



Content analysis of computer-mediated collaborative mathematical problem solving  
by Julia Chiyo Myers

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Education  
in Education

Montana State University

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Abstract:

This study provides a preliminary look into the process of mathematical problem solving within small groups in an online environment, and distinguishes the behaviors of individuals and groups involved in effective collaborative problem solving via computer-mediated communication.

The subjects of this study included four groups of three middle school teachers enrolled in a mathematical content course in which the primary format was group problem solving with communications mediated by computer. Data was collected by observing and performing a content analysis on transcripts of online problem-solving protocols, and coding messages along four dimensions - (a) participative, (b) social, (c) interactive, and (d) heuristic episode - using a framework developed by the researcher. Problem-solving success was measured by group achievement on the assigned problem-solving task as determined by an analytical rubric.

Several important conclusions emerged concerning computer-mediated collaborative problem solving. The primary observations related to the ways in which groups communicated and participated. The data indicated that the most effective group not only participated more, but also socialized less, interacted more, with more task-specific content and more interactions related to solving the problem. Additionally, this group collaborated throughout the problem-solving process in a recursive nature, working to understand the problem, develop a plan, execute that plan, and verify the results, and revisiting each of these heuristic stages. This study also illuminated several issues related to the use of technology for collaborative problem solving. Evidence pointed to the choice between asynchronous and synchronous modes for communication as the primary determinant in the amount of participation and the amount and type of interactivity amongst members. While asynchronous messages tended to contain more lengthy messages with multiple units of meaning, the messages were few, and the communication between group members was highly non-interactive and appeared to do little to advance group problem-solving efforts. In contrast, the synchronous mode was much more interactive in nature and appeared to promote much greater collaboration.

In conclusion the author includes several suggestions for promoting effective collaboration and successful problem-solving groups; they include communication requirements and guidelines, and considerations for group composition and task characteristics. Additionally, the author suggests several ideas for future research.

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MATHEMATICAL PROBLEM SOLVING

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MONTANA STATE UNIVERSITY  
Bozeman, Montana

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APPROVAL

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This dissertation has been read by each member of the dissertation committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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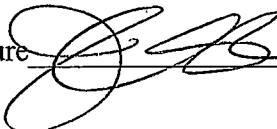
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This is dedicated to my two amazing sons, Spencer, 3, and Weston, 1, who patiently spent many long hours at my feet or in my lap, and passed the time building huge block buildings, racing trains, and staging dinosaur fights, as I worked at the computer. Stay curious, follow your dreams, and know that anything is possible.

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## ABSTRACT

This study provides a preliminary look into the process of mathematical problem solving within small groups in an online environment, and distinguishes the behaviors of individuals and groups involved in effective collaborative problem solving via computer-mediated communication.

The subjects of this study included four groups of three middle school teachers enrolled in a mathematical content course in which the primary format was group problem solving with communications mediated by computer. Data was collected by observing and performing a content analysis on transcripts of online problem-solving protocols, and coding messages along four dimensions – (a) participative, (b) social, (c) interactive, and (d) heuristic episode – using a framework developed by the researcher. Problem-solving success was measured by group achievement on the assigned problem-solving task as determined by an analytical rubric.

Several important conclusions emerged concerning computer-mediated collaborative problem solving. The primary observations related to the ways in which groups communicated and participated. The data indicated that the most effective group not only participated more, but also socialized less, interacted more, with more task-specific content and more interactions related to solving the problem. Additionally, this group collaborated throughout the problem-solving process in a recursive nature, working to understand the problem, develop a plan, execute that plan, and verify the results, and revisiting each of these heuristic stages. This study also illuminated several issues related to the use of technology for collaborative problem solving. Evidence pointed to the choice between asynchronous and synchronous modes for communication as the primary determinant in the amount of participation and the amount and type of interactivity amongst members. While asynchronous messages tended to contain more lengthy messages with multiple units of meaning, the messages were few, and the communication between group members was highly non-interactive and appeared to do little to advance group problem-solving efforts. In contrast, the synchronous mode was much more interactive in nature and appeared to promote much greater collaboration.

In conclusion the author includes several suggestions for promoting effective collaboration and successful problem-solving groups; they include communication requirements and guidelines, and considerations for group composition and task characteristics. Additionally, the author suggests several ideas for future research.

## CHAPTER I

## PROBLEM STATEMENT AND REVIEW OF THE LITERATURE

Introduction

Though the nature of educational activities and discourse distinguishes a variety of learning environments, perhaps no form of education is so significantly defined by its choice of communication medium, as is distance learning. As the principal link between students and their instructors, communication technologies play a central role in shaping the practice and character of distance education. While past advancements in communication have affected the delivery of course materials, nothing has so dramatically altered the relationships between students, teachers, and the activities that take place in distance learning as the advent of online technologies (Kahle, 2000). The traditional model of distance learning was a one-way approach that placed emphasis on independent study, supported by well-developed learning materials (Verdejo & Cerri, 1994). The isolation of this model excluded learners from valuable learning activities such as information sharing, interaction and collaboration. In a discipline such as mathematics where conceptual knowledge and mathematical know-how are often difficult to acquire from textbook alone, this isolation was quite detrimental. Fortunately, the rapidly increasing availability and use of online technologies such as computer-mediated communication have provided a solution to this problem. Computer-mediated communication as defined by Ebbelink (1999), refers to the exchange of information, including text, graphics, images, audio and video, between persons by way of computer

networks. The exchange of information can be synchronous (real-time), or asynchronous (people communicating at different times), and includes, among many others, systems such as e-mail, bulletin boards, and computer conferencing. Computer-mediated communication facilitates the interaction between spatially separated learners so that students in remote places can potentially benefit as if they were in a classroom with peers. It offers new facilities and opportunities to improve the collaborative aspects of learning without reducing individual flexibility (Kaye, 1992). As a result of this technology, valuable learning activities such as collaborative problem solving have seen growing use and attention in teaching mathematics at a distance.

While extensive research has shown that collaborative learning and problem solving are effective means for increasing student knowledge in mathematics, and developing key problem-solving skills such as metacognition and cognition (Curcio & Artzt, 1998; Schoenfeld, 1987), research on *online* collaborative activities is still relatively limited. Mason (1992), in a review of research and evaluation methodologies for computer-conferencing applications, found that a majority of the literature consists of descriptions of applications, and that most existing studies stop with quantitative analyses of things such as who sent messages, number and time of logons, and message maps showing the links between messages and the number of replies. In addition, even fewer studies have investigated the nature of learning *mathematics* collaboratively online. Unfortunately, the ability to communicate via computer does not guarantee a worthwhile learning experience or even collaboration. In fact, theories of what constitutes true collaboration and quality work in this medium still remain uncommon (Newman, Webb,

& Cochrane, 1995) and must be considered. For this reason, current research efforts are beginning to focus on the analysis of online conference content, specifically to identify characteristics of quality learning experiences in areas such as collaborative mathematical problem solving. This study contributes to the critical and still small body of research on this topic.

### Statement of Purpose

This study provides a preliminary look into the process of mathematical problem solving within small groups in an online environment, with the goal of distinguishing the behaviors of individuals and groups involved in effective collaborative problem solving via computer-mediated communication. Specifically, the purpose of this study is to develop and utilize a framework for coding the participative, social, and interactive behaviors of students in these groups throughout different stages of the problem-solving process, and to analyze, characterize, and describe the impact of these behaviors on the quality of the collaborative learning experience and the success in problem solving.

The aim of the study may be succinctly expressed through three main questions:

1. What behaviors do small groups, using computer-mediated communication, exhibit in terms of participation, and interaction – social and otherwise – while involved in collaborative mathematical problem solving?
2. Is there evidence of characteristic online interactive behaviors that enhance or diminish the quality of collaboration within a group?

3. Is there evidence of characteristic structures in the groups' collaborative behavior that lead to problem-solving success or problem-solving failure?

### Significance of the Study

As we enter the 21<sup>st</sup> century, the use of online technologies in education continues to grow rapidly. The Internet has quickly changed from a convenient tool for gathering resources to an opportune medium for delivering courses to countless students in distant places. Computer-mediated communication has provided an ideal solution for overcoming time and space gaps in distance education (Kaye, 1992). Along with the benefits it provides, however, computer-mediated communication changes the nature of learning activities. In fact, research has revealed that face-to-face communication and interactions differ significantly from asynchronous and even synchronous computer-mediated communication and interactions (Levin, Kim, & Riel, 1990). This evidence points to potentially critical implications for activities in which communication and interaction are essential practices. For example, in education, the verbal communication in peer discussions is the key feature of collaborative learning strategies (Higgins, 1991). Within the new medium, communication is most often written. In addition, peer interaction in collaborative learning activities, which is naturally synchronous in the face-to-face mode, is oftentimes asynchronous with the use of computer-mediated communication. Synchronicity may be a critical feature of peer interaction and an important component in the developing theories of the social construction of knowledge as they pertain to collaborative learning. On the other hand, asynchronous

communication may produce exchanges of richer intellectual quality than those resulting from immediate face-to-face dialogue (Levinson, 1989), and may improve group problem solving (Higgins, 1991). Consequently, educators are finding it necessary to reevaluate theories of teaching and learning within this new environment.

In the realm of educational computer-mediated communication, most existing studies have merely investigated general issues of collaboration. Surface questions relating to online collaboration, such as amount or patterns of participation and participant satisfaction, have been successfully answered (Levin et al., 1990). Although important, these questions mask important aspects of what is happening in a computer-mediated communication environment. For example, message maps can reveal the existence of extensive and complex webs of interactivity within a conference, but have only a limited significance without information about the content of those messages and the perceptions and intentions of the users at the time the messages were sent (Kaye, 1992). Indeed, few studies have focused on the types and quality of interactions and the quality of the learning experience in online collaborative groups. Mason (1992) suggested expanding research to include educational goals such as critical thinking, deep understanding of content or broad awareness of issues, and sent out an appeal for evaluators "to take up the challenge of content analysis both as a key to increasing the professionalism of the field and as the essence of the educational value of the activity" (p. 115). Mathematics educators have much work to do in this arena. Very few studies have explored the process of computer-mediated collaborative learning in mathematics, with

even fewer focusing on the quality of collaborative mathematical problem solving in this medium.

A specific concern to mathematics educators is the impact of computer-mediated communication on collaborative problem solving in the online environment. Since the publication of the National Council of Teachers of Mathematics' (NCTM, 1989) *Curriculum and Evaluation Standards for School Mathematics*, mathematics educators have continued to move toward a more collaborative approach. Research on the effects of collaborative learning on achievement outcomes has largely been positive (Webb, 1991). In particular, the collaborative approach has proven to be quite effective for improving mathematical problem-solving ability. Still, while a large amount of literature exists on mathematical problem solving and collaborative mathematical problem solving, nearly all of the existing studies focus on this process in a face-to-face environment. With the inevitable continuing growth in the use of online courses, researchers must now focus on identifying the nature of any similarities or differences that the process in the online medium might have with that in the face-to-face environment in order to make the most effective use of distance technologies and to determine the best ways to support and facilitate effective collaborative problem solving within the changed environment. First, though, fundamental questions about the nature of computer-mediated collaboration for mathematical problem solving must be addressed. Central to these questions is the need to determine how students collaborate or even if they collaborate during online group mathematical problem-solving exercises. It is quite possible that students might come together only to delegate individual subtasks, then again only to compile results. This

type of behavior would not be considered collaboration. On the other hand, students may work jointly through each stage of problem solving – analyzing the problem together, planning, executing, and verifying the solution. While student behavior probably falls somewhere between these two extremes, it is important to determine where it falls and also the effect of this behavior on the success of group problem-solving efforts. Indeed, prior to this study, there were no studies that analyzed and classified the participative, social, and interactive behaviors occurring during computer-mediated collaborative mathematical problem solving, thereby bridging the gap between the existing knowledge of collaborative mathematical problem solving in the face-to-face mode and the newest findings on collaborative learning in the distance-learning realm.

In the context of these issues and problems, this thesis represents an effort to provide initial insight into the process of computer-mediated collaborative mathematical problem solving, and to address the fundamental issues regarding the definition of effective online collaboration, by furthering the understanding of individual and group behaviors occurring in small groups solving mathematics problems. Specifically, it examines the collaborative characteristics, in terms of participative, social and interactive behaviors of these students in relation to the problem solving process, and considers the implications of these behaviors on the success and failure in problem solving and the quality of collaborative learning experiences, thereby shedding light on how to instruct and promote more successful collaborative mathematical problem-solving groups within the online medium.

### Definition of Terms

For the purpose of this study, the following terms are used:

1. Computer-mediated communication: Communication between persons by way of computer networks.
2. Collaborative problem solving: A mutual engagement of participants in a coordinated effort to solve a problem together.
3. Computer-mediated collaborative problem solving: Collaborative problem solving via computer-mediated communication.
4. Problem-solving groups: Small groups of students, joined for the purpose of collaborating toward the solution of a mathematical problem.
5. Message: A communication sent from one member of a group to other members of that group.
6. Unit of Meaning: Statement(s) or question(s) to communicate the thing one intends to convey.
7. Participative Behavior: The total contribution of an individual or group in the process of collaborative mathematical problem solving. This behavior is quantified by the number of messages sent and units of meaning contributed.
  - a. Overall Participation: The total number of units of meaning contributed.
  - b. Task-Specific Participation: The total number of units of meaning related to formal content or process of online problem solving.

8. Social Behavior: Contributions of an individual or group not related to formal content, subject matter or collaborative process of the group.
9. Interactive Behavior: Mutual and reciprocal contributions of an individual or group (e.g. statements, questions, or comments that receive response, or that are made in response to other statements, questions or comments) during the process of collaborative mathematical problem solving.
  - a. Task-specific Interactions: Interactive statements, questions or comments directly related to the learning activity. Within this study, task-specific interactions include problem-solving interactions and other interactions.
    - i. Problem-Solving Interactions: Interactions related to the formal content or process of problem solving.
    - ii. Other Interactions: Interactions related to the use of the technology, or to the process of online collaboration.
10. Non-Interactive contributions: Statements or questions relating to the subject under discussion, but which are not in response to another message, and which do not lead to any further statements or questions.
11. Heuristic episode: A period of time during which an individual or group is engaged in one large task, such as analyzing the problem, exploring the problem, or verifying the solution. For the purpose of this study, there are four heuristic episodes: orientation, organization, execution, and verification.

- a. Orientation: The stage of problem solving during which strategic behavior to assess and understand a problem occurs.
- b. Organization: The stage of problem solving during which planning of behavior and choice of actions occurs.
- c. Execution: The stage of problem solving during which regulation of behavior to conform to plans occurs.
- d. Verification: The stage of problem solving during which evaluation of decisions made and of outcomes of executed plans occurs.

### Review of the Literature

This study investigates the process of collaborative mathematical problem solving in an online environment in order to characterize the participative, social, and interactive behaviors of small groups of students throughout the problem-solving process, and to describe the impact of these behaviors on the quality of the collaborative experience and the success or failure in problem solving.

The following discussion reviews the research literature that provides a theoretical background for the study and informed the stated purpose of inquiry. The areas of research include mathematical problem solving, collaborative mathematical problem solving, computer-mediated communication, and collaboration via computer-mediated communication.

## Research on Mathematical Problem Solving

Problem solving has a special importance in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems. So, what is problem solving? "Problem solving," although a frequently used term, and a practice that continues to see increasing use and recognition for its importance in mathematics education, remains without a clear, universal definition. In fact, most exercises of problem-solving research have made some attempt to define mathematics problem solving. Yet, sometimes words fail. Most people resort to a few examples and a few non-examples (Wilson, Fernandez, & Hadaway, 1994). For this reason, when two people talk about mathematics problem solving, they may not be talking about the same thing. A problem, as defined by Krulik and Rudnick (1988), is a situation, quantitative or otherwise, which an individual or a group of individuals confronts, that requires resolution, and for which a readily apparent solution path does not exist. Typical activities that are considered as mathematical problem solving include finding solutions to word problems, creating patterns, interpreting figures, developing mathematical models, developing constructions or, proving mathematical theorems. Problem solving can also be defined by its inherent challenge created by the lack of a universal problem-solving method. Unlike other mathematical learning activities that require an application of specific procedures or algorithms, there are a myriad of different problem-solving methods, some of which only work for specific problems or in particular domains or under special conditions while others are more general and apply to large classes of problems. Furthermore, there may be several

different ways of solving the same problem and two students may reach the same solution by totally different routes or apply the same method in quite different ways. All of this makes teaching, learning and researching problem solving a challenging task.

Although the teaching and learning of mathematical problem solving has received time and attention throughout most of the 20<sup>th</sup> century, a special surge of research began in the late 1970s with the growing realization that problem solving is an extremely complex mental activity. Until this time most researchers were interested only in analyzing the cognitive aspects of problem solving (Days, Kulm, & Wheatley, 1979; Lucas, 1974; Polya, 1957; Vos, 1976). These researchers were concerned about characteristics of problem solvers, problem-solving performance, in terms of success and failure, and categorizing strategies employed by problem solvers. Studies placed much emphasis on studying the cognitive strategies used by expert problem solvers, with the goal of identifying processes, or heuristics, which might be taught to novice or below-average problem solvers (Lucas, 1974; Polya, 1957; Schoenfeld, 1979a; Vos, 1976). Numerous useful problem-solving strategies were developed, mostly based on Polya's (1957) four phases of problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back. However, attempts at teaching those "heuristics" yielded only marginal success. Schoenfeld (1979b) provided several reasons for why the instruction in these strategies was ineffective. They included the following: The strategies were not yet described in sufficient detail; they were descriptive, rather than prescriptive; and there were too many potentially useful strategies. By insufficient detail he implied that labels given to strategies categorized a range of behaviors and were not

detailed enough to specify how the strategy might be used. In the distinction between descriptive and prescriptive, Schoenfeld contended that knowing about strategies does not increase the likelihood that one will use them. He stated that "for a problem-solving scheme to be useful...it must suggest how and when to use particular problem-solving techniques" (Schoenfeld, 1979b, p. 3). Finally, Schoenfeld believed that a long list of strategies is of little value unless it is part of a manageable scheme.

He designed such a scheme or model, based upon observations of "expert" problem solvers (professional mathematicians), and used the model in an attempt to train students to think like experts. This model included five well-described and prescriptive stages: (a) analysis, (b) design, (c) exploration, (d) implementation, and (e) verification. Results indicated that by using this method students became more fluent at generating plausible approaches to problems. Yet, educators found that even with extensive training of this type, many students were still incompetent at solving problems. Researchers traced this inability to a lack of awareness and control of knowledge and skills (Flavell, 1976; Garofalo & Lester, 1982; Schoenfeld, 1981).

Schoenfeld (1981) characterized the "managerial" aspects of expert and novice problem-solving behavior, by describing the impact of "executive" actions on the success in problem solving. He defined two types of decisions, "tactical" and "strategic," which are necessary in mathematical problem solving. Tactics, as defined by Schoenfeld, are "things to implement" (p. 5) such as algorithms and heuristics. Strategic or managerial decisions, in contrast, have a major impact on the direction that a solution will take, and on allocation of problem-solving resources. Schoenfeld saw the lack of success in

teaching problem solving to be the result of neglecting the essential aspect of managerial behaviors. He stated that “heuristic fluency is of little value if the heuristics are not ‘managed’ properly,” (Schoenfeld, 1979b, p. 3) and labeled these managerial actions “metaheuristics.” Schoenfeld analyzed these executive decisions by developing a framework to look at the major “episodes” of problem solving, periods of time during which problem solvers were engaged in a single stage such as analysis, or exploration. He focused on decision making between episodes, as he considered those times as critical make-or-break points for success in finding a solution. His findings showed that expert problem solvers tended to have rather keen managerial skills, and strived for efficiency and accuracy. In contrast, novices wasted their problem-solving resources because they lacked such managerial skills.

These managerial or self-regulation skills soon became known as “metacognition.” Translated into everyday language, metacognition means “thinking about your own thinking.” As one of the early researchers to stress the importance of metacognition in cognitive performance, Flavell (1976) described it as follows:

I am engaging in metacognition...if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact...metacognition refers, among other things, to the active monitoring and consequent regulation and organization of these processes to the cognitive objects on which they bear. (p. 232)

Palinscar and Brown (1984) divided metacognition into two related but distinct categories, *knowledge of cognition* and *regulation of cognition*. Knowledge of cognition includes what a person knows about one’s own cognitive processes, abilities, and resources as they pertain to specific cognitive tasks. This category also includes beliefs

as a type of knowledge and an important influence on cognitive behavior. This category was divided further as the result of Garofalo and Lester's (1985) extension of Flavell and Wellman's (1977) person, task, and strategy categories of metacognitive knowledge to the realm of mathematics. In this context Garofalo and Lester described person

*knowledge as:*

One's assessment of one's own capabilities and limitations with respect to mathematics in general and also with respect to particular mathematical topics or tasks. Such knowledge also includes one's particular beliefs concerning the nature of mathematical ability, the relationship of performance in mathematics to performance in other areas, and the effects of affective variables such as motivation, anxiety, and perseverance. (p. 167)

*Mathematical task knowledge* is defined as:

One's beliefs about the subject of mathematics as well as beliefs about the nature of mathematical tasks. This knowledge also includes an awareness of the effects of task features such as content, context, structure, and syntax on task difficulty. (p. 167)

*Mathematical strategy knowledge* includes:

Knowledge of algorithms and heuristics, but it also includes a person's awareness of strategies to aid in comprehending problem statements, organizing information or data, planning solution attempts, executing plans, and checking results. (p. 168)

Regulation of cognition refers to a variety of decisions and strategic activities that one might engage in during cognitive tasks. Examples of such decisions and activities include planning courses of action, selecting appropriate tools or strategies to complete a task, evaluating outcomes of strategies and plans, and, when necessary, choosing to abandon nonproductive plans. The depth of one's metacognitive knowledge can influence the types of strategies he or she uses for managing and regulating cognitive plans and decisions, and hence affect his or her level of success in problem solving.

Researchers confirmed that successful problem solving depends on having not only adequate knowledge but also sufficient metacognitive ability (Flavell, 1976; Garofalo & Lester, 1982; Schoenfeld, 1981, 1982a). They also revealed that metacognitive beliefs, decisions, and actions are important determinants of success or failure in a wide variety of activities (Garofalo & Lester, 1985; Schoenfeld, 1981, 1982a). As a result, much of the focus of research in mathematical problem solving shifted to the role of metacognition (Goos & Galbraith, 1996; Swanson, 1990; Wong, 1989).

Several researchers developed cognitive-metacognitive frameworks for the purpose of looking at problem-solving behaviors. In 1984, Schoenfeld further developed his ideas of metacognition and his framework from 1981. He believed that consideration of three distinct levels was necessary for accurate insight into one's problem-solving performance. These three levels were (a) resources (knowledge that a person brings with them to a particular problem), (b) control (the ability to make decisions about the selection of strategies and implementation), and (c) belief system (one's beliefs about oneself, the environment, and mathematics that will affect behavior).

Garofalo and Lester (1985) also developed a framework related to Polya's (1957) four phases of problem solving. The four categories in Garofalo and Lester's framework were: (a) orientation, (b) organization, (c) execution, and (d) verification. These categories were more broadly defined than Polya's phases, and specified key points where metacognitive decisions were likely to influence cognitive actions.

Biggs (1987) chose to break down metacognitive strategies into three different types based on his theory that the depth of one's metacognitive knowledge can influence

the type of strategies one uses for monitoring and regulating cognition. His three categories were: (a) surface strategies (reproduced through rote learning), (b) deep strategies (meaningful, requiring the involvement of previous relevant knowledge), and (c) achieving strategies (involving organizing one's time and working space).

The importance of the above frameworks is that they all allowed for a variety of possible behaviors, whether cognitive or metacognitive. In particular, they highlighted aspects of behavior where the presence or absence of metacognitive actions can be seen. Although the frameworks developed by Schoenfeld, Garofalo and Lester, and Biggs had differences, they all emphasized the importance of self-regulation in problem solving, and have been used as tools for analyzing metacognitive aspects of mathematical performance.

Other studies investigated the relationship between metacognition and academic and problem-solving ability. Wong (1989) examined whether students from different academic settings differed in their usage of metacognitive strategies in mathematical problem solving. Specifically, he looked at students from different academic streams (academic performance levels), academic tracks (e.g. science, arts, and general), and grade levels. He sought to determine how frequently students employed metacognitive strategies, and if strategies differed based upon academic setting. Using a questionnaire based on Garofalo and Lester's (1985) cognitive-metacognitive framework, Biggs' (1987) framework, and a section which measured students' beliefs in problem-solving strategies, Wong found that although most students reported practicing some metacognitive activities, students from the lowest academic level reported using fewer

metacognitive strategies than students from higher academic levels. He also found that students in the lowest level used surface strategies more often than deep strategies, and as level increased the use of deep strategies also increased.

Wong's (1989) research suggested that academic ability influences metacognition in three ways:

1. The lack of academic ability hinders a student's knowledge of strategies.
2. The lack of knowledge of metacognition results in poor academic performance.
3. Teaching styles can encourage or impede the development of metacognitive abilities.

In another study, Swanson (1990) analyzed the relationship between metacognitive knowledge and problem-solving aptitude. His primary interest was in finding whether high levels of metacognitive knowledge about problem solving could compensate for low overall aptitude. To test his hypothesis he considered four groups: those with high aptitude and high metacognitive ability; those with low aptitude and low metacognitive ability; and those with low aptitude and high metacognitive ability. Aptitude was determined by standardized cognitive ability and achievement tests and a questionnaire was used to assess metacognition in the general domain of problem solving. In order to analyze the children's use of heuristics and strategies, think-aloud protocols were utilized.

The results suggested that the high aptitude/high metacognitive ability students had a greater array of heuristic and strategy tools than students of other abilities. Furthermore, analysis suggested that the advantage in problem solving for high

metacognitive ability students might be the result of mental representations that reflect if-then thinking and the prioritization of strategies. More importantly, however, findings showed the high metacognitive individuals significantly outperformed low metacognitive individuals regardless of aptitude.

The importance of metacognitive knowledge as a determinant in mathematical problem-solving success is clearly well established (Schoenfeld, 1981; Swanson, 1990; Wong, 1989). Determining the existence of metacognitive behaviors in problem-solving processes, however, has proven to be more difficult to ascertain.

Methods of Protocol Analysis for Mathematical Problem-Solving Episodes. As recognition of the complexity of problem solving grew, the need for new methods of analyzing problem-solving protocols also increased. While experimental methodologies of the 1960s and 70s satisfied behaviorists and scientists concerned with the lack of replicable or verifiable data associated with the analysis of verbal reports once used to research the nature of human cognitive processes, it became evident that controlling the multitude of variables involved, and extrapolating results from well-designed laboratory studies to complex cognitive phenomena were complicated tasks that were difficult to complete with just statistical methodologies (Schoenfeld, 1987). For this reason, the late 1970s marked resurgence in the use of verbal data gathered by retrospection or introspection of the subjects through clinical interviews, and through speak aloud problem-solving sessions.

Even with the return of verbal methods, though, many researchers became concerned for the previous lack of consideration of important mental processes that may

or may not have physical manifestations during problem solving, but which play a major role in the success or failure of a problem solver (Lester, 1979; Schoenfeld, 1981, 1982a). In 1979, Lester designed a procedure for studying the cognitive processes used during problem solving. He believed that existing verbal methodologies such as introspection or retrospection by the problem solver held serious weaknesses, because many problem solvers are themselves not aware of the thought processes which they use while attempting to solve problems; and even if they are it may be difficult to achieve an accurate account for those processes as it is often difficult to reconstruct or accurately verbalize them. Lester removed the weaknesses by devising problem-solving tasks that forced the subject to manifest mental processes through the physical manipulation of objects, thereby enabling an observer to record the processes being used. The physical moves made reflected several aspects of cognitive strategies including trial and error, heterogeneous groupings, local classifications, partial global classifications, and global classifications.

In 1981, as Schoenfeld investigated episodes and executive decisions in problem solving, he became aware that there were no substantive systems for protocol analysis that focused on strategic decisions. Typical existing systems, at that time, used symbols to represent observable, disjoint problem-solving behaviors, and problem-solving sequences were recorded with a horizontal string of symbols that represented a collection of events in chronological order. Schoenfeld developed a framework for keeping track of not only the overt strategies, but also the critical executive decisions that were made between stages of problem solving. He categorized two components, objective and

subjective, to the framework for analyzing protocols. The objective part consisted of identifying the loci of potential managerial decisions, and the subjective part consisted of characterizing the nature of the decision making process at those managerial decision points, and describing the impact of those decisions on the problem-solving process. This was accomplished by partitioning a protocol into chunks of consistent behavior called "episodes." Schoenfeld defined the managerial decision points as places between episodes where the direction or nature of the problem solution changes significantly, at the arrival of new information or suggestion of new tactics, and at a point where a series of tactical failures indicates that strategic review might be appropriate. Thus, by partitioning the protocol into episodes (reading, analysis, planning, implementation, exploration, verification, and transition), and asking questions about each episode once it was labeled, the protocol could be characterized.

With the belief that tangible cognitive actions are a result of the interaction between beliefs held about a task, the social environment within which the task takes place, and the problem solvers' perceptions about self, and his or her relation to the environment and task, Schoenfeld (1982a) attempted to consider cognitive behaviors within the broad social-cognitive and metacognitive matrix in which they reside. In order to do this he analyzed student verbal behavior at three levels: (a) analysis of tactical knowledge of facts, procedures, domain-specific knowledge and local heuristics; (b) analysis of control knowledge, strategic behavior and conscious metacognition knowledge; and (c) analysis of consciously and unconsciously held belief systems that drive problem-solving behavior. Schoenfeld contended that the researcher must have

insight into all three of these levels to prevent misinterpreting the analysis. He used, for the purpose of illustration, a dialogue from Cole, Gay, Glick, and Sharp's 1971 study of cognition in a Liberian people called the Kpelle.

*Experimenter:* Flumo and Yakpalo always drink can juice (rum) together. Flumo is drinking can juice. Is Yakpalo drinking can juice?

*Subject:* Flumo and Yakpalo drink can juice together, but the time Flumo was drinking the first one Yakpalo was not there on that day.

*Experimenter:* But I told you that Flumo and Yakpalo always drink can juice together. One day Flumo was drinking can juice. Was Yakpalo drinking can juice that day?

*Subject:* The day Flumo was drinking the can juice Yakpalo was not there on that day.

*Experimenter:* What was the reason?

*Subject:* The reason is that Yakpalo went to his farm on that day and Flumo remained in town that day. (pp. 187-188)

Schoenfeld made the point that had the experimenter declared the subject unintelligent based upon his answers to the questions, without considering the social context, he would have missed the point. The subject obviously held different beliefs about the ground rules of the exchange, which were essential to consider for accurate interpretation of the data (Schoenfeld, 1982a).

In this study Schoenfeld also discussed the "legitimacy" of verbal data, considering the intrusiveness of various experimental methods. Although he cited a study by Ericsson and Simon (as cited in Schoenfeld, 1982a), which concluded that simply instructing students to talk aloud while solving problems, and not to interpret or explain, would not affect students' performance, he acknowledged that there are other concerns with the related methodology. First, it is generally acknowledged that asking

subjects to analyze their problem-solving processes while they work has measurable effects on performance. Secondly, Schoenfeld had witnessed, through his own analysis of singles and pairs in problem-solving sessions, that single-student protocols were not a true reflection of their "typical" cognition, but rather a pathological materialization of the experimental setting itself. Students, feeling pressure under the circumstances of being observed, responded by doing the first thing that came to mind (Schoenfeld, 1982a). Another concern expressed by Schoenfeld in this study was in extending the results of clinical studies to educational settings. Again, he emphasized the importance of recognizing and considering the social context in which cognitions are embedded when transferring data to the socially rich contexts of the classroom.

In conjunction with his previous study, Schoenfeld (1982c) developed a special framework for analyzing two-person problem-solving protocols. His interest was more in analyzing the managerial decisions of college mathematics students, and not in their application of heuristics. Although the prevailing assumption was that single-person protocols created the purest cognitions, Schoenfeld selected groups of two to prevent the pathology that occurred as he observed individuals problem solving, and because certain behaviors, such as decision-making, become more prominent and easier to observe with more than one subject. He chose not to look at larger groups, because although they provide greater and more interesting idea exchange, the larger the groups, the greater the social dynamics. Other variables Schoenfeld considered when designing this methodology were: the degree of intervention (experimenter obtrusiveness); the nature and degrees of freedom in instructions and intervention (the kinds of instructions subjects

are given has an effect on their behavior); the nature of the environment and how comfortable the subject feels in it; and task variables (are manipulatives provided?). Schoenfeld emphasized the importance of keeping obtrusiveness and interventions to a minimum in this type of study. He videotaped each of his pairs and provided only instructions to work as a team to solve the problem. Each student was provided with only a pencil, paper, and necessary tools for geometric constructions. Although the nature of the environment was artificial, efforts were made to put subjects at ease. Nevertheless, the stressfulness of an unnatural setting may have affected students to some extent.

The analysis of the protocol took the form of Schoenfeld's (1981) previous analysis framework. Protocols were first partitioned into episodes; the researcher then examined transition points between episodes and new information points for executive or strategic decisions. Schoenfeld found the new framework informative, straightforward and reliable. Working with three students, their inter-coder reliability rate exceeded 85%. However, he cautioned that it only offered one perspective on the problem-solving process. To get a comprehensive look, Schoenfeld suggested that it should be employed with other methods such as paper-and-pencil tests, clinical interviews, and different levels of analysis.

In 1990 Artzt and Armour-Thomas developed another framework for the protocol analysis of group problem solving by adapting Schoenfeld's (1982c) analysis of two person protocols. The new framework differed from Schoenfeld's by the explicit delineation of the roles of metacognition and cognition within the mathematics heuristics of problem solving. Whereas Schoenfeld did not differentiate between cognitive and

metacognitive strategies at each of his episodes (reading, analysis, planning, implementation, exploration, and verification), but rather looked for metacognitive strategies at the transition stage and at new information points, Artzt and Armour-Thomas assigned each of the episodic categories with a metacognitive or cognitive label, based on Garofalo and Lester's (1985) distinctions of the two. The categories and their labels were: read (cognitive), understand (metacognitive), analyze (metacognitive), plan (metacognitive), explore (cognitive or metacognitive), verify (cognitive or metacognitive), and watch-and-listen (undetermined cognitive level). Although it is usually difficult to determine the level of cognition during watch and listen behaviors, the researchers emphasized that the role of this behavior in the group problem-solving process cannot be underestimated. The differences in the above frameworks do not necessarily point to weaknesses in any of the methodologies. Instead it is a reminder, as Schoenfeld (1982c) stated, that "each methodology is a lens (or filter, if you will) through which intellectual performance is being viewed. Thus the selection of any particular methodology for investigation may well determine what the experimenter does or does not see" (pp. 4-5).

### Research on Collaborative Mathematical Problem Solving

The term "collaboration," though widely used in education, is quite nebulous in its meaning. Through subtle differences in individual definitions, it has come to represent a broad variety of activities in which people work together. In the words of Kaye (1992), "etymologically, to collaborate (co-labor) means to work together, which implies a concept of shared goals, and an explicit intention to 'add value' – to create

something new or different through the collaboration, as opposed to simply exchanging information or passing on instructions" (p. 2). By this definition, "collaborative learning" deals with instructional methods that seek to promote learning through collaborative efforts among students working on a given learning task. However, Kaye (1992) also asserts that learning is, inherently an individual process, rather than a shared process, which is influenced by a variety of external factors, including group and inter-personal interactions. Therefore, collaborative learning does not necessarily imply learning in a group, but rather the possibility of being able to rely on other people to support one's own learning and to give feedback, as and when necessary, within the context of a non-competitive environment.

The term "cooperative learning" is frequently used interchangeably with the term collaborative learning, although in actuality there are subtle distinctions in their definitions, and significant debate about those differences (Higgins, 1991; Panitz, 1996). Higgins (1991) addressed this matter through his reference to literature by Damon and Phelps (as cited in Higgins, 1991) that distinguished the two types of peer education and others based on levels of "mutuality (i.e., mutual responsibility for and contribution to the tasks) and equality (i.e., equal in status, skills and knowledge) among group members" (p. 6). Cooperative learning, as described by Damon and Phelps, is high on equality, but varied in mutuality, implying equal individual responsibility and contribution, but subdivided tasks that require participants to work individually. On the other hand, collaborative learning is described as high on both equality and mutuality, implying equal skill and joint work on the task. For the purpose of this study, then, these characteristics

which define collaborative learning, and which separate it from some definitions of cooperative learning and other types of peer education are used: that all participants are equally unskilled in the task at hand, and that they work jointly on the same problem at all times. This means that there is no deliberate expert/novice relationship between members of the group, and that the task is not intended for subdivision into subtasks completed by different group members. Nevertheless, while the proposed study focuses on collaborative learning, it will also include select pieces of literature that include the term cooperative learning, yet that are still consistent with strategies considered collaborative by this researcher.

Collaborative learning is not a new practice. A variety of collaborative approaches have been used in classrooms since the early 1900s (Johnson & Johnson, 1993). In addition, insight into the importance of social interaction on cognitive growth can be traced back to the work of influential educational philosophers and theorists such as Dewey, Piaget, and Vygotsky. Kumar (1996) summarized the philosophies of the three in the following way: Dewey characterized education as a social process by which groups transmit and renew the meanings of shared experiences. Piaget viewed children's interactions with their peers as a crucial source of cognitive stimulation and development, especially of logical reasoning. Similarly, for Vygotsky social interaction was considered at the core of the developmental process and learning. Dillenbourg and Schneider (1995) identified three different theories of learning underlying collaborative learning systems: (a) socio-constructivist theory, (b) socio-cultural theory, and (c) shared cognition theory. All of these approaches are classified as cognitive developmental approaches, which

focus on interactions among peers around appropriate tasks in a given environment, and that increase the mastery of critical concepts (Kumar, 1996). Learning, within these theories, is considered something a learner does, not something that is done to the learner. It occurs through active participation by the student versus passive acceptance of information presented by an expert. Socio-constructivist theory, an extension of Piaget's theory, asserts that all knowledge is constructed, discovered and transformed by students (Panitz, 1996), as they interact with others. Within this theory, the emphasis is placed on interactions rather than actions themselves. Socio-cultural theory focuses on the causal relationship between social interaction and the individual's cognitive development (Kumar, 1996). This approach originated from Vygotsky's Zone of Proximal Development, a theory that asserts that each internal cognitive change can be traced to a causal effect of a social interaction. Within the socio-cultural theory it is through collaborative efforts that individuals internalize knowledge and techniques that will be used in future attempts to solve problems independently (Kumar, 1996).

Shared cognition theory is different from the other two theories in the sense that the environment in which learning takes place is given the focus rather than the environment-independent cognitive processes. While the previous two theories attributed the learning only to the physical presence of collaborators, the shared cognition approach focuses on the social content that is claimed to make the collaborations happen. Shared cognition views collaboration as a process of building and maintaining a shared conception of a problem, and is aimed at letting peers learn knowledge and skills in contexts where they are applicable (Kumar, 1996).

Collaborative learning proponents argue that group activity is superior to solo activity, allowing students opportunities to learn about “group process, personal strategies in contrast to those of others, multiple perspectives on the same topic, content falling out of complex projects, leadership, management of group tasks and communication” (Guzdial et al., 1997, p. 12). In fact, much research has confirmed the value of collaborative approaches (Gokhale, 1995; Johnson & Johnson, 1993). In a meta-analysis of 120 existing studies comparing cooperative, competitive, and individualistic learning on individual achievement of college students, Johnson and Johnson (1993) concluded that cooperative learning promoted higher individual achievement than did competitive (effect size = 0.54) or individualistic (effect size = 0.51) learning. These results held for verbal tasks such as reading, writing, and orally presenting, mathematical tasks, and procedural tasks. The meta-analysis of research on achievement also found cooperation to promote greater intrinsic motivation to learn, more frequent use of cognitive processes such as reconceptualization, higher-level reasoning, metacognition, cognitive elaboration, and networking, and greater long-term maintenance of skills learned (Johnson & Johnson, 1993). In addition to achievement, the studies included in the meta-analysis also found other important outcomes such as student retention, creating a learning community, and building positive relationships among diverse students. Within the studies (n=38) that included interpersonal attraction (including esprit-de-corps, cohesiveness, trust, and other relationship variables) as a dependent variable, cooperative efforts promoted greater liking among subjects than did competing with others (effect size = 0.68) or working individually (effect size = 0.55). Furthermore, in the studies that

focused on social support ( $n = 24$ ), students learning cooperatively felt more social support (both academically and personally) from peers and professors than did students working competitively (effect size = 0.60) or individualistically (effect size = 0.51). When students worked cooperatively, positive and supportive relationships tended to develop, even among students from different ethnic, cultural, language, social class, ability and gender groups (Johnson & Johnson, 1993). Finally, studies on psychological health issues, such as self-esteem ( $n = 13$ ), found that cooperation promotes higher self-esteem than did competitive (effect size = 0.47) or individualistic (effect size = 0.29) efforts.

A more recent study by Gokhale (1995) confirmed the benefits of collaborative techniques for enhancing critical thinking and problem-solving skills. The study analyzed the effectiveness of collaborative learning on learning outcomes at the college level for students in technology. Specifically, Gokhale compared the effectiveness of individual learning and collaborative learning for enhancing the drill-and-practice skills and critical-thinking skills of forty-eight students studying series and parallel dc circuits in a basic electronics course. Each group (the individual learners and collaborative learners) attended a common lecture and received a common worksheet consisting of drill-and-practice items and critical-thinking items. However, the individual learners worked in isolation, while the collaborative learners worked in groups of four students. A t-test was conducted on pretest scores to determine the equity of the two groups, and results concluded pretest differences among treatment groups were not significant

( $t = 1.62, p > 0.05$ ). Posttest scores were analyzed to determine the effects using a t-test. In addition, an analysis of covariance procedure was used to reduce the error variance by an amount proportional to the correlation between the pre- and posttests. The correlation between the pretest and the posttest was significant ( $r = 0.21, p < 0.05$ ). In this approach, the pretest was used as a single covariate in a simple ANCOVA analysis. The t-test did not show a significant difference in drill-and-practice skills between the two groups ( $t = 1.73, p > 0.05$ ). An analysis of covariance procedure yielded a F-value that was not statistically significant ( $F = 1.91, p > 0.05$ ). However, a t-test showed a significant difference, at the 0.001 alpha level, in critical thinking skills between the two groups, with participants in the collaborative group performing better than those that studied individually ( $t = 3.53, p < 0.001$ ). An analysis of covariance yielded a F-value that was significant at the same alpha level ( $F = 3.69, p < 0.001$ ).

Too often, however, collaborative learning advocates erroneously assume that learning is automatically a by-product of group projects. In reality, students who enthusiastically engage in collaborative activities often fail to pay attention to important learning lessons that are embedded in that activity. In other words, experience alone does not ensure learning. Researchers have explored the mechanisms that account for knowledge acquisition through collaboration. The evidence points to eight such mechanisms that lead to the successful acquisition of knowledge: (a) conflict or disagreement, (b) the alternative proposal, (c) self-explanation, (d) internalization, (e) appropriation, (f) shared cognitive load, (g) mutual regulation, and (h) social grounding. (Dillenbourg & Schneider, 1995).

Within the socio-constructivist theory, conflict or disagreement within a collaborative environment promotes learning, as group members must confront their differences to find a common solution. The disagreements need not be large; a simple disagreement or slight misunderstanding is sufficient for learning. However, the conflicts must be discussed. Indeed, it is the verbal interactions generated to solve the conflict that are related to positive learning outcomes (Dillenbourg & Schneider, 1995).

The second mechanism is similar to the first. It is related to what psychologists refer to as "confirmation bias," a term that describes the tendency of subjects to only design experiments to confirm their hypotheses, or to disregard any empirical findings that contradict their hypotheses (Dillenbourg & Schneider, 1995). Within a collaborative setting confirmation bias is reduced through the conflict mechanism, which forces group members to consider, and quite possibly adopt alternative hypotheses.

As with the others, the self-explanation mechanism is a natural and spontaneous benefit of collaborative learning. This mechanism can be described as the means from which both persons involved in giving and receiving an explanation gain benefits. It is well documented that, through the interactive process in which two partners try to understand one another, not only does the explainee benefit from the explanation, but also that the explainer often receives even higher benefits (Dillenbourg & Schneider, 1995). This can be explained by the fact that through explanation students are required to reorganize or clarify the material in new ways (Webb, 1991), proceduralize their declarative knowledge of a concept, and make explicit some implicit problem-solving steps, thereby reinforcing their knowledge and becoming more efficient in solving similar

problems. Additionally, the process of developing explanations aids the explainer in recognizing gaps in understanding (Webb, 1991). Webb (1991), however, observed that the cognitive benefits are restricted to elaborated explanations, and that less sophisticated messages from the explainer do not produce effects. This confirms that the effect of explanation is related to the cognitive activity of building the explanation.

It has also been determined that those on the receiving end of explanations benefit as well (Webb, 1991), though the benefits are complicated depending on several conditions: (a) whether the student receiving explanation needs help, (b) the relevance of the explanation to the need for help, (c) the timeliness of the explanation, (d) whether the explanation is understood, (e) whether the target student has an opportunity to use the explanation to solve the problem, and (f) whether the target student uses the opportunity.

Vygotsky (as cited in Kumar, 1996) labeled the mechanisms of learning that occur through participation in conversations 'internalization'. The concepts conveyed by the interactions with more able peers are progressively integrated into a listener's existing knowledge structures, provided that the concepts are within the listener's zone of proximal development, and that the listener is not passive, but actively participates in the problem-solving effort. It is the internalization of the new concepts and strategies that provides the listener with new knowledge and tools for use in his own reasoning.

Similar to internalization, appropriation focuses on how one learns from watching and working with a more skilled partner. This mechanism is central to the apprenticeship method. Dillenbourg and Schneider (1995) provided the following example, which makes clear the process of appropriation:

Let us consider two agents A and B such that B is more skilled on the task to be performed. When one agent A has performed some action, his or her partner B attempts to integrate A's action into his own plan, i.e. to appropriate A's action. Learning occurs when A reinterprets his actions with respect to how B appropriated it. Agent A learns progressively how to assemble the elementary piece of action that he is able to perform into a coherent problem strategy. (p. 7)

The shared cognitive load mechanism can be explained as the sharing of cognitive responsibilities implied by a task during collaboration. This does not imply a systematic division of labor or tasks, but a mutual, synchronous and interactive attempt to reach a joint solution. This spontaneous division of the cognitive responsibilities enables each partner to devote more resources to the task allocated to him or her and prevents redundancies in the group process.

During collaborative problem solving, group members must also justify their actions to one another. These explanations make explicit the strategies and knowledge that would otherwise remain implicit. Additionally, it is through these discussions that partners mutually regulate the activities of the others in the group. This mutual regulation mechanism is often the catalyst for other mechanisms such as conflict, explanation, internalization and appropriation (Dillenbourg & Schneider, 1995), and it is these mechanisms that encourage improved individual regulation skills in partners after collaborative problem solving (Schoenfeld, 1987).

The last mechanism, social grounding, refers to an individual's attempt to maintain the belief that his partner has understood what he meant, at least to an extent that is sufficient to carry out the task at hand (Dillenbourg & Schneider, 1995). By means of this mechanism the speaker monitors the listener's understanding, through

verbal and non-verbal clues, and in case of misunderstanding, attempts to repair communication. Repairs involve disambiguating dialogues, pointing to shared reminders and to visible objects, drawing a schema, and so forth. Through this mechanism, both partners progressively build a shared understanding of the problem.

Based on prior research, Dillenbourg and Schneider (1995) also identified conditions for effective collaborative learning. They are grouped into three categories: (a) group composition, (b) task features, and (c) communication media. Regarding the composition of the group, Dillenbourg & Schneider (1995) conclude that small groups seem to function better than large groups, because of the tendency for some members to exclude themselves or to be excluded from interactions, and because most of the mechanisms described in the previous paragraphs (e.g. mutual regulation, social grounding, etc.) can only occur between a few participants. In addition, research points to an optimal level of heterogeneity in terms of variables such as age, intelligence, performance, and academic achievement, since some difference in viewpoints is required to trigger interactions and conflicts, but within the boundaries of mutual interest and intelligibility (Dillenbourg & Schneider, 1995).

The effects of collaboration also vary according to task. Some tasks promote the activation of the mechanisms described above, while other tasks diminish them. For example, some tasks are naturally distributed and lead group members to work independently. Interaction occurs when assembling partial results, but not during each individual's reasoning process. Without interaction, none of the described mechanisms can be activated. Some tasks are so straightforward that they do not leave any

opportunity for disagreement or misunderstanding. Some tasks do not involve any planning and hence create no need for mutual regulation. Thus, optimal tasks must include features such as adequate difficulty that require group members to interact on a continual basis, reason together, regulate the thinking of other group members, and work towards a mutual solution.

Finally, the medium used for communication plays a role in quality of collaboration as well. However, this matter will be discussed later in the context of introducing alternate forms of communication such as computer-mediated communication.

Collaborative Learning in Mathematics. The findings on the use of collaborative learning in mathematics have also been largely positive (Stacey & Gooding, 1993; Owens, Perry, Conroy, Geoghegan, & Howe, 1998). Owens and her colleagues (1998) investigated the effects of an interactive approach for learning mathematics that used principles of cooperative learning and problem-solving activities. The intent of the researchers was to examine how students' responsiveness and interactions in the classroom influenced and, in turn, were influenced by affective states. Among the questions considered were whether there was an improvement in the students' knowledge as a result of participation, and how the learning approach influenced students' thinking and affects. Results from a two-tailed t-test conducted to compare pretest and posttest scores on a worksheet consisting of problems similar to those covered in class showed a significant difference in mean scores ( $t = 11.8, p < .0001$ ), with a mean gain of 47 out of 100 points. The authors contend that the increased gain in scores suggests that students

improved in their approaches to solving problems presented in the subject. Case studies were used to determine how the learning approach influenced students' thinking and affects. Data for the case studies came from student journals, records of observations taken from the video-recorded sessions, and comments made by students during classes, informally to researchers, and during formal interviews conducted at the end of the semester. From analysis of video recordings and student journals Owens and colleagues identified critical change points where development in the students' mathematical schema occurred or where specific challenges met in the problem-solving process were overcome. The analysis indicated that critical changes were associated with (a) personal characteristics such as humor; (b) empathy within the group, (c) separation of the wording of the problem from the mathematics, (d) recognition of the group of a useful strategy, (e) realization that they have useful knowledge already, and (f) collaborative validation of their ideas. The collaborative nature of the activity clearly promoted quality communication about mathematics, a safe environment for risk taking during early anxieties and fluctuating feelings, the ability to face problems with success, humor and confidence, the desire to solve problems, changes in beliefs about mathematics and one's own mathematical ability, increased confidence, changes in responsiveness, metacognition and the application of useful heuristics, and collaborative validation.

Stacey and Gooding (1993) investigated the patterns of oral communication associated with successful learning in small groups. The learning task was an activity designed to correct misconceptions about division and the ways to write about it. The criterion for successful learning was improvement on a written test given before and after

the activity. The task focused not on teaching new information or skills, but on eliminating or reducing misconceptions. Questions Stacey and Gooding (1993) attempted to answer in their study included: Do all children learn in groups? Is it possible to identify characteristics of effective and ineffective groups as well as to distinguish between individuals who will improve and those who will not? What are the nature and content of the children's talk and how do they relate to what the children learn? How do the children interact when they teach each other? How do they help each other learn mathematics? The procedure involved videotaping groups of four children while they worked on the assigned task. Two coding schemes were used to analyze the discourse. A pretest was administered one week before the activity and a posttest was administered three weeks after it. Coded discourse was then related to the results of the tests. The coding scheme was developed by modifying a scheme developed by Sharan and Shachar (as cited in Stacey & Gooding, 1993), that was derived empirically from the data, and that described how students used language in interaction rather than what they said about the subject. Turns talking were coded as focused interactions (e.g. a request for clarification, agreement, or disagreement) and cognitive strategies, (e.g., explanation with evidence, unstructured idea or thinking out loud, concrete examples). Stacey and Gooding modified the coding by combining and selecting the relevant categories of focused interactions and cognitive strategies according to statements made during discussion of the mathematics task. The categories included: (a) asking questions (of a previous speaker, from own thinking or working, reading a word problem), (b) responding (to a request for clarification, agreeing, disagreeing, repeating); (c) directing;

(e) explaining with evidence; (f) thinking aloud; (g) proposing ideas; (h) commenting (affective); and (i) refocusing discussion. The unit of analysis for this coding procedure was an utterance of any length and grammatical form that was produced by one speaker without interruption. A second scheme was also used to analyze the discourse to capture the content and content difficulties of the task. In this situation, short continuous episodes of speech, about one decision, was the unit of analysis.

Five of seven groups of students were designated effective because their members increased their scores. The other two groups were designated ineffective. The learning outcomes for individuals were mixed. Each effective group had at least two improvers. Every group had at least one non-improver. One ineffective group had completed the task correctly, and one, incorrectly. The researchers noted that it was interesting that the only group in which no one made gains contained the only member of the class who was able to complete a relevant procedure from the beginning. He did not explain his results to the others in his group, and they worked with the least discussion.

The two ineffective groups had students taking the fewest number of turns talking, 174 turns and 297 turns, respectively. In contrast, the number of turns talking in effective groups ranged from 362 to 820 turns. Among the effective groups, there was no correlation found between the number of turns talking and the amount of improvement. The mean percentage of turns talking in each category were given both for individuals and for the groups. The children in the ineffective groups interacted less than did those in the effective groups. The lower level of interaction for ineffective groups was reflected in the small overall amount of talk, the greater percentage of talk in the thinking aloud

category, and the generally lower percentages in the categories such as responding, that indicated intellectual interaction. Similar observations held when the comparison was between individuals (improvers and non-improvers), although a little less strongly than for the groups. Members from the effective groups were also found to give more explanations with evidence and repeated one another's statements more frequently.

Almost no explicit mathematical discussion took place in the ineffective groups. Comparatively the ineffective group made only five calculations of any sort, while the effective groups made at least twenty-seven calculations. The number of instances of talk in each category for the two ineffective groups was, with only one exception, always less than the corresponding number of instances for each of the effective groups. The probability that the low results of the ineffective groups would have been obtained if there were no real difference between the groups was calculated by the Mann-Whitney test to be less than 5% ( $n_1 = 5$ ,  $n_2 = 2$ ,  $U = 10$ ,  $p < .05$ ). At the individual, rather than the group level, the results were similar. In that case the researchers used a t-test on means rather than the Mann-Whitney test on medians.

This study confirmed the broad patterns of interaction than been associated with higher achievement in groups in previous studies. Members of effective groups interacted more. Generally they helped one another more by responding and explaining to a greater extent during the task. Improvers engaged in the activity by explicitly working out the division problems in the task. They used more specific mathematical talk than non-improvers for every category. The coding of mathematical content and the coding of interaction patterns provided evidence that giving help to others at a high level

of elaboration is related to achievement. In summary effective groups – talked more with more mathematical content, explicitly discussed the central idea, worked together by reading the questions on the cards out loud and repeating one another's statements; proposed ideas, gave explanations with evidence, and refocused discussion more often; and responded to the questions of others more.

Collaborative Mathematical Problem Solving. Collaborative learning strategies have been found to be particularly effective when used for mathematical problem solving (Dees, 1991; Forman, 1989; Goos & Galbraith, 1996; Hart, 1993). There is evidence that collaborative problem-solving efforts are often more effective than individual efforts for enhancing problem-solving performance and abilities (Hart, 1993). Hart (1993) suggested three characteristics of collaborative learning that serve to enhance mathematical problem-solving performance: (a) the collective experience of a group often supplies information that individuals in that group do not possess; (b) peers function as external monitors when self-monitoring is absent; and (c) the group setting encourages time for reflection, an important component of problem solving that is often neglected in individual problem solving. Lambdin (1993) focused on the external monitoring that occurs in a collaborative environment. Her study analyzed, described, and categorized the monitoring moves and roles of three pairs of college-aged women as each pair worked together during a series of problem-solving sessions. Utilizing an episodic parsing scheme developed by Schoenfeld in 1985 (as cited in Lambdin, 1993) and the cognitive-metacognitive framework developed by Garofalo and Lester (1985), Lambdin divided the problem-solving protocols for each pair into episodes and identified

the occurrence of monitoring events. The analysis revealed monitoring moves and roles that were clearly different from those that were present with individual efforts, and the successful accomplishment of problems that could not be solved individually. In particular, Lambdin found that the most successful pair informally adopted complementary monitoring roles thereby simplifying the conceptual demands of the task and promoting increased regulation of actions.

In another study, Curcio & Artzt (1998) used the Artzt and Amour-Thomas (1992) framework to examine the effects of small group communication on graph data-interpretation by fifth graders. In this task, small groups of students were given a broken line graph depicting the average time of sunset from June to December. They were asked to draw a picture of what the graph would look like if it were to continue from January through May. The researchers coded student problem-solving behaviors and statements in one-minute intervals. The analysis consisted of a chart of coded behaviors and statements, detailed excerpts and from the problem-solving protocol, and commentary about the problem-solving process of the small group. Curcio & Artzt discussed the behaviors of one group that successfully completed the task, and confirmed the findings of Artzt & Amour-Thomas that revealed the significance of the interplay of cognitive, metacognitive, and watch and listen behaviors. Of 80 behaviors coded during a 7-minute interval, 47.5% were metacognitive, which indicates a substantial amount of control and regulation on the part of the group members in helping to guide and direct the task to a successful completion (Curcio & Artzt, 1998). Thirty-three percent of the behaviors coded were watch and listen behaviors, which led the researchers to conclude that a high

degree of communication is necessary for effective small group functioning. The analysis of the interaction also revealed how students' metacognitive behaviors facilitated the building of ideas. Cognitive behaviors accounted for 20% of the total coded behaviors, the smallest amount of coded behaviors. The researcher stated that this implies that the group worked efficiently, listening to, monitoring, and evaluating the different ideas presented for accomplishing the task without working mindlessly and going off on individual tangents. The results of the analysis also illustrated the recursive nature of problem-solving processes that lead to success. The members of the group began by reading the directions and attempting to understand the task at hand. Next some exploration and analysis led to a plan that was implemented. The plan was then revised as the students continued to implement the revision. Finally, the group went on to verify a solution. The results of this study confirmed the effectiveness of the small group setting for communicating about mathematics and solving problems. It also confirmed that the nature of the communication and behaviors that occur within this setting mirrors the monitoring and regulating strategies that experts reveal during problem solving.

It also appears that through collaboration students learn the strategies essential to effective problem solving (Artzt & Amour-Thomas, 1992) and are able to transfer these strategies to individual problem-solving efforts (Forman, 1989). In 1989 Forman observed a pair of students working collaboratively to solve a mathematical problem. The observations illustrated how two subjects of comparable intellectual ability but with somewhat different stances toward a problem-solving task can help each other incorporate new reasoning and strategies into their repertoire.

On a larger scale, Dees (1991) conducted a study with 77 students in a college remedial mathematics course to determine whether cooperative learning would help students increase their problem-solving skills in mathematics. All students in the course were encouraged to work together and help each other on all tasks except for tests and exams. However, the students in the control group were not organized with respect to cooperation while the students in the treatment group were organized in such a way to require cooperation, as well as given continual and active encouragement to work together. Tests of means on all seven pretest variables revealed no significant differences among the four laboratory sections or between the two groups of students. Instruments for evaluation of achievement included multiple-choice skills tests and open-ended tests constructed by the teacher. Students using cooperative learning performed as well as or better than students in the control group on every measure. Outcome variables showed significant differences in favor of cooperative learning were solving word problems in algebra ( $t = 2.29, p < .05$ ) and proof writing in geometry ( $t = 2.92, p < .01$ ), thus confirming the theory that cooperative methods are significantly better than independent methods for tasks involving complex thinking and requiring higher cognitive skills.

While the aforementioned studies illustrate the benefits of collaboration for mathematical problem solving, the use of this educational practice is not without limitations. Individual actions as well as the internal dynamics of small groups are critical factors in the success or failure of the collaborative process. In 1996 Artzt and Amour-Thomas developed an assessment system based on their earlier work (Artzt & Amour-Thomas, 1992). The system was designed to examine, in part, the interplay of

cognitive processes observed in the problem-solving behaviors of individual students working in small groups. Their new assessment instrument, which attempted to clarify the distinction between cognitive and metacognitive processes, included the following categories of behaviors: talking about the problem, doing the problem, watching and listening, and off-task. The framework was further divided with *talking about the problem* including the following metacognitive processes: understanding, analyzing, exploring, planning, implementing, and verifying; and *doing the problem* including the following cognitive processes: reading, exploring, implementing, and verifying. Using videotaped protocols and stimulated-recall interviews the researchers coded the behaviors of six groups of students throughout the problem solving process and categorized the interactions of each small group on a continuum ranging from students who work independently with little communication to students who continually interact with one another in the solution of the problem. They found four broad categories summarizing the behaviors of the six groups: cooperative workers, independent workers, one-person show, and combination. The data revealed the importance of the interplay of talking about the problem, doing the problem and watching and listening for effective problem solving and group functioning. Indeed, the only group that failed to solve the problem exhibited behaviors of "independent workers" throughout most of the problem-solving process with a low percentage of *talking about the problem* and *watching and listening* behaviors.

In another study, Chiu (1997) analyzed the individual actions of students involved in small group mathematical problem-solving exercises to determine how these actions

facilitated or hindered the collaborative process. Chiu's framework for analysis included three dimensions: (a) evaluation of the previous action (supportive, critical, or unresponsive); (b) problem knowledge content (contribution, repetition, and null); and (c) interactive form (command, question, and statement). Through the analysis and coding of problem-solving protocols Chiu found that, as well as yielding rich benefits during collaborative learning, differences among students could pose difficult problems. While students can draw upon their diverse experiences and abilities to construct new knowledge and solutions, they also face the potential pitfalls of entrenching themselves in opposition to different views, dismissing other perspectives as less important or invalid, and misinterpreting other people. Similar findings were established by Wilczenski, Brontrager, Ventrone, and Correia (1999) who found that groups that exhibited behaviors such as not paying attention, distracting, interrupting, monopolizing, making personal attacks, not contributing, and making incorrect assumptions resulted in problem-solving accuracy and individual problem-solving achievement no higher than that observed with controls who did not have the opportunity for group collaboration.

#### Research on Computer-Mediated Communication

As the term alludes to, computer-mediated communication refers to human communication via computer. In this mode of communication, computers may serve as "central repositories" (Higgins, 1991, p. 1) for messages, or they may provide a network of links over which messages are transferred. Within this medium, there are various manners in which to communicate. Although computers and communications technology continue to grow in sophistication and capability, most current applications still focus on

text-based communication. Hence much of the research lies in this area.

There are numerous forms of text-based computer-mediated communication. In his 1991 study, Higgins described four of the most prevalent forms: computer conferencing, electronic bulletin board systems, email, and chat. Primary among the asynchronous types is computer conferencing. Computer conferencing as defined by Higgins (1991) are those systems "that reside on large central computers to which numerous users can connect directly via in-house terminals or indirectly via home computers, modems and telephone lines" (p. 5). Within these systems, messages are created and saved to files that allow multi-user access, thereby enabling groups of people to establish conferences in which they send messages they wish to share with one another. In this manner, the conference resembles an ongoing discussion or meeting that does not necessitate physical proximity, or synchronized participation. Rather, all members can link into the discussion at their own convenience, when they log on.

Electronic bulletin board systems (EBBSs, or BBSs) are another form of computer-mediated communication that facilitates group discussions. At their conception, BBSs were intended to serve as conventional community bulletin boards where public messages could be posted and read. In the early 1980s, computer user groups began establishing BBSs for the primary purpose of online distribution of software and related information, but the messaging feature rapidly grew in popularity as members found that posted messages received responses and resulted in interactive discussions. This capability for group discussions led to broadening interest and use in BBSs within the general public, and the influx of home computers, and the availability of

inexpensive communications software soon transformed BBSs from a medium geared just toward computer enthusiasts to forums for educational activities, special interest groups, and academic discussions (Higgins, 1991).

Electronic mail (EMAIL) is the most widely used form of text-based computer-mediated communication. It is asynchronous in nature, however unlike computer-conferencing and BBSs, messages are not group-accessible with EMAIL. Instead, messages are sent to individuals in the same way that conventional mail is sent to individual addresses. Still, there is ability to deliver bulk mailings by individuals or by automated message handling software. An example of this capability is the LISTSERVER. The primary function of the LISTSERVER is to receive messages from individuals and then distribute them according to mailing lists. Members of these mailing lists can then carry on asynchronous discussions much the same way as users of computer conferencing systems and BBSs do (Higgins, 1991).

The CHAT mode of interaction is an example of text-based computer-mediated communication that allows synchronous or real-time communication between two or more participants. The various forms of this type of synchronous computer-mediated communication differ primarily in the way dialogue is presented on the screen, and in the number of users who can participate simultaneously in a group activity. The user interface for synchronous text-based interaction often involves some partitioning of the terminal screen. While more advanced systems can allow for multiple windows of variable size and location to appear on the screen with any or all of them being used to receive or send messages, the most common design for two simultaneous users involves

splitting the screen horizontally, where one half is used for receiving messages, and the other half is used for composing and sending messages. For multiple simultaneous users contributing to the same discussion there is most often a smaller window for composing and sending messages and a larger window for receiving messages. The exchange of dialogue in this mode is best described by Higgins (1991) as "character by character, line by line, or in multiple-line blocks" (p. 6). In the character-by-character mode, recipients see each character as it is typed, while in line-by-line and multi-line blocks modes, the messages are not seen until the sender composes one or more lines and presses the send key. In all cases, however, the transaction occurs immediately, thereby enabling a real time exchange. While messages are not retained in a mail system or conferencing space for future retrieval, transcripts of the interaction are logged and can be collected for future use.

Advantages and Disadvantages of Computer-Mediated Communication. As a vehicle for group communication, computer-mediated communication possesses unique features that make it qualitatively different from either spoken or written communication, and, as a result, introduces various opportunities and problems (McDonald, 1998). Computer-mediated interactions can vary from rapid-fire exchanges of short messages typed online on the spur of the moment, to carefully edited and relatively lengthy texts prepared off-line over a period of several days before being uploaded into the system (Kaye, 1992). Asynchronous forms of text-based computer-mediated communication such as computer conferencing and email provide a quick, convenient means of worldwide communication, without the constraints of time and location necessary for

face-to-face communication, thereby enabling important learning activities, such as peer discussions and collaborative activities, amongst participants in multiple remote locations at different times. Other advantages of asynchronous text-based computer-mediated communication are:

1. It facilitates the thoughtful exchange of ideas by creating the ability to reflect on previous messages and compose articulate and thought provoking responses.
2. Discussions are continuous and there is no loss of relationship between group members in the periods between meetings (McConnell, 1994).
3. Activities need not be discrete; members can take part in discussions about multiple issues at the same time (McConnell, 1994).
4. It promotes the development of written skills.
5. The limitation to written communication encourages a clearer, more explicit, and more organized articulation of ideas than spoken communication (McComb, 1993).
6. Messages can be retained for continued reference and reflection.
7. Files can be attached to messages easing the communication of technical information.
8. It has a democratic and equalizing tendency and it facilitates interaction among people with different abilities, genders, etc.
9. As equality between people increases, more people are allowed to have their say; there is less risk of one single person dominating (McComb, 1993).

Turoff (1989) and Kaye (1992) focused primarily on the benefits of computer conferencing because of its properties particularly suited for support of collaborative activities. Turoff (1989) contended that the primary benefits were not related to the freedom of time and place made possible by the asynchronous mode of communication but rather because "the potential for real improvement in the group process lies in the fact that individuals can deal with the part of the problem they can contribute to at a given time, regardless of where other individuals are in the process" (p. 11). Kaye (1992) included as advantages of computer conferencing, the ability to provide shared working and learning spaces and to create both formal and informal environments. He also commented on the positive aspects of the cumulative record of message contributions in a conference, and the tools provided by the conferencing system for retrieving and organizing messages. In face-to-face environment, contributions and ideas may be fleeting. Subsequent review is dependent on partial recollections or selective note taking. Furthermore discussions must often be halted and resumed at a later date, prohibiting continuous group analysis and reflection. In this way, computer-mediated communication is superior in that it provides a greater potential for reflective and thoughtful analysis and review of earlier contributions, and hence for mutual elaboration and development than with face-to-face communication.

Although synchronous forms of text-based computer-mediated communication such as CHAT are subject to time constraints, they still provide a convenient means of communication without the limitations of location necessary for face-to-face communication. Additionally, because of the sense of "real communication" within a

synchronous computer-mediated communication environment, benefits of this mode include the pressure to react and respond, and the ability to receive direct and immediate feedback.

On the other hand, computer-mediated communication can create serious obstacles to effective collaboration. Kaye (1992) lists among the disadvantages, the lack of control over turn-taking, the difficulties providing contextual cues for framing different discussion genres, and the frequent development of multiple threads of discussion with the same message space. Additionally, text-based computer-mediated communication excludes the use of non-verbal gestures and facial cues that are useful for monitoring a partner's understanding, and eye contact, which appears to be related to metacognitive aspects (Dillenbourg & Schneider, 1995).

### Research on Computer-Mediated Collaborative Learning

Educational Computer-Mediated Communication. The rapidly growing use of computer-mediated communication in education continues to increase the need for developing evaluation methodologies to assess the value of this medium. Many of the standard evaluation techniques have been applied to computer-mediated communication – surveys, questionnaires, interviews, empirical experimentation, participant observation, and case study methodology. Research using these techniques indicates that the nature of instructional interaction among students involved in electronic networking is different than that exhibited by students engaged in typical classroom instruction (Levin, Kim, & Riel, 1990; Riel, 1990; Waugh, Miyake, Levin, & Cohen, 1988).

Waugh, Miyake, Levin, and Cohen (1988) found the nature of students' problem-solving efforts in the online medium quite different than typical classroom-based problem-solving efforts of students. The study analyzed the problem-solving activity of students in remote places involved in an Inter Cultural Learning Network in 1985 and 1986. The authors used four analytical techniques to examine the message interactions of students working on the *Water Problem-Solving Project* via electronic networks. The four analyses used were the Intermassage Reference analysis, the Message Flow analysis, the Message Act analysis, and the Semantic Trace analysis.

The Intermassage Reference developed by Levin, Kim and Riel (1990) analyzed the connections between interactions, determining whether reference was made to previous messages, and if so to what messages. The analysis indicated that the interaction during the *Water Problem-Solving Project* was not simple (Waugh et al., 1988). The interaction was not dominated by key individuals but rather was a free exchange of questions and responses among *all* participants. The authors contend that this occurrence was possibly due to the fact that messages were exchanged as electronic mail, rather than through real-time methods or that each of the participants was fully committed to participation in the activity.

The Message Flow analysis also developed by Levin et al. (1990) plotted the density of messages per unit time, followed across time. The data from the collaborations from the *Water Problem-Solving Project* generated a graphic pattern which indicated that the problem-solving activity "evolved" much as do the other types of activities conducted in the online medium. That is the level of activity began from a single message that

triggered extensive interaction among participants. This initial information exchange shifted to off-line data analysis, and the final messages exchanged concerned student interpretations of the shared idea.

The Message Act analysis, designed by Levin et al. (1990) to identify the functions that each message is to accomplish (e.g. initiation, reply, evaluation), revealed a pattern that indicated that the activity was composed of a single initiation with numerous responses and formative evaluations. Referring to research by Mehan (as cited in Levin et al., 1990), the authors contend that this pattern is not typical of traditional face-to-face classroom interactions. In addition, this analysis showed several of the remote sites exchanging summary data for cross comparisons among themselves.

Finally, the Semantic Trace analysis, developed by Miyake (Waugh et al., 1988), attempted to focus on the nature of the content of student interactions. Miyake began by constructing a collective overview consisting of all the ideas that were contributed by the students within the *Water Problem-Solving Project* and other related concepts that were established as important to the problem of obtaining drinking water. Within this framework, she traced the course of the development of the students' ideas and graphically represented that pattern in chart form. Using the chart as an activity map, information such as where a particular part of the interaction arose, how it became integrated into the previous discussion, and how the focus of the discussion shifted as a result of a particular communication, became apparent. The results of this analysis indicated that the network-based activity resulted in significant contributions to the problem-solving activity from multiple points of view. Based upon these results the

authors' suggest that electronic networking provides a medium that is qualitatively superior to the traditional classroom for conducting certain types of problem-solving exercises.

In another study, McConnell (1994) examined the way in which "time" is perceived and used online, and the dynamics of mixed gender group work online based upon perceptions and experiences of staff and students from a computer-mediated masters degree program at Lancaster University. The students and staff were all members of small groups made up of three to five participants and a tutor. From interviews with students, McConnell concluded that computer-mediated communication produced outcomes superior to face-to-face communication for group work, based upon the following factors: since time and location are not issues for online meetings and time does not necessarily limit computer-mediated groups, there is less, if any, sense of loss of relationship between participants between meetings than in face-to-face meetings; increased contact between group members using computer-mediated communication increases the sense of commitment and ownership concerning group purposes, processes and outcomes; the permanent nature of computer-mediated group work allows members to "re-join" at any time, encouraging continuous thought about the work; various activities of groups using computer-mediated communication are not discrete in the way that they often are in a face-to-face group, thus enabling members to work on multiple issues simultaneously; given the continual and extended nature of computer-mediated collaboration, the groups using computer-mediated communication usually worked more

slowly, were more concerned with the interpersonal relationships of the group, and produced higher quality outcomes than students in a face-to-face setting.

Not all evidence suggests differences between educational interactions in the face-to-face mode and those mediated by computer however. Using a simple turn taking analysis, McConnell found results about dynamics of mixed gender groups using computer-mediated communication similar to those in a face-to-face setting (females being cited as being more "chatty" than males, and often taking more turns than males at talking, and males being cited as saying more or talking the longest).

Furthermore, not all evidence points to the superiority of computer-mediated communication for educational interactions. Hiltz, Coppola, Rotter and Turoff (2000) found mixed evidence indicating that when students actively collaborate, learning and achievement outcomes can be as good as or better than those for traditional classes, but when individuals simply obtain posted material and send back individual work, the results are poorer than in traditional classrooms (Hiltz, Coppola, Rotter & Turoff, 2000).

Although these findings are important to our general knowledge about educational computer-mediated communication, alone they fail to reveal important aspects of what is happening as individuals collaborate in a computer-mediated communication environment.

Evaluation Methodologies for Computer-Mediated Collaboration. Research analyzing the content of educational computer-mediated communication was extremely limited until the early 1990's. The published literature consisted mostly of application-oriented descriptions (Mason, 1992), and very few researchers took on the time-

consuming and challenging task of analyzing the educational quality of conference interactions. The reasons for this were: that there existed no means for dealing with the abundance of information contained in the messages, nor any tried and tested methodologies for assessing educational value (Mason, 1992) or of interpreting the elements of meaning that had significance for the learning process (Henri, 1992); and even with the growing acceptance of qualitative approaches to research, there was still "strong pressure to measure in some way the elements of 'success' or 'learning' or even 'educational exchange'" (Mason, 1992, p. 107). To prevent subjectivity most research on computer conferencing was limited to quantitative analyses of messages sent, who sent them, number and time of logons, and message maps showing message chains and the number of replies. The unfortunate consequence was that those aspects most susceptible to evaluation were often taken as the whole; for example the volume of messages came to signify the efficiency, success and productivity and value of educational exchanges. Conclusions as to the potential of computer-mediated communication were often drawn without a mention of the actual content, much less the value of the interactions (Mason, 1992). In her paper, *Evaluation Methodologies for Computer Conferencing Applications*, Mason (1992) used a quote from Worthen and colleagues to describe modern educational evaluation as "...a multidimensional, pluralistic, situational, and political activity that encompasses much more than simple application of the skills of the empirical scientist" (p. 106). Fortunately, the field of educational evaluation has developed very rapidly over the same period that computer conferencing has been used in educational settings (Mason, 1992).

In 1987 Beckwith attempted to create one of the first interaction analysis instruments with three categories of group interactive behaviors. Category one, creative behaviors, was comprised of two sub-categories, synergistic behaviors (those behaviors which converge toward consensual solution) and synectic behaviors (those behaviors which diverged toward alternative possibilities). The second category, the debilitating category, comprised of self-referencing behaviors (those which direct individual angst and insecurities toward premature task resolution) and polarizing behaviors (those which tend to effect group dissolution through individual isolationism), represents individual actions (Beckwith, 1987). Lastly, category three, the facilitative behaviors consist of group-referencing behaviors (those behaviors which may be applied to transform debilitating behaviors into creative behaviors). Beckwith suggested using the interaction analysis instrument to provide feedback for problem-solving groups, to monitor, control and improve group progress, and to research the group problem-solving process. There was no mention, however, of the validity or reliability of the instrument, nor of whether Beckwith had used it himself. In fact, little is mentioned about this analysis model in most recent studies.

Henri (1992) led the way for content analysis, by developing a framework and analytical model to investigate the learning process and other significant information present in the content of messages exchanged by way of computer-mediated communication. Henri's method of analysis used a qualitative approach and had two characteristics: *a priori* criteria, and a cognitive view of learning (Henri, 1992). Henri used criteria established in advance in order to provide focus for the analysis; an attempt

to analyze all dimensions of computer-mediated communication would have led to an abundance of data, much of it irrelevant to the learning process. The analysis was consistent with cognitive theories of learning, which focus on the process of learning rather than the product, on "what and how the learner understands, rather than on what should have been understood" (Henri, 1992, p. 123), and how the learner manages the knowledge and skills that he or she brings to the cognitive activity (metacognition). The model, which Henri designed to identify the learning processes and strategies selected or developed by learners, highlighted the following five dimensions of the learning process: participation, interaction, social, cognitive, and metacognitive. Henri established operational definitions of each, and identified indicators that would allow evaluators to recognize the occurrences of each element. The dimension of participation was divided into two categories, (a) overall and (b) active participation in learning process, each with separate indicators. Henri's social dimension was limited in depth of analysis and had only one category, for statements not related to formal content or subject matter. Interactivity, included the following seven categories: (a) explicit interaction, (b) direct response, (c) direct commentary, (d) implicit interaction, (e) indirect response, (f) indirect commentary, and (g) independent statements (statements relating to the subject, but which is neither an answer nor a commentary and which does not lead to further statements). Within the cognitive dimension, Henri (1992) focused on the cognitive skills related to critical reasoning activities, and grouped into five categories: (a) elementary clarification, (b) in-depth clarification, (c) inference, (d) judgment, and (e) strategies. Furthermore, to look beyond the presence and frequency of these skills, Henri

distinguished “surface processing” from “in-depth processing” for each category. This analysis enabled Henri to identify the skills linked to critical reasoning and then to evaluate the level of information processing applied by learners in each of the skills. When analyzing the metacognitive dimension, Henri (1992) made a distinction between metacognitive knowledge (“declarative knowledge concerning the person, the task, and the strategies” [p. 131]) and metacognitive skills (“procedural knowledge relating to evaluation, planning, regulation and self-awareness” [p. 131]). Thus, the categories (a) person, (b) task, and (c) strategies were classified as metacognitive knowledge, while the categories (d) evaluation, (e) planning, (f) regulation, and (g) self-awareness fell under metacognitive skills. While the messages were not expected to reveal the totality of the metacognitive process, as metacognitive activity is often not observable, included information in this kind was certainly worthy of consideration. Henri did caution, however, that one could not conclude that the lack of metacognitive activity was due to student weaknesses in this area.

Henri’s (1992) analysis provided a thorough look at five important dimensions of computer-mediated communication in collaborative learning. Since its origin this framework has received enormous attention and has provided a solid basis for other such frameworks.

Gunawardena, Lowe, and Anderson (1997) developed an *Interaction Analysis Model for Examining Social Construction of Knowledge in Computer Conferencing* after analyzing interactions that occurred in a Worldwide Global Online Debate. The online debate gave professionals and graduate students in the field of distance education an

opportunity to come together to contribute knowledge, negotiate meaning, and come to an understanding about an important issue in the theory and practice of distance education. Although Gunawardena and her colleagues found models by Hiltz (as cited in Gunawardena, Lowe & Anderson, 1997), Levin et al. (1990) and Henri (1992) useful as a starting point for analyzing interactions in computer-mediated communication, none of the methods provided enough specificity on how to evaluate the negotiation of meaning and the co-construction of knowledge in computer-mediated communication.

Furthermore, the definitions of interaction provided by these models were not suitable for the pattern of interaction observed in a debate. Gunawardena et al. agreed with Henri (1992) that examination of the actual content of messages is the appropriate means of evaluating the quality of the learning experience, and found Henri's model to be the most promising starting point for interaction analysis. The model created by Gunawardena et al. evolved through an analysis of the entire transcript for (a) type of cognitive activity performed by participants (questioning, clarifying, negotiating, synthesizing, etc.); (b) types of arguments advanced throughout the debate; (c) resources brought in by participants for use in exploring their differences and negotiating new meanings, such as reports of personal experience, literature, citations, and data collected; and (d) evidence of changes in understanding or the creation of new personal constructions of knowledge as a result of interactions within the group (Gunawardena et al., 1997). Based upon the observations Gunawardena and her colleagues developed an outline of the process of negotiation that appears to occur in the co-construction of knowledge. The researchers observed that as the group interacted together more effectively and learned from each

other, the successive stages they went through could be considered forms of higher mental functions. The outline, then, consisted of the five following phases, from lower mental functions to higher mental functions: (a) Sharing/Comparing, (b) Dissonance, (c) Negotiation/Co-construction, (d) Testing Tentative Constructions, and (e) Statement/Application of Newly-Constructed Knowledge. Through applying the analysis to the online debate, Gunawardena et al. found the division of phases somewhat imprecise, and noted that the construction of knowledge could occur in more or fewer phases than described above, and that different phases could possibly occur at the same time among group members. Nonetheless, the researchers were satisfied that this analysis model would provide the means to determine that knowledge construction occurred within a group by means of the exchanges.

In 1995, Newman, Webb and Cochrane also developed a content analysis method to evaluate the quality of group learning via computer-mediated communication. Specifically, the method of analysis was created to measure and compare the critical thinking in face-to-face and online seminars. Newman and colleagues followed Mason's (1992) recommendations to widen the net and include other educational goals such as "critical thinking, deep understanding of course material or broad awareness of issues," and created a detailed content analysis "by breaking these [goals] down into examples of behavior and written work which display these characteristics" (as cited in Newman et al., 1995, p. 59). Their work focused on the cognitive dimension defined by Henri (1992), and specifically the area of critical thinking since "successful group problem-solving processes require critical thinking, leading to the critical understanding needed

for deep learning (Newman et al., 1995). The authors asserted, "Critical thinking is not just limited to the on-off assessment of a statement for its correctness, but a dynamic activity, in which critical perspectives on a problem develop through both individual analysis and social interaction" (p.10). For this reason the focus of their study was on looking for signs of critical thinking in a social context.

In 1996 Newman, Johnson, Cochrane and Webb used the critical thinking indicators for the purpose of content analysis. Similar to Henri (1992), the authors used paired opposites to indicate critical versus uncritical thinking. However, they attempted to address two perceived downfalls that they found with Henri's indicators: (a) Henri's indicators of cognitive skills were fairly broad and therefore could include a number of different activities; and (b) Henri's indicators did not attempt to evaluate the depth of the cognitive skills. Newman and colleagues developed the indicators by simplifying Henri's pairs, looking for indicators in all of Garrison's stages of critical thinking (as cited in Newman et al., 1995), and from prior experience of using similar techniques for assessing student work in computer conferences. The indicators were grouped according to (a) relevance, (b) importance, (c) novelty, (d) bringing outside knowledge/experience to bear on problem, (e) ambiguities (clarified or confused), (f) linking ideas, interpretation, (g) justification, (h) critical assessment, (i) practical utility, and (j) width of understanding, and included specific actions and their relative opposites (e.g. relevant statements vs. irrelevant statements or diversions, and justifying solutions or judgments vs. offering judgments or solutions without explanations or justification.)

The authors used tape-recorded transcripts of face-to-face seminars, and automatically stored transcripts of computer conferences, to classify each statement containing one unit of meaning and illustrating at least one, but possibly more, indicators, for each discussion. Once the scripts were marked, the totals for each critical versus uncritical thinking indicator were counted, and a critical thinking ratio calculated for each, converting the counts to a -1, (all uncritical, all surface actions) to +1 (all critical, all deep actions) scale. The authors claimed that this technique produced a measure that was independent of the quantity of participation, and instead reflected only the quality of the messages (Newman et al., 1995). The content analysis revealed similar depths of critical thinking on several different indicators. However, the investigation found more new ideas emerged in face-to-face seminars, while there were notably more positive ratios in computer conferences for justified and important statements and linking ideas. The authors suggested this occurrence might be explained by the lack of typical brainstorming activities in an asynchronous environment, such as that offered in the online seminar, and the propensity for less spontaneous and more thoughtful contributions within this environment.

The authors stated that the content analysis worked well for the transcripts to which it was applied. Still, they found some practical problems. One was the need for persons with subject specific knowledge to score transcripts. Second, the authors found few obvious statements in the categories of ambiguity and clarity/confusion, practical utility, and width of understanding. They reasoned that it might have been an effect of the nature of the task being a general discussion rather than more specific problem-

solving activities. Finally, some find picking out examples of uncritical thinking difficult. The authors suggest that this content analysis technique is a skill that evaluators need to develop, similar to marking an exam.

Newman et al. attempted to shorten the process of content analysis by choosing to focus only on cognitive factors, in particular, critical thinking, and by attempting to identify fairly obvious indicators. In addition they attempted to reduce the amount of subjectivity within the process, through the choice of indicators, through the analytical method and by the use of control groups. The authors contended that this analysis method is applicable to any group-learning environment, where transcripts of group discussions are obtainable and where the object is to evaluate the learning process taking place, rather than assessing individual student performance.

### Summary

The reviewed studies on mathematical problem solving, collaborative problem solving, computer-mediated communication and computer-mediated collaboration provide a solid foundation on which to investigate the process of computer-mediated collaborative mathematical problem solving. The studies on problem solving and collaborative learning establish the importance of self-regulation or metacognitive abilities to problem-solving ability, and the effectiveness of collaborative techniques for promoting these abilities, as well as other problem-solving skills and problem-solving achievement (Curcio & Artzt, 1992; Schoenfeld, 1988; Stacey & Gooding, 1993). The research indicates that certain behaviors must occur, though, in order for collaboration to be effective for this purpose. It is through task-related discussions, where individuals

justify why something is done and provide explanations for one another, that partners mutually regulate the problem solving activities of others in the group (Curcio & Artzt, 1992; Dillenbourg & Schneider, 1995; Stacey & Gooding, 1993). Dillenbourg and Schneider (1995) identified conditions, including group composition and task, which promote this type of behavior. Their research points to the importance of small groups, and tasks that require a continual group effort for understanding and planning, for effective collaboration.

The research on mathematical problem solving and collaborative problem solving also provides the basis for evaluating collaborative problem-solving protocols. The studies by Schoenfeld (1981), and Garofalo and Lester (1985) were particularly influential in establishing the practice of analyzing problem solving protocols, and their work combined with the study by Stacey and Gooding (1993) lays the groundwork for content analysis of group problem solving interactions.

The research on computer-mediated communication and computer-mediated collaboration reveals the differences in face-to-face communication and online communication, and the difficulties and benefits associated with online learning and collaboration (Kaye, 1992; Levin et al., 1990; McDonald, 1998). The research also confirms the need for methods of assessing the educational value of online interactions and interpreting the elements of meaning that have significance for the learning process (Henri, 1992; Mason, 1992). The Henri study opened the door for content analysis of online learning processes and established a framework for analyzing computer-mediated collaborative efforts. In combination with existing frameworks for evaluating

mathematical problem-solving protocols, it provides a good starting point for the look into the basics of computer-mediated collaborative mathematical problem solving.

## CHAPTER II

## DESIGN OF THE STUDY

Conceptual Framework

The investigation and research methodology used in this study were based on the theories of group learning and problem solving, the evaluation of mathematical problem solving, and the evaluation of online collaboration.

Mathematical problem solving received the majority of its attention in the 1980s, with researchers such as Schoenfeld (1979a, 1979b, 1981, 1982a, 1982b, 1982c, 1987), Garofalo and Lester (1985) focusing on analyzing the behaviors of individuals and groups in the mathematical problem-solving process. Polya (1957) was the first to formally conceive of mathematical problem solving as a four-phase heuristic process (understanding, planning, carrying out the plan, and looking back). From this premise came several other similar theories. Schoenfeld (1981) devised a model for analyzing the problem-solving process that included five heuristic episodes: reading, analysis, exploration, planning/implementation, and verification. He observed that, for successful problem solvers, key metacognitive or self-regulating behaviors occurred between each of the five episodes. Garofalo and Lester (1985) built upon this model by developing a framework for analyzing metacognitive aspects of performance on mathematical tasks. Their framework, consisting of four processes – orientation, organization, execution, and verification – combined the reading and analysis episodes of Schoenfeld's model, as well as omitted the exploration phase found in that model. These processes are related to

Polya's four phases yet are more broadly defined (Artzt & Armour-Thomas, 1992). Artzt and Armour-Thomas developed the most recent framework for protocol analysis of mathematical problem solving in 1992. This framework synthesized the problem-solving steps identified in mathematical research by Garofalo and Lester, Polya, and Schoenfeld, and cognitive and metacognitive levels of problem solving. It focused on collaborative mathematical problem solving, and outlined the interactive relationship between the highly important metacognitive and cognitive processes of problem solving. Episodes defined by this framework include reading, understanding, analyzing, planning, exploring, implementing, verifying, watching and listening, with each episode associated with a level or levels of cognition. Although research in this area has experienced a virtual halt in recent years, current studies continue to use the frameworks of Polya, Schoenfeld, Garofalo and Lester, and Artzt and Armour-Thomas as means to analyze the process of individual and group mathematical problem solving.

Collaborative learning has a long history of research in education (Collis, 1994). With extensive research on small group learning beginning in the 1920s, there were enough studies by 1981 to generate a sample of 122 with characteristics appropriate for meta-analysis (Collis, 1994). The general consensus of this study was that all types of small group learning are more effective than individualistic or competitive learning (Johnson, Maruyama, Johnson, Nelson, & Skon, 1981). Criticism that the methodology used by Johnson and colleagues ignored, among other things, relevant interactions among important variables (Collis, 1994), and the completion of additional studies with differing conclusions, led to a new meta-analysis by Slavin (1987), which used a best-evidence

synthesis. Slavin included the method of group learning involved as a variable of analysis, and selected seven of the most widely used methods found in educational research literature for the study. While a group goal was present, individual accountability was also stressed within all of these methods. Variation within the methods included the equal opportunity for success, team competition, task specialization per group member, and adaptation of instruction to individual needs (Collis, 1994). Slavin's analysis of 68 studies reinforced Johnson's and colleagues' earlier conclusion concerning the overall effectiveness of group learning, but added the importance of the method used by the teacher to organize the group learning experience.

Other researchers have added to the evidence that the teacher's implementation of a group learning strategy is a critical variable on student achievement. For example, Webb (1991) found that group activities structured by the teacher to include the giving and receiving of explanations resulted in significant achievement gains compared to those without this structure. In addition, Johnson and Johnson (1984) provided evidence that criteria related to group activity, such as positive inter-dependence relative to goals, task assignment, resources, and roles, were essential to the success of group learning. Specific to collaborative learning in mathematics, Stacey and Gooding (1993), who investigated the patterns of oral communication associated with successful learning in small groups, found that members of effective groups interacted more, and helped one another more by responding and explaining to a greater extent during tasks.

Educational research appears to have reached a certain level of consensus regarding small group learning, at least in traditional classroom settings. However, with

distance learning and electronically networked collaboration, a new variable has been introduced. The area of research involving computer-mediated communication and small group learning with participants possibly unknown to one another and at a distance from one another is still developing. Studies have shown both similarities and differences in collaborative learning activities taking place within classrooms and those taking place across electronic networks (Cohen & Riel 1989; Riel, 1990; Riel & Levin 1990;). Riel and Levin (1990) concluded that the teacher remains a major factor in the organization and execution of activities. Waugh et al. (1988) found that the nature of instructional interaction among students involved in this medium is different than that exhibited by students engaged in typical classroom instruction. Their research states that, in their view, computer-mediated communication “provides a medium which is qualitatively superior to the traditional classroom for conducting certain types of problem-solving exercises” (p. 1). Still, evidence in this area is quite limited. Most studies have been restricted to surveys, user interviews, empirical experimentation, and computer generated statistical measurements (Mason, 1992). Mason (1992) describes the reason for this as “there are no tried and true methodologies for assessing educational value, and despite the growing acceptance of qualitative approaches to research, there is still strong pressure to measure in some way the elements of ‘success’ or ‘learning’ or even educational exchange” (p. 107). Mason adds that the unfortunate consequence of this is that “aspects most susceptible to evaluation are often taken to define the whole” (p. 107). Even the case study methodology has proven insufficient as a means for providing information about the educational value of online collaborative learning. Although this method gives

a rich and wide-ranging picture of a conferencing scenario, providing multiple points of view and untested theory, the results cannot be generalized to other instances and contexts. As a result, existing evidence is inadequate, and can be misleading (Mason, 1992).

Within the last decade researchers have begun to recognize the importance of looking at the quality of student learning taking place (Mason, 1992; Kaye, 1992). There has been a call for more detailed study of the social and psychological aspects of groups as they work electronically (Collis, 1994), and a deeper look into educational goals such as the social construction of knowledge, cognitive development, and critical thinking. Mason (1992) suggested breaking these goals down into examples of behavior or written work which displays these characteristics, so that it is possible to analyze conference content and draw conclusions about the educational value of a particular online activity. In addition, Mason recommended progressing from description to analysis by using quantitative data to show the extent of student participation, or what percent of the total activity the educationally valuable interactions represent.

The shift to research focusing on content analysis of computer-mediated collaboration has begun. A number of evaluators have attempted to categorize messages according to basic types. Haile (1986) used the categories of organizational issues, technical help, social, and content specific. Similarly, Kaye (1989) categorized messages as technical, social or content-specific. Henri (1992) took content analysis a step further by analyzing the participative, social, interactive, cognitive and metacognitive dimensions of online collaboration. Mason (1992) pursued a different line of content

analysis by attempting to create a typology of conference messages related to the educational values they display. The method involved discovering what, if any, skills and abilities participants were displaying or developing. Questions considered during her analysis included: "Do the participants build on previous messages? Do they draw on their own experience? Do they refer to course material? Do they refer to relevant material outside the course? Do they initiate new ideas for discussion?" (p. 114)

Gunawardena et al. (1997) followed this work by developing an analysis model that examined the social construction of knowledge in computer conferencing by categorizing (a) types of cognitive activity, (b) types of arguments advanced, (c) resources brought in by participants, and (d) evidence of changes in understanding, or the construction of new knowledge. Likewise, Newman et al. (1995) created an analysis method to measure critical thinking by categorizing messages in terms of (a) relevance, (b) importance, (c) novelty, (d) bringing in outside knowledge, (e) ambiguities, (f) linking ideas and interpretation, (g) justification, (h) critical assessment, (i) practical utility, and (j) width of understanding. Although, subtle differences exist between each of these methods of analysis, there remains considerable agreement amongst educational researchers that content analysis is the most effective method of evaluating the quality of online collaborative activities.

The questions stated in Chapter 1 address the study of computer-mediated collaborative mathematical problem solving. It was the goal of this researcher to develop and utilize a framework for coding individual and group behaviors of small groups engaged in online mathematical problem solving; to perform a content analysis to

examine the basics of their collaboration, specifically the participative, social, interactive dimensions throughout the problem-solving process; and to determine the quality of collaboration and its effect on problem-solving success. Through this, the researcher hoped to gain additional insights into collaborative problem solving in the distance setting as well as in the face-to-face setting, in order to help further instructional techniques aimed at improving the collaborative mathematical problem-solving experience in every environment.

#### Description of the Population

The population of this study was twelve current New York City middle school mathematics teachers enrolled in an online mathematics course offered by New Visions for Public Schools, a non-profit educational organization in New York City, in conjunction with the City University of New York (CUNY). The course, entitled *Mathematics for the Middle School Teacher*, occurred from July 2, 2001 through August 10, 2001, and focused on mathematical problem solving in the area of "Number & Operation." The teachers who enrolled in this course were quite diverse in terms of mathematical backgrounds and ability, although a majority had little post-high school education in mathematics or mathematics education. Additionally, only an extremely small percentage was certified to teach mathematics. All teachers in this course elected to enroll in the course to receive graduate credits through CUNY and/or to improve their mathematical knowledge. All members of the population were included in the study. All groups had the same instructor, same materials, same syllabus and same course format.

### Description of Treatments

The primary format for the course was collaborative problem solving, in which student groups worked together to complete a variety of mathematical problems. Students met face-to-face for seven three-hour periods throughout a six-week period to receive instructional materials and technical training, to solve mathematical problems, and to discuss problem-solving solutions and mathematical content. During each of the six weeks of the course student groups were assigned one problem for which online communication tools provided the sole medium for participant communication in the collaborative problem-solving task. The groups were directed to complete all online group tasks in a collaborative manner, with each student contributing ideas, responding to ideas, proposing strategies and procedures, demonstrating solutions, and contributing to summarizing and reporting their groups' findings. Assignments were the same for every group, thereby ensuring identical mathematical content and level of difficulty for all groups. The instructor did not provide assistance to any group during online problem-solving activities.

Online communication took place by way of messages posted on the bulletin board (asynchronous communication), or through the use of the chat room (synchronous communication), both features of the software package Blackboard. Each group had an individual online folder to which all bulletin board messages and chats were automatically saved, so that a complete record of each groups' deliberations were recorded. Additionally, students within each group were able to exchange files with members of their own group within these folders. Students were directed to

communicate about group tasks only through the use of bulletin boards and chat rooms provided by Blackboard. Furthermore, in order to ensure that all information pertinent and relevant to solving the group task was recorded, students were directed to document and reiterate all face-to-face, phone, or email contributions within the bulletin board and chat room communications of the group.

Although regular participation was required in terms of attendance at face-to-face meetings, completion of assignments, and involvement with group problem solving tasks, and students were encouraged to contribute to group collaborations as much as possible, students were not required to contribute a set amount of postings per week, nor required to be present for all chat room meetings. Within weekly time limits, each group was instructed to submit a single solution to the instructor via email, once the group completed the assigned problem to their satisfaction. Groups were responsible for making decisions concerning the write-ups for the solutions. Groups were not allowed access to the deliberations of any other group nor to the submitted solutions. Once all solutions were submitted to the instructor they were discussed as a class within the weekly face-to-face meeting. Written solutions were submitted for each assigned problem-solving activity, and were graded by the instructor and included in the students' final grades. Grading was quantified using a rubric designed by the Northwest Regional Educational Laboratory (NWREL) (refer to Appendix A), and all members of each group received the same grade on each task.

### Sampling Procedures

The subjects of this study were assigned to groups consisting of three members. In order to help eliminate extraneous variables and to diminish variability among the groups, a matching technique was used to assign group members according to mathematical ability, technological experience and gender. Technological experience was assessed by means of a survey included as part of the course application packet (refer to Appendix B). Mathematical ability was determined through the use of a twenty-item test consisting of questions taken from a mathematics placement exam that was created and used by the Department of Mathematics at Montana State University. The questions were designed to assess the students' knowledge of concepts and basic mathematical skills relating to "Number and Operation" and Algebra (refer to Appendix C). The test was administered during the introductory face-to-face meeting, after which time groups were created. Group composition remained the same throughout the study. The limitations associated with the sample are addressed in the sectioned entitled *Limitations and Delimitations*.

### Questions to be Answered

The questions addressed in this study were as follows:

1. What behaviors did small groups, using computer-mediated communication, exhibit in terms of participation and interaction – social and otherwise – while involved in collaborative mathematical problem solving?

2. Was there evidence of characteristic online interactive behaviors that enhanced or diminished the quality of collaboration within a group?
3. Was there evidence of characteristic structures in a group's collaborative behavior that lead to problem-solving success or problem-solving failure?

### Research Design

This study investigated the process of collaborative mathematical problem solving in small groups in an online environment, in order to characterize the participative, social, and interactive problem-solving behaviors of students in these groups throughout the problem-solving process, and to describe the impact of these behaviors on the quality of collaboration and the groups' success or failure in problem solving.

As described earlier, the students in the course were placed into groups of three students, with students heterogeneously matched on mathematical ability, technological experience and gender. Course content and assignments were identical for all groups, as well as their method of communication with the instructor and one another. Although students attended seven face-to-face meetings in order to receive training on the technology and instructional materials, to complete face-to-face tasks, and to discuss problem solutions and mathematical content, online collaborative problem-solving activities were conducted solely using asynchronous or synchronous features of the software package Blackboard. All interactions were recorded on the system, either automatically, in the case of bulletin board message and chat-room contributions, or by a

simple process of saving and/or documenting, for any email. As mentioned previously, all students were assessed in the same manner. For each group assignment a common grade was given to all group members. The course instructor assigned grades with scores ranging from 5-20, based upon criteria from a problem-solving rubric designed by NWREL.

During the third week of the course the researcher performed a content analysis of protocols for each group throughout the process of solving one problem. Refer to Appendix D for the problem and solution. The analysis included all discourse that occurred over a six-day period, from the time the assignment was given on Monday evening to the time that it was submitted to the instructor on Sunday evening. The content analysis was completed using a framework created by the researcher and intended to examine the participative, social and interactive dimensions of behavior in relation to the problem-solving process, particularly throughout the following heuristic episodes: (a) orientation, (b) organization, (c) execution, and (d) verification (refer to Appendix E). Following is a rationale for the analytical framework as well as a description of each of its components.

### Rationale

The framework was an amalgamation of an existing framework by Garofalo and Lester (1985) that categorized the behaviors of students involved in small group mathematical problem-solving activities, a framework by Stacey and Gooding (1993) that focused on coding interactive aspects of collaboration on a mathematical task, and a framework by Henri (1992) that analyzed the participative, social, interactive, cognitive,

and metacognitive dimensions of collaborative learning in the online environment. The need for this framework existed because of the failure of present models to address important issues, aspects, and characteristics specific to the domain of collaborative mathematical problem solving in an online environment. The frameworks by Garofalo and Lester (1985), and Stacey and Gooding (1993), though quite straightforward and effective in a face-to-face environment, did not consider behaviors in the online environment, which lacks the typical immediacy and verbal communication of the face-to-face setting. Similarly, Henri's (1992) model only considered the activity of computer-mediated collaboration and did not consider the particular domain of mathematical problem solving in this environment. There was also a need for a narrowed, more in-depth focus on the fundamentals of collaborative problem solving, namely the dimensions of participative, social, and interactive behavior. Accordingly, this led to the elimination of the cognitive and metacognitive dimensions defined by the Garofalo and Lester (1985), and Henri (1992) frameworks.

Even so, the contribution of each existing model was fundamental to the framework. For example, while on the surface, a combination of the frameworks by Stacey and Gooding, and Henri might seem sufficient to address the aim of the study – to investigate the participative, social and interactive behaviors during computer-mediated collaborative problem solving, the information acquired from those frameworks would lack important aspects of what is known about mathematical problem solving, such as the variations in participation and interaction that can occur throughout the heuristic episodes (orientation, organization, execution and verification) of the problem-solving process.

Likewise, considerations of the online collaborative problem solving behaviors without the components of the Stacey and Gooding model would lack critical details about the types of interactions that occur during the process.

As with all of these existing approaches, the method of analysis used by the researcher included *a priori* criteria. Assigning criteria in advance allowed the researcher to concentrate on a single aspect of computer-mediated collaborative mathematical learning – the problem-solving process as revealed in the messages. The details of the framework, including the theoretical foundation for each of its dimensions follow.

### The Analytical Framework

Within a computer-mediated learning environment there are three facets to discourse, as described by Henri (1992): what is said on the subject under discussion; how it is said; and the processes and strategies adopted in dealing with it. Analysis of the discourse can be done on each of these levels with *what is said on the subject* focusing on understanding and judging the quality of what is being said on exactitude, logic, coherence, relevance and clarity; *how it is said* allowing us to measure how actively the learner acquires information and uses it to produce new knowledge, through the analysis of participation, social presence and interactivity; and *processes and strategies* centering on the processes and strategies adopted by the learners as they reveal them in dealing with the subject.

Henri (1992) contended that the first level of analysis, “what is said on the subject,” deals essentially with the ‘product’ of learning, versus the latter two levels, which reveal more of the ‘process’ of learning. While this study was intended to focus

on process rather than product, the goal was to investigate the essentials of collaboration; therefore the framework was established using only the second level of analysis, and included the following three dimensions: *participative*, *social*, and *interactive*. It is these dimensions that shed light on the basics of *if* and *how* students collaborate to solve mathematical problems in an online environment. In addition, there was a need to know how participation and interactions related to the problem solving process, namely the heuristic episodes. The question was whether or not group members interacted throughout the entire problem solving process, from trying to understand the problem, to establishing and executing a plan, and finally to verifying the solution. Therefore, *heuristic episode* was added to the framework as a fourth dimension. The analytical framework, which provides operational definitions and indicators for each dimension, can be found in Appendix E. The following is an explanation of the theoretical foundation of each.

The Participative Dimension. As stated previously, a mere count of the number of messages is not sufficient to present an accurate picture of student participation. Nevertheless, such quantitative data can be useful in content analysis if it is not the only factor considered, and if it is analyzed in conjunction with data from the analysis of the other dimensions, such as those defined by Henri (1992). This study followed Mason's (1992) recommendation to progress from description to analysis by using quantitative data to show the extent of student participation in order to determine what percent of the total activity educationally valuable interactions represented.

As it is well known that computer-mediated communication environments give rise to various types of conferencing, not all of which are directly concerned with learning activities (Henri, 1992), a distinction was made between two categories of quantitative data: data covering the totality of the statements and questions issued by all participants (overall participation), and data concerning the participation of the learners in the learning activity (task-specific participation).

The Social Dimension. Although the social dynamics of conferencing are considered a factor in participation, social cohesion within a group, and the feeling of belonging (Gunawardena, 1995), as with Henri (1992), this researcher did not want to replicate the sufficient number of studies offering in-depth analyses of social dynamics. Therefore, this study limited its consideration of this aspect to identifying the occurrences of the expression of the social factor.

The Interactive Dimension. Interaction amongst participants is clearly a necessary and critical element of collaborative problem solving. The internalization of cognitive and metacognitive processes implicit in the interaction and communication that occurs while collaborating can directly help to develop general mathematical problem-solving skills and strategies. The strengths of collaborative learning through discussion and conversation include the sharing of different perspectives, the obligation to communicate explanations to others and to make explicit one's own knowledge and understandings, and the motivational value of being a member of a well-functioning group (Kaye, 1992). However, as mentioned previously, communication is not enough.

Especially in an online environment, the presence of communication does not ensure collaboration or even interaction. A conference without interactivity would comprise a series of one-way statements, or monologues linked only by the theme or subject under discussion (Henri, 1992). Existing literature conveys the importance of participants interacting with one another by asking questions and giving explanations and assistance (Webb, 1991; Stacey & Gooding, 1993). In fact, it is clear from the literature that when interaction patterns of groups that learn effectively are compared with those of ineffective groups, students interact more with one another and more with the task (Stacey & Gooding, 1993). In Henri's (1992) framework for content analysis of computer conferencing, she defined interaction as a three-step process:

Step 1: communication of information

Step 2: a first response to this information

Step 3: a second response relating to the first

Her model allowed for interactivity, and for its absence as well. The framework qualified non-interactive messages or statements as "independent." The framework also differentiated between explicit interaction (statements containing specific reference to another message, person or group) and implicit interaction (statements containing an implicit reference to another message, person or group). Thus, the distinctions made for the interactive dimension of her framework were as follows: interactive versus non-interactive, and explicit versus implicit interaction.

Although the researcher for this study found these distinctions important for describing the actual structure of interaction for computer conferences, she did not find

them sufficient for providing insight into what types of interactions occur, and thus the effect of these interactions on the quality of the collaborative problem-solving process. In addition, it was the researcher's opinion that only one response to a statement or question is necessary in order for interaction to have occurred. Henri (1992) defines this as quasi-interactive. However, for the purpose of this study, the researcher defined interactive statements/questions as statements/questions that are connected implicitly or explicitly to one or more other statements/questions within other messages.

As mentioned above, the benefits of collaborative learning occur through the sharing of different perspectives, through asking questions, and through giving and receiving explanations and thus making explicit one's own knowledge and understandings. Stacey & Gooding (1993) developed a framework expressly for coding interactions of small groups in the discussion of a mathematical task. The categories included: asking questions, responding, directing, explaining with evidence, thinking aloud, commenting, and refocusing discussion. Since this research focused specifically on the collaborative process in mathematical problem solving, the researcher of the study considered these categories most appropriate. However, she also made a distinction between interactive and non-interactive statements and questions. Additionally, the category of "other" was added to classify interactive/non-interactive units of meaning that focused on the use of the technology or the process of online collaboration. An explanation for the addition of this category can be found in the section entitled "Considerations of Reliability and Validity."

The Heuristic Episode Dimension. While the existence of communication and interaction between participants remains the most significant element of the collaborative process, the importance of the nature of that communication and interaction, in terms of how, what and when, cannot be underestimated, especially in the realm of problem solving. As stated previously, group behaviors could range from coming together only to delegate individual subtasks, then again only to compile results, to working jointly through each stage of problem solving – analyzing the problem together, planning, executing, and verifying the solution. Questions and explanations between students are valuable, but this interaction can be of little benefit to collaboration and learning if used outside the process of problem solving. It is the communication throughout each problem-solving heuristic episode that promotes the scaffolding of knowledge, facilitates the monitoring and regulation of one another's knowledge and use of strategies, and that contributes to successful problem solving and the internalization of cognitive and metacognitive processes related to the development of mathematical problem-solving skills and strategies. In fact, it is these types of interactions that differentiate collaboration from mere group work.

As noted in the review of literature and conceptual framework, there are several frameworks specifically intended for analyzing the mathematical problem-solving processes of individuals and groups. Although there are slight differences between each of these models, all of the frameworks categorize mathematical problem-solving behaviors into heuristic episodes of similar nature. In fact, all agree that the problem-solving process includes a stage (or stages) at which students analyze and orient

themselves with the problem, one at which they plan a strategy, one at which they execute that strategy, and finally one at which they verify execution of the strategy and/or the solution. The researcher chose to use the heuristic episodes defined by Garofalo & Lester (1985), as they uphold this theory while remaining simple and clear-cut. The episodes and their descriptions are as follows:

1. Orientation: The stage during which individuals or groups exhibit strategic behavior to assess and understand a problem. This stage includes comprehension strategies; analysis of information and conditions of the problem; assessment of the familiarity and level of difficulty of the task; and the development of initial and subsequent mathematical representations of the problem.
- 2) Organization: The stage during which individuals or groups make plans concerning their behavior, and choose their actions. This stage includes the identification of goals and subgoals, and global and local planning.
- 3) Execution: The stage during which individuals or groups regulate their behavior to conform to plans. This stage includes the performance of local actions, monitoring of progress of local and global plans, and trade-off decisions such as speed versus accuracy.
- 4) Verification: The stage during which individuals or groups evaluate decisions made and of outcomes of executed plans. Verification includes the evaluation of elements of orientation and organization such as the adequacy of representation, adequacy of organizational decisions, the consistency of local plans with global plans, and the consistency of global plans with goals. It also includes the

evaluation of elements of execution and its subsequent outcomes such as the adequacy of performance of actions, the consistency of actions with plans, the adequacy of local or final results, the consistency of local results with plans and problem conditions, and the consistency of final results with problem conditions.

### Considerations of Reliability and Validity

There were several factors necessary to consider regarding the reliability and the validity of the study. First, there were concerns related to the analytical framework. Since the data analysis of content in the transcripts of small group discussions involved a complex coding scheme, improper use of the instrument posed a possible threat to reliability and validity. In addition, there is always a certain amount of subjectivity within the process of content analysis. The researcher attempted to reduce such subjectivity and establish validity through her choice of clearly defined indicators based on an extensive literature review, and through a process of piloting and revising the instrument prior to its use in the study. The pilot study, which was completed two months prior to the study and consisted of coding random sections of the online problem-solving transcripts from the first week of the course, revealed the need for only one significant adjustment. Through the preliminary analyses, the researcher recognized that certain messages could not be categorized as problem solving related. These messages concerned the use of the technology or the process of collaborating online rather than the actual process of solving the problem, and would clearly not occur in a face-to-face collaborative effort. Still they were directly concerned with learning activities and critical to the progression of the group effort. For that reason, the researcher added the

category of "other", as indicated briefly in the section entitled "The Interactive Dimension."

The pilot study also revealed the need for a way of coding messages such as "ummm," "...," or "ok," which were without meaning. Accordingly, for the primary study, these messages were marked NC (not categorized) and were included in the total number of messages, but not counted as units of meaning. Furthermore, observations from the pilot study revealed that once a unit of meaning was classified as "social" it was meaningless to code it within the categories of "interactive vs. non-interactive", "problem-solving vs. other," "interaction type," and "heuristic episode." Similarly, since units of meaning labeled as "other," and those categorized as "commenting" were either affective in nature, or did not relate directly to solving the mathematics problem, it was judged unnecessary to code them along the dimension of "heuristic episode."

Since the use of additional coders presented an immense training task that was not justified in terms of the volume of data to be coded, only the researcher coded all "official" transcripts. However, for the purpose of determining inter-coder reliability a randomly selected section of the transcript containing 50 messages was coded by both the researcher and an additional trained coder. Each unit of meaning within the messages was coded along six characteristics – participant, social vs. not social, interactive vs. non-interactive, problem solving vs. other, interactive category, and heuristic episode – and hence had six points at which coding agreement or disagreement could occur. For each coding agreement, one point was given. For example, if there were 20 units of meaning, total agreement in coding would result in 180 points. Inter-rater reliability was then

determined by dividing the points given for coding agreement by the number of points possible for total coding agreement. In this way, the inter-rater reliability for the framework was determined to be 81%. The researcher determined stability over time in the same manner, by re-coding a random section of the transcript two months after its first coding. Test-retest reliability was determined to be 86%.

Another concern was that the nature of the online medium could hamper the ability to achieve full awareness of all individual problem-solving behaviors, such as reading the problem and performing computations. Though this could be considered a threat to validity, the researcher did not consider it a weakness in the methodology. Rather, she felt it served to elucidate the differences between collaborative problem solving in a face-to-face environment and that mediated by computer, because in the end those individual behaviors seen by the observer are also all that is seen by the group members.

There were additional concerns related to the subjects and the treatment of the study. The lack of mathematics courses offered in an online environment with a focus on collaborative problem solving, the small number of students in the total population, and the nature of enrollment (self-selection) prevented randomization of subjects. However, the primary aim of this research was not to generalize results, but rather to take a preliminary and in-depth look at the processes of computer-mediated collaborative mathematical problem solving in order to develop theories about the relationship between the participative, social and interactive dimensions throughout the problem-solving process and the effect of these dimensions on problem-solving achievement.

Nonetheless, matching of subjects based upon gender, mathematical ability, and experience with online technology was used in an attempt to balance the abilities of all groups. To further ensure the internal validity of this study, the possible effects of history, maturation, and selection were addressed. The analysis of data showed no evidence of effects of history, and maturation. The greatest threat to the internal validity of this study was selection. Although efforts were made to equate groups on ability, there does exist the possibility that the groups, after being assigned, were different in ability levels at the beginning of the study. This must be considered when interpreting the results of this study, and therefore it is listed as a limitation of this study.

Extraneous variability created by the medium for course delivery was also a possibility. Difficulties such as the inability to use the technology or a lack of comfort with the technology might have affected the participation of group members. Efforts were made to address this variable by requiring attendance at a technology orientation prior to the course, and by providing online technical assistance to all students. Furthermore, studies have shown that once students become familiar with the technology, they tend to become more active participants (Beaudrie, 2000). For this reason, the researcher delayed content analysis until midway through the course, with the goal of diminishing variability caused by this factor.

A final possible threat to validity was the Hawthorne Effect, which is the tendency of subjects under study to exhibit atypical behavior because of their awareness of being studied, or because they are receiving special treatment. Although this was indeed a possibility with the subjects in this study, the fact that participants enrolled in

the course voluntarily, and that no changes were made to the structure of the course, the occurrence of this effect was minimized. Nonetheless, as with all of the concerns above it was necessary to consider and therefore acknowledged as a limitation of the study.

### Methods of Data Collection

#### Individual and Group Behavior

Data on individual and group behaviors was collected by observing and performing a content analysis on transcripts to all online problem-solving dialogues, and coding every unit of meaning contained within the messages along the participative, social, interactive and heuristic episode dimensions, using a framework developed by the researcher. The framework, which was a synthesis of frameworks by Garofalo and Lester (1985), Stacey and Gooding (1993), and Henri (1992), was designed to analyze the participative, social, and interactive behaviors of small groups involved in computer-mediated collaborative mathematical problem solving during each of four heuristic episodes (see Appendix E). Indicators were categorized under the following categories:

1. Participative
  - a) Overall
  - b) Task-specific
2. Social
3. Interactive

Included under this category were the following types of interactions:

- a) Problem Solving

- i. Questions
- ii. Responses
- iii. Directions
- iv. Explanations
- v. Think aloud
- vi. Proposal of ideas
- vii. Refocus of discussion
- viii. Comments

b) Other

#### 4. Heuristic episode

Included under this category were the following heuristic episodes:

- c) Orientation
- d) Organization
- e) Execution
- f) Verification

For a detailed explanation regarding the use of this framework for analysis of content refer to the section entitled *Analytical Techniques*.

#### Group Problem-Solving Achievement

Group problem-solving achievement was measured simply by the score earned by each group on the assigned weekly task. Only one score was given to all members of each group regardless of individual contributions to the solving of that task. Based upon the experience of Beaudrie (2000), who analyzed online problem-solving behavior and

found that a holistic rubric resulted in little variation in group problem-solving scores, scores were determined using a detailed analytic rubric developed by the Northwest Regional Educational Laboratory (NWREL). The researcher chose this rubric based upon its clear delineation of the criteria for evaluating five different traits of problem solving: (a) conceptual understanding, (b) strategies and reasoning, (c) computation and execution, (d) insights, and (e) communication. The researcher believed that the division of the rubric into five separate well-defined categories adequately addressed the aforementioned issue.

As described in the rubric, students exhibited good conceptual understanding of the mathematics in a problem if they chose appropriate representations, used relevant information, used mathematical terms precisely and selected applicable mathematical procedures. Students demonstrated their ability to use strategies and reasoning by investigating and selecting appropriate problem-solving strategies and conducting a logical, well-planned and supported process that led to a reasonable solution; all forms of representations were consistent and integrated into their solution, progress was self-monitored and adjustments were made as needed, and work was verified or a proof of its correctness was provided. Students exhibited mastery of computation by accurately executing all procedures, correctly applying and labeling all visual representations (charts, tables, graphs, etc.) of the problem and demonstrating the correct use of available technology or manipulatives. Students showed insight into a problem when they recognized the significance of the problem in its relationship to other problems or in its connections to other disciplines or "real-world" applications; recognized patterns

embedded in the problem, discovered multiple approaches and/or solutions, or created a general rule or formula. Finally, students demonstrated good communication when they described clearly what they did and why they did it; their explanation flowed in a logical, fluent sequence, and their message was direct, purposeful, and well organized. Each of the five above described traits was given a point value ranging from 1 (low - emerging) to 4 (high - exemplary) based upon the descriptors provided in the rubric (see Appendix A). A total score for each problem was calculated by totaling the scores for each of the five traits. As a result, scores for each problem ranged from 5 to 20. The matters of validity and reliability of the rubric were addressed previously by the creators of the NWREL rubric. Content validity was supported by a review of the literature and existing scoring guides, and extensive pilot testing. Inter-rater reliability for the instrument was 85%.

#### Analytical Techniques

The primary undertaking of this research consisted of analyzing the message transcripts of four collaborative groups throughout one mathematical problem-solving task. Although the questions from the section entitled "Questions to be Answered" were addressed principally through a qualitative description of individual and group behaviors, the use of quantitative data was used to show the extent of student participation and to provide essential details concerning the amount of educationally valuable interactions. This approach was used for this study in order to provide a richer picture of the nature of computer-mediated collaborative mathematical problem solving than would be possible with the use of a single method. While numbers proved useful for looking at overall

patterns of individual and group participative, social and interactive behaviors, and for identifying and expressing possible relationships between these behaviors and problem-solving achievement, a rich description offered a more holistic picture of the interplay between individual and group behaviors within the problem-solving groups and of the effect of these behaviors on the problem-solving process and the level of collaboration that took place.

Quantitative data regarding student behavior was categorized as follows: data covering the totality of participation by all participants, and data concerning the participation of the learners in the learning activity (task-specific). Total participation was quantified as the total number of messages sent by individuals and groups. Additionally, however, there was a need to distinguish between the participation of individuals who sent few messages that contained multiple ideas, trains of thought, and questions and/or statements regarding various matters, and those that conveyed only one thought or idea over the course of two or more messages. For this reason, the researcher used Henri's (1992) method of dividing messages into units of meaning (refer to definition in chapter 1). In so doing, the total number of units of meaning for each individual and group provided additional enumeration of total participation and became the principal unit for coding. Each of the units of meaning was then analyzed and coded along the social and interactive dimensions, and for units of meaning classified as problem solving related along the heuristic episodes of problem solving, using the *Matrix for Message Analysis* found in Appendix F. For clarity, the process for coding a message

is illustrated in the Message Coding Flowchart in Figure 1. Using the flowchart, the following message example can be coded:

“Hi guys, I’m back! I’m having real problems with my Internet connection. Doesn’t the problem say that we are supposed to find the largest rectangular number? Why don’t we eliminate all of the prime numbers and then make a chart with the rest of the numbers.”

Within the example message the participant makes a greeting, makes a comment about the technology, asks a question during the heuristic episode of orientation, and then proposes an idea as part of the organization stage. The message must first be divided into three units of meaning. The first unit of meaning is social in nature, therefore an X is placed in column four of row one in the matrix in Figure 2, and the process is continued with the second unit of meaning. The second unit of meaning concerns the use of technology. Assuming that it is not interactive, the coding is as shown in row two of the matrix in Figure 2. Notice that since the unit of meaning has been categorized as “other,” it is no longer categorized along the “interactive category” or “problem solving heuristic episode.” The third unit of meaning is clearly a question and relates to understanding the problem, and within the fourth unit of meaning the participant proposes an idea to solve the problem. Assuming the second and third units of meaning were both interactive, the coding in this case is presented in Figure 2.

Figure 1. Message Coding Flowchart

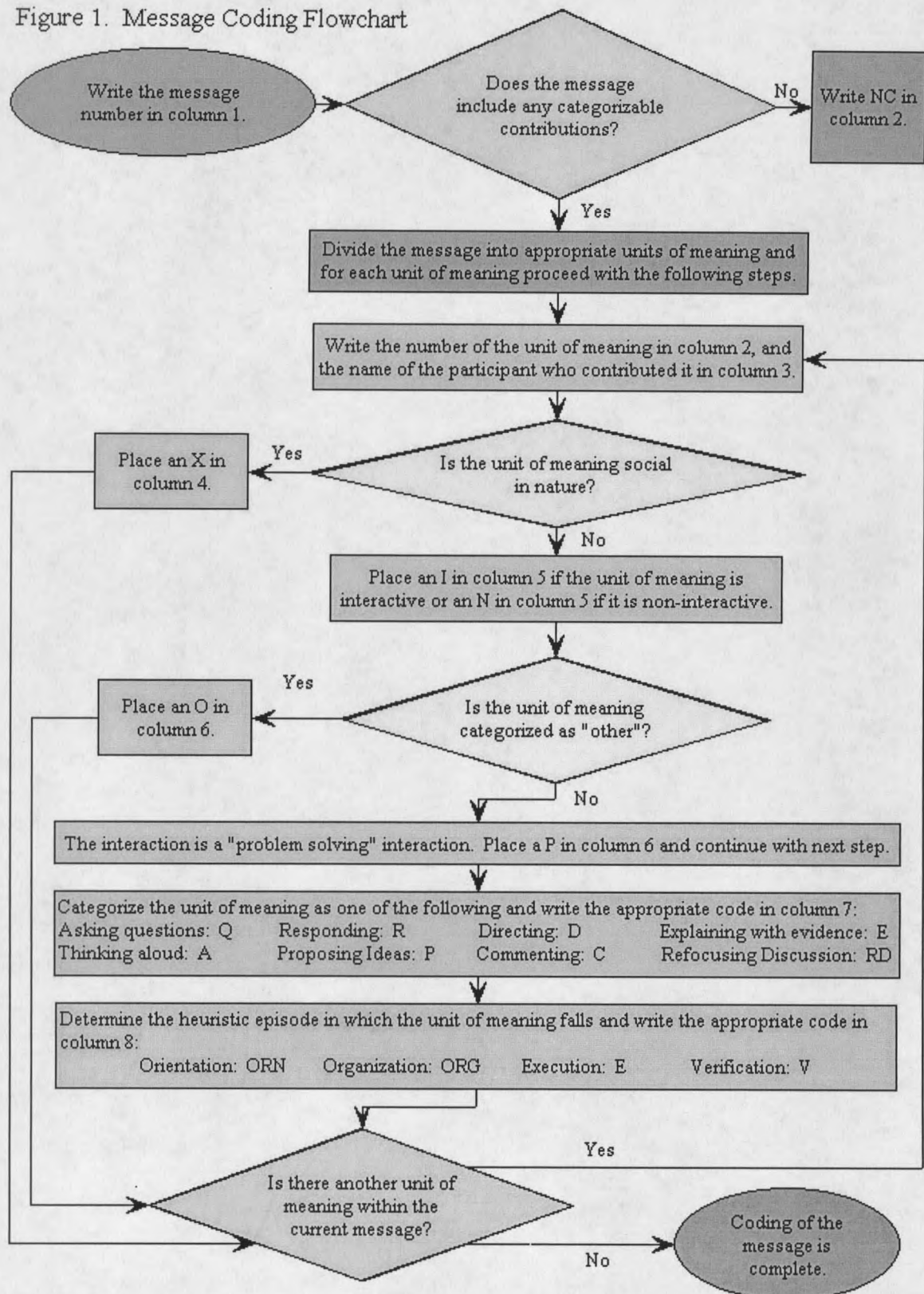


Figure 2. Sample Coded Message Matrix

MESSAGE	UNIT OF MEANING	PARTICIPANT	SOCIAL DIMENSION (X MARKS OCCURRENCE)	INTERACTIVE (I) VS. NON-INTERACTIVE (N)	PROBLEM-SOLVING (P) VS. OTHER (O)	INTERACTIVE CATEGORY	PROBLEM-SOLVING HEURISTIC EPISODE
#1	1	Name	X				
	2			N	O		
	3			I	P	Question	Orientation
	4			I	P	Proposing Ideas	Organization
#2	5						
	6						
	7						
	8						
	9						

Total participation was measured using the total number of units of meaning issued, and task-specific participation was measured using the number of units of meaning directly related to the learning activity. Task-specific participation was further separated into two categories: units of meaning directly related to solving the assigned mathematics problem (labeled “problem-solving”), and units of meaning related to the use of online technology and to the process of online collaboration (labeled “other”). The percentage of *social* contributions was defined as the number of occurrences of the expression of the social factor, divided by the total participation. Percentages for *problem-solving* participation and *other* contributions were calculated by dividing the number of units of meaning defined as problem solving and other, respectively, by the total participation. Percentages for interactions classified as questions, responses, directions, explanations, think aloud, proposals of ideas, commenting, and refocusing of discussion were calculated similarly.

### Limitations & Delimitations

The limitations of this study were as follows:

1. The groups of this study were limited to students taught by one teacher, in one course offered by New Visions for Public Schools and City University of New York.
2. The population of the study was limited to twelve students.
3. The researcher was not able to control for effects of history, maturation and selection on students.

The delimitations of the study were as follows:

1. The period of the study was limited to one six-week course.
2. Content analysis was limited to dialogue occurring during one week of the course and related to the collaborative effort to solve one mathematical problem.
3. The population of the study was enrolled in a course focusing on mathematics concepts surrounding "Number & Operation."

## CHAPTER III

## DATA ANALYSIS AND FINDINGS

Introduction

As discussed previously, the preliminary focus of this research consisted of analyzing the message transcripts to all online problem-solving dialogues from four collaborative groups throughout one mathematical problem-solving task. By coding messages along six dimensions, using a framework developed by the researcher, data on the participative, social, and interactive behaviors for each group and group member, throughout the problem-solving process, was collected. This chapter contains three sections. The first section, entitled "Description," includes a general account of the collaborative problem-solving effort of each group and is aimed at establishing a context for data reported, analyzed and synthesized within the subsequent sections. The next section, entitled "Analysis," includes quantitative data for and qualitative descriptions of individual and group behaviors, and reflects the findings of the content analysis in terms of individual elements of collaboration. It includes the following areas:

1. Total participation of the group during the entire problem-solving process
2. Social contributions
3. Task-specific participation of the group during the entire problem-solving process
4. Task-specific participation of the group during each heuristic episode
5. Problem-solving interactions throughout the entire problem-solving process

6. Other interactions throughout the entire problem-solving process
7. Problem-solving achievement

The final section, entitled "Synthesis," examines the connections and interdependence of the elements from above in order to address the question of how the collaborative system functioned as a whole, and to present a holistic picture of the joint problem-solving effort and achievement of each group within the online environment.

### Description

#### Blue Group

The blue group's participation was extremely limited with a total of only three asynchronous messages exchanged between two members of the group. The units of meaning within these messages were limited to a short series of thoughts and questions about the problem, and a message regarding the inability to utilize the chat room within the virtual classroom. There appeared to be a little connection between the group members, as interpreted by the lack of interactive units of meaning and related responses. All task-specific units of meaning for this group fell within the orientation stage of problem solving, and there was no evidence that suggested whether the task was delegated or divided, whether it was completed or even if any progress was made.

#### Green Group

The green group made use of both types of communication, exchanging initial comments and questions about the problem through bulletin board messages, completing a considerable amount of collaboration on the problem synchronously, and then once

again communicating asynchronously as they completed the problem. Asynchronous messages within this group also included a series of thoughts and questions about the problem. However, in addition there were also messages regarding when the group would meet in the chat room. As with the blue group, there was a noticeable lack of related responses and interactions between group members when using asynchronous communication. However, the presence of true teamwork was evident for this group as they used the chat room extensively throughout the problem-solving process in an attempt to understand the problem, brainstorm solutions and even to perform the mathematics. Synchronous collaboration within the green group was divided into three different sessions. The first session began with all three members present, but moved to two members after one member became frustrated and departed the chat room. The following two sessions were limited to two members – the member who had withdrawn earlier and one of the others. During these two sessions much of the dialogue consisted of one member providing clarification and mathematical explanations to the other member. Still collaborative efforts of this pair seemed to result in progress toward finding the solution to the problem. At the conclusion of the third session a plan for solving the problem had been produced and execution of the plan had begun. The final two messages exchanged by the group were asynchronous in nature and focused on verifying the answer and logistics related to submitting the assignment. Unlike the asynchronous messages sent at the beginning, these messages were interactive and well related.

Red Group

The red group also utilized both the asynchronous and synchronous modes of communication, although not as extensively or as productively as the green group. The members of this group met for several very brief chat room sessions and accomplished little in the way of collaborative problem solving, focusing primarily on discussing matters of completing and submitting the assignment and using the technology to do so. Dialogue during many of these sessions related to individual work that had been posted in the file exchange. The following remarks during the first session exemplify the independent efforts that occurred throughout the entire process:

Mara: Mary, I put my files in the exchange if you want to look at them.

Mary: I'll take a look at your file. I also have a response in the exchange.

Mara: Mary, I can only be on for 20 more minutes so let me know what you think of the file. I'll be on tomorrow night as well.

Mara: I have two files on the exchange.

*A short time later*

Jane: Hello ladies. I finally got on also.

Mary: Mara I saw your files. I feel that 999,960 is a better number since it has the factors 2, 5, 3 and it is very close to 1 million.

Mary: Jane what's happening? Why don't we meet again tomorrow night since Mara has to go now?

Jane: Mary: 999,960 is interesting. What determines the rectangular properties?

Jane: Ok with. I will look at the exchange files and add what I can contribute.

Jane: I am going to look in the exchange files now. Is anyone still there?

Mara: Hey guys, sorry, I didn't really understand the multiple personalities problem. 750 was just an example, I'm still going through numbers.

Mara: ok, look over my files, I'm going to be on again tomorrow.

In the same way, asynchronous communication consisted of only two messages, both exchanged at the end of the process and both focusing largely on verifying the solution to the problem.

### Yellow Group

The yellow group limited their participation to the synchronous mode of communication. The majority of the discourse for this group consisted of social dialogue interspersed with discussions related to finding a quick way to solve the problem, whether by formula or through online resources. While there were some collaborative efforts aimed at understanding the problem they were always related to the attempt to apply a formula. Near the end of the transcripts messages revealed a sudden possible solution to the problem followed by a hasty decision to write up the solution with no sign of verification.

### Analysis

#### Total Participation

It is evident from the above descriptions that participation varied significantly amongst the four groups. The blue group, whose participation was limited to use of the group bulletin board system, sent a mere three messages and a total of nine units of

meaning within those messages. Conversely, the yellow group limited its dialogue to synchronous messaging, but exchanged a total of 371 messages (347 categorized messages) and 366 units of meaning. The green and red groups used a combination of the two modes of communication to send a total of 652 messages (587 categorized messages) and 479 units of meaning, and 103 messages (96 categorized messages) and 121 units of meaning, respectively. Individual participation within the groups varied greatly as well, with no contributions for one member in the blue group and 240 units of meaning for one member in the green group.

As suggested in the previous paragraph, participation included two elements: the number of messages sent and the number of units of meaning contributed. Both components are important to our understanding of online group communication. While some individuals communicated many important ideas within a single message, others used several messages to convey only one thought, and still other messages were without meaning. A ratio of the number of units of meaning to the number of messages provides a more precise picture of how individual group members communicated with one another. The blue group had a ratio of 3 units of meaning per message; the green group's ratio was 0.73; the red group's ratio was 1.17, and the ratio for the yellow group was 0.99 units of meaning per message. Clearly ratios above one indicate longer messages with more than one unit of message, and ratios less than one signify messages that do not contain a complete thought.

On the other hand, although the number of messages sheds significant light on how individuals communicated, the number of ideas shared remains the most important

quantity for determining individual contributions to the group. Therefore, in order to provide an accurate representation of group and individual involvement and for calculating percentages of different types of contributions, the numerical value assigned to all future discussions of *total participation* for individuals and groups will be the number of units of meaning contributed.

### Social Contributions

This study limited its consideration of the social aspect to identifying the occurrences of the expression of the social factor. Their importance is only to the extent that they existed and therefore accounted for a certain percentage of the total units of meaning exchanged within the groups. The percentage of social contributions within each group was calculated by dividing the number of units of meaning classified as social by the total participation. The yellow group had the highest percentage of social contributions with 36.6%, followed by the red group with 27.3%, the blue group with 22.2%, and finally by the green group with 7.3%.

### Task-Specific Participation

Task-specific participation refers to all units of meaning that were directly related to the learning activity. The percentages for task-specific participation of each group were as follows: blue group: 77.8%; green group: 92.7%; red group: 72.7%; and yellow group: 63.4%. Since not all messages were interactive in nature (refer to section entitled "Analytical Framework" for definition), task-specific participation was separated into two categories: interactive and non-interactive. Within the blue group, 42.9% of

task-specific participation was interactive in nature. Likewise, the percentages for interactive task-specific participation for the green, red and yellow groups were 84.5%, 61.4%, and 88.4%, respectively. These percentages were calculated by dividing the number of interactive task-specific units of meaning by the number of task-specific units of meaning.

Task-specific participation was further divided into the following two categories: units of meaning directly related to *problem solving*, and *other* units of meaning related to the use of online technology and to the process of online collaboration. The percentages for units of meaning classified as *problem solving* and those classified as *other* are below. A distinction has been made between interactive and non-interactive units of meaning for both categories. The percentage of units of meaning classified as interactive problem solving was calculated by dividing the total number of units of meaning classified as interactive problem solving by the total number of task related units of meaning. Percentages for units of meaning classified as non-interactive problem solving, interactive other, and non-interactive other were calculated in the same way.

Table 1. Task-Specific Units of Meaning.

Group	Problem Solving		Other	
	Interactive	Non-Interactive	Interactive	Non-Interactive
Blue Group	42.9%	42.9%	0%	14.3%
Green Group	71.2%	12.6%	13.3%	2.9%
Red Group	13.6%	13.6%	47.7%	25.0%
Yellow Group	39.7%	5.6%	48.7%	6.0%

Problem-Solving Contributions. Problem-solving contributions included interactive and non-interactive statements and questions related to the process of solving the mathematical problem. The percentages of problem-solving contributions for each heuristic episode were calculated by dividing the number of units of meaning in each heuristic episode by the total number of task related units of meaning. The figures for each group are shown in the Table 2.

Table 2. Percentage of Task Related Participation for Each Heuristic Episode.

Group	Orientation	Organization	Execution	Verification
Blue Group	85.7%	0%	0%	0%
Green Group	42.8%	11.5%	16.7%	4.1%
Red Group	5.7%	1.1%	0%	10.2%
Yellow Group	14.7%	13.0%	11.6%	5.2%

Problem-solving contributions were further sorted into the following subcategories: questioning, responding, directing, explaining, thinking aloud, proposing ideas, refocusing discussion, and commenting. The overall percentage for each type of problem-solving interaction was calculated by dividing the total number of units of meaning for that category by the total number units of meaning related to problem solving. Since these categories were intended to describe types of group interactions, only units of meaning classified as "interactive" were considered within this calculation. Even so, the existence and importance of non-interactive statements should not be overlooked, and therefore will be discussed in the section entitled "Synthesis." The breakdown can be seen in Table 3.

Table 3. Total Percentage of Interactive Problem-Solving Contributions.

Interaction Category	Blue Group	Green Group	Red Group	Yellow Group
Questioning	33.3%	25.0%	8.3%	29.3%
Responding	33.3%	29.7%	41.7%	29.3%
Directing	0.0%	4.1%	0.0%	3.3%
Explaining	0.0%	8.5%	16.7%	0.0%
Thinking Aloud	0.0%	13.6%	8.3%	26.1%
Proposing Ideas	0.0%	8.5%	16.7%	3.3%
Refocusing Discussion	0.0%	0.6%	0.0%	1.1%
Commenting	33.3%	9.5%	8.3%	7.6%

Other Contributions. The ability to successfully collaborate in the online environment was dependent on more than just the group and individual behaviors and interactions related to problem solving. Units of meaning related to the use of the technology and the process of online collaboration were found to be important to the functioning of the group, as the dependence on technology to collaborate required group members to communicate about issues such as when to meet, how to communicate, and the difficulties of doing both. The percentage of "other" units of meaning was calculated by dividing the number of units of meaning classified as other by the number of task related units of meaning. These units of meaning accounted for a notable percentage of task related interactions within almost every group, with the blue group having 14.3%, the green group having 16.2%, the yellow group having 54.7%, and the red group having the highest percentage at 72.7%.

### Problem-Solving Achievement

Group problem-solving achievement refers to the scores assigned to each group on the assigned weekly task. Scores were determined using an analytic rubric developed by the Northwest Regional Educational Laboratory (NWREL), and reflected the evaluation of five different traits of problem solving: 1) conceptual understanding, 2) strategies and reasoning, 3) computation and execution, 4) insights, and 5) communication. Each of the five above described traits was given a point value ranging from 1 (low - emerging) to 4 (high - exemplary) based upon the descriptors provided in the rubric (see Appendix A). A total score for each problem was calculated by totaling the scores for each of the five traits. As a result, scores for each problem ranged from 5 to 20. The total scores for each group were as follows: blue group – 13 points; green group – 20 points; red group – 19 points; and yellow group – 15 points.

### Synthesis

The above descriptive accounts and analysis make quite apparent the variation on every level in the collaborative problem-solving behaviors within the four groups. Still, the numbers alone can be misleading. For example, while certain percentages for the blue group suggest constructive behaviors (e.g. 77.8% task-specific participation), the extreme lack of overall participation within the group gives a different account of their collaborative efforts. Indeed, collaborative problem solving must be considered a whole system that is both greater than and different from its parts. As such it cannot “validly be divided into independent parts as discrete entities of inquiry because the effects of the

behavior of the parts on the whole depend on what is happening in the other parts” (Patton, 1990, p. 79). The following sections address this issue by providing a synthesis of the variables that shaped the complete picture of online collaborative mathematical problem solving in this study.

The first and most visible difference between the groups was the mode of online communication that each group used. The blue group limited their communications to the asynchronous mode, while the yellow group limited their collaborations to synchronous communication and the green and red groups employed both modes of communication. The most significant consequence of this variable was the types of messages sent by group members. This can be understood by taking a closer look at the ratios of units of meaning to messages, for the four groups in the Table 4.

Table 4. Average Units of Meaning per Message.

Group	Asynchronous	Synchronous
Blue Group	3.0	
Green Group	5.25	.71
Red Group	1.5	1.17
Yellow Group		.99

In general, the data confirms existing research that computer-mediated interactions can range from rapid and spontaneous exchanges of short messages, to relatively lengthy texts. The ratios above suggest that the asynchronous messages tended to contain the more lengthy messages with multiple units of meaning, while the shorter messages with oftentimes-incomplete thoughts were more prevalent in the synchronous mode. Transcripts within the synchronous mode appeared to imitate collaboration in the

face-to-face mode, with a mutual, continual, and spontaneous exchange of ideas and feedback. The number of messages that were not categorized within the chat room transcripts (e.g. "hmmm," "ok," "...") of each group is evidence of the unplanned nature of synchronous collaboration.

It is not clear why groups chose to use synchronous or asynchronous modes of communicating. Although it may have been discussed during face-to-face meetings, there was extremely little online dialogue about the options and no evidence that the use of one or both types of communication was an explicit decision made within the groups. In fact, the only group to discuss the options did so only briefly during their third chat session and showed signs of confusion about the use of the asynchronous option, as illustrated by the following exchange of messages:

Shawn: Ann glad to have you. Long distance learning is going really slow. Are we going to try the discussion board?

Cindy: Why do you want to go to the discussion board?

Ann: This only works if you have all of the information that you need and we are ready to make a decision on how to present it.

Shawn: Isn't that the place for actual discussions?

The last message received no response and the short discussion ended abruptly as group members returned to communicating synchronously about other matters.

It was apparent, however, that asynchronous communication was very limited even within the groups that used it. The blue group exchanged a total of only three messages, the green group exchanged four asynchronous messages and the red group

exchanged only two asynchronous messages. Furthermore, in the case of all three groups, the asynchronous communication between group members seemed to do little to advance the group problem-solving efforts. In fact, the number of non-interactive units of meaning within this mode points to little consideration of previously posted messages. Four of seven of the blue group's asynchronous task related units of meaning were non-interactive in nature. Similarly, the majority (13 of 17) of the green group's asynchronous communications were non-interactive. Although the majority of task related units of meaning for the red group were not non-interactive in nature, the percentage was still quite high with one of three units of meaning being non-interactive. Consequently many questions remained unanswered, thinking aloud received little feedback, and proposed ideas were not pursued. Interestingly, most of the asynchronous non-interactive units of meaning occurred within messages that were sent at the beginning of the problem-solving process. Like one might imagine the first stages of collaborative problem solving in the face-to-face environment, the first set of asynchronous messages in the blue and green groups seemed to represent an initial brainstorming phase where students simply contributed thoughts and ideas without reflecting on or critiquing those of others. Conversely, the asynchronous messages at the end of the online collaborative process proved to be much more interactive and related in nature. During these exchanges group members verified the solution to the problem and discussed technological issues related to completing or submitting the problem.

The synchronous mode appeared to promote much greater collaboration and mutual elaboration and development within the groups that used it. Foremost, there were

a much smaller percentage of non-interactive units of meaning within this mode. The percentages of non-interactive contributions for each group were as follows: 13.1% for the green group, 38.8% for the red group, and 11.6% for the yellow group. Perhaps the sense of "real communication" within the synchronous environment, which is documented to create a pressure to react and respond, encouraged immediate feedback. This meant that questions received responses and explanations, and thinking aloud received feedback, both essential elements of collaboration. Additionally, the data indicated that, on the whole, synchronous communication facilitated group work throughout the entire problem-solving process, from trying to understand the problem, to brainstorming and organizing ideas, to executing plans, and finally to verifying solutions. This was quite different from asynchronous communication, in which the groups that used it communicated interactively only during a single heuristic episode (orientation for the blue group, and verification for the green and red groups).

Still, some obstacles related to using synchronous communication for collaborative problem solving were evident. These difficulties included lack of control over turn taking, and the frequent development of multiple threads of discussion within the same message space, and they manifested themselves with delayed responses or even a failure to respond in some instances. Comparable to confusion that might occur in a face-to-face environment in which all members of a group spoke at the same time and about different topics, without the ability to weed out extraneous dialogue, the researcher found these issues to create some disorder within the groups, typically when all three

members were involved in the dialogue. This is well illustrated in the following message transcript:

Karen: What we need to do first, is find all the composite numbers, then find the factors of each number.

Erin: Lets just work with whatever we understand.

Erin: You are right.

Peter: Look at the formula at the bottom of the page.

Karen: But how do we do that?

Erin: But we also need to find a formula, because a million numbers are too much.

Erin: ok

Peter: Define composite # so we can agree on what we are looking for.

Erin: Let's look at the formula.

Erin:  $N=p_1p_2^{n_2}\dots p_k^{n_k}$

Erin: Help me in determining what each means.

Erin: I do agree that we have to find the number, composite, that has the greatest number of factors.

Karen: I'm trying to find something.

Peter: n is the product of all the # that are applicable from 1-1,000,000.

Erin: it cannot be a prime number.

Karen: Why not?

Erin: What is p?

Peter: Because it does not have a rectangular personality.

Erin: because prime numbers only have 1 rectangular personality.

Erin: meaning only one set of factors.

Peter: ok

Karen: Is  $n =$  to prime numbers?

Peter: no

Erin: is  $=$  to any number

Erin: right?

Karen: Because it says, that  $N$  is broken down into the product of its prime powers.

Erin: which I guess it is just an example of its prime numbers.

Erin: sorry.

Erin: it is just an example of its factors.

Peter:  $n$  is the total of all the composite #s from 1-1,000,000

Erin: is  $p =$  factors?

Karen: no

Karen: It can't be.

Erin: and 1 next to the  $p$  the power of the exponent?

Erin: So is  $p =$  product?

Karen: I'll be back Peter only 5 minutes.

Peter: Let us agree on the terms first, then we can continue.

Erin: I agree

Peter: A prime # is a whole # greater than 1 whose only factors are 1 and itself.

Erin: correct

Erin: we need to find the factors of composite #

Peter: right.

Erin: and we need to find the # less than a million that has the greatest # of factors

Peter: So the best way to do this is to go through it step by step.

Karen: I'm back.

Peter: What is the highest composite number that is less than 1,000,000.

Erin: Yeah but that is a whole lot of numbers, 1000000.

Karen: p has to be prime number, because every integer can be written as a product of powers of prime #s.

Erin: minus the prime, of course.

Peter: is it 999,000,999?

Erin: I can agree to that.

Erin: How did u get that #?

Peter: It is 1 less than 1,000,000 and it is a composite #

Delays in responses and even a failure to respond in some instances are quite apparent within the aforementioned sample of messages. While it is difficult to discern the level of confusion caused by these types of behaviors, the continued failure to respond led to eventual frustration and withdrawal on the part of one group member as seen in the next section of the transcript:

Peter: please explain what you just did Karen.

Erin: I think you are right Karen.

Karen: I'm right.

Karen: I just don't know a formula for that, and it's impossible to do that for all composite numbers.

Erin: so, let's see how can we find that #

Karen: Let's do it for 1-100 first.

Erin: There must be a way so that we do not have to go through all those #

Erin: ok

Peter: Am I a part of this discussion?

Karen: I know.

Karen: Yes you are.

Karen: What's your question?

At that moment Peter left the chat room. Though his decision to withdraw from participation was not typical, at times groups seemed to cope with the aforementioned issues by informally limiting synchronous dialogue to two members at a time. If all three members of a group were online at the same time, it appeared that one contributed little or nothing, while two communicated with one another. From time to time the conversation would move to an alternate pair and once again a different member decreased involvement or sat silent. Though this practice was not discussed and overtly decided upon in any case, it appeared to be a natural evolution of group interactions.

Although the occurrence of questions or remarks that remained unanswered or even acknowledged appears to be a weakness with the online medium, their mere existence is likely to have served a positive purpose for the groups. The permanence of messages within the online bulletin board and chat transcripts provided detailed notes of all contributions, a benefit not found in the face-to-face environment. It was evident that members referred to these messages, though to what extent is not known.

Other notable differences relate to the ways in which groups participated and members interacted with one another. While it is difficult to determine a causal relationship between the groups' behaviors and their success with solving the mathematics problem, it is possible to compare the groups' achievement on the problem-solving assignment while contrasting the details of their behaviors. The blue group, which was the lowest scoring group (13 points), not only had the least amount of participation amongst the groups, it also exhibited the largest number of non-interactive units of meaning and limited its effort to the orientation stage of the problem-solving process. Additionally, only two members of the group participated in solving the problem. The interactive problem-solving behaviors of this group were divided equally between three different types of interactions – questioning, responding and commenting. Unfortunately much of these units of meaning were unrelated and therefore led to no apparent progress for the group. In the end, online transcripts showed no evidence of a solution or even completion of the problem.

The yellow group earned the second lowest score on the problem-solving assignment with a total of 15 points. While this group demonstrated considerably greater

participation than the blue group (the second highest amount at 366 units of meaning), it had the highest percentage of social interactions and a very limited number of problem-solving related interactions. Although a large percentage of the yellow group's task-specific units of meaning were interactive in nature (88.4%), over half of these were not related to solving the mathematical problem. Ultimately, collaborations concluded with an incorrect answer during the second to last session, and dialogue during the last session revealed that one of the group members had "found the solution." There was no effort made by the group to verify the answer, as the focus immediately moved to writing up the solution.

The red group earned a total of 19 points on the assignment. This group's participation was the second lowest with a total of only 121 units of meaning. Additionally this group had the second lowest percentage of interactive task-specific participation and the lowest percentage of problem-solving interactions. Even so, this group achieved success with this problem. A review of transcripts and the data related to "other" contributions revealed the explanation for this success. The red group had the highest percentage of "other" contributions at 72.7%, with a majority of these units of meaning making reference to depositing, retrieving and reviewing files using the file exchange feature of the virtual classroom. It is apparent that this group pursued a different approach to completing the assignment, choosing not to collaboratively solve the problem, but rather to work independently and to reserve communication for a small amount of verification, and for discussions about completing and submitting the assignment.

The highest scoring group was the green group, earning a score of 20 points on the assignment. The green group clearly exhibited the greatest amount and highest level of collaboration while solving the problem, as evident from the highest amount of participation amongst the groups, the lowest percentage of social contributions, the highest percentage of task-specific participation, and the highest percentage of interactive units of meaning directly related to solving the problem. Additionally, the green group collaborated throughout the problem, working to understand the problem, develop a plan, execute that plan and verify the results. Furthermore, this process was recursive in nature, as the members of the group continually assessed, verified and revised their understanding, goals, plans and execution, and thus revisited each heuristic episode.

Other differences between the groups were much more subtle and had less perceptible influence on the groups' success. These variations related to the types of problem-solving interactions each group exchanged and the heuristic episodes during which they were exchanged. The numbers of specific interactions that occurred during each problem-solving episode substantiate the above-described lack of collaboration in the blue and red groups and help to contrast their behavior with that of the green and yellow groups. While the extremely limited interaction within the blue group necessitates no additional need for analysis, a look at the numbers for the red group provides details such as the fact that interactions occurred primarily during the verification episode, less-so during orientation, and only minimally during the organization stage. This pattern confirms the overall lack of collaboration within this group, which instead worked independently and exchanged problem-solving interactions

primarily for the purpose of verification. On the other hand, the green and yellow groups worked together throughout all of the heuristic episodes. Even so, the data indicated contrasting behavior between these two groups, with the green group exhibiting percentages of interactions in the orientation and execution episodes that were higher than those of the yellow group, and the yellow group exhibiting higher percentages of interactions in the organization and verification episodes. Another difference between the behaviors of the green and yellow groups was the existence of "explanations" within the green group and the lack thereof in the yellow group. These facts suggest that the green group focused more on collaborative problem-solving behaviors, working together to understand the problem and to execute the mathematics. Finally, although the data provided few clear-cut patterns within the groups, it was evident that within the green group the majority of questions, responses and explanations occurred in the orientation stage, the majority of ideas were proposed during the organization stage, and the majority of contributions labeled "thinking aloud" occurred in the execution stage. A similar but less dramatic pattern was found for the yellow group, except for the category of "thinking aloud," which was more equally distributed throughout all of the episodes for the yellow group. In view of the fact that contributions labeled as "thinking aloud" were often incomplete and reflected members' thoughts as they occurred rather than thoughtfully constructed contributions, this information provides a potential explanation for the disjointed nature of discourse within the yellow group.

## CHAPTER IV

## CONCLUSION

Introduction

Collaborative mathematical problem solving is a complex process with many facets and interdependent parts, into which computer-mediated communication introduces entirely new dimensions. The goal of this study was to shed light on this system and reveal the details of individual and group behaviors of small groups solving a mathematical problem within the online environment. By means of content analysis, the researcher attempted to bridge the gap between case study methodology, which cannot be generalized to other instances and contexts, and methods such as surveys, user interviews, empirical experimentation, and statistical measurements, which lack a detailed picture of collaborative problem solving. This was accomplished through the development and use of a detailed framework to categorize participative, social and interactive behaviors, which enabled the researcher to obtain a holistic and descriptive picture of computer-mediated collaborative mathematical problem solving while still acquiring some measure of the elements of educational exchange within the new environment. The following sections provide a summary of the findings of this study; its conclusions and implications; remarks about the study and suggestions for future action; and recommendations for further studies.

### Summary

This study examined the process of computer-mediated collaborative mathematical problem solving within four small groups of students in order to characterize their individual and group participative, social, and interactive behaviors throughout the problem solving process, and to describe the impact of these behaviors on the quality of collaboration and the groups' success or failure in problem solving. Data on individual and group behaviors was collected by observing and performing a content analysis on transcripts of online problem-solving protocols, and coding messages along four dimensions using a framework developed by the researcher. Problem-solving success was measured by group achievement on the assigned problem-solving task as determined by an analytical rubric.

Content analysis revealed noticeable variations in the ways that the groups collaborated. Differences between groups ranged from simple and obvious ones, such as the amount of total participation, to more complex and subtle dissimilarities, such as the types of interactions exchanged and the phases of the problem solving process during which they were exchanged. The blue group was the only group that limited its communication to use of the bulletin board system. It had the least amount of participation amongst the groups, with only two members contributing to the effort, and only three messages being sent with a total of nine units of meaning within those messages. This group also exhibited the largest number of non-interactive units of meaning and limited its effort to the orientation stage of the problem solving process. The transcripts for this group revealed no evidence that suggested whether the task was

delegated or divided, and no progress was made online toward solution of the problem. This group received the lowest score on the assignment, with a total score of 13 out of 20 points on the problem-solving task.

At the other extreme, the yellow group was the only group that limited its dialogue to synchronous messaging, exchanging a total of 371 messages and 366 units of meaning by means of chat room discussions. Although this group demonstrated considerably greater participation than the blue group, it had the highest percentage of social interactions and a very limited number of interactions related to problem solving. Additionally, while there were some collaborative efforts aimed at understanding the problem, the majority of the discourse for this group consisted of social dialogue interspersed with discussions related to finding a formula to solve the problem or to finding the answer online. This group appeared to lack focus and was unable to progress through the problem solving process, as was evidenced with the circular nature of their discussions and the relatively large percentage of talk in the thinking aloud category throughout each heuristic episode. Near the end of the transcripts, messages revealed the discovery of a possible solution to the problem followed by a hasty decision to write up the solution with no sign of verification. The yellow group earned the second lowest score on the problem solving assignment, with a total of 15 points.

The green and red groups used a combination of the two modes of communication, sending a total of 652 messages (with 479 units of meaning) and 103 messages (with 121 units of meaning), respectively. The green group exhibited the greatest amount and highest level of collaboration while solving the problem, as

demonstrated by the highest amount of participation amongst the groups, the lowest percentage of social contributions, the highest percentage of task-specific participation, and the highest percentage of interactive units of meaning directly related to solving the problem. Additionally, the green group collaborated throughout the problem, working to understand the problem, develop a plan, execute that plan and verify the results. This group exchanged initial comments and questions about the problem through bulletin board messages, completed a considerable amount of collaboration on the problem synchronously, and returned to asynchronous communication to complete the problem. They achieved the highest score, earning 20 points on the assignment.

The red group took a different approach than the other three groups, demonstrating very little collaborative problem solving online. Rather, the red group worked independently and reserved online communication for a small amount orientation about the problem, verification, and for discussions about completing and submitting the assignment. This was demonstrated by the fact that this group had the second lowest amount of participation, the second lowest percentage of interactive task-specific participation and the lowest percentage of problem solving interactions. Additionally, the members of the group had the highest percentage of "other contributions," with a majority of these units of meaning making reference to depositing, retrieving and reviewing files using the file exchange feature of the virtual classroom. The red group earned a total of 19 points on the assignment.

Evidence pointed to the choice between asynchronous and synchronous modes for communication as the primary determinant in the amount of participation and the amount

and type of interactivity amongst members. The number of asynchronous messages sent was minimal in all groups that used this mode. While the data confirmed that the asynchronous messages tended to contain the more lengthy messages with multiple units of meaning the communication between group members was highly non-interactive in nature and appeared to do little to advance group problem solving efforts. In fact, the number of non-interactive units of meaning within this mode suggests little consideration of previously posted messages. Consequently many questions remained unanswered, thinking aloud received little feedback, and proposed ideas were not pursued.

In contrast, the synchronous mode was much more interactive in nature and appeared to promote much greater collaboration as evidenced by the low percentages of non-interactive contributions for each group that used it. This mode produced shorter messages and appeared to imitate collaboration in the face-to-face mode, with a mutual, continual, and spontaneous exchange of ideas and feedback. This meant that questions received responses and explanations and thinking aloud received feedback, both essential elements of collaboration. Additionally, the data indicated that synchronous communication effectively facilitated group work throughout the entire problem solving process, from orientation to verification. This was quite different from asynchronous communication, in which the groups that used it communicated interactively only during a single heuristic episode. There were some obstacles related to using synchronous communication, however, including lack of control over turn taking, and the frequent development of multiple threads of discussion within the same message space. These

difficulties manifested themselves with delayed responses, disorder within the group, and even a failure to respond in some instances.

Lastly, the online medium introduced a new variable to collaboration, namely the dependence on technology for communication. The analysis revealed the significance of "other" contributions in online collaborative problem solving. Although these units of meaning, which were related to the use of the technology and the process of online collaboration, might be considered secondary to the problem solving interactions, they accounted for a noteworthy percentage of task related interactions within almost every group and were found to be important to the functioning of the groups as they discussed issues related to when and how to communicate.

### Conclusions and Implications

Through the process of content analysis several important conclusions emerged concerning computer-mediated collaborative problem solving. The primary observations related to the ways in which groups communicated and participated. The evidence reveals various levels of collaboration, from simply exchanging files with little group effort, to participation and cooperation throughout the entire problem solving process. While participation was important to collaboration, the interaction between group members was found to be even more so. Though the group that exchanged files was successful in completing the task with little interaction, the lack of true collaboration within that group was quite evident. Within the remaining groups, the results of this study support existing research which indicates the importance of interaction to group

success in collaborative problem solving (Webb, 1991; Dillenbourg & Schneider, 1995; Stacey & Gooding, 1993). The data indicate that the most effective group not only participated more, but also socialized less, interacted more, with more task-specific content and more interactions related to solving the problem. Additionally, this group collaborated throughout the problem-solving process in a recursive nature – working to understand the problem, develop a plan, execute that plan, and verify the results, and revisiting each of these heuristic stages – thereby lending substantiation to previously defined problem-solving processes that lead to success (Curcio & Artzt, 1998).

Conversely, the participants in the ineffective groups clearly exhibited little participation, a large number of non-interactive and non-problem related contributions, and more social interaction. Furthermore, these groups failed to create a cohesive effort that advanced throughout the entire problem-solving process. Although no clear patterns were detected connecting specific types of problem-solving interactions to group success or failure, the data from the group collaborations did show the highest scoring group as the only one that had problem-solving interactions of all types. Interestingly, this group also exhibited a notable percentage of talk in the thinking aloud category, which in a previous face-to-face study was reported to be a predictor of low achieving groups (Stacey & Gooding, 1993). The researcher proposes that this may shed light on an important difference between the online medium and the face-to-face environment. Since the majority of this “thinking aloud” occurred during the heuristic episode of execution, it is likely that changes in the medium require or compel group members to communicate

about things such as the mathematical procedures they are executing, which in the face-to-face mode might be written on paper or simply completed in one's own mind.

There were also no detectable patterns regarding individual participation and interactions within the groups, though it did appear as if an informal leader emerged in the highest scoring group. This group member contributed a majority of the interactions within her group, was the only member of her group that was involved throughout the entire process, and was responsible for nearly all (93%) of her group's explanations. Additionally, she appeared to initiate all productive exchanges of ideas and informally focus and direct the group's discussions and behaviors through her contributions. The researcher considers her leadership as a primary reason for the progress and success of that group, and the lack of this type of leadership within the other groups as a key explanation for their failure to advance toward a solution during online collaboration.

This study also illuminated several issues related to the use of technology for collaborative problem solving. As discussed previously, the use of asynchronous and synchronous communication appeared to play a major factor in how and how much groups collaborated. Asynchronous communications proved to be disappointingly sparse. Data failed to support past research that found extensive and thoughtful asynchronous interactions amongst participants that were triggered from a single message (Levin et al., 1990). Instead group members communicated very little and seemed to do so without regard for the ideas and thoughts of other group members. The researcher suggests these findings are similar to what is most often found in communication via written letters. As letters are exchanged their writers often fail to address every thought

that is conveyed or question that has been asked in the letters that they have received, but rather communicate their own thoughts and what is important from their perspective. In view of this example, it is the researcher's opinion that it is the delay in communication that is responsible for the non-interactive and disconnected nature of discourse in the asynchronous mode. Thus, while the data still suggests the asynchronous mode may promote longer, well-formed messages, and may be a good means of sharing ideas, the lack of immediate feedback appears to hinder group progress in problem solving.

On the other hand, the findings revealed that synchronous communication facilitates increased and more continuous participation and free flowing interactions that imitate face-to-face collaboration. Evidently, as with face-to-face communication, the immediate nature of chat room discourse promotes immediate feedback for thoughts, ideas and questions, an essential element of collaborative problem solving. The synchronous mode is not completely without problems, however. The transcripts exposed the disorder and confusion that can be created by delayed responses and failures to respond and the resulting tendency for some members to exclude themselves or to be excluded from interactions. In addition, the requirement to meet simultaneously with other group members lessened the convenience of online communication and instead introduced problems associated with scheduling and the dependence on technology.

This study also revealed additional hindrances to collaboration associated with using the technology. First, there were small deterrents such as the difficulty of expressing mathematical ideas because of the lack of ease or the inability to type mathematical symbols and items such as superscripts and subscript. Larger obstacles to

collaboration were found to include the inability to connect for online discussions because of server or computer problems, or problems with Blackboard itself. Finally, the inconvenience of using technology for communication, and the lack of mandatory participation, that is a natural part of being present for face-to-face collaboration, appeared to discourage and even deter participation in online collaborative problem-solving efforts.

Regarding the analytical framework, the researcher was unable to conclude that it is a valid predictor of problem-solving success. While it is tempting to conclude that the green group achieved the highest score because of highly collaborative behaviors, there are other variables, such as individual problem-solving ability, that might have influenced the group's problem-solving success. Indeed, the very different behaviors within the green and red groups (highly collaborative vs. non-collaborative), and relatively comparable success, suggest that there are quite different paths to successful completion of group problem-solving tasks in the online environment. Nevertheless, the framework provided an effective means for examining the nature of computer-mediated group problem solving and revealing the presence and absence of collaboration, a process that is essential for activating learning mechanisms such as assimilation and accommodation and the construction of new knowledge.

The above findings introduce several implications for mathematics educators as they develop computer-mediated collaborative problem-solving opportunities. The issues are grouped into three categories: (a) communication medium, (b) task features, and (c) group composition. The first consideration is the communication medium.

Since communication is essential for collaboration, the communication medium is undoubtedly the underlying factor affecting group process. Although the use of email and chat rooms has become quite commonplace in today's society, the level of communication obtainable by online channels is quite different than face-to-face interaction. Obstacles such as the difficulty of expressing mathematical notation and mathematical thoughts with typewritten communication, and the inability to see the facial gestures and cues of one another during online communication are clearly shortcomings of computer-mediated communication. Additionally, an inadvertent consequence of the benefits of flexibility of time and location brought by computer-mediated communication is the decreased occurrence of whole-group meeting time. Although, the type of compulsory meeting time that occurs in face-to-face environments is not the key for successful collaboration, without the communication that occurs during these meetings no collaboration would be possible; this simple reason appears to be the cause of failure for many online collaborative efforts. Lastly, computer-mediated communication introduces an entirely new facet to collaborative problem solving, an already difficult and complex process. This study revealed groups communicating aimlessly, either failing to connect with one another or struggling to get focused and remain focused both in terms of communication and process. Even the most successful group in this study did not appear to have a plan for collaboration and appeared to struggle to initiate collaboration. This lack of direction and disjointedness appears to hinder groups from making progress toward solution of a problem. This evidence illuminates a need for procedures and protocol for computer-mediated communication and online collaboration. Established

guidelines and suggestions for communicating, proceeding through the problem-solving process, and collaborating online would be a valuable tool for assisting groups in initiating and maintaining focus and effective team efforts toward problem solving. However, this will require additional research to determine what procedures and techniques are most effective for online collaborative problem solving.

The effects of collaboration also vary according to task. For example, some tasks are naturally distributed and lead group members to work independently. Interaction occurs when assembling partial results, but not during each individual's reasoning process. While this may lead to group success in solving the problem, it does not activate the mechanisms such as conflict or disagreement, self-explanation, and mutual regulation, which account for the successful knowledge acquisition through collaboration. Thus, optimal tasks must include features such as adequate difficulty that require group members to interact on a continual basis, reason together, regulate the thinking of other group members, and work towards a mutual solution. Even with these features, however, tasks assigned for completion in the online medium will undoubtedly be more prone to distribution and independent work. For this reason mathematics educators must find additional ways of designing tasks or procedures that will necessitate continual interaction throughout the entire problem solving process.

The final issues relate to group composition. This study reveals the confusion and exclusion of members that can occur during synchronous dialogue even within small groups such as those in this study. Perhaps it was the odd number of members (three per group) that created this confusion. If this was the case, groups of four may be more

successful. Additionally, techniques such as "think, pair, and share," in which group members communicate in pairs prior to sharing their ideas with the entire group, might solve the problem of the lack of structured communication amongst larger groups.

Nonetheless, the matter of optimal group size will need further investigation. The other issue related to group composition concerns the roles of individuals within groups. This study provides anecdotal evidence that the existence of leadership, formal or informal, is beneficial to the functioning of the collaborative group. Assuming that this is so, collaborative groups will certainly benefit from some type of role assignment, especially that of a leader. Additional role assignments, such as those used successfully in face-to-face collaborative problem solving are also in order. It is possible that with these assignments issues of group size may become less significant.

In the end it is difficult to say whether any one variable has greater influence on computer-mediated group collaboration and mathematics problem-solving success. The participative, social, and interactive behaviors of collaborative problem solving are so interconnected and interdependent that any simple cause-effect analysis distorts more than it illuminates. Additionally, while task features and group composition are intended to influence participation and interaction, the behavior of a group is still subject to human nature and the dynamics of individuals within that group. Indisputably, participation is a primary requirement, and without there would be no interactions. However, devoid of thoughtful and organized task related interactions, communication is confusing and participation fruitless. Changes in one part lead to changes among all parts and the

system itself. The following analogy by Gharajedaghi and Ackoff (as cited in Patton, 1990) demonstrates the intricacy of this collaborative system remarkably well:

Because the effects of the behavior of the parts of a system are interdependent, it can be shown that if each part taken separately is made to perform as efficiently as possible, the system as a whole will not function as effectively as possible. For example, if we select from all the automobiles available the best carburetor, the best distributor, and so on for each part required for an automobile, and then try to assemble them, we will not even obtain an automobile, let alone the best one, because the parts will not fit together. The performance of a system is not the sum of the independent effects of its parts; it is the product of their interactions. Therefore, effective management of a system requires managing the interaction of its parts, not the actions of its parts taken separately. (p. 79)

In conclusion, mathematics educators must bear in mind the above quote and the findings of this study when attempting to design effective collaborative problem solving opportunities for students. Simply improving participation or increasing interactions is unlikely to create effective collaborative groups. Instead, we must take a closer look at the many elements in the collaborative system and work to improve the product of their interactions so that groups can perform as efficiently and effectively as possible.

#### Remarks and Suggestions for Future Action

It must be noted that while the goal of this study was analyze the educational quality of conference interactions the intention was to do so by looking at the types of interactions, and how and when they occurred. Therefore, the content of the interactions was not analyzed for correctness, quality, or value. Moreover, this study attempted to take a objective look at the process of online collaboration problem solving without the knowledge of why students participated or failed to participate, their likes or dislikes in

terms of online learning, or of their opinions or self-described experiences. Therefore no subject interviews were conducted.

While the researcher was unable to conclude that the analytical framework developed for this study is a valid predictor of problem-solving success, it does provide a reasonably objective and unambiguous means for dealing with the abundance of information contained in computer-mediated dialogue and for interpreting the elements of meaning that have significance for the learning process. The researcher not only suggests continued use of this framework for future studies of computer-mediated collaborative mathematical problem solving, but also suggests using the interaction analysis instrument to provide feedback for problem solving groups, to monitor, control and improve group progress.

The researcher also proposes several suggestions for promoting more effective collaboration and successful problem-solving groups based upon the findings of this study. These recommendations are as follows:

1. Groups should have requirements concerning the amount of participation and the frequency of meetings, and the use of both asynchronous and synchronous modes of communication should be mandatory.
2. Protocol and procedures for communication and collaboration should be developed to give participants guidelines for effective communication within the online medium, suggestions for exchanging ideas and interacting with one another, and techniques for collaborating such as "think, pair, share." These guidelines would include appropriate uses for asynchronous and synchronous

modes of communication, and strategies for problem solving as a group in the online environment.

3. Roles should be assigned to promote focused and smoothly functioning collaborative groups (Johnson & Johnson, 1993, Kumar, 1996). Role assignment can be as simple as designating a group leader, to assignments for each of the participants, such as manager, recorder/checker, skeptic, and energizer/summarizer. These roles should be rotated so that every group member benefits from assuming each of the roles.
4. Group size should remain as small as possible to lessen the confusion and exclusion created by larger groups, and so that all members are encouraged and even obligated to participate.
5. Tasks should be of adequate difficulty to prevent group members from distributing responsibilities, working independently, and then assembling partial results. An additional suggestion is to separate the task into several segments or formative tasks that would require all students to interact on a continual basis, reason together, and to regulate the thinking of other group members.

#### Recommendations for Future Studies

There is much more to learn about computer-mediated collaborative problem solving and the similarities or differences that it may have with collaborative problem solving in face-to-face environments. With the existing evidence, it is still difficult to

connect what occurs online with existing knowledge of face-to-face collaborative problem solving. While it seems likely that similarities exist, perhaps the online medium produces a completely different entity that cannot be compared or related to the face-to-face process. This reinforces the importance of continued investigation into the process of computer-mediated collaborative problem solving and particularly the use of content analysis. Additional studies, which utilize this approach, and specifically the analytical framework developed in this study, should be completed in order to shed further light on patterns of behavior, to assess the educational value of group interactions, and to provide a measure for the elements of success or educational exchange within small groups involved in computer-mediated collaborative mathematical problem solving.

The researcher makes the following recommendations for future studies:

1. Several studies, in which various variables related to the collaborative process are altered or compared, are needed. Variations to the existing study should include:
  - a. Structure of Material and/or Exercises
    - Highly structured: work broken down into a relatively large number of specific and detailed steps
    - Moderately structured: statements or descriptions of problem or major part of problem; relatively little direction for solutions
    - Little structure: high level, but precise statement of problem; guidance for milestones, but little direction for how to proceed
    - Open ended: gives statement of problem, with only final deadline

- Unstructured: identifies options available, and leaves groups to determine their own way
- b. Structure of Group and/or Collaborative Procedures
  - Group member role assignment
  - Think pair and share
- c. Group Size
  - Small: 2 or 3 people per group
  - Moderate: 4 or 5 people per group
  - Large: 6 to 10 people per group
- d. Selection of Groups
  - At Random: by student choice
  - At Random: by instructor selection
  - By ability: by instructor selection
    - Equal abilities together
    - Stronger students help weaker ones
  - By other factors: by instructor selection
    - Maintenance of gender balance
    - Combination of specific backgrounds
  - Groups may or may not change during the semester
- 2. A formal study to compare the participative, social and interactive behaviors of individuals and groups restricted to the asynchronous or synchronous communication for collaboration problem solving should be completed.

3. If possible, the above studies should include a larger number of subjects so that patterns can be ascertained, results may be generalized, and so as to establish the validity of framework for problem-solving success.
4. A study that investigates the cognitive and metacognitive behaviors of individuals in conjunction with the participative, social and interactive behaviors should be completed.
5. A study to analyze the role of, and positive or negative effects related to non-interactive questions or remarks during online collaboration needs to be completed.

Finally, while this study provides a detailed picture of the behaviors of groups that successfully collaborate utilizing computer-mediated communication, we are unable to assume that learning is an automatic by-product of these groups. For this reason, future studies should investigate the relationship between the participative, social and interactive behaviors and individual learning and achievement.

REFERENCES CITED

References Cited

- Artzt, A. F., & Armour-Thomas, E. (1990). *Protocol analysis of group problem solving in mathematics: a cognitive-metacognitive framework for assessment*. Paper presented at the Annual Meeting of American Education Research Association, Boston, MA. (ERIC Document Reproduction Service No. ED 320 927).
- Artzt, A. F., & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for potocol analysis of mathematical problem solving in small groups. *Cognition and Instruction*, 9(2), 137-175.
- Artzt, A.F., & Armour-Thomas, E. (1996). *Assessing the internal dynamics of mathematical problem solving in small groups*. Paper presented at the Annual meeting of the American Educational Research Association, New York, NY. (ERIC Document Reproduction Service No. ED 395 787).
- Beaudrie, B. (2000). *Analysis of group problem-solving tasks in a geometry course for teachers using computer-mediated conferencing*. Unpublished doctoral dissertation, Montana State University, Bozeman.
- Beckwith, D. (1987). Group problem solving via computer conferencing: The realizable potential. *Canadian Journal of Educational Communication*, 16(2), 89-106.
- Biggs, J. (1987). *Student approaches to learning and studying*. Hawthorn, Australia: Australian Council for Educational Research Limited.
- Chiu, M. (1997). *Building on diversity: A moment-to-moment analysis of students collaboratively solving mathematics problems*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL. (ERIC Document Reproduction Service No. ED 410 325).
- Cohen, M. & Riel, M. (1989). The effect of distant audiences on students' writing. *American Educational Research Journal*, 26, 143-159.
- Cole, M., Gay, J., Glick, J., & Sharp, D. (1971). *The cultural context of learning and thinking*. New York, NY: Basic Books.
- Collis, B. A. (1994). Collaborative learning and CSCW: research perspectives for interworked educational environments. In R. Lewis & P. Mendelsohn (Eds.), *Lessons from learning (A-46)* (pp. 81-104). North-Holland, Elsevier Science B. V.

- Curcio, F.R., & Artzt, A.F. (1998). Students communicating in small groups: Making sense of data in graphical form. In H. Steinbring, M.G. Bartolini Bussi, & A. Sierpiska (Eds.), *Language and Communication in the Classroom* (pp. 179-190). Reston, VA: National Council of Teachers of Mathematics.
- Days, H. C., Kulm, G., & Wheatley, G. H. (1979). Problem structure, cognitive level, and problem-solving performance. *Journal for Research in Mathematics Education*, 135-145.
- Dees, R. L. (1991). The role of cooperative learning in increasing problem-solving ability in a college remedial course. *Journal for Research in Mathematics Education*, 22(5), 409-421.
- Dillenbourg, P., & Schneider, D. (1995). *Collaborative learning and the internet*. Retrieved November 1, 2000, from [http://tecfa.unige.ch/tecfa/research/CMC/colla/iccai95\\_1.html](http://tecfa.unige.ch/tecfa/research/CMC/colla/iccai95_1.html)
- Ebbelink, I. (1999) *Computer Mediated Communication*. University of Twente Netherland. Retrieved November 1, 2000, from <http://huizen.dds.nl/~inki/>
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. Resnick (Ed.), *The nature of intelligence* (pp. 231-236). Hillsdale, NJ: Erlbaum.
- Flavell, J. H., & Wellman, H. M. (1977). Metamemory. In R. V. Kail & J. W. Hagen (Eds.), *Perspectives on the development of memory and cognition* (pp. 3-33). Hillsdale, NJ: Erlbaum.
- Forman, E. (1989). The role of peer interaction in the social construction of mathematical knowledge. In N. Webb (Ed.), *Peer Interaction, Problem-Solving and Cognition: Multidisciplinary Perspectives*. *International Journal of Educational Research*, 13(1), 55-70.
- Garofalo, J., & Lester, F.K. Jr. (Eds.). (1982). *Mathematical problem solving: Issues in research*. Philadelphia, PA: Franklin Institute Press.
- Garofalo, J., & Lester, F. K. Jr. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16(3), 163-175.
- Gokhale, A. A. (1995). Collaborative learning enhances critical thinking [Electronic Version]. *Journal of Technology Education*, 7 (1).

Goos, M., & Galbraith, P. (1996). Do it this way! Metacognitive strategies in collaborative mathematical problem solving. *Educational Studies in Mathematics*, 30, 229-260.

Gunawardena, C. N. (1995). Social presence theory and implications for interaction and collaborative learning in computer conferences. *International Journal of Educational Telecommunications*, 1(2/3), 147-166.

Gunawardena, C. N., Lowe, C. A., & Anderson, T. (1997). Analysis of a global online debate and the development of an interaction analysis model for examining social construction of knowledge in computer conferencing. *Journal of Educational Computing Research*, 17 (4), 397-431.

Guzdial, M., Hmelo, C., Hübscher, R., Nagel, K., Newstetter, W., Puntembakar, S., Shabo, A., Turns, J., & Kolodner, J. (1997). Integrating and Guiding Collaboration: Lessons learned in computer-supported collaboration learning research at Georgia Tech. *Proceedings of Computer-Supported Collaborative Learning*, Toronto, Ontario, CANADA, 87, 91-100.

Haile, P. (1986, April). *An analysis of computer conferences supporting the distance learner*. Paper presented at the Annual Conference of the American Educational Research Association. San Francisco, CA.

Hart, L.C. (1993). Some factors that impede or enhance performance in mathematical problem solving. *Journal for Research in Mathematics Education*, 24(2), 167-171.

Henri, F. (1992). Computer conferencing and content analysis. In A. R. Kaye (Ed.) *Collaborative learning through computer conferencing. The Najaden Papers. NATO ASI Series, Computer & systems sciences, Vol. 90* (pp. 118-136). Berlin Heidelberg Germany: Springer-Verlag.

Higgins, R. (1991). Computer-mediated cooperative learning: Synchronous and asynchronous communication between students learning nursing diagnosis. Unpublished doctoral dissertation, University of Toronto, Canada. Retrieved November 1, 2000, from <http://www.cybercorp.net/rhiggins/thesis/>

Hiltz, S. R., Coppola, N., Rotter, N., & Turoff, M. (2000). Measuring the importance of collaborative learning for the effectiveness of ALN: A multi-measure, multi-method approach. *Journal of Asynchronous Learning Networks*, 4 (2). Retrieved February 1, 2002, from [http://www.aln.org/alnweb/journal/Vol14\\_issue2/le/hiltz/le-hiltz.htm](http://www.aln.org/alnweb/journal/Vol14_issue2/le/hiltz/le-hiltz.htm)

- Johnson, D.W. and Johnson, R.T. (1984). Cooperative small-group learning. *Curriculum Report, 14*, 1-6.
- Johnson, D.W., and Johnson, R.T. (1993). What we know about cooperative learning at the college level. *Cooperative Learning, 13* (3).
- Johnson, D.W., Maruyama, G., Johnson, R.T., Nelson, D. & Skon, L. (1981). Effects of cooperative, competitive, and individualistic goal structures on achievement: A meta-analysis. *Psychological Bulletin, 89*, 47-62.
- Kahle, D. (2000). *Computer mediated communication in distance education: An annotated bibliography*. Retrieved February 1, 2002, from <http://www.sou.edu/ecp/disted/resource/cmcbibli.htm>
- Kaye, A. (1992). Learning together apart. In A. R. Kaye (Ed.) *Collaborative learning through computer conferencing. The Najaden Papers. NATO ASI Series, Computer & systems sciences, Vol. 90* (pp. 1-24). Berlin Heidelberg Germany: Springer-Verlag.
- Krulik, S. & Rudnick, J. (1988). *Problem solving: A handbook for elementary school teachers*. Needham Heights, MA: Allyn & Bacon / Logwood Division.
- Kumar, V. S. (1996). *Computer-supported collaborative learning: Issues for research*. Retrieved November 1, 2000, from <http://www.cs.usask.ca/grads/vsk719/academic/890/project2/project2.html>
- Lambin, D. V. (1993). Monitoring moves and roles in cooperative mathematical problem solving. *Focus on learning problems in mathematics, 15*(2&3), 48-64.
- Lester, F. K. Jr. (1979). *A procedure for studying the cognitive processes used during problem solving: an exploratory study*. San Francisco, CA: Paper presented at the Annual Meeting of American Education Research Association (ERIC Document Reproduction Service No. ED 171 590).
- Levin, J. A., Kim, H. & Riel, M. M. (1990). Analyzing instructional interactions on electronic message networks. In L. Harasim (Ed.), *Online Education: Perspectives on a New Environment* (pp. 185-213). New York: Praeger.
- Lucas, J. F. (1974). The teaching of heuristic problem-solving strategies in elementary calculus. *Journal for Research in Mathematics Education, 36*-45.

- Mason, R. (1992). Evaluation Methodologies for Computer Conferencing Applications. In A. R. Kaye (Ed.), *Collaborative learning through computer conferencing. The Najaden Papers. NATO ASI Series, Computer & systems sciences, Vol. 90* (pp. 105-116). Berlin Heidelberg Germany: Springer-Verlag.
- McComb, M. (1993). Augmenting a group discussion course with computer-mediated communication in a small college setting. *Interpersonal Computing and Technology: An Electronic Journal for the 21<sup>st</sup> Century*, 1(3). Retrieved November 1, 2000, from <http://jan.ucc.nau.edu/~ipct-j/1993/n3/mccomb.txt>
- McConnell, D. (1994). Learning in groups: Some experiences of online work. In M. F. Verdejo & S. A. Cerri (Eds.), *Collaborative Dialogue Technologies in Distance Learning NATO ASI Series F #133* (pp. 46-59). Berlin Germany: Springer-Verlag.
- McDonald, J. (1998). *Interpersonal group dynamics and development in computer conferencing: the rest of the story*. (ERIC Document Reproduction Service No. ED 422 864).
- National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Newman, D., Johnson, C., Cochrane, C. & Webb, B. (1996). An experiment in group learning technology: Evaluating critical thinking in face-to-face and computer-supported seminars. *Interpersonal Computing and Technology: An Electronic Journal for the 21<sup>st</sup> Century*, 4(1). Retrieved November 1, 2000, from <http://jan.ucc.nau.edu/~ipct-j/1996/n1/newman.txt>
- Newman, D., Webb, B., and Cochrane, C. (1995). A content analysis method to measure critical thinking in face-to-face and computer supported group learning. *Interpersonal Computing and Technology: An Electronic Journal for the 21<sup>st</sup> Century*, 3(2). Retrieved November 1, 2000, from <http://jan.ucc.nau.edu/~ipct-j/1995/n2/newman.txt>
- Owens, K., Perry, B., Conroy, J., Geoghegan, N. & Howe, P. (1998). Responsiveness and affective processes in the interactive construction of understanding in mathematics. *Educational Studies in Mathematics*, 35, 105-127.
- Palincsar, A. S. & Brown, A. L. (1984). Reciprocal teaching of comprehension-fostering and comprehension-monitoring activities. *Cognition and Instruction*, 1(2), 117-175.

- Panitz, T. (1996). *A definition of collaborative vs cooperative learning*. Retrieved November 1, 2000, from <http://www.lgu.ac.uk/deliberations/collab.learning/panitz2.html>
- Patton, M. (1990). *Qualitative Evaluation and Research Methods* (2<sup>nd</sup> ed.). Newbury Park, CA: Sage Publications, Inc.
- Polya, G. (1957). *How to solve it*. Princeton, NJ: Princeton University Press.
- Riel, M. (1990). Cooperative learning across classrooms in electronic learning circles. *Instructional Science*, 19, 445-466.
- Riel, M. & Levin, J.A. (1990). Building electronic communities: Successes and failures in computer networking. *Instructional Science*, 19, 145-169.
- Schoenfeld, A. H. (1979a). Explicit heuristic training as a variable in problem-solving performance. *Journal for Research in Mathematics Education*, 10, 173-187.
- Schoenfeld, A. H. (1979b). *Presenting a model of mathematical problem solving*. Paper presented at the Annual Meeting of American Education Research Association, San Francisco, CA. (ERIC Document Reproduction Service No. ED 171 536).
- Schoenfeld, A. H. (1981). *Episodes and executive decisions in mathematical problem solving*. Paper presented at the Annual Meeting of American Education Research Association, Los Angeles, CA. (ERIC Document Reproduction Service No. ED 201 505).
- Schoenfeld, A. H. (1982a). *Beyond the purely cognitive: metacognition and social cognition as driving forces in intellectual performance*. Paper presented at the Annual Meeting of American Education Research Association, New York, NY. (ERIC Document Reproduction Service No. ED 219 433).
- Schoenfeld, A. H. (1982b). Measures of problem-solving performance and of problem-solving instruction. *Journal for Research in Mathematics Education*, 13(1), 31-48.
- Schoenfeld, A. H. (1982c). *On the analysis of two-person problem-solving protocols*. Paper presented at the Annual Meeting of American Education Research Association, New York, NY. (ERIC Document Reproduction Service No. ED 219 434).

Schoenfeld, A. H. (1987). *Cognitive science and mathematics education*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Slavin, R.E. (1987). *Cooperative learning and student achievement* (Report No. OERI-G-86-006). Washington, DC: Office of Educational Research.

Stacey, K. & Gooding, A. (1993). Characteristics of Small Group Discussion Reducing Misconceptions. *Mathematics Education Research Journal*, 5(1), 60-73.

Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude of problem solving. *Journal of Educational Psychology*, 82(2), 306-314.

Turoff, M. (1989). *Computer mediated communication requirements for group support*. Paper presented at the Conference on Organizational Computing, Coordination and Collaboration Theories and Technologies for Computer Supported Work, Austin, TX.

Verdejo, M. & Cerri, S. (1994). Collaborative dialogue technologies in distance learning. In M. F. Verdejo & S. Cerri (Eds.), *Proceedings of the NATO advanced research workshop on collaborative dialogue technologies in distance learning, Series F: Computer and systems sciences, Vol. 133* (pp.1-7). Segovia, Spain

Vos, K. E. (1976). The effects of three instructional strategies on problem-solving behaviors in secondary school mathematics. *Journal for Research in Mathematics Education*, 264-275.

Waugh, M., Miyake, N., Levin, J. & Cohen, M. (1988). *Problem-solving interactions on electronic networks*. A paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Webb, N. M. (1991). Task related verbal interaction and mathematics learning in small groups. *Journal for Research in Mathematics Education*, 22, 366-89.

Wilczenski, F.L., Bontrager, T., Ventrone, P., Correia, M. (1999). *Assessing group process during collaborative problem solving*. Paper presented at the Annual Meeting of the American Psychological Association, Boston, MA. (ERIC Document Reproduction Service No. ED 434 156).

Wilson, J. W., Fernandez, M. L., and Hadaway, N. (1994). Mathematics problem solving. In P.S. Wilson (Ed.), *Survey of Research in Mathematics Education: Secondary School*, Reston, VA: National Council of Teachers of Mathematics.

Wong, P. S. K. (1989). *Students' metacognition in mathematical problem solving*. Paper presented at the Australian Association for Research in Education, Australia. (ERIC Document Reproduction Service No. ED 328 438).

Worthen, B. and J. Sanders (1991). The changing face of educational evaluation. *Theory Into Practice*, 30 (1), 3-12.

APPENDICES

APPENDIX A

PROBLEM SOLUTION RUBRIC

PROBLEM SOLUTION RUBRIC**Conceptual Understanding**

<b>1 point</b>	<b>2 points</b>	<b>3 points</b>	<b>4 points</b>
<ul style="list-style-type: none"> <li>▪ Your mathematical representations of the problem were incorrect.</li> <li>▪ You used the wrong information in trying to solve the problem.</li> <li>▪ The mathematical procedures you used would not lead to a correct solution.</li> <li>▪ You used mathematical terminology incorrectly.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Your choice of forms to represent the problem was inefficient.</li> <li>▪ Your response was not completely related to the problem.</li> <li>▪ The mathematical procedures you used would lead to a partially correct solution.</li> <li>▪ You used mathematical terminology imprecisely.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Your choice of mathematical representations of the problem was appropriate.</li> <li>▪ You used all relevant information from the problem in your solution.</li> <li>▪ The mathematical procedures you chose would lead to a correct solution.</li> <li>▪ You used mathematical terminology correctly.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Your choice of mathematical representations helped clarify the problem's meaning.</li> <li>▪ You uncovered hidden or implied information not readily apparent.</li> <li>▪ You chose mathematical procedures that would lead to an elegant solution.</li> <li>▪ You used mathematical terminology precisely.</li> </ul>

## Strategies &amp; Reasoning

1 point	2 points	3 points	4 points
<ul style="list-style-type: none"> <li>▪ Your strategy was not appropriate for the problem.</li> <li>▪ You didn't seem to know where to begin.</li> <li>▪ Your reasoning did not support your work.</li> <li>▪ There was no apparent relationship between your representations and the task.</li> <li>▪ There was no apparent logic to your solution.</li> <li>▪ Your approach to the problem would not lead to a correct solution.</li> </ul>	<ul style="list-style-type: none"> <li>▪ The strategy you chose was not efficient.</li> <li>▪ You used an oversimplified approach to the problem.</li> <li>▪ You offered little or no explanation of your strategies.</li> <li>▪ Some of your representations accurately depicted aspects of the problem.</li> <li>▪ You sometimes made leaps in your logic that were hard to follow.</li> <li>▪ Your process led to a partially complete solution.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You chose an appropriate, efficient strategy for solving the problem.</li> <li>▪ You justified each step of your work.</li> <li>▪ Your representation(s) fit the task.</li> <li>▪ The logic of your solution was apparent.</li> <li>▪ Your process would lead to a complete, correct solution of the problem.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You chose an innovative, elegant or unusual strategy for solving the problem.</li> <li>▪ You proved your solution was correct and your approach was valid.</li> <li>▪ You provided examples and/or counter examples to support your solution.</li> <li>▪ You used a sophisticated approach to solve the problem.</li> </ul>

### Computation & Execution

1 point	2 points	3 points	4 points
<ul style="list-style-type: none"> <li>▪ Errors in computation were serious enough to flaw your solution.</li> <li>▪ Your mathematical representations were inaccurate.</li> <li>▪ You labeled incorrectly.</li> <li>▪ Your solution was incorrect.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You made minor computational errors.</li> <li>▪ Your representations were essentially correct but not accurate or labeled completely.</li> <li>▪ Your inefficient choice of procedures impeded your success.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Your computations were essentially accurate.</li> <li>▪ All visual representations were complete and accurate.</li> <li>▪ Your solution was essentially correct.</li> </ul>	<ul style="list-style-type: none"> <li>▪ All aspects of your solution were completely accurate.</li> <li>▪ You used multiple representations for verifying your solution.</li> <li>▪ You showed multiple ways to compute your answer.</li> </ul>

## Insights

1 point	2 points	3 points	4 points
<ul style="list-style-type: none"> <li>▪ You were unable to recognize patterns and relationships.</li> <li>▪ You found a solution and then stopped.</li> <li>▪ You found no connections to other disciplines or mathematical concepts.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You recognized some patterns and relationships.</li> <li>▪ You found multiple solutions but not all were correct.</li> <li>▪ Your solution hinted at a connection to an application or another area of mathematics.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You recognized important patterns and relationships in the problem.</li> <li>▪ You found multiple solutions using different interpretations of the problem.</li> <li>▪ You connected your solution process to other problems, areas of mathematics or applications.</li> </ul>	<ul style="list-style-type: none"> <li>▪ You created a general rule or formula for solving related problems.</li> <li>▪ You related the underlying structure of the problem to other similar problems.</li> <li>▪ You noted possible sources of error or ambiguity in the problem.</li> <li>▪ Your connection to a real-life application was accurate and realistic.</li> </ul>

### Communication

1 point	2 points	3 points	4 points
<ul style="list-style-type: none"> <li>▪ I couldn't follow your thinking.</li> <li>▪ Your explanation seemed to ramble.</li> <li>▪ You gave no explanation for your work.</li> <li>▪ You did not seem to have a sense of what your audience needed to know.</li> <li>▪ Your mathematical representations did not help clarify your thinking.</li> </ul>	<ul style="list-style-type: none"> <li>▪ I had to make inferences about what you meant in places.</li> <li>▪ Your solution was hard to follow in places.</li> <li>▪ You weren't able to sustain your good beginning.</li> <li>▪ Your explanation was redundant in places.</li> <li>▪ Your mathematical representations were somewhat helpful in clarifying your thinking.</li> </ul>	<ul style="list-style-type: none"> <li>▪ I understood what you did and why you did it.</li> <li>▪ Your solution was well organized and easy to follow.</li> <li>▪ Your solution flowed logically from one step to the next.</li> <li>▪ You used an effective format for communicating.</li> <li>▪ Your mathematical representations helped clarify your solution.</li> </ul>	<ul style="list-style-type: none"> <li>▪ Your explanation was clear and concise.</li> <li>▪ You communicated concepts with precision.</li> <li>▪ Your mathematical representations expanded on your solution.</li> <li>▪ You gave an in-depth explanation of your reasoning.</li> </ul>

APPENDIX B

TECHNOLOGICAL EXPERIENCE SURVEY

TECHNOLOGICAL EXPERIENCE SURVEY

1. What is your age?

- 1 20 OR YOUNGER
- 2 21-30
- 3 31-40
- 4 41-50
- 5 51-60
- 6 OVER 60

2. What is your gender?

- 1 FEMALE
- 2 MALE

3. Do you own a computer?

- 1 NO (IF NO, PROCEED TO QUESTION 5)
- 2 YES

4. Is your computer connected to the Internet?

- 1 NO
- 2 YES

5. Do you have access to a computer through your school?

- 1 NO (IF NO, PROCEED TO QUESTION 7)
- 2 YES

6. Is there an Internet connection for that computer?

- 1 NO
- 2 YES

7. How would you rank your experience with computers?

- 1 NO EXPERIENCE
- 2 LITTLE EXPERIENCE
- 3 UNSURE OF EXPERIENCE
- 4 SOME EXPERIENCE
- 5 EXTENSIVE EXPERIENCE

8. How would you rank your knowledge of computers?

- 1 NO KNOWLEDGE
- 2 LITTLE KNOWLEDGE
- 3 UNSURE OF KNOWLEDGE
- 4 SOME KNOWLEDGE
- 5 EXTENSIVE KNOWLEDGE

9. Have you taken any distance learning courses in the past?

- 1 NO (IF NO, SURVEY IS COMPLETE)
- 2 YES

10. How many distance learning courses have you taken?

- 1 ONE
- 2 TWO
- 3 THREE OR MORE

11. In what types of distance learning have you taken part?

- 1 AUDIOCONFERENCING
- 2 VIDEOCONFERENCING
- 3 COMPUTER NETWORKING
- 4 OTHER (PLEASE SPECIFY) \_\_\_\_\_

Thank you

APPENDIX C

MATHEMATICAL ABILITY GROUP PLACEMENT INSTRUMENT

MATHEMATICAL ABILITYGROUP PLACEMENT INSTRUMENT

1. Which of the given numbers is farthest from zero?
  - (a) 1
  - (b)  $\frac{4}{3}$
  - (c)  $-\frac{3}{2}$
  - (d)  $-\frac{1}{8}$
  - (e)  $\frac{1}{32}$
  
2. The first operation performed in simplifying the expression  $[-6 + 3(4)]^2/2$  is:
  - (a) division
  - (b) addition
  - (c) multiplication
  - (d) squaring
  
3. Simplify:  $3a - [5b - 4a]$ 
  - (a)  $7a - 5b$
  - (b)  $-a - 5b$
  - (c)  $-15ab - 12a^2$
  - (d)  $-15ab + 12a^2$
  - (e)  $-6a^2b$
  
4. If  $x/2 = y$  and  $y = z - 4$  what is the value of  $x$  when  $z = 28$ ?
  - (a)  $x = 12$
  - (b)  $x = 48$
  - (c)  $x = 24$
  - (d)  $x = 40$
  - (e) none of these

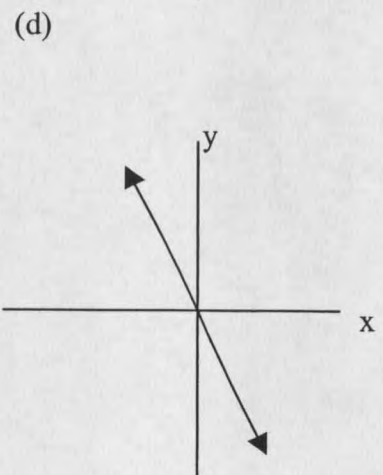
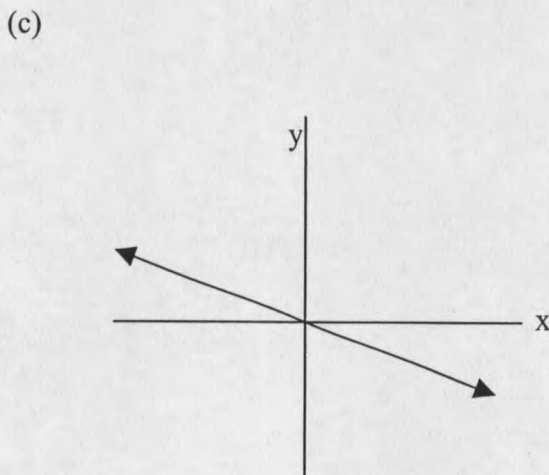
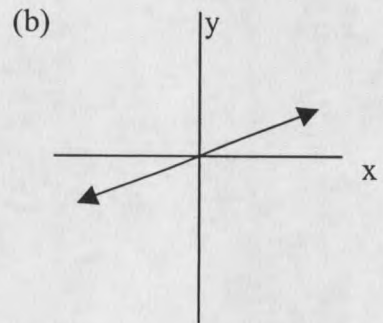
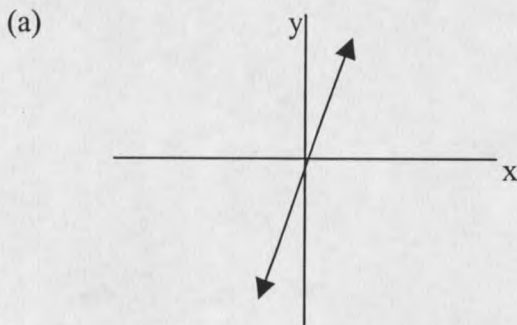
5. Find the value of  $[(3w)^2/10](r+l)$  when  $r = -3$ ,  $l = -1/3$ , and  $w = -2$ .
- (a) 3
  - (b) 0
  - (c) 12
  - (d) -4
  - (e) none of these
6. The least common denominator used to compute  $1/6 + -7/15 + 1/4$  is:
- (a) 60
  - (b) 360
  - (c) 20
  - (d) 4
  - (e) none of these
7. Twenty-five is 12.5% of what number?
- (a) 2
  - (b) 3.125
  - (c) 200
  - (d) 312.5
  - (e) none of these
8. At the canning factory, one machine can fill 180 jars in 15 minutes. How many jars can this machine fill in 50 minutes?
- (a) 710
  - (b) 600
  - (c) 558
  - (d) 240
  - (e) none of these

9. True or false:  $[(8t+3t) / t] = 11$  for all nonzero values of  $t$
- (a) true
  - (b) false
10. Solve  $4(k - 2) = (k/3) + 5$ . The solution is:
- (a) less than  $-3$
  - (b) between  $-3$  and  $0$
  - (c) between  $0$  and  $3$
  - (d) greater than  $3$
11. Solve  $5pq - 10p = 7q$  for  $p$ .
- (a)  $p = 2q / -5$
  - (b)  $p = 7q / (5q - 10)$
  - (c)  $p = (7q + 10) / 5q$
  - (d)  $p = 7q / (q - 2)$
  - (e) none of these
12. Solve:  $12 \geq 8 + 2t$
- (a)  $t \geq 10$
  - (b)  $t \leq 10$
  - (c)  $t \geq 2$
  - (d)  $t \leq 2$
  - (e)  $t \leq -2$

13. Which statement is true regarding the two lines whose equations are:

$$x - 2y = 4 \quad \text{and} \quad 2x + y = 2$$

- (a) the lines are parallel
  - (b) the lines are perpendicular
  - (c) the lines coincide
  - (d) the lines intersect at a point on the  $y$ -axis
  - (e) the lines intersect at a point on the  $x$ -axis
14. Which of the following graphs could have a slope of 3?  
(Assume the scales on the  $x$  and  $y$  axes are the same.)





20. Susie gets an allowance of \$1.00 per week. Spends some of it and saves the rest. The first weeks she saves 60 cents, the second week she saves 40 cents, but the third week she only saves 20 cents. At the end of the fourth week, Susie has saved an average of 45 cents per week. How much did Susie save the fourth week?

- (a) 40 cents
- (b) 45 cents
- (c) 50 cents
- (d) 60 cents
- (e) Not enough information is provided to answer the question.

APPENDIX D

PROBLEM-SOLVING TASK AND SOLUTION

PROBLEM-SOLVING TASK AND SOLUTIONNumbers with Multiple Personalities

All numbers have personalities. In this problem, we examine the rectangular personalities of numbers. The *rectangular personalities* of a number  $N$  are the rectangular arrays that can be formed from  $N$  dots. For example, the number 4 has one such array as does the number 6, and 12 has two arrays. We shall assume that the 2-by-3 array for 6 is not different from the 3-by-2 array because one can be formed from the other by rotation of the figure. Also, we shall say that the number 2 has no rectangular arrays because we will interpret the arrangement \*\* as a linear array, not a rectangular array. In fact, all prime numbers are like 2 in this regard; they do not have rectangular arrays.

*Question:* Of all numbers less than 1 million, which has(have) the most rectangular personalities?

*Hint:* You might use the fact that the formula for the number of factors (trivial & non-trivial) of a natural number is as follows:

If the number  $N$  is broken down into the product of its prime powers like this:  $N = p_1^{n_1} p_2^{n_2} \dots p_k^{n_k}$ , then the total number of factors is  $(n_1+1)(n_2+1)\dots(n_k+1)$ .

Solution: Numbers with Multiple Personalities

The chart below lists the composite numbers up to 50 along with the number of different rectangular personalities. The winners for the largest number of personalities are the numbers 36 and 48, with four apiece.

Number N	Rectangular Personalities	Number N	Rectangular Personalities
4	1	28	2
6	1	30	3
8	1	32	2
9	1	33	1
10	1	34	1
12	2	35	1
14	1	36	4
15	1	38	1
16	2	39	1
18	2	40	3
20	2	42	3
21	1	44	2
22	1	45	2
24	3	46	1
25	1	48	4
26	1	49	1
27	1	50	2

One may notice that the dimensions of the rectangles associated with a number are just the nontrivial factors of that number. As can be seen for the number 48, the dimensions come in pairs:  $2 \times 24$ ,  $3 \times 16$ ,  $4 \times 12$ , and  $6 \times 8$ . The product of each pair of numbers is 48. The trivial factors 1 and 48 do not form a rectangular array. So the number of rectangular personalities of 48 is half the number of nontrivial factors. Things are not quite as simple as this for 36, however. Notice that for the number 36, although the dimensions also come in pairs,  $2 \times 18$ ,  $3 \times 12$ ,  $4 \times 9$ , and  $6 \times 6$ , one of the factors 6, is used twice. Thus for 36, the number of nontrivial factors is odd, and to get the number of rectangular personalities, one must add 1 to the nontrivial factors before taking one-half of the total. This is true for all numbers that are perfect squares.

Now, use the formula for the number of factors (trivial & non-trivial) of a natural number:

If the number  $N$  is broken down into the product of its prime powers like this:  $N = p_1^{n_1} p_2^{n_2} \dots p_k^{n_k}$ , then the total number of factors is  $(n_1+1)(n_2+1)\dots(n_k+1)$ .

For example,  $48 = 2^4 3^1$ ; so the number of factors is  $(4+1)(1+1) = 10$ . Now two of these factors, 1 and 48, are trivial; so eight are nontrivial, one-half of 8 is 4 – this is the answer we have already found.

The question about multiple rectangular personalities comes down to a question of the quantity of factors of a number.

With some work one will find that we have three numbers all with 240 factors, 238 nontrivial factors, and 119 rectangular personalities. They are:

720,720

831,600

997,920

The above numbers have the most rectangular personalities of all the numbers less than 1 million.

APPENDIX E

ANALYTICAL FRAMEWORK

### THE ANALYTICAL FRAMEWORK

DIMENSION	DEFINITION
Participative	The total contribution of an individual or group in terms of number of messages or units of meaning transmitted
Social	Statement/question or part of statement/question not related to formal content of subject matter
Interactive	Statement, question, or comment that receives response, or that is made in response to other statements, questions or comments
Heuristic Episode	Phase in the problem-solving process such as orientation, organization, execution or verification

#### DIMENSION 1: PARTICIPATION

CATEGORY	DEFINITION	INDICATORS
Overall	Total number of learner messages or "units of meaning"	Quantitative data regarding the total number of messages or "units of meaning" made
Task-specific	Number of "units of meaning" made by learners directly related to learning	Quantitative data regarding the total number of "units of meaning" related to the formal content or process of online problem solving

#### DIMENSION 2: SOCIAL

CATEGORY	DEFINITION	INDICATORS
Social	Statement or question not related to formal content, subject matter or collaborative process of the group	Self-introduction Greetings "I'm feeling great..."

## DIMENSION 3: INTERACTIVE

CATEGORY	DEFINITION	INDICATORS
Interactive	Any statement, question, or comment that receives response, or that is made in response to other statements, questions or comments	
<i>Problem-Solving Interactions</i>	<i>Interactive statements, questions or comments related to the formal content or process of problem solving</i>	
Asking questions <ul style="list-style-type: none"> <li>▪ Of another group member</li> <li>▪ From own thinking or working</li> <li>▪ From the problem</li> </ul>	Any question that is asked in response to a previous statement or question, or that is followed by one or more responding statements or questions.	<p>"Ted, what exactly do you mean by add?"</p> <p>"Do I multiply or divide by 7 now?"</p> <p>"Split 20 students into 5 groups. Must the groups be equal in number?"</p>
Responding <ul style="list-style-type: none"> <li>▪ To a question</li> <li>▪ To a request for clarification</li> <li>▪ Agreeing</li> <li>▪ Disagreeing</li> <li>▪ Repeating</li> </ul>	Any statement that is made in response to a previous statement or question.	<p>"(in response to above question) I would multiply by 7."</p> <p>"I meant that 7 is a prime number."</p> <p>"I agree with Jim's opinion..."</p> <p>"You're wrong!"</p> <p>"I said 8 goes here."</p>
Directing	Any statement providing directions	"Make sure that we added correctly."
Explaining with evidence	Any statement providing an explanation with evidence	"It's 3 because you invert and multiply."
Thinking aloud while working	A statement exhibiting one's thoughts, actions or processes while working	"Now I will multiply the five numbers together..."
Proposing Ideas	Any statement proposing an idea	"What about separating them into two groups Jill?"
Refocusing discussion	Any statement refocusing the direction of the discussion	"That's what we're supposed to be doing."
Commenting	Statements of an affective nature	"I don't understand this problem!"

*Other Interactions  
(Technology & Online  
Group Process  
Related)*

*Interactive questions, statements or  
comments related to the use of the  
technology, or to the process of online  
collaboration.*

"I couldn't get online  
yesterday."  
"Let's discuss this online  
tomorrow at 4 o'clock."

**Non-interactive**

**Any statement, question or comment  
relating to the subject under  
discussion, but which is not in  
response to another message, and  
which does not lead to any further  
statements**

#### DIMENSION 4: HEURISTIC EPISODE

PROBLEM-SOLVING STAGE	DEFINITION	INDICATORS
Orientation	Strategic behavior to assess and understand a problem	<ul style="list-style-type: none"> <li>▪ Comprehension strategies</li> <li>▪ Analysis of information and conditions</li> <li>▪ Assessment with familiarity of task</li> <li>▪ Initial and subsequent representation</li> <li>▪ Assessment of level of difficulty and chances of success</li> </ul>
Organization	Planning of behavior and choice of actions	<ul style="list-style-type: none"> <li>▪ Identification of goals and subgoals</li> <li>▪ Global planning</li> <li>▪ Local planning (to implement global plans)</li> </ul>
Execution	Regulation of behavior to conform to plans	<ul style="list-style-type: none"> <li>▪ Performance of local actions</li> <li>▪ Monitoring of progress of local and global plans</li> <li>▪ Trade-off decisions (e.g., speed vs. accuracy, degree of elegance)</li> </ul>
Verification	Evaluation of decisions made and of outcomes of executed plans	<ul style="list-style-type: none"> <li>▪ Evaluation of orientation and organization               <ul style="list-style-type: none"> <li>➢ Adequacy of</li> </ul> </li> </ul>

- 
- representation
  - Adequacy of organizational decisions
  - Consistency of local plans with global plans
  - Consistency of global plans with goals
  - Evaluation of execution & outcomes
    - Adequacy of performance of actions
    - Consistency of actions with plans
    - Adequacy of local results or final results
    - Consistency of local results with plans and problem conditions
    - Consistency of final results with problem conditions
-

APPENDIX F

MATRIX FOR MESSAGE ANALYSIS



APPENDIX G

SAMPLE  
MESSAGE TRANSCRIPT

SAMPLE  
MESSAGE TRANSCRIPT

**Archive for Jul 12, 2001**

---

Karen has entered. [ 03:43:54 PM ]

**Karen** > hello

Karen has left. [ 03:46:56 PM ]

---

Karen has entered. [ 03:50:24 PM ]

**Karen** > hi

Karen has left. [ 03:58:03 PM ]

---

Karen has entered. [ 03:59:39 PM ]

Karen has left. [ 04:01:05 PM ]

---

Peter has entered. [ 04:22:13 PM ]

**Peter** > Hi

Peter has left. [ 04:22:59 PM ]

---

Karen has entered. [ 04:56:09 PM ]

Peter has entered. [ 04:58:44 PM ]

**Peter** > Hi

**Karen** > hello

**Karen** > I still don't understand the problem

**Peter** > I don't either

**Peter** > I was hoping that you could explain

**Karen** > I know that prime numbers do not have rect. arrays

**Karen** > So we need to look at the composite numbers

**Peter** > that was stated in the problem

**Karen** > but, how can we find all consecutive composite numbers

**Peter** > do you understand the formula that's given?

**Peter** > that is the question that if answered will win the lottery

**Karen** >  $p =$  prime numbers

**Peter** > ok

**Karen** >  $n =$  any pos. numbers

**Karen** > but then how do we apply the formula?

**Karen** > do we need to multiply the prime numbers?

**Peter** > it is the same method of calculating each stage

**Karen** >  $p$  increases every time

**Peter** > I think so

**Karen** > I don't know what else to do

**Karen** >  $N =$  number of factors

**Peter** > if you look at the formula, you keep adding the next  $n$ , and then multiply it by

the previous answer

**Karen** > Do we need to find the number of factors for each composite number?

**Peter** > we just lost some of the discussion

**Karen** > what?

**Karen** > If  $p_1 =$  a prime number, can we start with the smallest prime number?

**Peter** > what are the prime # from one to 1 million?

**Karen** > We need the primes or the composite numbers?

**Peter** > It is stated that prime # do not have rectangular personalities

**Karen** > Because, prime numbers are easier to get

**Peter** > sorry array

**Karen** > Peter, we don't need the prime numbers, they don't have rect. arrays

Erin has entered. [ 05:12:48 PM ]

**Erin** > hello group

**Karen** > hi

**Karen** > HELP

**Erin** > hi Karen, Hi Peter

**Peter** > hi. I agree

**Erin** > let me see your previous

**Karen** > ok

**Erin** > i do not know guys, but this is a difficult problem

**Karen** > What we need to do first, is find all the composite numbers, then find the factors of each number

**Erin** > lets just work first with whatever we understand

**Erin** > you are right..

**Peter** > Look at the formula at the bottom of the page

**Karen** > but how do we do that?

**Erin** > but we also need to find a formula, because million numbers are too much

**Erin** > ok

**Peter** > Define composite # so we can agree on what we are looking for

**Erin** > lets look at the formula

**Erin** >  $N = p_1 p_2^{n_2} \dots p_k^{n_k}$

**Erin** > help me in determining what each means

**Erin** > I do agree that we have to find the number, composite, that has the greatest number of factors

**Karen** > I'm trying to find something

**Peter** > n is the product of all the # that are applicable from 1-1,000,000

**Erin** > it can not be a prime number

**Karen** > Why not?

**Erin** > what is p?

**Peter** > because it does not have rectangular personality

**Erin** > because prime numbers only have 1 rectangular personality

**Erin** > meaning only one set of factors

**Peter** > ok

**Karen** > Is  $n =$  to prime numbers?

**Peter** > no  
**Erin** > is to any number  
**Erin** > right?  
**Karen** > Because it says, that N is broken down into the product of its prime powers  
**Erin** > which i guess it is just an example o its prime numbers  
**Erin** > sorry  
**Erin** > it is just an example of its factors  
**Peter** > n is the the total of all the composite #s from 1-1,000,000  
**Erin** > Is p+ factors?  
**Erin** > p=factors?  
**Karen** > no  
**Karen** > it can't be  
**Erin** > and 1 next to the p the power of the exponent?  
**Erin** > so is p=product?  
**Karen** > I'll be back Peter only 5 minutes  
**Peter** > Let us agree on the terms first, then we can continue  
**Erin** > i agree  
**Peter** > a prime # is a whole # graeter than 1 whose only factors are 1 and itself  
**Erin** > correct  
**Erin** > we need to find the factors of composite #  
**Peter** > right  
**Erin** > and we need to find the # less than a million that has the greatest # of factors  
**Peter** > so the best way to do this is to go through it step by step  
**Karen** > i'm back  
**Peter** > what is the highest composite number that is less than 1,000,000  
**Erin** > yeah but that is a whole lot of numbers, 1000000  
**Karen** > p has to be prime numbers, because every integer can be written as a product of powers of prime #s  
**Erin** > minus the prime, of course  
**Peter** > is it 999,000,999  
**Erin** > i can agree to tjahat  
**Erin** > to that  
**Erin** > how did u get that #?  
**Peter** > it is 1 less than 1,000,000, and it is a composite #  
**Peter** > hello  
**Karen** > we are thinking  
**Peter** > I can hear you  
**Karen** > no you not  
**Peter** > think  
**Peter** > but how do we use the formula to find the total # of factors  
**Erin** > let me think for a couple of minutes...i am looking at the end of the page to:  
 $(n_1+1)(n_2+1)\dots(n_k+1)$ , this is how you find how many factors a # have  
**Erin** > let me think  
**Erin** > I think this means that after finding each prime with its exponent, we need to

follow the formula that I just posted...correct me if i am wrong?

**Erin** > but how are we going to solve for something like this 48 which is equal to  $2^4 \cdot 3$ ????

**Peter** > but how will prime #s help when they don't have rectangular personalities?

**Erin** > the prime the talk about is like #7

**Erin** > it only have  $1 \cdot 7$

**Erin** > but all the composite numbers will be broken down into prime numbers

**Peter** > I know, but it is stated that prime #s do not have this quality

**Peter** > how

**Erin** > I think we first have to practice finding the personalities of some simple #, before we move on to more complex

**Peter** > give me an example

**Erin** > example #48 is a composite #, its factorization is  $= 2^4 \cdot 3$

**Erin** > the 2 to the fourth power and the 3 are prime #s

**Erin** > but the # we started with is not, 48

**Peter** > very good

**Karen** > ok

**Peter** > so will we have to do this to all composite#s ?

**Karen** > there are 74 composite numbers from 1-100

**Karen** > I don't think we can do that like that

**Erin** > What do u think about the rectangular arrays...I think that that is how many rectangles can we form with a given #, ....Am I right?

**Karen** > yes

**Peter** > what is a rectangular array?

**Erin** > so for example with #6, we can do 2,  $2 \cdot 3$  and  $6 \cdot 1$ , right?

**Karen** > no

**Erin** > why not?

**Karen** > you can't make a rectangular array with  $6 \cdot 1$

**Karen** > that's a linear array

**Erin** > ok, because it is a linear

**Peter** > Excuses me, but, what is a rectangular array ??????????

**Erin** > got it, so 6 only have one

**Karen** > the number six has four factors

**Erin** > it is how many rectangles you can form with a given #

**Karen** > 1, 2, 3, and 6

**Peter** > thank you

**Karen** > sorry 1, 2, 3, 6

**Karen** > 1, 2, 3, 6

**Erin** > ok, but since we do not use 1 and 6, it means that it only have one rectangular array, correct?

**Karen** > so we don't count 1 and 6 as part of an array

**Karen** > correct

**Peter** > why not 6?

**Karen** > ...

**Karen** > I'M SORRY

**Erin** > ok, so I guess we need to find out which number has the greatest number of rectangular arrays

**Karen** > yes

**Karen** > with the number 6 make a 2 by 3 rectangle, that's the only rectangle you can make

**Erin** > Peter, because if the number is 6, and what other number you need to multiply it times six to have a product of 6?...1 is the answer so the rectangle will be  $1*6$ ...but if we use one, that is linear and not a rectangular

**Erin** > gus, give me a couple minutes, let me digest this

**Erin** > the question

**Karen** > factors of 6 are ( 1, 2, 3, 6) don't use 1, 6 you have (2, 3) left only two factors  $2 \text{ factors} / 2 = \# \text{ of rect arrays}$

**Karen** > = 1 only one rect. array can be made from the # 6

**Karen** > the same thing with the # 12. Factors (1, 2, 3, 4, 6, 12) total of 6 factors take away 1, 6 you have 4 factors left. Divide 4 by  $2 = 2 \text{ rect. arrays}$

**Peter** > please explain what you just did Karen

**Erin** > I think u are right Karen

**Karen** > I'm right

**Karen** > I just don't know a formula for that, and it's impossible to do that for all composite numbers

**Erin** > so, lets see how can we find that #

**Karen** > let's do it for 1-100 first

**Erin** > There must be a way so that we do not have to go through all those #

**Erin** > ok

**Peter** > Am I a part of this discussion ?

**Karen** > I know

**Karen** > Yes you are

**Karen** > What's your question?

**Peter** has left. [ 06:07:36 PM ]

**Karen** > Erin, he left, is he mad?

**Erin** > i do not know, maybe he got disconnected

**Karen** > are we looking at the first 100 #'s? Maybe we will find a pattern

**Erin** > Karen, what do you think if we work on the first 100 on our own, and come back and compare to see if we can come up with a formula, by looking at the pattern

**Erin** > so far I am up to 15

**Karen** > ok

**Erin** > Karen, what about 16, how many it has, because  $4 * 4$

**Karen** > that's not rect

**Erin** > why?

**Karen** > because it is a square

**Erin** > but squares are special kinds of rect

**Erin** > rectangles+right angles

**Erin** > rect=right angles

**Karen** > ok

**Karen** > you're right

**Erin** > are you sure?

**Karen** > yes

**Erin** > ok

**Karen** > Erin are you there?

**Erin** > yes

**Karen** > this is too much...

**Erin** > yeah

**Erin** > 53 is prime?

**Karen** > I don't see any pattern

**Karen** > yes

**Erin** > neither do I

**Erin** > there must be a way

**Karen** > 36 and 100 have 4 rect arrays

**Erin** > i do not think it is possible for anyone to sit and look for this # one by one

**Karen** > Do you have something greater than 4?

**Erin** > no

**Karen** > I'm not doing this, let's do some research first, there has to be a formula for this.

**Erin** > where is Peter?

**Erin** > i agree

**Erin** > agree

**Karen** > I called his house, but there was no answer

**Karen** > I think he is mad

**Erin** > why do you think he got upset?

**Karen** > because he said, he was also part of the conversation

**Erin** > he has to support us...3 is better than 2

**Erin** > I did not see that?

**Karen** > but i was talking to both of you

**Erin** > i know

**Karen** > go up so you can see it

**Erin** > we were talking a

**Erin** > ok

**Erin** > i saw it, but we were all talking

**Karen** > Erin, let's find more information about this, and then we log in tomorrow

**Karen** > do you agree?

**Erin** > Karen, i agree, lets do some research and get back,,when is it good for you?...I know you have to study for the test

**Karen** > If I find something I will put it on the discussion board

**Erin** > i will do the same

**Karen** > tomorrow is good, tell me the best time for you

**Karen** > not early in the morning please

**Erin** > so see you ...lets see if we can take this bull by its horn!

Erin > any time

Karen > tomorrow at 3:00

Erin > ok

Karen > call Peter

Erin > ok

Karen > bye

Erin > bye

Karen has left. [ 06:40:59 PM ]

Erin has left. [ 06:41:15 PM ]

Erin has entered. [ 06:57:43 PM ]

Erin has left. [ 06:57:45 PM ]

### Archive for Jul 13, 2001

Karen has entered. [ 02:55:01 PM ]

Karen has left. [ 02:55:17 PM ]

Karen has entered. [ 03:00:46 PM ]

Karen > Peter, I'll be right back

Karen has left. [ 03:03:00 PM ]

Peter has entered. [ 03:06:51 PM ]

Peter > hi

Karen has entered. [ 03:19:49 PM ]

Karen > hi

Karen > Ok, whatlet's start

Peter > I hhave ten minutes

Karen > sorry

Peter > how far did you guys get

Karen > the first important thing is that the number of rect. arrays can be formed by the # of divisors the # has

Karen > for example, the # 6 has 4 divisors

Karen > 1,2,3,6

Peter > ok

Karen > and if you express 6 as a product of pairs you get

Karen > 1\*6

Karen > 6\*1

Karen > 2\*3

Karen > 3\*2

Peter > ok

Karen > we have to discard any product that has one in it because that is considered a linear array

**Peter** > so we have 4 rec. arrays?  
**Karen** > futhermore,  $2*3$  and  $3*2$  are just rotations of each other  
**Karen** > so we only have one rect. personality  
**Karen** > or one rect. array  
**Karen** > is the same thing  
**Karen** > questions?  
**Peter** > so  $3*2$  is the only rec. array?  
**Karen** > 30, on the other hand has 3 rect. personalities;  $2*5$ ,  $3*10$ ,  $5*6$   
**Karen** > yes that's the only one in the #6  
**Peter** > ok  
**Karen** > so now we need to look at the formula  
**Peter** > can we use the formula to work this out?  
**Karen** > the formula is for the # of divisors a number has  
**Karen** > so I think we can use that formula  
**Peter** > so how do we find the arrays  
**Karen** > let me look for the formula  
**Karen** > why do you have to go? is getting interesting  
**Karen** > Ok  
**Peter** > in about 10 minutes  
**Karen** > p represents a prime # and  
**Karen** has left. [ 03:31:30 PM ]  
**Peter** > could we login back at about 4:15  
**Peter** has left. [ 03:34:07 PM ]

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**Karen** has entered. [ 03:35:33 PM ]  
**Karen** > hello  
**Karen** has entered. [ 03:51:30 PM ]  
**Karen** has left. [ 03:51:37 PM ]  
**Karen** has entered. [ 04:18:40 PM ]  
**Karen** has left. [ 04:21:33 PM ]  
**Karen** has left. [ 04:21:33 PM ]

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**Erin** has entered. [ 11:58:17 PM. ]  
**Erin** has left. [ 11:58:26 PM ]

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## Archive for Jul 14, 2001

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**Peter** has entered. [ 09:55:27 AM ]  
**Peter** > welcome to math pro  
**Karen** has entered. [ 09:57:57 AM ]  
**Karen** > hi Peter  
**Karen** > Did you talk to Erin?  
**Peter** > i was out and i forgot

**Karen** > I saw in the discussion board that she is having problems to get into the v. classroom

**Karen** > Do you know how to find the rectangular arrays?

**Karen** > As we did yesterday

**Peter** > so we are going to have to go on without her then post it on the discussion board for her.

**Karen** > ok

**Peter** > using your theory we can do it , but it would take a very long time

**Karen** > ok, let's try the formula

**Peter** > At least I now understand. Thanks to you

**Karen** >  $(n1+1)(n2+2)$ ...

**Peter** > ok

**Karen** > the # 6 has 2 and 3 as its prime factors

**Peter** > therefore it has 1 rec. array

**Karen** >  $n1 =$  the first prime power so it will be = 1

**Karen** >  $n2 = 1$

**Karen** >  $(1+1)(1+1) = 4$

**Peter** > so what does the 4 represent?

**Karen** >  $4/2$ , because there are numbers that are rotations of each other like  $2*3$  and  $3*2$

**Karen** >  $4/2 = 2$

**Karen** > know, we need to subtract 1 because we have to discard any product that has 1 in it

**Karen** >  $2-1=1$

**Karen** > The answer is 1

**Peter** > I understand the calculations, but why  $N1$  &  $n2$  are both 1

**Karen** > because the prime factors are 2 and 3

**Karen** > and you are only repeating 2 one time

**Karen** > 3 one time

**Karen** > so the exponents of 2 and 3 is one

**Karen** > if the prime factors were 2, 3, 3, then

**Karen** > the exponents were 1, 2

**Karen** >  $n1$  will be =1

**Peter** > so  $n1$  is prime factor 1 &  $n2$  is prime factor 2

**Karen** >  $n2 = 2$

**Karen** > got it?

**Karen** > It depends on how many times the number is repeated in the prime factorization

**Karen** > Peter?

**Peter** > yes just one question look on the formula

**Karen** > take out a pen and let's work it together

**Peter** > that's what I am doing

**Karen** > let's find the prime factors of the # 18

**Peter** >  $(n1+1)(n2+2)$

**Karen** > ok

**Peter** > but you had  $(1+1)(1+1)$

**Karen** > I'm doing 18 now

**Karen** > before I did #6

**Karen** > which are the prime factors of 18?

**Peter** > yes. But why wasn't it  $(1+2)$  for the second part?

**Peter** > 6, 3, 9, 1

**Peter** > and 2

**Karen** > because you can't put the number, you have to use the power

**Peter** > ok

**Karen** > just take the prime numbers

**Peter** > 2, 3, 1

**Peter** > ok?

**Karen** > 18..... $2*9$  (2 is a prime #, so you have 9 left)  $9=3*3$  you are finish because 3 is also a prime #

**Karen** > so your prime factorization is 2, 3, 3.

**Karen** > 1 is not a prime #

**Karen** > now,  $n1=2$

**Karen** > sorry

**Karen** >  $n1=1$  (because you only have 1 #2)

**Peter** > ok

**Karen** >  $n2=2$  (because you have 2 # 3's)

**Peter** > you are a genius

**Peter** > so if we had a third factor it would have been  $n3$

**Karen** > if you get 2, 2, 2, 2, 3, 3 as an answer then  $n1$  will = 4 and  $n2=2$

**Karen** > If you a third factor =  $n3$

**Karen** > are you ok, now?

**Peter** > very good teacher

**Karen** > thank you

**Peter** > so what would be the rec. array.3?

**Karen** > now that we know how to work with the formula, we need to find a # whose exponent will give us a # less than 1,000,000

**Karen** > and the most rect. arrays

**Karen** > so, let's start with the lowest prime #

**Karen** > the smallest prime # is the #2

**Karen** > right?

**Peter** > yes. Because you have taught me that 1 is not a prime#

**Karen** > Very good

**Karen** > I know (and you should know) that  $2^{20} = 1,048,576$

**Karen** > that's more than 1,000,000

**Peter** > yes

**Karen** > so my exponent should be less than 20

**Karen** > let's try  $2^{19}$

**Peter** > maybe 18

Peter > ok  
 Karen >  $2^{19} = 524,288$   
 Peter > i am getting my calculator  
 Karen > in this case  $p(1) = 2$   
 Karen >  $n(1) = 19$   
 Peter > yes  
 Karen > and  $N = 524,288$   
 Peter > os far so good  
 Peter > so  
 Karen > to get the total of factors I go back to the formula  $(n1+1)$   
 Karen >  $TF = 19+1=20$   
 Karen > # rect arrays =  $TF/2 - 1$   
 Karen >  $20/2-1=9$   
 Karen > so this huge # only have 9 rect. arrays  
 Karen > that's the way we do it  
 Peter > you just lost me. I htoiught it would have been  
 Peter > what about  $n+524,288$ ?  
 Karen > what happened/  
 Peter >  $n=524,288$ . what do we do with that  
 Karen > remember, we are using the exponents of the factors  
 Karen > Is capital N  
 Karen > we have n and N  
 Peter > so we substitute the number with the exponent?  
 Karen >  $N =$  the number that we found  
 Peter > cleary understood young lady  
 Karen >  $2^{19} = 524,288$   
 Peter > so 20 is the answer.  
 Karen >  $2=p(1)$   $19=n1$  and  $524,288 = N$   
 Karen >  $20 =$  the # of divisors or the # of factors that  $524,288$  has  
 Peter > sorry  
 Peter > go on  
 Karen > now to find the rect. arrays we need to divide the # of factors in this case 20 by  
 2  
 Karen > Why do we divide by 2?  
 Peter > ???  
 Karen > Because there are numbers that are rotations of each other  
 Karen > remeber  $2*3$  is the rotation of  $3*2$   
 Peter > Got.  
 Karen > we are repeating the same rectangle  
 Peter > and why do we subtract 1?  
 Karen > so, when we divide  $20/2 = 10$   
 Karen > now, we need to subtract 1  
 Peter > why?  
 Karen > why do we subtract 1?

**Peter** > the suspense is killing me  
**Karen** > because we need to discard any product that has one in it, because that is considered  
**Karen** > a linear array  
**Peter** > wow  
**Karen** >  $10-1=9$   
**Peter** > excellent  
**Peter** > so the answer is 9  
**Karen** > the # 524,288 has 9 rect. arrays  
**Karen** > got it?  
**Peter** > very clear  
**Karen** > but,  
**Karen** > this is not quite right for all the numbers  
**Peter** > oh no  
**Karen** > because this doesn't work for the numbers that are perfect squares  
**Karen** > example 36 has  $6*6$   
**Peter** > so what do we do now?  
**Karen** >  $6*6$  will not produce a rect array  
**Karen** > that's consider a square array  
**Karen** > but there is nothing we can do with those numbers  
**Karen** > because if we also take one more out for the number 36  
**Karen** > the answer will not be an integer  
**Karen** > it will be a decimal  
**Karen** > so I think, we have to keep that and at the end just round our answer  
**Peter** > You taught me well, now you are confusing me.  
**Karen** > are you following me?  
**Karen** > This is just a clarification  
**Peter** > painfully so  
**Karen** > we are not going to do anything with the perfect square numbers  
**Karen** > is just for you to know  
**Karen** > and son't tell me you are painfully listening to me  
**Peter** > or just for you to tortue me  
**Karen** > shut up  
**Peter** > that is not nice  
**Karen** > be quiet then  
**Peter** > so the answer is still 9  
**Karen** > so know what we do? yes the answer is 9 for that particular #  
**Karen** > we need Erin  
**Karen** > let's try  $2^{18}$   
**Karen** > do you have your calc  
**Peter** > I think you have done an excellent job, and you should explain it and send it to Dr. Gaston  
**Peter** > ok  
**Karen** > but we don't have the answer yet

Peter > I understand  
Peter > so lets go on  
Karen >  $2^{18} = 262,144$   
Peter > yes  
Karen > can we mult. that # by any prime # and still get less than 1,000,000?  
Peter > 2  
Karen > if we mult. by 2 we will be back to our previous #  
Peter > or 3  
Karen > do it by 3  
Peter > 786,432  
Karen > ok  
Karen > we can't mult that # by 2  
Karen > it will be greater than one million  
Peter > ok  
Karen > so in this case  $n1=18$   
Karen >  $n2=1$   
Karen > because we are mult by 3  
Karen > and 3 has 1 as an exponent  
Peter > ok  
Karen >  $N = 2^{18} * 3 = 786,432$   
Peter > ok  
Karen > now how do we get the factors again?  
Karen > got it  
Karen >  $(n1+1)(n2+2)$   
Karen >  $(18+1)(1+2)$   
Peter > i am trying to get Erin  
Karen > I made a mystake  
Karen >  $(n1+1)(n2+1)$   
Karen >  $(18+1)(1+1)$   
Karen >  $(19)(2)$   
Karen >  $=38$   
Karen >  $38/2=19-1=18$   
Karen > rect. arrays =18  
Peter > Erin cannot login  
Karen > what happened?  
Karen > can she read our discussion/  
Peter > her internet is not working  
Karen > ok  
Peter > so she has to use her boyfrind's own  
Peter > and he is not home  
Karen > oh God!  
Peter > lets not get dramatic now  
Karen > ja ja  
Peter > english please

**Karen** > read what I got for that big #  
**Karen** > does it makes sence?  
**Peter** > 18  
**Peter** > It is logical, but why did you X the factors by 2  
**Peter** > 3 of factors  
**Karen** > I think we need to try some more and see if we find a pattern  
**Peter** > # of factors. sorry  
**Karen** > where?  
**Karen** > let's do it tgether  
**Peter** >  $(18+1)(1+1)$   
**Karen** > we have  $2^{18} * 3$ , is that clear?  
**Peter** > yes  
**Karen** >  $n1=18$   
**Karen** > right?  
**Peter** > yes  
**Karen** >  $n2=1$   
**Karen** > yes?  
**Peter** > yes  
**Karen** > formula  $(n1+1)(n2+1)$   
**Karen** >  $(18+1)(1+1)$   
**Karen** >  $(19)(2)$   
**Karen** >  $=38$   
**Karen** > are you ok?  
**Peter** > did we do this when the exponent was 19?  
**Karen** > yes darling  
**Karen** > don't you remember?  
**Karen** > that the answer was 20  
**Karen** > then we /2 and subtract one  
**Karen** > that's how we got 9  
**Peter** > so it would have been  $40/2-1=19$   
**Karen** > where you get that 40 from?  
**Karen** > we only had n1  
**Karen** > we didn't have n2  
**Peter** >  $(19+1)(1+1)=40$   
**Peter** > ok, ok  
**Karen** > in this case we have n2 because we are mult by 3  
**Karen** > and 3 has an invisible exponent 1  
**Peter** > ok  
**Peter** > so let us move to 17  
**Peter** > let me try and do this one  
**Karen** > so do you agree that the answer for  $2^{18} * 3 = 38/2-1 = 18$   
**Peter** > yes  
**Karen** > know  $2^{17}$ , you do it  
**Peter** >  $2^{17}=131,072$

**Karen** > i got tha  
**Peter** >  $131,072 \times 7 = 917,504$   
**Peter** >  $2^{17} \times 7 = 917,504$   
**Karen** > ok  
**Peter** >  $(17+1)(1+1) = 36$   
**Karen** > ok  
**Peter** >  $36/2 - 1 = 17$   
**Peter** > rec array is 17  
**Karen** > ok  
**Peter** > right?  
**Karen** > yes  
**Peter** > so there is a pattern  
**Karen** > we need to make a table  
**Karen** >  $2^{19} = 9$   
**Peter** > for exponents 2-19?  
**Karen** > what/  
**Karen** > I'm just going back to record our answers  
**Peter** > using exponents 2-19 so cover all the bases  
**Peter** > should  
**Karen** > I don't understand  
**Peter** > we used 19, 18, and 17  
**Karen** > ok  
**Peter** > shouldn't we keep going to 2  
**Karen** > what you mean?  
**Peter** > where should we stop so as to establish the answer  
**Peter** > am i confusing you?  
**Karen** > I don't know, I think we should work out all the # 2's  
**Karen** > let's try  $2^{16}$   
**Peter** > with exponents 16, 15, 14, 13, 12, 11, 10,.....  
**Karen** > yes  
**Karen** > and then check to see which one has the most rct arrays  
**Peter** > you do 16, and i will do 15  
**Karen** > ok  
**Peter** > yes  
**Karen** > aul, when you're mult and you get a # greater tahn 500,000 stop, because you can't mult any further  
**Peter** >  $2^{15} = 32,769$ .  $32,769 \times 30 = 983070$ .  $(15+1)(1+1) = 32/2 - 1 = 15$   
**Karen** > I mean Peter  
**Karen** > I don' think tha's right,  
**Karen** > because you X by 30  
**Karen** > 30 is not a prime #  
**Peter** > The pattern seems to be that after 19, the array will be = to the exponent  
**Karen** > i didn't get that  
**Karen** > you need to muly by a prime #

**Karen** > mult

**Peter** > It does not seem to matter how much you multiply by, only that you can multiply

**Karen** > what is  $2^{15}$ ?

**Peter** > 32768

**Karen** > ok

**Karen** > you pt 32,769 before

**Peter** > after that it does not matter how much you multiply by. That may have been a mistake

**Karen** >  $2^{15} = 32,768$

**Karen** >  $32,768 * 3 = 98,304$

**Peter** > ok

**Karen** >  $*3 = 294,912 * 3 = 884,736$

**Karen** > i can't mult any more because the # is greater than 500,000

**Peter** > ok

**Karen** > ans.  $2^{16} * 3^2$

**Karen** >  $n_1=16$   $n_2= 2$

**Karen** >  $(16+1)(2+1)$

**Karen** >  $(17)(3)$

**Karen** > I'm sorry

**Karen** > i was doing mines

**Peter** > continue

**Karen** > let's go back

**Karen** > your is  $2^{15} * 3^3$

**Peter** > yes

**Karen** >  $n_1 = 15$   $n_2=3$

**Karen** >  $(15+1) (3+1)$

**Karen** >  $(16) (4)$

**Karen** > 64

**Karen** >  $64/2 = 32$

**Karen** >  $32-1 = 31$

**Karen** > we didn't get the same ans

**Karen** > Peter, are you there?

**Peter** > It means that the one I did before that is incorrect also

**Karen** > which one?

**Karen** >  $2^{17}$

**Peter** > i am trying to find it now

**Karen** > it was  $2^{17}$ , that one is correct, because you multiplied by 7 and 7 is a prime number

**Karen** > in this case  $2^{15}$  is not correct because you X by 30

**Peter** > but you use 3 then 3

**Karen** > 30 is not a prime #

**Karen** > you can do that

**Peter** > but that will make all the answers different

**Karen** > do you know how to find the prime factorization?  
**Karen** > let me try to do it in the blackboard or whiteboard  
**Karen** > how do you write there?  
**Peter** > i do not know  
**Karen** > can you see what I put there?  
**Peter** > 5,7  
**Karen** > ok  
**Karen** > that's a mystake  
**Peter** > ok  
**Peter** > some of what you wrote went off the screen and I did not get it  
**Karen** > What I wrote was that 18 is not a prime # so we need to get the prime fact of 18  
**Peter** > ok  
**Karen** > Do you understand why the prime factors of 18 are 2,2,3,3?  
**Karen** > and if you mult  $2*2*3*3 = 36$   
**Karen** > in this case  
**Karen** >  $n1=2$   
**Karen** >  $n2=2$   
**Peter** > shoudn't it be 2,3,3  
**Karen** > why?  
**Peter** > we are doing 18  
**Karen** > Look up in the blackboard I started with 36  
**Peter** > ok  
**Karen** > so, this is the reason why you can't mult by 30  
**Peter** > ok  
**Karen** > we need to mult by prime #'s  
**Karen** > Peter, we are doing the right thing, but I don't think we can finish this and the other project by monday  
**Karen** > I'm working plus  
**Peter** > this one has to be posted tommorrow  
**Karen** > tomorrow i'll be taking the CTS  
**Karen** > I'll be there all morning  
**Karen** > I'll be getting home by 2  
**Peter** > what time later today can you lodin  
**Karen** > Today I have to go study for my test tomorrow  
**Karen** > can we keep working on the tables?  
**Peter** > so we are gonna have to do it after your test then  
**Karen** > and then compare our results  
**Karen** > after we finish with  $p1(2)$   
**Peter** > yes  
**Karen** > we have to do the same thing for  $p(3)$   
**Karen** > but this time  $p3$  is not going to have  $n1=19$   
**Karen** > because it will be greater than a million  
**Karen** > we need to find another exponent

**Karen** > ok

**Karen** > then we do the same for 5

**Karen** > we don't do it for 4, because 4 is not a prime #

**Peter** has left. [ 12:00:30 PM ]

**Karen** > then for 7

**Karen** has left. [ 12:00:50 PM ]

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**Peter** has entered. [ 12:00:58 PM ]

**Peter** has left. [ 12:01:12 PM ]

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**Peter** has entered. [ 12:13:25 PM ]

**Peter** has left. [ 12:14:03 PM ]

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APPENDIX H

SAMPLE  
CODED TRANSCRIPT MATRIX

SAMPLE  
CODED TRANSCRIPT MATRIX

MESSAGE	UNIT OF MEANING	PARTICIPANT	SOCIAL DIMENSION (X MARKS OCCURRENCE)	INTERACTIVE (I) vs. NON-INTERACTIVE (N)	PROBLEM SOLVING (P) vs. OTHER (O)	INTERACTIVE CATEGORY	PROBLEM SOLVING HEURISTIC EPISODE
1	1	MARA	X				
2	2	MARY	X				
3	3	MARA		I	O		
4	4	MARY		I	O		
5	5	MARY		I	O		
5	6	MARY		I	O		
6	7	MARA		N	O		
7	8	MARA		I	O		
7	9	MARA		N	O		
8	10	MARA		I	O		
9	11	JANE	X				
9	12	JANE		I	O		
10	13	MARY		I	O		
10	14	MARY		I	P	R	V
11	15	MARY	X				
11	16	MARY		I	O		
12	17	JANE		N	P	Q	V
13	18	JANE		I	O		
14	19	JANE		N	O		
14	20	JANE		N	O		
15	21	MARA		N	P	C	

15	22	MARA		N	P	T	V
16	23	MARA		I	O		
17	24	JANE		N	P	C	
17	25	JANE		I	P	T	ORN
18	25						
19	26	MARY		I	P	E	ORN
20	27	MARA		I	P	Q	V
21	28	JANE		I	P	R	ORN
21	29	JANE	X				
22	30	MARY		N	P	C	
22	31	MARY		N	O		
23	32	MARA		I	O		
23	33	MARA		I	O		
24	34	MARY		I	O		
24	35	MARY		I	P	C	
24	36	MARY		I	P	R	V
25	37	MARA		I	O		
25	38	MARA	X				
26	39	MARY		N	O		
26	40	MARY		N	O		
27	41	JANE	X				
28	42	JANE	X				
28	43	JANE		N	O		
29	44	MARY	X				
29	45	MARY		N	O		
29	46	MARY		I	P	P	V

29	47	MARY		N	P	C	
30	48	JANE		I	P	R	ORN
31	49	JANE		N	P	P	ORG
32	50	MARY		N	O		
32	51	MARY		N	O		
33	52	JANE		N	O		
33	53	JANE		N	O		
34	NC						
35	54	JANE		N	O		
36	55	JANE		N	O		
37	56	MARY		N	P	T	ORN
37	57	MARY	X				
38	58	JANE	X				
39	59	MARA	X				
40	60	MARA	X				
41	NC						
42	61	MARA		N	O		
43	62	MARA		N	O		
44	63	MARA		N	O		
45	64	JANE	X				
46	65	JANE		N	P	C	
46	66	JANE		N	P	C	
47	67	JANE		N	O		
47	68	JANE		N	O		
47	69	JANE	X				
48	70	MARY	X				

49	71	MARA	X				
50	72	JANE	X				
51	73	MARA	X				
52	74	MARA		N	O		
53	75	MARA		I	O		
54	76	JANE		I	O		
54	77	JANE		I	O		
55	78	MARA		I	O		
56	79	JANE		I	O		
57	80	JANE		I	O		
58	81	MARA		I	O		
59	82	JANE		I	P	P	V
60	NC						
61	81						
62	81						
63	83	MARY	X				
64	84	JANE		I	O		
65	85	JANE	X				
66	86	MARA	X				
67	87	JANE		N	P	C	
67	88	JANE	X				
68	89	MARY		I	O		
69	90	MARA		I	O		
70	91	JANE		I	O		
71	92	MARA	X				
72	NC						

73	93	MARY		I	O		
74	94	JANE	X				
74	95	JANE	X				
75	96	MARA	X				
76	97	MARA		I	O		
77	98	MARY	X				
78	NC						
79	99	JANE		I	O		
79	100	JANE		I	O		
80	101	MARA		I	O		
81	102	JANE	X				
82	103	MARY		I	O		
83	104	MARA		I	O		
84	105	JANE		I	O		
85	106	MARA	X				
86	107	MARY		I	O		
87	108	JANE		N	P	C	
88	109	MARA		I	O		
89	109						
90	110	MARA		I	O		
91	111	MARA		I	O		
92	111						
93	112	MARY	X				
94	113	JANE		I	O		
95	114	MARA		I	O		
96	115	MARY		I	O		

97	116	MARA		I	O		
98	117	JANE	X				
99	NC						
100	118	MARA	X				
101	NC						

APPENDIX I

SAMPLE  
GROUP BEHAVIOR TALLY



APPENDIX J

SAMPLE  
INDIVIDUAL BEHAVIOR TALLY



MONTANA STATE UNIVERSITY - BOZEMAN



3 1762 10359843 7