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Date August 3, 1972

INDIVIDUAL DIFFERENCES -- A SEMI-INDEPENDENT STUDY PROGRAM FOR SECONDARY MATHEMATICS

by

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ABSTRACT

This study involved the development of a semi-independent study program in algebra I and geometry in the Manhattan High School, Manhattan, Montana. The primary purpose of this investigation was to evaluate the semi-independent study program as opposed to a conventional program.

Data for this study was obtained from pre- and post-achievement tests of match-paired students in each of the two courses. The two sets of data were treated separately. The null hypothesis, which stated there was no significant difference in the mean achievement of mathematical skills between students subjected to the two methods of instruction, was retained or rejected on the basis of a t-distribution. The level of significance was .05 for a one-tailed test.

The study indicated there was significantly different results in algebra I between the experimental and control groups. This meant the students in the semi-independent study program achieved at a higher level than did the students in the conventional classroom. However, in the geometry course, the achievement results of the two groups were not significantly different.

In reading the following recommendation, keep in mind that the results of this study are applicable only to those groups of students in the study. To reduce the danger of acceptance of a false conclusion arising from chance errors, the experiment should be repeated several times under similar conditions and in varied locations.

Chapter 1

INTRODUCTION

As one thinks of individual differences in the classroom, he wonders how students learn as much in school as they do. There has been an increasing interest in recent years in the "independent study" teaching methods, and hopefully, more and more educators will place some of their professional goals in this direction--for the sake of education.

Human variability, in the light of reality, must be dealt with in a constructive manner. It is the individual pupil and not the group that learns or fails to learn.

There has been mention of two reasons for individualizing instruction. There is the existence of individual differences in the classroom. Teachers in the conventional mathematics classroom make the assumption that all students are at the same place at the same time. It is quite rare if one or two students will be ready at the same time to learn the same things. Secondly, teaching young people by the "old-school" traditional method fosters the learning of values and styles that are not in line with real life. When the students are passive and the teacher does all the work to make them learn, this is not according to reality.[1]

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[1]Opinion expressed by Dr. Marshall B. Rosenberg in conducting a workshop at the Bozeman public schools (Montana), January 26, 1970.

During the past three years, this researcher has been developing a mathematics curriculum centered around a "semi-independent" study program in the Manhattan High School to fit into each student's range of learning. There has been many hours spent in planning, experimenting, evaluating, revising, and reevaluating this program. Now, the time has come for a formal appraisal of the techniques designed and demonstrated to adjust the learning of mathematics on the secondary school level.

Statement of the Problem

There was a very definite need for academic programs to fit in with the individual differences of members in any class of instruction. The individualizing of instruction, in the realm of independent study, had reference to the steps taken to meet the needs of the students. Since the students were unique individuals, the problem was in constructing a program of variable teaching techniques to give them the best formal education possible. However, the primary problem of this study was to compare the semi-independent study program to the conventional method of teaching mathematics and to determine the merits of a semi-independent study program.

Purpose of the Study

This study was important because of the need to know if a semi-independent study program was characterized by qualities that are more advantageous to learning. A purpose of this study was to lay the ground rules for a semi-independent study program in two different areas of secondary mathematics. Secondly, there was the desire to stimulate an evaluation of a semi-independent study program as opposed to a conventional study program. Thirdly, there was the desire to furnish information to those involved in the development of such a program, and lastly, to determine the areas of the program in need of revision so as to improve the method of instruction.

General Questions to be Answered

The results of this professional paper, hopefully, have answered to some degree these general questions: (1) Will better results be attained with the semi-independent study program than with the conventional program? and (2) What value will the program have for the students?

General Procedures

The general scope and sequence for this professional paper was: (1) the development of a semi-independent study program in secondary mathematics, and (2) the organization and evaluation of an experimental situation involving the program cited in part one.

The first part was a study of individual differences which involve the readings of primary and secondary sources of pertinent material. This information was used to set the foundation for building an experimental situation. The collected information was incorporated to form part of the body of this paper so as to inform any reader of the necessity of individualized education.

The remaining portion of part one was the features of the semi-independent study program. The format of this program was:

1. The sequence of events in a semi-independent study program
2. The sequence of events in the course of study
3. The sequence of events in each unit study

Another area of this investigation discussed the techniques of teaching used during the period of learning in each unit study.

The second part of the body of this paper discussed the organization and evaluation of an experimental situation involving semi-independent study as opposed to conventional study of secondary mathematics. The evaluation of learning during the experimental situation was derived from levels of achievement based on results of post-testing as compared to the data from the pre-testing.

Limitations

This study was limited to a small group of students at the ninth and tenth grade levels. The information obtained and conclusions drawn related specifically to that sample and it was not intended that the results of this investigation be generalized to fit all other students in the same courses of study.

Definition of Terms

The language used in this paper was composed of terms commonly used in any area of education. The following terms were listed to eliminate any possible ambiguities in order that the reader could interpret in the way intended.

Individual Differences. Individual differences refer to the dissimilarities among the various members of a class or age group in any characteristics that can be identified. It is common to speak of the existence of individual differences in such things as problem-solving, reading ability, visual and auditory acuity, interest, emotional stability, and motivation, to name a few.[2]

Secondary Mathematics. Secondary mathematics in this paper is a curriculum composed of different courses of mathematics taught in the secondary school, grade levels nine through twelve.

Conventional Study. Conventional study is a variety of learning and teaching procedures of mathematics that are outgrowth of tradition, custom, or common practice, such as group lecture.

Independent Study. Independent study is a variety of learning

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[2]Theodore Clymer and Nolan C. Kearney, "Curricular and Instructional Provisions for Individual Differences," Individualizing Instruction, Sixty-first Yearbook, Part I.

and teaching procedures in which the student assumes the major responsibility for his education and shows significant independence in some or all aspects of the learning process.

Semi-independent Study. Semi-independent study in this paper is a derivative of independent study which has been incorporated with an element of more teacher planning and control.

Control Group. The control group, (Y), in this study is a sampling of algebra I and geometry students who were subjected to the conventional methods of teaching.

Experimental Group. The experimental group, (X), in this study is a sampling of algebra I and geometry students who were subjected to semi-independent study methods of teaching.

SUMMARY

"INDIVIDUAL DIFFERENCES -- A Semi-independent Study Program for Secondary Mathematics" was a "stimulus-response" reaction that this researcher must become more aware of individual differences to improve the effectiveness of the art in teaching.

This study involved the development of a semi-independent study program in secondary mathematics. The primary problem of this study was to compare the semi-independent study program to the conventional method of teaching and to determine the merits of a semi-independent study program. The evaluation of this experiment played an important role in determining if the innovated program was characterized by qualities that are more advantageous to the learning of mathematics.

Chapter 2

REVIEW OF THE LITERATURE

The review of available literature serves as a guide in the construction or modification of a semi-independent study program. The areas of concern in this literary study were: (1) ways of considering individual differences which are significant in learning mathematics, (2) ways of recognizing individual differences in learning potential and performance, (3) ways of providing for individual differences in learning mathematics, and (4) ways in which other educators have tried to individualize instruction in secondary mathematics.

In relating to individual differences in the mathematics classroom, Johnson and Rising stated that human beings are complex, unpredictable, and unequal. No two individuals are exactly alike in appearance, in ability, in personality, or in any other trait. Therefore, as soon as there is a class of two or more students, there exists a problem of providing for individual differences. When considering variances that are significant in learning mathematics, the educator must consider mental ability, mathematical ability, mathematical knowledge, motivation, maturity, special talents or deficiencies, and learning habits.

Variability in mental ability, ability to reason or think, and ability to solve problems is usually determined by intellectual aptitude tests and is reported in terms of an intelligence quotient. Ideally, measures of ability should indicate difference in the learning rates, as well as in quality of response. Educators must recognize one result of the difference in learning rates--the differences between the students' achievements should be expected to diverge rather than converge.

Differences in mathematical ability, ability to use symbols, ability to do logical reasoning and ability to compute are often measured by a mathematical aptitude test, a test on quantitative thinking, or a prognostic test. Teacher-constructed achievement tests administered during the early part of the school year give results that compare with those tests which are standardized.

The knowledge of mathematical concepts, structures, and processes is related to the previous educational experience of the learner and greatly determines the readiness of the student for the material in the next course. The mathematical background of a learner is often measured by his past achievement record or by an achievement test. Pre-tests of achievement may be used as general measures and as indicators of specific areas of difficulties.

While motivations, interests, attitudes, and appreciations are potential factors in the classroom, they cannot be measured well at the present time. Teacher ratings are often used but cannot be judged entirely reliable due to the fluctuations of these behaviorial controls.

The measures of physical, emotional, and social maturity are also at best subjective evaluations of previous teachers or counselors.

Student variation in cultural, social, and emotional adjustment traits are great and are highly significant for the placement of students in special classes. A student's physical disability should be noted and its effect on his emotional reaction should be watched. Minor defects in sensory perceptions such as sight or hearing should also be discovered.

The ability to verbalize is closely related to mathematical achievement. Special talents or deficiencies such as creativity, lack of reading skills, or retention span are usually identified by observation or interview. Although, tests of specific traits are becoming available.

Differences in learning habits, self-discipline, attention and retention span, and organization of written work bear a significant relationship to the student's home environment and previous education.

If students are unequal in the traits mentioned, it is not reasonable to give them all the same mathematics courses, the same assignments, the same instruction, the same achievement requirements. Therefore, individual differences should first be provided for by gathering as much information about the student as possible. To do this will mean the use of a variety of tests and a study of the student's cumulative information folder. It will mean the continued collection of information based on the classroom performance, learner products, and careful observation of and thoughtful conferences with the learner.[1]

There are certain distinguishing characteristics which should be helpful in recognizing the slow learner in mathematics and the mathematically gifted in contradistinction to each other and to the average group. Although there may be many exceptions to any given pattern, the following by Potter and Mallory might be safely listed as characteristics of the slow learner:

1. Intelligence quotient below 90
2. Has little drive
3. Has short span of attention
4. Has weak association memory
5. Is a poor reader
6. Has difficulty with abstractions
7. Is not logical
8. Lacks imagination
9. Is unable to detect his own errors
10. Has little power to transfer training
11. Is not creative in his thinking[2]

In contrast, the Secondary School Curriculum Committee gives the following general and special characteristics which mark the mathematically gifted:

1. General characteristics are (a) makes associations readily and retains them indefinitely, (b) recognizes similarities and

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[1]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics (Wadsworth Publishing Co., 1967) pp. 179-181.

[2]Mary Potter and Virgil Mallory, Education in Mathematics for the Slow Learner (Washington, D.C.: National Council of Teachers of Mathematics, 1958), pp. 11-12.

> differences quickly; (c) has excellent memory, good vocabulary, broad attention span, and high reading ability; (d) has a relatively mature sense of values; (e) pursues interests with tremendous energy and drive; and (f) uses his spare time productively.
>
> 2. Special characteristics are (a) recognizes patterns readily and enjoys speculating on generalizations; (b) prefers to think on higher levels of abstraction; (c) classifies particular cases as special cases of more general situations with relative ease; (d) follows a long chain of reasoning, frequently anticipating and contributing; (e) frequently asks profound questions; (f) may be reading mathematics books years ahead of his class; and (g) is frequently impatient with drill and details that he thinks are not important.[3]

Although there are a variety of ways of organizing the mathematics program and a variety of materials for use in meeting individual needs, the teacher is still the key to the success of the program. The teacher's success in dealing with individual differences will be determined by how he accepts the learner, how he uses materials, how he varies his assignments. The teacher must recognize differences and provide for them by offering different experiences for different students--varying content, language, rate of learning, materials of instruction, and the goals of learning according to individual differences. Some practical ways of providing for individual differences might include a multiple-track curriculum, modified classroom activities, varied instructional materials, and evaluation appropriate to the group.

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[3]Secondary School Curriculum Committee, "The Report of the Secondary School Curriculum Committee," The Mathematics Teacher, Number 5, LII (May, 1959), p. 410.

A multiple-track curriculum with learners assigned to classes according to certain abilities or interests might include: (a) accelerated classes for gifted students; (b) advanced placement classes for gifted students; (c) remedial instruction courses; (d) vocationally oriented courses; (e) courses designed for students of average ability; and (f) special courses for the low-ability student. The multiple-track curriculum would be most effective in high schools with large enrollments as opposed to smaller high schools who may only offer one or two sections for each course of study.

The classroom activities modified according to the needs of the student seems most fitting in any school situation, especially the low enrollment high school. The following is a list of ideas involved in modified classroom activities.

1. The daily learning assignment can be varied according to ability or achievement levels. Some system is needed to provide many easy exercises for the slow learner and more challenging exercises for the talented.
2. The class can be organized into small groups according to ability and each group can be given special instructing and assignments. It may be appropriate to have capable students act as group leaders. The capable student then receives experience in the leadership role and reinforces his learning by having to teach the material.
3. The instruction can be enriched with student reports, demonstration, projects, and creative writing.
4. Supervised study time can be provided so that the work of individual students can be observed and help given when needed.
5. Each student needs to feel that he has a place in the class and can participate in some activities without frustration. Students should be involved in many of the classroom activities, such as writing on the chalkboard, collecting and distributing papers, correcting assignments, to name a few.

6. The teacher can establish a level of participation which is related to the student's ability. Capable students are given difficult discussion questions and are not given undue praise for an average performance. Slow learners are given relatively easy questions and their performance is judged according to their ability.[4]

Many innovations in teaching methods have found application in mathematics classes. Those techniques involving innovations such as grouping arrangements, resource centers, non-graded instruction, programmed instruction, computer mathematics, television, independent study, use of teacher aids and flexible scheduling are all aimed at individualizing the approach to learning.[5]

To provide for individual rates of learning in mathematics, educators at Matawan Regional High School in Matawan, New Jersey, introduced a special program in the eleventh year. The entire program was based on the belief that the student must be involved. He is advised to read the text, work through sample exercises and begin the recommended problems. On the first day, each student received a list of suggested goals for the school year, in addition to a grading period requirement list. Each student independently decides how he will achieve the stated objectives. He must study one unit at a time and

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[4]Johnson and Rising, op. cit., pp. 181-182.

[5]Wallace H. Geisz, Leroy Sachs, and Robert Wendt, "Modern Teaching Methods for Modern Mathematics," The Continuing Revolution in Mathematics, (Washington, D.C.: National Council of Teachers of Mathematics, 1968), pp. 129-140.

is not permitted to begin a new chapter until he has passed a test on the preceding one. The teacher's role is to answer questions, set reasonable goals, offer constructive criticism, encourage, guide, and listen. The results of most standard tests in this school are very near the national norms, with scores about evenly distributed. In ninth and tenth year mathematics, scores on a standard test in 1967 were evenly divided about the national median, again with fairly even distribution throughout the range. The results of the eleventh year mathematics standard test place a majority of the students in the upper half of the decile distribution. The quartile distribution indicated that the students were evenly placed in the first, second, and third quartile.[6]

A relatively new technique in teaching is making use of educational programming and teaching machines. A study was made in 1965 on the effectiveness of a variety of education programming and teaching machines in mathematical achievement. Experiences of the study by May suggest:

> . . . the teacher should remain the central component of mathematics at all levels, directing a variety of media but devoting more time than ever to working with individuals . . . adherence to any one style of programming should be avoided in favor of eclectic and hybrid styles determined by the particular teaching task . . . programmed materials should be written by

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[6]Nellie L. Noddings, "Providing for Individual Rates of Learning in Mathematics," The Mathematics Teacher, Number 7, XLII, (November, 1969), pp. 543-545.

mathematics teachers on the same basis as textbooks, judged by the same standards, and sold at comparable prices . . . experiments in automation should concentrate on devices for the single student and, for group instruction, extend the teacher's range and the degree of student involvement.[7]

A study was made to determine if students could learn to capacity without ability grouping in the area of algebra. As a result, there was the feeling that individualized instruction in heterogeneously grouped classes was possible. Some of the advantages of this independent study were:

1. The bright students could move ahead while the slow students were under little pressure to maintain another student's pace.
2. The teachers were more aware of the progress that was being made by each student than they had been before.
3. The students appreciated the opportunity to share in their learning experiences, to take individual tests upon their request, to select their own topics from the state syllabus, and to work at a rate commensurate with their ability.
4. A greater sense of responsibility was apparent as the result of democratic procedures and a general atmosphere.
5. Self-motivation was the stimulus for learning, rather than the usual extrinsic motivation of grades.[8]

This study revealed that the students in the individual study program learned at least as well as their contemporaries in the classes employing teacher-presentation, student-recitation type of instruction. However, some of the disadvantages noted in individualized instruction

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[7]Kenneth O. May, "Programming and Automation," The Mathematics Teacher, Number 5, LIX (May, 1966), p. 452.

[8]Gladys Crosby and Herbert Fremont, "Individualized Algebra," The Mathematics Teacher, Number 2, LIII (February, 1960), p. 111.

are:

1. The independent study program required a tremendous amount of work, time, and energy.
2. There was a lack of discovery material and that which was available was inadequate.
3. There was a need of a period of adjustment to make the proper transition from behavior patterns developed in previous classes.
4. The group did not cover the materials as fast as had been expected.[9]

SUMMARY

The review of the literature was organized in such a way as to accumulate knowledge necessary to construct an effective semi-independent study program. The areas of concern in this literary study were: (1) ways of considering individual difference which are significant in learning mathematics; (2) ways of recognizing individual differences in learning potential and performance; (3) ways of providing for individual differences in learning mathematics; and (4) ways in which other educators have tried to individualize instruction in secondary mathematics.

When considering variances that are significant in learning mathematics, the educator must consider mental ability, mathematical ability, mathematical knowledge, motivation, maturity, special talents or deficiencies, and learning habits.

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[9] Ibid., p. 112.

Individual differences should first be provided for by gathering as much information about the student as possible. To do this will mean the use of a variety of tests and a study of the student's cumulative information folder. It will mean the continued collection of information based on the classroom performance, learner products, and careful observation of and thoughtful conferences with the learner.

Some practical ways of providing for individual differences might include a multiple-track curriculum, modified classroom activities, varied instructional materials, and evaluation appropriate to the group.

Educators from the different areas of the United States have experimented with individualized instruction in mathematics. They have utilized methods of instruction involving innovations such as programmed instruction, independent study, computer mathematics, flexible scheduling, television, non-graded instruction, and resource centers, to name a few. Generally speaking, the results of those research programs on individualized instruction published in the journals include the advantages or disadvantages of the techniques used. Some of the researchers have suggested there was no significant difference in the achievement of mathematical concepts when students are enrolled in an independent study program as opposed to a conventional study program. There was very little commitment as to the level of academic achievement in two program studies.

Chapter 3

PROCEDURES

The primary problem of this study was to compare the semi-independent study program to the conventional ways of teaching secondary mathematics, namely algebra I and geometry.

The type of research was a controlled experimental study where several groups of students in secondary mathematics were subjected to an experimental situation. An objective evaluation of the variables studied was analyzed to determine whether one treatment was better than the other in mathematical achievement. This evaluation used a variety of measurements and tools of inferential statistics for the data analysis. The results were used to test the null hypothesis that the treatments were the same.

Since this study was two-fold, the researcher discussed only the procedures for the algebra I situation. The procedures for the geometry group were synonymous to those of the algebra I, with the exception of the specific standardized tests. Those tests were listed in the methods of collecting data section.

Format of Program

The format for the semi-independent study program was:

1. The sequence of events in the semi-independent study included: (a) the listing of objectives, and (b) the follow-up

statistical evaluation of the experimental group.

2. The sequence of events in the courses of study, algebra I and geometry, included: (a) the listing of course objectives; (b) the prognostic testing using an appropriate prognostic test; (c) the student pre-course evaluation using a specific course achievement test; (d) the ability tracking of students within the course on the basis of the individual pre-course evaluation and prognosis testing; (e) the measuring of post-course student achievement using the same testing instrument as in the pre-test; and (f) the post-course student evaluation based on semestrial achievement.

3. The sequence of events within each unit in the course of study included: (a) the unit pretest upon which to base unit objectives; (b) the determination of unit objectives by the teacher and students; (c) the determination of maximum time limit for the unit study by the student and teacher; (d) the period of learning involved with various techniques of teaching; (e) the post testing for achievement on unit materials; (f) the remedial instruction when necessary; and (g) the student evaluation based on logical consequences.

In the unit study, during the period of learning, the researcher concentrated on various techniques of teaching the students in the experimental groups. Since the students were at different places within the unit, depending on their ability and interest, the researcher used the following:

1. Individual attention on a one-to-one basis
2. Small group discussion limited to six students.
3. Minimum amount of class discussion
4. Individual placement in tracking system within the unit study
5. Uses of supplementary materials
6. Uses of enrichment materials
7. Uses of mathematical projects
8. Remedial instruction
9. Evaluation by logical consequences

In contrast, during this same period of time, the researcher was teaching the students in the control group using the conventional method. This was a routine approach using group lecture and discussion followed by a supervised study period for the students to solve mathematical problems related to the daily lecture. There was time made available for students to check their homework and ask questions.

Population and Sampling

The students involved in this study were members of the student body at Manhattan High School, Manhattan, Montana. This school, which had as enrollment of approximately one hundred fifty students, was a public-supported institution located in the Gallatin Valley. The populace in the Manhattan school district was mainly employed in farming, ranching, business, education, and industry, which comprises lumber and cement.

The sampling for this research was students in the two required courses of secondary mathematics, algebra I and geometry. The school's administration and guidance and counseling department arranged the scheduling of classes and assigned each student enrolled to a section in the courses of study selected by each student. There were two sections of algebra I and two sections of geometry. One section of algebra I and one section of geometry was selected as an experimental group and the alternate classes were the control groups. Since there was no effort made for ability grouping the students in those sections, they were assumed to be heterogeneous in nature. Thus, a cluster sampling was utilized to establish the experimental and the control in the study. Both the experimental and the control classes were evaluated by a series of pre-tests to determine the characteristics of each group and the individual members in each group.

Ten students in the experimental class were matched and paired with ten students on the control class to obtain data for a statistical analysis. The students were matched on the basis of their results on the Otis-Lennon Mental Test and the three arithmetic sections in the Stanford Achievement Test. The specific details of match-pairing were reported later in this paper.

Experimental Treatment

There was only one school involved in this study for an eight-month period with the researcher being the only teacher involved in teaching both the experimental and control classes. A cluster sampling was necessary to establish the experimental and control classes in the study. The size of each group was approximately the same and was from fifteen to twenty students. Since this study involved individual differences with the two methods of teaching, a match-pair sampling was used to help control the variables and direct the study primarily at the methods of instruction.

Factors such as motivation, assignments, unit tests, enrichment, and discipline were assumed to be uniform in all experimental and control classes. Other variables to be considered in both of the groups were: (1) the students' work was divided into five or six units per semester; (2) the researcher planned the course without consulting the students to eliminate any variance from the unit by either group; (3) the immediate aim on which all students focused was successful performances on the unit and final examination; and (4) the person evaluating the students' performance was the researcher.

Method of Collecting Data

The methods of collecting data for this study included the procurement of testing results from the student accumulative information folder and the administration of standardized prognostic and achievement tests. All of the tests used in this study were commercial instruments and were listed in Table 1 with their respective coefficients of reliability and validity.

Table 1

Evaluation Instruments Used in Study

Name of Test	Reliability	Validity
Otis-Lennon Mental Test	.95	
Stanford Achievement Test		Not presently known
Arithmetic Computation	.90	
Arithmetic Concepts	.86	
Arithmetic Application	.81	
Orleans-Hanna Algebra Prognosis Test	.95	.50 (predictive)
Lankton First-Year Algebra Test (revised	.86	High degree (content)
Orleans-Hanna Geometry Prognosis Test	.86	.50 (predictive)
Howell Geometry Test (revised)	.82	High degree (content)

To determine the past performances in mathematical achievement, the results of the arithmetic computation, arithmetic concepts, and arithmetic application sections in the Stanford Achievement Test were available in the student accumulation information folder. The data were used in evaluating the mathematical ability of each member in the experimental group and the control group to aid in match-pairing.

Each member of the sampling was given an algebra prognosis test from which the results were used by the researcher as an instructional aid for planning lessons and assignments that would meet the needs of students of different abilities.

The evaluative instrument used for testing the null hypothesis was the revised Lankton First-Year Algebra Test for the algebra I classes and the revised Howell Geometry Test for the geometry classes. These achievement tests were used in this study as a pre-test and a post-test.

The content validity of the Lankton First-Year Algebra Test and the Howell Geometry Test was determined by the researcher following an item analysis for content. The content of these tests was closely related to the objectives of the respective course.

The tests mentioned are group tests which were administered simply by following the directions in the accompanying handbook. Since the number of students being tested was small, a special answer sheet was used so the researcher could hand score them.

Method of Organizing Data

After the testing was completed, the data of the pre-test and post-test were organized in tabular form and placed in tables. These tables were put in the appendix of this paper. The difference between the post-test and pre-test scores was then calculated and put into predetermined pairs on the basis of deviation intelligence quotients and previous scores on the three arithmetic sections in the Stanford Achievement Tests. The data were then used to determine the measures of central tendency and variability.

Statistical Hypothesis

The hypothesis on which this study was based dealt with each subject area individually. Verbally, the null hypothesis was: There is no difference in the mean achievement of algebraic skills as measured by an achievement test between students who were in the semi-independent study program and students who were subjected to the conventional way of teaching. And, the corresponding alternate hypothesis was: There is greater mean achievement of algebraic skills as measured by an achievement test when students are in the semi-independent study program than when students are subjected to the conventional way of teaching.

Algebraically, the null hypothesis and the alternate hypothesis are

$$H_0 : \mu_X = \mu_Y$$

$$H_1 : \mu_X > \mu_Y$$

where μ_X is the mean achievement of the experimental class and μ_Y is the mean achievement of the control class.

Analysis of the Data

The analysis of the data was done in two steps. The first step, match-pairing, equated the experimental and control groups. The second step, t-distribution, determined whether or not those results were statistically significant.

Match-pairing. The data received from the Otis-Lennon Mental Test and the Stanford Achievement Test were used to match a student from the experimental group with a student from the control group. A student from the experimental group was paired with a student from the control group provided they had approximately the same score on each of the four respective variables: deviation intelligence quotient, arithmetic computation, arithmetic concepts, and arithmetic application. In matching a student from the experimental group with a student from the control group, the limits on the deviation intelligence quotient score were set at plus or minus eight points and the limits on each of the three arithmetic achievement scores were approximately plus or minus 1.5 grade equivalence points. The data used for match-pairing can be found in Tables 5, page 44; 6, page 45; 9, page 49; and 10, page 50 in the Appendix.

t-Distribution. Using the ten match-pairs of students, the researcher assumed the difference between the mathematical ability of the paired individuals to be zero. The achievement scores of all students were determined by calculating the difference between the post-test and the pre-test which were taken during the study.

The researcher first calculated the differences between the scores made by the paired individuals. The statistical analysis to follow determined the mean of the distribution of differences, the variance of the differences, the variance of the mean, the standard error of the mean, and the t-distribution. The theory of each part of the statistical analysis was:

Difference between the scores of paired students, D,

$$D = X_i - Y_i$$

Mean of the distribution of differences, $\overline{D}$,

$$\overline{D} = \frac{\Sigma D}{N}$$

Variance of the differences, s_D^2

$$s_D^2 = \frac{N \Sigma D^2 - (\Sigma D)^2}{N (N-1)}$$

Variance of the mean, $s_{\overline{D}}^2$,

$$s_{\overline{D}}^2 = \frac{s_D^2}{N}$$

Standard error of the mean, $s_{\overline{D}}$,

$$s_{\overline{D}} = \sqrt{s_{\overline{D}}^2}$$

t-distribution, t,

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

where:

N = the number of match-pairs of students

X_i = the student from the experimental group (i^{th} pair)

Y_i = the student from the control group (i^{th} pair)

D = the difference in the scores of a pair of students (experimental minus control)

The final step in the analysis was to compare the calculated value of 't' with the value of 't' in the table. The test of the t-distribution was restricted to a one-tailed test with (N-1) degrees of freedom and set at the $\alpha = .05$ level of significance.

SUMMARY OF THE PROCEDURES

A semi-dependent study, used as a teaching method in two required courses of secondary mathematics at Manhattan High School, was investigated and analyzed by means of controlled experimental research. The study was two-fold; meaning, the algebra I students were treated separately from the geometry students, but in the same manner. The students in the experimental and control groups were tested to determine the differences of their mathematical achievement over an eight-month period of time.

A majority of the students in the samplings were members of the working, middle class of people. The high school classes were heterogeneously group through chance by the school's administration and counseling center. Ten members of the experimental group were match-paired with members of a corresponding control group.

The variables were controlled in a consistent manner. The study was organized in one school with one teacher controlling such factors as motivation, assignments, tests, and discipline in both groups.

The data for the statistical analysis were derived from scores made on an achievement test in a pre- and post-evaluation. All tests used in this study were commercial standardized tests.

The null hypothesis and the alternate hypothesis were

$$H_0\colon \mu_X = \mu_Y \quad \text{and} \quad H_1\colon \mu_X > \mu_Y$$

respectively, where μ_X is the mean achievement of the experimental class and μ_Y is the mean achievement of the control class.

The 't' test was used to determine if the null hypothesis was rejected or accepted. The level of significance was $\alpha = .05$ for a one-tailed test.

Chapter 4

FINDINGS

In this chapter, the results of the one-tailed t-distribution were reported. Those results were used to test the null hypothesis of each subject area, algebra I and geometry. The data, along with an outline of the computation of the inferential statistics, were placed in Table 2, pages 32 and 33, and Table 3, pages 34 and 35.

The significance of the difference in the mean achievement of the students in the experimental and control classes was determined by a comparison of mathematical achievement scores and evaluated on the basis of a t-distribution. When the probability of obtained differences was smaller than the $\alpha = .05$ level of significance, the differences were considered to be statistically significant. Conclusions were then stated on this basis, with the recognition that chance differences were considered significant 5 per cent of the time. Under those conditions, the researcher accepted the risk of being wrong five times in one hundred when he assigned significant differences to the experimental variable rather than to chance. Stating that the obtained difference was statistically significant means that the null hypothesis was rejected. Test results that were not significantly different indicate the null hypothesis remains in doubt. The reader must remember a statement cannot be accepted as true, no matter how many verifications were found. However, the probability level indicates the likelihood of

Table 2

Calculations for Tests of Significance of Difference in Paired Groups in Algebra I

Pair Number	Lankton First-Year Algebra Achievement Score		Difference	
	Experimental (X)	Control (Y)	D	D^2
(1)	(2)	(3)	(4)	(5)
I	78	25	53	2,809
II	43	41	2	4
III	53	40	13	169
IV	19	3	16	256
V	41	23	18	324
VI	51	64	-13	169
VII	28	21	7	49
VIII	68	9	59	3,481
IX	41	36	5	25
X	56	4	52	2,704
Total	478	266	212	9,990

Table 2 (continued)

$$\overline{D} = \frac{212}{10} = 21.2$$

N = 10

$$s_D^{\ 2} = \frac{(10)(9990) - (212)^2}{(10((9)} \doteq 610.6$$

$$s_{\overline{D}}^{\ 2} = \frac{610.6}{10} \doteq 61.1$$

$$s_{\overline{D}} = \sqrt{61.1} \doteq 7.8$$

$$t = \frac{21.2}{7.8} \doteq 2.72$$

<u>Restrictions on 't':</u>

One-tailed 't' test

Degress of freedom
d.f. = N-1 = 9

Level of significance
α = .05

Table 3

Calculations for Tests of Significance of Difference in Paired Groups in Geometry

Pair Number	Howell Geometry Achievement Score		Difference	
	Experimental (X)	Control (Y)	D	D^2
(1)	(2)	(3)	(4)	(5)
I	53	23	30	900
II	19	30	-11	121
III	40	18	22	484
IV	57	63	- 5	25
V	40	32	8	64
VI	52	26	26	676
VII	78	64	14	196
VIII	74	48	26	676
IX	18	45	-27	729
X	49	81	-32	1,024
Total	480	429	51	4,895

Table 3 (continued)

$$\overline{D} = \frac{51}{10} = 5.1 \qquad N = 10$$

$$s_D^{\;2} = \frac{(10)(4895) - (51)^2}{(10)(9)} \doteq 515.0$$

$$s_{\overline{D}}^{\;2} = \frac{515.0}{10} \doteq 51.5$$

$$s_{\overline{D}} = \sqrt{51.5} \doteq 22.7$$

$$t = \frac{5.1}{22.7} \doteq .22$$

Restrictions of 't':

One-tailed 't' test

Degrees of freedom
d.f. = N-1 = 9

Level of significance
α = .05

real differences.

Algebra I Findings

For a one-tailed test, the calculated value of 't', $t \doteq 2.22$, was greater than 1.833 from the table of 't'. That indicated the results of the two methods were significantly different. Hence, the null hypothesis, H_0: $\mu_X = \mu_Y$, was rejected at the $\alpha = .05$ level of significance. The researcher concluded that since the mean scores of the two groups of algebra I students were significantly different at the $\alpha = .05$ level, there was at least a 95 per cent chance the mean of the algebra I experimental group was significantly greater than the mean of the algebra I control group. The alternate hypothesis, H_1: $\mu_X > \mu_Y$, was accepted.

Geometry Findings

For a one-tailed test, the calculated value of 't', $t \doteq 0.22$, was less than 1.833 from the table of 't'. That indicated the results of the two methods were not significantly different. Hence, the null hypothesis, H_0: $\mu_X = \mu_Y$, was accepted at the $\alpha = .05$ level of significance. The researcher concluded that there was no significant difference in the mean achievement of geometric skills as measured by an achievement test between students who were in the semi-independent study program and those students who were subjected to the conventional way of teaching.

Other Findings

To shed more light on the findings in this study, the researcher investigated the amount of overall mathematical achievement that was indicated on the post-test. The merits and demerits of the semi-independent study program were also listed in this section.

The Algebra I Findings and the Geometry Findings gave the reader information about the comparison of two methods of teaching. The data in Table 4 presents an idea as to the level of achievement that was attained by the students in the experimental and control groups.

Table 4

Percentage of Students in a Percentile Range on the Basis of the Post-test Percentile Rank

Course	Group	Percentile Ranges			
		99-75	74-50	49-25	24-0
Algebra I	Experimental	61%	17%	11%	11%
Algebra I	Control	33%	42%	17%	8%
Geometry	Experimental	24%	36%	21%	21%
Geometry	Control	42%	26%	21%	11%

The discussion of the merits and demerits of the semi-independent study program was a subjective evaluation by the researcher. The program revolved around each individual student with an emphasis on learning. The student assumed more responsibility for his own learning and in the process he spent less time on materials that were easy for him and more time on the difficult material. The slower student became an independent learner and freed himself from a predominance of failure. The high achiever was given the opportunity to advance ahead of other members in the class. The student seemed to become more involved in the learning process and competed with himself as well as other individuals of the class. The class time was devoted primarily to helping the individual students and students in small groups.

Several problems arose in the semi-independent study program. After the first few weeks, the students were working on different units or parts of units which created a problem of reduced group interaction on mathematical materials. Also, the student that was so conditioned to his previous classroom experiences had a difficult time in accepting the non-routine approach to learning. The large amount of clerical work and preparation of classroom materials for the students kept the teacher very busy. Those duties limited the amount of time the teacher could spend on curriculum management.

SUMMARY

The data and calculations for the tests of significance of difference in paired groups in algebra I and geometry were presented in Chapter Four. The one-tailed t-distribution was used to determine the probability (P-value) upon which the null hypothesis was accepted or rejected.

In the algebra I study, the researcher rejected the null hypothesis and accepted the alternate hypothesis. Hence, there was a greater mean achievement of algebraic skills as measured by an achievement test when students are in the semi-independent study program than when students are subjected to the conventional way of teaching.

In the geometry study, the researcher accepted the null hypothesis. Hence, there was no difference in the mean achievement of geometric skills as measured by an achievement test between students who were in the semi-independent study program and the students who were subjected to the conventional way of teaching.

Chapter 5

SUMMARY AND RECOMMENDATIONS

"INDIVIDUAL DIFFERENCES -- A Semi-Independent Study Program for Secondary Mathematics" was a stimulus-response reaction that this researcher must become more aware of individual differences to improve the effectiveness of the art in teaching. This study involved the development of a semi-independent study program in secondary mathematics, algebra I and geometry. The semi-independent study program in this paper was a derivative of independent study which was incorporated with an element of more teacher planning and control.

The primary problem of this study was to compare the semi-independent study program to the conventional method of teaching and to determine the merits of the semi-independent study program.

The review of the literature was organized in such a way as to accumulate knowledge to serve as a guide in the construction or modification of an effective semi-independent study program. The areas of concern in this literary study were: (1) ways of considering individual difference which are significant in learning mathematics; (2) ways of recognizing individual differences in learning potential and performance; (3) ways of providing for individual differences in learning mathematics; and (4) ways in which other educators have tried to individualize instruction in secondary mathematics.

The semi-independent study, used as a teaching method in two required courses of secondary mathematics at Manhattan High School, Manhattan, Montana, was investigated and analyzed by means of controlled experimental research. The students in the experimental and control groups were tested to determine the differences of their mathematical achievement over an eight-month period of time. A majority of the students in the samplings were members of the working, middle class of people. The high school classes were heterogeneously grouped through chance by the school's administration and counseling center. Ten members of the experimental group were match-paired with members of a corresponding control group. The variables were controlled in a consistent manner. The study was organized in one school with one teacher controlling such factors as motivation, assignments, tests, and discipline in both groups. The data for the statistical analysis were derived from scores made on an achievement test in a pre- and post-evaluation. All tests used in this study were commercial standardized tests. The null hypothesis and the alternate hypothesis were

$$H_0\colon \mu_X = \mu_Y \quad \text{and} \quad H_1\colon \mu_X > \mu_Y$$

respectively, where μ_X is the mean achievement of the experimental class and μ_Y is the mean achievement of the control class. The 't' test was used to determine if the null hypothesis was rejected or retained. The level of significance was .05 for a one-tailed test.

In the algebra I study, the researcher rejected the null hypothesis and accepted the alternate hypothesis. Hence, there was a greater mean achievement of algebraic skills as measured by an achievement test when students are in the semi-independent study program than when students are subjected to the conventional way of teaching.

In the geometry study, the researcher accepted the null hypothesis. Hence, there was no difference in the mean achievement of geometric skills as measured by an achievement test between students who were in the semi-independent study program and students who were subjected to the conventional way of teaching.

RECOMMENDATIONS

In reading the following recommendations, keep in mind that the results of this study are applicable only to those groups of students in the study. To reduce the danger of acceptance of a false conclusion arising from chance errors, the experiment should be repeated several times under similar conditions and in varied locations. This replication would add precision to the experimental results and provide a means of estimating the experimental errors.

This researcher also recommends that each teacher, irregardless of grade level or subject area, take time to develop a unit of work along the pattern of the independent study, use it, and analyze the results.

APPENDIX

Table 5

Data for Match-pairing Students in Algebra I
(Experimental Group)

Student Number	D.I.Q.	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application
2	120	11.5	12.0	9.1
3	94	8.4	6.3	7.4
5	100	10.0	11.1	7.7
6	104	8.8	8.5	8.2
7	103	5.8	7.2	6.7
9	109	12.3	12.6	11.3
10	148	12.9	12.9	11.3
11	88	6.2	6.3	7.4
12	106	7.8	8.8	8.2
13	111	9.2	8.5	7.7
14	120	8.4	8.5	10.4
15	124	12.6	11.4	11.1
16	102	8.8	8.8	7.9
18	118	11.3	11.1	8.2
19	128	12.1	11.8	12.1
21	106	8.2	10.7	7.9
23	106	7.2	6.0	11.1
24	104	9.2	7.8	7.4

Table 6

Data for Match-pairing Students in Algebra I
(Control Group)

Student Number	D.I.Q.	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application
1	98	8.0	9.9	6.7
3	109	5.1	7.2	3.6
5	114	10.4	8.5	7.7
7	110	9.2	9.3	9.1
8	93	--	6.9	7.7
9	102	8.2	10.7	11.9
10	95	11.2	7.8	7.4
11	95	10.0	8.8	7.7
13	94	6.4	7.8	7.4
15	121	10.8	10.3	9.1
17	101	8.0	7.2	7.4
18	102	8.8	7.8	7.9

Table 7

Paired Students in Algebra I

Pair Number	Student Identification Number	
	Experimental Group	Control Group
I	13	5
II	21	1
III	5	11
IV	11	13
V	3	8
VI	24	17
VII	6	7
VIII	16	18
IX	23	9
X	2	15

Table 8

Data from Lankton First-Year Algebra Test

Experimental Group			Control Group		
Student Number	Percentile Score		Student Number	Percentile Score	
	Pre	Post		Pre	Post
1	7	41	1	14	55
2	36	92	2	41	94
3	22	63	3	7	41
5	41	94	5	96	88
6	50	78	7	67	88
7	31	22	8	22	45
9	45	88	9	55	91
10	59	99	10	14	50
11	3	22	11	27	67
12	31	71	13	4	7
13	14	92	14	3	7
14	14	45	15	55	59
15	67	96	16	14	50
16	18	86	17	14	78
17	86	99	18	50	59

Table 8 (continued)

Experimental Group			Control Group		
Student Number	Percentile Score		Student Number	Percentile Score	
	Pre	Post		Pre	Post
18	41	93	20	14	55
19	78	94	22	22	63
21	7	50			
22	59	71			
23	4	45			
24	45	96			
25	4	41			
26	22	14			

Table 9

Data for Match-pairing Students in Geometry
(Experimental Group)

Student Number	D.I.Q.	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application
1	99	10.4	10.7	7.2
2	103	10.3	9.5	8.8
3	105	8.0	8.5	7.9
5	113	12.9	12.7	11.6
7	103	6.6	9.2	6.3
8	88	6.0	6.0	6.3
9	93	6.6	8.2	7.4
10	90	8.9	7.6	7.9
11	98	7.8	9.9	8.2
12	100	8.2	9.6	10.8
13	117	10.8	10.3	8.2
14	125	11.9	12.5	11.3
15	106	10.4	9.6	9.8
17	106	8.2	9.2	9.8

Table 10

Data for Match-pairing Students in Geometry
(Control Group)

Student Number	D.I.Q.	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application
1	131	12.1	12.9	12.5
2	121	12.3	12.6	12.3
3	114	10.8	8.2	9.1
4	133	12.9	12.9	12.3
5	133	12.3	12.2	10.8
6	119	9.6	11.3	10.5
8	110	11.5	11.1	7.4
10	110	10.8	11.4	10.4
11	138	12.7	12.9	12.8
12	97	8.6	8.2	9.1
13	120	10.4	12.2	11.3
14	123	10.8	11.4	9.8
15	113	8.4	10.7	10.8
16	124	9.6	9.2	11.1
17	100	9.4	10.1	9.6
18	105	8.3	8.7	7.2
20	129	11.2	11.8	12.5
21	95	4.2	8.2	6.7
22	115	10.5	9.8	9.6

Table 11

Paired Students in Geometry

Pair Number	Student Indentification Number	
	Experimental Group	Control Group
I	12	17
II	9	18
III	10	12
IV	1	8
V	2	10
VI	15	3
VII	17	15
VIII	5	2
IX	3	22
X	14	20

Table 12

Data from Howell Geometry Test

Experimental Group			Control Group		
Student Number	Percentile Score		Student Number	Percentile Score	
	Pre	Post		Pre	Post
1	5	62	1	62	99
2	5	45	2	51	99
3	5	23	3	2	28
4	2	45	4	67	99
5	23	97	5	15	80
7	5	57	6	2	57
8	2	5	8	5	67
9	0	19	10	7	39
10	5	45	11	19	99
11	51	39	12	5	23
12	9	62	13	28	72
13	2	90	14	9	76
14	23	72	15	12	76
15	15	67	16	15	45
16	19	84	17	28	51
17	2	80	18	3	33
			19	3	23
			20	15	96
			22	12	57

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