



Optimal Economic Control Strategies in Forest Resource Management
by AFFENDI ANWAR

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Applied Economics
Montana State University
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Abstract:

The investment decision is the most complex and difficult problem in forestry management because it involves large lapses of time and uncertainty of the future outcome resulting from the decision. This study attempts to analyze the forest production decision problem in general and management aspects related to investment, particularly those related to optimal thinning and rotation decisions. The decision model for this study is based on the application of modern control theory to the forest management problem. An appropriate line of approach to solve the control problem in forest production systems is stochastic dynamic programming which is capable of capturing the stochastic behavior of the system and incorporate risk factors of the decision explicitly. Another advantage of the method is that it provides a powerful systematic search for obtaining numerical solutions to the problem, provided the required dynamic interrelationships of the system and transition probabilities can be estimated empirically. Based on this method a criterion function is formulated as maximization of the sum of expected discounted net returns to soil site as a modification of the Faustmann criterion in a statistical sense.

To approach a specific problem in forest management, however, this study pursues another method to solve the control problem of a disease-infected lodgepole pine stand due to lack of available basic data. The control model for this specific problem is based on a modified simulation model which was previously developed by Myers et al. [1971]. By a modification of this model, the redirected computer simulation program resulted in an economic model which has a specific objective function consistent with the problem of managing a diseased timber stand. The criterion function of the model is generalized present value as an extension of the Faustmann criterion.

By virtue of the certainty equivalence principle, the simulation model which is deterministic can be considered as a good approximation for solving stochastic problems in managing a forest production system due to the fact that the decision processes based on this model are carried out sequentially. The computer simulation model can serve as a management aid to help the forest manager in making decisions related to investment problems for any forest stand condition and economic factors which influence the system. Since the simulation results, in general, show a consistency with a priori logical reasoning, it is concluded that the approach can be expanded for application to other species and areas larger than the Rocky Mountain Region where this study was carried out.

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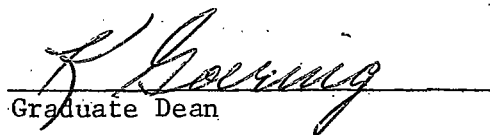
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ABSTRACT

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CHAPTER I
INTRODUCTION

1.1. Motivation of Study

Among the most complex and difficult problems confronting a forest management authority are those involving investment decisions. Growing timber on forest land is an investment asset which has a changing value over time. Growing a timber stand can be considered as a long term investment decision where a long gestation period is needed between initial investment outlay and realization of all expected earnings. On the other hand, cutting timber at final harvest is an act of disinvestment in which capital represented by the value of timber is released and the forest land is made available for a subsequent timber crop or other uses. Hence, there is opportunity cost attached to the land presently occupied by standing timber.

Between the date of planting trees and final harvest, the forest manager may undertake a series of thinning actions and other stand improvement activities such as clearing, fertilizing, pruning, disease control, etc. which involve additional investment decisions, in order to improve the future stand condition and hence to obtain a higher value of final harvest. The land committed to timber production, growing stock of timber, and cumulated stand improvement outlays constitute the investment in a forest at any point in time. The long gestation period between investment outlays and receipt of revenues, combined with the

complex biological interrelationships of all management activities over the economic life of the forest, is what makes forest investment decisions a difficult problem.

Throughout the development of the principles of traditional forest resource management, various theoretical and empirical studies have been made to serve as guides for sound investment decisions in forestry. In an era of rapidly changing technology, economic decisions grow in frequency and complexity, and wrong decisions increase in costs. With this additional impetus and advances in applied mathematics in recent years, quantitative methods for handling complex economic problems have been introduced to the forestry field. Among others are those which are related to determination of forest management, including simultaneous optimization of rotation and thinning decisions (see Näslund, 1969 and Schreuder, 1971). However, both authors have mainly concentrated their studies on the discussion of a general analytical framework for determining optimal rotations and thinnings without any empirical application of the results.

Determination of optimal rotation and thinning decisions in forestry, by and large, is influenced by physical-biological aspects of the forest growth which constitute an "engineering" side of forest production systems. Hence the optimal decision will be specific to a given forest environmental condition. Therefore, if there are any

extraneous factors which influence this "engineering" aspect, the optimal decision will be affected.

Another factor which may influence the optimal decision is economic elements, including supply and demand of the commodity produced, capital, and input factors within the production system. Conceptually, optimal investment decisions in forest production systems can be approached from a more general equilibrium theory, but in practice, a model giving the general equilibrium solution would often be extremely unwieldy. Hence, in order to formulate an operationally manageable model, we must often be willing to settle the problem for something less. Thus the investment problem in forestry for this study will be limited to partial equilibrium analysis where the price and cost structure of the commodity, capital, and other input factors are determined outside the system of forest production. One specific aspect of the study will deal with the formulation of conceptual and empirical solutions to the forest investment problem so that it will help to give a better understanding of the overall problem.

Since the major portion of forest investment decisions is dominated by determination of the rotation and thinning policy, for clarity of further discussion, throughout the course of this study the term investment will refer to both rotation and thinning decisions interchangeably. Moreover, even though there are many intangible benefits that could be derived from the forest stand, this study will be limited to the measurable monetary benefits from wood production.

1.2. Objectives and Plan of Analysis

There are two general objectives to be pursued in this study. The first and more theoretical objective is to formulate and to evaluate the optimal control strategy in an operational model suitable for analysis of decision making involved in the choice of a sound forest management policy in cases where complete basic data can be obtained-- for example, the basic data which could be obtained from a continuous forest inventory (C.F.I.). To obtain an optimal control strategy in forest production systems requires a multistage decision process; that is, a sequence of decisions that will optimize some objective function or criterion function. The appropriate criterion function for forest management policy is maximum discounted net returns with respect to soil site, but some method must be devised to account for the random occurrence of some measurable exogenous factors to the production system, such as disease, insects, and other factors that may affect the forest growth during evolution of the system. Therefore, the decision criterion of this study will be maximization of expected soil value over the entire planning horizon in a statistical sense--a modification of the well-known Faustmann criterion (Faustmann, 1849).

The strategy or conditional decision rule is applied sequentially as new information unfolds through time and the rule is chosen to maximize the above stated criterion. Mathematical analysis of this type of decision processes led to the development of dynamic program-

ming, a la Bellman [1957]. 1/ This type of model provides more than just a computational procedure for obtaining numerical results. It is also a logical framework and in some cases, analytical results can be derived for specific problems, such as in optimal control of natural resources (see Burt, 1964, 1966 and Burt and Cummings, 1970). In fact, analytically, dynamic programming is one of the modern approaches for solving general control problems (see Intriligator, 1971 and Robert and Schulze, 1973).

The realm of modern control theory also includes the maximum principle of Pontryagin et al. [1962] as an extension of the classical calculus of variations. This method will also be briefly reviewed in formulating the concept of optimal forest resource management.

The second objective, which is more applied, is to analyze forest management decisions regarding an optimal control strategy for dwarf mistletoe (Arceuthobium spicies). This disease has adverse effects on forest growth and constitutes one of the exogenous factors which affects the production system, and hence will affect the optimal investment decision. However, due to lack of available basic data,

1/ Sometimes dated versions of linear programming are also called "dynamic programming" which is intrinsically not dynamic because the optimization is made once and for all.

a somewhat different approach for controlling diseased stands will be applied, as an approximate solution to more general control problems. The criterion function is still based on a stochastic version of the Faustmann criterion, but with some modification.

The physical-biological aspects of the production system for this study are obtained from a computer simulation program which has been developed by Myers et al. [1971]. This simulation model includes the dynamic physical-biological interrelationships between the level of disease infestation and growth of the forest which can be altered by thinning decisions at various stages of maturity of the forest. This simulation study provides the necessary information which enables us to construct a dynamic decision model for controlling forest production.

In order to build a decision model based on economic criteria, the simulation model required some modifications by inserting the computation of some economic elements into the model. By varying some of the parameters of the system, results could then be obtained by calculating a performance measure of the system. The performance measure for this study is present value of the stream of net returns from wood production over an infinite planning horizon. A search procedure on the parameters of the modified simulation model provides an approximation to the optimal decision rule. Information derived from this analysis could be of assistance to the forest management authority which is

responsible for the execution of current forest management policy in the Rocky Mountain Region where the study was carried out.

Chapter II of the study discusses the basic structure of the forestry investment decision, and a simple investment decision model is also described. In Chapter III, the general optimal control problem in relation to forest investment decisions is dealt with. Then a more operational model of the investment problem with a dynamic programming solution is presented. In Chapter IV the empirical model of the study using the simulation model as an approximate solution to the general control problem is presented. Finally, Chapter V contains an interpretation of the results of the empirical study and some conclusions.

CHAPTER II
INVESTMENT MODEL FOR FORESTRY

2.1. The Concept of Forest Production

Forestry, the business of growing timber, has characteristics that distinctly give it special status among agricultural production activities. First, the long gestation period between initial input and the first harvested output is quite unique. Second, there is an extended period of autonomous growth in value associated with the initial investment decision. Then eventually, there is a gradual decline in the productive capacity of trees to grow over an extended time due to the biological aging process.

To produce a wood crop, one has to plant the trees and care for them through several years before there is any opportunity to receive revenues. There is a long interval between the date when growing timber reaches a minimum size of marketability and the date when trees finish their growth. At any time between these two dates the timber may be judged ready for cutting on the basis of an economic criterion. The land has many alternative uses, but the trees on the land have virtually no use other than production of standing timber. Thus, an understanding of resources committed on a long-run basis is important to any consideration of timber production.

Let us assume there exists an expected intertemporal production function with many inputs and many outputs. The production function

associated with initial time t is assumed to be twice differentiable:

$$\phi_t(Q) = \phi_t(q_1, q_2, \dots, q_n) \quad (1)$$

where q_j ($j = 1, 2, \dots, n$) are discrete dated inputs and outputs and n is the total number of dated inputs and outputs over some arbitrarily long but finite planning horizon. The condition $q_j < 0$ signifies an input and $q_j > 0$ indicates an output. A given set of dated inputs will result in a unique set of dated outputs which the firm values at v_j ($j = 1, 2, \dots, n$). The v_j are prices adjusted by the time discount structure.

Assuming profit maximizing behavior of the firm and perfect competition in the factor and product markets, an optimal production plan can be derived by means of the Lagrangian function:

$$L(Q, \lambda) = \sum_{j=1}^n v_j q_j - \lambda \phi_t(Q). \quad (2)$$

The first order necessary conditions for a maximum of this Lagrangian will give a solution for output supply and input derived demand functions:

$$q_j^* = q_j^*(v_1, v_2, \dots, v_n) \quad , \quad j = 1, 2, \dots, n. \quad (3)$$

Given initial expectations, the demand function for inputs shows how the firm would desire to allocate inputs over time to maximize

profit. It also shows the stream of outputs which the firm expects as a result.

The t -subscript on Φ implies that a different production function exists at each time period t when a set of decisions are made. This production function is determined by all previous decisions and reflects the number of acres of standing timber of various ages at the point in time t , as well as any other fixed conditions resulting from all previous decisions of the firm.

Moreover, because of some changes in production conditions, it is assumed that the firm makes periodic revisions of its plans based on new expectations. Thus each period the firm maximizes an appropriate Lagrangian of the above form. This maximization may in some periods call for the firm to invest in planting new trees expressed as derived demand for land.

When prices were sufficiently favorable in some previous period trees were planted. This decision process occurs recursively where in each period the forest manager uses updated expectations as timber becomes one period older. As long as reasonable economic stability is maintained, we may assume that there exists a wide span of periods where it is more profitable to keep the trees, rather than to cut them for timber, until it comes the time where additional net returns from presently standing timber are equal to opportunity cost of the land for other uses. One use of the land which may determine its opportunity cost is growing a new stand of trees.

Each optimization problem faced by the firm at a particular time period t involves variables associated with time t and all future periods in the firm's planning horizon. The future variables must be considered jointly with the current variables in the optimization problem to determine optimum levels of the current variables, but these future variables are then ignored until the perpetually recurring optimization problem is faced in the next time period, say $t+1$. At that time the entire process is repeated to get optimal levels of the variables associated with period $t+1$. In a sense, the variables in the optimization which are associated with future time periods are merely instruments or artificial variables used to derive optimal levels of current variables.

The underlying justification for this sequential dynamic optimization approach is certainty equivalence decision theory of Simon [1955] which was later expanded by Theil [1957] and Madansky [1960]. Although the theory only applies to quadratic criterion functions subject to linear constraints, it would appear to be a reasonable approximation to the general case of decision under uncertainty.

The above description of the concept of forest production is approached mainly from neo-classical theory of production related to investment decisions. However, in order for the concept to be meaningful in practice it must be operationally feasible. The following section

will discuss a more realistic model of investment decisions consistent with the above concept.

2.2. Multiperiod Investment Decisions

Before we can go further to a formal investment decision model it is necessary to describe the procedure which must be employed to compare present and future receipts and outlays. Hence, a discussion of the discounting or present value concept is important to any investment decision model.

In real world situations, most decisions are periodic; hence, in facing economic problems which deal with time, the time span or planning horizon can be divided into several periods usually with equal intervals. Multiperiod investment decisions are characterized by a situation where factors of production invested during one time period will affect the level of output during subsequent periods. Therefore, there exists a functional relationship between input and output which has different dating. Further, we assume that there exists a market of capital where capital can be borrowed or lent at some given rate of interest. By using this interest rate, outlays or revenues incurred during different time periods can be made comparable by discounting (or compounding) them to one common period.

Suppose P dollars were invested or borrowed for one time period at the rate r . The value of r expresses the proportion of the amount of capital borrowed or lent which constitutes a cost of capital to the

borrower or income to the lender (investor) per period. Hence at the end of one period the investor would expect rP dollars in interest, and with return of the principal P , this would give the sum $S_1 = P + rP = P(1+r)$ dollars. If the investment rate does not change over time, the investment would yield $S_2 = S_1(1+r) = P(1+r)^2$ dollars at the end of the subsequent period after the first, or in general, it would yield $S_t = S_{t-1}(1+r) = P(1+r)^t$ after t time periods. Therefore, an amount of P dollars expected to be received after t time periods can always be exchanged, or equals $P(1+r)^{-t}$ at the present, because the firm or individual who expects to receive the amount of P dollars after t time periods can borrow the amount of $P(1+r)^{-t}$ dollars at the present and he can repay with the amount $P(1+r)^{-t}(1+r)^t = P$ dollars at the later period.

The expected future receipts $S_t = P(1+r)^t$ from investment are called the future or compounded value of P , while the amount $P = S_t(1+r)^{-t}$ is called present value or discounted value of S_t after t periods of investment. Hence, the factor $(1+r)^{-t}$ by which the future value should be multiplied to get the present value P is called the discount factor.

If we assume that within one time period we have more than one time compounding, say m times, then the future value S_t becomes:

$$S_t = P \left(1 + \frac{r}{m}\right)^{mt}. \quad (4)$$

Let us designate $\frac{m}{r} = k$, then

$$S_t = P \left(1 + \frac{1}{k}\right)^{rkt} = P \left(1 + \frac{1}{k}\right)^k{}^{rt}. \quad (5)$$

As compounding within the time period becomes more and more frequent, in the limit as $m \rightarrow \infty$, $k \rightarrow \infty$, hence:

$$\lim_{m \rightarrow \infty} S_t = \lim_{k \rightarrow \infty} P \left(1 + \frac{1}{k}\right)^k{}^{rt} = P \left[\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k\right]^{rt}. \quad (6)$$

Since by definition $\lim_{k \rightarrow \infty} \left(1 + \frac{1}{k}\right)^k = e$, where e is the base of natural logarithm, then

$$\lim_{m \rightarrow \infty} S_t = P e^{rt} \quad (7)$$

or

$$P = \left(\lim_{m \rightarrow \infty} S_t\right) e^{-rt}. \quad (8)$$

Discounting provides a method of transforming future incomes and outlays so that they are commensurate with the present. Through discounting, a future stream of net returns can be expressed as a single number which is called present value of the entire set of future returns. For example, an entrepreneur expects to receive returns from an investment equal to R_0, R_1, \dots, R_T during time periods 0, 1, 2, ..., T, respectively, and the corresponding discount factors are $\beta^t = 1/(1+r)^t$, $t = 0, 1, 2, \dots, T$. Present value of this revenue stream is:

$$R = \sum_{t=0}^T R_t \beta^t, \quad t = 0, 1, 2, \dots, T \quad (9)$$

or for instantaneous discounting,

$$R = \int_0^T R(t) e^{-rt} dt, \quad 0 \leq t \leq T.$$

While static theory assumes that the entrepreneur will maximize his profit in a single time period, in dynamic multiperiod cases, it is assumed that the entrepreneur will maximize present value of the difference between future incomes and outlays; or in short, he will maximize present value of the streams of future net returns from the investment. ^{2/}

Having established the concept of multiperiod investment decisions, now we are in a position to apply this concept to forestry enterprises. The following discussion will deal with a very simple case of investment decisions concerning a growing stand of timber.

2.3. An Investment Decision for Timber Stands

In the forestry enterprise, the investment decision is characterized by a relatively large amount of initial outlays for timber

^{2/} If the decision maker is a public agency who is concerned with public investment projects, the same criteria still can be applied. A primary difference between private and public investment criteria lies in the rate of discount. The private rate reflects private opportunity cost of capital, while the social rate reflects the social opportunity cost of public projects as the value to society of the next best alternative use to which the funds employed on the project could have been put.

stand establishment, and receipts are dominated by the income from terminal harvest as contrasted to the flow of annual costs of upkeep and returns from thinnings. In order to formulate a criterion function for the investment decision, let us use the following notation:

T = length of the rotation cycle;

$B(T)$ = present value of all net returns;

$R(t)$ = net returns associated with management of the timber stand at the moment t , except for net sale value of final harvest;

$H(T)$ = net sale value of final harvest;

r = interest rate; and

$\beta^t = 1/(1+r)^t$ is the discount factor for any period t .

Present value of the stream of net returns from the timber stand for a single rotation cycle can be expressed as

$$B(T) = \sum_{t=0}^T R(t)\beta^t + H(T)\beta^T, \quad (10)$$

or for the continuous time discounting case: 3/

$$B(T) = \int_0^T R(t)e^{-rt} dt + H(T)e^{-rT} \quad (11)$$

However, formulation of the present value criterion in (10) and (11) considers only one rotation cycle, which is not applicable to the

3/ In some cases, a continuous version of the present value criterion is preferable, especially for analytical treatment of the model.

forestry enterprise based on a sustained yield principle. Based on this principle, the enterprise would evaluate the income stream not only from one rotation cycle, but from all future cycles within the planning horizon. Hence, the appropriate present value criterion for investment decisions in forestry should be based on an expected income stream over an infinite planning horizon. Therefore, present value of net returns from growing stock of timber over an infinite chain of rotations can be expressed as follows:

$$B(T) = \left[\int_0^T R(t)e^{-rt} dt + H(T)e^{-rT} \right] [1 + e^{-rT} + e^{-2rT} + \dots + e^{-irT} + \dots] = \frac{1}{1 - e^{-rT}} \left[\int_0^T R(t)e^{-rt} dt + H(T)e^{-rT} \right]. \quad (12)$$

The expression in (12) is essentially the Faustmann criterion. ^{4/}

Having established the criterion function, we can analytically deduce a decision rule for determining optimal length of the rotation cycle. For simplicity, we assume that other decisions such as the thinning level are given; the forest entrepreneur is only concerned with determination of the optimal rotation. The object is to maximize

^{4/} Faustmann's solution to the problem, was later confirmed by G. A. D. Preinreich, who was apparently unaware of Faustmann's work in forestry nearly a century earlier, in "The Economic Life of Industrial Equipment," *Econometrica*, VIII (January, 1940), pp. 12-44.

present value of the infinite series of discounted future returns with respect to rotation length T . Therefore, the problem of investment becomes:

$$\text{Max}_T B(T) = \text{Max}_T \left[\frac{1}{1 - e^{-rT}} \left\{ \int_0^T R(t) e^{-rt} dt + H(T) e^{-rT} \right\} \right]. \quad (13)$$

The necessary condition for the above problem can be obtained by differentiating (13) with respect to T and equating the results to zero, which gives the following:

$$\frac{dB}{dT} = - \frac{re^{-rT}}{(1 - e^{-rT})^2} \left\{ \int_0^T R(t) e^{-rt} dt + H(T) e^{-rT} \right\} + \frac{1}{(1 - e^{-rT})} \left\{ R(T) e^{-rT} + \frac{dH}{dt} e^{-rT} - rH(T) e^{-rT} \right\} = 0. \quad (14)$$

Multiplying (14) by $e^{rT}(1 - e^{-rT})$ yields

$$-rB(T) + R(T) + \frac{dH}{dT} - rH(T) = 0 \quad (15)$$

or

$$R(T) + \frac{dH}{dT} = r[H(T) + B(T)]. \quad (16)$$

A literal economic interpretation of (16) is that under an optimal rotation policy, we shall cut the timber when the current return $R(T)$ for having the stand one more year plus the increase in the harvest value $\frac{dH}{dT}$ equals the interest on the sum of harvest value and soil rent. Soil rent is equal to $B(T)$ which is present value of net returns

starting with bare timber land, following an optimal rotation policy over an infinite planning horizon.

The above analytical method, using elementary calculus for determining the optimal rotation in forestry, gives some insight as to the optimal decision rule to be followed by the forest entrepreneur. However as the problem becomes more and more realistic, and consequently more complicated, more sophisticated mathematical methods are required. The next chapter will examine some more comprehensive models for forest investment decisions. Before going into the analysis of more realistic models for investment, we will discuss the problem of uncertainty which is an important difficulty faced by any decision maker in a real world situation.

2.4. Consideration of Risk and Uncertainty

No discussion of economic decisions--such as determination of optimal investment in growing timber--can be complete without recognizing and taking into account the effect of uncertainty of future events. The analytical model of investment in the last section was simplified by assuming perfect knowledge of input-output relations and prices in the production system within the firm's planning horizon. However a good economic decision model should be capable of providing managers with guides to take action when they face choices between alternative courses of action where the results are uncertain. Introduction of risk and uncertainty into the analytical model will

complicate the problem, but from a practical decision viewpoint, it is justified by the additional realism obtained in approaching a real world problem.

Although no one knows the exact outcome of future events, a real world problem of investment could be treated as if the probability distribution of the future outcomes were known rather than pretending to know the exact future outcomes. Knight [1921] distinguished risk and uncertainty as two different phenomena. Following his ideas, risk refers to the situation when parameters of probability distributions of future outcomes can be estimated so as to be actuarially insurable; or in other words, the variability of future outcomes can be empirically measured. Uncertainty, on the other hand, refers to a situation in which the parameters of the probability distribution of the outcomes cannot be empirically or quantitatively estimated.

The importance of risk factors affecting forest management decisions has long been recognized by forest economists, and most forest land managers are aware of the advantage in maintaining a flexible forest management program to reduce the difficulties of changing output of the forest in response to changes in the market. Markowitz made an important contribution to the theory of investment under risk when he developed his methods for determining an optimally diversified portfolio [1959]. He developed a logical framework to guide investment decisions by explicitly recognizing the risk factor. His model

provides a formal method for evaluating the advantage of diversifying investments as a means of reducing risk associated with the expected returns from a number of investments. Dowdle [1962] has applied the EV investment guide of Markowitz to diversify cost expenditures in forest investment activities. ^{5/} However this method only applies to static investment decisions in forestry.

A promising method applicable to dynamic investment in forestry which recognizes risk factors has been developed by Burt [1965]. He analyzes the problem of investment in the context of the general replacement model under risk. His discussion is related to a special case of Howard's model (Howard, 1960), for which an analytical solution is derived. The replacement model is concerned primarily with the economic decision problem of asset life under conditions of chance failure or loss, i.e., possible loss of the asset due to exogenous influences outside the system. Though his model pertains to dynamic decisions of investment in forestry, the exogenous factors described

^{5/} E and V refer to expected income and variance of income, respectively. The efficiency criterion of investment decisions can be considered as a combination of individual investments such that the variance for the entire set of investments cannot be reduced without also reducing the mean. All possible combinations of such investments traces out what is commonly called the EV efficiency frontier.

in the analysis refer to a sudden extraneous shock, such as insect attack or fire which may cause a sudden destruction of the asset in a short period of time. This model is not applicable to our problem where the exogenous influence of dwarf mistletoe on the standing timber system is chronic in nature.

In the next chapter we present a stochastic dynamic programming solution to the control problem which can incorporate the risk factor into the investment decision model.

CHAPTER III

OPTIMAL CONTROL IN FOREST MANAGEMENT

3.1. Management Problem in Forestry

Sound management practices in forestry require knowledge about physical-biological aspects of the forest resource. If the primary objective in forest management is to produce timber, the basic physical resource with which a forest manager is concerned is land and the timber growing on it. The productivity of timber as a growing "machine" where the wood product is part of that "machine" depends on many factors. First, growth of timber depends on the site quality which reflects the capacity of a given area of land to grow a certain timber species. Another important factor to be considered in growing timber is stocking (tree density) which affects the rate of growth of timber at a given age and site quality. Genetic make-up of a tree species determines the capacity of timber to grow and withstand any exogenous influences which might affect its development. Those exogenous factors to be accounted for in growing timber are disease, insects, adverse climate, etc. For a certain tree species, there exists an optimum physical-biological combination among the elements comprising the forest environment which give an optimum growth of timber. Hence, a good knowledge of how to use those resources effectively is essential in forest management.

In order to obtain maximum benefit from the forest resource, full advantage of knowledge, skill, and art of silviculture and forest measurement should be taken. Those technical aspects of forestry which are important should be considered in making any decision to solve management problems. However the concern of this study is investment decisions related to the economic problem of how to grow a better timber stand which will result in maximum benefits over a given time horizon. In order to achieve this objective, the study will focus on economic decisions, particularly the ones that are associated with the intensity of thinning and length of rotation cycle.

Thinning the standing stock of timber can be beneficial for the following reasons: (1) to eliminate undesirable trees and thereby concentrate growth on the desirable trees which produce a high value of timber, (2) to prevent excessive competition on site resources among trees which would result in wasting energies needed for total growth, (3) to obtain revenues before final harvest. The fourth reason often advanced is to increase the net growth of timber. However, evidence on this aspect of thinning is quite inconclusive, at least for most timber species.

Timber production involves a large lapse of time between planting and harvesting. Hence time is an important variable in management decisions of forestry. In a sense, decisions must be made in the forestry enterprise at each moment in time over the entire planning

horizon of the firm. A mathematical model adapted to this kind of problem is commonly called "control theory."

3.2. Formulation of General Control Problem

The essence of an optimal control problem is to choose a feasible time path for the control (decision) variables which will maximize the objective function. The choice of time paths for the control variables implies a set of differential equations called equations of motion, and also determines time paths for certain additional variables called state variables which describe the state of the system. Employing this concept in the forest management problem, let us consider a forest production system which can be described by a set of state variables x_1, x_2, \dots, x_r . Hence the state of the system at any given time t can be represented by the state vector $\underline{X}(t) = [x_1(t), x_2(t), \dots, x_r(t)]$.

Suppose the objective of forest management is to maximize total discounted net benefits over a given planning horizon. In order to achieve this objective, a sequence of decisions is required that will maximize the objective function. Let us designate the set of managerial decisions by decision variables u_1, u_2, \dots, u_s , so that at any time t , the related decision is represented by the vector of decision $\underline{U}(t) = [u_1(t), u_2(t), \dots, u_s(t)]$.

From the definition of state and decision variables, there should exist a relation such that the rate of change in a state variable at

any time t is a function of the present state of the system, the date, and the decision taken which is implied by the decision vector. This relation can be expressed in the form of a system of differential equations:

$$\dot{\underline{X}}(t) = \frac{d\underline{X}}{dt} = \omega(\underline{X}(t), \underline{U}(t), t). \quad (17)$$

Expression (17) constitutes the equations of motion which can trace values of the state variables in a dynamic process.

Decisions influence the system in two different ways: (1) the rate at which net returns are earned at that moment in time, and (2) the rate at which the growing stock of timber is changing, or in more general terms, the future path of the state vector.

Let us designate $R(\underline{X}(t), \underline{U}(t), t)$ as the marginal return with respect to time generated from a stand of timber at time t , where the state is described by state vector $\underline{X}(t)$, and $\underline{U}(t)$ implies the decision taken at that time. Further we define $H(\underline{X}(T), \underline{U}(T), T)$ as net harvest value at the end of rotation cycle T resulting from the decision implied by $\underline{U}(T)$, and when the state of the system is described by $\underline{X}(T)$. The objective of timber management can be translated into the following problem:

$$\begin{aligned} \text{Max}_{\underline{d}} J(\underline{X}_0, \underline{d}) = & \text{Max}_{T, \underline{U}(t)} \sum_{i=0}^{\infty} e^{-irT} \left[\int_0^T R(\underline{X}(t), \underline{U}(t), t) e^{-rt} dt \right. \\ & \left. + H(\underline{X}(T), \underline{U}(T), T) e^{-rT} \right] \end{aligned}$$

$$\begin{aligned}
&= \text{Max}_{T, \underline{U}(t)} \left[\int_0^T R(\underline{X}(t), \underline{U}(t), t) e^{-rt} dt \right. \\
&\quad \left. + H(\underline{X}(T), \underline{U}(T), T) e^{-rT} \right] / (1 - e^{-rT}).
\end{aligned} \tag{18}$$

Both T and the vector function of time $\underline{U}(t)$ are variables in the maximization subject to the following constraints:

$$\begin{aligned}
\dot{\underline{X}}(t) &= \omega(\underline{X}(t), \underline{U}(t), t) \\
\underline{U}(t) &\in \Psi(\underline{X}(t)).
\end{aligned} \tag{19}$$

If the initial state of the control problem is included in the system and the state vector is confined to a given region, then

$$\begin{aligned}
\underline{X}(0) &= \underline{X}_0 \\
\underline{X}(t) &\in S
\end{aligned} \tag{20}$$

where \underline{X}_0 is the initial state of the standing timber system and S is the set of possible states of the system.

The problem of (18) subject to (19) and (20) is a model in optimal control theory which can be applied to timber management decisions in the continuous time case $0 \leq t \leq \infty$. Theoretically this problem can be solved by either the classical calculus of variations or the maximum principle of Pontryagin *et al* (see Intriligator, 1971). An application of the maximum principle to solve the optimal thinning and rotation problem in timber management has been presented by Näsrlund [1969], and an

excellent interpretation of the principle in an economic context has been given by Dorfman [1969]. Mathematically, this principle is a generalization of the Lagrange multiplier method to solve optimization problems over time. Derivation of the necessary conditions for an optimal solution according to this principle boils down to finding the Hamiltonian function, so to speak, which underlies the basic theory of the principle. ^{6/}

However, for the discrete time situation, any optimal control problem can be formulated in a dynamic programming framework, because in that situation, both the maximum principle and dynamic programming give equivalent solutions. Hence for practical purposes in which the decision is taken periodically like the one in timber management decisions, dynamic programming often has some advantages, especially in getting numerical solutions. The following section will discuss a more concrete and detailed formulation of optimal control to approach the investment problem in timber management. The numerical procedure for the problem will be described in a dynamic programming framework.

^{6/} The name of the function is after the astronomer-physicist W. R. Hamilton who first used such a function to unify mechanics and optics. See D. J. Wilde and C. S. Brightler, Foundations of Optimization (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967), p. 431.

3.3. Dynamic Programming Solution

In order to formulate optimal control in a dynamic programming context, first we have to specify a stage of the decision process which in timber management may be taken as a year or some longer interval of time. A stage of one year gives the most sensitive decision rule, but little precision is lost in timber management applications by using a somewhat longer period.

Next we have to determine the state variables which are capable of describing the future state of the system completely. Since the decision process of our problem is growing a stand of timber, age of the stand and a density index can be considered as state variables, because once we know the present age of the stand and the stand density, the future condition (state) of the stand can be predicted by the present state of growing timber. However, any exogenous factors such as the disease of dwarf mistletoe which influences growth conditions of the stand will change the future state of the forest system also. Hence, the level of disease infestation will constitute another state variable.

Then we have to determine the appropriate decision variables that are going to be used to control future states of the standing timber system. Since we know that there is a relationship between thinning action and the future level of mistletoe infestation, and of course the stand density (see Hawksworth and Hinds, 1964; Baranyay and Safranyik, 1970), the level of thinning constitutes a decision or control variable.

Another decision variable which will change the future state of the forest system is the harvest variable which is a dichotomous variable which specifies to cut or let the stand grow. In fact, both of these decision variables can be considered as one decision variable because the harvest variable is an extreme case of the thinning decision.

One important aspect of the dynamic programming method requires estimation of biological relationships which express each state variable in period $t+1$ as a function of state and control variables in the previous period t . These relationships constitute first-order difference equations in the state variables and are discrete counterparts of the differential equations in (17).

In order to be more specific, let us introduce the following notation:

- x_1 = age of the stand;
- x_2 = stand density index;
- x_3 = dwarf mistletoe rating; and
- u = thinning level, including harvest as an extreme.

The set of dynamic relationships will be three in number:

$$\begin{aligned} x_{1,t+1} &= f(u_t, x_{1t}, x_{2t}, x_{3t}) = x_{1t} + 1 \\ x_{2,t+1} &= g(u_t, x_{1t}, x_{2t}, x_{3t}) \\ x_{3,t+1} &= h(u_t, x_{1t}, x_{2t}, x_{3t}) \end{aligned} \quad (21)$$

The equation of x_1 happens to be extremely simple, so its specific algebraic form was written out in the above set of equations. The

relationships in the last two equations of (21) can be estimated by some kind of regression analysis, if adequate data could be found. These three equations in (21) also constitute a discrete version of the equations of motion which trace the dynamic process over time, starting from some initial state.

Now we define a strategy or conditional decision rule (not necessarily optimal) which states how to control the standing stock system at any given stage for each set of values of the state variables. Mathematically, this rule expresses each decision variable as a function of all state variables, at any given stage. Following the above notation and applying it to our problem, this rule can be expressed as follows:

$$u_t = d_t(x_{1t}, x_{2t}, x_{3t}). \quad (22)$$

If we could estimate the relationship of (21) and set up the rule of (22), we would be able to trace the value of each state variable through time starting from an initial state of the process \underline{x}_0 by iterating the equations in (21). Therefore, the strategy expressed in (22) can be evaluated at any stage and state of the process. Complete enumeration of all possible decision rules given by (22) is not feasible except in the simplest of problems; dynamic programming provides a powerful method to systematically determine the optimal decision rule.

Since some of the state variables are subjected to random fluctuation, the decision rule of (22) which is a function of random

variables is also a random variable. The fact that the decision rule is random, makes it difficult to evaluate a strategy by the above iterative method. In cases where a decision process involves some random state variable, a Markov decision model solved by dynamic programming can overcome the difficulty. In addition, stochastic dynamic programming provides a systematic method to obtain the optimal policy, i.e., the one that maximizes the criterion function.

Since the decision to be made involves some random variables, the appropriate model for optimal control in the discrete case will be a stochastic decision model solved by dynamic programming to obtain the optimal policy. The following discussion will first present the deterministic version of dynamic programming and then modify it into a stochastic framework by incorporating a Markov chain into the model.

A characteristic of the dynamic programming method is that decisions are made sequentially in a multi-stage process. One important aspect of this multi-stage decision process is the effect of a decision at a given stage upon the state variables in the immediately following stage. A particular decision within the set of admissible alternatives will change the state of the process in the passage from one stage to the next. If the optimal decision to be made at a particular stage of the process depends only on the state of the process at that

stage, then the decision process satisfies the Markovian requirement. 7/
 In a deterministic case, the Markovian requirement is equivalent to the difference equation of (21) being first order instead of second or higher order difference equations, and also assuming that all relevant state variables have been included in the set.

The implied change of the state of the process from one stage to the next resulting from a decision is referred to as state transition. Transition from one state to another can be made deterministically (with certainty) or stochastically--according to a probability distribution of the future state of the process. The latter is called a stochastic decision process.

Now let us introduce dynamic programming with a deterministic model. In order to be able to formulate the dynamic decision model, the first order difference equation of (21) must be shifted into dynamic programming notation in terms of stages as follows:

$$\begin{aligned}
 x_{1,n-1} &= f(u_n, x_{1n}, x_{2n}, x_{3n}) = x_{1n-1} \\
 x_{2,n-1} &= g(u_n, x_{1n}, x_{2n}, x_{3n}) \\
 x_{3,n-1} &= h(u_n, x_{1n}, x_{2n}, x_{3n})
 \end{aligned}
 \tag{21'}$$

7/ A formal definition of the Markovian requirement can be found in Richard E. Bellman, Adaptive Control Process (New Jersey: Princeton University Press, 1961), p. 54.

where n refers to the number of stages remaining in the planning horizon. The relation between stages and chronological time can be illustrated by the following diagram:

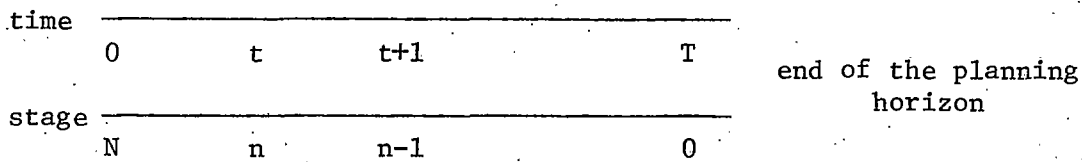


Figure 1. The Relation Between Stages and Chronological Time.

Bellman's principle of optimality states "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman, 1957). ^{8/} By application of this principle, the entire problem can be summarized by the following recurrence relation:

$$F_n(x_1, x_2, x_3) = \text{Max}_u [R_n(u, x_1, x_2, x_3) + \beta F_{n-1}\{(x_1-1), g(\), h(\)\}] \quad (23)$$

where: $f_n(x_1, x_2, x_3)$ is present value of total net returns from soil site in the next n stages under an optimal policy, when the initial state is that implied by the variables x_1, x_2, x_3 .

^{8/} A direct mathematical derivation of the recurrence relation as an alternative to the application of the principle of optimality can be found in R. E. Bellman and S. E. Dreyfus, Applied Dynamic Programming (New Jersey: Princeton University Press, 1962), p. 15.

$R_n(u, x_1, x_2, x_3)$ is the immediate net return resulting from decision u in stage n .

$F_{n-1}\{(x_1-1), g(\cdot), h(\cdot)\}$ is the future net returns from the site in the next $n-1$ stages to go. The functions $g(\cdot)$ and $h(\cdot)$ are evaluated with their arguments set equal to values of the variables in stage n , i.e., the arguments are as given in (21').

$\beta = 1/(1+r)$ is a discount factor with interest rate r .

The above procedure describes how to formulate the multi-stage decision model to control the production system in growing timber in a deterministic framework. If we want to incorporate stochastic elements into the model which would more closely represent real world phenomena, the dynamic equation (21') and the decision model (23) need slight modifications as follows:

$$\begin{aligned} x_{2,n-1} &= g(u_n, x_{1n}, x_{2n}, x_{3n}, e_n) \\ x_{3,n-1} &= h(u_n, x_{1n}, x_{2n}, x_{3n}, e'_n) \end{aligned} \quad (24)$$

where e and e' are random variables. Also (23) becomes an expected discounted net returns function:

$$\begin{aligned} F_n(x_1, x_2, x_3) &= \text{Max}_u E[R_n(u, x_1, x_2, x_3) + \beta F_{n-1}\{(x_1-1), g(\cdot), h(\cdot)\}] \\ &= \text{Max}_u [\bar{R}_n(u, x_1, x_2, x_3) + \beta E F_{n-1}\{(x_1-1), g(\cdot), h(\cdot)\}] \end{aligned} \quad (25)$$

where E is the mathematical expectation operator. The function $\bar{R}_n(u, x_1, x_2, x_3)$ is now defined as an expected value measure of immediate net returns and $F_n(x_1, x_2, x_3)$ is an expected value measure of discounted net returns.

Since the decision model (25) is a dynamic stochastic decision model, it can be approximated as a Markov decision model with a finite number of states. The stochastic difference equations in (24) determine the random transition from state to state in the Markov process. The Markov process can be expressed in matrix form, where the matrix is composed of transition probabilities of changing from any stage i to another state j when the stage of the process moves from the n -th stage to the $(n-1)$ -th stage. The state of a forest system can be represented by some meaningful set of state variables, such as the ones that were defined in (21) for the problem.

Let us define the probability of going from state i to state j at the n -th stage under decision alternative k as p_{ij}^k . Then let us consider a decision rule which specifies k for any given state i , and let that rule be denoted as $k(i)$. This decision rule is a discrete variable approximation to the functional relationship (22). If there are M states possible, the $M \times M$ matrix with element $\{p_{ij}^{k(i)}\}$ is the transition matrix of a Markov chain.

Because a Markov chain decision process can be considered as a discrete approximation to the stochastic difference equations in (24), Howard [1960] was able to formulate the Markov decision model in a dynamic programming framework. This approximation to the decision model of (25) is expressed as follows:

$$F_i(n) = \text{Max}_k [R_i^k(n) + \beta \sum_{j=1}^M p_{ij}^k F_j(n-1)] \quad , \quad n = 1, 2, \dots, N \quad (26)$$

$$F_j(0) = \theta_j$$

where: $F_i(n)$ is the maximum expected present value of total net returns from soil site when the process is initially in state i and n stages remain in the planning horizon.

$R_i^k(n)$ is expected immediate net returns, given the i -th state, k -th decision alternative, and the n -th stage of the process.

The $\{\theta_j\}$ are terminal values of the decision process which can be set equal to zero if the planning horizon is sufficiently long.

There are two solution methods for obtaining the optimal policy, i.e., the value iterative and policy iterative methods (Howard, 1960). However, the value iterative method appears to be the most feasible and efficient for problems with a large number of states. The value iterative method solves the optimization problem by starting at the end of the planning horizon with only one stage remaining; then works back to the present by utilizing the principle of optimality at each step. Applying this method, the general formula of (26) then becomes:

$$F_i(1) = \max_k [R_i^k(1) + \beta \sum_{j=1}^M p_{ij}^k F_j(0)] = \max_k [R_i^k(1) + \beta \sum_{j=1}^M p_{ij}^k \theta_j]$$

Or if $\theta_j = 0$, $j = 1, 2, \dots, M$, we have

$$F_i(1) = \max_k R_i^k(1).$$

By using the results of $F_i(1)$ we can calculate the following:

$$\begin{aligned} F_i(2) &= \max_k [R_i^k(2) + \beta \sum_{j=1}^M p_{ij}^k F_j(1)] \\ F_i(3) &= \max_k [R_i^k(3) + \beta \sum_{j=1}^M p_{ij}^k F_j(2)] \\ &\vdots \\ F_i(n+1) &= \max_k [R_i^k(n+1) + \beta \sum_{j=1}^M p_{ij}^k F_j(n)] \end{aligned} \quad (27)$$

The iterative procedure of (27) could be used as a computational algorithm to solve the optimal control problem of a production system in growing timber. A FORTRAN program and guide for its application using the above algorithm is now available and was written by Burt et al. (undated).

If $R_n(\cdot)$ and consequently $R_i^k(n)$ are independent of the stage n as $n \rightarrow \infty$, then it can be shown that the recurrence relation in (26) converges to a constant decision rule for large n . Therefore the solution of the problem for n large enough that convergence has taken place provides an optimal policy for all values of n . A criterion to

verify convergence of the optimal policy is given in Burt and Allison [1963]. Hence the computational burden and time for obtaining an optimal policy could possibly be reduced.

By using the above algorithm at each stage of the decision process, an optimal decision rule could be obtained with respect to the level of thinning control and final harvest of a timber stand, which underlies the basic principle of forest investment decisions, for every possible state of the timber production system. Derivation and application of an optimal control policy to a disease infected stand of timber in a particular forest region requires detailed specification of forest conditions in that region. If the forest region consists of several management blocks which in turn consist of several forest stands, then a different optimal decision rule must be derived for each of the various stand conditions which have different characteristics. In order to be able to accommodate changes in parameter values and changes in price and cost structure for making economic projections, a sensitivity analysis in dynamic programming can be applied without much difficulty.

The general model of dynamic programming expressed in (26) consists of two major ingredients for obtaining optimal policies. Specifically it requires information related to expected immediate returns $\{R_i^k(n)\}$ and the transition probabilities $\{p_{ij}^k\}$. The estimate of expected immediate net returns can be obtained via estimation of

conditional expected yield of timber. This expected yield should adequately describe total cubic feet of yields by class of timber so that the conversion can be made to market value as a function of all related decision and state variables involved in the model. In economic jargon this relationship constitutes a multi-product joint production function where generalized input variables are represented by decision and state variables, and output is comprised of the various classes of timber products which have market value.

The second kind of information needed for the model pertains to stochastic behavior of the standing timber system which can be represented by two interacting random variables, dwarf mistletoe rating and stand density index. A third interacting variable in the system is age of the stand, but this variable is non-stochastic. Hence, estimated transition probabilities should be based on the joint probability distribution of two state variables with age entering the joint density function as a parameter.

Let us first discuss estimation of the conditional yield relationship to estimate expected immediate returns. In the estimation we assume wood yield to be similar to that in the past. Yield varies because of many factors; among them are disease, insects, and climatic factors which behave stochastically. Another group of factors which have more "systematic" influence are site characteristics and age of the stand at harvest. The relationship of yield and the above factors

is assumed to be independent of the stage of the process; hence this relationship will constitute a function of state and decision variables at any given stage if all factors enter the function as either state or decision variables.

Let us simplify the discussion by assuming that a single variable (total marketable volume if you please) can be used to measure timber production, and thus we are dealing with only one equation which describes yield in terms of various factors mentioned above. Factors which remain unchanged over the growth cycle of timber, site characteristics in particular, do not need to be entered in the timber yield equation because their values can be treated as merely parameters in the dynamic optimization problem.

Certain stochastic variables which one might want to delete for the sake of simplicity cause no problem if they are not stochastically dependent with any of the included factors of the yield equation, or at least the dependency should be quite weak. For example, a study such as this one which focuses on the influence of dwarf mistletoe upon timber growth would be simplified a great deal if we could delete climatic and insect variables from the timber yield equation, and essentially pool the random influences of these variables in the error term of the regression equation.

The regression equation for this simplified view of the timber yield relationship would take the following form:

$$q_t = \psi(u_t, x_{1t}, x_{2t}, x_{3t}) + \varepsilon_t, \quad t = 0, 1, 2, \dots, T$$

$$E(\varepsilon_t) = 0 \quad (28)$$

$$E(\varepsilon_t^2) = \sigma^2, \quad E(\varepsilon_t, \varepsilon_{t+\tau}) = 0, \quad \tau \neq t,$$

where: q_t is the quantity of merchantable volume of timber in cubic feet for period t .

u_t is thinning level, including harvest.

x_{1t} is age of the stand for period t .

x_{2t} is stand density index for period t .

x_{3t} is dwarf mistletoe rating for period t .

ε_t is a random disturbance which represents climatic factors and other factors which individually are relatively minor.

The above physical relationship would be used for estimating expected net returns at each period resulting from the decision implied by u_t .

Expected immediate net returns are defined as the difference between value of gross returns and variable costs which can be expressed as follows:

$$R(u_t, x_{1t}, x_{2t}, x_{3t}) = G(u_t, x_{1t}, x_{2t}, x_{3t}) - C(u_t, x_{1t}, x_{2t}, x_{3t}) \quad (29)$$

where: $R(u_t, x_{1t}, x_{2t}, x_{3t})$ is expected immediate net returns in period t .

$G(u_t, x_{1t}, x_{2t}, x_{3t})$ is expected immediate gross returns in period t .

$C(u_t, x_{1t}, x_{2t}, x_{3t})$ is expected variable cost of thinning for period t .

Expected timber yield can be obtained by estimating the conditional expectation of yield from equation (28) for any decision implied by u_t at period t . This conditional expected yield can be expressed as follows:

$$E(q_t | u_t, x_{1t}, x_{2t}, x_{3t}) = \psi(u_t, x_{1t}, x_{2t}, x_{3t}) \quad (30)$$

where E is the expectation operator.

Assuming perfect competition in the output market, expected immediate net returns can be calculated as follows:

$$\begin{aligned} R(u_t, x_{1t}, x_{2t}, x_{3t}) &= P_w E(q_t | u_t, x_{1t}, x_{2t}, x_{3t}) - C(u_t, x_{1t}, x_{2t}, x_{3t}) \\ &= P_w \psi(u_t, x_{1t}, x_{2t}, x_{3t}) - C(u_t, x_{1t}, x_{2t}, x_{3t}) \end{aligned} \quad (31)$$

where P_w is the price of merchantable wood per cubic foot.

In more general terms, we would have several timber yield equations, one for each of several dependent variables which would estimate quantities of various classes of wood harvested. This would then be converted into total gross value by an appropriate set of prices for classes of timber.

As explained earlier, the transition probabilities could be estimated by means of a joint probability distribution of two state variables. Hence the joint probability density function can be expressed as:

$$(\xi_{1t}, \xi_{2t} | u_t, x_{1t}, x_{2t}, x_{3t}) \quad (32)$$

where ξ_{1t} and ξ_{2t} are specific values of the random variable $x_{2,t+1}$ and $x_{3,t+1}$. Note that u_t , x_{1t} , x_{2t} , and x_{3t} enter as parameters in the joint density function for $x_{2,t+1}$ and $x_{3,t+1}$.

The transition probabilities defined as $\{p_{ij}^k\}$ explain stochastic changes in the stand density index and mistletoe rating as the movement occurs from one stage to the next. Specifically, the basic concern is the probabilities associated with future states of the forest system, which in this problem can be represented by the three state variables, age, stand density and mistletoe rating. But the change from one age to the other is with probability unity, so that age enters the stochastic transition from stage to stage in a trivial way.

Consider the range of values for each stand density and dwarf mistletoe rating as having been divided up into discrete intervals as follows:

$$\begin{array}{ll} x_{21} \leq x_2 \leq x_{22} & x_{31} \leq x_3 \leq x_{32} \\ x_{22} \leq x_2 \leq x_{23} & x_{32} \leq x_3 \leq x_{33} \\ \vdots & \vdots \\ x_{2H} \leq x_2 \leq \infty & x_{3L} \leq x_3 \leq \infty \end{array} \quad (33)$$

Also, consider the thinning variables as taking a few discrete values u_1, u_2, \dots, u_k and use the integer k to denote the value of u_k . Age is rather naturally defined for discrete values on an annual basis. However, the stage of the decision process is arbitrary and could constitute a period of time longer than a year, for example, 5 or 10 years. Age is defined as an integer to correspond with the time period used as a stage, for example, age equal to 5 would imply 25 years of age if the stage is a 5-year period of time.

With the above conventions, we can now define the transition probabilities in (26). The i -th state implies a combination of three conditions: (1) x_1 equal to an integer which specifies age, (2) x_2 lies in one of the intervals as outlined above, and (3) x_3 lies in a particular interval as outlined above. Therefore, p_{ij}^k is the probability that x_2 goes from its interval associated with state i to its interval associated with state j , that simultaneously x_3 goes from its interval associated with state i to its interval associated with state j , that simultaneously x_1 goes from the integer value associated with state i to the integer value associated with state j , and also simultaneously that the decision variable u is equal to u_k . We note that the only value of j for which p_{ij}^k can have a positive value is where x_1 is equal to its value at state i plus one, otherwise the probability must be zero. This observation follows from the non-stochastic nature of age as a state variable.

The actual calculation of the transition probabilities $\{p_{ij}^k\}$ would be accomplished by analysis of the residuals in the regression equations used to estimate (24). The classical assumptions of the regression model would provide a base from which to begin. However, the almost certain stochastic dependence between the residuals in the equations for x_2 and x_3 should not be overlooked.

Estimation of these two major components of the model, namely (24) and (30), requires physical-biological information which is possibly obtainable from continuous forest inventory or basic data which were specifically collected for this purpose. However, in a situation where basic data are difficult to obtain, we will try to pursue another method to approach the control problem in forest management. The last two chapters of this study will use a simulation method to approximate a solution to the problem.

CHAPTER IV

SIMULATION APPROACH TO CONTROL PROBLEM

4.1. The Setting

The control problem in this chapter will deal with the management of forest production systems in which the timber stands involved are affected by an exogenous factor which is the disease dwarf mistletoe (*Arceuthobium* species). This disease causes major losses of pine and other coniferous stands, especially in Western United States and Canada (see, e.g., Baranyay and Safranyik, 1970; Hawksworth, 1961; Shea and Stewart, 1972; Graham, 1961; Korstian and Long, 1922). In the Western United States it has been estimated that the annual loss to dwarf mistletoes due to growth reduction and mortality is over 3 billion board feet (Hawksworth, 1972).

Biologically, these diseases can be quite easily controlled in conjunction with silvicultural practices, because (1) they are generally host specific, (2) they are obligate parasites which means that they die as soon as the host dies, and (3) their rate of spread through a tree or stand is relatively slow. The effective silvicultural controls available now are physical, i.e., through thinning or clear cutting. Thus, disease control for improving a timber production system can be incorporated with management decisions related to determination of thinning level and the length of rotation cycle.

Since we are interested in a control problem in which the stand of timber is influenced by an extraneous factor of disease, we begin with a discussion of the nature of the disease and its affect on timber growth, together with possible methods of control. Then the economic model for disease control is developed.

4.2. The Impact of Disease Infestation

Literature dealing with research on the disease of dwarf mistletoe infestation is voluminous (see Hawksworth and Wein, 1972). Yet, only a few studies are directly related to quantitative measurement which can be used to evaluate the impact of the disease on stand development. Hence, the following review will discuss only the research which has an important bearing on the problem of disease control in relation to timber management.

An early study by Korstian and Long [1922] measured the impact of dwarf mistletoe on stand growth of ponderosa pine by using a stem analysis method on felled trees. They concluded from observing various degrees of mistletoe infection that there was little or no reduction in the growth rate of lightly infected trees, but that there was a marked falling off of current growth of heavily infected trees. Radial increment of a 5-year period in heavily infected trees was only 12 to 14 percent of that of uninfected trees. The reduction in cubic foot increments was somewhat less, ranging from 15 to 31 percent of that for disease free trees. Based on the individual estimates, the authors

aggregated the impact on the whole stand. The influence of the disease infestation on timber quality was also briefly discussed. However, this study did not reveal the total loss caused by the parasite because the researchers did not measure the effect of disease on mortality.

Hawksworth and Hinds [1964] improved the estimation of dwarf mistletoe impact on the standing trees by taking into account the volume loss caused by mortality, in addition to the effects of mistletoe infestation on survival trees. They estimated the impact of infestation over a longer period of time by devising a measurement scale of dwarf mistletoe infection which represented the state of disease infestation in an infected tree. This dwarf mistletoe rating (DMR) was expressed as the sum of ordinal values assigned by an integer number (0, 1, or 2) to different parts of the infested tree crown which reflects the intensity level of the disease infestation. 9/ It was found that this rating was highly correlated with the time since infestation started.

9/ The actual way the infection was rated: first the live crown is divided into thirds, and each third is rated as 0, no mistletoe; 1, light mistletoe; and 2, heavy mistletoe. The rating of each third is added to obtain a total for the tree. Based on the individual rating of an infected tree in the sample plot, then the rating for the whole stand was averaged on each rating.

In order to evaluate the effect of dwarf mistletoe on individual trees Hawksworth and Hinds [1964] performed a regression analysis between percent volume reduction, stand age when infected, and time since infection. It was reported that the time since infection made a relatively higher contribution to explanation of the variation in percent volume reduction than the influence of age of the stand when infected. Hence, indirectly the dwarf mistletoe rating had performed well as a measure of dynamic impact of the disease infestation.

Mortality rate was estimated from the number of dead standing trees around the infected lodgepole pine stand. It was found that mortality in the infected lodgepole pine stands was higher than the plots in the stand with no mistletoe. Further, the mortality rates based on cubic volumes showed low correlation with time since infection. Presumably, this is because most of the trees died before they reached the minimum size for which cubic foot volume was estimated.

The authors then used their results for individual trees to estimate the impact for the whole stand. By creating a measure of dwarf mistletoe rating for infection intensity and estimating its impact on percent reduction over time, this research revealed an important piece of information needed for dynamic control processes on stand development. No decision alternative other than the classical replacement decision was allowed, however, because the study

assumed that there was no treatment (control) during stand development. 10/

Myers, Hawksworth and Stewart [1971] used results of the previous study done by Hawksworth and Hinds [1964] to incorporate dwarf mistletoe effects into a stand management simulation study for lodgepole pine in the Rocky Mountain Region. This simulation study was primarily concerned with development of a computer program to calculate certain key variables associated with managed dwarf mistletoe infested lodgepole pine stands. The simulation model is capable of generating information on yields pertaining to different levels of the state of mistletoe infestation and thinning decision alternatives. Thus, the research provides physical-biological information which should enable us to develop a decision model for controlling production systems of disease-infested timber stands of lodgepole pine under specific forest management objectives. However, this study like most other research based on simulation does not try to search for an explicit formal optimal policy for controlling the production process in growing timber stands. Nevertheless, the simulation program does contain some

10/ For articles dealing with economic replacement models, see Oscar R. Burt, "Economic Replacement," SIAM Review, V, No. 3 (1963), pp. 203-208, and Oscar R. Burt, "Optimal Replacement Under Risk," Journal of Farm Economics, XLVII, No. 2 (1965), pp. 324-346.

potentially useful dynamic interrelationships which could be used for dynamic decision processes related to our problem.

Although the simulation study provides information pertaining to dynamic decision processes, all input and output variables in the model are expressed in physical-biological terms. If the decision is to be based on economic criteria, the model must be expanded to contain price and cost elements to introduce the role of economics in achieving an optimum decision rule. The simulation model which will be discussed in the next section will include economic factors, and the decision criteria to guide management decisions will be formulated explicitly in a formal decision model based on widely accepted investment criteria for forestry.

4.3. Simulation Model

Naylor [1971] has stated that simulation is a numerical technique of "last resort" to be used only when analytical methods are not available for obtaining solutions to a given model. A realistic investment problem in forestry deals with a complex probabilistic production system, including physical-biological relationships coupled with the price system over the entire economic life of the forest, which makes an analytical solution of the investment problem extremely difficult, if not impossible. Because of this complexity and the available computer simulation model of Myers, et al. [1971], simulation plays

an important role in the economic model to be used here. The simulation study is redirected into a more specific objective to solve the economic problem of optimal management of diseased stands of lodgepole pine. Even though the simulation study is based on a deterministic model, the certainty equivalence principle suggests that the deterministic solution to the problem will give a good approximation to the stochastic problem of the investment decision if the decision rule based on this model is carried out sequentially (see, e.g., Simon, 1955; Theil, 1957; Madansky, 1960; and Burt, 1971).

Discussion on the simulation model will begin with an explanation of the physical-biological relationships of the timber production system as described by Myers *et. al.* [1971]. Then after modifications of this model into an economic system, an economic criterion for investment decisions will be formulated. Specification of the objective function of the investment model will be consistent with a dynamic control model of disease-infected lodgepole pine stands faced by a forest management authority in the Rocky Mountain Region.

The simulation program on physical-biological aspects of managing disease-infected stands is similar to that of a previous study done by Myers [1967] in constructing a yield table for healthy stands of lodgepole pine, except additional information is calculated on the exogenous

factor of dwarf mistletoe infestation in the stand. 11/ Myers et. al. [1971] used several basic working relationships for their simulation program in the form of equations (among them are difference equations) and tables. Parameters in the equations were estimated from data obtained in semi-permanent plots of lodgepole pine stands. Those stands represented various site qualities with indices ranging from 30 to 85, and dwarf mistletoe infestation varied from none to very heavy. From each sample plot, several variables were measured and the field data were then converted to calculate volume and other values for each plot on a per acre basis. Basal area and other per acre values, including average stand diameter (DBH) 12/, were used as dependent variables to obtain predictive equations to be used in the computer program called LPMIST. 13/ This computer program was written in FORTRAN language (see appendix).

11/ A yield table would show values of physical yield of wood products and other related state variables of the forest stand for each period of time during the rotation cycle, for a given thinning policy.

12/ The abbreviation DBH is "Diameter at Breast Height" where the diameter of a tree is usually measured.

13/ Apparently, this abbreviation stands for "Lodgepole Pine Mistletoe."

The LPMIST program consists of the main program and four sub-routines. The main program performs most of the computations and writes the yield table. Three subroutines compute average stand diameter and stand density after thinning. The fourth subroutine computes factors that are used in the main program to convert total cubic feet of wood to other units. The modifications of the original LPMIST program into LPMIST2--where an additional subroutine is called for economic calculations--does not change the operation of the main program. The description of the modified program is essentially the same as in the LPMIST program. Only the order of data input is somewhat modified. The calculations necessary in simulation operations of a production system in disease-infected stands are outlined in Figure 2 which shows operation of the main program.

The program first reads in the values of stand parameters and state variables. Then, it calculates the dependent variables by means of predictive equations just prior to the thinning decision. Based on the predictive values of the dependent variables, total cubic feet is computed. This total cubic feet value can be converted to other values by means of a special subroutine. If a thinning decision is required, then another subroutine computes new values of the dependent variables explained below. Based on these new values, total cubic feet and associated values are calculated by a new subroutine after the thinning decision occurred. One of the subroutines uses thinning standards

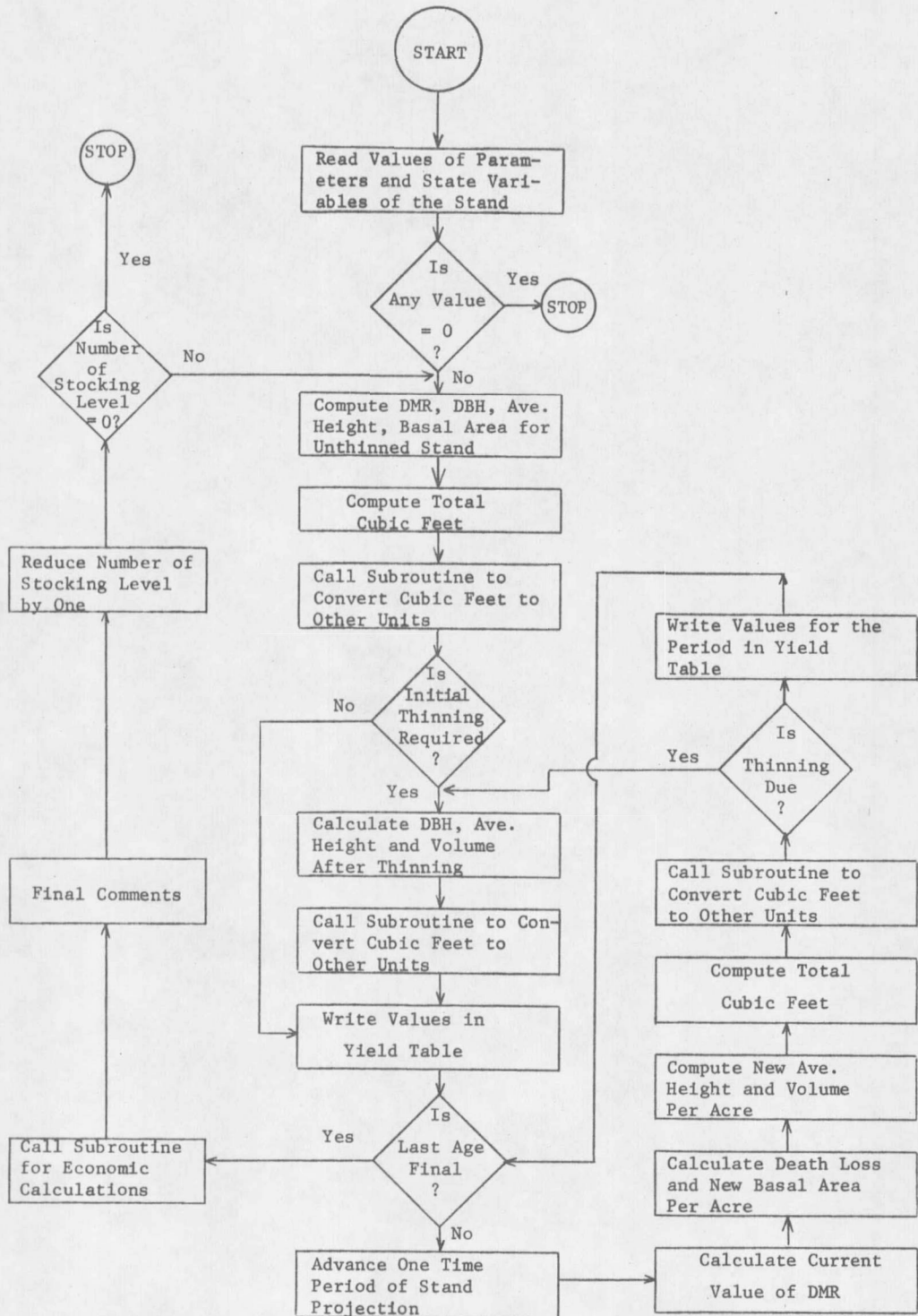


Figure 2. Flow Chart of Main Program LPMIST2.

based on the goal of sanitation thinning. The reduced infection rating due to this sanitation thinning then is computed by the main program as a function of average stand diameter.

In order to make successive projections on stand variables over several periods of time, a special loop is used. Another loop calculates the impact of thinning decisions for different thinning cycles. Before calculations reach the terminal operation of the program, a subroutine for economic calculations is called and final results are printed in a table of LPMIST2. Economic calculations are directed toward achieving the objective of a control process applied to disease-infected stands, details of which are described in the following section.

4.4. Objective of the Control Process

Since the decisions in forest management are periodic, the discussion of the control process in this section will be in the context of discrete time. It is assumed that the decisions involved in the control process are sequential, where the management planning horizon is divided into several time periods of say 10-year intervals. The criterion function, or management objective for the disease-infected stand, is assumed to be maximization of the sum of discounted net returns over an infinite planning horizon.

To clarify the decision problem, let us assume that at a certain time period in timber stand development the forest manager faces a

disease-infected stand which is to be controlled. When the stand has grown up to that period, say time period $t = s$, the condition of the timber stand (including the state of the disease infestation) can be described by a state vector \underline{X}^s . Based on technical knowledge of effective methods for controlling disease-infected stands, the forest manager has several decision alternatives from which to choose; let a particular decision be implied by decision \underline{U}^s . For this particular control problem, the decision vector may take the form of $\underline{U}^s = [u_1^s, u_2^s]$ where u_1^s indicates whether or not to harvest the timber and $u_2^s = y_s$ implies a thinning decision.

The state vector \underline{X}^t which enters the periodic net return function in time period t is somewhat redundant in a nonstochastic model since an initial value of \underline{X} and thinning policy y_1, y_2, \dots, y_T completely determine $\underline{X}^1, \underline{X}^2, \dots, \underline{X}^T$. However, since the state vector at any period will give additional descriptive value to the return function and simplify the mathematical model, the simulation model uses \underline{X}^t directly in making numerical evaluations on the return and cost functions.

The forest manager may assume that after the first cycle of the control process, the bare land resulting from clear cutting of a disease-infected stand will grow the subsequent timber crop in a disease-free state continually, cycle after cycle, into perpetuity. This assumption is justified from the disease control viewpoint as

explained by Hawksworth [1972], particularly on the obligate parasitic nature of dwarf mistletoe in pine stands. Of course, the diseased stand must be completely cut with no diseased trees left standing.

Now, let us define the following notation:

T = length of the rotation cycle;

y_t = combination of actions constituting a thinning decision in stand age period t ;

$R_t(y_t, \underline{X}^t)$ = periodic net returns generated from the timber stand as a result of thinning decisions implied by y_t at age period t , when the stand can be described by the state vector \underline{X}^t ;

$H(T, \underline{X}^T)$ = net harvest value at the end of the rotation cycle when the state vector is \underline{X}^T ;

$\beta = 1/(1+r)$ = the discount factor associated with the age interval t .

We have made a change in the notation where the period of time has been replaced with an age period, such as on the superscript t of the state vector \underline{X}^t . This switch to age as a substitute for time assumes that time per se does not affect net returns and harvest value. The subscript t on $R_t(y_t, \underline{X}^t)$ and argument T in $H(T, \underline{X}^T)$ was used to make the relationships more explicitly a function of stand age, which implies that age is not included in the state vector \underline{X}^t , $t = 1, 2, \dots, T$.

Note that if we did not enter the state vector into the return function $R_t(\cdot)$ and $H(\cdot)$, these functions would have to be written as $R_t(y_1, y_2, \dots, y_t)$ and $H(T, y_1, y_2, \dots, y_T)$. This observation

clarifies the advantage of including the state vector in those functions since it makes the return function conditionally independent of earlier thinnings—like Bellman's Markovian requirement on the net returns in a dynamic programming framework.

Given an existing condition of the forest described by bare land, the opportunity cost of the land will be equal to the maximum soil rent associated with a new stand of timber starting from bare land and all subsequent rotations over an infinite planning horizon:

$$K^* = \underset{\substack{T, y_i \\ i=1 \text{ to } T}}{\text{Max}} \left[\sum_{t=1}^T R_t(y_t, \underline{X}^t) \beta^t + H(T, \underline{X}^T) \beta^T \right] / (1 - \beta^T) \quad (34)$$

where net returns are assumed to be received at the end of each period.

Note that \underline{X}^t reflects a disease-free stand at $t = 1, 2, \dots, T$.

Because we can expect that after controlling the disease we will have a disease-free stand, then the objective of the control process can be expressed by the following criterion function when the current diseased stand is of age t :

$$\begin{aligned} f(\underline{X}^t) &= \underset{\substack{T, y_i \\ i=1 \text{ to } T}}{\text{Max}} \pi[T, y_t, y_{t+1}, \dots, y_T, \underline{X}^t] \\ &= \underset{\substack{T, y_i \\ i=1 \text{ to } T}}{\text{Max}} \left[\sum_{j=t}^T R_j(y_j, \underline{X}^j) \beta^j + H(T, \underline{X}^T) \beta^T + K^* \beta^{T-t} \right]. \end{aligned} \quad (35)$$

Note that \underline{X}^t reflects the state of the disease and each subsequent \underline{X}^j , $j > t$, also reflects some level of disease, depending on $y_t, y_{t+1}, \dots, y_{j-1}$.

In a sense the objective function of the control process in (35) can be considered as a generalization of the Faustmann criterion. Since the maximization of (35) has to be done with respect to two sets of arguments, T and $\{y_j\}$, the maximization operation can be carried out in two phases. First, we maximize the criterion function (35) with respect to T which gives the following:

$$\text{Max}_T \pi(T, y_t, y_{t+1}, \dots, y_T, \underline{X}^t) = \gamma(y_t, y_{t+1}, \dots, y_T, \underline{X}^t) \quad (36)$$

Then, in the second phase we maximize with respect to y_t, y_{t+1}, \dots, y_T to obtain the final solution. Hence, in short we have:

$$\begin{aligned} f(\underline{X}^t) &= \text{Max}_{y_i} \gamma(y_t, y_{t+1}, \dots, y_T, \underline{X}^t) \\ &\quad i=t \text{ to } T \\ &= \text{Max}_{y_i} [\text{Max}_T \pi(T, y_t, y_{t+1}, \dots, y_T, \underline{X}^t)] \\ &\quad i=t \text{ to } T \\ &= \text{Max}_{T, y_i} \pi(T, y_t, y_{t+1}, \dots, y_T, \underline{X}^t) \end{aligned} \quad (37)$$

The above maximization operation is essentially what we use in the computer program for this study. First we find K^* , then $\gamma(y_t, y_{t+1}, \dots,$

y_T, \underline{X}^t) for a limited set of thinning policies, and finally we maximize over thinning policies to get $f(\underline{X}^t)$. Each of the last two maximizations must be made with initial disease condition, stand age, etc., taken as fixed.

CHAPTER V

APPLICATION TO LODGEPOLE PINE

5.1. Data Description and Assumptions

The present value functions in (34) and (35) for determining optimal thinning and rotation policies consist of two important components to be estimated; (1) net returns associated with timber management decisions at each period, and (2) the discount rate associated with opportunity cost of capital invested in the project. In order to obtain numerical solutions to the investment problem, we have to quantify and evaluate these variables.

To derive estimates of net returns, the physical yields of wood, generated by the computer program in a yield table for each age period of the timber, are multiplied by an appropriate price of wood products which then gives the stream of gross returns. The stream of gross returns adjusted by variable costs related to a particular timber management decision then gives the stream of net returns.

In order to clarify the calculations involved, let us use the following notation:

$R_t(y_t, \underline{X}^t)$ = expected net returns associated with managing a timber stand where the decision is implied by y_t and the state of the system at period t is described by \underline{X}^t .

$G_t(y_t, \underline{X}^t)$ = expected gross returns associated with managing a timber stand where the decision is implied by y_t and the state of the system at period t is described by \underline{X}^t .

$C_t(y_t, \underline{X}^t)$ = expected variable costs associated with managing a timber stand where the decision is implied by y_t and the state of the system at period t is described by \underline{X}^t .

$E(q_t | y_t, \underline{X}^t, t) = T(y_t, \underline{X}^t, t)$ is expected physical wood yield whose estimate can be generated by a computer program in a yield table associated with the management decision implied by y_t when the state of the system at period t is described by \underline{X}^t .

Assuming a perfectly competitive wood product market, the expected net returns for any period t can be calculated by the following method:

$$R_t(y_t, \underline{X}^t) = P_w E(q_t | y_t, \underline{X}^t, t) - C_t(y_t, \underline{X}^t) = P_w T(y_t, \underline{X}^t, t) - C_t(y_t, \underline{X}^t). \quad (38)$$

Assume further that the price of wood products, P_w , is determined by interaction of supply and demand forces in the wood product market outside the timber production system. Then an estimate of wood product price can be based on time series of wood price data for related wood species which reflects the equilibrium prices that have been prevailing in the market as the result of the interacting forces.

The most closely related wood product price for timber production decisions is stumpage price. The stumpage price in a perfectly competitive market at any specific location will equal the price of sawlogs at the mill gate minus the cost of availability (including harvesting and transfer costs). A general model of stumpage price formation has been described by Gregory [1972] and a stumpage price response to

changes in volume of timber sold has been studied by Hamilton [1970]. The former author explained the uniqueness of the stumpage price formation where the location factor has an important role in determining the equilibrium price. The second author concluded that in the short-run, stumpage prices have fluctuated considerably. Moreover, an examination of the region's stumpage market indicates that shifts in demand for wood products, rather than quantity of stumpage offered for sale, are the primary determinants of this short-term price fluctuation.

One source of wood price data used in this study originated from stumpage price data which was compiled by Holt [1973] at the Pacific Northwest forest research station. The published stumpage price data are categorized into several subgroups according to wood species and the regions where they are formed. One of the wood species in the publication is pertinent to this study, namely lodgepole pine (Pinus concorta) and the regional coverage of the price report includes the Rocky Mountain region. From this price data we can determine the current stumpage price, the minimum price which has ever prevailed, and the highest price projected for the future. These different levels of estimated price are used in making sensitivity analyses in the simulation study.

Another component of the stream of net returns in (38) is cost variables. The cost of timber production has time and spatial dimensions, where the costs may vary from period to period and from one stand to another. Hence, the forest manager should be aware of possible cost

variations associated with each timber stand during each period of stand management. Therefore, an attempt must be made to obtain representative costs which can accommodate these cost differences. Whenever possible, cost should be related to physical input, state of the standing timber system, and the decision taken at each timber management period.

In relation to the cost structure, we know that production economics distinguishes two kinds of costs, i.e., fixed and variable costs, as components of total cost. However, the costs that directly relate to production decisions are variable costs; that is, the costs that vary with the state of the timber stand system, investment decision, and age period. A source of cost information that virtually meets the above requirement has been developed by Wikstrom and Alley [1967]. These authors derived predictive cost equations, originally for the purpose of cost control in timber management decisions, by using multiple regression techniques. They constructed cost equations related to each timber management activity that incurred costs such as site preparation costs, including slash piling, burning, and land scarification costs, planting costs--in case natural regeneration failed to establish a new stand--and thinning costs. By applying proper adjustments to local conditions, labor wage and equipment costs, this study employs these cost equations in combination with the computer simulation program which, at its final stage, is capable of generating a performance measure based on the economic criteria as discussed in the last chapter.

The most difficult parameter to be estimated in measuring present value of net returns in an investment model is the discount rate which reflects the opportunity cost of capital, either as viewed by private enterprise, or social opportunity cost as viewed by public decision making agencies. Although we are dealing with investment decisions related to timber land which belongs to the public domain, private investment criteria may help throw some light on public investment decisions related to forest land resources (see Lutzés, 1951; Masse, 1962; and Hirschleifer, 1970). In discussing difficulties of assessing social opportunity cost of capital, Feldstein [1964] concluded that the only way to assess the social opportunity cost is to discount at social time preference. But the search for a perfect formula to specify a social rate of time preference is futile. It is often the case that the rate is set solely as a matter of social policy, taking into account certain ethical principles as well as economic opportunities. Hence, various lines of approach attempting to find a rational choice for the discount rate all appear to be highly subjective. Therefore for purposes of the analysis of this study, the discount rate used in the investment decision is specified at several values rather than a single value. Another advantage of selecting a set of different values of discount rates is that it enables us to evaluate their impact on the optimal solution of the investment problem.

5.2. Model Experimentation

The purpose of model experimentation with a simulation program is to evaluate the impact of changes in parameters and initial states on the optimal decision rule for the problem. This analysis sometimes is called sensitivity or parameterization analysis. The computer simulation model provides a basis for making such analysis and to evaluate those changes on the performance measure of the system under study, and hence on the optimal decision rule for the problem. The procedures for selecting the initial state and parameter values of the standing timber system which are related to the physical-biological aspects in the simulation program are well explained by Myers, et al. [1971] in their manual. However, we do not attempt to select an exhaustive set of combinations of state variables and parameter values of the timber stand system; rather the choice is made in a systematic way to obtain a meaningful economic range of results and avoid redundancy in the analysis. Thus we do not attempt to explore a complete set of parameters and initial conditions to generate a complete surface of the output of the system's performance because such an attempt would be too costly.

For example, we choose only the most typical site quality for the timber stand which has an index of 70, with which we choose a limited range of other parameter and state variable values of the timber stand system including stand age, initial time of disease infestation, stand

density at the initial management period, stand age at initial thinning, the level of thinning goal, and thinning interval. The desired level of thinning goal and thinning interval are combined to constitute a particular thinning policy. Among eight combinations of thinning policies we selected seven combinations of thinning policies where each of them is a combination of sanitation thinning, two levels of desired goal of stocking at subsequent thinning with a given initial level of thinning, and two thinning intervals. ^{14/} For the decision of no thinning, we set up the initial stocking and subsequent levels with very large values so that no thinning will occur. The original program always entails sanitation thinning when a thinning decision is specified. The authors-- among them is a forest pathologist--apparently believed that sanitation thinning is so obviously necessary in managing disease-infected stands that its economic evaluation would be a waste of time.

Selection of values of predetermined variables related to economics, such as price levels of outputs, were based primarily on time series data and also using other sources of information. On the other hand, the cost variables related to thinning and harvesting activities are endogenously built into the computer simulation program because these costs are a function of the state variables of the system. However, the

^{14/} A more detailed explanation of thinning combinations is discussed in the next section.

cost of establishing a new stand is estimated by pertinent cost equations outside the system.

Results of the analysis showed some of the present values calculated to be negative. This result is mainly due to high current establishment costs prevailing in the region. Then we tried to reduce establishment cost by about one-third by excluding some components of establishment costs which seemed unimportant. There is a realistic possibility of reducing the establishment costs since the seed for lodgepole pine species is capable of establishing its new stand naturally without dozer piling and land scarification, if prescribed burning is practiced properly. With proper burning of the slash, the resin that usually seals the scale of the pine cone will melt at about 113°F. ^{15/} When this occurs the scale is free to flex and spread apart to release the seed to expose it to mineral soil which is the primary requirement for natural regeneration of a new stand. Even though this can occur under natural regeneration without fire, the prescribed burning at least will guarantee establishment of a fairly good new stand (Tockle, 1961). Based on current knowledge available, however, reduction of establishment cost would result in a sparser tree distribution in the stand. Hence, for this kind of situation, we set up a lower initial stand density as compared to the one with full establishment cost.

^{15/} The closeness of the pine cone, which causes difficulty in spreading its seed, is referred to as cone serotiny (Lotan, 1967).

In addition to predetermined price levels estimated from time series data, we also use an estimate of the future price of wood mass--instead of lumber price. The assumption behind the projected future price is based on the good prospect of new development in wood technology to produce a man-made wood, so to speak, without regard to the dimension of the log. A preliminary report on fiberboard and particle board as substitutes for lumber and plywood by Western Timber Industry showed that the possibility is promising. 16/

The discount rate chosen by a public decision agency to analyze an investment problem such as that in this study is seldom based on a purely financial objective for timber production. Nevertheless, the interest rate, like other factor prices, is an important element in determining allocation of resources. Some approaches to the selection of discount in practice have been discussed by Johnston, et al. [1967], including the long-term government borrowing rate. Gregory [1972] argued that the accepted view today is that the appropriate interest rate for public investment is the opportunity cost of capital taken from private investors by the tax system. This cost and the government borrowing rate both are typically lower than any encountered in the private

16/ "Particle Board to Outdo Lumber Coming Next?," XXII, No. 3 (Western Timber Industry, 1974).

capital market. The rate in question is in the neighborhood of 3 percent if the influence of inflation is accounted for. It is interesting to compare this rate with an aggregate physical growth rate for timber, the growth rate in Pacific Coast Forests has been estimated to average only 0.44 percent. 17/

However, as explained in the previous section, interest rates chosen for this study are a set of values ranging from very low to 5 percent. This range, with half a percent between each value, should give sufficient information for making a sensitivity analysis and yet maintain a positive present value in the optimization model which gives an admissible optimal solution for the investment problem.

Having selected the appropriate values of timber stand parameters and initial conditions for each set of these values, the computer program is run in two stages. First, the present value function for a disease-free stand is calculated and this value is maximized over an infinite planning horizon by a search procedure to obtain the value of K^* as expressed in the formula of (34). Then this optimal value resulting from the first stage was inserted into the second stage.

17/ U. S. Department of Agriculture, Forest Service, "Timber Trends in the United States," Forest Report No. 17 (Washington, D. C.: Government Printing Office, 1965).

program to search the optimal value of a generalized present value criterion as expressed in formula (35). This optimization provides a solution to the investment problem for managing a disease-infected stand.

A stage in the decision process with a shorter time period would in general give a more sensitive decision rule. However, the original computer program was constructed in such a way that the program can permit only time intervals larger than 10 years. Therefore, a quadratic function was fitted to three values of the criterion function in the neighborhood of the optimal solution with respect to the decision variable T in order to give a more sensitive decision rule. This final analysis gives a more accurate estimate of the optimal rotation age.

5.3. Simulation Results

Results of the simulation computations are summarized in Tables 1, 2, 3, 4, 5, and 6. From Table 1 we can visually observe the optimal present value, or discounted net returns (DNR), for a perpetual timber crop which is disease-free for various price levels and interest rates. The other five tables show optimal DNR with respect to the sum of soil site value and existing timber stand value for various price levels and interest rates, as well as the values in relation to different timber stand conditions. The impact of variation in economic elements and physical-biological conditions of the stand on optimal solutions to the investment problem, i.e., the optimal rotation length and thinning combinations, can also be evaluated.

TABLE 1. OPTIMAL DECISION RULES STARTING FROM BARE LAND UNDER AN INFINITE PLANNING HORIZON.*

Interest Rate	Decision Variables		Criterion
	Rotation Length	Thinning Decision	Present Value
<u>Pct.</u>	<u>Years</u>		<u>\$/Acre</u>
<u>Current Price</u>			
0.5	120	II	2,149
2.0	80	II	159
3.0	70	II	26
4.0	70	II	-22
5.0	70	II	-43
<u>Highest Price</u>			
0.5	120	II	6,266
2.0	80	II	661
3.0	70	II	247
4.0	70	II	82
5.0	70	II	7
<u>Lowest Price</u>			
0.5	120	II	1,696
2.0	80	II	125
3.0	70	II	-13
4.0	70	II	-29
5.0	70	II	-48
<u>Advanced Technology Price</u>			
0.5	90	I	1,761
2.0	70	I	214
3.0	60	I	87
4.0	40	I	41
5.0	40	I	19

*All timber stands are assumed to be disease free.

TABLE 2. OPTIMAL DECISION RULES FOR CURRENT PRICE.

Interest Rate	State Variables			Decision Variables		Criterion Present Value
	Density	Initial Age	Time Since Infection	Rotation Length	Thinning Decision	
Pct.	Trees/ Acre	Years	Years	Years		\$/Acre
0.5	2000	20	10	20	*	2,137
0.5	2000	30	20	103	II	2,337
0.5	2000	40	30	104	II	2,226
0.5	2000	50	40	50	I	2,137
0.5	6000	20	10	20	*	2,134
2.0	2000	20	10	94	VI	229
2.0	2000	30	20	102	II	317
2.0	2000	40	30	103	II	292
2.0	2000	50	40	50	I	176
2.0	6000	20	10	20	*	180
3.0	2000	20	10	85	VI	86
3.0	2000	30	20	94	IV	127
3.0	2000	40	30	94	IV	122
3.0	2000	50	40	90	I	49
3.0	6000	20	10	91	V	27
4.0	2000	20	10	84	VI	33
4.0	2000	30	20	93	IV	51
4.0	2000	40	30	93	IV	51
4.0	2000	50	40	92	I	23
4.0	6000	20	10	105	I	5
5.0	2000	20	10	76	I	6
5.0	2000	30	20	79	I	18
5.0	2000	40	30	87	I	15
5.0	2000	50	40	92	I	13
5.0	6000	20	10	104	I	2

*Independent of thinning decisions.

TABLE 3. OPTIMAL DECISION RULES FOR THE HIGHEST PRICE.

Interest Rate	State Variables			Decision Variables		Criterion Present Value
	Density	Initial Age	Time Since Infection	Rotation Length	Thinning Decision	
Pct.	Trees/ Acre	Years	Years	Years		\$/Acre
0.5	2000	20	10	20	*	6,258
0.5	2000	30	20	99	II	6,756
0.5	6000	20	10	20	*	6,238
2.0	2000	20	10	91	VI	757
2.0	2000	30	20	102	II	971
2.0	6000	20	10	20	*	651
3.0	2000	20	10	85	VI	332
3.0	2000	30	20	94	IV	446
3.0	6000	20	10	20	*	207
4.0	2000	20	10	84	VI	162
4.0	2000	30	20	84	III	238
4.0	6000	20	10	80	VII	76
5.0	2000	20	10	83	VI	77
5.0	2000	30	20	82	III	128
5.0	6000	20	10	80	VII	26

*Independent of thinning decisions.

TABLE 4. OPTIMAL DECISION RULES FOR THE LOWEST PRICE.

Interest Rate	State Variables			Decision Variables		Criterion Present Value
	Density	Initial Age	Time Since Infection	Rotation Length	Thinning Decision	
Pct.	Trees/ Acre	Years	Years	Years		\$/Acre
0.5	2000	20	10	20	*	1,669
0.5	2000	30	20	103	II	1,834
0.5	6000	20	10	20	*	1,666
2.0	2000	20	10	96	VI	169
2.0	2000	30	20	102	II	142
2.0	6000	20	10	20	*	126
3.0	2000	20	10	85	VI	60
3.0	2000	30	20	102	II	90
3.0	6000	20	10	84	I	15
4.0	2000	20	10	79	I	21
4.0	2000	30	20	93	IV	31
4.0	6000	20	10	105	I	3
5.0	2000	20	10	77	I	11
5.0	2000	30	20	80	I	13
5.0	6000	20	10	105	I	1

*Independent of thinning decisions.

TABLE 5. OPTIMAL DECISION RULES FOR ADVANCED TECHNOLOGY PRICE.*

Interest Rate	State Variables		Decision Variables		Criterion Present Value
	Initial Age	Time Since Infection	Rotation Length	Thinning Decision	
Pct.	Years	Years	Years		\$/Acre
0.5	20	10	63	I	1,852
0.5	30	20	61	I	1,932
2.0	20	10	57	I	323
2.0	30	20	56	I	385
3.0	20	10	45	I	186
3.0	30	20	44	I	245
4.0	20	10	41	I	133
4.0	30	20	41	I	192
5.0	20	10	38	I	101
5.0	30	20	39	I	161

*This price is associated with a reduced establishment cost and a lower initial density of 500 trees per acre. There is also a substantial change in the way wood is assumed to be utilized.

TABLE 6. OPTIMAL DECISION RULES FOR ADVANCED TECHNOLOGY PRICE WITH CURRENT ESTABLISHMENT COST.*

Interest Rate	Decision Variables		Criterion
	Rotation Length	Thinning Decision	Present Value
<u>Pct.</u>	<u>Years</u>	<u>Years</u>	<u>\$/Acre</u>
0.5	40	I	1,997
2.0	40	I	408
3.0	40	I	258
4.0	40	I	184
5.0	40	I	140

*This current establishment cost is associated with initial stand density of 2,000 trees per acre, initial age of 20 years, and infection time of 10 years.

In order to clarify some notation and possible implications of the results, we should first explain the meaning and definitions of the key words in the tables and the related variables which were used in the simulation operation. First, thinning combinations which constitute alternative thinning decisions are defined below:

- I = no thinning;
- II = thinning for sanitation only;
- III = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 120, and a thinning interval of 20 years;
- IV = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 120, and a thinning interval of 30 years;
- V = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 80, and a thinning interval of 20 years;
- VI = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 80, and a thinning interval of 30 years;
- VII = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 50, and a thinning interval of 20 years;
- VIII = combination of sanitation thinning with stocking level goal at subsequent thinnings equal to 50, and a thinning interval of 30 years.

All thinning decisions start with an initial stocking level index of 120. The stocking level refers to a stand density index, expressed as basal area per acre where the index is based on 10 inches of stand diameter; while stocking goal indicates the desired level of the stocking index expected to result after a thinning has been executed.

Thinning interval refers to the time interval between two consecutive thinning decisions during the growth period of the timber stand.

In the computation of a simulation run, we used three price levels based on time series data where each of them consists of prices of two components of the wood harvested; one is saw-timber price and the other merchantable timber (pole) price. The current price for saw timber is \$50.00 per thousand board feet and the current price for merchantable timber is \$100.00 per thousand cubic feet. The highest price is based on current price plus average increase of timber prices during the previous ten years, which reached 275 percent of current price so that this price is \$137.50 per thousand board feet for saw timber and \$275.00 per thousand cubic feet for merchantable timber. The lowest timber price which ever prevailed is 80 percent of the current price, so that the lowest price of saw-timber is \$40.00 per thousand board feet and \$80.00 per thousand cubic feet for the lowest merchantable timber price.

Besides the above price levels we also use a future price projection for advanced technology in wood utilization, based on the assumption that raw wood price will reach 150 percent of the current price. This price is \$150.00 per thousand cubic feet. The technology assumed to exist for this price is such that only volume of wood is important and the current bias of the price system toward saw timber does not exist. We have a simplified situation where value of the harvested timber is simply the product of price and volume of timber.

Current establishment cost for site preparation was estimated at \$59.47 per acre, while the reduced establishment cost used was \$20.00 per acre. The computer program was run for the reduced establishment cost that is associated with a lower initial stand density. Based on current knowledge available, this lower stand density will be around 500 trees per acre. Also, the computation for the advanced technology price with current establishment cost was run to evaluate the impact of this price projection on the optimal decision rules.

The state variable which measures the degree of disease infestation is time since infection. The reason for this representation is that time since infection is highly correlated with timber volume reduction caused by the disease. Thus time since infection is a surrogate measure of dwarf mistletoe rating which is used as an index to measure the degree of disease infection (see Hawksworth and Hinds, 1964).

Let us now interpret the results of the simulation experiments and examine the economic implications. First, in Table 1 we can observe the impact of interest rates and price levels on optimal DNR, rotation length, and thinning decisions in cases when we plant a disease-free stand starting from bare land. Discounted net returns (DNR) in Table 1 are equal to K^* in equation (34). In this table we notice that based on the current price level, the optimal DNR for higher interest rates than 3 percent is negative. For the lowest price, negative values even start with a 3 percent interest rate.

In comparing optimal DNR of the advanced technology price with the ones of the first three price levels, the results show that even though the optimal DNR of the advanced technology price are all positive, DNR are lower at the upper interest rate levels than with the other price levels. The reason behind this result is that at lower interest rates with lighter discounting, the advanced technology price is associated with the reduced establishment cost which interacts with a lower density as a consequence of less intensive site preparation. The final result then yields a lower optimal DNR for the advanced technology price.

Further observation of Table 1 shows that the first three price levels and variations in interest rates, in general, do not significantly change the optimal decision rules, i.e., the optimal rotation is the same within the 10-year approximation allowed and the optimal thinning decision is of the Type II which implies thinning for sanitation only. The impact of the lowest interest rate consistently gives a longer optimal rotation length for every price level. This implies that when the cost of capital expressed as the interest rate becomes very low, there is a general tendency to postpone cutting timber in order to obtain the higher value associated with more mature trees which yield more saw timber.

In a diseased timber stand, it is important to realize that the maximization operation on the generalized present value, or generalized discounted net returns (GDNR), for controlling the disease-infected

stand implies that we maximize the sum of present value of the timber stand in its current state (if it is infected) and discounted soil site value for the bare land, K^* . Hence, there is a close relationship between GDNR and K^* . In fact, the difference, $GDNR - K^*$, can be considered as a measure of the value of the presently standing timber independently of the land itself.

Tables 2, 3, 4, 5 and 6 show optimal values of GDNR and related decision rules for controlling disease infected stands for various prices and interest rates under different initial stand conditions. From Tables 2, 3, and 4 we can evaluate the impact of different interest rates and price levels on the optimal GDNR and optimal decision rules, namely the optimal rules related to thinning and rotation length. Each of these three tables can also be used to evaluate the impact of different interest rates and stand conditions on the optimal GDNR and decision rules at each given price level.

The first impression of the results is that when the interest rate becomes higher, the resulting optimal GDNR is lower. This result is merely a logical consequence of the interest rates being inversely related to the weights used in the discounting of future net returns. Hence the optimal GDNR decreases with an increase in the interest rate.

Variation in the interest rate does change the optimal decision rules. The change in interest rates from 2 to 5 percent at the same stand density usually causes a shorter optimal rotation length in a

diseased stand. The logical basis for this result is essentially the same as given above for a disease-free stand. The exception to this general influence of interest rates on the optimal rotation length occurs when the diseased stand is in a state for which the best action is to replace it immediately with a new rotation.

The first row of Table 2 illustrates a case where the optimal action is to cut the diseased stand immediately, which is associated with an interest rate of 0.5 percent. But the same stand and a rate of 2.0 percent (row 6) indicates an optimal action much different; namely, wait 74 years before replacement (a rotation of 94 years instead of 20 for the diseased stand). An explanation for this apparent anomaly is that at a lower interest rate, we can afford to immediately plant a more highly productive disease-free stand and wait for it to mature, where at the higher interest rate, this delay in receipt of the returns costs too much and we are better off to keep the less productive diseased stand until maturity.

Examination of the impact of the first three price levels shows that a change in price does change the optimal rotation length and thinning decisions. However, the change of optimal rotation is within the range of 20 years so that from a practical management viewpoint it is not very significant. Hence, we may say that the price levels do not greatly influence the optimal solution to the investment problem for this particular lodgepole pine stand.

Price changes from the current price level to the advanced technology price for a given stand condition decrease the optimal GDNR about 7 percent and extend the rotation length at very low interest rate. However, at the higher interest rate, advanced technology price increases the optimal GDNR and decreases the rotation length, ceteris paribus. This result implies that advanced technology requires earlier cutting of timber in order to obtain a better wood-mass production and higher optimal GDNR without regard to the size of the timber and also requires no thinning.

Comparing Tables 5 and 6, the impact of the establishment cost can be evaluated for the same advanced technology price. The reduction of that cost causes a lower optimal GDNR and longer rotation length. This result shows that the reduction of the cost interacts with less volume of the wood due to the lower stand density and the final result yields a lower optimal GDNR. For obtaining an optimal GDNR the sparser stand has to wait a longer period to yield more wood production. However, since advanced technology does not require good timber for lumber, different establishment cost requires no thinning decision.

Comparing the optimal rotation length at different infection times, the results show that when time since infection becomes longer, the optimal rotation length in general becomes longer. This result signifies that when the intensity of infection is heavier, the diseased

stand needs a longer growth period in order for the stand to reach saw-timber size. However, since the impact of infection length is confounded with the influence of different initial ages, the result does not give an unequivocal conclusion with regard to the impact of infection time due to limitation from biological nature of the disease.

There is a biological limitation in interpreting the economic impact of longer disease infection time. First, dwarf mistletoe is a chronic disease which is difficult to observe visually in the field before the infection time reaches 10 years, except by a microscope intercellular investigation on the wood tissues infected. Because of the chronic nature of the disease, any visible degree of infection by the disease must have been developed from an earlier date when the infection started. Hence, we cannot assume that any visible mistletoe has started at a time less than 10 years prior to its appearance in experimental data.

Another problem is the assumption within the model that the entire stand of timber is infected in some average or uniform way. About the only way that this situation can occur is when the seedlings of a new stand are infected by an overstory of diseased trees. This is apparently the reason for the computer program of Myers et al. [1971] being written such that stand age and time since infection must be chosen in a systematic way; namely, age must exceed time since infection by a 10-year interval.

At a given price and starting at a sufficiently low interest rate, an increase in stand density appears to reduce the optimal rotation age. But at sufficiently high interest rate, the converse seems to be true. The breaking point on interest rates where the reversal takes place depends on the price level assumed--see the tables.

The tendency is not very defensible because of limited data provided by the analysis, but the evidence is clearest in Table 4, next clearest in Table 2, and the reversal does not appear in Table 3 but supposedly would at a high enough interest rate.

The impact of interest rate and price levels on the optimal thinning combinations depends on the condition of the timber stand. A low initial stand density with a short infection time shows that the majority of optimal thinning combinations are of Type VI; that is, the thinning combination of sanitation with desired stocking level at subsequent thinning with index 80 and thinning interval 30 years. In addition, it is noticed that the shortest optimal rotation length (20 years) is necessarily associated with a situation where thinning is irrelevant because the current stand is replaced immediately.

CHAPTER VI

SUMMARY AND CONCLUSION

6.1. General

This study attempts to analyze a forest production decision problem in general and the management aspects related to investment decisions. An application is made to a thinning and rotation decision for a disease infected timber stand in particular. Since production decisions in forestry involve large lapses of time--where large amounts of capital outlays are spent at the initial period of production and receipts are dominated by the income from terminal harvest (point-input-output case of investment)--an appropriate line of approach to the problem is capital theory because the essence of the theory is concerned with the economics of time. In an earlier chapter we presented an elementary notion of capital theory and applied the basic structure to an investment model for a simple decision problem in forestry which can be solved analytically. However, as the problem becomes more realistic, a more sophisticated method is required to analyze and solve the problem. Dorfman [1969], in interpreting modern control theory from an economic viewpoint, has asserted that optimal control theory is identical with capital theory. Hence a more realistic, but more complicated analytical framework for approaching forest production decision problems in this study is that of optimal control strategies.

We define a strategy as a conditional decision rule which states how to control the forest production system at any given stage of timber stand development for each set of values of state variables describing the system. The strategy is applied sequentially as new information on the state of the system unfolds through time and the rule is chosen to maximize a certain criterion function.

Forest production constitutes a complex probabilistic system, where some of its state variables exhibit random fluctuations over time. Hence a decision model for controlling the system should be capable of capturing the stochastic behavior of the system. Among various methods available to approach the control problem in forest production, stochastic dynamic programming seems to be the most appropriate method for solving the problem. An advantage of stochastic dynamic programming is in obtaining numerical solutions to the problem since it provides a method to search for the optimal policy in a systematic way. In addition, a decision model based on this method can recognize and explicitly incorporate risk aspects of the decision into the model so that the uncertainty of future outcomes resulting from any decision can be evaluated.

Dynamic programming requires the definition of a stage of the process, which serves as a basis for the multistage decision model, and the state variables, which can capture the essence of the state of the system. Then we formulate the state transformation functions which can predict or

trace the dynamic process through time. Prediction of the future state of the system can be made deterministically, with certainty, or stochastically according to the probability distribution of the future state of the system. In a stochastic version of the decision process, we incorporate a Markov chain into the decision model, where the transition probability matrix of the chain can be estimated from the conditional probability distribution of the future state of the process by means of an analysis of the residuals of regression equations.

Having specified the variables of the system and estimated the parameters required in the dynamic relationships, a criterion function for the problem is formulated as maximization of the sum of expected discounted net returns with respect to soil site--the Faustmann criterion in a statistical sense. In order for a criterion function to be applied in a dynamic programming framework, the criterion function must be decomposable by stages. A multistage problem which can be solved by the dynamic programming procedure is characterized by a separable criterion function and satisfies a first-order Markovian dependence structure (see Bellman, 1961). This condition is met in the investment problem in forestry because the net returns are assumed to occur at each stage and to be additive.

An essential step in formulating the control problem using dynamic programming is application of the "principle of optimality" which yields the fundamental recurrence relation. This recurrence relation connects

the different sub-problems at each stage of the overall problem, and in effect, permits the control problem to be solved in terms of a number of smaller optimization problems. With such algorithms, the optimal decision rule for the investment problem in forestry can be obtained. If there are variations in stand parameters due to different locations, and in case an economic projection is needed using different values of predetermined economic variables, a sensitivity analysis can be applied.

Besides the advantages of stochastic programming for solving the control problem in forestry, there are some obstacles to successful implementation of this method. The first obstacle is related to computer capacity, where for each set of values of the state variables, the algorithm must perform an optimization operation and store the result. Hence, dynamic programming is usually impractical when there are more than two state variables per stage unless computing resources are sufficient to permit the reduced speed of computations associated with partial storage on disc files. In large computer systems with plenty of disc storage for spillover, problem size is largely limited by the budget the analyst can justify to solve the problem.

Another obstacle is related to empirical measurement of parameters to estimate transition probabilities in the Markov chain. This problem, like that in every area of research in agricultural economics, requires some ingenuity by the researcher. Nevertheless, the importance of this aspect in research using stochastic dynamic programming is crucial, so

that Burt (undated) has made this admonishment: "Do not get too excited about solving any economic question until you check the feasibility of estimating these transition probabilities."

6.2. Specific Problem

As explained in the first part of this study, the basic data for estimating the dynamic relationships which constitute state transformation functions in dynamic programming, which are needed to approach the control problem in forest production, are not readily available. Hence, in the second part of this study we pursue another method to solve a specific problem in forest management.

The specific problem of this study deals with the control problem of managing a disease-infected lodgepole pine stand caused by dwarf mistletoe. The disease constitutes one of the exogenous factors affecting timber growth with an adverse effect, particularly on lodgepole pine stands. Since the relationship between the level of disease infestation and its effect on timber growth can be predicted over time, Myers et al. [1971] have incorporated its impact in a computer program to be used in simulating management problems of diseased stands. In a sense, the exogenous influence of the disease has been internalized into a stand management system to account for its impact in evaluating a timber stand for management purposes. The simulation program is capable of generating information on yields per-

taining to different levels of the state of the disease infestation, which can be altered by thinning decisions. However, this simulation model does not try to search for an explicit optimal policy for controlling the production process in growing timber; neither does it contain economic elements to be used for an economic decision. Hence in order for the simulation program to be used to build a decision model based on economic criteria, the program had to be modified somewhat.

Consistent with the control problem of managing a diseased timber stand, we modified the simulation program into an economic model with a more specific objective function to solve the economic problem of optimal management of diseased stands. The modified program contains economic elements such as price and cost components, which at the final stage of its development is capable of generating a performance measure of the forest management system based on economic criteria. The criterion function of the modified program is generalized present value as an extension of the Faustman criterion.

Although the economic decision model for this specific problem is deterministic in nature, by virtue of the certainty equivalence principle, the model can be considered as a good approximation for solving a stochastic problem in managing forest production systems because the decision process based on the model is carried out sequentially. The search procedure used to find the optimal solution for the investment problem specified in this study is consistent with the nature of the

the technical and physical-biological aspects of the problem in managing a diseased stand.

The modified computer program will help a forest management authority in making decisions related to investment problems in any forest stand conditions--whether a bare land condition, a healthy stand, or a disease-infected stand. In other words, for any given initial state of the forest production system and appropriate estimates of economic factors outside the system, optimal decision rules can be derived which are similar to results like the ones that are presented in Tables 1, 2, 3, 4, 5, and 6. Even though these tables were constructed from a limited number of data input combinations for the computer program, conceptually these tables can be expanded with a large number of data input combinations so that the tables will consist of complete information needed by the forest manager to make appropriate decisions. In practice, the forest manager does not have to construct such large tables if he has access to a computer. With a computer available, he can calculate the optimal decision for a given situation by using physical-biological data related to the forest system and the appropriate economic information. Further, with the availability of a computer, or "complete" tables, the forest manager at any time period of management, say period t , can always take an appropriate decision from among many possible courses of investment alternatives which affect the future state of the system. When he arrives at the

next period, say $t+1$ period, he can again make an optimal decision after observing the state of the system at that period. In short, he can proceed through the decision process sequentially where the consequence of a decision at any period is carried over to the next, and the next period decision is extended to the following period, and so on into perpetuity.

In general, the model gave results which are consistent with a priori logical reasoning applied to the investment decision problem. Hence we believe that the approach can be expanded for application to other species and larger areas than the Rocky Mountain Region. However, in order for the effort to be successful, many improvements of the model should be made, such as validation of the physical-biological results of the simulation by comparison with actual observations in the field by examining forest inventory data. Estimation of prices can be improved by additional supply and demand analysis for the interrelated timber products.

For the particular timber species of lodgepole pine in the Rocky Mountain Region, in general, economic elements of price levels and interest rates do not have a very substantial influence on the optimal investment decision, especially with respect to rotation length. Hence, optimal economic management decisions for diseased timber stands in this particular situation depend mostly on physical-biological aspects of the forest.

In addition, it is noted that the low interest rates necessary to obtain positive discounted net returns in forestry projects, especially in tree planting activities, are not attractive for private enterprise, particularly for low wood quality like lodgepole pine. The one exception to this conclusion is where an advanced technology in wood utilization was assumed, in which case, mere production of wood in any form becomes the dominant consideration and lodgepole pine can then produce a high value product. Rotations are substantially shorter for this kind of wood usage because saw-log size is not a dominant consideration any longer.

APPENDIX

C PROGRAM LPMIST

C C
C C
C TO COMPUTE AND PRINT YIELD TABLES FOR EVEN-AGED STANDS OF LODGEPOLE
C PINE INFECTED BY DWARF MISTLETOE.

C C
C DEFINITIONS OF VARIABLES.

C ADDHT = INCREASE OR DECREASE IN AVERAGE STAND HEIGHT BY THINNING.
C AGE0 = INITIAL AGE IN YIELD TABLE.
C BASC = BASAL AREA CUT PER ACRE.
C BAS0 = BASAL AREA PER ACRE BEFORE THINNING.
C BAS1 = BASAL AREA PER ACRE AFTER THINNING.
C BCFC = BOARD FEET CUT PER ACRE.
C BCFO = BOARD FEET PER ACRE BEFORE THINNING.
C BCFT = BOARD FEET PER ACRE AFTER THINNING.
C CFMC = MERCHANTABLE CU. FT. CUT PER ACRE.
C CFMO = MERCH. CU. FT. PER ACRE BEFORE THINNING.
C CFMT = MERCH. CU. FT. PER ACRE AFTER THINNING.
C DBH0 = AVERAGE STAND D.B.H. BEFORE THINNING.
C DBH1 = AVERAGE STAND D.B.H. AFTER THINNING.
C DENC = TREES CUT PER ACRE.
C DENO = TREES PER ACRE BEFORE THINNING.
C DENT = TREES PER ACRE AFTER THINNING.
C DIE = TREES LOST IN DISEASED STANDS IN 10 YEARS, IN PERCENT.
C DLEV = GROWING STOCK LEVEL FOR INTERMEDIATE CUTS AFTER THE FIRST.
C DMR = DWARF MISTLETOE INFECTION RATING.
C DMRT = MAXIMUM INFECTION EXPECTED IN STANDS AFTER THINNING. GOAL
C FOR STANDS NOT ALREADY BEYOND DMR OF 3.0.
C DSTY = LOWEST VALUE OF DLEV USED IN A TEST.
C HTSO = TREE HEIGHT BEFORE THINNING.
C HTST = TREE HEIGHT AFTER THINNING.
C JCYCL = INTERVAL BETWEEN INTERMEDIATE CUTS.
C KSTEP = INDICATOR WITH VALUE OF ONE IF CURRENT THINNING IS FROM
C BELOW AND TWO IF CURRENT THINNING IS FROM ABOVE.

C KTR = INDICATOR WITH VALUE GREATER THAN ZERO IF A SCHEDULED
 C THINNING HAS BEEN SKIPPED BECAUSE MISTLETOE INDEX IS TOO HIGH
 C OR BECAUSE STAND IS ALREADY TO SPECIFIED STOCKING.
 C MIX = NUMBER OF STOCKING LEVELS EXAMINED PER TEST.
 C NFLAG = INDICATOR WITH VALUE GREATER THAN ZERO IF A THINNING FROM
 C ABOVE HAS BEEN MADE AT ANY TIME.
 C NTSTS = NUMBER OF TESTS PER BATCH.
 C CLT = PERCENT MORTALITY IN HEALTHY STANDS.
 C PCT = PERIODIC HEIGHT INCREASE IN INFESTED STAND, AS A PERCENTAGE
 C OF THE INCREASE IN COMPARABLE HEALTHY STANDS.
 C PRET = PERCENTAGE OF TREES RETAINED AFTER THINNING.
 C REDT = PERCENTAGE REDUCTION IN NUMBER OF TREES WHEN DMR IS
 C REDUCED TO DMRT BY THINNING.
 C RINT = NUMBER OF YEARS FOR WHICH A SINGLE PROJECTION IS MADE.
 C ROTA = FINAL AGE IN YIELD TABLE.
 C SITE = SITE INDEX.
 C START = STAND AGE AT TIME OF INITIAL INFECTION.
 C TEM = PERIODIC D.B.H. INCREASE IN INFESTED STAND, AS A PERCENTAGE
 C OF THE INCREASE IN COMPARABLE HEALTHY STANDS.
 C THIN = GROWING STOCK LEVEL FOR INITIAL THINNING.
 C TCTC = TOTAL CUBIC FEET CUT PER ACRE.
 C TOTO = TOTAL CUBIC FEET PER ACRE BEFORE THINNING.
 C TOTT = TOTAL CUBIC FEET PER ACRE AFTER THINNING.

100

C COMMON BA, BAST, DBHO, DBHT, DEMO, DMR, DMRT, FCTR, PRET, PROD, REST, VCM
 C DIMENSION VAR(9), TEMH(2)
 C DIMENSION SAGE(20), SAWTIM(20), MERCHVOL(20), NTREES(20)
 C *, CVAL(20), MB(20), CSAWTIM(20), CMERCHVOL(20), DIS(8), CONSTANT(8)
 C *, TVOL(20), CTVOL(20)

C READ ESTABLISHMENT COST, LUMBER PRICES, INTEREST RATES, AND CONSTANTS
 C
 C READ(5,15) CESTR, VSAWTIM, VMERCHVOL, VTVOL
 C READ(5,10) NRATES, OPTIGN
 C REPEAT 3, FOR I=(1, NRATES)

```

      3 READ(5,15) DIS(2*I-1),CONSTAN(2*I-1),DIS(2*I),CONSTANT(2*I)
C
C READ NUMBER OF TESTS PER BATCH FROM CARD TYPE ONE.
C
      READ (5,5) NTSTS
      5 FORMAT (I4)
      IF(NTSTS .LE. 0) GO TO 310
C
C EXECUTE PROGRAM ONCE FOR EACH SET OF INITIAL VALUES OF INTEREST.
C
      DO 300 I=1,NTSTS
C
C READ INITIAL VALUES, ONE TEST AT A TIME, FROM CARD TYPES 2 AND 3.
C
      READ (5,10,END=350) JCYCL,MIX
      10 FORMAT (2I4)
      IF(JCYCL .LE. 0 .OR. MIX .LE. 0) GO TO 310
      READ (5,15) AGE0,DBH0,DEN0,DS1Y,RINT,ROTA,SITE,THIN,START
      15 FORMAT (9F8.3)
      VAR(1) = AGE0
      VAR(2) = DBH0
      VAR(3) = DEN0
      VAR(4) = DS1Y
      VAR(5) = RINT
      VAR(6) = ROTA
      VAR(7) = SITE
      VAR(8) = THIN
      VAR(9) = START
      DO 20 L=1,9
      IF(VAR(L) .LE. 0.0) GO TO 310
      20 CONTINUE
      DLEV = 0.0
C
C PROVIDE FOR SEVERAL GROWING STOCK LEVELS PER TEST.

```

C

DC 300 M=1,MIX
A = M
ACDHT = 0.0
BCFO = 0.0
BCFT = 0.0
CFMO = 0.0
CFMT = 0.0
DMR = 0.0
DMRT = 0.0
FTCUM = 0.0
KSTEP = 1
KTR = 0
NFLAG = 0
TIME = 0.0
DLEV = (DSTY + (A * 10.0)) = 10.0
BASO = DENO * 0.0054542 * DBHQ * DBHQ
KR=0

102

C

C

C

COMPUTE CURRENT DWARF MISTLETOE RATING, UNTHINNED STANDS.

TIME = AGE0 = START
IF(TIME .LE. 0.0) GO TO 25
DMR = 0.31572 + 0.08654 * TIME = 0.00016 * DENO
IF(DMR .LT. 0.0) DMR = 0.0
IF(DMR .GT. 6.0) DMR = 6.0

C

C

C

OBTAIN AVERAGE HEIGHT AND VOLUMES PER ACRE.

25 IF(AGE0 .GT. 45.0) GO TO 30
HTSO = 3.26111 = 0.05979 * AGE0 + 0.01215 * AGE0 * SITE
GO TO 35
30 HTSO = 0.33401 = 33.2866 / AGE0 + 0.92341 * ALOG10(SITE) + 6.27811
1* ALOG10(SITE) / AGE0
HTSO = 10.0 ** HTSO

35 PCT = 1.0 = 0.0165 * DMR * DMR
HTSO = HTSO * PCT

C
C COMPUTE TCTAL CU. FT. AND CONVERT TO OTHER UNITS.
C

D2H = DBHD * DBHO * HTSO
IF(D2H .GT. 7000.0) GO TO 40
TCTO = (0.00276 * D2H + 0.00059 * BASO + 0.00577) * DENO
GO TO 45
40 TCTO = (0.00248 * D2H + 1.96336) * DENO
45 IF(DBHO .LT. 5.0) GO TO 50
VDM = DBHO
BA = BASO
CALL LPVCL
BCFO = TCTO * PROD
CFMO = TCTO * FCTR
50 REST = THIN

C
C ENTER LOOP FOR REMAINING COMPUTATIONS AND PRINTOUT.
C

DO 250 K=1,100
IF(AGED .GE. ROTA) GO TO 90

C
C COMPUTE D.B.H. AFTER THINNING.
C

IF(DMR .LT. 3.0) GO TO 55
EAST = BASO
DBHT = DBHO
DMRT = DMR
HTST = HTSO
KTR = 1
GO TO 75
55 IF(DMR .EQ. 0.0) GO TO 63
IF(INFLAG .GT. 0) GO TO 62
DMRT = 0.25 * DBHO + 0.50

```

IF(DMRT .LT. 0.0) DMRT = 0.0
IF(DMRT .GE. DMR) GO TO 62
CALL LPCUT2
NFLAG = 1
KSTEP = 2
GO TO 65
62 CALL LPCUT3
IF(PRET .GE. 100.0) GO TO 67
KSTEP = 1
DMRT = DMR + 0.0275 * PRET = 2.79
GO TO 65
63 DMRT = DMR
CALL LPCUT1
IF(PRET .GE. 100.0) GO TO 67
KSTEP = 1
65 IF(BAST .LT. BASO) GO TO 70
67 BAST = BASO
DBHT = DBHO
DMRT = DMR
HTST = HTSO
KTR = 1
GO TO 75

```

C
C COMPLETE HEIGHT AND VOLUMES AFTER THINNING.

C

```

70 GO TO (71,72), KSTEP
71 ADDHT = 6.79950 = 3.41979 * ALOG10(PRET)
GO TO 73
72 ADDHT = 3.76362 * ALOG10(PRET) = 7.97347
73 HTCUM = HTCUM + ADDHT
HTST = HTSC + ADDHT
75 JDENT = (BAST / (0.0054542 * DBHT * DBHT)) + 0.5
DENT = JDENT
BAST = 0.0054542 * DBHT * DBHT * DENT
D2H = DBHT * DBHT * HTST

```

```
IF(D2H .GT. 7000.0) GO TO 80
TCTT = (0.00276 * D2H + 0.00059 * BAST + 0.00577) * DENT
GO TO 85
```

```
80 TCTT = (0.00248 * D2H + 1.96336) * DENT
```

```
C
C CONVERT TCTAL CU. FT. TO OTHER UNITS.
C
```

```
85 IF(DBHT .LT. 5.0) GO TO 90
```

```
VDM = DBHT
```

```
BA = BAST
```

```
CALL LPVCL
```

```
BDFT = TCTT * PROD
```

```
CFMT = TCTT * FCTR
```

```
C
C CHANGE MODE AND ROUND OFF FOR PRINTING.
C
```

```
90 JAGED = AGEO
JSITE = SJTE
JCENC = DENO + 0.5
JHTSC = HTSO + 0.5
JTOTC = (TOTO * 0.1) + 0.5
JTOTO = JTOTO * 10
JBASC = BASO + 0.5
JCFMO = (CFMO * 0.1) + 0.5
JCFMO = JCFMO * 10
JBDFO = (BDFO * 0.01) + 0.5
JBDFO = JBDFO * 100
JHTST = HTST + 0.5
JTOTT = (TOTT * 0.1) + 0.5
JTOTT = JTOTT * 10
JCFMT = (CFMT * 0.1) + 0.5
JCFMT = JCFMT * 10
IF(JCFMT .GT. JCFMO) JCFMO = JCFMT
JBDFT = (BDFT * 0.01) + 0.5
JBDFT = JBDFT * 100
```

```
IF(JBDFT .GT. JBDFO) JBDFO = JBDFT
JBAST = BAST + 0.5
JDENC = JDENO = JDENT
JBASC = JBASO = JBAST
JTOTC = JTOTO = JTOIT
JCFMC = JCFMO = JCFMT
IF(JCFMC .LE. 0) JCFMC = 0
JBDFC = JBDFO = JBDFT
IF(JBDFC .LE. 0) JBDFC = 0
```

```
C
C WRITE HEADINGS FOR YIELD TABLE.
```

```
IF(K .GE. 2) GO TO 120
WRITE (6,96) JSITE,THIN,DLEV
95 FORMAT (1H1,///,39X,53HYIELDS PER ACRE OF EVEN-AGED STANDS OF LODG
1EPOLE PINE/1H,57X,11HSITE INDEX,13/1H,38X,29HTHINNING INTENSITY
2= INITIAL-,F5.0,2X,12HSUBSEQUENT=,F5.0)
WRITE (6,100)
100 FORMAT (1H0,25X,38HENTIRE STAND BEFORE AND AFTER THINNING,28X,26HP
1ERIODIC INTERMEDIATE CUTS)
WRITE (6,105)
105 FORMAT (1H0,9X,5HSTAND,10X,5HBASAL,3X,7HAVERAGE,2X,7HAVERAGE,3X,5H
1TOTAL,3X,9HMERCHANT=,3X,9HSAWTIMBER,9X,5HBASAL,4X,5HTOTAL,3X,9HMER
2CHANT=,3X,9HSAWTIMBER)
WRITE (6,110)
110 FORMAT (1H,10X,3HAGE,4X,5HTREES,3X,4HAREA,4X,6HD.B.H.,3X,6HHEIGHT
1,2X,6HVOLUME,2X,11HABLE VOLUME,4X,6HVOLUME,3X,5HTREES,3X,4HAREA,3X
2,6HVOLUME,2X,11HABLE VOLUME,4X,6HVOLUME)
WRITE (6,115)
115 FORMAT (1H,8X,7H(YEARS),3X,3HNO.,3X,6HSG.FT.,4X,3HIN.,6X,3HFT.,4X
1,6HCU.FT.,5X,6HCU.FT.,6X,6HRD.FT.,4X,3HNO.,3X,6HSG.FT.,2X,6HCU.FT.
2,5X,6HCU.FT.,6X,6HRD.FT.)
```

```
C
C WRITE TABLE ENTRIES OF DIAMETER, VOLUMES, ETC.
```

```

120 CONTINUE
    KB=KB+1
    MB(KB)=1
    SAGE(KB)=JAGEO
    MERCHVOL(KB)=JCFMO
    SAWTIM(KB)=JBDFD
    NTREES(KB)=JDENC
    CMERCHVOL(KB)=JCFMC
    CSAWTIM(KB)=JBDFC
    CVAL(KB)=2*DBHO*DBHT
    CTVOL(KB)=JTOTC
    TVOL(KB)=JTOTO
    WRITE (6,125) JAGEO,JDENO,JBASO,DBHO,JHTSO,JTCTO,JCFMO,JBDFD
125  FORMAT (1H0,9X,I4,4X,I5,2X,I4,5X,F5.1,5X,I3,4X,I5,6X,I5,6X,I6)
    IF(AGEO .GE. ROTA) GO TO 255
    WRITE (6,130) JAGEO,JDENT,JBAST,DBHT,JHTST,JTOTT,JCFMT,JBDFD,JDENC
    1,JBASC,JTOTC,JCFMC,JBDFC
130  FORMAT (1H ,9X,I4,4X,I5,2X,I4,5X,F5.1,5X,I3,4X,I5,6X,I5,6X,I6,4X,I
    15,3X,I3,5X,I4,6X,I4,8X,I5)

C
C  COMPUTE VALUES FOR EACH PERIOD. THIN AS SPECIFIED.
C
    IRINT = RINT
    IK = JCYCL / IRINT
    DO 200 L=1,IK
    AGE0 = AGE0 + RINT
    IF(AGE0 .GT. ROTA) GO TO 255

C
C  COMPUTE CURRENT DWARF MISTLETOE RATING.
C
    TIME = AGE0 - START
    IF(DMR .GT. 0.0) GO TO 135
    IF(TIME .LE. 0.0) GO TO 150
    DMR = 0.31572 + 0.08654 * TIME - 0.00016 * DENT
    GO TO 145
135  IF(DMRT .LE. 1.0) GO TO 140

```

```

    DMR = DMRT + 0.07 * RINT
    GO TO 145
140 DMR = DMRT + (0.03 + 0.038 * DMRT) * RINT
    IF(L .LE. 2) GO TO 145
    DMR = DMR + 0.07 * RINT
145 IF(DMR .LT. 0.0) DMR = 0.0
    IF(DMR .GT. 6.0) DMR = 6.0

```

C
C COMPUTE NEW D.B.P. BEFORE THINNING AND ROUND OFF TO 0.1 INCH.

```

150 DBHO = 1.0222*DBHT + 0.0151*SITE = 1.2417*ALOG10(BAST) + 2.1450
    IF(DMRT .LE. 3.9) GO TO 155
    TEM = (DBHO - DBHT) * (1.0 - (0.192 * DMRT - 0.754))
    DBHO = DBHT + TEM
155 IDBHO = DBHO * 10.0 + 0.5
    DBHO = IDBHO
    DBHO = DBHO * 0.1
    IF(DENT .GT. 1000.0) GO TO 160
    DIE = (3.81 * DMRT - 6.63) * 0.01
    IF(DIE .LT. 0.0) DIE = 0.0
    GO TO 165
160 DIE = (8.04 + 3.28 * DMRT) * 0.01
165 OUT = 0.0
    IF(DBHT .GE. 10.0) GO TO 170
    OUT = 0.05285 = 0.01346 * DBHT + 0.00226 * DBHT * DBHT + 0.0000066
    1 * BAST * BAST = 0.0001931 * DBHT * BAST
    IF(OUT .LT. 0.0) OUT = 0.0
170 IF(DIE .LT. OUT) DIE = OUT
    JDENO = (DENT * (1.0 - DIE)) + 0.5
    DENO = JDENO
    BASO = DENO * (0.0054542 * DBHO * DBHO)

```

C
C OBTAIN AVERAGE HEIGHT AND VOLUMES PER ACRE.

C

```
DC 180 J=1,2
LUB = J
GC TO (172,174), LUB
172 YARS = AGE0
GC TO 176
174 YARS = AGE0 * RINT
176 IF(YARS .GT. 45.0) GO TO 178
TEMH(J) = 3.86111 * C.05979 * YARS + 0.01215 * YARS * SITE
GC TO 180
178 TEMH(J) = 0.33401 * 33.2866 / YARS + 0.92341 * ALOG10(SITE) + 6.27
1811 * ALCG10(SITE) / YARS
TEMH(J) = 10.0 ** TEMH(J)
180 CONTINUE
PCT = 1.0 * 0.0028 * DMRT * DMRT * DMRT
CHNG = (TEMH(1) - TEMH(2)) * PCT
HTSC = HTST + CHNG
```

C

C COMPUTE TOTAL CU. FT. AND CONVERT TO OTHER UNITS.

C

```
D2H = DBFO * DBHO * HTSC
IF(D2H .GT. 7000.0) GO TO 185
TOT0 = (0.00276 * D2H * 0.00059 * BASC * 0.00577) * DENO
GO TO 190
185 TOT0 = (0.00248 * D2H + 1.96336) * DENO
190 IF(DBHO .LT. 5.0) GO TO 195
VDM = DBFO
BA = BASC
CALL LPVOL
BCFO = TOT0 * PRCD
CEMO = TOT0 * FCTR
```

C

C CHANGE MODE AND ROUND OFF FOR PRINTING.

C

```
195 IF(L .EQ. IK) GO TO 205
```

KDENC = DENC + 0.5
KAGEC = AGECE
KHTSC = HTSC + 0.5
KBASC = BASC + 0.5
KTOTC = (TOTC * 0.1) + 0.5
KTOTC = KTOTC * 10
KCFMC = (CFMC * 0.1) + 0.5
KCFMC = KCFMC * 10
KBDFC = (BDFC * 0.01) + 0.5
KBDFC = KBDFC * 100

C
C WRITE VALUES FOR THE PERIOD IF THINNING IS NOT DUE.
C

KB=KB+1
SAGE(KB)=KAGEC
MERCHVOL(KB)=KCFMC
SAWTIM(KB)=KBDFC
MB(KB)=0
TVOL(KB)=KTOTC
WRITE (6,125) KAGEC,KDENC,KBASC,DBHC,KHTSC,KTOTC,KCFMC,KBDFC
DBHT = DBHC
BAST = BASC
DENT = DENC
DMRT = DMR
HTST = HTSC

200 CONTINUE

C
C PREPARE TO START LOOP AGAIN FOR NEXT THINNING.
C

205 REST = DLEV

250 CONTINUE

255 CONTINUE

CALL LPCOST(KB,MB,SAGE,MERCHVOL,SAWTIM,NTPEES,
*CMERCHVOL,CSAWTIM,CVAL,DIS,CONSTANT,CESTR,VSAWTIM
*,VMERCHVOL,VRATES,TVOL,CTVOL,VTVOL,OPTIGN)

```
IF(START .GE. ROTA) GO TO 265
WRITE (6,260) START,DMR,ROTA
260 FORMAT (1H0,25X,41HDWARF MISTLETOE INFECTION STARTED AT AGE ,F4.0,
116H AND RATING WAS ,F5.1,8H AT AGE ,F4.0)
GO TO 275
265 WRITE (6,270) ROTA
270 FORMAT (1H0,25X,63HDWARF MISTLETOE INFECTION DID NOT OCCUR DURING
1THE ROTATION OF ,F4.0,7H YEARS.)
275 IF(KTR .EQ. C) GO TO 285
WRITE (6,280)
280 FORMAT (1H0,25X,52HNOTE THAT NOT ALL SCHEDULED THINNINGS WERE POSS
1IBLE.)
285 WRITE (6,290)
290 FORMAT (1H0,25X,66HMERCH. CU. FT. = TREES 6.0 INCHES D.B.H. AND LA
1RGER TO 4-INCH TOP.)
WRITE (6,295)
295 FORMAT (1H0,25X,59HBD. FT. = TREES 6.5 INCHES D.B.H. AND LARGER TO
1 6-INCH TOP.)
```

III

```
C
C PREPARE FOR NEXT TABLE OF THE TEST.
```

```
C
    AGED = VAR(1)
    DBHD = VAR(2)
    DENO = VAR(3)
300 CONTINUE
    GO TO 350
```

```
C
C PROGRAM CONTROL GOES HERE IF ANY ZEROS IN DATA DECK.
```

```
C
310 WRITE (6,320)
320 FORMAT (1H1,///,10X,64HEXECUTION STOPPED BECAUSE OF NEGATIVE OR ZE
1RO ITEM ON DATA CARD.)
350 CALL EXIT
    END
    SUBROUTINE LPVOL
```

```
C
```

C TO CONVERT TOTAL CU. FT. TO MERCH. CU. FT. AND TO BD. FT.

C

COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM
FCTR = C.0
PROD = 0.0
IF(VDM .LT. 5.0) GO TO 10

C

C OBTAIN CONVERSION FACTORS FOR MERCH. CU. FT. = VOLUMES TO 4.0-INCH TOP
C IN TREES 6.0 INCHES D.B.H. AND LARGER.

C

IF(VDM .GT. 6.7) GO TO 2
FCTR = 0.31963 * VDM = 1.42291
GO TO 6
2 IF(VDM .GT. 9.8) GO TO 4
FCTR = 3.68255 = 0.14007 * VDM = 13.54644 / VDM
GO TO 6
4 FCTR = 0.99503 = 0.58018 / VDM
6 IF(VDM .LT. 8.0) GO TO 10

C

C OBTAIN CONVERSION FACTORS FOR BD. FT. = VOLUMES TO 6-INCH TOP IN TREES
C 6.5 INCHES D.B.H. AND LARGER.

C

IF(VDM .GT. 10.0) GO TO 8
PROD = 2.08874 + 0.18091 * VDM + 0.00045 * BA
GO TO 10
8 PROD = 0.16583 + 3.74174 * ALOG10(VDM)
10 RETURN
END
SUBROUTINE LPCUT1

C

C TO ESTIMATE INCREASE IN AVERAGE D.B.H. DUE TO THINNING LODGEPOLE PINE
C IF DWARF MISTLETOE RATING EQUALS ZERO.

C

COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM
IF(DBHO .LT. 9.5) GO TO 30

C
C COMPLETE D.B.H. IF DBHQ IS LARGE ENOUGH FOR BASAL AREA TO REMAIN CONSTANT.
C

```
PRET = 100.0
CC 21 KJ=1,100
IF(PRET .LT. 50.0) GO TO 5
DBHE = 0.44222 + 1.03170 * DBHQ * 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
GO TO 11
5 PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHQ)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHQ)
DBHE = 10.0 ** PDBHE
11 IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * PRET * 0.01
NDENE = DENE + 0.5
CENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
TMPY = 0.0054542 * DBHE * DBHE
TEM = BASE = REST
IF(KJ .EQ. 1 .AND. TEM .LT. 0.0) GO TO 90
IF(TEM .LE. TMPY) GO TO 70
IF(TEM .LT. 4.0) GO TO 20
PRET = PRET = 1.0
GO TO 21
20 PRET = PRET = 0.3
21 CONTINUE
GO TO 70
```

C
C COMPLETE D.B.H. IF BASAL AREA INCREASES WITH D.B.H.
C

```

30 PRET = 40.0
   IF(DBHO .GT. 7.0) PRET = 70.0
   CC 65 J=1.100
   IF(PRET .GE. 50.0) GO TO 40
   PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.75921 * ALOG10(DBHO)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHO)
   DBHE = 10.0 ** PDBHE
   GO TO 45
40 DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
45 IDBHE = DBHE * 10.0 + 0.5
   DBHE = IDBHE
   DBHE = DBHE * 0.1
   DENE = DENO * (PRET * 0.01)
   NDENE = DENE + 0.5
   DENE = NDENE
   BASE = 0.0054542 * DBHE * DBHE * DENE
   NBASE = BASE * 10.0 + 0.5
   BASE = NBASE
   BASE = BASE * 0.1
   BREAK = 49.9 * REST / 80.0
   IF(BASE .GT. BREAK) GO TO 50
   DBHP = (80.0 / REST) * (0.08682 * BASE) + 0.94636
   GO TO 52
50 BUST = 66.2 * (REST / 80.0)
   IF(BASE .GT. BUST) GO TO 51
   DBHP = (80.0 / REST) * (0.10938 * BASE) = 0.17858
   GO TO 52
51 TMPY = BASE * (80.0 / REST)
   TEM = TMPY * TMPY
   DBHP = 19.04740 * TMPY = 0.26673 * TEM + 0.0012539 * TEM * TMPY
1 = 448.76833
   IF(TMPY .GT. 80.0) DBHP = DBHO + 0.8
52 IDBHP = DBHP * 10.0 + 0.5
   DBHP = IDBHP

```

```
DBHP = DBHP * 0.1
IF(DBHP = DBHE) 60,70,61
60 PRET = PRET * 1.02
IF(PRET .GT. 100.0) GO TO 90
GO TO 65
61 PRET = PRET * 0.98
65 CONTINUE
70 DBHT = DBHE
```

C

C COMPLETE PCST=THINNING BASAL AREA.

C

```
IF(DBHT .GT. 5.0) GO TO 75
SQFT = 11.58495 * DBHT = 11.09724
GO TO 76
75 IF(DBHT .GE. 10.0) GO TO 77
TEM = DBHT * DBHT
SQFT = 7.76226 * DBHT + 0.85289 * TEM = 0.07952 * TEM * DBHT = 3.45624
76 BAST = (REST / 80.0) * SQFT
GO TO 80
77 BAST = REST
80 RETURN
90 PRET = 100.0
RETURN
END
SUBROUTINE LPCUT2
```

C

C TO ESTIMATE CHANGE IN AVERAGE D.B.H. DUE TO THINNING LODGEPOLE PINE
C IF DWARF MISTLETOE RATING DETERMINES THE STANDARDS.

C

COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM

C

C COMPUTE STAND DENSITY AFTER A THINNING THAT REDUCES THE INDEX.

C

```
IF(DMR .LT. 2.0) GO TO 5
REST = 77.5 = 8.5 * DBHO + 10.0 * DMR
GO TO 10
```

```

5 REDT = 15.5 = 5.5 * DBHO + 41.0 * DMR
10 PRET = 100.0 = REDT
   DENT = DENO * (PRET * 0.01)
   IDENT = DENT * 0.5
   DENT = IDENT

```

```

C
C COMPUTE D.B.H. AFTER THINNING TO DESIRED DENSITY.
C

```

```

   IF (PRET .LT. 50.0) GO TO 15
   DBHT = 0.96559 * DBHO + 0.00668 * (PRET - 50.0) + 0.00015 * (PRET
1 = 50.0) * (PRET - 50.0) = 0.50568
   GO TO 20
15 DBHT = 0.33478 * ALOG10(PRET) + 1.42477 * ALOG10(DBHO) = 0.21199 *
1 ALOG10(PRET) * ALOG10(DBHO) = 0.67651
   DBHT = 10.0 ** DBHT
20 IDBHT = DBHT * 10.0 + 0.5
   DBHT = IDBHT
   DBHT = DBHT * 0.1
   BAST = 0.0054542 * DBHT * DBHT * DENT
   RETURN
   END
   SUBROUTINE LPCUT3

```

```

C
C TO ESTIMATE INCREASE IN AVERAGE D.B.H. DUE TO THINNING FROM BELOW IF
C DWARF MISTLETOE RATING IS GREATER THAN ZERO.
C

```

```

   COMMON BA, BAST, DBHO, DBHT, DENO, DMR, DMRT, FCTR, PRET, PROD, REST, VDM
   IF (DBHO .LT. 9.5) GO TO 30

```

```

C
C COMPUTE D.B.H. IF DBHO IS LARGE ENOUGH FOR BASAL AREA TO REMAIN CONSTANT.
C

```

```

   PRET = 100.0
   DO 21 KJ=1,100
   IF (PRET .LT. 50.0) GO TO 5
   DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET - 50.0) = 0.0000
19 * (PRET - 50.0) * (PRET - 50.0)

```

```

GO TO 11
5 PDBHE = 0.37321 * 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHO)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHO)
DBHE = 10.0 ** PDBHE
11 TEM = DBHE = DBHO
CBHE = DBHO + TEM * 0.5
IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * PRET * 0.01
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
TMPY = 0.0054542 * DBHE * DBHE
TEM = BASE = REST
IF(KJ .EQ. 1 .AND. TEM .LT. 0.0) GO TO 90
IF(TEM .LE. TMPY) GO TO 70
IF(TEM .LT. 4.0) GO TO 20
PRET = PRET * 1.0
GO TO 21
20 PRET = PRET * 0.3
21 CONTINUE
GO TO 70

```

C
C COMPUTE D.B.H. IF BASAL AREA INCREASES WITH D.B.H.
C

```

30 PRET = 40.0
IF(DBHO .GT. 7.0) PRET = 70.0
DO 65 J=1,100
IF(PRET .GE. 50.0) GO TO 40
PDBHE = 0.37321 * 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHO)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHO)

```

```

DBHE = 10.0 ** PDBHE
GO TO 45
40 DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
45 TEM = DBHE + DBHO
DBHE = DBHO + TEM * 0.5
IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * (PRET * 0.01)
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
BREAK = 49.9 * REST / 80.0
IF(BASE .GT. BREAK) GO TO 50
DBHP = (80.0 / REST) * (0.08682 * BASE) + 0.94636
GO TO 52
50 BUST = 66.2 * (REST / 80.0)
IF(BASE .GT. BUST) GO TO 51
DBHP = (80.0 / REST) * (0.10938 * BASE) = 0.17858
GO TO 52
51 TMPY = BASE * (80.0 / REST)
TEM = TMPY * TMPY
DBHP = 19.04740 * TMPY = 0.26673 * TEM + 0.0012539 * TEM * TMPY
1 = 448.76833
IF(TMPY .GT. 80.0) DBHP = DBHO + 0.8
52 IDBHP = DBHP * 10.0 + 0.5
DBHP = IDBHP
DBHP = DBHP * 0.1
IF(DBHP = DBHE) 60,70,61
60 PRET = PRET * 1.02
IF(PRET .GT. 100.0) GO TO 90

```

```
GO TO 65
61 PRET = PRET * 0.98
65 CONTINUE
70 DBHT = DBHE
```

```
C
C COMPUTE POST-THINNING BASAL AREA.
```

```
C
      IF (DBHT .GT. 5.0) GO TO 75
      SQFT = 11.58495 * DBHT = 11.09724
      GO TO 76
75  IF (DBHT .GE. 10.0) GO TO 77
      TEM = DBHT * DBHT
      SQFT = 7.76226 * DBHT + 0.85289 * TEM = 0.07952 * TEM * DBHT = 3.45624
76  BAST = (REST / 80.0) * SQFT
      GO TO 80
77  BAST = REST
80  RETURN
90  PRET = 100.0
      RETURN
      END
```

```
      SUBROUTINE LPCOST(KB, MB, SAGE, MERCHVOL, SAWTIM, NTREES,
*CMERCHVOL, CSAWTIM, CVAL, DIS, CONSTANT, CESTB, VSAWTIM,
*VMERCHVOL, NRATES, TVOL, CTVOL, VTVOL, OPTION)
```

```
C
C SUBROUTINE TO CALCULATE RETURNS ASSOCIATED WITH DWARF
C MISTLETOE INFESTATIONS AND CONTROL PROCEDURES
```

```
C
      DIMENSION MB(20), SAGE(20), MERCHVOL(20), SAWTIM(20),
*NTREES(20), CMERCHVOL(20), CVAL(20), OUF(12), DIS(8), YNR(20),
*CRPVAL(20), CSAWTIM(20), CONSTANT(8), TVOL(20), CTVOL(20)
      WRITE(6,1101)
1101  FORMAT(///1X, 'STAND', 3X, 'VALUE', 2X, 'ESTABLISH=', 2X,
* 'VALUE OF', 4X, 'COST', 4X, 'NET RETURN', 2X, 'PERIODIC', /
* 2X, 'AGE', 2X, 'OF TREE', 2X, 'MEND COST', 4X, 'CRCP', 7X, 'OF', 7X,
* 'FROM', 8X, 'NET'/' (YEARS)', 2X, 'CRCP', 15X,
* 'THINNED', 3X, 'THINNING', 2X, 'THINNING', 4X, 'RETURNS')
```

```

YNR(1)=- (CESTB)
CROPVAL(1)=0
WRITE(6,1103) 0,0,CESTB,0,0,0,- (CESTB)
1103 FORMAT(2X,I3,F9.2,F10.2,F11.2,F10.2,F10.2,F12.2)
DO 1005 I=1,KB
VTHIN=CTHIN=RTHIN=0
CROPVAL(I+1)=VMERCHVOL*MERCHVOL(I)
IF(SAWTIM(I).NE.0) CROPVAL(I+1)=VSAWTIM*SAWTIM(I)
IF(OPTICN.EQ.1) CROPVAL(I+1)=VTVOL*TVOL(I)
IF (MB(I).EQ.0.OR.I.EQ.KB) GO TO 1002
VTHIN=VMERCHVOL*CMERCHVOL(I)
IF(CSAWTIM(I).NE.0) VTHIN=VSAWTIM*CSAWTIM(I)
IF(OPTICN.EQ.1) VTHIN=VTVOL*CTVOL(I)
CTHIN=((648.143+0.033*NTREES(I)*CVAL(I)**2)*1.502)/51.9
IF(NTREES(I).EQ.0) CTHIN=0
RTHIN=VTHIN+CTHIN
1002 YNR(I+1)=RTHIN
WRITE(6,1103) SAGE(I),CROPVAL(I+1),0,VTHIN,CTHIN,RTHIN,YNR(I+1)
1005 CONTINUE
DO 1010 M=1,NRATES
WRITE(6,1105) DIS(2*M-1),DIS(2*M)
1105 FORMAT(/,1X,'STAND',3X,'PERIODIC',2X,2('DISCOUNT',3X,
*'DISCOUNTED',3X,'CUMULATIVE',3X,'VALUE',4X,
*'CONTROLLED',1X)/2X,'AGE',
*6X,'NET',4X,2(2X,'RATE',4X,'PERIODIC NET',2X,'DISCOUNTED',4X,
*'TO',10X,'TO',6X)/('YEARS)'),2X,'RETURNS',2X,2(1X,F5.3,6X,'RETURNS',
*,4X,'NET RETURNS',2X,'INFINITY',3X,'INFINITY',3X)/)
SUM1=SUM2=0
DO 1008 I=1,KB+1
IF(I.EQ.1) OUT(1)=0
IF(I.GT.1) OUT(1)=SAGE(I-1)
OUT(2)=YNR(I)
OUT(3)=1.0/((1+DIS(2*M-1))**OUT(1))
OUT(4)=OUT(3)*OUT(2)
OUT(5)=SUM1+CROPVAL(I)*OUT(3)

```

```

SUM1=SUM1+OUT(4)
CUT(6)=OUT(5)*(1.0/(1.0+(1.0/(1.0+DIS(2*M=1))))**OUT(1))
CUT(7)=CUT(5)+CONSTANT(2*M=1)*OUT(3)
CUT(8)=1.0/((1+DIS(2*M))*OUT(1))
CUT(9)=CUT(8)*OUT(2)
CUT(10)=SUM2+CROPVAL(I)*OUT(8)
SUM2=SUM2+OUT(9)
CUT(11)=CUT(10)*(1.0/(1.0+(1.0/(1+DIS(2*M))))**OUT(1))
CUT(12)=CUT(10)+CONSTANT(2*M)*OUT(8)
1008 WRITE(6,1107) (OUT(J),J=1,12)
1107 FORMAT(3X,I3,F10.2,2X,2(F7.3,4X,F8.2,4X,F8.2,F12.2,3X,F8.2,3X))
1010 CONTINUE
      WRITE(6,1109) VSAWTIM*1000,VMERCHVOL ,VTVOL
1109 FORMAT('0',25X,'VALUE OF SAWTIMBER IS',F6.2,'/M',
*5X,'VALUE OF MERCHANTABLE VOLUME IS',F6.2,'/CU. FT.',
*26X,'VALUE OF TOTAL VOLUME IS',F6.2,'/CU. FT.(/)
      REPEAT 1015, FOR I=(1, NRATES)
1015 WRITE(6,1111) DIS(2*I=1),CONSTANT(2*I=1),DIS(2*I),CONSTANT(2*I)
1111 FORMAT(21X,2(5X,'FOR DISCOUNT RATE OF',1X,F4.3,1X,
*'CONSTANT IS',F7.2))
      RETURN
      END

```

C PROGRAM LPMIST2

C TO COMPUTE AND PRINT YIELD TABLES FOR EVEN-AGED STANDS OF LODGEPOLE
C PINE INFECTED BY DWARF MISTLETOE.

C DEFINITIONS OF VARIABLES.

C ADDHT = INCREASE OR DECREASE IN AVERAGE STAND HEIGHT BY THINNING.
C AGED = INITIAL AGE IN YIELD TABLE.
C BASC = BASAL AREA CUT PER ACRE.
C BASO = BASAL AREA PER ACRE BEFORE THINNING.
C BAST = BASAL AREA PER ACRE AFTER THINNING.
C BDFC = BCARD FEET CUT PER ACRE.
C BDFO = BCARD FEET PER ACRE BEFORE THINNING.
C BDFT = BCARD FEET PER ACRE AFTER THINNING.
C CFMC = MERCHANTABLE CU. FT. CUT PER ACRE.
C CFMO = MERCH. CU. FT. PER ACRE BEFORE THINNING.
C CFMT = MERCH. CU. FT. PER ACRE AFTER THINNING.
C DBHO = AVERAGE STAND D.B.H. BEFORE THINNING.
C DBHT = AVERAGE STAND D.B.H. AFTER THINNING.
C DENC = TREES CUT PER ACRE.
C DENO = TREES PER ACRE BEFORE THINNING.
C DENT = TREES PER ACRE AFTER THINNING.
C DIE = TREES LOST IN DISEASED STANDS IN 10 YEARS, IN PERCENT.
C DLEV = GROWING STOCK LEVEL FOR INTERMEDIATE CUTS AFTER THE FIRST.
C DMR = DWARF MISTLETOE INFECTION RATING.
C DMRT = MAXIMUM INFECTION EXPECTED IN STANDS AFTER THINNING. GOAL
C FOR STANDS NOT ALREADY BEYOND DMR OF 3.0.
C DSTY = LOWEST VALUE OF DLEV USED IN A TEST.
C HTSO = TREE HEIGHT BEFORE THINNING.
C HTST = TREE HEIGHT AFTER THINNING.
C JCYCL = INTERVAL BETWEEN INTERMEDIATE CUTS.
C KSTEP = INDICATOR WITH VALUE OF ONE IF CURRENT THINNING IS FROM
C BELOW AND TWO IF CURRENT THINNING IS FROM ABOVE.
C KTR = INDICATOR WITH VALUE GREATER THAN ZERO IF A SCHEDULED

```

C          THINNING HAS BEEN SKIPPED BECAUSE MISTLETOE INDEX IS TOO HIGH
C          OR BECAUSE STAND IS ALREADY TO SPECIFIED STOCKING.
C          MIX = NUMBER OF STOCKING LEVELS EXAMINED PER TEST.
C          NFLAG = INDICATOR WITH VALUE GREATER THAN ZERO IF A THINNING FROM
C          ABOVE HAS BEEN MADE AT ANY TIME.
C          NTSTS = NUMBER OF TESTS PER BATCH.
C          CUT = PERCENT MORTALITY IN HEALTHY STANDS.
C          FCT = PERIODIC HEIGHT INCREASE IN INFESTED STAND, AS A PERCENTAGE
C          OF THE INCREASE IN COMPARABLE HEALTHY STANDS.
C          PRET = PERCENTAGE OF TREES RETAINED AFTER THINNING.
C          REDT = PERCENTAGE REDUCTION IN NUMBER OF TREES WHEN DMR IS
C          REDUCED TO DMRT BY THINNING.
C          RINT = NUMBER OF YEARS FOR WHICH A SINGLE PROJECTION IS MADE.
C          RCTA = FINAL AGE IN YIELD TABLE.
C          SITE = SITE INDEX.
C          START = STAND AGE AT TIME OF INITIAL INFECTION.
C          TEM = PERIODIC D.B.H. INCREASE IN INFESTED STAND, AS A PERCENTAGE
C          OF THE INCREASE IN COMPARABLE HEALTHY STANDS.
C          THIN = GROWING STOCK LEVEL FOR INITIAL THINNING.
C          TOTC = TOTAL CUBIC FEET CUT PER ACRE.
C          TOTO = TOTAL CUBIC FEET PER ACRE BEFORE THINNING.
C          TOTT = TOTAL CUBIC FEET PER ACRE AFTER THINNING.
C
C          COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM
C          DIMENSION VAR(9),TEMH(2)
C          DIMENSION SAGE(20),SAWTIM(20),MERCYVOL(20),NTREES(20)
C          *,CVAL(20),MB(20),CSAWTIM(20),CMERCYVOL(20),DIS(8),CONSTANT(8)
C          *,TVOL(20),CTVOL(20)
C
C          READ ESTABLISHMENT COST, LUMBER PRICES, INTEREST RATES, AND CONSTANTS
C
C          READ(5,15) CESTB, VSAWTIM, VMERCYVOL, VTVOL
C          READ(5,10) NRATES, OPTION
C          REPEAT 3, FOR I=(1, NRATES)
C          3 READ(5,15) DIS(2*I-1), CONSTANT(2*I-1), DIS(2*I), CONSTANT(2*I)
C

```

```

C READ NUMBER OF TESTS PER BATCH FROM CARD TYPE ONE.
C
  READ (5,5) NTSTS
  5 FORMAT (I4)
  IF(NTSTS .LE. 0) GO TO 310
C
C EXECUTE PROGRAM ONCE FOR EACH SET OF INITIAL VALUES OF INTEREST.
C
  DO 300 I=1,NTSTS
C
C READ INITIAL VALUES, ONE TEST AT A TIME, FROM CARD TYPES 2 AND 3.
C
  READ (5,10,END=350) JCYCL,MIX
  10 FORMAT (2I4)
  IF(JCYCL .LE. 0 .OR. MIX .LE. 0) GO TO 310
  READ (5,15) AGE0,DBHO,DENO,OSTY,RINT,ROTA,SITE,THIN,START
  15 FORMAT (9F8.3)
  VAR(1) = AGE0
  VAR(2) = DBHO
  VAR(3) = DENO
  VAR(4) = OSTY
  VAR(5) = RINT
  VAR(6) = ROTA
  VAR(7) = SITE
  VAR(8) = THIN
  VAR(9) = START
  DO 20 L=1,9
  IF(VAR(L) .LE. 0.0) GO TO 310
  20 CONTINUE
  DLEV = 0.0
C
C PROVIDE FOR SEVERAL GROWING STOCK LEVELS PER TEST.
C
  DO 300 M=1,MIX

```

```

A = M
ADDHT = 0.0
BDFO = 0.0
BDFT = 0.0
CFMO = 0.0
CFMT = 0.0
DMR = 0.0
DMRT = 0.0
HTCUM = 0.0
KSTEP = 1
KTR = 0
NFLAG = 0
TIME = 0.0
DLEV = (DSTY + (A * 10.0)) - 10.0
BASC = DENO * 0.0054542 * DBHO * DBHO
KB=0

```

```

C
C COMPUTE CURRENT DWARF MISTLETOE RATING, UNTHINNED STANDS.
C

```

```

TIME = AGE0 - START
IF (TIME .LE. 0.0) GO TO 25
DMR = 0.31572 + 0.08654 * TIME - 0.00016 * DENO
IF (DMR .LT. 0.0) DMR = 0.0
IF (DMR .GT. 6.0) DMR = 6.0

```

```

C
C OBTAIN AVERAGE HEIGHT AND VOLUMES PER ACRE.
C

```

```

25 IF (AGE0 .GT. 45.0) GO TO 30
HTSQ = 3.86111 - 0.05979 * AGE0 + 0.01215 * AGE0 * SITE
GO TO 35
30 HTSQ = 0.33401 - 33.2866 / AGE0 + 0.92341 * ALOG10(SITE) + 6.27811
1 * ALOG10(SITE) / AGE0
HTSQ = 10.0 ** HTSQ
35 PCT = 1.0 - 0.0165 * DMR * DMR
HTSQ = HTSQ * PCT

```

```

C
C COMPUTE TOTAL CU. FT. AND CONVERT TO OTHER UNITS.

```

C

```
D2H = DBFG * DBHO * HTSO
IF(D2H .GT. 7000.0) GO TO 40
TOTO = (0.00276 * D2H + 0.00059 * BASC + 0.00577) * DENO
GO TO 45
40 TOTO = (0.00248 * D2H + 1.96336) * DENO
45 IF(DBHO .LT. 5.0) GO TO 50
VDM = DBFG
BA = BASC
CALL LPVCL
BDFC = TOTO * PROD
CFMO = TOTO * FCTR
50 REST = THIN
```

C

C ENTER LOOP FOR REMAINING COMPUTATIONS AND PRINTOUT.

C

```
DO 250 K=1,100
  IF(AGEC .GE. ROTA) GO TO 90
```

C

C COMPUTE D.B.H. AFTER THINNING.

C

```
IF(DMR .LT. 3.0) GO TO 55
BAST = BASO
DRHT = DBHO
DMRT = DMR
HTST = HTSO
KTR = 1
GO TO 75
55 IF(DMR .EQ. 0.0) GO TO 63
IF(NFLAG .GT. 0) GO TO 62
DMRT = 0.25 * DBHO + 0.50
IF(DMRT .LT. 0.0) DMRT = 0.0
IF(DMRT .GE. DMR) GO TO 62
IF(THIN .EQ. 900.) GO TO 67
CALL LPCUT2
```

```

NFLAG = 1
KSTEP = 2
GO TO 65
62 CALL LPCUT3
IF(PRET .GE. 100.0) GO TO 67
KSTEP = 1
DMRT = DMR + 0.0279 * PRET = 2.79
GO TO 65
63 DMRT = DMR
CALL LPCUT1
IF(PRET .GE. 100.0) GO TO 67
KSTEP = 1
65 IF(BAST .LT. BASO) GO TO 70
67 BAST = BASO
DBHT = DBHO
DMRT = DMR
HTST = HTSO
KTR = 1
GO TO 75

```

C
C COMPLETE HEIGHT AND VOLUMES AFTER THINNING.
C

```

70 GO TO (71,72), KSTEP
71 ADDHT = 6.79950 = 3.41979 * ALOG10(PRET)
GO TO 73
72 ADDHT = 3.76362 * ALOG10(PRET) = 7.97347
73 HTCUM = HTCUM + ADDHT
HTST = HTSO + ADDHT
75 JDENT = (BAST / (0.0054542 * DBHT * DBHT)) + 0.5
DENT = JDENT
BAST = 0.0054542 * DBHT * DBHT * DENT
D2H = DBHT * DBHT * HTST
IF(D2H .GT. 7000.0) GO TO 80
TOTT = (0.00276 * D2H = 0.00059 * BAST = 0.00577) * DENT
GO TO 85

```

80 TOTT = (0.00248 * D2H + 1.96336) * DEAT

C

C CONVERT TOTAL CU. FT. TO OTHER UNITS.

C

85 IF (DBHT .LT. 5.0) GO TO 90

VDM = DBHT

BA = BAST

CALL LPVOL

BDFT = TOTT * PROD

CFMT = TOTT * FCTR

C

C CHANGE MODE AND ROUND OFF FOR PRINTING.

C

90 JAGEO = AGEO

JSITE = SITE

JDENO = DENO + 0.5

JHTSO = HTSO + 0.5

JTOTO = (TOTO * 0.1) + 0.5

JTOTO = JTOTO * 10

JBASO = BASO + 0.5

JCFMO = (CFMO * 0.1) + 0.5

JCFMO = JCFMO * 10

JBDFO = (BDFO * 0.01) + 0.5

JBDFO = JBDFO * 100

JHTST = HTST + 0.5

JTOTT = (TOTT * 0.1) + 0.5

JTOTT = JTOTT * 10

JCFMT = (CFMT * 0.1) + 0.5

JCFMT = JCFMT * 10

IF (JCFMT .GT. JCFMO) JCFMO = JCFMT

JBDFT = (BDFT * 0.01) + 0.5

JBDFT = JBDFT * 100

IF (JBDFT .GT. JBDFO) JBDFO = JBDFT

JBAST = BAST + 0.5

JDENC = JDENO = JDENT

```
JBASC = JBASO = JBAST
JTOTC = JTOTO = JTOTT
JCFMC = JCFMO = JCFMT
IF(JCFMC .LE. 0) JCFMC = 0
JBDFC = JBDFO = JBDFT
IF(JBDFC .LE. 0) JBDFC = 0
```

C

C WRITE HEADINGS FOR YIELD TABLE.

C

```
IF(K .GE. 2) GO TO 120
WRITE (6,95) JSITE,THIN,DLEV
95 FORMAT (1H1,///,39X,53HYIELDS PER ACRE OF EVEN-AGED STANDS OF LOGG
1EPOLE PINE/1H ,57X,11HSITE INDEX ,13/1H ,38X,29HTHINNING INTENSITY
2= INITIAL ,F5.0,2X,12HSUBSEQUENT= ,F5.0)
WRITE (6,100)
100 FORMAT (1H0,25X,38HENTIRE STAND BEFORE AND AFTER THINNING,28X,26HP
1ERIODIC INTERMEDIATE CUTS)
WRITE (6,105)
105 FORMAT (1H0,9X,5HSTAND,10X,5HBASAL,3X,7HAVERAGE,2X,7HAVERAGE,3X,5H
1TOTAL,3X,9HMERCHANT=,3X,9HSAWTIMBER,9X,5HBASAL,4X,5HTOTAL,3X,9HMER
2CHANT=,3X,9HSAWTIMBER)
WRITE (6,110)
110 FORMAT (1H ,10X,3HAGE,4X,5HTREES,3X,4HAREA,4X,6HD.B.H.,3X,6HHEIGHT
1,2X,6HVOLUME,2X,11HABLE VOLUME,4X,6HVOLUME,3X,5HTREES,3X,4HAREA,3X
2,6HVOLUME,2X,11HABLE VOLUME,4X,6HVOLUME)
WRITE (6,115)
115 FORMAT (1H ,8X,7H(YEARS),3X,3HNO.,3X,6HSG.FT.,4X,3HIN.,6X,3HFT.,4X
1,6HCU.FT.,5X,6HCU.FT.,6X,6HBD.FT.,4X,3HNO.,3X,6HSG.FT.,2X,6HCU.FT.
2,5X,6HCU.FT.,6X,6HBD.FT.)
```

C

C WRITE TABLE ENTRIES OF DIAMETER, VOLUMES, ETC.

C

```
120 CONTINUE
KB=KB+1
MB(KB)=1
```

```

SAGE(KB)=JAGEO
MERCHVOL(KB)=JCFMO
SAWTIM(KB)=JBDFO
NTREES(KB)=JDENC
CMERCHVOL(KB)=JCFMC
CSAWTIM(KB)=JBDFC
CVAL(KB)=2*DBHO=DBHT
CTVOL(KB)=JTOTC
TVOL(KB)=JTOTO
WRITE (6,125) JAGEO,JDENO,JBASO,DBHO,JHT50,JTOTO,JCFMO,JBDFO
125 FORMAT (1H0,9X,I4,4X,I5,2X,I4,5X,F5.1,5X,I3,4X,I5,6X,I5,6X,I6)
IF(AGEO .GE. ROTA) GO TO 255
WRITE (6,130) JAGEO,JDENT,JBAST,DBHT,JHTST,JTOTT,JCFMT,JBDFC,JDENC
1,JBASC,JTOTC,JCFMC,JBDFC
130 FORMAT (1H ,9X,I4,4X,I5,2X,I4,5X,F5.1,5X,I3,4X,I5,6X,I5,6X,I6,4X,I
15,3X,I3,5X,I4,6X,I4,8X,I5)

```

```

C
C COMPUTE VALUES FOR EACH PERIOD. THIN AS SPECIFIED.
C

```

```

IRINT = RINT
IK = JCYCL / IRINT
DO 200 L=1,IK
AGEO = AGE0 + RINT
IF(AGE0 .GT. ROTA) GO TO 255

```

```

C
C COMPUTE CURRENT DWARF MISTLETOE RATING.
C

```

```

TIME = AGE0 = START
IF(DMR .GT. 0.0) GO TO 135
IF(TIME .LE. 0.0) GO TO 150
DMR = 0.31572 + 0.08654 * TIME + 0.00016 * DENT
GO TO 145
135 IF(DMRT .LE. 1.0) GO TO 140
DMR = DMRT + 0.07 * RINT
GO TO 145

```

```
140 DMR = DMRT + (0.03 + 0.038 * DMRT) * RINT
    IF(L .LE. 2) GO TO 145
    DMR = DMR + 0.07 * RINT
145 IF(DMR .LT. 0.0) DMR = 0.0
    IF(DMR .GT. 6.0) DMR = 6.0
```

C
C
C

COMPUTE NEW D.B.H. BEFORE THINNING AND ROUND OFF TO 0.1 INCH.

```
150 DBHO = 1.0222*DBHT + 0.0151*SITE + 1.2417*ALOG10(BAST) + 2.1450
    IF(DMRT .LE. 3.9) GO TO 155
    TEM = (DBHO - DBHT) * (1.0 - (0.192 * DMRT - 0.754))
    DBHO = DBHT + TEM
155 IDBHO = DBHO * 10.0 + 0.5
    DBHO = IDBHO
    DBHO = DBHO * 0.1
    IF(DENT .GT. 1000.0) GO TO 160
    DIE = (3.31 * DMRT - 6.63) * 0.01
    IF(DIE .LT. 0.0) DIE = 0.0
    GO TO 165
160 DIE = (3.64 + 3.28 * DMRT) * 0.01
165 CUT = 0.0
    IF(DBHT .GE. 10.0) GO TO 170
    CUT = 0.05285 + 0.01346 * DBHT + 0.00226 * DBHT * DBHT + 0.0000066
    * BAST * BAST + 0.0001931 * DBHT * BAST
    IF(CUT .LT. 0.0) CUT = 0.0
170 IF(DIE .LT. CUT) DIE = CUT
    JDENO = (DENT * (1.0 - DIE)) + 0.5
    DENO = JDENO
    BASO = DENO * (0.0054542 * DBHO * DBHO)
```

131

C
C
C

OBTAIN AVERAGE HEIGHT AND VOLUMES PER ACRE.

```
DO 180 J=1,2
LUB = J
GO TO (172,174), LUB
```

```

172 YARS = AGEQ
    GO TO 176
174 YARS = AGEQ = RINT
176 IF(YARS .GT. 45.0) GO TO 178
    TEMH(J) = 3.86111 = 0.05979 * YARS + 0.01215 * YARS * SITE
    GO TO 180
178 TEMH(J) = 0.33401 = 33.2866 / YARS + 0.92341 * ALOG10(SITE) + 6.27
    1811 * ALOG10(SITE) / YARS
    TEMH(J) = 10.0 ** TEMH(J)
180 CONTINUE
    PCT = 1.0 = 0.0028 * DMRT * DMRT * DMRT
    CHNG = (TEMH(1) = TEMH(2)) * PCT
    HTSO = HTST + CHNG

```

```

C
C COMPUTE TOTAL CU. FT. AND CONVERT TO OTHER UNITS.
C

```

```

    D2H = DBFO * DBHO * HTSO
    IF(D2H .GT. 7000.0) GO TO 185
    TOTO = (0.00276 * D2H = 0.00059 * BASO = 0.00577) * DENO
    GO TO 190
185 TOTO = (0.00248 * D2H + 1.96336) * DENO
190 IF(DBHO .LT. 5.0) GO TO 195
    VDM = DBFO
    BA = BASO
    CALL LPVCL
    BDFO = TOTO * PROD
    CFMO = TOTO * FCTR

```

```

C
C CHANGE MODE AND ROUND OFF FOR PRINTING.
C

```

```

195 IF(L .EQ. IK) GO TO 205
    KDENG = DENO + 0.5
    KAGEQ = AGEQ
    KHTSO = HTSO + 0.5
    KBASO = BASO + 0.5

```

```
KTOTO = (TOTO * 0.1) + 0.5
KTOTC = KTOTO * 10
KCFMC = (CFMC * 0.1) + 0.5
KCFMO = KCFMC * 10
KBDFC = (BDFC * 0.01) + 0.5
KBDFO = KBDFC * 100
```

```
C
C WRITE VALUES FOR THE PERIOD IF THINNING IS NOT DUE.
C
```

```
KB=KB+1
SAGE(KB)=KAGEO
MERCHVOL(KB)=KCFMO
SAWTIM(KB)=KBDFC
MB(KB)=0
TVOL(KB)=KTOTO
WRITE (6,125) KAGEO,KDENO,KBASO,DBHO,KHTSO,KTOTO,KCFMO,KBDFC
DBHT = DBHO
BAST = BASO
DENT = DENO
DMRT = DMR
HTST = HTSO
```

```
200 CONTINUE
```

```
C
C PREPARE TO START LOOP AGAIN FOR NEXT THINNING.
C
```

```
205 REST = DLEV
```

```
250 CONTINUE
```

```
255 CONTINUE
```

```
CALL LPCOST(KB,MB,SAGE,MERCHVOL,SAWTIM,NTREES,
*CMERCHVOL,CSAWTIM,CVAL,DIS,CONSTANT,CESTB,VSAWTIM
*,VMERCHVOL,NRATES,TVOL,CTVOL,VTVOL,OPTION)
```

```
IF(START *GE* ROTA) GO TO 265
```

```
WRITE (6,260) START,DMR,ROTA
```

```
260 FORMAT (1H0,25X,41H DWARF MISTLETOE INFECTION STARTED AT AGE ,F4.0,
116H AND RATING WAS ,F5.1,8H AT AGE ,F4.0)
GO TO 275
```

```
265 WRITE (6,270) ROTA
270 FORMAT (1H0,25X,63H'DWARF MISTLETOE INFECTION DID NOT OCCUR DURING
1THE ROTATION OF ,F4.0,7H YEARS'.)
275 IF(KTR .EQ. 0) GO TO 285
WRITE (6,280)
280 FORMAT (1H0,25X,52H'NOTE THAT NOT ALL SCHEDULED THINNINGS WERE POSS
1IBLE'.)
285 WRITE (6,290)
290 FORMAT (1H0,25X,66H'MERCH. CU. FT. = TREES 6.0 INCHES D.B.H. AND LA
1RGER TO 4-INCH TOP'.)
WRITE (6,295)
295 FORMAT (1H0,25X,59H'BD. FT. = TREES 6.5 INCHES D.B.H. AND LARGER TO
1.6-INCH TOP'.)
```

```
C
C PREPARE FOR NEXT TABLE OF THE TEST.
```

```
C
```

```
AGED = VAR(1)
DBHO = VAR(2)
DENO = VAR(3)
300 CONTINUE
GO TO 350
```

```
C
```

```
C PROGRAM CONTROL GOES HERE IF ANY ZEROS IN DATA DECK.
```

```
C
```

```
310 WRITE (6,320)
320 FORMAT (1H1,///,10X,64H'EXECUTION STOPPED BECAUSE OF NEGATIVE OR ZE
1RO ITEM ON DATA CARD'.)
350 CALL EXIT
END
SUBROUTINE LPVOL
```

```
C
```

```
C TO CONVERT TOTAL CU. FT. TO MERCH. CU. FT. AND TO BD. FT.
```

```
C
```

```
COMMON BA,BAST,DBHO,DBHT,DENO,DNR,DMRT,FCTR,PRET,PROD,REST,VDM
FCTR = 0.0
```

```

      PROD = 0.0
      IF(VDM .LT. 5.0) GO TO 10
C
C OBTAIN CONVERSION FACTORS FOR MERCH. CU. FT. = VOLUMES TO 4.0-INCH TOP
C IN TREES 6.0 INCHES D.B.H. AND LARGER.
C
      IF(VDM .GT. 6.7) GO TO 2
      FCTR = 0.31963 * VDM - 1.42291
      GO TO 6
2 IF(VDM .GT. 9.8) GO TO 4
      FCTR = 3.68255 - 0.14007 * VDM - 13.54644 / VDM
      GO TO 6
4 FCTR = 0.99503 - 0.58018 / VDM
6 IF(VDM .LT. 8.0) GO TO 10
C
C OBTAIN CONVERSION FACTORS FOR BD. FT. = VOLUMES TO 6-INCH TOP IN TREES
C 6.5 INCHES D.B.H. AND LARGER.
C
      IF(VDM .GT. 10.0) GO TO 8
      PROD = 2.08874 + 0.18091 * VDM + 0.00045 * BA
      GO TO 10
8 PROD = 0.16583 + 3.74174 * ALOG10(VDM)
10 RETURN
      END
      SUBROUTINE LPCUT1
C
C TO ESTIMATE INCREASE IN AVERAGE D.B.H. DUE TO THINNING LODGEPOLE PINE
C IF DWARF MISTLETOE RATING EQUALS ZERO.
C
      COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM
      IF(DBHO .LT. 9.5) GO TO 30
C
C COMPUTE D.B.H. IF DBHO IS LARGE ENOUGH FOR BASAL AREA TO REMAIN CONSTANT.

```

C

```
PRET = 100.0
DO 21 KJ=1,100
IF(PRET .LT. 50.0) GO TO 5
DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
GO TO 11
5 PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHO)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHO)
DBHE = 10.0 ** PDBHE
11 IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * PRET * 0.01
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
TMPY = 0.0054542 * DBHE * DBHE
TEM = BASE = REST
IF(KJ .EQ. 1 .AND. TEM .LT. 0.0) GO TO 90
IF(TEM .LE. TMPY) GO TO 70
IF(TEM .LT. 4.0) GO TO 20
PRET = PRET * 1.0
GO TO 21
20 PRET = PRET * 0.3
21 CONTINUE
GO TO 70
```

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C

C COMPUTE D.B.H. IF BASAL AREA INCREASES WITH D.B.H.

C

```
30 PRET = 40.0
IF(DBHO .GT. 7.0) PRET = 70.0
```

```

DO 65 J=1,100
IF(PRET .GE. 50.0) GO TO 40
PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHO)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHO)
DBHE = 10.0 ** PDBHE
GO TO 45
40 DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
45 IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * (PRET * 0.01)
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
BREAK = 49.9 * REST / 80.0
IF(BASE .GT. BREAK) GO TO 50
DBHP = (80.0 / REST) * (0.08682 * BASE) + 0.94636
GO TO 52
50 BUST = 66.2 * (REST / 80.0)
IF(BASE .GT. BUST) GO TO 51
DBHP = (80.0 / REST) * (0.10938 * BASE) = 0.17858
GO TO 52
51 TMPY = BASE * (80.0 / REST)
TEM = TMPY * TMPY
DBHP = 19.04740 * TMPY = 0.26673 * TEM + 0.0012539 * TEM * TMPY
1 * 448.76833
IF(TMPY .GT. 80.0) DBHP = DBHP + 0.8
52 IDBHP = DBHP * 10.0 + 0.5
DBHP = IDBHP
DBHP = DBHP * 0.1
IF(DBHP = DBHE) 60,70,61

```

```
60 PRET = PRET * 1.02
   IF(PRET .GT. 100.0) GO TO 90
   GO TO 65
61 PRET = PRET * 0.98
65 CONTINUE
70 DBHT = DBHE
```

```
C
C COMPLETE POST-THINNING BASAL AREA.
```

```
C
   IF(DBHT .GT. 5.0) GO TO 75
   SQFT = 11.58495 * DBHT = 11.09724
   GO TO 76
75 IF(DBHT .GE. 10.0) GO TO 77
   TEM = DBHT * DBHT
   SQFT = 7.76226 * DBHT + 0.85289 * TEM = 0.07952 * TEM * DBHT = 3.45624
76 BAST = (REST / 80.0) * SQFT
   GO TO 80
77 BAST = REST
80 RETURN
90 PRET = 100.0
1000 RETURN
   END
   SUBROUTINE LPCUT2
```

```
C
C TO ESTIMATE CHANGE IN AVERAGE D.B.H. DUE TO THINNING LODGEPOLE PINE
C IF DWARF MISTLETOE RATING DETERMINES THE STANDARDS.
```

```
C
   COMMON BA, BAST, DBHO, DBHT, DENO, DMR, DMRT, FCTR, PRET, PROD, REST, VDM
```

```
C
C COMPUTE STAND DENSITY AFTER A THINNING THAT REDUCES THE INDEX.
```

```
C
   IF(DMR .LT. 2.0) GO TO 5
   REDT = 77.5 = 8.5 * DBHO + 10.0 * DMR
   GO TO 10
5 REDT = 15.5 = 8.5 * DBHO + 11.0 * DMR
```

```
10 PRET = 100.0 = REDT
   DENT = DENO * (PRET * 0.01)
   IDENT = DENT + 0.5
   DENT = IDENT
```

C
C
C

COMPUTE D.B.H. AFTER THINNING TO DESIRED DENSITY.

```
   IF(PRET .LT. 50.0) GO TO 15
   DBHT = 0.96559 * DBHO + 0.00668 * (PRET = 50.0) + 0.00015 * (PRET
1  = 50.0) * (PRET = 50.0) = 0.00568
   GO TO 20
15 DBHT = 0.33478 * ALOG10(PRET) + 1.42477 * ALOG10(DBHO) = 0.21199 *
1 ALOG10(PRET) * ALOG10(DBHO) = 0.67651
   DBHT = 10.0 ** DBHT
20 IDBHT = DBHT * 10.0 + 0.5
   DBHT = IDBHT
   DBHT = DBHT * 0.1
   BAST = 0.0054542 * DBHT * DBHT * DENT
   RETURN
   END
   SUBROUTINE LPCUT3
```

139

C
C
C

TO ESTIMATE INCREASE IN AVERAGE D.B.H. DUE TO THINNING FROM BELOW IF
DWARF MISTLETOE RATING IS GREATER THAN ZERO.

```
COMMON BA,BAST,DBHO,DBHT,DENO,DMR,DMRT,FCTR,PRET,PROD,REST,VDM
```

```
IF(DBHO .LT. 9.5) GO TO 30
```

C
C
C

COMPUTE D.B.H. IF DBHO IS LARGE ENOUGH FOR BASAL AREA TO REMAIN CONSTANT.

```
PRET = 100.0
DO 21 KJ=1,100
IF(PRET .LT. 50.0) GO TO 5
DBHE = 0.44222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
```

```

GO TO 11
5 PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHQ)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHQ)
DBHE = 10.0 ** PDBHE
11 TEM = DBHE * DBHQ
DBHE = DBHQ + TEM * 0.5
IDBHE = DBHE * 10.0 + 0.5
DBHE = IDBHE
DBHE = DBHE * 0.1
DENE = DENO * PRET * 0.01
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
TMPY = 0.0054542 * DBHE * DBHE
TEM = BASE - REST
IF(KJ .EQ. 1 .AND. TEM .LT. 0.0) GO TO 90
IF(TEM .LE. TMPY) GO TO 70
IF(TEM .LT. 4.0) GO TO 20
PRET = PRET * 1.0
GO TO 21
20 PRET = PRET * 0.3
21 CONTINUE
GO TO 70

```

C
C COMPUTE D.B.H. IF BASAL AREA INCREASES WITH D.B.H.
C

```

30 PRET = 40.0
IF(DBHQ .GT. 7.0) PRET = 70.0
DO 65 J=1,100
IF(PRET .GE. 50.0) GO TO 40
PDBHE = 0.37321 = 0.17274 * ALOG10(PRET) + 0.79921 * ALOG10(DBHQ)
1 + 0.09315 * ALOG10(PRET) * ALOG10(DBHQ)

```

```

DBHE = 10.0 ** PDBHE
GO TO 45
40 DBHE = 0.74222 + 1.03170 * DBHO = 0.00816 * (PRET = 50.0) = 0.0000
19 * (PRET = 50.0) * (PRET = 50.0)
45 TEM = DBHE * DBHO
DBHE = DBHO + TEM * 0.5
IOBHE = DBHE * 10.0 + 0.5
DBHE = IOBHE
DBHE = DBHE * 0.1
DENE = DENO * (PRET * 0.01)
NDENE = DENE + 0.5
DENE = NDENE
BASE = 0.0054542 * DBHE * DBHE * DENE
NBASE = BASE * 10.0 + 0.5
BASE = NBASE
BASE = BASE * 0.1
BREAK = 49.9 * REST / 80.0
IF (BASE .GT. BREAK) GO TO 50
DBHP = (80.0 / REST) * -(0.08682 * BASE) + 0.94636
GO TO 52
50 BUST = 66.2 * (REST / 80.0)
IF (BASE .GT. BUST) GO TO 51
DBHP = (80.0 / REST) * (0.10938 * BASE) = 0.17858
GO TO 52
51 TMPY = BASE * (80.0 / REST)
TEM = TMPY * TMPY
DBHP = 19.04740 * TMPY = 0.26673 * TEM + 0.0012539 * TEM * TMPY
1 = 448.76833
IF (TMPY .GT. 80.0) DBHP = DBHO + 0.8
52 IOBHP = DBHP * 10.0 + 0.5
DBHP = IOBHP
DBHP = DBHP * 0.1
IF (DBHP * DBHE) 60,70,61
60 PRET = PRET * 1.02

```

```
IF(PRET .GT. 100.0) GO TO 90
GO TO 65
61 PRET = PRET * 0.98
65 CONTINUE
70 DBHT = DBHE
```

```
C
C COMPLTE POST-THINNING BASAL AREA.
C
```

```
IF(DBHT .GT. 5.0) GO TO 75
SQFT = 11.58495 * DBHT = 11.09724
GO TO 76
75 IF(DBHT .GE. 10.0) GO TO 77
TEM = DBHT * DBHT
SQFT = 7.76226 * DBHT + 0.85289 * TEM * 0.07952 * TEM * DBHT = 3.45624
76 BAST = (REST / 80.0) * SQFT
GO TO 80
77 BAST = REST
80 RETURN
90 PRET = 100.0
```

```
RETURN
END
```

```
SUBROUTINE LPCOST(KB,MB,SAGE,MERCHVOL,SAWTIM,NTREES,
*CMERCHVOL,CSAWTIM,CVAL,DIS,CONSTANT,CESTB,VSAWTIM,
*VMERCHVOL,NRATES,TVOL,CTVOL,VIVOL,OPTION)
```

```
C
C SUBROUTINE TO CALCULATE RETURNS ASSOCIATED WITH DWARF
C MISTLETOE INFESTATIONS AND CONTROL PROCEDURES
C
```

```
DIMENSION MB(20),SAGE(20),MERCHVOL(20),SAWTIM(20)
*,NTREES(20),CMERCHVOL(20),CVAL(20),OUT(12),DIS(8),YNR(20)
*,CROPVAL(20),CSAWTIM(20),CONSTANT(8),TVOL(20),CTVOL(20)
WRITE(6,1101)
1101 FORMAT(///1X,'STAND',3X,'VALUE',2X,'ESTABLISH=',2X,
*'VALUE OF',4X,'COST',4X,'NET RETURN',2X,'PERIODIC'/
*2X,'AGE',2X,'OF TREE',2X,'MENI COST',4X,'CROP',7X,'OF',7X,
```

```

* 'FROM', 8X, 'NET' / '(YEARS)', 2X, 'CROP', 15X,
* 'THINNED', 3X, 'THINNING', 2X, 'THINNING', 4X, 'RETURNS' )
YNR(1) = (CESTB)
CROPVAL(1) = 0
WRITE(6, 1103) 0, 0, CESTB, 0, 0, 0, (CESTB)
1103 FORMAT(2X, I3, F9.2, F10.2, F11.2, F10.2, F10.2, F12.2)
DO 1005 I=1, KB
VTHIN=CTHIN=RTHIN=0
CROPVAL(I+1)=VMERCHVOL*MERCHVOL(I)
IF(SAWTIM(I).NE.0) CROPVAL(I+1)=VSAWTIM*SAWTIM(I)
IF(OPTICK.EQ.1) CROPVAL(I+1)=VTVOL*TVOL(I)
IF (MB(I).EQ.0.OR.I.EQ.KB) GO TO 1002
VTHIN=VMERCHVOL*CMERCHVOL(I)
IF(CSAWTIM(I).NE.0) VTHIN=VSAWTIM*CSAWTIM(I)
IF(OPTICK.EQ.1) VTHIN=VTVOL*CTVOL(I)
CTHIN=((648.143+0.033*NTREES(I)*CVAL(I)**2)*1.502)/51.9
IF(NTREES(I).EQ.0) CTHIN=0
RTHIN=VTHIN=CTHIN
1002 YNR(I+1)=RTHIN
WRITE(6, 1103) SAGE(I), CROPVAL(I+1), 0, VTHIN, CTHIN, RTHIN, YNR(I+1)
1005 CONTINUE
DO 1010 M=1, NRATES
WRITE(6, 1105) DIS(2*M-1), DIS(2*M)
1105 FORMAT(/ / 1X, 'STAND', 3X, 'PERIODIC', 2X, 2('DISCOUNT', 3X,
* 'DISCOUNTED', 3X, 'CUMULATIVE', 3X, 'VALUE', 4X,
* 'CONTROLLED', 1X) / 2X, 'AGE',
* 6X, 'NET', 4X, 2(2X, 'RATE', 4X, 'PERIODIC NET', 2X, 'DISCOUNTED', 4X,
* 'TO', 10X, 'TO', 6X) / '(YEARS)', 2X, 'RETURNS', 2X, 2(1X, F5.3, 6X, 'RETURNS'
* , 4X, 'NET RETURNS', 2X, 'INFINITY', 3X, 'INFINITY', 3X) / )
SUM1=SLF2=0
DO 1008 I=2, KB+1

```

```

IF(I.EQ.1) OUT(1)=0
IF(I.GT.1) OUT(1)=SAGE(I-1)
OUT(2)=YNR(I)
OUT(3)=1.0/((1+DIS(2*M=1))*+(OUT(1)=SAGE(1)))
OUT(4)=OUT(3)*OUT(2)
OUT(5)=SUM1+CROPVAL(I)*OUT(3)
SUM1=SUM1+OUT(4)
OUT(6)=OUT(5)*(1.0/(1.0=(1.0/(1.0+DIS(2*M=1)))*+(OUT(1)=SAGE(1))))
IF(I.EQ.2)OUT(6)=OUT(5)*(1.0/(1.0=(1.0/(1.0+DIS(2*M=1))))
OUT(7)=OUT(5)+CONSTANT(2*M=1)*OUT(3)
OUT(8)=1.0/((1+DIS(2*M))*+(OUT(1)=SAGE(1)))
OUT(9)=OUT(8)*OUT(2)
OUT(10)=SUM2+CROPVAL(I)*OUT(8)
SUM2=SUM2+OUT(9)
OUT(11)=OUT(10)*(1.0/(1.0=(1.0/(1+DIS(2*M)))*+(OUT(1)=SAGE(1))))
IF(I.EQ.2)OUT(11)=OUT(10)*(1.0/(1.0=(1.0/(1+DIS(2*M))))
OUT(12)=OUT(10)+CONSTANT(2*M)*OUT(8)
1008 WRITE(6,1107) (OUT(J),J=1,12)
1107 FORMAT(3X,13,F10.2,2X,2(F7.3,4X,F8.2,4X,F8.2,F12.2,3X,F8.2,3X))
1010 CONTINUE
WRITE(6,1109) VSAWTIM*1000,VMERCHVOL ,VTVOL
1109 FORMAT('0',25X,'VALUE OF SAWTIMBER IS',F6.2,'/M',
*5X,'VALUE OF MERCHANTABLE VOLUME IS',F6.2,'/CU. FT..'/
*26X,'VALUE OF TOTAL VOLUME IS',F6.2,'/CU. FT. '/')
REPEAT 1015, FOR I=(1,NRATES)
1015 WRITE(6,1111) DIS(2*I-1),CONSTANT(2*I-1),DIS(2*I),CONSTANT(2*I)
1111 FORMAT(21X,2(5X,'FOR DISCOUNT RATE OF',1X,F4.3,1X,
*'CONSTANT IS',F7.2))
RETURN
END

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