ESTIMATION OF MONTANA WINTER WHEAT RESPONSE TO NITROGEN AND PRECIPITATION: AN APPLICATION OF THE FOURIER FLEXIBLE FUNCTIONAL FORM

by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Applied Economics

MONTANA STATE UNIVERSITY
Bozeman, Montana

June 1990

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of a thesis submitted by

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ACKNOWLEDGMENTS

I wish to express my appreciation and thanks to my thesis advisors, Professor Bruce Beattie and Dr. Michael Frank, for their invaluable advice throughout the development of this study. Additionally, I would like to express my gratitude to the other members of my graduate committee, Professors M. Steve Stauber and Myles J. Watts, for their assistance. Credit also is due to Drs. Carol Bittinger and Jeffrey T. LaFrance for their guidance and advice.

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ABSTRACT

The principal focus of this study was to use a Fourier function, a specific flexible functional form, to test and reexamine two-dimension yield-input relationships which have been explored previously by agricultural economists. Based on previous literature, three debated crop response functions, i.e., von Liebig, quadratic and Mitscherlich-Baule, their estimation, and their pros and cons were reviewed regarding the matter of whether plant nutrients (including water) are substitutable factors in crop production response functions. The fundamental issue is the nature of the isoquant pattern exhibited.

A flexible functional form was used in this research. Global flexibility, Sobolev-flexibility, appears to be a more attractive criterion than local flexibility, Diewert-flexibility. A Fourier-flexible form is the only form that has been shown to be the Sobolev-flexible which can endow a parametric statistical methodology with the semi-non-parametric property.

In the estimation process of the Fourier functional form, establishing an acceptable relationship for yield response of winter wheat to fertilizers and various soil and climatic factors failed because of the high noise level in the 900-observation data set. The attempt to delete ill-conditioned plots did not improve the performance of the model. Therefore, the last step in estimation was based on a "stylized" data set containing 388 arbitrarily-trimmed observations.

Since the Fourier function is very sensitive to the data, one should use this highly flexible form only when a data set with normal statistical properties is available. Once armed with a good data set, the Fourier function should perform a fairly good approximation to a differentiable function. Finally, the results using the stylized data set seem to support the hypothesis of the Mitscherlich-Baule for nitrogen, i.e., a smooth upward curve followed by a plateau as suggested by Frank, Beattie and Embleton (1989). However, given that the data were stylized, nothing definitive can be said in this regard.

CHAPTER 1

INTRODUCTION

The production function is a physical/biological concept. However, it was largely developed and, until recently, used mainly by economists. The most important reason for estimating production functions is to provide basic scientific knowledge. Historically, plant scientists' interest in production response function research was designed and conducted on the basis of discrete phenomena wherein two or a few treatments were used to provide point estimates of crop output resulting from input of factors. Typically, the experimental designs and statistical procedures used have only permitted testing for statistically significant differences between the yield or output level of two or three discrete treatments or input levels. These designs and approaches have proved useful for this purpose and may continue to do so under certain conditions.

In many cases, research workers in biological fields have been concerned only with estimating the output from a specific quantity of an innovative new material. Here the goal of the research often has been to answer the question: Does the material or resource, used at any level whatsoever, give a response? Much early research on fertilization fell in this realm. In some cases, the practice or treatment under consideration represents a resource or material of discrete and limitational nature. The optima selected within this framework of assumed physical relationships always occurred "at the corners," represented by the point estimates.

There are many biological and physical areas, however, where continuous relationships are involved and the data lend themselves to formal production function analysis. In the past decade, an increasing number of physical and biological scientists in agriculture have become acquainted with production economic principles and their use to make recommendations to farmers (Box, 1954). This increased interest and activity parallels advances in the field of agricultural economics. Pioneer agricultural economists, like other agricultural scientists, were concerned with physical production function relationships. However, since economists were particularly concerned with conditions of profit maximization and competing economic alternatives, the central interest has been concepts such as marginal productivity, marginal rates of substitution, isoquants, and isoclines and other constructs derived from continuous production functions.

Numerous algebraic equation forms can be used to specify production functions. No single form seems to adequately characterize agricultural production under all environmental conditions. The algebraic form of the function and the magnitude of coefficients vary with soil, climate, crop specie, variable resources under consideration, tillage methods, and magnitude of other inputs in "fixed quantity" to the firm. Hence, principal focus of this study is the issue of model selection; that is, choosing an algebraic functional form which appears or is known to be consistent with the phenomena under investigation. Insights regarding appropriate algebraic form come from previous investigations and economic and plant growth theory. Selection of any specific equation to express production phenomena imposes certain restraints or assumptions regarding the relationships involved and the optimum resource quantities to be determined. However, some functional forms are more flexible than others.

Problem Statement

For many years, the manner in which a plant responds to nutrients has been debated. Many alternative models have been hypothesized. Three of particular interest are: (1) a smooth, concave production function, such as a quadratic polynomial function; (2) a linear response followed by a plateau, i.e., a von Liebig model; and (3) an exponential response plus a plateau, i.e., a Mitscherlich-Baule function. Researchers have been trying to answer the following questions: What is the shape of yield response to an input(s)? What is the relationship between pairs of inputs: are they substitutable or not? If substitutable, what is the nature and degree of the substitution relationship? The debate focuses on two main issues: (1) do crop response functions to plant nutrients tend to be von Liebig in character, i.e., linear response followed by a plateau and no factor substitution (right angle isoquants), or (2) are response relationships smooth and differentiable, exhibiting some (limited to considerable) factor substitution? This study intends to explore both issues but with particular attention to the matter of factor substitution.

When alternative crop response models are compared, the generality of parametric statistical inference is inevitably limited by restrictions imposed by the model. If the true model does not, in fact, belong to the assumed parametric family of models, then estimators can be seriously biased due to specification error. In a testing situation, the true rejection probability of a statistical test can greatly exceed its nominal rejection probability, since the usual consequence of specification error is to cause the non-centrality parameter of a test statistic to be positive even if the null hypothesis is true. In hopes of increasing the generality of an inference, that is, reducing biases induced by specification error, richer parametric families of models called flexible functional forms (FFF) are finding increasing use in empirical economic research. The idea is to assign to a consumer, to a firm, to an industry (or to whomever), a utility function, a cost function, a profit function (or whatever), that is parametrically rich enough so as not to impose extraneous behavioral restrictions that limit the generality of an inference (Gallant, 1984).

Purpose of the Study

In brief, the primary goal of this study is to use a Fourier function, a specific FFF, to test and reexamine two-dimension yield-input relationships which have been explored previously by agricultural economists. Specifically, the project serves to expand upon previous crop response research by Frank et al. (1989). More precisely, the objectives of this project are as follows:

1. To investigate the applicability of theory and practice of flexible functional forms (FFF) in the study of crop response.

2. To estimate and compare the parameters of the Fourier functional form using Montana experimental data on wheat response to water and fertilizer nutrients and to evaluate the consistency of the parameter estimates with accepted economic theory.

3. To determine the substitutability of nitrogen and water by calculating the elasticity of factor substitution (σ values) generated from the first and second derivatives of the chosen Fourier function. The motivation for this objective is to determine whether or not the no factor substitution implication of a von Liebig model is valid.

As it turns out, severe data problems were encountered in the study. Had the data chosen for analysis not been so noisy, and had there been a discernable relationship between yield and inputs, the empirical results would have been compared with those of other functional forms. The motivating hypothesis was that a Fourier flexible functional form would enable reliable estimation of the elasticity of substitution between pairs of inputs. However, given the data problems encountered, the conclusions focus more on the mechanism of how this flexible functional form works and the prerequisites for successful application of a Fourier function rather than the original intentions of the study.

Outline of Study

The remainder of the thesis is organized as follows. Chapter 2 presents the theory of factor substitution and reviews the relevant previous literature. The empirical methodology appropriate to the study is developed in Chapter 3. The data set and empirical results are presented in Chapter 4. Chapter 5 summarizes the research findings and conclusions and offers suggestions for further research.

CHAPTER 2

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

The first section of this chapter reviews previous literature, mainly concerning the three debated crop response functions, their estimation, and their pros and cons. In the second section, an economic framework for dealing with isoquant patterns and elasticity of factor substitution is developed and designed to clarify the nature of previous debate. The purpose of this chapter is to pave a path for the empirical work presented in the balance of the thesis.

Previous Literature

Scholars have conducted extensive research concerning the yield response of crops to a variety of agronomic factors. Generally, Dr. Justus von Liebig is credited with the first scientific attempt to explain the relationship between plant growth and nutrient levels. His model,

$$Y = \min (Y^*, \beta_1 + \beta_2 X_1, \beta_3 + \beta_4 X_2), \qquad (2.1)$$

where Y^* is maximum crop yield, is consistent with the idea that input X_1 and input X_2 perform different biochemical functions in plant growth. Further, the function implies that the plant responds (in a linear fashion) only to the most limiting nutrient. After some level of application (say, X^*_1 and X^*_2) the plant will no longer respond to the applied nutrients. The von Liebig model a priori imposes both $\sigma = 0$ (no factor substitution) for all levels of inputs and a growth plateau after X^*_1 and X^*_2 (Frank et al., 1989). This original von Liebig model related the factors of growth linearly to yield, and restricted plant growth by any single factor depending on the level of the most limiting factor, which von Liebig referred to as the "law of the minimum" (Heady and Dillon, 1961). This concept has endured, and has been cited by plant scientists for more than a century.

The founding work of von Liebig and his colleagues has fostered the development of more sophisticated physiological plant growth models. In 1976, Smith presented a polynomial model which simulated nitrogen, phosphorus, and potassium utilization in the plant-soil system for a variety of crops including corn, beans, oats, rutabagas, and pine trees. To paraphrase Smith, the model is a measure of general physiology of many species subjected to a wide range of nutrient availability.

Along with the advancements in biological and physiological knowledge of crop growth, a general awareness of the technical relationship between inputs and products has increased. Review of the contemporary yield-response literature suggests that a variety of agronomic factors affect yield levels (Hucklesby et al., 1971). Commonly, yield studies include at least one explanatory variable that represents a plant nutrient such as soil nitrate (NO_3-N), soil phosphorus (in soil solution, organic, and inorganic forms), or soil moisture, and often these studies attempt to analyze interactions among nitrogen, phosphorus, and potassium.[1]

It is worthwhile to note that considerable controversy exists among soil scientists as to the accuracy of soil testing. Most soil scientists profess that procedures for measuring soil nutrients are accurate (Haby and Larson, 1976). However, others believe that no precise

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[1] Extensive soil potassium tests conducted in Montana from 1972 through 1974 reveal that nearly all major agricultural soils of Montana are high in potassium. Because crop responses to potassium fertilizer would occur only infrequently in Montana, potassium is not often considered in studies of crop fertilization (Skogley, 1977).

tests exist for certain chemicals and micronutrients (*Illinois Agronomy Handbook*, 1983-84). Thus one must maintain discretion when undertaking research involving soil nutrients.

In addition to soil characteristics and nutrient influences on crop yields, other factors such as soil moisture, precipitation, and temperature are important for successful crop production (Baier and Robertson, 1968; Black, 1970; Singh et al., 1975).[2] Canadian greenhouse experiments demonstrate that increased wheat yields may be obtained from large quantities of soil moisture (up to three-quarters of field capacity) in loam but not in loamy sand (Dubetz, 1961). Wheat and barley field experiments show that yield responses to nitrogen fertilizer on nonfallowed soils are significantly affected by growing season precipitation and available stored soil moisture at seeding. In fact, the sum of growing season precipitation and soil moisture comprised 40.3 percent of the yield response to nitrogen (Bauer et al., 1965).

In 1924, Fisher used linear regression techniques to predict wheat yields from rainfall distribution at Rothamsted, England. Since then, many researchers have utilized Fisher's methods on different crops in dissimilar environments. One such study, completed in India, concluded that 75 percent of the yield variation was due to rainfall distribution (Gangopadhyaya and Sarker, 1965). Furthermore, the India study presented response curves depicting expected changes in yield caused by marginal increases in precipitation at any point in time. These response curves show that precipitation exceeding the average during the period of a month prior to seeding and during the period of germination is generally beneficial to the crop, while precipitation at tillage is damaging.

A Montana study (Black, 1982) examining the long term fertilizer and climatic influences

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[2]The literature review in this and the following paragraph closely follows Wessells (1984).

on morphology and yield of spring wheat showed that water use efficiency increases with increasing rates of applied nitrogen and phosphorus. Based on the work of these above-mentioned authors, Wessells (1984) studied optimal fertilization of Montana winter wheat assuming a quadratic response relationship.

The century-old "law of the minimum" has been recently verified and advanced by Ackello-Ogutu et al. (1985), Grimm et al. (1987), and Paris and Knapp (1989). In their studies, they empirically tested certain technical aspects of crop response using non-nested hypothesis tests. In 1985, this research led to a discovery that a response model based on a von Liebig specification was seldom rejected in favor of a polynomial (Ackello-Ogutu et al., 1985). In 1987, the von Liebig was compared to three commonly used polynomial functions (quadratic, three-halves and square root) for five independent data sets (Grimm et al., 1987). Their results support a growth plateau and zero elasticity of factor substitution, σ, between water and nitrogen (right angle isoquants). These empirical findings provide strong appeal for the von Liebig model on both theoretical and empirical grounds for modelling crop response to macronutrients (Frank et al., 1989).

Shortcomings of the work of Paris and others were pointed out by Frank et al. (1989). They argued that it is not possible to "disentangle" the separate issues of plateau growth and factor substitution using the "Paris approach." By limiting their model comparisons to the quadratic, three-halves and/or square root versus the von Liebig, Paris and co-workers effectively allowed choice of either the plateau growth plus zero factor substitution or nonplateau growth plus nonzero factor substitution. But, in fact, there is a third choice, viz. plateau growth and factor substitution that is perhaps more interesting.

Since von Liebig's pioneering studies in 1840, scientists have discovered that crop response to nutrients may be more complicated than a simple von Liebig specification. In addition to the von Liebig function, several differentiable functions, e.g. polynomials (mainly the quadratic) and Mitscherlich functions, have been included in comparisons of alternative crop response models.

Mitscherlich first attempted to define the algebraic nature of the fertilizer-crop production function in 1909. He perhaps was the first agriculturist to suggest a nonlinear production function relating nutrient input and crop output. With the aid of Baule, a mathematician, he proposed the equation

$$\log A - \log (A-Y) = cX \tag{2.2}$$

to explain fertilizer response allowing diminishing marginal productivity (Heady and Dillon, 1961, p. 11). In this equation, A is total yield when the nutrient, X, is not deficient (i.e., A is the maximum yield attainable from addition of X) and c is a proportionality constant, defining the rate at which marginal yields decline. Mitscherlich proposed that coefficient c was a constant for all crops, unaffected by type of crop, climate, or other environmental factors. A debate of the Mitscherlich-Baule vs. von Liebig theories involved Briggs, Rippel, Balmurkand, and Mitscherlich.

Baule generalized the Mitscherlich equation to n variables. His equation was based on the "percentage sufficiency" concept for individual nutrients. Evolved from the original Mitscherlich function, the Mitscherlich-Baule function has the following form:

$$Y_i = \beta_0 [1 - \exp(-\beta_1(\beta_2+X_1))] [1 - \exp(-\beta_3(\beta_4+X_2))]. \tag{2.3}$$

For the last three decades, most agricultural economists have advocated the use of polynomial functions (quadratic and square root) for representing crop response to fertilizer nutrients. Proponents of polynomial functions argue that since the exact mathematical nature of crop response is unknown, approximation using polynomials is preferable in view of their computational simplicity and good fit in many applications (Koster and Whittlesey, 1971; Heady and Hexem, 1978).

The quadratic function,

(2.4) $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X^2_1 + \beta_4 X^2_2 + \beta_5 X_1 X_2$

where Y is yield, X_1 and X_2 are applied fertilizer nutrients and the β_i are parameters, imposes factor substitution and no growth plateau. That is, for β_1, $\beta_2 > 0$ and β_3, $\beta_4 < 0$, yield decreases as X_1 and X_2 levels become large ceteris paribus; the function exhibits diminishing marginal productivity and input substitution for all X_1 and $X_2 > 0$. These properties plus the fact that the function is linear in all parameters contributed to its historical popularity (Frank et al., 1989).

In 1978, Heady and Hexem published *Water Production Functions for Irrigated Agriculture* to introduce agronomists to the economic principles of water allocation and production function estimation. They illustrated their proposed methodology using experimental data on various combinations of irrigation water and nitrogen fertilizer for different sites, years and crops. The functional forms were restricted to quadratic, three-halves, and square root polynomial functions. In this book, the authors presented estimates of marginal rates of substitution for input levels where the isoquants were convex to the origin.

During the last decade, however, researchers have pointed out the weaknesses of polynomial crop response functions (Grimm et al., 1987; Perrin, 1976; Frank et al., 1989). Polynomials tend to consistently overestimate the maximum yield and the optimum fertilizer levels. Additionally, there is little empirical support for yield decreases due to excessive application of many plant nutrients as implied by the quadratic specification.

In 1985, Ackello-Ogutu et al. tested a von Liebig crop response function against polynomial specifications. From the derived statistical results, they claimed that the presence of plateaus was evidenced not only by the fact that the nonsubstitution hypothesis could not be rejected in any of the years examined, but also by the fact that the quadratic hypotheses were rejected at significance levels higher than those of the square root hypotheses. The square root function has a slightly flatter surface than that of the quadratic. They concluded that the use of polynomial approximations leads to costly biases whenever plateaus are significantly manifested, so the polynomial specifications should be abandoned.

In 1989, Frank et al. compared the quadratic function with the von Liebig and Mitscherlich-Baule functions. Both the von Liebig and Mitscherlich-Baule functions have a plateau. The results demonstrate that the quadratic function does not appear to fit the data as well as either the von Liebig or Mitscherlich-Baule functions.

Theoretical Framework

As noted in Chapter 1, the principal focus of this study is the matter of whether plant nutrients (including water) are substitutable factors in crop production response functions. The fundamental issue is the nature of the isoquant pattern exhibited. Three alternative isoquant patterns are presented in Figure 1 for the case of convex isoquants (to the origin). Case (a) shows perfect factor substitutability; case (b), imperfect factor substitutability; and case (c), no factor substitutability.

An alternative and convenient way of characterizing the isoquant pattern is the concept of elasticity of factor substitution, σ. As traditionally defined in the theory of the firm,

$$\text{(2.5)} \qquad \sigma = [d(X_2/X_1)/(X_2/X_1)]/(dRTS/RTS)$$

$$= [d(X_2/X_1)/(X_2/X_1)]/[d(f_1/f_2)/(f_1/f_2)]$$

$$= \frac{f_1/f_2}{X_2/X_1} \cdot \frac{d(X_2/X_1)}{d(f_1/f_2)}$$

where RTS is the rate of technical substitution of input X_1 for X_2 and f_1 and f_2 are the first derivatives of the production function with respect to X_1 and X_2, respectively. Carrying out the differentiation in (2.5) yields

$$\text{(2.6)} \qquad \sigma = \frac{f_1 f_2 (f_1 X_1 + f_2 X_2)}{X_1 X_2 (2 f_1 f_2 f_{12} - f_1^2 f_{22} - f_2^2 f_{11})}$$

The value of σ is always between 0 and positive infinity for the case of convex isoquants within the ridge lines. The elasticity of substitution measures the ease with which the inputs can be substituted for each other; σ is inversely proportional to the curvature of the isoquant. In Figure 1, σ is positive infinity for case (a), positive for case (b), and equals zero for case (c).

When inputs must be used in fixed proportions, and hence cannot be substituted for each other, $\sigma = 0$, the isoquant appears as a right angle, and the optimal input combination is at the vertex; other input combinations on the isoquant would cost more but yield no more <u>and</u> no less output. The von Liebig model is such a case. With zero σ-value and a linear-response plus plateau, the von Liebig production function and its isoquant are demonstrated in Figure 2.

When the input being increased substitutes for successively smaller amounts of the input being replaced, the σ-value will lie between zero and positive infinity, so long as the

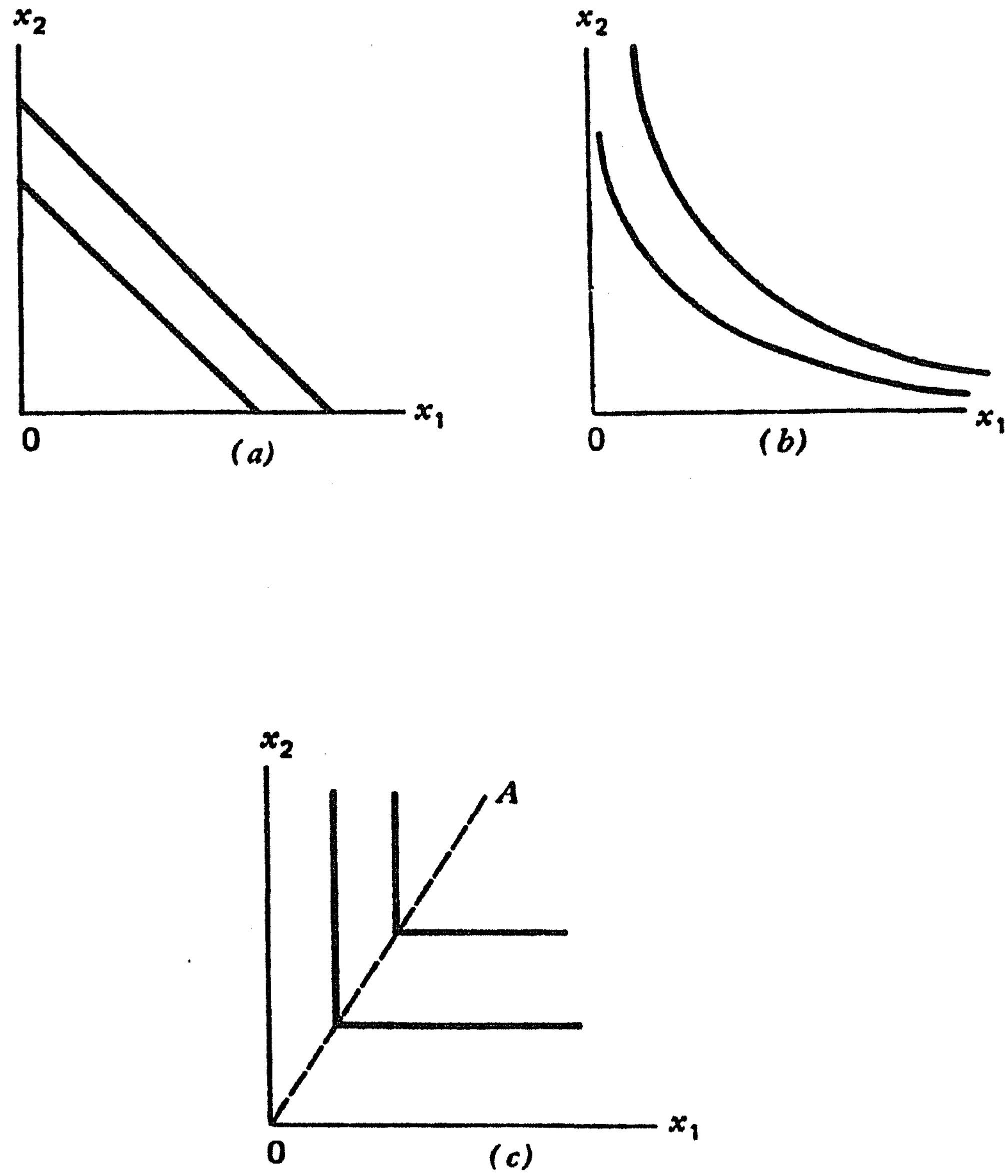
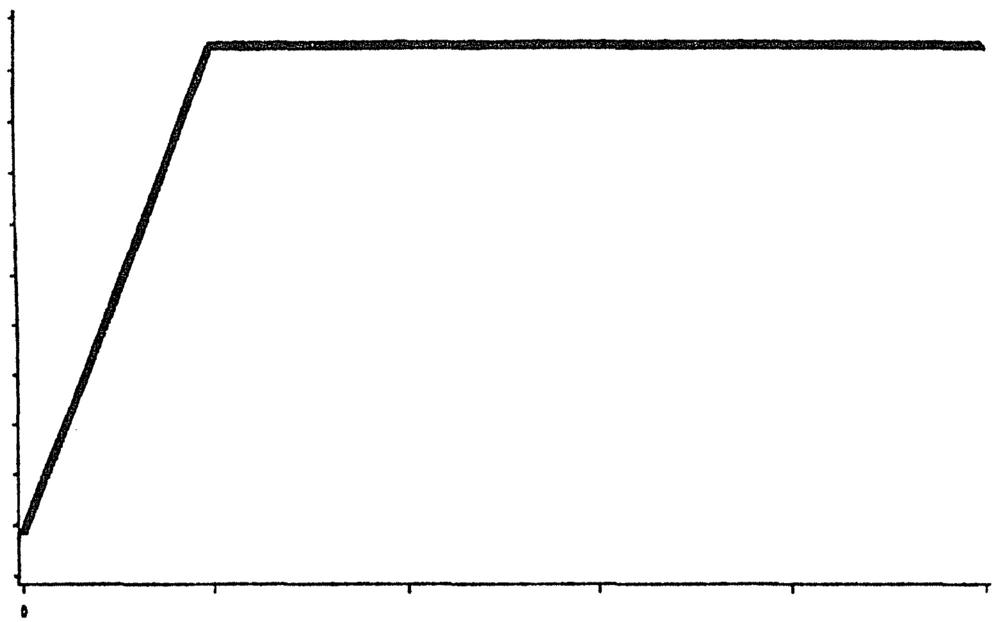


Figure 1. Three Cases of Isoquant Convex to Origin. (Source: Beattie and Taylor, 1985, p. 27)

von Liebig Function



Corresponding Isoquant

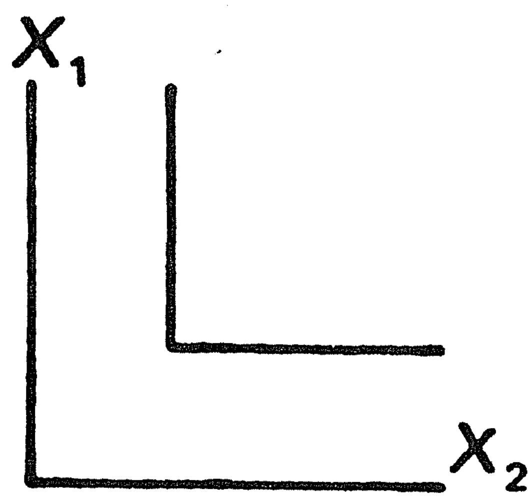


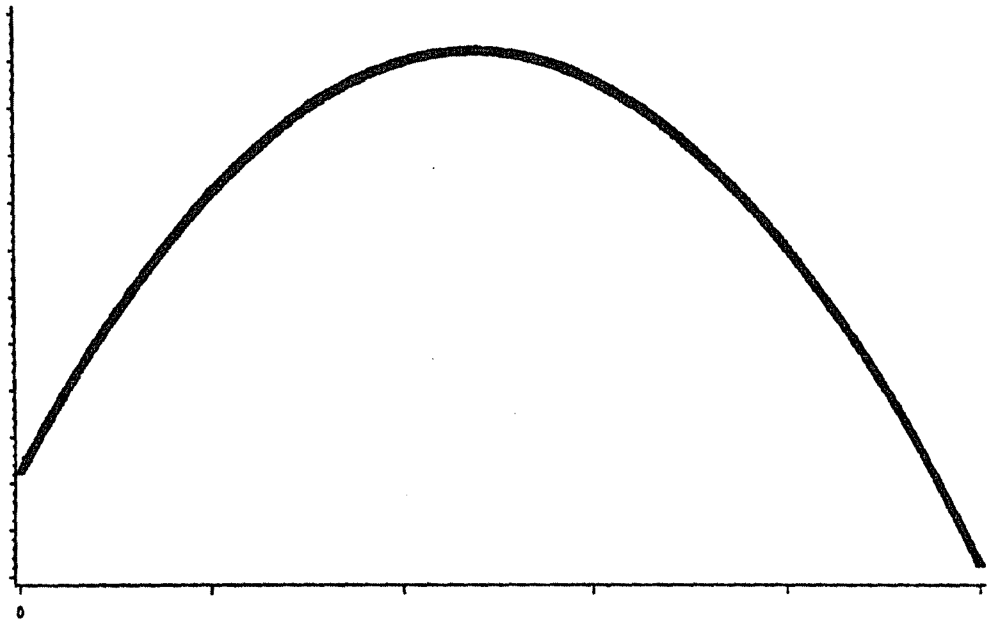
Figure 2. The von Liebig Function and Its Isoquant.

isoquant is negatively sloped and convex to the origin. This case applies to the quadratic function. However, for the quadratic function the isoquants may attain a positive slope, suggesting that if too much of one input is used, increasing amounts of the other must be applied to maintain output. The zone of economic relevance is, of course, that portion of the isoquant which has negative slope and is convex to the origin. The isoquant pattern and single-factor representation of the quadratic function and its isoquant are demonstrated in Figure 3.

For the Mitscherlich-Baule function, $0<\sigma<\infty$, the isoquants of this function become parallel to the input axes as the amount of either input is increased indefinitely. In this case the inputs substitute within limits, but after one input decreases to a certain low level, the second can be added in extremely large amounts without causing significant changes in either output or the amount of the first input. The Mitscherlich-Baule function and its isoquant pattern are demonstrated in Figure 4.

As mentioned previously, factor substitution is one of the principal issues in the von Liebig vs. differentiable functional form debate. Accordingly, the next chapter focuses on presenting an empirical methodology for estimating the parameters of a flexible functional form to ascertain the elasticity of factor substitution (isoquant pattern) for nitrogen vs. water for winter wheat in south central Montana. The proposed procedure is particularly useful for accurately estimating the derivatives of a function. Of course, first and second order derivatives of the production function are the key elements in equation (2.6), the usual formula for calculating the elasticity of substitution.

Quadratic Function



Corresponding Isoquant

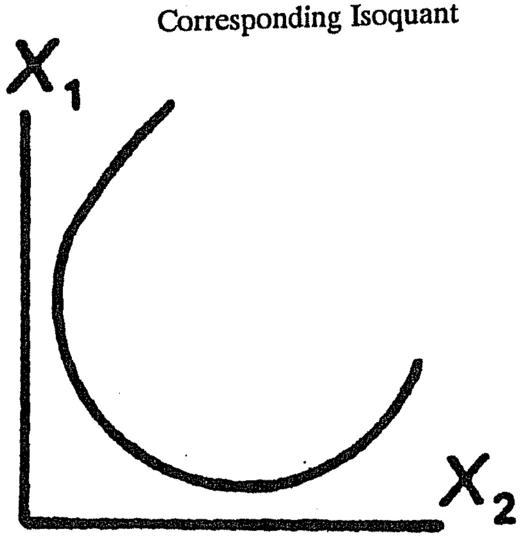
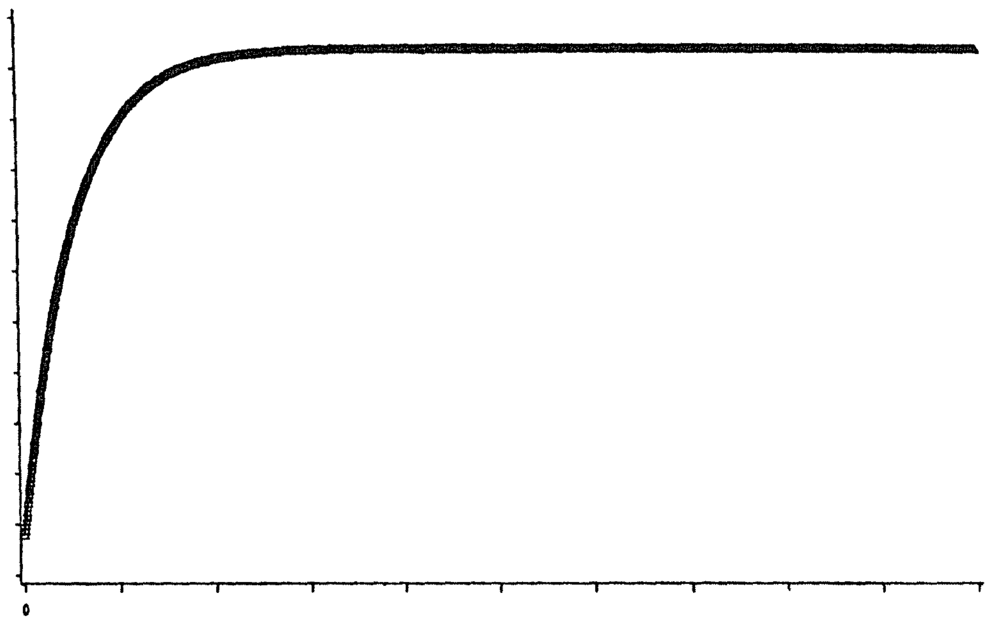


Figure 3. The Quadratic Function and Its Isoquant.

Mitscherlich-Baule Function



Corresponding Isoquant

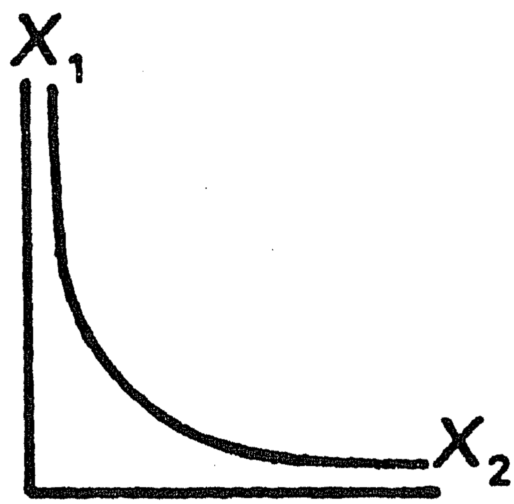


Figure 4. The Mitscherlich-Baule Function and Its Isoquant.

CHAPTER 3

EMPIRICAL METHODOLOGY

In recent years, a wealth of different production models have been proposed by applied econometricians. Contemporary emphasis on multi-input and multi-output firms has required more flexible functional forms than the traditional Cobb-Douglas and CES production functions. The applied analyst is faced with a wide array of possible functional forms for cost and production function relationships, each with its own set of advocates and "desirable" approximation properties (Rossi, 1985).

Gallant (1981) has suggested that asymptotical methods of functional approximation may be the most appropriate for the approximation of cost functions. Gallant proved that a multivariate Fourier series approximation can asymptotically approximate a true function and its derivatives. Increased flexibility is achieved through a significant increase in the number of parameters compared to less flexible functional forms. Thus, the Fourier function does not provide a practical approximation method for small samples. Following the theoretical work pioneered by Gallant, empirical applications by economists include studies by Wohlgenant (1984) and Rossi (1985).

Wohlgenant (1984) estimated two-good demand functions with the generalized Leontief, translog, and Fourier forms using U.S. time-series food consumption data. The Fourier form dominated the translog and generalized Leontief forms. Rossi (1985) compared the translog and logarithmic Fourier for a three-input cost share system in a production study using the Berndt and Wood data on aggregate U.S. manufacturing. The logarithmic Fourier was preferred.

The Quest for Flexibility

Recent advances in developing new functional forms have been dominated by efforts to discover "flexible" forms, and different technical definitions of flexibility have arisen as a result of these pursuits. In summary, there are two concepts cited in the literature regarding flexibility--local flexibility and global flexibility. The most fundamental comparison of flexible functional forms (FFF) can be made according to the two commonly used definitions of flexibility. Griffin et al. (1987) offered a comprehensive treatment of flexibility criteria. The following brief discussion of flexibility definitions is intended to highlight the differences between notions of local and global flexibility.

Diewert (1971) formalized the notion of flexibility in functional forms by defining a second-order approximation to an arbitrary function. In descriptive terms, Diewert's definition of a flexible functional form requires that an FFF have parameter values such that the FFF and its first-and second-order derivatives are equal to the arbitrary function and its first-and second-order derivatives, respectively, for any particular point in the domain. Thus, Diewert's flexibility definition refers to a local property. Any FFF which satisfies this definition may be denoted Diewert-flexible. Of course, the derivative function of any indirect function (indirect utility, cost, or profit function) will be approximated only up to its first derivative evaluated at any point (Chambers, 1982). Second-order Taylor series expansions have dominated the field of locally flexible forms, although they are not unique in their ability to offer local flexibility.

Gallant (1981) has proposed the Sobolev norm as a more attractive measure of flexibility than Diewert's definition. The appeal of the Sobolev norm is due to its measure of average error of approximation over a chosen order of derivatives. Sobolev-flexibility is a global property, and any functional form displaying this property will yield elasticities closely approximating the true ones (Elbadawi et al., 1983). Sobolev-flexibility appears to be a more attractive criterion than Diewert-flexibility in mathematical and statistical terms, because it confers on the empirical model nonparametric properties: (a) small average bias approximations (Gallant, 1981); (b) consistent estimators of substitution elasticities (Elbadawi et al., 1983); and (c) asymptotically size α-testing procedures (Gallant, 1981). The advantages of using FFF are several. First, with the satisfaction of regularity conditions such as convexity (concavity), monotonicity, and homogeneity, duality results preclude the need for self-dual functions. Further, the use of derivative properties--Hotelling's and Shephard's lemmas--allows for derivation of demand and supply (or share) functions without solving analytically for those functions. Comparative statics are easily derived from the properties of the parent indirect functions. Finally, FFF have gained popularity because of the enhanced capacity of nonlinear estimation procedures for nonlinear-in-parameters equation systems.

However, empirical use of FFF, especially the Fourier form, has certain drawbacks. First is the problem of collinearity due to numerous terms involving transformations of the same variables and interaction among variables; second, failure to satisfy the regularity conditions over the entire range of sample observations is a problem; third, difficulty is encountered in interpreting initial parameter estimates; finally, the considerable flexibility is often overly sensitive to outliers in data sets. This drawback was, although not very explicitly, pointed out in the literature by Thompson (1988). He stated,

> If the data are not consistent with the behavioral assumptions, one would have to judge whether the inconsistencies are caused by measurement error or to cross-sectional and time-series heterogeneity. In the case of measurement error inconsistencies, the violating data points might be adjusted or the sample might be censored to purge the inconsistent data points. Cross-sectional and time-series heterogeneity, such as differing from endowments, regional differences, and technical progress, might call for alternative models which account for the inconsistencies (p. 179).

Furthermore, estimation of nonlinear-in-parameters systems may involve problems with convergence and statistical theory (Lau, 1986), while interpretation of FFF as approximations to arbitrary functions also may cause bias from an estimation standpoint (White, 1980; Byron and Bera, 1983). For estimating the Fourier, theoretical issues regarding choice of sample size rules for specifying the number of parameters to estimate as well as the order of expansion are not yet settled.

The Fourier Model

The Fourier flexible form can asymptotically approximate a true function in the sense of the Sobolev norm. By increasing J, the length of the Fourier series expansions and k, the number of directions in which the expansions are taken, the Fourier function can be made as close as possible to the true function as measured by the Sobolev norm. The Sobolev norm measures how close the derivatives of a function, as well as the function itself, are approximated. The ability of the Fourier form to closely approximate the first and second-order derivatives of a function is attractive for purposes of this research, since these derivatives are key elements in the calculation of the elasticity of factor substitution. In contrast, the local series expansion methods which are used to derive the translog and other quasi-quadratic forms do not have these asymptotic approximation properties. However, the Fourier function is characterized by a large number of parameters even for low-dimensional systems.

When approximating a nonperiodic function g(x) by a Fourier series expansion, a linear and a quadratic term are often appended to the sine-cosine representation of the Fourier function. With this modification, Gallant's general Fourier flexible form is written as

(3.1) $g_k(X|\theta) = \mu_0 + b'X + ½\, x'CX$

$$+ \sum_{\alpha=1}^{A} \{\mu_{0\alpha} + 2 \sum_{j=1}^{J} [\mu_{j\alpha} \cos(jk'_{\alpha}X) - v_{j\alpha}\sin(jk'_{\alpha}X)]\}$$

where the matrix of the quadratic form, C, is given by

$$C = \sum_{\alpha=1}^{A} \mu_{0\alpha} k_{\alpha} k_{\alpha}'$$

and X is a scaled vector consisting of input variables. The multi-index, $\{k_\alpha\}$, is an N-vector with integer components. The idea is to construct a sequence of multi-indexes $\{k_\alpha\}$ and choose values of A and J such that

$$\{k: |k|_* \leq K\} \subset jk_\alpha: \alpha=1, ..., A; j=0,1,...,J\}$$

where $|k|_* = \Sigma|k_i|$ is defined as the length of a multi-index, and K is its order (Gallant, 1984). The vector of parameters is

$$\theta = (\mu_{0\alpha}, \mu_{1\alpha}, v_{1\alpha}, ..., \mu_{J\alpha}, v_{J\alpha}, b_i,)'.$$

The Fourier function can be viewed as a quadratic approximation with an appended set of univariate Fourier expansions along the directions, k_α. Note that the matrix of the quadratic form consists of A parameters instead of the usual M(M+1)/2 parameters. This parameterization reduces the number of parameters and simplifies the expressions for the derivatives of the Fourier function.

In this thesis, the Fourier functional specification for yield takes the following form:

(3.2) $Y = f(X_1, X_2) = b_1 + b_2X_1 + b_3X_2 - ½\, [b_4X_1^2 + b_7X_2^2 + b_{10}(X_1-X_2)^2]$

$$+b_4+2[b_5\cos(X_1)-b_6\sin(X_1)]+b_7+2[b_8\cos(X_2)-b_9\sin(X_2)]$$

$$+b_{10}+2[b_{11}\cos(X_1-X_2)-b_{12}\sin(X_1-X_2)]+2[b_{13}\cos(2X_1)-b_{14}\sin(2X_1)]$$

$$+2[b_{15}\cos(2X_2)-b_{16}\sin(2X_2)]+2[b_{17}\cos(X_1+2X_2)-b_{18}\sin(X_1+2X_2)]$$

$$+2[b_{19}\cos(2X_1+X_2)-b_{20}\cos(2X_1+X_2)]$$

where

Y = total wheat yield

b_1 = intercept term

X_1 = total nitrogen, and

X_2 = summer precipitation.

It is not possible to analytically/definitively sign the parameters of equation (3.2) to ensure that the usual regularity/concavity conditions are satisfied. Consider the simplified single-variable version of (3.2), viz.,

$$Y = f(X_1) = a + bX_1 + ½cX_1^2 + 2[u\cos(X_1)-v\sin(X_1)]$$

Taking the first derivative with respect to X_1 to obtain the marginal physical productivity (MPP_1) yields

$$df/dX_1 = MPP_1 = b + cX_1 - 2[u\sin(X_1) + v\cos(X_1)],$$

and the second derivative with respect to X_1 is given by

$$d^2f/dX_1^2 = dMPP_1/dX_1 = c - 2[u\cos(X_1) - v\sin(X_1)].$$

Regularity conditions require that $MPP_1>0$, i.e.,

$$b + cX_1 - 2[u\sin(X_1) + v\cos(X_1)] > 0 \text{ or } b + cX_1 > 2[u\sin(X_1) + v\cos(X_1)];$$

and that $dMPP_1/dX_1 < 0$, i.e.,

$$c - 2[u\cos(X_1) - v\sin(X_1)] < 0 \text{ or } c < 2[u\cos(X_1) - v\sin(X_1)].$$

Clearly, validation of the above depends on the signs and relative magnitudes of the parameters b, c, u, and v and the sin-cos terms. Since the value of X_1 is constrained between 0 and 2π, the value of sin and cos terms must lie between 0 and 1, which means the sin-cos terms per se have only a minor effect on the function -- the heavier weight of effect rests with the values of u and v. If the values of u and v are small enough or off-setting so that b and c dominate the function, then the sign expectations which apply to a quadratic function may apply to the Fourier function as well. If this is the case, then sign expectations for the Fourier functional form in equation (3.2) are for b_2 and $b_3>0$, $(b_4+b_{10})>0$ and $(b_7-b_{10})>0$.

Estimation Technique

Given any m-times differentiable g(x), it is possible to choose coefficients θ_j such that $\lim_{K\to\infty} ||g-g_K||_m = 0$; one can choose θ_j independently of knowledge of order m as it turns out (Edmunds and Moscatelli, 1975). While such coefficients θ_j may exist, which specific statistical estimation procedure should be used? This question was answered by Elbadawi et al. (1983). They found that coefficients computed using any of the common estimation methods--least squares, maximum likelihood, two- or three- stage least squares, etc.--all have the desired properties of asymptotical unbiasness, efficiency and consistency. Therefore, choice of estimation method is merely a matter of convenience or preference. Elbadawi et al. further argued that a multivariate Fourier series expansion is a very convenient choice from a mathematical point of view. High-order derivatives are easily obtained, and it is easy to deduce the parameter restrictions that are necessary and sufficient for homotheticity, constant returns, etc.

An important issue in Fourier model estimation is computational considerations. The following steps are necessary in using Gallant's procedure:

1. Scaling the data. A Fourier series is a periodic function in each of its arguments and most production functions are not. A Fourier series approximation of a true function $g^*(x)$ can be made as accurate as desired on a region, say V, which is completely within the cube $x_{i=1}^{N}$ $[0, 2\pi]$, but the approximation will diverge from $g^*(x)$ for $X<0$ or $X>2\pi$ due to its

periodic nature, the so-called Gibb's phenomenon. One way to compensate for this feature of Fourier series expansions is by rescaling the independent variable observations so that the values of the observations are between 0 and 2π. Should the need occur to use the original units when reporting results, it can be accomplished by reverse scaling after the coefficients of the expansion have been estimated. The scaling formula used in this thesis was

$$X_i^* = \frac{2\pi \times X_i}{MAX.X_i}$$

2. Choosing k, the number of directions. The requisite sequence of multi-indexes,

$$\{k_\alpha: \alpha = 1, 2, ..., A\},$$

may be constructed from the set

$$\mathcal{K} = \{k: |k|^* \leq K\}$$

as follows. First, delete from $\mathcal{K}$ the zero vector and any k whose first non-zero element is negative; i.e., (0,-1,1) would be deleted but (0,1,-1) would not. Second, delete any k whose components have a common integral divisor; i.e., (0,2,4) would be deleted but (0,2,3) would remain. Third, arrange the k remaining vectors into a sequence

$$\{k_\alpha: \alpha = 1, 2, ..., A\},$$

such that $|k_\alpha|^*$ is non-decreasing in α and such that $k_1, k_2,..,k_N$ are the elementary vectors. The sequence $\{k_\alpha\}$ used in this thesis is presented as (1,0) (0,1) (1,-1) (2,0) (0,2) (1,2) (2,1). for N=2, J=1, k=7 and K=3. In general, it can be expected that the approximation improves as n and k increase together. As an extreme case, when n tends to infinity the curve derived from the Fourier function will coincide with the curve derived from the approximated function over the interval estimated, provided k increases with n.

3. Choosing J. A longer length, J, of the Fourier expansion may not ensure a better approximation. This is an important unresolved problem. However, in practice that J is chosen for which the model fit is judged "best" in some overall sense.

In econometric literature, there appears to have been a strong urge by many to settle on some single statistic or index to gauge the "goodness" of an econometric model. One natural competitor for this honor is the coefficient of multiple determination, R^2, which is a measure of the proportion of the total variance accounted for by the linear influence of the explanatory variables. As a basis for model choice, the R^2 measure has an obvious fault; it can be increased by increasing the number of explanatory variables. Since there are only two explanatory variables used in the Fourier function in this study [see eq. (3.1)], the main flaw in using R^2 as a basis for model choice can be avoided.

The Akaike information criterion (AIC) seeks to incorporate in model selection the divergent considerations of accuracy of estimation and the "best" approximation to reality. Thus, use of this criterion involves a statistic that incorporates a measure of the precision of the estimate and a measure of the rule of parsimony in the parameterization of a statistical model. The detailed procedures of the AIC test are presented as follows.

Let $L(\beta|y)$ be the likelihood function and consider the statistical model,

$$y = X\beta + e = X_1\beta_1 + X_2\beta_2 + e, \tag{3.3}$$

where the design matrix X is partitioned into components X_1 and X_2 with corresponding partitioning of the β vector. Under the hypothesis $R\beta = (0_{(k2\times k1)}, I_{k2})\beta = 0$, i.e., the last $k-k_1=k_2$ elements of β are assumed to be zero, the AIC is

$$AIC = -\frac{2}{T} \ln L(b_1^*|y) + \frac{2K_1}{T} \tag{3.4}$$

Under the AIC criterion, equation (3.4) is to be minimized among all of the possible linear hypotheses $R\beta = 0$. For the statistical model $y=X\beta+e$, the criterion reduces to

(3.5) $$AIC_{(R\beta=0)}=\ln \frac{y'M_1y}{n} + \frac{2K_1}{n}$$

where K_1(number of the parameters of the first partitioned ß vector) and n is the number of observations. Thus $M_1=I - X_1(X_1'X_1)^{-1}X_1'$ is chosen so as to numerically minimize the AIC value. Therefore, as K_1 increases, $y'M_1y$ decreases and the value of the likelihood function increases; thus the trade-off between parsimony and precision is clear, and the penalty for increasing the number of parameters is explicit. Practically, in this thesis the following formula for AIC was used:

(3.6) $$AIC = \ln SSE - \ln n + 2K/n$$

where SSE is Sum of Square Error.

Results obtained using the model and estimation procedures presented in this chapter are the topic of Chapter 4.

CHAPTER 4

ESTIMATION OF THE FOURIER FLEXIBLE FUNCTIONAL FORM: RESULTS

It is common knowledge, of course, that noisy data -- data for which there is essentially no relationship between the explained and the explanatory variables -- reduces to impotence the results of attempts to estimate any functional form. Unfortunately, this plague was encountered in this study. This chapter begins with a description and discussion of the data set; then in the following two sections, inherent problems in the original and reduced data sets are revealed and discussed. Finally, in the last two sections, given estimation failures and data problems encountered, the question of whether the Fourier functional form can be successfully used to approximate a response production function is addressed. In each section, the empirical results obtained are presented and discussed.

The Data Set

The data used to fit the yield response functions were generated from a research project of the Southern Agricultural Research Center, Huntley, Montana, on experimental plots located on 18 farms in south central Montana. A total of 30 treatments of various combinations of nitrogen, phosphorus, and potassium were applied to each of 30 total sites over five years (1976-1980). Periodically the experiment was replicated some years on the 18 farms, hence accounting for 30 total experiments. Furthermore, among the 30 fertilizer treatments per site, there were seven replications. The product of 30 treatments and 30 site-years resulted in 900 observations. A complete discussion of the data set is presented in Appendix 1 of Wessells (1984).

Site Consideration and Experimental Procedures

Selection of experimental sites was based on a specific set of characteristics. Namely, the locations chosen were adjacent sites in a crop-fallow rotation having uniform soil conditions and unhindered by insects, weeds, and other detrimental factors. In further efforts to achieve consistent soil types, surface color and texture, and strata concentrations of soil nitrate, elemental phosphorus, and potassium were considered (Haby and Larson, 1976). Sites which received hail damage to crops or suffered from unusually high saline content were excluded.

In the fall, prior to seeding, soil samples were analyzed for each plot. In the spring, before applications of nitrogen, soil nutrients were measured again to check for any appreciable change over winter.

Applications of nitrogen, phosphorus, and potassium were each calibrated precisely at five rates. The nitrogen fertilizer, ammonium nitrate, was top dressed in the spring at rates varying from 0 to 89 kg/ha in increments of 22.25 kg. Phosphorus, in the form of triple superphosphate, was banded with seed at rates ranging from 0 to 35.7 kg/ha in increments of 8.925 kg.

Centurnk winter wheat was seeded in 30 cm row widths at a rate of 50 kg/ha. Weeds were sprayed with 0.42 kg/ha of (2,4-dichlorophenoxy) acetic acid (2,4-D) amine plus 0.125 kg/ha Banvel.

Measurements of weather conditions were made for each site. Summer precipitation (April through July), temperature, and estimates of evapotranspiration were the primary factors subject to observation. Any missing data were substituted with similar data from the nearest weather station. Researchers did not record winter precipitation (September through March) at each site. Consequently, winter precipitation was taken from records of the nearest weather station with the same average precipitation gradient as the site in question.

Scatter Diagrams and Data Problems

The two-dimension relationships of yield vs. input(s) in the original data set are presented in Figures 5 through 14. From these scatter diagrams, the following problems are revealed.

First, there is clearly little relation between yield and total phosphorus. Figure 8 is essentially dense with data points distributed everywhere on the plate. No doubt, results obtained from any attempt to estimate a phosphorus-yield response function would be meaningless. For this reason, phosphorus was not included as an explanatory variable in the model.

Second, as mentioned in the previous section, the fertilizer inputs, both nitrogen and phosphorus, were applied at only five different levels and the observed yield levels ranged widely for each application level (see Figure 5 and 7). For these reasons and because plants respond to both applied and soil nitrogen, total nitrogen, X_1, was included as an input instead of just applied nitrogen in the Fourier model.

Third, since the data for winter precipitation were taken from the nearest weather station, the summer precipitation, X_2, is probably more relevant for response function estimation than winter precipitation. Poorer estimation results in terms of R^2, t-ratios and AIC confirmed this allegation when total precipitation was included in the model as an explanatory variable. The models with summer precipitation only had higher R^2 and lower AIC than those that included total precipitation for those cases where "correct" parameter

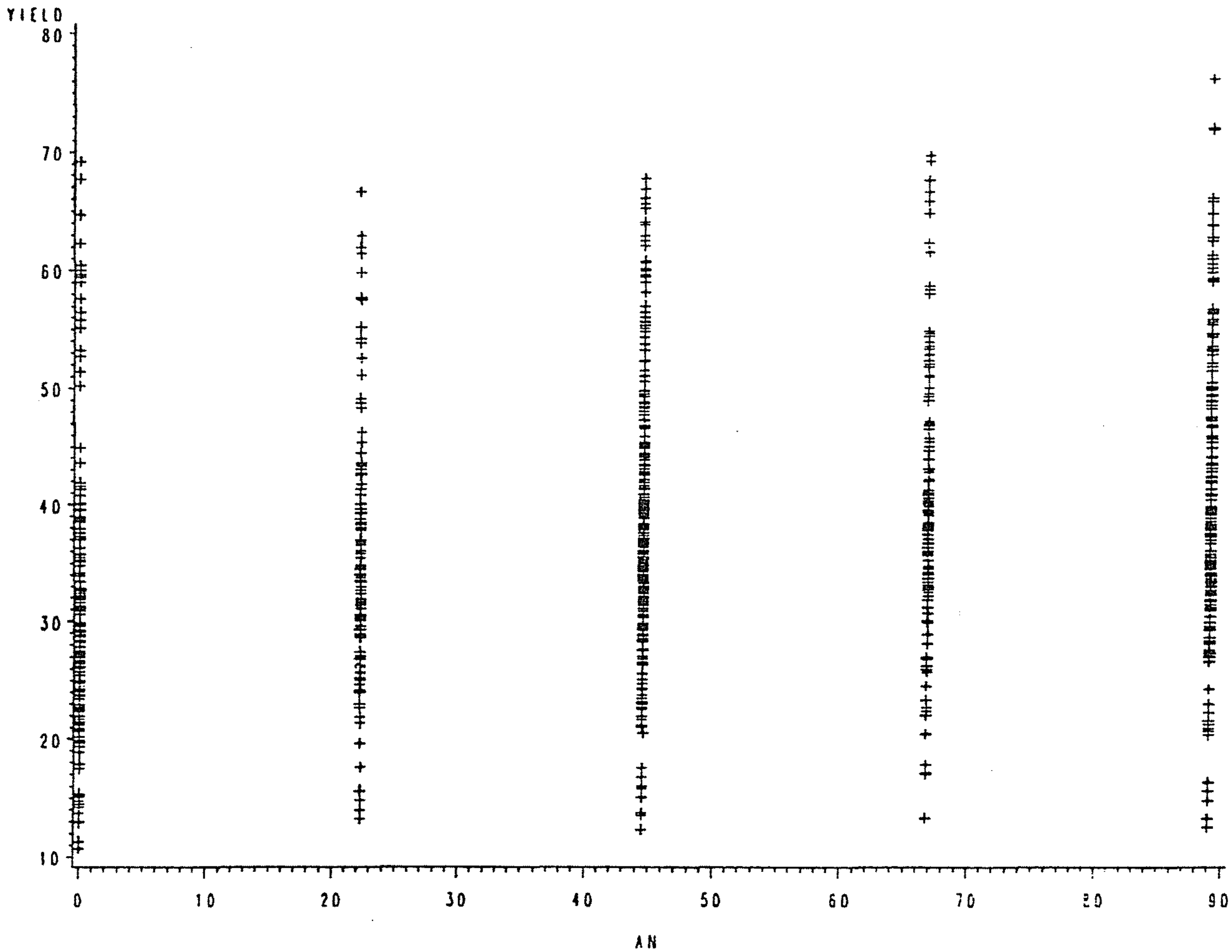


Figure 5. Yield vs. Applied Nitrogen (AN) Using 900 Observations.

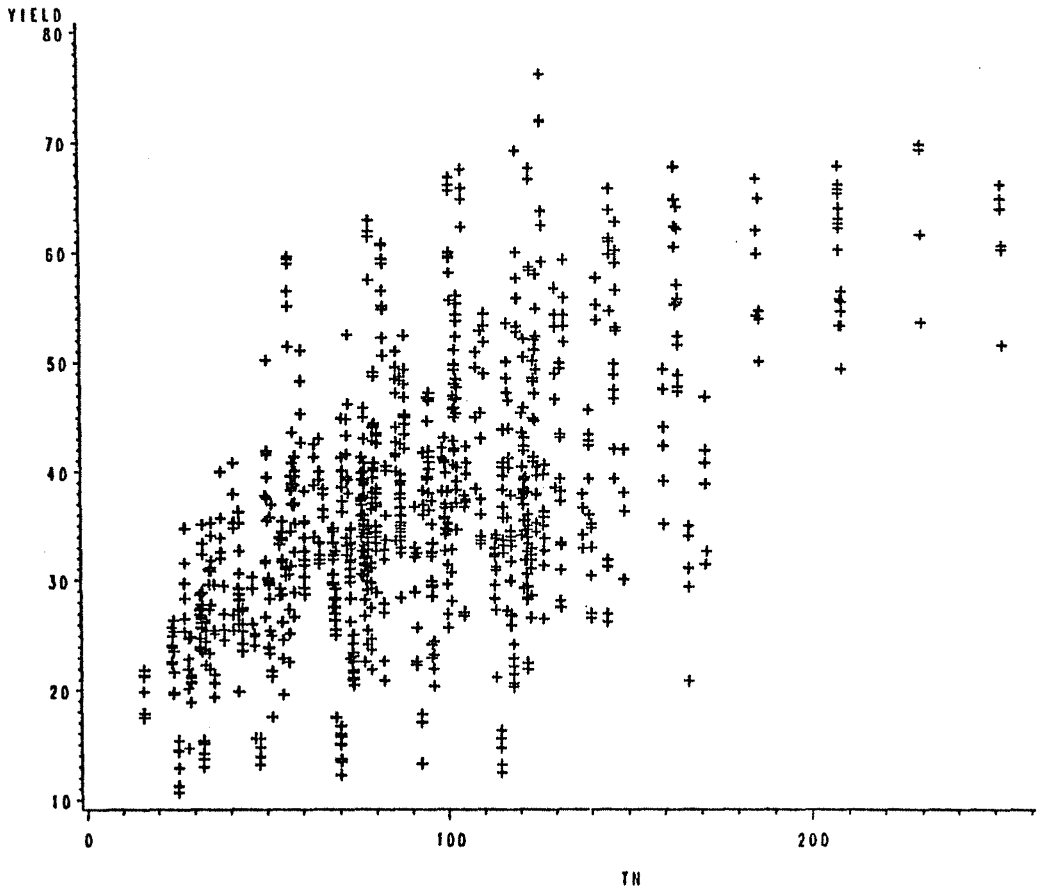


Figure 6. Yield vs. Total Nitrogen (TN) Using 900 Observations.

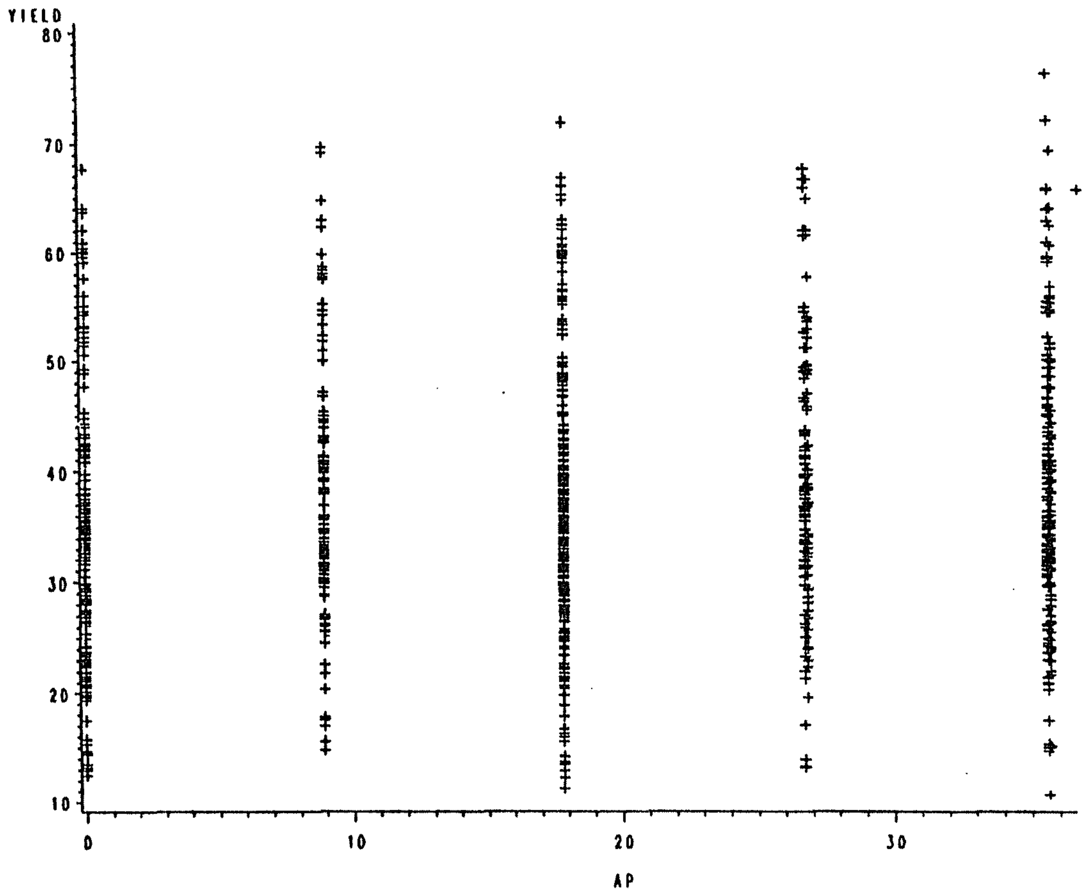


Figure 7. Yield vs. Applied Phosphorus (AP) Using 900 Observations.

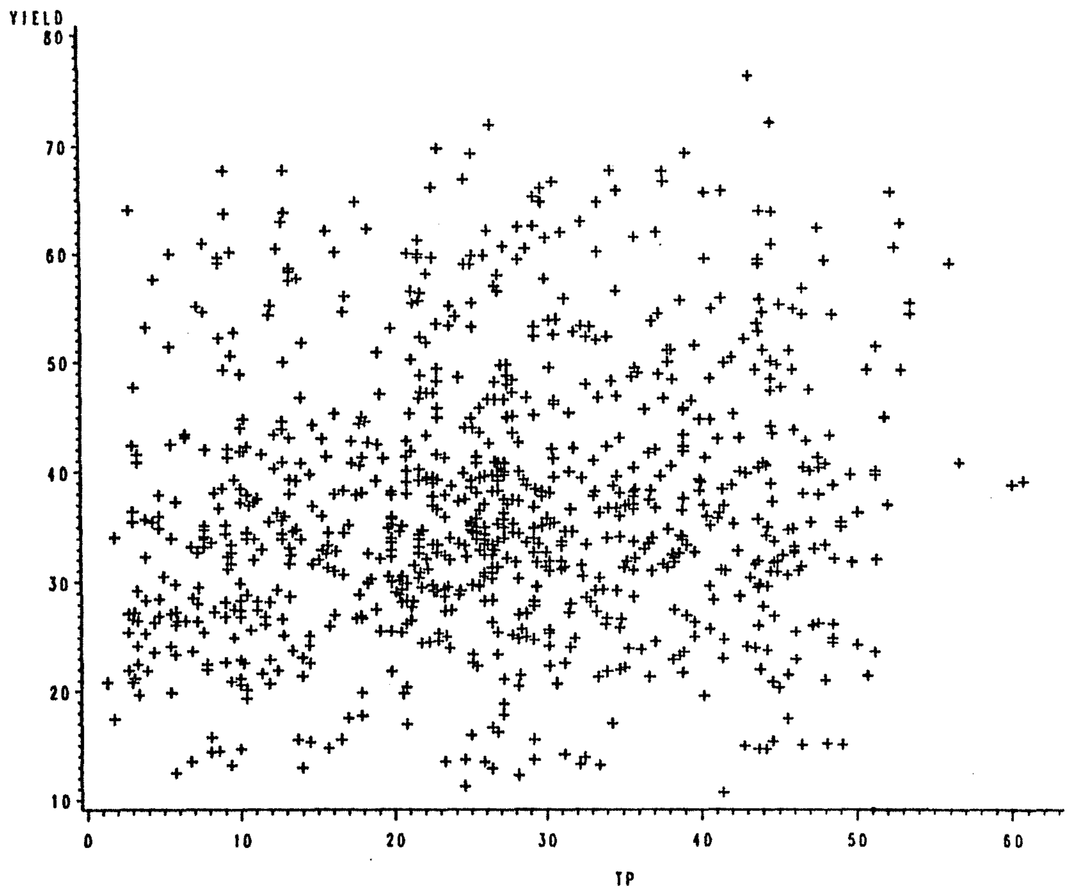


Figure 8. Yield vs. Total Phosphorus (TP) Using 900 Observations.

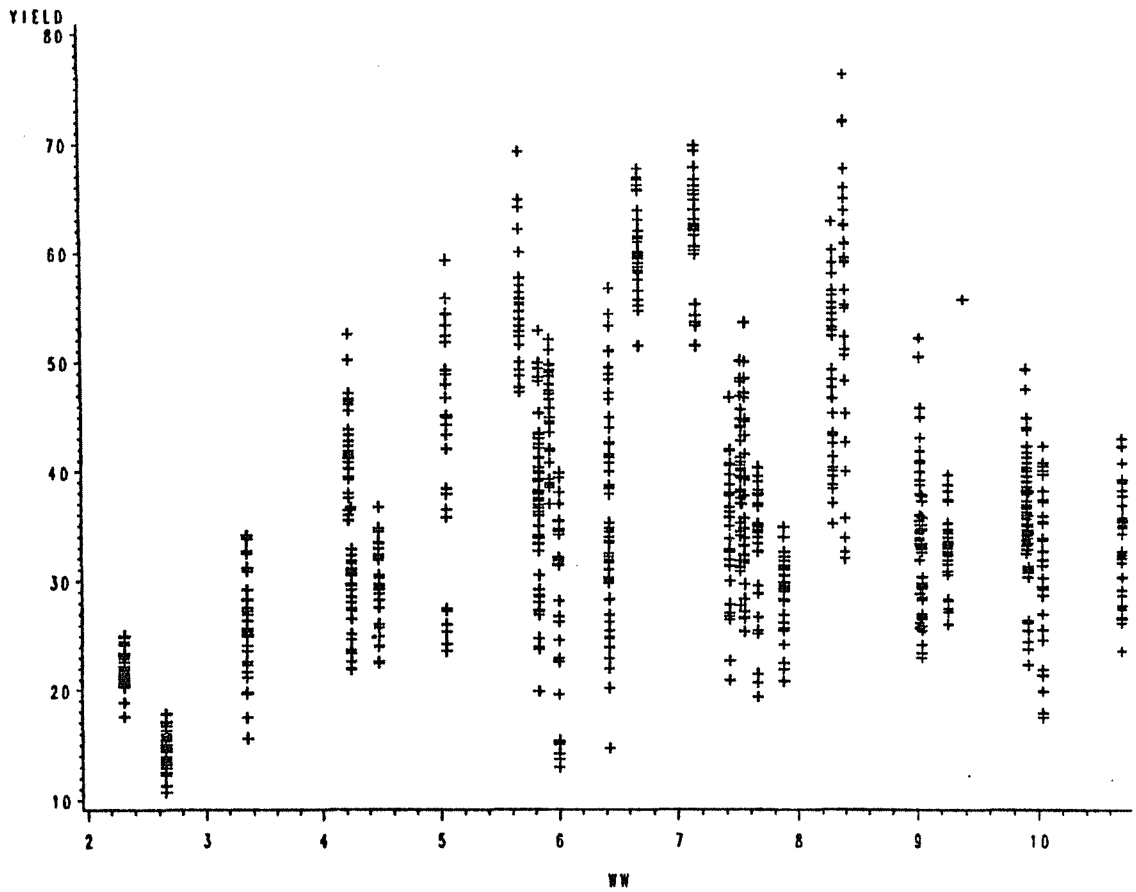


Figure 9. Yield vs. Winter Precipitation (WW) Using 900 Observations.

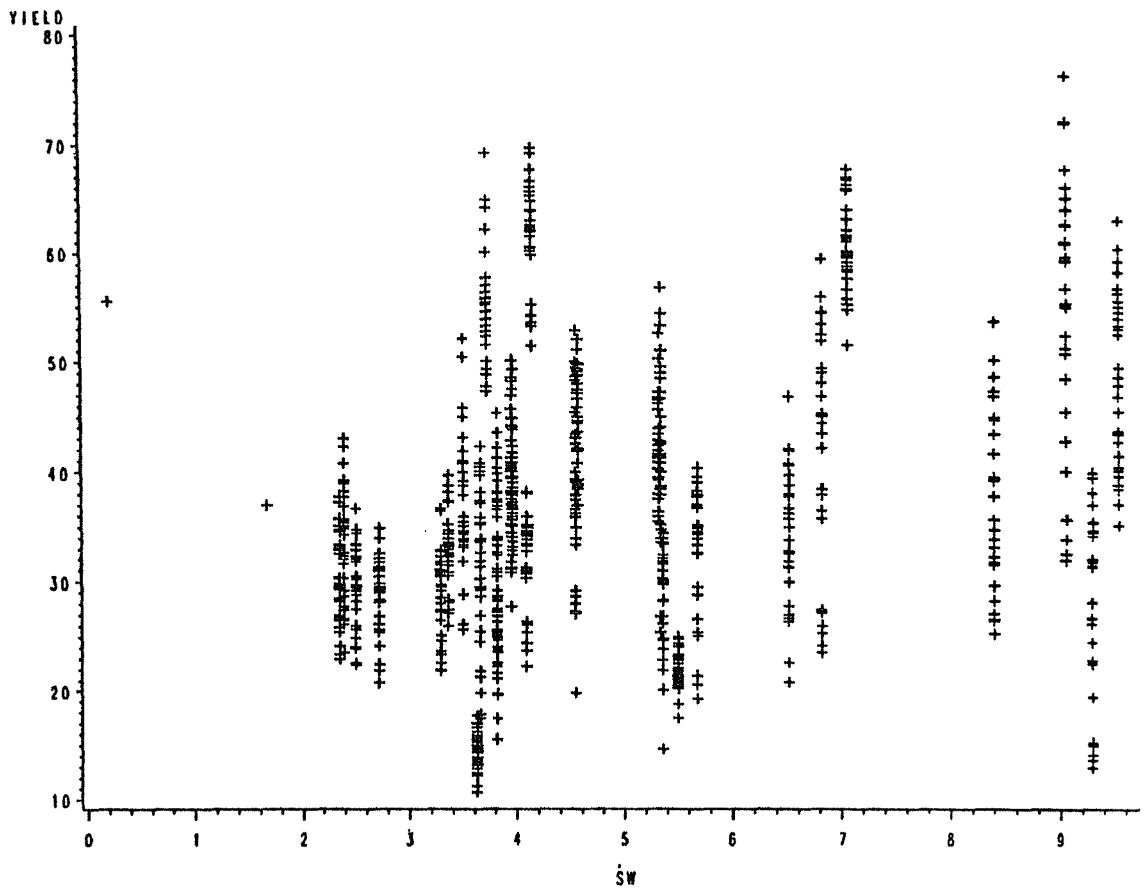


Figure 10. Yield vs. Summer Precipitation (SW) Using 900 Observations.

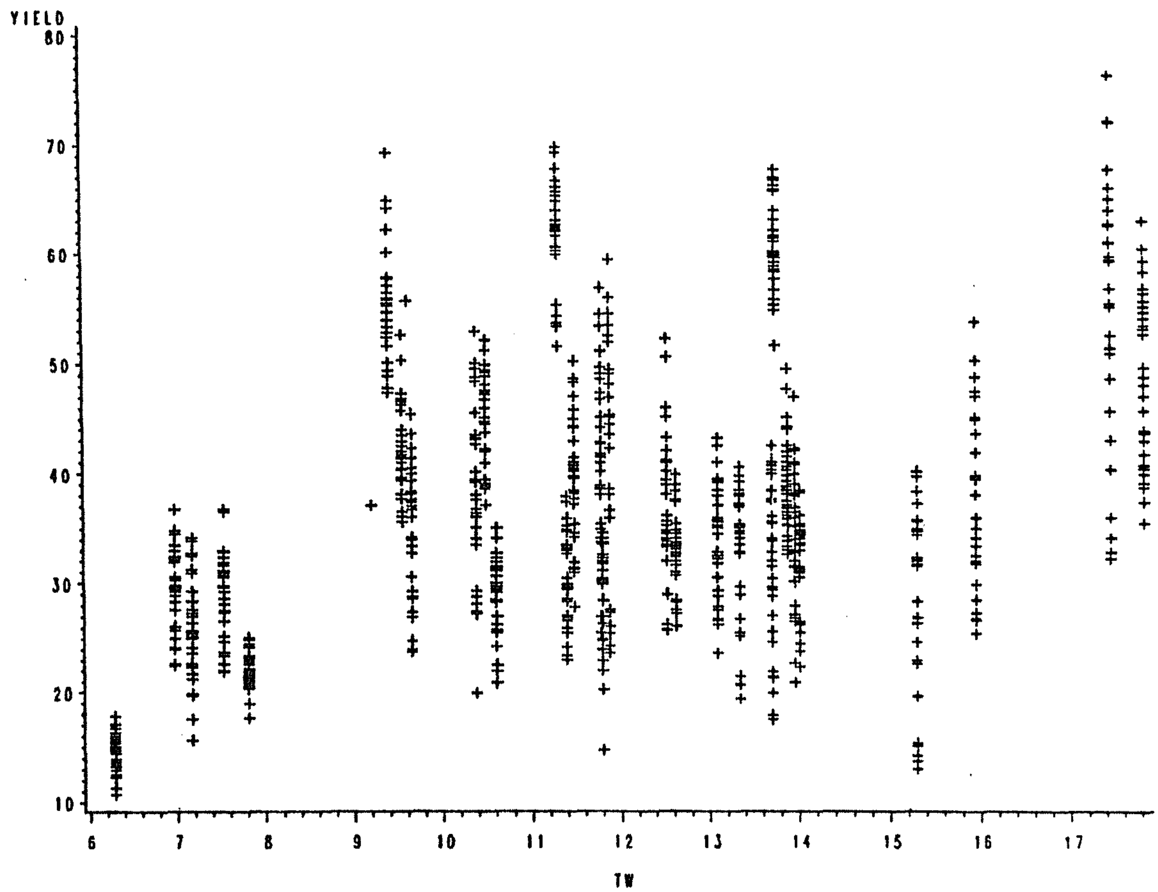


Figure 11. Yield vs. Total Precipitation (TW) Using 900 Observations.

signs were obtained for the linear and quadratic terms of the explanatory variables.

In summary, based on the scatter diagrams (Figure 5-11) and preliminary estimation results, it was decided that the best hope was to specify the Fourier model with just two explanatory variables, total nitrogen (X_1) and summer precipitation (X_2) -- see equation (3.2). Results reported in this chapter are limited to this two-variable version of the models.

Results: Full Data Set (900 Observations)

The estimated parameters and related statistics with estimation based on the full data set are presented in Table 1. Problems include: (1) the signs of parameters on X_2, both linear and quadratic terms, are not correct according to economic theory, i.e., the global concavity requirement that $f_{22}<0$ is violated; (2) R^2 was not particularly high considering the large sample of 900 observations (see also Figures 12 and 13); (3) the average level of the estimated values of yield are much higher than those observed in the original data set, which suggests that the estimated model is probably not acceptable.

Results: Reduced Data Set (450 Observations)

Since the results using the full data set (with phosphorus excluded) were untenable, a careful examination of the data set was conducted. Eighteen separate scatter diagrams (one for each plot/site) were screened and several troublesome plots were discovered. Most noisy observations were focused in 10 of 18 plots, accounting for 450 observations. In an attempt to rid the estimated equation of the noisy effect, the 450 data points for the 10 troublesome plots were deleted from the original data set. Deleted plots were those where the individual scatter diagrams depicted an abnormal pattern. Scatter plots for total nitrogen vs. yield and summer precipitation vs. yield are presented in Figures 14 and 15, respectively.

Table 1. Parameter Estimates and Related Statistics, 900 Observations.

Parameter	Estimate	t-ratios	Interpretation
b_1	61.7482	2.2297	Intercept term
b_2	18.8479	1.6783	Linear effect for X_1
b_3	-31.5602	-2.6178	Linear effect for X_2
b_4	3.0334	0.9000	Quadratic effect for X_1
b_5	5.1836	1.5627	
b_6	-3.5049	-3.2937	
b_7	-8.4447	-2.4816	Quadratic term of X_2
b_8	-2.1726	-0.7532	
b_9	9.6555	5.7116	
b_{10}	2.3486	4.5395	Interaction effect for X_1 & X_2
b_{11}	-0.8288	-2.3836	
b_{12}	-1.2754	-2.1759	
b_{13}	0.3897	0.5661	
b_{14}	0.0406	0.0919	
b_{15}	0.5803	0.8727	
b_{16}	4.4796	8.7178	
b_{17}	-1.6028	-4.5184	
b_{18}	0.4764	1.4166	
b_{19}	0.0437	0.1658	
b_{20}	0.3272	1.1716	

Multiple R_2 = 0.5282. AIC value = 4.3161.

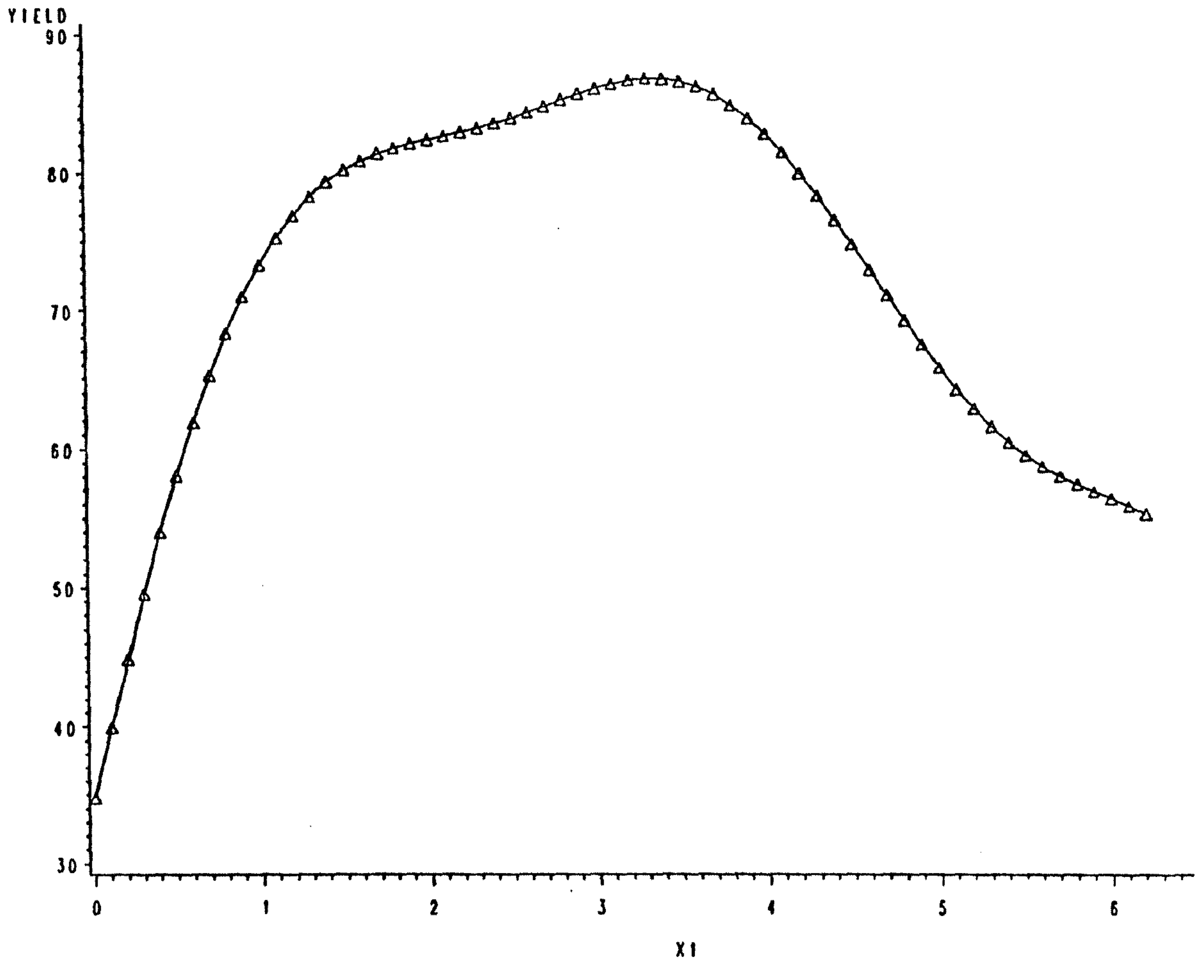


Figure 12. Estimated Yield vs. Nitrogen X_1 (Full Data Set).

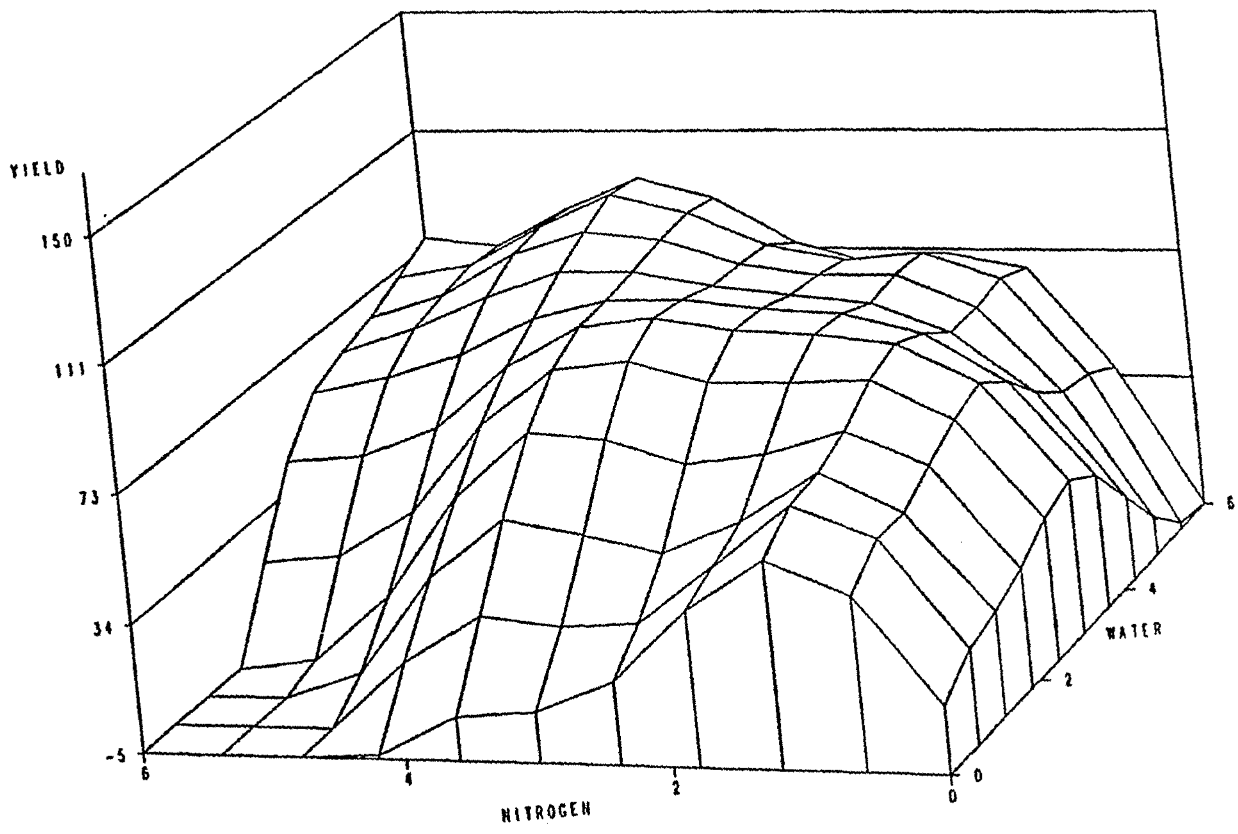


Figure 13. Estimated Response Surface Based on the 900 Observation-Derived Parameters.

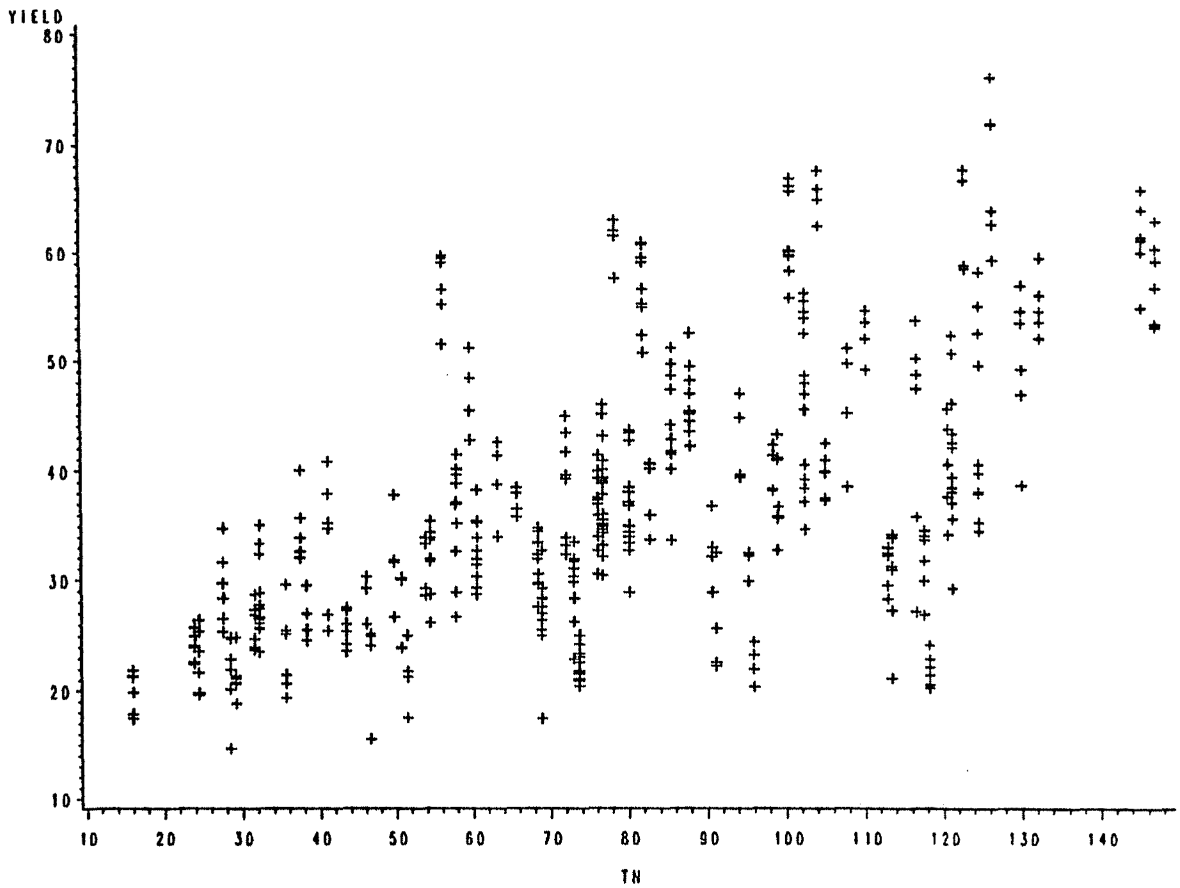


Figure 14. Yield vs. Total Nitrogen (TN) Using 450 Observations.

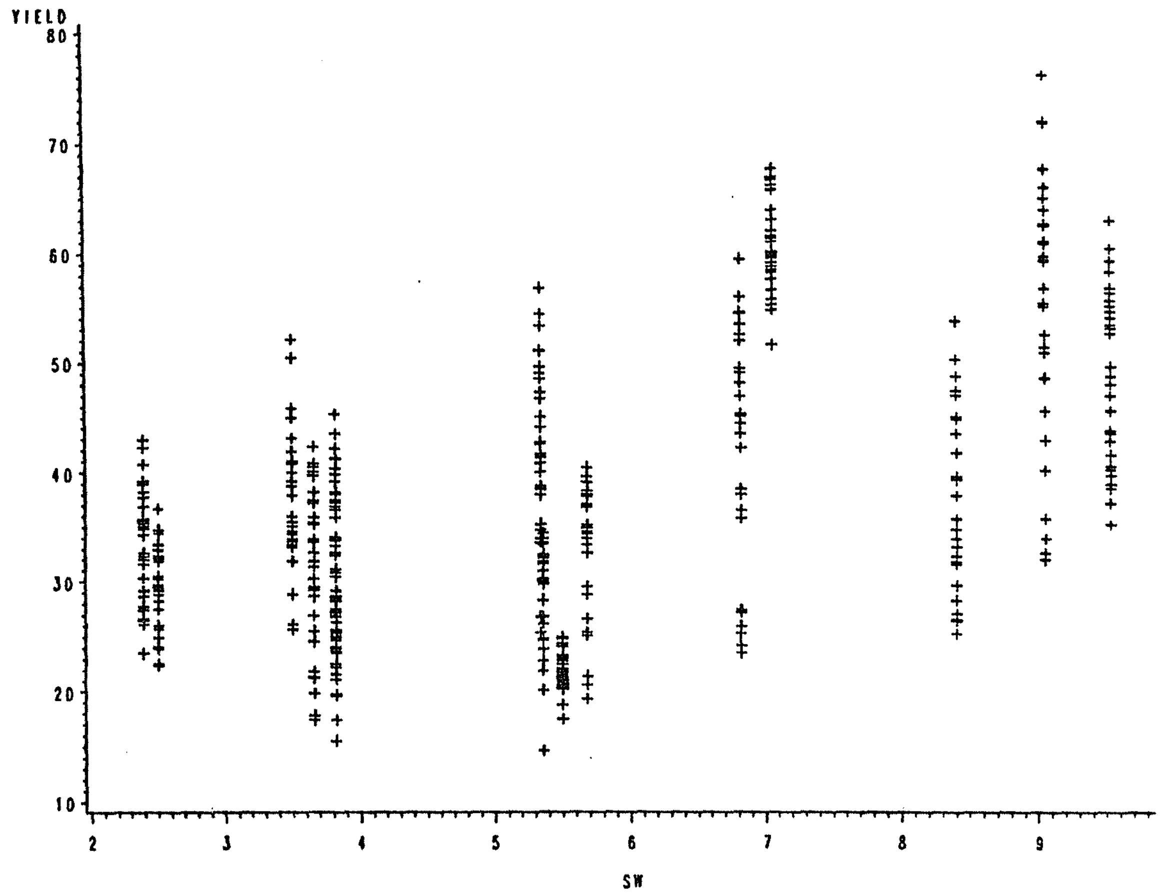


Figure 15. Yield vs. Summer Precipitation (SW) Using 450 Observations.

The estimated parameters and related statistics for the reduced data set are presented in Table 2. Unfortunately, the reduced data set (450 observations) exclusive of the 10 questionable plots, shows only a slight improvement in statistical results and the signs of estimated parameters. The coefficient on the quadratic term for summer precipitation (b_7) is positive, while R^2 increased from 0.5282 to 0.6740 and AIC value decreased from 4.3161 to 3.9976. Estimated yield values increase at a decreasing rate at first, and then increase but at an increasing rate as the level of total nitrogen increases, ceteris paribus (see Figures 16 and 17). This result departs far from the "diminishing return" hypothesis.

Results: Stylized Data Set (388 Observations)

Seemingly every reasonable effort, including elimination of plots and adjusting the combination of the multi-indices k matrix, was made to obtain "well-behaved" results, but to no avail. Finally, just for the heck of it, all outliers in the original 900 observation data were deleted. This exercise yielded a much reduced data set consisting of only 388 observations (see Figures 18 and 19). In view of the estimation problems discussed in the two preceding sections, the purpose of this exercise was to see if the Fourier function would produce a well-behaved fit given a hand-selected or stylized data set.

Not surprisingly, results using this stylized data set were much "improved." The estimated parameters based on the 388 observations include correct signs on both linear and quadratic terms of the Fourier function. Of course, the R^2 and AIC improved significantly compared with those derived from the full data set. The statistical results are summarized in Table 3. Given the manner in which the data were selected, it should be emphasized that the usual inferences and interpretations of statistics presented in Table 3 should be avoided.

Given that a good data set is available, the Fourier function should be a powerful

Table 2. Parameter Estimates and Related Statistics, 450 Observations.

Parameter	Estimate	t-ratios	Interpretation
b_1	2.3440	1.6700	Intercept term
b_2	15.8977	2.1828	Linear effect for X_1
b_3	18.9649	1.4331	Linear effect for X_2
b_4	4.5055	2.4899	Quadratic effect for X_1
b_5	0.0123	3.7725	
b_6	-6.4037	-2.1087	
b_7	-2.5958	-1.4479	Quadratic effect for X_2
b_8	0.0710	1.4548	
b_9	0.5376	1.4607	
b_{10}	4.8342	0.2132	Interaction effect for X_1 & X_2
b_{11}	0.6659	0.9010	
b_{12}	-0.7742	-0.9205	
b_{13}	-0.4008	2.8920	
b_{14}	-5.3200	-0.1166	
b_{15}	1.6270	-1.0495	
b_{16}	0.9002	1.5653	
b_{17}	1.0203	-0.0626	
b_{18}	-1.8591	-1.0265	
b_{19}	-2.4059	-1.1629	
b_{20}	-2.0993	-1.0621	

Multiple R_2 = 0.6740. AIC value = 3.9976

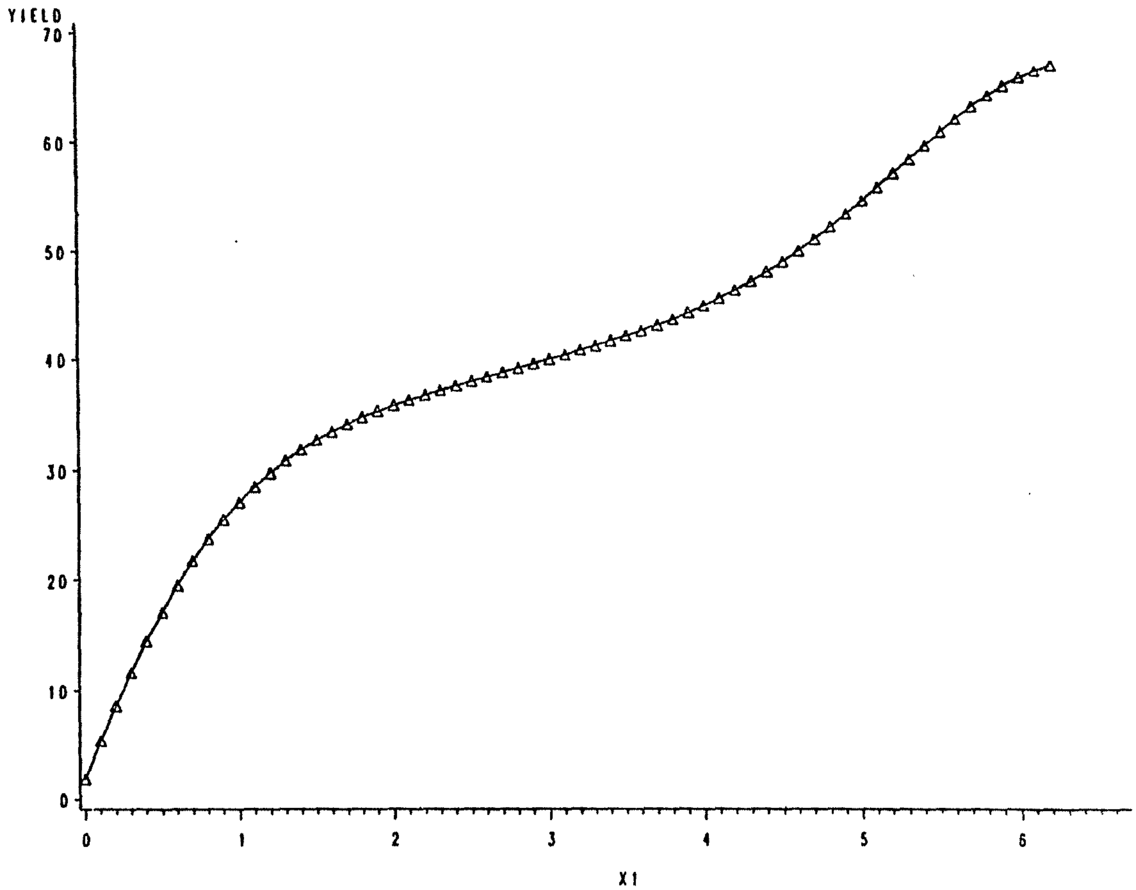


Figure 16. Estimated Yield vs. Nitrogen X_1 (Reduced Data Set).

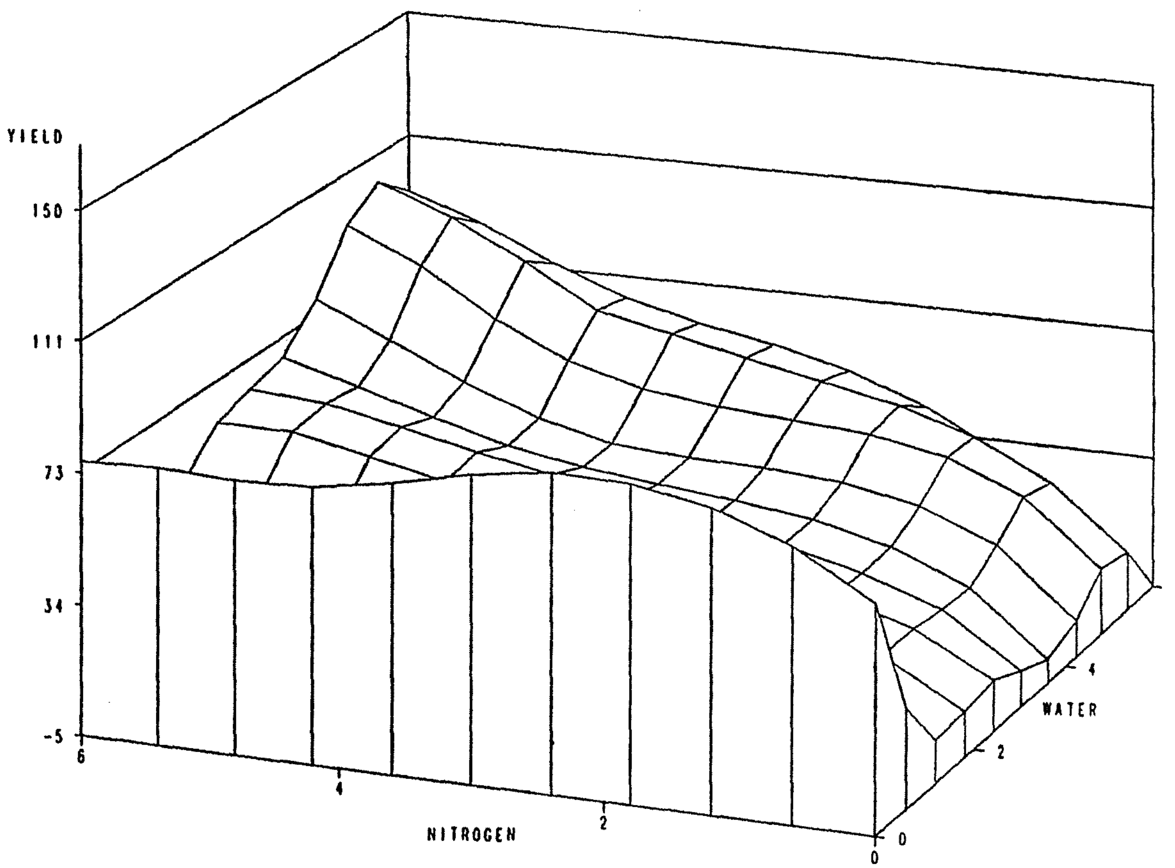


Figure 17. Estimated Response Surface Based on the 450 Observation-Derived Parameters.

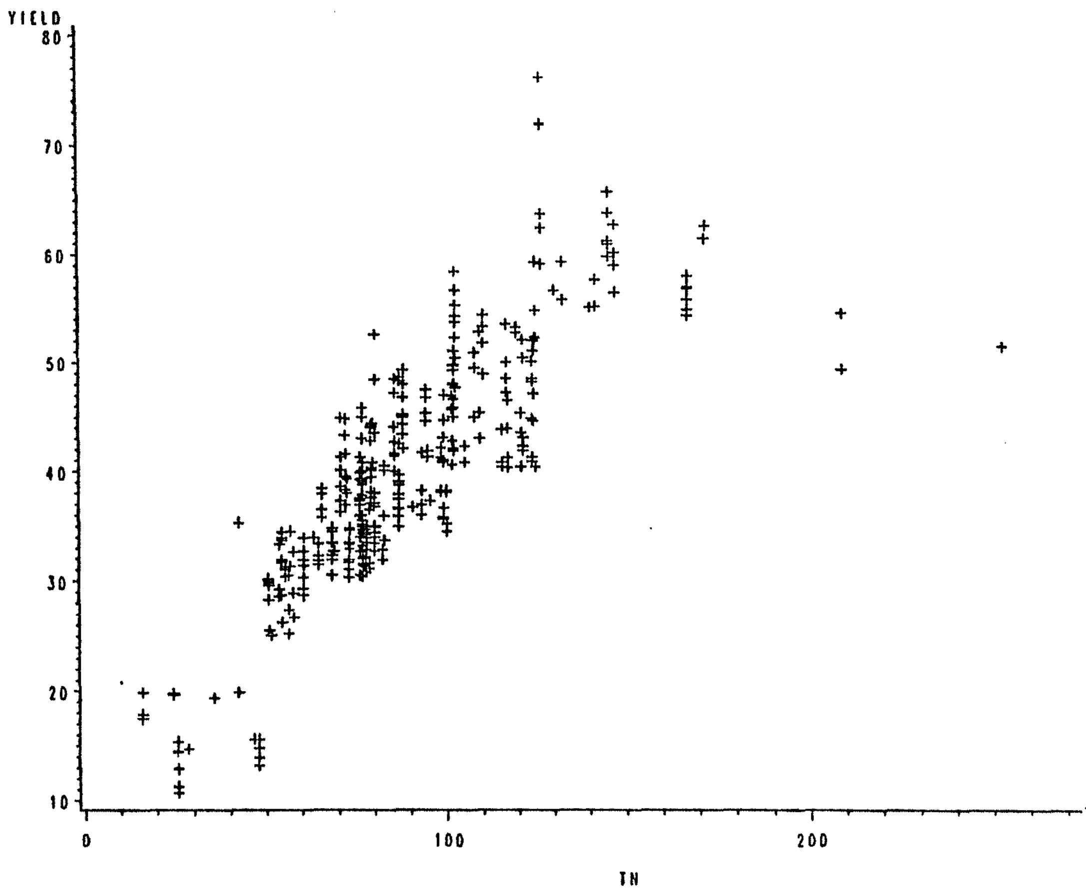


Figure 18. Yield vs. Total Nitrogen (TN) Using 388 Observations.

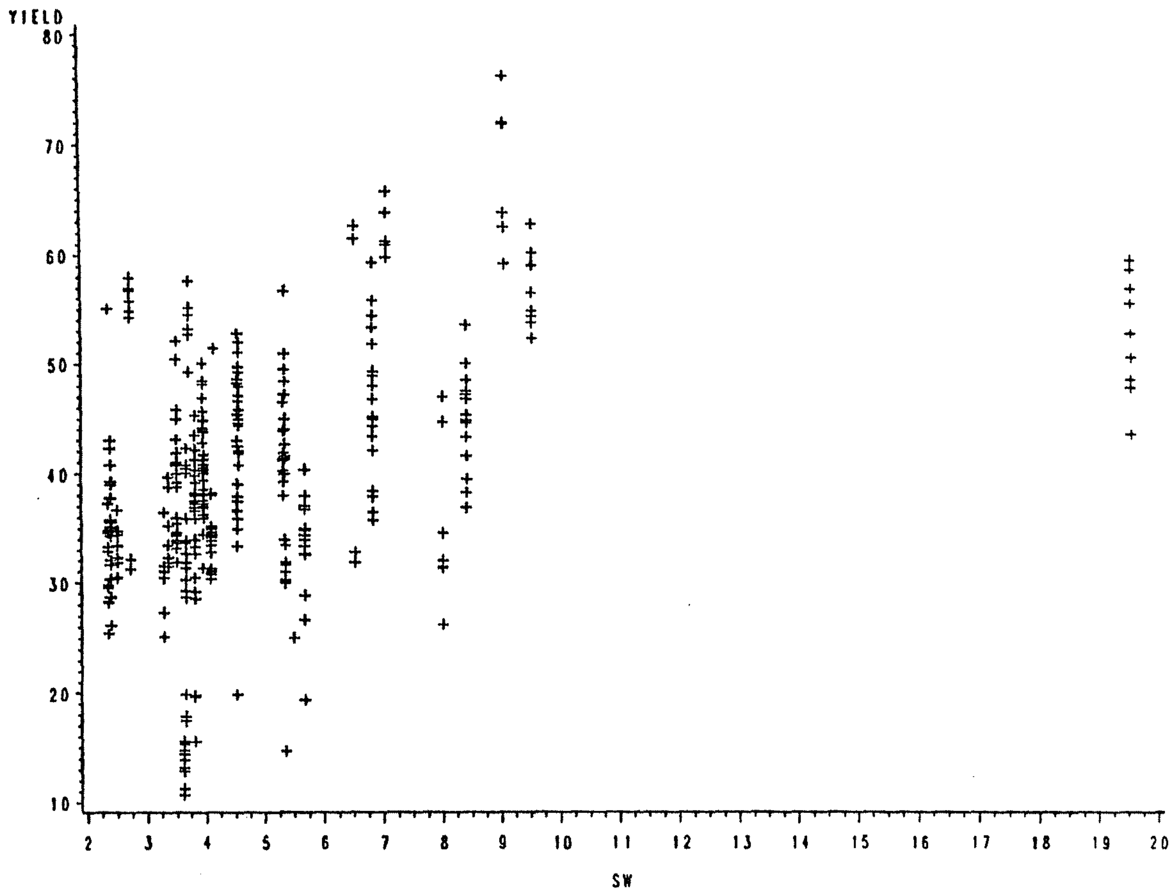


Figure 19. Yield vs. Summer Precipitation (SW) Using 388 Observations.

Table 3. Parameter Estimates and Related Statistics, 388 Observations.

Parameter	Estimate	t-ratios	Interpretation
b_1	-435.1966	-3.3556	Intercept Term
b_2	41.2015	2.7576	Linear effect for X_1
b_3	220.8155	2.9649	Linear effect for X_2
b_4	10.1792	2.4078	Quadratic effect for X_1
b_5	8.4421	2.2856	
b_6	-4.2827	-2.4994	
b_7	53.1899	2.8409	Quadratic effect for X_2
b_8	29.0494	2.6365	
b_9	-38.4526	-3.1183	
b_{10}	0.8616	2.4163	Interaction effect for X_1 & X_2
b_{11}	-0.0306	-0.0688	
b_{12}	0.5343	1.4201	
b_{13}	1.4873	2.1693	
b_{14}	-1.6183	-2.6343	
b_{15}	-5.5102	-6.6895	
b_{16}	-6.5070	-2.4780	
b_{17}	-0.1159	-0.4056	
b_{18}	-0.1897	-0.6195	
b_{19}	0.0272	0.0959	
b_{20}	-0.0806	-0.2913	

Multiple R_2 = 0.8394. AIC value = 3.0520.

instrument to approximate any known differentiable function. Figure 20 shows that the two-dimensional yield response curve based on estimates using the 388 point data set has a quadratic relationship at first until a certain stage (maximum yield), and then plateaus -- a "quasi" not strict plateau due to the Gibb's phenomena. The three-dimensional estimated response function is presented in Figure 21.

Final Thoughts/Conclusions

This chapter has claimed that data problems of this study were severe and compared the results from the original data set with two alternative "trimmed" data sets. These three data sets represent three different attempts to fit the Fourier function form to experimental data on winter wheat response to nitrogen and precipitation in south central Montana. The last data set, with 388 observations, is stylized in terms of its statistical properties, i.e., independent identical distribution (iid). Potentially, both independent (X) and dependent (Y) variables are stochastic. If Y could be assumed to be iid and covariance stationary, then the least square estimates could asymptotically converge to the estimates θ. This is a necessary condition for using Gallant's assumption: $E(y|x) = f(x)$ and $\min\{f(x)-f_k(x)\}$ (minimize the average prediction error). In other words, the strategy, i.e., reducing and stylizing the original data set, is equivalent to making the form and properties of the noise known prior to estimation.

Based on the results from previous sections, the conclusions are: (1) since the Fourier function is very sensitive to the data, one should use this highly flexible functional form only when a data set with normal statistical properties is available; (2) once armed with a good data set, the Fourier function should perform a fairly good approximation to a differentiable function, *ceteris paribus*; and (3) the results using the stylized data set seem to support the hypothesis of the Mitscherlich-Baule for nitrogen, i.e., a smooth upward curve followed by a plateau as suggested by Frank et al. (1989). The strange relationship implied by Figure 21 for increasing precipitation is inexplicable.

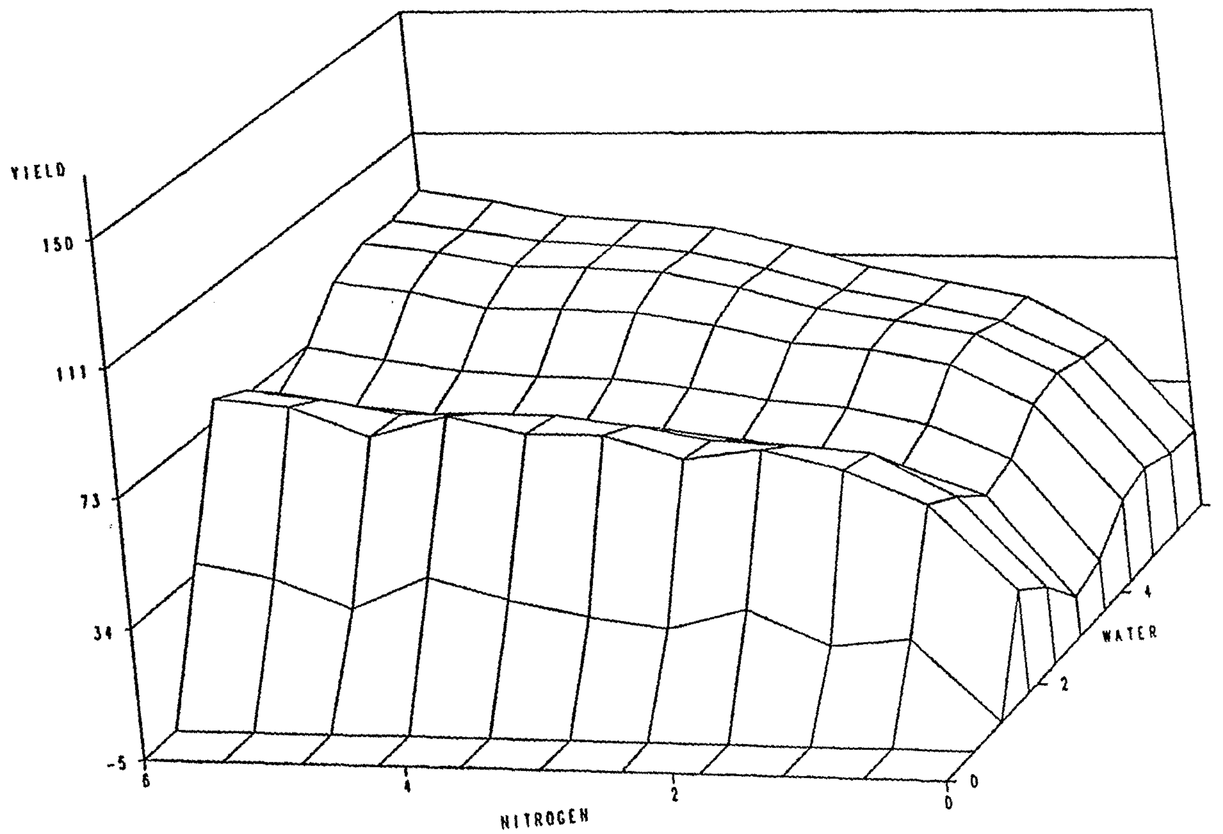


Figure 21. Estimated Response Surface Based on the 388 Observation-Derived Parameters.

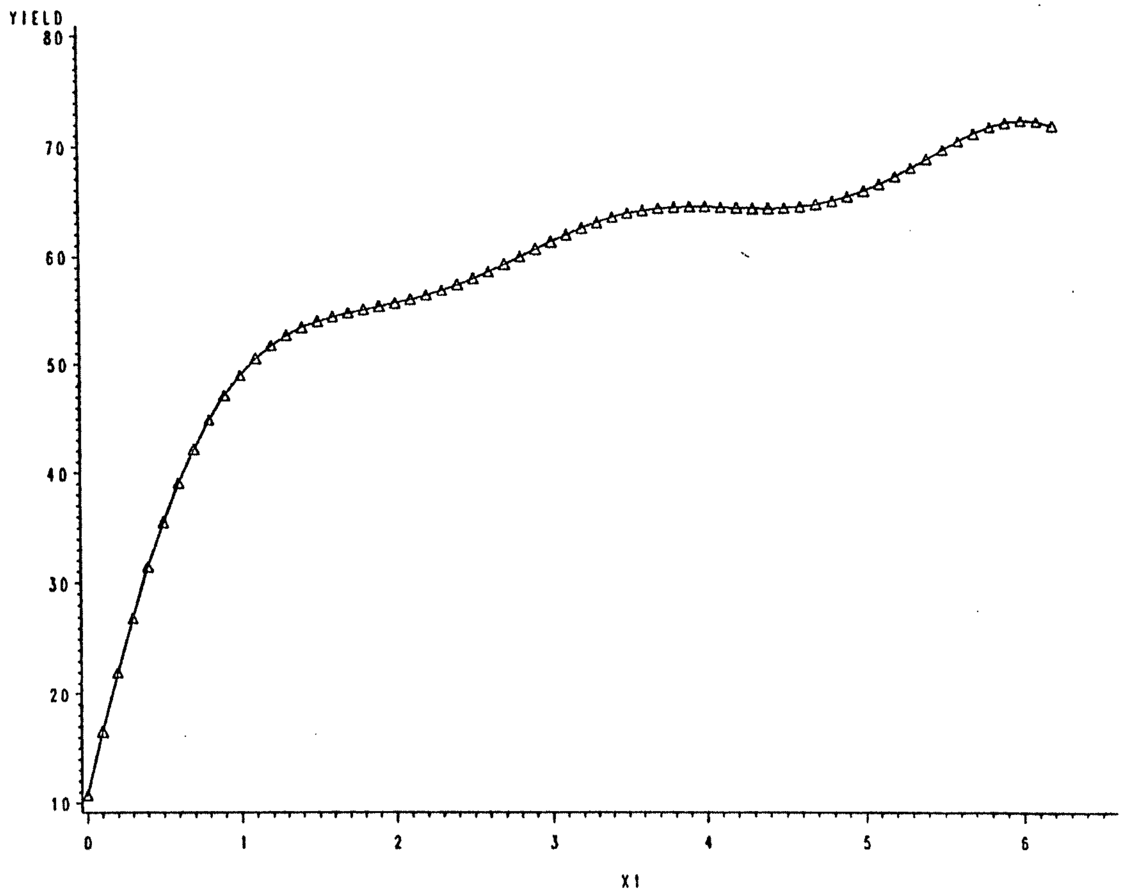


Figure 20. Estimated Yield vs. Nitrogen X_1 (Stylized Data Set).

CHAPTER 5

CONCLUDING REMARKS

Summary and Limitations

The purpose of this study was to use a Fourier flexible functional form to address the problem of yield response when faced with several alternative or competing models. Among those models, three functional forms representing observed and maintained differences in plant yield response to nutrient inputs were reviewed and compared. They included the quadratic, von Liebig and Mitscherlich-Baule functions. The theory of flexible functional forms and the Fourier functional form then were reviewed. The application methodology and procedures for estimating the Fourier function were designed. Finally, the empirical results were presented and discussed.

There were three specific objectives for this research. The first was to investigate the applicability of theory and practice of flexible functional forms in the study of crop response. The second was to estimate and compare the parameters of the Fourier functional form using Montana experimental data on wheat response to fertilizer nutrients and water and to evaluate the consistency of the parameter estimations with accepted economic theory. The third was to determine the substitutability of nitrogen and water by calculating the elasticity of factor substitution (σ values) generated from the first and second derivatives of the chosen Fourier function.

The data problems were unveiled by scatter diagrams. The noise, unknown stochastic distribution, in the data existed not only in certain plots but also in the whole data set.

Estimation was carried out using a nonlinear squares procedure and a procedure for selecting a multi-index K, i.e. the number of directions. Because of the data problems, three variants of the full data were developed; the full data set and two revised reduced data sets were utilized and model parameters estimated in each case. Coherent results could not be obtained using the full data set or a reduced data set obtained by deleting selected plot/sites. Thus, as a final exercise, model parameters were estimated based on a "stylized" or trimmed data set. The statistical tests for model choice were conducted using R^2 and AIC. The procedure for the latter involved incorporating a measure of the precision of the estimate and a measure of the rule of parsimony in the parameterization of a statistical model.

It seems that the Fourier form is very sensitive to ill-conditioned data; a "well-behaved" data set is seemingly essential for using the Fourier flexible function form. Because of its periodicity in fitting a production function, the Fourier functional form could be much more sensitive to the chosen data set than any other flexible functional forms. This characteristic seems to limit the breadth of possible applications where the Fourier FFF will yield useful results.

The prerequisite for using the Fourier function is, according to Gallant (1981), to assume that the form and properties of the data noise are known prior to estimation. Since, in the real world, there are more unknown forms and properties of data noise than known, the Fourier function is less attractive for approximating many continuous functions.

Suggestions for Further Research

The Fourier functional form did not work well in this production function application. However, insights into the problem of model specification were gained. During the course of this study, a number of possibilities for further research became evident.

First, further studies need to examine the empirical criteria for judging flexible functional forms before any FFF is used. There are four groups of empirical criteria: (a) Monte Carlo studies, (b) parametric models, (c) Bayesian analysis, and (d) nonnested hypothesis testing (Thompson, 1988). In Monte Carlo studies, the ability of an FFF to approximate a known underlying function is measured. Parametric methods have been proposed to test a special form of a generalized function through hypothesis tests of the appropriate parameter restrictions. However, these methods fail to discriminate the Fourier function from other generalized functions. To compare the Fourier function with other competing functional forms where the underlying function is unknown, it is suggested that a data-generated measure, either (c) or (d), be used.

A Bayesian approach would allow a posterior comparison of FFFs. The attraction of this method is that it allows comparison of fundamentally different models on the basis of actual data. Discussions and details for using this method are presented by Berndt et al. (1977), Wolhlgenant (1984) and Rossi (1985).

Nonnested hypothesis testing allows the researcher to make pair-wise comparisons between the Fourier function and its competitors. A conceptual difference between nonnested testing and Bayesian analysis is that the nonnested testing allows for all proposed models to be rejected on the basis of the data. The extent to which nonnested testing results coincide with the Bayesian results is a potential methodological and empirical question. The applications of this method were demonstrated by Embleton (1984) and Frank et al. (1989).

In addition to the examination of empirical criteria, there are some technical considerations of concern involving the Fourier functional form. In the process of empirical work, the main confusing question is what is the best way to choose the multi-index, K? This is an open issue, although more so in estimation than in testing. To date, what practitioners have done is to use specification tests much as one does in conventional parametric statistical inference. Additional terms are added to or deducted from the Fourier series when a test rejects the current specification. Gallant (1984) and his students tried the Projection Pursuit idea of Friedman and Stuetzle (1981) but failed to make it work.

Another computational research problem of the Fourier FFF is how to overcome the Gibb's phenomenon. According to Gallant (1989), if the second derivative of g(X) with respect to X is constructed as

$$d^2g(X)/dX^2 = -2\pi\exp[-0.5(X-\pi)^2]$$

and dg(X)/dX and g(X) are obtained by integration, then the Gibb's phenomenon will be eliminated. Obviously, this reverse methodology is impractical. The first and second derivatives cannot be produced until all the parameters have been estimated from the Fourier function. Thus, additional research aimed at eliminating or reducing the Gibb's phenomenon is worthy of further study.

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