Freedericksz transitions in nematic liquid crystal
by Phillip Alexander Himmer

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Physics
Montana State University

Abstract:
Liquid crystal macromolecules have many potential uses in various devices such as displays. These types of applications require liquid crystals with specific elastic constants in order to work effectively. The Freedericksz transition is one method of measuring these constants. In this experiment the Freedericksz transition for the bend geometry is studied in order to obtain the bend and splay elastic constants. Using the silane agent DMOAP a perpendicular orientation of the molecules was obtained at the cell boundaries. The gap thickness was measured using the He-Ne back reflection technique. The magnetic susceptibility for this particular liquid crystal was never measured and measuring it was not possible with the available equipment. Therefore a value which agreed within an order of magnitude of published values was used. This limits the accuracy of the fitted constants to an order of magnitude also. These constants were found to be 1.64x10 -11 for K33 and 8.64x10-12 for K11

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ABSTRACT

Liquid crystal macromolecules have many potential uses in various devices such as displays. These types of applications require liquid crystals with specific elastic constants in order to work effectively. The Freedericksz transition is one method of measuring these constants. In this experiment the Freedericksz transition for the bend geometry is studied in order to obtain the bend and splay elastic constants. Using the silane agent DMOAP a perpendicular orientation of the molecules was obtained at the cell boundaries. The gap thickness was measured using the He-Ne back reflection technique. The magnetic susceptibility for this particular liquid crystal was never measured and measuring it was not possible with the available equipment. Therefore a value which agreed within an order of magnitude of published values was used. This limits the accuracy of the fitted constants to an order of magnitude also. These constants were found to be 1.64×10^{-11} for K_{33} and 8.64×10^{-12} for K_{11}.

CHAPTER 1

INTRODUCTION

Liquid crystals are macromolecules that have varying degrees of orientational and positional order. There are three basic phases that liquid crystals can form: isotropic, nematic, and smectic. The isotropic phase is the least ordered of the three with random orientational and positional order. The nematic phase has orientational order such that the molecules are pointing in the same direction on the average but it has no positional order. In the smectic phase there is positional as well as orientational order.

The physics of liquid crystals was initially discussed in 1933 by various scientists. The Freedericksz transition, in which an external field distorts the liquid crystal molecules from some prearranged geometry, was noticed at this time but little importance was attached to this phenomenon. Also during this time Oseen[1] presented a paper detailing a theory of liquid crystal elastic interactions. In 1958 a paper by Frank[2] expanded upon this original theory. The resulting formulation for the free energy is known as the Oseen-Frank free energy. The Freedericksz transition has since been done for, in the language of the theory, the bend, splay, and twist cases (Figure 1). Most of the work has been done with nematic liquid crystals since they are the simplest to describe and the data are easier to interpret. The theory assumes that the liquid crystal molecules are uniaxial which is a good assumption for most nematic liquid

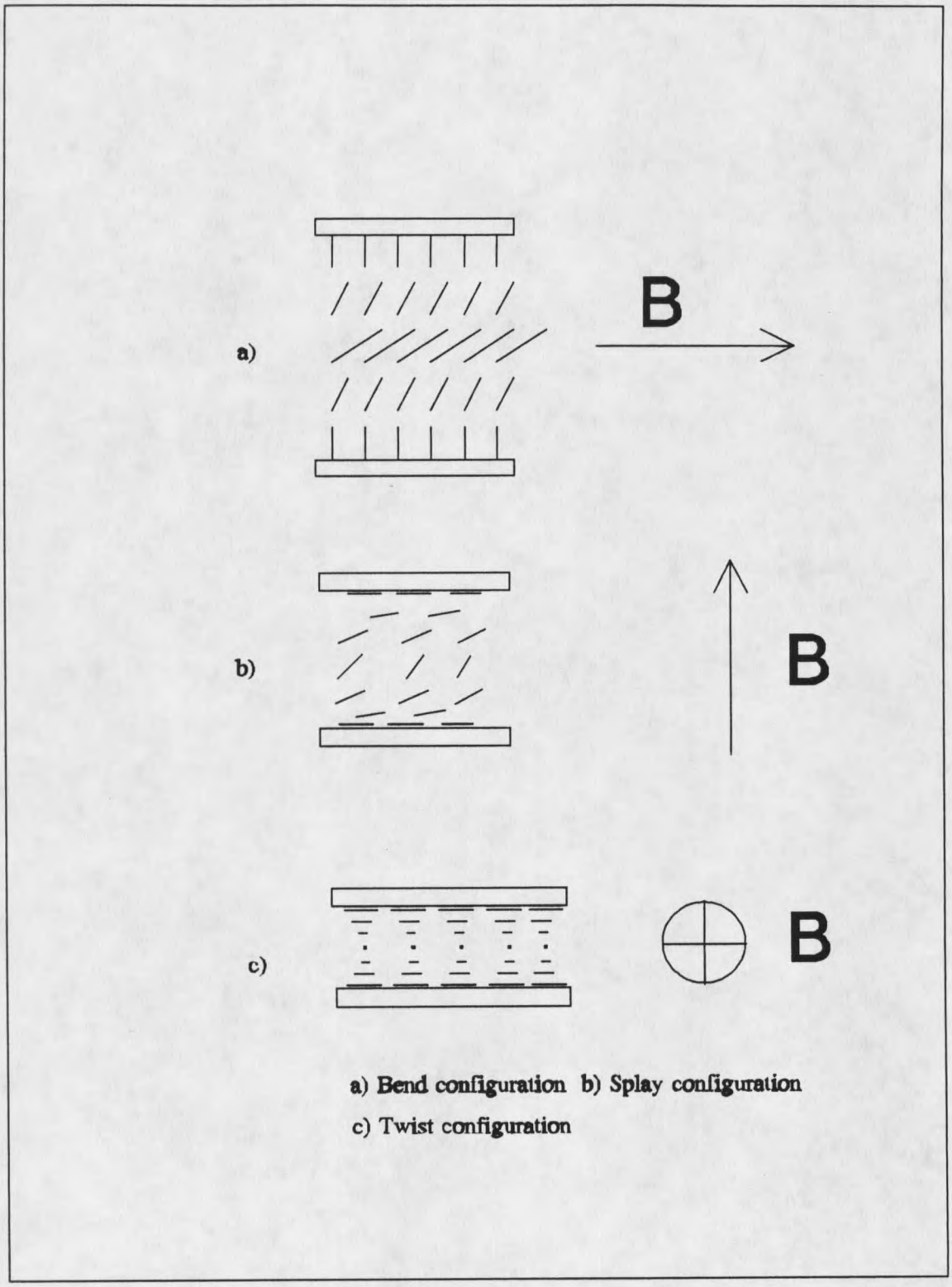

Figure 1. Alignment geometries.

crystals. In this thesis the nematic liquid crystal E-7, a type of cyanobiphenyl liquid crystal, made by BDH is studied with the goal of determining the splay elastic constant K_{11} and the bend elastic constant K_{33}. E-7 has one phase transition between isotropic and nematic at 60°C. The principal use for this material is in twisted nematic cells, such as would be used for watch displays.

CHAPTER 2

SAMPLE PREPARATION

In order to measure the bend and splay elastic constants by means of the Freedericksz effect I used a magnetic field perpendicular to the direction of the aligned liquid crystals (Figure 2). This gives the bend geometry as seen in Figure 1.

To make a sample, two pieces of optically flat glass, obtained from Edmunds Scientific, were used to form a gap that the liquid crystal will be inserted into. This glass was first cleaned with a 1M potassium hydroxide solution for approximately one hour in a water bath at 50° C. Then the glass was put into a solution of one part concentrated nitric acid,one part concentrated sulfuric acid, and two parts distilled water and allowed to soak for approximately one hour in a 60° C water bath. To remove any residue the glass was then put in isopropanol alcohol, soaking for at least 4 hours. The cleaning procedure called for a hot vapor of isopropanol condensing but the apparatus needed to do this was not available. Between each step of the procedure the glass was rinsed with distilled water[3].

To align the liquid crystal the surface of the glass was treated with a silane bonding agent called N,N-dimethyl-N-octadecyl-3-aminopropyltrimethoxysilyl chloride (DMOAP) made by Dow Chemical. This silane agent adheres to the glass surface with a long trailing end pointing perpendicular to the glass surface. The cleaned glass was put in

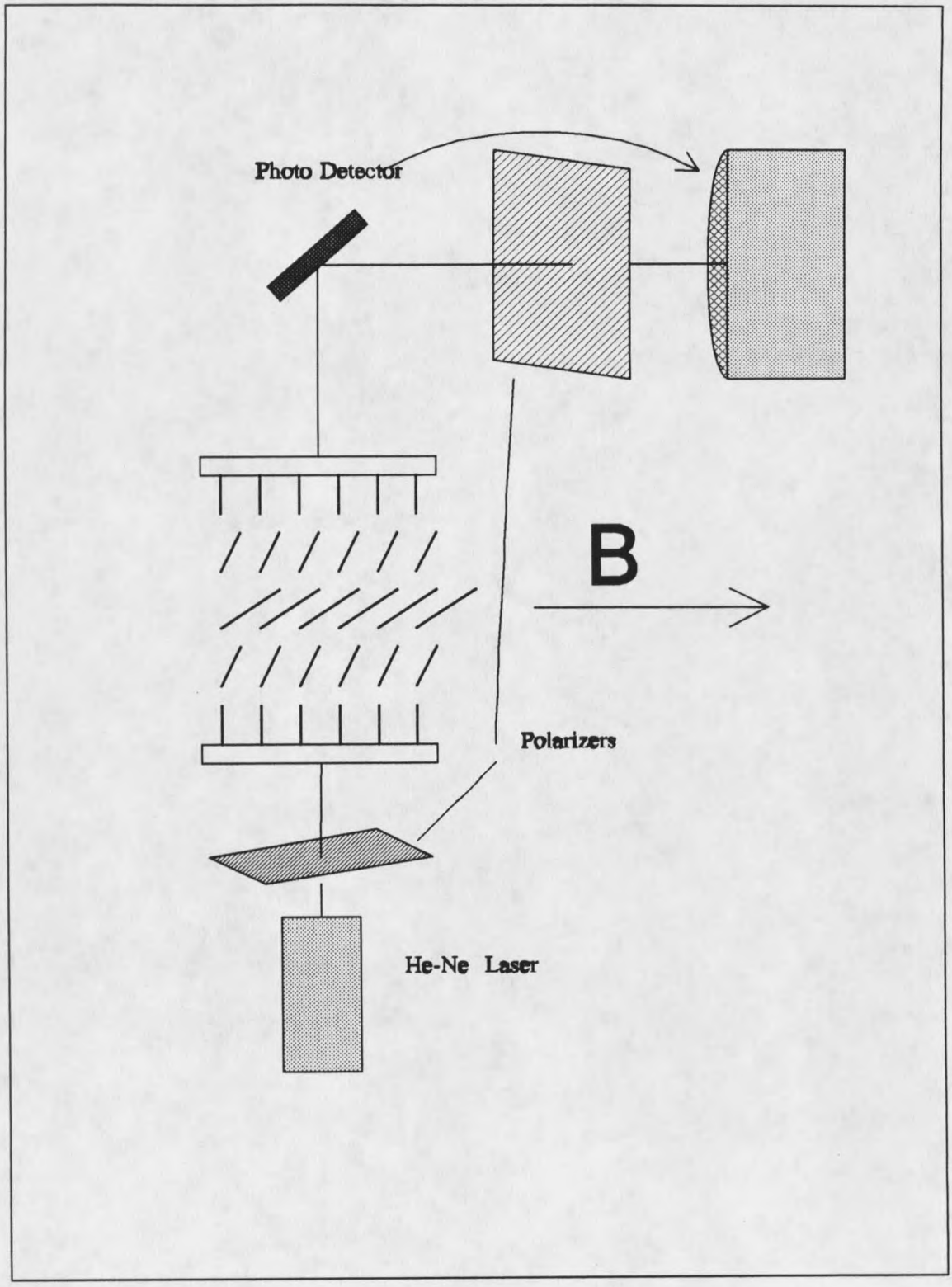

Figure 2. Experimental setup

a 0.1% (by volume) solution of DMOAP and distilled water and agitated for five minutes in an ultrasonic cleaner. The glass was then rinsed with distilled water and dried with nitrogen gas. To cure the coating the glass flats were heated under nitrogen for one hour at 110° C.[4] Initially, alignment of the liquid crystal was attempted by rubbing the glass surfaces in two perpendicular directions with diamond paste but poor data resulted[5]. This could be due to the rubbing or to the fact that the glass flats used were microscope slides and not the optically flat glass finally used.

The glass flats were held by nylon set screws in aluminum plates separated by eight beryllium copper springs under compression (See Figure 3). The thickness was adjusted by tightening or loosening four brass screws which run through the center of four of the springs.

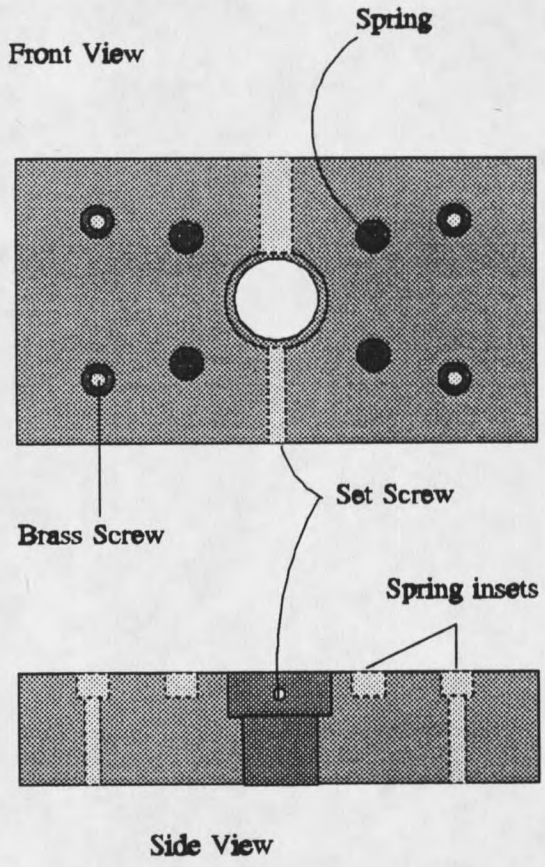

Figure 3. Sample holder

CHAPTER 3

EXPERIMENTAL TECHNIQUES

He-Ne Back Reflection Technique

To align the glass flats to a desired thickness and to set them parallel a He-Ne back reflection technique was used[6]. The cell was illuminated with He-Ne laser light that was passed through a pinhole, then collimated with a 250 mm focal length lens (See Figure 4). Four spots appeared from back reflections; one from each surface of the two glass flats. To see these spots clearly, two mirrors were used to reflect the beam allowing the small deviation, due to the fact that the surfaces of a flat were not exactly parallel, to separate the reflections. The two spots that represent the inside surfaces of the two glass flats could be found through trial and error. The interference pattern representing the gap changed more slowly than other interference patterns such as that from the outside surface of one flat and the inside surface of the other flat, because the gap was thinner. The cell was attached to an Aerotech 301R rotation table in order to accurately rotate the sample. As the cell was rotated through an angle one could observe interference fringes moving outward from the center of the combined spot. Starting at a known angle, typically three degrees, the fringes could be counted up to another known angle. From the equation

$$2d\cos(\theta) = (m)\lambda \ , \tag{2.1}$$

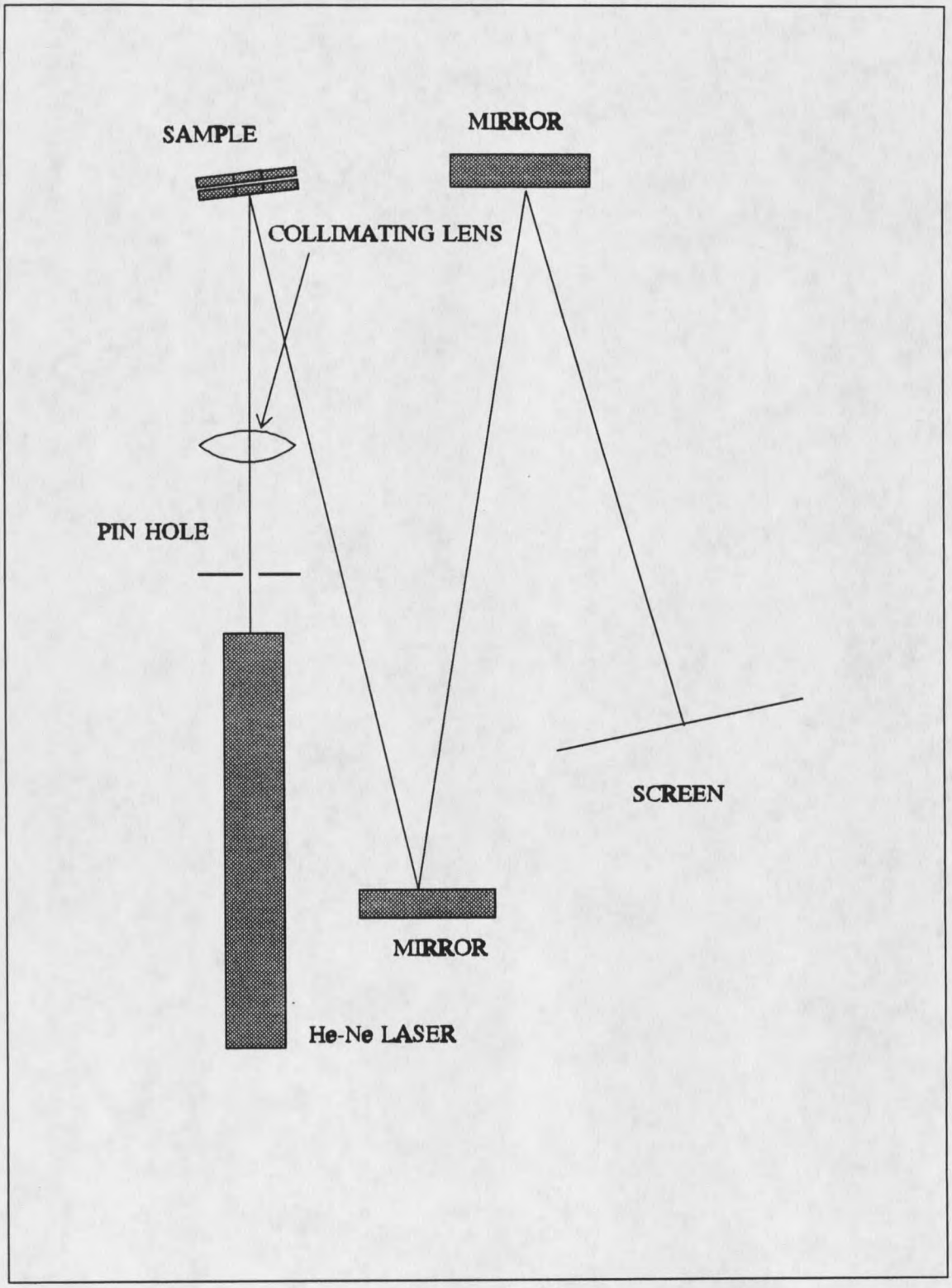

Figure 4. He-Ne back reflection apparatus

the number of fringes that one saw as the table was rotated from θ_1 to θ_2 allowed the thickness of the sample to be calculated by taking the difference of the order number at the two different angles. To align θ=0, the back reflection of the fixed plate was lined up onto the pinhole. Ideally one would see spots as with a Fabry-Perot interferometer but ¼λ glass flats have too much variation to give spots, so an interference pattern was seen. This could be remedied by using better optical flats.

While inserting the liquid crystal into the gap of the cell, the liquid crystal should ideally be in the isotropic phase so that it can move more freely.[3] I twice attempted to heat the liquid crystal into the isotropic phase after the liquid crystal was in the cavity, and then let it cool to room temperature, but poor data resulted each time. I suspect that it was due to a interaction between the liquid crystal and the DMOAP, or perhaps the sample holder became distorted. Therefore the liquid crystal was left in the room temperature nematic phase when it was inserted and was kept at room temperature for the duration of the experiment. After a day it appeared that the samples settled down so that the data were reproducible. In the future it would be nice to be able to insert the liquid crystal in the isotropic phase to reduce the time needed for the molecules to orient themselves and to insure better anchoring.

Data Acquisition

The data were taken with a Hewlett Packard 34401A multimeter and a Keithley 195A multimeter connected via GPIB ports to a Zenith Data Systems 286 computer.

I distinguish between the HP and Keithley because the software that read in the data assumed that the Keithley was connected to a Varian V-FR 2703 magnet with 5" poles and when the voltage at the Keithley was 15 volts the data file was closed and the program terminated. To take data, the cell was inserted into the holder, positioning it between the magnet poles, and the holder was adjusted so the beam entered the photodiode at the center. To keep the intensity below the saturation level of the photocell, two polarizers were placed on the laser and turned until the intensity was low enough. To polarize the beam 45 degrees from the magnetic field axis the two polarizers on the laser were rotated as a unit until the top polarizer was oriented at 45° with respect to the direction of the magnetic field. After the light passed through the cell, the polarizer in front of the photocell was turned so as to minimize the intensity. Thus the oscillations began from a minimum[7].

The data was stored in the computer as an ASCII file. The program accepted twenty readings from the multimeters and averaged them to give a data point. This was done to help reduce random fluctuations in the voltages representing the magnetic field and the intensity. The number of data points taken ranged from 763 to 788 for a 50-minute scan. The variation occurred because the field sweep percent dial could not be started exactly at zero each time. The program used the voltage from the sweep circuit which varied from 0 to 15 volts to obtain the magnetic field, given the center field and the range of the field (Appendix A).

CHAPTER 4

THEORY

Oseen Frank Free Energy

The Oseen Frank free energy for liquid crystals is given by the expression

$$F_f = \frac{1}{2}\int [K_{11}(\nabla\cdot\hat{n})^2 + K_{22}(\hat{n}\cdot\nabla\times\hat{n})^2 + K_{33}(\hat{n}\times\nabla\times\hat{n})^2]\, d\tau \tag{3.1}$$

where $\hat{n}$ is the direction of the director. The director is the average direction of the liquid crystal molecules. When the liquid crystal is put in a magnetic field an additional term given by

$$F_m = -\frac{1}{2}\frac{\Delta\chi}{\mu_o}\int (B\cdot\hat{n})^2 d\tau \tag{3.2}$$

is introduced[8]. For the magnetic field in the $\hat{x}$ direction and the liquid crystal aligned perpendicular to the glass with the director $\hat{n}$ pointed in the $+\hat{z}$ direction, $\hat{n}$ will have the form $\hat{n} = \sin(\theta)\hat{x} + \cos(\theta)\hat{y}$. The free energy can then be written as

$$F_f + F_m = AK_{33}\int_0^{\frac{d}{2}} [(1-\kappa)\,Sin^2(\theta)\left(\frac{\partial\theta}{\partial z}\right)^2 - \frac{\Delta\chi B^2}{\mu_o K_{33}}Sin^2(\theta)]\, dz \tag{3.3}$$

where

$\kappa = 1 - \frac{K_{11}}{K_{33}}$, A= Area of the sample, d=sample thickness,

and θ is the angle between the director and the $+\hat{z}$ axis. Now it is necessary to find the function $\theta(z)$ that minimizes the free energy. This is done with variational calculus which says that the function $\theta(z)$ must satisfy the differential equation

$$\frac{d}{dz}\left(\frac{\partial \phi}{\partial \theta}\right) - \frac{\partial \phi}{\partial \theta} = 0 \tag{3.4}$$

where ϕ is the integrand of the integral representing the free energy. After making the substitution $U = \frac{\partial \theta}{\partial z}$ the resulting differential equation becomes separable. Using the limits that θ goes from θ to θ_m and U goes from U to 0 one obtains

$$\frac{\partial \theta}{\partial z} = \left(\frac{\Delta \chi B^2}{\mu_o K_{33} \kappa}\right)^{\frac{1}{2}} \left(1 - \frac{(1 - \kappa Sin^2(\theta_m))}{(1 - \kappa Sin^2(\theta))}\right)^{\frac{1}{2}} \tag{3.5}$$

Separating variables and integrating gives

$$\int_{\theta}^{\theta_m} \frac{(1 - \kappa Sin^2(\theta))^{\frac{1}{2}} d\theta}{(Sin^2(\theta_m) - Sin^2(\theta))^{\frac{1}{2}}} = \left(\frac{\Delta \chi B^2}{\mu_o K_{33}}\right)^{\frac{1}{2}} \int_{z}^{\frac{d}{2}} dz \ . \tag{3.6}$$

Setting the lower limits on the integral to 0 gives an expression relating the magnetic field to the maximum angle θ_m. This expression can subsequently be used to find the

magnetic field for a given phase shift. To relate equation 3.6 to the observed phase shift an expression for the index of refraction vs. angle of the director is needed.

Index of Refraction Calculation

Using Maxwell's equations an expression for the index of refraction can be found.

$$\nabla \times \vec{E} = \mu_o \frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \epsilon_o \frac{\partial}{\partial t} (\vec{\vec{\epsilon}}_r \cdot \vec{E}) \tag{3.7}$$

where $\vec{\vec{\epsilon}}_r = \begin{pmatrix} \epsilon_{ro} & 0 & 0 \\ 0 & \epsilon_{ro} & 0 \\ 0 & 0 & \epsilon_{re} \end{pmatrix}$, such that ϵ_{ro} is the dielectric constant perpendicular to the director and ϵ_{re} is the dielectric constant along the director. Since the externally applied magnetic field is in the $\hat{x}$ direction the extraordinary axis of the liquid crystal will be in the $\hat{x}$ $\hat{z}$ plane. Assuming the laser beam is propagating in the $\hat{z}$ direction the condition at the boundary is that the $\hat{z}$ components of the displacement field at z=0 and at z=d are zero. This comes from the assumption that the electric field is of the form

$$\vec{E} = \vec{E}_{eo} e^{i(kz - \omega t)} \tag{3.8}$$

so that the only spatial dependence is in the $\hat{z}$ direction. Writing the displacement vector in terms of the $\hat{x}'$ and $\hat{z}'$ coordinates gives (See Figure 5)

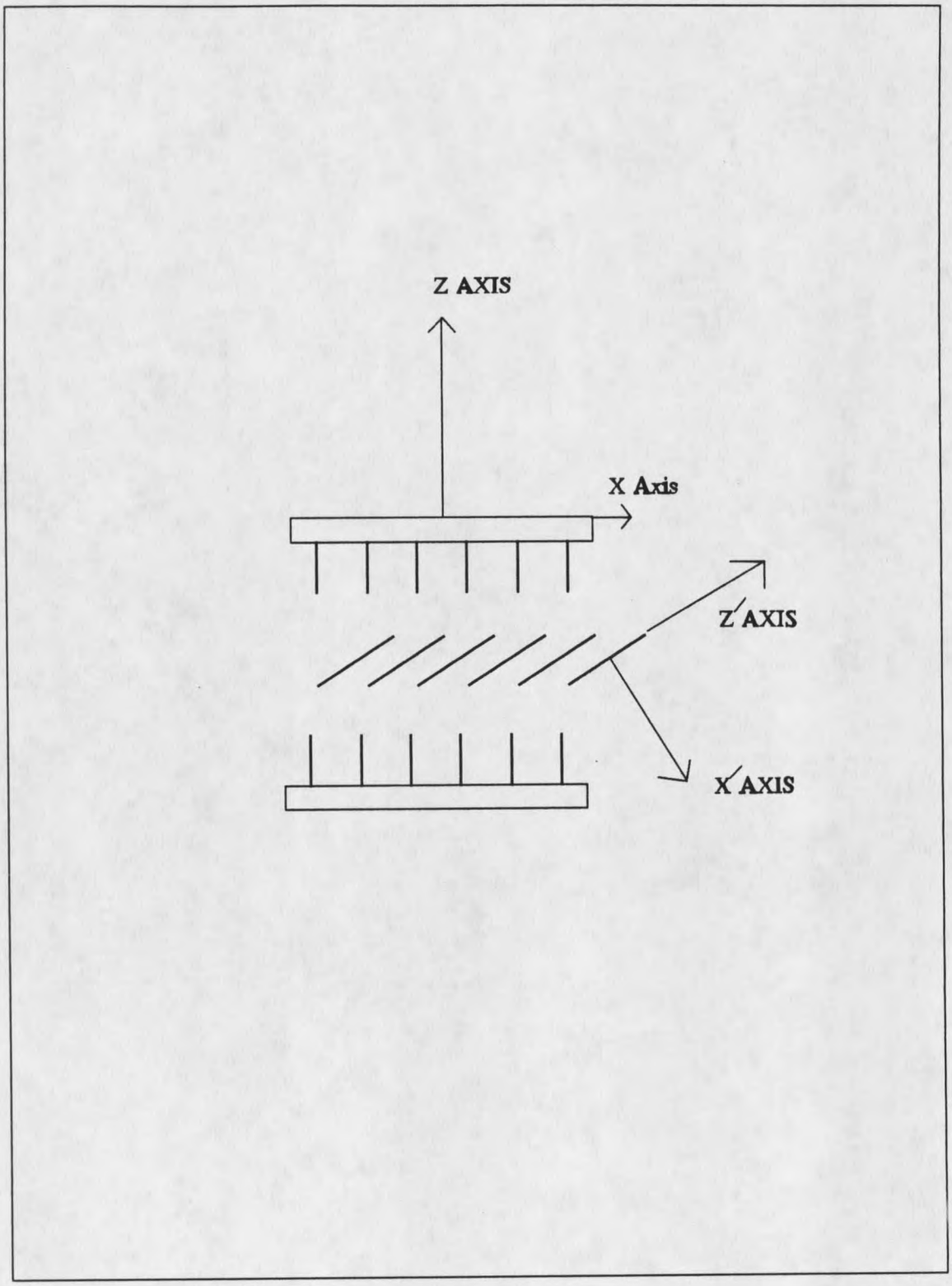

Figure 5. Coordinates for index of refraction calculation

$$\vec{\vec{\epsilon}}_r \cdot \vec{E}_{eo} = \epsilon_{re} E_{eo} Sin(\theta - \alpha)\, \hat{z}' + \epsilon_{ro} E_{eo} Cos(\theta - \alpha)\, \hat{x}' \tag{3.9}$$

The boundary condition $\vec{D}_z = 0$ gives the relation

$$Tan(\theta - \alpha) = \frac{\epsilon_{ro}}{\epsilon_{re}} Tan(\theta) \tag{3.10}$$

After a bit of work the expression

$$\vec{\vec{\epsilon}}_r \cdot \vec{E}_{eo} = \frac{E_{eo} n_o^2 n_e^2}{(n_e^4 Cos^2(\theta) + n_o^4 Sin(\theta))}\, \hat{x} \tag{3.11}$$

is found where n_e and n_o are the extraordinary and ordinary indices of refraction respectively. Putting equation 3.8 into the first of Maxwell's equations and then substituting the result into the second leads to the result

$$\vec{\vec{\epsilon}}_r \cdot \vec{E}_{eo} = n^2(z)\, E_{eo} Cos(\alpha)\, \hat{x} \tag{3.12}$$

where n(z) is the index of refraction at some distance z through the thickness of the sample. After rewriting cos(α) in terms of θ the result

$$n(z) = \frac{n_e n_o}{(n_o^2 Sin^2(\theta(z)) + n_e^2 Cos^2(\theta(z)))^{\frac{1}{2}}} \tag{3.13}$$

is found. Taking the difference in the optical path length between the ordinary and extraordinary axis and integrating over half the thickness of the sample gives the expression[3]

$$\delta = \left(\frac{4\pi}{\lambda}\right)\int_0^{\frac{d}{2}} [n(z) - n_o]\, dz \tag{3.14}$$

Using equation 3.5 you can replace the integral over z with a integral over θ. This can then be numerically integrated to give the phase shift as a function of the magnetic field.

Intensity Modulation

Once the phase shift is known the intensity can be calculated by letting

$$\vec{E} = \frac{E_o}{\sqrt{2}}\left(\hat{1}_f e^{-i\omega t} + \hat{1}_s e^{-i\omega t}\right) \quad \text{at z=0} \tag{3.15}$$

so that after passing through the sample the electric field along the $\hat{1}_f$ axis undergoes a phase change of φ_f while the field along the $\hat{1}_s$ undergoes a phase shift of φ_s. Letting $\varphi_f = \varphi_s + \delta$ and changing the axis to the $\hat{x}$ and $\hat{y}$ axis as defined in Figure 6 gives the equation

$$\vec{E} = \frac{E_o}{2} e^{-i(\omega t + \varphi)}\left(\hat{x}(1 - e^{-i\delta}) + \hat{y}(1 + e^{-i\delta})\right) \tag{3.16}$$

After passing through the polarizer which is in the $\hat{x}$ direction and then squaring the complex conjugate the expression

$$I = \frac{E_o}{4} Cos^2(\omega t + \varphi_s)\,(1 - 2\,Cos(\delta) + Cos(\delta)^2) \tag{3.17}$$

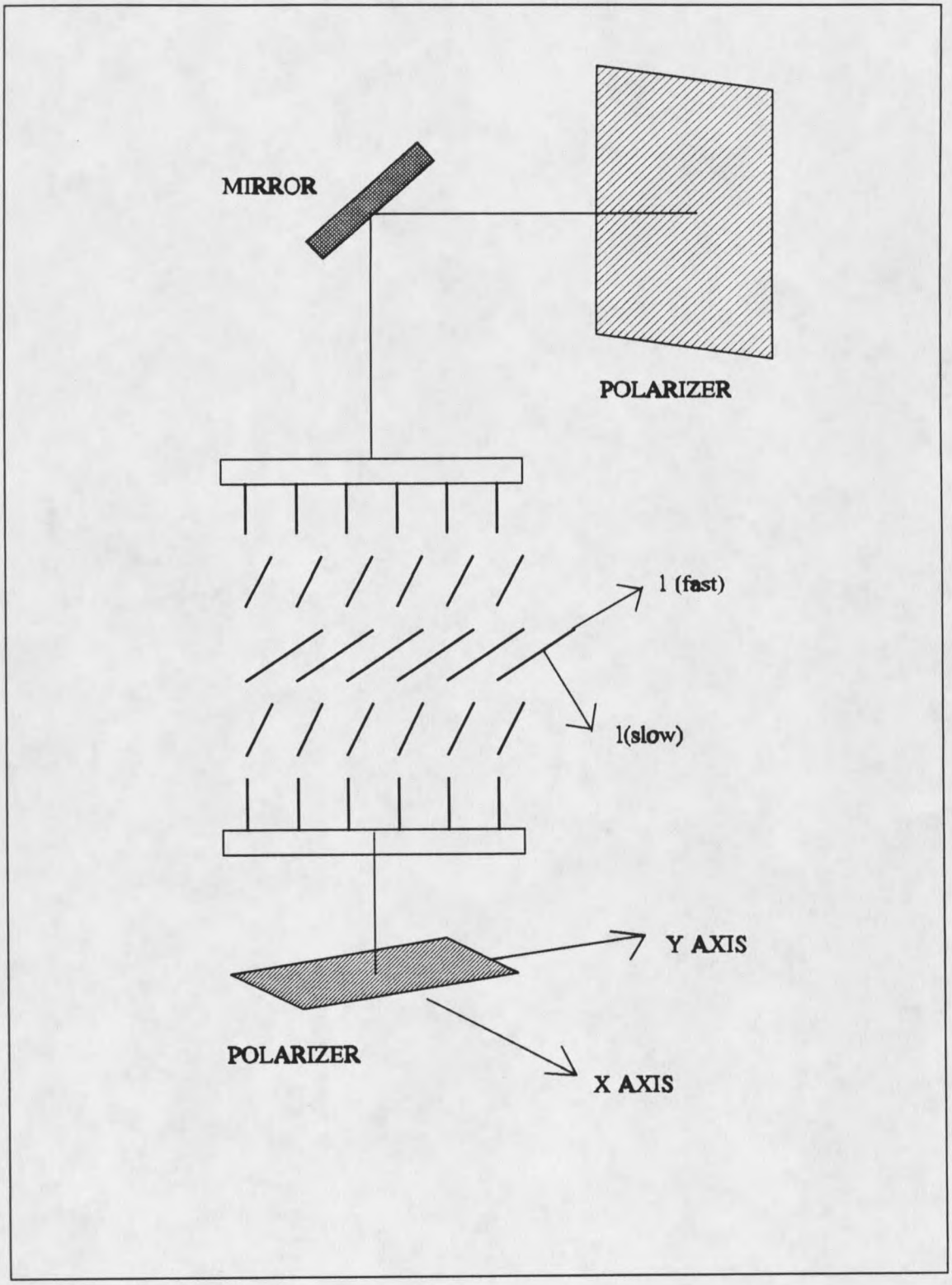

Figure 6. Coordinate system for the intensity calculation

is obtained.

Numerical Calculations

The numerical integrations were carried out using canned routines in Fortran from Numerical Recipes[9] run on the HP Apollo. The left side of equation 3.6 is integrated using the method QROMO which is a Romberg integration with an open interval. This is important because the integrand is undefined at $\theta=\theta_m$. The integral however remains finite as can be seen by doing an expansion around θ_m. The four parameters fit are the thickness, k_{11}, k_{33}, and $\Delta\chi$. The integrations are performed for values of θ_m extending from 0.01 to 1.57 with a step size of 0.01. Thus for each value of θ_m the magnetic field which gives that angle is found by solving equation 3.6; then the phase shift is found using equations 3.5, 3.13, and 3.14.

The program was written in four sections. First is the main routine where values for the four parameters and θ_m are assigned. The next section is ROMB which contains the subroutines QROMO and QROMA. These routines set up the numerical integration tolerances. From here the subroutines MDPNT, MDPNTA and POLINT are called. The MDPNT section actually calculates a rough value for the integral and then POLINT takes these values and gives refined values (See Appendix B).

CHAPTER 5

DATA ANALYSIS

Theoretical Fitting Procedure

The data were fit by visually comparing how the theoretical graph and the measured graph fit. Because of the oscillations the fit is quite sensitive to any changes of the parameters. It is possible to compare data files and get a fitting value so as to compare with different data. This ought be done in the future.

It should be pointed out that the value given to $\Delta\chi$ (see table 2) is arbitrary. The manufacturer,(BDH), did not know the value nor could it be found in any publications. It is possible to measure the susceptibility with a SQUID detector.[10] Unfortunately there were persistent noise problems with the SQUID detector available to us which made the measurement of this small a susceptibility impossible. The value used was picked because it agrees within an order of magnitude with values quoted in the literature.[3]

To fit the data the first step was to match the threshold field. Once this was done the ratio of the square root of K_{33} to the thickness remains constant. The next step in my procedure was fitting the first oscillation by adjusting K_{11}. To increase the period, K_{11} was made larger and to reduce the period K_{11} was reduced. Once the best value of K_{11} is found, the ratio of K_{11} to K_{33} remains constant. If the theoretical oscillations led the experimental oscillations, the value used for the thickness was less than it should

have been, and if they lag, the value is greater than it should have been. These considerations could be used to automate the fitting of the data in any future experiment.

Comparison of Theory and Data

Data acquired from the samples was transferred over to the Hewlett Packard Apollo workstation where theory was compared to data. Due to inconsistencies in both theory and data the fits were not perfect.

The theory used does not consider the surface to liquid crystal interfacial energy so for thin films where this becomes important the fitted values will be incorrect[11]. For thick films, diffusion and other fluid effects that are not taken into account will also affect the fitted values. It seems that a thin film is considered anything under 10 μm while the definition for a thick film is less precise.

As seen in table 1, the largest value for thickness that was obtained theoretically was 27.7 μm. This is not a particularly thick sample so any bulk effects should be negligible. Except for sample 4 the average estimated relative error is 14% in the measurement technique. Of the six samples only in samples 1 and 6 did the theoretical fit yield a thickness within the estimated experimental error. The cause or causes of the abnormally large error in the thickness is unknown at this time. Any small theoretical thickness error could be due to the sample holder's not being rigid. In considering sample 4, assuming the theoretical thickness is correct, the discrepancy in the measurement could be accounted for by the change in the order number Δm, of equation 2.1, having the value 2 instead of 1.

In Figure 7 a "high frequency" signal is seen superimposed on the intensity oscillations. This is due to the laser not being thermally stable so the He-Ne laser hopped between modes with different polarizations. Otherwise the fit gives a thickness values within the experimental error.

The theoretical fit for Figures 8 and 10 show a different thickness than for sample 2 runs 3, 4, and 5. Comparing graphs 8 and 9 the difference in the thickness parameter causes a noticeable difference in the fit. Figure 11 shows the best fit obtainable when the thickness is set at 14μm for sample2.Rn2. Again the difference is significant. The theoretical curve obviously does not fit in Figure 11. This may be the result of the data being taken too soon after the first run or it might be the result of not inserting the liquid crystal in the isotropic phase although this should effect sample2.Rn1 more than seen in Figure 8. Figures 12, 13, and 14 contain the rest of the runs of sample 2 and all of them are very similar.

Figures 15 and 16 which show the sample 3 data have values that show good repeatability. The theoretical fit shown in Figure 18 could be improved upon. The poor fit could account for the large values of K_{11} and K_{33}. Sample 5 data shown on Figures 19, 20, and 21 give the most consistent results of all of the data with very good fits. Figure 22 is surprising in that the experimental and theoretical thicknesses are within experimental error even though as seen in the graph the theory fits only in the middle of the data and the elastic constants do not correspond with the other values very well.

The average value of K_{11} from table 1 is 8.64×10^{-12} and the average value of K_{33} is

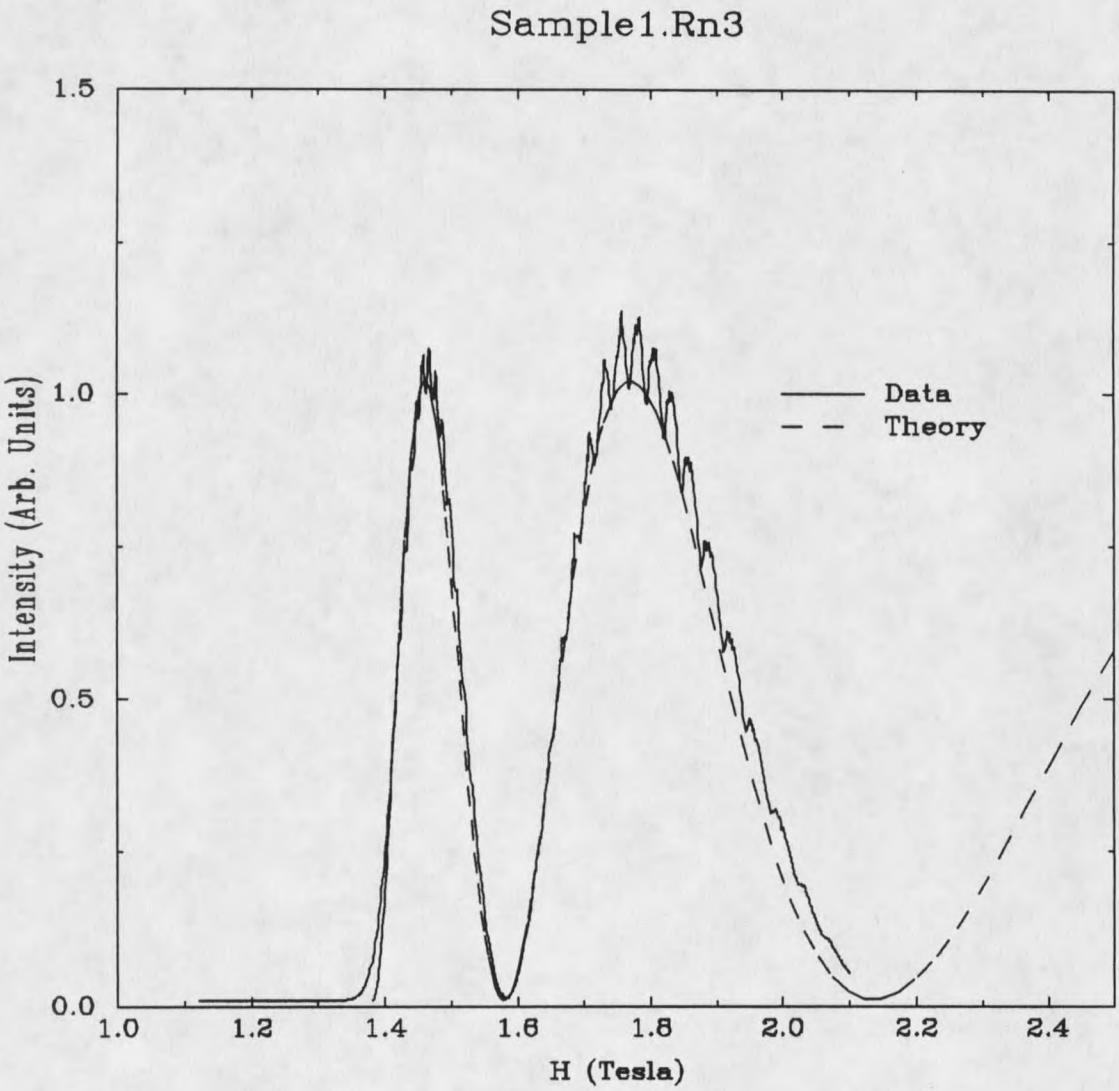

Figure 7 Freedericksz bend transition in magnetic field.

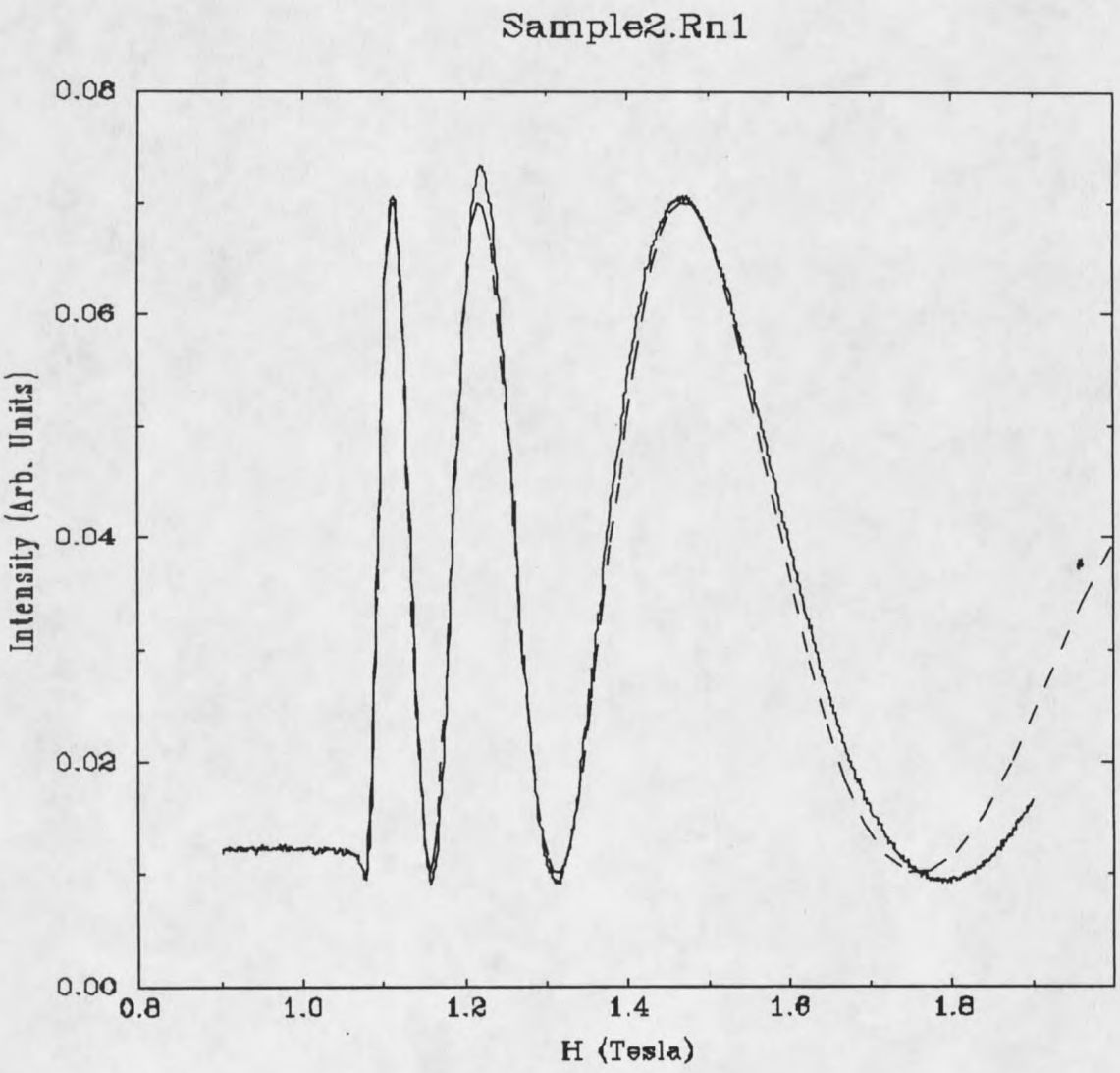

Figure 8 Freedericksz bend transition in magnetic field.

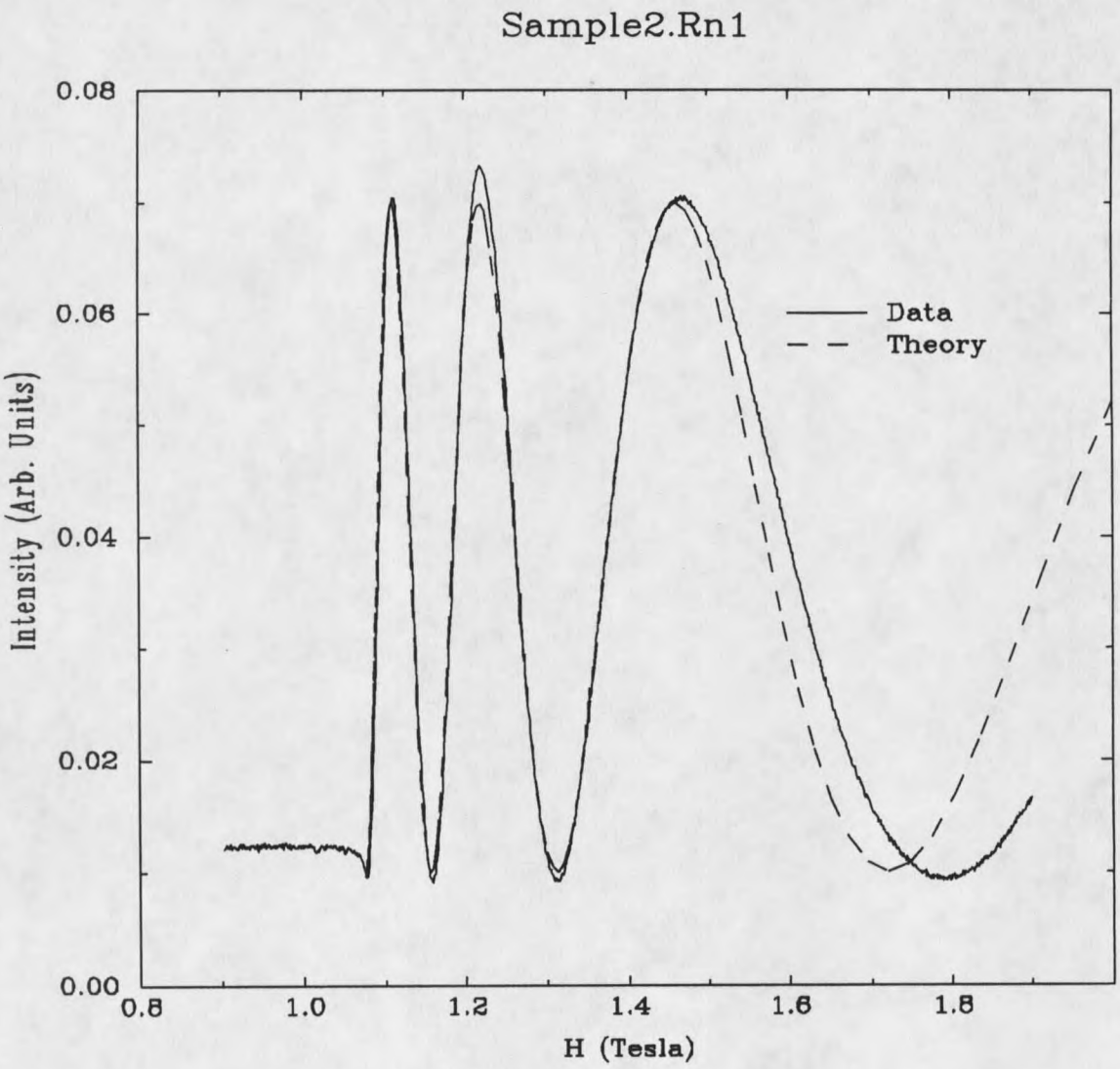

Figure 9 Freedericksz bend transition in magnetic field.

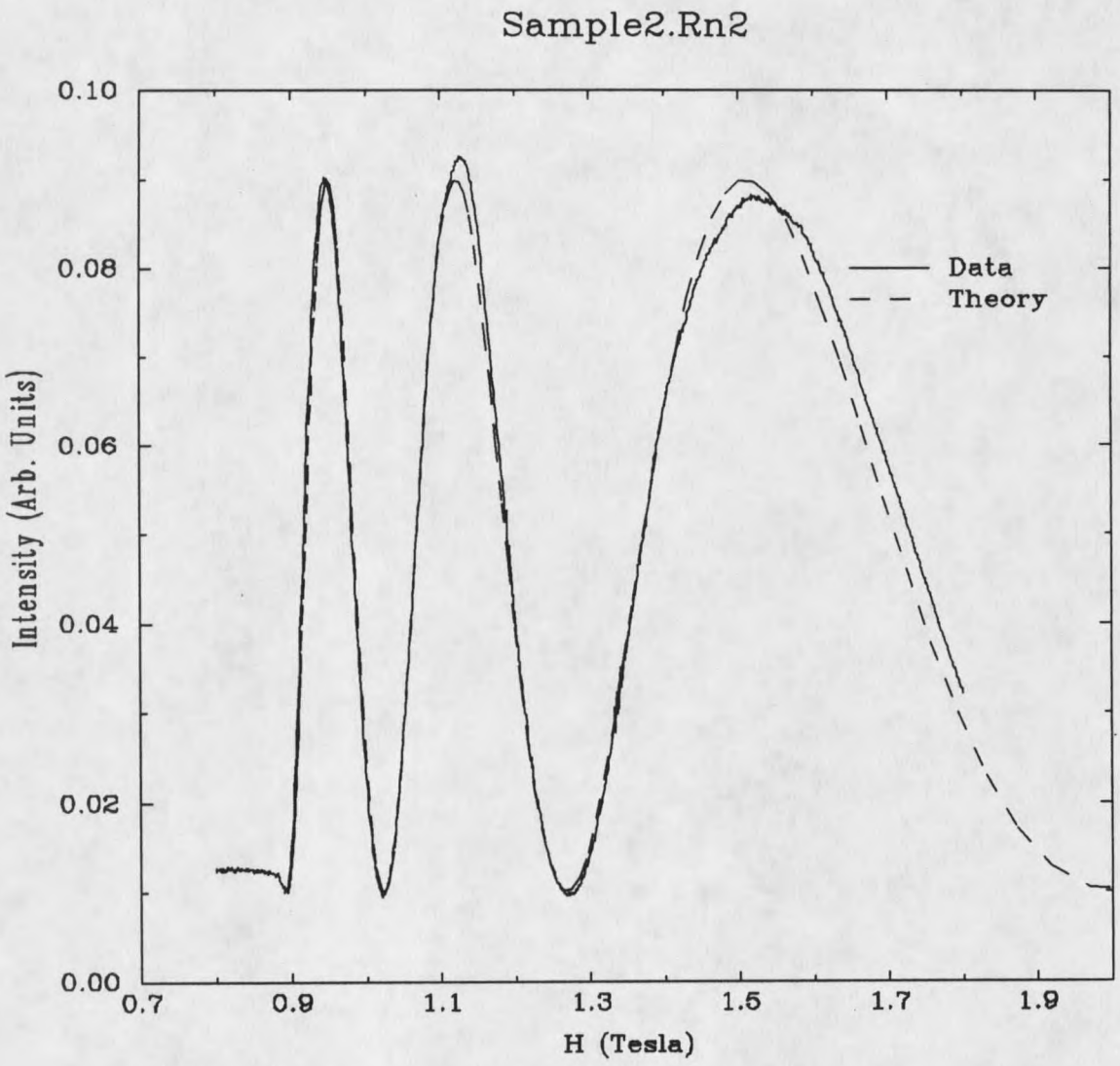

Figure 10 Freedericksz bend transition in magnetic field.

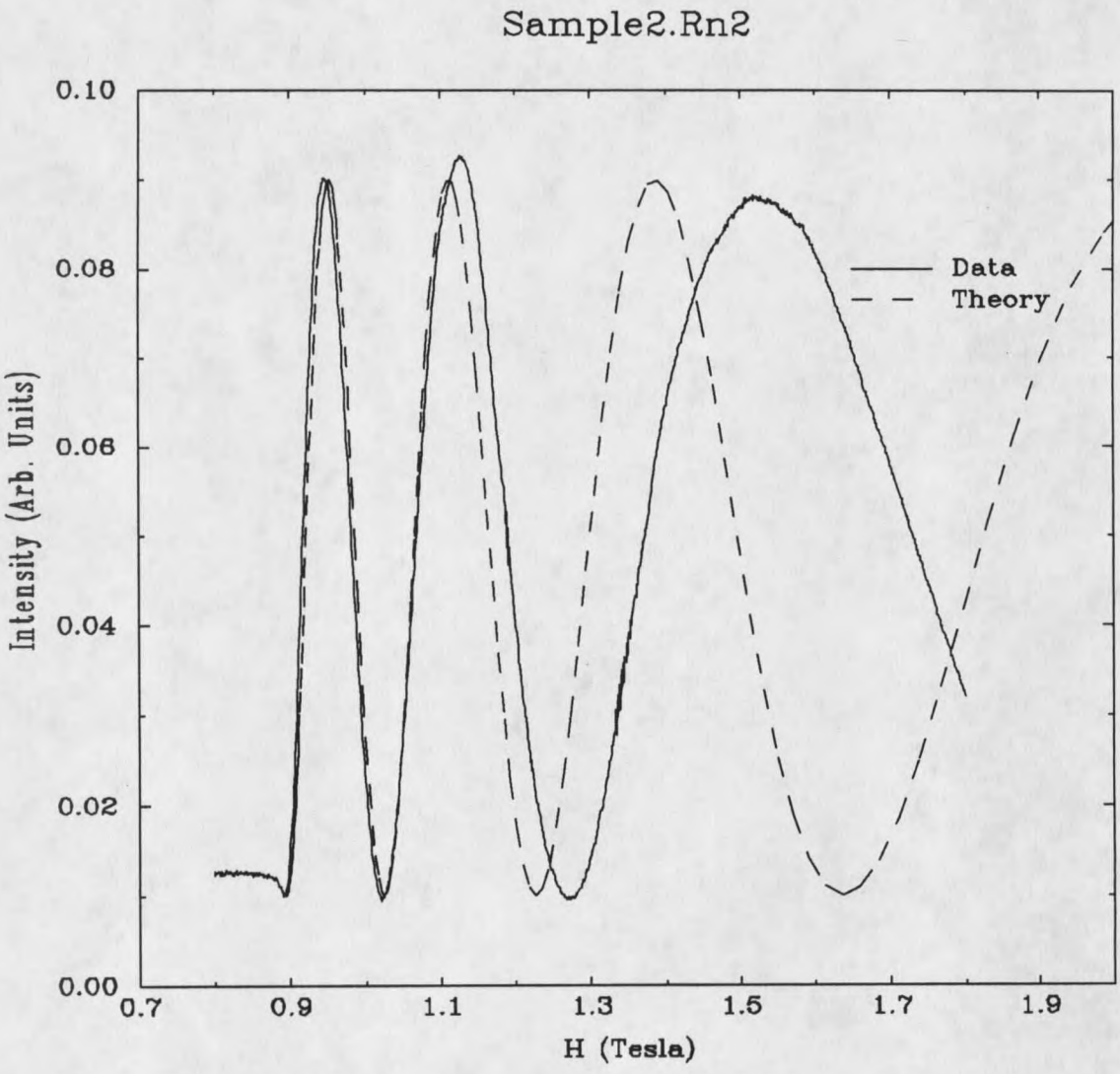

Figure 11 Freedericksz bend transition in magnetic field.
Sample fit at 14μm.

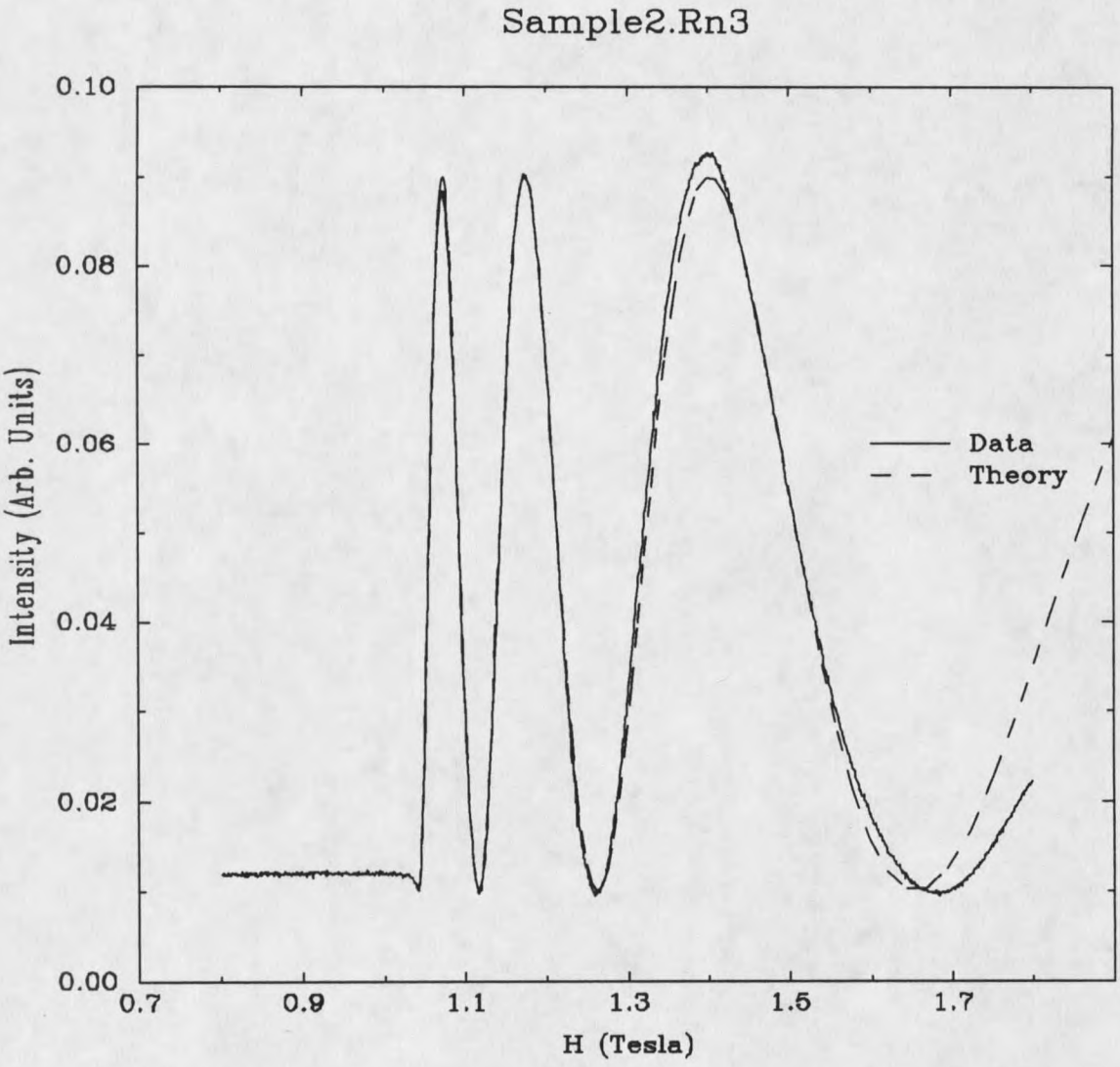

Figure 12 Freedericksz bend transition in magnetic field.

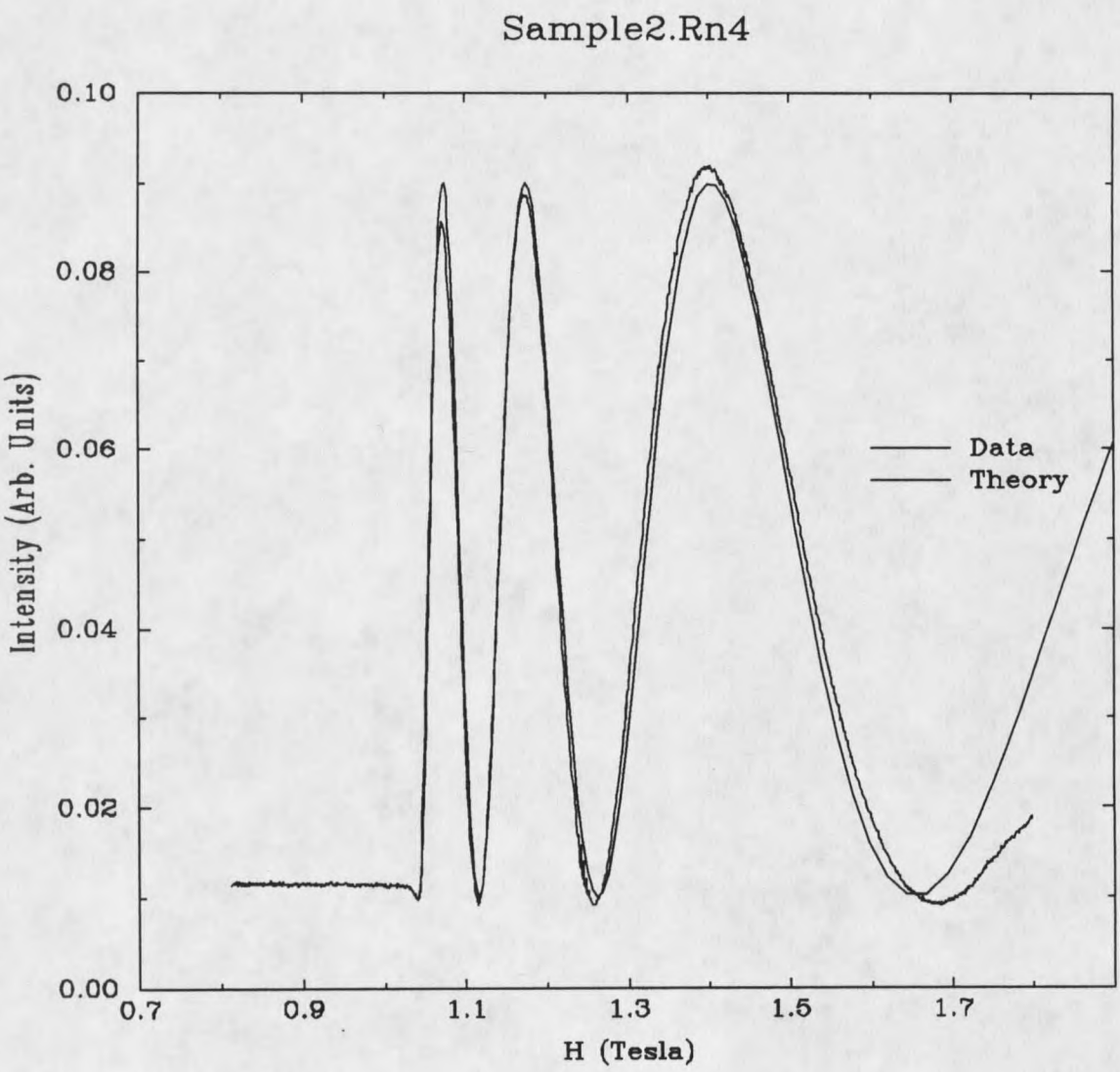

Figure 13 Freedericksz bend transition in magnetic field.

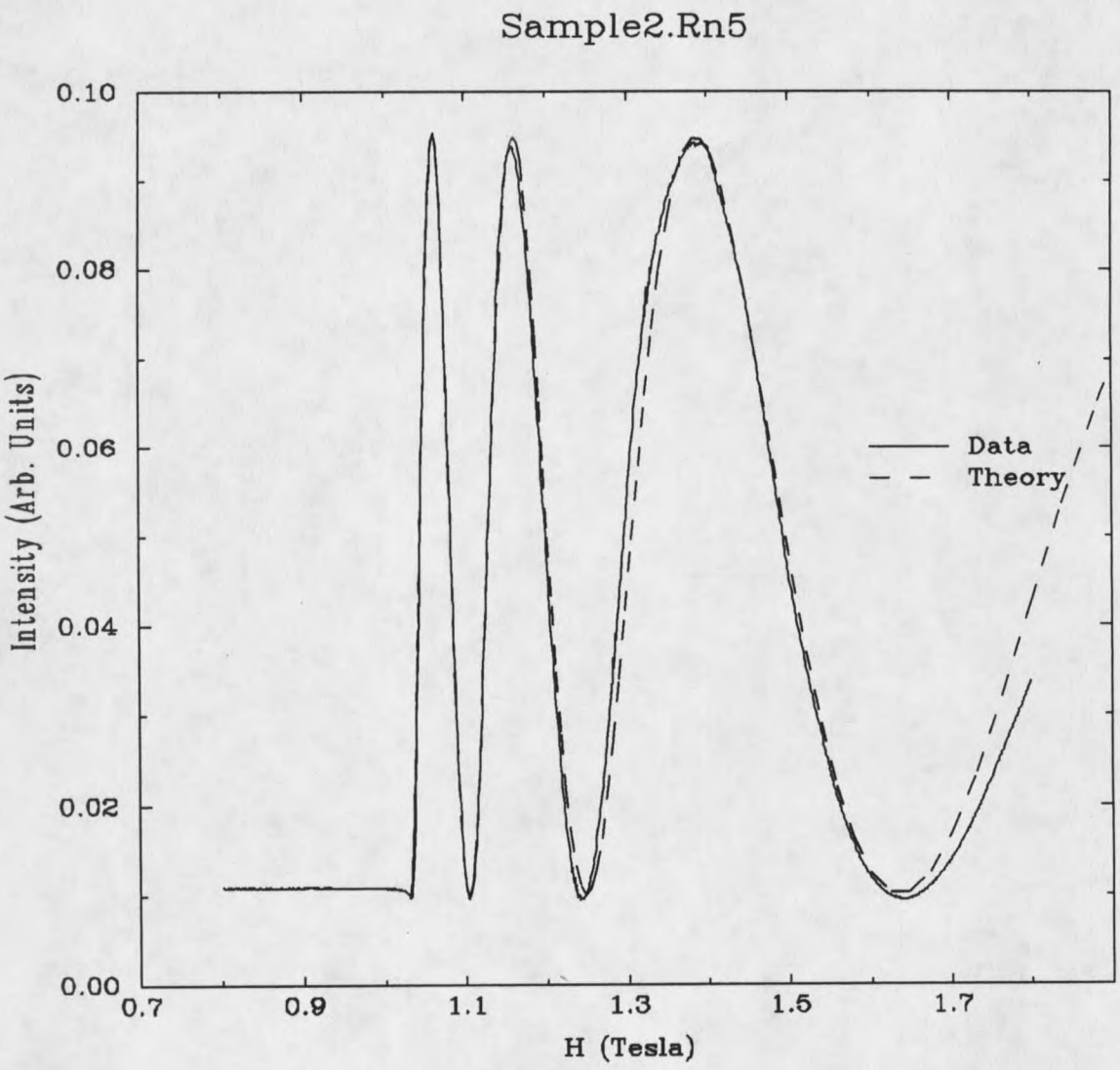

Figure 14 Freedericksz bend transition in magnetic field.

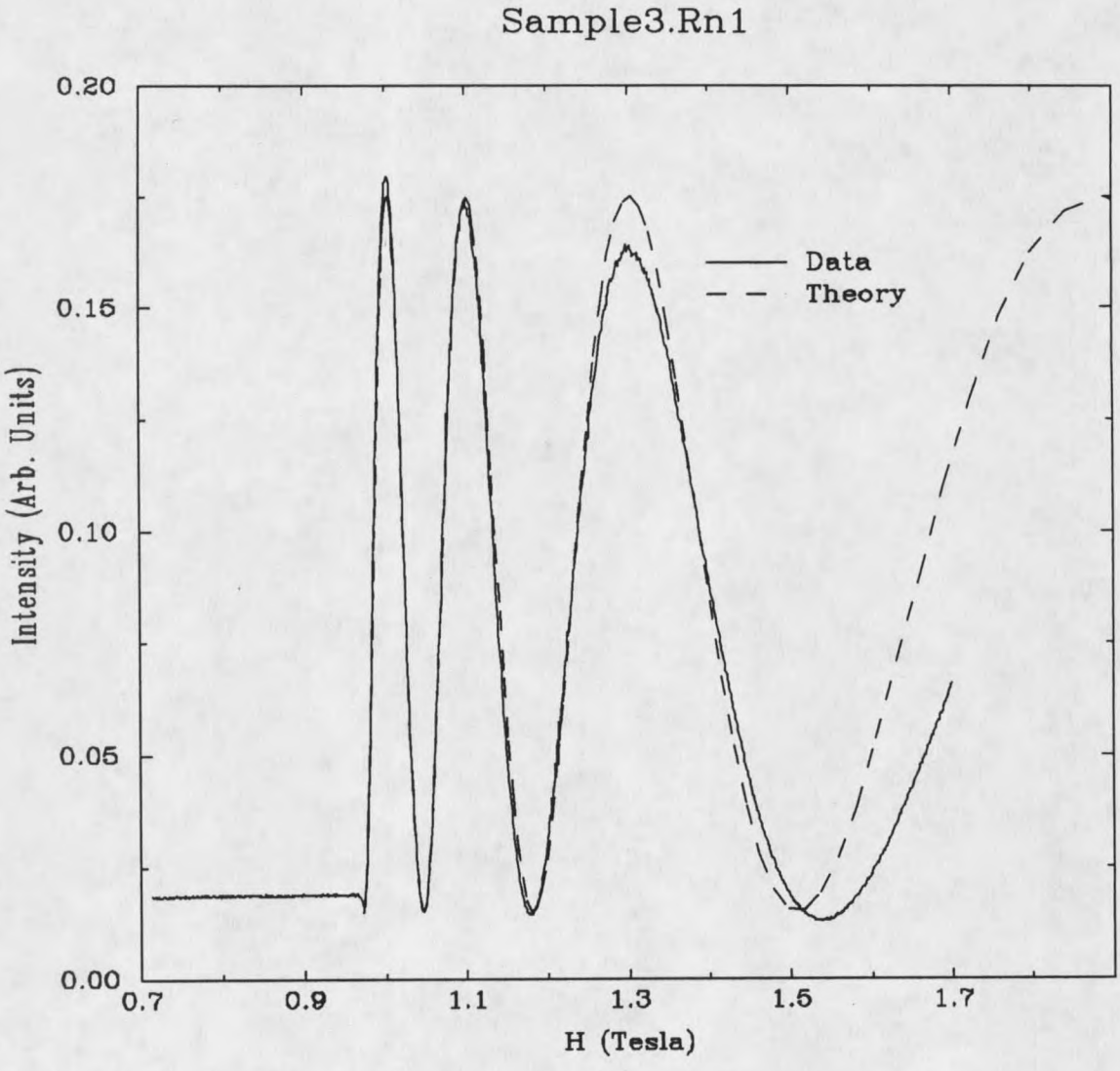

Figure 15 Freedericksz bend transition in magnetic field.

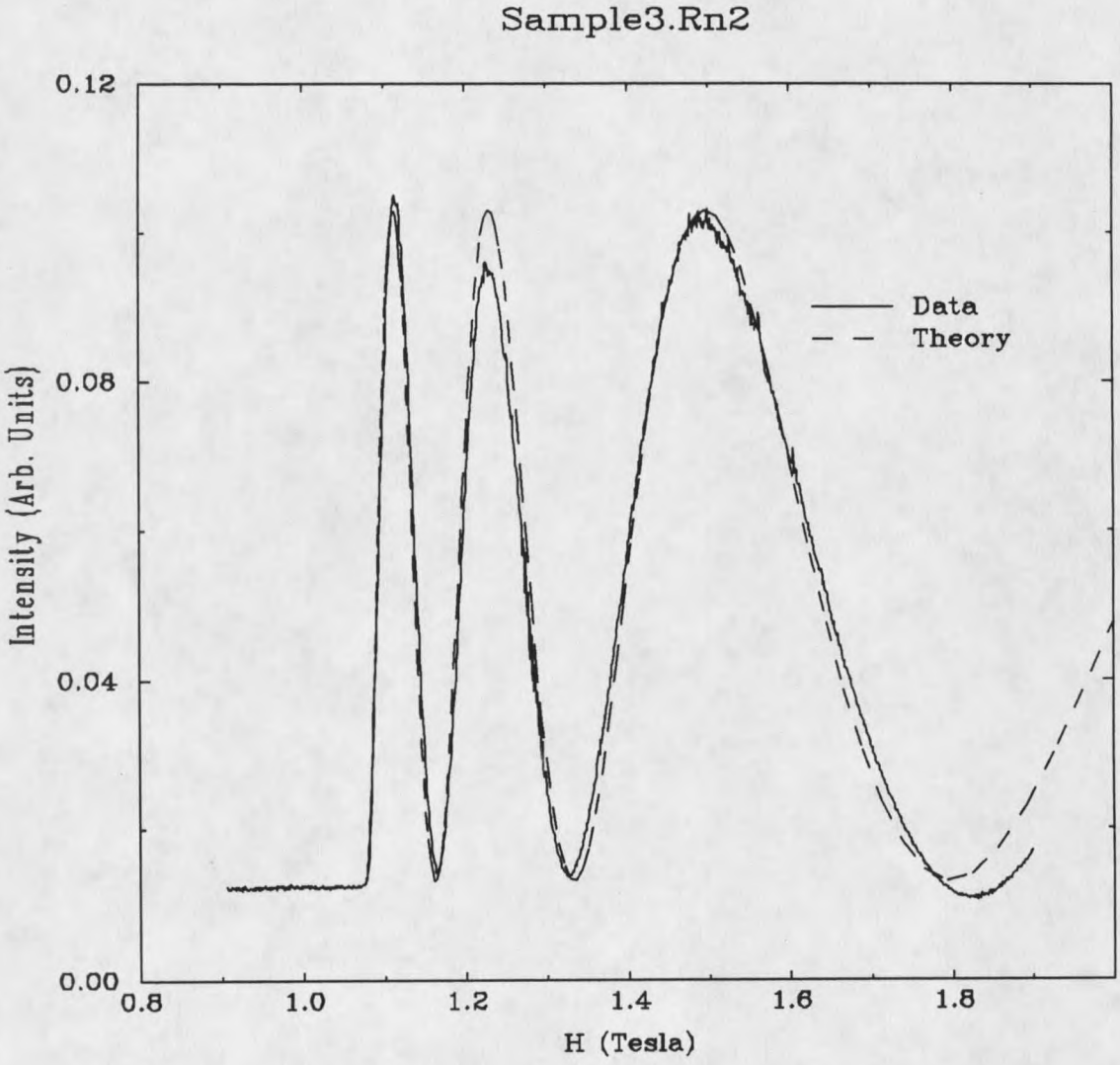

Figure 16 Freedericksz bend transition in magnetic field.

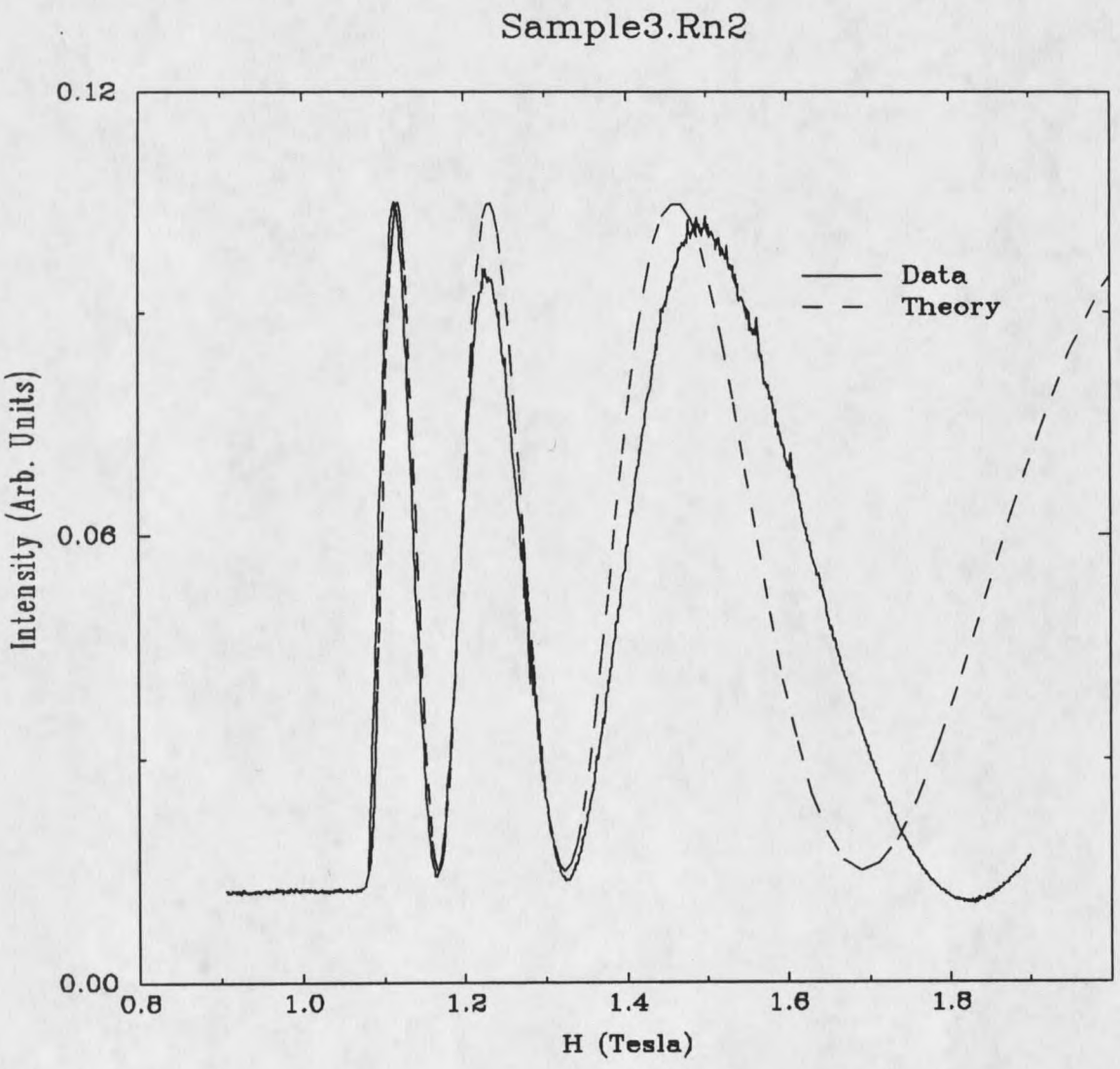

Figure 17 Freedericksz bend transition in magnetic field.
Sample fit at 14.5μm

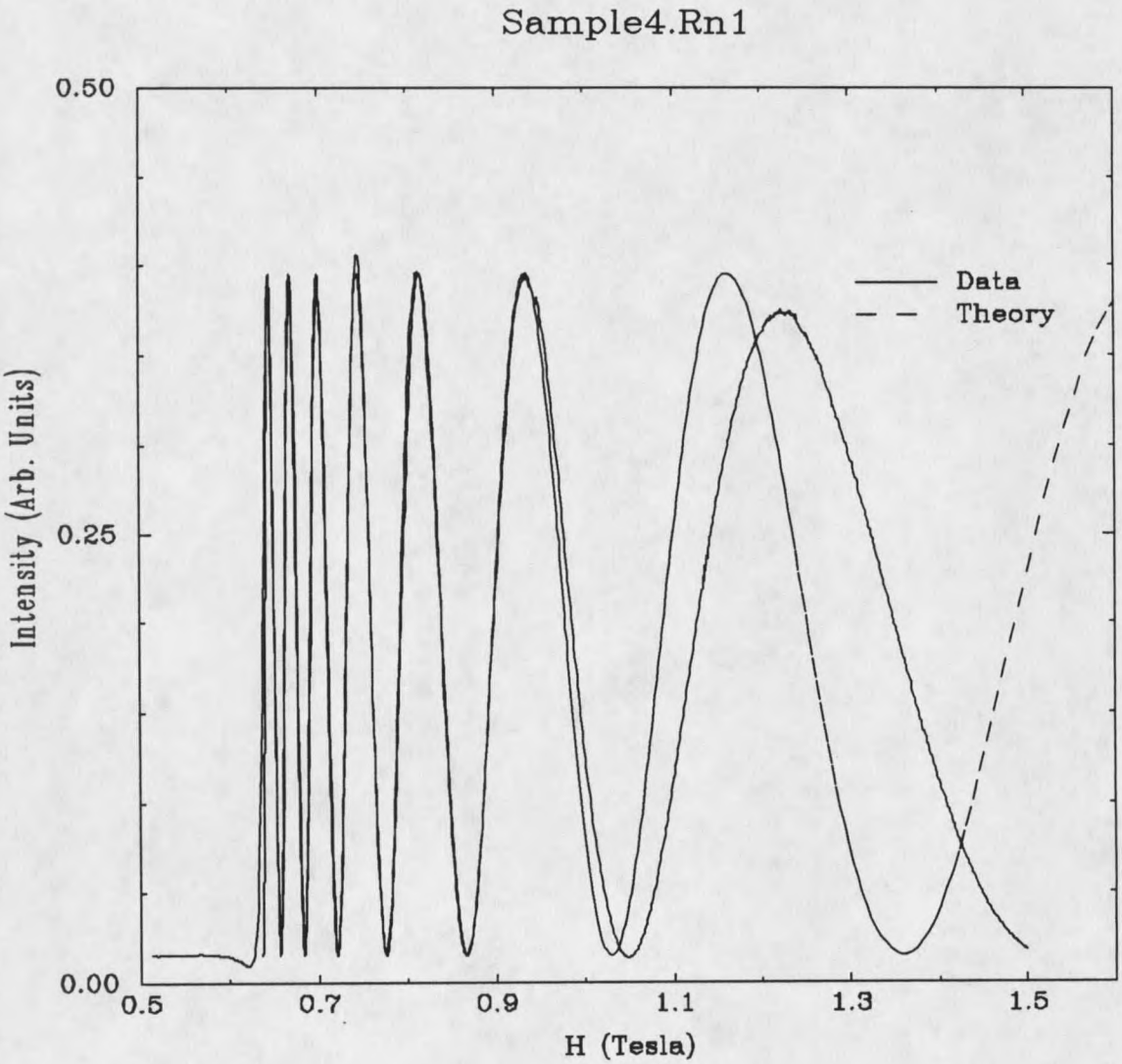

Figure 18 Freedericksz bend transition in magnetic field

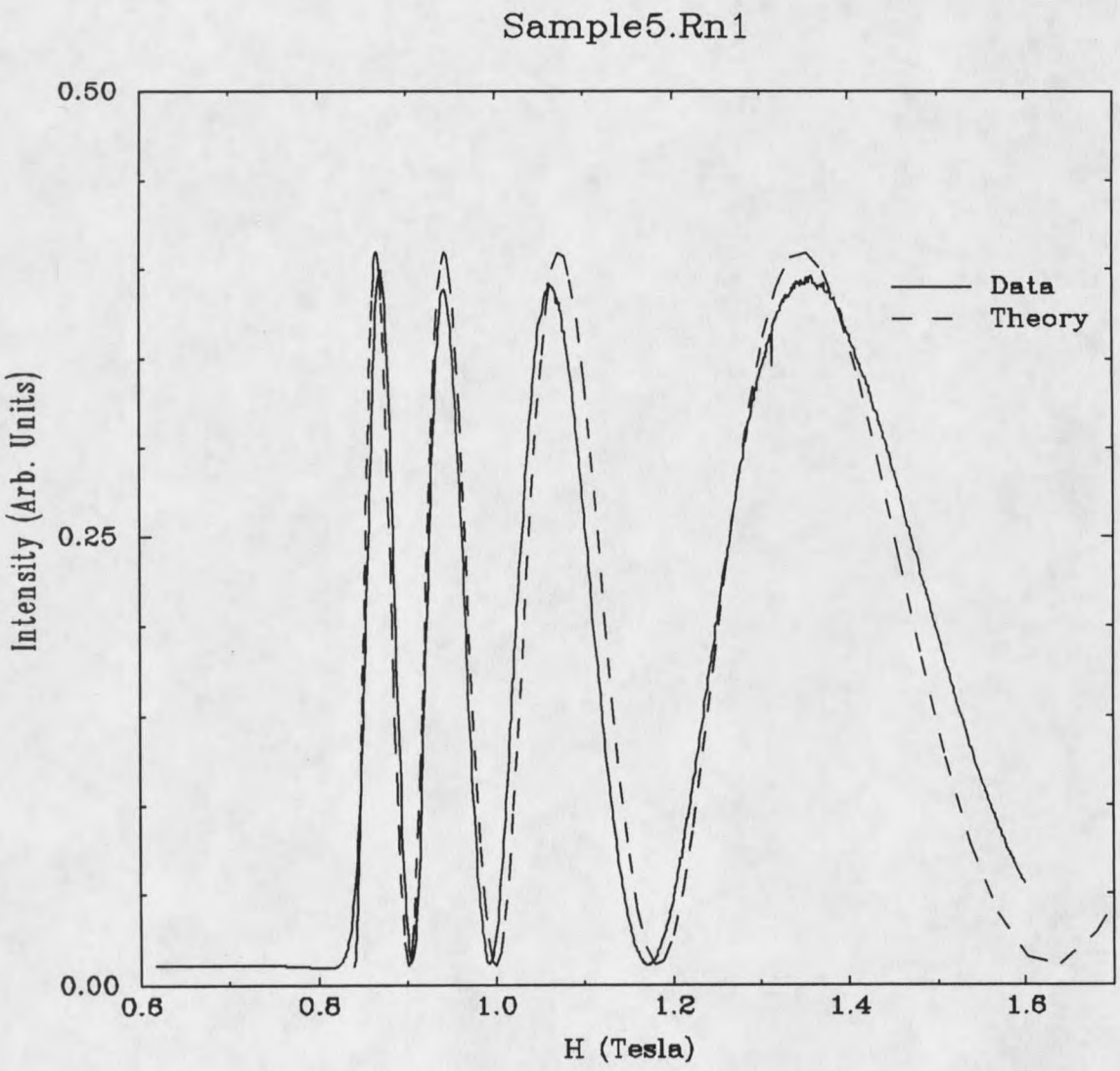

Figure 19 Freedericksz bend transition in magnetic field.

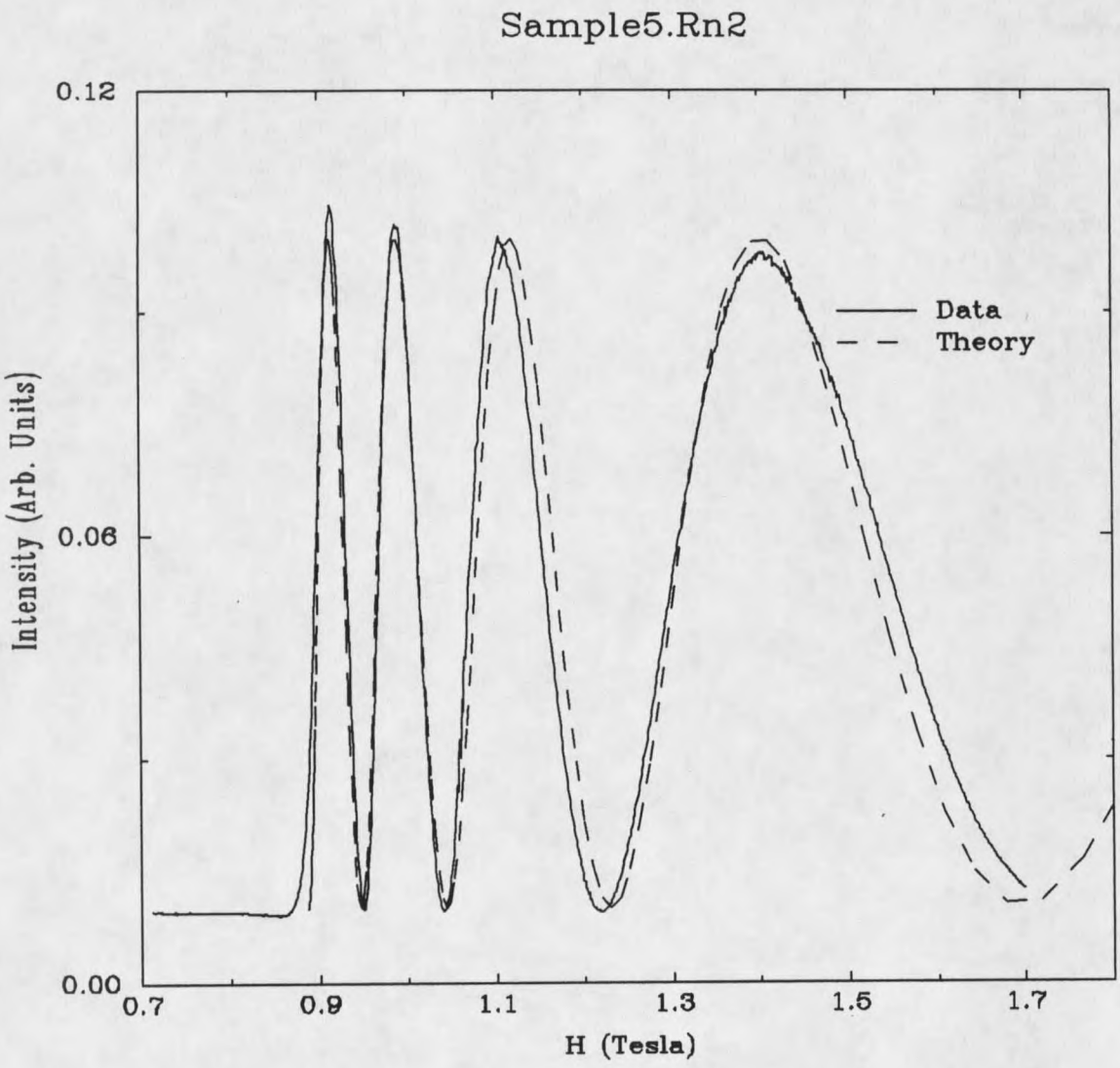

Figure 20 Freedericksz bend transition in magnetic field.

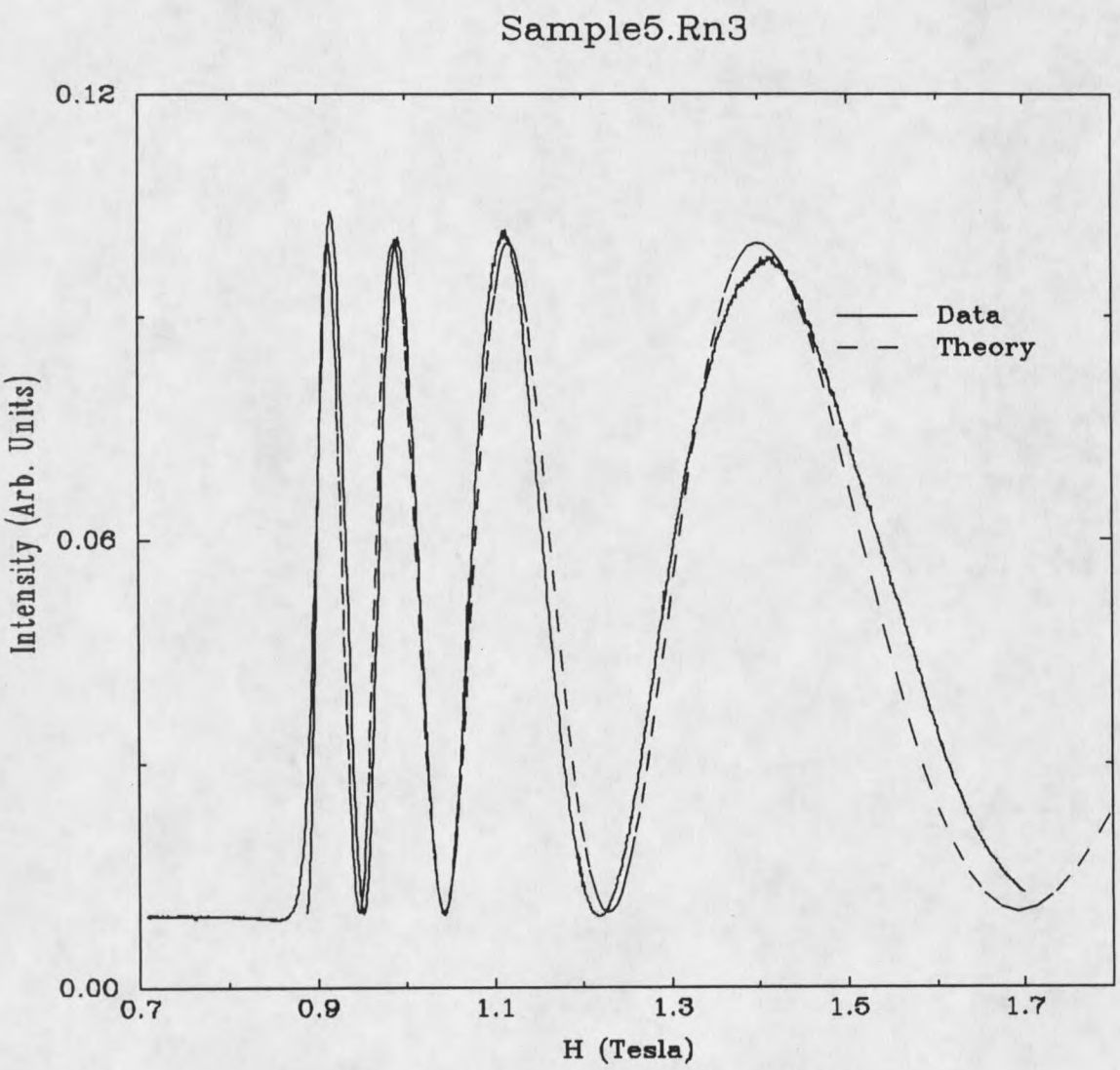

Figure 21 Freedericksz bend transition in magnetic field.

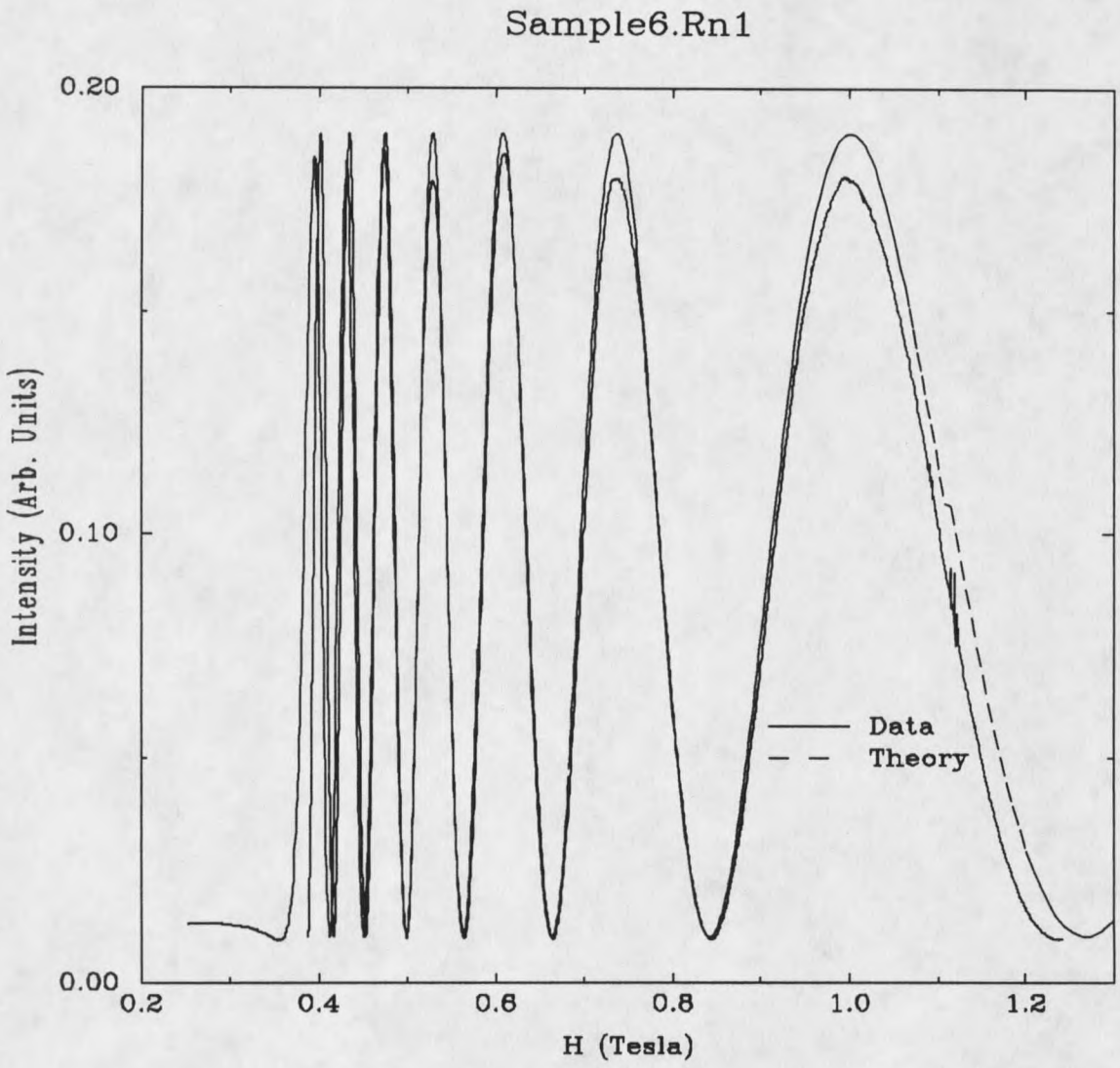

Figure 22 Freedericksz bend transition in magnetic field.

1.64×10^{-11} excluding sample 4. Theoretically K_{11} and K_{33} are constant and the data shows that there is little variation in the elastic constants between the thinnest sample at 9.9 μm and the thickest at 27.7 μm. The experimental results agree within 16% for the largest percent difference of K_{11} and K_{33} excluding sample 6 when looking at the largest deviation for K_{33} which differs by 51%.

Threshold Field Comparison

The Landau expansion of the free energy can be used to compute a value for the threshold magnetic field.[8] The expression is given as

$$H_{th} = \frac{\pi}{d}\left(\frac{\mu_o K_{33}}{\Delta\chi}\right)^{\frac{1}{2}} \tag{3.1}$$

where $\mu_o = 4\pi \times 10^{-7}$ Newton/Ampere and d is the thickness of the sample. From equation 3.1 the values in table 3 can be calculated and compared to the measured values. As can be seen these threshold values are very close together with small percent differences. This says that the two theories compare well as they should. The error could be due in part to the starting angle of θ_m=0.01 instead of zero in the computer code.

Data	Measured thickness(μm)	Theoretical thickness(μm)	Percent difference
Sample1.3	11	9.9	10%
Sample2.1	9.5	13.7	44%
Sample2.2	9.5	12	26%
Sample2.3	9.5	14	47%
Sample2.4	9.5	14	47%
Sample2.5	9.5	14	47%
Sample3.1	17.8	14.5	18.5%
Sample3.2	17.8	13.6	24%
Sample4.1	15	27.7	85%
Sample5.1	12.7	16.8	32%
Sample5.2	12.7	16.8	32%
Sample5.3	12.7	16.8	32%
Sample6.1	27	25	7.4%

Table 1 Gap Thickness Results

Data	$\Delta\chi$	Thick(μm)	K_{11}(N)	K_{33}(N)
Sample1.3	1.05xE-6	9.9	9xE-12	1.6xE-11
Sample2.1	1.05xE-6	13.7	8xE-12	1.86xE-11
Sample2.2	1.05xE-6	12	7.7xE-12	9.84xE-12
Sample2.3	1.05xE-6	14	8xE-12	1.82xE-11
Sample2.4	1.05xE-6	14	8xE-12	1.82xE-11
Sample2.5	1.05xE-6	14	7.8xE-12	1.78xE-11
Sample3.1	1.05xE-6	14.5	8.4xE-12	1.7xE-11
Sample3.2	1.05xE-6	13.6	8.8xE-12	1.83xE-11
Sample4.1	1.05xE-6	27.7	1.18xE-11	2.64xE-11
Sample5.1	1.05xE-6	16.8	1.0xE-11	1.7xE-11
sample5.2	1.05xE-6	16.8	1.0xE-11	1.9xE-11
Sample5.3	1.05xE-6	16.8	1.0xE-11	1.9xE-11
Sample6.1	1.05xE-6	25	7.99xE-12	8.0xE-12

Table 2 Values of the parameters fit in the theoretical model.

Data	Measured H_{th}(Tesla)	Theoretical H_{th}(Tesla)	Percent differences
Sample1.3	1.335	1.387	3.8%
Sample2.1	1.078	1.082	.4%
Sample2.2	0.894	0.898	.4%
Sample2.3	1.041	1.047	.6%
Sample2.4	1.041	1.047	.6%
Sample2.5	1.033	1.036	.3%
Sample3.1	0.973	0.977	.4%
Sample3.2	1.075	1.081	.6%
Sample4.1	0.622	0.638	2.6%
Sample5.1	0.815	0.843	3.4%
Sample5.2	0.859	0.892	3.8%
Sample5.3	0.857	0.892	4.1%
Sample6.1	0.358	0.389	8.7%

Table 3 Threshold Field Data

CHAPTER 6

CONCLUSION

In conclusion, the average values of K_{11}, and K_{33} were found to be 8.64×10^{-12} and 1.64×10^{-11} Newtons respectively which gives a ratio of 0.53 for K^{11}/K^{33}. One must remember in this discussion that $\Delta\chi$ was arbitrarily picked to have the value of 1.05×10^{-06}. This being off will affect the values of the parameters but it may not affect the ratios. That is something that needs to be looked into.

Another concern is the poor match of the theoretical and measured thicknesses. There are problems with the experimental technique used to measure the thickness and there is some error in the theory as previously discussed. The problems in the experimental technique are due to bad holder design in that micrometer adjustments ought to be used instead of springs. Also, there is a limit to how thin a gap can be measured. The smaller the gap the bigger the angle through which one must rotate the sample and at some point the interference spot becomes too distorted to see the fringes.

A future project should deal with inserting the liquid crystal in the isotropic phase, knowing the gap thickness accurately, and including a finite anchoring energy to the glass surface in the theoretical model. An electric field could be used instead of a magnetic field; then one could measure the saturation field more easily and the electric field susceptibility is a well known quantity. However since the electric susceptibility is so large dynamic effects occur at very small electric fields.

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REFERENCES

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APPENDICES

APPENDIX A

COMPUTER PROGRAM FOR DATA COLLECTION

Figure 23 Data Acquisition Program

```
10      CLEAR ,59000! : IBINIT1=59000! : IBINIT2=IBINIT1+3 : BLOAD
"bib.m",IBINIT1
20  CALL
IBINIT1(IBFIND,IBTRG,IBCLR,IBPCT,IBSIC,IBLOC,IBPPC,IBBNA,IBONL,IBRS
C,IBSRE,IBRSV,IBPAD,IBSAD,IBIST,IBDMA,IBEOS,IBTMO,IBEOT,IBRDF,IBW
RTF,IBTRAP)
30  CALL
IBINIT2(IBGTS,IBCAC,IBWAIT,IBPOKE,IBWRT,IBWRTA,IBCMD,IBCMDA,IBR
D,IBRDA,IBSTOP,IBRPP,IBRSP,IBDIAG,IBXTRC,IBRDI,IBWRTI,IBRDIA,IBWR
TIA,IBSTA%,IBERR%,IBCNT%)
40  INPUT" Enter file name :  ",H$
50 INPUT" FIELD CENTER(Tesla) :  ",FC
60 INPUT" SWEEP RANGE(Tesla) :  ",SW
70 INPUT"ARE YOU READY DO YOU THINK?  :  ",T$
80 IF T$="NO" THEN GOTO 70 ELSE
90  OPEN "o" , #1 , H$
100 DEV1$="hp34401"
110 DEV2$="keithly"
120 CALL IBFIND (DEV1$,VLT%)
130 CALL IBFIND (DEV2$,MAG%)
140 REM Telling HP to measure dc voltage and trigger
150 REM TELLING KEITHLY TO MEASURE DC VOLT ON AUTORANGE
160 A$= "meas:volt:dc? .5, 0.001"
170 REM ASSUMING C$(KEITHLEY) IS HOOKED TO MAGNET
180 F$="F0R0T0G1X"
190 CALL IBWRT(MAG%,F$)
200 REM LOOPS UNTIL SWEEP IS AT THIS VOLTAGE
210 IF VAL(C$) > 15 GOTO 390 ELSE
220 REM  AVERAGE VALUES BEFORE WRITING INTO FILE
230 SUMINTN=0 : SUMFLD=0
240 J=20
250 FOR I=1 TO J
260 CALL IBWRT(VLT%,A$)
270 B$= SPACE$(20)
280 C$= SPACE$(20)
290 CALL IBRD(VLT%,B$)
300 CALL IBRD(MAG%,C$)
310 REM AVERAGING 30 VALUES FOR OUTPUT DATA POINT
320 INTNSTY=VAL(B$) : FLD=VAL(C$)
330 SUMINTN= SUMINTN + INTNSTY : SUMFLD= SUMFLD + FLD
```

```
340 NEXT I
350 AVEINTN = SUMINTN/J : AVEFLD=(FC-SW/2)+ SW/15*SUMFLD/J
360 PRINT #1 , AVEFLD , AVEINTN
370 locate 1,1:print space$(79);:locate 1,1:PRINT AVEINTN , AVEFLD;
371  KEY OFF
372  XMIN=0:YMIN=0:XMAX=100:YMAX=100
373   SCREEN 2
374   WINDOW (XMIN,YMIN)-(XMAX,YMAX)
375   LINE (XMIN,YMIN)-(XMAX,YMIN)
376   LINE (XMIN,YMIN)-(XMIN,YMAX)
377 ' to set a data point given by ( x,y) do line 60
378 'pset (x,y)
380 GOTO 210
390 CLOSE 1
400 END
```

APPENDIX B
COMPUTER PROGRAM FOR NUMERICAL INTEGRATION

Figure 24 Theoretical Fitting Program

```
      REAL*8 thmax, thick, k11, k33, intgl2, mag, intgl1
     $     , shift,Z,myunot,wlngth,ord,exord,intensity,i,j
      CHARACTER name*11
      EXTERNAL MIDPNT
      EXTERNAL MIDPNTA
      ord=1.522
      exord=1.746
      wlngth=632.8E-9
      myunot=12.56637E-7
      Z=0.0
      PRINT *, 'NAME OF THE OUTPUT FILE.  11 CHARACTERS
      MAXIMUM'
      READ *, name
      PRINT *, 'intensity matching(0 to 1) Default=1'
      READ *, i
      PRINT *, 'DC offset of intensity'
      READ *, j
      PRINT *, 'Thickness of the sample is'
      READ *, thick
      PRINT *, 'k11 is  '
      READ *,k11
      PRINT *, 'k33 is '
      READ *,k33
      PRINT *, 'chi is '
      READ *,chi

      OPEN(ACCESS='SEQUENTIAL', UNIT=15,STATUS='NEW',
      FILE=name)

      DO 10 , thmax=.001, 1.57, .001
            CALL QROMO(Z,thmax,intgl1,MIDPNT,k11,k33)
                  mag=intgl1*2*SQRT(myunot*k33/chi)/thick
            CALL QROMOA(Z,thmax,intgl2,MIDPNTA,k11,k33)
                  shift=(1/mag)*(12.566*ord/wlngth)*
     $                        SQRT(myunot*k33/chi)*intgl2
        intensity= i*( 1-COS(shift))/2 +j
        WRITE(15,25) mag,intensity
25      FORMAT(F10.6,3X,F12.7)
10      CONTINUE
```

```
      CLOSE(15)
      END



      SUBROUTINE QROMO(A,B,SS,MIDPNT,k11,k33)
      REAL*8  A,B,SS,k11,k33,S,H,DSS,Z
      EXTERNAL MIDPNT
      PARAMETER (EPS=4.E-7,JMAX=14,JMAXP=JMAX+1,KM=4,K=KM+1)
      DIMENSION S(JMAXP),H(JMAXP)
      Z=0.0
      H(1)=1.
      DO 11 J=1,JMAX
        CALL MIDPNT(A,B,S(J),J,k11,k33)
        IF (J.GE.K) THEN
          CALL POLINT(H(J-KM),S(J-KM),K,Z,SS,DSS)
          IF (ABS(DSS).LT.EPS*ABS(SS)) RETURN
        ENDIF
        S(J+1)=S(J)
        H(J+1)=H(J)/9.
11    CONTINUE
      PAUSE 'Too many steps in qromo.'
      END
c *************************************************
      SUBROUTINE QROMOA(C,D,SS,MIDPNTA,k11,k33)
      REAL*8 C,D,SS,k11,k33,S,H,DSS,Z
      EXTERNAL MIDPNTA
      PARAMETER (EPS=4.E-7,JMAX=14,JMAXP=JMAX+1,KM=4,K=KM+1)
      DIMENSION S(JMAXP),H(JMAXP)
      Z=0.0
      H(1)=1.
      DO 11 J=1,JMAX
        CALL MIDPNTA(C,D,S(J),J,k11,k33)
        IF (J.GE.K) THEN
          CALL POLINT(H(J-KM),S(J-KM),K,Z,SS,DSS)
          IF (ABS(DSS).LT.EPS*ABS(SS)) RETURN
        ENDIF
        S(J+1)=S(J)
        H(J+1)=H(J)/9.
11    CONTINUE
      PAUSE 'Too many steps in qromoa.'
      END
```

```
      SUBROUTINE MIDPNT(A,B,S,N,k11,k33)
      REAL*8 A,B,S,k11,k33,X,SUM,DEL,DDEL
      INTEGER IT
      SAVE IT
           IF (N.EQ.1) THEN
        S=(B-A)* SQRT((1-(1-k11/k33)*SIN(0.5*(A+B))**2)/
     $             (SIN(B)**2 - SIN(0.5*(A+B))**2))
        IT=1
      ELSE
        TNM=IT
        DEL=(B-A)/(3.*TNM)
        DDEL=DEL+DEL
        X=A+0.5*DEL
        SUM=0.
        DO 11 J=1,IT
          SUM=SUM+ SQRT((1-(1-k11/k33)*SIN(X)**2)/
     $             (SIN(B)**2 - SIN(X)**2))
          X=X+DDEL
          SUM=SUM+ SQRT((1-(1-k11/k33)*SIN(X)**2)/
     $             (SIN(B)**2 - SIN(X)**2))
          X=X+DEL
11      CONTINUE
        S=(S+(B-A)*SUM/TNM)/3.
        IT=3*IT
      ENDIF
      RETURN
      END
C *************************************************************
      SUBROUTINE MIDPNTA(C,D,S,N,k11,k33)
      REAL*8 C,D,S,k11,k33,X,SUM,DEL,exord,ord,
     $       au,DDEL
      INTEGER IT
      SAVE IT
      exord=1.746
      ord=1.522
      IF (N.EQ.1) THEN
      au=   SQRT((ord**2)*SIN(0.5*(C+D))**2+   (exord**2)*
COS(0.5*(C+D))**2)
      S=(D-C)*((exord/au)-1)
     $
     $       *SQRT((1-(1-k11/k33)*SIN(0.5*(C+D))**2)/
```

```
$       (SIN(D)**2 - SIN(0.5*(C+D))**2))
        IT=1
       ELSE
        TNM=IT
        DEL=(D-C)/(3.*TNM)
        DDEL=DEL+DEL
        X=C+0.5*DEL
        SUM=0.
        DO 11 J=1,IT
          SUM=SUM+((exord/SQRT((ord**2)*SIN(X)**2+
$          (exord**2)*COS(X)**2))-1)
$          *SQRT((1-(1-k11/k33)*SIN(X)**2)/
$           (SIN(D)**2 - SIN(X)**2))
          X=X+DDEL
          SUM=SUM+((exord/SQRT((ord**2)*SIN(X)**2+
$          (exord**2)*COS(X)**2))-1)
$          *SQRT((1-(1-k11/k33)*SIN(X)**2)/
$           (SIN(D)**2 - SIN(X)**2))
          X=X+DEL
11       CONTINUE
        S=(S+(D-C)*SUM/TNM)/3.
        IT=3*IT
       ENDIF
       RETURN
       END
```

```
      SUBROUTINE POLINT(XA,YA,N,X,Y,DY)
      REAL*8 XA,YA,X,Y,DY,HO,HP,C,D,DEN,W,DIF,DIFT
      PARAMETER (NMAX=10)
      DIMENSION XA(N),YA(N),C(NMAX),D(NMAX)
      NS=1
      DIF=ABS(X-XA(1))
      DO 11 I=1,N
        DIFT=ABS(X-XA(I))
        IF (DIFT.LT.DIF) THEN
          NS=I
          DIF=DIFT
        ENDIF
        C(I)=YA(I)
        D(I)=YA(I)
11    CONTINUE
      Y=YA(NS)
      NS=NS-1
      DO 13 M=1,N-1
        DO 12 I=1,N-M
          HO=XA(I)-X
          HP=XA(I+M)-X
          W=C(I+1)-D(I)
          DEN=HO-HP
          IF(DEN.EQ.0.)PAUSE
          DEN=W/DEN
          D(I)=HP*DEN
          C(I)=HO*DEN
12      CONTINUE
        IF (2*NS.LT.N-M)THEN
          DY=C(NS+1)
        ELSE
          DY=D(NS)
          NS=NS-1
        ENDIF
        Y=Y+DY
13    CONTINUE
      RETURN
      END
```