Supplemental visual computer assisted instruction and student achievement in freshman college calculus
by Virgil Grant Fredenberg

A thesis submitted in partial fulfillment of the requirement for the degree of Doctor of Education
Montana State University

Abstract:
Society has become increasingly dependent upon technologically trained professionals which has increased the demand for such individuals. Most often these are white males, but female and minority students represent a largely untapped resource from which more highly trained individuals can be realized. College calculus can present an obstacle because it is prerequisite to a scientific career. This course has a low rate of success, which has prompted many instructors to search for methods of improving the achievement of their students. Teachers of college calculus have assigned students weekly computer lab work to supplement the traditional classroom instruction. Little well-designed research has been conducted into this use of computer-assisted instruction [CAI]. This study sought to explore such use of CAI in a traditional college calculus course.

For this study, the experimental group consisted of four sections of Math 181: Calculus and Analytic Geometry I, at Montana State University. Students in this group were administered five weekly, highly visual, supplemental computer labs during winter quarter 1991. The control group consisted of four other sections of the same course, and was administered five corresponding supplemental homework assignments. The independent and attribute variables were treatments, and learning style, gender, and minority group membership. The dependent variables were student attitudes, anxiety and achievement. These variables were assessed through changes in pretreatment and post-treatment surveys and student scores on homework, quizzes and exams, and course grades.

Results indicated little statistically significant change in student attitudes and anxiety, and no statistically significant change in achievement. Students receiving supplemental computer labs performed as well as students who received additional homework.

Recommendations for future research include: continued research into the use of supplemental CAI in the form of computer labs; research into the use of graphing calculators in college calculus; research into long-term effects of CAI in college calculus; and research into college calculus courses which have been restructured to take advantage of technology in the classroom.

SUPPLEMENTAL VISUAL COMPUTER ASSISTED INSTRUCTION AND STUDENT ACHIEVEMENT IN FRESHMAN COLLEGE CALCULUS

by

Virgil Grant Fredenberg

A thesis submitted in partial fulfillment
of the requirement for the degree

of

Doctor of Education

MONTANA STATE UNIVERSITY
Bozeman, Montana

August 1993

SUPPLEMENTAL VISUAL COMPUTER ASSISTED INSTRUCTION AND STUDENT ACHIEVEMENT IN FRESHMAN COLLEGE CALCULUS

by

Virgil Grant Fredenberg

Advisor: Maurice Burke, Ph.D.
Advisor: William Hall, Ed.D.

Montana State University
1993

Abstract

Society has become increasingly dependent upon technologically trained professionals which has increased the demand for such individuals. Most often these are white males, but female and minority students represent a largely untapped resource from which more highly trained individuals can be realized. College calculus can present an obstacle because it is prerequisite to a scientific career. This course has a low rate of success, which has prompted many instructors to search for methods of improving the achievement of their students. Teachers of college calculus have assigned students weekly computer lab work to supplement the traditional classroom instruction. Little well-designed research has been conducted into this use of

computer-assisted instruction (CAI). This study sought to explore such use of CAI in a traditional college calculus course.

For this study, the experimental group consisted of four sections of Math 181: Calculus and Analytic Geometry I, at Montana State University. Students in this group were administered five weekly, highly visual, supplemental computer labs during winter quarter 1991. The control group consisted of four other sections of the same course, and was administered five corresponding supplemental homework assignments. The independent and attribute variables were treatments, and learning style, gender, and minority group membership. The dependent variables were student attitudes, anxiety and achievement. These variables were assessed through changes in pretreatment and post-treatment surveys and student scores on homework, quizzes and exams, and course grades.

Results indicated little statistically significant change in student attitudes and anxiety, and no statistically significant change in achievement. Students receiving supplemental computer labs performed as well as students who received additional homework.

Recommendations for future research include: continued research into the use of supplemental CAI in the form of computer labs; research into the use of graphing calculators in college calculus; research into long-term effects of CAI in college calculus; and research into college calculus courses which have been restructured to take advantage of technology in the classroom.

APPROVAL

of a thesis submitted by

Virgil Grant Fredenberg

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

7/21/93 Date — Maurice Burke, Co-Chair, Graduate Committee

7/21/93 Date — William D Hall, Co-Chair, Graduate Committee

Approved for the Major Department

7/21/93 Date — Duane Mell, Head, Major Department

Approved for the College of Graduate Studies

7/30/93 Date — RL Brown, Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a doctoral degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. I further agree that copying of this thesis is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U. S. Copyright Law. Requests for extensive copying or reproduction of this thesis should be referred to University Microfilms International, 300 North Zeeb Road, Ann Arbor, Michigan 48106, to whom I have granted "the exclusive right to reproduce and distribute my dissertation for sale in and from microform or electronic format, along with the right to reproduce and distribute my abstract in any format in whole or in part."

Signature______________________

Date______________________

VITA

Virgil Grant Fredenberg was born October 15, 1954 in Kalispell, Montana. He graduated from Flathead High School in June, 1973.

He attended Flathead Valley Community College from September, 1973 to December 1975 when he joined the United States Air Force. He served in computer operations in the Strategic Air Command and the Alaskan Air Command. He was honorably discharged in July, 1980.

He returned to college and received a Bachelor of Science degree from Montana State University, Bozeman, Montana in 1983 with a major in Mathematics and a minor in Industrial Arts.

He taught mathematics, woodworking and computer science at Kenny Lake School, Copper River School District, Glennallen, Alaska from August, 1983 to May, 1985. From August, 1985 to June, 1986 he taught mathematics and computer science at Whitehall High School, Whitehall, Montana.

He returned to Montana State University as a graduate teaching assistant in the Department of Mathematical Sciences in September, 1986 and completed a Master of Science degree with a major in Applied Mathematics in June, 1988. He continued as a graduate teaching assistant in the Department of Mathematical Sciences from June, 1988 through June, 1991. During these five years, his wife completed her undergraduate degree in education and they had two beautiful daughters.

In August, 1991 he began teaching in the Division of Education at the University of Texas of the Permian Basin. He received his Doctor of Education degree from Montana State University in August, 1993.

ACKNOWLEDGEMENTS

This author would like to express his appreciation to the following people:

Dr. Maurice Burke, co-chairman, who gave his time, expertise and support despite his many other commitments.

Dr. William Hall, co-chairman and mentor, who gave advice and encouragement when it was needed.

Dr. Glenn Allinger, whose patience, concern and guidance led this author to become a mathematics teacher and who contributed his expert advice and assistance to this project.

Dr. David Thomas, who encouraged this author to keep going and provided needed advice all along the way.

Dr. Don Robson, who continued to provide support and serve on this author's committee despite other duties.

Dr. Kenneth Tiahrt, whose direction and assistance helped this author and his family make it through the graduate years.

This author is grateful to his stepson, Thomas, who provided support by doing more than his share of work. To his daughters, Elsa and Macey, who knew daddy could do it. And to his parents, Mr. and Mrs. David Fredenberg, and his in-laws, Mr. and Mrs. A C Moore, for their roles in all of this. But this author is especially indebted to his wife, Lori, whose love, encouragement and support made his dream a reality and to whom this dissertation is dedicated.

TABLE OF CONTENTS

TABLE OF CONTENTS--continued

TABLE OF CONTENTS--continued

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Society has become increasingly dependent upon technologically trained professionals which has increased the demand for such individuals. Most often these are white males, but female and minority students represent a largely untapped resource from which more highly trained individuals can be realized. College calculus can present an obstacle because it is prerequisite to a scientific career. This course has a low rate of success, which has prompted many instructors to search for methods of improving the achievement of their students. Teachers of college calculus have assigned students weekly computer lab work to supplement the traditional classroom instruction. Little well-designed research has been conducted into this use of computer-assisted instruction [CAI]. This study sought to explore such use of CAI in a traditional college calculus course.

For this study, the experimental group consisted of four sections of Math 181: Calculus and Analytic Geometry I, at Montana State University. Students in this group were administered five weekly, highly visual, supplemental computer labs during winter quarter 1991. The control group consisted of four other sections of the same course, and was administered five corresponding supplemental homework assignments. The independent and attribute variables were treatments, and learning style, gender, and minority group membership. The dependent variables were student attitudes, anxiety and achievement. These variables were assessed through changes in pretreatment and post-treatment surveys and student scores on homework, quizzes and exams, and course grades.

Results indicated little statistically significant change in student attitudes and anxiety, and no statistically significant change in achievement. Students receiving supplemental computer labs performed as well as students who received additional homework.

Recommendations for future research include: continued research into the use of supplemental CAI in the form of computer labs; research into the use of graphing calculators in college calculus; research into long-term effects of CAI in college calculus; and research into college calculus courses which have been restructured to take advantage of technology in the classroom.

CHAPTER I

INTRODUCTION

Introduction

Science and technology have become an integral part of our society. Our government, schools and industry are increasingly dependent upon individuals who are technologically and scientifically literate. A growing demand for such individuals has created a shortage of college graduates who have a high degree of scientific and technical literacy. The demand will become even more acute in the 1990's (Shapiro, 1987).

A common stumbling block for colleges interested in a technological or scientific area is the mathematics course requirements included in these curricula. Increasing the successful course completion by all students enrolled in college mathematics, including female and minority students, is the general problem motivating this study.

Advances in technology and its widespread uses created the demand for technologically and scientifically literate individuals and may hold the key to alleviating this demand as well. The computer has been used successfully to supplement mathematics courses at the elementary, secondary and college levels. Z. R. Mevarech (1985) reported computer-assisted instruction (CAI) improved the achievement and reduced the anxiety levels of disadvantaged third graders. Burns and Bozeman (1981) compared research showing students' mathematical comprehension is higher in classes where the instruction method is enhanced by the incorporation of CAI into the normal classroom methods of instruction. In their meta analysis of research, Kulik, Bangert and Williams (1983) compared studies which noted secondary students who received computer-based instruction (CBI) outperformed students who did not. They also reported these students develop better attitudes toward the subject, final exam scores are higher, follow-up exam scores are higher, and time needed to cover the material is usually much less than their counterparts in traditionally taught classrooms.

Other researchers have shown that CAI motivates students, cultivates better student attitudes towards mathematics and other subjects, and reduces levels of mathematics anxiety. Dugdale (1981), Reglin and Butler (1989), and Thomas (1979) believe CAI is successful in motivating students to perform at higher levels. Heid (1988) and Thomas (1979) found CAI improved student understanding of the concepts they were learning. Heid (1988) found CAI increased student confidence and Thomas

(1979) found CAI lowered student mathematics anxiety levels. The value of lowering mathematics anxiety levels and improving student levels of confidence is pointed out by Skemp (1987), "...reduce anxiety and build up confidence, and thereby improve the performance" (p. 94). Thus, computer-assisted instruction appears to improve student achievement by reducing anxiety levels and improving student confidence.

The graphing capabilities of micro-computers, and computers in general, allow faster and better visual illustrations of mathematical concepts than the traditional sketching of graphs by hand. In his book, "The Psychology of Learning Mathematics", Skemp (1987) cited examples where visualization conveys mathematical information more efficiently than verbal statements. He believes the graphic capabilities of the computer could be employed to visually illustrate many mathematical concepts to the advantage of both the student and the instructor. This view was also supported by Blackburn (1983). In their study, Eisenberg and Dreyfus (1989) concluded using visual images of mathematical concepts can deepen student understanding and help student progress in mathematics. Ethington and Wolfle (1984) reported that improving spatial visualization skills of females resulted in positive increases in achievement in mathematics. These results lead to the conjecture that using the computer to generate graphical images of functions may help calculus students visualize the mathematical concepts that are the basis of freshman level college calculus. Because the material is presented with a strong visual emphasis, computer generated graphs may also improve the achievement of female and minority calculus students, and calculus students whose preferred mode of learning is primarily visual.

Purpose of Study

The purpose of this study was to determine if supplementing a traditionally taught freshman college calculus course with CAI, that visually illustrated key mathematical concepts as they were covered in the course, would improve student achievement and attitudes towards mathematics and reduce mathematics anxiety levels. Because this study was interested in the improvement of student achievement in a traditional college calculus course, and because many college mathematics professors are wary of weakening the rigor and content of this course by the assimilation of the computer into the classroom, this researcher made the decision to supplement the course and not to alter it or to change the methods of instruction. In addition to the purpose mentioned above, it was the intent of this researcher to investigate three other facets dealing with the affects of CAI on the achievement, attitudes, and anxiety levels of female students, minority students, and students who were designated as primarily visual learners.

This study analyzed raw scores and success rates of students taking freshman level college calculus. It examined effects on attitudes and anxiety by analyzing student confidence in learning mathematics, attitude toward success in mathematics, view of the usefulness of mathematics, and level of mathematics anxiety as measured on the appropriate subscales of the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1986).

Need for the Study

Each autumn quarter over 400 students register at Montana State University (MSU) to take freshman level calculus. While the mathematics courses and the methods of instruction used at MSU are continually being revised and improved, many of the students in Math 181 receive grades lower than C (below 2.00 on a four-point scale) or withdraw from the class. For example, during autumn quarter, 1988, 420 students were enrolled in 14 sections of Math 181, freshman level calculus, at Montana State University. Sixty-seven students received a letter grade of A, 82 received a letter grade of B, 92 received a letter grade of C, 39 received a letter grade of D, 67 received a letter grade of F, 39 withdrew from the course with a passing grade, and 34 withdrew with a failing grade. Two hundred forty-one students received grades of C or higher. This means 197, or 43% of the students who enrolled in the course, withdrew or received grades lower than a C. Similar findings were reported by Cipra (1988), "at some institutions as many as 50% of the students enrolling [in] calculus either fail or withdraw from the course" (p.1491).

Positive results in college algebra programs where CAI has been used suggest the need for a study addressing the utilization of CAI in a college calculus course. Payton (1987) found that college students enrolled in algebra classes utilizing CAI scored higher than those who were enrolled in a traditional algebra course. College students enrolled in algebra made significant gains in solving systems of linear equations when they utilized the computer's assistance (Gronberg, 1987). In a study completed by Kiser (1986), CAI enhanced the students' ability to visualize linear and absolute-value inequalities.

A preliminary search conducted by this researcher found one study by Kiser (1986) which dealt directly with the effectiveness of CAI in visualization and mathematics. Eisenberg and Dreyfus (1989) reported the need for more research into the possible connection between mathematics and visualization. The research reported by Heid dealt with a college calculus course in which the method of instruction was altered to accommodate the use of computers (Heid, 1988). In this adapted course, Heid found college students better understood the concepts when the course utilized CAI.

These results are important, but this study investigated the possibility that the course does not need to be altered to fit the computer, that is, the computer is flexible enough to fit the course. No research was found during this preliminary review of literature in which the computer, and CAI, was used to supplement the existing course with graphical representations of calculus concepts. This researcher was interested in the affects such supplementary CAI might have on student achievement and attitude as described previously.

In concept, functions play an important role in calculus. Therefore, to be truly successful, a student must have a deep understanding of functions and their behaviors. In traditionally taught calculus courses functions are often represented using algebraic equations. But functions can also be represented visually through the use of graphs. Ayers, Davis and Lewin (1988), found students receiving CAI scored significantly higher on tests covering concepts dealing with functions. Although many functions are very difficult for an instructor to graph by hand, the computer can graph even difficult functions easily, and much more clearly. This makes the function easier to visualize, and visual symbols are often easier to understand than algebraic representations of the same material (Skemp, 1987). Skemp states, "...visual imagery is that most favorable to the integration of ideas" (p.80).

Questions to be Answered

The increasing availability of computer facilities in high schools and on college campuses, and the claims put forth about the capabilities of CAI, has tempted many mathematics instructors to send students to the "computer lab" as a means of increasing student achievement and supplementing their own instruction. Therefore, more research into the use of supplemental CAI is needed to establish what effects it may have on student achievement and attitude toward mathematics. With this in mind, the questions addressed by this study were: How does CAI affect student achievement when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?; How does CAI affect the achievement of students in underrepresented groups when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?; How does CAI affect student attitudes when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?; and How does CAI affect student mathematics anxiety levels when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

In Chapter III these four research questions are refined and divided into distinct questions dealing with their inherent sub-parts. They are then rewritten in null hypothesis form for analysis purposes. The number of female or minority students in the sample population is also considered in Chapter III.

General Procedures

The students in 12 sections of Math 181, Calculus and Analytic Geometry I, at Montana State University were given a researcher-created multiple-choice diagnostic test designed to determine how prepared they were to take the course. The concepts tested included topics in algebra, geometry, trigonometry and calculus. All included items were from topics judged necessary for adequate calculus student preparation by experienced math professors in the department.

The research design for this study was a matched group design (Drew & Hardman, 1985). The results of the diagnostic test were used to create four matched pairs of sections based upon their mean scores. The eight sections with the closest means were paired. The remaining four sections, from the original 12, were excluded.

One section from each of the four pairs was randomly placed into either the experimental treatment group or the control treatment group. Thus, the experimental treatment group consisted of four sections of Math 181 as did the control treatment group. A t-test of the means of the two groups, as measured on the pretest, was used to determine if this method of selection resulted in equivalent groups (see Chapter IV).

All eight sections used in the experiment were given a preliminary survey containing a page of background data and selected scales of the Fennema-Sherman Mathematics Attitudes Scales (see Appendix C). The selected scales were: the Confidence in Learning Mathematics Scale, the Attitude Toward Success in Mathematics Scale, the Usefulness of Mathematics Scale, and the Mathematics Anxiety Scale (Fennema & Sherman, 1986). The two groups were also given the Perceptual Response Subscale of the Learning Style Profile (LSP) created by a task force for the National Association of Secondary School Principals, in 1986, to differentiate an individual's primary mode of learning (Keefe, Monk, Letteri, Languis, & Dunn, 1986, see Appendix D).

The treatment for the experimental group consisted of five supplementary computer-assisted labs designed by the researcher to illustrate some of the major concepts covered in Math 181, the first quarter calculus course (see Appendix A). These labs were edited for content by professors and graduate teaching assistants with experience in teaching the course. No training on the use of the computer was given to the students or classroom instructors. The students received an instruction sheet. Any additional directions necessary for completion of a specific lab was included with the lab. The labs closely followed the daily lessons presented by the course instructors. The questions were designed to encourage the students to analyze the graphs created by the computer software. Students were required to complete labs outside of class. Labs were scored by the classroom instructors with guidance from the researcher. No training in the use of the software was given to course instructors.

Treatment for the control group consisted of five homework assignments similar to the treatments of the experimental group, but without the graphs of the functions (see Appendix A). Students answered questions similar to those answered by the experimental group, but without benefit of the function graphs generated by the computer. These assignments were also completed outside of the class and graded by the course instructors.

All sections of Math 181, including experimental and control treatment groups, were given two common hour exams and a final exam. All sections took the same exams at the same time. The first exam occurred after two of the treatments and the second exam was administered after the three remaining treatments were completed. After the second common hour exam the eight sections in the study were given the same survey of attitudes given before the treatments began. The difference between the results of this post-treatment survey and the pretreatment survey were used as data in the analysis. The final exam was given approximately two weeks after the second common hour exam.

A student's final course grade was determined by the total number of points accumulated in the course. Each common hour exam was worth 100 points. The final exam was worth 200 points. In addition to the exam scores, 100 points were given by each individual instructor for homework and quizzes. For those sections receiving the treatments of the study, 50 of the 100 total course points determined by each individual instructor were awarded for correct completion of the computer lab worksheets.

Student raw scores and course grades were used to examine differences in achievement and rate of success of experimental and control groups. All collected data were analyzed using analysis of variance. Analysis of variance was also used to determine if any interaction effects occurred between selected background variables, learning style preference, attitudes and the treatments.

Limitations and Delimitations

Limitations

The limitations of this study are as follows:

1. This study was limited to students enrolled in Math 181, Calculus and Analytic Geometry I, at Montana State University during winter quarter, 1991.

2. The number of minority students enrolled in the

above mentioned course may be too small to make meaningful inferences from the statistical analysis.

3. All exams given to the students in this study were created by one professor, the Math 181 course supervisor.

Delimitations

The delimitations of this study are as follows:

1. The treatment of the experimental group consisted of five computer lab activities over a six-week period.

2. The treatment of the control group consisted of five take-home exercise sheets over a six-week period.

3. The topics covered in the treatments closely followed the course syllabus during the time the treatments were given.

4. The computer software used in this study is designed for IBM compatible computers only.

Definition of Terms

For the purpose of this study, the following definitions were used:

1. Computer-assisted instruction (CAI) - employing a computer to assist in the teaching of a course. It may be used for remediation, tutorial, simulation, drill and practice, or other applications.

2. Computer-Based Instruction (CBI) - instruction that utilizes the computer as the primary source of teaching. It may or may not be used with teacher input to assist the

students with the lesson. An example would be a user interactive software package.

3. Computer lab worksheets (labs) - worksheets utilizing computer-assisted instruction in visualization to guide the students through the graphing of functions and the analyzing of those graphs to improve their understanding of the underlying mathematical concepts.

4. Fennema-Sherman Mathematics Attitudes Scales - a collection of nine scales developed by Fennema and Sherman to assess student attitudes toward many different items pertaining to the teaching and learning of mathematics. The five scales chosen to make up the survey given to the students in this study are (a) confidence in learning mathematics; (b) attitude toward success in mathematics; (c) usefulness of mathematics; and (d) mathematics anxiety scale (Fennema & Sherman, 1986).

5. Learning Style Profile - a test created by a task force for the National Association of Secondary School Principals, NASSP, in 1986 to differentiate those whose primary mode of learning is visual, auditory, or kinesthetic (Keefe et al., 1986).

6. Math 181 - Calculus and Analytic Geometry I. The first quarter college calculus course at Montana State University, using traditional methods and textbooks.

7. Raw score - the total number of a possible 500 points a student obtains in Math 181.

8. Satisfactory results - grades of "C" (2.00 on a four-point scale) or higher. Students who are enrolled in programs with courses that rely heavily upon the calculus sequence are advised that they need a grade of "C" (2.00 on a four-point scale) or better to continue on with the curriculum. Thus a satisfactory result is such a grade.

9. Success rate - a successful course completion rate. The decimal fraction of all the students in a group who complete the course with satisfactory results.

10. Underrepresented groups - groups not traditionally representative of those individuals who embark on careers in fields dependent upon technology such as engineering, mathematics, and sciences.

11. Visual learners - individuals whose preferred mode of learning is primarily visual. Such learners might prefer that information is presented visually as well as verbally.

12. Learning style (or modality) - "the sensory channel through which information is perceived and processed most efficiently" (George & Schaer, 1987, p. 1).

CHAPTER II

REVIEW OF LITERATURE

Introduction

In this study, computer-assisted instruction (CAI) was used to supplement traditional college calculus instruction through the use of graphical representations of several key mathematical concepts. Students in mathematics courses may benefit from the use of computer-generated graphics because the visual presentation of course concepts provides an alternative format through which the information could be processed (Bork, 1980). Bialo and Sivin (1990) suggested technology could be used successfully as a supplement to traditional instruction. Another advocate of the use of the microcomputer in the classroom, Caissy (1987) feels:

> The computer is a classroom tool that can enhance and improve instruction and learning. It should be used to complement and supplement the curriculum and should not require a revamping of the curriculum to fit the computer....The computer is intended to assist teachers with instruction and students with learning (p. 12).

The concerns of this study were the affect CAI had on mathematics achievement, attitudes towards mathematics, and level of mathematical anxiety of the sampled student population. It was the intent of this researcher that this study investigate and report on results for all students in the sampled student population including minority and female students, and students who were primarily visual learners. With this in mind, the review of literature pertained to: CAI and mathematics achievement; visualization and mathematics achievement; and CAI and the affective domain.

CAI and mathematics achievement as prescribed in this study dealt with the three subtopics: the affectiveness of CAI, CAI and female students, and CAI and minority students. Visualization and mathematics achievement dealt with the four subtopics: visual learners, visualization and computers; female students and visualization; and minority students and visualization. CAI and the affective domain dealt with the two topics: student attitudes towards mathematics and CAI, and CAI and its affects on mathematics anxiety.

Computer-Assisted Instruction and Mathematics Achievement

Using CAI has many enthusiastic supporters who believe that utilizing computer generated graphics could be a useful tool for mathematics instruction. Bork's (1980) article, "Learning Through Graphics," discusses the use of graphics in learning, especially in mathematics and science, and the critical role the computer plays in the use of graphics in teaching.

There are others who feel computers are of value in the mathematics curriculum: the graphics capabilities of the computer make it a powerful tool in the teaching of mathematics (Piele, 1983); using the microcomputer a student can illustrate mathematical concepts which may have been difficult and time consuming to do by hand (Blackburn, 1983); and the graphic capabilities of the computer could be employed to visually illustrate many mathematical concepts to the advantage of the student and the instructor (Skemp, 1987).

Kiser (1987) conducted research utilizing computer-enhanced instruction (CEI), a variation of computer-assisted instruction, which involved two intact college algebra classes and the teaching of solving linear inequalities. The treatment was conducted over a short, two-week period during the trimester. The control presentation consisted of a traditional expository approach where the overhead and chalkboard were used to graphically illustrate solution procedures without the use of computer support. In the experimental group, the computer presentation was designed as a highly visual treatment to utilize the spatial abilities of the students in the instruction. Both classes had the same objectives and were measured with the same criteria. Results indicated students in the CEI treatment classes performed significantly better and had a more positive attitude than students in traditionally taught classes. Students in the experimental treatment class with high spatial abilities significantly outperformed their counterparts in the control class. Kiser (1987) feels these results indicate CEI can improve the achievement and attitude of many students, "there is a possibility of higher student achievement in elementary, secondary, and college level by matching CEI presentations to students higher spatial abilities" (p. 39). While Kiser's study showed CEI to be effective in a college algebra course; it may be equally effective in a college calculus course.

The Effectiveness of Computer-Assisted Instruction

A review of literature dealing with the effectiveness of CAI in education has shown CAI improves achievement and attitudes of students at the elementary, secondary and college levels. In a study involving 204 disadvantaged third graders, Mevarech (1985) found CAI to facilitate achievement and reduce math anxiety. And in a previous study involving 376 disadvantaged elementary students in third, fourth and fifth grades, Mevarech and Rich (1985) found that CAI had a positive effect on mathematical achievement and attitudes. They also found CAI raised the math self-concepts of these students higher than those students in the control group.

Further evidence of the effectiveness of CAI at the elementary level appears in a study by Carrier, Post and Heck (1985) involving 144 fourth grade students. These researchers found "support for the claim that microcomputers can enhance some aspects of elementary school children's achievement in mathematics" (Carrier et al., 1985, p. 51).

In their meta-analysis of studies in which CAI was utilized, Burns and Bozeman (1981) found that CAI was effective in increasing mathematics achievement at both the elementary and secondary levels. The 40 studies dealing with CAI were included in the meta-analysis because they met the following criteria: CAI was used in conjunction with mathematics instruction; CAI was used as a supplement to the traditional instruction; the study was conducted at elementary or secondary grades; control group performance was compared to the treatment group performance; student achievement was the outcome variable; and the supplementary CAI took the form of drill and practice or tutorial assistance. Three important outcomes of this analysis were: student achievement was significantly increased through the use of supplementary CAI; high achieving and disadvantaged students at both the elementary and secondary levels achieved significantly higher when CAI was used to supplement traditional instruction; and the results found were independent of the design and length of the studies (Burns & Bozeman, 1981).

In a meta-analysis of 59 studies in which computer-based instruction (CBI), a form of computer-assisted instruction which utilizes the computer as a primary source of teaching, was employed at the college level, Kulik, Kulik and Cohen (1980) reported: CBI significantly improved the achievement of college students in a small positive direction; the attitudes of the students toward instruction and the subject were positively affected; and the time needed to learn the material was substantially reduced. Similar to the previously reported meta-analysis, the design of the experiments did not influence the outcome (Kulik et al., 1980).

In a study in which microcomputers were used, Rhoads (1986) obtained results similar to those found by Burns and Bozeman (1981) and Kulik et al. (1980). One hundred fourteen students in five high school algebra one classes were given a total of 51 minutes of instruction using microcomputers over the course of two days. The treatments differed in the amount of guidance given through the use of worksheets, and whether the students worked in same-sex pairs or alone. Although this study dealt with a very short length of time, the results indicated microcomputer instruction significantly promoted the learning of skills and concepts for high school students regardless of the conditions of the treatment.

In a third meta-analysis of CBI research dealing with secondary students, Kulik et al. (1983) reported on 51 studies involving students in grades six through 12. The studies included in this analysis met the following criteria: the study was carried out in a secondary classroom, grades 6-12; results of both experimental and control groups were reported in the study; and no possibly crippling methodology flaws were found in the study. The results obtained indicated secondary students who received computer-based teaching scored higher on final examinations, developed positive attitudes towards the courses they were taking, and learned the material in a shorter length of time. Results also indicated these "effects were greater in studies of shorter duration," (Kulik et al., 1983, p. 23).

At the post-secondary level, research as far back as 1976 encouraged the use of computers to supplement calculus instruction and called for more research in the use of CAI in college mathematics classes. In a study conducted by Lang (1976), the use of the computer as an extension to traditional instruction was found to be effective as a tool in teaching the basic concepts of introductory calculus. During Fall semester of 1972 at the University of Texas at Austin, two classes of introductory calculus were divided into eight discussion sections, four assigned to the experimental group and four to the control group. The experimental group received instruction supplemented by assignments in which students were to use prepared computer programs to investigate underlying mathematical concepts and the control group received supplementary assignments which did not utilize the computer. Students were to enter data into the computer using punch cards and wait for their results. Another important result indicated male students benefitted significantly from the use of the computer in this study. Lang concluded that use of computer extended instruction should be explored more fully as an aid in calculus classes. "Serious thought should be given toward implementing computer extended instruction in introductory calculus in our colleges and universities" (Lang, 1976, p. 279). It should be noted that in 1972, when this research was conducted, using the computer in a college mathematics course could have been exciting and novel to new freshmen students.

In 1977, two studies utilizing CAI, in the form of computer programming, to directly supplement the traditional instruction in college calculus were reported. In an experiment where college students in an introductory calculus course were invited to participate in a programming applications project, Berkey (1977) noted students responded positively to the use of the computer. And in a computer programming application reported by Sanger (1977), freshmen engineering students enrolling in the first year of calculus were allowed to choose the traditional course or the computer supplemented one. Students choosing the computer-assisted course were given one two-hour laboratory session each week instead of one hour of classroom instruction. During this laboratory session the students were taught the basics of computer programming and were assisted with the writing of these programs. The results were positive: "The students as a whole have enjoyed the courses, accepting the computer as one of the tools with which they must be familiar in today's society" (Sanger, 1977, p. 217).

In both of these studies students did the programming themselves and were taught the language as part of the course. However, with the introduction of the microcomputer and the development of more advanced user-friendly applications software, CAI is now affordable to most classrooms and much easier to implement. These advances allow teachers and students to have the power of the larger mainframe computers in smaller less expensive personal microcomputers.

More recent research utilizing microcomputers has shown similar results. When CAI was used in college mathematics courses other than calculus, higher levels of mathematics achievement were obtained by those students (Payton, 1987, Gronberg, 1987, and Ganguli, 1990).

In a study by Gronberg (1987), three sections of freshman business mathematics were taught to solve systems of linear equations using three different methods. The control group was taught in the traditional manner, one treatment used CAI as a supplement to the instruction, and one treatment used CAI as the primary mode of instruction. Based upon course grades, 23 students from each class were matched for the purpose of the study. The achievement level of the students in the treatment group which utilized CAI as a supplement to traditional instruction was significantly higher than the levels of both the other two groups (Gronberg, 1987).

In the study conducted by Payton (1987), four classes of basic mathematics were equally divided into the experimental group and control group. Of the 135 students on the final roster, complete data were obtained for only 87 of them. In addition to a pretest and a post-test, a survey of attitudes towards mathematics and computers was administered to each student before and after the treatment. The treatment for the experimental group consisted of supplementing the traditional lecture with computer-assisted instruction. The control group was taught using just the traditional lecture method. The experimental group was given demonstrations on the use of selected computer software and were then assigned worksheets to be completed using the demonstrated software. The software used in this study centered around function plotters and problem solvers. Results of the two-way analysis of covariance indicated significant gains in achievement for students in the experimental group on the post-test of knowledge of graphs, relations, and functions. No other significant results were found although a positive improvement in attitudes towards mathematics and computers was found for the experimental group (Payton, 1987).

The study conducted by Ganguli (1990) found students in the experimental group scored significantly higher on the comprehensive final than did the control group. Four sections, 118 students, of intermediate college algebra were matched based upon their pretest scores. One student from each pair was randomly assigned to the experimental treatment with the other assigned to the control group. Treatment for the experimental group was given over a five-week period during the semester and consisted of microcomputer demonstrations of four selected course topics. Students in the experimental group watched visual illustrations of the topics while students in the control group were taught using the traditional lecture/chalkboard method. Ganguli interpreted the positive results as an indication "that overall the students in the experimental group performed better than the students in the control group" (p. 158).

Not all studies attribute such results to the use of CAI. In a case study conducted by Wright (1989), where 28 college algebra students were given assignment worksheets and the option of using a computer to complete them, the higher averages of the students who opted to use the computer might be attributed to their higher level of motivation and not necessarily to the computer treatment. Although no change in student attitudes towards mathematics or computers was indicated when the students were surveyed, about half of the students involved believed the computer helped them better understand the material in the course. Because students were given the option of using the computers, Wright concluded "...it is not apparent from this experiment whether or not the use of computers significantly enhanced the teaching of college algebra" (p. 60).

In a college calculus study conducted by Ayers et al. (1988), CAI was used as a supplement to traditional instruction. A total of 30 college students in three sections of an optional first-year mathematics lab were divided into three treatment groups, two computer groups and a group which received a pencil and paper treatment. The study was conducted over a six-week period with all three groups receiving a weekly two-hour treatment session. During the first session all three groups were administered a pretest covering the mathematical concepts necessary for function and composition. During the following five weeks the students: completed assignments dealing with function and composition, were given a two-hour lecture on function and composition during the fourth session, were instructed on the connections between the lab exercises and the lecture, and were given a post-test during the sixth session. Results indicated students in the computer groups scored higher on the post-test than did students in the pencil and paper group:

> The scores on the post-test are consistent with our hypothesis that the computer experiences given to the

> students in the Computer groups were more effective in inducing the reflective abstractions involved in constructing the concepts of function and composition than was the traditional treatment given to the Paper-and-Pencil group (Ayers et al., 1988, p. 256).

In college calculus courses which were redesigned to utilize the computer, Heid (1988) noted, "There was evidence that the students in the experimental classes understood concepts as well as, and in most cases better than, the students in a traditional version of the course" (p. 22). Hickernell and Proskurowski (1985) reported students in the group receiving supplementary CAI achieved significantly higher final exam scores than those in the control group.

In the study conducted by Heid (1988) the sections of applied calculus were taught by the researcher in the study conducted by Heid (1988). These sections met three times each week for 50 minutes during the semester and used special materials developed by the instructor to incorporate CAI. Students in the experimental classes used the microcomputer to carry out most of the algorithms in the course and to find solutions to assigned problems and analyze the computer generated results. Students in a traditionally taught lecture section met twice a week for a 75-minute lecture and once a week for a 50-minute recitation section. Students in the traditionally taught lecture section were not given instruction incorporating CAI. Students in the experimental classes were expected to complete special assignments designed to be completed using microcomputers and attend the weekly class meetings. Fundamental ingredients in the course given to the experimental sections were the construction of computer generated graphs and the analysis of those graphs. While only the last three weeks of the course were used to work on the skills found in a traditionally taught course, the results indicate students in the experimental classes understood the concepts of the course as well as students in the traditionally taught class. Some students in the experimental classes were more confident in the results they obtained and felt the computer took some of the tedious calculation out of the course and allowed them to think more about the concepts and problem solving in the course (Heid, 1988).

Hickernell and Proskurowski (1985) reported on a college calculus course designed to incorporate CAI in the form of menu-driven software which students used to create graphs of mathematical concepts and for numerical applications covered in the course. Students were not taught programming, nor were they expected to have had previous experience with the use of computers. Hickernell and Proskurowski acknowledged:

> Comparing the students who used our computer system to those who took the traditional Calculus course is difficult because of several factors which cannot be easily measured. Examples of these are: the quality of instructors and teaching assistants, the effects of class size, and the academic ability and preparation of the students upon entering the course. Nevertheless, the difference between the final exam scores of these

> two groups of students, median 133 versus 119.5, is greater than a chance event. Therefore, a soft conclusion is that the students improve in a computer enhanced environment, at the expense of some extra work (p. 123-124).

They go on to recommend "Our experience suggests that the computer can be a valuable tool for teaching Calculus if it is used properly" (Hickernell & Proskurowski, 1985, p. 124).

In a study involving 78 college calculus students Palmiter (1991) reported a significantly higher achievement for the students in a calculus course designed around a specific application-package computer algebra system. The traditionally taught control group covered the integral calculus concepts in the normal ten-week period while the experimental group covered the same material in five weeks. Students in the experimental group were not presented with the techniques of integration which made up the largest part of the traditional course, but employed the computer software to perform the computation needed for the integrals found in the homework assignments and on the exams. At the end of the ten-week traditionally taught course and the five-week experimental course, both groups were administered the same traditionally created exams. Students in the experimental course scored significantly higher than the traditionally taught control group. In addition, the students who continued on in subsequent calculus courses performed as well as the students in the control group who continued. While these results are impressive, Palmiter (1991) noted:

> Although the ...[experimental] group significantly outscored the traditional group on both the conceptual and computational exams, several underlying factors may have influenced the positive results. The students in the ...[experimental] group were actively participating and were fully aware of their participation in the study. It is well understood that subjects in an experiment perform better by being a part of an experiment....Likewise the traditional class was being tested for concepts presented over ten weeks instead of only the 5 weeks of conceptual development in the ...[experimental] class. Thus, the traditional class may have scored lower because of the longer time period and the interference of computational work (p. 155).

These cautions are important to note and should be concerns for all educational researchers, especially those concerned with CAI research.

All of these studies and reports of research indicate the effectiveness of supplemental CAI. However, several researchers noted there may have been factors other than CAI which could have produced positive results. Perhaps stricter control for the teacher variable, for the unequal amounts of work given to the two groups, for the different lengths of time spent on the topics in the course, and for the effect knowing one is participating in an experiment may have on the outcome of that experiment should be established in CAI studies. Controlling these and other possibly contaminating variables was an important consideration in the present study.

Computer-Assisted Instruction and Female Students

In reviewing literature related to CAI and the mathematics achievement of female students, this researcher found results which were not restricted to female students alone. Mevarech and Rich (1985) reported higher achievement scores for all disadvantaged third, fourth, and fifth graders in their study, but no significant difference was found between boys and girls due to the treatment. Carrier et al. (1985) noted in their research report, where CAI was utilized as a supplement to traditional mathematics instruction, "The only sex difference was in the retention of division facts, where the girls made greater gains." While this is significant, it does not ensure CAI will benefit female students more than male students.

Accepting that males and females do not necessarily process mathematical information in the same way, Damarin (1988) believes designers of Intelligent Computer-Assisted Instruction (ICAI) should take these differences into account when writing curricula which utilize ICAI. ICAI might be used to correct some of the inequalities between males and females in education, since the computer shows none of the preconceived notions which may dictate teacher prejudice.

In studies involving college students, Reglin (1987, 1988), and Reglin and Butler (1989) reported CAI to be more effective than traditional instruction methods for both male and female students. The study conducted by Reglin (1987) was concerned with student mathematics achievement, locus of control, and academic self-concept. It began with 84 high school aged students enrolled in a vocational training program and ended with 76 of these students. Reglin (1987) described the design of the research as a "nonrandomized pretest-post-test experimental group design" (p. 12). The experimental and control groups finished with 50 male students, 17 white and 33 black, and 26 female students, ten white and 16 black. The difference in treatment of the two groups was that the experimental group received ten minutes of CAI in mathematics during each 60-minute session of the 12 weeks (60 sessions in all), while the control group did not receive CAI. The results of the analysis of covariance (using pretest scores as the covariate) indicated no significant difference in mathematics achievement, locus of control, or academic self-concept between the students in the experimental group and the control group. There was a significant change in academic self-concept and locus of control between male and female students (Reglin, 1988). While these results did not indicate an increase in mathematics achievement specific to female students, Reglin (1988) noted:

> The interesting difference which appeared on the measures of academic self-concept and locus of control between the males and females may also indicate the need for further research into the area of sex difference (p. 65).

The study conducted by Reglin and Butler (1989) consisted of 49 college students, 48 minority, enrolled in a mathematics seminar designed to prepare them for a required education program admissions examination. Students were randomly assigned to the experimental and control groups. Students in each group attended 18 sessions during the six weeks of the seminar. All of the students were administered the same pretest and post-test during the six weeks. The only difference in the treatments was the experimental group received 30 minutes of CAI in mathematics during each 60-minute session, while the control group did not receive any CAI. Results of this study indicated the students in the experimental group made significantly higher gains than did those students in the control group, but no significant differences were shown between the male and female students in either group (Reglin & Butler, 1989).

This present review of literature indicates a definite need for more research into what affect CAI may have on female students enrolled in college calculus. This study addressed that topic.

Computer-Assisted Instruction and Minority Students

The little research and information concerned with CAI and its affect on minority students indicates more research into this aspect of CAI is needed. Bork (1980) suggested the interactive graphics capabilities of the computer may be a useful aide in education and noted, "nonverbal modes are particularly important with students from minority backgrounds."

Reglin (1987) reported on research he conducted with minority college students where CAI was used to enhance instruction in a six-week seminar designed to help them pass the mathematics portion of an examination for entrance into the college's education program. Although minority group membership was not a variable in this study, all but one of the 49 students in the treatment groups were minority students. Results were positive, with the group that received CAI performing significantly better than the group that did not receive CAI (Reglin, 1987). Reglin and Butler (1989) reported on research into enhancing instruction with CAI. Their research also dealt with predominantly minority students. Their results indicated that the students who received CAI scored significantly higher on a mathematics post-test than students who did not receive CAI.

The lack of research dealing with mathematical achievement of minority students is evident. This study attempted to address that topic.

Visualization and Mathematics Achievement

A review of literature on visualization and learning, visualization of mathematical concepts, and visualization and mathematics achievement was included in this chapter because this researcher was interested in how CAI incorporating visualization of mathematical concepts may affect the achievement of college calculus students. While many mathematical concepts have visual interpretations and most educators feel students benefit from visual approaches, few utilize them in the classroom (Eisenberg & Dreyfus, 1989). They noted, "It seems as though building visual concepts images can be exploited to take students further into mathematics, and deeper" (Eisenberg & Dreyfus, 1989, p. 4) and called for more research into the role of visualization in mathematics (Eisenberg & Dreyfus, 1989). And, while visual illustrations are not to be considered mathematical proof, others also feel they supply an insight that is invaluable to the student. As Vinner (1989) stated "visual considerations are always illuminating even if not taken as a mathematical proof. They are indispensable in the calculus course" (p. 155).

Iben (1989) feels one possible explanation for why Japanese students do so well in mathematics may be that throughout their lives they are developing visual-spatial abilities through many paper folding experiences. He suggested more research into enhancing student achievement in mathematics should be conducted.

Harel (1989) reported on a successful program in which high school and college students learned linear algebra in a course which incorporated the use of visualization to intuitively teach some abstract theories. In the teaching experiment involving 56 high school students and 72 college students, students and teachers were very positive about this method of instruction. He went on to state:

> We believe that the idea we proposed in this paper of gradual abstraction of the concepts and their construction processes along with a firm intuitive visual base is applicable to other domains. We believe that similar treatment in teaching these theories would yield similar results. (Harel, 1989, p. 147-148)

And in their overview of research dealing with the enhancement of visualized instruction, Dwyer and Dwyer (1989) suggested:

> At present, educators have no way of knowing whether one type of visual is more effective than another in transmitting certain types of information, nor do they know whether instruction without visuals would be any more effective than the same instruction with visuals. (p. 117)

Visual Learners, Visualization and Computers

This researcher was concerned with the effects on student achievement of supplementing the traditional freshman college calculus course with computer generated illustrations of some of the mathematical topics covered, and as components of that concern, how this approach affected female and minority students, and students whose preferred mode of instruction is primarily a visual one.

Visual Learners. Students' preferred mode of learning, or learning style, is the manner of presentation from which they find the material easiest to comprehend and process. "The three modalities used most often for learning are visual, auditory, and kinesthetic" (George & Schaer, 1987, p. 1). Therefore, visual learners learn best when information is presented in a visual mode.

Because a large quantity of research has been conducted into learning styles and education, this review is restricted to literature dealing primarily with visual learning styles, or spatial visualization, and the interconnection with computers and CAI at the college level.

Canelos, Taylor, Dwyer and Belland (1988) reported on research which found simple visual instruction and visual feedback resulted in significant learning improvement for college students in an experiment designed to compare visual instruction and nonvisual instruction. The study involved 112 college freshmen who were given programmed instruction on parts of the human heart and how it operated. The programmed instruction simulated a computer display and presented material in two types of visual methods. The first type consisted of simple line drawings of the heart, while the second type consisted of more detailed line drawings. Both visual methods supplied the necessary information, but the second method had added detail. A recall test in different forms, two visual and three verbal, was then used to determine the effects of the two types of instruction. Results of the tests indicated that when visual feedback is paired with simple visual instruction the result is a significant improvement in learning (Canelos et al., 1988).

Similar results were found in an experiment by Akanbi and Dwyer (1989). Similar to the study by Canelos et al. (1988), their study involved 67 graduate and 139 undergraduate students and visual instruction methods about the human heart and its parts. Results indicated instruction with visualization was helpful to college students classified as having low prior knowledge of the material while students classified as having high prior knowledge benefitted from the non-visualized instruction. They noted, "the findings of this study emphasize the importance of the interrelatedness of variables associated with learning and the effective use of visualization in the teaching-learning process" (Akanbi & Dwyer, 1989, p. 9).

Dwinell and Higbee (1989) recommended instructors of high-risk college freshmen use a variety of teaching methods, including visual aids. And Hinterthuer (1984) found CAI could be effective for developmental studies students in college when it was matched to their learning styles. He noted, "An initial match of preferred style of learning to preferred mode of instruction may increase motivation and interest to work on basic skills" (Hinterthuer, 1984, p. 103). He found the use of CAI which matched the student learning style increased motivation.

Other researchers also recommended that the method of instruction should take into account the visual mode of learning as well as other learning modes. Bork (1980) noted, "many students, at all levels of education, are not this sophisticated in assimilating verbal information, and would be greatly aided by the use of other channels of communication" (p. 68). Shrum (1985) noted the anxiety, which results when the mode of the presentation and the learning style preferred by the student differ, could be reduced if, "teachers vary the mode of presentation within a lesson and from day to day" (p. 13). Keefe and Monk (1986) stated "students with strong visual response are likely to learn less effectively if instruction is strictly verbal (auditory)" (p. 15). And Schaalma (1989) recommended students be provided with activities that allow them to practice their spatial skills. She went on to state, "[mathematics] teachers must provide learning activities that cater to the learning styles of all students" (p. 34).

And in a study in which the preferred learning mode of the student was taken into account and the instructional method emphasized visualization, Miller (1988) found female minority college calculus students whose preferred mode of learning was primarily visual outperformed corresponding students whose preference was not primarily visual. Her research supports the idea that students who are visual learners perform better when the instruction includes presentation of materials in visual, or graphical, ways.

While the above researchers indicate visual-spatial abilities may affect mathematics achievement, Dick and Balomenos (1984) cautioned that success in programs designed to enhance visual abilities may not necessarily translate into improved achievement in calculus. Their study involved 268 first-semester college calculus students, 124 females and 144 males. Students filled out a background questionnaire and were administered a diagnostic test covering precalculus topics during the first week of the semester. In addition, they were surveyed about their attitudes towards mathematics and were given a test of visualization skills. The unit and final exams were used to determine student achievement for the semester. These researchers stated in their conclusions:

> Differences in attitudes toward mathematics or in cognitive skills such as spatial ability make little contribution to explaining variance in calculus achievement. This implies that special program [sic] designed to improve attitudes or spatial ability, even when successful, should not be expected to have a "transfer" effect to improvement in calculus achievement. (p. 19)

Visualization and Computers. Kiser (1987) found college algebra students who were taught with computer enhanced instruction designed for high visual-spatial ability students did significantly better than did students in the traditionally taught group. He found the microcomputer to be an effective instrument to assess and enhance mathematics instruction, "the microcomputer in the classroom does make a significant difference" (Kiser, 1987, p. 39).

Other educators feel computers and microcomputers should be effective in visually enhancing mathematics instruction through the use of the graphics technology these devices can provide (Elsner, 1983; Greenfield, 1987; Head & Moore, 1989; Laughbaum, 1989).

Laughbaum (1989) was impressed with the abilities of function plotting software and its effect on the way college algebra and calculus are being taught. With a graphing calculator or function plotter, visual images can be used in the classroom to enhance mathematics in the classroom. Seeing a 'picture' of the function helps the student visualize some of its characteristics and behaviors. The function plotter is a new tool to use to solve equations (Laughbaum, 1989). With this new tool students can graph almost any function, determine the domain and range of that function, and visually analyze the consequences of changing the parameters of the function.

Female Students and Visualization

A review of literature dealing with female students and visualization appears to indicate a difference in mathematical ability due to the gender of the student, and controlling for spatial visualization may close the gap in the differences in math achievement between males and females.

Using data from a national survey, Ethington and Wolfle (1984) utilized a "covariance-structures model of mathematics achievement" (p. 367) in an attempt to determine "why men exhibit higher average mathematics achievement scores than women" (p.362). They feel several variables contribute to the difference in mathematics achievement between men and women. In the area of spatial visualization they note, "women tend to have less spatial visualization ability than men, but the effects of this variable on mathematics achievement are greater for women" (p. 361), and conclude, "a unit increase in spatial ability produces greater increases in mathematics achievement for women than it does for men" (p. 375).

In a report of their research, Fennema and Tartre (1985) found females made use of pictures more often than did males when solving word problems. Their study involved 669 sixth grades students who were given tests to determine spatial visualization, verbal skills, and mathematics achievement. These students were retested in the eighth grade. In addition to this data students were asked to solve pretested word and fraction problems in the spring of of their sixth, seventh, and eighth grades. They suggested from their results, "the hypothesis that females are more debilitated than males by low spatial visualization skills should be investigated" (p. 204).

Research in spatial visualization at the college level was conducted by Ferrini-Mundy in 1987. A random sample of 334 first-semester calculus students, 167 male and 167 female, were selected and randomly assigned to seven groups, with equal numbers of each gender in each group. Pretests dealing with the calculus background and spatial visualization of the students were administered at the beginning of the semester. Students were asked to complete six worksheets which accompanied six researcher-designed modules emphasizing spatial visualization during eight weeks of the semester. Two unit tests of calculus achievement were used to assess student achievement in the course. A third unit test was analyzed separately. Results of the multi-variate analysis of covariance were mixed, however the researcher notes, "perhaps the most interesting finding of the study is that practice on spatial tasks enhanced women's ability and tendency to visualize while doing solid-of-revolution problems" (Ferrini-Mundy, 1987, p. 137).

In their report of research dealing with visual-spatial learning differences between males and females, and how such differences may affect the success in math and science, Baker and Belland (1988) suggested visual-spatial learning preference might be a factor in the choice and selection of a career in science and mathematics and might be one reason females are underrepresented in technology related careers. They go on to make the statement and ask the important question:

> It is surely too simplistic to claim that women are so underrepresented in science and technology because they perform less well in visual-spatial thinking, but if visual-spatial thinking is a factor, isn't it appropriate that educators and educational researchers should work to provide experiences for female learners to eliminate it? (p. 17)

In a meta-analysis of 43 doctoral dissertations dealing with visual spatial abilities, Druva-Roush and Wu (1989) found evidence the effects of spatial manipulation become more important as the age of the student increases. Their analysis contradicted other researchers findings on the differences between males and females. They suggested their results indicated the difference in spatial abilities between males and females is not as large as other articles have reported.

While these results appear to indicate a connection between visual abilities and mathematical achievement, especially in the case for female students, a need for more research into visualization and the mathematical achievement of female college students is indicated.

Minority Students and Visualization

It is a widely held belief that Native Americans learn better when the presentation contains a large amount of instruction utilizing spatial visualization techniques.

This review of literature has found little actual research in that area. Kleinfeld and Nelson (1988) reported that research concerning Native Americans and spatial visualization has resulted in inconsistent findings. In their review of educational research studying Native Americans and learning styles they stated, "What is clear is that visual-spatial abilities [of Native American students] are an area of relative cognitive strength" (p. 6), however they go on to note:

> Despite more than twenty years' discussion of the importance of adapting instruction to Native American students' visual learning style, research has not succeeded in demonstrating educational benefits. (p. 16)

While Kleinfeld and Nelson discuss research involving Native Americans and instructional methods involving visualization, none of the studies they reported dealt with mathematics and Native Americans.

In his work cited previously, Bork (1980) stated, "nonverbal modes are particularly important with students from minority backgrounds" (p. 68).

These results appear to indicate a void exists in research into visual presentations for minority students, especially in the area of mathematics education.

<u>Computer-Assisted Instruction and the Effective Domain</u>

<u>Student Attitudes Towards Mathematics and CAI</u>

In his report comparing the development of mathematical abstraction and spatial relations in U. S., Japanese, and Australian students, Iben (1989) noted "mathematics confidence and lack of anxiety are consistently significant predictors [of success in mathematics] for Australian and U.S. caucasian males and Japanese males" (p. 8). Confidence in one's ability to do mathematics is important to a student's success in mathematics, and female students are not as confident about their mathematical abilities as are their male counterparts. These statements are supported by Greenberg (1991) and Meyer (1989).

In a study conducted during the summer of 1990 involving college students enrolled in intermediate algebra, Greenberg found "successful students were more confident than unsuccessful students that they had learned the course material well" (p. 10). And in an article dealing with gender differences and mathematics achievement, which discusses the results of the National Assessment of Educational Progress, Meyer (1989) noted "... both males and females made significant gains from 1982 to 1986 is encouraging, but the gap between males and females remains a concern" (p. 151). She went on to state:

> Attitudes about mathematics and about oneself as a learner of mathematics are important not only because they are outcomes of learning mathematics but also because they potentially influence the learning of new mathematics. (p.154-155)

She pointed out that females continued to be less confident in their abilities to do mathematics than their male counterparts. Their attitudes towards the practical use of mathematics was on par with males, but they did not envision their future careers would be in areas where a good background in mathematics was necessary. This last statement was supported by Schaalma (1989), "Student attitudes may affect females' decisions about continuing in mathematics beyond the minimum requirements in high school" (p. 34).

Many educators have reported that computers motivate students and improve their attitudes toward mathematics. Bork (1980) suggested incorporating graphics and CAI into the traditional classroom could be motivating for the students. In their report, Bialo and Sivin (1990) concluded there is evidence that using microcomputers motivates students and improves their attitudes toward the subject they are studying. Dugdale (1981) and Reglin (1989/90) also reported that using the computer is a good method of motivating students in mathematics courses. Caissy (1987) feels just being allowed to use the computer motivates students "Computers themselves provide a form of motivation for students that other teaching aids cannot match" (Caissy, 1987, p. 14).

Other educators reported students developed positive attitudes toward instruction and subject matter when the computer and CAI are used to supplement traditional instruction. Where computers were used as part of the teaching process in a course, students developed positive attitudes toward the computer and the course (Kulik et al., 1980; Kulik et al., 1983). Land and Haney (1989) also found college students' attitudes were more positive toward the course and the professor when CAI was incorporated into the course.

Computer-Assisted Instruction and Mathematics Anxiety

One aspect of this study is how CAI affects the level of mathematics anxiety of students enrolled in a college calculus course. Success in mathematics courses and mathematics anxiety have been found to have an inverse relationship. In a study conducted to determine the effects of a highly interactive discovery method of teaching on the mathematics achievement of college students with varying degrees of mathematics anxiety, Clute (1984) found mathematics anxiety inversely affected the achievement of the students in college mathematics; the higher the anxiety level, the lower student mathematics achievement. The study involved 81 college students, 38 males and 43 females, in a mathematics survey course. Students were classified as having a low, medium, or high level of mathematics anxiety based on the results of a rating scale designed for that purpose. Results indicated students with high mathematics anxiety did significantly better when the method of instruction was a highly interactive discovery one. This indicated interaction and exploration might reduce mathematics anxiety. The computer also provides an environment where the students can explore and interact according to their abilities and inclination (Dugdale, 1981).

Buckley and Ribordy (1982) also supported the hypothesis that the higher the level of mathematics anxiety the lower the student's performance. They noted math anxiety may be a variable for sex-related differences in mathematics performance and enrollment in math courses. Dwinell and Higbee (1989) reported some high risk students prefer a hands-on learning style and learning through interaction and visual stimuli rather than through lecture and text. This supports the introduction of interactive, visual CAI into the mathematics classroom. Others also suggest the use of computers and CAI can reduce mathematics anxiety. Bialo and Sivin (1990) reviewed research showing microcomputers reduced mathematics anxiety in female algebra students.

Research reported by Crumb and Monroe (1988) found CAI produced a significant reduction in mathematics anxiety in college students enrolled in developmental studies courses. Their study involved 17 remedial students in basic mathematics courses. The experimental group consisted of five students and the control group of 12. While the experimental group received intensive individualized CAI, the control group logged at least ten hours of computer use during the semester. Each group made significant gains in their problem solving and computational abilities, however the experimental group receiving CAI also experienced a significant decrease in mathematics anxiety levels. They went on to suggest "using CAI may be an appropriate means of addressing ... math anxiety" (p. 16).

Summary

This review of literature finds support for the notion that CAI effectively increases the mathematical achievement of students at all levels of education when it is used to supplement the traditional methods of instruction. The cited research suggests students who receive CAI have a higher level of achievement in mathematics courses than students who do not receive it. It also suggests using the capabilities of the computer to generate graphical illustrations of mathematical concepts would be beneficial to all students.

Visual-spatial ability is a factor in mathematics achievement for male, female and minority students. However, the lack of research into the connection between visual-spatial abilities of minority students and mathematical achievement demonstrates a need for more research into this topic.

The literature demonstrates the effect student attitudes, confidence, and level of mathematics anxiety have on mathematics achievement. It also shows CAI is a positive influence on student attitudes toward mathematics, that it reduces the level of mathematics anxiety, and improves confidence in ability to do mathematics. It suggests CAI would have a positive effect on the achievement of college calculus students by improving their attitudes towards calculus, reducing their mathematics anxiety, and building their confidence to do calculus, but more research is needed to address this issue.

Using CAI to graphically illustrate mathematical concepts in college calculus appears to be a promising prospect in education. There is a need to improve the mathematics achievement of students in underrepresented groups and supplementing instruction with CAI appears to be a possible method of doing so. Evidence suggests using CAI to visually illustrate mathematical concepts in a freshman college calculus course may improve student achievement, result in positive improvement of attitudes towards the course, and reduce the level of mathematics anxiety in all students, including those underrepresented in technology related fields. This review demonstrates a need for research into this topic, also.

In high school algebra classes Rhoads (1986) found CAI significantly promoted learning skills and concepts in a two day application. In a study in which a highly visual form of computer-enhanced instruction (CEI) was used over a two-week period, Kiser (1987) found significant results and a positive change in attitude in college algebra classes. Ganguli (1990) found the experimental group scored significantly higher on the comprehensive final than did the control group after a five-week period of treatment in which the microcomputer was used to demonstrate four selected course topics. Palmiter (1991) found CAI helped students obtain significantly higher achievement in a college calculus course which was covered in five weeks instead of the usual ten-week period. And Reglin and Butler (1989) conducted 18 class sessions (three per week) during six weeks and found the students in the experimental group made significantly higher gains than did those students in the control group.

In a study by Ayers et al. (1988), CAI was used as a supplement to traditional instruction over a six-week period with a weekly two-hour session during the last five weeks of the study. Students in the experimental group used the two hours for CAI with instructor assistance. Payton (1987) also found significant gains in achievement for the college students on the post-test of knowledge of graphs, relations, and functions in a study consisting of seven assignments where CAI was used to supplement the traditional instruction.

The strength of supplementary CAI is apparent in the literature. In a meta-analysis of 40 studies where CAI was used as a supplement to traditional instruction, Burns and

Bozeman (1981) found CAI to be effective in mathematics applications regardless of the length of the study or the number of treatments. In another meta-analysis, one of 59 studies, Kulik et al. (1980) found computer-based instruction (CBI) significantly improved the achievement of college students and positively affected student attitudes toward the method of instruction and the subject. These findings were found to be independent of the length of the study, the number of assigned treatments, and the design of the study. And in a third meta-analysis of 51 studies, Kulik et al. (1983) found results which indicated secondary students who received computer-based teaching scored higher on final examinations, developed positive attitudes towards the courses they were taking, and needed less time to learn the material covered in the course. They actually found these "effects were greater in studies of shorter duration," (Kulik et al., 1983, p. 23). This seems to indicate the length of a study and the number of treatment assignments are not indications of a successful application of CAI, and that a shorter length study with a relatively small number of treatment assignments may be more successful than a longer study with more assignments. Such is the case with a one semester study conducted by Heid (1988) in which the results were mixed and no significant improvement in student achievement on the traditional material was noted.

This researcher agrees with Bell, who in 1978 noted:

> The use of computers in mathematics classes has seldom resulted in less effective teaching and learning. In many cases computers have helped to improve instruction, learning, and student interest in mathematics. (p.430)

While the literature cited in this review indicates supplementary CAI should affect student achievement, attitude, and mathematics anxiety in a positive manner, many of the above studies and reports suffer from some form of inherent weakness according to Schmitt (1989). Schmitt listed four major weaknesses in the design of many studies dealing with CAI: "small sample sizes, lack of identified criteria for determining 'quality' software, inappropriately used statistics, and inadequate time allocated to conducting the study." (Schmitt, 1989, p. 2) This study addresses these possible weaknesses in CHAPTER III.

CHAPTER III

METHODOLOGY AND PROCEDURES

Introduction

The purpose of this study was to determine the effects of supplemental computer-assisted instruction (CAI) based upon graphical representation of mathematical concepts on student achievement, attitudes, and anxiety in a college freshman level engineering calculus course. The purpose of this chapter is to describe: (a) the sample population; (b) the course supplemented with CAI; (c) the research design; (d) the treatments used; (e) the data collection methods; (f) the statistical hypotheses; and (g) the data analysis procedures employed.

The results of a pilot study conducted by this researcher (see Appendix E) and studies described in the review of literature influenced the design of this study. The pilot study results indicated a trend toward higher success rates and grades. Students in the CAI group obtained higher average scores on all three course exams and on the final exam. There was also a significant positive improvement in student attitude toward mathematics in the group which utilized CAI. Based on the pilot study extra controls were implemented for this study. The number of students in the sample was increased, raw scores were used to determine student achievement and success because they combine all possible points a student could obtain in the course, and tighter controls were utilized to determine if the use of CAI to supplement college calculus significantly improved student attitude toward mathematics or if some other variable could have done so.

The results and designs of the studies and meta-analyses reviewed in Chapter II influenced the design of this study. The length of this study was set at one ten-week quarter because several studies indicated a study of relatively short duration would have the most benefit on student achievement (Kulik et al., 1980; Burns & Bozeman, 1981; Kiser, 1987; Payton, 1987; Ganguli, 1990). The study by Heid (1988) indicated a connection between student attitudes and college calculus, so these variables were included in this study.

The possible flaws in other studies in which CAI was used which were indicated by Schmidt (1989) encouraged this researcher to impose more control on this study by doing the following:

1. Use a relatively large sample size;

2. Select a versatile, user-friendly graphing utility;

3. Employ an analysis of variance to analyze the data collected; and

4. Allow the students a week to complete the labs and reflect on the concepts they covered.

Other reports in the reviewed literature also influenced the amount of control for extraneous variables imposed on this study. Some studies did not employ comparable treatments for both the experimental and control groups as did this study (Gronberg, 1987; Payton, 1987; Reglin, 1987; Reglin & Butler, 1989; Ganguli, 1990; Palmiter, 1991). To minimize the influence of the instructor variable this study utilized eight intact classes, none being taught by this researcher. That was not the case in the study conducted by Heid (1988). And the hard to measure factors: instructor quality, class size, and student preparation reported by Hickernell and Proskurowski (1985) also influenced this researcher to utilize eight sections with eight instructors, use a relatively large sample size, and match the intact sections based upon their pretest means.

Sample Population

This study was conducted at Montana State University (MSU) during winter quarter of 1991. MSU is a state-funded university with an enrollment of approximately 10,000 students. The student body consisted mainly of Anglo-American students, with Native American students making up the largest minority group. Smaller numbers of other minority students were also enrolled.

The sample for this study consisted of 322 students enrolled in Math 181: Calculus and Analytic Geometry I (from now on referred to as Math 181). Twelve sections of Math 181 were taught during winter quarter 1991. Students were assigned to the twelve sections through the registration process used by the university. Each section was taught by a different instructor, either an experienced graduate teaching assistant or a professor. Because this researcher exercised no control over either process, eight of the twelve sections were matched using mean scores on a researcher designed pretreatment test of algebra and precalculus concepts. One section of each matched pair was randomly assigned to the experimental group, the other was assigned to the control group. Each treatment group was formed from four different sections of the course, with four different instructors. The experimental group began with a total of 165 students in its four sections, and the control group with a total of 157 students.

Course Description

The course which was used for this study was Math 181. This course is the first quarter college calculus course for freshmen majoring in an engineering, math, or science curriculum and covers the topics traditionally taught in such a course. It is a four-credit course, meeting four days a week for 50 minutes a day for the ten-week quarter. All sections follow the same master schedule and syllabus prepared by a supervising professor. Common hour and final examinations are administered following the master schedule. A student's grade is determined by the number of points accumulated on homework assignments and the exams. The two common hour exams are scored on a 100 point scale, the final exam is scored on a 200 point scale, and 100 points are awarded by the individual instructor for performance on homework and quizzes. Grades are determined by the percentage of the 500 points possible using the following scale:

A, for 90% or more of the 500 points;

B, for 80 - 89% ;

C, for 70 - 79% ;

D, for 60 - 69% ; and

F, for less than 60%.

Students were allowed to drop the course prior to the fifteenth day of instruction, and were allowed to withdraw with a passing grade prior to the twenty-fifth day. For the purpose of this study, a student who finished the course and received a C or better for a final grade was considered to have successfully completed the course. Because it would require retaking the course to receive credit towards graduation, a student who withdrew or finished the course with a grade lower than a C was considered to be unsuccessful.

Research Design

The design of this study was a quasi-experimental control-group design. It was a modified version of the nonequivalent control-group design frequently used in educational research when the researcher cannot randomly assign a student to the experimental or control group but must work with intact classes (Campbell & Stanley, 1963). In this study the modification consisted of matching intact classes based upon the pretest scores and then randomly assigning one of each matched pair to the experimental group, the other being assigned to the control group.

A student enrolling in Math 181 was assigned to one of 12 sections of the course offered during winter quarter through the regular registration process used by the university. The registration process made random assignment of individual students to either of the two groups unfeasible, and made necessary the use of intact classes. The eight sections involved in this study were chosen based upon the mean section scores on a researcher-created multiple-choice pretreatment test of algebra, trigonometry, and precalculus skills (see Appendix B). Some questions on this test were selected from the Mathematics Assessment Test (MAT) which was created and used for diagnostic purposes by the Department of Mathematical Sciences at Montana State University (MSU). Because the MAT includes many questions covering lower level mathematics, some questions on the pretest were developed from other algebra, trigonometry, and precalculus sources. The 20 questions included on this test were edited for content by experienced mathematics professors at MSU. The topics covered by the pretest were judged important for success in an introductory course in college calculus. To aid in experiment precision, it was important to establish a connection between the concepts tested and the topics covered in the course to help ensure students in sections with equal means possessed similar mathematical backgrounds (Box, Hunter, & Hunter, 1978).

The eight sections with the closest matched mean scores on the pretreatment test were selected to participate in this study, the remaining four were excluded as a control for the threat to internal validity caused by statistical regression. One section from each matched pair was randomly assigned to the experimental group, the other section was assigned to the control group. The use of this modification minimized some of the possible threat to internal validity caused by the selection process while imposing some control on the sensitization effects of the pretest (Cates, 1985).

Each section was assigned a different instructor, either a mathematics professor or an experienced graduate teaching assistant. These assignments were made by the head of the Department of Mathematical Sciences at MSU. This researcher exercised no control over the assignment of section instructors. To exercise some control of the teacher variable a choice was made to use eight complete sections of Math 181 and not to train the instructors in the use of the computer and software. This researcher was interested in keeping the instruction as traditional as possible and did not want instructor enthusiasm for the use of technology to influence the possible outcomes of this study. Therefore, the role of the instructors was limited to teaching the traditional course only.

Once the eight sections of Math 181 were selected, a pretreatment survey assessing attitudes toward mathematics and collecting student background data was administered in both the experimental and control groups during the second week of the quarter. A separate inventory was also administered to identify those students who were primarily visual learners.

The experimental and control groups received separate, though similar, treatments over the next six weeks. The treatments consisted of five weekly supplemental -homework assignments. These assignments were developed by the researcher and consisted of worksheets covering the topics being taught in the course during the week the assignments were given. For the experimental group, the worksheets were designed as computer laboratory assignments to be completed using CAI to graph selected functions using function graphing software. Supplemental homework assignments of the control group covered the same content as those of the experimental group, but without the aid of the computer generated graphical illustrations.

During the course of the treatments, all students involved in this study were given two common hour examinations. The first was administered during the week following completion of assignments one and two. The second common hour examination was administered after the completion of the remaining three assignments. The final examination for the course was administered during finals week, approximately two weeks after the second common hour examination. During the two weeks between the second common hour exam and the final examination, the same statements on the pretreatment attitude survey were rearranged and administered as a post-treatment attitude survey to assess possible changes in student attitudes towards mathematics.

Student achievement in the course was determined using criteria outlined in the above course description, with the scores given for treatment assignments included as part of the 100 points awarded for homework and quizzes. The total number of points a student received for homework, quizzes, and exams scores (raw score) was used to establish student achievement. The final grade awarded and completion of the course determined student success. Differences recorded between the pretreatment and post-treatment survey of attitudes were used to determine if changes in those attitudes occurred because of the treatments. The background information and learning style survey were used to distinguish sex, ethnicity and learning style preferences of the students involved in this study.

Control of Extraneous Variables

Seven extraneous variables might interact and interfere with the internal validity of an experiment and confound the effects of the experimental treatment when using the nonequivalent control-group design (Campbell & Stanley, 1963). These variables are: history, maturation, testing, instrumentation, statistical regression, selection, and experimental mortality. The modified nonequivalent control-group design employed in this study exercises some control over these contaminating variables.

The impact of the history of the experiment on the treatments effects was controlled to some degree in two ways. The first was the design of the experiment itself. Since a control group was utilized and both treatments were administered concurrently, any events which might cause a difference in the effects of one treatment would most likely cause a similar difference in the effects of the other. The second control was the relatively short length of this experiment. History is usually not considered a problem in studies shorter than a semester (Cates, 1985).

Three controls for the variable of maturation, the change in the subjects which may occur because of the passing of time, were at work in this study. The first was the length of this experiment, the second was the age of the students in this study, and the third, and strongest, was the design itself. The use of a control group in the design of this study controlled the effects of maturation because the treatments were administered concurrently.

Controlling for the effects that taking one test may have on the taking of a second test, was also through the nonequivalent control-group design used in this study. Because both groups were administered the same tests at the same times, any confounding by the testing variable should have been similar for both groups. The variable of instrumentation was controlled in a fashion similar to that of the testing variable. Students' raw scores and success rates were determined using the same criteria for both groups, so any changes in these two dependent variables were not likely to be caused by the instrumentation variable.

The design of this study was modified to include matching of the intact classes and randomly dividing the matched pairs into one or the other of the two groups. This was done to assist in controlling the possible confounding variables of statistical regression, selection, and experimental mortality.

Statistical regression was controlled in three ways: the random division of the pairs matched with the closest mean scores on the pretest produced groups similar in general composition; the use of the section means for the matching process reduced the impact of any extremely high or low pretest scores; and the elimination of four intact classes whose mean scores did not match well with those of the other sections. Matching the intact classes provided some control over the possible threat of the selection process and the sensitization effects of the pretest (Cates, 1985). The selection variable was also controlled through the random assignment of the matched pairs to the two groups. The use of matching to supplement randomization in this fashion can improve statistical precision (Campbell & Stanley, 1963).

Experimental mortality was a difficult variable to control because of the nature of the course and its past history. First-year college calculus has traditionally suffered from a large number of students who drop or withdraw from the course (Cipra, 1988). The use of eight intact sections meant a relatively high number of students began the study. While this did not eliminate the possible effects of mortality, any loss of students in one group was supposed similar in the other which offered some form of control over this variable.

According to Campbell and Stanley (1963), possible threats to the external validity, or the ability to generalize the findings, of a study employing a modified nonequivalent control-group design, might be: interaction of testing and the treatments; interaction of selection and the treatments; and reactive arrangements, or experimental settings. The interaction of testing and treatment for the dependent variable of student achievement was controlled in this study through the use of a multiple-choice pretest over topics which, although judged important to success in college calculus by qualified individuals, were not covered directly in the course during the quarter. This helped reduce any sensitization toward future tests since the common hour and final exams were not multiple-choice tests and the tested topics were new to the students. For the pretreatment and post-treatment attitude surveys, some control for interaction of testing and treatments was achieved through the random rearrangement of the items used in the pretreatment survey in the creation of the post-treatment survey.

Partial control for the interaction of selection and the treatments used in this study was provided by the initial use of all 12 sections of Math 181 and using the relatively large number of eight intact classes to create the experimental and control groups. The number of minority students enrolled in Math 181 during winter quarter of 1991 was very small. While it was implied in the previous chapters that this researcher would include the effects of CAI on minority students enrolled in Math 181 during winter quarter, 1991, the low number of minority students receiving raw scores and success rates make the utilization of the collected data difficult.

Under the category of reactive arrangements (Campbell & Stanley, 1963), one might find the effect of being included in a study itself. A partial control for this variable, sometimes called the Hawthorne effect, was the use of a separate treatment for the control group which was similar to that of the experimental group. Another possible confounding variable which might be classified as a reactive arrangement is the teacher variable. This researcher exercised no control over the assignment of section instructors. However some control of the teacher variable was provided by using eight sections of Math 181 with eight different instructors.

Treatments

Treatments for this study consisted of five weekly supplemental homework assignments created by this researcher. These assignments were administered over a six-week period in the form of worksheets which the student completed during the week. Forms of each lab and worksheet were used in the pilot study (see Appendix E). Changes in the forms were edited for content by a course instructor, a mathematics education professor, and a mathematics professor (see Appendix A). For the experimental group, assignments were designed as computer laboratories to be completed using function graphing computer software. For the control group, supplemental homework assignments covered the same content as those of the experimental group, but without the aid of the computer generated graphical illustrations.

The choice of supplementing course instruction with five weekly assignments over a six-week period was supported in the review of literature conducted for this study. Computer-assisted instruction appeared to be more effective on achievement, attitudes, and mathematics anxiety in applications with a small number of treatments given over short periods of time (Ayers et al., 1988; Ganguli, 1990; Kiser, 1987; Palmiter, 1991; Payton, 1987; Reglin & Butler, 1989; Rhoads, 1986). CAI is effective in mathematics regardless of the design or length of the study (Burns & Bozeman, 1981; Kulik et al., 1980; Kulik et al., 1983). Thus, the length of a study and the number of treatment assignments are not indications of a successful application of CAI, and a shorter study with a relatively small number of treatment assignments may be more successful than a longer study with more assignments (Heid, 1988).

Experimental Treatment

The treatment for the experimental group consisted of five specially designed computer assignments (called labs) utilizing CAI (see Appendix A). Students in this group were encouraged to use the graphing utility "Master Grapher" (Waits & Demana, 1990) to graph the selected functions and relations included with each assignment, but could use any function plotting software or graphing calculator if they so desired. Each student was supplied with a simple set of instructions explaining the basic operations of the Master Grapher software. Those students who chose to utilize other means of plotting the graphs of the functions in the labs obtained no specific instructions from this researcher. Assistance in the use of the computers and the graphing software was not designed into this study because computers were located in several locations on campus, some students may have had access to computers located off campus, others may have utilized graphing calculators, and it was hoped that the students would benefit from the exploration which may have occurred during the use of the software.

Weekly labs guided the students through a series of exercises and questions designed to illustrate specific concepts being covered in the course during that week. Each lab was edited for content and pedagogy by a course instructor, a mathematics education professor, and a mathematics professor. Students were expected to complete the labs in a one-hour session with the computer. As students became more familiar with the graphing utility, the amount of time to complete a lab was expected to decrease.

The Software

The Master Grapher interactive graphing utility was chosen for several reasons. The advantages of using the Master Grapher graphing utility as the function plotting software are: (a) the program can be used on most microcomputers; (b) the program is user-friendly; (c) it uses the basic notation found in other mathematics software many students are familiar with; (d) it can graph several functions on the same axes; (e) it is versatile and can graph many different types of functions easily; and (f) it is menu driven, which allows the user to see the options available and access those options by typing a single key.

Master Grapher was designed by Bert K. Waits and Franklin Demana of The Ohio State University, and developed and programmed by David B. New (Waits & Demana, 1990). The Master Grapher graphing utility is pedagogically sound with a design well-tested by mathematics educators and it performs to the design standards (Haase, Marion & Mestre, 1985).

The disadvantage of using the Master Grapher graphing utility is the same disadvantage most function plotting software programs have: the need to write functions in a form slightly different from that used in the mathematics classroom. For example, $x^2 + 2x - 1$ would be written x^2 + 2*x - 1 .

Control Treatment

The treatment for the control group closely corresponded to the treatment given the experimental group. It consisted of five weekly supplemental homework assignments to be completed outside of the classroom. The concepts covered were the same concepts covered in the experimental treatments, but without the benefit of CAI (see Appendix A). These assignments controlled for differences in treatments other than the use of CAI. Without such assignments, any change in student achievement could be attributed to the additional homework assigned to the students in the experimental group and not to the use of CAI.

The weekly out-of-class assignments were given to students in the control group using the same schedule as was used for the experimental group. Traditional classroom instruction consisting of lectures and chalkboard presentations were given in both groups. Therefore, the only difference was the highly visual treatment of the topics given to students in the experimental group through the use of CAI.

Homework Assignments and Computer Labs

These assignments were developed by the researcher and consisted of worksheets covering topics judged important to an introductory course in college calculus by experienced mathematics professors. The first assignment dealt with the definition of the derivative and the product rule for taking derivatives, the second dealt with composite functions and the chain rule, the third dealt with the Mean Value Theorem and its applications, the fourth dealt with approximating the maximum and/or minimum of a function graphically and using the derivative to find the exact value, and the fifth assignment covered applications of the local maximum and minimum values of a function (see Appendix A).

In the first assignment students in the control group were asked to find the difference quotient for the function: $f(x) = x^2$. Students in the experimental group were asked to find the difference quotient for the function: $f(x) = x^2/4$. Dividing the function by four was for illustration purposes and not to complicate the function. Each group was asked to find the derivative using the definition of the derivative as the limit of a difference quotient. The experimental group was then asked to use the computer to graph the function and its derivative. Both groups were asked about what information the derivative of a function tells about that function. The experimental group was given another function and its derivative to graph on the computer while the control group was not. The reason for this was because of the speed, ease, and accuracy of graphing a function on the computer. Some students in the control group may have attempted to graph the first set using pencil and paper, which could lengthen the time needed to complete the assignment.

The first assignment concluded with each group exploring the product rule for taking the derivative, answering questions about the slope of the function, and what information the derivative provided. Students in the experimental group viewed graphs of the functions and their derivatives, and used those graphs to help them answer the questions on Computer Lab 1.

In the second assignment, students in both groups were asked to consider two functions, find the domains of the functions, and then find the domain of the composite of those two functions. The experimental group was asked to graph both functions, the composite of the two functions, and composites using the function derivatives. Students in both groups answered questions designed to encourage them to analyze the information they had discovered. However, the control group did not have the computer-generated graphs to consult. Assignment two ended with students completing the "Chain Rule" for taking the derivative of composite functions.

In assignment three, students explored the Mean Value Theorem and its applications. Students in both groups were asked for the conditions necessary in order to apply the

Mean Value Theorem. Students were given a function and the points where its slope was zero. They were given a second function and then asked to take the derivatives of both the first and second functions. The derivatives of both these functions were the same because the functions differed by only a constant. From this information they were asked to draw conclusions about the second function based on the application of the Mean Value Theorem. Students in the experimental group used the graphing utility to graph both functions and the derivatives. These graphs were to help them apply the Mean Value Theorem and make some conclusions. Students in the experimental group were requested to sketch the graph they expected the computer to generate.

The fourth assignment emphasized the first and second derivative tests. Students in both groups were given the same continuous function: $f(x)=(x^3 -6x^2 -36x +216) /27$. They were then given a series of questions designed to establish that this function met the conditions necessary to apply the first derivative test. Through the use of selective questioning, students were led through the first derivative test and then asked to state it in their own words. Similarly, selective questioning also led them through the second derivative test and the statement of it. The experimental group had computer generated graphs of the functions and derivatives to view and analyze the first and second derivative tests while the control group did not have these visual images.

The last assignment dealt with applying the derivative to find the maxima, or minima, of a function. Students in both groups were given the word problem:

> A farmer wishes to build a pigpen along one side of her barn. She is going to have to fence three sides. The pigpen is going to be in the shape of an isosceles trapezoid. Each side will be of equal length. If the total length of the three sides that she needs to fence is 30 feet with two of the sides meeting the barn side at an angle of x, what angle should x be so that the area of the pigpen is the greatest?

Students in both groups were also provided with a sketch of the problem (see Worksheet 5, Appendix A). In addition to the sketch and the problem, students were given an eight step process for solving this type of problem (see Worksheet 5, Appendix A). Students were then asked to follow each step of the process and answer questions about the function which was to be maximized. Students in the experimental group were asked to use the computer to graph the function being maximized and allowed to use that graph to assist them in finding the solution. They may have created a graph of the function by hand, but did not have the benefit of the computer generated one. The solution to the problem was given later in the assignment so that students could check their results and the process they had used. Each student was then given a problem to solve on their own. Those students in the experimental group were encouraged to use the computer to graph the function in the second problem and use that graph to help them find a solution (see Computer Lab 5, Appendix A).

Pedagogy of Treatments

Each of the five worksheet assignments and computer labs was edited for content by this researcher, a course instructor, a mathematics education professor, and a mathematics professor. The pedagogical basis for supplementary instruction as utilized in the homework worksheets and the computer labs was supported in the literature:

1) The assignment worksheets and the computer labs both provided a collection of functions and open-ended questions which were designed to allow students to analyze the concepts being covered in the course, encouraging them to use higher order thinking skills (Skemp, 1987; Northwest Regional Educational Laboratory, 1985).

2) Allowing a week for students to complete the assignments gave them time to reflect on the concepts covered in the assignments and how the assignments related to the topics covered in the course (Skemp, 1987).

The use of CAI as supplemental instruction as was utilized in this study is also pedagogically supported in the literature:

1) The computer labs supplemented the traditional course structure and helped stimulate higher order thinking through the use of carefully designed questions (Haase et

al., 1985; Olson & Eaton, 1986).

2) The computer labs used in this study supplemented the traditional instruction which allowed the capabilities of the computer to enrich the course materials and the experiences of the students (Northwest Regional Educational Laboratory, 1985; Olson & Eaton, 1986; Keuper, 1985).

3) The functions graphically illustrated in the computer labs were "...relevant to the topic and content of the lesson" (Alesandrini, 1985, p. 5). The graphs of these functions were presented clearly, without additional information which could be distracting to the student (Alesandrini, 1985; Bialo & Erickson, 1984).

4) The computer labs expressed the concepts covered in the classroom visually through the use of graphs of functions. Many ideas and concepts are expressed and understood better through pictures and graphs (Bork, 1980; Skemp, 1987; Laughbaum, 1989).

5) Students were required to interact with the computer software and the computer labs by entering and graphing the requested functions, interpreting the graphs produced by the computer, and answering the questions requested by the computer labs (Bork, 1980; Haase et al., 1985; Keuper, 1985). This interaction allowed students to control some of the aspects of their own learning (Bialo & Erickson, 1984), and assisted them in reaching an appropriate level of understanding of the material (Haase et al., 1985).

6) The computer generated graphics were also used to supply students with a form of feedback after they answered the assigned questions which could lead to improved student learning (Alesandrini, 1985).

7) The graphing capabilities of the computer allowed students to view the functions, change the parameters, and re-graph the functions. This allowed students to build their intuitive knowledge of the mathematical concepts covered in the assignments and labs (Bork, 1980).

8) Students receiving CAI were encouraged to manipulate the parameters of the functions, resulting in a more open-ended form of instruction. This allowed students to analyze the material more critically (Olson & Eaton, 1986), it also required the use of higher order thinking skills on the part of the student (Northwest Regional Educational Laboratory, 1985).

9) Students in the experimental group were indirectly encouraged to utilize the computer software to graph other functions illustrating other topics which is a move closer to an inquiry/discovery method of instruction (Olson & Eaton, 1986).

Data Collection Methods

The data collected from students consisted of raw scores, success rate in the course, responses to a pretreatment survey (consisting of a background information sheet and a mathematics attitudes survey), responses to a post-treatment mathematics attitude survey, and responses to the learning style assessment.

The background information sheet consisted of information pertinent to this study: name, identification number, sex, and ethnicity (see Appendix C). Included with the background information sheet was a mathematics attitudes survey. The questions on this pretreatment survey were randomly rearranged and used as a post-treatment survey to assess changes in students' attitudes towards mathematics. These surveys were made up of four scales of the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1986). The learning style assessment instrument employed was one section of the Learning Style Profile (LSP) developed by the National Association of Secondary School Principals to determine a student's primary learning style preference (Keefe et al., 1986). A student's raw score was made up of the total number of points awarded to the student on the two common hour examinations, the final examination, homework and quizzes. Student success rate was based on completion of the course with a grade of C or better.

Mathematics Attitudes Survey

The mathematics attitudes survey instrument (see Appendix C), used as a pretreatment survey and post-treatment survey, consisted of four scales selected from the Fennema-Sherman Mathematics Attitudes Scales: the Confidence in Learning Mathematics Scale, the Attitude Toward Success in Mathematics Scale, the Usefulness of Mathematics Scale, and the Mathematics Anxiety Scale (Fennema & Sherman, 1986).

The Fennema-Sherman Mathematics Attitudes Scales are a collection of nine Likert-type scales developed by Elizabeth Fennema and Julia A. Sherman in 1976 to assess student attitudes toward many different items pertaining to the teaching and learning of mathematics. These scales were developed through a grant from the National Science Foundation and consist of 12 statements, six positively stated and six negatively stated. Each statement is scored on a scale of 1-5 points, with a 5 awarded for the most positive response and a 1 awarded for the most negative response. Thus a student's total score on any one of the scales ranged from 12 to 60.

The Confidence in Learning Mathematics Scale measures a student's confidence in her, or his, ability to learn and use mathematics. The Attitude Toward Success in Mathematics Scale was developed to measure the magnitude of student anticipation of positive or negative consequences attached to his or her success in mathematics. The Mathematics Usefulness Scale measures how students feel about mathematics and its relationship to their future activities. The Mathematics Anxiety Scale was developed to establish a level of mathematics anxiety in a student.

The 48 statements which made up the four scales employed in this study were randomly distributed in the pretreatment survey following the recommendations of the scale developers (see Appendix C). For the post-treatment survey the items were randomly rearranged in an attempt to control for pretreatment survey sensitivity.

Validity studies of the Fennema-Sherman Mathematics Attitudes Scales were conducted by the developers (Fennema & Sherman, 1986) and by Broadbooks, Elmore, Pedersen and Bleyer (1981). Broadbooks et al. (1981) noted "... there is evidence to support the theoretical structure of the Fennema-Sherman Mathematics Attitudes Scales" (p. 556). Reliability has been established through results from studies which have utilized these scales to determine student attitudes towards mathematics (Dick & Balomenos, 1984; Iben, 1989; Schaalma, 1989).

Learning Style Assessment Instrument

The survey used to determine a student's primary learning style preference was a section of the Learning Style Profile (LSP) (Keefe et al., 1986). It consisted of a list of 20 words. The student was to read and then circle a response denoting what the word evoked in the mind of the student: a picture, a sound, or a feeling (see Appendix D). Reliability and validity of this instrument were established by the developers and are reported in the LSP Examiner's Manual (Keefe & Monk, 1986).

A student who responded that 12 or more of the 20 words evoked a picture was classified, for the purpose of this study, as being primarily visual learners following the guidelines in the LSP Examiner's Manual (Keefe & Monk, 1986). A score of 12 was one standard deviation above the mean score established by the developers of the LSP (Keefe & Monk, 1986).

Course Examinations

All course examinations given in Math 181 were created by the professor supervising the course based upon his experience and the traditional format. They were administered to students in all twelve sections of the course at specified times, in specified locations, on specified days during the quarter. The grading of these examinations was divided equally between all 12 section instructors, each was assigned certain problems from one of the three exams and graded those problems for students in all 12 sections of the course. A student's grade for an examination was the sum of the points awarded for each problem on the exam.

Statistical Hypotheses

The questions addressed by this study as stated in Chapter I were:

(1) How does CAI affect student achievement when it is used to supplement traditional methods of instruction in a

college freshman level engineering calculus class?

(2) How does CAI affect the achievement of students in underrepresented groups when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

(3) How does CAI affect student attitudes when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class? and

(4) How does CAI affect student mathematics anxiety levels when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

These questions dealt with student achievement in college calculus, student attitudes toward mathematics, and student mathematics anxiety levels. The methods of assessing achievement in this study were student raw score and success rate. Student attitudes were divided into: confidence in learning mathematics; attitudes towards success in mathematics; beliefs about the usefulness of mathematics; and level of mathematics anxiety. These questions also dealt with students whose preferred mode of instruction was visual, female students, and students who belonged to a minority group. It was implied earlier in this study that this researcher would include the effects of supplemental CAI on minority students enrolled in Math 181 during winter quarter, 1991. The lack of research into minority student achievement and the use of CAI in college mathematics courses demonstrated the importance of studies dealing with minority issues in education. However, to the disappointment of this researcher, the low number of minority students who received raw scores and success rates, and from whom attitude and anxiety data was collected, make the utilization and analysis of such data difficult. While the data which was collected was analyzed and included in the tables found in Chapter IV, the analysis is for the reader's information and interest only. Thus question (2) was changed to: How does CAI affect the achievement of female students when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class? And the hypotheses which would have dealt with minority group membership were not addressed in this study.

For the purpose of this study, the four research questions were then separated into 18 distinct questions based upon the independent and dependent variables. Stated as null hypotheses for analysis purposes, the 18 questions were:

1. There is no statistically significant difference in achievement between the experimental and control groups due to treatment as measured by the mean of the total raw score course points.

2. There is no statistically significant interaction

effect between treatment and learning style on achievement as measured by the mean of the total raw score course points.

3. There is no statistically significant interaction effect between treatment and gender on achievement as measured by the mean of the total raw score course points.

4. There is no statistically significant difference in achievement between the experimental and control groups due to treatment as measured by the mean of the success rate.

5. There is no statistically significant interaction effect between treatment and learning style on achievement as measured by the mean of the success rate.

6. There is no statistically significant interaction effect between treatment and gender on achievement as measured by the mean of the success rate.

7. There is no statistically significant difference in the mean change in score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

8. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

9. There is no statistically significant interaction effect between treatment and gender in the mean change in

score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

10. There is no statistically significant difference in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

11. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

12. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

13. There is no statistically significant difference in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

14. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

15. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

16. There is no statistically significant difference in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

17. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales.

18. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales.

Data Analysis Procedures

The independent variables for this study were:

1. treatment - experimental with CAI and control without CAI;

2. learning style - primarily visual and primarily not visual;

3. gender - male and female; and

4. minority group - member and non-member.

It is acknowledged that learning style, gender, and minority group membership may be more appropriately classified as attribute variables. However, in this study, they were considered as independent variables following accepted practice. The minority group variable is included here as explanation of the tables in Chapter IV and was not included in the hypotheses due to the extremely small number of minority students in the sample.

The dependent variables for this study were:

1. achievement - total point accumulation, raw score;
2. success - successful course completion, success rate;
3. confidence - confidence in learning mathematics;
4. attitude - attitude toward success in mathematics;
5. usefulness - usefulness of mathematics; and
6. anxiety - mathematics anxiety level.

The statistical analysis package SPSS was used to perform an analysis of variance on the collected data.

Equivalence of Experimental and Control Groups

The eight sections used in this study were matched by section means on the pretest as a way to ensure equivalent groups. After being matched and randomly assigned, the group mean on the pretest for the experimental group was 62.51515 with a standard deviation of 17.79193. The group mean on the pretest for the control group was 62.54777 with a standard deviation of 18.46513. Thus the means of the experimental and control groups were within thirty-three one thousandths of one point and the standard deviations were within sixty-eight one hundredths of one point on a twenty-question multiple-choice test. The test scores within one and two standard deviations of the means ranged from 45 to 80 and 30 to 95, respectively, for both groups. Thus ninety-five percent of the student scores of both groups fell between 30 and 95. Results of a t-test to determine if these means differed significantly are given in Chapter IV.

Choice of Alpha Level

For this study, the commonly chosen alpha of .05 ($\alpha = .05$) was selected over the sometimes used alpha of .01 ($\alpha = .01$). The selection of a larger alpha level increases the possibility of making a Type I error. A Type I error occurs when a null hypothesis which is true is rejected, while a Type II error occurs when a null hypothesis is retained when it is actually false.

A Type I error could lead to the expenditure of time and money creating supplementary instruction for a calculus course, purchasing computer equipment and software, and training instructors and students in the use of that equipment and software when the results of such supplementary instruction may not merit such expenditures.

If a Type II error were to occur, no time or money would be expended purchasing equipment and software, creating supplementary instruction, or training instructors and students although such a move could have had positive effects upon student achievement. If the instruction methods remained those of a traditional calculus course, student achievement levels would likely remain at the traditional levels. As partial control for a Type II error, a relatively large sample size of 322 students (165 in the experimental group and 157 in the control group) was utilized in this study. Increasing the sample size lowers the probability of a Type II error (Ferguson & Takane, 1989).

A Type II error would most likely cost less in time and money, and cause the least amount of inconvenience, while the making of a Type I error could result in unmerited expenditures of time and money. Thus, the larger alpha of .05 was more appropriate for this study.

Method of Analysis

There were four independent variables identified for this study. The first was the treatments administered, the second was the learning style of the student, the third was the gender of the student, and the fourth was student membership in a minority group.

This study was interested in the effects the independent variables had on the six dependent variables: student achievement; rate of success; confidence in learning mathematics; attitude toward success in mathematics; perceived usefulness of mathematics; and mathematics anxiety level. In addition to the effects of the individual independent variables on the dependent variables, this study was also interested in the possible interactive effects the treatment and the other independent variables had on the dependent variables. This study investigated what interactive effects, if any, the treatment administered and the learning style of the student had on the dependent variables, the treatment and the gender of the student had on the dependent variables, and the treatment and students' membership in a minority group had on the dependent variables. Because of the desire to investigate the interaction effects of two independent variables on the dependent variables, a two-way analysis of variance (ANOVA) was chosen as the method of statistical analysis. A factorial ANOVA is employed when a researcher is interested in both the direct effects and the interactive effects of two or more independent variables on one, or more, dependent variables (Kerlinger, 1986).

CHAPTER IV

ANALYSIS OF DATA

Introduction

The purpose of this study was to look at the possible effects of supplemental computer-assisted instruction in visualization in a traditionally taught college calculus course. Students in eight sections of Math 181: Calculus and Analytic Geometry I were used as the sample population. The sections were matched using mean scores on a researcher designed pretreatment test of algebra and precalculus concepts. One section of each matched pair was randomly assigned to the experimental group, the other was assigned to the control group. Thus, four sections were randomly placed into the experimental treatment group (165 students) and the other four sections were placed in the control treatment group (157 students). Both groups received the pretreatment learning style survey, classroom instruction adhering to the same course syllabus and the same text, and the same pretreatment and post-treatment mathematics attitude surveys. Both groups were administered the same common hour and final exams.

The experimental group received five treatments over a six-week period during Winter Quarter, 1991. These treatments supplemented the traditional instruction with computer-assisted instruction designed to graphically illustrate some of the central concepts covered in the course.

The control group received five treatments during the same period of time. These treatments consisted of supplementary exercises covering the same concepts given to the experimental group, but without the computer generated illustrations.

This chapter will describe the method of forming the treatment groups, restate the research questions and corresponding null hypotheses, and report the statistical analyses of the collected data. It is divided into the sections:

1. Pretreatment equivalence of treatment groups;
2. effects of treatments on student achievement;
3. effects of treatments on student attitudes and level of anxiety; and
4. discussion of findings.

Pretreatment Equivalence of Treatment Groups

Pilot Study Results

Results of a pilot study (see Appendix E) conducted by this researcher during the Winter Quarter, 1990, indicated students who received computer-assisted instruction as a supplement to traditional instruction achieved higher average grades, higher success rates, and a greater improvement in their attitude towards mathematics than did those students who did not receive supplemental instruction. Although this was an interesting trend, the results of an analysis of variance found only the improvement in attitude to be statistically significant.

These results were encouraging and indicated improvements in mathematics achievement and attitudes might be obtained through the use of CAI as a supplement to traditional instruction. To determine if these positive results were attributable to the supplemental CAI, more control of possible extraneous variables was necessary. Possible extraneous variables identified as a result of the pilot study were: nonequivalent treatment groups, sample size, completion of treatments, and instructor differences.

One partial control for the possibility of the experimental and control groups not being similar was to use groups made up of eight matched sections of Math 181 randomly assigned to the groups. The number of students in this study was almost twice that used in the pilot study, thus providing some control for the sample size being too small. To help control for uncompleted assignments, each lab activity was given a point value of ten. This made completion of the labs and worksheets valuable in calculation of the final grades. To control for instructor differences the number of sections utilized in each group was increased to four. This doubled the number of instructors used in each group in the pilot study for each group. While these procedures helped establish more control, it was still not possible to completely control these variables and others which may have existed but were not identified.

Pretreatment Equivalence Methods

Results of the pretest were used to equate the two treatment groups. The section means were used to select eight sections of Math 181 to be used in this study. The 12 means by section number were: (1) 58.72; (2) 55.79; (3) 62.21; (4) 57.94; (5) 64.50; (6) 60.68; (7) 51.20; (8) 66.05; (9) 64.87; (10) 64.63; (11) 59.17; and (12) 53.67. The sections were then matched with the two sections having the closest means making up the first pair, the next two - the second pair, the next two - the third pair, and the next two - the fourth pair. Thus, the pairs of sections were: (5) and (10), (1) and (11), (8) and (9), and (3) and (6). After randomly assigning the sections of each pair to the experimental and control groups, the experimental group was made up of sections (1), (3), (9) and (10), and contained 165 students; and the control group was made up of sections (11), (6), (8) and (5), and contained 157 students. A t-test was performed using the individual student scores on the pretest to determine if the two treatment groups were significantly different. Results of the t-test are recorded in Table 1. The t-ratio obtained indicated the difference in the mean of the control group (62.5478) and the experimental group (62.5152) was not significant.

Table 1. Results of t-test Analysis of Pretest Scores

	Experimental Group	Control Group
Sample size	N = 165	N = 157
Mean	X = 62.5152	X = 62.5478
Variance	s^2 = 316.5528	s^2 = 340.9610
Standard Deviation	s = 17.7919	s = 18.4651
Estimate of Variance	s^2 = 328.4518	
Degrees of Freedom	df= 320	
Standard Error	S_e = 2.0206	
Difference in Means	X = 0.0326	
t-ratio	t = 0.016	

Effects of Treatments on Student Achievement

Student Achievement as Measured by Raw Score

The number of students in both the experimental and control groups who received a total raw score was 126 and the number of visual learners in each group who received a total raw score was 29. Because it was surprising that these two numbers turned out to be the same, this researcher rechecked the collected data to determine how this could have occurred. Students were allowed to drop out of the course up to 25 days after the beginning of the quarter without receiving a grade or total raw score. They were allowed to withdraw from the course up to ten days before the end of the quarter, but were assigned a withdrawing with a passing grade or withdrawing with a failing grade depending upon their individual situation and circumstance. Thirty-two students in the experimental group dropped the course and seven withdrew without receiving a final raw score. In the control group, 22 students dropped the course and nine withdrew without receiving a final raw score. Students who dropped the course were not assigned a grade and were thus excluded from this study and the analysis of raw scores. Thus, the numbers were correct and occurred by coincidence only.

The means of the raw scores are recorded in Table 2. They are categorized by the treatment groups and dependent variables. The mean raw score of the different divisions of the sample population are given along with the number of students in each division. This information was used in the Analysis of Variance to determine if the following null hypotheses were retained or rejected:

1. There is no statistically significant difference in achievement between the experimental and control groups due to treatment as measured by the mean of the total raw score course points.

2. There is no statistically significant interaction effect between treatment and learning style on achievement

as measured by the mean of the total raw score course points.

3. There is no statistically significant interaction effect between treatment and gender on achievement as measured by the mean of the total raw score course points.

Table 2. Student Achievement - Mean Raw Scores

	Experimental Group	Control Group
Treatment	349.31 (N =126)	350.00 (N =126)
Visual Learner Mean	382.00 (N = 29)	382.55 (N = 29)
Other Learner Mean	339.54 (N = 97)	340.27 (N = 97)
Male Student Mean	346.72 (N = 96)	350.44 (N = 97)
Female Student Mean	357.60 (N = 30)	348.52 (N = 29)
Minority Student Mean	296.50 (N = 4)	370.00 (N = 9)
Majority Student Mean	351.04 (N =122)	348.46 (N =117)

Results of the Analysis of Variance for null hypotheses 1 to 3 are summarized in Table 3. No significant difference was found between the treatments, and no significant interaction effect was found between treatment and learning style, or between treatment and gender. From these results null hypotheses 1, 2, and 3 were retained.

Table 3. Analysis of Variance - Raw Scores

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	62.44	1	62.44	0.007	.935
Two-way Interactions					
Treatment x Learning Style	16.54	1	16.54	0.002	.967
Treatment x Gender	3429.94	1	3429.94	0.364	.547
Treatment x Minority Grouping	8584.24	1	8584.24	0.910	.341

Student Achievement as Measured by Success Rate

The success rates for the categorized divisions are recorded in Table 4. The mean success rates of the different divisions of the sample population are given along with the number of students in each division. Because students who withdrew from the course received a grade which only a retaking of the course can remove from their records, they were classified as unsuccessful and were included in this analysis. Thus, more students were given success rates than were given total raw scores. An analysis of variance performed on this data was used to determine if the following null hypotheses were retained or rejected:

4. There is no statistically significant difference in achievement between the experimental and control groups due to treatment as measured mean success rate.

5. There is no statistically significant interaction effect between treatment and learning style on achievement

as measured by mean success rate.

6. There is no statistically significant interaction effect between treatment and gender on achievement as measured by mean success rate.

Table 4. Student Achievement - Mean Success Rates

	Experimental Group	Control Group
Treatment	0.59 (N =133)	0.59 (N =135)
Visual Learner Mean	0.76 (N = 29)	0.73 (N = 30)
Other Learner Mean	0.55 (N =104)	0.54 (N =105)
Male Student Mean	0.57 (N =103)	0.61 (N =103)
Female Student Mean	0.67 (N = 30)	0.50 (N = 32)
Minority Student Mean	0.20 (N = 5)	0.67 (N = 9)
Majority Student Mean	0.61 (N =128)	0.58 (N =126)

Results of the Analysis of Variance for null hypotheses 4, 5, and 6 are summarized in Table 5. Results indicated no significant differences between the treatment groups on the variable of success rate, therefore null hypothesis 4 was retained. No significant interaction effects were indicated between the treatments and learning style or gender; therefore, null hypotheses 5 and 6 were also retained.

Table 5. Analysis of Variance - Success Rates

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	0.004	1	0.004	0.015	.903
Two-way Interactions					
Treatment x Learning Style	0.073	1	0.073	0.307	.580
Treatment x Gender	0.663	1	0.663	2.792	.096
Treatment x Minority Grouping	0.407	1	0.407	1.714	.192

Effects of Treatments on Student Attitudes and Anxiety

The research questions dealing with student attitudes can be broken down into the three categories: those questions concerned with confidence in learning mathematics; those concerned with attitude toward success in mathematics; and those concerned with usefulness of mathematics. The dependent variables for the statistical hypotheses corresponding to these questions are recorded as: Confidence Level; Attitude Level; and Usefulness Level.

Only 88 students in the experimental group and 79 in the control group could be identified with enough accuracy to match the pretreatment and post-treatment surveys to the correct student. This problem is attributable to several factors: a number of students dropped or withdrew from the course prior to completion of the treatments and the post-treatment survey; several students switched sections between the administration of the pretreatment and post-treatment surveys; several students did not complete the survey response forms correctly, or did so incompletely; and several filled out only one of the two surveys due to absenteeism. For whatever the reason, the number of students appearing in the cells in the following tables dealing with the survey data collected was disappointing.

Change in Confidence Level

The mean changes in Confidence Level are recorded in Table 6. They are categorized by the treatment groups and dependent variables. The data was analyzed using an analysis of variance and the results were used to determine if the following null hypotheses were retained or rejected:

7. There is no statistically significant difference in the mean change in score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

Table 6. Student Attitude - Mean Confidence Levels

	Experimental Group	Control Group
Treatment	-0.10 (N = 88)	-0.01 (N = 79)
Visual Learner Mean	-0.09 (N = 22)	0.54 (N = 24)
Other Learner Mean	-0.11 (N = 66)	-0.25 (N = 55)
Male Student Mean	-0.06 (N = 62)	0.20 (N = 59)
Female Student Mean	-0.19 (N = 26)	-0.65 (N = 20)
Minority Student Mean	-4.00 (N = 4)	-2.00 (N = 6)
Majority Student Mean	0.08 (N = 84)	0.15 (N = 73)

8. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

9. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Confidence in Learning Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

Results of the analysis of variance for hypotheses 7 to 9 are recorded in Table 7. These results indicated that no significant change in confidence level occurred between the two treatment groups because of the treatments. Thus, null hypothesis 7 was retained. No significant interaction effects were found for the dependent variables. Therefore, hypotheses 8 and 9 were also retained.

Table 7. Analysis of Variance - Confidence Levels

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	0.976	1	0.976	0.084	.772
Two-way Interactions					
Treatment x Learning Style	5.593	1	5.593	0.483	.488
Treatment x Gender	5.587	1	5.587	0.483	.488
Treatment x Minority Grouping	6.286	1	6.286	0.543	.462

Change in Attitude Level

The following null hypotheses were concerned with students' attitude toward success in mathematics. The dependent variable for these questions was labeled Attitude Level. Table 8 contains the mean changes in attitude level for each cell:

10. There is no statistically significant difference in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

Table 8. Student Attitude - Mean Attitude Levels

	Experimental Group	Control Group
Treatment	-0.17 (N = 88)	-0.19 (N = 79)
Visual Learner Mean	-1.59 (N = 22)	0.71 (N = 24)
Other Learner Mean	0.30 (N = 66)	-0.58 (N = 55)
Male Student Mean	0.11 (N = 62)	-0.05 (N = 59)
Female Student Mean	-0.85 (N = 26)	-0.60 (N = 20)
Minority Student Mean	-2.00 (N = 4)	0.33 (N = 6)
Majority Student Mean	-0.08 (N = 84)	-0.23 (N = 73)

11. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

12. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

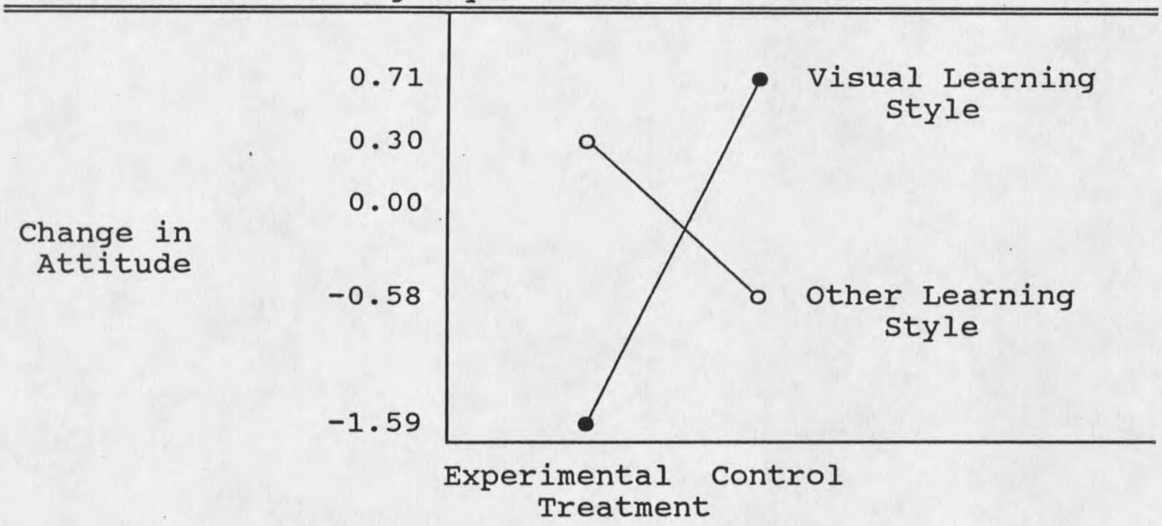
Results of the analysis of variance are summarized in Table 9.

Table 9. Analysis of Variance - Attitude Levels

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	0.017	1	0.017	0.001	.973
Two-way Interactions					
Treatment x Learning Style	92.139	1	92.139	6.293	.013
Treatment x Gender	9.945	1	9.945	0.679	.411
Treatment x Minority Grouping	6.231	1	6.231	0.425	.515

Results of the analysis indicate that no significant difference in the change of attitude level occurred between the two treatments. Thus, hypothesis 10 was not rejected. One interaction effect appeared to be significant while the other did not. Thus, hypothesis 11 was rejected and hypothesis 12 retained. The interaction effect between treatment and learning style on the change in attitude level was significant ($\alpha = 0.05$ level). To determine which specific interaction was significant, the mean changes involved in attitude level from Table 9 were plotted in Figure 1. A simple main effects test was performed to determine which interaction was significant.

Figure 1. Plot of Significant Interaction Treatment x Learning Style on Attitude Levels



Results of this test are recorded in Table 10. Significant interaction exists for visual learners between the experimental and control groups on the dependent variable, Attitude Level. Visual learners in the experimental group experienced a negative change in attitude toward success in mathematics, while those in the control group experienced a positive change in attitude toward success in mathematics. Implications of these findings are discussed in Chapter V.

Table 10. Main-effects Test for Significant Interaction

Source of Variance	SS	df	MS	F	Sig. of F
Visual Learning Style	60.680	1	60.680	4.081	.049
Other Learning Style	23.489	1	23.489	1.655	.201

Change in Usefulness Level

The mean changes in the students' view of the Usefulness of Mathematics are recorded in Table 11. Results of this analysis of variance were used to determine if the following null hypotheses were rejected or retained:

13. There is no statistically significant difference in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

14. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

15. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Usefulness of Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

Table 11. Student Attitude - Mean Usefulness Levels

	Experimental Group	Control Group
Treatment	0.03 (N = 88)	-0.05 (N = 79)
Visual Learner Mean	-0.23 (N = 22)	0.00 (N = 24)
Other Learner Mean	0.12 (N = 66)	-0.07 (N = 55)
Male Student Mean	-0.37 (N = 62)	-0.42 (N = 59)
Female Student Mean	1.00 (N = 26)	1.05 (N = 20)
Minority Student Mean	5.25 (N = 4)	0.33 (N = 6)
Majority Student Mean	0.12 (N = 84)	-0.07 (N = 73)

Results in Table 12 were used to determine whether the hypotheses were to be rejected or retained.

Table 12. Analysis of Variance - Usefulness Levels

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	0.486	1	0.486	0.036	.850
Two-way Interactions					
Treatment x Learning Style	3.224	1	3.224	0.239	.626
Treatment x Gender	1.109	1	1.109	0.082	.775
Treatment x Minority Grouping	2.799	1	2.799	0.207	.649

Results of the Analysis of Variance indicated that no significant change in usefulness level occurred between the two treatment groups because of the treatments. Thus, null hypothesis 13 was not rejected. Also, no significant interaction effects were found between the treatments and the dependent variables. Therefore, hypotheses 14 and 15 were also retained.

Change in Anxiety Level

The mean changes in anxiety level are recorded in Table 13. This information was used in the analysis of variance used to determine if the null hypotheses below were retained or rejected. The dependent variable in the following null hypotheses was called Anxiety Level:

16. There is no statistically significant difference in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales between the experimental and control groups due to treatment.

Table 13. Mathematics Anxiety - Mean Anxiety Levels

	Experimental Group	Control Group
Treatment	0.05 (N = 88)	0.95 (N = 79)
Visual Learner Mean	-0.77 (N = 22)	1.29 (N = 24)
Other Learner Mean	0.32 (N = 66)	0.80 (N = 55)
Male Student Mean	-0.26 (N = 62)	0.90 (N = 59)
Female Student Mean	0.77 (N = 26)	1.10 (N = 20)
Minority Student Mean	1.40 (N = 4)	0.67 (N = 6)
Majority Student Mean	-0.01 (N = 84)	0.97 (N = 73)

17. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales.

18. There is no statistically significant interaction effect between treatment and gender in the mean change in score on the Mathematics Anxiety Scale of the Fennema-Sherman Mathematics Attitudes Scales.

Results of the Analysis of Variance for hypotheses 16 to 18 indicated that no significant change in anxiety level occurred between the two treatment groups. Also, no significant interaction effects are evident. Therefore all three hypotheses were retained based upon the results.

Table 14. Analysis of Variance - Anxiety Levels

Source of Variance	SS	df	MS	F	Sig. of F
Between Treatments	35.778	1	35.778	2.380	.125
Two-way Interactions					
Treatment x Learning Style	14.085	1	14.085	0.937	.335
Treatment x Gender	0.602	1	0.602	0.040	.842
Treatment x Minority Grouping	5.764	1	5.764	0.383	.537

Summary and Discussion

The pretest results were used to match and randomly create the experimental and control groups. Results of a t-test indicated that the two groups did not differ significantly on the group mean scores on the pretest. This appears to indicate the two groups were similar in knowledge and abilities prior to the course and the two treatments.

Results of the analyses of variance indicated no significant difference in student achievement between the two treatment groups as measured by raw score and success rate. No significant interaction effects between the treatments and learning style or gender were indicated on student raw scores, nor were any found on student success rates.

Only one significant interaction effect was found between the treatments and learning style; a significant negative change in student attitude toward success in mathematics due to the experimental treatment for students who were classified as being primarily visual learners. No other significant interaction effects were found on student attitudes, nor were any significant interaction effects found on student mathematics anxiety levels.

CHAPTER V

SUMMARY, RESULTS AND RECOMMENDATIONS

Introduction

The purpose of Chapter V is to provide an overall summary of this study and its results. This chapter is divided into four sections: a summary of the previous chapters, the results of the analysis, a listing of recommendations for further research, and a conclusion summarizing this study and its possible contributions to the field of education. Included in the summary of the previous chapters is a restatement of the problem, a summary of the literature review, a brief description of the procedures utilized, and a list of the possible limitations of this study. Included in the section discussing the results of the analysis is a description of how those results answer the original four questions asked in Chapter I.

The Problem

The overall problem of this study was to investigate the use of supplemental CAI in the form of computer generated graphics in a traditional freshman level college calculus course. This study was conducted to determine what effects this type of CAI had on the number of students who successfully completed the course. Because of the highly visual nature of the CAI treatment employed in this study, the achievement of students who could be classified as having a visual learning style was investigated. The interaction of the treatment and gender was also investigated.

In this study, student success was determined by whether the student completed the course and the student's course grade. The grade of the student was determined by raw score homework, quizzes, and exams assigned and administered during the quarter. A student was said to have completed the course if that student received a final grade. A student was classified as being successful if the final grade was a "C", or better.

Summary of Literature

The review of literature in Chapter II showed CAI has effectively increased the achievement of students in all levels of mathematics education. This review suggested CAI could effectively improve student achievement when used as a supplement to college calculus where students were required to program the computer (Lang, 1976; Sanger, 1977; Berkey, 1977), and where students accessed and utilized pre-programmed application software (Hickernell & Proskurowski, 1985; Ayers et al., 1988; Palmiter, 1991).

Visualization has been shown to be effective in improving student achievement in college calculus (Vinner, 1989), and a highly visual form of CAI has been reported to improve student achievement in college algebra (Kiser, 1987). There is also some indication that visualization and visual-spatial presentations may be beneficial to females and minorities (Bork, 1980; Kleinfeld & Nelson, 1988).

The literature review indicated other factors influenced the mathematical achievement of college students, including attitudes toward mathematics, level of mathematics anxiety, and confidence in ability to do mathematics. CAI influenced student conceptual understanding and attitudes at the college level in a positive manner (Heid, 1988).

The review of literature appeared to indicate that a highly visual form of CAI, in which the concepts covered in calculus were illustrated for student analysis, would effectively supplement and increase achievement in a traditionally taught college calculus course. The effects of this form of CAI should positively influence the abilities and attitudes of male and female students, minority and majority students, and students whose mode of learning is primarily visual.

Summary of Procedures

The sample population for this study consisted of students in eight sections of Math 181: Calculus and Analytic Geometry I at a single university in the Northwest.

Mean scores on a researcher designed pretreatment test of algebra and precalculus concepts were used to create four matched pairs of sections from eight of the original twelve sections, the remaining four sections were excluded. One section of each matched pair was randomly assigned to the experimental group, the other was assigned to the control group. Both groups received pretreatment surveys to distinguish visual learners from students with other learning styles and to assess student attitudes and mathematics anxiety levels.

The treatment for the experimental group consisted of supplemental CAI in the form of computer labs. These labs supplemented the traditional instruction with highly visual computer-assisted instruction designed to graphically illustrate selected topics being covered in the course. Students answered questions requiring the analysis of the computer generated illustrations. The treatment the control group received consisted of supplementary homework exercises covering the same topics as the experimental groups' computer labs. These homework assignments did not involve the analysis of computer generated graphics.

During the course both groups adhered to the same course syllabus and the same traditonal textbook. They were given the same post-treatment attitude and mathematics anxiety survey and were administered the same common hour and final exams.

Student raw scores, course grades and success rates were calculated and statistically analyzed to determine how the treatments affected student achievement and success. The differences in student attitudes towards mathematics and anxiety levels were also analyzed for changes due to the treatments.

Limitations

This study, like most research studies, had a number of limitations related to the setting in which it was conducted, the population which was sampled and the design of the research. Readers must carefully consider these limitations before they generalize any of the reported results to other schools, student populations or situations.

The setting for this research imposes some important limitations on the ability to generalize the findings of this study:

1. This study was conducted in a freshman level college calculus course at Montana State University. Montana has a small, mostly rural population, with a small minority population composed primarily of Native Americans. The character of this population was reflected in the student body of the university. The small number of minority students in the sample population made exclusion of consideration of this variable a necessity.

2. At the time this research was conducted it is possible computers and calculators were not widely used in

many communities in Montana, thus some students enrolled in Math 181 may not have had opportunity to become familiar with computers and their applications prior to beginning college. The use of calculators in the introductory calculus course was prohibited on the exams at MSU when this study was conducted, as was their use in many high school mathematics courses in the state.

The population sampled also imposes some limitations on the application of the results of this study to other students and schools:

1. A small number of minority students were enrolled in the freshman level college calculus. This made the number of students in some cells of the analysis extremely small. While the lack of research in this area indicates a need for inclusion of the analysis of what data was collected, this study did not include hypotheses dealing with this variable.

2. Students sampled in this study may have had relatively little experience with computers and computer applications. This may make generalization of the findings of this study difficult, since computers are becoming more and more available and utilized inside and outside of many high school mathematics classrooms.

3. Students wishing to take college courses at Montana State University can enroll in a course up to ten working days after the course begins and drop the course up to 25 working days after it begins. This feature allowed some

students in the sections studied to start the course after the pretest and pretreatment surveys were administered. Some students also were able to switch sections after the pretest and pretreatment surveys were given. Consequently, some of the collected data was not usable and had to be excluded.

4. Students were allowed to withdraw from the course up to ten days prior to end of the quarter. While the failure of these students to complete the course contributed to the overall success rate scores, data concerning their attitudes towards mathematics and mathematics anxiety had to be excluded from analysis.

The design of this study may place some limitations on the ability to generalize its findings. The reader should take into account the following when attempting to apply the results reported in this paper to other populations and other courses:

1. It was necessary to utilize intact classes for this study. This meant eight different instructors with eight individual teaching styles and preferences taught the eight sections of Math 181 used for this study. The inherent characteristics of the classes and methods of instruction could have had some effect on the implementation of the treatments and some influence on the results.

2. The period of time over which this study was conducted and the treatments were administered was a factor

in the interpretation and the generalization of the results. A longer, or shorter, period could possibly result in different findings.

3. This study measured the effects of supplemental CAI. This is an extremely important consideration when attempting to generalize the findings of this research. Results of this study should not be generalized to situations where CAI is integrated into the course structure. Results reported are only applicable to similar courses where CAI is used to supplement traditional instruction.

4. Although students were free to use the computer and software of their choice, that use was restricted to their time outside of class; class time was not taken up with completion of the computer labs.

5. Student assistance in the computer lab was not designed into the study. Nor was a record of student computer use kept.

Analysis Results

Results of the analysis of variance were reported in Chapter IV. This section summarizes and discusses the results of that analysis and answers the four questions put forth in Chapter I and restated in Chapter III:

1. How does CAI affect student achievement when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

2. How does CAI affect the achievement of female students when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

3. How does CAI affect student attitudes when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

4. How does CAI affect student mathematics anxiety levels when it is used to supplement traditional methods of instruction in a college freshman level engineering calculus class?

These four questions are concerned with how supplemental CAI, of the form described in the previous chapters, affected the following: (a) student achievement, as measured by raw score and success rate; (b) student attitude, as measured by the student's confidence in learning mathematics, attitude toward success in mathematics and usefulness of mathematics; and (c) student mathematics anxiety level. They also pertain to the affects this form of CAI may have on those variables for visual learners and female students enrolled in the course.

<u>Student Achievement</u>

The first six null hypotheses dealt with the effects upon student achievement of the three independent variables as measured by the two dependent variables of raw score and success rate.

The first three null hypotheses were retained because no significant differences were found between the raw scores of students in the experimental group and those students in the control group. Nor were significant differences found which could be attributed to possible interactions between the treatments and student learning style or gender.

Null hypotheses 4 through 6 were concerned with the effects of the independent variables on the success rate of the students. No significant differences were found in this variable for students in the experimental and control groups which could be attributed to the treatments alone. No significant differences were found due to interactions between the treatments and student learning style or gender. These three null hypotheses were also retained.

Retaining null hypotheses 1 through 3 indicates any differences in the raw scores of students in the experimental and control groups could have occurred by chance. The differences between the raw scores of visual learners and non-visual learners, and male and female students could also have occurred by chance and, therefore, can not be attributed to any differences in the two treatments. These results indicate supplemental CAI as discussed in this study is no better than traditional homework for students in a traditional freshman level introductory college calculus course.

It may be possible raw scores of students did not reflect an actual significant difference in achievement between students in the experimental and control groups where one existed. In other words, it is possible a Type II error would be committed if these hypotheses are retained. This type of error occurs when a false null hypothesis is retained. If such an error occurs it may be for the following reasons:

1. The impact of the computer labs and additional homework assignments may have contributed little to the overall raw scores since each lab and assignment accounted for only ten points, or 50 points for the five treatments, of the total 500 points possible.

2. The items tested on the common hour and final exams may not have required the analysis of graphically represented material as had the computer labs. Graphing utilities, computers, and calculators are not traditionally used in this course; therefore, because the exams were written by the course supervisor and followed the traditional format, and this researcher had no direct input into their creation, the common hour and final exams may not have tested any new abilities or insights the students in the experimental group may have picked up during the study.

3. Because of the time and location of this research, these students may have been relatively inexperienced with computers and graphing utilities. They may have spent much of their time learning to use the computer and not an

adequate amount of time analyzing the graphs produced by the computer. Adequate support may not have been available in the lab situations.

4. It is possible the labs and worksheets did not adequately address the concepts and topics covered in the course.

Retaining the null hypotheses 4 through 6 indicates any difference in the success rates of students in the two treatment groups just as likely occurred by chance as by any difference in the treatments. It is also just as likely any differences in the success rates between visual and non-visual learners, and male and female students occurred by chance as by any difference in the treatments.

Because student success rates were based upon grades, which were based upon raw scores, any nonsignificant differences in raw scores would contribute to similar nonsignificant differences in success rates. As was the case in null hypotheses 1 through 3, a Type II error could be made by retaining null hypotheses 4, 5, and 6. If such an error was committed, it may have been for the same reasons as mentioned for the independent variable in the previous null hypotheses, or for one of the following additional reasons:

1. Quantifying the data for success rates, students were given a 1 if they were successful and a 0 otherwise. This may have reduced the amount of variation in the data,

causing any differences in student achievement to go undetected through analysis.

2. The deadline for dropping the course made it possible for some students, who would mostly be classified as unsuccessful, to drop the course early enough to be excluded from analysis.

Assuming no Type II errors occurred in the analysis of null hypotheses 1 through 6, the answer indicated by this study to question one is: the use of a highly visual form of CAI as a supplement to the traditional methods of instruction in a freshman level college calculus course has the same effect as additional homework on the overall achievement of students in the course. The answer indicated by this study to the second question is: the use of CAI, of the form employed in this study, is as effective for student achievement as the use of additional homework exercises for students identified as visual learners and female students in a freshman level college calculus course.

Student Attitudes

The null hypotheses dealing with student confidence in learning mathematics, hypotheses 7 through ten were retained based upon the analyses of variance as reported in Chapter IV. No significant change in this dependent variable was found due to the treatments, nor were significant differences found due to interactions between the treatments and the other two independent variables;

learning style and gender. This indicates students in both groups were equally confident in their ability to do mathematics before and after the treatments (Fennema & Sherman, 1986). If a Type II error was made by the retention of these null hypotheses, it may have been that the instrument used to assess this variable was not effective for students in this study. It may be these students have a level of confidence in their ability to do mathematics which is so strongly established the treatments did not affect it.

No significant difference in student attitudes toward success in mathematics was found due to the treatments, nor was a significant difference found due to interaction between treatments and gender. Based upon these results, null hypotheses 10 and 12 were retained. This indicates anticipation of positive consequences because success in mathematics was similar for students in both groups (Fennema & Sherman, 1986). Reasons for the possibility of making a Type II error in this case are similar to those given for such an error in assessing student confidence in ability to do mathematics: the attitude these college calculus students have about the consequences of being successful in mathematics may be so strongly established that the instrument could not properly identify any changes in this variable which could be attributed to the difference in the treatments.

No significant differences in the student view of the usefulness of mathematics were found which could be attributed to the treatments, nor were any found due to interaction between the treatments and the other two independent variables. Thus, null hypotheses 13 through 15 were also retained. This indicates students in both groups share similar beliefs about how useful mathematics might be to them and their future (Fennema & Sherman, 1986). A Type II error could be made in retaining these hypotheses for reasons similar to those given for the two previous dependent variables: students' feelings about the usefulness of mathematics may be so firm that the treatments did not affect them and the pre- and post-surveys did not detect any significant changes.

While the above null hypotheses were retained based upon the analyses of variance, the following null hypothesis was not:

11. There is no statistically significant interaction effect between treatment and learning style in the mean change in score on the Attitude Toward Success in Mathematics Scale of the Fennema-Sherman Mathematics Attitudes Scales.

This indicates the differences in the change in attitude toward success which occurred between visual learners in the experimental and control groups most probably did not occur by chance, but were likely caused by the treatments. Thus, visual learners who analyze computer generated illustrations of the topics in an introductory calculus course are more likely to experience a drop in their level of anticipation of positive consequences as a result of success in mathematics than are visual learners who receive additional homework assignments (Fennema & Sherman, 1986). This result seems highly unlikely considering the research reported in the review of literature.

To this researcher, it appears more likely that a Type I error would be made if null hypothesis 11 is rejected. A Type I error occurs when a true null hypothesis is not retained. One reason for making such an error could be the instrument used to measure change in this variable indicated a difference between visual learners in the experimental and control groups where none existed. Student attitudes toward success in mathematics may be so strongly established that this instrument could not accurately access the subtle changes which actually occurred. Another reason may be that data could not be collected for all the visual learners, but only for those who correctly completed both the pretreatment and post-treatment surveys. Thus, the analysis may have been distorted because the data collected was not representative of sample. Still a third reason may be because of the number of hypotheses tested. "When more than one ANOVA [analysis of variance], or any other univariate test, is performed on the same dependent variables the experimentwise Type I error probability is increased exponentially," (Schmitt, 1989, p. 9). It may be that such an error occured in this instance.

If no Type I error was made, the answer indicated by this study to question three is: supplemental CAI of the form utilized in this study affects the attitude toward success of visual learners in a negative manner, but does not seem to similarly affect students who prefer a different mode of instruction. If a Type I error was made, the answer to question three is: the type of CAI utilized in this study affects student attitudes in the same way as the use of additional homework exercises for all freshman college calculus students.

Mathematics Anxiety

Results of the data analysis concerning changes in the level of mathematics anxiety of students in the experimental and control groups indicates the differences between the two groups as likely occurred by chance as by any difference in the treatments. There were no significant differences due to interaction between treatments and learning style or gender. Results of this analysis indicate null hypotheses 16 through 18 should be retained. Students in the two groups did not experience a significant difference in their change in level of mathematics anxiety because of the treatments, or because of any interaction between the treatments and the other independent variables (Fennema & Sherman, 1986).

As in the previous cases dealing with student attitudes, a Type II error occurring by retaining hypotheses 16 through 18 is a possibility. This error could occur for the reason mentioned in the previous cases: the instrument used could not delineate the changes because the mathematics anxiety levels were so strongly established in these students.

Using this information, the answer indicated by this study to question four is: supplementary CAI of the form employed in this study does not significantly differ in its effect on the mathematics anxiety level of students than does assigning additional homework. This result may have some significance and merit further study because some may expect the anxiety levels of students in the experimental group to rise since they were required to utilize the computer without direct supervision during the treatments.

Interpretations, Implications and Recommendations

This study measured the effects of CAI on overall achievement and success of students in a freshman level introductory college calculus course. It sought to determine those effects when CAI was used as a supplement to the course. Many mathematics instructors send students to computer labs once a week thinking they are helping their students. Are they? In this study the course was the traditional freshman level engineering calculus course taught at Montana State University, Winter Quarter, 1991. For this research, that course was not altered from the traditional course, but was supplemented by computer labs designed to illustrate concepts covered in the course. These CAI assignments were completed by the students outside the classroom.

In this study, the standard methods of assigning raw points were used and success rates were determined by the successful completion of the course. While it may be important to establish the effectiveness of CAI at teaching CAI topics, this study was not designed to do that. The goal of this study was to determine if CAI was effective in improving student success and achievement in a course often the prerequisite for future math, science, and technological courses.

With this goal in mind, this study was designed to exercise control of many extraneous variables. This part of Chapter V gives this researcher's interpretations of the results previously described and discusses possible implications for math educators and those teaching college calculus.

Interpretations

When interpreting the results of this study, it should be noted the students in the freshman level engineering calculus course who received a highly visual form of supplementary CAI did as well in the course as did students who received additional homework. The use of the computer affected student achievement in the same manner that additional homework did. The computer also affected student success as well as additional homework did. This was true for all students including visual learners and female students.

Supplemental CAI had no adverse effects on student attitudes and anxiety levels. Students were as confident in their ability to do mathematics no matter which treatment they received. Their beliefs about the usefulness of mathematics were the same; students who were not classified as visual learners felt the same about possible consequences of being successful in mathematics. And supplemental CAI did not raise the level of mathematics anxiety of students in the experimental group, including visual learners, nonvisual learners, males, and females.

One result appeared inconsistent with the other findings: the visual learners who received the CAI treatments experienced a drop in their level of anticipation of positive consequences as a result of success in mathematics. If true, this means highly visual CAI as a supplement to a traditionally taught calculus course could be detrimental to such a student's attitude, which may affect his or her performance. Eisenberg and Dreyfus (1989) indicate one possible problem with introducing concepts in more than one way, such as incorporating visual interpretations, could be that the complexity of such an introduction confuses more than clarifies. Visual learners may prefer their own methods of visualizing the mathematical concepts presented in the course and possibly reacted against the need to analyze the computer generated graphical illustrations they were required to perform.

Implications

Several implications may be extracted from the results of this study. Those which appear to this researcher as having the most importance to introductory college calculus instructors and other math educators are:

1. Supplementing classroom assignments with time in a computer lab is no more effective on student achievement, success, and attitude than assigning additional homework.

2. Supplementing the traditionally taught course with CAI completed outside of the classroom is no more effective on student achievement, success, and attitude than assigning additional homework.

3. In many cases, supplementing the traditionally taught course with CAI is not economically practical when

additional homework is just as effective on student achievement, success, and attitude.

Two other implications which may be of importance are: the attitude toward the usefulness of mathematics of visual learners may be negatively affected when a graphical form of CAI is used to supplement the traditional introductory college calculus course, and assigning homework which requires the interpretation of graphically represented material is as effective in a traditional college calculus course as traditional homework assignments for most students.

Recommendations for Future Research

Several interesting related topics surfaced during the time this study was conducted. The following are presented to the reader as recommendations for further research:

1. A study should be conducted in which a college calculus course is altered to make use of graphing calculators. In the time since this study was begun the use of graphing calculators and computers in mathematics courses has continued to grow, and the state of Montana has become a leader in the use of technology in the classroom. With these applications of technology expanding into education, many students from Montana are beginning college having had more experience with computers and graphing calculators than did students sampled in this study. The potential of

graphing calculators has just begun to be realized and new research into how effective they are in a college mathematics courses is needed.

2. A study should be conducted into the actual differences graphical representations of the topics in a college calculus course have on the understanding of the topics and not only their effect on overall student achievement. Understanding of the topics in a calculus course is important to applying calculus to problems in future mathematics, science, and in courses and careers in related technology fields. However, the role of visual presentations in the understanding of the concepts in college mathematics courses has not been adequately addressed in the literature. More research into that role is needed.

3. A study should be conducted in which the learning styles of students in a college calculus course are closely matched to the type of supplementary CAI used in the course. As elementary and secondary teachers become aware of learning style and the role it may play in the acquisition of mathematical knowledge, more students will enter college mathematics classes having been taught in different styles. The role learning styles may play in the mathematical success of college students should be investigated.

4. The lack of research suggests more studies be conducted into the role learning style plays in the

mathematical achievement of minority and female college students. While it is a widely held belief minority students prefer a visual mode of instruction, little formal research into this belief has been reported in the literature.

5. A study should be conducted investigating the effect CAI has on the mathematical achievement of minority and female college students. The review of literature conducted for this study demonstrated a lack of research into this topic. Because of the need for more engineers and scientists, research into methods which may improve the mathematics achievement of students belonging to these underrepresented groups is extremely important. Female and minority students represent a largely untapped source of future engineers and scientists.

6. More long-term research into CAI in college mathematics courses and its effects on student achievement and the understanding of mathematical concepts should be conducted. The potential of CAI is great; but without more research into its effect over a longer period of time, the best applications of its use may not be discovered.

7. Research should be conducted to develop a method of assessing the attitudes and mathematics anxiety level of college students. Student attitudes and mathematics anxiety levels play an important role in their success and achievement in college. While appropriate in some

instances, the instrument utilized in this study may not have had enough sensitivity to determine the subtle changes which may have occurred in this variable. The development of a more sensitive instrument should be investigated.

Conclusion

In concluding this chapter, it should be noted this study was concerned with the need to increase the number of students enrolled in college calculus who successfully complete the course, including female and minority students. Thus, this study was directed at the overall student achievement and success in traditional freshman college calculus courses and not at the understanding of specific topics in the course. This researcher used the standard measures of raw score and success rate to determine the achievement and success of students in this study. Because of the emphasis on success in college calculus, this researcher did not look for differences in students' answers on individual laboratories and worksheets, nor did this researcher attempt to have any exam questions written in the same form as was used on the labs and worksheets.

While the results of this study do not provide support for any particular method of increasing the number of students who successfully complete freshman level engineering calculus, the results of this study demonstrate the viability of using visually oriented CAI as homework instead of the traditional homework in such a course. Students whose traditional instruction is supplemented with a highly visual form of CAI perform as well in the course as do students whose instruction is supplemented by additional homework. This appears to be true for all types of students; visual learners and those with other learning style preferences, and male and female students.

Equally important is the result: the use of supplemental CAI, in a once-per-week computer lab session, does not affect student achievement in a traditional college calculus course. Instructors who believe weekly computer sessions will increase student achievement may be making incorrect assumptions about the capabilities of the computer and CAI.

Computer-assisted instruction and the use of graphing calculators are changing the way mathematics is taught at all levels of education. It is the hope of this researcher that the findings reported in this paper will demonstrate the need for further research into the effects of CAI and graphing calculators in college calculus courses, the effects of graphically represented topics in mathematics at all levels, and the effects of CAI and graphing calculators on the mathematical achievement of female and minority college students.

REFERENCES CITED

References Cited

Akanbi, M. R. & Dwyer, F. M. (1989). The effect of inductive and deductive instructional strategies on students' ability to profit from visualized instruction. In J. Boca (Ed.), About Visuals: Research, Teaching and Applications. Readings From the 20th Annual Conference of the International Visual Literacy Association (pp. 1-10). Blacksburg, Virginia: Virginia Tech University.

Alesandrini, K. (1985). The instructional graphics checklist: A look at the design of graphics in courseware. In M. R. Simonson & M. Triemer (Ed.), Proceedings of the Selected Research Paper Presentation of the Association for Educational communications and Technology (pp. 1-13). Anaheim, CA. (ERIC Document Reproduction Service No. ED 256 303).

Ayers, T., Davis, G., & Lewin, P. (1988). Computer experiences in learning composition of functions. Journal for Research in Mathematics Education, 19, 246-259.

Baker, P. R. & Belland, J. C. (1988, January). Visual-spatial learning: Issues in equity. Paper presented at the Annual Meeting of the Association for Educational Communications and Technology, New Orleans, LA. (ERIC Document Reproduction Service No. ED 295 626).

Bell, F. H. (1978). Can computers really improve school mathematics? Mathematics Teacher, 71, 428-433.

Berkey, D. D. (1977, June). A method for the use of computing in a conventional calculus course. Paper presented at the Conference on Computers in the Undergraduate Curricula, East Lansing, MI. (ERIC Document Reproduction Service No. ED 156 206).

Bialo, E. & Erickson, L. B. (1984, April). Microcomputer courseware: Characteristics and design trends. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED 244 606).

Bialo, E. & Sivin, J. (1990). Report on the effectiveness of microcomputers in schools. Washington, D. C.: Software Publishers Association. (ERIC Document Reproduction Service No. ED 327 177).

Blackburn, K. (1983, Fall). Issues in education resulting from the "computer revolution". Centroid, 9, 15-18. (ERIC Document Reproduction Service No. ED 255 197).

Bork, A. (1980). Learning through graphics. In R. G. Taylor (Ed.), The computer in the school: Tutor, tool, tutee. (pp. 67-72). New York: Teachers College Press.

Box, G. E. P., Hunter, W. G., & Hunter, J. S. (1978). Statistics for experimenters: An introduction to design, data analysis, and model building. New York: Wiley.

Broadbooks, W. J., Elmore, P. B., Pedersen, K., & Bleyer, D. R. (1981). A construct validation study of the Fennema-Sherman mathematics attitudes scales. Educational and Psychological Measurement, 41, 551-557.

Buckley, P. A. & Ribordy, S. C. (1982, May). Mathematics anxiety and the effect of evaluative instructions on math performance. Paper presented at the Midwestern Psychological Association, Minneapolis, MN. (ERIC Document Reproduction Service No. ED 222 334).

Burns, P. K. & Bozeman, W. C. (1981, October). Computer-assisted instruction and mathematics achievement: is there a relationship? Educational Technology, pp. 32-39.

Caissy, G. A. (1987). Microcomputers and the classroom teacher. Phi Delta Kappa Educational Foundation, Bloomington, Indiana.

Campbell, D. T., & Stanley, J. C. (1963). Experimental and quasi-experimental designs for research on teaching. In N. L. Gage (Ed.) Handbook of research on teaching. (pp. 171-246). Chicago: Rand McNally.

Canelos, J., Taylor, W., Dwyer, F. & Belland, J. (1988, January). Cognitive style factors and learning from micro-computer based and programmed instructional materials: A preliminary analysis. Paper presented at the Annual Meeting of the Association for Educational Communications and Technology, New Orleans, LA. (ERIC Document Reproduction Service No. ED 295 630).

Carrier, C., Post, T. R. & Heck, W. (1985). Using microcomputers with fourth-grade students to reinforce arithmetic skills. Journal for Research in Mathematics Education, 16, 45-51.

Cates, W. M. (1985). A practical guide to educational research. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Cipra, B. A. (1988). Calculus: crisis looms in mathematics' future. Science, 239, 1491-1492.

Clute, P. S. (1984). Mathematics anxiety, instructional method, and achievement in a survey course in college mathematics. Journal for Research in Mathematics Education, 15(1), 50-58.

Crumb, G. H. & Monroe, E. E. (1988). CAI use by developmental studies students. Western Kentucky University, Center for Math, Science, and Environmental Education. (ERIC Document Reproduction Service No. ED 301 410).

Damarin, S. K. (1988, January). Issues of gender and computer assisted instruction. Paper presented at the Annual Meeting of the Association for Educational Communications and Technology, New Orleans, LA. (ERIC Document Reproduction Service No. ED 295 625).

Dick, T. P. & Balomenos, R. H. (1984, April). An investigation of calculus learning using factorial modeling. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED 245 933).

Drew, C. J. & Hardman, M. L. (1985). Designing and conducting behavioral research. New York: Pergamon Press.

Druva-Roush, C. A. & Wu, Z. J. (1989, August). Gender differences in visual Spatial Skills: A meta-analysis of doctoral theses. American Psychological Association. New Orleans, LA. (ERIC Document Reproduction Service No. ED 318 796).

Dugdale, S. (1981, August). Green globs: A microcomputer application for graphing of equations. (ERIC Document Reproduction Service No. ED).

Dwinell, P. L. & Higbee, J. L. (1989, March). The relationship of affective variables to student performance: Research findings. Paper presented at the

13th Annual Conference of the National Association of Developmental Education, Cincinnati, OH. (ERIC Document Reproduction Service No. 304614).

Dwyer, F. M. & Dwyer, C. A. (1989). Enhancing visualized instruction: A research overview. In J. Boca (Ed.) About Visuals: Research, Teaching and Applications. Readings From the 20th Annual Conference of the International Visual Literacy Association (pp. 117-127). Blacksburg, Virginia: Virginia Tech University.

Eisenberg, T. & Dreyfus, T. (1989). Spatial visualization in the mathematics curriculum. Focus on Learning Problems in Mathematics, 11(1), 1-5.

Elsner, T. E. (1983). The graphics microcomputer as a visual aid in the classroom. Collegiate Microcomputer, 1(1), 19-23.

Ethington, C. A. & Wolfle, L. M. (1984). Sex differences in a causal model of mathematics achievement. Journal for Research in Mathematics Education, 15, 371-377.

Fennema, E. & Sherman, J. A. (1986). Fennema-Sherman mathematics attitudes scales. Madison, WI: Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison. (Reprinted from JSAS Catalog of Selected Documents in Psychology, 1976, 6, 31. Ms. No. 1225).

Fennema, E. & Tartre, L. A. (1985). The use of spatial visualization in mathematics by girls and boys. Journal for Research in Mathematics Education, 16, 184-206.

Ferguson, G. A. & Takane, Y. (1989). Statistical analysis in psychology and education (6th ed.). New York: McGraw-Hill.

Ferrini-Mundy, J. (1987). Spatial training for calculus students: Sex differences in achievement and in visualization ability. Journal for Research in Mathematics Education, 18, 126-140.

Ganguli, A. B. (1990). The microcomputer as a demonstration tool for instruction in mathematics. Journal for Research in Mathematics Education, 21, 154-159.

George, Y. & Schaer, B. (1987, November). Program evaluation of a reading laboratory for a learning modality instructional approach. Paper presented at the Annual Meeting of the Mid-South Educational Research Association, Mobile, AL. (ERIC Document Reproduction Service No. ED 290 769).

Greenberg, R. L. (1991, January). Factors affecting achievement in algebra at a community college. Paper presented at the Annual Meeting of the Southwest Educational Research Association, San Antonio, TX. (ERIC Document Reproduction Service No. ED 327 261).

Greenfield, P. M. (1987). Electronic technologies, education, and cognitive development. In D. E. Berger, K. Pezdek, & W. P. Banks (Eds.). Applications of cognitive psychology: Problem solving, education, and computing. (pp. 17-32). Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Gronberg, S. M. H. (1987). Student achievement in solving systems of linear equations using traditional instruction and computer assisted instruction at the college level (Doctoral dissertation, The University of Texas at Austin. Dissertation Abstracts International, 48(05), 1140A.

Haase, H., Marion, R. & Mestre, J. (1985). Design features of pedagogically-sound software in mathematics. Amherst, MA: University of Massachusetts, Department of Physics and Astronomy. (ERIC Document Reproduction Service No. ED 290 627).

Harel, G. (1989). Learning and teaching linear algebra: Difficulties and an alternative approach to visualizing concepts and processes. Focus on Learning Problems in Mathematics, 11, 139-148.

Head, T. & Moore, M. (1989). The recall of quantitative data and graphic format: An experimental study. In J. Boca (Ed.) About Visuals: Research, Teaching and Applications. Readings From the 20th Annual Conference of the International Visual Literacy Association (pp. 201-206). Blacksburg, Virginia: Virginia Tech University.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. Journal for Research in Mathematics Education, 19, 3-25.

Hickernell, F. & Proskurowski, W. (1985) The use of microcomputers in the teaching of calculus. Collegiate Microcomputer, 3(2), 111-125.

Hinterthuer, R. J. (1984). The relationship of developmental college students' learning styles to computer assisted and programmed instruction. Unpublished doctoral dissertation, The University of Arkansas.

Iben, M. F. (1989, March). Cross-cultural investigations into student development of spatial relations and abstract mathematical thought: Some preliminary findings from Australia, Japan and the United States. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA (ERIC Document Reproduction Service No. ED 313 230).

Keefe, J. W. & Monk, J. S. (1986). Learning style profile examiner's manual. Reston, Va.: National Association of Secondary School Principals.

Keefe, J. W., Monk, J. S., Letteri, C. A., Languis, M. & Dunn, R. (1986). Learning style profile. Reston, Va.: National Association of Secondary School Principals.

Kerlinger, F. N. (1986). Foundations of behavioral research (3rd ed.). New York: Holt, Rinehart and Winston.

Keuper, S. M. (1985). An annotated bibliography of the effectiveness of the computer used as a tool to learn mathematics in secondary schools. Exit Research Project - S591. South Bend, IN: Indiana University. (ERIC Document Reproduction Service No. ED 257 679).

Kiser, L. (1987, November). Spatial-visual ability: can computer visualization facilitate achievement? Educational Technology, pp. 36-40.

Kleinfeld, J. & Nelson, P. (1988). Adapting instruction to native americans' "learning styles": An iconoclastic view. Fairbanks: University of Alaska. (ERIC Document Reproduction Service No. ED 321 952).

Kulik, J. A., Bangert, R. L. & Williams, G. W. (1983). Effects of computer-based teaching on secondary school students. Journal of Educational Psychology, 75(1), 19-26.

Kulik, J. A., Kulik, C. C. & Cohen, P. A. (1980). Effectiveness of computer-based college teaching: A meta-analysis of findings. Review of Educational Research, 50(4), 525-544.

Land, W. A. & Haney, J. J. (1989). The academic achievement of junior college students and computer assisted instruction. Educational Media Center Bureau of Educational Research and Evaluation, Mississippi State University, MS (ERIC Document Reproduction Service No. ED 317 191).

Lang, M. T. (1976). The effectiveness of using computer extended instruction to teach basic concepts of introductory calculus. Proceedings of the 1976 Association for Educational Data Systems International Convention (pp. 277-282). Phoenix, AZ.

Laughbaum, E. D. (1989). A graphical view of domain and range inequalities and equations. Columbus State Community College, Columbus, OH (ERIC Document Reproduction Service No. ED 315 307).

Mevarech, Z. R. & Rich, Y. (1985). Effects of computer-assisted mathematics instruction on disadvantaged pupils' cognitive and affective development. Journal of Educational Research. 79(1), 5-11.

Mevarech, Z. R. (1985). Computer-assisted instruction methods: a factorial study within mathematics disadvantaged classrooms. Journal of Experimental Education, 54(1), 22-27.

Meyer, M. R. (1989). Gender differences in mathematics. In M. M. Lindquist (Ed.), Results from the fourth mathematics assessment of the national assessment of educational progress. (pp. 149-159) Reston, VA: The National Council of Teachers of Mathematics, Inc.

Miller, C. A. (1988, April). Do left or right brain training exercises have the greater effect upon college calculus achievement? Paper presented at the annual meeting of The National Council of Teachers of Mathematics, Chicago, IL. (ERIC Document Reproduction Service No. ED 312 122).

Northwest Regional Educational Laboratory. (1985). The future of educational computers. Portland, OR; Computer Technology Program. (ERIC Document Reproduction Service No. ED 268 962).

Olson, J. & Eaton, S. (1986). Case studies of microcomputers in the classroom: Questions for curriculum and teacher education. Toronto, Ontario, Canada: The Ontario Institute for Studies in Education. (ERIC Document Reproduction Service No. ED 278 386).

Palmiter, J. R. (1991). Effects of computer algebra systems on concept and skill acquisition in calculus. Journal for Research in Mathematics Education, 22, 151-156.

Payton, J. N. (1987). The effects of selected computer software on achievement and attitude toward mathematics and computers of college students in basic mathematics (Doctoral dissertation, University of Virginia, 1987). Dissertation Abstracts International, 48, 2827-28A.

Piele, D. (1983). Computer-assisted mathematics. In M. T. Grady & J. D. Gawronski (Eds.). Computers in curriculum and instruction. (pp. 118-123). City: Association for Supervision and Curriculum Development.

Reglin, G. L. (1987). Effects of a computer assisted instruction on mathematics and locus of control. Unpublished manuscript. Charlotte, NC: University of North Carolina, Department of Curriculum and Instruction. (ERIC Document Reproduction Service No. ED 310 919).

Reglin, G. L. (1988). Effects of a computer assisted remediation program on basic skills mathematics achievement, academic self-concept, and locus of control of students in a selected dropout retrieval program in an urban setting. Unpublished doctoral dissertation, University of Florida. (ERIC Document Reproduction Service No. ED 317 423).

Reglin, G. L. (1989/90). CAI effects on mathematics achievement and academic self-concept seminar. Journal of Educational Technology Systems, 18(1), 43-48.

Reglin, G. L. & Butler, D. (1989). Effects of a computer assisted instruction EEE seminar on mathematics achievement and academic self-concept of students at a predominantly black college in a rural community in the south. Paper presented at the Annual Meeting of the Eastern Educational Research Association, Savannah, GA. (ERIC Document Reproduction Service No. ED 306 949).

Rhoads, C. (1986). The relationship between conditions and outcomes of microcomputer instruction. Journal of Computers in Mathematics and Science Teaching, 5(3), 48-50.

Sanger, R. C. (1977). A transfer level computer calculus sequence. The Two-Year College Mathematics Journal, 8(4), 216-218.

Schaalma, D. (1989, July). Gender differences in mathematics achievement: Do they exist? If so, what are their causes? S591--Exit Project. (ERIC Document Reproduction Service No. ED 309 958).

Schmitt, D. (1989). Methodological weaknesses with CAI research. Paper presented at the Annual Meeting of the Eastern Educational Research Association, Savannah, GA. (ERIC Document Reproduction Service No. ED 321 729).

Shapiro, R. J. (1987, September 7). The great jobs mismatch. U. S. News & World Report, pp. 42-44.

Shrum, J. L. (1985). The integration of modalities: Research and instructional implications. Unpublished manuscript. Virginia Tech, Division of Curriculum and Instruction and College of Education, Blacksburg, VA. (ERIC Document Reproduction Service No. ED 255 045).

Skemp, R. (1987). The psychology of learning math. (expanded American Ed.). Hillsdale, NJ : L. Erlbaum Associates.

SPSS Inc. (1991). SPSS release 4.1. Chicago: SPSS Inc.

Thomas, D. B. (1979). The effectiveness of computer-assisted instruction in secondary schools. AEDS Journal, 12, 103-116.

Vinner, S. (1989). The avoidance of visual considerations in calculus students. Focus of Learning Problems in Mathematics, 11(2), 149-156.

Waits, B. & Demana, F. (1990). Master grapher and 3D grapher. New York: Addison-Wesley Publishing Company.

Wright, J. K. (1989). Case study: Computer-enhanced college algebra. The Journal of Computers in Mathematics and Science Teaching, 8(4), 60-63.

APPENDICES

APPENDIX A

COMPUTER LABS AND HOMEWORK ASSIGNMENTS

COMPUTER LAB 1 - MASTER GRAPHER NAME:

The exercises given in this lab deal with the definition of the derivative and the product rule for finding derivatives. This lab is designed to illustrate concepts covered in the text in Chapter 3, sections 1 and 2.

I. Consider the function $f(x) = x^2/4$.

a). Graph this function using the Master Grapher program.

Enter in index 1: x^2/4

Remember to toggle the display of index 1 on by typing a 1 if it is not on already. Toggle any other indices off by typing their numbers.

Enter **9** to return to the previous menu and **G** to draw the graph.

b). The difference quotient $\frac{f(x + .1) - f(x)}{.1}$

gives a close approximation to the derivative.

Graph this function in index 2.

Enter: (((x + .1)^2)/4 - x^2/4)/.1

The value of the difference quotient for x = 2 looks to be 1 . The slope of the line tangent to the graph at the point (2,1) looks like it must be close to 1.

c). The derivative will give the slope of a function at a particular point. We can use our approximation to graph the line tangent to the graph at the point (2,1).

Enter in index 3:

((((2 + .1)^2)/4 - (2)^2/4)/.1)*(x-2) + 1

d). Solve for the derivative of f(x) and see how close the approximation is to the actual tangent line at the point (2,1) by obtaining the equation of the tangent line and graphing it in index 4.

II. Look at the graphs of the two functions:

$$f(x) = x^2 /4 , \quad \text{and} \quad f'(x) = x/2 .$$

Note that f'(x) is the derivative of f(x). What does f'(x) tell us about the function f(x)? It gives the slope of the line tangent to the function for any value of x. The slope of the tangent line at the point (6,9) is 3 , this means the graph of f(x) is increasing at the value x = 6.

For x = -2, f'(x) = -1, thus the slope of the tangent line is negative and goes down 1 for every 1 it goes to the right.

III. Graph the function $f(x) = 1/x$ and its derivative function
$f'(x) = -1/x^2$.

IV. Answer the following questions:

1). Is the graph of the derivative you approximated in I.b). similar to that of $f'(x) = x/2$ in II? If you used $h = .01$ instead of $h = .1$, do you think they would be closer?

____

2). For what values of x does the line tangent to the function
$f(x) = x^2/4$ have

positive slope?____________, negative slope?____________

3). For what values of x does the line tangent to the function
$f(x) = 1/x$ have

positive slope?____________, negative slope?____________

4). What does the graph of the derivative of a function tell you about the graph of that function in terms of its steepness, or slope?

____

____

____

____

____

V. Consider the product: $f(x)g(x) = (x^2/2 + 2)(x^3 - x)$.

To find the derivative of the product of these two functions we could multiply the two together and then use the "Power Rule", but let's not so that we can illustrate the "Product Rule".

a). Graph the product $f(x)g(x)$

Enter in index 1: (x^2/2 + 2)*(x^3 - x)

b). Note from the graph that f(x)g(x) is increasing for the values of x < -1 . This means that the slope of the lines tangent to the values of x < -1 should be positive.

c). Using the power rule, etc. find the derivative of f(x) and g(x) separately. Enter the product f′(x)g′(x) into index 2. Do not graph it at this time. Go on to d).

d). Using what you learned in the first part of this lab, graph the function $\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$

with an h = .1

Enter the entire line below in index 3:

```
(((x+.1)^2/2+2)*((x+.1)^3-(x+.1))-(x^2/2+2)*(x^3-x))/.1
```

Graph both index 2 and 3 by toggling them on.

e). Now take the derivative of f(x)g(x) using the "Product Rule" and graph it in index 4.

VI. Answer the following questions about your graphs:

1). Use words to describe the graph of the slope of the product f(x)g(x). Is it positive for some values of x and negative for others?

____

____

____

____

____

2). <u>True or False</u>: The derivative of the product is the product of the derivatives.

____

3). Write the product rule for taking the derivative of the product f(x)g(x):

$[f(x)g(x)]' =$

COMPUTER LAB 2 - MASTER GRAPHER NAME:____________________

The exercises given in this lab deal with the derivatives of composite functions and the chain rule for finding those derivatives. This lab is designed to illustrate concepts covered in the text in Chapter 3, section 3.5.

NOTE: ONCE YOU ARE IN MASTER GRAPHER, BUT BEFORE YOU BEGIN, SET THE SPEED OF THE GRAPHING TO 400 (TYPE **"S"** AND THEN ENTER **400** AND HIT RETURN), THEN CHANGE THE VIEWING WINDOW BY TYPING **"V"** AND THEN **"5"**, SET L = **-5**, R = **5**, B **-5**, AND T = **5**. RETURN TO PREVIOUS MENU AND ENTER **"F"** TO ENTER THE FUNCTIONS.

I. Graph the function $f(x) = \sqrt{x}$.

Enter in index 1: sqr(x)

Remember to toggle the display of index 1 on by typing a 1 if it is not on already. Toggle any other indices off by typing their numbers.

Enter **9** to return to the previous menu and **G** to draw the graph.

1). What is the domain of this function?________________

II. Graph the function $g(x) = -(x+1)(x-3)$ in index 2.

Enter: -(x+1)*(x-3)

2). What is the domain of g(x)?____________________

III. Consider the composite function f(g(x)).

What would you guess it's domain might be?_______________

Graph the composite function f(g(x)) in index 3. Turn off the display of indices 1 and 2.

Enter: sqr(-(x+1)*(x-3))

3). What is the domain of f(g(x))?____________________

4). Where does it appear that the graph of f(g(x)) is

increasing?________________, decreasing?_______________.

With these intervals in mind, sketch what the graph of the derivative function might look like on the set of axes at right.

IV. Take the derivative of $f(x)$ and $g(x)$.

5). They are:

$f'(x)$ = ________________ and $g'(x)$ = ________________

6). Find $f'(g'(x))$ = ______________________________

7). Find $f'(x)g'(x)$ = ______________________________

V. Graph $f'(g'(x))$.

8). Is the domain of this composition different than that of $f(g(x))$?______________________________

9). Does it appear that $f'(g'(x))$ would be a good choice for the derivative of $f(g(x))$?______________________

If not, why not?______________________________

VI. Graph $f'(x)g'(x)$.

10). Is the domain of this product different than that of $f(g(x))$?______________________________

11). Does it appear that $f'(x)g'(x)$ would be a good choice for the derivative of $f(g(x))$?______________________

If not, why not?______________________________

VII. Use the chain rule to find the derivative of $f(g(x))$.

12). What is it?______________________________

Graph $[f(g(x))]'$.

13). Is its domain different than that of $f(g(x))$?______

VIII. State the Chain Rule for finding the derivative of a composite function.

If the function $z = g(x)$ has the derivative $g'(x_o)$ at x_o, and the function $y = f(z)$ has the derivative $f'(z_o)$ at $z_o = g(x_o)$, then the composite function $f(g(x))$ is differentiable at $x = x_o$, and

14). $[f(g(x_o))]' =$ ____

COMPUTER LAB 3 - MASTER GRAPHER NAME:

The exercises given in this lab deal with The Mean Value Theorem and one of its consequences,(that two functions with the same derivative on an open interval must differ by only a constant on that interval). This lab is designed to illustrate concepts covered in the text in Chapter 4, section 4.1.

I. Enter the following functions into the indices indicated:

1. Enter: x^3/6+x^2/2-3*x/2+3

2. Enter: x^3/6+x^2/2-3*x/2

3. Enter: x^3/6+x^2/2-3*x/2-6

4. Enter: x^2/2+x-3/2

5. Enter: x^5/2-x^4/2-2*x^3+2*x/3+2

6. Enter: x^5/2-x^4/2+2*x/3+2

II. Graph the function in index 1, (toggle to display the function and toggle the others off). It looks like there may be two points where the slope of the tangent line is probably 0. The Mean Value Theorem can be used to prove that those two points must exist.

Graph the line y = 4 by entering the draw option, choosing a specified horizontal line, entering 4 and hitting return.

Enter: D 4 4 Return

Enter: 6 to return to previous menu

III. As can be seen from the graph, the line y = 4 intersects the graph of f(x) in three points, call the x-value of these three points a,b and c from left to right. The conditions necessary to apply the Mean Value Theorem have been met by these functions.

1. What are those conditions?

So, from the Mean Value Theorem, there must be two points, call them p and q, in the intervals (a,b) and (b,c), respectively, such that:

and

2. What then are the values of f′(p) and f′(q) ?

3. Type F and toggle the display of the function in index 5. Turn the display of index 1 off. Go back by entering 9 and regraph by entering G. How many points are there where the derivative appears to be zero?

4. Without graphing the function with index 6 above, how many points would you guess there are where the derivative appears to be zero? Why?

5. Look at the graph of the function in index 6. How many points
are there where the derivative would appear to be zero?

6. What is the derivative of the function in index 5 above?

7. What is the derivative of the function in index 6 above?

8. Compare the derivatives you found in questions 4 and 5. What conclusion can you draw about the functions in index 5 and index 6
from their derivatives?

IV. Look at the graphs of the functions in indices 1, 2 and 3 all at the same time. They all have the same derivative.

9. What is that derivative?

Display the function in index 4. Notice that it goes through the x-axis at the two points where the other functions have slopes of zero. This is the derivative of the three functions.

10. What conclusion can you draw about the three functions in indices 1, 2 and 3 from their derivative?

COMPUTER LAB 4 - MASTER GRAPHER NAME: ____________________

The exercises given in this lab deal with the first and second derivative tests. This lab is designed to illustrate concepts covered in the text in Chapter 4, section 4.2.

I. Enter the following functions into the indices indicated:

1. Enter: (x^3-6*x^2-36*x+216)/27

2. Enter: (3*x^2-12*x-36)/27

3. Enter: (6*x-12)/27

4. Enter: sin(x)+x

5. Enter: cos(x)+1

6. Enter: -sin(x)

II. Graph the function f(x). It is in index 1, (toggle it to be displayed and toggle the others off).

1. Where is the function in index 1 continuous? How do you know?

____

2. Sketch the graph of f(x) that the computer gives.

3. Indicate on the number line below where the slope of f(x) is positive by marking + above the line, where it is negative by marking -, and where it is zero or does not exist by marking the point.

```
<--|----|----|----|----|----|----|----|----|----|----|----|----|-->
      -10   -8   -6   -4   -2    0    2    4    6    8   10
```

Graph the function in index 2 on the same graph as 1 by using the OVERLAY option. Enter 0 and then enter 2. Do not rotate it, just hit RETURN .

This function is f′(x), the derivative of the function in index 1.

4. For what values of x is:

f′(x) < 0 ?________________, f′(x) > 0 ?____________________.

5. At what values is f′(x) = 0 ?_____________________________

6. From this information, a point x=c is a maximum if the slope

left of c is________________and right of c is________________

7. A point x=c is a minimum if the slope of the function to the

left of c is________________and right of c is________________

8. Using your answers in questions 6 and 7, give a brief explanation, in your own words, of the First Derivative Test.

____

____

III. Now overlay the graph in index 3 on the same graph as those of index 1 and index 2. This is f''(x), the second derivative of f(x), the function in index 1

9. Where is this function continuous? How do you know?

____

10. Is f''(-2) positive or negative?______________

Is f''(6) positive or negative?______________

11. If x=c is a maximum, then is f''(c) positive or negative?

____

12. If x=c is a minimum, then is f''(c) positive or negative?

____

13. Use questions 11 and 12 to give a brief explanation of the

second derivative test.______________________________

____

____

14. There are certain hypotheses that must be met for both the first and second derivative tests. What are these hypotheses?

____

IV. There are instances where the first derivative test is more useful than the second derivative test. Clear the screen and look at the graph of the function, g(x), in index 4. You will want to turn the display of index 1 off and 4 on to graph it. Graph the first derivative, g'(x), of this function. It is in index 5. Use the overlay procedure you used above.

15. Using the graphs of g(x) and g′(x), fill in the number line below like you did the one for f(x) previously, marking +, −, or a point where the slope of g(x) is positive, negative, or zero.

```
<--|----|----|----|----|----|----|----|----|----|----|----|----|-->
     -10   -8   -6   -4   -2    0    2    4    6    8   10
```

16. From the first derivative test, how many maximums or minimums are there for g(x)? Refer to your explanation above.

V. Graph the second derivative, g′′(x). It is in index 6. Use the overlay procedure you used above.

17. From the second derivative test, how many maximums or minimums are there for g(x)?

18. Give another function where the first derivative equals zero for some value of x that is not a maximum nor a minimum?

COMPUTER LAB 5 - MASTER GRAPHER NAME:

The exercises given in this lab deal with applications of the derivative to find the maxima and/or minima of functions. This lab is designed to illustrate concepts found in the text in Chapter 4, section 4.3.

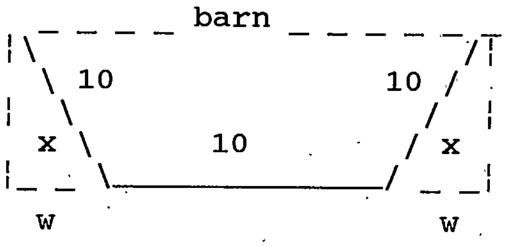
A critical step in finding the maxima and/or minima of a function is determining from the problem the function that must be maximized and/or minimized. One possible strategy is given in the following steps. You may use them or find your own. Eventually you should be able to do away with any such list, but that comes with experience in doing this type of problem.

To do maximum/minimum application problems one should follow steps similar to these:

1. Read the problem completely before starting.
2. Make a sketch of the problem if necessary.
3. Reread the problem identifying, and assigning names to, the variables and the quantity, or item, that is to be maximized or minimized.
4. Look for equations that relate the different variables to each other and a function that relates the item to be maximized or minimized to the variables.
5. Eliminate all but one variable from the function using the equation you found in step 4. Determine the domain of this function in relation to the problem.
6. Differentiate the function found in step 5 and identify all possible critical points, including endpoints.
7. Test the critical points to determine the maxima or minima.
8. Solve the problem and recheck your answer, making sure that it seems reasonable.

Once you have the function that needs to be maximized or minimized, MASTER GRAPHER can be used to determine where the maxima or minima are located, or if the answer that you get is reasonable.

Here is a problem:



A farmer wishes to build a pigpen along one side of her barn. She is going to have to fence three sides. The pigpen is going to be in the shape of an isosceles trapezoid Each side will be of equal length. If the total length of the three sides that she needs to fence is 30 feet with two of the sides meeting the barn side at an angle of x, what angle should x be so that the area of the pigpen is the greatest?

<u>First</u>: Read the problem completely.

<u>Second</u>: A sketch of the problem has already been given.

<u>Third</u>: Let **x** be the angle. Let **f(x)** be the area of the pigpen; this is the item to be maximized. Let **w** be half of the difference between the length of the barn and the length of the opposite side of the pigpen (see the figure).

<u>Fourth</u>: To determine the area, a formula for the area of a trapezoid is needed. The formula for the area of a trapezoid is: $A = h(b_1 + b_2)$, where A is the area, h is the height, b_1 is one base, and b_2 is the other base.

1). What is the function to be maximized?

f(x) = ______________________________

Now, relate x to the height and the bases. $b_1 = 10$ feet, and $b_2 = 10 + 2w$, which is the length of the barn side. (Using trigonometry, an angle can be related to the sides of a right triangle.)

2). What is the height of the trapezoid in terms of the angle x ?

___

3). What is the length of w in terms of angle x ?

___

Fifth: Now eliminate all variables except x in the function f(x).

4). What is the resulting f(x) ?

___

Simplifying should give:

$$f(x) = 200[\sin(x) + \sin(x)\cos(x)]$$

5). What is the domain of f(x) in terms of the angle x ?

___

Sixth: Differentiate f(x) .

6). What is the result?

___

Converting this to a function dealing with just cos(x) gives:

$$f'(x) = 200[\ 2\cos^2(x) + \cos(x) - 1]$$

7). Using the possible values of x and algebraic techniques, find the critical points for this function? (Hint: there are three of them.)

___

MASTER GRAPHER can now be used to give you a graph of the function f(x). From the graph you can tell whether the endpoints of the domain of x are reasonable answers or if there should be an answer at all.

Use MASTER GRAPHER to graph the function f(x). Once you are in MASTER GRAPHER, change the view by entering **V**, then **8** (which will switch the scale to $\pi/2$), then **5**. I suggest that you set:

L = 0	R = 3.142
B = -100	T = 300

Now go back to the previous menu and enter **F** to graph f(x). In index 1 enter:

200*(sin(x) + sin(x)*cos(x))

From the graph you can see that there is a maximum between 0 and π, and that the endpoints themselves are not maxima. You can also see that the maximum occurs about a third of the way from 0 to π. This should give you an idea of the values to check when using the first derivative test.

<u>Seventh</u>: Check critical points for maxima or minima.

8). What value of x will give a pigpen of maximum area?

____

9). How do you know that the value you found in question 8) is a maximum? (Looking at the graph is not proof, it is only a check.)

____

____

Your answer should be that the maximum area of the pigpen is $f(x) = 150(\sqrt{3})$, when $x = \pi/3$. That this is a maximum can be checked by using the first derivative test: $f'(\pi/4) > 0$, and $f'(\pi/2) < 0$. The graph goes up to the point $(\pi/3, 150(\sqrt{3}))$ and then goes down. Thus it reaches a maximum when $x = \pi/3$.

<u>Eighth</u>: Rechecking the problem. Just checking the graph shows that this value of x is a reasonable answer.

MASTER GRAPHER can also be used in some instances to make a sketch of a problem if none is given. Use it to make a sketch in the following problem. Remember to switch the settings back by entering **V**, then **6** ,for the default settings, and then **9** to return to the previous menu. Now enter **F**, and enter the function, etc.

Problem: Find the point, or points, on the graph of $y = \sqrt{x}$ that is closest to the point (1,0).

10). The variables are identified for you. But what is to be maximized or minimized?

____

11). Since you are to find the distance between two points, it seems likely that you will need the distance formula. What is the distance formula in terms of (x_1,y_1) and (x_2,y_2) ?

____

Pick an arbitrary point (x,y) on the graph of the function and find the distance from that point to the point (1,0).

12). What is the function we are to minimize?

f(x) = ______________________________

13). What function relates variables x and y ?

____

14). What is the function we are to minimized in terms of one variable?

____

15). What is the domain of this variable?

____

16). What is the f′(x) ?____

Use MASTER GRAPHER to graph f′(x), also.

17). What is(are) the critical point(s) for this function?

____

18). What is the value of x where this distance is a minimum?

____

19). How do you know that this answer is the minimum? Give the results of your test.

____

____

20). Does this answer seem reasonable? Why?

____

WORKSHEET 1 MATH 181 NAME: ______________________

The exercises given in this worksheet deal with the definition of the derivative and the product rule for finding derivatives. This lab is designed to give practice utilizing concepts covered in the text in Chapter 3, sections 1 and 2.

I. Consider the function $f(x) = x^2$

The derivative of f(x) is given by $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

a). Find the quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2$.

ans: ______________________

b). Using the definition of the derivative of a function, find the derivative of the function above.

ans: ______________________

c). For negative values of x, the derivative of f(x) is negative. What does this tell you about the function

$f(x) = x^2$?

ans: ______________________

II. Use the definition of the derivative as the limit of a difference quotient to find the derivative of the function

$$f(x) = \frac{x}{x-1}$$

a). What is the derivative of the function?

ans: ______________________________

b). What does this derivative tell you about the function f(x)?

ans: ______________________________

____

____

____

III. Consider the product: $f(x)g(x) = (x^2/2 + 2)(x^3 - x)$.

To find the derivative of the product of these two functions we could multiply the two together and then use the "Power Rule", but let's not so that we can illustrate the "Product Rule".

a). Using the power rule, etc. find the derivative of

$f(x) = (x^2/2 + 2)$ and $g(x) = (x^3 - x)$.

What is the derivative of f(x) ? ______________________________

What is the derivative of g(x) ? ______________________________

b). Multiply the two derivatives in a) together.

What is this product? ______________________________

If this were the derivative of the product of the two functions f(x) and g(x), what would you think the slope of the product would be doing for negative values of x?

ans: ______________________________

c). Using the product rule, find the derivative of f(x)g(x).

What is the slope of the product doing for negative values of x?

ans: ______________________________

WORKSHEET 2 MATH 181 NAME: ______________________

The exercises given in this worksheet deal with the derivatives of composite functions and the chain rule for finding those derivatives. This worksheet is designed to illustrate concepts covered in the text in Chapter 3, section 3.5.

I. Consider the functions: $f(x) = \sqrt{x}$ and $g(x) = -(x+1)(x-3)$

1). What is the domain of f(x)? ______________________

2). What is the domain of g(x)? ______________________

II. Consider the composite function f(g(x)).

3). What is the domain of f(g(x))? ______________________

III. Find the derivative of f(x) and g(x).

4). $f'(x) =$ ______________ and $g'(x) =$ ______________

5). Find $f'(g'(x)) =$ ______________________

6). What is the domain of this composition? ______________

7). Why might $f'(g'(x))$ be a poor choice for the derivative of f(g(x))? ______________________

IV. Find $f'(x)g'(x)$.

8). What is the domain of this product? ______________

9). Why might $f'(x)g'(x)$ be a poor choice for the derivative of f(g(x))? ______________________

V. Use the chain rule to find the derivative of f(g(x)).

10). What is it? ______________________

11). Is the domain of the derivative different than that of $f(g(x))$?____

VII. State the Chain Rule for finding the derivative of a composite function.

If the function $z = g(x)$ has the derivative $g'(x_o)$ at x_o, and the function $y = f(z)$ has the derivative $f'(z_o)$ at $z_o = g(x_o)$, then the composite function $f(g(x))$ is differentiable at $x = x_o$, and

12). $[f(g(x_o))]' =$ ____

WORKSHEET 3 MATH 181 NAME: ____________________

The exercises given in this lab deal with The Mean Value Theorem and one of its consequences,(that two functions with the same derivative on an open interval must differ by only a constant on that interval). This lab is designed to illustrate concepts covered in the text in Chapter 4, section 4.1.

I. Consider the function $f(x) = -x^3 + 5x^2 - 2x - 5$

f(x) is a polynomial and, from previous work, we know that polynomials are continuous over all of the real numbers. That means, for any closed bounded interval [a,b], f(x) is continuous on that interval.
We also know that the derivative of f(x) exists on any open interval (a,b).
So the conditions needed to apply the Mean Value Theorem are met by this function.

Look at the line g(x) = 3. This is a horizontal line that intersects f(x) in three points. You do not need to find these three points, but you may wish to check that the three points are (2,3), (-1,3), and (4,3).

Since f(x) and g(x) intersect at these three points, then the Mean Value Theorem says that there are values of x, call them c and d, in the intervals (-1,2) and (2,4) such that:

$$f'(c) = \frac{f(2) - f(-1)}{2 - -1} \qquad f'(d) = \frac{f(4) - f(2)}{4 - 2}$$

which results in

$$f'(c) = \frac{3 - 3}{2 - -1} = 0 \qquad f'(d) = \frac{3 - 3}{4 - 2} = 0$$

which means that there are at least two places where the derivative of f(x) is equal to zero.

II. Now, look at the function $h(x) = -x^3 + 5x^2 - 2x + 2$

This function differs from f(x) by only a constant. Look at f'(x) and h'(x). They are the same. That means that there must be at least two places where the derivative of h(x) equals zero. In fact, these two values are the same two places where f'(x) equals zero, since $f'(x) = h'(x)$.

You could find those two values of x by finding f'(x) and solving the equation $f'(x) = 0$ by any means possible, (i.e. the quadratic formula, completing the square, whatever).

III. Consider the function: $f(x) = -x^3 - 2x^2 + 11x + 10$ and the horizontal line $g(x) = -2$.

There is at least one point where these two function intersect and its x-value is an integer. Find it by setting

$f(x) = g(x) = -2$, or $f(x) - g(x) = f(x) + 2 = 0$

Here is a hint, if you find it, call it c, then factor (x - c) out of f(x) and then factor the result or use the quadratic formula to find any more.

The point, or points, of intersection are:

____

How many points are there where $f'(x) = 0$, if any?

____

If you found that there are points where $f'(x) = 0$, what are they?

____

Give one function that has the same derivative as f(x).

____

At how many points does the derivative of this new function equal zero, if any?

____

If there are any, what are they?

____

What conclusion, if any, can you make about two functions that have the same derivative, other than they have the same derivative? ______________________________

____

Note: If you think that this is true for all cubic polynomials functions, it is not. Look at the cubic polynomial function

$f(x) = x^3 + 2x^2 + 2x + 2$,it is different than those above.

WORKSHEET 4 MATH 181 NAME: ______________________

The exercises given in this lab deal with the first and second derivative tests. This lab is designed to illustrate concepts covered in the text in Chapter 4, section 4.2.

I. Consider the function

1. Where is f(x) continuous? How do you know?______________

____

2. What is f′(x) ?______________________________

3. For what values of x does f′(x) = 0 ?______________
(Hint: there are two points, call them a and b such that a < b)

4. Choose a point c such that c < a. Is f′(c) positive or negative?

____

5. Choose a point d such that a < d < b. Is f′(d) positive or negative?

____

6. Choose a point e such that b < e. Is f′(e) positive or negative?

____

7. What does the First Derivative test tell you about the point x=a?

____

How do you know?______________________________

8. What does the First Derivative test tell you about the point x=b?

____

How do you know?______________________________

9. What is $f''(x)$?______________________________

Where is $f''(x)$ continuous? How do you know?

____

10. Is $f''(a)$ positive or negative?__________________

Is $f''(b)$ positive or negative?__________________

11. Some point $x=c$ is a maximum if $f'(c) = 0$ and $f''(c)$ is

____

12. Some point $x=c$ is a minimum if $f'(c) = 0$ and $f''(c)$ is

____

13. Use questions 11 and 12 to give a brief explanation of the

second derivative test.______________________________

____

____

14. There are certain hypotheses that must be met for both the first and second derivative tests. What are these hypotheses?

____

____

II. There are instances where the first derivative test gives different results than the second derivative test. Consider the function:

15. What is $g'(x)$?______________________________

16. For what values of x is $g'(x)$ equal to zero?_____________

____

(Hint: there are many of them, just write a general solution)

17. For what values of x is $g'(x) < 0$?______________________

____

What does this say about maximum and minimum values of x?____

____

18. What is $g''(x)$?______________________________________

19. Are there values of x where $g''(x)$ is negative?, positive?, zero?

____

20. Give another example of a function for which there are values of x where the derivative is zero, but where the second derivative is also zero.

____

WORKSHEET 5 MATH 181 NAME:

The exercises given in this worksheet deal with applications of the derivative to find the maxima and/or minima of functions. This lab is designed to illustrate concepts found in the text in Chapter 4, section 4.3.

A critical step in finding the maxima and/or minima of a function is determining from the problem the function that must be maximized and/or minimized. One possible strategy is given in the following steps. You may use them or find your own. Eventually you should be able to do away with any such list, but that comes with experience in doing this type of problem.

To do maximum/minimum application problems one should follow steps similar to these:

1. Read the problem completely before starting.
2. Make a sketch of the problem if necessary.
3. Reread the problem identifying, and assigning names to, the variables and the quantity, or item, that is to be maximized or minimized.
4. Look for equations that relate the different variables to each other and a function that relates the item to be maximized or minimized to the variables.
5. Eliminate all but one variable from the function using the equation you found in step 4. Determine the domain of this function in relation to the problem.
6. Differentiate the function found in step 5 and identify all possible critical points, including endpoints.
7. Test the critical points to determine the maxima or minima.
8. Solve the problem and recheck your answer, making sure that it seems reasonable.

Here is a problem to work through:

A farmer wishes to build a pigpen along one side of her barn. She is going to have to fence three sides. The pigpen is going to be in the shape of an isosceles trapezoid Each side will be of equal length. If the total length of the three sides that she needs to fence is 30 feet with two of the sides meeting the barn side at an angle of x, what angle should x be so that the area of the pigpen is the greatest?

First: Read the problem completely.

Second: A sketch of the problem has already been given.

Third: Let **x** be the angle. Let **f(x)** be the area of the pigpen; this is the item to be maximized. Let **w** be half of the difference between the length of the barn and the length of the opposite side of the pigpen (see the figure).

Fourth: To determine the area, a formula for the area of a trapezoid is needed. The formula for the area of a trapezoid is: $A = h(b_1 + b_2)$, where A is the area, h is the height, b_1 is one base, and b_2 is the other base.

1). What is the function to be maximized?

f(x) = ______________________________

Relate x to the height and the bases. $b_1 = 10$ feet, and $b_2 = 10 + 2w$, which is the length of the barn side. (Use trigonometry to relate angel x to h and w.)

2). What is the height of the trapezoid in terms of the angle x ?

____

3). What is the length of w in terms of angle x ?

____

Fifth: Now eliminate all variables except x in the function f(x).

4). What is the resulting simplified f(x) ?

____

5). What is the domain of $f(x)$ in terms of the angle x ?

____

Sixth: Differentiate $f(x)$.

6). What is the result?______________________________

Using an identity gives: $f'(x) = 200[2\cos^2(x) + \cos(x) - 1]$

7). Using the possible values of x and algebraic techniques, find the critical points for this function? (Hint: there are three of them.)

____

Seventh: Check critical points for maxima or minima.

8). What value of x will give a pigpen of maximum area?

____

9). How do you know that the value you found in question 8) is a maximum?

____

____

Your answer should be that the maximum area of the pigpen is $f(x) = 150(\sqrt{3})$, when $x = \pi/3$.

Eighth: Rechecking the problem. $f(\pi/3) = 150(\sqrt{3})$ which is approximately 260 square feet, so $x = \pi/3$ appears to be a reasonable answer.

Here is a problem to do own your own.

Problem: Find the point, or points, on the graph of $y = \sqrt{x}$ that is closest to the point (1,0).

10). The variables are identified for you. But what is to be maximized or minimized?

____

11). Since you are to find the distance between two points, it seems likely that you will need the distance formula. What is the distance formula in terms of (x_1, y_1) and (x_2, y_2) ?

____

Pick an arbitrary point (x,y) on the graph of the function and find the distance from that point to the point (1,0).

12). What is the function we are to minimize?

f(x) = ____

13). What function relates variables x and y ?

____

14). What is the function we are to minimized in terms of one variable?

____

15). What is the domain of this variable?

____

16). What is the $f'(x)$?______________________________

17). What is(are) the critical point(s) for this function?

____

18). What is the value of x where this distance is a minimum?

____

19). How do you know that this answer is the minimum? Give the results of your test.

20). Does this answer seem reasonable? Why?

APPENDIX B

PRETREATMENT TEST

MATH 181 Diagnostic Test of Algebra Knowledge

Use a number 2 lead pencil and be sure to completely fill in the appropriate circle

On your answer sheet enter your:
Name: Last name, First namd, Middle initial
Student I.D. Number
Section Number: Enter this in the box marked Univ. Code

1). $\sqrt{x^2} =$

(a). $-x$ (b). $|x|$ (c). x (d). $\sqrt{x}$ (e). None of the others.

2). Solve: $-3x+\frac{5}{3}\le 6$

(a). $x\le -13$ (b). $x\ge -\frac{13}{9}$ (c). $x\ge -13$ (d). $x\le 7$ (e). None of the others.

3). Determine the slope of the line with the equation: $6x-2y=8$

(a). 6 (b). -4 (c). 3 (d). $\frac{1}{2}$ (e). None of the others.

4). Multiply and simplify: $\frac{w^2-2w-8}{w^2-9}\times\frac{w-3}{w-4}$

(a). $\frac{w-2}{w+3}$ (b). $\frac{w^2+2w-8}{w^2-w-12}$ (c). $\frac{w-2}{w-3}$ (d). $\frac{-2}{3}$ (e). None of the others.

5). Multiply: $(x+3)(x^2-2x+3)$

(a). x^3-6x+3 (b). x^2-x+6 (c). x^3+5x^2+9x+9
(d). x^3+x^2-3x+9 (e). None of the others.

6). Solving $2x^2+9x-5$ for x, gives $x=$

(a). 1,5 (b). $\frac{1}{2},-5$ (c). 0,5 (d). $\frac{1}{2},5$ (e). None of the others.

7). Evaluate $-\sqrt{25}$

(a). ± 5 (b). -5 (c). 5 (d). 5i (e). None of the others.

8). Simplify $\left(8^{\frac{4}{3}}\right)^{\frac{3}{2}}$

(a). $\sqrt[3]{16}$ (b). $\sqrt{2}$ (c). 64 (d). 4 (e). None of the others.

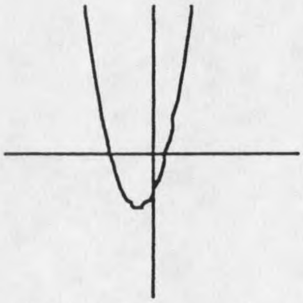
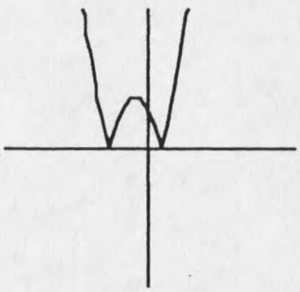
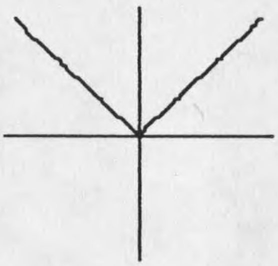
9). Find the distance between the points $(1,2)$ and $(-7,-4)$.

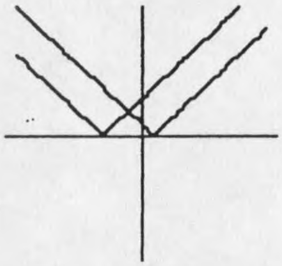
(a). -15 (b). 14 (c). 26 (d). 10 (e). None of the others.

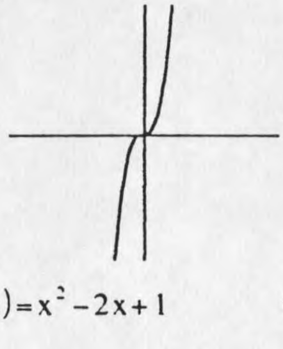
10). If triangle ABC is an equilateral triangle, then the measure of angle BAC is

(a). 45° (b). 90° (c). 180° (d). 60° (e). None of the others.

11). Which of the following is the graph of $y=\left|x^2+2x-3\right|$?

(a).  (b).  (c). 

(d).  (e). None of the others.

12). The graph  could be the graph of which function?

(a). $f(x)=x^2-2x+1$ (b). $f(x)=\sin(x)$ (c). $f(x)=x^3$

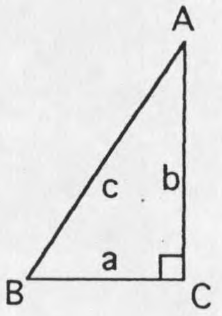
(d). $f(x)=x+7$ (e). None of the others.

13). $\frac{x^2}{4}+\frac{y^2}{9}=1$ is the equation of ...

(a). a circle (b). a hyperbola (c). an ellipse

(d). a parabola (e). None of the others

14). In the given right triangle, find cos(B)



(a). $\frac{b}{c}$ (b). $\frac{c}{a}$ (c). $\frac{c}{b}$ (d). $\frac{a}{c}$ (e). None of the others.

15). For what values of x is $f(x)=\frac{x^2+5x+6}{x^2-x-2}$ discontinuous?

(a). 0,6 (b). −3, − 2 (c). −1,2 (d). −5, 2 (e). None of the others.

16). If $f(x)=2x^2+3$ and $g(x)=x+2$, then for $x=-1$; $f(g(x))=$

(a). 21 (b). 6 (c). 5 (d). 7 (e). None of the others.

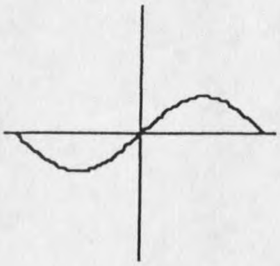
17). The slope of the function $f(x)=x^2-2$ at the point $(1,-1)$ is ...

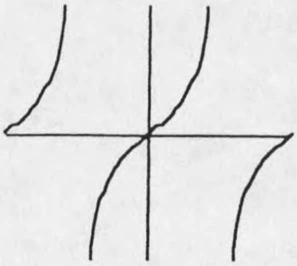
(a). 2 (b). −1 (c). 1 (d). 0 (e). None of the others.

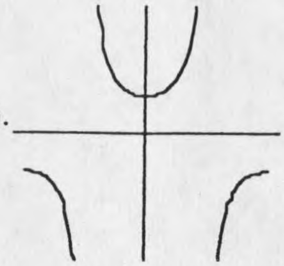
18). Given $f(x)=2x-3$, then $f^{-1}(4)=$

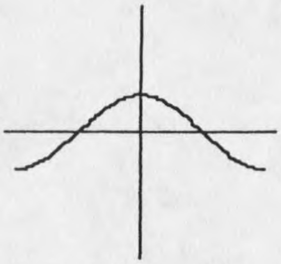
(a). $\frac{7}{2}$ (b). 2 (c). 5 (d). 14 (e). None of the others.

19). Which of these graphs is the picture of $\tan(\theta)$ on the interval $(-\pi, \pi)$?

(a).


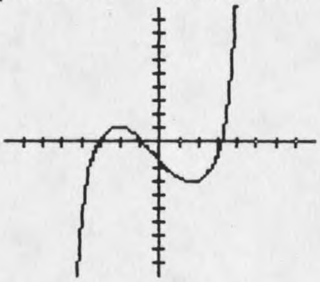
(b).


(c).


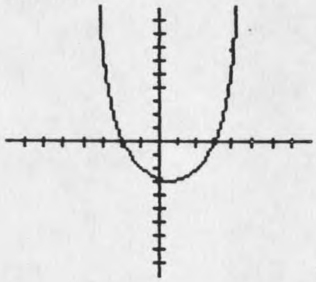
(d).


(e). None of the others.

20). If the graphs of the two functions $f(x)$ and $g(x)$ look like

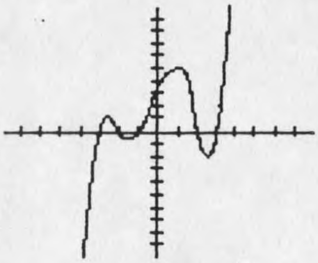


$f(x)$

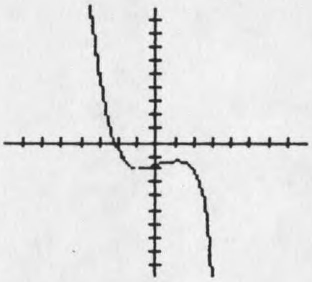


and $g(x)$

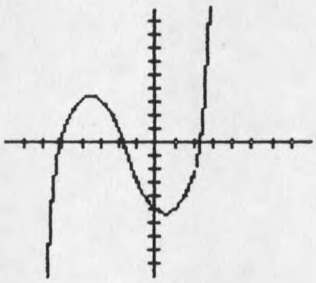
then the graph of $f(x)+g(x)$ would probably look like which of these ?



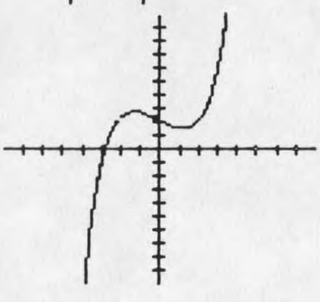
a).



(b).



(c).



(d).

(e). None of the others.

APPENDIX C

ATTITUDE SURVEYS

MATHEMATICS ATTITUDE SURVEY

This inventory is being used for research purposes only. It is being used to gather information about people's attitudes toward mathematics. Your responses will be kept confidential. Only the survey director will have access to this information.

BE SURE TO INCLUDE YOUR NAME AND ID NUMBER ON YOUR ANSWER SHEET
This is to assure future results can be matched to your responses.

Fill in the appropriate response to the following on your answer sheet.

I). Name: Enter your name; Last, first, M.I.
II). Student I.D. No.: Enter your I.D. number
III).Sex: Enter the appropriate information.
IV). Special Codes: enter your race as follows:
0 White
1 Native American
2 African Am./Black
3 Hispanic
4 Other

Please be sure to fill in your name and your student I.D. number on your answer sheet.

FILL IN THE APPROPRIATE CIRCLE ON YOUR ANSWER SHEET USING THE FOLLOWING SCALE:

(A)	(B)	(C)	(D)	(E)
strongly agree	agree	neither agree nor disagree	disagree	strongly disagree

1. It wouldn't bother me at all to take more math courses.
2. I'll need mathematics for my future work.
3. Being regarded as smart in mathematics would be a great thing.
4. Mathematics will not be important to me in my life's work.
5. I see mathematics as a subject I will rarely use in my daily life as an adult.
6. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.
7. I will use mathematics in many ways as an adult.
8. I am sure that I can learn mathematics.
9. Most subjects I can handle O.K., but I have a knack for flubbing up math.

10. Mathematics is a worthwhile and necessary subject.
11. I have a lot of self-confidence when it comes to math.
12. My mind goes blank and I am unable to think clearly when working math.
13. I'll need a firm mastery of mathematics for my future work.
14. I usually have been at ease during math tests.
15. I can get food grades in mathematics.
16. It would make people like me less if I were a really good math student.
17. I don't think I could do advanced mathematics.
18. Winning a prize in mathematics would make me feel unpleasantly conspicuous.
19. It would be really great to win a prize in mathematics.
20. Mathematics is of ;no relevance to my life.
21. People would think I was some kind of a grind if I got A's in math.
22. I am sure I could do advanced work in mathematics.
23. It would make me happy to be recognized as an excellent student in math.
24. I'd be happy to get top grades in mathematics.
25. I expect to have little use for mathematics when I get out of school.
26. Generally I have felt secure about attempting mathematics.
27. I usually have been at ease in math classes.
28. I haven't usually worried about being able to solve math problems.
29. In terms of my adult life it is not important for me to do well in mathematics.
30. A math test would scare me.
31. I think I could handle more difficult mathematics.
32. Mathematics makes me feel uneasy and confused.
33. I almost never have gotten shook up during a math test.
34. Math doesn't scare me at all.
35. Math has been my worst subject.
36. I'm not the type to do well in math.
37. Being first in a mathematics competition would make me pleased.
38. I'm no good in math.
39. For some reason even though I study, math seems unusually hard for me.
40. If I had good grades in math, I would try to hide it.
41. I'd be proud to be the outstanding student in math.
42. If I got the highest grade in math I'd prefer no one knew.
43. Taking mathematics is a waste of time.
44. Mathematics usually makes me feel uncomfortable and nervous.
45. Knowing mathematics will help me earn a living.
46. I don't like people to think I'm smart in math.
47. I study mathematics because I know how useful it is.
48. I get a sinking feeling when I think of trying hard math problems.

<u>Please be sure to fill in your name and your student I.D. number on your answer sheet.</u>

FILL IN THE APPROPRIATE CIRCLE ON YOUR ANSWER SHEET USING THE FOLLOWING SCALE:

(A)	(B)	(C)	(D)	(E)
strongly agree	agree	neither agree nor disagree	disagree	strongly disagree

1. I get a sinking feeling when I think of trying hard math problems.
2. I study mathematics because I know how useful it is.
3. I don't like people to think I'm smart in math.
4. Knowing mathematics will help me earn a living.
5. Mathematics usually makes me feel uncomfortable and nervous.
6. Taking mathematics is a waste of time.
7. If I got the highest grade in math I'd prefer no one knew.
8. I'd be proud to be the outstanding student in math.
9. If I had good grades in math, I would try to hide it.
10. For some reason even though I study, math seems unusually hard for me.
11. I'm no good in math.
12. Being first in a mathematics competition would make me pleased.
13. I'm not the type to do well in math.
14. Math has been my worst subject.
15. Math doesn't scare me at all.
16. I almost never have gotten shook up during a math test.
17. Mathematics makes me feel uneasy and confused.
18. I think I could handle more difficult mathematics.
19. A math test would scare me.
20. In terms of my adult life it is not important for me to do well in mathematics.
21. I haven't usually worried about being able to solve math problems.
22. I usually have been at ease in math classes.
23. Generally I have felt secure about attempting mathematics.
24. I expect to have little use for mathematics when I get out of school.
25. I'd be happy to get top grades in mathematics.
26. It would make me happy to be recognized as an excellent student in math.
27. I am sure I could do advanced work in mathematics.
28. People would think I was some kind of a grind if I got A's in math.
29. Mathematics is of ;no relevance to my life.
30. It would be really great to win a prize in mathematics.

31. Winning a prize in mathematics would make me feel unpleasantly conspicuous.
32. I don't think I could do advanced mathematics.
33. It would make people like me less if I were a really good math student.
34. I can get food grades in mathematics.
35. I usually have been at ease during math tests.
36. I'll need a firm mastery of mathematics for my future work.
37. My mind goes blank and I am unable to think clearly when working math.
38. I have a lot of self-confidence when it comes to math.
39. Mathematics is a worthwhile and necessary subject.
40. Most subjects I can handle O.K., but I have a knack for flubbing up math.
41. I am sure that I can learn mathematics.
42. I will use mathematics in many ways as an adult.
43. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.
44. I see mathematics as a subject I will rarely use in my daily life as an adult.
45. Mathematics will not be important to me in my life's work.
46. Being regarded as smart in mathematics would be a great thing.
47. I'll need mathematics for my future work.
48. It wouldn't bother me at all to take more math courses.

APPENDIX D

LEARNING MODE SURVEY

Department of Mathematics
Room 214 Wilson Hall
Montana State University
Bozeman, Montana 59717
November 20, 1990

Dr. Thomas Koerner
Director of Publications
National Association of Secondary School Principals
1904 Association Drive
Reston, Virginia 22091

Dear Dr. Koerner:

I am a doctoral candidate at Montana State University, Bozeman, Montana. I am currently conducting research utilizing computer graphics in first quarter calculus at the university. A recent addition to my research deals with identifying those students who learn better when visual methods are employed. To identify those students, I have discovered the Learning Style Profile(LSP) created by the task force for NASSP in 1986.

I would like permission to use the Perceptual Response Subscale to identify those students in both my experiment and control groups that tend to be more visually oriented. Because of time constraints and very limited resources I cannot afford to give the entire LSP to the over 300 subjects in my study. Your expeditious permission to use just this subscale of 20 items would be of great benefit to my work. I have a copy of the LSP, so you would not have to supply one.

Please fax to me your response and any pertinent details or constraints as soon as possible.

FAX # (406) 586-0396

Thank you very much.

My supervising professor for this project is Dr. Maurice Burke, who has signed below.

Sincerely,

Virgil G. Fredenberg

Maurice Burke

Dr. Maurice Burke

PERMISSION IS GRANTED FOR YOUR USE OF NASSP MATERIALS AS SPECIFIED ABOVE. THIS IS A ONE-TIME ONLY PERMISSION. FUTURE REQUESTS MUST BE REPEATED. PLEASE CREDIT MATERIAL APPROPRIATELY.

T. KOERNER, EDITOR
NASSP
11/30/90

The following is a self-test to determine if your primary mode of perception is visual, auditory, or emotive.

PLEASE FILL IN THE FOLLOWING INFORMATION:

Name:________________________I.D. Number:____________________

Section:______________________Instructor:_____________________

Circle **A** if you see a *picture*, **B** if you hear a *sound*, and **C** if you have a *feeling* about the word.

1.	SUMMER	A. Picture	B. Sound	C. Feeling
2.	CHICKEN	A. Picture	B. Sound	C. Feeling
3.	LIAR	A. Picture	B. Sound	C. Feeling
4.	BEAUTIFUL	A. Picture	B. Sound	C. Feeling
5.	FIVE	A. Picture	B. Sound	C. Feeling
6.	READ	A. Picture	B. Sound	C. Feeling
7.	BABY	A. Picture	B. Sound	C. Feeling
8.	ENEMY	A. Picture	B. Sound	C. Feeling
9.	STORY	A. Picture	B. Sound	C. Feeling
10.	OCEAN	A. Picture	B. Sound	C. Feeling
11.	DOWN	A. Picture	B. Sound	C. Feeling
12.	RUNNING	A. Picture	B. Sound	C. Feeling
13.	LAW	A. Picture	B. Sound	C. Feeling
14.	FRIEND	A. Picture	B. Sound	C. Feeling
15.	SWIM	A. Picture	B. Sound	C. Feeling
16.	POOL	A. Picture	B. Sound	C. Feeling
17.	GOD	A. Picture	B. Sound	C. Feeling
18.	KILL	A. Picture	B. Sound	C. Feeling
19.	HOUSE	A. Picture	B. Sound	C. Feeling
20.	HAPPY	A. Picture	B. Sound	C. Feeling

APPENDIX E

PILOT STUDY

INTRODUCTION

Introduction

Computer-assisted instruction has been shown to improve college students understanding of concepts in algebra classes. In 1987 Payton,(1987), found that college students enrolled in algebra classes utilizing computer-assisted instruction scored higher than those who were enrolled in traditional algebra classes. Likewise, Gronberg,(1987), found significantly higher gains were made by college students solving systems of linear equations when they made use of the computer's abilities in their college algebra classes. In a study completed by Kiser,(1986), computer-assisted instruction enhanced the students ability to visualize linear and absolute-valued inequalities. Heid,(1988), found that college students had a better understanding of the concepts that were covered when the calculus course they were taking was altered to utilize computer-assisted instruction. And, Ayers, et. al.,(1988), found that computer-assisted instruction resulted in significantly higher scores for college students learning about composite functions. This study attempted to answer some questions on how to incorporate more visualization into the first quarter calculus class at MSU without jeopardizing the integrity of the course.

These findings suggested that Computer-assisted instruction, CAI, could be utilized in the first quarter calculus course, denoted as Math 181 at Montana State University, to improve the achievement and success of students in that course. Of the over four hundred students registered to take first quarter calculus at Montana State University during a typical fall quarter, many complete the course with grades lower than a "C", or 2.00 on a scale where 4.00 is an "A", or withdraw from the class before they receive their grade, which is usually lower than 2.00. For this study students receiving such a grade, or withdrawing, will be classified as having been unsuccessful in the course. Students who are unsuccessful in the course may be forced to retake it in order to continue with the academic program they have chosen to pursue or change their program entirely. Such results can set the student back several quarters or, in some cases, a year or more.

Problem

The problem then becomes "how do we help these unsuccessful students improve their results without watering down the course or lengthening the time required to finish the calculus sequence?" To investigate one possible solution, I attempted to incorporate CAI into the course during Winter quarter, 1990. Without changing the course content or structure, I created computer labs that were designed specifically to coincide with the concepts that were being covered in Math 181. These labs attempted to give the students visual representation to the concepts being studied in the course without altering the course. The labs were to be completed outside of class as supplements to the homework normally assigned in a traditionally taught first quarter calculus course.

Questions addressed by this study

This study also addressed several questions that could arise from this type of research. Because students with better attitudes toward success in mathematics, more confidence in learning mathematics, and those with lower levels of mathematics anxiety perform better in mathematics classes, some of the questions that I attempted to answer dealt with these items. Other questions arose because the students in this study needed to use the computer to complete the labs. This lead me to attempt to answer questions about their attitude toward computers, their confidence in using the computer, their likability of computers, and how useful they thought computers were. Other questions that I attempted to answer dealt with whether the incorporation of guided computer-assisted instruction into a first quarter college calculus class in the form of graphing of functions and functional concepts results in more students receiving a grade of 2.00 or better, helps students perform better on the courses final exam, and results in higher grades for the students who complete the course?

The specific questions that this study attempted to address were:

1. How does computer-assisted instruction, CAI, affect the achievement of students in the first quarter college calculus course at Montana State University?
2. How does CAI affect the percent of students who successfully complete the first quarter of college calculus at Montana State University?
3. How does CAI affect the students' confidence in learning mathematics?
4. How does CAI affect the students' attitudes toward success in mathematics and calculus?
5. How does CAI affect the students' level of mathematics anxiety?
6. How does CAI affect the students' attitudes toward computers?
7. How does CAI affect the students' confidence in using computers?
8. How does CAI affect the students' likability of computers?
9. How does CAI affect how useful the students believe computers are?

Overview of procedures and treatments

To answer these questions six of the seven sections of Math 181 were used in this study. The seventh section was taught by the researcher, so it was excluded from the study. The six remaining sections were randomly assigned to three groups. Two were assigned to experiment group one, two were assigned to experiment group two, and the remaining two were to be the control group. The students in experiment group one were given six specially designed computer labs to complete during six weeks of the quarter, the students in experiment group two were given six worksheets requiring them to make their own graphs, and the students in the control group were given a treatment that consisted of the normal course and homework given in the course.

The computer labs consisted of functions that the students were to graph using computer software designed for that purpose. These students were then asked questions about the concepts that the graphs were illustrating. The worksheets consisted of functions that the students were to graph without the assistance of the computer and the software package. The students in experimental group two were to graph the functions on their own. These students were then asked to answer questions about the concepts that their graphs were attempting to illustrate. Those in the control group were not given any supplementary graphing exercises, but were given the homework traditionally assigned in a first quarter college calculus class.

Limitations and delimitations

There were some limitations and delimitations to this study. Some limitations were that the course could not be altered to better use the computer as a graphing tool, the instructor was free to score the labs or worksheets for grade as they saw fit, the attitude of the teachers toward the study, toward mathematics, and toward computers could not be controlled, the students in the study were those enrolled in Math 181 during Winter quarter, 1990, the time period was for one quarter (thirty-seven instructional days), and the exams given to the students in this study were created by the Math 181 course supervisor. Some of the delimitations were that many students in the study were repeating the course after obtaining unsatisfactory results during the same course given the previous quarter, the treatments of the two experimental groups consisted of six computer labs or six worksheets over a period of six weeks, there were only seven sections of Math 181 to include in the study, and the computer software used in the study was designed for IBM compatible computers only.

Summary

This study was an attempt to determine if guided computer-assisted instruction could improve student achievement in first quarter calculus as taught at Montana State University during Winter quarter, 1990. Two different treatments were given to two different experimental groups of Math 181 students. Each experimental group consisted of approximately 60 students. A third group of approximately 60 students in Math 181 was used as the control group. The first experimental group received a treatment consisting of six computer labs designed to graphically illustrate the calculus concepts being covered in the class. The second experimental group received six worksheets where the students created their own graphs. The control group received only the instruction and homework found in the traditionally taught course.

PROCEDURES USED IN THIS STUDY

Introduction

During Winter quarter, 1990, students in six sections of Math 181 at Montana State University were randomly assigned to one of the three groups, experimental group one, experimental group two, or the control group. Those students in experimental group one received the computer labs, those in experimental group two received the worksheets, and those in the control group did not receive any treatment. All three groups were surveyed prior to the beginning of treatments and, again, after the treatments were completed to determine if there were any changes in their attitudes towards mathematics and computers. Many students were questioned after the completion of the treatments to elicit their feelings about the calculus course and the treatments.

The treatment for experimental group one consisted of six guided, computer-assisted labs designed by the researcher to illustrate some of the major concepts covered in the course. The labs were designed to closely follow the course outline created by the course supervisor. The students were expected to complete the labs on their own outside of class. The individual instructors graded and scored the labs as they saw fit.

The treatment for experimental group two consisted of six worksheets designed by the researcher to illustrate the same concepts that the computer labs did. The students were to create the graphs by hand and complete the worksheets from those graphs. These worksheets were graded by the instructors and scored similarly to the computer labs of experimental group one.

The control group did not receive any specially designed assignments or homework supplements. They completed only the homework that was assigned by their instructors.

Each lab and each corresponding worksheet were given to the students early in the week, on a Monday or a Tuesday, and collected the following Friday.

All sections of Math 181, including those in the two experimental groups and in the control group, were given three common hour exams and a final exam. All sections took the same exams at the same time. The first exam was given after the first two treatments of the labs and worksheets, the second after the third and fourth treatments, and the third exam was given after treatments five and six. The final was given approximately two weeks after the third common hour exam.

A student's final course grade was made up of the number of points they accumulate by their performance in all aspects of the course. Each common hour exam was worth one hundred points with the final exam worth two hundred points. In addition to the exam scores, one hundred points was given by each individual instructor for homework, quizzes, and the labs or worksheets if they wanted to count them.

Before the treatments were started and after the third common hour exam the students in the Math 181 sections involved in the experiment were given two surveys. One of the surveys was designed to assess the students' mathematics attitudes and the other was designed to assess the students' computer attitudes as previously described to determine if those attitudes changed due to the treatments.

The differences between the first, second, third, and final common hour exams' scores and course grades between the three groups were analyzed using one-way analysis of variance methods. The differences in the information in the surveys was also analyzed for any significant changes that occur.

Design of surveys

The Fennema-Sherman Mathematics Attitudes Scales, FSMAS, survey developed by E. Fennema and J. A. Sherman to assess attitudes about mathematics and the Survey Of Attitudes About And Working With Computers, SAAWC, survey developed by B. H. Loyd and C. P. Gressard for computer attitudes were used to survey the students in this study (Appendix A). Each of these surveys consisted of several parts which assessed different aspects of students attitudes toward mathematics and computers.

The FSMAS consists of nine parts. Each part measures a different item. The nine items deal with mathematics confidence, parental attitudes, teacher attitude, student's attitude, student's view of mathematical roles, mathematical anxiety, motivation, and usefulness of mathematics. The items used in the survey of mathematics attitudes used in this study were confidence in doing mathematics, attitude toward success in mathematics, and mathematical anxiety.

The SAAWC consists of four parts dealing with computer anxiety, computer confidence, how well the student likes computers, and how useful the student feel computers are. All four of these subsections were used to survey the students in this study about their attitudes toward computers.

The information obtained from the surveys was scored for each student by given the answers a numerical value based on the Likert type scales and scores given with the surveys themselves. These scores were then entered into the computer and analyzed.

Design of treatments

The treatment design for experimental group one was that of a computer lab (Appendix B). The students were given a one or two page list of instructions to carry out with the assistance of the computer function graphing program Master Grapher. The computer software graphed the functions that the students entered as per the instructions that were given in the lab. The students were then asked to answered questions about the graphs that the computer displayed. The students were then asked to make inferences from those graphs. These inferences were to illustrate particular mathematical concepts that were being learned. The treatment design for experimental group two was that of a worksheet designed to have the students graph selected functions and draw inferences from those graphs similar to those for experimental group one (Appendix B). The control group was not given any special treatment.

The topics that the labs and worksheets were to illustrate were:

Lab 1 covered the concepts of domain and range of a function, x- and y-intercepts of a function, absolute value, reciprocals of functions, and discontinuity.

Lab 2 covered the concepts of limits and equality of functions. It also was designed to help the student to determine what seemingly simple changes to a function may do to the limit at a point. It was also designed to help them get an intuitive idea of what a limit is and how it relates to the concept of continuity.

Lab 3 covered the concept of continuity. It was designed to give the students some different types of discontinuous functions to analyze graphically. From these graphs the student needed to find the actual discontinuity and classify it. This lab also gave the students some work with Bolzano's Theorem (one way of determining the roots of an equation).

Lab 4 covered the use of the definition of the derivative to find the slope of the tangent line of a graph at a point. It gave the student a visual representation of the function and the tangent line so that they could get an intuitive idea of what the slope of a tangent line might show. It also gave them a visual representation of how one can determine a maximum or minimum point from the slope of the tangent line at that point and the points to either side.

Lab 5 covered using the derivative to demonstrate all of the information that can be found from analyzing the graph of the derivative. The preliminaries to the first derivative test were illustrated to give the students an intuitive idea of that test. In addition, the student was given some illustrative work with composite functions.

Lab 6 covered the First Derivative Test and the Second Derivative Test. The students used the graphical representations of the functions and the first and second derivatives of those functions to analyze and determine characteristics of those functions. From that analysis they were to determine information about those functions and what information might be given by the first and second derivatives.

<u>Questions asked of students after treatments</u>

After the completion of the treatments several students were questioned individually by the researcher concerning their attitudes toward the labs and the treatments. The questions they were asked included the following:

1. Did you complete all of the labs, or worksheets?

2. Did you feel the labs, or worksheets were too simple or too difficult?

3. Could you tell the concepts the labs, or worksheets, were trying to illustrate?

4. Were the labs, or worksheets helpful to you?

5. What suggestions would you give to improve this course, assuming that it needs to be improved?

STATISTICAL ANALYSIS

Hypotheses

1. Students who receive CAI do not have a higher rate of success in first quarter college calculus than students who receive traditional homework.

2. Students who receive CAI do not have a higher rate of success in first quarter college calculus than students who do similar work by hand.

3. Students who receive CAI do not score higher on exams than students who receive traditional homework.

4. Students who receive CAI do not score higher on exams than students who do similar work by hand.

5. Students who receive CAI do not achieve higher grades in the course than students who receive traditional homework.

6. Students who receive CAI do not achieve higher grades

in the course than students who do similar work by hand.

7. There is no difference in the change in student attitude toward mathematics between students who receive CAI and students who receive traditional homework.

8. There is no difference in the change in student attitude toward mathematics between students who receive CAI and students who do similar work by hand.

9. There is no difference in the change in the level of mathematics anxiety between students who receive CAI and students who receive traditional homework.

10. There is no difference in the change in the level of mathematics anxiety between students who receive CAI and students who do similar work by hand.

11. There is no difference in the change in student confidence to do mathematics between students who receive CAI and students who receive traditional homework.

12. There is no difference in the change in student confidence to do mathematics between students who receive CAI and students who do similar work by hand.

13. There is no difference in the change in student attitude toward computers between students who receive CAI and students who receive traditional homework.

14. There is no difference in the change in student attitude toward computers between students who receive CAI and students who do similar work by hand.

15. There is no difference in the change in student

confidence in using computers between students who receive CAI and students who receive traditional homework.

16. There is no difference in the change in student confidence in using computers between students who receive CAI and students who do similar work by hand.

17. There is no difference in the change in student likability of computers between students who receive CAI and students who receive traditional homework.

18. There is no difference in the change in student likability of computers between students who receive CAI and students who do similar work by hand.

19. There is no difference in the change in student perception of the usefulness of computers between students who receive CAI and students who receive traditional homework.

20. There is no difference in the change in student perception of the usefulness of computers between students who receive CAI and students who do similar work by hand.

Hypotheses to be Tested

1. $H_{o1} : S_{e1} \leq S_{e2} \leq S_c$

   $H_{a1} : S_{e1} > S_{e2} > S_c$

S_{e1} is the percentage of students finishing the course who received a grade of 2.00 in experimental group one.

S_{e2} is the percentage of students finishing the course who received a grade of 2.00 in experimental group two.

S_c is the percentage of students finishing the course who received a grade of 2.00 in the control group.

2. H_{o2} : $MX1_{e1} \leq MX1_{e2} \leq MX1_c$

H_{a2} : $MX1_{e1} \rangle MX1_{e2} \rangle MX1_c$

$MX1_{e1}$ is the mean score on the first common hour exam of experimental group one.

$MX1_{e2}$ is the mean score on the first common hour exam of experimental group two.

$MX1_c$ is the mean score on the first common hour exam of the control group.

3. H_{o3} : $MX2_{e1} \leq MX2_{e2} \leq MX2_c$

H_{a3} : $MX2_{e1} \rangle MX2_{e2} \rangle MX2_c$

$MX2_{e1}$ is the mean score on the second common hour exam of experimental group one.

$MX2_{e2}$ is the mean score on the second common hour exam of experimental group two.

$MX2_c$ is the mean score on the second common hour exam of the control group.

4. H_{o4} : $MX3_{e1} \leq MX3_{e2} \leq MX3_c$

H_{a4} : $MX3_{e1} \rangle MX3_{e2} \rangle MX3_c$

$MX3_{e1}$ is the mean score on the second common hour exam of experimental group one.

$MX3_{e2}$ is the mean score on the second common hour exam of experimental group two.

$MX3_c$ is the mean score on the second common hour exam of the control group.

5. H_{o5} : $MFX_{e1} \leq MFX_{e2} \leq MFX_c$

H_{a5} : $MFX_{e1} \rangle MFX_{e2} \rangle MFX_c$

MFX_{e1} is the mean score on the common hour final exam of experimental group one.

MFX_{e2} is the mean score on the common hour final exam of experimental group two.

MFX_c is the mean score on the common hour final exam of the control group.

6. H_{o6} : $G_{e1} \leq G_{e2} \leq G_c$

H_{a6} : $G_{e1} \rangle G_{e2} \rangle G_c$

G_{e1} is the mean average course grade of experimental group one.

G_{e2} is the mean average course grade of experimental group two.

G_c is the mean average course grade of the control group.

7. H_{o7} : $MAtt_{e1} \leq MAtt_{e2} \leq MAtt_c$

H_{a7} : $MAtt_{e1} \rangle MAtt_{e2} \rangle MAtt_c$

$MAtt_{e1}$ is the mean score of experimental group one on the attitude toward mathematics portion of the survey.

$MAtt_{e2}$ is the mean score of experimental group two on the attitude toward mathematics portion of the survey.

$MAtt_c$ is the mean score of the control group on the attitude toward mathematics portion of the survey.

8. H_{o8} : $MAnx_{e1} \leq MAnx_{e2} \leq MAnx_c$

H_{a8} : $MAnx_{e1} \rangle MAnx_{e2} \rangle MAnx_{c}$

$MAnx_{e1}$ is the mean score of experimental group one on the mathematics anxiety portion of the survey.

$MAnx_{e2}$ is the mean score of experimental group two on the mathematics anxiety portion of the survey.

$MAnx_{c}$ is the mean score of the control group on the mathematics anxiety portion of the survey.

9. H_{o9} : $MC_{e1} \leq MC_{e2} \leq MC_{c}$

H_{a9} : $MC_{e1} \rangle MC_{e2} \rangle MC_{c}$

MC_{e1} is the mean score of experimental group one on the confidence in doing mathematics portion of the survey.

MC_{e2} is the mean score of experimental group two on the confidence in doing mathematics portion of the survey.

MC_{c} is the mean score of the control group on the confidence in doing mathematics portion of the survey.

10. H_{o10} : $CAtt_{e1} \leq CAtt_{e2} \leq CAtt_{c}$

H_{a10} : $CAtt_{e1} \rangle CAtt_{e2} \rangle CAtt_{c}$

$CAtt_{e1}$ is the mean score of experimental group one on the attitude toward computers portion of the survey.

$CAtt_{e2}$ is the mean score of experimental group two on the attitude toward computers portion of the survey.

$CAtt_{c}$ is the mean score of the control group on the attitude toward computers portion of the survey.

11. H_{o11} : $CCon_{e1} \leq CCon_{e2} \leq CCon_{c}$

H_{a11} : $CCon_{e1} \rangle CCon_{e2} \rangle CCon_{c}$

$CCon_{e1}$ is the mean score of experimental group one on the confidence in using computers portion of the survey.

$CCon_{e2}$ is the mean score of experimental group two on the confidence in using computers portion of the survey.

$CCon_{c}$ is the mean score of the control group on the confidence in using computers portion of the survey.

12. H_{o12} : $CL_{e1} \leq CL_{e2} \leq CL_{c}$

    H_{a12} : $CL_{e1} \rangle CL_{e2} \rangle CL_{c}$

CL_{e1} is the mean score of experimental group one on the likability of computers portion of the survey.

CL_{e2} is the mean score of experimental group two on the likability of computers portion of the survey.

CL_{c} is the mean score of the control group on the likability of computers portion of the survey.

13. H_{o13} : $CU_{e1} \leq CU_{e2} \leq CU_{c}$

    H_{a13} : $CU_{e1} \rangle CU_{e2} \rangle CU_{c}$

CU_{e1} is the mean score of experimental group one on the usefulness of computers portion of the survey.

CU_{e2} is the mean score of experimental group two on the usefulness of computers portion of the survey.

CU_{c} is the mean score of the control group on the usefulness of computers portion of the survey.

RESULTS OF STUDY

All hypotheses were tested at a significance level of $\alpha=0.05$. After entering all of the data into a spreadsheet in the computer, the following means were given:

S_{e1} = 47.37%, S_{e2} = 36.84%, S_c = 34.92%;

$MX1_{e1}$ = 61.74, $MX1_{e2}$ = 60.88, $MX1_c$ = 59.17;

$MX2_{e1}$ = 60.81, $MX2_{e2}$ = 55.95, $MX2_c$ = 58.65;

$MX3_{e1}$ = 65.19, $MX3_{e2}$ = 60.17, $MX3_c$ = 61.20;

MFX_{e1} = 142.93, MFX_{e2} = 133.10, MFX_c = 132.29; and

G_{e1} = 1.53, G_{e2} = 1.32, G_c = 1.16.

The following values refer to the mean gains in the areas of the surveys.

$MAtt_{e1}$ = 2.58, $MAtt_{e2}$ = 0.35, $MAtt_c$ = 0.39;

$MAnx_{e1}$ = 1.01, $MAnx_{e2}$ = -1.38, $MAnx_c$ = -2.96;

MC_{e1} = 1.27, MC_{e2} = -1.15, MC_c = -1.05;

$CAtt_{e1}$ = 0.22, $CAtt_{e2}$ = -0.16, $CAtt_c$ = -1.10;

$CCon_{e1}$ = -0.39, $CCon_{e2}$ = -0.33, $CCon_c$ = -0.77;

CL_{e1} = 0.45, CL_{e2} = -1.31, CL_c = -0.69; and

CU_{e1} = -0.33, CU_{e2} = -0.04, CU_c = -0.78.

These results appear to be encouraging, although only minor results were actually significant when Analysis of Variance was used to analyze these means. A plotting of the data showed that it was normally distributed, so Analysis of Variance is an adequate procedure to use for analysis in this case. The computer package MSUSTAT was employed to compare these means for significance at an $\alpha=0.05$.

Experimental group one, those students utilizing CAI, had a higher success rate and a higher average grade than did the other two groups in the study, but this result was not significant and hypotheses $H_{o1} : S_{e1} \leq S_{e2} \leq S_c$ and $H_{o6} : G_{e1} \leq G_{e2} \leq G_c$ could not be rejected. Experimental group one also had higher average scores on all three exams and the final exam, but, again, these results were not significant at the level set and the hypotheses

$H_{o2} : MX1_{e1} \leq MX1_{e2} \leq MX1_c$, $H_{o3} : MX2_{e1} \leq MX2_{e2} \leq MX2_c$, $H_{o4} : MX3_{e1} \leq MX3_{e2} \leq MX3_c$, and $H_{o5} : MFX_{e1} \leq MFX_{e2} \leq MFX_c$ could not be rejected.

However, there were two areas where the gains in the mean scores of the students in experimental group one were significant. Those were in the students' attitudes toward mathematics and the students' likability of computers. Thus, $H_{o7} : MAtt_{e1} \leq MAtt_{e2} \leq MAtt_c$ and $H_{o12} : CL_{e1} \leq CL_{e2} \leq CL_c$ were rejected, and

$H_{a7} : MAtt_{e1} \rangle MAtt_{e2} \rangle MAtt_c$ and $H_{a12} : CL_{e1} \rangle CL_{e2} \rangle CL_c$ were retained. The p-values obtained from the analyses were .0281 and .0399 , respectively.

None of the other results were determined to be significant at the level that was set, so all of the null hypotheses were retained and the alternative hypotheses were rejected.

Comments on the questions asked after treatments

The students' answers to those questions asked by the researcher after the completion of the treatments did not give much insight into the treatments. It did confirm the concern that some of the students did not complete all of the labs, or worksheets. More than 75% of those asked said that they did complete all of the worksheets, or labs. Only one individual thought some of labs were too difficult and only one thought some were too simple. All said they could tell which concepts were being illustrated by the labs and that they found them helpful in general.

Other comments about the labs and ways to improve the course were: they preferred more work using the computer and less analysis of the concepts being illustrated; they would like to be given additional problems that they could work, they found it beneficial to do as many problems as possible; perhaps video tapes of problems being worked that could be placed in the library so that the students could view them and review them; and perhaps the material is covered too quickly and should be gone over at a slower pace.

INTERPRETATION OF RESULTS, COMMENTS, AND RECOMMENDATIONS

The lack of significance of most of the differences in means might be attributable to the lack of control of several variables that appeared once the study was begun. One variable that could not be controlled was that the individual instructors each weighted the labs and worksheets by their own scale. Some even made completing the labs voluntary. This meant that several students did not complete all or part of the labs and worksheets. This was caused partly by the researcher wanting to have as little impact upon the classes and course as possible. Another variable that could not be controlled was that of the number of students that dropped the course once the treatments had begun. This meant that the number of students in each group decreased during the treatments, some more than others.

The higher percentage of students who succeeded in the course and the higher average grade of the students in experimental group one, those utilizing CAI, is encouraging even though the results could not be shown to be statistically significant. The researcher feels that this study was successful in that it showed CAI could be used to enhance the course by improving students' attitude toward mathematics and their likability of computers.

From the questions asked of a sample of the students after the completion of the treatments, it appears to the researcher that the CAI labs were helpful but that the students might benefit from other forms and/or more similar forms of CAI.

More research should be conducted into the use of the computer to improve student achievement in Math 181 at Montana State University. Perhaps a course altered to utilize CAI as much as possible and to the best advantage could be developed and studied to determine how beneficial CAI would be to Math 181, first quarter calculus, here at Montana State University.

REFERENCES CITED

REFERENCES CITED

Ayers, T., Davis, G., & Lewin, P. (1988). Computer experiences in learning composition of functions. Journal for Research in Mathematics Education, 19, 246-259.

Fennema, E. & Sherman, J. A. (1986). Fennema-Sherman mathematics attitudes scales. Madison, WI: Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison. (Reprinted from JSAS Catalog of Selected Documents in Psychology, 1976, 6, 31. Ms. No. 1225).

Gronberg, S. M. H. (1987). Student achievement in solving systems of linear equations using traditional instruction and computer assisted instruction at the college level (Doctoral dissertation, The University of Texas at Austin. Dissertation Abstracts International, 48(05), 1140A.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. Journal for Research in Mathematics Education, 19, 3-25.

Kiser, L. (1987, November). Spatial-visual ability: can computer visualization facilitate achievement? Educational Technology, pp. 36-40.

Loyd, B., & Gressard, C. (1984). Reliability and factorial validity of computer attitude scales. Educational and Psychological Measurement, 44, 639-646.

Payton, J. N. (1987). The effects of selected computer software on achievement and attitude toward mathematics and computers of college students in basic mathematics (Doctoral dissertation, University of Virginia, 1987). Dissertation Abstracts International, 48, 2827-28A.

Waits, B. & Demana, F. (1990). Master grapher and 3D grapher. New York: Addison-Wesley Publishing Company.

APPENDICES

Appendix A

Surveys Used to Obtain Information About Students Attitudes

MATHEMATICS ATTITUDE SURVEY AND ANSWER SHEET

THIS INVENTORY IS BEING USED FOR RESEARCH PURPOSES ONLY THE SURVEY DIRECTOR WILL KNOW WHAT YOUR RESPONSES ARE.

NAME(optional): ______________________ ID #: ______________________

Please fill in the appropriate circle.

SA - Strongly Agree with the statement.
A - Agree but with reservations.
N - Neither agree not disagree with the statement.
D - Disagree with the statement.
SD - Strongly Disagree with the statement.

Work fast but carefully. Be sure to answer every question. There are no "right" or "wrong" answers. The only correct responses are those that are true for you.

SA	A	N	D	SD	
( )	( )	( )	( )	( )	1. It wouldn't bother me at all to take more math courses.
( )	( )	( )	( )	( )	2. I'm not the type to do well in math.
( )	( )	( )	( )	( )	3. For some reason even though I study, math seems unusually hard for me.
( )	( )	( )	( )	( )	4. Being regarded as smart in mathematics would be a great thing.
( )	( )	( )	( )	( )	5. Math doesn't scare me at all.
( )	( )	( )	( )	( )	6. It would be really great to win a prize in mathematics.
( )	( )	( )	( )	( )	7. I'd be happy to get top grades in mathematics.
( )	( )	( )	( )	( )	8. If I had good grades in math, I would try to hide it.
( )	( )	( )	( )	( )	9. I'd be proud to be the outstanding student in math.
( )	( )	( )	( )	( )	10. If I got the highest grade in math I'd prefer no one knew.
( )	( )	( )	( )	( )	11. I think I could handle more difficult mathematics.
( )	( )	( )	( )	( )	12. I'm no good in math.
( )	( )	( )	( )	( )	13. Most subjects I can handle O.K., but I have a knack for flubbing up math.
( )	( )	( )	( )	( )	14. Being first in a mathematics competition would make me pleased.
( )	( )	( )	( )	( )	15. I usually have been at ease in math classes.
( )	( )	( )	( )	( )	16. Mathematics makes me feel uneasy and confused.
( )	( )	( )	( )	( )	17. People would think I was some kind of a grind if I got A's in math.
( )	( )	( )	( )	( )	18. Mathematics usually makes me feel uncomfortable and nervous.
( )	( )	( )	( )	( )	19. I haven't usually worried about being able to solve math problems.
( )	( )	( )	( )	( )	20. Math has been my worst subject.
( )	( )	( )	( )	( )	21. I almost never have gotten shook up during a math test.
( )	( )	( )	( )	( )	22. I don't think I could do advanced mathematics.
( )	( )	( )	( )	( )	23. I can get good grades in mathematics.
( )	( )	( )	( )	( )	24. Generally I have felt secure about attempting mathematics.
( )	( )	( )	( )	( )	25. I don't like people to think I'm smart in math.
( )	( )	( )	( )	( )	26. I usually have been at ease during math tests.
( )	( )	( )	( )	( )	27. It would make me happy to be recognized as an excellent student in math.
( )	( )	( )	( )	( )	28. It would make people like me less if I were a really good math student.
( )	( )	( )	( )	( )	29. A math test would scare me.
( )	( )	( )	( )	( )	30. Winning a prize in mathematics would make me feel unpleasantly conspicuous.
( )	( )	( )	( )	( )	31. I am sure that I can learn mathematics.
( )	( )	( )	( )	( )	32. I am sure I could do advanced work in mathematics.
( )	( )	( )	( )	( )	33. I have a lot of self-confidence when it comes to math.
( )	( )	( )	( )	( )	34. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.
( )	( )	( )	( )	( )	35. I get a sinking feeling when I think of trying hard math problems.
( )	( )	( )	( )	( )	36. My mind goes blank and I am unable to think clearly when working math.

SURVEY OF ATTITUDES TOWARD LEARNING ABOUT AND WORKING WITH COMPUTERS

The purpose of this survey is to gather information concerning people's attitudes toward learning about and working with computers. It should take about five minutes to complete this survey. All responses are kept confidential. Please return the survey to your instructor when you are finished.

Please check the blank which applies to you.

1. Age: ( ) 22 or less ( ) 23-25 ( ) 26-30
   ( ) 31-35 ( ) 36-40 ( ) 41-45
   ( ) 46-50 ( ) 51-55 ( ) 55+

2. College level completed: ( ) 1st year ( ) 2nd year ( ) 3rd year ( ) 4th year
   ( ) Bachelors ( ) Masters ( ) Doctorate

3. Major area of study: ____________________

4. Sex: ( ) Male ( ) Female

5. Experience with learning about or working with computers:
   ( ) 1 week or less ( ) 6 months to 1 year
   ( ) 1 week to 1 month ( ) 1 year or more
   ( ) 1 month to 6 months

   Briefly state the type of computer experience: ____________________

COMPUTER ATTITUDE SCALE

Below are a series of statements. There are no correct answers to these statements. They are designed to permit you to indicate the extent to which you agree or disagree with the ideas expressed. Place a check mark in the parentheses under the label which is closest to your agreement or disagreement with the statements.

	Strongly Agree	Slightly Agree	Slightly Disagree	Strongly Disagree
1. Computers do not scare me at all.	( )	( )	( )	( )
2. I'm no good with computers.	( )	( )	( )	( )
3. I would like working with computers.	( )	( )	( )	( )
4. I will use computers many ways in my life.	( )	( )	( )	( )
5. Working with a computer would make me very nervous.	( )	( )	( )	( )
6. Generally I would feel OK about trying a new problem on the computer.	( )	( )	( )	( )
7. The challenge of solving problems with computers does not appeal to me.	( )	( )	( )	( )
8. Learning about computers is a waste of time.	( )	( )	( )	( )
9. I do not feel threatened when others talk about computers.	( )	( )	( )	( )

		Strongly Agree	Slightly Agree	Slightly Disagree	Strongly Disagree
10.	I don't think I would do advanced computer work.	( )	( )	( )	( )
11.	I think working with computers would be enjoyable and stimulating.	( )	( )	( )	( )
12.	Learning about computers is worthwhile.	( )	( )	( )	( )
13.	I feel aggressive and hostile toward computers.	( )	( )	( )	( )
14.	I am sure I could do work with computers.	( )	( )	( )	( )
15.	Figuring out computer problems does not appeal to me.	( )	( )	( )	( )
16.	I'll need a firm mastery of computers for my future work.	( )	( )	( )	( )
17.	I wouldn't bother me at all to take computer courses.	( )	( )	( )	( )
18.	I'm not the type to do well with computers.	( )	( )	( )	( )
19.	When there is a problem with a computer run that I can't immediately solve, I would stick with it until I have the answer.	( )	( )	( )	( )
20.	I expect to have little use for computers in my daily life.	( )	( )	( )	( )
21.	Computers make me feel uncomfortable.	( )	( )	( )	( )
22.	I am sure I could learn a computer language.	( )	( )	( )	( )
23.	I don't understand how some people can spend so much time working with computers and seem to enjoy it.	( )	( )	( )	( )
24.	I can't think of any way that I will use computers in my career.	( )	( )	( )	( )
25.	I would feel at ease in a computer class.	( )	( )	( )	( )
26.	I think using a computer would be very hard for me.	( )	( )	( )	( )
27.	Once I start to work with the computer, I would find it hard to stop.	( )	( )	( )	( )
28.	Knowing how to work with computers will increase my job possibilities.	( )	( )	( )	( )
29.	I get a sinking feeling when I think of trying to use a computer.	( )	( )	( )	( )
30.	I could get good grades in computer courses.	( )	( )	( )	( )
31.	I will do as little work with computers as possible.	( )	( )	( )	( )
32.	Anything that a computer can be used for, I can do just as well some other way.	( )	( )	( )	( )

		Strongly Agree	Slightly Agree	Slightly Disagree	Strongly Disagree
33.	I would feel comfortable working with a computer.	( )	( )	( )	( )
34.	I do not think I could handle a computer course.	( )	( )	( )	( )
35.	If a problem is left unsolved in a computer class, I would continue to think about it afterward.	( )	( )	( )	( )
36.	It is important to me to do well in computer classes.	( )	( )	( )	( )
37.	Computers make me feel uneasy and confused.	( )	( )	( )	( )
38.	I have a lot of self-confidence when it comes to working with computers.	( )	( )	( )	( )
39.	I do not enjoy talking with others about computers.	( )	( )	( )	( )
40.	Working with computers will not be important to me in my life's work.	( )	( )	( )	( )

Appendix B

Computer Labs and Worksheets Used In This Study

Description of Concepts Covered by Individual Questions

Math lab/worksheet 1 is designed to give student some experience using the Master Grapher software and at the same time work on the concepts of domain and range of a function, x- and y-intercepts of a function, absolute value, reciprocals of functions, and discontinuity.

Question 1 is designed to help the student explore the concepts of domain and range of a function. The student also is to get some idea of what the x- and y-intercepts are.

Question 2 is designed to illustrate how the absolute value of a function alters the original function. It also emphasizes how the domain and range change, if they do at all. Again the student finds the x- and y-intercepts if they exist.

Question 3 is designed to illustrate how taking the reciprocal changes a function, its domain and range, and the intercepts. It also intuitively explores asymptotes and discontinuities.

MATH181 LAB 1 - Using Master Grapher:

1). Graph the function: x^2+2x-3

by entering x∧2+2*x-3 in equation 1. Now display equation 1.

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

2). Graph the function: $| x^2+2x-3 |$

by entering abs(x∧2+2*x-3) in equation 2. Now display equation 2 but not equation 1 (just type 1 to toggle its display off).

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

What does the absolute value appear to do to the y-values of the function?

3). Graph the function $\dfrac{1}{x^2+2x-3}$

by entering 1/(x∧2+2*x-3) in equation 3. Now display equation 3, toggle equations 1 and 2 off.

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

Are there values of x where the function appears not to exist? If so, where?

MATH181 LAB 1

1). On a separate piece of paper graph the function: x^2+2x-3 .

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

2). Graph the function: $| x^2+2x-3 |$.

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

What does the absolute value appear to do to the y-values of the function?

3). Graph the function $\frac{1}{x^2+2x-3}$.

Where does this function appear to cross the x-axis? The y-axis?

What appears to be the function's domain? It's range?

Are there values of x where the function appears not to exist? If so, where?

Math lab/worksheet 2 is designed to give the students a graphical representation of the concepts of limits and equality of functions. It also is to help the student to determine what seemingly simple changes to a function may do to the limit at a point. It is also designed to help them get an intuitive ideas of what a limit is and how it relates to the concept of continuity.

Question 1 is designed to give the student some experience with finding the limit of a function that is continuous. It is to help their intuitive understanding of the concept of limit.

Question 2 is designed to help the student realize that the graphs of two functions may have the same value at every point except one, so that the two functions are not equal. It is also to help them understand that where a function is not defined, the limit may still exist.

Question 3 is designed to show that the limit of a function at one point may exist and equal a value other than zero even if the limits of separate parts of the function do equal zero at the point.

Question 4 is designed to show the student what happens to the limit when a small change is made to the function. It also helps with the concept of continuity.

Questions 5(This is question 8 of the lab designed for use with the computer. Questions 5, 6, and 7 of that lab deal with changing the size of the picture to help with the geometric representation of the concept developed in question 4.) is designed to illustrate the connection between the concepts of continuity and limit.

Question 6(Question 9 on the computer users lab.) is designed to help the student get a geometric representation of the epsilon-delta idea of limits. It is designed to help the student make the transition of the intuitive idea of limit with the formal definition of limit.

MATH181 LAB 2 PAGE 1 OF 2 - Using Master Grapher:

1). Graph the function: sin(x)

by entering sin(x) in equation 1. Now display equation 1.

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at x=0 ?

2). Now graph the function: $\frac{x\sin(x)}{x}$

by entering (x*sin(x))/x in equation 2 and displaying it.

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at x=0 ?

Does the function exist at x=0 ?

3). Now graph the function: $\frac{\sin(x)}{x}$

by entering sin(x)/x in equation 3 and displaying it.

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at x=0?

4). Now graph the function: $\frac{\sin(x)}{x^2}$

by entering sin(x)/(x^2) in equation 4 and displaying it.

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at x=0?

MATH181 LAB 2 PAGE 2 OF 2 - Using Master Grapher:

5). After selecting FUNCTION GRAPHER, select option S .

Enter: 200

6). Select option V .

Select option 5

L=? Enter -.1
R=? Enter 1.9
B=? Enter -.1
T=? Enter 1.9

This will set the viewing window to give a closer picture of the function.

7). Selecti option 9. Previous Menu.

8). Graph the function: $\frac{2x^2-3x+1}{x(x-1)}$

by entering (2*x∧2-3*x+1)/(x*(x-1)) in equation 1. Now display equation 1.

Note that there is a jump in the graph at x=1. This function is not defined at x=1.

What do you think the limit as x goes to 1 of this function is?

9). To check your guess, select F again and change the following equations:

Equation 2 to: .9
Equation 3 to: 1.1
Equation 4 to: .95
Equation 5 to: 1.04 Display these equations.
Equation 6 to: .99
Equation 7 to: 1.01

As the band get smaller around the value y=1, what has happened to the interval around the value x=1?

Was your guess about the limit correct?

This is meant to be a geometrical representation of the ϵ-δ argument in the text.

MATH181 LAB 2 PAGE 1 OF 2

1). On a sheet of paper graph the function: sin(x)

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at $x=0$?

2). Now graph the function: $\frac{x\sin(x)}{x}$

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at $x=0$?

Does the function exist at $x=0$?

3). Now graph the function: $\frac{\sin(x)}{x}$

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at $x=0$?

4). Now graph the function: $\frac{\sin(x)}{x^2}$

As $x \to 0$ what value does the function appear to approach?

What appears to be the limit of the function at $x=0$?

5). On a sheet of paper graph the function: $\dfrac{2x^2-3x+1}{x(x-1)}$

What do you think the limit as $x \to 1$ of this function is?

6). To check your guess graph the following equations as accurately as possible:

y= .9
y= 1.1
y= .95
y= 1.04
y= .99
y= 1.01

As the band get smaller around the value y=1, what has happened to the interval around the value x=1?

Was your guess about the limit correct?

This is meant to be a geometrical representation of the ϵ-δ argument in the text.

Math lab/worksheet 3 is designed to give the student a geometrical view of the concept of continuity. It gives them some different types of discontinuous functions that they need to analyze graphically. From these graphs they need to go to the abstract to find the actual discontinuity and classify it. This lab also gives the students some work with Bolzano's Theorem, one way of determining the roots of an equation.

Questions 1-4(1-6 on the computer lab) are designed to illustrate the different types of discontinuities. The student is to supply information about the type of discontinuity and, if it is a removable one, redefine the function to fill in the hole.

Questions 5-8(7-10 on the computer lab) are designed to give the student practice with using Bolzano's Theorem to find roots and with functions as well.

MATH181 LAB 3 - Using Master Grapher:

Graph the following functions. You may wish to change the speed to 200. Then find all the x-values where the following functions are discontinuous and identify the type of discontinuity. If the discontinuity is removable give the points, both the x- and y-values that will fill in the hole. Let the domain be every x for which the function is defined.

1) $f(x) = \dfrac{1}{x^2+2x+1}$

Enter: 1/(x^2+2*x+1)

2) $f(x) = \begin{cases} x^{1/2}, & x>0 \\ -(-x)^{1/2}, & x<0 \end{cases}$

Enter: x^(1/2) for one function and -(-x)^(1/2) for another

3) $f(x) = \dfrac{(x-3)(x+1)}{(x-1)(x+3)}$

Enter:((x-3)*(x+1))/((x-1)*(x+3))

4) $f(x) = \dfrac{x^2}{2x}$

Enter: (x^2)/(2*x)

5) $f(x) = \dfrac{x^2+x-2}{x+1}$

Enter: (x^2+x-2)/(x+1)

6) $f(x) = \dfrac{x^2-2x}{x^2-5x+6}$

Enter: (x^2-2*x)/(x^2-5*x+6)

Try to predict which of the following functions have roots in the given intervals. Then graph them to see how well you did.

7) $f(x) = x^2-3x+7$, [-4,4].

Enter: x^2-3*x+7

8) $f(x) = x^2+3x-2$, [-2,2].

Enter: x^2+3*x-2

9) $f(x) = x^3-x^2-3x+3$, [-4,4].

Enter: x^3-x^2-3*x+3

10) $f(x) = x^2-5$, [-2,2].

Enter: x^2-5

Look up Bolzano's Theorem and see if you could apply it to these functions now that you have seen their graphs.

MATH181 LAB 3

Find all the x-values where the following functions are discontinuous and identify the type of discontinuity. If the discontinuity is removable give the points, both the x- and y-values that will fill in the hole. Let the domain be every x for which the function is defined.

1) $f(x) = \dfrac{1}{x^2+2x+1}$

2) $f(x) = \begin{cases} x^{1/2} & , x>0 \\ -(-x)^{1/2} & , x<0 \end{cases}$

3) $f(x) = \dfrac{(x-3)(x+1)}{(x-1)(x+3)}$

4) $f(x) = \dfrac{x^2}{2x}$

Which of the following functions have roots in the given intervals? You might like to try Bolzano's Theorem to find the roots.

7) $f(x) = x^2-3x+7$, [-4,4].

8) $f(x) = x^2+3x-2$, [-2,2].

9) $f(x) = x^3-x^2-3x+3$, [-4,4].

10) $f(x) = x^2-5$, [-2,2].

Math lab/worksheet 4 is designed to give the student experience with using the definition of derivative to find the slope of the tangent line of a graph at a point. It gives the student a visual representation of the function and the tangent line so that they can get an intuitive idea of what the slope of a tangent line can show. It also gives them a visual representation of how one can determine a maximum or minimum point from the slope of the tangent line at that point and the points to either side.

Question 1 is designed to illustrate the relationship of the slopes of tangent lines to either side of a minimum point to the slope of the tangent line at that point. The student is given a graphical view of the notion that all the other points near a minimum point are above that point and that the slopes of the tangent lines go from being negative to zero to positive through that point.

Question 2 is designed to illustrate the relationship of the slopes of tangent lines to either side of a maximum point to the slope of the tangent line at that point. The student is given a graphical view of the notion that all the other points near a maximum point are above that point and that the slopes of the tangent lines go from being positive to zero to negative through that point.

MATH181 LAB 4 - Using Master Grapher:

1). Let $y = f(x) = -x^2+6x-5$

a). Graph and display the function by entering -x∧2+6*x-5 in equation 1.

b). Find y'

c). Find the equation of the line tangent to the graph at the point (2,3).

d). Graph and display this line as well as the previous function by entering it in equation 2.

e). Find the point on the graph where the tangent line has the slope zero.

f). Graph the tangent line at this point.
What relation does this point have to other points on the graph?

2). Let $y = f(x) = x^2+4x+7$

a). Graph and display the function by entering x∧2+4*x+7 in equation 1.

b). Find y'

c). Find an equation of the line tangent to the graph at the point (-1,4).

d). Graph and display this line as well as the previous function by entering it in equation 2.

e). Find the point on the graph where the tangent line has the slope zero.

f). Graph the tangent line at this point.
What relation does this point have to other points on the graph?

MATH181 LAB 4

1). Let $y = f(x) = -x^2+6x-5$

a). Graph the function.

b). Find y'

c). Find the equation of the line tangent to the graph at the point (2,3).

d). Graph this line on the same graph as the function.

e). Find the point on the graph where the tangent line has the slope zero.

f). Graph the tangent line at this point.
What relation does this point have to other points on the graph?

2). Let $y = f(x) = x^2+4x+7$

a). Graph the function.

b). Find y'

c). Find an equation of the line tangent to the graph at the point (-1,4).

d). Graph this line on the same graph as the function.

e). Find the point on the graph where the tangent line has the slope zero.

f). Graph the tangent line at this point.
What relation does this point have to other points on the graph?

Math lab/worksheet 5 is designed to give the student some work with the derivative and to demonstrate all of the information that can ge gleaned from analyzing the graph of the derivative. The preliminaries to the first derivative test are illustrated to give the students an intuitive idea of that test. In addition, the student is given some illustrative work with composite functions.

Question 1 is designed to illustrate the information one can determine about a function from the graph of its derivative function. It is designed to give the students an intuitive idea of the first derivative test so that when that idea is introduced the student will have a better of what the test is all about.

Question 2 deals with composite functions and the derivative of a composite function. Preliminary ideas of the chain rule is illustrated in this question.

MATH181 LAB 5 - Using Master Grapher

1). Let $f(x) = \frac{2(x^3+1)^{\frac{5}{2}}}{15}$

a). Find f'(x).

b). Graph f(x) by entering (2/15)*(x^3+1)^(5/2) in equation 1.

c). Graph f'(x) in equation 2. Display both equations.

d). Change View by entering V. Now Set window by entering 5.
Now enter for:

L = -1
R = 1
B = -1
T = 1

e). Now go to previous menu by entering 9, and regraph by entering G.

f). The function f'(x) is the slope of f(x). Answer the following questions.

Where is f'(x) = 0 ? What is the slope of f(x) at that point?

Is f'(x) increasing or decreasing to the left of the y-axis?

Is the slope of f(x) increasing or decreasing to the right of the y-axis?

2). Let $f(z) = z^2$ and g(x) = sin(x).

a). Find f(g(x)).

b). Find D f(g(x)).

c). Graph f(g(x)) and D f(g(x)) at the same time. First change the view back by entering V then entering 6. Now enter f(g(x)) in equation 1 and D f(g(x)) in equation 2.

d). The function D f(g(x)) is the slope of f(g(x)). Answer the following questions.

Where is the slope of f(g(x)) increasing?

Where is the slope of f(g(x)) decreasing?

Where is the slope of f(g(x)) = 0 ?

MATH181 LAB 5

1). Let $f(x) = \frac{2x^3+x^2}{3}$

a). Find $f'(x)$.

b). Graph $f(x)$.

c). Graph $f'(x)$ on the same graph.

d). The function $f'(x)$ is the slope of $f(x)$. Answer the following questions.

Where is $f'(x) = 0$? What is the slope of $f(x)$ at that point?

Is $f'(x)$ increasing or decreasing to the left of the y-axis?

Is the slope of $f(x)$ increasing or decreasing to the right of the y-axis?

2). Let $f(z) = z^2$ and $g(x) = \sin(x)$.

a). Find $f(g(x))$.

b). Find $D\, f(g(x))$.

c). Graph $f(g(x))$ and $D\, f(g(x))$ on the same graph.

d). The function $D\, f(g(x))$ is the slope of $f(g(x))$. Answer the following questions.

Where is the slope of $f(g(x))$ increasing?

Where is the slope of $f(g(x))$ decreasing?

Where is the slope of $f(g(x)) = 0$?

Math lab/worksheet 6 is designed to give a visual representation of the First Derivative Test and the Second Derivative Test. The students use the graphical representations of the functions and the first and second derivatives of those functions to analyze the functions. From that analysis they can determine information about the functions and what information is given by the first and second derivatives.

Question 1 is designed to illustrate the first derivative test. The student graphs the function and its first derivative. From the graphs the student answers questions designed to help them think about what information the first derivative gives about the function.

Question 2 gives the student another function, or more functions, to apply the first derivative test to.

Question 3 is designed to illustrate the second derivative test. The student graphs the function, its first derivative, and its second derivative. From the graphs the student answers questions designed to help them think about what information the first and second derivatives give about the function.

Question 4 gives the student another function, or more functions, to apply the first derivative test to.

MATH 181 LAB 6 - First and Second Derivative Tests

1). Let $f(x) = x^2 + 2x + 1$ Graph $f(x)$.

Find $f'(x)$.

Graph it.

Where is $f'(x)$ positive? (right or left of an x-value?)

Where is $f'(x)$ negative? (right or left of an x-value?)

At what x-value is $f'(x)$ zero?

Note that if $f'(x)$ is negative to the left of an x and positive to the right, that $f(x)$ has a minimum where $f'(x)$ is zero and if $f'(x)$ is positive and then negative then $f(x)$ has a maximum at that point. This is the **First Derivative Test.**

2). Follow the same steps with the functions.

$f(x) = -x^2 + 6x - 5$

3). Let $f(x) = x^3 - 6x^2 + 9x + 2$.

Find $f'(x)$.

Where does $f'(x)$ equal zero?

Find $f''(x)$.

Is $f''(x)$ positive or negative at the x-values for which $f'(x)$ is zero. Are they maximums or minimums?

If $f''(x)$ is positive at a local extreme then that extreme is a minimum. If $f''(x)$ is negative at a local extreme then the extreme is a maximum. This is the Second Derivative Test.

4). Try the same steps with the following functions:

$f(x) = x^2 + 5x + 6$

MATH 181 LAB 6 - Master Grapher - First and Second Derivative Tests

1). Let $f(x) = x^2 + 2x + 1$ Graph f(x). [Enter x^2+2*x+1.]

Find $f'(x)$.

Enter it into equation 2 and display it.

Where is $f'(x)$ positive? (right or left of an x-value?)

Where is $f'(x)$ negative? (right or left of an x-value?)

At what x-value is $f'(x)$ zero?

Note that if $f'(x)$ is negative to the left of an x and positive to the right, that f(x) has a minimum where $f'(x)$ is zero and if $f'(x)$ is positive and then negative then f(x) has a maximum at that point. This is the **First Derivative Test.**

2). Follow the same steps with the functions. You may wish to change the window on the third one. I would suggest L= -10, R=10, B=-50, T=50. That should give you both points where $f'(x)$ is zero. Also, the third function has a point where $f'(x)$ is zero but it is not a maximum nor a minimum.

$f(x) = -x^2 + 6x - 5$ $\quad$ $f(x) = 5\sin(x) + 2$ $\quad$ $f(x) = \dfrac{x^3 + 3x + 6}{x + 2}$

3). Let $f(x) = x^3 - 6x^2 + 9x + 2$. Display the graph of f(x).

Find $f'(x)$.

Display its graph.

Where does $f'(x)$ equal zero?

Find $f''(x)$.

Display its graph. [You may wish to graph f(x) and $f''(x)$ on the same set of axes without $f'(x)$.]

Is $f''(x)$ positive or negative at the x-values for which $f'(x)$ is zero. Are they maximums or minimums?

If $f''(x)$ is positive at a local extreme then that extreme is a minimum. If $f''(x)$ is negative at a local extreme then the extreme is a maximum. This is the **Second Derivative Test.**

4). Try the same steps with the following functions:

$f(x) = 5\sin(x/3)$ $\quad$ $f(x) = x^2 + 5x + 6$ $\quad$ $f(x) = x + 4/x$