

MATHEMATICAL CONCEPTS OF NURSERY SCHOOL CHILDREN

by

ALICEANN L. HULLER

A problem submitted to the Graduate Faculty in partial
fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

HOME ECONOMICS


Major Professor

MONTANA STATE COLLEGE
Bozeman, Montana

June, 1962

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION.....	1
The Problem.....	1
Procedures.....	2
Limitations.....	2
II. REVIEW OF LITERATURE ON MATHEMATICAL CONCEPTS.....	3
III. TEST PROCEDURE AND RESULTS.....	8
The Tests.....	8
Method.....	9
Results.....	11
IV. OBSERVATIONS RECORDED DURING TESTING.....	15
V. SUMMARY AND CONCLUSIONS.....	17
APPENDIX.....	18
Appendix A: Reproduction of Number Recognition Charts.....	19
Appendix B: Number Recognition Data Sheet.....	20
Appendix C: Number Production Data Sheet.....	21
LITERATURE CONSULTED.....	22

LIST OF TABLES

Table	Page
1. Comparison of Counting Behavior of children ages 48 to 57 months and 58 to 66 months.....	12
2. Comparison of Correct Responses of Numbers 1 through 4 and 5 through 8.....	13

LIST OF FIGURES

Figure	Page
1. Frequency and Configurations of Numbers on Recognition Test.....	10
2. Percentage of Correct Recognition and Production Responses Made by All of the Children for Numbers 1 through 8.....	13

CHAPTER I

INTRODUCTION

It is a source of pride to parents when their young children begin to use numbers. They try to teach counting and rote learning of age, address, phone number, and any other numbers of which the child may be aware. The ability of a pre schooler to count may represent a status symbol to his parents. Impressive though this verbalization is, it does not necessarily denote true knowledge of numbers.

Of the formation of mathematical concepts, Piaget writes:

It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth.¹

This study has been designed to compare the ability to recognize numbers with the ability to produce numbers, in pre school children.

The Problem

When children have grasped the essential idea of number and have developed an ability to manipulate numbers accurately they are ready for teaching. The writer has set up a test which compared the capacity of the children with

¹Piaget, Jean, "How Children Form Mathematical Concepts", reprint from Scientific American, Nov. 1953, p. 2.

whom she works to: recognize numbers of eight or less when presented in varied spatial arrangements, and to produce the same numbers, when requested, with wooden cubes.

The practical consequences of such a test might be to assess readiness for use of numbers in order to: (1) teach children when they are ready to profit from teaching, and (2) to provide practice and experience with numbers which will help prepare for success in school.

Procedures

The first procedure used in investigation was a survey of the literature on mathematical concept formation, available in the Montana State College Library. Second, a test was developed and given to a group of nursery school children which was adapted from generally accepted methods of testing number concepts. Third, the results of these tests were analyzed and discussed.

Limitations

Because of the amount of time available and the accessibility of subjects this study was limited to a group of four and five year old children regularly enrolled in the Montana State College Nursery School. All had shown some counting ability and had had exposure to use of numbers.

CHAPTER II

REVIEW OF LITERATURE ON MATHEMATICAL CONCEPTS

In order to measure mathematical concepts in nursery school children, one must first determine what constitutes a number concept and recognize the aspects of the concept. Further, one needs to become aware of the implications of these concepts for the child, his parents, and teachers.

Wagoner¹ proposes that the child's knowledge of number develops in two distinct ways: recognition of groups, and construction of series-counting. She reports that Binet found that the 4 year old's perception of number included four and often, with some practice, five, while a 2 year old could not recognize, with assurance, numbers beyond three.

Brenner², in his study of school readiness, stated that most three year olds have a concept of two, four year olds a concept of three, and five year olds a concept of four. He suggested that through observations of pluralities, the child develops a concept of 2, 3, 4, 5, and learns how to use these concepts in a meaningful fashion. It is possible that a child may acquire the meaning of 4 prior to that of 3, if he has had more experience with fours than with threes.

Much of the study of number concepts in human children has been based on Piaget's book The Child's Conception of Number.³ Perhaps the most comprehensive of these was the work

¹Wagoner, L. C., "The Development of Ideas--Number," The Development of Learning in Young Children, McGraw-Hill, New York, 1933, p. 267.

²Brenner, Anton, "Reality Perception, Perceptual Differentiation and Readiness for School," The Merrill-Palmer Quarterly, 1958, 4, p. 207.

³Piaget, Jean, The Child's Conception of Number, The Humanities Press, London, 1952, 248 pp.

of Wohwill⁴ on the premise that the development of a concept has its origin in an essentially discriminative function of abstraction, proceeding by a process of gradual elaboration to an eventual state in which the concept exists as a symbolic representation. His findings bore out Piaget's⁵ theory that the child's conception of numbers developed in three stages.

Number concepts are not unique to human beings. In experiments with animals, Wesley⁶ found that rats could be taught a concept of two and three, and Hicks⁷ developed in Rhesus monkeys a concept of threeness.

During the pre-school years, ages four and five particularly, children begin to use numbers as they go about their daily activities. They are interested in numbers and number ideas.⁸ This is not to say that they understand the full meaning of numbers, but rather, that they are becoming acquainted with the language of numbers, grouping objects and counting them, matching one-to-one, and in some cases exhibiting addition

⁴Wohwill, J. F., "A Study of the Development of the Number Concept by Scalogram Analysis," Journal of Genetic Psychology, 1960, 97, pp. 345-377.

⁵Piaget, op. cit., p. 40.

⁶Wesley, Frank, The Number Concept: a Phylogenetic Review and Two Experiments Testing Its Formation in the Rat, Doctoral Dissertation, State College of Washington, 1958, 102 pp.

⁷Hicks, L. H., "An Analysis of Number-Concept Formation in the Rhesus Monkey," Journal of Comparative and Physiological Psychology, 1956, 49, pp. 212-218.

⁸Deans, Edwina, Arithmetic--Children Use It!, Association for Childhood Education International, Washington, D. C. 1954, Bul. No. 94, p. 5.

and multiplication.⁹

At this age a child may have been taught to name numbers from 1 to 10. If ten objects are laid out in a row he can count them correctly. If, however, they are rearranged into a pattern or piled up he cannot count them with consistent accuracy. Although he knows the names of numbers he has not grasped the essential idea of number: "...that the number of objects in a group remains the same, is 'conserved' no matter how they are shuffled or arranged."¹⁰ Children must grasp the principle of conservation of quantity before they can develop the concept of number.

Elkind¹¹ reported that by studying the child's responses to quantity comparisons of increasing difficulty Piaget sought to trace the developmental changes in the form and content of the child's quantitative thought. His results showed that children's success in comparing quantity increased with age.

In their experiment on development of conservation, Wohlwill and Lowe¹² found that the child who lacks conservation will assert that numerical equality between two collections of objects no longer holds following a change in the volume, area, or length over which they extend. They found support for the notion that "...the attainment of the level of conservation marks a clearly defined stage in the formation of the number concept."¹³

⁹Ibid., p. 6.

¹⁰Piaget, Jean, "How Children Form Mathematical Concepts," reprint from Scientific American, Nov. 1953, p. 2.

¹¹Elkind, David, "The Development of Quantitative Thinking: a Systematic Replication of Piaget's Studies," Journal of Genetic Psychology, 1961, 98, p. 37.

¹²Wohlwill, J. F. and R. C. Lowe, "Experimental Analysis of the Development of the Conservation of Number," Child Development, 1962, 33, p. 153.

¹³Ibid., p. 154.

As a part of the Gestalt Test for measuring readiness for school¹⁴ Brenner¹⁵ included two tasks using number concepts. The task of number recognition gives the child a visual external stimulus. The child is asked to show 2, 4, 5, or 3 dots and must choose from the varied configuration on three charts.¹⁶

In the number producing activity there is no visual external stimulus. The child must produce the quantity first in his imaginative thinking and then visibly in the act of taking 3 of 20.¹⁷

A similar number producing activity is introduced in the Stanford-Binet Intelligence Scale¹⁸ at the six-year level. On this test the child is given twelve one-inch cubes and asked to place on a sheet of paper 3, 10, 6, 9, 7.

Through most of the writings on mathematical concepts runs the theory that development of number concepts is a function of age and experience.

Three theoretical views of number conservation are: the reinforcement hypothesis—as the child obtains experience in counting numerical collections of different types and in different arrangements, he gradually learns that alterations in the

¹⁴Developed during the Research Program on Readiness for School, The Merrill-Palmer School.

¹⁵Brenner, Anton, "A New Gestalt Test for Measuring Readiness for School," Merrill-Palmer Quarterly, 1959, 6, pp. 27-51.

¹⁶Ibid., p. 38.

¹⁷Ibid., p. 37.

¹⁸Terman, L. M. and M. A. Merrill, The Stanford-Binet Intelligence Scale. Houghton Mifflin Co., Cambridge, Mass., 1960, p. 84.

perceptual dimensions of a set do not change its number. The differentiation hypothesis - this interprets lack of conservation as a response to an irrelevant but visible cue (length) which typically shows substantial correlation with number. Repeated experience will weaken the association and facilitate conservation responses. The inference hypothesis - from cumulative exposure to the effects of adding an element to a collection, on subtracting one from it, the child may be led to infer conservation as the result of a change involving neither addition nor subtraction.¹⁹

¹⁹Wohlwill and Lowe, op. cit., p. 155.

CHAPTER III

TEST PROCEDURE AND RESULTS

In an attempt to measure the extent of mathematical concepts held by a group of nursery school children, tests of number recognition and of number production were devised. These were adaptations of the Brenner Gestalt Test and the number producing task from the Stanford-Binet Intelligence Scale.

The Tests

Number recognition. The number recognition task was presented as four charts on which quantities one through eight were symbolized. The numbers 2, 3, 4, and 5 appeared with the greatest frequency.

Number recognition probably is less demanding than number producing. In the number recognition activity the child was given a visual stimulus which evoked a recollection of the child's concept of the number requested. The process required not only the selection of a "3" out of the varied constellations but also the recognition of 3 as symbolized in four different constellations.



It was assumed that the task increased in difficulty as the numbers became larger. However, not all possible groups of 3 are equally easy or difficult for an individual child or for all children. A particular 4 may be easier for one child than a particular 3, although in general difficulties increase with the increase in elements which constitute a number. The reason is that each number value which is higher than the previous one

allows for an increased variability of symbolic representation.¹

Number producing. One inch wood cubes were used for the number producing task. The child was asked to give the experimenter a specified number of the cubes. In some cases, after the child had produced two groups of blocks the experimenter asked how many she now had altogether.

The same number of trials were given in this task as in the number recognition task. The numbers appeared in the same frequency but in a somewhat different order.

When a child is able to project his concept of 3 or 4 into the unstructured group of cubes and structure it by taking out 3 or 4, then the child must have a clear concept of these numbers. The child must really produce the number. This is a high degree of abstraction for preschool children.²

Recording. During the testing all the childrens' number responses were recorded on data sheets.³ Indication was made whether the child operated on the level of simultaneity or counting by "S" and "C", respectively. Observational notes were made by the recorder and remarks made by the child, considered pertinent to the test, were inserted.

Method

Subjects. All children enrolled in the afternoon session at the Montana State College Nursery School, who were present during the week that the testing was conducted, were included in the study. One male, a victim of cerebral palsy, with

¹Brenner, Anton, "A New Gestalt Test for Measuring Readiness for School," Merrill-Palmer Quarterly, 1959, 6, pp. 38-39.

²Ibid., p. 36.

³Sample data sheets are included in the appendix.

vision and hearing loss, was tested but not included in the statistical analysis. The age range of the subjects was from 48 months to 66 months.

Apparatus. The experiment was carried on in the room adjoining the nursery school playroom which is used as the teacher's office and was familiar to all the children. They have been, on occasion, permitted to play in the room. Earlier experiments were done here using the same table and chairs.

The stimuli apparatus for the number recognition test consisted of four 10 by 15 inch white charts, each with five different groups of $3/4$ inch green felt dots. The groups were arranged in varied configurations of one to eight dots. The frequency and arrangement is shown in Figure 1. A reproduction of the charts is included in the Appendix.

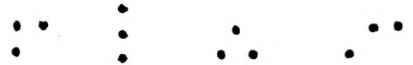
Number	Frequency	Configurations
1	1	
2	3	
3	4	
4	4	
5	3	
6	2	
7	2	
8	1	

Figure 1. Frequency and Configurations of Numbers on Recognition Test

Twelve natural finish, one inch wooden cubes were used in the number producing test.

Design and procedure. The children were divided into two groups which were equated, as nearly as was possible, by age and sex. One group was given the recognition test and then the producing test. The other group was given first the producing and then the recognition test. This was done merely to eliminate the possibility of learning skewing the results of one or the other test. The results are treated as one group of subjects.

The charts used in the number recognition test were presented one at a time. The child was asked "Where are there three spots?" "Show me two." "How many are here?" The sequence was the same for all children.

During the number producing test the child was given twelve cubes and asked to "give me 3." "Hand me 2." The predominance of numbers below five was based on the assumption held by Brenner⁴ that the five year old has a concept of numbers up to and including four. Numbers up to eight were included in an attempt to assess the child's number range.

It was believed that there would be more correct responses on the recognition test than on the producing test. It was further believed that a higher percentage of numbers between one and four would be conceptualized than would numbers between five and eight.

Results

Data were statistically analyzed to determine significance by means of the chi square (χ^2) test. This statistic

⁴Brenner, Anton, "Reality Perception, Perceptual Differentiation and Readiness for School," Merrill-Palmer Quarterly, 1958, 4, p. 196.

is a very useful one in research because no assumptions must be made about the shape of the distributions of the frequencies being tested. It is most commonly used when data are in frequencies such as in the number of responses in different categories.⁵

A comparison by χ^2 revealed no difference between ability to recognize numbers and ability to produce numbers. When results of both tests were compared for boys and girls, there was no significant difference between sexes.

For comparison of younger with older children the group was divided at the median age and the performance of the older group on number recognition and production was significantly better at the .05 level.

Table 1 shows the counting behaviors of the children responding to the various items on the tests.

TABLE 1. COMPARISON OF COUNTING BEHAVIOR OF CHILDREN AGES 48-57 MONTHS AND 58-66 MONTHS

Counting Behavior								
Ages	Simultaneous				Counting			
	Correct		Incorrect		Correct		Incorrect	
	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.	Obs.	Exp.
48-57 months	89	101	47	32	73	89	31	16
58-66 months	165	151	34	48	151	134	10	24
χ^2 .001=16.268 df=3 χ^2 =41.101								

The chi square value of 41.101 is significant at the

⁵Downie, N. M. and R. W. Heath, "Chi Square", Basic Statistical Methods, Harper Brothers, New York, 1959, p. 147.

.001 level. Examination of Table 1 shows that in all cases children in the age group 58-66 months had fewer errors and more correct responses than was expected. Children in the 48-57 months group had more errors and fewer correct responses than was expected.

In order to investigate any difference between simultaneous recognition and production of numbers and counting out of the numbers, responses were divided into two categories, as shown in Table 2. These categories were: numbers 1 through 4 and 5 through 8.

TABLE 2. COMPARISON OF CORRECT RESPONSES OF NUMBERS 1 THROUGH 4 AND 5 THROUGH 8

Numbers	1-4		5-8	
	Obs.	Exp.	Obs.	Exp.
Simultaneous	214	174	38	78
Counting	115	155	109	69

$$\chi^2 .001=10.827$$

$$df=1$$

$$\chi^2=92.19$$

Analysis of Table 2 shows that numbers four and smaller were simultaneously recognized more often than expected while larger numbers were recognized simultaneously less often than expected. The reverse was true for counted numbers. Numbers one through four were counted less often than expected and numbers five through eight were counted more often than expected. These results were also significant at the .001 level.

Figure 2 shows the percentage of correct responses for each number. One hundred percent of all responses were correct for number one. More than 90% of all responses for numbers 2 and 3 were correct. In general, percentage of correct re-

sponses decreased as size of numbers increased. Less than 50% of responses were correct for number 8.

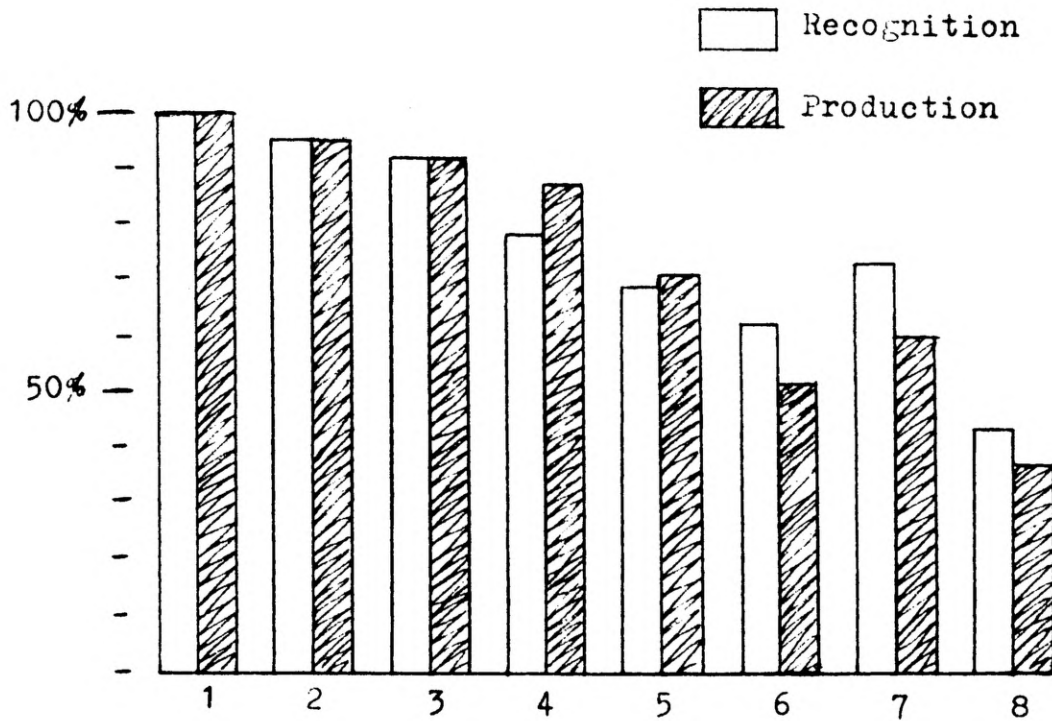


Figure 2. Percentage of correct recognition and production responses made by all of the children for numbers 1 through 8

The data supported the belief that numbers one through four would be more fully conceptualized than numbers five through eight. The assumption that number producing would be more difficult than number recognition was not supported.

CHAPTER IV

OBSERVATIONS RECORDED DURING TESTING

During the testing periods remarks made by the children which pertained to the test, and notations concerning their overt behavior were recorded by a student teacher. Some of the observations probably revealed as pertinent information regarding the counting behaviors of the children as did their actual scores.

In the instances of counting numbers there was great variation in the ability of individual children. A few were able to count accurately to themselves without fingering the dots or blocks, a high degree of counting ability for preschool children. One could count to himself by twos. Another child chanted numbers but did not seem to assign the number to an object or associate numbers with objects. His concept of numbers had not gone beyond the stage of becoming acquainted with the language. He had not grasped the idea of conservation, nor were numbers symbolic representations.

Several children were able to count numbers up to and including five without difficulty. When asked to produce larger numbers they made no attempt to count but rather gave all the blocks. One commented, "Seven makes the whole thing."

One male was able to count only to five. When asked for seven cubes he chose the right number but said, "See if I can count that far. Five and two more. Can't count these two more." About a week later this child spontaneously exhibited ability to count to eight. When questioned he said, "I can count to eight now. Donnie (brother) taught me. I knew by myself, but I forgot, and now he taught me again." During the period of these observations this child had had number experiences, had practiced, and had expanded his working concept of numbers.

Brenner¹ has stated that speed on a test is a rather reliable indicator of a high level of maturity and ability. This generally held true. However, some good performers are slow. Some may find their own ways of using the test apparatus which are not what the researcher intended. Also, immature and uncritical children may run through the test in a very short time.

One notable example was the child with the lowest score who went through the tests very quickly. She mouthed words as though counting to herself and pointed to a group of dots. When asked to produce numbers she never gave more than five and generally two or three.

Another child, with a very high score, stacked her blocks, built towers and bridges and could not be hurried. She became so involved in building that she forgot what number she was asked for.

¹Brenner, Anton, "A New Gestalt Test for Measuring Readiness for School," Merrill-Palmer Quarterly, 1959, 6, p. 38.

CHAPTER V

SUMMARY AND CONCLUSIONS

This study represents an attempt to determine the extent of mathematical concepts held by a group of nursery school children. Fifteen children, ages 48 to 66 months were given number recognition and number producing tests.

It was believed that number recognition, picking out a requested number from a visual stimulus chart would be less demanding and yield higher scores than would number producing. A variety of configurations were presented in order to ascertain whether the subjects had grasped the essential idea that the number of objects in a group remains the same no matter how they are arranged.

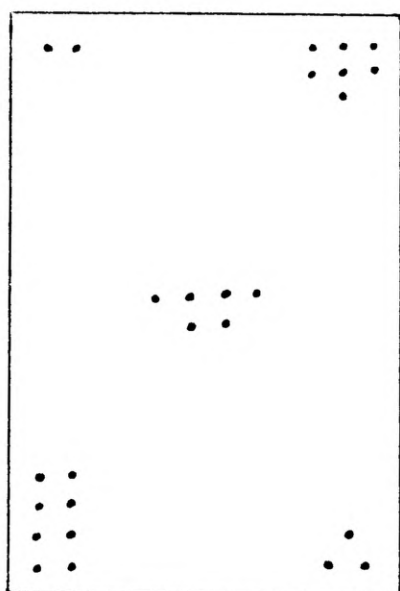
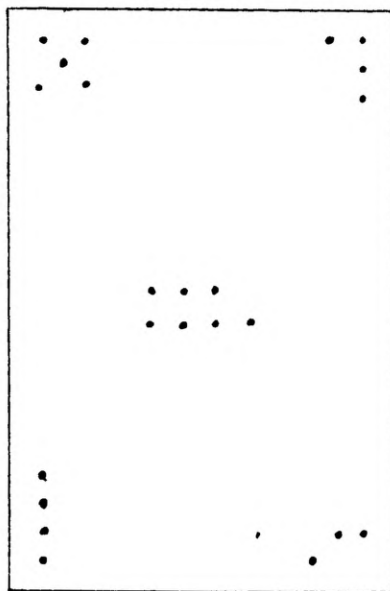
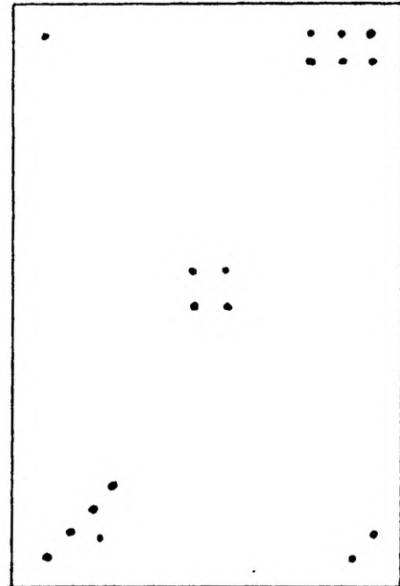
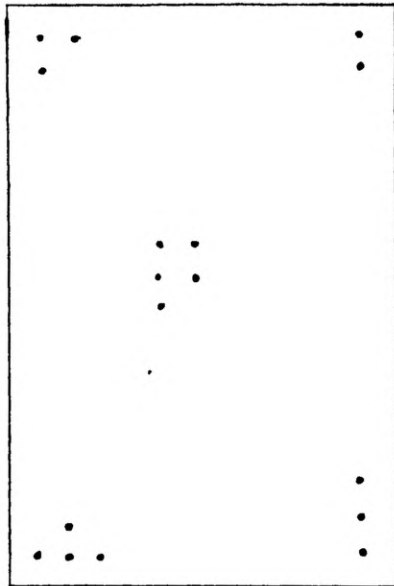
The number producing test contained the same numbers in the same frequency as the recognition test. The child was asked to give the experimenter a certain number from a group of twelve cubes and in some cases was asked to add two groups.

The results indicate no difference in difficulty of the two tests. Numbers one through four were more familiar than numbers five through eight. Older children made significantly better scores over all numbers than did younger children.

The results of this study might not hold true for a larger group. These children had been in a nursery school environment where, while they were not taught, numbers were used whenever an occasion arose. The two children with the lowest scores were newest to the group. One of these was also the youngest tested.

In addition, nearly all the children had been included in previous psychological experiments and were not naive to testing situations.

APPENDIX



Appendix A: Reproduction of Number Recognition Charts

Test I

Name _____ Date _____

Age _____ Time _____

	S	C
3		
2		
5		
4		
3		

4		
5		
6		
1		
2		

4		
3		
5		
7		
4		

6		
8		
3		
2		
7		

Test II

Name _____ Date _____
 Age _____ Time _____

3	2	5
4		
3		

4	1	5
2		
6		

4	3	7
4		
5		

6	2	8
3		
7		

LITERATURE CONSULTED

LITERATURE CONSULTED

- Brenner, Anton, "Reality Perception, Perceptual Differentiation and Readiness for School," Merrill-Palmer Quarterly, 1958, 4, 196-209.
- Brenner, Anton, "A New Gestalt Test for Measuring Readiness for School," Merrill-Palmer Quarterly, 1959, 6, 27-51.
- Deans, Edwina, Arithmetic--Children Use It!, Association for Childhood Education International, Washington D. C., 1954, Bul. No. 94, 56 pp.
- Downie, N. M. and Heath, R. W., "Chi Square," Basic Statistical Methods, Harper & Brothers, New York, 1959, 289 pp.
- Elkind, David, "The Development of Quantitative Thinking: A Systematic Replication of Piaget's Studies," Journal of Genetic Psychology, 1961, 98, 37-46.
- Hicks, Leslie H., "An Analysis of Number Concept Formation in the Rhesus Monkey," Journal of Comparative and Physiological Psychology, 1956, 49, 212-218.
- Mussen, Paul H. (Ed.), Handbook of Research Methods in Child Development, John Wiley & Sons, New York, 1960, 1059 pp.
- Piaget, Jean, The Child's Conception of Number, The Humanities Press, London, 1952, 248 pp.
- Piaget, Jean, "How Children Form Mathematical Concepts," reprint from Scientific American, Nov. 1953, 6 pp.
- Terman, L. M., and Merrill, M. A., The Stanford-Binet Intelligence Scale, Houghton Mifflin Co., Cambridge, Mass., 1960, 363 pp.
- Wagoner, Lovisa C., "The Development of Ideas--Number," The Development of Learning in Young Children, McGraw-Hill, New York, 1933, 267-270.
- Wesley, Frank, The Number Concept: A Phylogenetic Review and Two Experiments Testing Its Formation in the Rat, Doctoral Dissertation, State College of Washington, 1958, 102 pp.
- Wohwill, Joachim F., "A Study of the Development of the Number Concept by Scalogram Analysis," Journal of Genetic Psychology, 1960, 97, 345-377.

Wohwill, Joachim F. and Lowe, R. C., "Experimental Analysis
of the Development of the Conservation of Number,"
Child Development, 1962, 33, 153-167.