

LONG-TERM VARIABILITY OF THE SUN IN THE CONTEXT OF
SOLAR-ANALOG STARS

by

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DEDICATION

To those of our ancestors who diligently chronicled Nature.

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I thank God, for His inspiration on a starry night in Brazil, for His wondrous creation, and for giving us some capacity to understand it. Thanks to my parents John and Bonnie for their care and wisdom. Thanks to my loving wife Patricia for her support, as well as her patience and confidence, which at times were greater than my own. Thanks to my daughter Emilie, for being a constant joy. Thanks to Dr. David DeMuth, who gave me the opportunity to work in physics research and computer programming at a time when I knew nothing. Thanks to my undergraduate advisor and supervisor, Dr. Roger Rusack, to my early mentor in solar physics, Dr. David McKenzie, and to my mentor and colleague Dr. Travis S. Metcalfe. Thanks to my excellent teachers in physics, Dr. Joseph Kapusta and Dr. Paul Crowell of the University of Minnesota, and Dr. Carla Riedel, Dr. Charles Kankelborg, and Dr. Dana Longcope of Montana State University. Thanks to Dr. Willie Soon, Dr. Sallie L. Baliunas, Dr. Jeffrey C. Hall, Gregory W. Henry, Dr. Alexei Pevtsov, Dr. Christoffer Karoff, and Dr. Timothy W. Henry for graciously sharing and explaining the various datasets used in this work. Thanks to Dr. Robert A. Donahue and Dr. Douglas K. Duncan for helpful conversations on the technical details and history of the Mount Wilson HK project. Thanks to my advisors Dr. Petrus C. H. Martens and Dr. Phillip G. Judge for guiding me toward a research topic that I found so stimulating, and for the many useful discussions.

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ABSTRACT

The Sun is the best observed object in astrophysics, but despite this distinction the nature of its well-ordered generation of magnetic field in 11-year activity cycles remains a mystery. In this work, we place the solar cycle in a broader context by examining the long-term variability of solar analog stars within 5% of the solar effective temperature, but varied in rotation rate and metallicity. Emission in the Fraunhofer H & K line cores from singly-ionized calcium in the lower chromosphere is due to magnetic heating, and is a proven proxy for magnetic flux on the Sun. We use Ca H & K observations from the Mount Wilson Observatory HK project, the Lowell Observatory Solar Stellar Spectrograph, and other sources to construct composite activity time series of over 100 years in length for the Sun and up to 50 years for 26 nearby solar analogs. Archival Ca H & K observations of reflected sunlight from the Moon using the Mount Wilson instrument allow us to properly calibrate the solar time series to the S -index scale used in stellar studies. We find the mean solar S -index to be 5–9% lower than previously estimated, and the amplitude of activity to be small compared to active stars in our sample. A detailed look at the young solar analog HD 30495, which rotates 2.3 times faster than the Sun, reveals a large amplitude ~ 12 -year activity cycle and an intermittent short-period variation of 1.7 years, comparable to the solar variability time scales despite its faster rotation. Finally, time series analyses of the solar analog ensemble and a quantitative analysis of results from the literature indicate that truly Sun-like cyclic variability is rare, and that the amplitude of activity over both long and short timescales is linearly proportional to the mean activity. We conclude that the physical conditions conducive to a quasi-periodic magnetic activity cycle like the Sun’s are rare in stars of approximately the solar mass, and that the proper conditions may be restricted to a relatively narrow range of rotation rates.

CHAPTER ONE

INTRODUCTION

1.1 The Corrupt Sun is a Nearby Star

One of the most profound truths yet learned by mankind is that the stars are the same kind of object as our Sun, only very far away. This idea was proposed by philosophers Anaxagoras (c.a. 490 B.C.) and Aristarchus of Samos (c.a. 290 B.C.), but scientific proof and popular acceptance had to wait over two millenia. Huygens (1698) assumed that the brightest star, Sirius, had the same intrinsic magnitude of the Sun, and by measuring the relative magnitude of the star he estimated Sirius' distance to be about $\sim 28,000$ AU. This underestimate already places the star incredibly far away!¹ The great distance of the stars was later proven with Bessel's geometric parallax measurement to 61 Cygni in 1838. Following laboratory work by Kirchhoff and Bunsen, photographic spectroscopy by Huggins and Miller (1864) would later show that the stars are made of the same chemical elements as the Sun, essentially completing the proof that the stars are distant suns. Realizing that the Sun is a star immediately implies two things: first, it begins to set the distance scale of the Universe. Second, it allows us to put our Sun, apparently a singular object, into a much wider context. It then allows us to ask: is the Sun *typical*?

For nearly two millenia Western thought held to the Aristotelian belief that the heavenly bodies, which included everything from the Moon, to the Sun, out to the

¹Had Huygens known that Sirius is 25 times more luminous than the Sun, his distance measurement would still fall below the modern, accurate measurements by a factor of ~ 4 .

stars, were unchanging, blemishless spheres moving in perfect circles (Aristotle 350 B.C). The contrary view began to be appreciated with Tycho’s 1572 supernovae and the use of the telescope to better observe the heavens. The telescope made it possible to regularly observe small dark spots crossing the solar disk. Evidence of the frequent appearance of such blemishes on the solar surface led Galileo to conclude that *all* objects in the heavens are thus likely to be dynamic, complicated places like our Earth:

“...it would be impossible ...not to be convinced by the [above] proofs that [sunspot] material is necessarily contiguous to the Sun and undergoes generations and dissolutions so great that nothing of comparable size has ever occurred on Earth. And if the generations and *corruptions* occurring on the very globe of the Sun are so many, so great, and so frequent, while this can reasonably be called the noblest part of the heavens, then what argument remains that can dissuade us from believing that others take place on the other globes?”

— Galileo “Dialogue Concerning the Two Chief World Systems” (1632; translated by Stillman Drake 1953). Emphasis added.

Indeed, the Sun is a dynamic, complicated place, full of *corruptions*. It was not until Schwabe (1844) that the underlying *order* in these corruptions began to be appreciated. Schwabe found the occurrence of spot groups and spotless days repeated with a period of about 10 years. Spot group counting has since continued until the present day, and reconstructions using earlier historical records extend the time series back to the time of Galileo (see Figure 1.3 and Svalgaard and Schatten (2016)). Carrington (1858) and later Spoerer (1861) noted that the spots migrate toward the equator throughout the cycle. Maunder (1904) visualized this phenomenon beautifully in the 2D time-latitude histogram of spot occurrences, later named the “butterfly diagram” (Figure 1.2). Hale (1908), observing at the famous Mount Wilson Observatory, which he founded, photographed line doublets from various elements

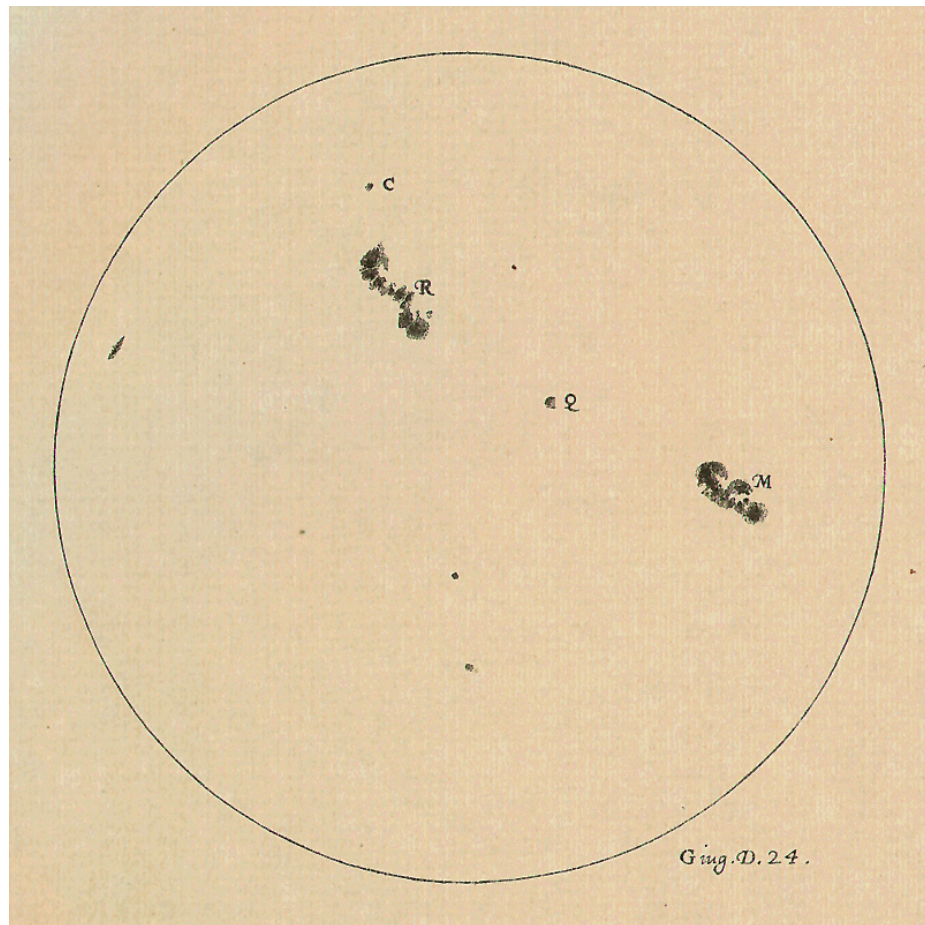


Figure 1.1 Drawing of the Sun by Galileo on June 24, 1613. Source: “The Galileo Project”, Rice University web page.

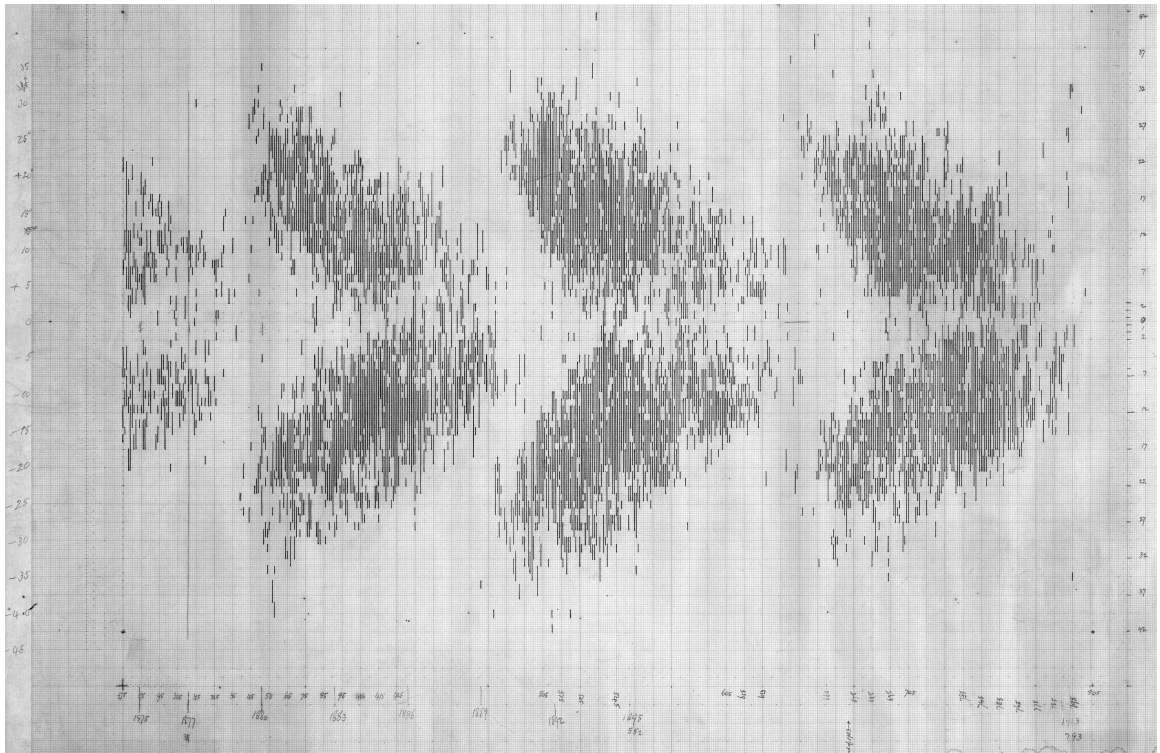


Figure 1.2 Time-latitude “butterfly diagram” drawn by Annie S.D. Maunder and E. Walter Maunder. The longitude-averaged sunspot data goes from 1875 to 1913, covering solar cycles 11 (partial) through 14. Source: “Annie Maunder, a Pioneer of Solar Astronomy”, High Altitude Observatory web page.

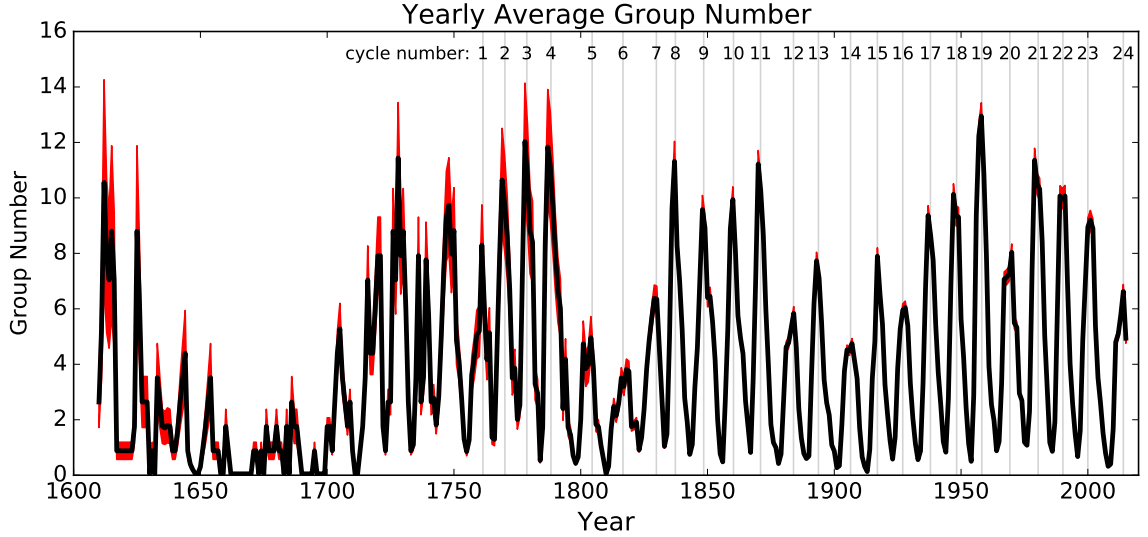


Figure 1.3 Yearly mean sunspot group number record since 1610 from the reconstruction of historical observations by Svalgaard and Schatten (2016). Red areas indicate the 1σ uncertainty. Data provided by SILSO, Royal Observatory of Belgium, Brussels.

which conclusively showed (though Hale urged caution with this result) that the spots were magnetic in origin:

“Photographs like these seemed to leave no doubt that the components of the spot doublets are circularly polarized in opposite directions. Since the only known means of transforming a single line into such a doublet is a strong magnetic field, it appeared probable that a sunspot contains such a field, and that the widening and doubling of the lines in the spot spectrum result from this cause. But much remained to be done before the proof could be regarded as complete.”

Hale later goes on to discount instrumental and atmospheric effects, leaving only the hypothesis that the spots are magnetic phenomena. He furthermore estimates the field strength of a few kilogauss based on Zeeman splitting of iron, titanium, and chromium lines from a “brilliant spark” made under known magnetic fields the laboratory. Thus it was established “the probable existence of a magnetic field in sun-spots”, and by induction the magnetic character of the solar cycle. In Hale et al. (1919), the authors devised a method of determining the angle between the field

vector on the Sun and the line-of-sight, and therefore the polarity of spots. From repeated observations, Hale et al. found several interesting patterns in the behavior of sunspot groups:

1. Spots frequently appear in pairs, with the western or preceding member forming first, although there are exceptions.
2. The axis of a binary spot group forms a small angle with the equator, with the leading spot nearer to the equator. This angle is greater on average for spots forming at higher latitudes (Joy's Law).
3. The two principle members of a binary spot group are almost invariably of opposite magnetic polarity. The polarity for the leading/following spot is the same for bipolar regions in the same hemisphere, and opposite for bipolar regions in opposing hemispheres (Hale's Law).
4. After the cycle minimum, the polarity for leading/following spots in bipolar regions reverses in each hemisphere (The Hale Cycle).

These observations demonstrated for the first time the remarkable level of order in the solar cycle. These features are shown in greater detail in the magnetogram snapshot of Figure 1.4, and the magnetic butterfly diagram of Figure 1.5, covering nearly four Schwabe sunspot cycles, or two Hale magnetic cycles. Sunspot cycles are numbered from cycle 1, which peaked in 1761, to cycle 24 at present, which peaked in early 2014 (Figure 1.3). Cycles have a variable duration from about 9 to 14 yr with a mean of ~ 11 yr (Hathaway, 2015). Their amplitude is also variable, with a standard deviation about the mean amplitude of $\sim 45\%$ (Hathaway, 2015). The most striking feature of the sunspot group number record is the period of low activity from 1660 to 1700 (40 yr). Note that earlier estimations find the extent of this period, known as

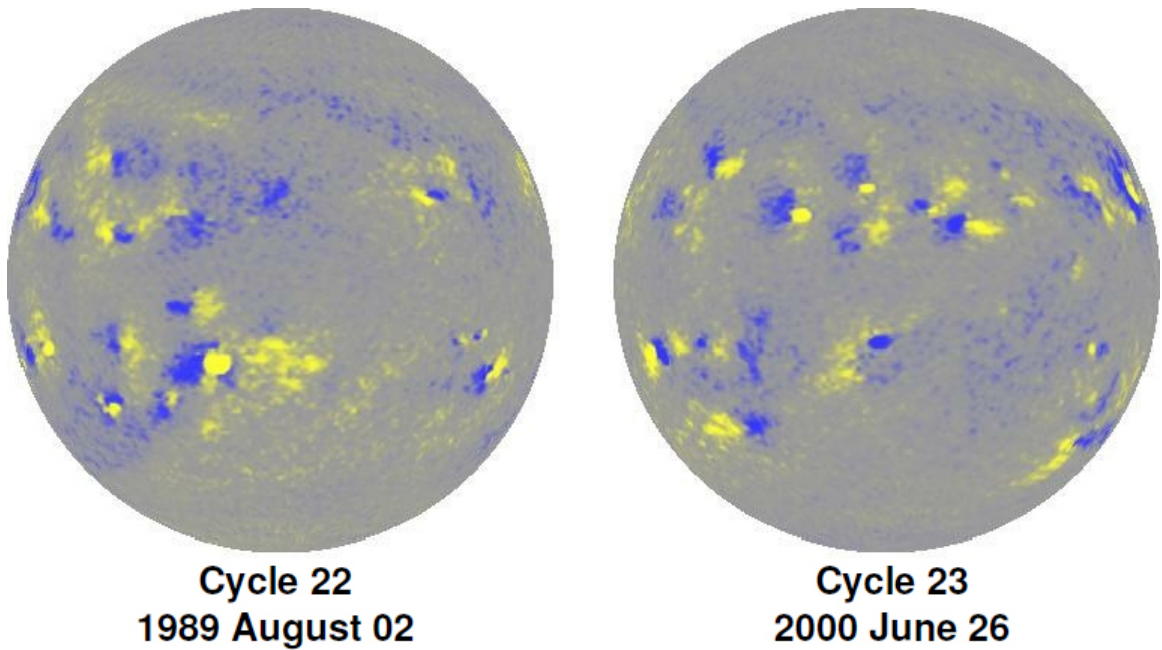


Figure 1.4 Magnetogram snapshots from dates in two consecutive solar cycles, showing Hale’s and Joy’s laws. Source: Hathaway (2015).

the “Maunder Minimum”, to be from 1645 to 1715 (70 yr) (Eddy, 1976). The more recent reconstructions are based on newly uncovered historical sources and a critical assesment of the records (Vaquero and Trigo, 2014; Vaquero et al., 2015). Regardless of the duration, there is substantial first-hand evidence that there was an extended period of time in the late 1600s during which sunspots were scarce, and proxy records indicate that this kind of “grand minimum” has occurred multiple times in the past several millenia (for a review, see Usoskin, 2013).

1.2 The Solar Dynamo Problem

Given all of the above behavior, the underlying fundamental question to ask of the Sun’s variability is this: what processes are responsible for the recurrent and orderly organization of magnetic field in the Sun? Evidence and theory have pointed

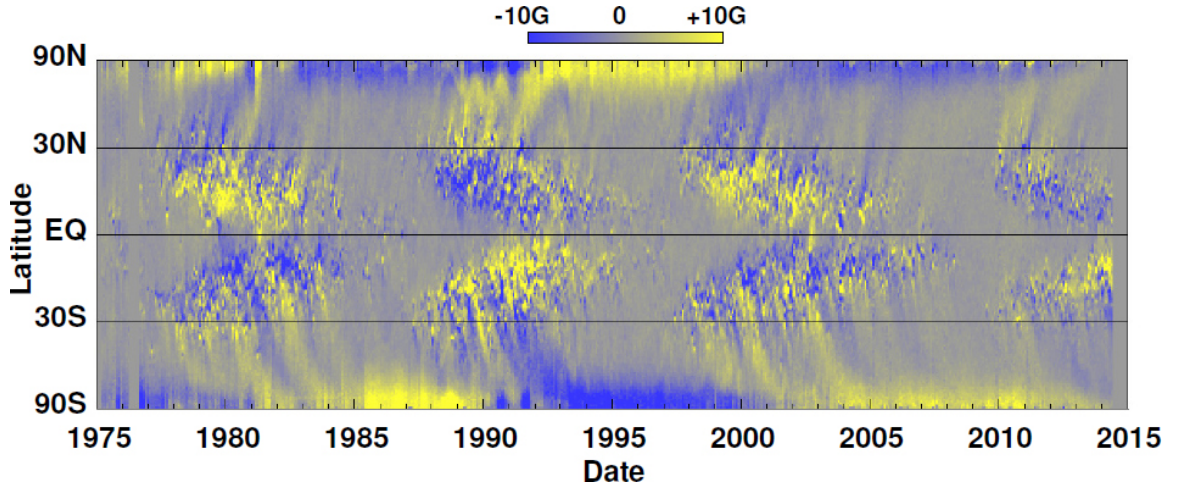


Figure 1.5 Magnetic butterfly diagram combining data obtained from Kitt Peak and SOHO, covering the four most recent solar cycles. Notice that the global polar fields ($\pm 90^\circ$) change sign every ~ 11 years. Source: Hathaway (2015).

to a global sub-surface magnetohydrodynamic *dynamo* as the origin of the global large-scale magnetic field in the Sun. Dynamo theory is a vast and active topic. In this section we summarize the field drawing from numerous excellent reviews and texts: Cameron et al. (2016); Charbonneau (2010, 2013, 2014); Miesch and Toomre (2009); Ossendrijver (2003); Rempel (2009).

Larmor (1919) first suggested that the magnetism in the Sun may be due to induction coupled with the differential rotation known to be present on the surface. This idea could be expanded upon with Alfvén’s development of magnetohydrodynamics (MHD) (Alfvén, 1942b) and the concept of “frozen in” magnetic fields that are inseparable from the plasma motions (Alfvén, 1942a). In particular, for non-relativistic, quasi-neutral plasmas in which the length scales of interest are much larger than collisional mean-free-path of electrons and the electron/ion gyroradius, we can treat the plasma as a conducting fluid and use Ohm’s Law to combine Maxwell’s equations into a single MHD induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \quad (1.1)$$

where $\mathbf{B}$ is the magnetic field vector, $\mathbf{u}$ is the fluid velocity, and η is the magnetic diffusivity. Applying the outside curl and using vector identities gives:

$$\frac{\partial \mathbf{B}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{u} - \mathbf{B}(\nabla \cdot \mathbf{u}) + \eta \nabla^2 \mathbf{B}, \quad (1.2)$$

where the first term is the advection of magnetic field, the second term is the amplification of field by shear, the third term is the amplification of field by compression, and the final term is the dissipation of field through diffusion. Differential rotation in Larmor's proposal can amplify field via the shear term $(\mathbf{B} \cdot \nabla) \mathbf{u}$.

The MHD induction equation is the starting point for Parker's axisymmetric kinematic dynamo model in which plasma flows $\mathbf{u}$ are sought which can generate toroidal ($\hat{\phi}$ direction) and poloidal ($\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}$ plane) fields $\mathbf{B}_t, \mathbf{B}_p$ supported against field-destroying magnetic diffusivity η (Parker, 1955). Parker showed with the induction equation that a purely poloidal field acted upon by a sheared, but purely toroidal flow would generate a toroidal field. In order to close the loop and produce a global cycle such as seen in Figure 1.5, another mechanism must transform toroidal field back to poloidal. For this, Parker postulated a process through which toroidal field fixed in cylindrical convective cells are deflected by the Coriolis force, providing a non-axisymmetric flow necessary to circumvent the anti-dynamo theory of Cowling (1933) and generate a poloidal field using the induction equation in an axisymmetric formulation. It was also shown that Parker's dynamo equations admitted wave-like solutions, which allow for the propagation of dynamo action. This attractive feature gave a potential explanation for the observed equatorward latitudinal migration of sunspots, through a propagating *dynamo wave* of toroidal field in the interior.

1.2.1 Mean-Field Dynamo Models

Dynamo models studied today are of two general types. In the first type, Parker’s kinematic dynamo model is refined with the mean-field electrodynamics formulation (Steenbeck et al., 1966) which decomposes the fields and flows into average ($\langle \mathbf{B} \rangle, \langle \mathbf{u} \rangle$) and fluctuating ($\mathbf{b}', \mathbf{u}'$) components. These two-component fields and flows are inserted into the induction equation (1.1) and averaged in such a way that $\langle \mathbf{b}' \rangle$ and $\langle \mathbf{u}' \rangle$ vanish, leaving a new form of the induction equation:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \langle \mathbf{u}' \times \mathbf{b}' \rangle - \eta \nabla \times \langle \mathbf{B} \rangle) \quad (1.3)$$

The new term in this equation, $\langle \mathbf{u}' \times \mathbf{b}' \rangle$, corresponds to a mean electromotive force, $\boldsymbol{\varepsilon}$, which is then expressed as a series expansion of the mean field $\langle \mathbf{B} \rangle$:

$$\boldsymbol{\varepsilon} = \boldsymbol{\alpha} \cdot \langle \mathbf{B} \rangle + \boldsymbol{\beta} \cdot \nabla \times \langle \mathbf{B} \rangle + \dots, \quad (1.4)$$

where $\cdot$ indicates a tensorial contraction. The crucial toroidal-to-poloidal process is captured in the $\boldsymbol{\alpha}$ tensor, where Parker’s helical twisting mechanism can be expressed as one possible functional form for $\boldsymbol{\alpha}$. The simplest and most commonly used implementation of kinematic mean-field dynamo models reduce $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to scalar values and fold β into a net “turbulent diffusivity” term in the induction equation, $\eta_T \nabla \times \mathbf{B}$, with $\eta_T = \eta + \beta$. In an axisymmetric formulation, equation (1.3) with the $\boldsymbol{\varepsilon}$ term (1.4) can be decomposed into two equations concerning the time evolution of the purely toroidal magnetic field B , and the poloidal vector magnetic potential A . Transforming those expressions into a non-dimensional form reveals two dimensionless numbers which govern the axisymmetric mean-field $\alpha\Omega$ dynamo models:

$$C_\alpha = \frac{\alpha_0 R_\star}{\eta_0}, \quad C_\Omega = \frac{(\Delta\Omega)_0 R_\star^2}{\eta_0} \quad (1.5)$$

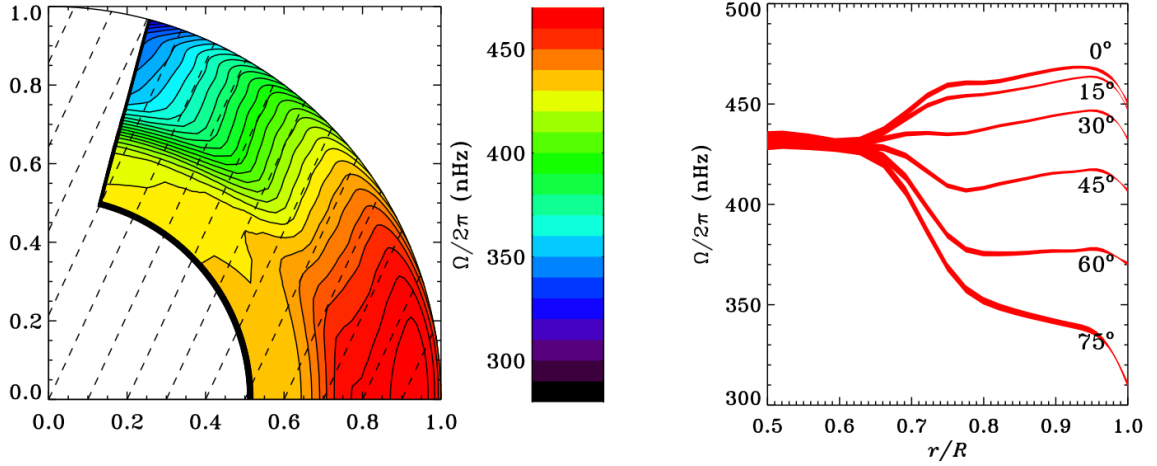


Figure 1.6 Internal rotation profile of the Sun from helioseismology. Source: Howe (2009).

where α_0 and η_0 are “typical” values of the turbulent convection and diffusivity terms, $R_\star$ is the radius of the star, and $(\Delta\Omega)_0$ is a typical value of large-scale rotational shear.

Kinematic mean-field dynamo models prescribe the mean interior flow field $\mathbf{u}$, which early on were only justified *a posteriori*. Axisymmetric kinematic models specify the mean toroidal component $u_\phi = \Omega(r, \theta)r \sin \theta$ and the poloidal components u_r and u_θ separately. The functional form of the flow field together with the sign of the α parameter determine the direction of dynamo wave propagation according to the Parker-Yoshimura rule (Yoshimura, 1975). For equatorward propagation such as is seen in the butterfly diagram (Figures 1.2 and 1.5), it is required that $\alpha \partial\Omega/\partial r < 0$ in the northern hemisphere and $\alpha \partial\Omega/\partial r > 0$ in the south. Early modelers had the freedom to assume any of α and $\Omega(r, \theta)$ which produced equatorward propagation, often assuming cylindrical isorotation contours (e.g. Stix, 1976).

The observational understanding of the solar internal rotation advanced incrementally throughout the 1980s and 1990s (for a review, see Howe, 2009) and

resulted in a conclusive rejection of the previous picture of cylindrical contours of isorotation. The definitive blow came with the helioseismic inversions of data from GONG (Thompson et al., 1996) and SOHO/MDI (Kosovichev et al., 1997). The mean toroidal component of the interior flow $u_\phi(r, \theta)$ is now well constrained by helioseismology, as shown in Figure 1.6. The general features are (1) faster rotation at the equator, slower at the poles; (2) a rigidly rotating core; (3) conical contours of isorotation ($\partial\Omega/\partial r \sim 0$) in the mid convection zone; (4) a region of strong radial shear near the base of the convection zone, or the *tachocline*, which is positive at low latitudes and negative at high latitudes, and (5) a region of strong negative radial shear near the surface.

These measurement of the interior rotation of the Sun threw existing mean-field dynamo theory into disarray due to the requirement of a negative alpha effect in the northern hemisphere. One *ad hoc* solution to this problem is to bodily transport toroidal flux with an equatorward meridional flow at the base of the convection zone. Models incorporating this feature are known as flux-transport dynamo models. The structure of the slow deep interior meridional flow is just beyond the reach of helioseismology today, with several conflicting measurements in the literature pointing to a single circulation cell (Jackiewicz et al., 2015), multiple cells in radius (Zhao et al., 2013), or even more complex patterns (Schad et al., 2013).

Babcock (1961) proposed an alternative mechanism for toroidal-to-poloidal flux conversion based on surface observations, which was elaborated upon and placed on firm mathematical footing in Leighton (1964, 1969). The systematic tilt of bipolar active regions (Joy’s Law) preferentially places the leading polarity closer to the equator and the following polarity closer to the pole. Through diffusion or advection by the meridional flow, the polarity of the following spot is transported to the poles, which cancels and replaces the oppositely-signed polar flux from the previous cycle.

This effect is strongly suggested in the magnetic butterfly diagram of Figure 1.5. Flux-transport dynamos replacing the α -effect with a Babcock-Leighton surface source term are gaining dominance in dynamo modeling due to their successful reproduction of the observed surface magnetic butterfly diagram (e.g. Miesch and Teweldebirhan, 2016; Yeates and Muñoz-Jaramillo, 2013). Furthermore, Cameron and Schüssler (2015) provide a compelling argument based on solar magnetogram observations and Stokes’ theorem that the Babcock-Leighton mechanism is operating in the Sun.

Like the $\alpha\Omega$ flux-transport dynamos, the deep meridional flow and magnetic diffusivity are free parameters which can be tuned to reproduce solar observations. The Babcock-Leighton source term is somewhat more constrained than the analogous α parameter, due to its location near the surface and relation to solar observations. The period of the dynamo cycle in flux transport models is strongly coupled to the meridional flow speed. This, coupled with the lack of observational constraints on the meridional flow, makes it possible to arbitrarily tune these models to the solar cycle period. Cycle amplitude is also specified in an *ad hoc* fashion through the so-called “ α -quenching” term which truncates the exponential growth of toroidal fields in these models. The preponderance of unconstrained parameters related to turbulent interior flows $\mathbf{u}'$ is the principle drawback of this class of models. We lack a robust theory of turbulence in stratified atmospheres from which to calculate the α and β tensors directly.

1.2.2 Global Convective MHD Dynamo Models

The second principal approach to the dynamo problem is to solve the full set of MHD equations, augmenting the induction equation (1.1) with equations governing the evolution of mass, momentum, and energy:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1.6)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \boldsymbol{\tau} - 2\boldsymbol{\Omega} \times \mathbf{u} + \mathbf{g} + \frac{1}{\mu_0 \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} \quad (1.7)$$

$$\frac{\partial e}{\partial t} + (\gamma - 1)e \nabla \cdot \mathbf{u} = \frac{1}{\rho} \{ \nabla \cdot [(\chi + \chi_r) \nabla T] + \phi_u + \phi_B \} \quad (1.8)$$

where ρ is the plasma density, e is the internal energy, p is the pressure, $\boldsymbol{\tau}$ is the viscous stress tensor, $\mathbf{g}$ is the gravity, χ and χ_r are the kinetic and radiative thermal conductivities, and ϕ_u and ϕ_B are the viscous and Ohmic dissipation functions. An equation of state (usually the perfect gas law) and appropriate boundary conditions must also be specified, and the solenoidal constraint $\nabla \cdot \mathbf{B} = 0$ must be enforced. This consists of a set of coupled non-linear advection and diffusion equations.

Solving the full MHD equations is obviously much more computationally intensive than the mean-field approach described in the previous section. The anelastic approximation, whereby a static density is defined so that $\partial \rho / \partial t = 0$, is commonly taken to reduce the load demanded by small timesteps associated with sound waves. However, this approach still demands massive parallel computing resources and the resolution of the spherical grid is far coarser than what is required to capture convection at the smallest scales.

The effects of rotation on convective flow are determined by the Rossby number,

$$\text{Ro} \equiv \frac{u'_{\text{rms}}}{2\Omega \ell} \equiv \frac{P_{\text{rot}}}{\tau_c}, \quad (1.9)$$

where u'_{rms} and ℓ are the typical velocity and length scale of the interior convection, Ω is the angular velocity, and τ_c is the convective turnover timescale. Deep interior convection is difficult to measure in the Sun, making the value of the internal solar Rossby number a matter of debate. At the surface, $\text{Ro} \gg 1$ owing to the short length and time scales of solar granulation ($L \sim 2$ Mm; $T \sim 5$ minutes; Title et al. 1989). The simulations discussed below which produce solar-like differential rotation and giant cells suggest that $\text{Ro} \lesssim 1$ in the interior, in agreement with lower limits of convective velocity $u'_{\text{rms}} > 8 \text{ m s}^{-1}$ and characteristic length scales $\ell \gtrsim 5.5\text{--}30$ Mm argued for by Miesch et al. (2012) as necessary to sustain the observed differential rotation and meridional circulation.

Global convective MHD simulations began with Gilman (1983) and Glatzmaier (1984, 1985a,b). These early simulations produced a rapidly rotating equatorial region (although with cylindrical instead of conical contours as shown by later observations), as well as magnetic field generation and latitudinal propagation (although poleward instead of the observed equatorward). Recent simulations using much larger computational resources have achieved more success in reproducing features resembling solar observations. Simulations using the EULAG-MHD code (Ghizaru et al., 2010; Racine et al., 2011) have achieved an approximately solar-like conical differential rotation profile, as well as regular hemispheric polarity reversals and equatorward migration of field generation with a period of 30 years. Simulations with the ASH code (Brown et al., 2011) also achieve cycling polarity reversals with a period of about 4 years, with poleward migration of dynamo activity. The ASH simulations also generate large “magnetic wreath” structures throughout the convection zone, with strong flux regions experiencing a buoyant rise to the surface of the simulation (Nelson et al., 2013). Later ASH simulations by Augustson et al. (2015) show equatorward propagation of the dynamo action, and temporary lapse

in field generation analogous to the solar Maunder Minimum. Simulations with the PENCIL code (Käpylä et al., 2010, 2012, 2013) show both a low-latitude equatorward and a stronger high-latitude poleward-propagating bands of toroidal flux, and a cycle period of 33 years. The common feature of these simulations is the generation of magnetic flux in the bulk of the convection zone according to processes which, remarkably, appear to correspond to the kinematic $\alpha\Omega$ dynamos in the mean-field parlance. However, unlike flux transport dynamos, the meridional flow does not play an important role in the dynamics of these simulations. Furthermore, the ASH and PENCIL simulations do not include a stable stratified shear layer at the base of the convection zone (tachocline), a critical component for storage of toroidal flux and equatorward propagation in mean-field $\alpha\Omega$ dynamos. These results bring into question the importance of the tachocline as a source of toroidal field for the solar dynamo.

While global MHD simulations produce encouraging solar-like results in some aspects, it remains an open question whether they are really demonstrating behavior which occurs in the solar interior. The principal problems are (1) the limited resolution of the models fails to capture small scale convection and magnetic field which may have an impact on large-scale dynamics, and (2) numerical diffusion is orders of magnitude higher than the expected diffusion for the Sun, forcing these models to operate in non-solar parameter regimes. Hotta et al. (2016) showed the impact of grid scale on a convective simulation, with the generation of large-scale magnetic field disappearing as grid resolution is increased, then appearing again as it is increased further. To achieve convection in the face of higher diffusion, some models enhance the energy input by increasing the luminosity by multiples of the solar value. The resulting large-scale convection can be orders of magnitude more vigorous than indicated by helioseismology and surface measurements (Hanasoge et al., 2012;

Hathaway, 2012). However, see Greer et al. (2015) for helioseismic measurements which are in agreement with convective MHD models. The rotation in simulations featuring polarity reversals are faster than the solar rotation. For example, the ASH simulations cycle when run at $5 \Omega_{\odot}$, but not at $3 \Omega_{\odot}$ (Brown et al., 2011), except when diffusion is reduced (Nelson et al., 2013), while simulations run at the solar rotation do not generate global-scale field at all (Charbonneau, 2014).

Parker (2009) summarizes the progress of solar dynamo theory over the recent half-century, but cautions against complacency:

“It is gratifying to see that this exploratory approach over the years has provided a variety of circumstances that might provide the actual magnetic fields seen on the Sun. So there has been great progress, and some have been emboldened to apply the same $\alpha\Omega$ -dynamo concepts to others stars, to accretion disks, and to the Galaxy. However, it must be recognized that these gratifying achievements are really only the first major step in establishing a scientific theory of the origin of the magnetic fields of the Sun, and by implication, of the other magnetic objects to be found in the astronomical universe. The physics implied by the appropriate values of the parameters must also be understood. That there exist values of the parameters such that a mathematical simulation can be made to conform to the observational facts is not sufficient for a scientific understanding of the solar dynamo. . .

“Finally, we must keep in mind that until the theory of the solar dynamo can be raised above the level of conjecture, we should restrain our enthusiasm for extrapolating the $\alpha\Omega$ -dynamo concept to distant unresolved objects.”

In this work we advocate a reverse approach, to use observations of stellar magnetism as additional constraints to the general dynamo problem. Each star is its own self-contained instance of a *dynamo experiment*, and by carefully characterizing these objects and the behavior of their magnetic fields, we hope to find discriminating evidence to reject classes of dynamo models. This effort can guide us toward the correct *phenomenological* understanding of the stellar dynamo, even while a first-principles description is still lacking.

1.3 Stellar “Dynamo Experiments”

Another fundamental question one can ask of the solar dynamo is this: how are the characteristics of the solar cycle determined by the global properties of the Sun? For example, is the ~ 11 year period of the cycle uniquely determined by the solar mass, rotation, and composition? This question is difficult to approach studying the Sun in isolation, for it would require a complete and correct description of the dynamo from first principles. While this is ultimately the goal in the quest for the physics of the Sun, dynamo models at this point involve a number of free or uncertain parameters which are “tuned” to the solar cycle. With only one data point to fit, degeneracies in the parameters cannot be broken. More *dynamo experiments* can help resolve this problem, or perhaps, if nature chooses to be so kind, empirical relationships between stellar properties and dynamo behavior may be found. Furthermore, with the knowledge of many stars – some of which are like the Sun was in the distant past, and others as it will be in the distant future – we can begin to appreciate changes in magnetic behavior on the *time scale of stars*, compared to which the human lifespan, and indeed, the whole history of humanity, is insignificant.

Wilson expressed the idea of the utility of “stellar experiments” in his 1968 paper which introduced the Mount Wilson Observatory (MWO) HK project:

“...A vast amount of observational data and theoretical speculation relating to the cyclical solar variation has been accumulated. Nevertheless, it seems very likely that understanding has been severely hampered because all this material relates to a single star with a fixed set of parameters such as age, mass, and surface temperature. It is a reasonable supposition that if analogous cycles could be detected in other stars with different values of the fundamental stellar parameters, the results would be of considerable value in sharpening the theoretical attack on the whole problem.”

Spectroheliograms in the Fraunhofer H (3968.47 Å) and K (3933.66 Å) K lines have been taken at MWO and other observatories since the early 1900s (Hale and Ellerman, 1904; Tlatov et al., 2009). These images reveal a diversity of structures on the Sun, with high-intensity *plage* regions surrounding dark sunspots, and an enhanced *network* in the “quiet Sun” areas away from active regions. Recognizing the large variation of appearance in HK spectroheliograms from solar minimum to solar maximum, Wilson began examining a set of stars in these lines to see if similar variations in integrated HK emission could be observed. While no unambiguous variations in HK flux were observed in the one year of observations presented in Wilson (1968), variations were found from the decade of observations of 91 main-sequence stars in Wilson (1978). Wilson reported that about a dozen of these stars appeared to have completed a cycle in HK flux variations, with most of them of later spectral type (i.e. less massive) than the Sun.

The Fraunhofer H & K lines are prominent absorption features in the far violet continuum from singly-ionized calcium (Ca II) in the photosphere. Figure 1.7 shows that in the cores of these lines there is a double reversal feature. These reversals are emission formed in the lower chromosphere due to magnetic heating processes (Athay, 1970; Linsky and Avrett, 1970). Figure 1.8 shows the spatial correspondence between a Ca II K spectroheliogram and regions of enhanced magnetic field in a line-of-sight magnetogram. The correspondence between HK emission and magnetic flux in the Sun is well studied (Leighton 1959, Skumanich et al. 1975, Schrijver et al. 1989, Harvey and White 1999, Sheeley et al. 2011, Pevtsov et al. 2016), and it is this feature which makes Ca II HK emission a good proxy for magnetism on the surface of stars. Distant stars are not spatially resolved, but flux in the H & K lines over the whole surface of the star is additive. In contrast, the solenoid condition $\nabla \cdot B = 0$ makes most of the magnetic signature due to the Zeeman effect

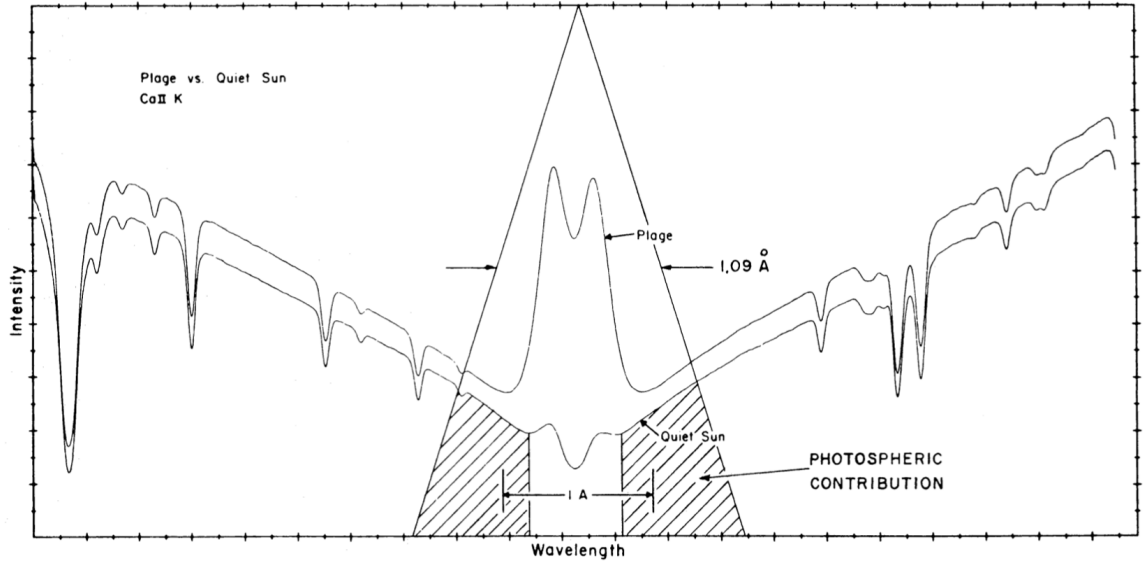


Figure 1.7 Disk-integrated Calcium K-line core for quiet Sun and plage regions from NSO Kitt Peak (White and Livingston, 1981). The overlying triangle shows the bandpass of the Mount Wilson HKP-2 instrument, with regions of photospheric contribution shaded. Source: Hartmann et al. (1984).

cancel. With nearly as much positive magnetic polarity on the stellar surface as negative, there will be almost equal contributions of left circularly polarized light as right. In integrated light these contributions approximately cancel, and therefore spectropolarimetric measurements of extreme sensitivity are required to measure any imbalance. Ca II H & K flux, on the other hand, has no sign or sensitivity to polarity, and thus no cancellation effect, making it a good proxy for total unsigned magnetic flux. The practical utility of Ca II HK proxies as an activity measure is demonstrated by the fact that time-averaged 1 Å K flux varies by nearly 20% over the course of the solar cycle (White and Livingston, 1981), while total solar irradiance (integrated light over all wavelengths) varies by only $\sim 0.1\%$ (Kopp, 2016; Kopp and Lean, 2011). Wilson (1968) estimated a 0.001 magnitude ($\approx 0.1\%$) change in solar luminosity due to the passage of spots covering about 1400 millionths of the solar surface, and judged

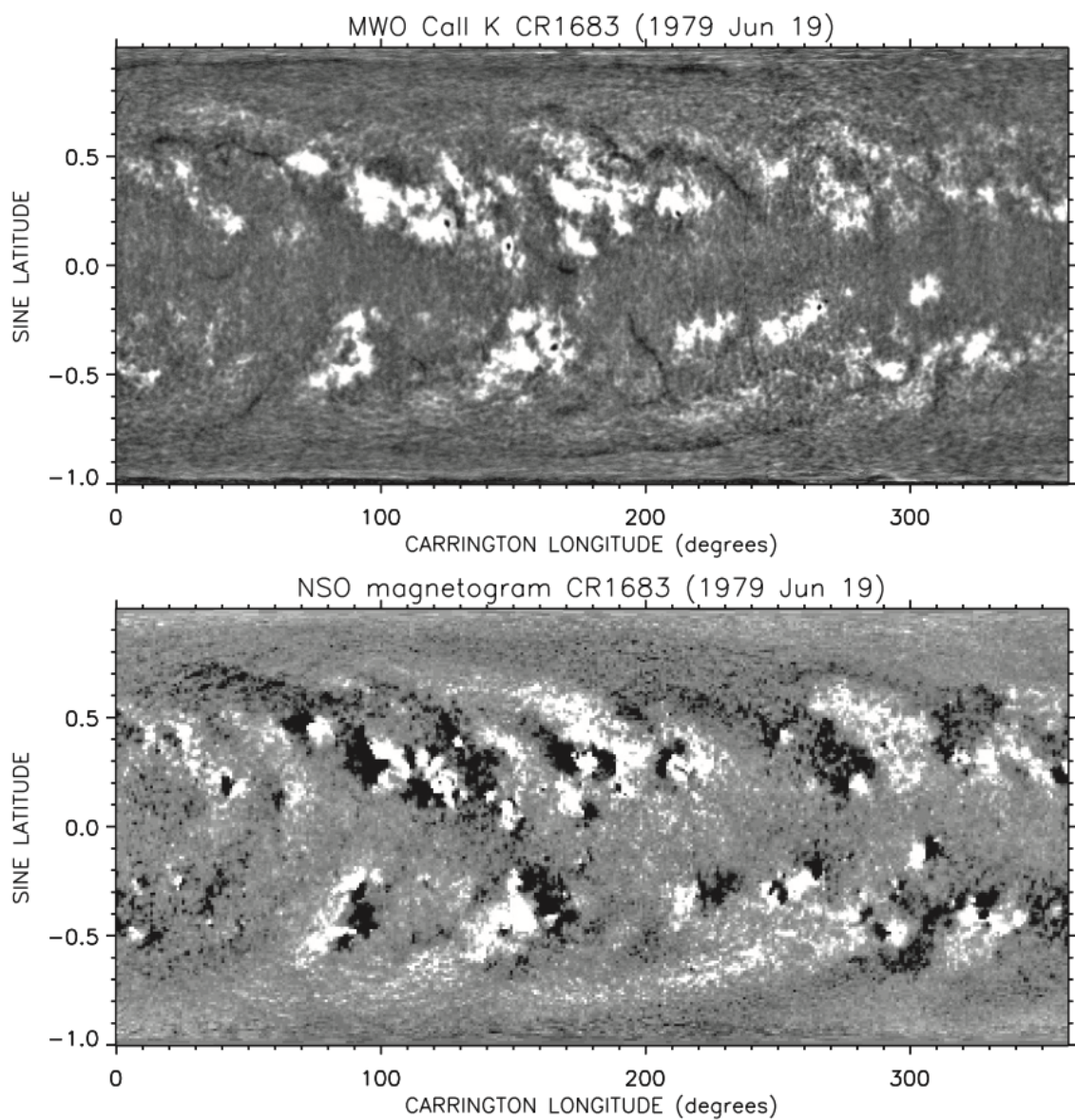


Figure 1.8 Carrington map of Ca K intensity and magnetic flux. Note the correspondence between Ca K intensity and magnetic flux of either polarity. Source: Sheeley et al. (2011).

that detecting stellar cycles through luminosity measurements, or more specifically, broad-band visible observations, to be impractical at the time.

1.3.1 Indices of Chromospheric Activity

With the successful detection of stellar long-term variability in Ca II H & K using the Coudé scanner on the 100-inch telescope at Mount Wilson, the HK project built a new instrument to increase efficiency (Vaughan et al., 1978). The new instrument was installed at the 60-inch telescope which was now dedicated to HK observations. The “HK Photometer 2”, or HKP-2, came to define the now standard S -index of chromospheric activity:

$$S = \alpha \frac{N_H + N_K}{N_R + N_V} \quad (1.10)$$

where N_H and N_K are the counts in 1.09 Å triangular bands centered on Ca II H & K, N_R and N_V are counts in 20 Å reference bandpasses in the nearby continuum region, and α is a calibration constant (Vaughan et al., 1978). The HKP-2 instrument consisted of a flat-field Ebert spectrometer with a multi-slit exit and a chopper wheel to measure the fluxes in the four bands sequentially using a single photometer. Calibration of α was done nightly to maintain long-term stability by comparison with a standard lamp and observation of standard (“non-variable”) stars. In practice, α was typically found to be about 2.40 and stable to within 1% (Baliunas et al., 1995; Duncan et al., 1991; Vaughan et al., 1978).

For stars at equal distances, the denominator $N_R + N_V$ will be larger for bluer (more massive) stars and smaller for redder (less massive) stars, introducing a *color term* into the S -index which complicates its use for comparing stars of different spectral type. Furthermore, the color temperature of the star will affect the photospheric contribution to N_H and N_K , as the 1 Å band is wide enough to

admit light originating below the temperature minimum. To mitigate these biases, a derivative index was defined in Noyes et al. (1984a), the chromospheric emission ratio R'_{HK} :

$$R'_{HK} \equiv \frac{(\mathcal{F}_H + \mathcal{F}_K)_{\text{chromo}}}{\sigma T_{\text{eff}}^4} = R_{\text{HK}} - R_{\text{phot}} \quad (1.11)$$

where $\mathcal{F}$ represents an *absolute* flux in the 1 Å H and K bands *from the chromosphere* and σT_{eff}^4 is the bolometric flux. This ratio can be understood as the fraction of the total energy flux of the star that is caused by magnetic heating in the chromosphere leading to HK emission. An empirical relationship to convert color index $(B - V)$ and S to HK flux was obtained by Middelkoop (1982), which divided by the bolometric flux gives R_{HK} . For practical purposes, Noyes et al. (1984a) follows Hartmann et al. (1984) and considered all flux outside the H1 and K1 minima (see Figure 1.7) to be photospheric in origin, and used the high resolution spectra of four stars and the Sun to derive an empirical relationship for the fraction of photospheric flux, R_{phot} as a function of $(B - V)$ color. It was estimated that the derived relationship for R_{phot} is accurate to about 10%. The derivation for R'_{HK} does not take into account the effects of metallicity, which can affect the line blanketing of the N_R and N_V continuum bands, producing an additional bias in S .

The quantity R'_{HK} represents the efficiency of transforming energy to a specific class of magnetism. Hall et al. (2007b) advocates the use of physical ($\text{erg cm}^{-2} \text{s}^{-1}$) *excess* flux from the chromosphere, $\Delta\mathcal{F} \equiv \mathcal{F}_{\text{HK}} - \mathcal{F}_{\text{phot}} - \mathcal{F}_{\text{min}}$. The first two terms are equivalent to the numerator in the definition of R'_{HK} (equation 1.11). The final term, $\mathcal{F}_{\text{min}}$, represents the so-called “basal” flux, a temperature-dependent lower limit to chromospheric flux. Schrijver et al. (1989) found that the minimum observed chromospheric flux in the Sun, located in the center of supergranular cells,

corresponds to the minimum found in subgiants and giants of solar spectral type. This correspondence leads to the interpretation that stars with chromospheric flux of $\sim \mathcal{F}_{\min}$, or equivalently $\Delta\mathcal{F} = 0$, have surfaces that are 100% “quiet Sun”, devoid of all activity and consisting of the lowest observed magnetic flux densities. The quantity $\Delta\mathcal{F}$ is more appropriate when one is interested in the absolute amount of magnetic activity, as opposed to the efficiency of its generation.

This present work is focused on relative variations between stars of nearly the same surface temperature. This sample selection minimizes the effects of the “color term” in the S -index, and we therefore employ that basic measure of the Mount Wilson program as our fundamental datum. By avoiding the use of empirical relationships to transform the data to R'_{HK} or physical flux $\mathcal{F}$, we avoid introducing the uncertainties implicit in those relationships into our results. However, for ease of comparison of our results with a broad section of the literature, we transform some of our S values to R'_{HK} using the procedure described in Noyes et al. (1984a).

1.3.2 The Solar-Stellar Connection

With Wilson’s successful detection of stellar long-term variability in Ca II H & K, an entire field of study on the *solar-stellar connection* was born (Noyes, 1996). The principal goals are to understand the relationships between stellar activity and variability with fundamental stellar properties, and to understand the Sun’s place in this context. The MWO HK Project continued making observations with the new HKP-2 instrument until 2003, accumulating synoptic observations for 38 years and resulting in a number of advancements. In the following sections, we outline major results in this area of study.

1.3.3 The Vaughan-Preston Gap

Vaughan (1980); Vaughan and Preston (1980) noted the presence of two branches in a diagram of $\log(S)$ vs $(B - V)$ for 485 MWO stars. The deficit of stars at intermediate activity levels became known as the Vaughan-Preston gap. Originally suspected to be a selection effect or due to saturation defects in the S -index, the gap has since been seen in other large activity surveys using different instruments to find $\log(R'_{\text{HK}})$ (Gray et al., 2003, 2006; Henry et al., 1996; Pace, 2010) and absolute HK flux (Hall et al., 2007b). Gray et al. (2006) further noted that the gap disappears for metal-poor stars, $[M/H] < -0.02$. When activity is measured with the S -index, the gap appears at higher values of S for stars of lower mass. In $\log(R'_{\text{HK}})$, the gap is independent of mass and is centered at $\log(R'_{\text{HK}}) \sim -4.75$. While no convincing explanation for the Vaughan-Preston gap has been found, the existence of the gap is consistent with a rapid decrease in activity at a mass-dependent critical age (Pace, 2010). The gap has been used in the literature to divide stars into two groups, “active” vs. “inactive” or “young” vs. “old”, depending on which side of the gap they lie. Our Sun is found on the inactive, old side of the Vaughan-Preston gap.

1.3.4 Rotation and Differential Rotation

A major advance came with Vaughan et al. (1981) and Baliunas et al. (1983), who used autocorrelation analysis of high-cadence (nightly) observations to measure rotational modulations in the HK time series. These were the first measurements of the rotation period P_{rot} in stars by such a direct method. Stellar rotation had previously been known through the spectroscopic determination of projected rotational velocity, $v \sin i$. P_{rot} has the distinct advantage of independence of the usually unknown inclination angle, i . The technique also allowed for slower rotations to be measured, down to 1 km s^{-1} , which is beyond the sensitivity of the spectroscopic

method. Vaughan et al. (1981) and Baliunas et al. (1983) found that mean activity and range of activity both decrease with slower rotation for a small sample of K-type stars. No evidence of an hypothesized rotational discontinuity across the Vaughan-Preston activity gap was found. Vaughan et al. (1981) noted that “obvious 10–12 year activity cycles are found almost exclusively among stars with rotation periods longer than about 20 days, and the periods of these cycles are uncorrelated with the rotational velocities.”

Building on this, Donahue et al. (1996) used periodogram analysis and compiled rotation measurements in yearly bins (or *seasons*) to search for differences which could be interpreted as surface differential rotation. In the Sun, activity develops in latitudinal bands which migrate toward the equator during the cycle. Since the Sun rotates at a different rate at these different latitudes, measuring rotational modulations at different times in the cycle will indicate differential rotation. Donahue and Keil (1995) showed that this approach works for the Sun in disk-integrated Ca K observations, measuring a period difference $\Delta P_{\odot} = 4.0$ days and a mean sidereal rotation $P_{\text{rot},\odot} = 26.09$ days during cycle 22. The major limitation of these differential rotation measurements in stellar observations is that the latitude of spots causing the modulations is unknown, which means that any measurement is only a lower limit of the total equator-to-pole differential rotation. The sense of the rotation, whether it be solar-like with the equator faster than the poles, or the opposite, is also unknown. Nevertheless, Donahue et al. (1996) measured ΔP for 36 of ~ 100 MWO stars, and found a power law relationship $\Delta P \propto \langle P \rangle^{1.3 \pm 0.1}$, independent of mass, indicating that surface rotational shear is larger for slower rotating and older stars. Recall from above the important role rotational shear plays in toroidal field amplification according to dynamo theory.

1.3.5 Rotation-Activity-Age Relationships

Skumanich (1972) found a relationship between rotational velocity, Ca II H & K emission, and lithium abundance with age using observations of stellar cluster members with determined ages from stellar evolution models. The rotational decay law $P_{\text{rot}} \propto t^{1/2}$ is known as the Skumanich spin-down law, and estimating the age of a star based on rotation is now known as *gyrochronology*. The $t^{1/2}$ spin-down has been verified in subsequent rotation/activity/age studies from ever-larger and better calibrated samples of stars (Barnes, 2007; Guinan and Engle, 2009; Mamajek and Hillenbrand, 2008; Pace and Pasquini, 2004; Soderblom, 1983; Soderblom et al., 1991), see Soderblom (2010) for a comprehensive review. The constant of proportionality for the spin-down law is found to be mass dependent, with lower mass stars spinning down more rapidly than higher mass stars. The physical understanding for these relationships is via angular momentum loss through magnetized stellar winds (Mestel, 1968; Schatzman, 1962; Weber and Davis, 1967). Stellar surface magnetism is coupled to rotation through dynamo processes. The magnetism is also coupled to the stellar wind, whose ejection of mass tied to magnetic field is the principle lever-arm for angular momentum loss of the star. Since lower-mass stars have higher levels of surface magnetism (inferred from activity proxies), they presumably have stronger winds and higher mass loss, leading to larger losses in angular momentum over time. The existence of the empirical spin-down laws indicates that all stars of a given mass arrive at a common value of rotation at some young age; initial conditions do not matter. Recent evidence using older stars with ages determined from asteroseismology indicates that the spin-down mechanism is greatly weakened at a critical Rossby number, with the consequence that the gyrochronology techniques are not applicable to older stars (van Saders et al., 2016). The Sun is near this critical point.

Noyes et al. (1984a) studied the activity-rotation relationship in the MWO stars and found a tight mass-independent empirical relationship when using a semi-empirical Rossby number $Ro \equiv P_{\text{rot}}/\tau_c$ instead of P_{rot} . Recall from above the importance of the Rossby number to differential rotation and the dynamo. Noyes et al. obtain the mass-dependent convective turnover time τ_c from numerical hydrodynamic simulations of Gilman (1980), with α , the ratio of the convective mixing length to the pressure scale height, left as a free parameter. The turnover time of Gilman (1980) is that of convection one pressure scale height from the bottom of the convective zone. Noyes et al. uses empirical relationships to translate the simulation's mass-dependent turnover time, $\tau_c(M)$, to a function of color index, $\tau_c(B - V)$. An iterative fitting procedure was employed to determine the value of α which produces a cubic relationship of $\log(P_{\text{rot}}/\tau_c)$ as a function of $\log(R'_{\text{HK}})$ with the least scatter. The value $\alpha = 1.9$ produced the tightest fit, which agreed with independent estimates of $\alpha \approx 2$ obtained from calibrating stellar evolution codes to the physical parameters of the present-day Sun. The observational data were then used to invert the relationship giving a semi-empirical cubic form of $\tau_c(B - V)$. The tight mass-independent relationship found between Ro and R'_{HK} was taken as evidence that the Rossby number is fundamental to the nature of the dynamo responsible for producing chromospheric activity. The relationship between the Noyes et al. (1984a) semi-empirical Ro and the calculated Rossby numbers of modern global MHD simulations has not been addressed.

Figure 1.9 demonstrates the relationships of chromospheric activity with rotation and Rossby number using the Noyes et al. (1984a) formulation. Rotation data are gathered from the literature and presented later in Table 5.3. Activity data are taken from Baliunas et al. (1995) and converted to $\log(R'_{\text{HK}})$ following Noyes et al. (1984a). In the P_{rot} relationship, notice the vertical gradients in $(B - V)$. For stars of equal

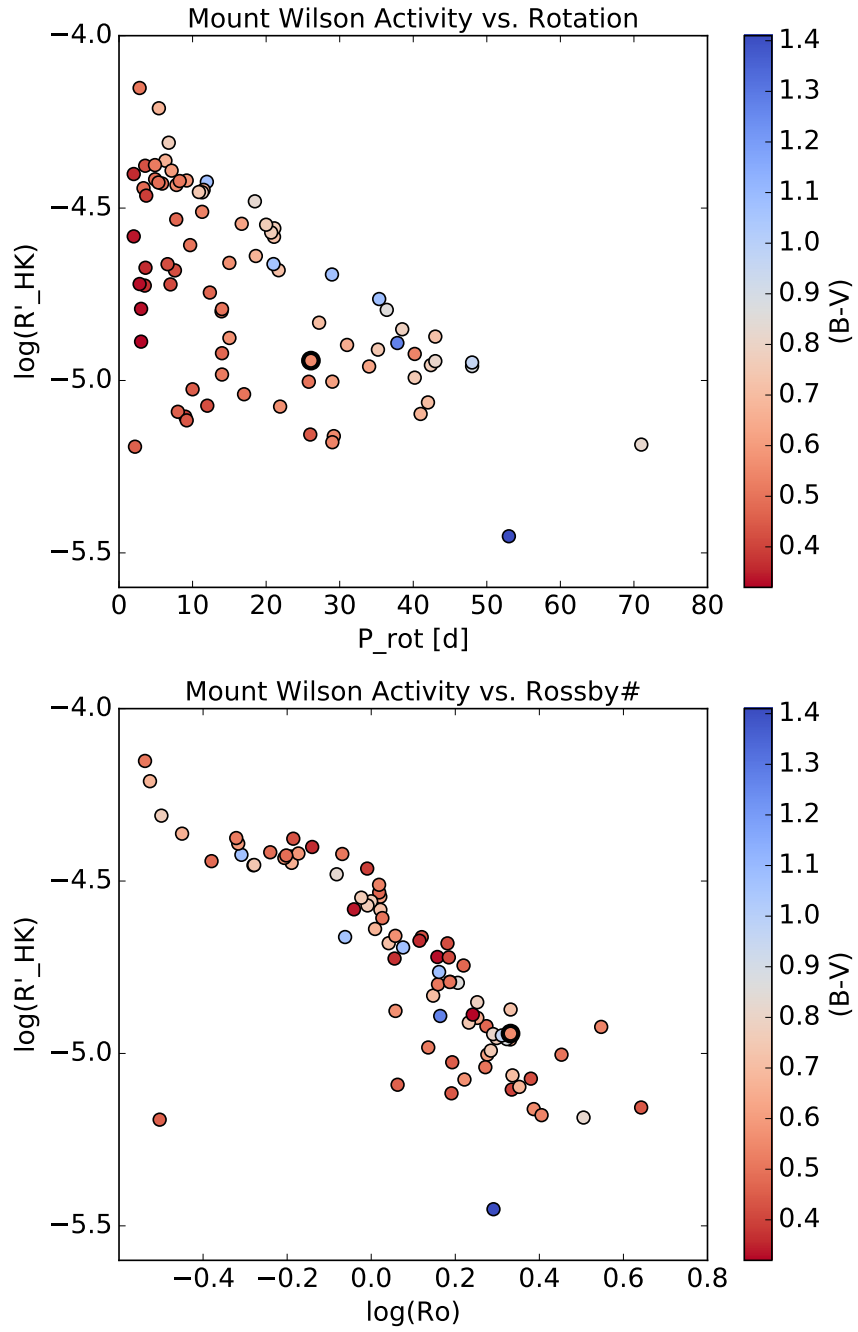


Figure 1.9 Activity vs P_rot (top) and Rossby number (bottom) for a sample of Mount Wilson stars with measured rotation periods (see Table 5.3). The Sun is indicated with a bold outline.

rotation, the lower mass stars tend to be more active. When activity is plotted against Ro , the relationship is tighter, but scatter increases below $\log(R'_{\text{HK}}) \approx -4.75$, the location of the Vaughan-Preston gap. Noyes et al. original dataset did not have this feature; we will discuss changes in long-term variability across this threshold in Chapter 5. One non-physical explanation for the large scatter is that it may be due to increased noise in both the chromospheric component and the R_{phot} term of equation (1.11) when the HK emission is low.

1.3.6 Patterns of Long-term Variability

A landmark in the Mount Wilson HK project came with Baliunas et al. (1995), a study of 25 years of S -index observations for 111 main sequence stars with spectral types ranging from F2 to M2. The work combined observations from Wilson’s original instrument (dubbed HKP-1) and the later HKP-2 instrument described in Vaughan et al. (1978). The long-term observation of several non-varying “standard” stars showed that the nightly measurement uncertainty was no larger than 1.2%. Using the Lomb-Scargle periodogram (Lomb 1976, Scargle 1982, Horne and Baliunas 1986) Baliunas et al. (1995) investigated the variability (or lack thereof) of their sample on timescales of 1–25 yr. They divided the stars into four variability classes: “Flat”, “Long”, “Var”, and cycling. “Flat” stars have a relative variability $\sigma_S / \langle S \rangle \leq 2\%$. “Long” stars have significant variability on timescales longer than 25 years. Cycling stars have statistically significant peaks in their periodogram, such that the false alarm probability (FAP) is less than 0.1%. The FAP is the probability that *any* peak in the periodogram, given the uneven sampling of that time series, is due to random Gaussian noise of the same variance as the data (Horne and Baliunas, 1986). Finally, erratically variable “Var” stars have $\sigma_S / \langle S \rangle > 2\%$, but no peaks with $\text{FAP} < 0.1\%$. The cycling stars were further classified into four quality groups, or “FAP Grades”:

“excellent”, “good”, “fair”, and “poor”, each with a progressively more stringent FAP threshold, but with some flexibility allowed based on the “visual appearance of the records.” The authors cautioned against taking FAP too literally, citing its basis on the assumption of pure sinusoidal signals with Gaussian noise, which is certainly not the case even for real stellar activity. The major results of this substantial work are enumerated below, mostly verbatim with clarification added in brackets:

1. “For stars of spectral type G0–K5 V... young stars exhibit high average levels of activity, rapid rotation rates, no Maunder minimum [“Flat”] phase, and rarely display a smooth, cyclic variation.”
2. “For stars of spectral type G0–K5 V... stars of intermediate age ($\sim 1\text{--}2$ Gyr for $1 M_{\odot}$) have moderate levels of activity and rotation rates, and occasional smooth cycles.”
3. “For stars of spectral type G0–K5 V... stars as old as the Sun and older have slower rotation rates, lower activity levels, and smooth cycles with occasional Maunder minimum phases.”
4. “K-type stars with low $\langle S \rangle$ values almost all have pronounced cycles.”
5. “F-type stars, especially those stars with low $\langle S \rangle$, generally have nearly constant records (flat) or slow, secular variations (long).”
6. “Among the G-type stars, very low amplitudes of chromospheric variation and levels of activity occur only in stars with low $\langle S \rangle$. Such low activity and flat variability may be similar to episodes of low magnetism such as the Maunder Minimum of the seventeenth century. The Sun and stars with *flat* records have slow rotation and are therefore old, suggesting that the Maunder minimum phase appears in old stars.”

7. “A few stars of all spectral types have two significant cycles. Those stars are located at intermediate values of $\langle S \rangle$, and are close to the Vaughan-Preston gap. The location of that group of double-period stars suggests that the gap may have physical significance. . . .”
8. “Stars with significant variability and no preferred timescale are generally young.”
9. “Roughly 52 [of 112] stars (including the Sun) show cycles, 31 are flat or have linear trends over the observing interval, and 29 show variability with no periodicity.”
10. “No period with a grade of “good” or “excellent” is shorter than 7 yr, although the measurements would reveal such short periods if they existed.”

The general conclusion to be drawn is that not all stars vary like the Sun, and the way that a star varies appears to change with stellar age as rotation slows down. Many of points above were not quantitatively demonstrated in Baliunas et al. (1995), though the data required to draw these conclusions was tabulated. In Chapter 5 we will quantitatively analyze the results of Baliunas et al. (1995).

1.3.7 “Maunder Minimum” Stars

It was assumed in Baliunas et al. (1995) that stars with flat records of activity over 25 years are in a “Maunder Minimum” state, analogous to the long period of infrequent sunspots during the seventeenth century (Eddy, 1976). It is unknown what the Sun’s Ca II H & K emission was like during that period, but so far as plage regions in the chromosphere remain associated with sunspots it is a reasonable assumption that a low number and small variability in sunspots would have a correspondingly low intensity and variability in Ca II H & K. Baliunas and Jastrow (1990) plotted the

distribution of individual S -index measurements for a sample of 74 solar-type MWO stars, finding a bimodal distribution with the lower activity peak occupied by the flat stars. The interpretation was that stars (and the Sun) in a temporary Maunder minimum state have lower activity than their minima when in a cycling state. This led to a number of low estimates for solar Ca II emission and total solar irradiance during the Sun’s Maunder minimum, with TSI 0.25% to 0.6% lower than modern cycle minimum (Baliunas and Soon, 1995; Lean et al., 1995; White et al., 1992; Zhang et al., 1994). Such a change would have potentially serious consequences for Earth’s climate. However, Hall and Lockwood (2004) did not observe a bimodal distribution in activity for their sample of 57 solar-type stars observed for 10 years, which included 10 flat-activity stars. Furthermore, Wright (2004) found that stars with low activity ($\log(R'_{\text{HK}}) < -5.1$) were almost always evolved off the main sequence and therefore not Sun-like, including most of the Maunder minimum stars identified in Baliunas and Jastrow (1990). Judge and Saar (2007) evaluated the MWO flat-activity stars, and found two candidates, τ Cet and ρ CrB, that are *not* evolved and therefore may be in a temporary state analogous to the Maunder minimum. Observations in the UV (*Hubble*) and X-rays (*ROSAT*) indicate that their chromospheric and coronal emission are comparable to the Sun during cycle minimum, which leads the authors to suggest that the solar Maunder minimum was magnetically not very much different than a prolonged cycle minimum.

1.3.8 Search for “Solar Twins”

Cayrel de Strobel (1996) reviews the search for “solar twins”, stars which are observationally very similar to the Sun-as-a-star. This of course requires that we understand the Sun in terms of common stellar properties, which is not trivial given that the Sun usually cannot be observed with the same calibrated instruments used

to define things like the Johnson color index ($B - V$). Gray (1992) demonstrated the difficulty by collecting literature estimates of $(B - V)_{\odot}$ ranging from 0.619 to 0.686, exceeding six times the typical error measurement for a common field star!

Cayrel de Strobel (1996) outlines a classification system, defining a “photometric solar analog” to be a star with $0.59 \leq (B - V) \leq 0.69$ and proceeds to use high-resolution spectroscopic analysis to compare 109 solar twin candidates to the Sun. The ultimate result was that “none came out to be a ‘real solar twin’, in the sense that always one or more of the physical parameters of a candidate-star differed significantly from those of the Sun... Real solar twins are ideal stars possessing fundamental physical parameters (mass, chemical composition, age, effective temperature, luminosity, gravity, velocity fields, magnetic fields, equatorial rotation, etc) very similar, if not identical to those of the Sun.”

“Identical” is taken here to mean “within observational uncertainties”, which may be too stringent and has the unfortunate property of making the solar twin class infinitesimal as observational precision improves over time. Nonetheless, it is clear that stars very similar to the Sun are surprisingly hard to come by. But there are a lot of stars. Porto de Mello and da Silva (1997) soon after presented 18 Sco as the closest ever solar twin, with atmospheric parameters, mass, chromospheric activity, and UBV colors indistinguishable from the Sun. Meléndez et al. (2014) found 18 Sco to be slightly hotter, more massive, and younger than the Sun with higher precision measurements, illustrating the conundrum of the solar twin definition. No longer the closest known solar twin (e.g. Meléndez et al., 2012), 18 Sco is nevertheless indisputably similar to the Sun in many respects. 18 Sco remains distinguished due to its inclusion in the MWO and Lowell Observatory Solar-Stellar Spectrograph synoptic programs, the latter which revealed an ~ 7 year Sun-like activity cycle (Hall et al., 2007a). Ongoing analyses are revealing more and more stars very similar to the Sun

(e.g. Mahdi et al., 2016; Porto de Mello et al., 2014), which will be interesting targets for future synoptic observations.

1.3.9 Variations in Visible Bands

Lowell Observatory began synoptic observations of Sun-like stars in the Strömgren b and y bands in 1984, and published results from their first decade of observations in Lockwood et al. (1997). The goal was to understand the response of the photosphere to stellar magnetism following the recent detection of variations in total solar irradiance (TSI) by as much as 0.2% from the *Solar Maximum Mission* (SMM) spacecraft (Willson et al., 1981), as large solar active regions transit the disk. Extended SMM measurements showed a $\sim 0.1\%$ variation of TSI in phase with the solar cycle (Willson and Hudson, 1991). To achieve this level of precision from ground-based observations hampered by atmospheric fluctuations, Lockwood et al. (1997) used *differential photometry*: the basic measurement consisted of the *difference* in a target star’s brightness with respect to a group of non-varying comparison stars. This approach negates the effects of night-to-night variations in seeing conditions, as well as changes in the instrument. The initial difficulty was that it was not known which stars were non-varying; only continued observations could reveal this.

Nonetheless, with some luck many of the target groups contained sufficiently stable comparison stars, and sensible results were obtained. Lockwood et al. (1997) measured short-term (inter-year) and long-term (year-to-year) rms amplitude ranging from 0.002 mag (0.2%) to 0.07 mag (7%) for about 41 program stars. Overlap with MWO targets allowed the comparison of photometric variability ($(b + y)/2$) to chromospheric activity R'_{HK} , and it was generally found that more active stars have larger variability. Radick et al. (1998) found the relationship between $(b + y)/2$ rms variability on short and long time scales with average activity $\langle R'_{\text{HK}} \rangle$ to be fairly well

described by a power law. They furthermore analyzed the correlations of yearly means in photometry and activity and found that stars can either be *faculae-dominated*, with positive correlations like for the Sun, or *spot-dominated*, with a negative correlation. Interestingly, the behavior transition appears near $\log(R'_{\text{HK}}) = -4.75$, the location of the Vaughan-Preston gap.

Observations of this kind continued with enhanced precision (down to 0.0001 mag seasonal means) using the Fairborne Observatory Automated Photometric Telescopes (APT) (Henry, 1999; Henry et al., 1995). Observations from the Lowell program were combined with the APT records in Lockwood et al. (2007), producing photometric time series up to 20 years long. This confirmed and improved the previous results of Radick et al. (1998) using the full range of decadal-scale variability for many stars. The Sun appears on the low-activity, low-photometric-variability side of the distribution, with lower photometric variability than would be expected considering other similar stars. Of note, Shapiro et al. (2014) extrapolated spot and faculae characteristics found on the Sun to extra-solar activity levels, and found that irradiance models designed to reproduce the solar TSI record given distributions of surface features could also reproduce the transition point from spot-dominated to faculae-dominated $(b + y)/2$. The remarkable long-term precision achieved with the technique of differential photometry encouraged a proposal to use this method to monitor the solar variability on century-long timescales by placing a well-characterized and stable reflecting body in space (Judge and Egeland, 2015).

1.3.10 Cycle Period Patterns and Dynamo Theory

With most dynamo models tailored to reproduce the solar cycle, it is perhaps not surprising that of all the phenomena observed in studies of stellar activity, it is stellar cycles that attract the most attention. Efforts to find empirical relationships between

stellar parameters and cycle periods began with Noyes et al. (1984b) who found $P_{\text{cyc}} \propto P_{\text{rot}}^n$ with $n \approx 1.25$ when stars are grouped by similar $(B - V)$. The sample at the time only contained 13 stars all rotating slower than 20 days. No correlation between P_{cyc} and P_{rot} was found using the entire sample, however a power law between cycle frequency P_{cyc}^{-1} and inverse Rossby number (Ro^{-1}) was found, independent of spectral type.

These early results were found to be in agreement with the mean field $\alpha\Omega$ dynamo models of Robinson and Durney (1982), which found that the cycle period is inversely proportional to the *dynamo number* N_D , expressed as the product of the dimensionless parameters of equation (1.5):

$$N_D = C_\alpha \cdot C_\omega = \frac{\alpha_0(\Delta\Omega)_0 R_\star^3}{\eta_0} \quad (1.12)$$

Approximating α_0 as proportional to the Coriolis force on convection, $\alpha \propto \Omega L$, with L the length scale of the convective velocity, magnetic diffusivity with $\eta_0 \approx L^2/\tau_c$ and the rotational shear $(\Delta\Omega)_0 \propto \Omega \equiv 2\pi/P_{\text{rot}}$ (as observed by Donahue et al. 1996), then $N_D \propto (\tau_c/P_{\text{rot}})^2 (R_\star/L)^4$. Therefore the $\alpha\Omega$ models predict $P_{\text{cyc}}^{-1} \propto N_D \propto Ro^{-2} \propto P_{\text{rot}}^{-2}$. Further assumptions of strong quenching due to magnetic buoyancy give $P_{\text{cyc}} \propto N_D^{-1/2}$, therefore $P_{\text{cyc}} \propto P_{\text{rot}}$, broadly compatible with the observations of Noyes et al. (1984b).

Saar and Baliunas (1992) studied cycle period for a larger sample of stars and found *no obvious trend between P_{cyc} and any single stellar parameter*, where $(B - V)$, convection zone depth, activity, rotation, and Rossby number were tried. A more complex parameterization inspired by $\alpha\Omega$ dynamo theory and dependent on model values for convection zone depth, radius, and magnetic diffusivity revealed two branches of cycle behavior with young, active, fast rotating stars on the “active”

branch and old, inactive, slow rotating stars on the “inactive” branch. A similar two-branch division was found in plots of $(P_{\text{cyc}}/P_{\text{rot}})^2$ versus age (Soon et al., 1993a). Using a more selective sample of only high-quality cycles, Brandenburg et al. (1998) and Saar and Brandenburg (1999) found the two branches in simpler dimensionless ($P_{\text{rot}}/P_{\text{cyc}}$ vs. Ro^{-1}), mixed ($P_{\text{rot}}/P_{\text{cyc}}$ vs. P_{rot}^{-1}), and dimensional ($1/P_{\text{cyc}}$ vs. $1/P_{\text{rot}}$) plots, finding a clear separation of the branches in the dimensionless plots, but a single relationship in the mixed and dimensioned plots. Furthermore, they noted that (1) all stars on the “active” branch were on the active side of the Vaughan-Preston gap ($\log(R'_{\text{HK}})$), and similarly for the “inactive” branch, (2) the “secondary cycles” of the active branch stars lie on the extrapolation of the inactive branch. Böhm-Vitense (2007) used a high-quality selection from the Saar and Brandenburg (1999) sample and plotted it in terms the simplest observables, (P_{cyc} vs P_{rot}), finding the two branches with the Sun inexplicably alone between them (see Figure 1.10). Böhm-Vitense speculated that the two branches may be due to different “modes” of dynamo operation (see also Durney et al., 1981), with both modes operating concurrently in the dual-cycle stars, and the Sun currently transitioning from one mode to another.

Following Noyes et al. (1984b), there have been several more applications of $\alpha\Omega$ dynamo theory to explain observations in one way or another parameterized by P_{cyc} and P_{rot} (e.g. Baliunas et al., 1996b; Brandenburg et al., 1998; Charbonneau and Saar, 2001; Dikpati et al., 2001; Saar and Brandenburg, 1999), (see Rempel, 2008, for a recent review). Three issues are at the core of these exercises:

1. Which cycles or other patterns of variability should the dynamo models explain?
2. What observational parameterization involving P_{cyc} and P_{rot} , and other stellar observables produce significant correlations for dynamo theory to explain?

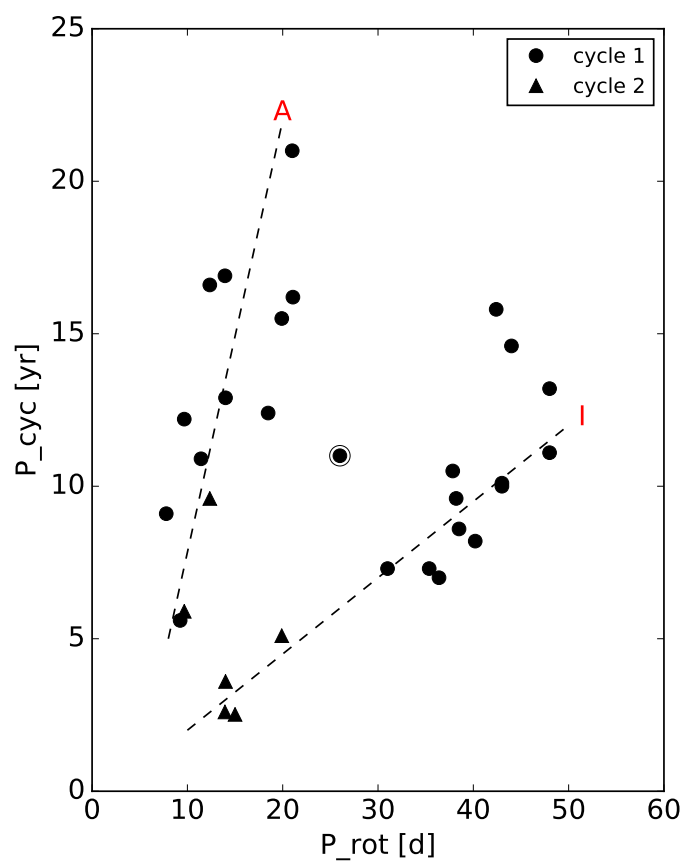


Figure 1.10 P_{cyc} versus P_{rot} for the Böhm-Vitense (2007) sample of high-quality cycles. The active branch is indicated by “A” and the inactive branch by “I”. The “cycle 2” points are another significant period of a star which already has a “cycle 1” point in the diagram. The Sun appears between the branches.

3. What are the valid physical assumptions to parameterize α_0 , η_0 , and $(\Delta\Omega)_0$ in terms of stellar observables (P_{rot} , $R_\star$, etc.)?

The first two points are observational considerations of cycle quality selection, primary vs. secondary or lower-order “cycles”, and spectral type, and the burden of proving that trends exist to be explained. The last point is at the crux of the matter of successfully explaining stellar observations with a model, in this case a particular class of model, but in general all models will have some unconstrained parameters. It is one thing to find parameterizations that work; but the underlying goal is to explain *why* the parameterization works. At this point, each of the above issues is in some dispute; several approaches have been tried, but the data have thus far been unsuccessful in rejecting specific dynamo models, nor have specific dynamo models been particularly bolstered by their success in explaining stellar cycle observations.

It deserves mention that Jouve et al. (2010) outlined a conundrum that is a fairly strong refutation of mean-field flux transport dynamos in light of the observed positive correlation between rotation period and cycle period. We will discuss this in Chapter 6, in light of our new observations.

1.3.11 Second-Generation Stellar Activity Surveys

Following Baliunas et al. (1995), no further comprehensive study of the Mount Wilson HK Project data has been done, though an additional 13 years of observations were obtained before the HK project ended in 2003. This includes the entirety of a new sample of 120 stars “close in mass and overall surface activity level to the Sun” for which observations began in 1991 (Baliunas et al., 1998). A few subsequent studies have employed new time-frequency techniques on the complete (~ 35 year) records for a subset of MWO stars with high-quality cycles (Baliunas et al., 2006; Frick et al.,

2004; Katsova et al., 2010; Oláh et al., 2009, 2016). Some of these results were described above.

The *de facto* successor to the HK Project is the Solar Stellar Spectrograph (SSS) designed and built at the High Altitude Observatory in Boulder and installed and operated at the 1.1 m telescope of Lowell Observatory in Flagstaff, Arizona (Hall et al., 2007b; SSS Webpage). The SSS is a medium resolution ($\lambda/\Delta\lambda \approx 10,000$ at $H\alpha$) dual-mode spectrograph which observes the far violet Ca II H & K region with a single-order Littrow configuration and passes the visible light to an Echelle grating, generating 19 usable orders covering 70% of $\lambda 5100$ to $\lambda 9000$. The SSS is also a dual-objective instrument; in addition to the 1.1 m telescope, the SSS can also observe the unresolved full-disk Sun through an optical fiber. The ability to observe the Sun and the stars with the same instrument is a unique capability of the SSS. The SSS regularly observes about 100 Sun-like stars, intentionally biased towards G-type solar analogs near the solar mass. Observations began in 1994 and continue to the present day, currently spanning 24 years. A sizable fraction of the SSS sample overlaps with MWO, and these stars are the focus of this work. Hall et al. (2007b) summarizes the first decade of Ca II H & K observations the SSS, just prior to the upgrade of its CCD. They found that 14 of their 99 stars exhibit flat-activity behavior, and identified one star (HD 140538) transitioning from a flat-activity state to a cycling state.

Other programs monitoring stellar activity are either short lived or serendipitous (activity is not the objective), and therefore limited in their usefulness for studying long-term variability. The SMARTS Southern HK Project obtained five years (Aug 2007 to Jan 2013) of low resolution ($R \approx 2,500$) spectra for 57 stars in the southern hemisphere (Metcalf et al., 2009, 2013). The high-cadence queue-scheduled observations are less obstructed by seasonal gaps than MWO or SSS, which made it possible to detect cyclic variations as short as 1.6 yr (Metcalf et al., 2010). The

TIGRE program (Schmitt et al., 2014) began observing Ca II H & K in 2014 and reported results from a rotation search of 95 stars in Hempelmann et al. (2016). In the serendipitous category, there is the California Planet Search, a radial velocity planet search whose medium resolution ($R \approx 50,000$) spectra cover the HK region. Wright (2004) and Isaacson and Fischer (2010) published chromospheric activity measurements for over 2,600 stars, but the observation cadence and time series duration for this program is highly variable. HARPS is another radial velocity planet search that covers Ca II H & K. Lovis et al. (2011) presents a comprehensive cycle search of 304 HARPS targets using 7 years of observations, reporting 99 stellar cycles. Many of these HARPS time series are extremely sparse, and the 1% False Alarm Probability threshold was less stringent than Baliunas et al. (1995), calling into question the reliability of many of the claimed cycles.² With careful calibration, observations from these short-lived or serendipitous programs can be combined to produce longer time series or improve sampling, as we demonstrate in Chapter 3.

1.4 Organization of this Work

The purpose of this dissertation is to quantitatively compare a well-characterized sample of solar-analog stars with long-term magnetic variability records to the Sun. In order to compare the variability of the stars to the Sun, we must have a record of solar variability on the same scale. This is done in Chapter 2, where we develop a composite time series of Ca II observations calibrated to the Mount Wilson Observatory *S*-index. In Chapter 3 we perform a detailed comparative analysis with a single star, HD 30495, a fast-rotating star with cyclic variability on similar time scales as the Sun. Next, in

²Lovis et al. (2011) never completed the peer-review process. The preprint is nonetheless highly cited, perhaps due to the rarity of comprehensive long-term variability studies. Some of the reported cycles are certainly valid; the work should be revisited, especially now that the HARPS observations have spanned 15 years.

Chapter 4 we analyze an ensemble of 26 solar analogs, examining the patterns of long term variability. In Chapter 5 we explore the statistics of the Baliunas et al. (1995) study to set a broader context for the Sun. We conclude in Chapter 6 with a discussion of our findings and the potential for further application of stellar observations to the dynamo problem.

1.4.1 Related Publications and Contributions

Portions of the work from this dissertation have been previously published in peer-reviewed journals and presented at conferences. These contributions are listed below in reverse chronological order.

R. Egeland, W. Soon, S. Baliunas, J. C. Hall, and G. W. Henry. Evolution of Long Term Variability in Solar Analogs. In D. Nandi, A. Valio, and P. Petit, editors, *Living Around Active Stars*, Proceedings of the 328th Symposium of the International Astronomical Union Held in Maresias, Brazil. Cambridge University Press, 2017, in press

R. Egeland, W. Soon, S. Baliunas, J. C. Hall, A. A. Pevtsov, and L. Bertello. The Mount Wilson Observatory S-index of the Sun. *ApJ*, 835(1), January 2017. doi:10.3847/1538-4357/835/1/25

R. Egeland, W. Soon, S. Baliunas, J. C. Hall, A. A. Pevtsov, and G. W. Henry. Dynamo Sensitivity in Solar Analogs with 50 Years of Ca II H & K Activity. In G. A. Feiden, editor, *Proceedings of the 19th Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun*. Zenodo, September 2016. doi:10.5281/zenodo.154118

T. S. Metcalfe, R. Egeland, and J. van Saders. Stellar Evidence That the Solar Dynamo May Be in Transition. *ApJ*, 826:L2, July 2016. doi:10.3847/2041-8205/826/1/L2

D. Salabert, R. A. García, P. G. Beck, R. Egeland, P. L. Pallé, S. Mathur, T. S. Metcalfe,

J.-D. do Nascimento, Jr., T. Ceillier, M. F. Andersen, and A. Triviño Hage. Photospheric and chromospheric magnetic activity of seismic solar analogs. Observational inputs on the solar-stellar connection from Kepler and Hermes. *A&A*, 596:A31, November 2016. doi:10.1051/0004-6361/201628583

R. Egeland, T. S. Metcalfe, J. C. Hall, and G. W. Henry. Sun-like Magnetic Cycles in the Rapidly-rotating Young Solar Analog HD 30495. *ApJ*, 812:12, October 2015. doi:10.1088/0004-637X/812/1/12

P. G. Judge and R. Egeland. Century-long monitoring of solar irradiance and Earth’s albedo using a stable scattering target in space. *MNRAS*, 448:L90–L93, March 2015. doi:10.1093/mnrasl/slv004

In particular, Chapter 2 is contained in Egeland et al. (2017), and Chapter 3 in Egeland et al. (2015). At the time of writing, chapters 4 and 5 are being prepared for submission to the *Astrophysical Journal*.

CHAPTER TWO

THE SUN AS A STAR: THE SOLAR S-INDEX TIME SERIES

2.1 The S-index Scale

The usefulness of the solar-stellar comparison is only as good as the knowledge of the Sun's properties in terms of measurable quantities commonly used in stellar studies. The stellar magnetic activity proxy established by the MWO HK project is the S -index:

$$S = \alpha \frac{N_H + N_K}{N_R + N_V}, \quad (2.1)$$

where N_H and N_K are the counts in 1.09 Å triangular bands centered on Ca II H & K in the HKP-2 spectrophotometer, and N_R and N_V are 20 Å reference bandpasses in the nearby continuum region, and α is a calibration constant (Vaughan et al., 1978).

The HKP-2 instrument is distinct from the coudé scanner used by Olin Wilson at the 100-inch telescope at MWO, later designated HKP-1 in Vaughan et al. (1978). HKP-1 was a two-channel photometer, with one 1 Å channel centered on either the H- or K-line and the other channel measuring two 25 Å bands separated by about 250 Å from the HK region (Wilson, 1968). HKP-1 measurements were therefore:

$$\begin{aligned} F_H &= \frac{N_H}{N_R + N_V} \\ F_K &= \frac{N_K}{N_R + N_V} \\ F &= \frac{1}{2}(F_H + F_K), \end{aligned} \quad (2.2)$$

where we use $\mathcal{R}$ and $\mathcal{V}$ to distinguish the difference between the reference channels in the two instruments, with HKP-1 $\mathcal{R}$ and $\mathcal{V}$ being 5 Å wider than HKP-2 R and V . The α parameter of equation (2.1) was determined nightly with the standard lamp and standard stars such that *on average* $S = F$ (Duncan et al., 1991; Vaughan et al., 1978). However, differences are expected given that the two instruments are not identical, and Vaughan et al. (1978) derived the following relation with coincident measurements on 13 nights in 1977:

$$F = 0.033 + 0.9978 S - 0.2019 S^2. \quad (2.3)$$

It is important to stress that S is an *instrumental* flux scale of the HKP-2 spectrophotometer that cannot be independently measured without cross-calibration using overlapping Mount Wilson targets. The consequence of this is that there are only two methods of placing the solar activity cycle on the S -index scale: directly measuring solar light with the HKP-2 instrument, or calibrating another measurement to the S -index scale using some proxy. Previously, only the latter method has been possible. In this work we analyze hitherto unpublished observations of the Moon with the HKP-2 instrument and determine the placement of the Sun on the S -index scale. We review past calibrations of solar S in Section 2.2. In Section 2.3 we describe the observations used in the determination of solar S , and the analysis procedure in Section 2.4. In Section 2.5 we empirically explore the assertion that S is linear with Ca K-line emission. We conclude in Section 2.6 with a discussion on the implications of our results, and future directions for establishing the solar-stellar connection.

2.2 Previous Solar S Proxies

Sun-as-a-star Ca II H & K measurements have been made at Kitt Peak National Observatory (NSO/KP) on four consecutive days each month from 1974–2013 (White and Livingston, 1978), and at Sacramento Peak (NSO/SP) daily, albeit with gaps, from 1976–2016. (Keil and Worden, 1984; Keil et al., 1998). Results from these observations for three solar cycles are given in Livingston et al. (2007). From the NSO/KP and NSO/SP spectrographs, the K emission index (hereafter K) is computed as the integrated flux in a 1 Å band centered on the Ca II K-line normalized by a band in the line wing. The principal difference of K with respect to S is (1) K includes flux only in the K line, while S measures flux in both H and K, (2) the reference bandpass is in the line wing for K , as opposed to two 20 Å bands in the nearby pseudo-continuum region for S . These differences could be cause for concern relating S to K , however (1) H and K are a doublet of singly ionized calcium and thus are formed by the same population of excited ions, therefore the ratio K/H cannot vary except by changes in the optical depth of the emitting plasma (Linsky and Avrett, 1970), which is unlikely to vary by a large amount (2) the far wing of the K line does vary somewhat over the solar cycle, however only to a level of 1% (White and Livingston, 1981). Therefore we can expect *a priori* that there should be a simple linear relationship between the K and S indices. We investigate this assumption in detail in Section 2.5.

Several authors have developed transformations from the K to MWO S -index, and the results are summarized in Table 2.1. The earliest attempt was from Duncan et al. (1991), who used spectrograms of 16 MWO stars taken between 1964 and 1966 at the coudé focus of the Lick 120 inch telescope to estimate a stellar K , and thereby establish a relationship with an average of S for those stars from MWO. The Sun

Table 2.1. $S(K)$ Transformations

Reference	$S(K)$	$S_{23,\min}$	$S_{23,\max}$	ΔS_{23}	$\langle S_{23} \rangle$
Duncan et al. (1991)	$(1.58 \pm 0.33) K + (0.040 \pm 0.002)^a$	0.179	0.194	0.0151	0.187
White et al. (1992), original	$1.69 K + 0.016^b$	0.165	0.181	0.0162	0.173
White et al. (1992), mean	$(1.64 \pm 0.07) K + (0.028 \pm 0.007)^b$	0.172	0.188	0.0156	0.180
Baliunas et al. (1995)	$2.63 K - 0.066^c$	0.166	0.191	0.0251	0.178
Radick et al. (1998)	$(1.475 \pm 0.070) K + (0.041 \pm 0.013)$	0.171	0.185	0.0141	0.178
Hall and Lockwood (2004)	$1.359 K + 0.0423^d$	0.162	0.175	0.0130	0.168
This Work	$(1.50 \pm 0.13) K + (0.031 \pm 0.013)$	0.163	0.178	0.0143	0.170

Note. — ^a: Duncan’s r_{HK} replaced by $r_K \equiv K$ using $r_{\text{HK}}/r_K = 0.089/0.087$ from their paper. Uncertainties are from a formal linear regression done in White et al. (1992), who noted the slope uncertainty is unrealistically large. ^b: Original calibration used the NSO/KP measurements; this version is transformed to use the NSO/SP measurements using Equation 2.4 ^c: Calculated using the published intervals and $\langle S \rangle$ in Donahue and Keil (1995), along with the NSO/SP K data. ^d: Calculated from the published yearly mean values of S at cycle 23 minimum and maximum.

was also a data point used in determining the relationship, with its K determined by NSO/KP and S from Wilson (1978)’s observations of the Moon at cycle 20 maximum and cycle 20-21 minimum. Radick et al. (1998) revisited the Duncan et al. (1991) calibration using longer time averages of S for the stars and K for the Sun with updated observations, and adjusting the zero-point of the regression to force the Sun’s residual to zero. These approaches neglect the potential difference in scaling among the NSO/SP or NSO/KP K -indices and the K -index derived from the Lick Spectrograph, but such a cross correlation is not feasible in any case, due to the lack of common targets from the different spectrographs. Furthermore, this method only uses a single measurement of the K -index for the stellar sample, while the S -index is a decades-long average from Mount Wilson data. This poor sampling of K will result in large scatter due to rotational and cycle-scale activity for the active stars in the sample, increasing the uncertainty in the determination of the $S(K)$ scaling relation.

White et al. (1992) took another approach, leveraging Olin Wilson’s observations of the Moon with the original HKP-1 instrument during cycle 20 (Wilson, 1978, Table 3). However, because the HKP-1 Moon observations did not overlap with the NSO K -index programs, those time series had to be projected back in time using an intermediary solar activity proxy, the 10.7 cm radio flux measurements (hereafter abbreviated $F_{10.7}$). White et al. (1992) was discouraged that the result from this method was discrepant with the Duncan et al. (1991) calibration, and citing the validity of both approaches, they chose to average the two results.

The White et al. (1992) $S(K)$ relationship was based on the NSO/KP data, which is on a slightly different flux scale than the NSO/SP data we use in this work. White et al. (1998) determined a linear relationship between the two instruments to be $K_{\text{KP}} = 1.1 K_{\text{SP}} - 0.01$. Using a cycle shape model fit (see Section 2.4.2), we determined the cycle minima preceding cycles 22, 23, and 24, and the maxima of cycles 21, 22, and 23 in both data sets. Then, using a ordinary least squares regression on these data, we obtained:

$$K_{\text{KP}} = 1.143 K_{\text{SP}} - 0.0148, \quad (2.4)$$

which is in agreement with the White et al. (1998) relationship to the precision provided. We substitute (2.4) in the White et al. (1992) original and mean transformations and the results are shown in Table 2.1. Note that in the mean with Duncan et al. (1991) transformation we do *not* use equation (2.4) in the latter, as it was determined from stellar observations using the Lick spectrograph, and therefore not specific to NSO/KP data.

Baliunas et al. (1995) observed that the White et al. (1992) result failed to cover the cycle 20-21 minima values of Wilson (1978). Furthermore, they noted that their

calibration resulted in maxima for cycles 21 and 22 that were approximately equal to the amplitude of cycle 20 measured by Wilson with HKP-1, while other activity proxies (the sunspot record and $F_{10.7}$) show cycle 20 to be significantly weaker than cycles 21 and 22. With these problems in mind, Baliunas et al. (1995) derived a new transformation $S(K)$ which smoothed the cycle 20 to 21 minima transition from the Wilson measurements to the $S(K)$ proxy and preserved the relative amplitudes found in $F_{10.7}$ and sunspot records. This transformation was not published, however it was used again in Donahue and Keil (1995) who published mean S values from this transformation for several intervals. Using these intervals and mean values along with the NSO/SP record we computed the Baliunas et al. (1995) $S(K)$ relationship, which is shown in Table 2.1.

The Solar-Stellar Spectrograph (SSS) at Lowell Observatory synoptically observes the Ca H & K lines for ~ 100 FGK stars, as well as the Sun (Hall and Lockwood, 1995; Hall et al., 2007b). The spectra are placed on an absolute flux scale, and the MWO S -index is determined using an empirically calibrated relationship (Hall and Lockwood, 1995). The calibration is shown to be consistent with actual MWO observations to a level of 7% rms for low-activity stars (see Figure 4 Hall et al., 2007b). In Hall and Lockwood (2004), mean values of S for cycle 23 minimum and maximum were published, which we used with the NSO/SP data to derive an $S(K)$ transformation, shown in Table 2.1. Hall and Lockwood (2004) remarked that their cycle 23 S amplitude was “noticeably less” than the Baliunas et al. (1995) amplitude for the stronger cycle 22 (as evidenced from other proxies, such as the sunspot record). Subsequent re-reductions of the SSS solar time series in Hall et al. (2007b) and Hall et al. (2009) reported mean values of 0.170 and 0.171 for cycle 23 observations, $\sim 5\%$ lower than the mean value of 0.179 reported in Baliunas et al. (1995) for cycles 20–22.

We used the cycle shape model fit (see Section 2.4.2) to determine the values of the cycle 22-23 minima and cycle 23 maxima in the NSO/SP K timeseries. We also computed the mean value $\langle K \rangle$ of cycle 23 from this data set. The conversion of these K values to S using the previously published relationships is shown in Table 2.1. Each relationship arrives at different conclusions about the placement of solar minimum, maximum, and mean value for this cycle. The relative range (max - min)/mean of minima positions is $\approx 10\%$, amplitudes $\approx 81\%$, and cycle means $\approx 11\%$. These discrepancies are significantly larger than the uncertainty of the determination of these values from K , which we estimate has an daily measurement uncertainty of $\sim 1\%$ (see Section 2.4) and a cycle amplitude of 6% above minimum. The largest discrepancy, as Hall and Lockwood (2004) noted, is in the cycle amplitude, with the Baliunas et al. (1995) $S(K)$ proxy estimate being more than double their measurement. Considering the variety and magnitude of these discrepancies, we conclude that the solar S -index has so far not been well understood.

2.3 Observations

2.3.1 Mount Wilson Observatory HKP-1 and HKP-2

The Mount Wilson HK Program observed the Moon with both the HKP-1 and HKP-2 instruments. After removing 11 obvious outliers there are 162 HKP-1 observations taken from 2 Sep 1966 to 4 Jun 1977 with the Mount Wilson 100-inch reflector, covering the maximum of cycle 20 and the cycle 20-21 minimum. Wilson (1968) and Duncan et al. (1991) published mean values from these data, with the latter shifted upward by about 0.003 in S . Our HKP-1 data is under the same calibration as in Duncan et al. (1991) and Baliunas et al. (1995).

As mentioned in Baliunas et al. (1995), observations of the Moon resumed in 1993 with the HKP-2 instrument. After removing 10 obvious outliers there are 75 HKP-2

observations taken from 27 Mar 1994 to 23 Nov 2002 with the Mount Wilson 60-inch reflector, covering the end of cycle 22 and the cycle 23 minimum, extending just past the cycle 23 maximum. The end of observations coincides with the unfortunate termination of the HK Project in 2003. These observations were calibrated in the same way as the stellar HKP-2 observations as described in Baliunas et al. (1995), using the standard lamp and measurements from the standard stars. Long-term precision of the HKP-2 instrument was shown to be 1.2% using 25 years of observations in a sample of 13 stable standard stars.

The 75 HKP-2 lunar observations are the only observations of solar light with the HKP-2 instrument, and thus are the only means of directly placing the Sun on the instrumental S -index scale of equation (2.1). We assume that these observations measure S for the Sun to within the 1.2% precision determined for the HKP-2 instrument. Most HKP-1 and HKP-2 observations were taken within 5 days of full Moon, and all observations were taken within an hour of local midnight. We do not expect a significant alteration of the nearby spectral bands constituting S by reflection from the Moon, which is to first order a gray diffuse reflector.

2.3.2 NSO Sacramento Peak K-line Program

We seek to extend our time series of solar variability beyond cycle 23 by establishing a proxy to the NSO Sacramento Peak (NSO/SP) observations¹ taken from 1976–2016, covering cycles 21 to 24. The NSO/SP K-line apparatus is described in Keil and Worden (1984). Briefly, it consists of a $R \sim 150,000$ Littrow spectrograph installed at the John W. Evans Solar Facility (ESF) at Sacramento Peak Observatory, fed by a cylindrical objective lens which blurs the Sun into a $50 \mu\text{m}$ by 10 mm line image at the spectrograph slit. The spectral intensity scale is set by a integrating a

¹<ftp://ftp.nso.edu/idl/cak.parameters>

0.53 Å band centered at 3934.869 Å in the K-line wing and setting it to the fixed value of 0.162. The K emission index is then defined as the integrated flux of a 1 Å band centered at the K-line core (3935.662 Å) (White and Livingston, 1978). We estimated the measurement uncertainty to be 1.0% by calculating the standard deviation during the period 2008.30–2009.95, which is exceptionally flat in the sunspot record and 10.7 cm radio flux time series.

2.3.3 Kodaikanal Observatory Ca K Spectroheliograms

We extend the S -index record back to cycle 20 using the composite K time series of Bertello et al. (2016), which is available online (Pevtsov, 2016). This composite calibrates the NSO/SP data to K-line observations by the successor program at NSO, the Synoptic Optical Long-term Investigations of the Sun (SOLIS) Integrated Sunlight Spectrometer (ISS) (Bertello et al., 2011). The calibration used in Bertello et al. (2016) is

$$K_{\text{ISS}} = 0.8781 K_{\text{SP}} + 0.0062. \quad (2.5)$$

Bertello et al. (2016) calibrated the synoptic Ca II K plage index from spectroheliograms from the Kodaikanal (KKL) Observatory in India to the ISS flux scale using the overlapping portion of NSO/SP data, resulting in a time series of Ca II K emission from 1907 to the present. We transform this composite timeseries from the ISS flux scale to the NSO/SP flux scale by applying the inverse of equation (2.5):

$$K_{\text{KKL}(\text{SP})} = 1.1388 K_{\text{KKL}(\text{ISS})} - 0.0071. \quad (2.6)$$

We prefer this homogeneous chromospheric K-line proxy (hereafter denoted simply K_{KKL}) over proxies based on photospheric phenomena such as $F_{10.7}$ or the sunspot number. In particular, Pevtsov et al. (2014) found that the correlation between K and $F_{10.7}$ is non-linear and varies with the phase of the solar cycle, with strong correlation during the rising and declining phases, and poor correlation at maximum and minimum, precisely the sections of the solar cycle of most interest in this work.

2.3.4 Lowell Observatory Solar Stellar Spectrograph

We compare our results to a new reduction of observations from the Lowell Observatory Solar-Stellar Spectrograph (SSS), which is running a long-term stellar activity survey complementary to the MWO HK Project. The SSS observes solar and stellar light with the same spectrograph, with the solar telescope consisting of an exposed optical fiber that observes the Sun as an unresolved source (Hall and Lockwood, 1995; Hall et al., 2007b). The basic measurement of SSS is the integrated flux in 1 Å bandpasses centered on the Ca II H & K cores from continuum-normalized spectra, ϕ_{HK} , which can then be transformed to the S -index using a combination of empirical relationships derived from stellar observations:

$$S_{\text{SSS}} = \frac{10^{14} \mathcal{F}_{\text{c}, \lambda 3950}}{K_{\mathcal{F}} C_{\text{cf}} T_{\text{eff}}^4} \phi_{\text{HK}}, \quad (2.7)$$

where $\mathcal{F}_{\text{c}, \lambda 3950}$ is the continuum flux scale for the Ca II H & K wavelength region, which converts ϕ_{HK} to physical flux ($\text{erg cm}^{-2} \text{s}^{-1}$). $\mathcal{F}_{\text{c}, \lambda 3950}$ is a function of Strömgren ($b-y$) and is taken from Hall (1996). $K_{\mathcal{F}}$ (simply K in other works) is the conversion factor between the MWO HKP-2 H & K flux (numerator of equation 2.1) to physical flux (Rutten, 1984). C_{cf} a factor that removes the color term from S , and is a function of Johnson ($B - V$) (Rutten, 1984). Finally, T_{eff} is the effective temperature. See Hall

et al. (2007b) and Hall and Lockwood (1995) for details on the extensive work leading to this formulation. What is important to realize about this method of obtaining S is that it requires three measurements of solar properties, $(b-y)_{\odot}$, $(B-V)_{\odot}$, and $T_{\text{eff},\odot}$, along with the determination of one constant, $K_{\mathcal{F}}$. The solar properties are taken from best estimates in the literature, which vary widely depending on the source used, and can dramatically affect the resulting S_{SSS} for the Sun. Hall et al. (2007b) used $(b-y)_{\odot} = 0.409$, $(B-V)_{\odot} = 0.642$, and $T_{\text{eff},\odot} = 5780$ K. The constant $K_{\mathcal{F}}$ was empirically determined to be 0.97 ± 0.11 erg cm⁻² s⁻¹ in Hall et al. (2007b) as the value which provides the best agreement between S_{SSS} and S_{MWO} from Baliunas et al. (1995) for an ensemble of stars and the Sun. This combination of parameters resulted in a mean S_{SSS} of 0.170 for the Sun using observations covering cycle 23. A slightly different calibration of SSS data in Hall and Lockwood (2004) used a flux scale $\mathcal{F}_{c,\lambda 3950}$ based on Johnson $(B-V)$, set to 0.65 for the Sun, and $T_{\text{eff},\odot} = 5780$ K. In Table 2.1 we estimated that this calibration resulted in a mean $S = 0.168$ for cycle 23. Hall et al. (2009), which included a revised reduction procedure and one year of data with the upgraded camera (see below), found $\langle S \rangle = 0.171$.

As mentioned previously, the three solar properties $(b-y)_{\odot}$, $(B-V)_{\odot}$, and $T_{\text{eff},\odot}$ used in the SSS flux-to- S conversion are not accurately known. The fundamental problem is that instruments designed to observe stars typically cannot observe the Sun. Cayrel de Strobel (1996) studied this problem, and collected $(B-V)_{\odot}$ from the literature ranging from 0.62 to 0.69. Meléndez et al. (2010) compiled literature values $(b-y)_{\odot}$ ranging from 0.394 to 0.425. $T_{\text{eff},\odot}$ is more accurately known, which is fortunate given that it appears in equation (2.7) to the fourth power. However, a 0.01 change in $(B-V)_{\odot}$ or $(b-y)_{\odot}$ results in an approximately 2% or 10% change in S_{SSS} , respectively. The sensitivity of S_{SSS} to these properties makes it especially important to use the best known values.

More recent photometric surveys of solar analogs have resulted in improved determinations of the solar properties by way of color-temperature relations. We are therefore motivated to update S_{SSS} for the Sun using these measurements: $(B-V)_{\odot} = 0.653 \pm 0.003$ (Ramírez et al., 2012), $(b-y)_{\odot} = 0.4105 \pm 0.0015$ (Meléndez et al., 2010), and $T_{\text{eff},\odot} = 5772.0 \pm 0.8$ (IAU General Assembly 2015 Resolution B3). The latter lower value for the effective temperature follows from the recent lower estimate of total solar irradiance in Kopp and Lean (2011). The constant $K_{\mathcal{F}}$ is kept at 0.97 as determined in Hall et al. (2007b). The SSS data analyzed here now include data taken after upgrading the instrument CCD to an Andor iDus in early 2008 (Hall et al., 2009). This new CCD has higher sensitivity in the blue and reduced read noise. The reduction procedure remains the same as described in Hall et al. (2007b) and on the SSS web site², albeit with updated software and two additional steps: (1) high S/N spectra from the new camera are used as reference spectra for the old camera data, which improves stability of the older data and avoids discontinuity at the camera upgrade boundary (2) an additional scaling correction is applied to the continuum-normalized spectra so that the line wings are shifted to match the normalized intensity of the Kurucz et al. (1984) solar spectrum. Despite these efforts, a non-negligible discontinuity was apparent across the CCD upgrade boundary. We suspect this may be due to minute differences in CCD pixel size, and the slightly different wavelength sampling at the continuum normalization reference points. The discontinuity is corrected *post facto* by multiplying the CCD-1 S data by 0.9710, determined by the ratio of the medians for the last year of CCD-1 data to the first year of CCD-2 data. Finally, due to tape degradation, raw CCD data from 1998-2000 were lost, preventing reduction using the updated routines. Continuum normalized spectra from that period still exist, though introducing them into the updated pipeline

²<http://www2.lowell.edu/users/jch/sss/tech.php>

results in significant discontinuities in S which had to be corrected. The correction consisted of applying an additional scaling factor to the lost data region such that the region median falls on a linear interpolation of the cycle across the region. Keeping or removing the “corrected” data in this region does not affect our conclusions. Finally, we estimated the measurement uncertainty to be 1.6% for CCD-1, and 1.3% for CCD-2, by computing the standard deviation of observations from each device in the long minimum from 2007–2010.

2.4 Analysis

Our goal in this work is to use the 75 HKP-2 observations to determine the S -index for the Sun. Specifically, we seek to measure the minimum, maximum, and mean value of S over several solar cycles. We proceed first by directly measuring these quantities using our cycle 23 HKP-2 data, and then establish a proxy with the NSO/SP measurements to extend our measurements to other cycles.

2.4.1 Cycle 23 Direct Measurements

The HKP-2 data can be used to measure the minimum, maximum and mean of cycle 23 directly, but first the time of minimum and maximum must be established by some other means. We choose to use the NSO/SP K record for this purpose. Applying a 1-yr boxcar median filter to the NSO/SP time series, we find the absolute minimum between cycles 22-23 at decimal year 1996.646, and absolute maximum at 2001.708. In this interval defining cycle 23, there are 56 HKP-2 measurements, with the remaining 19 points belonging to cycle 22. Next, using the HKP-2 time series, we take the median of a 2-yr wide window centered at the minima and maxima times to find $S_{23,\min} = 0.1643$ ($N = 17$) and $S_{23,\max} = 0.1755$ ($N = 12$), for a cycle 23 amplitude $\Delta S_{23} = 0.0112$. The mean value for the cycle $\langle S_{23} \rangle = 0.172$ ($N = 56$). We

choose the median over the mean when measuring fractions of a cycle because we do not expect Gaussian distributions in that case. We choose a 2-yr wide window to be about as wide as we can reasonably go without picking up another phase of the solar cycle. Still, the number of points in each window is low, which does not give much confidence that the cycle can be precisely measured in this way. We shall investigate the uncertainty of this method in the next section. For the cycle mean, besides the problem of low sampling, we would expect this to be an overestimate since no data exists for the longer declining phase of the cycle.

Nonetheless, even with these simple estimates we find discrepancies with the previous work shown in Table 2.1. The measured minimum is appreciably lower than the Duncan et al. (1991), White et al. (1992) mean, and Radick et al. (1998) values, and the amplitude is lower than all other estimates. Our cycle mean is also lower than all but the Hall and Lockwood (2004) estimate, indicating that the previous work has overestimated the S -index of the Sun. In the next section we will attempt to improve our precision using a method in which more of the data are used.

2.4.2 Cycle Shape Model Fit for Cycle 23

Due to the limited data we have from HKP-2 for cycle 23, the results of the previous section are susceptible to appreciable uncertainties from unsampled short-timescale variability due to rotation and active region growth and decay. By fitting a cycle model to our data, we can reduce the uncertainty in our minimum and maximum point determinations. We use the skewed Gaussian cycle shape model of Du (2011) for this purpose:

$$f(t) = A \exp \left(-\frac{(t - t_m)^2}{2B^2 [1 + \alpha(t - t_m)]^2} \right) + f_{\min} , \quad (2.8)$$

where t is the time, A is the cycle amplitude, t_m is approximately the time of maximum, B is roughly the width of the cycle rising phase, and α is an asymmetry parameter. $f_{\min}$, which was not present in the Du model, is an offset which sets the value of cycle minimum. We also tried the quasi-Planck function of Hathaway et al. (1994) for this purpose and obtained similar results, however we prefer the above function due to the simplicity of interpreting its parameters and developing heuristics to guide the fit. We fit the model to the data using the Python `scipy` library `curve_fit` routine with bounds. This function uses a Trusted Region Reflective (TRR) method with the Levenberg-Marquardt (LM) algorithm applied to trusted-region subproblems (Branch et al., 1999; Moré, 1978). The fitting algorithm searches for the optimum parameters from a bounded space within $\pm 50\%$ of heuristic values obtained using a 1-yr median filter of the data, with the exception of $f_{\min}$ which has a lower bound set at the lowest data point in order to prevent the fitting procedure from underestimating the minimum.

The 56 HKP-2 data points contained in cycle 23 only cover the rising phase and are not enough to constrain a least-squares fitting procedure. To circumvent this problem, we first fit the NSO/SP data and assume that the parameters which determine the cycle shape t_m , B , and α are the same in the two chromospheric time series. The only way this assumption could fail is if the Ca II H band, or the 20 Å continuum bands of S (equation 2.1) varied in such a way as to distort the cycle shape with respect to K . Soon et al. (1993b) explored the long-term variability of the C_{RV} index based on the 20 Å reference bands, finding it to be generally quite small. We therefore assume for the moment that the H, R, and V bands are linear with K or constant, and later confirm these assumptions in Section 2.5. Fitting equation (2.8) to the NSO/SP K for cycle 23, we find $t_m = 2001.122$, $B = 2.154$, and $\alpha = 0.0343$. We

then hold these parameters fixed and fit equation (2.8) to the 56 HKP-2 observations for cycle 23, finding the remaining free parameters $A = 0.0150$, $f_{\min} = 0.163$.

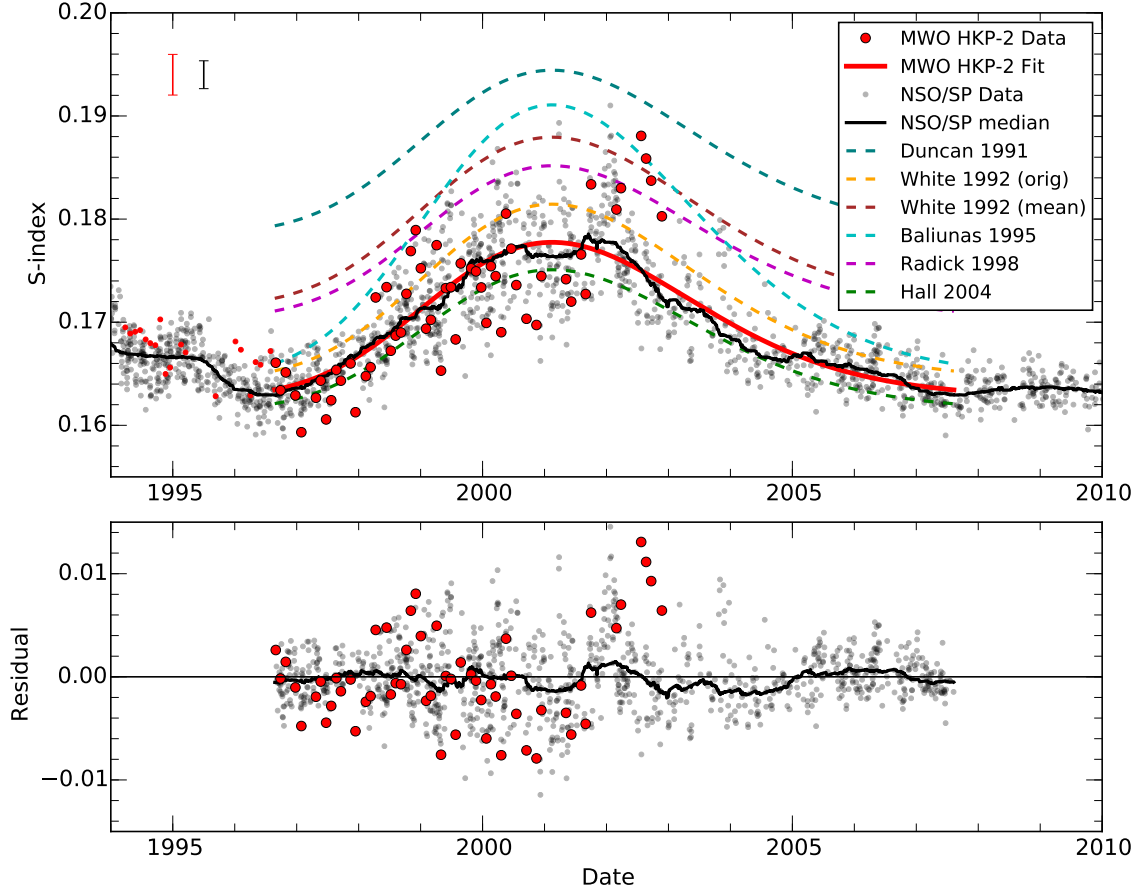


Figure 2.1 The MWO HKP-2 Moon observations are shown in red, with the larger points used in the cycle shape model fit, which is shown as a thick red curve. NSO/SP K -index data are transformed to the S scale using equation (2.12). A 1-yr wide median filter applied to the NSO/SP data is shown in the solid black line. $S(K)$ transformations of found in the literature (see Table 2.1) of a cycle shape model fit to the NSO/SP K data are shown as colored curves for comparison. The bottom panel shows the residual difference of the cycle shape model curve and the data. Error bars in the top left show the estimated measurement uncertainty for MWO HKP-2 (red) and NSO/SP (black).

The cycle model fit and the HKP-2 data are shown as the red curve in Figure 2.1. The reduced χ^2 of the fit is 6.45, which we find acceptable given the model does

not seek to explain all the variation in the data (e.g. rotation, active region growth and decay), only the mean cycle. We find an RMS residual of the fit of $\sigma_{\text{res}} = 0.0047$, which is a bit more than double the estimated individual measurement uncertainty $\sigma = 0.0020$.

Using the cycle model fit, we find the minimum $S_{23,\text{min}} = 0.1634 \pm 0.0008$, maximum $S_{23,\text{max}} = 0.1777 \pm 0.0010$, and amplitude $\Delta S_{23} = 0.0143 \pm 0.0012$. The mean of the cycle model curve is $\langle S_{23} \rangle = 0.1701 \pm 0.0005$. The uncertainties are determined from a Monte Carlo experiment in which we build a distribution of cycle model fits from only 56 points of NSO/SP K data during the rising phase of cycle 23, and comparing that to the “true” cycle model fit using all 1087 observations throughout the cycle (see Appendix A for details). The Monte Carlo experiment shows that the cycle model fit method with only 56 randomly selected points finds true minima and maxima with a 1σ standard deviation of $\approx 0.5\%$, while the method of direct means finds minima equally well, but maxima with about double the uncertainty due to the increased variability at that phase of the cycle.

Our results for cycle 23 are shown in the final row of Table 2.1, and are discrepant with most of the previous literature values. Hall and Lockwood (2004) comes closest to our result, though their amplitude is 9% lower. The Radick et al. (1998) amplitude is only 0.002 S -units (1.4%) lower than ours, but the mean is 4.7% higher due to the higher value for solar minimum. The Baliunas et al. (1995) relation finds a minimum only 1.8% higher than ours, but the amplitude is substantially larger (75%), leading to a 4.7% higher estimate of the solar cycle mean.

The cycle shape model fit to the NSO/SP K data is transformed using the literature relations in Table 2.1 and are shown as colored curves in Figure 2.1. Here we see that no transformation matches the MWO HKP-2 observations in a satisfactory way, though the Hall and Lockwood (2004) curve comes close. In the next section

we will construct a new $S(K)$ transformation that exactly matches our cycle shape model curve in Figure 2.1.

2.4.3 $S(K)$ Proxy Using NSO/SP K Emission Index

We now seek a transformation between the NSO/SP K emission index and the Mount Wilson S -index. We assume a linear relationship:

$$S(K) = a + b K . \quad (2.9)$$

Now we write K as a function of time using the cycle shape model (equation 2.8), obtaining

$$S(t) = (a + b f_{\min,K}) + b A_K E(t; t_m, B, \alpha) , \quad (2.10)$$

where $E(t; t_m, B, \alpha)$ is the exponential function from (2.8). This is precisely the same form as (2.8), which is made more clear by defining $f_{\min,S} = (a + b f_{\min,K})$ and $A_S = b A_K$. These definitions may be used as the solutions for $\{a, b\}$ when cycle shape model fits have been done for both $K(t)$ and $S(t)$, as was done in the previous section, giving

$$\begin{aligned} a &= f_{\min,S} - (A_S/A_K) f_{\min,K} \\ b &= A_S/A_K . \end{aligned} \quad (2.11)$$

Note that the cycle shape parameters $\{t_m, B, \alpha\}$ are not explicitly related to $\{a, b\}$. Using the cycle 23 fit parameters described in the previous section, we arrive at the following linear transformation:

$$S(K) = (1.50 \pm 0.13)K + (0.031 \pm 0.013). \quad (2.12)$$

The uncertainty in the slope and intercept are calculated using standard error propagation methods on equation (2.11):

$$\begin{aligned} \sigma_b^2 &= b^2 ((\sigma_{A_S}/A_S)^2 + (\sigma_{A_K}/A_K)^2) \\ \sigma_{f_K \cdot b}^2 &= (f_K \cdot b)^2 \cdot [(\sigma_b/b)^2 \\ &\quad + (\sigma_{f_K}/f_K)^2 + 2(\sigma_b/b)(\sigma_{f_K}/f_K)\rho(f_K, b)] \\ \sigma_a^2 &= \sigma_{f_S}^2 + \sigma_{f_K \cdot b}^2 - 2\sigma_{f_S}\sigma_{f_K \cdot b}\rho(f_S, f_K \cdot b), \end{aligned} \quad (2.13)$$

where we have simplified the notation with $f_K \equiv f_{\min, K}$ and $f_S \equiv f_{\min, S}$. Correlation coefficients $\rho(x, y)$ for quantities x and y are defined as $\rho(x, y) = \text{cov}(x, y)/(\sigma_x \cdot \sigma_y)$. We approximate $\rho(f_K, b) \sim \rho(f_K, A_K)$ and $\rho(f_S, f_K \cdot b) \sim \rho(f_S, A_S)$, which likely overestimates the correlation between these quantities. The uncertainties of each of the curve fit parameters are obtained from Monte Carlo experiments described in Appendix A. Correlation between A_S and A_K through the cycle shape parameters $\{t_m, B, \alpha\}$ is included in our estimate of σ_{A_S} due to the setup of the Monte Carlo experiment (see Appendix A). The error budget is dominated by $(\sigma_{A_S}/A_S)^2$, $(\sigma_b/b)^2$, and $\sigma_{f_S}^2$, with all other terms accounting for less than 5% of the total in their respective equations.

The estimated uncertainty of the cycle shape model cross-calibration method described above is significantly less than was achieved by linear regression of coincident measurements. The latter method had formal uncertainties in the scale factor in excess of 100%. Linear regression failed because we have few coincident

measurements and the individual measurement uncertainty of $\approx 1\%$ in both K and S is roughly 10% the amplitude of the variability over the cycle.

We transformed the NSO/SP K -index time series using equation (2.12) and plotted it with the MWO HKP-2 data and cycle shape model fits in Figure 2.1. A 1-yr boxcar median filter of the data is also plotted as a black line, which is in good agreement with the cycle shape model curve.

2.4.4 Comparison with SSS Solar Data

As an additional check of our transformation of the NSO/SP data to the S -index scale, we compare our result to the independently calibrated Solar-Stellar Spectrograph (SSS) observations described in Section 2.3.4.

In Figure 2.2 we overplot the Lowell Observatory SSS solar S -index data on our NSO/SP $S(K)$ proxy for cycles 22, 23 and 24. A 1-yr median filter line is plotted for both time series, as well as the cycle shape model curves fit to the NSO/SP $S(K)$ data using the TRR+LM algorithm described in Section 2.4.2. In general, we find excellent agreement between all three curves. We use the cycle shape model curves as a reference to compare the NSO/SP $S(K)$ proxy with SSS. For cycle 23, which is covered by the SSS CCD-1 data, the mean of the residual is -0.00031 S -units with a standard deviation of 0.0040 S . This may be compared to the NSO/SP data for the same period used to constrain the model fit, with an essentially zero mean and a standard deviation of 0.0032 S . Similarly for cycle 24, SSS CCD-2 data have a residual mean of -0.000097 S and standard deviation of 0.0026, compared to the NSO/SP residual mean of -0.000047 S and standard deviation 0.0028 S . In the case of cycle 24, SSS observations have a lower residual with the cycle model than the NSO/SP data used to define it! The difference of the SSS and NSO/SP running medians is shown as an orange line in Figure 2.2, and rarely exceeds ± 0.002 S .

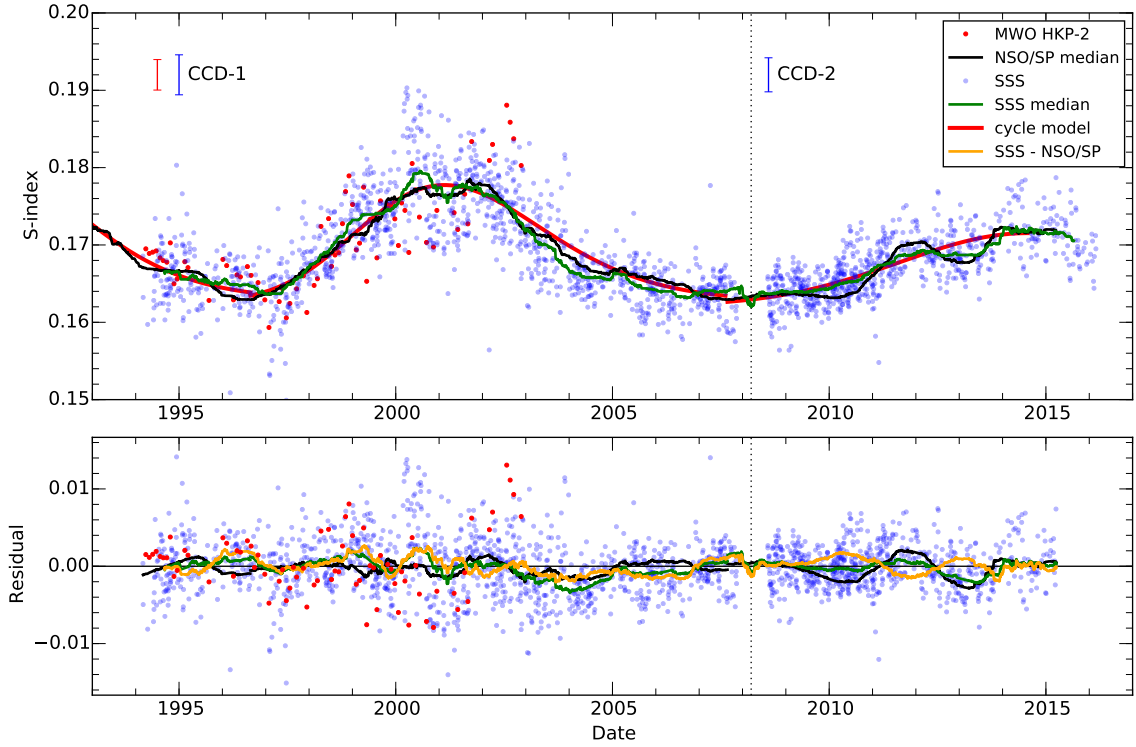


Figure 2.2 SSS solar observations (blue) compared to the MWO HKP-2 measurements (red) and the NSO/SP $S(K)$ proxy, shown here as a running 1-yr median (black). Cycle shape model curves fit to the NSO/SP data are shown in red. The vertical dotted line denotes the upgrade of the SSS CCD. The bottom panel shows the residual difference of the data with the cycle shape model, while the orange line is the difference between the SSS (green) and NSO/SP (black) running medians. Error bars in the top panel show the estimated measurement uncertainty for SSS CCD-1 and CCD-2 observations (blue), and MWO HKP-2 (red).

We now consider whether the remarkable agreement is confirmation of the true solar S -index, or mere coincidence. As discussed in Section 2.3.4, the computation of S from SSS spectra is sensitive to the measurement of solar color indices and the effective temperature. We have recalibrated the data with the best available measurements of these quantities. The resulting calibration results in a 3% scaling difference between CCD-1 and CCD-2 data, which we choose to remove by rescaling CCD-1 data to the CCD-2 scale, which resulted in the excellent agreement with

NSO/SP $S(K)$. There is good reason for this choice, since CCD-2 is a higher quality detector. However, *if there were* a significant offset between the NSO/SP $S(K)$ and SSS, we would be justified in applying a small scaling factor to reconcile differences, citing uncertainties in the solar properties or the conversion factor $K_{\mathcal{F}}$. The fact that this was not necessary might be considered coincidence. However, the fact that *only* a scaling factor would be required, and not an absolute offset, can *not* be coincidence. From our determination of the $S(K)$ scaling relation (2.12) using equation (2.11), we see that using a different amplitude A_S would change the scale of the conversion, b , *and* the offset, a , which could not be confirmed by these SSS data using a scaling factor alone. Therefore, the agreement between SSS and the NSO/SP $S(K)$ proxy can be taken as confirmation of the latter. The agreement between SSS and the MWO HKP-2 data points (red points in Figure 2.2) is confirmation that S_{SSS} is properly calibrated for the Sun.

2.4.5 Calibrating HKP-1 Measurements

A simple inspection of the HKP-1 data alongside the HKP-2 data reveals that the calibration of HKP-1 data to the HKP-2 S -index scale could be improved. Notably, the HKP-1 data appear higher than the HKP-2 data. Here we perform a simple analysis analogous to that of Section 2.4.1 to illustrate the problem. Using the cycle boundaries and times of maxima of Hathaway et al. (1999) based on the sunspot record, we take the cycle 20 maximum to be 1969/03 and the cycle 20-21 minimum to be 1976/03. We compute the median of a 2-yr window of the MWO HKP-1 data at these points giving $S_{20,\text{max}} = 0.182$ (N=35), and $S_{21,\text{min}} = 0.168$ (N=54). Taking the difference we find $\Delta S_{20} = 0.014$.

The amplitude ΔS_{20} is slightly less than that of cycle 23, which agrees with the relative amplitudes of other activity proxies such as sunspot number and $F_{10.7}$

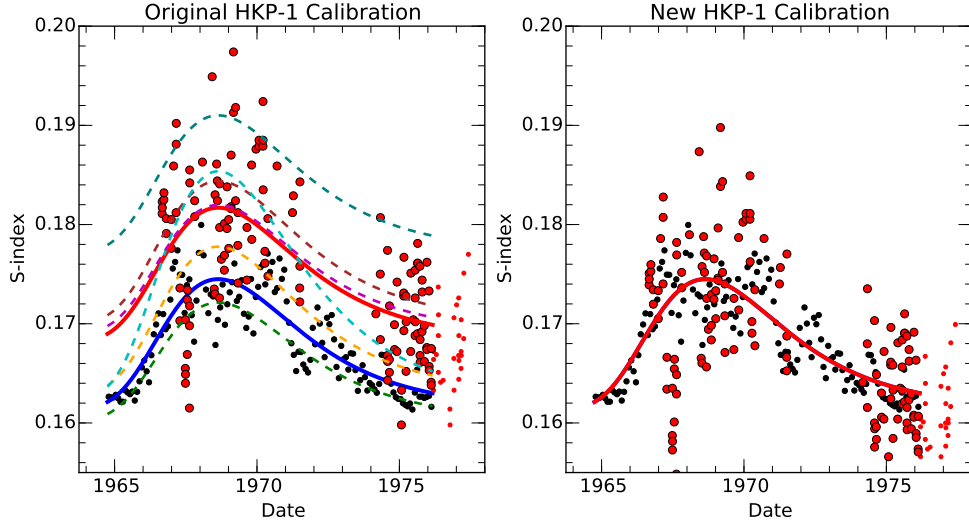


Figure 2.3 *Left*: The original HKP-1 data are plotted as red circles, with a cycle shape model fit to those data as a red line. Monthly averaged K_{KKL} data transformed to the S -index scale using equation (2.12) are shown as black points, and a cycle shape model fit to those data as a blue line. Transformations of the K_{KKL} curve to S using relationships found in the literature (see Table 2.1) are shown as colored curves for comparison, using the same color scheme as in Figure 2.1. *Right*: HKP-1 data calibrated to the HKP-2 scale using equation (2.14).

(Hathaway, 2015). However, the directly measured minimum is 0.005 S -units higher than the cycle 23 value computed in the same way (Section 2.4.1) or with the cycle shape model fit (Section 2.4.2). This discrepancy, while small in an absolute sense, is over 1/3 of the cycle amplitude.

The discrepancy in the minima is not unexpected when one considers the uncertainties involved in calibrating the HKP-1 and HKP-2 instruments. They were calibrated using near-coincident observations for a sample of stars resulting in equation (2.3), but with individual stellar (F , S) means scattered about the calibration curve. Figure 5 of Vaughan et al. (1978) shows the calibration data and regression curve for HKP-1 F and HKP-2 S . Near the solar mean S of ~ 0.170 , scatter about the calibration curve of $\sim 5\%$ is apparent. As a result, for any given star (or

the Moon), a shift of the mean of about $\sim 5\%$ may be required to achieve continuity in the time series.

We apply such a correction to the HKP-1 Moon data to place it on the same scale as HKP-2. We use the K_{KKL} data as a proxy to tie the two datasets together, following a similar procedure described in Section 2.4.2.

First, we define the boundaries of cycle 20 as 1964/10 to 1976/03 as in Hathaway et al. (1999). We then fit a cycle shape model curve to the K_{KKL} data in this interval, obtaining the parameters $\{A_{\text{KKL}}, t_m, B, \alpha, f_{\text{min,KKL}}\}$. Holding the shape parameters $\{t_m, B, \alpha\}$ fixed, we fit another curve to the HKP-1 data to find $\{A_1, f_{\text{min},1}\}$. Transforming the K_{KKL} curve amplitude and offset parameters $\{A_{\text{KKL}}, f_{\text{min,KKL}}\}$ to the HKP-2 scale with equation (2.12), we obtain the HKP-2 parameters $\{A_2, f_{\text{min},2}\}$. Finally, using the analog of equation (2.11) for the HKP-1 and HKP-2 amplitude and offset parameters, we then obtain the transformation:

$$S_{\text{HKP-2}} = 0.9738 S_{\text{HKP-1}} - 0.0025. \quad (2.14)$$

Figure 2.3 shows the results of this new calibration. The left panel shows the HKP-1 data with the original calibration. Comparing the red curve (cycle shape model fit to the HKP-1 data) and the blue curve (fit to the $S(K_{\text{KKL}})$ data with equation (2.12)), we see clearly the $\sim 0.007 S$ offset with the HKP-2 scale determined above. Literature relationships from Table 2.1 are also shown, as applied to a K_{KKL} cycle shape model curve. Agreement here is generally better than to the HKP-2 data, especially in the case of the Radick et al. (1998) calibration. Applying the HKP-1 to HKP-2 transformation of (2.14) to the MWO data, the blue and red curves coincide, as shown in the right panel.

The ~ 0.007 offset between the original HKP-1 calibration to S and our HKP-2 calibration demonstrates the principal reason for the discrepancy between our results and the generally higher values for $\langle S \rangle$ in previous works summarized in Table 2.1. Without the advantage of HKP-2 measurements of reflected sunlight from the Moon, previous authors seeking an $S(K)$ relationship using only cycle 20 HKP-1 data for the Sun (Baliunas et al., 1995; Duncan et al., 1991; Radick et al., 1998) were susceptible to this systematic offset of $\sim 0.007 S$. White et al. (1992), on the other hand, used the Wilson (1978) published data on the F scale. Coincidentally, those data had a lower value with $\langle F \rangle = 0.171$ which puts their original $S(K)$ (actually, an $F(K)$ relationship) estimate closer to ours (see Figure 2.1). However, the offset error was partially introduced when they chose to average with the Duncan et al. (1991) result. The remaining differences are due to the myriad of problems associated with coupling the NSO/KP or SP K measurements to the HKP-1 measurements, either using proxy time series (Baliunas et al., 1995; White et al., 1992) or stellar observations with the Lick spectrograph (Duncan et al., 1991; Radick et al., 1998), which we discussed in Section 2.2.

The scatter of the HKP-1 measurements is somewhat larger than those from HKP-2. The reader will also notice a cluster of unusually low measurements in 1967. We investigated these points in more detail, but could not find any anomaly with respect to Moon phase at time of measurement, or anything in the MWO database that would suggest a problem with the observations. With no strong basis for removal of these observations we keep them in our analysis.

2.4.6 Composite MWO and NSO/SP $S(K)$ Time Series

We have now calibrated both the NSO/SP data and the MWO HKP-1 data to the HKP-2 scale. The complete composite time series covering cycles 20–24 is

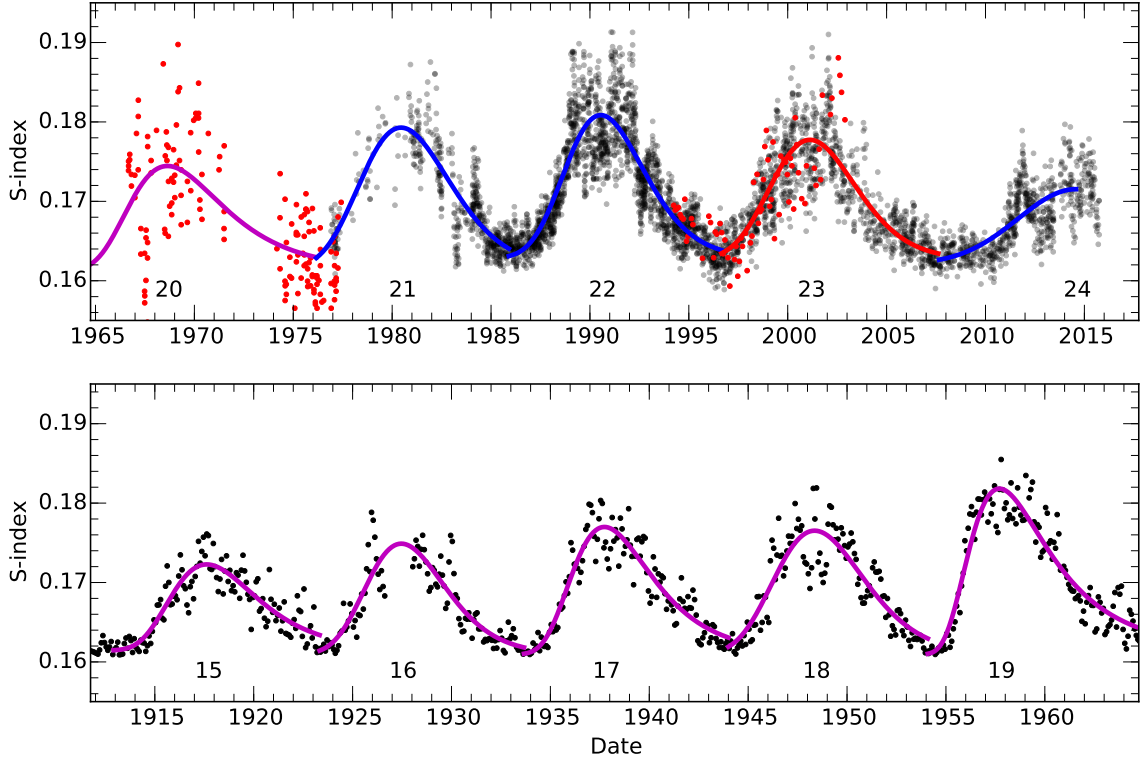


Figure 2.4 *Top*: composite time series of nightly MWO Moon measurements (red) with daily NSO/SP data (black) converted to the MWO HKP-2 scale. *Bottom*: Monthly averaged K_{KKL} data transformed to the MWO HKP-2 scale. Cycle shape model curves are shown in red when fit using MWO data, blue when fit using only NSO/SP data, and magenta when fit using K_{KKL} data. Cycle numbers are shown below each cycle.

shown in top panel of Figure 2.4. The KKL composite (Bertello et al., 2016) allows us to further extend S back to cycle 15, as shown in the bottom panel the figure. Note that the KKL data are monthly means, while the NSO/SP and MWO series are daily measurements. For each cycle we have determined the cycle duration using the absolute minimum points of a 1-yr median filter on the NSO/SP data (cycles 21–24) or using Hathaway et al. (1999) values (cycles 15–20). We have fit each cycle with a cycle shape model (equation 2.8), with the best fit parameters shown in the left portion of Table 2.2. Cycle 24 is a special case. Because we only have observations for

Table 2.2. Solar Cycle Shape Parameters and S -index Measurements

Cycle	t_{start}	t_m	B	$\alpha \times 10^5$	f_{min}	A	S_{min}	S_{max}	ΔS_{cyc}	$\langle S_{\text{cyc}} \rangle$
15	1912.917	1917.617	2.145	7657	0.1615	0.01083	0.1615	0.172	0.0108	0.1671
16	1923.333	1927.465	1.992	5080	0.1610	0.01391	0.1614	0.175	0.0135	0.1678
17	1933.667	1937.731	1.991	9018	0.1610	0.01604	0.1610	0.177	0.0160	0.1690
18	1944.000	1948.359	2.253	4314	0.1611	0.01552	0.1619	0.177	0.0146	0.1696
19	1954.083	1957.705	1.965	12379	0.1610	0.02086	0.1610	0.182	0.0208	0.1716
20	1964.750	1968.639	2.334	7743	0.1614	0.01314	0.1621	0.174	0.0124	0.1684
21	1976.167	1980.447	2.275	3929	0.1615	0.01778	0.1629	0.179	0.0164	0.1717
22	1985.900	1990.548	2.003	4394	0.1629	0.01801	0.1631	0.181	0.0178	0.1713
23	1996.646	2001.122	2.154	3426	0.1627	0.01504	0.1634	0.178	0.0143	0.1701
24	2007.654	2014.577	2.942	1329	0.1621	0.00945	0.1626	0.172	0.0089	0.1670
$\langle 15 - 24 \rangle$							0.1621	0.177	0.0145	0.1694
σ_{measure}							0.0008	0.001	0.0012	0.0005
σ_{scatter}							0.0008	0.003	0.003	0.002

half of the cycle, the optimizer has difficulty obtaining a reasonable fit. This problem was resolved by constraining α to be a function of t_m using the relationship found in Du (2011). From these curve fits, we determine the cycle minimum, maximum, max - min amplitude, and mean value. These results are summarized in the right portion of Table 2.2 and represent our best estimate of chromospheric variability through the MWO S -index over ten solar cycles.

The uncertainty in the cycle minima and maxima for cycle 23 were found to be $\approx 0.5\%$ (Section 2.4.2), which are summed in quadrature along with the covariance term give the uncertainty in the amplitude of $\approx 10\%$. These uncertainties are then propagated into equation (2.12) which determines the uncertainties of the other cycles. The uncertainty in the cycle means is about 0.3%, as determined by the Monte Carlo experiment (see Appendix A). These relative uncertainties are applied to the cycle 15–24 mean in Table 2.2 to compute σ_{measure} , the typical uncertainty cycle measurements

on the S -index scale. The standard deviation of the minima, maxima, amplitudes and means are given as σ_{scatter} .

2.4.7 Conversion to $\log(R'_{\text{HK}})$

The S -index reference bands R and V (equation 2.1) vary with stellar surface temperature (and metallicity; see (Soon et al., 1993b)), and furthermore temperature-dependent flux from the photosphere is present in the H and K bands. These effects lead to a temperature dependence or “color term” in S which limits its usefulness when comparing stars of varied spectral types. The activity index R'_{HK} (Noyes et al. (1984a)) seeks to remove the aforementioned color dependence of S and is widely used in the literature. We therefore compute $\log(R'_{\text{HK}})$ of the Sun for the purpose of inter-comparison with stellar magnetic activity variations of other Sun-like stars from the MWO HK project or elsewhere. In Table 2.3 we used the procedure of Noyes et al. (1984a) to calculate $\log(R'_{\text{HK}})$ from the S measurements of Table 2.2. In this calculation, we adopted the solar color index $(B - V)_{\odot} = 0.653 \pm 0.003$ (Ramírez et al., 2012). The typical uncertainty of the cycle measurements on the $\log(R'_{\text{HK}})$ scale, σ_{measure} , and standard deviation for the ten cycles, σ_{scatter} , are also presented in Table 2.3. The cycle amplitude expressed as $\log(\Delta R'_{\text{HK}})$, and the fractional amplitude $\Delta R'_{\text{HK}} / \langle R'_{\text{HK}} \rangle$ have been used in stellar amplitude studies in the literature (Baliunas et al., 1996a; Saar and Brandenburg, 2002; Soon et al., 1994).

2.5 Linearity of S with K

In the above analysis we assumed that the ratio of 1 Å emission in the cores of the H & K lines, H and K are linearly related such that S (equation 2.1) and K are also linear as in equation (2.12). It is also assumed that the pseudo-continuum bands V and R are constant. These assumptions were also implicit in the derivation

Table 2.3. Solar Cycle Measurements in $\log(R'_{\text{HK}})$

Cycle	$\log(R'_{\text{HK,min}})$	$\log(R'_{\text{HK,max}})$	$\log(\Delta R'_{\text{HK}})$	$\log(\langle R'_{\text{HK}} \rangle)$	$\Delta R'_{\text{HK}} / \langle R'_{\text{HK}} \rangle$
15	-4.9882	-4.927	-5.807	-4.9552	0.141
16	-4.9885	-4.913	-5.712	-4.9513	0.174
17	-4.9909	-4.903	-5.638	-4.9443	0.203
18	-4.9856	-4.905	-5.676	-4.9413	0.184
19	-4.9909	-4.879	-5.523	-4.9305	0.256
20	-4.9846	-4.916	-5.748	-4.9481	0.158
21	-4.9797	-4.891	-5.626	-4.9300	0.201
22	-4.9783	-4.884	-5.592	-4.9319	0.219
23	-4.9763	-4.899	-5.686	-4.9385	0.179
24	-4.9811	-4.931	-5.892	-4.9558	0.116
$\langle 15 - 24 \rangle$	-4.9844	-4.905	-5.690	-4.9427	0.183
σ_{measure}	0.0087	0.008	0.068	0.0072	0.032
σ_{scatter}	0.0049	0.016	0.101	0.0093	0.038

of $S(K)$ relationships in the literature shown in Table 2.1, but we are not aware of any observational evidence in support of them.

We use emission indices from the SOLIS/ISS instrument (K , H) and SSS (K , H , V , R) to examine the trends with respect to the K band. SOLIS/ISS emission indices are derived from spectra normalized to a single reference line profile obtained with the NSO Fourier Transform Spectrometer as described in Pevtsov et al. (2014). SSS indices are derived from continuum normalized and wavelength calibrated spectra with intensity points at 3909.3770 Å set to 0.83 and 4003.2688 Å at 0.96, and the rest of the spectrum normalized according to the line defined by those points.

Figure 2.5 (top) shows the relationship between the 1-Å emission index in the H and K line cores from the NSO SOLIS/ISS and Lowell SSS instruments. SOLIS/ISS data (black points) are from the beginning of observations until the middle of 2015, when the instrument moved from Kitt Peak to Tuscon and resulted in a discontinuous

shift in the K/H time series which is still under investigation. SSS data (green points) includes only the observations after the camera was upgraded in 2008, which is significantly less noisy than with the previous CCD. Differences in resolution and systematics such as stray light are likely responsible for the offsets in both K and H between the two instruments. However, in both instruments the $H(K)$ relationship is linear with a slope $b < 1$ and a zero point $a > 0$. The slopes found from each instrument, ≈ 0.8 , agree to within the uncertainties. Linearity of H with K is compatible with the assumption of linearity of S with K . The $H(K)$ linearity further implies that the emission ratio K/H has the form

$$\frac{K}{H} = \frac{K}{a + bK}. \quad (2.15)$$

With nonzero a the ratio must vary over the solar cycle. This ratio is a diagnostic of the optical depth of the surface integrated chromosphere (Linsky and Avrett, 1970). The data show that the ratio increases by $\sim 2\%$ over the rising phase of solar cycle 24.

Figure 2.5 (bottom) shows V (3891.0–3911.0 Å) and R (3991.0–4011.0) Å integrated emission indices versus K from continuum normalized SSS solar spectra. The data show these 20 Å pseudo-continuum bands to be nearly constant with activity, with the V and R bands having RMS variability of 0.6% and 0.4%, respectively. This is in agreement with the small variance in the C_{RV} index based on V and R found by Soon et al. (1993b). Linear regression of the $V(K)$ and $R(K)$ gives uncertainties in the slope (see Figure 2.5) that place them 3.2σ and 1.9σ from zero, respectively, giving them marginal statistical significance. The relative increase in flux indicated by these slopes for $V(K)$ and $R(K)$ over the full range of K are 0.3% and 0.1% respectively.

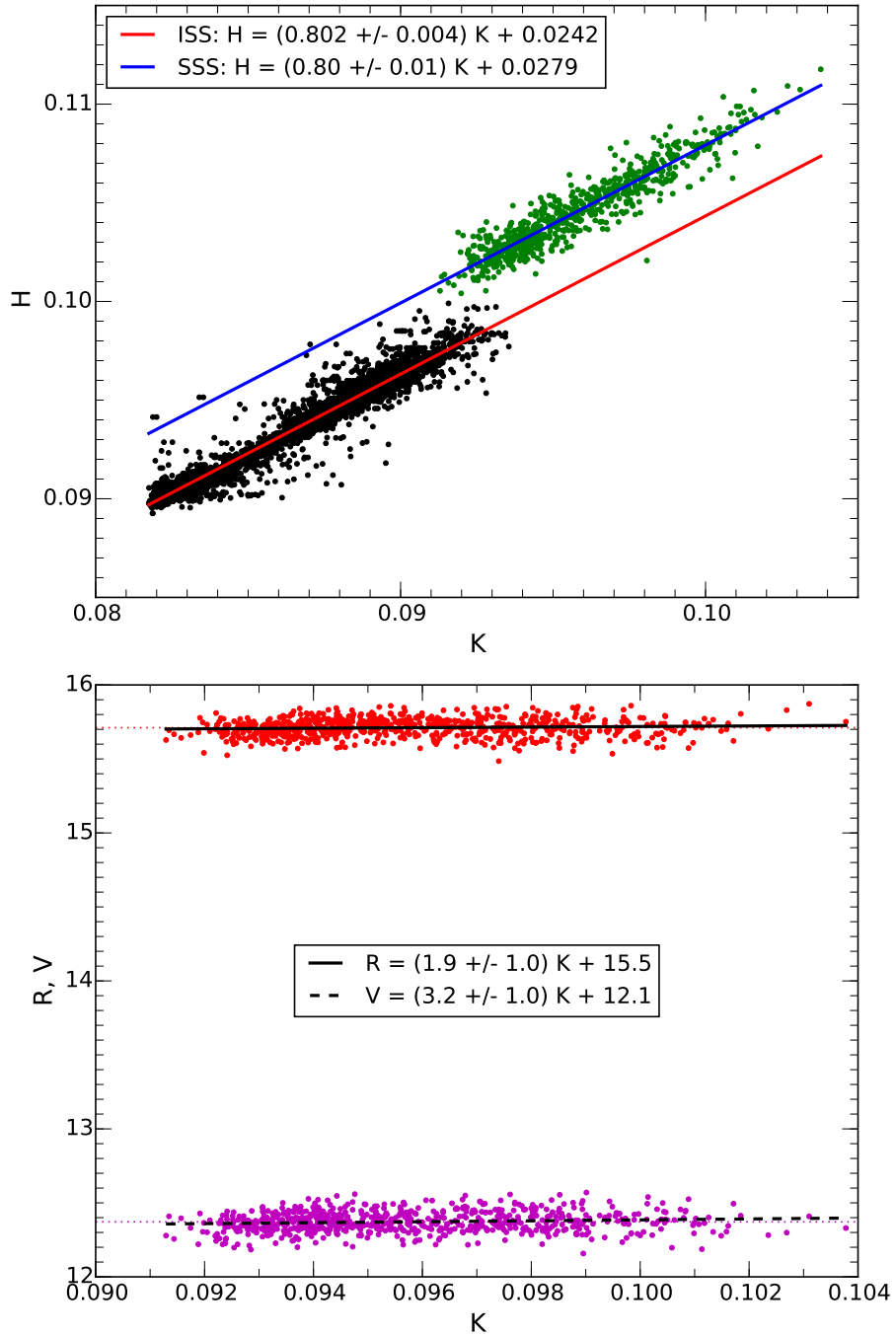


Figure 2.5 *Top:* Relationship between calcium H & K 1-Å emission indices in SOLIS/ISS (black) and SSS (green). *Bottom:* V and R band emission indices versus K from SSS spectra.

To conclude, the data show H to be linear with K , and V and R to be nearly constant, which is compatible with the model that $S = a + bK$ as assumed in the previous sections.

2.6 Discussion

We have used the observations of the Moon with the MWO HKP-2 instrument to accurately place solar cycle 23 on the S -index scale. By deriving a proxy with the NSO/SP K -index data we extend the solar record to include cycles 21–24. We found that our cross-calibration method using a cycle shape model has lower uncertainty than averaging and regression methods when the number of observations is small. The Kodaikanal Observatory plage index data calibrated to the Ca K 1-Å emission index were used to calibrate the MWO HKP-1 measurements of cycle 20 to the HKP-2 scale, as well as to extend the S -index record back to cycle 15. The full composite time series from the KKL, HKP-1, NSO/SP, and HKP-2 instruments forms a record of chromospheric variability of the Sun over 100 years in length which may be compared to the stellar observations of the MWO and SSS programs, as well as other instrumental surveys which have accurately calibrated their data to the HKP-2 S scale.

We find a mean value of $\langle S \rangle = 0.1694 \pm 0.0005$ for the Sun that is 4%–9% lower than previous estimates in the literature (see Table 2.1) that used MWO HKP-1 data or stellar observations for their calibration. We believe the discrepancy is due to (1) a systematic +0.007 offset in the previous S calibration of the HKP-1 solar data that is within the scatter of the F and S relationship of Vaughan et al. (1978), and (2) uncertainties introduced in coupling those HKP-1 measurements to the non-overlapping NSO/KP and SP K time series using either proxy activity time series or stellar measurements with the Lick spectrograph. This relatively small change in S

on the stellar activity scale (where S ranges from 0.13 to 1.4 (Baliunas et al., 1995)) is a rather large fraction of the cycle amplitude, which we estimate to be 0.0145 ± 0.0014 in S , on average. Our results are consistent with previous estimations from the SSS (Hall and Lockwood, 2004; Hall et al., 2007b, 2009), as well as our new reduction of that dataset (see Section 2.4.4). Our results are also consistent with parallel work by Freitas et al. (2016), who calibrate HARPS Ca II H & K observations to the MWO S -index scale using an ensemble of solar twins, and found $\langle S \rangle = 0.1686$ for the Sun using observations of asteroids during cycles 23 and 24.

We do not expect our change in solar S to have large consequences on previous studies that used the Sun as just another star in stellar ensembles covering a wide range of rotation periods and spectral types. However, studies which used the Sun as an absolutely known anchor point in activity relationships could be significantly affected by this shift (e.g. Mamajek and Hillenbrand, 2008, who compromises and adopts the mean of Baliunas’ and Hall’s solar S -index in their activity-age relationship). Detailed comparisons of the Sun to solar twins will also be sensitive to this change. For example, MWO HKP-2 observations of famous solar twin 18 Sco from 1993–2003 gives a mean S -index of 0.173, which we find to be $\sim 2\%$ higher than the Sun, but using other estimates from the literature we would conclude that the activity is 2%–3% lower than the Sun. This star has a measured mean rotation period of 22.7 days (Petit et al., 2008), slightly faster than the Sun, which would lead us to expect a higher activity level if in all other respects the star really is a solar twin.

Cycle amplitude is another quantity that can be compared to stellar measurements to put the solar cycle in context. So far as Ca II H & K emission is proportional to surface magnetic flux, this gives an estimate of how much the surface flux changes over a cycle period. This dB/dt is the surface manifestation of the induction equation at the heart of the dynamo problem. Soon et al. (1994) was the first to study this

for the Sun and an ensemble of stars using the fractional amplitude $\Delta R'_{\text{HK}} / \langle R'_{\text{HK}} \rangle$. They found an inverse relationship between fractional amplitude and cycle period, which is also seen in the sunspot record. The fractional solar amplitude measured here has a mean value of 0.18 ± 0.03 and ranges from 0.116 (cycle 24) to 0.255 (cycle 19). Our fractional amplitudes for cycles 21 and 22 are close to the value of 0.22 found in Soon et al. (1994) using NSO/KP data and the White et al. (1992) $S(K)$ transformation, and our range over cycles 15–24 largely overlaps with their range of values 0.06–0.17 found by transforming the sunspot record to S . Egeland et al. (2015) recently measured four cycle amplitudes for the active solar analog $((B - V) = 0.632)$ HD 30495, with fractional amplitudes $\Delta R'_{\text{HK}} / \langle R'_{\text{HK}} \rangle$ ranging from 0.098 to 0.226 comparable to our solar measurements despite the 2.3 times faster rotation of this star. However, when using absolute amplitudes the largest HD 30495 cycle has $\Delta S = 0.047$, which is 2.3 times the *largest* solar amplitude of 0.0211 for cycle 19. This illustrates that the use of fractional amplitudes obscures the fact that the more active, faster rotators in general have a much larger variability than the Sun, which indicates a much more efficient dynamo. Indeed, Saar and Brandenburg (2002) studied cycle amplitudes for a stellar ensemble and found that $\Delta R'_{\text{HK}} / \langle R'_{\text{HK}} \rangle \propto \langle R'_{\text{HK}} \rangle^{-0.23}$, fractional amplitude *decreases* with increased activity. In Chapter 4 we will use the longer timeseries available today to reexamine cycle properties such as amplitude and period for an ensemble of solar analogs.

Our value of S is significantly higher than the basal flux estimate of Livingston et al. (2007) of 0.133 ± 0.006 using the disk center H + K index values from NSO/KP, transformed to the S -index scale using the flux relationships of Hall and Lockwood (2004). The disk center measurements integrate flux from a small, quiet region near disk center where little or no plage occurs, and the derived S estimate purported to be indicative of “especially quiescent stars, or even the Sun during prolonged episodes of

relatively reduced activity, as appears to have occurred during the Maunder Minimum period”. If the latter assumption were true, we estimate it would reflect an 18% reduction in S from current solar minima to Maunder Minimum, or about twice the amplitude of the solar cycle. Total solar irradiance varies by $\sim 0.1\%$ over the solar cycle (Yeo et al., 2014), so further assuming a linear relationship between S and total solar irradiance this would translate into a 0.2% reduction in flux at the top of the Earth’s atmosphere. The validity of these assumptions are uncertain. Precise photometric observations of a star transitioning to or from a flat activity phase would greatly aid in determining the relationship between grand minima in magnetic activity and irradiance.

Schröder et al. (2012) measured the S -index for the Sun from daytime sky observations using the Hamburg Robotic Telescope (HRT) during the extended solar minimum of cycle 23–24 (2008–2009). Their instrumental S -index was calibrated to the MWO scale using 29 common stellar targets and published MWO measurements. They report an average $\langle S \rangle = 0.153$ over 79 measurements during the minimum period, and discuss their absolute minimum S -index of 0.150 on several plage-free days, comparing this “basal” value to the activity of several flat-activity stars presumed to be in a Maunder Minimum-like state. Our NSO/SP $S(K)$ proxy covers the same minimum period and has an absolute minimum measurement of 0.1596 on 9 Oct 2009, while the lowest Moon measurement from the HKP-2 instrument is 0.1593 on 28 Jan 1997. Inspection of SOHO MDI magnetograms on those dates shows no significant magnetic features on the former date, while one small active region is present in the latter. The uncertainty in the calibration scale parameter from S_{HRT} to S_{MWO} (HKP-2) is about 2% Schröder et al. (2012), making their absolute minimum $S \sim 2\sigma$ below ours. Baliunas et al. (1995) found a systematic error in the value and amplitude of the solar S -index measured from daytime sky observations, which they

attributed to Rayleigh scattering in the atmosphere, ultimately deciding to omit those observations from their analysis. While Schröder et al. (2012) applied a correction for atmospheric scattering and estimated a small 0.2–1.8% error for it, we suspect that some systematic offset due to scattering remains that explains their lower S -index measurements.

Our results compare favorably with the independently calibrated SSS instrument at Lowell Observatory. While the MWO, NSO/SP, and NSO/KP programs have ceased solar observations, SSS continues to take observations of Ca H & K, as does the SOLIS/ISS instrument which began H & K-line observations in Dec 2006 (Bertello et al., 2011). Combining equations (2.5) and (2.12), we obtain $S(K_{\text{ISS}}) = 1.71 K_{\text{ISS}} + 0.02$, which can be used to transform data on the SOLIS/ISS scale to S , including the composite K -index dataset from 1907–present (Bertello et al., 2016; Pevtsov, 2016).

This work illustrates once again the complexities of comparing and calibrating data on several different instrumental flux scales. Some of this confusion could be avoided if more effort were put toward calibrating instruments to physical flux ($\text{erg cm}^{-2} \text{ s}^{-1}$). Should this be done, discussions of discrepancies would be more about the validity of the methods used to achieve the absolute calibration rather than the details of the chain of calibrations used to place measurements on a common scale. We believe the former path is preferable, though not without its own substantial difficulties. Given the encouraging agreement between the SSS and MWO HKP-2 solar and stellar data, the S to flux relationships presented in Hall et al. (2007b) are a good starting point for placing the MWO data on an absolute scale. Further work in this area should carefully evaluate the S to flux relationship, so that calibrated Ca II H & K flux measurements by future instruments may immediately be comparable to the extensive and pioneering Mount Wilson Observatory observations.

CHAPTER THREE

CASE STUDY: THE YOUNG SOLAR ANALOG HD 30495

3.1 Solar and Stellar Short-Period Variations

In addition to the 11 yr solar cycle, short-term quasi-periodic variability has been observed in a number of solar phenomena. Various manifestations of the so-called quasi-biennial oscillation (QBO) of 0.6-4 yrs are reviewed in Bazilevskaya et al. (2014) and McIntosh et al. (2015). QBOs are found in records of sunspot number and area, magnetic field measurements, solar irradiance, and in magnetically-sensitive phenomena such as filaments in H-alpha and variations of field-sensitive lines such as Ca II and Mn I. The QBO also appears in eruptive phenomena – flares, coronal mass ejections, and solar energetic particle events – which arise from magnetically active regions. Short period variations have also been observed in the stars: Baliunas et al. (1995) reported nine stars with significant “secondary cycles”. Six of these (and one new addition) were part of the high-quality activity cycles of the Saar and Brandenburg (1999) sample, with the secondary cycle falling on the “inactive” branch. Oláh et al. (2009) performed a time-frequency analysis of multi-decadal photometry and Ca II emission for 20 stars and found 15 of them to exhibit multiple cycles. High-cadence SMARTS HK observations have found short-period variations (1.6 yr) on ι Horologi (Metcalf et al., 2010) and ϵ Eridani (2.95 yr), another dual-cycle star with a long-term cycle of 12.7 yr (Metcalf et al., 2013). Fares et al. (2009) used Zeeman Doppler Imaging to observe a polarity-flipping cycle of ~ 2 years in the fast-rotating F6 star τ Bootis, which had weak indications of a long-period activity cycle of 11.6

yr in Baliunas et al. (1995). This short-period cycle is distinct from the solar QBO phenomena, which does not reverse magnetic polarity.

The origin of the solar QBO and its relation to the solar cycle is not understood. The discovery of periodic variations of 1.3 yrs in the differential rotation of the deep interior revealed by helioseismology (Howe et al., 2000) suggests that the QBO is sub-surface in origin and may be an additional feature of a deep-interior dynamo process responsible for the 11-year cycle (see Bazilevskaya et al., 2014, and references therein). McIntosh et al. (2015) also points to a deep interior process, inferring that this short-period variability is driven by the interaction of two oppositely-signed magnetic activity bands deep in the interior of each hemisphere. Another possibility is the distinct-dynamo scenario described above as an explanation for the two activity branches in the Saar and Brandenburg (1999)/Böhm-Vitense (2007) stellar sample. Fletcher et al. (2010) find a ~ 2 yr variation in the frequency shift of solar p-mode oscillations, and locate the origin of the variations to be *below* the source of the 11-yr signal in the data. They hypothesize that spatially distinct dynamo processes may be responsible for this phenomenon. Chowdhury et al. (2009) suggested a non-dynamo origin for QBOs: an instability caused by Rossby waves interacting with the tachocline.

3.2 HD 30495 Properties and Previous Observations

While stellar observations cannot match the level of detail in which the solar QBO is observed, the varied physical conditions present in other stars may have an impact on the manifestations of these short-period oscillations that sheds additional light on their origins. In this work, we present new observations of variability in HD 30495 (58 Eri), a nearby young solar analog that demonstrates both a long-term activity cycle and a short-period oscillation, which may be analogous to the solar cycle

and QBO. The upper section of Table 3.1 summarizes the measured global properties of HD 30495. Photometry and spectroscopy show the star to be essentially solar-like, leading previous authors to study this star as a potential solar twin (Cayrel de Strobel, 1996; Porto de Mello et al., 2014). Gaidos et al. (2000) found rotational modulations in high-cadence Strömgren b and y photometry to determine a rotation period of 11.3 days, roughly 2.3 times faster than the Sun. Due to the process of magnetic braking, older stars have slower rotations, giving the well-known $P_{\text{rot}} \propto t^{1/2}$ age-rotation relationship (Skumanich, 1972). The faster rotation of HD 30495 thereby implies it is younger than the Sun, and by the age-rotation relationship given in Barnes (2007) we obtain an age of about ~ 1 Gyr. Observations of excess infrared flux attributed to a diffuse and distant debris disk of ~ 73 Earth-masses leftover from formation (Habing et al., 2001) give further evidence for a young age. Spectroscopic searches of similar “Vega-like” main-sequence objects with excess infrared emission have ruled out the possibility of dense concentrations of gas close to the star (Habing et al., 2001; Liseau, 1999). Based on these studies, in the discussions that follow, we shall assume that the disk is not a contributing factor to the observed magnetic signatures.

Baliunas et al. (1995) previously searched for cyclic variability in HD 30495 using the Lomb-Scargle periodogram on the Mount Wilson S time series from 1966–1992, but classified it as “Var”, defined as “significant variability without pronounced periodicity” and $\langle \sigma_S/S \rangle \gtrsim 2\%$. The mean activity level was found to be high, $\langle S \rangle = 0.297$, as expected for fast rotators (compare to the solar value from the same study $\langle S_{\odot} \rangle = 0.179$). Hall et al. (2007b) S -index measurements from the Solar Stellar Spectrograph (SSS) again found stronger-than-solar activity levels, with $\langle S \rangle = 0.309$. Hall et al. (2009) used twelve years of precise Strömgren b and y photometry from the Fairborn Automated Photometric Telescope (APT) program

Table 3.1. HD 30495 Properties

Property	Value	Reference
Spectral type	G1.5 V	(1)
V	5.49 ± 0.01	(2)
B-V	0.632 ± 0.006	(2)
Parallax	75.32 ± 0.36 mas	(2)
$v \sin i$	4.1 ± 0.8 km s ⁻¹	(3)
T_{eff}	5826 ± 48 K	(4)
$\log g$	4.54 ± 0.012 dex	(4)
[Fe/H]	$+0.005 \pm 0.029$ solar	(4)
Mass	$1.02 \pm 0.01 M_{\odot}$	(4)
Radius	$0.898 \pm 0.013 R_{\odot}$	(4*)
Luminosity	$0.837 \pm 0.037 L_{\odot}$	(4*)
Age	970 ± 120 Myr	(5*)
$\bar{P}_{\text{rot}}$	11.36 ± 0.17 days	
ΔP	0.59 ± 0.05 days	
$\Delta \Omega$	$\gtrsim 1.67 \pm 0.15$ deg/day	
$\sin i$	1.0 ± 0.2	
i	$\gtrsim 55.4^{\circ}$	
$P_{\text{cyc,long}}$	$12.2 \pm [3.0]$ yr	
$A_{\text{cyc,long}}$	$0.118 \pm [0.044]$	
P_{short}	$1.67 \pm [0.35]$ yr	
A_{short}	$0.066 \pm [0.028]$	

Note. — References: (1) Gray et al. (2006) (2) Hipparcos, Perryman et al. (1997); van Leeuwen (2007) (3) Gaidos et al. (2000) (4) Baumann et al. (2010) (4*) Derived from Baumann et al. (2010) measurements (5*) Derived from Barnes (2007) age-rotation relationship. The lower section of the table are measurements found in this work Quantities in brackets represent one half of the observed range of values.

(Henry, 1999), finding a photometric variability for HD 30495 roughly six times solar. Furthermore, brightness was shown to decrease with increased chromospheric activity, indicating that variability in the star’s brightness is dominated by dark spots, typical of fast rotators in that study.

Naively, and based solely on its similarity to the Sun and faster rotation and, hence, presumably greater differential rotation, we might expect HD 30495 to have a more vigorous dynamo, leading to higher magnetic activity and a shorter activity cycle. As we shall see, the former is borne out by observations, while the latter is not.

3.3 Activity Analysis

We analyze a combined 47-year time-series of the Mount Wilson S -index shown in Fig 3.1. This dimensionless index is defined as the ratio of the core emission in the Fraunhofer H and K lines of Ca II with the nearby continuum regions, as measured by the HKP-1 and later HKP-2 photometers at Mount Wilson Observatory (MWO) (Vaughan et al., 1978; Wilson, 1968). Ca II H & K global-scale emission reversals are a signature of departure from radiative equilibrium, a defining feature of a chromosphere, and must be due to magnetic non-thermal heating mechanisms. Due to the unsurpassed duration and breadth of the Mount Wilson survey, S , a measure on an *instrumental scale*, has become the *de facto* standard for measuring stellar magnetic activity, and subsequent surveys have been calibrated to the Mount Wilson scale. S is dependent on stellar properties such as temperature, surface gravity, and composition, which precludes its use for directly comparing activity levels of a heterogeneous ensemble. As we focus our analysis on a single star, conversion to a corrected quantity (e.g., R'_{HK} ; Noyes et al. (1984a)) is not necessary.

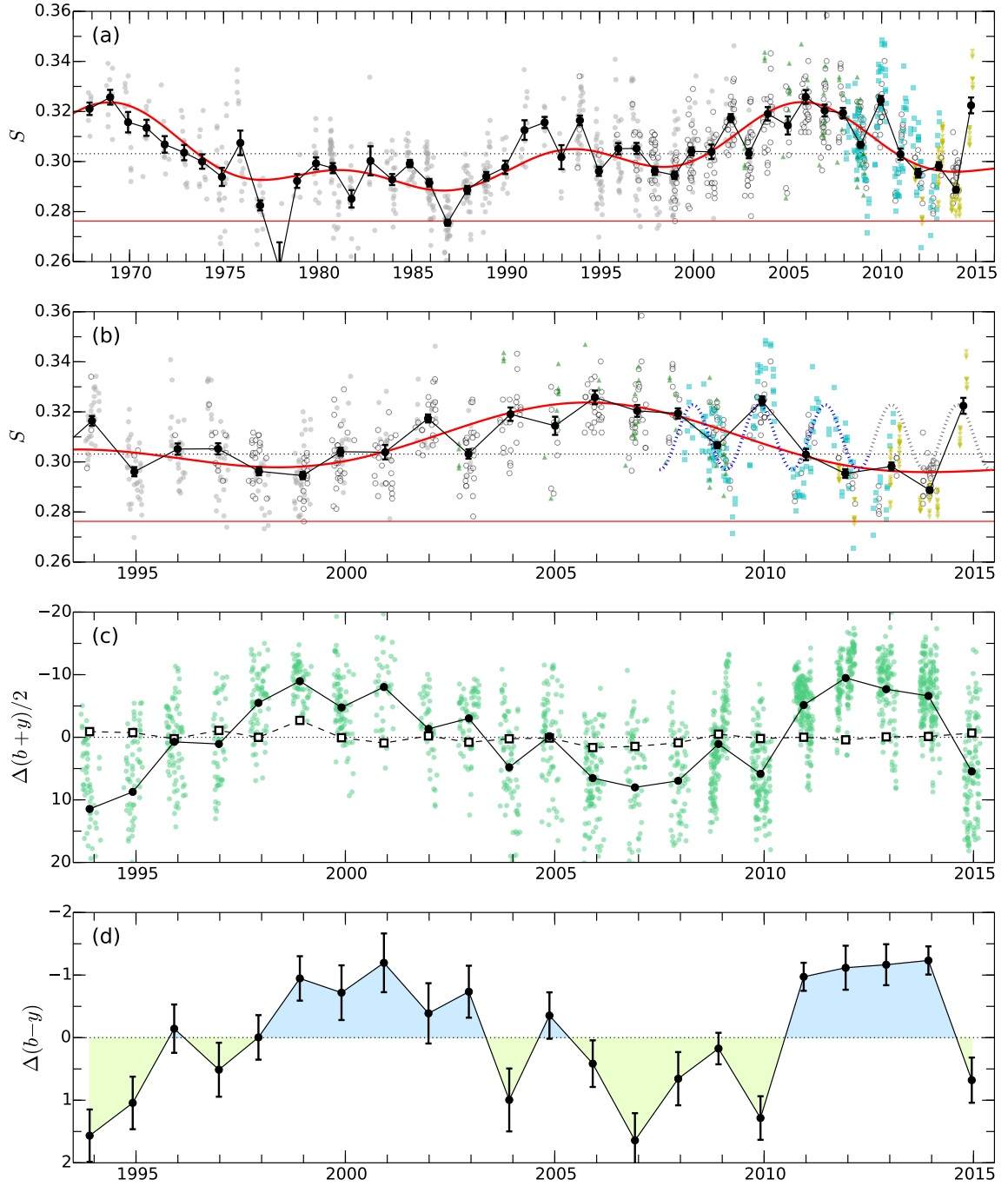


Figure 3.1 (a) HD 30495 combined S -index time series, including data from MWO ($\bullet$), SSS ($\circ$), SMARTS ($\blacksquare$), CPS ($\blacktriangle$), and HARPS ($\blacktriangledown$), along with seasonal means ($\bullet$). (b) Zoomed portion highlighting higher-cadence SMARTS data. (c) APT differential photometry brightness measurements in millimagnitudes. (d) Seasonal mean differential brightness difference in the b and y bands.

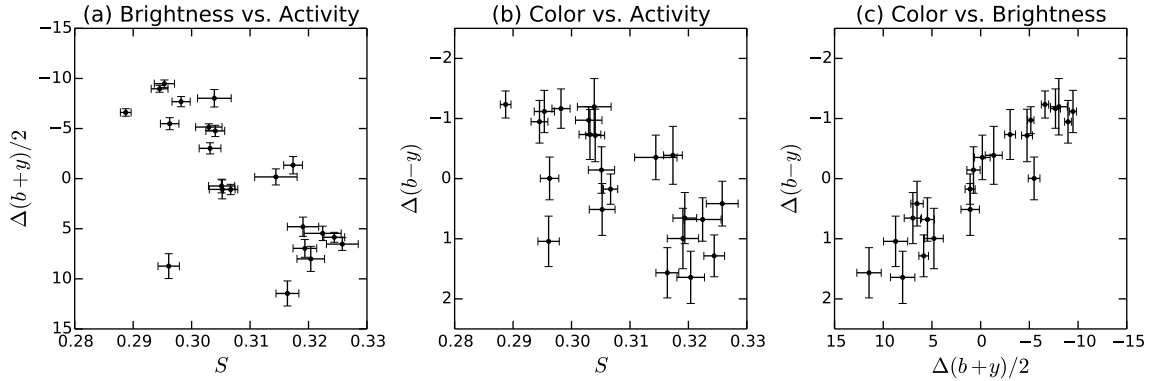


Figure 3.2 Correlations among seasonal means of activity, brightness and color from the time series of Figure 3.1. Error bars indicate the error in the mean. Magnitude scales are in milli-magnitudes and are inverted such that brightness increases in the upward/rightward directions.

The combined S time series in Figure 3.1(a) contains 1285 measurements from five different instruments. The largest set of measurements (624 measurements from 1967-2003) come from the original Mount Wilson survey, calibrated as described in Baliunas et al. (1995). We assumed a uniform measurement error of 3% of the mean for this time series, near the upper limit quoted by Wilson (1968). The next largest portion of the measurements are from the Solar-Stellar Spectrograph (SSS) at Lowell Observatory (364 measurements from 1993-2014) (Hall and Lockwood, 1995), taking the time series from the beginning of the Mount Wilson survey to present day. SSS obtains $R \approx 10,000$ at $H\alpha$ spectra, and S is derived by approximating the Ca II H, K and continuum bandpasses used by the MWO instrument. These data are then calibrated to the Mount Wilson instrumental scale using long-duration means of common targets. A typical measurement error of 2.4% was estimated using photon statistics in the K line core and detector properties. Observations from the SMARTS Southern HK survey using the *RC Spec* $R \approx 2500$ spectrograph at 1.5-m telescope at CTIO are the third largest contribution (140 measurements from 2008–2013), and

though of shorter duration, this queue-scheduled time series is not plagued by the large seasonal gaps of the other surveys, allowing short-period variation to be better determined. These data were calibrated to the Mount Wilson scale via common observations with SSS targets, as described in Metcalfe et al. (2010). An additional 108 measurements from 2011–2015 derived from HARPS $R \approx 120,000$ spectra from a solar twin planet search (Bedell et al., 2015; Ramírez et al., 2014), again calibrated to the MWO scale using common targets, as described in (Lovis et al., 2011). Finally, we add 49 observations from 2002–2008 derived from $R \approx 55,000$ spectra of the Hamilton Spectrometer at Lick Observatory. These observations¹ are part of the California Planet Search (CPS) and were similarly calibrated to the MWO scale using common targets (Isaacson and Fischer, 2010).

Though each of these time series used a global calibration to the Mount Wilson scale using long-term means of commonly observed targets, visual inspection of the combined time series revealed obvious discontinuities and differences in scale. This is likely due to the fact that the *global* calibration involves a compromise linear fit among all targets, while scatter about that fit reveals error in the calibration that would result in a discontinuity in any individual target. We applied a simple calibration that assumes overlapping periods of two different time series ought to agree on the mean for that period. To calibrate S to the scale of S_0 , the mean value over the period of overlap, $\bar{S}$ and $\bar{S}_0$ were calculated, and a scaling factor $C = \bar{S}_0/\bar{S}$ was derived. The resulting calibrated time series $S' = CS$ then has an equivalent mean value over the overlapping period to the base series S_0 . The resulting scaling factors were $C(\text{SSS} \rightarrow \text{MWO}) = 1.015$, $C(\text{SMARTS} \rightarrow \text{SSS}') = 1.067$, $C(\text{HARPS} \rightarrow \text{SSS}') = 1.074$, $C(\text{CPS} \rightarrow \text{SSS}') = 1.098$. The SMARTS, HARPS, and CPS time series were scaled using overlapping portions of the post-calibration SSS time series, therefore

¹<http://cdsarc.u-strasbg.fr/viz-bin/Cat?J/ApJ/725/875>

their overall scaling is multiplied by $C(\text{SSS} \rightarrow \text{MWO})$. This calibration removed obvious discontinuities in the combined time series and reduced the standard deviation by 3.8%. The final combined time series has a grand mean $\bar{S} = 0.303$ and a standard deviation $\sigma = 0.0167$. Seasonal means for the combined time series are shown as black circles in Figure 3.1(a). Following these seasonal means, clear cyclic behavior is visible, emphasized by the cycle model (red curve) described below.

We also examined the 22-yr time series of differential photometry acquired with the T4 0.75 meter Automatic Photoelectric Telescope (APT) at Fairborn Observatory (Henry, 1999), shown in Figure 3.1(c). These measurements, made in the Strömgren b (467 nm) and y (547 nm) bands, are a difference with respect to the mean brightness of two stable comparison stars, HD 31414 and HD 30606. The differential measurements in the b and y bands are then averaged to $(b+y)/2$ to create a “ by ” band that increases the signal to noise ratio. The unimportant mean difference is subtracted from the time series. The stability of the comparison stars is demonstrated in the seasonal mean of their brightness difference in the by band, shown as white squares in Figure 3.1(c), with a standard deviation $\sigma = 0.00093$ mag. HD 30495 by brightness is strongly variable ($\sigma = 0.0065$ mag) and out-of-phase with the S -index shown in Figure 3.1(b). A rank correlation test between S and by seasonal means shows 99.98% significance in the correlation, which is plotted in Figure 3.2(a). This is interpreted as evidence the star’s brightness variations are dominated by dark spots, which are more prevalent during times of activity maximum. Figure 3.1(d) plots the $\Delta(b-y)$ color index where blue shading indicates negative color index and green shading indicates positive color index. Comparing panels (b), (c) and (d) of Figure 3.1, we see that HD 30495 gets bluer as it gets brighter (activity minimum) and redder as it gets fainter (activity maximum). This is shown again more clearly in Figure 3.2 (b) and (c), in particular the remarkably tight color-brightness correlation. We interpret this color shift as an

increase in surface temperature during times of activity minima, due to the reduction of cool spots on the surface.

We computed the Lomb-Scargle periodogram (Scargle (1982), Horne and Baliunas (1986)) from our time series to find statistically significant periodicities in the data, with the results shown in Figure 3.3. To verify the robustness of the peaks, we compare the periodogram of the combined time series (thick black line) to those of the individual MWO, SSS, and SMARTS series over shorter intervals, as well as to the periodogram of the $(b + y)/2$ photometry. Not shown in the figure are the large peaks beyond 25 years in the MWO and combined S -index periodograms, which are most likely due to the windowing of the entire time series, not true physical variation. The hatched regions of $P < 1.1$ on the left side of periodograms contain a number of large peaks near 1 year, which are aliases due to the seasonal sampling in our time series. We verified these are all aliases by obtaining a least-squares fit of the data to a sine wave with a period set by one of the ~ 1 yr peaks, then subtracting that signal from the time series and re-computing the periodogram. The new periodogram would no longer contain the ~ 1 yr peak, and a corresponding low-frequency peak would be removed as well. This established a symmetry between the low-frequency peaks and these ~ 1 year peaks, and as a result we do not consider any peak < 1.1 years to be physical. (See also Figure 3.4(c), in which spurious ~ 1 yr peaks are found in the periodogram of a signal of pure sine waves of lower frequency.)

Following Horne and Baliunas (1986), we calculate the “False Alarm Probability” (FAP) threshold:

$$z = -\ln \left(1 - (1 - F)^{1/N_i} \right), \quad (3.1)$$

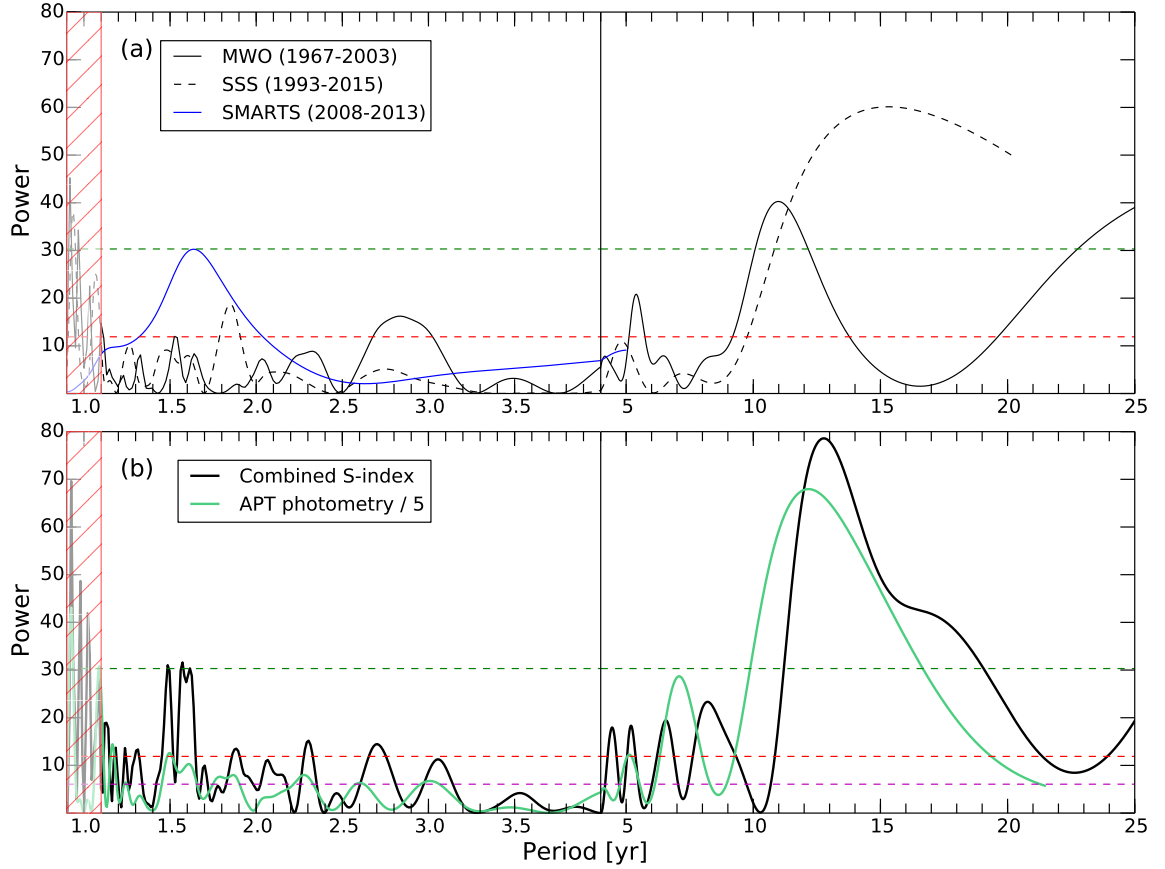


Figure 3.3 Lomb-Scargle periodograms from the time series of Figure 3.1. Panel (a) contains the result from single-instrument S-index surveys and panel (b) the combined S-index time series, as well as the APT photometry of Figure 3.1(c). Note the division in the period scale. The hatched region near $P \approx 1$ yr contains artifacts of the seasonal sampling. The green and red horizontal dashed lines are the “excellent” and “poor” significance thresholds for the S-index periodograms, as defined in Baliunas et al. (1995). Note that the APT periodogram is scaled down by a factor of five for easy comparison and the magenta horizontal line is the “excellent” threshold for that series.

where F is the probability that there exists a peak of height z at any frequency due to random Gaussian noise in the signal, and N_i is the number of independent frequencies in the time series. We computed N_i by generating 5000 random time series with the same sampling times of our data, generating a probability distribution for the maximum peak z , and fitting this distribution to equation (3.1) inverted for F , with N_i as the free parameter. The upper threshold (green line) shown in Figure 3.3 corresponds to $F = 10^{-11}$, the threshold for an “excellent” cycle in Baliunas et al. (1995), and the lower threshold (red line) is for $F = 10^{-3}$ (99.9% significance), the minimum requirement for a “poor” cycle in that work. The FAP thresholds shown are those computed for the combined time series, however they are similar to those obtained for the individual component time series, being only slightly more stringent.

The uncertainties in peak positions were estimated using a Monte Carlo method. In each trial, each time series measurement is randomly sampled from a Gaussian distribution defined by that measurement’s value and uncertainty. Then, a periodogram is computed and the new peak position saved. By running 5000 trials, an approximately Gaussian distribution of peak positions is obtained, and the uncertainty is estimated as its standard deviation.

In the combined time series we found four resolved peaks above the “excellent” threshold: a long period peak $P_{\text{long}} = 12.77 \pm 0.09$ yr, and a cluster of three short-period peaks at $P_{\text{short},1} = 1.572$, $P_{\text{short},2} = 1.486$, and $P_{\text{short},3} = 1.615$ yr ($\sigma_{\text{short}} \approx 0.003$). The P_{long} peak is found between nearby significant peaks found in the MWO ($P_{\text{long,MWO}} = 10.7$ yr) and SSS ($P_{\text{long,SSS}} = 15.3$ yr) time series, and nearby the $P_{\text{long,APT}} = 12.2$ peak from *by* photometry. The spread in periods from the earlier MWO data to the later SSS data indicates that, like the Sun, the long-term cycle is only quasi-periodic, and the duration of each individual cycle varies. This increase in period was confirmed in a wavelet analysis of the S seasonal means (not shown), which

showed the period increasing from ~ 10 to ~ 14 years over the duration of the time series. The ~ 13 yr peak in Figure 3.3(b) has a protruding “shoulder” on its right side, which is due to an unresolved peak near 17 yr. This peak was resolved in $\sim 25\%$ of the Monte Carlo trials done to determine the uncertainty in peak positions, allowing us to measure a mean value of 16.9 yr. This ~ 17 year period and another large peak at ~ 37 yr were found to be artifacts of the amplitude structure and/or the duration of the time series (data window). We verified this by computing periodograms of various fractions of the data window (e.g. $2/3$ to $1/2$ of the total duration) at various offsets and noting that the ~ 17 yr peak disappears in all cases and the ~ 37 yr peak shifts close to the duration of the new window.

Figure 3.3(b) shows the short-period peaks are almost perfectly matched by two peaks (1.49 yr and 1.61 yr) in the APT periodogram. We also find a broad corresponding “excellent”-class peak at 1.63 yr in the SMARTS time series and again in the HARPS series at 1.75 yr. A less significant “poor”-class peak at 1.85 yr is in the SSS data and 1.53 yr in the MWO data. The spread in values indicates that these short-period variations are not of a constant frequency, which we investigate in detail in Section 3.5.

We find that both the long and short-period signals are found consistently in several distinct *S*-index time series of different time intervals, as well as in the APT differential photometry, a measurement using very different observation methods to sample a physically distinct region of the star. This is strong evidence for the co-existence of variability on different time scales and in distinct regions of the stellar atmosphere, analogous to the solar observations of the 11-year cycle and quasi-biennial oscillation.

3.4 Simple Cycle Model

In the Sun, each occurrence of rising and falling activity is numbered and, somewhat confusingly, referred to as a “cycle”, with the current episode denoted as “cycle 24”. Properties of each cycle such as duration, amplitude, and shape are measured and found to vary. We wish to similarly decompose the ~ 12 yr periodic signal of HD 30495 into individual cycles and measure their properties. For the Sun, this decomposition is typically done by identifying cycle minima in a smoothed time series of a proxy such as sunspot number, e.g. the 13-month boxcar smoothing of the monthly averages, and then using the minima as delimiters for each cycle (Hathaway, 2010). The seasonal gaps in stellar time series do not allow us to use this same prescription. Instead, we construct an idealized smoothed model of the time series as a superposition of low-frequency sine waves:

$$S(t)_i = A_i \sin \left(\frac{2\pi}{P_i} (t + \phi_i) \right) + y_i, \quad (3.2)$$

where P_i is a low-frequency period from a periodogram analysis, and the amplitude A_i , phase ϕ_i and offset y_i are found using a least squares optimization of this model to the mean-subtracted data. The final model is simply:

$$S(t) = \sum_i^{N_P} S(t)_i + \bar{S}, \quad (3.3)$$

where N_P is the number of component sine waves and $\bar{S}$ is the original mean S -index.

We obtain the parameters of equation (3.2) by iteratively finding the lowest-frequency period in a Lomb-Scargle periodogram of the composite S -index time series, fitting the sine to the data using a least-squares optimization, and subtracting the result from the data before computing the next periodogram. We carry out three

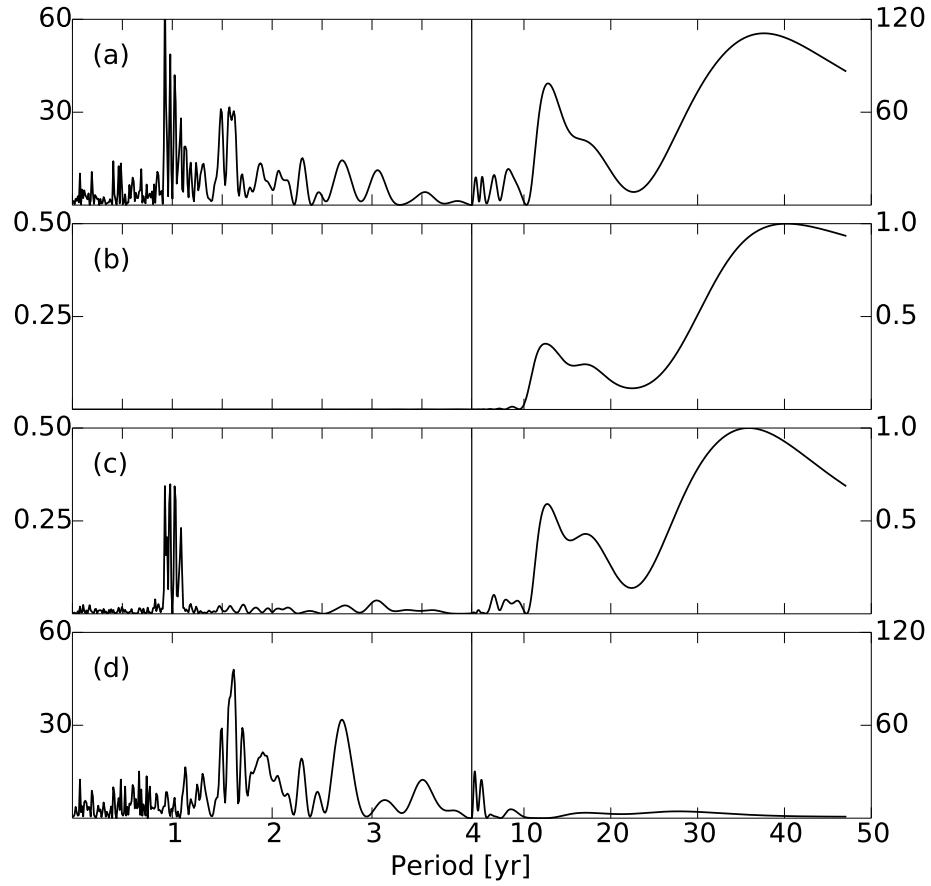


Figure 3.4 Lomb-Scargle periodogram of (a) the original composite S -index time series (b) 3-component cycle model with equal-spaced sampling (c) 3-component cycle model with the same sampling as the original data (d) the residual of the original data minus the cycle model. Note that period and power scales change at the 4-yr mark; the left and right y-axis give the power scale for that side. Periodograms of the cycle model are normalized to 1.

Table 3.2. HD 30495 Cycle Properties

Cycle	Start	Max	Duration	$S_{\max}$	A_{cyc}
0	(1961.7)	1969.0	(15.3)	0.324	0.156
1	1977.1	1981.2	9.6	0.297	0.067
2	1986.7	1993.7	11.7	0.305	0.095
3	1998.4	2005.8	15.5	0.324	0.156
4	2013.9	—	—	—	—

such iterations in order to find the three low-frequency components revealed by the periodogram of the original dataset. The resulting model is akin to the result of a low-pass filter on the data. Indeed, subtracting the model from the data and computing the periodogram reveals that we have effectively removed most of the low-frequency power, as can be seen in Figure 3.4. The parameter set obtained from the fits of equation (3.2) were $P = \{37.6, 18.8, 12.2\}$, $A = \{11.4, 4.68, 6.64\} \times 10^{-3}$, $\phi = \{-0.103, 5.93, 5.45\}$, and $y = \{-2.47, -0.0519, 1.26\} \times 10^{-3}$.

Using this model we characterized the individual cycles of HD 30495 as shown in Table 3.2. Quantities in parenthesis are based on an extrapolation beyond the data, and should be treated with caution. We see a spread in cycle durations from 9.6 to 15.5 yr, with a mean value of the fully observed cycles 1–3 of 12.2 yr. This value is close to the $P_{\text{long}} = 12.77$ yr peak in the periodogram of Figure 3.3(b). We shall adopt the mean value of 12.2 yr as our best estimate for the mean cycle period for HD 30495. The spread of individual cycle durations from the model gives us an estimate of the variability of cycle durations, $\Delta P_{\text{long}}/2 = 3.0$ yr. The increase in cycle duration from cycle 1 to 3 obtained from our model agrees with the trend observed in the periodograms, with the earlier MWO series which includes cycles 1 and 2 having $P_{\text{long,MWO}} = 10.7$, and the later SSS series which only includes cycle 3

having $P_{\text{long,SSS}} = 15.3$ (see Figure 3.3(a)). The cycle model components at $P = 37.6$ and 18.8 yr are artifacts of the amplitude structure in the time series and the data window, as discussed above. These components are necessary, however, to reconstruct the amplitude and cycle duration variability present in the original time series. For example, the amplitude $A = 6.64 \times 10^{-3}$ from the $P = 12.2$ yr least-squares fit to the original data is not representative of the amplitude of the individual cycles seen in Figure 3.1(a), being at least a factor 2 too low. Only when the other two low-frequency components are included does the model amplitude more closely match the data.

The relative cycle amplitudes from the model are shown in column 6 of Table 3.2, defined as $A_{\text{cyc}} = \Delta S / \bar{S}$. Determining $\Delta S = S_{\text{max}} - S_{\text{ref}}$ requires us to select a reference point that approximates the lowest possible activity level for this star. Our cycle model does not effectively model the depth of the minima, as can be seen in Figure 3.1(a), which precludes its use for setting S_{ref} . Instead, we choose S_{ref} to be the lowest seasonal median with more than 3 measurements, $S_{\text{ref}} = 0.2763$ from the 1986-1987 season. This definition of activity minimum avoids too much sensitivity to outliers, while being low enough that only 3.6% of the data lie below this point. With our choice of S_{ref} and $\bar{S} = 0.303$ we find A_{cyc} ranging from 0.067 to 0.156, and a mean amplitude $\overline{A_{\text{cyc}}} = 0.118$. We find an increase in cycle amplitude from cycles 1 to 3, which occurs in parallel with the increase in cycle duration. Note that for the Sun the amplitude of a subsequent cycle is negatively correlated with cycle period (Hathaway, 2010). The two transitions of fully observed cycles for HD 30495 indicate a positive correlation, though obviously no firm conclusions can be drawn from so little data.

Robustness of the cycle model was examined by running Monte Carlo simulations with the nightly measurements resampled from within their estimated uncertainties

for each trial, then repeating the iterative procedure described above to compute the cycle model periods, amplitudes, phases, and offsets. The resulting minima positions, maxima positions, and amplitudes were gathered and the standard deviation of those distributions computed. The times of minima and maxima were found to be relatively robust forming a Gaussian distribution with a standard deviation of $\sigma = 0.25$ yr. The amplitudes were also fairly robust, forming a Gaussian distribution with $\sigma = 8.6 \times 10^{-4}$. We use this to estimate the uncertainty in ΔS to be about 7%, which dominates the uncertainty of our relative amplitudes $A_{\text{cyc}} = \Delta S / \bar{S}$.

3.5 Short-Period Time-Frequency Analysis

A sine fit with $P = 1.58$ was found for the combined time series for a 5-year window centered at 2010.0 is shown in Figure 3.1(b), with the curve colored cyan within the fitting window and colored gray outside of the window. Comparing this sine wave to the data visually demonstrates the short-period variation, especially visible during the epoch of SMARTS measurements (cyan points). Even in this short segment we notice that the data goes out-of-step with the sine curve after 2012. This shows by example the quasi-periodic nature of the short-period variations, which was also evident in the triplet of peaks near $P \approx 1.6$ yr in Figure 3.3(b).

To investigate possible coupling between the short- and long-period variations found above, we performed a periodogram analysis on a 5-yr moving window of our combined S -index time series, the results of which are shown in Figure 3.5. First, we removed the low-frequency components from the data by subtracting the cycle model described in Section 3.4. Comparing Figure 3.4 panels (a) and (d), we see that this procedure amplifies the high-frequency peaks in the residual periodogram. However, we found the results of this analysis are qualitatively the same when working with the original composite time series. Next, the residual time series is divided into 1-yr

bins, which contain the entirety of the MWO/SSS seasonal observations. Every set of five consecutive bins are then subjected to a Lomb-Scargle periodogram analysis as described in Section 3.3, searching for periods from 1.1 to 2.5 years. Five year windows were chosen as a sufficient time period to capture multiple oscillations of the $P \approx 1.6$ yr variability detected in Figure 3.3(b). The period search cutoff at 1.1 yr is to avoid aliasing issues related to the seasonal sampling, and the 2.5 yr cutoff is to avoid strong signals associated with the duration of the window. In total, 44 periodograms were calculated as well as a 0.001 (99.9% significance) false alarm probability threshold based on a Monte Carlo analysis as described in Section 3.3. Up to two significant peaks were extracted from these periodograms, and a least-squares fit of a sine function to the data window is done to determine the amplitude of that signal. In Figure 3.5 the normalized periodogram power is plotted as a contour area plot in the time-period plane, and the positions of significant peaks are plotted as open data points. The 99.9% significance contour is plotted as a dotted white line.

It is important to note that higher periodogram power is possible in time series with a larger number of data points, so that periodogram power does not scale equally from one window to the next. To check this behavior, we generated unevenly sampled time series of a sine function by randomly removing 50% of the data points and calculated the Lomb-Scargle periodogram, finding that periodogram power is proportional to the number of data points. To correct for this, we normalized the periodogram power by the number of data points in the window. Differences in periodogram power between windows are then due to differences in amplitude of the underlying signal. The number of data points in each window is plotted in the top panel of Figure 3.5 for reference. The 99.9% significance threshold is computed separately for each window, so the contour position is correct despite the difference in the number of data points.

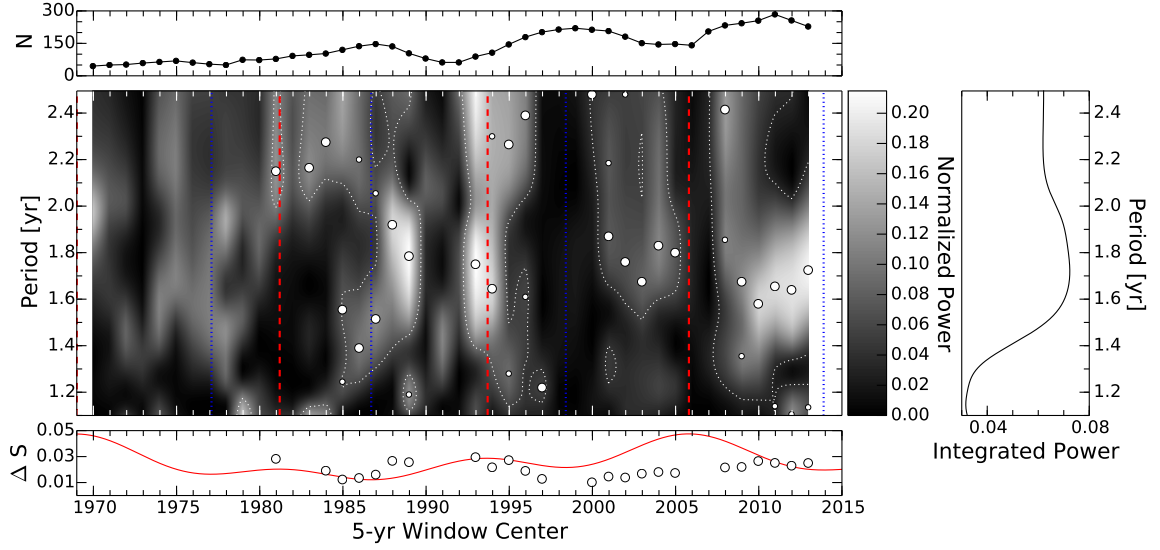


Figure 3.5 A Lomb-Scargle periodogram is computed for 5-yr moving windows in 1 yr increments of the composite S -index time series for HD 30495. The contour plot gives the periodogram power normalized by the number of data points as a function of time and period, with the 99.9% significance contour highlighted as a dotted white line. The indicated time is for the center of the 5-yr window. In each window, the highest peak above the 99.9% threshold is found and plotted as a large open circle, while a secondary significant peak, if present, is plotted as a small open circle. Vertical blue and red dashed lines indicate the minima and maxima in the long-term cycle model. The top panel indicates the number of data points in each 5-yr window. The right panel gives the integrated normalized power for all windows. The bottom panel plots the amplitude ΔS of sine fits of the significant peak periods in the windowed data as open circles, with the amplitude of the 3-component cycle model in red.

Figure 3.5 reveals that variability near $P_{\text{short}} \approx 1.6$ yr is intermittent with peak periods occurring across the full range of periods analyzed. Significant peak variability in the 1.4-1.8 yr range begins in 5-yr windows centered at 1985, 1993, 2000, and 2009, with each episode lasting for 3-5 years. The 1985 and 2009 episodes precede the cycle 2 and 4 minima in the long-period cycle, respectively, while the 1993 and 2002 episodes roughly coincide with cycle maxima. In contrast to the minima of cycles 2 and 4, the cycle 1 and 3 minima are devoid of short period variability. Peaks are often found from ~ 2.2 to ~ 2.4 yr as well, which is confirmed in the plot of the power integrated over all windows. Here the broad peak at ~ 1.7 yr is notably shifted from the cluster of narrow peaks in the periodogram of Figure 3.3(b). This may be due to varying phase in intermittent short- period signals leading to interference effects in the full-time-series periodogram. We will take the mean of the detected peak periods < 2.0 yr as our best estimate of the short-period signal, $P_{\text{short}} = 1.67 \pm [0.35]$, where the quantity in brackets is half the observed range of peak periods.

We analyzed the peak-to-peak amplitudes ΔS of sine fits to the data with the significant peak periods in the range of 1.1 to 2.0 yr and found them to range from 0.012 to 0.030, with a mean of 0.020. The average short-period relative amplitude $A_{\text{short}} = \Delta S / \bar{S}$ is then $0.066 \pm [0.028]$, roughly half of the average long-period amplitude but nearly equal to relatively low amplitude of cycle 1, as deduced from our cycle model. We performed a rank-correlation test between the short-period amplitudes and the long-period cycle model, but no significant correlation was found.

From the above observations we conclude that there is no clear association between the long-period cycle and the episodic short-period variations. The presence or absence of the short-period variations are found in all phases of the long-term cycle, and the amplitudes are not correlated with the long-term cycle amplitude.

3.6 Rotation

We repeated the rotation measurements done for six seasons of APT photometry in Gaidos et al. (2000) using the current 22-season record. The dense sampling of the APT program allows the detection of rotational modulations due to the transit of spots on the stellar photosphere. The time series is broken into individual seasons containing 55–185 measurements over the course of 150–200 days. From each season’s time series a Lomb-Scargle periodogram was computed looking for rotational periods from 2–25 days. Peaks passing the 99.9% significance level are taken as a signal of rotation. Uncertainty in the peak position was calculated using the Monte Carlo method described in Section 3.3 and ranged from 0.014 to 0.2 days. In total, rotational periods were detected in 17 of 22 seasons. Taking the mean, we find the average $P_{\text{rot}} = 11.36 \pm 0.17$ d. This result is within the uncertainty of the previous Gaidos et al. (2000) measurement, as well as that of Baliunas et al. (1996a) who obtained rotation from a densely-sampled season of MWO *S*-index observations. The range of the precisely determined rotations are from 10.970 ± 0.028 to 11.560 ± 0.023 days, giving $\Delta P = 0.59 \pm 0.05$ d. This is reduced from the Gaidos et al. (2000) value of $\Delta P = 1.0$ d, due to the fact that we could not reproduce a significant 10.5 d detection in season 6. ΔP is interpreted as a sign of surface differential rotation, due to transiting spots at various latitudes. Our measurements range over one and a half stellar cycles, which increases the confidence that we have explored the full range of rotational periods which can be sampled with this technique. However, due to the unknown latitude ranges of the spots on the star, the measurement provides only a lower bound on the true equator-to-pole surface differential rotation.

The time series of rotation measurements and the seasonal activity-rotation relationship are shown in Figure 3.6. P_{rot} vs. S is a kind of pseudo-butterfly diagram

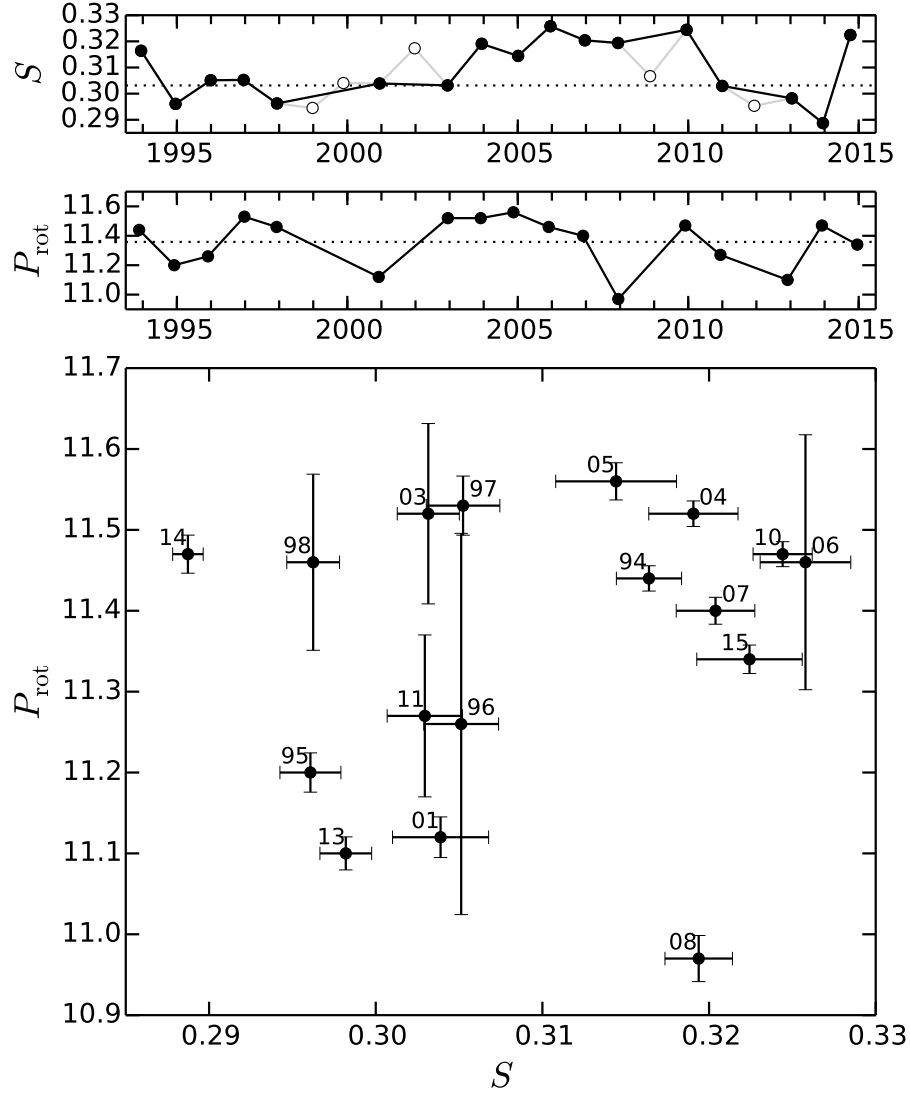


Figure 3.6 Top two panels: time series of seasonal rotation period measurements beneath seasonal mean S -index time series, for comparison. Filled (open) circles are measurements for seasons in which a rotation period detection was successful (unsuccessful). A horizontal dotted line marks the grand mean for the whole time series. Bottom panel: seasonal activity-rotation correlation plot, with error bars representing 1- σ uncertainty in the rotation period and seasonal mean S value. Data points are annotated with the two-digit year.

in the absence of spot latitude information, which would demonstrate different morphologies under different migration patterns (Donahue, 1993). For example, in the Sun, if rotation were measured by tracking spots during the cycle we would expect a long rotation period and low activity at solar minimum (high latitude spots), transitioning to shorter rotation periods and higher activity until solar maximum (mid-latitude spots), and finishing with still shorter rotation periods as activity wanes (near-equator spots). A variety of morphologies were observed in Donahue (1993), including the anti-solar case. From Figure 3.6, we see that for HD 30495 in general rotation is slow when activity is high (with the exception of the 2008 season), but when activity is low both long and short rotation periods are seen. Tracing points in chronological order reveals no clear pattern, with seasons transitioning from quadrant to quadrant in the diagram. Near the maxima of cycle 3 ($t = 2005.8$, ref Table 3.2) there is a cluster of four seasons with slow rotation, 2004-2007. The 1997-1998 rotation was also slow, which occurred just before the minimum at the start of cycle 3 ($t = 1998.4$), however the 2011 and 2013 rotation before the start of cycle 4 ($t = 2013.9$) was relatively fast. The lack of coherent structure in Figure 3.6 may be an indication that large spots which make the detection of P_{rot} possible are not restricted to a narrow range of latitudes as for the Sun.

Line-of-sight inclination is an important factor for interpreting the stellar rotation and activity data. We estimated the inclination using the $v \sin i$ measurement of Gaidos et al. (2000), together with $v = 2\pi R/P_{\text{rot}}$, where R was derived using $g \propto M/R^2$ and the mass and surface gravity estimates of Baumann et al. (2010) (see Table 3.1), as well as our rotation measurement. We obtained $\sin i = 1.0 \pm 0.2$, indicating an equator-on view of the star, but the large uncertainty giving a one-sigma range of $i \gtrsim 55.4^\circ$. This perspective provides a best-case scenario for measuring rotation with spot transits, as well as a “solar-like” view of the activity cycles.

Finally, using the age-rotation relationship given in equation (3) of Barnes (2007) along with our mean rotation period, we obtain an age of 970 ± 120 Myr. This revises the t_{gyro} from Table 3 of that work, which was based on a lower estimate for the rotation period.

3.7 Discussion

We have observed quasi-periodic signals with representative values of $P_{\text{long}} = 12.2 \pm [3.0]$ yr and $P_{\text{short}} = 1.67 \pm [0.35]$ yr in the chromospheric activity of the fast-rotating ($P_{\text{rot}} = 11.36 \pm 0.17$ days) solar analog HD 30495. A simple three-component sine cycle model shows three full cycles in the time series, each with varying duration and amplitude. This, combined with the improved signal-to-noise from our longer time series, has allowed us to demonstrate the “pronounced periodicity” that was previously lacking to classify this star as cycling in the Baliunas et al. (1995) study.

Taking the ratio of these periodicities to the rotation period, $n = P_{\text{cyc}}/P_{\text{rot}}$, we find $n_{\text{long}} \approx 400$ rot/cyc and $n_{\text{short}} \approx 50$ rot/cyc respectively, which closely correspond to the “active” and “inactive” sequences found in the stellar sample of Böhm-Vitense (2007). What is remarkable from that work is that the Sun appeared to be a unique outlier in that sample, with $n_{\text{long},\odot} \approx 150$, and taking the solar QBO period as 2 years, $n_{\text{short},\odot} \approx 30$. Why does the Sun appear as an outlier with respect to the stars? One explanation could be that the relatively small sample of 21 stars in Böhm-Vitense (2007) is insufficient to show the full picture of the relation between P_{cyc} and P_{rot} , and further data will simply erase the observed trends. Indeed, the sample of Saar and Brandenburg (1999), which is a superset of the Böhm-Vitense sample, includes a few neighboring points for the Sun on log plots of quantities proportional to P_{cyc} and P_{rot} . Our datum for HD 30495, however, is decidedly not a neighbor of the Sun on these plots, while it agrees well with the trend set by stars on the active

branch. Both the Sun and this star have a similar time scale for the observed long and short periodicities, yet rotation, $\bar{\Omega}_*/\bar{\Omega}_\odot \approx 2.3$, is very different. This poses a serious problem for Babcock-Leighton flux transport dynamo models whose time scale is determined by the meridional flow. 3D hydrodynamic models show that meridional flow speed decreases with increased rotation rate (Ballot et al., 2007; Brown et al., 2008), but kinematic mean-field dynamos with one meridional flow cell have cycle times proportional to the flow speed and hence slower cycles with faster rotation. Jouve et al. (2010) investigated this problem, finding that multiple meridional flow cells in latitude were needed to make cycle period proportional to rotation. Unfortunately, observations to determine whether this indeed occurs in fast-rotating stars such as HD 30495 are nearly beyond imagination due to the extremely slow flow speeds, of order ~ 10 m/s on the Sun.

HD 30495 is nearly rotation and cycle-degenerate with the K2 star ϵ Eridani, which has $P_{\text{rot}} = 11.1$ days and $P_{\text{long}} = 12.7$ yr, but a longer $P_{\text{short}} = 2.95$ yr (Metcalf et al., 2013). In this comparison, the experimental variables include the depth of the convection zone, with stellar structure models predicting a deeper convective region for the cooler K star, as well as the average convective velocity. Apparently, these factors alone are not enough to prescribe a substantially different long-term cycle period.

In most dynamo theories, differential rotation is the driving force of the Ω -effect, responsible for turning poloidal magnetic flux into toroidal flux (e.g. Babcock, 1961). Stars with greater differential rotation would be expected to “wind up” the field faster, and we should reasonably expect a shorter cycle time, as at least half of the process would be faster (the other half being the return of toroidal field to poloidal). Using our range of rotation period detections, the total measured surface rotational shear is $\Delta\Omega = 1.67 \pm 0.15$ degrees/day. Then, using the the solar surface differential rotation

result of Snodgrass and Ulrich (1990), we calculate the equator-to-pole total shear for comparison, finding $\Delta\Omega_*/\Delta\Omega_\odot \geq 0.40 \pm 0.03$, which is a lower bound due to the unknown latitude ranges causing the rotation signal on the star. However, if one is prepared to assume that the spot latitudes of HD 30495 never form above 45° , as for the Sun, then differential rotation can be compared in terms of the solar shear from the equator to 45° , giving $\Delta\Omega_*/\Delta\Omega_{45,\odot} \geq 1.02 \pm 0.09$. Both results allow the possibility that HD 30495 and the Sun have equivalent surface differential rotation, which might help to explain their similar cycle characteristics. Asteroseismic measurements of HD 30495 may be able to put tighter limits on $\Delta\Omega$ (Gizon and Solanki, 2004; Lund et al., 2014).

Our time-frequency analysis does not indicate coupling between the intermittent short-period variability and the long-period cycle for this star. This is in contrast to the solar QBO, whose amplitude is strongly modulated by the 11-yr cycle (Bazilevskaya et al., 2014). For HD30495, the amplitudes of the short-period variations are at times larger than the amplitude of the long-term cycle, bringing into question which component of the variability is more fundamental to its dynamo. The absence of correlation between the two periodicities may be an indication that they are of a fundamentally different nature. It would be interesting to know at what time scale global magnetic field polarity reverses in this star, if indeed it does reverse. The S time series during the densely-sampled SMARTS era shows a convincing sinusoidal variation, perhaps as convincing as the long-term trend. A campaign of Zeeman Doppler Imaging measurements spaced over at least a four year period should be able to determine if the short-period variability is polarity-reversing as well.

To the best of our knowledge, this is the first work to separate and characterize the amplitudes and durations of individual cycles from a stellar activity proxy. There are rich opportunities in this direction to explore the variability *of the cycles*

themselves, as well as differences in stellar behavior during times of minima and maxima, which in turn can provide additional constraints for dynamo models. Already, the periodic signals measured here, together with the global properties collected in Table 3.1, present a well-characterized object to study with dynamo models. The existence of two significant time scales of variability in activity poses an additional modeling constraint. This bright object is also a prime candidate for future asteroseismic observations, which can further constrain its mass, radius, rotation profile, and depth of the convection zone (Metcalf, 2009). Successful modeling of such well-described targets will hopefully lead to improved understanding of the dynamo process.

CHAPTER FOUR

LONG-TERM VARIABILITY OF SOLAR-ANALOG STARS

In the study of HD 30495 we have seen some of the advantages of analyzing a longer time series than the 25-year records available in Baliunas et al. (1995) (B95 hereafter). In this case, new data led us to reclassify the magnetic variability pattern from that of an erratic variable (“var” in B95) to a two-period variable, the longer period appearing to be cyclic and similar to the Sun in both its period and the variability of that period. On the other hand, the shorter period signal showed signs of intermittency that would be abuse to the term “cycle” if applied to this phenomenon. While the time scale of this short-period variation is similar to that of the solar quasi-biennial oscillation reported in the literature (Bazilevskaya et al., 2014), in HD 30495 the relative amplitude of the signal is comparable to the long-period cycle (see Figure 3.5), while for the Sun it is not. This observation illustrates one way in which the variabilities of HD 30945 and the Sun are *not* alike, which can be overlooked when considering the period of variability alone.

We wish to find other cycles which are like the Sun’s in all respects that can be measured in order to determine which stellar properties are conducive to Sun-like cycles. To that end, in this chapter we will examine composite time series up to 50 years in length using MWO and SSS observations for a set of stars similar to the Sun. We will examine the ways in which the extended record can change our understanding of the variability of these stars, and address the problem of estimating *cycle quality*. We will uniformly characterize our sample and attempt to understand how stellar properties determine the patterns of long-term variability.

4.1 Sample Selection and Characterization

We compared target lists from the MWO HK Project and the SSS program in order to find additional stars for which long composite time series could be constructed. We shall restrict our study to stars very similar to the Sun in order to limit the range of differences in stellar structure that may also play a role in determining long-term magnetic behavior. Our sample selection criteria are (1) must have overlapping periods of observation in both MWO and SSS, (2) must have $0.59 \leq (B - V) \leq 0.69$, *or* a spectral type classification from G0V to G5V. The $(B - V)$ restriction is the definition of a *solar analog* used in Cayrel de Strobel (1996) as the starting point for the search for *solar twins*. The spectral type restriction is approximately equivalent according to Allen (1973), and is meant to capture additional Sun-like stars which may be border cases in $(B - V)$. We used the target tables in B95, Duncan et al. (1991), and Hall et al. (2007b) to select this sample, finding 27 stars.

4.1.1 Binary Stars and Characterizing HD 81809

We searched for these stars in the Washington Double Star Catalog (WDS; Mason et al. 2001) and Ninth Spectroscopic Binary Catalog (SB9; Pourbaix et al. 2004) in order to avoid problematic binaries where the secondary may contaminate either the chromospheric activity record or the stellar property measurements we wish to use to characterize our sample. We identified two such stars: HD 178428 and HD 81809. HD 178428 has a short orbital period of 22 d and a maximum orbit-induced radial velocity $K_1 = 13.34 \text{ km s}^{-1}$ (Pourbaix, 2000). This translates to a Doppler shift of the Ca K-line of $0.18 \text{ \AA}$. This line shift was not compensated for in the MWO observations with the HK photometer (see Appendix B of B95) and was reported as

a rotation period in Baliunas et al. (1996b), clearly demonstrating that the orbital radial velocity has impacted the data. For this reason we have removed HD 178428 from this study.

HD 81809 is of special interest, as it was once thought to be a good solar analog (G2V; $(B - V) = 0.64$) with a Sun-like “excellent” cycle, but later found to be a binary (see B95 Appendix A & B, and Duquennoy and Mayor 1988). Due to the regularity of its S -index cycle (Figure B.12), it is clear that the activity variation is dominated by only one star of the pair. Duquennoy and Mayor (1988) disentangled the blended spectra and estimated a mass sum of $2.3 M_{\odot}$, and component masses of $M_A = 1.5 \pm 0.5 M_{\odot}$ and $M_B = 0.8 \pm 0.2 M_{\odot}$, which, together with the long rotation period of $P_{\text{rot}} \sim 40$ d and moderate S -index, led B95 to conclude that the lower mass component is responsible for the variation. Based on these considerations B95 estimated the spectral type to be $\sim K0$. However, Duquennoy and Mayor (1988) noted the discrepancy between their estimated spectral types and luminosity classes (G0V and G9V) and the dynamical parallax, which indicated a high absolute magnitude consistent with an evolved A component. Spectroscopic binaries that have been optically resolved can have their full orbital parameters and component masses determined. Pourbaix (2000) used the visual magnitude estimates obtained from speckle interferometry ($m_A = 5.80$, $m_B = 6.60$) to constrain the orbital solution of HD 81809, which leads to mass estimates of $M_A = 1.7 \pm 0.64 M_{\odot}$ and $M_B = 1.0 \pm 0.25 M_{\odot}$. The orbital period was estimated to be $P_{\text{orb}} = 34$ yr. The revised *Hipparcos* parallax estimate of Söderhjelm (1999), $\pi = 29.1 \pm 1.1$ mas, leaves little doubt that at least the A component is evolved. Photometry from the Tycho Double Star Catalog (Fabricius et al., 2002) converted to the Johnson B & V colors gives ($V_A = 5.56 \pm 0.01$ mag, $(B - V)_A = 0.64 \pm 0.01$ mag) and ($V_B = 7.45 \pm 0.01$ mag, $(B - V)_B = 0.65 \pm 0.02$ mag). Using the Söderhjelm (1999) parallax the

absolute magnitude of the A component is $M_{V,A} = 2.88 \pm 0.08$ mag. Using the Flower (1996) empirical bolometric corrections and color-temperature relationship and the Stephan-Boltzman law we find the fundamental properties for the A component to be $T_{\text{eff},A} = 5731 \pm 47$ K, $L/\mathcal{L}_{\odot}^{\text{N}} = 6.00 \pm 0.46$, and $R/\mathcal{R}_{\odot}^{\text{N}} = 2.5 \pm 0.1$, where the ratios are with respect to the nominal solar values of Prša et al. (2016). The high luminosity for a near-solar effective temperature proves that the A component is evolved.

Next, we argue that the chromospheric record of HD 81809 is dominated by the evolved A component. The mean rotation period $P_{\text{rot}} = 40.2$ d from Donahue et al. (1996) was obtained from spot modulation of the MWO S -index time series. We can obtain an estimate of the equatorial rotation period using the equation:

$$\frac{P_{\text{eq}}}{\sin i} = \frac{2\pi R}{v \sin i}, \quad (4.1)$$

where $v \sin i$ is the projected rotational velocity measured from Doppler broadening. We use $v \sin i = 2.9 \pm 0.3$ km s⁻¹ from Ammler-von Eiff and Reiners (2012). Next, assuming the rotational axis is aligned with the orbital axis, we use orbital inclination $i = 85.4^\circ \pm 0.1^\circ$ from the binary solution of Tokovinin et al. (2015). Using these values and the radius of the A component from above in equation (4.1) we find $P_{\text{eq},A} = 43 \pm 30$ d. The large uncertainty is dominated by the uncertainty in $v \sin i$, however the best value is close to the measurement of Donahue et al. (1996). Since the Doppler broadening of the blended spectra should be dominated by the much brighter A component ($\Delta m_V = 1.89$ implies a flux ratio $F_A/F_B = 5.70$), and the rotation period from the S -index record agrees with P_{eq} , we conclude that the S -index modulations of HD 81809 are dominated by the evolved A component. Using similar arguments for the B component, we find $P_{\text{eq},B} = 18 \pm 13$ d, such that the angle between the orbital axis and the rotational axis of B would have to be 58° in order for $P_{\text{eq},B}$ to

agree with $P_{\text{rot}} = 40.2$ d, which is physically challenging assuming both stars formed from the same disc of gas with some common initial angular momentum. Furthermore, the long rotation period is consistent with our expectations from conservation of angular momentum as an evolved star expands. We will keep the binary HD 81809 in our sample under the assumption that the chromospheric emission variation is due to the evolved A component.

4.1.2 Properties of the Sample

For the remaining 25 stars in our sample we obtained absolute magnitudes M_V , effective temperatures T_{eff} , and metallicities $[\text{Fe}/\text{H}]$ from the Third Geneva Copenhagen Survey (GCS3; Holmberg et al. 2009). Absolute magnitudes in GCS3 were derived using *Hipparcos* parallaxes (van Leeuwen, 2007) and precision is better than 1% for all of these nearby stars. Photometric effective temperatures are determined using a calibration from Strömgren photometry to effective temperature in stars with T_{eff} determined directly from the Stefan-Boltzmann law, $T_{\text{eff}} = (4F_{\text{bol}}/\sigma\theta^2)^{1/4}$, where F_{bol} is the bolometric flux, θ is the angular diameter, and σ is the Stefan-Boltzmann constant (Holmberg et al., 2007). We take the uncertainty in these temperatures to be a fixed 57 K, the standard deviation of the difference of GCS3 T_{eff} with measurements using angular diameters from the CHARA and SUSI interferometers (Holmberg et al., 2009).

With M_V and T_{eff} from GCS3, we obtain the bolometric magnitude $M_{\text{bol}} = M_V + \text{BC}$ using the bolometric corrections (BC) of Flower (1996) and Torres (2010). From there it is straightforward to calculate the luminosity and radius:

$$\frac{L}{\mathcal{L}_{\odot}^{\text{N}}} = \frac{L_0}{\mathcal{L}_{\odot}^{\text{N}}} \times 10^{-0.4 M_{\text{bol}}}, \quad \frac{R}{\mathcal{R}_{\odot}^{\text{N}}} = \frac{\sqrt{L/\mathcal{L}_{\odot}^{\text{N}}}}{(T/T_{\odot}^{\text{N}})^2}$$

The zero-point luminosity is that defined in IAU Resolution B2 at the 2015 General Assembly, $L_0 = 3.0128 \times 10^{28}$ W. This value is consistent for the Sun when we use the nominal solar values $\mathcal{L}_\odot^N = 3.828 \times 10^{26}$ W, $\mathcal{R}_\odot^N = 6.957 \times 10^8$ m, and $\mathcal{T}_\odot^N = 5772$ K defined in IAU Resolution B3 (Prša et al., 2016). These nominal values are chosen to be close to current best estimates of the respective quantities. The absolute magnitude derived for the Sun when using these nominal values is $M_{V,\odot} = 4.82$.

Effective temperatures, radii, luminosities, and metallicities for the stellar sample are tabulated in Table 4.1. Luminosity is plotted against effective temperature in Figure 4.1a. We see that while all stars are very near the Sun in effective temperature, a number of stars have evolved away from the main sequence and are in the subgiant phase. There is no widely accepted limit in absolute magnitude, luminosity, or radius for the subgiant phase, which in evolutionary models is typified by a short horizontal progression (decreasing temperature; fixed luminosity) on the HR diagram as hydrogen core burning ceases and hydrogen shell burning begins. The location of the subgiant branch depends on model calibration and input physics. Bressan et al. (2012) computes evolutionary tracks for $1 M_\odot$ and solar composition that place the subgiant branch at $L/L_\odot \approx 2.0$ – 2.5 , depending on the mixing length parameter and chemical mixing physics. We shall take $L/\mathcal{L}_\odot^N = 2.0$ as our working definition for the subgiant phase, which includes five stars. In order of increasing luminosity they are HD 141004, HD 142373, HD 9562, HD 81809, and HD 6920.

Note that a $1 M_\odot$ star on the horizontal subgiant branch would have already cooled out of our temperature domain, therefore any star in our sample with $L/L_\odot > 2.0$ is more massive than the Sun, and has cooled into our temperature domain. HD 81809 is the best example, with the estimated mass of the binary A component from

Table 4.1. Solar Analog Ensemble Properties

HD	$B - V$	M_V	$T/T_{\odot}^N$	$R/R_{\odot}^N$	$L/L_{\odot}^N$	[Fe/H]	P_{rot} (d)	Ref.
Sun	0.65	4.82	1.00	1.00	1.00	0.00	26.09	D96
1835	0.66	4.80 ± 0.02	0.99 ± 0.01	1.05 ± 0.02	1.03 ± 0.02	-0.02	7.81	G00
6920 ⁺	0.60	2.08 ± 0.03	1.01 ± 0.01	3.42 ± 0.08	12.34 ± 0.35	-0.14	14.0	B96
9562 ⁺	0.64	3.41 ± 0.02	1.01 ± 0.01	1.85 ± 0.04	3.62 ± 0.07	+0.13	29.0	B96
20630	0.68	5.04 ± 0.01	0.99 ± 0.01	0.93 ± 0.02	0.83 ± 0.01	0.00	9.2	G00
30495	0.63	4.87 ± 0.01	1.00 ± 0.01	0.97 ± 0.02	0.95 ± 0.01	-0.08	11.36	E15
39587	0.59	4.72 ± 0.01	1.02 ± 0.01	0.99 ± 0.02	1.08 ± 0.01	-0.16	5.36	D96
43587	0.61	4.28 ± 0.02	1.02 ± 0.01	1.24 ± 0.03	1.62 ± 0.03	-0.11	≥ 21.0	M10
71148	0.63	4.59 ± 0.02	1.00 ± 0.01	1.11 ± 0.02	1.24 ± 0.02	-0.10	≥ 21.6	M14
72905	0.62	4.86 ± 0.01	1.00 ± 0.01	0.98 ± 0.02	0.97 ± 0.01	-0.25	4.89	G00
76151	0.67	4.81 ± 0.01	0.98 ± 0.01	1.05 ± 0.02	1.03 ± 0.01	-0.04	15.0	D97
78366	0.60	4.53 ± 0.01	1.02 ± 0.01	1.08 ± 0.02	1.28 ± 0.01	-0.10	9.67	D96
81809 ⁺	0.64	2.88 ± 0.08	1.00 ± 0.01	2.50 ± 0.10	6.00 ± 0.46	-0.29	40.2	D96
97334	0.61	4.71 ± 0.02	1.01 ± 0.01	1.03 ± 0.02	1.10 ± 0.02	-0.09	8.25	G00
114710	0.57	4.46 ± 0.00	1.03 ± 0.01	1.09 ± 0.02	1.36 ± 0.01	-0.06	12.35	D96
115043	0.60	4.80 ± 0.02	1.00 ± 0.01	1.01 ± 0.02	1.02 ± 0.02	-0.22	5.861	H16
126053	0.63	5.09 ± 0.02	0.98 ± 0.01	0.92 ± 0.02	0.79 ± 0.02	-0.39	26.28	H16
141004 ⁺	0.60	4.01 ± 0.01	1.02 ± 0.01	1.38 ± 0.03	2.07 ± 0.02	-0.01	25.8	D96
142373 ⁺	0.56	3.60 ± 0.01	1.01 ± 0.01	1.73 ± 0.04	3.06 ± 0.03	-0.50	≥ 36.4	F97
143761	0.60	4.22 ± 0.01	1.00 ± 0.01	1.31 ± 0.03	1.74 ± 0.02	-0.25	17.0	B96
146233	0.65	4.79 ± 0.01	1.00 ± 0.01	1.02 ± 0.02	1.03 ± 0.01	-0.02	22.7	P08
176051	0.59	4.36 ± 0.01	1.01 ± 0.01	1.20 ± 0.02	1.51 ± 0.02	-0.19	≥ 15.2	S83
190406	0.61	4.55 ± 0.01	1.01 ± 0.01	1.10 ± 0.02	1.27 ± 0.01	-0.12	13.94	D96
197076	0.76	4.83 ± 0.02	1.00 ± 0.01	1.00 ± 0.02	0.99 ± 0.02	-0.22	≥ 18.7	B84
206860	0.59	4.72 ± 0.02	1.02 ± 0.01	1.00 ± 0.02	1.08 ± 0.02	-0.19	4.91	G00
217014	0.67	4.49 ± 0.01	0.99 ± 0.01	1.19 ± 0.02	1.37 ± 0.02	+0.12	21.9	S10
224930	0.67	5.32 ± 0.06	0.95 ± 0.01	0.91 ± 0.03	0.67 ± 0.04	-0.78	15.01	H16

Note. — P_{rot} written as a lower limit are $P_{\text{rot}}/\sin i$ estimated using $v \sin i$ in the reference and the listed radius. Rotation references are S83=Soderblom (1983); B84=Benz and Mayor (1984); B96=Baliunas et al. (1996b); D96=Donahue et al. (1996); F97=Fekel (1997); D97=Donahue et al. (1997); G00=Gaidos et al. (2000); P08=Petit et al. (2008); S10=Simpson et al. (2010); M10=Martínez-Arnáiz et al. (2010); M14=Marsden et al. (2014); E15=Egeland et al. (2015) (Chapter 3); H16=Hempelmann et al. (2016). HD numbers marked with ⁺ are subgiants.

the orbital solution $M_A = 1.7 \pm 0.6 M_{\odot}$ (Pourbaix, 2000). In the text and tables that follow, we will follow their HD number with ⁺ to distinguish them as subgiants.

Rotation periods shown in Table 4.1 and Figure 4.1b were gathered from the literature. These rotations are measured from spot-induced modulations of Ca H & K or photometric time series, with the exception of HD 146233 (18 Sco). Its rotation is estimated from an inversion spectropolarimic observations, where it is assumed that the observed variation in the polarimetric signal is due to stellar rotation (Petit

et al., 2008). Where rotations were available from multiple references, preference was given to the source that analyzed a longer time series. This essentially ranks the sources in chronological order, as most sources used the MWO observations. The most reliable rotations are those from Donahue et al. (1996), who required that rotation modulations in the MWO time series were detected in at least five observing seasons. The 26.09 d rotation period for the Sun was obtained from a windowed periodogram analysis of the NSO/SP Sun-as-a-star Ca K-line record, and is therefore derived in the same way as most of the stellar rotation periods (Donahue and Keil, 1995). This rotation period corresponds to a latitude of about 33.5° according to the differential rotation formula of Snodgrass and Ulrich (1990).

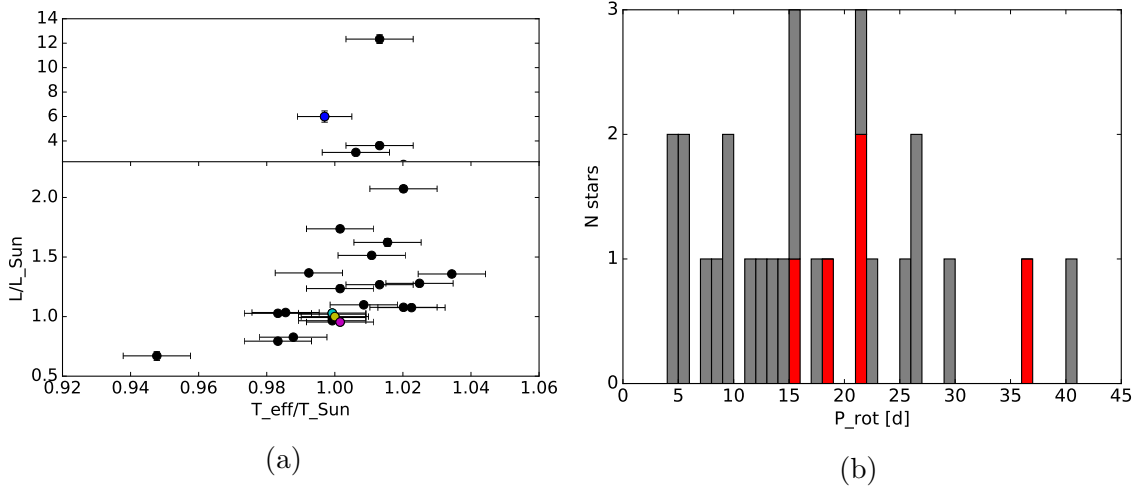


Figure 4.1 (a) Luminosity versus effective temperature in solar units. Luminosity error bars are too small to be seen for most of the stars. Four stars are indicated by colored markers: the Sun (yellow), HD 30495 (magenta), 18 Sco (cyan), and HD 81809+ (blue). (b) Distribution of rotation periods in 1-day bins. Stars with only a lower limit estimate ($P_{\text{rot}}/\sin i$) are indicated in red.

The rotation period of HD 224930 is in some doubt. Baliunas et al. (1996b) was the first to report a rotation for this star, finding $P_{\text{rot}} = 33$ d. Huber et al. (2009) performed a detailed spot modeling analysis using high-cadence space-

based photometry from *MOST*, finding a faster rotation period of 11.6 d. These authors urged caution for the result and advised follow-up observations to confirm the measurement, which Hempelmann et al. (2016) appears to have done, finding $P_{\text{rot}} = 15$ d in one season of Ca H & K measurements from TIGRE. We searched 47 seasons of *S*-index measurements of HD 224930 for rotational modulations using a periodogram analysis and found no statistically significant (99% confidence level) signals between 1 and 50 d in the data. Hempelmann et al. (2016) raised the possibility that their single-season detection, roughly half the Baliunas et al. (1996b) value, was due to an occurrence of spots on opposite latitudes. It is unlikely, however, that the same problem occurred for both Hempelmann et al. and Huber et al., who observed at different epochs. We therefore prefer the fast rotation period measurement due to the rough agreement from two separate groups using two very different datasets and techniques. Additional confirmation would be helpful, and should come with further TIGRE observations.

Where rotation periods were not available in the literature we computed a lower limit from an estimation of the projected rotational velocity, $v \sin i$, derived from the observed effect of rotational broadening of stellar spectral lines and dependent on the inclination angle i . We can then compute a lower limit for the equatorial rotation period using equation (4.1) and our estimation for the stellar radius. We note that rotations from these lower limits and the inversion of Petit et al. (2008) for 18 Sco are qualitatively different than those obtained from spot modulation. This latter case is of particular interest, as 18 Sco is a well-known “solar twin” in the literature, photometrically and spectroscopically very similar to the Sun (Meléndez et al., 2014; Porto de Mello and da Silva, 1997). Rotations obtained from spot modulation give the rotation period of the unknown latitude at which spots reside. In the case of the Sun, spots are rarely seen on the equator (see Figures 1.2 and 1.5). However,

rotations obtained from Doppler broadening of spectral lines and from the inversion of Petit et al. (2008) refer specifically to rotation at the equator, P_{eq} . Assuming solar-like differential rotation, these estimates are lower limits compared to what would be observed from mid- to high-latitude spot modulation. The difference is small for the Sun, where spots appear at a maximum of $\approx 35^\circ$ latitude, giving a maximum relative shear $(P_{35^\circ} - P_{\text{eq}})/P_{\text{eq}}$ of only 7% using the differential rotation formula of Snodgrass and Ulrich (1990). It is unknown what the spot distributions and surface differential rotation is for these stars with P_{rot} expressed in terms of P_{eq} . The distinction is unimportant for the stars where we only have a lower limit $P_{\text{eq}}/\sin i$ in any case, but for 18 Sco it means that the difference in rotation with respect to the Sun is even less than the 3.4 d indicated in Table 4.1. Comparing equatorial rotation of the Sun $P_{\text{eq},\odot} = 24.47$ d (Snodgrass and Ulrich, 1990) to 18 Sco, we find $\Delta P_{\text{eq}} = 1.8 \pm 0.5$ d, where the uncertainty is dominated by 18 Sco.

Our ensemble can be roughly characterized by the median values. In this work we denote the median of quantity x as $\widehat{x}$. From Table 4.1 we have $\widehat{T/\mathcal{T}_\odot^N} = 1.00$, $\widehat{R/\mathcal{R}_\odot^N} = 1.06$, $\widehat{L/\mathcal{L}_\odot^N} = 1.16$, $\widehat{[Fe/H]} = -0.11$, and $\widehat{P_{\text{rot}}} = 15.0$ d. Therefore, we may say that our ensemble is very nearly the solar temperature, but slightly more luminous and metal poor than the Sun. The distribution in rotation (Figure 4.1b is roughly uniform from $P_{\text{rot}} = 4$ to 15 d, with sparser sampling for $P_{\text{rot}} > 15$ d.

4.2 SSS Calibration to the MWO S -index Scale

Hall et al. (2007b) describes the S -index calibration of the SSS targets to the MWO scale, which we summarized in Section 2.3.4. We found from the study of HD 30495 (Chapter 3) that an additional scaling factor was required to smooth out a discontinuity in the time series where SSS and MWO overlap. In Section 2.4.5 we noted how the MWO HKP-1 to HKP-2 cross-instrument calibration curve of Vaughan

et al. (1978) had a residual scatter of $\approx 5\%$, which means that for any *individual star* on the curve a composite time series would have a discontinuity in the mean with a typical value of about 5%. We refer to the per-star distance from an instrument-to-instrument calibration curve as the *calibration anomaly*. Hall et al. (2007b) found after their calibration that the rms scatter with respect to the flat-activity stars was $0.011 S$. While this is very small compared to the whole range of S among all target stars, for low-activity stars like the Sun with $S \approx 0.170$, this calibration anomaly is comparable to the amplitude of the solar cycle ($\langle \Delta S \rangle = 0.0145$). Therefore, it is clear that to construct composite time series without discontinuities *each star will require its own calibration factor*. In the case of the SSS the calibration anomaly may simply be a consequence of uncertainty in the stellar parameters $(b - y)$, $(B - V)$, and T_{eff} which are used in the transformation of SSS HK flux to S (see equation 2.7).

The SSS data in the present study are from a large data reduction done in August 2015. The reduction procedure was described in Section 2.3.4. Both the MWO and SSS time series contained a number of outlying points which complicate our analyses. We removed outlier points by applying a median absolute deviation (MAD) filter, where:

$$\text{MAD}(X) = \text{median}(|X_i - \text{median}(X)|)$$

for a univariate data set X . MAD is rank-based measure of statistical dispersion analogous to the standard deviation from the mean, but less sensitive to outliers. In the special case of normally distributed data, the standard deviation σ is related to the MAD by $\sigma \approx 1.4826 \text{ MAD}$. We use this formulation, denoted σ_{MAD} , to remove data $4\sigma_{\text{MAD}}$ from the median of the entire time series *or* $4\sigma_{\text{MAD}}$ from the seasonal median of the point in question.

Our present task is to further calibrate these data to the MWO S -index scale using coincident observations. We consider an SSS observation “coincident” with an MWO observation if it was taken the same night or one night earlier or later. In Figure 4.2 we plot the S -indices from MWO and SSS for 573 coincident measurements among the 26 stars in our sample. For most of the stars, the calibration is well clustered about the $y = x$ identity line. However, we find three stars with measurements dispersed far from the identity line: HD 1835, 39587, and HD 206860. Measurements from these stars are marked as “neglected” in Figure 4.2. These are some of the faster rotating stars in our sample, with $P_{\text{rot}} = 7.81$ d, 5.36 d, and 4.91 d respectively, leading to a suspicion that our relaxed definition of “coincident” may have resulted in significant rotation and appearance of stellar active regions on the limb between measurements. However, this cannot be the explanation, since (1) other stars rotating just as fast (e.g. HD 72905 with $P_{\text{rot}} = 4.89$ d) do not demonstrate this problem, and (2) the dispersion of measurements do not span the identity line, as one would expect if the appearance of active regions and order of the measurements were random. The offset from the identity line for these stars appears to be systematic, and its cause is unknown at this time. We proceed by simply removing these stars from our calibration procedure.

We used a linear least-squares regression to fit a straight line to coincident measurements of the 23 well-behaved stars in our sample. We find:

$$S_{\text{SSS}} = f(S_{\text{MWO}}) = 0.907 S_{\text{MWO}} + 0.024 \quad (4.2)$$

The residual of the nightly measurements to this fit has a standard deviation of 0.012 S . In Figure 4.2 we also plot the median centroid of coincident measurements for each star, $(\widehat{S_{\text{SSS}}}, \widehat{S_{\text{MWO}}})$. The residual of these centroids to the fit has a standard deviation of 0.014 S . To produce composite time series free from discontinuities, these

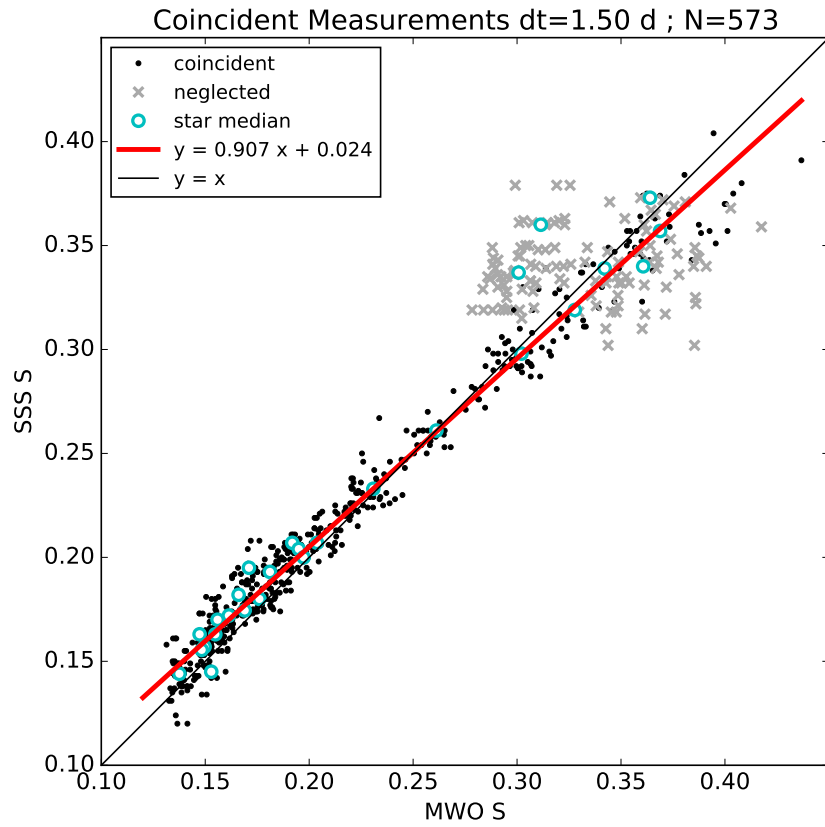


Figure 4.2 Semi-coincident measurements ($\Delta t < 1.5$ d) from MWO and SSS. Data from three anomalous stars were neglected due to the large scatter in the coincident measurements.

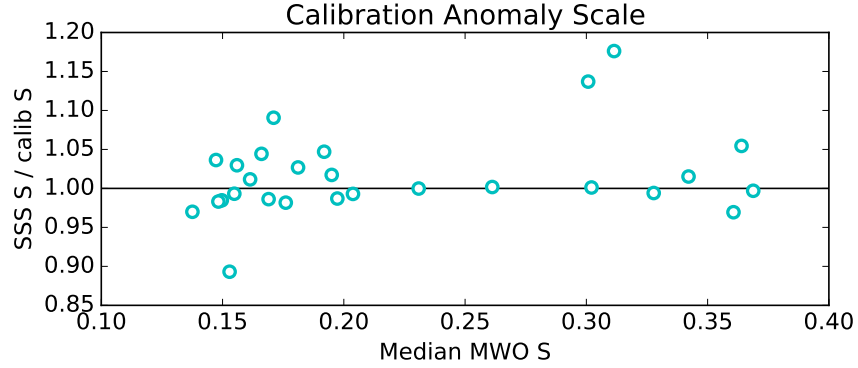


Figure 4.3 Calibration anomaly for the stars in Figure 4.2.

calibration anomalies must be taken to zero. We accomplish this by inverting equation (4.2) and additionally scaling each time series by the ratio of $f(\widehat{S_{\text{MWO}}})$ to $\widehat{S_{\text{SSS}}}$:

$$S'_{\text{SSS}} = (1.102 S_{\text{SSS}} - 0.026) \cdot \frac{f(\widehat{S_{\text{MWO}}})}{\widehat{S_{\text{SSS}}}} \quad (4.3)$$

This approach combines an ensemble average calibration with a per-star scaling to force the median values to lie on the calibration line. We choose a scaling factor as opposed to an additive correction because we suspect uncertainties in the observational quantities used in equation 2.7 are largely to blame for the calibration anomalies. Figure 4.3 shows the scaling factors applied as a function of $\widehat{S_{\text{MWO}}}$. Most of the scalings applied are less than 5%, the notable exceptions being the three misbehaving stars mentioned above, as well as HD 178428.

The results of the calibration can be seen in Figure 4.5 and Appendix B, where MWO data are plotted in red and SSS in blue. For HD 20630 and HD 76151 the data points are in good agreement during the period of overlap. HD 9562⁺ has very sparse data from SSS during the period of overlap, and a suspicious upward trend in the SSS CCD-1 data before the transition to CCD-2 in 2008. Similar problems are seen in other low-variability time series HD 43587 and HD 143761, while others

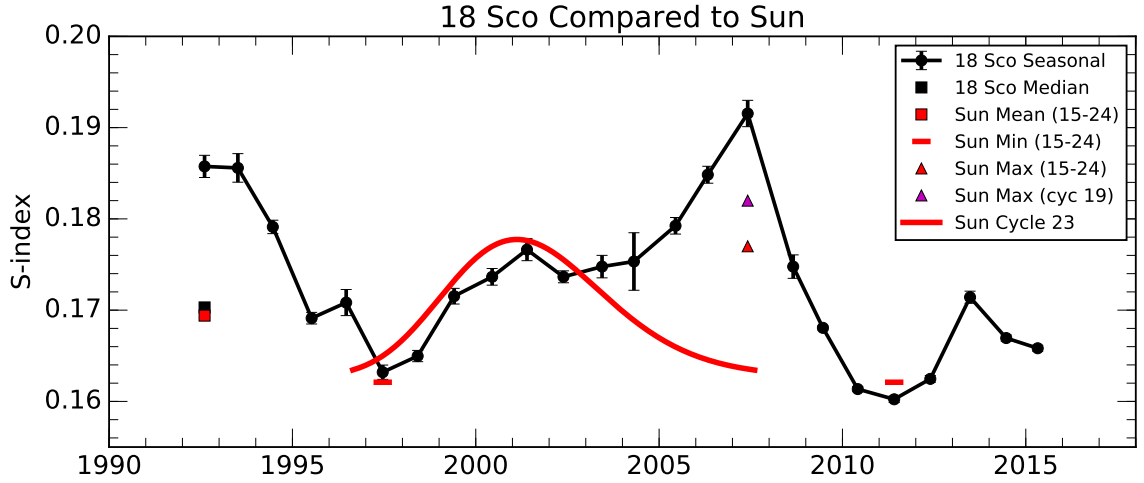


Figure 4.4 Calibrated composite MWO + SSS seasonal median time series for solar twin 18 Sco compared to the new calibration of the Sun from Chapter 2.

remain flat throughout (e.g. HD 126053 and HD 142373⁺). These trends are small and short-lived, and do not have a significant effect on our conclusions.

4.3 Activity of 18 Sco Compared to the Sun

Figure 4.4 shows an interesting result from our ensemble calibration and the new calibration of the solar S -index from Chapter 2. The figure shows the seasonal medians of the solar twin 18 Sco (HD 146233) with the solar cycle 23 model fit from Figure 2.1 overplotted. Error bars on the seasonal medians are $\sigma_{\text{MAD}}/\sqrt{N}$. The nightly measurements of 18 Sco can be seen in Figure B.20. The mean solar minimum from Table 2.2, $0.1621 \pm 0.0008 S$, is also plotted next to the minima of 18 Sco at decimal years 1997.5 and 2011.4. The cycle minimum S -index of these two stars are remarkably close. The mean of the two minima in the 18 Sco record is 0.1617 ± 0.0004 , which is within uncertainty of the solar value. The 18 Sco maximum at 2007.4 is at $S_{\text{max}} = 0.192 \pm 0.001$. This is 8.2% higher than the mean solar maximum, and 5.4% higher than the maximum of cycle 19, the strongest

cycle in our solar record. Consequently, the median activity of 18 Sco is marginally higher, by 0.5%. The close agreement shows that stars with nearly identical physical properties have nearly identical magnetic activity. This may well be expected, but is not necessarily guaranteed given the nonlinear nature of the MHD equations which govern the dynamo. This result required accurate calibration to the common MWO *S*-index scale, which was attained by tying the SSS data to the MWO record during the period of overlap from 1997–2002.

4.4 Amplitude of Variability

Figure 4.5 shows the calibrated MWO+SSS composite time series for the Sun and three other stars. No shifts have been applied to the data; the plot illustrates the range of mean values, amplitudes, and patterns of variability in our sample. The Sun’s approximately regular, low amplitude variability is in stark contrast to the subdued behavior of HD 9562⁺ as well as the erratic variability of the active stars HD 76151 and HD 20630. This latter star demonstrates three clear instances of a rise and fall in activity from about 1977 to 1995, which was followed by a sharp rise to levels not seen since 1974, where it remained until the present, punctuated by occasional dips and jumps. In the case of HD 76151 and HD 20630, it is clear that the variability is significantly more complicated than that of the Sun, which can be reasonably well described by a single amplitude and the times of minimum and maximum. Finding structures analogous to the solar cycle in these active star time series is a stretch of the imagination.

Noting that the mean level and the amplitude of variability on both long and short time scales is significantly different than the Sun for these stars, we will now quantify these aspects of the stellar variability in our ensemble. We will continue to use rank-based estimators for this task, as we have no expectations that the underlying

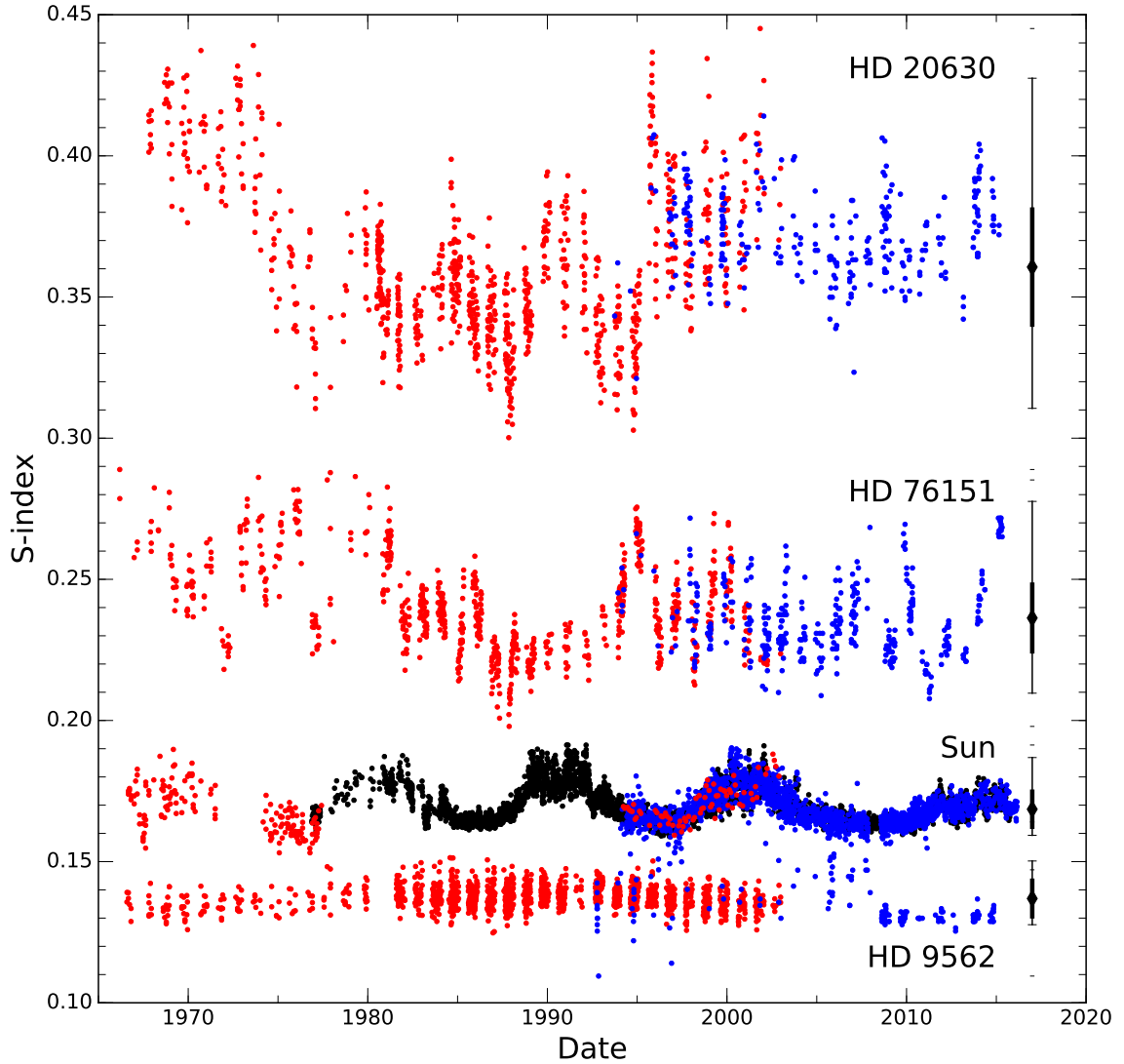


Figure 4.5 Calibrated composite MWO (red) + SSS (blue) time series for the Sun and three solar analogs. The relative offsets of each time series are real. Data from the Sun are those described in Chapter 2. The black bar symbol on the right of each time series indicates four quantities: (1) the middle diamond is at the median S for the complete time series, (2) the thin capped bar indicates the location of the 1st and 99th percentile of the data (3) the small dashes indicate the minimum and maximum points and (4) the thick bar is the median seasonal inner-98% amplitude.

distributions are Gaussian. We use the median to estimate a typical value for both the whole time series and seasonal bins. We use the inter-98% range, denoted A_{98} , to estimate the amplitude of both the whole time series and seasonally binned data. We prefer this measure of scale over estimates of smaller spans like the interquartile range or the MAD because it closely captures the full range of variation without being unduly impacted by rare extreme values.

Statistics for each star are listed in Table 4.2. Columns 2–4 give statistics on the duration of the time series and the number of measurements. The median duration of observation is 48.4 yr, while the median number of seasons with 10 or more measurements is lower, 37 seasons. The median number of measurements in a time series is 1077, and the median number of measurements in an observing season is 25. The distribution of durations is bimodal, with most stars having over 45 years of observation, six stars with under 35 years, and no intermediate cases. The stars with shorter records had a later start date for observations in the MWO program.

Figure 4.6 shows the dependence of median activity, total amplitude A_{98} , and median seasonal amplitude $\widehat{A}_{98,s}$ on rotation. Mean activity and activity amplitude roughly increases with faster rotation, although there is significant scatter. The diagram can be divided into three regions: fast rotating stars with $P_{\text{rot}} \lesssim 10$ d, intermediate rotation $10 \lesssim P_{\text{rot}} \lesssim 15$ d, and slow rotation $P_{\text{rot}} \gtrsim 15$ d. The fast rotators have the largest mean activity and amplitudes, with no obvious trend in this region. The intermediate rotators show a decreasing trend of activity and amplitude, while the slow rotators all have roughly similar activity and amplitude.

The amplitude of variability shows significant scatter with respect to rotation, but has a tighter relationship with the median level of activity, as shown in Figure 4.7a. A linear trend $\widehat{A}_{98,s} = (0.13 \pm 0.01) \widehat{S} - 0.00093$ (Pearson $r = 0.97$) is seen for the median seasonal amplitude over the whole range of activity. For the total amplitude,

Table 4.2. Solar Analog Ensemble Statistics

HD	T	N_s	N_{obs}	$\widehat{N}_{\in s}$	$\widehat{S}$	$\min(\widehat{S}_s)$	$\max(\widehat{S}_s)$	A_s	A_{98}	$\widehat{A}_{98,s}$
Sun	49.5	46	6349	154	0.1686 ± 0.0001	0.1603	0.1806	0.0203	0.0275	0.0124
1835	48.3	40	1242	23	0.3517 ± 0.0007	0.3019	0.3974	0.0955	0.1108 ⁺	0.0433 ⁺
6920 ⁺	48.2	27	771	17	0.1931 ± 0.0003	0.1836	0.2129	0.0293	0.0385	0.0144
9562 ⁺	48.3	32	1605	44	0.1369 ± 0.0001	0.1299	0.1459	0.0159	0.0226	0.0126
20630	47.3	42	1232	22	0.3606 ± 0.0007	0.3285	0.4187	0.0902	0.1169 ⁺	0.0408 ⁺
30495	47.3	37	987	25	0.3020 ± 0.0005	0.2763	0.3264	0.0502	0.0708 ⁻	0.0330 ⁻
39587	49.1	41	1288	25	0.3126 ± 0.0004	0.2832	0.3452	0.0620	0.0815 ⁻	0.0295 ⁻
43587	48.4	35	934	18	0.1552 ± 0.0001	0.1524	0.1636	0.0111	0.0267	0.0111
71148	22.4	17	294	13	0.1567 ± 0.0004	0.1492	0.1637	0.0145	0.0236	0.0089
72905	31.6	17	430	24	0.3708 ± 0.0010	0.3533	0.4040	0.0507	0.0815 ⁻	0.0337 ⁻
76151	49.1	41	1167	28	0.2363 ± 0.0005	0.2165	0.2741	0.0576	0.0679	0.0236
78366	49.2	37	1123	28	0.2535 ± 0.0006	0.2240	0.2907	0.0667	0.0795	0.0236
81809 ⁺	49.1	39	1077	20	0.1746 ± 0.0003	0.1579	0.1892	0.0313	0.0453	0.0158
97334	49.2	39	928	19	0.3409 ± 0.0006	0.3160	0.3795	0.0635	0.0795 ⁻	0.0305 ⁻
114710	49.3	42	1455	25	0.2036 ± 0.0003	0.1892	0.2204	0.0313	0.0439	0.0174
115043	32.0	18	479	20	0.3232 ± 0.0010	0.2947	0.3507	0.0559	0.0792 ⁻	0.0322 ⁻
126053	49.3	33	754	16	0.1663 ± 0.0002	0.1601	0.1735	0.0134	0.0246	0.0107
141004 ⁺	49.3	40	1121	23	0.1573 ± 0.0002	0.1510	0.1688	0.0178	0.0253	0.0112
142373 ⁺	49.3	41	1666	29	0.1470 ± 0.0001	0.1418	0.1503	0.0085	0.0161	0.0073
143761	49.3	40	1801	39	0.1505 ± 0.0001	0.1484	0.1627	0.0143	0.0200	0.0092
146233	34.0	24	740	27	0.1703 ± 0.0003	0.1602	0.1915	0.0313	0.0414	0.0157
176051	34.2	28	877	28	0.1793 ± 0.0002	0.1714	0.1871	0.0157	0.0311	0.0141
190406	49.1	42	1601	29	0.1944 ± 0.0002	0.1832	0.2189	0.0357	0.0489	0.0163
197076	23.0	17	359	19	0.1732 ± 0.0006	0.1631	0.1912	0.0282	0.0402	0.0133
206860	48.4	37	1619	32	0.3212 ± 0.0005	0.2987	0.3585	0.0598	0.0771 ⁻	0.0309 ⁻
217014	47.2	31	986	29	0.1493 ± 0.0001	0.1445	0.1563	0.0118	0.0220	0.0092
224930	48.3	37	1049	22	0.1858 ± 0.0003	0.1771	0.2100	0.0329	0.0424	0.0121

Note. — Columns are (1) HD number (2) duration of time series in years (3) number of seasons with more than 10 observations (4) total number of observations (5) median number of observations in a season (6) grand median and estimated uncertainty (7) minimum seasonal median (8) maximum seasonal median (9) seasonal median amplitude, $\max(\widehat{S}_s) - \min(\widehat{S}_s)$ (10) inner-98% of whole time series (11) median seasonal inner-98%. Amplitudes marked with a ⁺ lie near the linear relationship with median activity shown in Figure 4.7, while those with a ⁻ lie below it. HD numbers marked with ⁺ are subgiants. In the electronic version of this document the first column is hyperlinked to the time series.

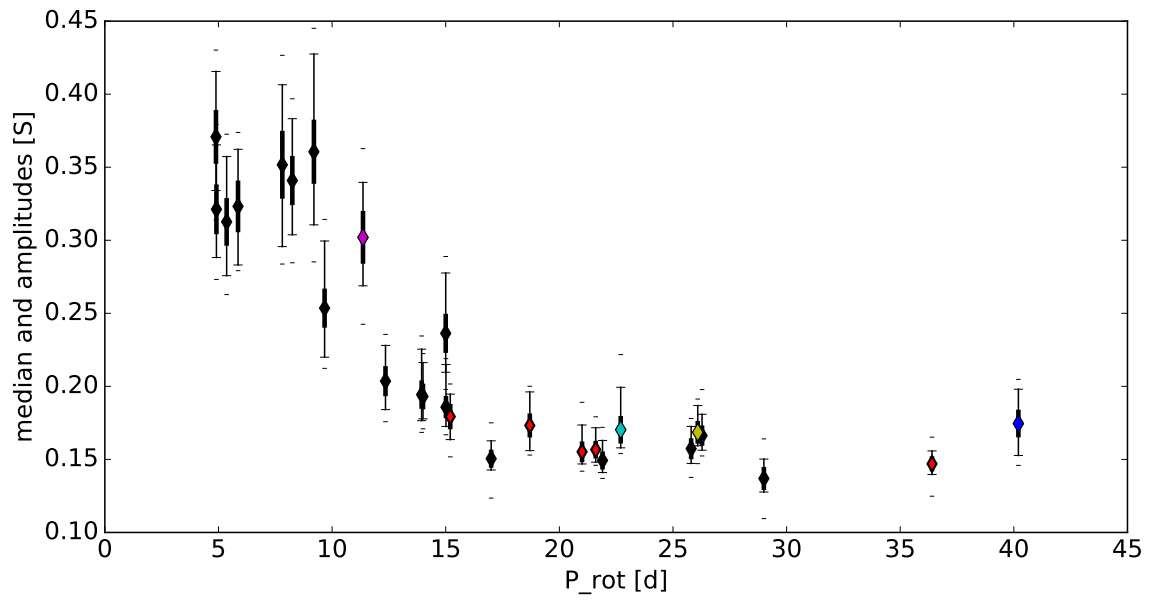


Figure 4.6 Activity versus rotation for the stellar ensemble. The bar symbols indicate the median activity and amplitude, as described in Figure 4.5. Stars with only a lower limit estimate ($P_{\text{rot}}/\sin i$) are indicated in red. Four stars are indicated by colored markers: the Sun (yellow), HD 30495 (magenta), 18 Sco (cyan), and HD 81809⁺ (blue).

a linear trend $A_{98} = (0.52 \pm 0.04) \hat{S} - 0.056$ (Pearson $r = 0.95$) for $0.13 \leq \hat{S} \leq 0.26$ reasonably fits 19 of the 27 stars, however six stars with $\hat{S} > 0.27$ fall far away from the linear trend, with A_{98} staying below about 0.08. Two other stars have much higher amplitudes ($\approx 0.115 S$) at this activity level, but still not quite at the level of the trend defined by the lower activity stars (see dashed red line extrapolation in Figure 4.7a). Figure 4.7b shows a different estimate of amplitude: the seasonal median range $A_s = \max(\hat{S}_s) - \min(\hat{S}_s)$ plotted against $\min(\hat{S}_s)$. This figure gives an estimate of how far the time series of seasonal medians rises from its lowest point. For the Sun, this would give a simple estimate of the amplitude of the largest cycle in the record. Here we see again that stars with high $\min(\hat{S}_s) > 0.25$ are segregated into two groups: two stars with $A_s \approx 0.095$ and six stars with $A_s \approx 0.06$. In this figure, however, the two high activity, high amplitude stars straddle the trend defined by the lower activity stars.

The two stars with high amplitudes for their median activity level are marked with a $+$ next to the amplitude value in Table 4.2, while the six stars with low amplitude are marked with a $-$. The high amplitude stars have about 45% higher amplitude than the low amplitude stars. Looking to Table 4.1, the only distinguishing feature between the properties two groups is that the two high amplitude stars have higher metallicity than the six low amplitude stars, with mean values of -0.01 dex and -0.14 dex respectively. This difference in metallicity seems inadequate to explain the amplitude difference, as the star with the lowest metallicity ($[\text{Fe}/\text{H}] = -0.78$ dex), HD 224930, does not appear to be likewise deficient in amplitude. One possible explanation that is not intrinsic to the star are projection effects due to inclination. Shapiro et al. (2014) estimated that the S -index changes by -5% when perspective changes from the equator to the poles for a solar-like mid-latitude distribution of activity. On the other hand, for a polar distribution of activity the same change of

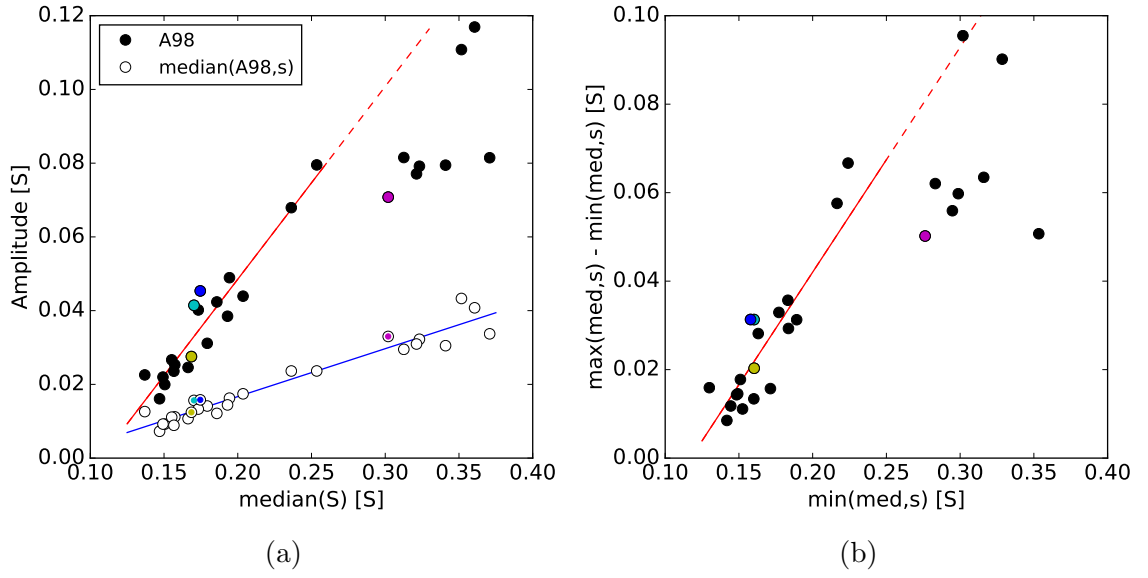


Figure 4.7 Amplitudes versus activity. Panel (a) black points are the inner-98% amplitude of the entire time series, while white points are the median seasonal inner-98% amplitude. Panel (b) shows the amplitude of seasonal medians versus the minimum seasonal median. Four stars are indicated by colored markers: the Sun (yellow), HD 30495 (magenta), 18 Sco (cyan), and HD 81809⁺ (blue). Solid lines are least-squares fit to the data along their extent, while dashed lines are an extrapolation of the fit.

perspective results in a +30% increase in S . The S -index is thought to be dominated by projection effects of the plage areas, as opposed to the center-to-limb variation in intensity. The larger difference for a polar distribution is due to the opposite hemisphere becoming completely obscured as the inclination angle decreases. This effect can apparently explain most of the amplitude differences seen here. To test this, We use the equation $\sin i = P_{\text{eq}} v \sin i / 2\pi R_*$ to estimate the inclination of these stars. Mishenina et al. (2012) finds $v \sin i = 5.20 \text{ km s}^{-1}$ for HD 1835 and 6.51 km s^{-1} for HD 20630. Assuming an uncertainty of 0.1 km s^{-1} in these measurements, and 0.5 d for the rotation period, we obtain $i = 49^\circ \pm 21^\circ$ for HD 1835, and $\sin i = 1.27 \pm 0.17$ for HD 20630, which is out of the range of the sine function. We stress again that P_{rot} in Table 4.1 is likely not the rotation of the equator, and if $P_{\text{rot}}/P_{\text{eq}} = 1.27$ the inclination calculation would be valid with $i = 90^\circ$. This rotational shear is not outside the limits of solar differential rotation, therefore it is plausible that HD 20630 has high inclination. In Chapter 3 (Table 3.1) we used the same method to find a high inclination for HD 30495, $\sin i = 1.0 \pm 0.2$. While the uncertainties are large, the indications are that HD 1835 has a lower inclination than HD 20630 and HD 30495, yet its activity amplitude is roughly equivalent to HD 20630 and $\approx 45\%$ higher than HD 30495. It is also unlikely that the other five low-amplitude stars have the same inclination as HD 30495. Therefore, we find it unlikely that the amplitude differences discussed here are due to an effect of inclination.

Another possibility for the amplitude differences at high median activity is that the six low amplitude stars have not been observed long enough to capture their full range of variability. Perhaps with another 50 years of observation they would move closer to the extrapolated line. Another factor to consider is the saturation of the Ca H & K emission in regions of strong magnetic field $|B| \gtrsim 300 \text{ G}$ (Harvey and White, 1999; Pevtsov et al., 2014). If the low-amplitude stars are somehow more efficient at

producing regions of strong field which saturate H & K emission, the result could be to dampen the amplitude due to less space available on the surface for H & K emitting regions. This is a delicate argument, because one would expect the same phenomenon to also reduce the median activity level. One would also expect this problem to affect the short-duration amplitude $\widehat{A_{98,s}}$. While this quantity does appear to retain a linear relationship at high median activity, we note that the two high-amplitude stars are still separated above the low amplitude stars in this measure as well, although to a lesser degree ($\approx 32\%$ higher compared to 45%).

Radick et al. (1998) and Lockwood et al. (2007) found log-linear relationships between mean activity $\log(\langle R'_{\text{HK}} \rangle)$ and long-term (decadal) and short-term (seasonal) rms variation in activity, $\log(\text{rms}(R'_{\text{HK}}))$ for a larger sample of stars with a larger range of effective temperatures. Our result is similar to this, but is a stronger statement on the relationship between amplitude and mean activity due to the reduced scatter which precludes the need to use logarithms. The clear distinction between the high-amplitude and low-amplitude stars at high-activity disappears when logarithms are used. Furthermore, in those previous works, the Sun appeared to have a large rms amplitude ($\approx +30\%$) compared to stars of similar mean activity, while we find the solar amplitude very near but slightly below the trend line. The important difference is the inclusion of a wider range of stellar masses: our comparison of the Sun to other solar analog stars makes its amplitude appear normal, while when including F, late G, and K-type stars the amplitude appears to be large.

Figure 4.7 demonstrates an important feature of the dynamos of these stars. Insofar as the S -index remains a good proxy for total unsigned magnetic flux in stars with higher activity than the Sun, it tells us that the variation of total unsigned magnetic flux is proportional to the mean level on two separate time scales: about a half a year and half a century:

$$\Delta \int |B| dA \propto \left\langle \int |B| dA \right\rangle \quad (4.4)$$

where dA is surface area element and Δ can represent a min-to-max variation like A_{98} , or a smaller range of like the root mean square variation or MAD. All such measures of activity scale show similar relationships in our data.

We are aware of one dynamo study of the effect of rotation on activity amplitude with which we may compare our results. Karak et al. (2014) studied a parameterization of rotation in the Babcock-Leighton flux transport dynamo which resulted in a power law relationship between the amplitude of toroidal flux variations over the cycle, f_m , and the Rossby number: $f_m^2 \propto Ro^{-1.3}$. The power of 2 applied to f_m was a guess at the scaling of Ca H & K emission with magnetic surface flux based on “naive considerations”, but which happens to coincide with the observational results of Harvey and White (1999). Hence the prediction is that HK flux $F_{\text{HK}} \propto Ro^{-1.3} \propto P_{\text{rot}}^{-1.3}$. A power law fit to our decadal scale amplitudes gives an exponent of -0.8 ± 0.1 (Pearson $r = 0.81$). Though the exponents are discrepant, Karak et al. noted that their result could be tuned by assuming a different functional form of the Babcock-Leighton source term dependence on rotation period. Therefore, it is demonstrated that this class of model can adapt to this particular observation, but without an understanding of the physical nature of the relationship between rotation and the Babcock-Leighton source term. Furthermore, their model results in regular cycles for all parameter regimes and a rigid inverse relationship between rotation period and cycle period, neither of which are seen in our data.

4.5 A New Cycle Quality Metric

While all the stars of our sample have a measurable activity amplitude, not all stars have a clearly discernable cycle. The three stars in Figure 4.5 all clearly have patterns of variability quite different than the Sun. While the Lomb-Scargle periodogram analysis is appropriate for finding statistically significant periods of variability in any time series, evaluating whether those variations are in fact *Sun-like* requires further analysis. We refer to this as the question of judging *cycle quality*, where the solar cycle is the primary example of a high-quality cycle with which we wish to compare other stars.

Baliunas et al. (1995) used the False Alarm Probability (FAP; F in equations) developed in Scargle (1982) as both a means to determine the statistical significance of signals in the MWO time series *and* a means to determine cycle quality. Four FAPs defined periodogram power thresholds (equation 3.1) which determined the “FAP Grade”: “poor” $\equiv 10^{-4} < F \leq 10^{-3}$, “fair” $\equiv 10^{-7} < F \leq 10^{-4}$, “good” $\equiv 10^{-11} < F \leq 10^{-7}$, and “excellent” $\equiv F \leq 10^{-11}$. When we use the methods and cycle quality scale of B95 on our twice longer time series, we find that *nearly every star has an “excellent” cycle*, which is clearly not the case from visual inspection of the time series. A new metric of cycle quality is evidently necessary, but first we will show why using FAP as a cycle quality metric can be problematic.

Scargle (1982) noted that a redeeming quality of the periodogram is that the power spectral density, and therefore the peak detection signal-to-noise ratio, increases linearly with the number of observations, N :

$$P_X = N(A_X/2)^2, \tag{4.5}$$

where P_X is the periodogram power of a sinusoidal signal X with amplitude A_X . Horne and Baliunas (1986) showed that the periodogram power must be normalized by the total variance of the time series σ^2 in order to properly compute the FAP of peaks in the periodogram using equation (3.1):

$$P_N = P_X / \sigma^2 = N(A_X / 2\sigma)^2 \quad (4.6)$$

Scargle (1982) also found that for small FAP ($F \ll 1$), the peak power threshold z which has a FAP less than or equal to F can be estimated with $z \approx \ln(N/F)$, which grows slowly with N . Therefore we should expect that when using the Lomb-Scargle periodogram and a cycle quality metric based on FAP, as more data are added signals that were previously rated “poor” will quickly become “excellent”, even when the time series remains qualitatively the same. This is demonstrated in Figure 4.8, where a sinusoidal signal buried in noise ($\text{SNR} = 0.05$) is analyzed every decade. Stellar sampling was simulated by creating 0.5 yr seasonal gaps and randomly removing 75% of the remaining nightly observations. In the first two decades this star would progress from a FAP grade of “poor” to “fair”, and each decade afterwards it would be called “excellent”.

The progression toward higher quality ratings would occur regardless of whether the noise were due to instrumental uncertainty or other types of intrinsic stellar variability. In the case of the MWO H & K data, we are certainly in the latter regime for a star like the Sun. The estimated uncertainty of a nightly measurement in MWO is 1.2% (B95), while the mean amplitude of the solar cycle from Chapter 2 (Table 2.2) is $\langle \Delta S \rangle / \langle S \rangle = 8.5\%$, giving an instrumental $\text{SNR}_i \approx 7.1$. The intrinsic noise of the Sun, due to rotation and active region growth and decay, can be estimated from the median variance of one year bins, from which we obtain an intrinsic signal-

to-noise ratio $\text{SNR}_\star = \langle \Delta S \rangle / \sigma_{1\text{yr}} = 2.3$. Since $\text{SNR}_\star < \text{SNR}_i$, we work in a regime where we are trying to recognize the solar cycle in the midst of other higher-frequency variation generated by the Sun. If such higher frequency variation were comparable to or dominating a signal we would like to call a “cycle”, then that signal, and thus this “cycle”, could not be said to be Sun-like.

We will now develop a cycle quality metric that is sensitive to the signal-to-noise ratio of the period under question. To do this, we simply rearrange equation (4.6) and divide by $\sqrt{2}$:

$$\frac{(A_X/\sqrt{2})}{\sigma} = \sqrt{\frac{2P_N}{N}} \equiv A_N. \quad (4.7)$$

The normalized amplitude spectral density A_N is the root mean square amplitude of a sine signal, $A_X/\sqrt{2}$, normalized by the standard deviation of the time series. This quantity has three useful features for judging cycle quality: (1) it is independent of the number of observations, (2) it has a maximum value of 1, and (3) it is a measure of signal purity; the amplitude at a given period, A_1 , is relative to the variance of all other contributions to the time series, $\sigma^2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots)$. We can convert the FAP grade system B95 to normalized amplitude using:

$$A_N(F, N) = \sqrt{\frac{2z(F, N)}{N}}, \quad (4.8)$$

where $z(F, N)$ is the FAP threshold of equation (3.1), wherein we estimate the number of independent frequencies N_i using the empirical relationship of Horne and Baliunas (1986). We computed the mean number of observations in B95 from our stellar sample with $t_{\text{obs}} < 1992$, finding $N = 562$. The “FAP Grade” system of B95 therefore converts to “poor” $\equiv 0.22 \leq A_N < 0.24$, “fair” $\equiv 0.24 \leq A_N < 0.29$, “good” $\equiv 0.28 \leq A_N < 0.34$, “excellent” $\equiv A_N \geq 0.34$. Note that this is only a rough

estimate, as the number of observations taken up to 1992 varied significantly from star to star. In our sample, the standard deviation is $\sigma_N = 200$.

Figure 4.8 (bottom panel) shows the normalized amplitude spectral density A_N for sinusoidal signal plus SNR=0.05 Gaussian noise. Unlike P_N , as additional data is collected, A_N stays relatively constant. It can be shown that the expected amplitude for this signal is $A_N = 1/\sqrt{1 + \text{SNR}^{-1}} = 0.218$. The deviations from this constant amplitude are due to the randomness introduced by the simulated stellar sampling.

In Figure 4.9 we evaluate A_N using the solar time series developed in Chapter 2. In the top panel, we have computed A_N using the entire time series, where the black curve is the periodogram of the composite and the colored curves are for the MWO (red) and SSS (blue) time series alone. The top three statistically significant periods (FAP $< 10^{-3}$) are shown with green vertical lines. The 11-year cycle is prominently featured with $A_N = 0.68$. The broad peak at 31.7 yr is an artifact due to the variation in cycle amplitudes. The small peak at 5.9 yr is a harmonic due to the non-sinusoidal shape of the solar cycle. These statistically significant features are examples of the kinds of periods we must avoid calling a “cycle” in the analysis of stellar time series. The bottom panel of Figure 4.9 shows the solar time series degraded to simulate typical stellar sampling. Seasonal gaps of 0.5 yr were created and observations were removed at random until the total number of observations reached 1000. We notice that the form of the periodogram remains the same, although the periods have shifted slightly. The amplitude of the 11 yr peak remains roughly the same, showing again the independence of this quantity to the number of measurements.

To complete our quality metric, we will add a factor to weigh cycles according to how many periods have been observed:

$$Q_{\text{cyc}} = 100 \left(1 - 0.5 \frac{P_{\text{var}}}{T} \right) A_N, \quad (4.9)$$

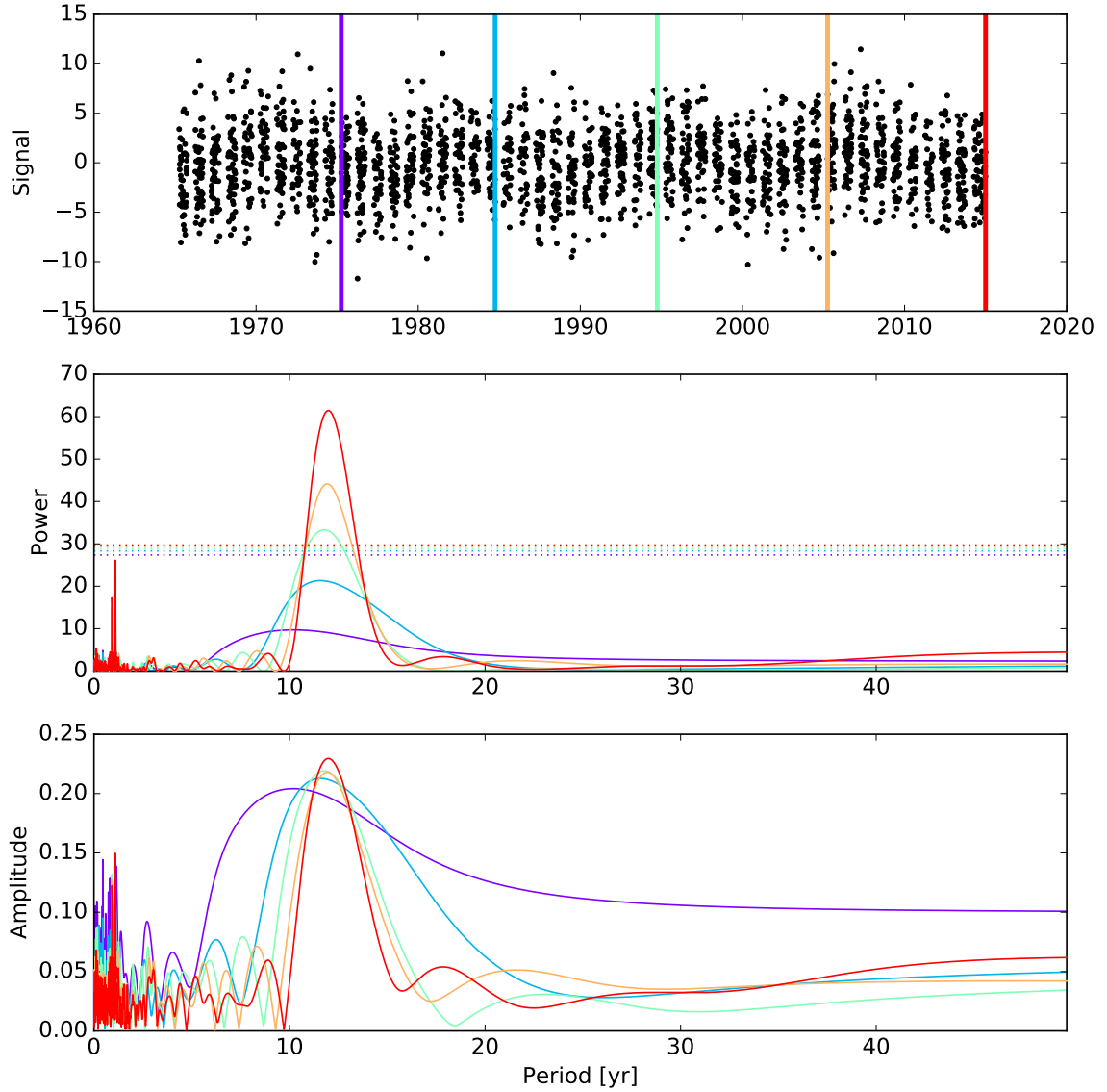


Figure 4.8 Experiment in which a sinusoidal signal with a period of 12 years is buried in noise ($\text{SNR}=0.05$) and analyzed each decade (top panel) with a Lomb-Scargle periodogram using power spectral density (middle) and amplitude spectral density (bottom). Each periodogram curve is colored according to the endpoint used in the time series, indicated by a vertical line in the top panel. The colored horizontal lines in the power spectrum (middle) show the per-endpoint threshold for an “excellent” cycle ($\text{FAP} < 10^{-9}$) in Baliunas et al. (1995).

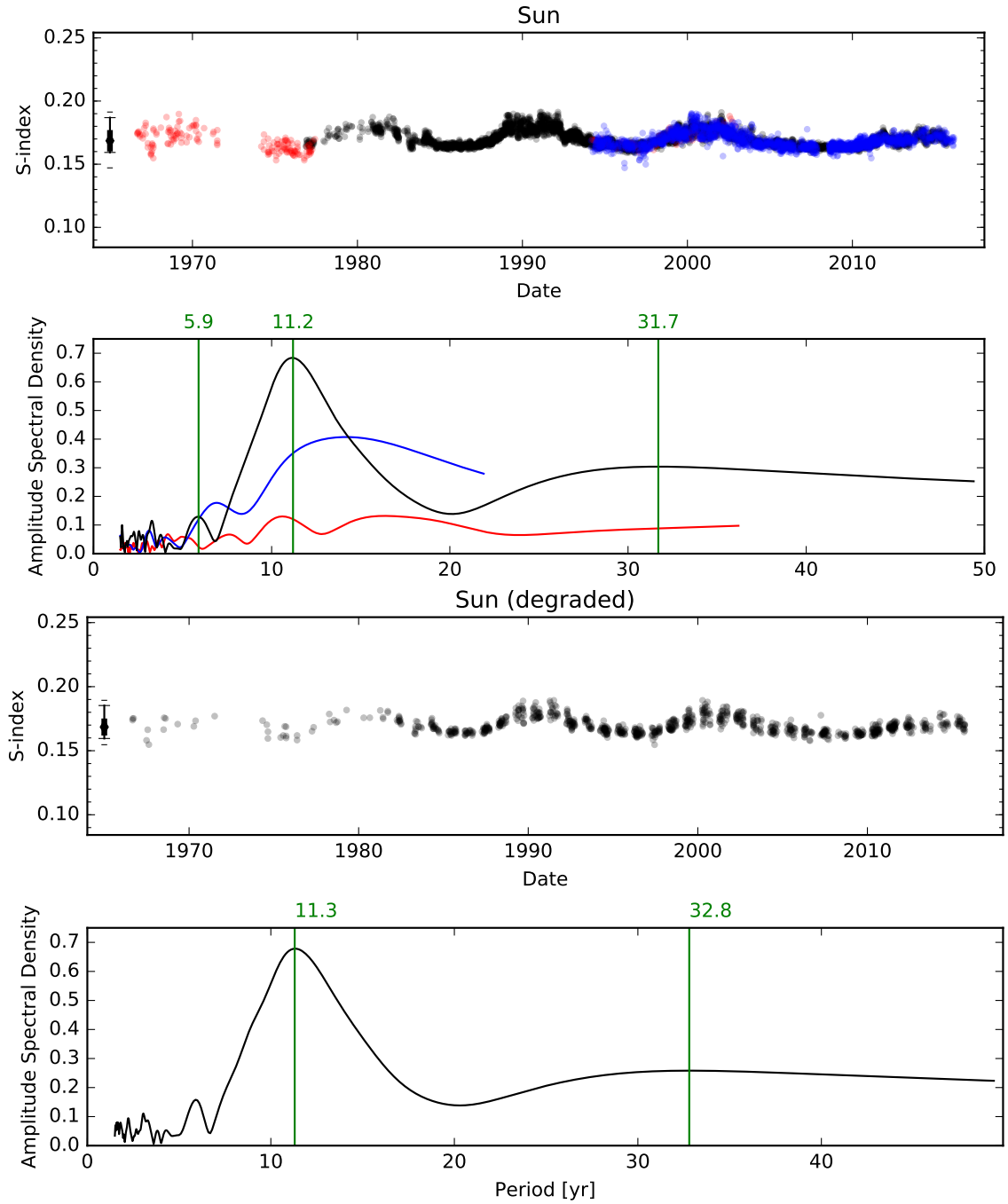


Figure 4.9 (*Top*) S -index time series and amplitude spectral density periodogram for the calibrated solar time series of Chapter 2. (*Bottom*) The same time series degraded to simulate stellar sampling.

Table 4.3. Cycle Quality Metric Test Cases

Description	T [yr]	P_{var} [yr]	A_N	Q_{cyc}
pure sine	∞	any	1.00	100
pure sine	$= P_{\text{var}}$	$= T$	1.00	50
pure sine	50.0	11.0	1.00	89
repeated cycle 23 model	50.0	11.0	0.99	88
cycle models 20–24	50.0	10.9	0.86	82
$\dots + \text{noise SNR}=2.3$	50.0	10.9	0.72	64
real solar data	49.5	11.2	0.68	61

where P_{var} is the period of variability we are ranking, and T is the duration of the time series. The factor of 100 is applied for aesthetic reasons to put the metric on a 100-point scale.

In order to get a sense of how Q_{cyc} works in practice, we will consider several test cases which are summarized in Table 4.3. The Q_{cyc} metric can only reach 100 for an infinite time series of a pure sine signal. A pure sine signal observed for only one period has $Q_{\text{cyc}} = 50$, while a solar-like pure sine signal with a period of 11 yr observed for 50 years has $Q_{\text{cyc}} = 89$. The skewed-Gaussian solar cycle shape model of cycle 23 (as in Figure 2.1) repeated for 50 years has a near-identical $Q_{\text{cyc}} = 88$. The slight deviation from sinusoidal makes almost no difference to A_N . Using the cycle shape models for cycles 20–24 (see Figure 2.4) reduces A_N due to the variations in cycle amplitudes, durations, and shapes which increases the variance, reducing Q_{cyc} to 82. Adding Gaussian noise with the intrinsic $\text{SNR}=2.3$ estimated above reduces Q_{cyc} further to 64. This is close to the $Q_{\text{cyc}} = 61$ obtained for the real solar time series data in Figure 4.9. The three principal factors contributing to the solar Q_{cyc} are (1) the intrinsic “noise” of high-frequency processes occurring on the Sun, (2) the non-stationary nature of the solar cycle, (3) the finite time series.

The above exercise illustrates the types of complications to an otherwise clean signal which can reduce Q_{cyc} . Real stellar data with Q_{cyc} lower than the solar value are primarily due to the presence of variability on multiple time scales with comparable amplitudes. The reduction of the intrinsic SNR, such as for the flat time series, also dramatically reduces Q_{cyc} . In conclusion, we find that Q_{cyc} is appropriately diminished when time series qualitatively diverge from the relatively clean, monop periodic behavior of the solar cycle.

4.6 Analysis of Cyclic Variability

We have computed periodograms of the normalized amplitude spectral density A_N for each time series of our stellar ensemble. Periodograms for each time series are shown in Appendix B. We examined periods from 1.5 yr to the duration of the time series. The lower limit is set to avoid the problem of aliasing due to the seasonal sampling. The top three statistically significant peaks with $\text{FAP} < 10^{-3}$ were selected and Q_{cyc} was computed using equation (4.9). We only consider $Q_{\text{cyc}} > 20$ as worthy of being termed a “cycle”, and we stress that this is a very generous threshold. The largest $Q_{\text{cyc}} > 20$ for every star is reported in Table 4.4 as a “primary” cycle. A “secondary” cycle is reported if $Q_{\text{cyc},2} > Q_{\text{cyc},1} - 10$. This rule only chooses secondary cycles if the quality is comparable to the primary cycle. We furthermore found it necessary to remove all $P_{\text{var}} > 0.85T$ from consideration. This is because many periodograms have high-amplitude, broad peaks at $P > 40$ yr for which we do not believe we have sufficient data to call these cycles (see Figures B.1, B.5, and B.17 for examples).

We compare our results in Table 4.4 to those of B95 which used time series half as long. In general, we confirm most of the cycle periods of B95 with period shifts less than about 2 yr. Some short-period primary cycles of B95 become secondary

Table 4.4. Solar Analog Ensemble Variability Periods

HD	$P_{\text{var},1}$	$P_{N,1}$	$A_{N,1}$	$Q_{\text{cyc},1}$	$P_{\text{var},2}$	$P_{N,2}$	$A_{N,2}$	$Q_{\text{cyc},2}$	B95	P_{B95}	$P_{\text{B95},2}$
Sun	11.2	1483.8	0.68	61	—	—	—	—	excl*	11.0	—
1835	7.8	89.5	0.38	35	20.8	119.1	0.44	34	fair*	9.1	—
6920 ⁺	12.0	91.3	0.49	43	—	—	—	—	var	—	—
9562 ⁺	—	—	—	—	—	—	—	—	long	—	—
20630	35.8	200.7	0.57	36	5.7	91.1	0.38	36	fair*	5.6	—
30495	12.5	64.7	0.36	31	(1.7)	(30.0)	(0.22)	(21)	var	—	—
39587	16.7	114.1	0.42	35	8.2	53.2	0.29	26	var	—	—
43587	—	—	—	—	—	—	—	—	flat	—	—
71148	8.5	32.6	0.47	38	—	—	—	—	—	—	—
72905	4.0	23.3	0.33	31	—	—	—	—	var	—	—
76151	5.0	61.8	0.33	31	20.9	74.3	0.36	28	fair*	2.52	—
78366	19.8	102.7	0.43	34	10.9	81.9	0.38	34	good*	12.2	5.9
81809 ⁺	8.2	242.4	0.67	61	—	—	—	—	excl*	8.17	—
97334	5.6	41.3	0.30	28	1.9	18.4	0.20	20	var	—	—
114710	26.4	166.2	0.48	35	6.5	65.5	0.30	28	good*	16.6	9.6
115043	10.3	53.7	0.47	40	—	—	—	—	var	—	—
126053	20.1	38.3	0.32	25	—	—	—	—	poor	22.0	—
141004 ⁺	6.7	34.1	0.25	23	—	—	—	—	long	—	—
142373 ⁺	19.6	54.6	0.26	21	—	—	—	—	flat	—	—
143761	11.9	54.4	0.25	22	—	—	—	—	long	—	—
146233	13.9	112.1	0.55	44	6.5	87.8	0.49	44	—	—	—
176051	15.6	67.6	0.39	30	3.8	25.1	0.24	23	poor	10.0	—
190406	18.7	250.9	0.56	45	2.6	121.4	0.39	38	fair	2.6	16.9
197076	4.7	61.8	0.59	53	—	—	—	—	—	—	—
206860	5.4	67.7	0.29	27	18.1	79.8	0.31	26	poor	6.2	—
217014	12.1	62.0	0.35	31	—	—	—	—	var	—	—
224930	37.6	284.8	0.74	45	—	—	—	—	poor	10.2	—

Note. — The Lomb-Scargle periodograms used in this analysis are shown in Appendix B. In the electronic version of this document the first column is hyperlinked to the periodogram. Columns 2–9 are the variability periods and quality metrics discussed in this analysis. Cols 10–12 are the FAP grade and cycle periods of B95. Secondary P_{var} of HD 30495 is not detected in this analysis, but is added from the results of Chapter 3. HD numbers marked with ⁺ are subgiants. Starred (*) B95 FAP grades indicate the star was included in the sample of Böhm-Vitense (2007).

cycles in our classification system due to their lower Q_{cyc} . Notable differences include HD 76151 (Figure B.10), where the B95 period of 2.52 yr is the third largest peak in the periodogram, and possibly a harmonic of the 5.0 yr period we report as the primary cycle. For HD 114710 B95 reported a 16.6 yr cycle which does not appear as a peak in our composite record, although an unresolved feature at ≈ 14 yr is seen in the periodogram. Our primary period for this star is 26.4 yr, and is roughly visible in the time series as the long-term swing between relatively low and high activity,

with a higher frequency variation superimposed. For HD 176051, The “poor” period of 10 yr in B95 was based on a relatively short time series of 12 yr. We instead find a moderately high-quality period of 15.6 yr with a shorter 3.8 yr signal superimposed.

Eight stars were previously classified as “var”, “long”, or “flat” in B95 and have credible cycle periods in our analysis. Some of the previous “var” stars now show cycles of relatively high quality, especially HD 6920⁺ and 115043. Those that were previously classified as “long” or “flat” are HD 141004⁺, HD 142373⁺, and HD 143761, all have relatively low quality cycles barely above our $Q_{\text{cyc}} > 20$ threshold. HD 141004⁺ in particular does appear to have a very long term trend which dominates the variability. The decrease in activity around 2008 is unfortunately coincident with the CCD upgrade in SSS, which is a difficult calibration issue that leaves considerable uncertainty in low-amplitude variations at that date. Judging from the MWO data alone, it appears possible that a cycle period in excess of 40 yr may exist in this star.

We find evidence for long-period cycles of greater than 25 yr that were not observable in the shorter duration B95 data. The highest quality of these is the 37.6 yr cycle of HD 224930 with $Q_{\text{cyc}} = 45$ (see Figure B.26). This star has the highest normalized amplitude in our sample, $A_N = 0.74$, indicating that the time series is relatively free of higher frequency variations. The reduction in Q_{cyc} is due to the small ratio of this period to the duration of the time series. Indeed, we have only seen this “cycle” once. However, it is like the solar cycle in form, with a fast rise and a very long decay, making it an interesting case for further study. The other two long-period cycles are from HD 20630 and HD 114710, which are both superimposed with a significant shorter-period variation.

Few stars have a cycle quality comparable to the Sun. The notable exception is the subgiant HD 81809⁺ ($Q_{\text{cyc}} = 61$; see Figure B.12) whose exquisite 8.2 yr cycle identified in B95 continued uninterrupted to the present day. Coming in third place

is the short 4.7 yr cycle of HD 197076 ($Q_{\text{cyc}} = 53$; Figure B.23). This star was not analyzed in B95, so this is a newly reported cycle. In fourth place are the remarkably distinct dual cycles of HD 190406 ($Q_{\text{cyc},1} = 45, Q_{\text{cyc},2} = 38$; Figure B.22). Both the long (18.7 yr) and short (2.6 yr) periods are easily seen in the time series and were reported in B95.

The fifth highest quality cycle is that of the famous solar twin 18 Sco (HD 146233). This star was not analyzed in B95, but Hall et al. (2007b) reported a clear cycle of 7.1 yr using SSS data from 1997–2006. Our addition of the MWO data and the ongoing SSS observations have increased the duration of the time series to 34 years. We find a primary cycle of 13.9 yr and a secondary cycle at 6.5 yr with equal quality ($Q_{\text{cyc}} = 44$). Looking closely at Figure 4.4, the origin our secondary cycle appears to be related to the widely separated double peak in the 2001 and 2007 seasons. This feature was not present in the Hall et al. (2007b) data; in particular their 2004 seasonal mean was nearly as low as the 1997 minima, while in our data it is significantly higher and activity similar to the 1997 minima are not reached until the 2011 season – hence the 14 yr cycle. The 2004 season data is entirely from SSS, and the difference between our data and Hall et al. (2007b) stems from the updated data reduction procedure that uses the less noisy CCD-2 spectra (post-2008) as a reference for the continuum normalization of the CCD-1 spectra. This change has shifted the 2004 season upward. Although the 2004 season has a larger uncertainty in the median than the other seasons, it still lies $3.8 \sigma_{\text{MAD}}$ above the activity minimum of 1997, making it very unlikely that the true activity in 2004 could be so low. The new reduction procedure significantly lowers the scatter in S for all stars, and is a clear improvement over the previous situation when only CCD-1 data were available. Therefore, we must accept this new characterization of the long-term variability of 18 Sco as our best estimate. A few more years of observations will help confirm the true

Table 4.5. Solar Analog Ensemble Cycle Quality Statistics

$Q_{\text{cyc},1}$ Bin	N Stars	N also with $Q_{\text{cyc},2}$
$Q_{\text{cyc}} \geq 50$	3	0
$40 \leq Q_{\text{cyc}} < 50$	5	2
$30 \leq Q_{\text{cyc}} < 40$	11	8
$20 \leq Q_{\text{cyc}} < 30$	6	2
$Q_{\text{cyc}} < 20$	2	—

character of this star. A local maximum has already been seen in the 2013 season; if the activity returns to the 2011 minimum levels by 2018 or 2019, more weight will be given to the ~ 7 yr cycle, likely switching the order of “primary” and “secondary” in our classification scheme.

Looking again to Figure 4.4, we see that the form of 18 Sco’s variability is unlike the Sun in that the rise to 2007 maximum was very prolonged (~ 10 years), followed by a fast decay to the 2011 minimum (~ 4 years). The decay from the previous maximum of 1993 was similarly rapid (~ 5 years). Though we have found that the activity of 18 Sco is quantitatively very similar to the Sun, its cyclic behavior is qualitatively quite different. The cycle of the subgiant HD 81809⁺ is solar-like in form, as is the primary cycle of HD 190406 and the extremely long cycle of HD 224930. All other stars have patterns of variability quite far removed from what we see from the Sun.

Table 4.5 gives statistics for bins of Q_{cyc} with a width of 10. As we mentioned before, there are few stars with a cycle quality comparable to the Sun. Setting the threshold at $Q_{\text{cyc}} \geq 40$ to be “passably Sun-like” we find 8 of 27 stars in this category, including the Sun. The most populated bin is in the middle, $30 \leq Q_{\text{cyc}} < 40$. Most of these stars have a secondary cycle of comparable amplitude and in general their variability is quite complicated. Stars with $Q_{\text{cyc}} < 30$ should probably not be

considered in cycle period studies; their variability is either so complex (e.g. HD 97334) or so flat (e.g. HD 142373⁺) that comparison to other cycling stars is likely unwarranted, despite the fact that the periods detected are statistically significant. Recall from above that B95 “FAP Grade” of “good” have $A_N \gtrsim 0.29$, which translates to $Q_{\text{cyc}} \gtrsim 29$ when $P_{\text{var}}/T \ll 1$. Therefore $Q_{\text{cyc}} \geq 30$ is roughly the mid-point of the B95 “FAP Grade” system, and deliniates the high-quality cycles from the questionable cases.

4.7 Cycle Period and Rotation Period

We now take a look at the impact of this new analysis on the previous understanding of the dependency of cycle period on rotation. Brandenburg et al. (1998) selected a sample of 22 high-quality cycles (“good” and “excellent” class) from Baliunas et al. (1995) and found two branches on log-log plots of dimensionless quantities $\omega_{\text{cyc}}/\Omega$ and $Ro^{-1} \equiv \tau_c/P_{\text{rot}}$. Saar and Brandenburg (1999) went on to “add carefully additional stars with less certain cycles and/or trends from the Mount Wilson database and other sources,” and ultimately confirmed the previous results and the existence of the two branches. Böhm-Vitense (2007) chose a high-quality subset from the sample of Saar and Brandenburg (1999) and found the two branches in a linear plot of dimensioned quantities P_{cyc} versus P_{rot} (see Figure 1.10). We see that there were multiple judgments about cycle quality at play in each of these works; we will now add our own to the mix.

Figure 4.10a shows the addition of our better cycles with $Q_{\text{cyc}} \geq 40$ to the Böhm-Vitense selection which are *not* in our sample. See Table 4.4 to see which stars are included in both works. The new stars added to the diagram are two cycles of 18 Sco, the short cycle of HD 197076, and the very long cycle of HD 224930. For this last star, both of the alternate rotation periods discussed above shown. The inclusion of

our highest quality cycles does not substantially change the picture shown in Böhm-Vitense (2007). If the 33 d rotation period of HD 224930 is the correct one, this star appears to be on a distant extension of the active branch.

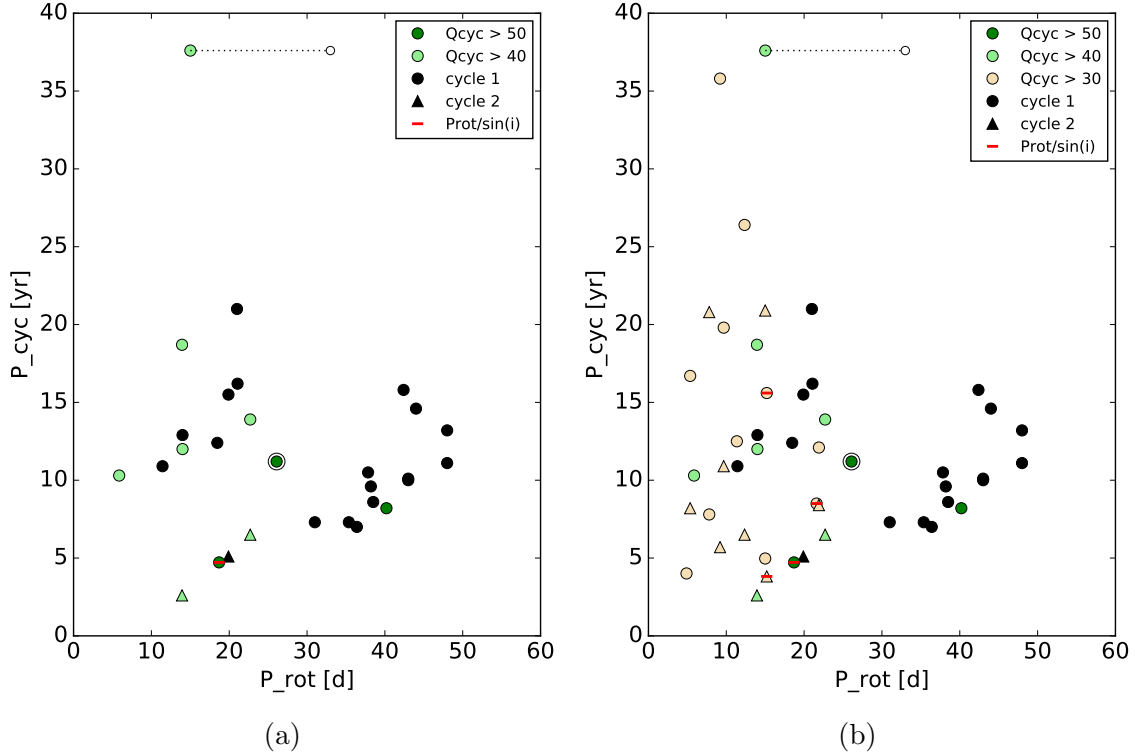
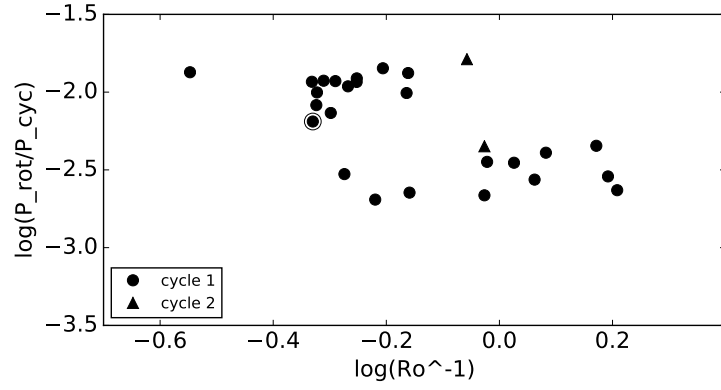
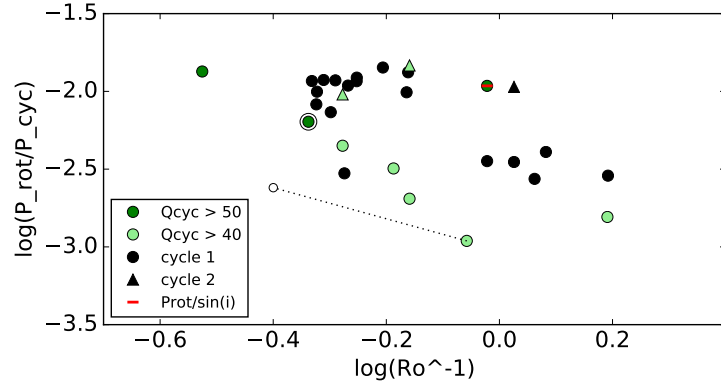


Figure 4.10 New cycles added to the P_{cyc} versus P_{rot} diagram (compare to Figure 1.10) satisfying (a) $Q_{\text{cyc}} \geq 40$ (b) $Q_{\text{cyc}} \geq 30$.

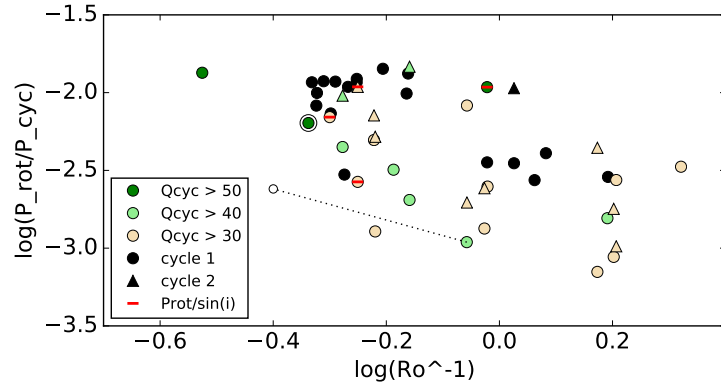
However, the picture becomes more complicated if we lower the quality threshold to $Q_{\text{cyc}} \geq 30$, as shown in Figure 4.10b. A number of primary and secondary cycles appear both above and below the previously distinct active branch. No correlation is apparent in the region of the diagram with $P_{\text{rot}} < 30$ d. The “active branch” no longer exists when these low-quality cycles are included, but some of these low-quality cycles come from stars for which the cycle is quite apparent, such as HD 30495. The “inactive branch” is unchanged, as it consists of stars with slower rotation than those studied here, with the exception of HD 81809.



(a)



(b)



(c)

Figure 4.11 $\omega_{\text{cyc}}/\Omega_{\text{rot}}$ versus Ro^{-1} with new cycles

The same sensitivity to cycle quality is seen in Figure 4.11, where we have plotted the same sample using the non-dimensional quantities $P_{\text{rot}}/P_{\text{cyc}} \equiv \omega_{\text{cyc}}/\Omega_{\text{rot}}$ versus the inverse Rossby number τ_c/P_{rot} , where τ_c is computed using the formula of Noyes et al. (1984a). In this view the Sun is an extreme point near low Ro^{-1} (slow rotation) inactive branch, but not a curious outlier as in Figures 4.10 and 1.10. Here HD 81809⁺ appears as a low $Ro^{-1} = -0.55$ outlying point because we have used $(B - V) = 0.64$ as in the Tycho Double Star Catalog (Fabricius et al., 2002), whereas Saar and Brandenburg (1999) assumed this was a K-type star with $(B - V) = 0.80$. HD 224930 again has a very different position depending on whether we take its 33 d or 15 d rotation period measurement. The 14 yr cycle of 18 Sco (HD 146233) falls in the space between the active and inactive branches, with $P_{\text{rot}}/P_{\text{cyc}} = -2.35$. When we lower the quality threshold to $Q_{\text{cyc}} \geq 30$ the distinction between the branches is largely erased, especially with the long cycle periods in fast rotating stars that show up on the low $P_{\text{rot}}/P_{\text{cyc}}$ portion of the diagram.

The question of the reality of our “cycles” with periods of 20–40 yr can only be resolved with much longer time series, on the order of a century or more. What is clear from our more limited data is that large amplitude variations on time scales of 20 years or more do exist – the question is whether they repeat in a regular fashion or if they are stochastic or mathematically chaotic in nature. This latter suggestion seems eminently possible given the nonlinearity of the governing MHD equations. For now, salvaging a picture in which cycle period is linearly dependent on either rotation alone or Rossby number requires a very unsatisfactory application of subjective judgement. We have made some progress in reducing the level of subjective judgement by introducing the cycle quality metric, Q_{cyc} , but the decision to place a threshold at $Q_{\text{cyc}} \geq 40$ or any other location remains subjective. Calculation of Q_{cyc} for the other lower-mass stars in the MWO and SSS databases could help by

putting all stars on a uniform quality scale. Finally, if decades-long time series from a much larger sample (e.g. 1,000 stars) were available, the activity branches may reveal themselves not as strict rules which cycle period obeys, but as an overdensity of points in the diagrams discussed above. As usual in observational work, we are left wanting more data.

CHAPTER FIVE

THE SOLAR CYCLE IN CONTEXT

In the previous chapter we found that clean, monoperiodic cycles similar to the Sun’s are rare in our small sample of G-type stars. Now we examine the question of what properties result in a cycling pattern similar to the solar cycle using results from the larger stellar sample of F, G, and K-type stars from Baliunas et al. (1995) (B95 hereafter). B95 commented that old, slow rotating stars tended to produce smooth cycles, and that K-type stars with low activity “almost all have pronounced cycles.” However, quantitative analysis of their variability classifications were limited to a division into just “active” versus “inactive” stars, and the major results were phrased in terms of imprecise stellar age. In this chapter, we examine the variability classifications of B95 according to spectral type, activity, rotation, and Rossby number in order to understand the solar cycle in a broad context. This study considers stars of all variability classes, and so is more general in its approach than previous work which focused only on stars with high-quality cycles (Böhm-Vitense, 2007; Brandenburg et al., 1998; Noyes et al., 1984b; Saar and Brandenburg, 1999). Here we examine which variability patterns are correlated with spectral type, rotation, activity, or the Rossby number.

5.1 Variability Classes and FAP Grade Cycle Quality

B95 analyzed S -index time series for 112 stars using statistical measures, namely mean activity $\langle S \rangle$ and standard deviation σ_S , and frequency-domain measures using the Lomb-Scargle periodogram. Statistical confidence in periodogram peaks was

Table 5.1. B95 Variability Class Definitions

Variability Class	$\sigma_S / \langle S \rangle$ Condition	FAP Condition	P_{cyc} Condition
<u>Non-cycling</u>			
Flat	$< 1.5\%$	—	—
Flat?	$< 2.0\%$	—	—
Long*	$\geq 2.0\%$	$\leq 0.1\%$	> 25 yr
Var	$\geq 2.0\%$	$> 0.1\%$	$\ll 25$ yr
<u>Cycling (FAP Grade)</u>			
Poor	$\geq 2.0\%$	$\leq 0.1\%$	1 to 25 yr
Fair	$\geq 2.0\%$	$\leq 10^{-2}\%$	1 to 25 yr
Good	$\geq 2.0\%$	$\leq 10^{-5}\%$	1 to 25 yr
Excellent	$\geq 2.0\%$	$\leq 10^{-9}\%$	1 to 25 yr

Note. — Taken from Baliunas et al. (1995). (*) Possibly cycling on timescales > 25 yr.

determined using the False Alarm Probability (FAP; Horne and Baliunas, 1986), which is defined as the probability that a given peak in the periodogram is due to Gaussian noise of the same variance as the time series under study. Periodogram peaks with a FAP less than 0.1% were considered significant, and a cycle period P_{cyc} was reported. A sine wave of period P_{cyc} was fit to the data using a least squares method and then subtracted from the data. A periodogram analysis of the residual time series searched for additional significant (FAP $< 0.1\%$) periods, and if found a second cycle period $P_{\text{cyc},2}$ was reported.

Table 5.1 summarizes the eight variability classes defined by B95 according to the relative standard deviation $\sigma_S / \langle S \rangle$ and the FAP. B95 pointed out that the FAP is formally defined for conditions that are not met in stellar time series; namely that the signals are sinusoidal and the noise is randomly drawn from a Gaussian distribution.

In fact, the solar cycle is a skewed Gaussian-like curve with a fast rise and a long decay, and the “noise” on timescales less than the cycle period are due to processes such as active region growth & decay and rotation, which will have a distinct structure in the time series. This led the authors of B95 to caution against taking FAP “too literally”, and instead developed the “FAP Grade” system based on FAP thresholds and the subjective “visual appearance of the records.”

Despite the lack of a rigorous, objective system of determining cycle quality, we find that the B95 “FAP Grade” classifications does effectively rank cycles in order of believability based on visual inspection of the time series. The “excellent” cycles (sometimes abbreviated “excl”) are clear and smooth, qualitatively similar to records of the solar cycle in sunspot number or Ca II K. The “good” cycles are nearly of that quality, but appear to have either somewhat increased noise, or feature only one rise and fall of the “cycle”. The “fair” and “poor” time series have noticeably increased noise, and picking out purported cycle period by eye is often impossible. The other classes are qualitatively similar to their name. “Var” time series are erratic, or chaotically variable. “Long” time series have a long-term trend in the time series, that may turn out to be a very long period (> 25 yr) cycle. Many “long” stars near the 2% amplitude threshold appear to be quite flat. Finally, “flat” time series are just that; generally featureless for the entire duration.

In this study we do not distinguish between “flat?” and “flat”, and we blur the distinction between “flat” and “long”, “fair” and “poor”, and “excellent” and “good” by using similar colors for these classes in the plots that follow. This is due to the qualitative similarities between time series of these classes in B95, and our desire to simplify the analysis by reducing the number of classes into “cycling” and “not cycling”, “good cycle” and “bad cycle”. Similar judgements were made in Saar and Brandenburg (1999), who developed a five-value weight w based on the B96 FAP

Grade and their own inspection of the time series. Most of the stars in their table of stars with “better determined behavior” are in the “good” or “excellent” class, with a small number of “fair” cycles admitted. Böhm-Vitense (2007) took the sub-sample of high-quality cycles from the former study, choosing those with weight $w \geq 2$. These quality judgements have a real impact on the interpretation of the data: as we will show, including all the detected cycles in B95 in a plot of P_{cyc} vs P_{rot} blurs the distinction of the two branches. Six stars in B95 were given an uncertain cycle period (e.g. “7?”) and no FAP Grade. These stars often had shorter time series, and the quoted cycle period was within a few years of the duration of the time series. We assigned a FAP Grade of “poor” to these stars. Finally, we note that many of the “long” time series in B95 appear quite flat, and thus qualitatively similar to the “flat” class, despite the possibility that the detected trend eventually turns around in a long-period cycle with $P_{\text{cyc}} > 25$ yr.

5.2 Binaries in the B95 Sample

It is important to note that the B95 sample contains some binary and higher multiple stellar systems. Some of these are widely separated long-period binaries resolved in the observations, e.g. HD 131156 (ξ Boo) A & B, HD 165341 (70 Oph) A & B, HD 219834 (94 Aqr) A & B. For these stars, their binary status will not affect their chromospheric observations or their color index ($B - V$) which is used in the computation of $\log(R'_{\text{HK}})$ and Ro . Other stars are spectroscopic binaries which may have blended Ca II H & K emission and ($B - V$), distorting our interpretation. The impact on our results depends on the magnitude difference of the A (primary) and B (secondary) components, $\Delta m = V_A - V_B$. Small values of Δm will lead to a blended ($B - V$) which is far from the value for either of the components, as well as blending in the Ca II H & K emission. Furthermore, close orbits will

result in large radial velocities which will bring more photospheric flux from the line wings into the S -index with a periodicity of the orbit. Extremely close orbits may cause tidal effects which impact the dynamo process. In Table 5.2 we have compiled Δm and minimum angular separations s from the Washington Double Star catalog (WDS; Mason et al. (2001)), and the orbital period P_{orb} and maximum calcium line shift $\Delta\lambda$ from the Ninth Spectroscopic Binary catalog (SB9; Pourbaix et al. (2004)) for problematic binaries which are removed from this study. The line shift $\Delta\lambda$ is computed from the maximum radial velocity of the primary component, K_1 in SB9, using $\Delta\lambda = (K_1/c)\lambda_K$, where λ_K is the wavelength of the calcium K line. We have removed stars from the B95 sample where $\Delta m < 2.5$ and $s < 3$ arcsec, or where $P_{\text{orb}} < 25$ yr and $\Delta\lambda > 0.1$ Å. The Δm threshold selects binary components where the V-band flux of the secondary is within an order of magnitude of the primary.

In total, 9 binaries satisfy the rejection criteria. This includes HD 81809, which we characterized in Section 4.1.1. We used *Tycho-2* observations to determine that *both* components are roughly the same ($B - V$), and find evidence from the rotation measurements that the Ca II H & K emission is dominated by the more luminous subgiant component. We therefore keep this star in our current study, and remove the remaining 8 problematic binaries that are listed in Table 5.2.

After removing the 8 problematic binaries from the B95 sample, we are left with 104 stars, including the Sun. It is still possible that some of these stars contain undetected problematic binaries which have resulted in characterization or blended Ca II H & K emission. We are not aware of a catalog which classifies stars as single due to a negative result in a search for binarity using radial velocity or photometric methods. Blended H & K emission would tend to result in a complicated time series that B95 would have classified as “var”, or perhaps as a “poor” cycle. Indeed, 5 of the 8 binaries we removed have such classifications. “flat” binaries with comparable

Table 5.2. B95 Binaries Removed from Sample

HD	$(B - V)$	$\log(R'_{\text{HK}})$	Class.	Binary ID	Criteria
3443	0.72	-4.90	var	WDS 00373-2446 SB9 1470	$\Delta m = 0.4$; $s = 0.5$ arcsec $P_{\text{orb}} = 25$ yr; $\Delta\lambda = 0.07$ Å
76572	0.43	-4.93	poor	WDS 08580+3014	$\Delta m = 0.0$; $s = 0$ arcsec
88355	0.46	-4.82	var	WDS 10116+1321	$\Delta m = 1.0$; $s = 0.2$ arcsec
106516	0.46	-4.65	var	WDS 12152-1019 SB9 1692	$\Delta m = 6.1$; $s = 43$ arcsec $P_{\text{orb}} = 2.3$ yr; $\Delta\lambda = 0.10$ Å
114378	0.45	-4.53	long	WDS 13100+1732	$\Delta m = 0.7$; $s = 0$ arcsec
137107	0.58	-4.83	var	WDS 15232+3017 SB9 842	$\Delta m = 0.3$; $s = 0.6$ arcsec $P_{\text{orb}} = 41.6$ yr; $\Delta\lambda = 0.06$ Å
158614	0.72	-5.03	flat	WDS 17304-0104 SB9 969	$\Delta m = 0.11$; $s = 0.6$ arcsec $P_{\text{orb}} = 46.3$ yr; $\Delta\lambda = 0.07$ Å
178428	0.70	-5.05	flat	WDS 19080+1651 SB9 1122	m_B unknown; $s = 0.2$ arcsec $P_{\text{orb}} = 22$ d; $\Delta\lambda = 0.18$ Å

H & K emission from both components would indicate that *both* of the components are flat-activity. In those cases, the only concern is the characterization of the color index $(B - V)$, which leads to an erroneous computation of the chromospheric emission fraction $\log(R'_{\text{HK}})$ and the Rossby number.

5.3 Properties of the MWO Sample

Table 5.3 gives the B95 variability class and observed properties for 104 stars of the B95 study that survived the binarity test described above. The color index $(B - V)$ is that used in B95. Rotation periods measurements were gathered from the literature, with most sources determining rotation from time series analysis of the same MWO S -index data of B95. Gaidos et al. (2000) looked for rotational modulations in the Fairborne Observatory photometric time series in the Strömgren b and y bands. The chromospheric emission fraction R'_{HK} and Rossby number Ro were calculated using the semi-empirical formulae of Noyes et al. (1984a).

Table 5.3: B95 Variability Classes with Rotations

HD	$(B - V)$ (mag)	$\langle S \rangle$	$\log(R'_{\text{HK}})$	P_{rot} (d)	Ref.	Ro	Var. Class	P_{cyc} (yr)	$P_{\text{cyc},2}$ (yr)
Sun	0.65	0.1694	-4.94	26.09	D96	2.15	excl	11.0	—
1835	0.66	0.349	-4.43	7.81	G00	0.62	fair	9.1	—
2454	0.43	0.17	-4.79	3.0	B96	1.54	var	—	—
3229	0.44	0.224	-4.58	2.0	B96	0.91	poor	4.9	—
3651	0.85	0.176	-4.99	40.2	S10	1.93	good	13.8	—
3795	0.70	0.156	-5.04	—	—	—	var	—	—
4628	0.88	0.23	-4.85	38.5	D96	1.79	excl	8.37	—
6920	0.60	0.194	-4.79	14.0	B96	1.54	var	—	—
9562	0.64	0.136	-5.18	29.0	B96	2.54	long	—	—
10476	0.84	0.198	-4.91	35.2	D96	1.71	excl	9.6	—
10700	0.72	0.171	-4.96	34.0	B96	2.14	flat	—	—
10780	0.81	0.28	-4.68	21.7	G00	1.10	poor*	7.0	—
12235	0.62	0.16	-4.98	14.0	B96	1.37	var	—	—
13421	0.56	0.131	-5.19	2.173	H16	0.31	flat	—	—
16160	0.98	0.226	-4.96	48.0	D96	2.10	excl	13.2	—
16673	0.52	0.215	-4.66	6.6	D97	1.32	var	—	—
17925	0.87	0.653	-4.31	6.76	D96	0.32	long	—	—
18256	0.43	0.185	-4.72	2.8	D97	1.44	fair	6.8	—
20630	0.68	0.366	-4.42	9.2	G00	0.67	fair	5.6	—
22049	0.88	0.496	-4.45	11.3	S10	0.52	long	—	—
22072	0.89	0.131	-5.21	—	—	—	var	—	—
23249	0.92	0.137	-5.19	71.0	B96	3.20	flat	—	—
25998	0.46	0.3	-4.40	2.0	B96	0.72	long	—	—
26913	0.70	0.396	-4.39	7.15	D96	0.48	fair	7.8	—
26923	0.59	0.287	-4.50	—	—	—	var	—	—
26965	0.82	0.206	-4.87	43.0	B96	2.15	excl	10.1	—
29645	0.57	0.14	-5.11	—	—	—	long	—	—
30495	0.63	0.297	-4.51	11.3	G00	1.04	var	—	—
32147	1.06	0.286	-4.95	48.0	D97	2.05	excl	11.1	—
33608	0.46	0.214	-4.63	—	—	—	long	—	—
35296	0.53	0.332	-4.38	3.56	D96	0.65	long	—	—
37394	0.84	0.453	-4.45	10.86	G00	0.53	poor	3.6	—
39587	0.59	0.325	-4.43	5.36	D96	0.63	var	—	—
43587	0.61	0.156	-5.00	—	—	—	flat	—	—
45067	0.56	0.141	-5.09	8.0	B96	1.16	flat	—	—
61421	0.42	0.171	-4.78	—	—	—	var	—	—
72905	0.62	0.367	-4.38	4.89	G00	0.48	var	—	—
75332	0.49	0.279	-4.46	3.7	D97	0.98	var	—	—
76151	0.67	0.246	-4.66	15.0	D97	1.56	fair	2.52	—
78366	0.60	0.248	-4.61	9.67	D96	1.06	good	12.2	5.9
81809	0.64	0.172	-4.92	40.2	D96	3.52	excl	8.17	—
82443	0.77	0.635	-4.21	5.42	G00	0.30	poor	2.8	—
82885	0.77	0.284	-4.64	18.6	D96	1.02	fair	7.9	12.6
88737	0.56	0.234	-4.62	—	—	—	poor*	24.0	—
89744	0.54	0.137	-5.12	9.2	S10	1.55	flat	—	—
95735	1.51	0.424	-5.45	53.0	B96	1.95	var	—	—
97334	0.61	0.335	-4.42	8.25	G00	0.85	var	—	—

Table 5.3 Continued

HD	$(B - V)$	$\langle S \rangle$	$\log(R'_{\text{HK}})$	P_{rot}	Ref.	Ro	Var. Class	P_{cyc}	$P_{\text{cyc},2}$
100180	0.57	0.165	-4.92	14.0	B96	1.88	fair	3.56	12.9
100563	0.46	0.202	-4.67	—	—	—	var	—	—
101501	0.72	0.311	-4.55	16.68	D96	1.05	var	—	—
103095	0.75	0.188	-4.90	31.0	B96	1.79	excl	7.3	—
107213	0.50	0.135	-5.10	9.0	B96	2.16	long	—	—
111456	0.46	0.3	-4.40	—	—	—	poor*	7.0	—
114710	0.57	0.201	-4.74	12.35	D96	1.66	good	16.6	9.6
115043	0.60	0.327	-4.43	5.861	H16	0.64	var	—	—
115383	0.58	0.313	-4.44	3.33	D96	0.42	var	—	—
115404	0.93	0.535	-4.48	18.47	D96	0.83	good	12.4	—
115617	0.71	0.162	-5.00	29.0	B96	1.89	var	—	—
120136	0.47	0.191	-4.72	3.5	D97	1.14	poor	11.6	—
124570	0.54	0.133	-5.16	26.0	B96	4.39	flat	—	—
124850	0.52	0.21	-4.68	7.6	N84	1.52	long	—	—
126053	0.63	0.165	-4.96	—	—	—	poor*	22.0	—
129333	0.61	0.544	-4.15	2.8	D96	0.29	var	—	—
131156B	1.17	1.381	-4.42	11.94	D96	0.49	long	—	—
131156A	0.76	0.461	-4.36	6.31	D96	0.35	var	—	—
136202	0.54	0.14	-5.09	—	—	—	long	—	—
141004	0.60	0.155	-5.00	25.8	D96	2.84	long	—	—
142373	0.56	0.147	-5.04	—	—	—	flat	—	—
143761	0.60	0.15	-5.04	17.0	B96	1.87	long	—	—
149661	0.82	0.339	-4.58	21.07	D96	1.05	good	17.4	4.0
152391	0.76	0.393	-4.45	11.5	G00	0.65	excl	10.9	—
154417	0.57	0.269	-4.53	7.78	D96	1.04	fair	7.4	—
155885	0.86	0.384	-4.56	21.11	D96	1.00	poor	5.7	—
155886	0.86	0.375	-4.57	20.69	D96	0.98	var	—	—
156026	1.16	0.77	-4.66	21.0	B96	0.87	good	21.0	—
157856	0.46	0.202	-4.67	—	—	—	fair	15.9	—
159332	0.48	0.144	-5.01	—	—	—	flat	—	—
160346	0.96	0.3	-4.80	36.4	D96	1.61	excl	7.0	—
161239	0.65	0.138	-5.16	29.2	D97	2.44	fair	5.7	11.8
165341A	0.86	0.392	-4.55	20.0	B96	0.95	fair	5.1	—
165341B	1.16	0.691	-4.71	—	—	—	var	—	—
166620	0.87	0.19	-4.96	42.4	D96	1.99	excl	15.8	—
176051	0.59	0.176	-4.87	—	—	—	poor*	10.0	—
176095	0.46	0.202	-4.67	3.6	D97	1.30	long	—	—
182101	0.44	0.216	-4.61	—	—	—	poor	5.1	—
182572	0.77	0.148	-5.10	41.0	B96	2.25	flat	—	—
185144	0.80	0.215	-4.83	27.2	D97	1.40	poor*	7.0	—
187013	0.47	0.154	-4.92	—	—	—	long	—	—
187691	0.55	0.148	-5.03	10.0	B96	1.56	fair	5.4	—
188512	0.86	0.136	-5.17	—	—	—	poor	4.1	—
190007	1.17	0.746	-4.69	28.95	D96	1.19	fair	13.7	5.3
190360	0.73	0.146	-5.11	—	—	—	flat	—	—
190406	0.61	0.194	-4.80	13.94	D96	1.44	fair	2.6	16.9
194012	0.51	0.198	-4.72	7.0	B96	1.53	poor	16.7	5.4
201091	1.18	0.658	-4.76	35.37	D96	1.45	excl	7.3	—
201092	1.37	0.986	-4.89	37.84	D96	1.46	good	11.7	—

Table 5.3 Continued

HD	$(B - V)$	$\langle S \rangle$	$\log(R'_{\text{HK}})$	P_{rot}	Ref.	Ro	Var. Class	P_{cyc}	$P_{\text{cyc},2}$
206860	0.59	0.33	-4.42	4.91	G00	0.58	poor	6.2	—
207978	0.42	0.152	-4.89	3.0	B96	1.74	flat	—	—
212754	0.52	0.14	-5.07	12.0	B96	2.40	long	—	—
216385	0.48	0.142	-5.02	—	—	—	flat	—	—
217014	0.67	0.149	-5.08	21.9	S10	1.67	var	—	—
219834A	0.80	0.155	-5.06	42.0	B96	2.17	good	21.0	—
219834B	0.91	0.204	-4.94	43.0	B96	1.95	excl	10.0	—
224930	0.67	0.184	-4.88	15.01	H16	1.14	poor	10.2	—

Notes: Rotation period references, in order of increasing preference: N84=Noyes et al. (1984a); B96=Baliunas et al. (1996b); H16=Hempelmann et al. (2016); D96=Donahue et al. (1996); D97=Donahue et al. (1997); G00=Gaidos et al. (2000); S10=Simpson et al. (2010). Boldface stars are $\sim$ G0–G5 solar analogs with continued observations from the SSS that were studied in Chapter 4. Variability class poor* were uncertain in B95 and promoted here to the “poor” class.

We divide the sample into three spectral groups based on the $(B - V)$ color index. Figure 5.1a shows the $(B - V)$ distribution and the separation into spectral groups. The F-group (35 stars) has $(B - V) < 0.58$, the G-group (33 stars) has $0.58 \leq (B - V) < 0.75$, and the K-group (36 stars) has $(B - V) \geq 0.75$. The boundaries were chosen to create three groups of roughly equal size and to approximately correspond with the F, G, and K spectral types.¹ However, some late F-type stars are in the G-group, and a few late G-type stars are in the K-group, which also includes two M-type stars.

Figure 5.2 shows distributions in $\log(R'_{\text{HK}})$, P_{rot} , and Ro broken down by spectral group. The overall distribution in $\log(R'_{\text{HK}})$ (Figure 5.1b) is relatively flat from -5.25 to -4.50, with fewer high-activity stars in with $\log(R'_{\text{HK}}) > -4.50$. Defining the location of the Vaughan-Preston gap $\log(R'_{\text{HK}}) = -4.75$ as the boundary between “active” and “inactive”, the F, G, and K spectral groups are roughly evenly divided into active and inactive stars. The distribution in rotation (Figure 5.1c) is weighted

¹In this chapter we use the MK spectral type catalog J/ApJ/457/L99 of the B95 sample available online from the *VizieR* catalog service (<http://vizier.u-strasbg.fr>). This catalog was prepared by E. E. Mamajek in 2015 from B. Skiff’s MK spectral type compilation (*VizieR* catalog B/mk).

towards fast rotators, with the maximum rotation period dependant on the spectral group. If the sampling were uniform in stellar ages, we would expect a bias toward slower rotators from a $P_{\text{rot}} \propto t^{1/2}$ spindown law. However, the B95 sample is biased toward faster rotators, or younger stars. Since the B95 sample contains only bright (nearby) northern stars, this may be an indication that our local region of space is biased toward younger objects. van Saders et al. (2016) observed that angular momentum loss $P_{\text{rot}} \propto t^{1/2}$ ceases after the Rossby number exceeds a critical value, $Ro \approx 2.16$. Note that $Ro_{\odot} = 2.15$ using the data from Table 5.3. Using the van Saders et al. (2016) critical Rossby number and our spectral group boundaries in the Rossby number formula of Noyes et al. (1984a), we compute the maximum expected rotation period for each group to be 15 d for the F-group, 34 d for the G-group, and 46 d for the K-group up to $(B - V) = 1.0$. The rotation distribution of Figure 5.1c falls within these limits with few exceptions. Where there are exceptions, some of the stars are classified as subgiants, for example HD 81809 of the G-group with $P_{\text{rot}} \approx 40$ d. The lone K-group star with an exceptionally long rotation period of 71 days is HD 23249, with a measured diameter of $2.27 \pm 0.06 R_{\odot}$ (Maldonado et al., 2013) and has recently been classified as transitioning to the giant branch (Gray et al., 2006). We note that there are only 5 other G-group stars in the Sun’s rotational bin of 20–30 d, and only 3 more with $P_{\text{rot}} > 30$ d. Finally, the distribution in Rossby number (Figure 5.1d) peaks at $Ro \approx 1.5$ – 2.0 , with significantly fewer stars with $Ro > 2.0$ like the Sun.

5.4 Variability Class by Spectral Group and Activity

We now examine the organization of the variability classes of Table 5.3. In Figure 5.2a, we show a histogram of the F, G, and K spectral groups divided by the activity class. Over half of the F-group (18/35) is in the “flat” or “long” classes, with

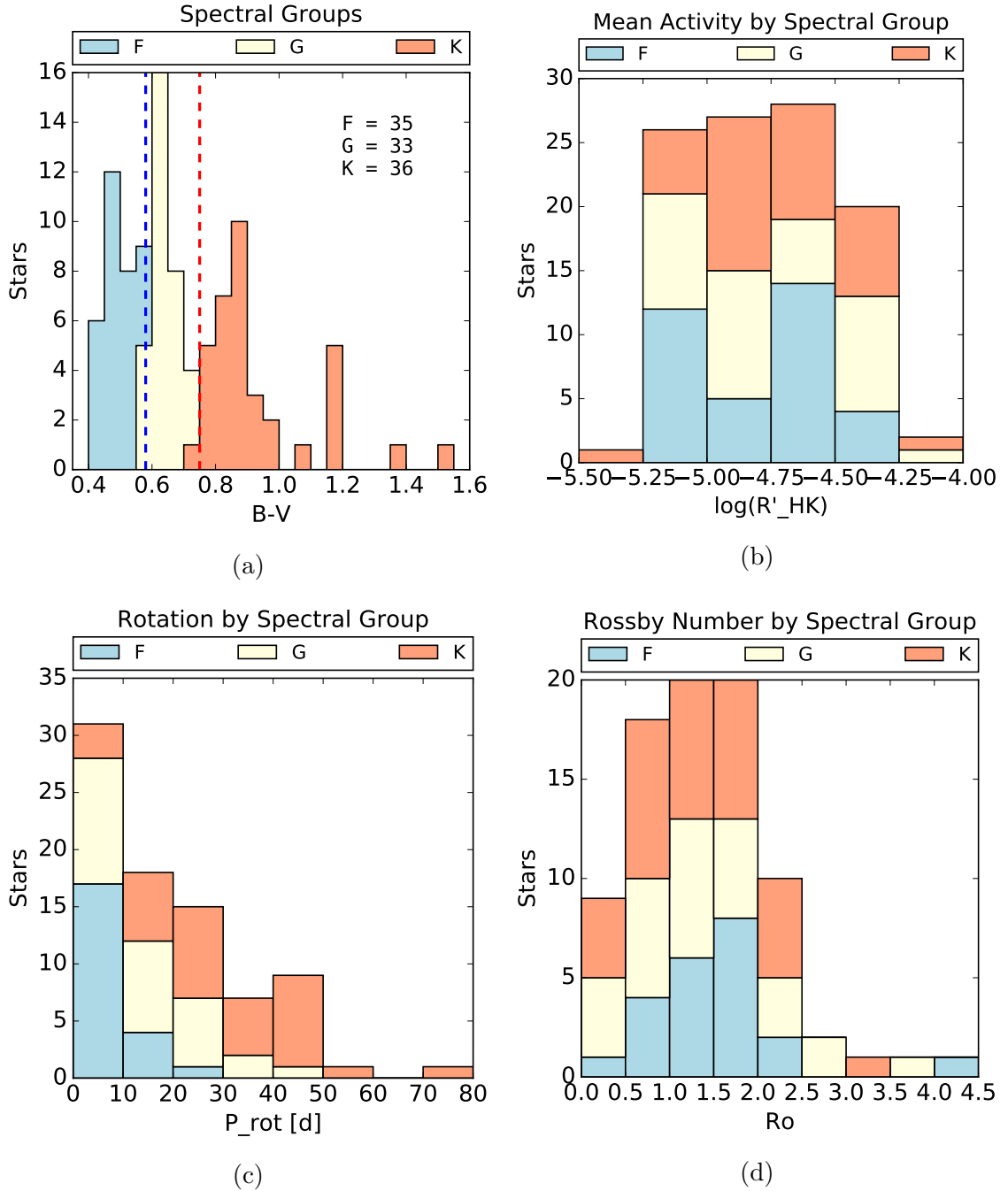


Figure 5.1 MWO sample from Baliunas et al. (1995). Histograms are shown for (a) spectral groups in $(B - V)$, (b) spectral groups by activity, (c) spectral groups by rotation, and (d) spectral groups by Rossby number. Panels (a) and (b) show all 104 stars, while (c) and (d) show 82 stars with measured rotation periods.

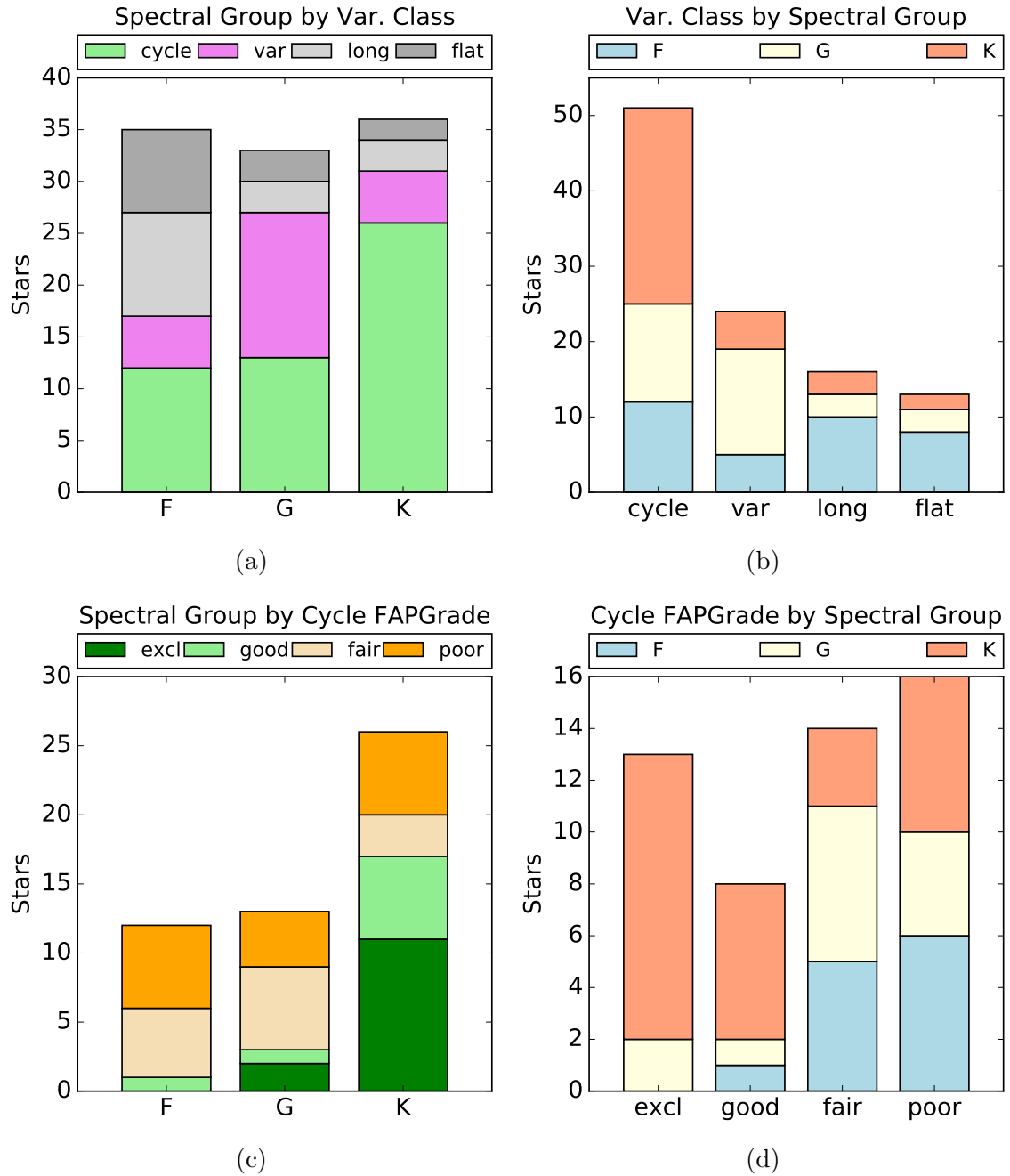


Figure 5.2 MWO sample variability class and cycle FAP Grades from Baliunas et al. (1995) divided into the spectral groups of Figure 5.1a. Panels (a) and (b) show 104 stars, while (c) and (d) show the 51 stars of the “cycle” class.

a smaller portion (12/35) in the “cycle” class, and the remaining (5/35) with erratic variability. In the G-group, few stars are flat+long (6/33), there are more erratic variables (14/33), and with nearly the same portion cycling (13/33). In the K-group, there are again few flat+long stars (5/36), even fewer erratic variables (2/36), and a large majority are cycling (26/36). Cycles are more common in K-group, and as can be seen in Figure 5.2b, about half (26/51) of the detected cycles are from that third of the sample. Similarly, about half (14/24) of the var class are from the G-group, and a little over half (18/29) of the flat+long stars are from the F-group.

In Figure 5.2c we now look at the cycling spectral groups divided by FAP grade. We will simplify the analysis somewhat by considering “good” and “excellent” to be high-quality cycles, and “fair” and “poor” to be low-quality cycles. The occurrence of high-quality cycles decreases with mass, with only (1/12) cycles for the F-group, (3/13) cycles for the G-group, and (17/26) cycles for the K-group. We see that not only is the K-group more likely than the other groups to have cycles, but its cycles are more likely to be of a high quality. Considering the overall occurrence of high-quality cycles, we have (1/35) in the F-group, (3/33) for the G-group, and (17/36) for the K-group. Figure 5.2d shows the FAP Grades broken down by spectral group. Notably, a strong majority (11/13) of the “excellent” cycles are from the K-group, the G-group only has 2 (including the Sun), and the F-group has none. Besides the Sun, the other “excellent” cycle from the G-group is from the subgiant HD 81809.

This analysis paints a rather different picture than the overall conclusion of B95 that about half of all stars have cycles. The F-group and G-group stars contribute significantly less to that result, and the K-group becomes even more dominant when one only considers the high-quality cycles that have typically been used in analyses of the cycle period.

In Figure 5.3 we examine the variability classes as a function of activity. We see that at very low activity ($\log(R'_{\text{HK}}) < -5.00$) “flat” and “long” classifications are the most common. Cycling is the most common behavior at intermediate activity ($-5.00 < \log(R'_{\text{HK}}) < -4.50$). At very high activity ($\log(R'_{\text{HK}}) > -4.50$), cycling, erratic variability, and “long” are about equally likely in this sample. Looking at cycle quality, there is a stark division with a majority (15/21) of the high-quality cycles appearing at low activity ($\log(R'_{\text{HK}}) < -4.75$) and low-quality cycles prevalent at higher activity.

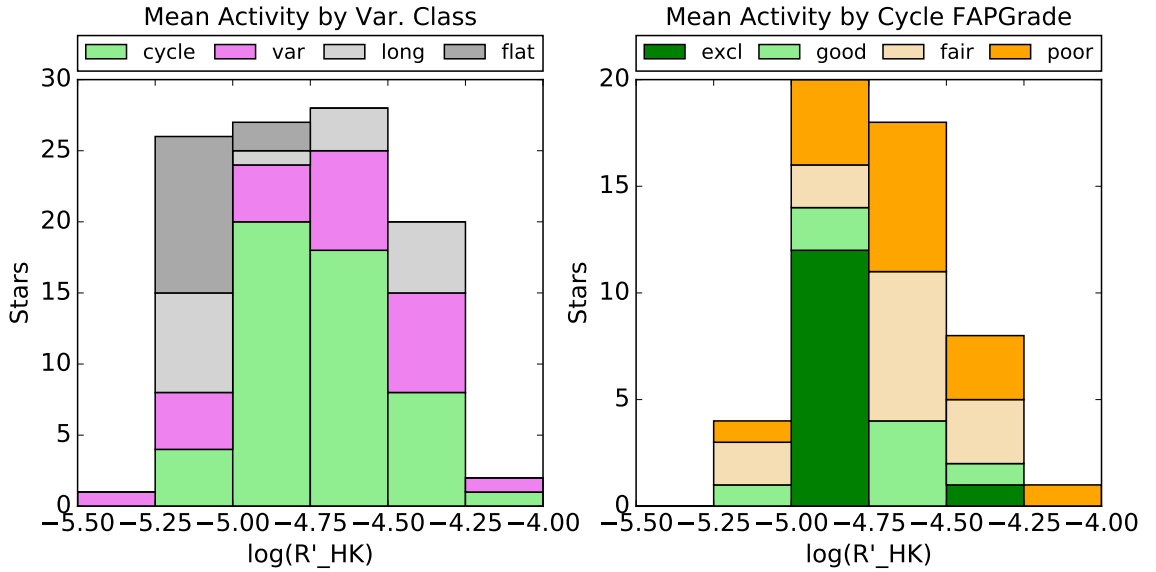


Figure 5.3 Variability class and cycle FAP Grade from Baliunas et al. (1995) divided according to activity.

Figure 5.4 shows the variability classes in a scatter plot of mean activity versus color index. Notice the large cluster of “long” and “flat” stars at very low activity ($\log(R'_{\text{HK}}) \lesssim -5.00$) and low $(B - V)$ (high mass). The Sun and HD 81809 share a similar activity level with the cluster of K-group stars with excellent cycles. The high-

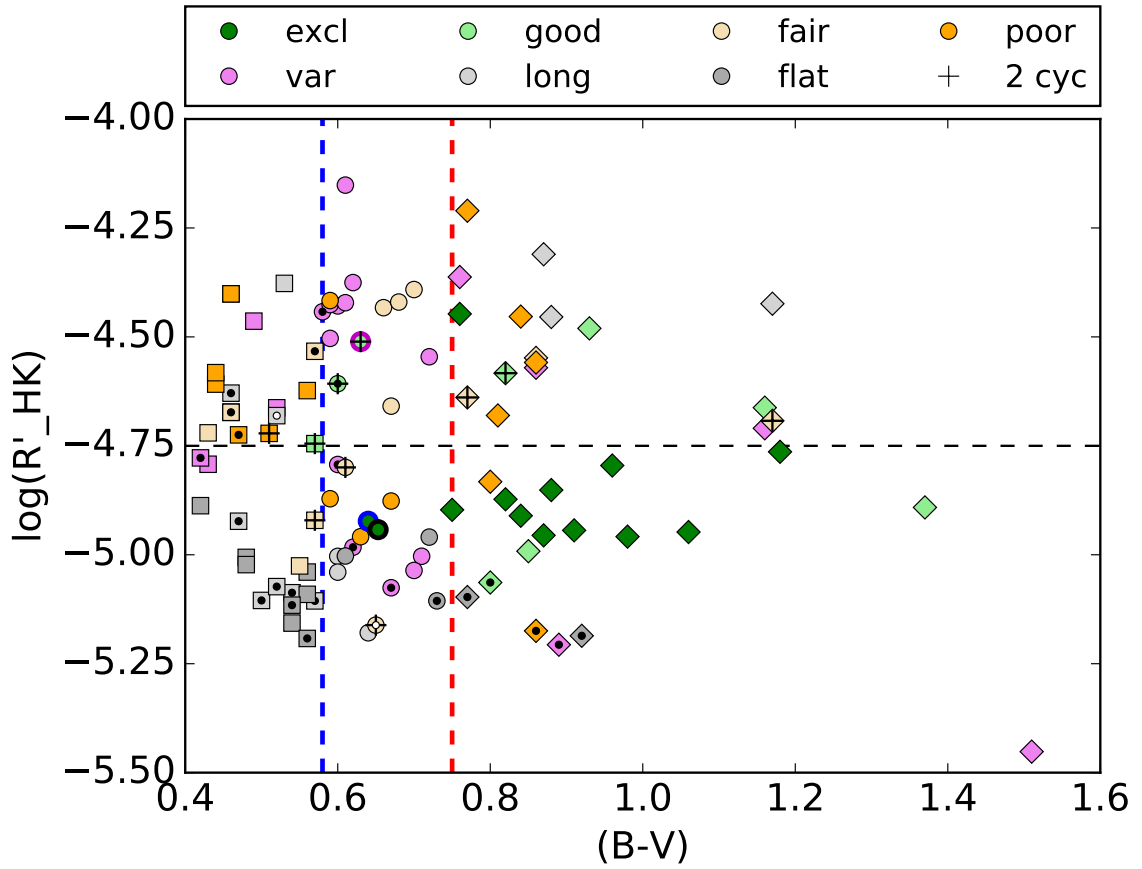


Figure 5.4 Scatter plot of activity versus color index with variability classes indicated by marker color for 104 stars in the B95 sample. The horizontal dashed line indicates the Vaughan-Preston gap, while the vertical dashed lines delineate the boundaries of the F, G, and K spectral groups, indicated by squares, circles, and diamonds, respectively. Subgiants have a black dot on the marker, and giants have a white dot. Three stars of interest are indicated by thick colored outlines: the Sun (black), HD 30495 (magenta) and HD 81809 (blue).

activity ($\log(R'_{\text{HK}}) > -4.75$) portion of the diagram consists of mostly low-quality cycles and erratic “var” class stars.

5.5 Variability Class by Rotation and Rossby Number

Mean activity is known to be dependent on rotation and mass, with both effects taken into account in the semi-empirical Rossby number (Noyes et al., 1984a). We will now examine the B95 sample by rotation and Rossby number. Figure 5.5 shows histograms of the B95 sample divided by rotation, with the variability classes indicated. We see the fraction of cycling stars increasing with rotation and the fraction of erratic variables highest at fast rotations ($P_{\text{rot}} < 10$ d). Somewhat surprisingly the number of “long” + “flat” stars is highest at fast rotations ($P_{\text{rot}} < 10$ d). However, this is due to a large contribution from the low-activity F-group stars (see Figure 5.4) which are mostly fast rotators (see Figure 5.1c). Mean activity is the strongest predictor of the “long” + “flat” stars (Figure 5.3), and low activity is a common feature of both fast-rotating F-group stars and slow rotating lower-mass stars.

Looking at the FAP Grades by rotation (Figure 5.5, right panel), we see a strong division between high- and low-quality cycles, with the dividing line at $P_{\text{rot}} = 30$ d. This agrees with our expectations based on the activity-rotation relationship and our previous observation of high-quality cycles at low activity (Figure 5.3). However, it is surprising to see that the vast majority of “excellent” cycles which are the most qualitatively Sun-like occur *almost exclusively* (11/13) in stars rotating slower than the Sun.

Turning now to the Rossby number in Figure 5.6, we see that the “var” stars and all low-quality cycles are found at $Ro < 2$. The “flat” and “long” stars are distributed among all Ro , indicating that Ro is not a good parameter in determining this classification. High-quality cycles are mostly found at $Ro > 1$, with the “excellent” cycles having $Ro > 1.5$ with only one exception. Excepting the G2III giant star HD 161239 with a “fair” cycle, it is notable that the Sun’s Rossby number $Ro_{\odot} = 2.15$

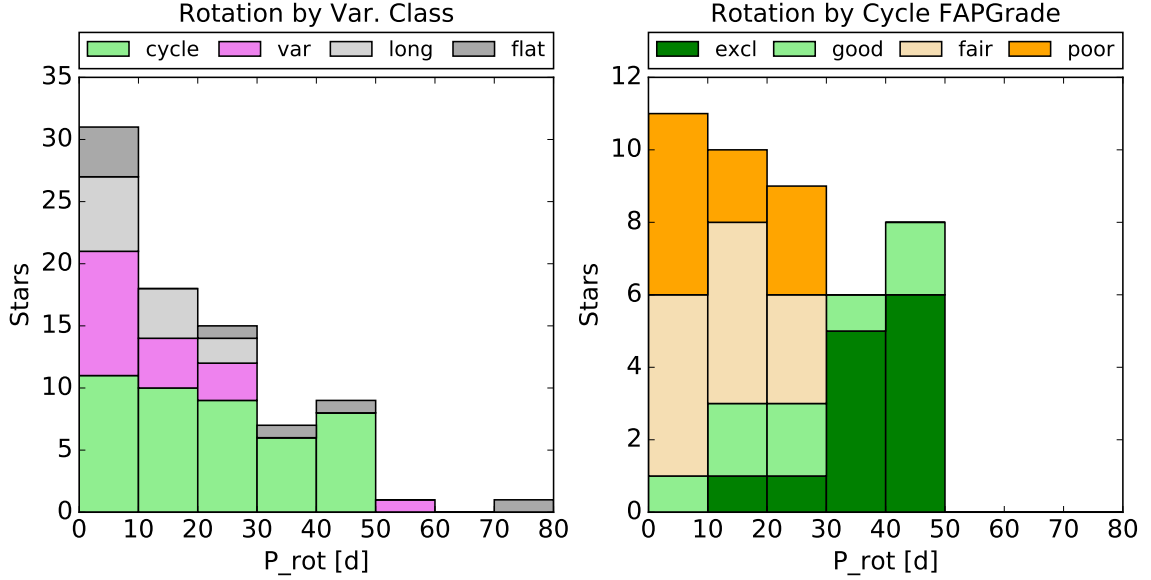


Figure 5.5 Variability class and cycle FAP Grade from Baliunas et al. (1995) divided according to rotation period.

falls into a bin containing *only* high-quality cycles and “flat” or “long” behavior. Considering that the Sun may have also demonstrated “flat” behavior during the Maunder minimum, we now have some evidence that the Sun’s behavior is not unusual in a stellar context. The “flat” and “long” stars in the Sun’s bin ($2.0 \leq Ro < 2.5$) consist of HD 107213 (F8IV-V; long), HD 212759 (F8IV-V; long), HD 182572 (G7IV; flat), and HD 10700 (G8V; flat). The last of these, also known as τ Ceti, is the only one firmly on the main sequence. Judge et al. (2004) and Judge and Saar (2007) performed extensive analyses of τ Ceti in UV and soft X-rays and presented this star as a likely stellar analog to the Sun’s Maunder-minimum phase.

Figure 5.7 shows a scatter plot of activity versus Rossby number. As discussed in Section 1.3.5, the log Rossby number of Noyes et al. (1984a) was found to produce a tight relationship with activity expressed as $\log(R'_{HK})$. With this larger sample, we see that the relationship is not very tight below $\log(R'_{HK}) > -4.75$, mostly due

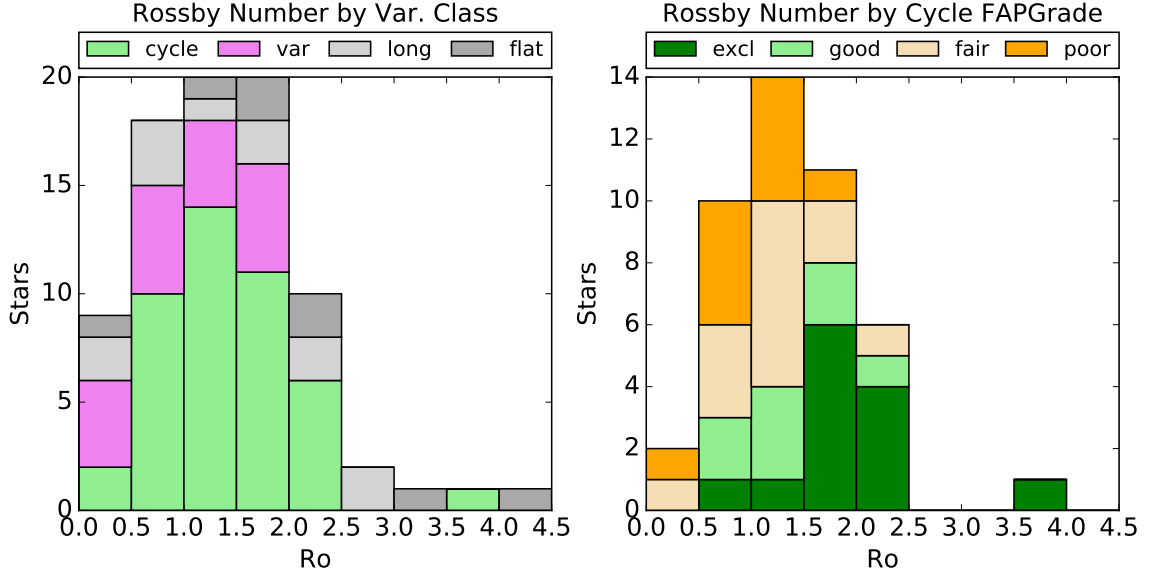


Figure 5.6 Variability class and cycle FAP Grade from Baliunas et al. (1995) divided according to Rossby number.

to subgiants and F-group stars. With the variability classes indicated, we now see some other interesting features. Many high-quality cycles are clustered around the Sun near $(Ro, \log(R'_{HK})) \approx (2.0, -4.90)$. Besides the Sun, all the stars in this cluster are members of the K-group.

There are two stars which have uncertain positions on the diagram, indicated by the dotted lines to an alternate position. HD 13421 has a P_{rot} measurement of 2.17 d in Hempelmann et al. (2016) with a 2σ confidence level. This measurement results in a very low Rossby number of 0.31, and making the star a severe outlier in the diagram. However, the authors found a second 1σ peak in their periodogram analysis at 4.5 d, which agrees to within the uncertainties with an earlier measurement by Stimets and Giles (1980) using an autocorrelation analysis of the MWO data. Using the slower rotation measurement gives $Ro = 0.65$, moving the star closer to the other “flat” stars in the diagram, but still an extreme value. The other star is HD 224930, which

also had its rotation period halved to 15.01 d in Hempelmann et al. (2016) compared to an existing literature value of 33 d (Baliunas et al., 1996b), though Huber et al. (2009) also found a similarly short rotation period of ~ 11 d. Both HD 13421 and HD 224930 need additional observations to more confidently determine their position on the diagram.

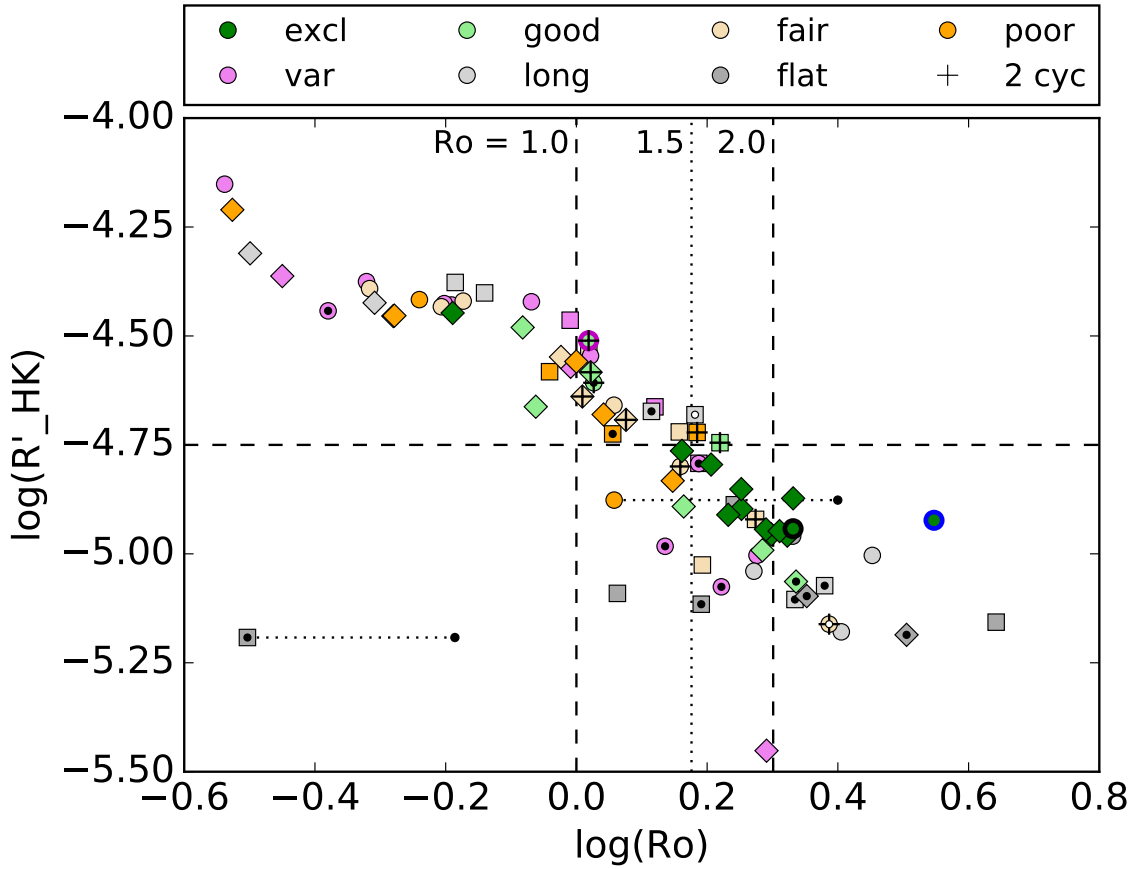


Figure 5.7 Scatter plot of activity versus Rossby number with variability classes indicated for 82 stars in the B95 sample with measured rotations. The horizontal dashed line indicates the Vaughan-Preston gap, while vertical lines show $Ro = 1$ and 2, and $Ro_{crit} = 1.5$. Marker shape indicates the F (squares), G (circles), and K (diamonds) spectral groups. Three stars of interest are indicated by thick colored outlines: the Sun (black), HD 30495 (magenta) and HD 81809 (blue). Dotted lines connecting points show alternative measurements which are discussed in the text.

5.6 Hypotheses on the Properties Resulting in Sun-like Cycles

We have identified that most of the high-quality cycles appear at $Ro > 1.5$ and cluster around the point $(Ro, \log(R'_{\text{HK}})) \approx (2.0, -4.90)$. Furthermore, the Ro versus $\log(R'_{\text{HK}})$ scatter plot (Figure 5.7) indicates that $Ro_{\text{crit}} = 1.5$ is approximately where stars transition the Vaughan-Preston gap at $\log(R'_{\text{HK}}) = -4.75$. This is evidence that the Vaughan-Preston gap has physical meaning; the patterns of variability observed depend on which side of the gap a star lies. This was previously observed in B95, we will now attempt to quantify the observation by testing the following hypothesis:

Hypothesis 1: Stars have clean monoperiodic cycles like the Sun’s, *or* flat activity analogous to the Sun’s Maunder minimum *if and only if* they have a Rossby number $Ro > 1.5$.

We phrase the hypothesis in terms of the Ro as opposed to the activity level because we seek a rule that links effects to causes, and the Ro is based on internal physical properties of the star that directly influence the generation of large-scale fields, while the activity level is a result dependent on internal physical properties. Specifically, a star is deemed to have a “clean monoperiodic cycle” if it has a FAP Grade of “excellent” or “good” and no reported $P_{\text{cyc},2}$ in B95. It is deemed to be “flat activity” if the variability class is “flat”.

As can be seen from the histogram in Figure 5.6, there are 16 counter-examples to Hypothesis 1. There are 19 stars that match the variability class and $Ro > 1.5$ conditions, 13 with high-quality monoperiodic cycles and 6 with flat activity. With only 19 of 35 stars with $Ro > 1.5$ satisfying Hypothesis 1, the conjecture must either be false or too general. A detailed analysis suggests the latter. In Table 5.4 we have classified the 16 exceptions into three groups: evolved stars, F-group main sequence

Table 5.4. Stars with $Ro > 1.5$ but without Sun-like variability

HD	$(B - V)$	$\log(R'_{\text{HK}})$	P_{rot} (d)	Ro	Var. Class	SpT
<u>Evolved Stars</u>						
124850	0.52	-4.68	7.6	1.52	long	F7III
6920	0.60	-4.79	14.0	1.54	var	F9IV
217014	0.67	-5.08	21.9	1.67	var	G2IV
107213	0.50	-5.10	9.0	2.16	long	F8IV-V
212754	0.52	-5.07	12.0	2.40	long	F8IV-V
161239	0.65	-5.16	29.2	2.44	fair (2 cyc)	G2IIIb
9562	0.64	-5.18	29.0	2.54	long	G1V (G2IV?)
<u>F-group Main-Sequence Stars</u>						
194012	0.51	-4.72	7.0	1.53	poor (2 cyc)	F7V
2454	0.43	-4.79	3.0	1.54	var	F5VSr
187691	0.55	-5.03	10.0	1.56	fair	F8V
114710	0.57	-4.74	12.35	1.66	good (2 cyc)	F9.5V
100180	0.57	-4.92	14.0	1.88	fair (2 cyc)	F9.5V
<u>G&K-group Main-Sequence Stars</u>						
143761	0.60	-5.04	17.0	1.87	long	G0+Va Fe-1
115617	0.71	-5.00	29.0	1.89	var (?)	G7V
95735	1.51	-5.45	53.0	1.95	var	M2V
141004	0.60	-5.00	25.8	2.84	long	G0-V

stars, and G & K group main-sequence stars. Stars in each group are ordered by the Rossby number. We discuss each of these groups in turn.

There are 7 evolved stars which are counter-examples to Hypothesis 1. The semi-empirical Rossby number of Noyes et al. (1984a) was not developed for evolved stars, so immediately we are uncertain whether the Ro has any meaning for this group; especially for the established giants. The first three evolved stars have Ro within 20% of the $Ro_{\text{crit}} = 1.5$ border, which may be within the (unquantified) uncertainty for these stars. The next four have $Ro > 2.0$ and generally have very low activity $\log(R'_{\text{HK}}) < -5.00$. Three of these 4 have a variability class of “long”, which possibly qualify as “flat activity analogous to the Maunder minimum” according to

Hypothesis 1. The most difficult exception is HD 161239, with $Ro = 2.44$ and two “fair” quality cycles at 5.7 yr and 11.8 yr. Keenan and McNeil (1989) classifies this star as G2IIIb, while Gray et al. (2003) reports G2IV. The slow rotation period of 29.2 d (Donahue et al., 1997) is consistent with an evolved G-type star. The Hipparcos parallax $\pi = 25.60 \pm 0.63$ and V magnitude 5.73 (van Leeuwen, 2007) result in an absolute magnitude of 2.77, also consistent with an evolved star. We must conclude that either the Noyes et al. (1984a) Ro is invalid for evolved stars, or Hypothesis 1 is false for evolved stars.

There are 5 F-group ($B - V < 0.58$) stars which are counter-examples to Hypothesis 1. The first four listed in Table 5.4 have Ro within 20% of the $Ro_{\text{crit}} = 1.5$ border, which may be within the Ro uncertainty for these stars. Taking HD 114710 ($Ro = 1.66$) as an example, Donahue (1993) reports an rms scatter of 0.71 d for the 12 seasonal rotation measurements, giving an error in the mean rotation of 0.2 d, or 1.6%. The uncertainty in $(B - V)$ is about 0.01 mag, which judging from the τ_c versus $(B - V)$ curve of Noyes et al. (1984a) makes the uncertainty in τ_c less than a day, or 8%. This results in an estimated uncertainty in Ro of about 8%, placing this star 1.2σ from the border. The P_{rot} uncertainties for the other F-group stars are not reported, so we shall assume they are worse than for HD 114710, and take them to be 5%. Then, for the worst case in this group, HD 100180, with $Ro = 1.88$, we estimate $\sigma_{Ro} \approx 0.18$, placing this star 2.1σ from the border. Therefore, there is a low, but non-negligible chance that each of these stars has a true $Ro < 1.5$. We could of course choose to move the $Ro_{\text{crit}} = 1.5$ limit of Hypothesis 1 to make this less of an issue, however we then have the problem of high-quality cycling stars with $Ro < Ro_{\text{crit}}$, which we discuss below. We shall conclude that the F-group main-sequence stars which lie near the $Ro_{\text{crit}} = 1.5$ border and are challenging to Hypothesis 1, but when considering the uncertainties there are no unquestionable counter-examples.

Finally, there are 4 main-sequence G or K group stars that are possible counter-examples to Hypothesis 1. Two of these, HD 143761 and HD 141004 are G-group stars with variability class “long” that were studied in Chapter 4. We find that their variability is in fact quite flat, with $\sigma_S/\langle S \rangle = 2.4\%$ and 3.5% , respectively. Furthermore, they are marginally evolved, with $L/L_\odot = 1.74$ and 2.07 , respectively. We may therefore rightly consider these stars to be in states analogous to the Maunder minimum, therefore satisfying Hypothesis 1, or justifiably be worried that their evolved state makes their Ro determination questionable. The star HD 95735 has $(B-V) = 1.51$, which is above the limit of $(B-V) = 1.40$ considered when developing Ro in Noyes et al. (1984a). Caution would advise against extrapolating the Ro to this regime, especially given the poor representation of only 7 low-mass stars $(B-V) > 1.0$ used in the development of the Ro -activity relationship of that study. The last star to consider is HD 115617. With $Ro = 1.89$ we estimate it is 2.2σ away from the $Ro_{\text{crit}} = 1.5$ border. Furthermore, this star was classified as “var” in B95 with only 10 seasons of data. Judging from the visual appearance of the published time series, a longer record could possibly reveal a high quality cycle. In summary, none of the G or K-group main sequence stars are unquestionable counter-examples to Hypothesis 1. We are motivated by the example of HD 95735 to set an upper limit to the $(B-V)$ for which Hypothesis 1 ought to be applied. This, together with our uncertainty in applying the Ro formula to evolved stars leads us to a more restricted hypothesis:

Hypothesis 2: Main-sequence stars with $0.4 \leq (B-V) \leq 1.4$ have clean monoperiodic cycles like the Sun’s, *or* flat activity analogous to the Sun’s Maunder minimum *if and only if* they have a Rossby number $Ro > 1.5$.

This modified hypothesis has no unquestionable counter-examples of stars satisfying the property conditions, but *not* satisfying the variability class conditions.

Table 5.5. Stars with $Ro < 1.5$ but with Sun-like variability

HD	$(B - V)$ (mag)	$\log(R'_{\text{HK}})$	P_{rot} (d)	Ro	Var. Class	P_{cyc} (yr)	SpT	$P_{\text{cyc},+}$ (yr)
<u>High-quality Monoperiodic Cycles</u>								
201092	1.37	-4.89	37.84	1.46	good	11.7	K7V	4.5
201091	1.18	-4.76	35.37	1.45	excl	7.3	K5V	None
156026	1.16	-4.66	21.0	0.87	good	21.0	K5V	4.3, 8.1
115404	0.93	-4.48	18.47	0.83	good	12.4	K2V	5.1, 3.4
152391	0.76	-4.45	11.5	0.65	excl	10.9	G8.5V	2.8
<u>Flat-Activity Stars</u>								
45067	0.56	-5.09	8.0	1.16	flat	—	F9V	—
13421	0.56	-5.19	2.2	0.31	flat	—	G0IV	—
			(4.5?)	(0.65?)				

Note. — Columns 2–6 are copied from Table 5.3. Column 8 gives the short-period variability detected by Oláh et al. (2016) which indicates the variability is not in fact Sun-like.

There are a few “edge cases” appropriately near the $(B - V) = 0.4$ and $Ro = 1.5$ boundaries which are of some concern. However, within the estimated uncertainties it remains possible that all of them satisfy Hypothesis 2.

Now we consider the exceptions to the “and only if” criterion of Hypotheses 1 and 2. Table 5.5 shows stars with high-quality monoperiodic cycles like the Sun’s or flat activity analogous to the Maunder minimum, but with a low Rossby number $Ro < 1.5$. We find 7 of 46 stars with $Ro < 1.5$ are possible counter-examples to Hypothesis 1. We have separated these stars into two groups: flat activity stars ($N = 2$) and high-quality monoperiodic cycles ($N = 5$), which we discuss separately below.

There are two flat activity stars with $Ro < 1.5$; both of them are in the F-group. The first, HD 13421, is classified as a subgiant. We discussed above the potential

problems with applying the Ro formula to subgiants. Furthermore, this star has an uncertain rotation period (see previous section). The Rossby number is either 0.31 or 0.65 depending on the choice of rotation measurement. The rotation period would have to be about 10 d to make $Ro = 1.5$, which would also better match our expectations for an expanding subgiant. We choose to ignore this contradiction to Hypothesis 1 until this fast rotation can be confirmed. The second star, HD 45067, has no evidence from the literature that it is a subgiant. The rotation period of 8.0 d was reported by Baliunas et al. (1996b). Using the same assumptions as above, we estimate it is 3.1σ away from $Ro_{\text{crit}} = 1.5$. This problematic case is again an F-group star, of which we found numerous others above. Additional rotation measurements for HD 45067 would be useful, but this looks to be a strong contradiction to Hypotheses 1 & 2.

The counter-example provided by HD 45067 could be handled in one of two ways. Either F-type flat activity should not be considered as “analogous to the Maunder minimum”, or we admit that Maunder minimum states can happen at any Rossby number. To help us decide, we looked at the 6 stars classified as “long” with $Ro < 1.5$ to see if more more low Ro stars are *roughly* flat. The subsample is divided evenly between the K- and F- groups, with Ro ranging from 0.32 to 1.3. Visual inspection of the time series for each of these stars in B95 shows large amplitude long-term trends that could not be confused with flat activity. The F-group “long” stars have fast rotation ranging from 2.0 to 3.6 d, which is 2 to 4 times faster than the flat-activity HD 45067. With no additional evidence for flat activity at $Ro < 1.5$, together with the difficulties posed by F-group stars with $Ro > 1.5$, we conclude that the F-group stars behave somewhat differently than the G- and K- groups.

We now examine the high-quality monoperiodic cycles with $Ro < 1.5$, which we have placed in order of increasing distance from $Ro_{\text{crit}} = 1.5$. The first two are

the binary pair HD 201091 and HD 201092 (61 Cyg A & B), very near Ro_{crit} with $Ro \approx 1.45$ for both stars. We estimate them to be 0.3σ from the Ro_{crit} , using same assumptions as above. Therefore, there is a possibility that these two stars are not in contradiction with Hypothesis 1 or 2.

The last three stars, HD 115404, HD 152391, and HD 156026 are indisputably outside the Rossby number selection of any of our hypotheses. The only way this could not be the case is if their color index or P_{rot} were to be in error. Rotation is the more difficult to measure, but both HD 115404 and HD 152391 have successful detections for multiple (7 and 11, respectively) observing seasons in Donahue et al. (1996). The latter star also has additional photometric confirmation in Gaidos et al. (2000). The rotation period of HD 156026 was reported in Noyes et al. (1984a) and Baliunas et al. (1996b), both in rough agreement. Its omission from the Donahue et al. study is presumably due to an inability to detect the rotation in five or more seasons, the threshold for inclusion in that work.

Assuming no errors in the rotation periods, we now turn to the quality of the monoprotic cycle itself for the low Ro stars. Oláh et al. (2016) examined the full ~ 35 yr MWO S -index record for each of these stars using a time-frequency analysis, and classified each of them as having “complex cycles” with significant short-duration variability which was not reported in B95. The periods of this short-duration variability are shown in Table 5.5. Four of the five stars have short-period variations reported, raising the possibility that these cycles are not truly Sun-like. However, to determine if this is the case a threshold should also be placed on the amplitude of the short-period variations. Oláh et al. (2016) applied their analysis technique to the sunspot record and Ca II K flux from Sacramento Peak, and report a 3.65 yr variation that is continuously present in both activity proxies. Although Oláh et al. display their time-frequency power spectra on equal scaling, it is likely that the solar spectrum

is biased due to the improved sampling, making it difficult to compare the Sun to the other stars. However, comparing these questionable stars to the other “simple cycle” stars in Oláh et al. (2016), there is a clear difference. The “simple cycle” stars, which satisfy hypotheses 1 & 2 without exception, have markedly less power at higher frequencies than the main cycle. They are unquestionably monoperiodic, as we usually consider the Sun.

The above analyses have caused us to further restrict our hypothesis:

Hypothesis 3: Main-sequence stars with $0.58 \leq (B - V) \leq 1.4$ have clean monoperiodic cycles like the Sun’s, *or* flat activity analogous to the Sun’s Maunder minimum *if and only if* they have a Rossby number $Ro > 1.5$.

This hypothesis has no unquestionable exceptions from the 20 G-group and 30 K-group stars on the main sequence with measured rotation periods in the B95 sample. The questionable cases are clustered around the critical Rossby number $Ro_{\text{crit}} = 1.5$ where the sample trend crosses the Vaughan-Preston gap. The omitted classes of evolved and F-group stars may be problematic due to a failure of the Rossby number formula of Noyes et al. (1984a) to properly characterize these stars, or perhaps due to the different physical conditions present in their respectively thinner and deeper convection zones. Considering the cycling stars, this result is similar and related to the conclusions of Oláh et al. (2016) and Baliunas et al. (1995), who found that smoothly cycling stars are older and rotate more slowly. Our parameterization in terms of the Rossby number has the advantage of mass-independence and easier application as a constraint to dynamo theory.

We observed in Section 5.4 that K-group stars in the B95 sample exhibit cyclic behavior with high frequency (26 of 36 stars) and high quality (17 of 26 cycles) when

compared with the G-group stars. For G-group stars, only 13 of 33 stars (including subgiants) are cycling, and 3 of those 13 cycles are high-quality. This *does not* appear to be due to an accidental bias in the Ro sampling. The G-group has $Ro > 1.5$ for (7/20) main-sequence members with rotation measurements, and the K-group has (11/30), nearly the same fraction. However, the G-group only has 1 or 2 cycles in this subsample (the Sun and one unconfirmed cycle; the rest are flat) while the K-group has 10 cycles, no flat stars, and the only non-cycling star is the M-dwarf HD 95735. On the low Ro side, the K-group has (14/19; 74%) stars with detected cycles while the G-group has (7/13; 54%). It appears that while Hypothesis 3 is satisfied for both groups, the G-group stars are more likely to be erratically variable on the low- Ro side, and flat on the high- Ro side. Supposing that this “trend” continues with the F-group stars to the point where high-quality cycles become nearly impossible (as appears to be the case; see Figures 5.2 and 5.4), it would explain the failure to apply Hypothesis 2 to the F-group.

5.6.1 Hypothesis Testing with the Solar Analog Ensemble

We computed the Rossby number for the 27 stars in our stellar ensemble in Chapter 4 using the data of Table 4.1 to see if there are any apparent exceptions to Hypothesis 3. We have 14 stars with $Ro > 1.5$. Five of these are evolved, with $L/\mathcal{L}_{\odot}^N > 2.0$, and one is in the F-group, and therefore Hypothesis 3 is not applicable. Four of the remaining eight have very flat activity, with $\sigma_{\text{MAD}}/\hat{S} < 3\%$, and therefore satisfy Hypothesis 3. Here we have used a higher threshold for “flat” than B95 due to the increased noise of SSS CCD-1 data. The remaining four stars are the Sun, HD 71148, HD 146233 (18 Sco), and HD 176051. From Table 4.4, we see that in addition to the Sun, HD 71148 has a single relatively high-quality cycle ($Q_{\text{cyc}} = 38$) that satisfies Hypothesis 3.

This leaves 18 Sco and HD 176051, both of which have two cycles reported and thus appear to contradict Hypothesis 3. Inspection of the periodogram of HD 176051 (Figure B.21) confirms that it has significant variability on multiple time scales. Its rotation period is uncertain, however, as it was obtained as a lower limit from $v \sin i$ and the estimated radius. A measurement from spot modulation is needed to firmly establish the rotation period and Rossby number, for which we find as a lower limit $Ro \geq 1.78$. We have discussed the time series of 18 Sco in depth in Section 4.3; refer to Figures 4.4 and B.20. The cycle of 18 Sco is fairly clean, but the widely separated double peak of the 14-year min-to-min cycle results in a strong secondary amplitude. The murky question is whether or not this is so far removed from solar variability to consider it a counter example to Hypothesis 3. The solar cycle maximum is also frequently double peaked, with peak separations on the order of 2 years (see Hathaway (2015) section 4.6 and Figure 2.2 cycles 23 and 24). However, the ~ 6 yr separation and large amplitude difference of the double peak seen in 18 Sco are of an order unseen in the Sun. Further observation of 18 Sco is needed to determine if the two periods are persistent. This star also lacks a rotation measurement from spot modulation, though it is doubtful that such a measurement would fall far enough below the estimate of $P_{\text{eq}} = 22.5$ d Petit et al. (2008) to place it below the $Ro_{\text{crit}} = 1.5$ threshold.

Furthermore, the uncertain rotation periods of HD 197076 and HD 224930 could make them counter examples in the other sense: they have high-quality monoperiodic cycles but lie below $Ro_{\text{crit}} = 1.5$. The former has only a lower limit rotation period from $v \sin i$, while the latter has conflicting measurements in the literature, as discussed previously.

The counter examples discussed above are not without uncertainties, but are nonetheless in contradiction to Hypothesis 3 according to the rules which we have defined.

5.7 FAP Grade and Cycle Period

We have now shown that high Ro , or slow rotation, results in the highest quality cycles. This is again reflected in Figure 5.8, where we have plotted P_{cyc} versus P_{rot} for all cycles of the B95 sample. Low-quality cycles have $P_{\text{rot}} < 30$ d and mostly $P_{\text{cyc}} < 10$ yr. Most high-quality cycles have either $P_{\text{rot}} > 30$ d or $P_{\text{cyc}} > 10$ yr. From equation 4.7 we see that the periodogram power P_N which is compared to the FAP threshold is not proportional to the cycle period, therefore there appear to be qualitatively different signals across the rotation boundary. This qualitative difference is likely due to the existence of multiple shorter-period signals in the data, which *does* reduce the FAP of the “main” (longer period) signal when all other factors are equal. Indeed, all of the secondary “cycle” detections are on the fast rotating side of the diagram.

The active and inactive branches of Böhm-Vitense (2007) (hereafter BV07) are not as apparent in Figure 5.8 (compare to Figure 1.10). There are several reasons for the differences between these two diagrams: (1) we have not applied the subjective cycle quality judgements of Saar and Brandenburg (1999) and subsequent high-quality selection of BV07; (2) Saar and Brandenburg changed the “primary” cycle to be the one with the longer period, while B95 took it to be the signal with the larger amplitude; (3) BV07 cycle periods differ by ≤ 1.2 yr, presumably due to Brandenburg et al. (1998) selecting unpublished “updates” originating from Donahue et al. (1996); (4) Saar and Brandenburg (1999) determined a cycle period of 15.5 yr for HD 165341A by visual inspection of the peaks in the B95 time series, while B95 reported 20 yr;

and finally (5) in seven instances we have selected different rotation periods from the literature, which differ by up to 3%².

The removal of the “poor” cycles helps somewhat to return us to the picture of BV07, but it is still clear that the existence of two cleanly separated activity branches is not a robust feature of the data. In particular, a line from the origin through the Sun, which previously was clear, now passes very near to several other stars. It does appear, however, that the density of points near such a line is lower than the previously indicated active and inactive branches. Only more data can remove all doubts, as with enough stars on the plot the “noise” due to the uncertainty of the cycle quality selection, as well as the intrinsic variability of stellar cycle periods and rotation measurements due to differential rotation could be averaged out, revealing the two branches if they indeed exist.

5.8 Summary of Results

Our analyses have quantitatively confirmed the statements made in Baliunas et al. (1995) that were reiterated in Section 1.3.6. Those results were phrased in terms of age and activity expressed by the S -index; here we iterate similar, but more precise points in terms of the Rossby number Ro and activity expressed by $\log(R'_{\text{HK}})$:

1. G- and K-group main-sequence stars ($0.58 \leq (B - V) \leq 1.40$) with $Ro < 1.5$ exhibit high average levels of activity ($\log(R'_{\text{HK}}) \gtrsim -4.75$). All these stars are variable; there are no instances of flat activity analogous to the solar Maunder minimum. They rarely have high-quality cycles, and when they do there is significant variability on multiple timescales, unlike the Sun. High-quality dual-cycle stars have $Ro \approx 1$.

²BV07 listed $P_{\text{rot}} = 38.2$ d for HD 10476; this appears to have been a typo. Saar and Brandenburg (1999) lists 35.2 d, as does the original source Donahue et al. (1996)

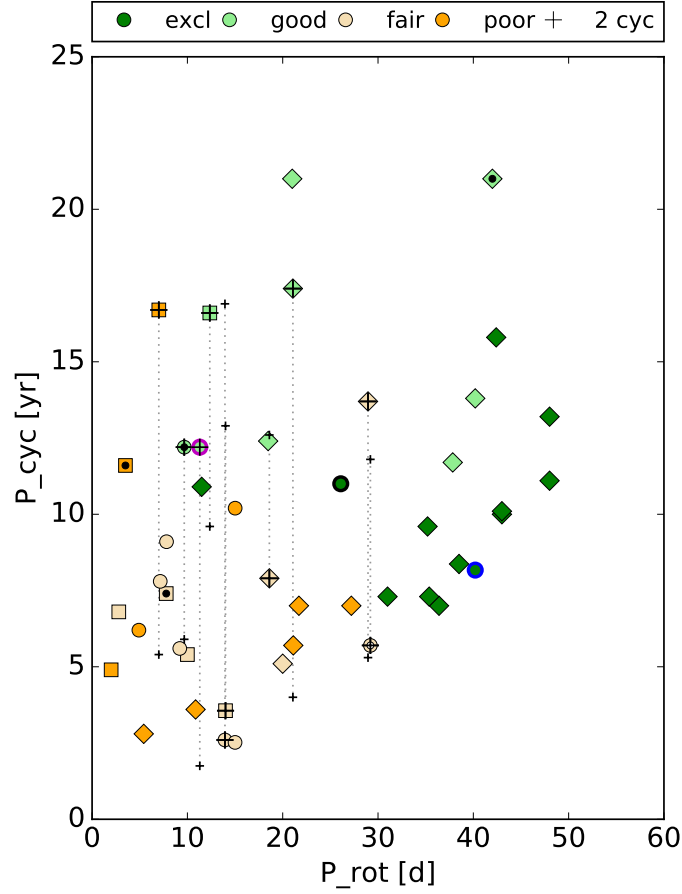


Figure 5.8 P_{cyc} versus P_{rot} for all cycles from Baliunas et al. (1995) with a measured rotation period. Compare to Figure 1.10.

2. G- and K-group main-sequence stars ($0.58 \leq (B - V) \leq 1.40$) with $Ro > 1.5$ have low levels of activity ($\log(R'_{\text{HK}}) \lesssim -4.75$), and either clean monoprotic cycles like the present Sun or flat activity that may be analogous to the Sun's Maunder minimum. We have found three exceptions to this rule which require further observation to confirm: HD 115617, HD 146233, and HD 176051.
3. K-group stars are more likely to have cycles and the cycles are more likely to be of a high quality when compared with the G-group stars. This does not appear to be due to a bias in the Ro sampling between the two groups; instead, erratic

variability is more likely in the G-group stars for $Ro < 1.5$ and flat activity is more likely for $Ro > 1.5$.

4. F-group stars are the least likely to have high-quality cycles and the most likely to be flat-activity. Their variability class does not appear to have a simple dependence on Rossby number.

Parameterizing the MWO sample in terms of Ro and $\log(R'_{\text{HK}})$ allows us to classify long-term variability with fewer exceptions and caveats than using other parameters, such as $(B - V)$, rotation, and age. However, exceptions and caveats still exist. Evolved stars are often involved in these exceptions, which may simply be due to the Rossby number formulation of Noyes et al. (1984a) being invalid for these stars. This is perhaps because that formulation does not properly handle the higher luminosity and deeper convection zone depth for these stars. Were we able to obtain the “true” Rossby number for evolved stars, we should expect the dynamo to have the same dependence on this property as it does for main sequence stars. The fundamental difference is the location of the nuclear fusion, with main sequence fusion taking place in the core, while evolved stars have fusion in shells surrounding an inert core. As long as the outer convective shell is sufficiently separated from the region where energy is released, it should not matter *where* the fusion takes place. The other problematic group are the F-group stars, whose vanishingly thin convective envelopes may prohibit the kind of dynamo mechanism at work in the lower-mass stars.

We note that in the B95 sample the G-group has few stars with $Ro > 1.5$, with the Sun as the *only* confirmed high-quality cycle from a main sequence star in this regime. For $(B - V) \approx 0.65$ like the Sun, this means stars with $P_{\text{rot}} \gtrsim 18$ d. Using Figure 5.7 as a guide, this also means G-group stars with $-5.00 < \log(R'_{\text{HK}}) < -4.75$.

Long-term observations of more G-group stars would serve to better test Hypothesis 3 with stars more like the Sun.

However, the simplest way to add data to test Hypothesis 3 is to obtain rotation periods for the 22 stars of Table 5.3 that are missing this measurement. This would allow the use of the existing decades-long activity records. The previous rotation period searches of the Mount Wilson HK project no doubt attempted to do this already, so it should be expected that the rotational signal is low for these stars, perhaps due to low surface activity or low inclination angle. An observation program of 2–5 years with relatively high cadence ($\gtrsim 50$ measurements per year) would have the best chance of achieving success.

CHAPTER SIX

CONCLUSION

In this dissertation we have examined the activity cycle of the Sun in chromospheric Ca II H & K emission in the context of other Sun-like stars. Our major results include the following:

1. We constructed a composite S -index time series of the Sun calibrated to MWO HKP-2 measurements of reflected sunlight of the Moon during solar cycle 23. For cycle 23, we find a minimum $S_{23,\min} = 0.1634 \pm 0.0008$, amplitude $\Delta S_{23} = 0.0143 \pm 0.0012$, and mean $\langle S_{23} \rangle = 0.1701 \pm 0.0005$. From the extended composite of cycles 15 through 24, we find mean values for the minimum $\langle S_{\min} \rangle = 0.1621 \pm 0.0008$, amplitude $\langle \Delta S \rangle = 0.0145 \pm 0.0012$, and mean $\langle S \rangle = 0.1694 \pm 0.0005$. Our mean value is 5–9% lower than previous estimates from the literature.
2. We constructed a ~ 50 -year composite S -index activity record for the solar-analog HD 30495 using data from five observation programs. The composite record revealed a ~ 12 yr cycle and an intermittent ~ 1.7 yr variation in a solar-mass star that rotates 2.3 times faster than the Sun. We found the amplitudes and durations of the three observed long-period cycles to vary, as they do in the Sun.
3. We developed a new method of estimating cycle quality that is insensitive to the number of observations present in a time series and sensitive to the purity of the underlying signal.

4. We conducted a variability study using composite S -index time series of up to ~ 50 yr in length for an ensemble of 27 solar temperature stars, 22 of which are firmly on the main sequence and analogous to the Sun. From this we have found:
 - a. instances of large-amplitude variability on long time scales > 25 yr;
 - b. a linear relationship between median activity and activity amplitude on time scales of ~ 1 yr, and also between median activity and activity amplitude on decadal time scales for $\hat{S} \lesssim 0.27$. This implies that mean unsigned magnetic flux is linearly proportional to the variation in the same quantity on both long and short time scales;
 - c. smooth, regular cycles as seen in the Sun appear to be relatively rare, occurring in only $\sim 30\%$ of our sample;
 - d. including cycles of moderate quality on diagrams relating cycle period to rotation period erases the previously identified active branch. We find no correlation between cycle period and rotation period in our sample.
5. The solar twin 18 Sco has an S -index activity minimum and grand median that are identical to the solar values within the uncertainties. The 18 Sco activity maximum is $\sim 5\%$ higher than the solar cycle 19 maxima, the strongest cycle in our record.
6. The variability in the S -index activity record of the binary HD 81809 appears to originate from the A component, which is a $L/\mathcal{L}_{\odot}^{\text{N}} = 6.0$ subgiant. The ~ 8.2 yr cyclic variability of this subgiant is qualitatively identical to the solar cycle.
7. We quantitatively analyzed the results on 103 F, G, and K-type stars from the Baliunas et al. (1995) study, finding:

- a. High-quality cycles among G-type stars are rare, occurring in only 3 of 32 stars (9%), which includes the Sun and the subgiant HD 81809.
- b. K-stars are most likely to cycle (72%) and most likely to have a high-quality cycle (47%).
- c. G- and K-type stars have qualitatively different behavior on either side of the Vaughan-Preston gap at $\log(R'_{\text{HK}}) = -4.75$, or equivalently Rossby number $Ro_{\text{crit}} = 1.5$. Complex patterns of variability occur when $Ro_{\text{crit}} < 1.5$, and smooth, monoperiodic cycles like the Sun or flat activity analogous to the Maunder Minimum occur when $Ro_{\text{crit}} > 1.5$. There are a few possible exceptions to this rule which require further observations to confirm their rotation and pattern of long-term variability.

In general, we find the solar cycle to be a rare and unusually clear phenomenon in the context of Sun-like stars. Most of the stellar time series we have studied are not nearly as well-behaved. This is emphatically *not* a fault of the observations. Our data have more than enough signal to noise to detect a clear and regular solar-like cycle if there were one to be found in these distant stars. Judging from the results of Baliunas et al. (1995), such regular behavior is easier to find in the K-type stars. Why should this be the case? Stellar structure models predict that lower mass stars transport energy via convection throughout a larger fraction of their radius than does the Sun. In mean field dynamo theory, convection both generates and destroys large-scale magnetic field through the α and β parameters (Equation 1.4). Perhaps the lower internal temperatures result in a “gentler” convection that tips the balance away from field-destroying β and in favor of α . This is idle speculation that requires theoretical and modeling effort to explore; but is an example of a way in which stellar observations can guide dynamo theory.

Another result which has immediate consequence for dynamo theory is the linear amplitude-activity relation seen in Figure 4.7. This implies that the more magnetic flux that a star has at the surface, the more that flux varies. This is related to the rotation of the star, since it has long been known that mean activity is proportional to rotation (Skumanich, 1972), however the relationship between amplitude and rotation has larger scatter which indicates that another factor is at work in determining the amplitude and the median values. Karak et al. (2014) found that a Babcock-Leighton flux-transport dynamo model could reproduce a power law trend in amplitude versus Rossby number. While this model can be tuned to agree with our observations, it lacks a physical mechanism to aid our understanding of the relationship between the Babcock-Leighton source term and the rotation period. Nonetheless, the result is encouraging and more theoretical work in this area could help improve understanding of the coupling between stellar rotation, mean activity, and the amplitude of activity variations.

Jouve et al. (2010) studied a conundrum that posed a significant difficulty for mean field flux-transport dynamos in the face of previous stellar cycle observations which generally indicate that $P_{\text{cyc}} \propto P_{\text{rot}}$ (e.g. Böhm-Vitense, 2007; Saar and Brandenburg, 1999). The cycle period in flux transport dynamos are determined by the meridional flow speed, with faster flows leading to a shorter cycle. However, the latest generation of numerical hydrodynamic simulations show that meridional flow speed is inversely related to rotation, so that faster rotation leads to *slower* meridional flows. Therefore, for a single-cell meridional flow, we would conclude that cycle period *increases* with faster rotation. This is contrary to previous observations, and to our results when we restrict cycle selection only to those of the highest quality. However, when the quality threshold is lowered and more erratic cycles are admitted, the relationship between cycle period and rotation period disappears. Note that this

applies only to the “active” branch; the “inactive” branch is defined entirely by slow-rotating K-stars that were not part of our study. The lack of correlation between cycle period and rotation period does not improve the situation for flux transport dynamo models, however, as they predict that a relationship does exist between the two quantities (see Karak et al., 2014). Instead, our results indicate that there is a relationship between the *existence* of cycles and rotation. This aspect is little explored in dynamo theory, which focuses on reproducing the ordered behavior of the solar cycle. In that context the inability to produce a regular 11-year cycle is a failure; our results indicate that in some parameter regimes “failure” is exactly what should happen.

Testing the observation of a critical Rossby number threshold $Ro_{\text{crit}} = 1.5$ for producing smooth, monop periodic cycles in numerical experiments is made difficult by the lack of a correspondence between the semi-empirical Rossby number of Noyes et al. (1984a) and the results of convective MHD simulations. Gastine et al. (2014) found that a wide variety of MHD and HD-only models produce solar-like differential rotation (fast equator; slow poles) when $Ro_c < 1$, and anti-solar differential rotation when $Ro_c > 1$, where Ro_c is the *convective* Rossby number defined in terms of dimensionless parameters which govern the MHD equations in the anelastic approximation. This is distinct from the *local* Rossby number, $Ro_\ell = u'_{\text{rms}}/2\Omega\ell$, which can only be measured from the resulting stable numerical solution. Gastine et al. (2014) finds that the switch to anti-solar differential rotation also occurs at $Ro_\ell = 1$. Now consider that the Sun’s Rossby number is 2.15 according to the formula of Noyes et al. (1984a), yet it clearly has solar-like differential rotation. A reconciliation of numerical and empirical Rossby numbers would aid the interpretation of our results.

Observationally, the $Ro_{\text{crit}} = 1.5$ threshold for regular cyclic behavior adds an interesting milestone in the qualitative activity evolution scenario of Metcalfe et al. (2016), which suggests that the solar dynamo may presently be in a transitional evolutionary phase. This follows from the observations of van Saders et al. (2016), who finds that rotational spindown is greatly diminished at $Ro \approx 2.16$, which is approximately the solar semi-empirical value according to Noyes et al. (1984a). The mechanism for the spindown transition is thought to be due to a shift in the topology of the large-scale magnetic field from a low-azimuthal order dipole configuration to higher orders *and/or* a sharp reduction in magnetic field strength. Metcalfe et al. propose that this transition begins at $Ro \sim 1$ when the nature of differential rotation begins to change from solar-like to anti-solar (Gastine et al., 2014). As discussed above, the relationship between the theoretical and semi-empirical determinations of Rossby number is at this point unclear. Metcalfe et al. (2016) cites evidence from Zeeman Doppler Imaging (ZDI) of nearly solar mass stars (Petit et al., 2008) that structure of stellar large-scale magnetic field does show a dependence on the semi-empirical Rossby number, with $Ro \lesssim 1$ stars able to generate significant toroidal surface fields and $Ro \gtrsim 1$ stars dominated by poloidal surface field, with a decreasing fraction in the dipole component as rotation slows and Rossby number increases. See et al. (2016) shows from a larger sample of ZDI measurements that the dominance of the surface poloidal field component is nearly universal for $Ro \gtrsim 1$, which is apparently true for the Sun when the harmonic decomposition is truncated at low spherical degree (Vidotto, 2016).¹ Is the transition to high-quality cycles related to a decrease in dipole field which reduces magnetic braking? For at least one example it

¹This result depends critically that unresolved magnetic fields in full 3D vector field inversions are properly accounted for so that sums over all pixels are equivalent to the response in integrated light as observed in distant stars. Applying the ZDI technique on an unresolved solar image would give a more direct estimate of the Sun-as-a-star magnetic topology.

is the case: using the ZDI results of Petit et al. (2008) compare HD 76151 ($Ro = 1.18$, $Q_{\text{cyc},1} = 31$, % dipole = 79 ± 13) to HD 146233 ($Ro = 1.92$, $Q_{\text{cyc},1} = 44$, % dipole = 34 ± 6). Further ZDI observation of the stars of our solar-analog sample would help further test this hypothesis.

Another possible connection between long-term variability and the transition in rotational braking may be the occurrence of flat activity states analogous to the Maunder minimum for stars with $Ro > 1.5$. We showed that the activity amplitude is lowest for slowly rotating stars, and many of these stars have a flat time series. If the flat activity state become more frequent and prolonged with decreasing rotation period, and if angular momentum loss is diminished while in such a state, stars would eventually end up permanently in a flat activity state with a constant rotation period. This would explain the observations of McQuillan et al. (2014), who measured rotation periods for 34,030 main-sequence stars with *Kepler* precision photometry. McQuillan et al. find a hard upper limit edge in the density of rotation period detections which increases with decreasing mass, with the solar rotation lying on this edge. McQuillan et al. commented “the position of the Sun close to the upper envelope is interesting, since this implies a lack of stars older than the Sun. More slowly rotating stars should still appear on the main sequence and would not have been removed by our exclusion of evolved stars.” The envelope edge could be an observational signature of a critical Rossby number beyond which the production of spots is greatly diminished. This hypothesis can be tested by monitoring the long-term activity of stars beyond the edge, or characterizing the spot modulation of their ~ 4 yr long *Kepler* light curves in detail.

There are a number of recent and upcoming observational programs dedicated to time-domain astronomy which will provide more data for the study of stellar variability, activity, and magnetism. The *Kepler* precision space-based photometry

will soon be joined by the Terrestrial Exoplanet Synoptic Survey (*TESS*), which will produce precision light curves of ≈ 45 d duration for the whole sky, and nearly constant coverage of stars near the ecliptic poles. This will produce a bounty of information on stellar rotation in particular, and could potentially provide more measurements of the elusive surface differential rotation. The Large Synoptic Survey Telescope is a ground-based synoptic survey which will obtain images of the entire night sky in five photometric bands approximately twice per week out to the ≈ 25 th magnitude with a target precision of 5 mmag (LSST Science Collaboration, 2009). This will *enormously* expand the catalog of photometric activity cycles for stars more variable than the Sun (~ 1 mmag; Lockwood et al. 2007); stars like HD 30495 with a variation of ~ 30 mmag.

Photometric surveys such as these must be complemented by observations of established proxies for magnetic flux in order to be of the most use for dynamo studies. With the termination of the Mount Wilson HK project in 2003, the Solar Stellar Spectrograph is the only dedicated activity survey with time series in excess of a decade. We encourage continued funding and operation of this instrument, which is the only near-term prospect for contributing additional data on the long-term variability of Sun-like stars. Fortunately, the SSS has recently been joined by the TIGRE survey (Schmitt et al., 2014), which is beginning to produce new results (Hempelmann et al., 2016). The Los Cumbres Observatory global telescope network (Brown et al., 2013) is expanding its capabilities at the time of this writing, and some fraction of the network is expected to be used for long-term synoptic observations of activity proxies. The challenging multi-instrument calibration problems encountered in this work demonstrate that in this area there is no substitute for a long time series. We hope that this work has demonstrated the value of diligent observation and the merit of a long-term vision in the pursuit of scientific understanding.

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APPENDICES

APPENDIX A

CYCLE MODEL MONTE CARLO EXPERIMENTS

In Chapter 2 (Section 2.4.2) we used fits to the NSO/SP data and 56 MWO HKP-2 data points during the rising phase of cycle 23 to determine the minimum, maximum, amplitude and average value on the HKP-2 S -index scale. These fits were used again to determine the $S(K)$ relationship of equation (2.12). We determine the uncertainty in these measurements using two Monte Carlo experiments described here.

The first experiment is aimed at understanding the uncertainties in the cycle shape fit parameters when fitting the NSO/SP K -index data. First, we determine the limits of cycle 23 using a 1-yr median filter on the data and taking the absolute minimum points before and after the maximum. There are 1087 NSO/SP measurements in this period. In each trial we select with replacement 80% of the measurements ($N=869$) and fit the cycle shape model (equation 2.12) parameters $\{A, t_m, B, \alpha, f_{\min}\}$ to those data using the TRR+LM algorithm. In addition to the fit parameters we measure the rise phase minimum (beginning of the cycle model) and maximum value, $\{K_{\min}, K_{\max}\}$.

We ran 50,000 Monte Carlo trials. The distributions for the amplitude and offset parameters, $\{A, f_{\min}\}$, and the cycle minima and maxima are shown in Figure A.1. We computed the correlation coefficients $\text{cov}(x, y)/(\sigma_x \cdot \sigma_y)$ for all five model parameters plus the two cycle measurements, obtaining the following symmetric correlation matrix:

$$\begin{array}{c}
A \quad t_m \quad B \quad \alpha \quad f_{\min} \quad K_{\min} \quad K_{\max} \\
\left[\begin{array}{ccccccc}
1 & 0.493 & 0.303 & -0.158 & -0.684 & -0.690 & 0.280 \\
& 1 & 0.314 & -0.122 & -0.933 & -0.753 & -0.645 \\
& & 1 & -0.846 & -0.450 & -0.710 & -0.234 \\
& & & 1 & 0.265 & 0.463 & 0.163 \\
& & & & 1 & 0.914 & 0.510 \\
& & & & & 1 & 0.389 \\
& & & & & & 1
\end{array} \right]
\end{array}$$

The amplitude A has a high negative correlation with the minima parameters, $f_{\min}$ and $K_{\min}$, indicating that high minima are compensated with low amplitudes, such that the maxima point is not too large. Indeed, there is a low correlation between A and $K_{\max}$. In general, the minima measurement $K_{\min}$ is more highly correlated with the fit parameters than the maxima measurement $K_{\max}$. The high correlation between the time of maximum t_m and $f_{\min}$ indicates that fits with early maxima have a lower minima. We are not interested in the correlations among the shape parameters t_m, B, α as they involve time scales in the cycle which are not studied in this work.

The percent standard deviation of each quantity x is $\sigma_x / \langle \mathbf{x} \rangle / 100 = (2.4, 4.2, 19, 6.7, 0.31, 0.22, 0.21)$, with the same ordering as in the above matrix. For t_m , we calculated σ relative to 1 year instead of the full decimal year of maximum. We find that the standard deviation is quite small for the quantities of most concern: $\{A, f_{\min}, K_{\min}, K_{\max}\}$.

In our second Monte Carlo experiment we determine the uncertainty in fitting a partial cycle using relatively few data points, as was done for the MWO HKP-2

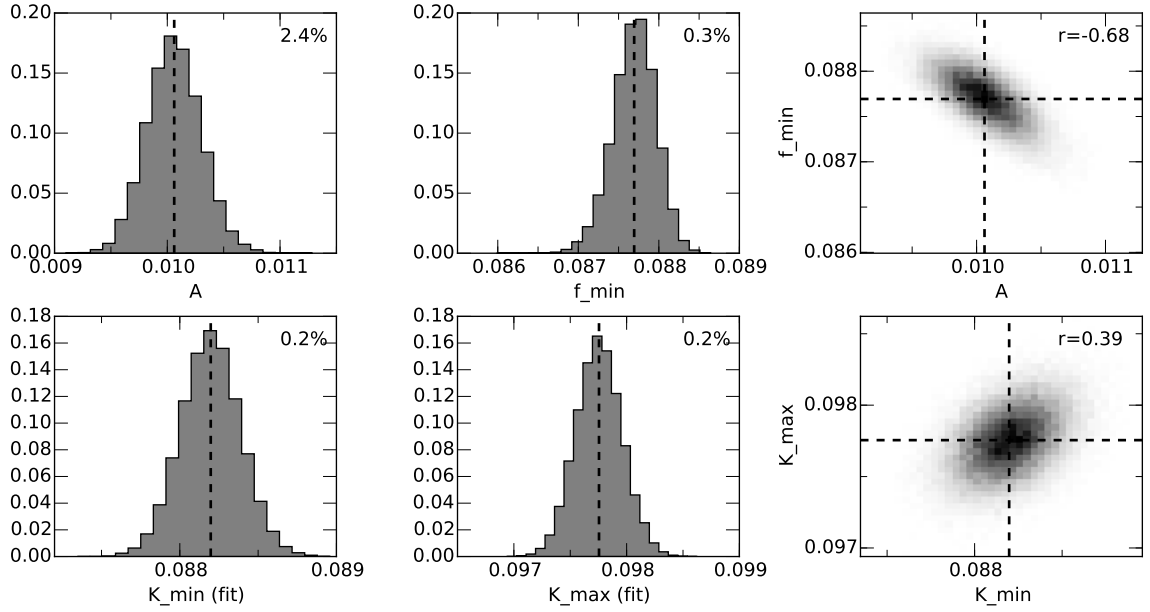


Figure A.1 Monte Carlo experiment to determine the uncertainty in cycle model amplitude and offset parameters, $\{A, f_{\min}\}$ (top row) and the cycle minimum and maximum $\{K_{\min}, K_{\max}\}$ (bottom row) using the NSO/SP K -index time series for cycle 23. Distributions show results of Monte Carlo trials sampling 80% of the 1087 measurements in cycle 23. The standard deviation of each 1D distribution is shown as a percentage in the top right corner. The correlation coefficient $r = \sigma_{xy}/\sigma_x\sigma_y$ is shown in the top right corner of the 2D distributions of the right column. Each histogram is normalized by the number of trials. The “true” value from a fit using all measurements is shown with a dashed line.

data for cycle 23. We take the fit to the full NSO/SP K -index dataset ($N=1087$) in cycle 23 to be the “true” cycle defined by the parameters $\{A, t_m, B, \alpha, f_{\min}\}$. We then ran 50,000 Monte Carlo trials in which we randomly selected only 56 data points from the rise phase of the cycle, up to the time of the last MWO HKP-2 measurement. The selections are drawn from bins according to an $N=10$ equal-density binning of the MWO data points in order to ensure each trial maintained a sampling relatively uniform in time. In each trial, we randomly draw a set of cycle shape parameters $\{t_m, B, \alpha\}$ from the previous Monte Carlo experiment. This is done in order to incorporate the uncertainty of the shape parameters into our results.

We use the same TRR+LM fitting procedure to find the remaining parameters $\{A, f_{\min}\}$. We compute the cycle minimum and maximum $\{K_{\min}, K_{\max}\}$ from each model fit. As a check on our uncertainty derivation in equation (2.13) we also compute distributions of linear fit parameters $\{a_i, b_i\}$ using equation (2.9) where the “true” values are $\{0, 1\}$.

The results from the second experiment are shown in Figure A.2. The percent standard deviation for the parameters $\{A, f_{\min}, K_{\min}, K_{\max}\}$ are $\{8.6, 0.60, 0.50, 0.58\}$. We find that using only 56 data points during the rise phase increases the uncertainty in A by a factor of 3.6 compared to the previous experiment. The offset parameter $f_{\min}$ is more robust, with the uncertainty increasing only by $\sim 65\%$. The relative standard deviation for the amplitude $\Delta K = K_{\max} - K_{\min}$ was 8.4%, and that of the cycle mean $\langle K \rangle$ was 0.29%. In Chapter 2, we use the relative standard deviations found in this experiment to estimate the uncertainty of $S_{\min}$, $S_{\max}$, ΔS , and $\langle S \rangle$ from the 56 HKP-2 measurements of cycle 23.

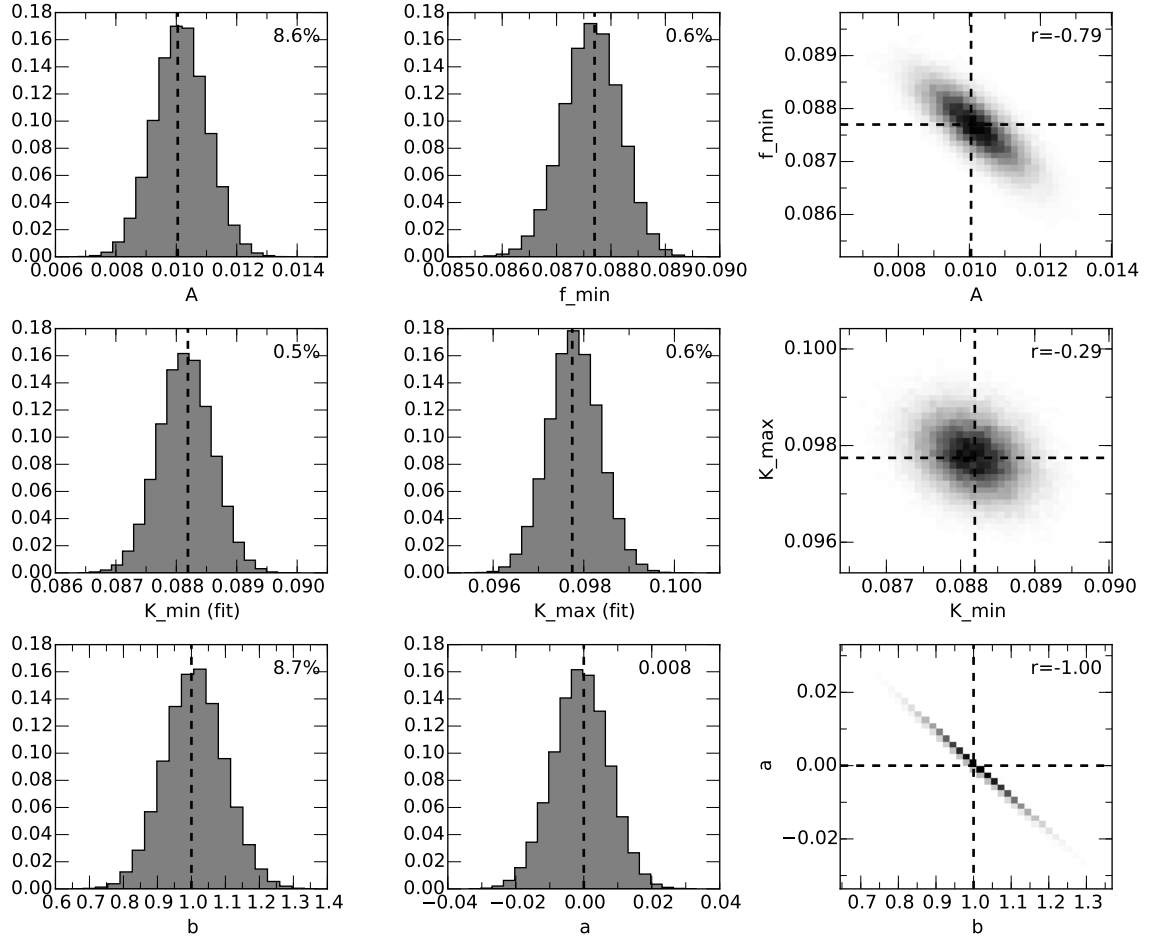


Figure A.2 Monte Carlo experiment to determine the uncertainty in the scale factor of $S(K)$ using only 56 points randomly drawn from the NSO/SP data for cycle 23. *Top row:* cycle shape model parameters A and $f_{\min}$. *Middle row:* cycle minima and maxima determined by the model fit. *Bottom row:* linear transformation parameters for $K_{\text{true}} = a + bK_i$ from the trial measurements K_i to the true scale, K_{true} . Each histogram is normalized by the number of trials. The “true” value from a fit using all measurements is shown with a dashed line.

APPENDIX B

ENSEMBLE TIME SERIES AND PERIODOGRAMS

Each figure below shows the composite time series (top panel) and Lomb-Scargle periodogram (bottom panel) for the stars in the ensemble study of Chapter 4. Each time series and periodogram is on the same scale so that the reader can easily understand the relative amplitudes. The composite time series show MWO data in red and SSS data in blue. At the left of each time series is an amplitude indicator that is reproduced in Figure 4.6. The amplitude indicator shows four quantities: (1) the middle diamond is the median S for the composite time series, (2) the thin capped bar indicates the location of the 1st and 99th percentile of the data (3) the small dashes indicate the minimum and maximum points and (4) the thick bar is the median seasonal inner-98% amplitude.

The bottom panel shows Lomb-Scargle periodograms for the MWO data alone (red), the SSS data alone (blue), and the composite time series (black). The periodogram is computed only up to the duration of the respective time series. The periodograms show amplitude spectral density in units of rms amplitude, (S/σ_{rms}) yr^{-1} , as defined in equation 4.7.

The top three statistically significant peaks in the periodogram are indicated by a green line. Significance is determined using the False Alarm Probability (FAP) method of Horne and Baliunas (1986), with a threshold of 0.1%. In this work we do not consider each statistically significant period of variability to be a "cycle". Statistically significant periods are rated according the cycle quality metric Q_{cyc} (equation 4.9) in Chapter 4.

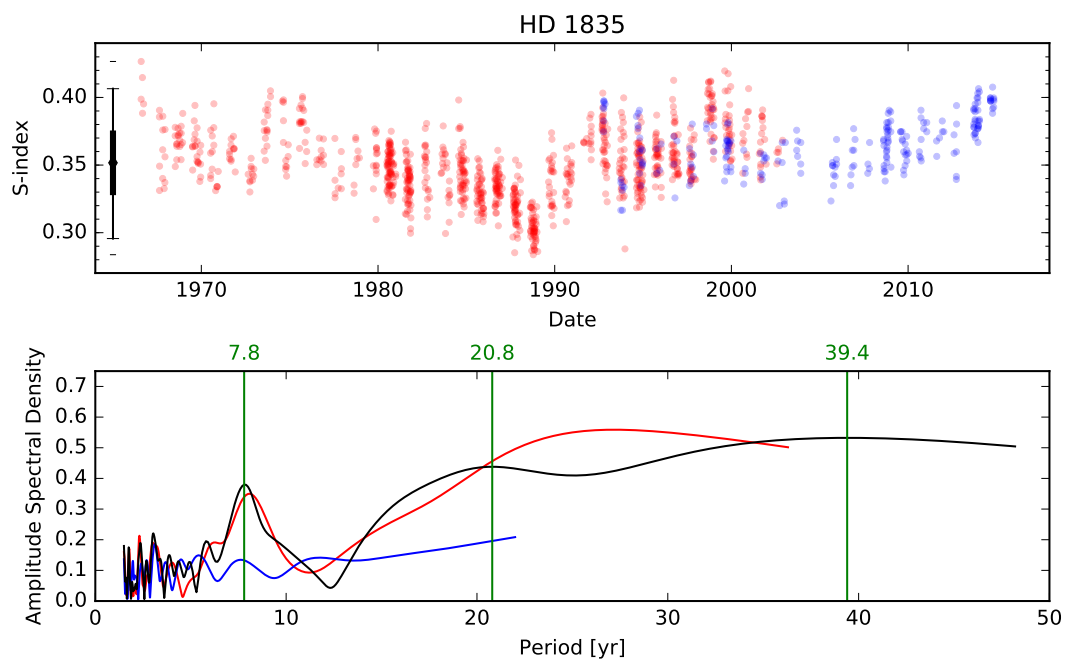


Figure B.1

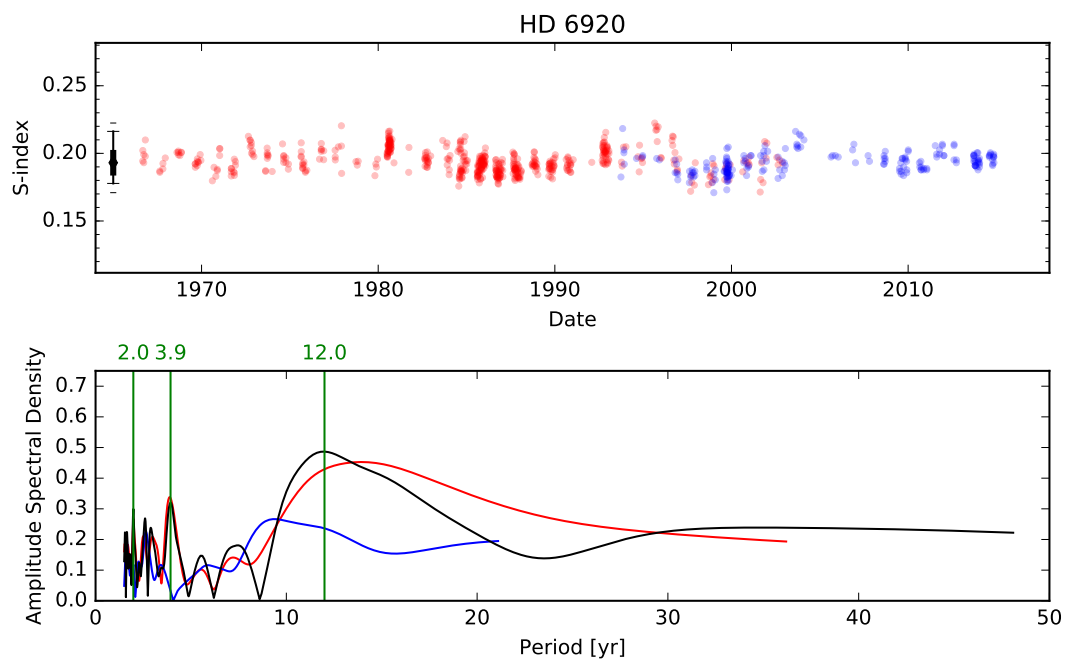


Figure B.2

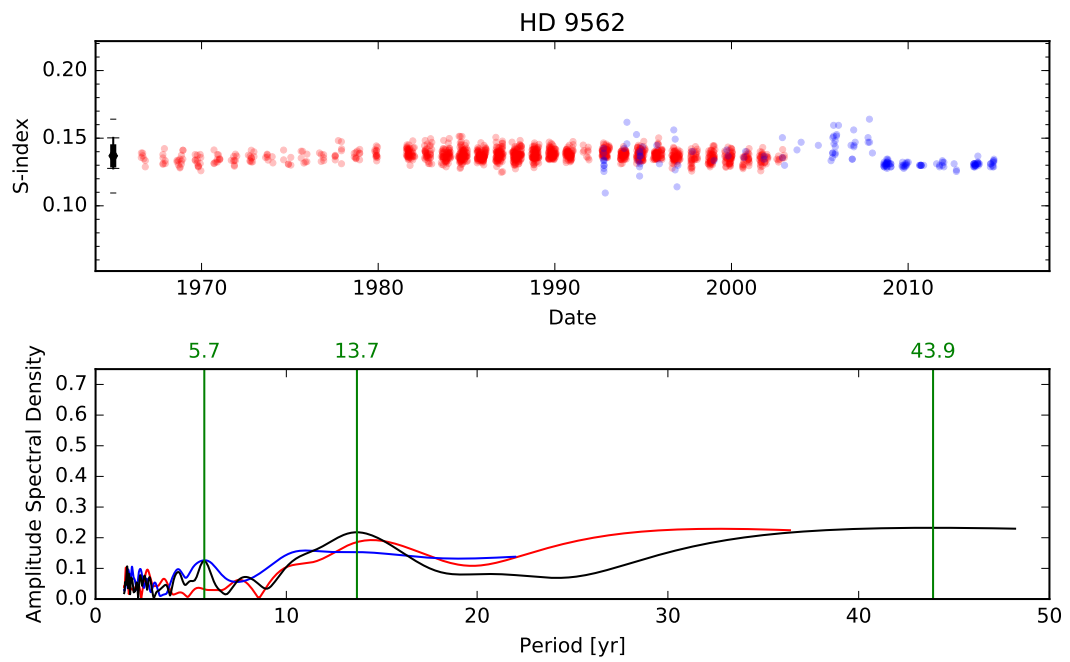


Figure B.3

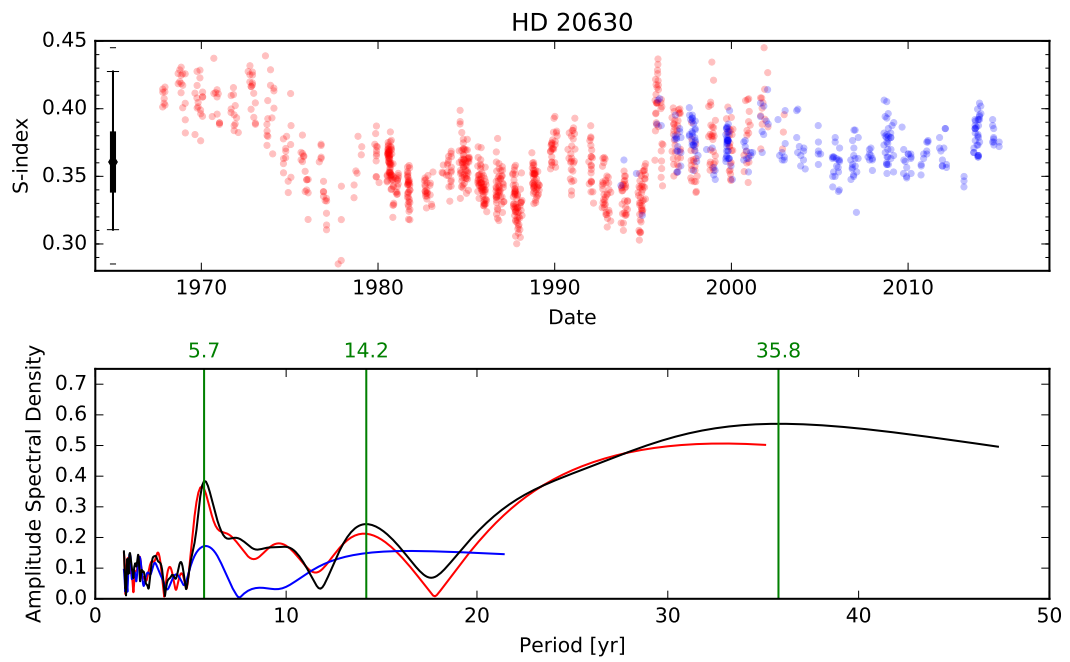


Figure B.4

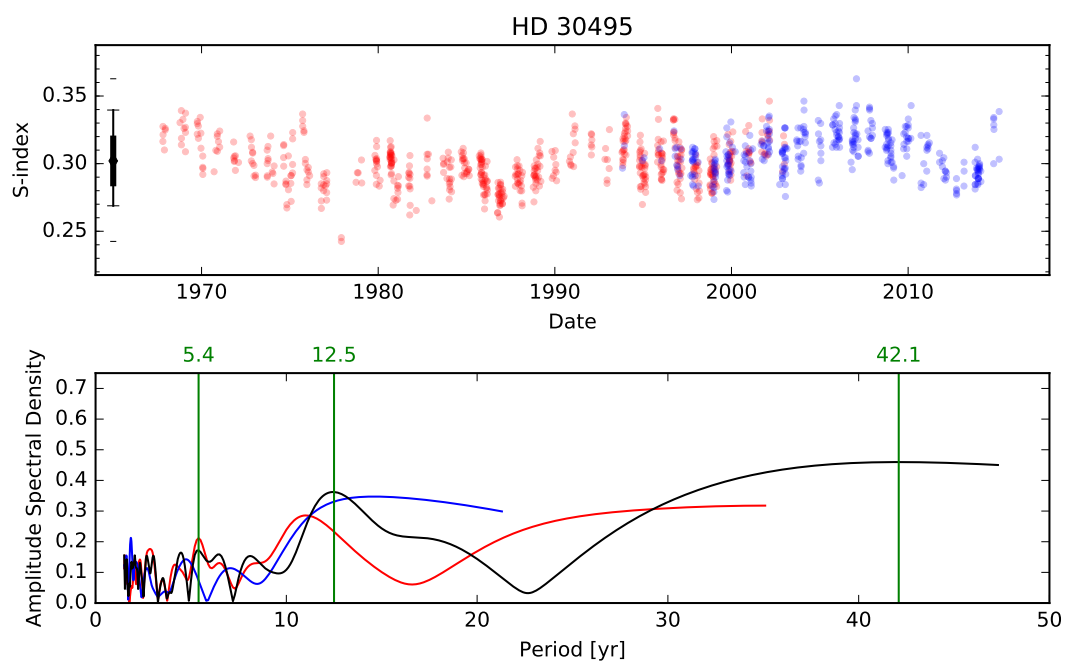


Figure B.5

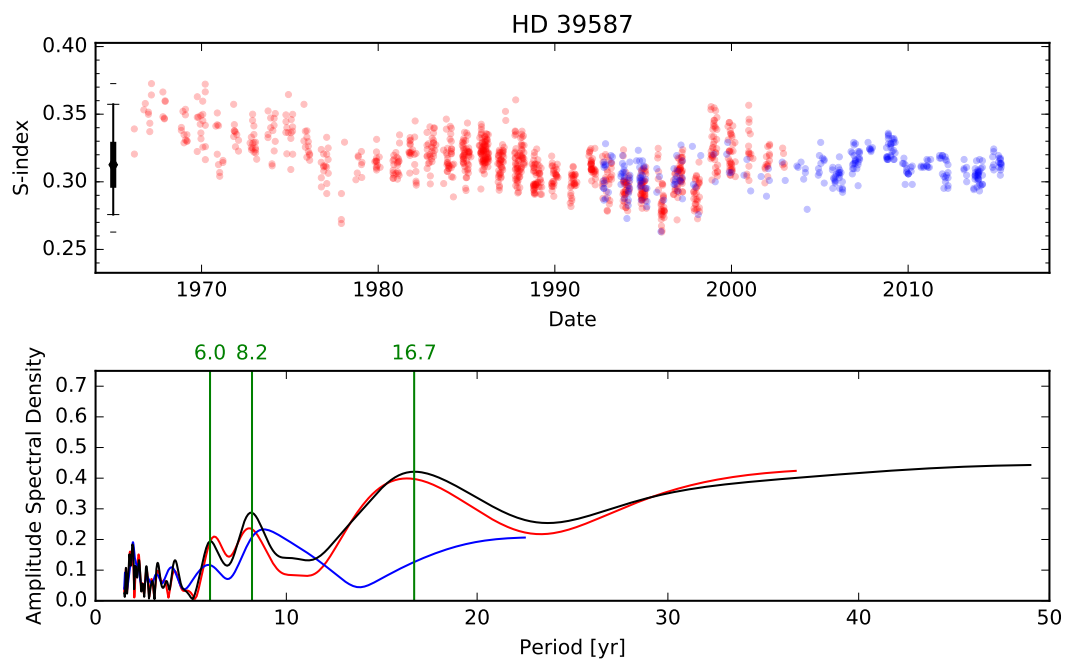


Figure B.6

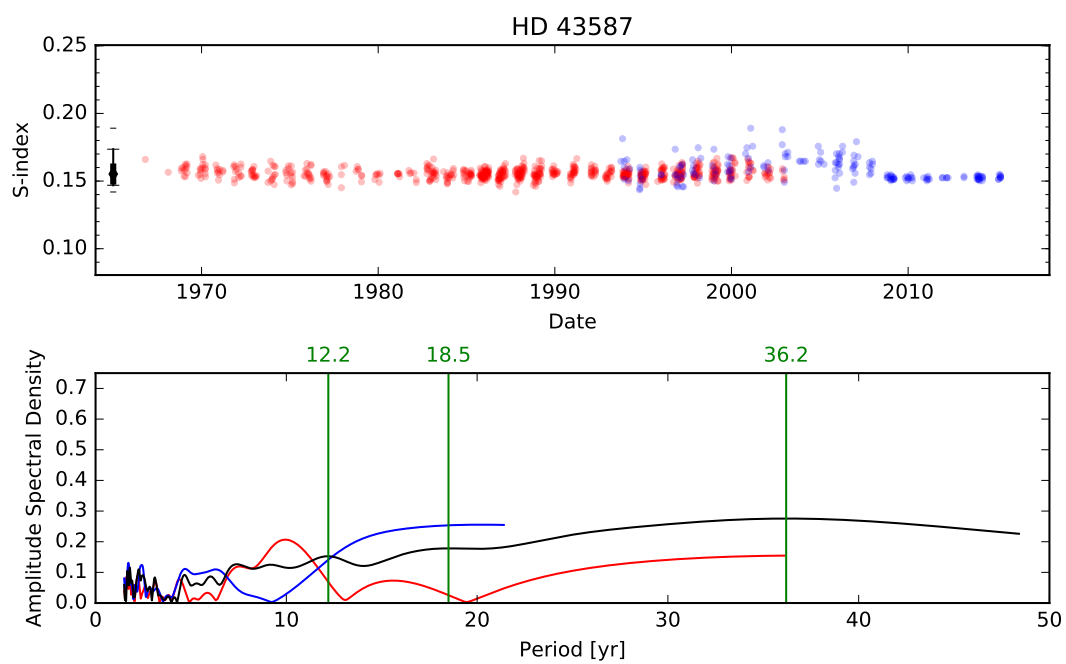


Figure B.7

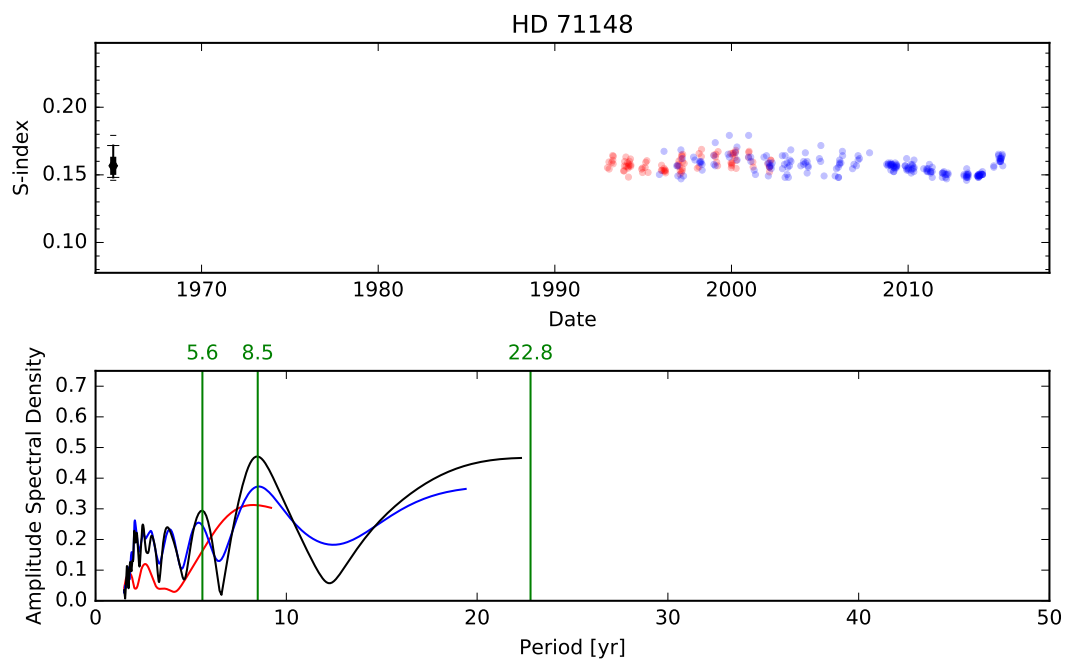


Figure B.8

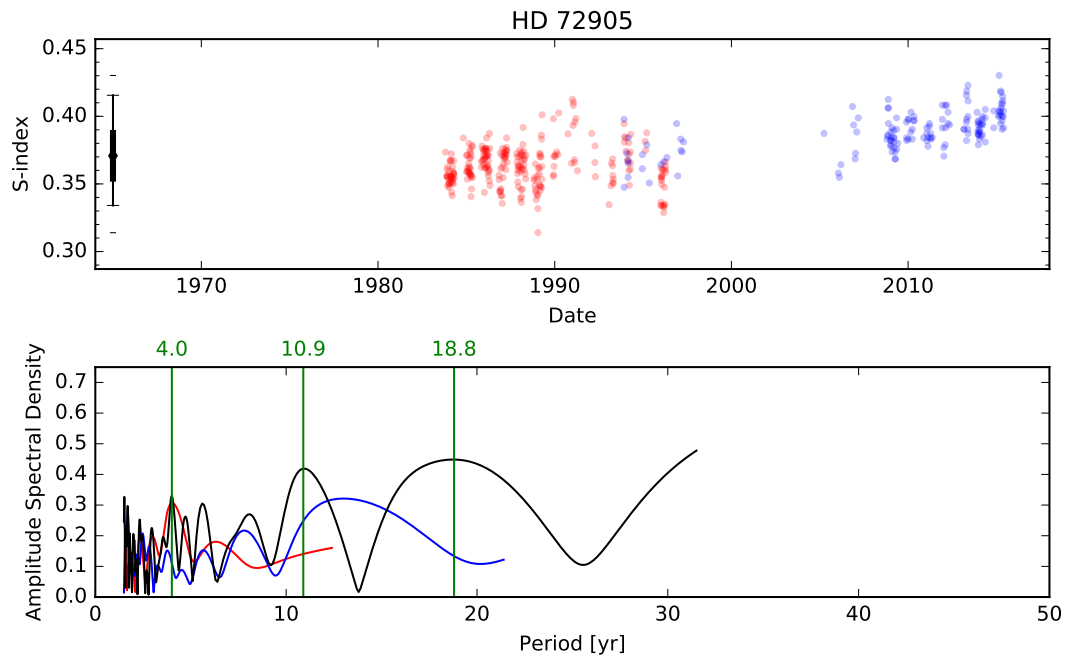


Figure B.9

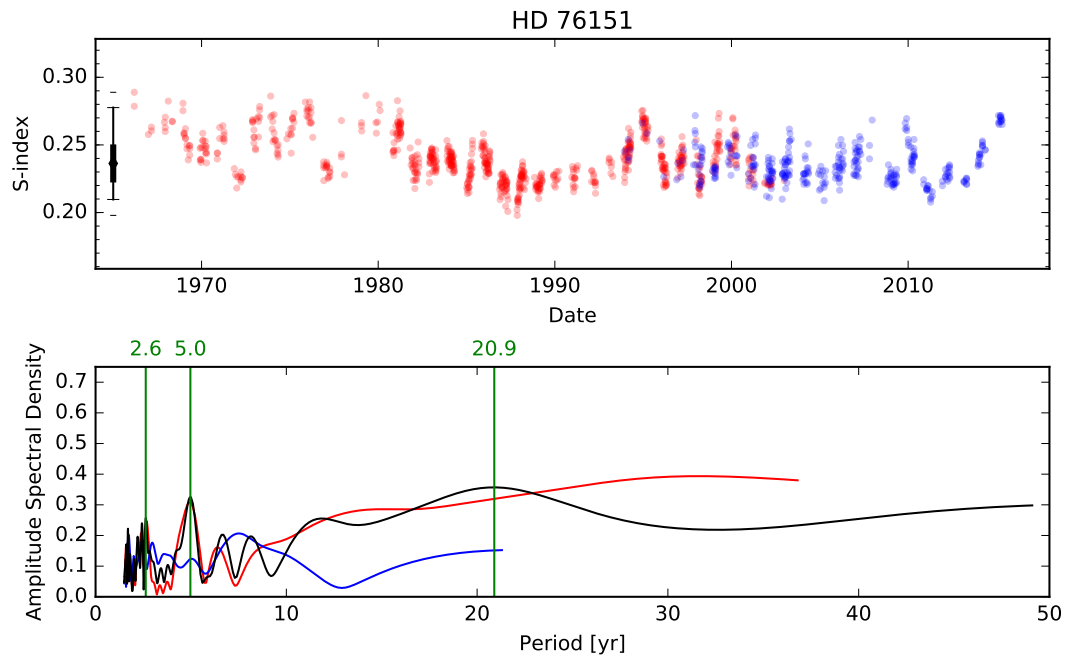


Figure B.10

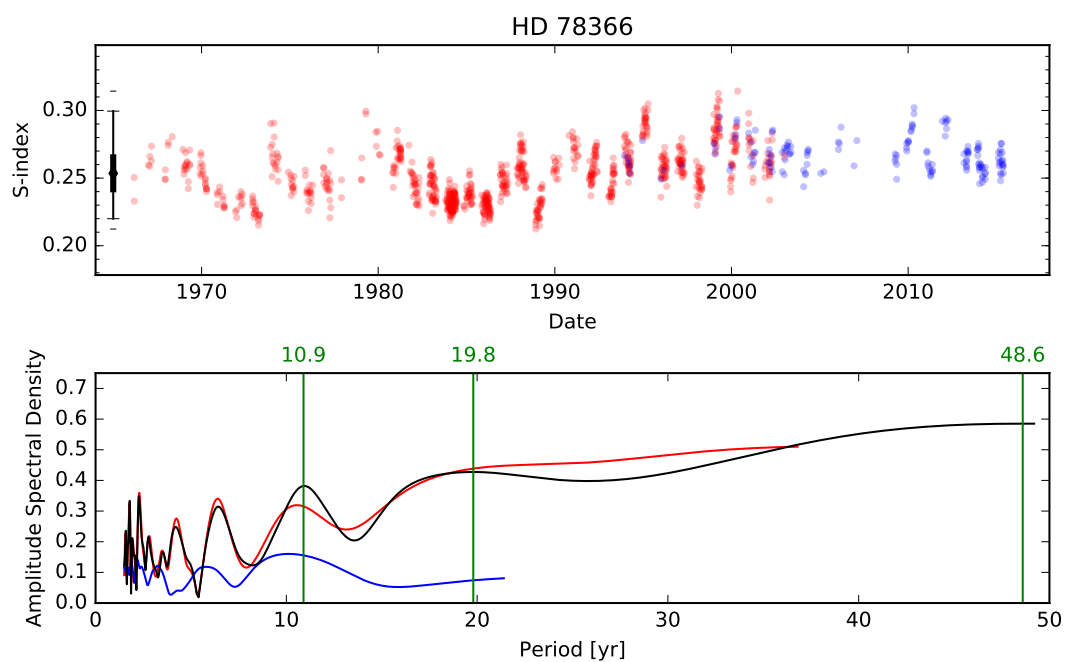


Figure B.11

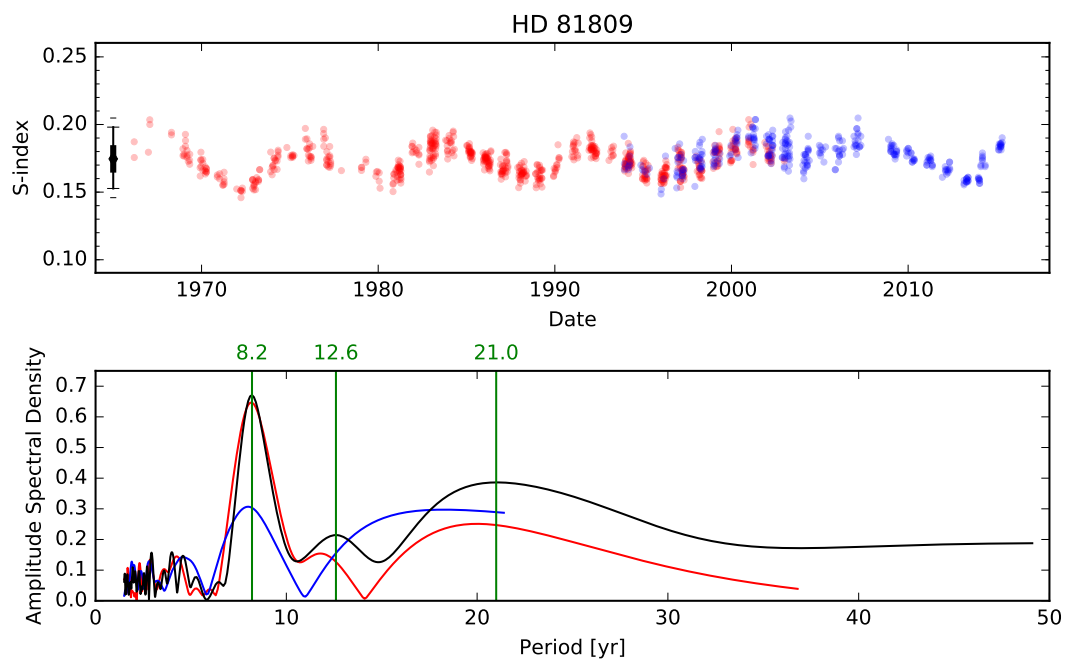


Figure B.12

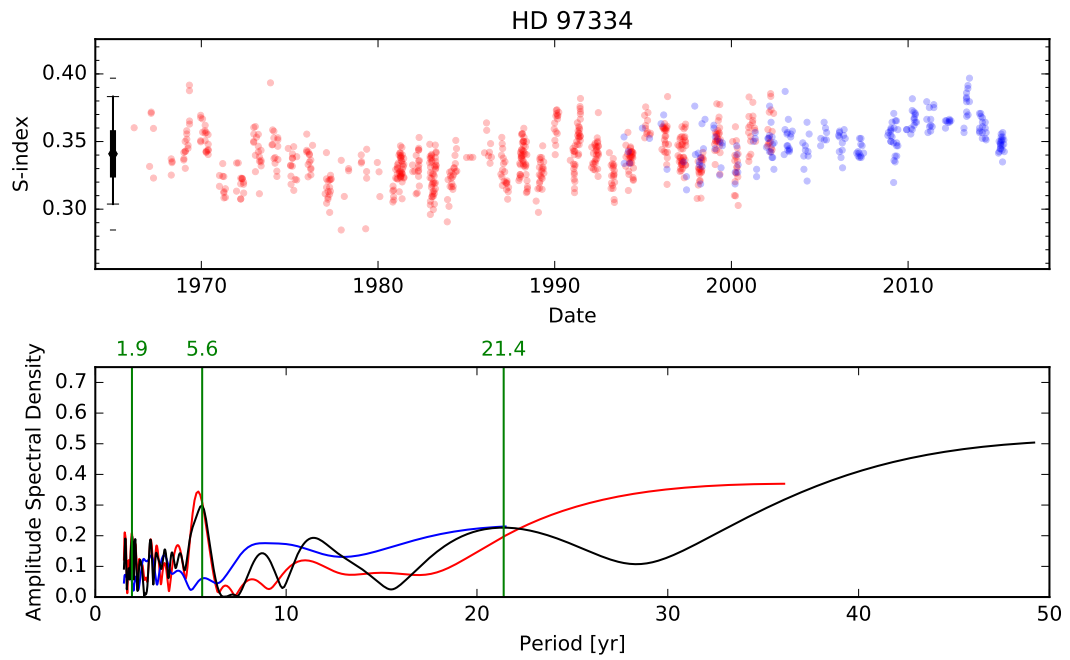


Figure B.13

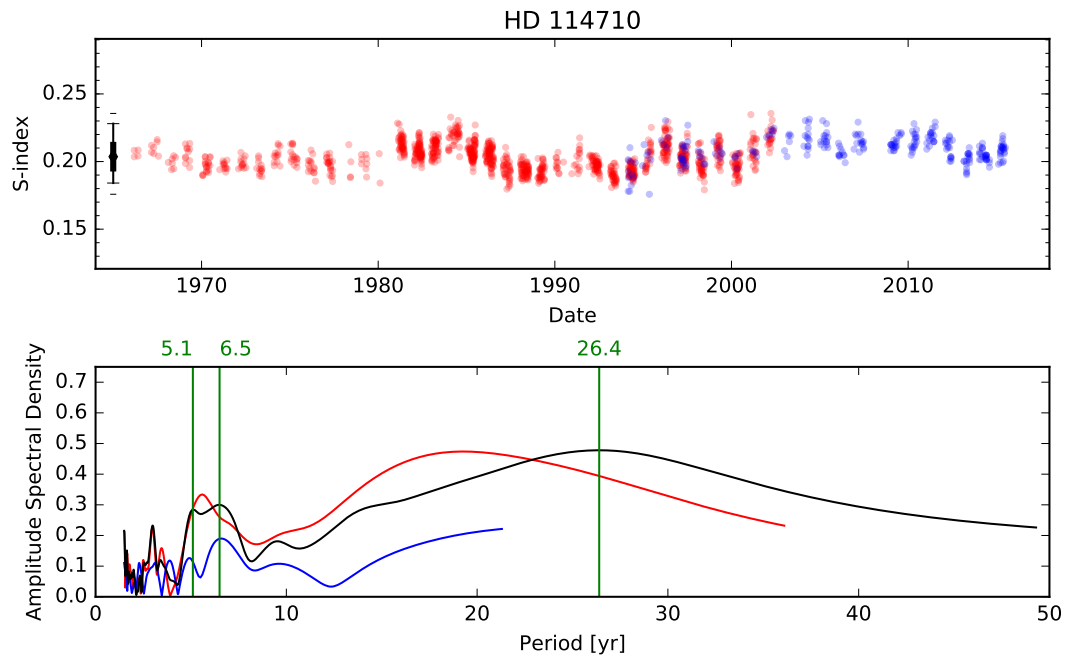


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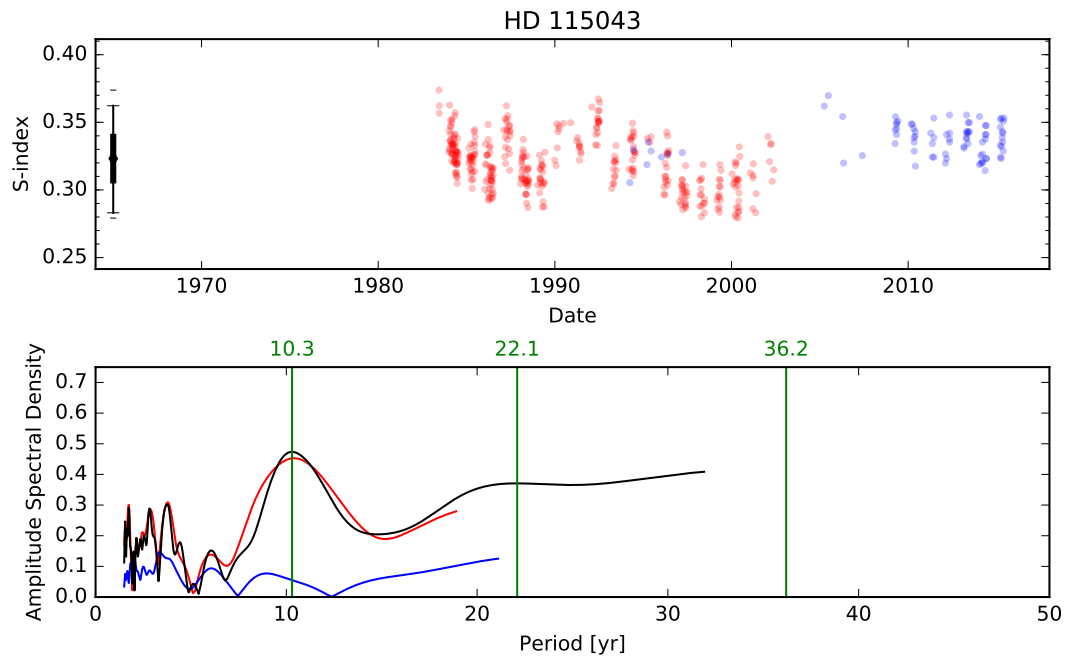


Figure B.15

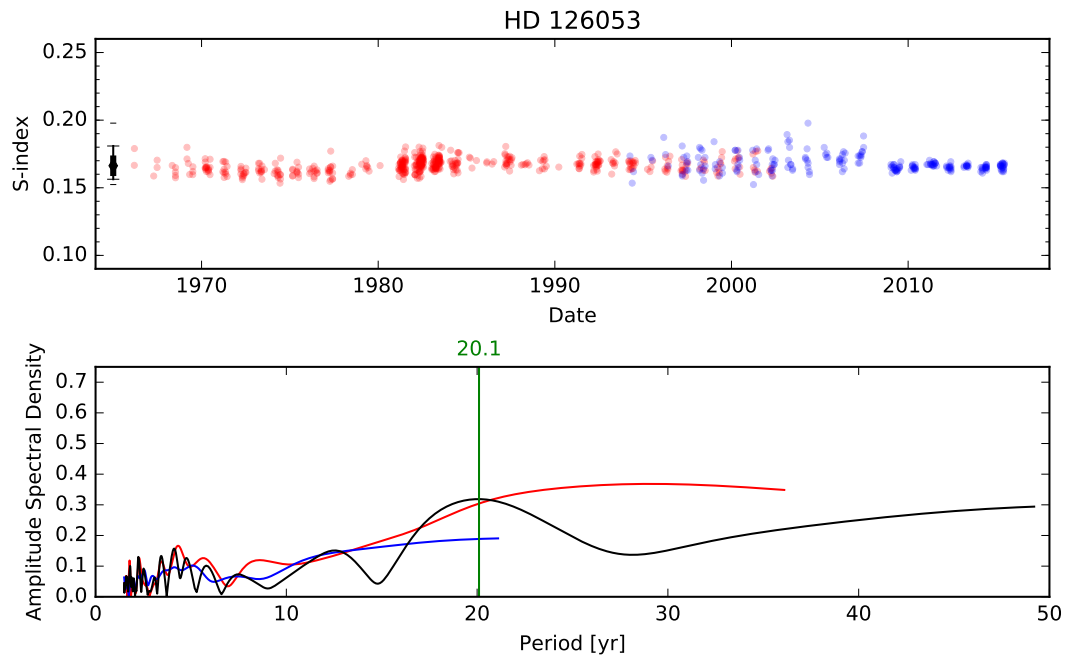


Figure B.16

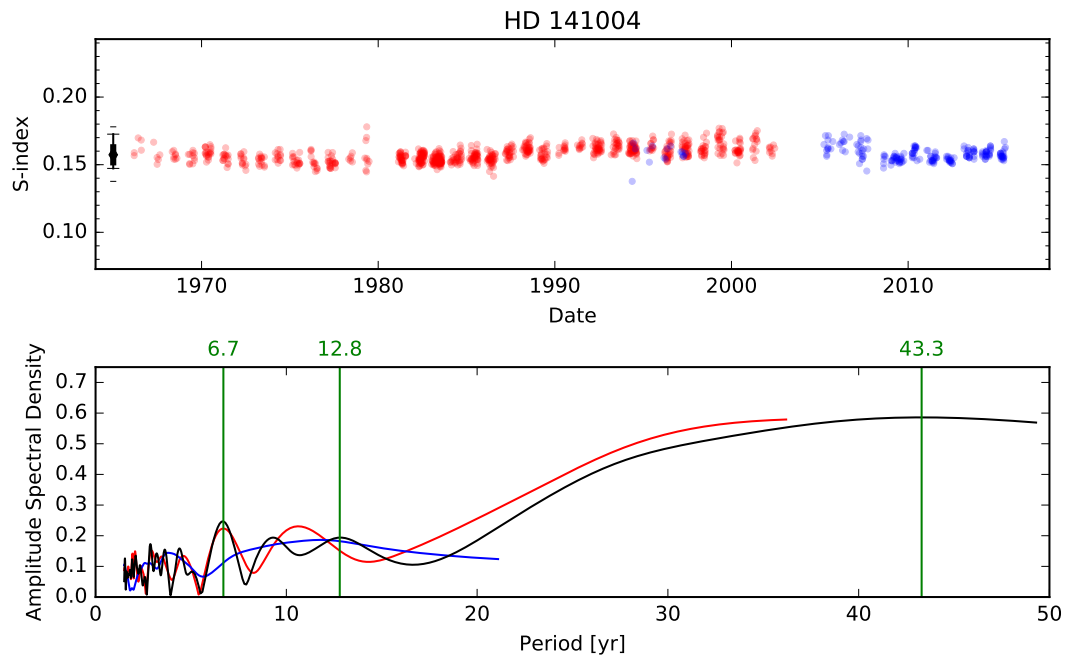


Figure B.17

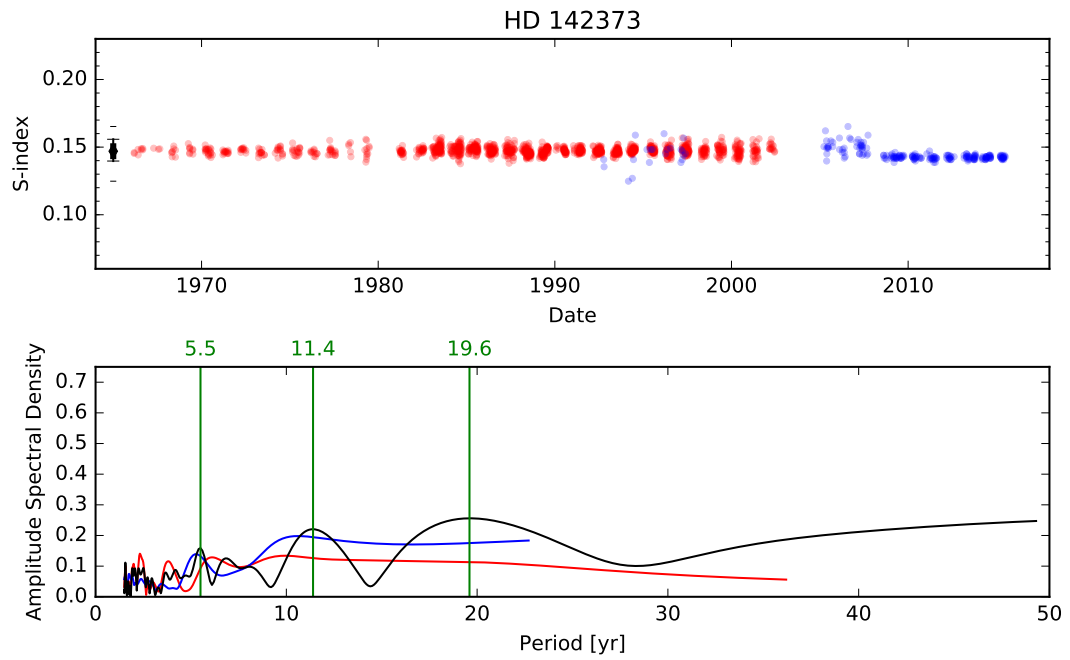


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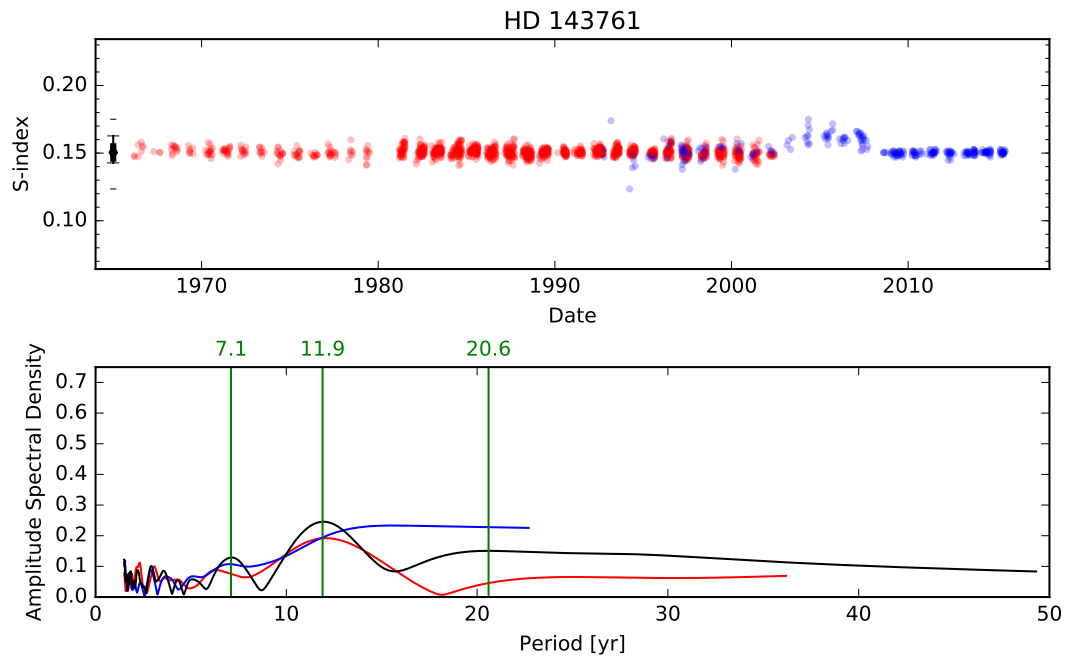


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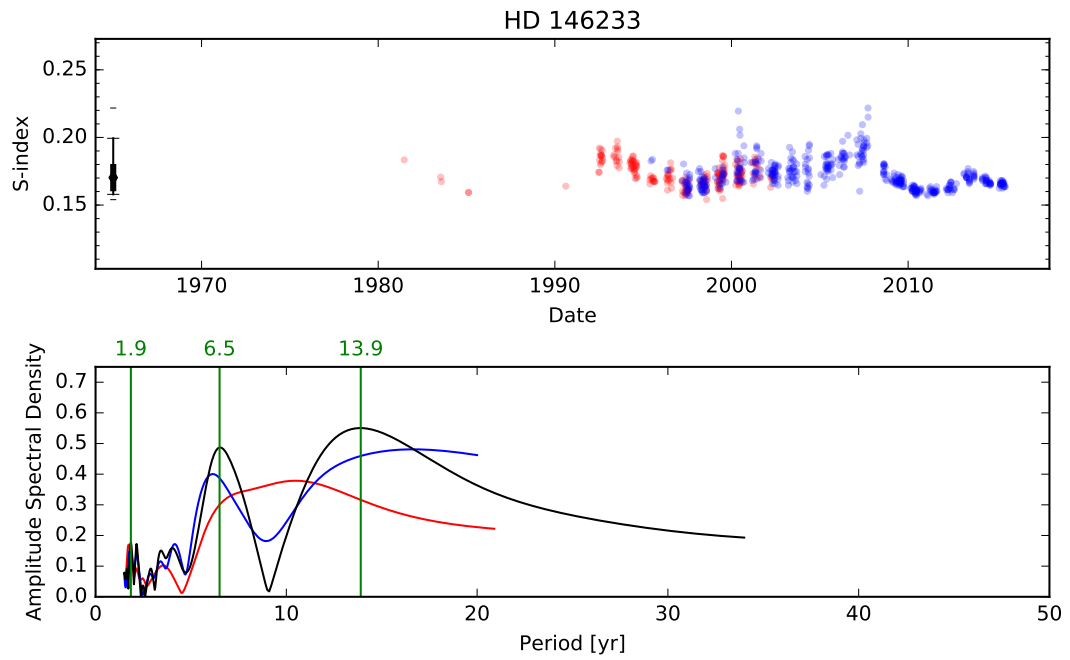


Figure B.20

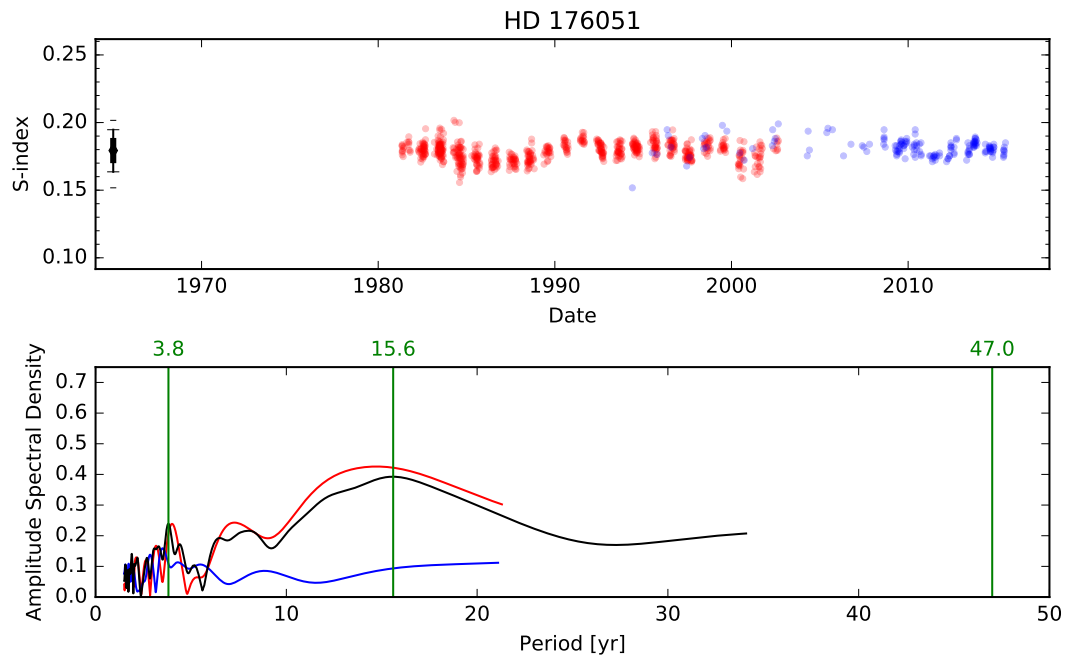


Figure B.21

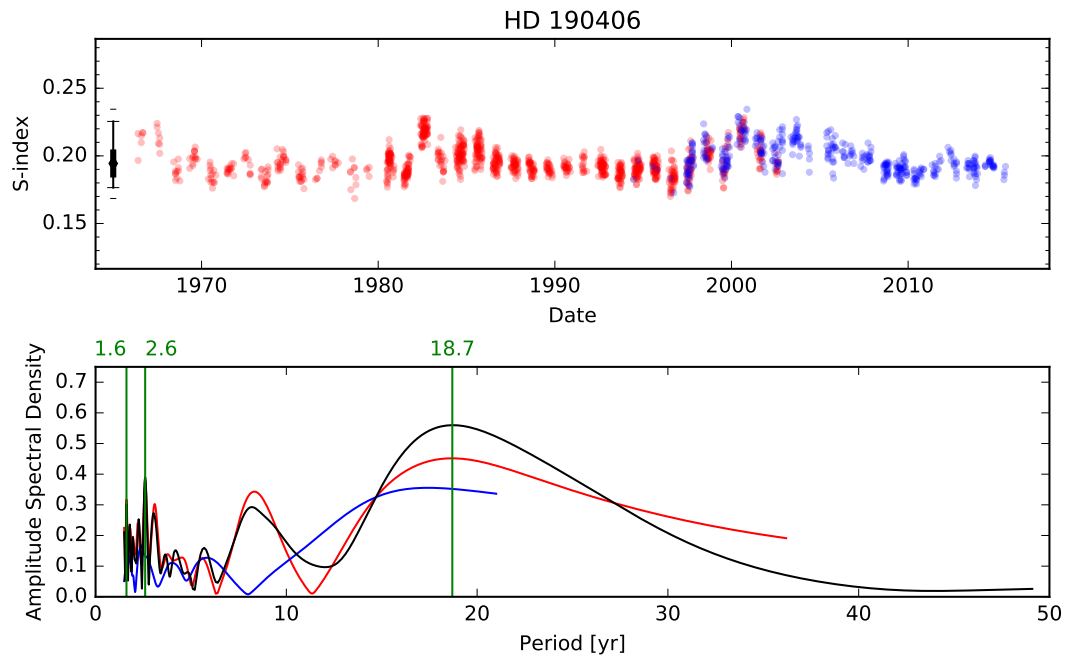


Figure B.22

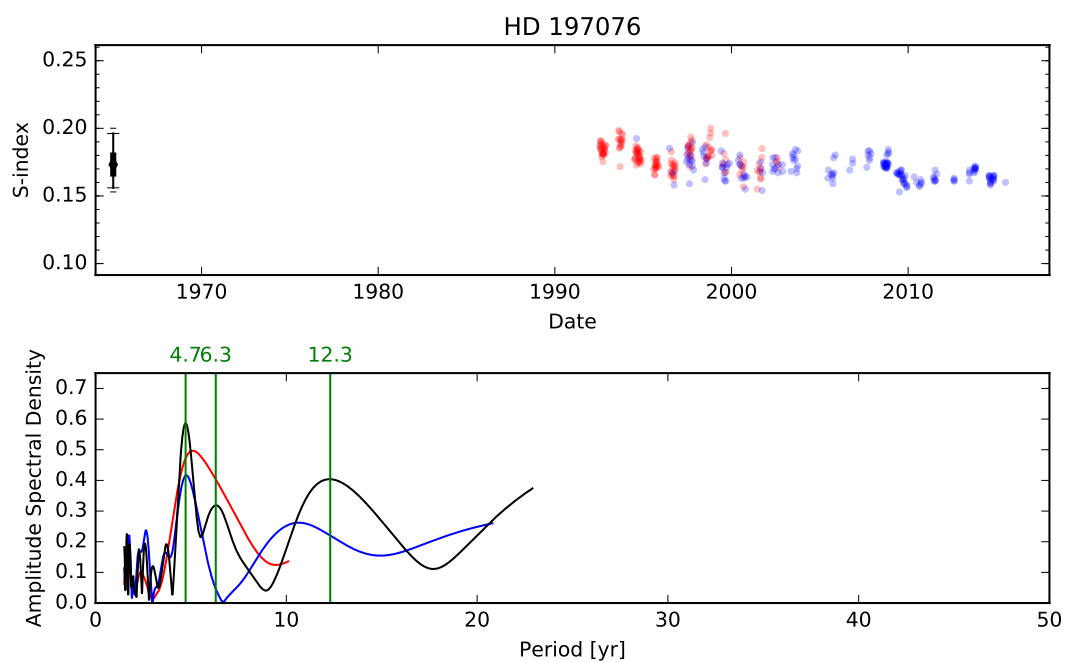


Figure B.23

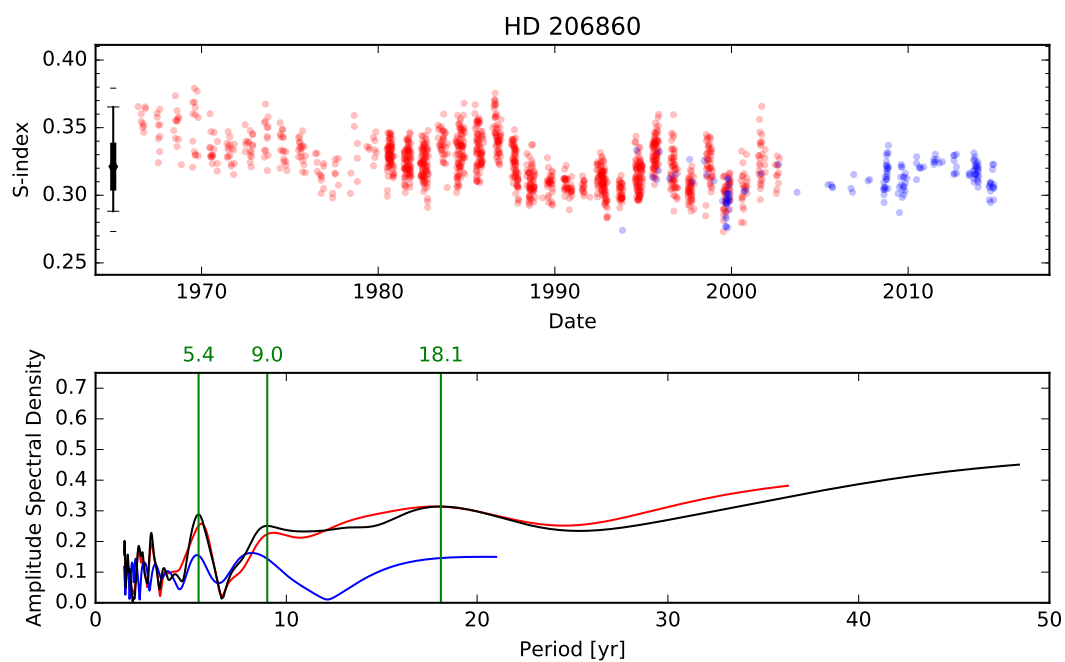


Figure B.24

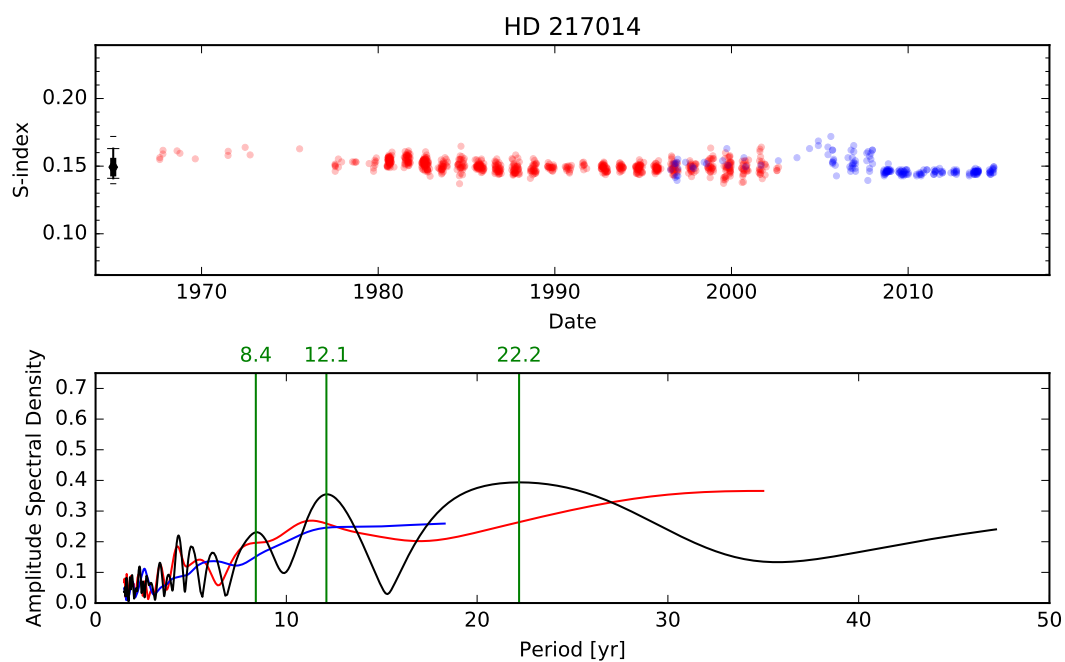


Figure B.25

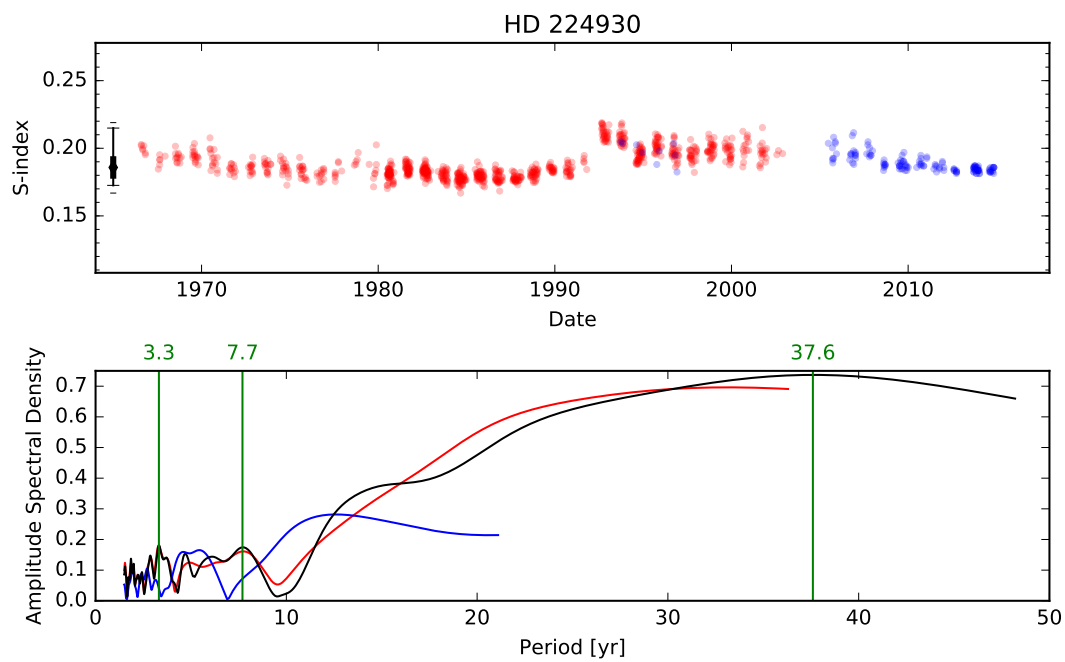


Figure B.26