(I_{ZZ}+I_{ZZ}) \\ &\quad -(-\Delta XEI)(-I_{YZ})^2 \\ &-(-\Delta YEI)(-I_{XY})(I_{ZZ}+I_{ZZ}) \\ &+(-\Delta YEI)(-I_{XZ})(-I_{YZ}) \\ &-(-\Delta ZEI)(-I_{XZ})(I_{YY}+I_{YY}) \\ &+(-\Delta ZEI)(-I_{XY})(-I_{YZ}) \end{aligned}$	$\begin{aligned} &-18.7EI \times 3.360 \times 10^5 \times 1.965 \times 10^5 \\ &18.7EI \times (-1.536 \times 10^5)^2 \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$	$\begin{aligned} &-123.46 \times 10^{10} EI \\ &+44.11 \times 10^{10} EI \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$			
$D_x$	$-79.35 \times 10^{10} EI$	$D_x$	$+381.87 \times 10^{10} EI$	$D_x$	0

SOLVING FOR NUMERATOR DETERMINANT ( $D_y$ )

$\begin{aligned} &-(-\Delta XEI)(-I_{XY})(I_{ZZ}+I_{ZZ}) \\ &+(-\Delta XEI)(-I_{XZ})(-I_{YZ}) \\ &+(-\Delta YEI)(I_{XX}+I_{XX})(I_{ZZ}+I_{ZZ}) \\ &\quad -(-\Delta YEI)(-I_{XZ})^2 \\ &-(-\Delta ZEI)(-I_{YZ})(I_{XX}+I_{XX}) \\ &+(-\Delta ZEI)(-I_{XY})(-I_{XZ}) \end{aligned}$	$\begin{aligned} &0 \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$	$\begin{aligned} &0 \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$			
$D_y$	0	$D_y$	0	$D_y$	$\begin{aligned} &86.7EI \times 5.686 \times 10^5 \times 1.965 \times 10^5 \\ &21.3EI \times (-1.536 \times 10^5) \times 5.686 \times 10^5 \\ &0 \end{aligned}$ $\begin{aligned} &+968.69 \times 10^{10} EI \\ &-186.92 \times 10^{10} EI \\ &+782.67 \times 10^{10} EI \end{aligned}$

SOLVING FOR NUMERATOR DETERMINANT ( $D_z$ )

$\begin{aligned} &-(-\Delta XEI)(-I_{XZ})(I_{YY}+I_{YY}) \\ &+(-\Delta XEI)(-I_{XY})(-I_{YZ}) \\ &-(-\Delta YEI)(-I_{YZ})(I_{XX}+I_{XX}) \\ &+(-\Delta YEI)(-I_{XY})(-I_{XZ}) \\ &+(-\Delta ZEI)(I_{XX}+I_{XX})(I_{YY}+I_{YY}) \\ &\quad -(-\Delta ZEI)(-I_{XY})^2 \end{aligned}$	$\begin{aligned} &0 \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$	$\begin{aligned} &0 \\ &0 \\ &0 \\ &0 \\ &0 \\ &0 \end{aligned}$			
$D_z$	0	$D_z$	0	$D_z$	$\begin{aligned} &-86.7EI \times (-1.536 \times 10^5) \times 5.686 \times 10^5 \\ &-21.3EI \times 5.686 \times 10^5 \times 3.360 \times 10^5 \\ &0 \end{aligned}$ $\begin{aligned} &+757.20 \times 10^{10} EI \\ &-4.06.93 \times 10^{10} EI \\ &+350.27 \times 10^{10} EI \end{aligned}$

NEUTRAL POINT FORCES AND MOMENT

$F_x = D_x \div D_D$	$F_x = -79.35 \times 10^{10} EI \div 24.113 \times 10^{15} = -3.291 \times 10^{-5} EI$	$F_x = 381.87 \times 10^{10} EI \div 24.113 \times 10^{15} = 1.584 \times 10^{-4} EI$	$F_x = 0$
$F_y = D_y \div D_D$	$F_y = 0$	$F_y = 0$	$F_y = 782.67 \times 10^{10} EI \div 24.113 \times 10^{15} = 3.246 \times 10^{-4} EI$
$F_z = D_z \div D_D$	$F_z = 0$	$F_z = 0$	$F_z = 350.27 \times 10^{10} EI \div 24.113 \times 10^{15} = 1.453 \times 10^{-4} EI$
$M_{XY} = EI \div 246 = 4.065 \times 10^{-3} EI$	$M_{XZ} = EI \div 240 = 4.167 \times 10^{-3} EI$	$M_{YZ} = EI \div 216 = 4.630 \times 10^{-3} EI$	

TABLE XI: NEUTRAL POINT FORCES AND MOMENT REQUIRED FOR A UNIT POSITIVE ROTATION OF JOINT B OF GFCB SYSTEM IN X-Y PLANE X-Z PLANE Y-Z PLANE

SOLVING FOR NUMERATOR DETERMINANT (D<sub>X</sub>)

$+(-\Delta XEI)(I_{YY}+I'_{YY})(I_{ZZ}+I'_{ZZ})$	$-12EI \times 79.124 \times 10^5 \times 60.858 \times 10^5$	$-5.778 \times 10^{14} EI$	$53.8EI \times 79.124 \times 10^5 \times 60.858 \times 10^5$	$+25.906 \times 10^{14} EI$	0	
$-(-\Delta XEI)(-I_{YZ})^2$	$12EI \times (-7.253 \times 10^5)^2$	$+0.63 \times 10^{14} EI$	$-53.8EI \times (-7.253 \times 10^5)^2$	$-0.283 \times 10^{14} EI$	0	
$-(-\Delta YEI)(-I_{XY})(I_{ZZ}+I'_{ZZ})$	$-225EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$+7.394 \times 10^{14} EI$	0		$-43.3EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$+1.423 \times 10^{14} EI$
$+(-\Delta YEI)(-I_{XZ})(-I_{YZ})$	$225EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$+3.951 \times 10^{14} EI$	0		$43.3EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$+0.760 \times 10^{14} EI$
$-(-\Delta ZEI)(-I_{XZ})(I_{YY}+I'_{YY})$	0		$224.2EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$	$-42.947 \times 10^{14} EI$	$11.3EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$	$-2.165 \times 10^{14} EI$
$+(-\Delta ZEI)(-I_{XY})(-I_{YZ})$	0		$-224.2EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$	$-0.878 \times 10^{14} EI$	$-11.3EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$	$-0.044 \times 10^{14} EI$
		D <sub>X</sub> $+5.630 \times 10^{14} EI$		D <sub>X</sub> $-18.202 \times 10^{14} EI$		D <sub>X</sub> $-0.026 \times 10^{14} EI$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>Y</sub>)

$-(-\Delta XEI)(-I_{XY})(I_{ZZ}+I'_{ZZ})$	$12EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$-0.394 \times 10^{14} EI$	$-53.8EI \times (-5.40 \times 10^5) \times 60.858 \times 10^5$	$+1.768 \times 10^{14} EI$	0	
$+(-\Delta XEI)(-I_{XZ})(-I_{YZ})$	$-12EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$-0.211 \times 10^{14} EI$	$53.8EI \times (-24.21 \times 10^5) \times (-7.253 \times 10^5)$	$+0.945 \times 10^{14} EI$	0	
$+(-\Delta YEI)(I_{XX}+I'_{XX})(I_{ZZ}+I'_{ZZ})$	$225EI \times 28.748 \times 10^5 \times 60.858 \times 10^5$	$+39.365 \times 10^{14} EI$	0		$43.3EI \times 28.748 \times 10^5 \times 60.858 \times 10^5$	$+7.576 \times 10^{14} EI$
$-(-\Delta YEI)(-I_{XZ})^2$	$-225EI \times (-24.21 \times 10^5)^2$	$-13.188 \times 10^{14} EI$	0		$-43.3EI \times (-24.21 \times 10^5)^2$	$-2.538 \times 10^{14} EI$
$-(-\Delta ZEI)(-I_{YZ})(I_{XX}+I'_{XX})$	0		$224.2EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$	$-4.675 \times 10^{14} EI$	$11.3EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$	$-0.236 \times 10^{14} EI$
$+(-\Delta ZEI)(-I_{XY})(-I_{XZ})$	0		$-224.2EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$	$-2.931 \times 10^{14} EI$	$-11.3EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$	$-0.148 \times 10^{14} EI$
		D <sub>Y</sub> $+25.572 \times 10^{14} EI$		D <sub>Y</sub> $-4.893 \times 10^{14} EI$		D <sub>Y</sub> $+4.654 \times 10^{14} EI$

SOLVING FOR NUMERATOR DETERMINANT (D<sub>Z</sub>)

$-(-\Delta XEI)(-I_{XZ})(I_{YY}+I'_{YY})$	$12EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$	$-2.299 \times 10^{14} EI$	$-53.8EI \times (-24.21 \times 10^5) \times 79.124 \times 10^5$	$+10.306 \times 10^{14} EI$	0	
$+(-\Delta XEI)(-I_{XY})(-I_{YZ})$	$-12EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$	$-0.047 \times 10^{14} EI$	$53.8EI \times (-5.40 \times 10^5) \times (-7.253 \times 10^5)$	$+0.211 \times 10^{14} EI$	0	
$-(-\Delta YEI)(-I_{YZ})(I_{XX}+I'_{XX})$	$-225EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$	$+4.691 \times 10^{14} EI$	0		$-43.3EI \times (-7.253 \times 10^5) \times 28.748 \times 10^5$	$+0.903 \times 10^{14} EI$
$+(-\Delta YEI)(-I_{XY})(-I_{XZ})$	$225EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$	$+2.941 \times 10^{14} EI$	0		$43.3EI \times (-5.40 \times 10^5) \times (-24.21 \times 10^5)$	$+0.566 \times 10^{14} EI$
$+(-\Delta ZEI)(I_{XX}+I'_{XX})(I_{YY}+I'_{YY})$	0		$-224.2EI \times 28.748 \times 10^5 \times 79.124 \times 10^5$	$-50.998 \times 10^{14} EI$	$-11.3EI \times 28.748 \times 10^5 \times 79.124 \times 10^5$	$-2.570 \times 10^{14} EI$
$-(-\Delta ZEI)(-I_{XY})^2$	0		$224.2EI \times (-5.40 \times 10^5)^2$	$+0.654 \times 10^{14} EI$	$11.3EI \times (-5.40 \times 10^5)^2$	$+0.033 \times 10^{14} EI$
		D <sub>Z</sub> $+5.286 \times 10^{14} EI$		D <sub>Z</sub> $-39.827 \times 10^{14} EI$		D <sub>Z</sub> $-1.068 \times 10^{14} EI$

NEUTRAL POINT FORCES AND MOMENT

$F_x = D_x \div D_D$	$F_x = 5.630 \times 10^{14} EI \div 86.872 \times 10^{18} = 0.0648 \times 10^{-4} EI$	$F_x = -18.202 \times 10^{14} EI \div 86.872 \times 10^{18} = -0.210 \times 10^{-4} EI$	$F_x = -0.026 \times 10^{14} EI \div 86.872 \times 10^{18} = -0.0003 \times 10^{-4} EI$
$F_y = D_y \div D_D$	$F_y = 25.572 \times 10^{14} EI \div 86.872 \times 10^{18} = 0.294 \times 10^{-4} EI$	$F_y = -4.893 \times 10^{14} EI \div 86.872 \times 10^{18} = -0.0563 \times 10^{-4} EI$	$F_y = +4.654 \times 10^{14} EI \div 86.872 \times 10^{18} = 0.0536 \times 10^{-4} EI$
$F_z = D_z \div D_D$	$F_z = 5.286 \times 10^{14} EI \div 86.872 \times 10^{18} = 0.0608 \times 10^{-4} EI$	$F_z = -39.827 \times 10^{14} EI \div 86.872 \times 10^{18} = -0.458 \times 10^{-4} EI$	$F_z = -1.068 \times 10^{14} EI \div 86.872 \times 10^{18} = -0.0123 \times 10^{-4} EI$
$M = -\theta EI \div \Sigma L$	$M_{XY} = EI \div 600 = 1.67 \times 10^{-3} EI$	$M_{XZ} = EI \div 594 = 1.68 \times 10^{-3} EI$	$M_{YZ} = EI \div 639 = 1.56 \times 10^{-3} EI$

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Table XII: Moments in Inch Pounds at Fixed End and Joint B of Each System Due to a Positive Unit Rotation of Joint B of Each System with no Translation of B

			$\theta_{xy}$	$\theta_{xz}$	$\theta_{yz}$
System AB	At B	$M_{xy}$	$+16.67 \times 10^{-3} EI$	0	0
		$M_{xz}$	0	$+16.67 \times 10^{-3} EI$	0
		$T_{yz}$	0	0	$+ 3.33 \times 10^{-3} EI$
	At A	$M_{xy}$	$+ 8.33 \times 10^{-3} EI$	0	0
		$M_{xz}$	0	$+ 8.33 \times 10^{-3} EI$	0
		$T_{yz}$	0	0	$- 3.33 \times 10^{-3} EI$
System EDB	At B	$T_{xy}$	$+ 4.68 \times 10^{-3} EI$	$- 2.96 \times 10^{-3} EI$	0
		$M_{xz}$	$- 2.96 \times 10^{-3} EI$	$+18.42 \times 10^{-3} EI$	0
		$M_{yz}$	0	0	$+29.7 \times 10^{-3} EI$
	At E	$M_{xy}$	$- 1.52 \times 10^{-3} EI$	$-12.24 \times 10^{-3} EI$	0
		$T_{xz}$	$- .987 \times 10^{-3} EI$	$+ .585 \times 10^{-3} EI$	0
		$M_{yz}$	0	0	$- 4.67 \times 10^{-3} EI$
System GFCB	At B	$M_{xy}$	$+ 8.21 \times 10^{-3} EI$	$- 1.02 \times 10^{-3} EI$	$+ 1.21 \times 10^{-3} EI$
		$M_{xz}$	$+ 1.01 \times 10^{-3} EI$	$+10.82 \times 10^{-3} EI$	$+ .274 \times 10^{-3} EI$
		$T_{yz}$	$+ 1.20 \times 10^{-3} EI$	$+ .274 \times 10^{-3} EI$	$+ 1.81 \times 10^{-3} EI$
	At C	$M_{xy}$	$- .164 \times 10^{-3} EI$	$+ 1.85 \times 10^{-3} EI$	$+ .405 \times 10^{-3} EI$
		$T_{xz}$	$+ .123 \times 10^{-3} EI$	$+ .102 \times 10^{-3} EI$	$+ .0905 \times 10^{-3} EI$
		$M_{yz}$	$+ 2.30 \times 10^{-3} EI$	$+ 4.41 \times 10^{-3} EI$	$- .886 \times 10^{-3} EI$

plane may give carry-over moments in the other two planes at the terminal B and in three planes at the other terminal. This produces a maximum of 5 carry-over factors when the terminal B rotates. Carry-over factors are computed using values listed in Table XII. For example, a moment of  $8.21 \times 10^{-3} EI''$  in the XY plane at B of system GFGB produces a moment of  $-1.01 \times 10^{-3} EI''$  in the XZ plane at B; therefore the carry-over factor from B in the XY plane to B in the XZ plane is

$$-1.01 \times 10^{-3} EI / 8.21 \times 10^{-3} EI = -.123$$

A tabulation of the carry-over factors is made in Table XIII.

#### Releasing Restraints

The restraints at joint B may be released by either of two methods as follows:

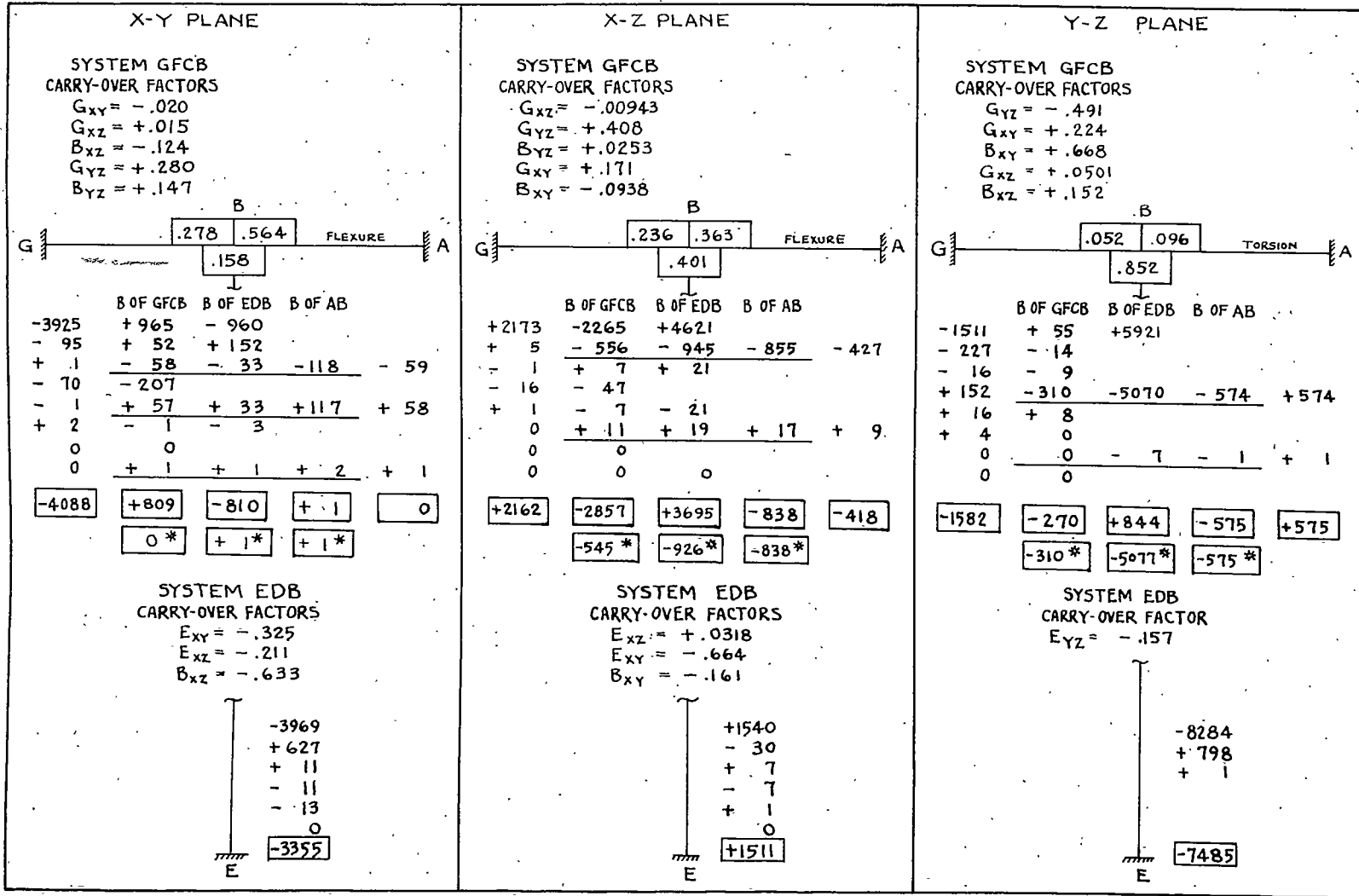
- (1) By first allowing rotational equilibrium to occur and then releasing the restraints against translation.
- (2) By releasing the restraints against translation first and then allowing rotational equilibrium to occur, but balancing shears each time joint B rotates.

Method I will be applied first.

#### Method I

The restraint against rotation is successively released in each plane by moment distribution. This is shown in Table XIII.

TABLE XIII: BALANCING FIXED END MOMENTS. DUE TO EXPANSION



NOTE: MULTIPLY ALL MOMENTS BY 100. \* INDICATES MOMENTS DUE TO ROTATION

Joint Restraint Determination

With the restraints against rotation released, the end reactions determined before rotation occurred must be corrected for this effect. The amount of moment used to determine the change in the end reactions for a given system is the algebraic sum of the moments added into the system due to moment balancing only. Carry-over moments are not included. The moments arising from balancing moments are a measure of the rotation that takes place. The moments and reactions at B and at the fixed end of each system are known for a unit rotation in a given plane. From these values the reactions for a given system due to a unit moment at B in a given plane are computed and tabulated in Table XIV.

The computation of change in reaction at G in the system GFCB due to rotation in the XZ plane is as follows. The algebraic sum of the balancing moments in the XZ plane at B of system GFCB as given in Table XIII is  $-54500''\#$ . The end reactions for system GFCB corresponding to a unit moment in the XZ plane are taken from Table XIV. Therefore, the changes in the reactions are

$$\begin{aligned} F_x &: (.00194''\# / ''\#) (-54500''\#) = -106\# \\ F_y &: (.00052''\# / ''\#) (-54500''\#) = -28\# \\ F_z &: (.00423''\# / ''\#) (-54500''\#) = -231\# \end{aligned}$$

In Table VII the changes in the reactions for each system are tabulated, and from these the restraints at B after rotation are  $F_y = +5762^{\#}$  and  $F_z = +15096^{\#}$ .

Table XIV: End Reactions in Pounds Due to a Positive Unit Moment\* at B of Each System with no Translation of B

		$F_x$	$F_y$	$F_z$
At A	$M_{xy} = +1$	0	+0.00625	0
	$M_{xz} = +1$	0	0	-0.00625
	$T_{yz} = +1$	0	0	0
At E	$T_{xy} = +1$	+0.00704	0	0
	$M_{xz} = +1$	-0.00860	0	0
	$M_{yz} = +1$	0	-0.01094	-0.00490
At G	$M_{xy} = +1$	+0.000789	-0.00358	-0.000741
	$M_{xz} = +1$	+0.00194	+0.00052	+0.00423
	$T_{yz} = +1$	+0.000017	-0.00297	+0.000681

\*Moment in Inch Pounds

Elimination of Joint Restraints

Since joint B has only two degrees of freedom, the existing restraints are eliminated by allowing B to translate a positive unit distance in the Y and Z directions. The change in the amount of joint restraint in the Y and Z directions produced by a positive unit translation in the Y direction is computed as follows. The joint B of each system is translated a positive unit distance in the Y direction while B is

restrained against rotation in each plane and restrained against translation in the remaining two directions. The end reactions and the fixed end moments due to this translation are obtained for each of the systems as already outlined. The end reactions are tabulated in Table XV.

The restraint against rotation is successively released in each plane until rotation equilibrium is attained while B is restrained against translation. This is shown in Table XVII. The reactions listed in Table XV are corrected for the amount of rotation which occurred as previously described. These reaction changes are listed in Table XV along with the corrected end reactions. The change in the joint restraint in the Y and Z directions due to a positive unit translation in the Y direction is equal but of opposite sign to the corrected end reactions. The same procedure is carried out for a unit translation in the Z direction which is the other degree of freedom. This information is given in Tables XVI and XVIII.

Since the change in restraints produced by a unit translation in the Y and Z directions are known and the restraints to be removed in the Y and Z directions are known, the following equations are written:

$$(1) \quad F_y + \Delta Y F_y^{\Delta Y=1} + \Delta Z F_y^{\Delta Z=1} = 0$$

$$(2) \quad F_z + \Delta Y F_z^{\Delta Y=1} + \Delta Z F_z^{\Delta Z=1} = 0$$

TABLE XV : End Reactions in Pounds Due to  $\Delta Y = +1''$  Corrected For Rotation

At A	Due to $\Delta Y = +1''$	Correction Due to Rotation*						Corrected F $\Delta Y = 1$
		$M_{xy} = +1$	$M_{xy} = +126100$	$M_{xz} = +1$	$M_{xz} = +18400$	$T_{yz} = +1$	$T_{yz} = -81200$	
$F_x$	0	0	0	0	0	0	0	+ 102
$F_y$	- 2200	+ .00825	+ 788	0	0	0	0	-1412
$F_z$	0	0	0	- .00625	- 115	0	0	- 115
At E	Due to $\Delta Y = +1''$	$T_{xy} = +1$	$T_{xy} = +35100$	$M_{xz} = +1$	$M_{xz} = +20400$	$M_{yz} = +1$	$M_{yz} = -720700$	
$F_x$	0	+ .00704	+ 247	- .00860	- 175	0	0	+ 72
$F_y$	-11750	0	0	0	0	- .01094	+7880	-3870
$F_z$	-9180	0	0	0	0	- .00490	+3530	-5650
At G	Due to $\Delta Y = +1''$	$M_{xy} = +1$	$M_{xy} = +62000$	$M_{xz} = +1$	$M_{xz} = +12000$	$T_{yz} = +1$	$T_{yz} = -44000$	
$F_x$	-147	- .000789	- 49	+ .00194	+ 23	+ .000017	- 1	- 174
$F_y$	-340	- .00358	-222	+ .00052	+ 6	- .00297	+131	- 425
$F_z$	- 99	- .000741	- 46	+ .00423	+ 51	+ .000381	+ 30	- 124

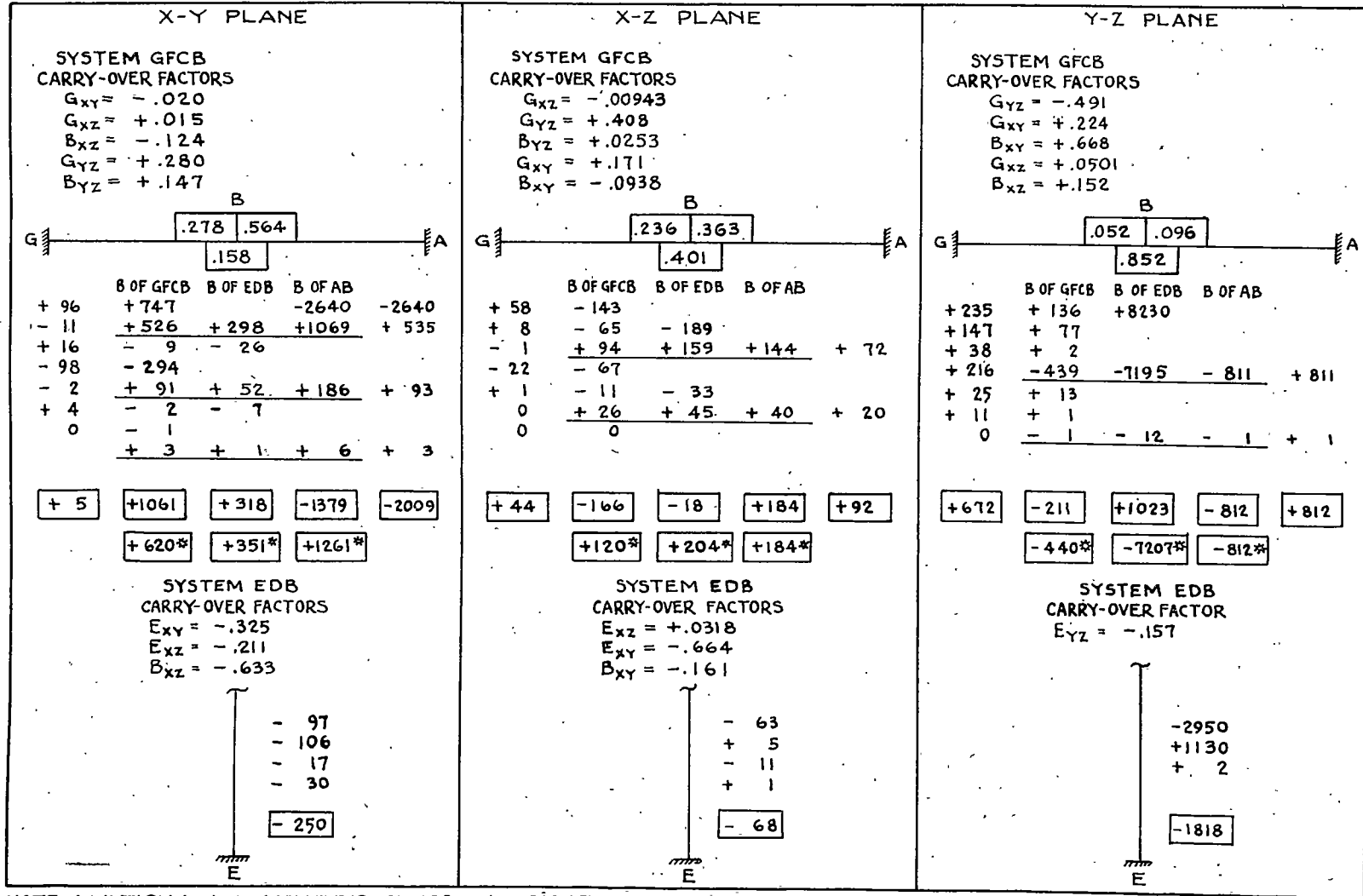
\* Rotation effect given in terms of moment in inch pounds.

TABLE XVI : End Reactions in Pounds Due to  $\Delta Z = +1'$  Corrected For Rotation

At A	Due to $\Delta Z = +1$	Correction Due to Rotation*						Corrected $\Delta Z = 1$
		$M_{xy} = +1$	$M_{xy} = -8800$	$M_{xz} = +1$	$M_{xz} = -53200$	$T_{yz} = +1$	$T_{yz} = -34900$	
$F_x$	0	0	0	0	0	0	0	+ 146
$F_y$	0	+ .00625	-55	0	0	0	0	- 55
$F_z$	-2200	0	0	- .00625	+333	0	0	-1867
At E	Due to $\Delta Z = +1$	$T_{xy} = +1$	$T_{xy} = -2400$	$M_{xz} = +1$	$M_{xz} = -58800$	$M_{yz} = +1$	$M_{yz} = -309600$	
$F_x$	0	+ .00704	-17	- .00860	+506	0	0	+ 489
$F_y$	- 9180	0	0	0	0	- .01094	+3387	-5793
$F_z$	-20100	0	0	0	0	- .00490	+1517	-18583
At G	Due to $\Delta Z = +1$	$M_{xy} = +1$	$M_{xy} = -4300$	$M_{xz} = +1$	$M_{xz} = -34500$	$T_{yz} = +1$	$T_{yz} = -18900$	
$F_x$	-571	- .000789	+ 3	+ .00194	- 67	+ .000017	neglect	-635
$F_y$	- 99	- .00358	+15	+ .00052	- 18	- .00297	+56	- 46
$F_z$	-657	- .000741	+ 3	+ .00423	-146	+ .000681	-13	-813

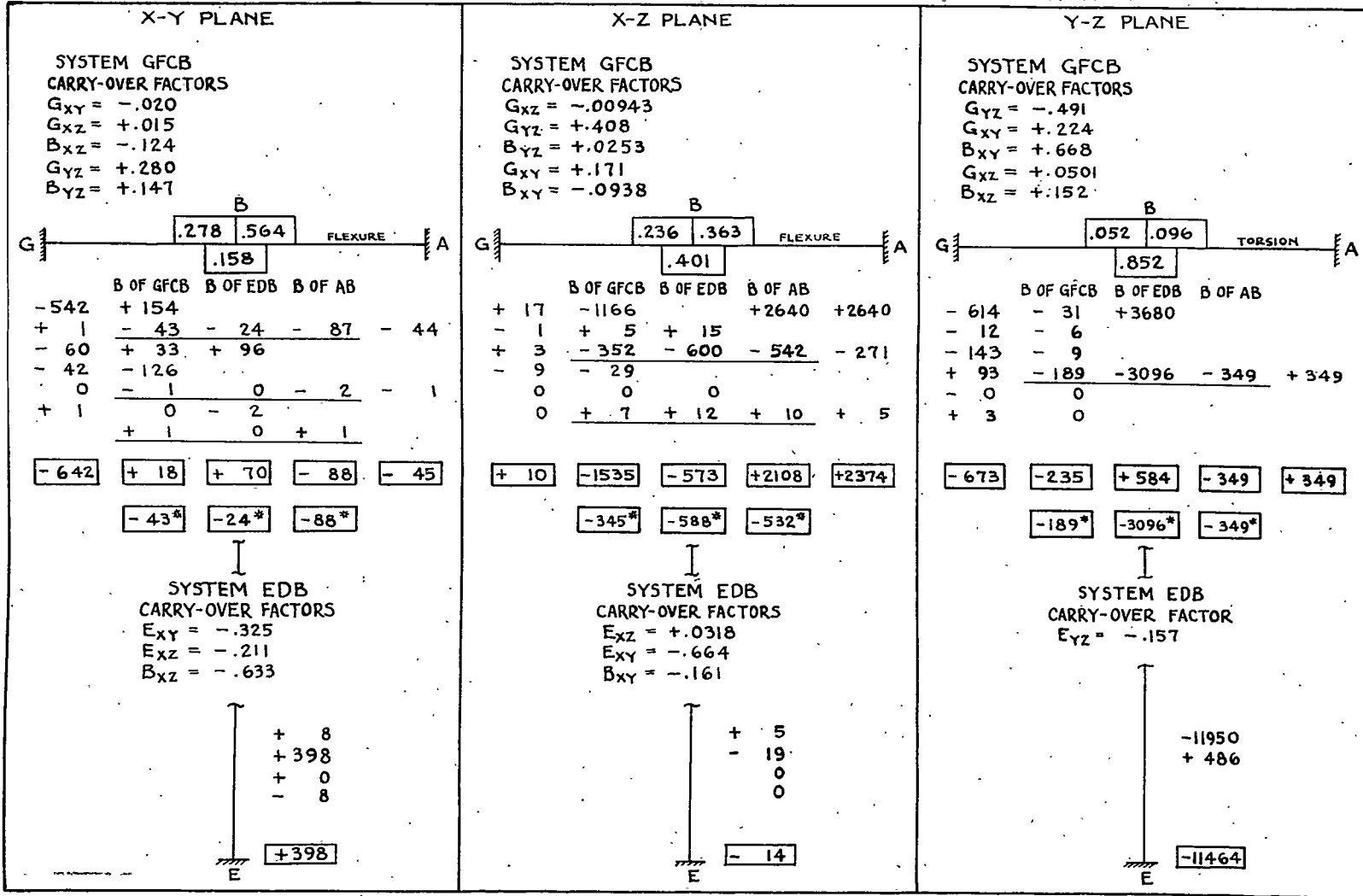
\* Rotation effect given in terms of moment in inch pounds.

TABLE XVII: BALANCING FIXED END MOMENTS DUE TO A UNIT TRANSLATION IN +Y DIRECTION AT B



NOTE: MULTIPLY ALL MOMENTS BY 100. \*INDICATES MOMENTS DUE TO ROTATION

TABLE XVIII BALANCING FIXED END MOMENTS DUE TO A UNIT TRANSLATION IN +Z DIRECTION AT B.



NOTE: MULTIPLY ALL MOMENTS BY 100. \* INDICATES MOMENTS DUE TO ROTATION

Substituting the value of the restraints in the equations gives

$$5762 + 5707\Delta Y + 5894\Delta Z = 0$$

$$15096 + 5889\Delta Y + 21263\Delta Z = 0$$

Which when solved yield  $\Delta Y = -.387$  and  $\Delta Z = -.603$  and correspond to the amount of translation of B in the Y and Z directions required to eliminate the restraints.

#### Final Reactions and Moments

The correction to the end reactions due to allowing joint B to translate are obtained when the values listed in Tables XV and XVI are multiplied by  $\Delta Y$  and  $\Delta Z$  respectively. Table VII shows the final end reactions.

The corrections to the moments shown in Table XIII, due to allowing joint B to translate, are obtained when the balanced moments shown in Tables XVII and XVIII are multiplied by  $\Delta Y$  and  $\Delta Z$  respectively. The final moments are tabulated in Table XIX.

#### Method II

From the reactions due to expansion, the restraints against translation at B are evaluated and are:

$$F_y = +11376\# \quad \text{and} \quad F_z = +17858\#$$

These restraints are eliminated by allowing B to translate in the Y and Z directions since joint B has only two degrees of freedom; the restraint against rotation is maintained. The change in the amount of joint restraint in the Y and Z directions produced by a positive unit translation in the Y

direction is computed as follows. The joint B of each system is translated a positive unit distance in the Y direction while B is restrained against rotation in each plane and restrained against translation in the remaining two directions. The end reactions and the fixed end moments due to this translation are obtained for each system as previously outlined. The end reactions are listed in Table XV and from these the restraints at B are:

$$F_y^{\Delta Y=1} = 14290\# \quad \text{and} \quad F_z^{\Delta Y=1} = 9279\#$$

Similarly, the change in the amount of joint restraint in the Y and Z direction produced by a positive unit translation in the Z direction are computed. They are;

$$F_y^{\Delta Z=1} = 9279\# \quad \text{and} \quad F_z^{\Delta Z=1} = 22957\#$$

Since the change in restraints produced by a unit translation in the Y and Z direction are known and the restraints to be removed in the Y and Z directions are known, these values are substituted into equations 1 and 2 and solved for  $\Delta Y$  and  $\Delta Z$ .

$$11376 + 14290\Delta Y + 9279\Delta Z = 0$$

$$117858 + 9279\Delta Y + 22957\Delta Z = 0$$

$$\Delta Y = -.397''$$

$$\Delta Z = -.615''$$

These are the translations required to remove the restraints at B while B is restrained against rotation.

Table XIX: Final Moments in Inch Pounds

		Moments from Table 12	Correction For Translation		Final Moments
			$\Delta Y M$	$\Delta Z M$	
At A	$M_{xy}$	00	+77700	+ 2710	+ 80410
	$M_{xz}$	- 41800	- 3560	-143200	-188560
	$T_{yz}$	+ 57500	-31400	- 21000	+ 5100
At E	$M_{xy}$	-335500	+ 9680	- 24000	-349820
	$T_{xz}$	+151100	+ 2630	+ 844	+154574
	$M_{yz}$	-748500	+70400	+691000	+ 12900
At G	$M_{xy}$	-408800	- 194	+ 38700	-370294
	$T_{xz}$	+216200	- 1700	- 600	+213900
	$M_{yz}$	-158200	-26000	+ 40600	-145600

Effect of Translation

The reactions due to expansion are corrected for these translations in Table XX. The moments at the fixed ends due to expansion are corrected for these translations in Table XXI.

Releasing Rotation Restraints

To remove the restraint against rotation at B in a given plane, the joint B is fixed against translation and allowed to rotate until rotational equilibrium is attained. After rotation, the restraints against translation which developed during the rotation are removed by letting the joint translate. The moments resulting from the translation are then applied to the structure as a correction for translation. The correction

TABLE XX : End Reactions in Pounds Due to Expansion Corrected For Translation of B With B  
Restrained Against Rotation

		Due to Expansion	Correction For Translation				Corrected For Translation
			$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
At A	$F_x$	+9209	0	0	0	0	+8800
	$F_y$	0	$-.0868 \times 10^{-5} EI$	+ 874	0	0	+ 874
	$F_z$	0	0	0	$-.0868 \times 10^{-5} EI$	+ 1354	+1354
			$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
At E	$F_x$	- 5130	0	0	0	0	-5130
	$F_y$	-10730	$-.463 \times 10^{-5} EI$	+4660	$-.362 \times 10^{-5} EI$	+ 5650	- 420
	$F_z$	-15870	$-.362 \times 10^{-5} EI$	+3650	$-.792 \times 10^{-5} EI$	+12360	+ 140
			$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
At G	$F_x$	-4079	$-.0058 \times 10^{-5} EI$	+ 58	$-.0225 \times 10^{-5} EI$	+351	-3670
	$F_y$	+ 646	$-.0134 \times 10^{-5} EI$	+135	$-.0039 \times 10^{-5} EI$	+ 61	- 450
	$F_z$	-1988	$-.0039 \times 10^{-5} EI$	+ 39	$-.0259 \times 10^{-5} EI$	+404	-1545

TABLE XXI : Fixed End Moments in Inch Pounds Due to Expansion Corrected For Translation with B Restrained Against Rotation

			Due to Expansion	Correction For Translation				Corrected For Translation
				$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
System AB	$\Delta t$ B	$M_{xy}$	0	-264000	+104800	0	0	+104800
		$M_{xz}$	0	0	0	+264000	-162400	-162400
		$T_{yz}$	0	0	0	0	0	0
	$\Delta t$ A	$M_{xy}$	0	-264000	+104800	0	0	+104800
		$M_{xz}$	0	0	0	+264000	-162400	-162400
		$T_{yz}$	0	0	0	0	0	0
				$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
System EDB	$\Delta t$ B	$T_{xy}$	-96000	0	0	0	0	-96000
		$M_{xz}$	+462100	0	0	0	0	+462100
		$M_{yz}$	+592100	+823000	-327000	+368000	-226000	+39100
	$\Delta t$ E	$M_{xy}$	-396900	0	0	0	0	-396900
		$T_{xz}$	+154000	0	0	0	0	+154000
		$M_{yz}$	-828400	-295000	+117100	-1195000	+735000	+23700
				$\Delta Y = +1''$	$\Delta Y = -.397''$	$\Delta Z = +1''$	$\Delta Z = -.615''$	
System GFCB	$\Delta t$ B	$M_{xy}$	+96500	+74700	-29700	+15420	-9500	+57300
		$M_{xz}$	-226500	-14270	+5670	-116600	+71700	-149100
		$T_{yz}$	+5510	+13600	-5400	-3140	+1930	+2040
	$\Delta t$ G	$M_{xy}$	-392500	+9610	-3820	-54200	+33300	-363000
		$T_{xz}$	+217300	+5770	-2290	+1675	-1030	+214000
		$M_{yz}$	-151100	+23500	-9330	-61400	+37800	-122700

moments required to eliminate the restraints against translation resulting from a unit moment change at joint B in a given plane are called shear correction factors.

Shear Correction Factors

To evaluate the shear correction factors, the joint B is allowed to rotate separately in each plane an amount corresponding to a unit moment while B is restrained against translation. This is shown in Table XXII. The resulting three sets of end reactions and three sets of restraints at B are evaluated as shown in Table XXIII.

The translations required to eliminate each set of restraints are determined by using equations 1 and 2. The translations required to eliminate the restraints due to  $M_{xy} = +1$  at B are determined as follows:

$$-.00253 + 14290\Delta Y + 9279\Delta Z = 0$$

$$+.000206 + 9279\Delta Y + 22957\Delta Z = 0$$

$$\Delta Y = +2.48 \times 10^{-7}$$

$$\Delta Z = -1.087 \times 10^{-7}$$

Table XXIV shows the translations required to eliminate each set of restraints. The moments corresponding to each pair of deflections are tabulated in Table XXV. These are the shear correction factors.

TABLE XXII: EFFECT OF A + UNIT MOMENT CHANGE AT JOINT B IN EACH PLANE

<p>X-Y PLANE</p> <p>CARRY-OVER FACTORS FOR</p> <table border="0"> <tr> <td>GFCB</td> <td>EDB</td> <td>AB</td> </tr> <tr> <td><math>G_{xy} = -.020</math></td> <td><math>E_{xy} = -.325</math></td> <td><math>A_{xy} = +.50</math></td> </tr> <tr> <td><math>G_{xz} = +.015</math></td> <td><math>E_{xz} = -.211</math></td> <td></td> </tr> <tr> <td><math>B_{yz} = -.124</math></td> <td><math>B_{xz} = -.633</math></td> <td></td> </tr> <tr> <td><math>G_{yz} = +.280</math></td> <td></td> <td></td> </tr> <tr> <td><math>B_{yz} = +.147</math></td> <td></td> <td></td> </tr> </table>	GFCB	EDB	AB	$G_{xy} = -.020$	$E_{xy} = -.325$	$A_{xy} = +.50$	$G_{xz} = +.015$	$E_{xz} = -.211$		$B_{yz} = -.124$	$B_{xz} = -.633$		$G_{yz} = +.280$			$B_{yz} = +.147$			<p>X-Z PLANE</p> <p>CARRY-OVER FACTORS FOR</p> <table border="0"> <tr> <td>GFCB</td> <td>EDB</td> <td>AB</td> </tr> <tr> <td><math>G_{xz} = -.0943</math></td> <td><math>E_{xz} = +.0318</math></td> <td><math>A_{xz} = +.50</math></td> </tr> <tr> <td><math>G_{yz} = +.408</math></td> <td><math>E_{xy} = -.664</math></td> <td></td> </tr> <tr> <td><math>B_{yz} = +.0253</math></td> <td><math>B_{xy} = -.161</math></td> <td></td> </tr> <tr> <td><math>G_{xy} = +.171</math></td> <td></td> <td></td> </tr> <tr> <td><math>B_{xy} = -.0938</math></td> <td></td> <td></td> </tr> </table>	GFCB	EDB	AB	$G_{xz} = -.0943$	$E_{xz} = +.0318$	$A_{xz} = +.50$	$G_{yz} = +.408$	$E_{xy} = -.664$		$B_{yz} = +.0253$	$B_{xy} = -.161$		$G_{xy} = +.171$			$B_{xy} = -.0938$			<p>Y-Z PLANE</p> <p>CARRY-OVER FACTORS FOR</p> <table border="0"> <tr> <td>GFCB</td> <td>EDB</td> <td>AB</td> </tr> <tr> <td><math>G_{yz} = -.491</math></td> <td><math>E_{yz} = -.157</math></td> <td><math>A_{yz} = -1</math></td> </tr> <tr> <td><math>G_{xy} = +.224</math></td> <td></td> <td></td> </tr> <tr> <td><math>B_{xy} = +.668</math></td> <td></td> <td></td> </tr> <tr> <td><math>G_{xz} = +.0501</math></td> <td></td> <td></td> </tr> <tr> <td><math>B_{xz} = +.152</math></td> <td></td> <td></td> </tr> </table>	GFCB	EDB	AB	$G_{yz} = -.491$	$E_{yz} = -.157$	$A_{yz} = -1$	$G_{xy} = +.224$			$B_{xy} = +.668$			$G_{xz} = +.0501$			$B_{xz} = +.152$		
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GFCB	EDB	AB																																																						
$G_{xz} = -.0943$	$E_{xz} = +.0318$	$A_{xz} = +.50$																																																						
$G_{yz} = +.408$	$E_{xy} = -.664$																																																							
$B_{yz} = +.0253$	$B_{xy} = -.161$																																																							
$G_{xy} = +.171$																																																								
$B_{xy} = -.0938$																																																								
GFCB	EDB	AB																																																						
$G_{yz} = -.491$	$E_{yz} = -.157$	$A_{yz} = -1$																																																						
$G_{xy} = +.224$																																																								
$B_{xy} = +.668$																																																								
$G_{xz} = +.0501$																																																								
$B_{xz} = +.152$																																																								
<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: .278 (top), .564 (right), .158 (bottom)</p> <p>Values at E: -.00556 (left), +.278 (top), +.158 (right), +.564 (bottom), +.282 (right), -.0514 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: +.00417 (left), -.0345 (top), -.100 (right)</p> <p>Value at E: -.0334 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: +.0779 (left), +.0409 (right)</p> <p>Value at E: -.0334 (bottom)</p>																																																						
<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: -.0403 (left), -.0221 (top), -.0645 (right)</p> <p>Value at E: -.266 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: -.00222 (left), +.236 (top), +.401 (right), +.363 (bottom)</p> <p>Value at E: +.0127 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: +.0964 (left), +.00596 (right)</p> <p>Value at E: -.0334 (bottom)</p>																																																						
<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: +.0116 (left), +.0347 (right)</p> <p>Value at E: -.0334 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: +.00260 (left), +.00791 (right)</p> <p>Value at E: -.0334 (bottom)</p>	<p>+UNIT MOMENT CHANGE AT B</p> <p>Values at B: -.0255 (left), +.052 (top), +.852 (right), +.096 (bottom)</p> <p>Value at E: -.134 (bottom)</p>																																																						

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TABLE XXIII : End Reactions and Restraints at B in Pounds Due to a Positive Unit Moment in Inch Pounds Applied at B with B Fixed Against Translation

	At A of AB		At E of EDB		At G of GFCB		Restraint at B	
	$M_{xy} = +1$	$M_{xy} = +.564$	$T_{xy} = +1$	$T_{xy} = +.158$	$M_{xy} = +1$	$M_{xy} = +.278$		
$M_{xy} = +1$	$F_x$	0	0	$+ .00704$	$+ .00111$	$- .000789$	$- .000219$	$- .00089$
	$F_y$	$+ .00625$	$+ .00353$	0	0	$- .00358$	$- .000995$	$- .00253$
	$F_z$	0	0	0	0	$- .000741$	$- .000206$	$+ .000206$
	At A of AB		At E of EDB		At G of GFCB		Restraint at B	
	$M_{xz} = +1$	$M_{xz} = +.363$	$M_{xz} = +1$	$M_{xz} = +.401$	$M_{xz} = +1$	$M_{xz} = +.236$		
$M_{xz} = +1$	$F_x$	0	0	$- .00860$	$- .00345$	$+ .00194$	$+ .000458$	$+ .00299$
	$F_y$	0	0	0	0	$+ .00052$	$+ .000123$	$- .000123$
	$F_z$	$- .00625$	$- .00227$	0	0	$+ .00423$	$+ .000998$	$+ .00127$
	At A of AB		At E of EDB		At G of GFCB		Restraint at B	
	$T_{yz} = +1$	$T_{yz} = +.096$	$M_{yz} = +1$	$M_{yz} = +.852$	$T_{yz} = +1$	$T_{yz} = +.052$		
$M_{yz} = +1$	$F_x$	0	0	0	0	$+ .000017$	$+ .00000086$	$- .00000086$
	$F_y$	0	0	$- .01094$	$- .00932$	$- .00297$	$- .000154$	$+ .00947$
	$F_z$	0	0	$- .00490$	$- .00417$	$+ .000681$	$+ .0000354$	$+ .00413$

Table XXIV: Translations Required to Eliminate the Restraints Due to a Unit Moment in Inch Pounds at B

	$M_{xy} = +1$ at B	$M_{xz} = +1$ at B	$M_{yz} = +1$ at B
$\Delta Y''$	$+2.48 \times 10^{-7}$	$+0.607 \times 10^{-7}$	$-7.40 \times 10^{-7}$
$\Delta Z''$	$-1.087 \times 10^{-7}$	$-0.796 \times 10^{-7}$	$+1.188 \times 10^{-7}$

#### Final Moments and Reactions

In Table XXVI the moments corrected for translation are balanced successively in each plane until equilibrium is attained. After joint B is allowed to rotate in a given plane, the shear correction is applied to eliminate the restraint against translation which developed during rotation. The final moments are obtained by adding the columns in Table XXVI.

The end reactions shown in Table XX must be corrected for the amount of rotation and for the amount of translation which occurred during the moment and shear balancing shown in Table XXVI. Since the moments at B for each system which corresponds to the amount of rotation are known and since the end reactions due to a unit moment at B are known, the corrections to be applied due to rotation are computed and tabulated in Table XXVIII. In Table XXVII are listed the end reactions corresponding to the translation required to release the restraint against translation due to a positive rotational unit moment change at joint B. These values, multiplied by the total rotational moment change which occurred, yields the

TABLE XXV Shear Correction Factors

		$M_{xy} = +1$ at B			$M_{xz} = +1$ at B			$M_{yz} = +1$ at B			
		$\Delta Y = +2.48 \times 10^{-7}$	$\Delta Z = -1.087 \times 10^{-7}$	Shear Correction	$\Delta Y = +.607 \times 10^{-7}$	$\Delta Z = -.796 \times 10^{-7}$	Shear Correction	$\Delta Y = -7.40 \times 10^{-7}$	$\Delta Z = +1.188 \times 10^{-7}$	Shear Correction	
System AB	at B	$M_{xy}$	-.0655	0	-.0655	-.01602	0	-.01602	+.1954	0	+.1954
		$M_{xz}$	0	-.0287	-.0287	0	-.0210	-.0210	0	+.0314	+.0314
		$T_{yz}$	0	0	0	0	0	0	0	0	0
	at A	$M_{xy}$	-.0655	0	-.0655	-.01602	0	-.01602	+.1954	0	+.1954
		$M_{xz}$	0	-.0287	-.0287	0	-.0210	-.0210	0	+.0314	+.0314
		$T_{yz}$	0	0	0	0	0	0	0	0	0
System EDB	at B	$T_{xy}$	0	0	0	0	0	0	0	0	0
		$M_{xz}$	0	0	0	0	0	0	0	0	0
		$M_{yz}$	+.204	-.040	+.164	+.0500	-.0293	+.0207	-.609	+.0437	-.565
	at E	$M_{xy}$	0	0	0	0	0	0	0	0	0
		$T_{xz}$	0	0	0	0	0	0	0	0	0
		$M_{yz}$	-.0732	+.1299	+.0567	-.01791	+.0951	+.0772	+.218	-.1420	+.076
System GFCB	at B	$M_{xy}$	+.01853	-.001676	+.01685	+.00453	-.00123	+.00330	-.0553	+.001832	-.0535
		$M_{xz}$	-.00354	+.01267	+.00913	-.000866	+.00928	+.00841	+.01056	-.01385	-.00329
		$T_{yz}$	+.00337	+.000341	+.00371	+.000826	+.000250	+.001076	-.01006	-.000373	-.01043
	at C	$M_{xy}$	+.00238	+.00589	+.00827	+.000583	+.00431	+.00489	-.00711	-.00644	-.01355
		$T_{xz}$	+.001431	-.000182	+.001249	+.000350	-.0001333	+.000217	-.00427	+.000199	-.00407
		$M_{yz}$	+.00583	+.00667	+.01250	+.001426	+.00489	+.00632	-.01739	-.00729	-.0247

TABLE XXVI: MOMENT AND SHEAR BALANCE

X-Y PLANE					X-Z PLANE					Y-Z PLANE																																																																																																																																																																																																																																																																																																																																																																																																																																						
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GFCB	EDB	AB			GFCB	EDB	AB			GFCB	EDB	AB																																																																																																																																																																																																																																																																																																																																																																																																																																				
$G_{XY} = -.020$	$E_{XY} = -.325$	$A_{XY} = +.50$			$G_{XZ} = -.00943$	$E_{XZ} = +.0318$	$A_{XZ} = +.50$			$G_{YZ} = -.491$	$E_{YZ} = -.157$	$A_{YZ} = -1$																																																																																																																																																																																																																																																																																																																																																																																																																																				
$G_{XZ} = +.015$	$E_{XZ} = -.211$				$G_{YZ} = +.408$	$E_{YZ} = -.664$				$G_{XY} = +.224$																																																																																																																																																																																																																																																																																																																																																																																																																																						
$B_{XZ} = -.124$	$B_{XZ} = -.633$				$D_{YZ} = +.0253$	$B_{XY} = -.161$				$B_{XY} = +.668$																																																																																																																																																																																																																																																																																																																																																																																																																																						
$G_{YZ} = +.280$					$G_{XY} = +.171$					$G_{XZ} = +.668$																																																																																																																																																																																																																																																																																																																																																																																																																																						
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<table border="1"> <tr><th colspan="2">B</th></tr> <tr><td>.278</td><td>.564</td></tr> <tr><td colspan="2">.158</td></tr> <tr><th>B of GFCB</th><th>B of EDB</th><th>B of AB</th><th colspan="2"></th></tr> <tr><td>-3630</td><td>+573</td><td>-960</td><td>+1048</td><td>+1048</td></tr> <tr><td>-61</td><td>+33</td><td>+97</td><td></td><td></td></tr> <tr><td>-7T</td><td>-5T</td><td></td><td>+24T</td><td>+24T</td></tr> <tr><td>+5</td><td>-225</td><td>-128</td><td>-457</td><td>-229</td></tr> <tr><td>-7T</td><td>-14T</td><td></td><td>+53T</td><td>+53T</td></tr> <tr><td>-2</td><td>-7</td><td></td><td></td><td></td></tr> <tr><td>+3T</td><td>+11T</td><td></td><td>-39T</td><td>-39T</td></tr> <tr><td>-5</td><td>+3</td><td>+9</td><td></td><td></td></tr> <tr><td>-1T</td><td>-0T</td><td></td><td>+2T</td><td>+2T</td></tr> <tr><td>-1</td><td>-3</td><td></td><td></td><td></td></tr> <tr><td>+1T</td><td>+6T</td><td></td><td>-22T</td><td>-22T</td></tr> 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AB			-3630	+573	-960	+1048	+1048	-61	+33	+97			-7T	-5T		+24T	+24T	+5	-225	-128	-457	-229	-7T	-14T		+53T	+53T	-2	-7				+3T	+11T		-39T	-39T	-5	+3	+9			-1T	-0T		+2T	+2T	-1	-3				+1T	+6T		-22T	-22T	-1	-2				+1T	+3T		-13T	-13T	0	-1				0	+2T		-7T	-7T	0	+6	+3	+11	+5	0	0		-1T	-1T	0	-1				0	+1T		-4T	-4T	0	-1				0	+1T		-2T	-2T	0	0		-1T	-1T	0	+2	+1	+5		-3705    +381    -978    +597    +814					-217*    -124*    -441*					<table border="1"> <tr><th colspan="2">B</th></tr> <tr><td>.236</td><td>.363</td></tr> <tr><td colspan="2">.401</td></tr> <tr><th>B of GFCB</th><th>B of EDB</th><th>B of AB</th><th colspan="2"></th></tr> <tr><td>+2140</td><td>-1491</td><td>+4621</td><td>-1624</td><td>-1624</td></tr> <tr><td>+3</td><td>-355</td><td>-604</td><td>-54T</td><td>-273</td></tr> <tr><td>0</td><td>-13T</td><td></td><td>+32T</td><td>+32T</td></tr> <tr><td>-3</td><td>+28</td><td>+81</td><td></td><td></td></tr> <tr><td>-1T</td><td>-7T</td><td></td><td>+23T</td><td>+23T</td></tr> 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<tr><td>-1227</td><td>+20</td><td>+391</td><td></td><td></td></tr> <tr><td>-145</td><td>-9</td><td></td><td></td><td></td></tr> <tr><td>-9T</td><td>-2T</td><td>-31T</td><td></td><td></td></tr> <tr><td>-63</td><td>-33</td><td></td><td></td><td></td></tr> <tr><td>-10T</td><td>-3T</td><td>-133T</td><td></td><td></td></tr> <tr><td>+5</td><td>-11</td><td>-170</td><td>-19</td><td>+19</td></tr> <tr><td>+5T</td><td>+2T</td><td>+113T</td><td></td><td></td></tr> <tr><td>-13</td><td>-1</td><td></td><td></td><td></td></tr> <tr><td>-1T</td><td>0T</td><td>-3T</td><td></td><td></td></tr> <tr><td>+2</td><td>-5</td><td>-95</td><td>-11</td><td>+11</td></tr> <tr><td>+3T</td><td>+1T</td><td>+63T</td><td></td><td></td></tr> <tr><td>+1</td><td>-3</td><td>-55</td><td>-6</td><td>+6</td></tr> <tr><td>+2T</td><td>+1T</td><td>+36T</td><td></td><td></td></tr> <tr><td>+1</td><td>-2</td><td>-32</td><td>-3</td><td>+3</td></tr> <tr><td>+1T</td><td>0</td><td>+21T</td><td></td><td></td></tr> 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AB			-1227	+20	+391			-145	-9				-9T	-2T	-31T			-63	-33				-10T	-3T	-133T			+5	-11	-170	-19	+19	+5T	+2T	+113T			-13	-1				-1T	0T	-3T			+2	-5	-95	-11	+11	+3T	+1T	+63T			+1	-3	-55	-6	+6	+2T	+1T	+36T			+1	-2	-32	-3	+3	+1T	0	+21T			-2	-1				0	0	+3T			0	-1	-20	-2	+2	0	0	+13			0	-1	-11	-1	+1	0	0	+7			0	0	-6	-1	+1	0	0	+4			0	-3	-1	+1		-1450    -48    +92    -44    +44					-23*    -392*    -44*				
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NOTE: MULTIPLY ALL MOMENTS BY 100. \* INDICATES MOMENTS DUE TO ROTATION. T INDICATES SHEAR CORRECTION

TABLE XXVII : End Reactions in Pounds Due to Translation Required to Eliminate Restraints when Joint B Rotates an Amount Corresponding to a Unit Moment in Inch Pounds

		$M_{xy} = +1$ at B			$M_{xz} = +1$ at B			$M_{yz} = +1$ at B		
		$\Delta Y = 2.46 \times 10^{-7}$	$\Delta Z = -1.087 \times 10^{-7}$	$\Sigma F$	$\Delta Y = +.607 \times 10^{-7}$	$\Delta Z = -.796 \times 10^{-7}$	$\Sigma F$	$\Delta Y = -7.40 \times 10^{-7}$	$Z = +1.188 \times 10^{-7}$	$\Sigma F$
At A	$F_x$	0	0	-.0000256	0	0	-.0000366	0	0	-.0000412
	$F_y$	-.000546	0	-.000546	-.0001337	0	-.0001337	+.001629	0	+.001629
	$F_z$	0	+.000240	+.000240	0	+.0001753	+.0001753	0	-.000261	-.000261
At E	$F_x$	0	0	0	0	0	0	0	0	0
	$F_y$	-.00291	+.000999	-.00191	-.000713	+.000731	+.000018	+.00869	-.00109	+.0076
	$F_z$	-.00228	+.00219	-.00009	-.000557	+.00160	+.00104	+.00679	-.00238	+.00441
At G	$F_x$	-.0000365	+.0000621	+.0000256	-.00000893	+.0000455	+.0000366	+.0001089	-.0000677	+.0000412
	$F_y$	-.0000843	+.0000108	-.0000735	-.0000206	+.00000788	-.0000127	+.000252	-.00001173	+.000024
	$F_z$	-.0000245	+.0000715	+.000047	-.00000301	+.0000523	+.0000463	+.0000732	-.0000780	-.0000048

TABLE XXVIII : Final End Reactions in Pounds

		Before Moment and Shear Balances	Correction For Rotation*			Correction For Translation*			Final Reactions
			$M_{xy} =$ -44100	$M_{xz} =$ -59400	$T_{yz} =$ -4400	$M_{xy} =$ -78200	$M_{xz} =$ -163400	$M_{yz} =$ -45900	
At A	$F_x$	+8800	+ 70	-488	0	+ 2	+ 6	+ 2	+8392
	$F_y$	+ 874	-275	0	0	+ 43	+ 22	- 75	+ 590
	$F_z$	+1354	0	+371	0	- 19	-29	+ 12	+1689
			$T_{xy} =$ -12400	$M_{xz} =$ -65500	$M_{yz} =$ -39200	$M_{xy} =$ -78200	$M_{xz} =$ -163400	$M_{yz} =$ -45900	
At B	$F_x$	-5130	- 87	+563	0	0	0	0	-4654
	$F_y$	- 420	0	0	+429	+149	- 3	-349	- 194
	$F_z$	+ 140	0	0	+192	+ 7	-170	-202	- 33
			$M_{xy} =$ -21700	$M_{xz} =$ -38500	$T_{yz} =$ -2300	$M_{xy} =$ -78200	$M_{xz} =$ -163400	$M_{yz} =$ -45900	
At C	$F_x$	-3670	+ 17	- 75	neglect	- 2	- 6	- 2	-3738
	$F_y$	- 450	+ 78	- 20	+7	+ 6	+ 2	-11	- 388
	$F_z$	-1545	+ 16	-163	-2	- 4	- 8	neglect	-1706

\* Rotation and Translation effect given in terms of moment in inch pounds.

correction due to translation and are tabulated in Table XXVIII along with the final end reactions.

### Conclusion

The solution of this type of structural problem is different from the usual case in that no attempt is made to obtain working stresses as near the allowable as possible. The piping arrangement is laid out for the shortest, most economical route. This problem then resolves itself into a matter of determining whether or not the pipe will be overstressed or the end reactions excessive. In a number of cases the determination of the initial fixed end moments and corresponding reactions are so low that it is evident that the piping layout is satisfactory. If this occurs no further analysis is necessary. The group relaxation procedure can be readily applied to structures subjected to concentrated loads providing the torsional properties of the members are available.

The subdivision of the structure into groups makes it possible for more than one person to work on the analysis, thereby expediting the solution.

The Method I probably is more advantageous as a procedure for analysis of a structure with 1 to 4 degrees of freedom for translation. If there are more than 4 degrees of freedom for translation, balancing moments and shears at the same time appears to be better.

APPENDIX I

- A : Elastic area. A subscript XY indicates the elastic area in the XY plane.
- E : Modulus of Elasticity in tension or compression
- F : Force. A subscript of X, Y or Z indicates the direction. A superscript  $\Delta Y = 1$  indicates force due to  $\Delta Y = 1$ ".
- G : Modulus of Elasticity in shear
- I : Moment of Inertia of pipe cross-section
- $I_{xx}, I_{xy}$  : Moment of Inertia and Product of Inertia of elastic area
- J : Polar Moment of Inertia of pipe cross-section
- L : Length of member
- M : Moment. A subscript of XY, XZ or YZ indicates the plane in which moment acts.
- T : Torque. A subscript of XY, XZ or YZ indicates the plane in which torque acts.
- X, Y, Z : Coordinate axes
- $\theta$  : Rotation. A subscript of XY, XZ or YZ indicates the plane in which rotation occurs.
- $\Delta$  : Translation. A suffix of X, Y or Z indicates direction of translation.

APPENDIX II

Development of the Neutral Point Equations

The three dimensional structure shown in Fig. 3a will be used to develop the equations required for the solution of end reactions and moments at the fixed end due to any loading by the Neutral Point Method.

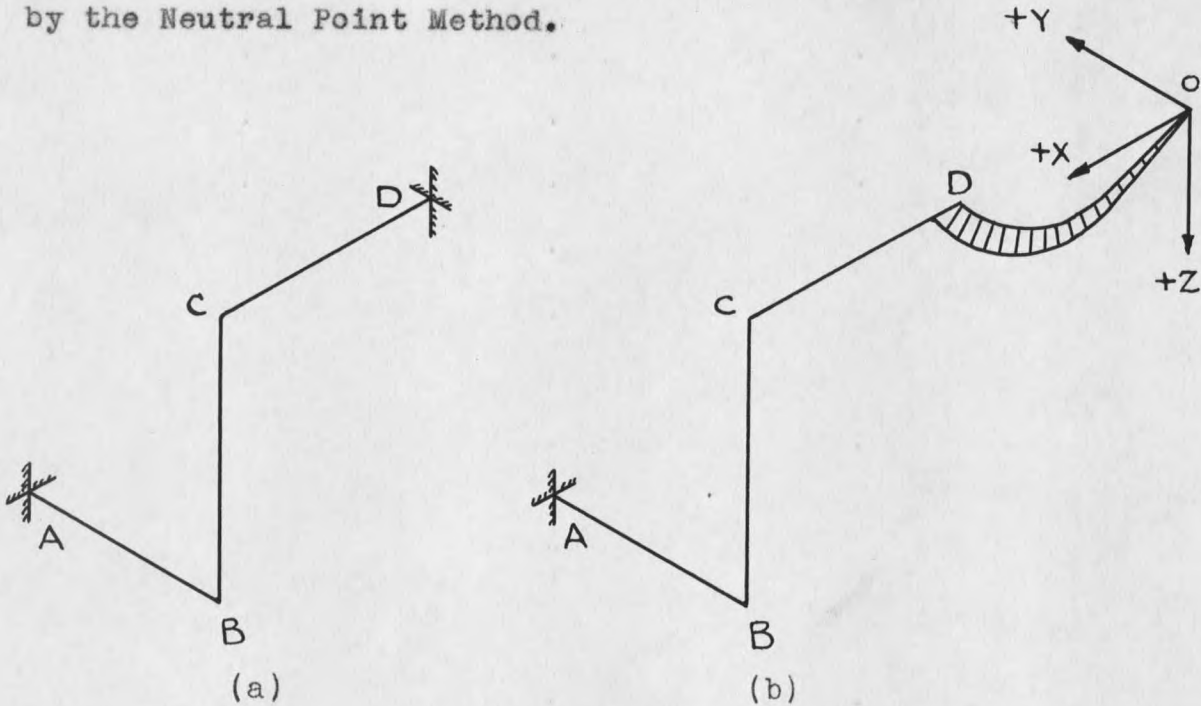


Fig. 3 Three-Dimensional Pipe Structure with Two Fixed Ends

The fixed end D is cut free; a rigid bracket is attached at D and is extended to an arbitrarily chosen point O, from which the coordinate axes originate as shown in Fig. 3b.

When any load is applied to the structure shown in

Fig. 3b, the rigid bracket at O will have deflections  $\Delta X^P$ ,  $\Delta Y^P$  and  $\Delta Z^P$  along with rotations  $\Delta\theta_{xy}^P$ ,  $\Delta\theta_{xz}^P$  and  $\Delta\theta_{yz}^P$ . These deflections and rotations of the rigid bracket at O imply that point D will also have identical deflections and rotations. The conditions shown in Fig. 3a indicate that D is unable to translate or rotate. Therefore it is required to find what forces  $F_x$ ,  $F_y$  and  $F_z$  and moments  $M_{xy}$ ,  $M_{xz}$  and  $M_{yz}$  will be required at O to prevent the deflections and rotations at D due to the loading.

The three deflections and the three rotations due to any load can be determined. The three forces and three moments at O are unknown, therefore if six equations can be written involving the unknowns, they will be determined.

Using the general method of indeterminate structures, the following six equations can be written:

$$3) \Delta X^P + Xdx^{X=1} + Ydx^{Y=1} + Zdx^{Z=1} + M_{xy}dx^{M_{xy}=1} + M_{xz}dx^{M_{xz}=1} + M_{yz}dx^{M_{yz}=1} = 0$$

$$4) \Delta Y^P + Xdy^{X=1} + Ydy^{Y=1} + Zdy^{Z=1} + M_{xy}dy^{M_{xy}=1} + M_{xz}dy^{M_{xz}=1} + M_{yz}dy^{M_{yz}=1} = 0$$

$$5) \Delta Z^P + Xdz^{X=1} + Ydz^{Y=1} + Zdz^{Z=1} + M_{xy}dz^{M_{xy}=1} + M_{xz}dz^{M_{xz}=1} + M_{yz}dz^{M_{yz}=1} = 0$$

$$6) \Delta\theta_{xy}^P + Xd\theta_{xy}^{X=1} + Yd\theta_{xy}^{Y=1} + Zd\theta_{xy}^{Z=1} + M_{xy}d\theta_{xy}^{M_{xy}=1} + M_{xz}d\theta_{xy}^{M_{xz}=1} + M_{yz}d\theta_{xy}^{M_{yz}=1} = 0$$

$$7) \Delta\theta_{xz}^P + Xd\theta_{xz}^{X=1} + Yd\theta_{xz}^{Y=1} + Zd\theta_{xz}^{Z=1} + M_{xy}d\theta_{xz}^{M_{xy}=1} + M_{xz}d\theta_{xz}^{M_{xz}=1} + M_{yz}d\theta_{xz}^{M_{yz}=1} = 0$$

$$8) \Delta\theta_{yz}^P + Xd\theta_{yz}^{X=1} + Yd\theta_{yz}^{Y=1} + Zd\theta_{yz}^{Z=1} + M_{xy}d\theta_{yz}^{M_{xy}=1} + M_{xz}d\theta_{yz}^{M_{xz}=1} + M_{yz}d\theta_{yz}^{M_{yz}=1} = 0$$

The explanation which follows for equation 3 holds similarly for the remaining five equations. As was previously stated, it is required to find forces  $F_x$ ,  $F_y$  and  $F_z$  and moments  $M_{xy}$ ,  $M_{xz}$  and  $M_{yz}$  which will eliminate the deflections and rotations due to the applied load. Each of the forces  $F_x$ ,  $F_y$  and  $F_z$  when applied at  $O$ , will not only affect the deflection in the direction in which it is applied but also in the other two directions, along with producing an angle change of the rigid bracket in each plane. The same holds for the application of  $M_{xy}$ ,  $M_{xz}$  and  $M_{yz}$  at  $O$ . Therefore  $\Delta X^P$  will be eliminated by the sum of the products made up of the actual forces or moments and the deflection in the  $X$  direction due to a unit load at  $O$ .

Using the principles of virtual work, the deflections and rotations due to a unit load and due to a unit moment at  $O$  can be evaluated, leaving six equations with six unknowns.

Figs. 4a through 4f show the application of a unit load or a unit moment at O. The moment diagrams, plotted on the compression side, are drawn on the figures. These are used with the equations of virtual work;

$$9) \quad 1 \times \Delta = \int Mm ds/EI \qquad 10) \quad 1 \times \theta = \int Mm ds/EI$$

To find the value of  $dx^{X=1}$ , a real load of unity is applied at O in the positive X direction, and a unit virtual load is also applied in the positive X direction. Fig. 4a can be used for each loading.

$$1 \times dx^{X=1} = \int_D^C Y_1^2 ds/EI + \int_C^B Y_1^2 ds/GJ + \int_B^A Y^2 ds/EI + \\ \int_D^C Z_1^2 ds/EI + \int_C^B Z^2 ds/EI + \int_B^A Z_2^2 ds/GJ$$

Note that each integral is of the form  $\int y^2 dA$  which represents a moment of inertia about the X axis. The first three integrals represent the moment of inertia about the X axis of the orthogonal projection of the elastic area,  $ds/EI$ , in the XY plane, if  $ds/EI$  is used as  $dA$ ;  $ds/GJ$  can be resolved to  $1.25ds/EI$ . Let the first three integrals be replaced by  $I_{xx}$ , the conventional symbol for moment of inertia about the X axis. The second three integrals represent the moment of inertia about the X axis of the orthogonal projection of the elastic area,  $ds/EI$ , in the XZ plane, which will be replaced by the

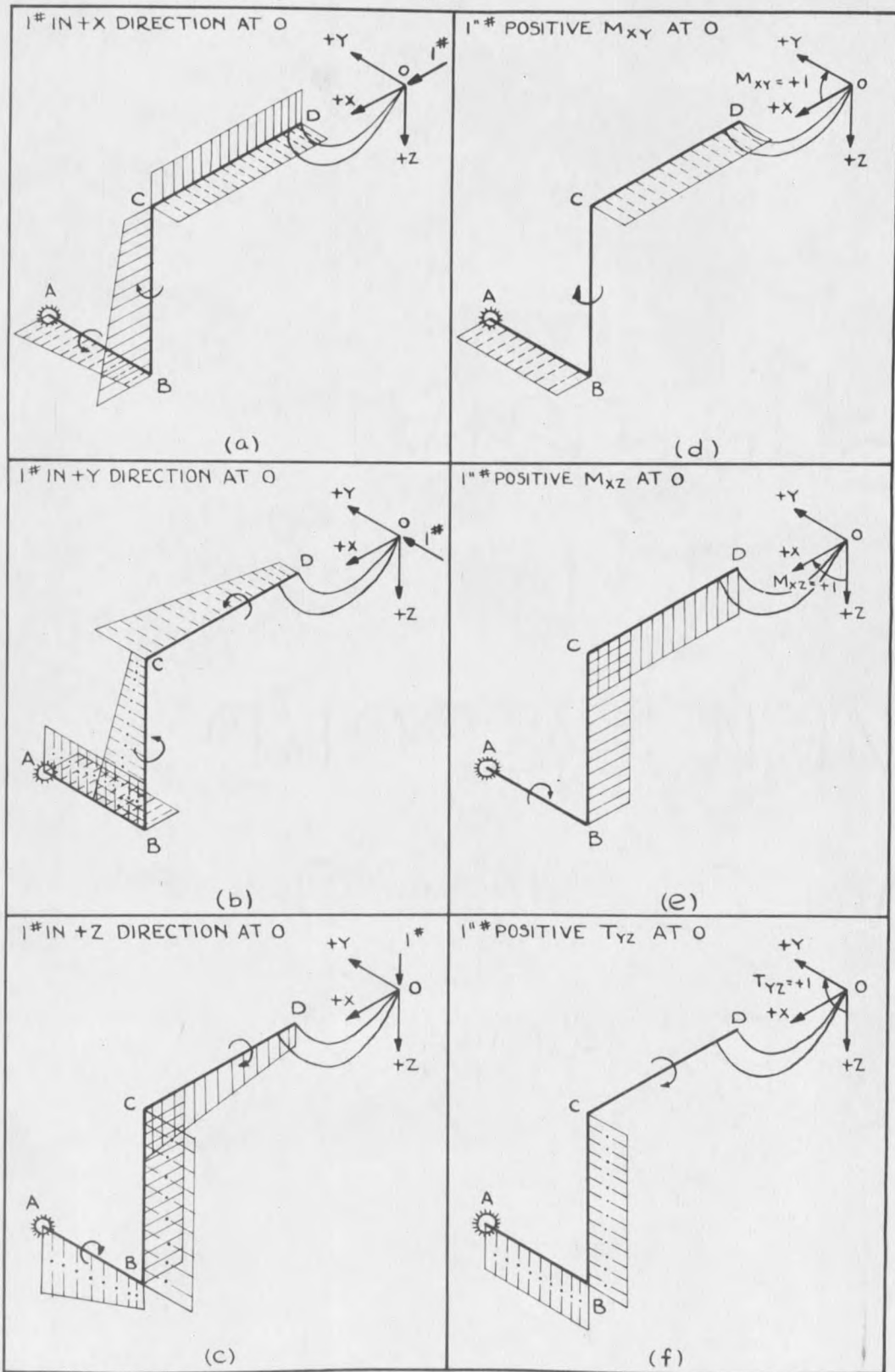


FIG. 4 Unit Load Applied at Origin of Coordinate Axes

symbol  $I'_{xx}$ . This will make

$$dx^{X=1} = I_{xx} - I'_{xx}$$

To find the value  $dx^{Y=1}$ , a real load of unity is applied at O in the positive X direction as shown in Fig. 4a and a unit virtual load is applied at O in the positive Y direction as shown in Fig. 4b. Then

$$1 \times dx^{Y=1} = - \int_D^O Y_1 X ds/EI - \int_C^B Y_1 X_1 ds/GJ - \int_B^A Y X_1 ds/EI$$

Each of these integrals takes the form of  $\int xy dA$  which represents the product of inertia of an area. Using the conventional symbol for the product of inertia,  $I_{xy}$ , the three integrals may be replaced by it, since they represent the product of inertia of the elastic area in the XY plane; and

$$dx^{Y=1} = -I_{xy}$$

To find the value of  $dx^{Z=1}$ , a real load of unity is applied at O in the positive X direction as shown in Fig. 4a and a unit virtual load is applied at O in the positive Z direction as shown in Fig. 4c. Then

$$1 \times dx^{Z=1} = - \int_D^O Z_1 X ds/EI - \int_C^B Z X_2 ds/EI - \int_B^A Z_2 X_2 ds/GJ$$

These integrals represent the product of inertia of the elastic area in the XZ plane. So

$$dx^{Z=1} = -I_{xz}$$

To find the value of  $dx^{M_{xy}=1}$ , a real load of unity is applied at O in the positive X direction as shown in Fig. 4a and a virtual unit positive moment is applied at O in the XY plane as shown in Fig. 4d.

Then

$$1 \times dx^{M_{xy}=1} = \int_D^C Y_1 ds/EI + \int_C^B Y_1 ds/GJ + \int_B^A Y ds/EI$$

This integral takes the form of  $\int ydA$  which is a moment of an area. Again letting  $ds/EI = dA$ , the integrals represent a moment of the elastic area in the XY plane about the X axis.

If  $Q_{xx}$  is used for the symbol to denote the moment of the area, then  $dx^{M_{xy}=1} = Q_{xx}$

To find the value of  $dx^{M_{xz}=1}$ , a real load of unity is applied at O in the positive X direction as shown in Fig. 4a and a virtual unit positive moment is applied at O in the XZ plane as shown in Fig. 4e. Then

$$1 \times dx^{M_{xz}=1} = - \int_D^C Z_1 ds/EI - \int_C^B Z ds/EI - \int_B^A Z_2 ds/GJ$$

This set of integrals, as in the previous paragraph, represent a moment of an area. Here they are the moment of the elastic area in the XZ plane about the X axis. Let  $Q'_{xx}$  represent this moment of the area, then

$$dx^{M_{xz}=1} = -Q'_{xx}$$

From a visual inspection of Fig. 4f it can be seen that  $\frac{dM_{yz}}{dx} = 0$  because the virtual unit positive moment applied at 0 in the YZ plane has no moment in the same planes as the positive unit load applied in the X direction shown in Fig. 4a. This would make the virtual work equation equal to zero.

The remaining deflections or rotations due to a unit load or unit moment may be found in a similar manner. Now equations 3 through 8 may be written as:

$$3a) \Delta X^P + X(I_{xx} + I'_{xx}) - YI_{xy} - ZI_{xz} + M_{xy}Q_{xx} - M_{xz}Q'_{xx} = 0$$

$$4a) \Delta Y^P - XI_{xy} + Y(I_{yy} + I'_{yy}) - ZI_{yz} - M_{xy}Q_{yy} - M_{yz}Q'_{yy} = 0$$

$$5a) \Delta Z^P - XI_{xz} - YI_{yz} + Z(I_{zz} + I'_{zz}) + M_{xz}Q_{zz} - M_{yz}Q'_{zz} = 0$$

$$6a) \Delta \theta_{xy}^P + XQ_{xx} - YQ_{yy} + M_{xy}A_{xy} = 0$$

$$7a) \Delta \theta_{xz}^P - XQ_{xx} + ZQ_{zz} + M_{xz}A_{xz} = 0$$

$$8a) \Delta \theta_{yz}^P - YQ_{yy} + ZQ_{zz} + M_{yz}A_{yz} = 0$$

The origin of the coordinate axis 0 was arbitrarily chosen. By choosing an origin such that the Q terms which represent the moment of an area are zero, the above equations can be simplified. The moment of an area will be zero if the moment is taken about a centroidal axis. Therefore if the origin 0 is taken as the centroid of the elastic area,  $ds/EI$ , the Q terms will be eliminated and the six equations will reduce to

$$3b) \Delta X^P + X(I_{xx} + I'_{xx}) - YI_{xy} - ZI_{xz} = 0$$

$$4b) \Delta Y^P - XI_{xy} + Y(I_{yy} + I'_{yy}) - ZI_{yz} = 0$$

$$5b) \Delta Z^P - XI_{xz} - YI_{yz} + Z(I_{zz} + I'_{zz}) = 0$$

$$6b) \Delta \theta_{xy}^P + M_{xy} A_{xy} = 0$$

$$7b) \Delta \theta_{xz}^P + M_{xz} A_{xz} = 0$$

$$8b) \Delta \theta_{yz}^P + M_{yz} A_{yz} = 0$$

These are the general equations of the Neutral Point Method for a three dimensional structure.

APPENDIX III

In Fig. 5 is shown a ring of elastic material which is cut and separated an amount  $R\phi$ . For small values of angle  $\phi$ , the arc  $R\phi$  will be equal to the chord, therefore by similar triangles,

$$\Delta X/R\phi = Y/R \quad \text{and} \quad \Delta Y/R\phi = X/R$$

or  $\Delta X = Y\phi$  and  $\Delta Y = X\phi$

Fig. 6 shows the projection in the YZ plane of system GFCB with a negative unit rotation applied at E. The corresponding deflections will be

$$+\Delta Z = Y\theta = 43.3''$$

$$-\Delta Y = Z\theta = -11.3''$$

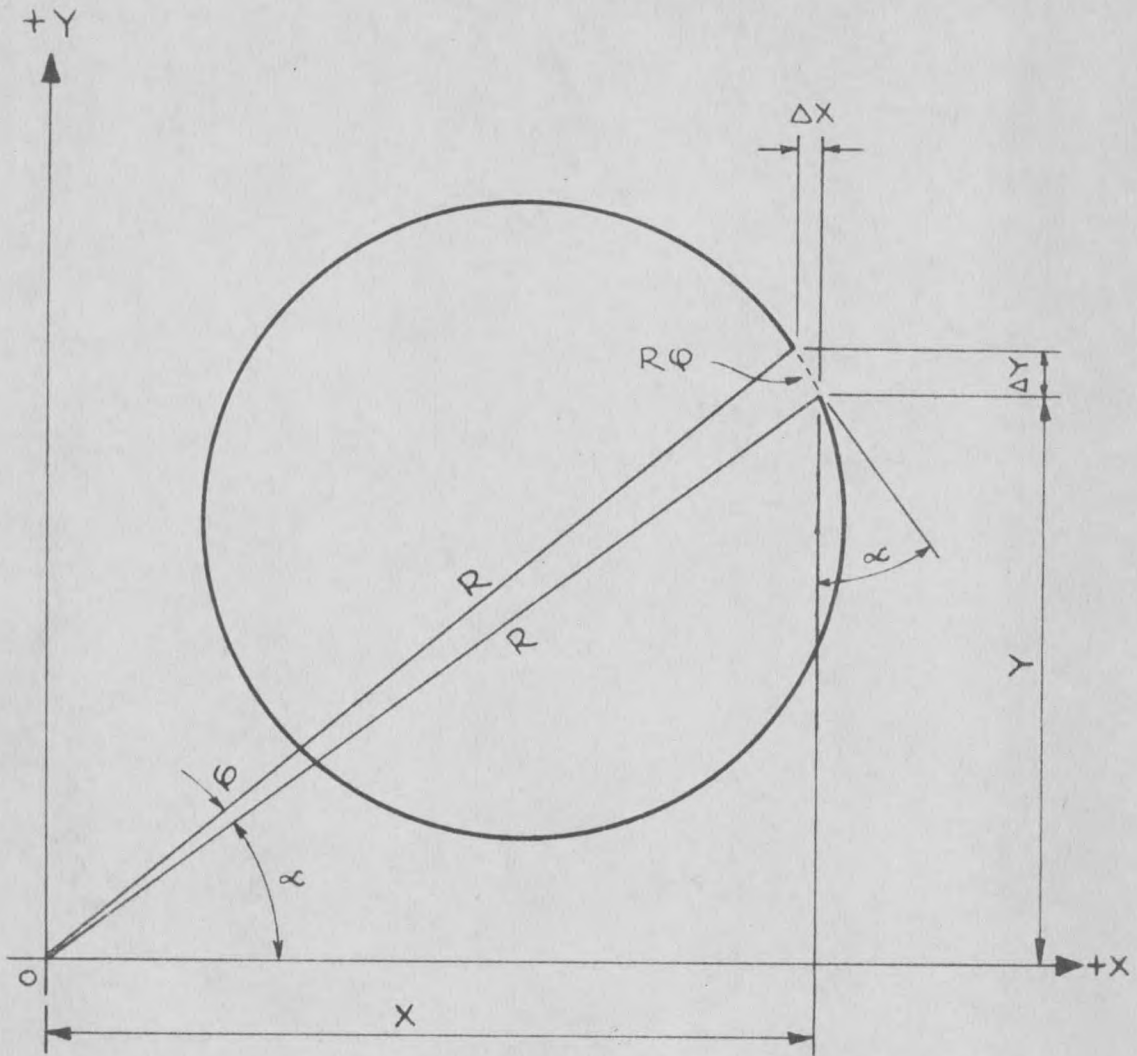


Fig. 5 Ring of Elastic Material Cut and Separated

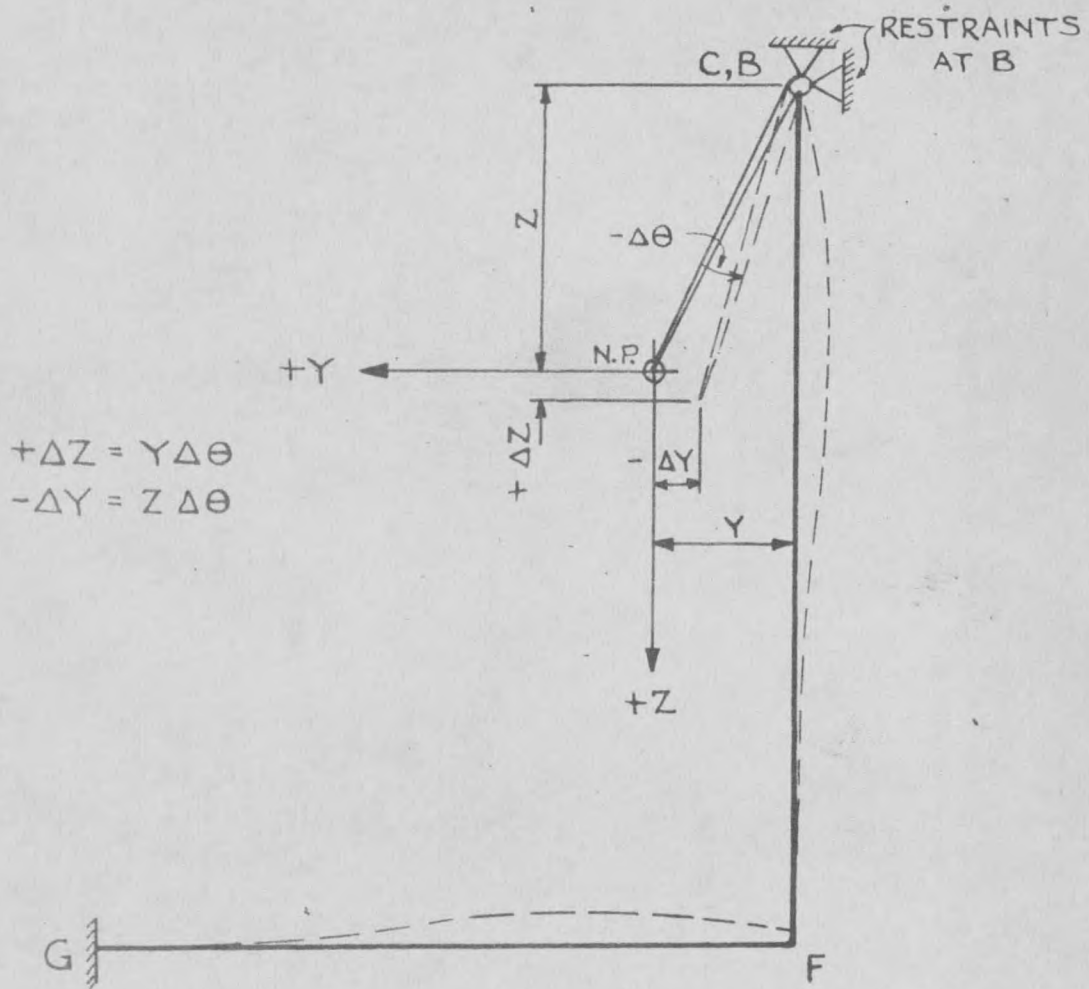


Fig. 6 Displacements of Neutral Point Due to Negative Unit Rotation of B in YZ Plane

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