Velocity profile measurements of laminar, turbulent and laminar pulsating flow
by Michael Graham Mower

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
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Velocities were measured with a hot wire anemometer which could be positioned within the pipe along a diameter by means of a microscope slide. Radial distance could be measured to within .0001 inch.

Laminar flow profiles were measured in a range of Reynolds numbers from 1000 - 20,000. For plots at Reynolds numbers below 5000, typical parabolic distributions were observed.

Turbulent flow profiles were measured in a range of Reynolds numbers from 10,000 to 25,000. Best fit curves were calculated by an IBM 1620 computer. They are for the buffer zone: (Formula not captured by OCR) and for the turbulent core: (Formula not captured by OCR) The flow was divided into three zones, Laminar Sub-layer (Formula not captured by OCR) 5
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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

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Turbulent flow profiles were measured in a range of Reynolds numbers from 10,000 to 25,000. Best fit curves were calculated by an IBM 1620 computer. They are for the buffer zone:

$$\frac{u}{V_*} = .445 + 4.30 \ln \left(\frac{V_* y \rho}{\mu}\right)$$

and for the turbulent core:

$$\frac{u}{V_*} = 5.23 + 2.53 \ln \left(\frac{V_* y \rho}{\mu}\right)$$

The flow was divided into three zones,

Laminar Sub-layer	$0 < \left(\frac{V_* y \rho}{\mu}\right) < 5$
Buffer Zone	$5 < \left(\frac{V_* y \rho}{\mu}\right) < 27$
Turbulent Core	$\left(\frac{V_* y \rho}{\mu}\right) < 27$

Laminar pulsating flow profiles were measured over a range of α parameters from 1.4 to 3.56. At an α of 3.56 definite flow reversal was observed, with a magnitude of approximately 25% of the mean flow. No turbulence at any time during a period was noticed. There was a maximum phase lag of the velocity relative to the pressure pulse at the pipe centerline of about 33° with α equal to 3.56. The profiles seemed to follow the theoretical curve reasonably well, except that four inflection points were noticed instead of two.

I. INTRODUCTION AND STATEMENT OF PROBLEM

With the rapid advances in medical science it has become increasingly important to find and define the flow characteristics of blood through the human artery. There have been a number of researchers who have attempted to investigate blood flow, with varying degrees of success. Investigation has been divided into two distinct classes, one in which a live subject, (i.e. rabbits or dogs) is used and blood flow is directly observed, and the second where blood flow is simulated by mechanical means. There are disadvantages connected with both methods of research. When a live subject is used, it is difficult to duplicate experimental observations and the instrumentation is, by necessity, limited. Consequently observations are usually made over a comparatively short period of time. With the second method, using simulated blood flow, the disadvantage is obvious, this being just how close the simulated flow is to real blood flow. Another problem with mechanically simulated flow arises when the question of viscosity is brought up. It is generally assumed that viscosity is a constant, however, in research performed by Taylor (2) there is evidence that in actual blood flow the viscosity varies in a radial direction. There have been a number of investigations concerning pulsital type flow with often contradictory findings. The majority of research has been carried out using an incompressible fluid and inferring from pressure measurements the configuration of the velocity profiles.

As a first step to the more complete understanding of pulsital type flow this investigation was concerned with the direct measurement of velocity profiles. Air was the working medium and hot wire anemometry technics were employed for velocity measurements. In this, the first part of a two part study, a flow system was designed and constructed. This system allowed accurate velocity measurements to be made. Using the recorded data varification of theoretical velocity profiles was made for laminar, turbulent, and laminar pulsating flow.

II. REVIEW OF LITERATURE

There has been a considerable amount of research done in the area of pulsital flow in recent years. Much of the work has been carried out in medical schools and hospitals with emphasis on arterial blood flow. McDonald (7) viewed blood flow through the aorta of a rabbit by injecting a blue dye in order to define the flow profiles. The resulting flow outline was filmed at 1500 frames per second. He observed a typical laminar parabolic profile during the early part of systole (rhythmical contraction of the heart). Toward the peak velocity some turbulence was noted, and then a brief period of flow reversal. Velocity was measured by timing the movement of dye in the axial direction. A critical Reynolds number of approximately 300 to 550 was calculated using the standard formula of:

$$Re = \frac{V \times D \times \rho}{\mu}$$

Velocity profiles were calculated from observed pressure gradients in the femoral artery of a dog by McDonald and Womersley (5). No turbulence was observed at any time during a systole. A marked difference in phase angle between the pressure gradient and the flow across the artery was observed with the maximum lag at the centerline. In a study by Help and McDonald (6) similiar to the previous two, a vortex ring of semi-turbulent flow was seen to exist with laminar flow on both sides of this turbulent vortex ring. It was postulated that turbulent flow in the human artery would develope at a Reynolds number of approximately 2000. Using mechanically produced flow in a 1 cm diameter glass tube, Stehbens

(10) observed the various flow regions. The study was aided by injecting dye to illuminate the streamlines. At velocities which produced Reynolds numbers in the vicinity of 2000 turbulent flow was seen to exist throughout the system. During flow reversal a critical Reynolds number of about 200 - 900 was observed in models of various configurations. (i.e. S shape, T shape).

From the review of literature it becomes increasingly clear that a number of contradictions concerning observed phenomenon existed. Also, although velocity profiles had been assumed, none had been measured directly. In the simulation of blood flow some simplifications are obviously necessary. For this study they include:

1. Rigid pipe walls,
2. Constant viscosity,
3. The working medium, air, is considered incompressible at low velocities.

III. EXPERIMENTAL APPARATUS

The pulsating flow system was designed and built with several design criteria in mind. They were:

1. Frequency parameter $= \sqrt{\omega/\nu}\ r_o$,
2. Ease in making velocity and pressure measurements,
3. Ease of changing Reynolds number, frequency of pulsation, and amplitude of pulsation,
4. The necessity for clean, dust free air when operating hot wire anemometers.

With these criteria in mind, the system was designed and consisted of the following components, 2/3 hp fan, 2 inch diameter aluminum pipe, pulse generator, entrance section, test section, pressure taps, cooling coils, and an orifice. (See figure 1).

Pipe for the flow system was selected for economy, availability, ease of handling and reference to the alpha parameter (see section on pulse generator). Aluminum irrigation pipe with an O.D. of 2.0 inches and an I.D. of 1.9 inches satisfied the design criteria and was chosen. Total length of run was 62 feet (372 diameters) which assured fully developed flow at the test section. The test section was 43.6 feet (281 diameters) from the entrance of the pipe.

The entrance section was designed to eliminate the incoming turbulence from the orifice and provide laminar flow at the entrance to the 2 inch diameter pipe. The entrance section consisted of two parts, a filter section and area reduction section. The filter section was 12 inch diameter aluminum tube with fiberglas furnace filters and 24 and 48 mesh copper screen. There were six screens, the first three were 24 mesh and the next three were 48 mesh. Filters were placed in front of each 24 mesh screen. A fiberglas reducing section was fastened to the back of the filter section to reduce the flow area from that of the 12 inch diameter screen section to the 2 inch diameter pipe. The area ratio between the 2 inch diameter pipe and the filter section was 36:1. The area reduction section tapered from a 12 inch diameter cylinder to a 11½ inch diameter cone. The cone decreased from a 11½ inch diameter to a 4 inch diameter in 9 inches and then in a smooth curve from a 4 inch diameter to a 2 inch diameter in a total distance of 3 inches. A mandle was cut to these specifications and liquid fiberglas was used to fabricate the area reduction section. The section was then fastened to the filter section and the junction sanded to a smooth inner surface. The end 2 inches of the transition section was machined to an outside diameter of 2.100 inches with an inside diameter of 1.9 inches. Schedule 80 aluminum pipe was used as a coupling between pipe sections. (See figure 2). All inside joints were smooth in order to eliminate turbulence.

The coupling sections were also sanded smooth. A picture of the completed entrance section is shown in figure 4. Design specifications are shown in figure 3.

The pulse generator was designed using the alpha parameter to determine frequency. A paper by J. R. Womersley (8) indicates that this parameter, which is dimensionless, is approximately 3.5

for a human artery. A short computer program was written giving the α factor and frequency as outputs (see appendix, program 1). The design frequency indicated was between .01 and .05 cps. The stroke necessary to provide the correct amplitude for the pulse was an unknown quantity, so it was decided to use the volume of ½ the system which was 1.38 ft.^{3}. A 16 inch diameter bore with a one foot stroke gives 1.39 cubic feet of displacement which is approximately ½ the volume of the system. To obtain the necessary speed reduction a motor was connected to a speed reducer through a series of pulleys and then to a 12 inch diameter driving wheel. The driving wheel was drilled at one inch intervals so the stroke could be varied from two to twelve inches, and connected to a bellows. Frequency could be varied from .009 to .07 cps.

Bellows of 16 inch diameter flexible ducting were fastened to a plywood frame, and a 2 inch hole was drilled to allow connection to the flow system. A cross head was used to convert circular motion to vertical motion. The bellows were then connected to the system just upstream of the entrance section (see figure 5).

An orifice that gave an area reduction of 16:1 was installed upstream from the pulse generator to dampen any pulses that possibly traveled upstream in the flow system.

The instrument section consisted mainly of a hot wire anemometer probe traversing mechanism. This traversing mechanism raised and lowered the hot wire probe as little as .001 inch and was able to repeat within $\pm$ .0001 inch. A used microscope base was modified by adding fiberglas blocks to the slide to hold the probe and then fastened to a flattened piece of aluminum 2½ inch bar stock which acted as a pipe coupling. A dial micrometer was mounted on top of the microscope slide to measure probe travel.

The probe entrance into the test section was through a .050 inch drilled and reamed hole. (See figure 6).

The inside of the coupling was machined and sanded in place providing a smooth inside surface throughout the test section. (See figure 9).

Pressures within the pipe were measured by a series of static pressure taps. These were positioned 6.0, 25.1, 43.6, and 62.0 feet from the entrance. Each tap consisted of four holes, .050 inch in diameter, drilled 90 degrees apart. Each hole was covered by a one inch square Plexiglass block drilled and tapped for a ¼ inch brass pipe fitting. The four blocks at a given location were connected together to give the average static pressure at the location. The four average static pressures were then ducted to a manifold by ¼ inch plastic hose. (See figure 8).

A .050 inch O.D. pitot tube was installed in the pipe one foot down stream from the test section. The tip was made to coincide with the centerline of the pipe. The total dynamic head pressure from this pitot tube was also run to the pressure tap manifold by a ¼ inch plastic tube, and used to determine the centerline velocity. (See figure 7).

The mean flow in the system was provided by a varible speed, 2/3 hp fan connected on the down stream side of the test section. Two coils of ¼ inch copper tubing approximately 30 inches long and ¼ inch in diameter were installed at the inlet and outlet of the fan for temperature control. Cooling liquid for air temperature control was water at approximately 50° F. Upstream from the fan a section of pipe one foot long was vented to atmosphere and covered with fiberglas filter to filter incoming air. (See figure 10).

Two inch black rubber exhaust hose was used to connect the main and return lines and also to connect the pulse generator to the mainline.

The main flow pipe was insulated with ½ inch armflex rubber pipe insulation to limit heat transfer.

Two thermometers were installed; one inside the flow system and one to record room temperature. These thermometers provided the basis for temperature control in the pipe.

IV. INSTRUMENTATION

The instrumentation necessary for recording the velocity profiles and pressure changes can be divided into two main areas depending on the flow type. They are:

1. Steady flow
    a. Laminar
    b. Turbulent
2. Pulsital flow
    a. Laminar

The instrumentation used to record steady laminar and turbulent flow was basically the same, except in turbulent flow it was necessary to average the fluctuating velocity to obtain a mean velocity. (See figures 11 and 12.)

The following instrumentation was used for steady flow velocity profile measurements:

1. Thermo-Systems model 1010A constant temperature anemometer,
2. Thermo-Systems model 1005B linearizer,
3. MKS Baratron Type 77 pressure meter,
4. Tektronix oscilloscope,
5. Hickok digital voltmeter.

A brief discussion of the anemometer, linearizer, and pressure meter is included for the reader's clarification. The hot wire anemometer is used to measure the velocity of a flowing fluid (in this case air) by measuring the heat transferred from a small heated wire, to the fluid. A small tungsten wire or sensor (diameter .00015 inch) is placed across a probe tip. (See figure 13). This probe or sensor is part of a feedback loop of the anemometer. Changes in environmental conditions around the probe result in increased or decreased heating current from the anemometer bridge, which in turn produces a change in output voltage from the anemometer. This change is caused by changes in the sensor heat transfer rate which increases or decreases the current necessary to maintain a constant wire temperature. Any parameter which then affects the heat transfer rate between the fluid and the sensor would affect the output voltage, i.e. both the fluid density and velocity, and any changes in surface conditions of the sensor. By maintaining all the systems parameters, except the velocity, relatively constant, it is then possible to obtain a relation between fluid velocity and the current through the hot wire sensor. For the case when the velocity is considered to be the only variable, the relation between bridge output and velocity is governed by an equation known as Kings Formula, written as follows:

$$CE_A^2 = (A + B\ u^{1/n})$$

Power output from the hot wire sensor is governed by the probe temperature which in turn is governed by the operating resistance. Cold resistance of the anemometer is determined (resistance of sensor at ambient temperature), and the running resistance is calculated and set on the anemometer.

$$R = Re \left[1 + \beta(ts - te)\right]$$

This equation if often eliminated by defining an overheat ratio.

$$\text{overheat ratio} = \frac{R}{Re}$$

so that: $R = Re$ (overheat ratio)

An overheat ratio of 1.7 was used to obtain all experimental data. It was noticed that the cold resistance would vary approximately $\pm$ 10% from run to run. This could possibly be attributed to the large temperature variations within the lab.

Since the anemometer output E_A varies as the one fourth power of the velocity, it is necessary to square the output twice to obtain a voltage which is linear with velocity past the hot wire. The linearizer accomplishes this by means of three analog function circuits. A linearizer is basically an analog device consisting of one squaring circuit, a difference circuit, and a variable power circuit. The linearizer can provide an output voltage which is an average or instantaneous value of an imput signal which has been squared, depressed by a constant voltage and raised to a variable power between 1.95 - 3.00.

This satisfies the following equations:

E_A = anemometer output

$C1 (E_A)^2$ = squaring circuit 1 output = $(A + B\, u^{1/n})$

$u^{1/n} = (C1\, (E_A)^2 - A)/\, B$

$U = C2\, (C1\, E_A^2 - A)^n$ = squaring circuit 2 output.

If the variable power n is set at the correct value, the output from squaring circuit 2 is linear with the velocity past the hot wire sensor.

A. Linearizer Calibration

The value of $1/n$ was taken as .45 (22) which gave n a value of 2.22. Since it was impossible to set the power of squaring circuit 2 directly to a predetermined value, it was necessary to find the value of the variable power setting which gave a power of 2.22. This was accomplished by trial and error as follows:

$$\text{output Sq2} = (\text{imput Sq2})^n \ (\text{constant})$$

$$\text{or} \quad y = C(x)^n$$

Variable power settings were selected in steps of 100 from 0 to 1000. Imput was supplied by a D-C source over the range of the anemometer output of 1.8 to 3.0 volts. Input and output were recorded. Taking the natural log of both sides gives the following equation which is a straight line:

$$\ln y = \ln C + n \ln (x)$$

The power is then the slope of the straight line. Using the theory of least squares to fit the curves (see appendix, program 2) the n power of each run was determined.

It was found that a variable power setting of 575 corresponded to a value of 2.22. A check of the assumed n value was also made by:

assuming: $$u = C2\,(C1\,E_A^2 - A)^{2.22}$$

but: $$\sqrt{u} \sim P^{1/4}$$

so: $$\sqrt{P} = C3(C1\,E_A^2 - A)^{2.22} = u$$

For different velocities through a calibration unit the dynamic head (P) and linearizer output (u) were recorded and plotted. (See figure 14).

From the plotted results it was concluded that n = .45 gave a linear velocity output.

The MKS Baratron Type 77 pressure meter is a very accurate pressure measuring device with a maximum pressure capability of 1 millimeter of mercury and with eight full scale ranges. These ranges are .0003, .003, .001, .03, .01, .3, .1 and 1 millimeter of mercury. The pressure measuring system consists of a measuring head, pressure indicator and connecting cables. The head contains a diaphragm which moves under pressure and unbalances an AC bridge. The voltage which is proportional to the pressure is amplified and used to deflect the meter. Read-out can be in one of three ways, directly from meter, from an external AC or DC voltmeter or by nulling (rebalancing the bridges).

In addition to the above mentioned equipment, additional equipment was necessary for measurement of pulsital flow. Included was:

8. An 8 track Sanborn FM tape recorder,
9. Hewlett-Packard model RMS voltmeter,
10. RC filter,
11. Timing light,
12, Stop watch,
13. DC source.

V. EXPERIMENTAL PROCEDURE

Experimental procedure differences between laminar and turbulent flow and laminar pulsating flow were primarily in the methods used to record data. In both cases the instrumentation and fan were turned on at least one hour before any testing was done to allow adequate warm-up and instrument stabilization. The fan speed was kept relatively constant by selecting orifices of different sizes depending on the flow region under consideration, i.e. small orifice for laminar flow. After allowing adequate warm-up time, the various instruments were adjusted to their optimum operating conditions. At frequent intervals it was necessary to check and re-adjust the instruments. Variations were caused primarily by changing temperatures within the test area.

For non-pulsating laminar flow the Reynolds number was varied by changing the fan speed and profiles of the velocity were obtained starting at the pipe center and moving toward the wall in .050 inch steps. At each step the linearizer output was recorded. The final step was .010 inch from the pipe wall. The centerline dynamic pressure for each run was recorded, so a relation between the linearizer output and the actual velocity could be calculated. Pressure difference between the second and third pressure tap was recorded, and also the static pressure at the third tap. Operating conditions within the test area were recorded at the beginning of each test session. Included were barometric pressure, ambiant air temperature and the temperature of the flowing air. At all times the linearizer output was monitored on an oscilloscope to verify the laminar flow conditions.

Nonpulsating turbulent flow was recorded in a similiar manner, except the linearizer output was put through a large RC filter to average the velocity fluctuations. To obtain turbulent flow, it was found necessary to provide a trigger in the form of a wire just downstream from the entrance section.

For laminar pulsating flow, test data was somewhat more difficult to obtain. After turning on the pulse generator, it was necessary to monitor the oscilloscope output to varify that the air flow remained laminar at all times during each period. As in laminar flow the probe was lowered in .050 inch steps.

On tape recorder channels 1 and 2 the non-steady, pressure meter and linearizer, voltage outputs were recorded. On channel 3 a one volt pulse was recorded each time the pulse generator passed top dead center. A fourth channel was used for voice comment. The data recorded on magnetic tape was converted to a useful form by an analog to digital converter coupled with an IBM 1620 computer. The output values were punched on cards. Since the input to the converter was limited to one volt, and the linearizer output had a maximum magnitude of about 2.5 volts, it was necessary to use a voltage divider to limit the output to 1 volt or less. A background level of about 30 millivolts was a characteristic of the taped material.

A calibration procedure was devised whereby the digitized output could be corrected to the original instrumentation output. Before each run the following calibrations were carried out.

A zero input (ground) was recorded on each channel for about 30 seconds. Directly following, a one volt input from a DC source was recorded for about the same time period. The 0 and 1 volt signals were averaged after being digitized to obtain levels corresponding to the 0 and 1 volt input. The following correction was then applied to each digitized value. (See appendix, program 5).

Tape input = A + B (OP)

So for 0 input: 0 = A + B (OP0)

and for 1 volt input: 1 = A + B (OP1)

Solving: A = -B (OP0)

B = 1/(OP1 - OP0)

Therefore: corrected (velocity or pressure) = A + B (digitizer velocity or pressure).

Two complete oscillations were recorded at each probe position. After each pulsating run was completed, the pulse generator was turned off, and a standard laminar test was performed.

VI. RESULTS

Extensive use was made of the IBM 1620 computer and plotter during this investigation. Comprehensive programs were written to correct and reduce recorded data to a meaningful form. (See appendix). As an example, with pulsating flow as many as 20,000 points were recorded during one data run of approximately 40 minutes. The results are divided into the following sections:

A. Laminar flow,

B. Turbulent flow,

C. Pulsating laminar flow.

A. Laminar Flow

As explained by Pao (15) in laminar flow the fluid appears to move in layers, with one layer sliding over another. For laminar flow the Hagen-Poiserille theory for incompresible fluids in round pipes is presented by Pao (15) as follows: assuming steady flow and a concentric cylinder as a free body, the momentum equation is:

$$P\pi r^2 - (P + \frac{\partial P}{\partial x}dx)\pi r^2 - \tau(2\pi r)dx = 0$$

Simplifing: $$\tau = -\frac{\partial P}{\partial x}\left(\frac{r}{2}\right) \qquad (1)$$

Newton's law of viscosity states (15):

$$\tau = \mu\frac{du}{dy} \qquad (2)$$

Substituting equation 2 into 1 and substituting for y give:

$$-u\frac{\partial u}{\partial r} = -\frac{\partial P}{\partial x}\left(\frac{r}{2}\right)$$

Solving: $$u = \frac{1}{4\mu}\frac{\partial P}{\partial x}(r_o^2 - r) \qquad (3)$$

Equation 3 is the equation of a standard parabola. For a circular pipe the maximum velocity occurs at the centerline, therefore:

$$u = u_{max}\left(1 - \left(\frac{r}{r_o}\right)^2\right) \qquad (4)$$

Experimental data is shown in graph form in figures 15, 16, 17. (See appendix, program 3). The plots are in a dimensionless form with r/r_o and $\frac{u}{u_{max}}$ as the abscissa and ordinant respectively, and show the theoretical curve as well as the experimental data. Figures 15 and 16 cover the range of Reynolds numbers expected in pulsital laminar flow. Tests were conducted offsetting the probe

$\pm$ 5, $\pm$ 10, $\pm$ 15 and $\pm$ 30 from the vertical to check for flow profile similarity. The offset profiles were indistinguishable from the vertical profiles. Laminar flow was maintained in the flow system to a Reynolds number greater than 20,000 with turbulence occurring only if triggered externally.

B. Turbulent Flow

Turbulent flow is characterized by very erratic motion of the fluid particles with large momentum exchanges between particles. Turbulent flow is believed to commence at a point, usually defined by the Reynolds number, when the ratio of the inertia forces to the viscous shear forces are about 2500 (14). The Reynolds number is a good indicator of the flow region, since it is the ratio of the inertia force over the viscous shear force. The larger the ratio, the more the inertia force predominates, and the more apt there is to be turbulent flow. At the present time there is no completely satisfactory theory explaining turbulent flow, however, a number of semiempirical theories have been presented which seem to agree fairly well with experimental evidence. Prandtl's mixing length theory is probably the most useful of the semiempirical theories developed for turbulent flow and is presented by Pao (15) as follows: based on the concept of momentum exchange, a mixing length L is postulated which is similiar in concept to the mean free path in the kinetic theory of gases. A particle is assumed to travel its entire mixing length with its original momentum and suddenly acquire new momentum at the end of its path. (See figure 19). The fluctuating velocity component u' has the same magnitude as the difference in x- direction mean velocities.

So:

$$u' = L\frac{\partial u}{\partial y} \qquad (5)$$

Multiplying both sides by ρ gives:

$$\rho u' = L \left(\frac{\partial u}{\partial y}\right) \qquad (6)$$

Rate of mass transfer in the y direction is $\rho v' dA$ and τ_t is equal to the rate of momentum exchange in the x- direction per unit area.

Giving:

$$_t = \frac{v' \; dA \;\; L(\partial u/\partial y)}{d A} = v' L \frac{\partial u}{\partial y} \qquad (7)$$

The combinations of equations 6 and 7 gives:

$$\tau_t = u' v' \qquad (8)$$

Which is known as Reynolds shear stress equation.

Assuming: u & v are of the same magnitude:

$$u \approx v = L \left(\frac{\partial u}{\partial y}\right),$$

the final equation is:

$$\tau_t = \rho L^2 \left(\frac{\partial u}{\partial y}\right)^2 . \qquad (9)$$

Also assuming that near the pipe wall,

$$L = ky$$

We have:

$$\tau_o = k^2 y^2 \left(\frac{d u}{d y}\right)^2 \qquad (10)$$

Solving for $\frac{du}{dy}$ gives:

$$\frac{du}{dy} = \sqrt{\tau_o/\rho} \; \frac{1}{ky}$$

Defining: $\sqrt{\tau_o/\rho}$ = V_* (shearing velocity) and solving the equation for u results in the equation:

$$u = \frac{V_*}{k} \, Ln(y) + c \tag{11}$$

Using the boundary conditions $u = u_{max}$

@ $y = r_o$ to solve for C,

$$u = u_{max} + \frac{1}{k} \; V_* \; ln(\frac{y}{r_o})$$

or:

$$\frac{u}{V_*} = \frac{u_{max}}{V_*} - \frac{1}{k} \; ln(\frac{r_o}{y}) \tag{12}$$

Equation 12 for the turbulent core of a smooth pipe becomes:

$$\frac{u}{V_*} = A + B \;\; Ln \; (\frac{V_* y \rho}{\mu}) \tag{13}$$

Three flow zones exist depending on the modified Reynolds number $(\frac{V_* y \rho}{\mu})$. They are:

1. laminar sub-layer,
2. buffer zone,
3. turbulent core.

The parameters $(\frac{u}{V_*})$ and $(\frac{V_* y \rho}{\mu})$ are usually used to define u^+ and y^+ respectively. The parameters u^+ and y^+ are dimensionless numbers that characterize the velocity and distance from the pipe wall.

Equation 13 was solved using the method of least squares to find constants A and B. (See appendix, program 4). The expected plus or minus deviation was also solved for. The following table gives the experimental results. It is noted that insufficient points were recorded in the laminar sub-layer for a curve to be fitted.

TABLE 1

Results of Turbulent Flow Tests

Reynolds Number	Buffer Zone		Turbulent Core	
	A	B	A	B
27,228			4.44 ± .100	2.53 ± .096
23,487			5.02 ± .078	2.47 ± .077
16,366	.409 ± .097	4.13 ± .188	4.79 ± .057	2.60 ± .067
11,206	.63 ± .16	4.26 ± .299	5.77 ± .097	2.51 ± .12
10,658	.296 ± .108	4.53 ± .19	6.11 ± .027	2.56 ± .037

A typical example of the turbulent plot is shown in figure 20. The variation within the curves from the mean at the 95% confidence level was about ± 5% in the turbulent core and about ± 10% in the buffer zone.

The final equations using average values were:

buffer zone: $$\frac{u}{V_*} = .445 + 4.30 \ln \left(\frac{V_* y \rho}{\mu}\right) \qquad (14)$$

turbulent core: $$\frac{u}{V_*} = 5.23 + 2.53 \ln \left(\frac{V_* y \rho}{\mu}\right) \qquad (15)$$

The flow zone divisions observed were as follows:

1. laminar sub-layer: $0 < y^+ < 5$
2. buffer zone: $5 < y^+ < 27$
3. turbulent core: $y^+ > 27$.

The zone divisions are shown in figure 20. The general shape of the plotted data points appears to agree very well with the results of Nikuradse and Reichardt as published by Pao (15). In figure 18, the accepted empirical formula derived by Nikuradse and Reichardt is plotted. The plotted equation is:

$$\frac{u}{V_*} = 5.5 + 2.5 \text{ Ln } \left(\frac{V_* y \rho}{\mu}\right)$$

Also plotted on figure 18 are experimental data points for a Reynolds number of 27,220.

C. Laminar Pulsating Flow

The results obtained from laminar pulsating flow consist primarily of computer drawn velocity profiles showing the relation (phase angle) between the velocity profile and pressure pulses. (See appendix, programs 5 & 6). The α factor was varied so velocity profiles would exhibit both flow reversing and non-flow reversing characteristics. The pressure and velocity tape channels were sampled at a rate of 10 times per second to obtain the pressure and velocity profiles. A typical profile showing flow reversal is shown in figure 21, and one with no flow reversal is shown in figure 22. Plots were also drawn of velocity versus centerline distance at specific points on each period (every 18°). Figure 23 shows a complete cycle including flow reversal and inflection points. Shown in figure 24 is a plot similiar to 23, however, there is no flow reversal. A theoretical solution for laminar pulsating flow is presented by Sarpkaya (11) and is reproduced in part as follows. From the Navier-Stokes equation of motion for an axially symmetrical flow the differential equation for a non-steady pressure gradient is:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial y}{\partial r}\right)$$

Assuming: $u = 0$ @ $r = r_o$ and $-\frac{1}{\rho}\frac{\partial P}{\partial x} = K \cos (nt)$

gives the following equation:

$$f''(r) + \frac{1}{r} f'(r) - \frac{in}{\nu} f(r) = \frac{-k}{\nu} \qquad (15)$$

where u(n, t) has the form f(r) e^{int}.

The solution for this differential equation has the following form:

$$u\ (r,t) = -i\ \frac{k}{n}\ e^{int}\ \left(1 - \frac{J_0\sqrt{\frac{-in}{\nu}}\,r}{J_0\sqrt{\frac{-in}{\nu}}\,r_0}\right) \qquad (16)$$

When solving this equation (14) several important parameters are defined. Included is the α parameter mentioned previously and a parameter λ. The parameter λ is defined as $\frac{V_p}{V}$. A graph of α versus λ was drawn by Sarpkaya (11) showing α necessary for flow reversal at a pipe wall (see figure 25). Figure 25 shows the theoretical curve with the data points from table 2 drawn in. The following table gives the calculated α and λ values for the test runs performed.

TABLE 2

& Values for Laminar Pulsating Flow Tests

Run	1	2	6	8	9	10
α	2.18	2.18	3.56	3.56	2.95	1.97
λ	.65	.302	1.24	1.17	1.19	1.035

In table 3 the phase lag between the pressure and velocity is shown for runs 6, 9, and 10. Distances are measured starting at the pipe centerline and moving toward the wall. Parenthesis indicate interpolated values.

The magnitude of the pressure differences measured between pressure taps 2 and 3 had a mean of about .1 mm of mercury, while the amplitude of the fluctuating component varied from .02 to .047 mm of mercury.

At an α factor of 3.56 and 2.95 definite flow reversal is observed. The reverse flow was approximately 25% of the maximum flow and occurred at the centerline of the pipe. No turbulence was noticed at any time during flow reversal. Flow reversal was seen to be dependent to a large extent on the frequency as shown by runs 9 and 10. Pressure in runs 9 and 10 was approximately the same, but the frequency of pulsation in 10 is decreased by ½. No flow reversal is observed in 10, and a very definite reversal is observed in 9. Phase lag also appears to be dependent on the frequency as seen in table 3 with the maximum lag observed at the centerline as expected. A phase lag of about 30 degrees was observed with $\alpha = 3.56$ and $\lambda = 1.17$.

TABLE 3

Phase Angles for Pulsating Laminar Flow

Distance	Run 6 $\alpha = 3.56$	Run 9 $\alpha = 2.95$	Run 10 $\alpha = 1.97$
.00	(33)	21.6	9.0
.05	30	19.8	8.4
.10	28.8	18.0	6.0
.15	28.2	16.7	(6.0)
.20	28.2	16.7	6.0
.25	24.0	(16.0)	(6.0)
.30	22.8	(15.2)	(6.0)
.35	21.0	14.4	6.0
.40	(20.7)	13.8	(4.8)
.45	20.4	10.8	(3.6)
.50	16.8	(10.5)	(2.4)
.55	(14.9)	(10.2)	1.2
.60	(12.8)	(9.9)	0
.65	(10.9)	9.6	0
.70	9.0	(8.1)	0
.75	(7.0)	(6.6)	0
.80	4.8	(5.1)	0
.85	3.6	3.6	0
.90	NA	NA	NA
.94	NA	NA	NA

NA = not measured.

VII. DISCUSSION OF RESULTS

In an analysis of the results produced by this investigation it is necessary to discuss and speculate about the following items:

1. Performance of flow system,
2. Laminar flow profiles,
3. Turbulent flow profiles,
4. Pulsital-laminar flow profile.

Over-all the flow system produced the results expected of it. There were, however, some good as well as poor points. The entrance section performed beyond expectation, which is shown by Reynolds numbers for laminar flow in excess of 20,000. The pulse generator produced a pressure wave which was somewhat uneven at times showing the need for a more rigid cylinder. Some difficulty was experienced with velocity profile shift from the centerline, due to temperature variations. This non-axial flow was minimized by careful temperature control.

Laminar flow profiles were as expected. They exhibited the typical parabolic velocity profile predicted by theory. An entrance length of 281 diameters proved adequate to produce fully developed laminar flow to a Reynolds number of about 5000. With higher Reynolds numbers the flow was obviously not fully developed. This could easily be seen in the "squared off" velocity profile of figure 17.

Turbulent flow profiles followed the experimentally derived empirical equations for turbulent flow very well. The turbulent flow equation given by Schlichting (21) has the form:

$$u^+ = A + B \ln y^+$$

The accepted values for A and B are 5.5 and 2.5 respectively. (15) Experimental data varified B within 5% at all times, however, the intercept A seemed somewhat dependent on the Reynolds number. The lower the Reynolds number the higher the B value seemed to be. This could possibly be due to the extremely smooth flow characteristics of the entrance section and pipe. This possibly could cause the buffer zone to shift closer to the wall, thereby causing the y intercept to shift upward. Also, the buffer zone was seen to be less by a factor of approximately 10% than the usually accepted value. This also could possibly shift the curve upward.

The results from pulsital laminar flow were not quite what had been predicted by several researchers. Sarpkaya (11) mentions the importance of inflection points and flow reversal as a criteria for determining the stability of the flow.

Definite flow reversal was evident and also four inflection points during each cycle, however, no turbulence was observed. This could possibly be due to the characteristics of the flow system, or the inflection points and flow reversal do not necessarily cause transition of the flow from laminar to turbulent.

Two laminar inflection rings (4 inflection points per diameter) were observed to exist during flow reversal. This is contrary to the theoretical development by McDonald and Womersley (5), which shows only one ring. McDonald (7) in previous work on live subjects noticed the existance of rings during flow reversal.

A theoretically predicted phase lag of about 60 degrees should have been noticed with $\alpha = 3.5$, however, the maximum lag observed was only 30 degrees, which occurred at the centerline of the pipe. There is no reasonable explanation for this phase discrepancy at this time.

VIII. SUMMARY

The purpose of this investigation was to measure velocity profiles in three flow areas. They were laminar, turbulent, and pulsating laminar flow. To accomplish this it was necessary to design and construct an experimental system that could cause a varying pressure gradient to be superimposed on a steady flow. Air was selected as a working fluid, and a hot wire anamometer was used to measure the velocity.

Laminar flow velocity profiles were drawn with Reynolds numbers varying from about 1000 to 20,000. With Reynolds numbers below 5000 the velocity profiles seemed to follow within 4% of the predicted parabolic shape.

Turbulent profiles were plotted and the equations of the lines computed. The equations were close to the emperically predicted ones, however, the buffer zone was noticed to run from y^+ of 5 to a y^+ of 27, which is somewhat less than the accepted limits.

Laminar pulsating profiles were drawn and definite flow reversal was noted at an α of 3.56 and a λ of 1.17. No turbulence was observed during peak or back flow. A maximum phase lag of 30° also occurred at an α of 3.56 and α of 1.17.

LIST OF SYMBOLS AND ABBREVIATIONS

A	Constant used in King's law, universal velocity profile, and the A to D converter correction equations.
B	Constant used in King's law, universal velocity profile, and the A to D converter correction equations.
C	Constant used in King's law.
C1	Constant used in King's law.
C2	Constant used in King's law.
C3	Constant used in linearizer calibration.
D	Diameter.
$\frac{du}{dy}$	Velocity gradient.
E_A	Anemometer output.
J_o	Bessels function, 1st kind, 0 order.
k	Constant used in universal velocity profiles.
L	Prandtl's mixing length.
n	Variable power in linearizer equations.
OP	A to D converter output.
OPO	A to D converter output at 0 volt input.
OP1	A to D converter output at 1 volt input.
P	Pressure.
r_o	Radius.
r	Any point along radius.
R	Running resistance of the anemometer.
R_e	Cold resistance of the anemometer.

t	Time.
t_s	Hot wire temperature.
t_e	Ambient temperature.
u	Velocity at any point in x- direction.
u'	Fluctuating velocity in x- direction.
u^+	Dimensionless velocity, $u^+ = \frac{u}{V*}$
y^+	Dimensionless distance, $y^+ = (\frac{y \rho V*}{\mu})$
V	Mean velocity.
$V*$	Shearing velocity, $V* = \sqrt{\tau_o / \rho}$
V_p	Amplitude of fluctuating velocity component.
v'	Fluctuating velocity in y- direction.

Greek Letters:

α	Frequency parameter.
β	Temperature coefficient.
μ	Dynamic viscosity.
ν	Kinematic viscosity.
ρ	Density.
ω	Angular velocity.
τ	Shear stress.
τ_o	Wall shear stress.
τ_t	Turbulent shear stress.

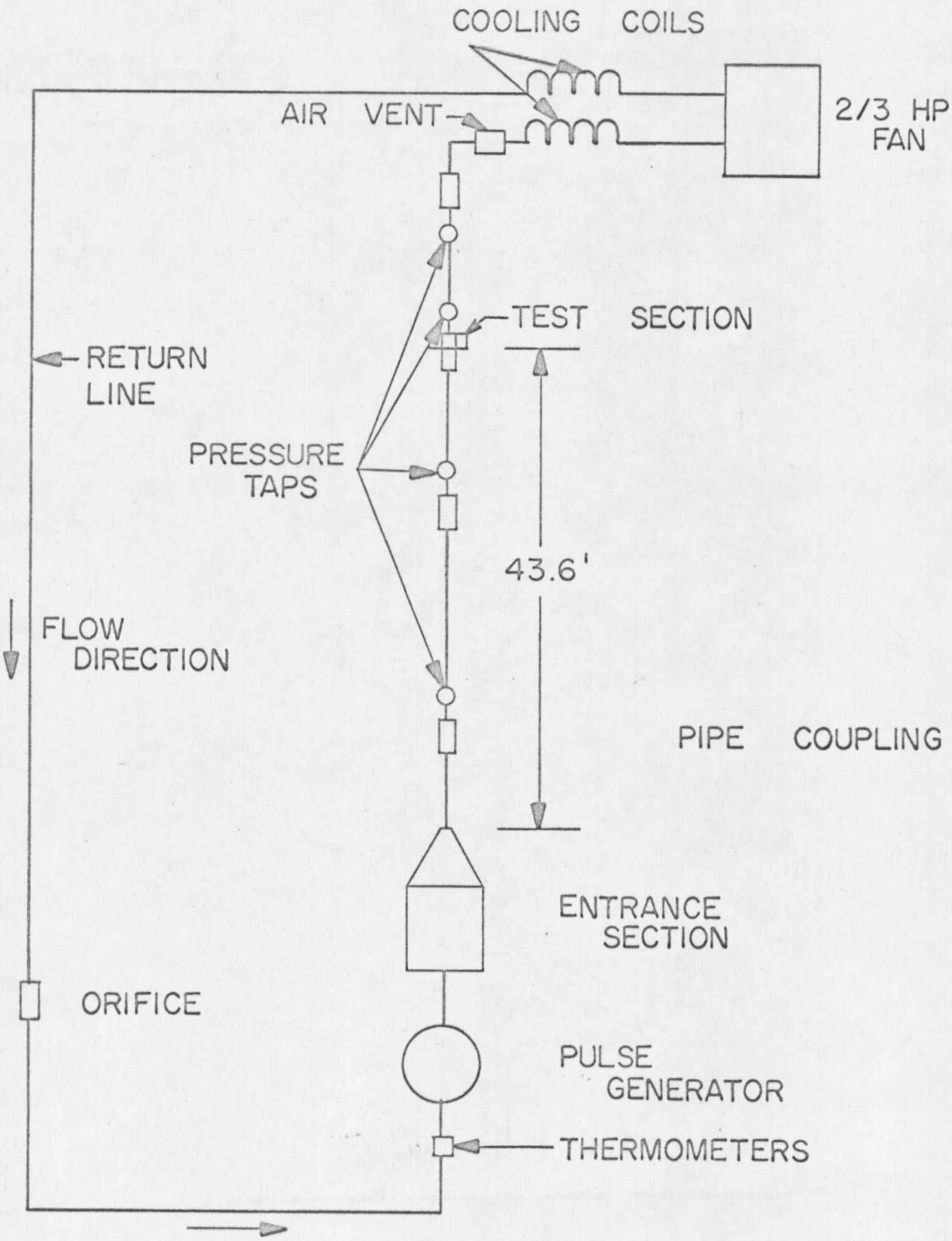

Fig. 1. Sketch of Overall System

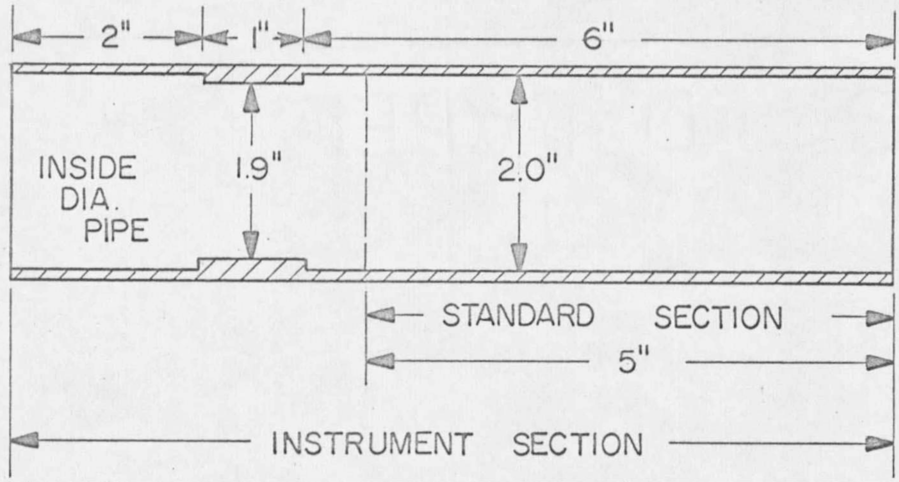

Fig. 2. Sketch of Pipe Coupling

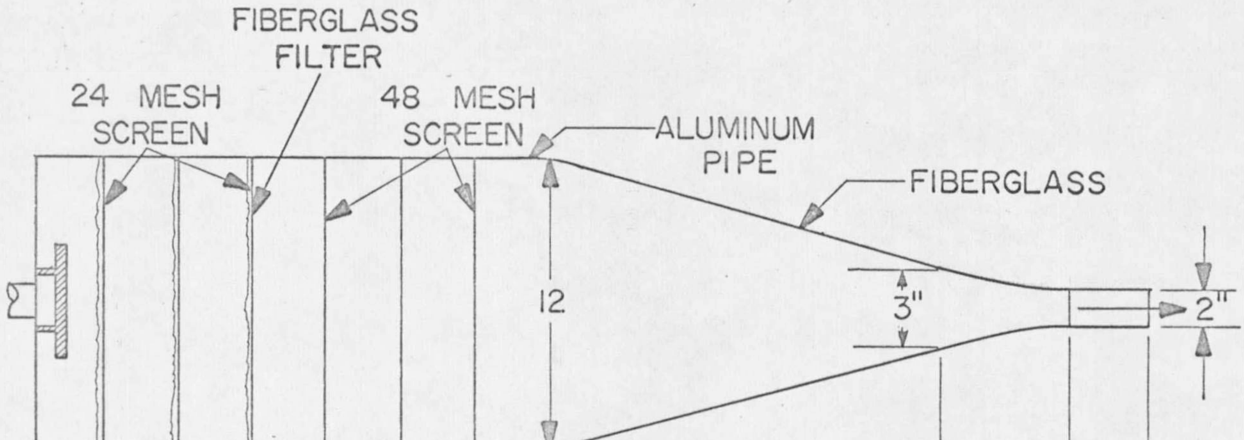

Fig. 3. Design Specifications of Entrance Section

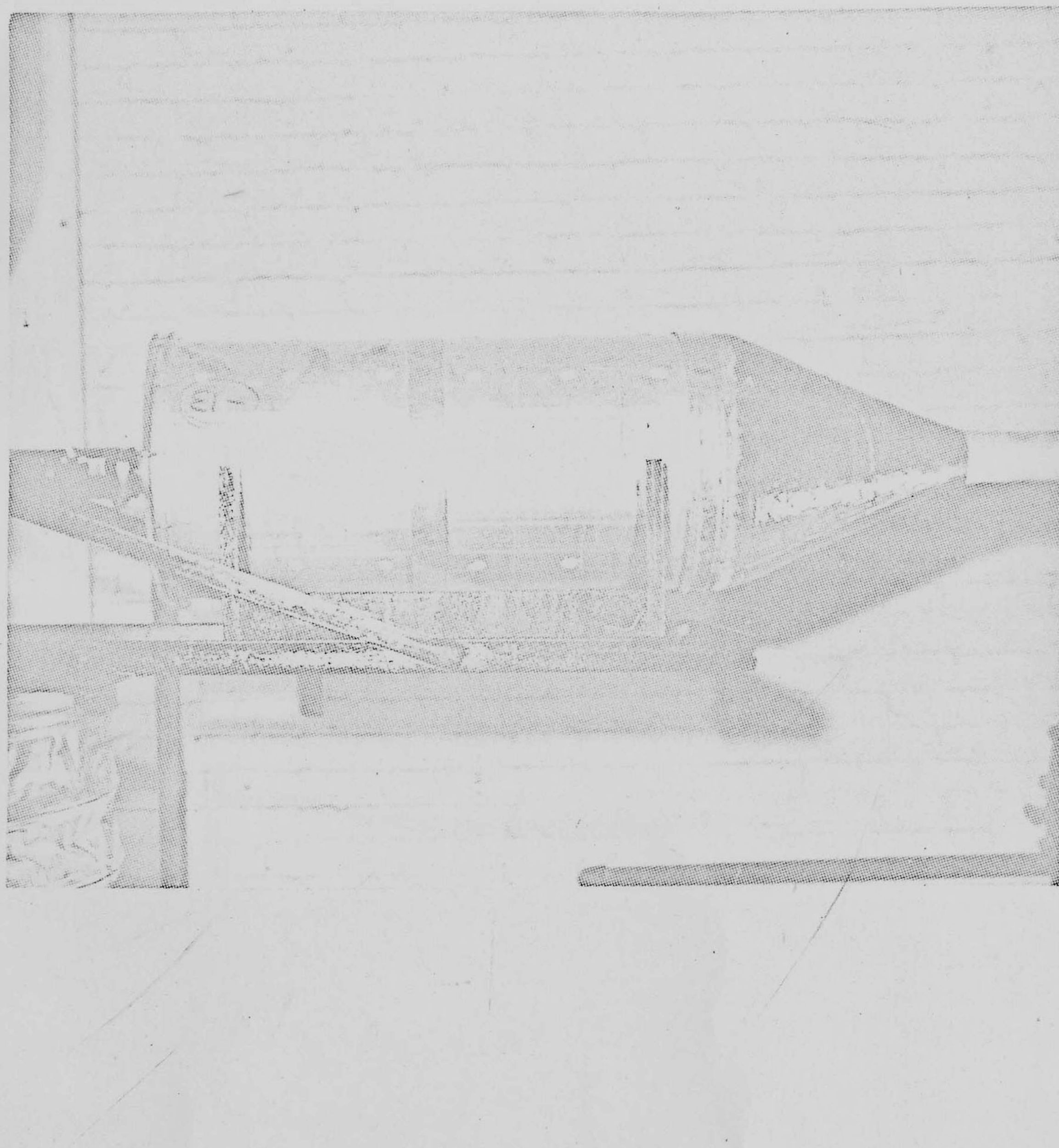

Fig. 4. Picture of Entrance Section

Fig. 5. Picture of Pulse Generator

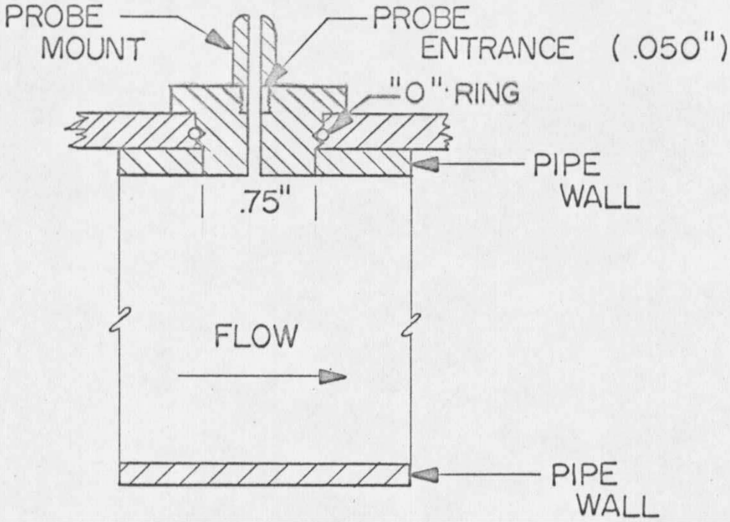

Fig. 6. Sketch of Entrance to Test Section

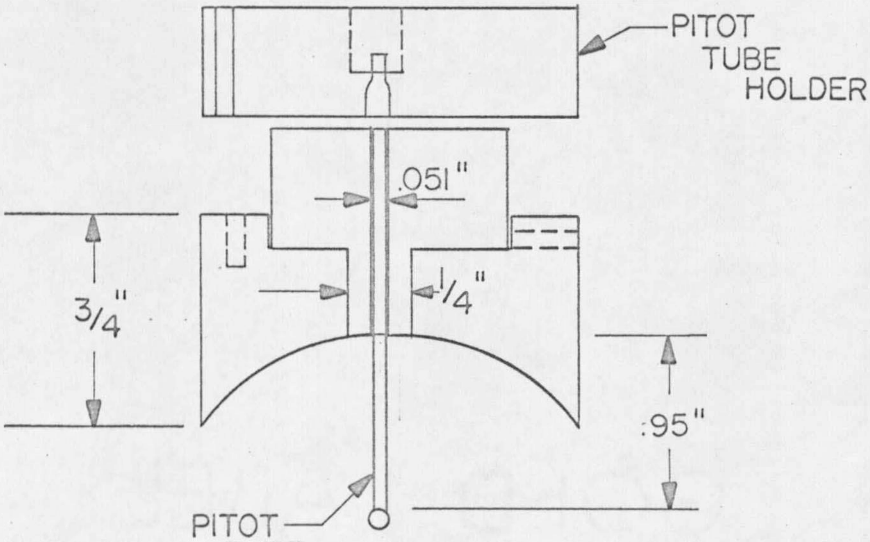

Fig. 7. Sketch of Pitot Tube

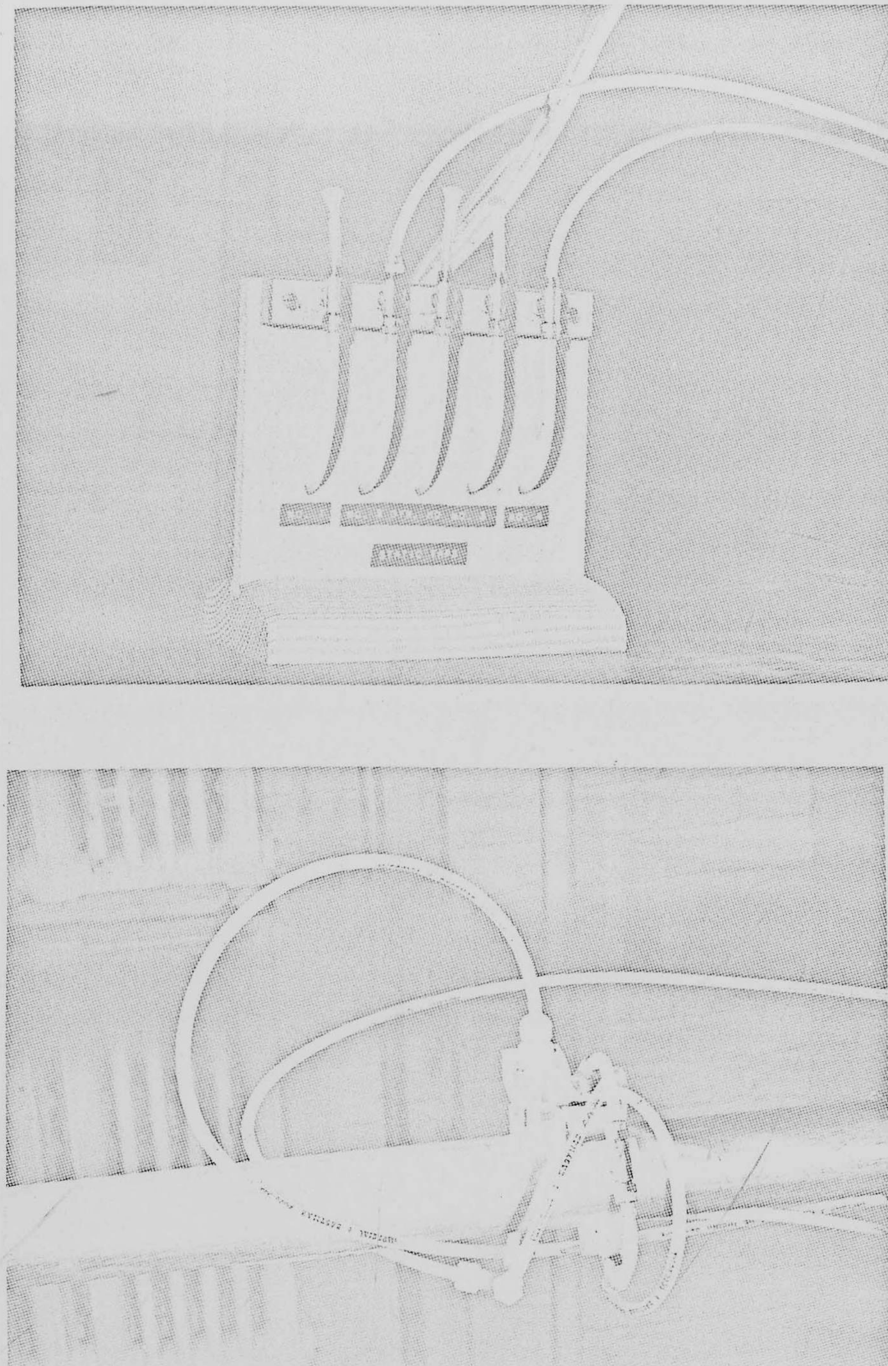

Fig. 8. Picture of Manifold and Pressure Tap

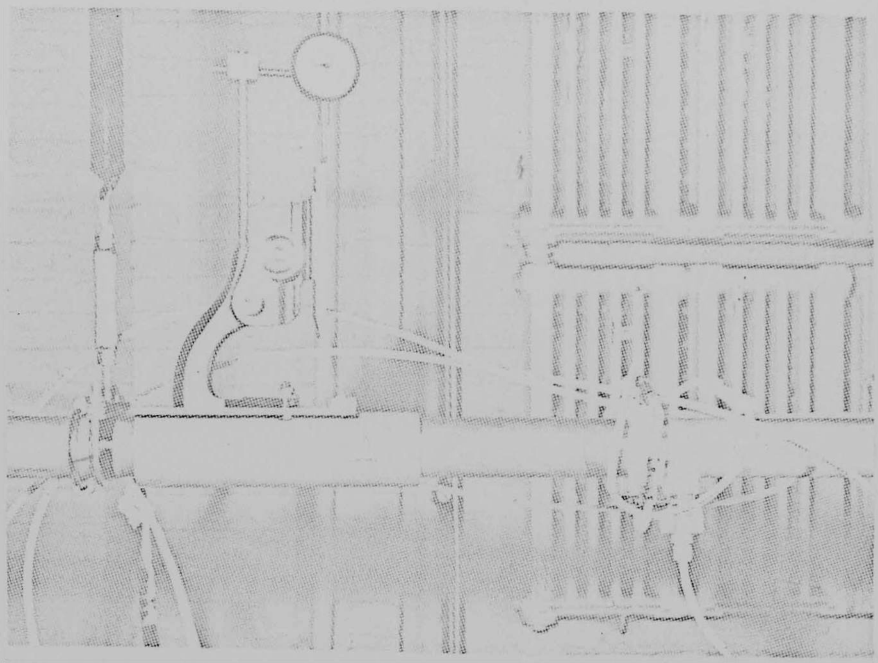

Fig. 9. Picture of Test Section

Fig. 10. Picture of Coils and Vent

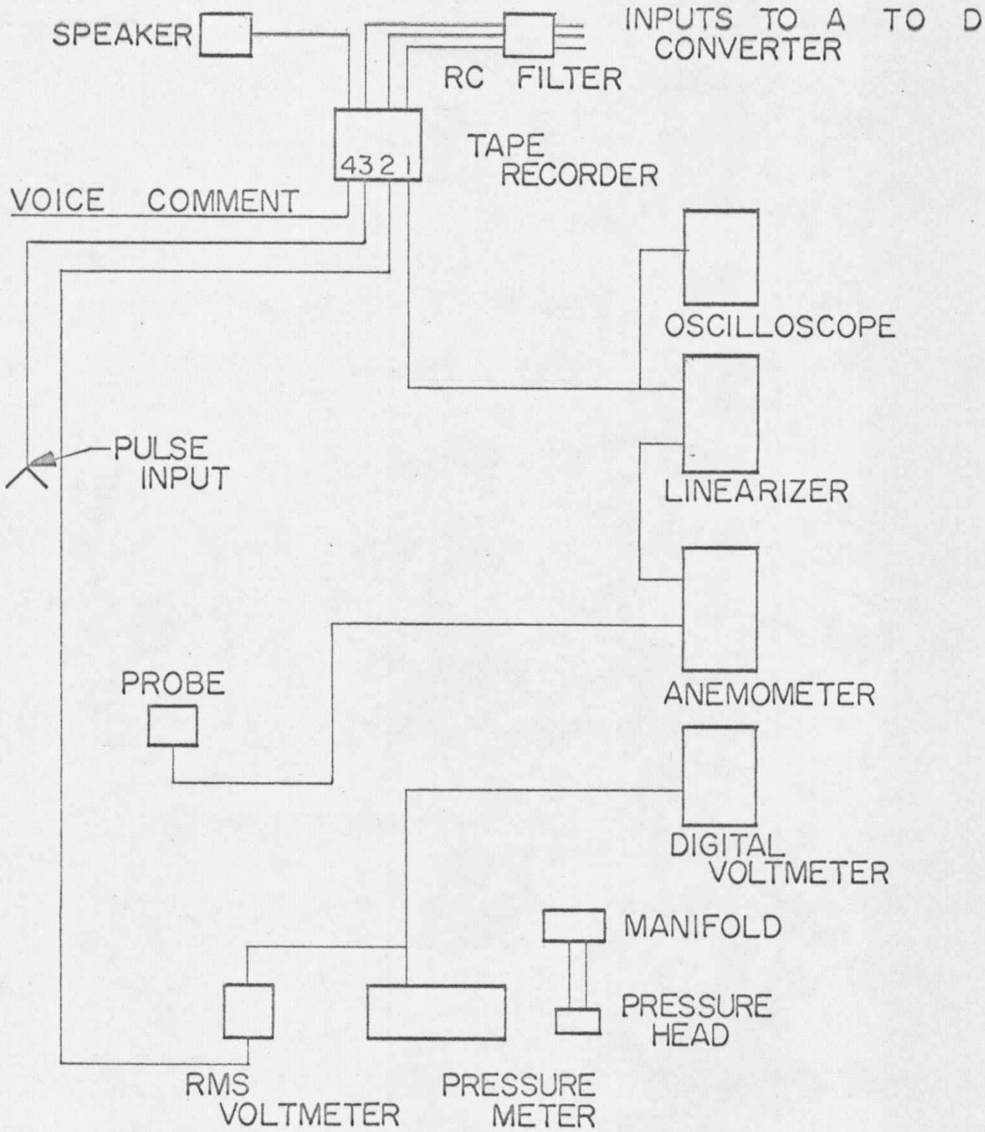

Fig. 11. Instrumentation Layout

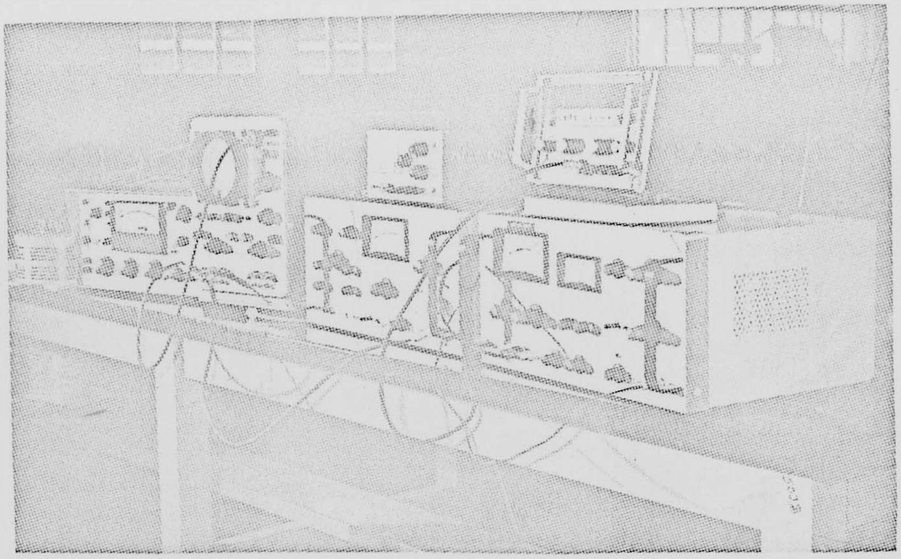

Fig. 12. Picture of Instrumentation Layout

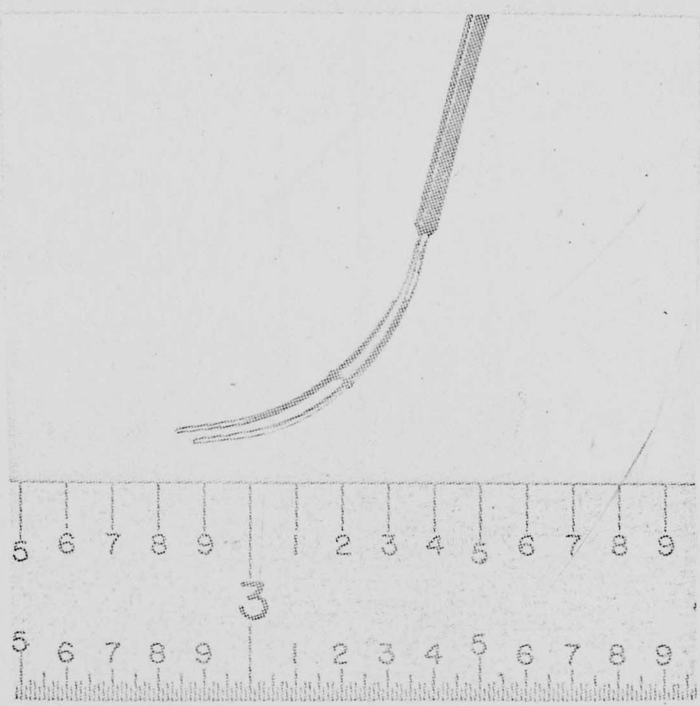

Fig. 13. Picture of Probe

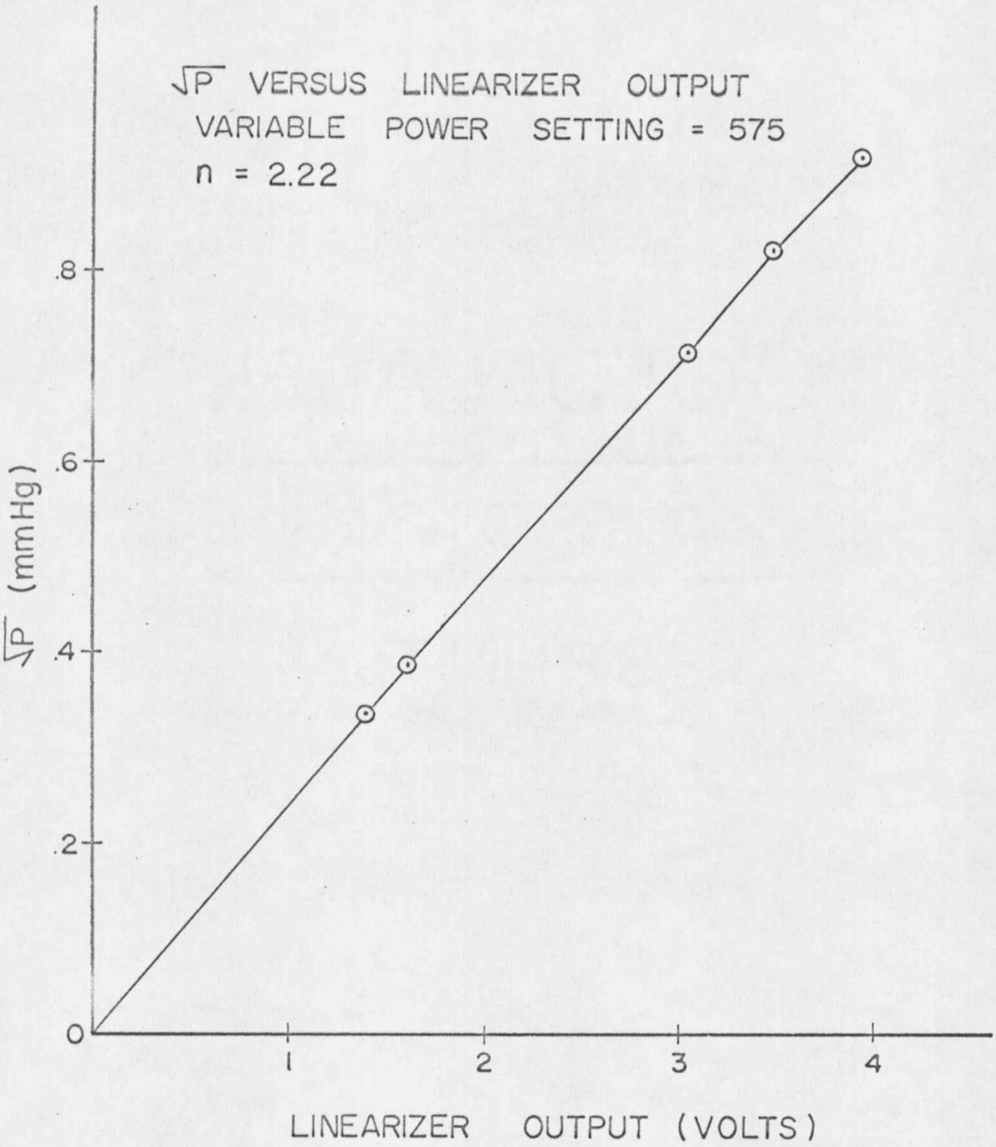

Fig. 14. $\sqrt{P}$ Versus Linearizer Output (P = Total Head)

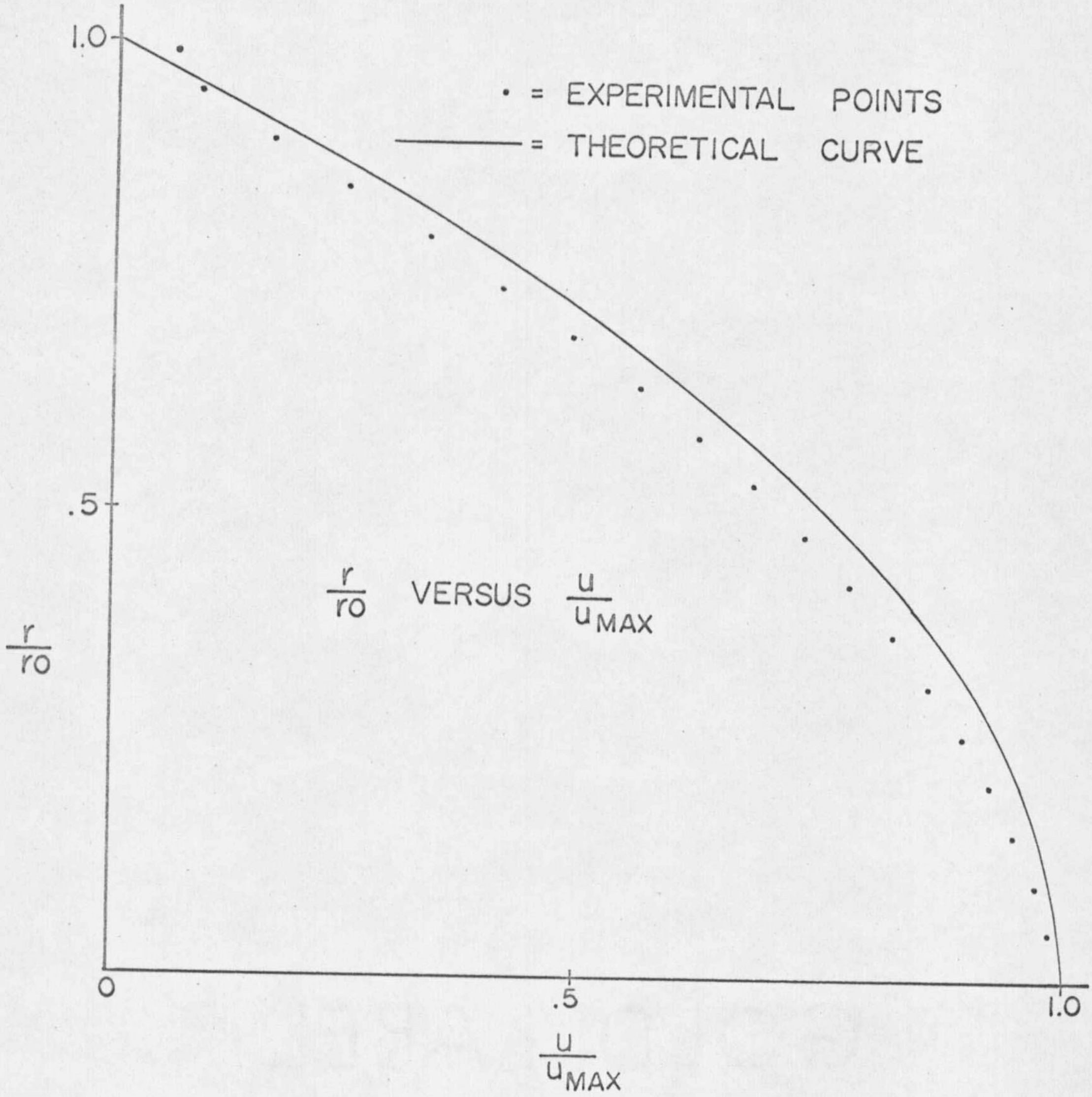

Fig. 15. Plot of Typical Laminar Profile (Re = 2588)

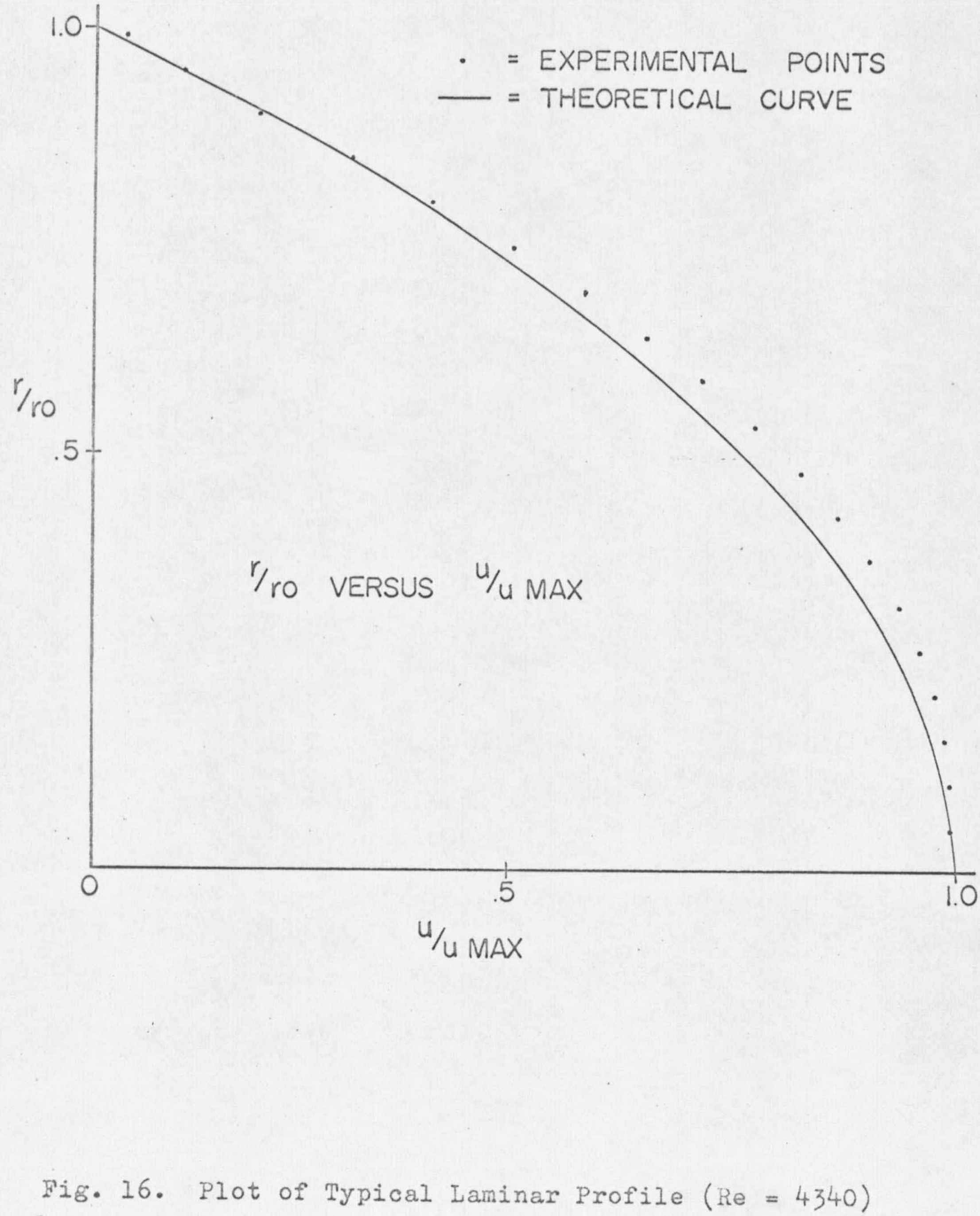

Fig. 16. Plot of Typical Laminar Profile (Re = 4340)

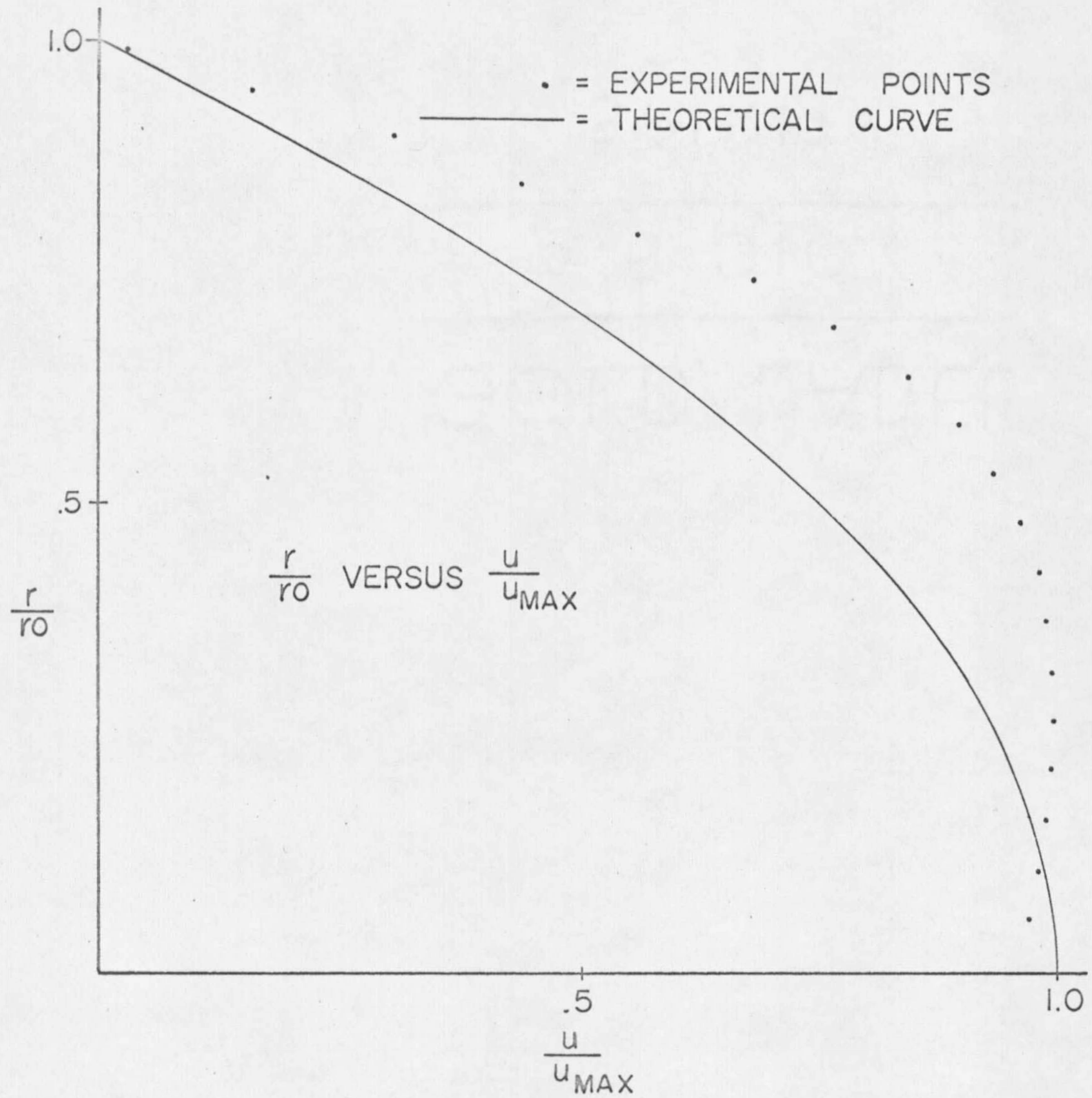

Fig. 17. Plot of High Reynold Number Laminar Profile (Re = 11,555)

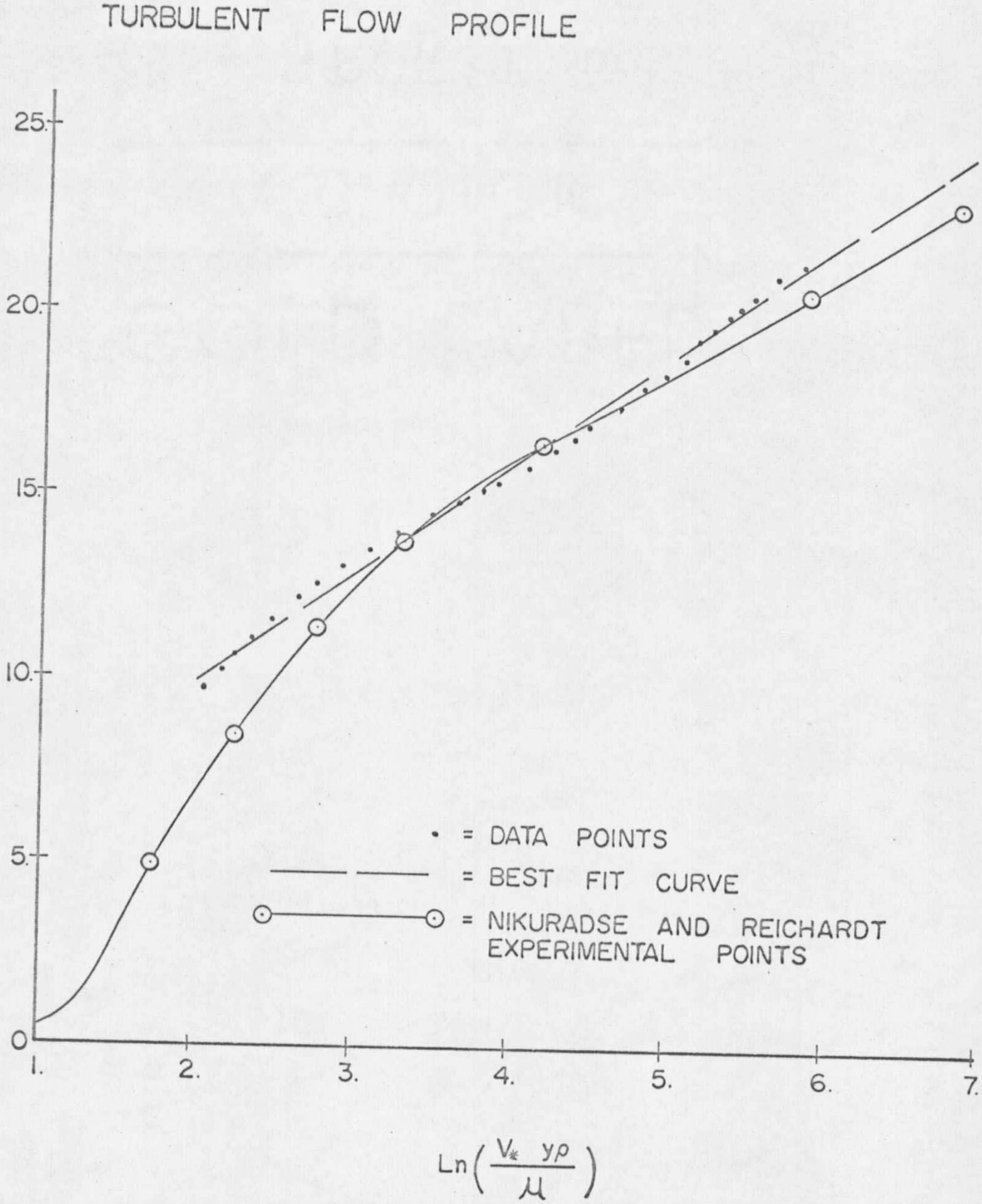

Fig. 18. Turbulent Flow Profile (Re = 27,220)

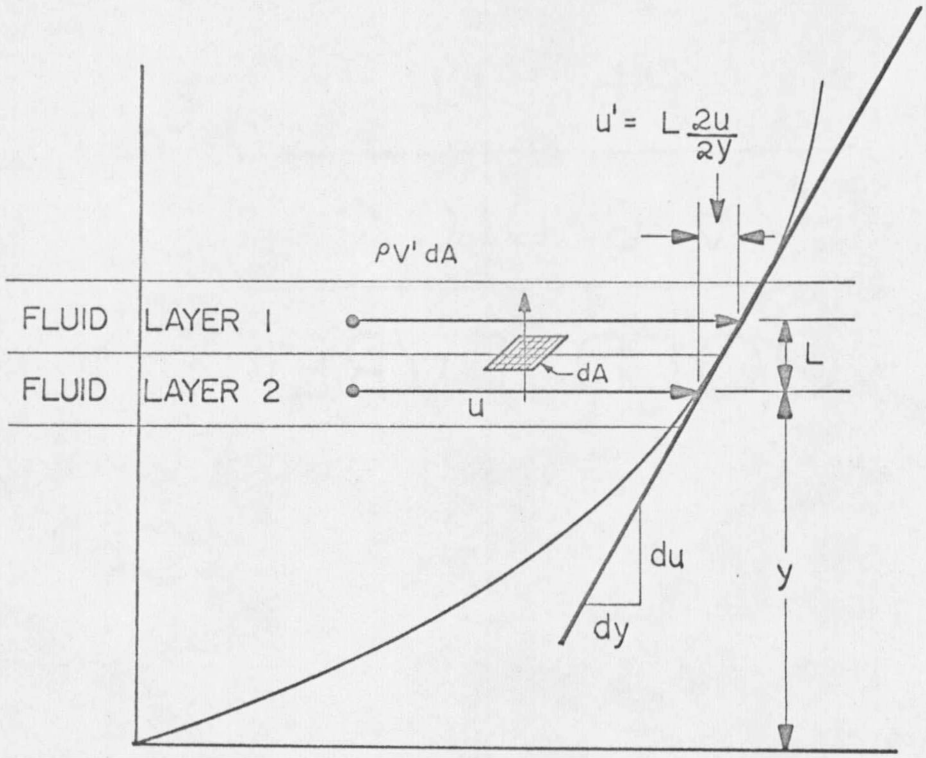

Fig. 19. Sketch for Prandtle's Mixing Length Development

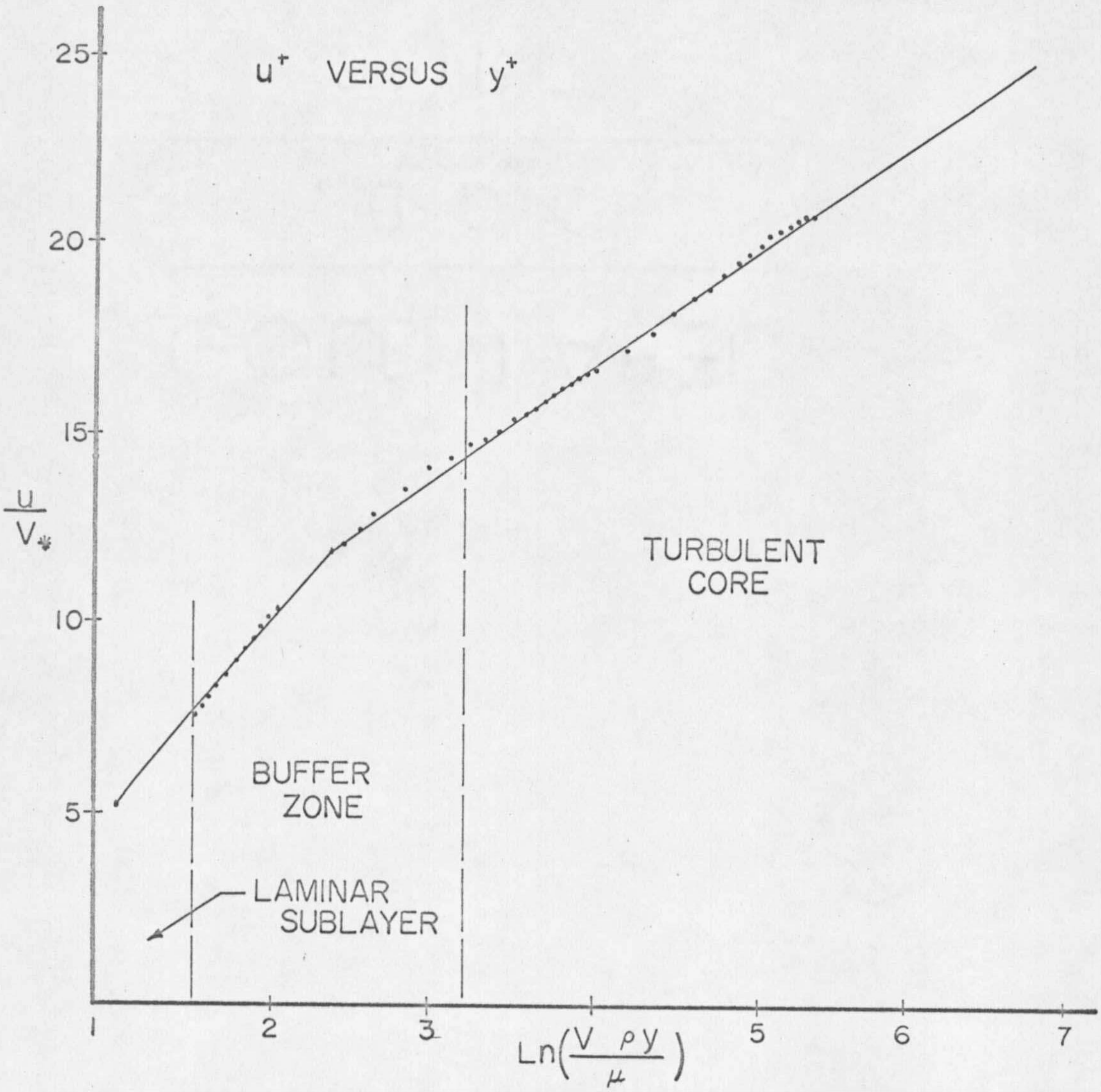

Fig. 20. Plot of Turbulent Profile Showing 3 Flow Zones (Re = 16,366)

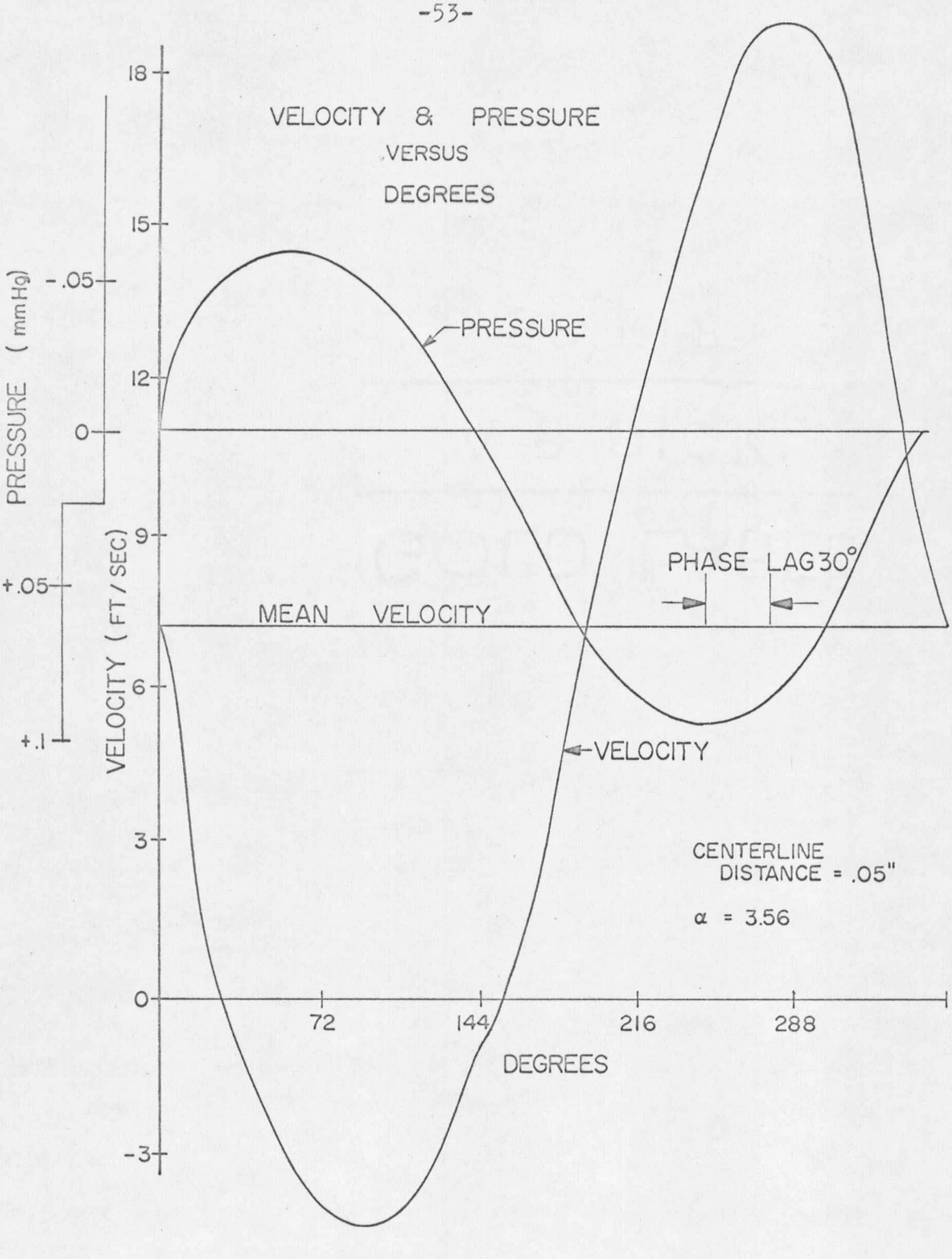

Fig. 21. Plot of Velocity Profile for One Period Showing Flow Reversal

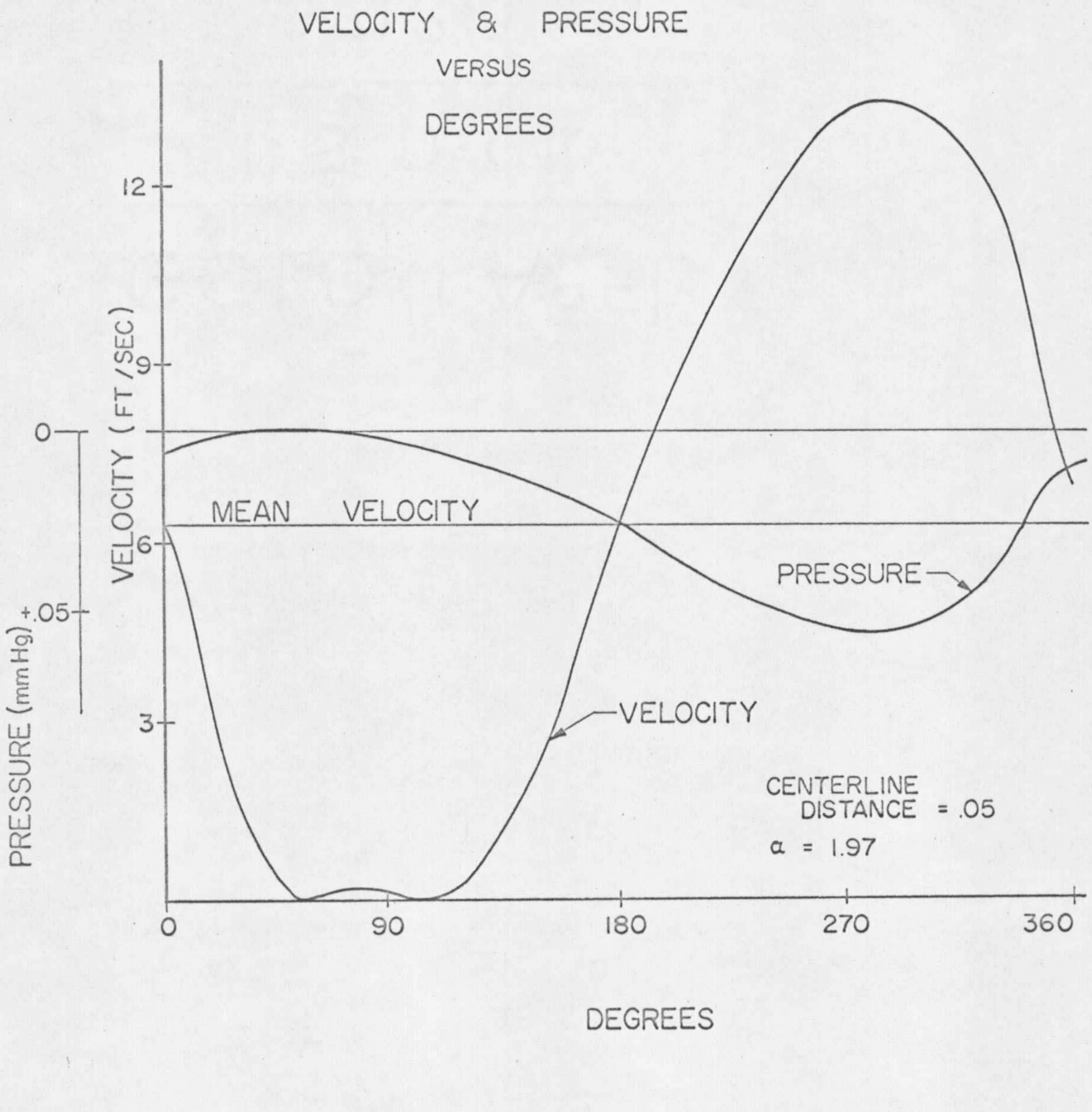

Fig. 22. Plot of Velocity Profile for One Period Showing Flow Reversal Just Starting.

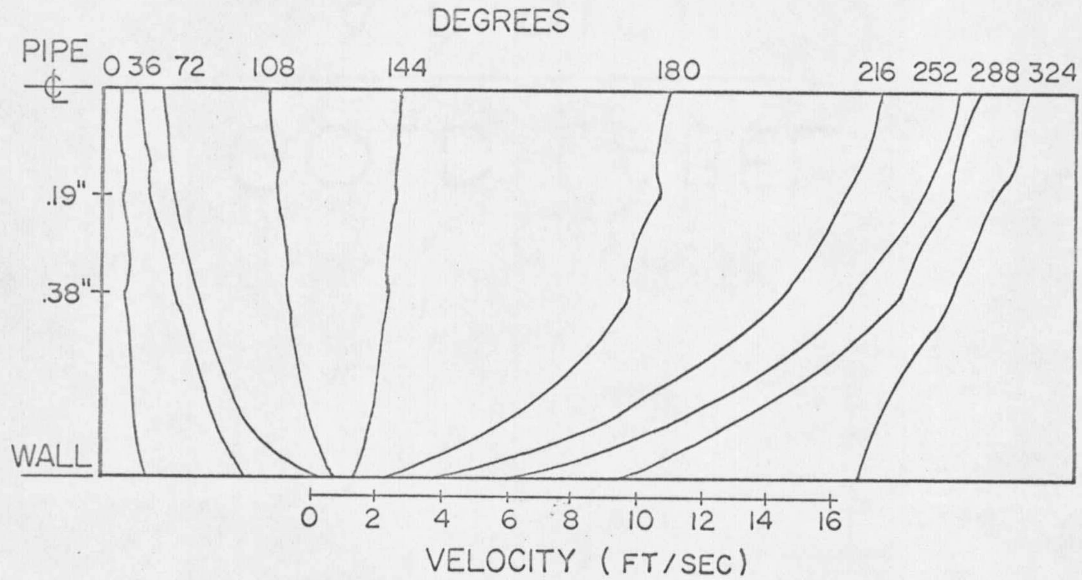

Fig. 23. Plot of Velocity Versus Centerline Distance At 36 Degree Intervals ($\alpha = 3.56$).

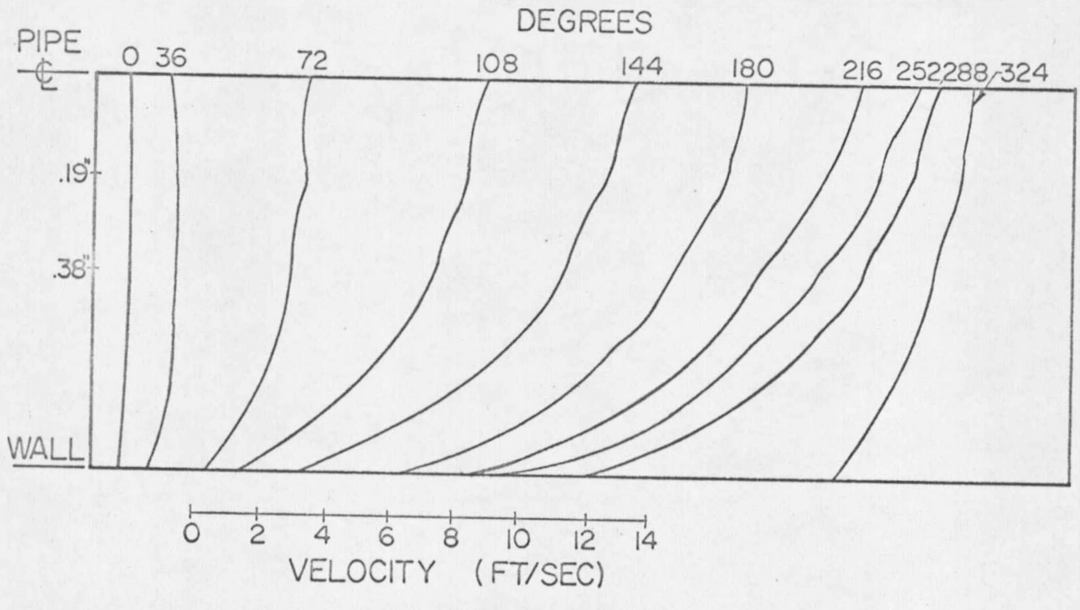

Fig. 24. Plot of Velocity Versus Centerline Distance At 36 Degree Intervals (α = 1.97).

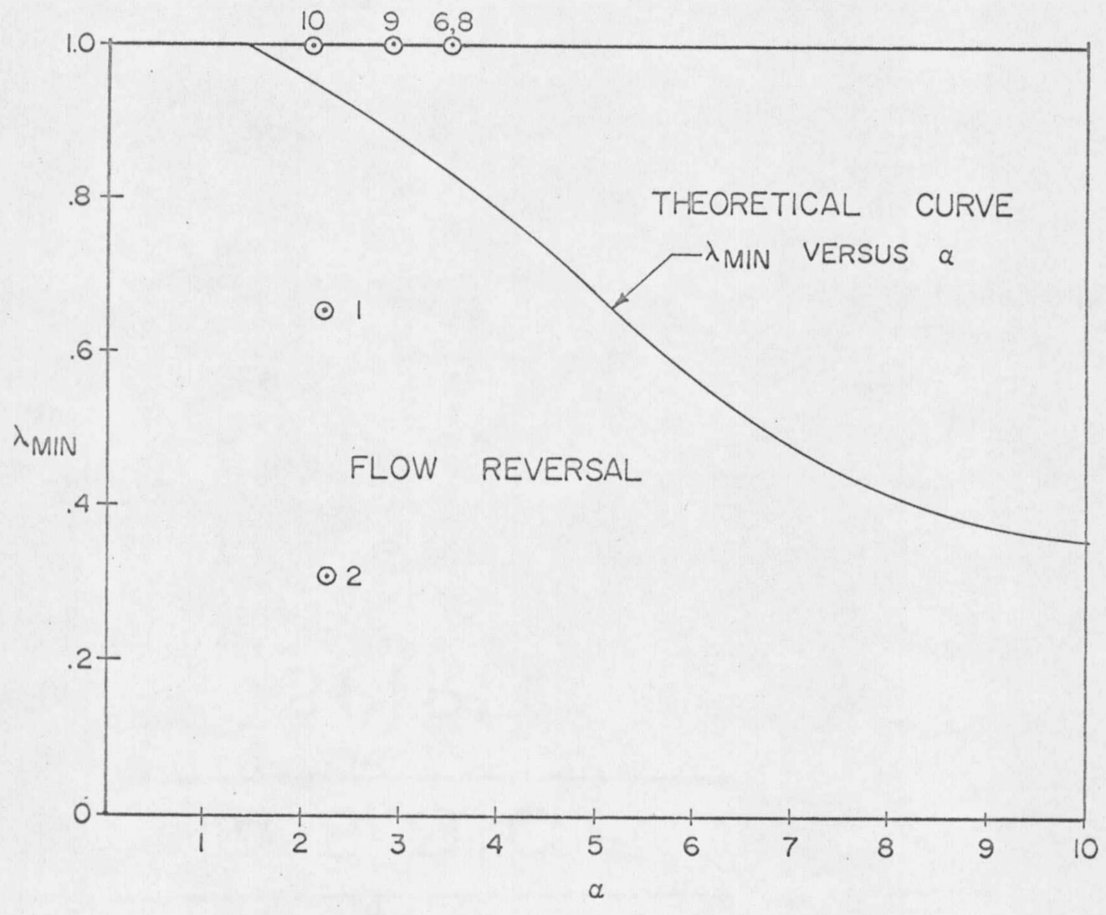

Fig. 25. Graph Showing Theoretical Flow Reversal and Experimental Data.

APPENDIX

Program 1

```
C        HUMAN FACTOR
      PRINT 20
      DO 10 I=100,1000,10
      AI=I
      X=AI*.1
      F=(X**2)*2.04E-5*144.0/(1.9*2.*3.14)
      PRINT 30,X,F
   10 CONTINUE
   20 FORMAT(5X,12HALPHA FACTOR,3X,8HFREQ CPS//)
   30 FORMAT (5X,E15.8,E15.8//)
      CALL EXIT
      END
```

Program 2

```
C     LEAST SQUARES REGRESSION CURVE FIT
C     CARD NUM        COLUMNS        INFORMATION
C     1               1-2    THE NUMBER OF CURVES TO BE FITTED (I2)
C     2               1-72       DISCRIPTION OF THE CURVE,DATE,ETC.
C     3-POINTS        1-15 AND16-30   THE DATE   (E15.10)
C     LAST            1      THE TYPE OF CURVE TO BE FITTED  (2    (I1)
C      1 STANDS FOR A STRAIGHT LINE FIT
C     2 FOR A POWER FIT Y=AX**B
C     3 FOR A EXPONENTIOAN FORM Y=A*(B**C)
      DIMENSION X(100),Y(100),DISC(18),T(100)
      DO 80 I = 1,30
   80 READ 81, T(I)
   81 FORMAT (1F10.5)
      READ 10,NCURVE
   10 FORMAT (I2)
      DO 99 IJ=1,NCURVE
      READ 9 ,DISC
    9 FORMAT (18A4)
      READ 7,NPTS
    7 FORMAT (I3)
      DO 1 I=1,NPTS
      READ 2,X(I),Y(I)
      Y(I)=SQRTF(Y(I))*53.8
    1 CONTINUE
    2 FORMAT (2F15.10)
      READ 11,IFIT
   11 FORMAT (I1)
      IF (IFIT-2) 12,13,14
   14 DO 16 I=1,NPTS
   16 Y(I)=LOGF(Y(I))
      GO TO 12
   13 DO 8 I=1,NPTS
      X(I)=LOGF(X(I))
    8 Y(I)=LOGF(Y(I))
   12 SUMX=0.0
      SUMY=0.0
      DO 3 I=1,NPTS
      SUMX=SUMX+X(I)
    3 SUMY=SUMY+Y(I)
C     AVERAGE VALUES
      ANPTS=NPTS
      XAVG=SUMX/ANPTS
      YAVG=SUMY/ANPTS
      A=YAVG
      SUMXM=0.0
      SUMSQ=0.0
      DO 4 I=1,NPTS
      SUMXM=SUMXM+((X(I)-XAVG)*(Y(I)-A))
    4 SUMSQ=SUMSQ+((X(I)-XAVG)**2)
      B=SUMXM/SUMSQ
```

```
      BETA=B
      ALPHA=YAVG-(B*XAVG)
C     THE EQUATION IS Y=ALPHA+BETA*X
C     COMPUTE THE CONFIDENCE INTERVALS
      SUMS=0.0
      DO 5 I=1,NPTS
    5 SUMS=SUMS+(X(I)*(X(I)-XAVG))
      SXSQ=SUMS/ANPTS
      SIGM=0.0
      DO 6 I=1,NPTS
    6 SIGM=SIGM+((Y(I)-ALPHA-(BETA*X(I)))**2)
      SIGMSQ=SIGM/ANPTS
C     COMPUTE THE VARIANCE
      VARBET=SIGMSQ/(ANPTS*SXSQ)
      VARALP=(SIGMSQ/ANPTS)*(1.0   +((XAVG**2)/SXSQ))
C     FOR THE CONFIDENCE LIMITS, DEPEND ON LEVEL AND NUM OF POINTS
C     CALCULATION OF SE IN PREDICTION EQUATION
      SUMX1= 0.0
      DO 53 I= 1, NPTS
   53 SUMX1= SUMX1 + X(I)**2
      SUMX2 = SUMX**2
      SXX = ANPTS*SUMX1- SUMX2
      SUMY1=0.0
      DO 54 I=1, NPTS
   54 SUMY1 = SUMY1 +  Y(I)**2
      SUMY2= SUMY**2
      SYY = ANPTS* SUMY1-SUMY2
      SUMXY1= 0.0
      DO 55  I=1, NPTS
   55 SUMXY1 = SUMXY1+ X(I)* Y(I)
      SUMXY2 = SUMX* SUMY
      SXY=  ANPTS* SUMXY1 - SUMXY2
      SE =((SXX * SYY - (SXY**2))/ (ANPTS*(ANPTS-2.0)* SXX))**.50
C     30 VALUES FROM TABLE
      IZ = NPTS -2
      IF(IZ-30)82,83,83
   82 GO TO 85
   83 IZ= 30
   85 CONTINUE
      AA =T(IZ) * SE * ((( SXX + ( ANPTS* XAVE)**2)/(ANPTS*SXX))**.5)
      BB = T(IZ) * SE *(( ANPTS /SXX )**.5)
      PRINT 98, DISC
   98 FORMAT (18A4)
      PRINT 200,SE
  200 FORMAT (5H SE =, E14.8)
      PRINT 2000, (X(I),Y(I),I=1,NPTS)
 2000 FORMAT (1H ,10F10.4//)
      PRINT 201, ALPHA, AA, BETA, BB
  201 FORMAT (6H Y = (,E14.8, 7H + OR -,E14.8,4H ) +,2H (E14.8,8H + OR
```

```
    1-,E14.8,4H ) X )
      PRINT 91, VARALP
      PRINT 92, VARBET
   91 FORMAT (1H0,1X,17HVARIENCE ALPHA =,E14.8)
   92 FORMAT (1H0,1X,16HVARIENCE BETA = ,E14.8///)
      PRINT 89
   89 FORMAT (1H0)
   99 CONTINUE
      CALL EXIT
      END
```

Program 3

```
C     DATA REDUCTION FOR 2  INCH PIPE
C     CALCULATION AND PLOT OF VELOCITY BEST FIT CURVES
C     INPUT DATA FORMATS
C     CARD NO. COLUMNS   FORMAT    VARIABLE       MEANING
C
C       1        1-5       F5.3       T1   MERCURY BAROMETER TEM- +++
C       1        6-10      F5.3       H1   BAROMETER HEIGHT INS HG
C       2        1-5       F5.3       T2   TEST SECTION TEMP DEG F
C       2        6-10      F5.3       T3   AMB TEMP DEG F AT TEST SECTION
C       3        1-2       I2              NUMBER OF CURVES
C     ABOVE DATA FOR EACH SERIES OF CURVES
C       4        1-80      20A4       DISCRIPTION OF THE DATA TO BE RUN
C       5        1-10      F10.5      P3   CENTERLINE DYNAMIC PRESSURE
C                                          MM HG AT 0 DEG C
C       5        16-20     F10.5      H2   PRESSURE DIFF BETWEEN ATM AN+
C                                          THE TEST SECTION MM HG
C       6        1-2       I2         NDP  THE NUMBER OF PRESSURE
C                                          DIFFERENCES MEASURED BETWEEN
C                                          STATIC PRESSURE TAPS
C       7        1-2       I2         I    NUMBER OF THE UPSTREAM PRESSU-E
C                                          TAP
C       7        3-4       I2         J    NUMBER OF THE DOWNSTREAM TAP
C       7        5-14      F10.3      PD(I,J) PRESSURE DROP STATIC BETWEEN
C                                          THE GIVEN TAPS IN MM HG
C       8        1-15      F15.5      U(I)    VELOCITY
C       8        16-30     F15.5      R(I)    DISTANCE FROM WALL IN
C       8        31-36     I5         ICHECK  1 IN COL 31-36 TO INDICATE LAS
      DIMENSION  X(100), Y(100),            T(40),R(100), U(100)
      DIMENSION  UPLUS(100), YPLUS(100)
      DIMENSION A(6,100),       B1(100),DISC(20),AL(100),B(6,100),
     1THETA(20),EPB(20),PD(7,7),II(7),JJ(7),DL(7,7),PDF(7,7)
C     NOW WANT THE ATMOSPHERIC CONDITIONS
C     T1 = TEMP OF MERCURY COLUMN, DEG F
C     H1 = HEIGHT OF MERCURY BAROMETER INCHES OF HG AT TEMP T1
      READ59,T1,H1
   59 FORMAT (2F5.3)
C     CHANGE T1 TO DEG C
      T1=(T1-32.0)*5.0/9.0
C     T1 IS NOW DEG C
C     CORRECT THE HEIGHT OF THE MECURY COLUMN FOR TEMPERATURE
      H1= (H1*(1.0-16.34E-05*T1/(1.0+18.18E-05*T1)))*25.4
      H11=H1/25.4
C     H1 IS NOW IN MM HG AT 0 DEG C, LAST EQ FROM ESCHBACH
C     NOW WANT THE CONDITIONS IN THE TEST SECTION
C     T2 = TEST SECTION TEMPERATURE DEG F
C     T3 = AMBIENT TEMP AT TEST SECTION DEG F
C     H2 = PRESSURE DIFFERENCE BETWEEN ATM AND TEST SECTION,INCHES H2O
      READ58,T2,T3
   58 FORMAT (2F5.3)
```

```
      READ 10, NCURVE
   10 FORMAT(I2)
      DO 99 IJ= 1,NCURVE
      READ61,DISC
   61 FORMAT (20A4)
      READ 37,P3,H2
   37 FORMAT (2F10.3)
      H2= (H2*(1.0-16.34E-05*T1/(1.0+18.18E-05*T1)))
C     H2 IS NOW IN MM HG AT 0 DEG C
C     P1 =TEST SECTION PRESSURE MM HG
      P1= H1 + H2
C     P1 IS NOW IN MM HG AT 0 DEG C
C     NOW WANT THE TEST SECTION FLUID PROPERTIES
C     CHANGE THE TEST SECTION PRESSURE TO LBF/ FT**2
      P2 = P1 * 2.785
      P=P2
C     USE THE PERFECT GAS EQUATION FOR THE SPECIFIC VOLUME
C     V = SPEC VOL CUFT/LBM
      RR= 53.36
      G = 32.17
      T2= T2+459.7
C     T2 IN DEG R
      V= RR*T2/P2
C     DEN IN LBM/CUFT
      DEN= 1.0/V
C     NOW COMPUTE THE VISCOSITY
C     VMU =ABSOLUTE VISCOSITY LBM/FT*SEC
      T2=T2-459.7
      VMU =(1.134+0.001396*T2)*1.0E-05
C     DYNAMIC VISCOSITY SQFT/SEC
      VNU=VMU/DEN
C     NOW PRINT OUT THE OPERATING CONDITIONS
      PRINT 14
   14 FORMAT(1H1)
      PRINT 15
   15 FORMAT (1X,32H TWO    INCH PIPE DATA REDUCTION//)
      PRINT 16,DISC
   16 FORMAT (1X,20A4/)
      PRINT 43,H11
   43 FORMAT (1X,25HATMOSPHERIC PRESSURE IS  ,F8.1,17H IN HG AT 0 DEG C/
     1)
      PRINT 17,T2
   17 FORMAT (21H TEST SECTION TEMP = ,F5.1,6H DEG F)
      PRINT 44,DEN,VNU
   44 FORMAT (1X,10HDENSITY = , F8.6,4X,20HDYNAMIC VISCOSITY = ,E10.3,10
     1H SQ FT/SEC//)
C     NOW TO GET THE FRICTION COEFFICIENT AND PRESSURE DROPS
C     P3 = MEASURED CENTERLINE DYNAMIC HEAD IN MM HG
C     CHANGE P3 TO LBF/FT**2
      P3 = P3* 2.785
```

```
C     VEL =CENTERLINE VELOCITY FT/SEC
      VEL = SQRTF (2.0*G*P3/DEN)
C     NEED THE PRESSURE DROPS FROM THE STATIC TAPS
C     NPD = NUMBER OF DIFFERENTIAL STATIC PRESSURE DROPS MEASURED
      READ 18,NDP
   18 FORMAT (   I2)
C     PD(I,J) = PRESSURE DROP BETWEEN I AND J NUMBER TAP
C     DL (I,J) = LENGTH IN FT BETWEEN I AND J TAP
      DO 19 K=1,NDP
      READ 57,I,J,PD(I,J)
   57 FORMAT (2I2,F10.3)
      II(K)=I
   19 JJ(K)=J
C     PD IN MM HG
C     NOW DEFINE THE DL(I,J) IN FT
      DL(1,2)=19.125
      DL(2,3)=18.500
      DL(3,4)=18.580
      DL(1,3)=37.625
      DL(1,4)=56.205
      DL(2,4)=37.080
C     NOW FIND PRESSURE DROP/FT
      DO 56 K=1,NDP
      I=II(K)
      J=JJ(K)
   56 PDF(I,J)=PD(I,J)/DL(I,J)
C     PDF ARE NOW IN MM HG/FT
      PRINT 24
C     PRINT OUT THE DATA
   24 FORMAT (    38H PRESSURE DROPS, MM HG/FT BETWEEN TAPS//)
      DO 22 K=1,NDP
      I=II(K)
      J=JJ(K)
   22 PRINT 23,I,J,PDF(I,J),PD(I,J),DL(I,J)
   23 FORMAT (1X,I1,1H-,I1,3X,E10.3,10X,2(2X,E10.3))
C     AVERAGE DROP SPD
      XNDP =NDP
      SPD =0.0
      DO 25 K=1,NDP
      I=II(K)
      J=JJ(K)
   25 SPD=SPD+PDF(I,J)/XNDP
C     PRINT SPD THE AVG PRESSURE DROP
      PRINT 26,SPD
   26 FORMAT (1X  30HTHE AVERAGE PRESSURE DROP IS   ,E10.3,9H MM HG/FT//)
      D= 1.90/ 12.0
      SPD=SPD* 2.785
C     SPD NOW IN(LBF/FT**2)/FT
C     VEL=MAX OR CENTERLINE VELOCITY
```

```
C     VM=MEAN VELOCITY
      VM = .5*VEL
      RE = VM* D/VNU
      F= 64. /RE
      P3 = P3 /2.785
      TO = SPD * D/ 4.0
C     PRINT OUT
      PRINT 28,P3,VEL
   28 FORMAT (1X,23H CENTERLINE DYN PRESS = ,E10.3,2X,    5HMM HG
     1   2X, 20H CENTERLINE VELOCITY,F6.2, 7H FT/SEC)
      PRINT 29,VM,F
   29 FORMAT (    15H MEAN VELOCITY ,F6.2, 7H FT/SEC,2X, 22H  FRICTION CO
     1EFFICIENT ,F7.5)
C     COMPUTE REYNOLDS NUMBER AND PREDICTED FRICTION COEFF
      PRINT 49, RE
   49 FORMAT ( 18H   REYNOLDS NUMBER,E10.1)
      PRINT 34,TO
   34 FORMAT (    26H THE WALL SHEAR STRESS IS ,E10.3 ,10H LBF/FT**2//)
      DO 75 I=1,200
      READ 76,U(I),R(I) ,ICHECK
   76 FORMAT (2F15.5, I5 )
      IF (ICHECK -1) 75,77,77
   75 CONTINUE
   77 NPTS=I
C     FIND MAX VELOCITY
      UMAX =U(1)
      DO 78 I=2,NPTS
      IF (U(I)-UMAX)78,78,79
   79 UMAX =U(I)
   78 CONTINUE
   11 CONTINUE
      XMIN =0.
      XMAX = 1.
      XL = 5.
      XD = 1.
      YMIN = 0.
      YMAX = 1.
      YL = 5.
      YD = 1.
      CALL PLOT (1, XMIN,XMAX,XL, XD,YMIN, YMAX,YL,YD)
      CALL PLOT (90,-.1,.5)
      CALL CHAR (0,.1,1)
  101 FORMAT (4HR/RO)
      CALL PLOT (90,.5,-.1)
      CALL CHAR (0,.1,0)
  102 FORMAT ( 4HV/VO)
      CALL PLOT (90,.3,.5)
      CALL CHAR (1,.1,0,RE )
   92 FORMAT ( 6H RE = ,E12.2)
      RAD = .95
```

```
      DO 40 I=1, NPTS
      VELL = U(I) /UMAX
      RADE =R(I) /RAD
      CALL PLOT (9,VELL ,RADE)
   40 CONTINUE
      CALL PLOT (99)
      DO 45 I=1,100
      AI =I
      RA = AI/100.
      UA = (1. -RA **2)
      CALL PLOT (90, UA,RA)
   45 CONTINUE
      CALL PLOT (99)
      CALL PLOT (7)
   99 CONTINUE
      CALL EXIT
      END
```

Program 4

```
C       DATA REDUCTION FOR 2  INCH-PIPE
C       CALCULATION AND PLOT OF VELOCITY BEST FIT CURVES
C       INPUT DATA FORMATS
C       CARD NO.  COLUMNS   FORMAT    VARIABLE       MEANING
C
C         1        1-5       F5.3      T1    MERCURY BAROMETER TEM- +++
C         1        6-10      F5.3      H1    BAROMETER HEIGHT INS HG
C         2        1-5       F5.3      T2    TEST SECTION TEMP DEG F
C         2        6-10      F5.3      T3    AMB TEMP DEG F AT TEST SECTION
C         3        1-2       I2              NUMBER OF CURVES
C       ABOVE DATA FOR EACH SERIES OF CURVES
C         4        1-80      20A4     DISCRIPTION OF THE DATA TO BE RUN
C         5        1-10      F10.5     P3    CENTERLINE DYNAMIC PRESSURE
C                                            MM HG AT 0 DEG C
C         5        16-20     F10.5     H2    PRESSURE DIFF BETWEEN ATM AN+
C                                            THE TEST SECTION MM HG
C         6        1-2       I2        NDP   THE NUMBER OF PRESSURE
C                                            DIFFERENCES MEASURED BETWEEN
C                                            STATIC PRESSURE TAPS
C         7        1-2       I2        I     NUMBER OF THE UPSTREAM PRESSU-E
C                                            TAP
C         7        3-4       I2        J     NUMBER OF THE DOWNSTREAM TAP
C         7        5-14      F10.3     PD(I,J) PRESSURE DROP STATIC BETWEEN
C                                            THE GIVEN TAPS IN MM HG
C         8        1-15      F15.5     U(I)     VELOCITY
C         8        16-30     F15.5     R(I)     DISTANCE FROM WALL IN
C         8        31-36     I5        ICHECK  1 IN COL 31-36 TO INDICATE LAS
      DIMENSION  X(100), Y(100),            T(40),R(100), U(100)
      DIMENSION  UPLUS(100), YPLUS(100)
      DO 80 I =1,30
   80 READ 81, T(I)
   81 FORMAT(1F10.5)
      DIMENSION A(6,100),        B1(100),DISC(20),AL(100),B(6,100),
     1THETA(20),EPB(20),PD(7,7),II(7),JJ(7),DL(7,7),PDF(7,7)
C       NOW WANT THE ATMOSPHERIC CONDITIONS
C       T1 = TEMP OF MERCURY COLUMN, DEG F
C       H1 = HEIGHT OF MERCURY BAROMETER INCHES OF HG AT TEMP T1
      READ59,T1,H1
   59 FORMAT (2F5.3)
C       CHANGE T1 TO DEG C
      T1=(T1-32.0)*5.0/9.0
C       T1 IS NOW DEG C
C       CORRECT THE HEIGHT OF THE MECURY COLUMN FOR TEMPERATURE
      H1= (H1*(1.0-16.34E-05*T1/(1.0+18.18E-05*T1)))*25.4
      H11=H1/25.4
C       H1 IS NOW IN MM HG AT 0 DEG C, LAST EQ FROM ESCHBACH
C       NOW WANT THE CONDITIONS IN THE TEST SECTION
C       T2 = TEST SECTION TEMPERATURE DEG F
C       T3 = AMBIENT TEMP AT TEST SECTION DEG F
```

```
C     H2 = PRESSURE DIFFERENCE BETWEEN ATM AND TEST SECTION,INCHES H2O
      READ58,T2,T3
   58 FORMAT (2F5.3)
      READ 10, NCURVE
   10 FORMAT(I2)
      DO 99 IJ= 1,NCURVE
      READ61,DISC
   61 FORMAT (20A4)
      READ 37,P3,H2
   37 FORMAT (2F10.3)
      H2= (H2*(1.0-16.34E-05*T1/(1.0+18.18E-05*T1)))
C     H2 IS NOW IN MM HG AT 0 DEG C
C     P1 =TEST SECTION PRESSURE MM HG
      P1= H1 + H2
C     P1 IS NOW IN MM HG AT 0 DEG C
C     NOW WANT THE TEST SECTION FLUID PROPERTIES
C     CHANGE THE TEST SECTION PRESSURE TO LBF/ FT**2
      P2 = P1 * 2.785
      P=P2
C     USE THE PERFECT GAS EQUATION FOR THE SPECIFIC VOLUME
C     V = SPEC VOL CUFT/LBM
      RR= 53.36
      G = 32.17
      T2= T2+459.7
C     T2 IN DEG R
      V=RR*T2/P2
C     DEN IN LBM/CUFT
      DEN= 1.0/V
C     NOW COMPUTE THE VISCOSITY
C     VMU =ABSOLUTE VISCOSITY LBM/FT*SEC
      T2=T2-459.7
      VMU =(1.134+0.001396*T2)*1.0E-05
C     DYNAMIC VISCOSITY SQFT/SEC
      VNU=VMU/DEN
C     NOW PRINT OUT THE OPERATING CONDITIONS
      PRINT 14
   14 FORMAT(1H1)
      PRINT 15
   15 FORMAT (1X,32H TWO    INCH PIPE DATA REDUCTION//)
      PRINT 16,DISC
   16 FORMAT (1X,20A4/)
      PRINT 43,H11
   43 FORMAT (1X,25HATMOSPHERIC PRESSURE IS  ,F8.1,17H IN HG AT 0 DEG C/
     1)
      PRINT 17,T2
   17 FORMAT (21H TEST SECTION TEMP = ,F5.1,6H DEG F)
      PRINT 44,DEN,VNU
   44 FORMAT (1X,10HDENSITY = , F8.6,4X,20HDYNAMIC VISCOSITY = ,E10.3,10
     1H SQ FT/SEC//)
C     NOW TO GET THE FRICTION COEFFICIENT AND PRESSURE DROPS
```

```
C     P3 = MEASURED CENTERLINE DYNAMIC HEAD IN MM HG
C     CHANGE P3 TO LBF/FT**2
      P3 = P3* 2.785
C     VEL =CENTERLINE VELOCITY FT/SEC
      VEL = SQRTF (2.0*G*P3/DEN)
C     NEED THE PRESSURE DROPS FROM THE STATIC TAPS
C     NPD = NUMBER OF DIFFERENTIAL STATIC PRESSURE DROPS MEASURED
      READ 18,NDP
   18 FORMAT (   I2)
C     PD(I,J)  = PRESSURE DROP BETWEEN I AND J NUMBER TAP
C     DL  (I,J) = LENGTH IN FT BETWEEN I AND J TAP
      DO 19 K=1,NDP
      READ 57,I,J,PD(I,J)
   57 FORMAT (2I2,F10.3)
      II(K)=I
   19 JJ(K)=J
C     PD IN MM HG
C     NOW DEFINE THE DL(I,J) IN FT
      DL(1,2)=19.125
      DL(2,3)=18.500
      DL(3,4)=18.580
      DL(1,3)=37.625
      DL(1,4)=56.205
      DL(2,4)=37.080
C     NOW FIND PRESSURE DROP/FT
      DO 56 K=1,NDP
      I=II(K)
      J=JJ(K)
   56 PDF(I,J)=PD(I,J)/DL(I,J)
C     PDF ARE NOW IN MM HG/FT
      PRINT 24
C     PRINT OUT THE DATA
   24 FORMAT (    38H PRESSURE DROPS, MM HG/FT BETWEEN TAPS//)
      DO 22 K=1,NDP
      I=II(K)
      J=JJ(K)
   22 PRINT 23,I,J,PDF(I,J),PD(I,J),DL(I,J)
   23 FORMAT (1X,I1,1H-,I1,3X,E10.3,10X,2(2X,E10.3))
C     AVERAGE DROP SPD
      XNDP =NDP
      SPD =0.0
      DO 25 K=1,NDP
      I=II(K)
      J=JJ(K)
   25 SPD=SPD+PDF(I,J)/XNDP
C     PRINT SPD THE AVG PRESSURE DROP
      PRINT 26,SPD
   26 FORMAT (1X  30HTHE AVERAGE PRESSURE DROP IS  ,E10.3,9H MM HG/FT//)
      D= 1.90/ 12.0
```

```
      SPD=SPD* 2.785
C     SPD NOW IN(LBF/FT**2)/FT
C     VEL=MAX OR CENTERLINE VELOCITY
C     VM=MEAN VELOCITY
      VM=VEL-1.43*SQRTF(2.0*G*D*SPD/DEN)
C     VM IS IN FT/SEC
C     FOR THE FRICTION COEFFICIENT, F
      F =((VEL/VM-1.0)/1.43)**2
      PRINT 27
   27 FORMAT (1HO)
C     CHANGG P3 BACK TO MM HG
      P3=P3/2.785
C     PRINT OUT
      PRINT 28,P3,VEL
   28 FORMAT (1X,23H CENTERLINE DYN PRESS = ,E10.3,2X,    5HMM HG,
     1   2X, 20H CENTERLINE VELOCITY,F6.2, 7H FT/SEC)
      PRINT 29,VM,F
   29 FORMAT (    15H MEAN VELOCITY ,F6.2, 7H FT/SEC,2X, 22H  FRICTION CO
     1EFFICIENT ,F7.5)
C     COMPUTE REYNOLDS NUMBER AND PREDICTED FRICTION COEFF
      RE = VM*D/VNU
C     FOR THE F1 USE FIRST BLASIUS,THEN PRANDTL
      F1 =0.3164/(RE**0.25)
      PRINT 45,F1
   45 FORMAT (25H BLASIUS FRICTION COEFF  ,F7.5)
      C1=2.0/LOGF(10.0)
      RF=SQRTF(F1)
      Q1 =C1*LOGF(RE*RF)-0.8
      Q2=1.0/RF
      IF (Q1-Q2) 47,48,46
   46 F1=F1-0.00001
      RF=SQRTF(F1)
      Q1=C1*LOGF(RE*RF)-0.8
      Q2=1.0/RF
      IF (Q1-Q2)48,48,46
   47 F1=F1+0.00001
      RF=SQRTF(F1)
      Q1=C1*LOGF(RE*RF)-0.8
      Q2=1.0/RF
      IF(Q1-Q2) 47,48,48
   48 CONTINUE
      PRINT 49, RE,F1
   49 FORMAT ( 18H   REYNOLDS NUMBER,E10.1,3X,  22HPRANDTL FRICTION C-++
     1F,F7.5)
C     FIND THE WALL SHEAR STRESS
      TO = SPD*D/4.0
      PRINT 34,TO
   34 FORMAT (    26H THE WALL SHEAR STRESS IS ,E10.3 ,10H LBF/FT**2//)
      DO 75 I=1,200
      READ 76,U(I),R(I) ,ICHECK
```

```
   76 FORMAT (2F15.5, I5 )
      IF (ICHECK -1) 75,77,77
   75 CONTINUE
   77 NPTS=I
C     FIND MAX VELOCITY
      UMAX =U(1)
      DO 78 I=2,NPTS
      IF (U(I)-UMAX)78,78,79
   79 UMAX =U(I)
   78 CONTINUE
      C =VEL/UMAX
      DO 300 I=1,NPTS
  300 U(I) =C*U(I)
C      TNOT = WALL SHEARING STRESS
C      SHEARING VELOCITY  = (TNOT/DENS)**.5
      UTAU = (32.3*TO /DEN )**.5
      DO 301 I=1, NPTS
      R(I) =R(I) /12.
      UPLUS(I) =U(I) / UTAU
  301 YPLUS(I) =R(I) *UTAU/VNU
C     PLOT DATA
      XMIN = 1.
      XMAX = 8.
      XL =5.
      XD= 1.142
      YMIN = 0.
      YMAX = 25.
      YL = 5.
      YD = 5.
      CALL PLOT (1,XMIN,XMAX,XL,XD,YMIN,YMAX,YL,YD)
      CALL PLOT (90,-.2,15.)
      CALL CHAR (0,.1,1)
  307 FORMAT (5H U/V*)
      CALL PLOT (90,3.,-1.)
      CALL CHAR (0,.1,0)
  308 FORMAT ( 11H LN(V*YP/U))
      CALL PLOT (90,5.,5.)
      CALL CHAR (1,.1,0,RE)
  309 FORMAT ( 5H RE = ,E12.2)
      DO  60 I= 1,NPTS
      C= UPLUS(I)
      AB= LOGF( YPLUS(I))
      CALL PLOT ( 9,AB,C)
   60 CONTINUE
      CALL PLOT(99)
C      SPLIT OF DATA INTO 3 FLOW REGIONS
      DO 30 I=1,NPTS
      IF( YPLUS(I) -5.)31,31,32
   31 IL=I
```

```
      GO TO 30
   32 IF(YPLUS(I)  -27.)33,33,30
   33 IB=I
   30 CONTINUE
      N= - (1)
      M= - (1)
      IX=1
   40 GO TO 35
   12 CONTINUE
      DO 304 I= IX,IL
      Y(I) = UPLUS(I)
  304 X(I)=LOGF(YPLUS(I))
      SUMX = 0.0
      SUMY = 0.0
   42 DO 3 I = IX,IL
      SUMX=SUMX+X(I)
    3 SUMY=SUMY+Y(I)
C     AVERAGE VALUES
      ANPTS = (IL -(IX-1))
      XAVG=SUMX/ANPTS
      YAVG=SUMY/ANPTS
      AB = YAVG
      SUMXM=0.0
      SUMSQ=0.0
      DO 4 I= IX,IL
      SUMXM=SUMXM+((X(I)-XAVG)*(Y(I)-AB))
    4 SUMSQ=SUMSQ+((X(I)-XAVG)**2)
      BA =SUMXM/SUMSQ
      BETA=BA
      ALPHA=YAVG-(BA*XAVG)
C     THE EQUATION IS Y=ALPHA+BETA*X
C     COMPUTE THE CONFIDENCE INTERVALS
      SUMS=0.0
      DO 5 I= IX,IL
    5 SUMS=SUMS+(X(I)*(X(I)-XAVG))
      SXSQ=SUMS/ANPTS
      SIGM=0.0
      DO 6 I= IX,IL
    6 SIGM=SIGM+((Y(I)-ALPHA-(BETA*X(I)))**2)
      SIGMSQ=SIGM/ANPTS
C     COMPUTE THE VARIANCE
      VARBET=SIGMSQ/(ANPTS*SXSQ)
      VARALP=(SIGMSQ/ANPTS)*(1.0   +((XAVG**2)/SXSQ))
C     FOR THE CONFIDENCE LIMITS, DEPEND ON LEVEL AND NUM OF POINTS
C     CALCULATION OF SE IN PREDICTION EQUATION
      SUMX1= 0.0
      DO 53 I=IX,IL
   53 SUMX1= SUMX1 + X(I)**2
      SUMX2 = SUMX**2
      SXX = ANPTS*SUMX1- SUMX2
```

```
      SUMY1=0.0
      DO 54 I=IX,IL
   54 SUMY1 = SUMY1 +  Y(I)**2
      SUMY2= SUMY**2
      SYY = ANPTS* SUMY1-SUMY2
      SUMXY1= 0.0
      DO 55 I=IX,IL
   55 SUMXY1 = SUMXY1+ X(I)* Y(I)
      SUMXY2 = SUMX* SUMY
      SXY=  ANPTS* SUMXY1 - SUMXY2
      SE =((SXX * SYY - (SXY**2))/ (ANPTS*(ANPTS-2.0)* SXX))**.50
      IZ = IL-2
      IF(IZ-2)51,51,52
   51 IZ=1
      GO TO 85
   52 IF(IZ-30)82,83,83
   82 GO TO 85
   83 IZ=30
   85 CONTINUE
  100 FORMAT ( 4F15.5)
      AA =T(IZ) * SE * ((( SXX + ( ANPTS* XAVE)**2)/(ANPTS*SXX))**.5)
      BB = T(IZ) * SE *(( ANPTS /SXX )**.5)
      PRINT 200,SE
  200 FORMAT (5H SE =, E14.8)
      PRINT 201, ALPHA, AA, BETA, BB
  201 FORMAT (6H Y = (,E14.8, 7H + OR -,E14.8,4H ) +,2H (E14.8,8H + OR
     1-,E14.8,4H ) X )
      PRINT 91, VARALP
   91 FORMAT (1H0,1X,17HVARIENCE ALPHA =,E14.8)
      PRINT 92, VARBET
   92 FORMAT (1H0,1X,16HVARIENCE BETA = ,E14.8///)
      PRINT 89
   89 FORMAT (1H0)
      IF (N)65,66,67
   65 Z= 4.
      S = 1.
      GO TO 69
   66 Z =7.
      S=5.
      GO TO 69
   67 Z =13.
      S =12.0
   69 DO 68 I=1,50
      AI= I
      YA = ( AI / 50.) * Z   +S
      XA = ( YA - ALPHA) / BETA
      CALL PLOT (90,XA,YA)
   68 CONTINUE
      IF (M) 35,36,96
```

```
35 IX=IL+1
   IL =IB
   IF(IL-IX) 71,71,72
71 GO TO 36
72 CONTINUE
   N=0
   M=0
   GO TO 12
36 IX = IB+1
   IL = NPTS
   N=1
   M=1
   GO TO 12
96 CONTINUE
   CALL PLOT (7)
99 CONTINUE
   CALL EXIT
   END
```

Program 5

```
C      B1 IS VELOCITY CONSTANT FROM LAMINAR PLOT PROGRAM
C      B2 IS THE PRESSURE CONSTANT
C      TIME = PERIOD OF OSCILATION
C      B1 COL1-10, B2 COL 11-20, TIME COL 21-30 ON CARD 1
       DIMENSION XVEL(1330),XPR(1330),XPULSE(1330),IPNT(3)
       DO 27 IJ=1,10
       READ 26, B1, B2, TIME
    26 FORMAT (3F10.3)
C      SENSE ON CALC CONSTANTS FOR EACH RUN
       IF (SENSE SWITCH 1) 60, 61
    60 DO 50  J= 1,2
       READ 21,NCARDS
    21 FORMAT( 36X,I5)
       NPTS= NCARDS*19
       READ 23, (XVEL(K),K=1,NPTS)
    23 FORMAT (19F4.0)
       READ 21,NCARDS
       READ 23,(XPR(K), K=1,NPTS)
       READ 21,NCARDS
       READ 23,(XPULSE(K), K=1,NPTS)
       GO TO (51,52),J
    51 V1SUM= 0
       P1SUM= 0
       DO 25 I=1,NPTS
       V1SUM = V1SUM + XVEL(I)
    25 P1SUM = P1SUM + XPR(I)
       POINTS= NPTS
       V1AVG =(V1SUM/ POINTS ) /1000.
       P1AVG =(P1SUM/ POINTS ) /1000.
       GO TO 50
    52 V2SUM=0
       P2SUM=0
       DO 53 I=1,NPTS
       V2SUM = V2SUM + XVEL(I)
    53 P2SUM =P2SUM + XPR(I)
       POINTS = NPTS
       V2AVG =(V2SUM /POINTS) / 1000.
       P2AVG =(P2SUM /POINTS  )  / 1000.
    50 CONTINUE
       C1= 1./ (V2AVG- V1AVG)
       C2= -C1*V1AVG
       C3= 1./ (P2AVG- P1AVG)
       C4= -C3 * P1AVG
       PRINT 65, C1,C2,C3,C4
    65 FORMAT (4HC1 =F9.3,5X,4HC2 =F9.3,5X,4HC3 =F9.3,5X,4HC4 =F9.3 )
    61 DO 27  L= 1,20
C      READ IN DATA FOR 1 POINT
       DO 7  J=1,3
       READ 1,ICODE, ICHNAL,NOPTS
     1 FORMAT (7X,I5,9X,I5,10X,I5)
```

```
      PRINT 2,ICODE, ICHNAL,NOPTS
    2 FORMAT(6H CODE=I5,5X,8HCHANNEL=I5,5X,20HTHE NUMBER OF CARDS=I5)
      NOPTS = 19* NOPTS
      NUM =0
      IF ( NOPTS - 1330 ) 85,85,86
   86 MI = NOPTS - 1330
      NUM = MI / 19
      NOPTS = 1330
   85 GO TO ( 3,5,6) ,J
    3 READ 4,(XVEL(K), K=1,NOPTS)
    4 FORMAT (19F4.0)
      IF ( NUM ) 82,82, 83
   83 DO 84 I= 1,NUM
   84 READ 4, DUM
   82 GO TO 7
    5 READ 4,(XPR(K), K=1,NOPTS)
      IF (NUM ) 90,90,91
   91 DO 92 I= 1, NUM
   92 READ 4, DUM
   90 GO TO 7
    6 READ 4,(XPULSE(K),K=1,NOPTS)
      IF (NUM )7,7,94
   94 DO 95  I= 1,NUM
   95 READ 4, DUM
    7 CONTINUE
      TOTAL =0.
      SUM   =0.
      IPNT(3) =0
      N=1
      DO 14 I=1,NOPTS
      GO TO (8,12,8)),N
    8 IF( XPULSE(I)+999.) 9,9,14
    9 J=I+1
      IF (XPULSE(J) +999.)10,10,14
   10 IPNT(N)= I
      IF(N-3) 11,15,11
   11 N=2
      GO TO 14
   12 IF ( XPULSE(I) + 800.) 14,13,13
   13 N=3
   14 CONTINUE
   15 IF (IPNT(3)) 17,16,17
   16 M=1
      M1= TIME * 10.
      GO TO 74
   17 M=IPNT(1)
      M1=IPNT(3)
   74 CONTINUE
C     RENUMBER POINTS STARTING AT 1 ,PTS BETWEEN PULSES
```

```
      IY = M1-M+1
      DO 30 I=1,IY
      K=M+I-1
      XVEL(I)= XVEL(K)
   30 XPR(I) = XPR(K)
C     CORRECT EACH POINT AND CHANGE TO VELOCITY FT/SEC
      DO 31 I= 1,IY
      XVEL(I)= B1*(C1* XVEL(I)+ C2) /1000.
   31 XPR(I) = B2*(C3* XPR(I) +C4)  /1000.
C     PUNCH CORRECTED VALUES OF VELOCITY AND PRESSURE
      PUNCH 75,ICODE,IY
   75 FORMAT(6H CODE=I6,5X,21HTHE NUMBER OF POINTS=I6)
      PUNCH 39,( XVEL(I),I=1,IY)
      PUNCH 39,( XPR(I) ,I=1,IY)
   39 FORMAT(13F6.2)
   27 CONTINUE
      CALL EXIT
      END
```

Program 6

```
      DIMENSION XPR(1000),XVEL(1000),PROFIL(20,20)
      DO 99 IJ=1,10
      DO 27 L=1,20
      READ 1, IY
    1 FORMAT (38X,I6)
      READ 2,(XVEL(I), I=1,IY )
    2 FORMAT (13F6.2 )
      READ 2, (XPR(I), I=1,IY )
C     FIND AVERAGE VALUE OF PRESSURE AND VELOCITY
      DO 11 I=1,IY
      SA= 0.
      N1 =I
      N2=I+13
      IF (N2-IY) 14,14,15
   14 DO 12 J=N1,N2
   12 SA =SA +XPR(J)
      AVE = SA/13.
      K= I+6
   11 XPR(K) = AVE
   15 CONTINUE
      DO 4 I= 1,IY
      SS =0.
      NN = I
      NM =I+5
      IF ( NM -IY)5,5,6
    5 DO 7 J=NN,NM
    7 SS = SS +XVEL(J)
      AVEV = SS/6.
      K= I+2
    4 XVEL(K) = AVEV
    6 CONTINUE
      SUM =0.
      TOTAL =0.
   18 DO 19 I=1,IY
      SUM = SUM+XVEL(I)
   19 TOTAL=TOTAL+XPR(I)
      U=IY
      AVGVEL= SUM/U
      AVGPR = TOTAL/U
      PRINT 20, AVGVEL, AVGPR
   20 FORMAT(21H THE AVG VELOCITY IS E12.6,10X,21H THE AVG PRESSURE IS E
     112.6)
C     MAKE VELOCITY AND PRESSURE AXIS SAME
      DIFF = AVGVEL -(AVGPR *60.)
      DO 43 I=1,IY
       XPR(I) =60. *XPR(I)
   43 XPR(I)=XPR(I)      + DIFF
C     FIND MIN PRESSURE AND MAX VELOCITY
      PMIN = XPR(1)
      DO 44 I=2,IY
```

```
      IF( PMIN-XPR(I)) 44,44,45
   45 PMIN = XPR(I)
      AI= I
   44 CONTINUE
      AMP =AVGVEL-PMIN
      VMAX = XVEL(1)
      DO 70  I=2,IY
      IF(XVEL(I)-VMAX)70,70,72
   72 VMAX=XVEL(I)
      BI= I
   70 CONTINUE
      AMP1 =VMAX - AVGVEL
      AMP2 = AMP/60.
      PRINT 8 , AMP2, AMP1
    8 FORMAT( 5X, 19H THE VELOCITY AMP = E12.6,13X,19H THE PRESSURE AMP
     1= E12.6)
C     IF PHASE ANGLE +,PRESSURE LEADS VELOCITY
      XX= AI-BI
      ANGLE = XX/U *360.
      PRINT 73,ANGLE
   73 FORMAT(5X,16H PHASE ANGLE IS F12.3)
      IF(SENSE SWITCH 3) 78,79
   78 CONTINUE
      PRINT 17, L
   17 FORMAT( 5X,16H POINT PLOTTED = I2 )
C     PLOT DATA
      CALL PLOT(1,0.0,10.0,5.0,2.5,0.0,15.0,5.0,3.0)
      DO 32  I=1,IY
      DI=I
      X= 10. * DI/ U
   32 CALL PLOT ( 90,X,XVEL(I))
      CALL PLOT(99)
      NI =IY -1
      DO 80 I=1,IY   ,NI
      DI= I
      X=  10. * DI /U
   80 CALL PLOT (90,X,AVGVEL)
      CALL PLOT(99)
      DO 55 I=1,IY
      DI= I
       X= 10.* DI /U
       P=     XPR(I)
   55 CALL PLOT(90,X,P)
      CALL PLOT(99)
      IF (SENSE SWITCH 1 )9,16
    9 DO 46  I= 1,IY,3
      DI= I
      THETA = ((DI/U) * 2.* 3.1416 + 3.1416 /16.)
      Y= (SINF(THETA)* AMP     ) +AVGVEL
```

```
      X= 10.*DI / U
   46 CALL PLOT (90,X,Y)
      CALL PLOT(99)
      DO 10 I=1,IY,3
      DI =I
      THETA = ((DI/U) * 2.* 3.1416 + 3.1416 *15./16.)
      Y= (SINF(THETA)* AMP1      ) +AVGVEL
      X= 10.*DI / U
   10 CALL PLOT (90,X,Y)
      CALL PLOT(99)
   16 CALL PLOT(7 )
C     STORE PLOT VALUES.
   79 CONTINUE
      DO 96 M= 1,20
      SUMV=0.
      AM =M
      N= AM * U / 20.
      N1= N-4
      DO 64  I= N1,N
   64 SUMV= SUMV + XVEL(I)
      VAVE= SUMV /5.
   96 PROFIL(M,L     ) = VAVE
   27 CONTINUE
      CALL PLOT(1,0.0,15.,5.,3.,0.0,20.,5.,5.0)
      DO 99 I=1,20
      DO 62 J=1,20
       S= PROFIL(I,J)
       T=21-J
   62  CALL PLOT( 90,S,T)
      CALL PLOT(99)
      IF ( I      -5)99,100,101
  101 IF (I      -10) 99,100,102
  102 IF (I      -15) 99,100,99
  100  CALL PLOT(7 )
      CALL PLOT(1,0.0,15.,5.,3.,0.0,20.,5.,5.0)
   99 CONTINUE
       CALL EXIT
       END
```

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